

# Calculus II

## Integrals of the form $\int \arctan(mx) dx$

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Integration by parts:  $\int u dv = uv - \int v du.$

## Example

Set  $w = 1 + x^2$ .

$$\begin{aligned}
 \int_0^1 \arctan x dx &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x d(\arctan x) \\
 &= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x \frac{1}{1+x^2} dx \\
 &= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2} d\left(\frac{x^2}{2}\right) \\
 &= \frac{\pi}{4} - \frac{1}{2} \int_{x=0}^{x=1} \frac{1}{1+x^2} d(1+x^2) \\
 &= \frac{\pi}{4} - \frac{1}{2} \int_{x=0}^{x=1} \frac{1}{w} dw = \frac{\pi}{4} - \frac{1}{2} [\ln |w|]_{x=0}^{x=1} \\
 &= \frac{\pi}{4} - \frac{1}{2} \left[ \ln(1+x^2) \right]_{x=0}^{x=1} \\
 &= \frac{\pi}{4} - \frac{1}{2} (\ln 2 - \ln 1) = \frac{\pi}{4} - \frac{1}{2} \ln 2.
 \end{aligned}$$