

# Offline Goal Conditioned Reinforcement Learning with Temporal Distance Representations

Anonymous Author(s)

Affiliation

Address

email

## Abstract

1 Learned successor features provide a powerful framework for learning goal-  
2 reaching policies. These representations are constructed such that similarity in  
3 the representation space predicts future outcomes, allowing goal-reaching policies  
4 to be extracted. Representations learned for forward inference have some practical  
5 limitations — stitching of behaviors does not arise naturally with forward  
6 objectives like contrastive classification, and additional regularization is required  
7 to enable valid policy extraction. In this work, we propose a new representation  
8 learning objective that enables extraction of goal-reaching policies. Our key insight  
9 is that rather than learning representations of the future, we should really learn  
10 representations that can associate outcomes with preceding states. We show that  
11 when combined with existing quasimetric network parameterization and the right  
12 invariances, these representations let us learn optimal goal-reaching policies from  
13 offline data. On existing offline GCRL benchmarks, the hindsight classification  
14 objective improves performance with a simpler algorithm and fewer independent  
15 networks/parameters to learn relative to past methods.

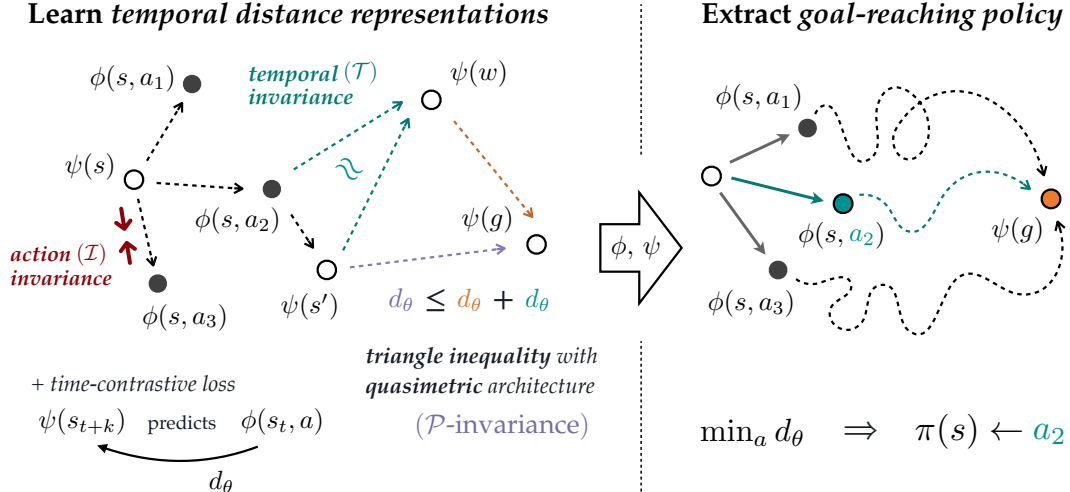


Figure 1: (Left) TMD learns a temporal distance  $d_\theta$  that satisfies the triangle inequality and action invariance. It does this by minimizing the distance between the learned distance and the distance between the successor features of the states and actions in the dataset. (Right) The learned distance is used to extract a goal-conditioned policy.

# 1 Introduction

Learning temporal distances lies at the heart of many important problems in both control theory and reinforcement learning. In control theory, such distances form important Lyapunov functions [1] and control barrier functions [2], and are at the core of reachability analysis [3] and safety filtering [4]. In reinforcement learning (RL), such distances are important not just for safe RL [5], but also for forming value functions in tasks ranging from navigation [6] to combinatorial reasoning [7] to robotic manipulation [8, 9]. Ideally, these learned distances have two important properties: (i) they can encode paths that are shorter than those demonstrated in the data (i.e., stitching); and (i) then can capture long-horizon distances with low variance.

Today, when users are selecting a method for learning temporal distances, they typically have to decide which of these desiderata they care more about and forgo the other. Methods based on Q-learning [10, 11, 12] use TD learning to stitch trajectories and find shortest paths, yet TD learning results in compounding errors that make it challenging to apply to long-horizon tasks [13]. In contrast, Monte Carlo methods [14] can directly learn to estimate the distances between temporally-separated states, yet their ability to find *shortest* paths remains limited.

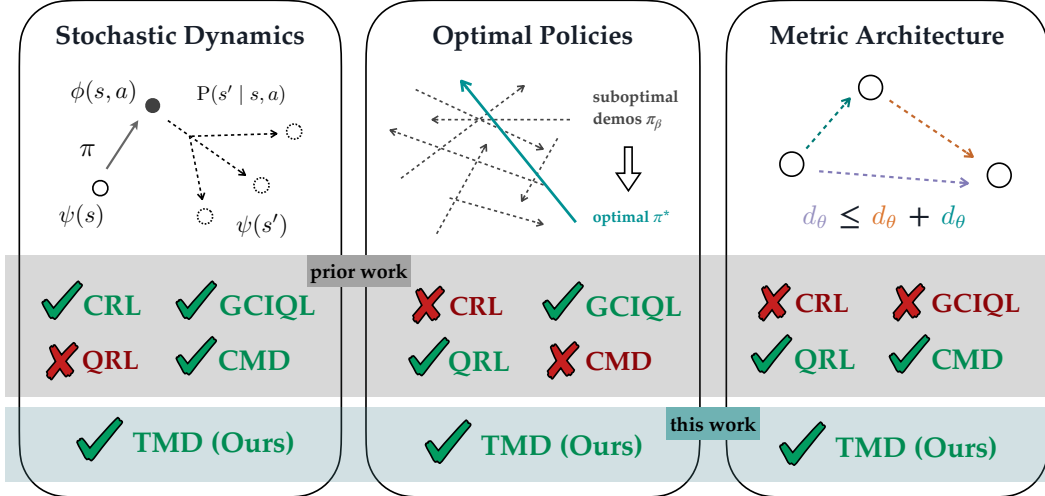


Figure 2: TMD enables key capabilities over prior work: (Left) handling stochastic transition dynamics, (Center) learning optimal policies from offline data, and (Right) stitching behaviors as a property of network architecture.

The aim of this paper is to build a method for learning temporal distances that retains the long-horizon estimation capabilities of Monte Carlo methods but nonetheless is able to compute shortest paths. We take an invariance perspective to do this. Temporal distances satisfy various invariance properties. Because they are value functions, they satisfy the Bellman equations. Prior work has also shown that they satisfy the triangle inequality, even in stochastic settings [15, 16]. The triangle inequality, also a form of invariance [17], is powerful because it lets us architecturally winnow down the hypothesis space of temporal distances by only considering neural network architectures that satisfy the triangle inequality [18, 19]. Importantly, the fact that temporal distances satisfy the triangle inequality holds for *any* temporal distance, including both optimal temporal distances and those learned by Monte Carlo methods. This raises an important question: might there be an *additional* invariance that is satisfied by optimal temporal distances, but not those learned by Monte Carlo methods? Identifying such invariance properties that would enable us to use Monte Carlo methods to architecturally winnow the hypothesis space, and then use this additional invariance property to identify optimal temporal distances within that space.

The key contribution of this paper is a method for learning temporal distances that relies primarily on Monte Carlo learning, but nonetheless provably converges to optimal shortest paths. To enable this, we identify two additional invariance properties that apply at the *transition* level. These invariance properties are structurally similar to the Bellman equations, but importantly forgo the need to define reward functions. We translate these invariance properties into a practical method for

learning temporal distances. We demonstrate an application of those distances to goal-conditioned RL tasks. On benchmark tasks up to 21-dimensions as well as visual observations, we demonstrate that our method achieves results that considerably outperforms that of similar baselines. Additional experiments reveal the importance of the enforced invariances and contrastive learning objective. Given the importance of long-horizon reasoning in many potential applications of RL today, we believe our work is useful for thinking about how to learn optimal temporal distances.

## 2 Related Work

We provide a unifying framework that connects metric learning to (optimal) offline goal-conditioned reinforcement learning (GCRL).

