

Precalculus

Quadratic trigonometric inequalities

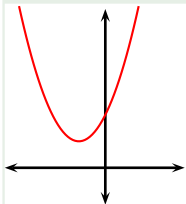
Todor Milev

2019

Example

- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2 \cos^2 \theta + 2 \cos \theta + 1 \leq \cos \theta + 2$ lying in $[-360^\circ, 360^\circ]$.

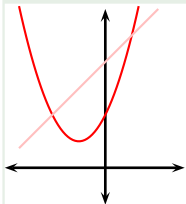
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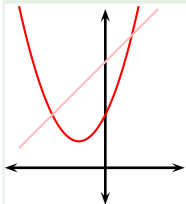
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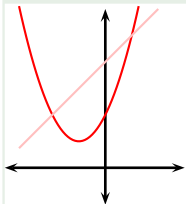


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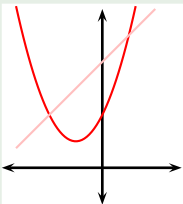


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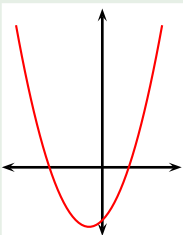
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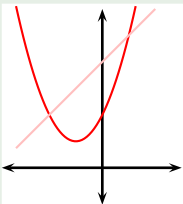
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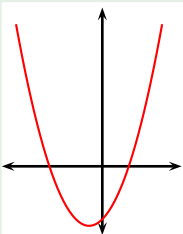


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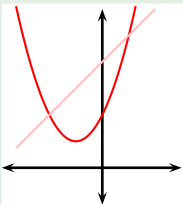
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$$2u^2 + u - 1 \leq 0$$

$$2(\text{?})(\text{?}) \leq 0$$



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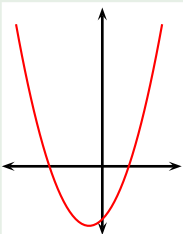


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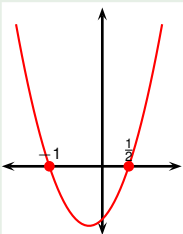
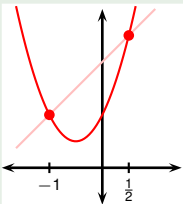
$$2u^2 + u - 1 \leq 0$$

$$2(u - \frac{1}{2})(u + 1) \leq 0$$



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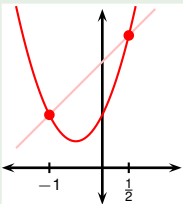
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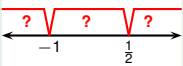
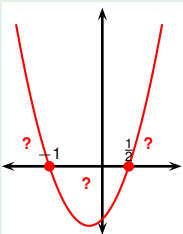
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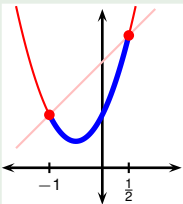


$$\begin{aligned}
 2u^2 + 2u + 1 &\leq u + 2 \\
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 2\left(u - \frac{1}{2}\right)(u + 1) &\leq 0 \\
 u &\in ?
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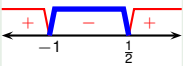
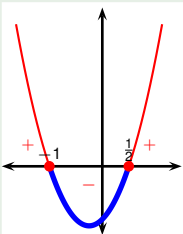


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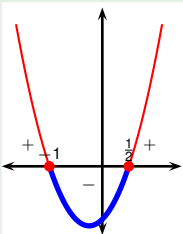
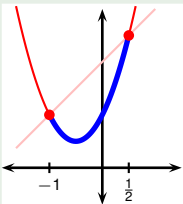


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 u &\in \left[-1, \frac{1}{2}\right]
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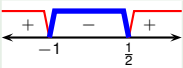
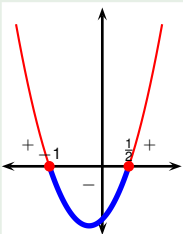
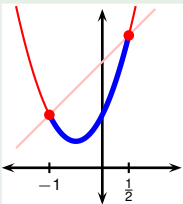
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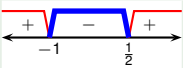
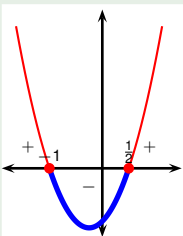
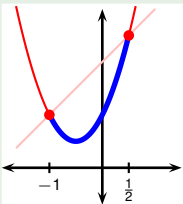
$$2\left(u - \frac{1}{2}\right)(u + 1) \leq 0$$

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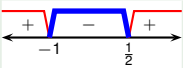
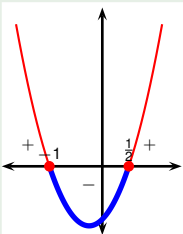
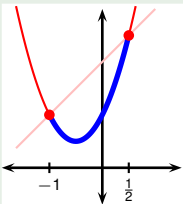
$$u \in [-1, \frac{1}{2}]$$

