Calculus I

Antiderivatives, indefinite integrals and the Evaluation Theorem

Todor Milev

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Outline

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Antiderivatives

Definition (Antiderivative)

A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

Example

- Let $f(x) = x^2$.

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- No. If $G(x) = \frac{1}{3}x^3 + 1$, then $G'(x) = x^2 = f(x)$.

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- No. If $G(x) = \frac{1}{3}x^3 + 1$, then $G'(x) = x^2 = f(x)$.
- $\frac{1}{3}x^3 + 2$ will also work.
- Any function of the form $H(x) = \frac{1}{3}x^3 + C$, where C is a constant, is an antiderivative of f.

Theorem

If F is an antiderivative of f on an interval I, then an arbitrary antiderivative of f on I is of the form

$$F(x) + C$$

where C is an arbitrary constant.

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Find all antiderivatives of each of the following functions.

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Find the most general antiderivative of $f(x) = \frac{1}{x}$.

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$$G(x) = \begin{cases} \ln|x| + C_1 & \text{if} \quad x > 0 \\ \ln|x| + C_2 & \text{if} \quad x < 0 \end{cases}$$

Every differentiation formula gives rise to an antidifferentiation formula. Suppose $F' = f$ and $G' = g$.

Function	Particular Antiderivative
$cf(x)$	
$f(x) + g(x)$	
$x^n (n \neq -1)$	
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$$= -4\cos x + \frac{2}{5}x^5 - 2\sqrt{x} + C$$

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$$
\begin{aligned}
f'(x) &= \frac{1}{x\sqrt{x}} = x^{-3/2} \\
f(x) &= \frac{x^{-1/2}}{-\frac{1}{2}} + C \\
&= -\frac{2}{\sqrt{x}} + C
\end{aligned}
$$

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$$-\frac{2}{\sqrt{1}} + C = 1$$

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Therefore

$$f(x) = -\frac{2}{\sqrt{x}} + 3.$$

Theorem (The Evaluation Theorem (FTC part 2))

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a),$$

where F is any antiderivative of f.

$\int_a^b f(x)dx$ exists for any continuous (over $[a, b]$) function f.

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Theorem

Let f be a continuous function on $[a,b]$. Then f is integrable over $[a,b]$.

In other words, $\int_a^b f(x)dx$ exists for any continuous (over $[a,b]$) function f.

Theorem (The Evaluation Theorem (FTC part 2))

If f is continuous on $[a,b]$, then

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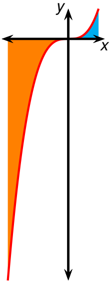
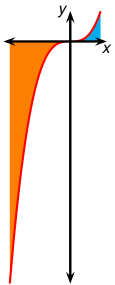
Example



Evaluate the integral $\int_{-2}^{1} x^3 \, dx$.

Example



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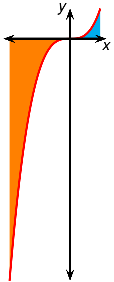

Evaluate the integral $\int_{-2}^{1} x^3 \, \mathrm{d}x$.

- x^3 is continuous on $[-2, 1]$ (in fact, it's continuous everywhere).
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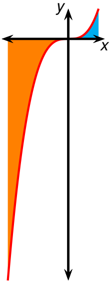
Example



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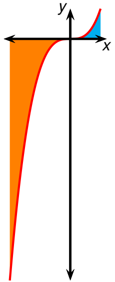

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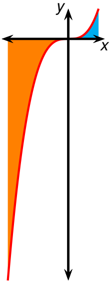

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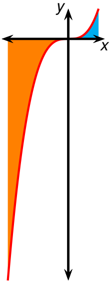

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$$\int_{-2}^{1} x^3 \, \mathrm{d}x = F(1) - F(-2) = \frac{1}{4}(1)^4 - \frac{1}{4}(-2)^4 = \frac{1}{4} - \frac{16}{4} = -\frac{15}{4}$$

We often use the notation

$$F(x)]_a^b = F(b) - F(a)$$

or

$$[F(x)]_a^b = F(b) - F(a)$$

Therefore we can write

$$\int_a^b f(x)\mathrm{d}x = F(x)]_a^b$$

or

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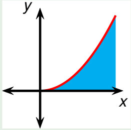
Example

Find the area under the parabola $y = x^2$ from 0 to 1.



