Calculus II

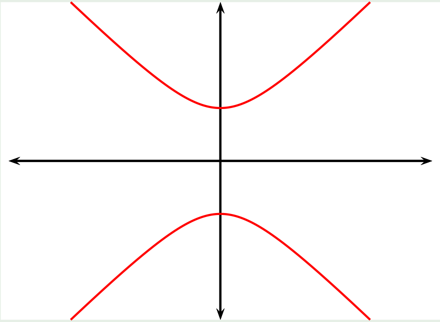
Definite integrals of the form $\int_p^q \sqrt{ax^2 + c}\mathbf{d}x$, $a, c > 0$

Todor Milev

2019

Example

Find the area locked b-n the hyperbolas $y = \pm\sqrt{x^2+1}$ and $x = \pm 2\sqrt{2}$.



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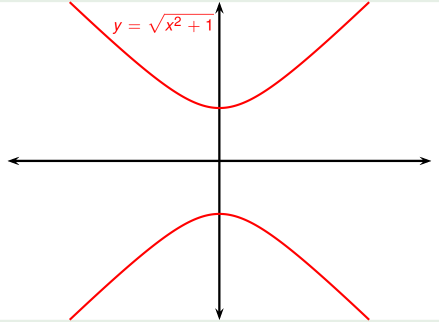

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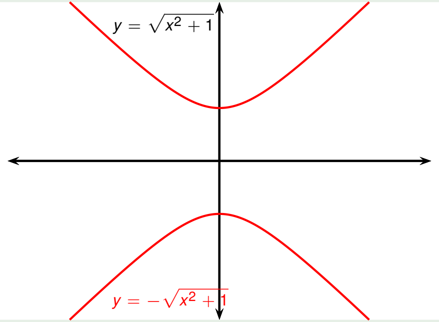

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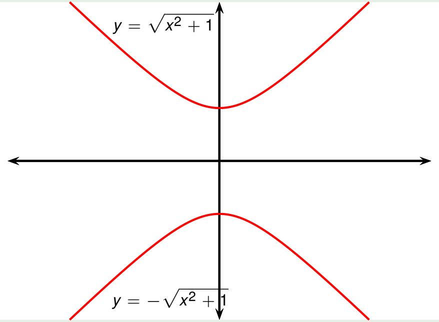

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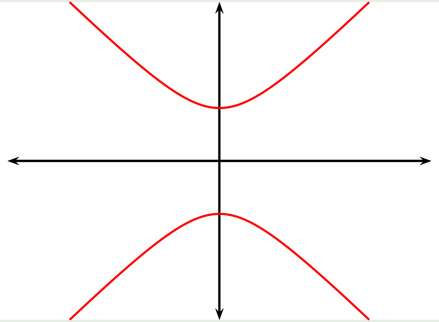

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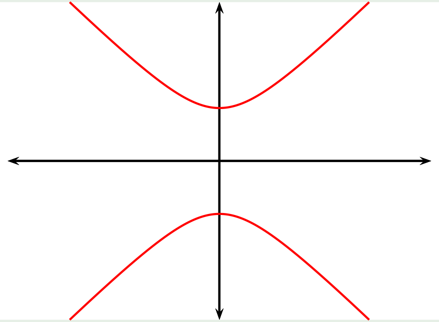

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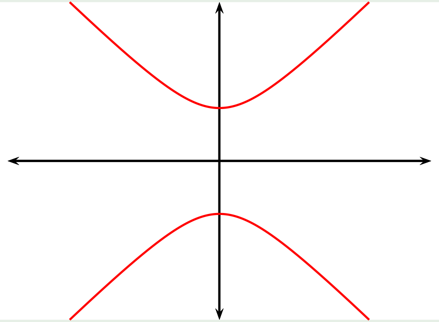

