

# Precalculus

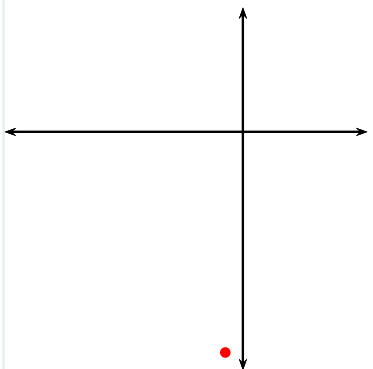
## Inverse of a quadratic function

Todor Milev

2019

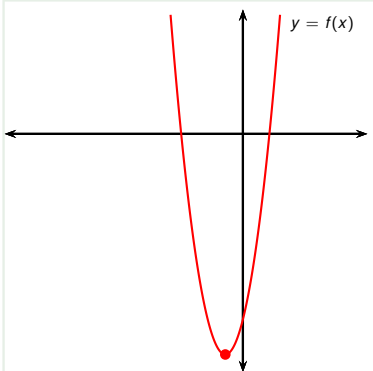
## Example ( )

Given:  $f(x) = 3x^2 + 4x - 7$  with domain  $x \geq -\frac{2}{3}$ . Find  $f^{-1}(x)$ .



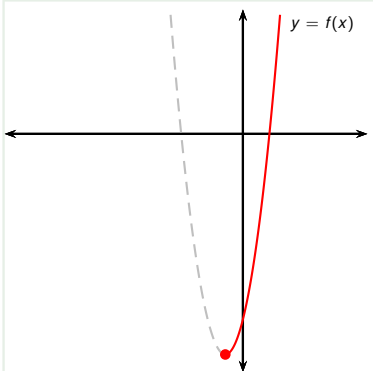
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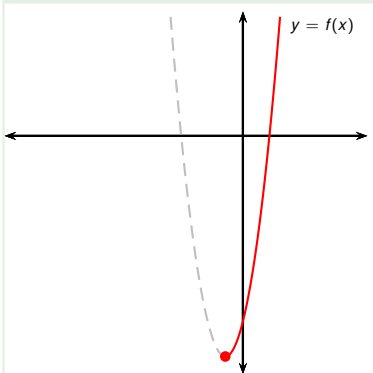
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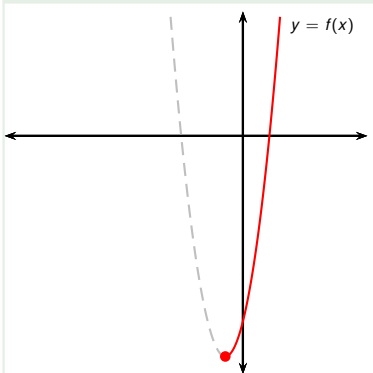
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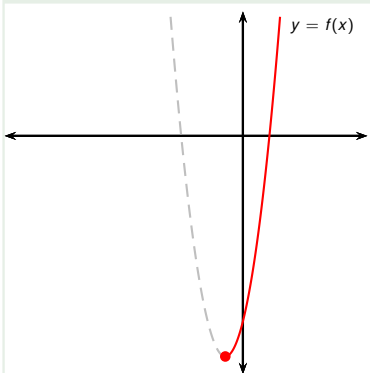
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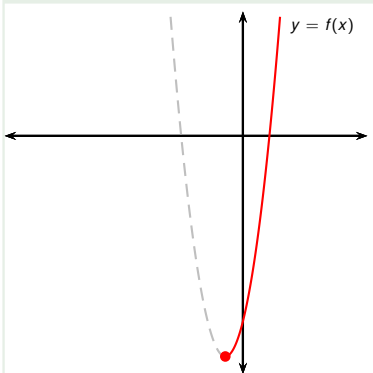
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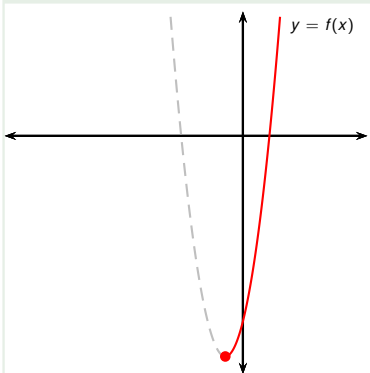
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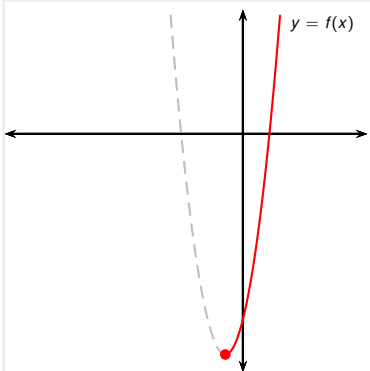
