Calculus II
Reference: strategy for integrating by parts

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Integration by Parts

Every differentiation rule

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$$(uv)' = u'v + uv' \qquad \Big| \text{ Product Rule}$$

Integration by Parts

Every differentiation rule corresponds to a differential form rule

$(uv)' = u'v + uv'$	Product Rule
$\mathrm{d}(uv) = v\mathrm{d}u + u\mathrm{d}v$	Differential Prod. Rule

Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

$$\begin{array}{rcl|l} (uv)' &=& u'v + uv' & \text{Product Rule} \\ \mathrm{d}(uv) &=& v\mathrm{d}u + u\mathrm{d}v & \text{Differential Prod. Rule} \\ \displaystyle\int \mathrm{d}(uv) &=& \displaystyle\int v\mathrm{d}u + \int u\mathrm{d}v & \text{integration of the above} \end{array}$$

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{\color{red}uv} & = & \int v\mathrm{d}u + \int u\mathrm{d}v & \text{rearrange} \\
\int u\mathrm{d}v & = & {\color{red}uv} - \int v\mathrm{d}u &
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We just proved the following.

Proposition ((Rule of) Integration by Parts)

$$\int u\mathrm{d}v = uv - \int v\mathrm{d}u$$

Integration by parts: strategy for applying

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Logs, Inverse trig, Polynomial, Exponential, Trig