Precalculus
Trigonometric functions with arguments translated by a multiple of $\frac{\pi}{2}$

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Example

Use the angle sum/difference formulas to simplify.

$$\cos\left(\frac{\pi}{2} - x\right)$$

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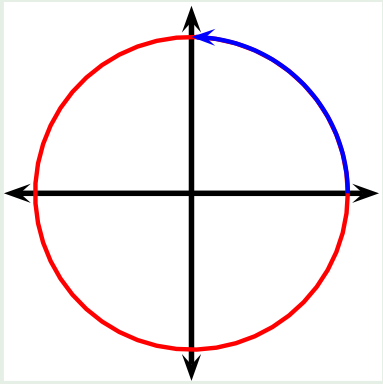

$$\begin{aligned}\cos\left(\frac{\pi}{2}-x\right) &= \cos\left(\frac{\pi}{2}\right)\cos x+\sin\left(\frac{\pi}{2}\right)\sin x\\ &= 0\cdot\cos(x)+1\cdot\sin x\\ &= \sin x\end{aligned}$$

Example

Use the angle sum/difference formulas to simplify.

$$\cot\left(\frac{3\pi}{2} + x\right)$$

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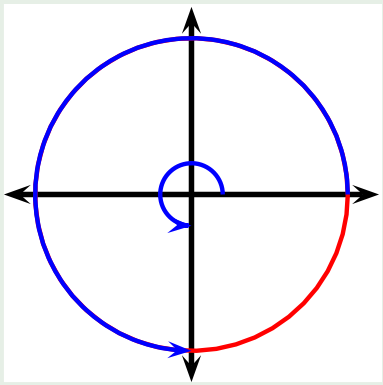
Use the angle sum/difference formulas to simplify.



$$\begin{aligned}
\cot\left(\frac{3\pi}{2}+x\right) &= \frac{\cos\left(\frac{3\pi}{2}+x\right)}{\sin\left(\frac{3\pi}{2}+x\right)} \\
&= \frac{\cos\left(\frac{3\pi}{2}\right)\cos x-\sin\left(\frac{3\pi}{2}\right)\sin x}{\sin\left(\frac{3\pi}{2}\right)\cos x+\cos\left(\frac{3\pi}{2}\right)\sin x} \\
&= \frac{0\cdot\cos x-(-1)\sin x}{(-1)\cos x+0\cdot\sin x} \\
&= \frac{\sin x}{-\cos x}=-\frac{\sin x}{\cos x} \\
&= -\tan x
\end{aligned}$$

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&= \frac{? \cdot \cos x + ? \cdot \sin x}{? \cdot \cos x - ? \cdot \sin x}
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&= \frac{-\sin x}{-\cos x} \\
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&= \tan x,
\end{aligned}$$

as desired.

Proposition ($\tan$, $\cot$ are π-periodic)

The tangent and cotangent functions are π-periodic, in other words,

$$\begin{aligned} \tan(\theta + \pi) &= \tan\theta \\ \cot(\theta + \pi) &= \cot\theta \end{aligned}$$