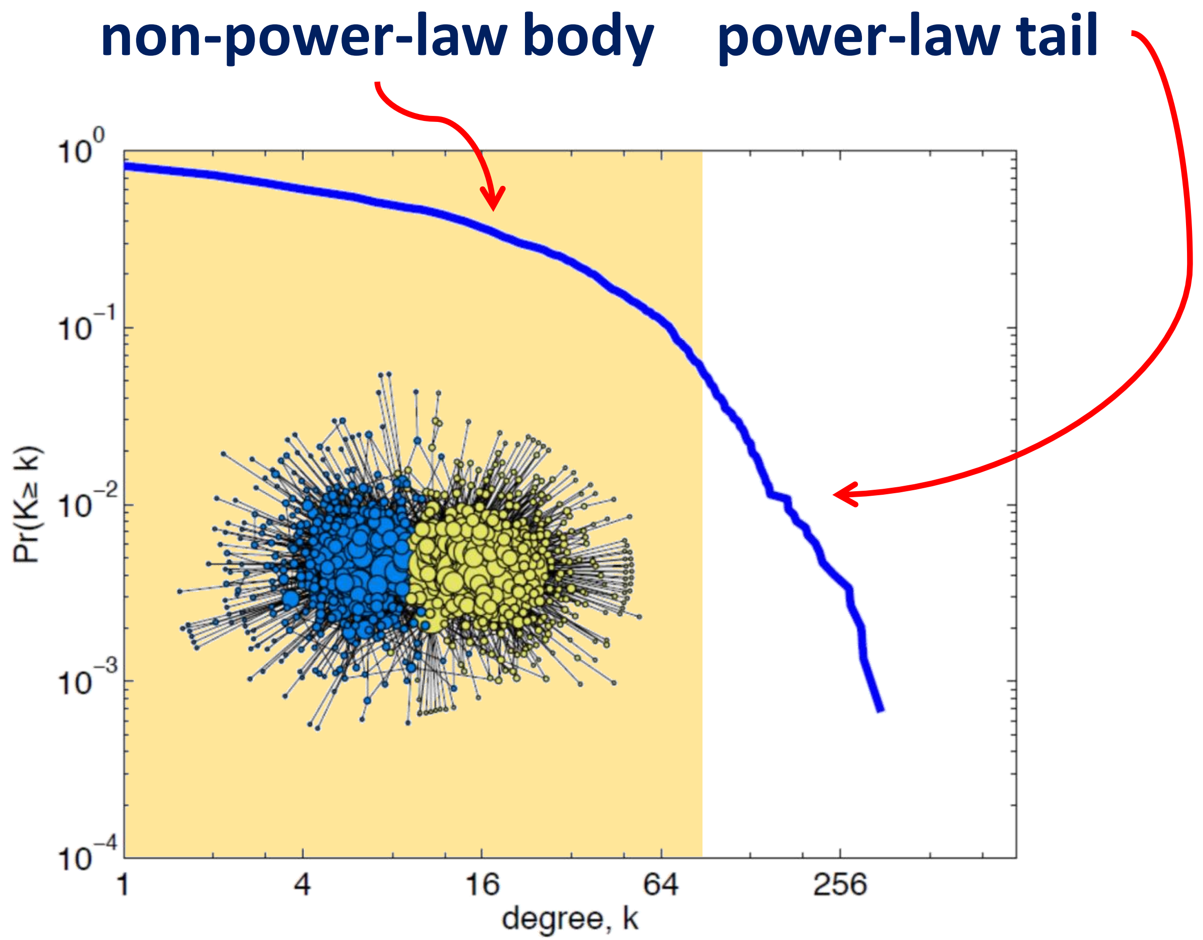
