

Calculus II

L'Hospital's rule

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Outline

- 1 Indeterminate Forms and L'Hospital's Rule
 - Indeterminate Products
 - Indeterminate Differences
 - Indeterminate Powers

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- We don't get any cancellation between top and bottom.
- We need new techniques.

Theorem (L'Hospital's Rule)

Suppose that f and g are differentiable and $g'(x) \neq 0$ on an open interval that contains a (except possibly at a). Suppose that

$$\lim_{x \rightarrow a} f(x) = 0 \qquad \text{and} \qquad \lim_{x \rightarrow a} g(x) = 0$$

$$\text{or that} \qquad \lim_{x \rightarrow a} f(x) = \pm\infty \qquad \text{and} \qquad \lim_{x \rightarrow a} g(x) = \pm\infty$$

(In other words, we have an indeterminate form of type $0/0$ or ∞/∞ .)
Then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}.$$

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If $\lim_{x \rightarrow a} f(x) = 0$ and $\lim_{x \rightarrow a} g(x) = \pm\infty$, then it isn't clear what $\lim_{x \rightarrow a} (fg)(x)$ will be.

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$$fg = \frac{f}{1/g} \quad \text{or} \quad fg = \frac{g}{1/f}.$$

This converts the given limit into an indeterminate form of type $0/0$ or ∞/∞ .

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• $\lim_{x \rightarrow 0^+} \ln x =$.

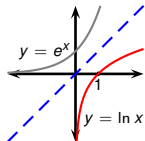
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• $\lim_{x \rightarrow 0^+} \ln x = ?$.

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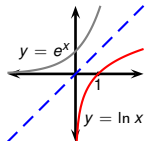


Example

Evaluate $\lim_{x \rightarrow 0^+} x \ln x$.

● $\lim_{x \rightarrow 0^+} \ln x = -\infty$.

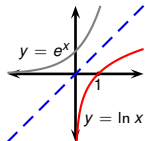
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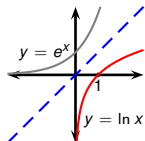
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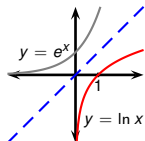
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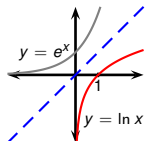
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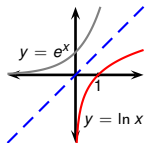


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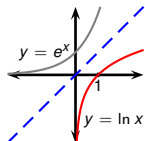


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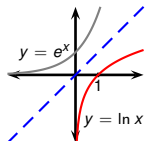


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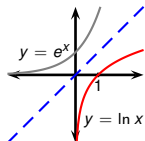


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 &= \lim_{x \rightarrow 0^+} \frac{?}{?}
 \end{aligned}$$

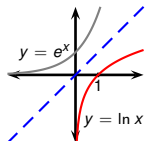


Example

Evaluate $\lim_{x \rightarrow 0^+} x \ln x$.

- $\lim_{x \rightarrow 0^+} \ln x = -\infty$.
- $\lim_{x \rightarrow 0^+} x = 0$.
- This is an indeterminate form of type $0(-\infty)$ (or $-\infty/(1/0)$).
- Apply L'Hospital's rule:

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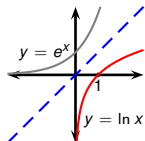


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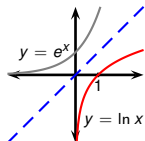


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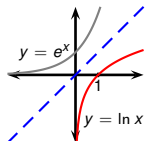


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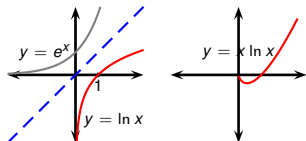


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Indeterminate Differences

If $\lim_{x \rightarrow a} f(x) = \infty$ and $\lim_{x \rightarrow a} g(x) = \infty$, then the limit

$$\lim_{x \rightarrow a} [f(x) - g(x)]$$

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To compute such a limit, try to convert it into a quotient (by using a common denominator, or by rationalizing, or by factoring out a common factor).

Example

Evaluate $\lim_{x \rightarrow (\pi/2)^-} (\sec x - \tan x)$.

