#### **Precalculus**

# Trigonometric equation that reduces to quadratic, masked by identity

**Todor Miley** 

2019

$$\sin \theta = \sin(2\theta)$$

$$\sin \theta = \sin(2\theta)$$
  
 $\sin \theta = ?$ 

$$\sin \theta = \sin(2\theta)$$
  
 $\sin \theta = 2\sin \theta \cos \theta$ 

```
\begin{array}{rcl}
\sin \theta & = & \sin(2\theta) \\
\sin \theta & = & 2\sin \theta \cos \theta \\
0 & = & 2\sin \theta \cos \theta - \sin \theta
\end{array}
```

```
\begin{array}{rcl}
\sin \theta & = & \sin(2\theta) \\
\sin \theta & = & 2\sin \theta \cos \theta \\
0 & = & 2\sin \theta \cos \theta - \sin \theta \\
0 & = & \sin \theta (2\cos \theta - 1)
\end{array}
```

 $\sin \theta = 0$ 

Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

$$\begin{array}{rcl}
\sin \theta & = & \sin(2\theta) \\
\sin \theta & = & 2\sin \theta \cos \theta \\
0 & = & 2\sin \theta \cos \theta - \sin \theta \\
0 & = & \sin \theta (2\cos \theta - 1) \\
2\cos \theta - 1 & = & 0
\end{array}$$

 $\sin \theta = 0$ 

Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

$$sin \theta = sin(2\theta)$$

$$sin \theta = 2 sin \theta cos \theta$$

$$0 = 2 sin \theta cos \theta - sin \theta$$

$$0 = sin \theta(2 cos \theta - 1)$$

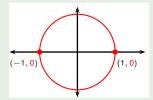
$$2 cos \theta - 1 = 0$$

Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

$$\sin \theta = 0$$

$$2\cos\theta - 1 = 0$$

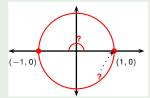


Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

$$\begin{array}{rcl}
\sin\theta & = & 0 \\
\theta & = & ?
\end{array}$$

$$2\cos\theta-1 = 0$$

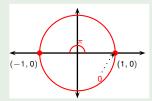


Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

$$sin \theta = 0 
\theta = 0 + 2k\pi 
or  $\pi + 2k\pi$$$

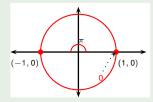
$$2\cos\theta-1 = 0$$



$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

$$\sin \theta = 0$$
 $\theta = 0 + 2k\pi$ 
or  $\pi + 2k\pi$ 

$$2\cos\theta-1 = 0$$

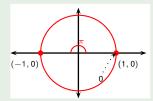


Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

$$sin \theta = 0 
\theta = 0 + 2k\pi 
or  $\pi + 2k\pi$$$

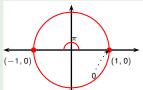
$$2\cos\theta-1 = 0$$



Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

$$\begin{array}{rcl}
\sin\theta &=& 0 \\
\theta &=& 0 + 2k\pi \\
&& \text{or } \pi + 2k\pi \\
\theta &=& 0 \text{ or } 2\pi \text{ or } \pi
\end{array}$$

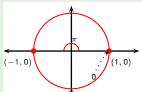


$$2\cos\theta - 1 = 0$$

Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

$$sin \theta = 0 
\theta = 0 + 2k\pi 
or  $\pi + 2k\pi$   
 $\theta = 0$  or  $2\pi$  or  $\pi$$$



$$2\cos\theta - 1 = 0$$

$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

$$\sin \theta = 0$$
 $\theta = 0 + 2k\pi$ 
 $\text{or } \pi + 2k\pi$ 
 $\theta = 0 \text{ or } 2\pi \text{ or } \pi$ 

$$2\cos\theta - 1 = 0$$

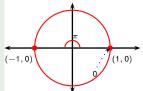
$$\cos\theta = \frac{1}{2}$$



Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

$$sin \theta = 0 
\theta = 0 + 2k\pi 
or  $\pi + 2k\pi$   
 $\theta = 0$  or  $2\pi$  or  $\pi$$$



$$\begin{array}{rcl} 2\cos\theta - 1 & = & 0\\ \cos\theta & = & \frac{1}{2} \end{array}$$

