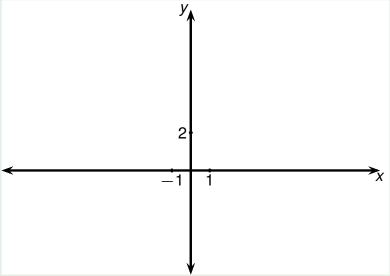
Calculus I
Intervals of increase and concavity, part 2

Todor Milev

2019

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



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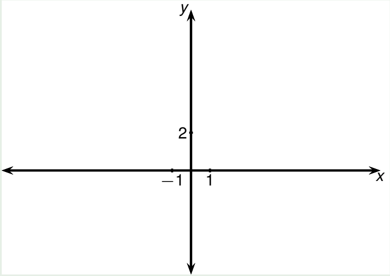

1. Domain

The domain of the function is
?

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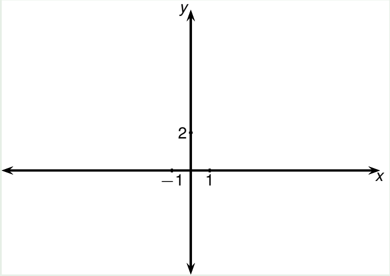

1. Domain

The domain of the function is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

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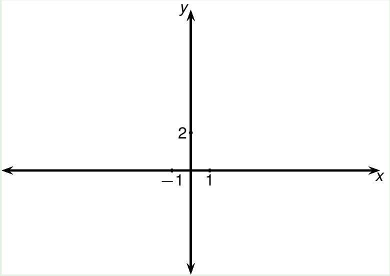

3. Intercepts

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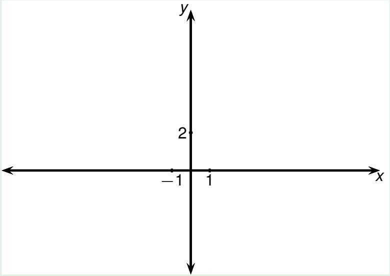

3. Intercepts
   - y-intercept: $f(0) = \mathbf{?}$.
   - x-intercept: $f(x) = 0$ when $x = \mathbf{?}$.

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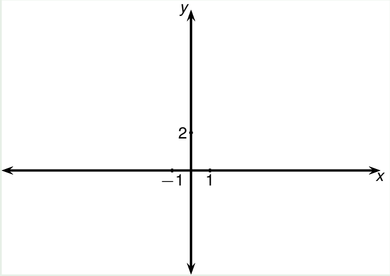

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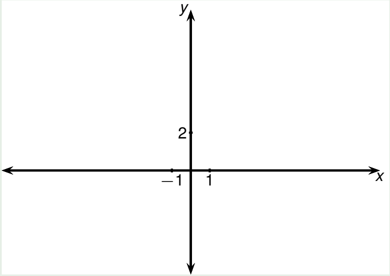

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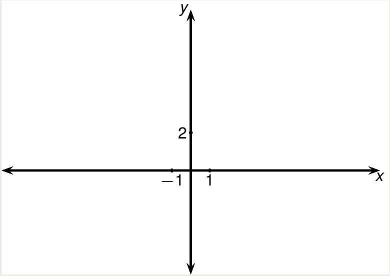

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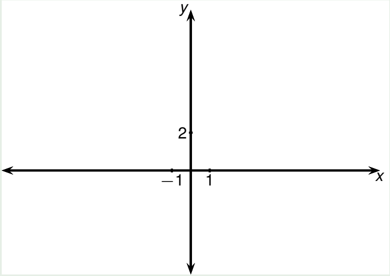

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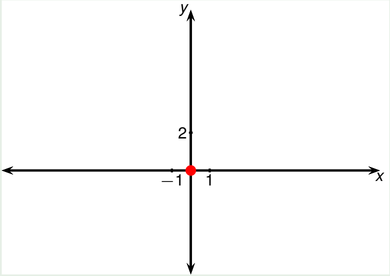

