EECE5666 (DSP) : Homework-3

Due on February 22, 2022 by 11:59 pm via submission portal.

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Instructions

1. You are required to complete this assignment using Live Editor.
2. Enter your MATLAB script in the spaces provided. If it contains a plot, the plot will be displayed after the script.
3. All your plots must be properly labeled and should have appropriate titles to get full credit.
4. Use the equation editor to typeset mathematical material such as variables, equations, etc.
5. After completeing this assignment, export this Live script to PDF and submit the PDF file through the provided submission portal.
6. You will have only one attempt to submit your assignment. Make every effort to submit the correct and completed PDF file the first time.
7. Please submit your homework before the due date/time. A late submission after midnight of the due date will result in loss of points at a rate of 10% per hour until 8 am the following day, at which time the solutions will be published.

Default Plot Parameters

```
set(0,'defaultfigurepaperunits','points','defaultfigureunits','points');
set(0,'defaultaxesfontsize',10); set(0,'defaultaxeslinewidth',1.5);
```

```
set(0,'defaultaxestitlefontsize',1.4,'defaultaxeslabelfontsize',1.2);
```

Problem 3.1

Text Problem 5.23 (Page 282)

An LTI system is described by the difference equation

$$y[n] = bx[n] + 0.8y[n-1] - 0.81y[n-2] \qquad (5.23.1)$$

```
clc; close all; clear;
```

(a) Determine the frequency response of the system in terms of b.

Solution:

First we solve the difference equation for the z-transform of the impulse response and then substitute $z = e^{j\omega}$

$y[n] - 0.8y[n-1] + 0.81y[n-2] = bx[n]$

$Y(z)(1 - 0.8z^{-1} + 0.81z^{-2}) = bX(z)$

$\frac{Y(z)}{X(z)} = H(z) = \frac{b}{1 - 0.8z^{-1} + 0.81z^{-2}}$ then substituting $z = e^{j\omega}$ we get the frequency response

$H(e^{j\omega}) = \frac{b}{1 - 0.8e^{-j\omega} + 0.81e^{-2j\omega}}$

We can expand this further using the euler's identities:

$e^{-j\omega} = \cos\omega - j\sin\omega$ and $e^{-2j\omega} = \cos 2\omega - j\sin 2\omega$

$H(e^{j\omega}) = \frac{b}{1 - 0.8(\cos\omega - j\sin\omega) + 0.81(\cos 2\omega - j\sin 2\omega)}$

$H(e^{j\omega}) = \frac{b}{1 - 0.8\cos\omega + 0.81\cos 2\omega + j(0.8\sin\omega - 0.81\sin 2\omega)}$

Then the Magnitude and Phase Responses would be

$|H(e^{j\omega})| = \frac{|b|}{\sqrt{(1 - 0.8\cos\omega + 0.81\cos 2\omega)^2 + (0.8\sin\omega - 0.81\sin 2\omega)^2}}$

$\angle H(e^{j\omega}) = \angle b - \tan^{-1}\left(\frac{0.8\sin\omega - 0.81\sin 2\omega}{1 - 0.8\cos\omega + 0.81\cos 2\omega}\right)$

(b) Determine b so that $|H(\mathrm{e}^{j\omega})|_{\max} = 1$. Plot the resulting magnitude response.

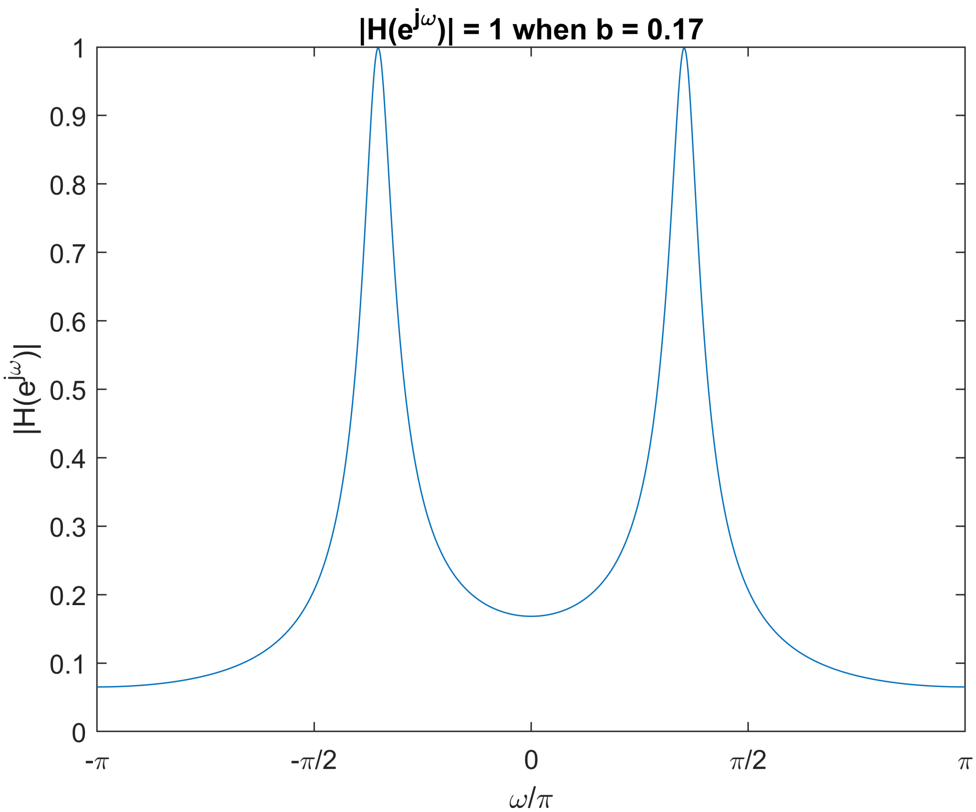
Solution:

If we want the maximum magnitude to be equal to one, then we choose b so that $|H(e^{j\omega})| = 1$

$\rightarrow b = \dfrac{1}{|H(e^{j\omega})|} = 0.17$

MATLAB script for the magnitude plot:

```
omega = linspace(-pi,pi,1000);
b = 0.17;
a = [1 -0.8 0.81];
H = freqz(b,a,omega);
figure
plot(omega/pi, abs(H))
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
xlabel('\omega/\pi')
ylabel('|H(e^{j\omega})|')
title('|H(e^{j\omega})| = 1 when b = 0.17')
```

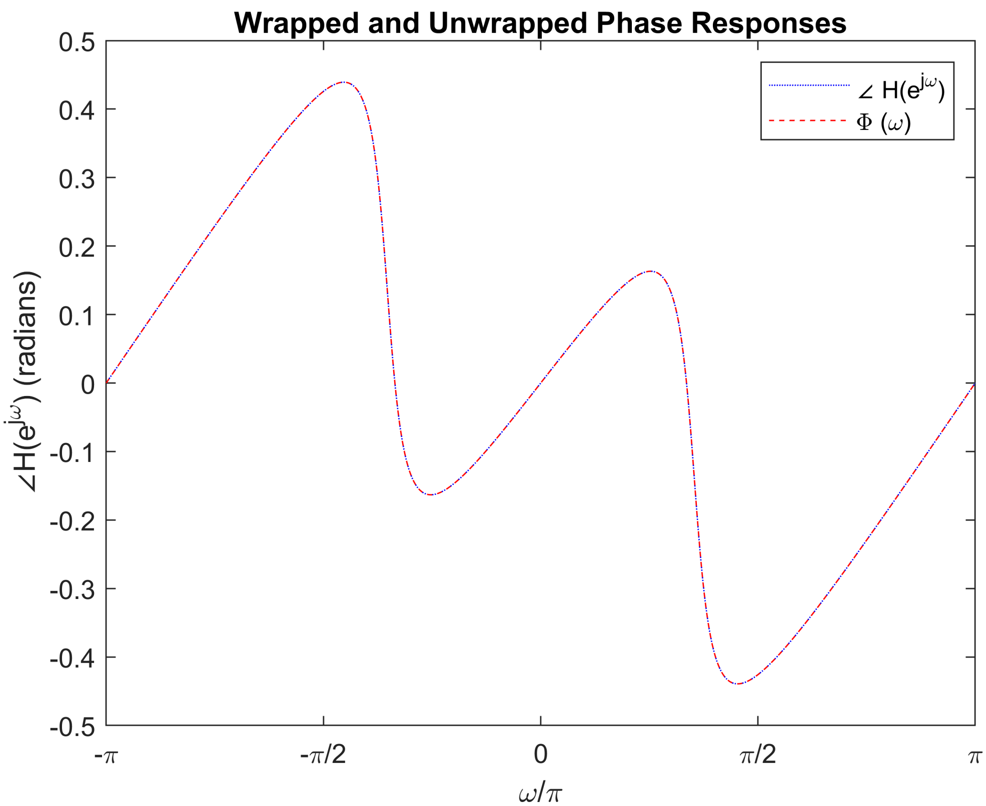


(c) Graph the wrapped and the unwrapped phase responses in one plot.

MATLAB script:

```
H_unwrapped = phasez(b,a,omega)';
figure
plot(omega/pi,angle(H)/pi,':b')
```

```
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
hold on
plot(omega/pi,H_unwrapped/pi,'--r')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
legend('\angle H(e^{j\omega})', '\Phi (\omega)')
hold off
title('Wrapped and Unwrapped Phase Responses')
xlabel('\omega/\pi')
ylabel('\angle{H(e^{j\omega})} (radians)')
```



(d) Determine analytically response $y[n]$ to the input $x[n] = 2\cos(\pi n/3 + 45^{\text{deg}})$. Plot the output sequence $y[n]$.

Solution:

Analyzing the input sequence: $x[n]$ we see that $\omega = \pi/3$, so to determine how the system will alter this input sequence, we solve the system's frequency response with $\omega = \pi/3$

Again,

$$H(e^{j\omega}) = \frac{0.17}{1 - 0.8e^{-j\omega} + 0.81e^{-2j\omega}}$$

$$H(e^{j\pi/3}) = \frac{0.17}{1 - 0.8e^{-j\pi/3} + 0.81e^{-j2\pi/3}} = 0.1229 - j0.1217$$

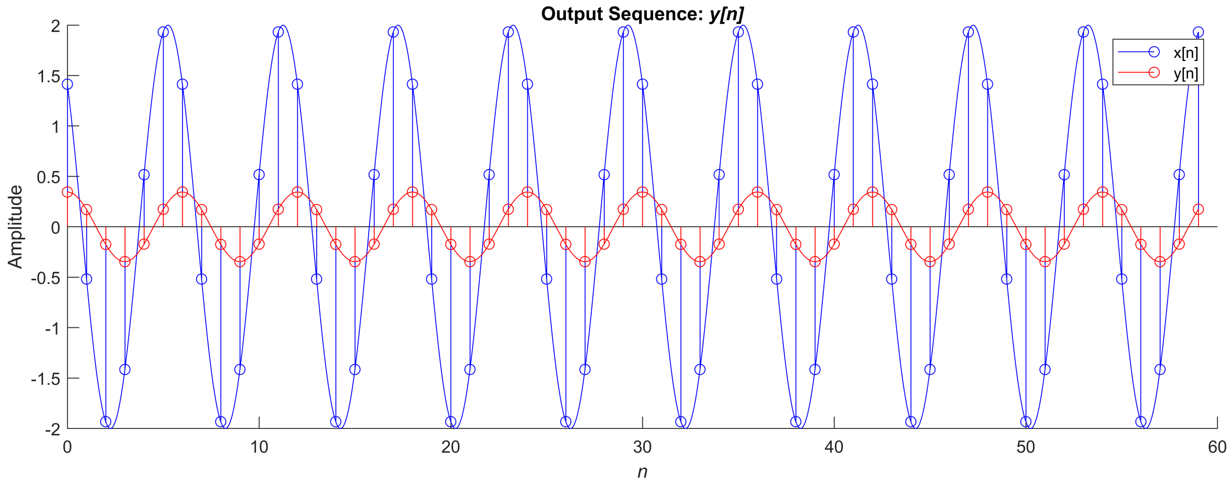
$H(e^{j\pi/3}) = 0.1729\angle{-44.715^\circ} \rightarrow$ thus the input sequence will be scaled by a factor of 0.1729 and have a phase shift of -44.715 degrees, resulting in:

$$y[n] = (0.1729)(2)\cos\left(\frac{\pi}{3}n + 45^\circ - 44.715^\circ\right)$$

$$y[n] = 0.2458\cos\left(\frac{\pi}{3}n + 0.285^\circ\right)$$

MATLAB script for the output sequence plot:

```
N = 6; w = (2*pi/N); b = 0.17;
h = b/(1 - 0.8*exp(-1i*w)+0.81*exp(-2*w));
H_mag = abs(h); H_ang = angle(h);
n = 0:10*N-1; x = 2.*cos((w).*n + (pi/4));
y = (H_mag).*2.*cos((pi/3).*n + (pi/4) + H_ang);
omega = linspace(0,10*N-1,1000);
X = 2.*cos((w).*omega + (pi/4)); Y = (H_mag).*2.*cos((w).*omega + (pi/4) + H_ang);
figure('Units','inches','Position',[0,0,12,4]); hold on
stem(n,x,'b'), stem(n,y,'r')
plot(omega,X,'b'), plot(omega,Y,'r')
legend('x[n]','y[n]'), hold off
title('Output Sequence: \it{y[n]}')
xlabel('\it{n}'), ylabel('Amplitude')
```

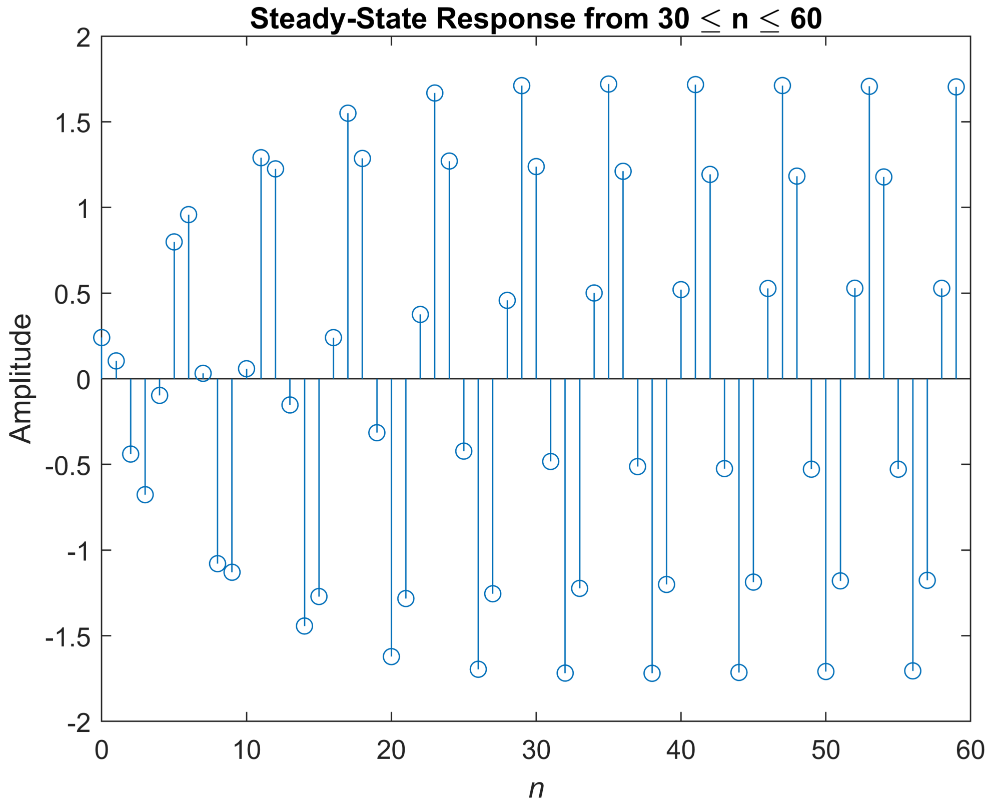


(e) Using MATLAB, compute the steady-state response to $x[n]$ above and verify your result.

