Cryptography 101
Addition and subtraction in base 10
calculator-algebra.org

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- In what follows we show integer addition by example.

- In what follows we show integer addition by example.
- We show a series of examples that lead to a complete algorithm for addition.

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= \\
2+2 &= \\
2+5 &= \\
9+2 &= \\
7+5 &= \\
9+7 &= \\
0+9 &=
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= \quad ? \\
2+2 &= \\
2+5 &= \\
9+2 &= \\
7+5 &= \\
9+7 &= \\
0+9 &=
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= 3 \\
2+2 &= \\
2+5 &= \\
9+2 &= \\
7+5 &= \\
9+7 &= \\
0+9 &=
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= 3 \\
2+2 &= ? \\
2+5 &= \\
9+2 &= \\
7+5 &= \\
9+7 &= \\
0+9 &=
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= 3 \\
2+2 &= 4 \\
2+5 &= \\
9+2 &= \\
7+5 &= \\
9+7 &= \\
0+9 &=
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= 3 \\
2+2 &= 4 \\
2+5 &= \textbf{?} \\
9+2 &= \\
7+5 &= \\
9+7 &= \\
0+9 &=
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= 3 \\
2+2 &= 4 \\
2+5 &= 7 \\
9+2 &= \\
7+5 &= \\
9+7 &= \\
0+9 &=
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= 3 \\
2+2 &= 4 \\
2+5 &= 7 \\
9+2 &= \textbf{?} \\
7+5 &= \\
9+7 &= \\
0+9 &=
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= 3 \\
2+2 &= 4 \\
2+5 &= 7 \\
9+2 &= 11 \\
7+5 &= \\
9+7 &= \\
0+9 &=
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= 3 \\
2+2 &= 4 \\
2+5 &= 7 \\
9+2 &= 11 \\
7+5 &= \textbf{?} \\
9+7 &= \\
0+9 &=
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= 3 \\
2+2 &= 4 \\
2+5 &= 7 \\
9+2 &= 11 \\
7+5 &= 12 \\
9+7 &= \\
0+9 &=
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= 3 \\
2+2 &= 4 \\
2+5 &= 7 \\
9+2 &= 11 \\
7+5 &= 12 \\
9+7 &= \textbf{?} \\
0+9 &=
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= 3 \\
2+2 &= 4 \\
2+5 &= 7 \\
9+2 &= 11 \\
7+5 &= 12 \\
9+7 &= 16 \\
0+9 &=
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= 3 \\
2+2 &= 4 \\
2+5 &= 7 \\
9+2 &= 11 \\
7+5 &= 12 \\
9+7 &= 16 \\
0+9 &= \textbf{?}
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= 3 \\
2+2 &= 4 \\
2+5 &= 7 \\
9+2 &= 11 \\
7+5 &= 12 \\
9+7 &= 16 \\
0+9 &= 9
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+3 &= \\
4+7 &= \\
2+8 &= \\
9+8 &= \\
5+5 &=
\end{aligned}
$$

Example

Add the one-digit numbers.

$$1+3 =$$
$$4+7 =$$
$$2+8 =$$
$$9+8 =$$
$$5+5 =$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

- To do one-digit addition quickly: make table with all possibilities.

Example

Add the one-digit numbers.

$$1+3 = \ ?$$
$$4+7 =$$
$$2+8 =$$
$$9+8 =$$
$$5+5 =$$

+	0	1	2	3	4	5	6	7	8	9
0										
1				?						
2										
3										
4										
5										
6										
7										
8										
9										

- To do one-digit addition quickly: make table with all possibilities.

Example

Add the one-digit numbers.

$$1+3 = 4$$
$$4+7 =$$
$$2+8 =$$
$$9+8 =$$
$$5+5 =$$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2										
3										
4										
5										
6										
7										
8										
9										

- To do one-digit addition quickly: make table with all possibilities.

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+3 &= 4 \\
4+7 &= \textbf{?} \\
2+8 &= \\
9+8 &= \\
5+5 &=
\end{aligned}
$$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2										
3										
4								?		
5										
6										
7										
8										
9										

- To do one-digit addition quickly: make table with all possibilities.