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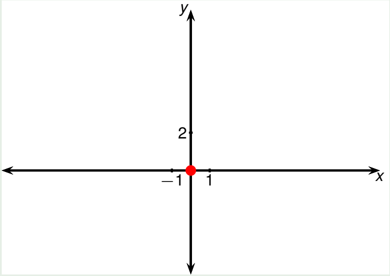
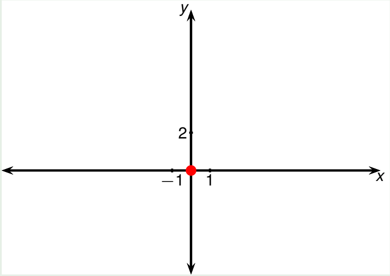


4. Symmetry

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



4. Symmetry

$$f(-x) = \frac{2(-x)^2}{(-x)^2 - 1}$$

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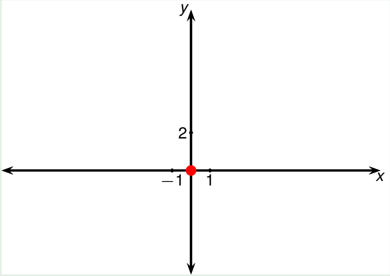

4. Symmetry

$$f(-x) = \frac{2(-x)^2}{(-x)^2 - 1} = \mathbf{?}$$

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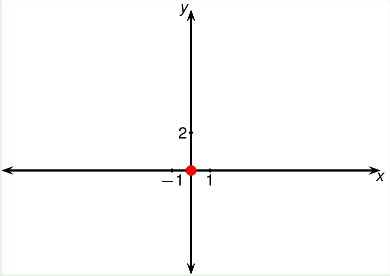

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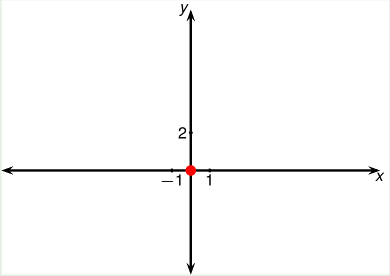

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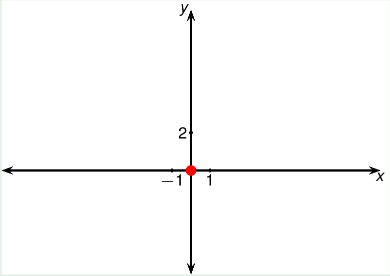

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Therefore f is **?** .

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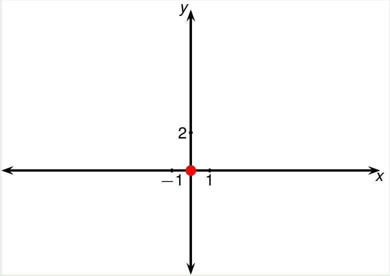

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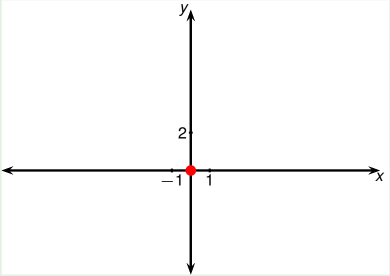
$$f(-x) = \frac{2(-x)^2}{(-x)^2 - 1} = \frac{2x^2}{x^2 - 1} = f(x)$$

Therefore f is even.

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



5. Asymptotes

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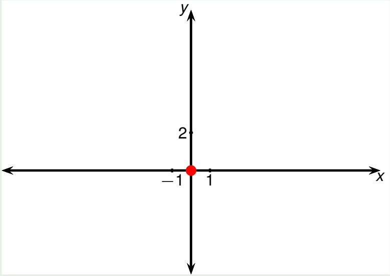

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$$\lim_{x\to\pm\infty} \frac{2x^2}{x^2-1}$$

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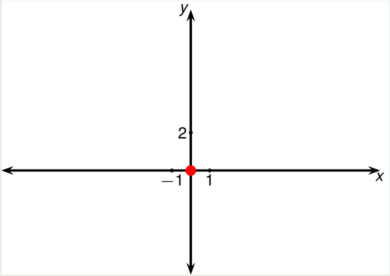

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$$\lim_{x\to\pm\infty} \frac{2x^2}{x^2-1} = \lim_{x\to\pm\infty} \frac{2}{1-1/x^2}$$

