Precalculus

Compute the trigonometric functions of an angle not in the first quadrant

Todor Milev

2019

To compute trigonometric functions from obtuse ($> 90°$) or negative angles, we can use the following visual aid.

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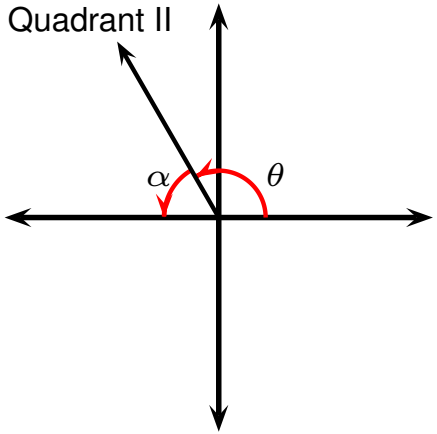
The computation of the reference angle α depends on the quadrant.



$\alpha = ?$

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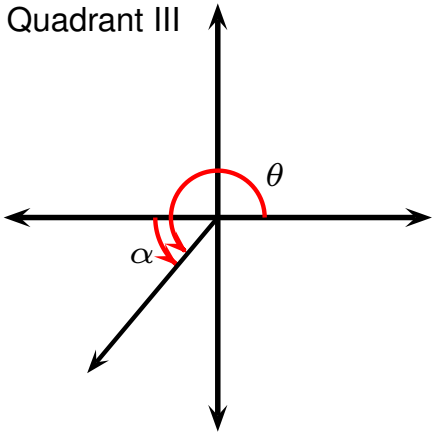

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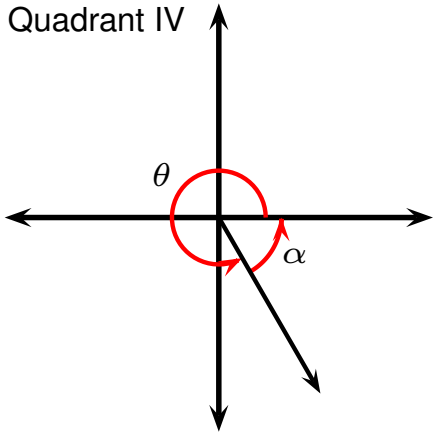

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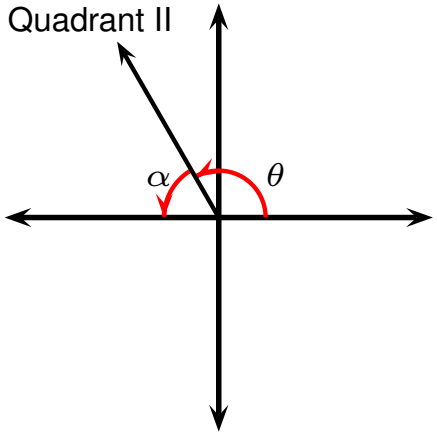	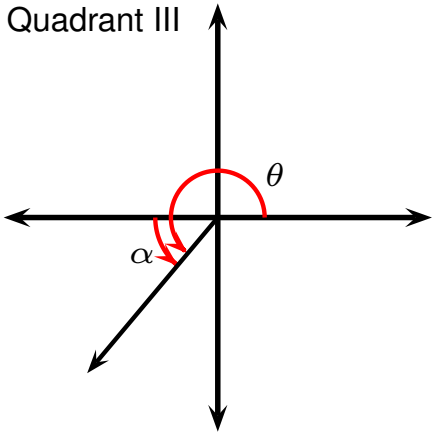	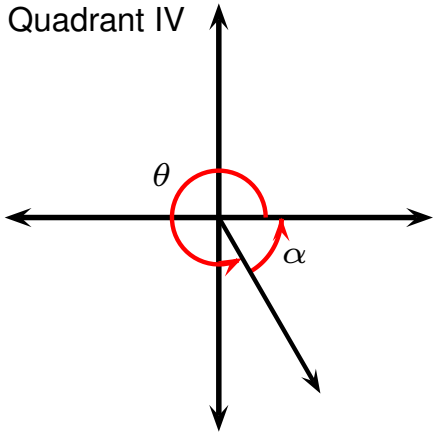
$\alpha = \pi - \theta$	$\alpha =$	$\alpha =$
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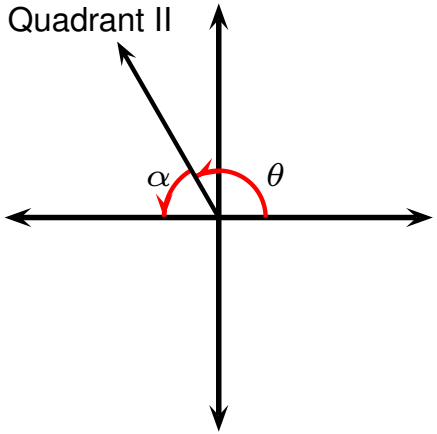	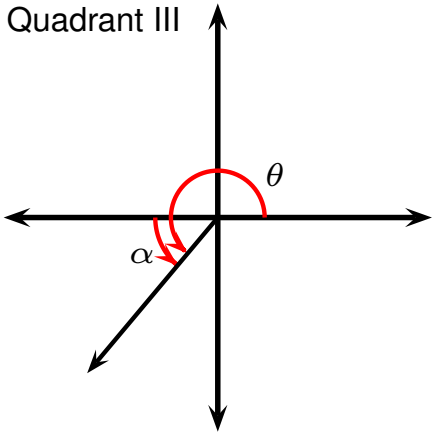	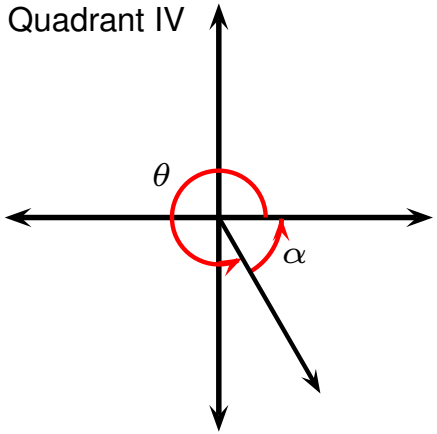
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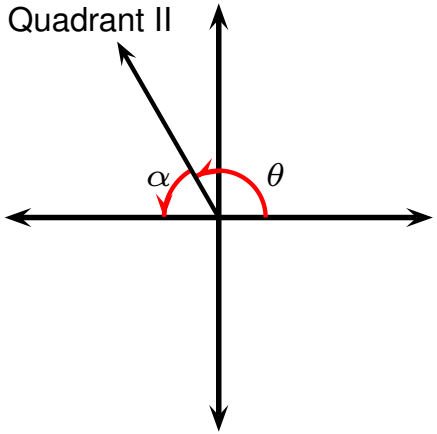	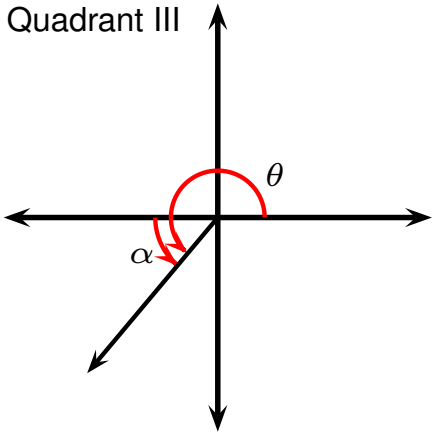	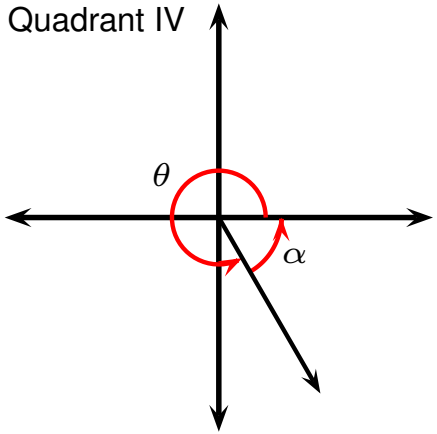
$\alpha = \pi - \theta$	$\alpha = \theta - \pi$	$\alpha =$
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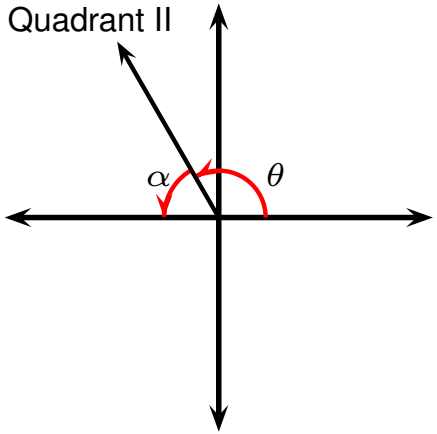

