EECE5666 (DSP) : Homework-7

Due on April 19, 2022 by 11:59 pm via submission portal.

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Instructions

1. You are required to complete this assignment using Live Editor.
2. Enter your MATLAB script in the spaces provided. If it contains a plot, the plot will be displayed after the script.
3. All your plots must be properly labeled and should have appropriate titles to get full credit.
4. Use the equation editor to typeset mathematical material such as variables, equations, etc.
5. After completeing this assignment, export this Live script to PDF and submit the PDF file through the provided submission portal.
6. You will have only one attempt to submit your assignment. Make every effort to submit the correct and completed PDF file the first time.
7. Please submit your homework before the due date/time. A late submission after midnight of the due date will result in loss of points at a rate of 10% per hour until 8 am the following day, at which time the solutions will be published.

Default Plot Parameters

```
set(0,'defaultfigurepaperunits','points','defaultfigureunits','points');
set(0,'defaultaxesfontsize',10); set(0,'defaultaxeslinewidth',1.5);
set(0,'defaultaxestitlefontsize',1.4,'defaultaxeslabelfontsize',1.2);
```

Problem 7.1

We want to design a lowpass analog Chebyshev-I filter that has a 0.5 dB or better ripple at 10 Hz and at least 45 dB of attenuation at 20 Hz.

```
clc; close all; clear;
```

(a) Using the design procedure on Page 640 of the textbook (or that in Example 11.3) obtain the system function in a rational function form.

Solution: Follow the following steps. Perform numerical calculations using MATLAB below each step.

Step-0: Determine the analog passband ripple parameter ϵ and stopband attenuation parameter A.

We can determine the values of parameters ϵ and A by

$$\epsilon = \sqrt{10^{Ap(0.1)} - 1} = \sqrt{10^{(0.5)(0.1)} - 1} = 0.3493$$

$$A = 10^{As(0.05)} = 10^{(45)(0.05)} = 177.83$$

```
Ap = 0.5; As = 45; Omegap = 2*pi*10; Omegas = 2*pi*20;
epsilon = sqrt(10^(Ap/10)-1), A = 10^(As/20)
```

```
epsilon = 0.3493
A = 177.8279
```

Step-1: Compute the parameters α and β using (11.50):

Using (11.50), we calculate the values of α and β to be

$$\alpha = \frac{\Omega_s}{\Omega_p} = \frac{2\pi(20)}{2\pi(10)} = 2$$

$$\beta = \frac{1}{\epsilon}\sqrt{A^2 - 1} = \frac{1}{0.3493}\sqrt{(177.83)^2 - 1} = 509.0734$$

```
alpha = Omegas/Omegap, beta = sqrt((A^2 -1)/epsilon^2)
```

```
alpha = 2
beta = 509.0734
```

Step-2: Compute order N using (11.49) and round upwards to the nearest integer:

$$N \geq \frac{\ln(\beta + \sqrt{\beta^2 - 1})}{\ln(\alpha + \sqrt{\alpha^2 - 1})} = \frac{\ln((509) + \sqrt{(509)^2 - 1})}{\ln(2 + \sqrt{2^2 - 1})}$$

```
N = ceil(log(beta + sqrt(beta^2 - 1))/(log(alpha + sqrt(alpha^2 - 1))))
```

```
N = 6
```

Step-3: Set $\Omega_c = \Omega_p$ and compute a and b using (11.44) and (11.45):

We have a passband frequency requirement of 10Hz, thus

$\Omega_c = \Omega_p \to 20\pi \, \frac{rad}{sec}$

We must first calculate the value of γ in order to compute values a and b

$$\gamma = \left(\frac{1}{\epsilon} + \sqrt{1 + \frac{1}{\epsilon^2}}\right)^{\frac{1}{N}} = \left(\frac{1}{0.3493} + \sqrt{1 + \frac{1}{(0.3493)^2}}\right)^{\frac{1}{6}} = 1.3441$$

$$a = \frac{1}{2}(\gamma - \gamma^{-1}) = \frac{1}{2}(1.3441 - 1.3441^{-1}) = 0.3$$

$$b = \frac{1}{2}(\gamma + \gamma^{-1}) = \frac{1}{2}(1.3441 + 1.3441^{-1}) = 1.044$$

```
gamma = (1/epsilon + sqrt(1+1/(0.3493)^2))^(1/N)
```

```
gamma = 1.3441
```

```
a = 0.5*(gamma - 1/gamma), b = 0.5*(gamma + 1/gamma)
```

```
a = 0.3000
b = 1.0440
```

Step-4: Compute the pole locations using (11.41) and (11.42):

We compute the pole locations using the formulas

$\sigma_k = [\Omega_c \sinh(\phi)]\cos(\theta_k)$ with $a \triangleq \sinh(\phi)$

$\Omega_k = [\Omega_c \cosh(\phi)]\sin(\theta_k)$ with $b \triangleq \cosh(\phi)$ with the angle being

$$\theta_k = \frac{\pi}{2} + \frac{2k-1}{2N}\pi, \quad \text{with } k = 1, 2, \ldots, 2N$$

Substituting Ω_c, a, and b into the first formulas we get:

$$\sigma_k = 20\pi(0.3)\cos(\theta_k)$$

$$\Omega_k = 20\pi(1.044)\sin(\theta_k)$$

```
Omegac = Omegap;
% Step-4: Calculations of Poles
k = 1:N; thetak = pi/2+(2*k-1)*pi/(2*N);
sigmak = (a*Omegac)*cos(thetak); Omegak = (b*Omegac)*sin(thetak);
sk = cplxpair(sigmak + 1j*Omegak)
```

```
sk = 1×6 complex
 -18.2085 -16.9782i -18.2085 +16.9782i -13.3295 -46.3853i -13.3295
+46.3853i ⋯
```

Taking only the first 6 poles calculated are the left-half elliptic shaped poles of the Chebyshev-I filter, which are

$s_1 = -4.8789 + j63.3635$

$s_2 = -13.3295 + j46.3853$

$s_3 = -18.2085 + j16.9782$

$s_4 = -18.2085 - j16.9782$

$s_5 = -13.3295 - j46.3853$

$s_6 = -4.8789 - j63.3635$

Step-5: Compute the filter gain G and the system function $H(j\Omega)$ from (11.43): Since N is even,

We can compute the filter gain G by the product of the first N poles by $\left(\frac{1}{\sqrt{1+\epsilon^2}}\right)$, which yields :

```
Rp = 1/sqrt(1+epsilon^2);
D = real(poly(sk));
G = D(end)*Rp
```

```
G = 5.5046e+09
```

Thus, the filter gain $G = 5.5046\,\text{e+09}$

Finally, determine the system function:

$$H_C(s) = \frac{5.5046e09}{(s+4.8789-j63.3635)(s+13.3295-j46.3853)(s+18.2085-j16.9782)(s+18.2085+j16.9782)(s+13.3295+j46.3853)(s+4.8789+j63.3}$$

```
s = tf('s');
Hc = G/((s + abs(sk(1)))*(s + abs(sk(2)))*(s + abs(sk(3)))*(s + abs(sk(4)))*(s
+ abs(sk(5)))*(s + abs(sk(6))))
```

```
Hc =
 
                                          5.505e09
  ------------------------------------------------------------------------------------
  s^6 + 273.4 s^5 + 3.039e04 s^4 + 1.752e06 s^3 + 5.511e07 s^2 + 8.935e08 s + 5.831e09
 
Continuous-time transfer function.
```

Thus, the system function $H(j\Omega)$ is described by the rational function above

(b) Verify your design using the **`cheb1ord`** and **`cheby1`** functions.

Solution:

```
[N,Omegac] = cheb1ord(Omegap,Omegas,Ap,As,'s')
```

```
N = 6
Omegac = 62.8319
```

```
Fc = Omegac/(2*pi)
```

```
Fc = 10
```

Thus, using the cheb1ord function yields the same result for $N = 6$ for the order of the filter with the cutoff frequency of

$F_c = 10$Hz

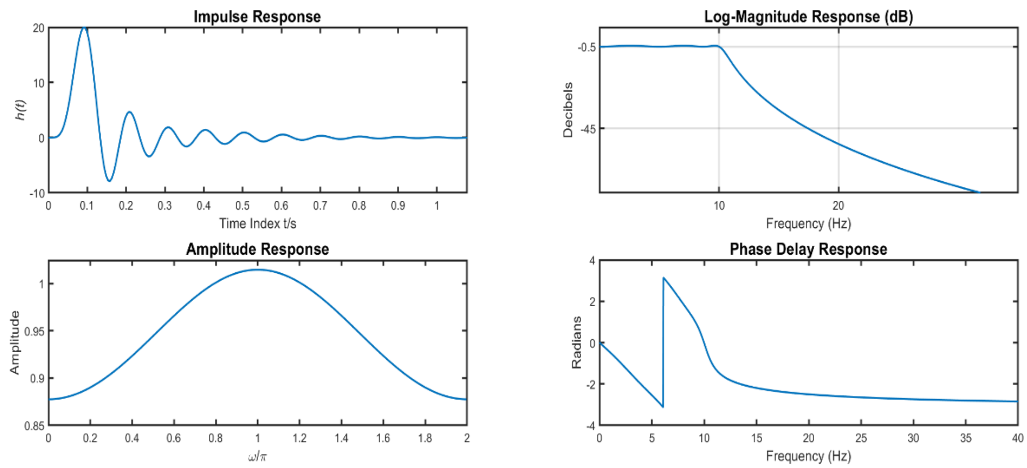
```
[C,D] = cheby1(N,Ap,Omegac,'s');
```

(c) Provide plots of impulse response, amplitude response, log-magnitude response in dB, and phase response in one figure using two rows and two columns.

