EECE-5666 : Midterm-2 Exam : 2022-SPRG

130 Minutes between Tuesday 3:30 pm and 6:30 pm

[40-Points]

Instructions:

1. You are required to read the NU Academic Integrity policy (given below) and sign below that the submitted work is your own work. (This is a COE requirement.)
2. You are required to complete this exam using Live Editor.
3. Use of the equation editor to typeset mathematical material such as variables, equations, etc., is strongly recommended. However, in the interest of time, a properly scanned image of a neatly hand-written fragment can be inserted as your work in the required space.
4. After completing this exam, export this Live script to PDF. If you encounter problems in the direct export to PDF, export to Microsoft Word and then print to PDF.
5. Submit the PDF file only. **DO NOT SUBMIT YOUR LIVE EDITOR (.MLX) FILE, IT WILL NOT BE GRADED**.
6. You should complete this exam in two hours with additional 10 minutes for submission activities.
7. The exam is made available for only one attempt.

Academic Integrity Policy

A commitment to the principles of academic integrity is essential to the mission of Northeastern University. The promotion of independent and original scholarship ensures that students derive the most from their educational experience and their pursuit of knowledge. Academic dishonesty violates the most fundamental values of an intellectual community and undermines the achievements of the entire University.

As members of the academic community, students must become familiar with their rights and responsibilities. In each course, they are responsible for knowing the requirements and restrictions regarding research and writing, examinations of whatever kind, collaborative work, the use of study aids, the appropriateness of assistance, and other issues. Students are responsible for learning the conventions of documentation and acknowledgment of sources in their fields. Northeastern University expects students to complete all examinations, tests, papers, creative projects, and assignments of any kind according to the highest ethical standards, as set forth either explicitly or implicitly in this Code or by the direction of instructors. The full academic integrity policy is available at

http://www.northeastern.edu/osccr/academic-integrity-policy/

Declaration

By signing (i.e., entering my name below) and submitting this exam through the submission portal, I declare that I have read the Academic Integrity Policy and that the submitted work is my own work.

Enter your name (Firstname MI Lastname): Tyler B McKean

If you do not enter your name, 2 points will be deducted.

Default Plot Parameters:

```
set(0,'defaultfigurepaperunits','points','defaultfigureunits','points');
set(0,'defaultaxesfontsize',10);
set(0,'defaultaxestitlefontsize',1.4,'defaultaxeslabelfontsize',1.2);
```

Problem-1 (15-points)   The Discrete Fourier Transform (DFT)

The following two parts, (a) and (b), are not related to each other.

(a) [9-points] Properties of the DFT

Consider a 4-point sequence $x[n] = \{\alpha, \beta, \delta, \gamma\}$ where α, β, δ, and γ are (possibly) complex-valued numbers. Let $X[k]$ be the 6-point DFT of $x[n]$. In the following sub-parts, you must provide analytical justifications to get full credit.

i. [3-points] Determine the finite-length sequence $y[n]$ whose 6-point DFT is

$$Y[k] = W_3^{2k} X[k].$$

Solution:

First we take the 6-point DFT to solve for $X[k]$

$$X[k] = \sum_{n=0}^{5} x[k] W_6^{kn} = \left[\alpha, \beta\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right), \delta\left(\frac{-1}{2} - j\frac{\sqrt{3}}{2}\right), -\gamma, 0, 0\right]$$

Now we take the IDFT of the sequence $Y[k]$ to obtain the sequence $y[n]$

$y[n] = \frac{1}{N}\sum_{k=0}^{5} Y[k] W_6^{-kn} = \frac{1}{N}\sum_{k=0}^{5} W_3^{2k} X[k] W_6^{-kn} = \left[\alpha, \beta W_3^1 W_3^2, \delta W_3^2 W_3^4, \gamma, W_3^3, 0, 0\right]$ after combing twiddle factors we get

$$Y[k] = [\alpha W_6^0, \beta W_6^5, \delta W_3^2, \gamma W_6^3, 0, 0] = [\alpha, \beta\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right), \delta\left(\frac{-1}{2} + j\frac{\sqrt{3}}{2}\right), -\gamma, 0, 0]$$

Then Taking the IDFT we get

$$y[n] = [\alpha, \beta\left(\frac{-1}{2} + j\frac{\sqrt{3}}{2}\right), \delta, \gamma, 0, 0]$$

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ii. [3-points] Determine the finite-length sequence $w[n]$ whose 6-point DFT is

$$W[k] = X[\langle k+3\rangle_6].$$

Solution:

$$W[k] = \left[-\gamma, 0, 0, \alpha, \beta\left(\frac{1}{2} - j\frac{\sqrt{3}}{2}\right), \delta\left(\frac{-1}{2} - j\frac{\sqrt{3}}{2}\right)\right]$$

