데이터사이언스응용
(Capstone design)

김응희

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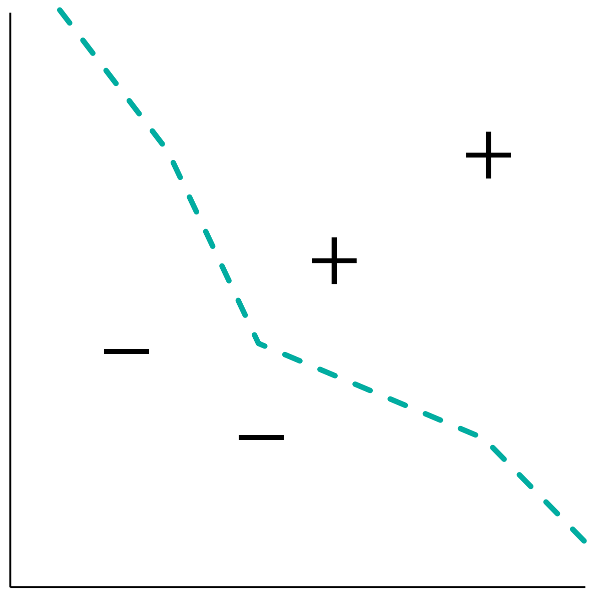
Week 08

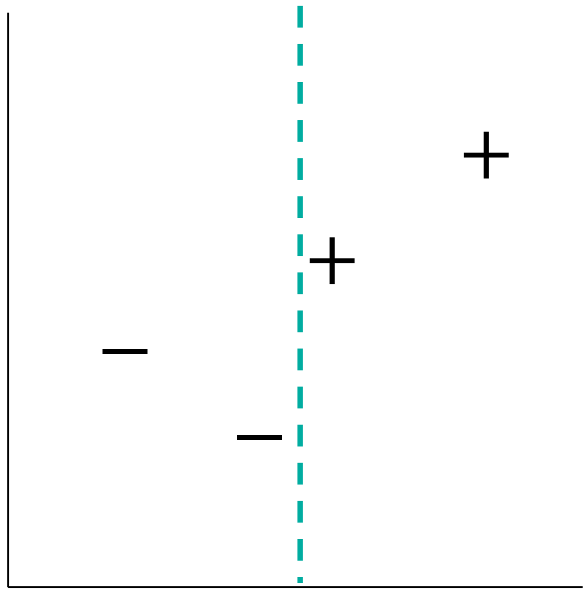
향후 수업 진행 방식 공지

방침	**격주** 등교
의도	등교 학생 수 **$^1/_2$선** 유지 → 코로나 **확산 방지**
예외	20명 이하 수강생 교과목
데이터사이언스응용 수업 요일	수요일 & 금요일
강의실	원화관 605호

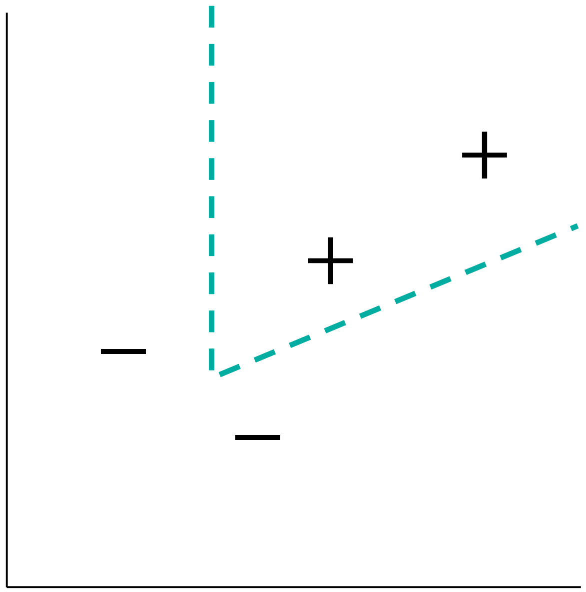
Ways classifying a given dataset



k −NN



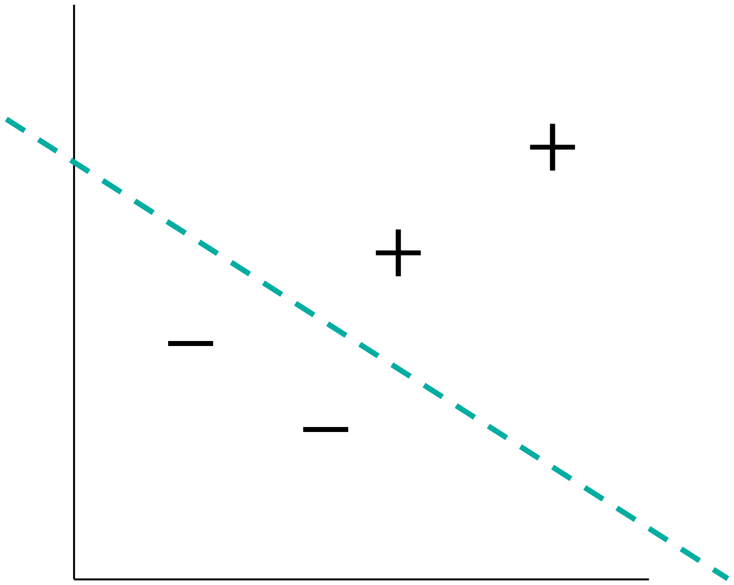
Decision tree/
Perceptron



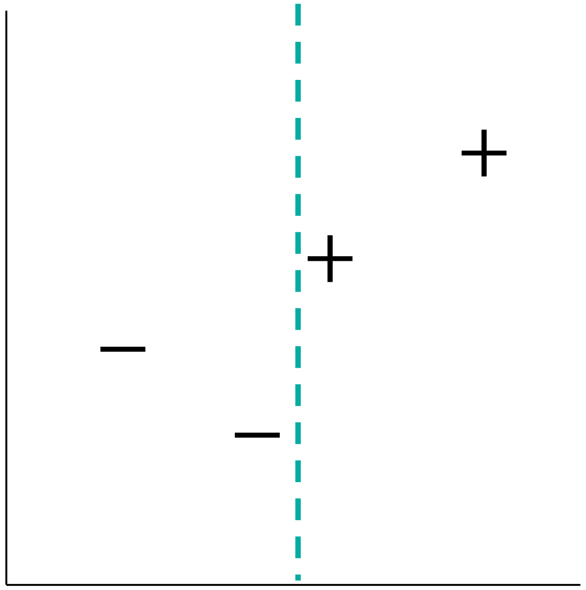
ANN

Support vector machine

Vladimir Vapnik



SVM



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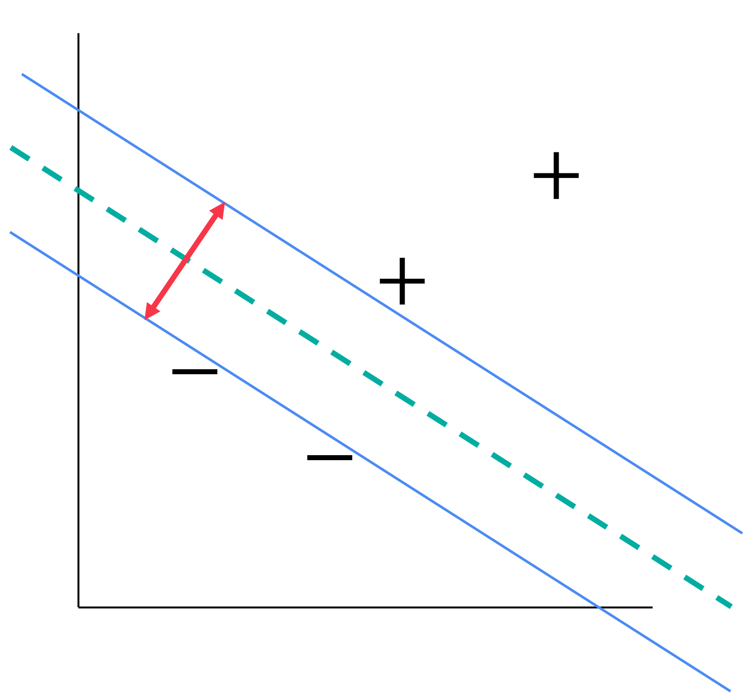

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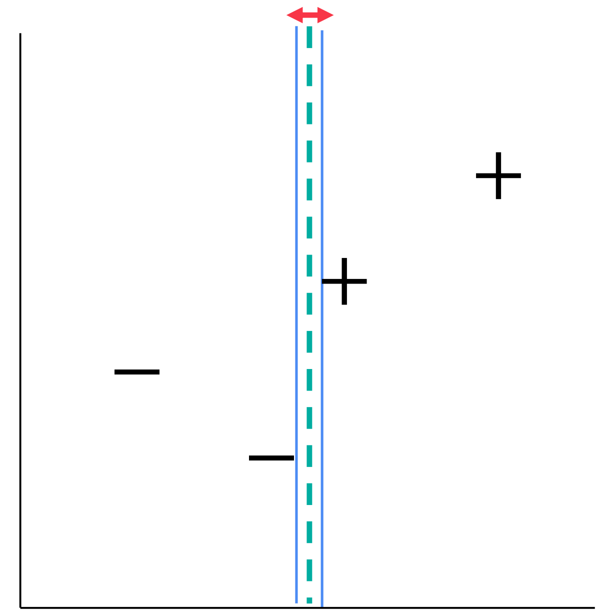

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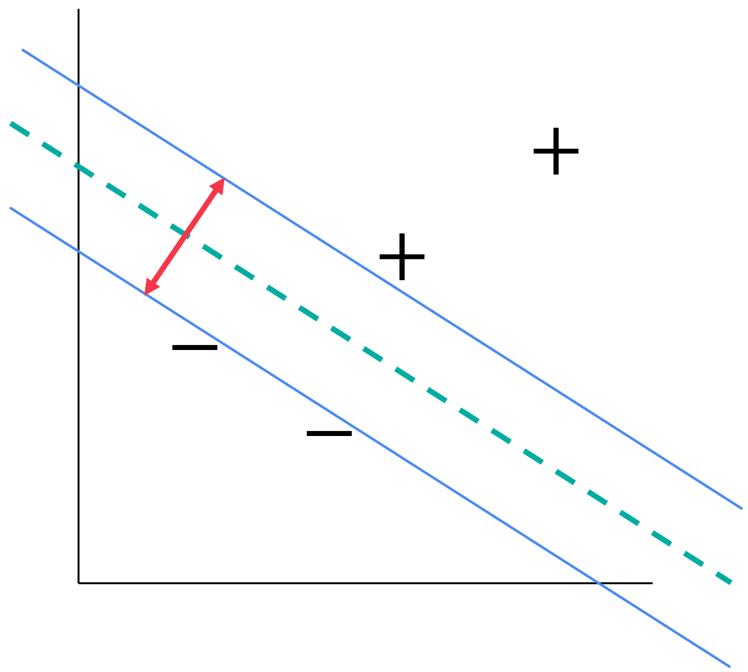

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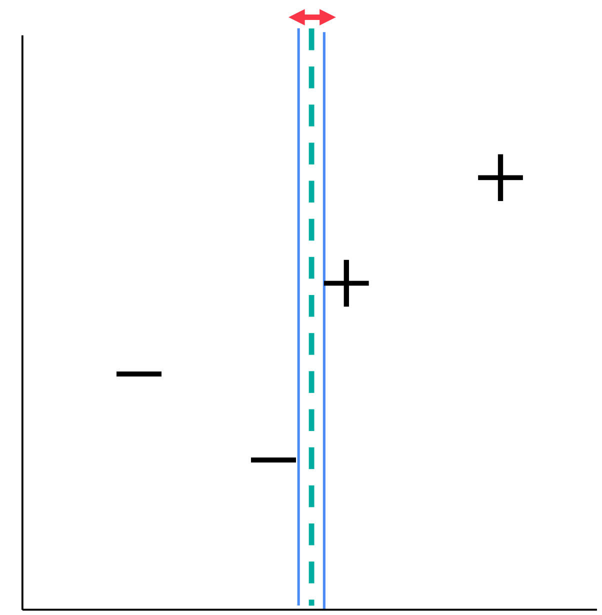
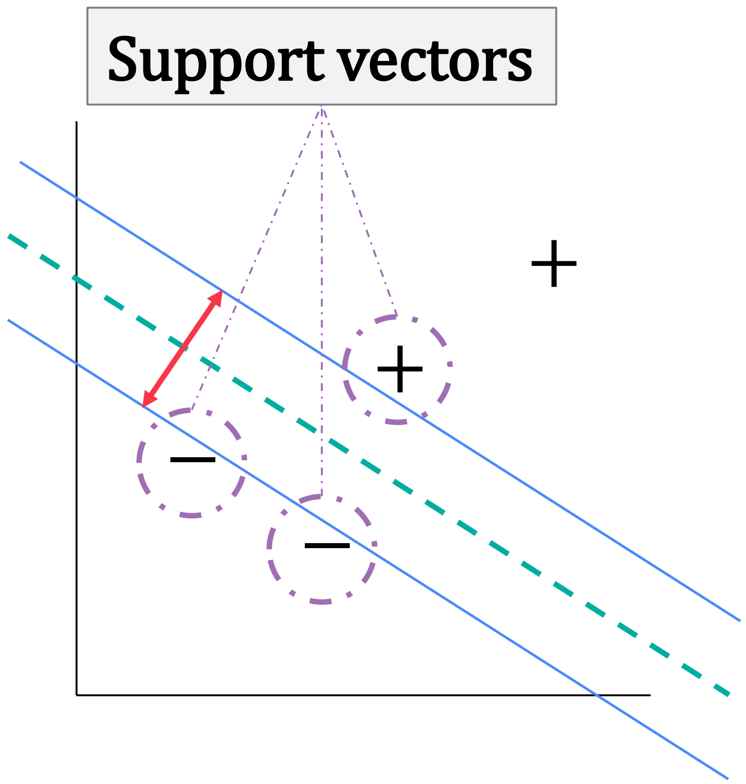


Decision tree/
Perceptron

Goal of machine learning

Optimization | **Generalization**

Support vector machine



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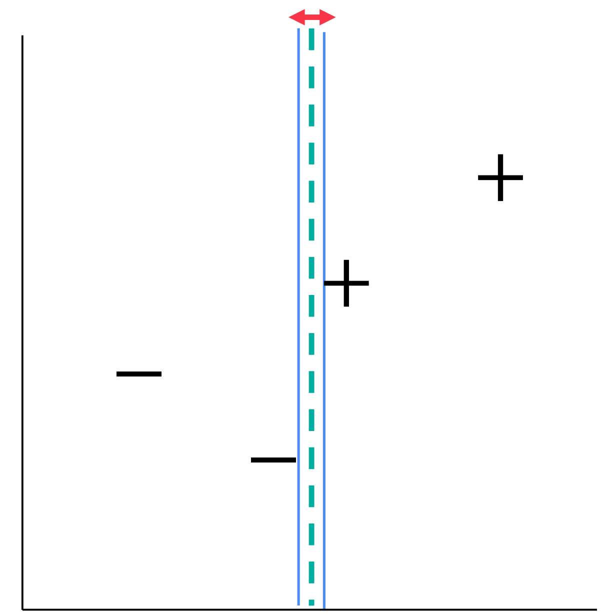

Decision tree/
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Optimization **Generalization**

Prerequisites for SVM

- Vector & dot product
- Equation of lines and planes
- Distance from a point to a plane
- Constrained optimization

Prerequisites for SVM

- Vector & dot product
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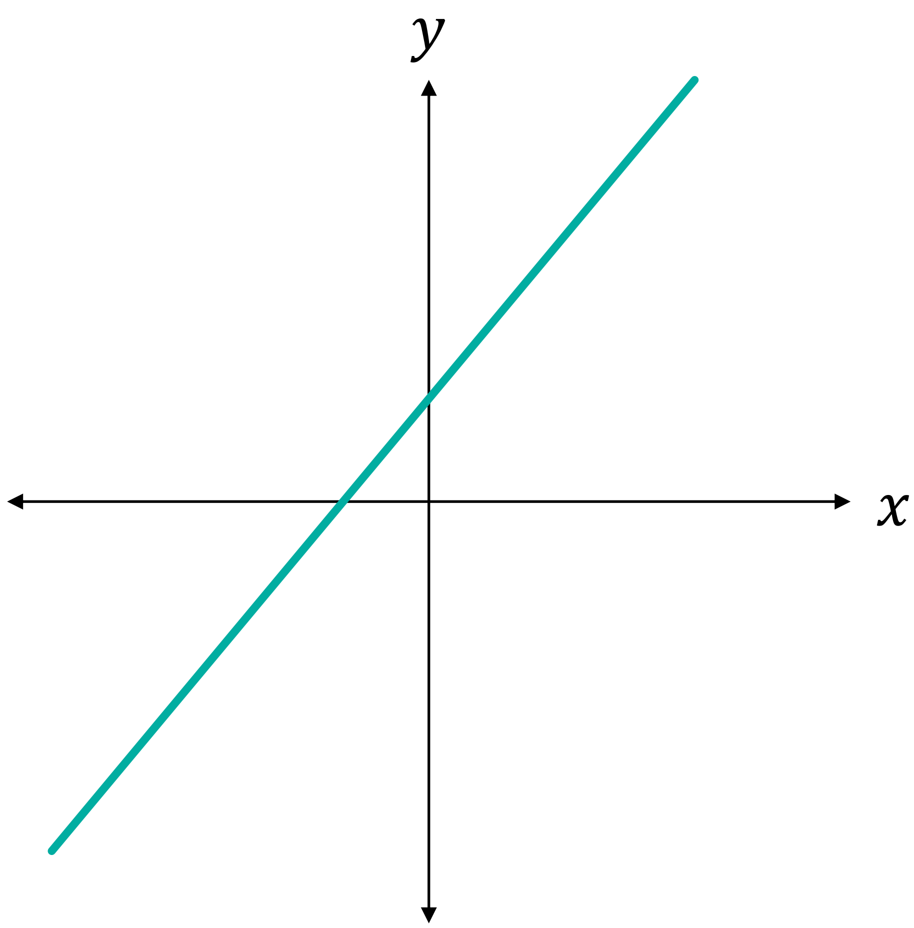
Contents of this week

- Equation of lines and planes
- Distance from a point to a plane
- Support vector machine part I
- Constrained optimization
- Support vector machine part II

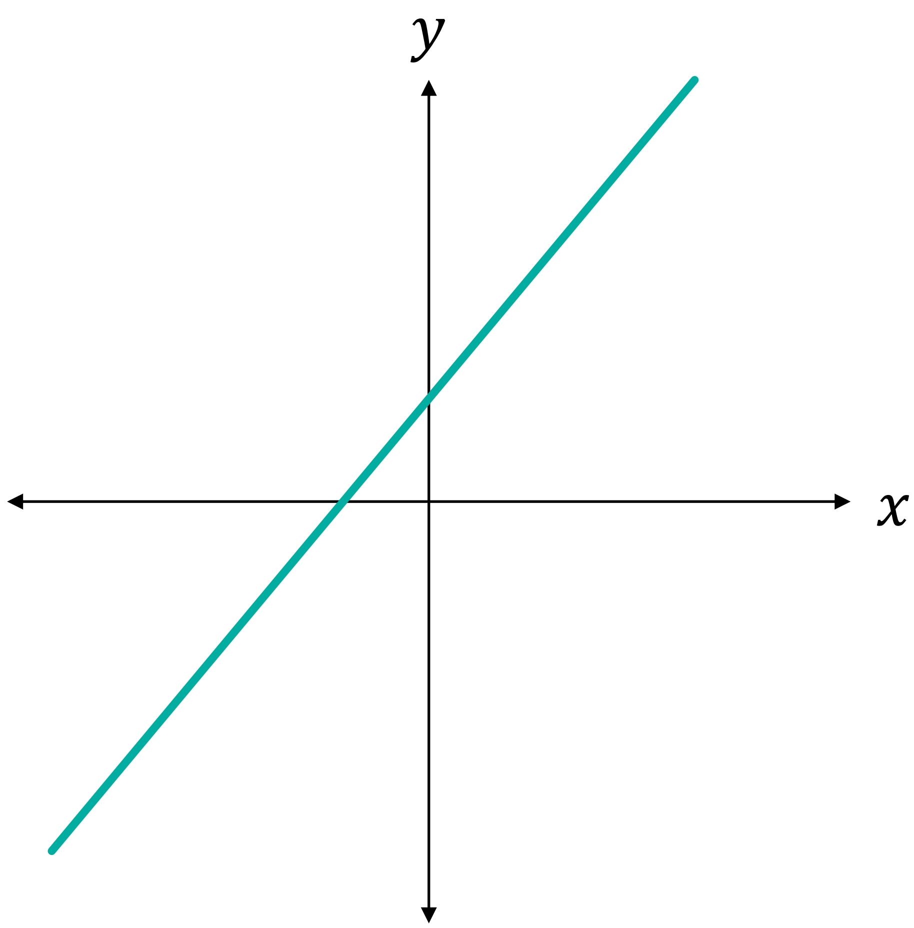
Equation of lines and planes

Forms of linear equations



Slope-intercept	$y = mx + b$	m: slope b: y-intercept
Point-slope	$y - y_0 = m(x - x_0)$	m: slope (x_0, y_0): a point on the line
Standard	$ax + by = c$	a, b, c: constants

Forms of linear equations



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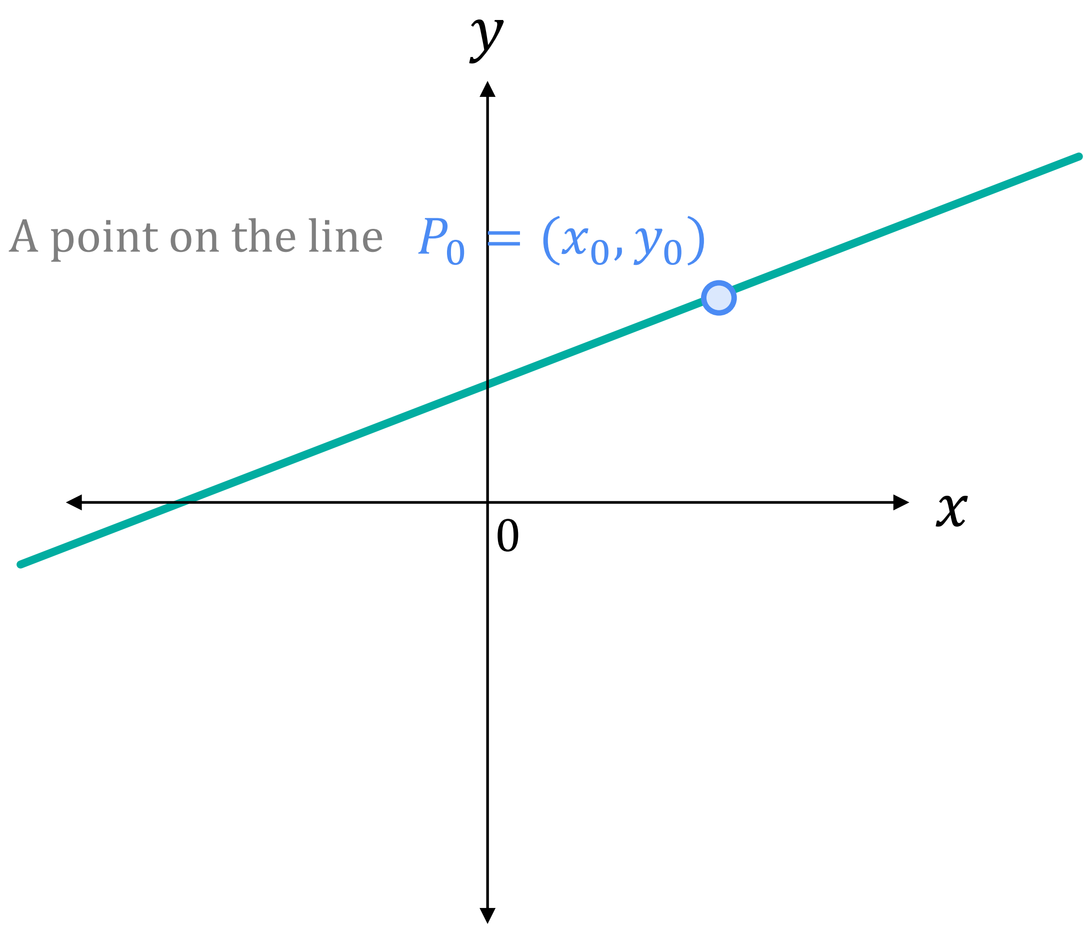
Vector equation of a line

Vector equation of a line

- Ingredients: 2 vectors
  - Position vector (point on the line)
  - Direction vector



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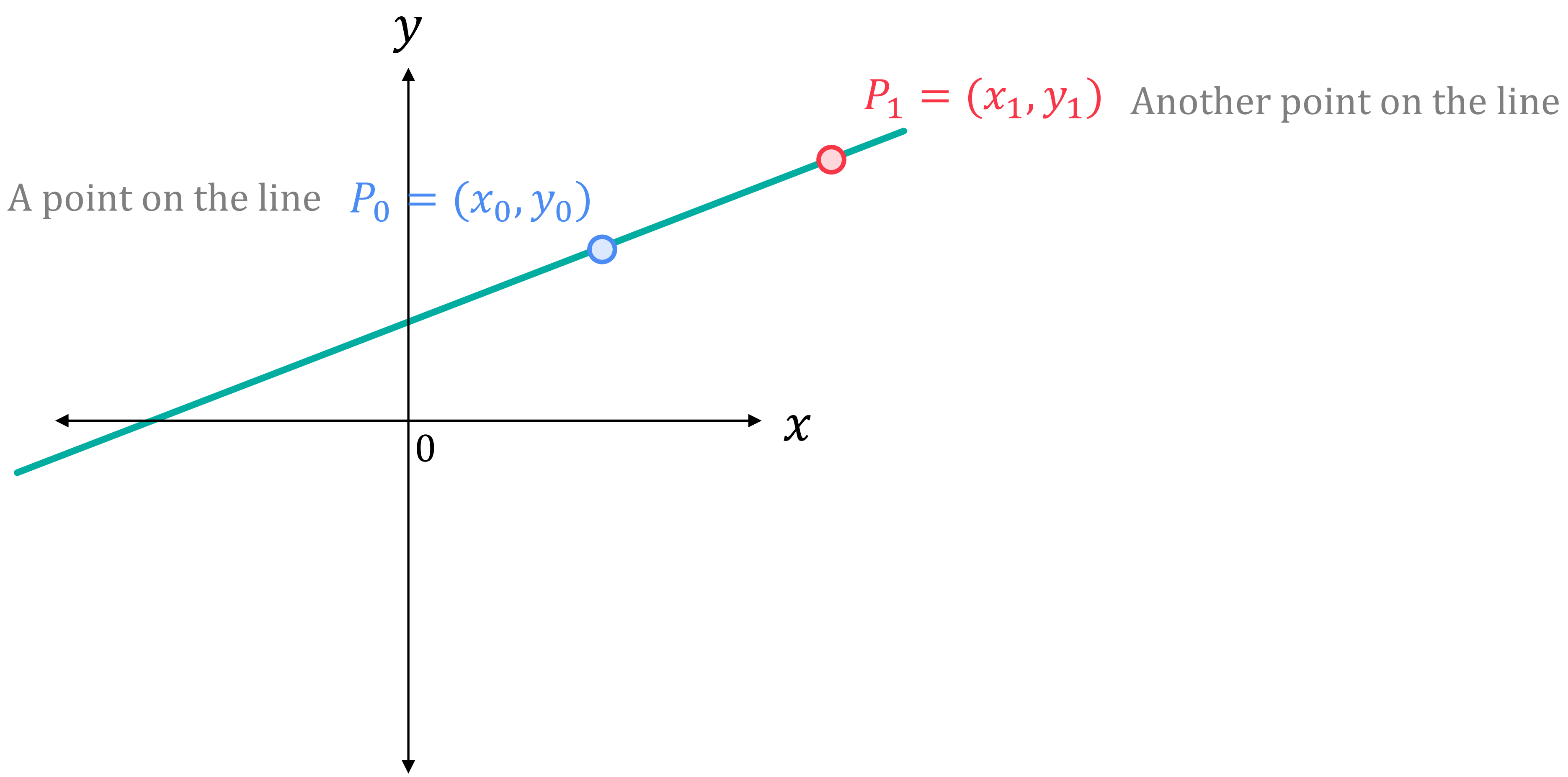

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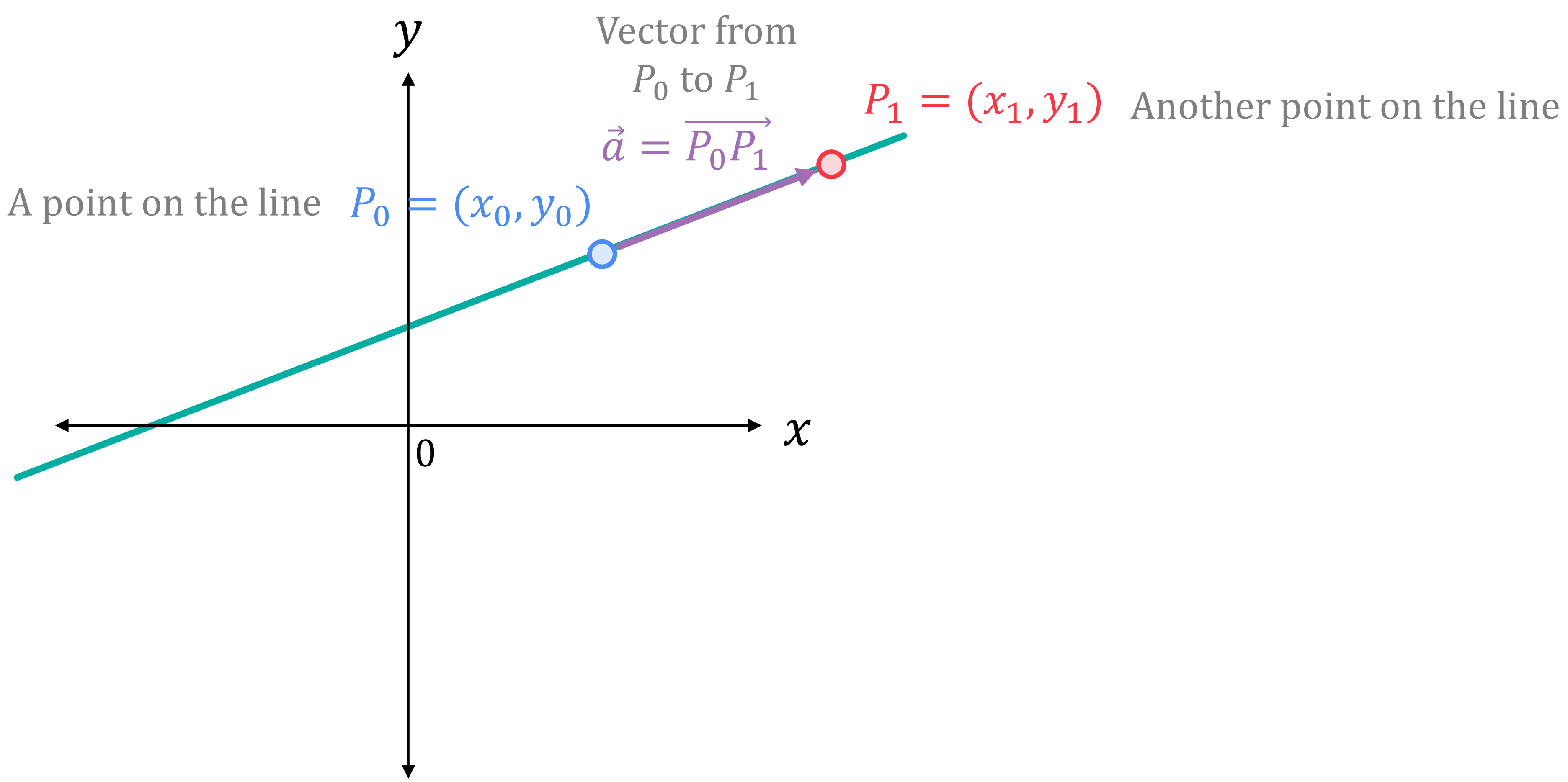

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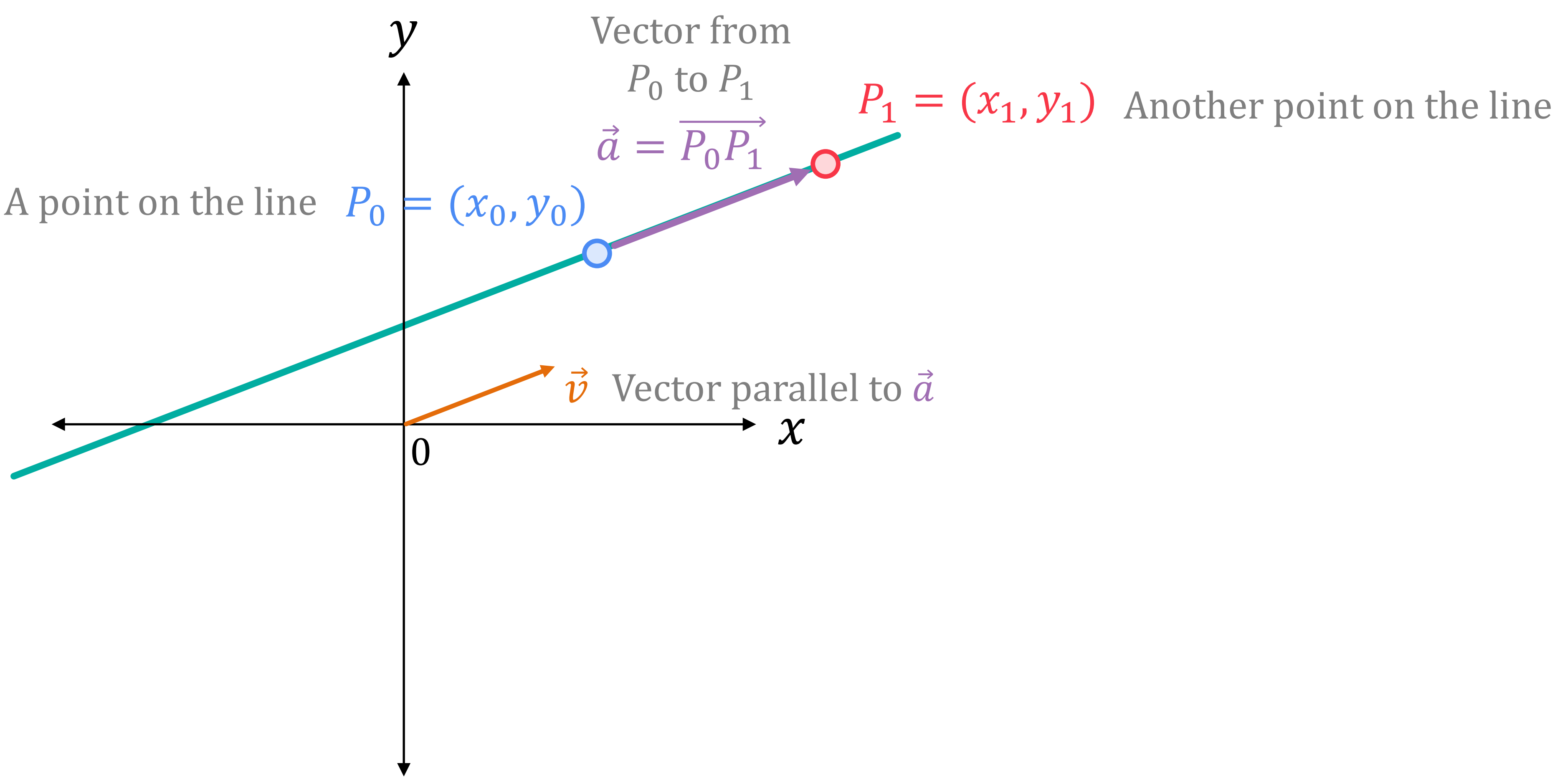

