Calculus II

Integrals of the form $\int \sin^n x \cos^m x \mathrm{d}x$, theory

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$$\begin{aligned}\int \sin^m x \cos^n x dx &= \int \sin^m x \cos^{n-1} x d(\sin x)\\ &= \int \sin^m x \left(1-\sin^2 x\right)^{\frac{n-1}{2}} d(\sin x)\\ &= \int u^m \left(1-u^2\right)^{\frac{n-1}{2}} du\end{aligned}$$	When $n-$ odd: $\cos x dx = d(\sin x)$ Express $\cos x$ via $\sin x$ Set $\sin x = u$
$$\begin{aligned}\int \sin^m x \cos^n x dx &= \int \sin^{m-1} x \cos^n x d(-\cos x)\\ &= -\int \left(1-\cos^2 x\right)^{\frac{m-1}{2}} \cos^n x d(\cos x)\\ &= -\int \left(1-u^2\right)^{\frac{m-1}{2}} u^n du\end{aligned}$$	When $m-$ odd: $\sin x dx = d(-\cos x)$ Express $\cos x$ via $\sin x$ Set $\cos x = u$

If both m, n- even, use $\left|\begin{array}{lll}\sin^2 x & = & \frac{1-\cos(2x)}{2}\\ \cos^2 x & = & \frac{\cos(2x)+1}{2}\end{array}\right.$ and substitute $s = 2x$ to lower trig powers. Repeat above considerations.