Precalculus
Cubic inequality

Todor Milev

2019

Example

Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality.

$$2x^3 - 5x^2 + x + 2 \quad > \quad 0$$

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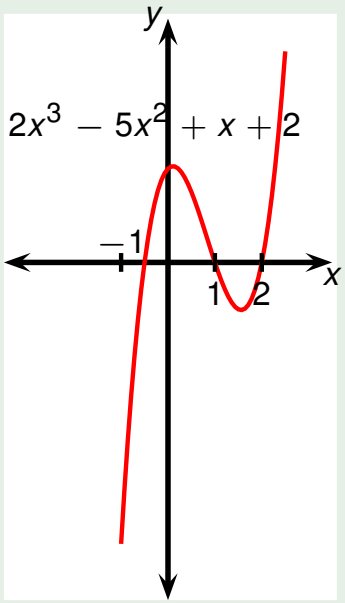

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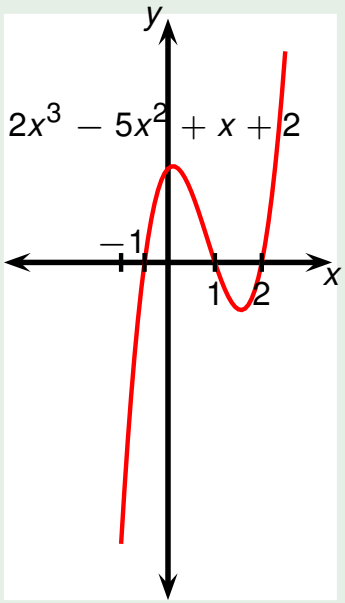

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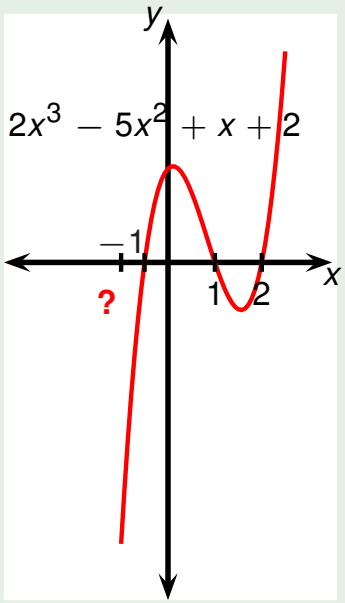

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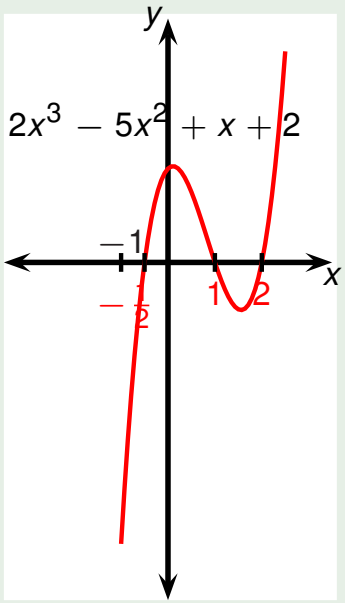

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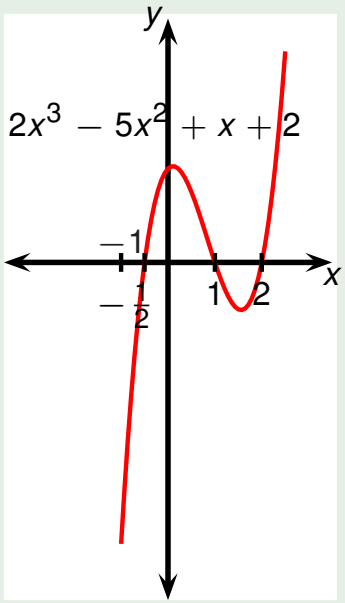

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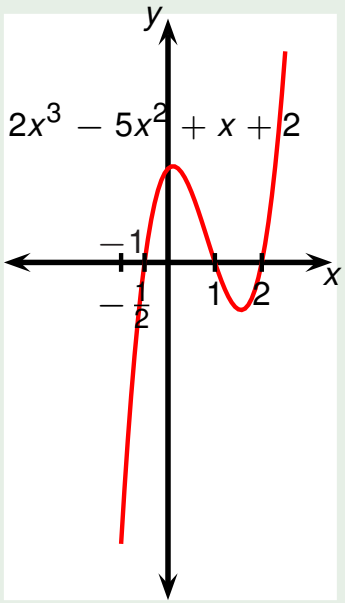

