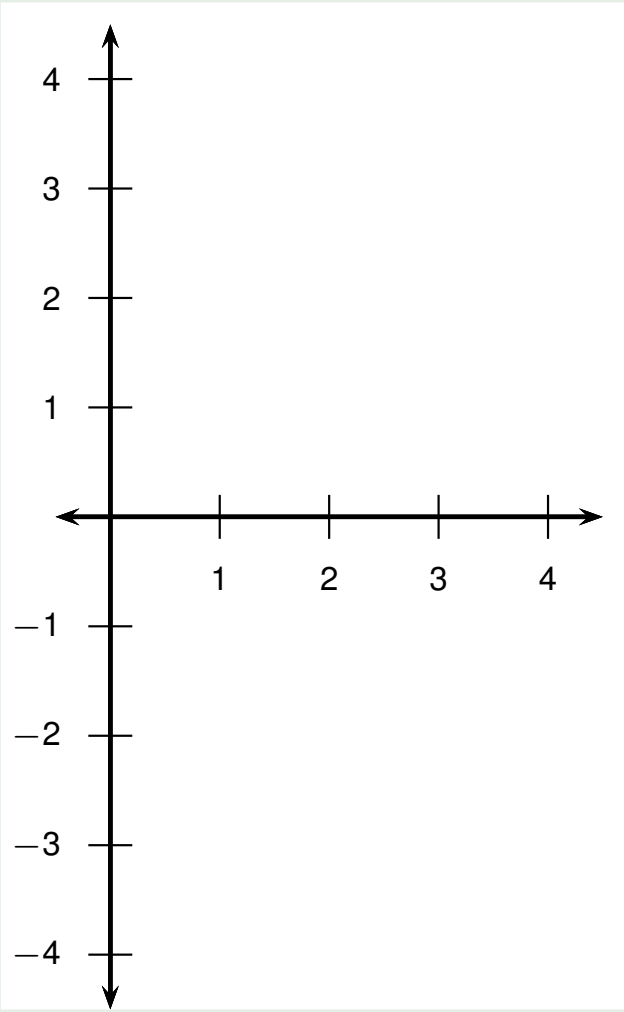
Calculus I
Maxima and minima of polynomials over closed intervals

Todor Milev

2019

Example

Find the maximum and minimum values of the function $f(x) = -x^3 + 2x^2 + 4x - 5$ on the interval $[1, 3]$.



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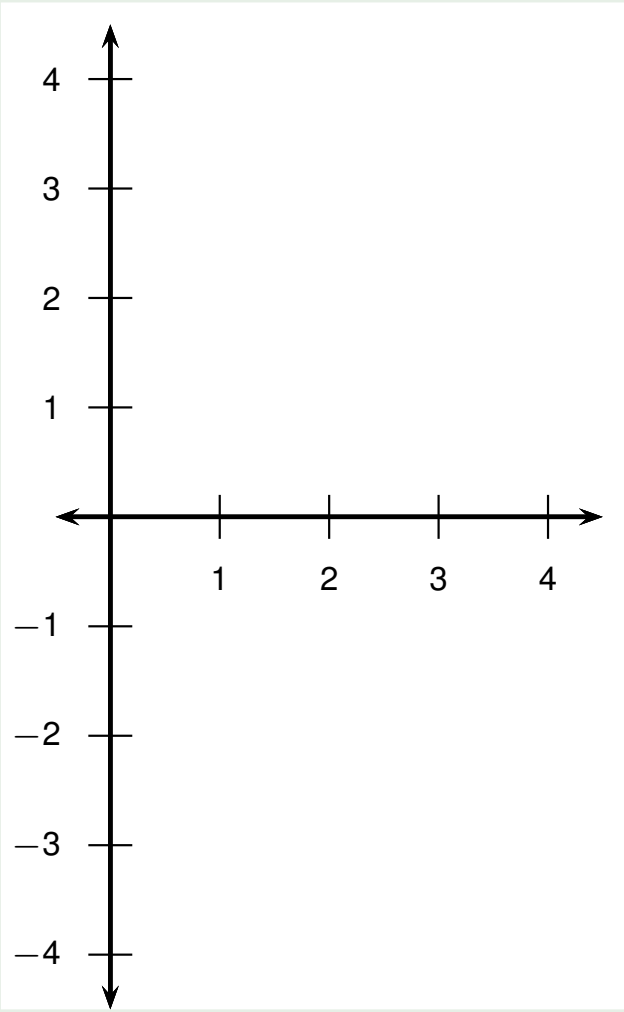

