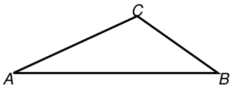
Precalculus
Solve triangle from two sides and an angle

Todor Milev

2019

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

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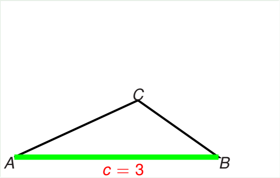

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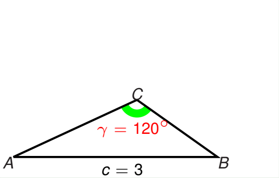

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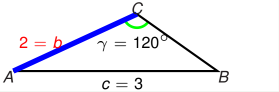

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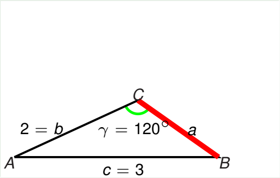

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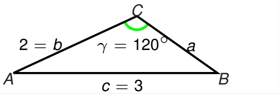

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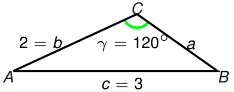
Law of cosines

Example



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$a^2 + b^2 - 2ab\cos\gamma = c^2$ | Law of cosines

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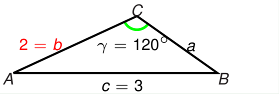

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$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

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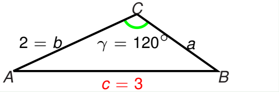

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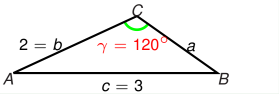

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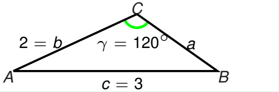

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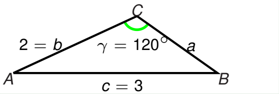

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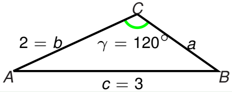

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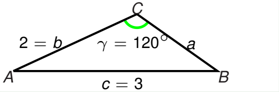

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Law of cosines

Solve for a:

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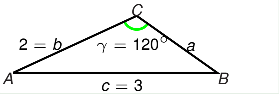

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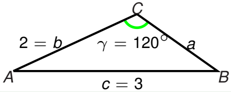

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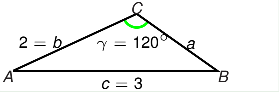

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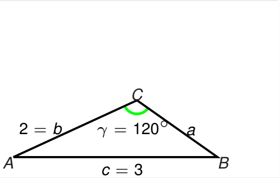

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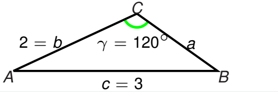

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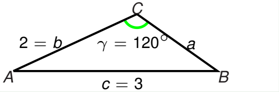

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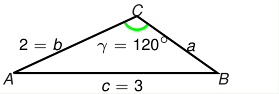

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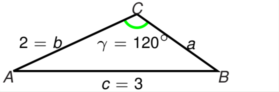

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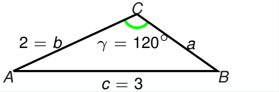

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a &= \frac{-2 \pm \sqrt{2^2 - 4\cdot(-5)\cdot 1}}{2\cdot 1} \\
&= \frac{-2 \pm \sqrt{24}}{2}
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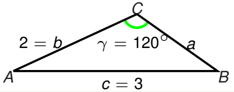

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a &= \frac{-2 \pm \sqrt{2^2 - 4\cdot(-5)\cdot 1}}{2\cdot 1}\\
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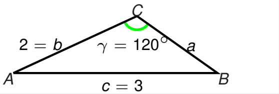

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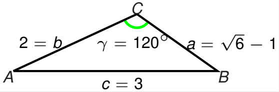

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$$\begin{aligned} a &= \frac{-2 \pm \sqrt{2^2 - 4\cdot(-5)\cdot 1}}{2\cdot 1} \\ &= \frac{-2\pm\sqrt{24}}{2} = \frac{-2+2\sqrt{6}}{2} && a > 0 \\ &= -1+\sqrt{6} \end{aligned}$$

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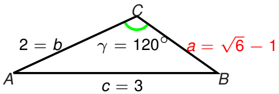

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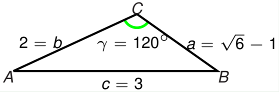

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$$= -1 + \sqrt{6}$$

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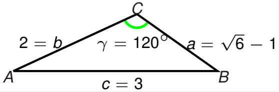

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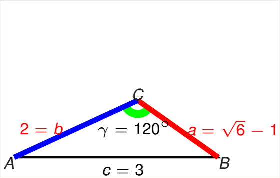
Area = ?

Example



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$$\textbf{Area} = \frac{ab\sin\gamma}{2}$$

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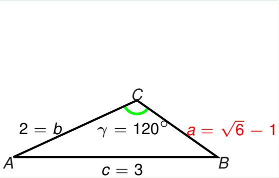

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$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1} = -1 + \sqrt{6}$$

$$\textbf{Area} = \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6} - 1\right)2}{2}\textbf{?}$$

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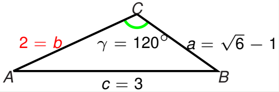

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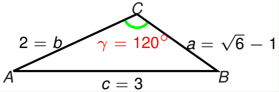
$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4\cdot(-5)\cdot 1}}{2\cdot 1} = -1 + \sqrt{6}$$

$$\textbf{Area} = \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6}-1\right)2}{2}\textbf{?}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

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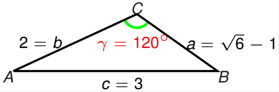

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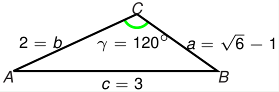

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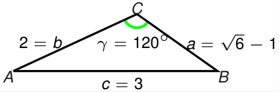

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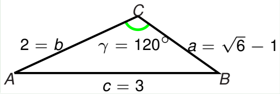

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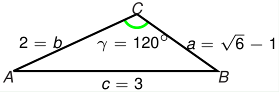

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