Precalculus
Trigonometry and triangles

Todor Milev

2019

Outline

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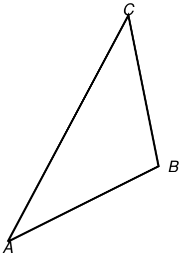
Triangle area = $\frac{1}{2}$base · height

Proposition (Triangle area)

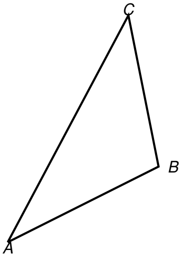
$$Area(\triangle ABC) = \textbf{?} \quad .$$



Triangle area = $\frac{1}{2}$base · height

Proposition (Triangle area)

$$Area(\triangle ABC) = \frac{1}{2} height \cdot base \quad .$$



Triangle area = $\frac{1}{2}$base · height

Let $\triangle ABC$ have side length a and height length h_a, as indicated - side a is opposite to vertex A and h_a starts at A

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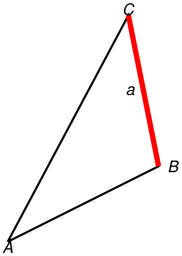
$$Area(\triangle ABC) = \frac{1}{2} height \cdot base = \frac{1}{2} h_a a .$$



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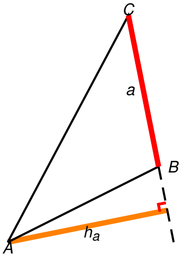

Triangle area = $\frac{1}{2}$base · height

Let $\triangle ABC$ have side lengths a, b and height lengths h_a, h_b, as indicated - side a is opposite to vertex A and h_a starts at A, and so on.

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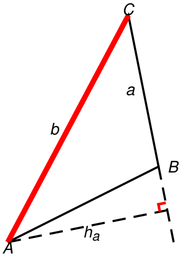

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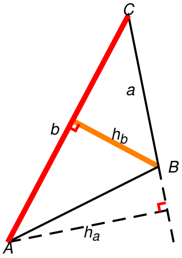

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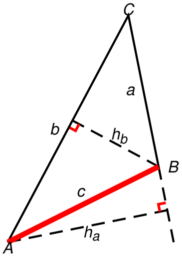

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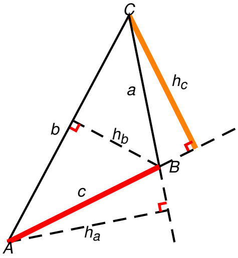


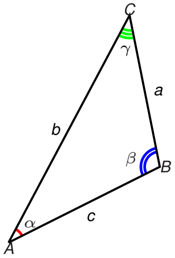
Triangle area from two sides and angle between them

Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ, as indicated: α is opposite to a, β is opposite to b, γ is opposite to c.

Proposition ($\triangle$ area from two sides and angle between them)

The area of a triangle is half the product of the lengths of two of its sides times the sine of the angle between them. In other words,

$$Area(\triangle ABC) = \frac{ab\sin\gamma}{2} = \frac{bc\sin\alpha}{2} = \frac{ca\sin\beta}{2}$$



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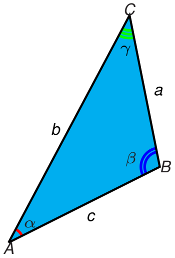

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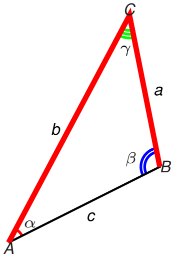

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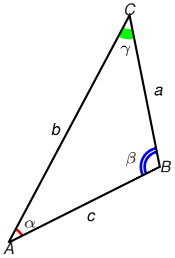

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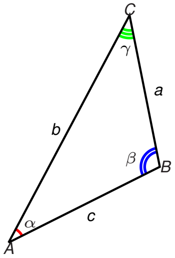

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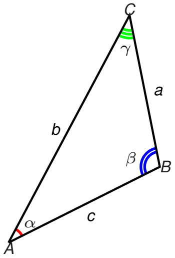

Proof.

$$\text{Area}(\triangle ABC) = \frac{\text{base}\cdot\text{height}}{2}$$

$\square$

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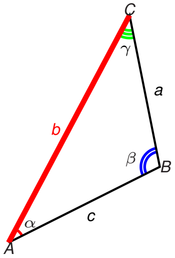

Proof.