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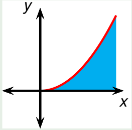

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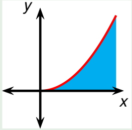

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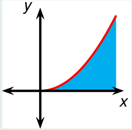

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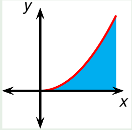

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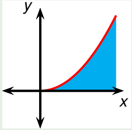

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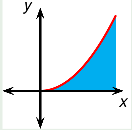

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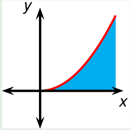

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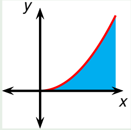

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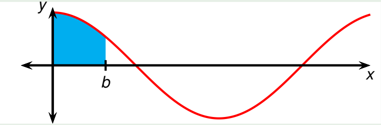

Find the area under the cosine curve from 0 to b, where $0 \leq b \leq \frac{\pi}{2}$.

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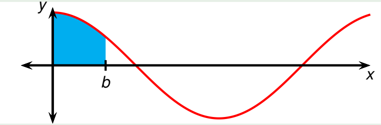

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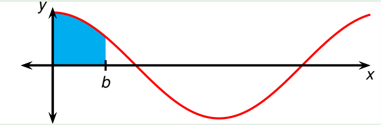

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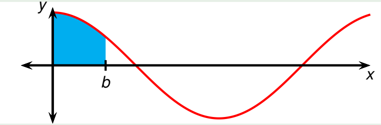

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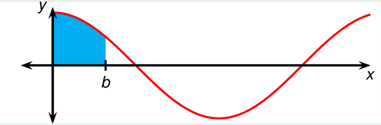

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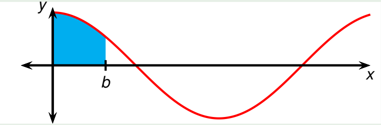

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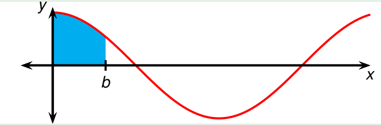

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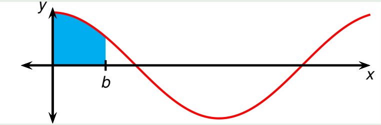

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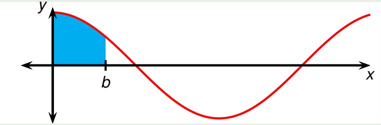

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Indefinite Integrals

- The Evaluation Theorem establishes a connection between antiderivatives and definite integrals.
- It says that $\int_a^b f(x)\mathrm{d}x$ equals $F(b) - F(a)$, where F is an antiderivative of f.
- We need convenient notation for writing antiderivatives.
- This is what the indefinite integral is.

Definition (Indefinite Integral)

The indefinite integral of f is another way of saying the antiderivative of f, and is written $\int f(x)\mathrm{d}x$. In other words,

$$\int f(x)\mathrm{d}x = F(x) \qquad \text{means} \qquad F'(x) = f(x).$$

Example

$$\int x^4 \mathrm{d}x = \textbf{?}$$

Example

$$\int x^4 \mathrm{d}x = \frac{x^5}{5}$$

Example

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Example

$$\int x^4 \mathrm{d}x = \frac{x^5}{5} + C$$

because

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^5}{5} + C\right) = x^4.$$

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- The indefinite integral represents a whole family of functions.

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- The indefinite integral represents a whole family of functions.
- Example: the general antiderivative of $\frac{1}{x}$ is

$$F(x) = \begin{cases} \ln|x| + C_1 & \text{if} \quad x > 0 \\ \ln|x| + C_2 & \text{if} \quad x < 0 \end{cases}$$

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- We adopt the convention that the constant participating in an indefinite integral is only valid on one interval.
- $\int \frac{1}{x}\mathrm{d}x = \ln|x| + C$, and this is valid either on $(-\infty, 0)$ or $(0, \infty)$.

Example

Compute the integral.

$$\int \left(2x^2 - x - 5\right) \mathrm{d}x$$

Example

Compute the integral.

$$\int \left(2x^2 - x - 5\right) \mathrm{d}x = 2\int x^2 \mathrm{d}x - \int x \mathrm{d}x - 5\int \mathrm{d}x$$

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$$
\begin{aligned}
\int \left(2x^2 - x - 5\right) \mathrm{d}x &= 2\int x^2 \mathrm{d}x - \int x \mathrm{d}x - 5\int \mathrm{d}x \\
&= 2\mathbf{?} \quad - \mathbf{?} \quad - 5\mathbf{?}
\end{aligned}
$$

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Example

Compute the integral.

$$\begin{aligned}\int \left(2x^2 - x - 5\right) dx &= 2\int x^2 dx - \int x dx - 5\int dx\\ &= 2\frac{x^3}{3} - \mathbf{?} \quad - 5\mathbf{?}\end{aligned}$$

