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Project 1-2, Block 1.6
A Titanic Space Odyssey!

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Submission:	06/26/2019 Room 0.002 BOU8-10
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Preface

The report is about the examination of a manned mission to reach Titan, one of Saturn's moons, and come back to Earth. Thereby different experiments and approaches are under closer investigation which are displayed in greater detail in later sections.

Summary

In this report several phases of a space mission are under investigation as well as the issues bound with such a travel, specifically to Titan, and from there, come back to Earth. Once the basics are layed out, experiments are performed on the developed methods. These show the performance and quality of the algorithms that are chosen. For instance, the implications of using a Runge-Kutta fourth order differential equation to simulate the solar system are discussed in greater detail. Furthermore, several landing module controllers are introduced: an open-loop controller, as well as one feedback controller with and one without a parachute. Afterwards, to optimise the travel, the fuel consumption has to be taken into account to analyse the methods mentioned above.

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1 Introduction

This report called 'A Titanic Space Odyssey!' deals with a spaceship travelling from Earth to Titan, a moon of Saturn. Why is this an interesting issue in current times? After the mission Apollo 11 July 1969 [1] exploring the moon of the Earth, space travel is knocking on our door. Today, visionary and controversial entrepreneurs are founding private companies, determined to travel space. An example is Elon Musk, who created the SpaceX company, committed to execute a manned mission to Mars as soon as possible. ´Our aspirational goal is to send our first cargo mission to Mars in 2022. The objectives for the first mission will be to confirm water resources, identify hazards, and put in place initial power, mining, and life support infrastructure. A second mission, with both cargo and crew, is targeted for 2024, with primary objectives of building a propellant depot and preparing for future crew flights.' [2].
Furthermore, Jeff Bezos, mainly known as founder of Amazon, is leaning towards space travel through his company Blue Origin. However his first steps are rather to land on Moon and in general closer destinations to access natural resources. According to him, these are needed so that the Earth habitat stays in pleasant and liveable conditions. "Blue Origin believes that in order to preserve Earth, our home, for our grandchildren's grandchildren, we must go to space to tap its unlimited resources and energy"[3].
Now might be the time where even conservative sceptics of science fiction might slowly realise that the technology featured in movies might not be far fetched and out of reach after all. Bezos and Musk have the power and resources necessary to discover space and enable humanity to benefit from it. Exciting projects like these are most likely just the beginning of human space travel in our galaxy. Hence a manned mission to Titan gets closer each day and in the current state of events it seems a few more decades might be all that's needed to reach it. After all, now is the time the get excited about space travel.

In the following section the problem is defined in detail. The section after places it in the historical, technical and theoretical background. Subsequently, the solar system and its implementation is explained, before going on to the actual journey to Titan. The sections after are concerned with the landing and the fuel consumption of the space shuttle. Then the experiments on the landing module are described, before stating the results and their discussion. Finally, the conclusion completes this report.

1.1 Project Definiton

Initially, data on the solar system and its celestial bodies is collected in order to create a mathematical model of it. The collected data (such as planet masses) is coupled to Newton's laws of motion, which govern motion through gravity, in order to create a physics engine that would be able to dynamically compute the orbits of the planets and the trajectory of the space probe. In addition to the model, different functions are implemented to perform a launch and landing of a spacecraft. In order to improve precision and reduce the error margins, a higher order differential equation solver was implemented. Through this implementation, it is already possible to simulate the flight of the spaceship from Earth to Titan.
Once the orbit of Titan is reached, the landing phase can be started. Firstly, there is the open-loop controller, which operates without taking the output data into account to compute the landing trajectory: the controller doesn't know about the altitude nor the velocity, it simply activates the thrusters continuously over a predefined amount of time. Secondly, there is a feedback controller, which takes the current conditions into account in order to adjust the position and velocity. In other words, if the landing module reaches an unsafe velocity at landing it uses the thrusters to slow down.
The last main objective is the mission from Titan back to Earth. Once reached, the next goal is to

complete the mission (travel to Titan, land on it and come back to Earth) using the smallest amount of fuel possible. Then the aim is again to complete the mission, but that time as fast as possible and so as we're not exceeding the minimum fuel consumption by more than 50%.

1.2 Background

Dutch astronomer Christiaan Huygens discovered Saturn's largest moon Titan on March 25, 1655. Nearly 300 years later, one of the characteristics that makes Titan so exceptional was discovered: this moon has an atmosphere. Since then, several space probes have been launched to visit and examine Titan: Pioneer 11, and Voyagers 1 and 2—studied Titan while flying by Saturn, and Cassini-Huygens flew close by Titan 127 times while in Saturn orbit for 13 years.

The Cassini spacecraft's numerous gravity measurements of Titan revealed that the moon is hiding an underground ocean of liquid water. Additionally, Titan's rivers, lakes and seas of liquid methane and ethane might serve as a habitable environment on the moon's surface, though any life there would likely be very different from Earth's. Thus, Titan could potentially harbour suitable life conditions. A human being wouldn't need a pressure suit to walk around on the surface, however, he would need an oxygen mask and protection against the cold—temperatures on Titan's surface, that reach minus 290 degrees Fahrenheit (minus 179 Celsius). Although there is so far no evidence of life on Titan, its complex chemistry and unique environments are certain to make it a destination for continued exploration. [5]

2 Algorithms

2.1 Solar System

In order to be able to plan different missions to Titan, a model of the solar system and its motion is established, including the Sun and the planets up to Neptune. Pluto is not included as it not considered a planet anymore by the scientific community, but also because of its distance from the Sun (its gravitational influence on the other celestial bodies is negligible). The main moons are added: Earth's Moon, Ganymede (orbiting around Jupiter and having a diameter 8% larger than Mercury's), and finally Titan. Once data on these bodies is collected from NASA's website [4], it is put together to create a physics engine able to compute the paths of the celestial bodies and trajectories of the spaceship.

2.1.1 Implementation

A class for the planets and moons was created: CelestialBody. Under that class, the different celestial bodies all share different variables: their name, mass, radius, position, velocity and acceleration. The different methods to update the positions of the planets implement the fourth-order Runge-Kutta differential equations solver, since the fourth order is a good compromise between computational costs and accuracy. The formulas of this differential equations solver are the following:

$$
\begin{aligned}
k_{i,1} &= h_i \cdot f(t_i, w_i) \\
k_{i,2} &= h_i \cdot f(t_i + \frac{1}{2}h_i, w_i + \frac{1}{2}k_{i,1}) \\
k_{i,3} &= h_i \cdot f(t_i + \frac{1}{2}h_i, w_i + \frac{1}{2}k_{i,2}) \\
k_{i,4} &= h_i \cdot f(t_i + h_i, w_i + k_{i,3}) \\
k_i &= \frac{1}{6}(k_{i,1} + 2k_{i,2} + 2k_{i,3} + k_{i,4}) \\
\\
w_{i+1} &= w_i + k_i
\end{aligned}
\tag{1}
$$

At regular time intervals, the positions of the planets and moons, as well as the space probe (if included in the simulation), are drawn on the GUI and updated by calling the relevant methods. Computing the positions of all the bodies in the simulation in the next iteration is divided in 4 main steps, which are the computing of the different k_i's. Each such step is again divided in two parts: In the first part, we set the positions of all the bodies to the position given as parameter to $f()$ in the expression of the k_i's. Then, in the second part, the forces applied on the body by all the other bodies in the system are computed, based on which the resulting acceleration caused by that force can be computed.
We then apply the computed acceleration to compute the new velocity, and then use the acceleration and the previous velocity to compute the new position, according to the following formulas:

$$v_{i+1} = v_i + a_{i+1} \cdot t \tag{2}$$

$$x_{i+1} = x_i + v_i \cdot t + \frac{1}{2}a_{i+1} \cdot t^2 \tag{3}$$

2.2 Travel to Titan

Travelling to Titan implies an efficient use of the available data: in order to send a probe millions of kilometres away to a moving body, the launching date and angle have to be accurately determined. Indeed, in such a system where all the bodies influence each other, a small error may have large repercussions. Moreover, as all of the bodies are moving, there might be some time windows that will make the path from Earth to Titan way more chaotic or even impossible and that may end up consuming more fuel. Fuel being relatively heavy and expensive, it is needed to avoid unnecessary consumption throughout the whole mission.

2.2.1 Angle Adjustment Search

For travelling to Titan, an algorithm was devised to search for an 'ideal' starting angle that would make the space probe arrive as close as possible to Titan, and if possible, reach the exact position of Titan. This method is based on the following idea:
In a 2-dimensional solar system, if the space probe arrives to the left of the target, the starting angle should be decreased in order to arrive more to the right. Note that this is independent of the gravitational forces of the planets on the space probe's trajectory. Similarly if the space probe arrives to the right of the target, the starting angle should be increased.
Thus, the algorithm launches a space probe from the origin body (Earth in this case) with a given launch angle and a given velocity. Then, once the space probe either crashes on a celestial body or travels the distance between the origin body and the destination planet, the simulation is terminated. Afterwards, the simulation is evaluated, meaning that the angle of the next iteration is either increased

or decreased, based on the position of the final position of the space probe.
Then, once a closed interval is found in which the target is comprised, we reduce the amount by which we modify the angle at each iteration. Thereby, we continue trying out different starting angles, until it cannot be further refined. This means that the found starting angle is the angle with which we will get closest to the target. Optimally, this results in arriving directly on the target.

