

Precalculus

Simplify $\sin(k \arcsin x)$, $\cos(k \arcsin x)$,
 $\sin(k \arccos x)$, $\cos(k \arccos x)$

Todor Milev

2019

Example

Rewrite $\sin(2 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$.

$$\sin(2 \arccos(x))$$

Example

Rewrite $\sin(2 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$.
To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$.

$$\sin(2 \arccos(x))$$

Example

Rewrite $\sin(2 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\sin(2 \arccos(x))$$

Example

Rewrite $\sin(2 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\sin(2 \arccos(x)) = \sin(2y)$$

$$| \text{ Set } y = \arccos x$$

Example

Rewrite $\sin(2 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}\sin(2 \arccos(x)) &= \sin(2y) \\ &= ?\end{aligned}$$

Set $y = \arccos x$
Express via $\sin y, \cos y$

Example

Rewrite $\sin(2 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}\sin(2 \arccos(x)) &= \sin(2y) \\ &= 2 \cos y \sin y\end{aligned}$$

Set $y = \arccos x$
Express via $\sin y, \cos y$

Example

Rewrite $\sin(2 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to **rewrite the expression only using the cos function**.

$$\begin{aligned}
 \sin(2 \arccos(x)) &= \sin(2y) \\
 &= 2 \cos y \sin y \\
 &= 2 \cos y \left(\pm \sqrt{1 - \cos^2 y} \right)
 \end{aligned}
 \quad \left| \begin{array}{l}
 \text{Set } y = \arccos x \\
 \text{Express via } \sin y, \cos y \\
 \text{Express } \sin y \text{ via } \cos y
 \end{array} \right.$$

Example

Rewrite $\sin(2 \arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \sin(2 \arccos(x)) &= \sin(2y) \\
 &= 2 \cos y \sin y \\
 &= 2 \cos y \left(\pm \sqrt{1 - \cos^2 y} \right) \\
 &= 2 \cos y \sqrt{1 - \cos^2 y}
 \end{aligned}$$

Set $y = \arccos x$
 Express via $\sin y, \cos y$
 Express $\sin y$ via $\cos y$
 $\sin y > 0$ because
 $0 \leq y \leq \pi$

Example

Rewrite $\sin(2 \arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \sin(2 \arccos(x)) &= \sin(2y) \\
 &= 2 \cos y \sin y \\
 &= 2 \cos y \left(\pm \sqrt{1 - \cos^2 y} \right) \\
 &= 2 \cos y \sqrt{1 - \cos^2 y}
 \end{aligned}$$

Set $y = \arccos x$

Express via $\sin y, \cos y$

Express $\sin y$ via $\cos y$

$\sin y > 0$ because

$0 \leq y \leq \pi$

Example

Rewrite $\sin(2 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \sin(2 \arccos(x)) &= \sin(2y) \\
 &= 2 \cos y \sin y \\
 &= 2 \cos y \left(\pm \sqrt{1 - \cos^2 y} \right) \\
 &= 2 \cos y \sqrt{1 - \cos^2 y} \\
 &= 2x \sqrt{1 - x^2}
 \end{aligned}$$

Set $y = \arccos x$

Express via $\sin y, \cos y$

Express $\sin y$ via $\cos y$

$\sin y > 0$ because

$0 \leq y \leq \pi$

use $x = \cos y$

Example

Rewrite $\sin(2 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$.
 To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \sin(2 \arccos(x)) &= \sin(2y) \\
 &= 2 \cos y \sin y \\
 &= 2 \cos y \left(\pm \sqrt{1 - \cos^2 y} \right) \\
 &= 2 \cos y \sqrt{1 - \cos^2 y} \\
 &= 2x \sqrt{1 - x^2}
 \end{aligned}$$

Set $y = \arccos x$
 Express via $\sin y, \cos y$
 Express $\sin y$ via $\cos y$
 $\sin y > 0$ because
 $0 \leq y \leq \pi$
 use $x = \cos y$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$.

$$\cos(3 \arccos(x))$$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$.
To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$.

