Calculus II
Curve length miscellaneous problem, part 2

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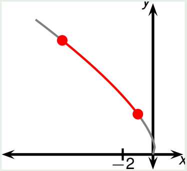
Example



Find the length of the curve γ.

$$\gamma : \left| \begin{array}{lcl} x(t) & = & \sqrt{t} - 2t \\ y(t) & = & \frac{8}{3} t^{\frac{3}{4}} \end{array} \right. , t \in [1, 4] \quad .$$

We have that $x'(t) = \frac{1}{2\sqrt{t}} - 2$ and $y'(t) = \frac{8}{3} \cdot \frac{3}{4} t^{-\frac{1}{4}} = 2t^{-\frac{1}{4}}$.

$$L(\gamma) = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2} \mathrm{d}t = \int_1^4 \sqrt{\left(\frac{1}{2\sqrt{t}} - 2\right)^2 + \left(2t^{-\frac{1}{4}}\right)^2} \mathrm{d}t$$

$$= \int_1^4 \sqrt{\frac{1}{4t} - \frac{2}{\sqrt{t}} + 4 + \frac{4}{\sqrt{t}}} \, \mathrm{d}t$$

$$= \int_1^4 \sqrt{\frac{1}{4t} + \frac{2}{\sqrt{t}} + 4} \, \mathrm{d}t = \int_1^4 \sqrt{\left(\frac{1}{2\sqrt{t}} + 2\right)^2} \mathrm{d}t$$

$$= \int_1^4 \left(\frac{1}{2\sqrt{t}} + 2\right) \mathrm{d}t = \left[\sqrt{t} + 2t\right]_1^4 = \sqrt{4} + 2 \cdot 4 - \left(\sqrt{1} + 2 \cdot 1\right) = 7.$$