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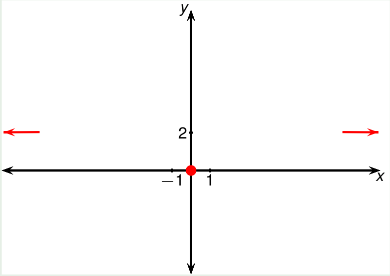

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$$\lim_{x\to\pm\infty} \frac{2x^2}{x^2-1} = \lim_{x\to\pm\infty} \frac{2}{1-1/x^2} = 2$$

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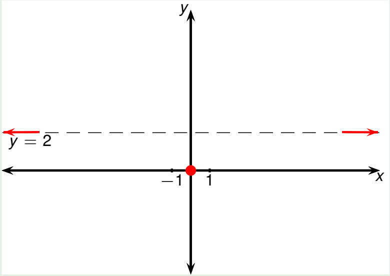

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$y = 2$ is a horizontal asymptote.

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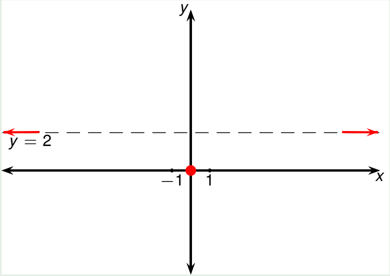

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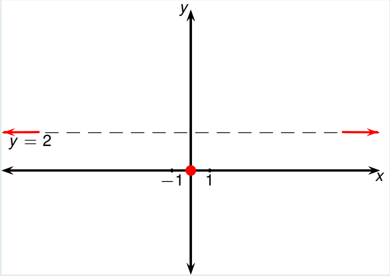

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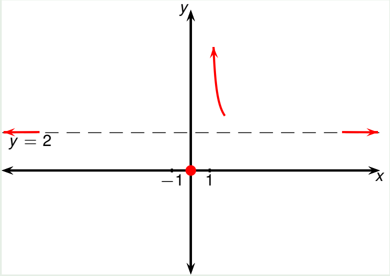

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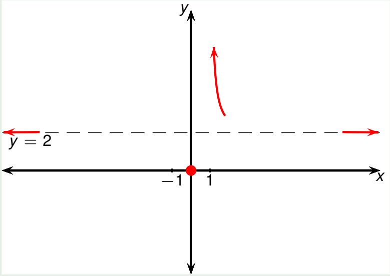

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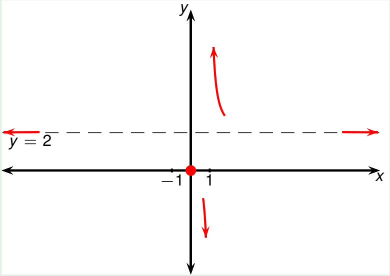

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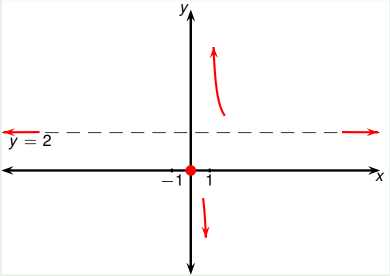

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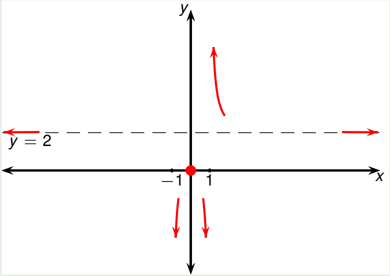

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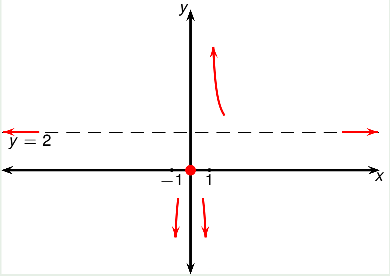

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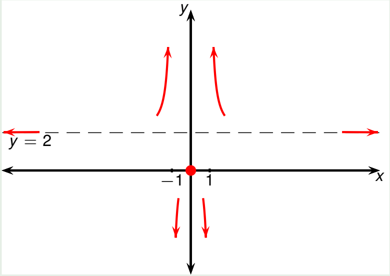

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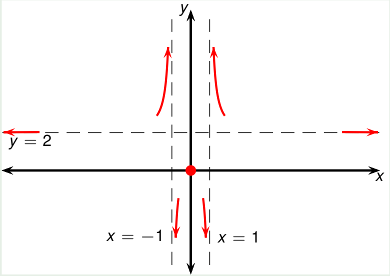

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$x = \pm 1$ are vertical asymptotes.

