Calculus II
Interval of convergence, part 3

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Example

Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$.

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- So it converges if $|x| < \frac{1}{3}$ and diverges if $|x| > \frac{1}{3}$.

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- The interval of convergence is $(-\frac{1}{3}, \frac{1}{3}]$.