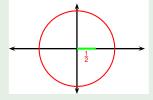


$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

$$\sin \theta = 0$$
 $\theta = 0 + 2k\pi$ 
 $\text{or } \pi + 2k\pi$ 
 $\theta = 0 \text{ or } 2\pi \text{ or } \pi$ 

$$2\cos\theta - 1 = 0$$

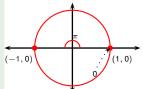
$$\cos\theta = \frac{1}{2}$$



Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

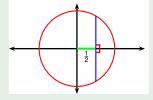
$$sin \theta = 0 
\theta = 0 + 2k\pi 
or  $\pi + 2k\pi$   
 $\theta = 0 \text{ or } 2\pi \text{ or } \pi$$$



$$2\cos\theta - 1 = 0$$

$$\cos\theta = \frac{1}{2}$$

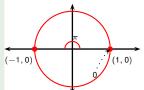
$$\theta = ?$$



Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

$$\begin{array}{rcl}
\sin \theta & = & \sin(2\theta) \\
\sin \theta & = & 2\sin\theta\cos\theta \\
0 & = & 2\sin\theta\cos\theta - \sin\theta \\
0 & = & \sin\theta(2\cos\theta - 1)
\end{array}$$

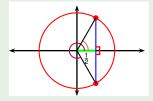
$$\begin{array}{rcl}
\sin\theta & = & 0 \\
\theta & = & 0 + 2k\pi \\
& & \text{or } \pi + 2k\pi \\
\theta & = & 0 \text{ or } 2\pi \text{ or } \pi
\end{array}$$



$$2\cos\theta - 1 = 0$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = ?$$



Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

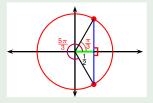
$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

$$\begin{array}{rcl} \sin\theta & = & 0 \\ \theta & = & 0 + 2k\pi \\ & & \text{or } \pi + 2k\pi \\ \theta & = & 0 \text{ or } 2\pi \text{ or } \pi \end{array}$$

$$2\cos\theta - 1 = 0$$

$$\cos\theta = \frac{1}{2}$$

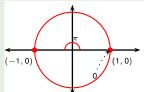
$$\theta = \frac{\pi}{3} + 2k\pi \text{ or } \frac{5\pi}{3} + 2k\pi$$



Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

$$\begin{array}{rcl} \sin\theta & = & 0 \\ \theta & = & 0 + 2k\pi \\ & & \text{or } \pi + 2k\pi \\ \theta & = & 0 \text{ or } 2\pi \text{ or } \pi \end{array}$$

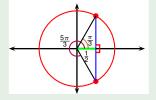


$$2\cos\theta - 1 = 0$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3} + 2k\pi \text{ or } \frac{5\pi}{3} + 2k\pi$$

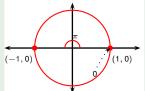
$$\theta = \frac{\pi}{3} \text{ or } \frac{5\pi}{3}$$



Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

$$\begin{array}{rcl}
\sin\theta & = & 0 \\
\theta & = & 0 + 2k\pi \\
& & \text{or } \pi + 2k\pi \\
\theta & = & 0 \text{ or } 2\pi \text{ or } \pi
\end{array}$$

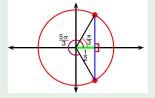


$$2\cos\theta - 1 = 0$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3} + 2k\pi \text{ or } \frac{5\pi}{3} + 2k\pi$$

$$\theta = \frac{\pi}{3} \text{ or } \frac{5\pi}{3}$$



Find all values of  $\theta$  in the interval  $\theta \in [0, 2\pi]$  for which  $\cos(2\theta) = \cos\theta$ 

$$\cos(2\theta) = \cos\theta$$
  
?  $-\cos\theta = 0$ 

$$cos(2\theta) = cos \theta$$
 $-cos \theta = 0$ 

$$\cos(2\theta) = \cos\theta$$
$$\cos^2\theta - \sin^2\theta - \cos\theta = 0$$

$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{ Express via } \cos\theta$$

$$\cos^2\theta - (?) - \cos\theta = 0$$

$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0$$

$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0$$

$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

$$2u^2 - u - 1 = 0$$

$$\cos(2\theta) = \cos\theta$$

$$\cos^{2}\theta - \sin^{2}\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^{2}\theta - (1 - \cos^{2}\theta) - \cos\theta = 0$$

$$2\cos^{2}\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

$$2u^{2} - u - 1 = 0$$

$$(?)(?) = 0$$

$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

Find all values of  $\theta$  in the interval  $\theta \in [0, 2\pi]$  for which

$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0$$

$$2u + 1 = 0$$

$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0 \qquad 2u + 1 = 0$$

$$\cos\theta = 1 \qquad \text{or}$$

$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0 \qquad 2u + 1 = 0$$

$$\cos\theta = 1$$

$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

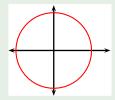
$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0 \qquad 2u + 1 = 0$$