MATLAB script:

```
% Impulse Response
trsys = tf(C,D);
[h t] = impulse(trsys);
% Frequency Response
Fmax = 40; F = linspace(0,Fmax,1001);
omega = 2*pi*F;
H = freqs(C,D,omega);
Hmag = abs(H);
Hdb = mag2db(Hmag);
% Phase Response
Hpha = angle(H);
Hgd = -diff(unwrap(Hpha))./diff(omega);
Hgd = [Hgd Hgd(end)];
% Amplitude Response
[Hr,wr] = zerophase(C,D,512,'whole');
% Plot Results
figure('Units','inches','Position',[0,0,12,4]);
% Impulse Response Plot
subplot(2,2,1),plot(t,h,'LineWidth',1.5),title('Impulse Response'),xlabel('Time
Index t/s'),ylabel('\it{h(t)}')
xlim([0 t(end)])
% Magnitude Response Plot
subplot(2,2,2),plot(F,Hdb,'LineWidth',1.5),title('Log-Magnitude Response
(dB)'), grid on
ylim([-80 10]), yticks([-45 -0.5]), xticks([10 20])
```

```
xlabel('Frequency (Hz)'),ylabel('Decibels')
% Amplitude Response Plot
subplot(2,2,3), plot(wr/pi,Hr,'LineWidth',1.5), title('Amplitude Response')
xlabel('\omega/\pi'), ylabel('Amplitude'), ylim([0.85 1.025])
% Phase Response Plot
subplot(2,2,4), plot(F,Hpha,'LineWidth',1.5), title('Phase Delay Response'),
xlabel('Frequency (Hz)')
ylabel('Radians')
```



Problem 7.2

Text Problem 11.21 (Page 694)

Consider a 9^{th}-order analog Butterworth lowpass filter $H_c(s)$ with 3-dB cutoff frequency of 10 Hz.

(a) Determine and graph pole locations of $H_c(s)$.

Solution: .

```
clc; close all; clear;
N = 9; Fc = 10; Omegac = 2*pi*Fc;
[C,D] = butter(N,Omegac,'s');
sk = roots(D)
```

```
sk = 9×1 complex
 -10.9106 +61.8773i
 -10.9106 -61.8773i
 -31.4159 +54.4140i
 -31.4159 -54.4140i
 -48.1320 +40.3875i
 -48.1320 -40.3875i
```

```
-62.8319 + 0.0000i
-59.0426 +21.4898i
-59.0426 -21.4898i
```

Thus, the 9 poles of the Butterworth lowpass filter are:

$s_{1,9} = -10.9101 \pm j61.8773$

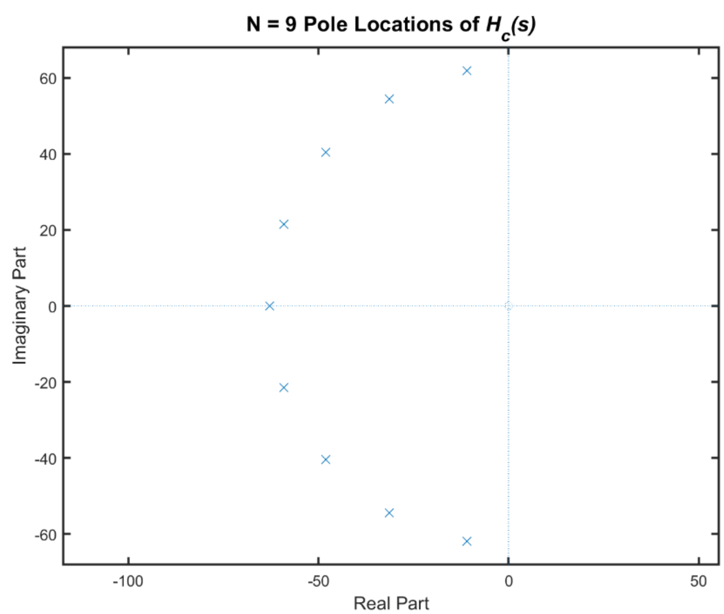
$s_{2,8} = -31.4159 \pm j54.414$

$s_{3,7} = -48.132 \pm j40.3875$

$s_{4,6} = -59.0426 \pm j21.489$

$s_5 = -62.8319$

```
figure, zplane(C,D), title('N = 9 Pole Locations of \it{H_c(s)}')
```

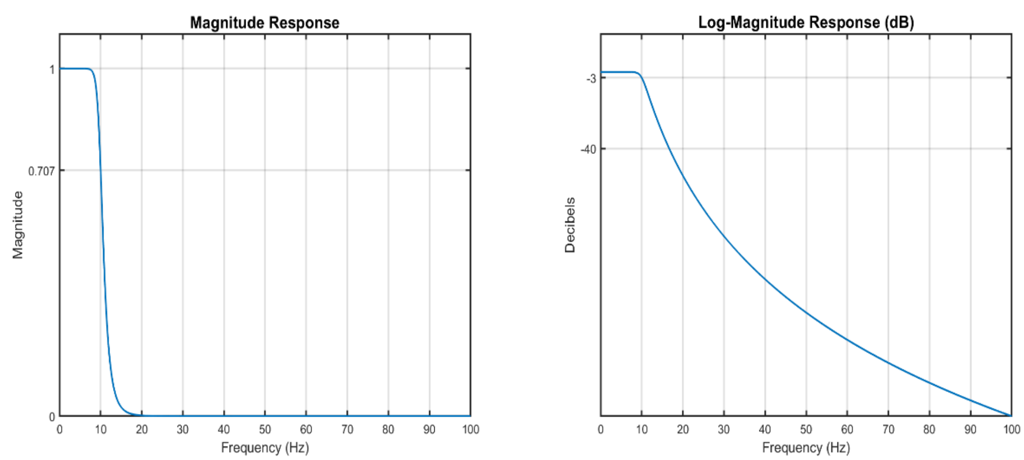


(b) Plot the magnitude and log-magnitude responses over $[0, 100]$ Hz range.

MATLAB script:

```
% Frequency Response
Fmax = 100; F = linspace(0,Fmax,2001);
omega = 2*pi*F;
H = freqs(C,D,omega);
Hmag = abs(H);
Hdb = mag2db(Hmag);
```

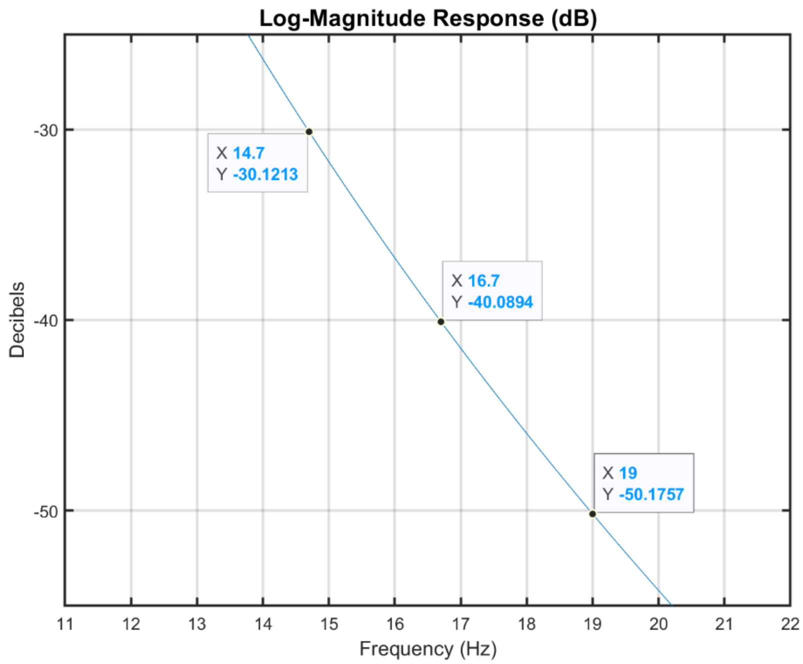
```
% Plot Results
figure('Units','inches','Position',[0,0,12,4]);
% Magnitude Response Plot
subplot(1,2,1),plot(F,Hmag,'LineWidth',1.5),title('Magnitude Response'), grid
on
xlim([0 100]), ylim([0 1.1]), yticks([0 0.707 1])
xlabel('Frequency (Hz)'),ylabel('Magnitude')
% Log-Magnitude Response Plot
subplot(1,2,2),plot(F,Hdb,'LineWidth',1.5),title('Log-Magnitude Response
(dB)'), grid on
xlabel('Frequency (Hz)'),ylabel('Decibels'), xlim([0 100])
yticks([-40 -3])
```



(c) Determine frequencies at which the attenuation is 30 db, 40 db, and 50 db.

