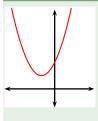
# Precalculus Quadratic trigonometric inequalities

**Todor Miley** 

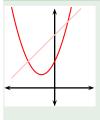
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- Solve the inequality  $2u^2 + 2u + 1 \le u + 2$ .
- Find all solutions of  $2\cos^2\theta + 2\cos\theta + 1 \le \cos\theta + 2$  lying in  $[-360^\circ, 360^\circ]$ .



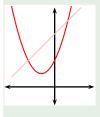
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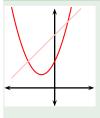
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Quadratic trigonometric inequalities

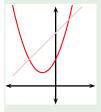
$$2u^2 + 2u + 1 \le u + 2$$
  
 $2u^2 + u - 1 \le 0$ 

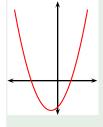


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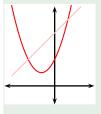
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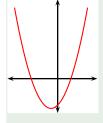




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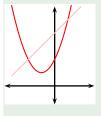
$$2u^2 + 2u + 1 \le u + 2$$
  
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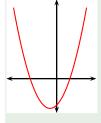




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$$\begin{array}{rcccc} 2u^2 + 2u + 1 & \leq & u + 2 \\ & 2u^2 + u - 1 & \leq & 0 \\ 2\left( \ref{eq:condition} \right) \left( \ref{eq:condition} \right) & \leq & 0 \end{array}$$



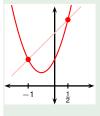


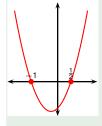
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$$2u^{2} + 2u + 1 \leq u + 2$$

$$2u^{2} + u - 1 \leq 0$$

$$2\left(u - \frac{1}{2}\right)\left(u + 1\right) \leq 0$$

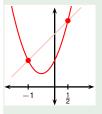


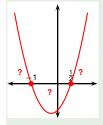




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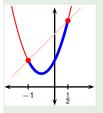
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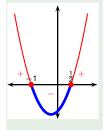




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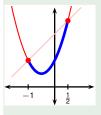


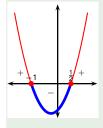


$$+$$
  $+$   $1$   $1$   $2$ 

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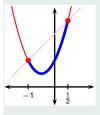


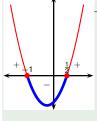


$$+$$
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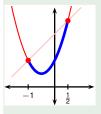


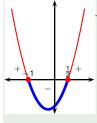
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$$2\cos^2\theta + 2\cos\theta + 1 < \cos\theta + 2$$



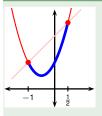


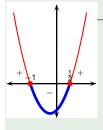
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$$2\cos^{2}\theta + 2\cos\theta + 1 \leq \cos\theta + 2 \quad \text{Set } \cos\theta = u$$
$$2u^{2} + 2u + 1 \leq u + 2$$





$$+$$
  $+$   $1$   $\frac{1}{2}$ 

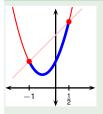
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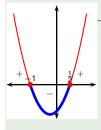
 $2u^2 + 2u + 1 \le u + 2$ 

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$$\begin{array}{rcl} 2u^{2} + u - 1 & \leq & 0 \\ 2\left(u - \frac{1}{2}\right)\left(u + 1\right) & \leq & 0 \\ & u & \in & \left[-1, \frac{1}{2}\right] \\ \hline 2\cos^{2}\theta + 2\cos\theta + 1 & \leq & \cos\theta + 2 & \text{Set } \cos\theta = u \\ 2u^{2} + 2u + 1 & \leq & u + 2 \end{array}$$

 $u \in \left[-1, \frac{1}{2}\right]$  (solved above)





$$+$$
  $+$   $1$   $\frac{1}{2}$ 

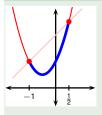
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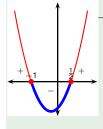
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$$2\cos^2\theta + 2\cos\theta + 1 \le \cos\theta + 2$$
 Set  $\cos\theta = u$ 

