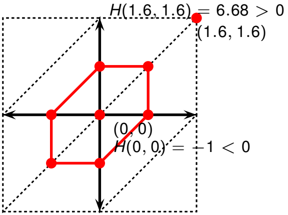
Calculus I
Plotting curves defined by implicit equations

Todor Milev

2019

(Elementary Computer algorithm for sketching graphs)

Let H-continuous; is there simple algorithm to sketch $H(x,y)=0$? Yes.



We illustrate the algorithm for:

$$x^2+2y^2=1$$
$$x^2+2y^2-1=0$$
Set $H(x,y)=x^2+2y^2-1$

- Elementary algorithm: fix large rectangle.
- Split the grid in triangular mesh. One strategy to do that is shown.
- For each triangle:
  - Fix two corners $P(x_P, y_P)$ and $Q(x_Q, y_Q)$.
  - If $H(x_P, y_P)$ and $H(x_Q, y_Q)$ have different sign then H must become zero somewhere on the segment between P and Q.
  - Select a point between P and Q and "guess" that H is zero there.
    - In our implementation, we select the midpoint (i.e., $\frac{1}{2}P+\frac{1}{2}Q$).
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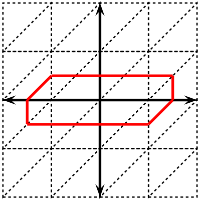

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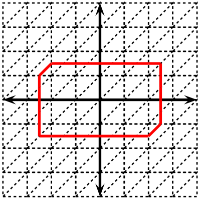

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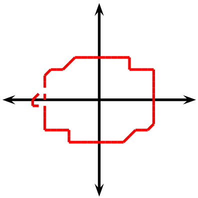

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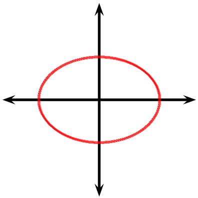

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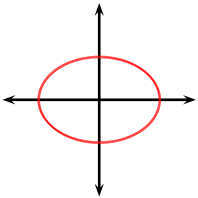

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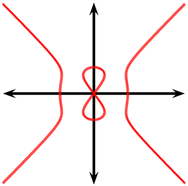

Illustrate the algorithm for:

$$y^2(y^2-3)=x^2(x^2-5)$$

$$H(x,y)=y^2(y^2-3)-x^2(x^2-5)$$

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