

Calculus II

Integrals of the form $\int \sin^n x \cos^m x dx$, both
powers even

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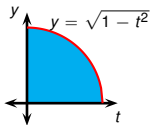
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Example

$$\begin{aligned}
 \int_0^{\frac{\pi}{2}} \sin^2 x \, dx &= \int_0^{\frac{\pi}{2}} \left(\frac{1 - \cos(2x)}{2} \right) dx && \left| \begin{array}{l} \text{express } \sin^2 x \\ \text{via } \cos(2x) \end{array} \right. \\
 &= \left[\frac{x}{2} - \frac{\sin(2x)}{4} \right]_0^{\frac{\pi}{2}} \\
 &= \left(\frac{\pi}{4} - \frac{\sin \pi}{4} \right) - \left(0 - \frac{\sin 0}{4} \right) = \frac{\pi}{4}.
 \end{aligned}$$

Example

Set $t = \cos x$, $x \in [0, \frac{\pi}{2}] \Rightarrow \sin x \geq 0$. Then
 $dt = d(\cos x) = -\sin x \, dx$.



$$\begin{aligned}
 \int_{t=0}^{t=1} \sqrt{1-t^2} \, dt &= - \int_{x=\frac{\pi}{2}}^{x=0} \sqrt{1-\cos^2 x} \sin x \, dx \\
 &= \int_{x=\frac{\pi}{2}}^{x=0} \sqrt{\sin^2 x} \sin x \, dx \\
 &= \int_0^{\frac{\pi}{2}} \sin^2 x \, dx = \frac{\pi}{4}.
 \end{aligned}$$