Calculus I
Reference: tangents to graphs of functions

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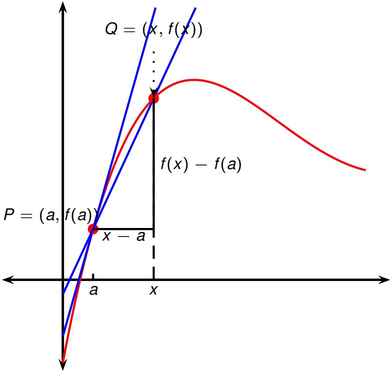


- How to find the tangent line to the curve $y = f(x)$ at $P = (a, f(a))$?
- Consider nearby point $Q = (x, f(x))$.
- Compute slope of secant line PQ: $m_{PQ} = \frac{f(x)-f(a)}{x-a}$.
- As x approaches a, the point Q approaches P.

Definition (Non-vertical tangent line)

Let $P = (a, f(a))$. Suppose the limit $m = \lim\limits_{x \to a} \frac{f(x) - f(a)}{x - a}$ exists. Define the tangent to $y = f(x)$ at P to be the line passing through P with slope m, in other words, the line with equation $y - f(a) = m(x - a)$.

Note. Even if the limit does not exist a reasonable notion of a tangent line may still exist.

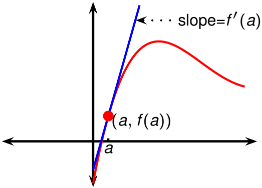
Derivatives

Definition (Derivative)

The derivative of a function f at a number a, denoted by $f'(a)$, is

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

if the limit exists.



- The two alternative formulas result in equivalent definitions.
- Equivalent formulation. The derivative $f'(a)$ is the slope of the tangent line to $y = f(x)$ at $(a, f(a))$, provided that tangent line exists and is non-vertical.