

Precalculus

Computing sine, cosine of a half-angle

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Recall the half angle formula $\cos \alpha = \pm \sqrt{\frac{1 + \cos(2\alpha)}{2}}$.

Example

Using radicals, find the exact value of the trigonometric expression.

$$\begin{aligned}
 \cos 105^\circ &= \pm \sqrt{\frac{1 + \cos(2 \cdot 105^\circ)}{2}} && \left| \cos 105^\circ < 0 \right. \\
 &= -\sqrt{\frac{1 + \cos(210^\circ)}{2}} \\
 &= -\sqrt{\frac{1 - \cos(30^\circ)}{2}} \\
 &= -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2 - \sqrt{3}}{2 \cdot 2}} \\
 &= -\frac{\sqrt{2 - \sqrt{3}}}{2}
 \end{aligned}$$