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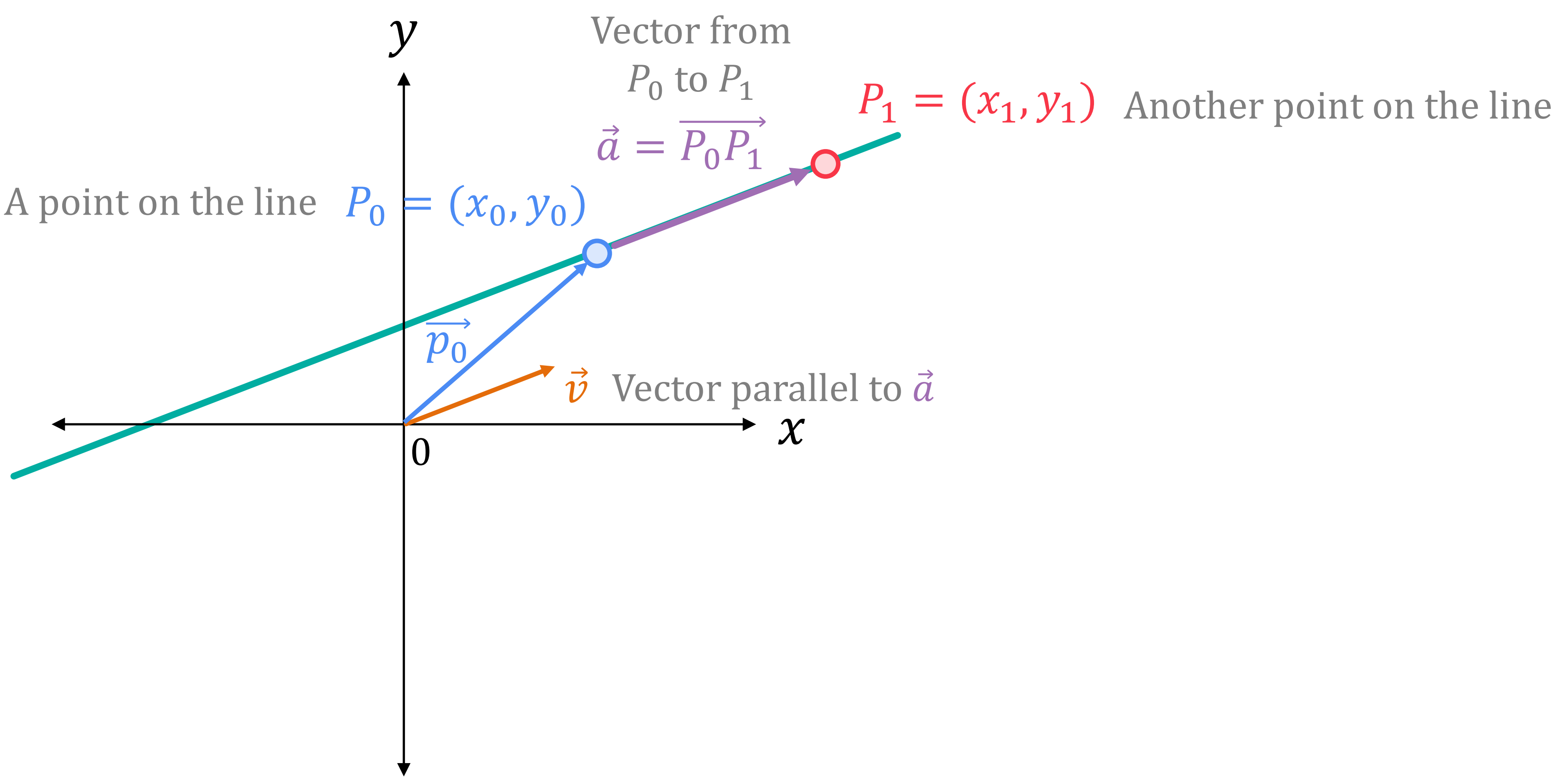

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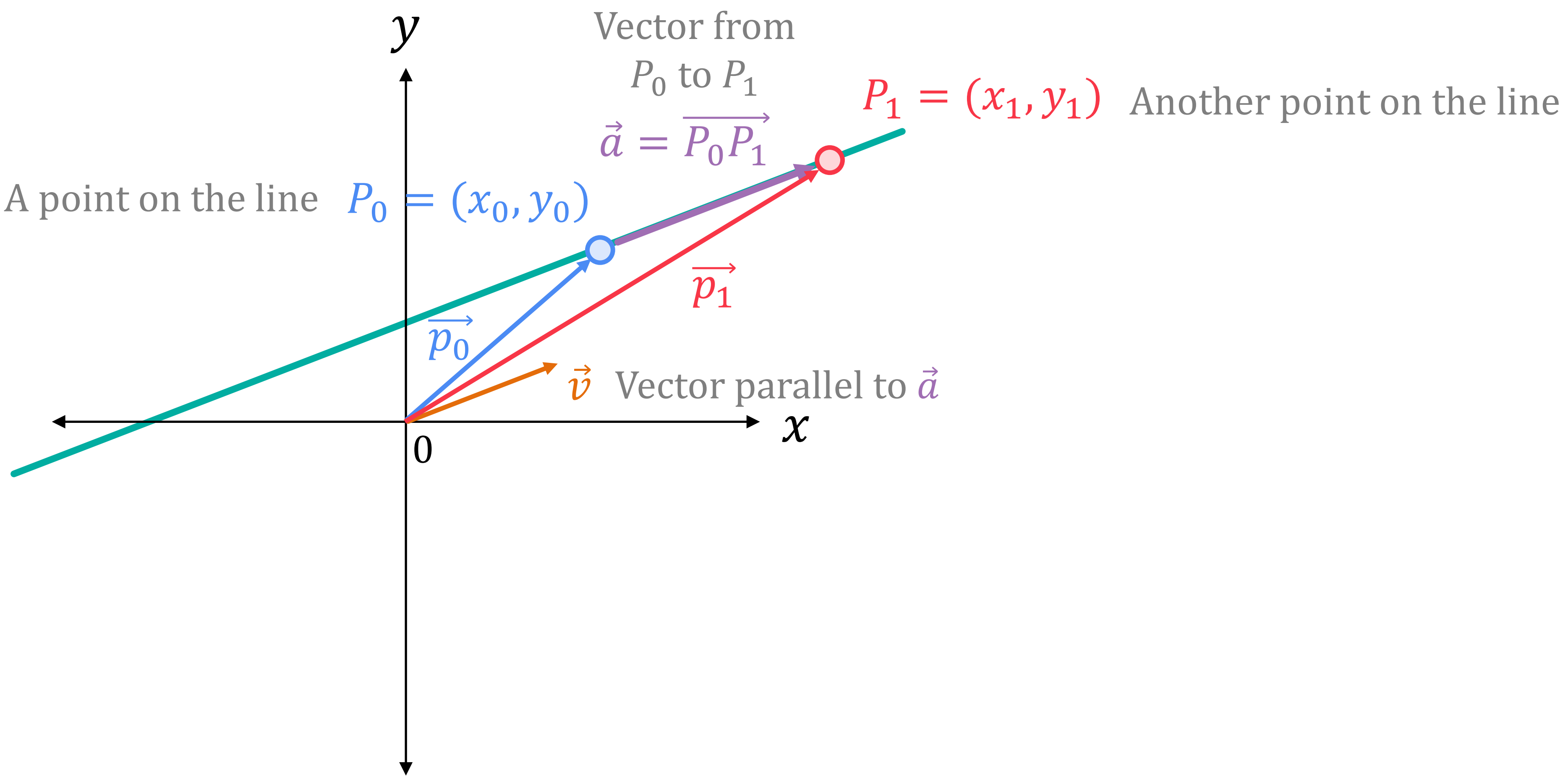

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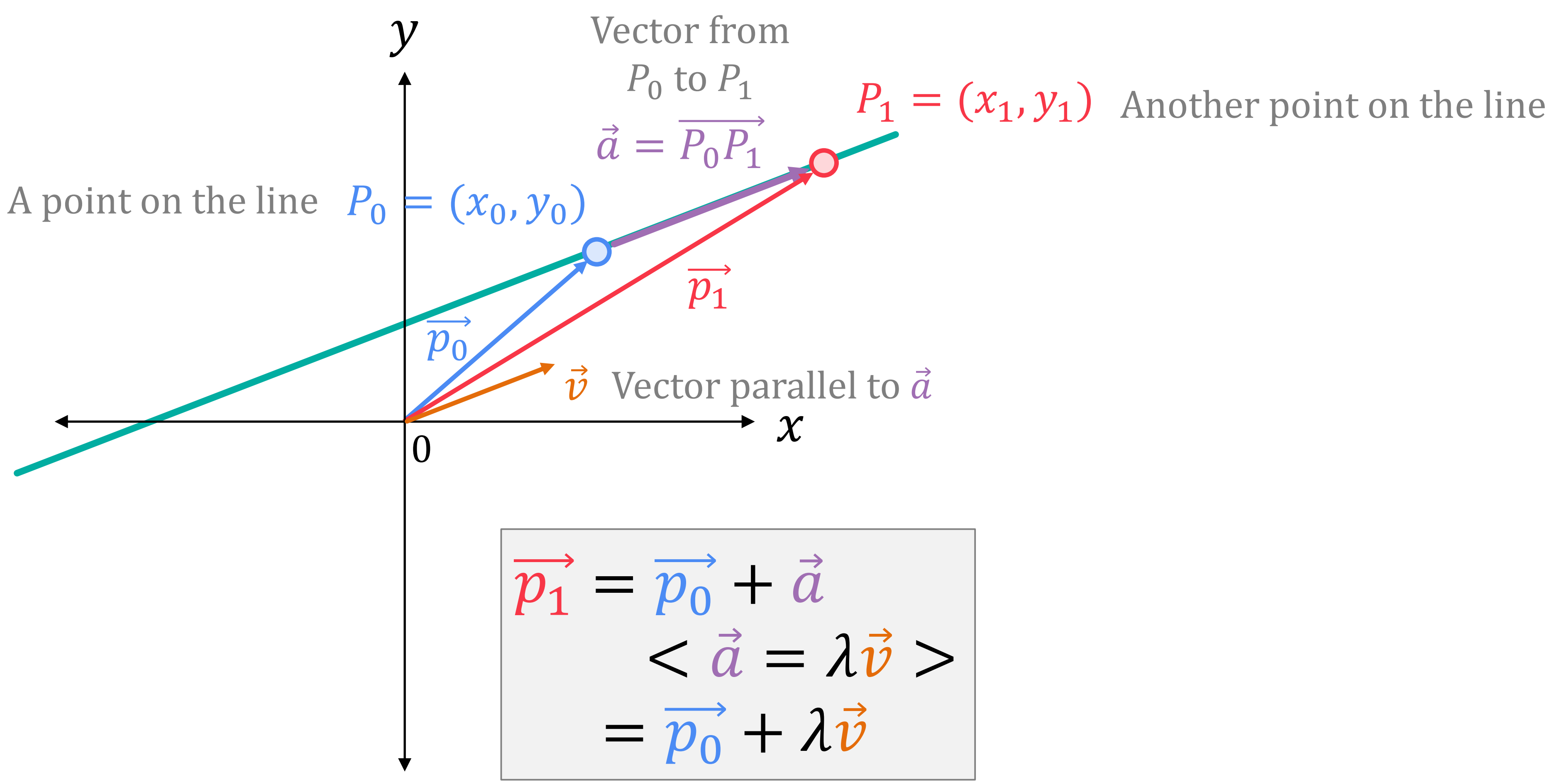

$$\overrightarrow{p_1} = \overrightarrow{p_0} + \vec{a}$$
$$< \vec{a} = \lambda\vec{v} >$$
$$= \overrightarrow{p_0} + \lambda\vec{v}$$

Vector equation of a line

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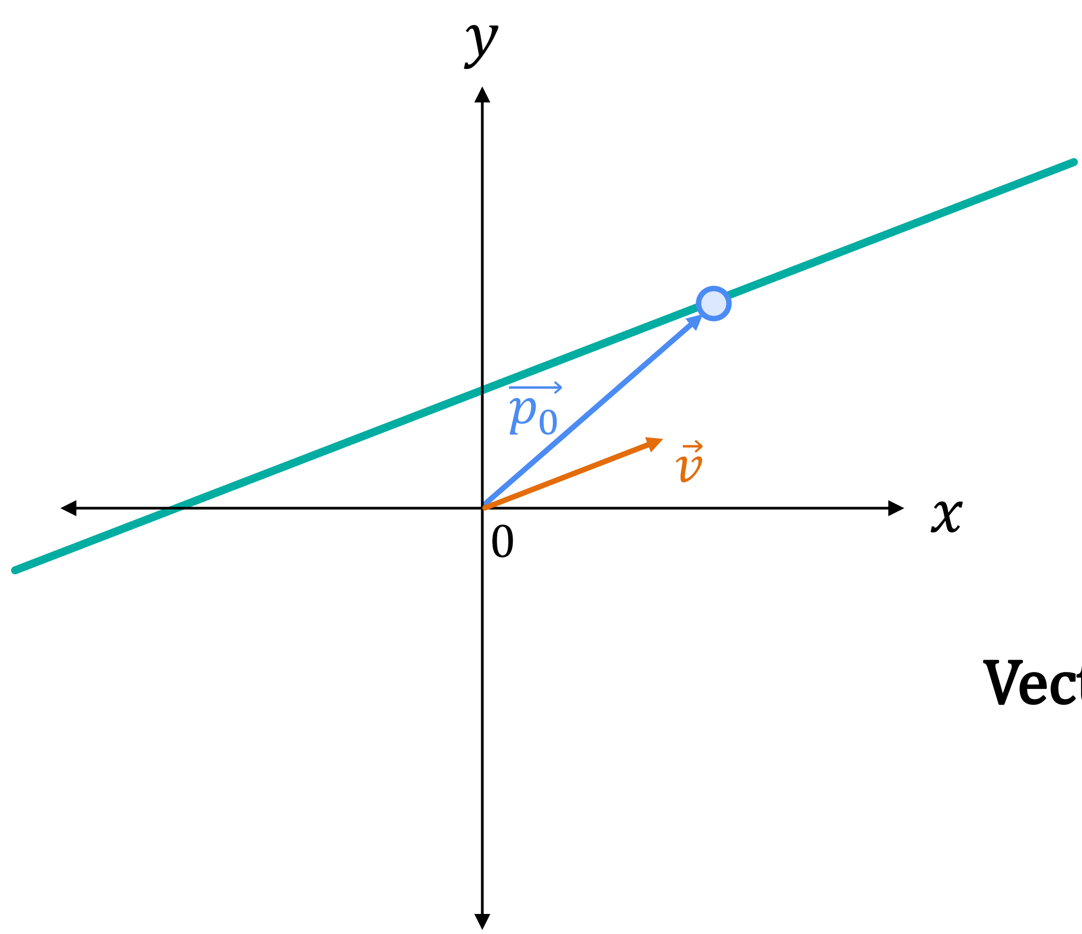

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$$\boxed{\vec{p} = \overrightarrow{p_0} + \lambda \vec{v}}$$

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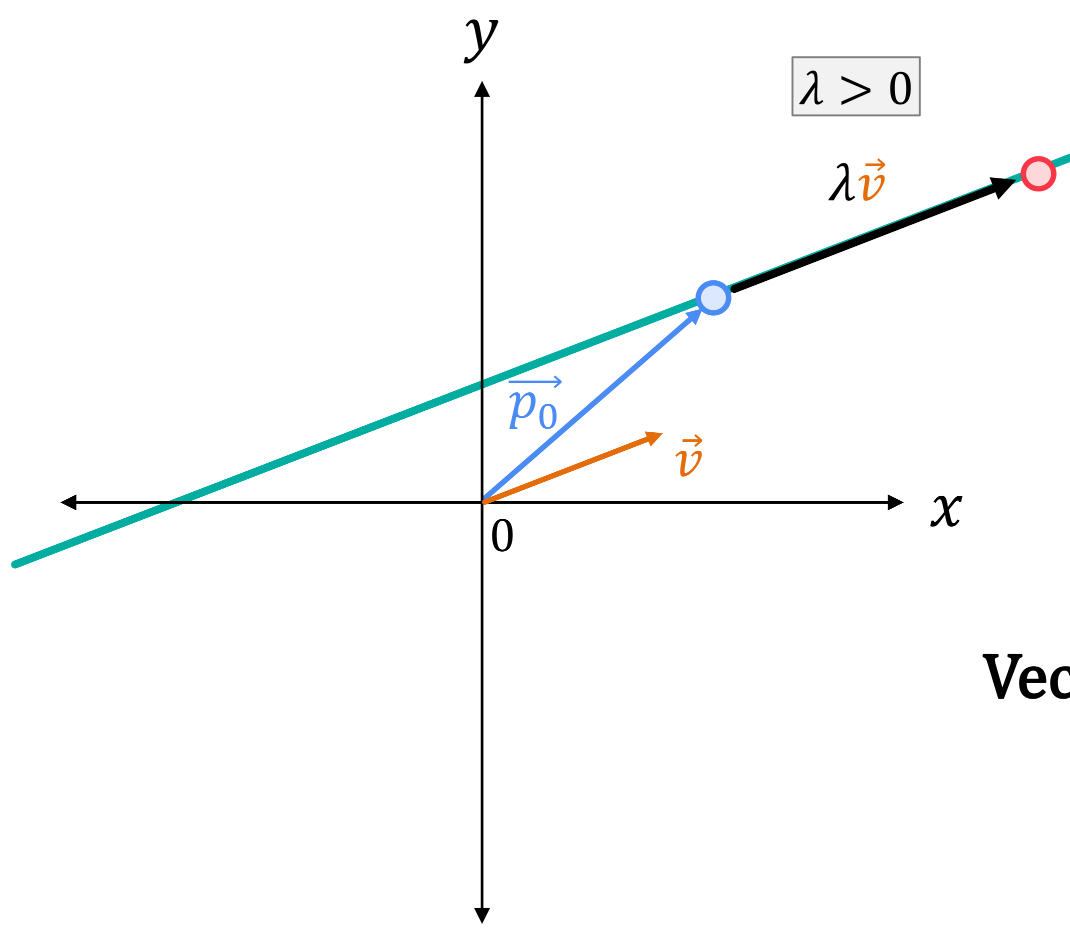

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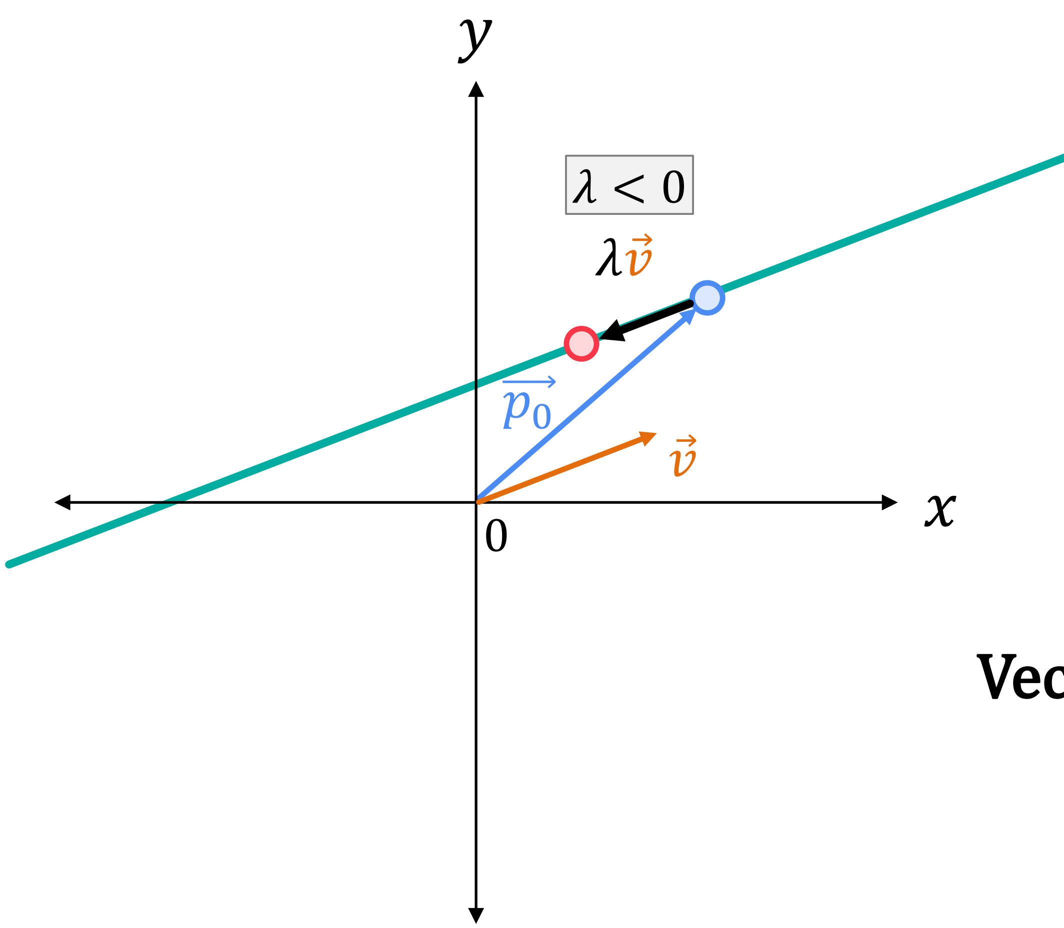

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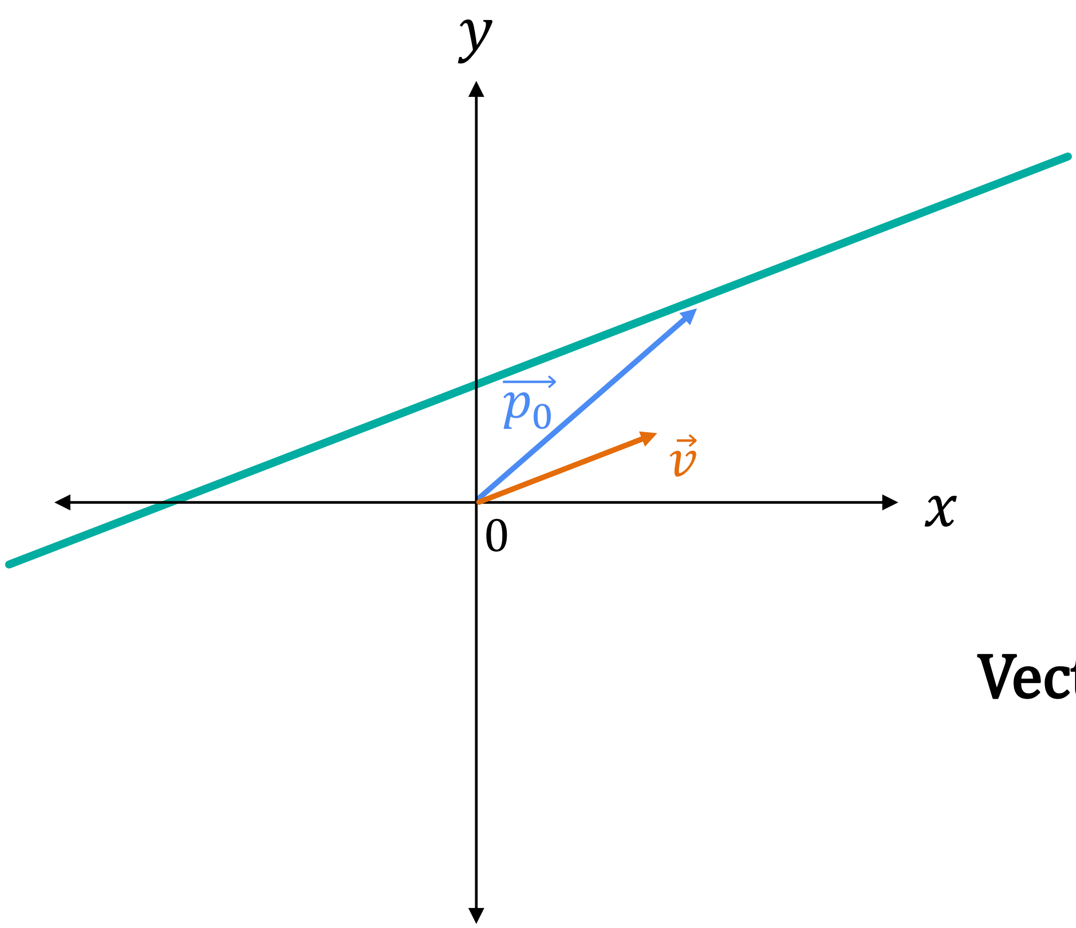

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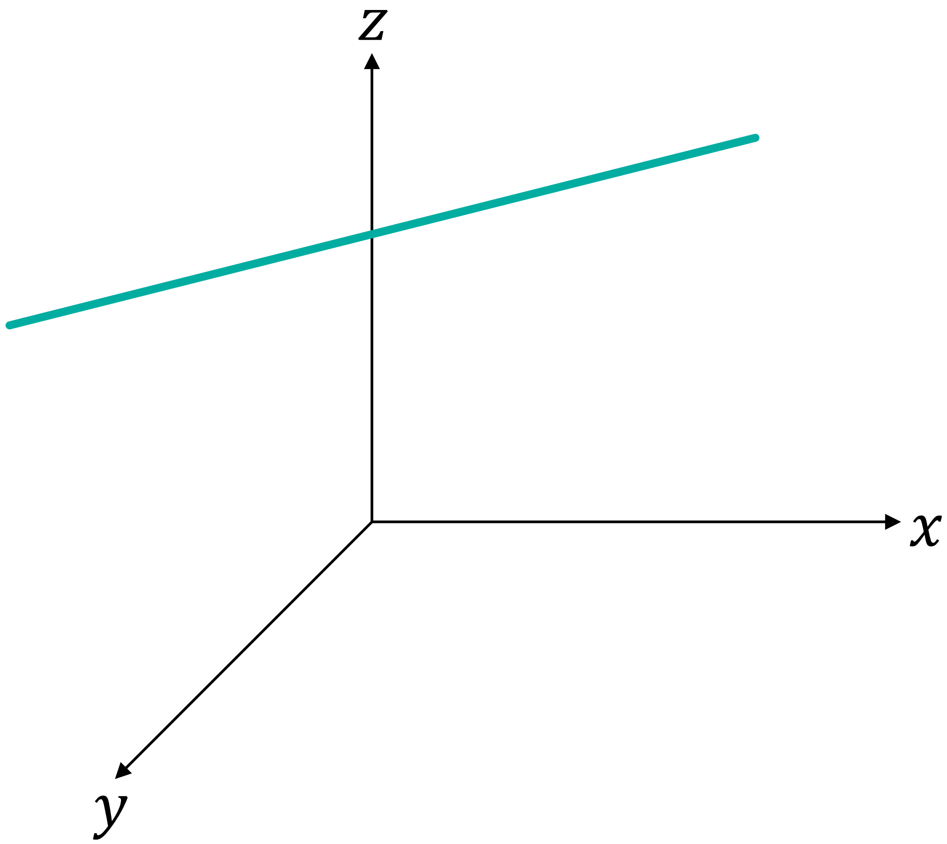

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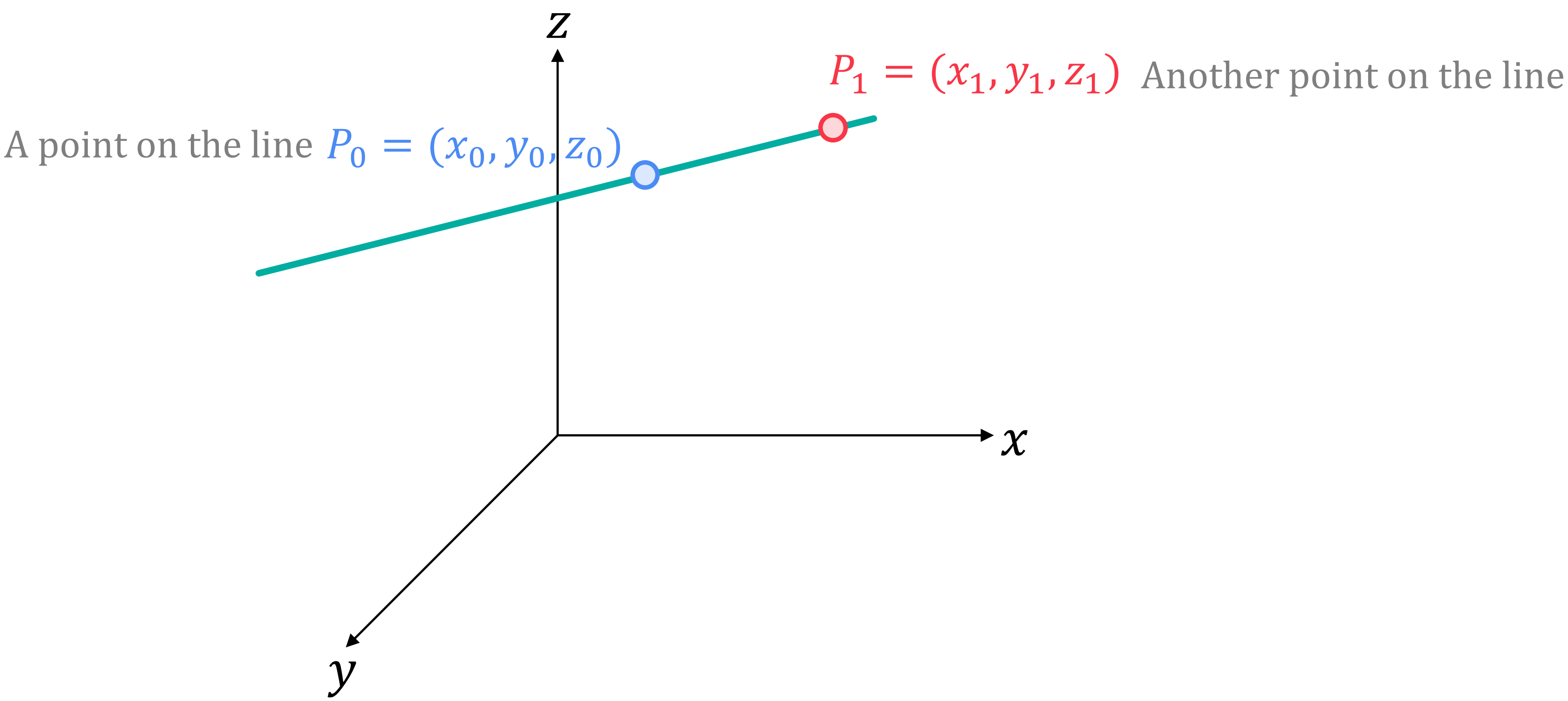

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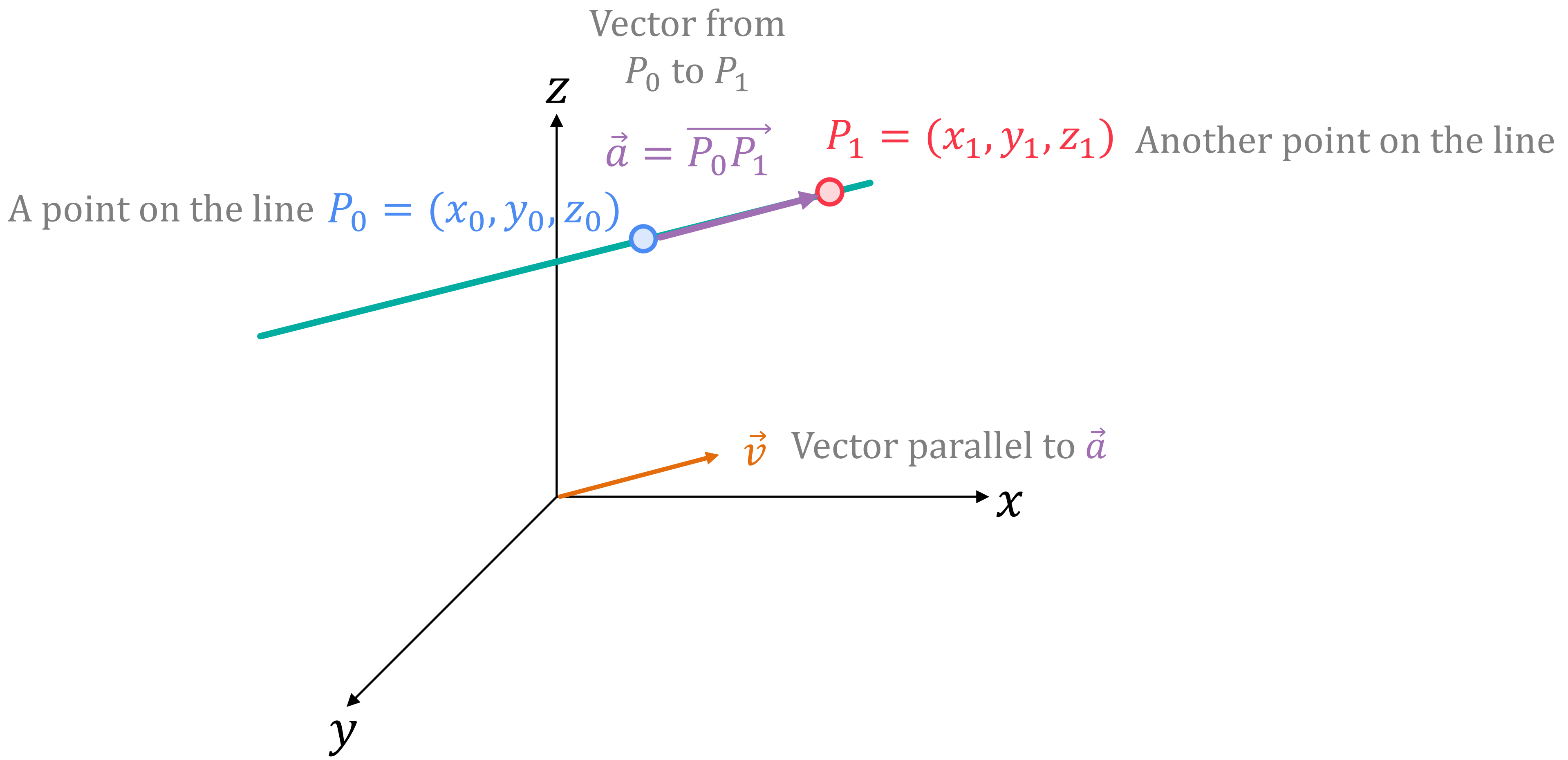