### 2.1 Metric Learning

We build on prior approaches to learning *temporal* distances, which reflect the reachability of states [16]. Temporal distances are usually defined as the expected number of time steps to transit from one state to another [20, 21]. Recent work has provided probabilistic definitions that are also compatible with continuous state spaces and stochastic transition dynamics [16]. A key consideration when thinking about temporal distance is *which* policy they reflect: is this an estimate of the number of time steps under our current policy or under the optimal policy? We will use *optimal temporal distance* to mean the temporal distance under the optimal (distance-minimizing) policy. Algorithmically, this choice is often reflected in the algorithm one uses for learning temporal distances. Methods based on Q-learning typically estimate optimal temporal distances [11, 22, 23], and are often structurally similar to popular actor-critic methods. Prior work has shown that temporal distance learning can be important for finding paths that are better than those demonstrated in the data, and can enable significantly more data efficient learning [24] (akin to standard results in the theory of Q-learning [25]). Methods based on Monte Carlo learning typically operate by sampling pairs of states that occur nearby in time (though not necessarily temporally-adjacent); distances are minimized for such positive pairs, and maximize for pairs of states that appear on different trajectories [26, 27]. These Monte Carlo methods typically estimate the temporal distance corresponding to the policy that collected the data. Methods for goal-conditioned behavioral cloning [28, 29], though not directly estimating temporal distances, are effectively working with this same behavioral temporal distance [7]. Despite the fact that these Monte Carlo methods do not estimate optimal temporal distances, they often outperform their Q-learning counterparts, suggesting that it is at least unclear whether the errors from learning the behavioral (rather than optimal) temporal distance are larger or smaller than those introduced by TD learning’s compounding errors. Our work aims to bridge these two types of temporal distances, providing a method that does learn optimal temporal distances while reducing the reliance on TD learning to propagate values (and accumulate errors).

### 2.2 Offline Reinforcement Learning

Our investigation into temporal distances closely mirrors discussions in the offline RL literature about 1-step RL methods [30], which often use Monte Carlo value estimation, versus multi-step RL methods [31], which often use Q-learning value estimation. These 1-step RL methods avoid the compounding errors of Q-learning, yet are limited by their capacity to learn  $Q^*$  rather than  $Q^\beta$ . However, their strong performance over the years [32] continues to suggest that it remains an open question whether the compounding errors of Q-learning outweigh the benefits from learning the behavioral value function, rather than the value function of the optimal policy.

## 3 Temporal Metric Distillation (TMD)

In this section, we formally define TMD in terms of the invariances it must enforce to recover optimal distances, and by extension, the optimal policy. In Section 4 we will then show how these invariances can be converted into losses which can be optimized with a quasimetric architecture that enforces the triangle inequality.

### 97 3.1 Notation

98 We consider a controlled Markov process  $\mathcal{M}$  with state space  $\mathcal{S}$ , action space  $\mathcal{A}$ , and dynamics  
 99  $P(s' | s, a)$ . The agent interacts with the environment by selecting actions according to a policy  
 100  $\pi(a | s)$ , i.e., a mapping from  $\mathcal{S}$  to distributions over  $\mathcal{A}$ . We further assume the state and action  
 101 spaces are compact.

102 Policies  $\pi \in \Pi$  are defined as distributions  $\pi(a | s)$  for  $s \in \mathcal{S}, a \in \mathcal{A}$ . When applicable, for a fixed  
 103 policy  $\pi$ , we can denote the state and action at step  $t$  as random variables  $\mathfrak{s}_t$  and  $\mathfrak{a}_t$ , respectively. We  
 104 will also use the shorthand

$$\mathfrak{s}_t^+ \triangleq \mathfrak{s}_{t+K} \text{ for } K \sim \text{Geom}(1 - \gamma). \quad (1)$$

105 We equip  $\mathcal{M}$  with an additional notion of *distances* between states. At the most basic level, a distance  
 106  $\mathcal{S} \times \mathcal{S} \rightarrow \mathbb{R}$  must be positive and have a zero diagonal. We will denote the set of all distances as  $\mathcal{D}$ ,  
 107 defined as

$$\mathcal{D} \triangleq \{d : \mathcal{S} \times \mathcal{S} \rightarrow \mathbb{R} : d(s, s) = 0, d(s, s') \geq 0 \text{ for each } s, s' \in \mathcal{S}\}.$$

108 A desirable property for distances to satisfy is the triangle inequality, which states that the distance  
 109 between two states is no greater than the sum of the distances between the states and a waypoint [15].  
 110 A distance satisfying this property is known as a *quasimetric*. Formally, we construct

$$\mathcal{Q} \triangleq \{d \in \mathcal{D} : d(s, g) \leq d(s, w) + d(w, g) \text{ for all } s, g, w \in \mathcal{S}\}.$$

111 If we further restrict distances to be symmetric ( $d(x, y) = d(y, x)$ ), we obtain the set of traditional  
 112 metrics over  $\mathcal{S}$ .

### 113 3.2 TMD Operators

114 TMD learns a distance parameterization that is made to satisfy two constraints: (i) the *triangle*  
 115 *inequality*,

$$d(x, z) \leq d(x, y) + d(y, z) \text{ for any } x, y, z \in \mathcal{S} \times \mathcal{A} \cup \mathcal{S}, \quad (2)$$

116 and (ii) *action invariance*,

$$d(s, (s, a)) = 0 \text{ for any } s \in \mathcal{S} \text{ and } a \in \mathcal{A}. \quad (3)$$

117 We will show that to ensure that we recover the optimal distance  $d_{SD}$  [16] given the learned (backward  
 118 NCE) contrastive critic distance, the missing additional constraint is the (exponentiated) SARSA  
 119 version of Bellman consistency. So, what TMD is doing with these additional constraints is weakening  
 120 the form of Bellman consistency that is required to recover the optimal distance from the standard  
 121  $\max_{a \in \mathcal{A}}$  Bellman operator to the weaker on-policy SARSA Bellman operator. TMD thus turns  
 122 on-policy SARSA into an off-policy algorithm through the metric constraints.

123 We can define this additional constraint as the fixed point of the following operator:

$$\mathcal{T}(d)(x, y) = \begin{cases} -\log \mathbb{E}_{P(s'|s,a)}[e^{-d(s',y)}] - \log \gamma & \text{if } x = (s, a) \in \mathcal{S} \times \mathcal{A}, \\ d(x, y) & \text{otherwise.} \end{cases} \quad (4)$$

124 The triangle inequality Eq. (2) and action invariance Eq. (3) properties can also be written in terms of  
 125 operator fixed points:

$$\mathcal{P}(d)(x, z) \triangleq \min_{y \in \mathcal{S}} [d(x, y) + d(y, z)] \quad (5)$$

$$\mathcal{I}d(s, x) \triangleq \begin{cases} 0 & \text{if } x = (s, a) \\ \mathcal{I}(d)(s, x) & \text{otherwise.} \end{cases} \quad (6)$$

### 126 3.3 Properties of path relaxation

127 Path relaxation  $\mathcal{P}$  [17] (Eq. 5) enforces invariance to the triangle inequality, i.e.,  $\mathcal{P}(d) = d$  if and  
 128 only if  $d \in \mathcal{Q}$ .  $\mathcal{P}$

129 **Theorem 1.** Take  $d \in \mathcal{D}$  and consider the sequence

$$d_n = \mathcal{P}^n(d).$$

130 Then,  $d_n$  converges uniformly to a fixed point  $d_\infty \in \mathcal{Q}$ .

131 In light of Theorem 1 we denote by  $\mathcal{P}_* = \lim_{n \rightarrow \infty} \mathcal{P}^n$  the fixed point operator of  $\mathcal{P}$ , and note that  
132  $\mathcal{P}_*$  is in fact a projection operator onto  $\mathcal{Q}$ .

