

## Precalculus

# A useful inverse hyperbolic function identity

Todor Milev

2019

The inverse hyperbolic function  $\operatorname{arcsinh} = \ln \left( x + \sqrt{1 + x^2} \right)$  is used when studying hyperbolas (types of curves in the plane).

### Example

Demonstrate that  $-\ln \left( \sqrt{1 + x^2} - x \right) = \ln \left( x + \sqrt{1 + x^2} \right)$ .

$$\begin{aligned}
 -\ln \left( \sqrt{1 + x^2} - x \right) &= \ln \left( \frac{1}{\sqrt{x^2 + 1} - x} \right) && \left| \text{rationalize} \right. \\
 &= \ln \left( \frac{\left( \sqrt{x^2 + 1} + x \right)}{\left( \sqrt{x^2 + 1} - x \right) \left( \sqrt{x^2 + 1} + x \right)} \right) \\
 &= \ln \left( \frac{\sqrt{x^2 + 1} + x}{x^2 + 1 - x^2} \right) \\
 &= \ln \left( \sqrt{x^2 + 1} + x \right) .
 \end{aligned}$$