Precalculus
Division of a polynomial by a linear polynomial

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Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$x - 1 \quad \overline{\left| x^3 + 2x^2 \qquad + 1 \right.}$$

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$x - 1 \quad \overline{\left| x^3 + 2x^2 \qquad + 1 \right.}$$

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r} \color{red}{?} \\ x - 1 \quad \overline{\left| x^3 + 2x^2 \qquad + 1 \right.} \end{array}$$

Divide x^3 by x.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r} x^2 \\ x-1 \quad \overline{\left| x^3+2x^2 \qquad +1 \right.} \end{array}$$

Divide x^3 by x.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
 & x^2 \\
x - 1 & \overline{\left| x^3 + 2x^2 \quad + 1 \right.} \\
 & ? \quad ? \\
 & \overline{}
\end{array}
$$

Multiply x^2 by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
 & x^2 \\
\cline{2-2}
x - 1 & \left| \begin{array}{l} x^3 + 2x^2 \quad\quad + 1 \\ x^3 - x^2 \end{array} \right. \\
\cline{2-2}
\end{array}
$$

Multiply x^2 by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
 & x^2 \\
x - 1 & \overline{\big| \; x^3 + 2x^2 \quad\quad + 1} \\
- & \underline{x^3 - \; x^2 \quad\quad\quad} \\
 & \quad ? \quad\quad\quad ?
\end{array}
$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
 & x^2 \\
x - 1 & \overline{\left| \begin{array}{l} x^3 + 2x^2 \quad\quad +1 \\ \underline{x^3 - x^2} \\ \quad 3x^2 \quad\quad +1 \end{array} \right.}
\end{array}
$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rr|l}
 & & x^2 \quad ? \\
\cline{2-3}
x - 1 & & x^3 + 2x^2 \quad\quad + 1 \\
- & & x^3 - x^2 \\
\cline{3-3}
 & & \quad 3x^2 \quad\quad + 1
\end{array}
$$

Divide $3x^2$ by x.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl|l}
 & & x^2 + 3x \\
\hline
x - 1 & & x^3 + 2x^2 \qquad\quad + 1 \\
- & & x^3 - x^2 \\
\hline
 & & 3x^2 \qquad\quad + 1
\end{array}
$$

Divide $3x^2$ by x.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
 & x^2 + 3x \\
x - 1 \quad \big) & \overline{x^3 + 2x^2 \qquad + 1} \\
- & x^3 - x^2 \\
\hline
 & \qquad 3x^2 \qquad + 1 \\
 & \quad ? \qquad ? \\
\hline
\end{array}
$$

Multiply $3x$ by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
 & x^2 + 3x \\
x - 1 \quad \big) & \overline{x^3 + 2x^2 \qquad + 1} \\
- & x^3 - x^2 \\
\hline
 & \qquad 3x^2 \qquad + 1 \\
 & \quad 3x^2 - 3x \\
\hline
\end{array}
$$

Multiply $3x$ by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rrl}
 & & x^2 + 3x \\
x - 1 & \big| & x^3 + 2x^2 \quad\quad + 1 \\
- & & x^3 - x^2 \\
\hline
 & & \quad 3x^2 \quad\quad + 1 \\
- & & \quad 3x^2 - 3x \\
\hline
 & & \quad\quad\quad ? \quad ?
\end{array}
$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl|l}
 & & x^2 + 3x \\
\hline
x - 1 & & x^3 + 2x^2 \qquad\quad + 1 \\
- & & x^3 - x^2 \\
\hline
 & & \qquad 3x^2 \qquad + 1 \\
- & & \qquad 3x^2 - 3x \\
\hline
 & & \qquad\quad 3x + 1
\end{array}
$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl|l}
 & & x^2 + 3x \quad \textbf{?} \\
\hline
x - 1 & & x^3 + 2x^2 \qquad + 1 \\
- & & x^3 - x^2 \\
\hline
 & & 3x^2 \qquad + 1 \\
- & & 3x^2 - 3x \\
\hline
 & & 3x + 1
\end{array}
$$

Divide $3x$ by x.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl|l}
 & & x^2 + 3x + 3 \\
\hline
x - 1 & & x^3 + 2x^2 \qquad\quad + 1 \\
- & & x^3 - x^2 \\
\hline
 & & \qquad 3x^2 \qquad + 1 \\
- & & \qquad 3x^2 - 3x \\
\hline
 & & \qquad\qquad\quad 3x + 1
\end{array}
$$

