Precalculus
Trig cofunction identities and angle-sum formulas

Todor Milev

2019

Outline

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Cofunction identities

Proposition (Cofunction identities)

$$\begin{array}{rclrcl}\sin\left(\frac{\pi}{2}-\alpha\right) & = & \cos\alpha & \sin\left(\frac{\pi}{2}+\alpha\right) & = & \cos\alpha\\ \cos\left(\frac{\pi}{2}-\alpha\right) & = & \sin\alpha & \cos\left(\frac{\pi}{2}+\alpha\right) & = & -\sin\alpha\end{array}$$

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- The proof each formula is broken into 4 cases depending on which quadrant contains α.

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- This makes a total of 4 formulas $\times$4 cases per formula = 16 cases.

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- This makes a total of 4 formulas $\times$4 cases per formula = 16 cases.
- We show only a few of the cases.
- The proof provides intuition why the formulas are true.
- The Quadrant I part of the proof serves as a visual aid for memorization.
- There is an algebraically simpler (but theoretically advanced) way to prove the above identities through the angle sum f-las, derived in turn from Euler's formula (studied later/in another course).

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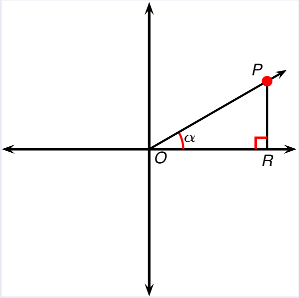
Part of Proof.



We are showing $\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$ when α is in quadrant I.

□

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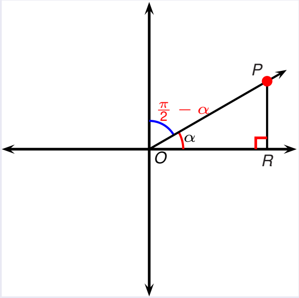

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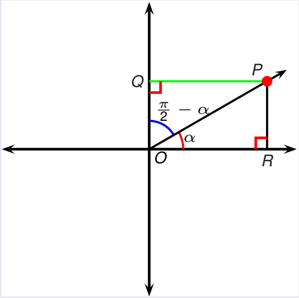

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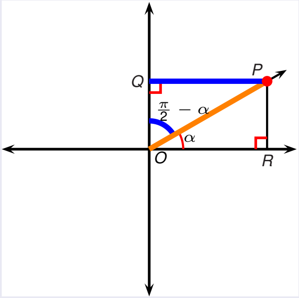

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$$\sin\left(\frac{\pi}{2}-\alpha\right) = \frac{|PQ|}{|OP|}$$

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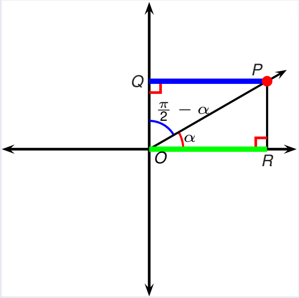

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$\square$

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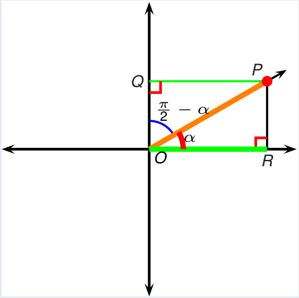

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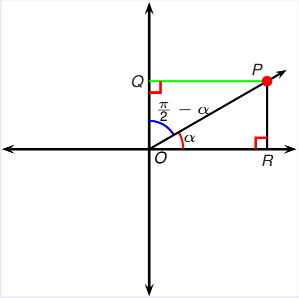

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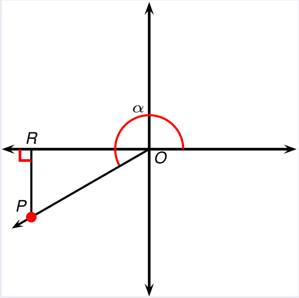

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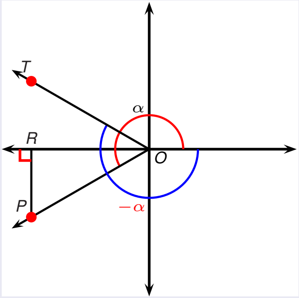

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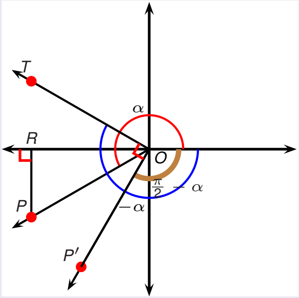

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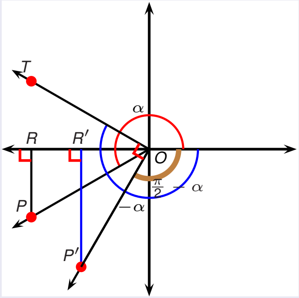

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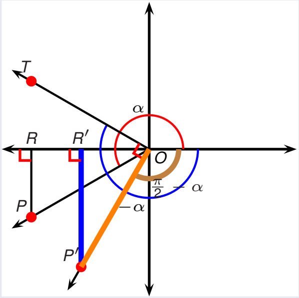

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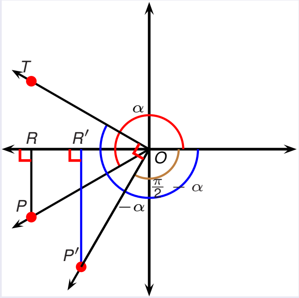

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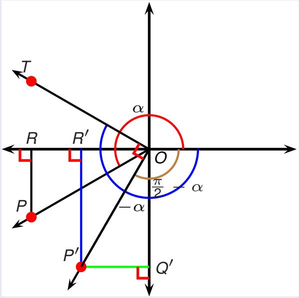

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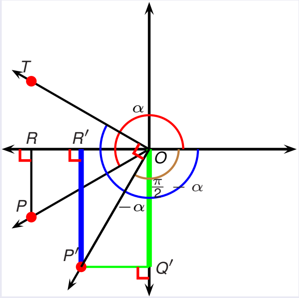

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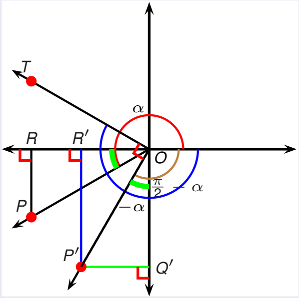

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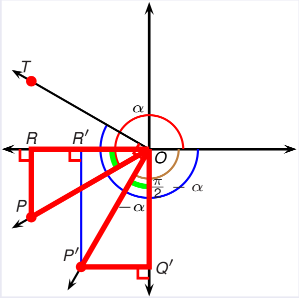

