# EEECE5666 (DSP) : Homework-2

**Due on February 8, 2022 by 11:59 pm via submission portal.**

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## Instructions

1. You are required to complete this assignment using Live Editor.
2. Enter your MATLAB script in the spaces provided. If it contains a plot, the plot will be displayed after the script.
3. All your plots must be properly labeled and should have appropriate titles to get full credit.
4. Use the equation editor to typeset mathematical material such as variables, equations, etc.
5. After completeing this assignment, export this Live script to PDF and submit the PDF file through the provided submission portal.
6. You will have only one attempt to submit your assignment. Make every effort to submit the correct and completed PDF file the first time.
7. Please submit your homework before the due date/time. A late submission after midnight of the due date will result in loss of points at a rate of 10% per hour until 8 am the following day, at which time the solutions will be published.

## Default Plot Parameters

```
set(0,'defaultfigurepaperunits','points','defaultfigureunits','points');
set(0,'defaultaxesfontsize',10); set(0,'defaultaxeslinewidth',1.5);
set(0,'defaultaxestitlefontsize',1.4,'defaultaxeslabelfontsize',1.2);
```

## Problem 2.1

### Text Problem 3.45, parts (b) and (d) only (Page 130)

Determine the $z$-transform and sketch the pole-zero plot with the ROC for each of the following sequences.

```
clc; close all; clear;
```

**(b)** $x[n] = (1/2)^n u[n+1] + 3^n u[-n-1]$:

**Solution**:

$$X(z) = \sum_{n=-\infty}^{\infty} \left((1/2)^n u[n+1] + 3^n u[-n-1]\right) z^{-n}$$

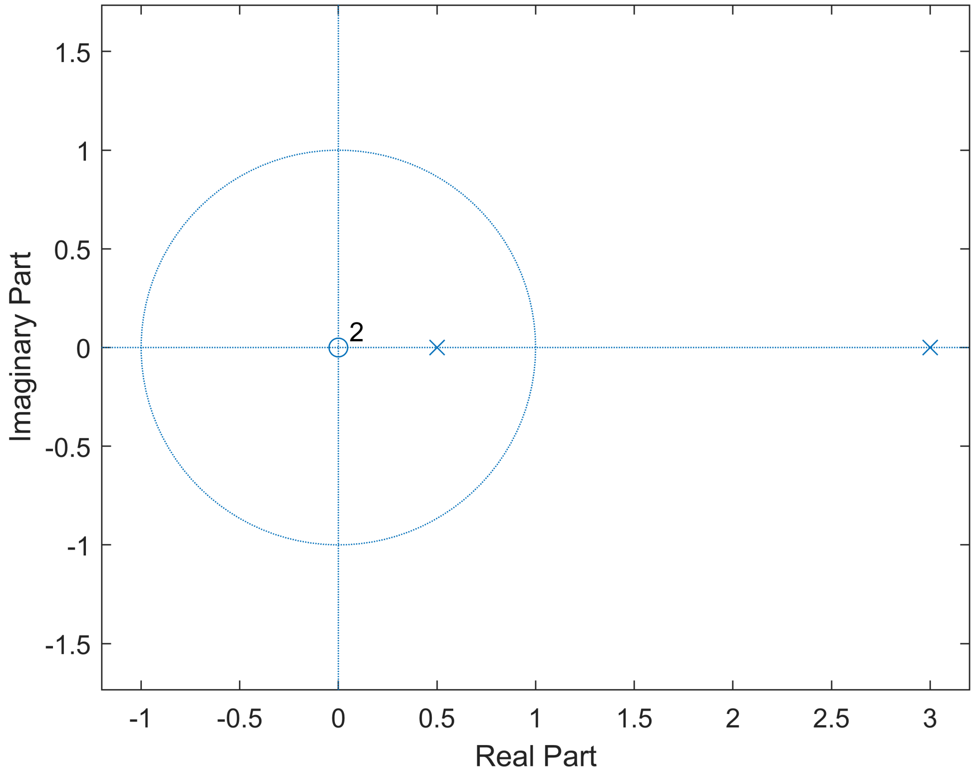
$$X(z) = \sum_{n=-\infty}^{\infty} (1/2)^n u[n+1] z^{-n} + \sum_{n=-\infty}^{\infty} 3^n u[-n-1] z^{-n}$$

$$X(z) = \sum_{n=-1}^{\infty} \left(\frac{1/2}{z}\right)^n + \sum_{n=-\infty}^{-1} \left(\frac{z}{3}\right)^n$$

$$X(z) = \frac{1}{1-0.5z^{-1}} - \frac{1}{1-3z^{-1}}\ , \quad \text{ROC: } \frac{1}{2} < |z| < 3$$

**MATLAB script for pole-zero and ROC plot**:

```
p = [0.5;3]; z = [0;0];
figure
zplane(z,p)
```




(d) $x[n] = |n|(1/2)^{|n|}$

**Solution**:

$$X(z) = \sum_{n=-\infty}^{\infty} (|n|(1/2)^{|n|})z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{-1} n\left(\frac{1}{2}\right)^{n} z^{-n} + 0 + \sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^{n} z^{-n}$$

$$X(z) = \frac{\frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)^2} + \frac{\frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)^2}$$

$$X(z) = \frac{z^{-1}}{1 - z^{-1} + \frac{1}{4}z^{-2}}$$

**MATLAB script for pole-zero and ROC plot**:

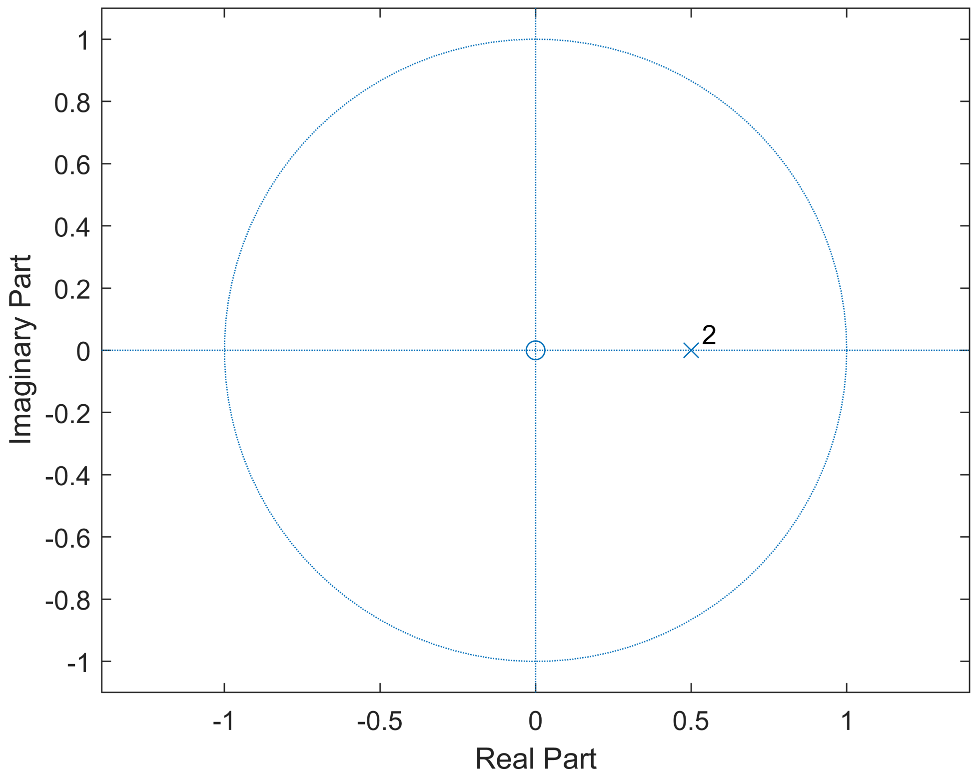
```
b = [0 1]; a = [1 -1 1/4];
[A,p,C] = residuez(b,a)
```

```
A = 2×1
```

```
   -2.0000
    2.0000
p = 2×1
    0.5000
    0.5000
C =

     []
```

```
zplane(b,a)
```




