Calculus I
Derivatives of arbitrary radicals, part 2

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2019

$$\frac{\mathrm{d}}{\mathrm{d}x}[h(x)]^n = n[h(x)]^{n-1} \cdot h'(x)$$

$$\begin{array}{rcll} (g(h(x)))' & = & g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\ (g(u))' & = & g'(u)u' & \text{where } u = h(x) & \text{(notation 2)} \\ \frac{\mathrm{d}y}{\mathrm{d}x} & = & \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)} \end{array} .$$

Example (Chain Rule, Notation 1, Power Rule)

Differentiate $f(x) = \frac{1}{\sqrt[3]{x^2+x+1}}$.

$$\frac{d}{dx}[h(x)]^n = n[h(x)]^{n-1} \cdot h'(x)$$

$$\begin{array}{rcll} (g(h(x)))' & = & g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\ (g(u))' & = & g'(u)u' & \text{where } u = h(x) & \text{(notation 2)} \\ \frac{dy}{dx} & = & \frac{dy}{du}\frac{du}{dx} & \text{where } y = g(u) & \text{(notation 3)} \end{array} .$$

Example (Chain Rule, Notation 1, Power Rule)

$$\begin{array}{rrcl} \text{Differentiate} & f(x) & = & \frac{1}{\sqrt[3]{x^2+x+1}}. \\ \text{Let} & h(x) & = & \textbf{?} \\ \text{Let} & g(u) & = & \textbf{?} \\ \text{Then} & f(x) & = & g(h(x)). \end{array}$$

$$\frac{d}{dx}[h(x)]^n = n[h(x)]^{n-1} \cdot h'(x)$$

$$\begin{array}{rcll} (g(h(x)))' &=& g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\ (g(u))' &=& g'(u)u' & \text{where } u = h(x) & \text{(notation 2)} \\ \dfrac{dy}{dx} &=& \dfrac{dy}{du}\dfrac{du}{dx} & \text{where } y = g(u) & \text{(notation 3)} \end{array} .$$

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$$\begin{array}{rrcl} \text{Differentiate} & f(x) &=& \frac{1}{\sqrt[3]{x^2+x+1}}. \\ \text{Let} & h(x) &=& x^2+x+1. \\ \text{Let} & g(u) &=& \textbf{?} \\ \text{Then} & f(x) &=& g(h(x)). \end{array}$$

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$$\frac{d}{dx}[h(x)]^n = n[h(x)]^{n-1} \cdot h'(x)$$

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(g(h(x)))' &= g'(h(x)) \cdot h'(x) && && \text{(notation 1)}\\
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\text{Then} \quad f(x) &= g(h(x)).\\
\text{Chain Rule:} \quad f'(x) &= g'(h(x))h'(x)
\end{aligned}$$

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$$\begin{array}{rrcl} \text{Differentiate} & f(x) & = & \frac{1}{\sqrt[3]{x^2+x+1}}. \\ \text{Let} & h(x) & = & x^2 + x + 1. \\ \text{Let} & g(u) & = & u^{-\frac{1}{3}}. \\ \text{Then} & f(x) & = & g(h(x)). \\ \text{Chain Rule:} & f'(x) & = & g'(h(x))h'(x) \\ & & = & \left(\textbf{?} \qquad \right) (\textbf{?} \qquad) \end{array}$$

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$$\frac{d}{dx}[h(x)]^n = n[h(x)]^{n-1} \cdot h'(x)$$

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& &=& \left(-\dfrac{1}{3}(h(x))^{-\frac{4}{3}}\right)(2x+1) \\
& &=& -\dfrac{2x+1}{3}(x^2+x+1)^{-\frac{4}{3}}.
\end{array}$$