Calculus II

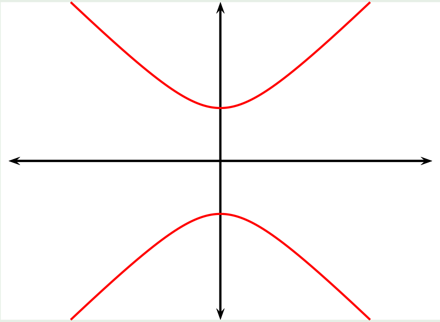
Definite integrals of the form $\int_p^q \sqrt{ax^2+c}\mathrm{d}x$, $a, c > 0$

Todor Milev

2019

Example

Find the area locked b-n the hyperbolas $y = \pm\sqrt{x^2+1}$ and $x = \pm 2\sqrt{2}$.



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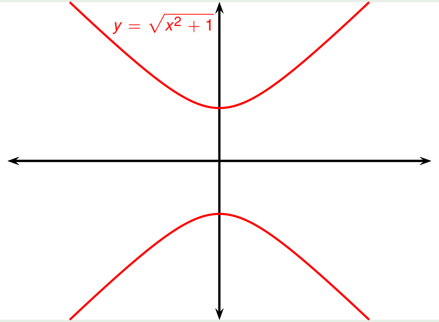

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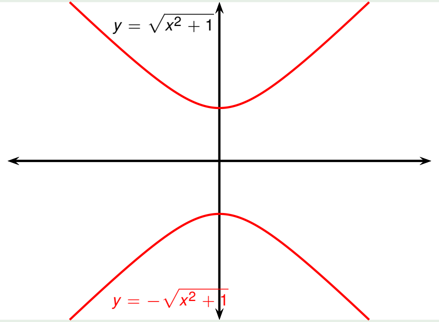

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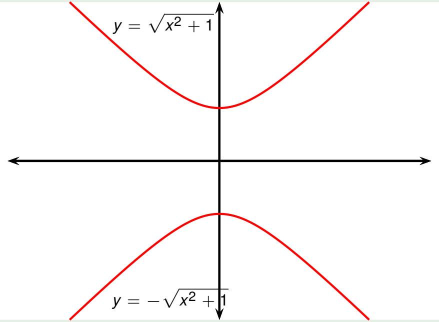

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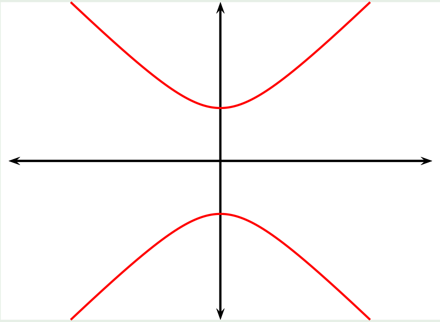

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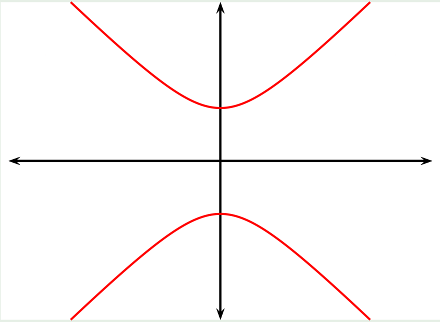

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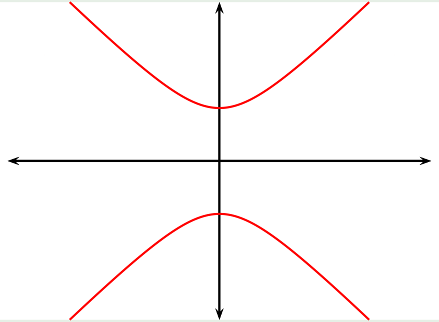

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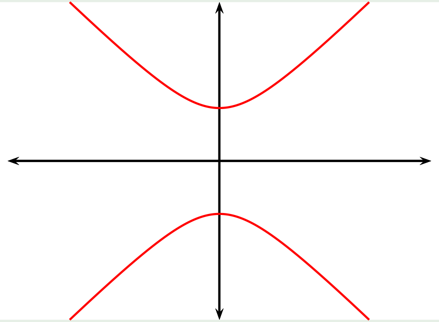