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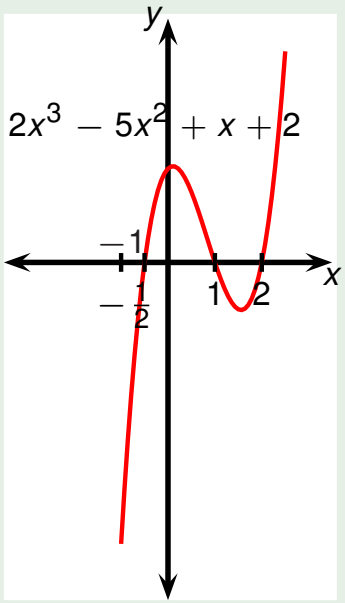

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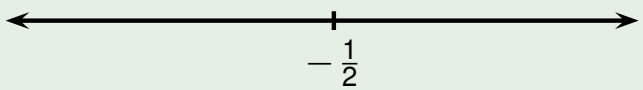

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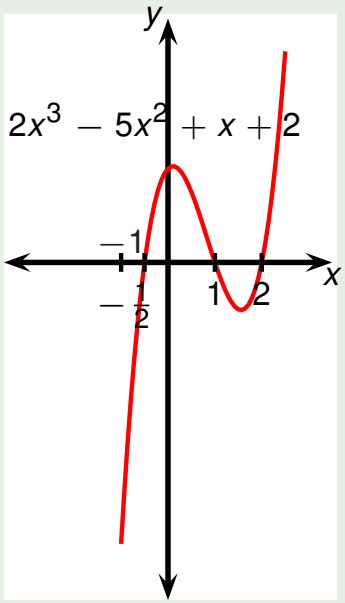

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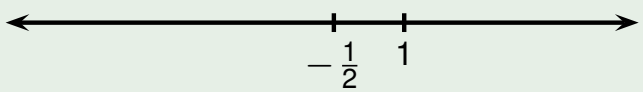

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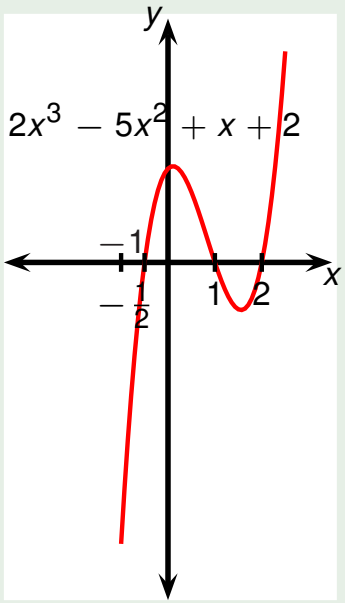

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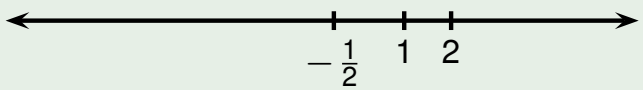

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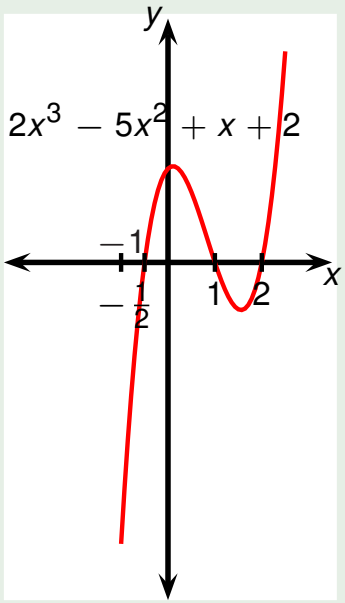

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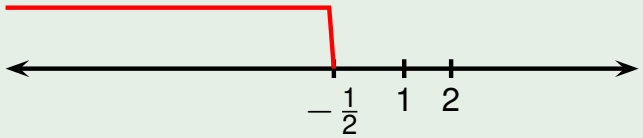