2.2.2 Hohmann Transfer

After testing the launch using the previous algorithm, we did not manage to get into orbit, rather we arrived directly on Titan. This is fast, but expensive energy-wise. In order to travel between celestial bodies using the lowest possible amount of energy, the Hohmann transfer is still nowadays one of the most efficient methods. It was first described by the German scientist Walter Hohmann in 1925 in his book *Die Erreichbarkeit der Himmelskörper* (The Attainability of Celestial Bodies).

The Hohmann Transfer orbit is an elliptical orbit which is used to transfer between two circular orbits that have a different radius. This would be achieved by applying a certain velocity change Δv_1 at the start of the transfer orbit to escape the first circular orbit, then another velocity change Δv_2 at the end of the transfer orbit to get into the other circular orbit. The transfer orbit can be applied at any point in time when the transfer is executed from a circular orbit to another circular orbit. However, in the solar system, the orbits of the planets are elliptical. Thus, to make up for it, we would have to start and end the transfer orbit at special points of the two orbits: either at the periapsis or apoapsis of the start orbit, and we need to arrive at either the periapsis or apoapsis of the end orbit. The periapsis is the point in an elliptical orbit where the body in orbit is closest to the central body and has the highest orbital velocity, whereas the apoapsis is the point where it's farthest from the central body and has the lowest orbital velocity. The following formulas could be used to compute the norm of Δv_1 and Δv_2:

$$a = \frac{(d_O + d_F)}{2} \qquad p_H = \sqrt{\frac{4 \cdot \pi^2}{G \cdot M} \cdot a^3} \qquad t = \frac{1}{2} \cdot p_H$$

$$v_O = \frac{2\pi \cdot d_O}{p_O} \qquad v_F = \frac{2\pi \cdot d_F}{p_F} \qquad v_p = \frac{2\pi \cdot a}{p_H} \cdot \sqrt{\frac{2a}{d_O} - 1} \qquad v_a = \frac{2\pi \cdot a}{p_H} \cdot \sqrt{\frac{2a}{d_F} - 1}$$

$$\Delta v_1 = v_p - v_O \qquad \Delta v_2 = v_F - v_a$$

where

- a is the semi-major axis of the Transfer orbit
- d_O is the distance between the body of the origin orbit and the central body at the start of the transfer orbit
- d_F is the distance between the body of the final orbit and the central body at the end of the transfer orbit
- p_H is the period of the Transfer orbit
- t is the time that the Transfer orbit will take
- p_O is the period of the origin orbit body's orbit

- p_F is the period of the final orbit body's orbit
- G is the gravitational constant
- M is the mass of the central body
- v_O is the velocity of the origin orbit's body
- v_F is the velocity of the final orbit's body
- v_p is the velocity at the periapsis of the Transfer orbit
- v_a is the velocity at the apoapsis of the Transfer orbit
- Δv_1 is the velocity change that is applied at the start of the Transfer orbit
- Δv_2 is the velocity change that is applied at the end of the Transfer orbit

However, the computation of the moment in time at which Earth would be at its apoapsis or periapsis, such that after the Transfer orbit, Saturn is at its apoapsis or periapsis, (for the travel to Titan) proved to be quite challenging. The exact minimal distance from Sun to Earth changes from year to year, so we have to iterate through a time of at least the orbiting period to know when the Earth is at periapsis or apoapsis. Once we computed the periapsis or apoapsis of the start planet, we still need to determine if the destination planet will be at periapsis or apoapsis once we will arrive in the destination orbit.

Furthermore, it was difficult to find a way to use the thrusters in order to reach the computed v_p, since we had to take the origin planet's gravitational attraction into account. We also did not find at which distance from Earth this gravitational attraction would be negligible, such that we didn't need to use thrusters anymore to stay at the desired velocity. After some research, we found the concept of the sphere of influence of a body [7], which is the area in space around a central celestial body in which the primary force exerted on another body is the gravitational attraction of the central body. Thus, starting from outside of the sphere of influence of the start planet would be an idea to reduce the gravitational attraction of Earth on the Hohmann Transfer, but after several tests, it turned out that it was not enough to render the Hohmann Transfer possible.

2.2.3 Launch Fuel Consumption

The angle adjustment search algorithm described above starts with an initial velocity from a certain distance from Earth. Thus, in order to compute the fuel consumption of the mission to Titan and back, an algorithm was needed in order to compute the quantity of fuel burnt in order to get to the given height with the initial velocity. This algorithm would have to compute the strength of the thruster force used and the fuel mass the space probe would need to contain at the start of the travel.
The method starts off by allowing the space probe to use up more fuel than it would contain, then uses the thruster at full power until it reaches the searched height (with a certain tolerance). This would then give an estimate of the mass of fuel needed for the launch.
Using this estimate, we can then run simulations in which we launch the space probe from the origin planet's surface, then using the thruster and updating the position of the space probe, as well as all planets and moons included in the solar system. Such a simulation would end in two cases: First case, the space probe runs out of fuel and starts falling back towards the origin planet. in this case, the mass of fuel the space probe starts with has to be increased. Second case, the space probe gets to the requested height or above. In this case, if the height is within the range we are searching for, then we need to increase or decrease the strength of the thruster depending on whether the velocity is lower or higher than the target velocity. Moreover, if the height is greater than the aimed height range, then

we will increase the strength of the thrusters, such that we might reach the requested height with the requested velocity with one less iteration than this simulation, unless
Then, once a simulation has managed to reach the given height at the given velocity, both within a certain error margin, then the algorithm returns the parameters of that simulation: the exact reached height as well as the exact reached velocity, the mass of fuel needed by the space probe, as well as the quantity of fuel that was burnt for the maneuver.
However, bugs remained in the method until the end, affecting the part where we make sure that the space probe does not fall back to Earth. Thus, we were not able to make that method work.

2.3 Landing on Titan

In order to land safely on a celestial body millions of kilometres away, the space probe needs to be able to withstand very harsh conditions, as there is no forecast that can be taken into account before the launch. Indeed, as Titan has an atmosphere, it entails that the probe is going to face extremely high temperatures due to friction when entering it, as well as winds. This simulation sets aside the friction, however a stochastic model of the wind was created. Two different types of controllers are devised so that the space probe lands safely at the desired position.

2.3.1 Landing Module Controllers

An open-loop controller is a type of continuous control system in which the output has no influence or effect on the control action of the input signal. That is to say that the open loop controller will perform the task it is assigned, regardless of changing conditions during the performance of the action. In our case, as it was needed to land with a very low velocity, the thrusters are used throughout the whole descent at an intensity that nearly counters Titan's gravity, allowing the space probe to land smoothly. However, this type of controller is not suited to handle wind as it is not aware of any change in position or velocity.

A closed-loop or feedback controller measures the output and accordingly modifies the input signal to drive the process variable towards the desired result. It is based on the idea of not using the thrusters at all in a first phase, thus building up a certain velocity towards the center of Titan. Then, in a second phase, it uses the main thruster (installed at the back of the landing module) at a certain constant power on the y-axis such that a safe landing on Titan is achieved.
To compute the point from which on the thruster should be used, the algorithm of the feedback controller uses a formula for computing the distance necessary for braking from the current height, with the current velocity, to a null velocity. That formula relies on another formula to compute the time needed to slow down. The two formulas are described in the following (4, 5):

$$\text{braking time} = \frac{\text{maximum velocity}}{\text{maximum acceleration}} \tag{4}$$

where the maximum velocity is considered to be the current velocity, given that we plan on constantly braking for the rest of the landing and the maximum acceleration is the difference between the gravity of Titan and the maximum acceleration of the thruster on the y-axis.

$$\text{braking distance} = \text{braking time} \cdot \text{maximum velocity} \cdot \frac{1}{2} \tag{5}$$

where the braking time is computed based on the previous formula and the maximum velocity is, again, considered to be the current velocity.

Based on these formulas, if the y-position is nearly within the braking distance, we start using the main thruster. However, if the velocity is bigger than a given speed on the y-axis (velocity oriented towards Titan is negative), we still don't use the thruster. Finally, once a certain distance from the ground is achieved, the thruster is used continuously, in order to make sure to slow down and aim for the y-velocity to be within the tolerance of 0.1 m/sec.

To make sure there is a constant y-acceleration, disregarding the inclination of the space probe, we set a maximum angle the space probe should have, and do not use the thrusters to incline the space probe more than that angle. When using the thruster, we compute the total force of the thruster to apply to have a certain constant thruster force on the y-axis, which we set to the maximum force of the thruster with the biggest inclination, which is the maximum angle mentioned before. This formula is applied to compute the thruster force to use:

$$\text{thruster force} = \text{minimum y-axis force} \cdot cos(\text{angle}) \tag{6}$$

After a closer look at the descent of the Huygens probe Nasa used to collect data on Titan, we noticed the use of a parachute to slow the probe down. Thus, a second feedback controller was implemented, based on the first one, but which would use a parachute in a first step, then in a second step, start using the thrusters. Similar to Huygens', this controller activates the parachute around 155 kilometres of Titan's ground, once the probe has been slowed down to 400 meters per second by the air resistance. Upon activation, the parachute would strongly decelerate the landing module in a first step, then only slowly afterwards.
Then, once the landing module arrives at an altitude of 50 kilometers of the surface of Titan, the module drops the parachute, and starts using the thrusters.

