Precalculus
Polynomial systems basics

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Outline

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- Typical polynomial systems have more than one variable/equation:
$$\left|\begin{array}{rcl} y^2 + xy - 4y - 2x + 4 & = & 0 \\ y^2 x - 2yx - y - 2x + 4 & = & 0. \end{array}\right.$$
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- The number of variables and equations need not be equal:
$$\left| \begin{array}{rcl} x + y + z + w & = & 2 \\ y + z^2 & = & 1 \\ y + zw^2 & = & 1. \end{array} \right.$$
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Here we have 4 variables (x, y, z, w), 3 equations.

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- Polynomial systems may have infinitely many solutions:

  $\left| \begin{array}{rcl} x &=& 0 \\ y + z &=& 1. \end{array} \right.$ If we set $x = 0$, $y = 1 - z$, we produce infinitely many solutions for every possible value of z.

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- Solving polynomial systems is an indispensable mathematical tool used in other branches of science and mathematics.
- Polynomial systems also have direct practical applications, for example kinematics - the configurations of a robotic arm can be parametrized with polynomials.

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- A system doable by hand would typically be solved easily using ad-hoc techniques.

Example

Solve the polynomial system. $$\left|\begin{array}{rcl} x-4y &=& 5\\ y^2+xy &=& 10\end{array}\right.$$

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$$\begin{array}{rcl} x &=& 5+4y\\ y^2+xy &=& 10\\ y^2+(5+4y)y &=& 10\end{array}\quad\left|\begin{array}{l}\text{Solve for } x \text{ in first eq-n.}\\ \text{Substitute } x \text{ away}\end{array}\right.$$

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$$\begin{aligned} x &= 5+4{\color{red}y} \\ &= 5+4({\color{red}-2}) \end{aligned}$$

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$$\begin{aligned} x &= 5+4y\\ &= 5+4(-2) = -3\end{aligned}$$

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$$\left|\begin{array}{rclcl} x-4y &=& (-3)-4(-2) &=& 5\\ y^2+xy &=& (-2)^2+(-3)(-2) &=& 10 \end{array}\right.$$

Check answer $y=1, x=9$:

$$\left|\begin{array}{rclcl} x-4y &=& 9-4\cdot 1 &=& 5\\ y^2+xy &=& 1^2+9\cdot 1 &=& 10. \end{array}\right.$$

Example

Solve the polynomial system. $\left|\begin{array}{rcl} x-4y & = & 5 \\ y^2+xy & = & 10 \end{array}\right.$

Final answer: $x=-3, y=-2$ or $x=9, y=1$.

Check answer $x=-3, y=-2$:

$$\left|\begin{array}{rcll} x-4y & = & (-3)-4(-2) & = & 5 \\ y^2+xy & = & (-2)^2+(-3)(-2) & = & 10 \end{array}\right.$$

Check answer $y=1, x=9$:

$$\left|\begin{array}{rcll} x-4y & = & 9-4\cdot 1 & = & 5 \\ y^2+xy & = & 1^2+9\cdot 1 & = & 10. \end{array}\right.$$

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

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The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$\begin{aligned} x + y &= 25 && |\text{Solve for } y \\ y &= 25 - x \\ x^2 + y^2 &= 313 \end{aligned}$$

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$$\begin{aligned} x + y &= 25 \qquad |\text{Solve for } y \\ y &= 25 - x \\ x^2 + y^2 &= 313 \\ x^2 + (25 - x)^2 &= 313 \end{aligned}$$

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The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$\begin{aligned} x + y &= 25 && \vert \text{Solve for } y \\ y &= 25 - x \\ x^2 + y^2 &= 313 \\ x^2 + (25 - x)^2 &= 313 \\ x^2 + \left(25^2 - 2 \cdot 25 \cdot x + x^2\right) - 313 &= 0 && \left\vert (a - b)^2 = a^2 - 2ab + b^2 \right. \end{aligned}$$