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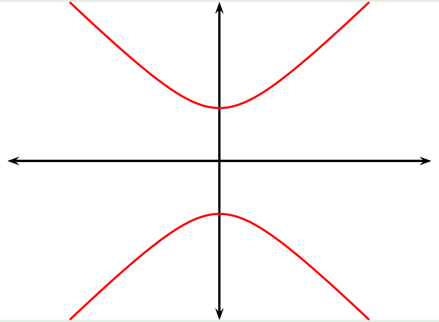

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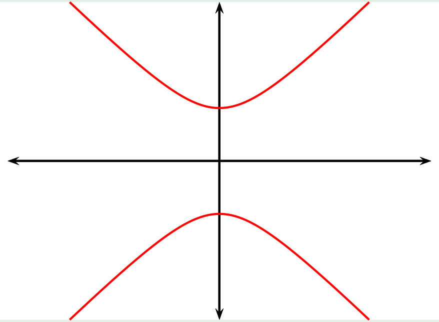

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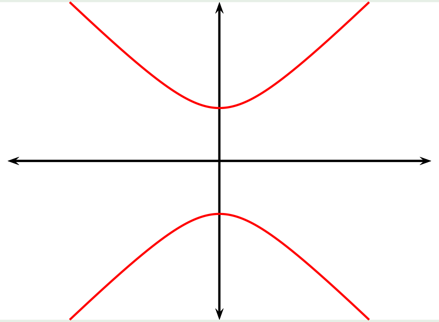

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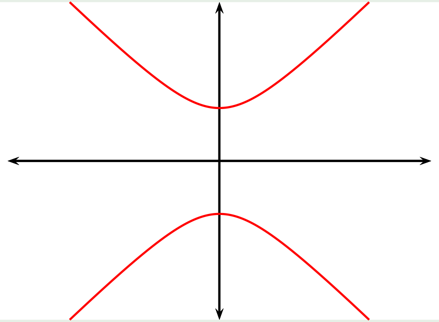

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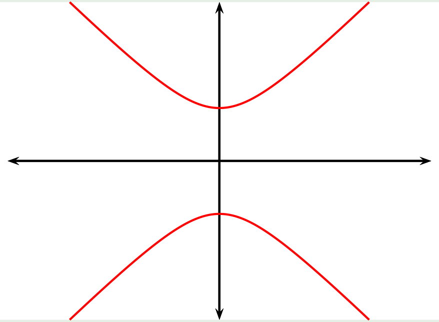

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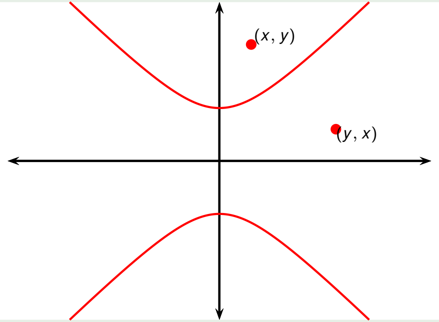

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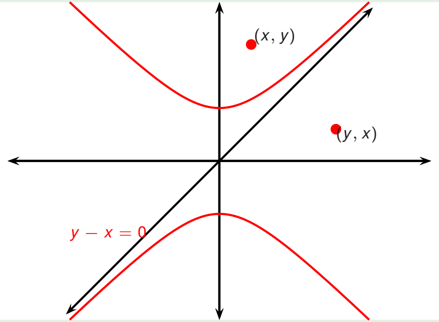

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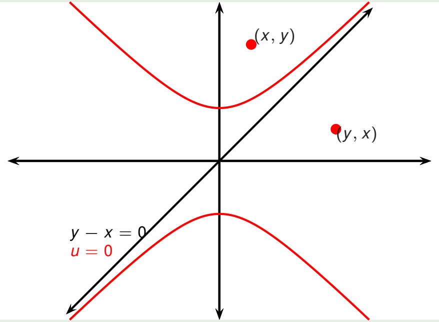

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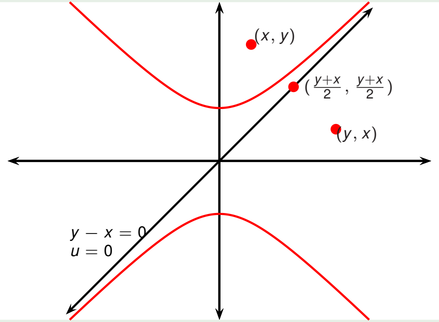

