Calculus II

Integrals of the form $\int \sin^n x \cos^m x dx$, theory

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$$\int \sin^m x \cos^n x dx = \int \sin^m x \cos^{n-1} x d(\sin x)$$
$$= \int \sin{}^m x \left(1-\sin{}^2 x\right)^{\frac{n-1}{2}} d(\sin x)$$
$$= \int u^m \left(1-u^2\right)^{\frac{n-1}{2}} du$$

When $n-$ odd: $\cos x dx = d(\sin x)$

Express $\cos x$ via $\sin x$

Set $\sin x = u$

$$\int \sin^m x \cos^n x dx = \int \sin^{m-1} x \cos^n x d(-\cos x)$$
$$= -\int \left(1-\cos{}^2 x\right)^{\frac{m-1}{2}} \cos{}^n x d(\cos x)$$
$$= -\int \left(1-u^2\right)^{\frac{m-1}{2}} u^n du$$

When $m-$ odd: $\sin x dx = d(-\cos x)$

Express $\cos x$ via $\sin x$

Set $\cos x = u$

If both m, n- even, use $\left| \begin{array}{ccc} \sin^2 x & = & \frac{1-\cos(2x)}{2} \\ \cos^2 x & = & \frac{\cos(2x)+1}{2} \end{array} \right.$ and substitute $s = 2x$ to lower trig powers. Repeat above considerations.