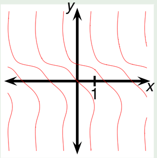
Calculus I
Implicit derivatives involving trigonometry, part 1

Todor Milev

2019

Example



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$$\sin(2(x+y)) = y^2 \cos(2x).$$

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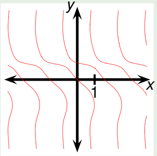

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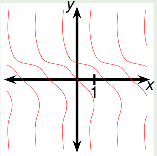

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? $$= \textbf{?}$$

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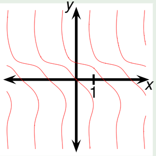

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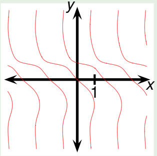

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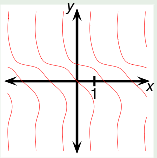

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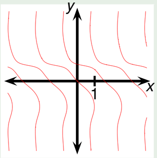

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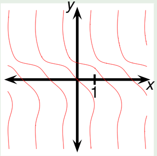

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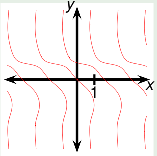

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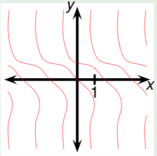

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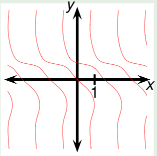

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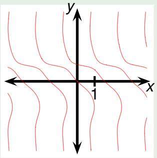

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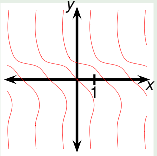

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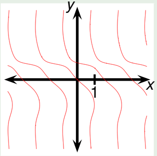

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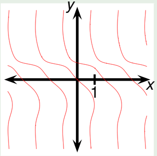

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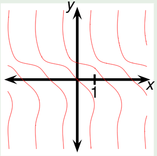

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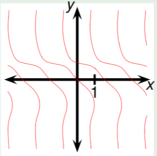

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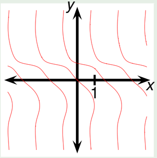

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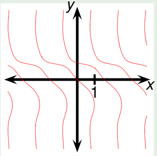

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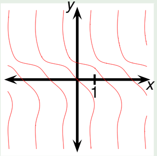

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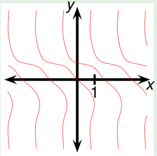

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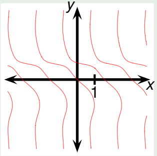

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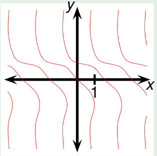

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$$
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\cos(2(x+y))\left(2+2y'\right) &= 2yy'\cos(2x)+y^2\left(-\sin(2x)\right)\frac{d}{dx}(2x)\\
2\cos(2(x+y))\left(1+y'\right) &= 2yy'\cos(2x)-y^2\sin(2x)2\\
\cos(2(x+y))+y'\cos(2(x+y)) &= yy'\cos(2x)-y^2\sin(2x)\\
y'\cos(2(x+y))-yy'\cos(2x) &= -\cos(2(x+y))-y^2\sin(2x)\\
y'(\cos(2(x+y))-y\cos(2x)) &= -\cos(2(x+y))-y^2\sin(2x)\\
y' &= \frac{-\cos(2(x+y))-y^2\sin(2x)}{\cos(2(x+y))-y\cos(2x)}.
\end{aligned}
$$