The ROC for this sequence would then be $\text{ROC: } |z| > \dfrac{1}{2}$

## Problem 2.2

Consider the $z$-transform expression:

$$X(z) = \frac{(z - 0.91)\left(z^2 + 0.3z + 0.4\right)}{\left(z + 1.5\right)\left(z^2 - 0.6z + 0.6\right)}.$$

```
clc; close all; clear;
```

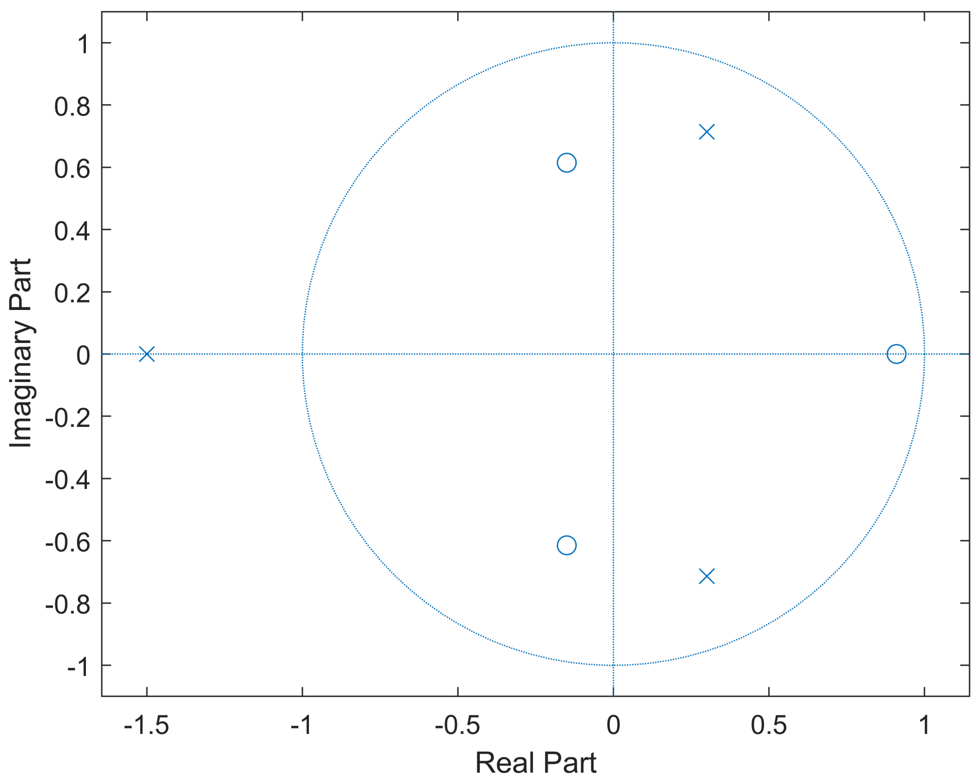
**(a)** Provide a zero-pole plot of $X(z)$.

**MATLAB script**:

```
b = [1 -0.61 0.127 -0.364]; a = [1 0.9 -0.3 0.9];
z = roots(b), p = roots(a)
```

```
z = 3×1 complex
   0.9100 + 0.0000i
  -0.1500 + 0.6144i
  -0.1500 - 0.6144i
p = 3×1 complex
  -1.5000 + 0.0000i
   0.3000 + 0.7141i
   0.3000 - 0.7141i
```

```
figure
zplane(z,p)
```




---

**(b)** List all possible regions of convergence (ROCs) for this $z$-transform.

**Solution:**

Given the calculated poles at $p_1 = -1.5, p_2 = 0.3 + j0.7141,$ and $p_3 = 0.3 - j0.7141$

There are 3 possible regions of convergence being:

$\text{ROC}_1 = |z| > 1.5$

$\text{ROC}_2 = 0.3 < |z| < 1.5$

$\text{ROC}_3 = |z| < 0.3$

---

**(c)** Determine the inverse $z$-transform so that the resulting sequence is absolutely summable. This sequence $x[n]$ should be a real-valued sequence. Provide a **`stem`** plot of $x[n]$.

**Solution**

**MATLAB script for sequence plot**:

```
[A,p,C] = residuez(b,a)
```

```
A = 3×1 complex
   0.9426 + 0.0000i
   0.2309 + 0.1643i
   0.2309 - 0.1643i
p = 3×1 complex
  -1.5000 + 0.0000i
   0.3000 + 0.7141i
   0.3000 - 0.7141i
C = -0.4044
```

```
Ma = (abs(A))', Mp = (abs(p))'
```

```
Ma = 1×3
    0.9426    0.2834    0.2834
Mp = 1×3
    1.5000    0.7746    0.7746
```

```
Aa = (angle(A)), Ap = (angle(p))
```

```
Aa = 3×1
         0
    0.6184
   -0.6184
Ap = 3×1
    3.1416
    1.1731
   -1.1731
```

