Calculus II
Homework
Improper integrals

1. Let $x \in (0, 1)$. Express the following using x and $\sqrt{1 - x^2}$.

   (a) $\sin(\arcsin(x))$.

   (b) $\sin(2\arcsin(x))$.

   (c) $\sin(3\arcsin(x))$.

   (d) $\sin(\arccos(x))$.

   (e) $\sin(2\arccos(x))$.

   (f) $\sin(3\arccos(x))$.

   (g) $\cos(2\arcsin(x))$.

   (h) $\cos(3\arccos(x))$.

2. Express as the following as an algebraic expression of x. In other words, "get rid" of the trigonometric and inverse trigonometric expressions.

   (a) $\cos^2(\arctan x)$.

   (b) $-\sin^2(\text{arccot } x)$.

   (c) $\dfrac{1}{\cos(\arcsin x)}$.

   (d) $-\dfrac{1}{\sin(\arccos x)}$.

3. Rewrite as a rational function of t. This problem will be later used to derive the Euler substitutions (an important technique for integrating).

   (a) $\cos(2\arctan t)$.

   (b) $\sin(2\arctan t)$.

   (c) $\tan(2\arctan t)$.

   (d) $\cot(2\arctan t)$.

   (e) $\csc(2\arctan t)$.

   (f) $\sec(2\arctan t)$.

   (g) $\cos(2\text{arccot } t)$.

   (h) $\sin(2\text{arccot } t)$.

   (i) $\tan(2\text{arccot } t)$.

   (j) $\cot(2\text{arccot } t)$.

   (k) $\csc(2\text{arccot } t)$.

   (l) $\sec(2\text{arccot } t)$.

4. Compute the derivative (derive the formula).

   (a) $(\arctan x)'$.

   (b) $(\text{arccot } x)'$.

   (c) $(\arcsin x)'$.

   (d) $(\arccos x)'$.

   (e) Let arcsec denote the inverse of the secant function. Compute $(\text{arcsec} x)'$.

5. (a) Let $a + b \neq k\pi$, $a \neq k\pi + \frac{\pi}{2}$ and $b \neq k\pi + \frac{\pi}{2}$ for any $k \in \mathbb{Z}$ (integers). Prove that

$$\frac{\tan a + \tan b}{1 - \tan a \tan b} = \tan(a + b) \quad .$$

   (b) Let x and y be real. Prove that, for $xy \neq 1$, we have

$$\arctan x + \arctan y = \arctan\left(\frac{x + y}{1 - xy}\right)$$

   if the left hand side lies between $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

6. Evaluate the indefinite integral. Illustrate the steps of your solutions.

(a) $\int x \sin x \mathrm{d}x.$

(b) $\int x e^{-x} \mathrm{d}x.$

(c) $\int x^2 e^x \mathrm{d}x.$

(d) $\int x \sin(-2x) \mathrm{d}x.$

(e) $\int x^2 \cos(3x) \mathrm{d}x.$

(f) $\int x^2 e^{-2x} \mathrm{d}x.$

(g) $\int x \sin(2x) \mathrm{d}x.$

(h) $\int x \cos(3x) \mathrm{d}x.$

(i) $\int x^2 e^{2x} \mathrm{d}x.$

(j) $\int x^3 e^x \mathrm{d}x.$

7. Evaluate the indefinite integral. Illustrate the steps of your solutions.

(a) $\int x^2 \cos(2x) \mathrm{d}x.$

(b) $\int x^2 e^{ax} \mathrm{d}x$, where a is a constant.

(c) $\int x^2 e^{-ax} \mathrm{d}x$, where a is a constant.

(d) $\int x^2 \frac{(e^{ax} + e^{-ax})^2}{4} \mathrm{d}x$, where a is a constant.

(e) $\int \frac{1}{\cos^2 x} \mathrm{d}x.$ (Hint: This problem does not require integration by parts. What is the derivative of $\tan x$?)