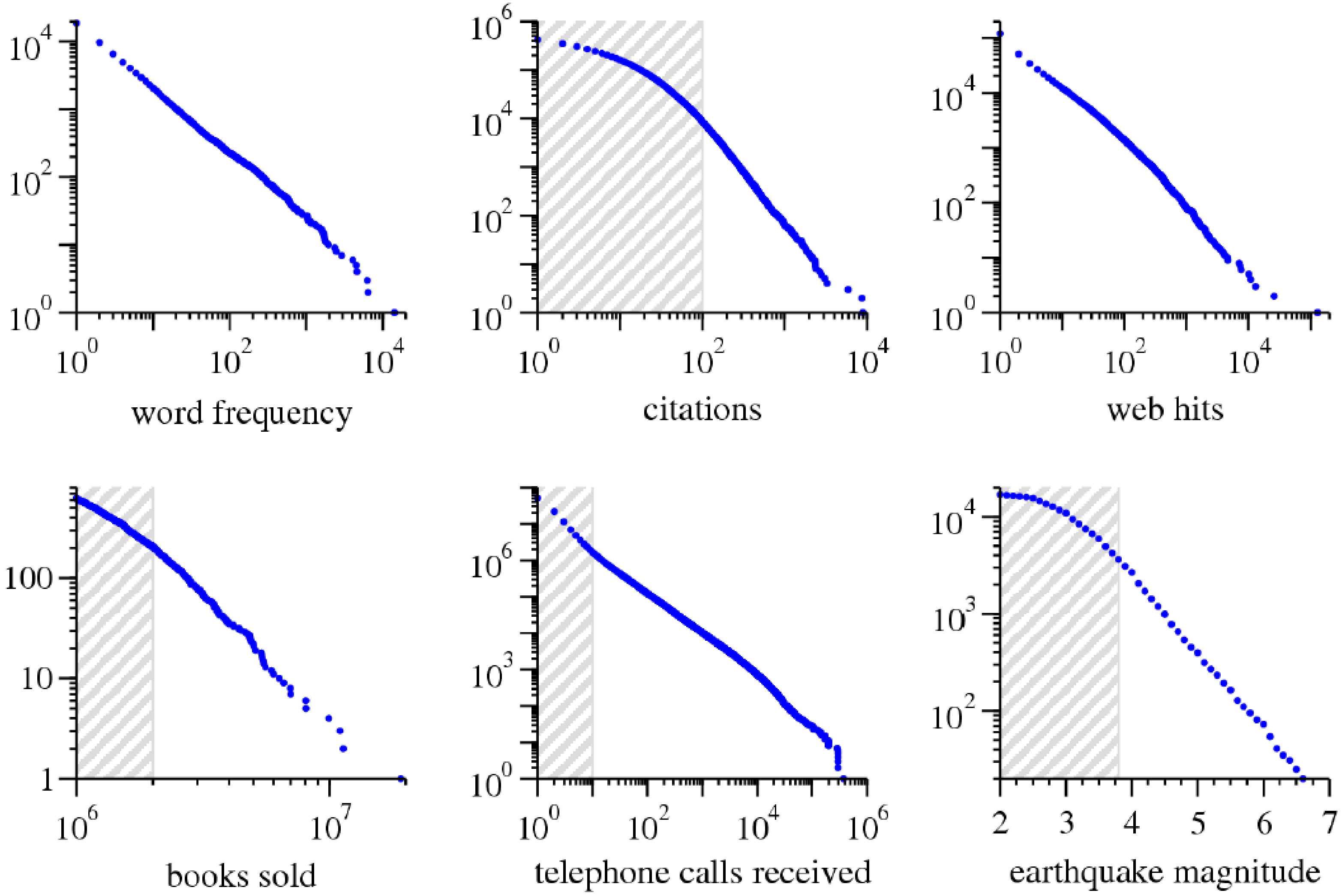
Degree Distribution in CCDF

