

# Precalculus

## Homework

### Logarithms basics

1. Use the definition of a logarithm to evaluate each of the following without using a calculator. The answer key has not been proofread, use with caution.

(a)  $\log_2 16$ .

ANSWER: 4

(e)  $\log_2(8\sqrt{2})$ .

ANSWER:  $\frac{9}{2}$

(b)  $\log_3 \left( \frac{1}{9} \right)$ .

ANSWER: -2

(f)  $\log_{\frac{1}{2}}(4)$ .

ANSWER:  $\frac{2}{7}$

(c)  $\log_{10} 1000$ .

ANSWER: -2

ANSWER: 3

(g)  $\log_{\frac{1}{9}}(\sqrt{3})$ .

ANSWER:  $\frac{1}{4}$

(d)  $\log_6 36^{-\frac{2}{3}}$ .

2. Find the exact value of each expression.

(a)  $\log_5 125$ .

ANSWER: 3

(h)  $\log_5 4 - \log_5 500$ .

ANSWER: -3

(b)  $\log_3 \frac{1}{27}$ .

ANSWER: -3

(i)  $\log_2 6 - \log_2 15 + \log_2 20$ .

ANSWER: 3

(c)  $\ln \left( \frac{1}{e} \right)$ .

ANSWER: -1

(j)  $\log_3 100 - \log_3 18 - \log_3 50$ .

ANSWER: -2

(d)  $\log_{10} \sqrt{10}$ .

ANSWER:  $\frac{1}{2}$

(k)  $e^{-2 \ln 5}$ .

ANSWER:  $\frac{25}{1}$

(e)  $e^{\ln 4.5}$ .

ANSWER: 4.5

(l)  $\ln \left( \ln e^{e^{10}} \right)$ .

ANSWER: 10

(f)  $\log_{10} 0.0001$ .

ANSWER: 4

(m)  $\log_7 \left( \frac{49^x}{343^y} \right)$

ANSWER:  $2x - 3y$

(g)  $\log_{1.5} 2.25$ .

ANSWER: 2

**Solution.** 2.m.

$$\begin{aligned} \log_7 \left( \frac{49^x}{343^y} \right) &= \log_7 49^x - \log_7 343^y \\ &= x \log_7 49 - y \log_7 343 \end{aligned}$$

$$\text{However } 49 = 7^2 \text{ and } 343 = 7^3, \text{ therefore } \log_7 \left( \frac{49^x}{343^y} \right) = 2x - 3y.$$

3. Using only the  $\ln$  operation of your calculator compute the indicated logarithm. Confirm your computation numerically by exponentiation.

(a)  $\log_5(13)$ .

(c)  $\log_{13}(101)$ .

(b)  $\log_{12}(9)$ .

(d)  $\log_{10}(2015)$ .

**Solution.**

$$\log_5(13) = \frac{\ln 13}{\ln 5} \approx \frac{2.564949357}{1.609437912} \approx 1.593693.$$

As a check of our computations, we compute by calculator:  $13 = 5^{\log_5 13} \approx 5^{1.593693} \approx 13.000007508$ , and our computations check out.

4. Express each of the following as a single logarithm. If possible, compute the logarithm without using a calculator. The answer key has not been proofread, use with caution.

(a)  $\ln 4 + \ln 6 - \ln 5$ .

(b)  $2 \ln 2 - 3 \ln 3 + 4 \ln 4$ .

(c)  $\ln 36 - 2 \ln 3 - 3 \ln 2$ .

(d)  $\log_2(24) - \log_4 9$ .

(e)  $\log_7(24) + \log_{\frac{1}{7}} 3 - \log_{49}(64)$ .

(f)  $\log_3(24) + \log_3\left(\frac{3}{8}\right)$ .

**Solution.** 4.b.

$$\begin{aligned} 2 \ln 2 - 3 \ln 3 + 4 \ln 4 &= \ln 2^2 - \ln 3^3 + \ln 4^4 \\ &= \ln 4 - \ln 27 + \ln 256 \\ &= \ln \left( \frac{4}{27} \right) + \ln 256 \\ &= \ln \left( \frac{4 \cdot 256}{27} \right) \\ &= \ln \left( \frac{1024}{27} \right). \end{aligned}$$

$\frac{1024}{27}$  is not a rational power of  $e$ , therefore  $\ln \left( \frac{1024}{27} \right)$  is not a rational number and there are no further simplifications of the answer (except possibly a numerical approximation with a calculator or equivalent).

**Solution.** 4.e

$$\begin{aligned}
\log_7(24) + \log_{\frac{1}{7}}(3) - \log_{49}(64) &= \log_7(24) + \frac{\log_7(3)}{\log_7\left(\frac{1}{7}\right)} - \frac{\log_7(64)}{\log_7(49)} && \left| \begin{array}{l} \text{common base} \\ \text{simplify logarithms} \end{array} \right. \\
&= \log_7(24) + \frac{\log_7(3)}{-1} - \frac{\log_7(64)}{2} \\
&= \log_7(24) - \log_7(3) - \frac{1}{2} \log_7(64) \\
&= \log_7\left(\frac{24}{3}\right) - \log_7\left(64^{\frac{1}{2}}\right) && \left| \begin{array}{l} \text{rule: } \log_a x - \log_a y = \log_a\left(\frac{x}{y}\right) \\ \text{rule: } \log_a x^r = r \log_a x \end{array} \right. \\
&= \log_7(8) - \log_7(\sqrt{64}) \\
&= \log_7 8 - \log_7 8 = 0 && \left| \begin{array}{l} \text{alternatively:} \end{array} \right. \\
&= \log_7\left(\frac{8}{8}\right) \\
&= \log_7(1) \\
&= 0.
\end{aligned}$$

5. Demonstrate the identity(s).

$$(a) \quad -\ln(\sqrt{1+x^2} - x) = \ln(x + \sqrt{1+x^2})$$