Precalculus
Angles

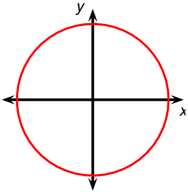
Todor Milev

2019

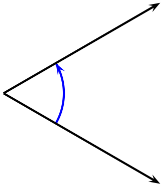
Outline

Definition

The *unit circle* is the circle with radius 1 and center at the center of the coordinate system.



Three Meanings of the Term Angle



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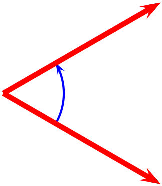

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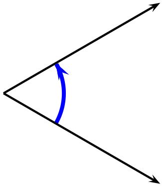

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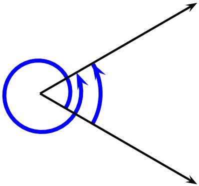

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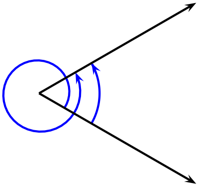

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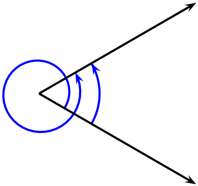

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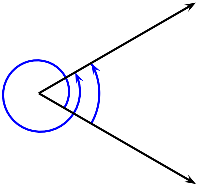

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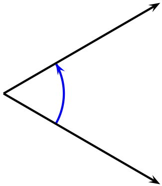
- Except for a few introductory slides, we take full advantage of this convention.

Geometric angle definition



Definition (Geometric angle)

A *geometric angle* (*angle* for short) is the figure formed by two rays, called arms, sharing a common endpoint called the vertex of the angle.

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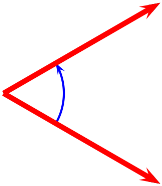

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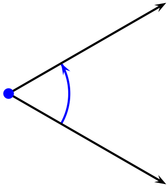

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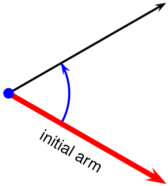

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A *geometric angle* (*angle* for short) is the figure formed by two rays, called arms, sharing a common endpoint called the vertex of the angle. The rays are ordered.

- The ray that comes first is called the initial arm (side) of the angle.

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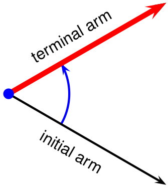

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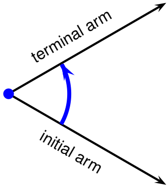

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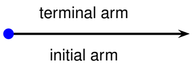

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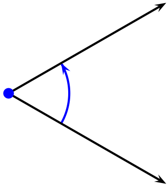
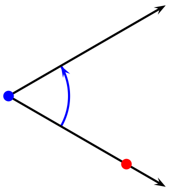
- The ray that comes second is called the terminal arm (side) of the angle.
- Angle measures are depicted as arcs pointing from the initial arm towards the terminal arm.
- By convention, the rays are allowed to coincide; the resulting angle is then called the *zero angle*.

Geometric angle definition



- A ray can be identified by its starting point and any other point on the ray.

Geometric angle definition



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- Therefore an angle can be identified by its vertex and one point on each of its arms.

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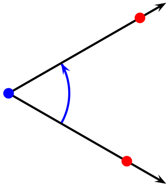

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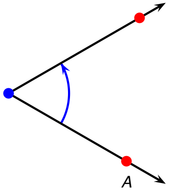

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- If A is pt. on the first ray and B on the second and O is the vertex, we denote the angle by $\angle AOB$.

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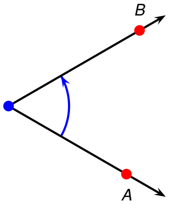

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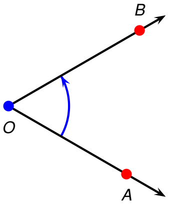

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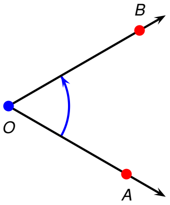

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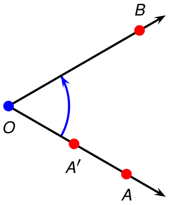

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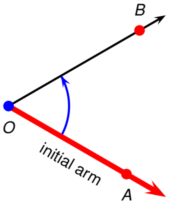

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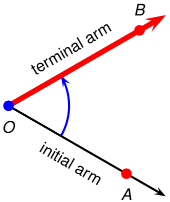

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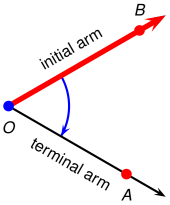

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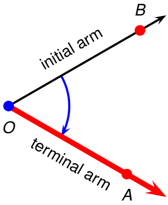

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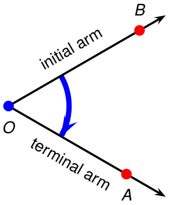

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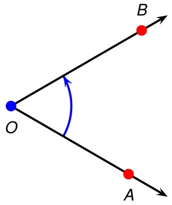

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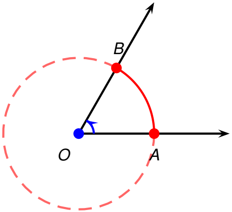
- In this way $\angle AOB \neq \angle BOA$.



Definition (Radian measure of geometric angle)

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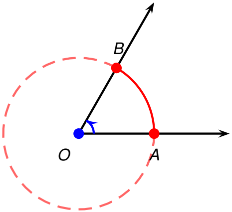

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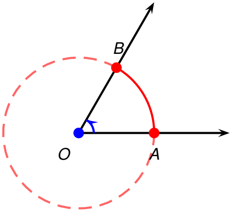

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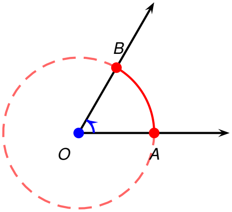

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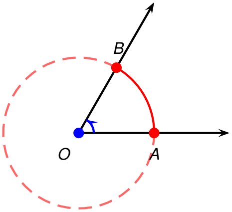

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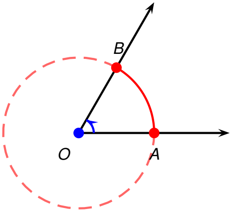

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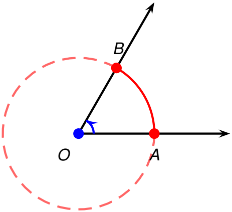

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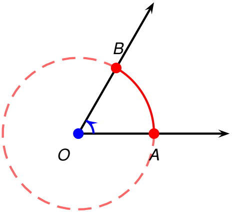

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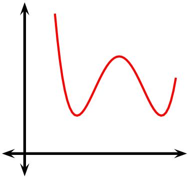
- Angle measures are frequently denoted by greek letters such as $\alpha, \beta, \gamma, \theta, \dots$.