$$f'(x) = -3x^2 + 4x + 4$$

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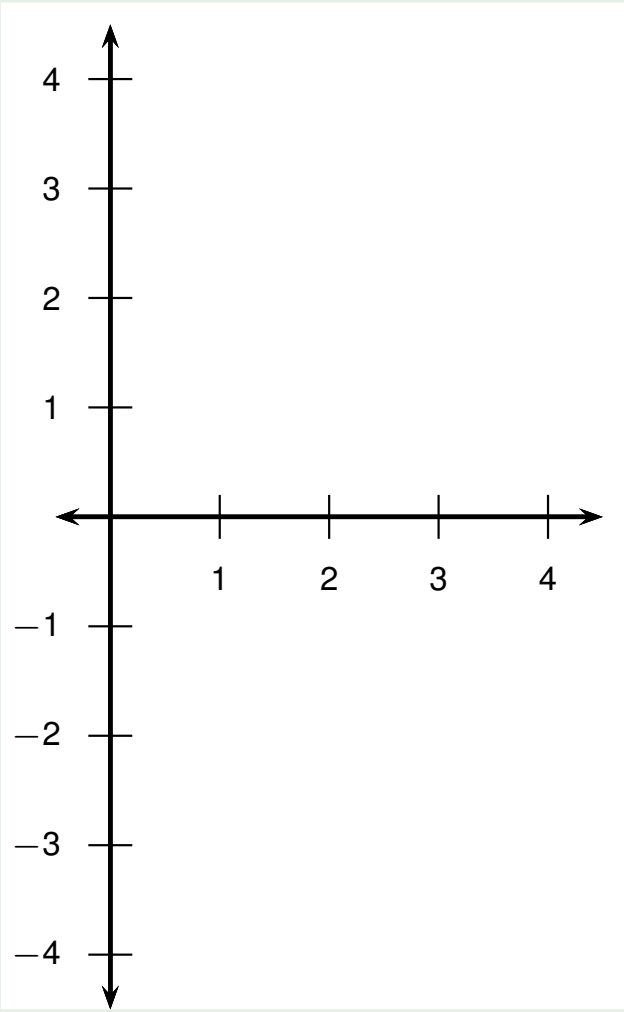

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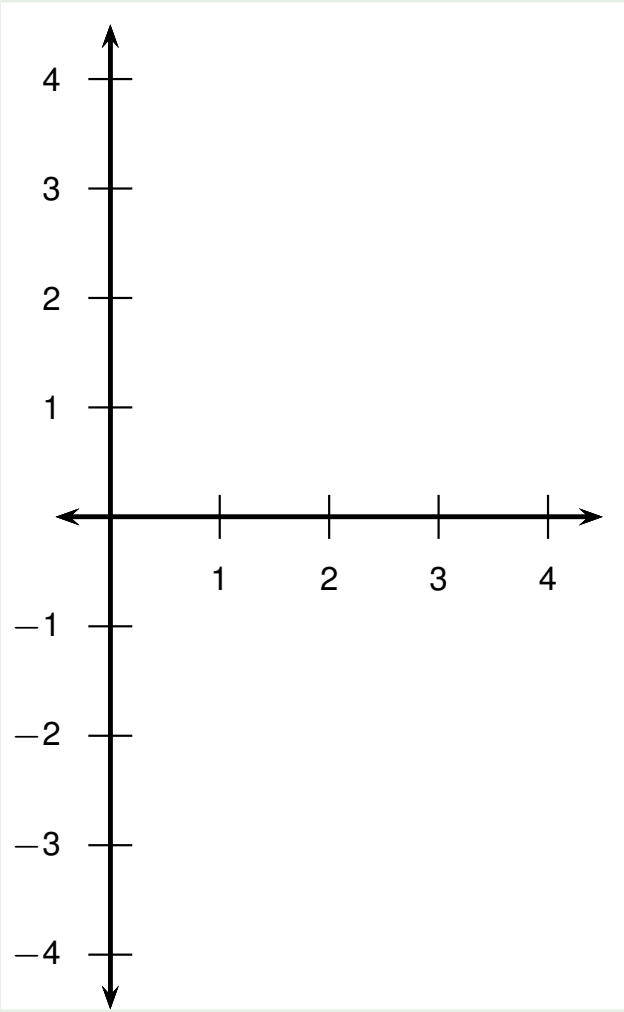

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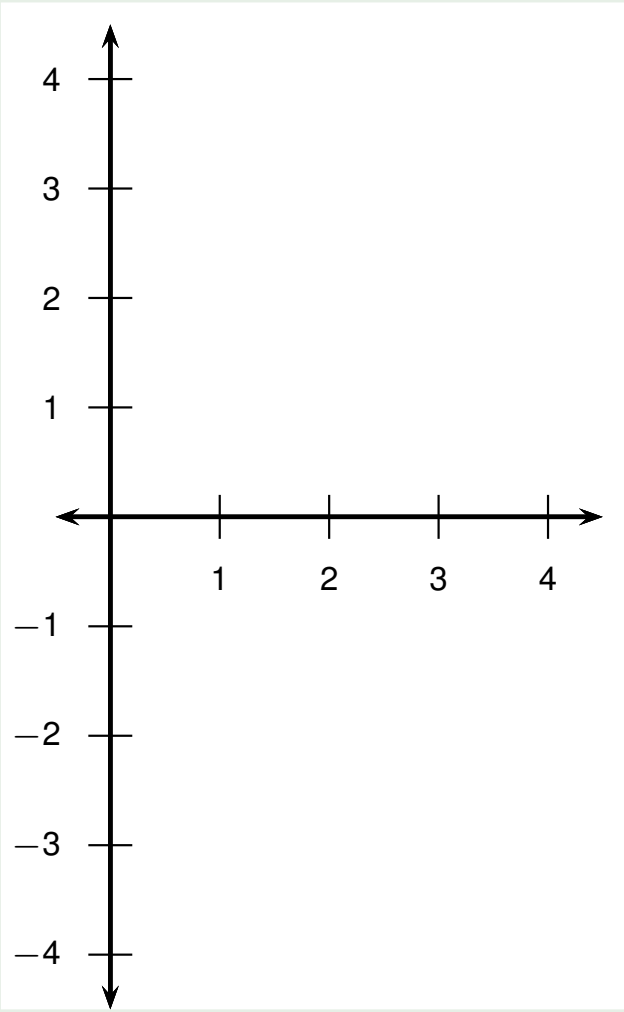

$$f'(x) = -3x^2 + 4x + 4$$
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Need to check:

1. The critical numbers of f in $[a, b]$.
2. The endpoints a and b.

x	$f(x)$

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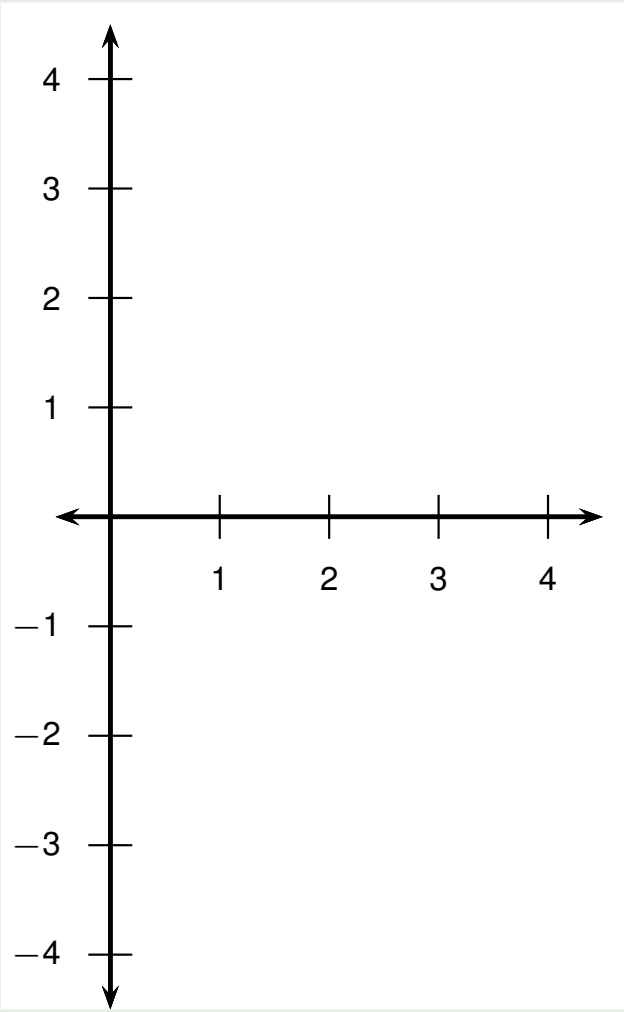

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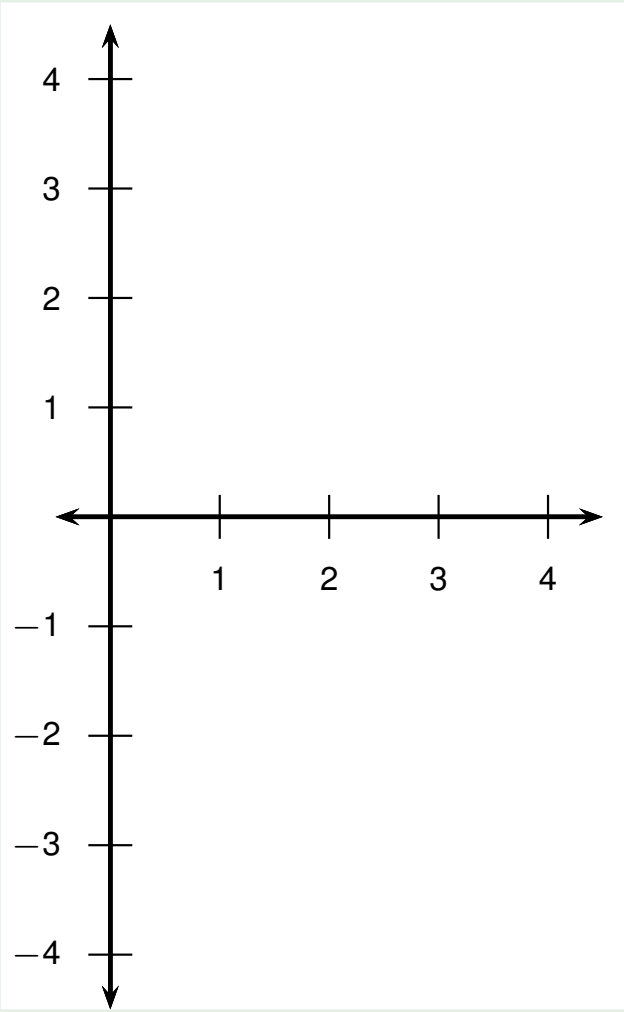