Solution:

```
figure
% Log-Magnitude Response Plot
plot(F,Hdb),title('Log-Magnitude Response (dB)'), grid on
xlabel('Frequency (Hz)'),ylabel('Decibels')
ylim([-55 -25]), yticks([-50 -40 -30]), xlim([11 22])
ax = gca;
chart = ax.Children(1);
datatip(chart,14.7,-30.1213,'Location','southwest');
datatip(chart,16.7,-40.0894,'Location','northeast');
datatip(chart,19,-50.1757,'Location','northeast');
```



Observing the data points of the Log-Magnitude Response, we can see that the corresponding frequencies are:

$F_1 = 14.7$ Hz with attenuation of -30dB

$F_2 = 16.7$ Hz with attentuation of -40dB

$F_3 = 19$ Hz with attenuation of -50dB

Problem 7.3

Text Problem 11.31 (Page 695)

A lowpass digital filter's specifications are given by:

$$\omega_\mathrm{p} = 0.4\pi, \quad A_\mathrm{p} = 0.5 \text{ dB}, \quad \omega_\mathrm{s} = 0.55\pi, \quad A_\mathrm{s} = 50 \text{ dB}.$$

(a) Using bilinear transformation and Chebyshev-I approximation approach obtain a system function $H(z)$ in the rational function form that satisfies the above specifications.

Solution:

```
clc; close all; clear;
Omegap = 0.4*pi; Omegas = 0.55*pi;
Ap = 0.5; As = 50; Td = 2;
Omegap = (2/Td)*tan(Omegap/2); Omegas = (2/Td)*tan(Omegas/2);
```

```
[N,Omegac] = cheb1ord(Omegap,Omegas,Ap,As,'s')
```

```
N = 8
Omegac = 0.7265
```

```
[C,D] = cheby1(N,Ap,Omegac,'s');
```

After using the Chebyshev-I approximation approach, we find that the CT system function $H_c(s)$ is:

$$H_c(s) = \frac{0.0874}{s^9 + 1.4358s^8 + 4.5838s^7 + 4.8208s^6 + 6.9361s^5 + 5.0495s^4 + 3.8733s^3 + 1.6865s^2 + 0.5853s + 0.0874}$$

Then using the bilinear transformation function along with an arbitrary factor of $T_d = 2$, we derive the system function $H(z)$ to be:

```
[B,A] = bilinear(C,D,1/Td)
```

```
B = 1×9
    0.0003    0.0028    0.0097    0.0194    0.0243    0.0194    0.0097 ⋯
A = 1×9
    1.0000   -3.8656    8.2625  -11.6939   11.7756   -8.5442    4.3559 ⋯
```

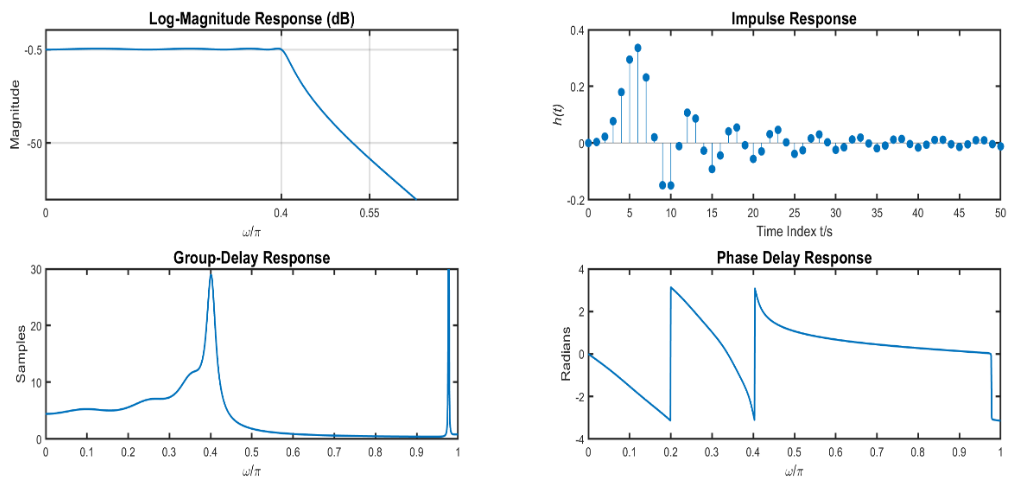
$$H(z) = \frac{0.0003 + 0.0028z^{-1} + 0.0097z^{-2} + 0.0194z^{-3} + 0.0243z^{-4} + 0.0194z^{-5} + 0.0097z^{-6} + 0.0028z^{-7} + 0.0003z^{-8}}{1 - 3.8656z^{-1} + 8.2625z^{-2} - 11.6939z^{-3} + 11.7756z^{-4} - 8.5442z^{-5} + 4.3559z^{-6} - 1.4343z^{-7} + 0.2381z^{-8}}$$

(b) Provide design plots in the form of log-magnitude, phase delay, group-delay, and impulse responses.

MATLAB script:

```
% Frequency Response
omega = linspace(0,pi,1001);
H = freqz(B,A,omega);
Hmag = abs(H);
Hdb = mag2db(Hmag);
% Impulse Response
N = 50; n = 0:N; x = (n==0); h = filter(B,A,x);
% Phase Response
Hpha = angle(H);
Hgd = -diff(unwrap(Hpha))./diff(omega);
Hgd = [Hgd Hgd(end)];
%[sos G] = tf2sos(B,A);
%Hgd = grpdelay(sos,1001)';
Hgd = medfilt1(Hgd,3);
% Plot Results
figure('Units','inches','Position',[0,0,12,4]);
% Magnitude Response Plot
```

```
subplot(2,2,1),plot(omega/pi,Hdb,'LineWidth',1.5),title('Log-Magnitude Response
(dB)'), grid on
xticks([0 0.4 0.55 1]), xticklabels({'0','0.4','0.55','1'}), yticks([-50 -
0.5]), ylim([-80 10]),
xlabel('\omega/\pi'),ylabel('Magnitude')
% Impulse Response Plot
subplot(2,2,2),stem(n,h,'filled'),title('Impulse Response'),xlabel('Time Index
t/s'),ylabel('\it{h(t)}')
% Group-Delay Plot
subplot(2,2,3), plot(omega/pi,Hgd,'LineWidth',1.5), title('Group-Delay
Response'), xlabel('\omega/\pi')
ylabel('Samples'), ylim([0 30])
% Phase Delay Plot
subplot(2,2,4), plot(omega/pi,Hpha,'LineWidth',1.5), title('Phase Delay
Response'), xlabel('\omega/\pi')
ylabel('Radians')
```



(c) Determine the exact band-edge frequencies for the given specifications.

Solution:

```
% Exact Band-Edge Frequencies
ind = find(Hdb > -Ap,1,'last'); w1 = omega(ind)/pi % Exact Passband Edge
```

```
w1 = 0.3990
```

```
ind = find(Hdb < -As,1,'first'); w2 = omega(ind)/pi % Exact Stopband Edge
```

```
w2 = 0.5220
```

Thus, the exact band-edge frequencies are

$\omega_p = 0.399\pi$ and $\omega_s = 0.548\pi$, which satisfy the design requirements.

Problem 7.4

Text Problem 11.38

A highpass filter specifications are given by:

$$\omega_s = 0.6\pi, \quad A_s = 40 \text{ dB}, \quad \omega_p = 0.8\pi, \quad A_p = 1 \text{ dB}.$$

(a) Using the Butterworth approximation obtain a system function $H(z)$ in the cascade function form that satisfies the above specifications.

