

Precalculus

Quadratic trigonometric inequalities

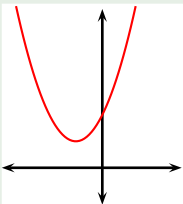
Todor Milev

2019

Example

- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2 \cos^2 \theta + 2 \cos \theta + 1 \leq \cos \theta + 2$ lying in $[-360^\circ, 360^\circ]$.

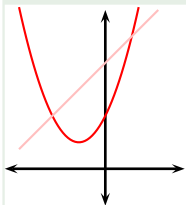
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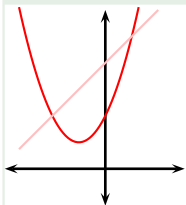
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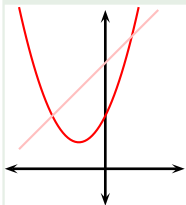


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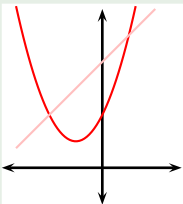


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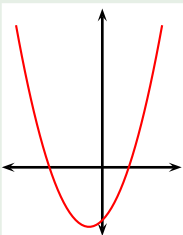
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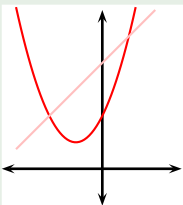
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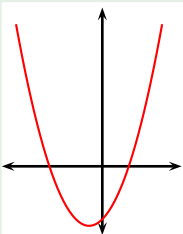


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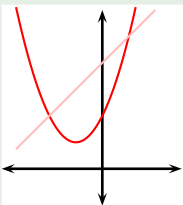
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$$2(\text{?})(\text{?}) \leq 0$$



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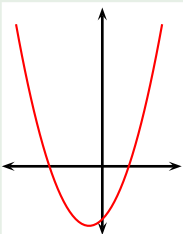


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$$2(u - \frac{1}{2})(u + 1) \leq 0$$



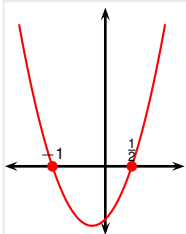
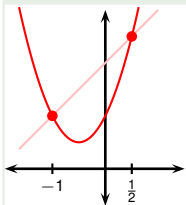
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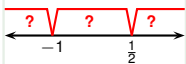
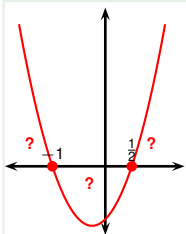
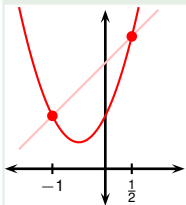
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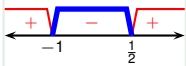
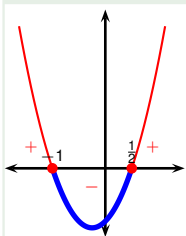
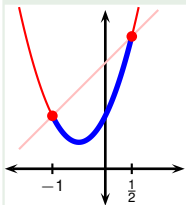
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$$u \in [-1, \frac{1}{2}]$$



