Calculus I
Reference: Indefinite integrals = antiderivatives

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Indefinite Integrals

- The Evaluation Theorem establishes a connection between antiderivatives and definite integrals.
- It says that $\int_a^b f(x)\mathrm{d}x$ equals $F(b) - F(a)$, where F is an antiderivative of f.
- We need convenient notation for writing antiderivatives.
- This is what the indefinite integral is.

Definition (Indefinite Integral)

The indefinite integral of f is another way of saying the antiderivative of f, and is written $\int f(x)\mathrm{d}x$. In other words,

$$\int f(x)\mathrm{d}x = F(x) \qquad \text{means} \qquad F'(x) = f(x).$$

Example

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- Example: the general antiderivative of $\frac{1}{x}$ is

$$F(x) = \begin{cases} \ln|x| + C_1 & \text{if} \quad x > 0 \\ \ln|x| + C_2 & \text{if} \quad x < 0 \end{cases}$$

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- We adopt the convention that the constant participating in an indefinite integral is only valid on one interval.
- $\int \frac{1}{x}\mathrm{d}x = \ln|x| + C$, and this is valid either on $(-\infty, 0)$ or $(0, \infty)$.