Calculus I
Implicit derivatives, related rates

Todor Milev

2019

Outline

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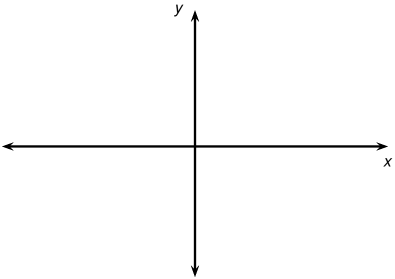
Implicit Differentiation

- So far, we have seen functions with formulas that express one varable explicitly in terms of the other.



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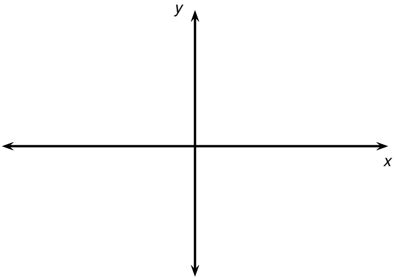

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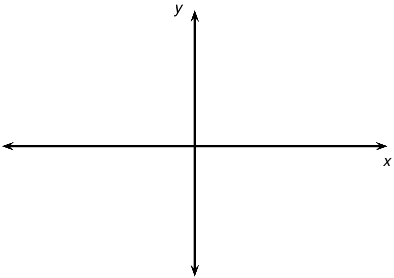

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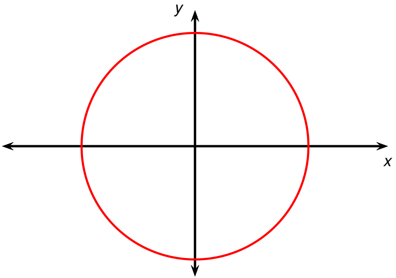

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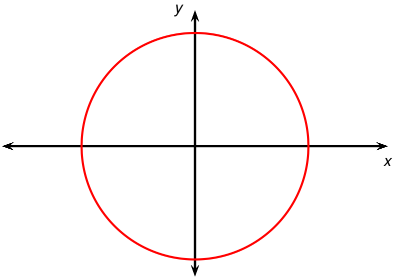

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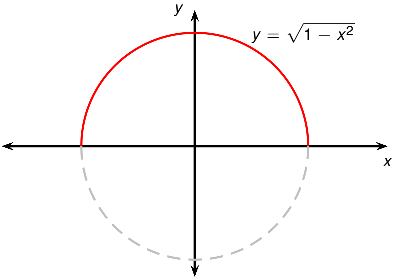

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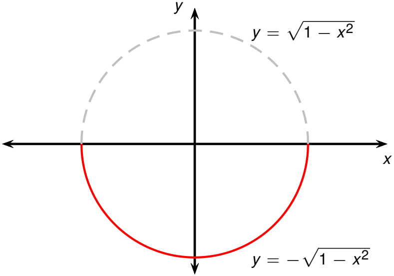

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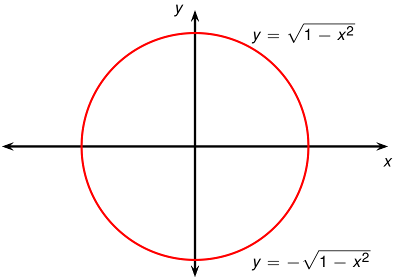

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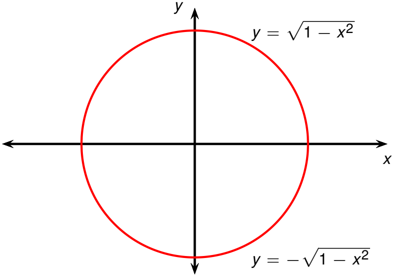
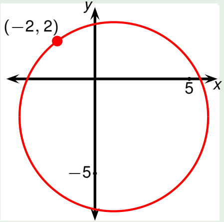
- Differentiate both sides with respect to x, and then solve for y'.



Example

Find an equation of the tangent line to $(x-1)^2+(y+2)^2=25$ at $(-2,2)$.