2.3.2 Wind Conditions

Along it's descent through Titan's atmosphere, Nasa's Huygens probe encountered strong winds, up to 120 meters per second. Thus, the stochastic wind generated in our model is oriented in a random direction in the 2D-plane, and with a random intensity, up to a certain limit maxWindForce of Newtons.

2.3.3 Getting back into orbit

For the return to Titan's orbit, we used a closed-loop controller which is in many ways the same as the one used for the landing on Titan. Just like with the landing, we have to take the forces of both gravity and wind into account. There is a certain reverse element to it; where the controller for the landing figured out the time when the landing module should start using the thruster, the controller for return to orbit determines the time it should stop thrusting such that it will end up at the desired position in orbit with a negligible velocity.

2.4 Fuel Consumption

The fuel consumption of using the thrusters during the travel from Earth to Titan's orbit, during the Landing, as well as for the travel back from Titan's orbit to Earth has to be tracked.

2.4.1 Implementation

To establish a link between the force used by the thrusters and the fuel consumed by it, formula 7 [8] is used, calculating the force of thrust:

$$F = \dot{m} \cdot V_e + (p_e - p_0) \cdot A_e \tag{7}$$

where

- $\dot{m}$ is the mass flow rate, the mass of exhaust gas produced by unit of time
- V_e is the exhaust velocity, the velocity of the exhaust gases produced by the combustion of the fuel with the oxidizer
- p_e is the exhaust pressure, the pressure under which the exhaust gases are
- p_0 is the free stream pressure, the ambient pressure outside the spacecraft
- A_e is the exit area determined by the output nozzle where the output gases come out of the spacecraft

As mentioned in [8], the exhaust pressure p_e and the exit pressure p_0 are equal at some design condition. Thus, the simplified version of the formula above, where p_e and p_0 are equal, leaves:

$$F = \dot{m} \cdot V_e \tag{8}$$

Then, we found the value of the exhaust velocity for RP-1 (highly refined kerosene) [9]. Thus, the mass flow rate could be set to an arbitrary value, then modulated to simulate the thruster being used at a stronger or weaker intensity and hence modify the force exerted by the thruster.

Once we have fixed a mass flow rate, we can multiply it by the time interval during which we apply it in order to get the mass of exhaust gas used up. Then, with the help of the following chemical equation, we can derive the quantity of exhaust gases produced.

$$\text{LOX} + \text{RP1} \rightarrow \text{exhaust gases} \tag{9}$$

And given the oxidizer-to-fuel ratio of RP-1 [10], we could get the mass of LOX (the oxidizer used with RP-1) in function of the mass of RP-1:

$$\text{oxidizer-to-fuel ratio} = \frac{\text{mass(LOX)}}{\text{mass(RP-1)}} \Leftrightarrow \text{mass(LOX)} = \text{oxidizer-to-fuel ratio} \cdot \text{mass(RP-1)}$$

Finally, this leaves

$$\begin{aligned} &\text{oxidizer-to-fuel ratio} \cdot \text{mass(RP1)} + \text{mass(RP1)} = \text{mass(exhaust gases)} \\ &\Leftrightarrow \text{mass(RP1)} = \frac{\text{mass(exhaust gases)}}{\text{oxidizer-to-fuel ratio} + 1} \end{aligned} \tag{10}$$

Thus, there is a link between the mass of exhaust gases and the fuel consumed by it.

3 Experiments

3.1 Solar System

Experiments were performed on the Solar System in which the accuracy of the Solar system simulation was compared to the NASA-data for Earth and Titan. This was done for x- and y position, as well as x- and y-velocity. The time step taken into account for our developed model was 1800 seconds. The comparison was done for 1 to 10 years and 15, 20 and 25 years.

Furthermore, experiments were done to compare the results of our model to the NASA-data for a different time step. This was done for x- and y-position, as well as x- and y-velocity. The time steps taken into account were 900, 1800, 3600, 5400 and 10800 seconds. The comparison was done for 1, 3, 5 and 10 years.

3.2 Landing on Titan

Experiments were performed on the Landing module with feedback controller, in which the resulting values for x- and y-position, as well as x- and y-velocity, the angle and the time on arrival on Titan were observed for various starting conditions. These starting conditions include the starting x- and y-positions, the x- and y-velocity, the strength of the main thruster, as well as whether to include the stochastic wind in the simulation or not. Below, some of the main experiments on the Landing module will be explained:

3.2.1 Feedback controller (without parachute)

1. First series of experiments executes the landing without modifying the thruster force of the space probe and with no influence of the wind:

   (a) Modified x-position
   The starting velocity is kept constant (x=0, y=0), as well as the starting angle=0, only modifying the starting position in x (y=500000).

   (b) Modified y-position
   The starting velocity is kept constant (x=0, y=0), as well as the starting angle=0, only modifying the starting position in y (x=0).

   (c) Modified angle
   The starting position is kept constant (x=0, y=500000), as well as the starting velocity (x=0, y=0), only modifying the angle.

   (d) Modified x-velocity
   The starting position is kept constant (x=0, y=500000), as well as the starting angle=0, only modifying the starting velocity in x (y=0).

   (e) Modified y-velocity
   The starting position is kept constant (x=0, y=500000), as well as the starting angle=0, only modifying the starting velocity in y (x=0).

   The results of this series of experiments were recorded in a table and shall be explained in the next section.

2. The second series of experiments executes the landing under the influence of the wind, with various strengths for the thruster of the space probe. The starting position is kept constant (x=0, y=500000), as well as the starting velocity (x=0, y=0) and the angle (0). The maximum wind force is changed (varying from 0.5 to 10) for each value of the thruster force (2500, 5000 and 10000).

3.2.2 Feedback controller with parachute

1. First series of experiments executes the landing without using the thruster force of the space probe and with no influence of the wind:

   (a) Modified x-position
   The starting velocity is kept constant (x=0, y=0), as well as the starting angle=0, only modifying the starting position depending on x (y=155000).

   (b) Modified y-position
   The starting velocity is kept constant (x=0, y=0), as well as the starting angle=0, only modifying the starting position depending on y (x=0).

(c) Modified angle
The starting position is kept constant (x=0, y=155000), as well as the starting velocity (x=0, y=0), only modifying the angle.

(d) Modified x-velocity
The starting position is kept constant (x=0, y=155000), as well as the starting angle=0, only modifying the starting velocity depending on x (y=0).

(e) Modified y-velocity
The starting position is kept constant (x=0, y=155000), as well as the starting angle=0, only modifying the starting velocity depending on y (x=0).

The results of this series of experiments were recorded in a table and shall be explained in the next section.

2. The second series of experiments executes the landing under the influence of the wind, with the use of the thruster force of the space probe. The starting position is kept constant (x=0, y=155000), as well as the starting velocity (x=0, y=0) and the angle (0). The maximum wind force is changed (varying from 0.5 to 10) for each value of the thruster force (2500, 5000 and 10000).

4 Results

4.1 Solar System

In the following few diagrams we compare the data from our model of the solar system and the data we gathered from NASA.

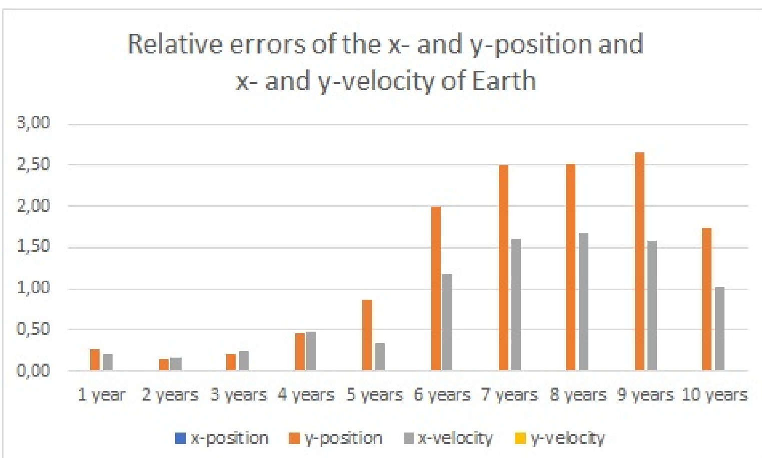

Figure 1: Relative error in the x- and y-position and the x- and y-velocity for the Earth for the first 10 years

It can be observed that for the y-position and x-velocity in the first five years, the error is always under 1. In the sixth year though, the error increases up to 200 % compared to before. The x-position and v-velocity error is constant zero.

Since the errors for the x-position and the y-velocity in figure 1 are extremely small compared to the other errors, figure 2 shows a detailed view for these two errors.

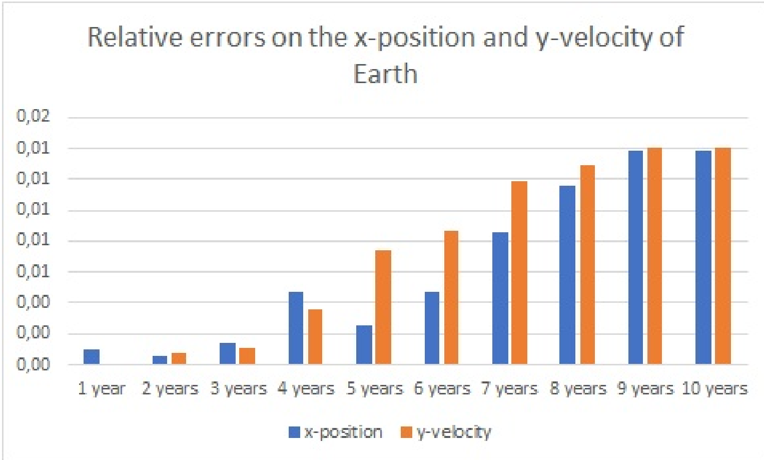

Figure 2: Relative error x-position and y-velocity Earth

It can be observed that the y-velocity increases in a more linear way than the errors in Figure 1. Similarly for the x-position, with an exception in year four which might be an anomaly.
We also look at errors on the longer term for Earth (Figure 3).