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+3 &= 4 \\
4+7 &= 11 \\
2+8 &= \\
9+8 &= \\
5+5 &=
\end{aligned}
$$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2										
3										
4								11		
5										
6										
7										
8										
9										

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Example

Add the one-digit numbers.

$$
\begin{aligned}
1+3 &= 4 \\
4+7 &= 11 \\
2+8 &= ? \\
9+8 &= \\
5+5 &=
\end{aligned}
$$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2									?	
3										
4								11		
5										
6										
7										
8										
9										

- To do one-digit addition quickly: make table with all possibilities.

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+3 &= 4 \\
4+7 &= 11 \\
2+8 &= 10 \\
9+8 &= \\
5+5 &=
\end{aligned}
$$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2									10	
3										
4								11		
5										
6										
7										
8										
9										

- To do one-digit addition quickly: make table with all possibilities.

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+3 &= 4 \\
4+7 &= 11 \\
2+8 &= 10 \\
9+8 &= \text{?} \\
5+5 &=
\end{aligned}
$$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2									10	
3										
4								11		
5										
6										
7										
8										
9									?	

- To do one-digit addition quickly: make table with all possibilities.

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+3 &= 4 \\
4+7 &= 11 \\
2+8 &= 10 \\
9+8 &= 17 \\
5+5 &=
\end{aligned}
$$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2									10	
3										
4								11		
5										
6										
7										
8										
9									17	

- To do one-digit addition quickly: make table with all possibilities.

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+3 &= 4 \\
4+7 &= 11 \\
2+8 &= 10 \\
9+8 &= 17 \\
5+5 &= \textbf{?}
\end{aligned}
$$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2									10	
3										
4								11		
5						?				
6										
7										
8										
9									17	

- To do one-digit addition quickly: make table with all possibilities.

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+3 &= 4 \\
4+7 &= 11 \\
2+8 &= 10 \\
9+8 &= 17 \\
5+5 &= 10
\end{aligned}
$$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2									10	
3										
4								11		
5						10				
6										
7										
8										
9									17	

- To do one-digit addition quickly: make table with all possibilities.

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+3 &= 4 \\
4+7 &= 11 \\
2+8 &= 10 \\
9+8 &= 17 \\
5+5 &= 10
\end{aligned}
$$

+	0	1	2	3	4	5	6	7	8	9
0	?	?	?	?	?	?	?	?	?	?
1	?	?	?	4	?	?	?	?	?	?
2	?	?	?	?	?	?	?	?	10	?
3	?	?	?	?	?	?	?	?	?	?
4	?	?	?	?	?	?	?	11	?	?
5	?	?	?	?	?	10	?	?	?	?
6	?	?	?	?	?	?	?	?	?	?
7	?	?	?	?	?	?	?	?	?	?
8	?	?	?	?	?	?	?	?	?	?
9	?	?	?	?	?	?	?	?	17	?

- To do one-digit addition quickly: make table with all possibilities.

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+3 &= 4 \\
4+7 &= 11 \\
2+8 &= 10 \\
9+8 &= 17 \\
5+5 &= 10
\end{aligned}
$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

- To do one-digit addition quickly: make table with all possibilities.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline \end{array} \qquad \begin{array}{r} 7 \\ +\,6 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ +\,2 \\ \hline \end{array}$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline ? \end{array} \qquad \begin{array}{r} 7 \\ +\,6 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ +\,2 \\ \hline \end{array}$$

$3 + 5 = ?$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						?				
4										
5										
6										
7										
8										
9										

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} 7 \\ +\,6 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ +\,2 \\ \hline \end{array}$$

$3 + 5 = 8$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7										
8										
9										

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} 7 \\ +\,6 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ +\,2 \\ \hline \end{array}$$

$3 + 5 = 8$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7										
8										
9										

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} 7 \\ +\,6 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ +\,2 \\ \hline \end{array}$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7										
8										
9										

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\ 5 \\ \hline 8 \end{array} \qquad \begin{array}{r} 7 \\ +\ 6 \\ \hline \end{array} \qquad \begin{array}{r} 9 \\ +\ 2 \\ \hline \end{array}$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7										
8										
9										

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle ?} \\ 7 \\ +\,6 \\ \hline ? \end{array} \qquad \begin{array}{r} 9 \\ +\,2 \\ \hline \end{array}$$

$7 + 6 = ?$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							?			
8										
9										

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\ 5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\ 6 \\ \hline 13 \end{array} \qquad \begin{array}{r} 9 \\ +\ 2 \\ \hline \end{array}$$

