Calculus I
Homework
Trigonometric derivatives

1. Compute the derivative.

   (a) $f(x) = 2x^3 - 3\cos x.$

   answer: $6x^2 + 3\sin x$

   (b) $f(x) = \sqrt{x}\cos x.$

   answer: $-x^{\frac{1}{2}}\sin x + \frac{1}{2}x^{-\frac{1}{2}}\cos x$

   (c) $f(x) = \sin x + \dfrac{1}{3}\cot x.$

   answer: $\frac{-\frac{1}{3}+\cos x\sin^2 x}{\sin^2 x} = \cos x - \frac{1}{3}\csc^2 x$

   (d) $y = 2\sec x - \csc x.$

   answer: $\frac{\cos^3 x+2\sin^3 x}{(\cos x\sin x)^2}$

   (e) $y = \dfrac{1+\sin^2\theta}{\cos^3\theta}.$

   answer: $y' = \frac{2\sin\theta\cos^2\theta+3\sin^3\theta+3\sin\theta}{\cos^4\theta}$

   (f) $g(t) = 4\sec t + \tan t - \csc t + 3\cot t.$

   answer: $4\sec t\tan t + \sec^2 t + \csc t\cot t - 3\csc^2 t$

   (g) $y = c\cos t + t^2\sin t.$

   answer: $-c\sin t + 2t\sin t + t^2\cos t$

   (h) $y = u(a\cos u + b\cot u).$

   answer: $\frac{-au\sin^3 u+a\cos u\sin^2 u-bu+b\cos u\sin u}{\sin^2 u}$

   (i) $y = \dfrac{x}{2-\tan x}.$

   answer: $\frac{x-\cos x\sin x+2\cos^2 x}{(2\cos x-\sin x)^2}$

   (j) $y = \sin\theta\cos\theta.$

   answer: $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

   (k) $f(\theta) = \dfrac{\sec\theta}{1+\sec\theta}.$

   answer: $\frac{\sin\theta}{(1+\cos\theta)^2}$

   (l) $y = \dfrac{\cos x}{1-\sin x}.$

   answer: $\frac{1}{1-\sin x}$

   (m) $y = \dfrac{t\sin t}{1+t}.$

   answer: $\frac{\sin t+t\cos t+t^2\cos t}{(1+t)^2}$

   (n) $y = \dfrac{1-\sec x}{\tan x}.$

   answer: $\frac{\cos x-1}{\sin^2 x}$

   (o) $h(\theta) = \theta\csc\theta - \cot\theta.$

   answer: $\frac{1+\sin\theta-\theta\cos\theta}{\sin^2\theta}$

   (p) $y = x^2\sin x\tan x.$

   answer: $\frac{2x\cos x\sin^2 x+2x^2\sin x\cos^3 x+x^2\sin^3(x)}{\cos^2 x}$

2. Differentiate.

   (a) $\tan x.$

   answer: $\sec^2 x$

   (b) $\cot x.$

   answer: $-\csc^2 x$

   (c) $\sec x.$

   answer: $\sec x\tan x = \frac{\sin x}{\cos^2 x}$

   (d) $\csc x.$

   answer: $-\csc x\cot x = -\frac{\cos x}{\sin^2 x}$

   (e) $\sec x\tan x.$

   answer: $\sec x\tan^2 x + \sec^3 x$

   (f) $\sec x + \tan x.$

   answer: $\sec x(\tan x + \sec x)$

   (g) $\sec^2 x.$

   answer: $2\tan x\sec^2 x$

   (h) $\csc^2 x.$

   answer: $-2\cot x\csc^2 x$

   (i) $f(x) = (\sec x)e^x.$

   answer: $\sec x\tan xe^x + \sec xe^x = (\tan x+1)(\sec x)e^x$

   (j) $f(x) = (\tan x)e^x.$

   answer: $\sec^2 xe^x + \tan xe^x$

   (k) $\dfrac{\sin x}{x}.$

   answer: $\frac{x\cos x-\sin x}{x^2}$

   (l) $\dfrac{\sin x}{e^x}.$

   answer: $\frac{\cos x-\sin x}{e^x}$

   (m) $x(\cos x)e^x.$

   answer: $e^x(x\cos x - x\sin x + \cos x)$

   (n) $\dfrac{e^x}{\tan x}.$

   answer: $e^x\left(\cot x - \csc^2 x\right)$

   (o) $\dfrac{e^x}{\sec x} + \sec x.$

   answer: $e^x(\cos x - \sin x) + \sec x\tan x$

Solution. 2i

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left((\sec x)e^x\right) &= \left(\frac{\mathrm{d}}{\mathrm{d}x}\left(\sec x\right)\right)e^x + \left(\sec x\right)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^x\right) \quad \Big| \text{ product rule} \\ &= \sec x\tan x e^x + \sec x e^x \\ &= \left(\tan x + 1\right)\sec x e^x\end{aligned}$$

Solution. 2j

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left((\tan x)e^x\right) &= \left(\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan x\right)\right)e^x + \left(\tan x\right)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^x\right) \quad \Big| \text{ product rule} \\ &= (\sec^2 x)e^x + (\tan x)e^x \\ &= (\sec^2 x + \tan x)e^x.\end{aligned}$$