Calculus II

Integrals of the form $\int \sin(nx)\cos(mx)\mathbf{d}x$

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To evaluate integrals of the form

1. $\int \sin(mx)\cos(nx)\mathrm{d}x$
2. $\int \sin(mx)\sin(nx)\mathrm{d}x$
3. $\int \cos(mx)\cos(nx)\mathrm{d}x$

use the corresponding identity:

1. $\sin A \cos B = \frac{1}{2}[\sin(A-B)+\sin(A+B)]$
2. $\sin A \sin B = \frac{1}{2}[\cos(A-B)-\cos(A+B)]$
3. $\cos A \cos B = \frac{1}{2}[\cos(A-B)+\cos(A+B)]$

Example

$$\int \sin(4x)\cos(5x)\mathrm{d}x$$

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$$\begin{aligned}\int \sin(4x)\cos(5x)\mathrm{d}x &= \int \frac{1}{2}[\sin(4x-5x)+\sin(4x+5x)]\mathrm{d}x \\ &= \frac{1}{2}\int(\sin(-x)+\sin(9x))\mathrm{d}x\end{aligned}$$

Example

$$\begin{aligned}\int \sin(4x)\cos(5x)\mathrm{d}x &= \int \frac{1}{2}[\sin(4x-5x)+\sin(4x+5x)]\mathrm{d}x \\ &= \frac{1}{2}\int(\sin(-x)+\sin(9x))\mathrm{d}x \\ &= \frac{1}{2}\int(-\sin x+\sin(9x))\mathrm{d}x\end{aligned}$$

Example

$$\begin{aligned}
\int \sin(4x)\cos(5x)\mathrm{d}x &= \int \frac{1}{2}[\sin(4x-5x)+\sin(4x+5x)]\mathrm{d}x \\
&= \frac{1}{2}\int(\sin(-x)+\sin(9x))\mathrm{d}x \\
&= \frac{1}{2}\int(-\sin x+\sin(9x))\mathrm{d}x \\
&= \frac{1}{2}(\cos x-\frac{1}{9}\cos(9x))+C
\end{aligned}$$