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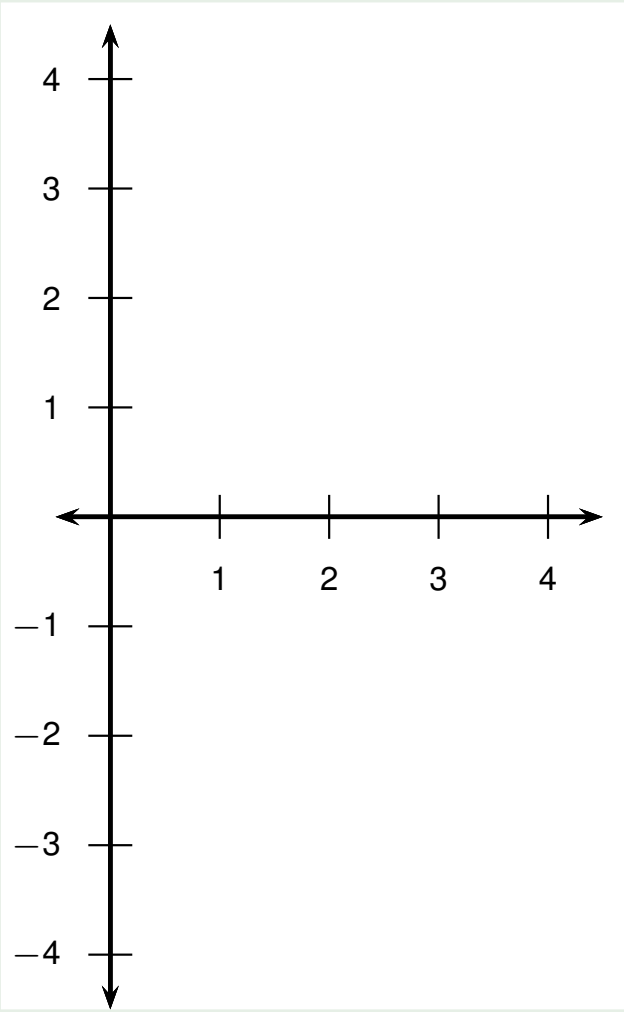

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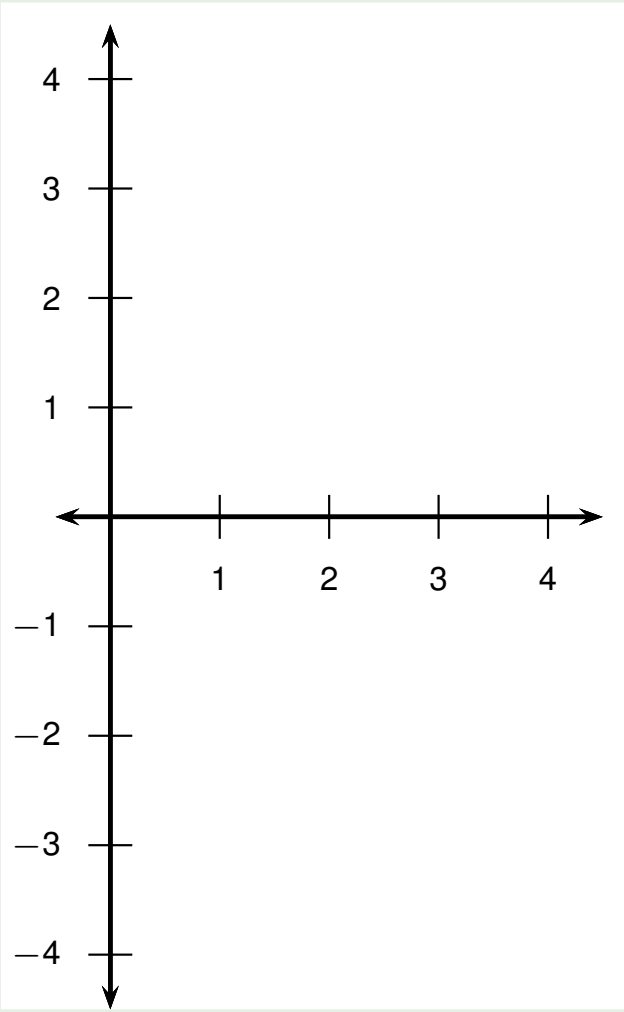

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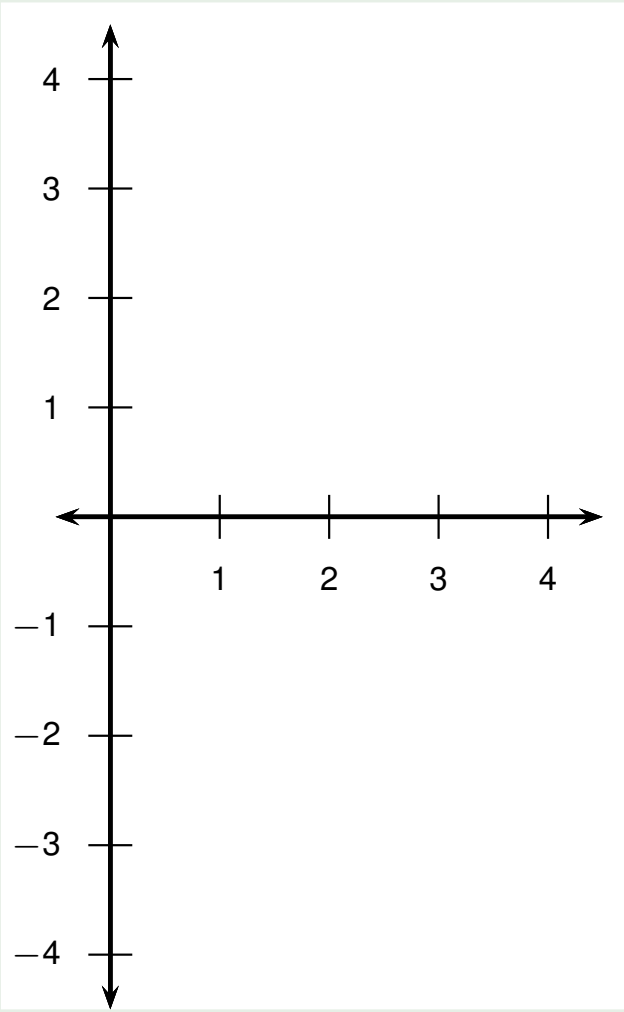