$$\text{Area}(\triangle ABC) = \frac{\text{base}\cdot\text{height}}{2} = \frac{bh_b}{2}$$

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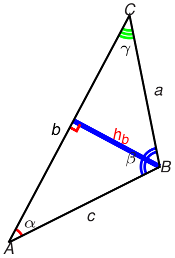

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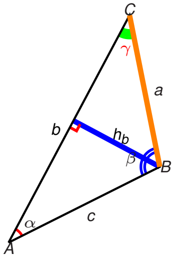

Proof.

$$\begin{aligned} \mathrm{Area}(\triangle ABC) &= \frac{\text{base} \cdot \text{height}}{2} = \frac{b h_b}{2} \\ &= \frac{b a \sin\gamma}{2}. \end{aligned}$$

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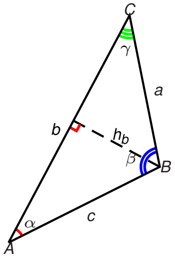

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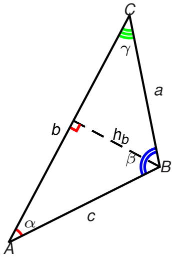

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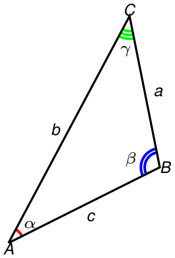
The proof of the other two cases is similar. □

Law of sines

Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ, as indicated: α is opposite to a, β is opposite to b, γ is opposite to c.

Proposition (Law of Sines)

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}.$$



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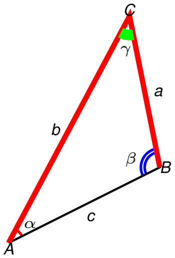

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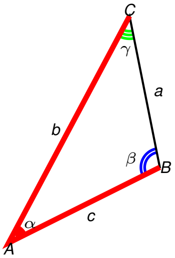

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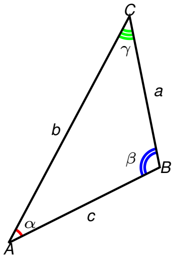

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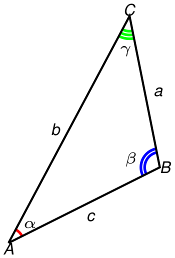

Proof.

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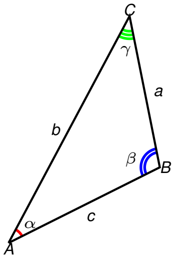

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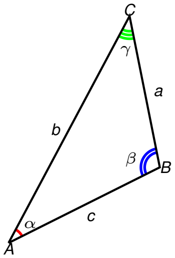

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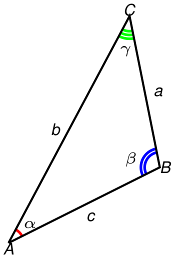

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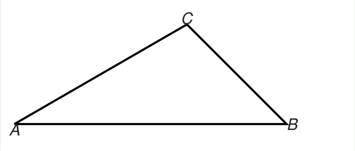
The remaining cases are similar. $\square$

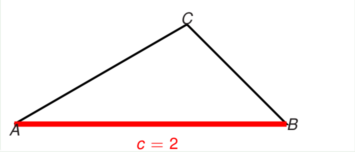
Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
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- Let the known side be $c = 2cm$.

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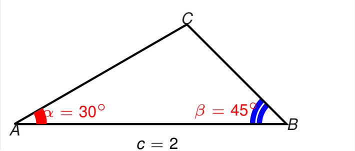

A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
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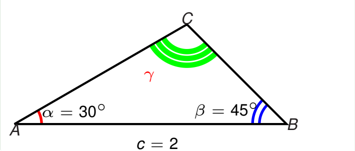
- Let the known side be $c = 2cm$.
- Let the known angles $30^\circ, 45^\circ$ be arranged as in the figure

Example



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- Find the other two sides of the triangle.
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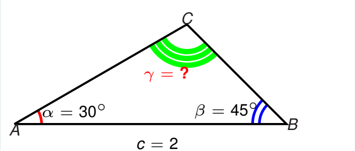
- Let the known side be $c = 2cm$.
- Let the known angles 30°, 45° be arranged as in the figure, and let the third angle be γ .

