

## Precalculus

# Division of a polynomial by a linear polynomial

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## Example (Polynomial long division)

Divide with quotient and remainder  $x^3 + 2x^2 + 1$  by  $x - 1$ .

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$$x - 1 \overline{) x^3 + 2x^2 \quad + 1}$$

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Divide with quotient and remainder  $x^3 + 2x^2 + 1$  by  $x - 1$ .

$$x - 1 \overline{) \overset{?}{x^3} + 2x^2 \quad + 1}$$

Divide  $x^3$  by  $x$ .

## Example (Polynomial long division)

Divide with quotient and remainder  $x^3 + 2x^2 + 1$  by  $x - 1$ .

$$x - 1 \overline{) \overset{x^2}{x^3 + 2x^2} + 1}$$

Divide  $x^3$  by  $x$ .

## Example (Polynomial long division)

Divide with quotient and remainder  $x^3 + 2x^2 + 1$  by  $x - 1$ .

$$\begin{array}{r}
 x^2 \\
 x - 1 \overline{) x^3 + 2x^2 \phantom{+ 0x} + 1} \\
 \underline{\phantom{x^3} ? \phantom{+ 0x} ?} \\
 \phantom{x^3 + 2x^2 + 0x}
 \end{array}$$

Multiply  $x^2$  by divisor.

## Example (Polynomial long division)

Divide with quotient and remainder  $x^3 + 2x^2 + 1$  by  $x - 1$ .

$$\begin{array}{r}
 x^2 \\
 x - 1 \overline{) x^3 + 2x^2 \phantom{+ 0x} + 1} \\
 \underline{x^3 - x^2} \phantom{+ 0x} \\
 \phantom{x^3 - } 3x^2 \phantom{+ 0x} + 1
 \end{array}$$

Multiply  $x^2$  by divisor.



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 \underline{x^3 - x^2} \phantom{+ 1} \\
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 \phantom{x^3} \underline{3x^2 - 3x} \phantom{+ 1} \\
 \phantom{x^3} \phantom{3x^2} 3x + 1 \\
 \phantom{x^3} \phantom{3x^2} \underline{3x - 3} \\
 \phantom{x^3} \phantom{3x^2} \phantom{3x} 4
 \end{array}$$

Subtract last two polynomials.

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 \underline{x^3 - x^2} \phantom{+ 1} \\
 3x^2 \phantom{+ 1}
 \end{array}$$

Divide  $3x^2$  by  $x$ .

## Example (Polynomial long division)

Divide with quotient and remainder  $x^3 + 2x^2 + 1$  by  $x - 1$ .

$$\begin{array}{r}
 x^2 + 3x \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \phantom{+ 1} \\
 3x^2 + 1
 \end{array}$$

Divide  $3x^2$  by  $x$ .

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 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \phantom{+ 1} \\
 3x^2 + 1 \\
 \phantom{3x^2} \underline{\phantom{+ 1} ? \phantom{+ 1} ?}
 \end{array}$$

Multiply  $3x$  by divisor.

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 \underline{3x^2 - 3x} \\
 \phantom{3x^2} 4x + 1
 \end{array}$$

Multiply  $3x$  by divisor.

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 \phantom{3x^2 - } 3x + 1 \\
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 \end{array}$$

Subtract last two polynomials.

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 \underline{x^3 - x^2} \phantom{+ 1} \\
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Divide with quotient and remainder  $x^3 + 2x^2 + 1$  by  $x - 1$ .

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 x^2 + 3x + 3 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \phantom{+ 1} \\
 3x^2 + 1 \\
 \underline{3x^2 - 3x} \\
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 \underline{x^3 - x^2} \phantom{+ 1} \\
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 \underline{3x^2 - 3x} \phantom{+ 1} \\
 3x + 1 \\
 \underline{\phantom{3x} ? \phantom{+ 1} ?}
 \end{array}$$

Multiply 3 by divisor.

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Divide with quotient and remainder  $x^3 + 2x^2 + 1$  by  $x - 1$ .

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 \underline{x^3 - x^2} \phantom{+ 1} \\
 3x^2 \phantom{+ 1} \\
 \underline{3x^2 - 3x} \phantom{+ 1} \\
 3x + 1 \\
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 \end{array}$$

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 3x^2 \phantom{+ 1} \\
 \underline{3x^2 - 3x} \phantom{+ 1} \\
 3x + 1 \\
 \underline{3x - 3} \\
 ?
 \end{array}$$

Subtract last two polynomials.

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Divide with quotient and remainder  $x^3 + 2x^2 + 1$  by  $x - 1$ .

$$\begin{array}{r}
 x^2 + 3x + 3 \\
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 \underline{3x^2 - 3x} \phantom{+ 1} \\
 3x + 1 \\
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 \underline{3x^2 - 3x} \phantom{+ 1} \\
 3x + 1 \\
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 4
 \end{array}$$

## Example (Polynomial long division)

Divide with quotient and remainder  $x^3 + 2x^2 + 1$  by  $x - 1$ .

$$\begin{array}{r}
 \text{Quotient: } x^2 + 3x + 3 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \phantom{+ 1} \\
 3x^2 \phantom{+ 1} \\
 \underline{3x^2 - 3x} \phantom{+ 1} \\
 3x + 1 \\
 \underline{3x - 3} \\
 4
 \end{array}$$

$$(\text{Dividend}) = (\text{Quotient}) \cdot (\text{Divisor}) + (\text{Remainder})$$

$$(x^3 + 2x^2 + 1) = (x^2 + 3x + 3) \cdot (x - 1) + 4$$



## Example (Polynomial long division)

Divide with quotient and remainder  $x^3 + 2x^2 + 1$  by  $x - 1$ .

$$\begin{array}{r}
 \text{Quotient:} \quad x^2 + 3x + 3 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{\phantom{x^3} x^3 - x^2} \phantom{+ 1} \\
 3x^2 \phantom{+ 1} \\
 \underline{\phantom{3x^2} 3x^2 - 3x} \phantom{+ 1} \\
 3x + 1 \\
 \underline{\phantom{3x} 3x - 3} \\
 4
 \end{array}$$

**Remainder:** 4

$$\begin{aligned}
 (\text{Dividend}) &= (\text{Quotient}) \cdot (\text{Divisor}) + (\text{Remainder}) \\
 (x^3 + 2x^2 + 1) &= (x^2 + 3x + 3) \cdot (x - 1) + 4
 \end{aligned}$$