University of Waterloo

Time Series - Forecasting Competition

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Presented to

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1 Scenario 1

The first task of the competition is to forecast the level of a body of water for the next 24 months, given the past 48 years of water level data at a monthly resolution. I started by plotting the series along with its auto-correlation function and Fourier transform powers.

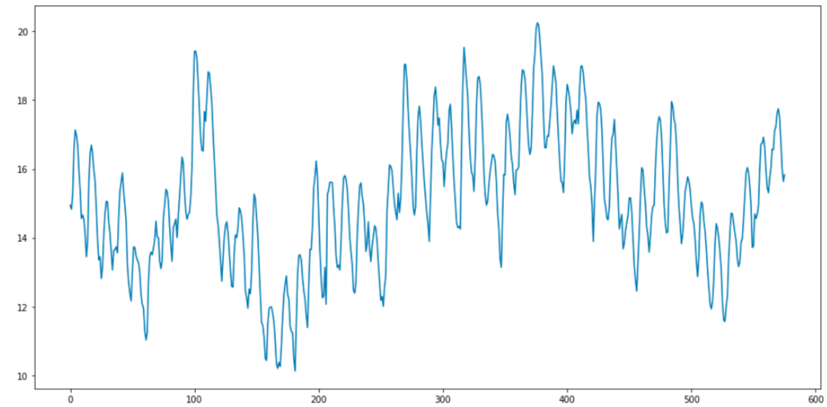

Figure 1: Plot of hydrological data

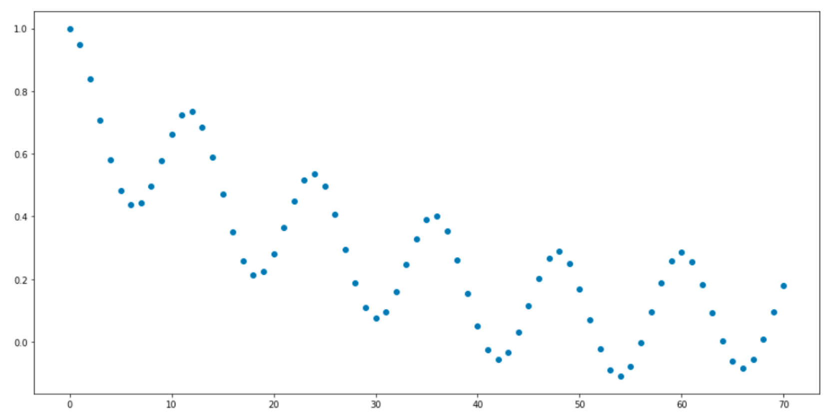

Figure 2: ACF of hydrological series

From the plots and the values of the Fourier coefficients I determined that the series has three main modes of oscillation. They are 12 months, 96 months, and 576 months. These correspond to 1 year, 8 years and the length of the entire sample. To get an idea of how well the first three modes of oscillation approximate the data I plotted the series along with various inverse Fourier transform filters.

The first three harmonics appear to capture the basic shape of the series. I proceeded to fit SARIMAX models using the slower oscillations as exogenous variables.

I fit two SARIMAX models with seasonality 12, one with both the long and medium term oscillations as explanatory variables and one with just the medium term oscillations. Both models failed the Box-ljung test

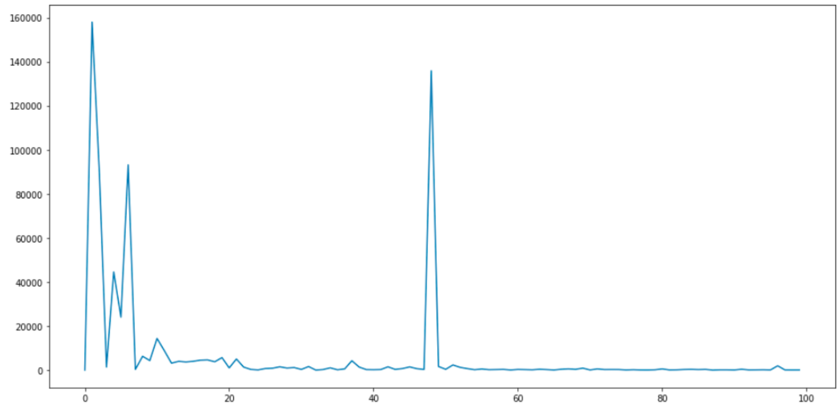

Figure 3: Squared magnitude of Fourier coefficients of hydrological series.