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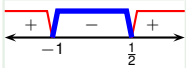
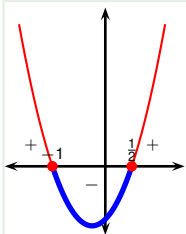
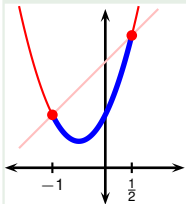
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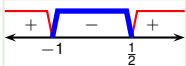
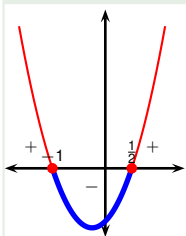
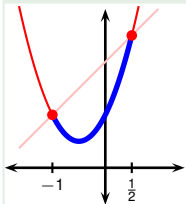
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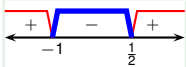
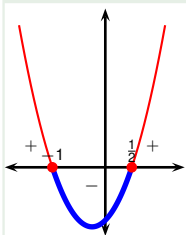
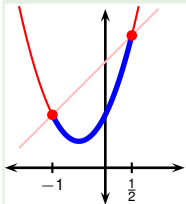
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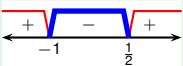
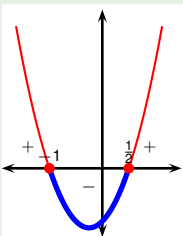
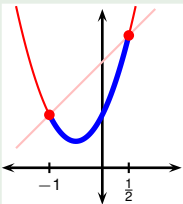
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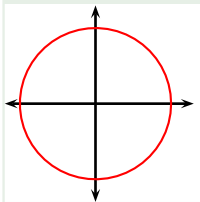
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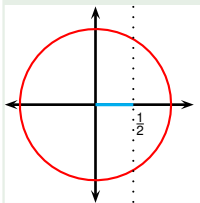


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$$\begin{aligned} \cos \theta &\in \left[-1, \frac{1}{2}\right] \\ -1 &\leq \cos \theta \leq \frac{1}{2} \end{aligned}$$

$$\theta \in \quad [? \quad , ? \quad]$$

Example



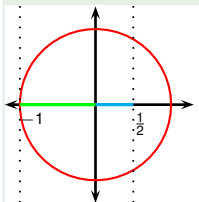
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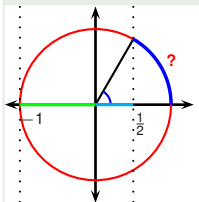
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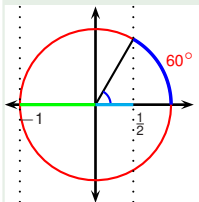
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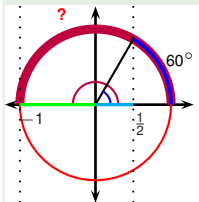
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$$\cos \theta \in \left[-1, \frac{1}{2}\right]$$

$$-1 \leq \cos \theta \leq \frac{1}{2}$$

$$\theta \in \quad [60^\circ, ?] \quad$$

Example

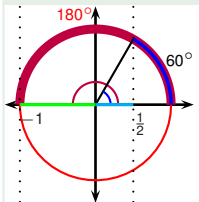


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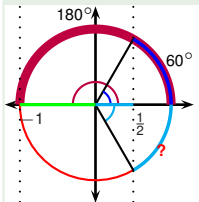
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$$\theta \in [60^\circ, 180^\circ]$$

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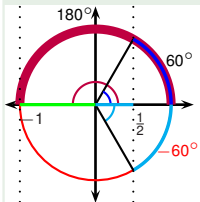


- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2 \cos^2 \theta + 2 \cos \theta + 1 \leq \cos \theta + 2$ lying in $[-360^\circ, 360^\circ]$.

$$\begin{aligned} \cos \theta &\in \left[-1, \frac{1}{2}\right] \\ -1 &\leq \cos \theta \leq \frac{1}{2} \end{aligned}$$

$$\theta \in [?, \text{?}] \cup [60^\circ, 180^\circ]$$

Example



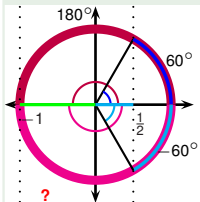
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Example



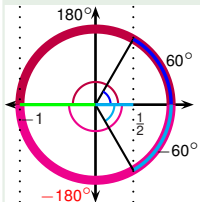
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Example



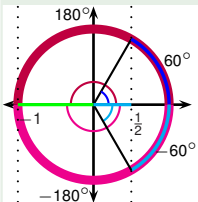
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Example

