

## Precalculus

**Express**  $\sin(kx)$ ,  $\cos(kx)$  **via**  $\sin x$ ,  $\cos x$

Todor Milev

2019

## Example

Express  $\sin(3x)$  and  $\cos(3x)$  via  $\cos x$  and  $\sin x$ .

$$\sin(3x)$$

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$$\sin(3x) = \sin(x + 2x)$$

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Recall the formulas  $\sin(\alpha + \beta) = ?$   
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