Precalculus
Inverse functions

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2019

Outline

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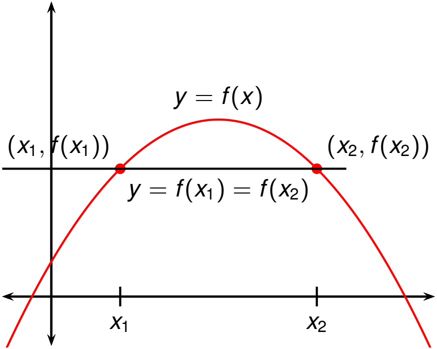
One-to-one Functions

Definition (One-to-one Function)

A function f is a one-to-one function if it never takes on the same value twice; that is,

$$f(x_1) \neq f(x_2) \text{ whenever } x_1 \neq x_2.$$

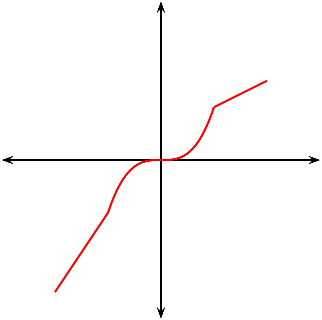


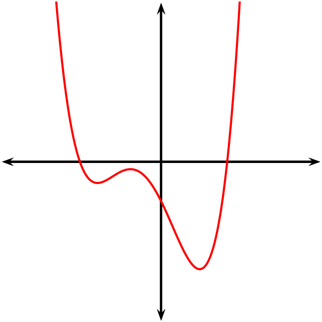
$\leftarrow$ This function is not one-to-one.

Question: How can we tell from the graph of a function whether it is one-to-one or not?
Answer: Use the horizontal line test.

The Horizontal Line Test.

A function is one-to-one if and only if no horizontal line intersects it more than once. □





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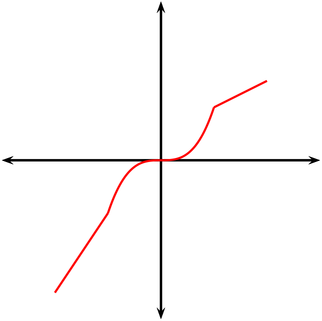

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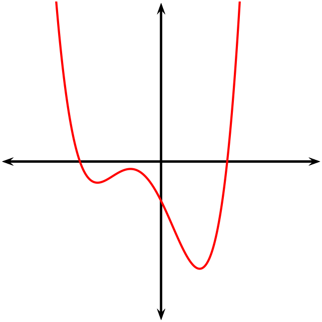

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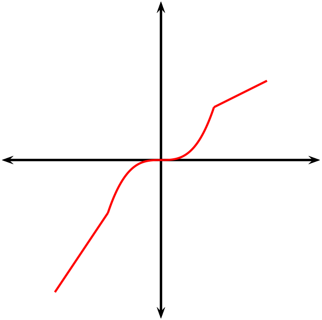

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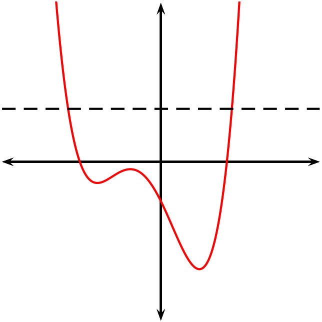

Not one-to-one

The Definition of the Inverse of f

Definition (f^{-1})

Let f be a one-to-one function with domain A and range B. Then the inverse of f is the function f^{-1} that has domain B and range A and is defined by

$$f^{-1}(y) = x \qquad \Leftrightarrow \qquad f(x) = y$$

for all y in B.

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Example ($f(x) = x^3$)

The inverse of $f(x) = x^3$ is $f^{-1}(x) = \sqrt[3]{x}$. This is because if $y = x^3$, then

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No one blamed English language of being logical.