Matlab script:

```
% N = 6; k = 0:N-1; w = (2*pi/N); x = 2.*cos((w).*k + (pi/4));
% ck_x = dtfs(x); H_mag = abs(freqz(b,a,k)); ck_y = H_mag.*ck_x; y = idtfs(ck_y);
y = filter(b,a,x);
figure
stem(n,y)
xlabel('\it{n}')
ylabel('Amplitude')
```

```
title('Steady-State Response from 30 \leq n \leq 60')
```



Verifying the results, we see that the steady-state response begins after the transient response has subsided, which occurs around $n = 30$. The amplitude of the input has been scaled and phase shifted as expected.

Problem 3.2

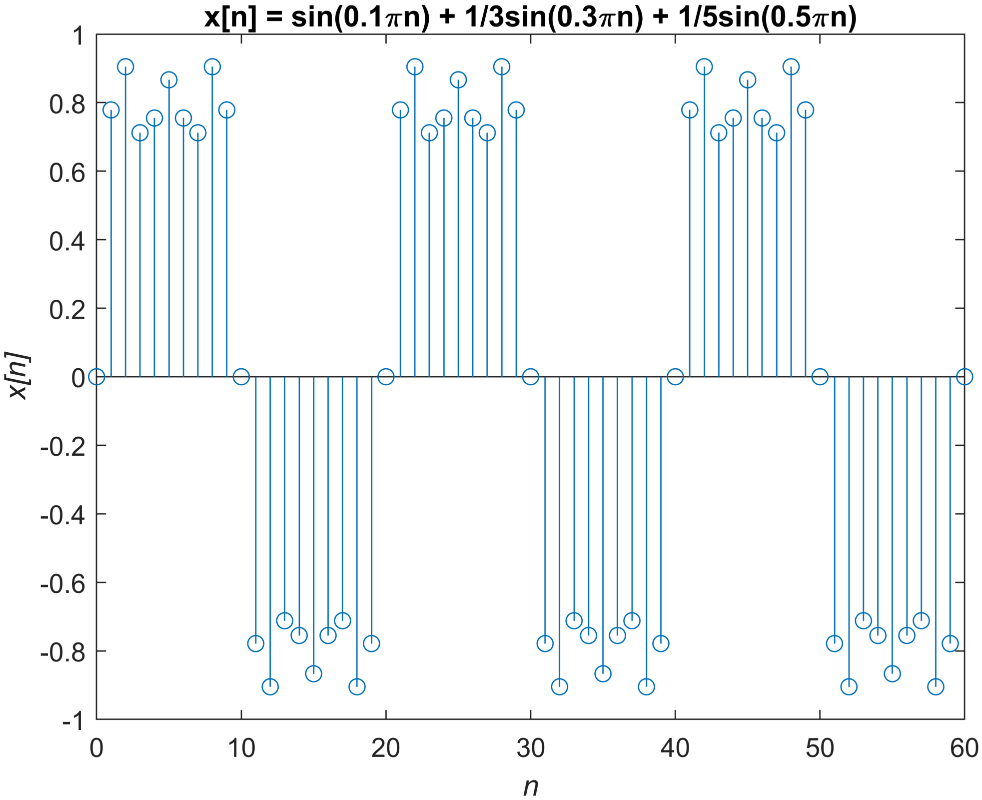
Text Problem 5.30, parts (a) and (e), (Page 283)

Consider a periodic signal

$$x[n] = \sin(0.1\pi n) + \frac{1}{3}\sin(0.3\pi n) + \frac{1}{5}\sin(0.5\pi n).$$

For each of the following systems, first plot magnitude and phase responses and then determine if the system imparts (i) no distortion, (ii) magnitude distortion, and/or (iii) phase (or delay) distortion. In each case, also graph the input and the steady state response for $0 \leq n \leq 60$.

```
clc; close all; clear;
n = 0:60;
x = sin(0.1.*pi.*n) + (1/3).*sin(0.3.*pi.*n) + (1/5).*sin(0.5.*pi.*n);
figure
stem(n,x)
title("x[n] = sin(0.1\pin) + 1/3sin(0.3\pin) + 1/5sin(0.5\pin)")
xlabel('\it{n}'), ylabel('\it{x[n]}')
```

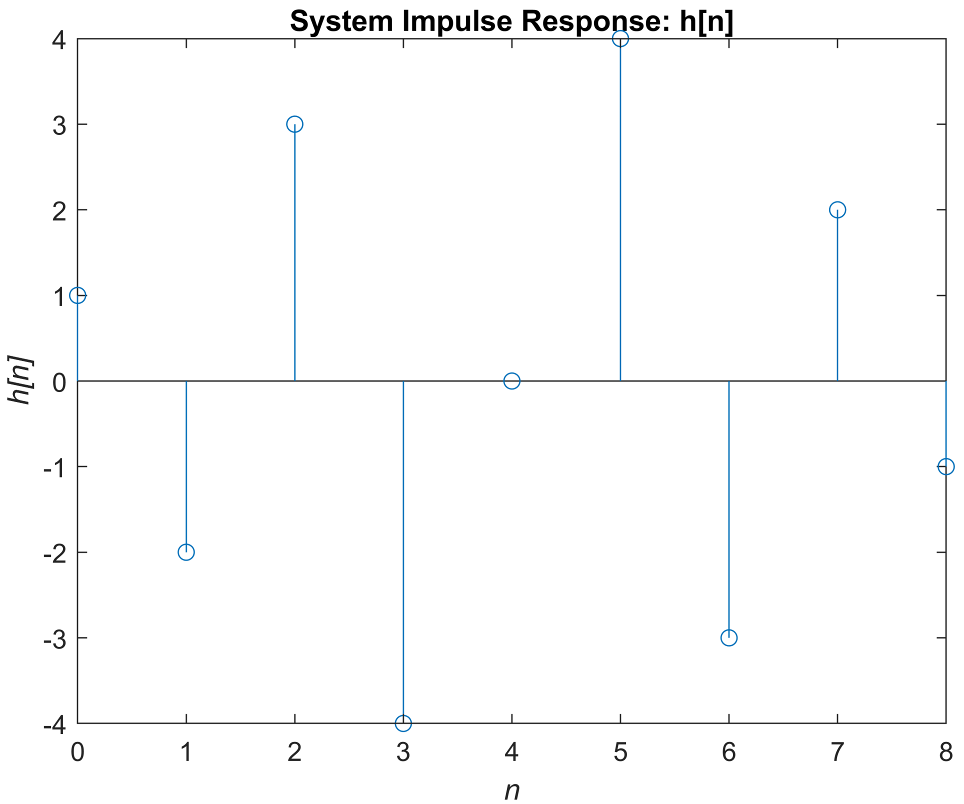


(a) $h[n] = \{\underset{\uparrow}{1}, -2, 3, -4, 0, 4, -3, 2, -1\}$:

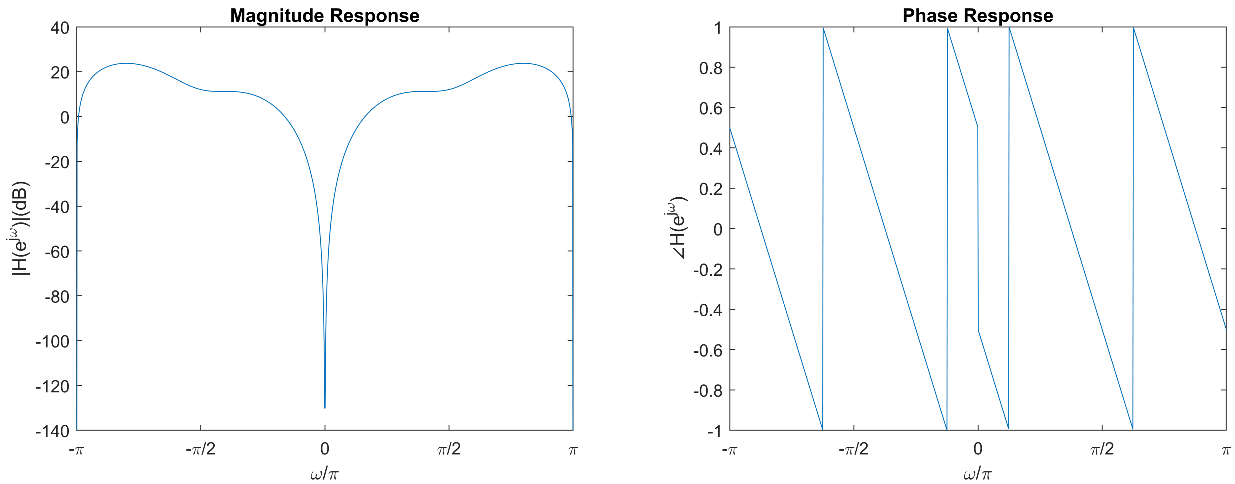
Solution:

MATLAB script for computation and plotting of magnitude and phase responses:

```
h = [1 -2 3 -4 0 4 -3 2 -1]; hn = 0:8;
figure
stem(hn,h)
title('System Impulse Response: h[n]')
xlabel('\it{n}')
ylabel('\it{h[n]}')
```




```
omega = linspace(-pi,pi,1000);
H = freqz(h,1,omega); H_mag = abs(H); H_ang = angle(H);
figure('Units','inches','Position',[0,0,12,4]);
subplot(1,2,1)
plot(omega/pi,20*log10(abs(H)));
title('Magnitude Response')
ylim([-140 40])
ylabel('|H(e^{j\omega})|(dB)')
xlabel('\omega/\pi')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
subplot(1,2,2)
plot(omega/pi,angle(H)/pi)
xlabel('\omega/\pi')
ylabel('\angle{H(e^{j\omega})}')
title('Phase Response')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
```



Answer for distortion:

A distortionless system can be described as

$y[n] = Gx[n - n_d], \quad G > 0$, where G and n_d are constants.

If the system magnitude response is not a constant, the system will provide magnitude distortion.

The impulse response of this system can be described as

$$h[n] = \delta[n] - 2\delta[n-1] + 3\delta[n-2] - 4\delta[n-3] + 4\delta[n-5] - 3\delta[n-6] + 2\delta[n-7] - \delta[n-8]$$

The system response can be found by taking the DTFT of the impulse response, which results in

$$H(e^{j\omega}) = 1 - 2e^{-j\omega} + 3e^{-j2\omega} - 4e^{-j3\omega} + 4e^{-j5\omega} - 3e^{-j6\omega} + 2e^{-j7\omega} - e^{-j8\omega}$$

Here we can see that when the frequency ω changes, the values of magnitude will also change

Thus, this system is not distortionless and experiences magnitude distortion.

This system also experiences a linear phase response before and after 4ω based on the system function, where there is a zero component in

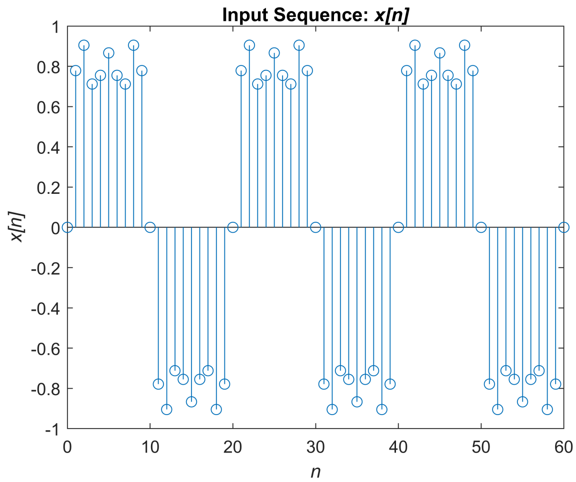
the impulse response.

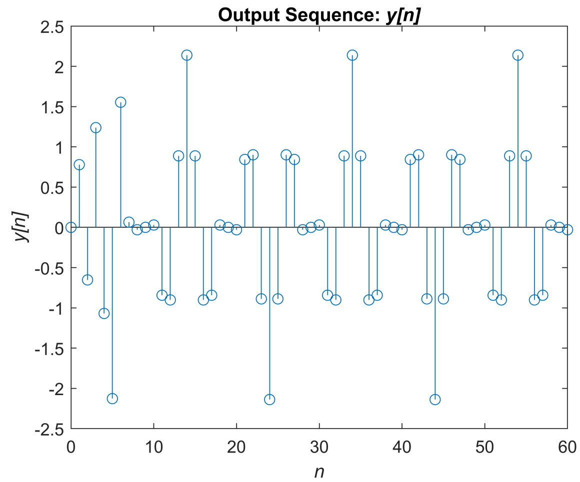
Thus, the phase response does not experience a truly linear-phase shift meaning this system also experiences phase distortion.