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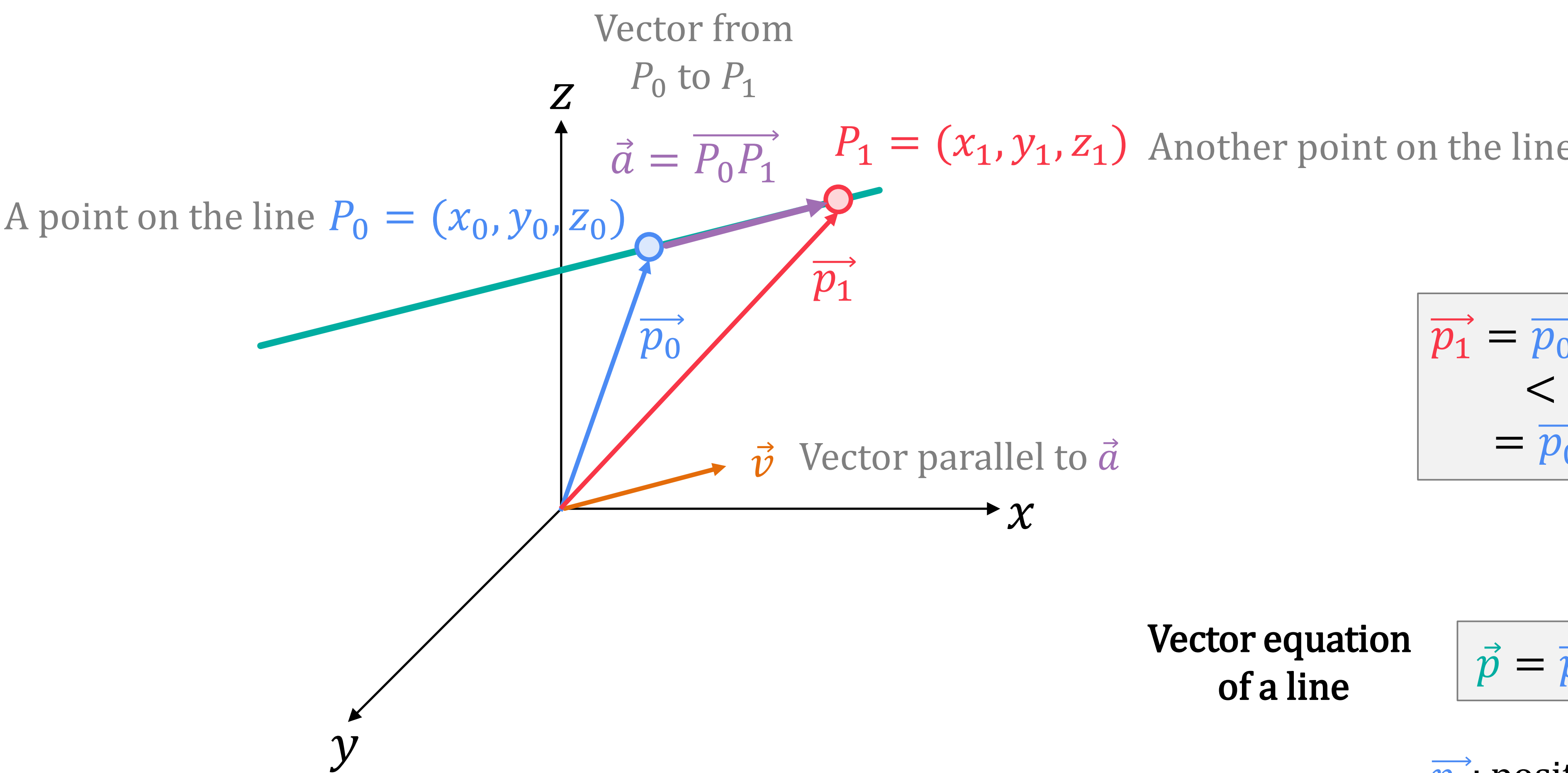

$$
\begin{aligned}
\overrightarrow{p_1} &= \overrightarrow{p_0} + \vec{a} \\
&\quad < \vec{a} = \lambda\vec{v} > \\
&= \overrightarrow{p_0} + \lambda\vec{v}
\end{aligned}
$$

Vector equation of a line

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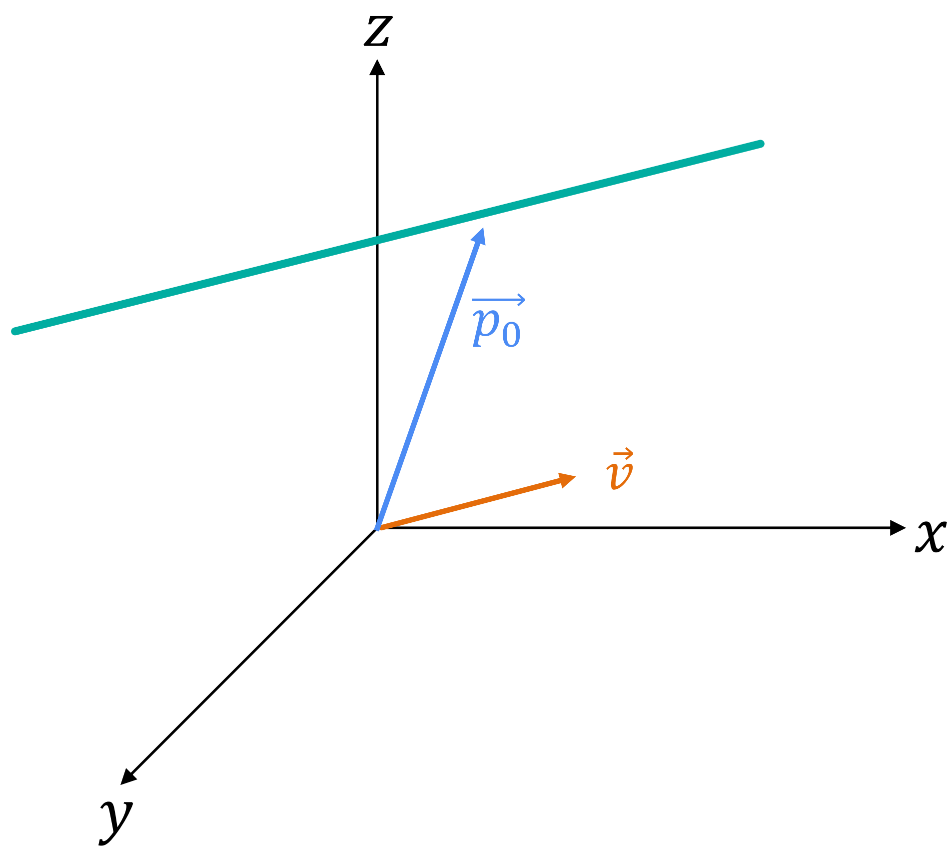

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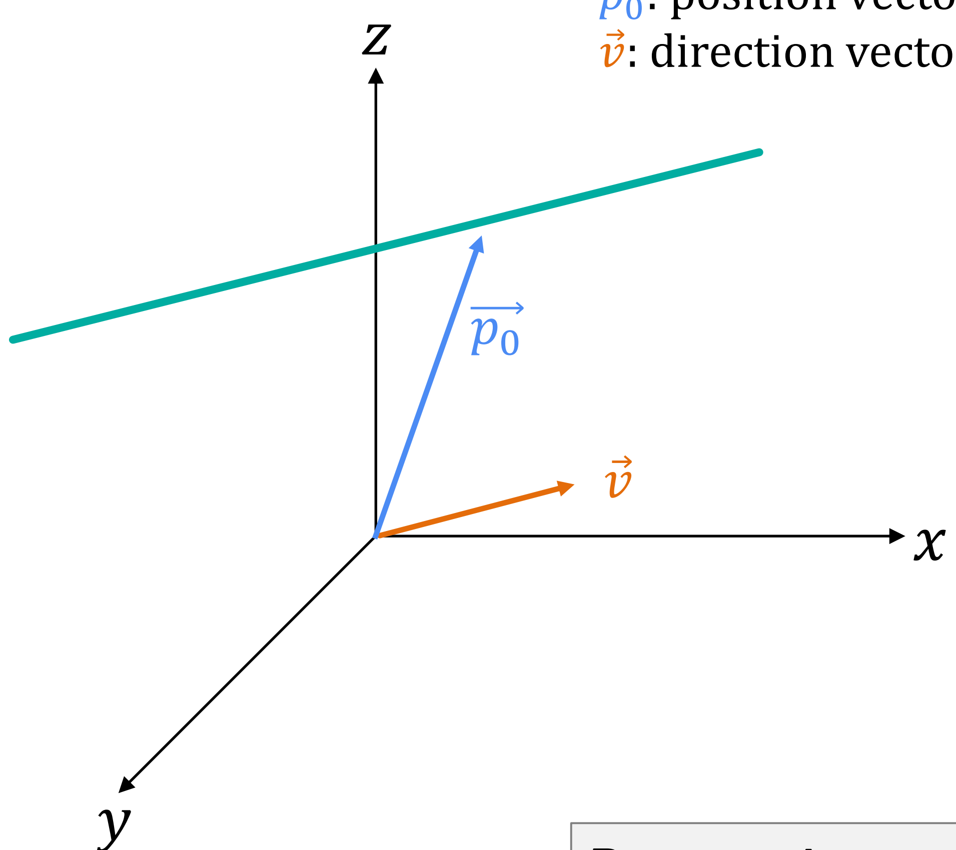


$$\vec{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad \overrightarrow{p_0} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \qquad \vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + \lambda \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_0 + \lambda a \\ y_0 + \lambda b \\ z_0 + \lambda c \end{bmatrix}$$

$$\downarrow$$

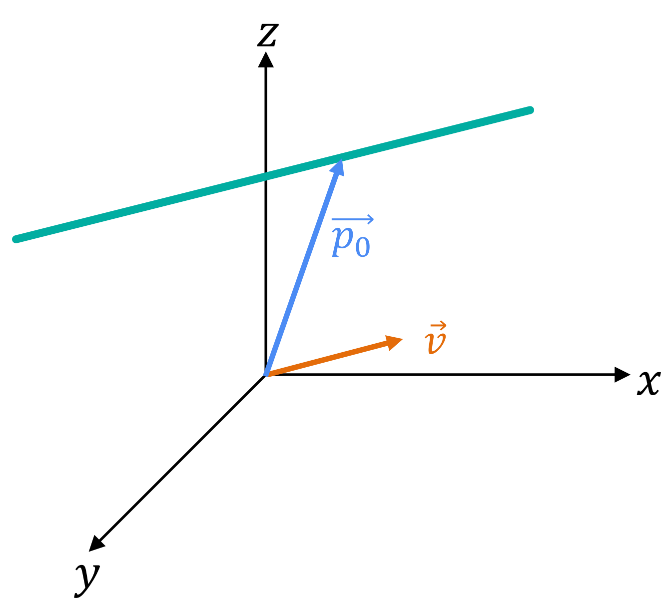
$$x = x_0 + a\lambda$$
$$y = y_0 + b\lambda$$
$$z = z_0 + c\lambda$$

Parametric equations for
a line through (x_0, y_0, z_0) and
parallel to direction vector $< a, b, c >$

From lines to planes

What we need to define a line
(in any dimensions)

1. Position vector (on the line)
2. Direction vector (parallel to the line)



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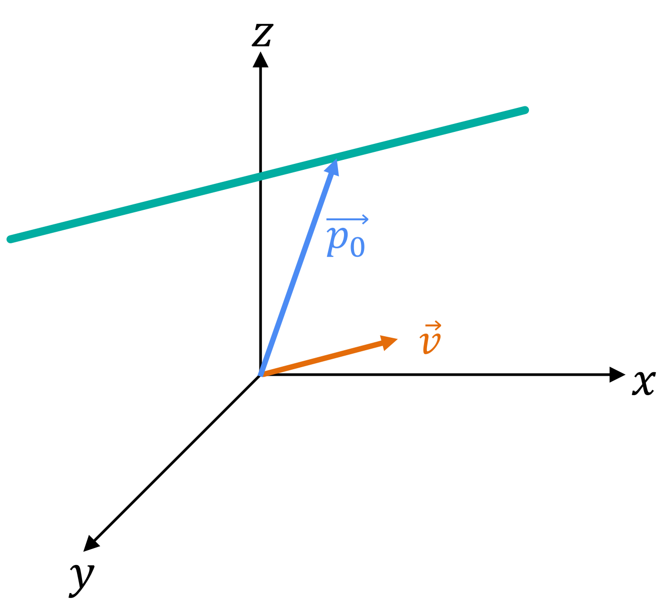
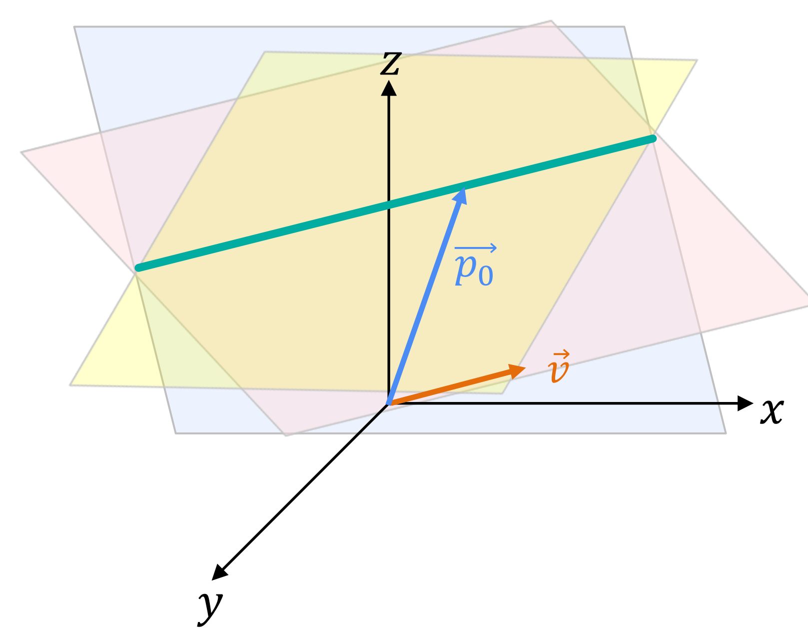


What we need to define a plane
(in any dimensions)

1. Position vector (on the plane)
2. Direction vector (parallel to the line)



Equations of a plane

What we need to define a plane
(in any dimensions)

1. Position vector (on the plane)
2. **Perpendicular vector (to the plane)**

Equation of a plane

- Position vector (point on the plane)
- Perpendicular vector (to the plane)

Equation of a plane

- Position vector (point on the plane)
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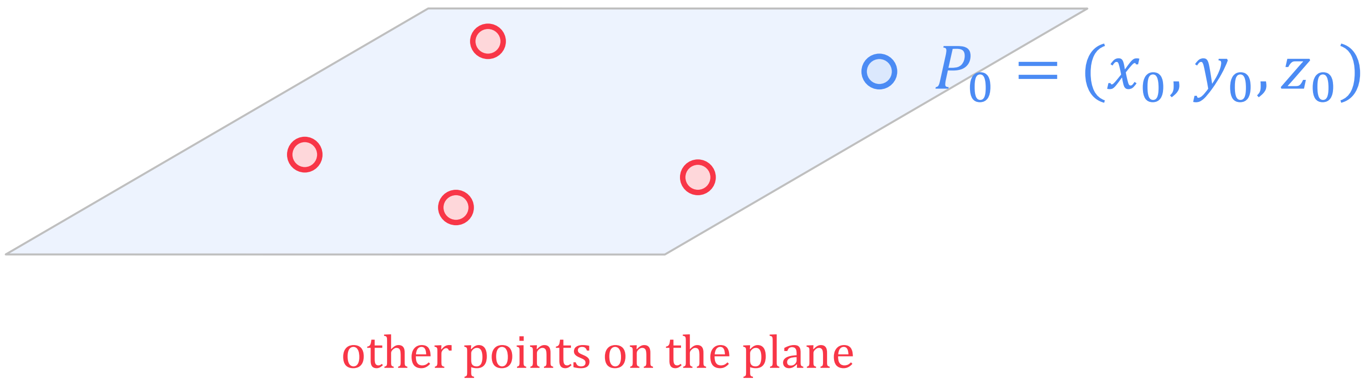
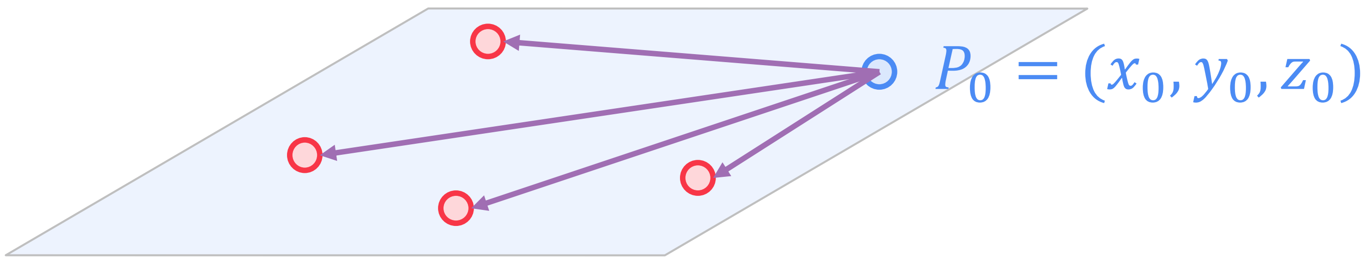


Equation of a plane

- Position vector (point on the plane)
- Perpendicular vector (to the plane)

Vectors from the position vector to other points on the plane



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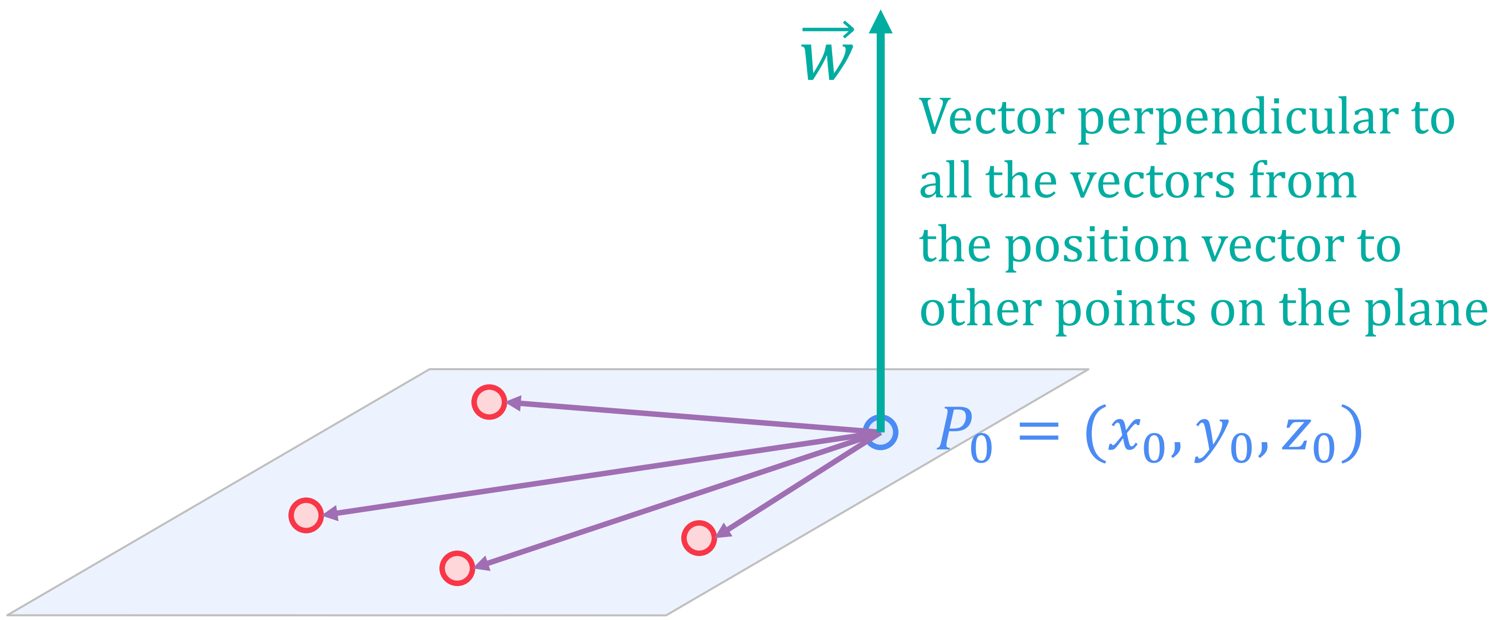

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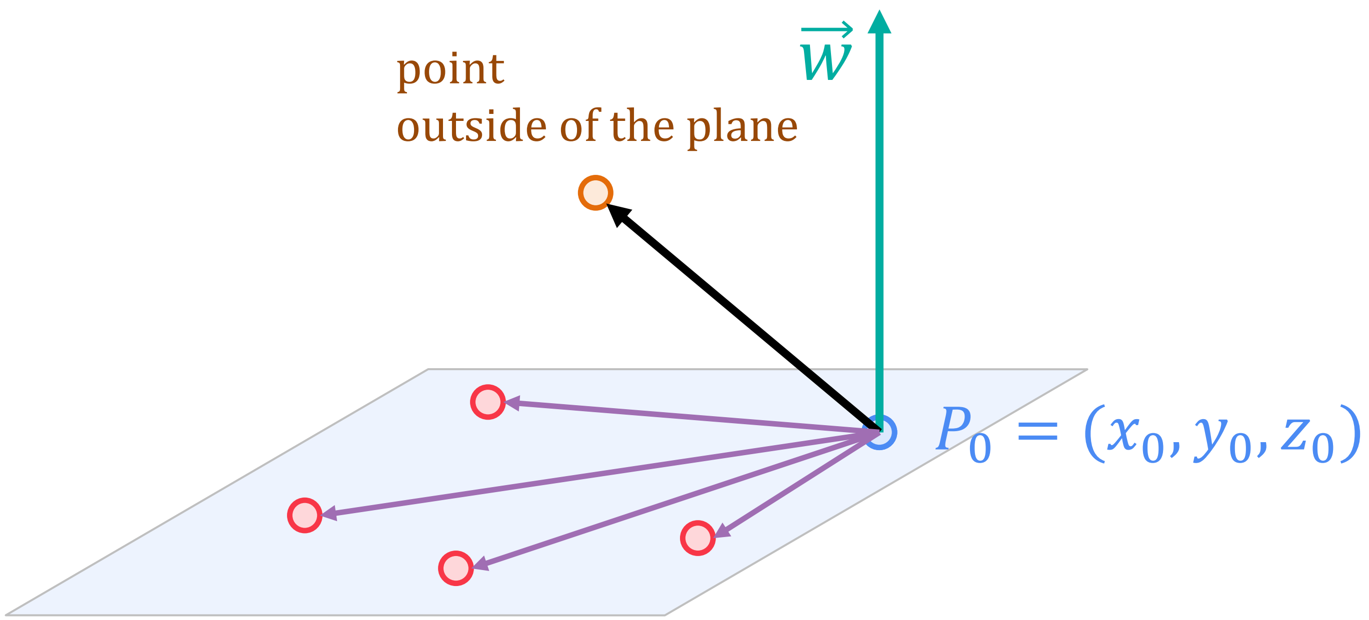

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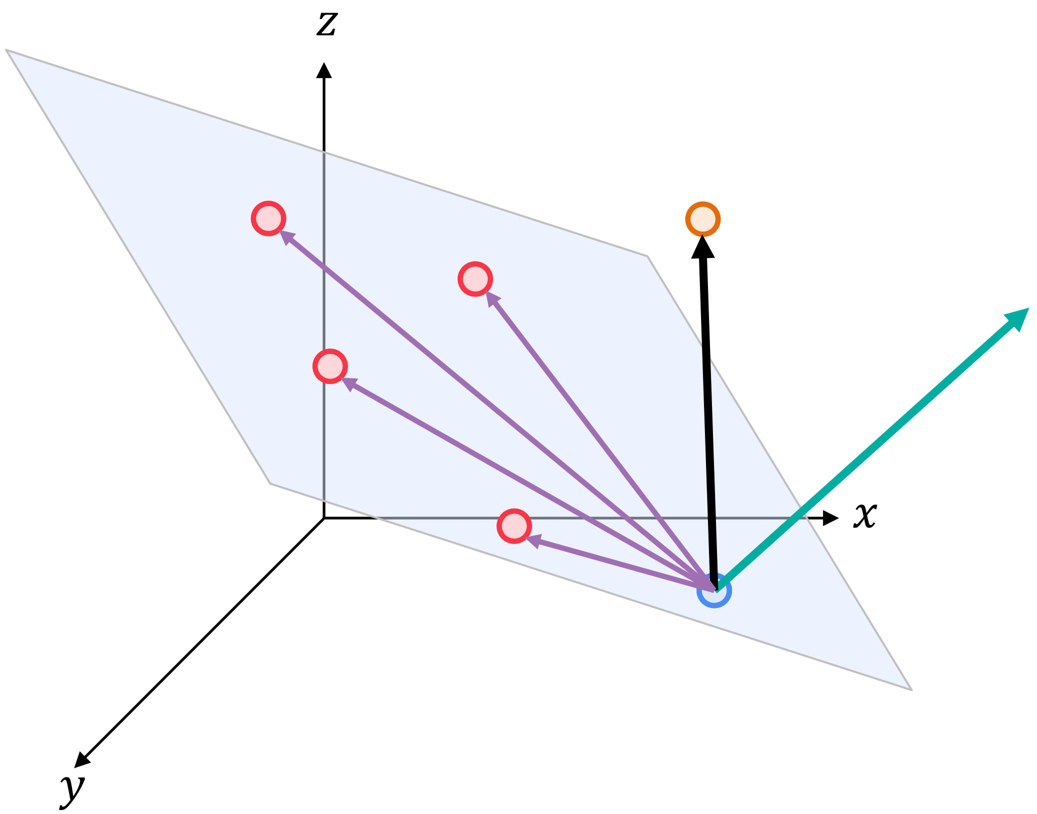

Equation of a plane

- Position vector (point on the plane)
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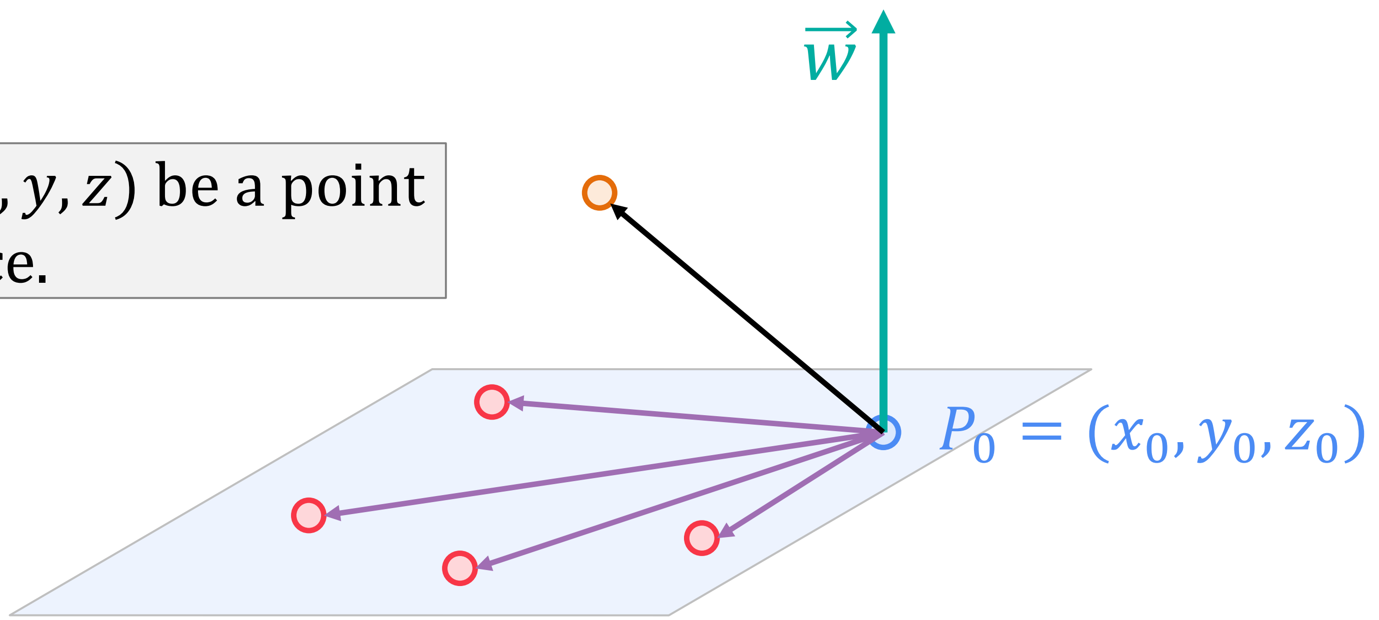
Let $Q = (x, y, z)$ be a point in our space.



Equation of a plane

- Position vector (point on the plane)
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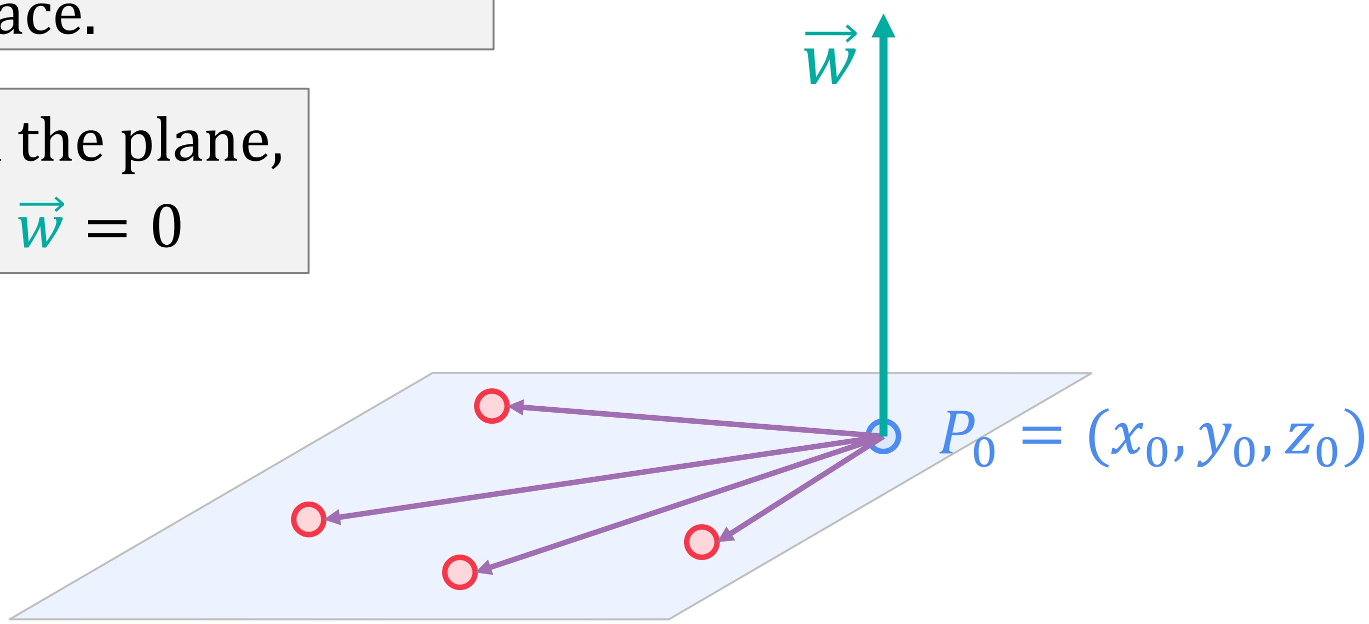
Let $Q = (x, y, z)$ be a point in our space.

If Q is on the plane, $\overrightarrow{P_o Q} \cdot \vec{w} = 0$



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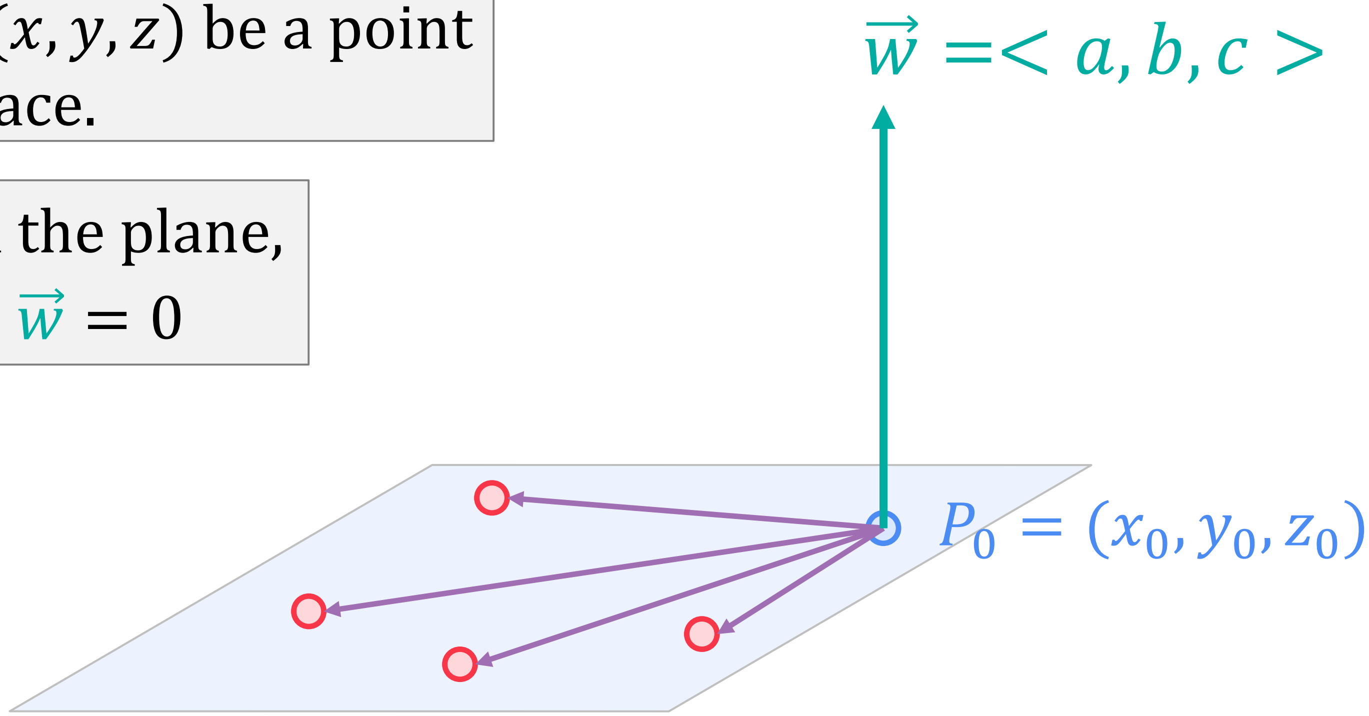

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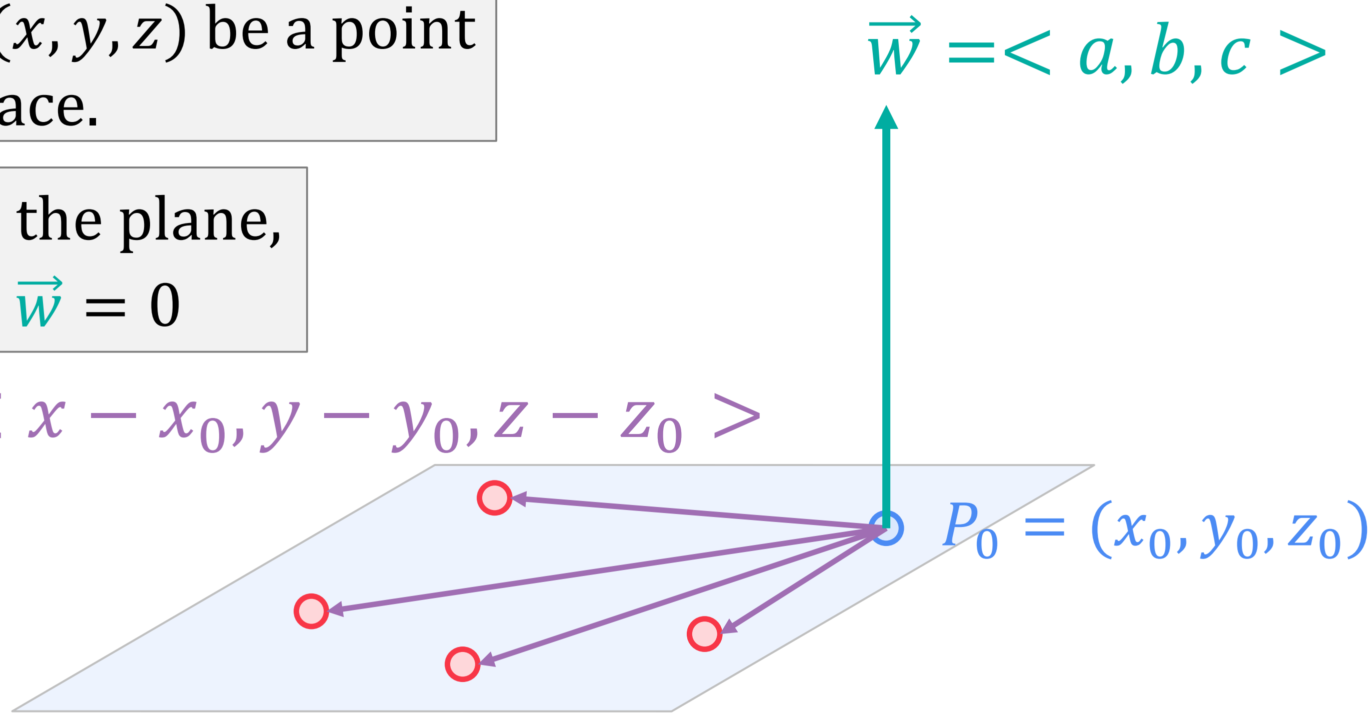
$$\overrightarrow{P_oQ} =< x - x_0, y - y_0, z - z_0 >$$



