Precalculus
Equations involving logarithms and exponents

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Outline

1 Equations involving logarithms

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1. Equations involving logarithms
2. Equations involving exponents

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2. Equations involving exponents
3. Inverse function problems and exponents

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1. Equations involving logarithms
2. Equations involving exponents
3. Inverse function problems and exponents
4. Basic exponential inequalities

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Example

Solve the equation.

$$\log_3(2x^2+1) = 2$$

Example

Solve the equation.

$$\begin{array}{rcl} \log_3(2x^2+1) & = & 2 \\ 3^{\log_3(2x^2+1)} & = & 3^2 \end{array}$$

| Exponentiate base 3

Example

Solve the equation.

$$
\begin{aligned}
\log_3(2x^2+1) &= 2 \\
3^{\log_3(2x^2+1)} &= 3^2 \\
2x^2+1 &= 9
\end{aligned}
$$

| Exponentiate base 3

Example

Solve the equation.

$$\begin{aligned} \log_3(2x^2+1) &= 2 \\ 3^{\log_3(2x^2+1)} &= 3^2 \\ 2x^2+1 &= 9 \\ 2x^2 &= 8 \end{aligned}$$

| Exponentiate base 3

Example

Solve the equation.

$$\begin{aligned} \log_3(2x^2+1) &= 2 && \Big| \text{ Exponentiate base 3} \\ 3^{\log_3(2x^2+1)} &= 3^2 \\ 2x^2+1 &= 9 \\ 2x^2 &= 8 \\ x^2 &= \frac{8}{2} = 4 \end{aligned}$$

Example

Solve the equation.

$$
\begin{aligned}
\log_3(2x^2+1) &= 2 && \Big| \text{ Exponentiate base 3} \\
3^{\log_3(2x^2+1)} &= 3^2 \\
2x^2+1 &= 9 \\
2x^2 &= 8 \\
x^2 &= \frac{8}{2} = 4 \\
x &= \pm\sqrt{4} = \pm 2
\end{aligned}
$$

Example

Solve the equation.

$$
\begin{array}{rcl|l}
\log_3(2x^2+1) & = & 2 & \text{Exponentiate base 3} \\
3^{\log_3(2x^2+1)} & = & 3^2 & \\
2x^2+1 & = & 9 & \\
2x^2 & = & 8 & \\
x^2 & = & \frac{8}{2} = 4 & \\
x & = & \pm\sqrt{4} = \pm 2 & \\
x = 2 & \text{or} & x = -2 & \text{final answer}
\end{array}
$$

The logarithmic property $\log_a(xy) = \log_a x + \log_a y$ holds only for positive x, y. Failure to check the positivity of x, y can result in extraneous (fake) solutions to logarithmic equations.

Example

Solve the equation.

$$\log_2(x+2) + \log_2(x-1) = 2$$

The logarithmic property $\log_a(xy) = \log_a x + \log_a y$ holds only for positive x, y. Failure to check the positivity of x, y can result in extraneous (fake) solutions to logarithmic equations.

Example

Solve the equation.

$$\begin{array}{rcll}
\log_2(x+2) + \log_2(x-1) & = & 2 & \Big| \text{ Domain: } x > 1 \\
\log_2\left((x+2)(x-1)\right) & = & 2 & \Big| \text{ Exponentiate base 2} \\
(x+2)(x-1) & = & 2^2 & \\
x^2 + x - 2 & = & 4 & \\
x^2 + x - 6 & = & 0 & \\
(x-2)(x+3) & = & 0 & \\
x = 2 & \text{or} & \cancel{x = -3} &
\end{array}$$

$x = -3$ not a solution (outside of domain)

Example (Solve exponential equation without logarithms)

Solve for t.

$$16^{4t} = 8^{t-2}$$

Example (Solve exponential equation without logarithms)

Solve for t.

$$
\begin{array}{rcl}
16^{4t} & = & 8^{t-2} \\
\text{Find a common base: } \left(\mathbf{?}\ \right)^{4t} & = & \left(\mathbf{?}\ \right)^{t-2}
\end{array}
$$

Example (Solve exponential equation without logarithms)

Solve for t.

$$16^{4t} = 8^{t-2}$$

Find a common base: $$\left(2^4\right)^{4t} = \left(2^3\right)^{t-2}$$

Example (Solve exponential equation without logarithms)

Solve for t.

$$\begin{array}{rrcl} & 16^{4t} & = & 8^{t-2} \\ \text{Find a common base:} & \left(2^4\right)^{4t} & = & \left(2^3\right)^{t-2} \\ & 2^{16t} & = & 2^{3t-6} \end{array}$$

Example (Solve exponential equation without logarithms)

Solve for t.

$$\begin{array}{rrcl}
 & 16^{4t} & = & 8^{t-2} \\
\text{Find a common base:} & \left(2^4\right)^{4t} & = & \left(2^3\right)^{t-2} \\
 & 2^{16t} & = & 2^{3t-6} \\
 & 16t & = & 3t-6
\end{array}$$

Example (Solve exponential equation without logarithms)

Solve for t.

$$
\begin{array}{rrcl}
 & 16^{4t} & = & 8^{t-2} \\
\text{Find a common base:} & \left(2^4\right)^{4t} & = & \left(2^3\right)^{t-2} \\
 & 2^{16t} & = & 2^{3t-6} \\
 & 16t & = & 3t-6 \\
 & 13t & = & -6
\end{array}
$$

Example (Solve exponential equation without logarithms)

Solve for t.

