

# Calculus II

## Summary of building block integrals

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# Building block integral summary

Type	a	b	Type a, lin. sub.	Type b, lin. sub
I	$\int \frac{1}{x} dx$	$\int \frac{1}{x^n} dx$	$\int \frac{A}{ax+b} dx$	$\int \frac{A}{(ax+b)^n} dx$
II	$\int \frac{x}{x^2+1} dx$	$\int \frac{x}{(x^2+1)^n} dx$	$\int \frac{A(x+\frac{b}{2a})}{ax^2+bx+c} dx$	$\int \frac{A(x+\frac{b}{2a})}{(ax^2+bx+c)^n} dx$
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  - Block IIa, IIIa, linear substitutions: done in full detail, by means of completing the square.
  - Block IIb, IIIb, linear substitutions: **done by means of completing the square;**

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  - Block IIb, IIIb, linear substitutions: done by means of completing the square; **computations are analogous and we leave them for exercise.**