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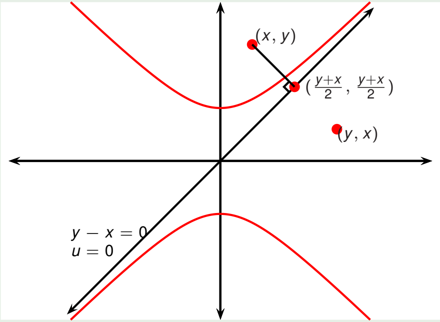

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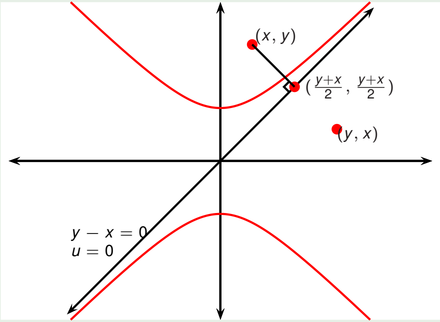

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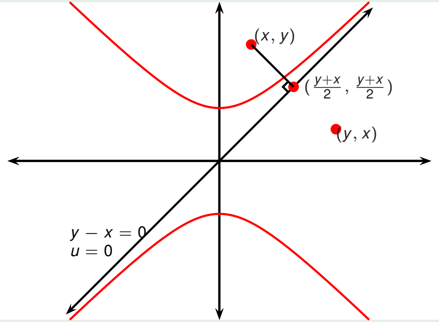

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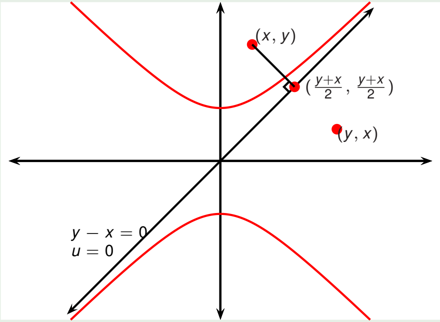

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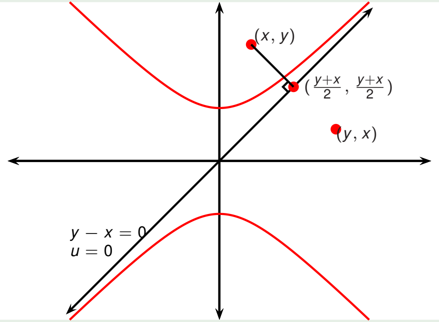

Signed distance b-n (x, y) and line $u = 0$ equals

$$\pm\sqrt{\left(x - \frac{(x+y)}{2}\right)^2 + \left(y - \frac{(x+y)}{2}\right)^2}$$
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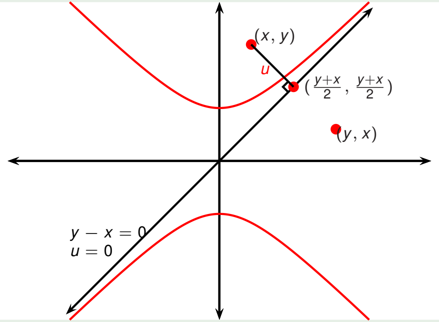

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$$\pm\sqrt{\left(x - \frac{(x+y)}{2}\right)^2 + \left(y - \frac{(x+y)}{2}\right)^2} = \pm\sqrt{\frac{1}{2}(y-x)^2} = \frac{\sqrt{2}}{2}(y-x) = u.$$

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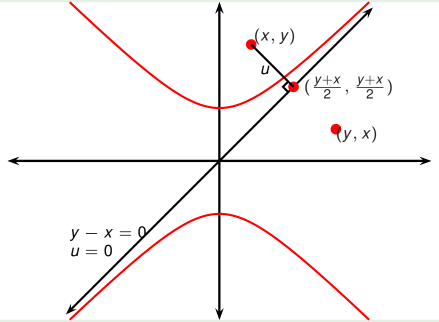

Signed distance b-n (x, y) and line $u = 0$ equals u.

