Calculus I
Miscellaneous chain rule problems, part 1

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2019

$$
\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) & & \text{(notation 1)}\\
(g(u))' &= g'(u)u' & \text{where } u = h(x) & \quad \text{(notation 2)}\\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \quad \text{(notation 3)} \quad .
\end{aligned}
$$

Example (Chain Rule, Notation 1, square root of a trigonometric function)

Differentiate $f(x) = \sqrt{\sin x + 2}$.

$$\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) && && \text{(notation 1)}\\
(g(u))' &= g'(u)u' && \text{where } u = h(x) && \text{(notation 2)}\\
\frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dx} && \text{where } y = g(u) && \text{(notation 3)}
\end{aligned} \quad .$$

Example (Chain Rule, Notation 1, square root of a trigonometric function)

$$\begin{aligned}
\text{Differentiate} \quad f(x) &= \sqrt{\sin x + 2}.\\
\text{Let} \quad h(x) &= \mathbf{?}\\
\text{Let} \quad g(u) &= \mathbf{?}\\
\text{Then} \quad f(x) &= g(h(x)).
\end{aligned}$$

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(g(h(x)))' &= g'(h(x)) \cdot h'(x) & & \text{(notation 1)}\\
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\frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dx} & \text{where } y = g(u) & \text{(notation 3)} \quad .
\end{aligned}$$

Example (Chain Rule, Notation 1, square root of a trigonometric function)

$$\begin{aligned}
\text{Differentiate} \quad f(x) &= \sqrt{\sin x + 2}.\\
\text{Let} \quad h(x) &= \sin x + 2.\\
\text{Let} \quad g(u) &= \textbf{?}\\
\text{Then} \quad f(x) &= g(h(x)).
\end{aligned}$$

$$\begin{array}{rcll}
(g(h(x)))' &=& g'(h(x)) \cdot h'(x) & & \text{(notation 1)}\\
(g(u))' &=& g'(u)u' & \text{where } u = h(x) & \text{(notation 2)}\\
\frac{dy}{dx} &=& \frac{dy}{du}\frac{du}{dx} & \text{where } y = g(u) & \text{(notation 3)}
\end{array} .$$

Example (Chain Rule, Notation 1, square root of a trigonometric function)

$$\begin{array}{rrcl}
\text{Differentiate} & f(x) &=& \sqrt{\sin x + 2}.\\
\text{Let} & h(x) &=& \sin x + 2.\\
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\text{Then} & f(x) &=& g(h(x)).
\end{array}$$

$$\begin{aligned} (g(h(x)))' &= g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\ (g(u))' &= g'(u)u' & \text{where } u = h(x) & \text{(notation 2)} \\ \frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dx} & \text{where } y = g(u) & \text{(notation 3)} \end{aligned} .$$

Example (Chain Rule, Notation 1, square root of a trigonometric function)

$$\begin{aligned} \text{Differentiate} \quad f(x) &= \sqrt{\sin x + 2}. \\ \text{Let} \quad h(x) &= \sin x + 2. \\ \text{Let} \quad g(u) &= \sqrt{u}. \\ \text{Then} \quad f(x) &= g(h(x)). \end{aligned}$$

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\text{Chain Rule:} \quad f'(x) &= g'(h(x))h'(x)
\end{aligned}
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&= \left(? \qquad \right) (? \quad)
\end{aligned}
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\text{Chain Rule:} & f'(x) & = & g'(h(x))h'(x) \\
 & & = & \left(\frac{1}{2\sqrt{h(x)}}\right) (\textbf{?} \quad)
\end{array}
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$$\begin{array}{rclll} (g(h(x)))' & = & g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\ (g(u))' & = & g'(u)u' & \text{where } u = h(x) & \text{(notation 2)} \\ \frac{\mathrm{d}y}{\mathrm{d}x} & = & \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)} \end{array} .$$

Example (Chain Rule, Notation 1, square root of a trigonometric function)

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\text{Chain Rule:} & f'(x) & = & g'(h(x))h'(x)\\
 & & = & \left(\frac{1}{2\sqrt{h(x)}}\right)(\cos x)
\end{array}$$

$$\begin{array}{rcll} (g(h(x)))' &=& g'(h(x))\cdot h'(x) & & \text{(notation 1)}\\ (g(u))' &=& g'(u)u' & \text{where } u=h(x) & \text{(notation 2)}\\ \frac{\mathrm{d}y}{\mathrm{d}x} &=& \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y=g(u) & \text{(notation 3)} \end{array} .$$

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$$\begin{array}{rrcl} \text{Differentiate} & f(x) &=& \sqrt{\sin x+2}.\\ \text{Let} & h(x) &=& \sin x+2.\\ \text{Let} & g(u) &=& \sqrt{u}.\\ \text{Then} & f(x) &=& g(h(x)).\\ \text{Chain Rule:} & f'(x) &=& g'(h(x))h'(x)\\ & &=& \left(\frac{1}{2\sqrt{h(x)}}\right)(\cos x)\\ & &=& \frac{\cos x}{2\sqrt{\sin x+2}}. \end{array}$$