Calculus I

Definite integral of $\frac{1}{(ax+b)^n}$

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2019

Example

Find $\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $\mathrm{d}u = -3\,\mathrm{d}x$.
- Therefore $\mathrm{d}x = -\frac{1}{3}\mathrm{d}u$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

$$\int_{x=1}^{x=2} \frac{\mathrm{d}x}{(2-3x)^2} = -\frac{1}{3}\int_{u=-1}^{u=-4} \frac{\mathrm{d}u}{u^2} = -\frac{1}{3}\int_{-1}^{-4} u^{-2}\mathrm{d}u$$

$$= -\frac{1}{3}\cdot\left[-\frac{1}{u}\right]_{-1}^{-4} = \frac{1}{3}\left[\frac{1}{u}\right]_{-1}^{-4}$$

$$= \frac{1}{3}\left(\frac{1}{-4} - \frac{1}{-1}\right) = \frac{1}{3}\left(1 - \frac{1}{4}\right) = \frac{1}{4}.$$