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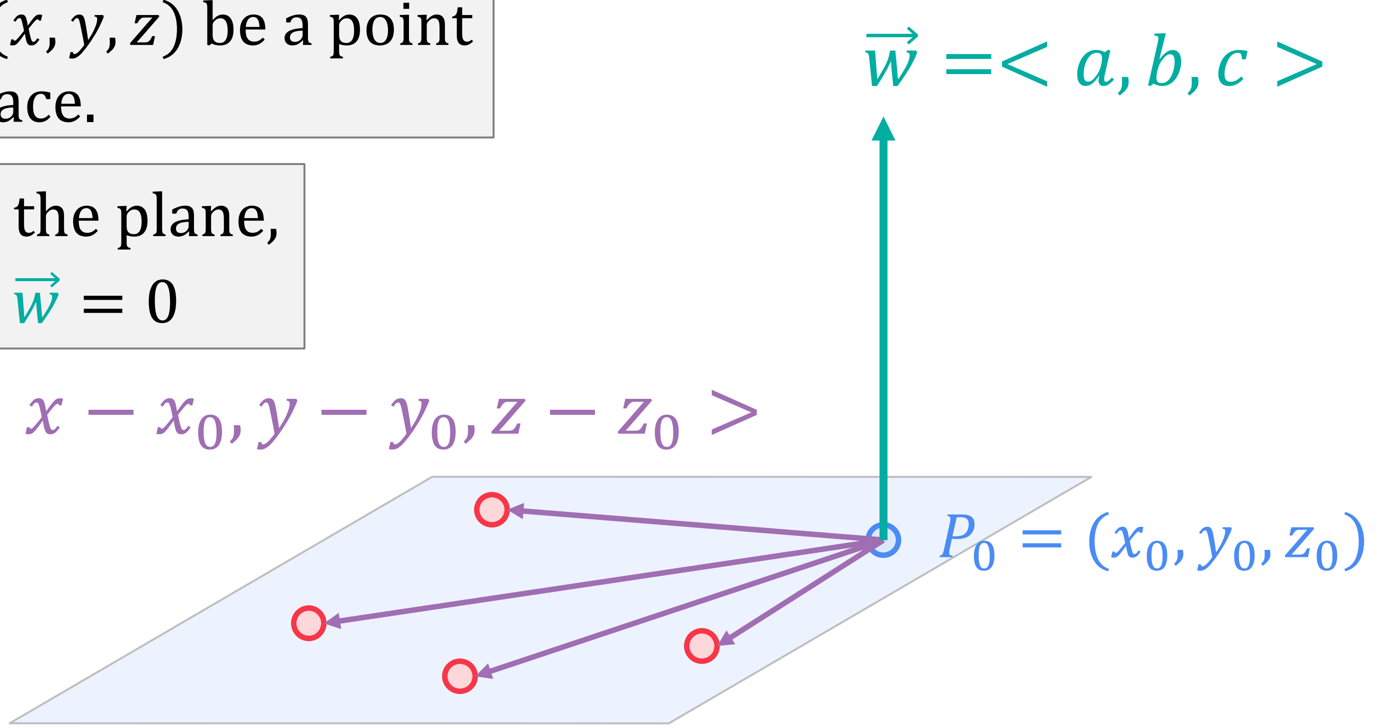

$$< x - x_0, y - y_0, z - z_0 > \cdot < a, b, c > = 0$$

Vector equation of a plane

Equation of a plane

- Position vector (point on the plane): $< x_0, y_0, z_0 >$
- Perpendicular vector (to the plane): $< a, b, c >$

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$$\boxed{< x - x_0, y - y_0, z - z_0 > \cdot < a, b, c > = 0}$$

Equation of a plane

- Position vector (point on the plane): $< x_0, y_0, z_0 >$
- Perpendicular vector (to the plane): $< a, b, c >$

Vector equation of a plane

$$\boxed{< x - x_0, y - y_0, z - z_0 > \cdot < a, b, c > = 0}$$

Scalar equation of a plane

$$\boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$$

Equation of a plane

- Position vector (point on the plane): $< x_0, y_0, z_0 >$
- Perpendicular vector (to the plane): $< a, b, c >$

Vector equation of a plane

$$< x - x_0, y - y_0, z - z_0 > \cdot < a, b, c > = 0$$

Scalar equation of a plane

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$ax + by + cz - (ax_0 + by_0 + cz_0) = 0$$

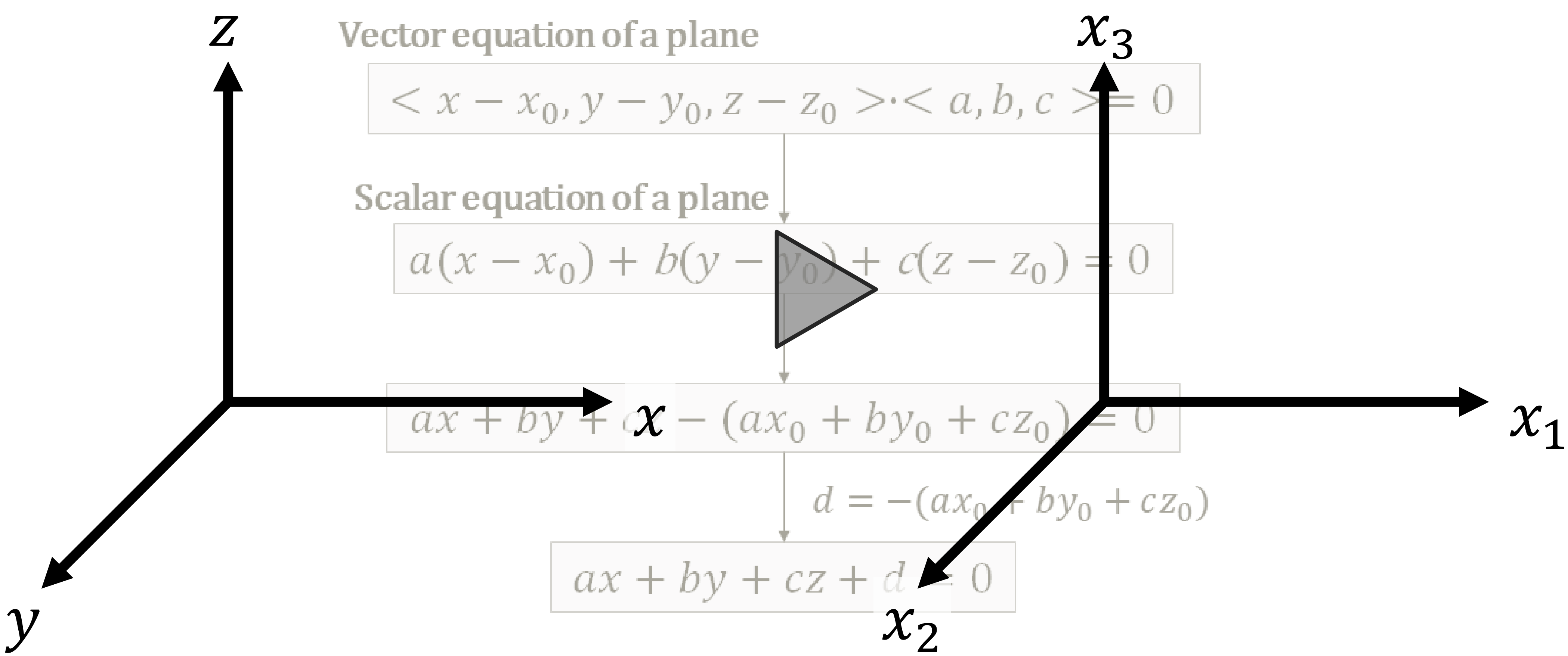
$d = -(ax_0 + by_0 + cz_0)$

$$ax + by + cz + d = 0$$

Linear equation of a plane

Equation of a plane

- Position vector (point on the plane): $< x_0, y_0, z_0 >$
- Perpendicular vector (to the plane): $< a, b, c >$



Vector equation of a plane

$$< x - x_0, y - y_0, z - z_0 > \cdot < a, b, c > = 0$$

Scalar equation of a plane

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$$d = -(ax_0 + by_0 + cz_0)$$

$$ax + by + cz + d = 0$$

Linear equation of a plane

Equation of a plane

- Position vector (point on the plane): $\vec{p} =< p_1, p_2, p_3 >$
- Perpendicular vector (to the plane): $\vec{w} =< w_1, w_2, w_3 >$

Vector equation of a plane

$$< x - x_0, y - y_0, z - z_0 > \cdot < a, b, c >= 0$$

$$\downarrow$$

Scalar equation of a plane

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$\downarrow$$

$$ax + by + cz - (ax_0 + by_0 + cz_0) = 0$$

$$\Big\downarrow \; d = -(ax_0 + by_0 + cz_0)$$

$$ax + by + cz + d = 0$$

Linear equation of a plane

Equation of a plane

- Position vector (point on the plane): $\vec{p} =< p_1, p_2, p_3 >$
- Perpendicular vector (to the plane): $\vec{w} =< w_1, w_2, w_3 >$

Vector equation of a plane

$$\boxed{< x_1 - p_1, x_2 - p_2, x_3 - p_3 > \cdot < w_1, w_2, w_3 > = 0}$$

Scalar equation of a plane

$$\boxed{w_1(x_1 - p_1) + w_2\,(x_2 - p_2) + w_3\,(x_3 - p_3) = 0}$$

$$\boxed{w_1x_1 + w_2x_2 + w_3x_3 - (w_1p_1 + w_2p_2 + w_3p_3) = 0}$$

$b = -(w_1p_1 + w_2p_2 + w_3p_3)$ $\quad$ $b = -(\vec{w} \cdot \vec{p})$

$$\boxed{w_1x_1 + w_2x_2 + w_3x_3 + b = 0} \longrightarrow \boxed{\vec{w} \cdot \vec{x} + b = 0}$$

Linear equation of a plane $\quad$ **plane for any dimension**

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- Position vector (point on the plane): $\vec{p} =< p_1, p_2, p_3 >$
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Vector equation of a plane

$$< x_1 - p_1, x_2 - p_2, x_3 - p_3 > \cdot < w_1, w_2, w_3 >= 0$$

Scalar equation of a plane

$$w_1(x_1 - p_1) + w_2\,(x_2 - p_2) + w_3\,(x_3 - p_3) = 0$$

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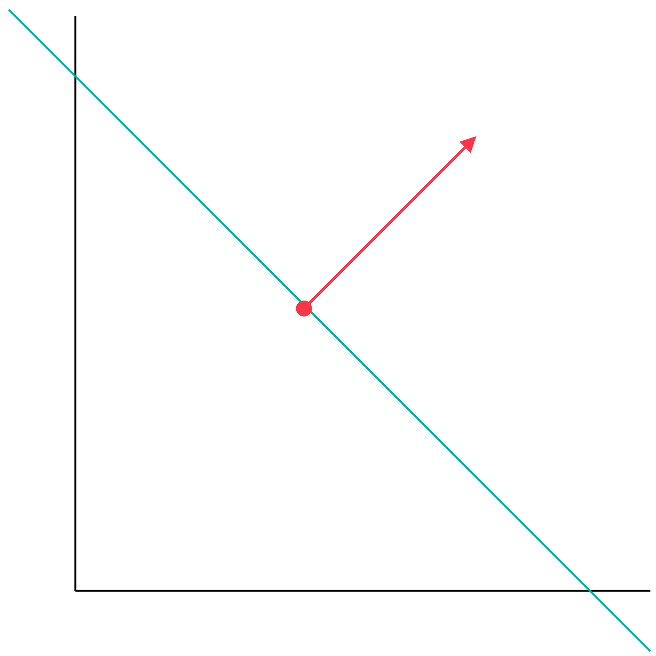
$b = -(w_1p_1 + w_2p_2 + w_3p_3)$ $\quad$ $b = -(\vec{w} \cdot \vec{p})$

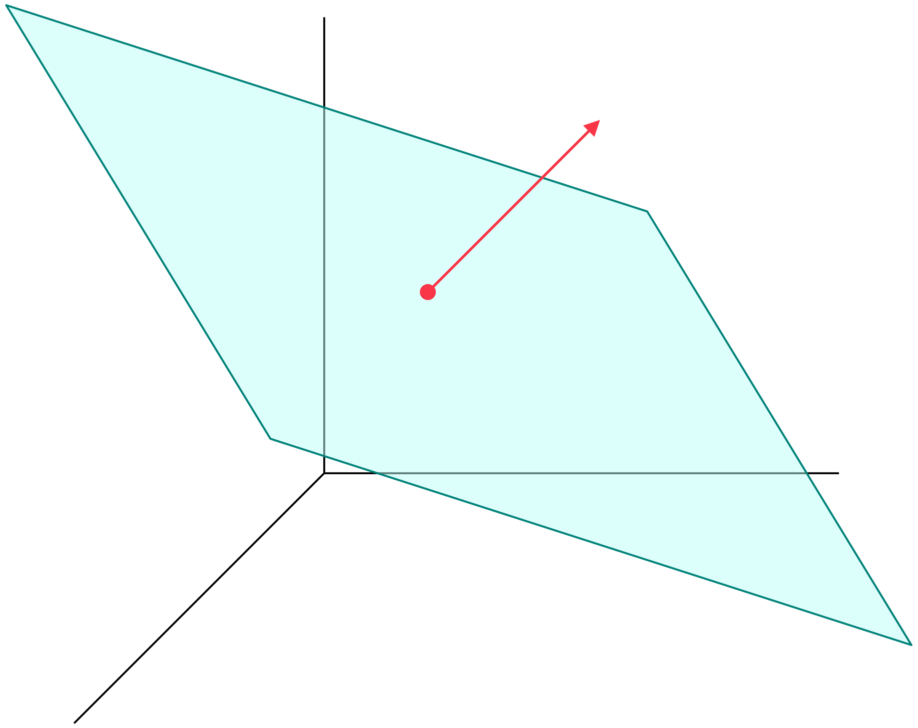
$$w_1x_1 + w_2x_2 + w_3x_3 + b = 0 \longrightarrow \vec{w} \cdot \vec{x} + b = 0 \longrightarrow w^T\vec{x} + b = 0$$

Linear equation of a plane $\quad$ **plane for any dimension**

Equation of a plane

- This is why people say..





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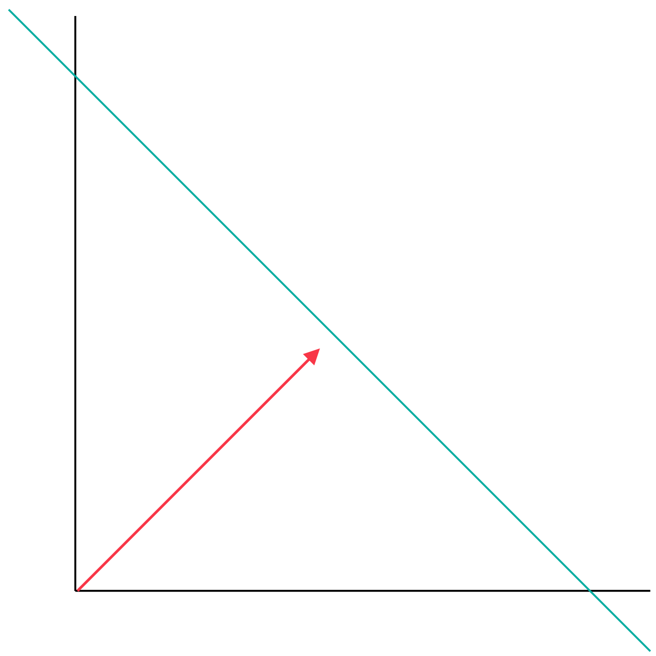

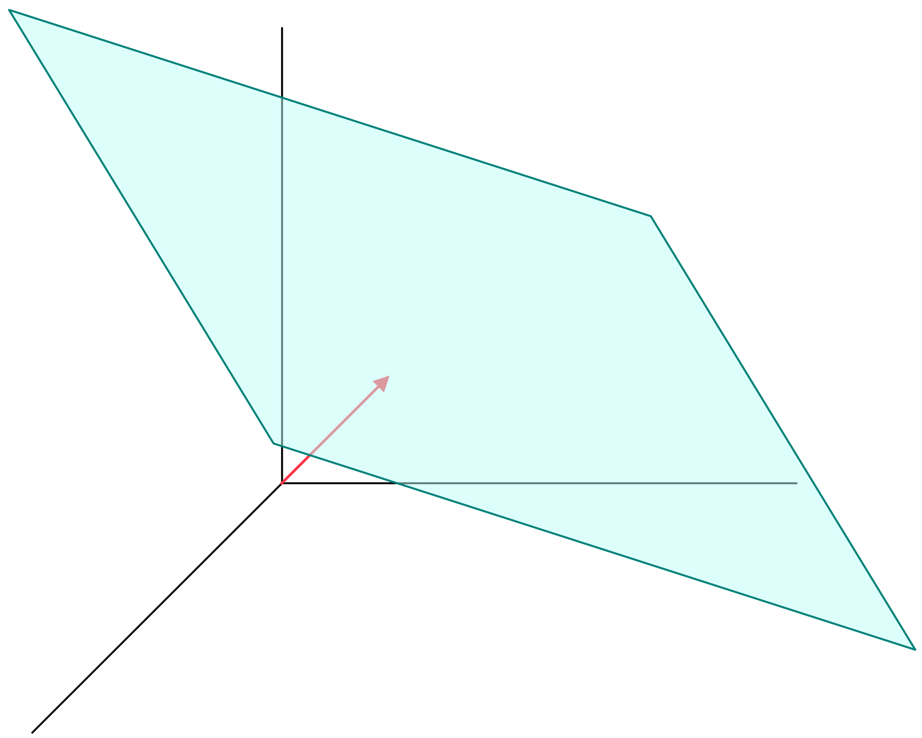

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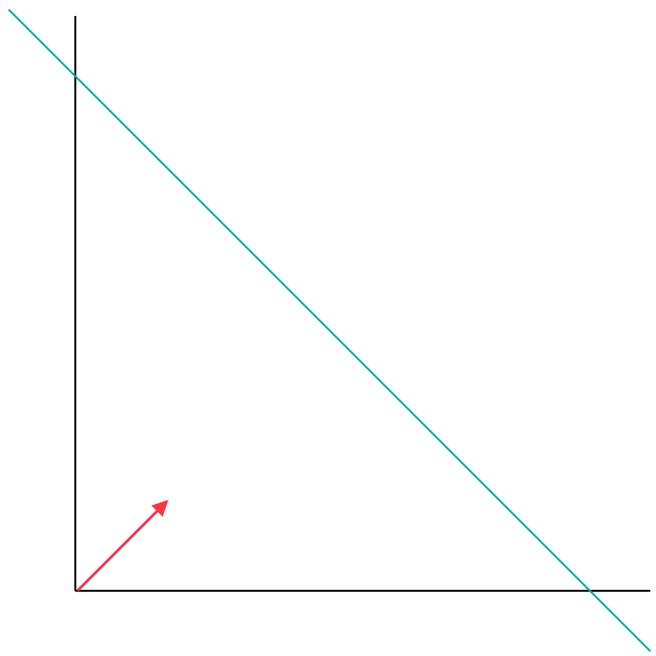

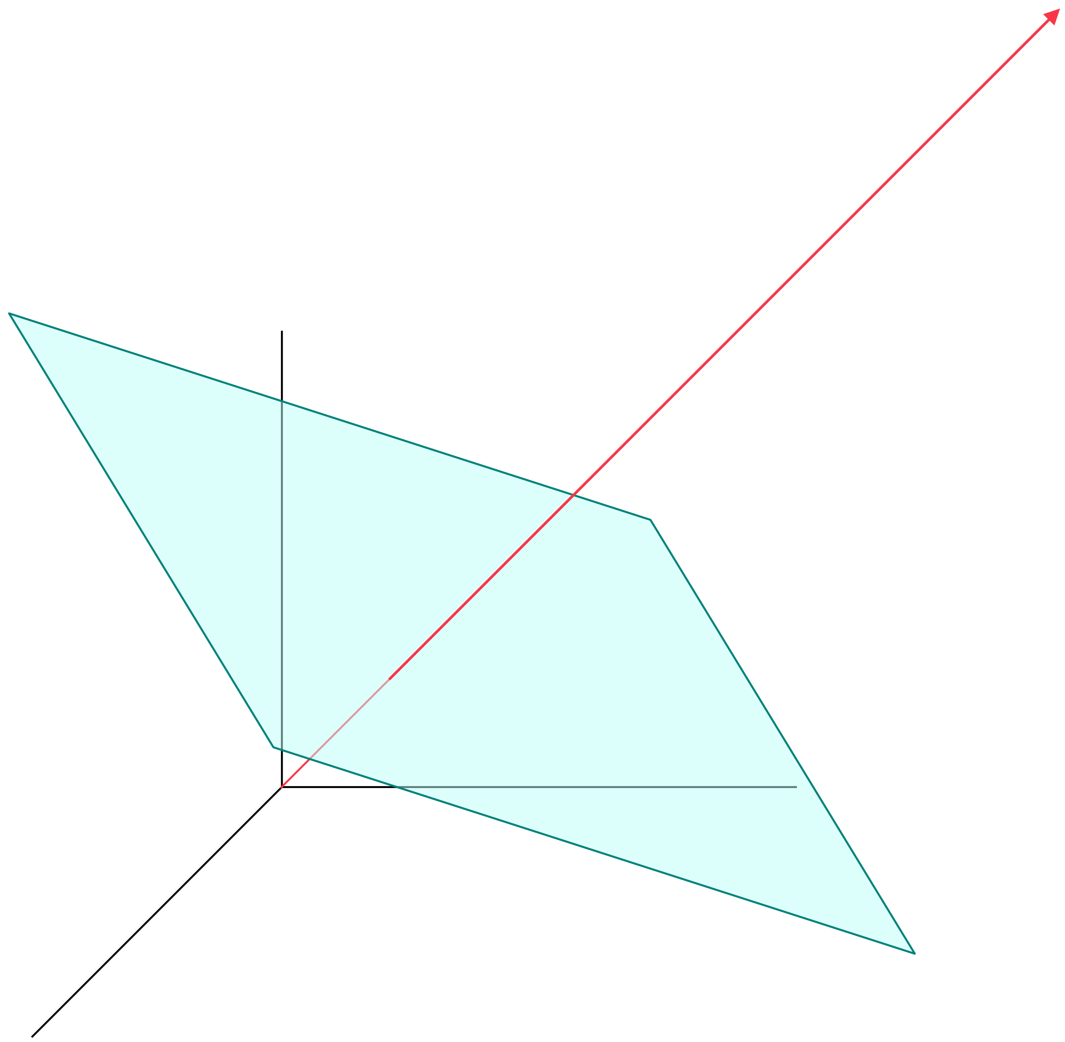

Distance from a point to a plane

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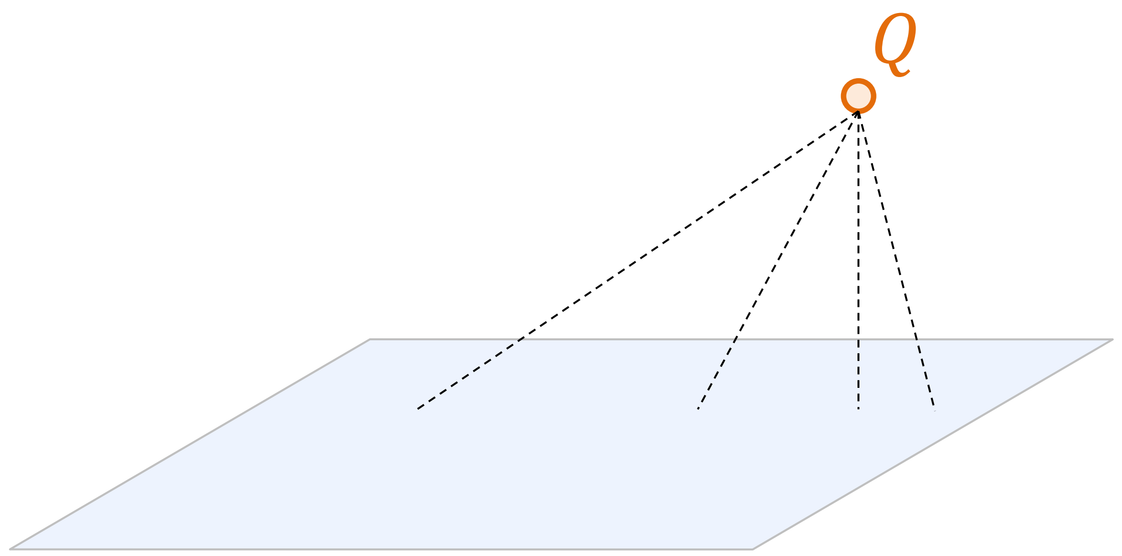

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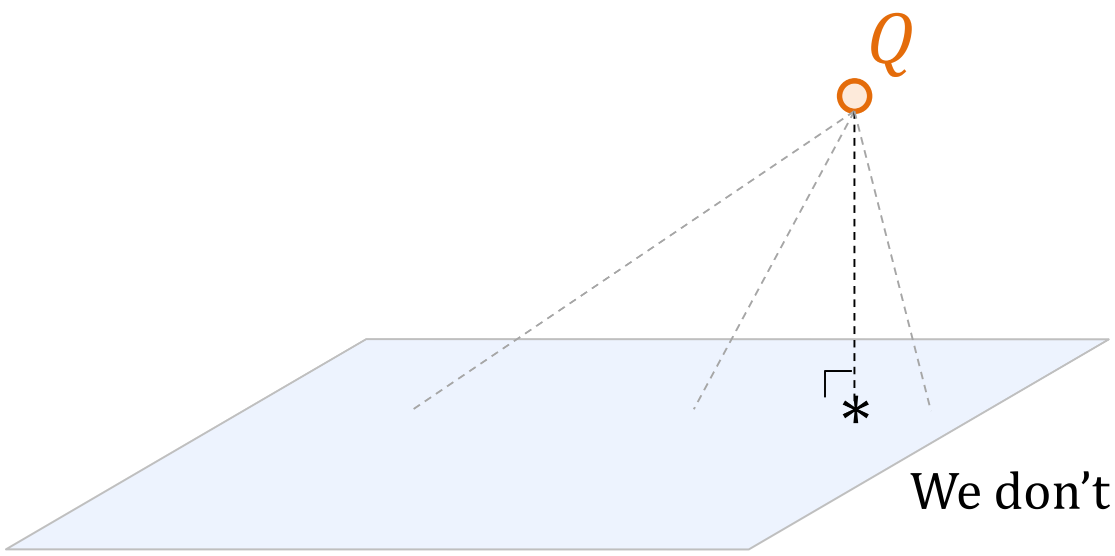
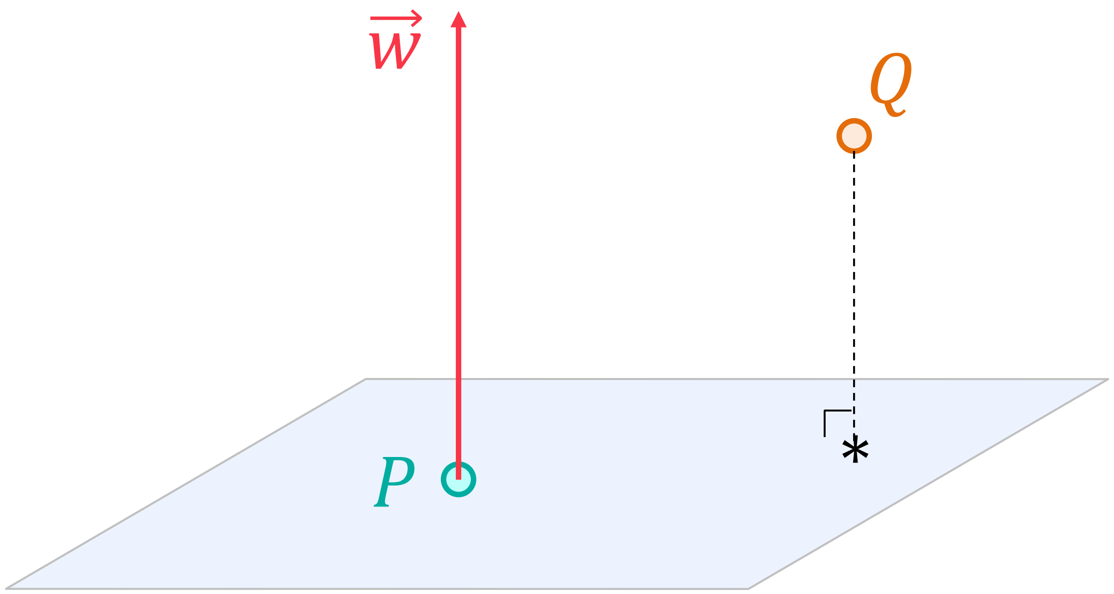


We don't know the '*'

Distance from a point to a plane



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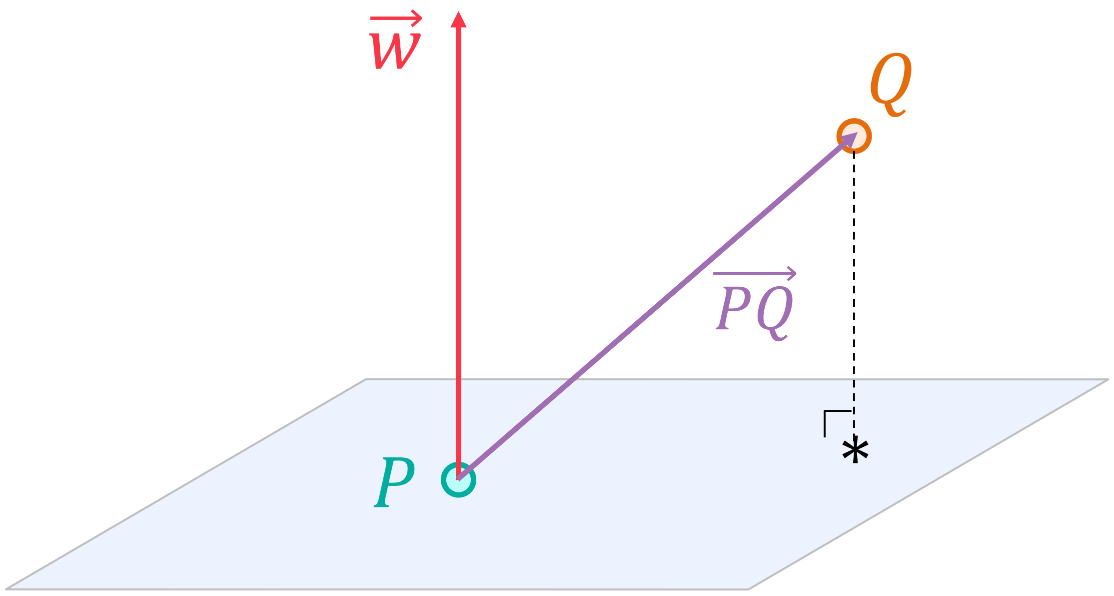
Distance from a point to a plane



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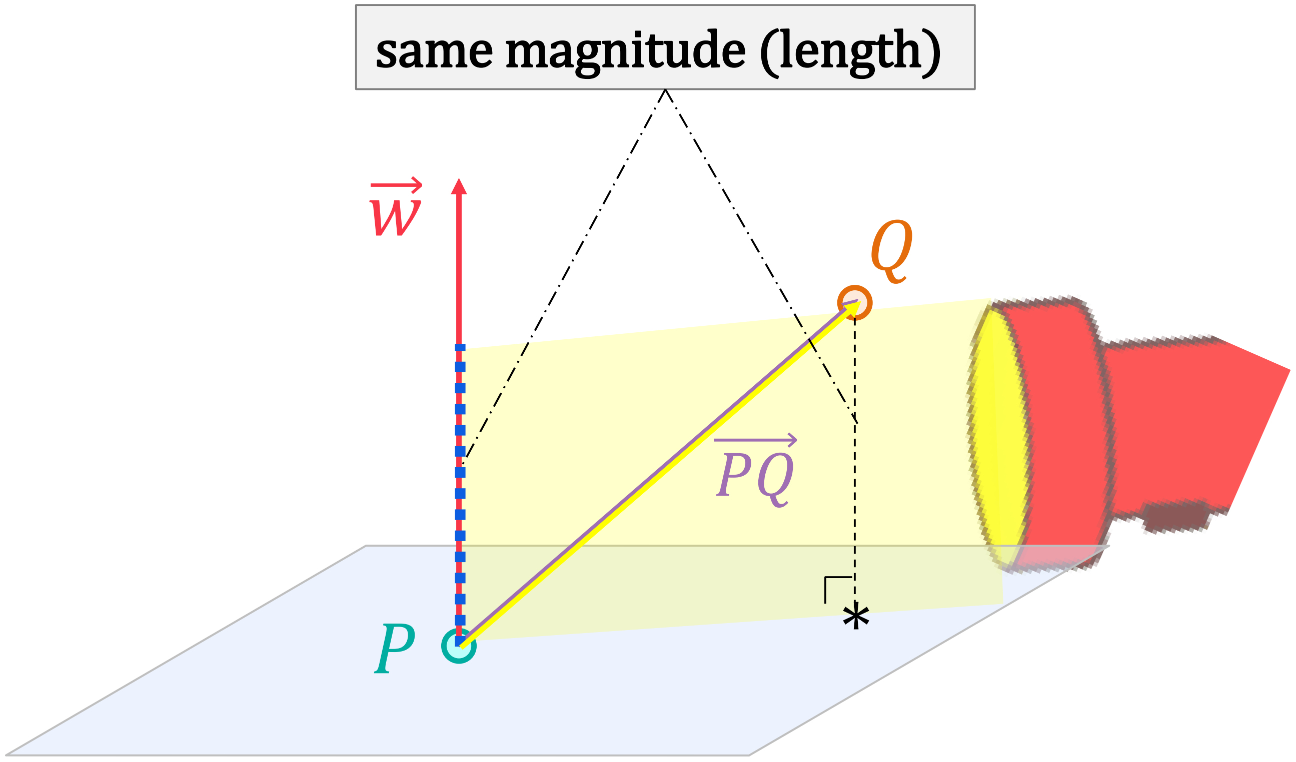

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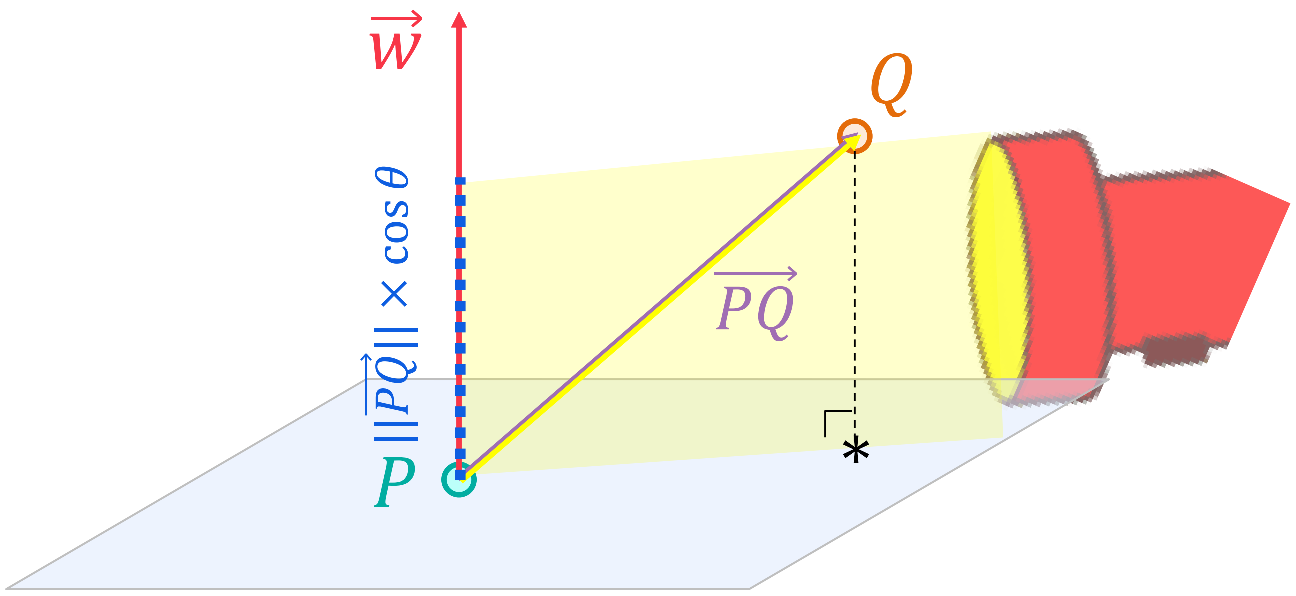
Distance from a point to a plane

$$\vec{w} \cdot \overrightarrow{PQ} = ||\vec{w}|| \times ||\overrightarrow{PQ}|| \times \cos\theta$$



