

Precalculus

**Find the area of a triangle from two sides and
an angle between them**

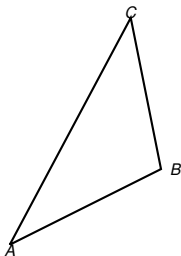
Todor Milev

2019

Triangle area = $\frac{1}{2}$ base · height

Proposition (Triangle area)

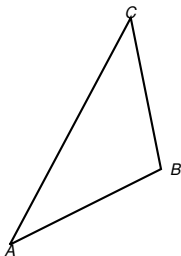
$$\text{Area}(\triangle ABC) = ?$$



Triangle area = $\frac{1}{2}$ base \cdot height

Proposition (Triangle area)

$$\text{Area}(\triangle ABC) = \frac{1}{2} \text{height} \cdot \text{base}$$

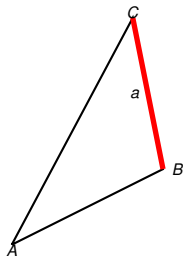


Triangle area = $\frac{1}{2}$ base \cdot height

Let $\triangle ABC$ have **side length a** and height length h_a , as indicated - **side a is opposite to vertex A** and h_a starts at A

Proposition (Triangle area)

$$\text{Area}(\triangle ABC) = \frac{1}{2} \text{height} \cdot \text{base} = \frac{1}{2} h_a a$$

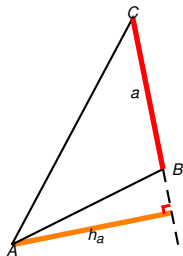


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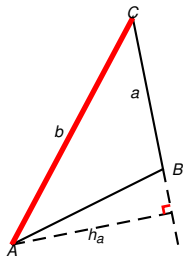


Triangle area = $\frac{1}{2}$ base \cdot height

Let $\triangle ABC$ have **side lengths** a, b and height lengths h_a, h_b , as indicated - side a is opposite to vertex A and h_a starts at A , and so on.

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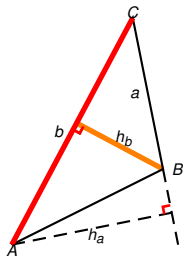


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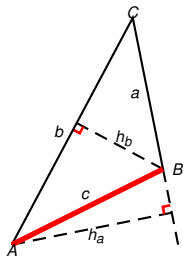


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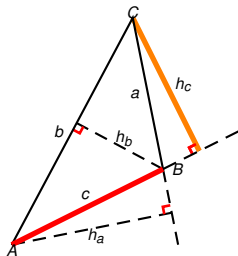


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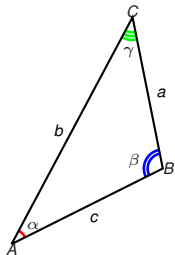
Triangle area from two sides and angle between them

Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ , as indicated: α is opposite to a , β is opposite to b , γ is opposite to c .

Proposition (\triangle area from two sides and angle between them)

The area of a triangle is half the product of the lengths of two of its sides times the sine of the angle between them. In other words,

$$\text{Area}(\triangle ABC) = \frac{ab \sin \gamma}{2} = \frac{bc \sin \alpha}{2} = \frac{ca \sin \beta}{2}$$



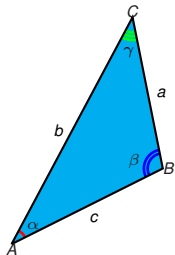
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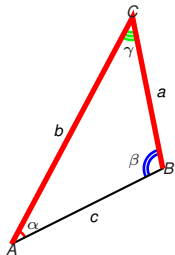
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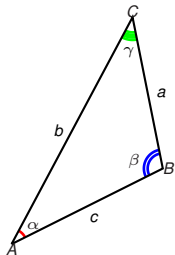
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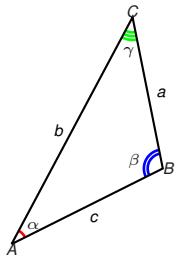
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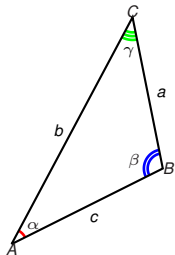
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Proof.

$$\text{Area}(\triangle ABC) = \frac{\text{base} \cdot \text{height}}{2}$$



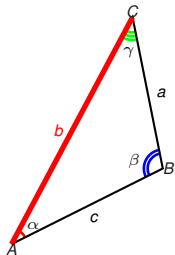
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Proof.

$$\text{Area}(\triangle ABC) = \frac{\text{base} \cdot \text{height}}{2} = \frac{bh_b}{2}$$



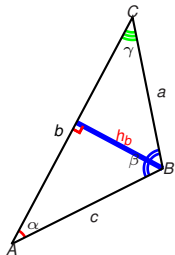
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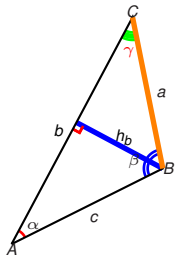
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Proof.

$$\begin{aligned} \text{Area}(\triangle ABC) &= \frac{\text{base} \cdot \text{height}}{2} = \frac{b h_b}{2} \\ &= \frac{b a \sin \gamma}{2}. \end{aligned}$$



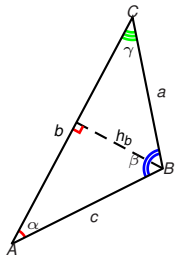
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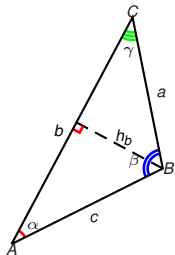
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The proof of the other two cases is similar. □