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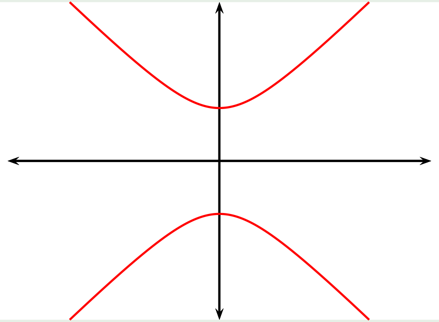

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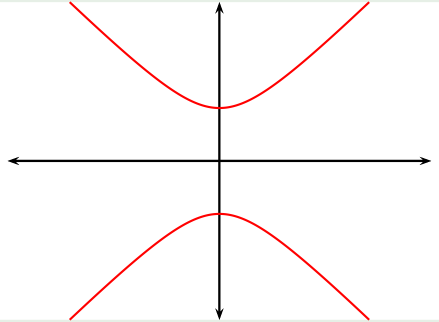

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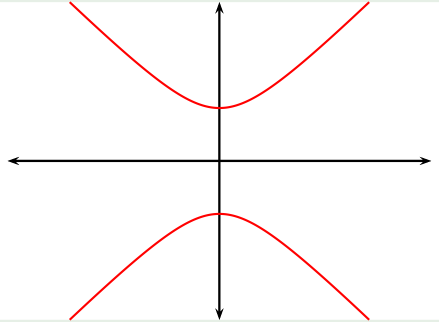

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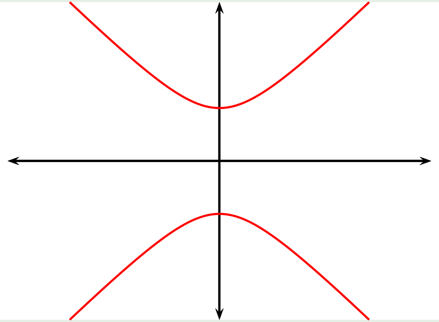

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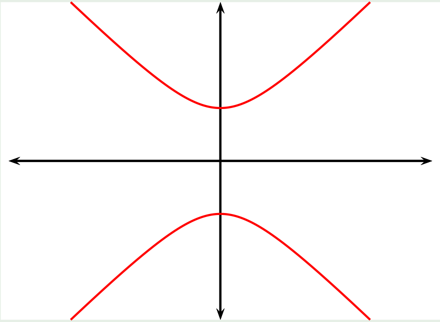

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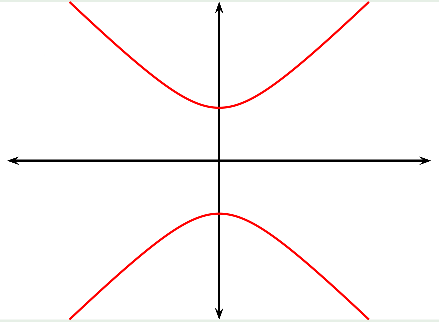

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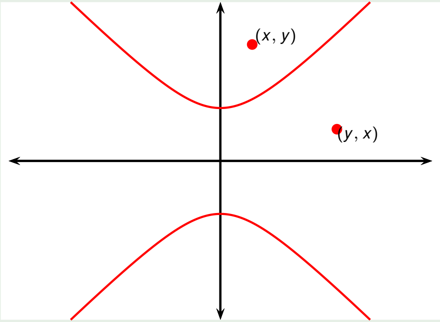

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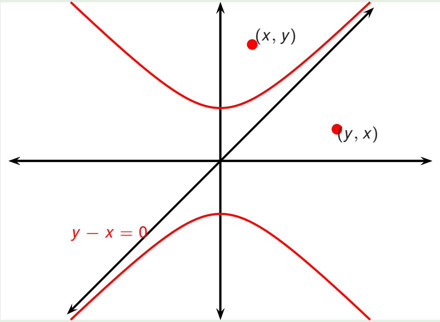