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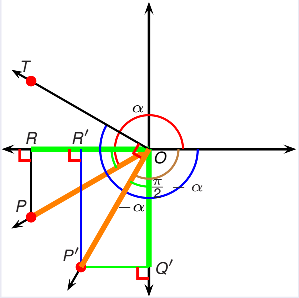

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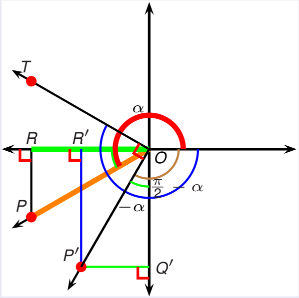

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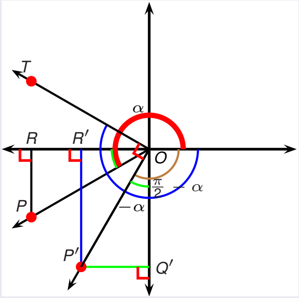

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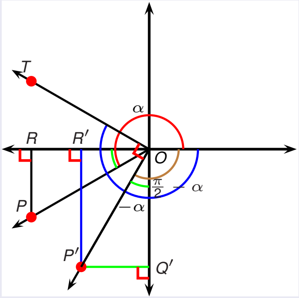

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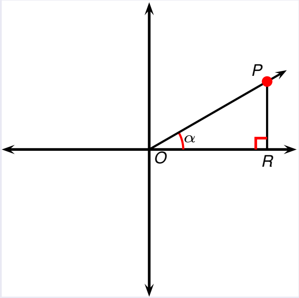

We show $\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$ when α is in Quadrant I.

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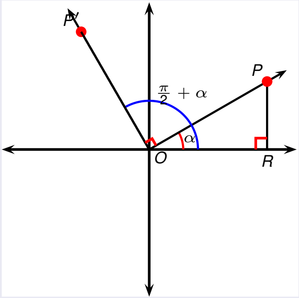

We show $\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$ when α is in Quadrant I.

$$\cos\left(\frac{\pi}{2}+\alpha\right) =$$

$\square$

Cofunction identities

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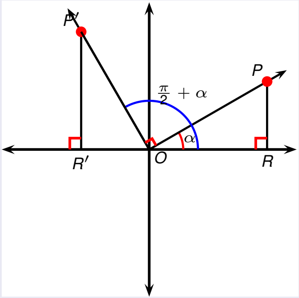

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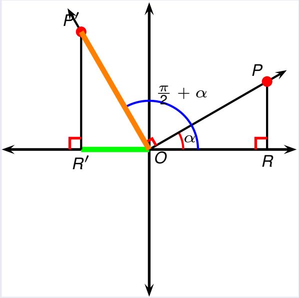

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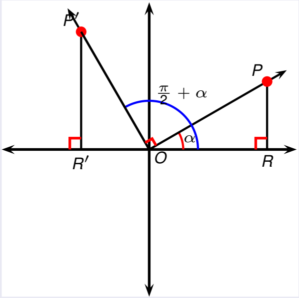

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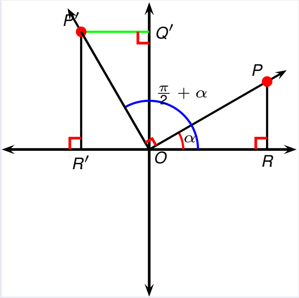

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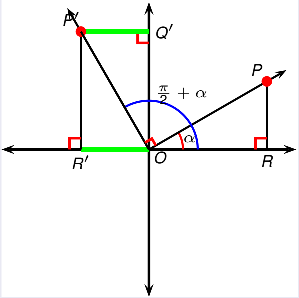

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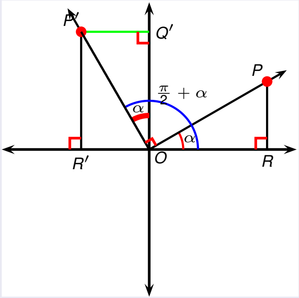

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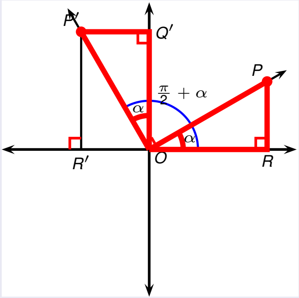

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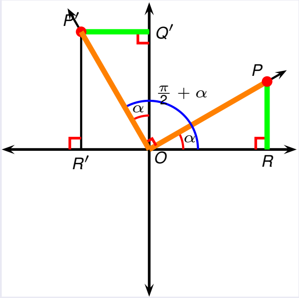

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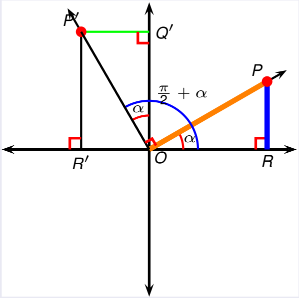

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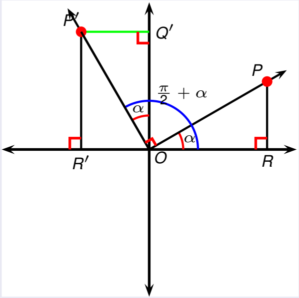

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□

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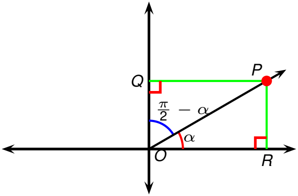
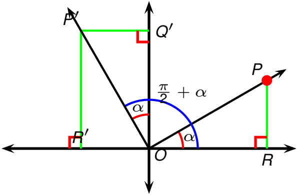
To memorize the cofunction identities it suffices to memorize the Quadrant I case via the two diagrams below.



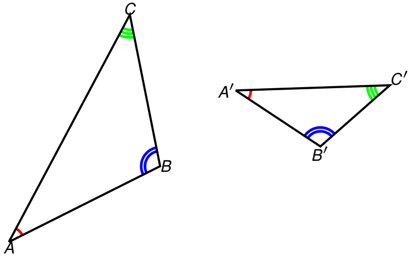
$$\begin{aligned}
\sin\left(\frac{\pi}{2}-\alpha\right) &= \frac{|PQ|}{|OP|}\\
\cos\left(\frac{\pi}{2}-\alpha\right) &= \frac{|OQ|}{|OP|}
\end{aligned}$$



$$\begin{aligned}
\sin\left(\frac{\pi}{2}+\alpha\right) &= \frac{|OQ'|}{|OP|}\\
\cos\left(\frac{\pi}{2}+\alpha\right) &= -\frac{|PQ'|}{|OP'|} = \frac{|PR|}{|OP|}
\end{aligned}$$

Definition (Similar triangles)

We say that a triangle $\triangle ABC$ is similar to a triangle $\triangle A'B'C'$ if the two triangles have equal angles.