133 Proofs of Lemmas 5 to 7 and Theorem 1 can be found in Appendix C.

### 134 3.4 The modified successor distance

135 The modified successor distance  $d_{\text{SD}}^\pi \in D$  can be defined by [16]: 11.5

$$d_{\text{SD}}^\pi(x, y) \triangleq \begin{cases} 0 & \text{if } x = y, \\ -\log p^\pi\left(\frac{\mathbb{P}(\mathfrak{s}^+ = g | \mathfrak{s} = s, \mathfrak{a} = a)}{\mathbb{P}(\mathfrak{s}^+ = g | \mathfrak{s} = g)}\right) \text{ for } K \sim \gamma & \text{if } x = (s, a) \in \mathcal{S} \times \mathcal{A}, y = g \in \mathcal{S} \\ -\log \mathbb{E}_{\pi(a|s)}[e^{-d_{\text{SD}}^\pi((s, a), g)}] - \log \gamma & \text{if } x = s \in \mathcal{S}, x \neq y \\ d_{\text{SD}}^\pi(s, g) - \log \pi(a | g) & \text{if } y = (g, a) \in \mathcal{S} \times \mathcal{A}. \end{cases} \quad (7)$$

136 The optimal successor distance  $d_{\text{SD}}^*$  can then be stated as

$$d_{\text{SD}}^*(x, y) \triangleq \min_{\pi \in \Pi} d_{\text{SD}}^\pi(x, y). \quad (8)$$

137 This distance is useful since it lets us recover optimal goal-reaching policies. For any  $s, g \in \mathcal{S}, a \in \mathcal{A}$ ,  
138 the distance is proportional to the optimal goal-reaching value function

$$d_{\text{SD}}^*((s, a), g) \propto_a -Q_g^*(s, a) \quad (9)$$

139 where  $Q_g^*$  is defined as the standard optimal  $Q$ -function for reaching goal  $g$  [27]:

$$Q_g^*(s, a) \triangleq \max_{\pi \in \Pi} \mathbb{E}_{\{\mathfrak{s}_i, \mathfrak{a}_i\} \sim \pi} \left[ \sum_{t=0}^{\infty} \gamma^t \mathbb{P}(\mathfrak{s}_t = g | \mathfrak{s}_0 = s, \mathfrak{a}_0 = a) \right]. \quad (10)$$

140 and

$$V_g^*(s) \triangleq \max_{a \in \mathcal{A}} Q_g^*(s, a). \quad (11)$$

141 In fact, we can equivalently define  $d_{\text{SD}}^*((s, a), g)$  in terms of  $Q^*$ :

$$d_{\text{SD}}^*((s, a), g) = \log V_g^*(g) - \log Q_g^*(s, a). \quad (12)$$

142 Similar to Myers et al. [16], we argue that contrastive learning can recover these distances, i.e.,

$$\mathcal{C}(\pi) = d_{\text{SD}} \quad (13)$$

143 Then, through the operators in Section 3.2, we will extend this to the optimal distance  $d_{\text{SD}}^*$ .

144 For convenience, we also define the set of realized successor distances

$$\tilde{\mathcal{D}} \triangleq \{d_{\text{SD}}^\pi : \pi \in \Pi\}. \quad (14)$$

145 Note that  $\tilde{\mathcal{D}}$  does not necessarily contain the optimal distance  $d_{\text{SD}}^*$ , as no single policy is generally  
146 optimal for reaching all goals.

147 **Remark 2.** The optimal successor distance  $d_{\text{SD}}^*$  satisfies

$$d_{\text{SD}}^*(s, (s, a)) = 0 \text{ for all } s \in \mathcal{S} \text{ and } a \in \mathcal{A}.$$

148

### 149 3.5 Convergence to the optimal successor distance

150 Applying the invariances in Section 3.2 to the contrastive distance Eq. (13), the TMD algorithm can  
151 be defined symbolically as

$$\mathcal{M}(\pi) \triangleq (\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})^\infty \mathcal{C}(\pi). \quad (15)$$

152 In other words, TMD computes the initial  $\pi^\beta$  distance  $\mathcal{C}(\pi)$ , and then enforces the invariance  
153 (architecturally or explicitly), as expressed with the iterative application of  $\mathcal{T} \circ \mathcal{I}$  followed by  
154 projection onto  $\mathcal{Q}$  by  $\mathcal{P}_*$ .

155 **Theorem 3.** *The TMD algorithm converges pointwise to the optimal successor distance  $d_{\text{SD}}^*$  for any*  
156 *policy  $\pi$  with full state and action coverage, i.e.,*

$$\lim_{n \rightarrow \infty} (\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})^n \mathcal{C}(\pi) = d_{\text{SD}}^*. \quad (16)$$

157 Our approach for proving Theorem 3 will be to analyze the convergence properties of  $(\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})$   
158 over the space of “suboptimal” distances  $\mathcal{D}_+^*$ , defined as

$$\mathcal{D}_+^* \triangleq \{d \in \mathcal{D} : d(x, y) \geq d_{\text{SD}}^*(x, y) \text{ for all } x, y \in \mathcal{S} \times \mathcal{A} \cup \mathcal{S}\}. \quad (17)$$

159 Unfortunately,  $(\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})$  is not a contraction on  $\mathcal{D}_+^*$ , so we cannot directly apply the Banach  
160 fixed-point theorem as we would for the standard Bellman (optimality) operator. Instead, we will  
161 show this operator induces a “more aggressive” form of tightening over  $\mathcal{D}_+^*$ , which will allow us to  
162 prove convergence to  $d_{\text{SD}}^*$ . We start by showing that  $d_{\text{SD}}^*$  is a fixed point of  $(\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})$  in Lemma 4.

163 **Lemma 4.** *The optimal successor distance  $d_{\text{SD}}^*$  is the unique fixed point of  $\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I}$  on  $\mathcal{D}_+^*$ .*

164 Proofs are in Appendix B.

## 165 4 Implementing TMD

166 We show that the backward NCE contrastive learning algorithm can recover an initial estimate of  $d_{\text{SD}}^{\pi\beta}$ .  
167 As justified by Theorem 3, we can then enforce the invariances to recover the optimal distance  $d_{\text{SD}}^*$ .

168 The algorithm learns a distance  $d_\theta$  parameterized by a quasimetric neural network  $\theta$  such as MRN [18].  
169 By construction, this distance is a quasimetric that is invariant to  $\mathcal{P}$ , i.e.,  $\mathcal{P}d_\theta = d_\theta$ .

### 170 4.1 Initializing the Distance with Contrastive Learning

171 Defining the critic

$$f(s, a, g) \triangleq -d_\theta((s, a), g),$$

172 the core contrastive objective is the backward NCE loss:

$$\mathcal{L}_{\text{NCE}}(\phi, \psi; \{s_i, a_i, s'_i, g_i\}_{i=1}^N) = \sum_{i=1}^N \log \left( \frac{e^{f(s_i, a_i, g_i)}}{\sum_{j=1}^N e^{f(s_j, a_j, g_i)}} \right) \quad (18)$$

173 which is enforced across batches of triplets  $\{s_i, a_i, s_{i+k}\}_{i=1}^N$  for  $k \sim \text{Geom}(1 - \gamma)$  sampled from  
174 the dataset generated by policy  $\pi_\beta$ .

175 The optimal solution to this objective is

$$f(s, a, g) = \log \left( \frac{\mathbb{P}(\mathfrak{s}^+ = g \mid \mathfrak{s} = s, \mathfrak{a} = a)}{\mathbb{P}(\mathfrak{s}^+ = g)C(g)} \right). \quad (19)$$

176 for some  $C(g)$  [33].

177 The parameterization  $f(s, a, g) = -d_\theta((s, a), g)$  where  $d_\theta$  is a quasimetric-enforcing parameteriza-  
178 tion (see [18, 15]) ensures that

$$C(g) = \frac{\mathbb{P}(\mathfrak{s}^+ = g \mid \mathfrak{s} = g)}{\mathbb{P}(\mathfrak{s}^+ = g)},$$

179 so the only valid quasimetric satisfying Eq. (19) is  $d_\theta = d_{\text{SD}}^{\pi_\beta}$ .

180 Optimality of  $\mathcal{L}$  in Eq. (18) implies that the learned distance  $d_\theta = \mathcal{C}(\pi_\beta) = d_{\text{SD}}^{\pi_\beta}$ .

181 The additional invariance constraints  $\mathcal{I}$  and  $\mathcal{T}$  can be directly enforced by regressing  $\|d_\theta - \mathcal{I}d_\theta\|_\infty$   
 182 and  $\|d_\theta - \mathcal{T}d_\theta\|_\infty$  to zero. Theorem 3 guarantees that if we can enforce those constraints and enforce  
 183 invariance to  $\mathcal{P}$  by using a quasimetric architecture, we can recover the optimal distance  $d_{\text{SD}}^*$ .

