Precalculus

§ Geometric-text problems leading to polynomial systems, part 2

Todor Miley

2019









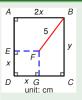












$$2\cdot(2x+y)=26$$



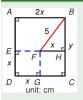
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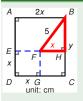


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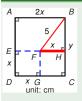


ABCD is a rectangle. Points E, F, G and D form a square as indicated. Given: |ED| = |DG| = x cm |AB| = 2x cm, |BC| = y cm, |BF| = 5 cm, |BF| = 5 cm, |BF| = 6 cm. Find |AB| = 6 so that |AB| = 6 so t

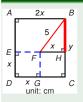
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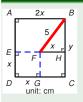
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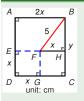
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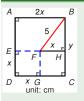
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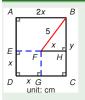
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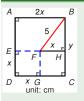
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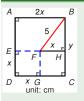
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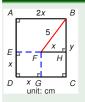
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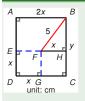
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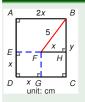
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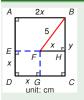
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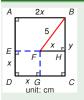
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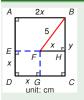
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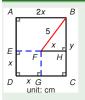
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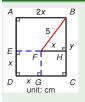


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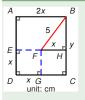
$$x_1, x_2 = ?$$



$$y = 13 - 2x$$

$$5x^{2} - 39x + 72 = 0$$

$$x_{1}, x_{2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}$$

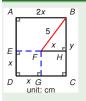


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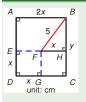


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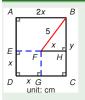
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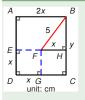
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$$x_1, x_2 = \frac{39 \pm 9}{10}$$



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Case 1. $x = \frac{39 + 9}{10}$



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Case 1. $x = \frac{39 + 9}{10} = \frac{48}{10}$



$$y = 13 - 2x$$

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Case 1. $x = \frac{39 + 9}{10} = \frac{48}{10} = 4.8$



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$$y = 13 - 2x = 13 - 2 \cdot 4.8 = 13 - 9.6$$



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Solution rejected as problem requires $y > x$.



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$$x = \frac{39 - 9}{10}$$



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$$x = \frac{39 - 9}{10} = 3$$



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 $y = 13 - 2 \cdot x$



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Case 2.
$$x = \frac{39 - 9}{10} = 3$$

 $y = 13 - 2 \cdot x = 13 - 2 \cdot 3 = 7$



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Solution is valid as $y > x$.