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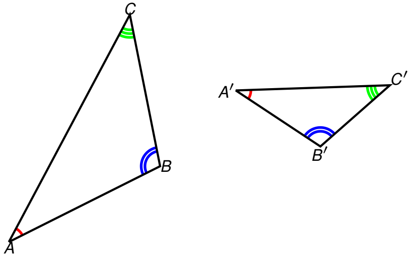
- The equal angles are assumed given in the same order for both triangles, that is, $\angle ABC = \angle A'B'C'$, $\angle BCA = \angle B'C'A'$, $\angle CAB = \angle C'A'B'$.



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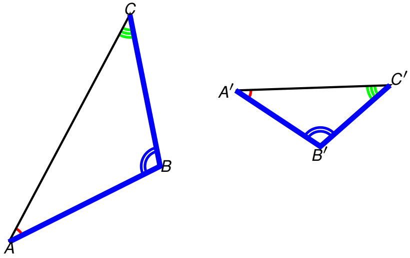

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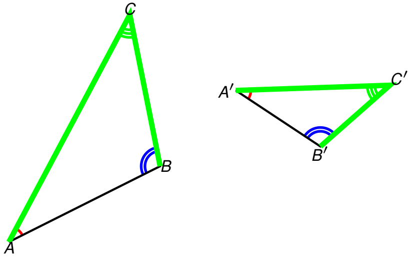

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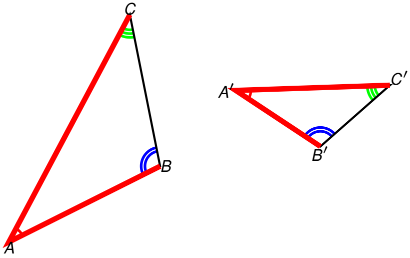
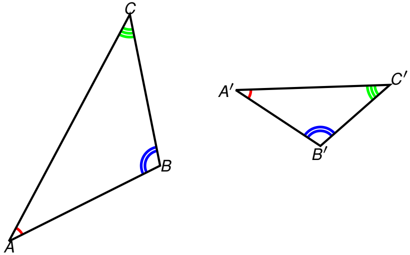


The following statement is proved in the subject of Euclidean (planar) geometry.

Theorem (Similar triangles have equal side ratios)

Let $\triangle ABC$ and $\triangle A'B'C'$ be two similar triangles. Then the ratios of the lengths of the sides of the two triangles are equal, that is

$$\frac{|AB|}{|BC|} = \frac{|A'B'|}{|B'C'|} \qquad \frac{|BC|}{|CA|} = \frac{|B'C'|}{|C'A'|} \qquad \frac{|CA|}{|AB|} = \frac{|C'A'|}{|A'B'|}$$



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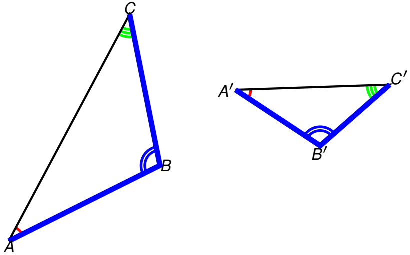

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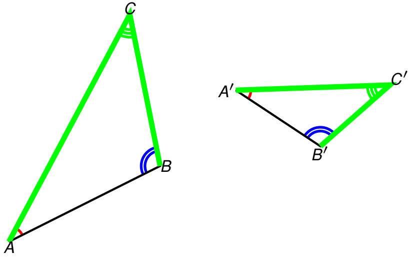

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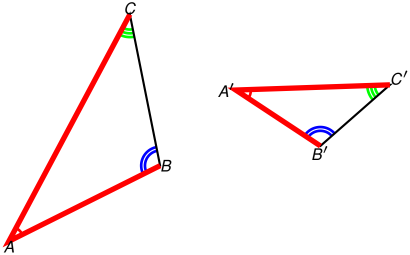

$\sin(\alpha+\beta), \cos(\alpha+\beta)$ via $\sin\alpha, \sin\beta, \cos\alpha, \cos\beta$

$\sin(\alpha+\beta) =$ **?**

$\cos(\alpha+\beta) =$ **?**

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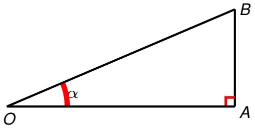

$\sin(\alpha + \beta) =$ **?**

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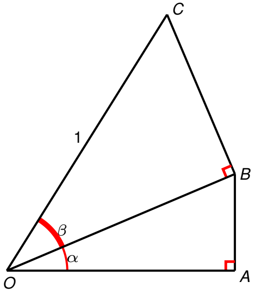

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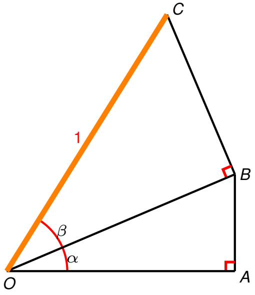

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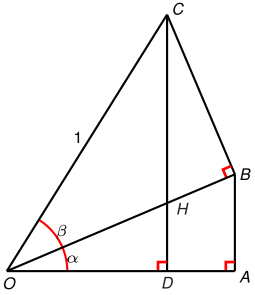

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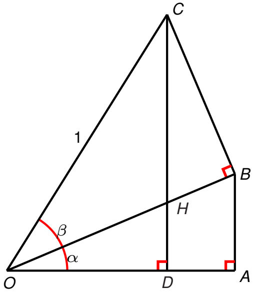

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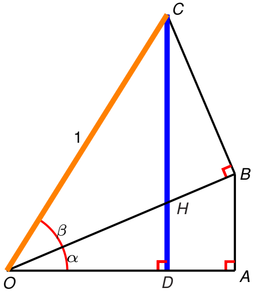

$$\sin(\alpha+\beta) = \frac{|CD|}{|OC|}$$

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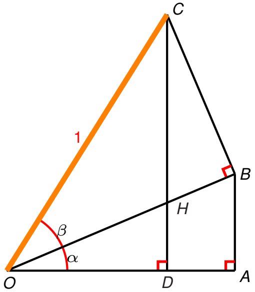

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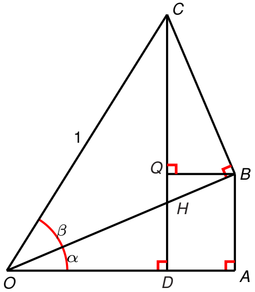

