

Calculus II

Ratio test basic

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Example

Test the series $\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$ for absolute convergence.

$$\begin{aligned} \left| \frac{a_{n+1}}{a_n} \right| &= \left| \frac{(-1)^{n+1} \frac{(n+1)^3}{3^{n+1}}}{(-1)^n \frac{n^3}{3^n}} \right| \\ &= \frac{(n+1)^3}{3^{n+1}} \cdot \frac{3^n}{n^3} \\ &= \frac{1}{3} \left(\frac{n+1}{n} \right)^3 \\ &= \frac{1}{3} \left(1 + \frac{1}{n} \right)^3 \\ &\rightarrow \frac{1}{3} < 1 \end{aligned}$$

Therefore the series is absolutely convergent by the Ratio Test.