

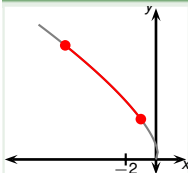
Calculus II

Curve length miscellaneous problem, part 2

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Example



Find the length of the curve γ .

$$\gamma : \begin{cases} x(t) = \sqrt{t} - 2 \\ y(t) = \frac{8}{3}t^{\frac{3}{4}} \end{cases}, t \in [1, 4]$$

We have that $x'(t) = \frac{1}{2\sqrt{t}} - 2$ and $y'(t) = \frac{8}{3} \cdot \frac{3}{4}t^{-\frac{1}{4}} = 2t^{-\frac{1}{4}}$.

$$\begin{aligned} L(\gamma) &= \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} dt = \int_1^4 \sqrt{\left(\frac{1}{2\sqrt{t}} - 2\right)^2 + \left(2t^{-\frac{1}{4}}\right)^2} dt \\ &= \int_1^4 \sqrt{\frac{1}{4t} - \frac{2}{\sqrt{t}} + 4 + \frac{4}{\sqrt{t}}} dt \\ &= \int_1^4 \sqrt{\frac{1}{4t} + \frac{2}{\sqrt{t}} + 4} dt = \int_1^4 \sqrt{\left(\frac{1}{2\sqrt{t}} + 2\right)^2} dt \\ &= \int_1^4 \left(\frac{1}{2\sqrt{t}} + 2\right) dt = \left[\sqrt{t} + 2t\right]_1^4 = \sqrt{4} + 2 \cdot 4 - (\sqrt{1} + 2 \cdot 1) = 7. \end{aligned}$$