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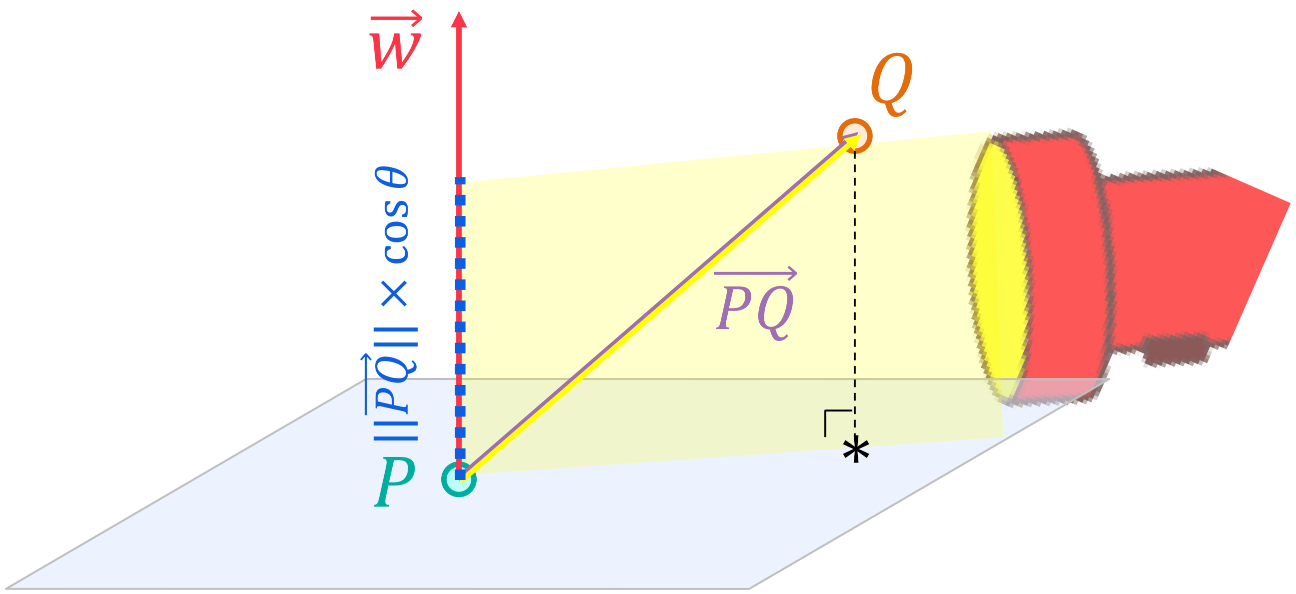

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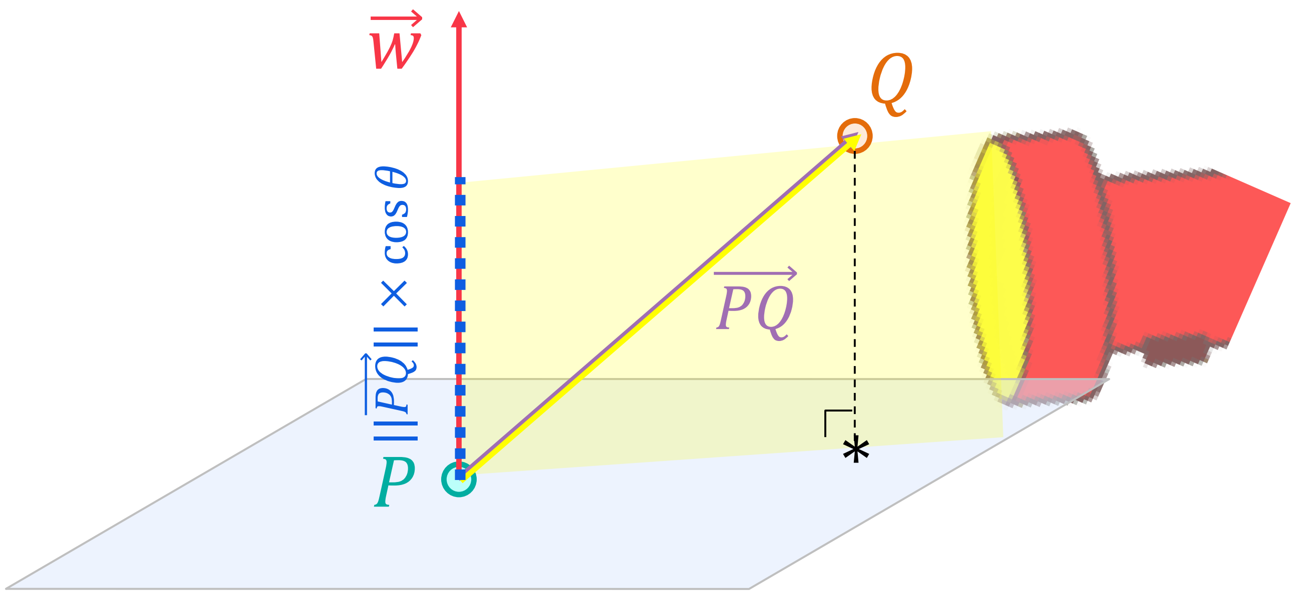

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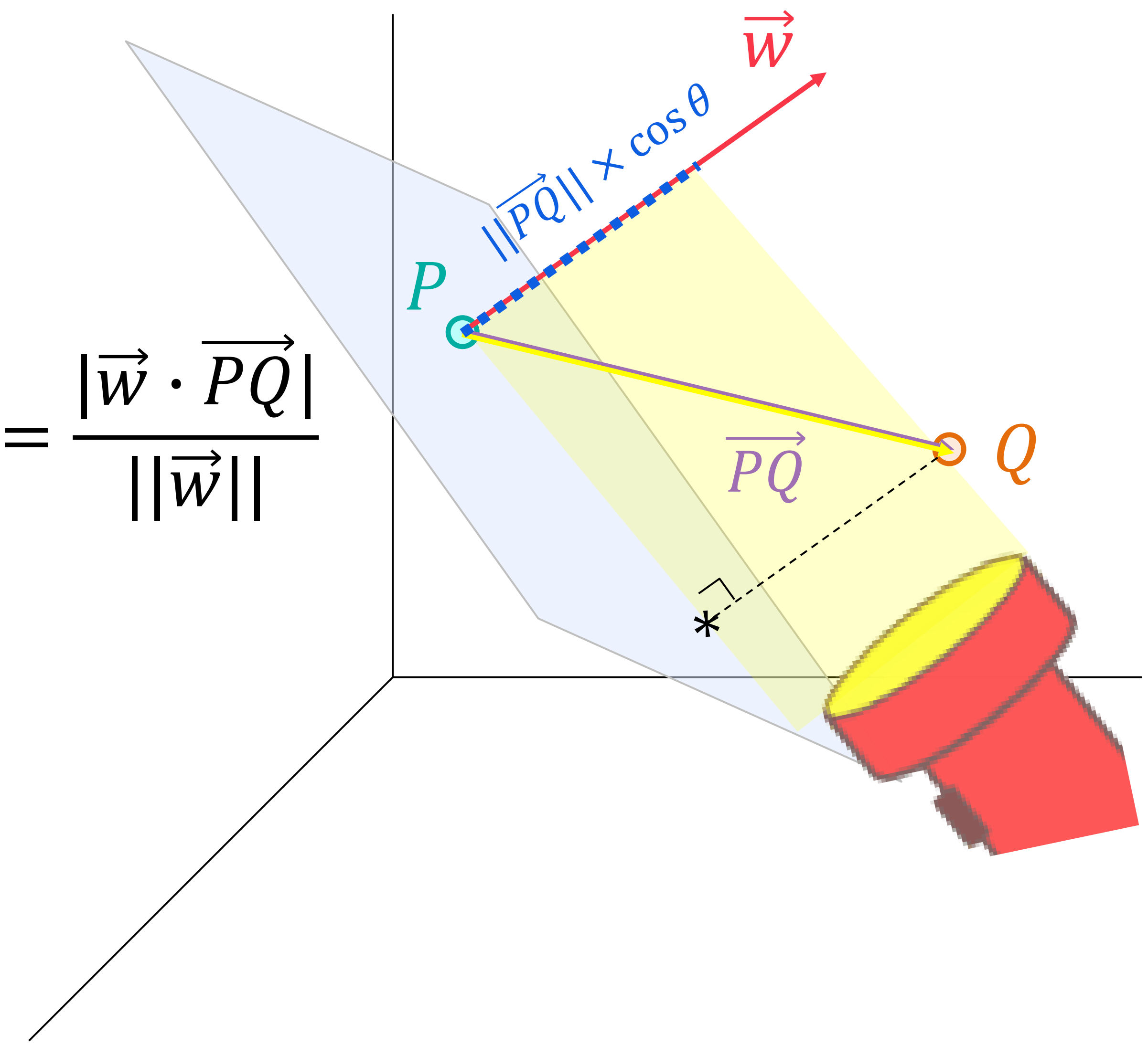
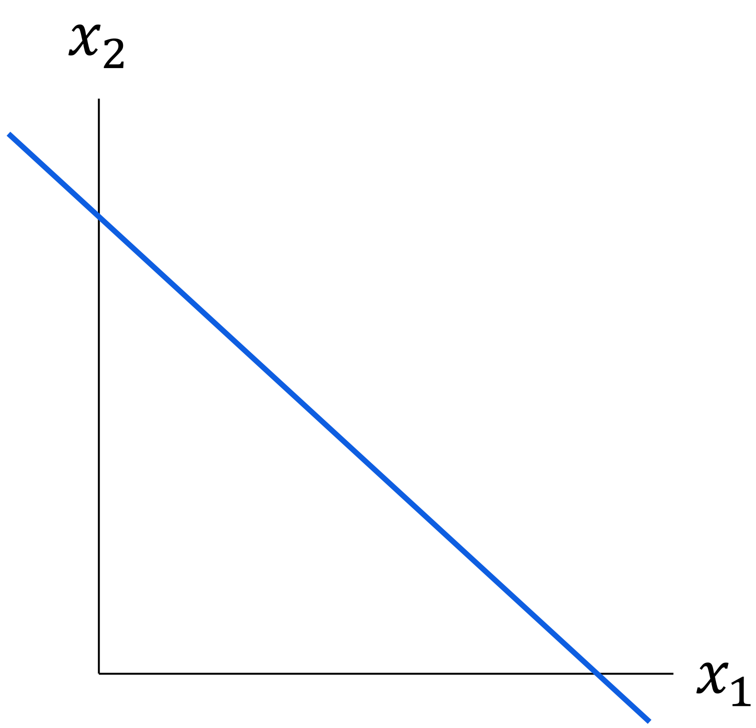


Contents of this week

- ~~Equation of lines and planes~~
- ~~Distance from a point to a plane~~
- Support vector machine part I
- Constrained optimization
- Support vector machine part II

Linear hyper plane

- Separates a D −dimensional space into two half-spaces



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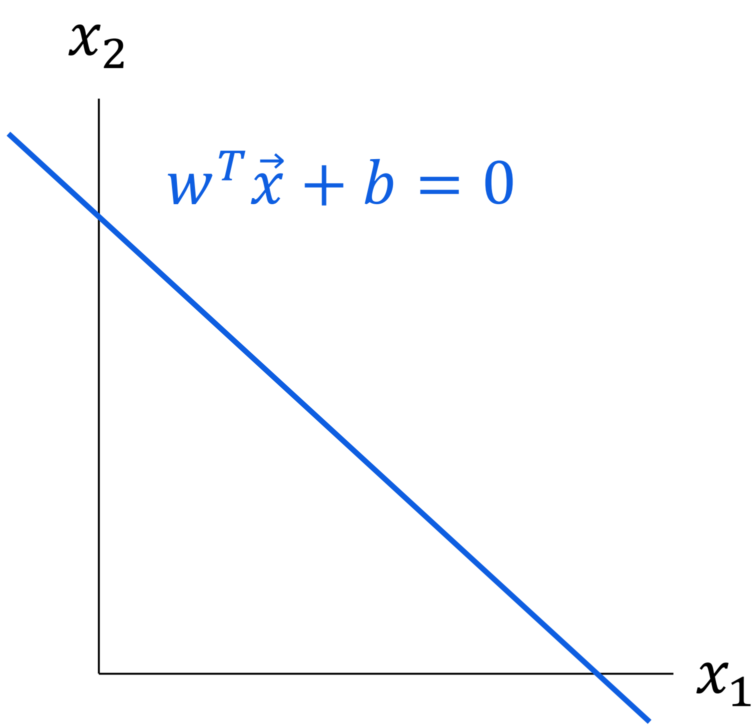

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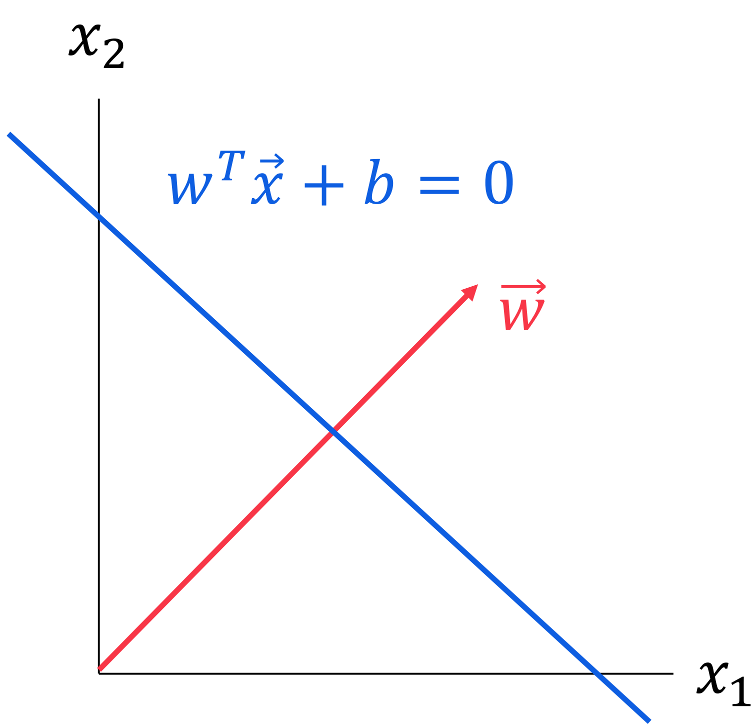

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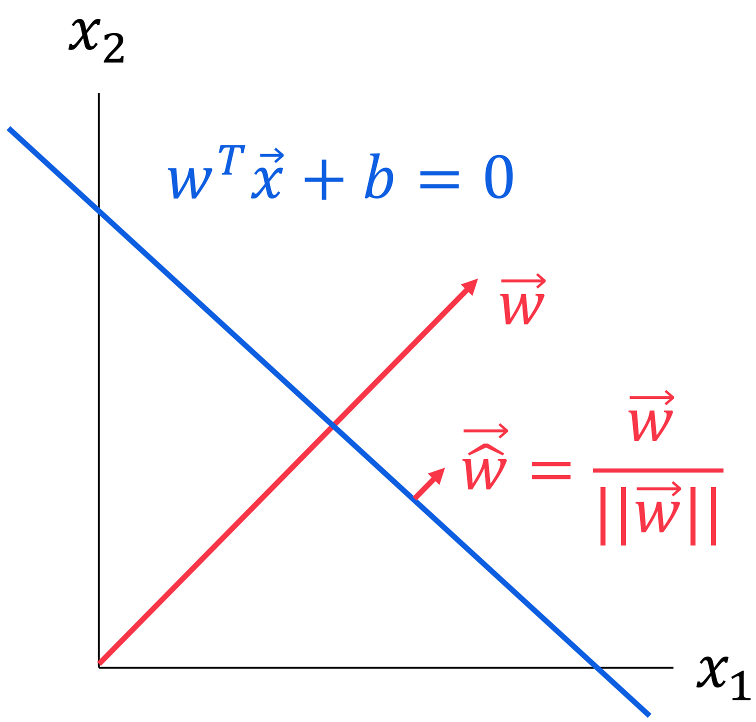

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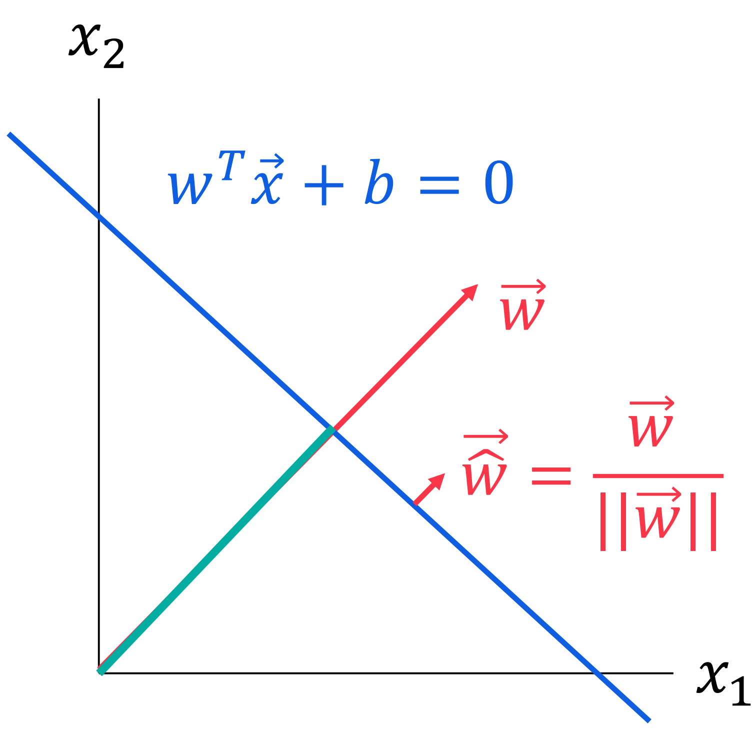

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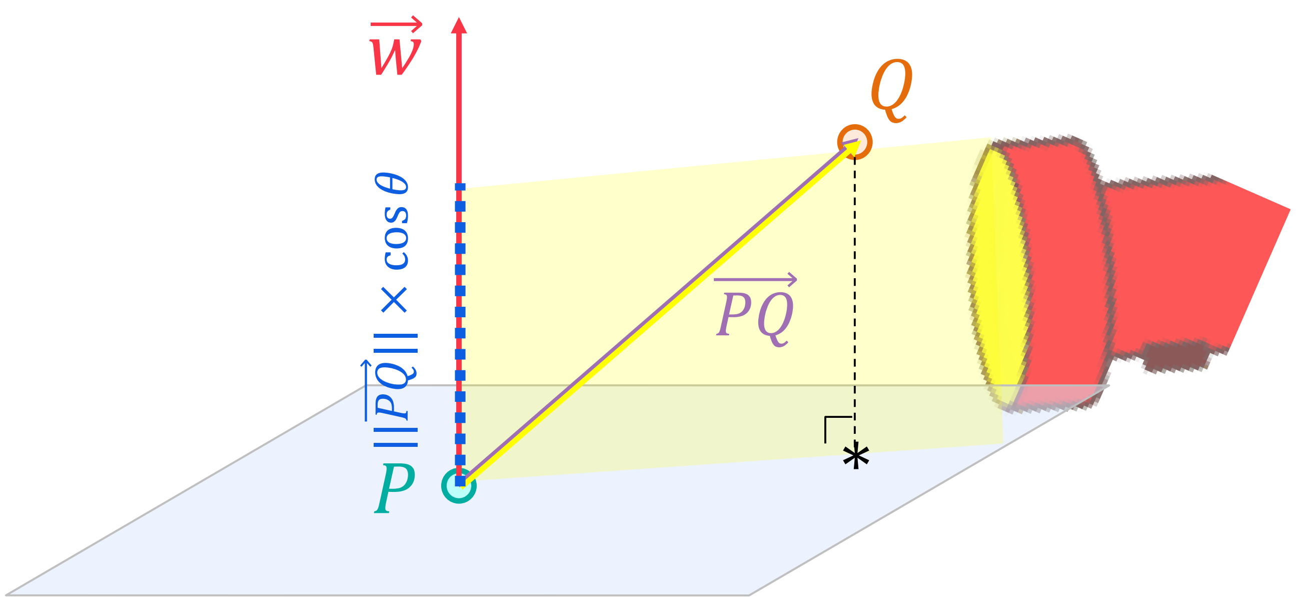

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Distance from a point to a plane

$$\frac{|\vec{w} \cdot \overrightarrow{PQ}|}{||\vec{w}||} = ||\overrightarrow{PQ}|| \times \cos\theta$$

- $\vec{p} =< p_1, p_2 >$
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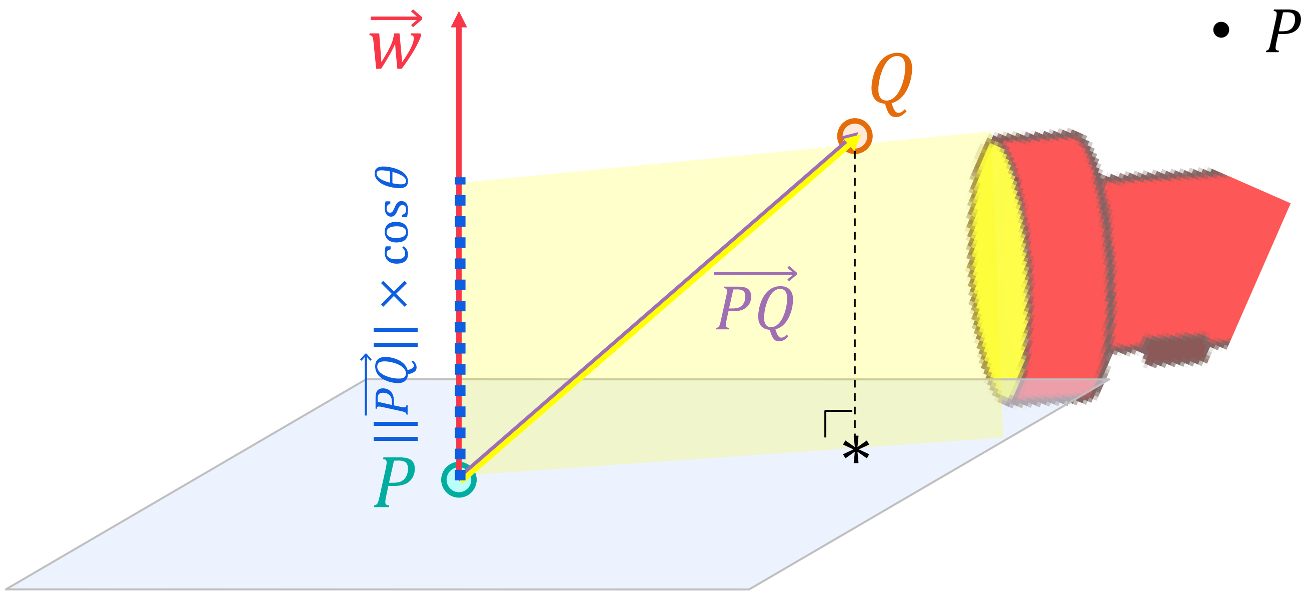

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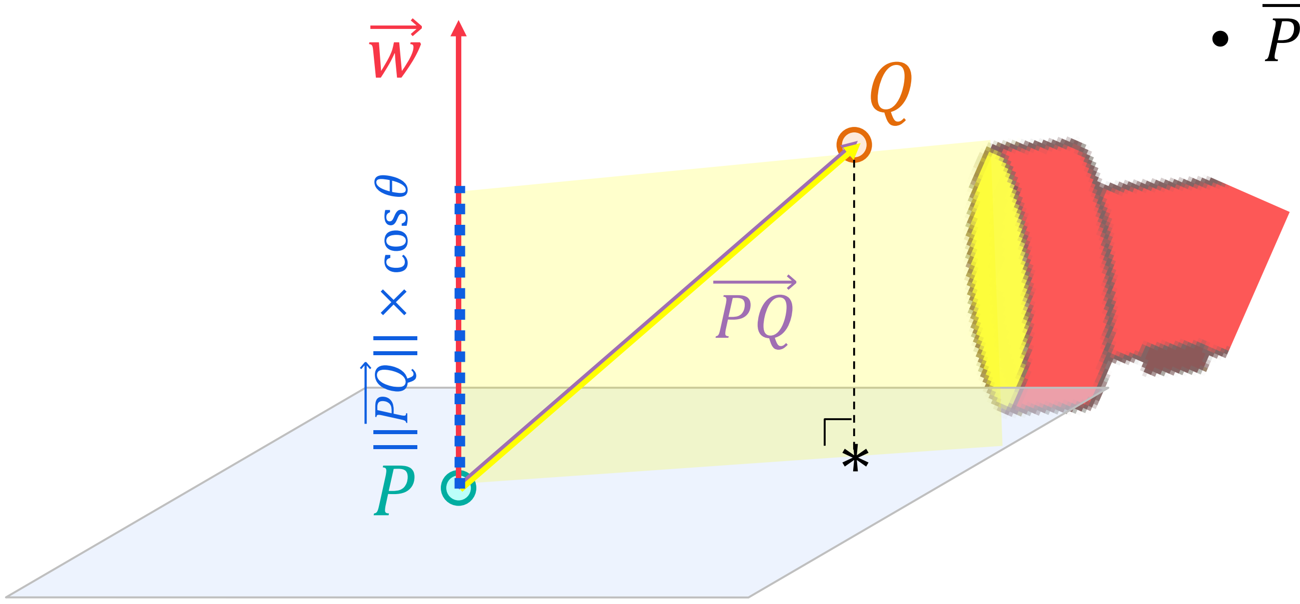

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Distance from a point to a plane

$$\frac{|-(\vec{w}\cdot\vec{p})|}{||\vec{w}||} = ||\overrightarrow{PQ}|| \times \cos\theta$$

- $\vec{p} =< p_1, p_2 >$
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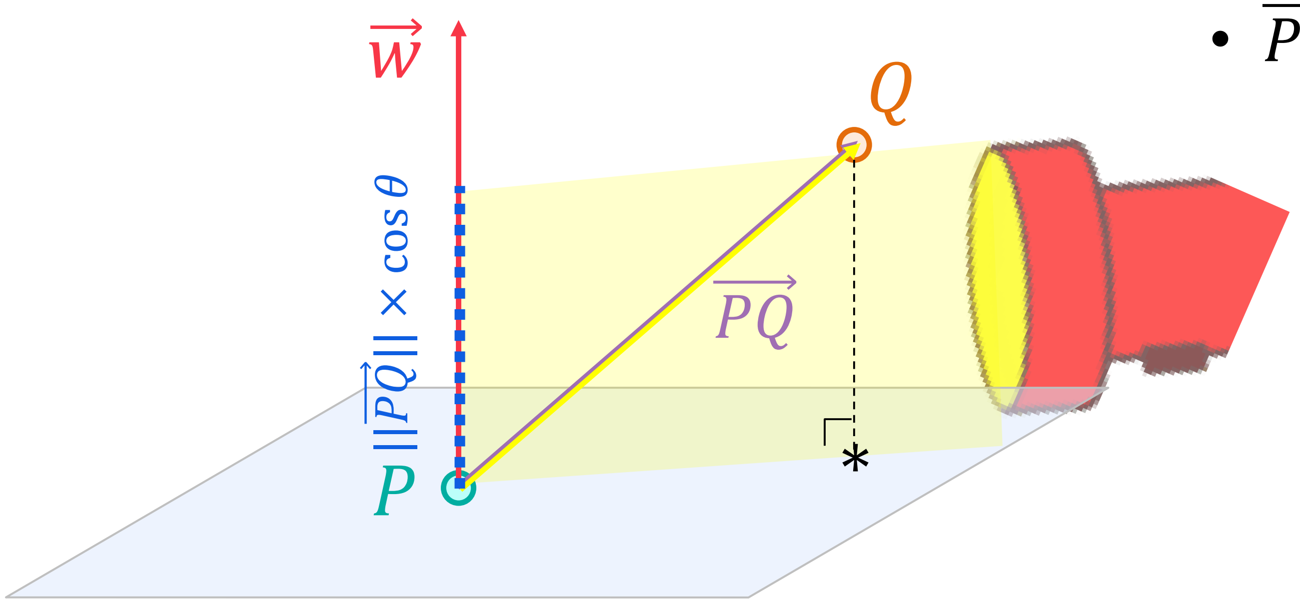

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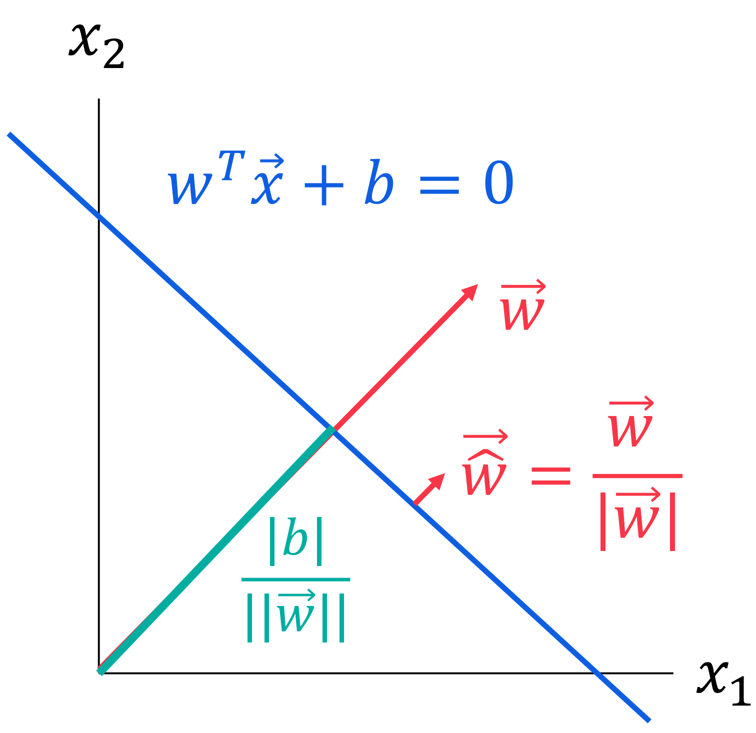

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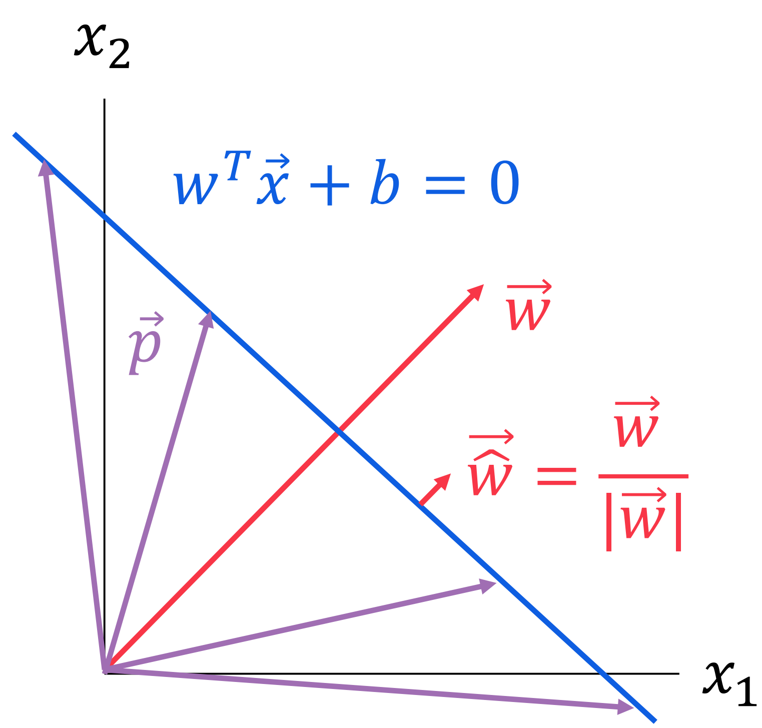

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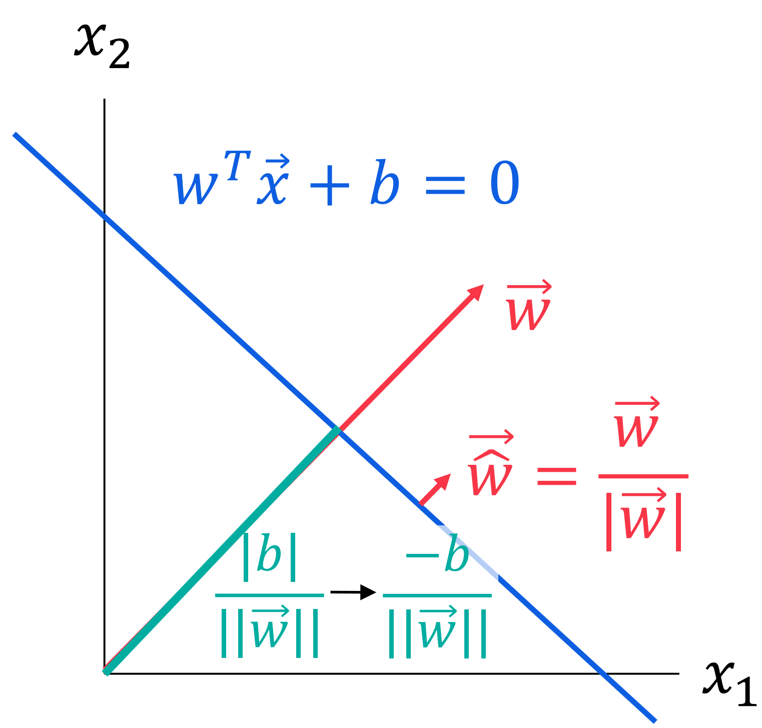

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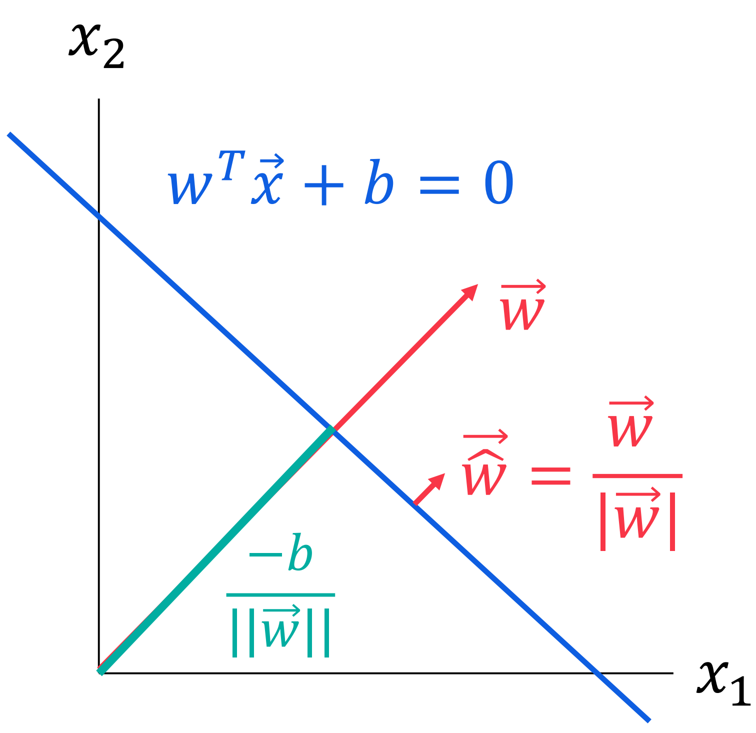

