Calculus I

$$\int x^3\sqrt{ax^2+b}\mathbf{d}x$$

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Example (Substitution Rule, more factors)

$$\text{Evaluate } \int 3x^5\sqrt{1+x^3}\mathrm{d}x = \int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x.$$

$$\text{Let } u = 1+x^3.$$

$$\text{Then } \mathrm{d}u = 3x^2\mathrm{d}x.$$

$$x^3 = u-1.$$

$$\begin{aligned}\int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x &= \int (u-1)\sqrt{u}\,\mathrm{d}u \\ &= \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u \\ &= \left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right) + C \\ &= \frac{2}{5}\left(1+x^3\right)^{\frac{5}{2}} - \frac{2}{3}\left(1+x^3\right)^{\frac{3}{2}} + C.\end{aligned}$$