Precalculus Homework **Exponent basics**

1. Convert from degrees to radians.

(a) 15° .

(h) 120° .

(o) 360° .

(b) 30° . (c) 36°.

(i) 135°. (j) 150° .

(p) 405° .

(d) 45° .

(e) 60° .

(k) 180° .

(q) 1200° .

(f) 75° .

(1) 225° . (m) 270° .

(r) -900° .

(g) 90° .

(n) 305° .

(s) -2014° .

2. Convert from radians to degrees. The answer key has not been proofread, use with caution.

(a) 4π .

(d) $\frac{4}{3}\pi$.

(g) 5.

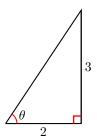
(b) $-\frac{7}{6}\pi$. (c) $\frac{7}{12}\pi$.

(e) $-\frac{3}{8}\pi$.

(f) 2014π .

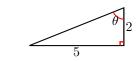
(h) -2014.

- 3. Find the indicated circle arc-length. The answer key has not been proofread, use with caution.
 - (a) Circle of radius 3, arc of measure 36°.
 - (b) Circle of radius $\frac{1}{2}$, arc of measure 100° .
 - (c) Circle of radius 1, arc of measure 3 (radians).
 - (d) Circle of radius 3, arc of measure 300°.
- 4. Find the 6 trigonometric functions of the indicated angle in the indicated right triangle.



(a)





(c) (d)



- 5. Find the exact value of the trigonometric function (using radicals).
 - (a) $\cos 135^{\circ}$.
 - (b) sin 225°.
 - (c) $\cos 495^{\circ}$.
 - (d) $\sin 560^{\circ}$.
 - (e) $\sin\left(\frac{3\pi}{2}\right)$.
 - (f) $\cos\left(\frac{11\pi}{6}\right)$.
 - (g) $\sin\left(\frac{2015\pi}{3}\right)$.
 - (h) $\cos\left(\frac{17\pi}{3}\right)$.
- 6. Find all solutions of the equation in the interval $[0, 2\pi)$. The answer key has not been proofread, use with caution.
 - (a) $\sin x = -\frac{\sqrt{2}}{2}$.
 - (b) $\cos x = \frac{\sqrt{3}}{2}$.
 - (c) $\sin(3x) = \frac{1}{2}$.
 - (d) $\cos(7x) = 0$.
 - (e) $\cos\left(3x + \frac{\pi}{2}\right) = 0.$
 - (f) $\sin(5x \frac{\pi}{3}) = 0$.
- 7. Use the known values of $\sin 30^\circ$, $\cos 30^\circ$, $\sin 45^\circ$, $\cos 45^\circ$, $\sin 60^\circ$, $\cos 60^\circ$, ..., the angle sum formulas and the cofunction identities to find an exact value (using radicals) for the trigonometric function.
 - (a) The six trigonometric functions of $105^{\circ} = 45^{\circ} + 60^{\circ}$:
 - $\sin{(105^{\circ})}$.
 - $\cos{(105^{\circ})}$. Should your answer be a positive or a negative number?
 - $\tan{(105^{\circ})}$.
 - $\cot (105^{\circ})$.
 - sec (105°).
 - $\csc{(105^{\circ})}$.

- (b) The six trigonometric functions of $\frac{\pi}{12} = \frac{\pi}{3} \frac{\pi}{4}$:
 - $\sin\left(\frac{\pi}{12}\right)$.
 - $\cos\left(\frac{\pi}{12}\right)$. Should $\sin\left(\frac{\pi}{12}\right)$ be larger or smaller than $\cos\left(\frac{\pi}{12}\right)$?
 - $\tan\left(\frac{\pi}{12}\right)$.
 - $\cot\left(\frac{\pi}{12}\right)$.
 - $\sec\left(\frac{\pi}{12}\right)$.
 - $\csc\left(\frac{\pi}{12}\right)$.
- 8. Simplify to a trigonometric function of the angle θ . The answer key has not been proofread, use with caution.
 - (a) $\sin\left(\frac{\pi}{2} \theta\right)$.

- (b) $\cos\left(\frac{13\pi}{2} \theta\right)$.
- (c) $\tan(\pi \theta)$
- (d) $\cot\left(\frac{3\pi}{2} \theta\right)$
- (e) $\csc\left(\frac{3\pi}{2} + \theta\right)$
- 9. Using the power-reducing formulas, rewrite the expression in terms of first powers of the cosines and sines of multiples of the angle θ .
 - (a) $\sin^4 \theta$.
 - (b) $\cos^4 \theta$.
 - (c) $\sin^6 \theta$.
 - (d) $\cos^6 \theta$.
- 10. Use the sum-to-product formulas to find all solutions of the trigonometric equation in the interval $[0, 2\pi)$.

Please note that typing a query such as "solve($\sin(x)+\sin(3x)=0$)" at www.wolframalpha.com will provide you with a correct answer and a function plot.

- (a) $\sin(x) + \sin(3x) = 0$.
- (b) $\cos(x) + \cos(-3x) = 0$.
- (c) $\sin(x) \sin(3x) = 0$.
- (d) $\cos(2x) \cos(3x) = 0$.
- 11. Find the inverse function. You are asked to do the algebra only; you are not asked to determine the domain or range of the function or its inverse.
 - (a) $f(x) = 3x^2 + 4x 7$, where $x \ge -\frac{2}{3}$.
 - (b) $f(x) = 2x^2 + 3x 5$, where $x \ge -\frac{3}{4}$.
 - (c) $f(x) = \frac{2x+5}{x-4}$, where $x \neq 4$.
 - (d) $f(x) = \frac{3x+5}{2x-4}$, where $x \neq 2$.
 - (e) $f(x) = \frac{5x+6}{4x+5}$, where $x \neq -\frac{5}{4}$.
 - (f) $f(x) = \frac{2x-3}{-3x+4}$, where $x \neq \frac{4}{3}$..
- 12. Find the inverse function and its domain.
 - (a) $y = \ln(x+3)$.