$$\cos(3 \arccos(x))$$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\cos(3 \arccos(x))$$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\cos(3 \arccos(x)) = \cos(3y) \quad \Big| \quad y = \arccos x$$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\cos(3 \arccos(x)) = \cos(3y) = \cos(2y + y) \quad | \quad y = \arccos x$$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\cos(3 \arccos(x)) = \cos(3y) = \cos(2y + y) \\ = ?$$

$y = \arccos x$
Angle sum f-la

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned} \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) \\ &= \cos(2y) \cos y - \sin(2y) \sin y \end{aligned} \quad \left| \begin{array}{l} y = \arccos x \\ \text{Angle sum f-la} \end{array} \right.$$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) \\
 &= \cos(2y) \cos y - \sin(2y) \sin y \\
 &= (\text{?} \quad \quad \quad) \cos y \\
 &\quad - \text{?} \quad \quad \quad \sin y
 \end{aligned}$$

$y = \arccos x$
 Angle sum f-la
 Express via
 $\sin y, \cos y$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) \\
 &= \cos(2y) \cos y - \sin(2y) \sin y \\
 &= (\cos^2 y - \sin^2 y) \cos y \\
 &\quad - \sin y \sin y
 \end{aligned}$$

$y = \arccos x$
 Angle sum f-la
 Express via
 $\sin y, \cos y$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) \\
 &= \cos(2y) \cos y - \sin(2y) \sin y \\
 &= (\cos^2 y - \sin^2 y) \cos y \\
 &\quad - \sin y
 \end{aligned}$$

$y = \arccos x$
 Angle sum f-la
 Express via
 $\sin y, \cos y$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) \\
 &= \cos(2y) \cos y - \sin(2y) \sin y \\
 &= (\cos^2 y - \sin^2 y) \cos y \\
 &\quad - 2 \sin y \cos y \sin y
 \end{aligned}$$

$y = \arccos x$
 Angle sum f-la
 Express via
 $\sin y, \cos y$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) \\
 &= \cos(2y) \cos y - \sin(2y) \sin y \\
 &= (\cos^2 y - \sin^2 y) \cos y \\
 &\quad - 2 \sin y \cos y \sin y \\
 &= \cos^3 y - \sin^2 y \cos y - 2 \sin^2 y \cos y
 \end{aligned}
 \quad \left| \begin{array}{l} y = \arccos x \\ \text{Angle sum f-la} \\ \text{Express via} \\ \sin y, \cos y \end{array} \right.$$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) \\
 &= \cos(2y) \cos y - \sin(2y) \sin y \\
 &= (\cos^2 y - \sin^2 y) \cos y \\
 &\quad - 2 \sin y \cos y \sin y \\
 &= \cos^3 y - \sin^2 y \cos y - 2 \sin^2 y \cos y
 \end{aligned}
 \quad \left| \begin{array}{l} y = \arccos x \\ \text{Angle sum f-la} \\ \text{Express via} \\ \sin y, \cos y \end{array} \right.$$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) \\
 &= \cos(2y) \cos y - \sin(2y) \sin y \\
 &= (\cos^2 y - \sin^2 y) \cos y \\
 &\quad - 2 \sin y \cos y \sin y \\
 &= \cos^3 y - \sin^2 y \cos y - 2 \sin^2 y \cos y
 \end{aligned}
 \quad \left| \begin{array}{l} y = \arccos x \\ \text{Angle sum f-la} \\ \text{Express via} \\ \sin y, \cos y \end{array} \right.$$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) \\
 &= \cos(2y) \cos y - \sin(2y) \sin y \\
 &= (\cos^2 y - \sin^2 y) \cos y \\
 &\quad - 2 \sin y \cos y \sin y \\
 &= \cos^3 y - \sin^2 y \cos y - 2 \sin^2 y \cos y \\
 &= \cos^3 y - 3 \sin^2 y \cos y
 \end{aligned}$$