Find a common base:

$$\begin{aligned} 16^{4t} &= 8^{t-2} \\ \left(2^4\right)^{4t} &= \left(2^3\right)^{t-2} \\ 2^{16t} &= 2^{3t-6} \\ 16t &= 3t-6 \\ 13t &= -6 \\ t &= -\frac{6}{13}. \end{aligned}$$

Example

Solve the equation.

$$2^{1-5x} = 12$$

Example

Solve the equation.

$$\begin{aligned} 2^{1-5x} &= 12 && \Big|\ \text{apply } \log_2 \\ \log_2(2^{1-5x}) &= \log_2 12 \end{aligned}$$

Example

Solve the equation.

$$\begin{array}{rcll} 2^{1-5x} & = & 12 & \Big|\ \text{apply } \log_2 \\ {\color{red}\log_2}({\color{red}2}^{1-5x}) & = & \log_2 12 & \\ 1-5x & = & \log_2 12 & \end{array}$$

Example

Solve the equation.

$$\begin{aligned} 2^{1-5x} &= 12 && \Big| \text{ apply } \log_2 \\ \log_2(2^{1-5x}) &= \log_2 12 \\ 1-5x &= \log_2 12 = \mathbf{?} \end{aligned}$$

Example

Solve the equation.

$$\begin{aligned} 2^{1-5x} &= 12 && \Big| \text{ apply } \log_2 \\ \log_2(2^{1-5x}) &= \log_2 12 \\ 1-5x &= \log_2 12 = \log_2(4\cdot 3) \end{aligned}$$

Example

Solve the equation.

$$\begin{array}{rcl} 2^{1-5x} & = & 12 \qquad\qquad\qquad \Big| \text{ apply } \log_2 \\ \log_2(2^{1-5x}) & = & \log_2 12 \\ 1-5x & = & \log_2 12 = \log_2(4\cdot 3) \\ 1-5x & = & \log_2 4 + \log_2 3 \end{array}$$

Example

Solve the equation.

$$\begin{array}{rcll} 2^{1-5x} & = & 12 & \Big|\ \text{apply } \log_2 \\ \log_2(2^{1-5x}) & = & \log_2 12 & \\ 1-5x & = & \log_2 12 = \log_2(4\cdot 3) & \\ 1-5x & = & \log_2 4 + \log_2 3 & \\ 1-5x & = & \mathbf{?} + \log_2 3 & \end{array}$$

Example

Solve the equation.

$$
\begin{array}{rcll}
2^{1-5x} & = & 12 & \Big| \text{ apply } \log_2 \\
\log_2(2^{1-5x}) & = & \log_2 12 & \\
1-5x & = & \log_2 12 = \log_2(4\cdot 3) & \\
1-5x & = & \color{red}{\log_2 4} + \log_2 3 & \\
1-5x & = & \color{red}{2} + \log_2 3 &
\end{array}
$$

Example

Solve the equation.

$$
\begin{aligned}
2^{1-5x} &= 12 && \Big|\ \text{apply } \log_2 \\
\log_2(2^{1-5x}) &= \log_2 12 \\
1-5x &= \log_2 12 = \log_2(4\cdot 3) \\
1-5x &= \log_2 4 + \log_2 3 \\
1-5x &= 2 + \log_2 3 \\
5x &= 1 - (2 + \log_2 3)
\end{aligned}
$$

Example

Solve the equation.

$$
\begin{aligned}
2^{1-5x} &= 12 && \Big|\ \text{apply } \log_2\\
\log_2(2^{1-5x}) &= \log_2 12\\
1-5x &= \log_2 12 = \log_2(4\cdot 3)\\
1-5x &= \log_2 4 + \log_2 3\\
1-5x &= 2+\log_2 3\\
5x &= 1-(2+\log_2 3)\\
x &= \frac{-1}{}
\end{aligned}
$$

Example

Solve the equation.

$$
\begin{array}{rcll}
2^{1-5x} &=& 12 & \Big| \text{ apply } \log_2 \\
\log_2(2^{1-5x}) &=& \log_2 12 & \\
1-5x &=& \log_2 12 = \log_2(4\cdot 3) & \\
1-5x &=& \log_2 4 + \log_2 3 & \\
1-5x &=& 2 + \log_2 3 & \\
5x &=& 1 - (2 + \log_2 3) & \\
x &=& \dfrac{-1 - \log_2 3}{} &
\end{array}
$$

Example

Solve the equation.

$$
\begin{array}{rcll}
2^{1-5x} & = & 12 & \Big| \text{ apply } \log_2 \\
\log_2(2^{1-5x}) & = & \log_2 12 & \\
1-5x & = & \log_2 12 = \log_2(4\cdot 3) & \\
1-5x & = & \log_2 4 + \log_2 3 & \\
1-5x & = & 2+\log_2 3 & \\
{\color{red}5}x & = & 1-(2+\log_2 3) & \\
x & = & \dfrac{-1-\log_2 3}{{\color{red}5}} &
\end{array}
$$

Example

Solve the equation.

