

Precalculus

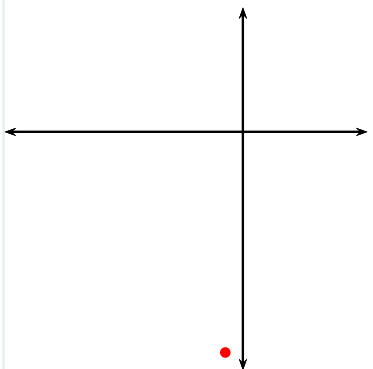
Inverse of a quadratic function

Todor Milev

2019

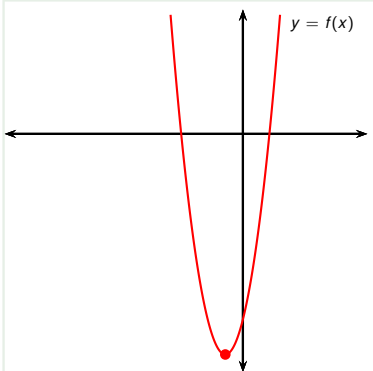
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Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



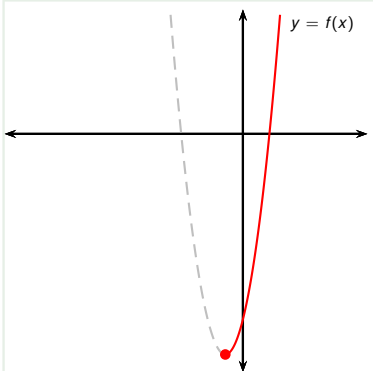
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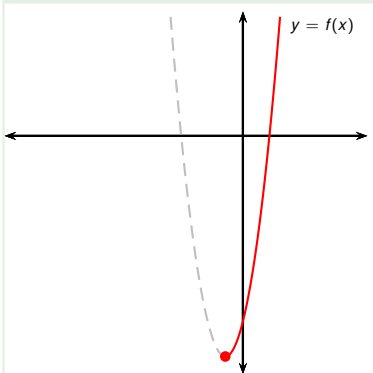
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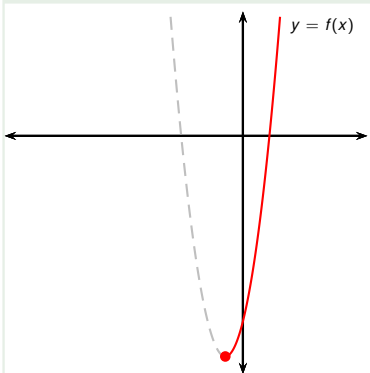
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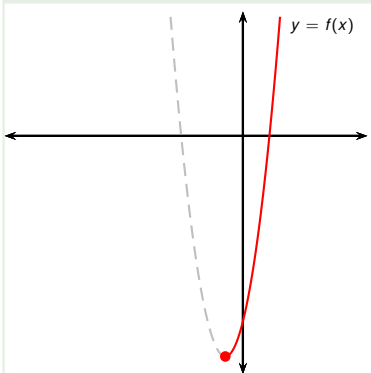
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That's a quadratic equation in x . Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

