

## Precalculus

**Find extremum of quadratic, text problem.**

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2019

# Maximum or minimum value of a quadratic function

- Let  $f(x) = ax^2 + bx + c$  - quadratic ( $a \neq 0$ ).
- Let  $D$  be the discriminant  $D = b^2 - 4ac$ .

$$f(x) = a \left( x - \left( -\frac{b}{2a} \right) \right)^2 - \frac{D}{4a} \quad \left| \text{complete the square} \right.$$

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Recall  $f(x) = ax^2 + bx + c = a \left( x - \left( -\frac{b}{2a} \right) \right)^2 - \frac{D}{4a}$ .

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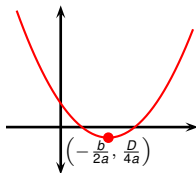
- If  $a > 0$  then  $f(x)$  has no maximum and has minimum at  $x = -\frac{b}{2a}$ .
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- In both cases, the extremal value (either maximum or minimum) is  $f\left(-\frac{b}{2a}\right) = -\frac{b^2-4ac}{4a} = -\frac{D}{4a}$ .

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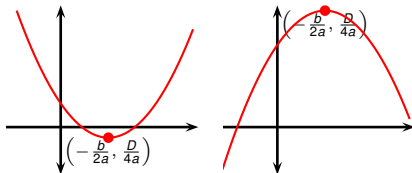


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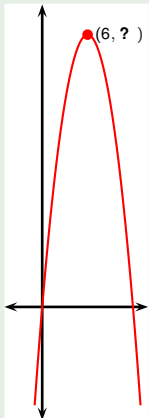
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Max. product  $= xz$

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