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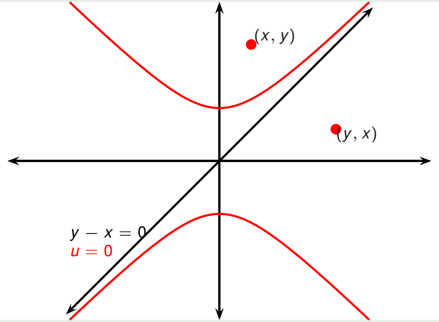

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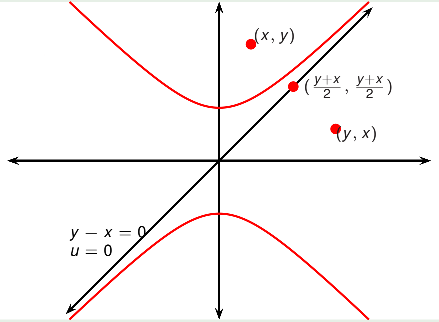

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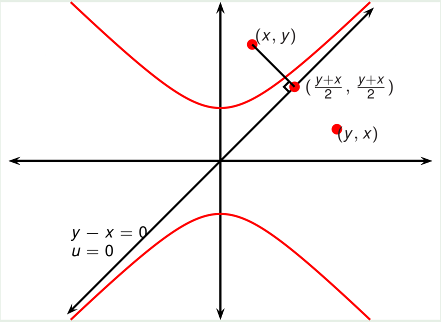

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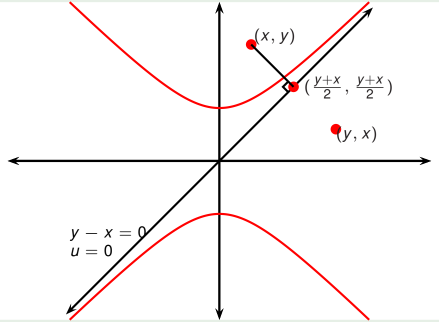

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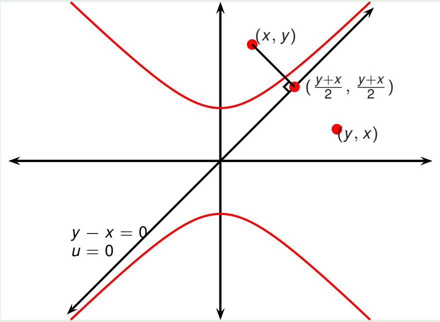

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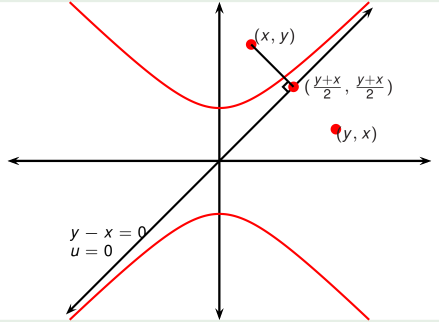

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distance b-n (x, y) and line $u = 0$ equals

$$\sqrt{\left(x - \frac{(x+y)}{2}\right)^2 + \left(y - \frac{(x+y)}{2}\right)^2}$$

$$= \sqrt{\frac{1}{2}(y-x)^2} = \pm\frac{\sqrt{2}}{2}(y-x)$$

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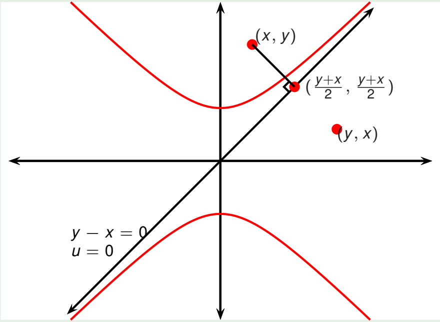