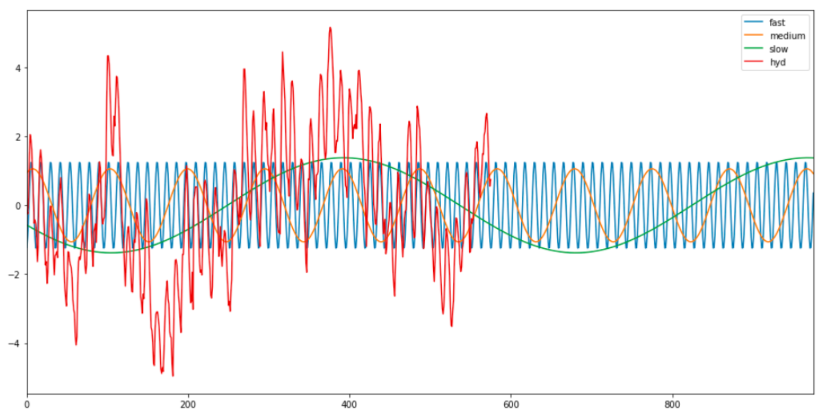

Figure 4: First three modes of oscillation.

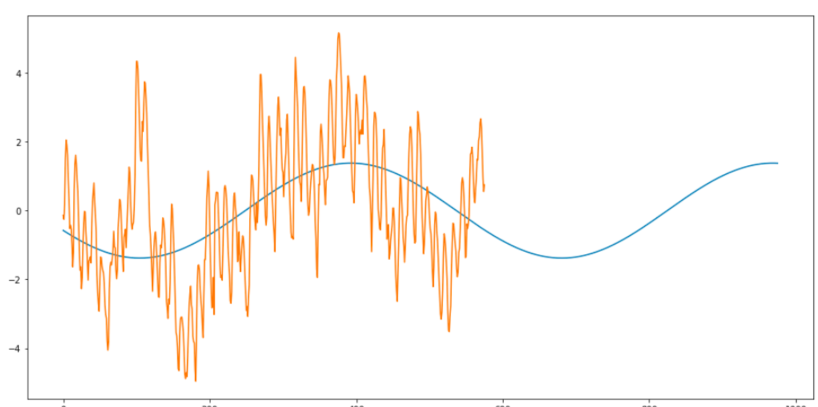

Figure 5: First mode of oscillation.

and the Jarque-Berra test. For my final forecast I chose the model with only the medium term component because the estimation error in the long term component was likely to be quite high and the long-term

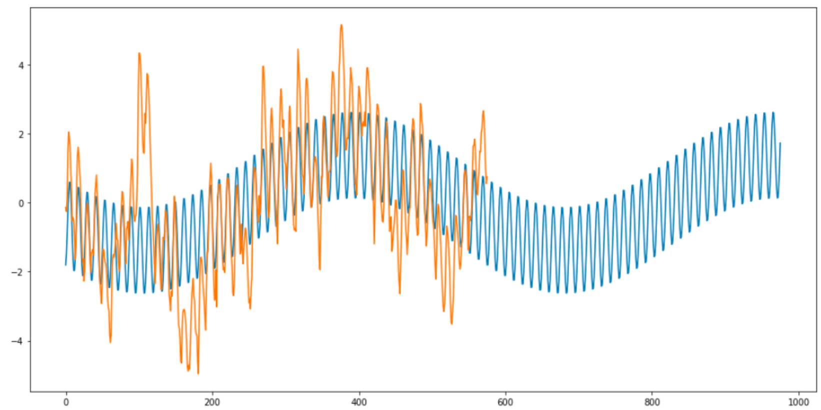

Figure 6: First and second modes of oscillation..