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Example

Integrate.

$$\int 5x^{\frac{3}{2}}\mathrm{d}x \quad = \quad 5$$

Example

Integrate.

$$\int 5x^{\frac{3}{2}}\mathrm{d}x = 5\textbf{?}$$

Example

Integrate.

$$\int 5x^{\frac{3}{2}}\mathrm{d}x = 5\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}$$

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$$\int 5x^{\frac{3}{2}}\mathrm{d}x = 5\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C$$

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Example

Integrate.

$$\int \frac{1}{\sqrt[3]{x^4}} \mathrm{d}x$$

Example

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$$\int \frac{1}{\sqrt[3]{x^4}} \mathrm{d}x = \int x^{?} \, \mathrm{d}x$$

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$$= \frac{x^{-\frac{4}{3}+1}}{-\frac{3}{4}+1}$$

Example

Integrate.

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Example

Integrate.

$$
\begin{aligned}
\int \frac{1}{\sqrt[3]{x^4}} \mathrm{d}x &= \int x^{-\frac{4}{3}} \mathrm{d}x \\
&= \frac{x^{-\frac{4}{3}+1}}{-\frac{3}{4}+1} + C \\
&= \frac{x^{-\frac{1}{3}}}{-\frac{1}{3}} + C
\end{aligned}
$$

Example

Integrate.

$$\begin{aligned}\int \frac{1}{\sqrt[3]{x^4}}\mathrm{d}x &= \int x^{-\frac{4}{3}}\mathrm{d}x\\ &= \frac{x^{-\frac{4}{3}+1}}{-\frac{3}{4}+1}+C\\ &= \frac{x^{-\frac{1}{3}}}{-\frac{1}{3}}+C\\ &= -3x^{-\frac{1}{3}}+C\end{aligned}$$

Example

Integrate.

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{x^4}} \mathrm{d}x &= \int x^{-\frac{4}{3}} \mathrm{d}x \\
&= \frac{x^{-\frac{4}{3}+1}}{-\frac{3}{4}+1} + C \\
&= \frac{x^{-\frac{1}{3}}}{-\frac{1}{3}} + C \\
&= -3x^{-\frac{1}{3}} + C
\end{aligned}$$

Example

Integrate.

$$\int \left(x^{\frac{3}{2}} - \frac{1}{x^{\frac{1}{3}}} \right)^2 \mathrm{d}x$$

Example

Integrate.

$$\int \left(x^{\frac{3}{2}} - \frac{1}{x^{\frac{1}{3}}} \right)^2 \mathrm{d}x \; = \; \int \left(x^{\frac{3}{2}} - x^{-\frac{1}{3}} \right)^2 \mathrm{d}x$$

Example

Integrate.

$$\begin{aligned}\int\left(x^{\frac{3}{2}}-\frac{1}{x^{\frac{1}{3}}}\right)^2\mathrm{d}x &= \int\left(x^{\frac{3}{2}}-x^{-\frac{1}{3}}\right)^2\mathrm{d}x\\ &= \int\left(\left(x^{\frac{3}{2}}\right)^2-2x^{\frac{3}{2}}x^{-\frac{1}{3}}+\left(x^{-\frac{1}{3}}\right)^2\right)\mathrm{d}x\end{aligned}$$

Example

Integrate.

$$\begin{aligned}\int \left(x^{\frac{3}{2}} - \frac{1}{x^{\frac{1}{3}}}\right)^2 \mathrm{d}x &= \int \left(x^{\frac{3}{2}} - x^{-\frac{1}{3}}\right)^2 \mathrm{d}x \\ &= \int \left(\left(x^{\frac{3}{2}}\right)^2 - 2x^{\frac{3}{2}}x^{-\frac{1}{3}} + \left(x^{-\frac{1}{3}}\right)^2\right) \mathrm{d}x\end{aligned}$$

Example

Integrate.

$$
\begin{aligned}
\int \left(x^{\frac{3}{2}} - \frac{1}{x^{\frac{1}{3}}}\right)^2 \mathrm{d}x &= \int \left(x^{\frac{3}{2}} - x^{-\frac{1}{3}}\right)^2 \mathrm{d}x \\
&= \int \left(\left(x^{\frac{3}{\cancel{2}}}\right)^{\cancel{2}} - 2x^{\frac{3}{2}} x^{-\frac{1}{3}} + \left(x^{-\frac{1}{3}}\right)^2 \right) \mathrm{d}x \\
&= \int \left(x^{3} - 2x^{\frac{3}{2}-\frac{1}{3}} + x^{-\frac{2}{3}} \right) \mathrm{d}x
\end{aligned}
$$

