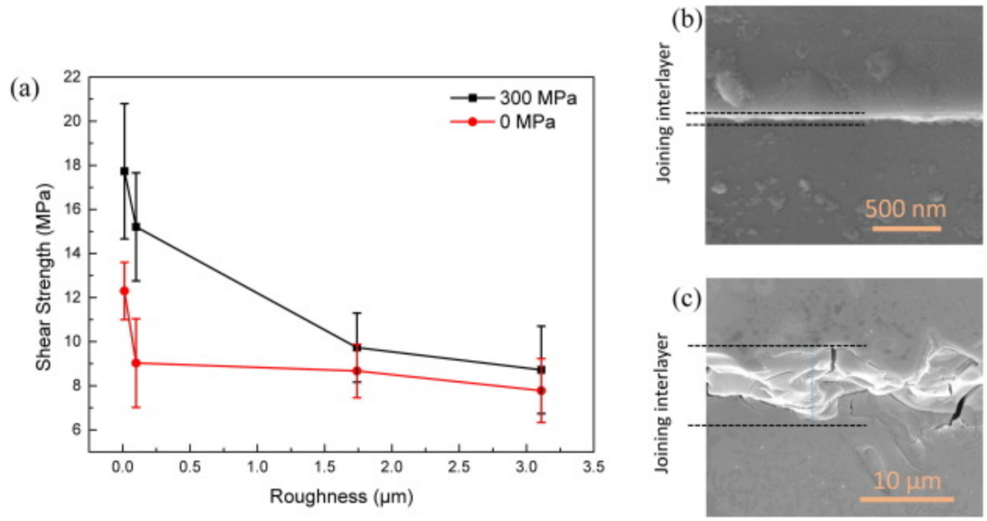

Fig. 2. a) Correlation between shear strength and surface roughness for samples joined with and without an instoatic pressure of 300 MPa. Representative SEM images of b) as received fused silica after CIJ and c) comparative image of pressure-less joined samples with 0.10 μm roughness.

Figure 1: From Grasso et al.[1]

Introduction

This note is to explore the striking relationship between shear strength, interfacial roughness, and pressure, that is shown for the cold fusing of silica in Fig 1. The variables and coefficients of interest are denoted as follows:

τ is shear strength
p is pressure
r is interfacial roughness
α_i are coefficients

Ansatz

Based on looking at Fig 1 from Grasso et al., let's guess an off-set log-linear relationship between shear strength and interfacial roughness,

$$\tau(r) = \alpha_0 + \exp\left(\alpha_1 + \alpha_2\, r\right) \; . \tag{1}$$

To include pressure, we can expand the coefficients. But there is limited data so simple is good. A few optionss are

1) expand all the coefs to first-order:

$$\tau(r,p) = \alpha_0 + \alpha_0^{'} p + \exp\left(\alpha_1 + \alpha_1^{'} p + \left(\alpha_2 + \alpha_2^{'} p\right)\, r\right)\,. \tag{2}$$

2) Alternatively, we could expand linear terms to second order and cut down on the exponential terms

$$\tau(r,p) = \alpha_0 + \alpha_0^{'} p + \alpha_0^{''} pr + \exp\left(\alpha_1 + \alpha_1^{'} p + \alpha_2\, r\right)\,. \tag{3}$$

2) Alternatively again, we could try

$$\tau(r,p) = \alpha_0 + \alpha_0^{'} p + \alpha_0^{''} pr + \exp\left(\alpha_1 + \left(\alpha_2 + \alpha_2^{'} p\right)\, r\right)\,. \tag{4}$$

Bayesian model specification

We can estimate the parameters in the model (α_0, etc) using MCMC, in order to get both expected values and credible intervals for the parameters.

For the model let's assume normally distributed uncertainty in what our estimate of the shear strength will be,

$$p(\tau|\alpha, M, D) \sim \mathrm{Normal}(\tau(r,p), \sigma)\,,$$

and give the standard deviation σ a prior of

$$\begin{aligned} \sigma &\sim \mathrm{Normal}(\sigma_s, 1)\ \in [0,\infty) \\ \sigma_s &\sim \mathrm{Normal}(3, 1)\ \in [0,\infty) \end{aligned} \tag{5}$$

which takes a hyperprior σ_s based on the empirical standard error of the measurements in Fig. 1.

The priors are

$$\begin{aligned} \alpha_0 &\sim \mathrm{Normal}(10, 3) \in [0,\infty) \\ \alpha_0^{'} &\sim \mathrm{Normal}(0, 0.1) \in [0,\infty) \\ \alpha_1 &\sim \mathrm{Normal}(2, 2) \in (-\infty,\infty) \\ \alpha_1^{'} &\sim \mathrm{Normal}(0, 0.1) \in (-\infty,\infty) && (6) \\ \alpha_2 &\sim \mathrm{Normal}(-1, 1) \in (-\infty, 0] && (7) \\ \alpha_2^{'} &\sim \mathrm{Normal}(0, 0.1) \in (-\infty,\infty) && (8) \end{aligned}$$

References

[1] Daoyao Ke, Peter Tatarko, Nan Luo, Gianmarco Taveri, Milad Kermani, Xiaoyi Wang, Chunfeng Hu, and Salvatore Grasso. Cold joining of fused silica: Bonding pressure and surface roughness effects. *Materials Letters*, 282:128836, 2021.