Precalculus
Quadratic inequality part 1

Todor Milev

2019

Example

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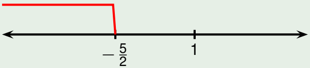

Interval	Factor signs	Final sign	
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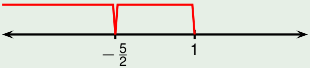

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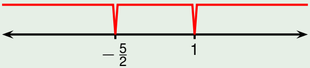

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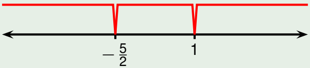

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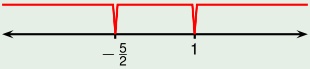

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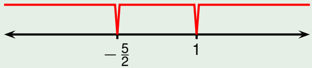

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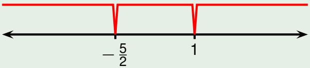

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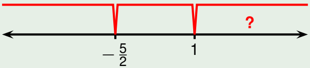

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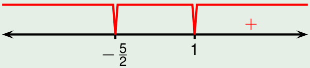

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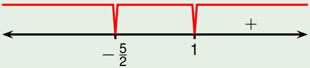

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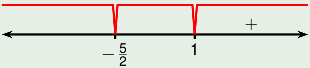

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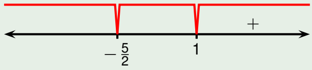

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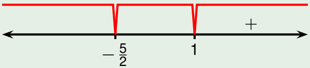

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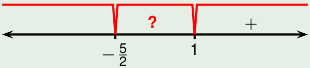

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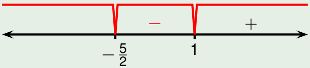

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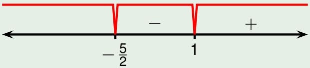

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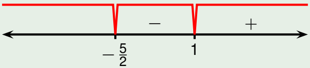

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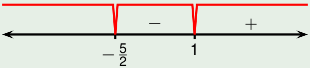

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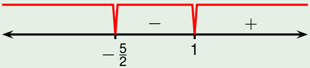

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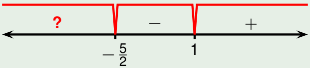

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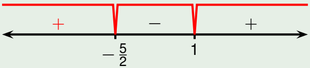

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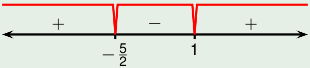

Interval	Factor signs	Final sign	Sample pt	Value at sample pt
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$\left(-\frac{5}{2}, 1\right)$	$(+)(-)$	$-$	0	$f(0) = -5 < 0$
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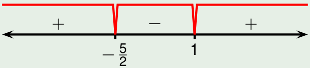

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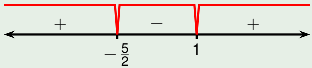

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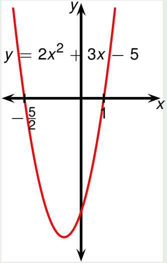

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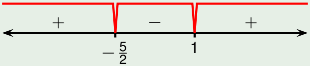

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