

Calculus II

Definition of complex numbers

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Definition (Complex numbers)

The set of complex numbers \mathbb{C} is defined as the set

$$\{a + bi \mid a, b - \text{real numbers}\},$$

where the number i is a number for which

$$i^2 = -1 \quad .$$

The number i is called the imaginary unit. By definition, $\sqrt{-1} = i$.

- Complex addition/subtraction

$$(a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i \quad .$$

- Complex multiplication

$$\begin{aligned} (a + bi)(c + di) &= ac + adi + bci + bdi^2 = ac + adi + bci - bd \\ &= (ac - bd) + i(ad + bc) \end{aligned}$$

Review of the basic types of numbers

- An integer, or whole number, is one of the numbers:

$$\dots, -2, -1, 0, 1, 2, \dots$$

- A rational number is the quotient of two integers, for example:

$$\frac{1}{2}, \quad \frac{2}{-3} = -\frac{2}{3}, \quad \frac{8}{12} = \frac{4}{6} = \frac{2}{3}.$$

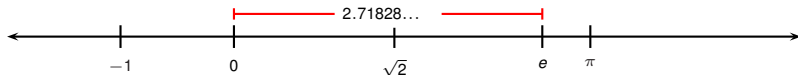
- A real number measures the location of a point on the real line:

$$\sqrt{2} = 1.414213562373095048801688724209698 \dots$$

$$\pi = 3.141592653589793238462643383279502 \dots$$

$$e = 2.718281828459045235360287471352662 \dots$$

$$-1$$



- A number is complex if it equals $a + bi$ with a, b - real, $\sqrt{-1} = i$:

$$2 + 3i, \quad -i, \quad 1 + \sqrt{2}i$$

- Geometric interpretation of complex numbers: beyond our scope.