Geometric Centrality – Harmonic Centrality

- A solution to the problem of the closeness centrality:
  - Take the *harmonic mean* of the shortest(geodesic) distances from i:

$$C_i = \frac{1}{n-1} \sum_{j(\neq i)} \frac{1}{d_{ij}}$$

  - The *harmonic centrality* handles disconnected components, since $d_{ij} = \infty$ does not affect the formulation any more.
  - Harmonic mean H :

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}} = \left(\frac{\sum_{i=1}^{n} x_i^{-1}}{n} \right)^{-1}$$

Geometric Centrality – Betweenness

- Measures how much a node falls "between" others
- How many node pairs have to go though a node i to reach one another in the minimum steps (shortest paths)?

$$C_i = \frac{1}{\binom{n-1}{2}} \sum_{st} \frac{n_{st}^i}{g_{st}},$$

- n_{st}^i: # of shortest paths from s to t passing through i
- g_{st}: total # of shortest paths from s to t
- $\binom{n-1}{2}$: normalized by # of pairs of vertices excluding the vertex i

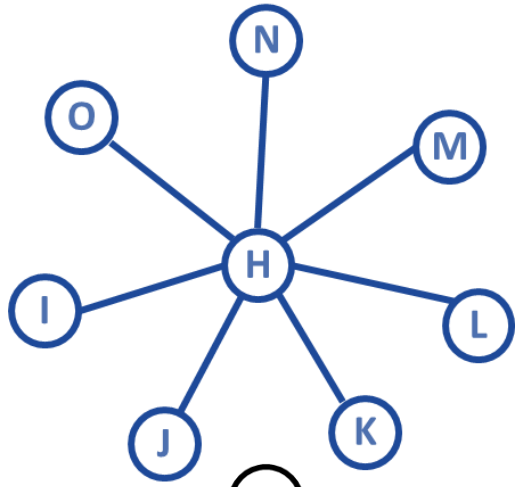
- where $\frac{n_{st}^i}{g_{st}} = 0$ if both $n_{st}^i = 0$ and $g_{st} = 0$.

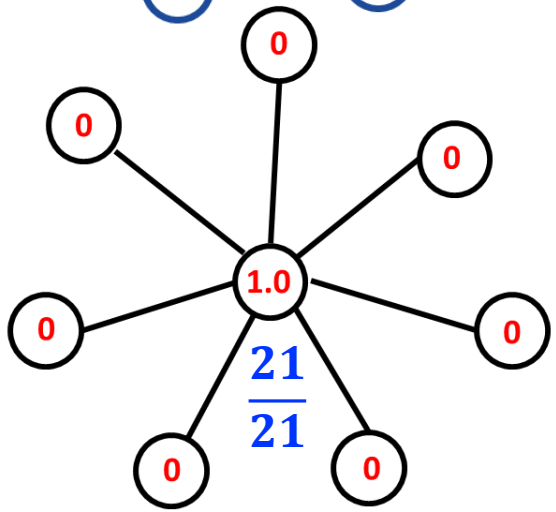
Geometric Centrality – Betweenness

- Calculate the normalized betweenness centrality.