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Indeterminate Powers

Several indeterminate forms arise from the limit $\lim_{x \rightarrow a} f(x)^{g(x)}$.

$$\lim_{x \rightarrow a} f(x) = 0 \quad \text{and} \quad \lim_{x \rightarrow a} g(x) = 0 \quad \text{type } 0^0$$

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These can all be solved either by taking the natural logarithm:

$$\text{let } y = [f(x)]^{g(x)}, \text{ then } \ln y = g(x) \ln f(x)$$

or by writing the function as an exponential:

$$[f(x)]^{g(x)} = e^{g(x) \ln f(x)}.$$

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$$\lim_{x \rightarrow 0^+} x^x = \lim_{x \rightarrow 0^+} e^{x \ln x} = e^0 = 1$$

Example

$$\lim_{x \rightarrow \infty} \left(1 + \frac{k}{x}\right)^x$$

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$$\lim_{x \rightarrow \infty} \left(1 + \frac{k}{x}\right)^x = \lim_{x \rightarrow \infty} e^{\ln\left(1 + \frac{k}{x}\right)^x}$$

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exponent= continuous f-n

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 &= \lim_{x \rightarrow \infty} \frac{?}{-\frac{1}{x^2}}
 \end{aligned}$$

Example

$$\begin{aligned}
 \lim_{x \rightarrow \infty} \left(1 + \frac{k}{x}\right)^x &= \lim_{x \rightarrow \infty} e^{\ln\left(1 + \frac{k}{x}\right)^x} & \text{exponent= continuous f-n} \\
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 &= \lim_{x \rightarrow \infty} \frac{\frac{d}{dx} \left(\ln\left(1 + \frac{k}{x}\right) \right)}{\frac{d}{dx} \left(\frac{1}{x} \right)} && \left| \text{form } \frac{0}{0}, \text{ use L'Hospital} \right. \\
 &= \lim_{x \rightarrow \infty} \frac{\frac{1}{1 + \frac{k}{x}} \left(1 + \frac{k}{x}\right)'}{-\frac{1}{x^2}}
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 &= \lim_{x \rightarrow \infty} \frac{k}{1 + \frac{k}{x}} = ?
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 \lim_{x \rightarrow \infty} \left(1 + \frac{k}{x}\right)^x &= \lim_{x \rightarrow \infty} e^{\ln\left(1 + \frac{k}{x}\right)^x} && \left| \text{exponent} = \text{continuous f-n} \right. \\
 &= e^{\lim_{x \rightarrow \infty} \ln\left(1 + \frac{k}{x}\right)^x} = ? \\
 \lim_{x \rightarrow \infty} \ln\left(1 + \frac{k}{x}\right)^x &= \lim_{x \rightarrow \infty} x \ln\left(1 + \frac{k}{x}\right) \\
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 \lim_{x \rightarrow \infty} \ln\left(1 + \frac{k}{x}\right)^x &= \lim_{x \rightarrow \infty} x \ln\left(1 + \frac{k}{x}\right) \\
 &= \lim_{x \rightarrow \infty} \frac{\frac{d}{dx} (\ln(1 + \frac{k}{x}))}{\frac{d}{dx} (\frac{1}{x})} && \left| \text{form "0/0", use L'Hospital} \right. \\
 &= \lim_{x \rightarrow \infty} \frac{\frac{1}{1 + \frac{k}{x}} \left(1 + \frac{k}{x}\right)'}{-\frac{1}{x^2}} \\
 &= \lim_{x \rightarrow \infty} \frac{\frac{1}{1 + \frac{k}{x}} \left(-\frac{k}{x^2}\right)}{-\frac{1}{x^2}} \\
 &= \lim_{x \rightarrow \infty} \frac{k}{1 + \frac{k}{x}} = k
 \end{aligned}$$

Example

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 \lim_{x \rightarrow \infty} \left(1 + \frac{k}{x}\right)^x &= \lim_{x \rightarrow \infty} e^{\ln\left(1 + \frac{k}{x}\right)^x} && \text{exponent= continuous f-n} \\
 &= e^{\lim_{x \rightarrow \infty} \ln\left(1 + \frac{k}{x}\right)^x} = e^k && \text{limit computed below} \\
 \lim_{x \rightarrow \infty} \ln\left(1 + \frac{k}{x}\right)^x &= \lim_{x \rightarrow \infty} x \ln\left(1 + \frac{k}{x}\right) \\
 &= \lim_{x \rightarrow \infty} \frac{\frac{d}{dx} (\ln(1 + \frac{k}{x}))}{\frac{d}{dx} (\frac{1}{x})} && \text{form "0/0", use L'Hospital} \\
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