Precalculus
Factoring quadratic polynomials

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Outline

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Definition ((Partial) Factorization)

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Example

$$\begin{aligned} x^2 - 1 &= (x-1)(x+1) \\ x^3 + 8 &= (x+2)\left(x^2 - 2x + 4\right) \end{aligned}$$

Theorem

The quadratic $ax^2 + bx + c$ factors as follows.

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

where x_1 and x_2 are the roots of the quadratic, given by:

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$$3x^2 + 8x - 11 \quad = \quad (3x + p)(x + q)$$

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Proposition (Vieta's formulas)

Let $ax^2 + bx + c$ be a quadratic functions with zeros x_1 and x_2. Then:

$$a(x - x_1)(x - x_2) = ax^2 + bx + c$$

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The last two formulas are called Vieta's formulas (after François Viète (1540-1603), Latinized name: Franciscus Vieta).

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

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Vieta's formulas

Example

Factor the quadratic.

$$x^2 + 5x + 6$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

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Vieta's formulas

Example

Factor the quadratic.

$$x^2 + 5x + 6 = (x + \mathbf{?})(x + \mathbf{?})$$

- The product of the two roots: $x_1 x_2 = 6$.

$$
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Vieta's formulas

Example

Factor the quadratic.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

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Factor the quadratic.

$$x^2 + 3x + 1$$

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$$ax^2+bx+c=a\left(x-x_1\right)\left(x-x_2\right),\qquad \left|\begin{array}{rcl} x_1x_2 & = & \frac{c}{a} \\ x_1+x_2 & = & -\frac{b}{a} \end{array}\right.$$

Example

Factor the quadratic.

$$x^2+3x+{\color{red}1}=\left(x+\textbf{?}\qquad\qquad\right)\left(x+\textbf{?}\qquad\qquad\right)$$

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Factor the quadratic.

$$x^2 + 3x + 1 = \left(x \quad - \left(\frac{-3+\sqrt{5}}{2}\right)\right)\left(x \quad - \left(\frac{-3-\sqrt{5}}{2}\right)\right)$$

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Example

Factor the quadratic, using complex numbers if needed.

$$x^2 + x + 1$$

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Example

Factor the quadratic, using complex numbers if needed.

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Example

Factor the quadratic, using complex numbers if needed.

$$x^2 + x + 1 = \left(x - \left(\frac{-1+\sqrt{3}i}{2}\right)\right)\left(x - \left(\frac{-1-\sqrt{3}i}{2}\right)\right)$$

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