$$
\begin{array}{rcll}
2^{1-5x} & = & 12 & \Big| \text{ apply } \log_2 \\
\log_2(2^{1-5x}) & = & \log_2 12 & \\
1-5x & = & \log_2 12 = \log_2(4\cdot 3) & \\
1-5x & = & \log_2 4 + \log_2 3 & \\
1-5x & = & 2+\log_2 3 & \\
5x & = & 1-(2+\log_2 3) & \\
x & = & \dfrac{-1-\log_2 3}{5} & \\
\text{Calculator: } \quad x & \approx & -0.516993. &
\end{array}
$$

Example

Solve the equation.

$$e^{x-3} = 2e^{2x-1}$$

Example

Solve the equation.

$$\begin{aligned} e^{x-3} &= 2e^{2x-1} \\ \frac{e^{x-3}}{e^{2x-1}} &= 2 \end{aligned}$$

Divide by e^{2x-1}

Example

Solve the equation.

$$
\begin{aligned}
e^{x-3} &= 2e^{2x-1} && \Big| \text{ Divide by } e^{2x-1} \\
\frac{e^{x-3}}{e^{2x-1}} &= 2 \\
e^{x-3-(2x-1)} &= 2
\end{aligned}
$$

Example

Solve the equation.

$$\begin{aligned} e^{x-3} &= 2e^{2x-1} && \Big|\ \text{Divide by } e^{2x-1} \\ \frac{e^{x-3}}{e^{2x-1}} &= 2 \\ e^{x-3-(2x-1)} &= 2 \end{aligned}$$

Example

Solve the equation.

$$\begin{aligned} e^{x-3} &= 2e^{2x-1} \\ \frac{e^{x-3}}{e^{2x-1}} &= 2 \\ e^{x-3-(2x-1)} &= 2 \\ e^{-x-2} &= 2 \end{aligned}$$

Divide by e^{2x-1}

Example

Solve the equation.

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\begin{aligned}
e^{x-3} &= 2e^{2x-1} && \Big| \text{ Divide by } e^{2x-1}\\
\frac{e^{x-3}}{e^{2x-1}} &= 2\\
e^{x-3-(2x-1)} &= 2\\
e^{-x-2} &= 2
\end{aligned}
$$

Example

Solve the equation.

$$\begin{aligned} e^{x-3} &= 2e^{2x-1} && \Big| \text{ Divide by } e^{2x-1} \\ \frac{e^{x-3}}{e^{2x-1}} &= 2 \\ e^{x-3-(2x-1)} &= 2 \\ {\color{red}e}^{-x-2} &= 2 && \Big| \text{ Apply } {\color{red}\ln} \\ -x-2 &= \ln 2 \end{aligned}$$

Example

Solve the equation.

$$\begin{aligned} e^{x-3} &= 2e^{2x-1} && \Big| \text{ Divide by } e^{2x-1} \\ \frac{e^{x-3}}{e^{2x-1}} &= 2 \\ e^{x-3-(2x-1)} &= 2 \\ e^{-x-2} &= 2 && \Big| \text{ Apply } \ln \\ -x-2 &= \ln 2 \end{aligned}$$

Example

Solve the equation.

$$\begin{aligned} e^{x-3} &= 2e^{2x-1} && \Big| \text{ Divide by } e^{2x-1} \\ \frac{e^{x-3}}{e^{2x-1}} &= 2 \\ e^{x-3-(2x-1)} &= 2 \\ e^{-x-2} &= 2 && \Big| \text{ Apply } \ln \\ -x-2 &= \ln 2 \\ -x &= \ln 2 + 2 \end{aligned}$$

Example

Solve the equation.

$$\begin{aligned}
e^{x-3} &= 2e^{2x-1} && \Big| \text{ Divide by } e^{2x-1}\\
\frac{e^{x-3}}{e^{2x-1}} &= 2\\
e^{x-3-(2x-1)} &= 2\\
e^{-x-2} &= 2 && \Big| \text{ Apply } \ln\\
-x-2 &= \ln 2\\
-x &= \ln 2 + 2\\
x &= -(\ln 2 + 2)
\end{aligned}$$

Example

Solve the equation.

$$
\begin{aligned}
e^{x-3} &= 2e^{2x-1} && \Big| \text{ Divide by } e^{2x-1}\\
\frac{e^{x-3}}{e^{2x-1}} &= 2\\
e^{x-3-(2x-1)} &= 2\\
e^{-x-2} &= 2 && \Big| \text{ Apply } \ln\\
-x-2 &= \ln 2\\
-x &= \ln 2 + 2\\
x &= -(\ln 2 + 2)\\
x &= -\ln 2 - 2 && \Big| \text{ Final answer}
\end{aligned}
$$

Example

Solve the equation.

$$
\begin{aligned}
e^{x-3} &= 2e^{2x-1} && \text{Divide by } e^{2x-1}\\
\frac{e^{x-3}}{e^{2x-1}} &= 2\\
e^{x-3-(2x-1)} &= 2\\
e^{-x-2} &= 2 && \text{Apply } \ln\\
-x-2 &= \ln 2\\
-x &= \ln 2 + 2\\
x &= -(\ln 2 + 2)\\
x &= -\ln 2 - 2 && \text{Final answer}\\
x &\approx -2.693 && \text{Calculator}
\end{aligned}
$$

Example

Solve.
$$3^{2x+5} = 5 \cdot 2^{-x+1}$$

Example

Solve.

$$3^{2x+5} = 5 \cdot 2^{-x+1}$$

| Common base

Example

Solve.

