Precalculus
Factoring quadratic polynomials

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Outline

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Definition ((Partial) Factorization)

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Example

$$\begin{aligned} x^2-1 &= (x-1)(x+1) \\ x^3+8 &= (x+2)\left(x^2-2x+4\right) \end{aligned}$$

Theorem

The quadratic $ax^2 + bx + c$ factors as follows.

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

where x_1 and x_2 are the roots of the quadratic, given by:

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Let $ax^2 + bx + c$ be a quadratic functions with zeros x_1 and x_2. Then:

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$$\begin{aligned} a(x-x_1)(x-x_2) &= ax^2+bx+c \\ ax^2 - axx_2 - ax_1x + a(-x_1)(-x_2) &= ax^2+bx+c \\ ax^2 - a(x_2+x_1)x + ax_1x_2 &= ax^2+bx+c \end{aligned}$$

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The last two formulas are called Vieta's formulas (after François Viète (1540-1603), Latinized name: Franciscus Vieta).

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

$$\begin{aligned} x_1 + x_2 &= -\frac{b}{a} \\ x_1 x_2 &= \frac{c}{a} \end{aligned}$$

Vieta's formulas

Example

Factor the quadratic.

$$x^2 + 5x + 6$$

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Example

Factor the quadratic.

$$x^2 + 5x + 6 = (x + \mathbf{?})(x + \mathbf{?})$$

- The product of the two roots: $x_1 x_2 = 6$.

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- The divisors of 6 are $\pm 1, \pm 2, \pm 3, \pm 6$.

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Vieta's formulas

Example

Factor the quadratic.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

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Factor the quadratic.

$$x^2 + 3x + 1$$

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$$x^2 + 3x + 1 = \left(x + ? \qquad\qquad \right)\left(x + ? \qquad\qquad \right)$$

- The product of the two roots: $x_1 x_2 = 1$.

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$$x^2 + 3x + 1 = \left(x \quad - \left(\frac{-3+\sqrt{5}}{2}\right)\right)\left(x \quad - \left(\frac{-3-\sqrt{5}}{2}\right)\right)$$

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Factor the quadratic, using complex numbers if needed.

$$x^2 + x + 1$$

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Factor the quadratic, using complex numbers if needed.

$$x^2 + x + 1 = \Big(x + \textbf{?} \qquad\qquad \Big)\Big(x + \textbf{?} \qquad\qquad \Big)$$

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Example

Factor the quadratic, using complex numbers if needed.

$$x^2 + x + 1 = \left(x \quad - \left(\frac{-1+\sqrt{3}i}{2}\right)\right)\left(x \quad - \left(\frac{-1-\sqrt{3}i}{2}\right)\right)$$

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