Calculus II

Integrals of the form $\int \frac{ax}{(bx^2+c)^n}\mathrm{d}x$

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Building blocks IIa and IIb

We solve building block IIb. For completeness, we solve block IIa again as well.

Example

$$\int \frac{x}{(x^2+1)^n}\mathrm{d}x$$

Building blocks IIa and IIb

We solve building block IIb. For completeness, we solve block IIa again as well.

Example

$$\int \frac{x}{(x^2+1)^n}\mathrm{d}x = \int \frac{1}{(x^2+1)^n}\frac{\mathrm{d}\left(x^2+1\right)}{2}$$

Building blocks IIa and IIb

We solve building block IIb. For completeness, we solve block IIa again as well.

Example

$$\begin{aligned}\int \frac{x}{(x^2+1)^n}\mathrm{d}x &= \int \frac{1}{(x^2+1)^n}\frac{\mathrm{d}\left(x^2+1\right)}{2} \\ &= \frac{1}{2}\int u^{-n}\mathrm{d}u\end{aligned}$$

where we used the substitution $u = x^2 + 1$.

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We solve building block IIb. For completeness, we solve block IIa again as well.

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$$\begin{aligned}\int \frac{x}{(x^2+1)^n}\mathrm{d}x &= \int \frac{1}{(x^2+1)^n}\frac{\mathrm{d}\left(x^2+1\right)}{2} \\ &= \frac{1}{2}\int u^{-n}\mathrm{d}u \\ &= \begin{cases} \textbf{?} & \text{if } n=1 \\ & \text{if } n\neq 1 \end{cases},\end{aligned}$$

where we used the substitution $u = x^2 + 1$.

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Example

$$\begin{aligned}
\int \frac{x}{(x^2+1)^n}\mathrm{d}x &= \int \frac{1}{(x^2+1)^n}\frac{\mathrm{d}\left(x^2+1\right)}{2} \\
&= \frac{1}{2}\int u^{-n}\mathrm{d}u \\
&= \begin{cases} \frac{1}{2}\ln(x^2+1)+C & \text{if } n=1 \\ \frac{1}{2}\frac{(x^2+1)^{-n+1}}{(-n+1)}+C & \text{if } n\neq 1 \end{cases},
\end{aligned}$$

where we used the substitution $u = x^2+1$.