$$\cos\theta = 1$$

$$\theta = 2 + 2k\pi$$



$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

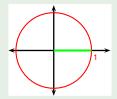
$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0 \qquad 2u + 1 = 0$$

$$\cos\theta = 1$$

$$\theta = ? + 2k\pi$$



$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

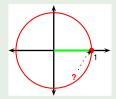
$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0 \qquad 2u + 1 = 0$$

$$\cos\theta = 1$$

$$\theta = \frac{2}{3} + 2k\pi$$



$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

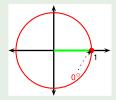
$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0 \qquad 2u + 1 = 0$$

$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi$$



$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

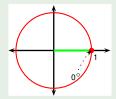
$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0 \qquad 2u + 1 = 0$$

$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi$$



$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

$$2u^2 - u - 1 = 0$$

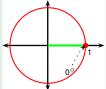
$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0 \qquad 2u + 1 = 0$$

$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi$$

$$\theta = 0 \text{ or } 2\pi$$



$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

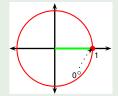
$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi$$

$$\theta = 0 \text{ or } 2\pi$$



$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

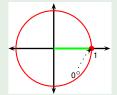
$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi$$

$$\theta = 0 \text{ or } 2\pi$$



$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

$$2u^2 - u - 1 = 0$$

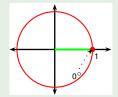
$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0$$

$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi$$

$$\theta = 0 \text{ or } 2\pi$$



Find all values of  $\theta$  in the interval  $\theta \in [0, 2\pi]$  for which

$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

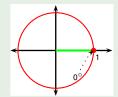
$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

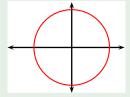
$$u - 1 = 0$$

$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi$$
or
$$\cos\theta$$



 $\theta = 0 \text{ or } 2\pi$ 



Find all values of  $\theta$  in the interval  $\theta \in [0, 2\pi]$  for which

$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \quad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \quad | \text{Set } \cos\theta = u$$

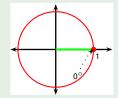
$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

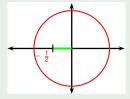
$$u - 1 = 0$$

$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi$$
or
$$\cos\theta$$



 $\theta = 0 \text{ or } 2\pi$ 



$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

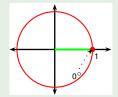
$$2u^2 - u - 1 = 0$$

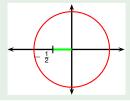
$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0$$

$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi \qquad \text{or} \qquad \theta = ?$$





$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

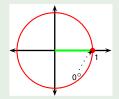
$$2u^2 - u - 1 = 0$$

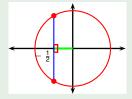
$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0$$

$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi \qquad \text{or} \qquad \theta = ?$$





$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

$$2u^2 - u - 1 = 0$$

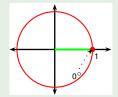
$$(u - 1)(2u + 1) = 0$$

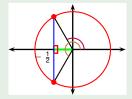
$$u - 1 = 0$$

$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi \qquad \text{or} \qquad \theta = \frac{1}{2}$$

$$\theta = 0 \text{ or } 2\pi$$





$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

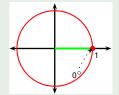
$$2u^2 - u - 1 = 0$$

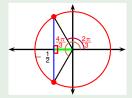
$$(u - 1)(2u + 1) = 0$$

$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi \qquad \text{or} \qquad \theta = \frac{2\pi}{3} + 2k\pi \text{ or } \frac{4\pi}{3} + 2k\pi$$

$$\theta = 0 \text{ or } 2\pi$$





$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

$$2u^2 - u - 1 = 0$$

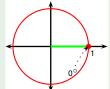
$$(u - 1)(2u + 1) = 0$$

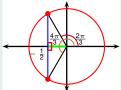
$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi \qquad \text{or}$$

$$\theta = 0 \text{ or } 2\pi$$

$$\cos\theta = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}$$





$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \qquad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \qquad | \text{Set } \cos\theta = u$$

$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0$$

$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi \qquad \text{or} \qquad \theta = \frac{2\pi}{3} + 2k\pi \text{ or } \frac{4\pi}{3} + 2k\pi$$

$$\theta = 0 \text{ or } 2\pi \qquad \theta = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}$$