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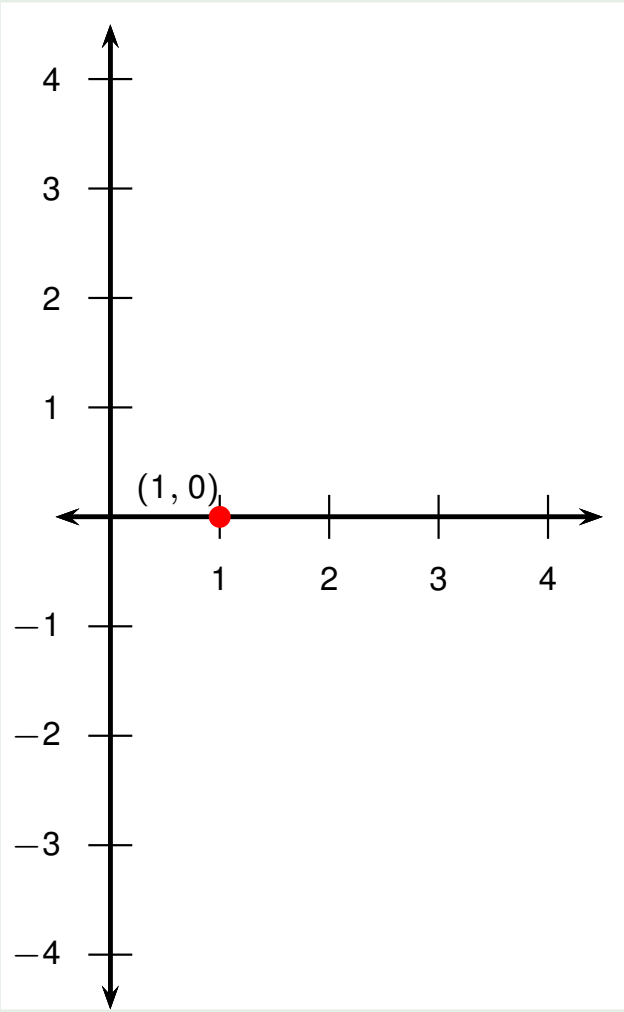

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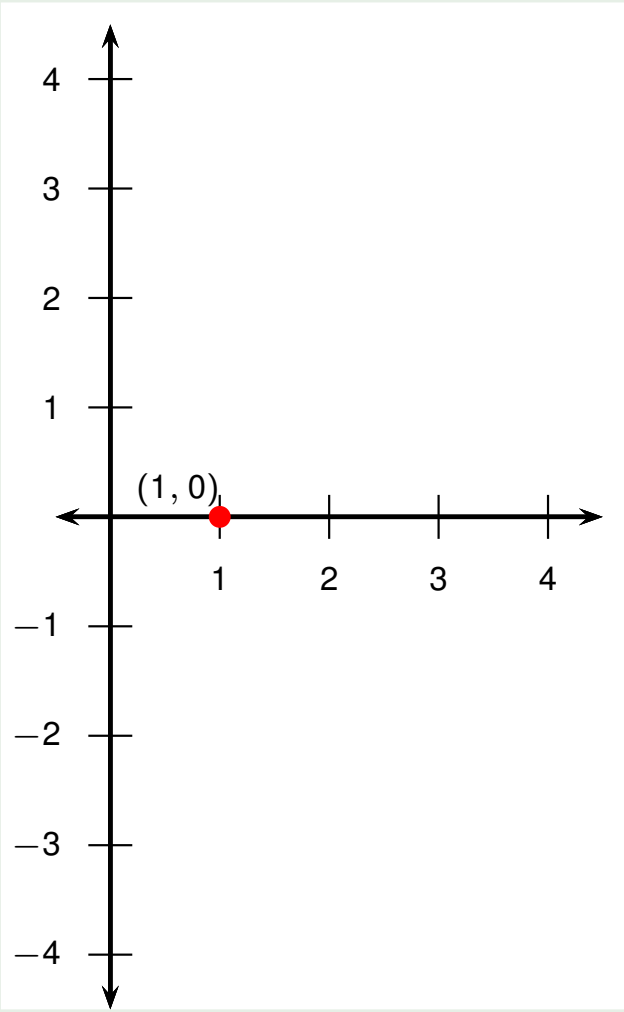