Example



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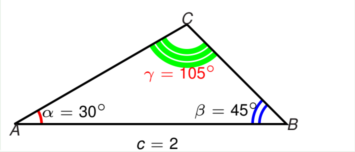
- Let the known angles 30°, 45° be arranged as in the figure, and let the third angle be $\gamma = ?$.

Example



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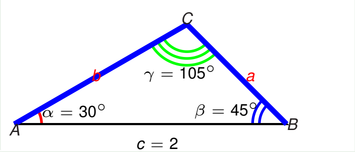
Example



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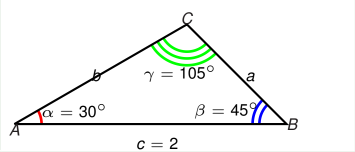
- Find the area of the triangle.

- Let the known side be $c = 2cm$.
- Let the known angles $30°$, $45°$ be arranged as in the figure, and let the third angle be $\gamma = 180° - 30° - 45° = 180° - 75° = 105°$.
- Label the unknown sides a, b as indicated.

Example



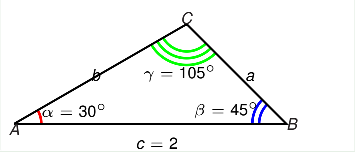
A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

|Law of sines

Example



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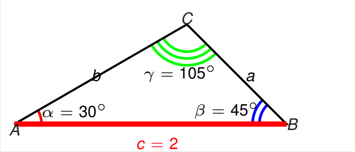
$$a = \frac{c\sin\alpha}{\sin\gamma}$$

Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
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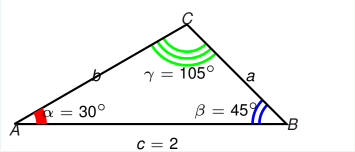
$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \qquad |\text{Law of sines}$$

$$a = \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ}$$

Example



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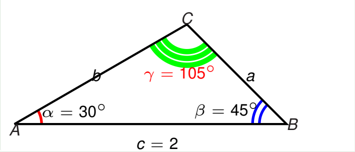

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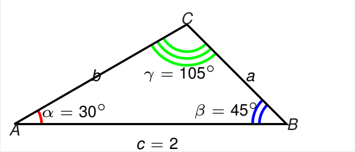

A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

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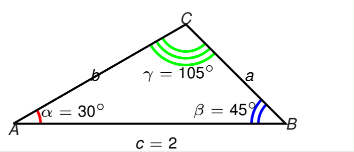
$\sin 105^\circ$

$$
\begin{aligned}
\frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} && |\text{Law of sines}\\
a &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ}
\end{aligned}
$$

Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

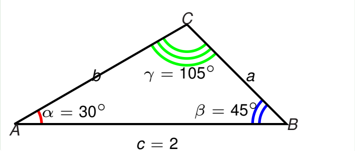
- Find the other two sides of the triangle.
- Find the area of the triangle.

$$\sin 105° = \sin(60° + 45°)$$

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \qquad |\text{Law of sines}$$

$$a = \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30°}{\sin 105°}$$

Example



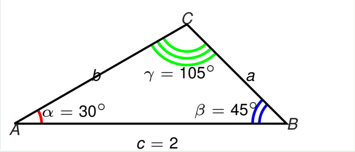
A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

$$\sin 105^\circ = \sin(60^\circ + 45^\circ) = \mathbf{?}$$

$$\begin{aligned} \frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} && |\text{Law of sines} \\ a &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ} \end{aligned}$$

Example



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$$\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

$$\begin{aligned} \frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} && \text{|Law of sines} \\ a &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ} \end{aligned}$$

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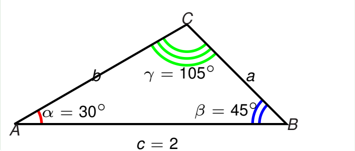

A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

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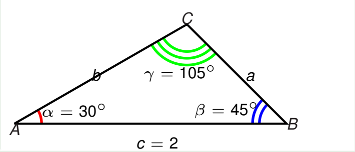
$$\begin{aligned}
\sin 105^\circ &= \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ \\
&= ? \quad ? \quad + ?\ ? \\
\frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} & &|\text{Law of sines} \\
a &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ}
\end{aligned}$$

Example



A triangle has a side of length 2*cm*; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

