

Precalculus

Homework

Trigonometric equations and inequalities

1. Convert from degrees to radians.

- | | | |
|------------------|-------------------|---------------------|
| (a) 15° . | (h) 120° . | (o) 360° . |
| (b) 30° . | (i) 135° . | (p) 405° . |
| (c) 36° . | (j) 150° . | (q) 1200° . |
| (d) 45° . | (k) 180° . | (r) -900° . |
| (e) 60° . | (l) 225° . | (s) -2014° . |
| (f) 75° . | (m) 270° . | |
| (g) 90° . | (n) 305° . | |

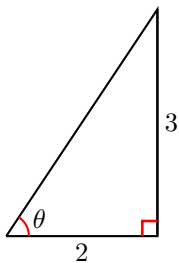
2. Convert from radians to degrees. The answer key has not been proofread, use with caution.

- | | | |
|-------------------------|-------------------------|---------------|
| (a) 4π . | (d) $\frac{4}{3}\pi$. | (g) 5. |
| (b) $-\frac{7}{6}\pi$. | (e) $-\frac{3}{8}\pi$. | |
| (c) $\frac{7}{12}\pi$. | (f) 2014π . | (h) -2014 . |

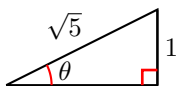
3. Find the indicated circle arc-length. The answer key has not been proofread, use with caution.

- (a) Circle of radius 3, arc of measure 36° .
- (b) Circle of radius $\frac{1}{2}$, arc of measure 100° .
- (c) Circle of radius 1, arc of measure 3 (radians).
- (d) Circle of radius 3, arc of measure 300° .

4. Find the 6 trigonometric functions of the indicated angle in the indicated right triangle.



(a)



(b)

- (c) 
- (d)



5. Find the exact value of the trigonometric function (using radicals).

- (a) $\cos 135^\circ$.
 (b) $\sin 225^\circ$.
 (c) $\cos 495^\circ$.
 (d) $\sin 560^\circ$.
 (e) $\sin\left(\frac{3\pi}{2}\right)$.
 (f) $\cos\left(\frac{11\pi}{6}\right)$.
 (g) $\sin\left(\frac{2015\pi}{3}\right)$.
 (h) $\cos\left(\frac{17\pi}{3}\right)$.

6. Find all solutions of the equation in the interval $[0, 2\pi)$. The answer key has not been proofread, use with caution.

- (a) $\sin x = -\frac{\sqrt{2}}{2}$.
 (b) $\cos x = \frac{\sqrt{3}}{2}$.
 (c) $\sin(3x) = \frac{1}{2}$.
 (d) $\cos(7x) = 0$.
 (e) $\cos\left(3x + \frac{\pi}{2}\right) = 0$.
 (f) $\sin\left(5x - \frac{\pi}{3}\right) = 0$.

7. Use the known values of $\sin 30^\circ$, $\cos 30^\circ$, $\sin 45^\circ$, $\cos 45^\circ$, $\sin 60^\circ$, $\cos 60^\circ$, \dots , the angle sum formulas and the cofunction identities to find an exact value (using radicals) for the trigonometric function.

- (a) The six trigonometric functions of $105^\circ = 45^\circ + 60^\circ$:
- $\sin(105^\circ)$.
 - $\cos(105^\circ)$. Should your answer be a positive or a negative number?
 - $\tan(105^\circ)$.
 - $\cot(105^\circ)$.
 - $\sec(105^\circ)$.
 - $\csc(105^\circ)$.
- (b) The six trigonometric functions of $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$:
- $\sin\left(\frac{\pi}{12}\right)$.
 - $\cos\left(\frac{\pi}{12}\right)$. Should $\sin\left(\frac{\pi}{12}\right)$ be larger or smaller than $\cos\left(\frac{\pi}{12}\right)$?
 - $\tan\left(\frac{\pi}{12}\right)$.
 - $\cot\left(\frac{\pi}{12}\right)$.
 - $\sec\left(\frac{\pi}{12}\right)$.
 - $\csc\left(\frac{\pi}{12}\right)$.

8. Simplify to a trigonometric function of the angle θ . The answer key has not been proofread, use with caution.

- (a) $\sin\left(\frac{\pi}{2} - \theta\right)$.

