# Precalculus
# Find all angles coterminal to a given one

Todor Milev

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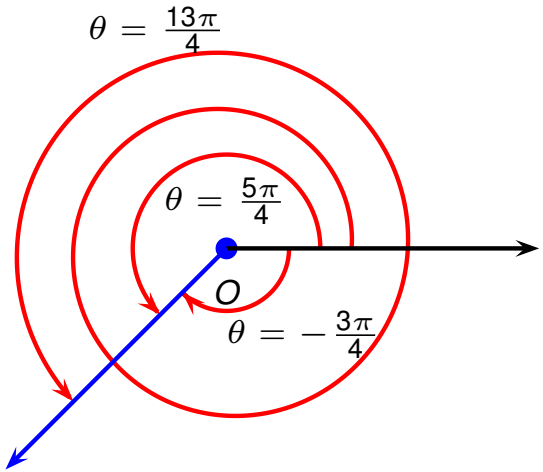
## Definition (Coterminal Angles)

Two angles (angle measures) are called coterminal if the corresponding geometric angles have the same initial and terminal sides.




## Observation

*The set of angles coterminal with $\alpha$ consists of the angles $\alpha + 2k\pi$, where k runs over the set of integers. In other words, the angles coterminal with $\alpha$ are the angles:*

$$\ldots, \alpha - 6\pi, \alpha - 4\pi, \alpha - 2\pi, \alpha, \alpha + 2\pi, \alpha + 4\pi, \alpha + 6\pi, \ldots.$$

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- Find all angles in the interval $[-2\pi, \pi]$ that are coterminal to $\frac{\pi}{4}$.

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Our final answer is $-\frac{7\pi}{4}, \frac{\pi}{4}$