(e) $y = (\ln x)^2, x \ge 1$.

- (b) $y = 4 \ln (x 3) 4$.
- (c) $y = 2 \ln (-2x + 4) + 1$

(f) $y = \frac{e^x}{1 + 2e^x}$.

(d) $f(x) = e^{x^3}$.

- (g) $f(x) = 2^{2x} + 2^x 2$.
- 13. Find each of the following values. Express your answers precisely, not as decimals.
 - (a) $\arcsin(\sin 4)$.
 - (b) $\arcsin(\sin 0.5)$.
 - (c) $\arcsin(\cos 120^{\circ})$.
 - (d) $\arccos(\cos(3))$.
 - (e) arccos(cos(-2)).
 - (f) arccos(sin(-4)).
 - (g) arctan(tan 5).
- 14. Express as the following as an algebraic expression of x. In other words, "get rid" of the trigonometric and inverse trigonometric expressions.

(a)
$$\cos^2(\arctan x)$$
.

(c)
$$\frac{1}{\cos(\arcsin x)}$$
.

(b)
$$-\sin^2(\operatorname{arccot} x)$$
.

(d)
$$-\frac{1}{\sin(\arccos x)}$$
.

15. Let $x \in (0,1)$. Express the following using x and $\sqrt{1-x^2}$.

(a)
$$\sin(\arcsin(x))$$
.

(e)
$$\sin(2\arccos(x))$$
.

(b)
$$\sin(2\arcsin(x))$$
.

(f)
$$\sin(3\arccos(x))$$
.

(c)
$$\sin(2\arcsin(x))$$
.

(g)
$$\cos(2\arcsin(x))$$
.

(d)
$$\sin(\arccos(x))$$
.

(h)
$$\cos(3\arccos(x))$$
.

16. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation.

(a)
$$2\cos x - 1 = 0$$
.

(b)
$$\sin(2x) = \cos x$$
.

(g)
$$2\cos^2 x - (1+\sqrt{2})\cos x + \frac{\sqrt{2}}{2} = 0.$$

(c)
$$\sqrt{3}\sin x = \sin(2x)$$
.

(h)
$$|\tan x| = 1$$
.

(d)
$$2\sin^2 x = 1$$
.

(i)
$$3\cot^2 x = 1$$
.

(e)
$$2 + \cos(2x) = 3\cos x$$
.

(j)
$$\sin x = \tan x$$
.

(f)
$$2\cos x + \sin(2x) = 0$$
.

17. Express each of the following as a single power.

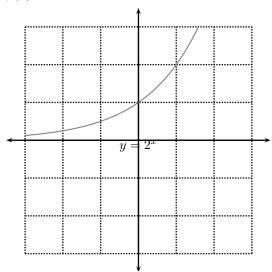
(a)
$$\frac{2^5 \cdot 2^7}{2\sqrt{2}}$$

(b)
$$\frac{3^2 \cdot 3^{-1}}{3^3 \cdot \sqrt{3^3}}$$

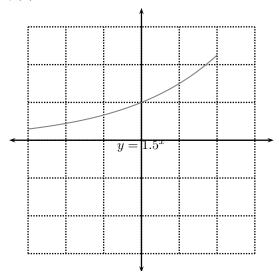
(c)
$$\frac{\pi^3}{\pi^{-1}\sqrt{\pi^5}}$$

18. Sketch by hand approximately the given function. The function is obtained by transforming linearly the graph of a known function. The known function has been sketched for you by computer.

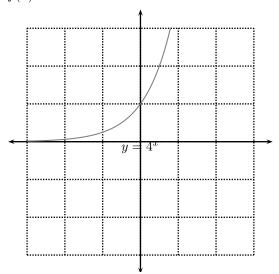
(a)
$$f(x) = 2^{x+1} - 1$$
.



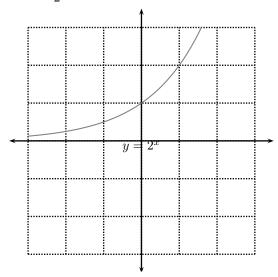
(b) $f(x) = 1.5^{x-2} + 2$.



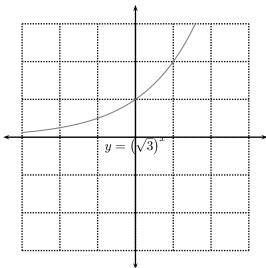
(c) $f(x) = 2^{2x-5}$.



(d) $f(x) = \frac{1}{2^{x-1}} + 1$.



(e) $f(x) = \frac{1}{3^{\frac{1}{2}x+1}} - 1$.



- 19. (a) A sum is held under a yearly compound interest of 1%. Make an approximation by hand (no calculators allowed) by what factor will have the money increased after 200 years. Can you do the computation in your head?
- (b) Decide, without using a calculator, which is more profitable: earning a yearly compound interest of 2% for 150 years or earning yearly simple interest of 11% for 150 years?