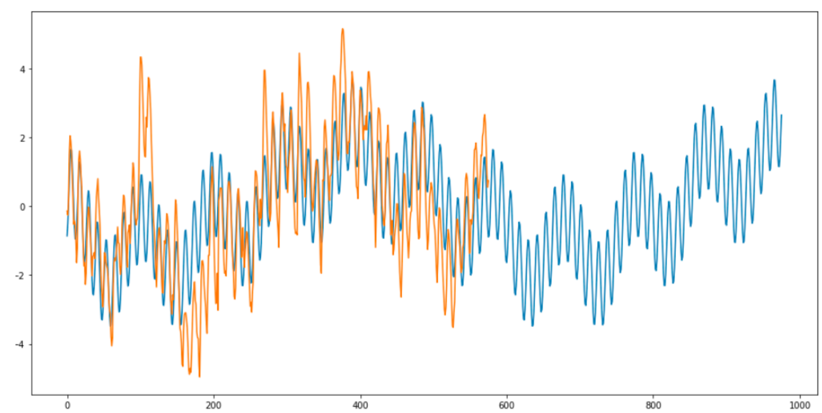

Figure 7: First three modes of oscillation.

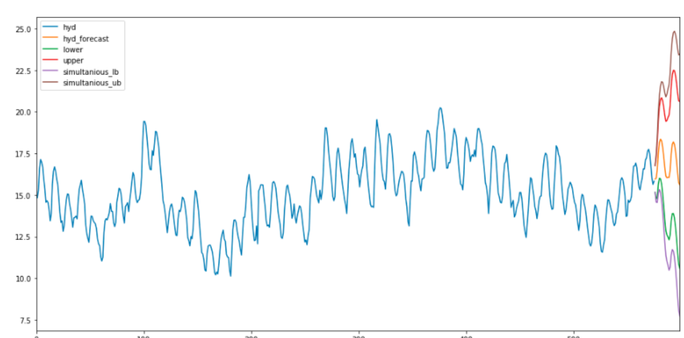

Figure 8: Final forecast with 95% prediction and (simultaneous) confidence bands for the forecasts.

component was very steep during the forcast period. These factors make the long term oscillation likely to throw off my forecast.

2 Scenario 2

The second task of the competition is to forecast 15% lower quantiles (85% confidence VaR) for the log returns of 40 different stocks over the next 10 days given 150 days of sample log returns.

I started my analysis with a visual check for trend and heteroscedasticity. The code in stock.ipynb produces plots of returns, rolling means, and rolling standard deviation. From the rolling means it looks like the mean is constant for all stocks implying the absence of trend. The rolling standard deviation seems to vary for some of the stocks. These checks are formalized later with tests for autocorrelation and ARCH effects. Figure 9 shows the visual check for stock1.

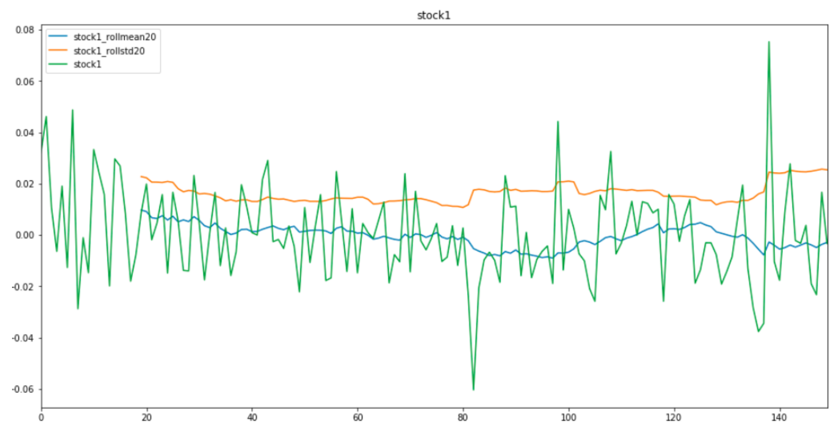

Figure 9: Visual check for trend and heteroscedasticity.