(f) $\int (\tan^2 x) \mathrm{d}x.$ (Hint: This problem does not require integration by parts. We can use $\tan^2 x = \frac{1}{\cos^2 x} - 1$ and the previous problem.)

(g) $\int x \tan^2 x \mathrm{d}x.$ (Hint: $\tan^2 x \mathrm{d}x = \mathrm{d}(F(x))$, where $F(x)$ is the answer from the preceding problem).

(h) $\int e^{-\sqrt{x}} \mathrm{d}x.$

(i) $\int \cos^2 x \, \mathrm{d}x.$

(j) $\int \frac{x}{1+x^2} \mathrm{d}x$ (Hint: use substitution rule, don't use integration by parts)

(k) $\int (\arctan x) \mathrm{d}x.$

(l) $\int (\arcsin x) \mathrm{d}x.$

(m) $\int (\arcsin x)^2 \mathrm{d}x.$ (Hint: Try substituting $x = \sin y$.)

(n) $\int \arctan \left(\frac{1}{x} \right) \mathrm{d}x.$

(o) $\int \sin x e^x \mathrm{d}x$

(p) $\int \cos x e^x \mathrm{d}x$

(q) $\int \sin(\ln(x)) \mathrm{d}x.$

(r) $\int \cos(\ln(x)) \mathrm{d}x.$

(s) $\int \ln x \mathrm{d}x$

(t) $\int x \ln x \, \mathrm{d}x.$

(u) $\int \frac{\ln x}{\sqrt{x}} \mathrm{d}x.$

(v) $\int (\ln x)^2 \mathrm{d}x.$

(w) $\int (\ln x)^3 \mathrm{d}x.$

(x) $\int x^2 \cos^2 x \mathrm{d}x.$ (This problem is related to Problem 7.d as $\cos x = \frac{e^{ix}+e^{-ix}}{2}$).

8. Compute $\int x^n e^x \mathrm{d}x$, where n is a non-negative integer.

9. Integrate. Illustrate the steps of your solution.

(a) $\int \frac{1}{x+1} \mathrm{d}x$

(b) $\int \frac{x-1}{x+1} \mathrm{d}x$

(c) $\int \frac{1}{(x+1)^2} \mathrm{d}x$

(d) $\int \frac{x}{(x+1)^2} \mathrm{d}x$

(e) $\int \frac{1}{(2x+3)^2} \mathrm{d}x$

(f) $\int \frac{x}{2x^2+3} \mathrm{d}x$

(g) $\int \frac{1}{2x^2+3} \mathrm{d}x$

(h) $\int \frac{x}{2x^2+x+1} \mathrm{d}x$.

(i) $\int \frac{x}{2x^2+x+3} \mathrm{d}x$

(j) $\int \frac{x}{x^2-x+3} \mathrm{d}x$

(k) $\displaystyle\int \frac{1}{(x^2+1)^2}\,\mathrm{d}x$

(m) $\displaystyle\int \frac{1}{(x^2+1)^3}\,\mathrm{d}x$

(l) $\displaystyle\int \frac{1}{(x^2+x+1)^2}\,\mathrm{d}x$

10. Let a, b, c, A, B be real numbers. Suppose in addition $a \neq 0$ and $b^2 - 4ac < 0$. Integrate

$$\int \frac{Ax+B}{ax^2+bx+c}\,\mathrm{d}x \quad .$$

The purpose of this exercise is to produce a formula in form ready for implementation in a computer algebra system.

11. Let a, b, c, A, B be real numbers and let $n > 1$ be an integer. Suppose in addition $a \neq 0$ and $b^2 - 4ac < 0$. Let

$$J(n) = \int \frac{1}{\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)^n}\,\mathrm{d}x \quad .$$

(a) Express the integral

$$\int \frac{Ax+B}{(ax^2+bx+c)^n}\,\mathrm{d}x$$

via $J(n)$.

(b) Express $J(n)$ recursively via $J(n-1)$

The purpose of this exercise is to produce a formula in form ready for implementation in a computer algebra system.