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 $u \in [-1, \frac{1}{2}]$   
 $\cos \theta \in [-1, \frac{1}{2}]$ 

(solved above)

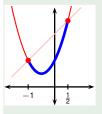


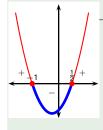


$$\begin{array}{c|c} + & - & + \\ \hline -1 & \frac{1}{2} \end{array}$$

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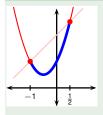


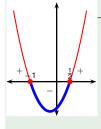


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$$2u^{2} + 2u + 1 \leq u + 2$$

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$$2(u - \frac{1}{2})(u + 1) \leq 0$$

$$u \in [-1, \frac{1}{2}]$$

$$2\cos^{2}\theta + 2\cos\theta + 1 \leq \cos\theta + 2 \text{ Set } \cos\theta = u$$

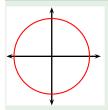
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$$u \in \left[-1, \frac{1}{2}\right] \quad \text{(solved above)}$$

$$\cos\theta \in \left[-1, \frac{1}{2}\right]$$

$$-1 \leq \cos\theta \leq \frac{1}{2}$$

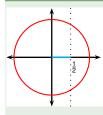


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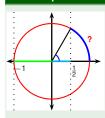


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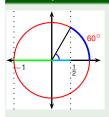


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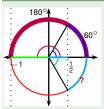
2019



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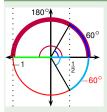


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$$\cos\theta \in \left[-1, \frac{1}{2}\right] \\
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$$, -60^{\circ}$$

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 ]  $\cup$  [60°

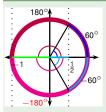


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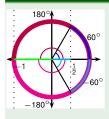
$$, -60^{\circ}$$
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$$\theta \in [-180^{\circ}]$$

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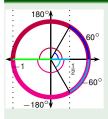
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$$\cos\theta \in \left[-1, \frac{1}{2}\right] \\
-1 \le \cos\theta \le \frac{1}{2}$$

$$\theta \in [-180^{\circ} + k360^{\circ}, -60^{\circ} + k360^{\circ}] \cup [60^{\circ} + k360^{\circ}, 180^{\circ} + k360^{\circ}]$$

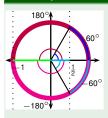


- Solve the inequality  $2u^2 + 2u + 1 \le u + 2$ .
- Find all solutions of  $2\cos^2\theta + 2\cos\theta + 1 \le \cos\theta + 2$  lying in  $[-360^\circ, 360^\circ]$ .

$$\cos \theta \in \left[-1, \frac{1}{2}\right] \\
-1 \le \cos \theta \le \frac{1}{2}$$

$$\theta \in [-180^{\circ} + k360^{\circ}, -60^{\circ} + k360^{\circ}] \cup [60^{\circ} + k360^{\circ}, 180^{\circ} + k360^{\circ}]$$

# Example<sup>1</sup>



- Solve the inequality  $2u^2 + 2u + 1 \le u + 2$ .
- Find all solutions of  $2\cos^2\theta + 2\cos\theta + 1 \le \cos\theta + 2$  lying in  $[-360^\circ, 360^\circ]$ .

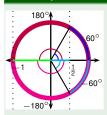
$$\begin{array}{rcl} \cos\theta & \in & \left[-1,\frac{1}{2}\right] \\ -1 \leq \cos\theta & \leq & \frac{1}{2} \end{array}$$

$$\theta \in [-180^{\circ} + \frac{k}{3}60^{\circ}, -60^{\circ} + \frac{k}{3}60^{\circ}] \cup [60^{\circ} + \frac{k}{3}60^{\circ}, 180^{\circ} + \frac{k}{3}60^{\circ}]$$

$$\theta \in$$

$$[-180^{\circ}, -60^{\circ}] \cup [60^{\circ}, 180^{\circ}]$$

k = 0



- Solve the inequality  $2u^2 + 2u + 1 \le u + 2$ .
- Find all solutions of  $2\cos^2\theta + 2\cos\theta + 1 \le \cos\theta + 2$  lying in  $[-360^\circ, 360^\circ]$ .