- (b) $\cos\left(\frac{13\pi}{2} - \theta\right)$.
- (c) $\tan(\pi - \theta)$
- (d) $\cot\left(\frac{3\pi}{2} - \theta\right)$
- (e) $\csc\left(\frac{3\pi}{2} + \theta\right)$

9. Using the power-reducing formulas, rewrite the expression in terms of first powers of the cosines and sines of multiples of the angle θ .

- (a) $\sin^4 \theta$.
- (b) $\cos^4 \theta$.
- (c) $\sin^6 \theta$.
- (d) $\cos^6 \theta$.

10. Use the sum-to-product formulas to find all solutions of the trigonometric equation in the interval $[0, 2\pi)$.

Please note that typing a query such as “solve($\sin(x)+\sin(3x)=0$)” at www.wolframalpha.com will provide you with a correct answer and a function plot.

- (a) $\sin(x) + \sin(3x) = 0$.
- (b) $\cos(x) + \cos(-3x) = 0$.
- (c) $\sin(x) - \sin(3x) = 0$.
- (d) $\cos(2x) - \cos(3x) = 0$.

11. Find the inverse function. You are asked to do the algebra only; you are not asked to determine the domain or range of the function or its inverse.

- (a) $f(x) = 3x^2 + 4x - 7$, where $x \geq -\frac{2}{3}$.
- (b) $f(x) = 2x^2 + 3x - 5$, where $x \geq -\frac{3}{4}$.
- (c) $f(x) = \frac{2x+5}{x-4}$, where $x \neq 4$.
- (d) $f(x) = \frac{3x+5}{2x-4}$, where $x \neq 2$.
- (e) $f(x) = \frac{5x+6}{4x+5}$, where $x \neq -\frac{5}{4}$.
- (f) $f(x) = \frac{2x-3}{-3x+4}$, where $x \neq \frac{4}{3}$.

12. Find the inverse function and its domain.

- (a) $y = \ln(x+3)$.
- (b) $y = 4 \ln(x-3) - 4$.
- (c) $y = 2 \ln(-2x+4) + 1$
- (d) $f(x) = e^{x^3}$.
- (e) $y = (\ln x)^2, x \geq 1$.
- (f) $y = \frac{e^x}{1+2e^x}$.
- (g) $f(x) = 2^{2x} + 2^x - 2$.

13. Find each of the following values. Express your answers precisely, not as decimals.

- (a) $\arcsin(\sin 4)$.
- (b) $\arcsin(\sin 0.5)$.
- (c) $\arcsin(\cos 120^\circ)$.
- (d) $\arccos(\cos(3))$.
- (e) $\arccos(\cos(-2))$.
- (f) $\arccos(\sin(-4))$.
- (g) $\arctan(\tan 5)$.

14. Express as the following as an algebraic expression of x . In other words, “get rid” of the trigonometric and inverse trigonometric expressions.

(a) $\cos^2(\arctan x)$.

(c) $\frac{1}{\cos(\arcsin x)}$.

(b) $-\sin^2(\operatorname{arccot} x)$.

(d) $-\frac{1}{\sin(\arccos x)}$.

15. Let $x \in (0, 1)$. Express the following using x and $\sqrt{1 - x^2}$.

(a) $\sin(\arcsin(x))$.

(e) $\sin(2 \arccos(x))$.

(b) $\sin(2 \arcsin(x))$.

(f) $\sin(3 \arccos(x))$.

(c) $\sin(3 \arcsin(x))$.

(g) $\cos(2 \arcsin(x))$.

(d) $\sin(\arccos(x))$.

(h) $\cos(3 \arccos(x))$.

16. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation.

(a) $2 \cos x - 1 = 0$.

(g) $2 \cos^2 x - (1 + \sqrt{2}) \cos x + \frac{\sqrt{2}}{2} = 0$.

(c) $\sqrt{3} \sin x = \sin(2x)$.

(h) $|\tan x| = 1$.

(d) $2 \sin^2 x = 1$.

(i) $3 \cot^2 x = 1$.

(e) $2 + \cos(2x) = 3 \cos x$.

(j) $\sin x = \tan x$.

(f) $2 \cos x + \sin(2x) = 0$.