Calculus I
Homework
Areas and integrals

1. Estimate the integral using a Riemann sum using the indicated sample points and interval length.

(a) $\int_0^4 \left(\sqrt{8x+1}\right)\,\mathrm{d}x$. Use four intervals of equal width, choose the sample point to be the left endpoint of each interval.

<div align="right" style="transform:rotate(180deg)">answer: $\Delta x = 1$ and $f(x) = \sqrt{8x+1}$. Thus $\int_4^0 f(x)\,\mathrm{d}x \approx 9 + \sqrt{17}$.</div>

(b) $\int_0^6 \dfrac{1}{x^2+1}\,\mathrm{d}x$. Use three intervals of equal width, choose the sample point to be the left endpoint.

<div align="right">answer: $\Delta x = 2$ and $f(x) = \dfrac{1}{x^2+1}$. Thus $\int_6^0 f(x)\,\mathrm{d}x \approx \dfrac{214}{85}$.</div>

(c) $\int_{-3.5}^{-0.5} \dfrac{\mathrm{d}x}{x^2+1}$. Use three intervals of equal width, choose the sample point to be the midpoint of each interval.

<div align="right">answer: $\Delta x = 1$ and $f(x) = \dfrac{1}{x^2+1}$. Thus $\int_{-0.5}^{-3.5} f(x)\,\mathrm{d}x \approx \Delta x\,(f(-3) + f(-2) + f(-1)) = \dfrac{4}{5} = 0.8$.</div>

(d) $\int_0^2 \dfrac{\mathrm{d}x}{1+x+x^3}$. Use $\Delta x = \tfrac{1}{2}$ and right endpoint sampling points.

<div align="right">answer: $\tfrac{1}{2}\left(\dfrac{13}{8} + \dfrac{3}{8} + \dfrac{47}{8} + \dfrac{11}{1}\right) = \dfrac{20163}{12197} \approx 0.604920$</div>

(e) $\int_{-2}^0 \dfrac{\mathrm{d}x}{1+x+x^2}$. Use $\Delta x = \tfrac{2}{3}$ and left endpoint sampling points.

<div align="right">answer: $\tfrac{2}{3}\left(\dfrac{3}{1} + \dfrac{13}{6} + \dfrac{7}{6}\right) = \dfrac{817}{252} \approx 1.549094$</div>

(f) $\int_0^2 \dfrac{\mathrm{d}x}{1+x^3}$. Use four intervals of equal width, choose the sample point to be the left endpoint of each interval.

<div align="right">answer: $\Delta x = 0.5$ and $f(x) = \dfrac{1}{1+x^3}$. Thus $\int_2^0 f(x)\mathrm{d}x \approx \Delta x\left(f(0) + f(1) + f\left(\dfrac{1}{2}\right) + f\left(\dfrac{3}{2}\right)\right) = \dfrac{1260}{649} \approx 1.30873$.</div>

(g) $\int_{-2}^0 \dfrac{\mathrm{d}x}{x^4+1}$. Use four intervals of equal width, choose the sample point to be the right endpoint.

<div align="right">answer: $\Delta x = 0.5$ and $f(x) = \dfrac{1}{x^4+1}$. Thus $\int_2^0 f(x)\mathrm{d}x \approx \Delta x\left(f\left(-\dfrac{3}{2}\right) + f(-1) + f\left(-\dfrac{1}{2}\right) + f(0)\right) = \dfrac{8595}{6596} \approx 1.303062$.</div>

(h) $\int_{-1}^0 \dfrac{1}{3x^2+1}\,\mathrm{d}x$. Use 3 **intervals** of equal width, choose the sampling points to be the **left endpoints** of each interval. Simplify your answer to a rational number (single fraction of two integers).
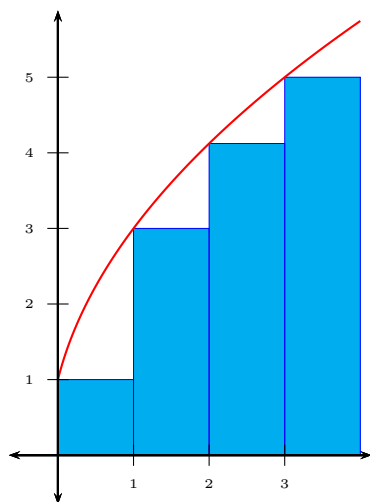
<div align="right">answer: $\tfrac{1}{3} = \Delta x$ and $f(x) = \dfrac{1}{3x^2+1}$. Thus $\int_0^{-1} f(x)\mathrm{d}x$ is approximated by $\Delta x\left(f(-1) + f\left(-\dfrac{2}{3}\right) + f\left(-\dfrac{1}{3}\right)\right) = \dfrac{10}{21}$.</div>

Solution. 1.a. The interval $[0,4]$ is subdivided into $n = 4$ intervals, therefore the length of each is $\Delta x = 1$. The intervals are therefore

$$[0,1], [1,2], [2,3], [3,4] \quad .$$

The problem asks us to use the left endpoints of each interval as sampling points. Therefore our sampling points are $0, 1, 2, 3$. Therefore the Riemann sum we are looking for is

$$\Delta x \left(f(0) + f(1) + f(2) + f(3)\right) = 1 \cdot \left(\sqrt{8\cdot 0 + 1} + \sqrt{8\cdot 1 + 1} + \sqrt{8\cdot 2 + 1} + \sqrt{8\cdot 3 + 1}\right) = 9 + \sqrt{17} \approx 13.1231$$

Solution. 1.c. The interval $[-3.5, -0.5]$ is subdivided into $n = 3$ intervals, therefore the length of each is $\Delta x = \frac{-0.5-(-3.5)}{3} = \frac{3}{3} = 1$. The intervals are therefore

$$[-3.5, -2.5], [-2.5, -1.5], [-1.5, -0.5] \quad .$$

The problem asks us to use the midpoint of each interval as a sampling point. Therefore our sampling points are $-3, -2, -1$. Therefore the Riemann sum we are looking for is

$$\Delta x \left(f(-3) + f(-2) + f(-1) \right) = 1 \cdot \left(\frac{1}{10} + \frac{1}{5} + \frac{1}{2} \right) = 0.8 \quad .$$
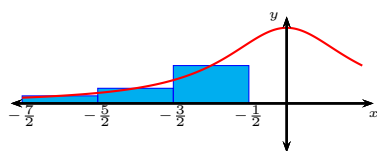
Solution. 1.h

$\Delta x = \frac{1}{3}$ and $f(x) = \frac{1}{3x^2+1}$. Thus $\displaystyle\int_{-1}^{0} f(x)\mathrm{d}x$ is approximated by $\Delta x \left(f(-1) + f\left(-\frac{2}{3}\right) + f\left(-\frac{1}{3}\right) \right) = \frac{10}{21}$.