Solution:

```
clc; close all; clear;
omegas = 0.6*pi; omegap = 0.8*pi;
As = 40; Ap = 1;
[N,Omegac] = buttord(omegap/pi,omegas/pi,Ap,As)
```

```
N = 7
Omegac = 0.7709
```

```
[bhp,ahp] = butter(N,Omegac,'high')
```

```
bhp = 1×8
    0.0002   -0.0014    0.0042   -0.0071    0.0071   -0.0042    0.0014 ⋯
ahp = 1×8
    1.0000    3.7738    6.5614    6.6518    4.2030    1.6437    0.3666 ⋯
```

```
[b0,B,A] = dir2cas(bhp,ahp)
```

```
b0 = 2.0235e-04
B = 4×3
    1.0000   -1.9907    0.9908
    1.0000   -2.0031    1.0031
    1.0000   -2.0135    1.0135
    1.0000   -0.9927         0
A = 4×3
    1.0000    1.3114    0.7441
    1.0000    1.0657    0.4174
    1.0000    0.9434    0.2547
    1.0000    0.4532         0
```

Thus, the rational system function $H(z)$ in its cascade form is:

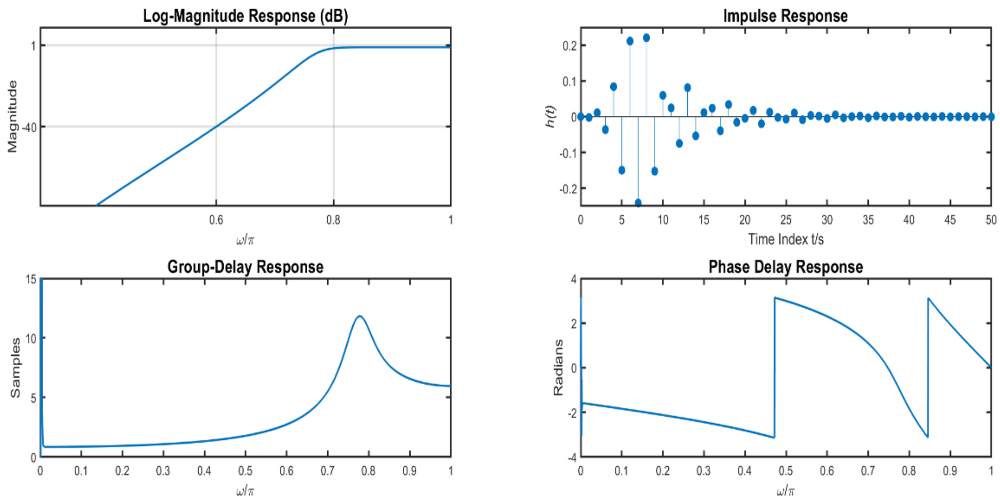
$$H(z) = \left(2.0235 \times 10^{-4}\right) \times \left(\frac{1 - 1.9907 + 0.9908z^{-2}}{1 + 1.3114z^{-1} + 0.7441z^{-2}}\right) \times \left(\frac{1 - 2.0031z^{-1} + 1.0031z^{-2}}{1 + 1.0657z^{-1} + 0.4174z^{-2}}\right) \times \left(\frac{1 - 2.0135z^{-1} + 1.0135z^{-2}}{1 + 0.9434z^{-1} + 0.2547z^{-2}}\right) \times \left(\frac{1 - 0.9927z^{-1}}{1 + 0.4532z^{-1}}\right)$$

```
[sos G] = tf2sos(bhp,ahp);
```

(b) Provide design plots in the form of log-magnitude, phase delay, group-delay, and impulse responses.

Solution:

```
% Frequency Response
omega = linspace(0,pi,2001);
H = freqz(bhp,ahp,omega);
Hmag = abs(H);
Hdb = mag2db(Hmag);
% Impulse Response
N = 50; n = 0:N; x = (n==0); h = filter(bhp,ahp,x);
% Phase Response
Hpha = angle(H);
Hgd = grpdelay(sos,2001)';
% Plot Results
figure('Units','inches','Position',[0,0,12,4]);
% Log-Magnitude Response Plot
subplot(2,2,1), plot(omega/pi,Hdb,'LineWidth',1.5),title('Log-Magnitude
Response (dB)'), grid on
xticks([0 0.6 0.8 1]),xticklabels({'0','0.6','0.8','1'}), yticks([-40 1]),
ylim([-80 10]),
xlabel('\omega/\pi'),ylabel('Magnitude')
% Impulse Response Plot
subplot(2,2,2),stem(n,h,'filled'),title('Impulse Response'),xlabel('Time Index
t/s'),ylabel('\it{h(t)}')
ylim([-0.25 0.25])
% Group-Delay Plot
subplot(2,2,3), plot(omega/pi,Hgd,'LineWidth',1.5), title('Group-Delay
Response'), xlabel('\omega/\pi')
ylabel('Samples'), ylim([0 15])
% Phase Delay Plot
subplot(2,2,4), plot(omega/pi,Hpha,'LineWidth',1.5), title('Phase Delay
Response'), xlabel('\omega/\pi')
ylabel('Radians')
```



(c) Determine the exact band-edge frequencies for the given specifications.

Solution:

```
% Exact Band-Edge Frequencies
ind = find(Hdb > -Ap,1,'first'); w1 = omega(ind)/pi % Exact Passband Edge
```

```
w1 = 0.7905
```

```
ind = find(Hdb < -As,1,'last'); w2 = omega(ind)/pi % Exact Stopband Edge
```

```
w2 = 0.5995
```

Thus, the exact band-edge frequencies are

$\omega_p = 0.79\pi$ and $\omega_s = 0.5995\pi$, which meets the design requirements for the highpass filter specifications.

Problem 7.5

Text Problem 11.43 (Page 698)

A digital filter is specified by the following band parameters:

$$\begin{aligned}
&\text{Band-1: } [0, 0.3\pi], && \text{Attn. } = 50 \text{ dB},\\
&\text{Band-2: } [0.4\pi, 0.5\pi], && \text{Attn. } = 1 \text{ dB},\\
&\text{Band-3: } [0.6\pi, \pi], && \text{Attn. } = 50 \text{ dB}.
\end{aligned}$$

(a) Using Chebyshev II approximation, obtain a system function $H(z)$ in the rational function form that satisfies the above specifications.

Solution:

```
clc; close all; clear;
omegas1 = 0.3*pi; omegap1 = 0.4*pi;
omegas2 = 0.6*pi; omegap2 = 0.5*pi;
As1 = 50; Ap = 1; As2 = 50; As = max(As1,As2);
omegas = [omegas1 omegas2];
omegap = [omegap1 omegap2];
[N,Omegac] = cheb2ord(omegap/pi,omegas/pi,Ap,As)
```

```
N = 4
Omegac = 1×2
    0.3000    0.6000
```

```
[b,a] = cheby2(N,As,Omegac,'bandpass')
```

```
b = 1×9
    0.0068   -0.0054    0.0053   -0.0055    0.0114   -0.0055    0.0053 ⋯
a = 1×9
    1.0000   -1.2238    3.5354   -2.8788    4.2798   -2.2237    2.1128 ⋯
```

With the numerator and denominator coefficients now calculated, the system function can be described as

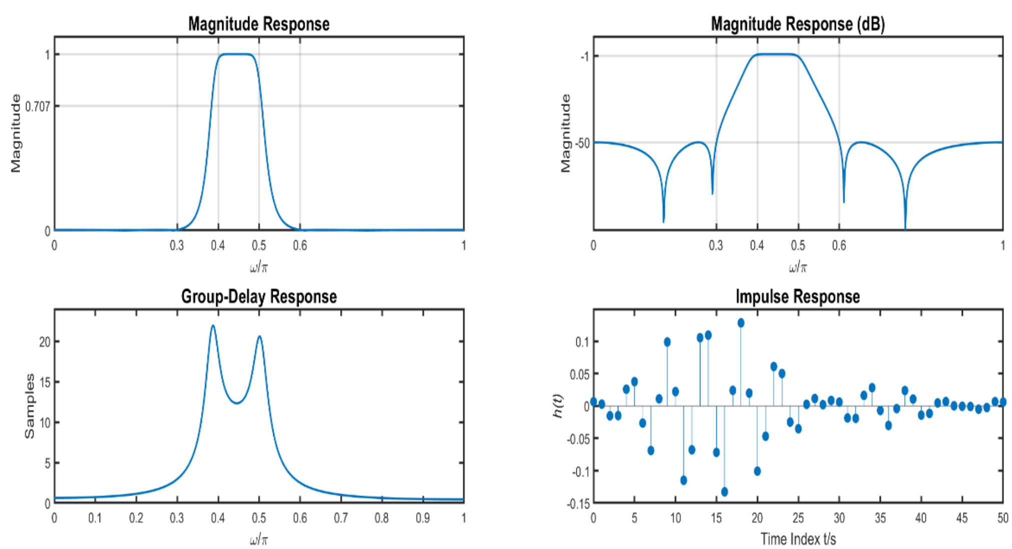
$$H(z) = \frac{0.0068 - 0.0054z^{-1} + 0.0053z^{-2} - 0.0055z^{-3} + 0.0114z^{-4} - 0.0055z^{-5} + 0.0053z^{-6} - 0.0054z^{-7} + 0.0068z^{-8}}{1 - 1.2238z^{-1} + 3.5354z^{-2} - 2.8788z^{-3} + 4.2798z^{-4} - 2.2237z^{-5} + 2.1128z^{-6} - 0.5604z^{-7} + 0.3536z^{-8}}$$

(b) Provide design plots in the form of magnitude, log-magnitude, group-delay, and impulse responses.

