Calculus I
Integral substitution rule

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Outline

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The Substitution Rule

- How do we integrate $\displaystyle\int 2x\sqrt{1+x^2}\,\mathrm{d}x$?

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- Substitute into the integral:

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Theorem (The Substitution Rule)

Let $u = g(x)$ be a differentiable function whose range is an interval I and let f be a function continuous on I. Then

$$\int f(g(x))g'(x)\,\mathrm{d}x = \int f(u)\,\mathrm{d}u$$

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This is the integration counterpart of the Chain Rule.

Example (Substitution Rule)

Find $$\int x^3 \cos(x^4 + 3)\mathrm{d}x.$$

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Let $u = \mathbf{?}$

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Example (Substitution Rule)

Find $\displaystyle\int x^3 \cos(x^4+3)\mathrm{d}x.$

Let $u = x^4 + 3.$

Then $\mathrm{d}u = 4x^3\mathrm{d}x$

Example (Substitution Rule)

Find $\displaystyle\int x^3 \cos(x^4+3)\mathrm{d}x.$

$$\text{Let} \quad u = x^4 + 3.$$

$$\text{Then} \quad \mathrm{d}u = 4x^3\mathrm{d}x$$

$$x^3\mathrm{d}x = \textbf{?}$$

Example (Substitution Rule)

Find $\displaystyle\int x^3 \cos(x^4+3)\mathrm{d}x.$

$$\text{Let} \quad u = x^4 + 3.$$

$$\text{Then} \quad \mathrm{d}u = 4x^3\mathrm{d}x$$

$$x^3\mathrm{d}x = \frac{1}{4}\mathrm{d}u.$$

Example (Substitution Rule)

Find $\displaystyle\int x^3 \cos(x^4+3)\mathrm{d}x.$

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$$x^3\mathrm{d}x = \frac{1}{4}\mathrm{d}u.$$

Substitute: $\displaystyle\int x^3 \cos(x^4+3)\mathrm{d}x = \int \quad \cos u$**?**

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Find $\displaystyle\int x^3 \cos(x^4+3)\mathrm{d}x.$

$$\begin{aligned} \text{Let} \quad u &= x^4+3. \\ \text{Then} \quad \mathrm{d}u &= 4x^3\mathrm{d}x \\ x^3\mathrm{d}x &= \frac{1}{4}\mathrm{d}u. \end{aligned}$$

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Substitute: $\displaystyle\int x^3 \cos(x^4 + 3) \mathrm{d}x = \int \frac{1}{4} \cos u \, \mathrm{d}u$

Example (Substitution Rule)

Find $\displaystyle\int x^3 \cos(x^4+3)\mathrm{d}x.$

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Substitute: $\displaystyle\int x^3 \cos(x^4+3)\mathrm{d}x = \int \frac{1}{4}\cos u\,\mathrm{d}u$

$$= \textbf{?}$$

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$$\text{Substitute:} \quad \int x^3\cos(x^4+3)\mathrm{d}x = \int \frac{1}{4}\cos u\,\mathrm{d}u$$

$$= \frac{1}{4}\sin u$$

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Substitute: $$\int x^3\cos(x^4+3)\mathrm{d}x = \int \frac{1}{4}\cos u\,\mathrm{d}u$$
$$= \frac{1}{4}\sin u + C$$

Example (Substitution Rule)

$$\text{Find} \quad \int x^3 \cos(x^4+3)\mathrm{d}x.$$

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\text{Let} \quad u &= x^4+3. \\
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x^3\mathrm{d}x &= \frac{1}{4}\mathrm{d}u.
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$$\begin{aligned}
\text{Substitute:} \quad \int x^3\cos(x^4+3)\mathrm{d}x &= \int \frac{1}{4}\cos u\,\mathrm{d}u \\
&= \frac{1}{4}\sin u + C \\
&= \frac{1}{4}\sin(x^4+3) + C.
\end{aligned}$$

Example (Substitution Rule)

$$\text{Find} \quad \int \sqrt{2x+1}\,\mathrm{d}x.$$

Example (Substitution Rule)

Find $\displaystyle\int \sqrt{2x+1}\,dx.$

Let $u = \textbf{?}$

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Find $$\int \sqrt{2x+1}\,dx.$$

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Example (Substitution Rule)

$$\text{Find} \quad \int \sqrt{2x+1}\,\mathrm{d}x.$$

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Example (Substitution Rule)

$$\text{Find} \quad \int \sqrt{2x+1}\,\mathrm{d}x.$$

$$\text{Let} \quad u = 2x + 1.$$

$$\text{Then} \quad \mathrm{d}u = 2\mathrm{d}x$$

Example (Substitution Rule)

Find $\displaystyle\int \sqrt{2x+1}\,dx.$

$$\text{Let} \quad u = 2x+1.$$
$$\text{Then} \quad du = 2dx$$
$$dx = \textbf{?}$$