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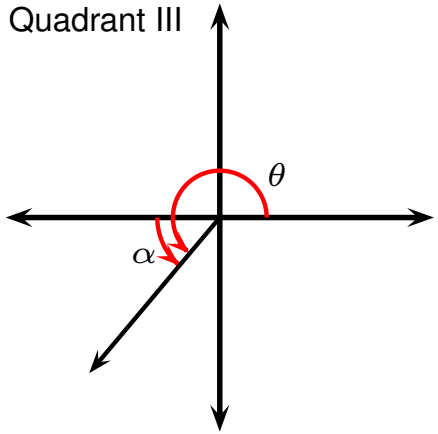
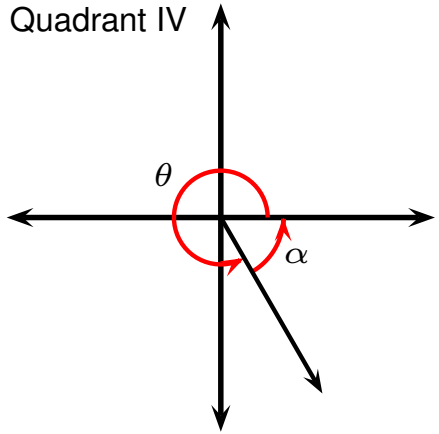


$$\alpha = \theta - \pi$$
$$\alpha = \theta - 180^\circ$$



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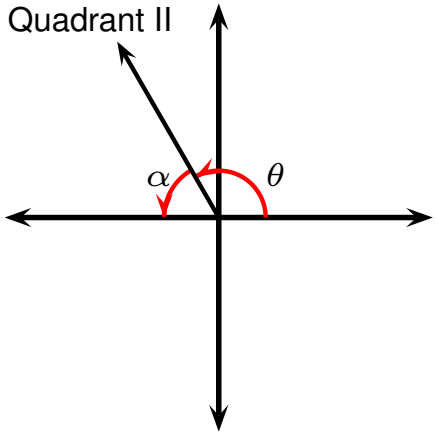

