# Precalculus
# Find extremum of quadratic, text problem.

Todor Milev

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# Maximum or minimum value of a quadratic function

- Let $f(x) = ax^2 + bx + c$ - quadratic ($a \neq 0$).
- Let $D$ be the discriminant $D = b^2 - 4ac$.

$$f(x) = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a} \quad \Big| \text{ complete the square}$$

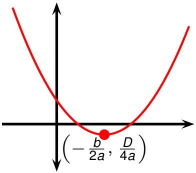
- Therefore if $a > 0$ then $f(x) = a(\text{square}) - \frac{D}{4a} \geq -\frac{D}{4a}$.
- Similarly if $a < 0$ then $f(x) = a(\text{square}) - \frac{D}{4a} \leq -\frac{D}{4a}$.

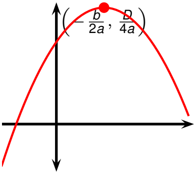
Recall $f(x) = ax^2 + bx + c = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a}$.

## Proposition

*Let $f(x) = ax^2 + bx + c$, $a \neq 0$ and let $D = b^2 - 4ac$.*

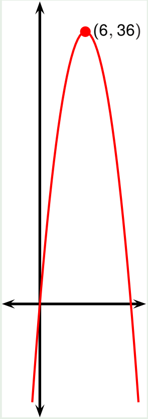
- *If $a > 0$ then $f(x)$ has no maximum and has minimum at $x = -\frac{b}{2a}$.*
- *If $a < 0$ then $f(x)$ has no minimum and has maximum at $x = -\frac{b}{2a}$.*
- *In both cases, the extremal value (either maximum or minimum) is $f\left(-\frac{b}{2a}\right) = -\frac{b^2-4ac}{4a} = -\frac{D}{4a}$.*



## Example

Let $x, z$ be two numbers that add to 12. Choose $x$ and $z$ so that the product $x \cdot z$ is maximal.




$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

Maximizing:

$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{12}{-2} = 6 \\ z &= 12 - x = 12 - 6 = 6 \\ \text{Max. product} &= xz = 6 \cdot 6 = 36. \end{aligned}$$