---

iii. [4-points] Determine the finite-length sequence $v[n]$ whose 3-point DFT is

$$V[k] = X[2k], \quad k = 0, 1, 2.$$

Solution:

$$v[n] = [\alpha, \delta W_3^2 W_3^{-1}, 0] = \left[\alpha, \delta\left(\frac{-1}{2} - j\frac{\sqrt{3}}{2}\right), 0\right]$$

(b) [6-points] Let $X[k]$ be an N-point DFT of an N-point sequence $x[n]$.

i. [3-Points] Using the analysis and synthesis equations of the DFT, show that the energy of an N-point sequence satisfies

$$\mathscr{E}_x \triangleq \sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X[k]|^2.$$

Solution:

---

ii. [3-Points] Using MATLAB, verify the above relation on the following 9-point sequence

$$x[n] = \{\underset{\uparrow}{1}, 2, 3, 4, 5, 4, 3, 2, 1\}.$$

MATLAB script:

```
clc; close all; clear;
x = [1,2,3,4,5,4,3,2,1]; N = 9;
sum(abs(x(:).^2))
```

```
ans = 85
```

```
X = fft(x,9); Xe = 1/N*sum(abs(X(:).^2))
```

```
Xe = 85.0000
```

Thus the energy is equivalent for the follwoing above relation

---

Problem-2 (13-Points) The Fast Fourier Transform (FFT)

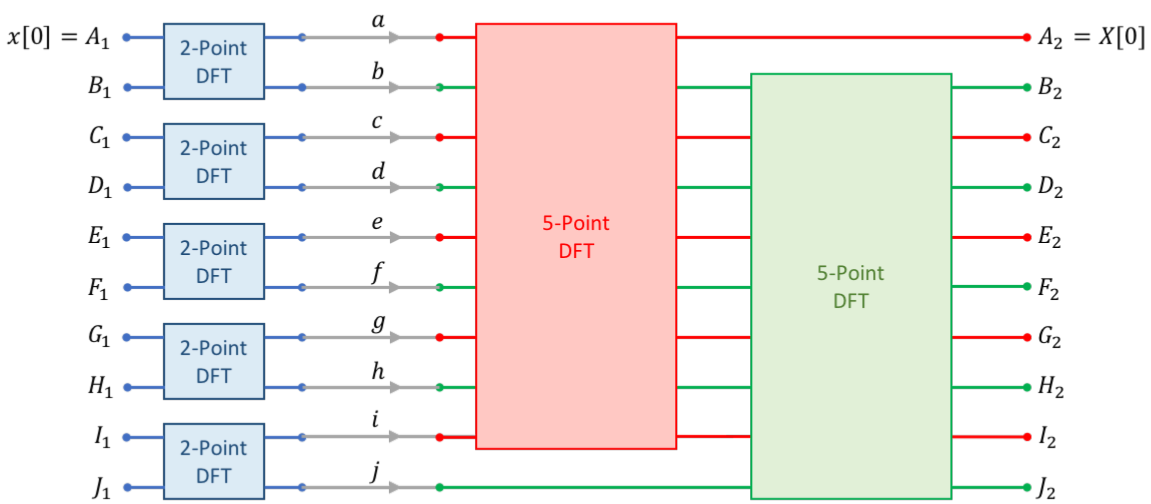
In this problem we will consider a mixed-radix fast Fourier transform algorithm for an $N = 10$ point DFT.

(a) [3-Points] Develop a FFT algorithm which computes five radix-2 DFTs followed by twiddle-factor merging and two radix-5 DFTs. That is, obtain a formula similar to equation (8.50) in the textbook for this 10-point DFT with $N = N_1 N_2$ where $N_1 = 2$ and $N_2 = 5$.

Solution:

$X[n_2 a + b] = \sum_{n=0}^{4}\left[\sum_{m=0}^{1} x[n_1 n + m] W_2^{an}\right] W_{10}^{bm} W_5^{mb}$ for $a = 0, 1, \ldots, 4$, $b = 0, 1$ and $n_1 = 0, 1$ and $n_2 = 0, 1, \ldots, 4$

(b) [6-points] The signal flow graph (SFG) for the formula in $(2a.1)$ can be drawn as follows.



Answer the following subparts based on the above SFG.

i. [2-Point] Determine the input signal samples in the nodes D_1 and G_1. For example, node A_1 has input sample $x[0]$.

Solution:

Enter your answers below by clicking next to the $=$ symbol:

$D_1 = x[6]$, $G_1 = x[3]$.

ii. [2-Point] Determine twiddle factors d and h.

Solution:

Enter your answers below by clicking next to the $=$ symbol:

$d = W_5^1 = 0.3090 - j0.9511$, $h = W_5^3 = -0.8090 + j0.5878$.

