

## Precalculus

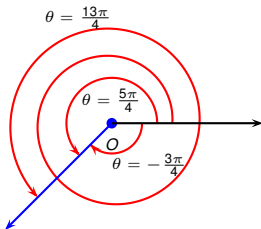
**Find all angles coterminal to a given one**

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## Definition (Coterminal Angles)

Two angles (angle measures) are called coterminal if the corresponding geometric angles have the same initial and terminal sides.



## Observation

*The set of angles coterminal with  $\alpha$  consists of the angles  $\alpha + 2k\pi$ , where  $k$  runs over the set of integers. In other words, the angles coterminal with  $\alpha$  are the angles:*

$$\dots, \alpha - 6\pi, \alpha - 4\pi, \alpha - 2\pi, \alpha, \alpha + 2\pi, \alpha + 4\pi, \alpha + 6\pi, \dots$$

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- Find all angles in the interval  $[-2\pi, \pi]$  that are coterminal to  $\frac{\pi}{4}$ .

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Our final answer is  $-\frac{7\pi}{4}, \frac{\pi}{4}$