Precalculus
Homework
Graphs of trig functions; inverse trig

1. Convert from degrees to radians.

   (a) $15°$.
   (b) $30°$.
   (c) $36°$.
   (d) $45°$.
   (e) $60°$.
   (f) $75°$.
   (g) $90°$.
   (h) $120°$.
   (i) $135°$.
   (j) $150°$.
   (k) $180°$.
   (l) $225°$.
   (m) $270°$.
   (n) $305°$.
   (o) $360°$.
   (p) $405°$.
   (q) $1200°$.
   (r) $-900°$.
   (s) $-2014°$.

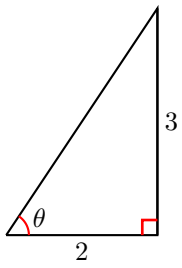
2. Convert from radians to degrees. The answer key has not been proofread, use with caution.

   (a) 4π.
   (b) $-\frac{7}{6}\pi$.
   (c) $\frac{7}{12}\pi$.
   (d) $\frac{4}{3}\pi$.
   (e) $-\frac{3}{8}\pi$.
   (f) 2014π.
   (g) 5.
   (h) -2014.

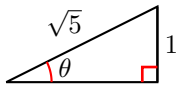
3. Find the indicated circle arc-length. The answer key has not been proofread, use with caution.

   (a) Circle of radius 3, arc of measure $36°$.
   (b) Circle of radius $\frac{1}{2}$, arc of measure $100°$.
   (c) Circle of radius 1, arc of measure 3 (radians).
   (d) Circle of radius 3, arc of measure $300°$.

4. Find the 6 trigonometric functions of the indicated angle in the indicated right triangle.

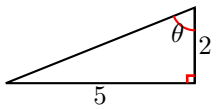


   (a)

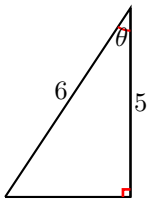


   (b)

(c)



(d)



5. Find the exact value of the trigonometric function (using radicals).

   (a) $\cos 135°$.

   (b) $\sin 225°$.

   (c) $\cos 495°$.

   (d) $\sin 560°$.

   (e) $\sin\left(\frac{3\pi}{2}\right)$.

   (f) $\cos\left(\frac{11\pi}{6}\right)$.

   (g) $\sin\left(\frac{2015\pi}{3}\right)$.

   (h) $\cos\left(\frac{17\pi}{3}\right)$.

6. Find all solutions of the equation in the interval $[0, 2\pi)$. The answer key has not been proofread, use with caution.

   (a) $\sin x = -\frac{\sqrt{2}}{2}$.

   (b) $\cos x = \frac{\sqrt{3}}{2}$.

   (c) $\sin(3x) = \frac{1}{2}$.

   (d) $\cos(7x) = 0$.

   (e) $\cos\left(3x + \frac{\pi}{2}\right) = 0$.

   (f) $\sin\left(5x - \frac{\pi}{3}\right) = 0$.

7. Use the known values of $\sin 30°, \cos 30°, \sin 45°, \cos 45°, \sin 60°, \cos 60°, \ldots$, the angle sum formulas and the cofunction identities to find an exact value (using radicals) for the trigonometric function.

   (a) The six trigonometric functions of $105° = 45° + 60°$:
   - $\sin(105°)$.
   - $\cos(105°)$. Should your answer be a positive or a negative number?
   - $\tan(105°)$.
   - $\cot(105°)$.
   - $\sec(105°)$.
   - $\csc(105°)$.

   (b) The six trigonometric functions of $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$:
   - $\sin\left(\frac{\pi}{12}\right)$.
   - $\cos\left(\frac{\pi}{12}\right)$. Should $\sin\left(\frac{\pi}{12}\right)$ be larger or smaller than $\cos\left(\frac{\pi}{12}\right)$?
   - $\tan\left(\frac{\pi}{12}\right)$.
   - $\cot\left(\frac{\pi}{12}\right)$.
   - $\sec\left(\frac{\pi}{12}\right)$.
   - $\csc\left(\frac{\pi}{12}\right)$.