Next, I did a check for autocorrelation and ARCH effects. The code in stock.ipynb produces autocorrelation plots for the returns and squared returns. It also produces the results of the Ljung-Box test for autocorrelation and Engle's Test for ARCH effects. The autocorrelation plots show that autocorrelation is not present and that autocorrelation of squared returns may be present for some stocks. All stocks were found to have no autocorrelation in log returns. ARCH effects were found to be present in stock2, stock27, stock32, stock37, and stock7. Figure 10 shows the acf plots for stock27.

Based on the above results pertaining to trend, autocorrelation and arch effects, it makes sense to model stock2, stock27, stock32, stock37, and stock7 with constant mean GARCH and the rest of the stocks with strong white noise.

For the strong white noise models I compared the fit and forecasts of four different distributions, namely the normal, student-t, non-central student-t and skewed normal distributions. The choice of distribution did not have a significant impact on the 15% lower quantile forecast. Nonetheless, I settled on the non-central student-t distribution because it best described the fat left tail of the log returns. Figure 11 shows fit diagnostics for stock10.

For the stocks exhibiting ARCH effects I fitted GARCH(1,1) models. The fits found all parameters to

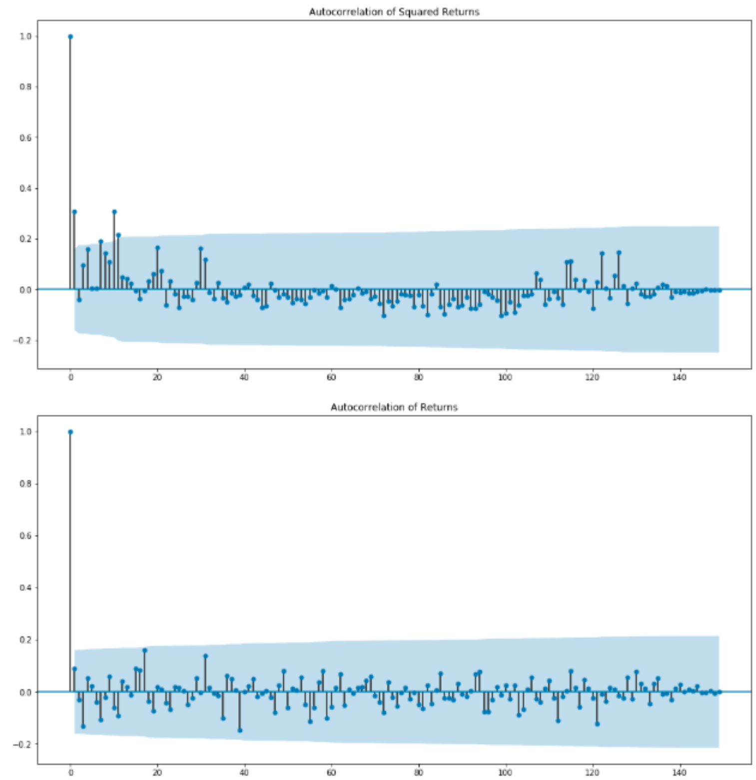

Figure 10: ACf plots for stock27.

be significant and passed standard mis-specification diagnostics such as the "Weighted Ljung-Box Test on Standardized Residuals", "Weighted Ljung-Box Test on Standardized Squared Residuals", and "Weighted ARCH LM Tests".

3 Scenario 3

The third task of the competition is to impute 30 months of Australian beer production data (Sept 1972 to Feb 1975) given beer production data surrounding the imputation period and several years of uninterrupted data on Australian car production, steel production, temperature, gas consumption, and electricity consumption.

