PrecalculusTangent angle sum formula

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Example

Prove the angle sum formula $tan(\alpha + \beta) = \frac{tan \alpha + tan \beta}{1 - tan \alpha tan \beta}$.

$$tan(\alpha + \beta) =$$

Example

Prove the angle sum formula $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$.

$$\tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)}$$

$$= \frac{(\sin \alpha \cos \beta + \cos \alpha \sin \beta) \frac{1}{\cos \alpha \cos \beta}}{(\cos \alpha \cos \beta - \sin \alpha \sin \beta) \frac{1}{\cos \alpha \cos \beta}}$$

$$= \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta}}$$

$$= \frac{\frac{\sin \alpha}{\cos \alpha} + \frac{\sin \beta}{\cos \alpha}}{\frac{\cos \alpha}{\cos \beta}}$$

$$= \frac{\frac{\sin \alpha}{\cos \alpha} \cdot \frac{\sin \beta}{\cos \beta}}{1 - \frac{\sin \alpha}{\cos \alpha} \cdot \frac{\sin \beta}{\cos \beta}}$$

$$= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$