

# Precalculus

## Cubic inequality

Todor Milev

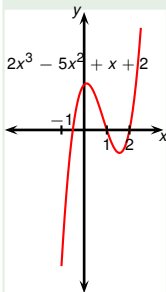
2019

## Example

Plot the function  $2x^3 - 5x^2 + x + 2$ . Solve the inequality.

$$2x^3 - 5x^2 + x + 2 > 0$$

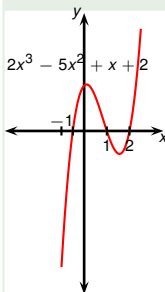
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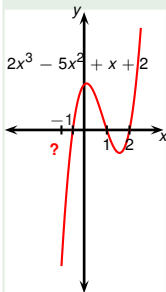


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$$? (x - \quad) (x - \quad) (x - \quad) > 0$$

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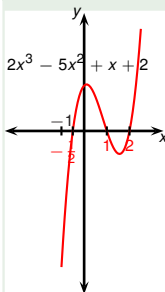


Plot the function  $2x^3 - 5x^2 + x + 2$ . Solve the inequality.

$$2x^3 - 5x^2 + x + 2 > 0$$

$$? (x - ?) (x - ?) (x - ?) > 0$$

## Example

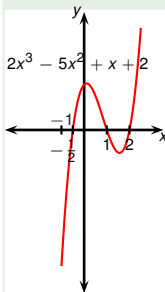


Plot the function  $2x^3 - 5x^2 + x + 2$ . Solve the inequality.

$$2x^3 - 5x^2 + x + 2 > 0$$

$$? (x - (-\frac{1}{2})) (x - 1)(x - 2) > 0$$

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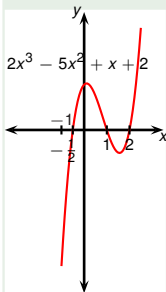


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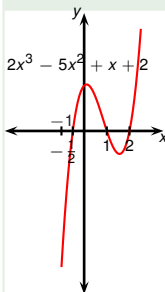
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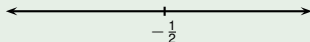


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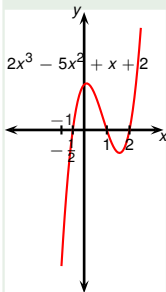
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Left hand side vanishes when  $x = -\frac{1}{2}$ , when  $x = 1$  and when  $x = 2$ .



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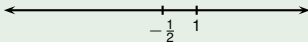


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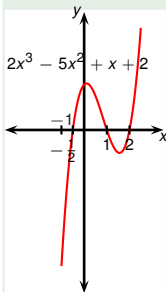
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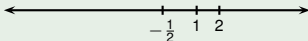


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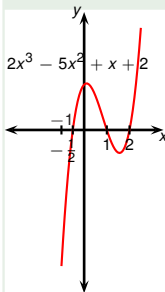
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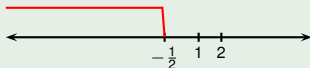


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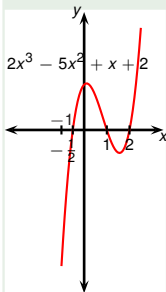
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Interval	Factor signs	Final sign from plot
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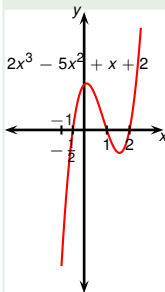
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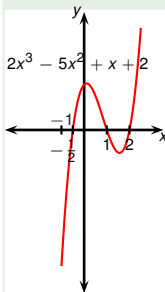
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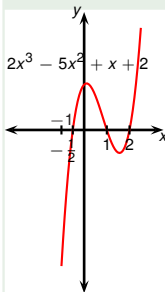
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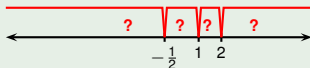


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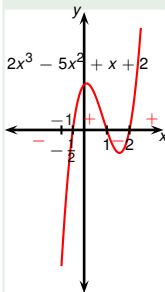
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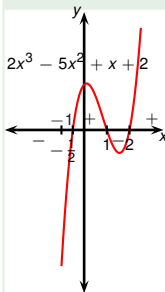
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$(1, 2)$	$(+)(+)(-)$	$-$
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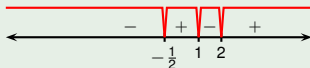
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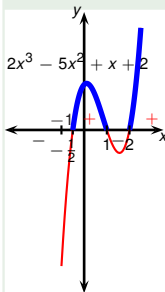
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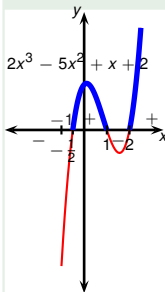
$$x \in (-\frac{1}{2}, 1) \cup (2, \infty)$$

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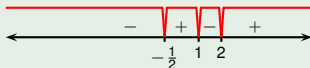
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