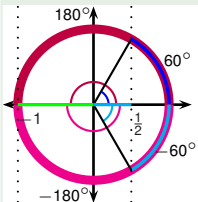


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Example



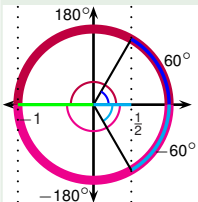
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Example



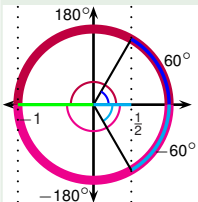
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$$\theta \in [-180^\circ, -60^\circ] \cup [60^\circ, 180^\circ] \quad k = 0$$

Example



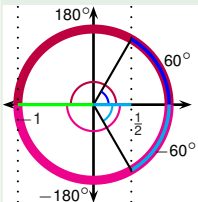
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Example



- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2 \cos^2 \theta + 2 \cos \theta + 1 \leq \cos \theta + 2$ lying in $[-360^\circ, 360^\circ]$.

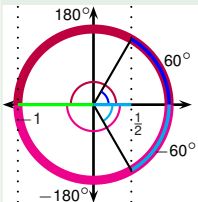
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...

Example



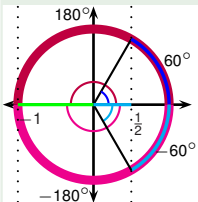
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Example



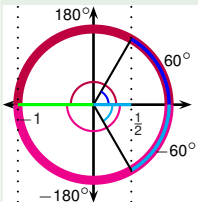
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Example



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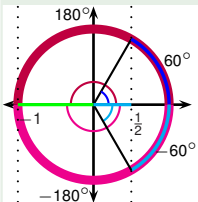
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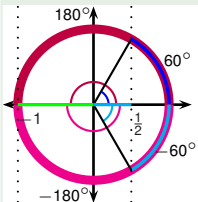
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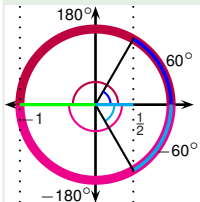
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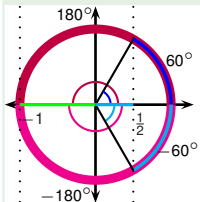
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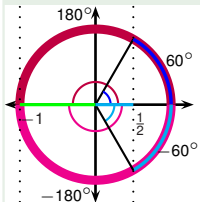
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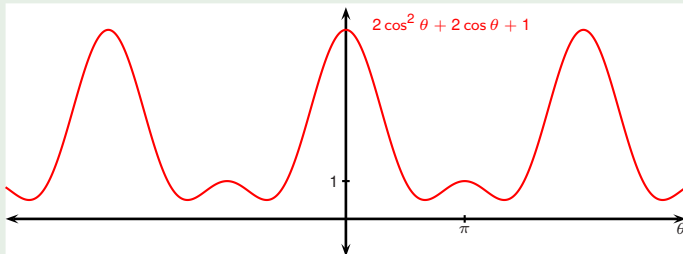


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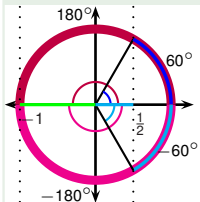
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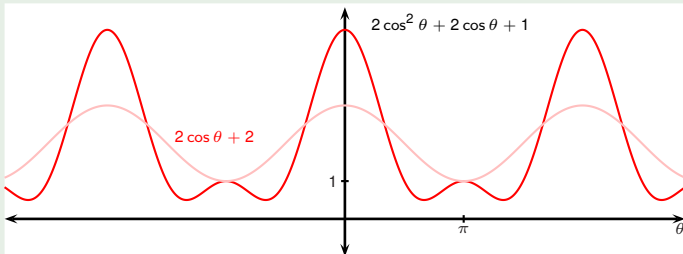


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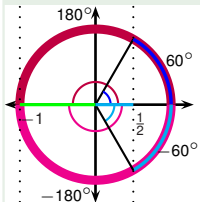
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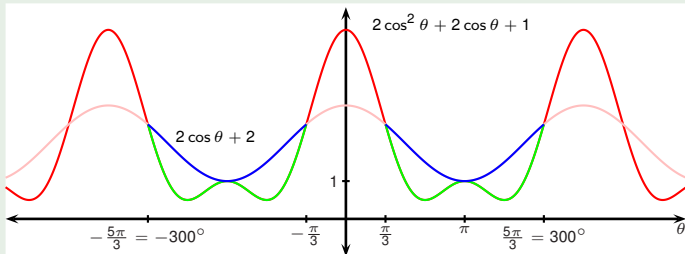


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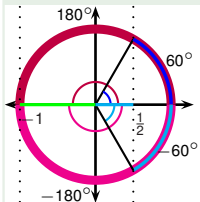
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