Cryptography 101
Addition and subtraction in base 10
calculator-algebra.org

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- In what follows we show integer addition by example.
- We show a series of examples that lead to a complete algorithm for addition.

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+2 &= 3 \\
2+2 &= 4 \\
2+5 &= 7 \\
9+2 &= 11 \\
7+5 &= 12 \\
9+7 &= 16 \\
0+9 &= 9
\end{aligned}
$$

Example

Add the one-digit numbers.

$$
\begin{aligned}
1+3 &= 4 \\
4+7 &= 11 \\
2+8 &= 10 \\
9+8 &= 17 \\
5+5 &= 10
\end{aligned}
$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

- To do one-digit addition quickly: make table with all possibilities.

Example

Add the one-digit numbers.

$$\begin{array}{r} 3 \\ +\,5 \\ \hline 8 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 7 \\ +\,6 \\ \hline 13 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 9 \\ +\,2 \\ \hline 11 \end{array}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

- Addition can also be written in columns.

Example

$$\begin{array}{r} 23 \\ +\,34 \\ \hline 57 \end{array}$$

Example

$$\begin{array}{r} {\scriptstyle 1} \\ 67 \\ +\ 8 \\ \hline 75 \end{array}$$

Example

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ } \\ 67 \\ +\ 962 \\ \hline 1029 \end{array}$$

Example

$$\begin{array}{r} {\scriptstyle 1\ 1\ \ \ 1\ \ } \\ 35461 \\ +\ 68072 \\ \hline 103533 \end{array}$$

We covered addition by example; algorithm follows. Feel free to skip.

Algorithm (Addition base 10)

1. *Set* **maxNumberOfDigits** *to the larger number of digits.*
2. *For each digit position* **i**, *starting at position* **0**:

2.1. - *Let* **topDigit** *and* **bottomDigit** *be the two digits in* $\mathbf{i}^{\mathbf{th}}$ *position. If smaller number has no digit at the position, set its digit to* **0**.

2.2. - *Set* **digitSum** *to* **topDigit** $+$ **bottomDigit**.

2.3. - *If* **digitSum** $\geq$ **10**, *set* **resultDigit** $=$ **digitSum** $-$ **10** *and* **carryOver** $=$ **1**.
- *Else* **digitSum** $<$ **10**, *so set* **resultDigit** $=$ **digitSum** *and* **carryOver** $=$ **0**.

2.4. - *Set the result's* $\mathbf{i}^{\mathbf{th}}$ *digit to* **resultDigit**.

3. *If after last step* **carryOver** *is* **1**, *set* **1** *as the result's* $(\mathbf{maxNumberOfDigits} + \mathbf{1})^{\mathbf{th}}$ *digit.*

- In what follows we show subtraction by example.
- We show a series of example that lead to a complete algorithm for subtraction.

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$5-3$	$=$	2	because $3+2=5$
$4-0$	$=$	4	because $0+4=4$
$7-4$	$=$	3	because $4+3=7$
$8-2$	$=$	6	because $2+6=8$
$9-7$	$=$	2	because $7+2=9$

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$\begin{array}{rcl|l} 6-1 & = & 5 & \text{because } 1+5=6 \\ 9-5 & = & 4 & \text{because } 5+4=9 \\ 8-2 & = & 6 & \text{because } 2+6=8 \end{array}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

- To do one-digit subtraction: guess from addition table.