- Nearly all real networks exhibit a *heavy-tailed degree distribution*.
- Very few networks exhibit perfect power-law degree distributions.



Degree Distributions in CCDF

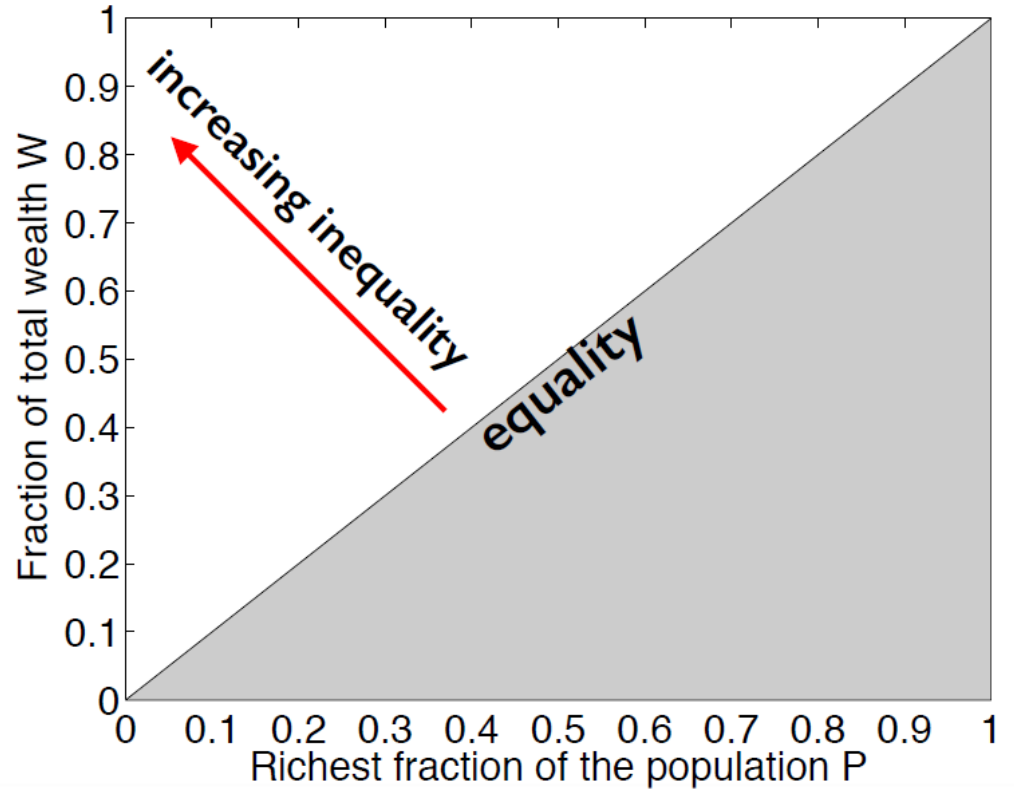


Equality

- Equality of ‘wealth’
  - How would you quantify the *equality of wealth* for each country?

Equality

- Equality of ‘wealth’
  - What fraction of total wealth (W) is owned by richest fraction (P)