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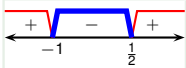
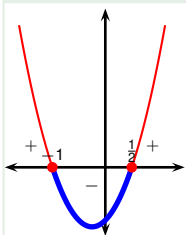
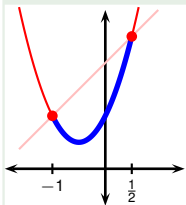
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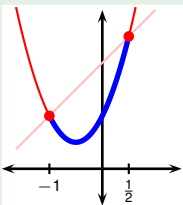
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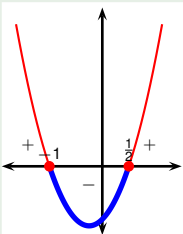
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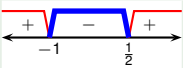
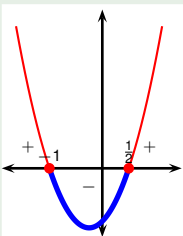
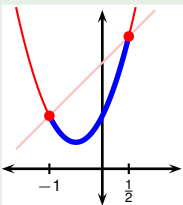
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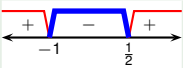
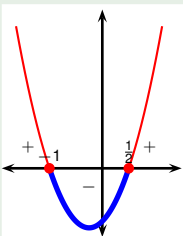
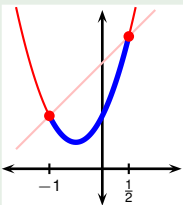
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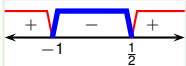
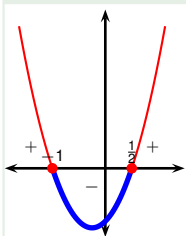
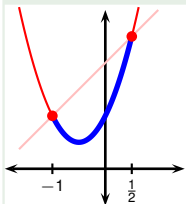
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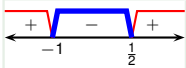
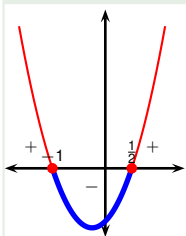
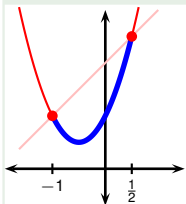
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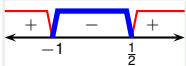
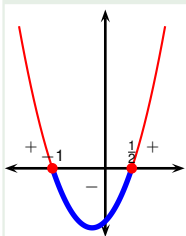
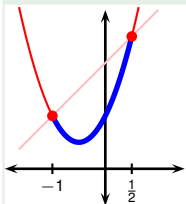
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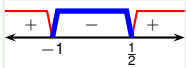
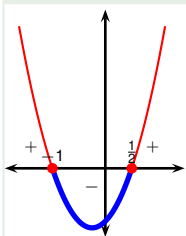
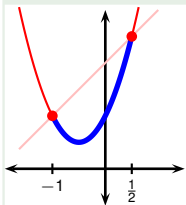
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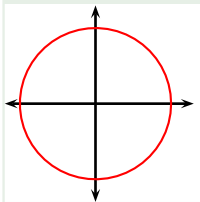
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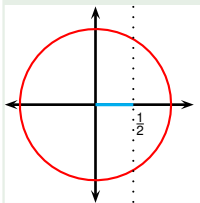


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$$\begin{aligned} \cos \theta &\in \left[-1, \frac{1}{2}\right] \\ -1 &\leq \cos \theta \leq \frac{1}{2} \end{aligned}$$

$$\theta \in \quad [? \quad , ? \quad]$$

Example



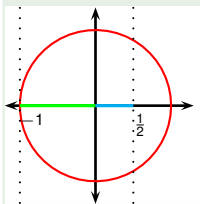
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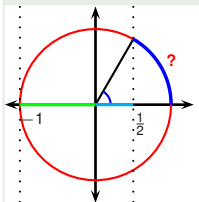
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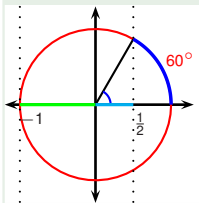
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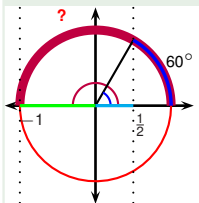
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$$\cos \theta \in \left[-1, \frac{1}{2}\right]$$

$$-1 \leq \cos \theta \leq \frac{1}{2}$$

$$\theta \in \quad [60^\circ \quad , ? \quad]$$

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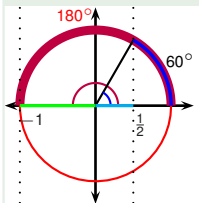
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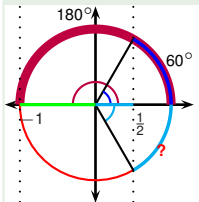
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$$\theta \in \quad [60^\circ \quad , 180^\circ \quad]$$

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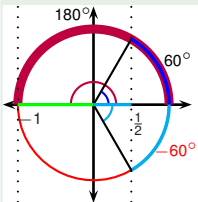
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$$-1 \leq \cos \theta \leq \frac{1}{2}$$

$$\theta \in [?, \quad , ? \quad] \cup [60^\circ \quad , 180^\circ \quad]$$

Example



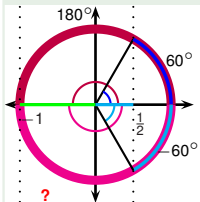
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Example



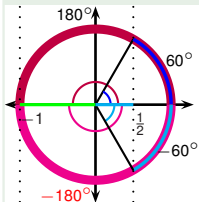
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Example



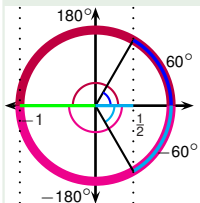
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Example

