

# Precalculus

## § Text problems leading to polynomial systems

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2019

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The two solution candidates are  $x = 12, y = 13$  and  $x = 13, y = 12$ . Since  $y \geq x$ , one of the solutions needs to be discarded and our final answer is  $x = 12, y = 13$ .