Calculus I
Homework
Antiderivatives, indefinite integrals and the Evaluation Theorem

1. Find all antiderivatives of the functions.

   (a) $f(x) = \sqrt{3} + \pi^2$.
   (b) $f(x) = x - 5$.
   (c) $f(x) = x^2 - 2x + 6$.
   (d) $f(x) = \dfrac{x(x+1)}{2}$.
   (e) $f(x) = x(x+1)(2x+1)$.
   (f) $f(x) = 7x^{\frac{2}{7}} + x^{-\frac{4}{7}}$.
   (g) $f(x) = x^{2.4} - 2x^{\sqrt{3}-1}$.
   (h) $f(x) = \dfrac{8}{x^7}$.
   (i) $f(x) = \dfrac{x+1}{x^3}$.
   (j) $f(x) = \dfrac{1}{x}$.
   (k) $f(x) = \dfrac{x^2+1}{x}$.
   (l) $f(x) = \dfrac{5 - 4x^3 + 2x^6}{x^4}$.
   (m) $g(x) = \dfrac{1 + \sqrt{x} + x}{\sqrt{x^3}}$.
   (n) $f(t) = 3\sin t - 4\cos t$.
   (o) $f(\theta) = \sec^2\theta$.
   (p) $f(\theta) = \csc^2\theta$.
   (q) $f(t) = \sec t \tan t + \csc t \cot t$.
   (r) $f(x) = \dfrac{2 + x\cos x}{x}$.

2. (a) Find $f(x)$ if $f'(x) = 3 + \frac{1}{x}$ and $f(1) = 2$.
   (b) Find $f(x)$ if $f'(x) = x - \sin x$ and $f(0) = 7$.

3. Verify by differentiation that the formula is correct.

   (a) $\displaystyle\int \sqrt{1+x^2}\mathrm{d}x = \frac{1}{2}\left(x\sqrt{1+x^2} + \ln\left(x + \sqrt{1+x^2}\right) + C\right)$.
   (b) $\displaystyle\int \sin^2 x \mathrm{d}x = -\frac{1}{4}\sin(2x) + \frac{1}{2}x + C$.
   (c) $\displaystyle\int \sin^3 x \mathrm{d}x = \frac{1}{3}\cos^3 x - \cos x + C$.
   (d) $\displaystyle\int \frac{x}{\sqrt{1+x}}\mathrm{d}x = \frac{2}{3}(x-2)\sqrt{1+x} + C$

4. Evaluate the integral (definite or indefinite).

   (a) $\displaystyle\int_{-2}^{3} \left(x^2 - 1\right)\mathrm{d}x$.
   (b) $\displaystyle\int_{1}^{2} \left(4x^3 + 3x^2 + 2x + 1\right)\mathrm{d}x$.
   (c) $\displaystyle\int_{0}^{2} (x-1)(x^2+1)\mathrm{d}x$.
   (d) $\displaystyle\int_{-1}^{1} \left(\frac{x(x+1)}{2}\right)^2 \mathrm{d}x$.
   (e) $\displaystyle\int_{0}^{1} (1+x^2)^3 dx$.
   (f) $\displaystyle\int_{1}^{2} \left(\frac{1}{x} - \frac{4}{x^2}\right)\mathrm{d}x$.
   (g) $\displaystyle\int_{1}^{4} \sqrt{x}(1+x)\mathrm{d}x$.
   (h) $\displaystyle\int_{1}^{4} \sqrt{\frac{6}{x}}\mathrm{d}x$.
   (i) $\displaystyle\int_{1}^{4} \frac{\frac{1}{\sqrt{x}} + 1 + x}{\sqrt{x}}\mathrm{d}x$.
   (j) $\displaystyle\int_{1}^{8} \frac{1+x}{\sqrt[3]{x}}\mathrm{d}x$.
   (k) $\displaystyle\int_{1}^{64} \frac{\frac{1}{\sqrt[3]{x}} + \sqrt[3]{x}}{\sqrt{x}}\mathrm{d}x$.
   (l) $\displaystyle\int_{0}^{1} \left(\sqrt[5]{x^6} + \sqrt[6]{x^5}\right)\mathrm{d}x$.
   (m) $\displaystyle\int_{1}^{2} \left(x + \frac{1}{x}\right)^2 \mathrm{d}x$.
   (n) $\displaystyle\int_{1}^{2} \left(x + \frac{1}{x}\right)^3 \mathrm{d}x$.
   (o) $\displaystyle\int_{1}^{2} \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 \mathrm{d}x$.
   (p) $\displaystyle\int_{1}^{2} \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^3 \mathrm{d}x$.
   (q) $\displaystyle\int_{0}^{2} |x-1|\mathrm{d}x$.
   (r) $\displaystyle\int_{0}^{1} \left|x - \frac{1}{2}\right|\mathrm{d}x$.

(s) $\displaystyle\int_{-1}^{1} (x - 3|x|)\mathrm{d}x.$

(t) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc^2\theta \mathrm{d}\theta.$

(u) $\displaystyle\int_{0}^{\frac{\pi}{4}} \frac{1-\cos^2\theta}{\cos^2\theta}\mathrm{d}\theta.$

(v) $\displaystyle\int_{0}^{\frac{\pi}{4}} \frac{\sin^2\theta}{\cos^2\theta}\mathrm{d}\theta.$

(w) $\displaystyle\int_{0}^{\frac{\pi}{4}} \tan^2\theta \mathrm{d}\theta.$

(x) $\displaystyle\int_{0}^{\frac{\pi}{3}} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\mathrm{d}\theta.$

(y) $\displaystyle\int_{0}^{\pi} (\sin\theta - \cos\theta)\mathrm{d}\theta.$

(z) $\displaystyle\int_{0}^{\pi} |\sin x|\mathrm{d}x.$

5. Integrate (definite or indefinite).

(a) $\displaystyle\int_{1}^{8} \frac{t - t^{\frac{1}{3}} + 2}{t^{\frac{4}{3}}}\mathrm{d}t \quad .$

(b) $\displaystyle\int_{1}^{4} \left(x + \sqrt{x}\right)^2 dx \quad .$

(c) $\displaystyle\int \frac{\sqrt[3]{x} - x^{\frac{1}{2}} + 1}{x}\mathrm{d}x.$

(d) $\displaystyle\int \frac{\sqrt[3]{x} - 1}{x}\mathrm{d}x.$