$$\begin{aligned} 3^{2x+5} &= 5 \cdot 2^{-x+1} \\ \left(2^{\log_2 3}\right)^{2x+5} &= 5 \cdot 2^{-x+1} \end{aligned}$$

Common base

$a = b^{\log_b a}$

Example

Solve.

$$\begin{aligned} 3^{2x+5} &= 5\cdot 2^{-x+1} \\ \left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\ 2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \end{aligned}$$

Common base

$a = b^{\log_b a}$

Example

Solve.

$$
\begin{aligned}
3^{2x+5} &= 5 \cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5 \cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5 \cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Example

Solve.

$$\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1}\\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1}\\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1}\\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5\\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5
\end{aligned}$$

Common base

$a = b^{\log_b a}$

Example

Solve.

$$\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1}\\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1}\\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1}\\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5\\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5
\end{aligned}$$

Common base

$a = b^{\log_b a}$

Example

Solve.

$$
\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
{\color{red}2}^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Apply ${\color{red}\log_2}$

Example

Solve.

$$\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5
\end{aligned}$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

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\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5 \\
x\,(\qquad + \quad)+ \qquad &= \log_2 5
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$
\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5 \\
x\,(2\log_2 3+\quad)+ &= \log_2 5
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$
\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5) + {\color{red}x} - 1 &= \log_2 5 \\
{\color{red}x}\,(2\log_2 3 + {\color{red}1}) + &= \log_2 5
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$\begin{aligned} 3^{2x+5} &= 5\cdot 2^{-x+1} \\ \left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\ 2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\ \frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\ 2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\ (\log_2 3)(2x+5)+x-1 &= \log_2 5 \\ x\left(2\log_2 3+1\right)+5\log_2 3 &= \log_2 5 \end{aligned}$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$
\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x {\color{red}-1} &= \log_2 5 \\
x\left(2\log_2 3+1\right)+5\log_2 3 &= \log_2 5 {\color{red}+1}
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5 \\
x\left(2\log_2 3+1\right)+5\log_2 3 &= \log_2 5+1 \\
x &= \frac{\log_2 5+1}{}
\end{aligned}$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$
\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5 \\
x\left(2\log_2 3+1\right)+{\color{red}5\log_2 3} &= \log_2 5+1 \\
x &= \frac{\log_2 5+1{\color{red}-5\log_2 3}}{}
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$
\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5 \\
x\left(2\log_2 3+1\right)+5\log_2 3 &= \log_2 5+1 \\
x &= \frac{\log_2 5+1-5\log_2 3}{2\log_2 3+1}
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} && \text{Common base} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} && a = b^{\log_b a} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 && \text{Apply } \log_2 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5 \\
x\left(2\log_2 3+1\right)+5\log_2 3 &= \log_2 5+1 \\
x &= \frac{\log_2 5+1-5\log_2 3}{2\log_2 3+1} \\
x &\approx -1.1038 && \text{Calculator}
\end{aligned}$$

Example

Solve the equation.

$$e^{5-3x} = 10$$

Example

Solve the equation.

$$\begin{aligned} e^{5-3x} &= 10 && \text{apply } \ln \\ \ln(e^{5-3x}) &= \ln 10 \end{aligned}$$

Example

Solve the equation.

$$\begin{aligned} e^{5-3x} &= 10 && \text{apply } \ln \\ \ln(e^{5-3x}) &= \ln 10 \\ 5-3x &= \ln 10 \end{aligned}$$

Example

Solve the equation.

$$
\begin{array}{rcll}
e^{5-3x} & = & 10 & \text{apply } \ln \\
\ln(e^{5-3x}) & = & \ln 10 & \\
5 - 3x & = & \ln 10 & \\
3x & = & 5 - \ln 10 &
\end{array}
$$

Example

Solve the equation.

$$\begin{aligned} e^{5-3x} &= 10 && \text{apply } \ln \\ \ln(e^{5-3x}) &= \ln 10 \\ 5-3x &= \ln 10 \\ 3x &= 5-\ln 10 \\ x &= \frac{5-\ln 10}{3} \end{aligned}$$

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Solve the equation.

$$\begin{array}{rcll} e^{5-3x} & = & 10 & \text{apply } \ln \\ \ln(e^{5-3x}) & = & \ln 10 & \\ 5-3x & = & \ln 10 & \\ 3x & = & 5-\ln 10 & \\ x & = & \dfrac{5-\ln 10}{3} & \\ \text{Calculator:} \quad x & \approx & 0.8991. & \end{array}$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

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2^{t+3} &= 2^{2t}\\
t+3 &= 2t\\
t &= 3.
\end{aligned}
$$

Therefore the chicken and rabbit populations are equal after 3 years.

