Mini-Project 2

One report/jupyter notebook per team must be submitted as soft copies to deepakns@iisc.ac.in. As before, please typeset it in jupyter notebook if you use Python or Julia or R, or latex (template is provided) if you use any other language or software. In the latter case, the codes should also be included with your submission. Place all files in a folder and submit the .zip (don't submit a tar.gz etc).

Please clearly specify the role of each team member in the solution. You must specify which problem was tackled by who all. Reference codes that you use from elsewhere.

Please keep the subject line of your email submission DS211:2019:MP2

Typically, this assignment should take your team between 12 to 20 person-hours.

Problem 1

Least Squares.

(a) Consider a data generating process $y = \exp(-rt) + \nu$, $0 \le t \le 5$. Use $r = 2.7$, $\nu \sim \mathcal{N}(0, 2)$ and $dt = 0.01$ to generate data. Henceforth work only with (t, y) as if someone had given you this data.

(b) You want to model the data as a deterministic exponential $y = \exp(-rt)$. Implement the Gauss Newton method and estimate r.

(c) Use the data and built-in levenberg marquadt algorithm to estimate r. Comment on the execution time and iteration counts of your implementation and built-in functions.

Problem 2

Quadratic Programming.

(a) Consider the problem

$$\begin{aligned} &\min x_1^2 + 2x_2^2 - 2x_1 - 6x_2 - 2x_1x_2\,, \\ &\text{s.t. } x_1 + x_2 \le 2, \quad -x_1 + 2x_2 \le 2\,, \quad x_1, x_2 \ge 0\,. \end{aligned}$$

Solve geometrically and find the solution. (Use Matlab or Python to plot the geometry etc.)

(b) Implement the active set method for quadratic programs (Algo 16.3 in Nocedal and Wright (2006)).

(c) Choose three initial starting points: one in the interior of the feasible region, one at the vertex, and one at a non-vertex point on the boundary of the feasible region. Comment on the convergence properties.

(d) Use CVXOPT and solve the above QP. What algorithm does CVXOPT uses to solve QP? Understand and comment in your own words -

Problem 3

Linear Programming.

(a) Your start-up will face the cash requirements shown in Table 1 in the next eight quarters (positive entries represent cash needs while negative entries represent cash surpluses). The company has three borrowing possibilities.

Table 1: Cash Flow (in Crores of INR)

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
100	500	100	-600	-500	200	600	-900

- a 2-year loan available at the beginning of Q1, with a 1% interest per quarter.
- The other two borrowing opportunities are available at the beginning of every quarter: a 6-month loan with a 1.8% interest per quarter, and a quarterly loan with a 2.5% interest for the quarter.

Any surplus can be invested at a 0.5% interest per quarter.

Formulate a LP that maximizes the wealth of the company at the beginning of Q9.

(b) Write a program that implements the revised simplex algorithm.

(c) Solve the above LP using your program and report the final results.

(d) Use built-in linear programming solvers (linprog in MATLAB and scipy.optimize.linprog in Python) with dual-simplex (in MATLAB) and revised-simplex (in Python) options to verify your answer.

(e) Use interior point method and solve the above problem. Report the execution times of your revised-simplex, and built-in algorithms that you tested so far. Comment on findings.

Problem 4

Penalty and Augmented Lagrangian.

(a) Consider the problem

$$\begin{aligned} &\min x_1 + x_2 \\ &\text{s.t. } x_1^2 + x_2^2 - 2 = 0. \end{aligned}$$

Write down the quadratic penalty function Q.

(b) Use $\mu_k = 1, 10, 100, 1000$ and solve $\min Q$ using any unconstrained optimization algorithm. Set $\tau_k = 1/\mu_k$ in Framework 17.1 (in Nocedal and Wright (2006)), and choose the starting point x_{k+1}^s for each minimization to the solution for the previous value of the penalty parameter. Report the approximate solution of each penalty function.

(c) Use the neos-server, and solve the above problem using AMPL and MINOS.