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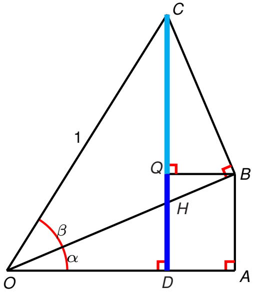

$$\sin(\alpha+\beta) = \frac{|CD|}{|OC|} = |CD| = |QD| + |CQ|$$

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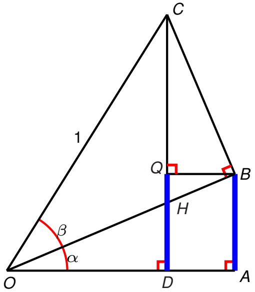

$|QD| = |BA|$ $\Big| \square DABQ$

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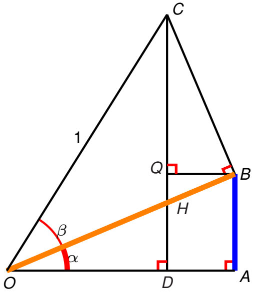
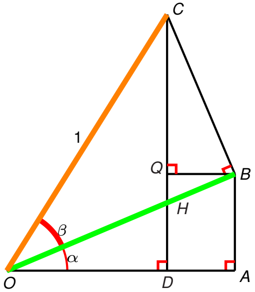


$$\sin(\alpha+\beta) = \frac{|CD|}{|OC|} = |CD|$$
$$= |QD| + |CQ|$$

$$\cos(\alpha+\beta) = \textbf{?}$$

$$|QD| = |BA| \qquad \Box DABQ$$
$$= \sin\alpha |OB| \qquad \triangle OAB$$

$\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$ via $\sin\alpha$, $\sin\beta$, $\cos\alpha$, $\cos\beta$

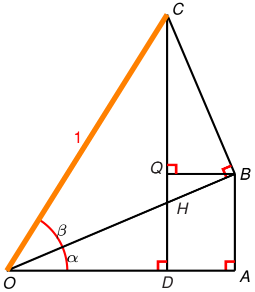


$$\begin{aligned}|QD| &= |BA| \\ &= \sin\alpha |OB| \\ &= \sin\alpha \cos\beta |OC|\end{aligned} \quad \left|\begin{array}{l}\square DABQ \\ \triangle OAB \\ \triangle OBC\end{array}\right.$$

$$\begin{aligned}\sin(\alpha+\beta) &= \frac{|CD|}{|OC|} = |CD| \\ &= |QD| + |CQ|\end{aligned}$$

$$\cos(\alpha+\beta) = \textbf{?}$$

$\sin(\alpha+\beta), \cos(\alpha+\beta)$ via $\sin\alpha, \sin\beta, \cos\alpha, \cos\beta$



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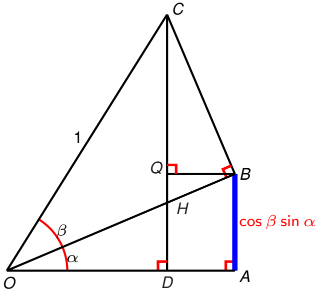

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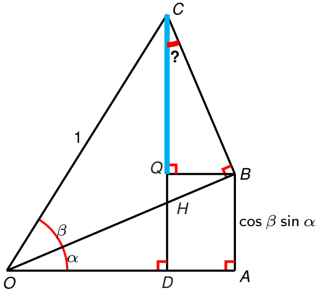

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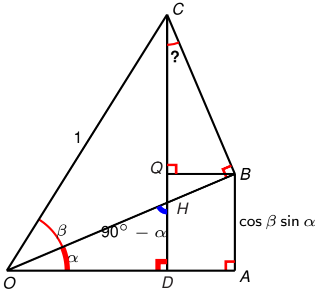

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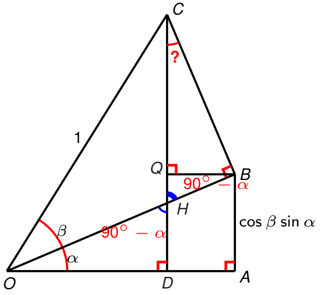

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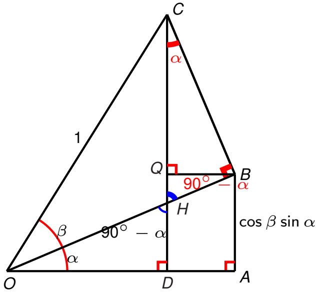

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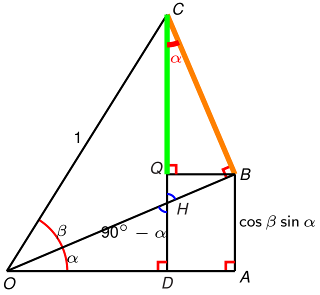

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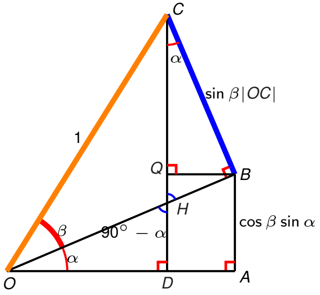

$$
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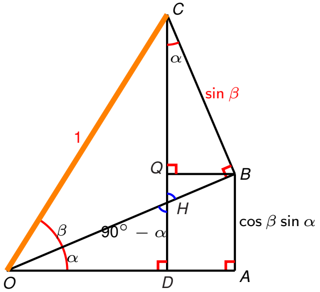

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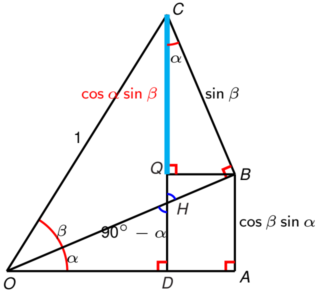

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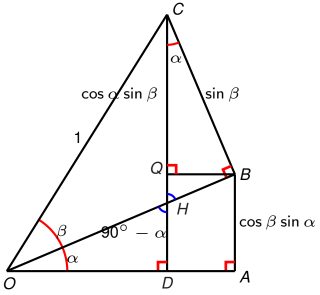

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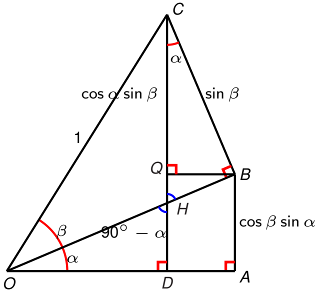

$$\sin(\alpha+\beta) = \frac{|CD|}{|OC|} = |CD| = |QD| + |CQ| = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

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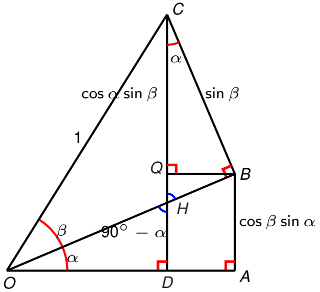

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Trig Functions of Sums and Differences of Angles

Theorem

$$\begin{aligned}\sin(\alpha+\beta) &= \sin\alpha\cos\beta+\cos\alpha\sin\beta\\ \cos(\alpha+\beta) &= \cos\alpha\cos\beta-\sin\alpha\sin\beta\end{aligned}$$

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- There is a theoretically more advanced (but algebraically simpler) proof using Euler's formula (to be studied later/in another course).

