

Precalculus

**Find the area of a triangle from two sides and
an angle between them**

Todor Milev

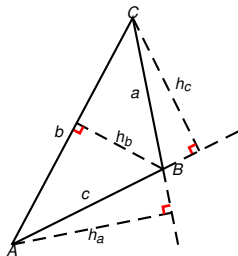
2019

Triangle area = $\frac{1}{2}$ base \cdot height

Let $\triangle ABC$ have side lengths a, b, c and height lengths h_a, h_b, h_c , as indicated - side a is opposite to vertex A and h_a starts at A , and so on.

Proposition (Triangle area)

$$\text{Area}(\triangle ABC) = \frac{1}{2} \text{height} \cdot \text{base} = \frac{1}{2} h_a a = \frac{1}{2} h_b b = \frac{1}{2} h_c c.$$



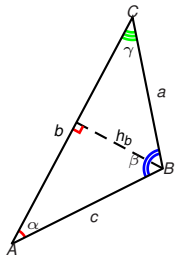
Triangle area from two sides and angle between them

Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ , as indicated: α is opposite to a , β is opposite to b , γ is opposite to c .

Proposition (\triangle area from two sides and angle between them)

The area of a triangle is half the product of the lengths of two of its sides times the sine of the angle between them. In other words,

$$\text{Area}(\triangle ABC) = \frac{ab \sin \gamma}{2} = \frac{bc \sin \alpha}{2} = \frac{ca \sin \beta}{2}$$



Proof.

$$\begin{aligned} \text{Area}(\triangle ABC) &= \frac{\text{base} \cdot \text{height}}{2} = \frac{bh_b}{2} \\ &= \frac{ba \sin \gamma}{2}. \end{aligned}$$

The proof of the other two cases is similar. □