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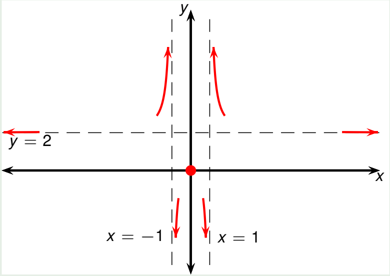
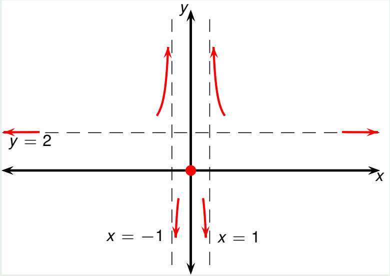


Interval	I/D	Concavity
$(-\infty, -1)$		
$(-1, 0)$		
$(0, 1)$		
$(1, \infty)$		

6. Intervals of increase or decrease

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



6. Intervals of increase or decrease

$$f'(x) \quad = \quad \textbf{?}$$

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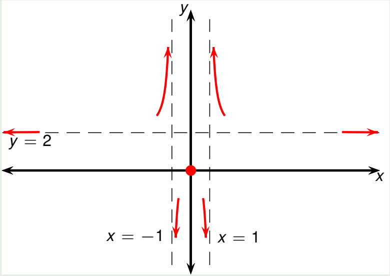

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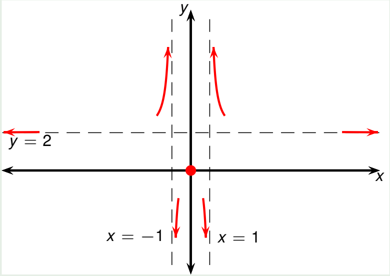

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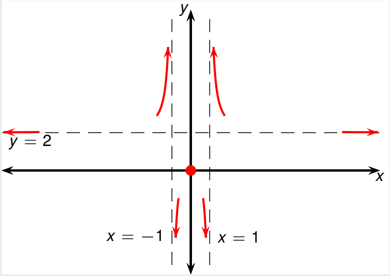
6. Intervals of increase or decrease

$$f'(x) = \frac{(x^2-1)(4x) - 2x^2(2x)}{(x^2-1)^2}$$

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



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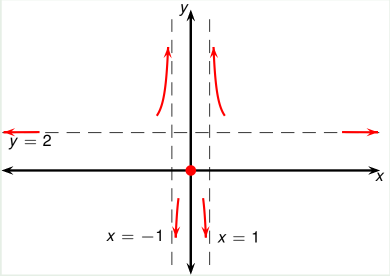
6. Intervals of increase or decrease

$$
\begin{aligned}
f'(x) &= \frac{(x^2-1)(4x) - 2x^2(2x)}{(x^2-1)^2} \\
&= \frac{-4x}{(x^2-1)^2}
\end{aligned}
$$

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



Interval	I/D	Concavity
$(-\infty,-1)$		
$(-1,0)$		
$(0,1)$		
$(1,\infty)$		

6. Intervals of increase or decrease

$$f'(x) = \frac{(x^2-1)(4x)-2x^2(2x)}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

	$-4x$	$(x^2-1)^2$	f'
$(-\infty,-1)$			
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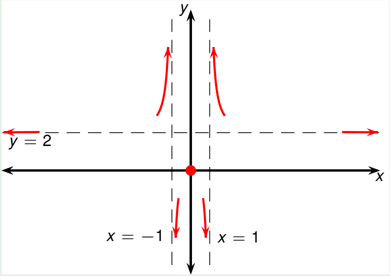

Interval	I/D	Concavity
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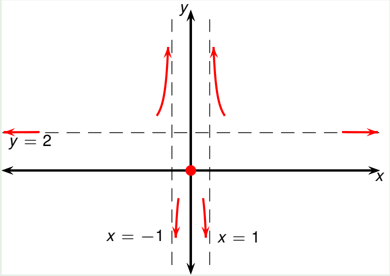
	$-4x$	$(x^2-1)^2$	f'
$(-\infty, -1)$	?		
$(-1, 0)$	?		
$(0, 1)$	?		
$(1, \infty)$	?		

