

Precalculus

Law of cosines theory

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Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ , as indicated.

Proposition (Law of Cosines)

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = c^2 + a^2 - 2ca \cos \beta$$

Proof if $\gamma < 90^\circ$.

Drop a perpendicular h from B to AC .

$$|CD| = a \cos \gamma$$

$$h = a \sin \gamma$$

$$|AD| = b - |CD| = b - a \cos \gamma$$

$$c^2 = |AD|^2 + h^2$$

$$= (b - a \cos \gamma)^2 + (a \sin \gamma)^2$$

$$= b^2 - 2ab \cos \gamma + a^2 \cos^2 \gamma + a^2 \sin^2 \gamma$$

$$= b^2 - 2ab \cos \gamma + a^2.$$

Pyth. thm.
 $\triangle BDA$