184 In practice, we will directly enforce the constraints across the batches used in our contrastive loss.  
 185 We will use the MRN parameterization for  $d_\theta$  for  $\theta = (\psi, \phi)$  on learned representations of states ( $\psi$ )  
 186 and state-action pairs ( $\phi$ ):

$$\begin{aligned} d_\theta(s, g) &\triangleq d_{\text{MRN}}(\psi(s), \psi(g)) & d_\theta((s, a), g) &\triangleq d_{\text{MRN}}(\phi(s, a), \psi(g)) \\ d_\theta(s, (s, a)) &\triangleq d_{\text{MRN}}(\psi(s), \phi(s, a)) & d_\theta((s, a), (s', a')) &\triangleq d_{\text{MRN}}(\phi(s, a), \phi(s', a')) \end{aligned}$$

187 where

$$d_{\text{MRN}}(x, y) \triangleq \frac{1}{K} \sum_{k=1}^K \max_{m=1 \dots M} \max(0, x_{kM+m} - y_{kM+m}) \quad (20)$$

## 188 4.2 Action $\mathcal{I}$ -Invariance

189 Invariance to the  $\mathcal{I}$  backup operator in Eq. (6) gives the following update across  $s, a \in \mathcal{S} \times \mathcal{A}$

$$d_\theta(\psi(s), \phi(s, a)) \leftarrow 0, \quad (21)$$

190 which can be directly enforced with the following loss across the batch:

$$\mathcal{L}_{\mathcal{I}}(\phi, \psi; \{s_i, a_i, s'_i, g_i\}_{i=1}^N) = \sum_{i,j=1}^N d_{\text{MRN}}(\psi(s_i), \phi(s_i, a_j)). \quad (22)$$

## 191 4.3 Temporal $\mathcal{T}$ -Invariance

192 Invariance to the  $\mathcal{T}$  backup operator in Eq. (4) corresponds to the following update performed with  
 193 respect to  $\phi(s, a)$ :

$$e^{-d_{\text{MRN}}(\phi(s, a), \psi(g))} \leftarrow \mathbb{E}_{P(s'|s, a)} e^{\log \gamma - d_{\text{MRN}}(\psi(s'), \psi(g))}. \quad (23)$$

194 This update is enforced by minimizing a divergence between the LHS and samples from the RHS  
 195 expectation. Classic approaches for backups in deep RL include the  $\ell_2$  distance to the target  
 196 (RHS) [34], or when values can be interpreted as probabilities, a binary cross-entropy loss [35].

197 We use the following Bregman divergence [36], which we find empirically is more stable for learning  
 198 the update in Eq. (23) (c.f. the Itakura-Saito distance [37]).

$$D_T(d, d') \triangleq \exp(d - d') - d. \quad (24)$$

199 We discuss this divergence and prove correctness in Appendix E. With the divergence, the  $\mathcal{T}$ -  
 200 invariance loss is:

$$\mathcal{L}_{\mathcal{T}}(\phi, \psi; \{s_i, a_i, s'_i, g_i\}_{i=1}^N) = \sum_{i,j=1}^N D_T(d_{\text{MRN}}(\phi(s_i, a_i), \psi(g_j)), d_{\text{MRN}}(\psi(s'_i), \psi(g_j)) - \log \gamma) \quad (25)$$

201 We minimize this loss only with respect to  $\phi$ , stopping the gradient through  $\psi$ . This avoids the moving  
 202 target that classically necessitates learning separate target networks in RL [34].

## 203 4.4 The Overall Distance Learning Objective

204 We can express the overall critic loss as:

$$\begin{aligned} \mathcal{L}_{\text{TMD}}(\phi, \psi; \bar{\psi}, \mathcal{B}) &= \mathcal{L}_{\text{NCE}}(\phi, \psi; \mathcal{B}) + \zeta \left( \mathcal{L}_{\mathcal{I}}(\phi, \psi; \mathcal{B}) + \mathcal{L}_{\mathcal{T}}(\phi, \bar{\psi}; \mathcal{B}) \right) \\ \text{for batch } \mathcal{B} &\sim p^{\pi_\beta} = \{s_i, a_i, s'_i, g_i\}_{i=1}^N \end{aligned} \quad (26)$$



Table 1: OGBench Evaluation

Task	Methods					
	TMD	CRL	QRL	GCBC	GCIQL	GCIVL
humanoidmaze_medium_stitch	<b>60.5</b> ( $\pm 1.6$ )	36.2( $\pm 0.9$ )	18.0( $\pm 0.7$ )	29.0( $\pm 1.7$ )	12.1( $\pm 1.1$ )	12.3( $\pm 0.6$ )
pointmaze_teleport_stitch	29.3( $\pm 2.2$ )	4.1( $\pm 1.1$ )	8.6( $\pm 1.9$ )	31.5( $\pm 3.2$ )	25.2( $\pm 1.0$ )	<b>44.4</b> ( $\pm 0.7$ )
humanoidmaze_large_stitch	<b>23.0</b> ( $\pm 1.5$ )	4.0( $\pm 0.2$ )	3.5( $\pm 0.5$ )	5.6( $\pm 1.0$ )	0.5( $\pm 0.1$ )	1.2( $\pm 0.2$ )
antmaze_teleport_explore	<b>49.6</b> ( $\pm 1.5$ )	19.5( $\pm 0.8$ )	2.3( $\pm 0.7$ )	2.4( $\pm 0.4$ )	7.3( $\pm 1.2$ )	32.0( $\pm 0.6$ )
antmaze_large_stitch	<b>20.9</b> ( $\pm 1.7$ )	10.8( $\pm 0.6$ )	<b>18.4</b> ( $\pm 0.7$ )	3.4( $\pm 1.0$ )	7.5( $\pm 0.7$ )	<b>18.5</b> ( $\pm 0.8$ )
scene_noisy	19.6( $\pm 1.7$ )	1.2( $\pm 0.3$ )	9.1( $\pm 0.7$ )	1.2( $\pm 0.2$ )	<b>25.9</b> ( $\pm 0.8$ )	<b>26.4</b> ( $\pm 1.7$ )
visual_antmaze_teleport_stitch	<b>38.5</b> ( $\pm 1.5$ )	31.7( $\pm 3.2$ )	1.4( $\pm 0.8$ )	31.8( $\pm 1.5$ )	1.0( $\pm 0.2$ )	1.4( $\pm 0.4$ )
visual_antmaze_large_stitch	<b>26.6</b> ( $\pm 2.8$ )	11.1( $\pm 1.3$ )	0.6( $\pm 0.3$ )	<b>23.6</b> ( $\pm 1.4$ )	0.1( $\pm 0.0$ )	0.8( $\pm 0.3$ )

We **bold** the best performance. Success rate (%) is presented with the standard deviation across six seeds.

We minimize Eq. (26) with respect to  $\phi$  and  $\psi$ , where  $\bar{\psi}$  is a separate copy of the representation network  $\psi$  (stop-gradient). Here,  $\zeta$  controls the weight of the contrastive loss and invariance constraints, and batches are sampled

$$\{s_i, a_i, s'_i, g_i\}_{i=1}^N \sim \pi_\beta,$$

for  $s'_i$  the state following  $s_i$ , and  $g_i$  the state  $K$  steps ahead of  $s_i$  for  $K \sim \text{Geom}(1 - \gamma)$ . In theory,  $\zeta^{-1}$  should be annealed between 1 at the start of training (to extract the distance  $\mathcal{C}(\pi)$ ), toward 0 at the end of training to enforce invariance to  $(\mathcal{T} \circ \mathcal{I})$ .

In practice, we pick  $\zeta$  based on how much stitching and stochasticity we expect in the environment — when  $\zeta$  is large, we more aggressively try and improve on the initial distance  $\mathcal{C}(\pi_\beta)$  describing the dataset policy  $\pi_\beta$ .

#### 4.5 Policy Extraction

We finally extract the goal-conditioned policy  $\pi(s, g) : \mathcal{S}^2 \rightarrow \mathcal{A}$  with the learned distance  $d_\theta$ :

$$\min_{\pi} \mathbb{E}_{\{s_i, a_i, s'_i, g_i\}_{i=1}^N \sim \pi_\beta} \left[ \sum_{i,j=1}^N d_\theta((s_i, \pi(s_i, g_j)), g_j) \right]. \quad (27)$$

For conservatism [38], we augment Eq. (27) with a behavioral cloning loss against  $\pi_\beta$  via behavior-constrained deep deterministic policy gradient [39]. Using additional goals  $g_i$  sampled from the same trajectory as  $s_i$  in Eq. (27) could also be done through an extra tuned parameter (cf. Bortkiewicz et al. [40], Park et al. [41]). Denoting these hyperparameters as  $\lambda$  and  $\alpha$  respectively, the overall policy extraction objective is:

$$\min_{\pi} \mathbb{E}_{\{s_i, a_i, s'_i, g_i\}_{i=1}^N \sim \pi_\beta} [\mathcal{L}_\pi(\pi; \phi, \psi, \{s_i, a_i, s'_i, g_i\}_{i=1}^N)] \quad (28)$$

$$\mathcal{L}_\pi \triangleq \sum_{i,j=1}^N (1 - \lambda) d_{\text{MRN}}(\phi(s_i, \hat{a}_{ij}), \psi(g_j), g_j) + \lambda d_{\text{MRN}}(\phi(s_i, \hat{a}_{ii}), \psi(g_i)) + \alpha \|\hat{a}_{ii} - a_i\|_2^2$$

$$\text{where } \hat{a}_{ij} = \pi(s_i, g_j). \quad (29)$$

Prior offline RL methods use similar  $\alpha$  and  $\lambda$  hyperparameters, which must be tuned per environment [41].