Trig Functions of Sums and Differences of Angles

Theorem

$$\begin{aligned}
\sin(\alpha+\beta) &= \sin\alpha\cos\beta+\cos\alpha\sin\beta\\
\sin(\alpha-\beta) &= \sin\alpha\cos\beta-\cos\alpha\sin\beta\\
\cos(\alpha+\beta) &= \cos\alpha\cos\beta-\sin\alpha\sin\beta\\
\cos(\alpha-\beta) &= \cos\alpha\cos\beta+\sin\alpha\sin\beta
\end{aligned}$$

- We gave a geometric proof of the sum formulas when the two angles are acute and their sum is less than $\pi = 90^\circ$.
- The theorem holds for all angles α, β without any restrictions.
- This can be shown by combining the preceding proof with identities such as $\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha$, $\cos\left(\frac{\pi}{2}+\alpha\right)=-\sin\alpha$.
- There is a theoretically more advanced (but algebraically simpler) proof using Euler's formula (to be studied later/in another course).
- The difference formulas are a consequence of the sum formulas and the fact that $\sin$ is an odd function and $\cos$ is even.

Trig Functions of Differences of Angles

Example

Prove the identities

$$\begin{aligned} \sin(\alpha-\beta) &= \sin\alpha\cos\beta-\cos\alpha\sin\beta \\ \cos(\alpha-\beta) &= \cos\alpha\cos\beta+\sin\alpha\sin\beta \end{aligned}$$

from the (already demonstrated) identities

$$\begin{aligned} \sin(\alpha+\beta) &= \sin\alpha\cos\beta+\cos\alpha\sin\beta \\ \cos(\alpha+\beta) &= \cos\alpha\cos\beta-\sin\alpha\sin\beta \end{aligned}$$

$$\begin{aligned} \sin(\alpha-\beta) &= \sin(\alpha+(-\beta)) \\ &= \sin\alpha\cos(-\beta)+\cos\alpha\sin(-\beta) && \left|\ \begin{array}{l} \cos \text{ is even },\\ \sin \text{ is odd} \end{array}\right. \\ &= \sin\alpha\cos\beta-\cos\alpha\sin\beta \\ \cos(\alpha-\beta) &= \cos(\alpha+(-\beta)) \\ &= \cos\alpha\cos(-\beta)-\sin\alpha\sin(-\beta) && \left|\ \begin{array}{l} \cos \text{ is even },\\ \sin \text{ is odd} \end{array}\right. \\ &= \cos\alpha\cos\beta+\cos\alpha\sin\beta \end{aligned}$$

Example

Find the exact value of the trigonometric function using radicals.

$\cos(105^\circ)$

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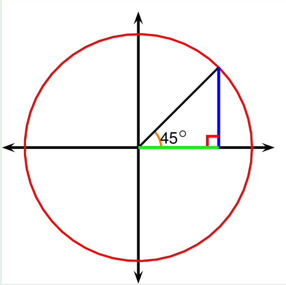
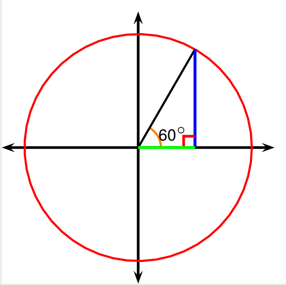
we know the trig f-ns of 45° and 60°





Example

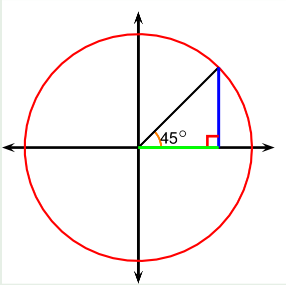
Find the exact value of the trigonometric function using radicals.

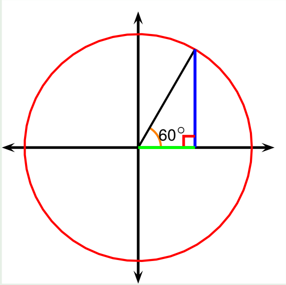
$\cos(105^\circ) = \cos(45^\circ + 60^\circ)$

$= \mathbf{?}$

we know the trig f-ns of 45° and 60°

Angle sum f-la





Example

Find the exact value of the trigonometric function using radicals.

$$\begin{aligned}\cos(105^\circ) &= \cos(45^\circ + 60^\circ) \\ &= \cos(45^\circ)\cos(60^\circ) - \sin(45^\circ)\sin(60^\circ)\end{aligned}$$

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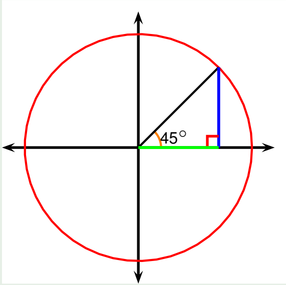
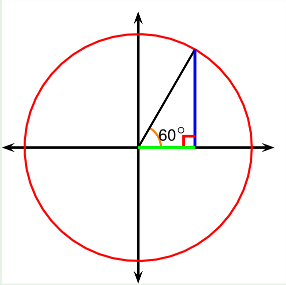
Angle sum f-la





