

## Proof of the derivative of sigmoid function

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$$f(x) = \frac{1}{1 + e^{-x}} \longrightarrow f'(x) = f(x)(1 - f(x))$$

Handwritten proof of the derivative of the sigmoid function:

$$\begin{aligned}\text{sigmoid}(x) &= \frac{1}{1+e^{-x}} \longrightarrow f'(x) = f(x)(1-f(x)) \\ (\text{미분공식}) \\ \frac{d}{dx} \text{sigmoid}(x) &= \frac{d}{dx} (1+e^{-x})^{-1} \\ &= (-1) \frac{1}{(1+e^{-x})^2} \frac{d}{dx} (1+e^{-x}) \\ &= (-1) \frac{1}{(1+e^{-x})^2} (0+e^{-x}) \frac{d}{dx} (-x) \\ &= (-1) \frac{1}{(1+e^{-x})^2} e^{-x} (-1) \\ &= \frac{e^{-x}}{(1+e^{-x})^2} \\ &= \frac{1+e^{-x}-1}{(1+e^{-x})^2} \\ &= \frac{(1+e^{-x})}{(1+e^{-x})^2} - \frac{1}{(1+e^{-x})^2} \\ &= \frac{1}{1+e^{-x}} - \frac{1}{(1+e^{-x})^2} \\ &= \frac{1}{1+e^{-x}} \left(1 - \frac{1}{1+e^{-x}}\right) \\ &= \text{sigmoid}(x)(1 - \text{sigmoid}(x)) \\ \therefore f'(x) &= f(x)(1-f(x))\end{aligned}$$