

Precalculus

Exponent equation that reduces to masked quadratic, part 2

Todor Milev

2019

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$3^{2x} = 2 + 63 \cdot 3^{-2x}$$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$3^{2x} = 2 + 63 \cdot 3^{-2x}$$

> 2 terms \Rightarrow
transfer one side

$$3^{2x} - 2 - 63 \cdot 3^{-2x} = 0$$

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 $3^{2x} = u$

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$$3^{-2x} = ?$$

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or **no real solution**

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$$2x = \log_3 9$$

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$$3^{2x} = 9 \quad \text{or} \quad \text{no real solution}$$

$$2x = \log_3 9$$

$$2x = 2$$

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$$2x = \log_3 9$$

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> 2 terms \Rightarrow
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$$2x = \log_3 9$$

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$$x = 1$$

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Multiply $\cdot u$