## 5 Experiments

In our experiments, we evaluate the performance of TMD on tasks from the OGBench benchmark [41]. We aim to answer the following questions:<sup>1</sup>

1. Do the invariance terms in Eq. (5) improve performance quantitatively in offline RL settings?
2. Is the contrastive loss in Eq. (18) necessary to facilitate learning these tasks?
3. What capabilities does TMD enable for compositional task learning?

<sup>1</sup>Anonymous code: <https://anonymous.4open.science/r/ogcrl-FCDC/>



## 5.1 Experimental Results

We evaluate TMD across evaluation tasks OGBench for the environments and datasets listed in Table 1, each containing 5 separate goal-reaching tasks, making a total of 40 tasks. We evaluate TMD with 6 seeds in all environments. Of particular interest are the “teleport” and “stitch” environments, which respectively test the ability to handle stochasticity and composition.

We compare against Goal-Conditioned Behavior Cloning (GCBC), Goal-Conditioned Implicit Q-Learning (GCIQL), Goal-Conditioned Implicit Value Learning (GCIVL), Contrastive RL (CRL), and Quasimetric RL (QRL) as our baselines. GCBC uses imitation learning to learn a policy that follows the given trajectories within a dataset [42]. CRL [27] performs policy improvement by fitting a value function via contrastive learning. QRL [15] learns a quasimetric value function via Bellman consistency. GCIQL and GCIVL use expectile regression to fit a value function [43].

Over stitching environments, TMD consistently outperforms QRL and CRL, especially in humanoidmaze, where it outperformed all baselines in comparison by up to a sixfold factor in the large humanoidmaze environment, and demonstrated the ability to solve the environment while the baselines displayed barely any positive success rate. In teleport environments, where there exists stochastic transitions, TMD outperforms both CRL and QRL by a considerable margin, especially in pointmaze\_teleport\_stitch, where TMD outperforms the other two methods by a factor of three fold. Although TMD is outperformed by GCIQL and GCIVL in some occasions, it shrinks the gap between it and these methods considerably than CRL and QRL. Finally, our results show that TMD performs consistently above baselines regardless of dataset composition, as it demonstrates strong performance in both explore and noisy datasets, where learning value functions become more important.

## 5.2 Ablation Study

We perform an ablation study on the pointmaze\_teleport\_stitch environment to evaluate the importance of the invariance terms and the contrastive initialization loss in TMD. We separately disable the contrastive,  $\mathcal{T}$  invariance, and  $\mathcal{I}$  invariance component during training and observe its effects. We also examine the empirical effects of stopping gradients when calculating  $\mathcal{L}_{\mathcal{T}}$ . We log the corresponding success rate for each of the ablations in 3.

Our ablation studies answer questions 2 and 3, in which we demonstrate that by removing some of the invariances or removing the contrastive loss, the performance of TMD decreases to levels similar to CRL and QRL. Similarly, we see the importance of keeping the contrastive objective, as the performance of TMD degrades even more despite the presence of other loss components. We also note the empirical performance of TMD is better when we stop gradients on  $\mathcal{L}_{\mathcal{T}}$ . We provide further ablation details in Appendix D.2.

## 6 Discussion

In this work, we introduce Temporal Metric Distillation (TMD), an offline goal-conditioned reinforcement learning method that learns representations which exploit the quasimetric structure of temporal distances. Our approach unifies quasimetric, temporal-difference, and Monte Carlo learning approaches to GCRL by enforcing a set of invariance properties on the learned distance function. To the best of our knowledge, TMD is the first method that can exploit the quasimetric structure of

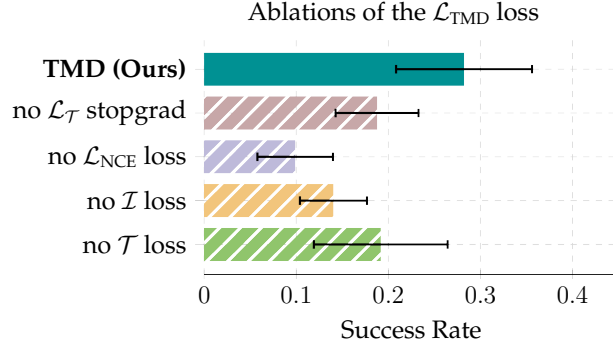


Figure 3: We ablate the loss components of TMD in the pointmaze\_teleport\_stitch environment.

temporal distances to learn optimal policies from offline data, even in stochastic settings (see Fig. 2). On a standard suite of offline GCRL benchmarks, TMD outperforms prior methods, in particular on long-horizon tasks that require stitching together trajectories across noisy dynamics and visual observations.

## 6.1 Limitations and Future Work

Future work could examine more principled ways to set the  $\zeta$  parameter in our method, or if there are ways to more directly integrate the contrastive and invariance components of the loss function. Future work could also explore integrating the policy extraction objective more directly into the distance learning to enable desirable properties (stitching through architecture, horizon generalization) at the level of the policy. While we used the MRN [18] architecture in our experiments, alternative architectures such as IQE [19] that enforce the triangle inequality could be more expressive. While the size of models studied in our experiments make them unlikely to pose any real-world risks, methods which implicitly enable long-horizon decision making could have unintended consequences or poor interpretability. Future work should consider these implications.

## References

- [1] Eduardo D. Sontag. A ‘Universal’ Construction of Artstein’s Theorem on Nonlinear Stabilization. *Systems & Control Letters*, 13(2):117–123, 1989.
- [2] Aaron D. Ames, Samuel Coogan, Magnus Egerstedt, Gennaro Notomista, Koushil Sreenath, and Paulo Tabuada. Control Barrier Functions: Theory and Applications. *European Control Conference*, pp. 3420–3431, 2019.
- [3] Matthias Althoff. Reachability Analysis and Its Application to the Safety Assessment of Autonomous Cars. 2010.
- [4] Kai-Chieh Hsu, Haimin Hu, and Jaime F Fisac. The Safety Filter: A Unified View of Safety-Critical Control in Autonomous Systems. *Annual Review of Control*, 7, 2023.
- [5] Shangding Gu, Long Yang, Yali Du, Guang Chen, Florian Walter, Jun Wang, and Alois Knoll. A Review of Safe Reinforcement Learning: Methods, Theory and Applications. arXiv:2205.10330, 2022.
- [6] Dhruv Shah, Blazej Osinski, Brian Ichter, and Sergey Levine. LM-Nav: Robotic Navigation With Large Pre-Trained Models of Language, Vision, and Action. *Conference on Robot Learning*, 2022.
- [7] Raj Ghugare, Matthieu Geist, Glen Berseth, and Benjamin Eysenbach. Closing the Gap Between TD Learning and Supervised Learning – a Generalisation Point of View. *International Conference on Learning Representations*, 2024.
- [8] Yecheng Jason Ma, Shagun Sodhani, Dinesh Jayaraman, Osbert Bastani, Vikash Kumar, and Amy Zhang. VIP: Towards Universal Visual Reward and Representation via Value-Implicit Pre-Training. *International Conference on Learning Representations*, 2023.
- [9] Suraj Nair, Aravind Rajeswaran, Vikash Kumar, Chelsea Finn, and Abhinav Gupta. R3m: A Universal Visual Representation for Robot Manipulation. *Conference on Robot Learning*, pp. 892–909, 2022.
- [10] Long-Ji Lin. Self-Improving Reactive Agents Based on Reinforcement Learning, Planning and Teaching. 1992.
- [11] Marcin Andrychowicz, Filip Wolski, Alex Ray, Jonas Schneider, Rachel Fong, Peter Welinder, Bob McGrew, Josh Tobin, OpenAI Pieter Abbeel, and Wojciech Zaremba. Hindsight Experience Replay. *Neural Information Processing Systems*, volume 30, 2017.
- [12] Vitchyr Pong, Shixiang Gu, Murtaza Dalal, and Sergey Levine. Temporal Difference Models: Model-Free Deep RL for Model-Based Control. *International Conference on Learning Representations*, 2018.
- [13] Aviral Kumar, Justin Fu, Matthew Soh, George Tucker, and Sergey Levine. Stabilizing Off-Policy Q-Learning via Bootstrapping Error Reduction. *Neural Information Processing Systems*, 32, 2019.
- [14] Alexey Dosovitskiy and Vladlen Koltun. Learning to Act by Predicting the Future. arXiv:1611.01779, 2016.