$$C_i = \frac{1}{\binom{n-1}{2}} \sum_{st} \frac{n_{st}^i}{g_{st}}$$

$\frac{5}{15}$ $\frac{8}{15}$ $\frac{9}{15}$

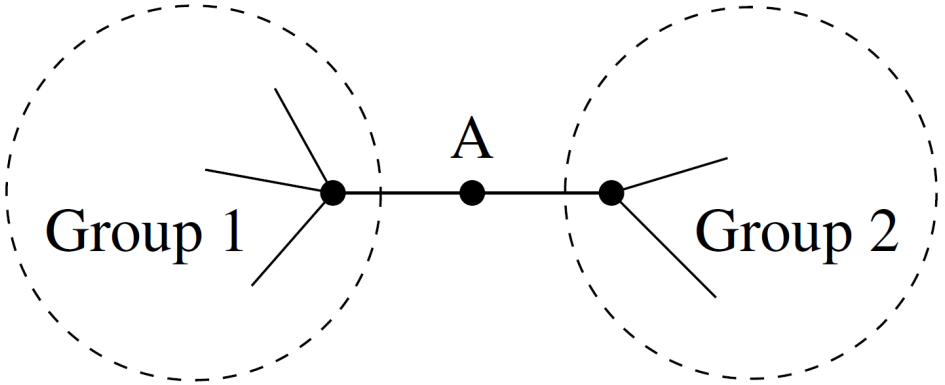




$\frac{21}{21}$

Geometric Centrality – Betweenness

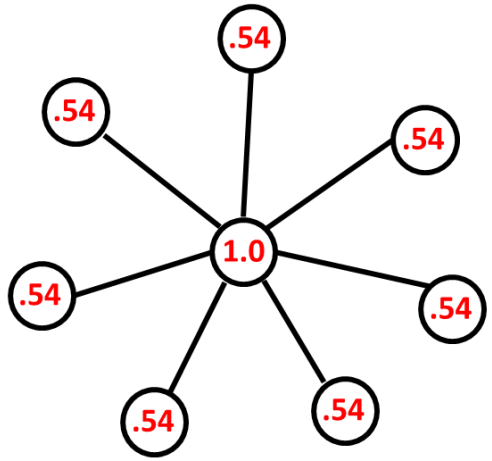
- Capture brokerage (bridge) roles:
  - Control over information flow between others – *social capital* (sociological literature)
  - Removal of nodes with highest betweenness will disrupt communications between other nodes.



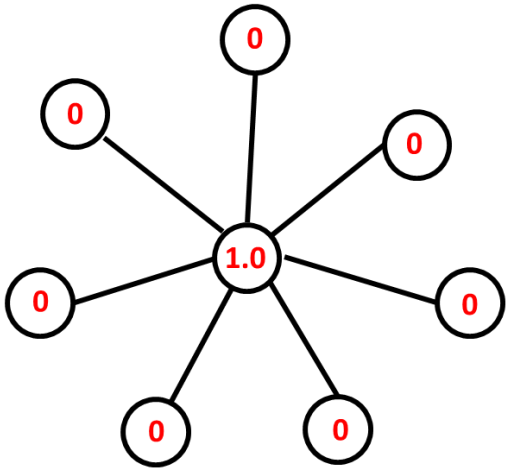
[A low-degree node with high betweeness]

