Calculus II
L'Hospital's rule

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Outline

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- We don't get any cancellation between top and bottom.
- We need new techniques.

Theorem (L'Hospital's Rule)

Suppose that f and g are differentiable and $g'(x) \neq 0$ on an open interval that contains a (except possibly at a). Suppose that

$$\begin{array}{llll} & \lim_{x\to a} f(x) = 0 & \textit{and} & \lim_{x\to a} g(x) = 0 \\ \textit{or that} & \lim_{x\to a} f(x) = \pm\infty & \textit{and} & \lim_{x\to a} g(x) = \pm\infty \end{array}$$

(In other words, we have an indeterminate form of type $0/0$ or ∞/∞.) Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

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This converts the given limit into an indeterminate form of type $0/0$ or ∞/∞.

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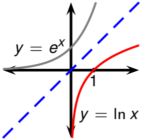

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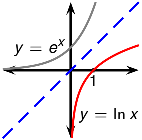

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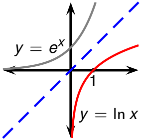

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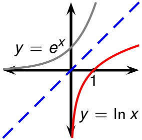

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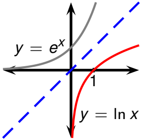

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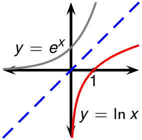

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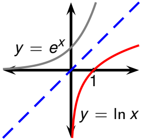

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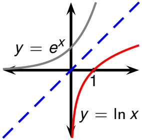

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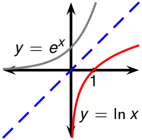

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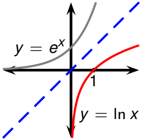

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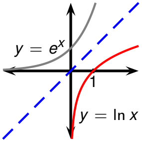

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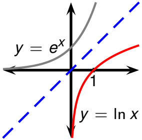

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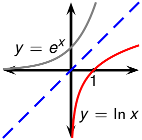

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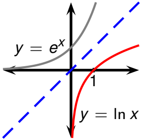

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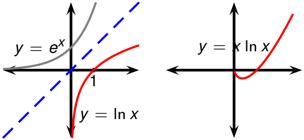

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If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$, then the limit

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To compute such a limit, try to convert it into a quotient (by using a common denominator, or by rationalizing, or by factoring out a common factor).

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Evaluate $\lim_{x\to(\pi/2)^-}(\sec x - \tan x)$.

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Indeterminate Powers

Several indeterminate forms arise from the limit $\lim_{x \to a} f(x)^{g(x)}$.

$$\lim_{x\to a} f(x) = 0 \quad \text{and} \quad \lim_{x\to a} g(x) = 0 \qquad \text{type } 0^0$$

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These can all be solved either by taking the natural logarithm:

$$\text{let} \quad y = [f(x)]^{g(x)}, \quad \text{then} \quad \ln y = g(x) \ln f(x)$$

or by writing the function as an exponential:

$$[f(x)]^{g(x)} = e^{g(x) \ln f(x)}.$$

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\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x &= \lim_{x\to\infty} e^{\ln\left(1+\frac{k}{x}\right)^x} && \text{exponent= continuous f-n}\\
&= e^{\lim\limits_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x}\\
\lim_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x &= \lim_{x\to\infty} x\ln\left(1+\frac{k}{x}\right)\\
&= \lim_{x\to\infty}\frac{\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(1+\frac{k}{x}\right)\right)}{\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{x}\right)} && \text{form ``}\tfrac{0}{0}\text{'', use L'Hospital}\\
&= \lim_{x\to\infty}\frac{?}{-\frac{1}{x^2}}
\end{aligned}$$

Example

$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} e^{\ln\left(1+\frac{k}{x}\right)^x}$$

exponent= continuous f-n

$$= e^{\lim\limits_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x}$$

$$\lim_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} x\ln\left(1+\frac{k}{x}\right)$$

$$= \lim_{x\to\infty}\frac{\frac{d}{dx}\left(\ln\left(1+\frac{k}{x}\right)\right)}{\frac{d}{dx}\left(\frac{1}{x}\right)}$$

form "$\frac{0}{0}$", use L'Hospital

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form "$\frac{0}{0}$", use L'Hospital

$$= \lim_{x\to\infty}\frac{\frac{1}{1+\frac{k}{x}}\left(1+\frac{k}{x}\right)'}{-\frac{1}{x^2}}$$

Example

$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} e^{\ln\left(1+\frac{k}{x}\right)^x} \quad \Big| \text{ exponent= continuous f-n}$$

$$= e^{\lim\limits_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x}$$

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$$= \lim_{x\to\infty}\frac{\frac{1}{1+\frac{k}{x}}\left(-\frac{k}{x^2}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty}\frac{k}{1+\frac{k}{x}}$$

Example

$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} e^{\ln\left(1+\frac{k}{x}\right)^x}$$ exponent= continuous f-n

$$= e^{\lim\limits_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x}$$

$$\lim_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} x\ln\left(1+\frac{k}{x}\right)$$

$$= \lim_{x\to\infty}\frac{\frac{d}{dx}\left(\ln\left(1+\frac{k}{x}\right)\right)}{\frac{d}{dx}\left(\frac{1}{x}\right)}$$ form "$\frac{0}{0}$", use L'Hospital

$$= \lim_{x\to\infty}\frac{\frac{1}{1+\frac{k}{x}}\left(1+\frac{k}{x}\right)'}{-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty}\frac{\frac{1}{1+\frac{k}{x}}\left(-\frac{k}{x^2}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty}\frac{k}{1+\frac{k}{x}} = \mathbf{?}$$

Example

$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} e^{\ln\left(1+\frac{k}{x}\right)^x} \quad \Big| \text{ exponent= continuous f-n}$$

$$= e^{\lim\limits_{x\to\infty} \ln\left(1+\frac{k}{x}\right)^x}$$

$$\lim_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} x\ln\left(1+\frac{k}{x}\right)$$

$$= \lim_{x\to\infty}\frac{\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(1+\frac{k}{x}\right)\right)}{\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{x}\right)} \quad \Big| \text{ form “}\tfrac{0}{0}\text{”, use L'Hospital}$$

$$= \lim_{x\to\infty}\frac{\frac{1}{1+\frac{k}{x}}\left(1+\frac{k}{x}\right)'}{-\frac{1}{x^2}}$$

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$$= \lim_{x\to\infty}\frac{k}{1+\frac{k}{x}} = k$$

Example

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exponent= continuous f-n

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form "$\frac{0}{0}$", use L'Hospital

$$= \lim_{x\to\infty}\frac{\frac{1}{1+\frac{k}{x}}\left(1+\frac{k}{x}\right)'}{-\frac{1}{x^2}}$$

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Example

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exponent= continuous f-n

$$= e^{\lim\limits_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x} = e^k$$

$$\lim_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} x\ln\left(1+\frac{k}{x}\right)$$

$$= \lim_{x\to\infty}\frac{\frac{d}{dx}\left(\ln\left(1+\frac{k}{x}\right)\right)}{\frac{d}{dx}\left(\frac{1}{x}\right)}$$

form "$\frac{0}{0}$", use L'Hospital

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exponent= continuous f-n

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limit computed below

$$\lim_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} x\ln\left(1+\frac{k}{x}\right)$$

$$= \lim_{x\to\infty}\frac{\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(1+\frac{k}{x}\right)\right)}{\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{x}\right)}$$

form "$\frac{0}{0}$", use L'Hospital

$$= \lim_{x\to\infty}\frac{\frac{1}{1+\frac{k}{x}}\left(1+\frac{k}{x}\right)'}{-\frac{1}{x^2}}$$

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