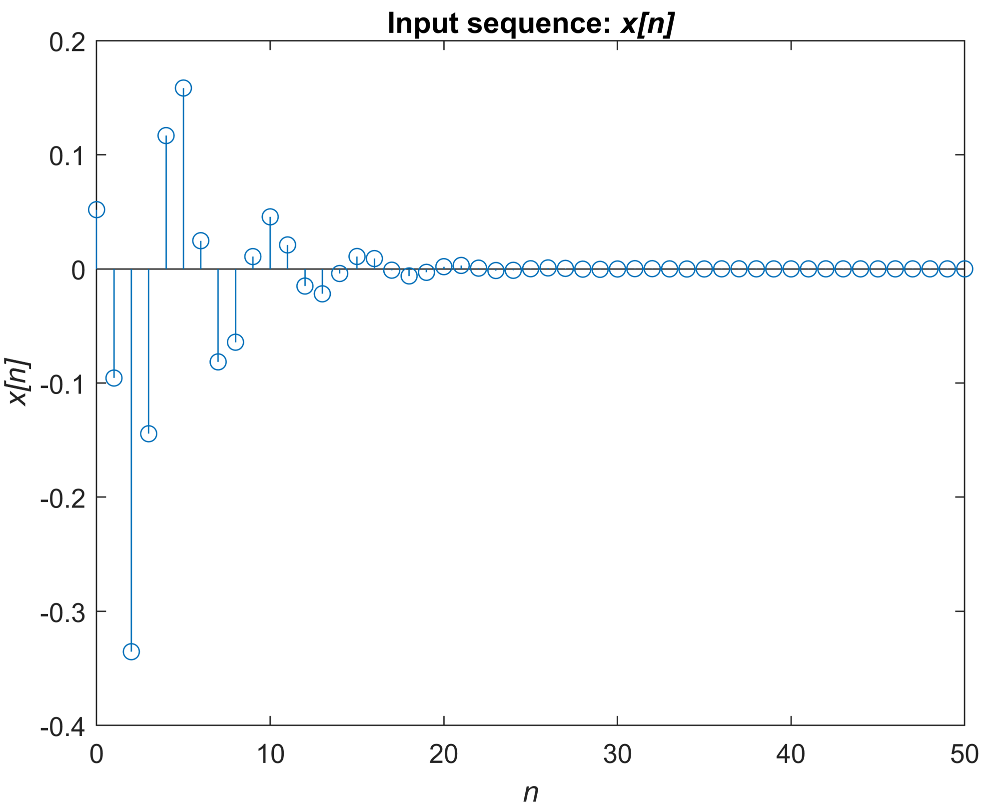
$$X(z) = -0.4044 + \frac{0.9426}{\left(1 + \frac{3}{2}z^{-1}\right)} + \frac{\left(0.2309 + j0.1643\right)}{1 - (0.3 + j0.7141)z^{-1}} + \frac{\left(0.2309 - j0.1643\right)}{1 - (0.3 - j0.7141)z^{-1}}$$ or

$$X(z) = -0.4044 + \frac{0.9426}{\left(1 + \frac{3}{2}z^{-1}\right)} + \frac{0.2834e^{j35.4^\circ}}{1 - 0.78e^{j67.2^\circ}z^{-1}} + \frac{0.2834e^{-j35.4^\circ}}{1 - 0.78e^{-j67.2^\circ}z^{-1}}$$

After computing the magnitude and phases of the complex poles and coefficients, we get the input sequence:

$$x[n] = -0.4044\delta[n] + 0.9426\left(-\frac{3}{2}\right)^n u[-n-1] + 0.56\left(0.78\right)^n \cos\left(67.2^\circ n + 35.4^\circ\right)u[n]$$

```
n = 0:50;
x1 = (n==0); x2 = (n>=0); x3 = (n<=-1);
x = -0.4044*x1 + 0.9426.*((-3/2).^(n)).*x3 + 0.56.*((0.78).^(n)).*cos(Ap(2).*n + Aa(2)).*x2;
figure
stem(n,x)
xlabel("\it{n}")
ylabel("\it{x[n]}")
title("Input sequence: \it{x[n]}")
```




## Problem 2.3

**Text Problem 3.47, parts (b) and (e), (Page 131)**

Given the $z$-transform pair $x[n] \leftrightarrow X(z) = z^{-1}/(1+0.8z^{-1})$ with ROC: $|z| > 0.8$, use the $z$-transform properties to determine the $z$-transform of the following sequences:

**(b)** $y[n] = x[3-n] = x[-(n-3)]$:

**Solution**:

The output signal $y[n]$ exhibits properties of the Time-Shift and Folding operations performed on the input signal $x[n]$ where,

$x[-n] \leftrightarrow X(1/z) = \frac{(z^{-1})^{-1}}{1+0.8(z^{-1})^{-1}} = \frac{z}{1+0.8z}$ would be the Folding operation performed on $x[n]$ and then

$x[n-k] \leftrightarrow z^{-k}X(z) = z^3\frac{z}{1+0.8z} = \frac{z^4}{1+0.8z}$ with $\text{ROC} = |z| < 0.8$

---

**(e)** $y[n] = x[n] * x[2-n]$:

**Solution:**

$x[2-n]$ is a folded and shifted sequence of the input signal $x[n]$ and can be derived from the following operations:

$x[2-n] \leftrightarrow z^2X(1/z) = z^2\frac{(z^{-1})^{-1}}{1+0.8(z^{-1})^{-1}} = \frac{z^3}{1+0.8z}$ then the operation

$y[n] = x[n] * x[2-n] \leftrightarrow X(z)X(1/z)z^2$ which means the z-transform is the product of the z-transforms for both input sequences giving:

$\left(\frac{z^{-1}}{1+0.8z^{-1}}\right)\left(\frac{z^3}{1+0.8z}\right) = \frac{z^2}{(1+0.8z^{-1})(1+0.8z)}$

---

## Problem 2.4

An LTI system described by the following impulse response

$$h[n] = n\left(\frac{1}{3}\right)^n u[n] + \left(-\frac{1}{4}\right)^n u[n].$$

```
clc; close all; clear;
```

---

**(a)** Determine the system function representation.

**Solution**:

The system function is defined as $H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$ so,

$H(z) = Z\{h[n]\} = Z\{n\left(\frac{1}{3}\right)^n u[n]\} + Z\{\left(-\frac{1}{4}\right)^n u[n]\}$

$H(z) = \frac{\frac{1}{3}z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)^2} + \frac{1}{\left(1+\frac{1}{4}z^{-1}\right)}$ which can be expanded to

$$H(z) = \frac{\frac{1}{3}z^{-1}}{\left(1 - \frac{2}{3}z^{-1} + \frac{1}{9}z^{-2}\right)} + \frac{1}{\left(1 + \frac{1}{4}z^{-1}\right)}$$

then in order to make it a single rational fraction we multiple the denominators together and the numerators by the other's denominator to get

$$H(z) = \frac{\frac{1}{3}z^{-1}\left(1 - \frac{2}{3}z^{-1} + \frac{1}{9}z^{-2}\right) + 1\left(1 - \frac{2}{3}z^{-1} + \frac{1}{9}z^{-2}\right)}{\left(1 - \frac{2}{3}z^{-1} + \frac{1}{9}z^{-2}\right)\left(1 + \frac{1}{4}z^{-1}\right)} = \frac{1 - \frac{1}{3}z^{-1} + \frac{7}{36}z^{-2}}{1 - \frac{5}{12}z^{-1} - \frac{1}{18}z^{-2} + \frac{1}{36}z^{-3}}$$ hence,

$$H(z) = \frac{1 - \frac{1}{3}z^{-1} + \frac{7}{36}z^{-2}}{1 - \frac{5}{12}z^{-1} - \frac{1}{18}z^{-2} + \frac{1}{36}z^{-3}}$$