MATLAB script:

```
% Frequency Response
omega = linspace(0,pi,1001);
H = freqz(b,a,omega);
Hmag = abs(H);
Hdb = mag2db(Hmag);
% Impulse Response
N = 50; n = 0:N; x = (n==0); h = filter(b,a,x);
% Phase Response
Hpha = angle(H);
Hgd = -diff(unwrap(Hpha))./diff(omega);
Hgd = [Hgd Hgd(end)];
Hgd = medfilt1(Hgd,3);
```

```
 % Plot Results
 figure('Units','inches','Position',[0,0,12,4]);
 % Magnitude Response Plot
 subplot(2,2,1),plot(omega/pi,Hmag,'LineWidth',1.5),title('Magnitude Response'),
grid on
 xticks([0 0.3 0.4 0.5 0.6 1]), yticks([0 0.707 1]), ylim([0 1.1]),
 xlabel('\omega/\pi'),ylabel('Magnitude')
 % Log-Magnitude Response Plot
 subplot(2,2,2),plot(omega/pi,Hdb,'LineWidth',1.5),title('Magnitude Response
(dB)'), grid on
 xticks([0 0.3 0.4 0.5 0.6 1]), yticks([-50 -1]), ylim([-100 10]),
 xlabel('\omega/\pi'),ylabel('Magnitude')
 % Group-Delay Plot
 subplot(2,2,3), plot(omega/pi,Hgd,'LineWidth',1.5), title('Group-Delay
Response'), xlabel('\omega/\pi')
 ylabel('Samples'), ylim([0 24])
 % Impulse Response Plot
 subplot(2,2,4),stem(n,h,'filled'),title('Impulse Response'),xlabel('Time Index
t/s'),ylabel('\it{h(t)}')
 ylim([-0.15 0.15])
```



(c) Determine the exact band-edge frequencies for the given attenuation.

Solution:

```
 % Exact Band-Edge Frequencies
 ind = find(Hdb > -Ap); wp1 = omega(ind)/pi; % Exact Passband Edges
 LowerPassEdge = wp1(1), UpperPassEdge = wp1(end)
```

```
LowerPassEdge = 0.3920
UpperPassEdge = 0.4970
```

```
inds1 = find(Hdb(1:401) < -As,1,'last'); % Exact Lower Stopband Edge
LowerStopEdge = omega(inds1)/pi
```

```
LowerStopEdge = 0.3000
```

```
inds2 = find(Hdb < -As); % Exact Upper Stopband Edge
LowerStopEdge = omega(602)/pi
```

```
LowerStopEdge = 0.6010
```

Thus the exact band-edge frequencies are:

$\omega_{s1} = 0.3\pi$, $\omega_{s2} = 0.601\pi$

$\omega_{p1} = 0.392\pi$, $\omega_{p2} = 0.497\pi$

Problem 7.6

Text Problem 11.66 (Page 701)

A digital filter is specified by the following band parameters:

$$\begin{array}{ll} \text{Band-1: } [0, 0.2\pi], & \text{Attn. } = 1 \text{ dB}, \\ \text{Band-2: } [0.35\pi, 0.5\pi], & \text{Attn. } = 50 \text{ dB}, \\ \text{Band-3: } [0.65\pi, \pi], & \text{Attn. } = 1 \text{ dB}. \end{array}$$

(a) Using Butterworth approximation, obtain a system function $H(z)$ in the cascade form that satisfies the above specifications.

Solution:

```
clc; close all; clear;
wp1 = 0.2; ws1 = 0.35; Ap1 = 1;
wp2 = 0.65; ws2 = 0.5; As = 50;
omegap = [wp1 wp2];
omegas = [ws1 ws2];
[N,Omegac] = buttord(omegap,omegas,Ap1,As)
```

```
N = 6
Omegac = 1×2
    0.2566    0.6130
```

```
[b,a] = butter(N,Omegac,'stop');
[bo,B,A] = dir2cas(b,a)
```

```
bo = 0.0982
```

```
B = 6×3
      1.0000   -0.4752    1.0001
      1.0000   -0.4766    0.9956
      1.0000   -0.4788    1.0045
      1.0000   -0.4815    0.9956
      1.0000   -0.4838    1.0044
      1.0000   -0.4851    0.9999
A = 6×3
      1.0000    0.5928    0.7650
      1.0000    0.3322    0.4315
      1.0000   -0.0441    0.2424
      1.0000   -0.5481    0.2885
      1.0000   -0.9404    0.5150
      1.0000   -1.2428    0.8133
```

Here the coefficents for the cascade form of $H(z)$ can be written as:

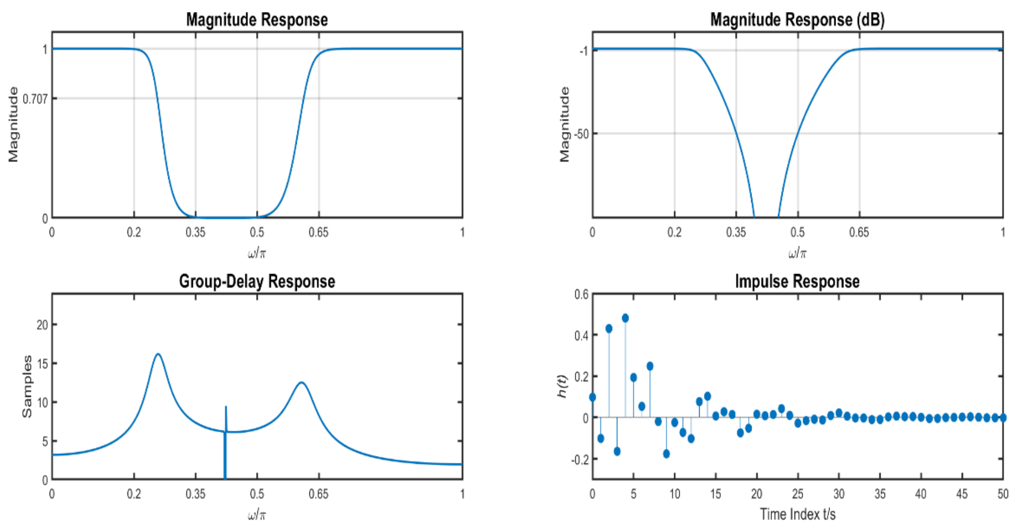
$$H(z) = (0.0982) \times \left(\frac{1-0.4752z^{-1}+1.0001z^{-2}}{1+0.5928z^{-1}+0.7650z^{-2}}\right) \times \left(\frac{1-0.4766z^{-1}+0.9956z^{-2}}{1-0.3322z^{-1}+0.4315z^{-2}}\right) \times \left(\frac{1-0.4788z^{-1}+1.0045z^{-2}}{1-0.0441z^{-1}+0.2424z^{-2}}\right) \times \left(\frac{1-0.4815z^{-1}+0.9956z^{-2}}{1-0.5481z^{-1}+0.2885z^{-2}}\right) \times \left(\frac{1-0.4838z^{-1}+1.0044z^{-2}}{1-0.9404z^{-1}+0.5150z^{-2}}\right) \times \left(\frac{1-0.4851z^{-1}+0.999z^{-2}}{1-1.2428z^{-1}+0.8133z^{-2}}\right)$$

(b) Provide design plots in the form of magnitude, log-magnitude, group-delay, and impulse responses.

Solution:

```
% Frequency Response
omega = linspace(0,pi,1001);
H = freqz(b,a,omega);
Hmag = abs(H);
Hdb = mag2db(Hmag);
% Impulse Response
L = 50; n = 0:L; x = (n==0); h = filter(b,a,x);
% Phase Response
Hpha = angle(H);
Hgd = -diff(unwrap(Hpha))./diff(omega);
Hgd = [Hgd Hgd(end)];
Hgd = medfilt1(Hgd,3);
% Plot Results
figure('Units','inches','Position',[0,0,12,4]);
% Magnitude Response Plot
subplot(2,2,1),plot(omega/pi,Hmag,'LineWidth',1.5),title('Magnitude Response'),
grid on
xticks([0 0.2 0.35 0.5 0.65 1]), yticks([0 0.707 1]), ylim([0 1.1]),
xlabel('\omega/\pi'),ylabel('Magnitude')
% Log-Magnitude Response Plot
subplot(2,2,2),plot(omega/pi,Hdb,'LineWidth',1.5),title('Magnitude Response
(dB)'), grid on
xticks([0 0.2 0.35 0.5 0.65 1]), yticks([-50 -1]), ylim([-100 10]),
```

```
xlabel('\omega/\pi'),ylabel('Magnitude')
% Group-Delay Plot
subplot(2,2,3), plot(omega/pi,Hgd,'LineWidth',1.5), title('Group-Delay
Response'), xlabel('\omega/\pi')
ylabel('Samples'),xticks([0 0.2 0.35 0.5 0.65 1]), ylim([0 24])
% Impulse Response Plot
subplot(2,2,4),stem(n,h,'filled'),title('Impulse Response'),xlabel('Time Index
t/s'),ylabel('\it{h(t)}')
ylim([-0.3 0.6])
```



(c) Determine the exact band-edge frequencies for the given attenuation.

