

Precalculus

Factor cubic with one rational and two real roots using its plot

Todor Milev

2019

Example

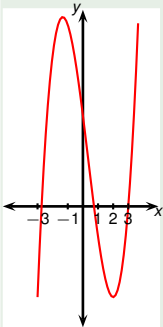
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$$x^3 - x^2 - 8x + 6 = 0$$

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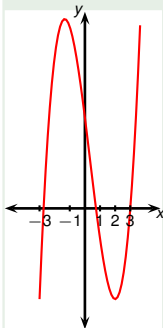


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The graph appears to intersect the x axis at:
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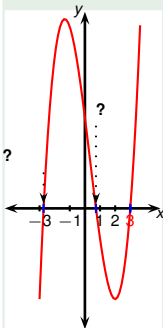


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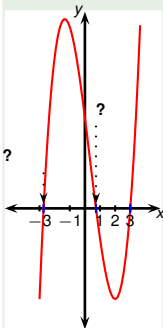


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The graph appears to intersect the x axis at:
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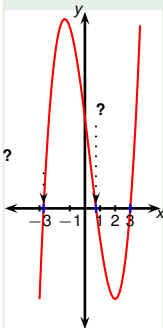
$$x - 3 \quad \overline{x^3 - x^2 - 8x + 6}$$

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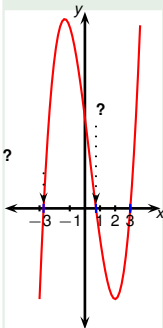
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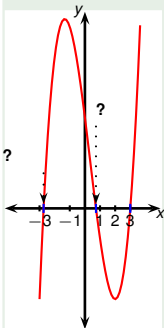
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$$x - 3 \overline{) \overset{?}{x^3} - x^2 - 8x + 6}$$

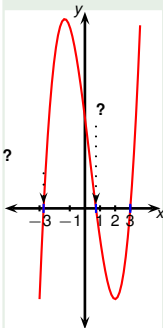
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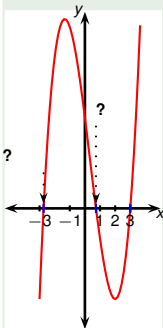
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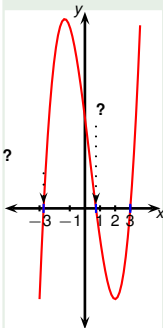
Multiply x^2 by divisor.

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 \end{array}$$

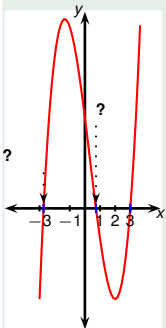
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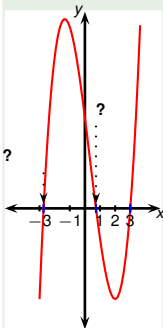
Subtract last two polynomials.

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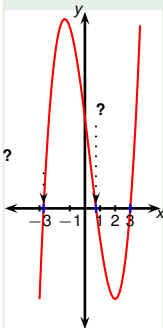
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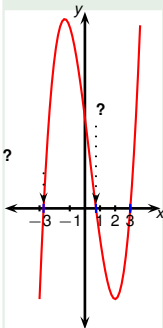
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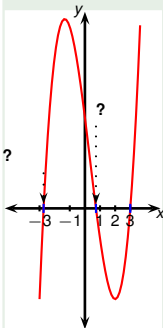
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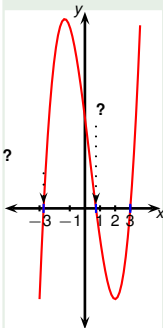
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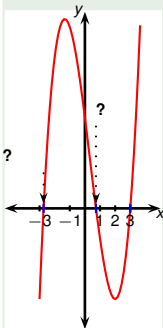
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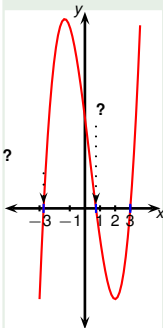
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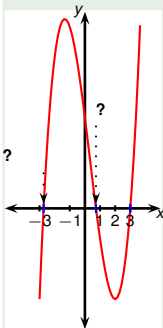
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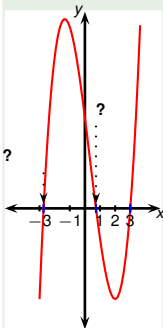
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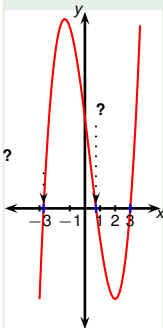
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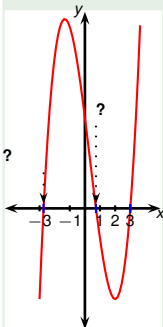
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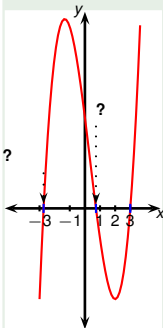
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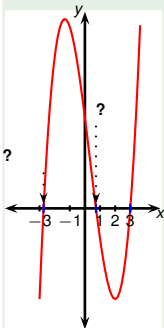
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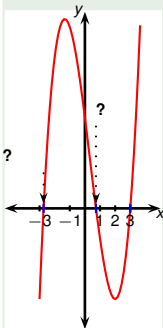
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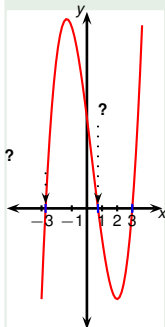
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$$(x - 3)(x^2 + 2x - 2) + 0 = 0$$