12. Integrate. Some of the examples require partial fraction decomposition and some do not. Illustrate the steps of your solution.

(a) $\displaystyle\int \frac{1}{4x^2+4x+1}\,\mathrm{d}x$

(h) $\displaystyle\int \frac{x}{3x^2+x-2}\,\mathrm{d}x$

(b) $\displaystyle\int \frac{1}{1-x^2}\,\mathrm{d}x$

(i) $\displaystyle\int \frac{x}{3x^2+x+2}\,\mathrm{d}x$

(c) $\displaystyle\int \frac{1}{5-x^2}\,\mathrm{d}x$

(j) $\displaystyle\int \frac{x}{2x^2+x+1}\,\mathrm{d}x$

(d) $\displaystyle\int \frac{x}{4x^2+x+\frac{1}{16}}\,\mathrm{d}x$

(k) $\displaystyle\int \frac{x}{2x^2+x-1}\,\mathrm{d}x$

(e) $\displaystyle\int \frac{x+1}{2x^2+x}\,\mathrm{d}x$

(l) $\displaystyle\int \frac{1}{x^2+x+1}\,\mathrm{d}x$

(f) $\displaystyle\int \frac{x}{4x^2+x+5}\,\mathrm{d}x$

(g) $\displaystyle\int \frac{x}{4x^2+x-5}\,\mathrm{d}x$

(m) $\displaystyle\int \frac{1}{2x^2+5x+1}\,\mathrm{d}x$

13. Evaluate the indefinite integral. Illustrate all steps of your solution.

(a) $\displaystyle\int \frac{x^3+4}{x^2+4}\,\mathrm{d}x$

(h) $\displaystyle\int \frac{15x^2-4x-81}{(x-3)(x+4)(x-1)}\,\mathrm{d}x$

(b) $\displaystyle\int \frac{4x^2}{2x^2-1}\,\mathrm{d}x$

(i) $\displaystyle\int \frac{x^4+10x^3+18x^2+2x-13}{x^4+4x^3+3x^2-4x-4}\,\mathrm{d}x$

(c) $\displaystyle\int \frac{x^3}{x^2+2x-3}\,\mathrm{d}x$

Check first that $(x-1)(x+2)^2(x+1) = x^4 + 4x^3 + 3x^2 - 4x - 4$.

(d) $\displaystyle\int \frac{x^3}{x^2+3x-4}\,\mathrm{d}x$

(j) $\displaystyle\int \frac{x^4}{(x^2+2)(x+2)}\,\mathrm{d}x$

(e) $\displaystyle\int \frac{x^3}{2x^2+3x-5}\,\mathrm{d}x$

(k) $\displaystyle\int \frac{x^5}{x^3-1}\,\mathrm{d}x$

(f) $\displaystyle\int \frac{x^2+1}{(x-3)(x-2)^2}\,\mathrm{d}x$

(l) $\displaystyle\int \frac{x^4}{(x^2+2)(x+1)^2}\,\mathrm{d}x$

(g) $\displaystyle\int \frac{x^4}{(x+1)^2(x+2)}\,\mathrm{d}x$

(m) $\displaystyle\int \frac{3x^2 + 2x - 1}{(x-1)(x^2+1)}\,\mathrm{d}x$

(n) $\displaystyle\int \frac{x^2 - 1}{x(x^2+1)^2}\,\mathrm{d}x$

14. Integrate

$$\int \frac{x^6 - x^5 + \frac{9}{2}x^4 - 4x^3 + \frac{13}{2}x^2 - \frac{7}{2}x + \frac{11}{4}}{x^5 - x^4 + 3x^3 - 3x^2 + \frac{9}{4}x - \frac{9}{4}}\,\mathrm{d}x \quad .$$

15. Integrate.

(a) $\displaystyle\int \frac{1}{3 + \cos x}\,\mathrm{d}x.$

(b) $\displaystyle\int \frac{1}{4 + \cos x}\,\mathrm{d}x.$

(c) $\displaystyle\int \frac{1}{3 + \sin x}\,\mathrm{d}x.$

(d) $\displaystyle\int \frac{1}{2 + \tan x}\,\mathrm{d}x.$ (Hint: this integral can be done simply with the substitution $x = \arctan t$.)