Signed distance b-n (x, y) and line $u = 0$ equals

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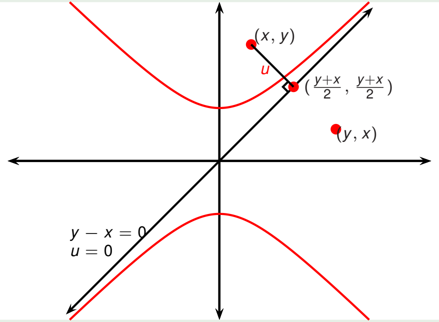

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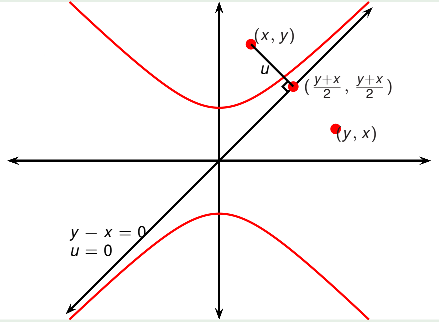

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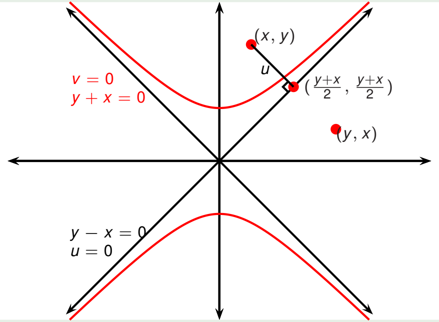

Signed distance b-n (x, y) and line $u = 0$ equals u. Similarly compute that signed distance b-n (x, y) and the line $v = 0$ equals v.

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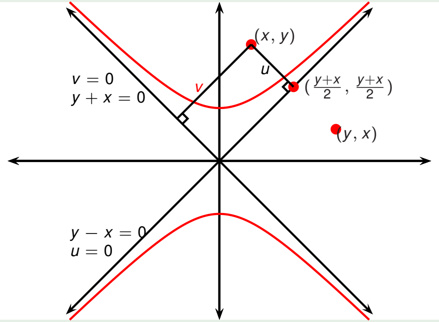

Signed distance b-n (x, y) and line $u = 0$ equals u. Similarly compute that signed distance b-n (x, y) and the line $v = 0$ equals v.

We studied $v = \frac{\frac{1}{2}}{u}$ is called a hyperbola: why do we call $y = \sqrt{x^2+1}$ hyperbola? Compute:

$$\begin{aligned} \sqrt{x^2+1} &= y \\ x^2+1 &= y^2 \\ y^2 - x^2 &= 1 \\ \frac{\sqrt{2}}{2}(y-x)\frac{\sqrt{2}}{2}(y+x) &= \frac{1}{2} \\ uv &= \frac{1}{2} \\ v &= \frac{\frac{1}{2}}{u}, \end{aligned}$$

where $\left| \begin{array}{l} u = \frac{\sqrt{2}}{2}(y-x) \\ v = \frac{\sqrt{2}}{2}(y+x) \end{array} \right.$. Consider an arbitrary point (x, y).

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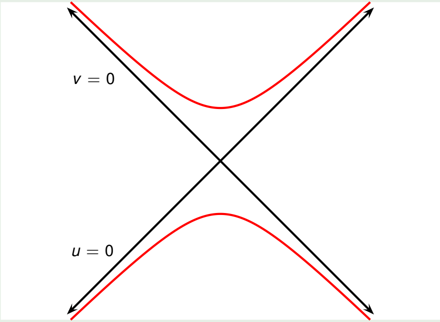
Find the area locked b-n the hyperbolas $y = \pm\sqrt{x^2+1}$ and $x = \pm 2\sqrt{2}$.



Signed distance b-n (x, y) and line $u = 0$ equals u. Similarly compute that signed distance b-n (x, y) and the line $v = 0$ equals v. $\Rightarrow y^2 - x^2 = 1$ is the hyperbola $v = \frac{1/2}{u}$ in the (u, v)-plane.

