Precalculus
Factor quadratic over the complex numbers

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$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

$$\left|\begin{array}{rcl} x_1 x_2 &=& \frac{c}{a} \\ x_1 + x_2 &=& -\frac{b}{a} \end{array}\right.$$

Example

Factor the quadratic, using complex numbers if needed.

$$x^2 + x + 1$$

$$ax^2+bx+c=a\left(x-x_1\right)\left(x-x_2\right),$$

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$$x^2 + x + 1 = \left(x - \left(\frac{-1+\sqrt{3}i}{2}\right)\right)\left(x - \left(\frac{-1-\sqrt{3}i}{2}\right)\right)$$

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