# Calculus II
# Comparison and limit-comparison tests, part 3

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Test the series $\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{7+n^5}}$ for convergence or divergence.

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- $\sum \frac{2}{n^{\frac{1}{2}}}$ is a constant multiple of a $p$-series with $p = \frac{1}{2}$.
- Therefore $\sum \frac{2}{n^{\frac{1}{2}}}$ is divergent, and so is $\sum \frac{2n^2+3n}{\sqrt{7+n^5}}$.