EECE-5666 : Midterm-2 Practice Exam : 2022-SPRG

Answers

Default Plot Parameters:

```
set(0,'defaultfigurepaperunits','points','defaultfigureunits','points');
set(0,'defaultaxesfontsize',10);
set(0,'defaultaxestitlefontsize',1.4,'defaultaxeslabelfontsize',1.2);
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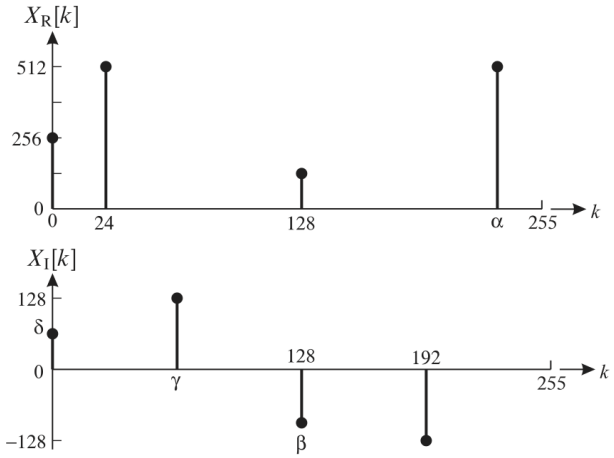
Problem-1 (10-points) The Discrete Fourier Transform (DFT)

The following two parts, (a) and (b), are not related to each other.

(a) [6-points] A real analog signal $x_c(t)$ has been sampled at a rate of 128 samples/sec and a total of 256 samples are obtained to form a 256-point sequence $x[n]$. A 256-point DFT is then taken to yield the real and imaginary plots shown below. ***Samples that are not shown below are zero-valued samples***.



i. [4-points] Determine the constants α, β, δ, and γ. You must provide clear explanations to get full credit.

Answers: $\alpha = 232,\ \beta = 0,\ \delta = 0,$ and $\gamma = 64.$

ii. [1-point] What is the digital frequency resolution (or separation), in cycles/sam, in the above plots?

Answer: 0.00391 cycles/sam.

iii. [1-point] What is the analog frequency in Hz corresponding to the sample $k = 24$ in the DFT above?

Answer: 12Hz.

(b) [5-points] Let $x_1[n]$ and $x_2[n]$ be two causal and finite-length sequences. Their values or their lengths are not known except that their lengths are ≤ 11. However, their linear convolution is known and is given by

$$x_1[n] * x_2[n] = \{\underset{\uparrow}{6}, 19, 40, 70, 110, 125, 140, 155, 170, 185, 184, 166, 130, 75\}.$$

i. [3-points] Let $y_1[n]$ be their 11-point circular convolution. Determine $y_1[n]$. You must provide clear reasoning, not just an answer, to get full credit.

Solution: $y_1[n] = \{\underset{\uparrow}{172}, 149, 115, 70, 110, 125, 140, 155, 170, 185, 184\}.$

ii. [2-points] Let $y_2[n]$ be their 18-point circular convolution. Determine $y_2[n]$. You must provide clear reasoning, not just an answer, to get full credit.

Solution: $y_2[n] = \{\underset{\uparrow}{6}, 19, 40, 70, 110, 125, 140, 155, 170, 185, 184, 166, 130, 75, 0, 0, 0, 0\}.$

Problem-2 (10-Points) The Fast Fourier Transform (FFT)

The following two parts are not related.

(a) [5-Points] A signal has length $N = N_1N_2N_3$ where N_1, N_2, and N_3 are prime (non-divisible except one and itself) numbers. Determine the total number of complex multiplications needed in computing its DFT using the divide-and-conquer approach. Assume that the twiddle factor $W_L^0 = 1 + \mathrm{J}0$ is a complex-valued number. You must provide clear reasoning or proof to get full credit.

Answer:

$$\underset{(N\text{-point DFT})}{\text{ComplexMults}} = N_1N_2N_3(N_1 + N_2 + N_3 + 2).$$

(b) [5-points] Consider a 16-point radix-2 decimation-in-time FFT algorithm similar to the textbook Figure 8.6 in structure.