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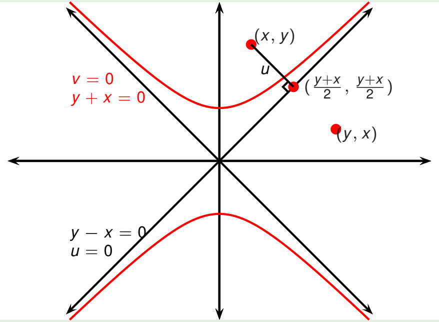

Signed distance b-n (x, y) and line $u = 0$ equals u. Similarly compute that signed distance b-n (x, y) and the line $v = 0$ equals v.

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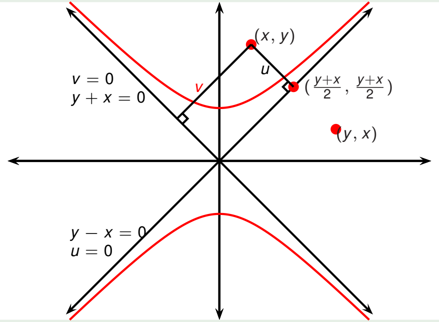

Signed distance b-n (x, y) and line $u = 0$ equals u. Similarly compute that signed distance b-n (x, y) and the line $v = 0$ equals v.

We studied $v = \frac{\frac{1}{2}}{u}$ is called a hyperbola: why do we call $y = \sqrt{x^2+1}$ hyperbola? Compute:

$$\begin{array}{rcl} \sqrt{x^2+1} &=& y \\ x^2+1 &=& y^2 \\ y^2 - x^2 &=& 1 \\ \frac{\sqrt{2}}{2}(y-x)\frac{\sqrt{2}}{2}(y+x) &=& \frac{1}{2} \\ uv &=& \frac{1}{2} \\ v &=& \frac{\frac{1}{2}}{u}, \end{array}$$

where $\left| \begin{array}{l} u = \frac{\sqrt{2}}{2}(y-x) \\ v = \frac{\sqrt{2}}{2}(y+x) \end{array} \right.$. Consider an arbitrary point (x, y).

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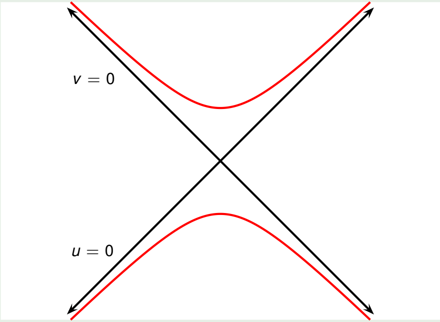

Signed distance b-n (x, y) and line $u = 0$ equals u. Similarly compute that signed distance b-n (x, y) and the line $v = 0$ equals v. $\Rightarrow y^2 - x^2 = 1$ is the hyperbola $v = \frac{1/2}{u}$ in the (u, v)-plane.

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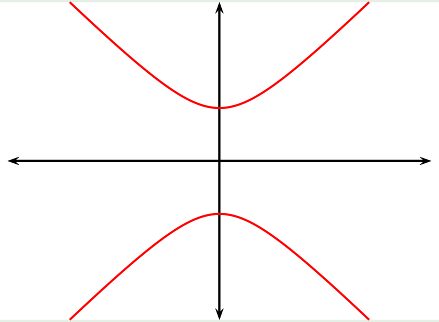

The area in question is:

$$\int_{?}^{?} 2\sqrt{x^2+1}\mathrm{d}x$$

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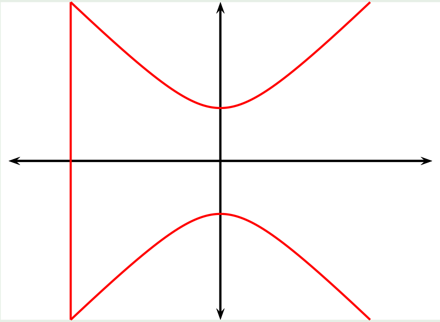

The area in question is:

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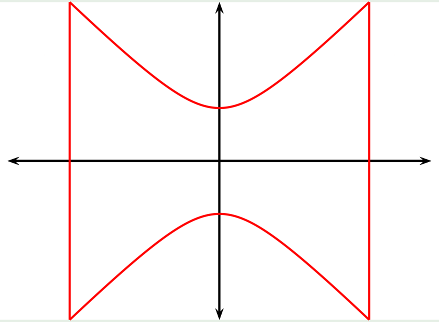

