

## Precalculus

# Trigonometric functions computable with algebraic numbers using special angles

Todor Milev

2019

## Example

Find the exact value of the trigonometric function using radicals.

$$\cos(105^\circ)$$

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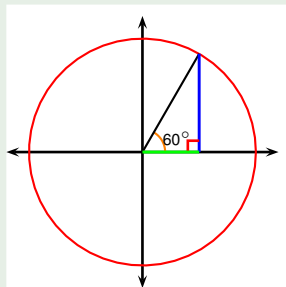
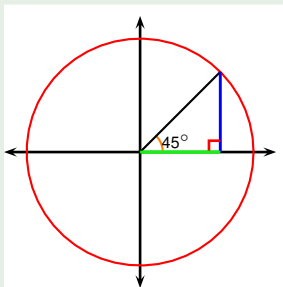
$$\cos(105^\circ) = \cos(45^\circ + 60^\circ)$$

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$$\cos(105^\circ) = \cos(45^\circ + 60^\circ)$$

we know the trig  
f-ns of  $45^\circ$  and  $60^\circ$

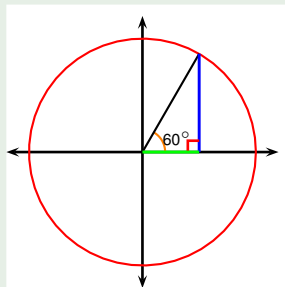
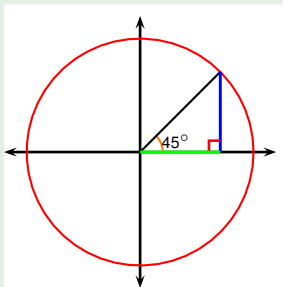


## Example

Find the exact value of the trigonometric function using radicals.

$$\cos(105^\circ) = \cos(45^\circ + 60^\circ) \\ = ?$$

we know the trig  
f-ns of  $45^\circ$  and  $60^\circ$   
Angle sum f-la



## Example

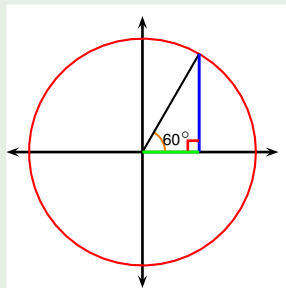
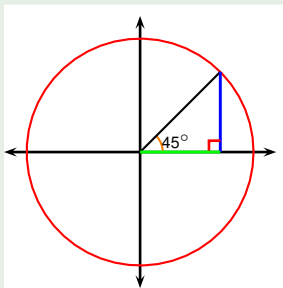
Find the exact value of the trigonometric function using radicals.

$$\cos(105^\circ) = \cos(45^\circ + 60^\circ)$$

$$= \cos(45^\circ) \cos(60^\circ) - \sin(45^\circ) \sin(60^\circ)$$

we know the trig  
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Angle sum f-la



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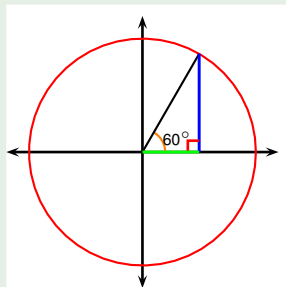
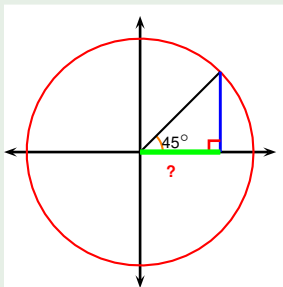
$$\cos(105^\circ) = \cos(45^\circ + 60^\circ)$$

$$= \cos(45^\circ) \cos(60^\circ) - \sin(45^\circ) \sin(60^\circ)$$

$$= ? \cdot ? - ? \cdot ?$$

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Angle sum f-la



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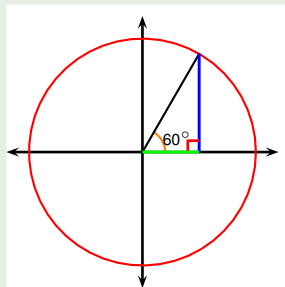
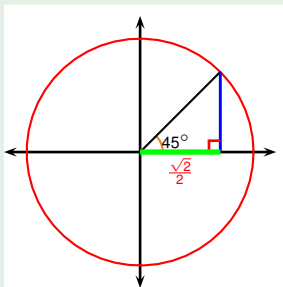
$$\cos(105^\circ) = \cos(45^\circ + 60^\circ)$$