We studied $v = \frac{\frac{1}{2}}{u}$ is called a hyperbola: why do we call $y = \sqrt{x^2+1}$ hyperbola? Compute:

$$\begin{array}{rcl} \sqrt{x^2+1} & = & y \\ x^2+1 & = & y^2 \\ y^2 - x^2 & = & 1 \\ \frac{\sqrt{2}}{2}(y-x)\frac{\sqrt{2}}{2}(y+x) & = & \frac{1}{2} \\ uv & = & \frac{1}{2} \\ v & = & \frac{\frac{1}{2}}{u}, \end{array}$$

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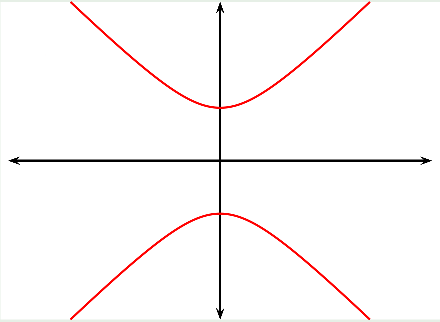

The area in question is:

$$\int_{?}^{?} 2\sqrt{x^2+1}\,\mathrm{d}x$$

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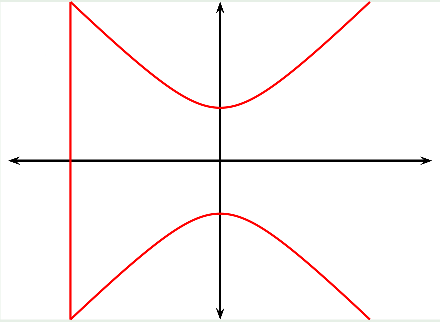

The area in question is:

$$\int_{-2\sqrt{2}}^{?} 2\sqrt{x^2+1}\mathrm{d}x$$

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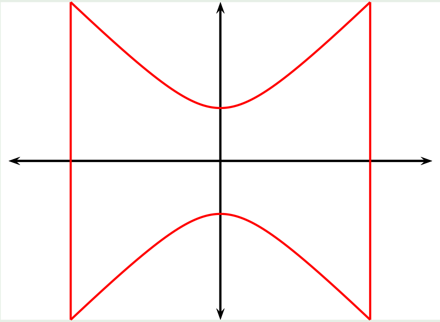

The area in question is:

$$\int_{-2\sqrt{2}}^{2\sqrt{2}} 2\sqrt{x^2+1}\mathrm{d}x$$

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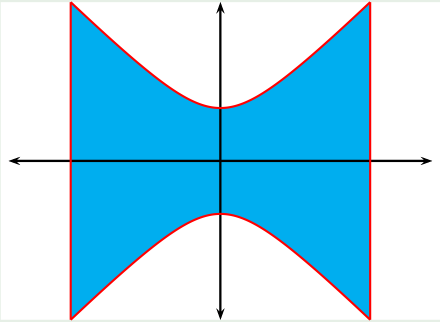

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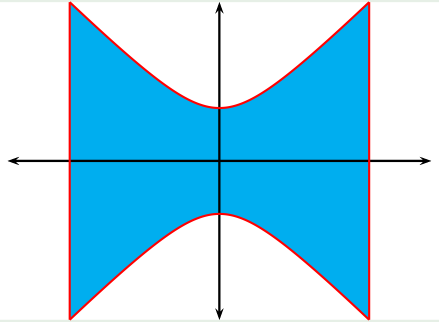

The area in question is:

$$\int_{-2\sqrt{2}}^{2\sqrt{2}} 2\sqrt{x^2+1}\mathrm{d}x$$

$$= \left[x\sqrt{x^2+1} + \ln\left(\sqrt{x^2+1}+x\right)\right]_{-2\sqrt{2}}^{2\sqrt{2}}$$

