EEECE5666 (DSP) : Homework-3 Solutions

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Default Plot Parameters

```
set(0,'defaultfigurepaperunits','points','defaultfigureunits','points');
set(0,'defaultaxesfontsize',10); set(0,'defaultaxeslinewidth',1.5);
set(0,'defaultaxestitlefontsize',1.4,'defaultaxeslabelfontsize',1.2);
```

Problem 3.1

Text Problem 5.23 (Page 282)

An LTI system is described by the difference equation

$$y[n] = bx[n] + 0.8y[n-1] - 0.81y[n-2] \qquad (5.23.1)$$

```
clc; close all; clear;
```

(a) Determine the frequency response of the system in terms of b.

Solution: Taking DTFT of both sides of $(5.23.1)$ and then solving for $H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega})$,we obtain

$$H(e^{j\omega}) = \frac{b}{1 - 0.8e^{-j\omega} + 0.81e^{-2j\omega}}.$$

(b) Determine b so that $|H(e^{j\omega})|_{\max} = 1$. Plot the resulting magnitude response.

Solution: To determine system poles we us the following script.

```
a = [1,-0.8,0.81]; Poles = roots(a); Poles_mag = abs(Poles.')
```

```
Poles_mag = 1×2
    0.9000    0.9000
```

```
Poles_pha = angle(Poles.')/pi, % Phase in pi units
```

```
Poles_pha = 1×2
    0.3534   -0.3534
```

Hence, the above system has poles at $re^{\pm j\phi} = 0.9e^{\pm j0.3534\pi}$. Using equation (5.119) in the text, we have

```
r = Poles_mag(1); phi = Poles_pha(1)*pi;
b = (1-r)*sqrt(1+r^2-2*r*cos(2*phi))
```

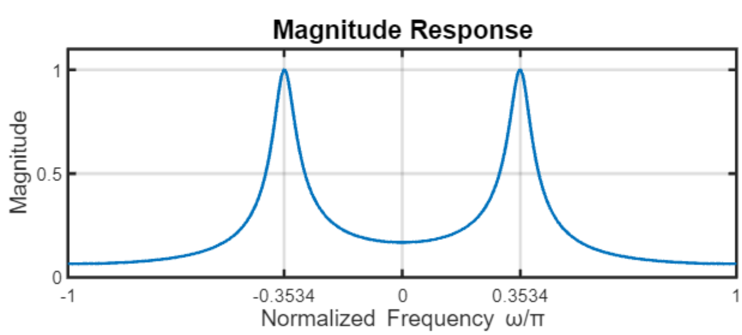
```
b = 0.1703
```

Hence

$$b_0 = b = (1-r)\sqrt{1+r^2-2r\cos(2\phi)} = 0.1703.$$

MATLAB script: The magnitude plot is computed below.

```
om = linspace(-1,1,1001)*pi;
H = b./(1+a(2)*exp(-1j*om)+a(3)*exp(-2j*om));
Hmag = abs(H);
figure('position',[0,0,8,3]*72);
plot(om/pi,Hmag,'linewidth',1.5);axis([-1,1,0,1.1]);
xlabel('Normalized Frequency \omega/\pi'); ylabel('Magnitude'); title('Magnitude Response');
set(gca,'xtick',[-1,Poles_pha(2),0,Poles_pha(1),1]); grid;
```

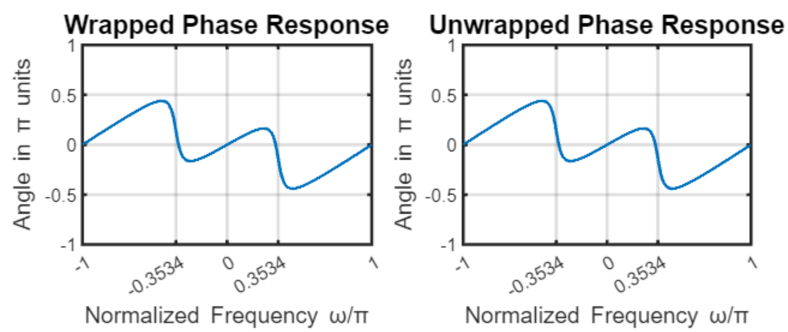


(c) Graph the wrapped and the unwrapped phase responses in one plot.

MATLAB script:

```
Hwpha = angle(H); % Wrapped phase in radians
```

```
Hupha = unwrap(Hwpha); % Unwrapped pahse in radians
figure('position',[0,0,8,3]*72);
subplot(1,2,1); plot(om/pi,Hwpha/pi,'linewidth',1.5);
xlabel('Normalized Frequency \omega/\pi'); axis([-1,1,-1,1]);
ylabel('Angle in \pi units'); title('Wrapped Phase Response');
set(gca,'xtick',[-1,Poles_pha(2),0,Poles_pha(1),1]);
set(gca,'ytick',(-1:0.5:1)); grid;
subplot(1,2,2); plot(om/pi,Hupha/pi,'linewidth',1.5);
xlabel('Normalized Frequency \omega/\pi'); axis([-1,1,-1,1]);
ylabel('Angle in \pi units'); title('Unwrapped Phase Response');
set(gca,'xtick',[-1,Poles_pha(2),0,Poles_pha(1),1]);
set(gca,'ytick',(-1:0.5:1)); grid;
```



Note that since the wrapped phase response is smooth and within the primary interval $[-\pi, \pi]$, both plots give the same result.

(d) Determine analytically response $y[n]$ to the input $x[n] = 2\cos(\pi n/3 + 45^{\text{deg}})$.

Solution: The input sinusoidal sequence has frequency of $\omega_0 = \pi/3$. We need the system response at $\pi/3$ given by

```
om0 = pi/3; Hom0 = b./(1+a(2)*exp(-1j*om0)+a(3)*exp(-2j*om0));
Hom0mag = abs(Hom0)
```

```
Hom0mag = 0.8723
```

```
Hom0pha = angle(Hom0)/pi*180, % Phase in degrees
```

```
Hom0pha = 2.5429
```

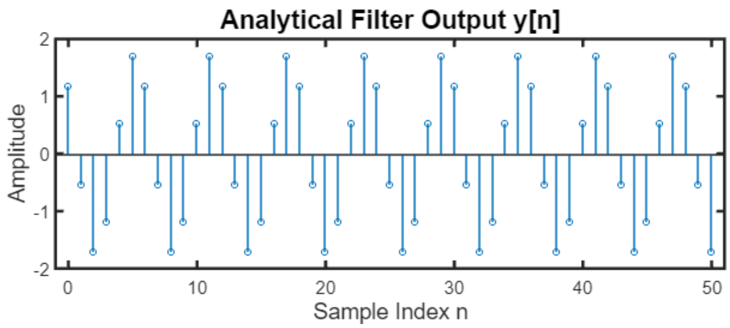
Now using the sinusoidal steady-state analysis, the output $y[n]$ is given by

$$y[n] = 2|H(e^{j\pi/3})|\cos\left[\frac{\pi}{3}n + 45^{\text{deg}} + \angle H(e^{j\pi/3})\right] = 2(0.8723)\cos\left[\frac{\pi}{3}n + 45^{\text{deg}} + 2.5429^{\text{deg}}\right]$$

$$= 1.7446\cos\left[\frac{\pi}{3}n + 47.5429^{\text{deg}}\right].$$

The plot is computed and plotted using the following script.

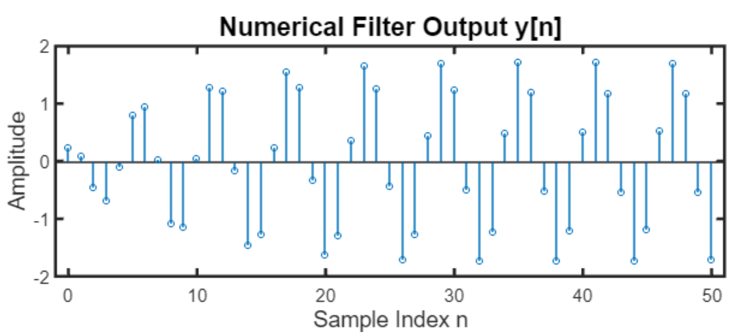
```
n = 0:50; y = 2*Hom0mag*cos(om0*n+45*pi/180+angle(Hom0));
figure('position',[0,0,8,3]*72);
stem(n,y,'markersize',3); axis([-1,51,-2,2]);
xlabel('Sample Index n'); ylabel('Amplitude');
title('Analytical Filter Output y[n]');
set(gca,'xtick',(0:10:50),'ytick',(-2:1:2));
```



(e) Using MATLAB compute the steady-state response to $x[n]$ above and verify your result.

