

Precalculus

Quadratic trigonometric inequalities

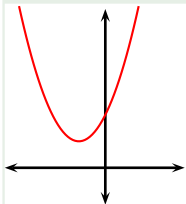
Todor Milev

2019

Example

- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2 \cos^2 \theta + 2 \cos \theta + 1 \leq \cos \theta + 2$ lying in $[-360^\circ, 360^\circ]$.

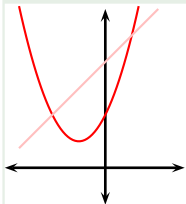
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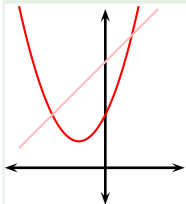
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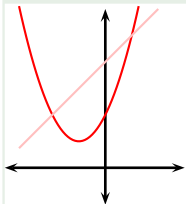


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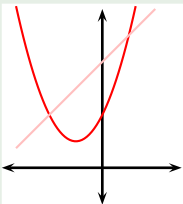


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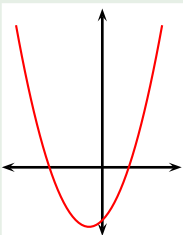
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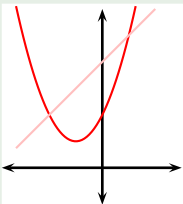
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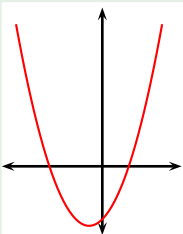


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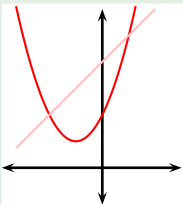
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$$2(\text{?})(\text{?}) \leq 0$$



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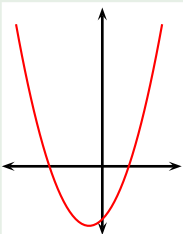


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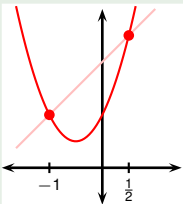
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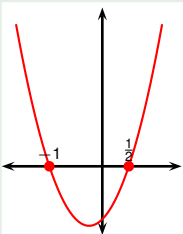
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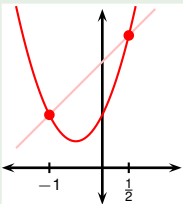
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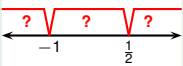
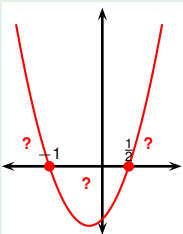


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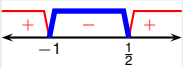
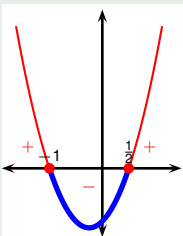
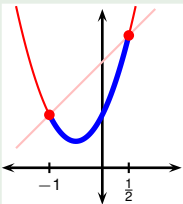