Arc-length of a circle arc

- There is a definition of arc-length of arbitrary smooth curve.



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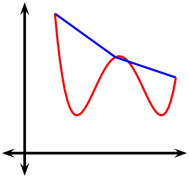

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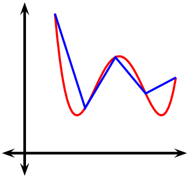

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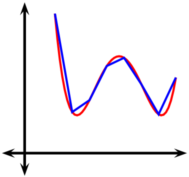

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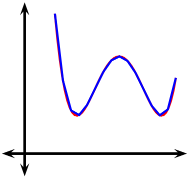

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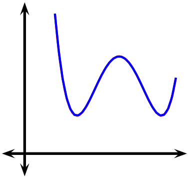
- The definition states that the arc-length of a smooth curve is the limit of the lengths of ever finer straight line approximations.



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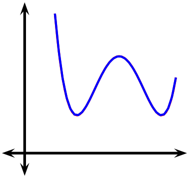

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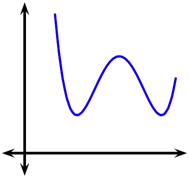
- Until then we ask the reader to think of arc-length of a curve as the quantity obtained by “aligning a rope along the curve” and measuring the “length of this rope”.



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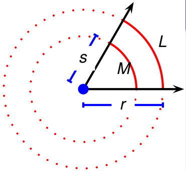

Proposition

Let two circles have common center and radii s and r. Suppose an arbitrary geometric angle with vertex at the common center of the circles cuts off short arcs of length M and L. Then $\frac{s}{r} = \frac{M}{L}$.

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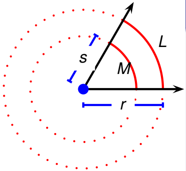

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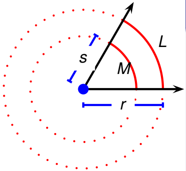

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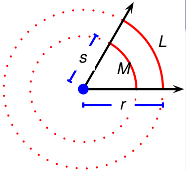

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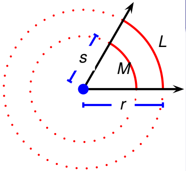

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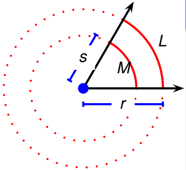

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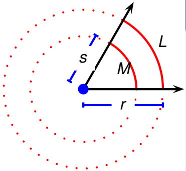

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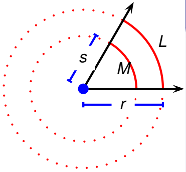

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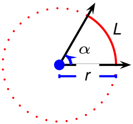
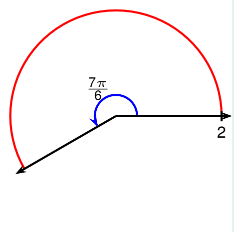


Corollary

The arc-length cut off by an angle with measure α from a circle of radius r equals αr.

Example



Find the length of an arc of a circle of radius 2 cut off by an angle of measure $\frac{7\pi}{6}$ $(= 210^\circ)$.

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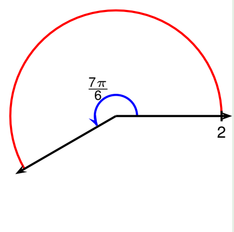

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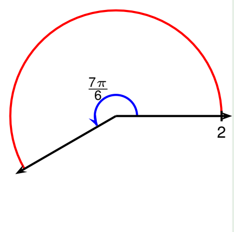

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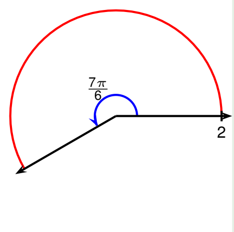

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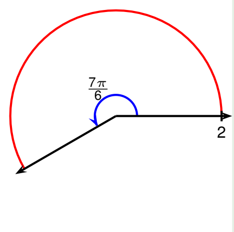

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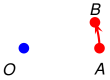

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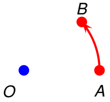

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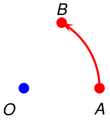

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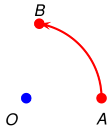

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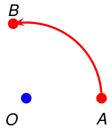

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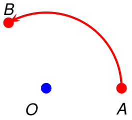

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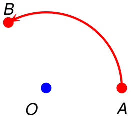

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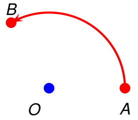

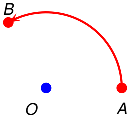

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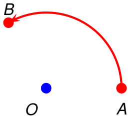

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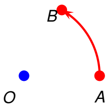
- Whether the term rotation refers to continuous rotation or to "instantaneous" rotation should be inferred from context.



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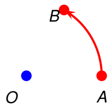

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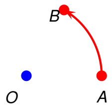

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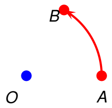

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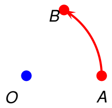

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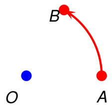

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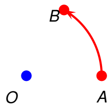

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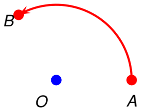

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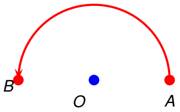

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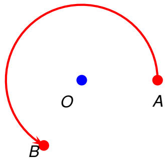

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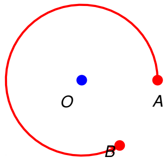

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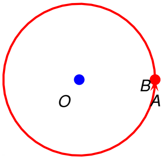

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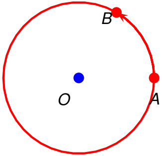

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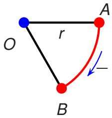

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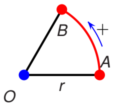

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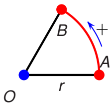

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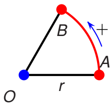

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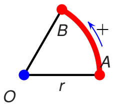

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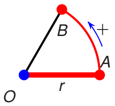

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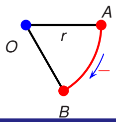

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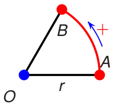

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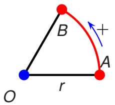

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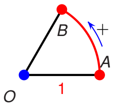

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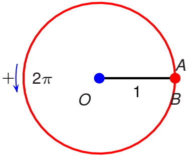

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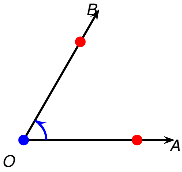
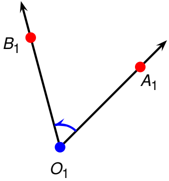
- A circle of radius 1 has circumference 2π, therefore a full counter-clockwise turn is measured by 2π radians.

