## **Precalculus**

## Trigonometric equation that reduces to quadratic, masked by identity

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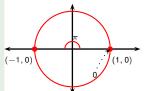
## Example

Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

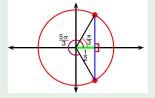
or

$$\begin{array}{rcl} \sin\theta & = & \sin(2\theta) \\ \sin\theta & = & 2\sin\theta\cos\theta \\ 0 & = & 2\sin\theta\cos\theta - \sin\theta \\ 0 & = & \sin\theta(2\cos\theta - 1) \end{array}$$

$$\begin{array}{rcl} \sin\theta & = & 0 \\ \theta & = & 0 + 2k\pi \\ & & \text{or } \pi + 2k\pi \\ \theta & = & 0 \text{ or } 2\pi \text{ or } \pi \end{array}$$



$$\begin{array}{rcl} 2\cos\theta-1&=&0\\ \cos\theta&=&\frac{1}{2}\\ \theta&=&\frac{\pi}{3}+2k\pi \text{ or } \frac{5\pi}{3}+2k\pi\\ \theta&=&\frac{\pi}{3} \text{ or } \frac{5\pi}{3} \end{array}$$



## Example

Find all values of  $\theta$  in the interval  $\theta \in [0, 2\pi]$  for which

$$\cos(2\theta) = \cos\theta$$

$$\cos^2\theta - \sin^2\theta - \cos\theta = 0 \quad | \text{Express via } \cos\theta$$

$$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta = 0$$

$$2\cos^2\theta - \cos\theta - 1 = 0 \quad | \text{Set } \cos\theta = u$$

$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0$$

$$\cos\theta = 1$$

$$\theta = 0 + 2k\pi \quad \text{or} \quad \theta = \frac{2\pi}{3} + 2k\pi \text{ or } \frac{4\pi}{3} + 2k\pi$$

$$\theta = 0 \text{ or } 2\pi \quad \theta = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}$$