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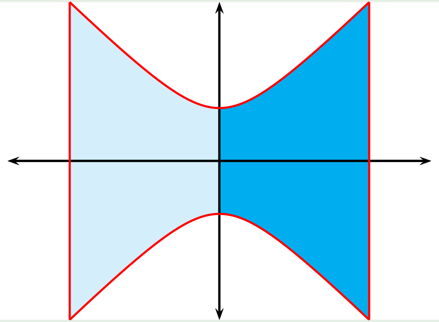

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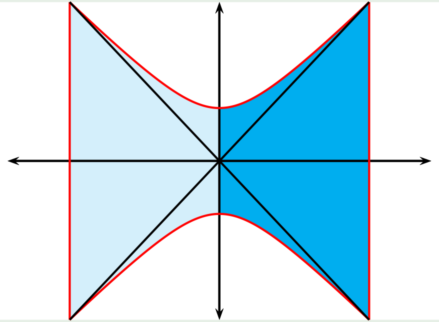

The area in question is:

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$$= 2\left[x\sqrt{x^2+1}\right.$$

$$\left.+\ln\left(\sqrt{x^2+1}+x\right)\right]_0^{2\sqrt{2}}$$

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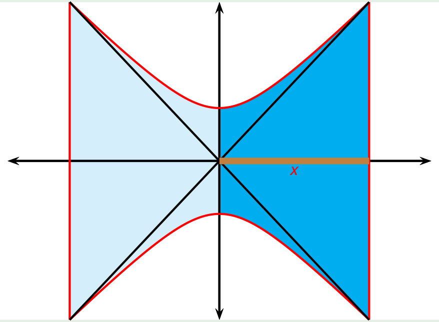

The area in question is:

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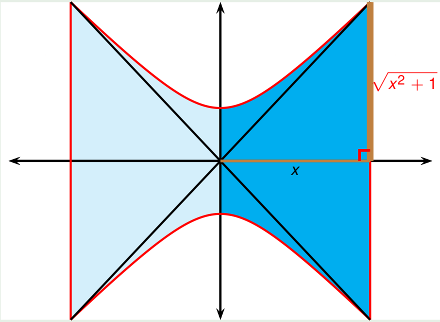

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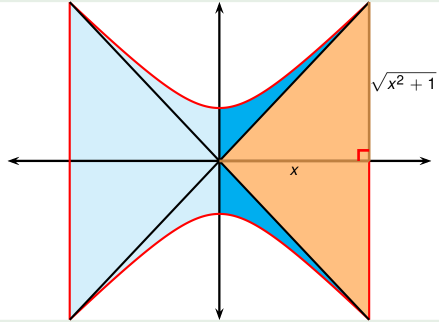

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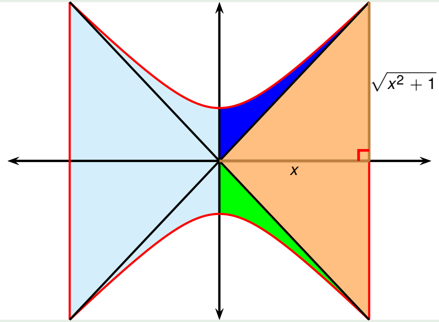

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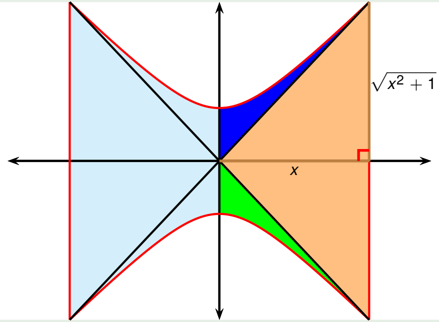

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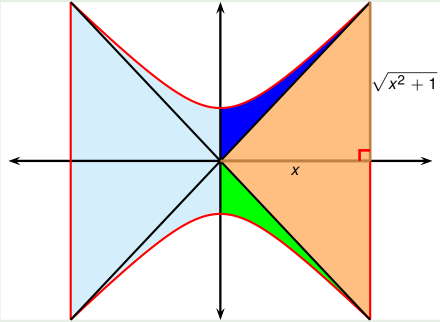

