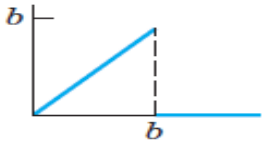
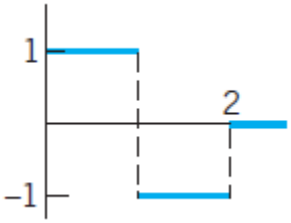
Q1. Find the transform. Show the details of your work. Assume that a, b,ω,θ are constants.

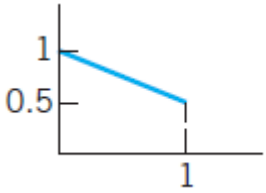
(i)



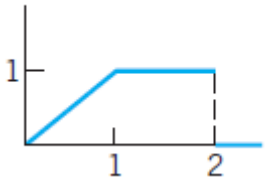
(ii)



(iii)



(iv)



Q2. Solve by the Laplace transform. If necessary, use partial fraction expansion. Show all details.

$y'' + 3y' + 2.25y = 9t^3 + 64$, $y(0)=1$, $y'(0)=31.5$

Q3. Solve the shifted data Problem by the Laplace transform. Show the details.

$y'' + 3y' - 4y = 6e^{2t-3}$, $y(1.5)=4$, $y'(1.5)=5$

Q4. Find $\mathcal{L}(f)$ if f(t) equals:

(i) te^{at}

(ii) $\cos^2 \omega t$

(iii) $\sinh^2 t$

Q5. Sketch or graph the given function, which is assumed to be zero outside the given interval. Represent it, using unit step functions. Find its transform. Show the details of your work.

(i) t-3 (t>3)

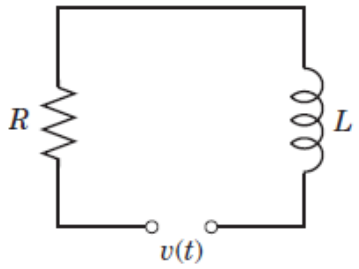
(ii)$2t^2$ (t> 5/2)

(iii) sin t($\pi/2 < t < \pi$)

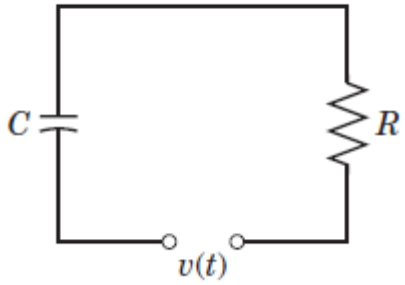
Q6. Find and sketch or graph f(t) if $\mathcal{L}(f)$ equals
(i) $4(1- e^{-\pi s})/(s^2+4)$
(ii) e^{-2s}/s^6
(iii)$(1+ e^{-2\pi(s+1)})(s+1) / ((s+1)^2+1)$

Q7. Using the Laplace transform and showing the details, solve
(i) $y''+ y'- 2y= 3\sin t - \cos t$, $(0< t< 2\pi)$, and $3\sin 2t - \cos 2t$, $(t> 2\pi)$; $y(0)=0, y'(0)= -1$
(ii)**Shifted data**. $y'' + 4y = 8t^2$ if $0< t < 5$ and 0 if $t > 5$; $y(1)=1+\cos 2$, $y'(1)= 4 - 2\sin 2$

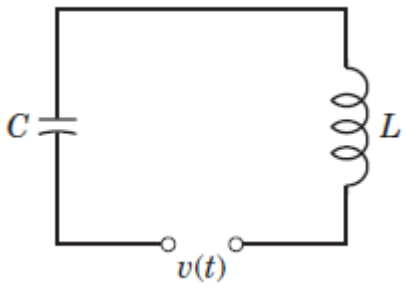
Q8. Using the Laplace transform and showing the details, find the current i(t) in the circuit in Fig. , assuming i(0)=0 And R =25 Ω, L = 0.1 H, v = 490 e^{-5t} V if $0 < t < 1$ and 0 if $t > 1$.