-Bjarne Stroustrup, creator of the programming language C++

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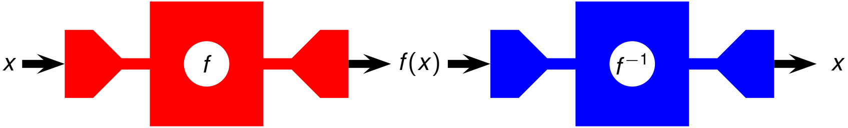

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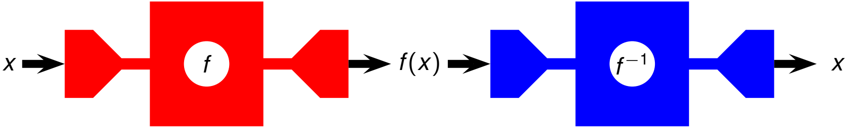

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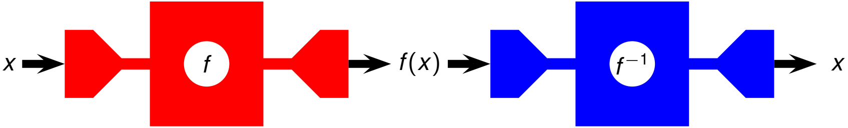

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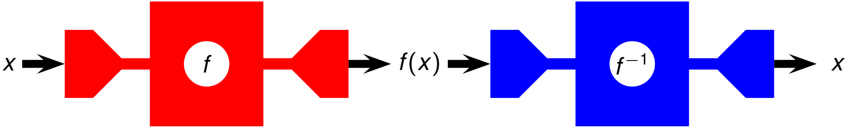

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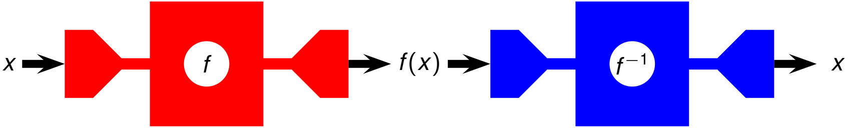

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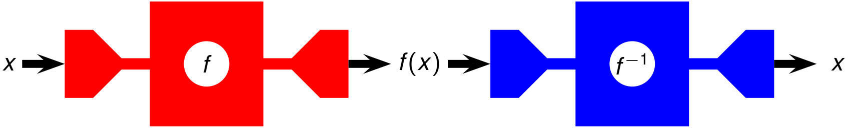

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Therefore $x = f^{-1}(y) = \sqrt[3]{y-2}$. Sometimes we relabel x and y and write $f^{-1}(x) = \sqrt[3]{x-2}$. Whenever in doubt, do not relabel anything.

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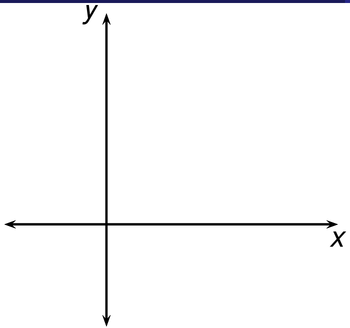
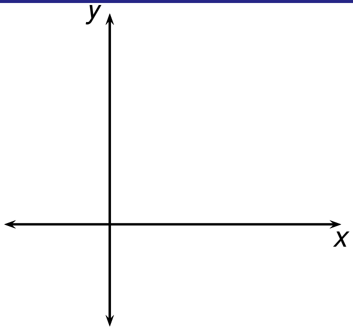
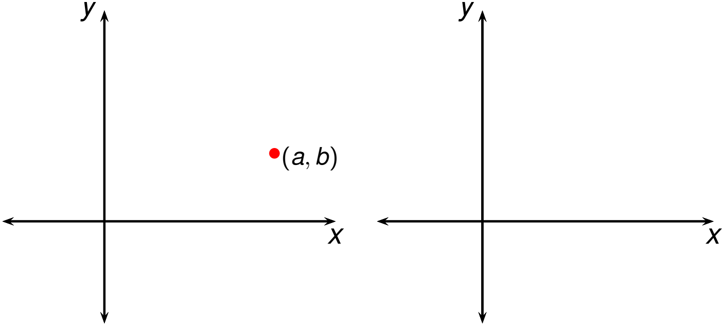




