Calculus I
Type 3: Exponent equation that reduces to quadratic

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Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$4u^2 - 4u - 3 = 0$$

$$(2u - 3)(2u + 1) = 0$$

$$2u - 3 = 0 \quad \text{or} \quad 2u + 1 = 0$$

$$u = \frac{3}{2} \quad \text{or} \quad u = -\frac{1}{2}$$

$$2^x = \frac{3}{2} \quad \text{or} \quad 2^x = -\frac{1}{2}$$

$$x = \log_2\left(\frac{3}{2}\right) = \frac{\ln\left(\frac{3}{2}\right)}{\ln 2} \approx 0.58496 \quad \text{or} \quad \text{no real solution}$$