MATLAB script:

```
x = 2*cos(pi*n/3+pi/4); y2 = filter(b,a,x);
figure('position',[0,0,8,3]*72);
stem(n,y2,'markersize',3); axis([-1,51,-2,2]);
xlabel('Sample Index n'); ylabel('Amplitude');
title('Numerical Filter Output y[n]');
set(gca,'xtick',(0:10:50),'ytick',(-2:1:2));
```



The above plot verifies the steady-state response when the transients have died down after approximately 30 samples. Alternatively, you can also display a plot after transients have died down to show a steady-state response.

Problem 3.2

Text Problem 5.30, parts (a) and (e), (Page 283)

Consider a periodic signal

$$x[n] = \sin(0.1\pi n) + \frac{1}{3}\sin(0.3\pi n) + \frac{1}{5}\sin(0.5\pi n).$$

For each of the following systems, determine if the system imparts (i) no distortion, (ii) magnitude distortion, and/or (iii) phase (or delay) distortion. In each case, graph the input and the steady state response for $0 \le n \le 60$.

```
clc; close all; clear;
```

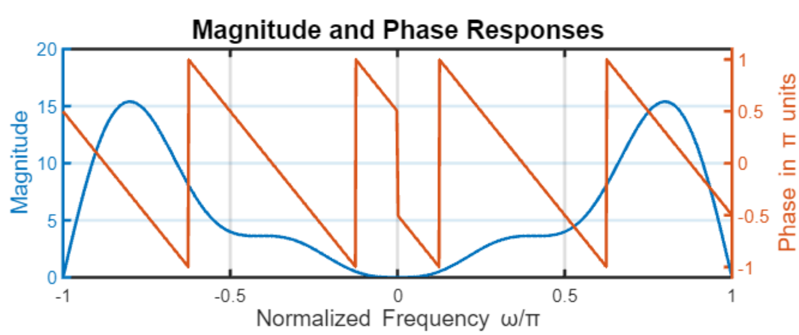
(a) $h[n] = \{\underset{\uparrow}{1}, -2, 3, -4, 0, 4, -3, 2, -1\}$:

Solution: The magnitude response is given by

$$\begin{aligned} H(e^{j\omega}) &= 1 - 2e^{-j\omega} + 3e^{-j2\omega} - 4e^{-j3\omega} + 4e^{-j5\omega} - 3e^{-j6\omega}2e^{-j7\omega} - e^{-j8\omega} \\ &= e^{j(\pi/2-4\omega)}\Big[2\sin(4\omega) - 4\sin(3\omega) + 6\sin(2\omega) - 8\sin(\omega)\Big]. \qquad (5.30.1) \end{aligned}$$

MATLAB script for computation and plotting of magnitude and phase responses:

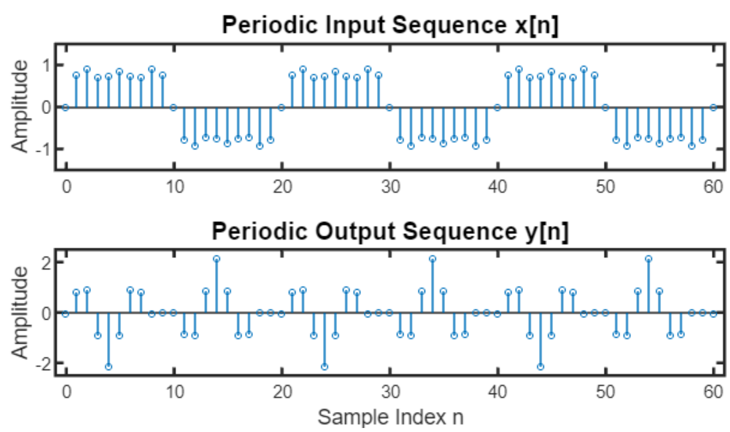
```
h = [1,-2,3,-4,0,4,-3,2,-1]; om = linspace(-1,1,1001)*pi;
H = freqz(h,1,om); Hmag = abs(H); Hpha = angle(H)/pi;
figure('position',[0,0,8,3]*72);
yyaxis left; plot(om/pi,Hmag,'linewidth',1.5); axis([-1,1,0,20]);
ylabel('Magnitude'); set(gca,'ytick',(0:5:20)); grid;
yyaxis right; plot(om/pi,Hpha,'linewidth',1.5);
ylabel('Phase in \pi units'); axis([-1,1,-1.1,1.1]);
xlabel('Normalized Frequency \omega/\pi');
title('Magnitude and Phase Responses');
set(gca,'xtick',(-1:0.5:1),'ytick',(-1:0.5:1));
```



Since the quantity in brackets in (1) is real, the system has a linear-phase response as can be seen from the above plot. However, the magnitude response is not constant. Therefore, the system imparts magnitude distortion but no phase distortion.

MATLAB script for the computation of input and output sequences:

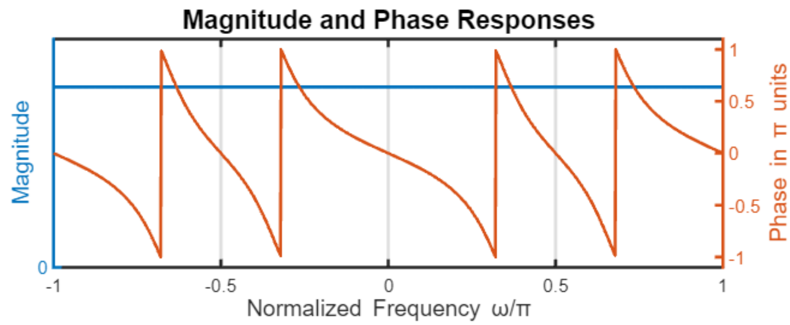
```
n = 0:60; om1 = 0.1*pi; om2 = 0.3*pi; om3 = 0.5*pi;
xn = sin(om1*n) + (1/3)*sin(om2*n) + (1/5)*sin(om3*n);
H = freqz(h,1,[om1,om2,om3]); % Freq responses at 3 input freq
y1 = abs(H(1))*sin(om1*n+angle(H(1))); % Resp due to 1st freq
y2 = abs(H(2))*(1/3)*sin(om2*n+angle(H(2))); % Resp due to 2nd freq
y3 = abs(H(3))*(1/5)*sin(om3*n+angle(H(3))); % Resp due to 3rd freq
yn = y1+y2+y3; % Total response
figure('position',[0,0,8,4]*72);
subplot(2,1,1); stem(n,xn,'markersize',3);
ylabel('Amplitude'); axis([-1,61,-1.5,1.5]);
title('Periodic Input Sequence x[n]');
subplot(2,1,2); stem(n,yn,'markersize',3);
xlabel('Sample Index n'); ylabel('Amplitude');
title('Periodic Output Sequence y[n]'); axis([-1,61,-2.5,2.5]);
```



(e) $H(z) = \dfrac{1 + 1.778z^{-2} + 3.1605z^{-4}}{1 + 0.5625z^{-2} + 0.3164z^{-4}}$:

Solution: The magnitude and phase responses are computed and plotted using the following script.

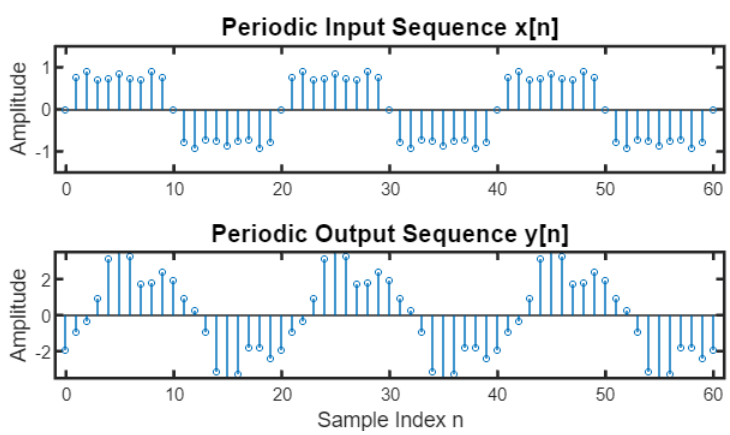
```
b = [1,0,1.778,0,3.1605]; a = [1,0,0.5625,0,0.3164];
H = freqz(b,a,om); Hmag = abs(H); Hpha = angle(H)/pi;
figure('position',[0,0,8,3]*72);
yyaxis left; plot(om/pi,Hmag,'linewidth',1.5); axis([-1,1,0,4]);
ylabel('Magnitude'); set(gca,'ytick',(0:5:20)); grid;
yyaxis right; plot(om/pi,Hpha,'linewidth',1.5);
ylabel('Phase in \pi units'); axis([-1,1,-1.1,1.1]);
xlabel('Normalized Frequency \omega/\pi');
title('Magnitude and Phase Responses');
set(gca,'xtick',(-1:0.5:1),'ytick',(-1:0.5:1));
```



From the above plot we observe that the magnitude response is (almost) constant but the phase response is nonlinear (the system is an IIR filter). Hence, although there is no magnitude distortion, the system imparts some phase distortion leading to the distorted input at the output of the filter.

