Precalculus
Solve triangle from two sides and an angle

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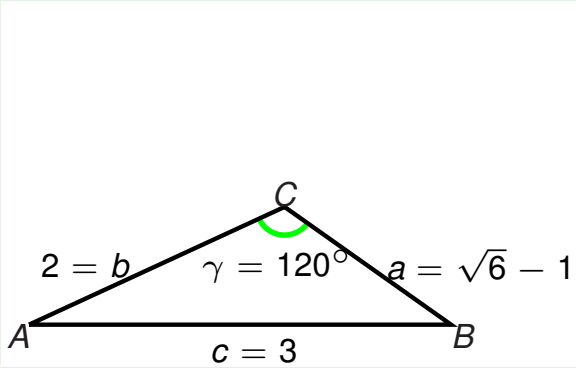
Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(-\frac{1}{2}\right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 + 2\sqrt{6}}{2} \qquad a > 0$$

$$= -1 + \sqrt{6}$$

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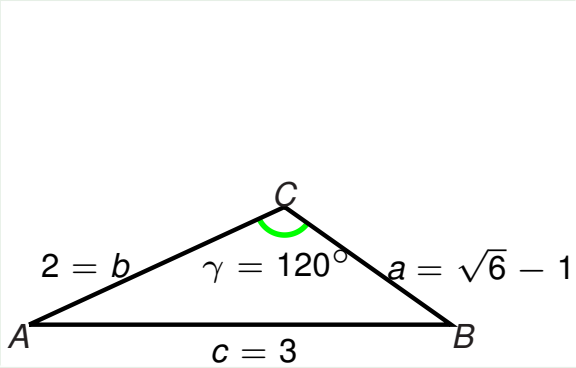

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$$\textbf{Area} = \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6}-1\right)\cancel{2}}{\cancel{2}}\frac{\sqrt{3}}{2}$$

$$= \frac{3\sqrt{2} - \sqrt{3}}{2}$$