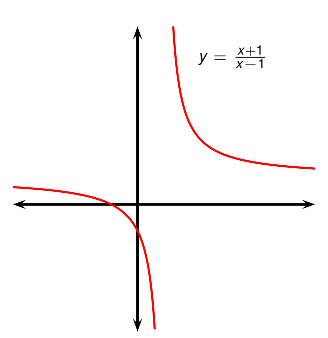
Calculus I
Inverse of fractional linear transformation

Todor Milev

2019

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.



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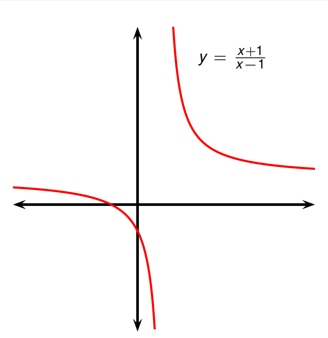

We deal with domains and ranges later:

$$y = \frac{x+1}{x-1}$$

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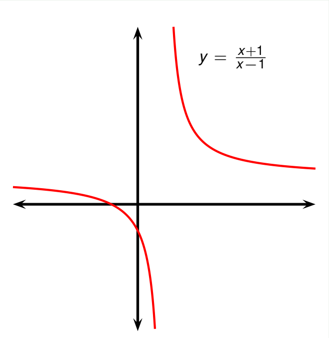

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$$\begin{array}{rcl|l} y &=& \dfrac{x+1}{x-1} & \text{mult. by } (x-1) \\ y(x-1) &=& x+1 & \end{array}$$

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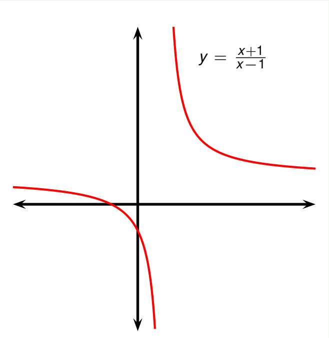

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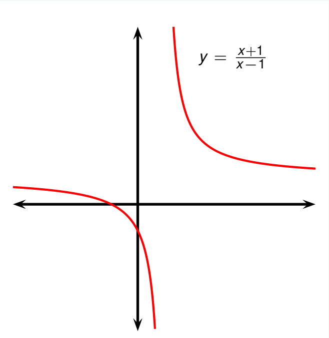

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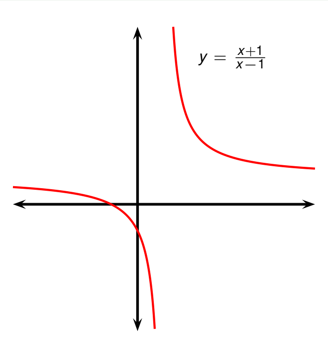

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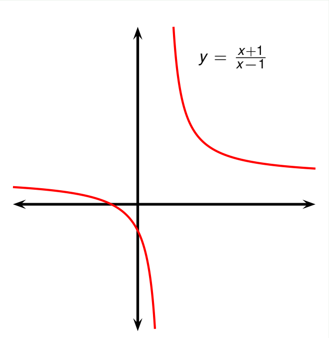

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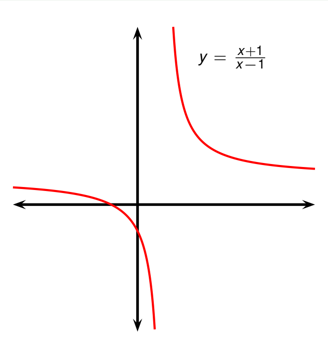

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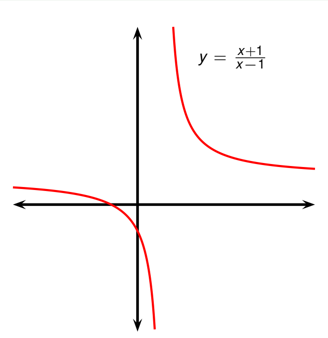

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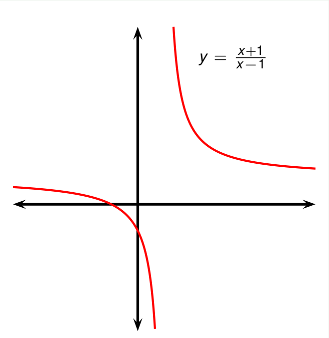

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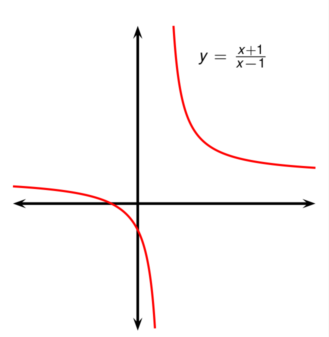

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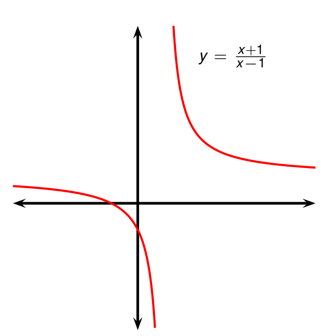

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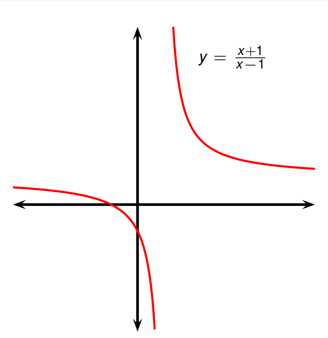

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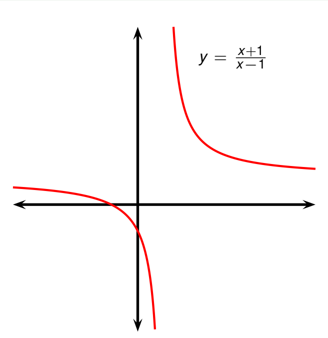

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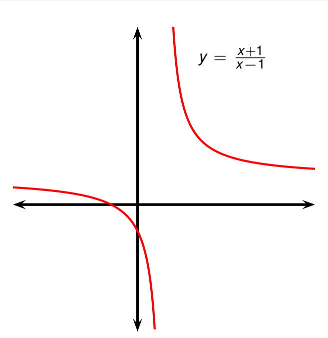

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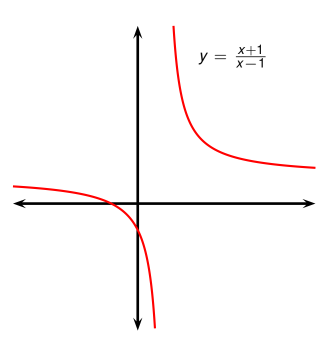

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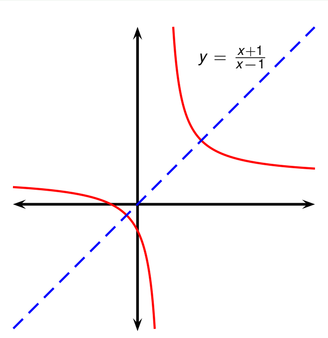

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Graph of f is symmetric across $y = x$.