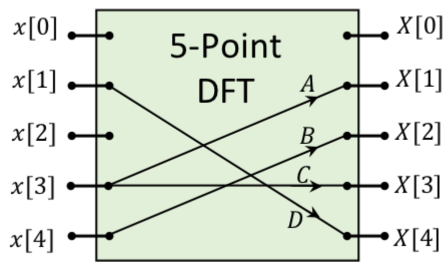
iii. [2-Point] Determine output signal samples in C_2 and H_2. For example, node A_2 has output sample $X[0]$.

Solution:

Enter your answers below by clicking next to the $=$ symbol:

$C_2 = X[2]$, $H_2 = X[7]$.

(c) [4-Points] The following diagram shows a partial SFG (to avoid clutter) for the butterfly in radix-5 (i.e., 5-point) DFT. Determine the reduced twiddle factors A, B, C, and D. For example, if the twiddle factor is W_{10}^{12}, then the reduced twiddle factor is $W_{10}^{10+2} = W_{10}^{10}W_{10}^{2} = W_{10}^{2}$.



Solution:

Enter your answers below by clicking next to the $=$ symbol:

$A = W_{10}^{6}$,

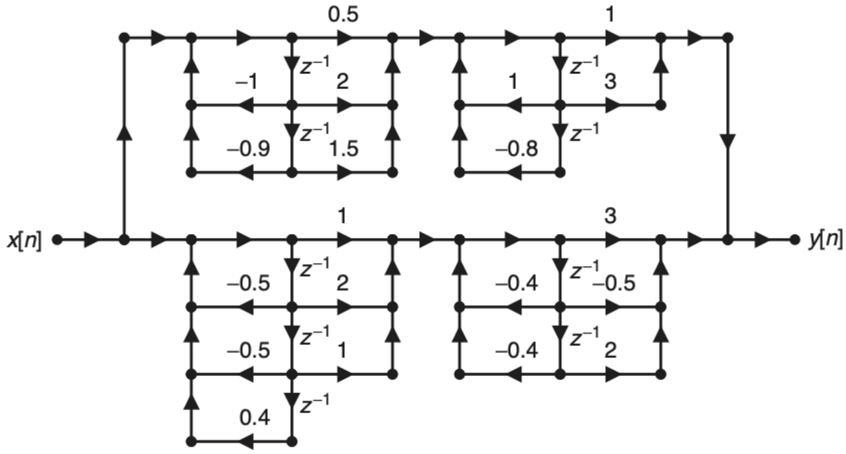
$B = -W_{10}^{8}$,

$C = -W_{10}^{6}$, and

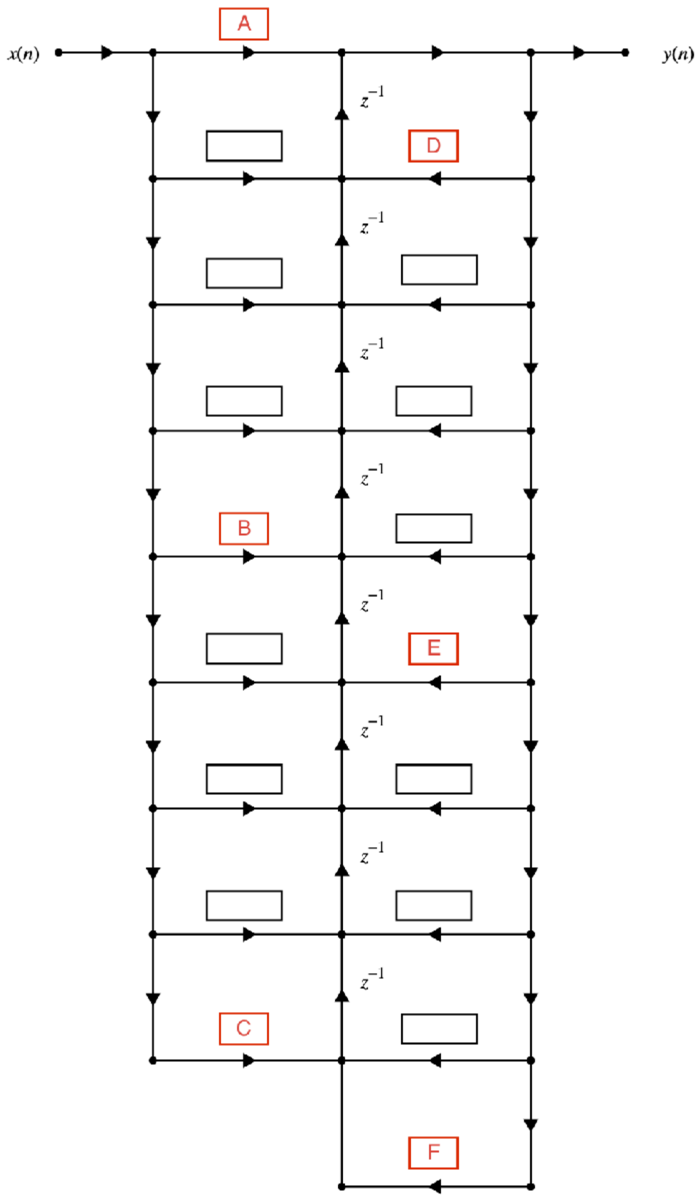
$D = W_{10}^{1}$.