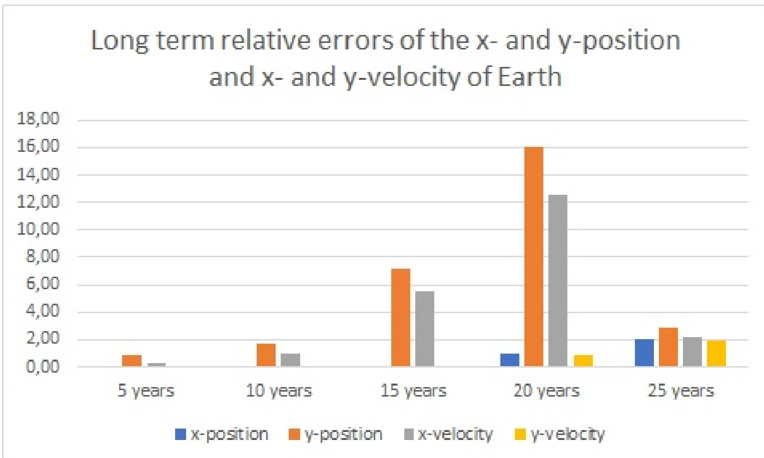

Figure 3: Relative errors the long term

Even though the errors don't not increase much in the first 10 years, a peak error of 16 occurs after 20 years for the y-position. After this, all errors drop down to approximately 2 again. Next, we looked at the relative errors for Titan, both for the first 10 years (Figure 4) and longer term (Figure 5).

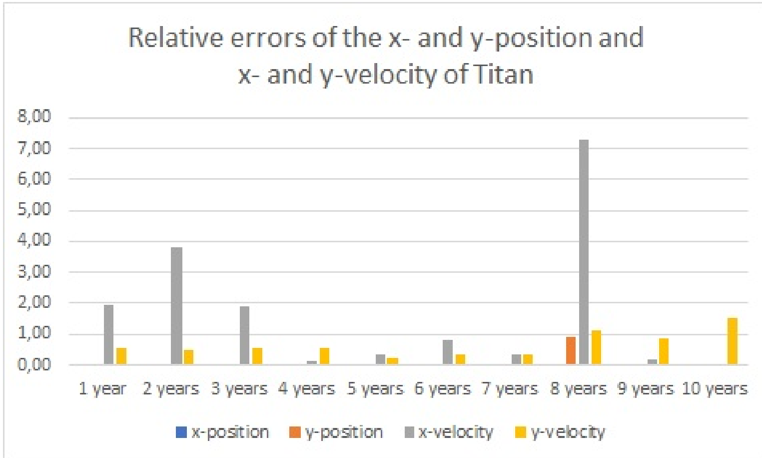

Figure 4: Relative error in the x- and y-position and the x- and y-velocity for Titan for the first 10 years

For the first ten years the error in x-velocity increases and decreases periodically, while the amplitude decreases. However there is one exception in year 8. Furthermore the other errors stay quite small, with the y-velocity error peaking in the tenth year with still under 2.

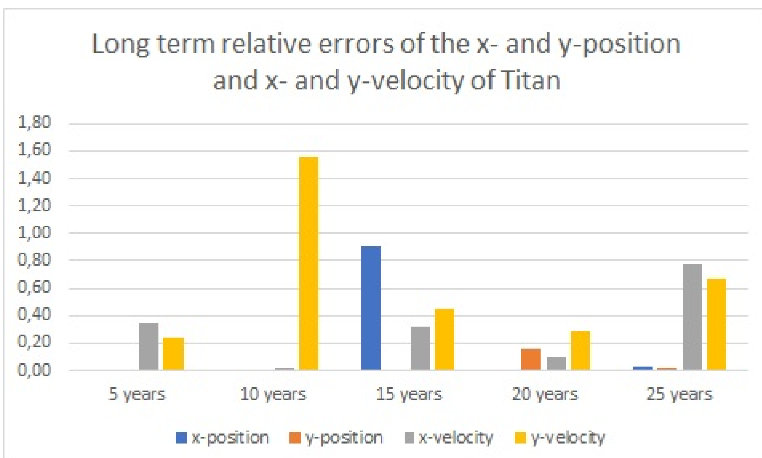

Figure 5: Error for the long term

Looking at the long term errors, they are much more diverse and it is difficult to spot a pattern for any of them. Finally, we tested our model with different time steps for Earth (Figure 6) and for Titan (7). There is no clear trend of the errors in those diagrams.

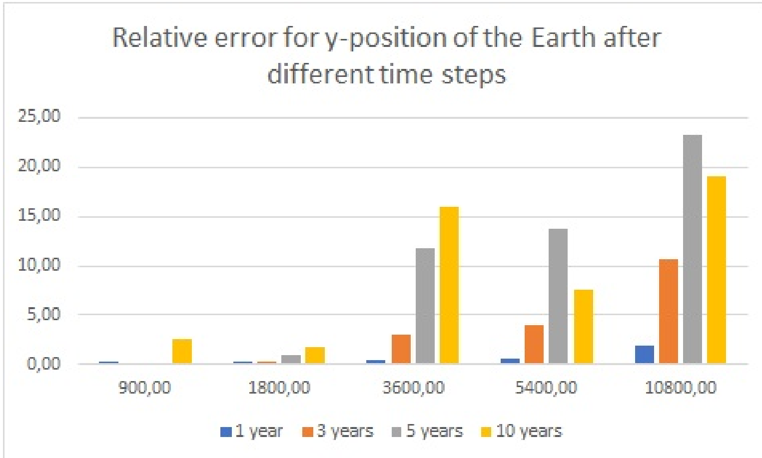

Figure 6: Relative error the y-position of the Earth after different time steps

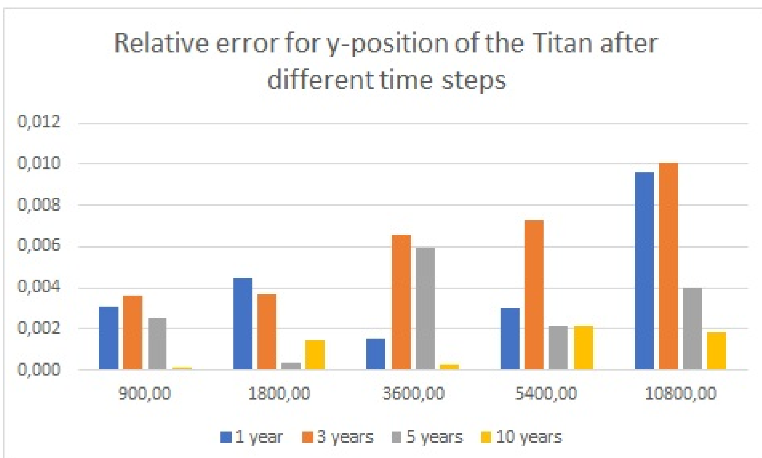

Figure 7: Relative error the x-position of the Titan after different time steps

4.2 Feedback controller without parachute

After executing the experiments with landing using Feedback Controller without parachute (explained in the section 3.2.1) the following results have been achieved:

1. Without the influence of the wind (see Figure 9 & Figure 10)
    - Modified x-position (1(a)) doesn't bring any changes to the Landing Time (1 day 2 hours 29 minutes and 26 seconds), and to the Landing Position and Landing Velocity of y. However, the Landing Position and Landing Velocity of x are in a constant change depending on the Starting Position of x, as well as the Landing Angle. The Burnt Fuel Mass and the Prize in euros are relatively similar, because the Landing Time is not changing.
    - Modified y-position (1(b)) changes the Landing Time with every modification. The Landing Position and Landing Velocity of y vary, while those of x stay constant (0). The Landing

Angle is stable (0), while the Burnt Fuel Mass and the Prize in euros have different values according to the Landing Time.

- Modified angle (1(c)) influences only the Landing Position and Landing Velocity of x, as well as the Landing Angle. The Landing Time and Landing Position and Velocity of y remain unmodified. Again, the Burnt Fuel Mass and the Prize have very small insignificant changes.
- Modified x-velocity (1(d)) has the same variations as Modified x-position (1(a)): Landing Time, Landing Position and Landing Velocity of y are constant, while the Landing Position and Landing Velocity of x, as well as the Landing Angle evolve depending on the Starting Velocity of x. The Burnt Fuel Mass and the Prize stay very similar.
- Modified y-velocity (1(e)) has the same changes as Modified y-position (1(b)): Landing Time, Landing Position and Landing velocity of y vary, while the Landing Position and Landing Velocity of x remain unchanged (0), as well as the Landing Angle (0). Again, the Burnt Fuel Mass and the Prize have minor modifications.