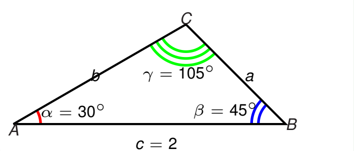
$$\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2}\,\mathbf{?} \quad + \mathbf{?}\;\mathbf{?}$$

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \qquad |\text{Law of sines}$$

$$a = \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ}$$

Example



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$$= \frac{\sqrt{3}}{2} ? \quad + ??$$

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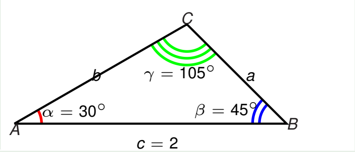
Example



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&= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \mathbf{??} \\
\frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} \qquad\qquad |\text{Law of sines} \\
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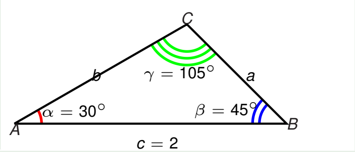
Example



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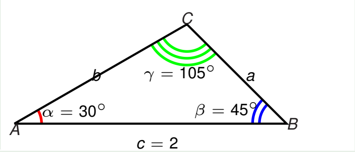

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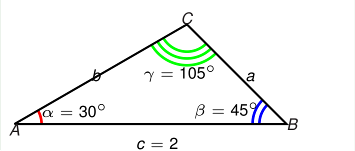
Example



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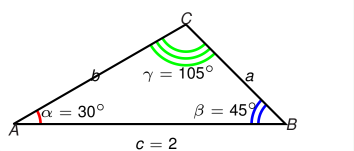
Example



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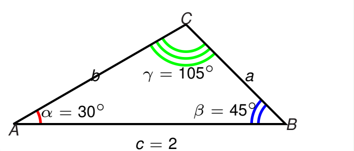
Example



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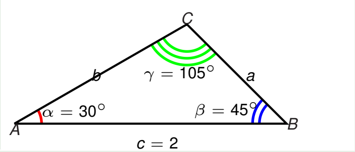
Example



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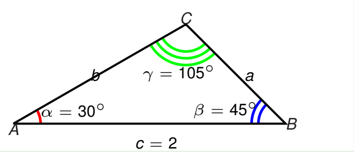
Example



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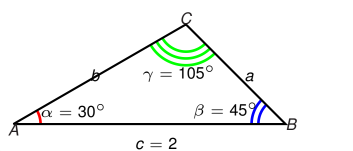
Example



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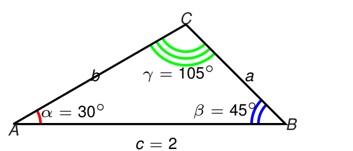
Example



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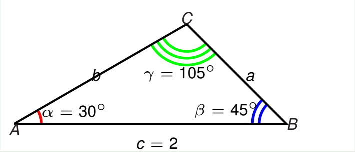
Example



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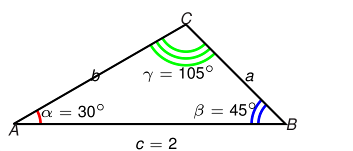

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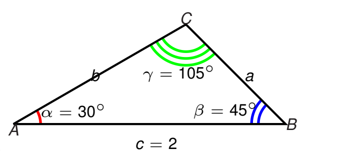
a &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ} = \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{4}{(\sqrt{6}+\sqrt{2})}
\end{aligned}$$

Example



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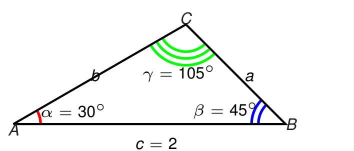
$$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \qquad |\text{Law of sines}$$

$$a = \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ} = \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{4(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})}$$

Example



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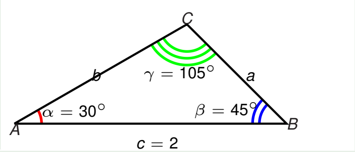
&= \frac{4(\sqrt{6}-\sqrt{2})}{6-2}
\end{aligned}$$

Example



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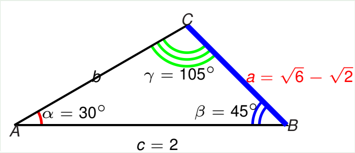
&= \frac{4(\sqrt{6}-\sqrt{2})}{6-2} = \sqrt{6}-\sqrt{2}
\end{aligned}$$

Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

$$\begin{aligned}
\sin 105^\circ &= \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ \\
&= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4} \\
\frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} \qquad |\text{Law of sines} \\
{\color{red}a} &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ} = \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{4(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})} \\
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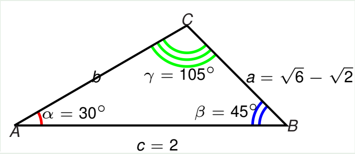
\end{aligned}$$

Example



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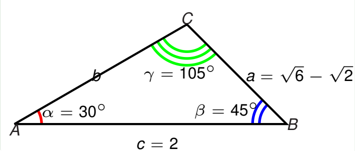
|Law of sines

Example



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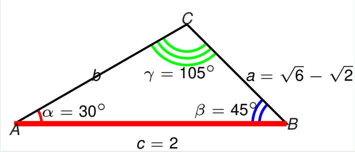

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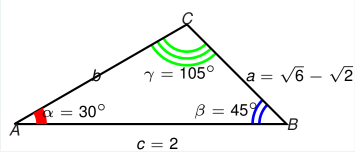
$$b = \frac{c\sin\beta}{\sin\gamma} = \frac{2\sin 45^\circ}{\sin 105^\circ}$$

Example



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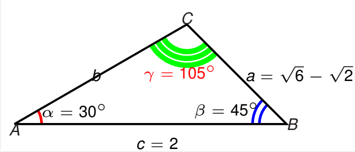

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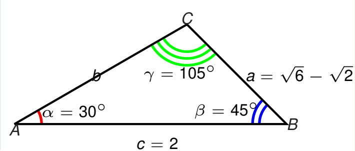

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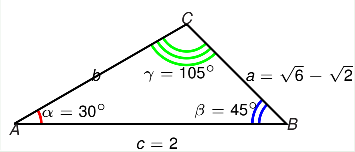
Example



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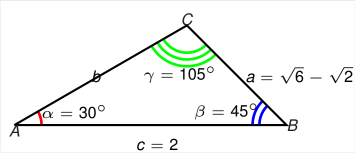

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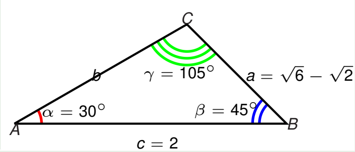
Example



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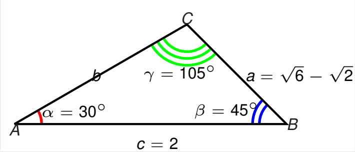

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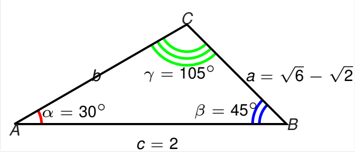

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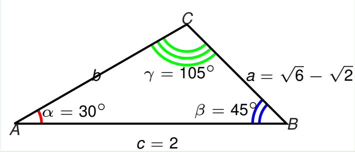
Example



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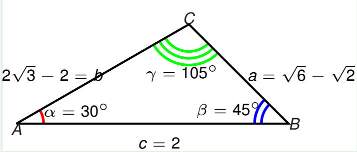

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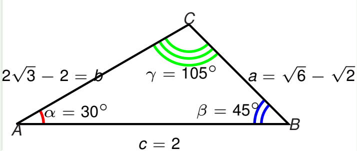
Example



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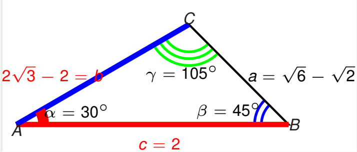

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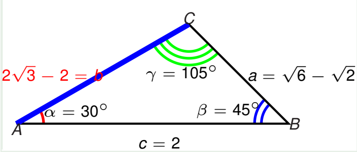
$$\text{Area} = \frac{bc\sin\alpha}{2}$$

Example



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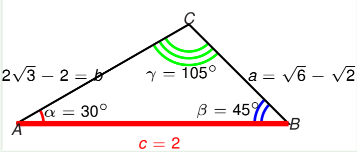
Example



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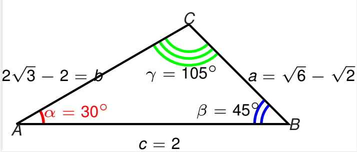

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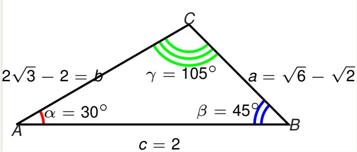

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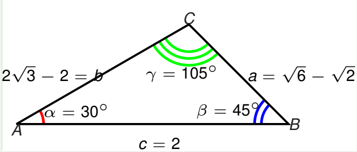

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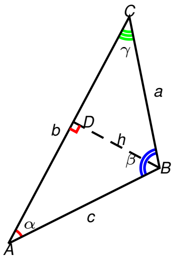
Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ, as indicated.