$$= \cos(45^\circ) \cos(60^\circ) - \sin(45^\circ) \sin(60^\circ)$$

$$= \frac{\sqrt{2}}{2} \cdot ? - ? \cdot ?$$

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Angle sum f-la





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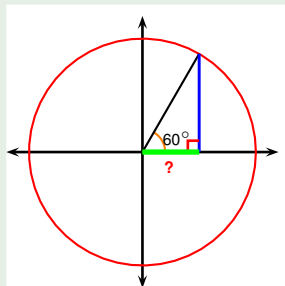
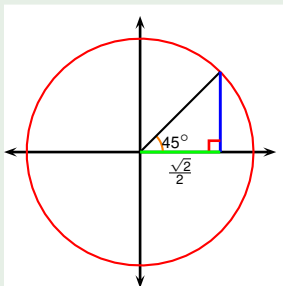
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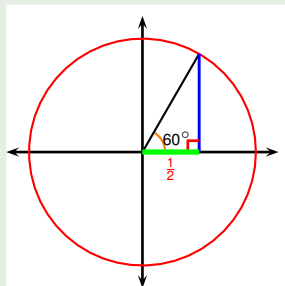
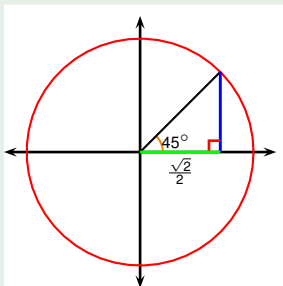
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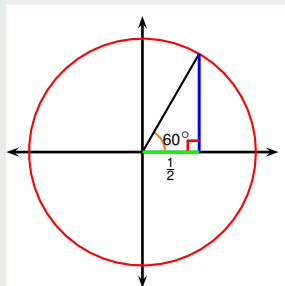
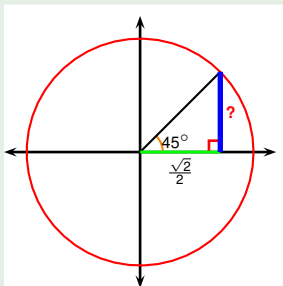
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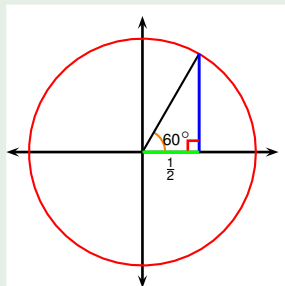
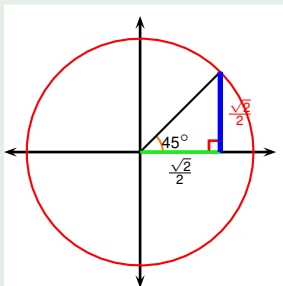
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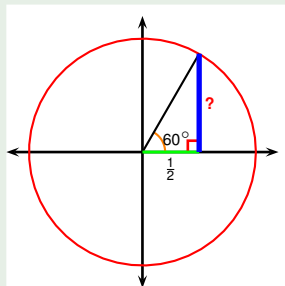
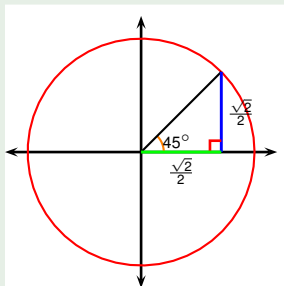
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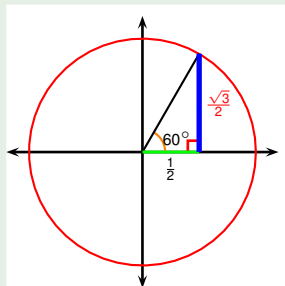
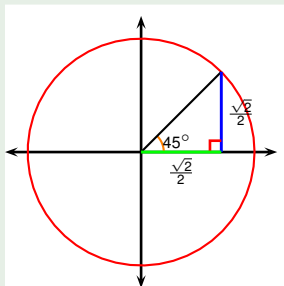
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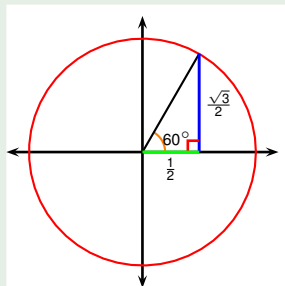
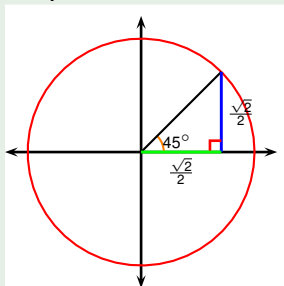
$$= \cos(45^\circ) \cos(60^\circ) - \sin(45^\circ) \sin(60^\circ)$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

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