2. With the influence of the wind using the thruster of the space probe (see Figure 11 & Figure 12)

   Thruster Force is varying between three values (2500, 5000 and 10000) and for each of these values different Wind force is executed (from 0.5 to 10). As a consequence the Landing Time ranges between 1 day 2 hours and 3 minutes and 1 day 2 hours and 9 minutes. The Landing Position and Landing Velocity fluctuate with every change, the average for Landing Position being x=193.54, y=-1.27 m, and for the Landing Velocity - x=-1.27, y=-1.78 m/s. The Landing Angle differs between -43.0 and 16.0 degrees, with the average of -4.0 degrees. The Burnt Fuel Mass and the Price in euros have minor changes, because the Landing Time is varying only in minutes and seconds, the most expensive landing being 7,556 euros.

4.3 Feedback controller with parachute

After executing the experiments with landing using Feedback Controller with parachute (explained in the section 3.2.2) the following results have been achieved:

1. Without the influence of the wind (see Figure 13 & Figure 14)

   - Modified x-position (1(a)) doesn't bring any changes to the Landing Time (9 hours 7 minutes and 4 seconds), and to the Landing Position and Landing Velocity of y. However, the Landing Position and Landing Velocity of x are in a constant change depending on the Starting Position of x, as well as the Landing Angle. The Burnt Fuel Mass and the Prize in euros are relatively similar, because the Landing Time is not changing.
   - Modified y-position (1(b)) changes the Landing Time with every modification. The Landing Position and Landing Velocity of y vary, while those of x stay constant (0). The Landing Angle is stable (0), while the Burnt Fuel Mass and the Prize in euros have different values according to the Landing Time.
   - Modified angle (1(c)) doesn't have any influence on any of the landing characteristics. The Landing Position, Landing Velocity, Landing Angle, the Burnt Fuel Mass and the total Price stay constant.
   - Modified x-velocity (1(d)) has the same variations as Modified x-position (1(a)): Landing Time, Landing Position and Landing Velocity of y are constant, while the Landing Position and Landing Velocity of x evolve depending on the Starting Velocity of x. The landing

angle has only two values (-45.0 and 45.0). The Burnt Fuel Mass and the Prize stay very similar.

- Modified y-velocity (1(e)) has the same changes as Modified y-position (1(b)): Landing Time, Landing Position and Landing velocity of y vary, while the Landing Position and Landing Velocity of x remain unchanged (0), as well as the Landing Angle (0). Again, the Burnt Fuel Mass and the Prize have minor modifications.

2. With the influence of the wind using the thruster of the space probe (see Figure 15 & Figure 16)

   Thruster Force is varying between three values (2500, 5000 and 10000) and for each of these values different Wind force is executed (from 0.5 to 10). As a consequence the Landing Time ranges between 1 hour and 12 minutes and 4 hours and 1 minute. The Landing Position and Landing Velocity fluctuate with every change, as well as the Landing Angle. The Burnt Fuel Mass and the Price in euros have minor changes, because the Landing Time is varying only in minutes and seconds, the most expensive landing being 1003.41 euros.

4.4 Burnt fuel mass

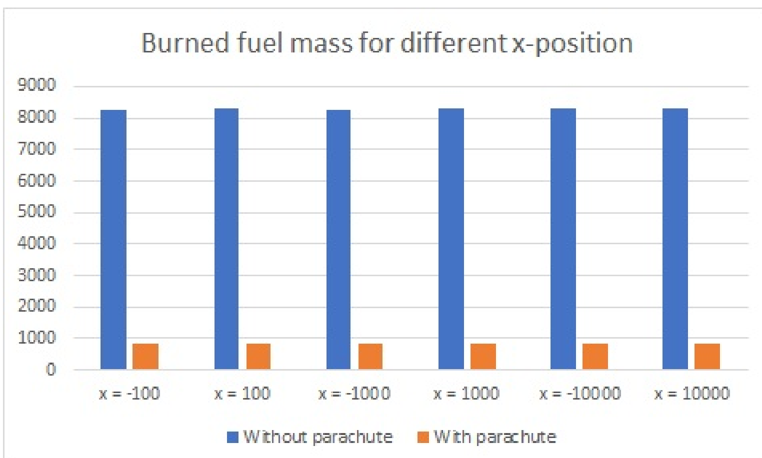

Figure 8: Burnt fuel mass for different x-position

The landing module uses a lot less fuel during the landing when it contains a parachute. Furthermore, the burnt fuel mass is around 8000, regardless of the x-position if the landing module does contain a parachute an below 1000 if it does contain a parachute.

5 Discussion

5.1 Solar System

From the diagrams showing the relative error of x- and y-position and the x- and y-velocity of the different bodies, we can see that the results are pretty good for the first couple of years, but there's overall an increasing gap between the computed values and the values gathered from NASA. This makes sense, as while the Runge-Kutta method will give a good approximation, every iteration will create a small error. Over a long period of time, these small errors will add up, which causes the bigger relative errors as the period of time increases.

5.2 Feedback Controller without Parachute

From the result from the experiments with the feedback controller without parachute it can be seen that if the starting x-position of the landing module is changed, the x-position on landing stays within reasonable bounds (around 10 meters) of the target (0 meters +/- 0.1). The case with a starting x-position of -100 meters is an exception, as the x-position after landing is unexpectedly high (9.24 meters), as well as the angle on landing (-11 degrees), compared to the other experiments with similar starting x-positions.
Furthermore, if the starting x-velocity is changed, the landing module still lands close (within ten meters) to the original landing destination.
When wind gets added to the model, the results become slightly less inaccurate. If the maximum thruster force was strong enough (10.000 N), the landing module is still able to land in a reasonable x-position. However, when the thruster force is set to a weaker value, the landing module does not manage to land with decent x-positions. This is probably due to the thruster force not being strong enough to completely counter the effect of the wind on the landing module.

5.3 Feedback Controller with Parachute

From the results from the experiments with the feedback controller with parachute, it can be seen that if the starting x-position of the landing module is changed, it does not affect the landing x-position a lot. It usually lands within ten meters of the original destination.
However, if the starting x-velocity is changed, the landing module is not able to land in an acceptable x-position. This is due to the fact that the thrusters are not used before the landing module comes down to an altitude of 50 kilometres from the ground of Titan, and the thrusters have to be used in order to correct the x-position. Therefore it gets moved on the x-axis without any correction to the position until the altitude is 50 kilometers above Titan. Moreover, in all test cases, the landing x-velocity is also above 1500 meters/second, in which case it is very unlikely to land safely.

5.4 Burnt Fuel Mass

The burnt fuel mass is higher for the landing module without a parachute compared to the landing module with a parachute. This result is as expected, since the module without the parachute uses the thrusters a lot more. The landing module with the parachute starts to use the thrusters 50 kilometres from the surface of Titan. Therefore, the time of using the thrusters is lower and less fuel mass is burned.

6 Conclusion

Several algorithms were devised in order to fulfill the assigned tasks. Moreover, experiments were performed to measure the performance of the algorithms and analyse the behaviour of the algorithms in different conditions. The landing module with feedback controller showed a greater resistance than the open-loop controller when starting conditions were modified, and a feedback controller using parachutes was tested and compared to the original feedback controller in order to analyse the gain in fuel consumption as well as the possible loss in precision. Indeed, while the parachute saves energy

(and money), it loses some precision in the landing position.
Naturally, the algorithms could be refined and improved even more, for example by searching for ways to make it possible, in practice, to execute a Hohmann Transfer, or find another method to enter orbit around Titan or Saturn in a first step, in order to reach Titan in a second step. Also, elaborating multiple algorithms for a given task would be a way to search for (better) alternatives, and get an idea of the performance of the various methods. A 3-dimensional model could also be implemented, as that is a more realistic depiction of reality, as most planets are slightly out of a plane.

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A Experimental Results

		Starting position (m)		Starting velocity (m/s)		Starting Angle	With wind?
		x	y	x	y	(degrees)	
1		0	500000	0	0	0	No
2	**modified x-position**	-100	500000	0	0	0	No
3		100	500000	0	0	0	No
4		-1000	500000	0	0	0	No
5		1000	500000	0	0	0	No
6		-10000	500000	0	0	0	No
7		10000	500000	0	0	0	No
8	**modified y-position**	0	5000	0	0	0	No
9		0	50000	0	0	0	No
10		0	5000000	0	0	0	No
11	**modified angle**	0	500000	0	0	15	No
12		0	500000	0	0	30	No
13		0	500000	0	0	45	No
14		0	500000	0	0	60	No
15	**modified x-velocity**	0	500000	100	0	0	No
16		0	500000	200	0	0	No
17		0	500000	-100	0	0	No
18		0	500000	-200	0	0	No
19	**modified y-velocity**	0	500000	0	100	0	No
20		0	500000	0	200	0	No
21		0	500000	0	-100	0	No
22		0	500000	0	-200	0	No

Figure 9: Landing using Feedback Controller without parachute (first half)