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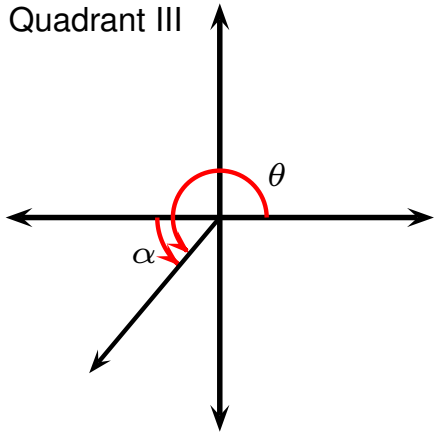

$$\alpha = \theta - \pi$$
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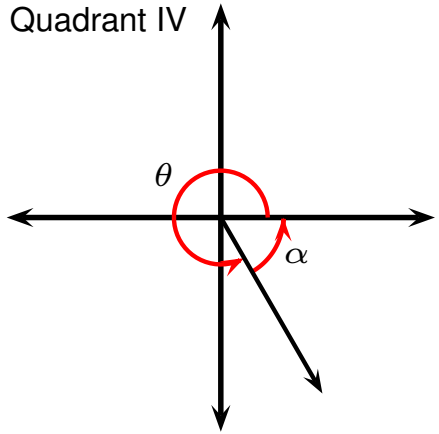

$$\alpha = 2\pi - \theta$$
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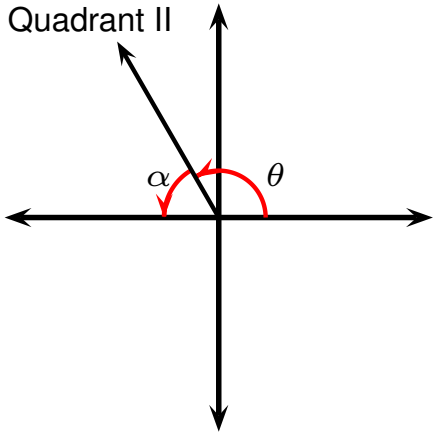

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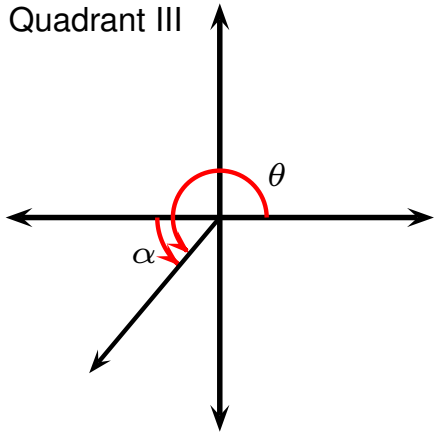

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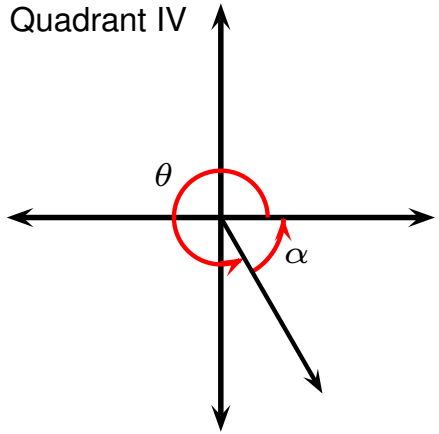

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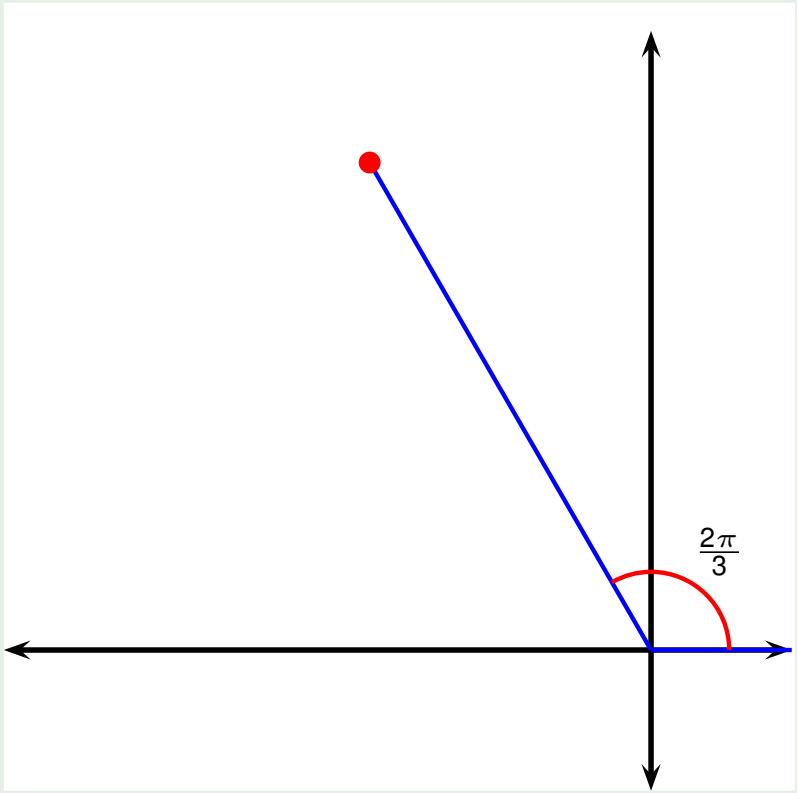
Observation

One can find the value of a trigonometric function of θ as follows.

- *Find the reference angle α associated to θ.*
- *Find the trig function of α.*
- *Use the quadrant in which θ lies to affix an appropriate sign to the function value.*

Example



Find the exact values of the trigonometric functions of $\theta = \dfrac{2\pi}{3} = 120°$.

$$\sin\left(\frac{2\pi}{3}\right) = \qquad \cos\left(\frac{2\pi}{3}\right) = \qquad \tan\left(\frac{2\pi}{3}\right) =$$

