Calculus I
Derivative of a^x

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2019

Example (Chain Rule, general exponential function)

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Let $u = x \ln 2.$

Then $y = e^u.$

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\text{Chain Rule:} \quad \frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dx} \\
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Theorem (The Derivative of a^x)

$$\frac{\mathrm{d}}{\mathrm{d}x}(a^x) = a^x \ln a.$$