Solution:

```
ind = find(Hdb(1:401) < -Ap1, 1,'first'); wplow = omega(ind)/pi;
ind = find(Hdb(1:401) > -As, 1,'last'); wslow = omega(ind)/pi;
ind = Hdb(500:502); wsupper = omega(502)/pi;
ind = find(Hdb(1:701) < -Ap1, 1,'last'); wpupper = omega(ind)/pi;
wplow,wslow,wsupper,wpupper
```

```
wplow = 0.2430
wslow = 0.3490
wsupper = 0.5010
wpupper = 0.6310
```

Thus, the exact band-edge frequencies for this Butterworth Bandstop IIR Filter are:

$\omega_{p1} = 0.243\pi$, $\omega_{s1} = 0.349\pi$, $\omega_{s2} = 0.501\pi$, and $\omega_{p2} = 0.631\pi$

Problem 7.7

Text Problem 11.70 (Page 702)

An analog signal $x_c(t) = 5\sin(2\pi 250t) + 10\sin(2\pi 300t)$ is to be processed using the effective continuous-time system of Figure 6.18 in which the sampling frequency is 1 kHz.

(a) Design a minimum-order IIR digital filter that will suppress the 300 Hz component down to 50 dB while pass the 250 Hz component with attenuation of less than 1 dB. The digital filter should have an equiripple passband and stopband. Determine the system function of the filter and plot its log-magnitude response in dB.

Solution:

Since the requirements call for a filter with an equiripple passband and stopband, the only filter that can achieve the design specs are an analog Elliptic filter

```
clc; close all; clear;
% Design Requirements
Fs = 1000; Fs2 = Fs/2;
fs = 300/Fs; As = 50;
fp = 250/Fs; Ap = 1;
omegas = 2*fs;
omegap = 2*fp;
[N,omegac] = ellipord(omegap,omegas,Ap,As)
```

```
N = 6
omegac = 0.5000
```

```
[b1,a1] = ellip(N,Ap,As,omegac)
```

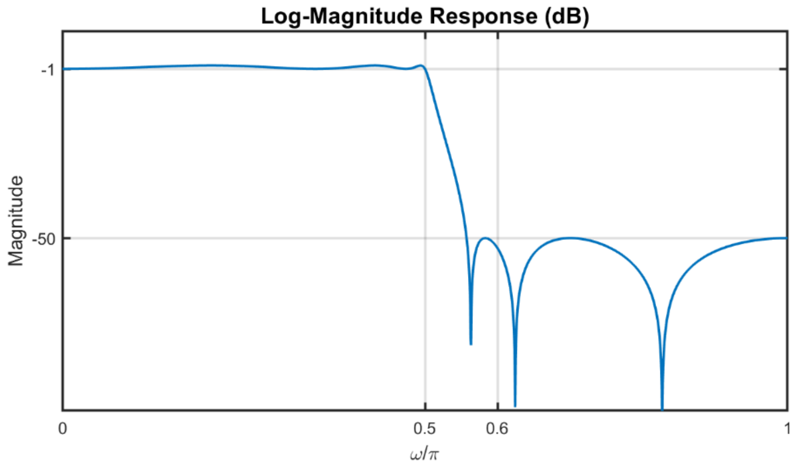
```
b1 = 1×7
    0.0481    0.1381    0.2542    0.3010    0.2542    0.1381    0.0481
a1 = 1×7
    1.0000   -1.1660    2.2689   -1.8296    1.5033   -0.6909    0.2401
```

Deriving the coefficients of the Digital Elliptic Lowpass IIR Filter shows that the system function is:

$$H(z) = \frac{0.0481 + 0.1381z^{-1} + 0.2542z^{-2} + 0.301z^{-3} + 0.2542z^{-4} + 0.1381z^{-5} + 0.0481z^{-6}}{1 - 1.166z^{-1} + 2.2689z^{-2} - 1.8296z^{-3} + 1.5033z^{-4} - 0.6909z^{-5} + 0.2401z^{-6}}$$

```
% Frequency Response
omega = linspace(0,pi,2001);
H = freqz(b1,a1,omega);
Hmag = abs(H);
Hdb = mag2db(Hmag);
% Log-Magnitude Response Plot
figure('Units','inches','Position',[0,0,6,3]);
```

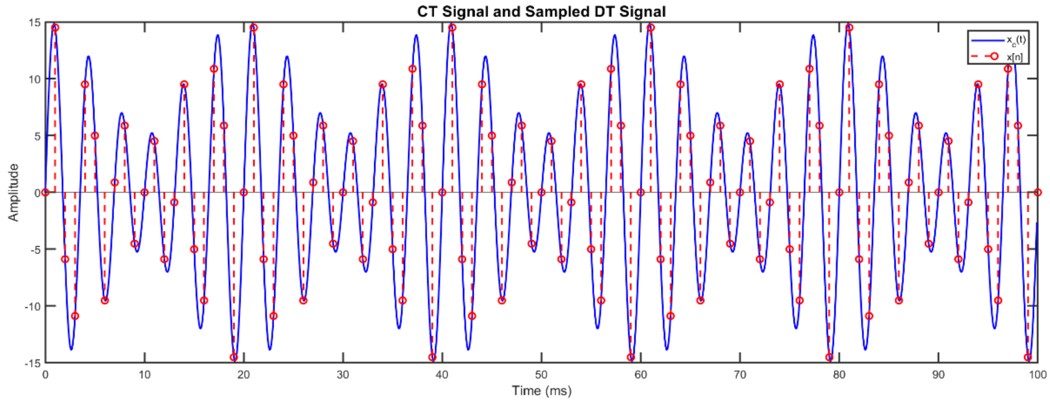
```
plot(omega/pi,Hdb,'LineWidth',1.5),title('Log-Magnitude Response (dB)'), grid
on
xticks([0 0.5 0.6 1]), yticks([-50 -1]), ylim([-100 10]),
xlabel('\omega/\pi'),ylabel('Magnitude')
```



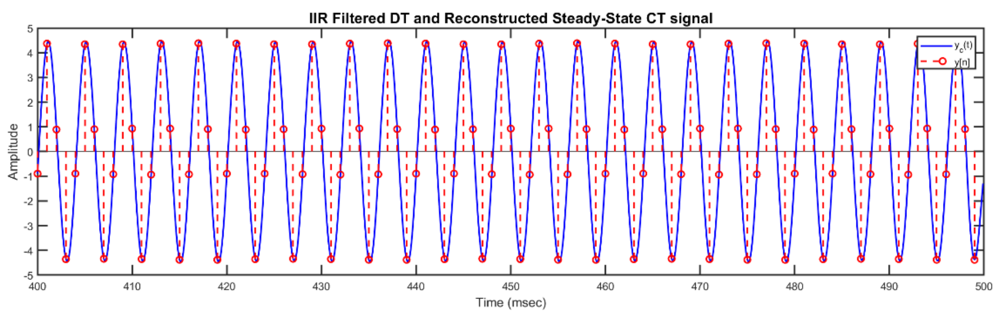
(b) Process the signal $x_c(t)$ through the effective analog system. Generate sufficient samples so that the output response $y_c(t)$ goes into steady-state. Plot the steady-state $y_{ss}(t)$ and comment on the filtering result.

Solution:

```
Ts = 1/Fs; t = 0:0.0001:.5;
% Continuous-Time Signal
xc = 5*sin(2*pi*250.*t)+10*sin(2*pi*300.*t);
% Discrete-Time Signal
n = 0:500; nTs = n*Ts;
xn = 5*sin(2*pi*250.*nTs)+10*sin(2*pi*300.*nTs);
% Plot the CT signal and Sampled DT signal
figure('Units','inches','Position',[0,0,12,4]);
plot(t(1:1000)*1000,xc(1:1000),'b','LineWidth',1.5), hold on,
stem(nTs(1:101)*1000,xn(1:101),'--r','LineWidth',1.5), xlabel('Time
(ms)'),ylabel('Amplitude')
title('CT Signal and Sampled DT Signal'), legend('x_c(t)','x[n]')
```




```
% Filter the Discrete-Signal x[n] through Elliptic Lowpass IIR Filter
yn = filter(b1,a1,xn);
% Reconstruct the analog signal through interpolation and analyze results
yt = yn * sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
figure('Units','inches','Position',[0,0,12,3]);
plot(t(4001:5000)*1000,yt(4001:5000),'b','LineWidth',1.5), hold on,
stem(n(401:500),yn(401:500),'--r','LineWidth',1.5)
xlabel('Time (msec)')
ylabel('Amplitude')
title('IIR Filtered DT and Reconstructed Steady-State CT signal'),
legend('y_c(t)','y[n]')
```



Comment:

After analyzing the the reconstructed signal, $y_c(t)$, we see that the resulting signal is a pure sinusoid with only one fundamental frequency. The higher frequency of 300 Hz has been filtered out by the IIR filter and only the 250 Hz component remains. Once the signal reaches a steady-state, there is a small difference in magnitude as the reconstructed signal has a peak amplitude of about 4.5, compared to the original sinusoidal component $5\sin(2\pi 250)$.

(c) Repear parts (a) and (b) by designing an equiripple FIR filter. Compare the orders of the two filters and their filtering results.