Proposition (Law of Cosines)

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos\gamma \\ a^2 &= b^2 + c^2 - 2bc\cos\alpha \\ b^2 &= c^2 + a^2 - 2ca\cos\beta \end{aligned}$$



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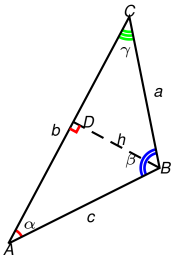

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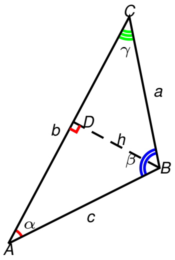

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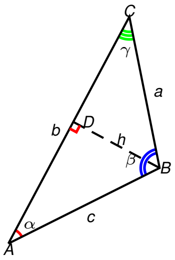

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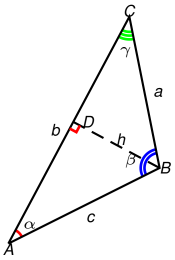


Proof if $\gamma < 90^\circ$.

Drop a perpendicular h from B to AC.

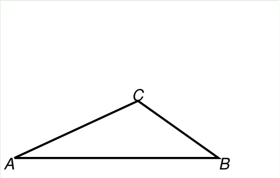
$$|CD| = a\cos\gamma$$
$$h = a\sin\gamma$$
$$|AD| = b - |CD| = b - a\cos\gamma$$

$$c^2 = |AD|^2 + h^2 \qquad \left|\ \text{Pyth. thm. } \triangle BDA\right.$$

$$\begin{aligned} &= (b - a\cos\gamma)^2 + (a\sin\gamma)^2 \\ &= b^2 - 2ab\cos\gamma + a^2\cos^2\gamma + a^2\sin^2\gamma \\ &= b^2 - 2ab\cos\gamma + a^2. \end{aligned}$$

$\square$

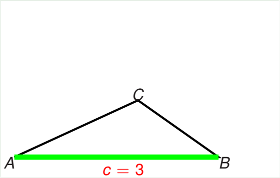
Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

Example



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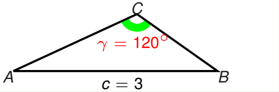

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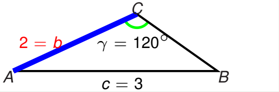
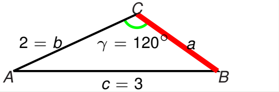
Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

Example



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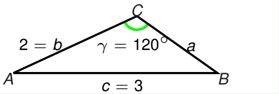

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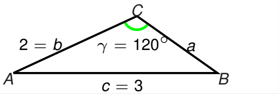
Law of cosines

Example



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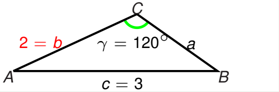
$$a^2 + b^2 - 2ab\cos\gamma = c^2 \quad \text{Law of cosines}$$

Example



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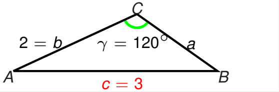

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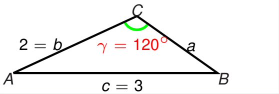

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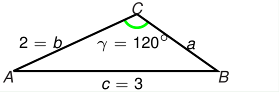

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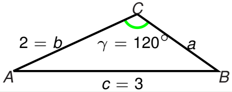
$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$	Solve for a :

Example



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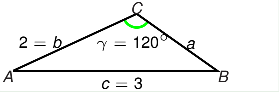
$$a^2 - 4a\left(\qquad \right) - 5 = 0$$

Example



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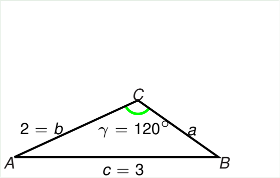

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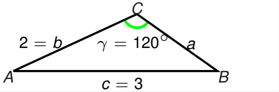

