

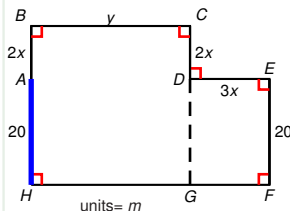
Precalculus

§ Geometric-text problems leading to polynomial systems, part 1

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Example



A field is enclosed by a wall AH and fencing at the rest of the boundary, as depicted. Given:
 $|EF| = |AH| = 20 \text{ m}$, $|BC| = y \text{ m}$,
 $|AB| = |DC| = 2x \text{ m}$, $|DE| = 3x \text{ m}$; fencing
length, excluding wall, is 130 m; area of $HBCG$ is
3 times that of $DEFG$. Find the length $|HF|$.

$$\text{Fence length} = 130 \text{ m}$$

$$2y + 2 \cdot 2x + 2 \cdot 3x + 20 = 130$$

$$10x + 2y = 110$$

| Div. by 2

$$5x + y = 55$$

$$y = 55 - 5x$$

$$\text{Area}(HBCG) = 3 \cdot \text{Area}(DEFG)$$

$$y \cdot (2x + 20) = 3 \cdot 3x \cdot 20$$

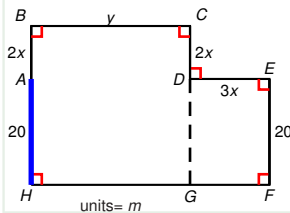
$$(55 - 5x)(2x + 20) - 180x = 0$$

$$110x + 1100 - 10x^2 - 100x - 180x = 0$$

$$-10x^2 - 170x + 1100 = 0$$

| Div. by -10

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$$\begin{aligned}
 y &= 55 - 5x \\
 -10x^2 - 170x + 1100 &= 0 & \left| \text{Div. by } -10 \right. \\
 x^2 + 17x - 110 &= 0 \\
 (x - 5)(x + 22) &= 0 \\
 x = 5 \text{ or } x = -22 & & \left| x > 0 \right. \\
 y &= 55 - 5x \\
 &= 55 - 5 \cdot 5 = 30 \\
 |HF| &= (y + 3x)m \\
 &= (30 + 3 \cdot 5)m \\
 &= 45m
 \end{aligned}$$