I start my analysis by plotting the normalized series on the same plot (Figure 12) to get an idea for what the trend, seasonality, and correlation of the various series look like. I then go on to plot scatter plots coloured according to time shown in figure 13. This gives a sense for how the relationships between beer and the exogenous variables are evolving over time.

From the plots it looks like all of the series have strong annual seasonality. We can also see that gas and electricity production feature strong increasing trends throughout. Beer has an increasing trend leading up to the imputation period but levels off shortly after.

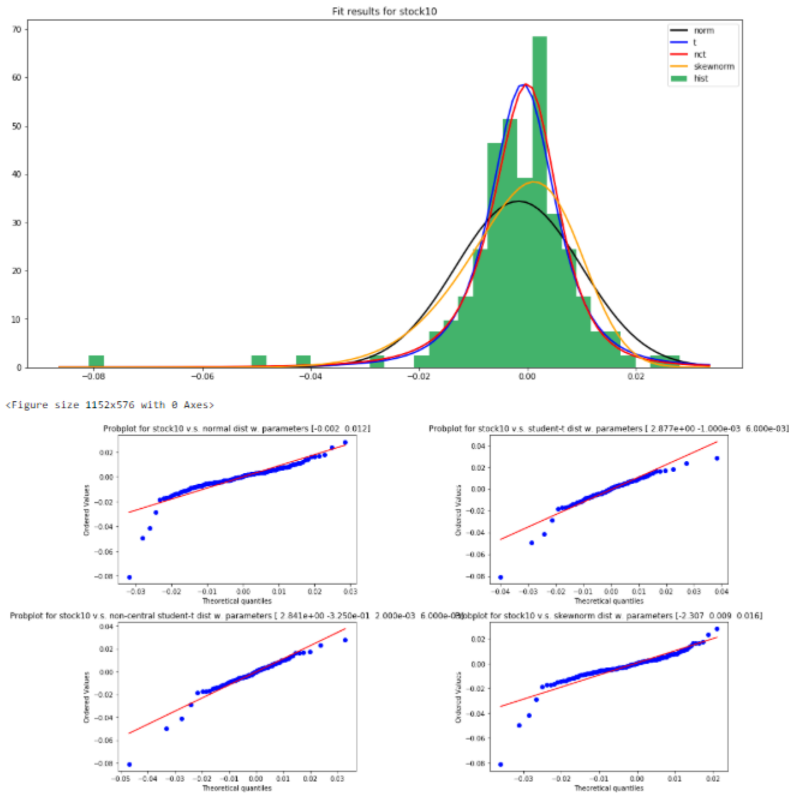

Figure 11: Strong white noise model fit comparison for Stock10.

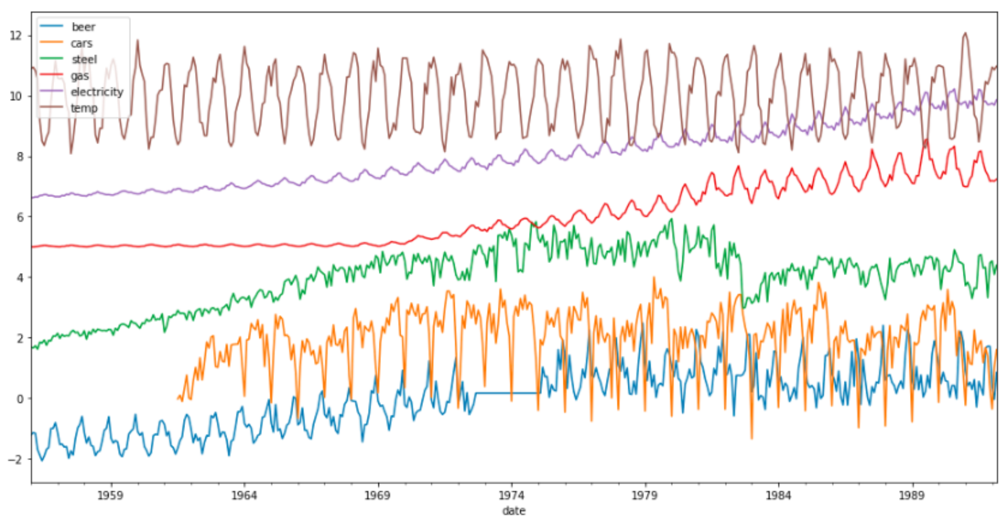

Figure 12: Scenario 3 Variables normalized and shifted for visual effect.

