Precalculus
Exponent equation that reduces to masked quadratic, part 2

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2019

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$3^{2x} = 2 + 63 \cdot 3^{-2x}$$

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Solve the equation.

$$3^{2x} = 2 + 63 \cdot 3^{-2x} \quad \Big| \quad > 2 \text{ terms} \Rightarrow \text{transfer one side}$$

$$3^{2x} - 2 - 63 \cdot 3^{-2x} = 0$$

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$3^{2x} = u$

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\begin{aligned}
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3^{2x} &= 2 + 63 \cdot 3^{-2x} && \text{> 2 terms $\Rightarrow$ transfer one side} \\
3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 && 3^{2x} = u \\
u - 2 - 63u^{-1} &= 0 && 3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1} \\
u^2 - 2u - 63 &= 0 && \text{Multiply } \cdot u
\end{aligned}
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3^{2x} &= 2 + 63 \cdot 3^{-2x} && \left| \begin{array}{l} > 2 \text{ terms} \Rightarrow \\ \text{transfer one side} \\ 3^{2x} = u \\ 3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1} \\ \text{Multiply } \cdot u \end{array} \right. \\
3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \\
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