Solution:

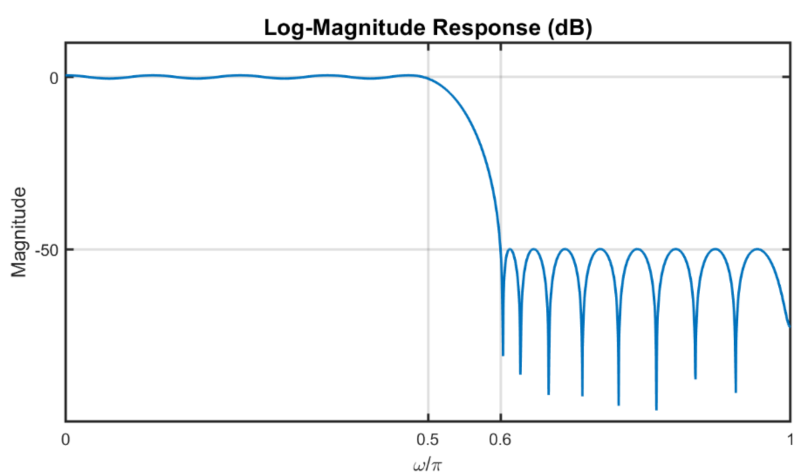
```
% Compute Absolute passband and stopband ripple values
[dp,ds] = spec_convert(Ap,As,'rel','abs')
```

```
dp = 0.0575
ds = 0.0033
```

```
% Estimate Filter using FIRPMORD function
f = [0.5 0.6]; % Band-edge array
a = [1 0]; % Band-edge desired gain
dev = [dp,ds]; % Band tolerance
[M,fo,ao,W] = firpmord(f,a,dev); M
```

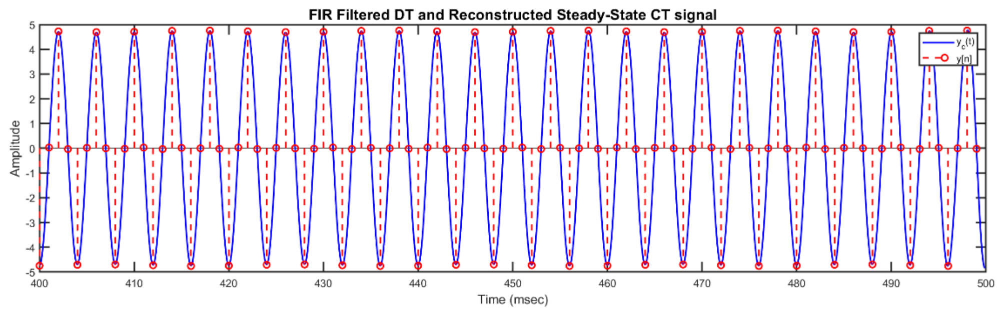
```
M = 34
```

```
% Filter Design using FIRPM function
[h,delta] = firpm(M,fo,ao,W);
% Frequency Response
omega = linspace(0,pi,2001);
H = freqz(h,1,omega);
Hmag = abs(H);
Hdb = mag2db(Hmag);
% Log-Magnitude Response Plot
figure('Units','inches','Position',[0,0,6,3]);
plot(omega/pi,Hdb,'LineWidth',1.5),title('Log-Magnitude Response (dB)'), grid
on
xticks([0 0.5 0.6 1]), yticks([-50 0]), ylim([-100 10]),
xlabel('\omega/\pi'),ylabel('Magnitude')
```




```
% Filter the Discrete-Signal x[n] through Equiripple Lowpass FIR Filter
yn = filter(h,1,xn);
% Reconstruct the analog signal through interpolation and analyze results
yt = yn * sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
figure('Units','inches','Position',[0,0,12,3]);
plot(t(4001:5000)*1000,yt(4001:5000),'b','LineWidth',1.5), hold on
```

```
 stem(n(401:500),yn(401:500),'--r','LineWidth',1.5)
 xlabel('Time (msec)')
 ylabel('Amplitude')
 title('FIR Filtered DT and Reconstructed Steady-State CT signal'),
legend('y_c(t)','y[n]')
```



Comment:

After drawing comparisons between the two filters, the Elliptic IIR filter had an order of $N = 6$, while the FIR filter had order $N = 34$, which is almost 6 times the order of the IIR Filter. When comparing the number of multiplications per output sample:

The IIR Filter had order $N = 6$, thus this filter had $4\left(\frac{N}{2}\right) \rightarrow 4\left(\frac{6}{2}\right) \rightarrow 4(3)$= 12 multiplications per output sample

The FIR Filter had order $M = 34$, thus the number of multiplications per output sample was $\frac{M}{2} \rightarrow \frac{34}{2} = 17$.

When comparing the exact band-edge frequencies, the IIR filter had a passband edge at $\omega_p = 0.4995\pi$ and a sharper transition band with a stopband edge at $\omega_s = 0.5575\pi$

Whereas the FIR filter had a narrower stopband width with a passband edge at $\omega_p = 0.505\pi$ and stopband edge at $\omega_s = 0.5995\pi$

The resulting reconstructed signals had a difference in amplitude, as the FIR filter produced a signal that was closer to the sinusoidal component of $5\sin(2\pi 250)$, whereas the IIR had a smaller peak amplitude due to its sharper transition band cutting off frequencies before the desired stopband-edge frequency.

Overall, the IIR filter achieved the design requirements using about 6 times less coefficients and 5 multiplcations/output sample less than the FIR filter while also providing a shorter transition band that provided the necessary attenuations in the corresponding frequency bands.

Problem 7.8

Text Problem 11.71 (Page 703)

Consider the following bandpass digital filter specifications:

$$\begin{aligned} \text{Stopband-1} &: [0, 0.4\pi], & \text{Attn.} &= 40 \text{ dB} \\ \text{Passband} &: [0.45\pi, 0.55\pi], & \text{Attn.} &= 0.5 \text{ dB} \\ \text{Stopband-2} &: [0.65\pi, \pi], & \text{Attn.} &= 50 \text{ dB} \end{aligned}$$

(a) Design a minimum order FIR filter to satisfy the above specifications. Plot its magnitude, log-magnitude (dB), and group-delay responses in one figure using 3 rows and 1 column.

MATLAB script:

```
clc; close all; clear;
omegas1 = 0.4*pi; omegap1 = 0.45*pi;
omegas2 = 0.65*pi; omegap2 = 0.55*pi;
As1 = 40; As2 = 50; As = max(As1,As2);
Ap = 0.5;
[dp,ds1] = spec_convert(Ap,As1,'rel','abs')
```

```
dp = 0.0288
ds1 = 0.0103
```

```
[dp,ds2] = spec_convert(Ap,As2,'rel','abs')
```

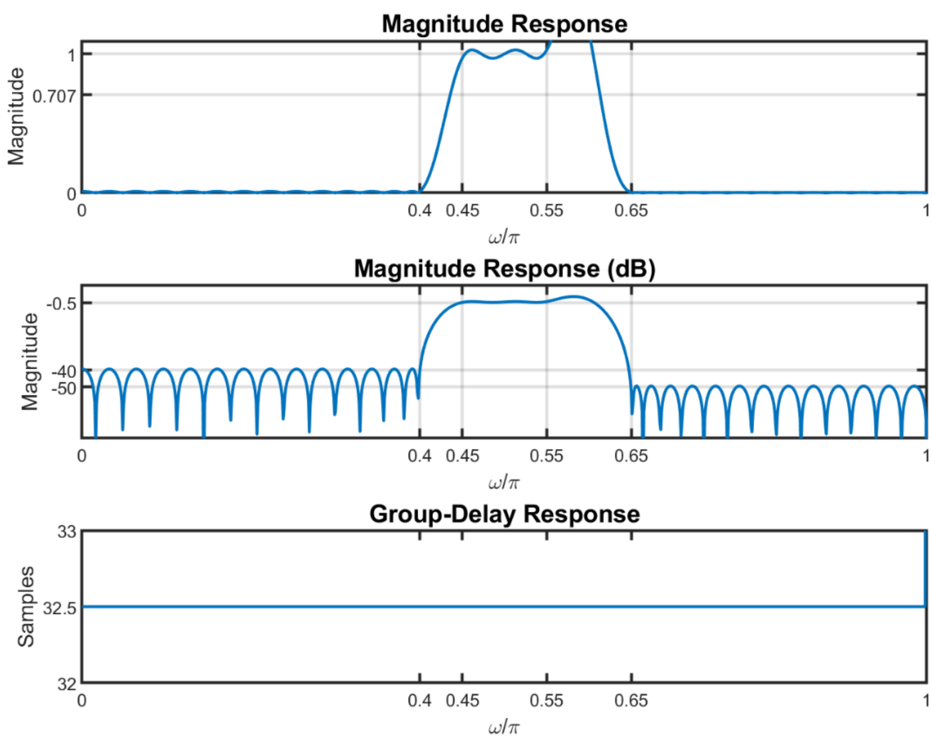
```
dp = 0.0288
ds2 = 0.0033
```

```
% Estimate Filter using FIRPMORD function
f = [omegas1,omegap1,omegap2,omegas2]/pi; % Band-edge array
a = [0 1 0]; % Band-edge desired gain
dev = [ds1 dp ds2]; % Band tolerance
[M,fo,ao,W] = firpmord(f,a,dev); M
```

```
M = 65
```

```
% Filter Design using FIRPM function
[h,delta] = firpm(M,fo,ao,W);
n = 0:M; omega = linspace(0,pi,1001);
% Magnitude Response
H = freqz(h,1,omega); Hmag = abs(H);
Hdb = mag2db(Hmag);
% Phase and Group-Delay
Hpha = angle(H);
Hgd = -diff(unwrap(Hpha))./diff(omega);
Hgd = [Hgd Hgd(end)];
Hgd = medfilt1(Hgd,3);
% Plot Results
figure
```

```
% Magnitude Response Plot
subplot(3,1,1),plot(omega/pi,Hmag,'LineWidth',1.5),title('Magnitude Response'),
grid on
xticks([0 0.4 0.45 0.55 0.65 1]), yticks([0 0.707 1]), ylim([0 1.1]),
xlabel('\omega/\pi'),ylabel('Magnitude')
% Log-Magnitude Response Plot
subplot(3,1,2),plot(omega/pi,Hdb,'LineWidth',1.5),title('Magnitude Response
(dB)'), grid on
xticks([0 0.4 0.45 0.55 0.65 1]), yticks([-50 -40 -0.5]), ylim([-80 10]),
xlabel('\omega/\pi'),ylabel('Magnitude')
% Group-Delay Plot
subplot(3,1,3), plot(omega/pi,Hgd,'LineWidth',1.5), title('Group-Delay
Response'), xlabel('\omega/\pi')
ylabel('Samples'),xticks([0 0.4 0.45 0.55 0.65 1]), ylim([32 33])
```