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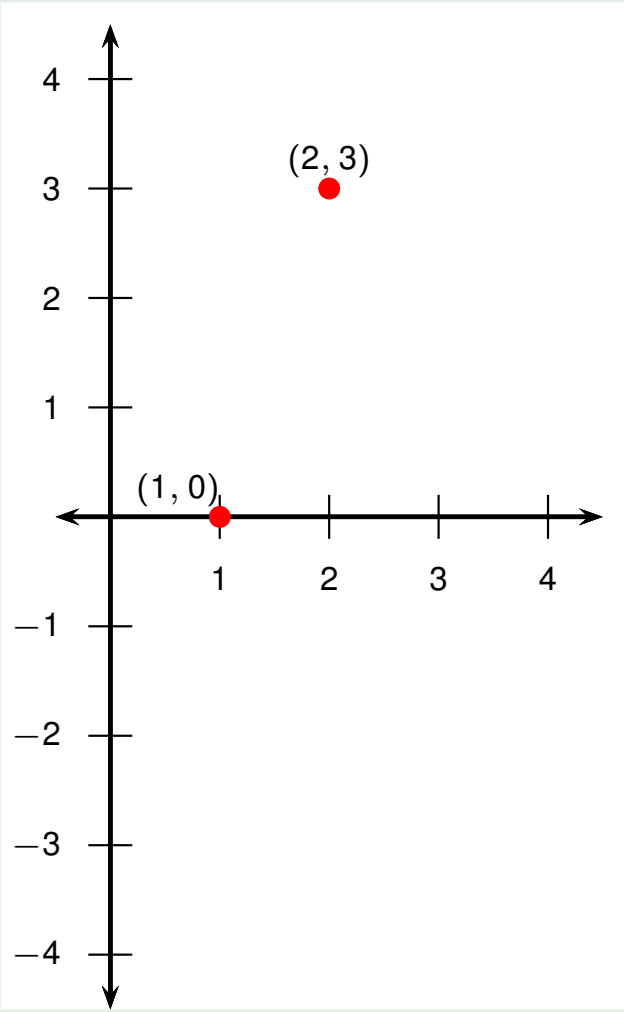

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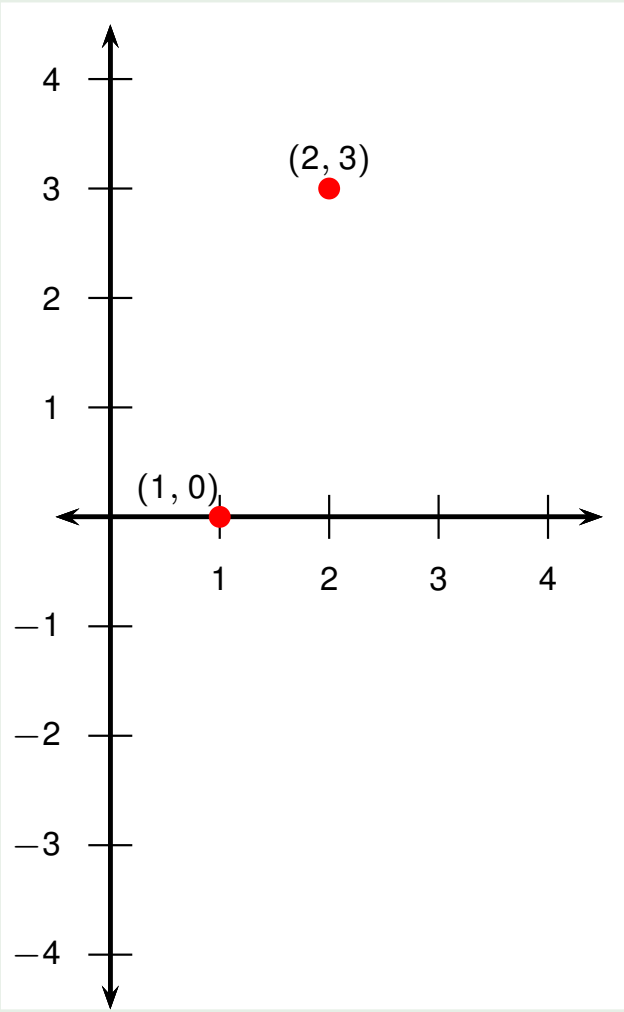

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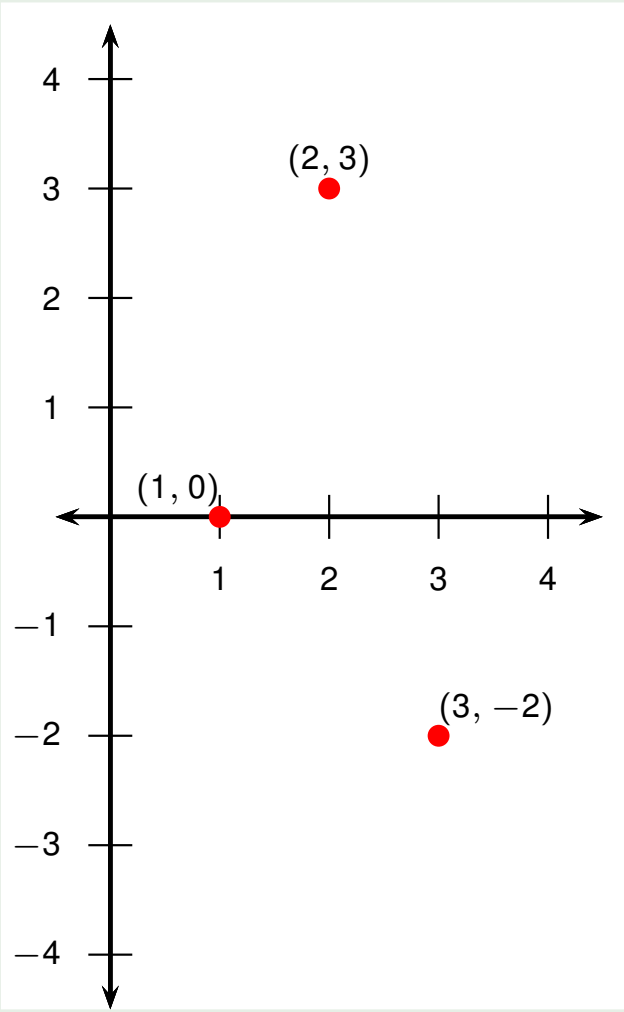