MATLAB script for the computation of input and output sequences:

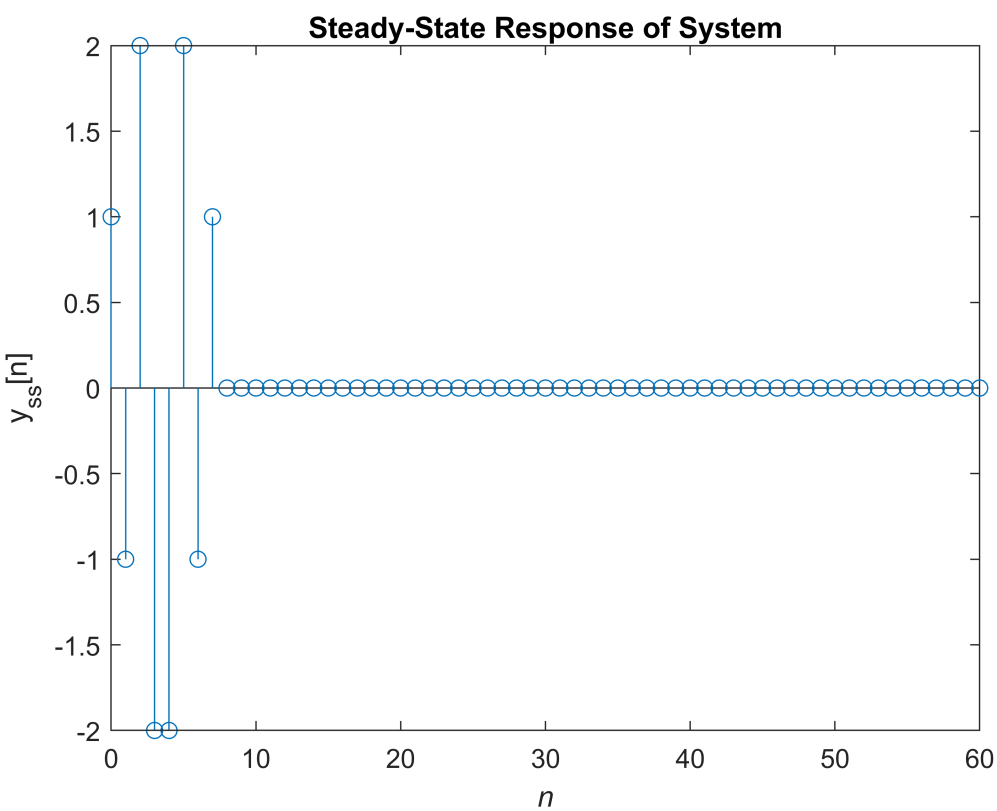
```
%y = filter(h,1,x);
y = conv(h,x);
figure('Units','inches','Position',[0,0,12,4]);
subplot(1,2,1)
stem(n,x);
title('Input Sequence: \it{x[n]}')
ylabel('\it{x[n]}')
xlabel('\it{n}')
subplot(1,2,2)
```

```
stem(n,y(1:61))
xlabel('\it{n}')
ylabel('\it{y[n]}')
title('Output Sequence: \it{y[n]}')
```






```
n = 0:60; u = (n>= 0);
y_ss = filter(h,1,u);
figure
stem(n,y_ss)
xlabel('\it{n}'), ylabel('y_{ss}[n]'), title('Steady-State Response of System')
```

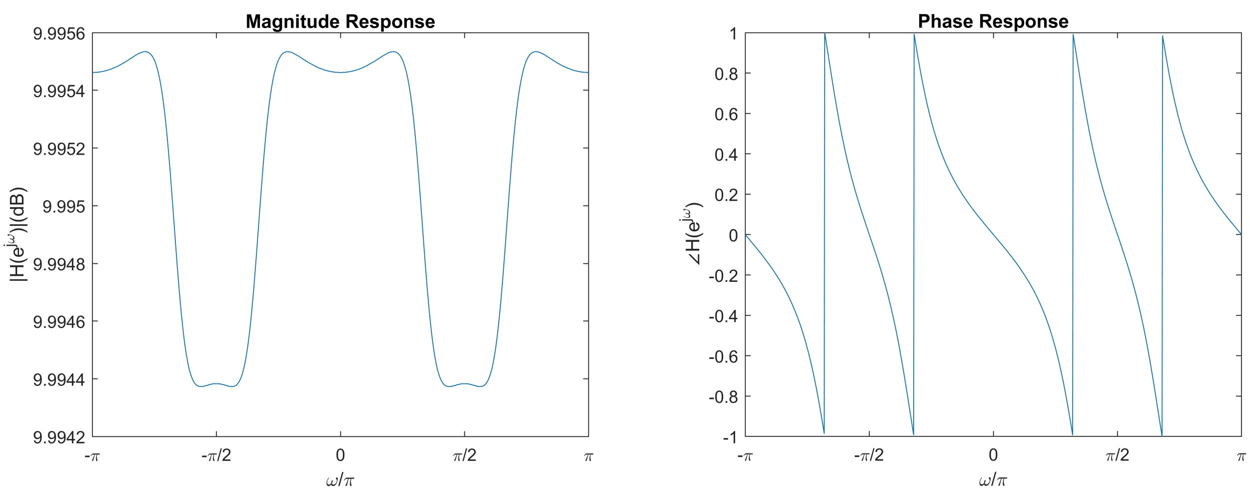


(e) $H(z) = \dfrac{1 + 1.778z^{-2} + 3.1605z^{-4}}{1 + 0.5625z^{-2} + 0.3164z^{-4}}$:

Solution:

MATLAB script for computation and plotting of magnitude and phase responses:

```
b = [1 0 1.778 0 3.1605]; a = [1 0 0.5625 0 0.3164];
omega = linspace(-pi,pi,1000);
H = freqz(b,a,omega);
figure('Units','inches','Position',[0,0,12,4]);
subplot(1,2,1)
plot(omega/pi,20*log10(abs(H)));
title('Magnitude Response')
ylabel('|H(e^{j\omega})|(dB)')
xlabel('\omega/\pi')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
subplot(1,2,2)
plot(omega/pi,angle(H)/pi)
xlabel('\omega/\pi')
ylabel('\angle{H(e^{j\omega})}')
title('Phase Response')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
```



Answer for distortion:

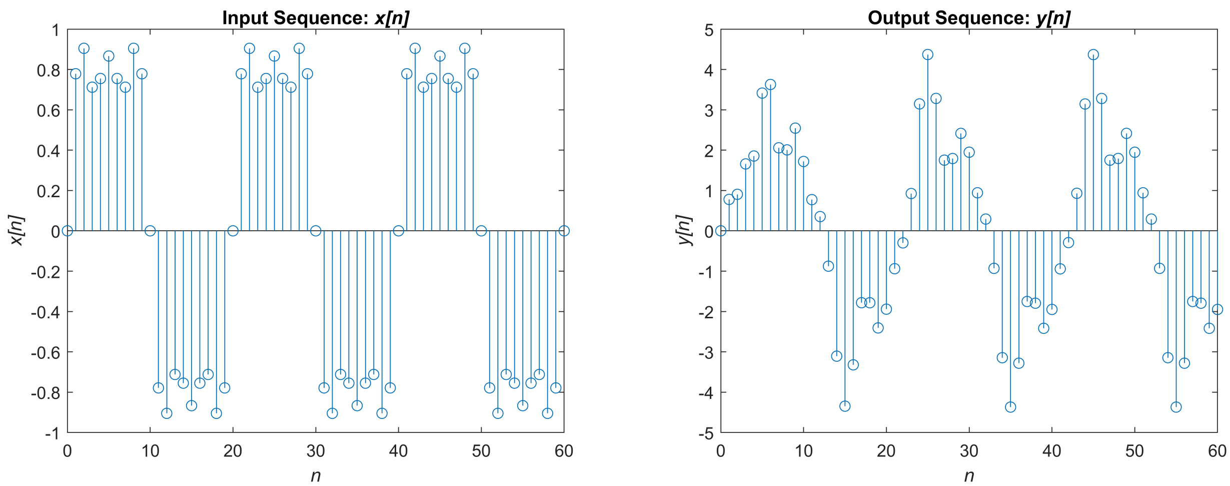
The response for this system resembles a bandreject filter with a gain factor around 3.16. Since the response is not a constant for all values of frequency $\omega \rightarrow$ system is not distortionless and experiences magnitude distortion for values around $\pm\frac{\pi}{2}$.

The system also experiences a phase response that does pass through the origin of $\omega = 0$, but it does so as a nonlinear function of frequency

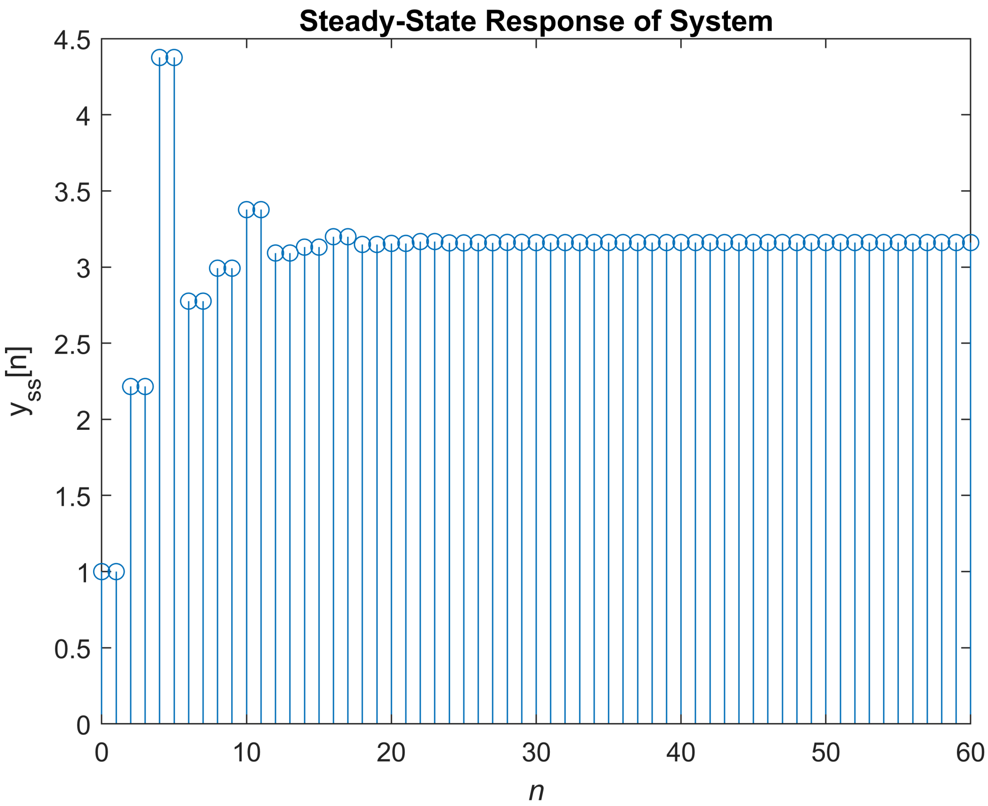
So, the system has not only a magnitude distortion but a phase distortion as well.

MATLAB script for the computation of input and output sequences:

```
y = filter(b,a,x);
figure('Units','inches','Position',[0,0,12,4]);
subplot(1,2,1)
stem(n,x);
title('Input Sequence: \it{x[n]}')
ylabel('\it{x[n]}')
xlabel('\it{n}')
subplot(1,2,2)
stem(n,y)
xlabel('\it{n}')
ylabel('\it{y[n]}')
title('Output Sequence: \it{y[n]}')
```




```
n = 0:60; u = (n>= 0);
y_ss = filter(b,a,u);
figure
stem(n,y_ss)
xlabel('\it{n}'), ylabel('y_{ss}[n]'), title('Steady-State Response of System')
```



Problem 3.3

Text Problem 5.37 (Page 284)

Compute and plot the phase response using the functions **`freqz`**, **`angle`**, **`phasez`**, **`unwrap`**, and **`phasedelay`** for the following systems.

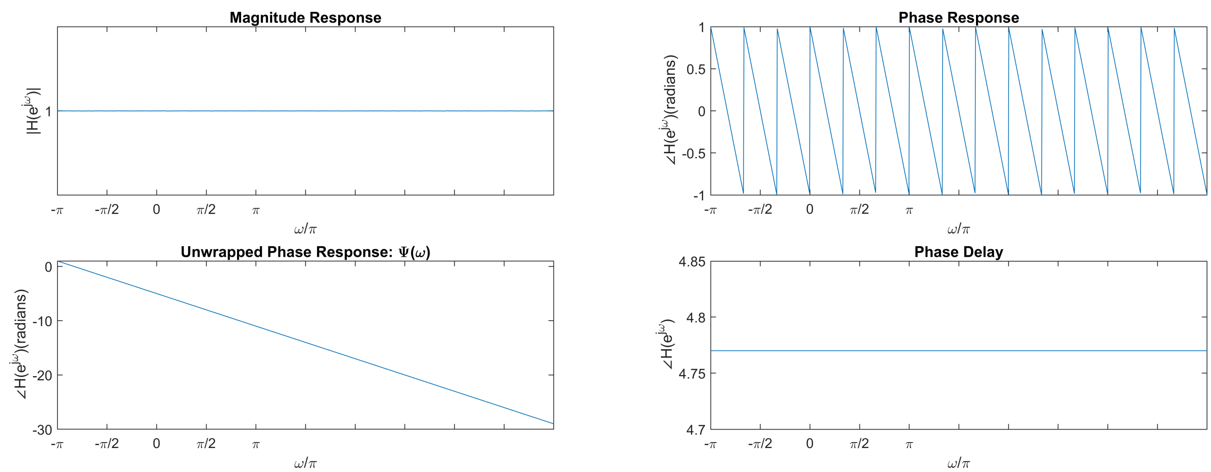
```
clc; close all; clear;
```

(a) The pure delay $y[n] = x[n-15]$.