Different Measures for Different Applications

- Closeness Centrality



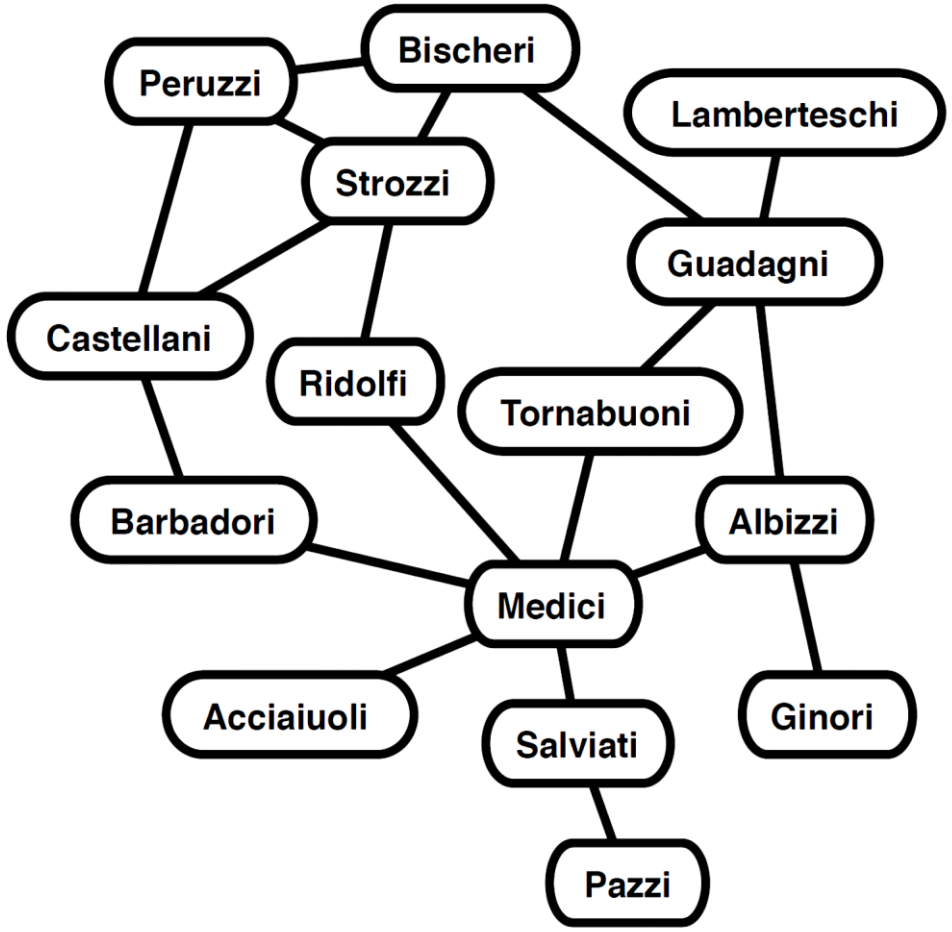
- Betweeness Centrality



In the Real World: Closeness Centrality

- Birthplace of the Italian Renaissance

In the Real World: Closeness Centrality

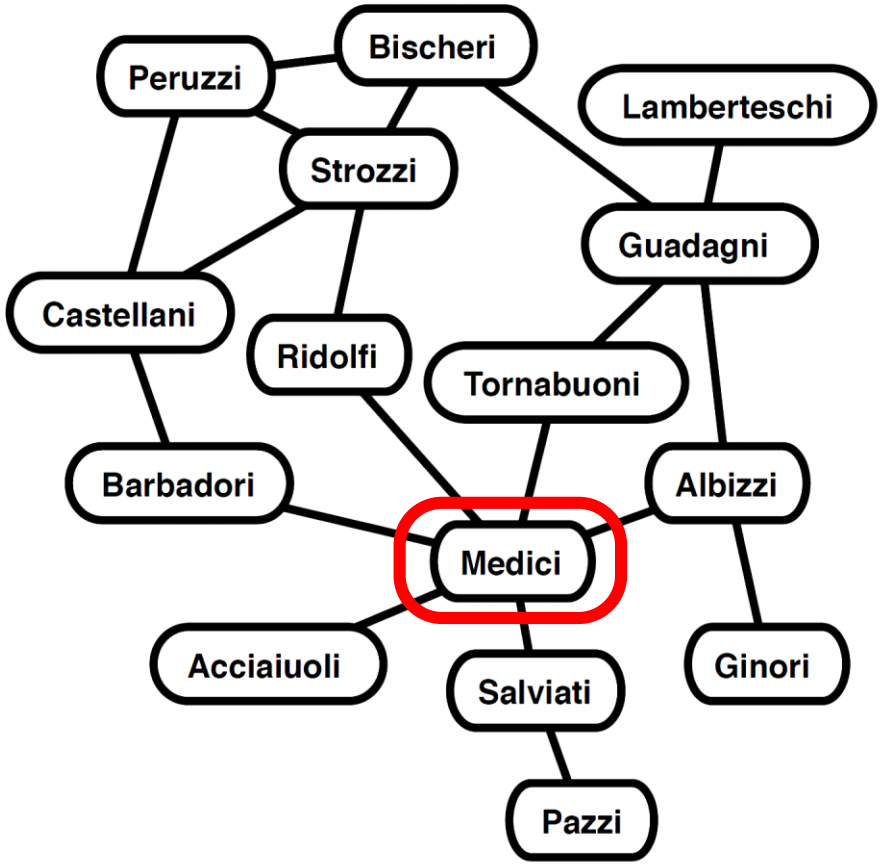


Florence families

inter-family marriages

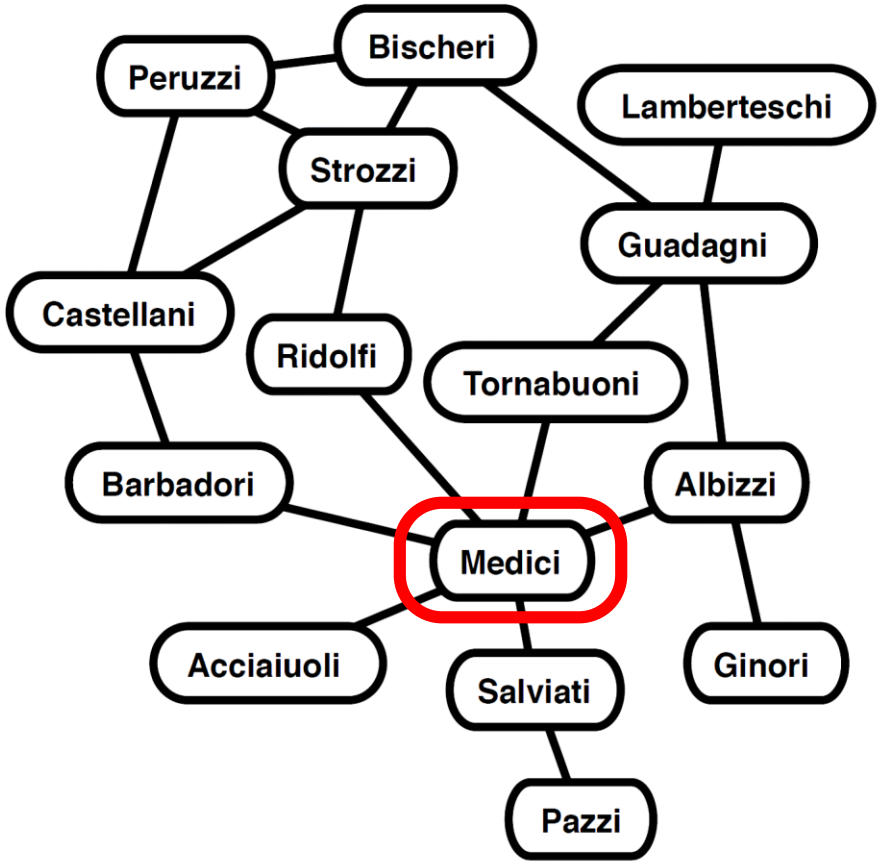
In the Real World: Closeness Centrality

- Who is in the center?



In the Real World: Closeness Centrality