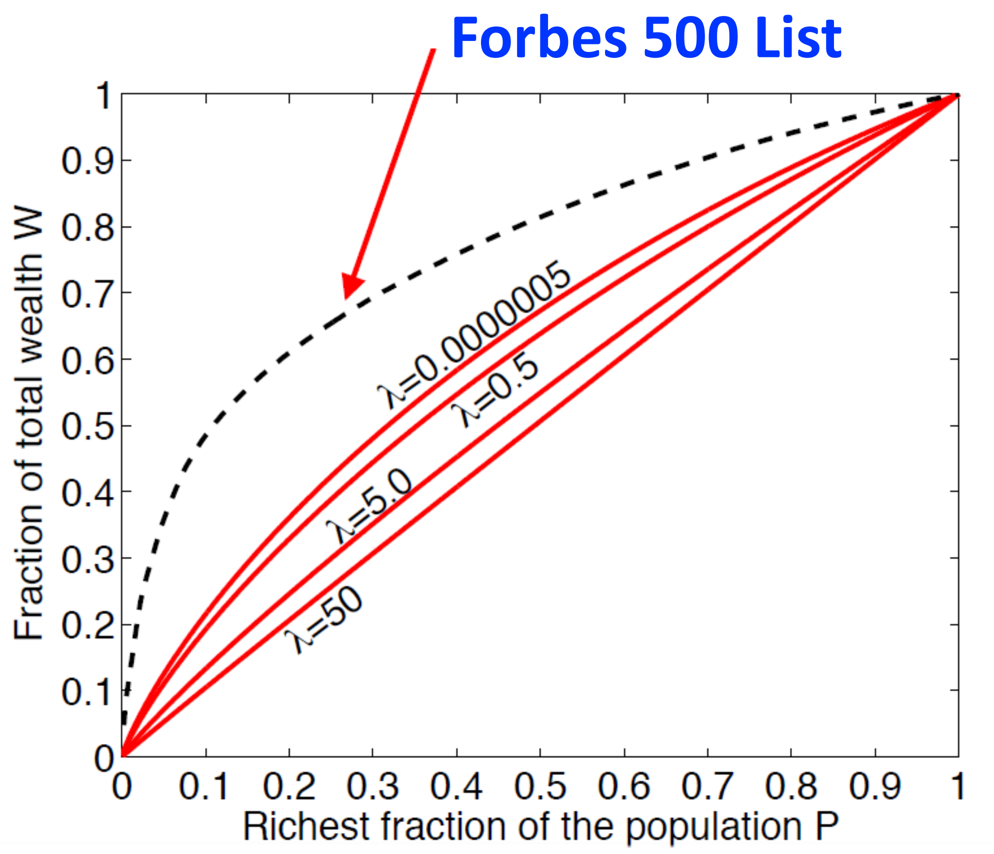
[Lorenz Curve]

Equality – Probability Density Function

- Equality of ‘wealth’
  - What fraction of total wealth (W) is owned by richest fraction (P)

$$p(x) \propto e^{-\lambda x}$$

exponential distribution



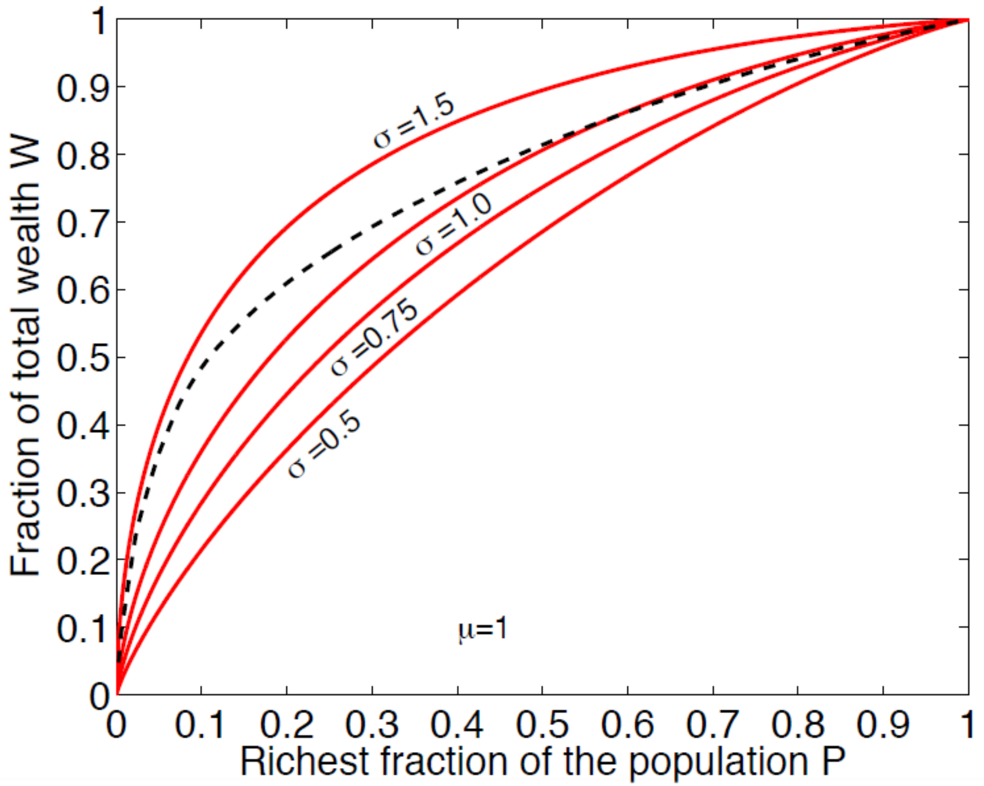
[Lorenz Curve]