Q9. Using the Laplace transform and showing the details, find the current $i(t)$ in the circuit in Fig. with R= 10Ω and $C=10^{-2}$F, where the current at t = 0 is assumed to be zero, and v=0 if $t < 2$ and 100(t-2)V if $t > 2$



Q10. Using the Laplace transform and showing the details, find the current i(t) in the circuit in Fig. , assuming zero initial current and charge on the capacitor and
(i) L=1 H, $C=10^{-2}$ F, v= -9900cos t V if $\pi < 1< 3\pi$ and 0 other wise
(ii) L = 0.5 H, C= 0.05F,v = 78 sin t V if $0< t < \pi$ and 0 if $t > \pi$



Q11. Find and graph or sketch.Show the details
(i) $y'' + 9y = \delta(t - \pi/2)$, $y(0)=2$, $y'(0)=0$
(ii) $4y'' + 16y' + 17y = 3e^{-t} + \delta(t - 1/4)$, $y(0)= 3/5$, $y'(0)= -3/5$
(iii) $y'' + 3y' + 2y = u(t-1) + \delta(t-2)$, $y(0)=0$, $y'(0)=1$

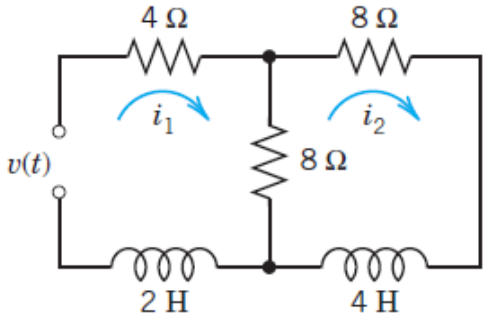
Q12. Showing details, find f(t) if $\mathcal{L}(f)$ equals:
(i) $2\pi s / (s^2 + \pi^2)^2$
(ii) $\omega / s^2(s^2 - \omega^2)$
(iii) $40.5 / s(s^2 - 9)$
(iv) $18s / (s^2 + 36)^2$

Q13. Using differentiation, integration, *s*-shifting, or convolution, and showing the details, find f(t) if $\mathcal{L}(f)$ equals:
(i) $s / (s^2 - 4)^2$
(ii) $\ln(s^2 + 1 / (s - 1)^2)$

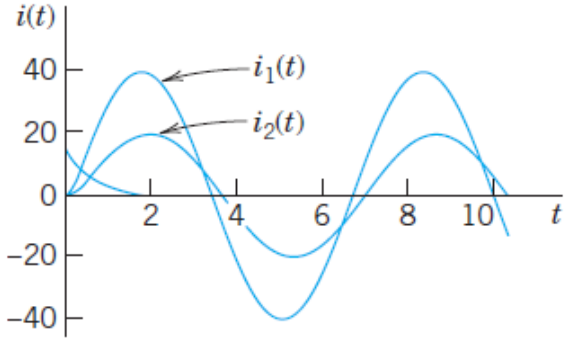
Q14. Using Laplace transform and showing the details of your work, solve :
(i) $y_1' - 2y_1 + 3y_2 = 0$, $y_2' - y_1 + 2y_2 = 0$, $y_1(0) = 1$, $y_2(0)=0$
(ii) $y_1' = 2y_1 - 4y_2 + u(t-1)e^t$, $y_2' = y_1 - 3y_2 + u(t-1)e^t$, $y_1(0) = 3$, $y_2(0)=0$
(iii) $y_1'' = y_1 + 3y_2$, $y_2'' = 4y_1 - 4e^t$, $y_1(0) = 2$, $y_1'(0) = 3$, $y_2(0)=1$, $y_2'(0)=2$
(iv) $-y_1' + y_2' = 2\cosh t$, $y_2' - y_3' = e^{-t}$, $y_3' + y_1' = 2e^{-t}$, $y_1(0) = 0$, $y_2(0)=0$, $y_3(0)=1$

Q15. **Mixing problem.** What will happen in Example 1(Chapter 6 , Section 6.7) if you double all flows (in particular, an increase to 12 gal/min containing 12 lb of salt from the outside), leaving the size of the tanks and the initial conditions as before? First guess, then calculate. Can you relate the new solution to the old one?

Q. 16 **Electrical network.** Using Laplace transforms, find the currents $i_1(t)$ and $i_2(t)$ in Fig. , where $v(t) = 390 \cos t$ and $i_1(0)=0$, $i_2(0)=0$. How soon will the currents practically reach their steady state?



Network



Currents