# Calculus I
# Derivative of $ax^3 + bx^2 + cx + d$

Todor Milev

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## Theorem (The Sum Rule)

*If $f$ and $g$ are both differentiable, then*

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)+g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}f(x) + \frac{\mathrm{d}}{\mathrm{d}x}g(x).$$

## Proof.

$$\begin{aligned}
\text{Let} \quad F(x) &= f(x)+g(x). \\
\text{Then} \quad F'(x) &= \lim_{h\to 0} \frac{F(x+h)-F(x)}{h} \\
&= \lim_{h\to 0} \frac{[f(x+h)+g(x+h)]-[f(x)+g(x)]}{h} \\
&= \lim_{h\to 0} \left[\frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h}\right] \\
\text{Limit Law 1:} \quad &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \\
&= f'(x)+g'(x).
\end{aligned}$$

□

The Sum Rule can be extended to any number of summands. For instance, using the theorem twice, we get

$$(f+g+h)' = [(f+g)+h]' = (f+g)'+h' = f'+g'+h'.$$

By writing $f-g$ as $f+(-1)g$ and applying the Sum Rule and the Constant Multiple Rule, we get

### Theorem (The Difference Rule)

*If $f$ and $g$ are both differentiable, then*

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)-g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}f(x) - \frac{\mathrm{d}}{\mathrm{d}x}g(x).$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

## Example (Derivative of a Polynomial)

$$\text{If} \quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$$

$$\begin{aligned}
\text{Then} \quad \frac{dy}{dx} &= \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right) \\
&= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5) \\
&= \frac{d}{dx}\left(x^{16}\right) + 2\sqrt{3}\frac{d}{dx}\left(x^7\right) - 4\frac{d}{dx}\left(x^3\right) + \frac{1}{8}\frac{d}{dx}(x) - \frac{d}{dx}(5) \\
&= (16x^{15}) + 2\sqrt{3}\left(7x^6\right) - 4\left(3x^2\right) + \frac{1}{8}(1) - (0) \\
&= 16x^{15} + 14\sqrt{3}x^6 - 12x^2 + \frac{1}{8}.
\end{aligned}$$