$$\begin{aligned}
 2u^2 + 2u + 1 &\leq u + 2 \\
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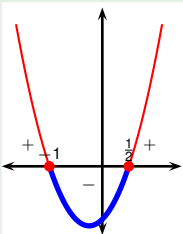
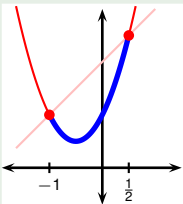
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 u &\in \left[-1, \frac{1}{2}\right]
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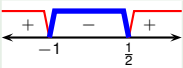
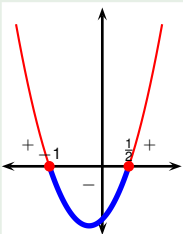
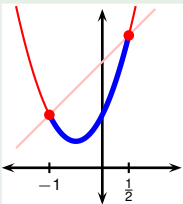
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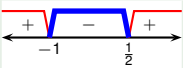
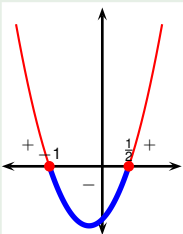
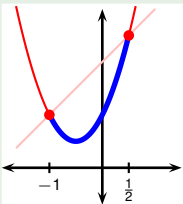
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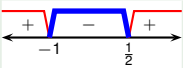
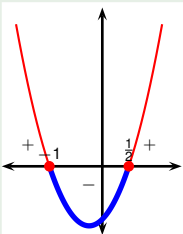
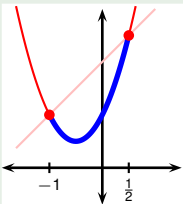
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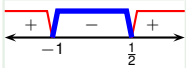
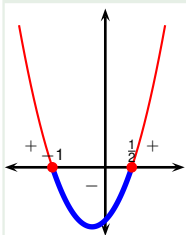
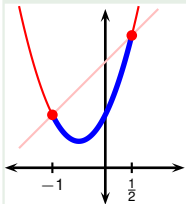
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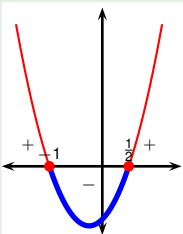
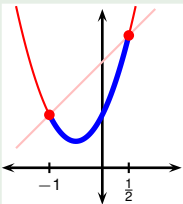
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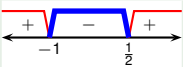
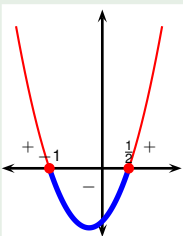
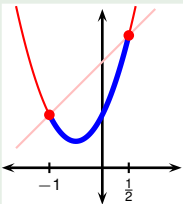
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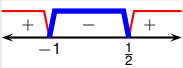
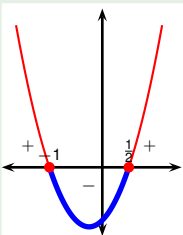
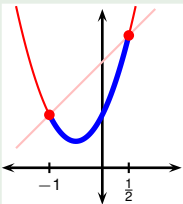
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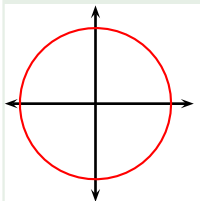
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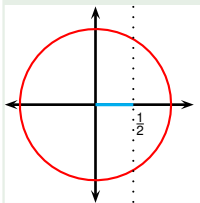


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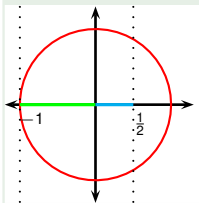
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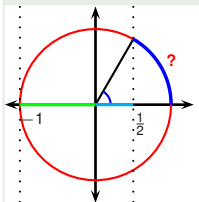
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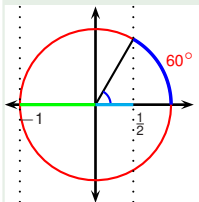
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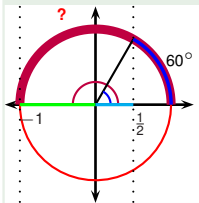
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$$\cos \theta \in \left[-1, \frac{1}{2}\right]$$

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$$\theta \in \quad [60^\circ \quad , ? \quad]$$

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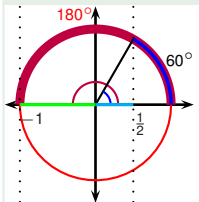


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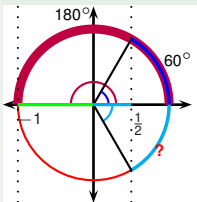
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$$\theta \in [60^\circ, 180^\circ]$$

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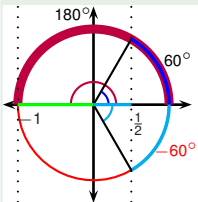


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$$\theta \in [?, \quad , ? \quad] \cup [60^\circ \quad , 180^\circ \quad]$$

Example

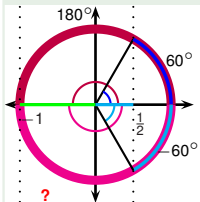


- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2 \cos^2 \theta + 2 \cos \theta + 1 \leq \cos \theta + 2$ lying in $[-360^\circ, 360^\circ]$.

$$\begin{aligned} \cos \theta &\in \left[-1, \frac{1}{2}\right] \\ -1 &\leq \cos \theta \leq \frac{1}{2} \end{aligned}$$

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Example



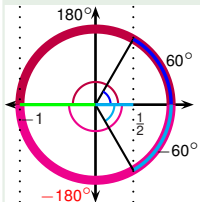
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Example



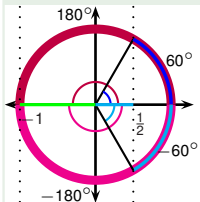
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Example

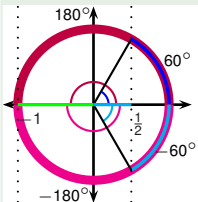


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Example



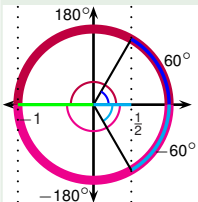
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Example