(e) $\displaystyle\int \frac{\mathrm{d}x}{2\sin x - \cos x + 5}.$

16. Integrate. The answer key has not been proofread, use with caution.

(a) $\displaystyle\int \sin(3x)\cos(2x)\,\mathrm{d}x.$

(b) $\displaystyle\int \sin x \cos(5x)\,\mathrm{d}x.$

(c) $\displaystyle\int \cos(3x)\sin(2x)\,\mathrm{d}x.$

(d) $\displaystyle\int \sin(5x)\sin(3x)\,\mathrm{d}x.$

(e) $\displaystyle\int \cos(x)\cos(3x)\,\mathrm{d}x.$

17. Integrate.

(a) $\displaystyle\int \sin^2 x \cos x\,\mathrm{d}x.$

(b) $\displaystyle\int \sin^2 x\,\mathrm{d}x.$

(c) $\displaystyle\int \cos^3 x\,\mathrm{d}x.$

(d) $\displaystyle\int \sin^3 x \cos^4 x\,\mathrm{d}x.$

18. Integrate.

(a) $\displaystyle\int \sec x\,\mathrm{d}x.$

(b) $\displaystyle\int \sec^3 x\,\mathrm{d}x.$

(c) $\displaystyle\int \tan^3 x\,\mathrm{d}x.$

(d) $\displaystyle\int \sec^2 x \tan^2 x\,\mathrm{d}x.$

19. Find a linear substitution (via completing the square) to transform the radical to a multiple of an expression of the form $\sqrt{u^2 + 1}$, $\sqrt{u^2 - 1}$ or $\sqrt{1 - u^2}$.

(a) $\sqrt{x^2 + x + 1}.$
(b) $\sqrt{-2x^2 + x + 1}.$

20. Compute the integral.

(a) $\displaystyle\int \frac{\sqrt{1 + x^2}}{x^2}\,\mathrm{d}x.$

21. Compute the integral using a trigonometric substitution.

(a) $\displaystyle\int \frac{\sqrt{9 - x^2}}{x^2}\,\mathrm{d}x$.

22. Compute the integral.

    (a) $\displaystyle\int \sqrt{x^2 + 1}\,\mathrm{d}x$

    (b) $\displaystyle\int \sqrt{x^2 + 2}\,\mathrm{d}x$

    (c) $\displaystyle\int \sqrt{x^2 + x + 1}\,\mathrm{d}x$

    (d) $\displaystyle\int \sqrt{(2x^2 + 2x + 1)}\,\mathrm{d}x$

    (e) $\displaystyle\int \sqrt{(3x^2 + 2x + 1)}\,\mathrm{d}x$

    (f) $\displaystyle\int \frac{\sqrt{x^2 + 1}}{x + 1}\,\mathrm{d}x$

23. Let $b^2 - 4ac < 0$ and $a > 0$ be (real) numbers. Show that

$$\int \sqrt{(ax^2 + bx + c)}\,\mathrm{d}x = \frac{\sqrt{a}D}{2}\left(\ln\left(\sqrt{\left(\frac{2xa+b}{2\sqrt{D}a}\right)^2 + 1} + \frac{2xa+b}{2\sqrt{D}a} \right) + \frac{2xa+b}{2\sqrt{D}a}\sqrt{\left(\frac{2xa+b}{2\sqrt{D}a}\right)^2 + 1} \right) + C,$$

where $D = \dfrac{4ac - b^2}{4a^2}$.

