

Precalculus

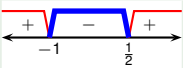
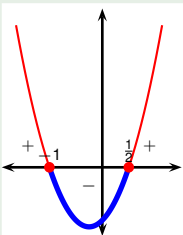
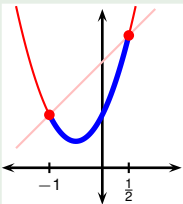
Quadratic trigonometric inequalities

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2019

Example

- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2\cos^2 \theta + 2\cos \theta + 1 \leq \cos \theta + 2$ lying in $[-360^\circ, 360^\circ]$.



$$2u^2 + 2u + 1 \leq u + 2$$

$$2u^2 + u - 1 \leq 0$$

$$2(u - \frac{1}{2})(u + 1) \leq 0$$

$$u \in [-1, \frac{1}{2}]$$

$$2\cos^2 \theta + 2\cos \theta + 1 \leq \cos \theta + 2 \quad \text{Set } \cos \theta = u$$

$$2u^2 + 2u + 1 \leq u + 2$$

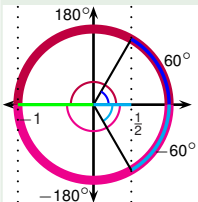
$$u \in [-1, \frac{1}{2}]$$

(solved above)

$$\cos \theta \in [-1, \frac{1}{2}]$$

$$-1 \leq \cos \theta \leq \frac{1}{2}$$

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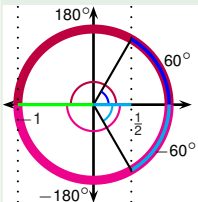
$$\cos \theta \in \left[-1, \frac{1}{2}\right]$$

$$-1 \leq \cos \theta \leq \frac{1}{2}$$

$$\theta \in [-180^\circ + k360^\circ, -60^\circ + k360^\circ] \cup [60^\circ + k360^\circ, 180^\circ + k360^\circ]$$

$$\begin{aligned} \theta \in & \quad \cup [-540^\circ, -420^\circ] \cup [-300^\circ, -180^\circ] & k = -1 \\ & \quad \cup [-180^\circ, -60^\circ] \cup [60^\circ, 180^\circ] & k = 0 \\ & \quad \cup [180^\circ, 300^\circ] \cup [420^\circ, 540^\circ] & k = 1 \\ \theta \in & \quad [-300^\circ, -60^\circ] \cup [60^\circ, 300^\circ] \end{aligned}$$

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$$\theta \in [-300^\circ, -60^\circ] \cup [60^\circ, 300^\circ]$$

