

Precalculus

Polynomial inequalities

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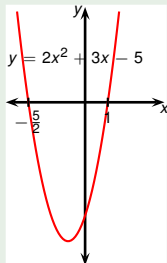
2019

Outline

1 Polynomial inequalities

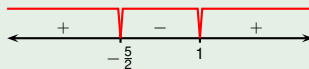
Example

Solve the inequality.



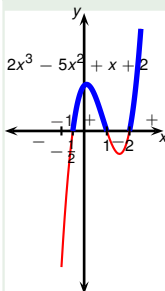
$$\begin{aligned}
 2x^2 + 3x - 5 &\geq 0 \\
 (2x + 5)(x - 1) &\geq 0 \\
 x &\in (-\infty, -\frac{5}{2}] \cup [1, \infty)
 \end{aligned}$$

Left hand side vanishes when $x = -\frac{5}{2}$ and when $x = 1$.
The two roots split the real line into three intervals:
 $(-\infty, -\frac{5}{2})$, $(-\frac{5}{2}, 1)$, $(1, \infty)$.



Interval	Factor signs	Final sign	Sample pt	Value at sample pt
$(-\infty, -\frac{5}{2})$	$(-)(-)$	+	-100	$f(-100) > 0$
$(-\frac{5}{2}, 1)$	$(+)(-)$	-	0	$f(0) = -5 < 0$
$(1, \infty)$	$(+)(+)$	+	100	$f(100) > 0$

Example



Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality.

$$2x^3 - 5x^2 + x + 2 > 0$$

$$2(x - (-\frac{1}{2}))(x - 1)(x - 2) > 0$$

$$x \in (-\frac{1}{2}, 1) \cup (2, \infty)$$

Left hand side vanishes when $x = -\frac{1}{2}$, when $x = 1$ and when $x = 2$. The two roots split the real line into four intervals: $(-\infty, -\frac{1}{2})$, $(-\frac{1}{2}, 1)$, $(1, 2)$, $(2, \infty)$.



Interval	Factor signs	Final sign from plot
$(-\infty, -\frac{1}{2})$	$(-)(-)(-)$	$-$
$(-\frac{1}{2}, 1)$	$(+)(-)(-)$	$+$
$(1, 2)$	$(+)(+)(-)$	$-$
$(2, \infty)$	$(+)(+)(+)$	$+$