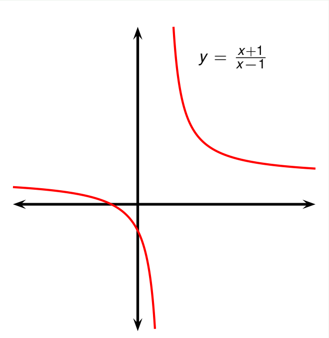
Precalculus
Inverse of fractional linear transformation

Todor Milev

2019

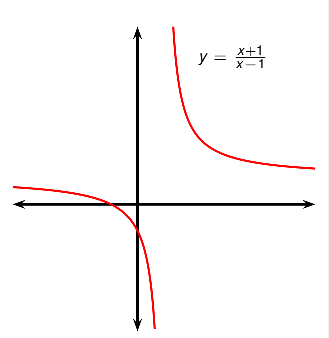
Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.



Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.

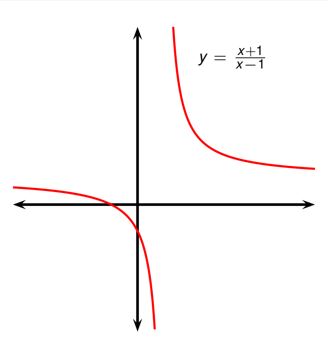


We deal with domains and ranges later:

$$y = \frac{x+1}{x-1}$$

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.

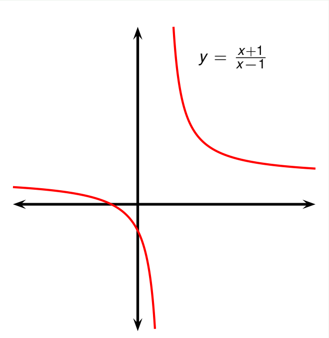


We deal with domains and ranges later:

$$\begin{aligned} y &= \frac{x+1}{x-1} && \Big| \text{ mult. by } (x-1) \\ y(x-1) &= x+1 \end{aligned}$$

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.

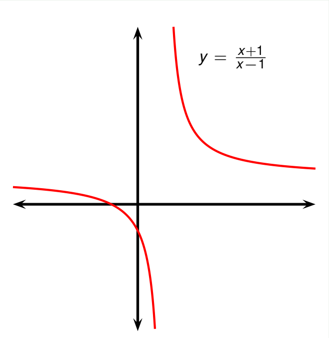


We deal with domains and ranges later:

$$\begin{array}{rcl|l} y &=& \dfrac{x+1}{x-1} & \text{mult. by } (x-1) \\ y(x-1) &=& x+1 & \\ x(y-1) &=& y+1 & \end{array}$$

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.

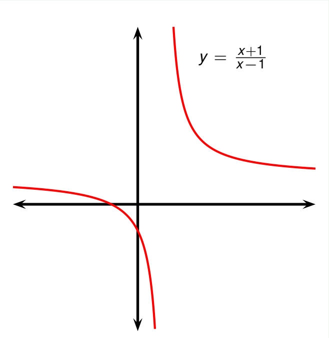


We deal with domains and ranges later:

$$\begin{array}{rcl|l} y & = & \dfrac{x+1}{x-1} & \text{mult. by } (x-1) \\ y(x-1) & = & x+1 & \\ x(y-1) & = & y+1 & \end{array}$$

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.

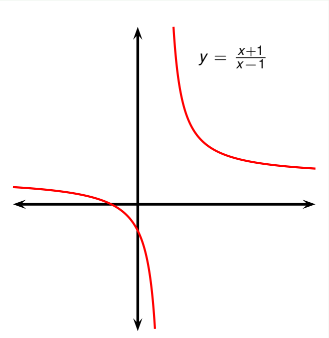


We deal with domains and ranges later:

$$
\begin{array}{rcll}
y &=& \dfrac{x+1}{x-1} & \Big| \text{ mult. by } (x-1) \\
{\color{red}y}(x {\color{red}-1}) &=& x+1 & \\
x(y-1) &=& {\color{red}y}+1 &
\end{array}
$$

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.

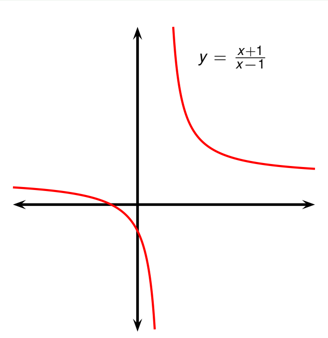


We deal with domains and ranges later:

$$\begin{array}{rcll} y &=& \dfrac{x+1}{x-1} & \Big|\ \text{mult. by } (x-1) \\ y(x-1) &=& x+1 & \\ x(y-1) &=& y+1 & \end{array}$$

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.