---

**(b)** Determine the difference equation representation.

**Solution**:

$$H(z) = \frac{B(z)}{A(z)} = \frac{1 - \frac{1}{3}z^{-1} + \frac{7}{36}z^{-2}}{1 - \frac{5}{12}z^{-1} - \frac{1}{18}z^{-2} + \frac{1}{36}z^{-3}}$$

$$Y(z) = -\sum_{k=1}^{N} a_k z^{-k} Y(z) + \sum_{k=0}^{M} b_k z^{-k} X(z)$$

$$Y(z) = \frac{5}{12}z^{-1}Y(z) + \frac{1}{18}z^{-2}Y(z) - \frac{1}{36}z^{-3}Y(z) + X(z) - \frac{1}{3}z^{-1}X(z) + \frac{7}{36}z^{-2}X(z)$$ then taking the inverse z-transform to solve for $y[n]$ we get:

$$y[n] = \frac{5}{12}y[n-1] + \frac{1}{18}y[n-2] - \frac{1}{36}y[n-3] + x[n] - \frac{1}{3}x[n-1] + \frac{7}{36}x[n-2]$$

---

**(c)** Determine the pole-zero plot.

**MATLAB script**:

```
b = [1, -1/3, 7/36]; a = [1, -5/12, -1/18, 1/36];
[A,p,C] = residuez(b,a)
```

```
A = 3×1 complex
  -1.0000 - 0.0000i
   1.0000 + 0.0000i
   1.0000 + 0.0000i
p = 3×1 complex
   0.3333 + 0.0000i
   0.3333 - 0.0000i
```

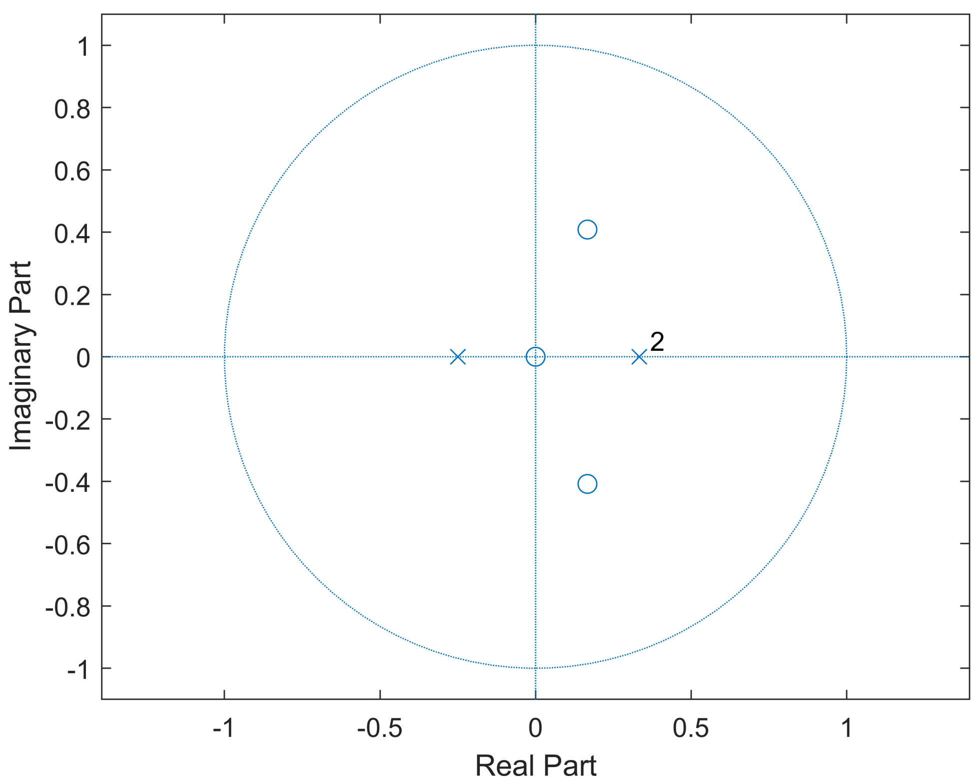
```
  -0.2500 + 0.0000i
C =

     []
```

```
zeros = roots(b), poles = roots(a)
```

```
zeros = 2×1 complex
   0.1667 + 0.4082i
   0.1667 - 0.4082i
poles = 3×1 complex
  -0.2500 + 0.0000i
   0.3333 + 0.0000i
   0.3333 - 0.0000i
```

```
figure
zplane(b,a)
```




Because this an LTI system the ROC will then be $\mathrm{ROC}\colon |z| > \frac{1}{3}$

**(d)** Determine the output sequence $y[n]$ when the input is $x[n] = \left(\frac{1}{4}\right)^n u[n]$.

**Solution**:

$X(z) = \frac{1}{1 - \frac{1}{4}z^{1}}$ and $H(z) = \frac{1 - \frac{1}{3}z^{-1} + \frac{7}{36}z^{-2}}{1 - \frac{5}{12}z^{-1} - \frac{1}{18}z^{-2} + \frac{1}{36}z^{-3}}$ then

$$Y(z) = X(z)H(z) = \left(\frac{1}{1 - \frac{1}{4}z^{-1}}\right)\left(\frac{1 - \frac{1}{3}z^{-1} + \frac{7}{36}z^{-2}}{1 - \frac{5}{12}z^{-1} - \frac{1}{18}z^{-2} + \frac{1}{36}z^{-3}}\right) = \frac{1 - \frac{1}{3}z^{-1} + \frac{7}{36}z^{-2}}{1 - \frac{2}{3}z^{-1} + \frac{7}{144}z^{-2} + \frac{1}{24}z^{-3} - \frac{1}{144}z^{-4}}$$

We can solve for $y[n]$ by taking the partial fraction expansion of $Y(z)$ followed by the inverse z-transform

```
b = [1, -1/3, 7/36]; a = [1, -2/3, 7/144, 1/24, -1/144];
[A,p,C] = residuez(b,a);
A = A', p = p'
```

```
A = 1×4
  -16.0000    4.0000    0.5000   12.5000
p = 1×4
    0.3333    0.3333   -0.2500    0.2500
```

Using the partial fraction coefficients along with the calculates poles we get an output signal of

$$y[n] = 12.5\left(\frac{1}{4}\right)^n u[n] + \frac{1}{2}\left(-\frac{1}{4}\right)^n u[n] + 4\left(\frac{1}{3}\right)^n u[n] - 16\left(\frac{1}{3}\right)^n u[n]$$

## Problem 2.5

### Text Problem 3.57 (Page 132)

Determine the impulse response of the system described by

$$y[n] + \frac{11}{6}y[n-1] + \frac{1}{2}y[n-2] = 2x[n].$$

for all possible regions of convergence.

