Precalculus
Homework
Trigonometric equations and inequalities

1. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation.

   (a) $2\cos x - 1 = 0$.

   answer: $x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$

   (b) $\sin(2x) = \cos x$.

   answer: $x = \frac{\pi}{2}, x = \frac{3\pi}{2}, x = \frac{\pi}{6}$, or $x = \frac{5\pi}{6}$

   (c) $\sqrt{3}\sin x = \sin(2x)$.

   answer: $x = \frac{\pi}{6}, \frac{11\pi}{6}, 0, \pi, 2\pi$

   (d) $2\sin^2 x = 1$.

   answer: $x = \frac{\pi}{4}, x = \frac{3\pi}{4}, x = \frac{5\pi}{4}$, or $x = \frac{7\pi}{4}$

   (e) $2 + \cos(2x) = 3\cos x$.

   answer: $x = 0, x = 2\pi, x = \frac{\pi}{3}$, or $x = \frac{5\pi}{3}$

   (f) $2\cos x + \sin(2x) = 0$.

   answer: $x = \frac{\pi}{2}, x = \frac{3\pi}{2}$

   (g) $2\cos^2 x - (1+\sqrt{2})\cos x + \frac{\sqrt{2}}{2} = 0$.

   answer: $x = \frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{4}$

   (h) $|\tan x| = 1$.

   answer: $x = \frac{\pi}{4}, x = \frac{3\pi}{4}, x = \frac{5\pi}{4}$, or $x = \frac{7\pi}{4}$

   (i) $3\cot^2 x = 1$.

   answer: $x = \frac{\pi}{3}, x = \frac{2\pi}{3}, x = \frac{4\pi}{3}$, or $x = \frac{5\pi}{3}$

   (j) $\sin x = \tan x$.

   answer: $x = 0, x = \pi$, or $x = 2\pi$

   Solution. 1.g Set $\cos x = u$. Then

$$2\cos^2 x - (1+\sqrt{2})\cos x + \frac{\sqrt{2}}{2} = 0$$

   becomes

$$2u^2 - (1+\sqrt{2})u + \frac{\sqrt{2}}{2} = 0.$$

   This is a quadratic equation in u and therefore has solutions

$$\begin{aligned} u_1, u_2 &= \frac{1+\sqrt{2} \pm \sqrt{(1+\sqrt{2})^2 - 4\sqrt{2}}}{4} \\ &= \frac{1+\sqrt{2} \pm \sqrt{1-2\sqrt{2}+2}}{4} \\ &= \frac{1+\sqrt{2} \pm \sqrt{(1-\sqrt{2})^2}}{4} \\ &= \frac{1+\sqrt{2} \pm (1-\sqrt{2})}{4} = \begin{cases} \frac{1}{2} \\ \frac{\sqrt{2}}{2} \end{cases} \quad \text{or} \end{aligned}$$

   Therefore $u = \cos x = \frac{1}{2}$ or $u = \cos x = \frac{\sqrt{2}}{2}$, and, as x is in the interval $[0, 2\pi]$, we get $x = \frac{\pi}{3}, \frac{5\pi}{3}$ (for $\cos x = \frac{1}{2}$) or $x = \frac{\pi}{4}, \frac{7\pi}{4}$ (for $\cos x = \frac{\sqrt{2}}{2}$).