8. Simplify to a trigonometric function of the angle θ. The answer key has not been proofread, use with caution.

   (a) $\sin\left(\frac{\pi}{2} - \theta\right)$.

(b) $\cos\left(\frac{13\pi}{2} - \theta\right)$.

(c) $\tan(\pi - \theta)$

(d) $\cot\left(\frac{3\pi}{2} - \theta\right)$

(e) $\csc\left(\frac{3\pi}{2} + \theta\right)$

9. Using the power-reducing formulas, rewrite the expression in terms of first powers of the cosines and sines of multiples of the angle θ.

(a) $\sin^4\theta$.

(b) $\cos^4\theta$.

(c) $\sin^6\theta$.

(d) $\cos^6\theta$.

10. Use the sum-to-product formulas to find all solutions of the trigonometric equation in the interval $[0, 2\pi)$.

Please note that typing a query such as "solve(sin(x)+sin (3x)=0)" at `www.wolframalpha.com` will provide you with a correct answer and a function plot.

(a) $\sin(x) + \sin(3x) = 0$.

(b) $\cos(x) + \cos(-3x) = 0$.

(c) $\sin(x) - \sin(3x) = 0$.

(d) $\cos(2x) - \cos(3x) = 0$.

11. Find the inverse function. You are asked to do the algebra only; you are not asked to determine the domain or range of the function or its inverse.

(a) $f(x) = 3x^2 + 4x - 7$, where $x \geq -\frac{2}{3}$.

(b) $f(x) = 2x^2 + 3x - 5$, where $x \geq -\frac{3}{4}$.

(c) $f(x) = \dfrac{2x+5}{x-4}$, where $x \neq 4$.

(d) $f(x) = \dfrac{3x+5}{2x-4}$, where $x \neq 2$.

(e) $f(x) = \dfrac{5x+6}{4x+5}$, where $x \neq -\frac{5}{4}$.

(f) $f(x) = \dfrac{2x-3}{-3x+4}$, where $x \neq \frac{4}{3}$..

12. Find the inverse function and its domain.

(a) $y = \ln(x+3)$.

(b) $y = 4\ln(x-3) - 4$.

(c) $y = 2\ln(-2x+4) + 1$

(d) $f(x) = e^{x^3}$.

(e) $y = (\ln x)^2$, $x \geq 1$.

(f) $y = \dfrac{e^x}{1+2e^x}$.

(g) $f(x) = 2^{2x} + 2^x - 2$.

13. Find each of the following values. Express your answers precisely, not as decimals.

(a) $\arcsin(\sin 4)$.

(b) $\arcsin(\sin 0.5)$.

(c) $\arcsin(\cos 120^\circ)$.

(d) $\arccos(\cos(3))$.

(e) $\arccos(\cos(-2))$.

(f) $\arccos(\sin(-4))$.

(g) $\arctan(\tan 5)$.

14. Express as the following as an algebraic expression of x. In other words, "get rid" of the trigonometric and inverse trigonometric expressions.

(a) $\cos^2(\arctan x)$.

(b) $-\sin^2(\operatorname{arccot} x)$.

(c) $\dfrac{1}{\cos(\arcsin x)}$.

(d) $-\dfrac{1}{\sin(\arccos x)}$.

15. Let $x \in (0,1)$. Express the following using x and $\sqrt{1-x^2}$.

(a) $\sin(\arcsin(x))$.

(b) $\sin(2\arcsin(x))$.

(c) $\sin(3\arcsin(x))$.

(d) $\sin(\arccos(x))$.

(e) $\sin(2\arccos(x))$.

(f) $\sin(3\arccos(x))$.

(g) $\cos(2\arcsin(x))$.

(h) $\cos(3\arccos(x))$.