MATLAB script for computation and plotting:

```
x = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1]; omega = linspace(-pi,pi,1000);
H = freqz(x,1,omega);
figure('Units','inches','Position',[0,0,18,6]);
subplot(2,2,1)
plot(omega/pi,abs(H));
title('Magnitude Response')
ylabel('|H(e^{j\omega})|')
xlabel('\omega/\pi')
yticks(1)
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
```

```
subplot(2,2,2)
plot(omega/pi,angle(H)/pi)
xlabel('\omega/\pi')
ylabel('\angle{H(e^{j\omega})(radians)}')
title('Phase Response')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
subplot(2,2,3)
plot(omega/pi,unwrap(angle(H))/pi)
xlabel('\omega/\pi')
ylabel('\angle{H(e^{j\omega})}(radians)')
title('Unwrapped Phase Response: \Psi(\omega)')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
subplot(2,2,4)
H_pd = phasedelay(x,1,omega)/pi;
plot(omega/pi,round(H_pd,2))
xlabel('\omega/\pi')
ylabel('\angle{H(e^{j\omega})}')
title('Phase Delay')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
ylim([4.7 4.85])
```

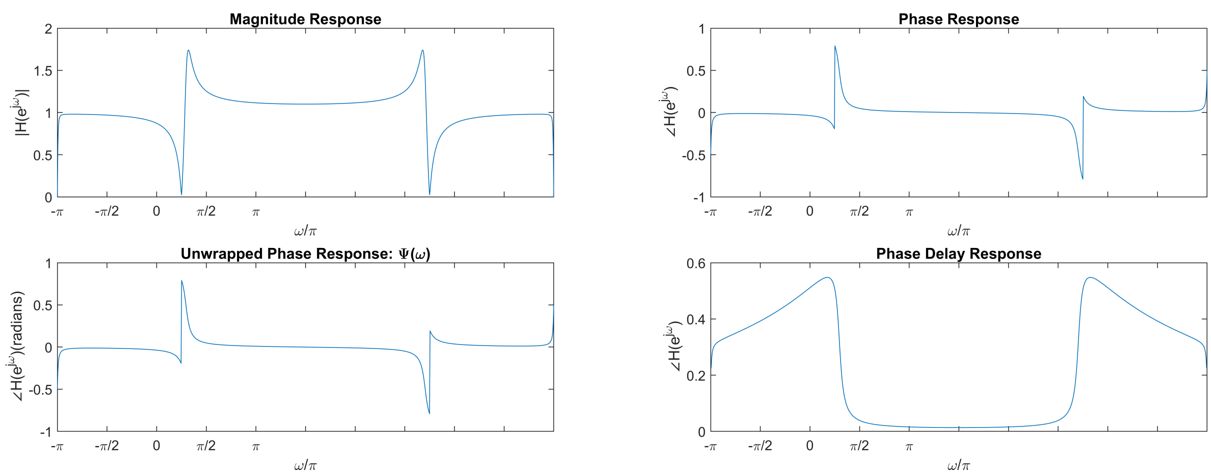


(b) The system defined by $H(z) = \dfrac{1 + z^{-1} + z^{-2} + z^{-3}}{1 + 0.9z^{-1} + 0.81z^{-2} + 0.927z^{-3}}$.

MATLAB script for computation and plotting:

```
b = [1 1 1 1]; a = [1 0.9 0.81 0.927]; omega = linspace(-pi,pi,1000);
H = freqz(b,a,omega);
figure('Units','inches','Position',[0,0,18,6]);
subplot(2,2,1)
plot(omega/pi,abs(H));
title('Magnitude Response')
ylabel('|H(e^{j\omega})|')
xlabel('\omega/\pi')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
subplot(2,2,2)
plot(omega/pi,angle(H)/pi)
```

```
xlabel('\omega/\pi')
ylabel('\angle{H(e^{j\omega})}')
title('Phase Response')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
subplot(2,2,3)
plot(omega/pi,unwrap(angle(H))/pi)
xlabel('\omega/\pi')
ylabel('\angle{H(e^{j\omega})}(radians)')
title('Unwrapped Phase Response: \Psi(\omega)')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
H_pd = phasedelay(b,a,omega);
subplot(2,2,4)
plot(omega/pi,H_pd/pi)
xlabel('\omega/\pi')
ylabel('\angle{H(e^{j\omega})}')
title('Phase Delay Response')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
```



Problem 3.4

Text Problem 5.40 (Page 285)

Consider a second-order IIR notch filter specification that satisfies the following requirements: (1) the magnitude response has notches at $\omega_{1,2} = \pm 2\pi/3$; (2) The maximum magnitude response is 1; (3) the magnitude response is approximately $1/\sqrt{2}$ at frequencies $\omega_{1,2} \pm 0.01$.

(a) Using the pole-zero placement approach determine locations of two poles and two zeros of the required filter and then compute its system function $H(z)$.

Solution:

The formula for a Notch Filter is

$$H(z) = b_0 \frac{(1 - e^{j\omega_0} z^{-1})(1 - e^{-j\omega_0} z^{-1})}{(1 - re^{j\omega_0} z^{-1})(1 - re^{-j\omega_0} z^{-1})}$$

Because we want to block the frequencies at $\pm\frac{2\pi}{3}$, we set are zeros to $z_1 = e^{j2\pi/3}$ and $z_2 = e^{-j2\pi/3}$

Then if we want the maximum magnitude response to be equal to 1 then b_0 must be equal to $\frac{1}{|H(e^{j\omega})|}$

Finally, for the magnitude response to be approximately $\frac{1}{\sqrt{2}}$ at frequencies $\omega_{1,2} \pm 0.01$, we set our poles equal

to $p_1 = 0.969e^{\frac{j2\pi}{3}}$ and $0.969e^{\frac{-j2\pi}{3}}$

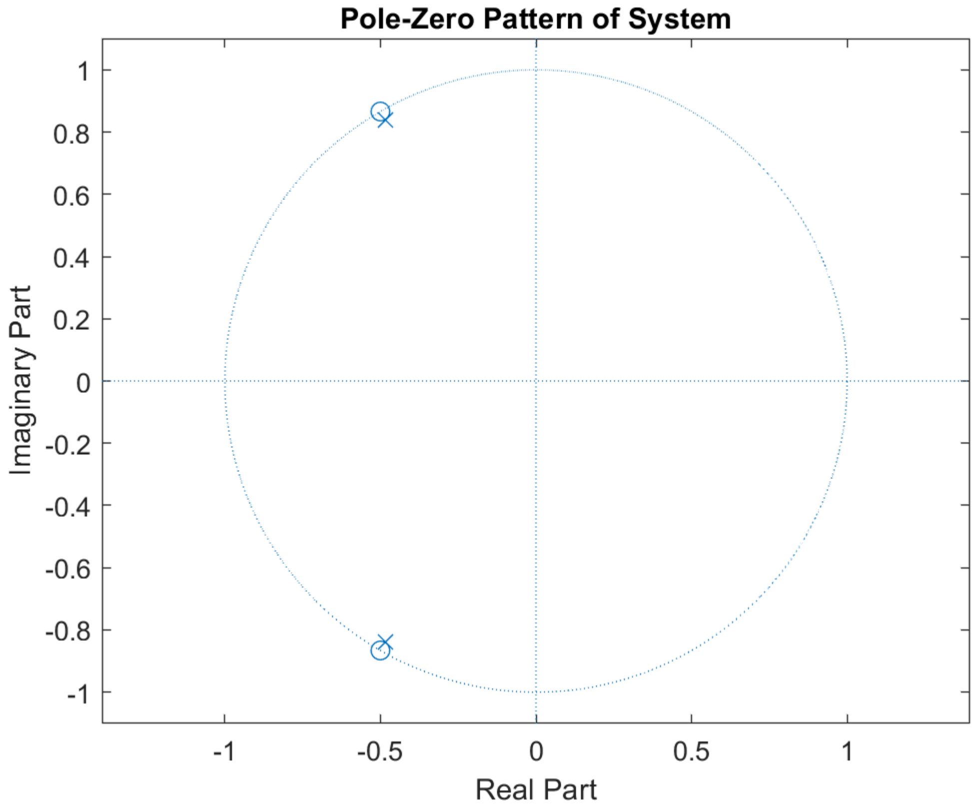
Which resolves into the system function:

$$H(z) = b_0 \frac{1 - 2\cos\left(\frac{2\pi}{3}\right) z^{-1} + z^{-2}}{1 - \left(1.938\cos\left(\frac{2\pi}{3}\right)\right) z^{-1} + 0.939 z^{-2}}$$

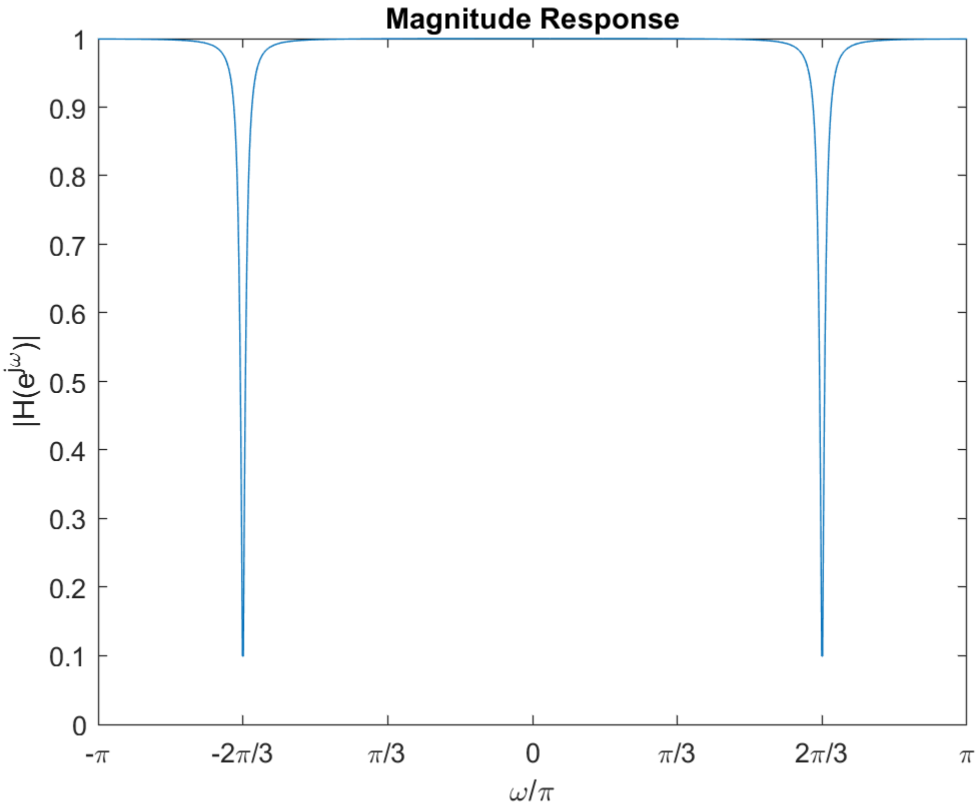
(b) Graph the magnitude response of the filter and verify the given requirements.

MATLAB script:

```
clc; close all; clear;
w0 = 2*pi/3; r1 = 0.969; %1/sqrt(2); %r2 = 1/sqrt(2);
 % a1 = [1 -r*exp(1i*0.01)]; a2 = [1 -r*exp(-1i*0.01)]; a = a.*conv(a1,a2);
%b1 = [1 -exp(1i*w0)]; b2 = [1 -exp(-1i*w0)]; a1 = [1 -r*exp(1i*0.01)]; a2 = [1 -r*exp(-1i*0.01
omega = linspace(-pi,pi,1000);
b = [1 -2*cos(w0) 1]; %b2 = [1 -2*cos(0) 1]; b = conv(b1,b2);
a = [1 -(2*r1)*cos(w0) r1^2]; %a2 = [1 -(2*r2)*cos(0) r2^2]; a = conv(a1,a2);
H = freqz(b,a,omega);
b0 = 1/max(abs(H)); H = b0.*H;
figure
zplane(b,a), title('Pole-Zero Pattern of System')
```

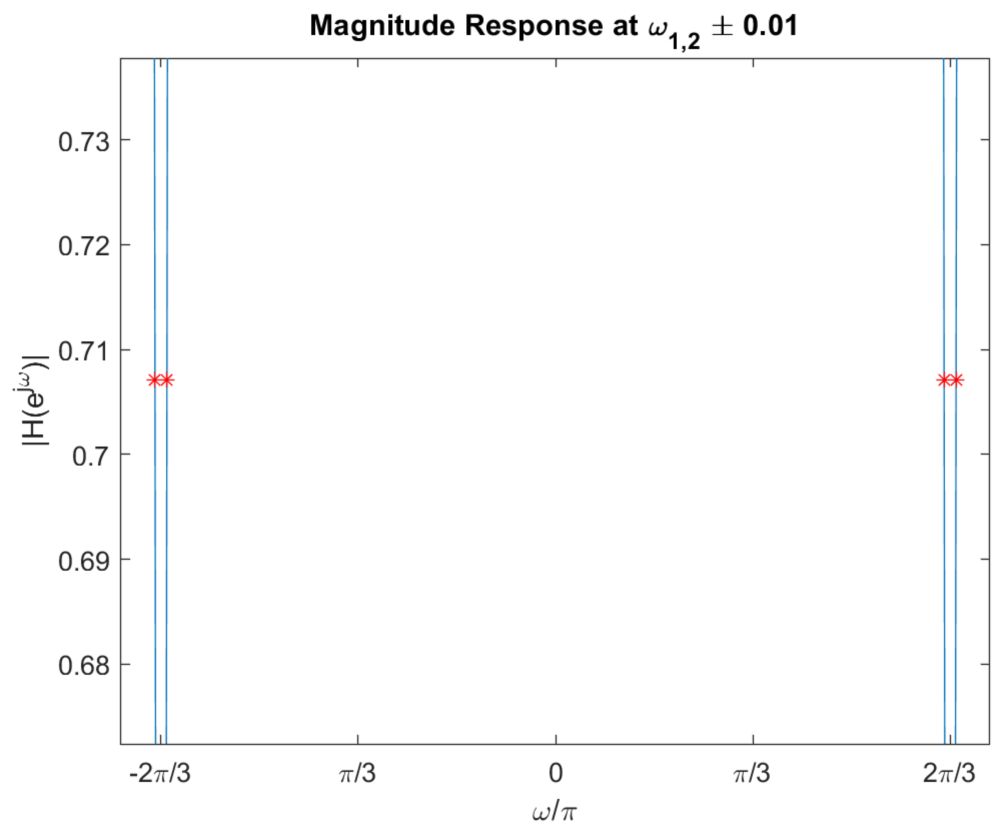



```
figure
plot(omega/pi, abs(H))
xticks(-1:1/3:1), xlabel('\omega/\pi'), ylabel('|H(e^{j\omega})|'), title('Magnitude Response')
xticklabels({'-\pi','-2\pi/3','\pi/3','0','\pi/3','2\pi/3','\pi'});
```



Verification: We will probe the magnitude responses at $\omega_{1,2} \pm 0.01$ and verify that it is approximates $1/\sqrt{2} = 0.7071$.

```
figure
x_pos = (2/3) + 0.01; x_min = (2/3) - 0.01; y_pos = 1/sqrt(2);
plot(omega/pi, abs(H)), hold on
plot(x_pos,y_pos,'r*'), plot(x_min,y_pos,'r*')
plot(-x_pos,y_pos,'r*'), plot(-x_min,y_pos,'r*')
xticks(-1:1/3:1), xlabel('\omega/\pi'), ylabel('|H(e^{j\omega})|'), title('Magnitude Response a
xticklabels({'-\pi','-2\pi/3','\pi/3','0','\pi/3','2\pi/3','\pi'});
hold off;
xlim([-0.734 0.734])
ylim([0.6724 0.7378])
```