MATLAB script for the computation of input and output sequences:

```
H = freqz(b,a,[om1,om2,om3]); % Freq responses at 3 input freq
y1 = abs(H(1))*sin(om1*n+angle(H(1))); % Resp due to 1st freq
y2 = abs(H(2))*(1/3)*sin(om2*n+angle(H(2))); % Resp due to 2nd freq
y3 = abs(H(3))*(1/5)*sin(om3*n+angle(H(3))); % Resp due to 3rd freq
yn = y1+y2+y3; % Total response
figure('position',[0,0,8,4]*72);
subplot(2,1,1); stem(n,xn,'markersize',3);
ylabel('Amplitude'); axis([-1,61,-1.5,1.5]);
title('Periodic Input Sequence x[n]');
subplot(2,1,2); stem(n,yn,'markersize',3);
xlabel('Sample Index n'); ylabel('Amplitude');
title('Periodic Output Sequence y[n]'); axis([-1,61,-3.5,3.5]);
```



Problem 3.3

Text Problem 5.37 (Page 284)

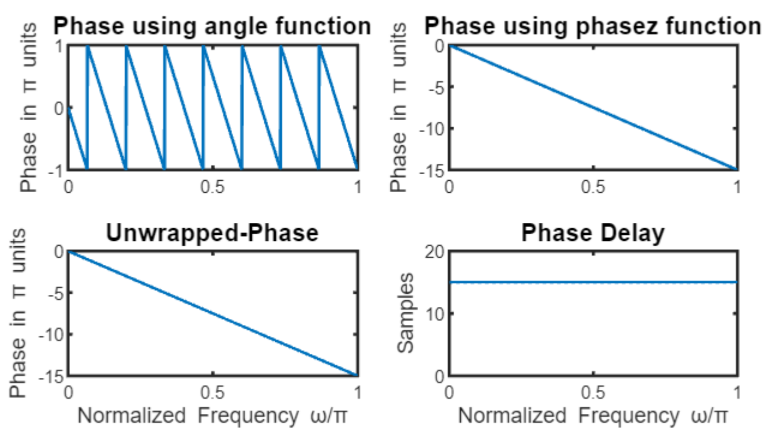
Compute and plot the phase response using the functions **freqz**, **angle**, **phasez**, **unwrap**, and **phasedelay** for the following systems.

```
clc; close all; clear;
```

(a) The pure delay $y[n] = x[n-15]$.

Solution: Various plots are computed and plotted using the following script.

```
b = [zeros(1,15),1]; a = 1; om = linspace(0,1,1001)*pi; H = freqz(b,a,om);
Hpha1 = angle(H); % Phase in radians
Hpha2 = phasez(b,a,om); % Phase in radians
Hpha3 = unwrap(Hpha1); % Phase in radians
Hpha4 = phasedelay(b,a,om); % Delay in samples
figure('position',[0,0,8,4]*72);
subplot(2,2,1); plot(om/pi,Hpha1/pi,'linewidth',1.5); axis([0,1,-1,1]);
ylabel('Phase in \pi units'); title('Phase using angle function');
subplot(2,2,2); plot(om/pi,Hpha2/pi,'linewidth',1.5); axis([0,1,-15,0]);
ylabel('Phase in \pi units'); title('Phase using phasez function');
subplot(2,2,3); plot(om/pi,Hpha3/pi,'linewidth',1.5); axis([0,1,-15,0]);
xlabel('Normalized Frequency \omega/\pi');
ylabel('Phase in \pi units'); title('Unwrapped-Phase');
subplot(2,2,4); plot(om/pi,Hpha4,'linewidth',1.5); axis([0,1,0,20]);
xlabel('Normalized Frequency \omega/\pi');
ylabel('Samples'); title('Phase Delay');
```

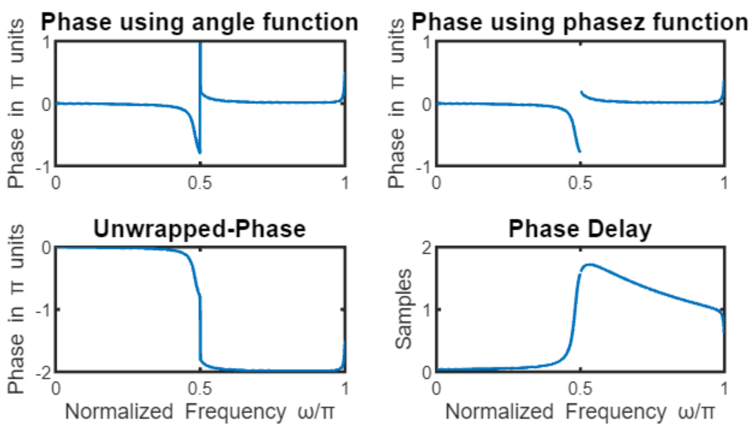


From the above plots observe that the **phasez** function also gives the unwrapped phase. The **phasedelay** function gives delay in samples.

(b) The system defined by $H(z) = \dfrac{1+z^{-1}+z^{-2}+z^{-3}}{1+0.9z^{-1}+0.81z^{-2}+0.927z^{-3}}$.

Solution: Various plots are computed and plotted using the following script.

```
b = [1,1,1,1]; a = [1,0.9,0.81,0.927]; H = freqz(b,a,om);
Hpha1 = angle(H); % Phase in radians
Hpha2 = phasez(b,a,om); % Phase in radians
Hpha3 = unwrap(Hpha1); % Phase in radians
Hpha4 = phasedelay(b,a,om); % Delay in samples
figure('position',[0,0,8,4]*72);
subplot(2,2,1); plot(om/pi,Hpha1/pi,'linewidth',1.5); axis([0,1,-1,1]);
ylabel('Phase in \pi units'); title('Phase using angle function');
subplot(2,2,2); plot(om/pi,Hpha2/pi,'linewidth',1.5); axis([0,1,-1,1]);
ylabel('Phase in \pi units'); title('Phase using phasez function');
subplot(2,2,3); plot(om/pi,Hpha3/pi,'linewidth',1.5); axis([0,1,-2,0]);
xlabel('Normalized Frequency \omega/\pi');
ylabel('Phase in \pi units'); title('Unwrapped-Phase');
subplot(2,2,4); plot(om/pi,Hpha4,'linewidth',1.5); axis([0,1,0,2]);
xlabel('Normalized Frequency \omega/\pi');
ylabel('Samples'); title('Phase Delay');
```



In this case, the **`phasedelay`** function gives a discontinuous response while the **`unwrap`** function gives a continuous response.

Problem 3.4

Text Problem 5.40 (Page 285)

Consider a second order IIR notch filter specification that satisfies the following requirements: (1) the magnitude response has notches at $\omega_{1,2} = \pm 2\pi/3$; (2) The maximum magnitude response is 1; (3) the magnitude response is approximately $1/\sqrt{2}$ at frequencies $\omega_{1,2} \pm 0.01$.

(a) Using the pole-zero placement approach determine locations of two poles and two zeros of the required filter and then compute its system function $H(z)$.

