

Precalculus
Homework
Equations involving logarithms and exponents

1. Convert from degrees to radians.

- | | | |
|------------------|-------------------|---------------------|
| (a) 15° . | (h) 120° . | (o) 360° . |
| (b) 30° . | (i) 135° . | (p) 405° . |
| (c) 36° . | (j) 150° . | (q) 1200° . |
| (d) 45° . | (k) 180° . | (r) -900° . |
| (e) 60° . | (l) 225° . | (s) -2014° . |
| (f) 75° . | (m) 270° . | |
| (g) 90° . | (n) 305° . | |

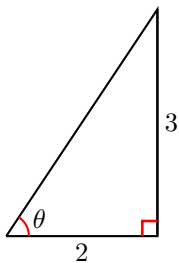
2. Convert from radians to degrees. The answer key has not been proofread, use with caution.

- | | | |
|-------------------------|-------------------------|---------------|
| (a) 4π . | (d) $\frac{4}{3}\pi$. | (g) 5. |
| (b) $-\frac{7}{6}\pi$. | (e) $-\frac{3}{8}\pi$. | |
| (c) $\frac{7}{12}\pi$. | (f) 2014π . | (h) -2014 . |

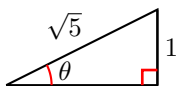
3. Find the indicated circle arc-length. The answer key has not been proofread, use with caution.

- (a) Circle of radius 3, arc of measure 36° .
- (b) Circle of radius $\frac{1}{2}$, arc of measure 100° .
- (c) Circle of radius 1, arc of measure 3 (radians).
- (d) Circle of radius 3, arc of measure 300° .

4. Find the 6 trigonometric functions of the indicated angle in the indicated right triangle.

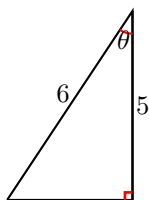
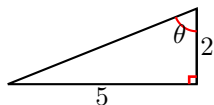


(a)



(b)

- (c)
(d)



5. Find the exact value of the trigonometric function (using radicals).

- (a) $\cos 135^\circ$.
 (b) $\sin 225^\circ$.
 (c) $\cos 495^\circ$.
 (d) $\sin 560^\circ$.
 (e) $\sin\left(\frac{3\pi}{2}\right)$.
 (f) $\cos\left(\frac{11\pi}{6}\right)$.
 (g) $\sin\left(\frac{2015\pi}{3}\right)$.
 (h) $\cos\left(\frac{17\pi}{3}\right)$.

6. Find all solutions of the equation in the interval $[0, 2\pi)$. The answer key has not been proofread, use with caution.

- (a) $\sin x = -\frac{\sqrt{2}}{2}$.
 (b) $\cos x = \frac{\sqrt{3}}{2}$.
 (c) $\sin(3x) = \frac{1}{2}$.
 (d) $\cos(7x) = 0$.
 (e) $\cos\left(3x + \frac{\pi}{2}\right) = 0$.
 (f) $\sin\left(5x - \frac{\pi}{3}\right) = 0$.

7. Use the known values of $\sin 30^\circ$, $\cos 30^\circ$, $\sin 45^\circ$, $\cos 45^\circ$, $\sin 60^\circ$, $\cos 60^\circ$, \dots , the angle sum formulas and the cofunction identities to find an exact value (using radicals) for the trigonometric function.

(a) The six trigonometric functions of $105^\circ = 45^\circ + 60^\circ$:

- $\sin(105^\circ)$.
- $\cos(105^\circ)$. Should your answer be a positive or a negative number?
- $\tan(105^\circ)$.
- $\cot(105^\circ)$.
- $\sec(105^\circ)$.
- $\csc(105^\circ)$.

(b) The six trigonometric functions of $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$:

- $\sin\left(\frac{\pi}{12}\right)$.
- $\cos\left(\frac{\pi}{12}\right)$. Should $\sin\left(\frac{\pi}{12}\right)$ be larger or smaller than $\cos\left(\frac{\pi}{12}\right)$?
- $\tan\left(\frac{\pi}{12}\right)$.
- $\cot\left(\frac{\pi}{12}\right)$.
- $\sec\left(\frac{\pi}{12}\right)$.
- $\csc\left(\frac{\pi}{12}\right)$.

8. Simplify to a trigonometric function of the angle θ . The answer key has not been proofread, use with caution.

- (a) $\sin\left(\frac{\pi}{2} - \theta\right)$.