(b) Design a minimum order IIR filter to satisfy the above specifications. Plot its magnitude, log-magnitude (dB), and group-delay responses in one figure using 3 rows and 1 column. From your plots determine the exact band-edge frequencies.

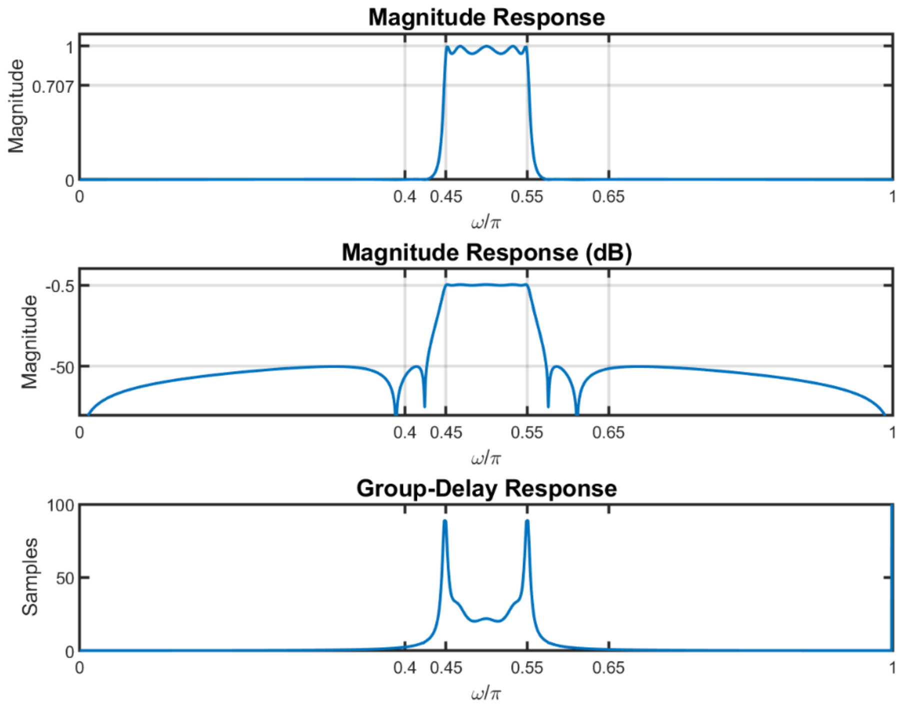
MATLAB script:

```
Omegas = [omegas1 omegas2];
```

```
Omegap = [omegap1 omegap2];
[N,Omegac] = ellipord(Omegap/pi,Omegas/pi,Ap,As); N
```

```
N = 5
```

```
[b,a] = ellip(N,Ap,As,Omegac,'bandpass');
omega = linspace(0,pi,1001);
% Magnitude Response
H = freqz(b,a,omega); Hmag = abs(H);
Hdb = mag2db(Hmag);
% Phase and Group-Delay
Hpha = angle(H);
Hgd = -diff(unwrap(Hpha))./diff(omega);
Hgd = [Hgd Hgd(end)];
Hgd = medfilt1(Hgd,3);
% Plot Results
figure
% Magnitude Response Plot
subplot(3,1,1),plot(omega/pi,Hmag,'LineWidth',1.5),title('Magnitude Response'),
grid on
xticks([0 0.4 0.45 0.55 0.65 1]), yticks([0 0.707 1]), ylim([0 1.1]),
xlabel('\omega/\pi'),ylabel('Magnitude')
% Log-Magnitude Response Plot
subplot(3,1,2),plot(omega/pi,Hdb,'LineWidth',1.5),title('Magnitude Response
(dB)'), grid on
xticks([0 0.4 0.45 0.55 0.65 1]), yticks([-50 -0.5]), ylim([-80 10]),
xlabel('\omega/\pi'),ylabel('Magnitude')
% Group-Delay Plot
subplot(3,1,3), plot(omega/pi,Hgd,'LineWidth',1.5), title('Group-Delay
Response'), xlabel('\omega/\pi')
ylabel('Samples'),xticks([0 0.4 0.45 0.55 0.65 1]),ylim([0 100])
```



(c) Compare the two filter designs in terms of their responses and orders.

Comparison:

The FIR filter design was designed using the Parks-McClellan algorithm which produced an equiripple FIR filter with

minimum order $M = 65$. The design produced a linear phase which resulted in a constant group-delay. The magnitude response was unsatisfactory towards the design specifications, as the first stopband did not meet the attenuation requirement of 40dB, due to difference between the lower and upper transition bands. The number of multiplications per output sample for the FIR filter was $\frac{M}{2} \rightarrow \frac{65}{2} = 32.5$

The IIR filter was designed using a digital Elliptic filter and achieved a minimum order of $N = 5$. This filter was able to achieve attenuation requirements in both stopbands and the passband, but resulted in a nonlinear phase and group-delay response. The number of multiplications per output sample for the IIR filter was $4\left(\frac{N}{2}\right) \rightarrow 4\left(\frac{5}{2}\right) \rightarrow 4(2.5) = 10$

Thus, the IIR filter was able to satisfy the design requirements with order $N = 5$, which resulted in only 10 multiplications per output sample. This is about 3 times less the number of multiplications, so the IIR filter not only satisfied the magnitude requirements, but is computationally quicker also.

```
function [A,B] = spec_convert(C,D,typein,typeout)
%  typein: 'abs' or 'rel' or 'ana'
% typeout: 'abs' or 'rel' or 'ana'
%     C,D: input specifications
%     A,B: output specifications

% Enter your function code below
if nargin > 4
    error('too many input arguments')
end

switch typein
    case 'abs'
        switch typeout
            case 'rel'
                Ap = 20*log10((1+C)/(1-C)); % Relative Output: Passband ripple
                As = floor(20*log10((1+C)/D)); % Relative Output: Stopband
Attenuation
                A = Ap; B = As;
            case 'ana'
                Ap = 20*log10((1+C)/(1-C)); % Relative Output: Passband ripple
                As = floor(20*log10((1+C)/D)); % Relative Output: Stopband
Attenuation
                A = sqrt(10^(Ap/10)-1); % Analog Output: Passband
                B = floor(10^(As/20)); % Analog Output: Stopband
        end
    case 'rel'
        switch typeout
            case 'abs'
                dp = (10^(C/20)-1)/(1+10^(C/20)); % Absolute Output: Passband
Error
                ds = (1+dp)/(10^(D/20)); % Absolute Output: Stopband Error
                A = dp; B = ds;
            case 'ana'
                epsilon = sqrt(10^(C/10)-1); % Analog Output: Passband
                B = floor(10^(D/20)); % Analog Output: Stopband
                A = epsilon;
        end
   case 'ana'
        switch typeout
            case 'rel'
                Ap = round(10*log10(C^(2)+1),2); % Relative Output: Passband
ripple (in dB)
                As = 20*log10(D); % Relative Output: Stopband Attenuation (in
dB)
```

```
                A = Ap; B = As;
            case 'abs'
                Ap = round(10*log10(C^(2)+1),2); % Relative Output: Passband
ripple (in dB)
                As = 20*log10(D); % Relative Output: Stopband Attenuation (in
dB)
                dp = (10^(Ap/20)-1)/(1+10^(Ap/20)); % Absolute Output: Passband
Error
                ds = (1+dp)/(10^(Ap/20)); % Absolute Output: Stopband Error
                A = dp; B = ds;
        end
 end
 end
```