Interchanging x and y suggests relation between the graphs of f^{-1} and f:



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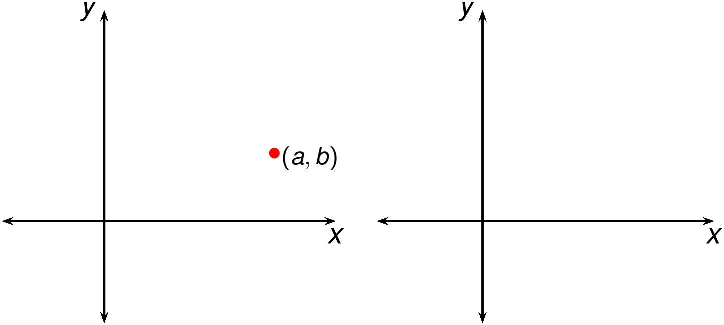

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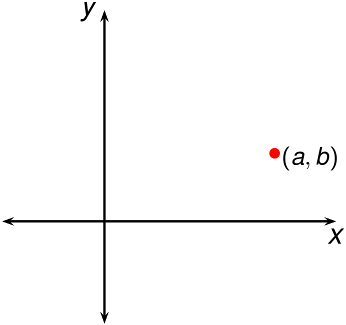

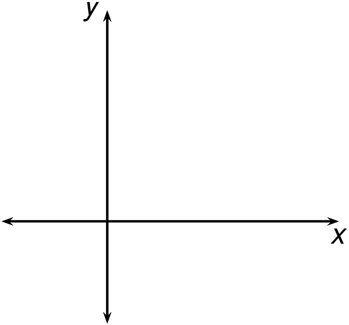

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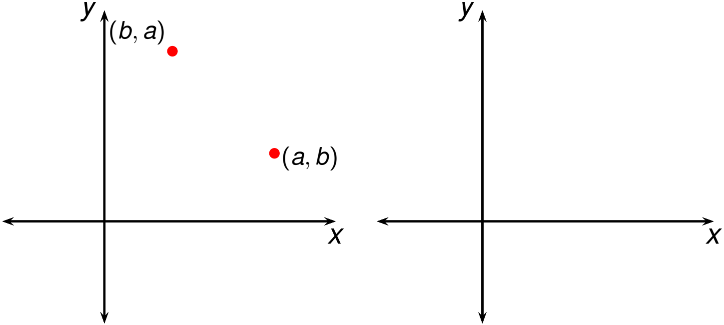

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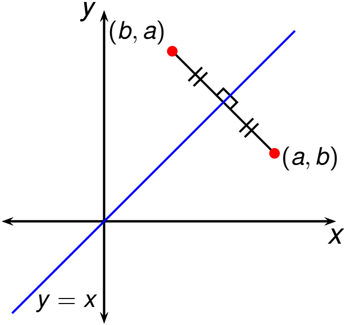

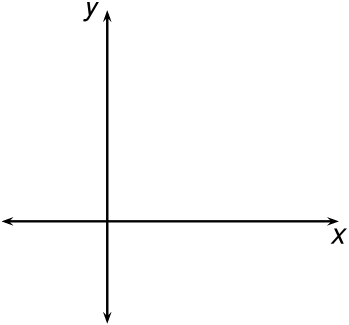

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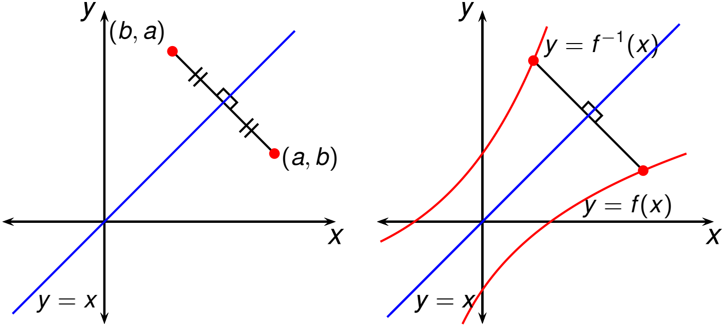
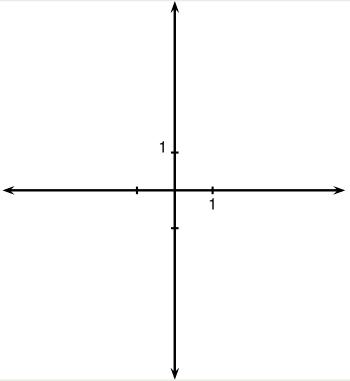
- Then (b, a) is on the graph of f^{-1}.
- (b, a) is the reflection of (a, b) in the line $y = x$.



