COLLEGE ALGEBRA SECTION 0.10 HOMEWORK

TIMOTHY HEATH

Date: February 6, 2023.

Question 1. *Find the complex conjugates of the following numbers:*
a. $\overline{11} = \boxed{11}$
b. $\overline{20i} = \boxed{-20i}$
c. $\overline{-95 + 88i} = \boxed{-95 - 88i}$
d. $\overline{-483i - 84} = \boxed{-84 + 483i}$
a.

$$\begin{aligned} \overline{11} \\ \overline{11 + 0i} \\ 11 - 0i \\ 11 \end{aligned}$$

b.

$$\begin{aligned} \overline{20i} \\ \overline{0 + 20i} \\ 0 - 20i \\ -20i \end{aligned}$$

c.

$$\begin{aligned} \overline{-95 + 88i} \\ -95 - 88i \end{aligned}$$

d.

$$\begin{aligned} \overline{-483i - 84} \\ \overline{-84 - 483i} \\ -84 + 483i \end{aligned}$$

Question 2. *You find the discriminant D of a quadratic equation and notice that $D = 0$. What does this tell you about the solution(s) to the equation?*
a. *The equation has one real solution.*
b. *The equation has two real number solutions.*
c. *The equation has two complex number solutions.*
d. *The equation has one real and one complex solution.*
The answer is a.

$$\begin{aligned} D &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-b \pm \sqrt{D}}{2a} \\ x &= \frac{-b \pm \sqrt{0}}{2a} \\ x &= \frac{-b \pm 0}{2a} \\ x &= \frac{-b}{2a} \end{aligned}$$

Question 3. *Add.*
$(-1 - 6i) + (6 + 10i) = \boxed{5 + 4i}$

$$\begin{aligned} (-1 - 6i) + (6 + 10i) = \\ -1 - 6i + 6 + 10i = \\ 6 - 1 + 10i - 6i = \\ 5 + 4i = \end{aligned}$$

Question 4. *Perform the indicated operations and simplify.*
Add: $(13-25i)+(6+7i)$
sum $=\boxed{19-18i}$
Subtract: $(13-25i)-(6+7i)$
difference $=\boxed{7-32i}$

$$\begin{aligned}(13-25i)+(6+7i)&=\\ 13+6-25i+7i&=\\ 19-18i&=\\ (13-25i)-(6+7i)&=\\ 13-6-25i-7i&=\\ 7-32i&=\end{aligned}$$

Question 5. *Simplify, write in the for a+bi.*
$(2+11i)(4-8i)=\boxed{96+28i}$

$$\begin{aligned}(2+11i)(4-8i)&=\\ 8-16i+44i-88i^2&=\\ 8+28i+88&=\\ 96+28i&=\end{aligned}$$

Question 6. *Perform the indicated operation and simplify. Express the answer as a complex number.*
$(11-12i)(-2+9i)=\boxed{86+123i}$

$$\begin{aligned}(11-12i)(-2+9i)&=\\ -22+99i+24i-108i^2&=\\ -22+123i+108&=\\ 86+123i&=\end{aligned}$$

Question 7. *Multiply the following complex number by it's conjugate, and simplify:* $(1-5i)$

$$\begin{aligned}&\boxed{26}\\ &(1-5i)\\ &(1-5i)(1+5i)\\ &1-25i^2\\ &1+25\\ &26\end{aligned}$$

Question 8. *In this problem you are going to investigate what happens when you multiply a complex number by its conjugate using the number* $1-i$.
First, multiply it by something that is not its conjugate:
$(1-i)(-9-5i)=\boxed{-14+4i}$
Now, multiply it by its conjugate:
$(1-i)(1+i)=\boxed{2}$
You should notice a difference in those two results. To test it, try another one:
$(-5+4i)(-5-4i)=\boxed{41}$
When you multiply a complex number by it's conjugate the result will be a real number.

Question 9.

$$\begin{aligned}&(-1+4i)^2\\ &1-8i-16\\ &-15-8i\end{aligned}$$

Question 10. *Let* $f(x) = x^2 + x - 4$.
$f(3+i) = \boxed{7+7i}$
$f(-i) = \boxed{-5-i}$

$$\begin{aligned} f(3+i) &= (3+i)^2 + (3+i) - 4 \\ &= 9 + 6i - 1 + 3 + i - 4 \\ &= 7 + 7i \\ f(-i) &= (-i)^2 + (-i) - 4 \\ &= -1 - i - 4 \\ &= -5 - i \end{aligned}$$

Question 11.

$$i^{14} = -1$$

Question 12.

$$i^{71} + i^{72} + i^{73} = 1$$

Question 13.

$$\frac{-2+2i}{6i} = \frac{1}{3} + \frac{1}{3}i$$

Question 14.

$$\frac{-1}{2i} = \frac{i}{2}$$

Question 15.

$$\frac{3}{4+i} = \frac{12-3i}{17}$$

Question 16.

$$\frac{i}{3-2i} = \frac{-2+3i}{13}$$

Question 17. *Find all complex solutions of the equation* $x^2 + 8x + 41 = 0$.
$x = \boxed{-4+5i, -4-5i}$

$$\begin{aligned} x^2 + 8x + 41 &= 0 \\ x^2 + 8x + 16 + 25 &= 0 \\ (x+4)^2 + 25 &= 0 \\ (x+4)^2 + 5^2 &= 0 \\ (x+4)^2 &= -5^2 \\ x + 4 &= \pm\sqrt{-5^2} \\ x + 4 &= \pm\sqrt{-25} \\ x + 4 &= \pm 5i \\ x &= -4 \pm 5i \end{aligned}$$

Question 18. *Solve via completing the square.*

$$\begin{aligned} p^2 - 12p + 17 &= 49 \\ p^2 - 12p + 36 &= 68 \\ (p-6)^2 &= 68 \\ p - 6 &= \pm 2\sqrt{17} \\ p &= 6 \pm 2\sqrt{17} \end{aligned}$$

Question 19.

$$\begin{aligned}
w^2 - 8w + 36 &= -60 \\
w^2 - 8w + 16 &= -80 \\
(w-4)^2 &= -80 \\
w - 4 &= \pm\sqrt{-80} \\
w - 4 &= \pm 4i\sqrt{5} \\
w &= 4 \pm 4i\sqrt{5} \\
m^2 + 4m - 38 &= 57 \\
m^2 + 4m + 4 &= 99 \\
(m+2)^2 &= 99 \\
m + 2 &= \pm\sqrt{99} \\
m + 2 &= \pm 3\sqrt{11} \\
m &= -2 \pm 3\sqrt{11}
\end{aligned}$$

Question 20.

$$\begin{aligned}
x^4 - 5x^2 - 6 &= 0 \\
(x^2)^2 - 5x^2 - 6 &= 0 \\
(x^2 - 6)(x^2 + 1) &= 0 \\
x^2 - 6 = 0,\ &or\ x^2 + 1 = 0 \\
(x - \sqrt{6})(x + \sqrt{6}) = 0,\ &or\ (x+i)(x-i) = 0 \\
\therefore x &= (-\sqrt{6}, +\sqrt{6}, -i, i)
\end{aligned}$$