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x + y &= 25 && \middle| \text{Solve for } y\\
y &= 25 - x\\
x^2 + y^2 &= 313\\
x^2 + (25 - x)^2 &= 313\\
{\color{red}x^2} + \left(25^2 - 2\cdot 25 \cdot x + {\color{red}x^2}\right) - 313 &= 0 && \left| (a-b)^2 = a^2 - 2ab + b^2 \right.\\
{\color{red}2x^2} - 50x + 625 - 313 &= 0
\end{aligned}$$

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The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

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\begin{aligned}
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2x^2 - 50x + 625 - 313 &= 0
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2x^2 - 50x + 625 - 313 &= 0\\
{\color{red}2}x^2 - {\color{red}50}x + {\color{red}312} &= 0 && |{\color{red}\text{Divide by 2}}
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2x^2 - 50x + 625 - 313 &= 0 \\
{\color{red}2}x^2 - {\color{red}50}x + {\color{red}312} &= 0 && {\color{red}\text{|Divide by 2}} \\
x^2 - {\color{red}25}x + {\color{red}156} &= 0
\end{aligned}$$

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2x^2 - 50x + 625 - 313 &= 0\\
2x^2 - 50x + 312 &= 0 && \text{|Divide by 2}\\
x^2 - 25x + 156 &= 0\\
x &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1}
\end{aligned}
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x^2 - 25x + 156 &= 0 \\
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&= \frac{25 \pm \sqrt{625 - 624}}{2}\\
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2x^2 - 50x + 625 - 313 &= 0\\
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x^2 - 25x + 156 &= 0\\
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x^2 - 25x + 156 &= 0 \\
x &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1} \\
&= \frac{25 \pm \sqrt{625 - 624}}{2} \\
&= \frac{25 \pm 1}{2} = \begin{cases} \frac{25+1}{2} = 13 \\ \frac{25-1}{2} \end{cases}
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x &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1}\\
&= \frac{25 \pm \sqrt{625 - 624}}{2}\\
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2x^2 - 50x + 625 - 313 &= 0\\
2x^2 - 50x + 312 &= 0 && |\text{Divide by } 2\\
x^2 - 25x + 156 &= 0
\end{aligned}
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x &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1}\\
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y &= 25 - x
\end{aligned}$$

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\begin{aligned}
x + y &= 25 \qquad |\text{Solve for } y\\
y &= 25 - x\\
x^2 + y^2 &= 313\\
x^2 + (25 - x)^2 &= 313\\
{\color{red}x} &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1}\\
&= \frac{25 \pm \sqrt{625 - 624}}{2}\\
&= \frac{25 \pm 1}{2} = \begin{cases} \frac{25+1}{2} {\color{red}= 13}\\ \frac{25-1}{2} {\color{red}= 12}\end{cases}\\
y = 25 - {\color{red}x} &= \begin{cases} 25 - {\color{red}13}\\ 25 - {\color{red}12}\end{cases}
\end{aligned}
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The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

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\begin{aligned}
x + y &= 25 \qquad |\text{Solve for } y\\
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y = 25 - x &= \begin{cases} 25 - 13 = 12\\ 25 - 12 = 13 \end{cases}
\end{aligned}
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Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

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\begin{aligned}
x + y &= 25 \qquad |\text{Solve for } y\\
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{\color{red}y} &= 25 - x = \begin{cases} 25 - 13 = 12\\ 25 - {\color{red}12} = 13 \end{cases}
\end{aligned}
$$

The two solution candidates are ${\color{red}x = 12, y = 13}$ and $x = 13, y = 12$.

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

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\begin{aligned}
x + y &= 25 \qquad |\text{Solve for } y\\
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The two solution candidates are $x = 12, y = 13$ and $x = 13, y = 12$.

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y = 25 - x &= \begin{cases} 25 - 13 = 12\\ 25 - 12 = 13 \end{cases}
\end{aligned}
$$

The two solution candidates are $x = 12, y = 13$ and $x = 13, y = 12$. Since $y \geq x$, one of the solutions needs to be discarded and our final answer is $x = 12, y = 13$.