Calculus II Basic divergence test

Todor Milev

2019

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2}{5n^2 + 4}$$

$$\lim_{n\to\infty}a_n=\lim_{n\to\infty}\frac{n^2}{5n^2+4}\cdot\frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{n^2}{5n^2 + 4} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \to \infty} \frac{1}{5 + \frac{4}{n^2}}$$

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{n^2}{5n^2 + 4} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \to \infty} \frac{1}{5 + \frac{4}{n^2}} = \frac{1}{5} \neq 0$$

Show that the series $\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$ diverges.

$$\lim_{n \to \infty} a_n = \lim_{n \to \infty} \frac{n^2}{5n^2 + 4} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim_{n \to \infty} \frac{1}{5 + \frac{4}{n^2}} = \frac{1}{5} \neq 0$$

Therefore, by the Divergence Test, the series diverges.