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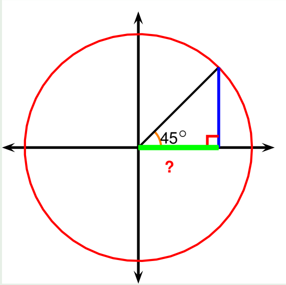
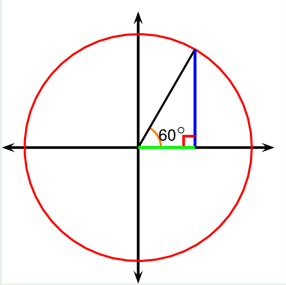
we know the trig f-ns of 45° and 60°

$$= \cos(45^\circ)\cos(60^\circ) - \sin(45^\circ)\sin(60^\circ)$$

Angle sum f-la

$$= \mathbf{?} \cdot \mathbf{?} - \mathbf{?} \cdot \mathbf{?}$$





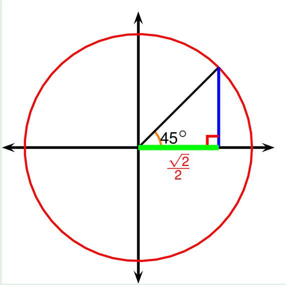
Example

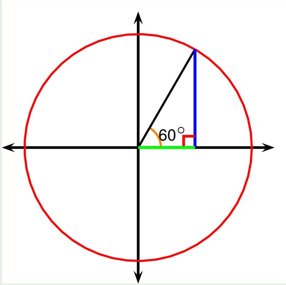
Find the exact value of the trigonometric function using radicals.

$$\cos(105^\circ) = \cos(45^\circ + 60^\circ)$$
$$= \cos(45^\circ)\cos(60^\circ) - \sin(45^\circ)\sin(60^\circ)$$
$$= \frac{\sqrt{2}}{2} \cdot \mathbf{?} - \mathbf{?} \quad \cdot \mathbf{?}$$

we know the trig f-ns of 45° and 60°

Angle sum f-la





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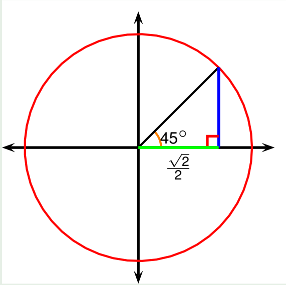
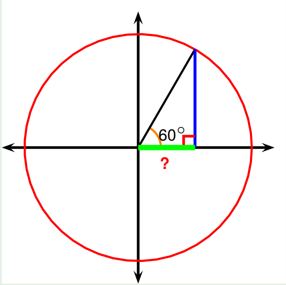




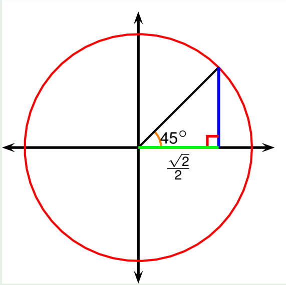
Example

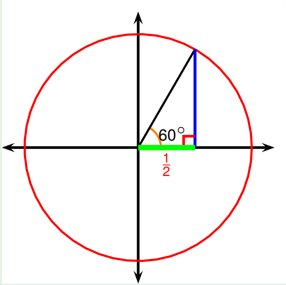
Find the exact value of the trigonometric function using radicals.

$$\begin{aligned} \cos(105^\circ) &= \cos(45^\circ + 60^\circ) \\ &= \cos(45^\circ)\, {\color{red}\cos(60^\circ)} - \sin(45^\circ)\sin(60^\circ) \\ &= \frac{\sqrt{2}}{2} \cdot {\color{red}\frac{1}{2}} - \textbf{?} \quad \cdot \textbf{?} \end{aligned}$$

we know the trig f-ns of 45° and 60°

Angle sum f-la





Example

Find the exact value of the trigonometric function using radicals.

$$\cos(105^\circ) = \cos(45^\circ + 60^\circ)$$
$$= \cos(45^\circ)\cos(60^\circ) - \sin(45^\circ)\sin(60^\circ)$$
$$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - ? \quad \cdot ?$$

we know the trig f-ns of 45° and 60°

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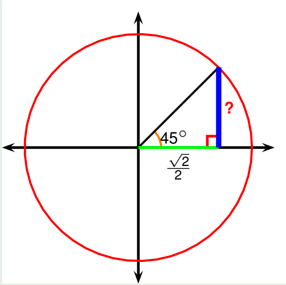

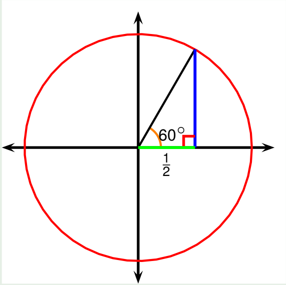

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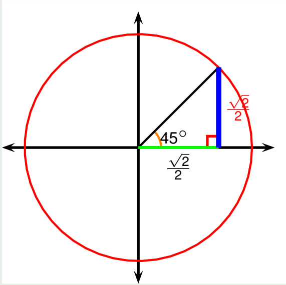

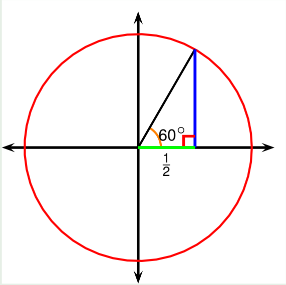

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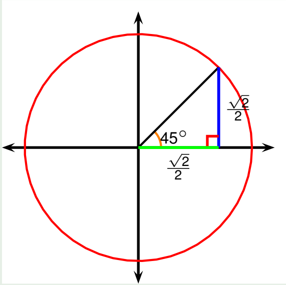

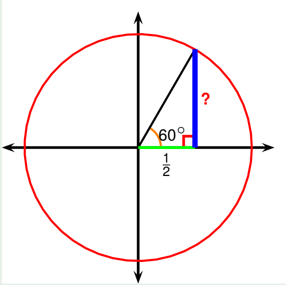

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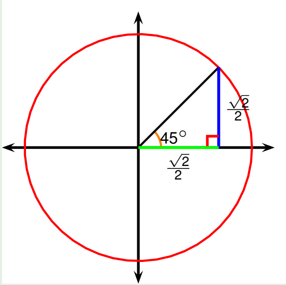

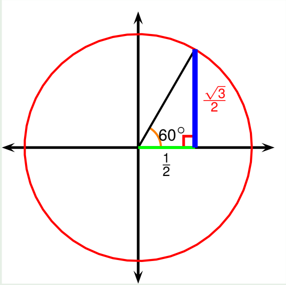

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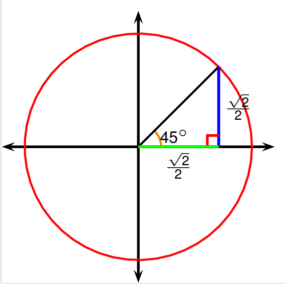