Example

Integrate.

$$\begin{aligned}\int\left(x^{\frac{3}{2}}-\frac{1}{x^{\frac{1}{3}}}\right)^2 \mathrm{d}x &= \int\left(x^{\frac{3}{2}}-x^{-\frac{1}{3}}\right)^2 \mathrm{d}x\\ &= \int\left(\left(x^{\frac{3}{\cancel{2}}}\right)^{\cancel{2}}-2x^{\frac{3}{2}}x^{-\frac{1}{3}}+\left(x^{-\frac{1}{3}}\right)^2\right)\mathrm{d}x\\ &= \int\left(x^3-2x^{\frac{3}{2}-\frac{1}{3}}+x^{-\frac{2}{3}}\right)\mathrm{d}x\end{aligned}$$

Example

Integrate.

$$\begin{aligned}
\int \left(x^{\frac{3}{2}} - \frac{1}{x^{\frac{1}{3}}} \right)^2 \mathrm{d}x &= \int \left(x^{\frac{3}{2}} - x^{-\frac{1}{3}} \right)^2 \mathrm{d}x \\
&= \int \left(\left(x^{\frac{3}{\cancel{2}}} \right)^{\cancel{2}} - 2x^{\frac{3}{2}} x^{-\frac{1}{3}} + \left(x^{-\frac{1}{3}} \right)^2 \right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{3}{2}-\frac{1}{3}} + x^{-\frac{2}{3}} \right) \mathrm{d}x
\end{aligned}$$

Example

Integrate.

$$\begin{aligned}\int\left(x^{\frac{3}{2}}-\frac{1}{x^{\frac{1}{3}}}\right)^2\mathrm{d}x &= \int\left(x^{\frac{3}{2}}-x^{-\frac{1}{3}}\right)^2\mathrm{d}x\\ &= \int\left(\left(x^{\frac{3}{\cancel{2}}}\right)^{\cancel{2}}-2x^{\frac{3}{2}}x^{-\frac{1}{3}}+\left(x^{-\frac{1}{3}}\right)^2\right)\mathrm{d}x\\ &= \int\left(x^3-2x^{\frac{3}{2}-\frac{1}{3}}+x^{-\frac{2}{3}}\right)\mathrm{d}x\\ &= \int\left(x^3-2x^{\frac{7}{6}}+x^{-\frac{2}{3}}\right)\mathrm{d}x\end{aligned}$$

Example

Integrate.

$$\begin{aligned}
\int \left(x^{\frac{3}{2}} - \frac{1}{x^{\frac{1}{3}}}\right)^2 \mathrm{d}x &= \int \left(x^{\frac{3}{2}} - x^{-\frac{1}{3}}\right)^2 \mathrm{d}x \\
&= \int \left(\left(x^{\frac{3}{\cancel{2}}}\right)^{\cancel{2}} - 2x^{\frac{3}{2}}x^{-\frac{1}{3}} + \left(x^{-\frac{1}{3}}\right)^2\right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{3}{2}-\frac{1}{3}} + x^{-\frac{2}{3}}\right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{7}{6}} + x^{-\frac{2}{3}}\right) \mathrm{d}x \\
&= \mathbf{?} \quad - 2\mathbf{?} \qquad + \mathbf{?}
\end{aligned}$$

Example

Integrate.

$$
\begin{aligned}
\int \left(x^{\frac{3}{2}} - \frac{1}{x^{\frac{1}{3}}}\right)^2 \mathrm{d}x &= \int \left(x^{\frac{3}{2}} - x^{-\frac{1}{3}}\right)^2 \mathrm{d}x \\
&= \int \left(\left(x^{\frac{3}{\cancel{2}}}\right)^{\cancel{2}} - 2x^{\frac{3}{2}}x^{-\frac{1}{3}} + \left(x^{-\frac{1}{3}}\right)^2\right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{3}{2}-\frac{1}{3}} + x^{-\frac{2}{3}}\right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{7}{6}} + x^{-\frac{2}{3}}\right) \mathrm{d}x \\
&= \frac{x^4}{4} - 2\mathbf{?} \qquad + \mathbf{?}
\end{aligned}
$$

Example

Integrate.