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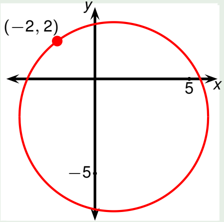

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, given $(x-1)^2 + (y+2)^2 = 25$:

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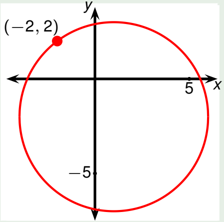

Find $\frac{dy}{dx}$, given $(x-1)^2 + (y+2)^2 = 25$:

$$\frac{d}{dx}\left((x-1)^2\right) + \frac{d}{dx}\left((y+2)^2\right) = \frac{d}{dx}(25)$$

$$\textbf{?} \qquad + \textbf{?} \qquad = \textbf{?}$$

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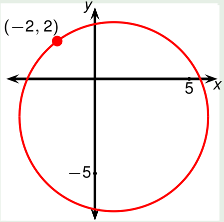

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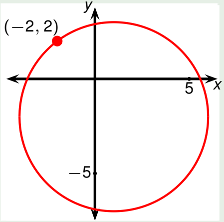

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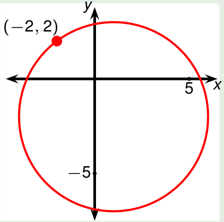

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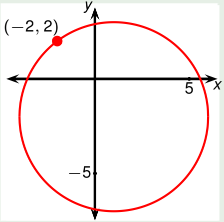

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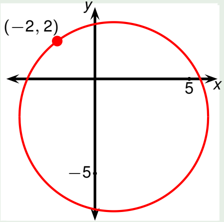

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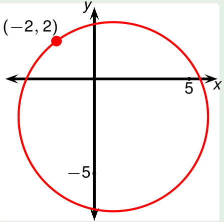

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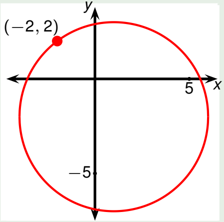

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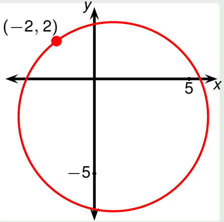

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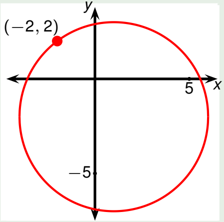

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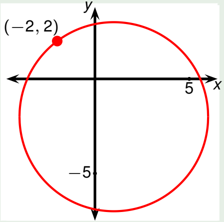

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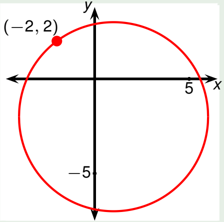

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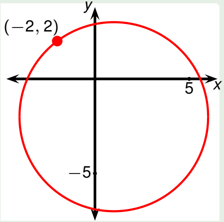

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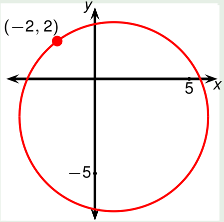

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Plug in $(-2,2)$:

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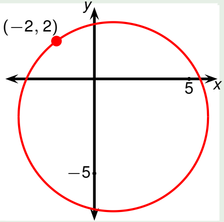

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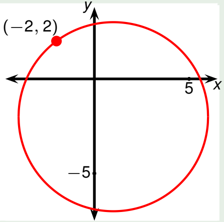

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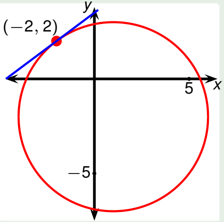

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Point-slope form:

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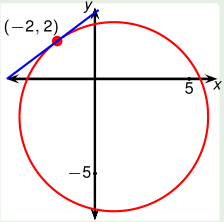

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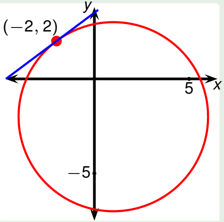

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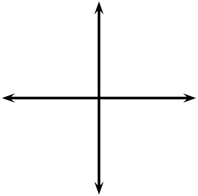
Point-slope form:

$$y - 2 = \frac{3}{4}(x+2)$$

(Elementary Computer algorithm for sketching graphs)

Let H-continuous; is there simple algorithm to sketch $H(x, y) = 0$?