Interchanging x and y suggests relation between the graphs of f^{-1} and f:

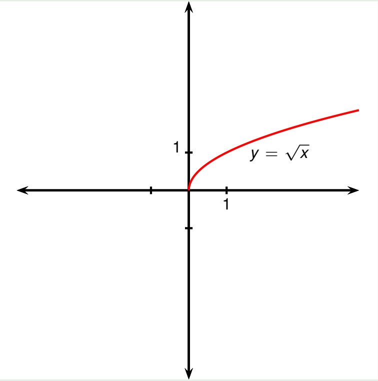
- Suppose (a, b) is on the graph of f.
- Then $f(a) = b$.
- Then $f^{-1}(b) = a$.
- Then (b, a) is on the graph of f^{-1}.
- (b, a) is the reflection of (a, b) in the line $y = x$.
- Thus the graph of f^{-1} is obtained by reflecting the graph of f in the line $y = x$.

Example



Sketch the graph of $f(x) = \sqrt{-x-1}$ and its inverse function.

Example



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- Draw the graph of $y = \sqrt{x}$.

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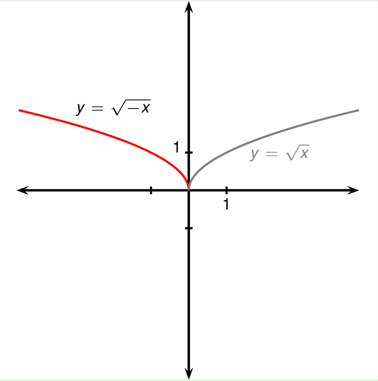
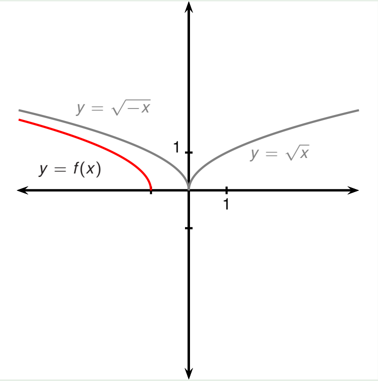


Sketch the graph of $f(x) = \sqrt{-x-1}$ and its inverse function.

- Draw the graph of $y = \sqrt{x}$.
- $y = \sqrt{-x}$ is the reflection of $y = \sqrt{x}$ in the y-axis.

Example



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- $y = f(x) = \sqrt{-(x+1)} = \sqrt{-x-1}$ is the shift of $y = \sqrt{-x}$ one unit to the left.

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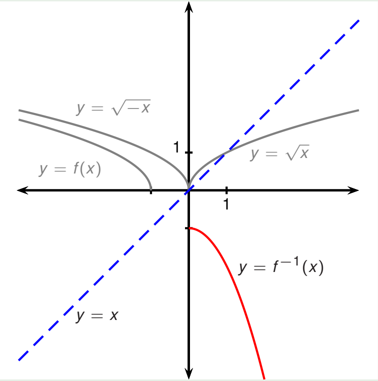

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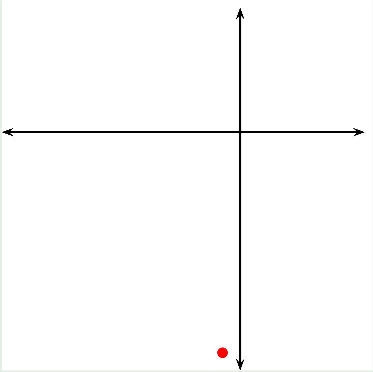
- Draw the graph of $y=\sqrt{x}$.
- $y=\sqrt{-x}$ is the reflection of $y=\sqrt{x}$ in the y-axis.
- $y=f(x)=\sqrt{-(x+1)}=\sqrt{-x-1}$ is the shift of $y=\sqrt{-x}$ one unit to the left.
- $y=f^{-1}(x)$ is the reflection of $y=f(x)$ across the line $y=x$.

Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



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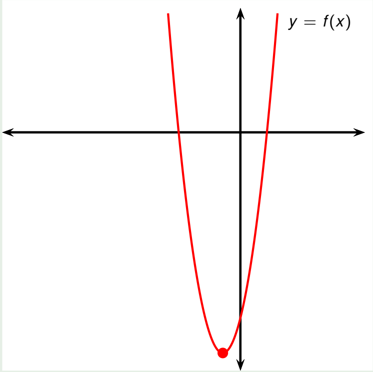

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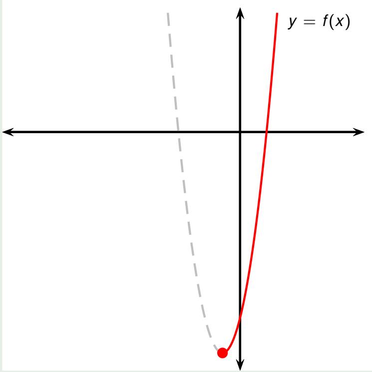

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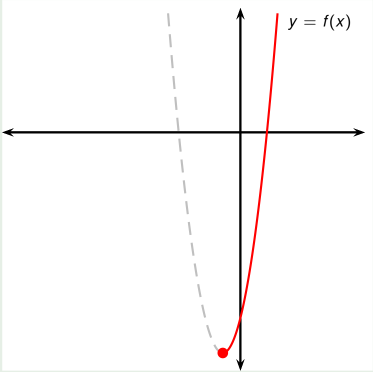

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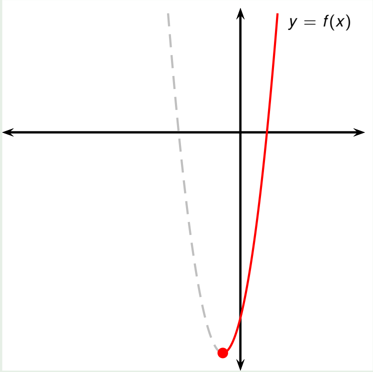

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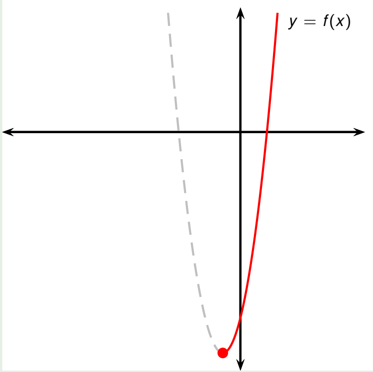

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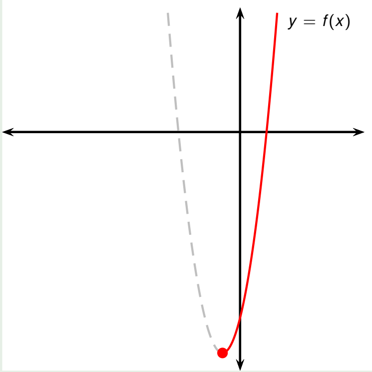

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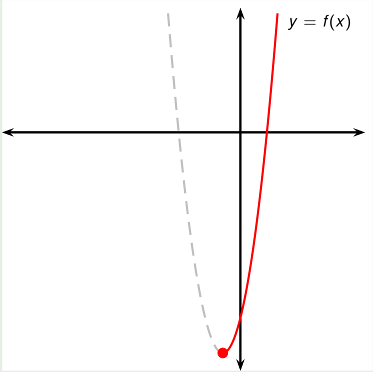