- [15] Tongzhou Wang, Antonio Torralba, Phillip Isola, and Amy Zhang. Optimal Goal-Reaching Reinforcement Learning via Quasimetric Learning. *International Conference on Machine Learning*, pp. 36411–36430, 2023.
- [16] Vivek Myers, Chongyi Zheng, Anca Dragan, Sergey Levine, and Benjamin Eysenbach. Learning Temporal Distances: Contrastive Successor Features Can Provide a Metric Structure for Decision-Making. *International Conference on Machine Learning*, 2024.
- [17] Vivek Myers, Catherine Ji, and Benjamin Eysenbach. Horizon Generalization in Reinforcement Learning. *International Conference on Learning Representations*, 2025.
- [18] Bo Liu, Yihao Feng, Qiang Liu, and Peter Stone. Metric Residual Network for Sample Efficient Goal-Conditioned Reinforcement Learning. *AAAI Conference on Artificial Intelligence*, volume 37, pp. 8799–8806, 2023.
- [19] Tongzhou Wang and Phillip Isola. Improved Representation of Asymmetrical Distances With Interval Quasimetric Embeddings. *NeurIPS 2022 NeurReps Workshop Proceedings Track*, 2022.
- [20] Junik Bae, Kwanyoung Park, and Youngwoon Lee. TLDR: Unsupervised Goal-Conditioned RL via Temporal Distance-Aware Representations. *Conference on Robot Learning*, 2024.
- [21] Stephen Tian, Suraj Nair, Frederik Ebert, Sudeep Dasari, Benjamin Eysenbach, Chelsea Finn, and Sergey Levine. Model-Based Visual Planning With Self-Supervised Functional Distances. *International Conference on Learning Representations*, 2021.
- [22] Scott Fujimoto, Herke van Hoof, and David Meger. Addressing Function Approximation Error in Actor-Critic Methods. *International Conference on Machine Learning*, 2018.
- [23] Leslie Pack Kaelbling. Learning to Achieve Goals. *International Joint Conference on Artificial Intelligence*, volume 2, pp. 1094–1098, 1993.
- [24] Chongyi Zheng, Ruslan Salakhutdinov, and Benjamin Eysenbach. Contrastive Difference Predictive Coding. *International Conference on Learning Representations*, 2023.
- [25] Alekh Agarwal, Nan Jiang, Sham M Kakade, and Wen Sun. Reinforcement Learning: Theory and Algorithms. *CS Dept.*, 32:96, 2019.
- [26] Kristian Hartikainen, Xinyang Geng, Tuomas Haarnoja, and Sergey Levine. Dynamical Distance Learning for Semi-Supervised and Unsupervised Skill Discovery. *International Conference on Learning Representations*, 2020.
- [27] Benjamin Eysenbach, Tianjun Zhang, Ruslan Salakhutdinov, and Sergey Levine. Contrastive Learning as Goal-Conditioned Reinforcement Learning. *Neural Information Processing Systems*, volume 35, pp. 35603–35620, 2022.
- [28] Dibya Ghosh, Abhishek Gupta, Ashwin Reddy, Justin Fu, Coline Devin, Benjamin Eysenbach, and Sergey Levine. Learning to Reach Goals via Iterated Supervised Learning. *International Conference on Learning Representations*, 2021.
- [29] Vivek Myers, Andre He, Kuan Fang, Homer Walke, Phillipe Hansen Estruch, Ching-An Cheng, Mihai Jalobeanu, Andrey Kolobov, Anca Dragan, and Sergey Levine. Goal Representations for Instruction Following: A Semi-Supervised Language Interface to Control. *Conference on Robot Learning*, 2023.
- [30] Benjamin Eysenbach, Matthieu Geist, Sergey Levine, and Ruslan Salakhutdinov. A Connection Between One-Step RL and Critic Regularization in Reinforcement Learning. *The International Conference on Machine Learning*, pp. 9485–9507, 2023.
- [31] John Schulman, Philipp Moritz, Sergey Levine, Michael Jordan, and Pieter Abbeel. High-Dimensional Continuous Control Using Generalized Advantage Estimation. *arXiv:1506.02438*, 2018.
- [32] Cassidy Laidlaw, Banghua Zhu, Stuart Russell, and Anca Dragan. The Effective Horizon Explains Deep RL Performance in Stochastic Environments. *International Conference on Learning Representations*, 2024.
- [33] Zhuang Ma and Michael Collins. Noise Contrastive Estimation and Negative Sampling for Conditional Models: Consistency and Statistical Efficiency. *Empirical Methods in Natural Language Processing*, pp. 3698–3707, 2018.
- [34] Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A. Rusu, Joel Veness, Marc G. Bellemare, Alex Graves, Martin Riedmiller, et al. Human-Level Control Through Deep Reinforcement Learning. *Nature*, volume 518, pp. 529–533, 2015.

- 387 [35] Dmitry Kalashnikov, Alex Irpan, Peter Pastor, Julian Ibarz, Alexander Herzog, Eric Jang,  
388 Deirdre Quillen, Ethan Holly, Mrinal Kalakrishnan, Vincent Vanhoucke, and Sergey Levine.  
389 Scalable Deep Reinforcement Learning for Vision-Based Robotic Manipulation. *Conference on*  
390 *Robot Learning*, pp. 651–673, 2018.
- 391 [36] L. M. Bregman. The Relaxation Method of Finding the Common Point of Convex Sets and  
392 Its Application to the Solution of Problems in Convex Programming. *USSR Computational*  
393 *Mathematics and Mathematical Physics*, 7(3):200–217, 1967.
- 394 [37] Fumitada Itakura. Analysis Synthesis Telephony Based on the Maximum Likelihood Method.  
395 *Reports of the 6th Int. Cong. Acoust*, 1968.
- 396 [38] Aviral Kumar, Aurick Zhou, George Tucker, and Sergey Levine. Conservative Q-Learning  
397 for Offline Reinforcement Learning. *Neural Information Processing Systems*, volume 33, pp.  
398 1179–1191, 2020.
- 399 [39] Scott Fujimoto and Shixiang Shane Gu. A Minimalist Approach to Offline Reinforcement  
400 Learning. arXiv:2106.06860, 2021.
- 401 [40] Michał Bortkiewicz, Włodek Pałucki, Vivek Myers, Tadeusz Dziarmaga, Tomasz Arczewski,  
402 Łukasz Kuciński, and Benjamin Eysenbach. Accelerating Goal-Conditioned RL Algorithms  
403 and Research. *International Conference on Learning Representations*, 2025.
- 404 [41] Seohong Park, Kevin Frans, Benjamin Eysenbach, and Sergey Levine. OGBench: Bench-  
405 marking Offline Goal-Conditioned RL. *International Conference on Learning Representations*,  
406 2025.
- 407 [42] Yiming Ding, Carlos Florensa, Pieter Abbeel, and Mariano Phielipp. Goal-Conditioned Imitation  
408 Learning. *Neural Information Processing Systems*, volume 32, 2019.
- 409 [43] Ilya Kostrikov, Ashvin Nair, and Sergey Levine. Offline Reinforcement Learning With Implicit  
410 Q-Learning. arXiv:2110.06169, 2021.
- 411 [44] Yim-Ming Wong and Kung-Fu Ng. On a Theorem of Dini. *Journal of the London Mathematical*  
412 *Society*, s2-11(1):46–48, 1975.
- 413 [45] Arindam Banerjee, Srujana Merugu, Inderjit Dhillon, and Joydeep Ghosh. Clustering With  
414 Bregman Divergences. *SIAM International Conference on Data Mining*, pp. 234–245, 2004.

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Question: For crowdsourcing experiments and research with human subjects, does the paper include the full text of instructions given to participants and screenshots, if applicable, as well as details about compensation (if any)?

Answer: [NA]

Justification: No crowdsourcing or human subjects were used in this research.

Guidelines:

- The answer NA means that the paper does not involve crowdsourcing nor research with human subjects.
- Including this information in the supplemental material is fine, but if the main contribution of the paper involves human subjects, then as much detail as possible should be included in the main paper.
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Question: Does the paper describe potential risks incurred by study participants, whether such risks were disclosed to the subjects, and whether Institutional Review Board (IRB) approvals (or an equivalent approval/review based on the requirements of your country or institution) were obtained?