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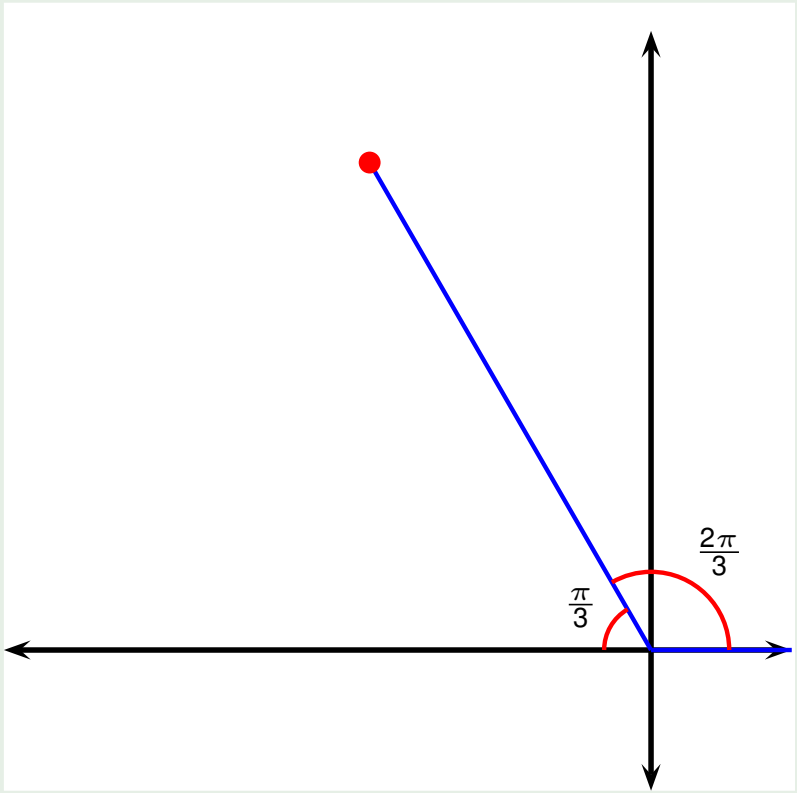

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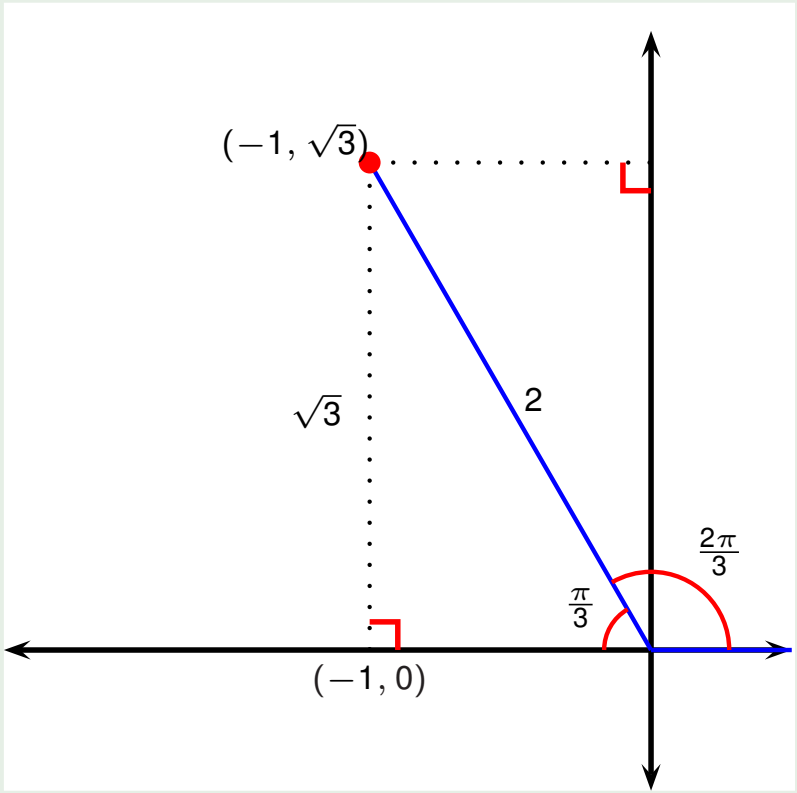

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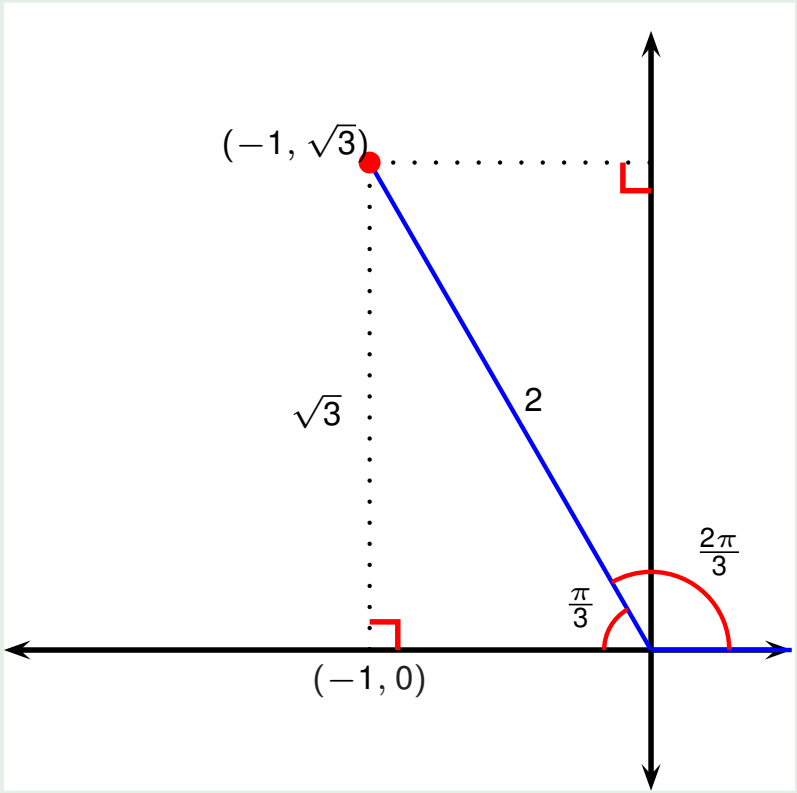