$7 + 6 = 13$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9										

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\,6 \\ \hline 3 \end{array} \qquad \begin{array}{r} 9 \\ +\,2 \\ \hline \end{array}$$

$7 + 6 = 13$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9										

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\,6 \\ \hline 3 \end{array} \qquad \begin{array}{r} 9 \\ +\,2 \\ \hline \end{array}$$

$7 + 6 = 13$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9										

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\,6 \\ \hline 13 \end{array} \qquad \begin{array}{r} 9 \\ +\,2 \\ \hline \end{array}$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9										

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\,6 \\ \hline 13 \end{array} \qquad \begin{array}{r} 9 \\ +\,2 \\ \hline \end{array}$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9										

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\,6 \\ \hline 13 \end{array} \qquad \begin{array}{r} {\scriptstyle ?} \\ 9 \\ +\,2 \\ \hline ? \end{array}$$

$9 + 2 = ?$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9			?							

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\ 5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\ 6 \\ \hline 13 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 9 \\ +\ 2 \\ \hline 1 \end{array}$$

$9 + 2 = 11$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9			11							

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\,6 \\ \hline 13 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 9 \\ +\,2 \\ \hline 1 \end{array}$$

$9 + 2 = 11$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9			11							

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\,6 \\ \hline 13 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 9 \\ +\,2 \\ \hline 1 \end{array}$$

$9 + 2 = 11$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9			11							

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\,6 \\ \hline 13 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 9 \\ +\,2 \\ \hline 11 \end{array}$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9			11							

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\,6 \\ \hline 13 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 9 \\ +\,2 \\ \hline 11 \end{array}$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9			11							

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\,6 \\ \hline 13 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 9 \\ +\,2 \\ \hline 11 \end{array}$$

+	0	1	2	3	4	5	6	7	8	9
0	?	?	?	?	?	?	?	?	?	?
1	?	?	?	?	?	?	?	?	?	?
2	?	?	?	?	?	?	?	?	?	?
3	?	?	?	?	?	8	?	?	?	?
4	?	?	?	?	?	?	?	?	?	?
5	?	?	?	?	?	?	?	?	?	?
6	?	?	?	?	?	?	?	?	?	?
7	?	?	?	?	?	?	13	?	?	?
8	?	?	?	?	?	?	?	?	?	?
9	?	?	11	?	?	?	?	?	?	?

- Addition can also be written in columns.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\,6 \\ \hline 13 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 9 \\ +\,2 \\ \hline 11 \end{array}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

- Addition can also be written in columns.

Example

$$\begin{array}{r} 23 \\ +\ 34 \\ \hline \end{array}$$

Example

$$\begin{array}{r} 23 \\ +\ 34 \\ \hline ? \end{array}$$

$3 + 4 = ?$

Example

$$\begin{array}{r} 23 \\ +\ 34 \\ \hline 7 \end{array}$$

$3 + 4 = 7$

Example

$$\begin{array}{r} 23 \\ +\ 34 \\ \hline 7 \end{array}$$

$3 + 4 = 7$

Example

$$\begin{array}{r} 23 \\ +\ 34 \\ \hline ?7 \end{array}$$

$2 + 3 = ?$

Example

$$\begin{array}{r} 23 \\ +\ 34 \\ \hline 57 \end{array}$$

$2 + 3 = 5$

Example

$$\begin{array}{r} 23 \\ +\ 34 \\ \hline 57 \end{array}$$

$2 + 3 = 5$

Example

$$\begin{array}{r} 23 \\ +\,34 \\ \hline 57 \end{array}$$

Example

$$\begin{array}{r} 23 \\ +\ 34 \\ \hline 57 \end{array}$$

Example

$$\begin{array}{r} 67 \\ +\ 8 \\ \hline \end{array}$$

Example

$$\begin{array}{r} {\scriptstyle ?}\\ 67\\ +\ 8\\ \hline ? \end{array}$$

$7 + 8 = ?$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 6\,7 \\ +\;\;8 \\ \hline \;\;5 \end{array}$$

$7 + 8 = 15$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 67 \\ +\ 8 \\ \hline 5 \end{array}$$

$7 + 8 = 15$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 67 \\ +\ 8 \\ \hline 5 \end{array}$$

$7 + 8 = 15$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 67 \\ +\ 8 \\ \hline ?5 \end{array}$$

$1 + 6 + = ?$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 67 \\ +\ 8 \\ \hline 75 \end{array}$$

