Precalculus
Polynomial inequalities

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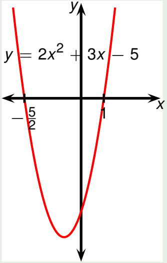
Outline

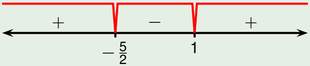
Example



Solve the inequality.

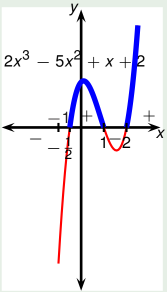
$$\begin{array}{rcl} 2x^2+3x-5 & \geq & 0 \\ (2x+5)(x-1) & \geq & 0 \\ x \in \left(-\infty, -\frac{5}{2}\right] \cup [1,\infty) & & \end{array}$$

Left hand side vanishes when $x = -\frac{5}{2}$ and when $x = 1$. The two roots split the real line into three intervals: $\left(-\infty, -\frac{5}{2}\right), \left(-\frac{5}{2}, 1\right), (1, \infty)$.



Interval	Factor signs	Final sign	Sample pt	Value at sample pt
$\left(-\infty, -\frac{5}{2}\right)$	$(-)(-)$	$+$	-100	$f(-100) > 0$
$\left(-\frac{5}{2}, 1\right)$	$(+)(-)$	$-$	0	$f(0) = -5 < 0$
$(1, \infty)$	$(+)(+)$	$+$	100	$f(100) > 0$

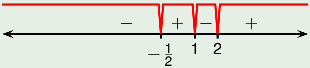
Example



Plot the function $2x^3-5x^2+x+2$. Solve the inequality.

$$\begin{aligned}2x^3-5x^2+x+2 &> 0\\ 2\left(x-\left(-\frac{1}{2}\right)\right)(x-1)(x-2) &> 0\\ x\in\left(-\frac{1}{2},1\right)&\cup(2,\infty)\end{aligned}$$

Left hand side vanishes when $x=-\frac{1}{2}$, when $x=1$ and when $x=2$. The two roots split the real line into four intervals: $\left(-\infty,-\frac{1}{2}\right),\left(-\frac{1}{2},1\right),(1,2),(2,\infty)$.



Interval	Factor signs	Final sign from plot
$\left(-\infty,-\frac{1}{2}\right)$	$(-)(-)(-)$	$-$
$\left(-\frac{1}{2},1\right)$	$(+)(-)(-)$	$+$
$(1,2)$	$(+)(+)(-)$	$-$
$(2,\infty)$	$(+)(+)(+)$	$+$