Equivalence of angles





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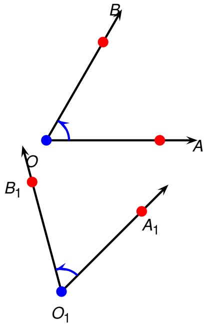

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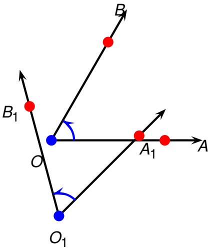

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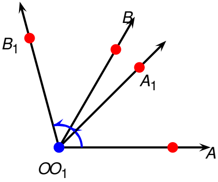

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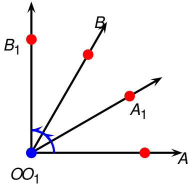

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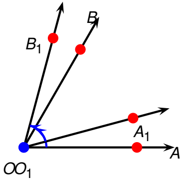

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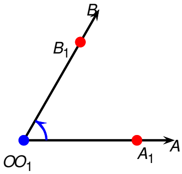

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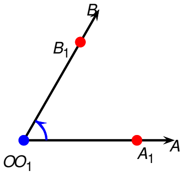

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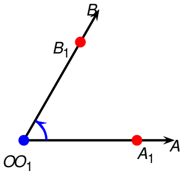

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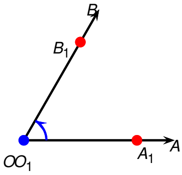

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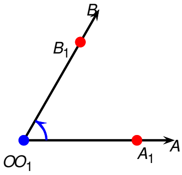

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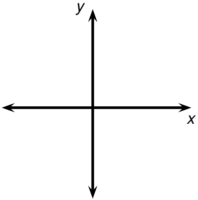
Angles and the coordinate system



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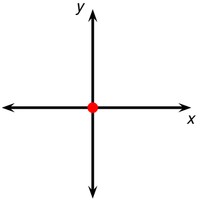

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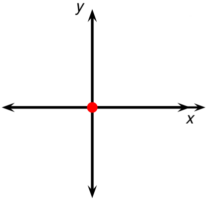

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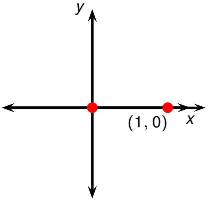

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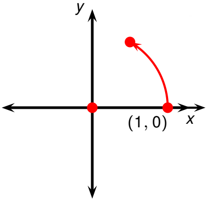

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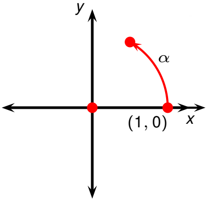

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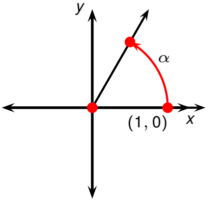

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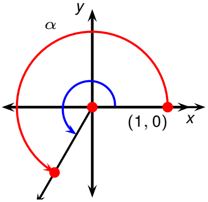

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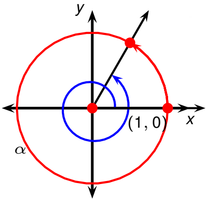

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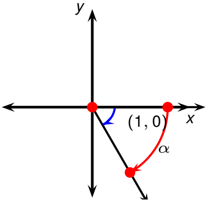

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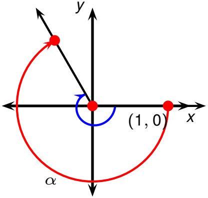

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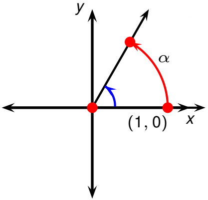

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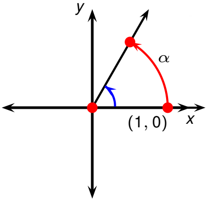

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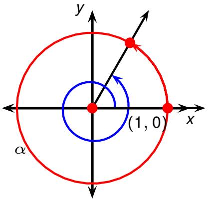

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- Degrees are useful because the most frequently encountered fractions of a half turn are measured by a whole number of degrees.

Degrees and radians

- Degrees is a unit for measuring angles, denoted by $^\circ$.
- The relationship between degrees and radians is:

$$\begin{aligned} \pi \text{ rad} &= 180^\circ \\ 1 \text{rad} &= \frac{180^\circ}{\pi} \approx 57.3^\circ \\ 1^\circ &= \frac{\pi}{180} \text{ rad} \approx 0.017 \text{ rad.} \end{aligned}$$

- In other words, a half-turn is measured by πrad or 180°.
- Degrees are useful because the most frequently encountered fractions of a half turn are measured by a whole number of degrees.
- If a measurement unit is not specified, it is implied to be radians. For example, in $\sin 5$, the number 5 stands for 5 radians.

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.							

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	**?**						

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$						

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	**?**					

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians).}$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$					

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	**?**				

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians).}$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$				

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	**?**			

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π			

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	**?**		

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π		

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians).}$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	?	

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians).}$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	**?**

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians).}$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.								

$$t^\circ = \frac{t}{180}\pi \text{ (radians).}$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	**?**							

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°							

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°	**?**						

$$t^\circ = \frac{t}{180}\pi \text{ (radians).}$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°	18°						

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°	18°	?					

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°	18°	330°					

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°	18°	330°	**?**				

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°	18°	330°	315°				

$$t^\circ = \frac{t}{180}\pi \text{ (radians).}$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°	18°	330°	315°	**?**			

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°	18°	330°	315°	$\frac{180}{7}^\circ \approx 25.7^\circ$			

$$t^\circ = \frac{t}{180}\pi \text{ (radians).}$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°	18°	330°	315°	$\frac{180}{7}^\circ \approx 25.7^\circ$	**?**		

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°	18°	330°	315°	$\frac{180}{7}^\circ \approx 25.7^\circ$	390°		

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°	18°	330°	315°	$\frac{180}{7}^\circ \approx 25.7^\circ$	390°	**?**	

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°	18°	330°	315°	$\frac{180}{7}^\circ \approx 25.7^\circ$	390°	-225°	

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°	18°	330°	315°	$\frac{180}{7}^\circ \approx 25.7^\circ$	390°	-225°	?

$$t^\circ = \frac{t}{180}\pi \text{ (radians)}.$$

Example

Convert from degrees to radians.

