

Calculus I

Type 5: Limits involving conjugate radicals.

Todor Milev

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Example

Find $\lim_{t \rightarrow 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}$

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Plug in 0: $\frac{\sqrt{(0)^2 + 9} - 3}{(0)^2} = \frac{?}{?}$

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Multiply top & bottom by (minus) the conjugate radical:

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Plug in 0: $= \frac{1}{\sqrt{(\textcolor{red}{0})^2 + 9} + 3}$

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