- (b) $\cos\left(\frac{13\pi}{2} - \theta\right)$.
- (c) $\tan(\pi - \theta)$
- (d) $\cot\left(\frac{3\pi}{2} - \theta\right)$
- (e) $\csc\left(\frac{3\pi}{2} + \theta\right)$

9. Using the power-reducing formulas, rewrite the expression in terms of first powers of the cosines and sines of multiples of the angle θ .

- (a) $\sin^4 \theta$.
- (b) $\cos^4 \theta$.
- (c) $\sin^6 \theta$.
- (d) $\cos^6 \theta$.

10. Use the sum-to-product formulas to find all solutions of the trigonometric equation in the interval $[0, 2\pi)$.

Please note that typing a query such as “solve(sin(x)+sin (3x)=0)” at www.wolframalpha.com will provide you with a correct answer and a function plot.

- (a) $\sin(x) + \sin(3x) = 0$.
- (b) $\cos(x) + \cos(-3x) = 0$.
- (c) $\sin(x) - \sin(3x) = 0$.
- (d) $\cos(2x) - \cos(3x) = 0$.

11. Find the inverse function. You are asked to do the algebra only; you are not asked to determine the domain or range of the function or its inverse.

- (a) $f(x) = 3x^2 + 4x - 7$, where $x \geq -\frac{2}{3}$.
- (b) $f(x) = 2x^2 + 3x - 5$, where $x \geq -\frac{3}{4}$.
- (c) $f(x) = \frac{2x+5}{x-4}$, where $x \neq 4$.
- (d) $f(x) = \frac{3x+5}{2x-4}$, where $x \neq 2$.
- (e) $f(x) = \frac{5x+6}{4x+5}$, where $x \neq -\frac{5}{4}$.
- (f) $f(x) = \frac{2x-3}{-3x+4}$, where $x \neq \frac{4}{3}$.

12. Find the inverse function and its domain.

- (a) $y = \ln(x+3)$.
- (b) $y = 4 \ln(x-3) - 4$.
- (c) $y = 2 \ln(-2x+4) + 1$
- (d) $f(x) = e^{x^3}$.
- (e) $y = (\ln x)^2, x \geq 1$.
- (f) $y = \frac{e^x}{1+2e^x}$.
- (g) $f(x) = 2^{2x} + 2^x - 2$.

13. Find each of the following values. Express your answers precisely, not as decimals.

- (a) $\arcsin(\sin 4)$.
- (b) $\arcsin(\sin 0.5)$.
- (c) $\arcsin(\cos 120^\circ)$.
- (d) $\arccos(\cos(3))$.
- (e) $\arccos(\cos(-2))$.
- (f) $\arccos(\sin(-4))$.
- (g) $\arctan(\tan 5)$.

14. Express as the following as an algebraic expression of x . In other words, “get rid” of the trigonometric and inverse trigonometric expressions.

(a) $\cos^2(\arctan x)$.

(c) $\frac{1}{\cos(\arcsin x)}$.

(b) $-\sin^2(\operatorname{arccot} x)$.

(d) $-\frac{1}{\sin(\arccos x)}$.

15. Let $x \in (0, 1)$. Express the following using x and $\sqrt{1-x^2}$.

(a) $\sin(\arcsin(x))$.

(e) $\sin(2 \arccos(x))$.

(b) $\sin(2 \arcsin(x))$.

(f) $\sin(3 \arccos(x))$.

(c) $\sin(3 \arcsin(x))$.

(g) $\cos(2 \arcsin(x))$.

(d) $\sin(\arccos(x))$.

(h) $\cos(3 \arccos(x))$.

16. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation.

(a) $2 \cos x - 1 = 0$.

(g) $2 \cos^2 x - (1 + \sqrt{2}) \cos x + \frac{\sqrt{2}}{2} = 0$.

(b) $\sin(2x) = \cos x$.

(h) $|\tan x| = 1$.

(c) $\sqrt{3} \sin x = \sin(2x)$.

(d) $2 \sin^2 x = 1$.

(i) $3 \cot^2 x = 1$.

(e) $2 + \cos(2x) = 3 \cos x$.

(j) $\sin x = \tan x$.

(f) $2 \cos x + \sin(2x) = 0$.