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



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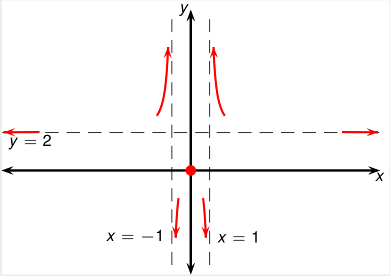
	$-4x$	$(x^2-1)^2$	f'
$(-\infty, -1)$	$+$		
$(-1, 0)$	$+$		
$(0, 1)$	$-$		
$(1, \infty)$	$-$		

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



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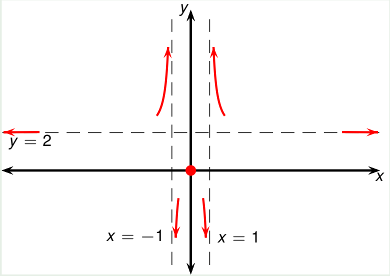
	$-4x$	$(x^2-1)^2$	f'
$(-\infty,-1)$	$+$	**?**	
$(-1,0)$	$+$	**?**	
$(0,1)$	$-$	**?**	
$(1,\infty)$	$-$	**?**	

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



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$(-\infty,-1)$	$+$	$+$	
$(-1,0)$	$+$	$+$	
$(0,1)$	$-$	$+$	
$(1,\infty)$	$-$	$+$	

Example

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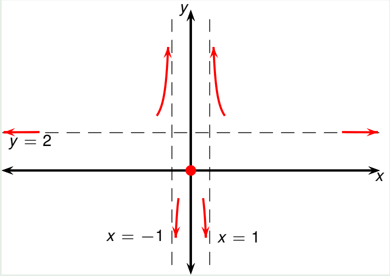

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$(-1, 0)$	$+$	$+$	$+$
$(0, 1)$	$-$	$+$	$-$
$(1, \infty)$	$-$	$+$	$-$

Example

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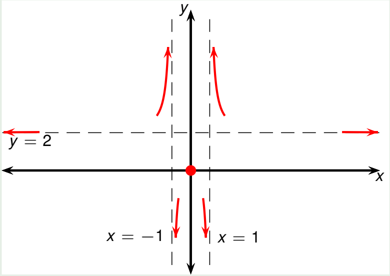

Interval	I/D	Concavity
$(-\infty, -1)$	I	
$(-1, 0)$	I	
$(0, 1)$	D	
$(1, \infty)$	D	

6. Intervals of increase or decrease

$$f'(x) = \frac{(x^2-1)(4x) - 2x^2(2x)}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

	$-4x$	$(x^2-1)^2$	f'
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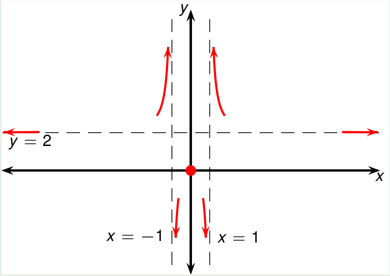
$(-1, 0)$	$+$	$+$	$+$
$(0, 1)$	$-$	$+$	$-$
$(1, \infty)$	$-$	$+$	$-$

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



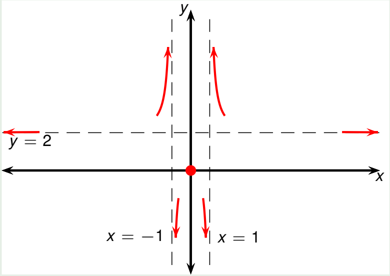
Interval	I/D	Concavity
$(-\infty, -1)$	I	
$(-1, 0)$	I	
$(0, 1)$	D	
$(1, \infty)$	D	