24. Integrate

    (a) $\displaystyle\int \sqrt{1 - x^2}\,\mathrm{d}x$

    (b) $\displaystyle\int \sqrt{2 - x^2}\,\mathrm{d}x$

    (c) $\displaystyle\int \sqrt{-x^2 + x + 1}\,\mathrm{d}x$

    (d) $\displaystyle\int \sqrt{2 - x - x^2}\,\mathrm{d}x$

    (e) $\displaystyle\int \frac{\sqrt{1 - x^2}}{1 + x}\,\mathrm{d}x$

    (f) $\displaystyle\int \frac{\sqrt{1 - x^2}}{2 + x}\,\mathrm{d}x$

25. Integrate

    (a) $\displaystyle\int \sqrt{x^2 - 1}\,\mathrm{d}x$

    (b) $\displaystyle\int \sqrt{x^2 - 2}\,\mathrm{d}x$

    (c) $\displaystyle\int \sqrt{2x^2 + x - 1}\,\mathrm{d}x$

    (d) $\displaystyle\int \sqrt{x^2 + x - 1}\,\mathrm{d}x$

26.   (a) Express $x, \mathrm{d}x$ and $\sqrt{x^2 + 1}$ via θ and $\mathrm{d}\theta$ for the trigonometric substitution $x = \cot\theta$, $\theta \in (0, \pi)$.

    (b) Express $x, \mathrm{d}x$ and $\sqrt{x^2 + 1}$ via t and $\mathrm{d}t$ for the Euler substitution $x = \cot(2\arctan t)$, $t > 0$. Express t via x.

27. Let the variables x and t be related via $\sqrt{x^2 + 1} = x + t$.

    (a) Express x via t.

    (b) Express $\sqrt{x^2 + 1}$ via t alone.

    (c) Express $\mathrm{d}x$ via t and $\mathrm{d}t$.

28.   (a) Express $x, \mathrm{d}x$ and $\sqrt{x^2+1}$ via θ and $\mathrm{d}\theta$ for the trigonometric substitution $x = \tan\theta$, $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

    (b) Express $x, \mathrm{d}x$ and $\sqrt{x^2+1}$ via t and $\mathrm{d}t$ for the Euler substitution $x = \tan(2\arctan t)$, $t \in (-1, 1)$. Express t via x.

29. Let the variables x and t be related via $\sqrt{x^2+1} = \frac{x}{t} - 1$.

    (a) Express x via t.

    (b) Express $\sqrt{x^2+1}$ via t alone.

    (c) Express $\mathrm{d}x$ via t and $\mathrm{d}t$.

30.   (a) Express $x, \mathrm{d}x$ and $\sqrt{1-x^2}$ via θ and $\mathrm{d}\theta$ for the trigonometric substitution $x = \cos\theta$, $\theta \in [0, \pi]$.

    (b) Express $x, \mathrm{d}x$ and $\sqrt{1-x^2}$ via t and $\mathrm{d}t$ for the Euler substitution $x = \cos(2\arctan t)$, $t \geq 0$. Express t via x.

31. Let the variables x and t be related via $\sqrt{-x^2+1} = (1-x)t$.

    (a) Express x via t.

    (b) Express $\sqrt{-x^2+1}$ via t alone.

    (c) Express $\mathrm{d}x$ via t and $\mathrm{d}t$.

32.   (a) Express $x, \mathrm{d}x$ and $\sqrt{1-x^2}$ via θ and $\mathrm{d}\theta$ for the trigonometric substitution $x = \sin\theta$, $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

    (b) Express $x, \mathrm{d}x$ and $\sqrt{1-x^2}$ via t and $\mathrm{d}t$ for the Euler substitution $x = \sin(2\arctan t)$, $t \in [-1, 1]$. Express t via x.

33. Let the variables x and t be related via $\sqrt{-x^2+1} = 1 - xt$.

    (a) Express x via t.

    (b) Express $\sqrt{-x^2+1}$ via t alone.

    (c) Express $\mathrm{d}x$ via t and $\mathrm{d}t$.

