# Calculus II Polar coordinates

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# Outline

Polar Coordinates

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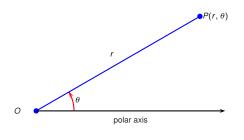
- Latest version of the .tex sources of the slides: https://github.com/tmilev/freecalc
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## **Polar Coordinates**

 The polar coordinate system is an alternative to the Cartesian coordinate system.

- Choose a point in the plane called *O* (the origin).
- Draw a ray starting at O. The ray is called the polar axis. This ray is usually drawn horizontally to the right.



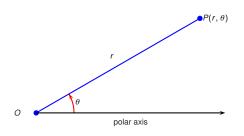
- Let *P* be a point in the plane.
- Let θ denote the angle between the polar axis and the line OP.
- Let *r* denote the length of the segment *OP*.
- Then P is represented by the ordered pair  $(r, \theta)$ .

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# Polar Coordinates

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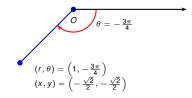
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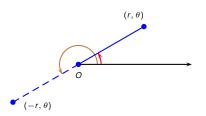


The letters (x, y) imply
 Cartesian coordinates and the letters (r, θ)- polar. When we use other letters, it should be clear from context whether we mean Cartesian or polar coordinates. If not, one must request clarification.

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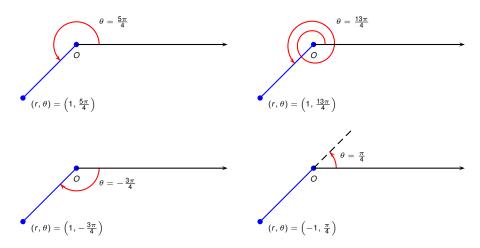
- **1** What if  $\theta$  is negative?
- What if r is negative?
- What if r is 0?





- Positive angles θ are measured in the counterclockwise direction from O. Negative angles are measured in the clockwise direction.
- Points with polar coordinates  $(-r, \theta)$  and  $(r, \theta)$  lie on the same line through O and at the same distance from O, but on opposite sides.
- If r = 0, then  $(0, \theta)$  represents O for all values of  $\theta$ .

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- There are many ways to represent the same point.
- We could use a negative  $\theta$ .
- We could go around more than once.
- We could use a negative r.

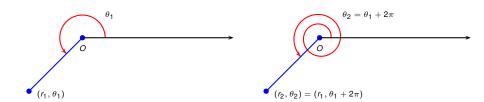
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- Let  $P_1$  be point with polar coordinates  $(r_1, \theta_1)$ .
- Let  $P_2$  be point with polar coordinates  $(r_2, \theta_2)$ .

#### Observation

 $P_1$  coincides with  $P_2$  if one of the three mutually exclusive possibilities holds:

- $r_1 = r_2 \neq 0$  and  $\theta_2 = \theta_1 + 2k\pi, k \in \mathbb{Z}$ ,
- $r_1 = -r_2 \neq 0$  and  $\theta_2 = \theta_1 + (2k+1)\pi, k \in \mathbb{Z}$ ,
- $r_1 = r_2 = 0$  and  $\theta$  is arbitrary.



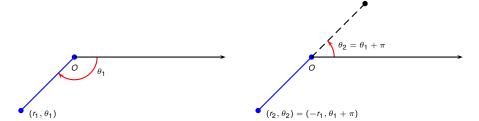
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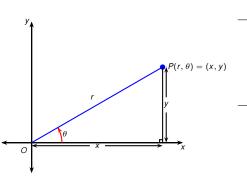
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• How do we go from polar coordinates to Cartesian coordinates?

- Suppose a point has polar coordinates  $(r, \theta)$  and Cartesian coordinates (x, y).
- How do we go from Cartesian coordinates to polar coordinates?



$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\cos \theta = \frac{x}{r}$$

$$\sin \theta = \frac{y}{r}$$

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \arcsin(\frac{y}{r}) \text{ if } x > 0$$

$$= \arccos(\frac{x}{r}) \text{ if } y > 0$$

$$= \arctan(\frac{y}{y}) \text{ if } x > 0$$

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#### Example

Convert the point  $(2, \frac{\pi}{3})$  from polar to Cartesian coordinates.

$$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2\left(\frac{1}{2}\right) = 1$$

$$y = r \sin \theta = 2 \sin \frac{\pi}{3} = 2 \left( \frac{\sqrt{3}}{2} \right) = \sqrt{3}$$

Therefore the point with polar coordinates  $(2, \frac{\pi}{3})$  has Cartesian coordinates  $(1, \sqrt{3})$ .

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## Example



Represent the point with Cartesian coordinates (1,-1) in terms of polar coordinates.

- Suppose r is positive.
- $\tan \theta = -1$  for  $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$ , and many other angles.
- (1, -1) is in the fourth quadrant.
- Of the two values above, only  $\theta = \frac{7\pi}{4}$  gives a point in the fourth quadrant.
- $\Rightarrow$  one representation of (1, -1) in polar coordinates is  $(\sqrt{2}, \frac{7\pi}{4})$ .
- $\left(\sqrt{2}, -\frac{\pi}{4}\right)$  is another.

$$r = \pm \sqrt{x^2 + y^2}$$
$$= \sqrt{1^2 + (-1)^2}$$
$$= \sqrt{2}$$

$$\tan \theta = \frac{y}{x}$$

$$= -$$