- Who is in the center?



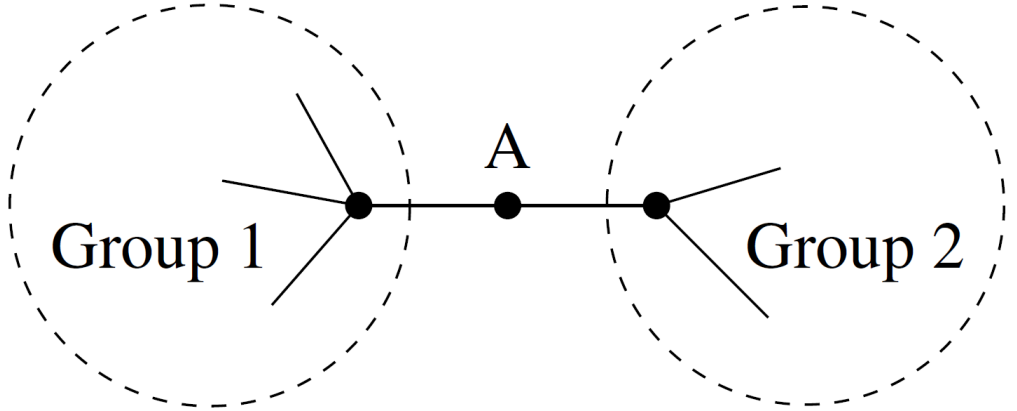
[Revisited]:

- Central vertices can be **connected to others with minimum steps**.
  - Efficient to exchange information with others or to **spread innovation**
  - Do not need brokers: **independent** and **autonomous**

In the Real World: Betweenness Centrality

Examples in the real world?

- Celebrities/orgs connecting different
  - countries
  - cultures
  - languages
  - branches of entertainment industry