$1 + 6 + = 7$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 67 \\ +\ 8 \\ \hline 75 \end{array}$$

$1 + 6 + = 7$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 67 \\ +\ 8 \\ \hline 75 \end{array}$$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 67 \\ +\ 8 \\ \hline 75 \end{array}$$

Example

$$\begin{array}{r} 67 \\ +\ 962 \\ \hline \end{array}$$

Example

$$\begin{array}{r} 67 \\ +\ 962 \\ \hline ? \end{array}$$

$7 + 2 = ?$

Example

$$\begin{array}{r} 67 \\ +\ 962 \\ \hline 9 \end{array}$$

$7 + 2 = 9$

Example

$$\begin{array}{r} 67 \\ +\ 962 \\ \hline 9 \end{array}$$

$7 + 2 = 9$

Example

$$\begin{array}{r} {\scriptstyle ?} \\ 67 \\ +\ 962 \\ \hline ?9 \end{array}$$

$6 + 6 = ?$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 67 \\ +\ 962 \\ \hline 29 \end{array}$$

$6 + 6 = 12$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 67 \\ +\ 962 \\ \hline 29 \end{array}$$

$6 + 6 = 12$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 67 \\ +\ 962 \\ \hline 29 \end{array}$$

$6 + 6 = 12$

Example

$$\begin{array}{r} {\scriptstyle ?\ 1\ \ } \\ 67 \\ +\ 962 \\ \hline ?29 \end{array}$$

$1 + 9 = ?$

Example

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 67 \\ +\ 962 \\ \hline 029 \end{array}$$

$1 + 9 = 10$

Example

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ } \\ 67 \\ +\ 962 \\ \hline 029 \end{array}$$

$1 + 9 = 10$

Example

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ } \\ 67 \\ +\ 962 \\ \hline 029 \end{array}$$

$1 + 9 = 10$

Example

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ } \\ 67 \\ +\ 962 \\ \hline 1029 \end{array}$$

Example

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ } \\ 67 \\ +\ 962 \\ \hline 1029 \end{array}$$

Example

$$\begin{array}{r} 35461 \\ +\ 68072 \\ \hline \end{array}$$

Example

$$\begin{array}{r} 35461 \\ +\ 68072 \\ \hline ? \end{array}$$

$1 + 2 = ?$

Example

$$\begin{array}{r} 35461 \\ +\ 68072 \\ \hline 3 \end{array}$$

$1 + 2 = 3$

Example

$$\begin{array}{r} 35461 \\ +\ 68072 \\ \hline 3 \end{array}$$

$1 + 2 = 3$

Example

$$\begin{array}{r} {\scriptstyle ?} \\ 35461 \\ +\ 68072 \\ \hline ?3 \end{array}$$

$6 + 7 = ?$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 35461 \\ +\ 68072 \\ \hline 33 \end{array}$$

$6 + 7 = 13$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 35461 \\ +\ 68072 \\ \hline 33 \end{array}$$

$6 + 7 = 13$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 35461 \\ +\ 68072 \\ \hline 33 \end{array}$$

$6 + 7 = 13$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 35461 \\ +\ 68072 \\ \hline ?33 \end{array}$$

$1 + 4 + 0 = ?$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 35461 \\ +\ 68072 \\ \hline 533 \end{array}$$

$1 + 4 + 0 = 5$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 35461 \\ +\ 68072 \\ \hline 533 \end{array}$$

$1 + 4 + 0 = 5$

Example

$$\begin{array}{r} {\scriptstyle ?\quad 1\quad\quad} \\ 35461 \\ +\ 68072 \\ \hline ?533 \end{array}$$

$5 + 8 = ?$

Example

$$\begin{array}{r} {\scriptstyle 1\quad 1\quad\ } \\ 35461 \\ +\ 68072 \\ \hline 03533 \end{array}$$

$5 + 8 = 13$

Example

$$\begin{array}{r} {\scriptstyle 1\quad 1\quad\ } \\ 35461 \\ +\ 68072 \\ \hline 103533 \end{array}$$

$5 + 8 = 13$

Example

$$\begin{array}{r} {\scriptstyle 1\quad 1\quad\ } \\ 35461 \\ +\ 68072 \\ \hline 3533 \end{array}$$

$5 + 8 = 13$

Example

$$\begin{array}{r} {\scriptstyle ?\ 1\quad 1} \\ 35461 \\ +\ 68072 \\ \hline ?3533 \end{array}$$

