

# Calculus II

## Integrals of the form $\int \frac{a}{bx + c} dx$

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# Building block Ia

Building block Ia:  $\int \frac{1}{x} dx$ .

## Example

Integrate building block Ia

$$\int \frac{1}{x} dx$$

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$$\int \frac{1}{x} dx = ?$$

# Building block Ia

Building block Ia:  $\int \frac{1}{x} dx$ .

## Example

Integrate building block Ia

$$\int \frac{1}{x} dx = \ln|x| + C$$

# Linear substitutions leading to building block Ia

Building block Ia:  $\int \frac{1}{x} dx = \ln |x| + C.$

## Example

Integrate

$$\int \frac{1}{-4x + 5} dx$$

# Linear substitutions leading to building block Ia

Building block Ia:  $\int \frac{1}{x} dx = \ln |x| + C.$

## Example

Integrate

$$\int \frac{1}{-4x+5} dx = \int \frac{1}{(-4x+5)} \frac{d(-4x)}{(-4)}$$

# Linear substitutions leading to building block Ia

Building block Ia:  $\int \frac{1}{x} dx = \ln |x| + C.$

## Example

Integrate

$$\begin{aligned} \int \frac{1}{-4x+5} dx &= \int \frac{1}{(-4x+5)} \frac{d(-4x)}{(-4)} \\ &= \int \frac{1}{(-4x+5)} \frac{d(-4x+5)}{(-4)} \end{aligned}$$

# Linear substitutions leading to building block Ia

Building block Ia:  $\int \frac{1}{x} dx = \ln |x| + C.$

## Example

Integrate

$$\begin{aligned}
 \int \frac{1}{-4x+5} dx &= \int \frac{1}{(-4x+5)} \frac{d(-4x)}{(-4)} \\
 &= \int \frac{1}{(-4x+5)} \frac{d(-4x+5)}{(-4)} && \left| \text{Set } u = -4x+5 \right. \\
 &= \int \frac{1}{u} \frac{du}{(-4)}
 \end{aligned}$$



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Building block Ia:  $\int \frac{1}{x} dx = \ln |x| + C.$

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 &= \int \frac{1}{u} \frac{du}{(-4)} \\
 &= -\frac{1}{4} \int u^{-1} du
 \end{aligned}$$

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 &= \int \frac{1}{(-4x+5)} \frac{d(-4x+5)}{(-4)} && \left| \text{Set } u = -4x+5 \right. \\
 &= \int \frac{\overset{1}{\cancel{1}} du}{\underset{\text{red}}{u} (-4)} \\
 &= -\frac{1}{4} \int \underset{\text{red}}{u}^{-1} du
 \end{aligned}$$

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 &= -\frac{1}{4} \ln | -4x+5 | + C .
 \end{aligned}$$

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Building block Ia:  $\int \frac{1}{x} dx = \ln |x| + C.$

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 &= \int \frac{1}{u} \frac{du}{(-4)} \\
 &= -\frac{1}{4} \int u^{-1} du = -\frac{1}{4} \ln |u| + C \\
 &= -\frac{1}{4} \ln |-4x+5| + C .
 \end{aligned}$$

# Lin. subst. leading to building block 1a: general case

Building block 1a:  $\int \frac{1}{x} dx = \ln |x| + C.$

## Example

Integrate

$$\begin{aligned}
 \int \frac{1}{ax+b} dx &= \int \frac{1}{(ax+b)} \frac{d(ax)}{a} \\
 &= \int \frac{1}{(ax+b)} \frac{d(ax+b)}{a} && \left| \text{Set } u = ax+b \right. \\
 &= \int \frac{1}{u} \frac{du}{a} \\
 &= \frac{1}{a} \int u^{-1} du = \frac{1}{a} \ln |u| + C \\
 &= \frac{1}{a} \ln |ax+b| + C.
 \end{aligned}$$