$$\begin{aligned}
\int \left(x^{\frac{3}{2}} - \frac{1}{x^{\frac{1}{3}}} \right)^2 \mathrm{d}x &= \int \left(x^{\frac{3}{2}} - x^{-\frac{1}{3}} \right)^2 \mathrm{d}x \\
&= \int \left(\left(x^{\frac{3}{\cancel{2}}} \right)^{\cancel{2}} - 2x^{\frac{3}{2}} x^{-\frac{1}{3}} + \left(x^{-\frac{1}{3}} \right)^2 \right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{3}{2}-\frac{1}{3}} + x^{-\frac{2}{3}} \right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{7}{6}} + x^{-\frac{2}{3}} \right) \mathrm{d}x \\
&= \frac{x^4}{4} - 2\textbf{?} \qquad + \textbf{?}
\end{aligned}$$

Example

Integrate.

$$
\begin{aligned}
\int \left(x^{\frac{3}{2}} - \frac{1}{x^{\frac{1}{3}}} \right)^2 \mathrm{d}x &= \int \left(x^{\frac{3}{2}} - x^{-\frac{1}{3}} \right)^2 \mathrm{d}x \\
&= \int \left(\left(x^{\frac{3}{\cancel{2}}} \right)^{\cancel{2}} - 2x^{\frac{3}{2}} x^{-\frac{1}{3}} + \left(x^{-\frac{1}{3}} \right)^2 \right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{3}{2}-\frac{1}{3}} + x^{-\frac{2}{3}} \right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{7}{6}} + x^{-\frac{2}{3}} \right) \mathrm{d}x \\
&= \frac{x^4}{4} - 2\,\frac{x^{\frac{7}{6}+1}}{\frac{7}{6}+1} + \mathbf{?}
\end{aligned}
$$

Example

Integrate.

$$
\begin{aligned}
\int \left(x^{\frac{3}{2}} - \frac{1}{x^{\frac{1}{3}}}\right)^2 \mathrm{d}x &= \int \left(x^{\frac{3}{2}} - x^{-\frac{1}{3}}\right)^2 \mathrm{d}x \\
&= \int \left(\left(x^{\frac{3}{\cancel{2}}}\right)^{\cancel{2}} - 2x^{\frac{3}{2}}x^{-\frac{1}{3}} + \left(x^{-\frac{1}{3}}\right)^2\right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{3}{2}-\frac{1}{3}} + x^{-\frac{2}{3}}\right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{7}{6}} + x^{-\frac{2}{3}}\right) \mathrm{d}x \\
&= \frac{x^4}{4} - 2\,\frac{x^{\frac{7}{6}+1}}{\frac{7}{6}+1} + \mathbf{?}
\end{aligned}
$$

Example

Integrate.

$$\begin{aligned}
\int \left(x^{\frac{3}{2}} - \frac{1}{x^{\frac{1}{3}}} \right)^2 \mathrm{d}x &= \int \left(x^{\frac{3}{2}} - x^{-\frac{1}{3}} \right)^2 \mathrm{d}x \\
&= \int \left(\left(x^{\frac{3}{\cancel{2}}} \right)^{\cancel{2}} - 2x^{\frac{3}{2}} x^{-\frac{1}{3}} + \left(x^{-\frac{1}{3}} \right)^2 \right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{3}{2}-\frac{1}{3}} + x^{-\frac{2}{3}} \right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{7}{6}} + x^{-\frac{2}{3}} \right) \mathrm{d}x \\
&= \frac{x^4}{4} - 2\,\frac{x^{\frac{7}{6}+1}}{\frac{7}{6}+1} + \frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1}
\end{aligned}$$

Example

Integrate.

$$
\begin{aligned}
\int\left(x^{\frac{3}{2}}-\frac{1}{x^{\frac{1}{3}}}\right)^2 \mathrm{d}x &= \int\left(x^{\frac{3}{2}}-x^{-\frac{1}{3}}\right)^2 \mathrm{d}x\\
&= \int\left(\left(x^{\frac{3}{\cancel{2}}}\right)^{\cancel{2}}-2x^{\frac{3}{2}}x^{-\frac{1}{3}}+\left(x^{-\frac{1}{3}}\right)^2\right)\mathrm{d}x\\
&= \int\left(x^3-2x^{\frac{3}{2}-\frac{1}{3}}+x^{-\frac{2}{3}}\right)\mathrm{d}x\\
&= \int\left(x^3-2x^{\frac{7}{6}}+x^{-\frac{2}{3}}\right)\mathrm{d}x\\
&= \frac{x^4}{4}-2\,\frac{x^{\frac{7}{6}+1}}{\frac{7}{6}+1}+\frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1}+C
\end{aligned}
$$

Example

Integrate.