Solution: From the given specifications we want the zeros at $z_1 = e^{\jmath 2\pi/3}$, and $z_2 = e^{-\jmath 2\pi/3}$. Since the 3-dB bandwidth is $2(0.01) = 2(1-r)$ we have $r = 0.99$. Then the poles should be at $p_1 = 0.99e^{\jmath(2\pi/3)}$, and $p_2 = 0.99e^{-\jmath(2\pi/3)}$. Hence the system function is:

$$H(z) = \frac{(1-e^{\jmath 2\pi/3}z^{-1})(1-e^{-\jmath 2\pi/3}z^{-1})}{(1-0.99e^{\jmath(2\pi/3)}z^{-1})(1-0.99e^{-\jmath(2\pi/3)}z^{-1})} = \frac{1+z^{-1}+z^{-2}}{1+0.99z^{-1}+0.9801z^{-2}}.$$

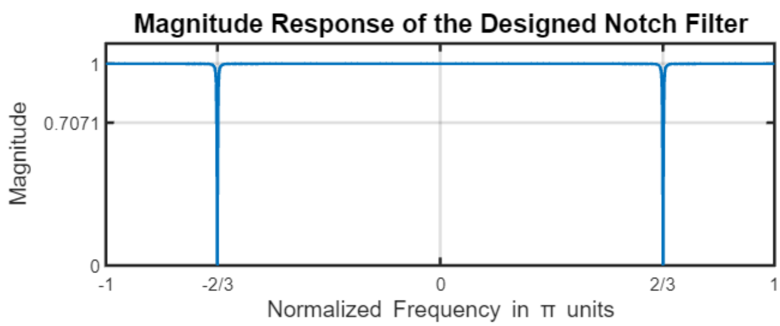
The difference equation of the notch filter is then given by

$$y[n] = x[n] + x[n-1] + x[n-2] - 0.99y[n-1] - 0.9801y[n-2], \quad n \geq 0.$$

(b) Graph the magnitude response of the filter and verify the given requirements.

MATLAB script:

```
clc; close all; clear;
om0 = 2*pi/3; z1 = exp(1j*om0); z2 = conj(z1); b = real(poly([z1;z2]));
p1 = 0.99*exp(1j*om0); p2 = conj(z1); a = real(poly([p1;p2]));
om = linspace(-1,1,1001)*pi; om = sort([om,-om0,om0]);
H = freqz(b,a,om); Hmag = abs(H); Hmagmax = max(Hmag); Hmag = Hmag/Hmagmax;
figure('position',[0,0,8,3]*72);
plot(om/pi,Hmag,'linewidth',1.5); axis([-1,1,0,1.1]);
xlabel('Normalized Frequency in \pi units'); ylabel('Magnitude');
title('Magnitude Response of the Designed Notch Filter');
set(gca,'xtick',[-1,-om0/pi,0,om0/pi,1],'ytick',[0,1/sqrt(2),1]); grid;
set(gca,'xticklabel',{'-1','-2/3','0','2/3','1'});
```



Verification: We will probe the magnitude responses at $\omega_{1,2} \pm 0.01$ and verify that it is approximates $1/\sqrt{2} = 0.7071$.

```
om1 = -om0-0.01; om2 = -om0+0.01; om3 = om0-0.01; om4 =om0+0.01;
H12 = freqz(b,a,[om1,om2,om3,om4]); Hmag12 = abs(H12);
Hmag12 = Hmag12/Hmagmax
```

```
Hmag12 = 1×4
```

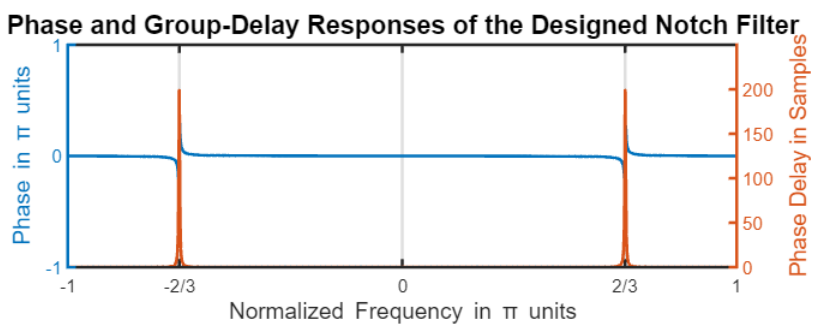
```
      0.8941      0.8930      0.8930      0.8941
```

There are two reasons for the discrepancies. First, the analytical derivation is an approximation and second, in MATLAB it is difficult to obtain exact numerical representation for $\omega_{1,2} \pm 0.01$ due to irrational numbers. You can verify this issue using the "Data Tip" feature of the plot above. Therefore, the above-obtained values are acceptable.

(c) Graph phase and group-delay responses in one plot.

MATLAB script: The responses are computed and plotted using the following script.

```
% om = linspace(-1,1,1001)*pi;
Hpha = phasez(b,a,om); Hgd = grpdelay(b,a,om);
figure('position',[0,0,8,3]*72);
yyaxis left; plot(om/pi,Hpha/pi,'linewidth',1.5); axis([-1,1,-1,1]);
ylabel('Phase in \pi units'); set(gca,'ytick',(-1:1:1));
yyaxis right; plot(om/pi,Hgd,'linewidth',1.5); axis([-1,1,0,250]);
ylabel('Phase Delay in Samples'); set(gca,'ytick',(0:50:200));
xlabel('Normalized Frequency in \pi units');
title('Phase and Group-Delay Responses of the Designed Notch Filter');
set(gca,'xtick',[-1,-om0/pi,0,om0/pi,1]);
set(gca,'xticklabel',{'-1','-2/3','0','2/3','1'},'xgrid','on');
```



From the above plot observe that all frequencies, except for those in the tiny band around the notch frequency, are not delayed. The frequencies in the notch band are delayed considerably but most of them are filtered out.

Problem 3.5

Text Problem 5.55, parts (c) and (d) (Page 288)

Determine the system function, magnitude response, and phase response of the following systems and use the pole-zero pattern to explain the shape of their magnitude response.

```
clc; close all; clear;
```

(c) $y[n] = x[n] - x[n-4] - 0.6561\,y[n-4]$

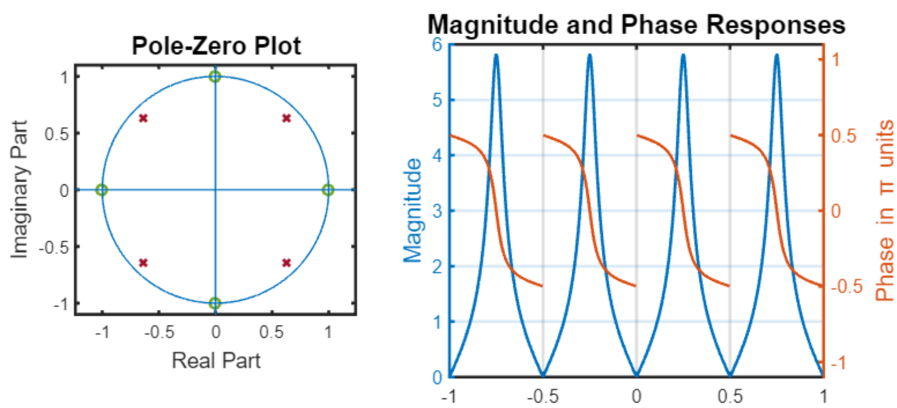
Solution: The system function is given by

$$H(z) = \frac{1 - z^{-4}}{1 + 0.6561z^{-4}}, \qquad |z| > \max(|p_k|),\ k = 1, 2, 3, 4.$$

Since the difference equation is run in the positive n direction, the system is causal and the ROC is outside of a circle with radius of largest pole magnitude.

MATLAB script: Various system responses and plots are obtained using the following script.

```
b = [1,0,0,0,-1]; a = [1,0,0,0,0.6561];
om = linspace(-1,1,1001)*pi; H = freqz(b,a,om);
Hmag = abs(H); Hpha = phasez(b,a,om);
figure('position',[0,0,9,4]*72);
subplot('position',[0.1,0.25,0.3,0.6]);
[Hz,Hp,Hl] = zplane(b,a);
set(Hz,'linewidth',1.5,'markersize',5,'color',[0.466,0.674,0.1880]);
set(Hp,'linewidth',1.5,'markersize',5,'color',[0.635,0.078,0.1840]);
set(Hl,'linestyle','-','linewidth',0.5); title('Pole-Zero Plot');
subplot('position',[0.5,0.1,0.4,0.8]);
yyaxis left; plot(om/pi,Hmag,'linewidth',1.5); axis([-1,1,0,6]);
ylabel('Magnitude'); set(gca,'ytick',(0:1:6)); grid;
yyaxis right; plot(om/pi,Hpha/pi,'linewidth',1.5);
ylabel('Phase in \pi units'); axis([-1,1,-1.1,1.1]);
xlabel('Normalized Frequency \omega/\pi');
title('Magnitude and Phase Responses'); set(gca,'ytick',(-1:0.5:1),'xgrid','on');
```



Observations: The system has four zeros on the unit circle and four poles very close to the unit circle at angles exactly in between the zero angles. Therefore, the magnitude response goes, alternately, from maximum (for frequencies near a pole) to 0 (for frequencies at a zero). Thus this is a **comb** filter. The phase response is overall nonlinear byt very nearly linear near the peak values (or in the passband regions).

