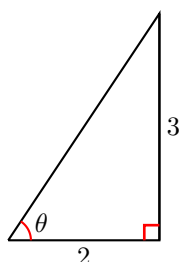


Precalculus

Homework

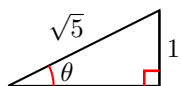
Definition of the trigonometric functions and basic computations

1. Find the 6 trigonometric functions of the indicated angle in the indicated right triangle.



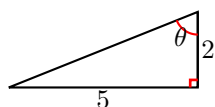
(a)

$$\frac{3}{\sqrt{13}} = \theta \csc \theta, \frac{2}{\sqrt{13}} = \theta \sec \theta = \frac{2}{3} \cot \theta = \frac{2}{3} \tan \theta = \frac{2}{3} \cos \theta = \frac{3}{2} \sin \theta = \frac{3}{\sqrt{13}}$$



(b)

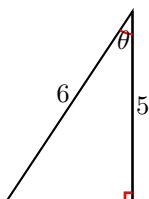
$$\frac{1}{\sqrt{5}} = \theta \csc \theta, \frac{2}{\sqrt{5}} = \theta \sec \theta = 2 \cot \theta = \frac{2}{1} \tan \theta = \frac{2}{\sqrt{5}} \cos \theta = \frac{1}{2} \sin \theta = \frac{1}{\sqrt{5}}$$



(c)

$$\frac{2}{\sqrt{29}} = \theta \csc \theta, \frac{5}{\sqrt{29}} = \theta \sec \theta = \frac{5}{2} \cot \theta = \frac{5}{2} \tan \theta = \frac{5\sqrt{29}}{2} \cos \theta = \frac{2\sqrt{29}}{5} \sin \theta = \frac{2\sqrt{29}}{5}$$

(d)



$$\frac{6}{11} = \theta \csc \theta, \frac{5}{11} = \theta \sec \theta = \frac{5}{6} \cot \theta = \frac{5}{6} \tan \theta = \frac{5\sqrt{11}}{6} \cos \theta = \frac{6\sqrt{11}}{5} \sin \theta = \frac{6\sqrt{11}}{5}$$

2. Find the exact value of the trigonometric function (using radicals).

(a) $\cos 135^\circ$.

(b) $\sin 225^\circ$.

(c) $\cos 495^\circ$.

(d) $\sin 560^\circ$.

(e) $\sin\left(\frac{3\pi}{2}\right)$.

(f) $\cos\left(\frac{11\pi}{6}\right)$.

(g) $\sin\left(\frac{2015\pi}{3}\right)$.

(h) $\cos\left(\frac{17\pi}{3}\right)$.

3. Find all solutions of the equation in the interval $[0, 2\pi)$. The answer key has not been proofread, use with caution.

(a) $\sin x = -\frac{\sqrt{2}}{2}$.

(b) $\cos x = \frac{\sqrt{3}}{2}$.

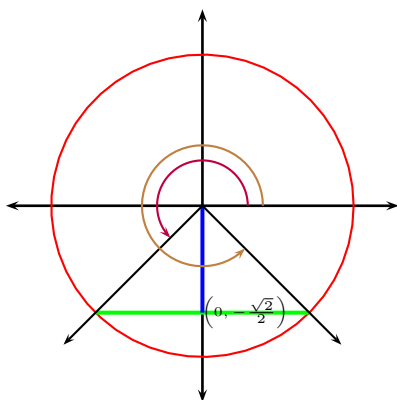
(c) $\sin(3x) = \frac{1}{2}$.

(d) $\cos(7x) = 0$.

(e) $\cos\left(3x + \frac{\pi}{2}\right) = 0$.

(f) $\sin\left(5x - \frac{\pi}{3}\right) = 0$.

Solution. 3.a



$$\sin x = -\frac{\sqrt{2}}{2}$$

Since $\sin x$ is negative it must be either in Quadrant III or IV. Therefore the angle x is coterminal either with $225^\circ = \frac{5\pi}{4}$ (Quadrant III) or $315^\circ = \frac{7\pi}{4}$ (Quadrant IV).