7. Local maxima and minima

	$-4x$	$(x^2-1)^2$	f'
$(-\infty, -1)$	$+$	$+$	$+$
$(-1, 0)$	$+$	$+$	$+$
$(0, 1)$	$-$	$+$	$-$
$(1, \infty)$	$-$	$+$	$-$

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



7. Local maxima and minima

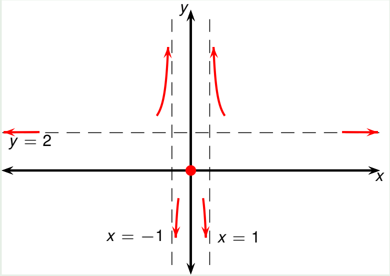
	$-4x$	$(x^2-1)^2$	f'
$(-\infty,-1)$	$+$	$+$	$+$
$(-1,0)$	$+$	$+$	$+$
$(0,1)$	$-$	$+$	$-$
$(1,\infty)$	$-$	$+$	$-$

- f' changes sign from $+$ to $-$ at 0.

Interval	I/D	Concavity
$(-\infty,-1)$	I	
$(-1,0)$	I	
$(0,1)$	D	
$(1,\infty)$	D	

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



Interval	I/D	Concavity
$(-\infty, -1)$	I	
$(-1, 0)$	I	
$(0, 1)$	D	
$(1, \infty)$	D	

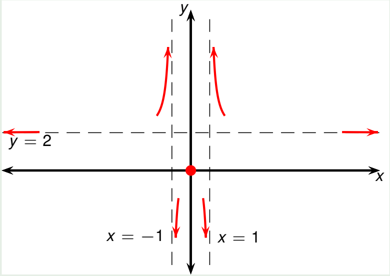
7. Local maxima and minima

	$-4x$	$(x^2-1)^2$	f'
$(-\infty, -1)$	$+$	$+$	$+$
$(-1, 0)$	$+$	$+$	$+$
$(0, 1)$	$-$	$+$	$-$
$(1, \infty)$	$-$	$+$	$-$

- f' changes sign from $+$ to $-$ at 0.
- Therefore $(0, 0)$ is a local maximum.

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.

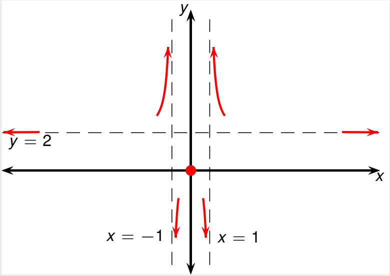


Interval	I/D	Concavity
$(-\infty, -1)$	I	
$(-1, 0)$	I	
$(0, 1)$	D	
$(1, \infty)$	D	

8. Concavity and points of inflection

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



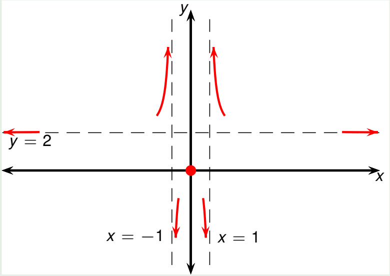
8. Concavity and points of inflection

$f''(x)$

Interval	I/D	Concavity
$(-\infty, -1)$	I	
$(-1, 0)$	I	
$(0, 1)$	D	
$(1, \infty)$	D	

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



Interval	I/D	Concavity
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$(1, \infty)$	D	

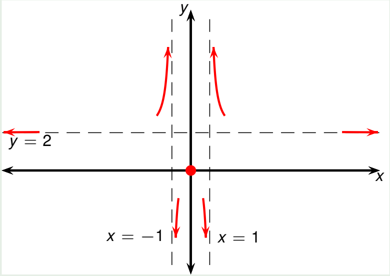
8. Concavity and points of inflection

$f''(x)$

$= \mathbf{?}$

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



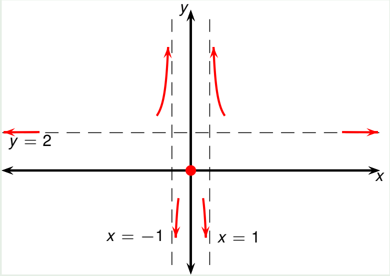
Interval	I/D	Concavity
$(-\infty, -1)$	I	
$(-1, 0)$	I	
$(0, 1)$	D	
$(1, \infty)$	D	