Equality – Probability Density Function

- Equality of ‘wealth’
  - What fraction of total wealth (W) is owned by richest fraction (P)

$$p(x) \propto \frac{1}{x} e^{-\left(\frac{\ln x - \mu}{\sigma\sqrt{2}}\right)^2}$$

log-normal distribution



[Lorenz Curve]

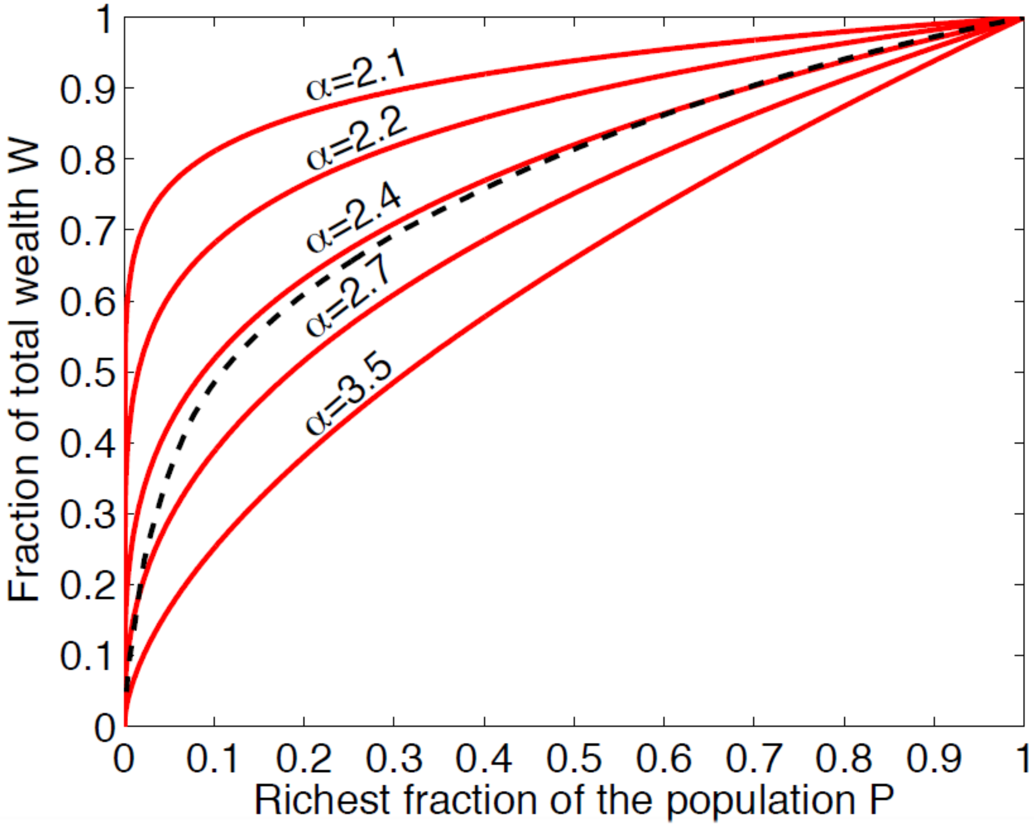
Equality – Probability Density Function

- Equality of 'wealth'
  - What fraction of total wealth (W) is owned by richest fraction (P)

