Calculus I
Trig functions with power notation and the chain rule

Todor Milev

2019

$$\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) & & \text{(notation 1)}\\
(g(u))' &= g'(u)u' & \text{where } u = h(x) & \text{(notation 2)}\\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)} \quad .
\end{aligned}$$

Example (Chain Rule, Notation 2)

Differentiate $\quad f(x) = \cos\left(x^3\right).$

$$\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) && && \text{(notation 1)}\\
(g(u))' &= g'(u)u' && \text{where } u = h(x) && \text{(notation 2)}\\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} && \text{where } y = g(u) && \text{(notation 3)}
\end{aligned}$$

.

Example (Chain Rule, Notation 2)

$$\begin{aligned}
\text{Differentiate} \quad f(x) &= \cos\left(x^3\right).\\
\text{Let} \quad u &= \mathbf{?}\\
\text{Let} \quad g(u) &= \mathbf{?}\\
\text{Then} \quad f(x) &= g(u).
\end{aligned}$$

$$
\begin{array}{rcllr}
(g(h(x)))' & = & g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\
{\color{red}(g(u))'} & {\color{red}=} & {\color{red}g'(u)u'} & \text{where } u = h(x) & \text{(notation 2)} \\
\frac{\mathrm{d}y}{\mathrm{d}x} & = & \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)}
\end{array} .
$$

Example (Chain Rule, Notation 2)

$$
\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos\left(x^3\right) . \\
\text{Let} & {\color{red}u} & {\color{red}=} & {\color{red}x^3.} \\
\text{Let} & g(u) & = & \textbf{?} \\
\text{Then} & f(x) & = & g(u).
\end{array}
$$

$$
\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) && && \text{(notation 1)}\\
(g(u))' &= g'(u)u' && \text{where } u = h(x) && \text{(notation 2)}\\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} && \text{where } y = g(u) && \text{(notation 3)}
\end{aligned} .
$$

Example (Chain Rule, Notation 2)

$$
\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos\left(x^3\right) .\\
\text{Let} & u & = & x^3 .\\
\text{Let} & g(u) & = & \textbf{?}\\
\text{Then} & f(x) & = & g(u).
\end{array}
$$

$$
\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) && && \text{(notation 1)}\\
(g(u))' &= g'(u)u' && \text{where } u = h(x) && \text{(notation 2)}\\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} && \text{where } y = g(u) && \text{(notation 3)}
\end{aligned} \quad .
$$

Example (Chain Rule, Notation 2)

$$
\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos\left(x^3\right). \\
\text{Let} & u & = & x^3. \\
\text{Let} & g(u) & = & \cos u. \\
\text{Then} & f(x) & = & g(u).
\end{array}
$$

$$\begin{array}{rclll}
(g(h(x)))' & = & g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\
{\color{red}(g(u))'} & {\color{red}=} & {\color{red}g'(u)u'} & \text{where } u = h(x) & \text{(notation 2)} \\
\frac{\mathrm{d}y}{\mathrm{d}x} & = & \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)}
\end{array} .$$

Example (Chain Rule, Notation 2)

$$\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos\left(x^3\right) . \\
\text{Let} & u & = & x^3. \\
\text{Let} & g(u) & = & \cos u. \\
\text{Then} & f(x) & = & g(u). \\
\text{Chain Rule:} & f'(x) & = & g'(u)u'
\end{array}$$

$$
\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\
(g(u))' &= g'(u)u' & \text{where } u = h(x) & \text{(notation 2)} \\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)} \quad .
\end{aligned}
$$

Example (Chain Rule, Notation 2)

$$
\begin{aligned}
\text{Differentiate} \quad f(x) &= \cos\left(x^3\right). \\
\text{Let} \quad u &= x^3. \\
\text{Let} \quad g(u) &= \cos u. \\
\text{Then} \quad f(x) &= g(u). \\
\text{Chain Rule:} \quad f'(x) &= g'(u)u' \\
&= (\textbf{?} \qquad) \left(\textbf{?} \quad \right)
\end{aligned}
$$

$$\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) & & \text{(notation 1)}\\
(g(u))' &= g'(u)u' & \text{where } u = h(x) & \text{(notation 2)}\\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)}
\end{aligned} .$$

Example (Chain Rule, Notation 2)

$$\begin{aligned}
\text{Differentiate} \quad f(x) &= \cos\left(x^3\right).\\
\text{Let} \quad u &= x^3.\\
\text{Let} \quad g(u) &= \cos u.\\
\text{Then} \quad f(x) &= g(u).\\
\text{Chain Rule:} \quad f'(x) &= g'(u)u'\\
&= (-\sin u)\left(\textbf{?} \quad \right)
\end{aligned}$$

$$
\begin{array}{rcll}
(g(h(x)))' & = & g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\
{\color{red}(g(u))'} & {\color{red}=} & {\color{red}g'(u)u'} & \text{where } u = h(x) & \text{(notation 2)} \\
\dfrac{\mathrm{d}y}{\mathrm{d}x} & = & \dfrac{\mathrm{d}y}{\mathrm{d}u}\dfrac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)} \quad .
\end{array}
$$