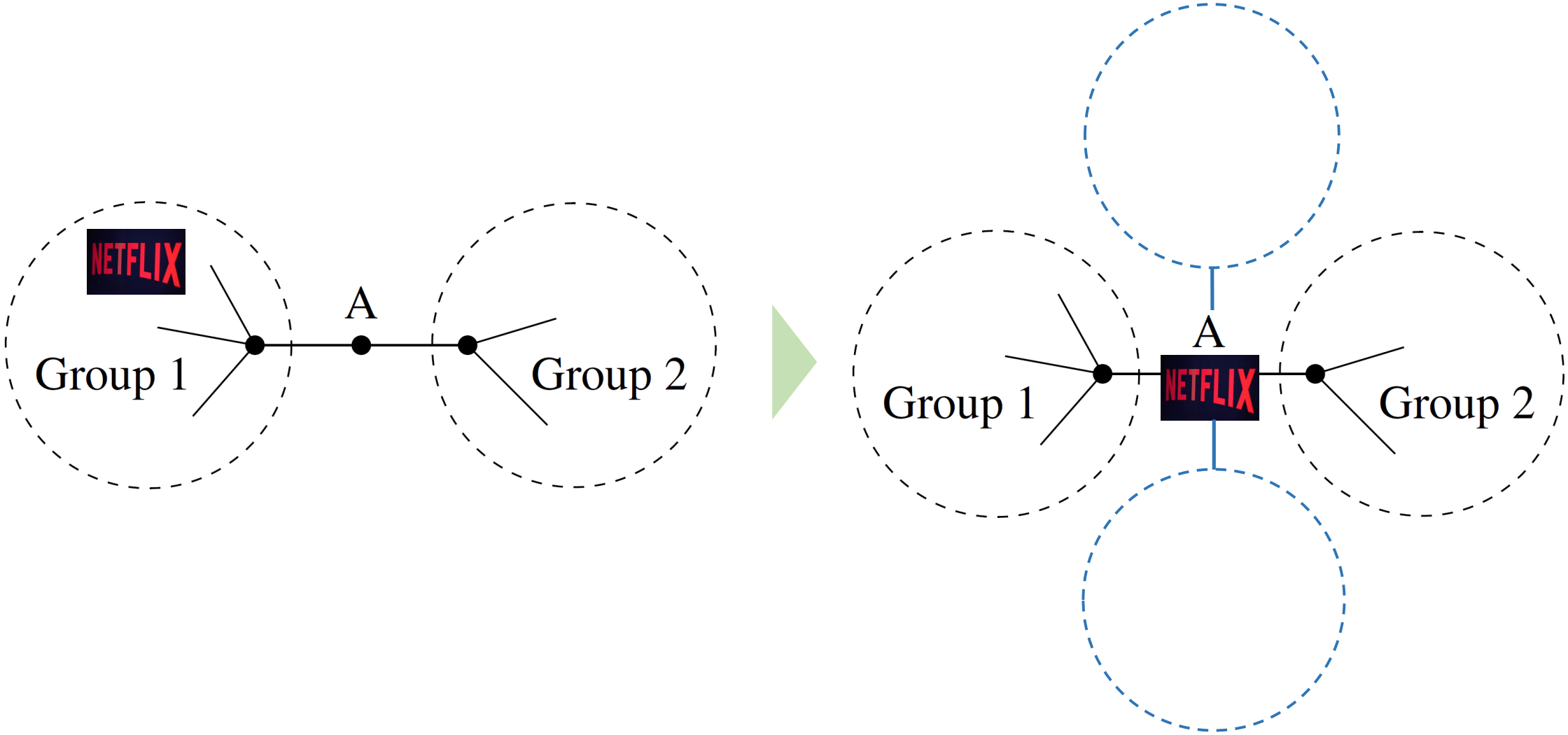


CUBE JYP ENTER LEADER IN ENT YG LOEN Big Hit

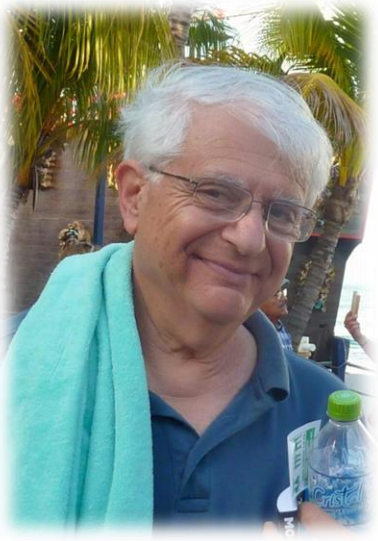
NETFLIX

LinkedIn

In the Real World: Betweenness Centrality



In the Real World: Betweenness Centrality



Prof. Mark Granovetter
@Stanford University

- "The Strength of Weak Ties"
  - *American Journal of Sociology* (1973)
  - The largest citations over 50,000 in social science
  - Most job seekers found their ultimate employment trough a weak tie rather than a close friend.

Information (*e.g.*, job opportunities) is **more frequently synchronized** between friends (*i.e.*, no novel information).

What do you see in the real world?