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- Depending on the country in which research is conducted, IRB approval (or equivalent) may be required for any human subjects research. If you obtained IRB approval, you should clearly state this in the paper.
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- For initial submissions, do not include any information that would break anonymity (if applicable), such as the institution conducting the review.

### 16. Declaration of LLM usage

725 Question: Does the paper describe the usage of LLMs if it is an important, original, or  
726 non-standard component of the core methods in this research? Note that if the LLM is used  
727 only for writing, editing, or formatting purposes and does not impact the core methodology,  
728 scientific rigorousness, or originality of the research, declaration is not required.

729 Answer: [NA]

730 Justification: No LLMs were used in this research.

731 Guidelines:

- 732 • The answer NA means that the core method development in this research does not
- 733 involve LLMs as any important, original, or non-standard components.
- 734 • Please refer to our LLM policy (<https://neurips.cc/Conferences/2025/LLM>)
- 735 for what should or should not be described.

## A Code

Code and videos are referenced here: <https://anonymous.4open.science/w/tmd-website-6E33/>. The evaluation and base agent structure follows the OGBench code-base [41]. The TMD agent is implemented in `agents/tmd.py`.

## B Analysis of TMD

This section provides the proofs of the main results in Section 3.

**Theorem 1.** Take  $d \in \mathcal{D}$  and consider the sequence

$$d_n = \mathcal{P}^n(d).$$

Then,  $d_n$  converges uniformly to a fixed point  $d_\infty \in \mathcal{Q}$ .

*Proof.* From Lemma 7, we have that  $d_{n+1}(s, g) \leq d_n(s, g)$  for all  $s, g \in \mathcal{S}$ . Thus, the sequence  $\{d_n\}$  is monotonically decreasing (and positive). By the monotone convergence theorem, the sequence converges pointwise to a limit  $d_\infty$ . Since  $\mathcal{S}$  is compact, by Dini’s theorem [44], the convergence is uniform, i.e.,  $d_n \rightarrow d_\infty$  under the  $L^\infty$  topology over  $\mathcal{D}$ .

To see that  $d_\infty$  is a fixed point of  $\mathcal{P}$ , we note that if  $\mathcal{P}d_\infty = d' \neq d_\infty$ , we can construct disjoint neighborhoods  $N$  of  $d_\infty$  and  $N'$  of  $d'$  (since  $L^\infty(\mathcal{D})$  is normed vector space and thus Hausdorff). By construction, the preimage  $\mathcal{P}^{-1}(N')$  contains  $d_\infty$  and is open by Lemma 6. Thus, we can define another, smaller open neighborhood  $N'' = N \cap \mathcal{P}^{-1}(N')$  of  $d_\infty$ . Now, since  $d_n \rightarrow d_\infty$ , there exists some  $k$  so  $d_k, d_{k+1} \in N'' \subset N$ . But then since  $d_k \in \mathcal{P}^{-1}(N')$ , we have that  $d_{k+1} \in N'$ . This is a contradiction as  $N$  and  $N'$  were disjoint by construction.

Thus, we have that  $d_\infty$  is a fixed point of  $\mathcal{P}$ . That  $d_\infty \in \mathcal{Q}$  follows from Lemma 5.  $\square$

**Theorem 3.** The TMD algorithm converges pointwise to the optimal successor distance  $d_{\text{SD}}^*$  for any policy  $\pi$  with full state and action coverage, i.e.,

$$\lim_{n \rightarrow \infty} (\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})^n \mathcal{C}(\pi) = d_{\text{SD}}^*. \quad (16)$$

*Proof of Theorem 3.* The initial distance  $\mathcal{C}(\pi) = d_{\text{SD}}^\pi \geq d_{\text{SD}}^*$  for any policy  $\pi$ . So,  $\mathcal{C}(\pi) \in \mathcal{D}_+^*$ . Define the sequence of distances  $d_n = (\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})^n \mathcal{C}(\pi)$  in  $\mathcal{D}_+^*$ . Note that  $\mathcal{P}_*$  and  $\mathcal{I}$  are monotone decreasing. So, the restriction  $(d_n)|_{\mathcal{X}}$  is monotonically decreasing on the domain  $\mathcal{X} = \mathcal{S} \times (\mathcal{S} \cup \mathcal{S} \times \mathcal{A})$ , and thus converges pointwise on  $\mathcal{X}$  as  $n \rightarrow \infty$ .

Since  $\mathcal{T}$  and  $\mathcal{P}_*$  are continuous operators (Lemma 6 and Eq. (4)), and  $\mathcal{T}$  is fully-determined by the restriction to  $\mathcal{X}$ , the sequence  $(\mathcal{P} \circ \mathcal{T})d_n = d_{n+1}$  converges pointwise on its full domain. The pointwise limit of  $d_n$  is a fixed point of  $(\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})$ , which must be the unique fixed point  $d_{\text{SD}}^*$  on  $\mathcal{D}_+^*$  by Lemma 4.  $\square$

**Lemma 4.** The optimal successor distance  $d_{\text{SD}}^*$  is the unique fixed point of  $\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I}$  on  $\mathcal{D}_+^*$ .

*Proof of Lemma 4.* For existence, we note

$$\begin{aligned} (\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})d_{\text{SD}}^* &= (\mathcal{P}_* \circ \mathcal{T})(\mathcal{I}d_{\text{SD}}^*) && \text{(Remark 2)} \\ &= (\mathcal{P}_* \circ \mathcal{T})d_{\text{SD}}^* && \text{(Bellman optimality of } Q_g^*) \\ &= \mathcal{P}_*d_{\text{SD}}^*. && \text{(Lemma 5)} \\ &= d_{\text{SD}}^*. && (30) \end{aligned}$$

For uniqueness, we need to show that  $(\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})$  has no fixed points besides  $d_{\text{SD}}^*$  in  $\mathcal{D}_+^*$ . Suppose there exists some  $d \in \mathcal{D}_+^*$  such that  $(\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})d = d$ . Then, we have for  $x \in \mathcal{S} \cup \mathcal{S} \times \mathcal{A}$ ,  $s, g \in \mathcal{S}$ , and  $a \in \mathcal{A}$ :

$$(\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})d(x, (g, a)) = d(x, (g, a)) = d(x, g). \quad (31)$$

Denote by  $Q(s, a) = e^{-d((s, a), g)}$ , and let  $\mathcal{B}$  be the goal-conditioned Bellman operator defined as

$$\mathcal{B}Q(s, a) \triangleq \mathbb{E}_{\mathcal{P}(s'|s, a)}[\mathbb{1}\{s' = g\} + \gamma Q(s', g)] \quad (32)$$

At any fixed point  $d \in \mathcal{D}_+^*$ , we have

$$(\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})d((s, a), g) = d((s, a), g) \quad (33)$$

This last expression implies that

$$\begin{aligned} Q(s, a) &= \exp[-d((s, a), g)] \\ &= \exp[-(\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})d((s, a), g)] \\ &\leq \mathbb{E}_{\mathcal{P}(s'|s, a)} \left[ \min_{a' \in \mathcal{A}} \exp d((s', a'), g) \right] - \log \gamma \\ &= \mathcal{B}Q(s, a). \end{aligned} \quad (34)$$

Since  $\mathcal{B}$  is a contraction on the exponentiated distance space, and  $d((s, a), g) \geq d_{\text{SD}}^*((s, a), g)$ ,

Eq. (34) is only consistent with  $Q(s, a) = Q_g^*(s, a)$ . This implies that

$$d((s, a), g) = d_{\text{SD}}^*((s, a), g). \quad (35)$$

We also know that at this fixed point,  $d(s, (s, a)) = 0$ , and thus from Eq. (35) we have

$$d(s, g) = d_{\text{SD}}^*(s, g). \quad (36)$$

So,  $d = d_{\text{SD}}^*$  must be the unique fixed point of  $(\mathcal{P}_* \circ \mathcal{T} \circ \mathcal{I})$ .

□

## C Path Relaxation and Quasimetric Distances

**Lemma 5.** We have  $\mathcal{P}(d) = d$  if and only if  $d \in \mathcal{Q}$ .

*Proof.*  $\mathcal{P}(d)(s, g) = \min_{w \in \mathcal{S}} [d(s, w) + d(w, g)]$  □

$$\begin{aligned} &\leq d(s, s) + d(s, g) \\ &= d(s, g). \end{aligned}$$

**Lemma 6.** The path relaxation operator  $\mathcal{P}$  is continuous with respect to the  $L^\infty$  topology over  $\mathcal{D}$ .

