PROOF FOR PROVING AN EQUALITY A FUNCTION — COLLEGE ALGEBRA

TIMOTHY HEATH

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Definition 1 (Relation)**.** *A relation shows how two quantities relate to each other. A relation may be either an equality or an inequality.*

Example 1.

$$x = 1$$

Example 2.

$$1 = 1$$

Example 3.

$$x + 5 = 1$$

Example 4.

$$x > 1$$

Example 5.

$$4x + 3 < 1$$

Definition 2 (Equation)**.** *An Equation is just a relation showing one quantity being the same as some other quantity.*

Example 6.

$$4x^2 + 3x = 7$$

Definition 3 (Inequality)**.** *An Inequality shows that two quantities are not the same.*

Example 7.

$$4x^2 + 3x \neq 7$$

Definition 4 (Set)**.** *A set is a collection of unique objects.*

Example 8.

$$\{1, 2, 3\}$$

Example 9.

$$\{1, 2, x\}$$

Example 10.

$$\{1, 2, \{1, 2, x\}\}$$

Example 11.

$$\{1, 1, 2\} = \{1, 2\}$$

Example 12.

$$(-\infty, \infty) = \mathbb{R}$$

Definition 5 (Expressed relation as set — graph data)**.** *Any relation may be expressed as a set of points.*

Example 13. *The set of all points from the relation.*

$$\{(x, y)|x^2 = y\}$$

Definition 6 (Domain)**.** *A domain of a relation shows the allowed input values.*

Example 14. *Let A be $\{(x, y)|x^2 = y\}$.*
Then the domain of A is...

$$D_A = \{x|(x, y) \in A\}$$

Definition 7 (Range)**.** *A range of a relation shows the possible output values.*

Example 15. *Let A be $\{(x, y)|x^2 = y\}$.*
Then the range of A is...

$$R_A = \{y|(x, y) \in A\}$$

Definition 8 (Function)**.** *A function is an equality where no two separate points share the same x value.*

Example 16.

$$f(x) = x^2$$

To prove an equality is a function you must demonstrate that there is at least a single x value that produces two solutions. I will demonstrate here that instead of proving directly, one may simply view the domain of an equality to determine whether or not it is a function.

Proof.

The equality

$\{(x, y)|x^2 = y\}$

In order to prove the equality to be a function, we must demonstrate that there are at least two points with the same y value.

If there are two x values that give the same y value, then there are more points then there are x values.

From a high level this may be shown via:

Let A be $\{(x, y)|x^2 = y\}$

If the relation is a function then:

$n(D_A) = n(A)$

$\therefore$ In order to prove an equality is a function, you only need to demonstrate that there are more points than there are x values.

In order to prove there are more points than x values, you must show that at least one x value gives two points.

$\square$