$$p(x) \propto x^{-\alpha}$$

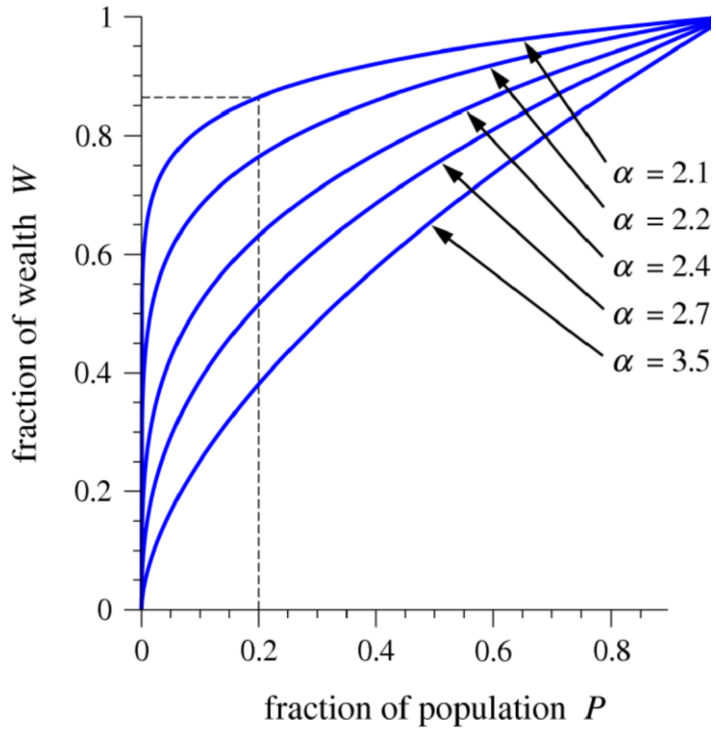
power-law distribution

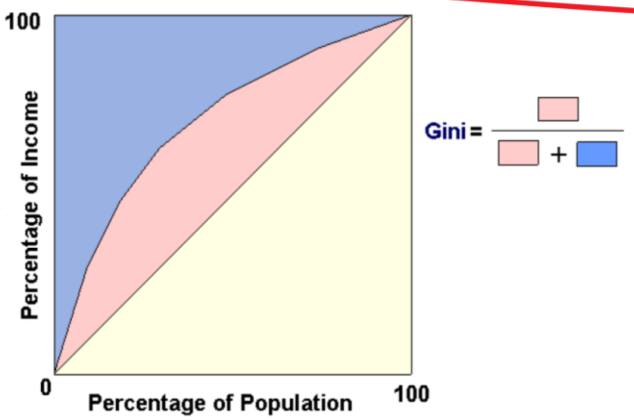
Pareto Principle
80/20 rule



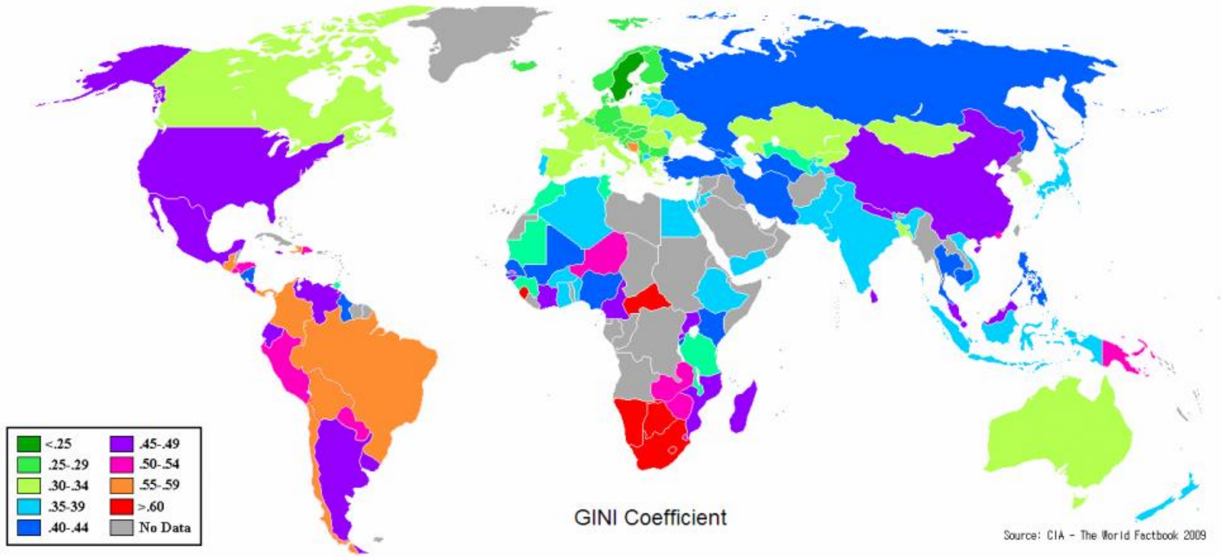
[Lorenz Curve]