17. Express each of the following as a single power.

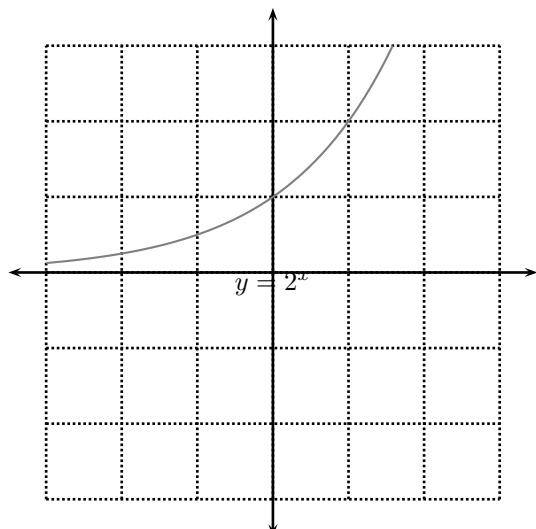
(a) $\frac{2^5 \cdot 2^7}{2\sqrt{2}}$

(b) $\frac{3^2 \cdot 3^{-1}}{3^3 \cdot \sqrt{3^3}}$

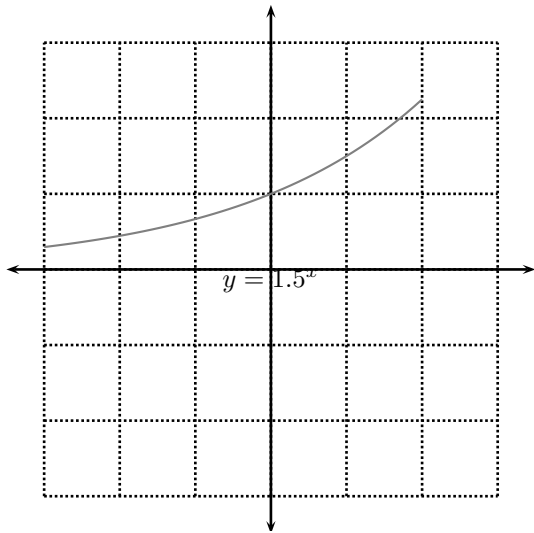
(c) $\frac{\pi^3}{\pi^{-1}\sqrt{\pi^5}}$

18. Sketch by hand approximately the given function. The function is obtained by transforming linearly the graph of a known function. The known function has been sketched for you by computer.

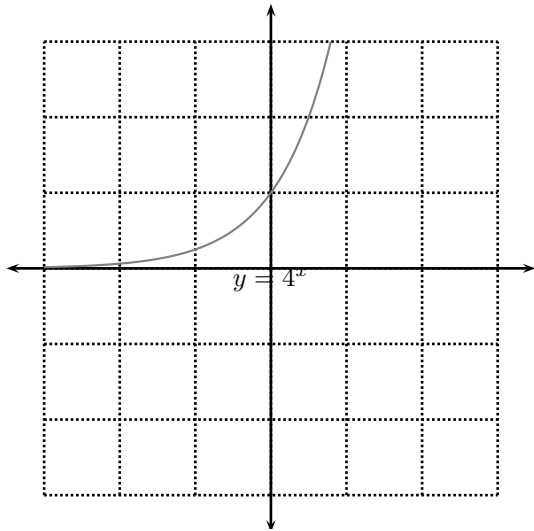
(a) $f(x) = 2^{x+1} - 1$.



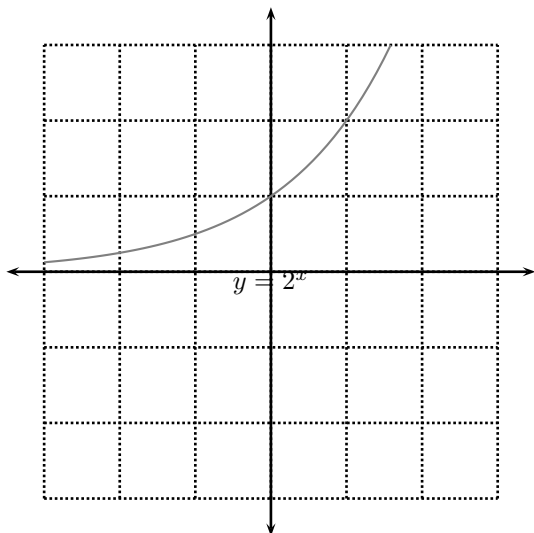
(b) $f(x) = 1.5^{x-2} + 2$.



