Precalculus Degree lowering formulas

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Proposition (Power-Reducing Formulas)

$$\sin^2 \alpha = \frac{1 - \cos(2\alpha)}{2} \quad \cos^2 \alpha = \frac{1 + \cos(2\alpha)}{2}$$

Proof.

$$\cos(2\alpha) = 1 - 2\sin^2\alpha \qquad \cos(2\alpha) = 2\cos^2\alpha - 1$$

$$2\sin^2\alpha = 1 - \cos(2\alpha) \qquad 2\cos^2\alpha = 1 + \cos(2\alpha)$$

$$\sin^2\alpha = \frac{1 - \cos(2\alpha)}{2} \qquad \cos^2\alpha = \frac{1 + \cos(2\alpha)}{2}$$

Corollary

$$\sin \alpha = \pm \sqrt{\frac{1 - \cos(2\alpha)}{2}}$$
 $\cos \alpha = \pm \sqrt{\frac{1 + \cos(2\alpha)}{2}}$

Corollary (Half-Angle Formulas)

$$\sin\left(\frac{\beta}{2}\right) = \pm\sqrt{\frac{1-\cos\beta}{2}} \cos\left(\frac{\beta}{2}\right) = \pm\sqrt{\frac{1+\cos\beta}{2}}$$

Proposition (Power-Reducing Formulas)

$$\sin^2 \alpha = \frac{1 - \cos(2\alpha)}{2} \cos^2 \alpha = \frac{1 + \cos(2\alpha)}{2}$$

- The power reducing formulas are used to express $\sin^k \alpha$ and $\cos^k \alpha$ via lower powers of the \sin and \cos functions (applied to angles other than α).
- This technique will play a key role in integration (studied later/in another course).

Recall the formulas: $\sin^2 \beta = \frac{1 - \cos(2\beta)}{2}$, $\cos^2 \beta = \frac{\cos(2\beta) + 1}{2}$.

Example

Rewrite $\sin^4 \alpha$ in terms of first powers of the cosines and sines of multiples of the angle α .

$$\sin^{4} \alpha = \left(\sin^{2} \alpha\right)^{2}$$

$$= \left(\frac{1 - \cos(2\alpha)}{2}\right)^{2}$$

$$= \frac{1}{4}\left(1 - 2\cos(2\alpha) + \cos^{2}(2\alpha)\right)$$

$$= \frac{1}{4}\left(1 - 2\cos(2\alpha) + \frac{\cos(2 \cdot 2\alpha) + 1}{2}\right)$$

$$= \frac{1}{4}\left(1 - 2\cos(2\alpha) + \frac{\cos(2 \cdot 2\alpha)}{2} + \frac{1}{2}\right)$$

$$= \frac{1}{4}\left(\frac{3}{2} - 2\cos(2\alpha) + \frac{\cos(4\alpha)}{2}\right)$$

$$= \frac{1}{8}\left(3 - 4\cos(2\alpha) + \cos(4\alpha)\right)$$