# Calculus I
# Derivatives of square root expressions

Todor Milev

2019

$$
\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) & & \text{(notation 1)} \\
(g(u))' &= g'(u)u' & \text{where } u = h(x) & \text{(notation 2)} \\
\frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dx} & \text{where } y = g(u) & \text{(notation 3)}
\end{aligned} .
$$

## Example (Chain Rule, Notation 2)

Differentiate $\quad f(x) \quad = \quad \sqrt{x^2+1}.$

$$
\begin{aligned}
(g(h(x)))' &= g'(h(x))\cdot h'(x) && && \text{(notation 1)}\\
{\color{red}(g(u))'} &{\color{red}=} {\color{red}g'(u)u'} && \text{where } u = h(x) && \text{(notation 2)}\\
\frac{dy}{dx} &= \frac{dy}{du}\frac{du}{dx} && \text{where } y = g(u) && \text{(notation 3)}
\end{aligned} \quad .
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## Example (Chain Rule, Notation 2)

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\begin{aligned}
\text{Differentiate} \quad f(x) &= \sqrt{x^2+1}.\\
\text{Let} \quad {\color{red}u} &{\color{red}=} {\color{red}\mathbf{?}}\\
\text{Let} \quad g(u) &= \mathbf{?}\\
\text{Then} \quad f(x) &= g(u).
\end{aligned}
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\color{red}{(g(u))'} &\color{red}{=} \color{red}{g'(u)u'} && \text{where } u = h(x) && \text{(notation 2)}\\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} && \text{where } y = g(u) && \text{(notation 3)} \quad .
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\text{Differentiate} & f(x) & = & \sqrt{x^2+1}.\\
\text{Let} & \color{red}{u} & \color{red}{=} & \color{red}{x^2+1.}\\
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