Deg.	45°	36°	-20°	360°	-720°	-225°	2015°
Rad.	$\frac{\pi}{4}$	$\frac{\pi}{5}$	$-\frac{\pi}{9}$	2π	-4π	$-\frac{5\pi}{4}$	$\frac{403}{36}\pi$

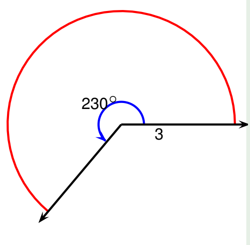
$$x = \frac{x}{\pi}180^\circ.$$

Example

Convert from radians to degrees.

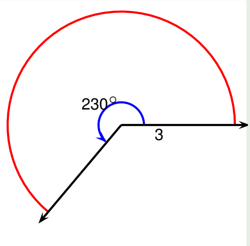
Rad.	$\frac{\pi}{3}$	$\frac{\pi}{10}$	$\frac{11\pi}{6}$	$\frac{7\pi}{4}$	$\frac{\pi}{7}$	$\frac{13\pi}{6}$	$-\frac{5\pi}{4}$	2
Deg.	60°	18°	330°	315°	$\frac{180}{7}^\circ \approx 25.7^\circ$	390°	-225°	$\frac{2}{\pi}180^\circ \approx 114.6^\circ$

Example



Find the length of an arc of a circle of radius 3 cut off by an angle of measure 230°.

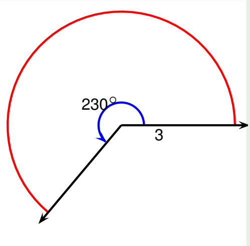
Example



Find the length of an arc of a circle of radius 3 cut off by an angle of measure 230°.

$$\text{arc-length} = \alpha r$$

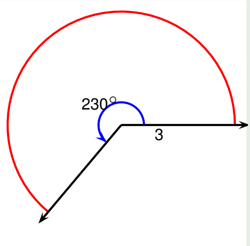
Example



Find the length of an arc of a circle of radius 3 cut off by an angle of measure 230°.

arc-length $= \alpha r = \textbf{?} \quad \cdot 3$

Example

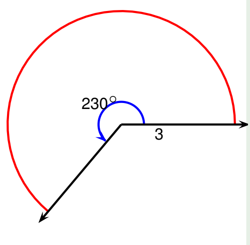


Find the length of an arc of a circle of radius 3 cut off by an angle of measure 230°.

$$\alpha = 230^\circ$$

$$\text{arc-length} = \alpha r = \textbf{?} \quad \cdot 3$$

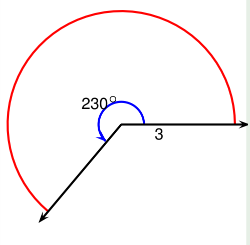
Example



Find the length of an arc of a circle of radius 3 cut off by an angle of measure 230°.

$$\begin{aligned} \alpha &= 230^\circ \\ &= \textbf{?} \qquad \Big| \text{ Convert to radians} \\ \text{arc-length} &= \alpha r = \textbf{?} \quad \cdot 3 \end{aligned}$$

Example

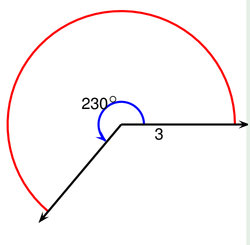


Find the length of an arc of a circle of radius 3 cut off by an angle of measure 230°.

$$\begin{aligned} \alpha &= 230^\circ \\ &= 230^\circ \frac{\pi \text{ rad}}{180^\circ} \\ \text{arc-length} &= \alpha r = \textbf{?} \quad \cdot 3 \end{aligned}$$

Convert to radians

Example

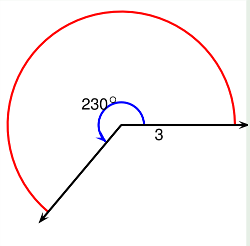


Find the length of an arc of a circle of radius 3 cut off by an angle of measure 230°.

$$\begin{aligned} \alpha &= 230^\circ \\ &= 230^\circ \frac{\pi \text{ rad}}{180^\circ} = \frac{23}{18}\pi \text{ rad} \\ \text{arc-length} &= \alpha r = \textbf{?} \quad \cdot 3 \end{aligned}$$

Convert to radians

Example



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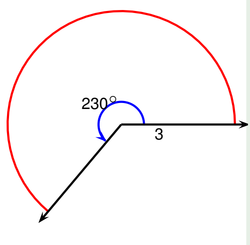
Convert to radians

Example



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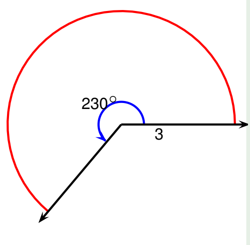
$$
\begin{aligned}
\alpha &= 230^\circ \\
&= 230^\circ \frac{\pi \text{ rad}}{180^\circ} = \frac{23}{18}\pi \text{ rad} \qquad \Big| \text{ Convert to radians} \\
\text{arc-length} &= \alpha r = \frac{23\pi}{18} \cdot 3 = \frac{23\pi}{6}
\end{aligned}
$$

Example



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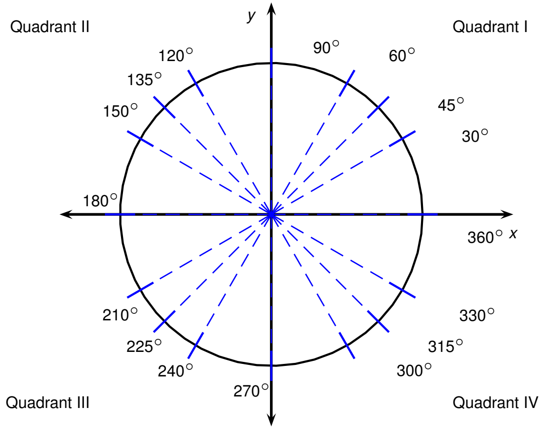
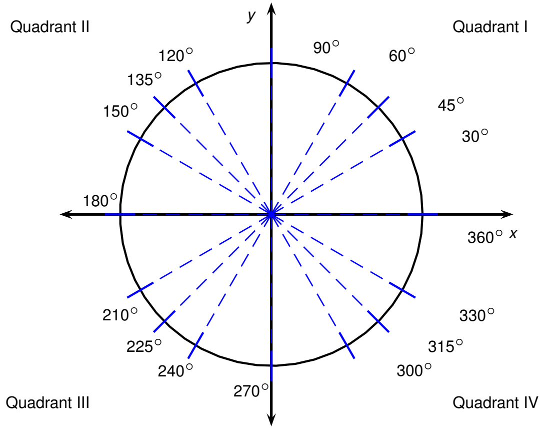
$$\begin{aligned} \alpha &= 230^\circ \\ &= 230^\circ \frac{\pi \text{ rad}}{180^\circ} = \frac{23}{18}\pi \text{ rad} && \Big| \text{ Convert to radians} \\ \text{arc-length} &= \alpha r = \frac{23\pi}{18} \cdot 3 = \frac{23\pi}{6} \approx 12.043 \end{aligned}$$