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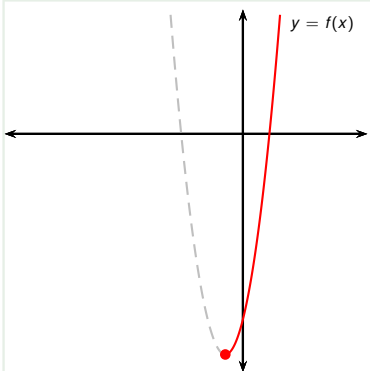
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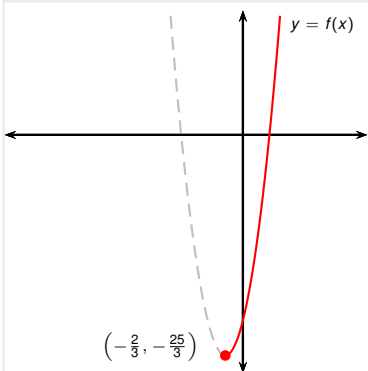
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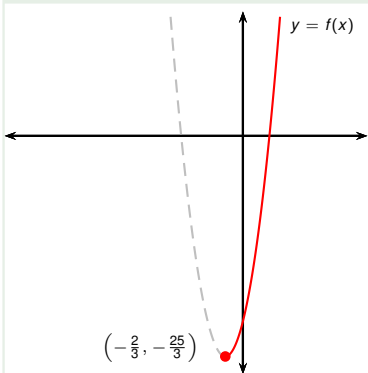
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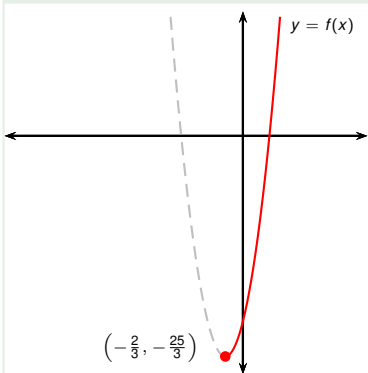
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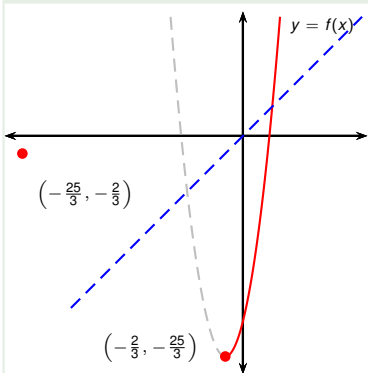
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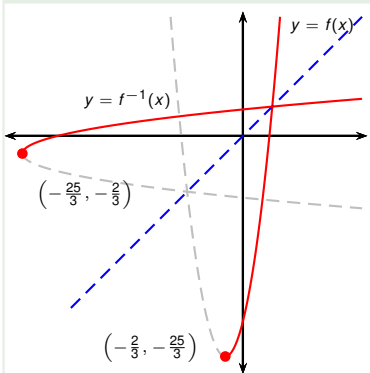
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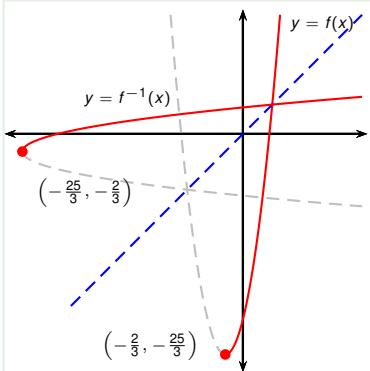
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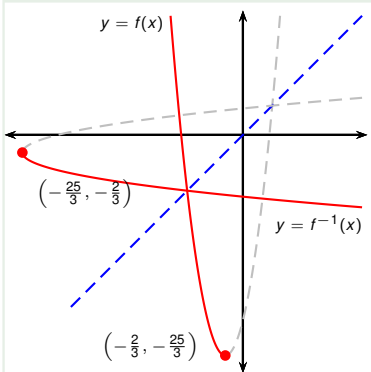
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Final answer, relabelled:

$$f^{-1}(x) = -\frac{2}{3} - \frac{\sqrt{25 + 3x}}{3}$$

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