Given the time varying characteristics of the series I decided to impute the data using SARIMAX models fitted to data close to the imputation period. I fitted a SARIMAX model to the data from 1962 to the start of the imputation period. I then fitted a backwards SARIMAX model to the data from the end of the

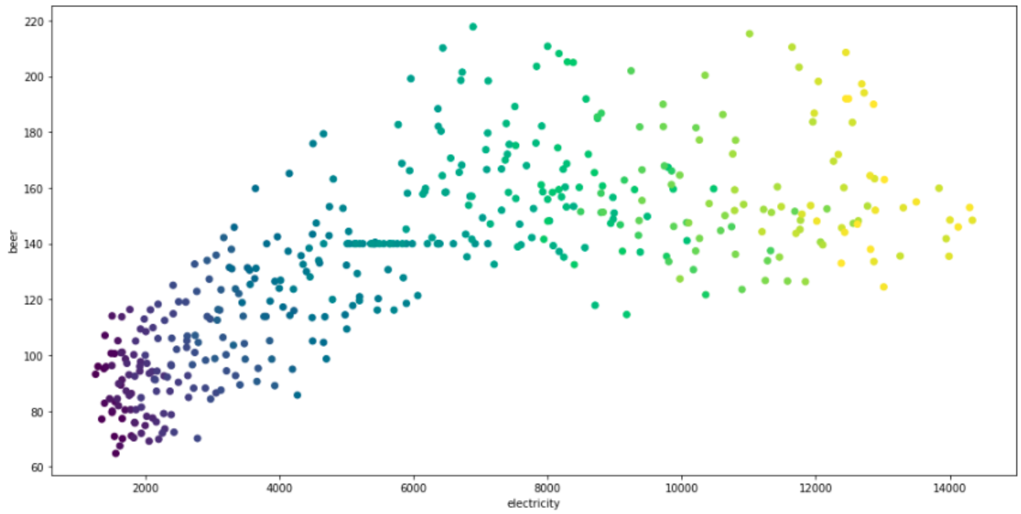

Figure 13: Electricity consumption versus beer production.

imputation period to 1985. I forecasted both models over the imputation period and took their average for the final imputation. The imputation result is show in Figure 14. The result is visually consistent with the neighboring beer production data.

I used auto_arima from python's pmdarima package to select and fit the SARIMAX models. The forward model was determined to be SARIMAX(1, 0, 0, 12) and the backward model was determined to be SARIMAX(1, 0, 0)x(1, 0, 2, 12). The backward model found gas, steel and electricity not to be significant so I dropped those regressors from it. Both fits passed the Jarque-Bera test for normality of residuals. The backwards model passed the Ljung-Box test for uncorrelated residuals, but the forward model did not.

