

Precalculus

Logarithmic equations involving quadratics

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Solve the equation.

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| Exponentiate base $\textcolor{red}{3}$

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 \log_3(2x^2 + 1) &= 2 && | \text{Exponentiate base 3} \\
 3^{\log_3(2x^2 + 1)} &= 3^2 \\
 2x^2 + 1 &= 9 \\
 2x^2 &= 8 \\
 x^2 &= \frac{8}{2} = 4 \\
 x &= \pm\sqrt{4} = \pm 2 \\
 x = 2 \text{ or } x = -2 && | \text{final answer}
 \end{aligned}$$