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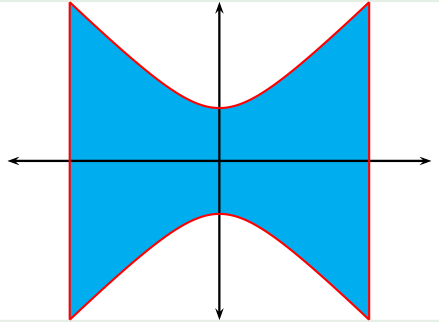

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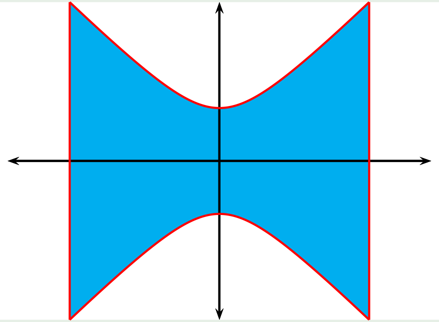

The area in question is:

$$\int_{-2\sqrt{2}}^{2\sqrt{2}} 2\sqrt{x^2+1}\,dx$$

$$= \left[x\sqrt{x^2+1} + \ln\left(\sqrt{x^2+1}+x\right)\right]_{-2\sqrt{2}}^{2\sqrt{2}}$$

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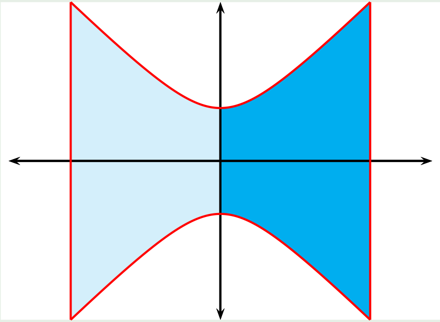

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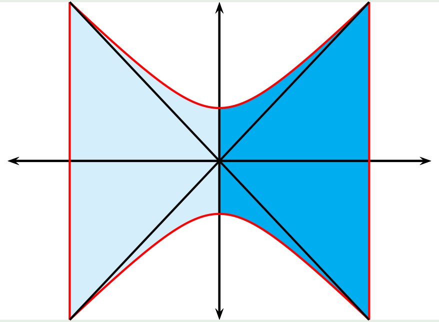

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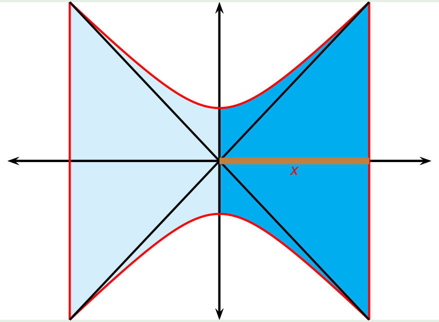

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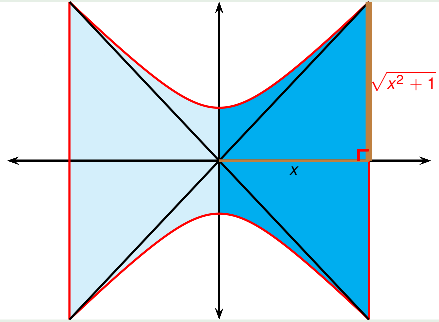

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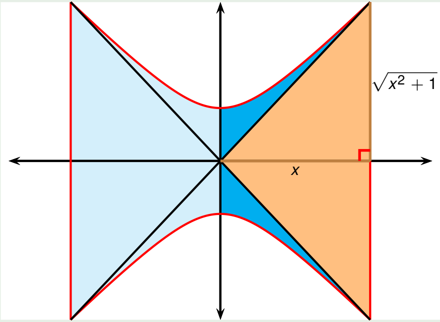

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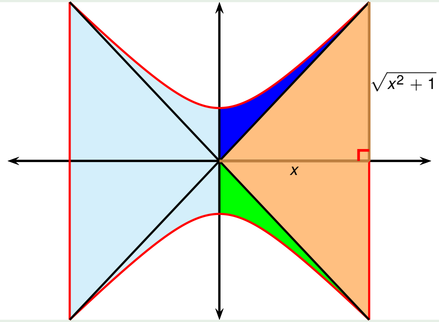