$$\begin{aligned}
\int \left(x^{\frac{3}{2}} - \frac{1}{x^{\frac{1}{3}}}\right)^2 \mathrm{d}x &= \int \left(x^{\frac{3}{2}} - x^{-\frac{1}{3}}\right)^2 \mathrm{d}x \\
&= \int \left(\left(x^{\frac{3}{\cancel{2}}}\right)^{\cancel{2}} - 2x^{\frac{3}{2}}x^{-\frac{1}{3}} + \left(x^{-\frac{1}{3}}\right)^2 \right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{3}{2}-\frac{1}{3}} + x^{-\frac{2}{3}}\right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{7}{6}} + x^{-\frac{2}{3}}\right) \mathrm{d}x \\
&= \frac{x^4}{4} - 2\,\frac{x^{\frac{7}{6}+1}}{\frac{7}{6}+1} + \frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1} + C \\
&= \frac{x^4}{4} - \frac{2x^{\frac{13}{6}}}{\frac{13}{6}} + \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + C
\end{aligned}$$

Example

Integrate.

$$
\begin{aligned}
\int \left(x^{\frac{3}{2}} - \frac{1}{x^{\frac{1}{3}}}\right)^2 \mathrm{d}x &= \int \left(x^{\frac{3}{2}} - x^{-\frac{1}{3}}\right)^2 \mathrm{d}x \\
&= \int \left(\left(x^{\frac{3}{\cancel{2}}}\right)^{\cancel{2}} - 2x^{\frac{3}{2}}x^{-\frac{1}{3}} + \left(x^{-\frac{1}{3}}\right)^2\right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{3}{2}-\frac{1}{3}} + x^{-\frac{2}{3}}\right) \mathrm{d}x \\
&= \int \left(x^3 - 2x^{\frac{7}{6}} + x^{-\frac{2}{3}}\right) \mathrm{d}x \\
&= \frac{x^4}{4} - 2\,\frac{x^{\frac{7}{6}+1}}{\frac{7}{6}+1} + \frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1} + C \\
&= \frac{x^4}{4} - \frac{2x^{\frac{13}{6}}}{\frac{13}{6}} + \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + C
\end{aligned}
$$

Example

Integrate.

$$
\begin{aligned}
\int\left(x^{\frac{3}{2}}-\frac{1}{x^{\frac{1}{3}}}\right)^2 \mathrm{d}x &= \int\left(x^{\frac{3}{2}}-x^{-\frac{1}{3}}\right)^2 \mathrm{d}x\\
&= \int\left(\left(x^{\frac{3}{\cancel{2}}}\right)^{\cancel{2}}-2x^{\frac{3}{2}}x^{-\frac{1}{3}}+\left(x^{-\frac{1}{3}}\right)^2\right)\mathrm{d}x\\
&= \int\left(x^3-2x^{\frac{3}{2}-\frac{1}{3}}+x^{-\frac{2}{3}}\right)\mathrm{d}x\\
&= \int\left(x^3-2x^{\frac{7}{6}}+x^{-\frac{2}{3}}\right)\mathrm{d}x\\
&= \frac{x^4}{4}-2\,\frac{x^{\frac{7}{6}+1}}{\frac{7}{6}+1}+\frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1}+C\\
&= \frac{x^4}{4}-\frac{2x^{\frac{13}{6}}}{\frac{13}{6}}+\frac{x^{\frac{1}{3}}}{\frac{1}{3}}+C\\
&= \frac{x^4}{4}-\frac{12x^{\frac{13}{6}}}{13}+3x^{\frac{1}{3}}+C
\end{aligned}
$$

Example

Integrate.

$$
\begin{aligned}
\int\left(x^{\frac{3}{2}}-\frac{1}{x^{\frac{1}{3}}}\right)^2 \mathrm{d}x &= \int\left(x^{\frac{3}{2}}-x^{-\frac{1}{3}}\right)^2 \mathrm{d}x\\
&= \int\left(\left(x^{\frac{3}{\cancel{2}}}\right)^{\cancel{2}}-2x^{\frac{3}{2}}x^{-\frac{1}{3}}+\left(x^{-\frac{1}{3}}\right)^2\right)\mathrm{d}x\\
&= \int\left(x^3-2x^{\frac{3}{2}-\frac{1}{3}}+x^{-\frac{2}{3}}\right)\mathrm{d}x\\
&= \int\left(x^3-2x^{\frac{7}{6}}+x^{-\frac{2}{3}}\right)\mathrm{d}x\\
&= \frac{x^4}{4}-2\,\frac{x^{\frac{7}{6}+1}}{\frac{7}{6}+1}+\frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1}+C\\
&= \frac{x^4}{4}-\frac{2x^{\frac{13}{6}}}{\frac{13}{6}}+\frac{x^{\frac{1}{3}}}{\frac{1}{3}}+C\\
&= \frac{x^4}{4}-\frac{12x^{\frac{13}{6}}}{13}+3x^{\frac{1}{3}}+C
\end{aligned}
$$