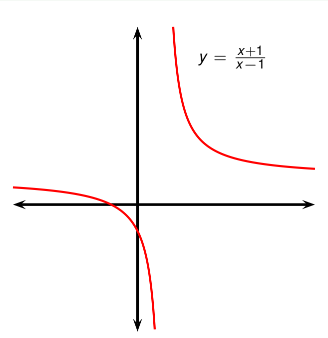


We deal with domains and ranges later:

$$\begin{aligned} y &= \frac{x+1}{x-1} && \Big| \text{ mult. by } (x-1) \\ y(x-1) &= x+1 \\ x(y-1) &= y+1 && \Big| \text{ div. by } (y-1) \\ x &= \frac{y+1}{y-1} \end{aligned}$$

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.

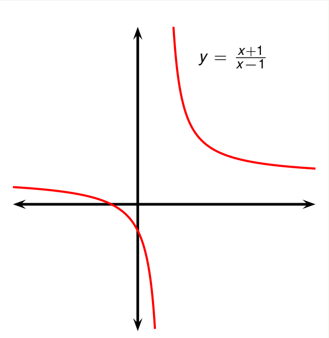


We deal with domains and ranges later:

$$\begin{array}{rcll} y &=& \dfrac{x+1}{x-1} & \Big| \text{ mult. by } (x-1) \\ y(x-1) &=& x+1 & \\ x(y-1) &=& y+1 & \Big| \text{ div. by } (y-1) \\ f^{-1}(y) = x &=& \dfrac{y+1}{y-1} & \end{array}$$

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.

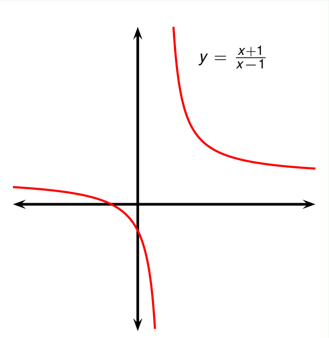


We deal with domains and ranges later:

$$
\begin{array}{rcll}
y &=& \dfrac{x+1}{x-1} & \Big| \text{ mult. by } (x-1) \\
y(x-1) &=& x+1 & \\
x(y-1) &=& y+1 & \Big| \text{ div. by } (y-1) \\
f^{-1}(y) = x &=& \dfrac{y+1}{y-1} & \Big| \text{ relabel } x, y \\
f^{-1}(x) &=& \dfrac{x+1}{x-1} &
\end{array}
$$

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.



We deal with domains and ranges later:

$$\begin{array}{rcll}
y &=& \dfrac{x+1}{x-1} & \Big| \text{ mult. by } (x-1) \\
y(x-1) &=& x+1 & \\
x(y-1) &=& y+1 & \Big| \text{ div. by } (y-1) \\
f^{-1}(y) = x &=& \dfrac{y+1}{y-1} & \Big| \text{ relabel } x, y \\
f^{-1}(x) &=& \dfrac{x+1}{x-1} &
\end{array}$$

Answer: $f^{-1}(x) = \frac{x+1}{x-1}$

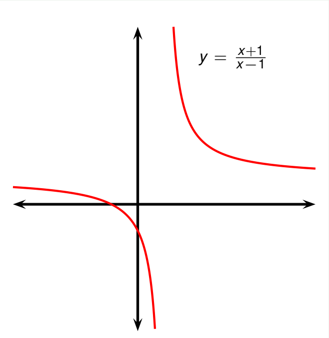
Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.

We deal with domains and ranges later:

$$\begin{array}{rcl|l} y &=& \dfrac{x+1}{x-1} & \text{mult. by } (x-1) \\ y(x-1) &=& x+1 & \\ x(y-1) &=& y+1 & \text{div. by } {\color{red}(y-1)} \\ f^{-1}(y) = x &=& \dfrac{y+1}{y-1} & \text{relabel } x, y \\ f^{-1}(x) &=& \dfrac{x+1}{x-1} & \end{array}$$

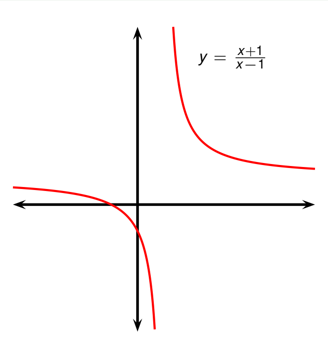
We divided by ${\color{red}y-1}$ so $y \neq 1$.



Answer: $f^{-1}(x) = \frac{x+1}{x-1}$

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.



We deal with domains and ranges later:

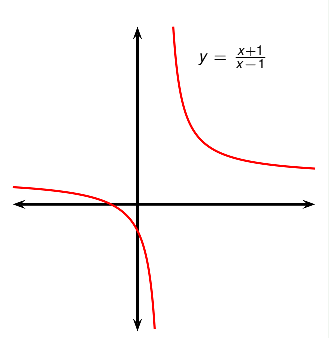
$$\begin{array}{rcl|l} y &=& \dfrac{x+1}{x-1} & \text{mult. by } (x-1) \\ y(x-1) &=& x+1 & \\ x(y-1) &=& y+1 & \text{div. by } (y-1) \\ f^{-1}(y) = x &=& \dfrac{y+1}{y-1} & \text{relabel } x, y \\ f^{-1}(x) &=& \dfrac{x+1}{x-1} & \end{array}$$

We divided by $y-1$ so $y \neq 1$. Therefore the domain of f^{-1} is all real numbers except 1.

