Calculus II
Homework
Integrals of involving radicals of quadratics

1. Find a linear substitution (via completing the square) to transform the radical to a multiple of an expression of the form $\sqrt{u^2+1}$, $\sqrt{u^2-1}$ or $\sqrt{1-u^2}$.

   (a) $\sqrt{x^2+x+1}$.

   (b) $\sqrt{-2x^2+x+1}$.

2. Compute the integral.

   (a) $\displaystyle\int \frac{\sqrt{1+x^2}}{x^2}\mathrm{d}x$.

3. Compute the integral using a trigonometric substitution.

   (a) $\displaystyle\int \frac{\sqrt{9-x^2}}{x^2}\mathrm{d}x$.

4. Compute the integral.

   (a) $\displaystyle\int \sqrt{x^2+1}\mathrm{d}x$

   (b) $\displaystyle\int \sqrt{x^2+2}\mathrm{d}x$

   (c) $\displaystyle\int \sqrt{x^2+x+1}\mathrm{d}x$

   (d) $\displaystyle\int \sqrt{(2x^2+2x+1)}\mathrm{d}x$

   (e) $\displaystyle\int \sqrt{(3x^2+2x+1)}\mathrm{d}x$

   (f) $\displaystyle\int \frac{\sqrt{x^2+1}}{x+1}\mathrm{d}x$

5. Let $b^2-4ac<0$ and $a>0$ be (real) numbers. Show that

   $$\int \sqrt{(ax^2+bx+c)}\mathrm{d}x = \frac{\sqrt{a}D}{2}\left(\ln\left(\sqrt{\left(\frac{2xa+b}{2\sqrt{D}a}\right)^2+1}+\frac{2xa+b}{2\sqrt{D}a}\right)+\frac{2xa+b}{2\sqrt{D}a}\sqrt{\left(\frac{2xa+b}{2\sqrt{D}a}\right)^2+1}\right)+C,$$

   where $D=\dfrac{4ac-b^2}{4a^2}$.

6. Integrate

   (a) $\displaystyle\int \sqrt{1-x^2}\mathrm{d}x$

   (b) $\displaystyle\int \sqrt{2-x^2}\mathrm{d}x$

   (c) $\displaystyle\int \sqrt{-x^2+x+1}\mathrm{d}x$

   (d) $\displaystyle\int \sqrt{2-x-x^2}\mathrm{d}x$

(e) $\int \frac{\sqrt{1-x^2}}{1+x}\mathrm{d}x$

(f) $\int \frac{\sqrt{1-x^2}}{2+x}\mathrm{d}x$

7. Integrate

   (a) $\int \sqrt{x^2-1}\mathrm{d}x$

   (b) $\int \sqrt{x^2-2}\mathrm{d}x$

   (c) $\int \sqrt{2x^2+x-1}\mathrm{d}x$

   (d) $\int \sqrt{x^2+x-1}\mathrm{d}x$

8. (a) Express $x, \mathrm{d}x$ and $\sqrt{x^2+1}$ via θ and $\mathrm{d}\theta$ for the trigonometric substitution $x = \cot\theta$, $\theta \in (0, \pi)$.

   (b) Express $x, \mathrm{d}x$ and $\sqrt{x^2+1}$ via t and $\mathrm{d}t$ for the Euler substitution $x = \cot(2\arctan t)$, $t > 0$. Express t via x.

9. Let the variables x and t be related via $\sqrt{x^2+1} = x + t$.

   (a) Express x via t.

   (b) Express $\sqrt{x^2+1}$ via t alone.

   (c) Express $\mathrm{d}x$ via t and $\mathrm{d}t$.

10. (a) Express $x, \mathrm{d}x$ and $\sqrt{x^2+1}$ via θ and $\mathrm{d}\theta$ for the trigonometric substitution $x = \tan\theta$, $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

    (b) Express $x, \mathrm{d}x$ and $\sqrt{x^2+1}$ via t and $\mathrm{d}t$ for the Euler substitution $x = \tan(2\arctan t)$, $t \in (-1, 1)$. Express t via x.

11. Let the variables x and t be related via $\sqrt{x^2+1} = \frac{x}{t} - 1$.

    (a) Express x via t.

    (b) Express $\sqrt{x^2+1}$ via t alone.

    (c) Express $\mathrm{d}x$ via t and $\mathrm{d}t$.

12. (a) Express $x, \mathrm{d}x$ and $\sqrt{1-x^2}$ via θ and $\mathrm{d}\theta$ for the trigonometric substitution $x = \cos\theta$, $\theta \in [0, \pi]$.

    (b) Express $x, \mathrm{d}x$ and $\sqrt{1-x^2}$ via t and $\mathrm{d}t$ for the Euler substitution $x = \cos(2\arctan t)$, $t \geq 0$. Express t via x.

13. Let the variables x and t be related via $\sqrt{-x^2+1} = (1-x)t$.

    (a) Express x via t.

    (b) Express $\sqrt{-x^2+1}$ via t alone.

    (c) Express $\mathrm{d}x$ via t and $\mathrm{d}t$.

14. (a) Express $x, \mathrm{d}x$ and $\sqrt{1-x^2}$ via θ and $\mathrm{d}\theta$ for the trigonometric substitution $x = \sin\theta$, $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

    (b) Express $x, \mathrm{d}x$ and $\sqrt{1-x^2}$ via t and $\mathrm{d}t$ for the Euler substitution $x = \sin(2\arctan t)$, $t \in [-1, 1]$. Express t via x.

15. Let the variables x and t be related via $\sqrt{-x^2+1} = 1 - xt$.

    (a) Express x via t.

    (b) Express $\sqrt{-x^2+1}$ via t alone.

    (c) Express $\mathrm{d}x$ via t and $\mathrm{d}t$.

16. (a) Express $x, \mathrm{d}x$ and $\sqrt{x^2-1}$ via θ and $\mathrm{d}\theta$ for the trigonometric substitution $x = \csc\theta$, $\theta \in \left[0, \frac{\pi}{2}\right] \cup \left[\pi, \frac{3\pi}{2}\right)$.

    (b) Express $x, \mathrm{d}x$ and $\sqrt{1-x^2}$ via t and $\mathrm{d}t$ for the Euler substitution $x = \sec(2\arctan t)$, $t \in (-\infty, -1) \cup [1, 0)$. Express t via x.

17. Let the variables x and t be related via $\sqrt{x^2-1} = (x+1)t$.

    (a) Express x via t.

    (b) Express $\sqrt{x^2-1}$ via t alone.

(c) Express $\mathrm{d}x$ via t and $\mathrm{d}t$.

18. (a) Express $x, \mathrm{d}x$ and $\sqrt{1-x^2}$ via θ and $\mathrm{d}\theta$ for the trigonometric substitution $x = \csc\theta$, $\theta \in \left[0, \frac{\pi}{2}\right] \cup \left[\pi, \frac{3\pi}{2}\right)$.

    (b) Express $x, \mathrm{d}x$ and $\sqrt{1-x^2}$ via t and $\mathrm{d}t$ for the Euler substitution $x = \csc(2\arctan t)$, $t \in (-\infty, -1) \cup [0, 1)$. Express t via x.

19. Let the variables x and t be related via $\sqrt{x^2-1} = \frac{1}{t} - x$.

    (a) Express x via t.

    (b) Express $\sqrt{x^2-1}$ via t alone.

    (c) Express $\mathrm{d}x$ via t and $\mathrm{d}t$.