Landing Time (Feedback)				Landing Position (m)		Landing Velocity (m/s)		Landing Angle	BFM	Prize
Days	Hours	Minutes	Seconds	x	y	x	y	(degrees)		€
1	2	29	26	0.0000	-0.0569	0.0000	-0.5396	0.0000	8254.7004	7268.4054
1	2	29	26	9.2429	-0.0569	-0.3251	-0.5396	-11.0000	8286.2660	7296.1995
1	2	29	26	3.8003	-0.0569	-1.5702	-0.5396	-2.0000	8286.1830	7296.1263
1	2	29	26	-3.8342	-0.0569	-1.9494	-0.5396	-2.0000	8286.4570	7296.3676
1	2	29	26	4.5001	-0.0569	-1.0234	-0.5396	-3.0000	8286.4387	7296.3515
1	2	29	26	-2.9583	-0.0569	0.8344	-0.5396	2.0000	8289.9608	7299.4528
1	2	29	26	-4.4556	-0.0569	-1.0500	-0.5396	1.0000	8288.7740	7298.4077
0	0	17	12	0.0000	-0.3905	0.0000	-0.9378	0.0000	89.2683	78.6023
0	2	34	40	0.0000	-0.1305	0.0000	-1.1450	0.0000	833.8310	734.2025
11	0	43	20	0.0000	-0.8408	0.0000	-1.3787	0.0000	82490.1323	72633.9774
1	2	29	26	-9.2470	-0.0569	0.3354	-0.5396	6.0000	8286.8139	7296.6818
1	2	29	26	-9.1165	-0.0569	0.5848	-0.5396	7.0000	8287.7766	7297.5295
1	2	29	26	-7.2786	-0.0569	-1.1630	-0.5396	3.0000	8289.0728	7298.6709
1	2	29	26	3.9918	-0.0569	-0.5045	-0.5396	-4.0000	8294.5504	7303.4940
1	2	29	26	1.9904	-0.0569	1.0092	-0.5396	0.0000	8326.6015	7331.7155
1	2	29	26	8.8474	-0.0569	-1.0820	-0.5396	-4.0000	8449.3877	7439.8309
1	2	29	26	8.1106	-0.0569	-0.9830	-0.5396	-7.0000	8323.0681	7328.6043
1	2	29	26	2.6994	-0.0569	1.1996	-0.5396	0.0000	8438.6328	7430.3610
1	2	42	23	0.0000	-0.1071	0.0000	-0.8688	0.0000	8315.5330	7321.9695
1	3	18	44	0.0000	-0.6622	0.0000	-1.2663	0.0000	8497.8893	7482.5374
1	2	13	43	0.0000	-0.9177	0.0000	-1.1686	0.0000	8179.4377	7202.1353
1	1	19	58	0.0000	-0.1775	0.0000	-1.2870	0.0000	7906.6813	6961.9686

Figure 10: Landing using Feedback Controller without parachute (second half)

thrusterForce	Starting Position (m)		Starting Velocity (m/s)		Starting Angle	max wind Force
	x	y	x	y	(degrees)	
2500	0	500000	0	0	0	0.5
2500	0	500000	0	0	0	1
2500	0	500000	0	0	0	2
2500	0	500000	0	0	0	5
5000	0	500000	0	0	0	1
5000	0	500000	0	0	0	2
5000	0	500000	0	0	0	5
5000	0	500000	0	0	0	10
10000	0	500000	0	0	0	1
10000	0	500000	0	0	0	2
10000	0	500000	0	0	0	5
10000	0	500000	0	0	0	10

Figure 11: Landing using Feedback Controller without parachute with wind (first half)

Landing Time (Feedback)				Landing Position (m)		Landing Velocity (m/s)		Landing Angle	BFM	Prize
Days	Hours	Minutes	Seconds	x	y	x	y	(degrees)		€
1	2	7	48	1,210.7507	-2.1579	-28.9078	-2.1386	-27.0000	8,519.2058	7,501.3070
1	2	5	58	-130.8781	-1.7783	-5.3383	-1.6666	16.0000	8,560.8803	7,538.0020
1	2	6	9	-64.8580	-0.1928	1.0078	-0.9588	2.0000	8,566.1460	7,542.6386
1	2	6	53	-119.2761	-0.6341	1.5396	-0.8783	2.0000	8,537.2474	7,517.1928
1	2	6	54	6.8315	-2.4694	2.0109	-2.2617	-1.0000	8,566.8944	7,543.2976
1	2	5	39	266.7322	-0.9229	-10.1574	-1.5713	-21.0000	8,513.8589	7,496.5989
1	2	6	10	1,476.5764	-0.4766	16.5698	-2.1685	-43.0000	8,541.9220	7,521.3089
1	2	5	21	-189.2656	-2.2860	-4.4597	-2.3460	19.0000	8,504.0923	7,487.9993
1	2	9	18	-15.6333	-0.8175	8.2646	-1.6165	3.0000	8,559.4529	7,536.7452
1	2	5	25	-17.6140	-1.8783	1.3372	-2.3412	2.0000	8,497.7814	7,482.4424
1	2	3	23	-132.2506	-1.4574	4.8118	-1.3489	2.0000	8,510.7700	7,493.8790
1	2	7	41	31.3709	-0.1963	-1.9707	-2.0723	-2.0000	8,581.6731	7,556.3104

Figure 12: Landing using Feedback Controller without parachute with wind (second half)

		Starting Position (m)		Starting Velocity (m/s)		Starting angle	With wind?
		x	y	x	y	(degrees)	
1		0	155000	0	0	0	No
2	modified x-position	-100	155000	0	0	0	No
3		100	155000	0	0	0	No
4		-1000	155000	0	0	0	No
5		1000	155000	0	0	0	No
6		-10000	155000	0	0	0	No
7		10000	155000	0	0	0	No
8	modified y-position	0	1550	0	0	0	No
9		0	15500	0	0	0	No
10		0	1550000	0	0	0	No
11	modified angle	0	155000	0	0	15	No
12		0	155000	0	0	30	No
13		0	155000	0	0	45	No
14		0	155000	0	0	60	No
15	modified x-velocity	0	155000	100	0	0	No
16		0	155000	200	0	0	No
17		0	155000	-100	0	0	No
18		0	155000	-200	0	0	No
19	modified y-velocity	0	155000	0	100	0	No
20		0	155000	0	200	0	No
21		0	155000	0	-100	0	No
22		0	155000	0	-200	0	No

Figure 13: Landing using Feedback Controller with parachute (first half)

Landing Time (Feedback)				Landing Position (m)		Landing Velocity (m/s)		Landing Angle	BFM	Price
Days	Hours	Minutes	Seconds	x	y	x	y	(degrees)		€
0	9	7	4	0.0000	-0.5843	0.0000	-1.1680	0.0000	833.5480	733.9534
0	9	7	4	-3.863312175	-0.5843	-0.8138	-1.1680	2.0000	838.3803	738.2082
0	9	7	4	4.5967	-0.5843	1.0430	-1.1680	-2.0000	839.4987	739.1931
0	9	7	4	8.6936	-0.5843	-1.0436	-1.1680	-5.0000	838.3799	738.2079
0	9	7	4	6.8475	-0.5843	0.6128	-1.1680	-6.0000	839.2432	738.9680
0	9	7	4	-3.5470	-0.5843	0.5937	-1.1680	2.0000	841.6181	741.0592
0	9	7	4	-0.8415	-0.5843	0.7855	-1.1680	2.0000	841.1836	740.6766
0	0	6	41	0.0000	-0.1604	0.0000	-0.7734	0.0000	34.6604	30.5191
0	0	50	59	0.0000	-0.6204	0.0000	-1.4030	0.0000	264.6925	233.0663
0	0	59	22	0.0000	-800.4974	0.0000	-1331.3796	0.0000	-	-
0	9	7	4	0.0000	-0.5843	0.0000	-1.1680	0.0000	833.5480	733.9534
0	9	7	4	0.0000	-0.5843	0.0000	-1.1680	0.0000	833.5480	733.9534
0	9	7	4	0.0000	-0.5843	0.0000	-1.1680	0.0000	833.5480	733.9534
0	9	7	4	0.0000	-0.5843	0.0000	-1.1680	0.0000	833.5480	733.9534
0	9	7	4	604270.7307	-0.5843	-2425.3979	-1.1680	-45.0000	1174.2680	1033.9631
0	9	7	4	4034002.7193	-0.5843	1841.4057	-1.1680	-45.0000	1174.2631	1033.9588
0	9	7	4	-604270.7307	-0.5843	2425.3979	-1.1680	45.0000	1174.2680	1033.9631
0	9	7	4	-4034002.7193	-0.5843	-1841.4057	-1.1680	45.0000	1174.2631	1033.9588
0	4	44	41	0.0000	-0.5659	0.0000	-1.1868	0.0000	831.9919	732.5831
0	4	41	5	0.0000	-0.9844	0.0000	-1.1331	0.0000	833.1236	733.5797
0	4	25	24	0.0000	-0.3265	0.0000	-0.7998	0.0000	829.5868	730.4655
0	5	1	9	0.0000	-1.1765	0.0000	-1.1737	0.0000	832.4163	732.9568

Figure 14: Landing using Feedback Controller with parachute (second half)

thrusterForce	Starting position (m)		Starting velocity (m/s)		Starting angle	max wind force
	x	y	x	y	(degrees)	
2500	0	155000	0	0	0	0.5
2500	0	155000	0	0	0	1
2500	0	155000	0	0	0	2
2500	0	155000	0	0	0	5
5000	0	155000	0	0	0	1
5000	0	155000	0	0	0	2
5000	0	155000	0	0	0	5
5000	0	155000	0	0	0	10
10000	0	155000	0	0	0	1
10000	0	155000	0	0	0	2
10000	0	155000	0	0	0	5
10000	0	155000	0	0	0	10

Figure 15: Landing using Feedback Controller with parachute with wind (first half)