Example (Chain Rule, Notation 2)

$$
\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos\left(x^3\right). \\
\text{Let} & {\color{red}u} & {\color{red}=} & {\color{red}x^3.} \\
\text{Let} & g(u) & = & \cos u. \\
\text{Then} & f(x) & = & g(u). \\
\text{Chain Rule:} & f'(x) & = & g'(u){\color{red}u'} \\
 & & = & (-\sin u)\left({\color{red}\mathbf{?}} \quad \right)
\end{array}
$$

$$
\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) && && \text{(notation 1)}\\
(g(u))' &= g'(u)u' && \text{where } u = h(x) && \text{(notation 2)}\\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} && \text{where } y = g(u) && \text{(notation 3)}
\end{aligned}
.
$$

Example (Chain Rule, Notation 2)

$$
\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos\left(x^3\right). \\
\text{Let} & u & = & x^3. \\
\text{Let} & g(u) & = & \cos u. \\
\text{Then} & f(x) & = & g(u). \\
\text{Chain Rule:} & f'(x) & = & g'(u)u' \\
 & & = & (-\sin u)\left(3x^2\right)
\end{array}
$$

$$\begin{array}{rclrr}
(g(h(x)))' & = & g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\
(g(u))' & = & g'(u)u' & \text{where } u = h(x) & \text{(notation 2)} \\
\dfrac{\mathrm{d}y}{\mathrm{d}x} & = & \dfrac{\mathrm{d}y}{\mathrm{d}u}\dfrac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)}
\end{array} .$$

Example (Chain Rule, Notation 2)

$$\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos\left(x^3\right) . \\
\text{Let} & u & = & x^3. \\
\text{Let} & g(u) & = & \cos u. \\
\text{Then} & f(x) & = & g(u). \\
\text{Chain Rule:} & f'(x) & = & g'(u)u' \\
 & & = & (-\sin u)\left(3x^2\right) \\
 & & = & -3x^2 \sin\left(x^3\right) .
\end{array}$$

$$\begin{array}{rcll} (g(h(x)))' &=& g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\ (g(u))' &=& g'(u)u' & \text{where } u = h(x) & \text{(notation 2)} \\ \frac{\mathrm{d}y}{\mathrm{d}x} &=& \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)} \end{array} .$$

Example (Chain Rule, Notation 2)

Differentiate $f(x) = \cos^3 x.$

$$\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) && && \text{(notation 1)}\\
(g(u))' &= g'(u)u' && \text{where } u = h(x) && \text{(notation 2)}\\
\frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dx} && \text{where } y = g(u) && \text{(notation 3)}
\end{aligned} .$$

Example (Chain Rule, Notation 2)

$$\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos^3 x. \\
\text{Let} & u & = & \textbf{?} \\
\text{Let} & g(u) & = & \textbf{?} \\
\text{Then} & f(x) & = & g(u).
\end{array}$$

$$
\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) && && \text{(notation 1)} \\
(g(u))' &= g'(u)u' && \text{where } u = h(x) && \text{(notation 2)} \\
\frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dx} && \text{where } y = g(u) && \text{(notation 3)}
\end{aligned} \quad .
$$

Example (Chain Rule, Notation 2)

$$
\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos^3 x. \\
\text{Let} & u & = & \cos x. \\
\text{Let} & g(u) & = & \textbf{?} \\
\text{Then} & f(x) & = & g(u).
\end{array}
$$

$$
\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) & & \text{(notation 1)}\\
(g(u))' &= g'(u)u' & \text{where } u = h(x) & \text{(notation 2)}\\
\frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dx} & \text{where } y = g(u) & \text{(notation 3)}
\end{aligned}
$$

Example (Chain Rule, Notation 2)

$$
\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos^3 x.\\
\text{Let} & u & = & \cos x.\\
\text{Let} & g(u) & = & \textbf{?}\\
\text{Then} & f(x) & = & g(u).
\end{array}
$$

$$
\begin{array}{rclrr}
(g(h(x)))' & = & g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\
(g(u))' & = & g'(u)u' & \text{where } u = h(x) & \text{(notation 2)} \\
\frac{\mathrm{d}y}{\mathrm{d}x} & = & \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)}
\end{array} .
$$