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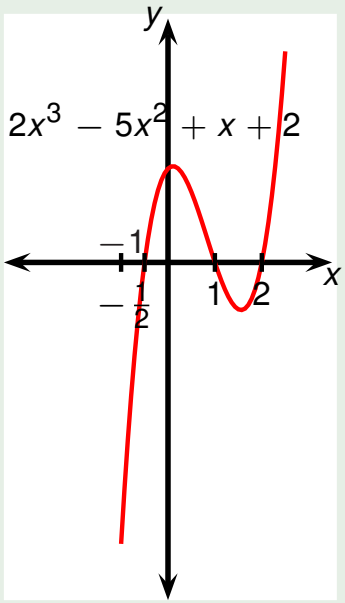

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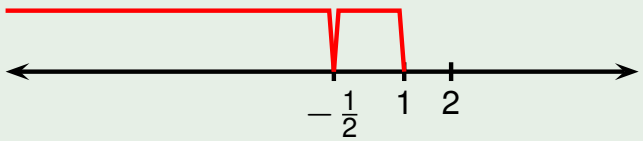

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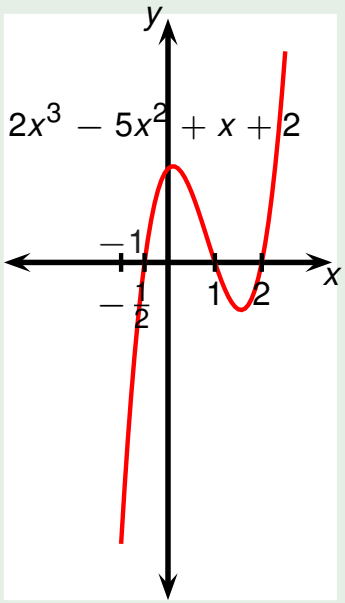

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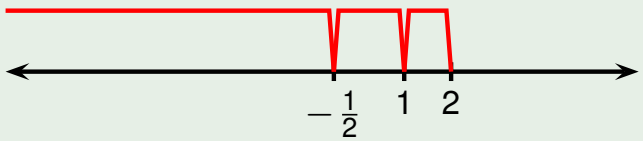

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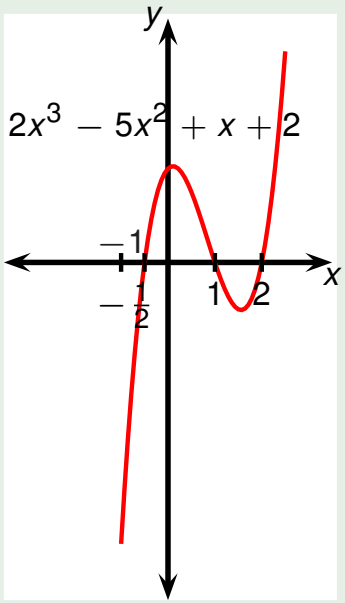

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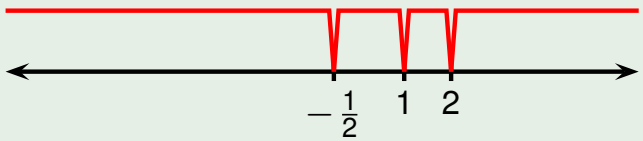

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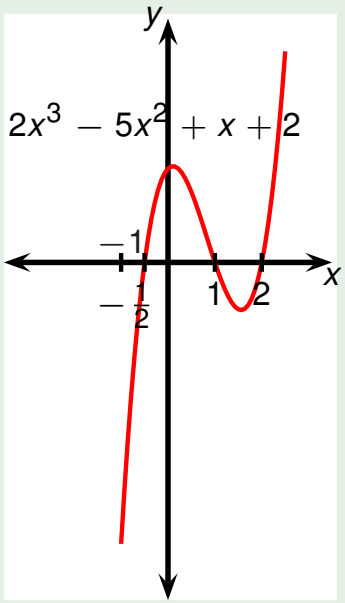

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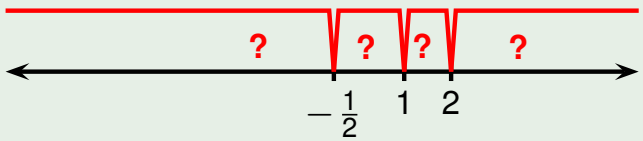

