

## Precalculus

# Exponent equation that reduces to quadratic, shifted exponents

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## Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set  $u = 2^x$ . Then  $4^{x+1} = 4u^2$ ,  $2^{x+2} = 4u$ .

$$4u^2 - 4u - 3 = 0$$

$$(2u - 3)(2u + 1) = 0$$

$$2u - 3 = 0 \quad \text{or} \quad 2u + 1 = 0$$

$$u = \frac{3}{2} \quad \text{or} \quad u = -\frac{1}{2}$$

$$2^x = \frac{3}{2} \quad \text{or} \quad 2^x = -\frac{1}{2}$$

$$x = \log_2 \left( \frac{3}{2} \right) = \frac{\ln \left( \frac{3}{2} \right)}{\ln 2} \approx 0.58496 \quad \text{or} \quad \text{no real solution}$$