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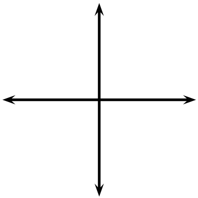

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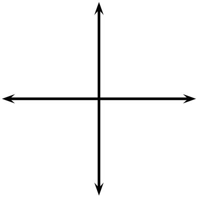

We illustrate the algorithm for:

$$x^2 + 2y^2 = 1$$

(Elementary Computer algorithm for sketching graphs)

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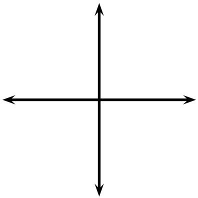

We illustrate the algorithm for:

$$x^2 + 2y^2 = 1$$
$$x^2 + 2y^2 - 1 = 0$$

(Elementary Computer algorithm for sketching graphs)

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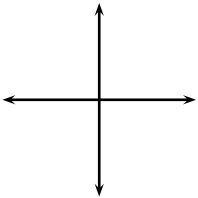


We illustrate the algorithm for:

$$x^2 + 2y^2 = 1$$

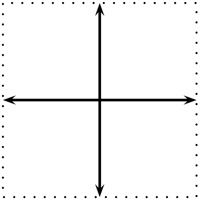
$$x^2 + 2y^2 - 1 = 0$$

Set $H(x, y) = x^2 + 2y^2 - 1$

(Elementary Computer algorithm for sketching graphs)

Let H-continuous; is there simple algorithm to sketch $H(x, y) = 0$? Yes.

- Elementary algorithm: fix large rectangle.



We illustrate the algorithm for:

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$$x^2 + 2y^2 - 1 = 0$$

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(Elementary Computer algorithm for sketching graphs)

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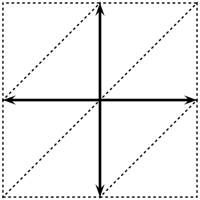

- Elementary algorithm: fix large rectangle.
- Split the grid in triangular mesh. One strategy to do that is shown.

We illustrate the algorithm for:

$$x^2 + 2y^2 = 1$$
$$x^2 + 2y^2 - 1 = 0$$

Set $H(x, y) = x^2 + 2y^2 - 1$

(Elementary Computer algorithm for sketching graphs)

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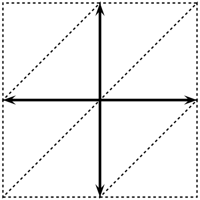

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- For each triangle:

We illustrate the algorithm for:

$$x^2 + 2y^2 = 1$$

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(Elementary Computer algorithm for sketching graphs)

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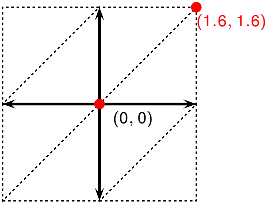

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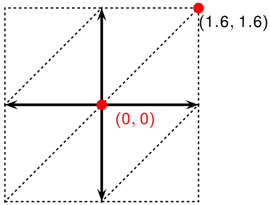

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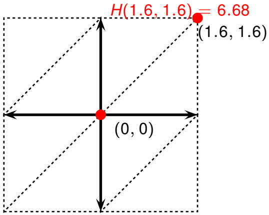

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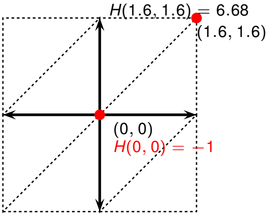

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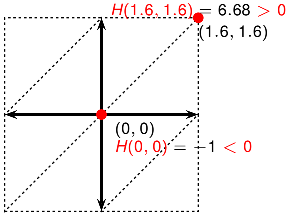

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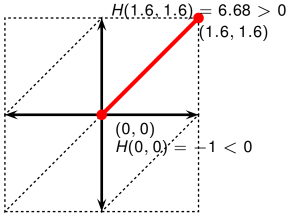

