

Calculus I

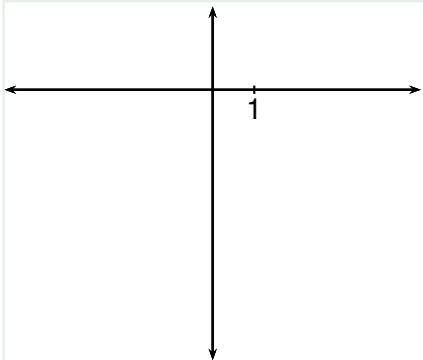
Type 1: $\lim_{x \rightarrow \infty}$ of rational function.

Todor Milev

2019

Example

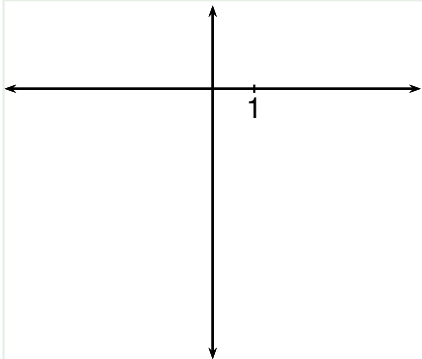
Evaluate $\lim_{x \rightarrow \infty} \frac{3x^2 - x - 2}{5x^2 + 4x + 1}$.



$$\lim_{x \rightarrow \infty} \frac{(3x^2 - x - 2)}{(5x^2 + 4x + 1)}$$

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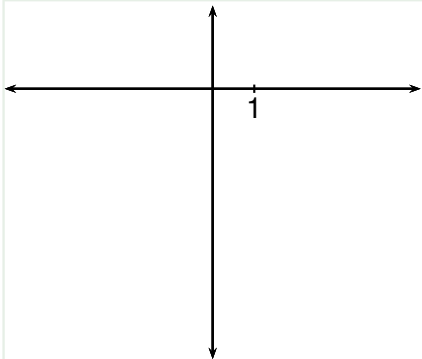


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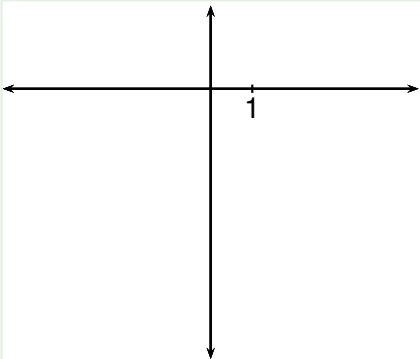


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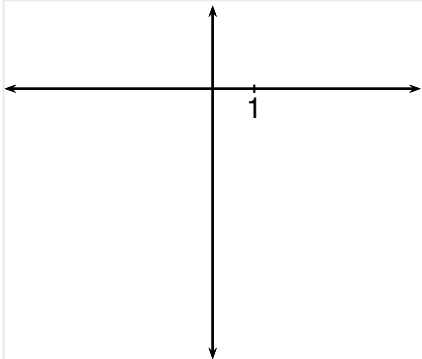
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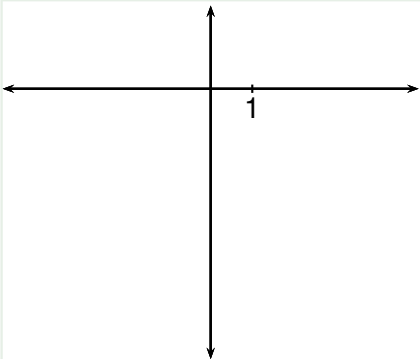


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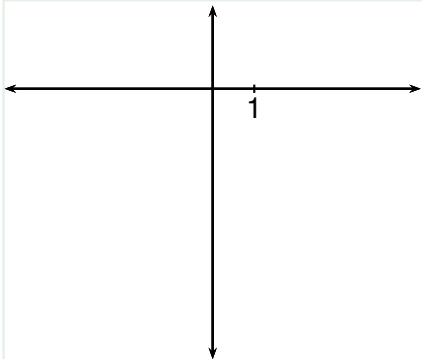