34.   (a) Express $x, \mathrm{d}x$ and $\sqrt{x^2-1}$ via θ and $\mathrm{d}\theta$ for the trigonometric substitution $x = \csc\theta$, $\theta \in \left[0, \frac{\pi}{2}\right] \cup \left[\pi, \frac{3\pi}{2}\right)$.

    (b) Express $x, \mathrm{d}x$ and $\sqrt{1-x^2}$ via t and $\mathrm{d}t$ for the Euler substitution $x = \sec(2\arctan t)$, $t \in (-\infty, -1) \cup [1, 0)$. Express t via x.

35. Let the variables x and t be related via $\sqrt{x^2-1} = (x+1)t$.

    (a) Express x via t.

    (b) Express $\sqrt{x^2-1}$ via t alone.

    (c) Express $\mathrm{d}x$ via t and $\mathrm{d}t$.

36.   (a) Express $x, \mathrm{d}x$ and $\sqrt{1-x^2}$ via θ and $\mathrm{d}\theta$ for the trigonometric substitution $x = \csc\theta$, $\theta \in \left[0, \frac{\pi}{2}\right] \cup \left[\pi, \frac{3\pi}{2}\right)$.

    (b) Express $x, \mathrm{d}x$ and $\sqrt{1-x^2}$ via t and $\mathrm{d}t$ for the Euler substitution $x = \csc(2\arctan t)$, $t \in (-\infty, -1) \cup [0, 1)$. Express t via x.

37. Let the variables x and t be related via $\sqrt{x^2-1} = \frac{1}{t} - x$.

    (a) Express x via t.

    (b) Express $\sqrt{x^2-1}$ via t alone.

    (c) Express $\mathrm{d}x$ via t and $\mathrm{d}t$.

38. Compute the limits. The answer key has not been fully proofread, use with caution.

    (a) $\displaystyle\lim_{x\to 0} \frac{\sin x}{x}$.

    (b) $\displaystyle\lim_{x\to 0} \frac{x}{\ln(1+x)}$.

    (c) $\displaystyle\lim_{x\to 0} \frac{x^2}{x - \ln(1+x)}$.

    (d) $\displaystyle\lim_{x\to 0} \frac{x^2}{\sin x \ln(1+x)}$.

    (e) $\displaystyle\lim_{x\to 0} \frac{\sin^2 x}{\left(\ln(1+x)\right)^2}$.

    (f) $\displaystyle\lim_{x\to 0} \frac{\cos x - 1}{\sin x \ln(1+x)}$.

    (g) $\displaystyle\lim_{x\to 0} \frac{\arctan x - x}{x^3}$.

    (h) $\displaystyle\lim_{x\to 0} \frac{\arcsin x - x}{x^3}$.

    (i) $\displaystyle\lim_{x\to 1} \frac{x}{x-1} - \frac{1}{\ln x}$.

    (j) $\displaystyle\lim_{x\to 0} \frac{\cos(nx) - \cos(mx)}{x^2}$.

(k) $\lim\limits_{x\to 0} \dfrac{\arcsin x - x - \frac{1}{6}x^3}{\sin^5 x}$.

(l) $\lim\limits_{x\to 1} \dfrac{\sin(\pi x)\ln x}{\cos(\pi x) + 1}$.

(m) $\lim\limits_{x\to 0} \dfrac{\sin x - x}{\arcsin x - x}$.

(n) $\lim\limits_{x\to 0} \dfrac{\sin x - x}{\arctan x - x}$.

(o) $\lim\limits_{x\to\infty} x\sin\left(\dfrac{2}{x}\right)$.

39. Compute the limit.

(a) $\lim\limits_{x\to\infty} \left(\dfrac{x-2}{x}\right)^x$.

(b) $\lim\limits_{x\to\infty} \left(\dfrac{x-2}{x}\right)^{2x}$.

(c) $\lim\limits_{x\to\infty} \left(\dfrac{x}{x+3}\right)^{2x}$.