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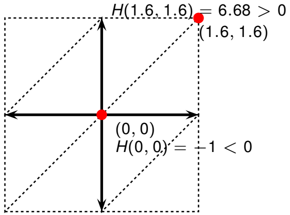

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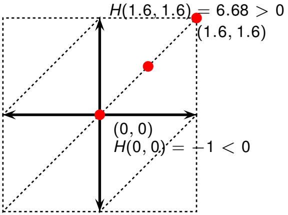

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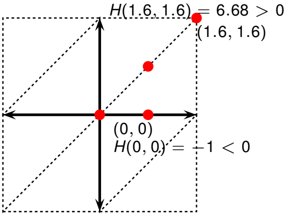

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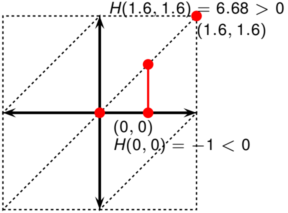

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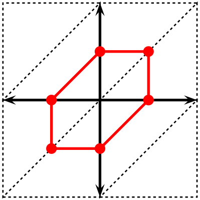

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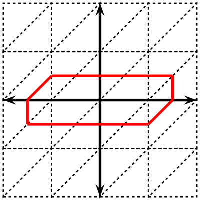

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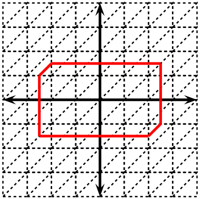

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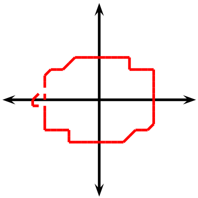

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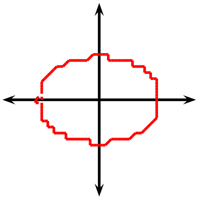

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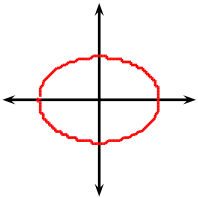

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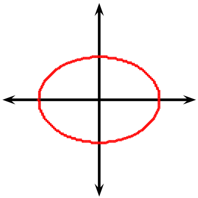

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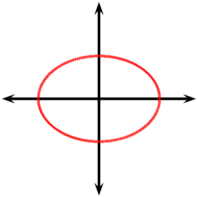

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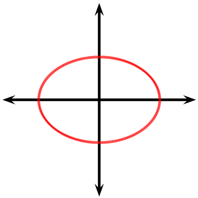

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Set $H(x, y) = x^2 + 2y^2 - 1$

- Elementary algorithm: fix large rectangle.
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- For each triangle:
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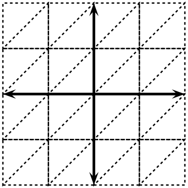

Illustrate the algorithm for:

$y^2(y^2 - 3) = x^2(x^2 - 5)$

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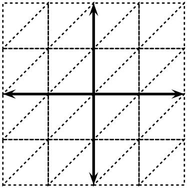

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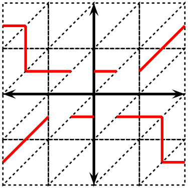

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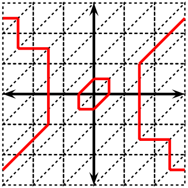

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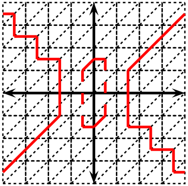

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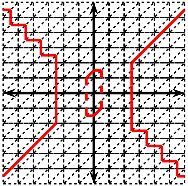

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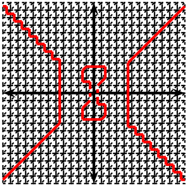

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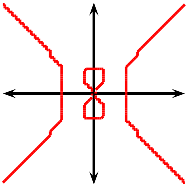

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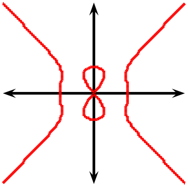

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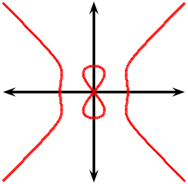