Example (Substitution Rule)

$$\text{Find} \quad \int \sqrt{2x+1}\,\mathrm{d}x.$$

$$\text{Let} \quad u = 2x+1.$$

$$\text{Then} \quad \mathrm{d}u = 2\mathrm{d}x$$

$$\mathrm{d}x = \frac{1}{2}\mathrm{d}u.$$

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$$\text{Let} \quad u = 2x+1.$$

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$$\mathrm{d}x = \frac{1}{2}\mathrm{d}u.$$

$$\text{Substitute:} \quad \int \sqrt{2x+1}\,\mathrm{d}x = \int \quad \sqrt{u}\,\textbf{?}$$

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Find $\displaystyle\int \sqrt{2x+1}\mathrm{d}x.$

$$\begin{aligned} \text{Let}\quad u &= 2x+1. \\ \text{Then}\quad \mathrm{d}u &= 2\mathrm{d}x \\ \mathrm{d}x &= \frac{1}{2}\mathrm{d}u. \end{aligned}$$

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$$\text{Substitute:} \quad \int \sqrt{2x+1}\mathrm{d}x = \int \frac{1}{2}\sqrt{u}\mathrm{d}u$$

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Substitute: $\displaystyle\int \sqrt{2x+1}\,\mathrm{d}x = \int \frac{1}{2}\sqrt{u}\,\mathrm{d}u$

$$= \textbf{?}$$

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$$\text{Substitute:}\quad \int \sqrt{2x+1}\mathrm{d}x = \int \frac{1}{2}\sqrt{u}\mathrm{d}u$$

$$= \frac{1}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}}$$

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\text{Substitute:} \quad \int \sqrt{2x+1}\mathrm{d}x &= \int \frac{1}{2}\sqrt{u}\mathrm{d}u \\
&= \frac{1}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C
\end{aligned}
$$

Example (Substitution Rule)

Find $\displaystyle\int \sqrt{2x+1}\,\mathrm{d}x.$

$$\text{Let}\quad u = 2x+1.$$

$$\begin{aligned} \text{Then}\quad \mathrm{d}u &= 2\mathrm{d}x \\ \mathrm{d}x &= \frac{1}{2}\mathrm{d}u. \end{aligned}$$

$$\begin{aligned} \text{Substitute:}\quad \int \sqrt{2x+1}\,\mathrm{d}x &= \int \frac{1}{2}\sqrt{u}\,\mathrm{d}u \\ &= \frac{1}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C \\ &= \frac{1}{3}(2x+1)^{\frac{3}{2}} + C. \end{aligned}$$

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$$\begin{aligned} \text{Substitute:} \quad \int \sqrt{2x+1}\,\mathrm{d}x &= \int \frac{1}{2}\sqrt{u}\,\mathrm{d}u \\ &= \frac{1}{\cancel{2}} \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{\cancel{2}}} + C \\ &= \frac{1}{3}(2x+1)^{\frac{3}{2}} + C. \end{aligned}$$

Example (Substitution Rule)

Find $$\int \frac{x}{\sqrt{3-4x^2}} \mathrm{d}x.$$

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Example (Substitution Rule)

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$$\text{Let} \quad u = 3 - 4x^2.$$

Example (Substitution Rule)

Find $\displaystyle\int \frac{x}{\sqrt{3-4x^2}}\mathrm{d}x.$

Let $u = 3 - 4x^2.$

Then $\mathrm{d}u = \mathbf{?}$

Example (Substitution Rule)

Find $$\int \frac{x}{\sqrt{3-4x^2}}\mathrm{d}x.$$

Let $u = 3 - 4x^2$.

Then $\mathrm{d}u = -8x\mathrm{d}x$

Example (Substitution Rule)

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$$\text{Find} \quad \int \frac{x}{\sqrt{3-4x^2}} \mathrm{d}x.$$

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$$\text{Find} \quad \int \frac{x}{\sqrt{3-4x^2}} \mathrm{d}x.$$

$$\begin{aligned} \text{Let} \quad u &= 3 - 4x^2. \\ \text{Then} \quad \mathrm{d}u &= -8x\mathrm{d}x \\ x\mathrm{d}x &= -\frac{1}{8}\mathrm{d}u. \end{aligned}$$

$$\begin{aligned} \text{Substitute:} \quad \int \frac{x}{\sqrt{3-4x^2}} \mathrm{d}x &= \int \left(-\frac{1}{8}\right) \frac{1}{\sqrt{u}} \mathrm{d}u \\ &= -\frac{1}{8} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} \end{aligned}$$

Example (Substitution Rule)

$$\text{Find} \quad \int \frac{x}{\sqrt{3-4x^2}} \mathrm{d}x.$$

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\text{Let} \quad u &= 3-4x^2. \\
\text{Then} \quad \mathrm{d}u &= -8x\mathrm{d}x \\
x\mathrm{d}x &= -\frac{1}{8}\mathrm{d}u.
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\text{Substitute:} \quad \int \frac{x}{\sqrt{3-4x^2}} \mathrm{d}x &= \int \left(-\frac{1}{8}\right) \frac{1}{\sqrt{u}} \mathrm{d}u \\
&= -\frac{1}{8} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C
\end{aligned}$$