Example (Solving a quadratic exponential equation)

Solve for x.

$$9^x = 2 \cdot 3^x + 63$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$\begin{aligned} 9^x &= 2 \cdot 3^x + 63 \\ 9^x - 2 \cdot 3^x - 63 &= 0 \end{aligned}$$

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(u-9)(u+7) &= 0
\end{aligned}
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$$u = 9 \quad \text{or} \quad u = -7$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$\begin{aligned} 9^x &= 2\cdot 3^x + 63 \\ 9^x - 2\cdot 3^x - 63 &= 0 \qquad \Big| \text{ Substitute } {\color{red}u = 3^x} \\ u^2 - 2u - 63 &= 0 \\ (u-9)(u+7) &= 0 \end{aligned}$$

$$\begin{aligned} {\color{red}u} &= 9 \quad \text{or} \quad {\color{red}u} = -7 \\ {\color{red}3^x} &= 9 \quad \text{or} \quad {\color{red}3^x} = -7 \end{aligned}$$

Example (Solving a quadratic exponential equation)

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9^x &= 2\cdot 3^x + 63 \\
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$$

$$
\begin{aligned}
u &= 9 \quad \text{or} \quad u = -7 \\
3^x &= 9 \quad \text{or} \quad 3^x = -7 \\
x &= 2
\end{aligned}
$$

Example (Solving a quadratic exponential equation)

Solve for x.

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$$\begin{array}{rclcrcl} u &=& 9 & \text{or} & u &=& -7 \\ 3^x &=& 9 & \text{or} & 3^x &=& -7 \\ x &=& 2 & & & & \text{no real solution} \end{array}$$

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Therefore $x = 2$ is the solution.

Example

Solve the equation

$$e^{2x} - 3e^{x} - 4 = 0$$

Example

Solve the equation

$$e^{2x} - 3e^{x} - 4 = 0$$

Set $e^{x} = u$.

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$$e^{2x} - 3e^x - 4 = 0$$

Set $e^x = u$. Then $e^{2x} = \mathbf{?}$.

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$$u^2 - 3u - 4 = 0$$

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Solve the equation

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Set $e^x = u$. Then $e^{2x} = u^2$.

$$u^2 - 3u - 4 = 0$$

$$(\textbf{?} \quad\quad)(\textbf{?} \quad\quad) = 0$$

Example

Solve the equation

$$e^{2x} - 3e^{x} - 4 = 0$$

Set $e^x = u$. Then $e^{2x} = u^2$.

$$u^2 - 3u - 4 = 0$$

$$(u - 4)(u + 1) = 0$$

Example

Solve the equation

$$e^{2x} - 3e^{x} - 4 = 0$$

Set $e^{x} = u$. Then $e^{2x} = u^2$.

$$u^2 - 3u - 4 = 0$$
$$(u - 4)(u + 1) = 0$$

$u = 4$ or $u = -1$

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Solve the equation

$$e^{2x} - 3e^{x} - 4 = 0$$

Set $e^x = u$. Then $e^{2x} = u^2$.

$$\begin{aligned} u^2 - 3u - 4 &= 0 \\ (u-4)(u+1) &= 0 \end{aligned}$$

$u = 4$	or	$u = -1$
$e^x = 4$	or	$e^x = -1$

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Solve the equation

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Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

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Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = \textbf{?}$.

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

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$$4^{x+1} - 2^{x+2} - 3 = 0$$

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$$4u^2 - 4u - 3 = 0$$

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$$2u - 3 = 0 \quad \text{or} \quad 2u + 1 = 0$$

$$u = \frac{3}{} \quad \text{or} \quad u =$$

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$$2u - 3 = 0 \quad \text{or} \quad 2u + 1 = 0$$

$$u = \frac{3}{2} \quad \text{or} \quad u = -\frac{1}{2}$$

$$2^x = \frac{3}{2} \quad \text{or} \quad 2^x = -\frac{1}{2}$$

$$x = \log_2\left(\frac{3}{2}\right)$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$\begin{aligned} 4u^2 - 4u - 3 &= 0 \\ (2u-3)(2u+1) &= 0 \end{aligned}$$

$$\begin{aligned} 2u - 3 = 0 &\quad \text{or} \quad 2u + 1 = 0 \\ u = \frac{3}{2} &\quad \text{or} \quad u = -\frac{1}{2} \\ 2^x = \frac{3}{2} &\quad \text{or} \quad 2^x = -\frac{1}{2} \\ x = \log_2\left(\frac{3}{2}\right) &\quad \text{or} \quad \text{no real solution} \end{aligned}$$

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Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

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$$2^x = \frac{3}{2} \quad \text{or} \quad 2^x = -\frac{1}{2}$$

$$x = \log_2\left(\frac{3}{2}\right) = \frac{\ln\left(\textbf{?}\right)}{\ln \textbf{?}} \quad \text{or} \quad \text{no real solution}$$

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4u^2 - 4u - 3 &= 0\\
(2u-3)(2u+1) &= 0\\
2u - 3 = 0 &\quad \text{or} \quad 2u + 1 = 0\\
u = \frac{3}{2} &\quad \text{or} \quad u = -\frac{1}{2}\\
2^x = \frac{3}{2} &\quad \text{or} \quad 2^x = -\frac{1}{2}
\end{aligned}$$

$$x = \log_2\left(\frac{3}{2}\right) = \frac{\ln\left(\frac{3}{2}\right)}{\ln 2} \quad \text{or} \quad \text{no real solution}$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$\begin{aligned} 4u^2 - 4u - 3 &= 0 \\ (2u - 3)(2u + 1) &= 0 \end{aligned}$$

$$\begin{aligned} 2u - 3 = 0 &\quad \text{or} \quad 2u + 1 = 0 \\ u = \frac{3}{2} &\quad \text{or} \quad u = -\frac{1}{2} \\ 2^x = \frac{3}{2} &\quad \text{or} \quad 2^x = -\frac{1}{2} \\ x = \log_2\left(\frac{3}{2}\right) = \frac{\ln\left(\frac{3}{2}\right)}{\ln 2} \approx 0.58496 &\quad \text{or} \quad \text{no real solution} \end{aligned}$$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$3^{2x} = 2 + 63 \cdot 3^{-2x}$$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$\begin{aligned} 3^{2x} &= 2 + 63 \cdot 3^{-2x} \\ 3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \end{aligned}$$