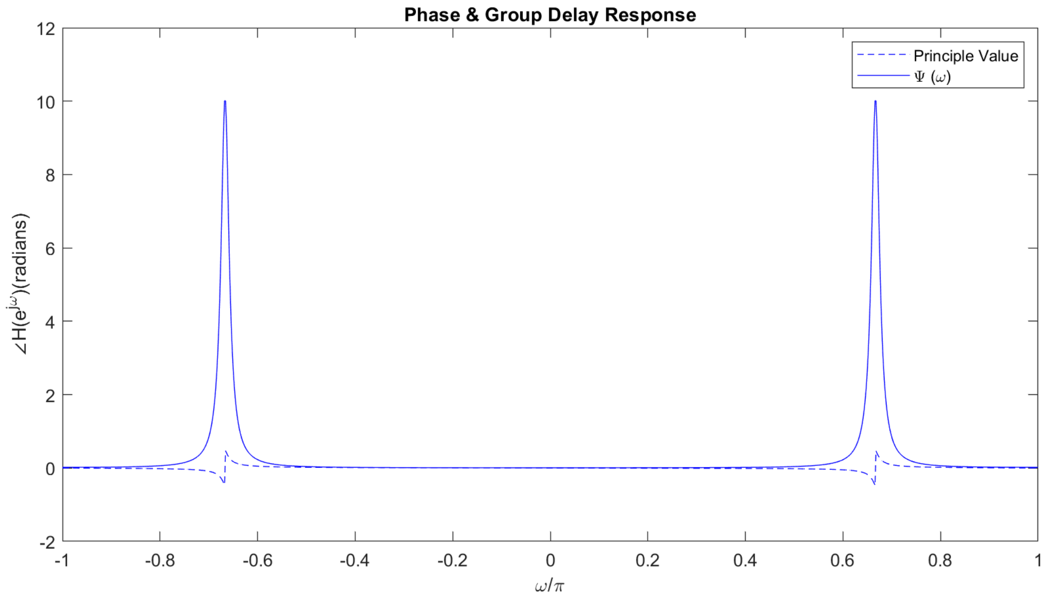


In the above plot, we can see the points at $\omega_{1,2} \pm 0.01$ where the values of $\frac{1}{\sqrt{2}}$ are being highlighted by red asterisks. This confirms that the magnitude response is equal to $\frac{1}{\sqrt{2}}$ at $\omega_{1,2} \pm 0.01$.

(c) Graph phase and group-delay responses in one plot.

MATLAB script:

```
H_phase = angle(H);
figure('Units','inches','Position',[0,0,10,5]);
plot(omega/pi,H_phase/pi, '--b')
xlabel('\omega/\pi')
ylabel('\angle{H(e^{j\omega})}(radians)')
title('Phase & Group Delay Response')
%xticklabels({'-\pi','-4\pi/5','-3\pi/5','-2\pi/5','-1\pi/5','0','1\pi/5','2\pi/5','3\pi/5','4\
hold on
H_gd = grpdelay(b,a,omega)';
plot(omega/pi,H_gd/pi,'-b'), legend('Principle Value', '\Psi (\omega)'), hold off
```



Problem 3.5

Text Problem 5.55, parts (c) and (d) (Page 288)

Determine the system function, magnitude response, and phase response of the following systems and use the pole-zero pattern to explain the shape of their magnitude response.

```
clc; close all; clear;
```

(c) $y[n] = x[n] - x[n-4] - 0.6561\,y[n-4]$

Solution:

Using the difference equation to derive the system function, we find that

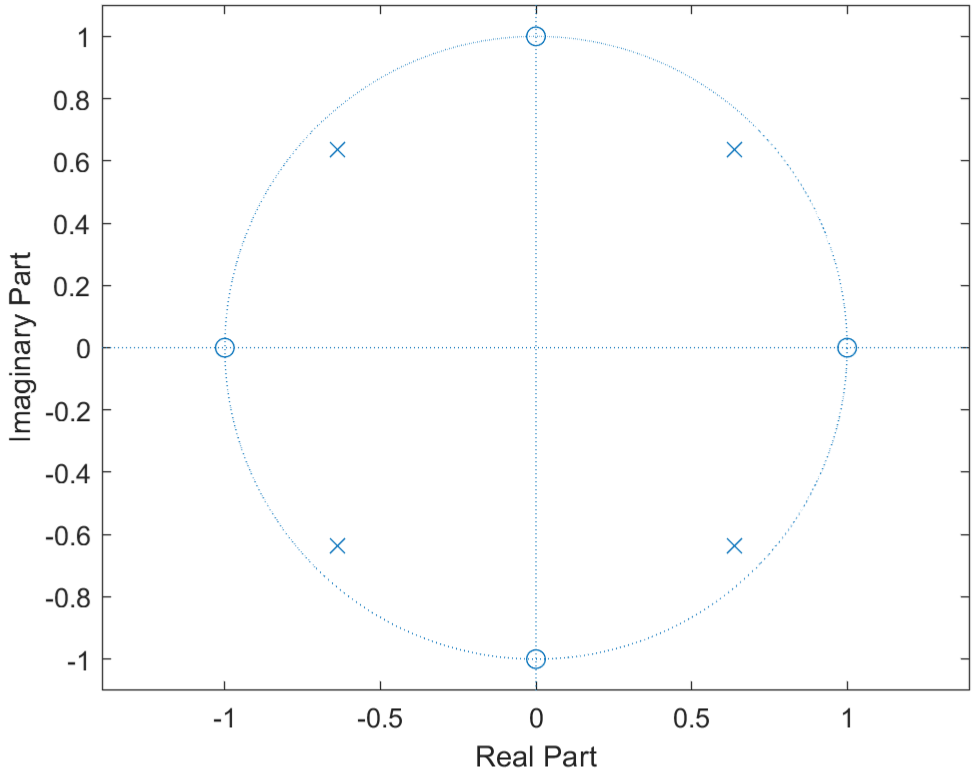
$$y[n] + 0.6561\,y[n-4] = x[n] - x[n-4] \rightarrow \frac{Y(z)}{X(z)} = H(z) = \frac{1 - z^{-4}}{1 + 0.6561z^{-4}}$$

The system function can then be described by substituting $z = e^{j\omega}$ from the above equation yielding:

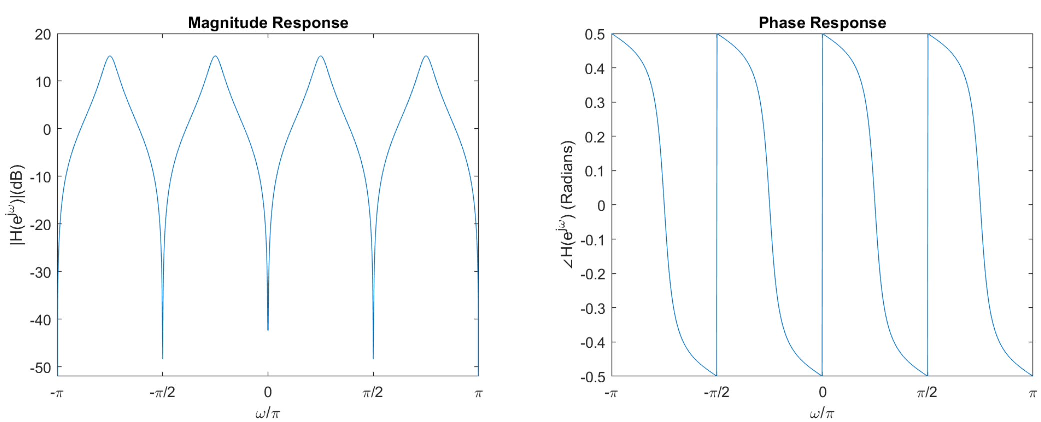
$$H(e^{j\omega}) = \frac{1 - e^{-j4\omega}}{1 + 0.6561e^{-j4\omega}}$$

MATLAB script for various system responses and plots:

```
b = [1 0 0 0 -1]; a = [1 0 0 0 0.6561]; omega = linspace(-pi,pi,1000);
H = freqz(b,a,omega); figure; zplane(b,a);
```




```
figure('Units','inches','Position',[0,0,12,4]);
subplot(1,2,1)
plot(omega/pi,20*log10(abs(H)));
title('Magnitude Response')
ylabel('|H(e^{j\omega})|(dB)')
xlabel('\omega/\pi')
ylim([-52 20])
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
subplot(1,2,2)
plot(omega/pi,angle(H)/pi)
xlabel('\omega/\pi')
ylabel('\angle{H(e^{j\omega})} (Radians)')
title('Phase Response')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
```



Explanation of plots:

From the pole-zero plots, we see that there are 4 poles at $\pm\frac{\pi}{4}$ and $\pm\frac{3\pi}{4}$ with a magnitude of 0.6561 for each pole location. This correlates to these frequencies being boosted in the magnitude response by a gain of about 15dB at each pole location. There were also 4 zero locations in

the pole-zero plot located at $\left[0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right]$, which results in a notch in the magnitude response at these corresponding frequencies, as seen in the magnitude response plot above.

(d) $y[n] = x[n] - x[n-1] + 0.99y[n-1] - 0.9801y[n-2]$

Solution:

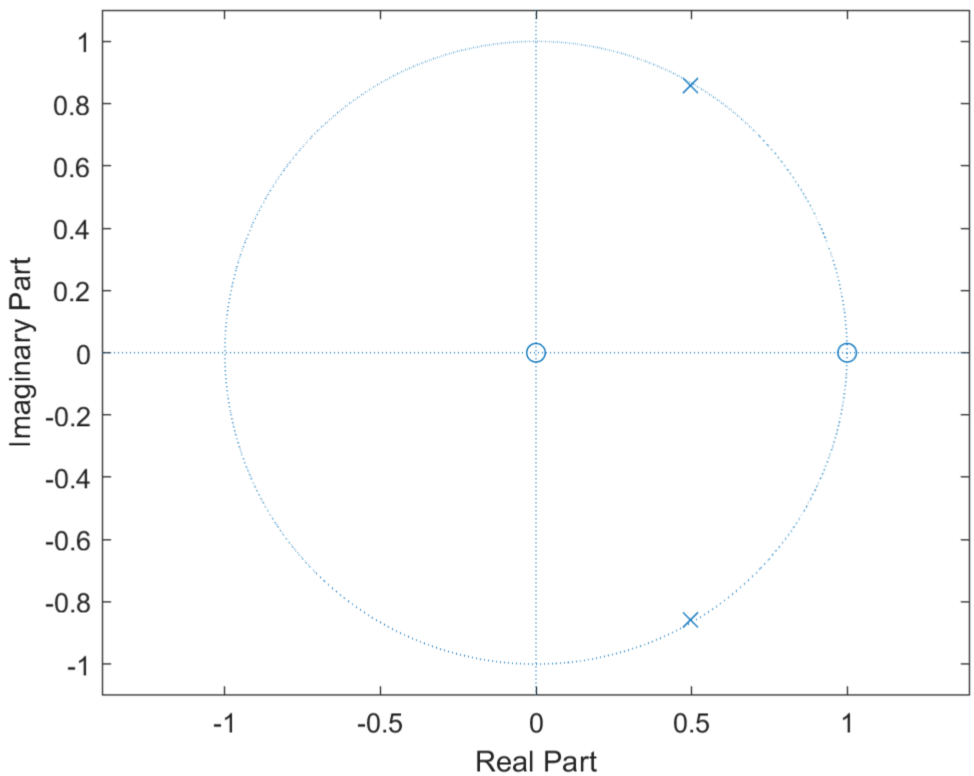
Using the difference equation to derive the system function, we find that

$$y[n] - 0.99y[n-1] + 0.9801y[n-2] = x[n] - x[n-1] \rightarrow \frac{Y(e^{j\omega})}{X(e^{j\omega})} = H(e^{j\omega})$$

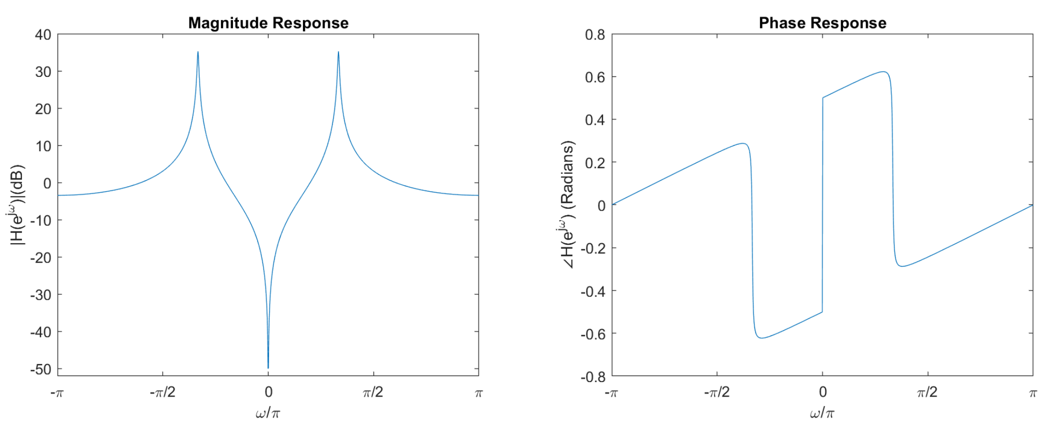
$$H(e^{j\omega}) = \frac{1 - e^{-j\omega}}{1 - 0.99e^{-j\omega} + 0.9801e^{-j2\omega}}$$

MATLAB script for various system responses and plots:

```
b = [1 -1]; a = [1 -0.99 0.9801]; omega = linspace(-pi,pi,1000);
H = freqz(b,a,omega); figure; zplane(b,a);
```




```
figure('Units','inches','Position',[0,0,12,4]);
subplot(1,2,1)
plot(omega/pi,20*log10(abs(H)));
title('Magnitude Response')
ylabel('|H(e^{j\omega})|(dB)')
xlabel('\omega/\pi')
ylim([-52 40])
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
subplot(1,2,2)
plot(omega/pi,angle(H)/pi)
xlabel('\omega/\pi')
ylabel('\angle{H(e^{j\omega})} (Radians)')
title('Phase Response')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
```



Explanation of plots:

The pole-zero plots show that there are two pole locations that have a magnitude of about 0.99 at frequencies $\omega_{1,2} = \pm\frac{\pi}{3}$, which results in the magnitude response have a large and sharp boost at these particular frequencies in the magnitude response plot. There is also a zero that is

present on the unit circle at an angle of $\omega = 0$, which corresponds to a notch at this frequency in the magnitude response. This zero placement

explains the large notch in magnitude when $\omega = 0$, respectively.

