Calculus II

Integrals of the form $\int \arctan(mx)\mathbf{d}x$

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Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

Set $w = 1 + x^2$.

$$\begin{aligned}
\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1}\frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1}\frac{1}{1+x^2}\mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1}\frac{1}{w}\mathrm{d}w = \frac{\pi}{4} - \frac{1}{2}\left[\ln|w|\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}\left[\ln\left(1+x^2\right)\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}(\ln 2 - \ln 1) = \frac{\pi}{4} - \frac{1}{2}\ln 2 \quad .
\end{aligned}$$