Example

Integrate.

$$
\begin{aligned}
\int\left(x^{\frac{3}{2}}-\frac{1}{x^{\frac{1}{3}}}\right)^2 \mathrm{d}x &= \int\left(x^{\frac{3}{2}}-x^{-\frac{1}{3}}\right)^2 \mathrm{d}x\\
&= \int\left(\left(x^{\frac{3}{\cancel{2}}}\right)^{\cancel{2}}-2x^{\frac{3}{2}}x^{-\frac{1}{3}}+\left(x^{-\frac{1}{3}}\right)^2\right)\mathrm{d}x\\
&= \int\left(x^3-2x^{\frac{3}{2}-\frac{1}{3}}+x^{-\frac{2}{3}}\right)\mathrm{d}x\\
&= \int\left(x^3-2x^{\frac{7}{6}}+x^{-\frac{2}{3}}\right)\mathrm{d}x\\
&= \frac{x^4}{4}-2\,\frac{x^{\frac{7}{6}+1}}{\frac{7}{6}+1}+\frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1}+C\\
&= \frac{x^4}{4}-\frac{2x^{\frac{13}{6}}}{\frac{13}{6}}+\frac{x^{\frac{1}{3}}}{\frac{1}{3}}+C\\
&= \frac{x^4}{4}-\frac{12x^{\frac{13}{6}}}{13}+3x^{\frac{1}{3}}+C
\end{aligned}
$$

Example

Find the indefinite integral.

$$\int (8x^3 - 3\sec^2 x) \mathrm{d}x$$

Example

Find the indefinite integral.

$$\int (8x^3 - 3\sec^2 x)\mathrm{d}x = 8\int x^3\mathrm{d}x - 3\int \sec^2 x\mathrm{d}x$$

Example

Find the indefinite integral.

$$\int (8x^3 - 3\sec^2 x)\mathrm{d}x = 8\int x^3 \mathrm{d}x - 3\int \sec^2 x \mathrm{d}x$$

$$= 8\textbf{?} \quad - 3\textbf{?}$$

Example

Find the indefinite integral.

$$\int (8x^3 - 3\sec^2 x)\mathrm{d}x = 8\int x^3\mathrm{d}x - 3\int \sec^2 x\mathrm{d}x$$
$$= 8\frac{x^4}{4} - 3\textbf{?}$$

Example

Find the indefinite integral.

$$\int (8x^3 - 3\sec^2 x)\mathrm{d}x = 8\int x^3 \mathrm{d}x - 3\int \sec^2 x \mathrm{d}x$$
$$= 8\frac{x^4}{4} - 3\mathbf{?}$$

Example

Find the indefinite integral.

$$\int (8x^3 - 3\sec^2 x)\mathrm{d}x = 8\int x^3\mathrm{d}x - 3\int \sec^2 x\mathrm{d}x$$
$$= 8\frac{x^4}{4} - 3\tan x$$

Example

Find the indefinite integral.

$$\begin{aligned}\int (8x^3 - 3\sec^2 x)\mathrm{d}x &= 8\int x^3 \mathrm{d}x - 3\int \sec^2 x \mathrm{d}x\\ &= 8\frac{x^4}{4} - 3\tan x + C\end{aligned}$$

Example

Find the indefinite integral.

$$\begin{aligned}\int (8x^3 - 3\sec^2 x)\mathrm{d}x &= 8\int x^3 \mathrm{d}x - 3\int \sec^2 x \mathrm{d}x\\ &= 8\frac{x^4}{4} - 3\tan x + C\\ &= 2x^4 - 3\tan x + C\end{aligned}$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta}\mathrm{d}\theta$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} \mathrm{d}\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) \mathrm{d}\theta$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} \mathrm{d}\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) \mathrm{d}\theta$$

$$= \int \mathbf{?} \quad \mathbf{?} \quad \mathrm{d}\theta$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} \mathrm{d}\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) \mathrm{d}\theta$$

$$= \int \csc\theta \textbf{?} \quad \mathrm{d}\theta$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} d\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) d\theta$$
$$= \int \csc\theta \, \mathbf{?} \quad d\theta$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} d\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) d\theta$$
$$= \int \csc\theta \cot\theta d\theta$$