The most frequently encountered angles are given in the table below.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	**?**										



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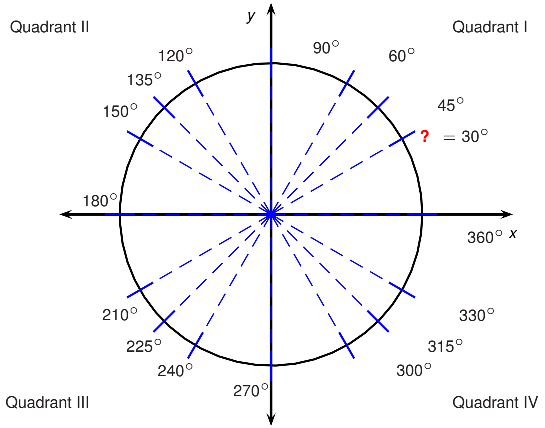
Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0										



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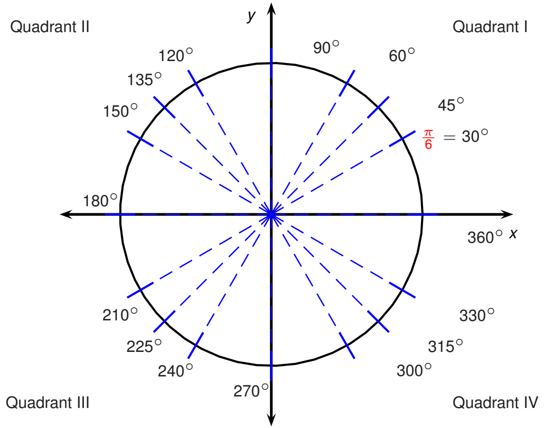

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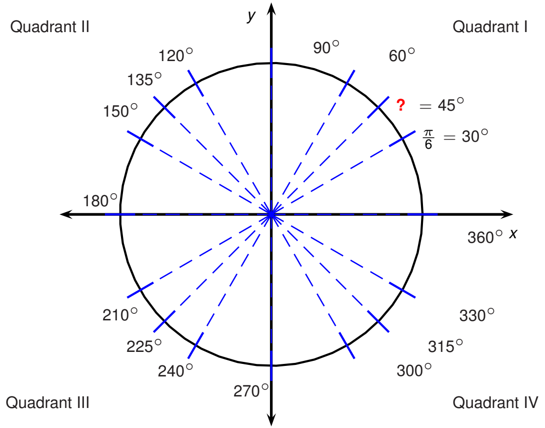

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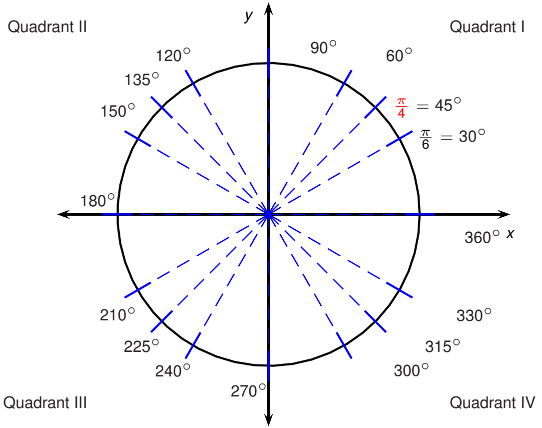

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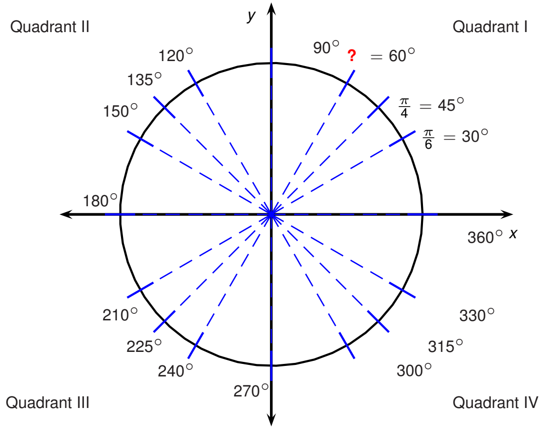
Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$								



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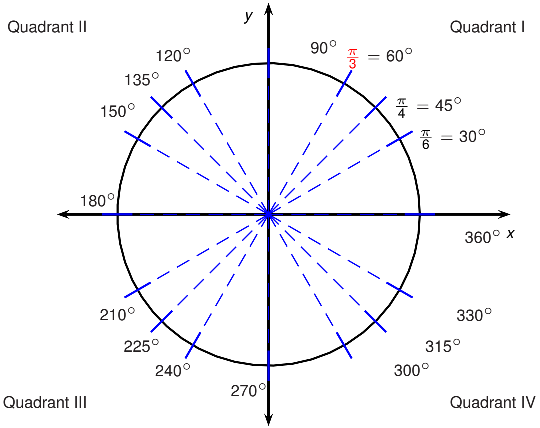
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	?							



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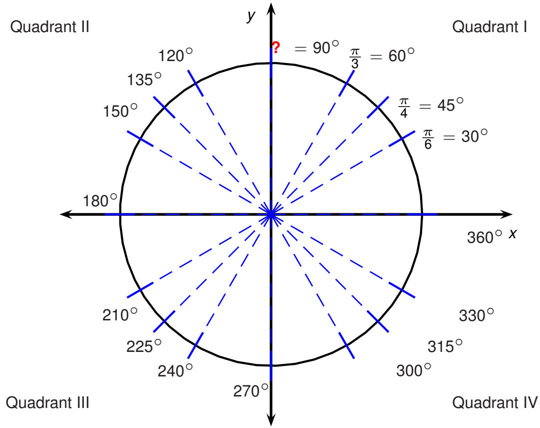
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$							



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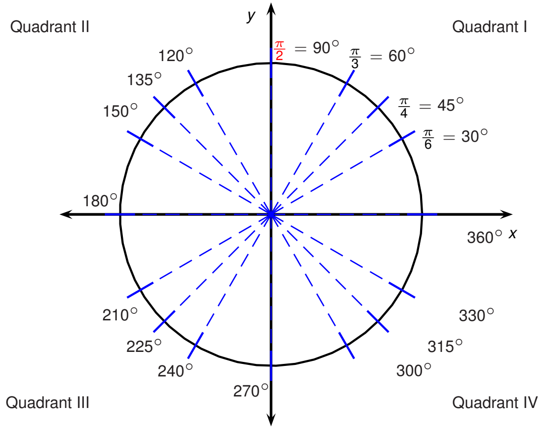