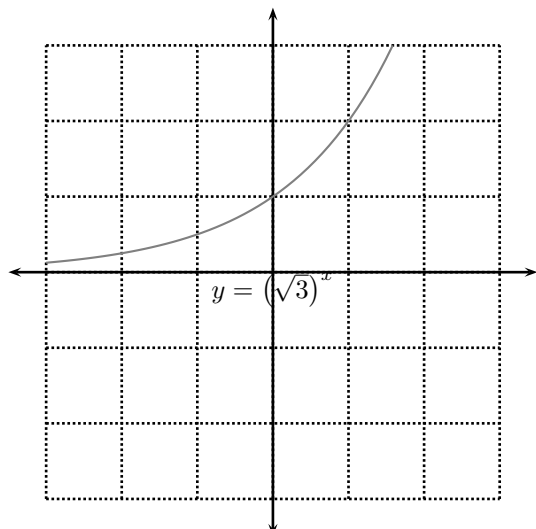
(c) $f(x) = 2^{2x-5}$.



(d) $f(x) = \frac{1}{2^{x-1}} + 1$.



(e) $f(x) = \frac{1}{3^{\frac{1}{2}x+1}} - 1$.



19. (a) A sum is held under a yearly compound interest of 1%. Make an approximation by hand (no calculators allowed) by what factor will have the money increased after 200 years. Can you do the computation in your head?
- (b) Decide, without using a calculator, which is more profitable: earning a yearly compound interest of 2% for 150 years or earning yearly simple interest of 11% for 150 years?
20. Use the definition of a logarithm to evaluate each of the following without using a calculator. The answer key has not been proofread, use with caution.
- (a) $\log_2 16$.
- (b) $\log_3 \left(\frac{1}{9} \right)$.
- (c) $\log_{10} 1000$.
- (d) $\log_6 36^{-\frac{2}{3}}$.
- (e) $\log_2 (8\sqrt{2})$.
- (f) $\log_{\frac{1}{2}} (4)$.
- (g) $\log_{\frac{1}{9}} (\sqrt{3})$.
21. Find the exact value of each expression.
- (a) $\log_5 125$.
- (b) $\log_3 \frac{1}{27}$.
- (c) $\ln \left(\frac{1}{e} \right)$.
- (d) $\log_{10} \sqrt{10}$.
- (e) $e^{\ln 4.5}$.
- (f) $\log_{10} 0.0001$.
- (g) $\log_{1.5} 2.25$.
- (h) $\log_5 4 - \log_5 500$.
- (i) $\log_2 6 - \log_2 15 + \log_2 20$.
- (j) $\log_3 100 - \log_3 18 - \log_3 50$.
- (k) $e^{-2 \ln 5}$.
- (l) $\ln \left(\ln e^{e^{10}} \right)$.
- (m) $\log_7 \left(\frac{49^x}{343^y} \right)$.
22. Using only the \ln operation of your calculator compute the indicated logarithm. Confirm your computation numerically by exponentiation.
- (a) $\log_5 (13)$.
- (b) $\log_{12} (9)$.
- (c) $\log_{13} (101)$.
- (d) $\log_{10} (2015)$.
23. Express each of the following as a single logarithm. If possible, compute the logarithm without using a calculator. The answer key has not been proofread, use with caution.
- (a) $\ln 4 + \ln 6 - \ln 5$.
- (b) $2 \ln 2 - 3 \ln 3 + 4 \ln 4$.

- (c) $\ln 36 - 2 \ln 3 - 3 \ln 2$.
- (d) $\log_2(24) - \log_4 9$.
- (e) $\log_7(24) + \log_{\frac{1}{7}} 3 - \log_{49}(64)$.
- (f) $\log_3(24) + \log_3\left(\frac{3}{8}\right)$.

24. Demonstrate the identity(s).

(a) $-\ln(\sqrt{1+x^2} - x) = \ln(x + \sqrt{1+x^2})$

25. Solve each equation for x . If available, use a calculator to give an (\approx) answer in decimal notation. If available, use a calculator to verify your approximate solutions.

(a) $e^{7-4x} = 7$.

(b) $\ln(2x - 9) = 2$.

(c) $\ln(x^2 - 2) = 3$.

(d) $2^{x-3} = 5$.

(e) $\ln x + \ln(x - 1) = 1$.

(f) $e^{2x+1} = t$.

(g) $\log_2(mx) = c$.

(h) $e - e^{-2x} = 1$.

(i) $8(1 + e^{-x})^{-1} = 3$.

(j) $\ln(\ln x) = 1$.

(k) $e^{e^x} = 10$.

(l) $\ln(2x + 1) = 3 - \ln x$.

(m) $e^{2x} - 4e^x + 3 = 0$.

(n) $e^{4x} + 3e^{2x} - 4 = 0$.

(o) $e^{2x} - e^x - 6 = 0$.

(p) $4^{3x} - 2^{3x+2} - 5 = 0$.

(q) $3 \cdot 2^x + 2\left(\frac{1}{2}\right)^{x-1} - 7 = 0$.