Problem 3.6

An ideal highpass filter is described in the frequency-domain by

$$H_d(e^{j\omega}) = \begin{cases} 1 \cdot e^{-j\alpha\omega}, & \omega_c < |\omega| \leq \pi \\ 0, & |\omega| \leq \omega_c \end{cases}$$

where ω_c is the cutoff frequency and α is called the phase delay.

(a) Determine the ideal impulse response $h_d[n]$ using the DTFT synthesis equation.

Solution:

The ideal impulse response $h_d[n]$ can be extracted using the IDTFT equation:

$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

$$h_d[n] = \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} e^{-j\alpha\omega}e^{j\omega n}d\omega + \int_{\omega_c}^{\pi} e^{-j\alpha\omega}e^{j\omega n}d\omega\right]$$

$$h_d[n] = \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} e^{j\omega(n-\alpha)}d\omega + \int_{\omega_c}^{\pi} e^{j\omega(n-\alpha)}d\omega\right]$$

$$h_d[n] = \frac{1}{2\pi}\left(\left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right]_{\omega=-\pi}^{\omega=-\omega_c} + \left[\frac{e^{j\omega(n-\alpha)}}{j(n-\alpha)}\right]_{\omega=\omega_c}^{\omega=\pi}\right)$$

$$h_d[n] = \frac{1}{\pi(n-\alpha)}\left[\frac{e^{-j\omega_c(n-\alpha)} - e^{-j\pi(n-\alpha)} + e^{j\pi(n-\alpha)} - e^{j\omega_c(n-\alpha)}}{2j}\right]$$

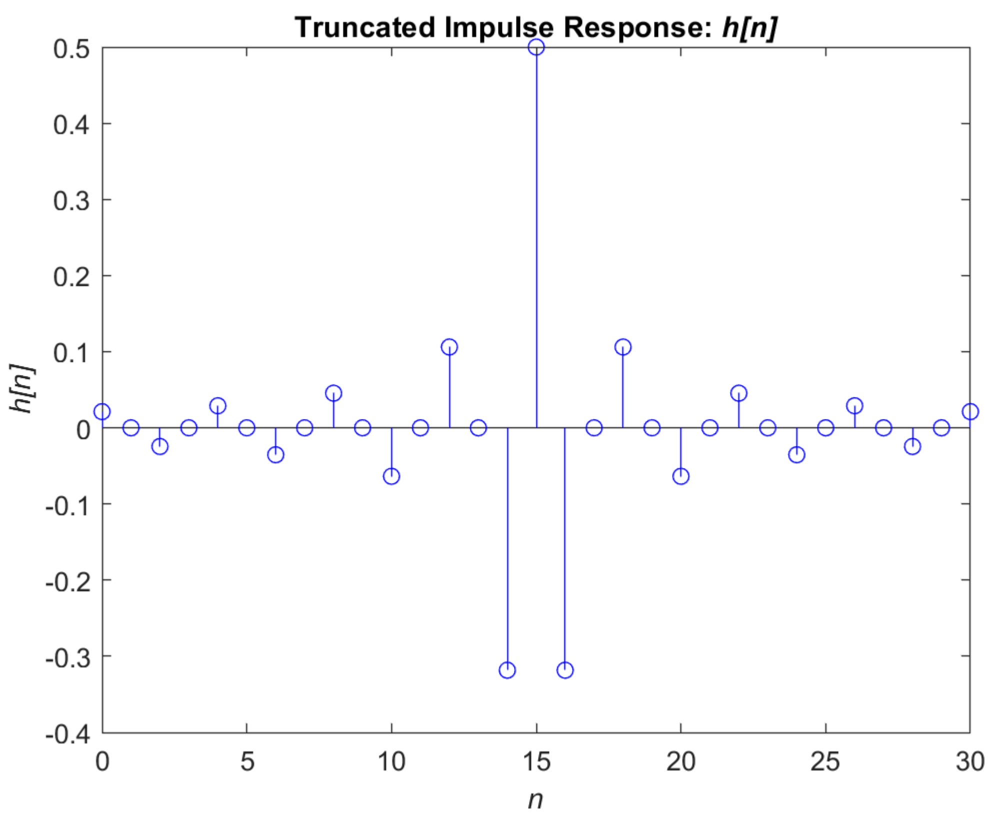
$$h_d[n] = \frac{1}{\pi(n-\alpha)}\left[\frac{e^{j\pi(n-\alpha)} - e^{-j\pi(n-\alpha)}}{2j} - \frac{e^{j\omega_c(n-\alpha)} - e^{-j\omega_c(n-\alpha)}}{2j}\right]$$

$$h_d[n] = \frac{\sin\pi(n-\alpha) - \sin\omega_c(n-\alpha)}{\pi(n-\alpha)}$$

(b) Determine and plot the truncated impulse response $h[n] = \begin{cases} h_d[n], & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$, for $N = 31$, $\alpha = 15$, and $\omega_c = 0.5\pi$.

MATLAB script for computation and plot:

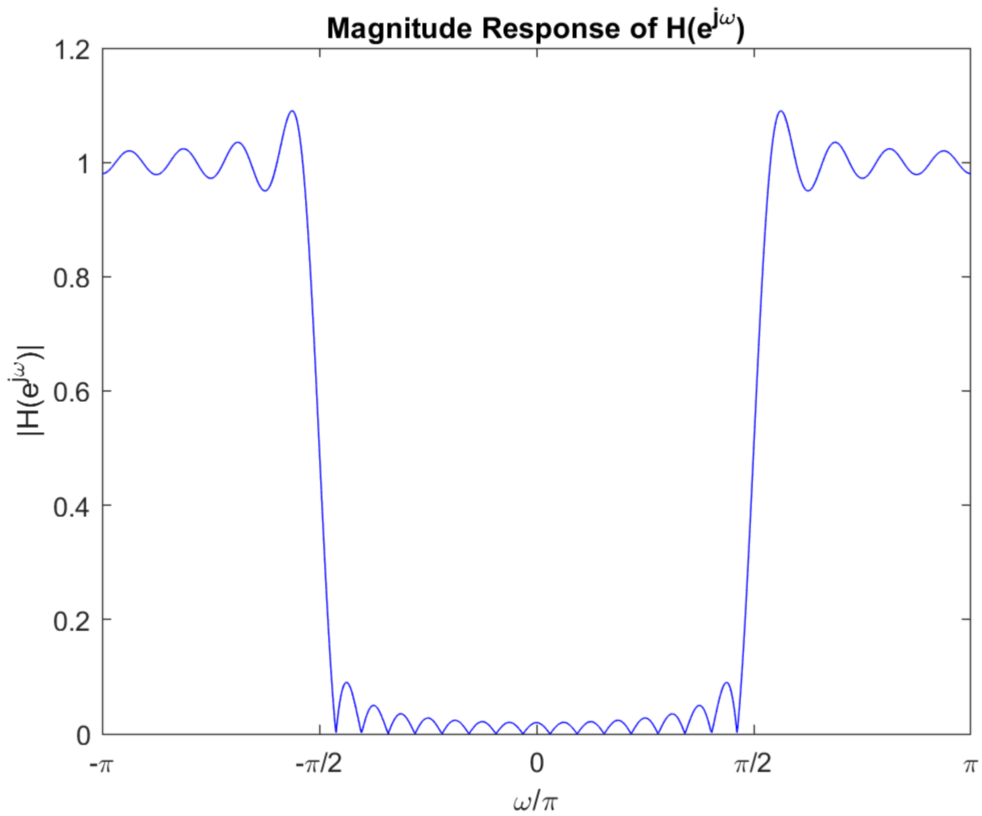
```
clc; close all; clear;
N = 31; alpha = 15; wc = 0.5*pi; n = 0:N-1;
a = sin((pi).*(n-alpha)); b = sin(wc.*(n-alpha)); c = pi.*(n-alpha);
h = (a-b)./(c); h(n==alpha)=(pi-wc)/pi;
figure
stem(n,h,'b')
xlabel('\it{n}')
ylabel('\it{h[n]}')
title('Truncated Impulse Response: \it{h[n]}')
```



(c) Determine and plot the frequency response function $H(e^{j\omega})$, and compare it with the ideal highpass filter response $H_d(e^{j\omega})$.

MATLAB script:

```
omega = linspace(-pi,pi,1000); H = freqz(h,1,omega);
figure
plot(omega/pi,abs(H),'b');
title('Magnitude Response of H(e^{j\omega})')
ylabel('|H(e^{j\omega})|')
xlabel('\omega/\pi')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
```



Comment on your observations:

We can observe that the frequency response function $H(e^{j\omega})$ is experiencing a phenomenon called the Gibb's phenomenon in which the Fourier sums overshoot at a jump discontinuity which results in the rippling effect in the magnitude response. This realized highpass filter cannot have the clear and distinct cutoff boundaries such as those in the ideal highpass filter $H_d(e^{j\omega})$. Since the cutoff frequency is $\omega_c = \pm\frac{\pi}{2}$ our realized highpass filter has a transition band that slowly rises/falls at the cutoff frequency, instead of having a clear and sharp frequency cut that occurs in the ideal highpass filter. The ideal highpass filter would require an infinite amount of harmonics to eliminate the Gibb's phenomenon, which makes it impractical for real-world applications.

Problem 3.7

Text Problem 6.22 part (b) (Page 346)

Signal $x_c(t) = 3 + 2\sin(16\pi t) + 10\cos(24\pi t)$ is sampled at a rate of $F_s = 20$ Hz to obtain the discrete-time signal $x[n]$.

```
clc; close all; clear;
```

(i) Determine the spectra $X(e^{j\omega})$ of $x[n]$.

Solution:

The CTFT of our original signal, $x_c(t)$ is described as

$$X_c(j2\pi F) = 3\delta - j\delta(F+8) + j\delta(F-8) + 5\delta(F+12) + 5\delta(F-12)$$

Our new sampled signal, can be obtained by

$$x[n] = x_c(nT) = x_c(n/F) = 3 + 2\sin(16\pi n/20) + 10\cos(24\pi n/20)$$

$$= 3 + 2\sin(0.8\pi n) + 10\cos(1.2\pi n) = 3 + 2\sin(0.8\pi n) + 10\cos(2\pi n - 0.8\pi n)$$

$$= 3 + 2\sin(0.8\pi n) + 10\cos(-0.8\pi n) \rightarrow 3 + 2\sin(0.8\pi n) + 10\cos(0.8\pi n)$$

Thus, $x[n] = 3 + 2\sin(0.8\pi n) + 10\cos(0.8\pi n)$

The spectra $X(e^{j\omega})$ of $x[n]$ can be obtained from the equation:

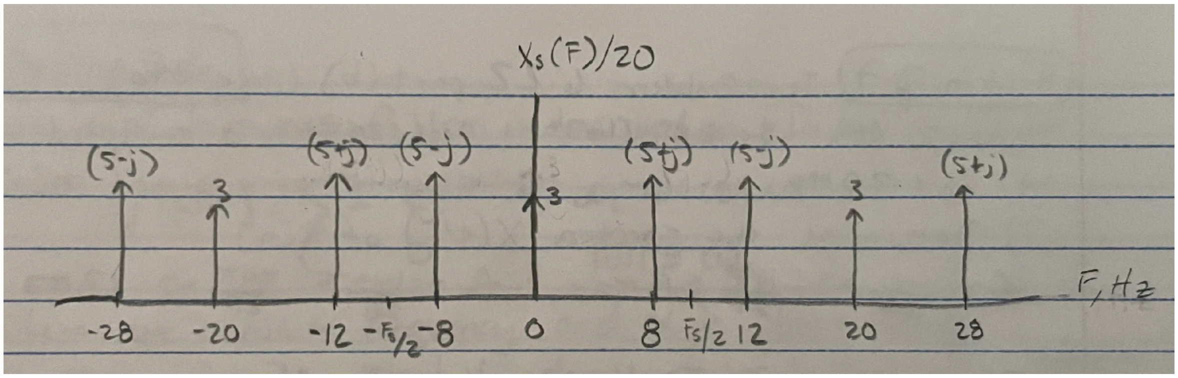
$$X_s(F) = F_s \sum X_c(F - kF_s) = 20 \sum X_c(F - 20k)$$

$$= 20 \sum [3\delta - j\delta(F + 8 - 20k) + j\delta(F - 8 - 20k) + 5\delta(F + 8 - 20k) + 5\delta(F - 8 - 20k)]$$

$$= 20 \sum [3\delta + (5 - j)\delta(F + 8 - 20k) + (5 + j)\delta(F - 8 - 20k)]$$

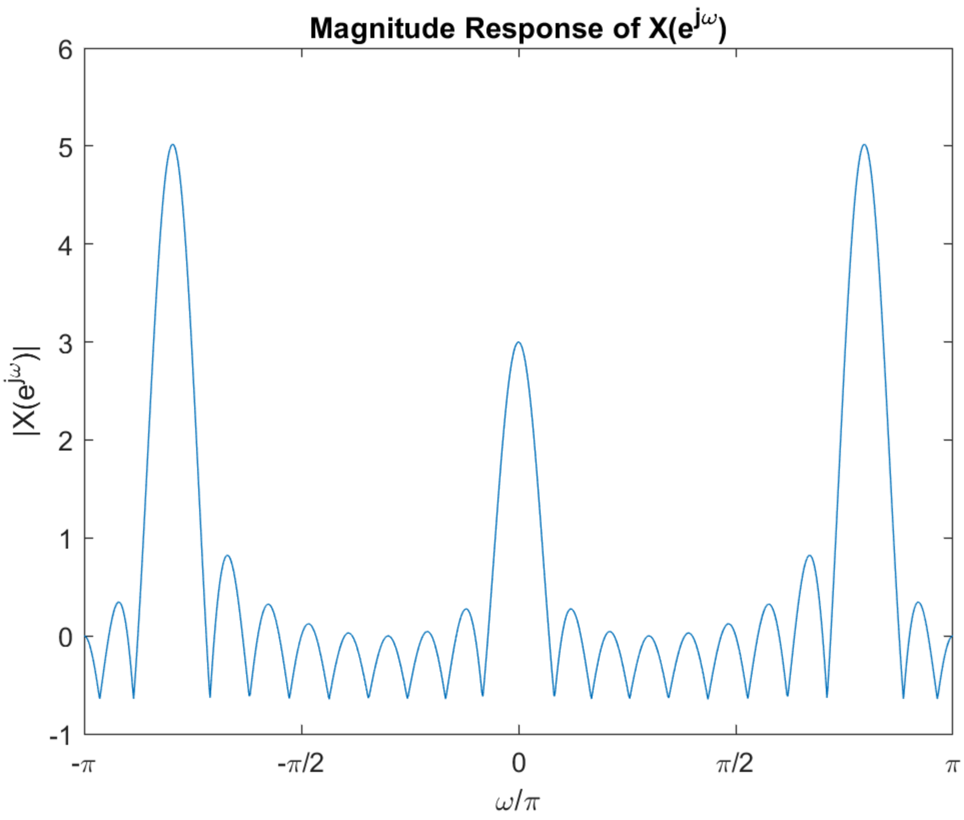
(ii) Plot magnitude of $X(e^{j\omega})$ as a function of ω in $\frac{\text{rad}}{\text{sam}}$ and as a function of F in Hz.