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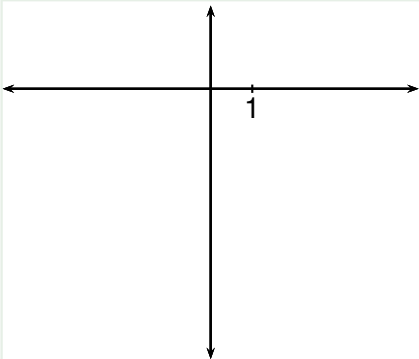


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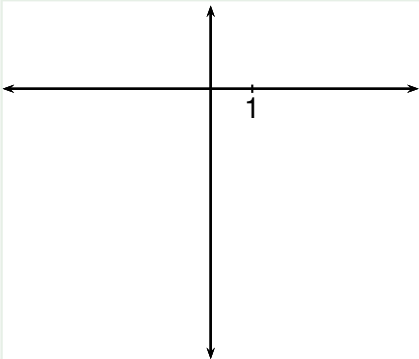


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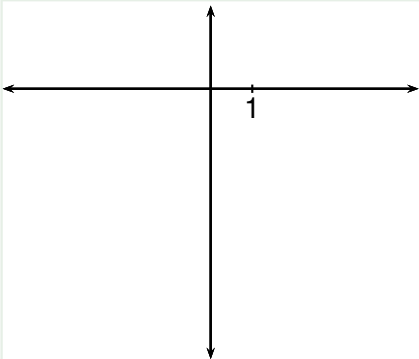


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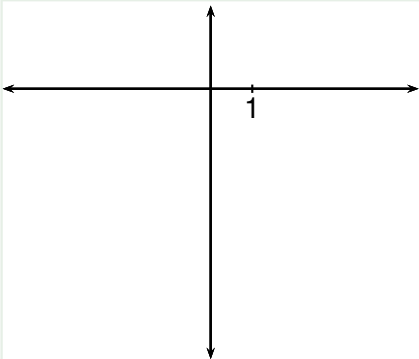


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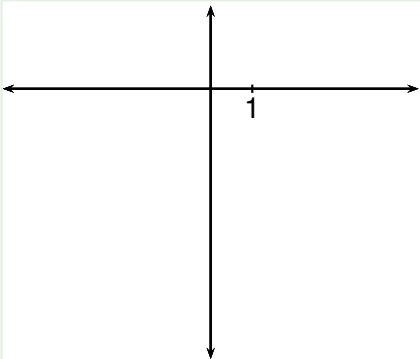


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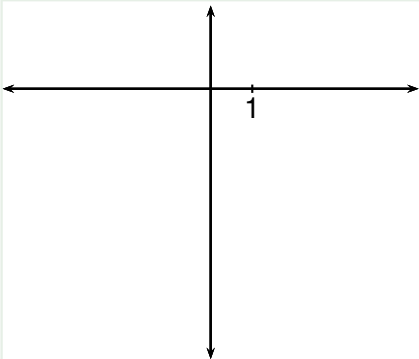


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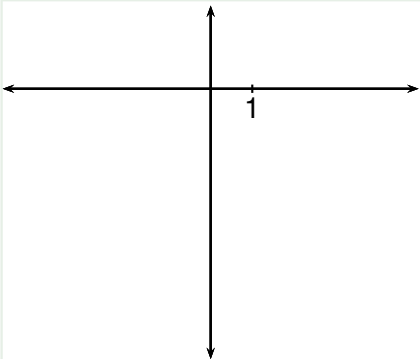


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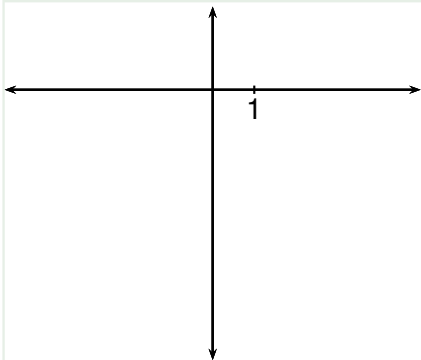