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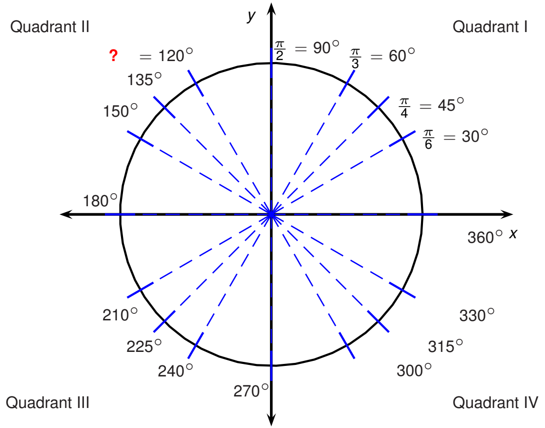

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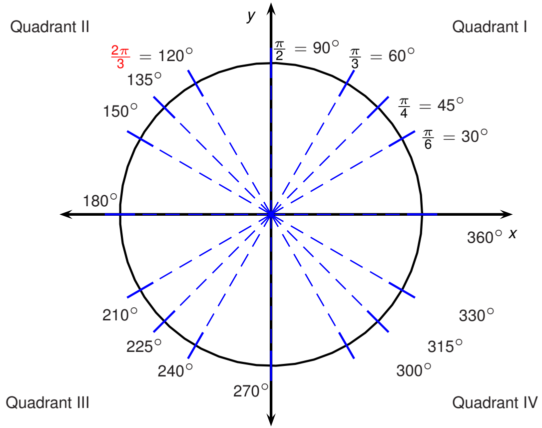

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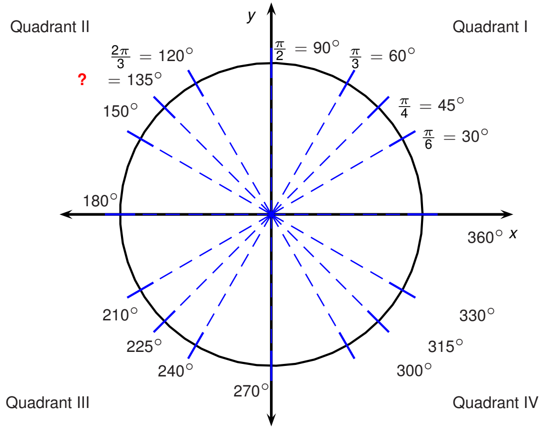

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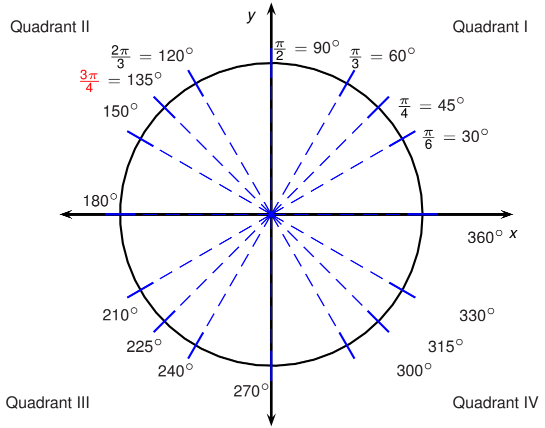
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	?				



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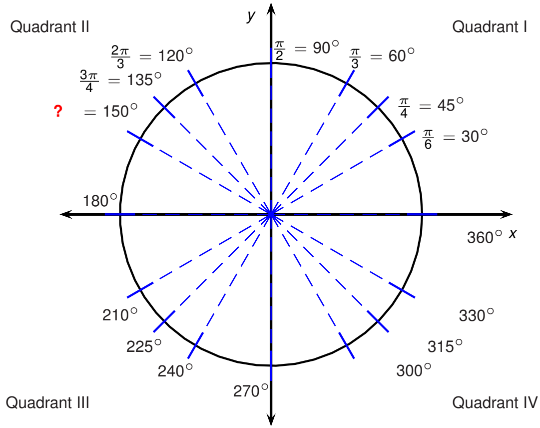

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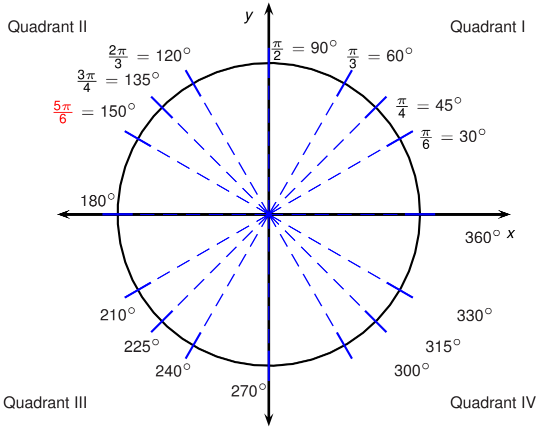

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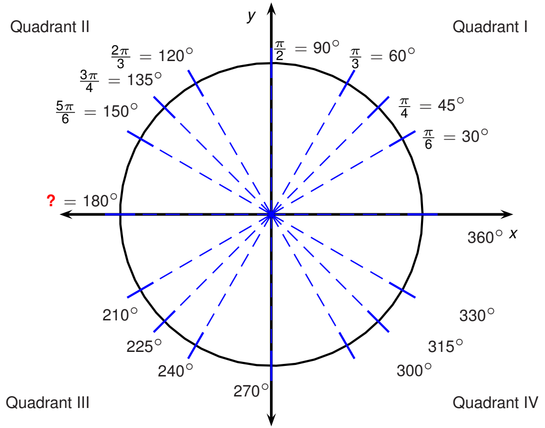

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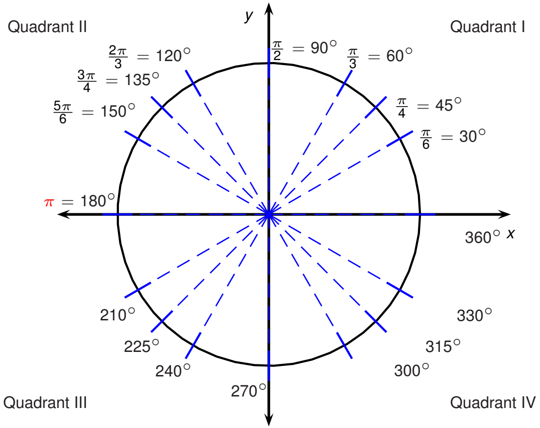