$1 + 3 + 6 = ?$

Example

$$\begin{array}{r} {\scriptstyle 1\ 1\quad 1} \\ 35461 \\ +\ 68072 \\ \hline 03533 \end{array}$$

$1 + 3 + 6 = 10$

Example

$$\begin{array}{r} {\scriptstyle 1\ 1\quad 1\ \ } \\ 35461 \\ +\ 68072 \\ \hline 03533 \end{array}$$

$1 + 3 + 6 = 10$

Example

$$\begin{array}{r} {\scriptstyle 1\ 1\quad 1} \\ 35461 \\ +\ 68072 \\ \hline 03533 \end{array}$$

$1 + 3 + 6 = 10$

Example

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ 1\ \ } \\ 35461 \\ +\ 68072 \\ \hline 103533 \end{array}$$

Example

$$\begin{array}{r} {\scriptstyle 1\ 1\quad 1} \\ 35461 \\ +\ 68072 \\ \hline 103533 \end{array}$$

We covered addition by example; algorithm follows. Feel free to skip.

Algorithm (Addition base 10)

1. *Set* **maxNumberOfDigits** *to the larger number of digits.*
2. *For each digit position* **i**, *starting at position* **0**:

2.1. - *Let* **topDigit** *and* **bottomDigit** *be the two digits in* $\mathbf{i}^{\mathbf{th}}$ *position. If smaller number has no digit at the position, set its digit to* **0**.

2.2. - *Set* **digitSum** *to* **topDigit** $+$ **bottomDigit**.

2.3. - *If* **digitSum** $\geq$ **10**, *set* **resultDigit** $=$ **digitSum** $-$ **10** *and* **carryOver** $=$ **1**.
- *Else* **digitSum** $<$ **10**, *so set* **resultDigit** $=$ **digitSum** *and* **carryOver** $=$ **0**.

2.4. - *Set the result's* $\mathbf{i}^{\mathbf{th}}$ *digit to* **resultDigit**.

3. *If after last step* **carryOver** *is* **1**, *set* **1** *as the result's* $(\mathbf{maxNumberOfDigits} + \mathbf{1})^{\mathbf{th}}$ *digit.*

- In what follows we show subtraction by example.

- In what follows we show subtraction by example.
- We show a series of example that lead to a complete algorithm for subtraction.

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$
\begin{aligned}
5 - 3 &= \\
4 - 0 &= \\
7 - 4 &= \\
8 - 2 &= \\
9 - 7 &=
\end{aligned}
$$

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$5 - 3 = \textbf{?} \quad \text{because } 3 + \textbf{?} = 5$$
$$4 - 0 =$$
$$7 - 4 =$$
$$8 - 2 =$$
$$9 - 7 =$$

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$\begin{aligned} 5-3 &= 2 \quad \big| \text{ because } 3+2=5 \\ 4-0 &= \\ 7-4 &= \\ 8-2 &= \\ 9-7 &= \end{aligned}$$

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$
\begin{array}{rcl|l}
5-3 & = & 2 & \text{because } 3+2=5 \\
4-0 & = & \textbf{?} & \text{because } 0+\textbf{?}=4 \\
7-4 & = & & \\
8-2 & = & & \\
9-7 & = & &
\end{array}
$$

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$
\begin{array}{rcl|l}
5-3 & = & 2 & \text{because } 3+2=5 \\
4-0 & = & 4 & \text{because } 0+4=4 \\
7-4 & = & & \\
8-2 & = & & \\
9-7 & = & &
\end{array}
$$

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$
\begin{array}{rcl|l}
5-3 & = & 2 & \text{because } 3+2=5 \\
4-0 & = & 4 & \text{because } 0+4=4 \\
7-4 & = & \textbf{?} & \text{because } 4+\textbf{?}=7 \\
8-2 & = & & \\
9-7 & = & &
\end{array}
$$

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$
\begin{array}{rcl|l}
5-3 & = & 2 & \text{because } 3+2=5 \\
4-0 & = & 4 & \text{because } 0+4=4 \\
7-4 & = & 3 & \text{because } 4+3=7 \\
8-2 & = & & \\
9-7 & = & &
\end{array}
$$