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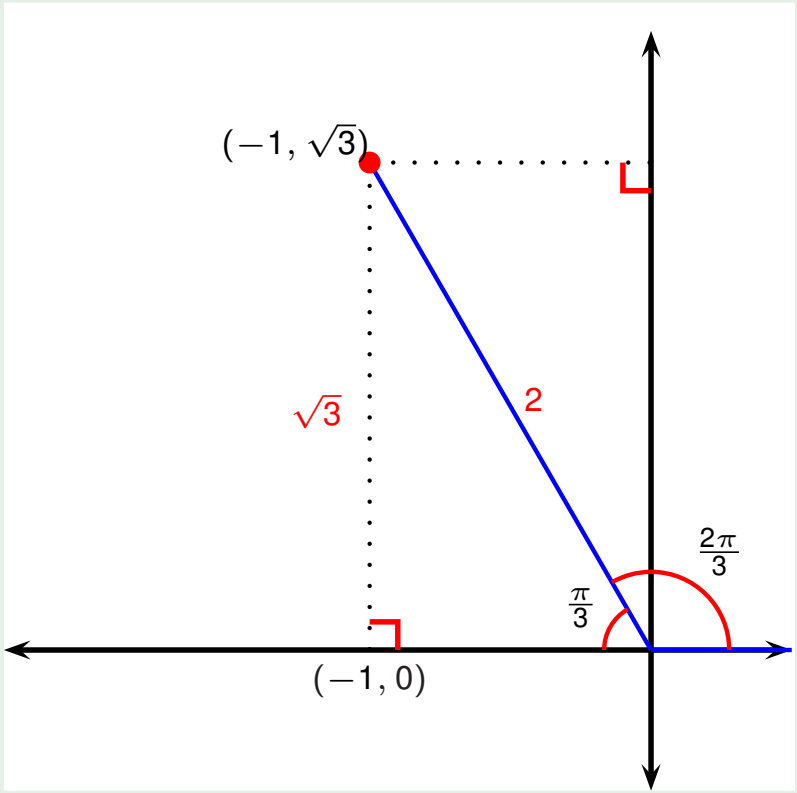

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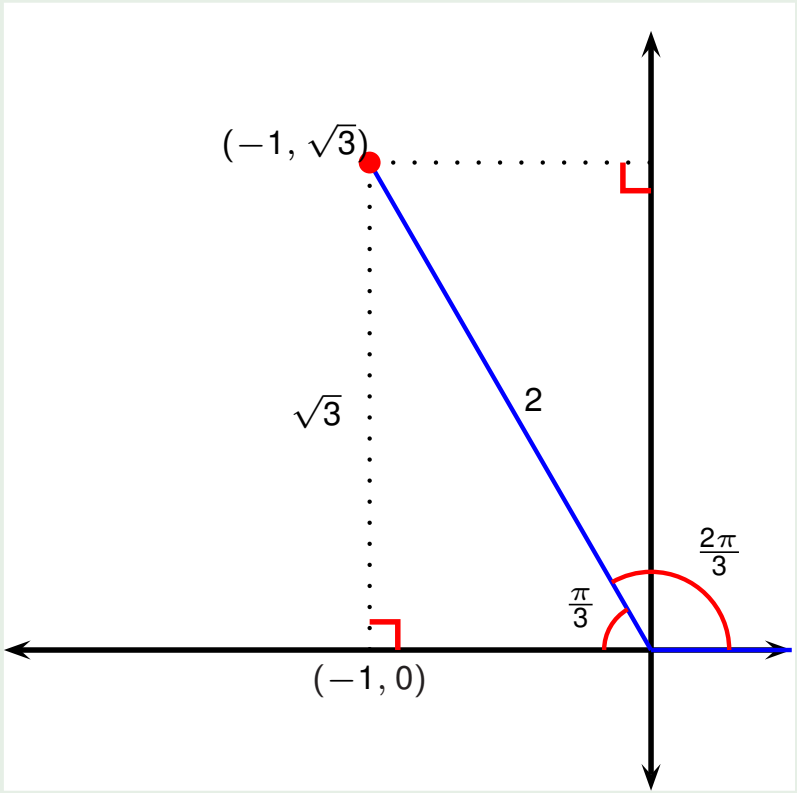

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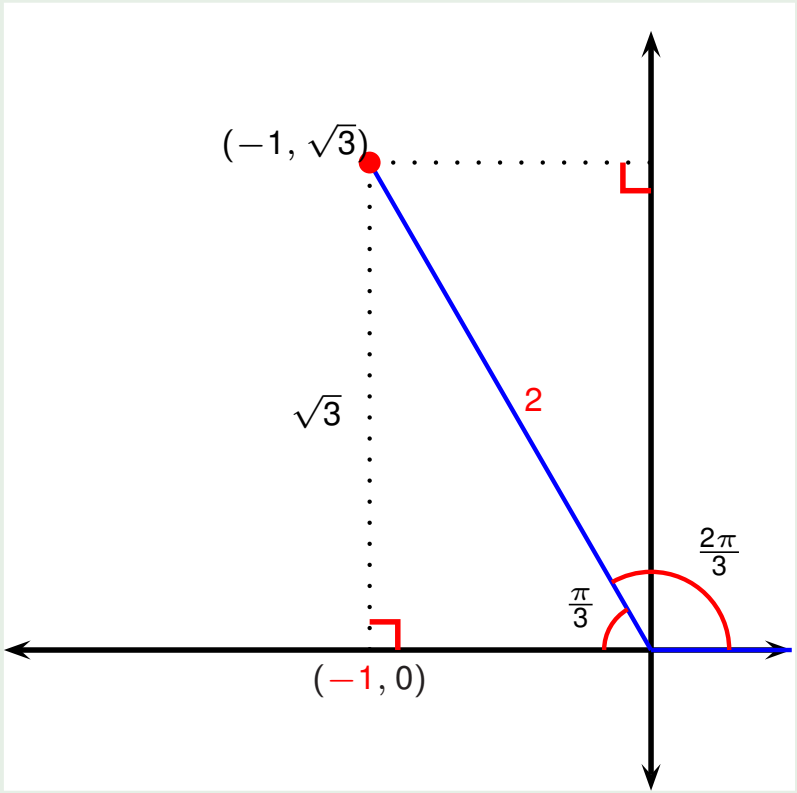