Landing Time (Feedback)				Landing Position (m)		Landing Velocity (m/s)		Landing Angle	BFM	Price
Days	Hours	Minutes	Seconds	x	y	x	y	(degrees)		€
0	3	27	50	-38,675.9597	-0.7144	1,212.7640	-1.3776	45.0000	1,112.8936	979.9219
0	2	11	25	345,680.7429	-0.0967	-418.5003	-2.3105	-45.0000	627.8972	552.8742
0	3	42	45	-507,651.5715	-0.9663	-612.2689	-1.6855	45.0000	1,070.2416	942.3661
0	2	49	22	-154,126.9942	-0.4970	977.9670	-2.1973	45.0000	879.2882	774.2284
0	2	39	51	-432,963.6080	-0.0349	292.4537	-1.8960	45.0000	820.9337	722.8462
0	3	28	26	-491,124.6062	-1.2522	681.9678	-1.1345	45.0000	1,086.3028	956.5083
0	1	12	16	-55,792.9474	-1.6326	-785.5732	-1.5887	25.0000	213.3221	187.8338
0	3	13	3	-446,792.2508	-1.4688	-610.0395	-2.4756	45.0000	1,035.4251	911.7096
0	4	0	56	-721,427.6514	-0.6275	-394.9442	-2.2327	45.0000	1,139.5693	1,003.4104
0	2	36	35	-78,173.9121	-0.9882	179.5878	-1.2906	45.0000	775.9122	683.2040
0	3	11	22	-562,603.8213	-0.8366	230.2245	-1.5258	45.0000	1,019.0704	897.3090
0	3	34	14	-379,701.2326	-0.6402	636.9014	-1.4795	45.0000	1,082.4780	953.1405

Figure 16: Landing using Feedback Controller with parachute with wind (second half)

	Earth											
	x	nx	REx	y	ny	REy	vx	nvx	REvx	vy	nvy	REvy
1 year	-1.49E+11	-1.49E+11	9.97E-04	7.98E+09	6.29E+09	2.69E-01	-2.12E+03	-1.75E+03	2.07E-01	-2.99E+04	-2.99E+04	2.98E-05
2 years	-1.49E+11	-1.49E+11	5.89E-04	7.99E+09	6.95E+09	1.50E-01	-2.17E+03	-1.87E+03	1.64E-01	-2.99E+04	-2.99E+04	7.34E-04
3 years	-1.48E+11	-1.49E+11	1.37E-03	9.20E+09	7.62E+09	2.07E-01	-2.51E+03	-2.01E+03	2.51E-01	-2.98E+04	-2.99E+04	1.11E-03
4 years	-1.48E+11	-1.49E+11	4.76E-03	1.21E+10	8.29E+09	4.59E-01	-3.18E+03	-2.15E+03	4.78E-01	-2.98E+04	-2.99E+04	3.58E-03
5 years	-1.48E+11	-1.49E+11	2.57E-03	7.95E+08	6.36E+09	8.75E-01	-1.15E+03	-1.75E+03	3.41E-01	-2.96E+04	-2.99E+04	7.46E-03
6 years	-1.48E+11	-1.49E+11	4.71E-03	-7.05E+09	7.04E+09	2.00E+00	3.48E+02	-1.90E+03	1.18E+00	-2.96E+04	-2.99E+04	8.63E-03
7 years	-1.47E+11	-1.49E+11	8.58E-03	-1.16E+10	7.70E+09	2.50E+00	1.22E+03	-2.03E+03	1.60E+00	-2.95E+04	-2.99E+04	1.19E-02
8 years	-1.47E+11	-1.49E+11	1.15E-02	-1.27E+10	8.34E+09	2.53E+00	1.45E+03	-2.14E+03	1.68E+00	-2.95E+04	-2.98E+04	1.29E-02
9 years	-1.47E+11	-1.49E+11	1.38E-02	-1.06E+10	6.43E+09	2.65E+00	1.05E+03	-1.78E+03	1.59E+00	-2.95E+04	-2.99E+04	1.41E-02
10 years	-1.47E+11	-1.49E+11	1.39E-02	-5.32E+09	7.09E+09	1.75E+00	4.86E+01	-1.90E+03	1.03E+00	-2.95E+04	-2.99E+04	1.41E-02
15 years	-1.30E+11	-1.49E+11	1.25E-01	6.39E+10	7.83E+09	7.16E+00	-1.36E+04	-2.06E+03	5.60E+00	-2.63E+04	-2.99E+04	1.19E-01
20 years	-7.74E+09	-1.49E+11	9.48E-01	1.46E+11	8.56E+09	1.61E+01	-2.98E+04	-2.21E+03	1.25E+01	-3.21E+03	-2.99E+04	8.93E-01
25 years	1.62E+11	-1.49E+11	2.09E+00	2.56E+10	6.69E+09	2.83E+00	-5.88E+03	-1.83E+03	2.21E+00	2.90E+04	-2.99E+04	1.97E+00

Figure 17: Earth data Timestep 1800s (extended)

	Titan											
	x	nx	REx	y	ny	REy	vx	nvx	REvx	vy	nvy	REvy
1 year	6.20E+11	6.23E+11	4.47E-03	-1.36E+12	-1.36E+12	1.31E-03	9.41E+03	3.19E+03	1.95E+00	1.05E+03	2.41E+03	5.62E-01
2 years	8.67E+11	8.70E+11	4.04E-03	-1.21E+12	-1.21E+12	2.57E-03	9.67E+03	2.02E+03	3.79E+00	3.53E+03	7.33E+03	5.18E-01
3 years	1.08E+12	1.08E+12	3.66E-03	-1.01E+12	-1.01E+12	3.50E-03	8.69E+03	3.01E+03	1.89E+00	5.41E+03	1.14E+04	5.27E-01
4 years	1.25E+12	1.25E+12	4.09E-03	-7.73E+11	-7.70E+11	3.19E-03	6.02E+03	5.30E+03	1.35E-01	6.31E+03	1.32E+04	5.23E-01
5 years	1.37E+12	1.37E+12	3.68E-04	-5.01E+11	-4.97E+11	9.55E-03	6.95E+03	5.18E+03	3.42E-01	1.03E+04	1.36E+04	2.44E-01
6 years	1.42E+12	1.42E+12	3.52E-03	-2.04E+11	-2.02E+11	7.19E-03	1.26E+03	5.95E+03	7.88E-01	7.82E+03	1.14E+04	3.16E-01
7 years	1.42E+12	1.42E+12	1.63E-03	9.63E+10	1.01E+11	4.39E-02	2.78E+03	4.11E+03	3.25E-01	1.09E+04	8.10E+03	3.52E-01
8 years	1.35E+12	1.34E+12	2.56E-03	3.95E+10	4.00E+11	9.01E-01	6.53E+02	-1.04E+02	7.27E+00	1.09E+04	5.17E+03	1.10E+00
9 years	1.20E+12	1.21E+12	2.48E-03	6.76E+11	6.80E+11	5.19E-03	-4.18E+03	-3.47E+03	2.05E-01	6.93E+03	3.76E+03	8.43E-01
10 years	1.01E+12	1.01E+12	1.46E-03	9.28E+11	9.26E+11	2.57E-03	-8.90E+03	-8.78E+03	1.33E-02	6.71E+03	2.63E+03	1.55E+00
15 years	-4.61E+11	-4.60E+12	9.00E-01	1.26E+12	1.27E+12	6.95E-03	-1.09E+04	-8.24E+03	3.23E-01	8.73E+02	1.59E+03	4.49E-01
20 years	-1.41E+12	-1.41E+12	1.46E-03	9.55E+10	1.13E+11	1.54E-01	-2.26E+03	-2.05E+03	9.95E-02	-1.02E+04	-1.44E+04	2.90E-01
25 years	-8.32E+11	-8.62E+11	3.53E-02	-1.22E+12	-1.21E+12	1.38E-02	9.49E+03	5.34E+03	7.76E-01	-1.33E+03	-7.96E+02	6.67E-01

Figure 18: Titan data Timestep 1800s (extended)

	Earth timestep 900s											
	x	nx	REx	y	ny	REy	vx	nvx	REvx	vy	nvy	REvy
1 year	-1.49E+11	-1.49E+11	1.31E-03	7.91E+09	6.29E+09	2.59E-01	-2.10E+03	-1.75E+03	1.95E-01	-2.99E+04	-2.99E+04	5.55E-04
3 years	-1.48E+11	-1.49E+11	1.73E-03	6.17E+09	7.62E+09	1.90E-01	-1.91E+03	-2.01E+03	5.04E-02	-2.99E+04	-2.99E+04	1.72E-03
5 years	-1.47E+11	-1.49E+11	8.98E-03	5.73E+09	6.36E+09	9.93E-02	-2.01E+03	-1.75E+03	1.53E-01	-2.99E+04	-2.99E+04	2.48E-04
10 years	-1.42E+11	-1.49E+11	4.48E-02	2.47E+10	7.09E+09	2.49E+00	-5.80E+03	-1.90E+03	2.06E+00	-2.93E+04	-2.99E+04	1.81E-02

