

Precalculus

Simplify linear combination of logarithms using all techniques

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Example

Compute as a rational number, without using calculator.

$$\begin{aligned}
 \log_7(24) + \log_{\frac{1}{7}}(3) - \log_{49}(64) &= \log_7(24) + \frac{\log_7(3)}{\log_7(\frac{1}{7})} - \frac{\log_7(64)}{\log_7(49)} \\
 &= \log_7(24) + \frac{\log_7(3)}{-1} - \frac{\log_7(64)}{2} \\
 &= \log_7(24) - \log_7(3) - \frac{1}{2} \log_7(64) \\
 &= \log_7\left(\frac{24}{3}\right) - \log_7\left(64^{\frac{1}{2}}\right) \\
 &= \log_7(8) - \log_7(\sqrt{64}) \\
 &= \log_7 8 - \log_7 8 = 0
 \end{aligned}$$

[alternatively:] $= \log_7\left(\frac{8}{8}\right) = \log_7(1) = 0.$

$$\left[\begin{array}{l} \log_a x - \log_a y = \log_a \left(\frac{x}{y}\right) \\ \log_a x^r = r \log_a x \end{array} \right]$$