Calculus II

Integrals of the form $\int \frac{ax}{bx^2+c}\mathbf{d}x$

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Linear substitutions leading to block IIa

Building block IIa: $\int \frac{x}{1+x^2}\mathrm{d}x = \frac{1}{2}\ln(1+x^2) + C$.

Example

$$\int \frac{x}{2x^2+3}\mathrm{d}x$$

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"Theoretical way" to solve example below: transform to IIa; this is slow.

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Example

$$
\begin{aligned}
\int \frac{x}{2x^2+3}\mathrm{d}x &= \int \frac{x}{3\left(\frac{2}{3}x^2+1\right)}\mathrm{d}x = \int \frac{x}{3\left(\left(\sqrt{\frac{2}{3}}x\right)^2+1\right)}\mathrm{d}x \\
&= \frac{3}{2}\int \frac{\sqrt{\frac{2}{3}}x}{3\left(\left(\sqrt{\frac{2}{3}}x\right)^2+1\right)}\mathrm{d}\left(\sqrt{\frac{2}{3}}x\right) \qquad \Big| \text{ Set } u=\sqrt{\frac{2}{3}}x \\
&= \frac{1}{2}\int \frac{u}{u^2+1}\mathrm{d}u = \frac{1}{4}\ln(1+u^2)+C \\
&= \frac{1}{4}\ln\left(\frac{1}{3}(2x^2+3)\right)+C \\
&= \frac{1}{4}\ln(2x^2+3)+\frac{\ln\left(\frac{1}{3}\right)}{4}+C \\
&= \frac{1}{4}\ln(2x^2+3)+K \quad .
\end{aligned}
$$

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The example below can be done directly, without transforming to block IIa.

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Example

$$\int \frac{x}{2x^2+3}\mathrm{d}x = \int \frac{1}{2x^2+3}\mathrm{d}\left(\frac{x^2}{2}\right)$$

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$$\begin{aligned}\int \frac{x}{2x^2+3}\mathrm{d}x &= \int \frac{1}{2x^2+3}\mathrm{d}\left(\frac{x^2}{2}\right)\\ &= \int \frac{1}{2x^2+3}\mathrm{d}\left(\frac{2x^2}{4}\right)\end{aligned}$$

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&= \int \frac{1}{2x^2+3}\mathrm{d}\left(\frac{2x^2+3}{4}\right) \quad \Big| \text{ Set } u = 2x^2+3 \\
&= \frac{1}{4}\int \frac{1}{u}\mathrm{d}u
\end{aligned}$$

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&= \frac{1}{4}\int \frac{1}{u}\mathrm{d}u \\
&= \frac{1}{4}\ln|u| + C \\
&= \frac{1}{4}\ln(2x^2+3) + C
\end{aligned}$$