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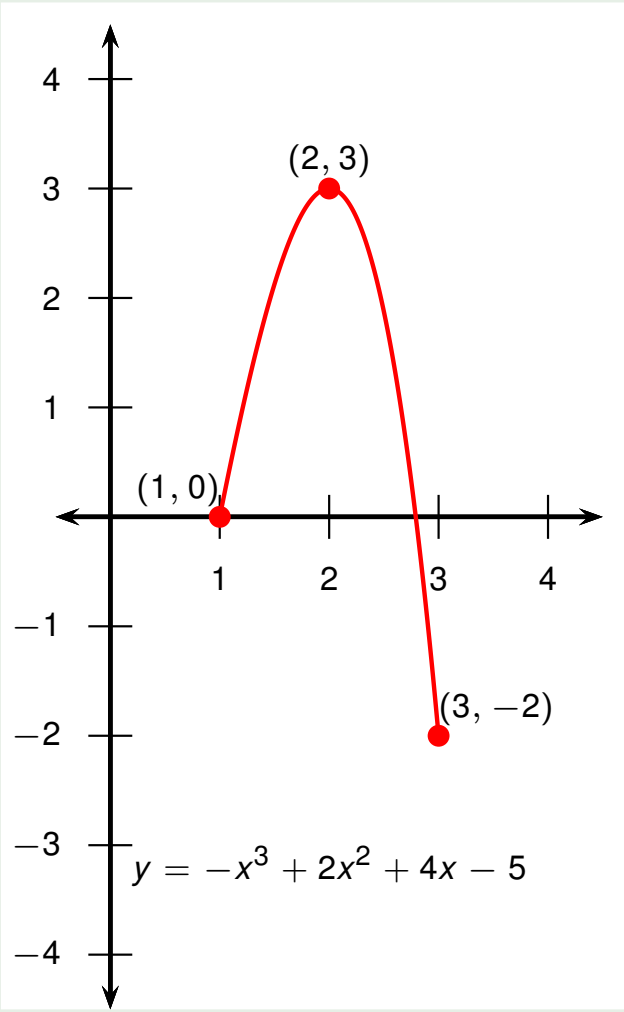

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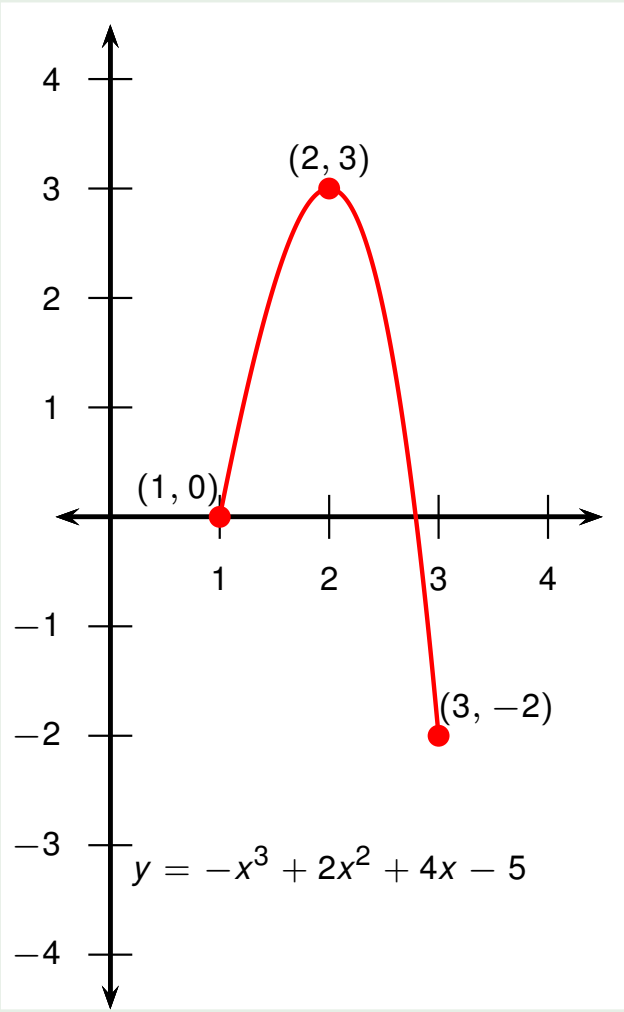

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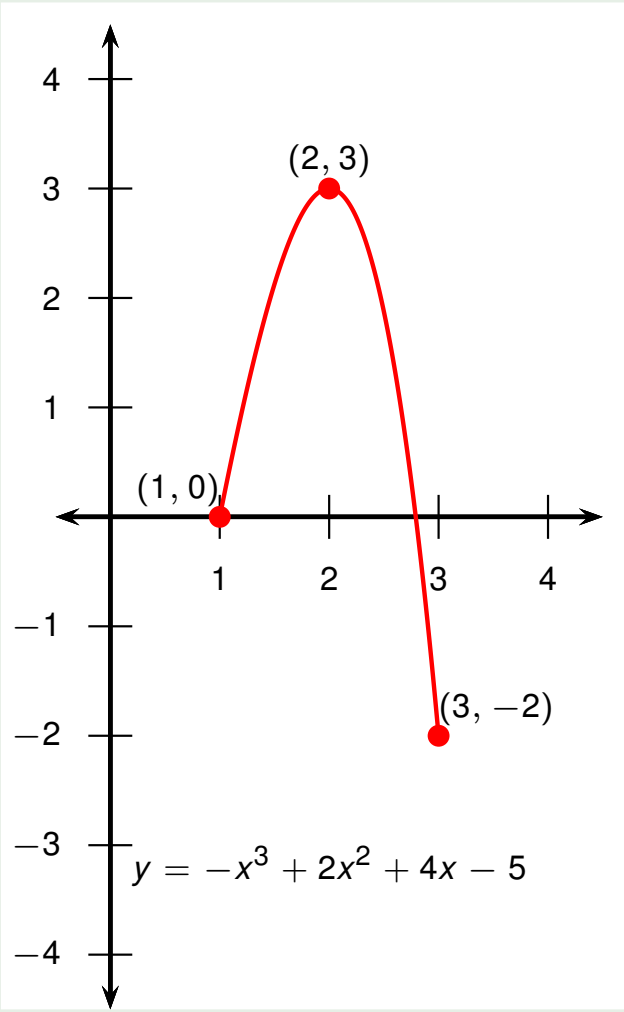