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$5-3$	$=$	2	because $3+2=5$
$4-0$	$=$	4	because $0+4=4$
$7-4$	$=$	3	because $4+3=7$
$8-2$	$=$	**?**	because $2+$ **?** $=8$
$9-7$	$=$		

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$
\begin{array}{rcl|l}
5-3 & = & 2 & \text{because } 3+2=5 \\
4-0 & = & 4 & \text{because } 0+4=4 \\
7-4 & = & 3 & \text{because } 4+3=7 \\
8-2 & = & 6 & \text{because } 2+6=8 \\
9-7 & = & &
\end{array}
$$

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$5 - 3$	$=$	2	because $3 + 2 = 5$
$4 - 0$	$=$	4	because $0 + 4 = 4$
$7 - 4$	$=$	3	because $4 + 3 = 7$
$8 - 2$	$=$	6	because $2 + 6 = 8$
$9 - 7$	$=$	$\mathbf{?}$	because $7 + \mathbf{?} = 9$

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$5-3$	$=$	2	because $3+2=5$
$4-0$	$=$	4	because $0+4=4$
$7-4$	$=$	3	because $4+3=7$
$8-2$	$=$	6	because $2+6=8$
$9-7$	$=$	2	because $7+2=9$

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$6 - 1 =$$
$$9 - 5 =$$
$$8 - 2 =$$

+										

- To do one-digit subtraction: guess from addition table.

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$6 - 1 = \textbf{?} \quad \text{because } 1 + \textbf{?} = 6$$
$$9 - 5 =$$
$$8 - 2 =$$

+		1								
?		6								

- To do one-digit subtraction: guess from addition table.

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$6 - 1 = 5 \quad | \quad \text{because } 1 + 5 = 6$$
$$9 - 5 =$$
$$8 - 2 =$$

+		1								
5		6								

- To do one-digit subtraction: guess from addition table.

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$\begin{array}{rcl|l} 6-1 & = & 5 & \text{because } 1+5=6 \\ 9-5 & = & \mathbf{?} & \text{because } 5+\mathbf{?}=9 \\ 8-2 & = & & \end{array}$$

$+$		1				5				
?						9				
5		6								

- To do one-digit subtraction: guess from addition table.

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$\begin{array}{rcl|l} 6-1 & = & 5 & \text{because } 1+5=6 \\ 9-5 & = & 4 & \text{because } 5+4=9 \\ 8-2 & = & & \end{array}$$

$+$		1				5				
4						9				
5		6								

- To do one-digit subtraction: guess from addition table.

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$\begin{array}{rcl|l} 6-1 & = & 5 & \text{because } 1+5=6 \\ 9-5 & = & 4 & \text{because } 5+4=9 \\ 8-2 & = & ? & \text{because } 2+?=8 \end{array}$$

+		1	2			5				
4						9				
5		6								
?			8							

- To do one-digit subtraction: guess from addition table.

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$\begin{array}{rcl|l} 6-1 & = & 5 & \text{because } 1+5=6 \\ 9-5 & = & 4 & \text{because } 5+4=9 \\ 8-2 & = & 6 & \text{because } 2+6=8 \end{array}$$

+		1	2			5				
4						9				
5		6								
6			8							

- To do one-digit subtraction: guess from addition table.

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$\begin{array}{rcl|l} 6-1 & = & 5 & \text{because } 1+5=6 \\ 9-5 & = & 4 & \text{because } 5+4=9 \\ 8-2 & = & 6 & \text{because } 2+6=8 \end{array}$$

+	?	1	2	?	?	5	?	?	?	?
?	?	?	?	?	?	?	?	?	?	?
?	?	?	?	?	?	?	?	?	?	?
?	?	?	?	?	?	?	?	?	?	?
?	?	?	?	?	?	?	?	?	?	?
4	?	?	?	?	?	9	?	?	?	?
5	?	6	?	?	?	?	?	?	?	?
6	?	?	8	?	?	?	?	?	?	?
?	?	?	?	?	?	?	?	?	?	?
?	?	?	?	?	?	?	?	?	?	?
?	?	?	?	?	?	?	?	?	?	?

- To do one-digit subtraction: guess from addition table.

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$\begin{array}{rcl|l} 6-1 & = & 5 & \text{because } 1+5=6 \\ 9-5 & = & 4 & \text{because } 5+4=9 \\ 8-2 & = & 6 & \text{because } 2+6=8 \end{array}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

- To do one-digit subtraction: guess from addition table.