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$$\text{Find} \quad \int \frac{x}{\sqrt{3-4x^2}} \mathrm{d}x.$$

$$\text{Let} \quad u = 3 - 4x^2.$$

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Example (Substitution Rule)

$$\text{Find} \quad \int e^{3x} \mathrm{d}x.$$

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$$\text{Let} \quad u = \mathbf{?}$$

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$$\text{Let} \quad u = 3x.$$

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$$\text{Substitute:} \quad \int e^{3x} \mathrm{d}x = \int \quad e^{u} \textbf{?}$$

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\text{Let} \quad u &= 3x. \\
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\text{Substitute:} \quad \int e^{3x} \mathrm{d}x &= \int \frac{1}{3} e^{u} \mathrm{d}u \\
&= \frac{1}{3} e^{u} {\color{red} + C}
\end{aligned}$$

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Example (Substitution Rule, more factors)

Evaluate $\displaystyle\int 3x^5\sqrt{1+x^3}\mathrm{d}x$.

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$$
\begin{aligned}
\text{Let } u &= 1+x^3.\\
\text{Then } \mathrm{d}u &= 3x^2\mathrm{d}x.\\
x^3 &= u-1.\\
\int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x &= \int (u-1)\sqrt{u}\,\mathrm{d}u\\
&= \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u
\end{aligned}
$$

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$$\text{Evaluate } \int 3x^5\sqrt{1+x^3}\mathrm{d}x = \int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x.$$

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&= \left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right)
\end{aligned}
$$

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&= \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u \\
&= \left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right) + C
\end{aligned}$$

Example (Substitution Rule, more factors)

Evaluate $\int 3x^5\sqrt{1+x^3}\mathrm{d}x = \int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x.$

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\text{Let } u &= 1+x^3.\\
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&= \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u\\
&= \left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right) + C\\
&= \frac{2}{5}\left(1+x^3\right)^{\frac{5}{2}} - \frac{2}{3}\left(1+x^3\right)^{\frac{3}{2}} + C.
\end{aligned}$$

Definite Integrals

There are two ways to find a definite integral with the Substitution Rule:

1. First evaluate the indefinite integral, then use the FTC.

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$$\int_0^4 \sqrt{2x+1}\,\mathrm{d}x = \left[\int \sqrt{2x+1}\,\mathrm{d}x\right]_0^4 = \left[\frac{1}{3}(2x+1)^{3/2}\right]_0^4$$

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$$
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&= \frac{1}{3}(2\cdot 4+1)^{3/2} - \frac{1}{3}(2\cdot 0+1)^{3/2} \\
&= \frac{1}{3}(9)^{3/2} - \frac{1}{3}(1)^{3/2} = \frac{1}{3}(27-1) = \frac{26}{3}
\end{aligned}
$$

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2. Change the limits of integration when the variable is changed.

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$$= \frac{1}{3}(9)^{3/2} - \frac{1}{3}(1)^{3/2} = \frac{1}{3}(27-1) = \frac{26}{3}$$

2. Change the limits of integration when the variable is changed.

Theorem (The Substitution Rule for Definite Integrals)

If g' is continuous on $[a,b]$ and f is continuous on the range of g, then letting $u = g(x)$ we get

$$\int_a^b f(g(x))g'(x)\,\mathrm{d}x = \int_{g(a)}^{g(b)} f(u)\,\mathrm{d}u$$

Example

Find $\int_0^4 \sqrt{2x+1}\,\mathrm{d}x$.

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- Let $u = \textbf{?}$.
- Then $\mathrm{d}u = \textbf{?}$.

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- Let $u = 2x + 1$.
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- Therefore $\mathrm{d}x =$ **?** .

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- When $x = 0$, $u = \mathbf{?}$.
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Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

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Find $\displaystyle\int_1^{2} \frac{dx}{(2-3x)^2}$.

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$$= -\frac{1}{3}\cdot\left[\ ?\ \right]_{-1}^{-4}$$

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$$= \frac{1}{3}\left(\frac{1}{-4} - \frac{1}{-1}\right)$$

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$$\begin{aligned}\int_{x=1}^{x=2} \frac{\mathrm{d}x}{(2-3x)^2} &= \ -\frac{1}{3}\int_{u=-1}^{u=-4}\frac{\mathrm{d}u}{u^2} = -\frac{1}{3}\int_{-1}^{-4} u^{-2}\mathrm{d}u \\ &= -\frac{1}{3}\cdot\left[-\frac{1}{u}\right]_{-1}^{-4} = \frac{1}{3}\left[\frac{1}{u}\right]_{-1}^{-4} \\ &= \frac{1}{3}\left(\frac{1}{-4} - \frac{1}{-1}\right) = \frac{1}{3}\left(1-\frac{1}{4}\right) = \frac{1}{4}.\end{aligned}$$