Precalculus
Factoring quadratic polynomials

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Outline

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Definition ((Partial) Factorization)

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Example

$$\begin{aligned} x^2 - 1 &= (x-1)(x+1) \\ x^3 + 8 &= (x+2)\left(x^2 - 2x + 4\right) \end{aligned}$$

Theorem

The quadratic $ax^2 + bx + c$ factors as follows.

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

where x_1 and x_2 are the roots of the quadratic, given by:

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Let $ax^2 + bx + c$ be a quadratic functions with zeros x_1 and x_2. Then:

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$$\begin{array}{rcl} a(x - x_1)(x - x_2) & = & ax^2 + bx + c \\ ax^2 - {\color{red}ax}x_2 - {\color{red}a}x_1{\color{red}x} + a(-x_1)(-x_2) & = & ax^2 + bx + c \\ ax^2 - {\color{red}a}(x_2 + x_1){\color{red}x} + ax_1x_2 & = & ax^2 + bx + c \end{array}$$

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a(x-x_1)(x-x_2) &= ax^2+bx+c\\
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ax^2-a(x_2+x_1)x+ax_1x_2 &= ax^2+bx+c\\
ax_1x_2 &= c\\
x_1x_2 &= \frac{c}{a}\\
-a(x_1+x_2) &= b\\
x_1+x_2 &= -\frac{b}{a}
\end{aligned}$$

The last two formulas are called Vieta's formulas (after François Viète (1540-1603), Latinized name: Franciscus Vieta).

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

$$\begin{aligned} x_1 + x_2 &= -\frac{b}{a} \\ x_1 x_2 &= \frac{c}{a} \end{aligned}$$

Vieta's formulas

Example

Factor the quadratic.

$$x^2 + 5x + 6$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

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Vieta's formulas

Example

Factor the quadratic.

$$x^2 + 5x + 6 = (x + \mathbf{?})(x + \mathbf{?})$$

- The product of the two roots: $x_1 x_2 = 6$.

$$
\begin{aligned}
ax^2 + bx + c &= a(x - x_1)(x - x_2)\\
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Vieta's formulas

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Factor the quadratic.

$$x^2 + 5x + 6 = (x + \mathbf{?})(x + \mathbf{?})$$

- The product of the two roots: $x_1 x_2 = 6$.
- The divisors of 6 are $\pm 1, \pm 2, \pm 3, \pm 6$.

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

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- The product of the two roots: $x_1 x_2 = 6$.
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- Therefore the pair x_1, x_2 is $\pm 1, \pm 6$ or $\pm 2, \pm 3$.

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Example

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Vieta's formulas

Example

Factor the quadratic.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

- The product of the two roots: $x_1 x_2 = 6$.
- The divisors of 6 are $\pm 1, \pm 2, \pm 3, \pm 6$.
- Therefore the pair x_1, x_2 is $\pm 1, \pm 6$ or $\pm 2, \pm 3$.
- The sum of the two roots: $x_1 + x_2 = -5$

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

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Example

Factor the quadratic.

$$x^2 + 3x + 1$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

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Example

Factor the quadratic.

$$x^2 + 3x + 1 = \left(x + \mathbf{?} \qquad\qquad\right)\left(x + \mathbf{?} \qquad\qquad\right)$$

- The product of the two roots: $x_1 x_2 = 1$.

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$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

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Example

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Factor the quadratic.

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Example

Factor the quadratic.

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$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

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Example

Factor the quadratic.

$$x^2 + {\color{red}3}x + 1 = \Big(x + \textbf{?} \qquad\qquad \Big)\Big(x + \textbf{?} \qquad\qquad \Big)$$

- The product of the two roots: $x_1 x_2 = 1$.
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- $\begin{array}{rcl} (x-1)(x-1) & = & (x-1)^2 = x^2 - 2x + 1 \\ (x+1)(x+1) & = & (x+1)^2 = x^2 + 2x + 1 \end{array}$ both don't work.
- $\Rightarrow$ No easy factorization; must use quadratic formula.

$$x_1, x_2 = \frac{-{\color{red}b} \pm \sqrt{{\color{red}b}^2 - 4ac}}{2a} = \frac{-{\color{red}3} \pm \sqrt{{\color{red}3}^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

$$\left| \begin{array}{rcl} x_1 x_2 & = & \frac{c}{a} \\ x_1 + x_2 & = & -\frac{b}{a} \end{array} \right.$$

Example

Factor the quadratic.

$$x^2 + 3x + 1 = \left(x + \textbf{?} \qquad\qquad \right)\left(x + \textbf{?} \qquad\qquad \right)$$

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Example

Factor the quadratic.

$$x^2 + 3x + 1 = \left(x - \left(\frac{-3+\sqrt{5}}{2}\right)\right)\left(x - \left(\frac{-3-\sqrt{5}}{2}\right)\right)$$

- The product of the two roots: $x_1 x_2 = 1$.
- Integer options: $x_1 = 1, x_2 = 1$ and $x_1 = -1, x_2 = -1$.
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Example

Factor the quadratic, using complex numbers if needed.

$$x^2+x+1$$

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Factor the quadratic, using complex numbers if needed.

$$x^2 + x + 1 = \left(x + \mathbf{?} \qquad\qquad \right)\left(x + \mathbf{?} \qquad\qquad \right)$$

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Example

Factor the quadratic, using complex numbers if needed.

$$x^2 + x + 1 = \left(x \quad - \left(\frac{-1+\sqrt{3}i}{2}\right)\right)\left(x \quad - \left(\frac{-1-\sqrt{3}i}{2}\right)\right)$$

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