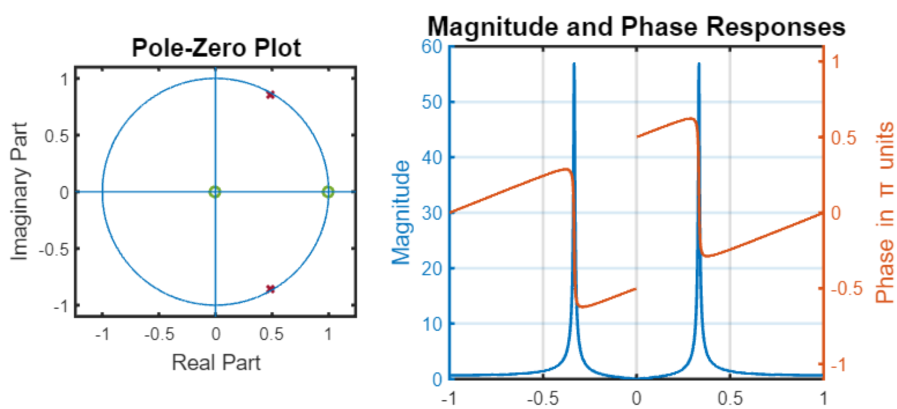
(d) $y[n] = x[n] - x[n-1] + 0.99y[n-1] - 0.9801y[n-2]$

Solution: The system function is given by

$$H(z) = \frac{1 - z^{-1}}{1 - 0.99z^{-1} + 0.9801z^{-2}}, \qquad |z| > \max(|p_k|),\ k = 1, 2.$$

MATLAB script: Various system responses and plots are obtained using the following script.

```
b = [1,-1]; a = [1,-0.99,0.9801];
om = linspace(-1,1,1001)*pi; H = freqz(b,a,om);
Hmag = abs(H); Hpha = phasez(b,a,om);
figure('position',[0,0,9,4]*72);
subplot('position',[0.1,0.25,0.3,0.6]);
[Hz,Hp,Hl] = zplane(b,a);
set(Hz,'linewidth',1.5,'markersize',5,'color',[0.466,0.674,0.1880]);
set(Hp,'linewidth',1.5,'markersize',5,'color',[0.635,0.078,0.1840]);
set(Hl,'linestyle','-','linewidth',0.5); title('Pole-Zero Plot');
subplot('position',[0.5,0.1,0.4,0.8]);
yyaxis left; plot(om/pi,Hmag,'linewidth',1.5); axis([-1,1,0,60]);
ylabel('Magnitude'); set(gca,'ytick',(0:10:60)); grid;
yyaxis right; plot(om/pi,Hpha/pi,'linewidth',1.5);
ylabel('Phase in \pi units'); axis([-1,1,-1.1,1.1]);
xlabel('Normalized Frequency \omega/\pi');
title('Magnitude and Phase Responses'); set(gca,'ytick',(-1:0.5:1),'xgrid','on');
```



Observations: Since there are complex-conjugate poles at $\pm\pi/3$ and zero at 0, the magnitude response goes through zero at 0 frequency and peaks at $\pm\pi/3$ frequencies. The phase response is very nearly linear around $\pm\pi/3$.

Problem 3.6

An ideal highpass filter is described in the frequency-domain by

$$H_d(e^{j\omega}) = \begin{cases} 1 \cdot e^{-j\alpha\omega}, & \omega_c < |\omega| \le \pi \\ 0, & |\omega| \le \omega_c \end{cases}$$

where ω_c is the cutoff frequency and α is called the phase delay.

(a) Determine the ideal impulse response $h_d[n]$ using the DTFT synthesis equation.

Solution: The ideal impulse response $h_d[n]$ using the IDTFT relation (4.91) in the text is given by

$$h_d[n] = \mathscr{F}^{-1}\left[H_d(e^{j\omega})\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_d(e^{j\omega})e^{j\omega n}d\omega = \frac{1}{2\pi}\int_{-\pi}^{-\omega_c} e^{-j\alpha\omega}e^{j\omega n}d\omega + \frac{1}{2\pi}\int_{\omega_c}^{\pi} e^{-j\alpha\omega}e^{j\omega n}d\omega$$

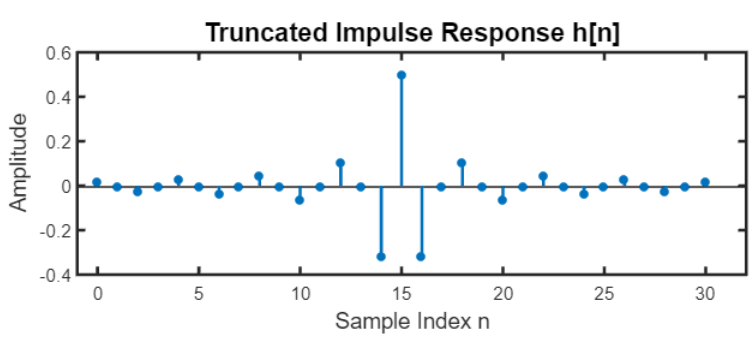
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(n-\alpha)\omega}d\omega - \frac{1}{2\pi}\int_{-\omega_c}^{\omega} e^{j(n-\alpha)\omega}d\omega = \frac{\sin[\pi(n-\alpha)]}{\pi(n-\alpha)} - \frac{\sin[\omega_c(n-\alpha)]}{\pi(n-\alpha)}$$

$$= \mathrm{sinc}(n-\alpha) - 2f_c\mathrm{sinc}\big(2f_c(n-\alpha)\big), \quad \omega_c = 2\pi f_c.$$

(b) Determine and plot the truncated impulse response $h[n] = \begin{cases} h_d[n], & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$, for $N = 31$, $\alpha = 15$, and $\omega_c = 0.5\pi$.

MATLAB script: Computation and plot is obtained using the following script.

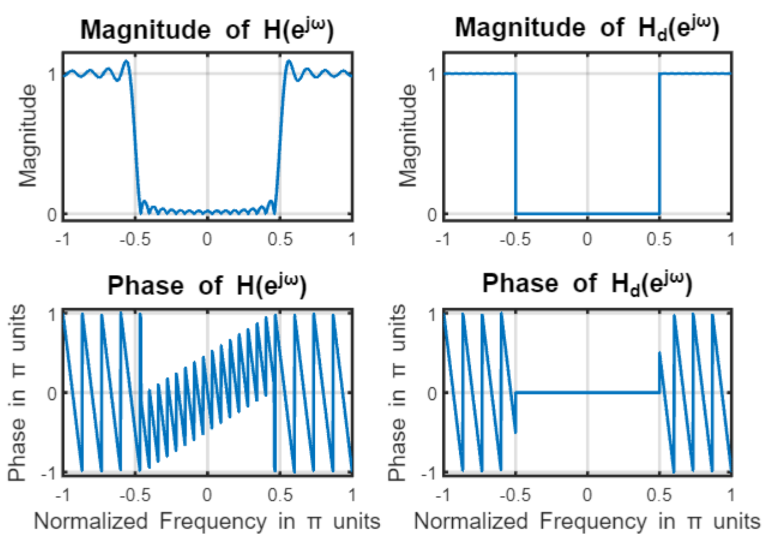
```
clc; close all; clear;
N = 31; n = 0:N-1; alpha = 15; omc = 0.5*pi; fc = omc/(2*pi);
h = sinc(n-alpha)-2*fc*sinc(2*fc*(n-alpha));
figure('position',[0,0,8,3]*72);
stem(n,h,'filled','markersize',3,'linewidth',1.5); axis([-1,N+1,-0.4,0.6]);
xlabel('Sample Index n'); ylabel('Amplitude');
title('Truncated Impulse Response h[n]');
set(gca,'xtick',0:5:N,'ytick',-0.4:0.2:0.6);
```



(c) Determine and plot the frequency response function $H(e^{j\omega})$, and compare it with the ideal highpass filter response $H_d(e^{j\omega})$. Comment on your observations.