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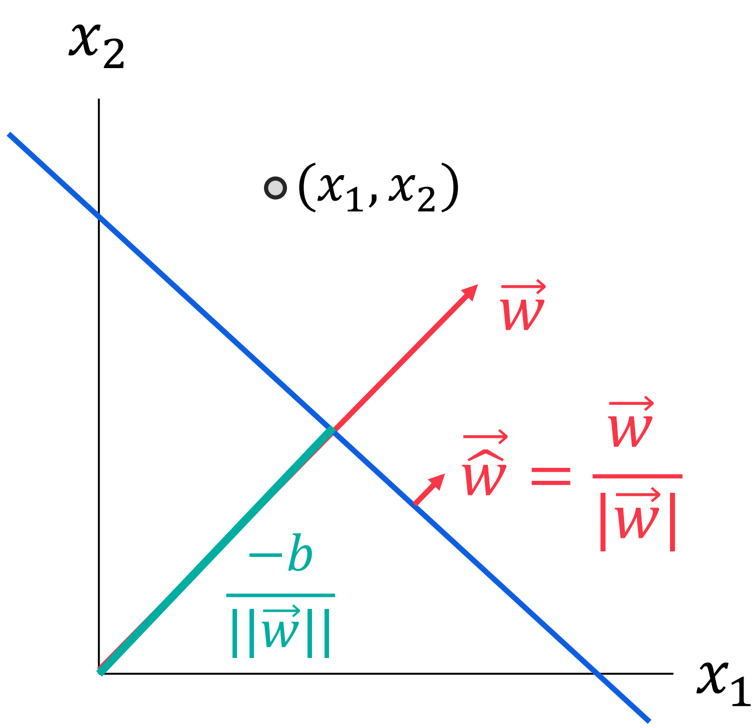

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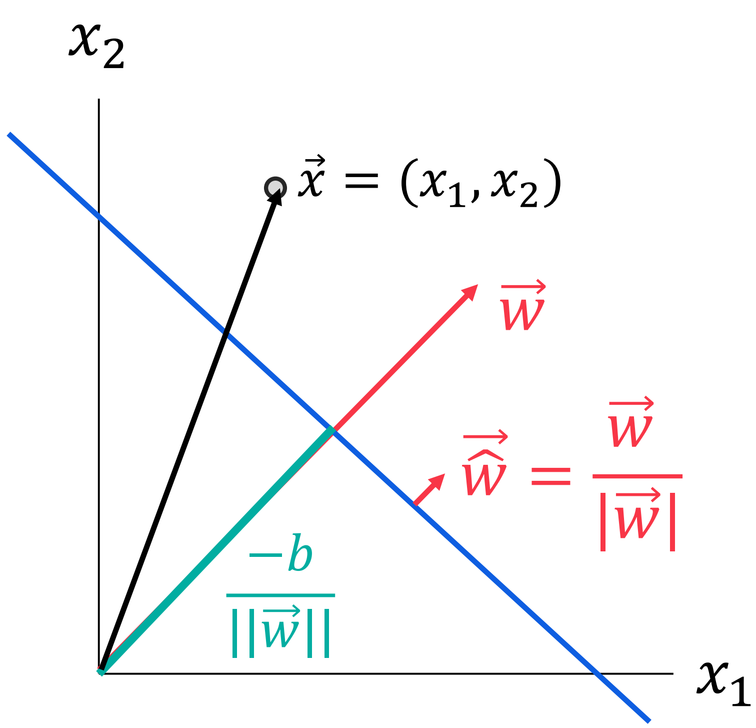

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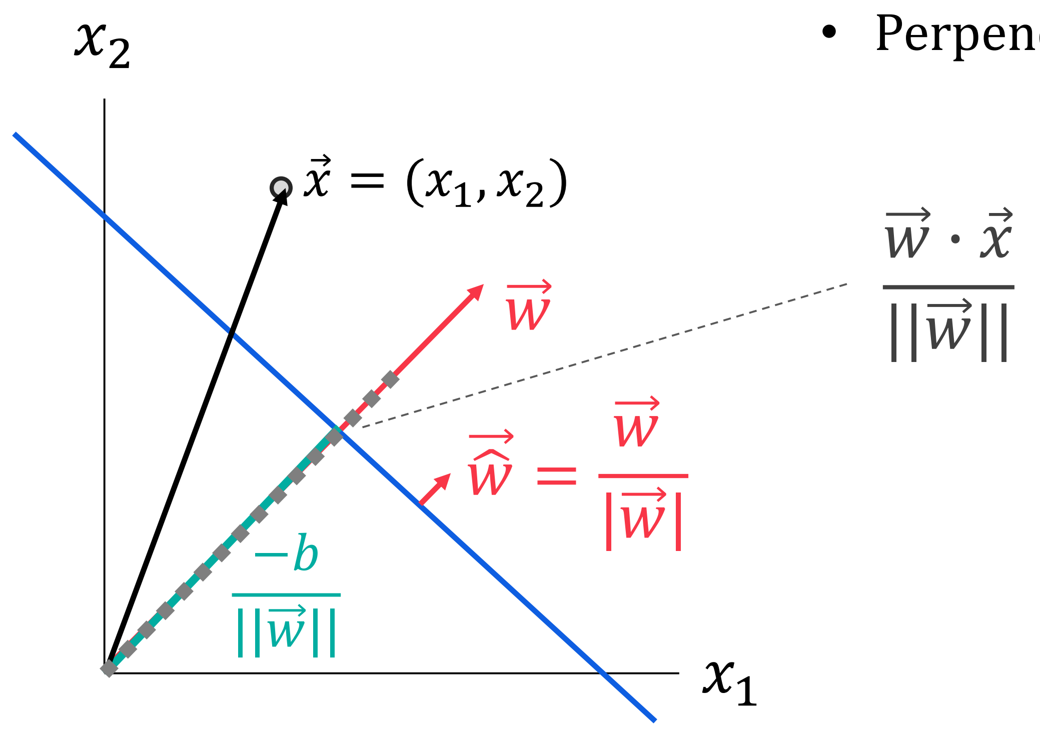

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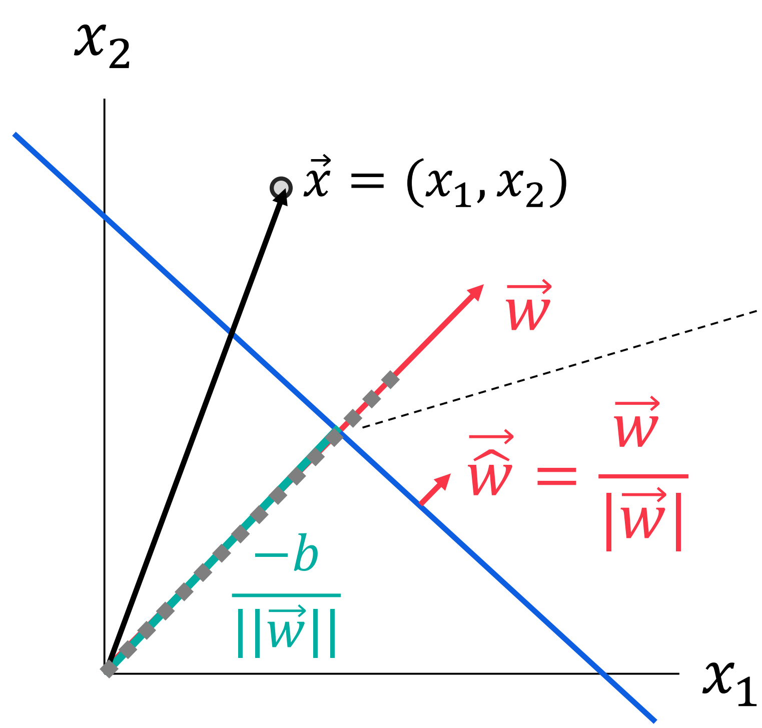

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$$\frac{\vec{w} \cdot \vec{x}}{||\vec{w}||} > \frac{-b}{||\vec{w}||}$$

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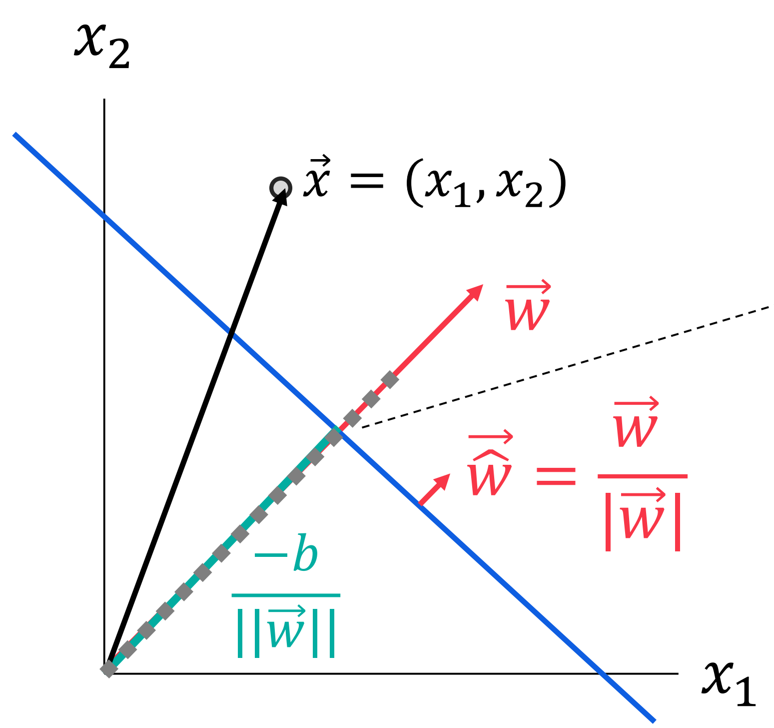

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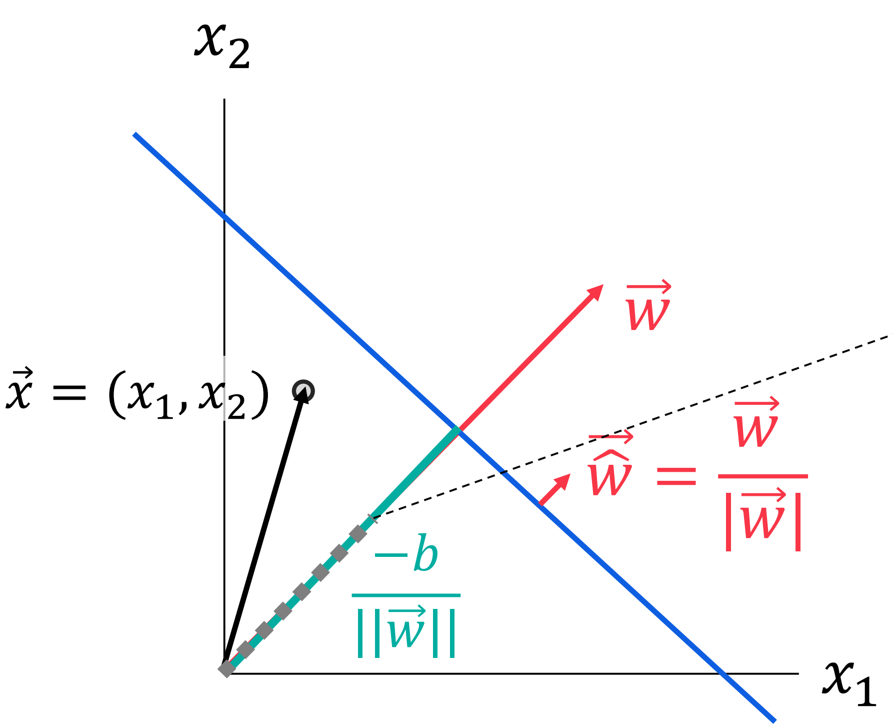

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$$\frac{\vec{w} \cdot \vec{x}}{||\vec{w}||} < \frac{-b}{||\vec{w}||}$$

▽

$$\vec{w} \cdot \vec{x} + b < 0$$

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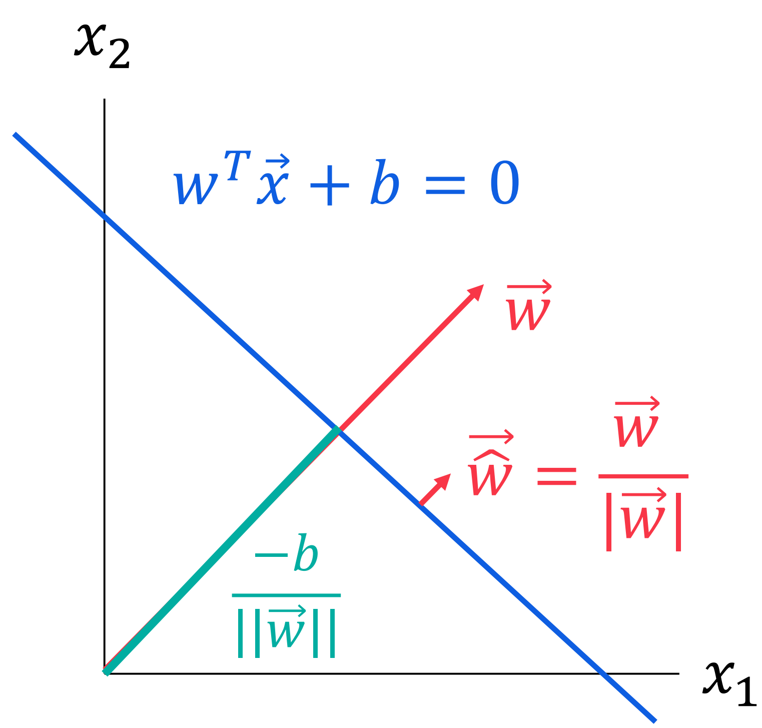

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Our decision rule

$$\begin{cases} \text{IF } \vec{w} \cdot \vec{x} + b > 0 \text{ THEN } \vec{x} \text{ is above the plane.} \\ \text{IF } \vec{w} \cdot \vec{x} + b < 0 \text{ THEN } \vec{x} \text{ is bellow the plane.} \end{cases}$$

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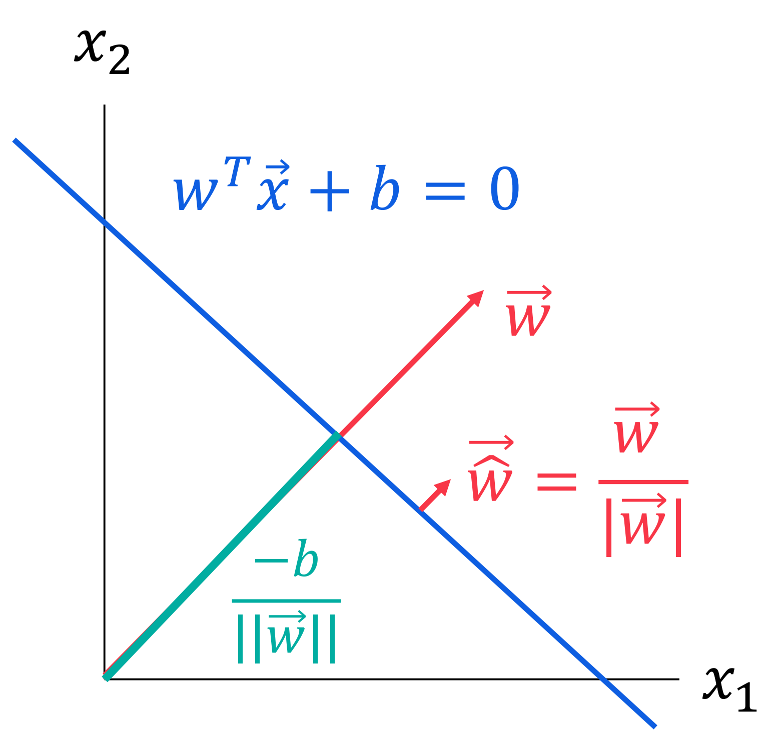

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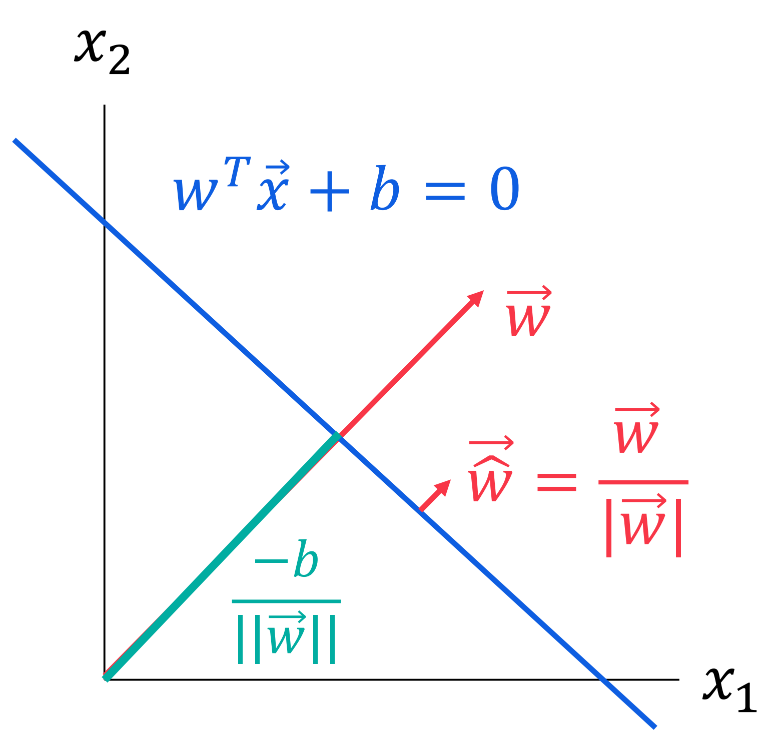

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- Perpendicular vector (to the plane): $\vec{w} =< w_1, w_2 >$

$$\boxed{w^T\vec{x} + b = 0}$$

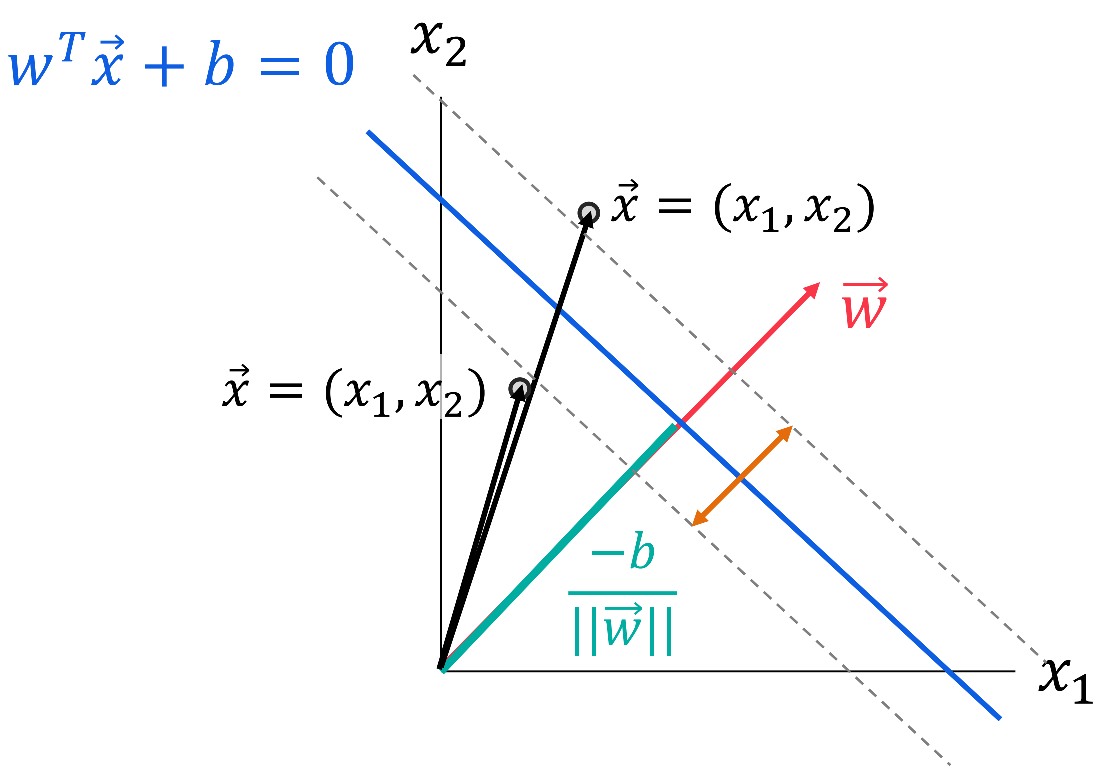
$$b = -(\vec{w} \cdot \vec{p})$$

Our decision rule

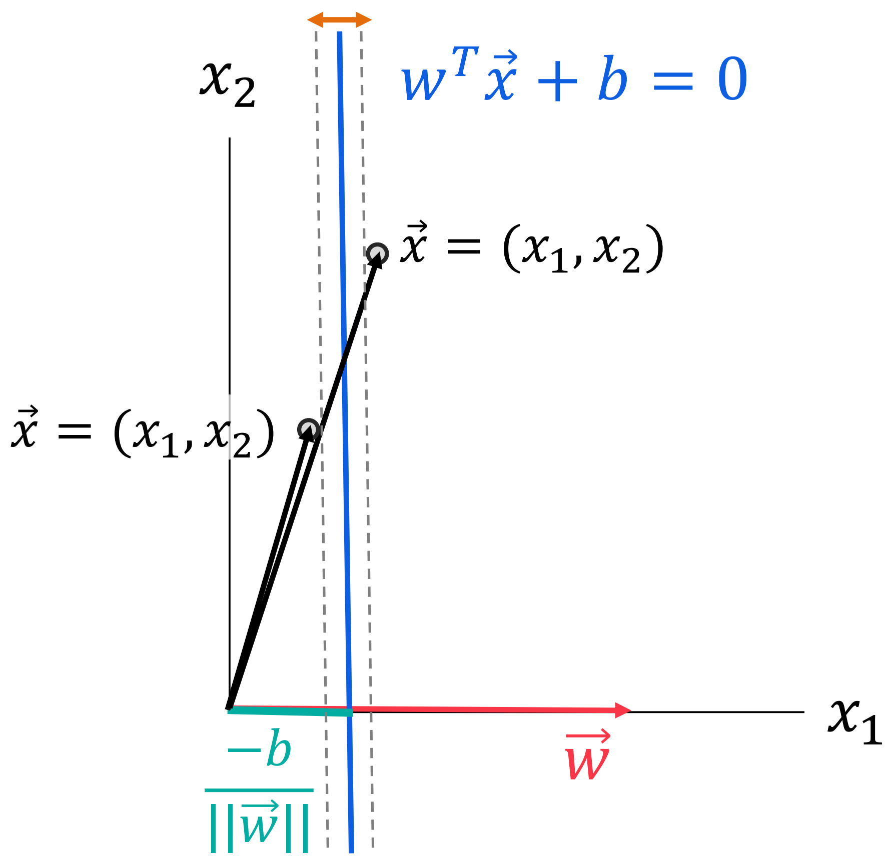
$$\begin{cases} \text{IF } w^T \cdot \vec{x} + b > 0 \text{ THEN } \vec{x} \text{ is above the plane.} \\ \text{IF } w^T \cdot \vec{x} + b < 0 \text{ THEN } \vec{x} \text{ is bellow the plane.} \end{cases}$$

$$sign(w^T \cdot \vec{x} + b) \begin{cases} \text{positive} \rightarrow +1 \\ \text{negative} \rightarrow -1 \end{cases}$$

Additional objective of SVM

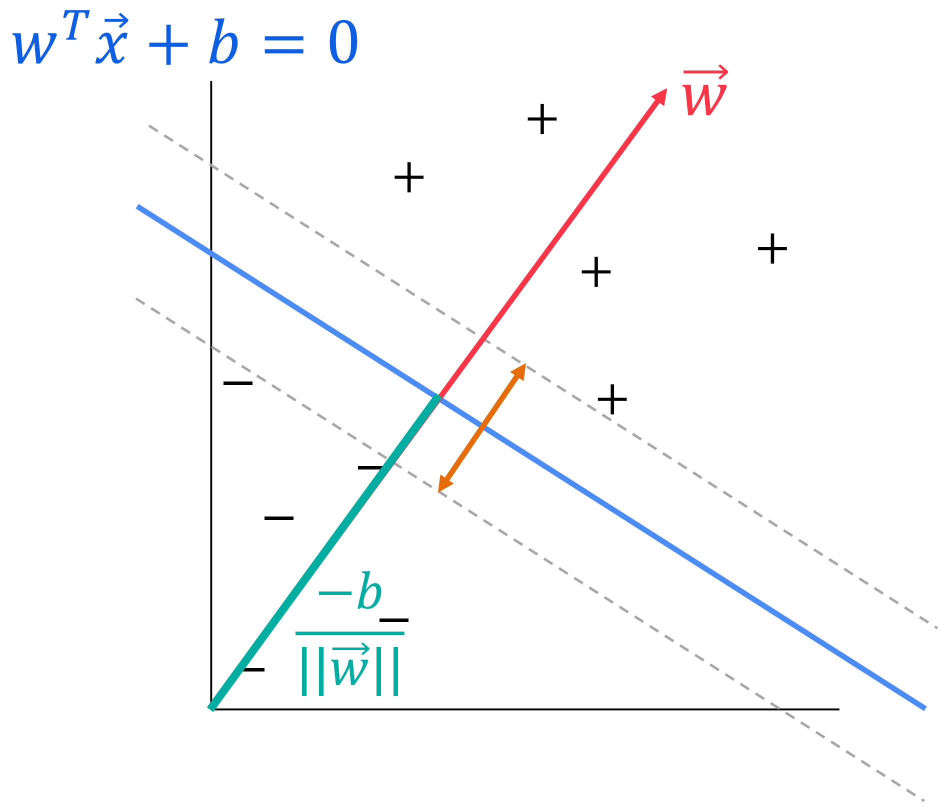


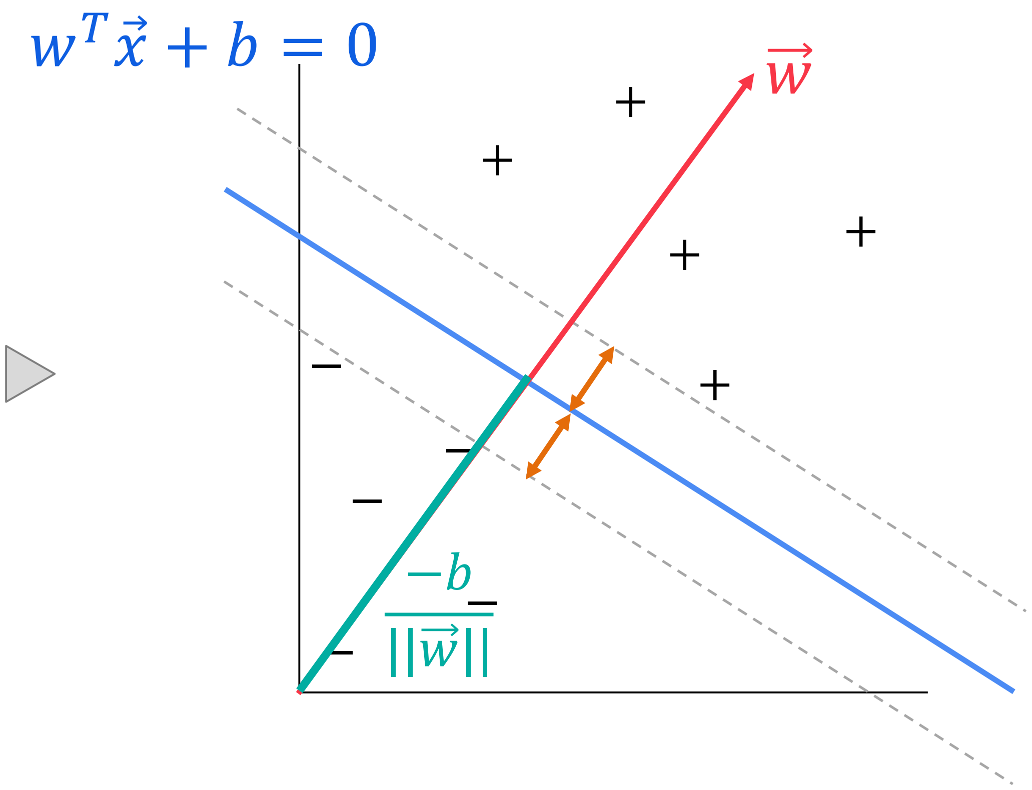
SVM



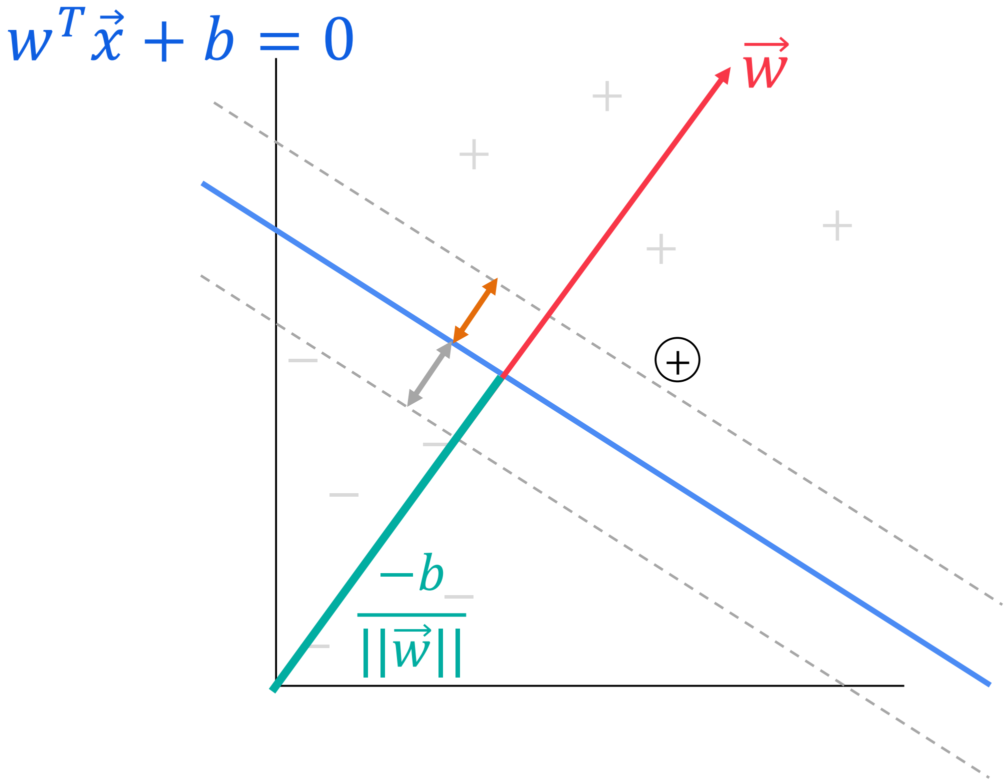
Perceptron

Additional objective of SVM

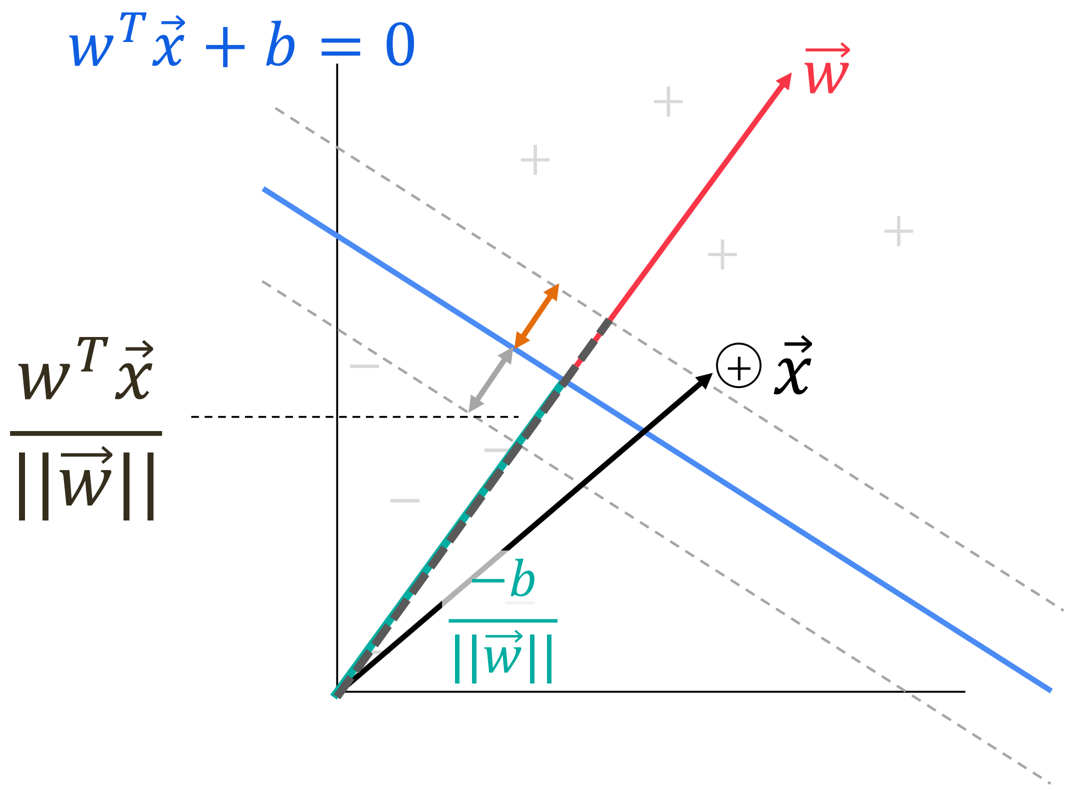




Additional objective of SVM

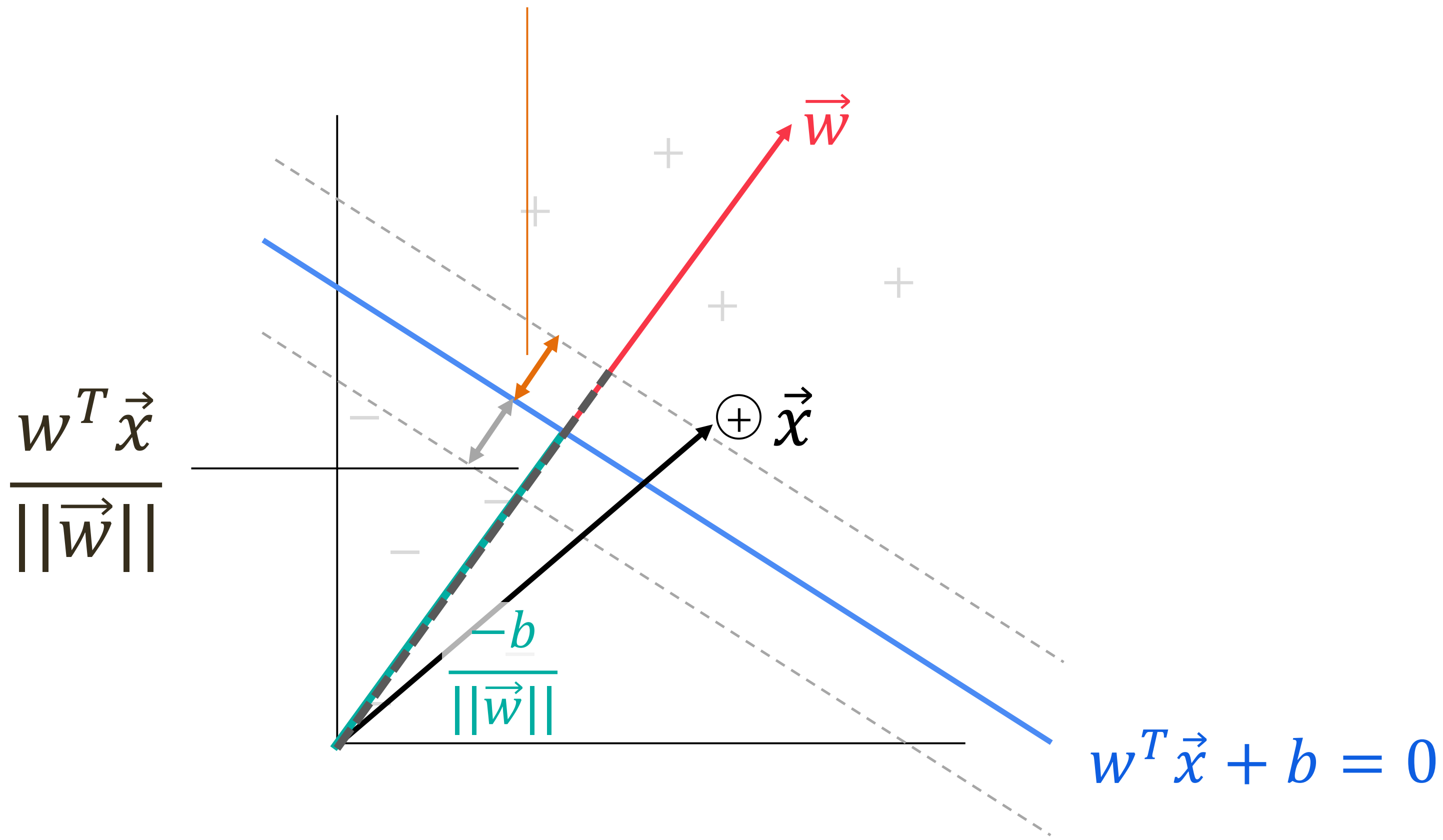


Additional objective of SVM

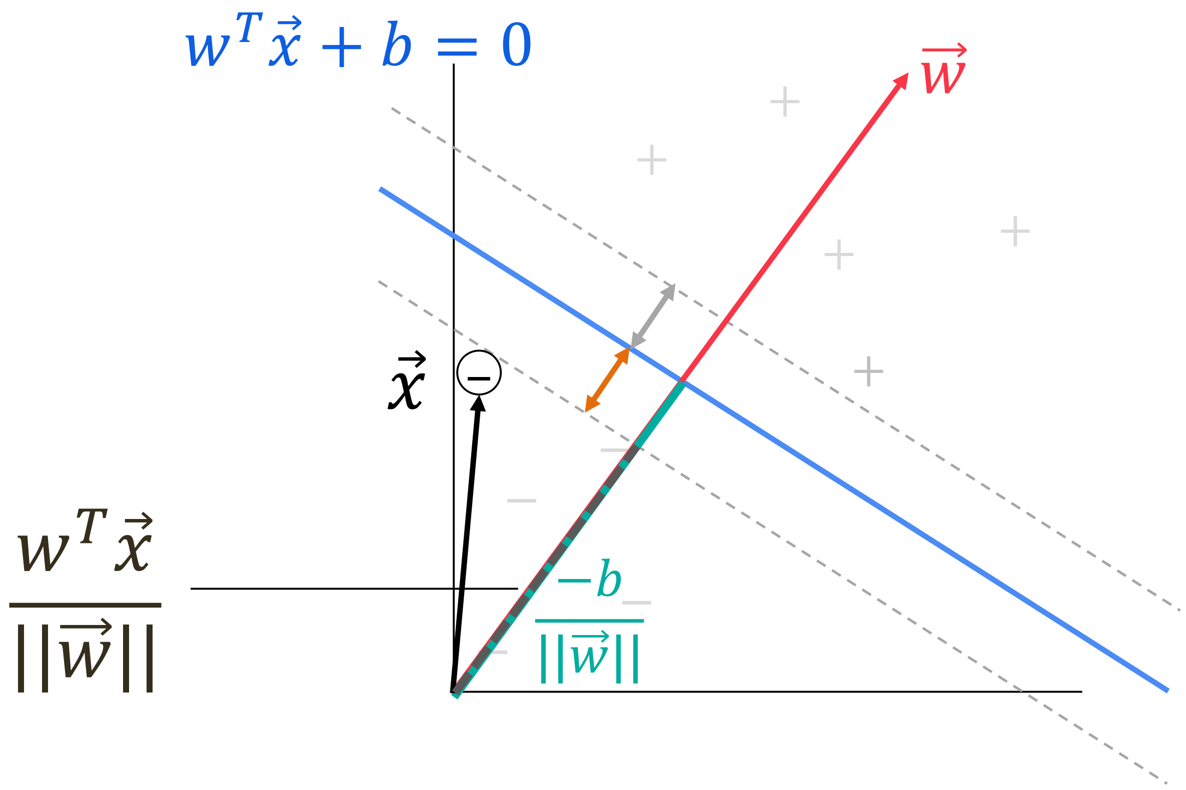


Additional objective of SVM

$$\frac{w^T\vec{x}}{||\vec{w}||} - \frac{-b}{||\vec{w}||} = \frac{w^T\vec{x}+b}{||\vec{w}||}$$