8. Concavity and points of inflection

$$f''(x) = \frac{-4(x^2-1)^2 + 4x \cdot 2(x^2-1)2x}{(x^2-1)^4}$$

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



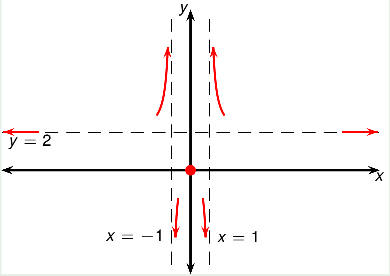
Interval	I/D	Concavity
$(-\infty, -1)$	I	
$(-1, 0)$	I	
$(0, 1)$	D	
$(1, \infty)$	D	

8. Concavity and points of inflection

$$f''(x)$$
$$= \frac{-4(x^2-1)^2 + 4x \cdot 2(x^2-1)2x}{(x^2-1)^4}$$
$$= \frac{12x^2+4}{(x^2-1)^3}$$

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



Interval	I/D	Concavity
$(-\infty,-1)$	I	
$(-1,0)$	I	
$(0,1)$	D	
$(1,\infty)$	D	

8. Concavity and points of inflection

$$f''(x) = \frac{-4(x^2-1)^2 + 4x\cdot 2(x^2-1)2x}{(x^2-1)^4} = \frac{12x^2+4}{(x^2-1)^3}$$

	$12x^2+4$	$(x^2-1)^3$	f''
$(-\infty,-1)$			
$(-1,1)$			
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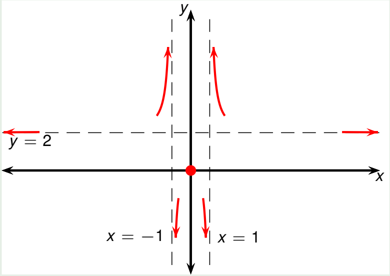
Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



Interval	I/D	Concavity
$(-\infty, -1)$	I	
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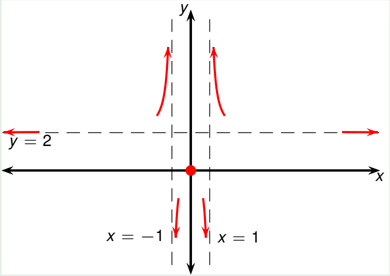
$$f''(x) = \frac{-4(x^2-1)^2 + 4x \cdot 2(x^2-1)2x}{(x^2-1)^4} = \frac{12x^2+4}{(x^2-1)^3}$$

	$12x^2+4$	$(x^2-1)^3$	f''
$(-\infty, -1)$	?	?	
$(-1, 1)$	?	?	
$(1, \infty)$	?	?	

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



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$$f''(x) = \frac{-4(x^2-1)^2 + 4x \cdot 2(x^2-1)2x}{(x^2-1)^4} = \frac{12x^2+4}{(x^2-1)^3}$$

	$12x^2+4$	$(x^2-1)^3$	f''
$(-\infty, -1)$	$+$	**?**	
$(-1, 1)$	$+$	**?**	
$(1, \infty)$	$+$	**?**	

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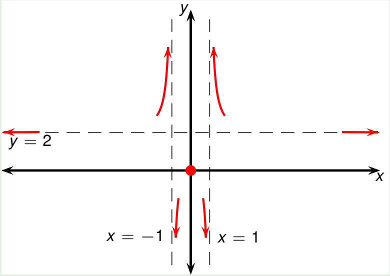

Interval	I/D	Concavity
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$(1, \infty)$	$+$	**?**	

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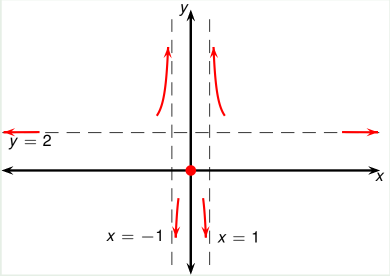

Interval	I/D	Concavity
$(-\infty, -1)$	I	
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$(1, \infty)$	D	

