Precalculus
Homework
Equations involving logarithms and exponents

1. Solve each equation for x. If available, use a calculator to give an ($\approx$) answer in decimal notation. If available, use a calculator to verify your approximate solutions.

   (a) $e^{7-4x} = 7$.

   answer: $\frac{7-\ln 7}{4} \approx 1.263522$

   (b) $\ln(2x-9) = 2$.

   answer: $\frac{e^2+9}{2} \approx 8.194528$

   (c) $\ln(x^2-2) = 3$.

   answer: $\pm\sqrt{e^3+2} \approx \pm 4.699525$

   (d) $2^{x-3} = 5$.

   answer: $\log_2 5 + 3 = \frac{\ln 5}{\ln 2} + 3 \approx 5.321928$

   (e) $\ln x + \ln(x-1) = 1$.

   answer: $\frac{1}{2}\left(1+\sqrt{1+4e}\right) \approx 2.223$

   (f) $e^{2x+1} = t$.

   answer: $\frac{\ln t - 1}{2}$

   (g) $\log_2(mx) = c$.

   answer: $\frac{2^c}{m}$

   (h) $e - e^{-2x} = 1$.

   answer: $-\frac{1}{2}\ln(e-1) \approx -0.271$

   (i) $8(1+e^{-x})^{-1} = 3$.

   answer: $-\ln\frac{5}{3} = \ln\frac{3}{5} \approx -0.510826$

   (j) $\ln(\ln x) = 1$.

   answer: $e^e \approx 15.154$

   (k) $e^{e^x} = 10$.

   answer: $\ln(\ln 10) \approx 0.834$

   (l) $\ln(2x+1) = 3 - \ln x$.

   answer: $\frac{-1+\sqrt{1+8e^3}}{4} \approx 2.92878$

   (m) $e^{2x} - 4e^x + 3 = 0$.

   answer: $x = \ln 3 \approx 1.098612,\ ,\ x = 0$

   (n) $e^{4x} + 3e^{2x} - 4 = 0$.

   answer: $x = 0$

   (o) $e^{2x} - e^x - 6 = 0$.

   answer: $x = \ln 3$

   (p) $4^{3x} - 2^{3x+2} - 5 = 0$.

   answer: $x = \frac{\log_2 5}{3}$

   (q) $3 \cdot 2^x + 2\left(\frac{1}{2}\right)^{x-1} - 7 = 0$.

   answer: $x = 0$ or $2 - \log_2 3 = 2 - \frac{\ln 3}{\ln 2}$

Solution. 1.d

$2^{x-3} = 5$	take $\log_2$
$x - 3 = \log_2(5)$	add 3 to both sides
$x = \log_2(5) + 3$	answer is complete
$= \dfrac{\ln 5}{\ln 2} + 3$	optional step: convert to $\ln$
≈ 5.321928095	calculator

Solution. 1.h

$$\begin{aligned} e - e^{-2x} &= 1 \\ e^{-2x} &= e - 1 && \text{apply } \ln \\ \ln e^{-2x} &= \ln(e-1) \\ -2x &= \ln(e-1) \\ x &= -\frac{1}{2}\ln(e-1) \\ &\approx -0.270662427 && \text{calculator} \end{aligned}$$

Solution. 1.e

$$\begin{array}{rcl}
\ln x + \ln(x-1) &=& 1\\
\ln\left(x^2-x\right) &=& 1\\
e^{\ln(x^2-x)} &=& e^1\\
x^2-x &=& e\\
x^2-x-e &=& 0\\
\text{Quadratic formula:}\quad x &=& \dfrac{-(-1)\pm\sqrt{(-1)^2-4(1)(-e)}}{2(1)}\\
&=& \dfrac{1\pm\sqrt{1+4e}}{2}.
\end{array}$$

However $\frac{1-\sqrt{1+4e}}{2}$ is negative, so $\ln\left(\frac{1-\sqrt{1+4e}}{2}\right)$ is undefined. Hence the only solution is $x=\frac{1+\sqrt{1+4e}}{2}\approx 2.2229$.

Solution. 1.p

$$\begin{array}{rcll}
4^{3x}-2^{3x+2}-5 &=& 0 &\\
4^{3x}-4\cdot 2^{3x}-5 &=& 0 & \left|\ \text{Set}\ \begin{array}{rcl}2^{3x}&=&u\\4^{3x}&=&u^2\end{array}\right.\\
u^2-4u-5 &=& 0 &\\
(u-5)(u+1) &=& 0 &\\
u=5 &\text{or}& u=-1 &\\
2^{3x}=5 && 2^{3x}=-1 &\\
3x=\log_2(5) && \text{no real solution} &\\
x=\dfrac{\log_2 5}{3} &&&\\
\text{Calculator: } x\approx 0.773976 &&&
\end{array}$$

Solution. 1.q

$$\begin{array}{rcll}
3\cdot 2^x+2\left(\dfrac{1}{2}\right)^{x-1}-7 &=& 0 &\\
3\cdot 2^x+2\left(\dfrac{1}{2}\right)^{x}\left(\dfrac{1}{2}\right)^{-1}-7 &=& 0 &\\
3\cdot 2^x+4\left(\dfrac{1}{2}\right)^{x}-7 &=& 0 & \Big|\ \text{Set } 2^x=u\\
3u+\dfrac{4}{u}-7 &=& 0 & \Big|\ \text{Multiply by } u\\
3u^2-7u+4 &=& 0 &\\
(u-1)(3u-4) &=& 0 &\\
u=1 &\text{or}& 3u-4=0 &\\
2^x=1 && u=\dfrac{4}{3} &\\
x=0 && 2^x=\dfrac{4}{3} &\\
&& x=\log_2\dfrac{4}{3}=\log_2 4-\log_2 3 &\\
&& x=2-\log_2 3 &\\
\text{Calculator:} && x\approx 0.415037 &
\end{array}$$