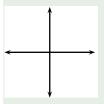
# Precalculus Computing sine, cosine of a half-angle

**Todor Miley** 

2019

Using radicals, find the exact value of the trigonometric expression.

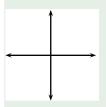
 $\cos 105^{\circ}$ 



# Example

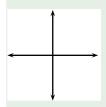
Using radicals, find the exact value of the trigonometric expression.

 $\cos 105^{\circ}$ 



Recall the half angle formula 
$$\cos \alpha = \pm \sqrt{\frac{1 + \cos(2\alpha)}{2}}$$
.

$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos (2 \cdot 105^{\circ})}{2}}$$

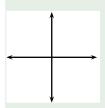


# Example

Using radicals, find the exact value of the trigonometric expression.

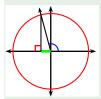
$$\cos 105^\circ = \pm \sqrt{\frac{1 + \cos \left(2 \cdot 105^\circ\right)}{2}}$$

cos 105°? 0

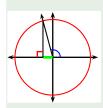


# Example

$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos (2 \cdot 105^{\circ})}{2}}$$
  $\cos 105^{\circ} < 0$ 



Recall the half angle formula 
$$\cos \alpha = \pm \sqrt{\frac{1 + \cos(2\alpha)}{2}}$$
.

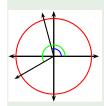


$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos (2 \cdot 105^{\circ})}{2}}$$
$$= -\sqrt{\frac{1 + \cos (210^{\circ})}{2}}$$



Recall the half angle formula 
$$\cos \alpha = \pm \sqrt{\frac{1 + \cos(2\alpha)}{2}}$$
.

Using radicals, find the exact value of the trigonometric expression.



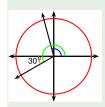
$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos (2 \cdot 105^{\circ})}{2}}$$
$$= -\sqrt{\frac{1 + \cos (210^{\circ})}{2}}$$

cos 105° <0

 $\cos 105^{\circ} < 0$ 

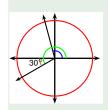
Recall the half angle formula 
$$\cos \alpha = \pm \sqrt{\frac{1 + \cos(2\alpha)}{2}}$$
.

# Example



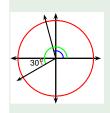
$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos (2 \cdot 105^{\circ})}{2}} \\ = -\sqrt{\frac{1 + \cos (210^{\circ})}{2}}$$

Recall the half angle formula 
$$\cos \alpha = \pm \sqrt{\frac{1 + \cos(2\alpha)}{2}}$$
.



$$\begin{aligned} \cos 105^\circ &= \pm \sqrt{\frac{1 + \cos \left(2 \cdot 105^\circ\right)}{2}} & & \left| \cos 105^\circ < 0 \right. \\ &= -\sqrt{\frac{1 + \cos \left(210^\circ\right)}{2}} \\ &= -\sqrt{\frac{1 - \cos \left(30^\circ\right)}{2}} \end{aligned}$$

# Example



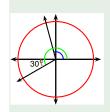
$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos(2 \cdot 105^{\circ})}{2}} \qquad \left| \cos 105^{\circ} < 0 \right|$$

$$= -\sqrt{\frac{1 + \cos(210^{\circ})}{2}}$$

$$= -\sqrt{\frac{1 - \cos(30^{\circ})}{2}}$$

$$= -\sqrt{\frac{1 - ?}{2}}$$

# Example



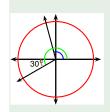
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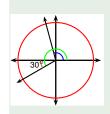
$$= -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$$

# Example



$$\begin{split} \cos 105^\circ &= \pm \sqrt{\frac{1 + \cos \left(2 \cdot 105^\circ\right)}{2}} \\ &= -\sqrt{\frac{1 + \cos \left(210^\circ\right)}{2}} \\ &= -\sqrt{\frac{1 - \cos \left(30^\circ\right)}{2}} \\ &= -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2 - \sqrt{3}}{2 \cdot 2}} \end{split}$$

# Example



$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos(2 \cdot 105^{\circ})}{2}} \quad | \cos 105^{\circ} < 0$$

$$= -\sqrt{\frac{1 + \cos(210^{\circ})}{2}}$$

$$= -\sqrt{\frac{1 - \cos(30^{\circ})}{2}}$$

$$= -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2 - \sqrt{3}}{2 \cdot 2}}$$

$$= -\frac{\sqrt{2 - \sqrt{3}}}{2}$$