$>$ 2 terms $\Rightarrow$ transfer one side

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$\begin{aligned} 3^{2x} &= 2 + 63 \cdot 3^{-2x} \\ {\color{red}3^{2x}} - 2 - 63 \cdot 3^{-2x} &= 0 \end{aligned}$$

$>$ 2 terms $\Rightarrow$
transfer one side
${\color{red}3^{2x} = u}$

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Solve the equation.

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\begin{aligned}
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u^2 - 2u - 63 &= 0
\end{aligned}
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Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$\begin{aligned} 3^{2x} &= 2 + 63 \cdot 3^{-2x} \\ 3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \\ u - 2 - 63u^{-1} &= 0 \\ u^2 - 2u - 63 &= 0 \\ (\mathbf{?} \quad)(\mathbf{?} \quad) &= 0 \end{aligned}$$

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u - 2 - 63u^{-1} &= 0 \\
u^2 - 2u - 63 &= 0 \\
(u-9)(u+7) &= 0
\end{aligned}
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 & \text{or} & \text{no real solution}
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3^{2x} = 9 \quad &\text{or} \quad \text{no real solution}
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3^{2x} = 9 & \text{or} & \text{no real solution} \\
2x = \log_3 9 & &
\end{array}
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2x = {\color{red}\log_3 9} & \\
2x = {\color{red}\mathbf{?}} &
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u = 9 \quad &\text{or} \quad u = -7 \\
3^{2x} = 9 \quad &\text{or} \quad \text{no real solution} \\
2x = \log_3 9 & \\
2x = 2 &
\end{aligned}
$$

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2x=\log_3 9 & & \\
2x=2 & & \\
x=1 & &
\end{array}
$$

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\end{array}$$

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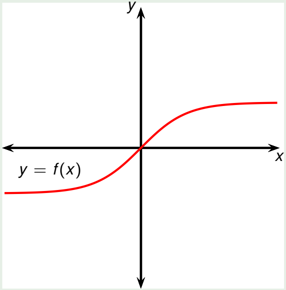
Multiply $\cdot u$

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

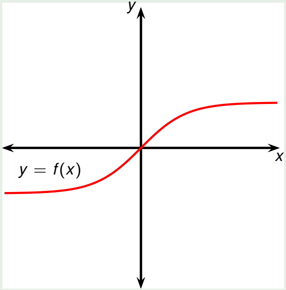


Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$



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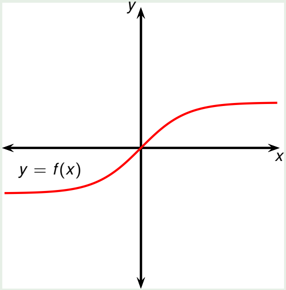
$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{\left(u - \textbf{?}\right)}{\left(u + \textbf{?}\right)} = y$$

Set $u = e^x$



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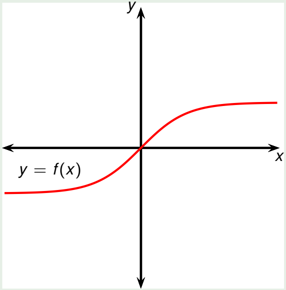
Set $u = e^x$
$e^{-x} = \mathbf{?}$



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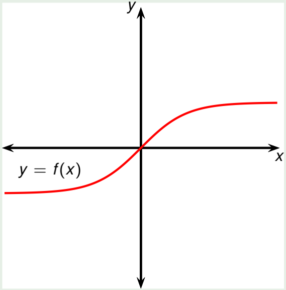

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

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Set $u = e^x$

$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

Example

Find $f^{-1}(x)$ for
$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

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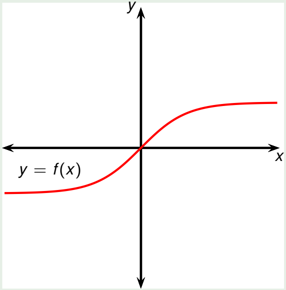
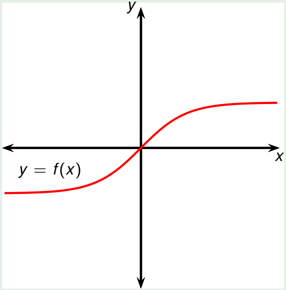


Example

Find $f^{-1}(x)$ for
$f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.



