Calculus I
Reference: The Evaluation Theorem
(Fundamental Theorem of Calculus, part 2)

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Theorem

Let f be a continuous function on [a, b]. *Then f is integrable over* [a, b].

In other words, $\displaystyle\int_a^b f(x)dx$ exists for any continuous (over [a, b]) function f.
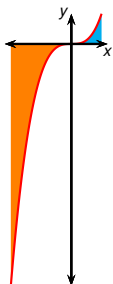
Theorem (The Evaluation Theorem (FTC part 2))

If f is continuous on [a, b], *then*

$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a),$$

where F is any antiderivative of f.

Example

Evaluate the integral $\displaystyle\int_{-2}^{1} x^3 \, dx$.

- x^3 is continuous on $[-2, 1]$ (in fact, it's continuous everywhere).
- An antiderivative is $F(x) = \frac{1}{4}x^4$.

$$\int_{-2}^{1} x^3 \, dx = F(1) - F(-2) = \frac{1}{4}(1)^4 - \frac{1}{4}(-2)^4 = \frac{1}{4} - \frac{16}{4} = -\frac{15}{4}$$