Example (Chain Rule, Notation 2)

$$
\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos^3 x. \\
\text{Let} & u & = & \cos x. \\
\text{Let} & g(u) & = & u^3. \\
\text{Then} & f(x) & = & g(u).
\end{array}
$$

$$\begin{array}{rcll} (g(h(x)))' &=& g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\ (g(u))' &=& g'(u)u' & \text{where } u = h(x) & \text{(notation 2)} \\ \dfrac{\mathrm{d}y}{\mathrm{d}x} &=& \dfrac{\mathrm{d}y}{\mathrm{d}u}\dfrac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)} \end{array} .$$

Example (Chain Rule, Notation 2)

$$\begin{array}{rrcl} \text{Differentiate} & f(x) &=& \cos^3 x. \\ \text{Let} & u &=& \cos x. \\ \text{Let} & g(u) &=& u^3. \\ \text{Then} & f(x) &=& g(u). \\ \text{Chain Rule:} & f'(x) &=& g'(u)u' \end{array}$$

$$
\begin{array}{rcll}
(g(h(x)))' & = & g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\
(g(u))' & = & g'(u)u' & \text{where } u = h(x) & \text{(notation 2)} \\
\frac{\mathrm{d}y}{\mathrm{d}x} & = & \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)}
\end{array} .
$$

Example (Chain Rule, Notation 2)

$$
\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos^3 x. \\
\text{Let} & u & = & \cos x. \\
\text{Let} & g(u) & = & u^3. \\
\text{Then} & f(x) & = & g(u). \\
\text{Chain Rule:} & f'(x) & = & g'(u)u' \\
 & & = & \left(\textbf{?} \quad \right) (\textbf{?} \qquad)
\end{array}
$$

$$
\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) && && \text{(notation 1)}\\
{\color{red}(g(u))'} &{\color{red}=} {\color{red}g'(u)u'} && \text{where } u = h(x) && \text{(notation 2)}\\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} && \text{where } y = g(u) && \text{(notation 3)} \quad .
\end{aligned}
$$

Example (Chain Rule, Notation 2)

$$
\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos^3 x. \\
\text{Let} & u & = & \cos x. \\
\text{Let} & {\color{red}g(u)} & {\color{red}=} & {\color{red}u^3.} \\
\text{Then} & f(x) & = & g(u). \\
\text{Chain Rule:} & f'(x) & = & {\color{red}g'(u)}u' \\
 & & = & {\color{red}\left(3u^2\right)} (\textbf{?} \qquad)
\end{array}
$$

$$
\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\
{\color{red}(g(u))'} &{\color{red}=} {\color{red}g'(u)u'} & \text{where } u = h(x) & \text{(notation 2)} \\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)} \quad .
\end{aligned}
$$

Example (Chain Rule, Notation 2)

$$
\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos^3 x. \\
\text{Let} & {\color{red}u} & {\color{red}=} & {\color{red}\cos x.} \\
\text{Let} & g(u) & = & u^3. \\
\text{Then} & f(x) & = & g(u). \\
\text{Chain Rule:} & f'(x) & = & g'(u){\color{red}u'} \\
 & & = & \left(3u^2\right) {\color{red}(\textbf{?} \qquad)}
\end{array}
$$

$$\begin{array}{rcll} (g(h(x)))' &=& g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\ (g(u))' &=& g'(u)u' & \text{where } u = h(x) & \text{(notation 2)} \\ \frac{dy}{dx} &=& \frac{dy}{du}\frac{du}{dx} & \text{where } y = g(u) & \text{(notation 3)} \end{array} .$$

Example (Chain Rule, Notation 2)

$$\begin{array}{rrcl} \text{Differentiate} & f(x) &=& \cos^3 x. \\ \text{Let} & u &=& \cos x. \\ \text{Let} & g(u) &=& u^3. \\ \text{Then} & f(x) &=& g(u). \\ \text{Chain Rule:} & f'(x) &=& g'(u)u' \\ & &=& \left(3u^2\right)(-\sin x) \end{array}$$

$$
\begin{array}{rcllr}
(g(h(x)))' & = & g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\
(g(u))' & = & g'(u)u' & \text{where } u = h(x) & \text{(notation 2)} \\
\frac{\mathrm{d}y}{\mathrm{d}x} & = & \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} & \text{where } y = g(u) & \text{(notation 3)}
\end{array} .
$$

Example (Chain Rule, Notation 2)

$$
\begin{array}{rrcl}
\text{Differentiate} & f(x) & = & \cos^3 x. \\
\text{Let} & u & = & \cos x. \\
\text{Let} & g(u) & = & u^3. \\
\text{Then} & f(x) & = & g(u). \\
\text{Chain Rule:} & f'(x) & = & g'(u)u' \\
 & & = & \left(3u^2\right)(-\sin x) \\
 & & = & -3\sin x(\cos x)^2.
\end{array}
$$