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Quotient: $x^2 + 2x - 2$

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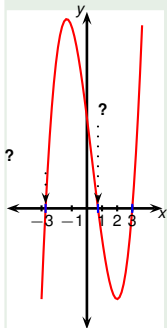
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$x - 3$	$\begin{array}{r} x^3 - x^2 - 8x + 6 \\ \underline{x^3 - 3x^2} \\ 2x^2 - 8x + 6 \\ \underline{2x^2 - 6x} \\ -2x + 6 \\ \underline{-2x + 6} \\ 0 \end{array}$
Remainder:	0

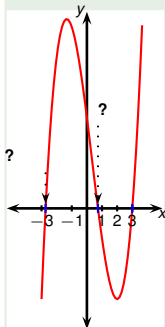
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	$\quad -2x + 6$
	$\quad \quad \quad 0$

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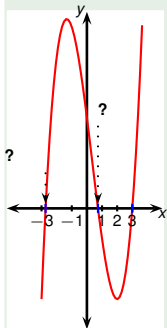
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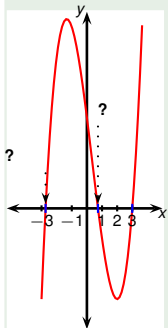
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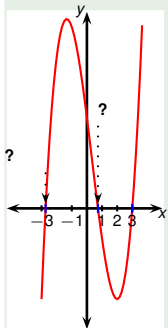
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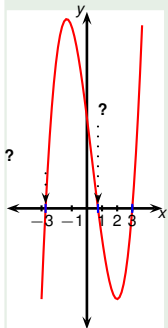
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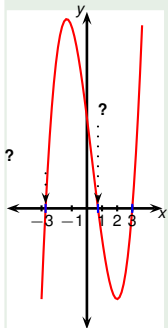
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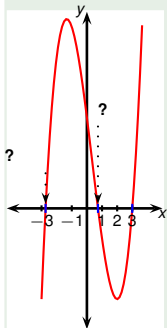
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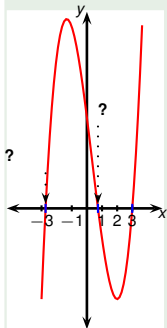
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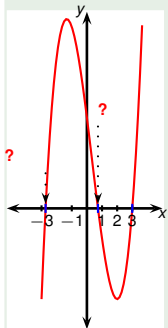
$$x = \frac{-2 \pm 2\sqrt{3}}{2}$$

The graph appears to intersect the x axis at:

?, ?, 3. What are the two roots besides 3?

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.



$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) = 0$$

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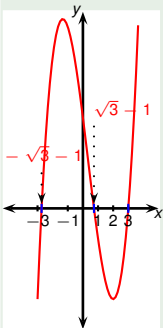
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The graph appears to intersect the x axis at:

$- \sqrt{3} - 1$, $\sqrt{3} - 1$, 3. What are the two roots besides 3?

Final answer:

$$x = 3 \quad \text{or} \quad x = -1 - \sqrt{3} \quad \text{or} \quad x = -1 + \sqrt{3}.$$