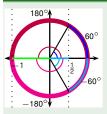
$$\cos\theta \in \left[-1, \frac{1}{2}\right] \\
-1 \le \cos\theta \le \frac{1}{2}$$

$$\theta \in [-180^{\circ} + k360^{\circ}, -60^{\circ} + k360^{\circ}] \cup [60^{\circ} + k360^{\circ}, 180^{\circ} + k360^{\circ}]$$

$$\theta \in$$

$$[-180^{\circ}, -60^{\circ}] \cup [60^{\circ}, 180^{\circ}] \cup [180^{\circ}, 300^{\circ}] \cup [420^{\circ}, 540^{\circ}]$$

$$k = 0$$
 $k = 1$ 



- Solve the inequality  $2u^2 + 2u + 1 \le u + 2$ .
- Find all solutions of  $2\cos^2\theta + 2\cos\theta + 1 \le \cos\theta + 2$  lying in  $[-360^\circ, 360^\circ]$ .

$$\cos\theta \in \left[-1, \frac{1}{2}\right] \\
-1 \le \cos\theta \le \frac{1}{2}$$

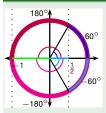
$$\theta \in [-180^{\circ} + k360^{\circ}, -60^{\circ} + k360^{\circ}] \cup [60^{\circ} + k360^{\circ}, 180^{\circ} + k360^{\circ}]$$

$$\theta \in$$

$$\begin{array}{l} [-180^{\circ}, -60^{\circ}] \ \cup \ [60^{\circ}, 180^{\circ}] \\ \cup \ [180^{\circ}, 300^{\circ}] \ \cup \ [420^{\circ}, 540^{\circ}] \end{array}$$

$$k = 0$$

$$k = 1$$



- Solve the inequality  $2u^2 + 2u + 1 \le u + 2$ .
- Find all solutions of  $2\cos^2\theta + 2\cos\theta + 1 \le \cos\theta + 2$  lying in  $[-360^\circ, 360^\circ]$ .

$$\cos\theta \in \left[-1, \frac{1}{2}\right] \\
-1 \le \cos\theta \le \frac{1}{2}$$

$$\theta \in [-180^{\circ} + k360^{\circ}, -60^{\circ} + k360^{\circ}] \cup [60^{\circ} + k360^{\circ}, 180^{\circ} + k360^{\circ}]$$

$$\theta \in$$

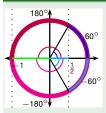
$$\begin{array}{c} [-540^{\circ}, -420^{\circ}] \ \cup \ [-300^{\circ}, -180^{\circ}] \\ \cup \ [-180^{\circ}, -60^{\circ}] \ \cup \ [60^{\circ}, 180^{\circ}] \\ \cup \ [180^{\circ}, 300^{\circ}] \ \cup \ [420^{\circ}, 540^{\circ}] \end{array}$$

. .

k = -1

k = 0

k = 1



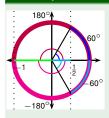
- Solve the inequality  $2u^2 + 2u + 1 \le u + 2$ .
- Find all solutions of  $2\cos^2\theta + 2\cos\theta + 1 \le \cos\theta + 2$  lying in  $[-360^\circ, 360^\circ]$ .

$$\begin{array}{rcl} \cos\theta & \in & \left[-1,\frac{1}{2}\right] \\ -1 \leq \cos\theta & \leq & \frac{1}{2} \end{array}$$

$$\theta \in [-180^{\circ} + k360^{\circ}, -60^{\circ} + k360^{\circ}] \cup [60^{\circ} + k360^{\circ}, 180^{\circ} + k360^{\circ}]$$

$$\theta \in$$

. .