MATLAB script: Various plots are computed and plotted using the following script.

```
om = linspace(-1,1,1001)*pi; H = freqz(h,1,om);
Hmag = abs(H); Hpha = angle(H); % Freq resp of truncated imp resp
omd1 = linspace(-1,-0.5,251)*pi; Hd1 = 1*exp(-1j*alpha*omd1);
omd2 = [-0.5,0.5]*pi; Hd2 = 0*exp(-1j*alpha*omd2);
omd3 = linspace(0.5,1,251)*pi; Hd3 = 1*exp(-1j*alpha*omd3);
Hd = [Hd1,Hd2,Hd3]; omd = [omd1,omd2,omd3];
Hdmag = abs(Hd); Hdpha = angle(Hd);
figure('position',[0,0,8,5]*72);
subplot(2,2,1); plot(om/pi,Hmag,'linewidth',1.5); axis([-1,1,-0.05,1.15]);
ylabel('Magnitude'); title('Magnitude of H(e^{j\omega})');
set(gca,'xtick',-1:0.5:1,'ytick',[0,1]); grid;
subplot(2,2,2); plot(omd/pi,Hdmag,'linewidth',1.5); axis([-1,1,-0.05,1.15]);
ylabel('Magnitude'); title('Magnitude of H_d(e^{j\omega})');
set(gca,'xtick',-1:0.5:1,'ytick',[0,1]); grid;
subplot(2,2,3); plot(om/pi,Hpha/pi,'linewidth',1.5); axis([-1,1,-1.05,1.05]);
xlabel('Normalized Frequency in \pi units');
ylabel('Phase in \pi units'); title('Phase of H(e^{j\omega})');
set(gca,'xtick',-1:0.5:1,'ytick',[-1,0,1]); grid;
subplot(2,2,4); plot(omd/pi,Hdpha/pi,'linewidth',1.5); axis([-1,1,-1.05,1.05]);
xlabel('Normalized Frequency in \pi units');
ylabel('Phase in \pi units'); title('Phase of H_d(e^{j\omega})');
set(gca,'xtick',-1:0.5:1,'ytick',[-1,0,1]); grid;
```



Observe that the frequency response of the truncated impulse response is a smeared or blurred version of the ideal frequency response.

Problem 3.7

Text Problem 6.22 part (b) (Page 346)

Signal $x_c(t) = 3 + 2\sin(16\pi t) + 10\cos(24\pi t)$ is sampled at a rate of $F_s = 20$ Hz to obtain the discrete-time signal $x[n]$.

```
clc; close all; clear;
```

(i) Determine the spectra $X(e^{j\omega})$ of $x[n]$.

Solution: The sampled signal $x[n]$ is given by

$$x[n] = 3 + 2\sin(16\pi n/20) + 10\cos(24\pi n/20)$$
$$= 3 + 2\sin(0.8\pi n) + 10\cos(1.2\pi n) = 3 + 2\sin(0.8\pi n) + 10\cos(0.8\pi n).$$

Using the following DTFT pairs

$$A \overset{\text{DTFT}}{\longleftrightarrow} 2\pi A\delta_{2\pi}(\omega)$$
$$A\cos(\omega_0 t) \overset{\text{DTFT}}{\longleftrightarrow} (\pi A)\left[\delta_{2\pi}(\omega+\omega_0) + \delta_{2\pi}(\omega-\omega_0)\right]$$
$$A\sin(\omega_0 t) \overset{\text{DTFT}}{\longleftrightarrow} (j\pi A)\left[\delta_{2\pi}(\omega+\omega_0) - \delta_{2\pi}(\omega-\omega_0)\right]$$

Hence the spectra or DTFT of $x[n]$ is given by

$$X(e^{j\omega}) = 6\pi\delta_{2\pi}(\omega) + 2\pi[\delta_{2\pi}(\omega+0.8\pi) + \delta_{2\pi}(\omega-0.8\pi)] + j10\pi[\delta_{2\pi}(\omega+0.8\pi) - \delta_{2\pi}(\omega-0.8\pi)]$$
$$= 6\pi\delta_{2\pi}(\omega) + 2\pi(1+j5)\delta_{2\pi}(\omega+0.8\pi) + 2\pi(1-j5)\delta_{2\pi}(\omega-0.8\pi)$$
$$= 6\pi\delta_{2\pi}(\omega) + 2\pi(\sqrt{26}\angle(\tan^{-1}(25))\delta_{2\pi}(\omega+0.8\pi) + 2\pi(\sqrt{26}\angle(-\tan^{-1}(25))\delta_{2\pi}(\omega-0.8\pi) \qquad (3.7.1)$$

where $\delta_{2\pi}(\omega)$ is a ***periodic impulse train*** with the fundamental period of 2π.

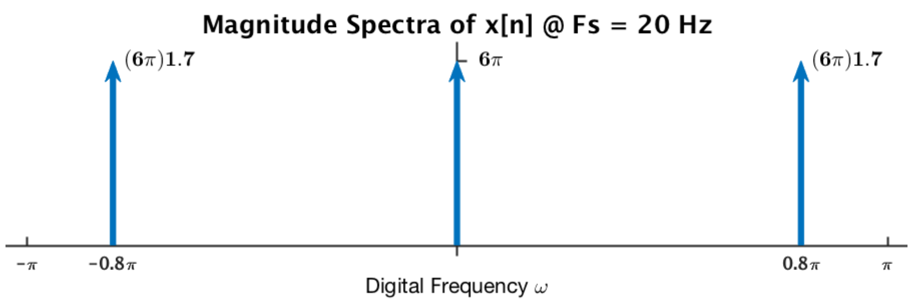
(ii) Plot magnitude of $X(e^{j\omega})$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ and as a function of F in Hz.

Solution: From $(3.7.1)$, the magnitude of $X(e^{j\omega})$ is given by

$$|X(e^{j\omega}| = (6\pi)\delta_{2\pi}(\omega) + \left(2\sqrt{26}\,\pi\right)\delta_{2\pi}(\omega+0.8\pi) + \left(2\sqrt{26}\,\pi\right)\delta_{2\pi}(\omega-0.8\pi)$$
$$= (6\pi)\delta_{2\pi}(\omega) + \left(10.198\pi\right)\delta_{2\pi}(\omega+0.8\pi) + \left(10.198\pi\right)\delta_{2\pi}(\omega-0.8\pi)$$
$$= (6\pi)\left[\delta_{2\pi}(\omega) + (1.7)\delta_{2\pi}(\omega+0.8\pi) + (1.7)\delta_{2\pi}(\omega-0.8\pi)\right] \qquad (2)$$

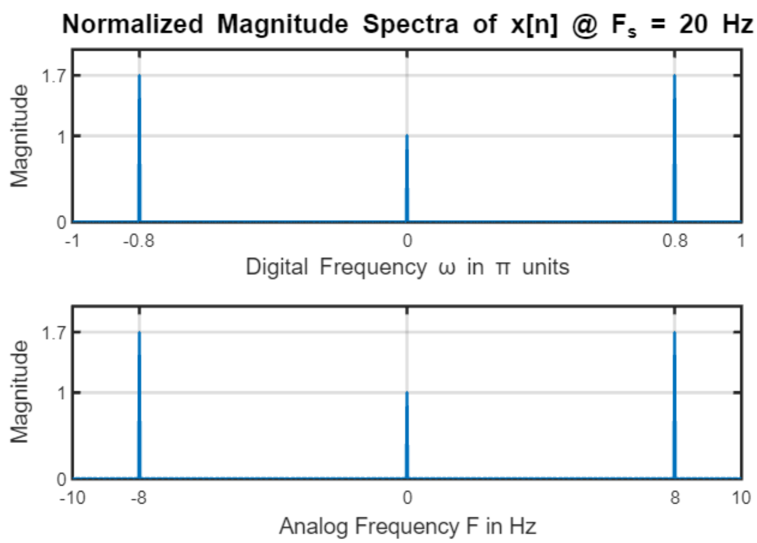
The plot of the magnitude $|X(e^{j\omega})|$ over $-\pi < \omega \leq \pi$ is an impulse plot with three impulses. The first one is at $\omega_1 = -0.8\pi$ rad/sam with area $1.7(6\pi)$, the second one at $\omega_2 = 0$ rad/sam with area (6π), and the third impulse at

$\omega_3 = +0.8\pi$ rad/sam with area of $1.7(6\pi)$. You can show it as a simple figure with three impulses along with area values next to their arrowheads.



