Precalculus
Basic algebraic properties of the logarithm

Todor Milev

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Theorem (Properties of Logarithmic Functions)

If $a > 1$, the function $f(x) = \log_a x$ is a one-to-one, continuous, increasing function with domain $(0, \infty)$ and range $\mathbb{R}$. If $x, y, a, b > 0$ and r is any real number, then

1. $\log_a(xy) = \log_a x + \log_a y.$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y.$
3. $\log_a(x^r) = r \log_a x.$
4. $\log_a(x) = \log_b x \log_a b = \frac{\log_b x}{\log_b a} = \frac{\ln x}{\ln a}.$