Example

Find the general indefinite integral.

$$\begin{aligned}\int \frac{\cos\theta}{\sin^2\theta} d\theta &= \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) d\theta \\ &= \int \csc\theta \cot\theta d\theta \\ &= \textbf{?}\end{aligned}$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} d\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) d\theta$$
$$= \int \csc\theta \cot\theta d\theta$$
$$= -\csc\theta$$

Example

Find the general indefinite integral.

$$\begin{aligned}\int \frac{\cos\theta}{\sin^2\theta} d\theta &= \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) d\theta \\ &= \int \csc\theta \cot\theta d\theta \\ &= -\csc\theta + C\end{aligned}$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x$$

Example

$$\int_{0}^{3}(x^3-6x)\mathrm{d}x = \left[\int (x^3-6x)\mathrm{d}x\right]_{0}^{3}$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x = \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3$$
$$= \left[\int x^3 \mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x = \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3$$

$$= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3$$

$$= \left[\textbf{?} \quad - 6\textbf{?} \quad\right]_0^3$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x = \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3$$
$$= \left[\int x^3 \mathrm{d}x - 6\int x \mathrm{d}x\right]_0^3$$
$$= \left[\frac{x^4}{4} - 6\,\mathbf{?}\quad\right]_0^3$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x = \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3$$

$$= \left[\int x^3\mathrm{d}x - 6\int {\color{red} x\mathrm{d}x}\right]_0^3$$

$$= \left[\frac{x^4}{4} - 6\,{\color{red}\mathbf{?}}\quad\right]_0^3$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x = \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3$$
$$= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3$$
$$= \left[\frac{x^4}{4} - 6\frac{x^2}{2}\right]_0^3$$

Example

$$
\begin{aligned}
\int_0^3 (x^3 - 6x)\mathrm{d}x &= \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3 \\
&= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3 \\
&= \left[\frac{x^4}{4} - 6\,\frac{x^2}{2}\right]_0^3 \\
&= \left(\frac{1}{4}\cdot 3^4 - 3\cdot 3^2\right) - \left(\frac{1}{4}\cdot 0^4 - 3\cdot 0^2\right)
\end{aligned}
$$

Example

$$
\begin{aligned}
\int_0^3 (x^3 - 6x)\mathrm{d}x &= \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3 \\
&= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3 \\
&= \left[\frac{x^4}{4} - 6\frac{x^2}{2}\right]_0^3 \\
&= \left(\frac{1}{4}\cdot 3^4 - 3\cdot 3^2\right) - \left(\frac{1}{4}\cdot 0^4 - 3\cdot 0^2\right)
\end{aligned}
$$

Example

$$
\begin{aligned}
\int_0^3 (x^3 - 6x)\mathrm{d}x &= \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3 \\
&= \left[\int x^3 \mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3 \\
&= \left[\frac{x^4}{4} - 6\,\frac{x^2}{2}\right]_0^3 \\
&= \left(\frac{1}{4}\cdot 3^4 - 3\cdot 3^2\right) - \left(\frac{1}{4}\cdot 0^4 - 3\cdot 0^2\right) \\
&= \frac{81}{4} - 27 - 0 + 0
\end{aligned}
$$

Example

$$
\begin{aligned}
\int_0^3 (x^3 - 6x)\mathrm{d}x &= \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3 \\
&= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3 \\
&= \left[\frac{x^4}{4} - 6\,\frac{x^2}{2}\right]_0^3 \\
&= \left(\frac{1}{4}\cdot 3^4 - {\color{red}3\cdot 3^2}\right) - \left(\frac{1}{4}\cdot 0^4 - 3\cdot 0^2\right) \\
&= \frac{81}{4} - {\color{red}27} - 0 + 0
\end{aligned}
$$

Example

$$
\begin{aligned}
\int_0^3 (x^3 - 6x)\mathrm{d}x &= \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3 \\
&= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3 \\
&= \left[\frac{x^4}{4} - 6\,\frac{x^2}{2}\right]_0^3 \\
&= \left(\frac{1}{4}\cdot 3^4 - 3\cdot 3^2\right) - \left(\frac{1}{4}\cdot 0^4 - 3\cdot 0^2\right) \\
&= \frac{81}{4} - 27 - 0 + 0 = -\frac{27}{4}.
\end{aligned}
$$

Example

Evaluate: $$\int_1^9 \frac{2t^3 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$$

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$$= 81 + 18 + \frac{1}{9} - 1 - \frac{2}{3} - 1 = \frac{868}{9}.$$