Precalculus
Find the area of a triangle from two sides and an angle between them

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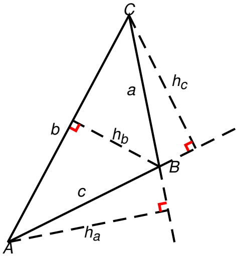
Triangle area = $\frac{1}{2}$base · height

Let $\triangle ABC$ have side lengths a, b, c and height lengths h_a, h_b, h_c, as indicated - side a is opposite to vertex A and h_a starts at A, and so on.

Proposition (Triangle area)

$$Area(\triangle ABC) = \frac{1}{2} height \cdot base = \frac{1}{2} h_a a = \frac{1}{2} h_b b = \frac{1}{2} h_c c.$$



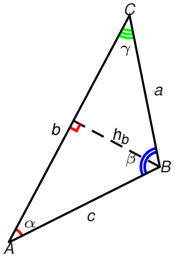
Triangle area from two sides and angle between them

Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ, as indicated: α is opposite to a, β is opposite to b, γ is opposite to c.

Proposition ($\triangle$ area from two sides and angle between them)

The area of a triangle is half the product of the lengths of two of its sides times the sine of the angle between them. In other words,

$$Area(\triangle ABC) = \frac{ab\sin\gamma}{2} = \frac{bc\sin\alpha}{2} = \frac{ca\sin\beta}{2}$$



Proof.

$$\begin{aligned}\text{Area}(\triangle ABC) &= \frac{\text{base}\cdot\text{height}}{2} = \frac{bh_b}{2}\\ &= \frac{ba\sin\gamma}{2}.\end{aligned}$$

The proof of the other two cases is similar. ∎