$y = \arccos x$
 Angle sum f-la
 Express via
 $\sin y, \cos y$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) \\
 &= \cos(2y) \cos y - \sin(2y) \sin y \\
 &= (\cos^2 y - \sin^2 y) \cos y \\
 &\quad - 2 \sin y \cos y \sin y \\
 &= \cos^3 y - \sin^2 y \cos y - 2 \sin^2 y \cos y \\
 &= \cos^3 y - 3 \sin^2 y \cos y \\
 &= \cos^3 y - 3(\text{?}) \cos y
 \end{aligned}$$

$y = \arccos x$
 Angle sum f-la
 Express via
 $\sin y, \cos y$

Express $\sin y$
 via $\cos y$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) \\
 &= \cos(2y) \cos y - \sin(2y) \sin y \\
 &= (\cos^2 y - \sin^2 y) \cos y \\
 &\quad - 2 \sin y \cos y \sin y \\
 &= \cos^3 y - \sin^2 y \cos y - 2 \sin^2 y \cos y \\
 &= \cos^3 y - 3 \sin^2 y \cos y \\
 &= \cos^3 y - 3(1 - \cos^2 y) \cos y
 \end{aligned}$$

$y = \arccos x$
 Angle sum f-la
 Express via
 $\sin y, \cos y$

Express $\sin y$
 via $\cos y$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) \\
 &= \cos(2y) \cos y - \sin(2y) \sin y \\
 &= (\cos^2 y - \sin^2 y) \cos y \\
 &\quad - 2 \sin y \cos y \sin y \\
 &= \cos^3 y - \sin^2 y \cos y - 2 \sin^2 y \cos y \\
 &= \cos^3 y - 3 \sin^2 y \cos y \\
 &= \cos^3 y - 3(1 - \cos^2 y) \cos y \\
 &= 4\cos^3 y - 3 \cos y
 \end{aligned}$$

$y = \arccos x$
 Angle sum f-la
 Express via
 $\sin y, \cos y$

 Express $\sin y$
 via $\cos y$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) \\
 &= \cos(2y) \cos y - \sin(2y) \sin y \\
 &= (\cos^2 y - \sin^2 y) \cos y \\
 &\quad - 2 \sin y \cos y \sin y \\
 &= \cos^3 y - \sin^2 y \cos y - 2 \sin^2 y \cos y \\
 &= \cos^3 y - 3 \sin^2 y \cos y \\
 &= \cos^3 y - 3(1 - \cos^2 y) \cos y \\
 &= 4\cos^3 y - 3\cos y
 \end{aligned}$$

$y = \arccos x$
 Angle sum f-la
 Express via
 $\sin y, \cos y$

 Express $\sin y$
 via $\cos y$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1 - x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) \\
 &= \cos(2y) \cos y - \sin(2y) \sin y \\
 &= (\cos^2 y - \sin^2 y) \cos y \\
 &\quad - 2 \sin y \cos y \sin y \\
 &= \cos^3 y - \sin^2 y \cos y - 2 \sin^2 y \cos y \\
 &= \cos^3 y - 3 \sin^2 y \cos y \\
 &= \cos^3 y - 3(1 - \cos^2 y) \cos y \\
 &= 4\cos^3 y - 3\cos y \\
 &= 4x^3 - 3x
 \end{aligned}$$

$y = \arccos x$
 Angle sum f-la
 Express via
 $\sin y, \cos y$

Express $\sin y$
 via $\cos y$

$x = \cos y$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\begin{aligned}
 \cos(3 \arccos(x)) &= \cos(3y) = \cos(2y + y) & y = \arccos x \\
 &= \cos(2y) \cos y - \sin(2y) \sin y & \text{Angle sum f-la} \\
 &= (\cos^2 y - \sin^2 y) \cos y & \text{Express via} \\
 &\quad - 2 \sin y \cos y \sin y & \sin y, \cos y \\
 &= \cos^3 y - \sin^2 y \cos y - 2 \sin^2 y \cos y \\
 &= \cos^3 y - 3 \sin^2 y \cos y & \text{Express } \sin y \\
 &= \cos^3 y - 3(1 - \cos^2 y) \cos y & \text{via } \cos y \\
 &= 4\cos^3 y - 3 \cos y \\
 &= 4x^3 - 3x & x = \cos y
 \end{aligned}$$