Precalculus
Factor quadratic with irrational real roots

Todor Milev

2019

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

$$\left| \begin{array}{rcl} x_1 x_2 & = & \dfrac{c}{a} \\ x_1 + x_2 & = & -\dfrac{b}{a} \end{array} \right.$$

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Factor the quadratic.

$$x^2 + 3x + 1$$

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