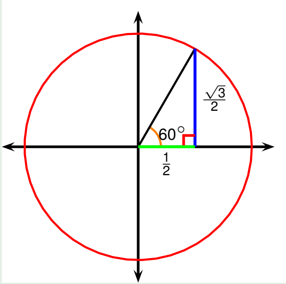

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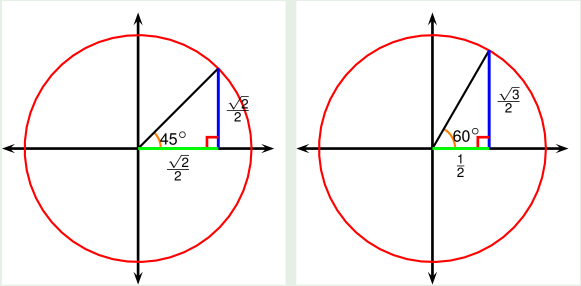

Example

Use the angle sum/difference formulas to simplify.

$$\cos\left(\frac{\pi}{2} - x\right)$$

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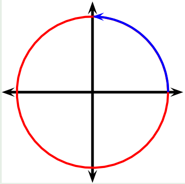


$$
\begin{aligned}
\cos\left(\frac{\pi}{2} - x\right) &= \cos\left(\frac{\pi}{2}\right)\cos x + \sin\left(\frac{\pi}{2}\right)\sin x \\
&= 0 \cdot \cos(x) + 1 \cdot \sin x \\
&= \sin x
\end{aligned}
$$

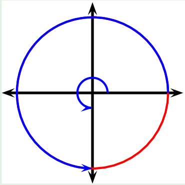
Example

Use the angle sum/difference formulas to simplify.

$$\cot\left(\frac{3\pi}{2}+x\right)$$

Example

Use the angle sum/difference formulas to simplify.



$$
\begin{aligned}
\cot\left(\frac{3\pi}{2}+x\right) &= \frac{\cos\left(\frac{3\pi}{2}+x\right)}{\sin\left(\frac{3\pi}{2}+x\right)}\\
&= \frac{\cos\left(\frac{3\pi}{2}\right)\cos x-\sin\left(\frac{3\pi}{2}\right)\sin x}{\sin\left(\frac{3\pi}{2}\right)\cos x+\cos\left(\frac{3\pi}{2}\right)\sin x}\\
&= \frac{0\cdot\cos x-(-1)\sin x}{(-1)\cos x+0\cdot\sin x}\\
&= \frac{\sin x}{-\cos x}=-\frac{\sin x}{\cos x}\\
&= -\tan x
\end{aligned}
$$

Example

Show that $\tan(\pi + x) = \tan x$ using the angle sum formulas.

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$$\begin{aligned}\tan(\pi + x) &= \frac{\sin(\pi + x)}{\cos(\pi + x)} \\ &= \frac{\sin\pi\cos x + \cos\pi\sin x}{\cos\pi\cos x - \sin\pi\sin x}\end{aligned}$$

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&= \frac{\textbf{?} \cdot \cos x + \textbf{?} \cdot \sin x}{\textbf{?} \cdot \cos x - \textbf{?} \cdot \sin x}
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$$

as desired.

Proposition ($\tan, \cot$ are π-periodic)

The tangent and cotangent functions are π-periodic, in other words,

$$\begin{aligned}\tan(\theta+\pi) &= \tan\theta\\ \cot(\theta+\pi) &= \cot\theta\end{aligned}$$

Recall the angle sum formula $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

Example

Show that the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ follows from the angle difference formula.

Recall the angle sum formula $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

Example

Show that the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ follows from the angle difference formula.

$$\begin{aligned} 1 &= \cos 0 \\ &= \cos(\theta - \theta) \\ &= \cos\theta\cos\theta + \sin\theta\sin\theta \\ &= \cos^2\theta + \sin^2\theta, \end{aligned}$$

as desired.

Example

Prove the angle sum formula $\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$.

$$\tan(\alpha+\beta) \quad =$$

Example

Prove the angle sum formula $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$.

$$
\begin{aligned}
\tan(\alpha+\beta) &= \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} \\
&= \frac{(\sin\alpha\cos\beta + \cos\alpha\sin\beta)\frac{1}{\cos\alpha\cos\beta}}{(\cos\alpha\cos\beta - \sin\alpha\sin\beta)\frac{1}{\cos\alpha\cos\beta}} \\
&= \frac{\frac{\sin\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}} + \frac{\cancel{\cos\alpha}\sin\beta}{\cancel{\cos\alpha}\cos\beta}}{\frac{\cancel{\cos\alpha\cos\beta}}{\cancel{\cos\alpha\cos\beta}} - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}} \\
&= \frac{\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}}{1 - \frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin\beta}{\cos\beta}} \\
&= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}
\end{aligned}
$$

Double angle formulas

Proposition (Double angle formulas)

$$
\begin{aligned}
\sin(2\alpha) &= 2\sin\alpha\cos\alpha \\
\cos(2\alpha) &= \cos^2\alpha - \sin^2\alpha \\
&= 2\cos^2\alpha - 1 \\
&= 1 - 2\sin^2\alpha
\end{aligned}
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- The double angle formulas play a special role in integration.

Example

Derive the double-angle formulas.

$$\sin(2\alpha) =$$

$$\cos(2\alpha) =$$

Example

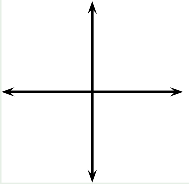
Derive the double-angle formulas.

$$
\begin{aligned}
\sin(2\alpha) &= \sin(\alpha+\alpha) \\
&= \sin\alpha\cos\alpha + \cos\alpha\sin\alpha \\
&= 2\sin\alpha\cos\alpha \\
\cos(2\alpha) &= \cos(\alpha+\alpha) \\
&= \cos\alpha\cos\alpha - \sin\alpha\sin\alpha \\
&= \cos^2\alpha - \sin^2\alpha \\
&= \cos^2\alpha - (1-\cos^2\alpha) \\
&= 2\cos^2\alpha - 1 \\
&= 1-\sin^2\alpha-\sin^2\alpha \\
&= 1-2\sin^2\alpha
\end{aligned}
$$

Example

Using radicals, find the exact value of the trigonometric expression.

$$\cos 105^\circ$$



Recall the half angle formula $\cos\alpha = \pm\sqrt{\frac{1+\cos(2\alpha)}{2}}$.