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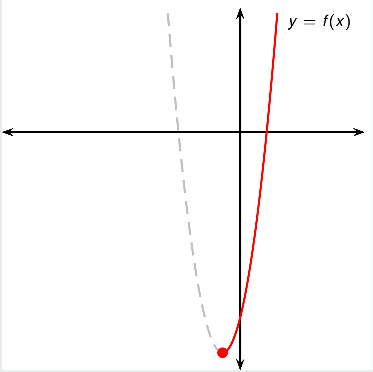

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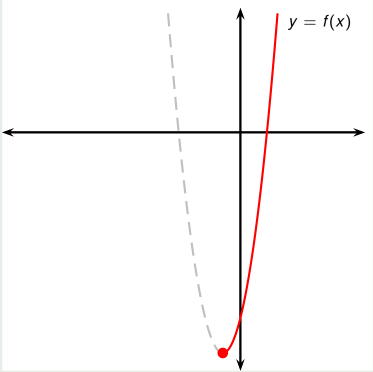

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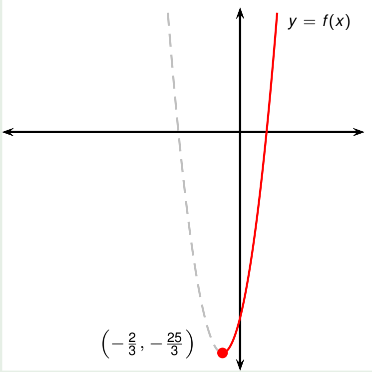

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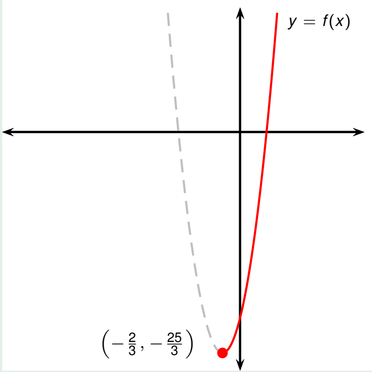

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answer

$$f^{-1}(y) = -\frac{2}{3} + \frac{\sqrt{25 + 3y}}{3}$$

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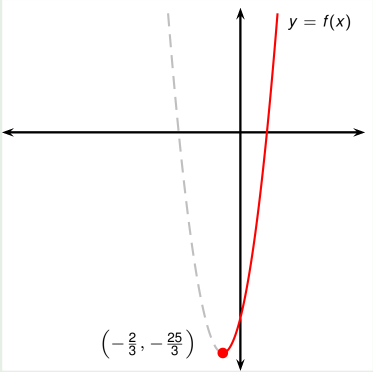

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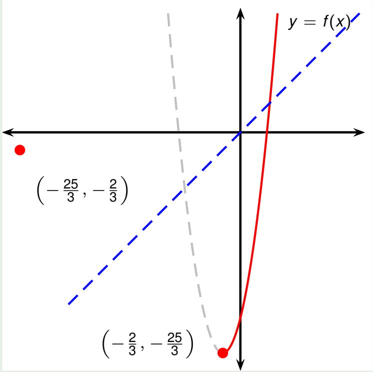

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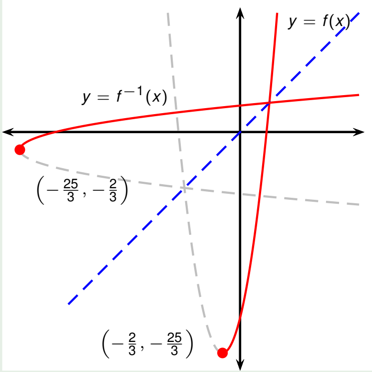

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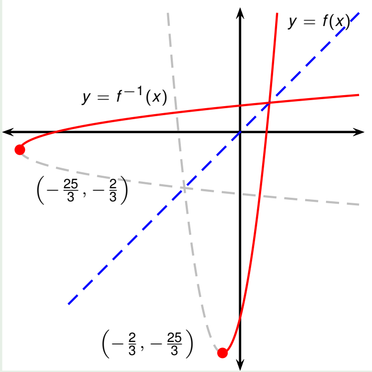

