MIDTERM TEST 1 - 2023.2 Course: **ODEs and Series** Course ID: MI1046 *Time duration: 30 minutes*		**Full name:** .... Student's ID: .... Order:..................... Class ID: ....
Full mark	Full name and signature of evaluators	Full name and signature of proctors

Exam number: 101 (The exam consists of 15 questions)

Remark: *No document is allowed.*

I. Tích chọn 01 đáp án đúng

Question 1. Calculate the sum of the series $\sum_{n\geq 0} \frac{1}{n^2+3n+2}$.

- ☐ $1/2$
- ☐ 1
- ☐ $1/3$
- ☐ $1/4$

Question 2. Which of the following claims is wrong?

- ☐ The series $\sum_{n\geq 0}(1+x)x^{2n}$ converges for all $x \in [0;1)$
- ☐ The series $\sum_{n\geq 0}(1+x)x^{2n}$ converges uniformly on $[0;1)$
- ☐ The series $\sum_{n\geq 0}(1+x)x^{2n}$ does not converges for all $x \in [0;1]$
- ☐ The series $\sum_{n\geq 0}(1+x)x^{2n}$ does not converges uniformly on $[0;1)$

Question 3. The domain of convergence of the series $\sum_{n\geq 0} \frac{(x+1)^n}{(x+1)^{2n}+1}$ is

- ☐ $\{x \in \mathbb{R},\ |x+1| < 1\}$
- ☐ $\mathbb{R} \setminus \{0;\ -2\}$
- ☐ $\mathbb{R}$
- ☐ $\{x \in \mathbb{R},\ |x+1| > 1\}$

Question 4. Which of the following series diverges?

- ☐ $\sum_{n\geq 0} \frac{\arctan(n^4+9)}{2n^2+3}$
- ☐ $\sum_{n\geq 0} \left(\frac{n+1}{n+2}\right)^n$
- ☐ $\sum_{n\geq 0} \frac{1}{e^n}$
- ☐ $\sum_{n\geq 0} \sin\frac{1}{n^2}$

Question 5. Which of the following series does not converge absolutely?

- ☐ $\sum_{n\geq 2} \frac{(-1)^n}{\sqrt[3]{n^4}+(-1)^n}$
- ☐ $\sum_{n\geq 2} \frac{\cos(n\pi)}{n(\ln n)^2}$
- ☐ $\sum_{n\geq 2} \sin\frac{1}{n^2}$
- ☐ $\sum_{n\geq 2}(-1)^n \arcsin\frac{1}{n}$

Question 6. Given the series $\sum_{n\geq 2} \frac{\cos(n\pi)}{n\ln n}$. Which of the following claims is true?

- ☐ This series is positive.
- ☐ This series is only conditionally but not absolutely convergent.
- ☐ This series is absolutely convergent.
- ☐ This series diverges

Question 7. The coefficient of x^5 in the Maclaurin expansion of $\sin(\tan x)$ is

- ☐ $3/5$
- ☐ $2/5$
- ☐ $2/3$
- ☐ $-1/40$

Question 8. The coefficient of $(x+4)^3$ in the Taylor expansion of $\frac{1}{x^2+5x+6}$ is

- ☐ $19/16$
- ☐ $14/16$
- ☐ $17/16$
- ☐ $15/16$

II. Tích chọn nhiều hơn một đáp án đúng (sinh viên phải chọn đủ các đáp án đúng)

Question 9. Given the series of functions $\sum_{n\geq 0} \frac{1}{3^n} \frac{(3x+4)^n}{(x+3)^n}$ on the interval $[-2;0]$. Which of the following claims hold true?

- ☐ This series converges for all $x \in [-2;0]$
- ☐ This series converges uniformly on $[-2;0]$
- ☐ This series does not converge uniformly on $[-2;0]$
- ☐ This series does not converges for some $x \in [-2;0]$

Question 10. Given two positive series $S_a := \sum_{n\geq 0} a_n$ and $S_b := \sum_{n\geq 0} b_n$. Which of the following claims hold true?