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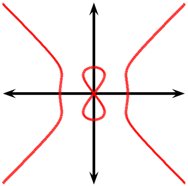

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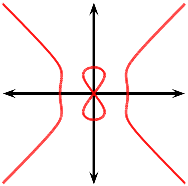

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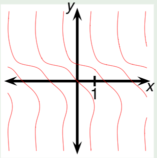
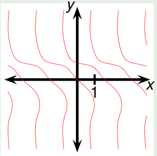


Find y' as an expression of x and y.

$$\sin(2(x+y)) = y^2 \cos(2x).$$

Example



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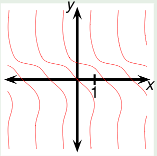

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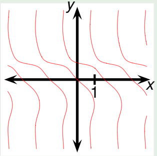

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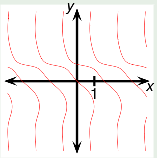

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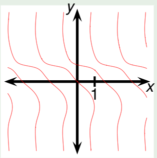

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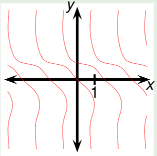

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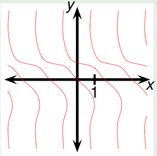

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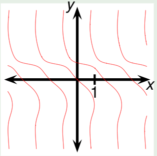

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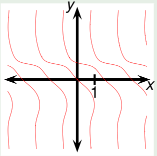

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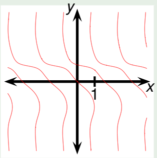

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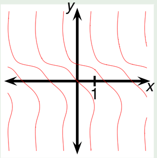

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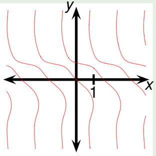

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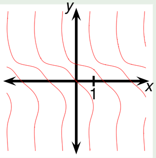

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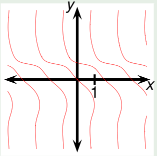

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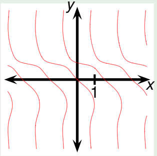

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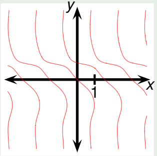

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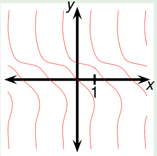

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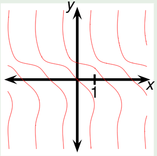

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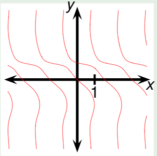

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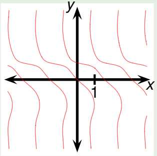

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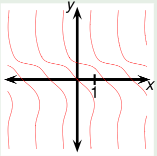

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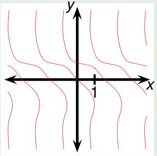

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$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{4\pi(25\mathrm{cm})^2}100\frac{\mathrm{cm}^3}{\mathrm{s}} = \frac{1}{25\pi}\mathrm{cm/s}$$

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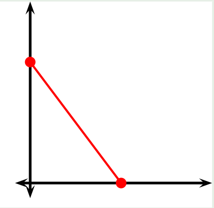

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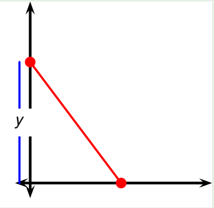

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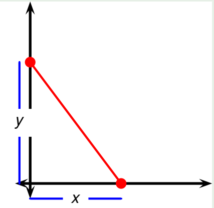

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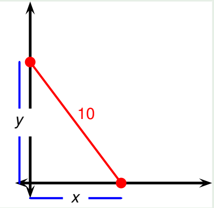

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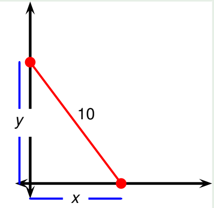

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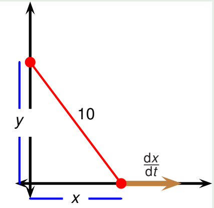

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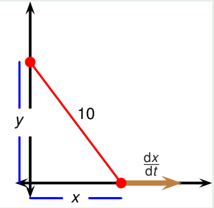
Example



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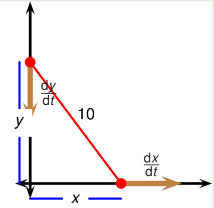

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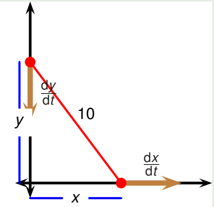

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- Relationship b/n quantities.
- Differentiate (use Chain Rule).