Alternatively, you can use MATLAB to numerically compute and plot the magnitude $|X(e^{j\omega})|$ over $-\pi < \omega \leq \pi$. The amplitudes will be approximate since we cannot synthesize a true impulse but should be proportional to the areas under the impulses. Therefore, we will plot normalized magnitudes. The important issues here are the location of spikes and their proportional heights.

```
N = 1000; n = 0:N; xn = 3+2*sin(0.8*pi*n)+10*cos(0.8*pi*n);
om = linspace(-1,1,N+1)*pi; k1 = find(om/pi == 0); k2 = find(om/pi == 0.8);
Xom = abs(freqz(xn,1,om)); Xom = Xom/Xom(k1); % Normalized wrt to 0 freq
figure('position',[0,0,8,5]*72);
subplot(2,1,1); plot(om/pi,Xom,'linewidth',1.5); axis([-1,1,0,2]);
title('Normalized Magnitude Spectra of x[n] @ F_s = 20 Hz');
xlabel('Digital Frequency \omega in \pi units'); ylabel('Magnitude');
set(gca,'ytick',[0,1,1.7],'xtick',[-1,-0.8,0,0.8,1]); grid;
subplot(2,1,2); plot(om/pi,Xom,'linewidth',1.5); axis([-1,1,0,2]);
xlabel('Analog Frequency F in Hz'); ylabel('Magnitude');
set(gca,'ytick',[0,1,1.7],'xtick',[-1,-0.8,0,0.8,1]); grid;
set(gca,'xticklabel',{-10,-8,0,8,10});
```



From the above plots we observe that the normalized plots agree with our analytical solution in (2) above.

(iii) Explain if $x_c(t)$ can be recovered from $x[n]$.

Solution: From the plots, it is obvious that the 12 Hz component has aliased into the 8 Hz component and hence $x_c(t)$ cannot be recovered from $x[n]$ when $F_s = 20$ Hz.

Problem 3.8

Text Problem 6.26 (Page 347)

Consider a continuous-time signal $x_c(t) = 3\cos(2\pi F_1 t + 45^{\text{deg}}) + 3\sin(2\pi F_2 t)$. It is sampled at $t = 0.001n$ to obtain $x[n]$ which is then applied to an ideal DAC to obtain another continuous-time signal $y_r(t)$.

```
clc; close all; clear;
```

(a) For $F_1 = 150$ Hz and $F_2 = 400$ Hz, determine $x[n]$ and graph its samples along with the signal $x_c(t)$ in one plot (choose few cycles of the $x_c(t)$ signal).

Solution: The sampled sequence $x[n]$ is:

$$x[n] = x_c(nT) = 3\cos(0.3\pi n + \pi/4) + 3\sin(0.8\pi n).$$

Since both digital radian frequencies were less that π, no corrections were needed.

(b) Determine $y_r(t)$ for the above $x[n]$ as a sinusoidal signal. Graph and compare it with $x_c(t)$.

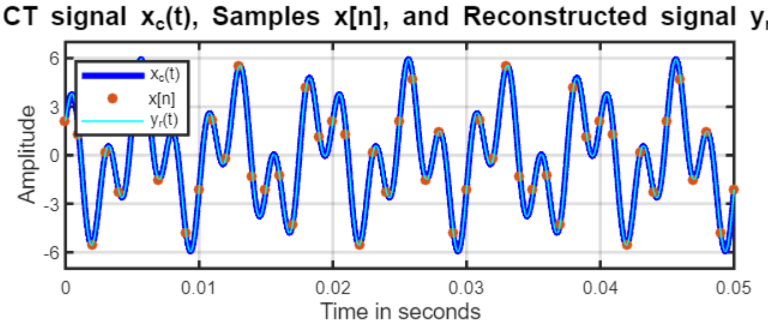
Solution: The reconstructed signal $y_r(t)$ is:

$$y_r(t) = 3\cos(300\pi t + \pi/4) + 3\sin(800\pi t)$$

which is same as the original signal $x_c(t)$.

MATLAB script:

```
t = linspace(0,0.05,10001); F1 = 150; F2 = 400;
xc = 3*cos(2*pi*F1*t+45*pi/180)+3*sin(2*pi*F2*t);
nT = 0:0.001:0.05; xn = 3*cos(2*pi*F1*nT+45*pi/180)+3*sin(2*pi*F2*nT);
yr = 3*cos(2*pi*F1*t+45*pi/180)+3*sin(2*pi*F2*t);
figure('position',[0,0,8,3]*72);
plot(t,xc,'b','linewidth',3); hold on;
plot(nT,xn,'*','markersize',5,'linewidth',1.5);
plot(t,yr,'c','linewidth',0.5); axis([0,0.05,-7,7]);
xlabel('Time in seconds'); ylabel('Amplitude');
title('CT signal x_c(t), Samples x[n], and Reconstructed signal y_r(t)');
set(gca,'xtick',0:0.01:0.05,'ytick',-6:3:6); grid;
legend('x_c(t)','x[n]','y_r(t)','location','best');
```



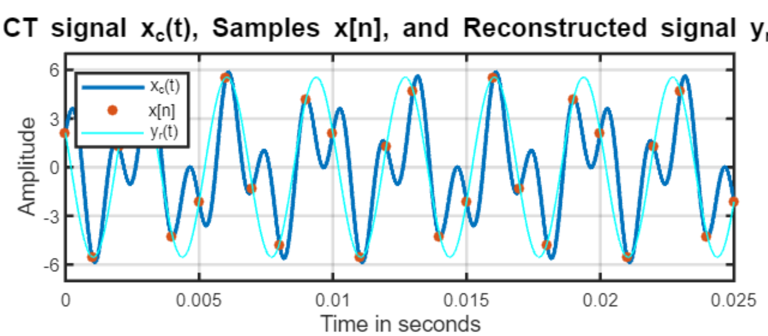
(c) Repeat (a) and (b) for $F_1 = 300$ Hz and $F_2 = 700$ Hz. Comment on your results.

Solution: The sampled sequence $x[n]$ is:

$$x[n] = x_c(nT) = 3\cos(0.6\pi n + \pi/4) + 3\sin(1.4\pi n) = 3\cos(0.6\pi n + \pi/4) - 3\sin(0.6\pi n).$$

Hence the reconstructed signal is $y_r(t) = 3\cos(600\pi t + \pi/4) - 3\sin(600\pi t)$. Evidently, $y_r(t) \neq x_c(t)$ due to aliasing of the 700 Hz component into a 300 Hz component.

```
t = linspace(0,0.025,10001); F1 = 300; F2 = 700;
xc = 3*cos(2*pi*F1*t+45*pi/180)+3*sin(2*pi*F2*t);
nT = 0:0.001:0.025; xn = 3*cos(2*pi*F1*nT+45*pi/180)+3*sin(2*pi*F2*nT);
yr = 3*cos(2*pi*F1*t+pi/4)-3*sin(2*pi*F1*t);
figure('position',[0,0,8,3]*72);
plot(t,xc,'linewidth',2); hold on; axis([0,0.025,-7,7]);
plot(nT,xn,'*','markersize',5,'linewidth',1.5);
plot(t,yr,'c','linewidth',0.5);
xlabel('Time in seconds'); ylabel('Amplitude');
title('CT signal x_c(t), Samples x[n], and Reconstructed signal y_r(t)');
set(gca,'xtick',0:0.005:0.025,'ytick',-6:3:6); grid;
legend('x_c(t)','x[n]','y_r(t)','location','best');
```



Observe that $y_r(t)$ goes through all samples of $x[n]$ but has the lower frequency.

Problem 3.9

Text Problem 6.23 part (c) (Page 348)

Signal $xc(t) = 5e^{j40t} + 3e^{-j70t}$ is sampled periodically with $T = 0.1$ sec to obtain the discrete-time signal $x[n]$. Determine the spectra $X(e^{j\omega})$ of $x[n]$ and plot its magnitude as a function of ω in $\frac{\text{rad}}{\text{sam}}$. Explain if $x_c(t)$ can be recovered from $x[n]$.