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Maximum on $[1, 3]$: 3. Minimum on $[1, 3]$:

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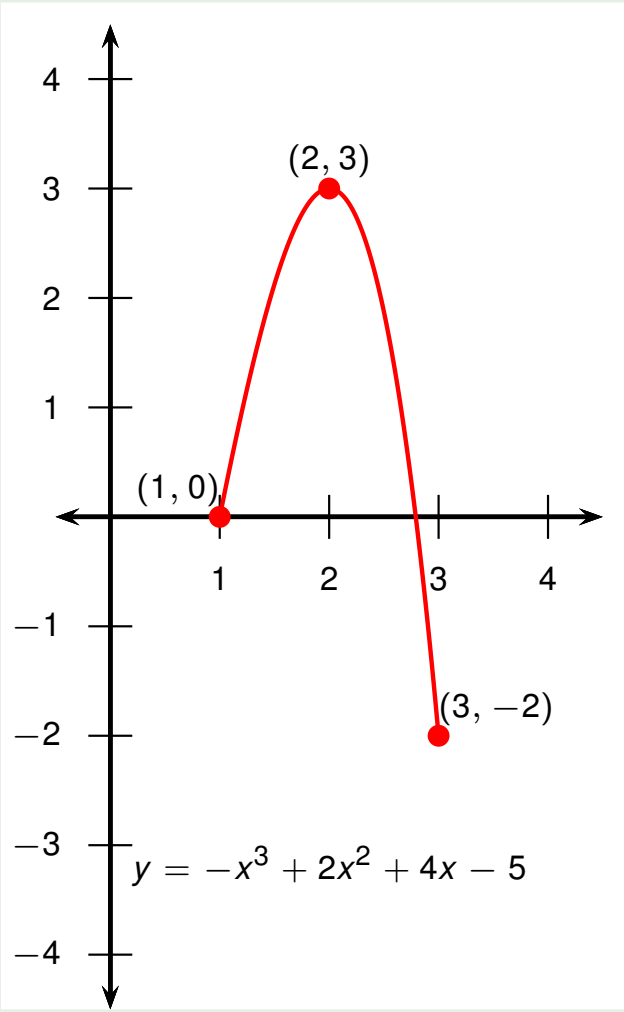

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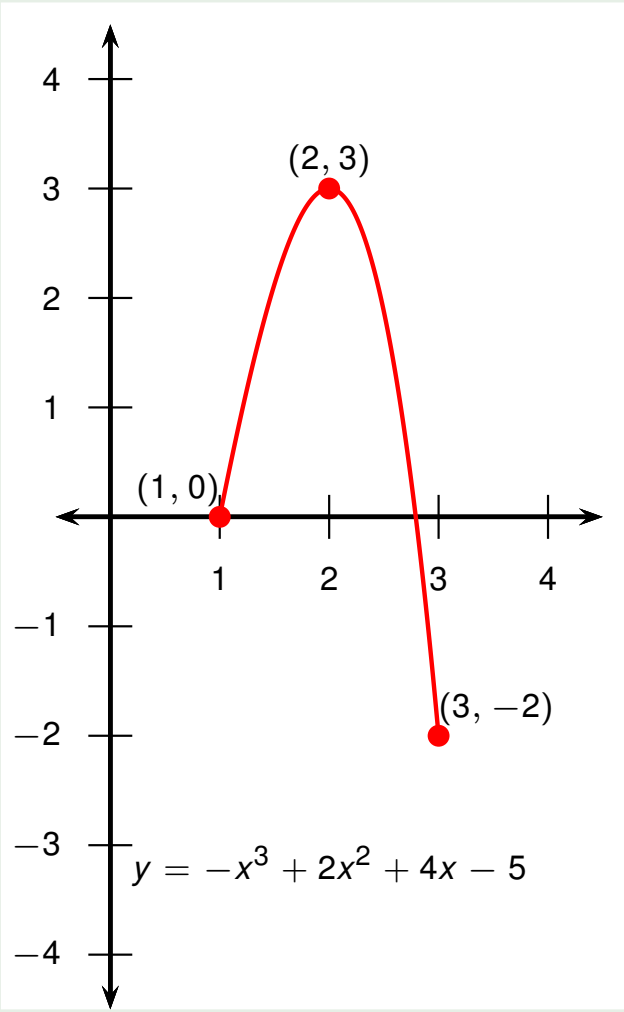

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