i. [1-Point] How many *complex multiplications and additions* would be required for direct evaluations of a 16-point DFT?

Answer: 240.

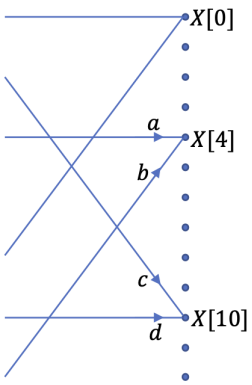
ii. [1-Point] How many *complex multiplications and additions* would be required in the 16-point Radix-2 DIT FFT algorithm?

Answer: 64.

iii. [1-Point] What time index samples will be put in the third, seventh, and twelfth position indices for implementing the 16-point, Radix-2 DIT FFT algorithm?

Answer: $x[12]$ is in position 3, $x[14]$ is in position 7, and $x[3]$ is in position $x[12]$.

iv. [2-Points] Part of the signal flow graph for the last stage is shown below. What are the twiddle factors a, b, c, and d?



Answer: $a = 1,\ b = -\mathrm{J},\ c = 1,$ and $d = -\frac{\sqrt{2}}{2} + \mathrm{J}\frac{\sqrt{2}}{2}$.

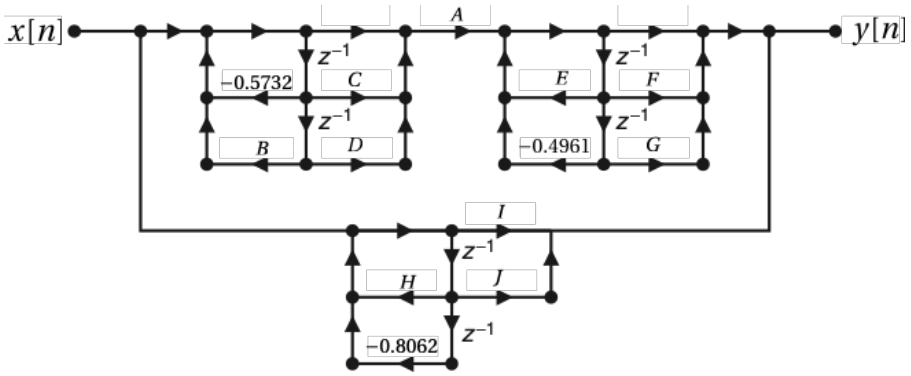
Problem-3 (10-Points) Digital Filter Structures

The following two parts are not related.

(a) [6-Points] An IIR system is given by the following system function:

$$H(z) = \frac{0.05 - 0.01z^{-1} - 0.13z^{-2} + 0.13z^{-4} + 0.01z^{-5} - 0.05z^{-6}}{1 - 0.77z^{-1} + 1.59z^{-2} - 0.88z^{-3} + 1.2z^{-4} - 0.35z^{-5} + 0.31z^{-6}}.$$

It is to be implemented using a signal flow graph of the form shown below:



The gains (or coefficient) values not shown or labeled are equal to 1.

i. [5-Points] Determine the coefficients A through J in the above signal flow graph.

Enter your coefficient values below:

$$A = 0.2939;\quad B = -0.7751;\quad C = 1.1389;\quad D = 1.0262;\quad E = 0.2611$$
$$F = -0.1592;\quad G = -0.2056;\quad H = 1,0821;\quad I = -0.2439;\quad J = 0.0964$$

ii. [1-Point] Is your solution unique? Explain clearly to get full credit.

Answer: No.

(b) [4-Points] Answer the following questions.

i. [2-Points] It is argued that the cascade form structure of a rational system function $H(z)$, is not always unique. Do you agree or disagree? Clearly explain your reason to get full credit.

Answer: Agree.

ii. [2-Points] It is argued that the parallel form structure of a rational system function $H(z)$, is always unique. Do you agree or disagree? Clearly explain your reason to get full credit.

Answer: Agree.