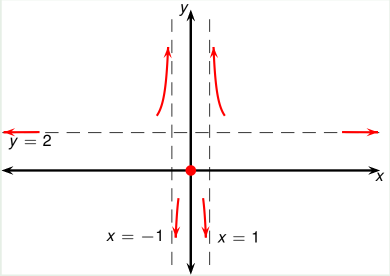
8. Concavity and points of inflection

$$f''(x) = \frac{-4(x^2-1)^2 + 4x \cdot 2(x^2-1)2x}{(x^2-1)^4} = \frac{12x^2+4}{(x^2-1)^3}$$

	$12x^2+4$	$(x^2-1)^3$	f''
$(-\infty, -1)$	$+$	$+$	
$(-1, 1)$	$+$	$-$	
$(1, \infty)$	$+$	$+$	

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



Interval	I/D	Concavity
$(-\infty,-1)$	I	
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$(-1,1)$	$+$	$-$	$-$
$(1,\infty)$	$+$	$+$	$+$

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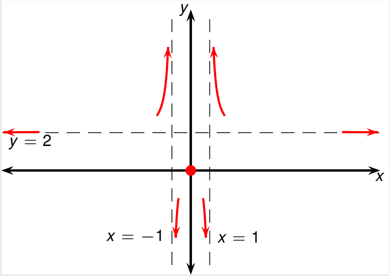


Interval	I/D	Concavity
$(-\infty, -1)$	I	up
$(-1, 0)$	I	down
$(0, 1)$	D	down
$(1, \infty)$	D	up

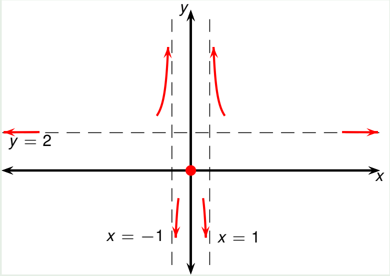
8. Concavity and points of inflection

$$f''(x) = \frac{-4(x^2-1)^2 + 4x \cdot 2(x^2-1)2x}{(x^2-1)^4} = \frac{12x^2+4}{(x^2-1)^3}$$

	$12x^2+4$	$(x^2-1)^3$	f''
$(-\infty, -1)$	$+$	$+$	$+$
$(-1, 1)$	$+$	$-$	$-$
$(1, \infty)$	$+$	$+$	$+$

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



Interval	I/D	Concavity
$(-\infty,-1)$	I	up
$(-1,0)$	I	down
$(0,1)$	D	down
$(1,\infty)$	D	up

8. Concavity and points of inflection

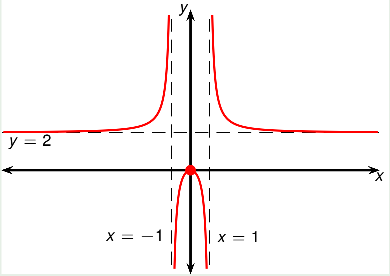
$$f''(x) = \frac{-4(x^2-1)^2 + 4x\cdot 2(x^2-1)2x}{(x^2-1)^4} = \frac{12x^2+4}{(x^2-1)^3}$$

	$12x^2+4$	$(x^2-1)^3$	f''
$(-\infty,-1)$	$+$	$+$	$+$
$(-1,1)$	$+$	$-$	$-$
$(1,\infty)$	$+$	$+$	$+$

No points of inflection because ± 1 are not in the domain of f.

Example

Sketch the curve $y = \frac{2x^2}{x^2-1}$.



Interval	I/D	Concavity
$(-\infty, -1)$	I	up
$(-1, 0)$	I	down
$(0, 1)$	D	down
$(1, \infty)$	D	up

8. Concavity and points of inflection

$$f''(x)$$

$$= \frac{-4(x^2-1)^2 + 4x \cdot 2(x^2-1)2x}{(x^2-1)^4}$$

$$= \frac{12x^2+4}{(x^2-1)^3}$$

	$12x^2+4$	$(x^2-1)^3$	f''
$(-\infty, -1)$	$+$	$+$	$+$
$(-1, 1)$	$+$	$-$	$-$
$(1, \infty)$	$+$	$+$	$+$

No points of inflection because ± 1 are not in the domain of f.