Precalculus
§ Geometric-text problems leading to polynomial systems, part 2

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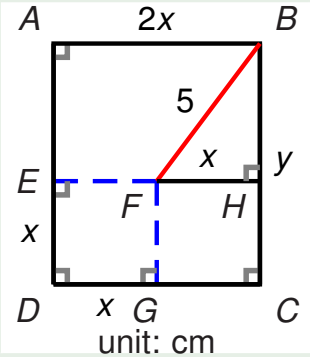
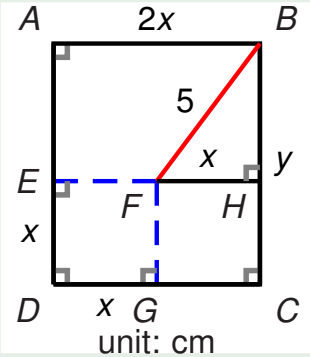
Example



ABCD is a rectangle. Points E, F, G and D form a square as indicated. Given: $|ED| = |DG| = x\ cm$ $|AB| = 2x\ cm$, $|BC| = y\ cm$, $|BF| = 5\ cm$, $Perimeter(ABCD) = 26\ cm$. Find x and y so that $y > x$.

$$
\begin{aligned}
2 \cdot (2x + y) &= 26 && \text{|Div. by 2} \\
2x + y &= 13 \\
y &= 13 - 2x \\
|FH|^2 + |BH|^2 &= |BF|^2 \\
x^2 + (y - x)^2 &= 5^2 \\
x^2 + y^2 - 2xy + x^2 - 5^2 &= 0 \\
2x^2 + y^2 - 2xy - 25 &= 0 \\
2x^2 + (13 - 2x)^2 - 2x(13 - 2x) - 25 &= 0 \\
2x^2 + 169 - 52x + 4x^2 - 26x + 4x^2 - 25 &= 0 \\
10x^2 - 78x + 144 &= 0 && \text{|Div. by 2} \\
5x^2 - 39x + 72 &= 0
\end{aligned}
$$

Example



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$$y = 13 - 2x$$

$$5x^2 - 39x + 72 = 0$$

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

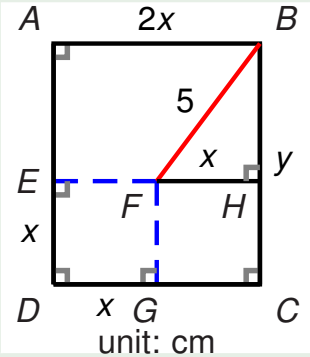
$$= \frac{-(-39) \pm \sqrt{39^2 - 4 \cdot 5 \cdot 72}}{2 \cdot 5}$$

$$= \frac{39 \pm \sqrt{1521 - 1440}}{10}$$

$$= \frac{39 \pm \sqrt{81}}{10}$$

$$= \frac{39 \pm 9}{10}$$

Example



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$$y = 13 - 2x$$

$$x_1, x_2 = \frac{39 \pm 9}{10}$$

Case 1. $x = \frac{39 + 9}{10} = \frac{48}{10} = 4.8$

$$y = 13 - 2x = 13 - 2 \cdot 4.8 = 13 - 9.6 = 3.4 < 4.8 = x$$

Solution rejected as problem requires $y > x$.

Case 2. $x = \frac{39 - 9}{10} = 3$

$$y = 13 - 2 \cdot x = 13 - 2 \cdot 3 = 7$$

Solution is valid as $y > x$.