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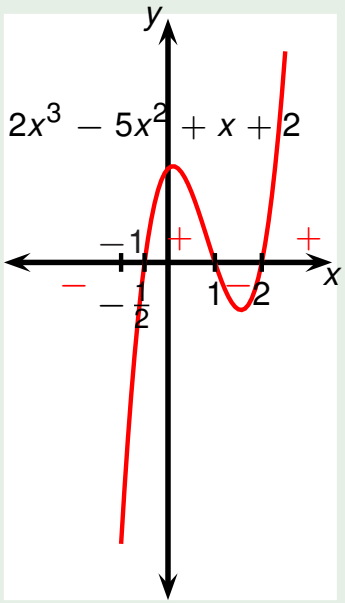

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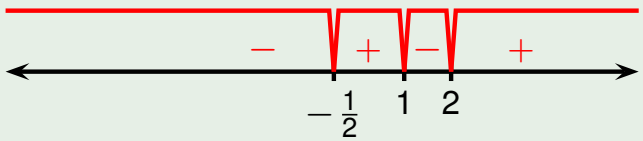

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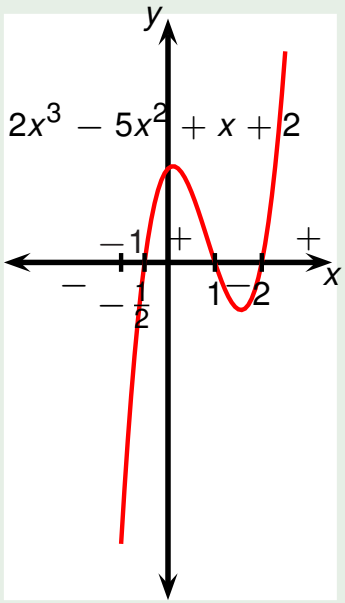

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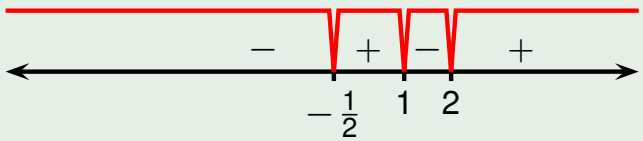

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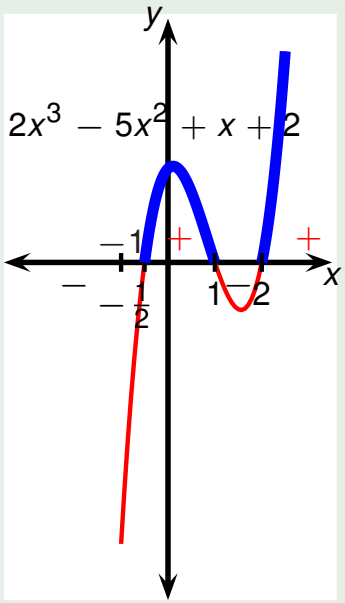

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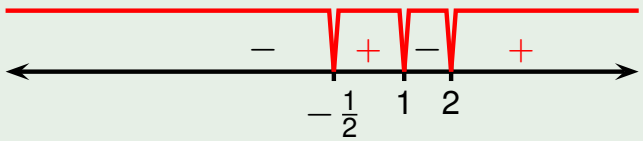

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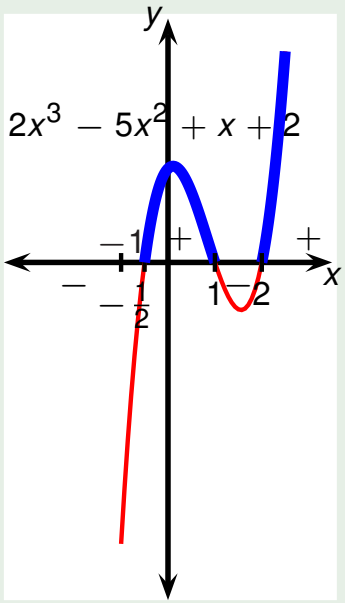

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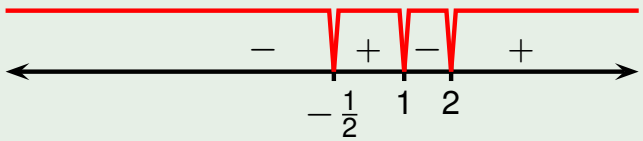

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