Precalculus
Quadratic inequality part 1

Todor Milev

2019

Example

Solve the inequality.

$$2x^2 + 3x - 5 \quad \geq \quad 0$$

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$$\begin{array}{rcl} 2x^2+3x-5 & \geq & 0 \\ (\textbf{?} \qquad\quad)(\textbf{?} \qquad\quad) & \geq & 0 \end{array}$$

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\begin{aligned}
2x^2 + 3x - 5 &\geq 0 \\
(2x + 5)(x - 1) &\geq 0
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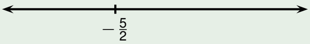
Left hand side vanishes when ${\color{red}x = -\frac{5}{2}}$ and when $x = 1$.



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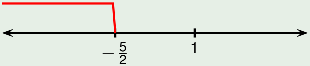

Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$			

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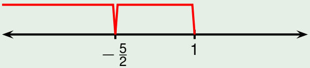

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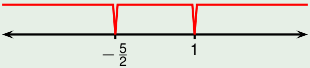

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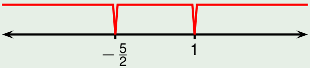

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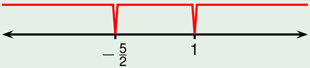

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${\color{red}(1, \infty)}$	$({\color{red}+})(\textbf{?}\quad)$		

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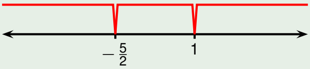

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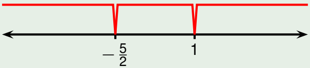

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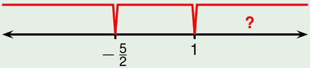

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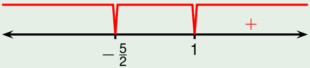

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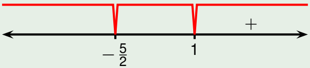

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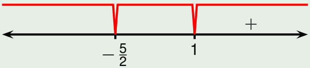

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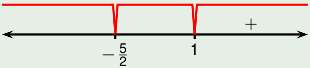

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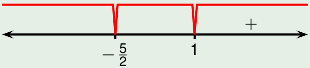

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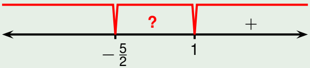

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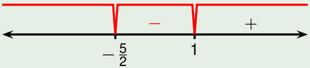

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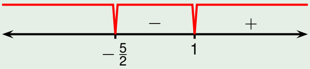

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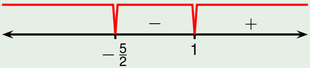

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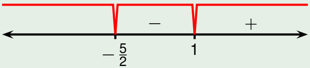

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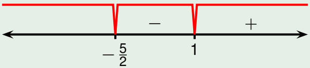

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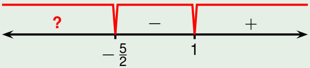

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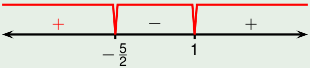

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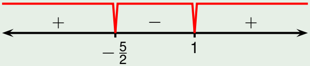

Interval	Factor signs	Final sign	Sample pt	Value at sample pt
$\left(-\infty,-\frac{5}{2}\right)$	$(-)(-)$	$+$	-100	$f(-100)>0$
$\left(-\frac{5}{2},1\right)$	$(+)(-)$	$-$	0	$f(0)=-5<0$
$(1,\infty)$	$(+)(+)$	$+$	100	$f(100)>0$

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$x \in$ **?**

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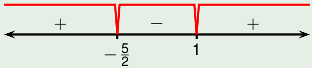

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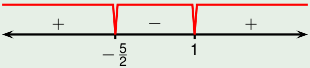

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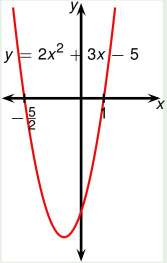

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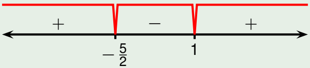

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