$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$
$$\frac{\left(u - \frac{1}{u}\right) u}{\left(u + \frac{1}{u}\right) u} = y$$
$$\frac{u^2 - 1}{u^2 + 1} = y$$

Set $u = e^x$
$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

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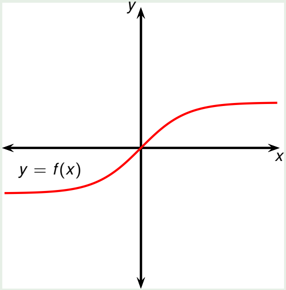

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$$u^2 - 1 = y(u^2 + 1)$$

Set $u = e^x$
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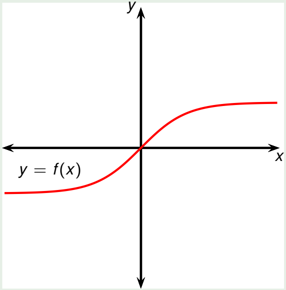

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$$\frac{u^2 - 1}{u^2 + 1} = y$$
$$u^2 - 1 = y(u^2 + 1)$$
$$u^2(1 - y) = 1 + y$$

Set $u = e^x$
$$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$$

Example

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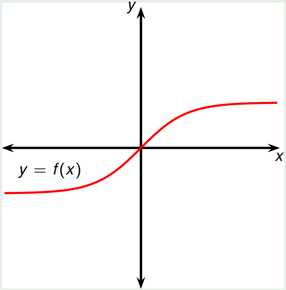

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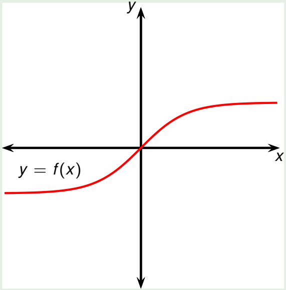

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$$u^2 - 1 = y(u^2 + 1)$$

$$u^2(1 - y) = 1 + y$$

$$u^2 = \frac{1 + y}{1 - y}$$

Set $u = e^x$

$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

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Find $f^{-1}(x)$ for
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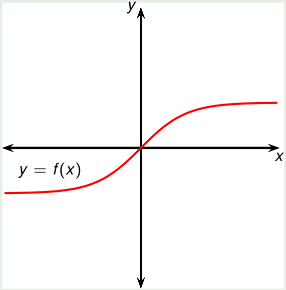

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$$u^2(1 - y) = 1 + y$$
$$u^2 = \frac{1 + y}{1 - y}$$
$$(e^x)^2 = \frac{1 + y}{1 - y}$$

Set $u = e^x$
$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

Example

Find $f^{-1}(x)$ for
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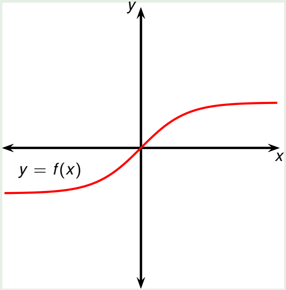
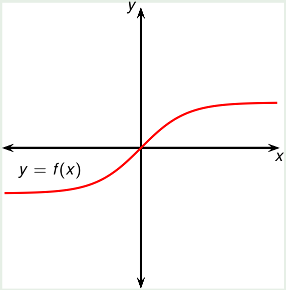


$$\begin{aligned}
\frac{e^x - e^{-x}}{e^x + e^{-x}} &= y \\
\frac{\left(u - \frac{1}{u}\right) u}{\left(u + \frac{1}{u}\right) u} &= y \\
\frac{u^2 - 1}{u^2 + 1} &= y \\
u^2 - 1 &= y(u^2 + 1) \\
u^2(1 - y) &= 1 + y \\
u^2 &= \frac{1 + y}{1 - y} \\
(e^{x})^{2} &= \frac{1 + y}{1 - y} \\
e^{2x} &= \frac{1 + y}{1 - y}
\end{aligned}$$

Set $u = e^x$
$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

Example

Find $f^{-1}(x)$ for
$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.



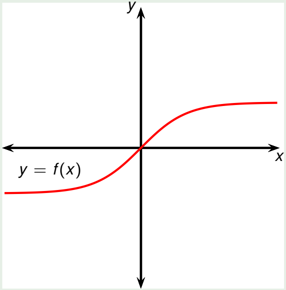
$$
\begin{aligned}
\frac{e^x - e^{-x}}{e^x + e^{-x}} &= y \\
\frac{\left(u - \frac{1}{u}\right) u}{\left(u + \frac{1}{u}\right) u} &= y \\
\frac{u^2 - 1}{u^2 + 1} &= y \\
u^2 - 1 &= y(u^2 + 1) \\
u^2(1 - y) &= 1 + y \\
u^2 &= \frac{1+y}{1-y} \\
(e^x)^2 &= \frac{1+y}{1-y} \\
e^{2x} &= \frac{1+y}{1-y} \\
2x &= \ln\left(\frac{1+y}{1-y}\right)
\end{aligned}
$$

Set $u = e^x$
$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

Take ln

Example

Find $f^{-1}(x)$ for
$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.



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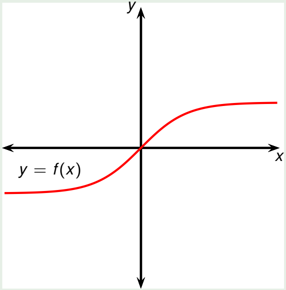
Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



answer

$$f^{-1}(y) = \frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)$$

$$\begin{aligned}
\frac{e^x - e^{-x}}{e^x + e^{-x}} &= y \\
\frac{\left(u - \frac{1}{u}\right)u}{\left(u + \frac{1}{u}\right)u} &= y \\
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u^2 - 1 &= y(u^2 + 1) \\
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(e^x)^2 &= \frac{1+y}{1-y} \\
e^{2x} &= \frac{1+y}{1-y} \\
x &= \frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)
\end{aligned}$$