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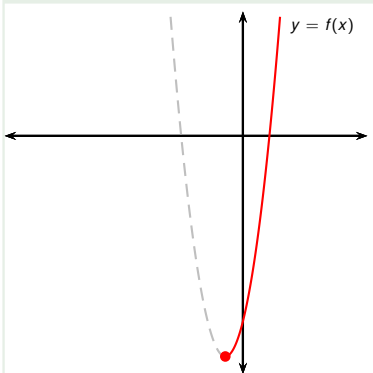
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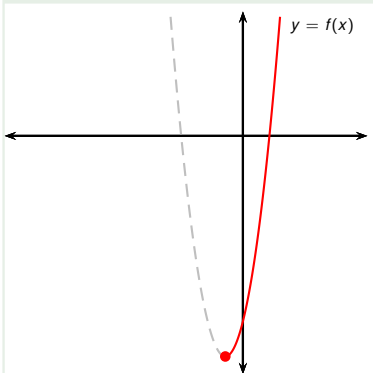
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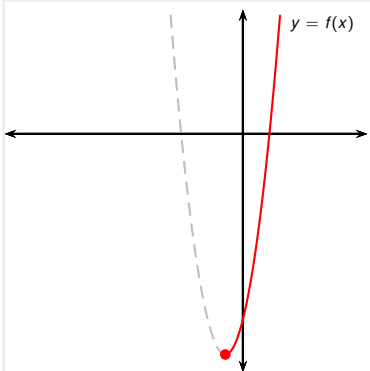
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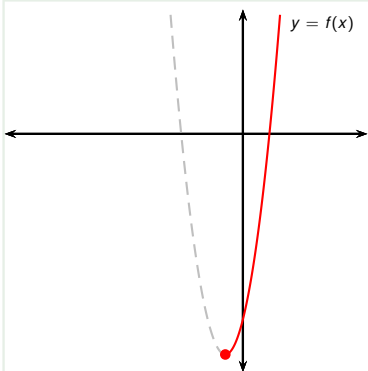
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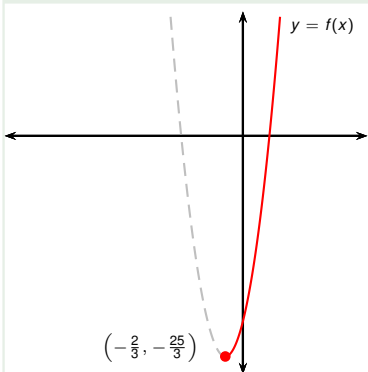
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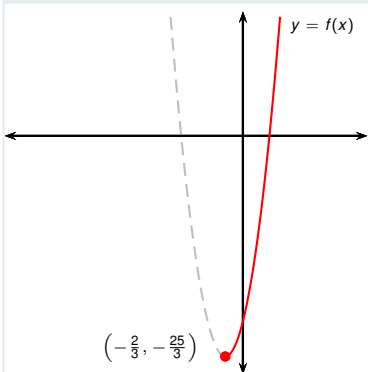
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$$x = -\frac{2}{3} + \frac{\sqrt{25 + 3y}}{3} = f^{-1}(y).$$

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answer

$$f^{-1}(y) = -\frac{2}{3} + \frac{\sqrt{25+3y}}{3}$$

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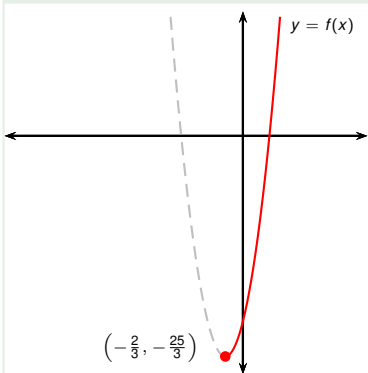
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Final answer, **relabelled**:

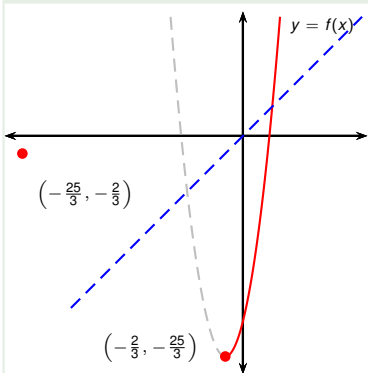
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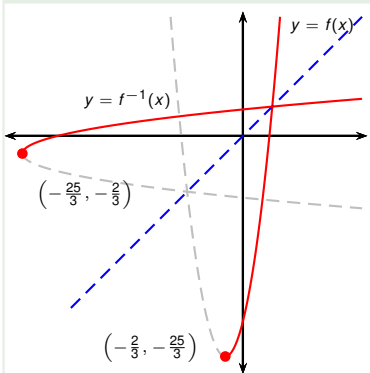
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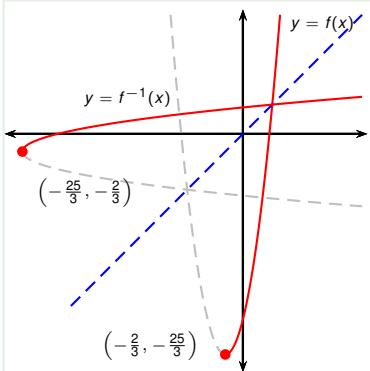
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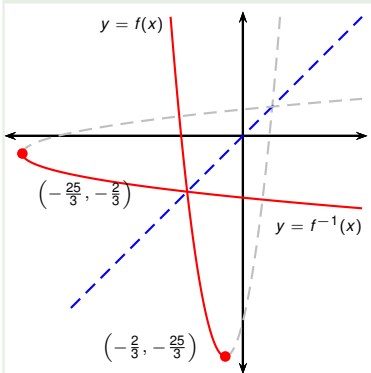
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Final answer, relabelled:

$$f^{-1}(x) = -\frac{2}{3} - \frac{\sqrt{25 + 3x}}{3}$$

We are given $x \leq -\frac{2}{3}$, therefore

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