Case 1. x is coterminal with $225^\circ = \frac{5\pi}{4}$. We can compute

$$\begin{aligned} x &= \frac{5\pi}{4} + 2k\pi & \left| \begin{array}{l} k \text{ is any integer} \end{array} \right. \\ x &= \frac{5\pi}{4} + \frac{8k\pi}{4} \\ x &= \frac{5\pi + 8k\pi}{4} \\ x &= \frac{\pi(5 + 8k)}{4} \end{aligned}$$

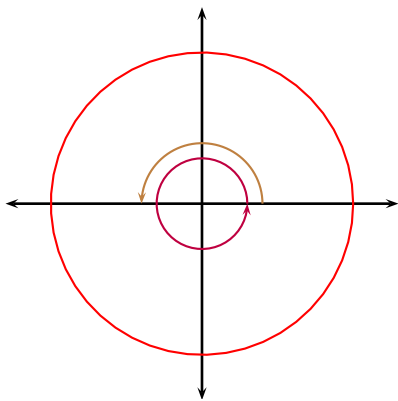
We are looking for solutions in the interval $[0, 2\pi)$ and so we must discard those values of the integer k for which $\frac{\pi(5+8k)}{4}$ is negative or is greater than or equal to 2π . Therefore the only solution in this case is $x = \frac{5\pi}{4}$.

Case 2.

$$\begin{aligned} x &= \frac{7\pi}{4} + 2k\pi \\ x &= \frac{7\pi}{4} + \frac{8k\pi}{4} \\ x &= \frac{7\pi + 8k\pi}{4} \\ x &= \frac{\pi(7 + 8k)}{4} \end{aligned}$$

We are looking for solutions in the interval $[0, 2\pi)$ and so we must discard those values of the integer k for which $\frac{\pi(7+8k)}{4}$ is negative or is greater than or equal to 2π . Therefore the only solution in this case is $x = \frac{7\pi}{4}$.

Solution. 3.f



$$\sin\left(5x - \frac{\pi}{3}\right) = 0$$

Since $\sin 0 = 0$ and $\sin 180^\circ = \sin \pi = 0$, the angle $5x - \frac{\pi}{3}$ must be coterminal with 0 or π .

Case 1. $5x - \frac{\pi}{3}$ is coterminal with 0. We compute

$$\begin{aligned}
5x - \frac{\pi}{3} &= 0 + 2k\pi \\
5x &= \frac{\pi}{3} + 2k\pi \\
x &= \frac{\frac{\pi}{3} + 2k\pi}{5} \\
x &= \frac{\frac{\pi}{3} + \frac{6k\pi}{3}}{5} \\
x &= \frac{\frac{\pi + 6k\pi}{3}}{5} \\
x &= \frac{\pi + 6k\pi}{15} \\
x &= \frac{\pi(1 + 6k)}{15} \\
x &= \cancel{\frac{\pi}{15}}, \frac{\pi[1 + 6(0)]}{15}, \frac{\pi[1 + 6(1)]}{15}, \frac{\pi[1 + 6(2)]}{15}, \frac{\pi(1 + 12)}{15}, \frac{\pi[1 + 6(3)]}{15}, \frac{\pi[1 + 6(4)]}{15}, \cancel{\frac{\pi}{15}}. \\
x &= \frac{\pi}{15}, \frac{7\pi}{15}, \frac{13\pi}{15}, \frac{19\pi}{15}, \frac{25\pi}{15}.
\end{aligned}$$

Discard other values of k as they yield angles outside of $[0, 2\pi)$

Case 2.

$$\begin{aligned}
5x - \frac{\pi}{3} &= \pi + 2k\pi \\
5x &= \pi + \frac{\pi}{3} + 2k\pi \\
5x &= \frac{4\pi}{3} + 2k\pi \\
x &= \frac{\frac{4\pi}{3} + 2k\pi}{5} \\
x &= \frac{\frac{4\pi}{3} + \frac{6k\pi}{3}}{5} \\
x &= \frac{\frac{4\pi + 6k\pi}{3}}{5} \\
x &= \frac{4\pi + 6k\pi}{15} \\
x &= \frac{2\pi(2 + 3k)}{15} \\
x &= \cancel{\frac{2\pi}{15}}, \frac{2\pi[2 + 3(0)]}{15}, \frac{2\pi[2 + 3(1)]}{15}, \frac{2\pi[2 + 3(2)]}{15}, \frac{2\pi[2 + 3(3)]}{15}, \frac{2\pi[2 + 3(4)]}{15}, \cancel{\frac{2\pi}{15}}. \\
x &= \frac{4\pi}{15}, \frac{10\pi}{15}, \frac{16\pi}{15}, \frac{22\pi}{15}, \frac{28\pi}{15}.
\end{aligned}$$

Discard other values of k as they yield angles outside of $[0, 2\pi)$

Our final answer (combined from the two cases) is $x = \frac{\pi}{15}, \frac{4\pi}{15}, \frac{7\pi}{15}, \frac{2\pi}{3}, \frac{13\pi}{15}, \frac{16\pi}{15}, \frac{19\pi}{15}, \frac{22\pi}{15}, \frac{5\pi}{3}$ or $\frac{28\pi}{15}$.