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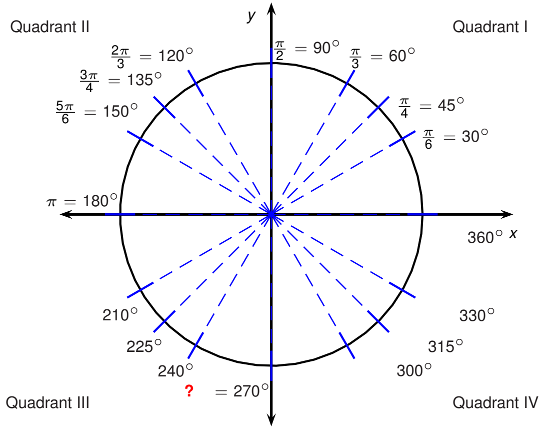

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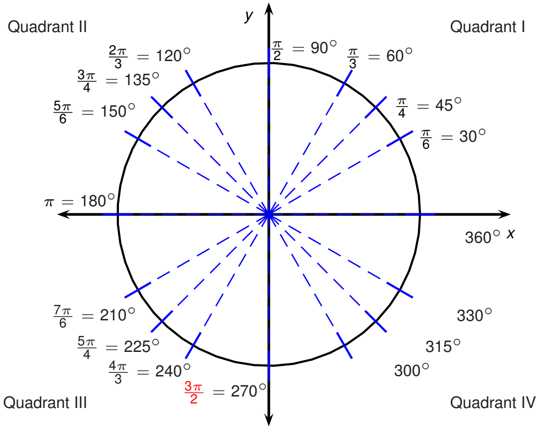

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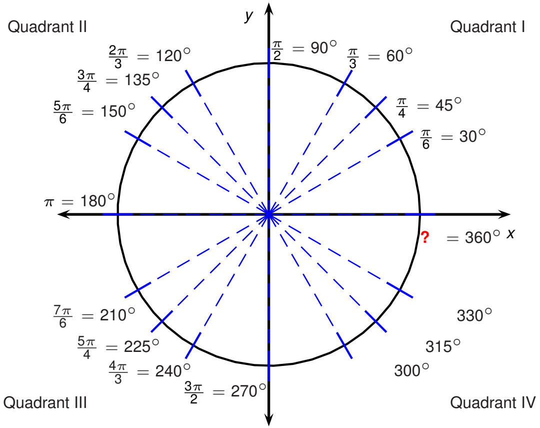

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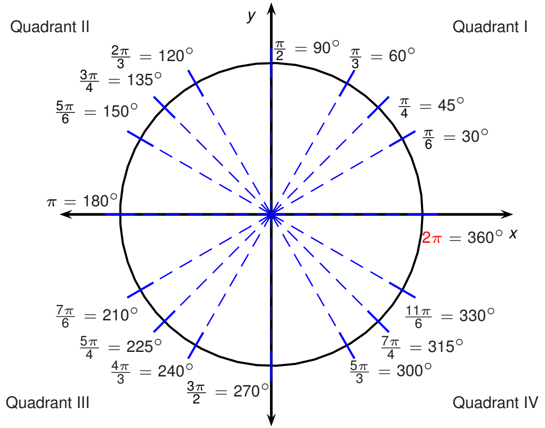

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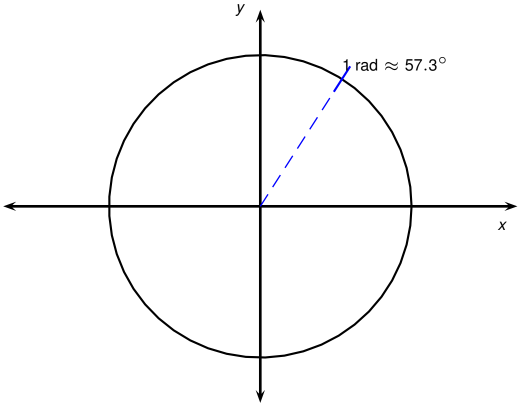
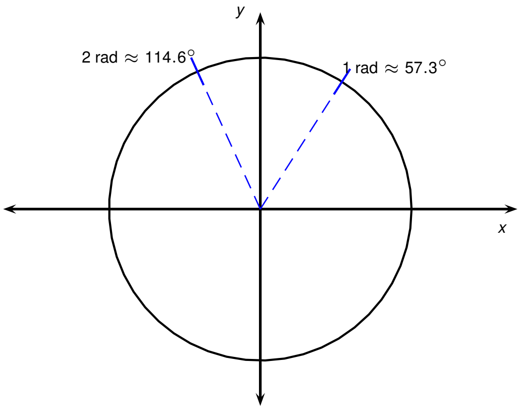


- Integer quantities of radians are not rational multiples of (the measure of) a half-turn and are not easy to compute with.



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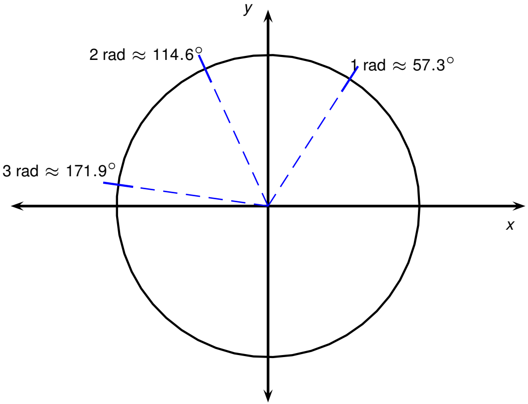

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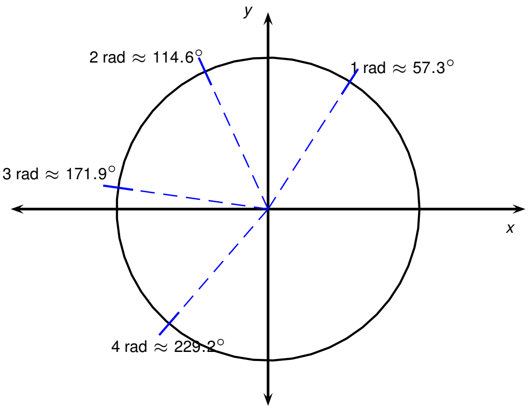

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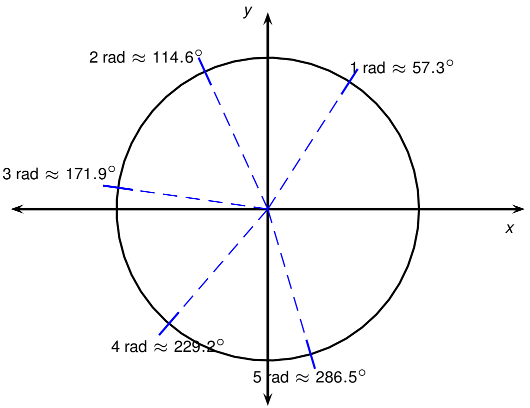

