Calculus II
Definition of complex numbers

Todor Milev

2019

Definition (Complex numbers)

The set of complex numbers $\mathbb{C}$ is defined as the set

$$\{a + bi | a, b - \text{real numbers}\},$$

where the number i is a number for which

$$i^2 = -1 \quad .$$

The number i is called the imaginary unit. .

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Review of the basic types of numbers

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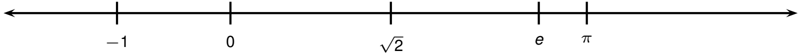
π

e

-1



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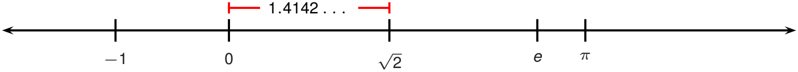

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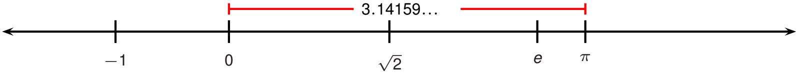

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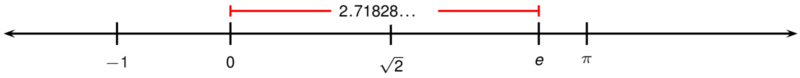

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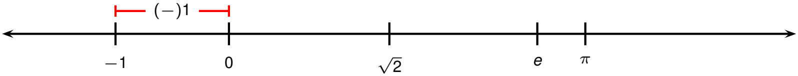

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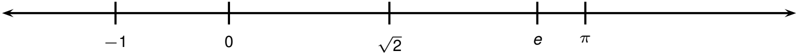

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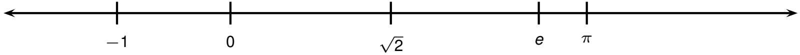

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- Geometric interpretation of complex numbers: beyond our scope.