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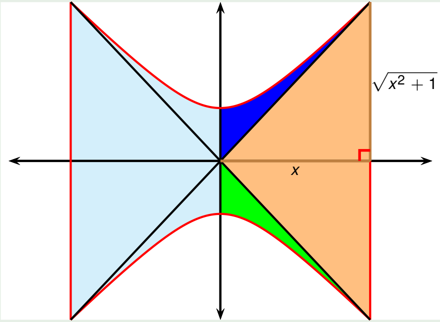

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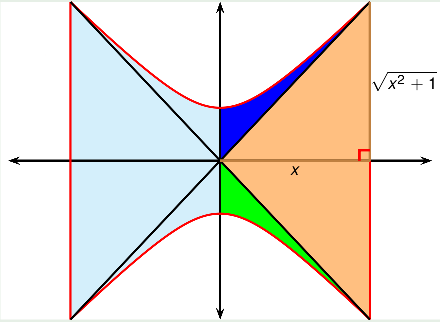

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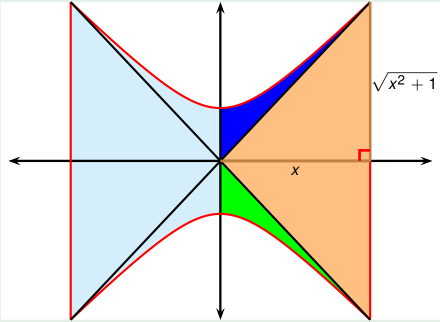

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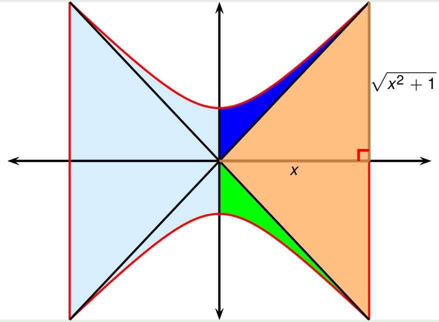

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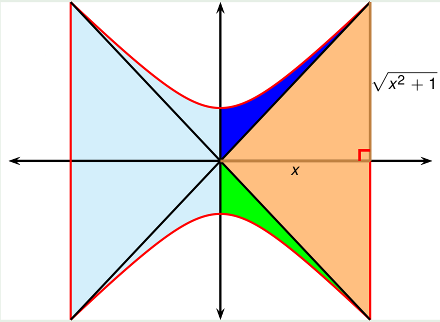

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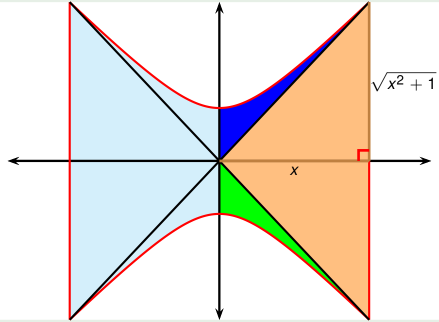

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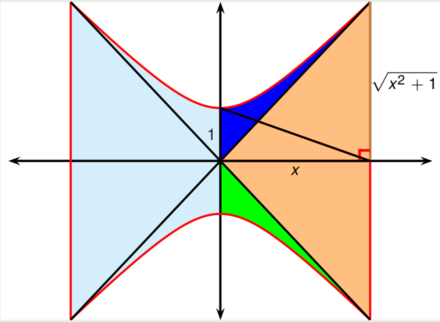

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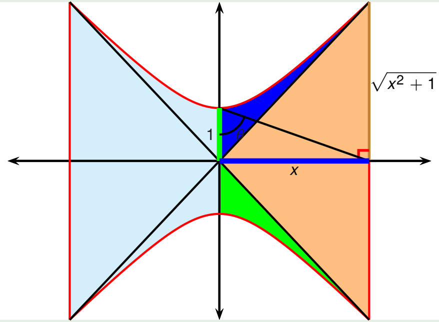

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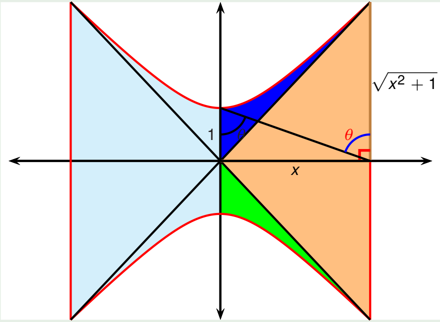

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