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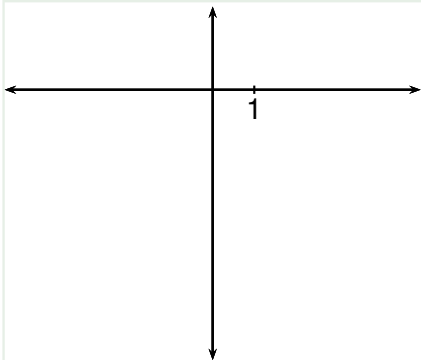


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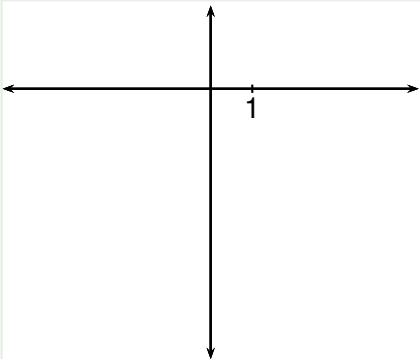


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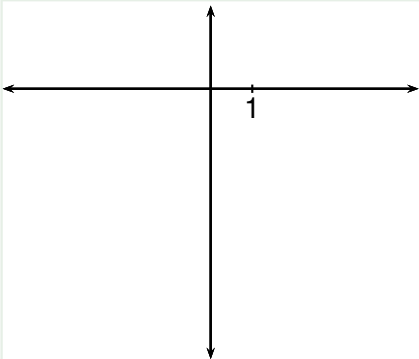


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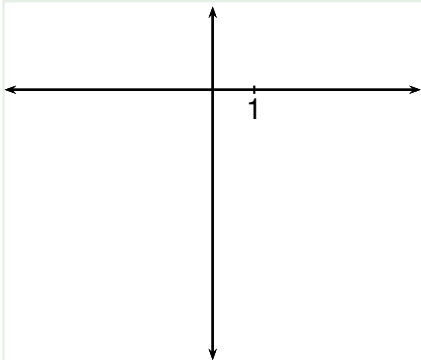


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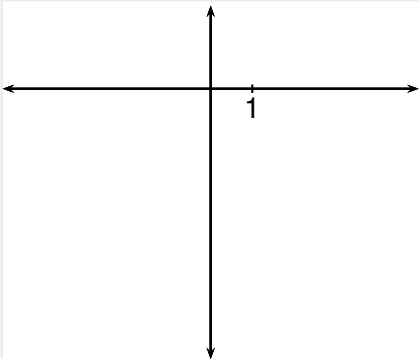


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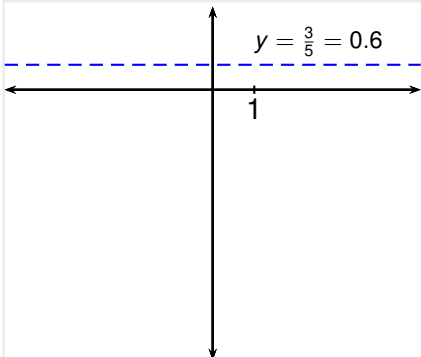


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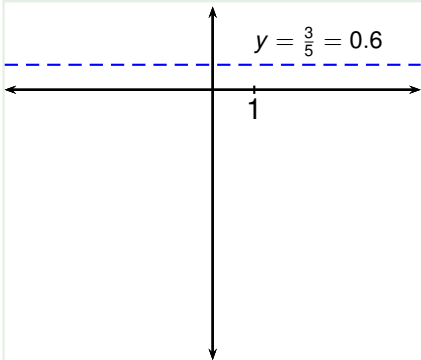


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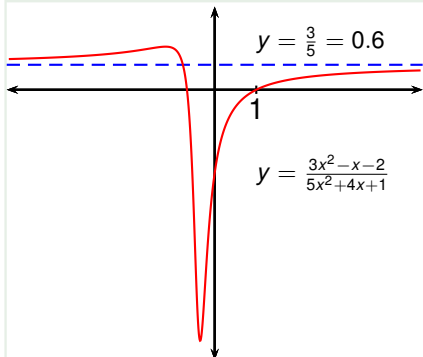
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