Calculus II
Summary of building block integrals

Todor Milev

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Building block integral summary

Type	a	b	Type a, lin. sub.	Type b, lin. sub
I	$\int \frac{1}{x}\mathrm{d}x$	$\int \frac{1}{x^n}\mathrm{d}x$	$\int \frac{A}{ax+b}\mathrm{d}x$	$\int \frac{A}{(ax+b)^n}\mathrm{d}x$
II	$\int \frac{x}{x^2+1}\mathrm{d}x$	$\int \frac{x}{\left(x^2+1\right)^n}\mathrm{d}x$	$\int \frac{A\left(x+\frac{b}{2a}\right)}{ax^2+bx+c}\mathrm{d}x$	$\int \frac{A\left(x+\frac{b}{2a}\right)}{\left(ax^2+bx+c\right)^n}\mathrm{d}x$
III	$\int \frac{1}{x^2+1}\mathrm{d}x$	$\int \frac{1}{\left(x^2+1\right)^n}\mathrm{d}x$	$\int \frac{B}{ax^2+bx+c}\mathrm{d}x$	$\int \frac{B}{\left(ax^2+bx+c\right)^n}\mathrm{d}x$

where A, B are arbitrary constants and a, b, c are constants with $b^2 - 4ac < 0$. The quadratics in the denominators have no real roots.

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  - Block IIb, IIIb, linear substitutions: done by means of completing the square; computations are analogous and we leave them for exercise.