Solution: The spectra of the continuous-time signal $x_c(t)$ is given by

$$X_c(j\Omega) = 5\delta(\Omega - 40) + 3\delta(\Omega + 70).$$

The discrete-time sequence $x[n]$ is given by

$$x[n] = x_c(nT) = 5e^{j40nT} + 3e^{-j70nT}$$

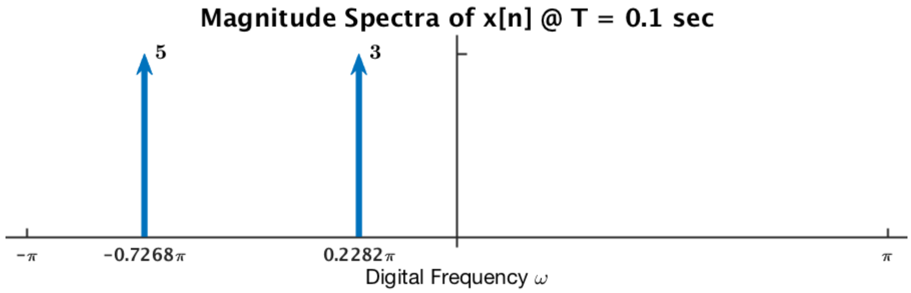
and, for $T = 0.1$sec., its spectra $X(e^{j\omega})$is given by

$$\begin{aligned}
X(e^{j\omega}) &= \frac{1}{T}\sum_{n=-\infty}^{\infty} x_c\left(j\frac{\omega - 2\pi n}{T}\right) = 10\sum_{n=-\infty}^{\infty} X_c(j10[\omega - 2\pi n]) \\
&= 50\sum_{n=-\infty}^{\infty}\delta(10[\omega - 2\pi n] - 40) + 30\sum_{n=-\infty}^{\infty}\delta(10[\omega - 2\pi n] + 70) \\
&= 5\sum_{n=-\infty}^{\infty}\delta(\omega - 4 - 2\pi n) + 3\sum_{n=-\infty}^{\infty}\delta(\omega + 7 - 2\pi n); \quad \text{using } \delta(a\omega) = \frac{1}{|a|}\delta(\omega)
\end{aligned}$$

Since for $n = 0$ the digital frequencies are outside $-\pi < \omega \le \pi$ there will be aliasing. The aliased frequencies are obtained by changing $n \to n - 1$in the first term and $n \to n + 1$in the second term.

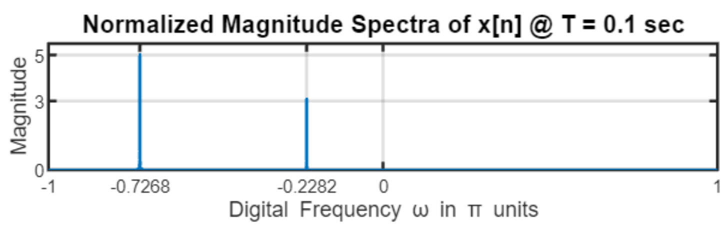
$$\begin{aligned}
X(e^{j\omega}) &= 5\sum_{n=-\infty}^{\infty}\delta(\omega - 4 + 2\pi - 2\pi n) + 3\sum_{n=-\infty}^{\infty}\delta(\omega + 7 - 2\pi - 2\pi n) \\
&= 5\sum_{n=-\infty}^{\infty}\delta(\omega + 2.2832 - 2\pi n) + 3\sum_{n=-\infty}^{\infty}\delta(\omega + 0.7168 - 2\pi n) \\
&= 5\delta_{2\pi}(\omega + 0.7268\pi) + 3\delta_{2\pi}(\omega + 0.2282\pi).
\end{aligned}$$

The aliased frequencies are $\omega_{a_1} = -0.7268\pi$ rad/sam and $\omega_{a_2} = -0.2282\pi$ rad/sam or $\Omega_{a_1} = -22.832$ rad/sec and $\Omega_{a_2} = -7.168$ rad/sec, respectively. The plot of the magnitude $|X(e^{j\omega})|$over $-\pi < \omega \le \pi$ is an impulse plot with one impulse at $\omega_{a_1} = -0.7268\pi$ rad/sam with area 5 and another impulse at $\omega_{a_2} = -0.2282\pi$ with area of 3. You can show it as a simple figure with two impulses along with area values next to their arrowheads.



Alternatively, you can use MATLAB to numerically compute and plot the magnitude $|X(e^{j\omega})|$ over $-\pi < \omega \le \pi$. The amplitudes will be approximate since we cannot synthesize a true impulse but should be proportional to the areas under the impulses. Therefore, we will plot normalized magnitudes. The important issues here are the location of spikes and proportional heights.

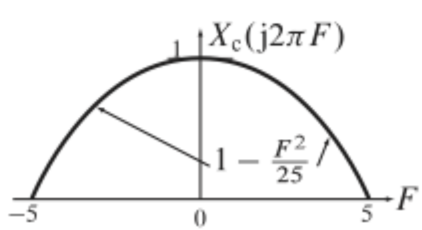
```
clc; close all; clear;
N = 10000; n = 0:N; T = 0.1; xn = 5*exp(40j*n*T)+3*exp(-70j*n*T);
om = linspace(-1,1,N+1)*pi; oma1 = (4-2*pi)/pi; oma2 = (2*pi-7)/pi;
Xom = abs(freqz(xn,1,om)); Xom = Xom/max(Xom);
figure('position',[0,0,8,2]*72);
plot(om/pi,Xom,'linewidth',1.5); axis([-1,1,0,1.1]);
title('Normalized Magnitude Spectra of x[n] @ T = 0.1 sec');
xlabel('Digital Frequency \omega in \pi units'); ylabel('Magnitude');
set(gca,'ytick',[0,0.6,1],'xtick',[-1,oma1,oma2,0,1]);
set(gca,'yticklabel',{'0','3','5'}); grid on;
```



Problem 3.10

Text Problem 6.37 (Page 348)

Signal $x_c(t)$ with spectra $X(j2\pi F)$ shown below is sampled at a rate of $F_s = 6$ Hz to obtain the discrete-time signal $x[n]$.




```
clc; close all; clear;
```

(i) Determine the spectra $X(e^{j\omega})$ of $x[n]$.

Solution: The spectra of the continuous signal $x_c(t)$ is:

$$X_c(j2\pi F) = \begin{cases} 1 - \dfrac{F^2}{25}, & |F| \le 5 \\ 0, & \text{otherwise} \end{cases}$$

The spectra of the sampled signal is then given by

$$X(e^{j\omega})\big|_{\omega=2\pi F/F_s} = F_s \sum_{k=-\infty}^{\infty} X_c\big(j2\pi(F - kF_s)\big) = 6 \sum_{k=-\infty}^{\infty} X_c\big(j2\pi(F-6k)\big)$$

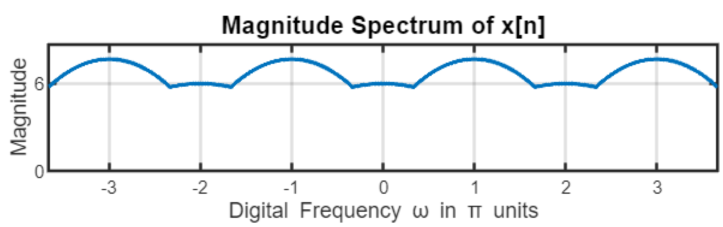
$$= 6 \sum_{k=-\infty}^{\infty} \begin{cases} 1 - \dfrac{(F-kF_s)^2}{25}, & |F - kF_s| \le 5 \\ 0, & \text{otherwise} \end{cases}$$

(ii) Plot it as a function of ω in rad/sam.

Solution: This plot can be best done using hand drawing or a drawing program. MATLAB can also be used as a drawing program as given below.

MATLAB script:

```
Fs = 6; FH = 5; % Highest frequency in xc(t)
Fmax = FH+Fs; F = linspace(-Fmax,Fmax,2001); % Freq range to plot over
X = zeros(1,2001);
for k = -5:5
    ind = abs(F-k*Fs) <= 5;
    X(ind) = X(ind)+(1-(F(ind)-k*Fs).^2/25)*Fs; % Amplitude scaling and sum
end
figure('position',[0,0,8,2]*72);
plot(2*F/Fs,abs(X),'linewidth',2); % Frequency scaling in the plot
xlabel('Digital Frequency \omega in \pi units');
ylabel('Magnitude'); title('Magnitude Spectrum of x[n]');
axis([-2*Fmax/Fs,2*Fmax/Fs,0,max(abs(X))+1]);
set(gca,'xtick',-3:1:3,'ytick',[0,Fs]); grid;
```



(iii) Explain if $x_c(t)$ can be recovered from $x[n]$.

Solution: From the above plot it is evident that $x_c(t)$ can be recovered from $x[n]$. The shape of the spectra in the primary interval $-\pi < \omega \leq \pi$ does not match with the figure given above.