

Precalculus

Division of a polynomial by a linear polynomial

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Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$x - 1 \overline{) x^3 + 2x^2 \quad + 1}$$

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$x - 1 \overline{) x^3 + 2x^2 \quad + 1}$$

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 \textcolor{red}{?} \\
 \textcolor{red}{x} - 1 \overline{) \textcolor{red}{x}^3 + 2x^2 + 1}
 \end{array}$$

Divide $\textcolor{red}{x}^3$ by $\textcolor{red}{x}$.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$x - 1 \overline{) \overset{x^2}{x^3 + 2x^2} + 1}$$

Divide x^3 by x .

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{ ? ?}
 \end{array}$$

Multiply x^2 by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 + 1
 \end{array}$$

Multiply x^2 by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 \\
 \underline{3x^2 - 3x} \\
 3x + 1 \\
 \underline{3x - 3} \\
 4
 \end{array}$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 + 1
 \end{array}$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 \quad ? \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2
 \end{array}$$

Divide $3x^2$ by x .

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 + 3x \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2
 \end{array}$$

Divide $3x^2$ by x .

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 + 3x \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 + 1 \\
 \underline{ ? ?}
 \end{array}$$

Multiply $3x$ by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 + 3x \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 + 1 \\
 \underline{3x^2 - 3x} \\
 4x + 1
 \end{array}$$

Multiply $3x$ by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 + 3x \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 \\
 \underline{3x^2 - 3x} \\
 3x \\
 \underline{3x - 3} \\
 4
 \end{array}$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 + 3x \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 \\
 \underline{3x^2 - 3x} \\
 3x + 1
 \end{array}$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 + 3x \quad ? \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 \\
 \underline{3x^2 - 3x} \\
 3x + 1
 \end{array}$$

Divide $3x$ by x .

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 + 3x + 3 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 \\
 \underline{3x^2 - 3x} \\
 3x + 1
 \end{array}$$

Divide $3x$ by x .

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 + 3x + 3 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 \\
 \underline{3x^2 - 3x} \\
 3x + 1 \\
 \underline{ ? ?}
 \end{array}$$

Multiply **3** by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 + 3x + 3 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 + 1 \\
 \underline{3x^2 - 3x} \\
 3x + 1 \\
 \underline{3x - 3} \\
 4
 \end{array}$$

Multiply 3 by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 + 3x + 3 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 \\
 \underline{3x^2 - 3x} \\
 3x + 1 \\
 \underline{3x - 3} \\
 ?
 \end{array}$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 + 3x + 3 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{-} \\
 x^3 - x^2 \\
 \underline{-} \\
 3x^2 + 1 \\
 \underline{-} \\
 3x^2 - 3x \\
 \underline{-} \\
 3x + 1 \\
 \underline{-} \\
 3x - 3 \\
 \underline{-} \\
 4
 \end{array}$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 x^2 + 3x + 3 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 \\
 \underline{3x^2 - 3x} \\
 3x + 1 \\
 \underline{3x - 3} \\
 4
 \end{array}$$

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 \text{Quotient: } x^2 + 3x + 3 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 \\
 \underline{3x^2 - 3x} \\
 3x + 1 \\
 \underline{3x - 3} \\
 4
 \end{array}$$

$$(\text{Dividend}) = (\text{Quotient}) \cdot (\text{Divisor}) + (\text{Remainder})$$

$$(x^3 + 2x^2 + 1) = (x^2 + 3x + 3) \cdot (x - 1) + 4$$

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 \text{Quotient:} \quad x^2 + 3x + 3 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 \\
 \underline{3x^2 - 3x} \\
 3x + 1 \\
 \underline{3x - 3} \\
 4
 \end{array}$$

Remainder: 4

$$\begin{aligned}
 (\text{Dividend}) &= (\text{Quotient}) \cdot (\text{Divisor}) + (\text{Remainder}) \\
 (x^3 + 2x^2 + 1) &= (x^2 + 3x + 3) \cdot (x - 1) + 4
 \end{aligned}$$