Solution: You can hand plot or use a plotting app or use MATLAB for the magnitude plot.



MATLAB script for the magnitude plot: You can use MATLAB to numerically compute and plot the magnitude $|X(e^{j\omega})|$ over $-\pi < \omega \le \pi$. The amplitudes will be approximate since we cannot synthesize a true impulse but should be proportional to the areas under the impulses. Therefore, plot normalized magnitudes. The important issues here are the location of spikes and their proportional heights.

```
F = 20; Ts = 1/20; n = -10:10; w1 = 16*pi; w2 = 24*pi;
% Discrete Time Signal
x = 3 + 2*sin(w1*(n*Ts)) + 10*cos(w2*(n*Ts));
% Discrete-time Fourier Transform
K = 500; k = 0:1:K; w = pi*k/K;
X = x * exp(-1i*n'*w); X = real(X);
w = [-fliplr(w), w(2:K+1)]; X = [fliplr(X), X(2:K+1)];
figure
plot(w/pi,abs(X/F)-0.65)
title('Magnitude Response of X(e^{j\omega})')
ylabel('|X(e^{j\omega})|')
xlabel('\omega/\pi')
xticklabels({'-\pi','-\pi/2','0','\pi/2','\pi'});
```



(iii) Explain if $x_c(t)$ can be recovered from $x[n]$.

Explanation:

If we were to derive a reconstructed signal $y_c(t)$ from the above figure after lowpass filtering, we would obtain

$y_c(t) = 3 + 2\sin(16\pi t) + 10\cos(16\pi t) \neq x_c(t)$

Because our sampling frequency, $F_s = 20Hz$ did not meet the Nyquist rate of twice the highest frequency component, the reconstructed signal experienced aliasing in the $10\cos(24\pi t)$ component and altered the original signal after sampling. If the sampling frequency was $F_s > 24Hz$ we would have been able to recover $x_c(t)$ from our sample signal $x[n]$.

Problem 3.8

Text Problem 6.26 (Page 347)

Consider a continuous-time signal $x_c(t) = 3\cos(2\pi F_1 t + 45^{\text{deg}}) + 3\sin(2\pi F_2 t)$. It is sampled at $t = 0.001n$ to obtain $x[n]$ which is then applied to an ideal DAC to obtain another continuous-time signal $y_r(t)$.

```
clc; close all; clear;
```

(a) For $F_1 = 150$ Hz and $F_2 = 400$ Hz, determine $x[n]$ and graph its samples along with the signal $x_c(t)$ in one plot (choose few cycles of the $x_c(t)$ signal).

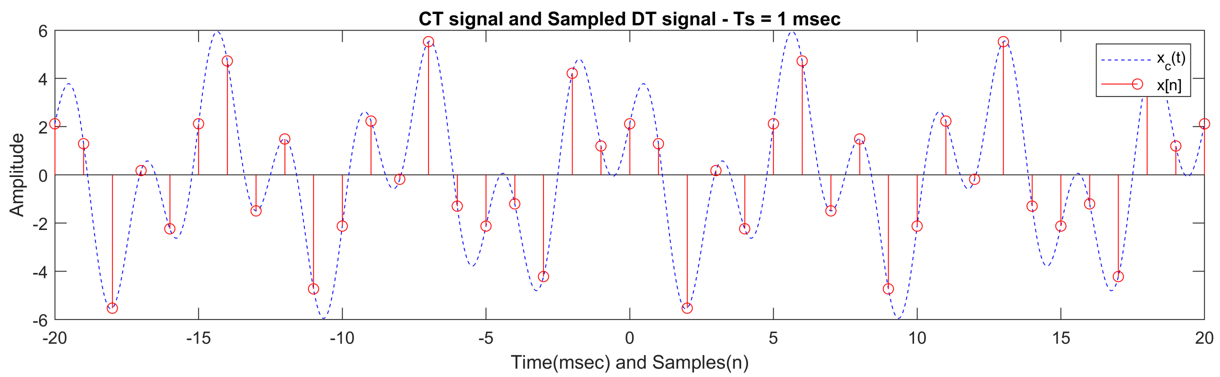
Solution:

$$x_c(t) = 3\cos(300\pi t + \pi/4) + 3\sin(800\pi t)$$

$$x[n] = x_c(nT) = 3\cos\left(\frac{300\pi n}{1000} + \frac{\pi}{4}\right) + 3\sin\left(\frac{800\pi n}{1000}\right)$$

$$x[n] = 3\cos\left(0.3\pi n + \frac{\pi}{4}\right) + 3\sin(0.8\pi n)$$

```
F1 = 150; F2 = 400; w1 = 2*pi*F1; w2 = 2*pi*F2;
Fs = 1000; Ts = 1/Fs; n = -20:20;
% Continuous-time Signal
Dt = 0.00001; t = -0.02:Dt:0.02; xc = 3.*cos(w1*t + pi/4) + 3*sin(w2*t);
% Discrete Time Signal
xn = 3.*cos(w1*(n*Ts) + pi/4) + 3*sin(w2*(n*Ts));
figure('Units','inches','Position',[0,0,12,3]);
plot(t*1000,xc,'--b'), hold on, stem(n*Ts*1000,xn,'r')
xlabel('Time(msec) and Samples(n)')
ylabel('Amplitude')
title('CT signal and Sampled DT signal - Ts = 1 msec'), legend('x_c(t)','x[n]')
```



(b) Determine $y_r(t)$ for the above $x[n]$ as a sinusoidal signal. Graph and compare it with $x_c(t)$.

Solution:

$$y_r(t) = x[n]|_{n=tFs} = x[n]|_{n=1000t}$$

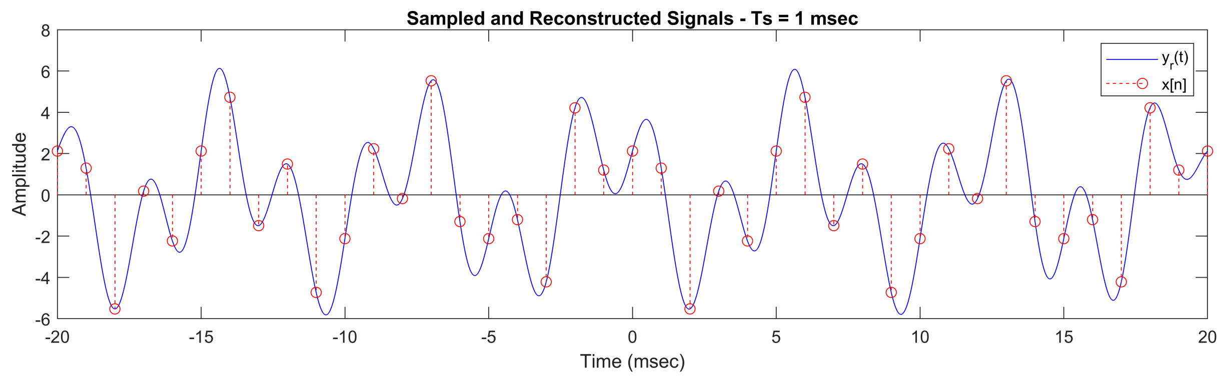
$$y_r(t) = 3\cos(0.3\pi(1000t) + \pi/4) + 3\sin(0.8\pi(1000t))$$

$$y_r(t) = 3\cos(300\pi t + \pi/4) + 3\sin(800\pi t) = x_c(t)$$

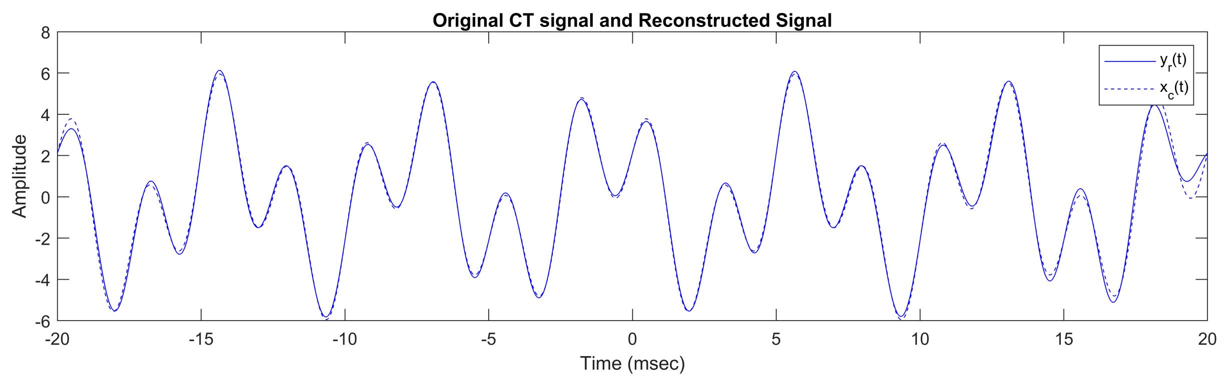
Thus, no aliasing occurred during the reconstruction process.

MATLAB script:

```
Fs = 1000; Ts = 1/Fs; n = -20:20; t= -0.02:Dt:0.02; nTs = n*Ts;
% Reconstructed Signal
yr = xn * sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
figure('Units','inches','Position',[0,0,12,3]);
plot(t*1000,yr,'b'), hold on, stem(n,xn,'--r')
xlabel('Time (msec)')
ylabel('Amplitude')
title('Sampled and Reconstructed Signals - Ts = 1 msec'), legend('y_r(t)','x[n]')
```




```
figure('Units','inches','Position',[0,0,12,3]);
plot(t*1000,yr,'b'), hold on, plot(t*1000,xc,'--b')
xlabel('Time (msec)')
ylabel('Amplitude')
title('Original CT signal and Reconstructed Signal'), legend('y_r(t)','x_c(t)')
```



(c) Repeat (a) and (b) for $F_1 = 300$ Hz and $F_2 = 700$ Hz.

Solution:

$$x_c(t) = 3\cos(600\pi t + \pi/4) + 3\sin(1400\pi t)$$

$$x[n] = x_c(nT) = 3\cos\left(\frac{600\pi n}{1000} + \frac{\pi}{4}\right) + 3\sin\left(\frac{1400\pi n}{1000}\right)$$

$$x[n] = 3\cos\left(0.6\pi n + \frac{\pi}{4}\right) + 3\sin(1.4\pi n)$$

$$x[n] = 3\cos\left(0.6\pi n + \frac{\pi}{4}\right) + 3\sin(2\pi n - 1.4\pi n) \rightarrow 3\sin(2\pi n - 1.4\pi n) = 3\sin(-0.6\pi n) \rightarrow -3\sin(0.6\pi n)$$ (odd symmetry)

$$x[n] = 3\cos\left(0.6\pi n + \frac{\pi}{4}\right) - 3\sin(0.6\pi n)$$

The high frequency component $F_2 = 700$ Hz experiences aliasing because $F_s \neq F_s > 2F_2$

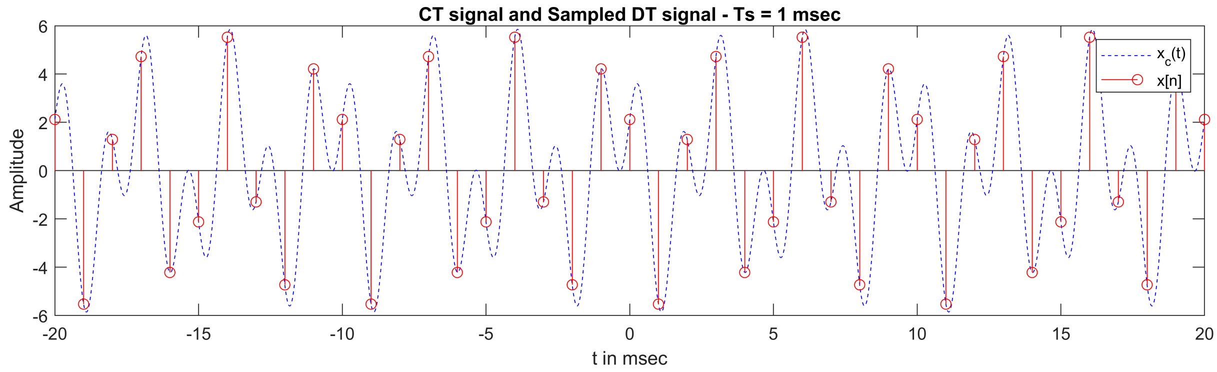
$$y_r(t) = x[n]|_{n=tFs} = x[n]|_{n=1000t}$$

$$y_r(t) = 3\cos(0.6\pi(1000t) + \pi/4) - 3\sin(0.6\pi(1000t))$$

$$y_r(t) = 3\cos(600\pi t + \pi/4) - 3\sin(600\pi t) \neq x_c(t)$$

Thus, aliasing occurred during the reconstruction process because of the sampling rate not meeting the Nyquist rate.