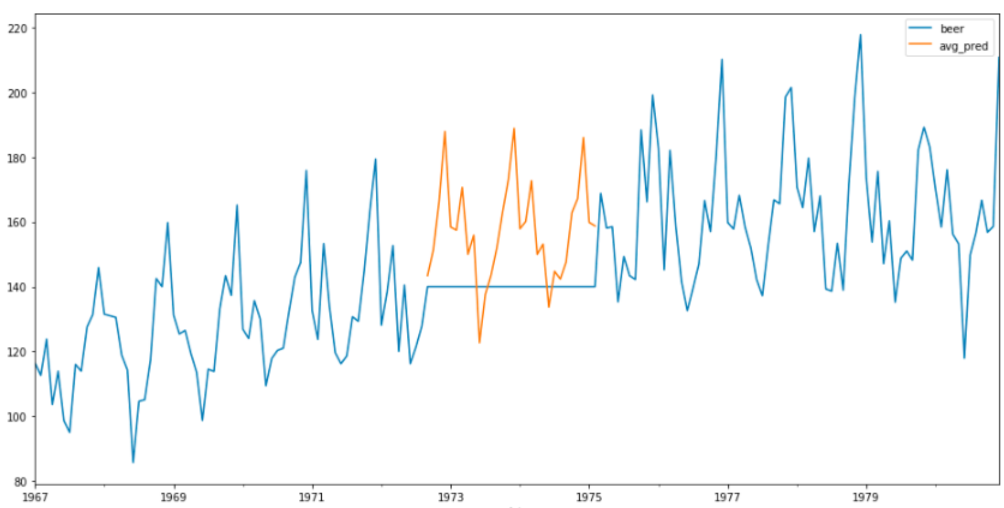

Figure 14: Beer production imputation results.

4 Scenario 4

The fourth and final task of the competition is to forecast the Australian beer production series 24 months into the future. The results on the structure of the time series in Scenario 3 motivate the use of a Vector auto-regression with an exogenous seasonal component.

Due to the non stationary nature of the series, only the period from 1980 and onward was used to fit the forecasting models. A trend was also incorporated into the vector auto-regression. The exogenous seasonal component is extracted in a manner similar to Scenario 1. The first five modes of oscillation approximate the series well. This is shown in Figure 15. I used VARMAX from python's statsmodels package to select and fit the VARMAX model. The final model was a VARMAX(1, 1)

The forecasts for the vector auto regression are not visually consistent with the series. For example the peaks of the forecasted beer production levels are not as high as the peaks leading up to the forecast period. I decided to compare these results to a simple SARIMA model with seasonality 12. Figure 17 shows the forecast of a SARIMA(1,0,0,12) model with 95% confidence prediction bands. Both models discussed in this scenario passed the Jarque-Berra test but not the Box-Ljung test. The simpler model produced results closer to what would be expected. The superiority of univariate methods in this context makes sense. The lag of the VARMAX model is one, thus only the one step beer forecast is being directly informed by the sample data of the other variables. Forecasts of beer production greater than one step ahead depend on forecasted values of the other variables. This leads to a situation where forecast errors compound. This makes sense as it should be more difficult to forecast several series simultaneously instead of just one.

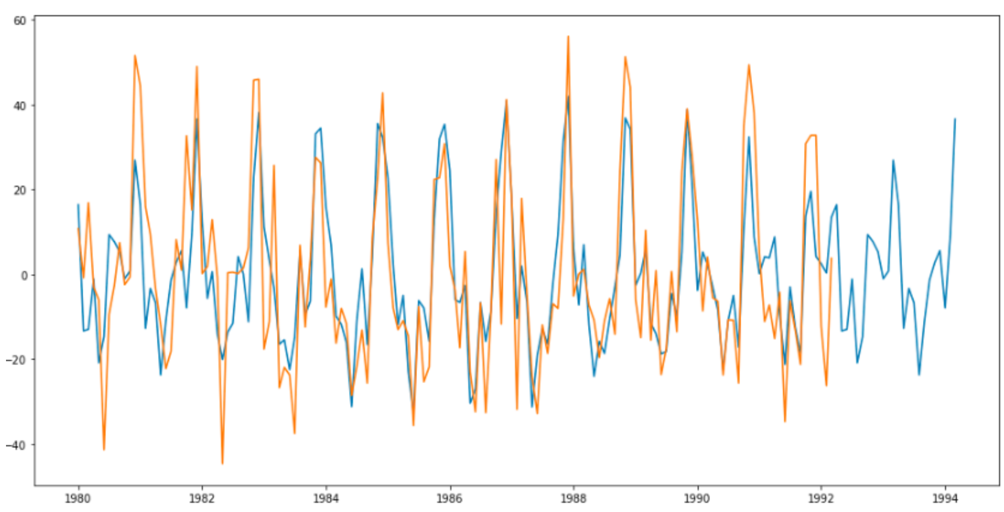

Figure 15: Seasonal Component for Vector auto-regression.

