Calculus II
Power series expansion related to geometric series

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Recall the geometric series formula:

$$\frac{1}{1-y} = \sum_{n=0}^{\infty} y^n = 1 + y + y^2 + y^3 + \dots \qquad \text{if \& only if } |y| < 1 \qquad .$$

Example

Write $\frac{1}{1+x^2}$ as a power series and find the interval of convergence.

Recall the geometric series formula:

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Write $\frac{1}{1+x^2}$ as a power series and find the interval of convergence.

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)}$$

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$$\frac{1}{1+x^2} = \frac{1}{1-\left(-x^2\right)} = \sum_{n=0}^{\infty} \left(-x^2\right)^n \qquad \text{if \& only if } |-x^2| < 1$$

Recall the geometric series formula:

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if & only if $|-x^2| < 1$

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&= \sum_{n=0}^{\infty} (-1)^n x^{2n}
\end{aligned}$$

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- This converges if and only if $\left|\begin{array}{rcl} |-x^2| & < & 1 \\ |x| & < & 1 \end{array}\right.$.
- Therefore the interval of convergence is $x \in (-1, 1)$.