

Precalculus

Simplify $\sin(k \arcsin x)$, $\cos(k \arcsin x)$,
 $\sin(k \arccos x)$, $\cos(k \arccos x)$

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Example

Rewrite $\sin(2 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \sin(2 \arccos(x)) &= \sin(2y) \\
 &= 2 \cos y \sin y \\
 &= 2 \cos y \left(\pm \sqrt{1 - \cos^2 y} \right) \\
 &= 2 \cos y \sqrt{1 - \cos^2 y} \\
 &= 2x \sqrt{1 - x^2}
 \end{aligned}$$

Set $y = \arccos x$
 Express via $\sin y, \cos y$
 Express $\sin y$ via $\cos y$
 $\sin y > 0$ because
 $0 \leq y \leq \pi$
 use $x = \cos y$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) & y = \arccos x \\
 &= \cos(2y) \cos y - \sin(2y) \sin y & \text{Angle sum f-la} \\
 &= (\cos^2 y - \sin^2 y) \cos y & \text{Express via} \\
 &\quad - 2 \sin y \cos y \sin y & \sin y, \cos y \\
 &= \cos^3 y - \sin^2 y \cos y - 2 \sin^2 y \cos y \\
 &= \cos^3 y - 3 \sin^2 y \cos y & \text{Express } \sin y \\
 &= \cos^3 y - 3(1 - \cos^2 y) \cos y & \text{via } \cos y \\
 &= 4\cos^3 y - 3 \cos y \\
 &= 4x^3 - 3x & x = \cos y
 \end{aligned}$$