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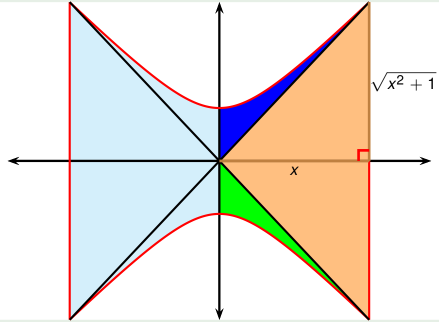

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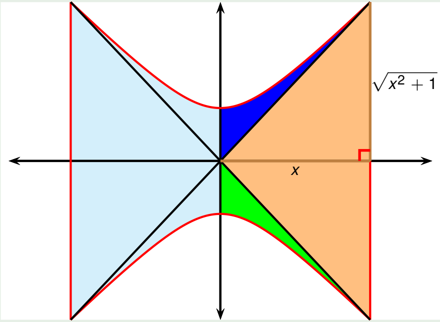

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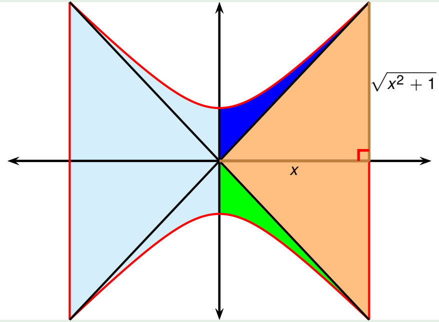

- Recall: integral can be solved via $x = \tan\theta$.

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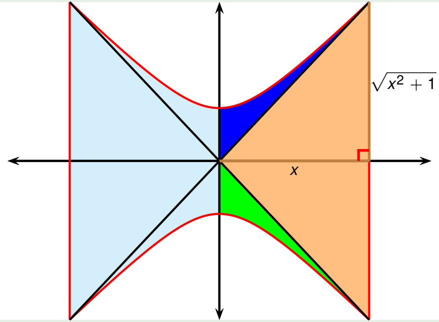

- Recall: integral can be solved via $x = \tan\theta$.
- Geometric interpretation of θ?

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$$\int_{-2\sqrt{2}}^{2\sqrt{2}} 2\sqrt{x^2+1}dx$$
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$$= 2\left(2\sqrt{2}\sqrt{(2\sqrt{2})^2+1}\right.$$
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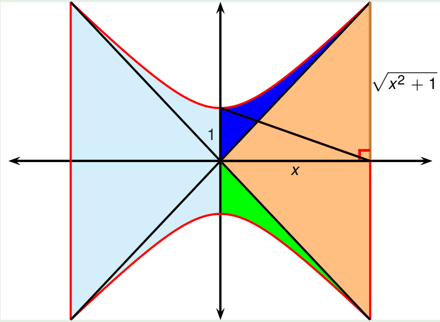

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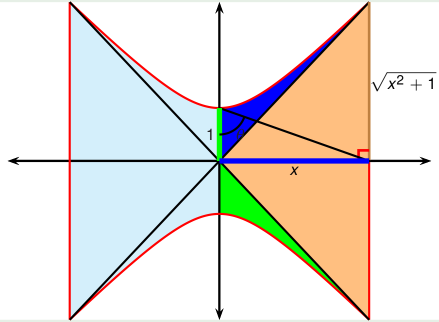

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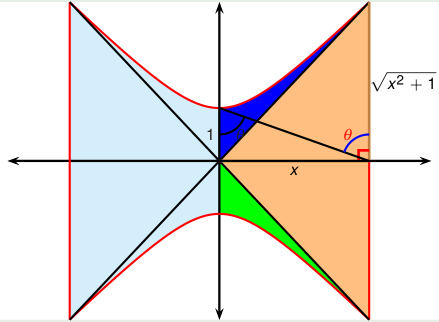

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