**Solution:**

Taking the z-transform of the expression above we get:

$$Y(z) + \frac{11}{6}z^{-1}Y(z) + \frac{1}{2}z^{-2}Y(z) = 2X(z)$$

$$(1 + \frac{11}{6}z^{-1} + \frac{1}{2}z^{-2})Y(z) = 2X(z)$$

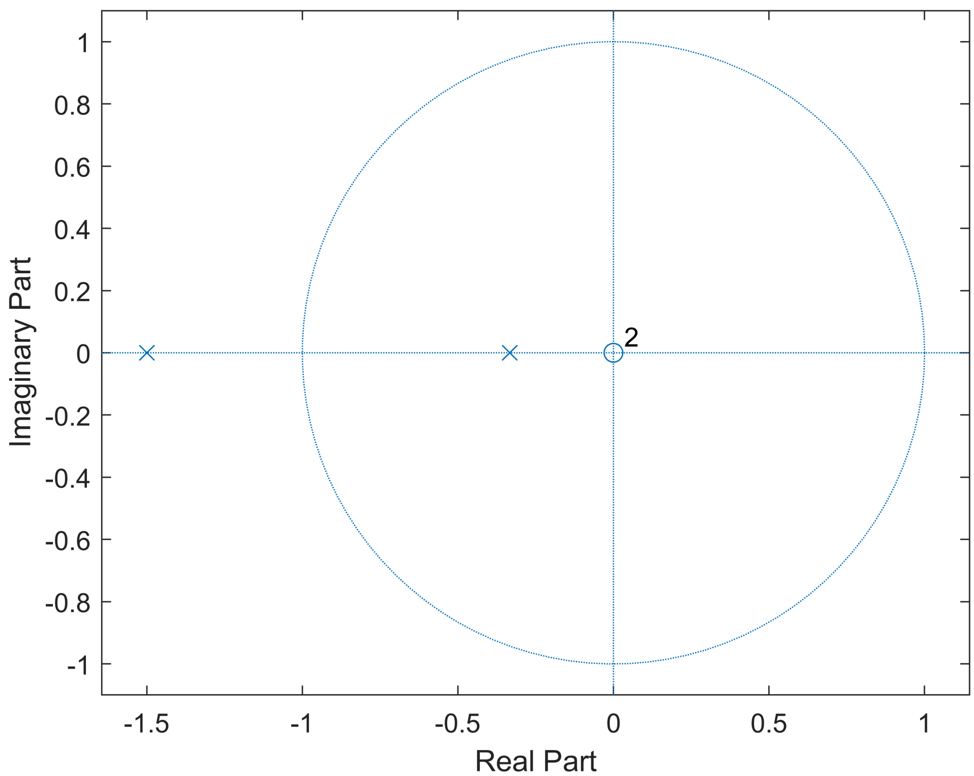
$$H(z) = \frac{Y(z)}{X(z)} = \frac{2}{1 + \frac{11}{6}z^{-1} + \frac{1}{2}z^{-2}}$$

Solving for the partial fraction coefficients and poles we get:

```
b = 2; a = [1, 11/6, 1/2];
[A,p,C] = residuez(b,a); A = A', p = p'
```

```
A = 1×2
    2.5714   -0.5714
p = 1×2
   -1.5000   -0.3333
```

```
zplane(b,a)
```




Which gives us coefficients 2.5714 and -0.5714 with poles $p_1 = \frac{1}{3}$ and $p_2 = \frac{3}{2}$

$H(z) = -\dfrac{0.5714}{1+\frac{1}{3}z^{-1}} + \dfrac{2.5714}{1+\frac{3}{2}z^{-1}}$ then by taking the inverse z-transform we get 3 possible regions of convergence

$$h[n] = 2.5714\left(-\frac{3}{2}\right)^n u[n] - 0.5714\left(-\frac{1}{3}\right)^n u[n], \quad |z| > \frac{3}{2} \text{ (causal)}$$

$$h[n] = -2.5714\left(-\frac{3}{2}\right)^n u[-n-1] + 0.5714\left(-\frac{1}{3}\right)^n u[-n-1], \quad |z| < \frac{1}{3} \text{ (anticausal)}$$

$$h[n] = -2.5714\left(-\frac{3}{2}\right)^n u[-n-1] - 0.5714\left(-\frac{1}{3}\right)^n u[n], \quad \frac{1}{3} < |z| < \frac{3}{2} \text{ (two-sided)}$$

## Problem 2.6

**Text Problem 3.63 (Page 133)**

Consider the following LCCDE

$$y[n] = 2\cos(\omega_0)y[n-1] - y[n-2]$$

with no input but with initial conditions $y[-1] = 0$ and $y[-2] = -A\sin(\omega_0)$.

---

**(a)** Show that the solution of the above LCCDE is given by $y[n] = A\sin[(n+1)\omega_0]u[n]$. This system is known as a *digital oscillator*.

**Solution:**

$y[n] = 2\cos(\omega_0)y[n-1] - y[n-2]$

We can apply the z-transform to the LCCDE to obtain $Y(z)$ :

$Y(z) - 2\cos(\omega_0)z^{-1}\left(Y(z) + y[-1]z\right) + z^{-2}\left(Y(z) + y[-1]z + y[-2]z^2\right) = 0$

$(1 - 2\cos(\omega_0)z^{-1} + z^{-2})Y(z) - A\sin(\omega_0) = 0$

$Y(z) = \dfrac{A\sin(\omega_0)}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}$ noticing the numerator doesn't have a $z$ term we can write the z-transform as if it was being time-shifted by +1 so,

$Y(z) = \dfrac{z^1\left(A\sin(\omega_0)z^{-1}\right)}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}$ which after taking the inverse transform to obtain $y[n]$ we observe

