Precalculus
Simplify linear combination of logarithms using all techniques

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Example

Compute as a rational number, without using calculator.

$$\log_7(24)+\log_{\frac{1}{7}}(3)-\log_{49}(64)=\log_7(24)+\frac{\log_7(3)}{\log_7\left(\frac{1}{7}\right)}-\frac{\log_7(64)}{\log_7(49)}$$

$$=\log_7(24)+\frac{\log_7(3)}{-1}-\frac{\log_7(64)}{2}$$

$$=\log_7(24)-\log_7(3)-\frac{1}{2}\log_7(64)$$

$$\left[\begin{array}{l}\log_a x-\log_a y=\log_a\left(\frac{x}{y}\right)\\ \log_a x^r=r\log_a x\end{array}\right]$$

$$=\log_7\left(\frac{24}{3}\right)-\log_7\left(64^{\frac{1}{2}}\right)$$

$$=\log_7(8)-\log_7\left(\sqrt{64}\right)$$

$$=\log_7 8-\log_7 8=0$$

[alternatively:]

$$=\log_7\left(\frac{8}{8}\right)=\log_7(1)=0.$$