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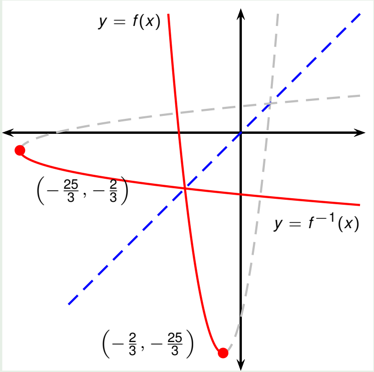

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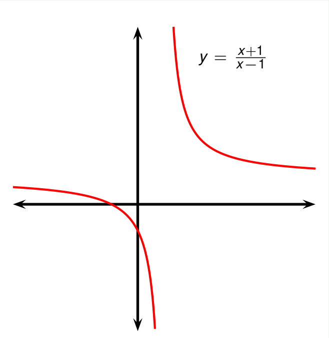
Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.



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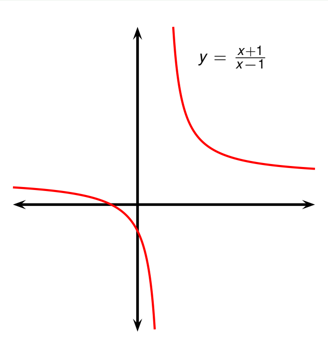

We deal with domains and ranges later:

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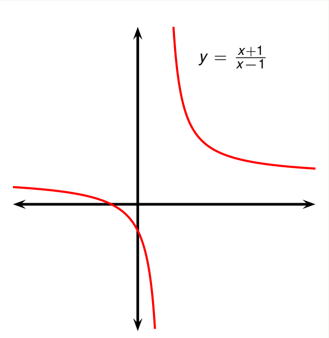

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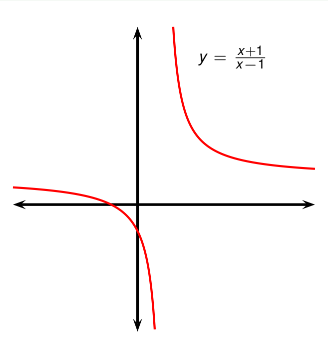

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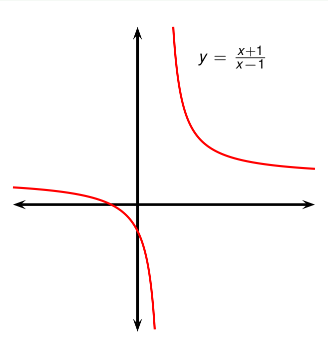

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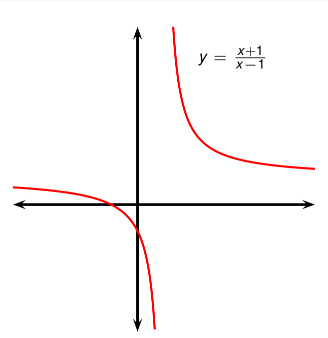

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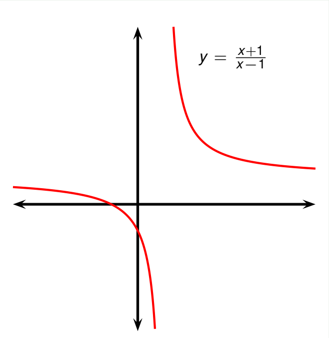

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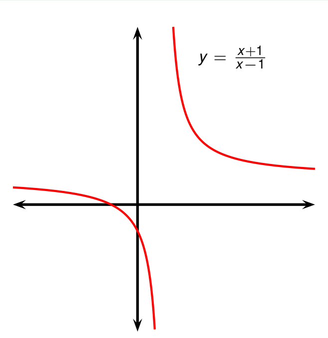

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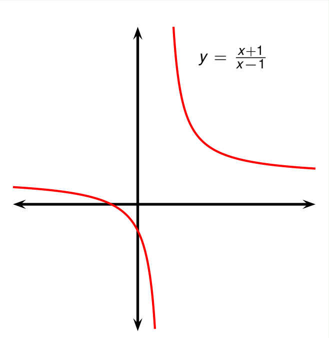

