

# Precalculus

## Factoring quadratic polynomials

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# Outline

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2 Vietas' formulas

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## Example

$$\begin{aligned}x^2 - 1 &= (x - 1)(x + 1) \\ x^3 + 8 &= (x + 2)(x^2 - 2x + 4)\end{aligned}$$

## Theorem

*The quadratic  $ax^2 + bx + c$  factors as follows.*

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

*where  $x_1$  and  $x_2$  are the roots of the quadratic, given by:*

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*Let  $ax^2 + bx + c$  be a quadratic functions with zeros  $x_1$  and  $x_2$ . Then:*

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The last two formulas are called Vieta's formulas (after François Viète (1540-1603), Latinized name: Franciscus Vieta).

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

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Vieta's formulas

## Example

Factor the quadratic.

$$x^2 + 5x + 6$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

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## Example

Factor the quadratic.

$$x^2 + 5x + 6 = (x + ?)(x + ?)$$

- The product of the two roots:  $x_1 x_2 = 6$ .

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Factor the quadratic.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

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$$x^2 + 3x + 1 = \left( x - \left( \frac{-3 + \sqrt{5}}{2} \right) \right) \left( x - \left( \frac{-3 - \sqrt{5}}{2} \right) \right)$$

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Factor the quadratic, using complex numbers if needed.

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## Example

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$$x^2 + x + 1 = \left( x - \left( \frac{-1 + \sqrt{3}i}{2} \right) \right) \left( x - \left( \frac{-1 - \sqrt{3}i}{2} \right) \right)$$

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