Problem-3 (12-Points) Digital Filter Structures

The filter structure shown below contains a parallel connection of cascade sections.



(a) [6-Points] The entire filter is to be implemented using a signal flow graph of the form shown below:



The coefficient not shown or labeled are equal to 1. The white rectangular boxes with black borders indicate hidden coefficients. Determine the coefficients A through F hidden behind the red boxes in the above signal flow graph.

MATLAB script for determination:

```
clear; close all;
G1 = 1; sos1 = [1 3 0 1 -1 0.8; 0.5 2 1.5 1 1 0.9]; [b1,a1] = sos2tf(sos1,G1)
```

```
b1 = 1×4
    0.5000    3.5000    7.5000    4.5000
a1 = 1×5
    1.0000         0    0.7000   -0.1000    0.7200
```

```
B1 = [1 3 0; 0.5 2 1.5]; A1 = [1 -1 0.8; 1 1 0.9];
[b1,a1] = cas2dir(1,B1,A1), a1 = [a1,0];
```

```
b1 = 1×5
    0.5000    3.5000    7.5000    4.5000         0
a1 = 1×5
    1.0000         0    0.7000   -0.1000    0.7200
```

```
B2 = [1 2 1]; A2 = [1 0.5 0.5 -0.4];
B3 = [3 -0.5 2]; A3 = [1 0.4 0.4];
B = conv(B2,B3); A = conv(A2,A3);
[b2,a2] = cas2dir(1,B,A)
```

```
b2 = 1×5
    3.0000    5.5000    4.0000    3.5000    2.0000
a2 = 1×6
    1.0000    0.9000    1.1000         0    0.0400   -0.1600
```

```
B4 = conv(b1,a2) + conv(b2,a1)
```

```
B4 = 1×10
    3.5000    9.4500   17.3000   22.1500   18.7300   11.0200    3.6700    1.3000 ···
```

```
A4 = conv(a1,a2)
```

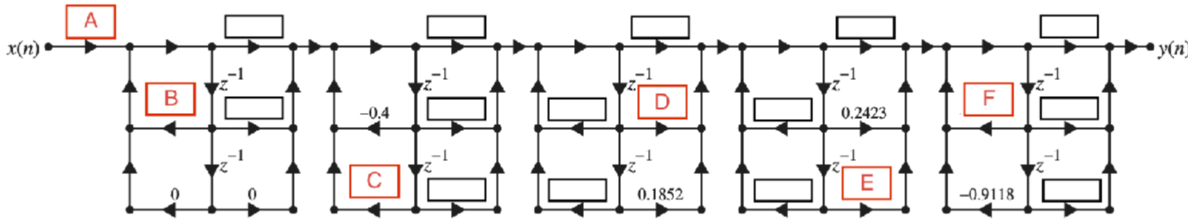
```
A4 = 1×11
    1.0000    0.9000    1.8000    0.5300    1.4400    0.3780    0.8200   -0.1160 ···
```

Enter your coefficient values (up to 4 decimals) below by clicking next to the $=$ symbol:

$A = 3.5$, $B = 18.73$, $C = 0.72$,

$D = -0.9$, $E = -0.378$, $F = 0.1152$.

(b) [6-Points] The entire filter is to be implemented using a signal flow graph of the form shown below:



The coefficient not shown or labeled are equal to 1. The white rectangular boxes with black borders indicate hidden coefficients. Determine the coefficients A through F hidden behind the red boxes in the above signal flow graph.

MATLAB script for determination:

```
[sos,G] = tf2sos(B4,A4)
```

```
sos = 5×6
    1.0000    1.0000         0    1.0000   -0.4387         0
    1.0000    0.7486         0    1.0000    0.4000    0.4000
    1.0000   -0.3323    0.1852    1.0000   -1.0000    0.8000
    1.0000    0.2423    1.5819    1.0000    1.0000    0.9000
    1.0000    1.0414    0.9377    1.0000    0.9387    0.9118
G = 3.5000
```

Enter your coefficient values (up to 4 decimals) below by clicking next to the $=$ symbol:

$A =$ 3.5 , $B = 0.4387$, $C = -0.4$,

$D =$ -0.3323 , $E =$ 1.5819 , $F =$ -0.9387 .