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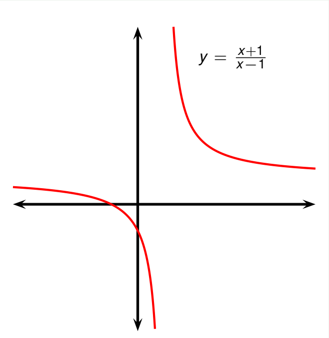

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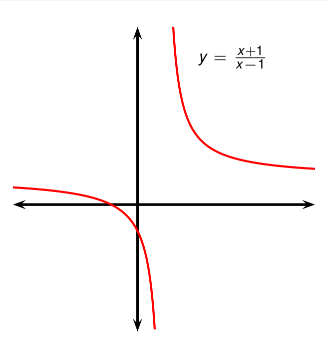
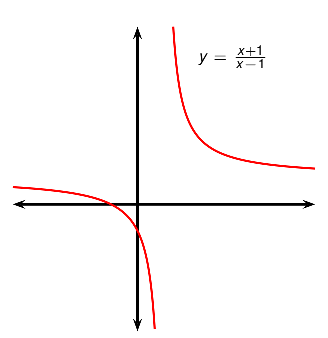
$$\begin{array}{rcl|l} y & = & \dfrac{x+1}{x-1} & \text{mult. by } (x-1) \\ y(x-1) & = & x+1 & \\ x(y-1) & = & y+1 & \text{div. by } (y-1) \\ f^{-1}(y) = x & = & \dfrac{y+1}{y-1} & \text{relabel } x, y \\ f^{-1}(x) & = & \dfrac{x+1}{x-1} & \end{array}$$



Answer: $f^{-1}(x) = \frac{x+1}{x-1}$

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.



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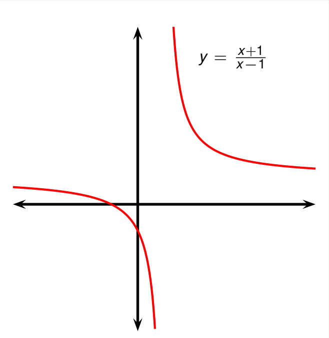

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Answer: $f^{-1}(x) = \frac{x+1}{x-1}$,
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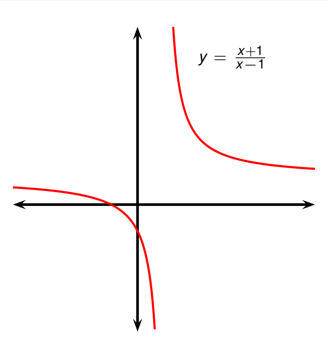

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Can a non-identity function be its own inverse?

Example

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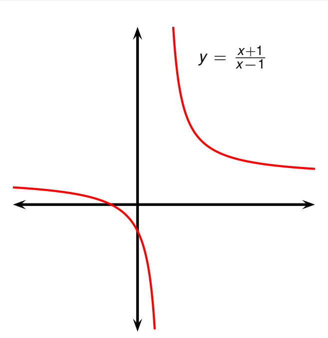

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Can a non-identity function be its own inverse? Yes, f is.

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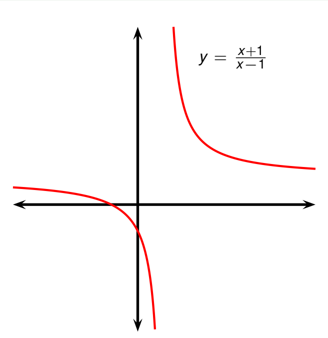

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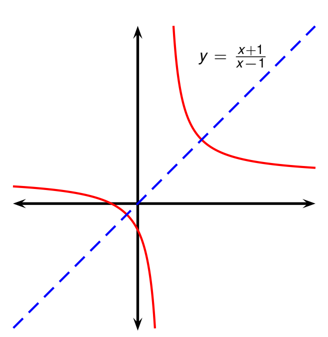

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Graph of f is symmetric across $y = x$.