40. Find the limit.

(a) $\lim\limits_{x\to\infty} \left(1 - \dfrac{2}{x}\right)^x$.

(b) $\lim\limits_{x\to 0} (1 - x)^{\frac{1}{x}}$.

(c) $\lim\limits_{x\to\infty} \left(\dfrac{x}{x-5}\right)^x$.

(d) $\lim\limits_{x\to\infty} \left(\dfrac{x}{x-2}\right)^{3x+2}$.

41. Determine whether the integral is convergent or divergent. Motivate your answer.

(a) $\displaystyle\int_2^\infty \dfrac{1}{(x-1)^{\frac{3}{2}}}\mathrm{d}x$.

(b) $\displaystyle\int_{-1}^1 \dfrac{1}{\sqrt[5]{1+x}}\mathrm{d}x$.

(c) $\displaystyle\int_1^\infty \dfrac{1}{\sqrt[5]{1+x}}\mathrm{d}x$.

(d) $\displaystyle\int_{-1}^\infty \dfrac{1}{\sqrt[5]{1+x}}\mathrm{d}x$.

(e) $\displaystyle\int_{-\infty}^0 \dfrac{1}{2-3x}\mathrm{d}x$.

(f) $\displaystyle\int_{-\infty}^0 \dfrac{1}{(2-3x)^2}\mathrm{d}x$.

(g) $\displaystyle\int_{-\infty}^0 \dfrac{1}{(2-3x)^{1.00000001}}\mathrm{d}x$.

(h) $\displaystyle\int_{-2}^{\frac{1}{2}} \dfrac{1}{2x-1}\mathrm{d}x$.

(i) $\displaystyle\int_{-1}^\infty e^{-3x}\mathrm{d}x$.

(j) $\displaystyle\int_{-\infty}^5 2^x\mathrm{d}x$.

(k) $\displaystyle\int_{-\infty}^\infty x^3\mathrm{d}x$.

(l) $\displaystyle\int_{-\infty}^\infty xe^{-x^2}\mathrm{d}x$.

(m) $\displaystyle\int_0^\infty \sqrt{x}e^{-\sqrt{x}}\mathrm{d}x$.

(n) $\displaystyle\int_0^\infty \sin^2 x\mathrm{d}x$.

(o) $\displaystyle\int_0^5 \dfrac{1}{x^2+x-2}\mathrm{d}x$.

(p) $\displaystyle\int_0^\infty \dfrac{1}{x^2+x+1}\mathrm{d}x$.

(q) $\displaystyle\int_2^\infty \dfrac{1}{x^2-x-1}\mathrm{d}x$.

(r) $\displaystyle\int_0^\infty \dfrac{1}{x^2-x-1}\mathrm{d}x$.

(s) $\displaystyle\int_{-\infty}^\infty \dfrac{x^2}{x^4+2}\mathrm{d}x$.

(t) $\displaystyle\int_{100}^\infty \dfrac{1}{x\ln x}\mathrm{d}x$.

(u) $\displaystyle\int_{100}^{\infty} \frac{1}{x(\ln x)^2}\,\mathrm{d}x.$

(v) $\displaystyle\int_{0}^{1} \ln x\,\mathrm{d}x.$

(w) $\displaystyle\int_{0}^{1} \frac{\ln x}{\sqrt{x}}\,\mathrm{d}x.$

(x) $\displaystyle\int_{0}^{2} x^3 \ln x\,\mathrm{d}x.$

(y) $\displaystyle\int_{0}^{1} \frac{e^{\frac{1}{x}}}{x^2}\,\mathrm{d}x.$

(z) $\displaystyle\int_{-1}^{0} \frac{e^{\frac{1}{x}}}{x^2}\,\mathrm{d}x.$

42. Determine whether the integral is convergent or divergent. Motivate your answer. The answer key has not been proofread, use with caution.

(a) $\displaystyle\int_{0}^{\infty} \sin x^2\,\mathrm{d}x$ (This problem is more difficult and may require knowledge of sequences to solve).