Answer: $f^{-1}(x) = \frac{x+1}{x-1}$, $x \neq 1$.

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.



Answer: $f^{-1}(x) = \frac{x+1}{x-1}$, $x \neq 1$.

We deal with domains and ranges later:

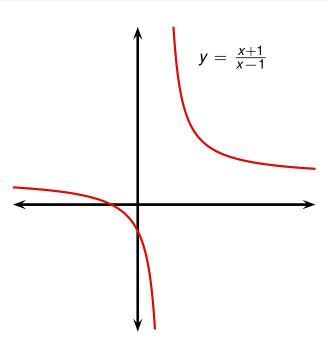
$$\begin{array}{rcl|l} y & = & \dfrac{x+1}{x-1} & \text{mult. by } (x-1) \\ y(x-1) & = & x+1 & \\ x(y-1) & = & y+1 & \text{div. by } (y-1) \\ f^{-1}(y) = x & = & \dfrac{y+1}{y-1} & \text{relabel } x, y \\ f^{-1}(x) & = & \dfrac{x+1}{x-1} & \end{array}$$

We divided by $y-1$ so $y \neq 1$. Therefore the domain of f^{-1} is all real numbers except 1.

Can a non-identity function be its own inverse?

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.



Answer: $f^{-1}(x) = \frac{x+1}{x-1}$, $x \neq 1$.

We deal with domains and ranges later:

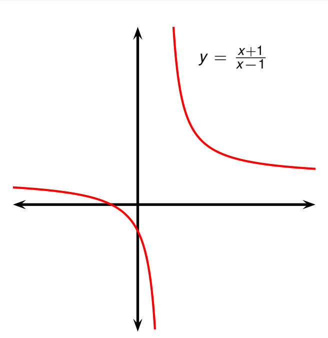
$$
\begin{array}{rcl|l}
y & = & \dfrac{x+1}{x-1} & \text{mult. by } (x-1) \\
y(x-1) & = & x+1 & \\
x(y-1) & = & y+1 & \text{div. by } (y-1) \\
f^{-1}(y) = x & = & \dfrac{y+1}{y-1} & \text{relabel } x, y \\
f^{-1}(x) & = & \dfrac{x+1}{x-1} &
\end{array}
$$

We divided by $y-1$ so $y \neq 1$. Therefore the domain of f^{-1} is all real numbers except 1.

Can a non-identity function be its own inverse? Yes, f is.

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.



Answer: $f^{-1}(x) = \frac{x+1}{x-1}$, $x \neq 1$.

We deal with domains and ranges later:

$$
\begin{array}{rcl|l}
y & = & \dfrac{x+1}{x-1} & \text{mult. by } (x-1) \\
y(x-1) & = & x+1 & \\
x(y-1) & = & y+1 & \text{div. by } (y-1) \\
f^{-1}(y) = x & = & \dfrac{y+1}{y-1} & \text{relabel } x, y \\
f^{-1}(x) & = & \dfrac{x+1}{x-1} &
\end{array}
$$

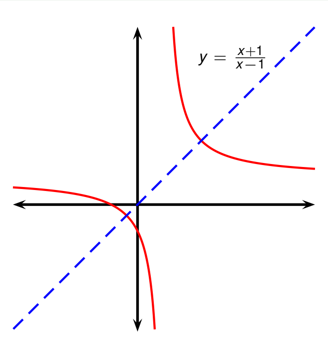
We divided by $y-1$ so $y \neq 1$. Therefore the domain of f^{-1} is all real numbers except 1.

Can a non-identity function be its own inverse? Yes, f is.

What does it mean for f to be its own inverse?

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.



Answer: $f^{-1}(x) = \frac{x+1}{x-1}$, $x \neq 1$.

We deal with domains and ranges later:

$$
\begin{array}{rcl|l}
y & = & \dfrac{x+1}{x-1} & \text{mult. by } (x-1) \\
y(x-1) & = & x+1 & \\
x(y-1) & = & y+1 & \text{div. by } (y-1) \\
f^{-1}(y) = x & = & \dfrac{y+1}{y-1} & \text{relabel } x, y \\
f^{-1}(x) & = & \dfrac{x+1}{x-1} &
\end{array}
$$

We divided by $y-1$ so $y \neq 1$. Therefore the domain of f^{-1} is all real numbers except 1.

Can a non-identity function be its own inverse? Yes, f is.
What does it mean for f to be its own inverse?
Graph of f is symmetric across $y = x$.