Figure 19: Earth data Timestep 900s

	Earth timestep 1800s											
	x	nx	REx	y	ny	REy	vx	nvx	REvx	vy	nvy	REvy
1 year	-1.49E+11	-1.49E+11	9.97E-04	7.98E+09	6.29E+09	2.69E-01	-2.12E+03	-1.75E+03	2.07E-01	-2.99E+04	-2.99E+04	2.98E-05
3 years	-1.48E+11	-1.49E+11	1.37E-03	9.20E+09	7.62E+09	2.07E-01	-2.51E+03	-2.01E+03	2.51E-01	-2.98E+04	-2.99E+04	1.11E-03
5 years	-1.48E+11	-1.49E+11	2.57E-03	7.95E+08	6.36E+09	8.75E-01	-1.15E+03	-1.75E+03	3.41E-01	-2.96E+04	-2.99E+04	7.46E-03
10 years	-1.47E+11	-1.49E+11	1.39E-02	-5.32E+09	7.09E+09	1.75E+00	4.86E+01	-1.90E+03	1.03E+00	-2.95E+04	-2.99E+04	1.41E-02

Figure 20: Earth data Timestep 1800s

	Earth timestep 3600s											
	x	nx	REx	y	ny	REy	vx	nvx	REvx	vy	nvy	REvy
1 year	-1.49E+11	-1.49E+11	1.31E-03	7.91E+09	6.29E+09	2.59E-01	-2.10E+03	-1.75E+03	1.95E-01	-2.99E+04	-2.99E+04	5.55E-04
3 years	-1.48E+11	-1.49E+11	1.73E-03	6.17E+09	7.62E+09	1.90E-01	-1.91E+03	-2.01E+03	5.04E-02	-2.99E+04	-2.99E+04	1.72E-03
5 years	-1.47E+11	-1.49E+11	8.98E-03	5.73E+09	6.36E+09	9.93E-02	-2.01E+03	-1.75E+03	1.53E-01	-2.99E+04	-2.99E+04	2.48E-04
10 years	-1.42E+11	-1.49E+11	4.48E-02	2.47E+10	7.09E+09	2.49E+00	-5.80E+03	-1.90E+03	2.06E+00	-2.93E+04	-2.99E+04	1.81E-02

Figure 21: Earth data Timestep 3600s

	Earth timestep 5400s											
	x	nx	REx	y	ny	REy	vx	nvx	REvx	vy	nvy	REvy
1 year	-1.49E+11	-1.49E+11	9.90E-04	1.04E+10	6.29E+09	6.60E-01	-2.61E+03	-1.75E+03	4.87E-01	-2.98E+04	-2.99E+04	2.75E-03
3 years	-1.45E+11	-1.49E+11	2.26E-02	3.77E+10	7.62E+09	3.95E+00	-8.07E+03	-2.01E+03	3.02E+00	-2.87E+04	-2.99E+04	3.75E-02
5 years	-1.16E+11	-1.49E+11	2.20E-01	9.42E+10	6.36E+09	1.38E+01	-1.91E+04	-1.75E+03	9.93E+00	-2.29E+04	-2.99E+04	2.35E-01
10 years	1.50E+11	-1.49E+11	2.01E+00	6.04E+10	7.09E+09	7.52E+00	-1.18E+04	-1.90E+03	5.23E+00	2.64E+04	-2.99E+04	1.89E+00

Figure 22: Earth data Timestep 5400s

	Earth timestep 10800s											
	x	nx	REx	y	ny	REy	vx	nvx	REvx	vy	nvy	REvy
1 year	-1.50E+11	-1.49E+11	6.02E-03	1.85E+10	6.29E+09	1.94E+00	-4.12E+03	-1.75E+03	1.35E+00	-2.95E+04	-2.99E+04	1.36E-02
3 years	-1.25E+11	-1.49E+11	1.61E-01	8.87E+10	7.62E+09	1.06E+01	-1.75E+04	-2.01E+03	7.70E+00	-2.38E+04	-2.99E+04	2.01E-01
5 years	7.19E+08	-1.49E+11	1.00E+00	1.55E+11	6.36E+09	2.33E+01	-2.93E+04	-1.75E+03	1.58E+01	-1.84E+02	-2.99E+04	9.94E-01
10 years	-1.08E+11	-1.49E+11	2.76E-01	-1.28E+11	7.09E+09	1.91E+01	2.06E+04	-1.90E+03	1.18E+01	-1.89E+04	-2.99E+04	3.68E-01

Figure 23: Earth data Timestep 10800s

	Titan timestep 900s											
	x	nx	REx	y	ny	REy	vx	nvx	REvx	vy	nvy	REvy
1 year	6.21E+11	6.23E+11	3.05E-03	-1.36E+12	-1.36E+12	8.30E-05	5.92E+03	3.19E+03	8.57E-01	1.45E+03	2.41E+03	3.97E-01
3 years	1.08E+12	1.08E+12	3.64E-03	-1.01E+12	-1.01E+12	2.42E-03	7.68E+03	3.01E+03	1.55E+00	4.50E+03	1.14E+04	6.06E-01
5 years	1.36E+12	1.37E+12	2.55E-03	-4.96E+11	-4.97E+11	6.22E-04	1.04E+03	5.18E+03	8.00E-01	7.51E+03	1.36E+04	4.47E-01
10 years	1.01E+12	1.01E+12	1.35E-04	9.27E+11	9.26E+11	1.11E-03	-9.24E+03	-8.78E+03	5.18E-02	6.53E+03	2.63E+03	1.48E+00

Figure 24: Titan data Timestep 900s

	Titan timestep 1800s											
	x	nx	REx	y	ny	REy	vx	nvx	REvx	vy	nvy	REvy
1 year	6.20E+11	6.23E+11	4.47E-03	-1.36E+12	-1.36E+12	1.31E-03	9.41E+03	3.19E+03	1.95E+00	1.05E+03	2.41E+03	5.62E-01
3 years	1.08E+12	1.08E+12	3.66E-03	-1.01E+12	-1.01E+12	3.50E-03	8.69E+03	3.01E+03	1.89E+00	5.41E+03	1.14E+04	5.27E-01
5 years	1.37E+12	1.37E+12	3.68E-04	-5.01E+11	-4.97E+11	9.55E-03	6.95E+03	5.18E+03	3.42E-01	1.03E+04	1.36E+04	2.44E-01
10 years	1.01E+12	1.01E+12	1.46E-03	9.28E+11	9.26E+11	2.57E-03	-8.90E+03	-8.78E+03	1.33E-02	6.71E+03	2.63E+03	1.55E+00

Figure 25: Titan data Timestep 1800s

	Titan timestep 3600s											
	x	nx	REx	y	ny	REy	vx	nvx	REvx	vy	nvy	REvy
1 year	6.24E+11	6.23E+11	1.52E-03	-1.36E+12	-1.36E+12	3.48E-04	5.25E+03	3.19E+03	6.46E-01	7.55E+03	2.41E+03	2.14E+00
3 years	1.08E+12	1.08E+12	6.61E-03	-1.01E+12	-1.01E+12	1.23E-03	6.44E+03	3.01E+03	1.14E+00	5.28E+03	1.14E+04	5.38E-01
5 years	1.36E+12	1.37E+12	5.96E-03	-4.96E+11	-4.97E+11	1.74E-03	2.25E+03	5.18E+03	5.66E-01	7.82E+03	1.36E+04	4.24E-01
10 years	1.01E+12	1.01E+12	3.06E-04	9.18E+11	9.26E+11	8.00E-03	-4.07E+03	-8.78E+03	5.37E-01	7.32E+03	2.63E+03	1.79E+00

Figure 26: Titan data Timestep 3600s

	Titan timestep 5400s											
	x	nx	REx	y	ny	REy	vx	nvx	REvx	vy	nvy	REvy
1 year	6.21E+11	6.23E+11	2.98E-03	-1.36E+12	-1.36E+12	1.46E-03	5.44E+03	3.19E+03	7.04E-01	3.84E+03	2.41E+03	5.93E-01
3 years	1.08E+12	1.08E+12	7.24E-03	-1.01E+12	-1.01E+12	2.47E-03	7.09E+03	3.01E+03	1.36E+00	5.51E+03	1.14E+04	5.18E-01
5 years	1.36E+12	1.37E+12	2.12E-03	-4.92E+11	-4.97E+11	8.36E-03	5.44E+02	5.18E+03	8.95E-01	9.69E+03	1.36E+04	2.86E-01
10 years	1.02E+12	1.01E+12	2.13E-03	9.30E+11	9.26E+11	4.31E-03	-9.28E+03	-8.78E+03	5.64E-02	8.51E+03	2.63E+03	2.24E+00

Figure 27: Titan data Timestep 5400s

	Titan timestep 10800s											
	x	nx	REx	y	ny	REy	vx	nvx	REvx	vy	nvy	REvy
1 year	6.17E+11	6.23E+11	9.64E-03	-1.36E+12	-1.36E+12	5.37E-04	8.45E+03	3.19E+03	1.65E+00	2.15E+03	2.41E+03	1.08E-01
3 years	1.07E+12	1.08E+12	1.01E-02	-1.01E+12	-1.01E+12	3.43E-03	5.31E+03	3.01E+03	7.65E-01	6.17E+03	1.14E+04	4.60E-01
5 years	1.36E+12	1.37E+12	3.99E-03	-4.90E+11	-4.97E+11	1.31E-02	9.51E+02	5.18E+03	8.16E-01	9.51E+03	1.36E+04	3.00E-01
10 years	1.01E+12	1.01E+12	1.80E-03	9.34E+11	9.26E+11	8.39E-03	-8.87E+03	-8.78E+03	9.80E-03	7.80E+03	2.63E+03	1.97E+00

Figure 28: Titan data Timestep 10800s