Calculus I
Type 1: Composing fractional linear transformations

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Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x - 1}{x + 2} \qquad\qquad\qquad\qquad\qquad\qquad\quad x \neq -2$$

$$g(x) = \frac{2x + 3}{5x - 7} \qquad\qquad\qquad\qquad\qquad\qquad\quad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x + 3}{5x - 7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{5x-7}}{\frac{2x+3+(10x-14)}{5x-7}} = \frac{-x + 13}{12x - 11} \quad \Big| \quad x \neq \tfrac{11}{12}, \tfrac{7}{5}$$

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{2x - 1}{x + 2}\right) = \frac{2\left(\frac{2x-1}{x+2}\right) - 1}{\frac{2x-1}{x+2} + 2}$$

$$= \frac{3x - 4}{4x + 3} \qquad\qquad\qquad\qquad\qquad\qquad x \neq -2, -\tfrac{3}{4}$$

$$(g \circ f)(x) = \frac{7x+4}{3x-19} \qquad\qquad\qquad\qquad\qquad\quad x \neq -2, \tfrac{19}{3}$$

$$(g \circ g)(x) = \frac{19x-15}{-25x+64} \qquad\qquad\qquad\qquad\qquad x \neq \tfrac{7}{5}, \tfrac{64}{25}$$