Precalculus
Find circle arclength from radius and angle

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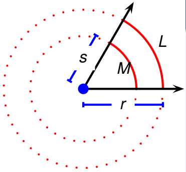
Arc-length of a circle arc



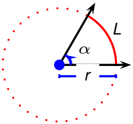
Proposition

Let two circles have common center and radii s and r. Suppose an arbitrary geometric angle with vertex at the common center of the circles cuts off short arcs of length M and L. Then $\frac{s}{r} = \frac{M}{L}$.

$$\begin{aligned} \frac{s}{r} &= \frac{M}{L} \\ \frac{1}{r} &= \frac{\alpha}{L} \\ L &= \alpha r \end{aligned}$$

Choose $s = 1$, relabel $M = \alpha$

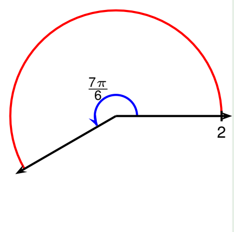
The angle-measure of a geometric angle is the arc-length cut off from a radius 1 circle, therefore we get the following.



Corollary

The arc-length cut off by an angle with measure α from a circle of radius r equals αr.

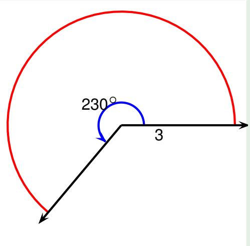
Example



Find the length of an arc of a circle of radius 2 cut off by an angle of measure $\frac{7\pi}{6}$ $(= 210^\circ)$.

$$\text{arc-length} = \alpha r = \frac{7\pi}{6} \cdot 2 = \frac{7\pi}{3} \approx 7.33038 \text{ (units)}$$

Example



Find the length of an arc of a circle of radius 3 cut off by an angle of measure 230°.

$$\begin{aligned} \alpha &= 230^\circ \\ &= 230^\circ \frac{\pi \text{ rad}}{180^\circ} = \frac{23}{18}\pi \text{ rad} && \Big| \text{ Convert to radians} \\ \text{arc-length} &= \alpha r = \frac{23\pi}{18} \cdot 3 = \frac{23\pi}{6} \approx 12.043 \end{aligned}$$