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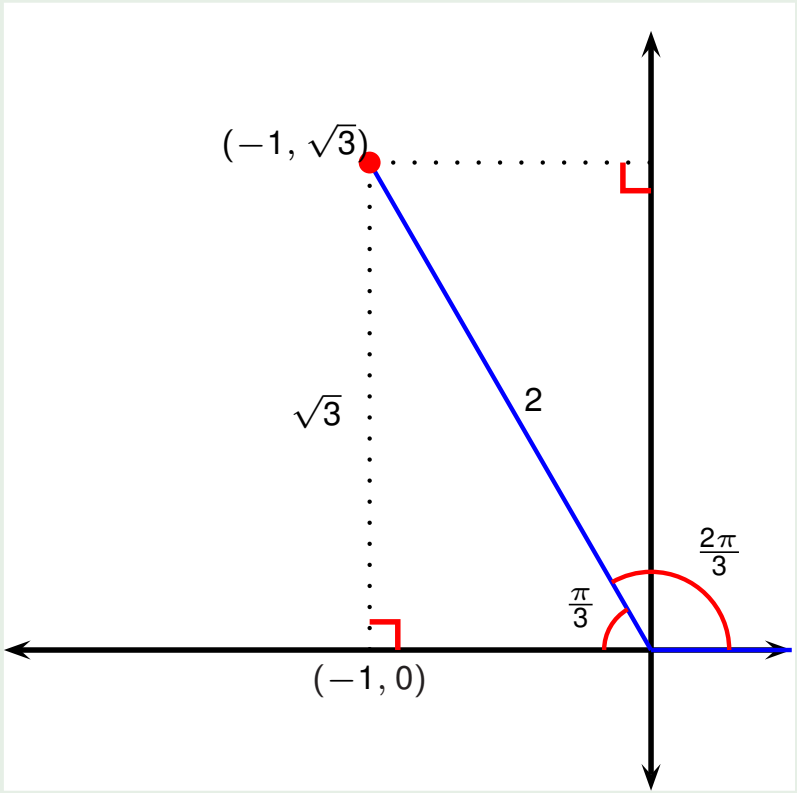

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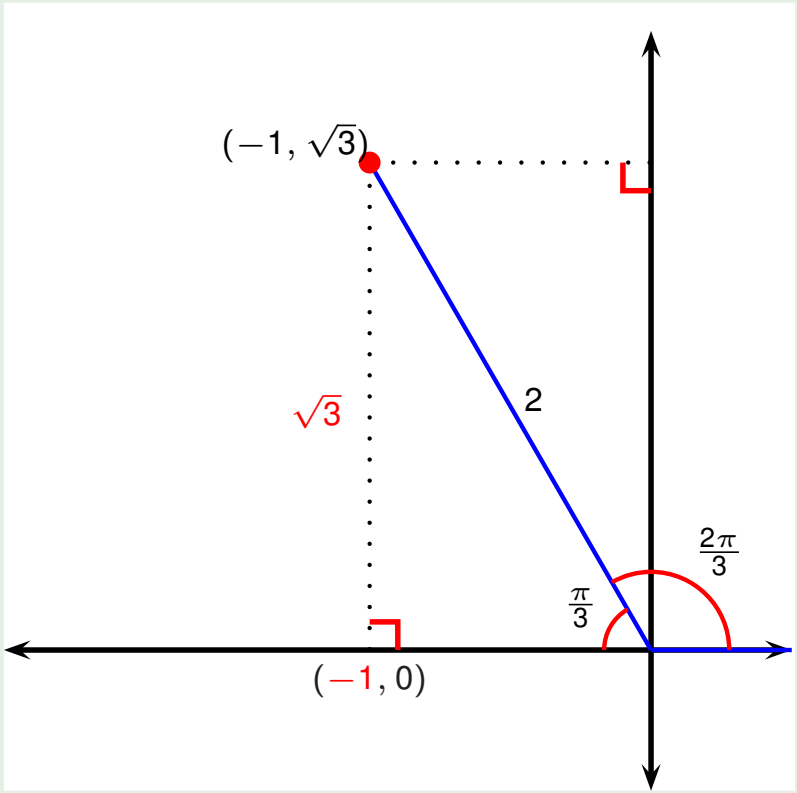

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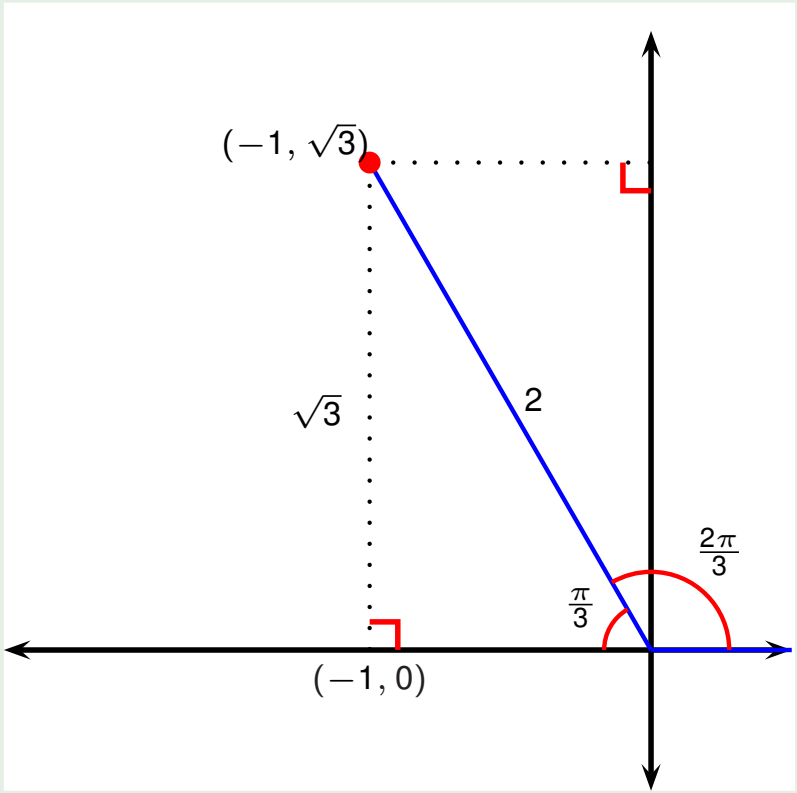