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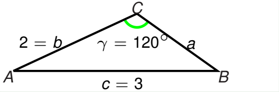
Example



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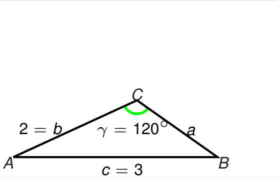

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| Law of cosines
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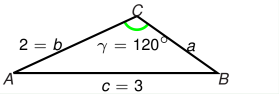
Example



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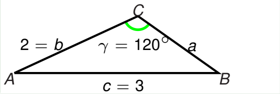

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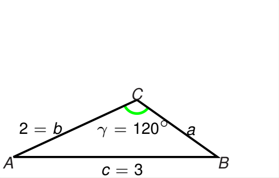

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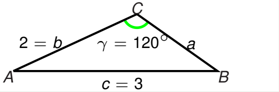

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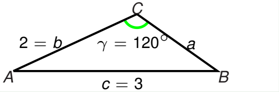

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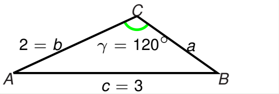

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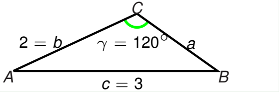
$$
\begin{aligned}
a &= \frac{-2\pm\sqrt{2^2-4\cdot(-5)\cdot 1}}{2\cdot 1}\\
&= \frac{-2\pm\sqrt{24}}{2}
\end{aligned}
$$

Example



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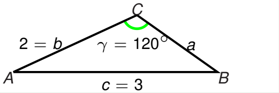
$$\begin{aligned} a &= \frac{-2 \pm \sqrt{2^2 - 4\cdot(-5)\cdot 1}}{2\cdot 1} \\ &= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 + 2\sqrt{6}}{2} \end{aligned}$$

Example



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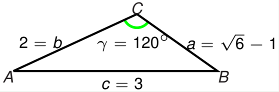

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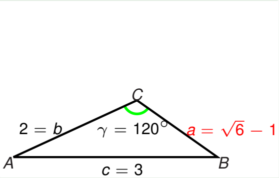
$$= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 + 2\sqrt{6}}{2} \qquad \Big| a > 0$$

$$= -1 + \sqrt{6}$$

Example



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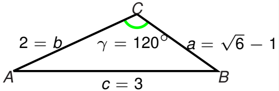

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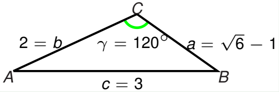
Example



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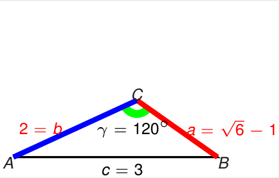
Area = ?

Example



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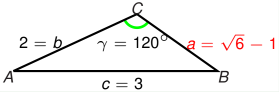
$$\textbf{Area} = \frac{ab\sin\gamma}{2}$$

Example



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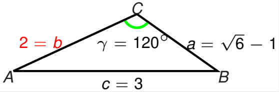
$$\textbf{Area} = \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6}-1\right)2}{2}\textbf{?}$$

Example



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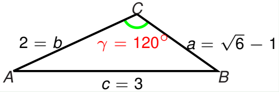
$$\textbf{Area} = \frac{a{\color{red}b}\sin\gamma}{2} = \frac{\left(\sqrt{6} - 1\right){\color{red}2}}{2}\textbf{?}$$

Example



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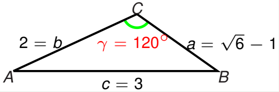
$$\textbf{Area} = \frac{ab \sin\gamma}{2} = \frac{\left(\sqrt{6} - 1\right) 2}{2} \textbf{?}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a\cdot 2\cdot\cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4\cdot(-5)\cdot 1}}{2\cdot 1} = -1 + \sqrt{6}$$

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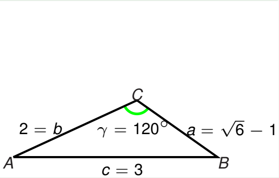

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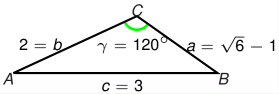

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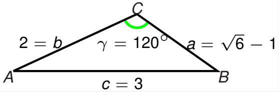

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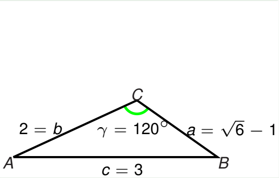

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