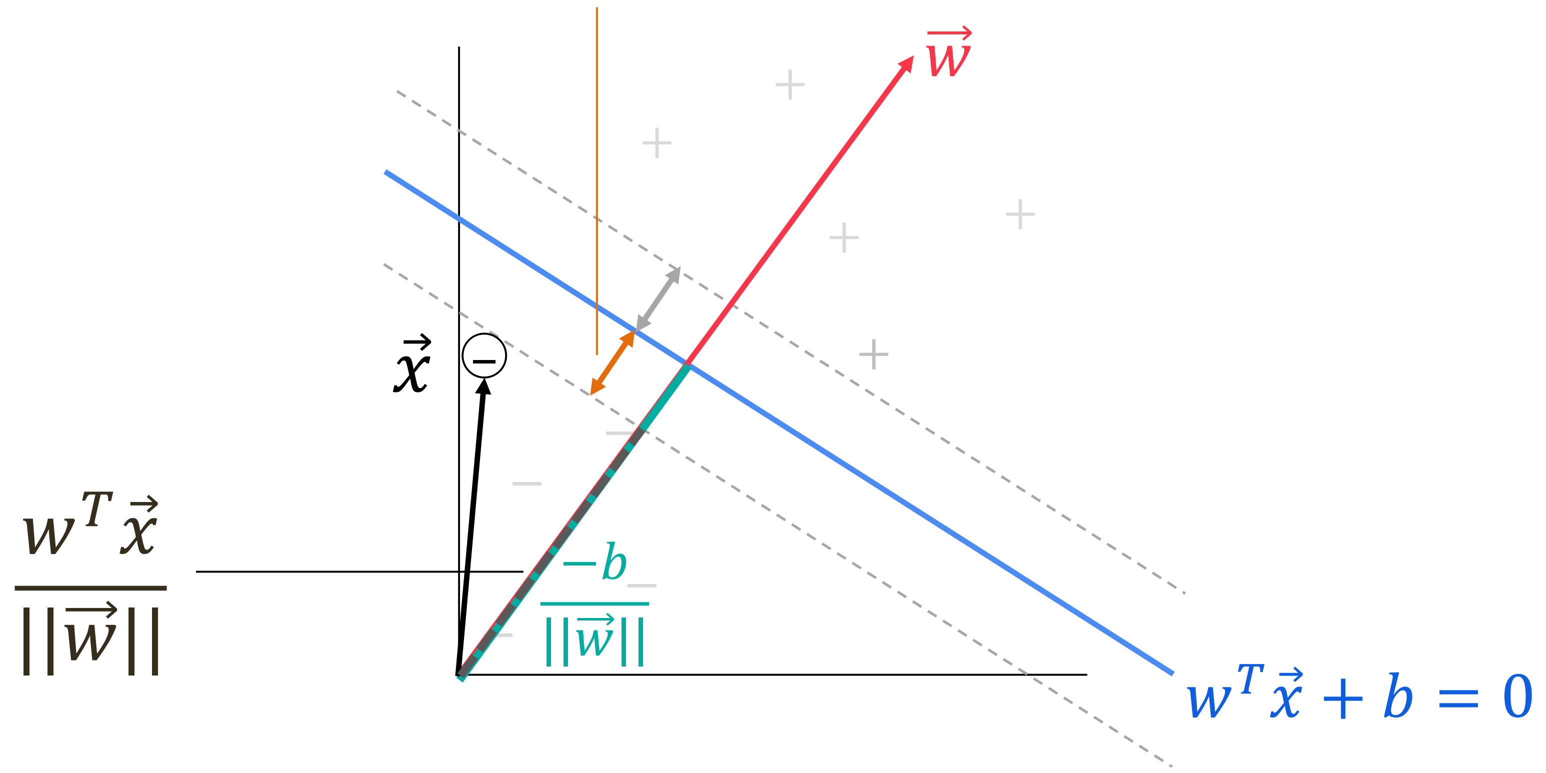


Additional objective of SVM

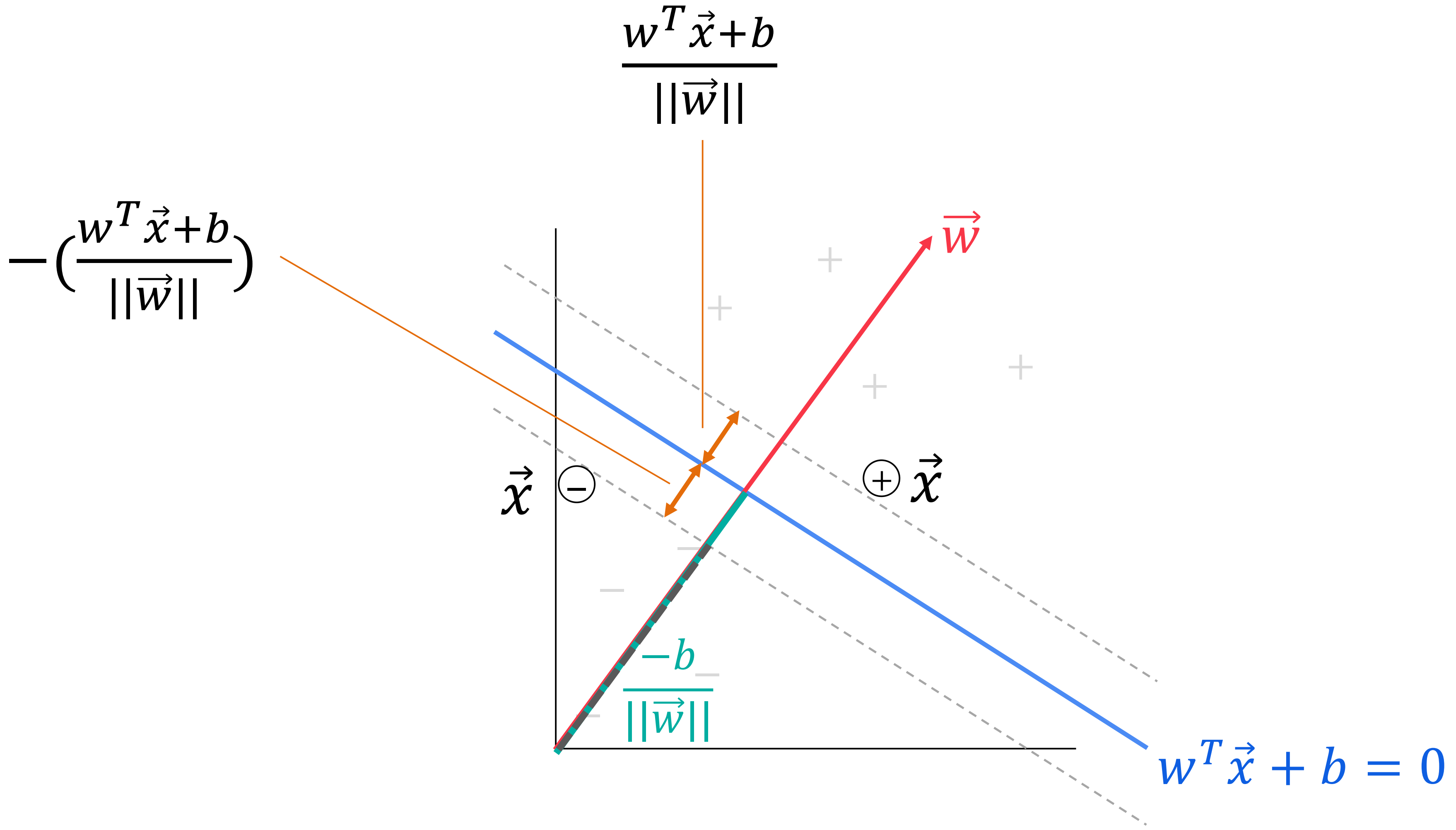


Additional objective of SVM

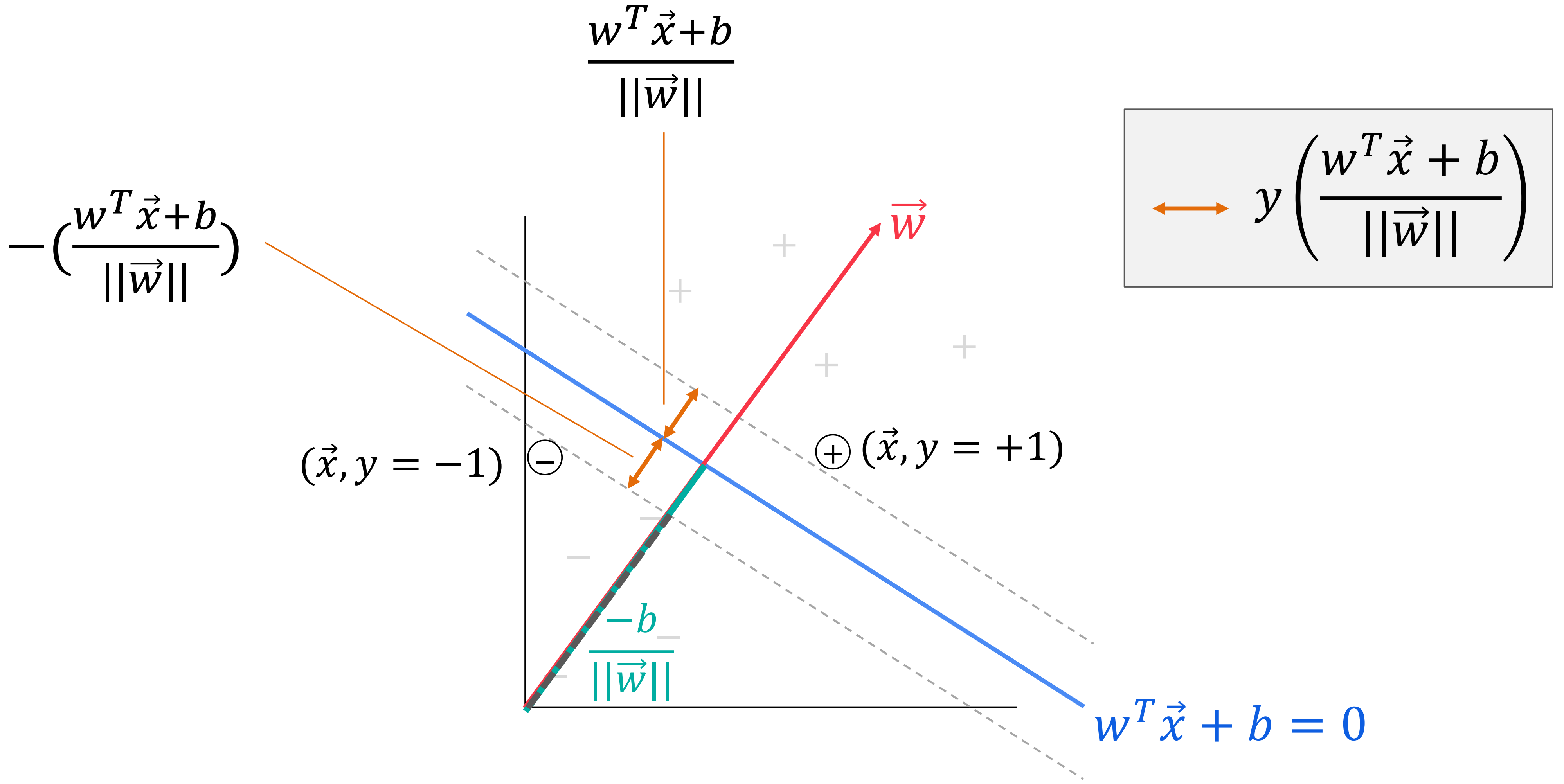
$$\frac{-b}{||\vec{w}||} - \frac{w^T\vec{x}}{||\vec{w}||} = -\left(\frac{w^T\vec{x}+b}{||\vec{w}||}\right)$$



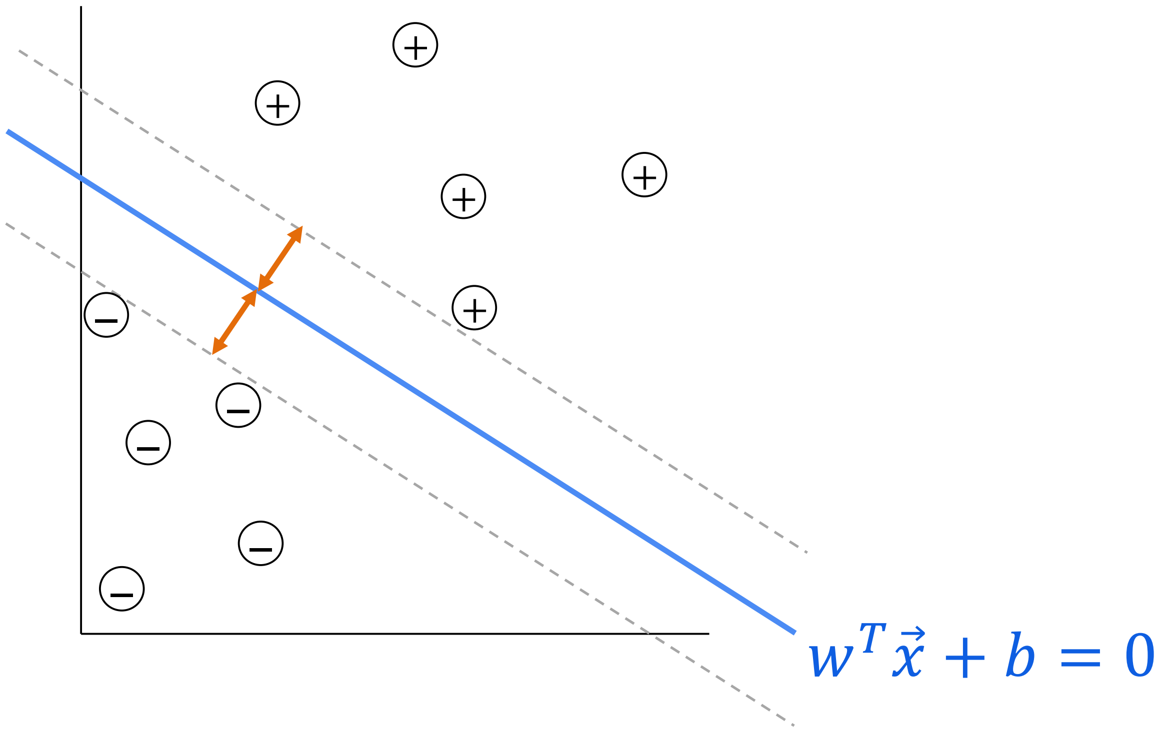
Additional objective of SVM



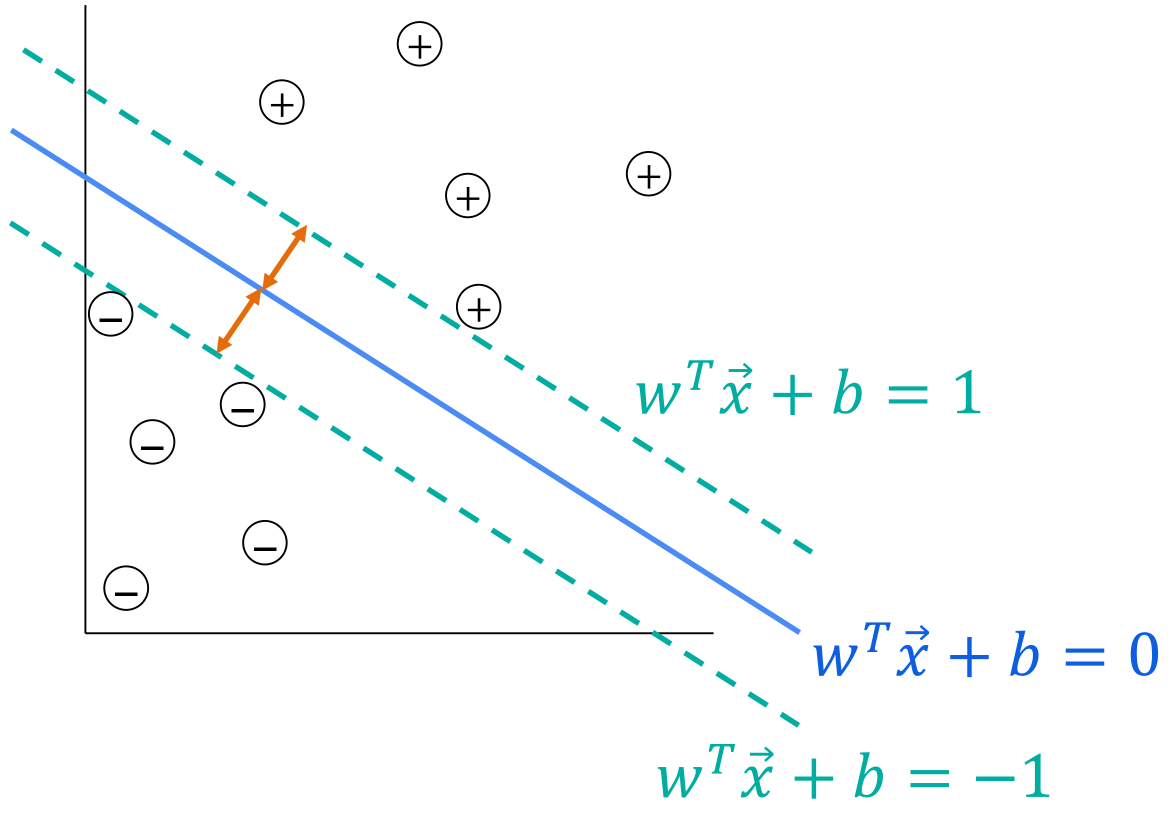
Additional objective of SVM



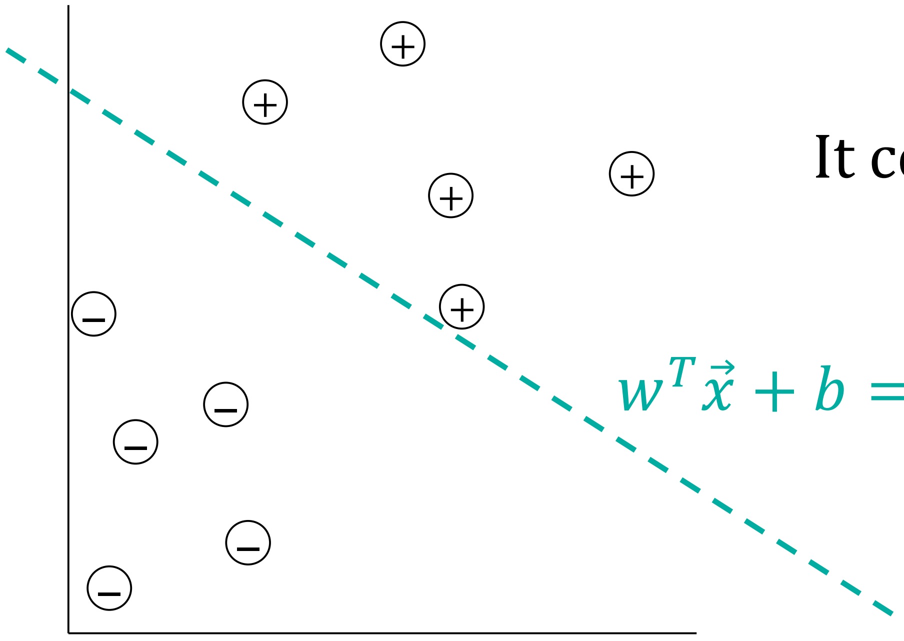
Additional objective of SVM



Additional objective of SVM



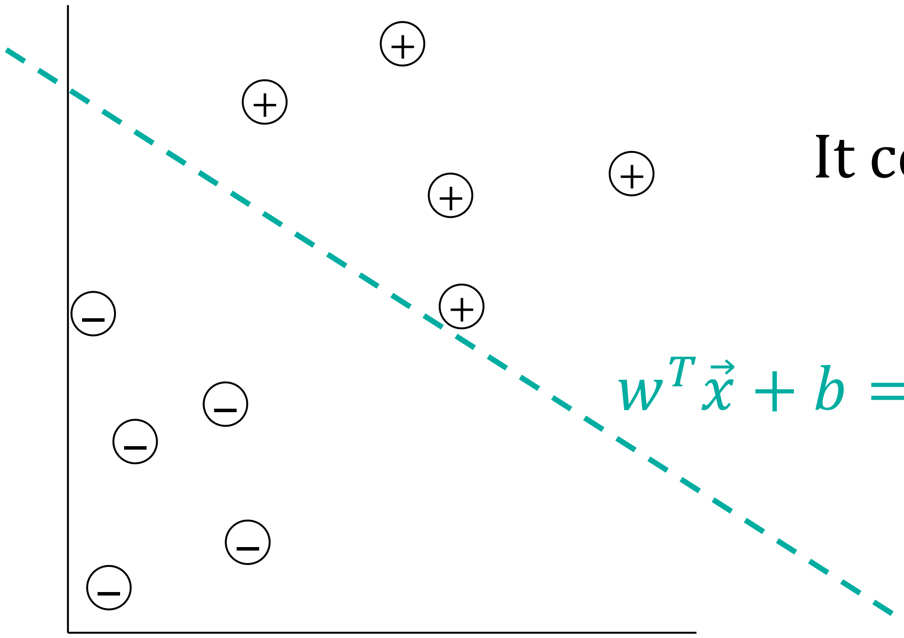
Additional objective of SVM



It could be any other value

$w^T \vec{x} + b = k$

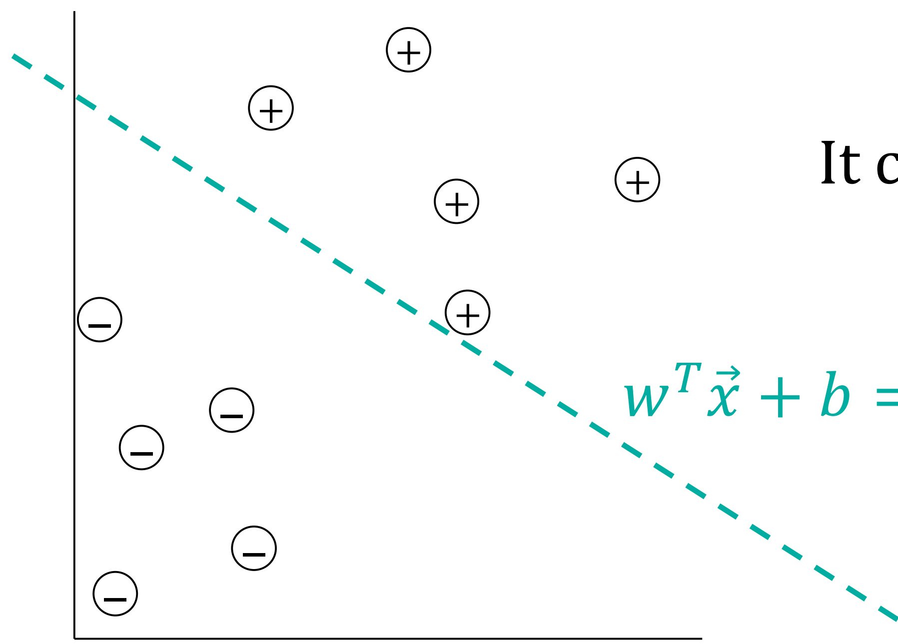
Additional objective of SVM



It could be any other value

$$w^T\vec{x} + b = k \quad \equiv \quad \vec{w}\cdot\vec{x} + b = k$$

Additional objective of SVM



It could be any other value

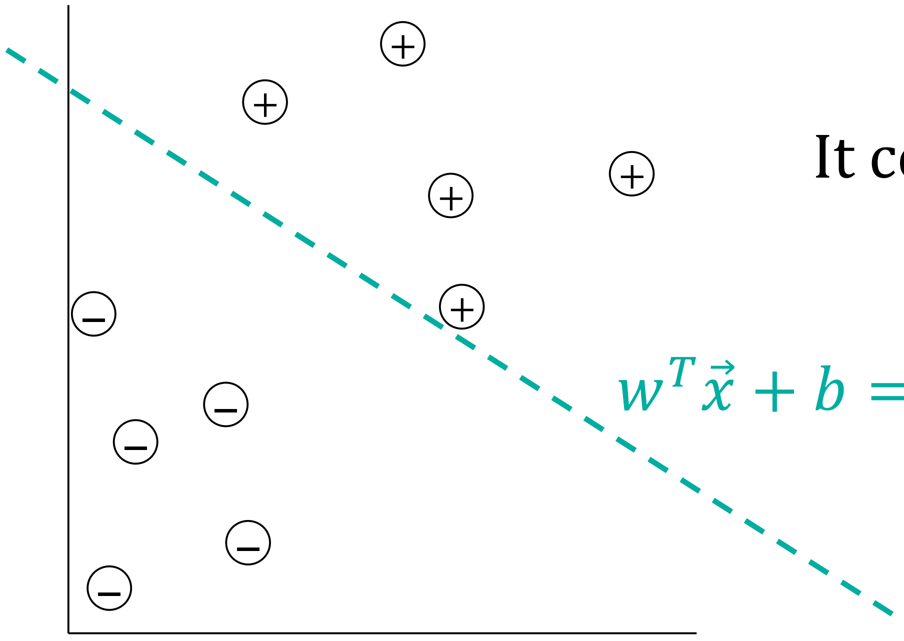
$$w^T\vec{x} + b = k \quad \equiv \quad \vec{w}\cdot\vec{x} + b = k$$

$$\equiv \quad \vec{w}\cdot\vec{x} + -(\vec{w}\cdot\vec{p}) = k$$

- Position vector (point on the plane): $\vec{p}$
- Perpendicular vector (to the plane): $\vec{w}$

$$b = -(\vec{w}\cdot\vec{p})$$

Additional objective of SVM



It could be any other value

$$w^T\vec{x} + b = k \equiv \vec{w}\cdot\vec{x} + b = k$$

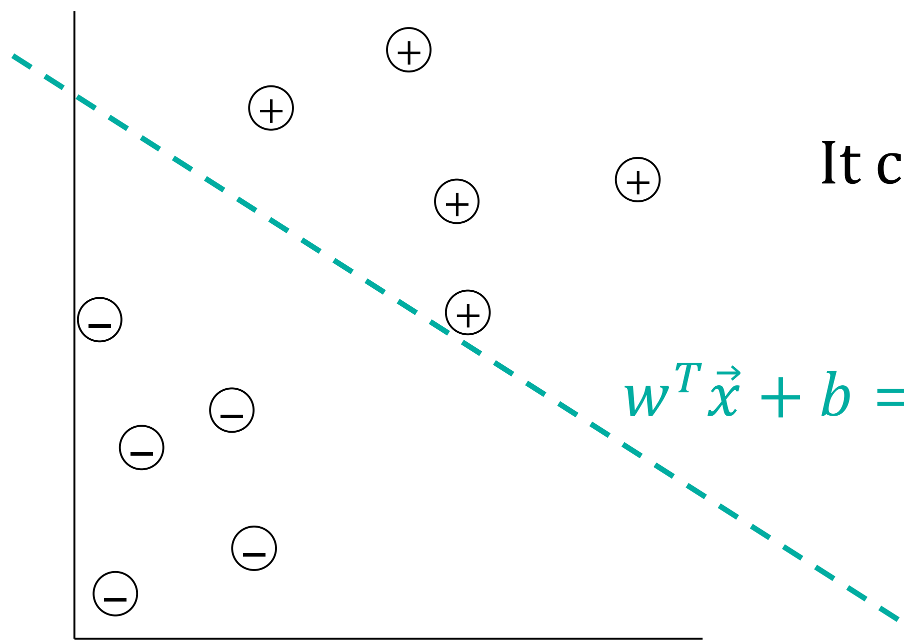
$$\equiv \vec{w}\cdot\vec{x} + -(\vec{w}\cdot\vec{p}) = k$$

$$\equiv \frac{1}{k}\vec{w}\cdot\vec{x} + -(\frac{1}{k}\vec{w}\cdot\vec{p}) = 1$$

- Position vector (point on the plane): $\vec{p}$
- Perpendicular vector (to the plane): $\vec{w}$

$$b = -(\vec{w}\cdot\vec{p})$$

Additional objective of SVM



It could be any other value

$$w^T\vec{x} + b = k \;\equiv\; \vec{w}\cdot\vec{x} + b = k$$

$$\equiv\; \vec{w}\cdot\vec{x} + -(\vec{w}\cdot\vec{p}) = k$$

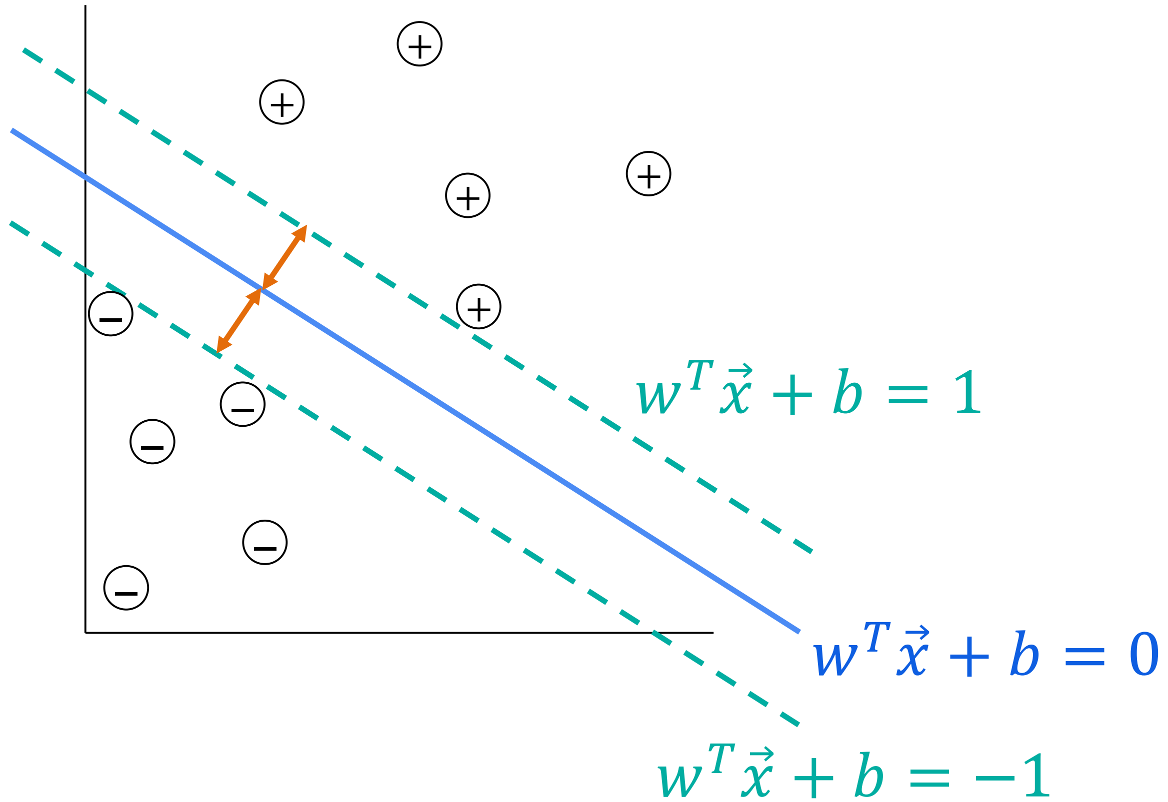
$$\equiv\; \frac{1}{k}\vec{w}\cdot\vec{x} + -(\frac{1}{k}\vec{w}\cdot\vec{p}) = 1$$