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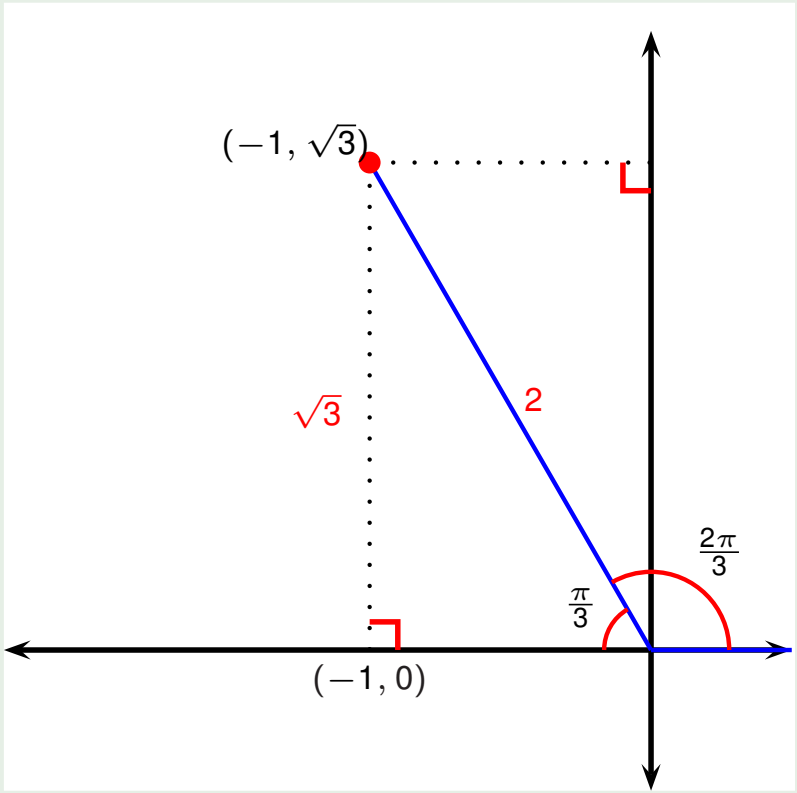

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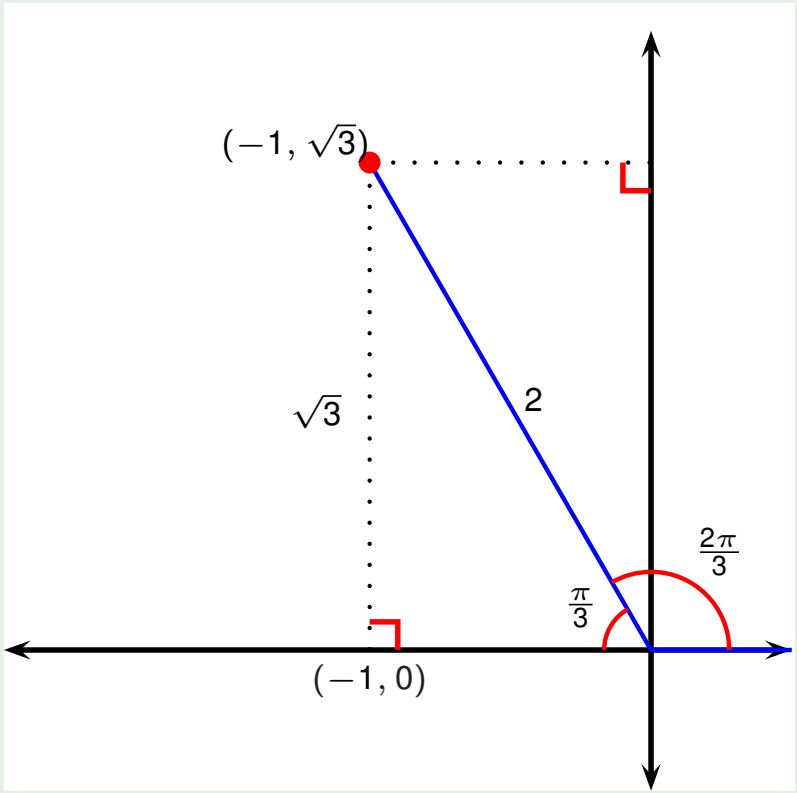