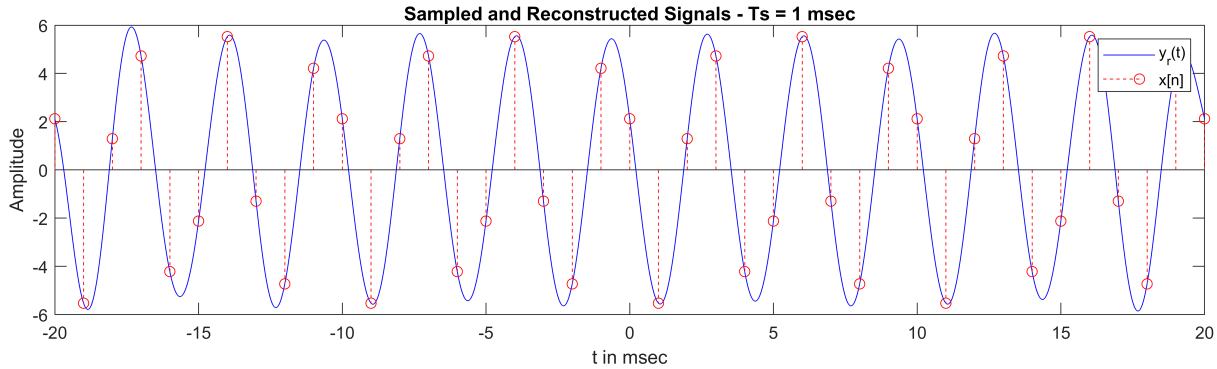
```
F1 = 300; F2 = 700; w1 = 2*pi*F1; w2 = 2*pi*F2;
Fs = 1000; Ts = 1/Fs; n = -20:20; nTs = n*Ts;
% Continuous-time Signal
Dt = 0.00001; t = -0.02:Dt:0.02; xc = 3.*cos(w1*t + pi/4) + 3*sin(w2*t);
% Discrete Time Signal
xn = 3.*cos(w1*(n*Ts) + pi/4) + 3*sin(w2*(n*Ts));
figure('Units','inches','Position',[0,0,12,3]);
plot(t*1000,xc,'--b'), hold on, stem(n*Ts*1000,xn,'r')
xlabel('t in msec')
ylabel('Amplitude')
title('CT signal and Sampled DT signal - Ts = 1 msec'), legend('x_c(t)','x[n]')
```



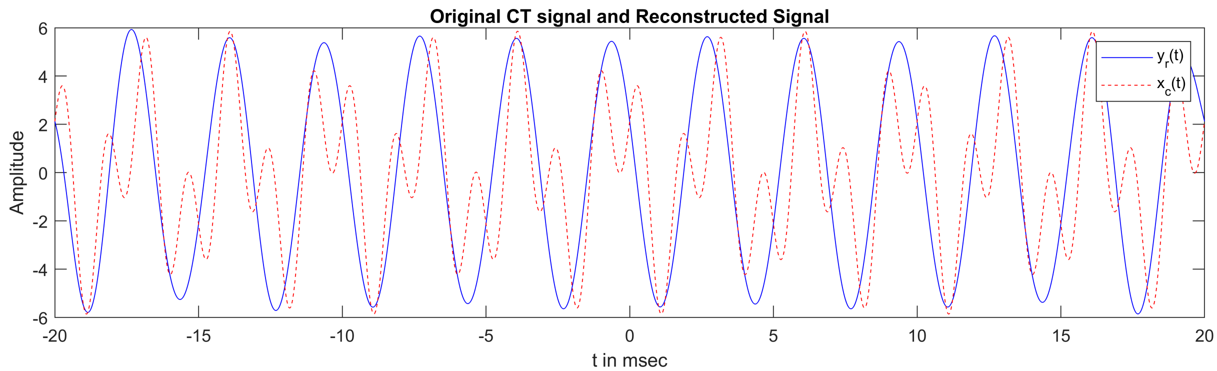
MATLAB script:

```
% Reconstructed Signal
yr = xn * sinc(Fs*(ones(length(n),1)*t-nTs'*ones(1,length(t))));
figure('Units','inches','Position',[0,0,12,3]);
plot(t*1000,yr,'b'), hold on, stem(n,xn,'--r')
xlabel('t in msec')
ylabel('Amplitude')
```

```
title('Sampled and Reconstructed Signals - Ts = 1 msec'), legend('y_r(t)','x[n]')
```




```
figure('Units','inches','Position',[0,0,12,3]);
plot(t*1000,yr,'b'), hold on, plot(t*1000,xc,'--r')
xlabel('t in msec')
ylabel('Amplitude')
title('Original CT signal and Reconstructed Signal'), legend('y_r(t)','x_c(t)')
```



Comment on your results:

Analyzing both the reconstructed plots, for parts (a) and (b), the sampling frequency $F_s = 1000$ satisfied the Nyquist rate, which was able to recover the original continuous signal after reconstruction. However, for parts (c) and (d), the sampling rate was not twice the value of the highest frequency component $F_2 = 700Hz$, which resulted in aliasing after reconstructing the signal from the sampled signal. The higher frequencies being aliased resulted in a lower frequency sinusoid, as seen in the plot above.

Problem 3.9

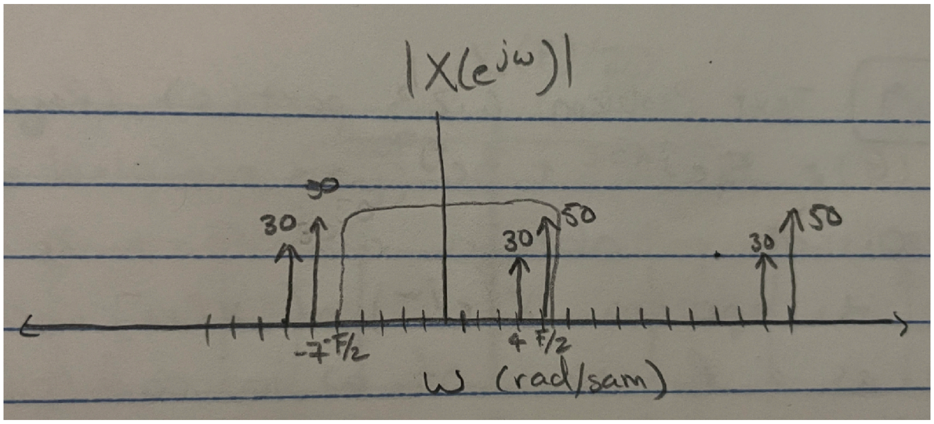
Text Problem 6.23 part (c) (Page 348)

Signal $xc(t) = 5e^{j40t} + 3e^{-j70t}$ is sampled periodically with $T = 0.1$ sec to obtain the discrete-time signal $x[n]$.

Determine the spectra $X(e^{j\omega})$ of $x[n]$ and plot its magnitude as a function of ω in $\frac{\text{rad}}{\text{sam}}$.

```
clc; close all; clear;
```

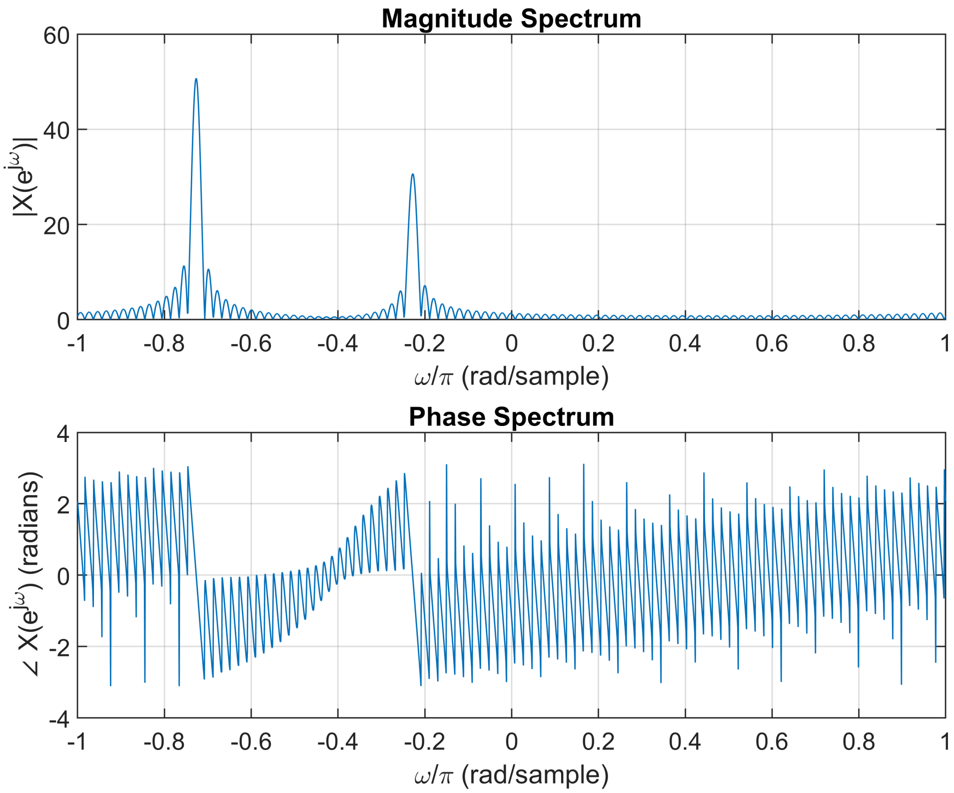
Solution: You can hand plot or use a plotting app or use MATLAB for the magnitude plot.



MATLAB script for the magnitude plot: You can use MATLAB to numerically compute and plot the magnitude $|X(e^{j\omega})|$ over $-\pi < \omega \le \pi$. The amplitudes will be approximate since we cannot synthesize a true impulse but should be proportional to the areas under the impulses. Therefore, plot normalized magnitudes. The important issues here are the location of spikes and their proportional heights.

```
Ts = 0.1; n = 0:100; nTs = n*Ts;
x = 5*exp(40i*nTs) + 3*exp(-70i*nTs);
w = -pi:pi/1000:pi;
X = x*exp(-1i*n'*w);
X_mag = abs(X);
X_phase = angle(X);

subplot(2,1,1)
plot(w/pi, X_mag/10);
grid on
xlabel('\omega/\pi (rad/sample)');
ylabel('|X(e^{j\omega})|');
title('Magnitude Spectrum');
subplot(2,1,2)
plot(w/pi, X_phase);
grid on
xlabel('\omega/\pi (rad/sample)');
ylabel('\angle X(e^{j\omega}) (radians)');
title('Phase Spectrum');
```



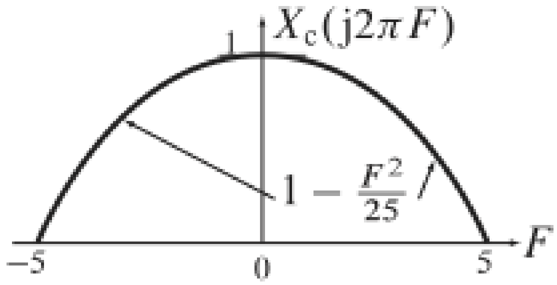
Explain if $x_c(t)$ can be recovered from $x[n]$:

The frequency components of the signal $x_c(t)$ are $F_1 = \dfrac{20}{\pi} \approx 6.4Hz$ and $F_2 = \dfrac{35}{\pi} \approx 11.1Hz$. With our sampling interval being $T_S = 0.1\mathrm{s}$ this means our sampling frequency is $F_s = 10Hz$. So, aliasing will occur during the sampling process and will ultimately lose the F_2 component because the sampling interval is too low. The lower frequency component F_1 will be sampled, but aliasing will occur that will make $x_c(t)$ be impossible to reconstruct from our sample signal $x[n]$.

Problem 3.10

Text Problem 6.37 (Page 348)

Signal $x_c(t)$ with spectra $X(j2\pi F)$ shown below is sampled at a rate of $F_s = 6$ Hz to obtain the discrete-time signal $x[n]$.




```
clc; close all; clear;
```

(i) Determine the spectra $X(e^{j\omega})$ of $x[n]$.

Solution:

First, we see that $X_c(j2\pi F) = 1 - \frac{|F|^2}{25}, \quad \text{for } |F| \leq 5$ and 0, everywhere else.

We can relate the spectra $X_c(j2\pi F)$ to $X_c(j\Omega)$ since $\Omega = 2\pi F$, then using the equation

$X(e^{j\omega})|_{\omega=\Omega T} = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\Omega - j\frac{2\pi}{T}k\right)$ along with our sample interval $F_s = 6Hz \rightarrow T_s = 1/F_s = \frac{1}{6}$

we can approximate our new sampled spectra $X(e^{j\omega})$ using the substitution $\omega = \Omega T$ to get:

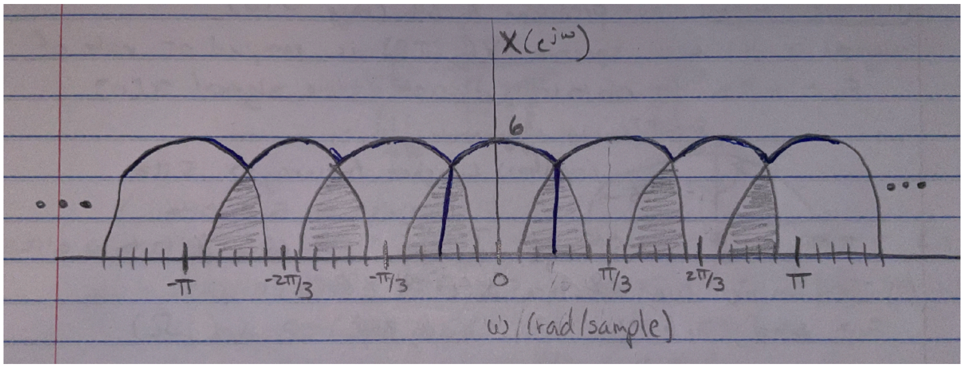
$$X(e^{j\omega}) = 6\sum_{k=-\infty}^{\infty} X_c[j(6\omega - \frac{\pi}{3}k)]$$

where

$$X(e^{j\omega}) = 6 - \frac{|\frac{\omega}{2\pi}|^2}{25}, \quad \text{for } |6\omega| \leq 10\pi$$

(ii) Plot it as a function of ω in rad/sam.

Solution: You can hand plot or use a plotting app or use MATLAB for the magnitude plot.



The plot of $X(e^{j\omega})$ results in an overlapped version of the original spectra. The highlighted rippling top-edges of the plot are the resulting spectrum shape.

(iii) Explain if $x_c(t)$ can be recovered from $x[n]$.

Solution:

We would not be able to recover the original signal $x_c(t)$ from the sampled signal $x[n]$, because the sampling frequency $F_s = 6Hz$ did not meet the Nyquist rate, which should have been $\geq 10Hz$. The original spectra starts to overlap because of the sampling process around $3Hz$, where edges of the semi-circles were summed together and produced a new aliased spectra shown in $X(e^{j\omega})$ above.