

## Precalculus

**Find the area of a triangle from two sides and  
an angle between them**

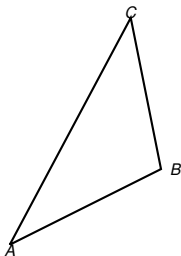
Todor Milev

2019

Triangle area =  $\frac{1}{2}$ base  $\cdot$  height

Proposition (Triangle area)

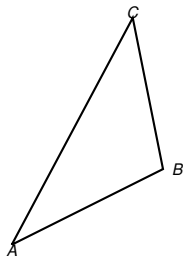
$$\text{Area}(\triangle ABC) = ?$$



Triangle area =  $\frac{1}{2}$ base  $\cdot$  height

Proposition (Triangle area)

$$\text{Area}(\triangle ABC) = \frac{1}{2} \text{height} \cdot \text{base}$$

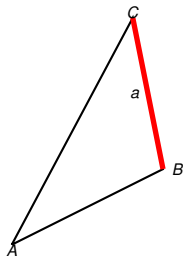


# Triangle area = $\frac{1}{2}$ base $\cdot$ height

Let  $\triangle ABC$  have **side length  $a$**  and height length  $h_a$ , as indicated - **side  $a$  is opposite to vertex  $A$**  and  $h_a$  starts at  $A$

## Proposition (Triangle area)

$$\text{Area}(\triangle ABC) = \frac{1}{2} \text{height} \cdot \text{base} = \frac{1}{2} h_a a$$

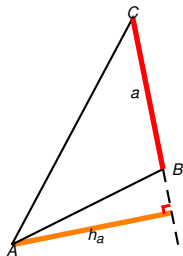


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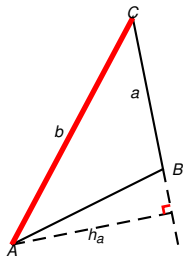


# Triangle area = $\frac{1}{2}$ base $\cdot$ height

Let  $\triangle ABC$  have **side lengths**  $a, b$  and height lengths  $h_a, h_b$ , as indicated - side  $a$  is opposite to vertex  $A$  and  $h_a$  starts at  $A$ , and so on.

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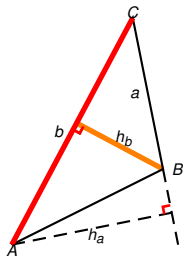


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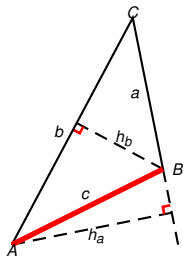


# Triangle area = $\frac{1}{2}$ base $\cdot$ height

Let  $\triangle ABC$  have **side lengths**  $a, b, c$  and height lengths  $h_a, h_b, h_c$ , as indicated - side  $a$  is opposite to vertex  $A$  and  $h_a$  starts at  $A$ , and so on.

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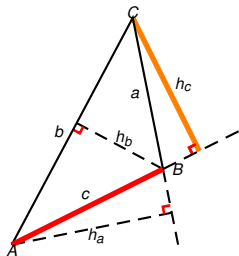


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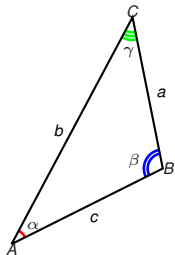
# Triangle area from two sides and angle between them

Let  $\triangle ABC$  have sides lengths  $a, b, c$  angles  $\alpha, \beta, \gamma$ , as indicated:  $\alpha$  is opposite to  $a$ ,  $\beta$  is opposite to  $b$ ,  $\gamma$  is opposite to  $c$ .

**Proposition ( $\triangle$  area from two sides and angle between them)**

*The area of a triangle is half the product of the lengths of two of its sides times the sine of the angle between them. In other words,*

$$\text{Area}(\triangle ABC) = \frac{ab \sin \gamma}{2} = \frac{bc \sin \alpha}{2} = \frac{ca \sin \beta}{2}$$



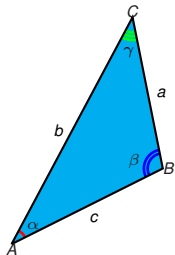
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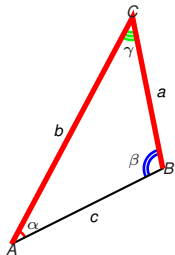
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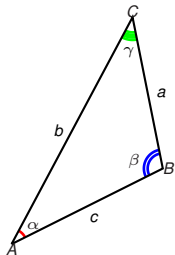
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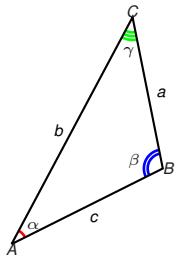
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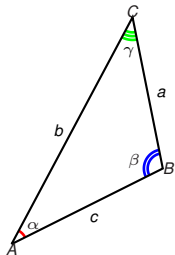
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**Proof.**

$$\text{Area}(\triangle ABC) = \frac{\text{base} \cdot \text{height}}{2}$$



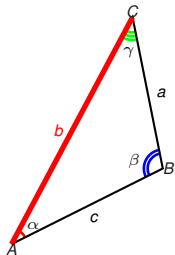
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$$\text{Area}(\triangle ABC) = \frac{\text{base} \cdot \text{height}}{2} = \frac{bh_b}{2}$$





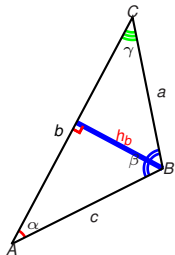
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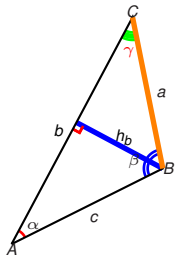
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**Proof.**

$$\begin{aligned} \text{Area}(\triangle ABC) &= \frac{\text{base} \cdot \text{height}}{2} = \frac{b h_b}{2} \\ &= \frac{b a \sin \gamma}{2}. \end{aligned}$$



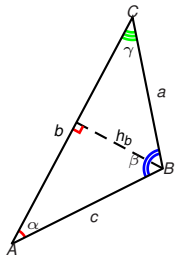
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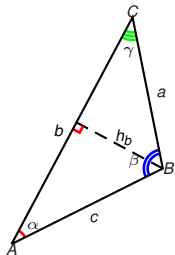
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The proof of the other two cases is similar. □