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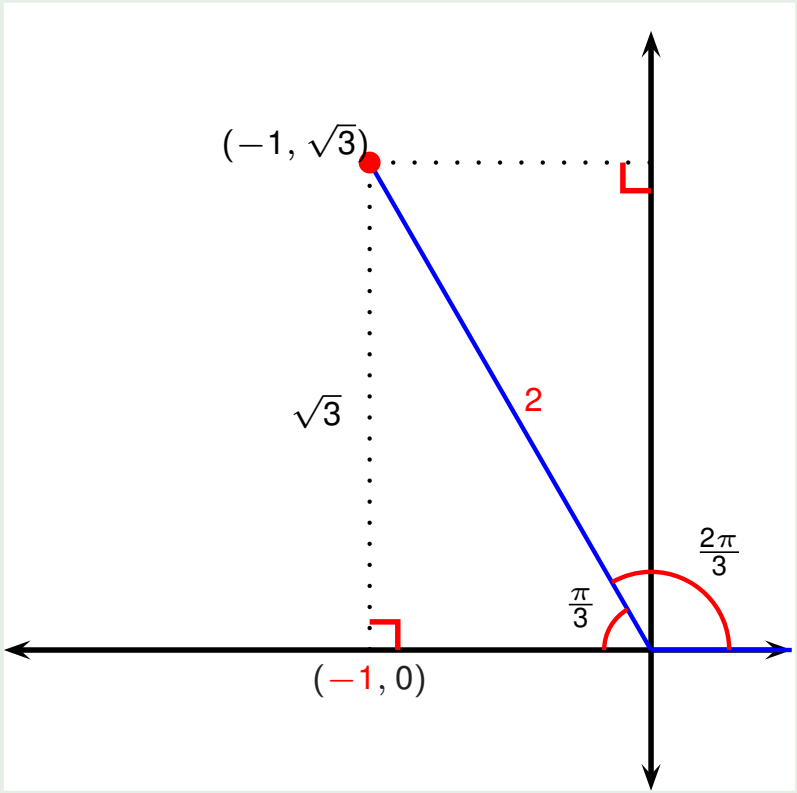

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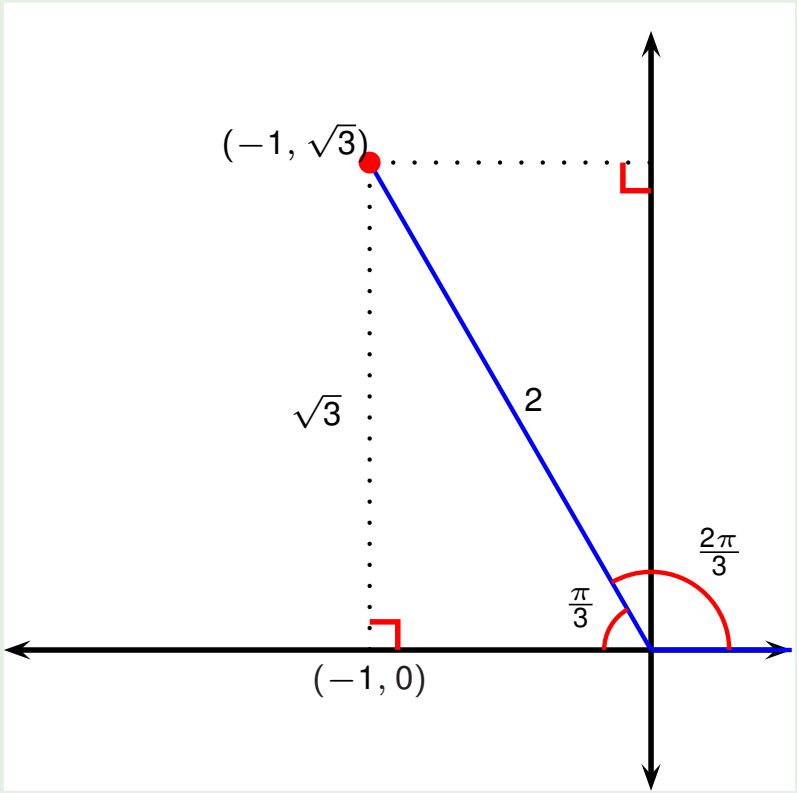

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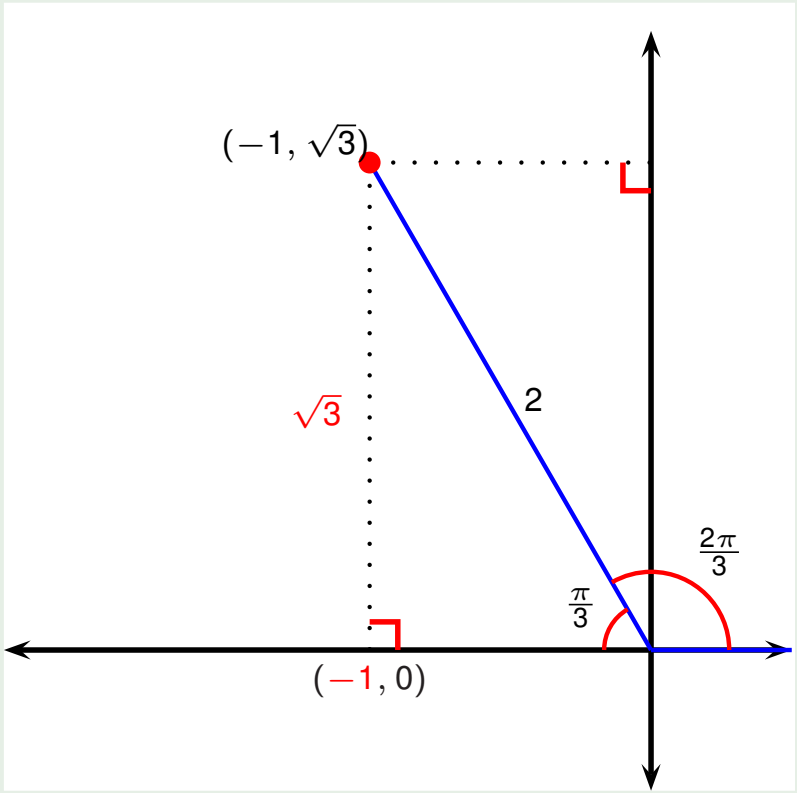

Find the exact values of the trigonometric functions of $\theta = \dfrac{2\pi}{3} = 120°$.

$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \qquad \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} \qquad \tan\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

$$\csc\left(\frac{2\pi}{3}\right) = \frac{2}{\sqrt{3}} \qquad \sec\left(\frac{2\pi}{3}\right) = -\frac{2}{1} = -2 \qquad \cot\left(\frac{2\pi}{3}\right) = -\frac{1}{\sqrt{3}}$$