Set $u = e^x$

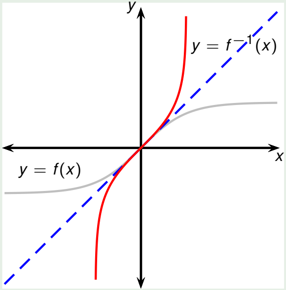
$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

Take ln

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



Final answer, relabeled:

$$f^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

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\frac{e^x - e^{-x}}{e^x + e^{-x}} &= y \\
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\end{aligned}$$

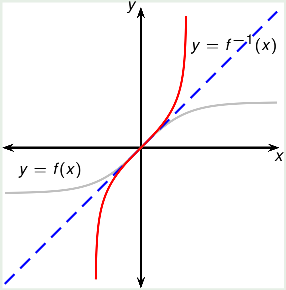
Set $u = e^x$

$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

Take ln

Example

Find $f^{-1}(x)$ for
$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.
$f = \mathtt{tanh}$ = hyperbolic tangent function.



Final answer, relabeled:
$$f^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$
Set $u = e^x$
$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$
$$\frac{\left(u - \frac{1}{u}\right)u}{\left(u + \frac{1}{u}\right)u} = y$$
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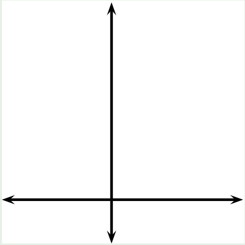
$$e^{2x} = \frac{1+y}{1-y}$$
Take ln
$$x = \frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)$$

Example

Solve the inequality.

$$2^{-3x-5} < 7$$



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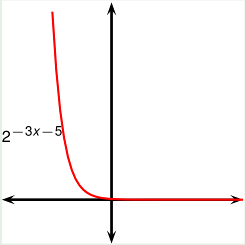
$$2^{-3x-5} < 7$$



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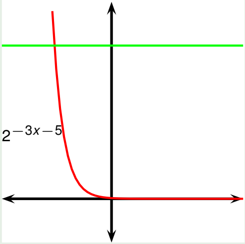


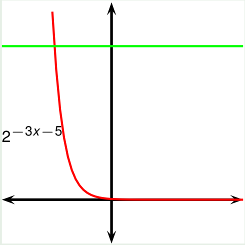
Example

Solve the inequality.

$$2^{-3x-5} < 7$$

$$\log_2 2^{-3x-5} < \log_2 7$$

Logarithms preserve inequalities: apply $\log_2$



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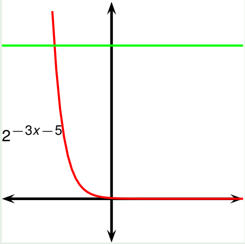


Example

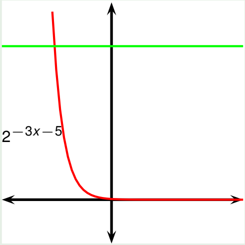
Solve the inequality.

$$2^{-3x-5} < 7$$

$$\textcolor{red}{\log_2 2^{-3x-5}} < \log_2 7$$

$$-3x-5 < \log_2 7$$

Logarithms preserve inequalities: apply $\log_2$

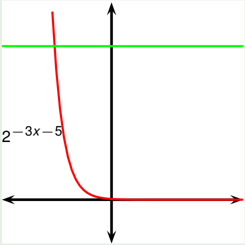


Example

Solve the inequality.

$$\begin{aligned} 2^{-3x-5} &< 7 \\ \log_2 2^{-3x-5} &< \log_2 7 \\ -3x-5 &< \log_2 7 \\ -3x &< \log_2 7 + 5 \end{aligned}$$

Logarithms preserve inequalities: apply $\log_2$



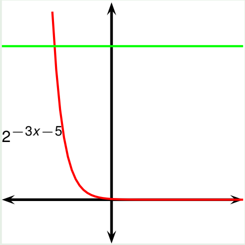
Example

Solve the inequality.

$$
\begin{aligned}
2^{-3x-5} &< 7 \\
\log_2 2^{-3x-5} &< \log_2 7 \\
-3x-5 &< \log_2 7 \\
-3x &< \log_2 7 + 5 \\
x &> -\frac{\log_2 7 + 5}{3}
\end{aligned}
$$

Logarithms preserve inequalities: apply $\log_2$

Division by negative number flips inequalities



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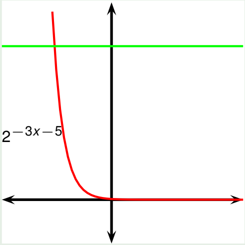

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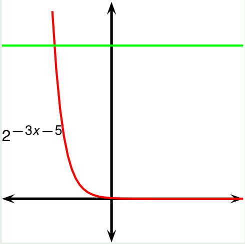
-3x-5 &< \log_2 7 \\
-3x &< \log_2 7 + 5 \\
x &> -\frac{\log_2 7 + 5}{3} \\
x &\in \left(-\frac{5+\log_2 7}{3}, \infty\right)
\end{aligned}
$$

Logarithms preserve inequalities: apply $\log_2$

Division by negative number flips inequalities



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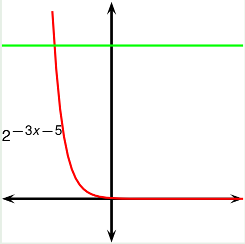

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