

Calculus I

Inverse of fractional linear transformation

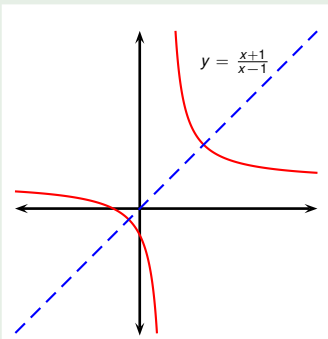
Todor Milev

2019

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.

We deal with domains and ranges later:



Answer: $f^{-1}(x) = \frac{x+1}{x-1}$,
 $x \neq 1$.

$$\begin{array}{rcll}
 y & = & \frac{x+1}{x-1} & \left| \begin{array}{l} \text{mult. by } (x-1) \\ \text{div. by } (y-1) \\ \text{relabel } x, y \end{array} \right. \\
 y(x-1) & = & x+1 & \\
 x(y-1) & = & y+1 & \\
 f^{-1}(y) = x & = & \frac{y+1}{y-1} & \\
 f^{-1}(x) & = & \frac{x+1}{x-1} &
 \end{array}$$

We divided by $y-1$ so $y \neq 1$. Therefore the domain of f^{-1} is all real numbers except 1.

Can a non-identity function be its own inverse? Yes, f is.

What does it mean for f to be its own inverse?
 Graph of f is symmetric across $y = x$.