**Lemma 7.** For any  $s, g \in \mathcal{S}$  and  $d \in \mathcal{D}$  we have that  $\mathcal{P}(d)(s, g) \leq d(s, g)$ .

*Proof.* Let  $d, d' \in \mathcal{D}$  and  $\epsilon > 0$ . We have

$$\begin{aligned} |\mathcal{P}(d)(s, g) - \mathcal{P}(d')(s, g)| &= \left| \min_{w \in \mathcal{S}} [d(s, w) + d(w, g)] - \min_{w \in \mathcal{S}} [d'(s, w) + d'(w, g)] \right| \\ &\leq \min_{w \in \mathcal{S}} |d(s, w) + d(w, g) - d'(s, w) - d'(w, g)| \\ &\leq \min_{w \in \mathcal{S}} |d(s, w) - d'(s, w)| + \min_{w \in \mathcal{S}} |d(w, g) - d'(w, g)| \\ &\leq \|d - d'\|_\infty + \|d - d'\|_\infty \\ &= 2\|d - d'\|_\infty. \end{aligned}$$

Thus, if  $\|d - d'\|_\infty < \epsilon/2$ , we have  $\|\mathcal{P}(d) - \mathcal{P}(d')\|_\infty < \epsilon$ . □

**Lemma 5.** We have  $\mathcal{P}(d) = d$  if and only if  $d \in \mathcal{Q}$ .

*Proof.*  $(\Rightarrow)$  Suppose  $\mathcal{P}(d) = d$ . Then, for all  $s, g, w \in \mathcal{S}$  we have

$$d(s, g) = \mathcal{P}(d)(s, g) = \min_{w \in \mathcal{S}} [d(s, w) + d(w, g)] \leq d(s, w) + d(w, g).$$

Thus,  $d \in \mathcal{Q}$ .

$(\Leftarrow)$  Suppose  $d \in \mathcal{Q}$ . Then, for all  $s, g \in \mathcal{S}$  we have

$$d(s, g) \leq \min_{w \in \mathcal{S}} [d(s, w) + d(w, g)] = \mathcal{P}(d)(s, g).$$

We also have  $\mathcal{P}(d)(s, g) \leq d(s, g)$  by Lemma 7. Thus,  $\mathcal{P}(d) = d$ .

□

Table 2: Hyperparameters for TMD

Hyperparameter	Value
batch size	256
learning rate	$3 \cdot 10^{-4}$
discount factor	0.995
invariance weight $\zeta$	0.2

Table 3: Network configuration for TMD.

Hyperparameter	Value
latent dimension size	512
encoder MLP dimensions	(512, 512, 512)
visual encoder (visual- envs)	impala-small
MRN components	8

## 788 D Experimental Details

789 We describe our experiment detail here. Our hyperparameters are entailed in Table 2.

790 We use JAX to implement our method, building upon code first released in OGBench [41]. For our  
 791 behavior cloning weight  $\alpha$ , we used the same value as implemented in OGBench for CRL, with  
 792 the exception for humanoidmaze-large-stitch, in which we used 0.2 instead of 0.1. To prevent  
 793 gradients from overflowing, we clip the  $\mathcal{T}$  invariance loss per component to be no more than 5. We  
 794 also found using the same batch size of 256 in CMD [16] is useful for good performance.

### 795 D.1 Implementation Details

796 The network architecture for TMD is described in Table 3, which are consistent with configurations  
 797 found in OGBench with the exception to the number of MRN components; we found 8 components  
 798 to be generally beneficial for our performance.

### 799 D.2 Ablations

800 We provide additional ablation details here. We examine whether it is useful for us to keep all  
 801 components in TMD, and we use pointmaze-teleport-stitch as a reference environment/dataset  
 802 pair for our ablation.

### 803 D.3 Computational Resources

804 Experiments were run using NVIDIA A6000 GPUs with 48GB of memory, and 4 CPU cores and 1  
 805 GPU per experiment. Each experiment took around 4 hours to run with these resources.

## 806 E Bregman Divergence in $\mathcal{T}$ -invariance

807 Recall the divergence used in Eq. (24):

$$D_T(d, d') \triangleq \exp(d - d') - d. \quad (24)$$

Table 4: Ablation Details..

Ablation	Success Rate
None	$29.3^{(\pm 2.2)}$
encoder MLP dimensions	(512, 512, 512)
visual encoder (visual- envs)	impala-small
MRN components	8



Comparison of Bregman divergences for fitting distances

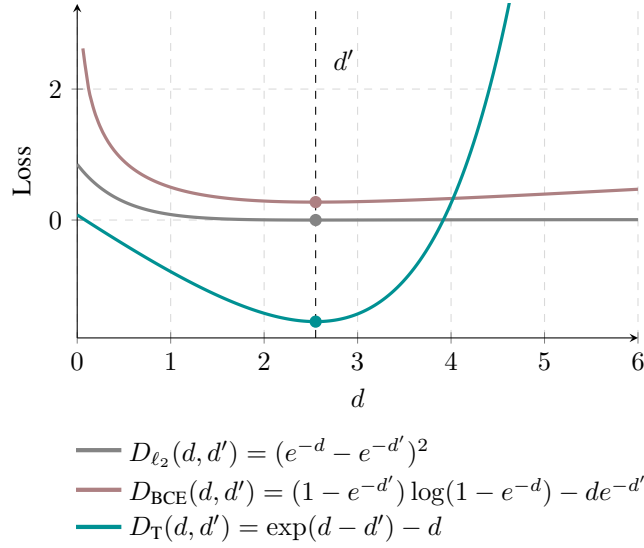


Figure 4: Comparison of Bregman divergences for  $e^{-d}$  onto  $e^{-d'}$  in expectation. All losses are minimized at  $d = d'$ , and share the property that they will be minimized in expectation when  $e^{-d} = \mathbb{E}[e^{-d'}]$ . But only the  $D_T(d, d')$  loss has non-vanishing gradients  $d \gg d'$  for large  $d'$ .

808 This divergence is proportional to the Bregman divergence [36] for the function  $F(x) = -\log(x)$ ,  
 809 similar to the Itakura-Saito divergence [37].

$$\begin{aligned}
 D_F(e^{-d'}, e^{-d}) &= F(e^{-d'}) - F(e^{-d}) - F'(e^{-d})(e^{-d'} - e^{-d}) \\
 &= d' - d + \frac{1}{e^{-d}}(e^{-d'} - e^{-d}) \\
 &= d' - d + \exp(d - d') - 1 \\
 &= D_T(d, d') + d' - 1.
 \end{aligned} \tag{37}$$

810 The minimizer of Eq. (24) satisfies

$$\arg \min_{d \geq 0} \mathbb{E}_{d'}[D_T(d, d')] = -\log \mathbb{E}_{d'}[e^{-d'}] \tag{38}$$

811 when  $d'$  is a random “target” distance [45]. In other words, using Eq. (24) as a loss function regresses  
 812  $e^{-d}$  onto the expected value of  $e^{-d'}$ .

813 The key advantage of this divergence when backing up temporal distances is that the gradients do not  
 814 vanish when either  $d$ ,  $d'$ , or the difference between them is small or large. This property is *not* shared  
 815 by more standard loss functions like the squared loss or binary cross-entropy loss when applied to the  
 816 probability space and the models (distances) are in log-probability space.

## 817 F Algorithm Pseudocode

---

**Algorithm 1:** Temporal Metric Distillation (TMD)

---

```
1: input: dataset  $\mathcal{D}$ 
2: initialize representations  $\phi, \psi$ , policy  $\pi$ 
3: while training do
4:   sample  $\mathcal{B} = \{s_i, a_i, s'_i, g_i\}_{i=1}^N \sim \mathcal{D}$ 
5:    $\bar{\psi} \leftarrow \psi$ 
6:    $(\phi, \psi) \leftarrow \nabla_{\phi, \psi} \mathcal{L}_{\text{TMD}}(\phi, \psi; \bar{\psi}, \mathcal{B})$   $\triangleright$  Eq. (26)
7:    $\pi \leftarrow \nabla_{\pi} \mathcal{L}_{\pi}(\phi, \psi, \pi; \mathcal{B})$   $\triangleright$  Eq. (27)
8: return  $\pi$ 
```

---