

# Calculus II

## Integrals with irreducible quadratic denominator

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# Building blocks IIa and IIIa

Building block IIa:  $\int \frac{x}{1+x^2} dx$ . Building block IIIa:  $\int \frac{1}{1+x^2} dx$ .

## Example (Block IIa)

$$\int \frac{x}{1+x^2} dx$$

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## Example (Block IIa)

$$\int \frac{x}{1+x^2} dx = \int \frac{1}{(1+x^2)} \frac{d(x^2)}{2}$$

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## Example (Block IIa)

$$\begin{aligned} \int \frac{x}{1+x^2} dx &= \int \frac{1}{(1+x^2)} \frac{d(x^2)}{2} \\ &= \int \frac{1}{1+x^2} \frac{d(1+x^2)}{2} \end{aligned}$$

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## Example (Block IIa)

$$\begin{aligned}
 \int \frac{x}{1+x^2} dx &= \int \frac{1}{(1+x^2)} \frac{d(x^2)}{2} \\
 &= \int \frac{1}{\overset{1+x^2}{\text{red}}} \frac{\overset{d(1+x^2)}{\text{red}}}{2} \\
 &= \int \frac{1}{\overset{u}{\text{red}}} \frac{\overset{du}{\text{red}}}{2}
 \end{aligned}
 \quad \left| \quad \text{Set } \overset{u}{\text{red}} = 1+x^2
 \right.$$

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## Example (Block IIa)

$$\begin{aligned}
 \int \frac{x}{1+x^2} dx &= \int \frac{1}{(1+x^2)} \frac{d(x^2)}{2} \\
 &= \int \frac{1}{1+x^2} \frac{d(1+x^2)}{2} && \left| \text{Set } u = 1+x^2 \right. \\
 &= \int \frac{1}{u} \frac{du}{2} \\
 &= \frac{1}{2} \ln |u| + C
 \end{aligned}$$

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 \int \frac{x}{1+x^2} dx &= \int \frac{1}{(1+x^2)} \frac{d(x^2)}{2} \\
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 &= \int \frac{1}{u} \frac{du}{2} \\
 &= \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln (1+x^2) + C .
 \end{aligned}$$

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### Example (Block IIa)

$$\begin{aligned}
 \int \frac{x}{1+x^2} dx &= \int \frac{1}{(1+x^2)} \frac{d(x^2)}{2} \\
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 &= \int \frac{1}{u} \frac{du}{2} \\
 &= \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln (1+x^2) + C .
 \end{aligned}$$

### Example (Block IIIa)

$$\int \frac{1}{1+x^2} dx$$



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## Example (Block IIa)

$$\begin{aligned}
 \int \frac{x}{1+x^2} dx &= \int \frac{1}{(1+x^2)} \frac{d(x^2)}{2} \\
 &= \int \frac{1}{1+x^2} \frac{d(1+x^2)}{2} && \left| \text{Set } u = 1+x^2 \right. \\
 &= \int \frac{1}{u} \frac{du}{2} \\
 &= \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln (1+x^2) + C .
 \end{aligned}$$

## Example (Block IIIa)

$$\int \frac{1}{1+x^2} dx = ?$$

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## Example (Block IIa)

$$\begin{aligned}
 \int \frac{x}{1+x^2} dx &= \int \frac{1}{(1+x^2)} \frac{d(x^2)}{2} \\
 &= \int \frac{1}{1+x^2} \frac{d(1+x^2)}{2} && \left| \text{Set } u = 1+x^2 \right. \\
 &= \int \frac{1}{u} \frac{du}{2} \\
 &= \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln (1+x^2) + C .
 \end{aligned}$$

## Example (Block IIIa)

$$\int \frac{1}{1+x^2} dx = \arctan x + C$$