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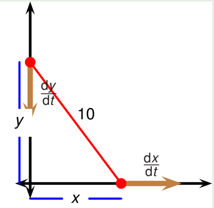

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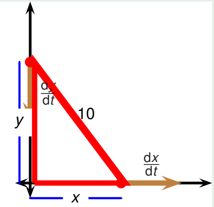

10 ft ladder rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 ft/s. How fast is the ladder top sliding down when the bottom is 6 ft from the wall?

$$x^2 + y^2 = 10^2 = 100$$

- Let y= dist. from top to ground.
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- Relationship b/n quantities.
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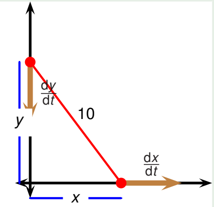

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$$x^2 + y^2 = 10^2 = 100$$

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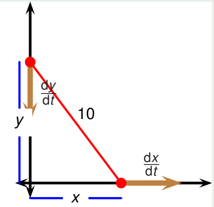

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- Relationship b/n quantities.
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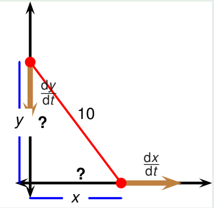
Example



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$$x^2 + y^2 = 10^2 = 100$$
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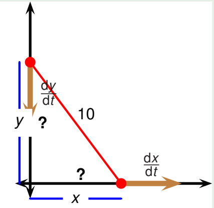

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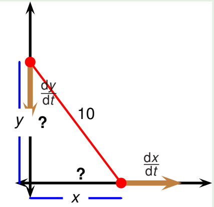

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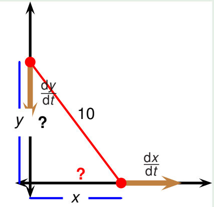

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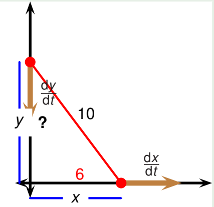

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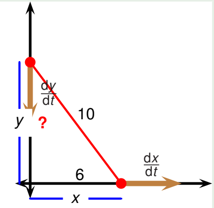

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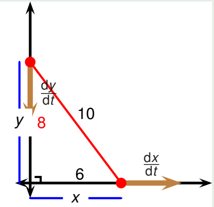

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- Pythagorean Therem: $y = \sqrt{10^2 - 6^2} = 8$.
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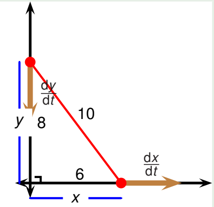

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- Differentiate (use Chain Rule).

$$x^2 + y^2 = 10^2 = 100$$

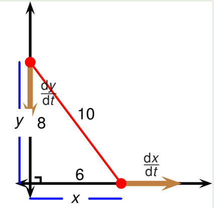
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{6 \text{ ft}}{8 \text{ ft}} \cdot 1 \text{ ft/s}$$

$$= -3/4 \text{ ft/s}.$$

Example



10 ft ladder rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 ft/s. How fast is the ladder top sliding down when the bottom is 6 ft from the wall?

- Let y= dist. from top to ground.
- Let x= dist. from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.
- Pythagorean Therem: $y = \sqrt{10^2 - 6^2} = 8$.
- Relationship b/n quantities.
- Differentiate (use Chain Rule).

$$x^2 + y^2 = 10^2 = 100$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$
$$\frac{dy}{dt} = -\frac{6\text{ ft}}{8\text{ ft}} \cdot 1\text{ ft/s}$$
$$= -3/4\text{ ft/s}.$$

Therefore the top of the ladder is falling at a rate of $3/4$ ft/s.