Divide $3x$ by x.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
 & x^2 + 3x + 3 \\
\cline{2-2}
x - 1 \;\big| & x^3 + 2x^2 \quad\quad + 1 \\
- & x^3 - x^2 \\
\cline{2-2}
 & \quad 3x^2 \quad\quad + 1 \\
- & \quad 3x^2 - 3x \\
\cline{2-2}
 & \quad\quad\quad 3x + 1 \\
 & \quad\quad\quad ? \quad ? \\
\cline{2-2}
\end{array}
$$

Multiply 3 by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
 & x^2 + 3x + 3 \\
\cline{2-2}
x - 1 \;\big| & x^3 + 2x^2 \quad\quad + 1 \\
- & x^3 - x^2 \\
\cline{2-2}
 & \quad 3x^2 \quad\quad + 1 \\
- & \quad 3x^2 - 3x \\
\cline{2-2}
 & \quad\quad\quad 3x + 1 \\
 & \quad\quad\quad 3x - 3 \\
\cline{2-2}
\end{array}
$$

Multiply 3 by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
 & x^2 + 3x + 3 \\
x - 1 \quad & \overline{\big| \; x^3 + 2x^2 \qquad + 1} \\
- & x^3 - x^2 \\
\hline
 & \qquad 3x^2 \qquad + 1 \\
- & \qquad 3x^2 - 3x \\
\hline
 & \qquad\qquad 3x + 1 \\
- & \qquad\qquad 3x - 3 \\
\hline
 & \qquad\qquad\quad ?
\end{array}
$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
 & x^2 + 3x + 3 \\
x - 1 \quad \big| & x^3 + 2x^2 \qquad + 1 \\
- & x^3 - x^2 \\
\hline
 & \qquad 3x^2 \qquad + 1 \\
- & \qquad 3x^2 - 3x \\
\hline
 & \qquad\qquad 3x + 1 \\
- & \qquad\qquad 3x - 3 \\
\hline
 & \qquad\qquad\quad 4
\end{array}
$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
 & x^2 + 3x + 3 \\
x - 1 \quad \big| & x^3 + 2x^2 \qquad + 1 \\
- & x^3 - x^2 \\
\hline
 & \qquad 3x^2 \qquad + 1 \\
- & \qquad 3x^2 - 3x \\
\hline
 & \qquad\qquad 3x + 1 \\
- & \qquad\qquad 3x - 3 \\
\hline
 & \qquad\qquad\qquad 4
\end{array}
$$

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
\textbf{Quotient:} & x^2 + 3x + 3 \\
x - 1 \;\big)\!\!\!\big) & x^3 + 2x^2 \qquad + 1 \\
- & x^3 - x^2 \\
\hline
 & \qquad 3x^2 \qquad + 1 \\
- & \qquad 3x^2 - 3x \\
\hline
 & \qquad\qquad 3x + 1 \\
- & \qquad\qquad 3x - 3 \\
\hline
 & \qquad\qquad\qquad 4
\end{array}
$$

$$
\begin{aligned}
\textbf{(Dividend)} &= \textbf{(Quotient)} \cdot \textbf{(Divisor)} + \textbf{(Remainder)} \\
(x^3 + 2x^2 + 1) &= (x^2 + 3x + 3) \cdot (x - 1) + 4
\end{aligned}
$$

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rr|l}
\textbf{Quotient:} & & x^2 + 3x + 3 \\
\hline
x - 1 & & x^3 + 2x^2 \qquad + 1 \\
 & - & x^3 - x^2 \\
\hline
 & & 3x^2 \qquad + 1 \\
 & - & 3x^2 - 3x \\
\hline
 & & 3x + 1 \\
 & - & 3x - 3 \\
\hline
\textbf{Remainder:} & & 4
\end{array}
$$

$$
\begin{array}{rcl}
\textbf{(Dividend)} & = & \textbf{(Quotient)} \cdot \textbf{(Divisor)} + \textbf{(Remainder)} \\
(x^3 + 2x^2 + 1) & = & (x^2 + 3x + 3) \cdot (x - 1) + 4
\end{array}
$$