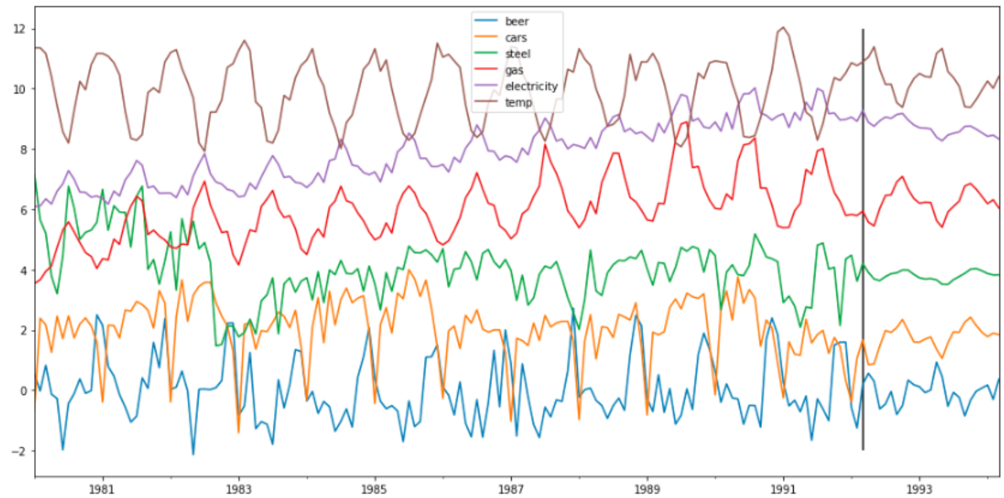

Figure 16: Vector auto-regression forecast. The black line indicates the start of the forecast period.

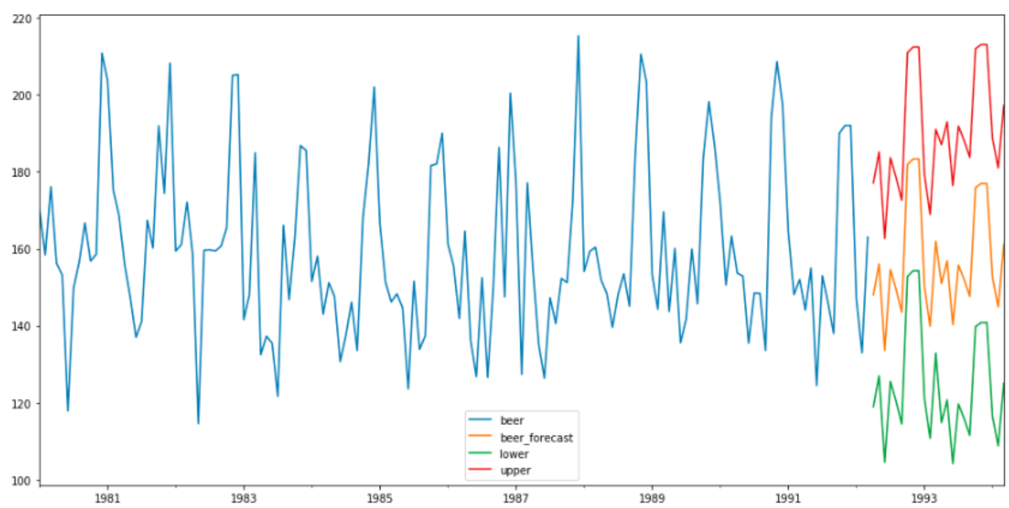

Figure 17: SARIMA model forecast with prediciton bands.