Calculus II
Power series expansion of the exponent

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Example

Find the Maclaurin series of $f(x) = e^x$ and its radius of convergence.

- $f^{(n)}(x) = e^x$.
- $f^{(n)}(0) = e^0 = 1$.
- Therefore the Maclaurin series is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = \sum_{n=0}^{\infty} \frac{1}{n!} x^n = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

- To find the radius of convergence, let $a_n = \frac{x^n}{n!}$.

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| = \lim_{n\to\infty} \frac{|x|}{n+1} = 0 < 1$$

- Therefore by the Ratio Test the series converges for all x.
- Therefore $R = \infty$.