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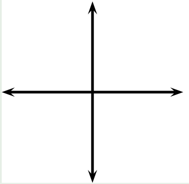

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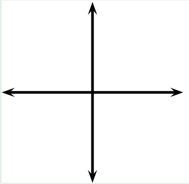
$$\cos 105^\circ = \pm\sqrt{\frac{1+\cos\left(2\cdot 105^\circ\right)}{2}}$$



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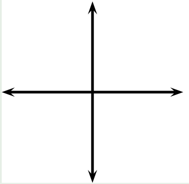
Using radicals, find the exact value of the trigonometric expression.

$$\cos 105^\circ = \pm\sqrt{\frac{1+\cos\left(2\cdot 105^\circ\right)}{2}} \qquad \Big|\ \cos 105^\circ \,\mathbf{?}\, 0$$



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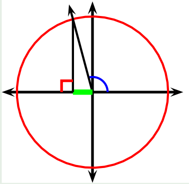
Using radicals, find the exact value of the trigonometric expression.

$$\cos 105^\circ = \pm\sqrt{\frac{1+\cos\left(2\cdot 105^\circ\right)}{2}} \qquad \Bigg| \; \cos 105^\circ < 0$$



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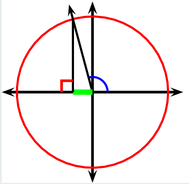

$$\begin{aligned}\cos 105^\circ &= \pm\sqrt{\frac{1+\cos\left(2\cdot 105^\circ\right)}{2}} \\ &= -\sqrt{\frac{1+\cos\left(210^\circ\right)}{2}}\end{aligned}$$

$\cos 105^\circ < 0$

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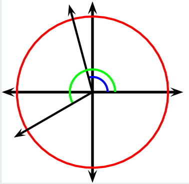

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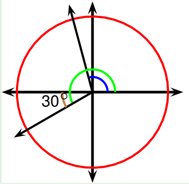

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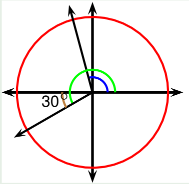
$$\begin{aligned}\cos 105^\circ &= \pm\sqrt{\frac{1+\cos(2\cdot 105^\circ)}{2}} && \Big|\ \cos 105^\circ < 0\\ &= -\sqrt{\frac{1+\cos(210^\circ)}{2}}\\ &= -\sqrt{\frac{1-\cos(30^\circ)}{2}}\end{aligned}$$



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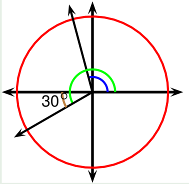


$$\begin{aligned}
\cos 105^\circ &= \pm\sqrt{\frac{1+\cos(2\cdot 105^\circ)}{2}} \\
&= -\sqrt{\frac{1+\cos(210^\circ)}{2}} \\
&= -\sqrt{\frac{1-\cos(30^\circ)}{2}} \\
&= -\sqrt{\frac{1-?}{2}}
\end{aligned}
\qquad \Bigg| \quad \cos 105^\circ < 0$$

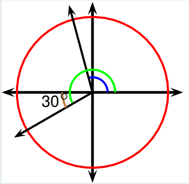
Recall the half angle formula $\cos\alpha = \pm\sqrt{\frac{1+\cos(2\alpha)}{2}}$.

Example

Using radicals, find the exact value of the trigonometric expression.

$$\begin{aligned}\cos 105^\circ &= \pm\sqrt{\frac{1+\cos(2\cdot 105^\circ)}{2}} \\ &= -\sqrt{\frac{1+\cos(210^\circ)}{2}} \\ &= -\sqrt{\frac{1-\cos(30^\circ)}{2}} \\ &= -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}\end{aligned}$$

$\cos 105^\circ < 0$



Recall the half angle formula $\cos\alpha = \pm\sqrt{\frac{1+\cos(2\alpha)}{2}}$.

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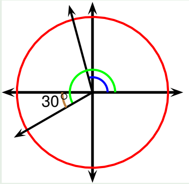

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&= -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2-\sqrt{3}}{2\cdot 2}}
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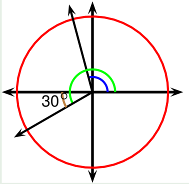

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Proposition (Power-Reducing Formulas)

$$\sin^2\alpha = \frac{1-\cos(2\alpha)}{2} \qquad \cos^2\alpha = \frac{1+\cos(2\alpha)}{2}$$

Proof.

$\square$

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$$\cos(2\alpha) = 1 - 2\sin^2\alpha$$

□

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$$\begin{aligned} \cos(2\alpha) &= 1 - 2\sin^2\alpha \\ 2\sin^2\alpha &= 1-\cos(2\alpha) \end{aligned}$$

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Corollary

$$\sin\alpha = \pm\sqrt{\frac{1-\cos(2\alpha)}{2}} \qquad \cos\alpha = \pm\sqrt{\frac{1+\cos(2\alpha)}{2}}$$

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Corollary (Half-Angle Formulas)

$$\sin\left(\frac{\beta}{2}\right) = \pm\sqrt{\frac{1-\cos\beta}{2}} \qquad \cos\left(\frac{\beta}{2}\right) = \pm\sqrt{\frac{1+\cos\beta}{2}}$$

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- The power reducing formulas are used to express $\sin^k \alpha$ and $\cos^k \alpha$ via lower powers of the $\sin$ and $\cos$ functions (applied to angles other than α).

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- The power reducing formulas are used to express $\sin^k\alpha$ and $\cos^k\alpha$ via lower powers of the $\sin$ and $\cos$ functions (applied to angles other than α).
- This technique will play a key role in integration (studied later/in another course).

Example

Rewrite $\sin^4 \alpha$ in terms of first powers of the cosines and sines of multiples of the angle α.

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$$\sin^{4} \alpha = \left(\sin^{2} \alpha\right)^{2}$$

Recall the formulas: $\sin^2\beta = \textbf{?}$, $\cos^2\beta = \textbf{?}$.

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Rewrite $\sin^4\alpha$ in terms of first powers of the cosines and sines of multiples of the angle α.

$$\begin{aligned}\sin^4\alpha &= \left(\sin^2\alpha\right)^2\\ &= \left(\textbf{?} \qquad\qquad\right)^2\end{aligned}$$

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$$\begin{aligned}\sin^4\alpha &= \left(\sin^2\alpha\right)^2\\ &= \left(\frac{1-\cos(2\alpha)}{2}\right)^2\\ &= \frac{1}{4}\left(\textbf{?} \qquad\qquad\qquad\qquad \right)\end{aligned}$$

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$$\begin{aligned}\sin^4\alpha &= \left(\sin^2\alpha\right)^2\\ &= \left(\frac{1-\cos(2\alpha)}{2}\right)^2\\ &= \frac{1}{4}\left(1-2\cos(2\alpha)+\cos^2(2\alpha)\right)\end{aligned}$$

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&= \frac{1}{8}\left(3-4\cos(2\alpha)+\cos(4\alpha)\right)
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