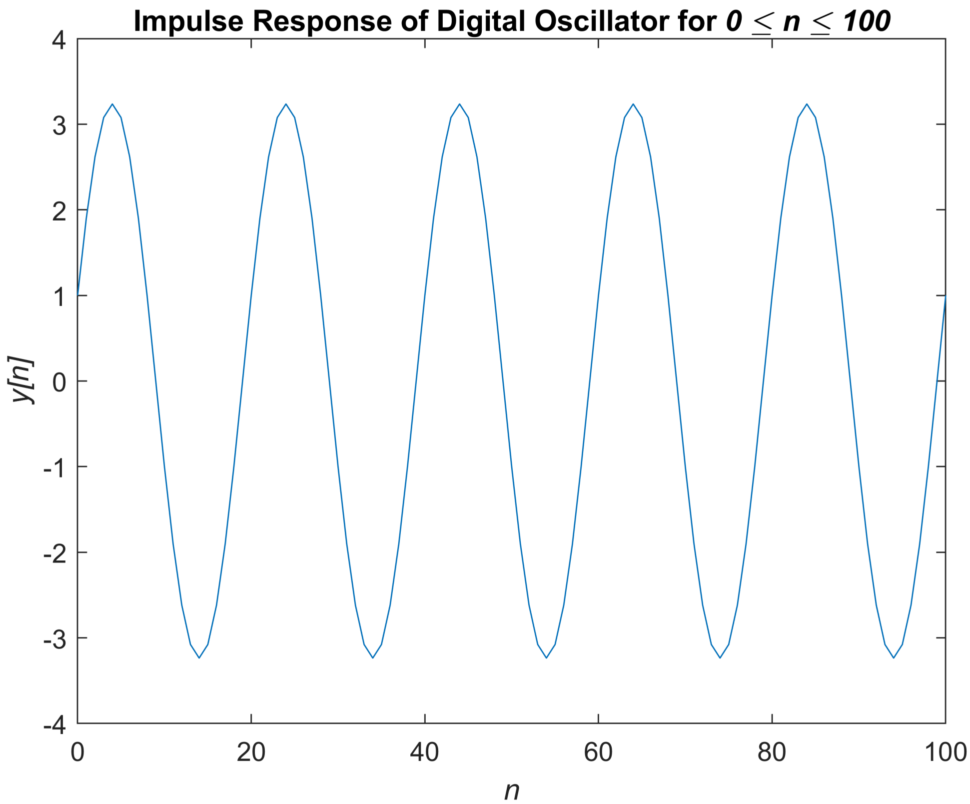
$y[n] = A\sin[(n+1)\omega_0]u[n]$ which verifies the solution to the LCCDE.

---

**(b)** For $A = 2$ and $\omega_0 = 0.1\pi$, verify the operation of the above digital oscillator using MATLAB.

**MATLAB script**:

```
clc; close all; clear;
A = 2; w0 = pi/10;
a = [1, -A*cos(w0),1]; n = 0:100; x = (n==0);
y = filter(1,a,x);
figure
plot(n,y)
xlabel("\it{n}")
ylabel("\it{y[n]}")
title("Impulse Response of Digital Oscillator for \it{0 \leq n \leq 100}")
```




## Problem 2.7

**Text Problem 4.38, parts (a) and (d) only, (Page 197)**

Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period.

**(a)** $x_1(t) = |\sin(7\pi t)|\cos(11\pi t)$

**Solution:**

Since $|\sin(7\pi t)|$ has only positive values, it becomes periodic from $0 < t < \pi$ instead of $2\pi$

Thus, $x_1(t) = \sin\left(\frac{7}{2}\pi t\right)\cos\left(11\pi t\right)$ then we can use a trigonometric identity for the product of these sinusoids giving

$$x_1(t) = \sin\left(\frac{7}{2}\pi t\right)\cos\left(11\pi t\right) = \frac{1}{2}\left[\sin\left(\frac{29\pi}{2}t\right) - \sin\left(\frac{15\pi}{2}t\right)\right]$$

The first sinusoid is periodic with period $T_1 = \frac{2\pi}{\omega_0} = \frac{2\pi}{29\pi/2} = \frac{4}{29}$ and then

The second sinusoid is periodic with period $T_2 = \frac{2\pi}{\omega_0} = \frac{2\pi}{15\pi/2} = \frac{4}{15}$

Using the fact that the sum of two periodic signals result in a periodic signal, hence $x_1(t)$ is thus periodic.

The fundamental period of $x_1(t)$ would then be the least common multiple of both $T_1$ and $T_2$

$$T = LCM(T_1, T_2) = LCM\left(\frac{4}{29}, \frac{4}{15}\right) = 4$$

Thus, the fundamental period of $x_1(t)$ is $T = 4$

---

**(d)** $x_4[n] = \mathrm{e}^{\jmath\pi n/7} + \mathrm{e}^{\jmath\pi n/11}$ :

**Solution:**

Observing the fundamental frequencies of each of the exponentials of $x_4[n]$ we see

$\mathrm{e}^{\jmath\pi n/7} \to \omega_0 = \frac{\pi}{7} \to f = \frac{\omega_0}{2\pi} = \frac{1}{14}\mathrm{Hz}$ and $\mathrm{e}^{\jmath\pi n/11} \to \omega_0 = \frac{\pi}{11} \to f = \frac{\omega_0}{2\pi} = \frac{1}{22}\mathrm{Hz}$

both frequencies are rational values of $\omega_0$ , thus the sequence $x_4[n]$ is periodic.

It's fundamental period can be calculated from the Least Common Multiple of each frequency, so

$T_1 = 1/F = 14Hz$ and $T_2 = 1/F = 22Hz$ then fundamental period would be

$N = LCM(14, 22) = 154$

Thus, the sequence $x_4[n]$ is periodic with fundamental period $= 154$

---

## Problem 2.8

### Text Problem 4.45, parts (c) and (d) only, (Page 198)

Given that $x[n]$ is a periodic sequence with fundamental period $N$ and Fourier coefficients $a_k$, determine the Fourier coefficients of the following sequences in terms of $a_k$.

Note that

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\mathrm{e}^{-\jmath 2\pi kn/N}. \qquad (2.8.1)$$

---

**(c)** $x_3[n] = 3\cos(2\pi 5n/N)x[-n],\ N > 5$ :

**Solution**:

Using the analysis equation of the Fourier Series, we find that

$$x_3[n] = 3F\{\cos(2\pi 5n/N)x[-n]\} = 3\left[\frac{1}{2}e^{j\left(\frac{2\pi}{N}5n\right)} + \frac{1}{2}e^{-j\left(\frac{2\pi}{N}5n\right)}\right]x[-n] = \left[\frac{3}{2}e^{j\left(\frac{2\pi}{N}5n\right)} + \frac{3}{2}e^{-j\left(\frac{2\pi}{N}5n\right)}\right]x[-n]$$

To obtain the Fourier coefficients of the sequence we can then use the following properties of the Fourier Series:

$$x[-n] \leftrightarrow a_{-k}$$

$$e^{j\left(\frac{2\pi}{N}Mn\right)} \leftrightarrow a_{k-M}$$

$$e^{-j\left(\frac{2\pi}{N}Mn\right)} \leftrightarrow a_{k+M}$$

then let $c_k$ be the Fourier coefficients in terms of $a_k$ we get

$$c_k = \frac{3}{2}(a_{-k-5} + a_{-k+5})$$

---

**(d)** $x_4[n] = x[n] + x^*[-n]$ :

**Solution**:

Using the conjugate and folding properties of the Fourier series, we obtain the coefficients represented by $c_k$ in terms of $a_k$

$$c_k = a_k + a_k^*$$

Since our sequence is an evenly symmetry sequence our final result of the Fourier coefficients would be

$$c_k = 2Re\{a_k\}$$

---

## Problem 2.9

**Text Problem 4.49, partsa (c) and (d) only, (Page 198)**

Determine sequences corresponding to each of the following Fourier transforms.

