

Calculus II

Integrals of the form $\int \frac{ax}{bx^2 + c} dx$

Todor Milev

2019

Linear substitutions leading to block IIa

Building block IIa: $\int \frac{x}{1+x^2} dx = \frac{1}{2} \ln(1+x^2) + C$.

“Theoretical way” to solve example below: transform to IIa; this is slow.

Feel free to skip slide, we will redo in next slide with a shortcut.

Example

$$\int \frac{x}{2x^2+3} dx = \int \frac{x}{3(\frac{2}{3}x^2+1)} dx = \int \frac{x}{3\left(\left(\sqrt{\frac{2}{3}}x\right)^2+1\right)} dx$$

$$= \frac{3}{2} \int \frac{\sqrt{\frac{2}{3}}x}{3\left(\left(\sqrt{\frac{2}{3}}x\right)^2+1\right)} d\left(\sqrt{\frac{2}{3}}x\right)$$

$$\left| \text{Set } u = \sqrt{\frac{2}{3}}x \right.$$

$$= \frac{1}{2} \int \frac{u}{u^2+1} du = \frac{1}{4} \ln(1+u^2) + C$$

$$= \frac{1}{4} \ln\left(\frac{1}{3}(2x^2+3)\right) + C$$

$$= \frac{1}{4} \ln(2x^2+3) + \frac{\ln(\frac{1}{3})}{4} + C$$

$$= \frac{1}{4} \ln(2x^2+3) + K$$

Linear substitutions leading to blocks IIa

Building block IIa: $\int \frac{x}{1+x^2} dx = \frac{1}{2} \ln(1+x^2) + C$.

The example below can be done directly, without transforming to block IIa.

Example

$$\begin{aligned}
 \int \frac{x}{2x^2+3} dx &= \int \frac{1}{2x^2+3} d\left(\frac{x^2}{2}\right) \\
 &= \int \frac{1}{2x^2+3} d\left(\frac{2x^2+3}{4}\right) && \left| \text{Set } u = 2x^2+3 \right. \\
 &= \frac{1}{4} \int \frac{1}{u} du \\
 &= \frac{1}{4} \ln|u| + C \\
 &= \frac{1}{4} \ln(2x^2+3) + C
 \end{aligned}$$