

# Calculus II

## Split partial fraction, general case

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## Example

Write out the form of the partial fraction decomposition of

$$\frac{x^3 + x^2 + 1}{x(x-1)(x^2 + x + 1)(x^2 + 1)^3}$$

$$= \frac{A}{x} + \frac{B}{x-1} + \frac{Cx + D}{x^2 + x + 1} + \frac{Ex + F}{x^2 + 1} + \frac{Gx + H}{(x^2 + 1)^2} + \frac{Ix + J}{(x^2 + 1)^3}.$$

For example of this size it makes sense to use a computer algebra system; one such system easily produces the decomposition:

$$= \frac{-1}{x} + \frac{\frac{1}{8}}{x-1} + \frac{-x-1}{(x^2 + x + 1)} + \frac{\frac{15}{8}x - \frac{1}{8}}{(x^2 + 1)} + \frac{\frac{3}{4}x + \frac{3}{4}}{(x^2 + 1)^2} + \frac{-\frac{x}{2} + \frac{1}{2}}{(x^2 + 1)^3}.$$

## $Q(x)$ has quadratic factors with multiplicity $> 1$

- Suppose  $Q(x)$  has the factor  $(ax^2 + bx + c)^r$ , where  $b^2 - 4ac < 0$  and  $r > 1$ .
- Then the partial fraction decomposition should include summands of the form

$$\frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{A_rx + B_r}{(ax^2 + bx + c)^r}$$