- ☐ If $a_n \leq b_n$ for all n and $\sum_{n\geq 0} a_n$ is divergent then so is $\sum_{n\geq 0} b_n$.
- ☐ If $a_n \leq b_n$ for all n and $\sum_{n\geq 0} a_n$ is convergent then so is $\sum_{n\geq 0} b_n$.
- ☐ If $a_n \leq b_n$ for all n and $\sum_{n\geq 0} b_n$ is divergent then so is $\sum_{n\geq 0} a_n$.
- ☐ If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ then they are both convergent or both divergent
- ☐ If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum_{n\geq 0} a_n$ is convergent then both series are convergent.
- ☐ If $\lim_{n\to\infty} \frac{a_n}{b_n} = k \neq 0$ then they are both convergent or both divergent

Question 11. Provided that the series $\sum_{n\geq 0} a_n x^n$ converges at $x = 6$ and diverges at $x = -8$. Which of the following claims always hold true?

- ☐ The series $\sum_{n\geq 0} a_n(-9)^n$ converges
- ☐ The series $\sum_{n\geq 0} a_n(-5)^n$ converges
- ☐ The series $\sum_{n\geq 0} a_n x^n$ converges uniformly on $[-5,5]$
- ☐ The series $\sum_{n\geq 0} a_n x^n$ converges uniformly on $[-6,6]$
- ☐ The series $\sum_{n\geq 0} a_n 8^n$ converges
- ☐ The series $\sum_{n\geq 0} \frac{a_n}{10^n}$ diverges

Question 12. Which of the following identities hold true?

- ☐ $\sum_{n\geq 0} nx^n = \frac{-x}{(1-x)^2}, x \in (-1,1)$.
- ☐ $\sum_{n\geq 0} nx^n = \frac{x}{(1-x)^2}, x \in (-1,1)$.
- ☐ $\sum_{n\geq 1} \frac{x^{4n-3}}{4n-3} = -\frac{1}{2}\arctan x + \frac{1}{4}\ln\frac{x+1}{1-x}, x \in (-1,1)$.
- ☐ $\sum_{n\geq 1} \frac{x^{4n-3}}{4n-3} = \frac{1}{2}\arctan x + \frac{1}{4}\ln\frac{x+1}{1-x}, x \in (-1,1)$.

III. Điền vào chỗ trống để được một phát biểu Toán học đúng

Question 13. The Fourier series of $f(x) = |x|$ on $(-\pi,\pi)$ is ________________________________.

Question 14. Sum of the series $\sum_{n\geq 0} \frac{x^{2n+2}}{2n+2}, x \in (-1;1)$ is ________________________________.

Question 15. Complete the Weierstraß theorem about the uniform convergence below.
If ____ then the series $\sum_{n\geq 0} u_n(x)$ converges uniformly on the set X.

Mã đề thi: 101

ĐÁP ÁN

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- ■ $\sum_{n\geq 1} \frac{x^{4n-3}}{4n-3} = \frac{1}{2}\arctan x + \frac{1}{4}\ln\frac{x+1}{1-x}$, $x \in (-1, 1)$.

III. Điền vào chỗ trống để được một phát biểu Toán học đúng

Question 13. The Fourier series of $f(x) = |x|$ on $(-\pi, \pi)$ is $\boxed{\frac{\pi}{2} + \sum_{n\geq 1} 2\frac{(-1)^n - 1}{\pi n^2}\cos(nx)}$.

Question 14. Sum of the series $\sum_{n\geq 0} \frac{x^{2n+2}}{2n+2}$, $x \in (-1; 1)$ is $\boxed{\sum_{n\geq 0} \frac{x^{2n+2}}{2n+2} = -\frac{1}{2}\ln(1-x^2)}$.

Question 15. Complete the Weierstraß theorem about the uniform convergence below.

If $\boxed{|u_n(x)| \leq a_n \forall n \in \mathbb{N},\ \forall x \in X, \text{ and the series } \sum_{n\geq 0} a_n \text{ converges}}$ then the series $\sum_{n\geq 0} u_n(x)$ converges uniformly on the set X.