Precalculus Law of cosines theory

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2019

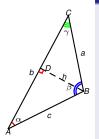
Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ , as indicated.

Proposition (Law of Cosines)

$$c^{2} = a^{2} + b^{2} - 2ab\cos \gamma$$

 $a^{2} = b^{2} + c^{2} - 2bc\cos \alpha$
 $b^{2} = c^{2} + a^{2} - 2ca\cos \beta$

Proof if γ < 90°.



Drop a perpendicular *h* from *B* to *AC*.

$$|CD| = a\cos\gamma$$

$$h = a\sin\gamma$$

$$|AD| = b - |CD| = b - a\cos\gamma$$

$$c^2 = |AD|^2 + h^2$$

$$= (b - a\cos\gamma)^2 + (a\sin\gamma)^2$$

Pyth. thm.
$$\triangle BDA$$

$$=(b - a\cos\gamma)^{2} + (a\sin\gamma)^{2}$$

$$=b^{2} - 2ab\cos\gamma + a^{2}\cos^{2}\gamma + a^{2}\sin^{2}\gamma$$

$$=b^{2} - 2ab\cos\gamma + a^{2}.$$