The 80-20 Rule





Can you find some patterns here?



- In general, countries with higher levels of economic development have a lower Gini value (*e.g.*, Europe).
- US is the richest but unequal country.

[In-Depth]: Power-law Distributions

$$p(k) = Ck^{-\alpha} \text{ for } k \geq k_{\min}$$

- Normalization (probability density function):

$$\int_{k_{\min}}^{\infty} p(k)\, dk = 1 \quad \Rightarrow \quad p(k) = \quad ?$$

- Complementary Cumulative Distribution Function:

$$P(K \geq k) = \int_{k}^{\infty} p(y)\, dy \quad \Rightarrow \quad P(K \geq k) = \quad ?$$

[In-Depth]: Power-law Distributions

$$p(k) = Ck^{-\alpha} \text{ for } k \geq k_{\min}$$

- Normalization (probability density function):

$$\int_{k_{\min}}^{\infty} p(k)\, dk = 1 \quad \Rightarrow \quad \left[\frac{C}{-\alpha+1} k^{-\alpha+1}\right]_{k_{\min}}^{\infty} = 1$$

$$\frac{C}{\alpha-1} {k_{\min}}^{-\alpha+1} = 1$$

$$C = (\alpha-1)\left(\frac{1}{k_{\min}}\right)^{-\alpha+1} = \left(\frac{\alpha-1}{k_{\min}}\right)\left(\frac{1}{k_{\min}}\right)^{-\alpha}$$

$$\therefore\ p(k) = \left(\frac{\alpha-1}{k_{\min}}\right)\left(\frac{k}{k_{\min}}\right)^{-\alpha}$$

[In-Depth]: Power-law Distributions

- Complementary Cumulative Distribution Function (CCDF):

$$P(K \geq k) = \int_k^\infty p(y)\, dy$$

$$P(K \geq k) = \frac{C}{\alpha - 1} k^{-(\alpha-1)}$$

$$\because\ P(X \geq x) = \frac{C}{\alpha - 1} x^{-(\alpha-1)}$$

$$= \left(\frac{1}{\alpha - 1}\right)\left(\frac{\alpha - 1}{k_{\min}}\right)\left(\frac{1}{k_{\min}}\right)^{-\alpha} k^{-(\alpha-1)}$$

$$\because\ C = \left(\frac{\alpha - 1}{k_{\min}}\right)\left(\frac{1}{k_{\min}}\right)^{-\alpha}$$

$$= \left(\frac{1}{k_{\min}}\right)^{-\alpha+1} k^{-(\alpha-1)}$$

$$= \left(\frac{k}{k_{\min}}\right)^{-\alpha+1} \Rightarrow \therefore\ P(K \geq k) = \left(\frac{k}{k_{\min}}\right)^{-\alpha+1}$$

[In-Depth]: Power-law Distributions

$$p(k) = Ck^{-\alpha} \text{ for } k \geq k_{\min}$$

- Normalization (probability density function):

$$\int_{k_{\min}}^{\infty} p(k)\, dk = 1 \quad \Rightarrow \quad p(k) = \frac{\alpha - 1}{k_{\min}} \left(\frac{k}{k_{\min}}\right)^{-\alpha}$$

- Complementary Cumulative Distribution Function:

$$P(K \geq k) = \int_{k}^{\infty} p(y)\, dy \quad \Rightarrow \quad P(K \geq k) = \left(\frac{k}{k_{\min}}\right)^{-\alpha+1}$$