

Precalculus

Cubic inequality

Todor Milev

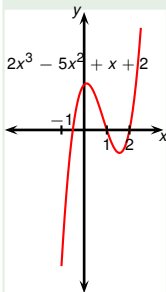
2019

Example

Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality.

$$2x^3 - 5x^2 + x + 2 > 0$$

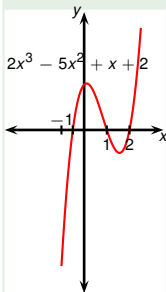
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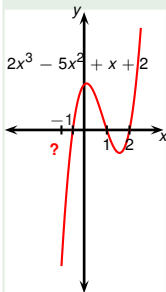


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$$? (x - \quad) (x - \quad) (x - \quad) > 0$$

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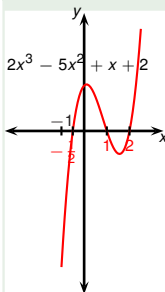


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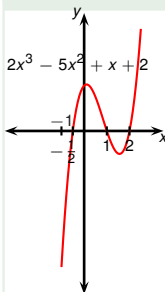


Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality.

$$2x^3 - 5x^2 + x + 2 > 0$$

$$? (x - (-\frac{1}{2})) (x - 1) (x - 2) > 0$$

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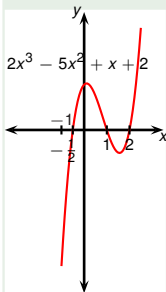


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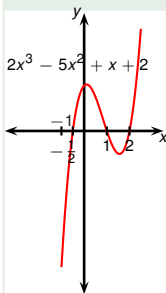


Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality.

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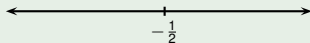


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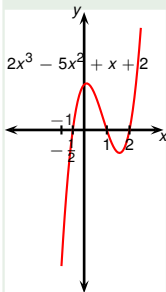
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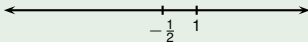


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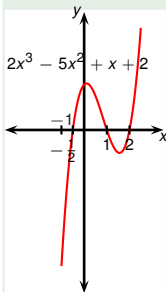
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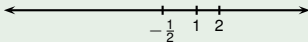


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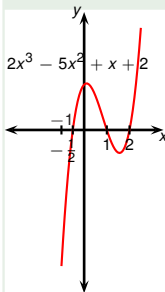
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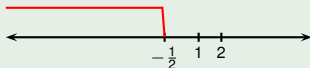


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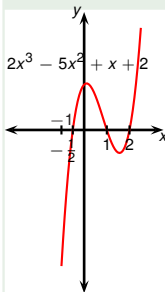
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Interval	Factor signs	Final sign from plot
$(-\infty, -\frac{1}{2})$		

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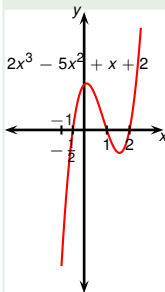
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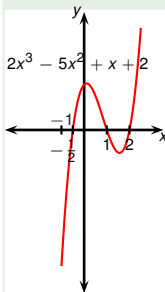
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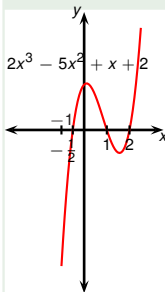
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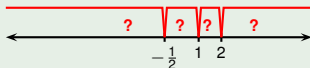


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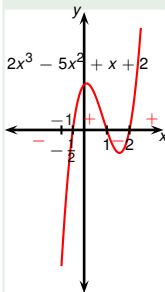
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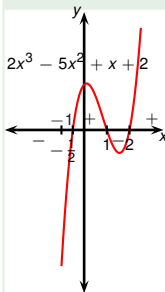
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$(-\frac{1}{2}, 1)$	$(+)(-)(-)$	+
$(1, 2)$	$(+)(+)(-)$	-
$(2, \infty)$	$(+)(+)(+)$	+

Example



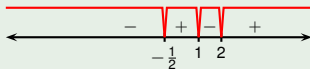
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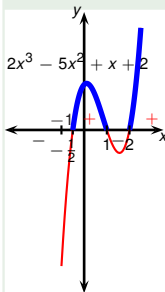
$$x \in ?$$

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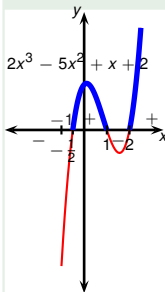
$$x \in (-\frac{1}{2}, 1) \cup (2, \infty)$$

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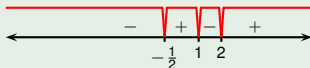
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