- Position vector (point on the plane): $\vec{p}$
- Perpendicular vector (to the plane): $\vec{w}$

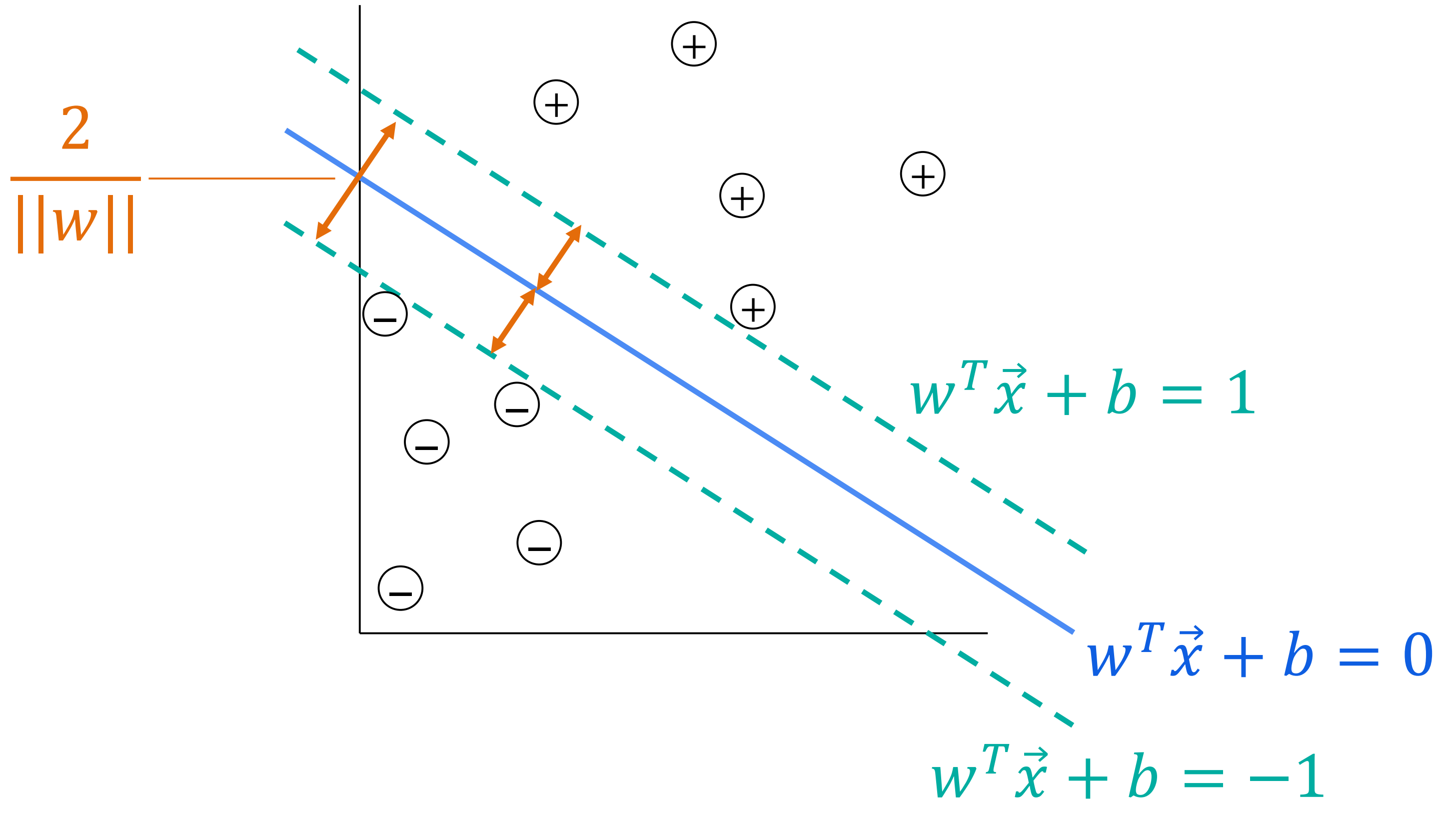
$$b = -(\vec{w}\cdot\vec{p})$$

$\frac{1}{k}\vec{w}$: perpendicular vector to the plane

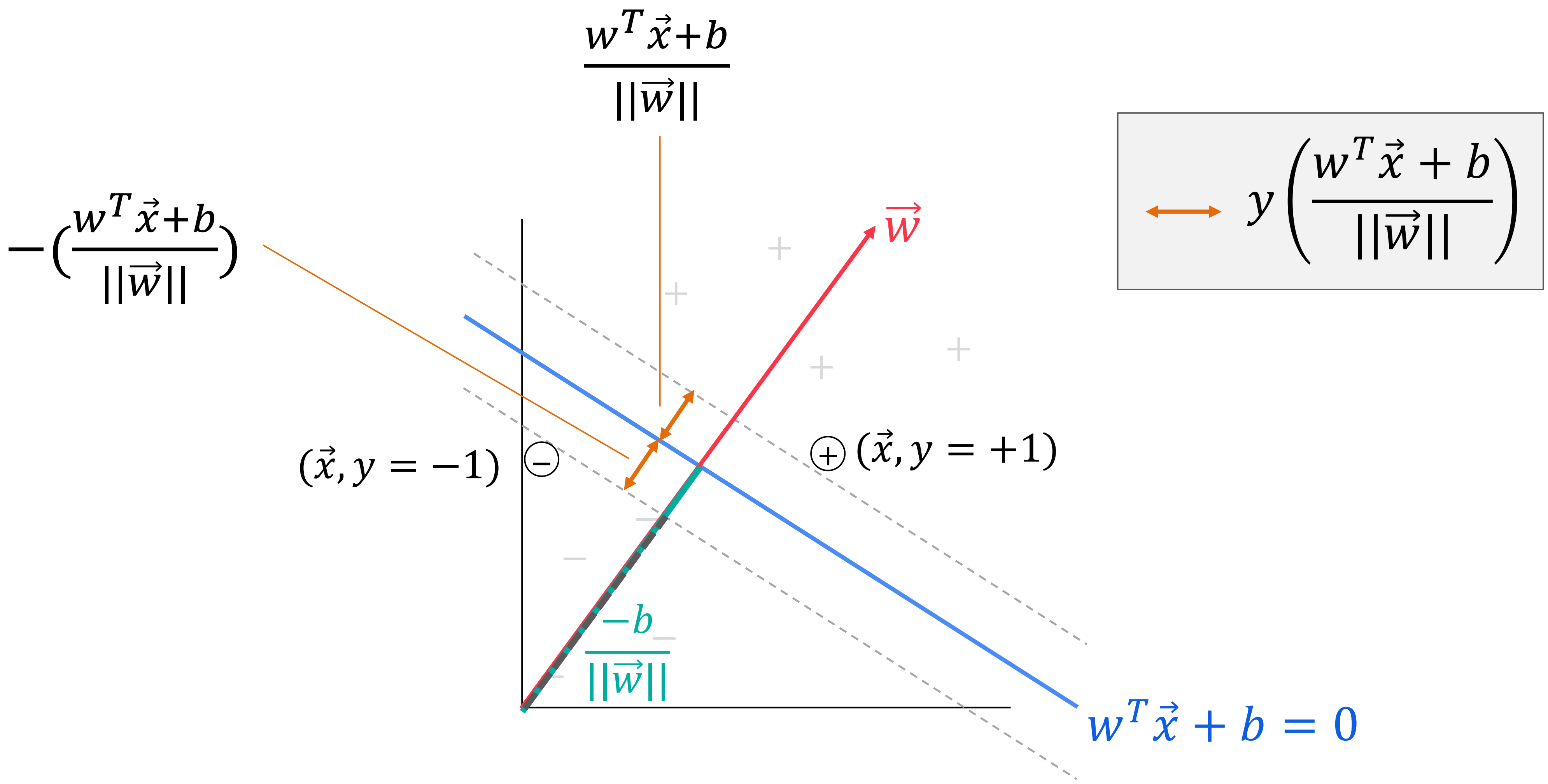
Additional objective of SVM



Additional objective of SVM

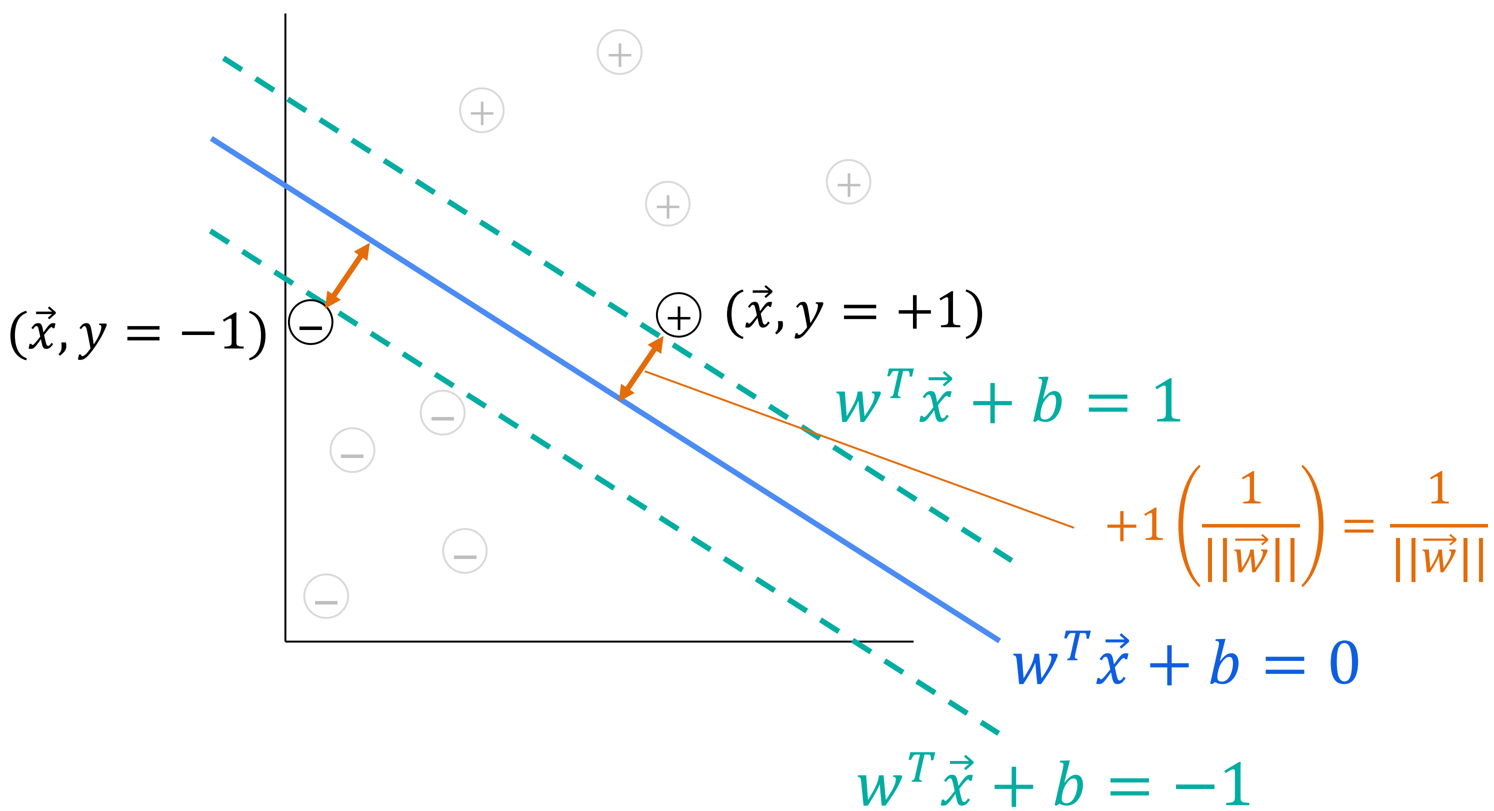


Additional objective of SVM



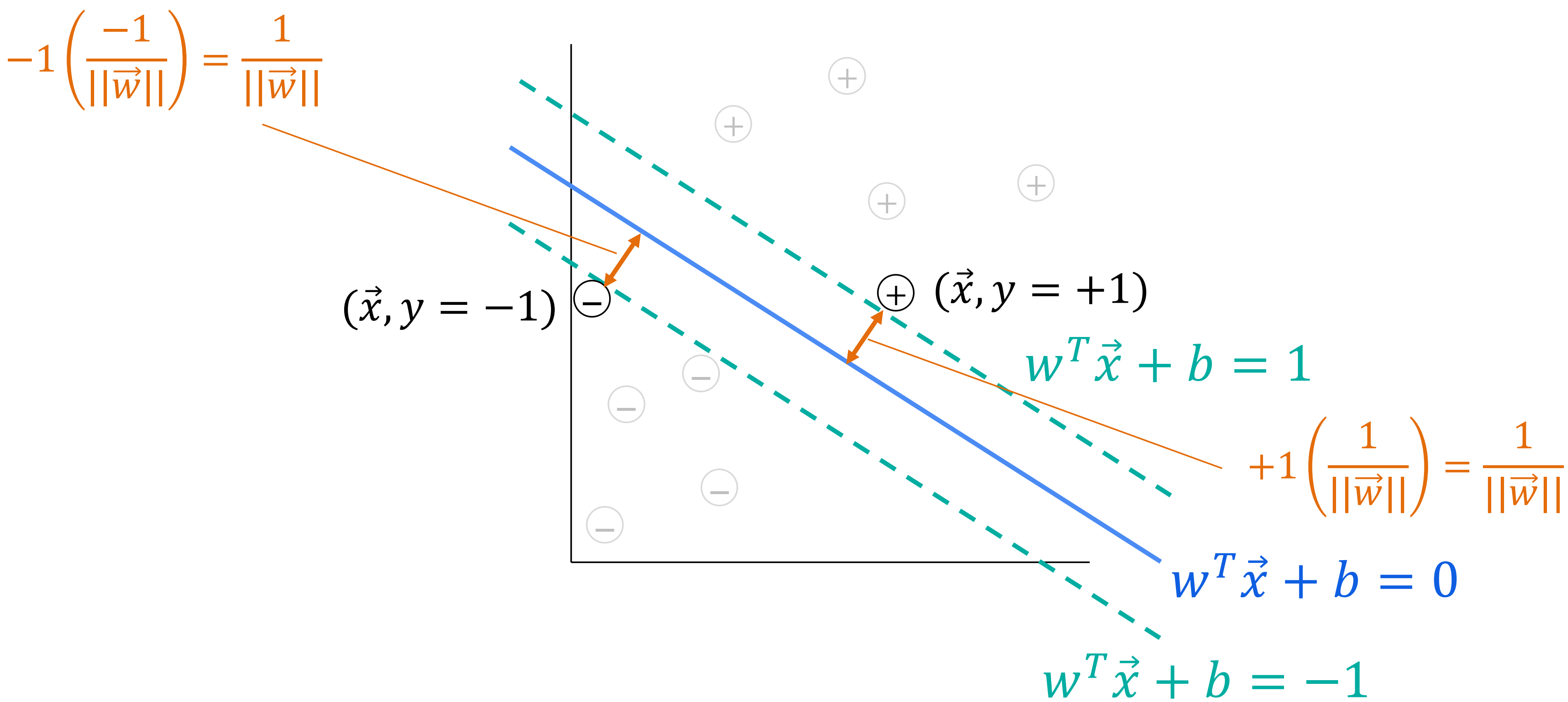
Additional objective of SVM

$$\longleftrightarrow \; y\left(\frac{w^T\vec{x}+b}{||\vec{w}||}\right)$$

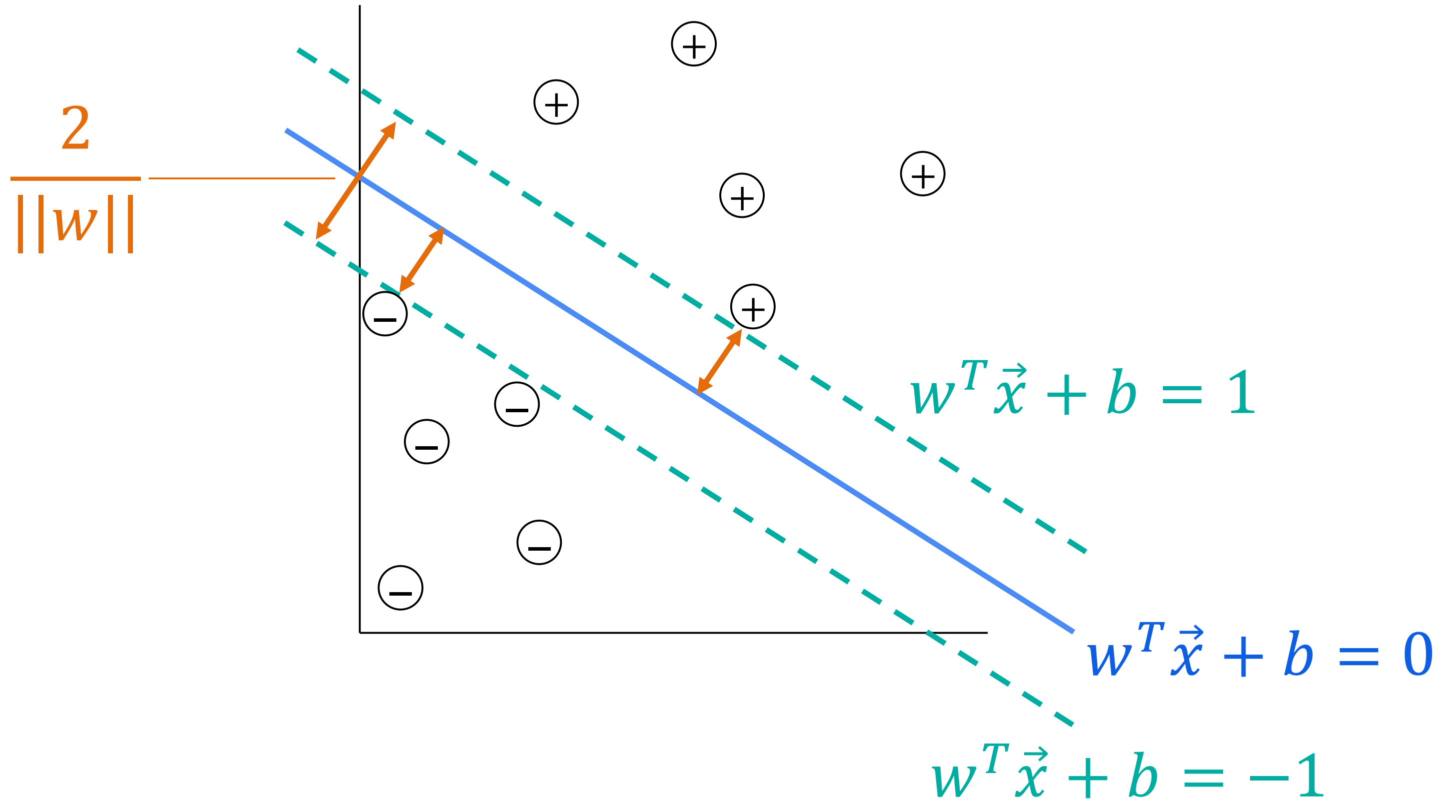


Additional objective of SVM

$$\longleftrightarrow \quad y\left(\frac{w^T\vec{x}+b}{||\vec{w}||}\right)$$

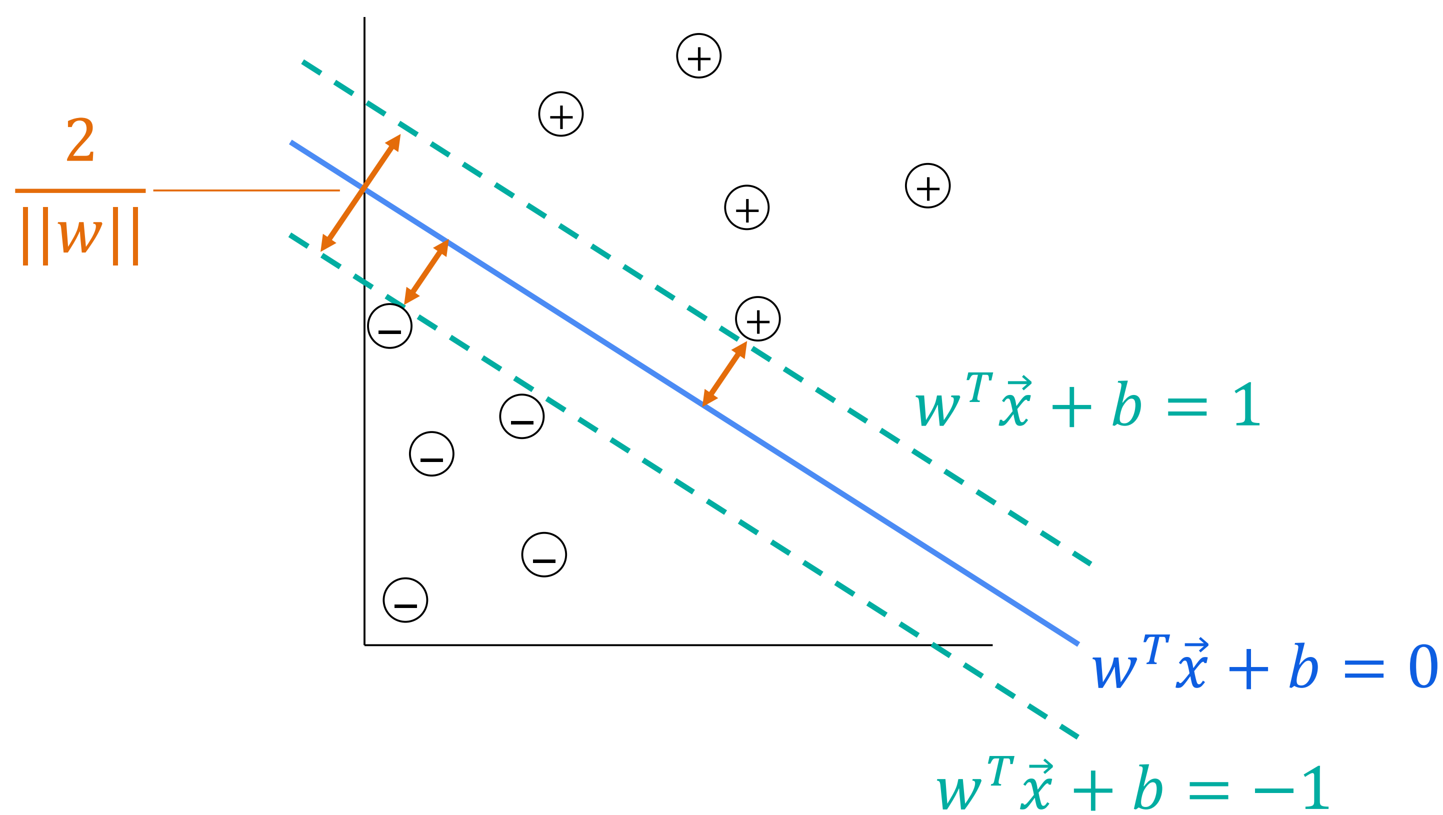


Additional objective of SVM



Objective of SVM

Maximize $\frac{2}{||w||}$
subject to $y_i(w^T\overrightarrow{x_i}) \geq\ 1, \forall i \in n$

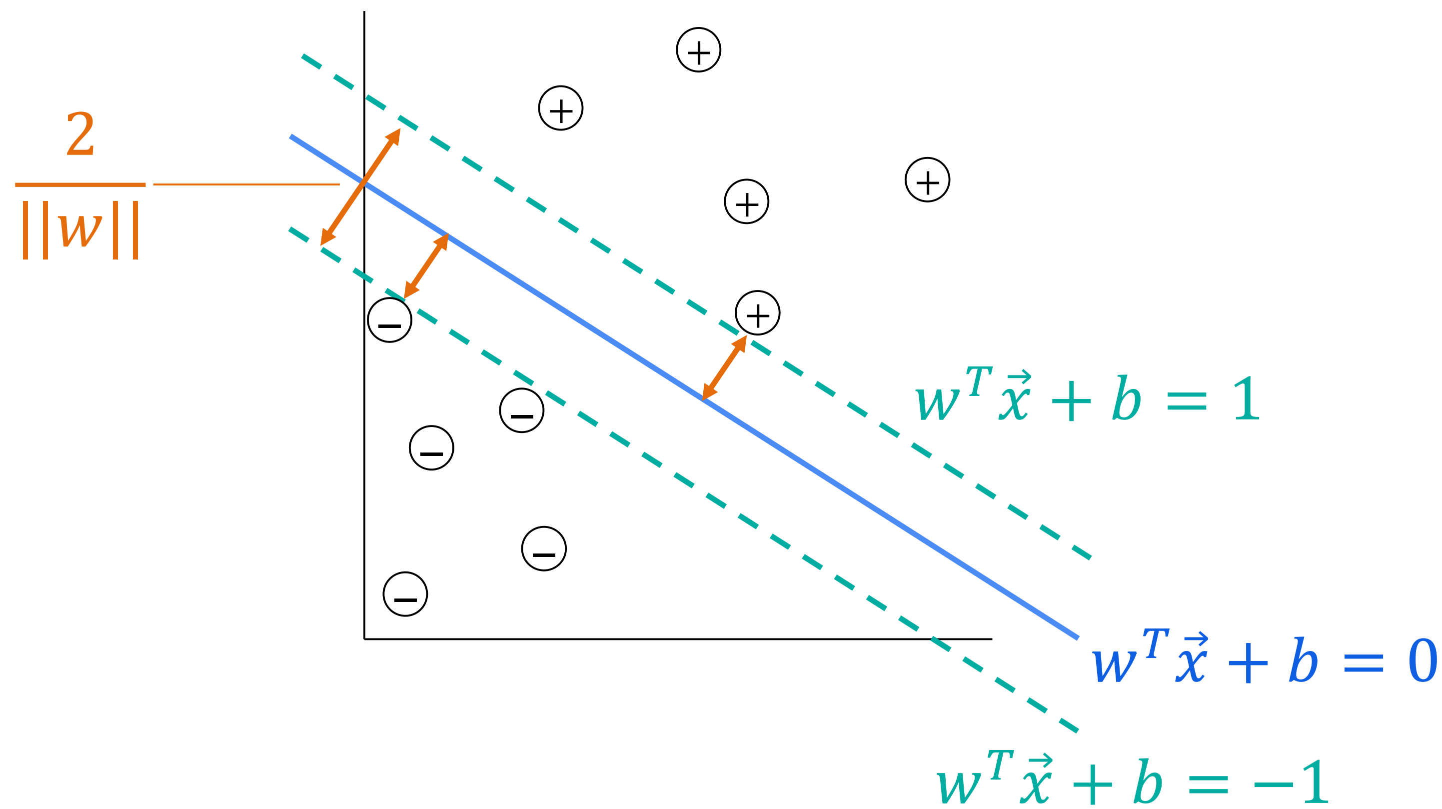


Objective of SVM

Maximize $\frac{2}{||w||}$
subject to $y_i(w^T\overrightarrow{x_i}) \geq 1, \forall i \in n$

$\equiv$

Minimize $||w||$
subject to $y_i(w^T\overrightarrow{x_i}) \geq 1, \forall i \in n$

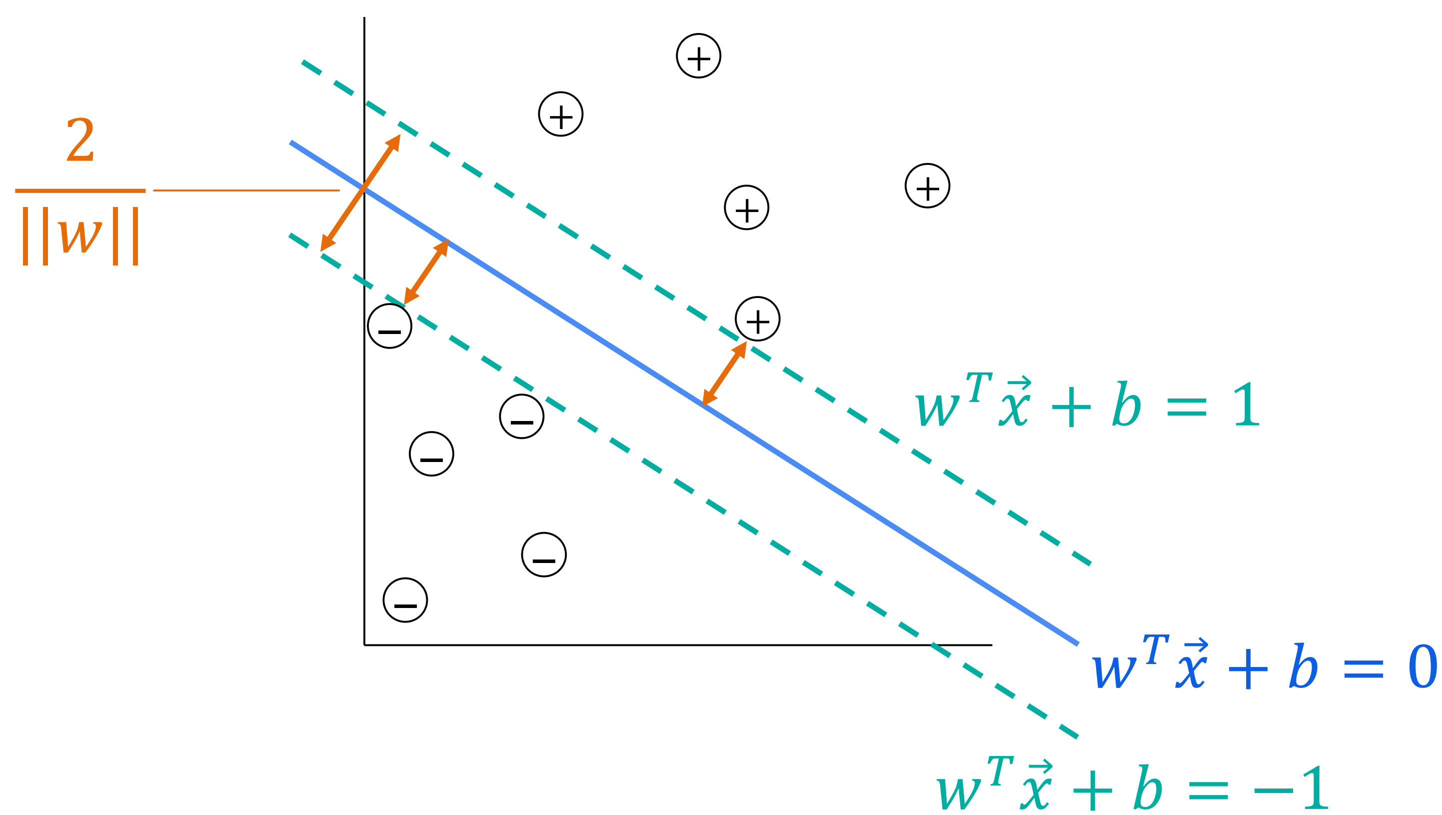


Objective of SVM

Minimize $||w||$
subject to $y_i(w^T\overrightarrow{x_i}) \geq 1, \forall i \in n$

$\equiv$

Minimize $||w||^2$
subject to $y_i(w^T\overrightarrow{x_i}) \geq 1, \forall i \in n$

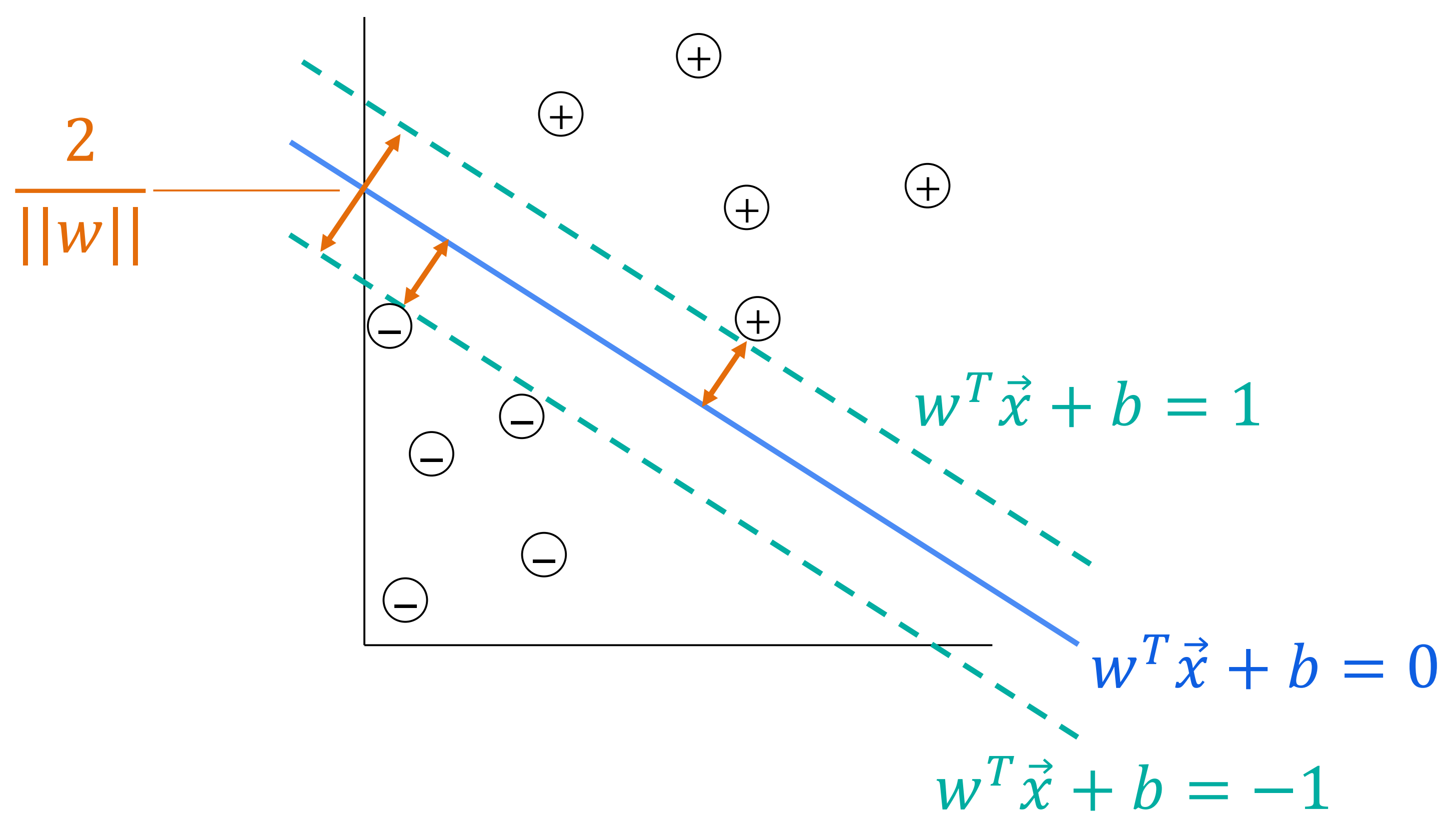


Objective of SVM

Minimize $||w||^2$
subject to $y_i(w^T\overrightarrow{x_i}) \geq 1, \forall i \in n$

$\equiv$

Minimize $\frac{1}{2}||w||^2$
subject to $y_i(w^T\overrightarrow{x_i}) \geq 1, \forall i \in n$



Contents of this week

- ~~Equation of lines and planes~~
- ~~Distance from a point to a plane~~
- ~~Support vector machine part I~~
- Constrained optimization
- Support vector machine part II

Thank you