---

**(c)** $X_3(e^{j\omega}) = je^{-j4\omega}[2 + 3\cos(\omega) + \cos(2\omega)]$ :

**Solution**:

Distributing the $je^{-j4\omega}$ term to the components inside the brackets we get:

$$X_3(e^{jw}) = 2je^{-j4\omega} + 3je^{-j4\omega}\left[\frac{e^{-j\omega}+e^{j\omega}}{2}\right] + je^{-j4\omega}\left[\frac{e^{-j2\omega}+e^{j2\omega}}{2}\right]$$

$$X_3(e^{jw}) = 2je^{-j4\omega} + \frac{3}{2}je^{-j5\omega} + \frac{3}{2}je^{-j3\omega} + \frac{1}{2}je^{-j6\omega} + \frac{1}{2}je^{-j2\omega}$$

Applying the inverse Discrete Fourer Transform to $X_3(e^{jw})$ results in:

$$x_3[n] = \frac{1}{2}j\delta(n-2) + \frac{3}{2}j\delta(n-3) + 2j\delta(n-4) + \frac{3}{2}j\delta(n-5) + \frac{1}{2}j\delta(n-6)$$

---

**(d)** $X_4(\mathrm{e}^{j\omega}) = \begin{cases} 2, & 0 \le |\omega| \le \pi/8 \\ 1, & \pi/8 \le |\omega| \le 3\pi/4 \\ 0, & 3\pi/4 \le |\omega| \le \pi \end{cases}$

**Solution**:

$$x_4[n] = \frac{1}{2\pi}\int_{<2\pi>} X(e^{j\omega})e^{j\omega n}d\omega$$

$$x_4[n] = \frac{1}{2\pi}\left[\int_{\frac{-\pi}{8}}^{0} 2e^{j\omega n}d\omega + \int_{0}^{\frac{\pi}{8}} 2e^{j\omega n}d\omega + \int_{-\frac{3\pi}{4}}^{\frac{\pi}{8}} e^{j\omega n}d\omega + \int_{\frac{\pi}{8}}^{\frac{3\pi}{4}} e^{j\omega n}d\omega\right]$$

$$x_4[n] = \frac{1}{2\pi}\left[2\left(\frac{1-e^{-j\frac{\pi}{8}n}}{jn}\right) + 2\left(\frac{e^{j\frac{\pi}{8}n}-1}{jn}\right) + \left(\frac{e^{j\frac{\pi}{8}} - e^{-j\frac{3\pi}{4}n}}{jn}\right) + \left(\frac{e^{j\frac{3\pi}{4}} - e^{j\frac{\pi}{8}n}}{jn}\right)\right]$$

$$x_4[n] = \frac{1}{2\pi}\left[\frac{2}{jn}(1-1) + \frac{2}{jn}\left(e^{j\frac{\pi}{8}n} - e^{-j\frac{\pi}{8}n}\right) + \frac{1}{jn}\left(e^{j\frac{\pi}{8}n} - e^{j\frac{\pi}{8}n}\right) + \frac{1}{jn}\left(e^{j\frac{3\pi}{4}n} - e^{-j\frac{3\pi}{4}n}\right)\right]$$

Using the euler's identity for $\sin$ and canceling some terms we obtain

$x_4[n] = \frac{1}{\pi n}\left[2\sin\left(\frac{\pi}{8}n\right) + \sin\left(\frac{3\pi}{4}n\right)\right]$ is the corresponding sequence.

---

## Problem 2.10

### Text Problem 4.53 (Page 199)

**Note**: There are two corrections in the errata sheet. Please follow them.

Let a sinusoidal pulse be given by $x(n) = (\cos\omega_0 n)\big(u[n] - u[n-M]\big)$.

---

**(a)** Using the frequency-shifting property of the DTFT, show that the real-part of DTFT of $x(n)$ is given by

$$X_R(e^{j\omega}) = \frac{1}{2}\cos\left[\frac{(\omega-\omega_0)(M-1)}{2}\right]\left[\frac{\sin\left(\frac{(\omega-\omega_0)M}{2}\right)}{\sin\left(\frac{\omega-\omega_0}{2}\right)}\right] + \frac{1}{2}\cos\left[\frac{(\omega+\omega_0)(M-1)}{2}\right]\left[\frac{\sin\left(\frac{(\omega+\omega_0)M}{2}\right)}{\sin\left(\frac{\omega+\omega_0}{2}\right)}\right] \tag{2.10.1}$$

**Solution**:

The frequency-shifting property of the DTFT says that $\rightarrow x[n]e^{j\omega_0 n} \leftrightarrow X\left(e^{j(\omega-\omega_0)}\right)$

Using euler's formula, we can represent the sinusoidal pulse $x(n)$ as

$$\cos(\omega_0 n) = \frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2} \rightarrow x(n) = \frac{1}{2}e^{j\omega_0 n}(u[n]-u[n-M]) + \frac{1}{2}e^{-j\omega_0 n}(u[n]-u[n-M])$$

Here, the square pulse is being frequency-shifted by a factor of $\omega_0$ so we can apply this when taking the DTFT of the pulse train, such that

$u[n]-u[n-M] \leftarrow \mathrm{DTFT} \rightarrow \sum_{n=0}^{M-1} e^{-j\omega n} = \frac{1-e^{-j\omega M}}{1-e^{-j\omega}}$ then apply the frequency shifts to get

$$X(e^{j\omega}) = \frac{1}{2}\left[\frac{1-e^{-jM(\omega-\omega_0)}}{1-e^{-j(\omega-\omega_0)}} + \frac{1-e^{-jM(\omega+\omega_0)}}{1-e^{-j(\omega+\omega_0)}}\right]$$