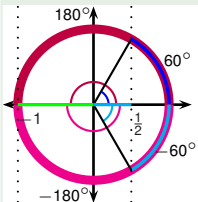


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Example



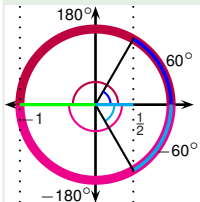
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Example



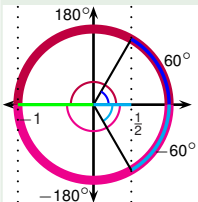
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$$\theta \in [-180^\circ, -60^\circ] \cup [60^\circ, 180^\circ] \quad k = 0$$

Example



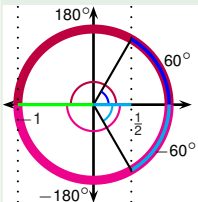
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Example



- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2 \cos^2 \theta + 2 \cos \theta + 1 \leq \cos \theta + 2$ lying in $[-360^\circ, 360^\circ]$.

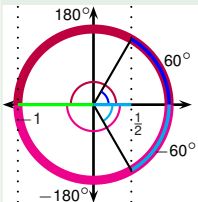
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...

Example



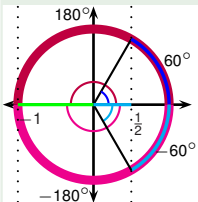
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Example



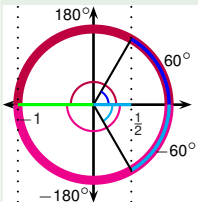
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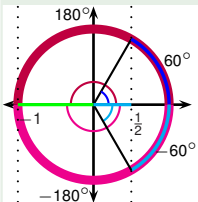
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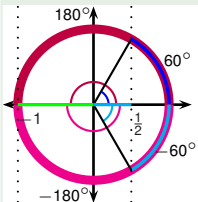
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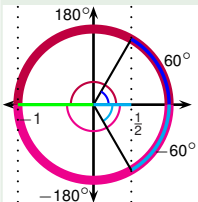
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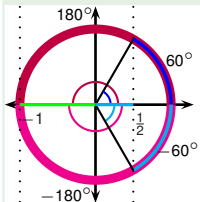
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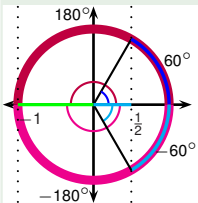


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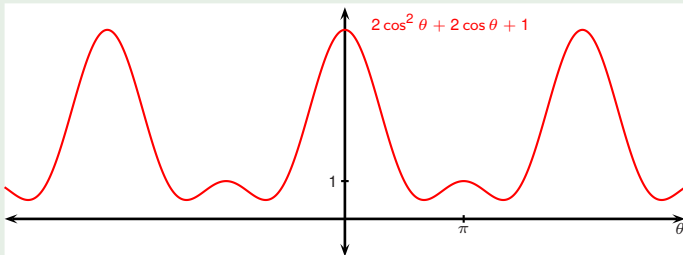
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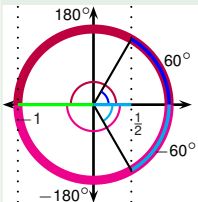
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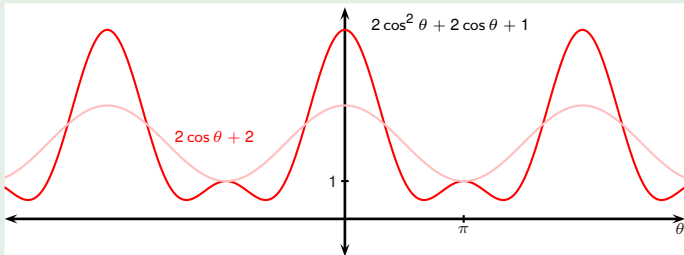


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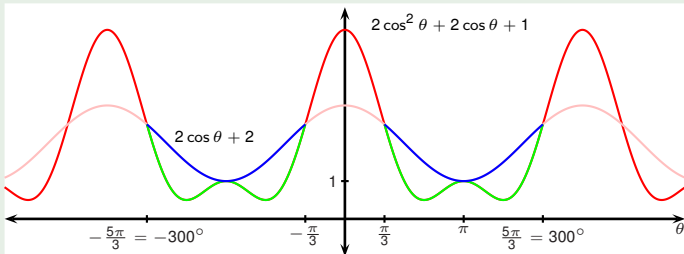
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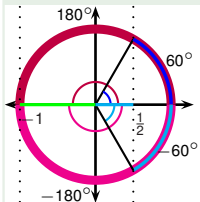
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