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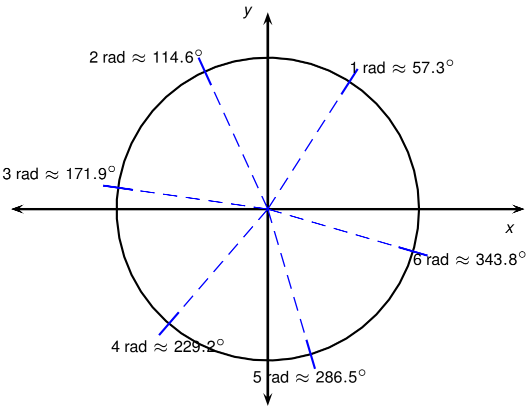

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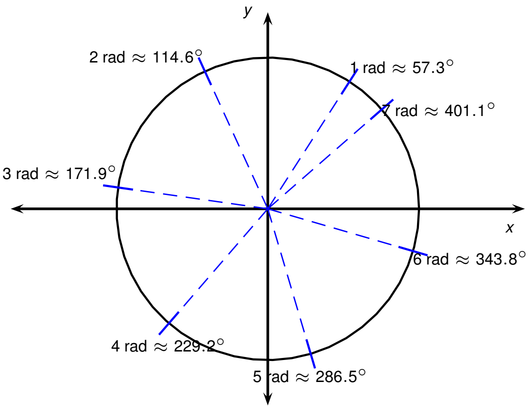

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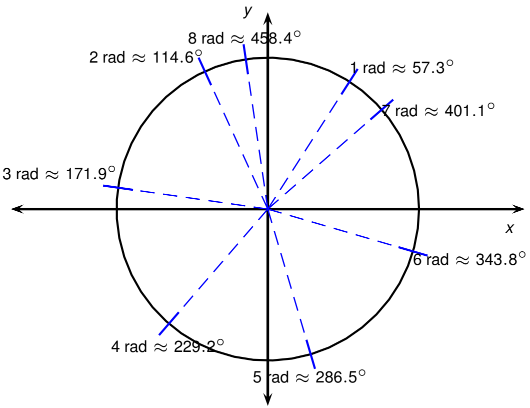

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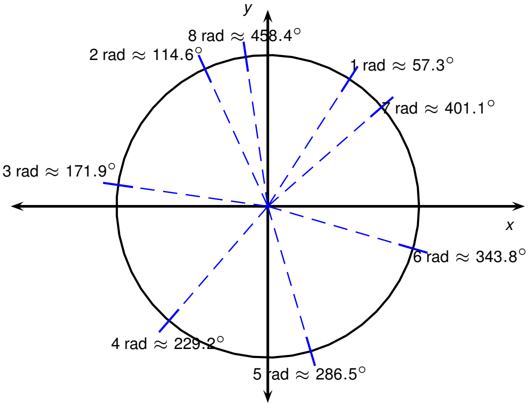
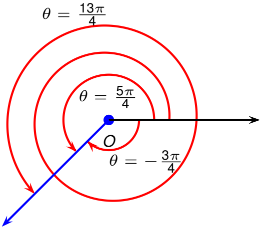


- Integer quantities of radians are not rational multiples of (the measure of) a half-turn and are not easy to compute with.
- For example to determine in which quadrant is an angle of k radians located one needs to know the numerical value of $\frac{k}{\pi}$, which requires knowledge of π with great numerical accuracy.

Definition (Coterminal Angles)

Two angles (angle measures) are called coterminal if the corresponding geometric angles have the same initial and terminal sides.



Observation

The set of angles coterminal with α consists of the angles $\alpha + 2k\pi$, where k runs over the set of integers. In other words, the angles coterminal with α are the angles:

$$\dots, \alpha - 6\pi, \alpha - 4\pi, \alpha - 2\pi, \alpha, \alpha + 2\pi, \alpha + 4\pi, \alpha + 6\pi, \dots .$$

Example

- Find all angles that are coterminal to $\frac{\pi}{4}$.
- Find all angles in the interval $[-2\pi, \pi]$ that are coterminal to $\frac{\pi}{4}$.

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By theory, the angles coterminal with $\frac{\pi}{4}$ are all angles of the form

$$\frac{\pi}{4} + 2k\pi.$$

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To find which among the angles $\frac{\pi}{4} + 2k\pi$ lie in the interval $[-2\pi, \pi]$, we write them as an infinite list

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$$\ldots, \frac{\pi}{4} - 4\pi, \frac{\pi}{4} - 2\pi, \frac{\pi}{4}, \frac{\pi}{4} + 2\pi, \frac{\pi}{4} + 4\pi, \ldots.$$

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$$\dots, \frac{\pi}{4} - 4\pi, \frac{\pi}{4} - 2\pi, \frac{\pi}{4}, \frac{\pi}{4} + 2\pi, \frac{\pi}{4} + 4\pi, \dots.$$

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$$\frac{\pi}{4} + 2k\pi.$$

To find which among the angles $\frac{\pi}{4} + 2k\pi$ lie in the interval $[-2\pi, \pi]$, we write them as an infinite list (we indicate the unboundedness of the list by ellipsis dots) and cross out the angles that lie outside of the desired interval.

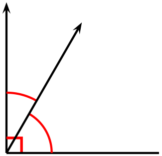
$$\cancel{\dots}, \cancel{\frac{\pi}{4} - 4\pi}, \frac{\pi}{4} - 2\pi, \frac{\pi}{4}, \cancel{\frac{\pi}{4} + 2\pi}, \cancel{\frac{\pi}{4} + 4\pi}, \cancel{\dots}.$$

Our final answer is $-\frac{7\pi}{4}, \frac{\pi}{4}$

Complementary angles

Definition

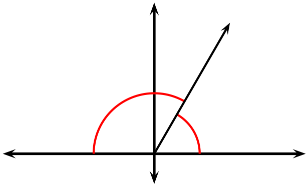
Two positive angles are called complementary when they sum to a right angle, i.e., an angle of measure $\frac{\pi}{2} = 90^\circ$.



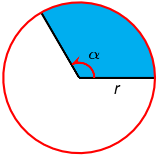
Supplementary angles

Definition

Two positive angles are called supplementary when they sum to $\pi = 180^\circ$.



A sector of a circle is the region cut off from a circle by an angle whose vertex is at the center of the circle.



Proposition (Area of a circle sector)

The area of a circle sector equals

$$\frac{1}{2}\alpha r^2,$$

where α is the angle of the sector.