We can reduce this term even further by factoring out exponential terms

$$\frac{1-e^{-jM(\omega-\omega_0)}}{1-e^{-j(\omega-\omega_0)}} \rightarrow \frac{e^{-jM\left(\frac{\omega-\omega_0}{2}\right)}}{e^{-j\left(\frac{\omega-\omega_0}{2}\right)}}\left[\frac{e^{jM\left(\frac{\omega-\omega_0}{2}\right)} - e^{-jM\left(\frac{\omega-\omega_0}{2}\right)}}{e^{j\left(\frac{\omega-\omega_0}{2}\right)} - e^{-j\left(\frac{\omega-\omega_0}{2}\right)}}\right] \rightarrow e^{-j\left(\frac{(\omega-\omega_0)(M-1)}{2}\right)}\left[\frac{2j\sin\left(\frac{(\omega-\omega_0)M}{2}\right)}{2j\sin\left(\frac{(\omega-\omega_0)}{2}\right)}\right]$$

Thus, $X(e^{j\omega}) = \frac{1}{2}e^{-j\left(\frac{(\omega-\omega_0)(M-1)}{2}\right)}\left[\frac{\sin\left(\frac{(\omega-\omega_0)M}{2}\right)}{\sin\left(\frac{(\omega-\omega_0)}{2}\right)}\right] + \frac{1}{2}e^{-j\left(\frac{(\omega+\omega_0)(M-1)}{2}\right)}\left[\frac{\sin\left(\frac{(\omega+\omega_0)M}{2}\right)}{\sin\left(\frac{(\omega+\omega_0)}{2}\right)}\right]$

Then if we only want to take the $Re\{X(e^{j\omega})\}$ we can expand out the exponential terms to cosine terms resulting in

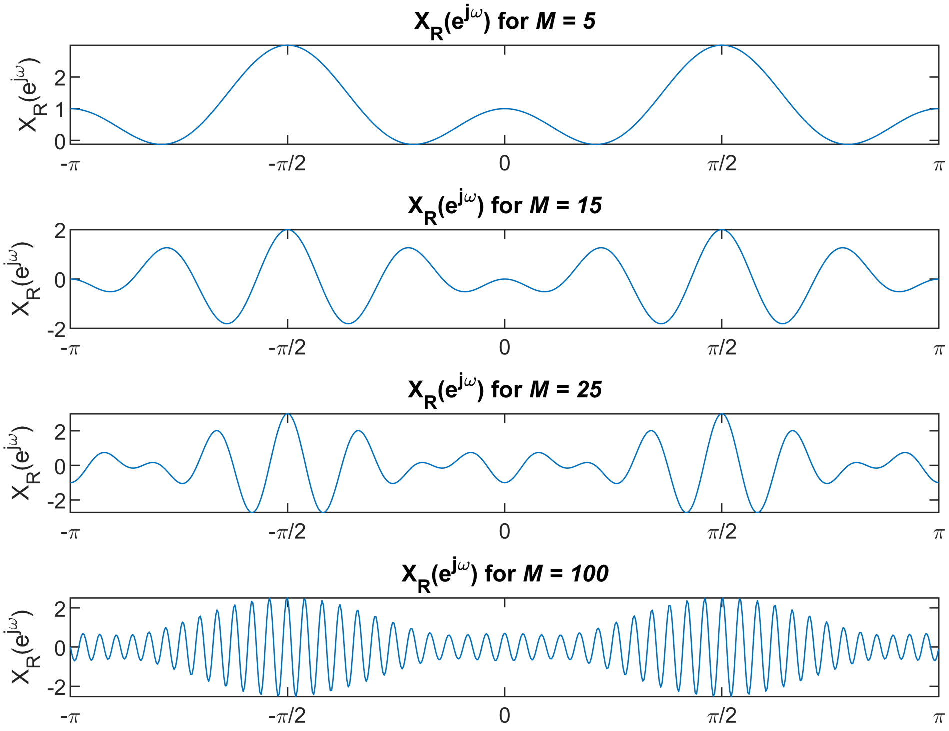
$$Re\{X(e^{j\omega})\} = X_R(e^{j\omega}) = \frac{1}{2}\cos\left(\frac{(\omega-\omega_0)(M-1)}{2}\right)\left[\frac{\sin\left(\frac{(\omega-\omega_0)M}{2}\right)}{\sin\left(\frac{(\omega-\omega_0)}{2}\right)}\right] + \frac{1}{2}\cos\left(\frac{(\omega+\omega_0)(M-1)}{2}\right)\left[\frac{\sin\left(\frac{(\omega+\omega_0)M}{2}\right)}{\sin\left(\frac{(\omega+\omega_0)}{2}\right)}\right]$$

---

**(b)** Compute and plot $X_R(e^{j\omega})$ for $\omega_0 = \pi/2$ and $M = 5, 15, 25, 100$. Use the plotting interval of $[-\pi, \pi]$. Comment on your results.

**MATLAB script**:

```
clc; close all; clear;
Xr = zeros(4,629);
w0 = pi/2; omega = -pi:1/100:pi; M = 5;
A = sin(((omega-w0).*M)/2)./sin((omega-w0)./2); B = sin(((omega+w0).*M)./2)./sin((omega+w0)./2)
Xr(1,:) = 0.5.*((cos(((omega-w0).*(M-1))./2)).*A + (cos(((omega+w0).*(M-1))/2)).*B);
M = 15;
Xr(2,:) = 0.5.*((cos(((omega-w0).*(M-1))./2)).*A + (cos(((omega+w0).*(M-1))./2)).*B);
M = 25;
Xr(3,:) = 0.5.*((cos(((omega-w0).*(M-1))./2)).*A + (cos(((omega+w0).*(M-1))./2)).*B);
M = 100;
Xr(4,:) = 0.5.*((cos(((omega-w0).*(M-1))./2)).*A + (cos(((omega+w0).*(M-1))./2)).*B);
figure
subplot(4,1,1)
plot(omega,Xr(1,:))
xlim([-pi pi])
xticks([-pi -pi/2 0 pi/2 pi]);
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
ylabel("X_R(e^{j\omega})"), title("X_R(e^{j\omega}) for \it{M} = 5");
subplot(4,1,2)
plot(omega,Xr(2,:))
xlim([-pi pi])
xticks([-pi -pi/2 0 pi/2 pi]);
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
ylabel("X_R(e^{j\omega})"), title("X_R(e^{j\omega}) for \it{M} = 15");
subplot(4,1,3)
plot(omega,Xr(3,:))
xlim([-pi pi])
xticks([-pi -pi/2 0 pi/2 pi]);
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
ylabel("X_R(e^{j\omega})"), title("X_R(e^{j\omega}) for \it{M} = 25");
subplot(4,1,4)
plot(omega,Xr(4,:))
xlim([-pi pi])
xticks([-pi -pi/2 0 pi/2 pi]);
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
ylabel("X_R(e^{j\omega})"), title("X_R(e^{j\omega}) for \it{M} = 100");
```




Comparing the results for $M = 5, 15, 25, 100$, we observe the real signals are symmetrical across zero and are oscillating at higher frequencies as M increases. Then since the cosine term is multiplying with a square-pulse train, we end up with symmetrical, frequency-shifted sinc functions.