

# Calculus II

## Integrals of the form $\int \frac{ax}{bx^2 + c} dx$

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# Linear substitutions leading to block IIa

Building block IIa:  $\int \frac{x}{1+x^2} dx = \frac{1}{2} \ln(1+x^2) + C.$

## Example

$$\int \frac{x}{2x^2+3} dx$$

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“Theoretical way” to solve example below: transform to IIa; this is slow.

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## Example

$$\int \frac{x}{2x^2+3} dx = \int \frac{x}{3\left(\frac{2}{3}x^2+1\right)} dx = \int \frac{x}{3\left(\left(\sqrt{\frac{2}{3}}x\right)^2+1\right)} dx$$

$$= \frac{3}{2} \int \frac{\sqrt{\frac{2}{3}}x}{3\left(\left(\sqrt{\frac{2}{3}}x\right)^2+1\right)} d\left(\sqrt{\frac{2}{3}}x\right)$$

$$\left| \text{Set } u = \sqrt{\frac{2}{3}}x \right.$$

$$= \frac{1}{2} \int \frac{u}{u^2+1} du = \frac{1}{4} \ln(1+u^2) + C$$

$$= \frac{1}{4} \ln\left(\frac{1}{3}(2x^2+3)\right) + C$$

$$= \frac{1}{4} \ln(2x^2+3) + \frac{\ln\left(\frac{1}{3}\right)}{4} + C$$

$$= \frac{1}{4} \ln(2x^2+3) + K.$$

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The example below can be done directly, without transforming to block IIa.

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## Example

$$\int \frac{x}{2x^2+3} dx = \int \frac{1}{2x^2+3} d\left(\frac{x^2}{2}\right)$$

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## Example

$$\begin{aligned} \int \frac{x}{2x^2+3} dx &= \int \frac{1}{2x^2+3} d\left(\frac{x^2}{2}\right) \\ &= \int \frac{1}{2x^2+3} d\left(\frac{2x^2}{4}\right) \end{aligned}$$



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## Example

$$\begin{aligned}
 \int \frac{x}{2x^2+3} dx &= \int \frac{1}{2x^2+3} d\left(\frac{x^2}{2}\right) \\
 &= \int \frac{1}{\color{red}{2x^2+3}} d\left(\frac{\color{red}{2x^2+3}}{4}\right) && \left| \text{Set } \color{red}{u} = \color{red}{2x^2+3} \right. \\
 &= \frac{1}{4} \int \frac{1}{\color{red}{u}} d\color{red}{u}
 \end{aligned}$$

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 &= \int \frac{1}{2x^2+3} d\left(\frac{2x^2+3}{4}\right) && \left| \text{Set } u = 2x^2+3 \right. \\
 &= \frac{1}{4} \int \frac{1}{u} du \\
 &= \frac{1}{4} \ln |u| + C
 \end{aligned}$$

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 \int \frac{x}{2x^2+3} dx &= \int \frac{1}{2x^2+3} d\left(\frac{x^2}{2}\right) \\
 &= \int \frac{1}{2x^2+3} d\left(\frac{2x^2+3}{4}\right) && \left| \text{Set } u = 2x^2 + 3 \right. \\
 &= \frac{1}{4} \int \frac{1}{u} du \\
 &= \frac{1}{4} \ln |u| + C \\
 &= \frac{1}{4} \ln(2x^2 + 3) + C
 \end{aligned}$$