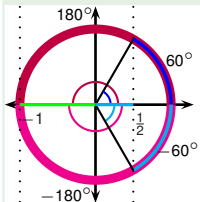
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$$\theta \in [-180^\circ, -60^\circ] \cup [60^\circ, 180^\circ] \quad k = 0$$

Example



- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
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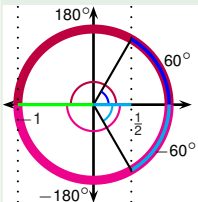
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$$\theta \in \begin{aligned} &[-180^\circ, -60^\circ] \cup [60^\circ, 180^\circ] && k = 0 \\ &\cup [180^\circ, 300^\circ] \cup [420^\circ, 540^\circ] && k = 1 \end{aligned}$$

Example



- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
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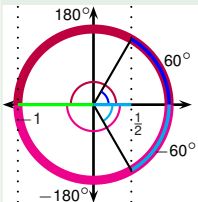
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...

Example



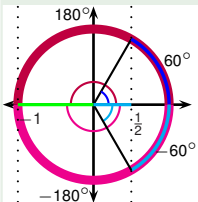
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Example



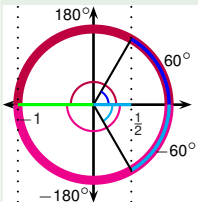
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Example



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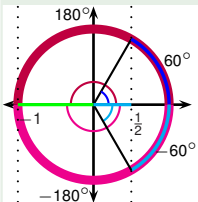
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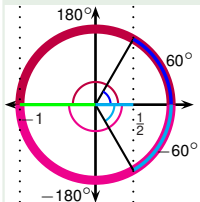
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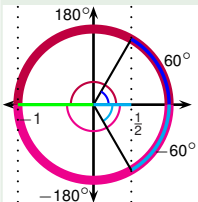
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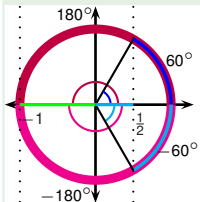
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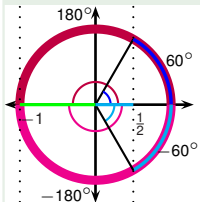


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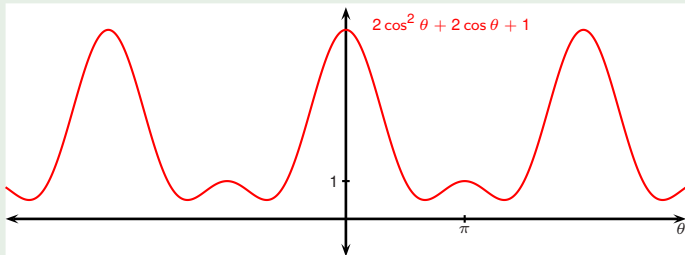


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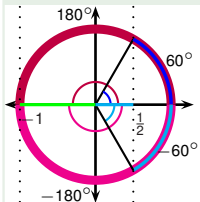
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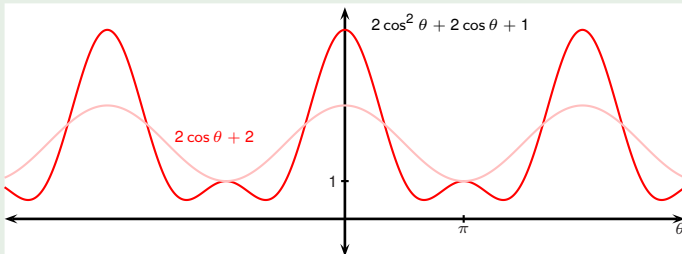


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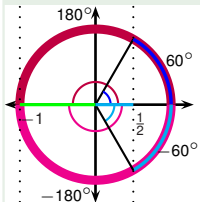
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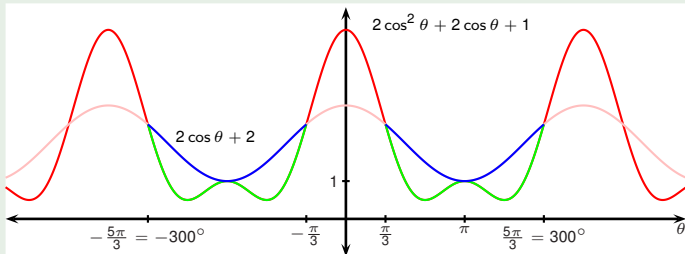


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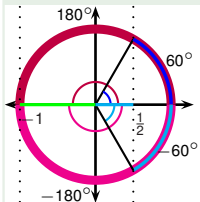
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