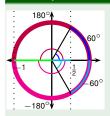


- Solve the inequality  $2u^2 + 2u + 1 \le u + 2$ .
- Find all solutions of  $2\cos^2\theta + 2\cos\theta + 1 \le \cos\theta + 2$  lying in  $[-360^\circ, 360^\circ]$ .

$$\cos\theta \in \left[-1, \frac{1}{2}\right] \\
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$$\theta \in [-180^{\circ} + k360^{\circ}, -60^{\circ} + k360^{\circ}] \cup [60^{\circ} + k360^{\circ}, 180^{\circ} + k360^{\circ}]$$

$$\theta \in$$



- Solve the inequality  $2u^2 + 2u + 1 \le u + 2$ .
- Find all solutions of  $2\cos^2\theta + 2\cos\theta + 1 < \cos\theta + 2$ lying in  $[-360^{\circ}, 360^{\circ}]$ .

$$\cos\theta \in \left[-1, \frac{1}{2}\right] \\
-1 \le \cos\theta \le \frac{1}{2}$$

$$\theta \in [-180^{\circ} + k360^{\circ}, -60^{\circ} + k360^{\circ}] \cup [60^{\circ} + k360^{\circ}, 180^{\circ} + k360^{\circ}]$$

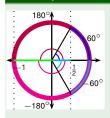
$$\theta \in \begin{array}{c} \cup \left[-540^{\circ}, -420^{\circ}\right] \cup \left[-300^{\circ}, -180^{\circ}\right] \\ \cup \left[-180^{\circ}, -60^{\circ}\right] \cup \left[60^{\circ}, 180^{\circ}\right] \\ \cup \left[180^{\circ}, 300^{\circ}\right] \cup \left[420^{\circ}, 540^{\circ}\right] \end{array}$$

 $\theta \in$ 

$$k = -1$$

$$k = 0$$

$$k = 1$$



- Solve the inequality  $2u^2 + 2u + 1 \le u + 2$ .
- Find all solutions of  $2\cos^2\theta + 2\cos\theta + 1 \le \cos\theta + 2$  lying in  $[-360^\circ, 360^\circ]$ .

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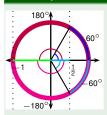
$$k = -1$$

$$k = 0$$

$$k = 1$$

$$\theta \in$$

$$[-300^{\circ}, -60^{\circ}] \cup [60^{\circ}, 300^{\circ}]$$



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$$\theta \in [-180^{\circ} + k360^{\circ}, -60^{\circ} + k360^{\circ}] \cup [60^{\circ} + k360^{\circ}, 180^{\circ} + k360^{\circ}]$$

$$\theta \in$$

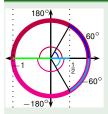
$$k = -1$$

$$k = 0$$

$$k = 1$$

$$\theta \in$$

$$[-300^{\circ}, -60^{\circ}] \cup [60^{\circ}, 300^{\circ}]$$

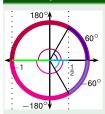


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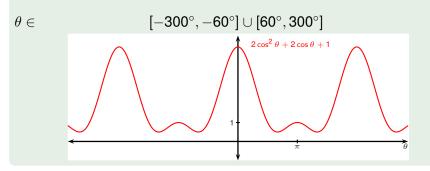
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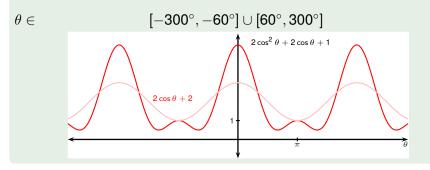
$$\cos\theta \in \left[-1, \frac{1}{2}\right] \\
-1 \le \cos\theta \le \frac{1}{2}$$

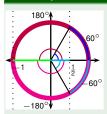




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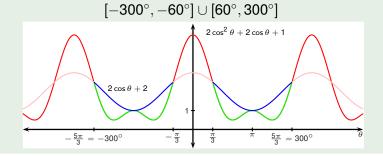


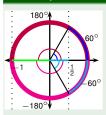


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