

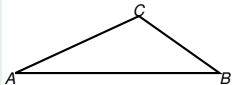
## Precalculus

# Solve triangle from two sides and an angle

Todor Milev

2019

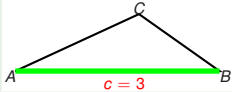
## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

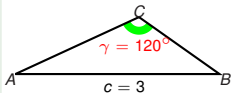
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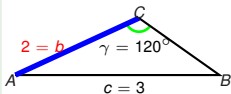
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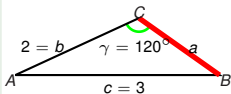
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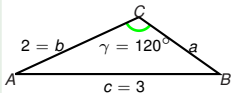
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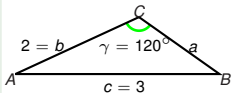


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| Law of cosines

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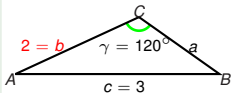
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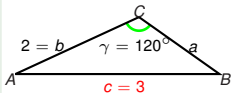
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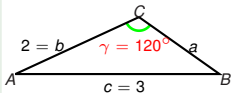
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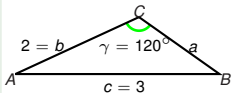
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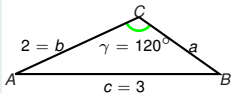
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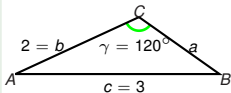
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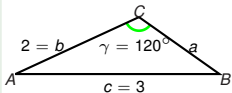
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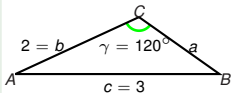
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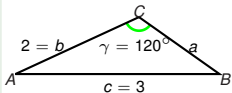
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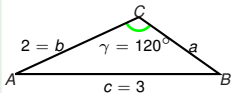
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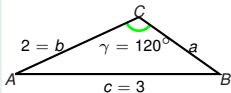
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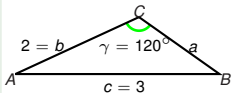
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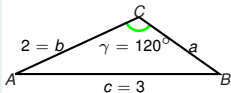
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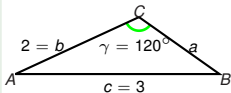
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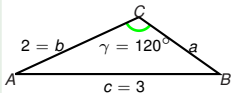
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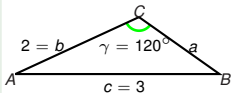
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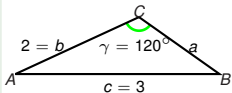
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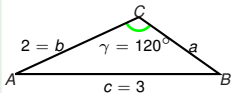
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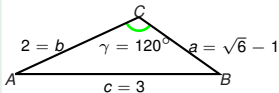
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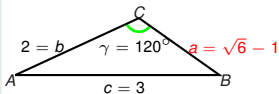
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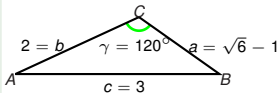
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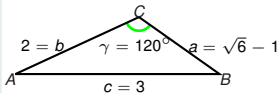
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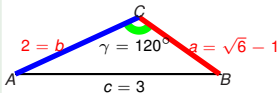
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**Area = ?**

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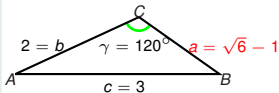
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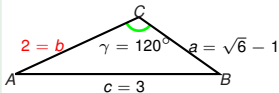
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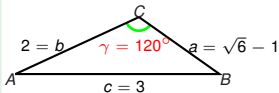
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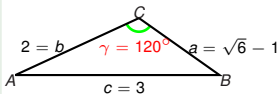
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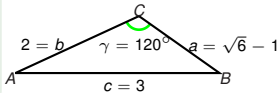
Law of cosines  
Solve for  $a$  :

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$$= -1 + \sqrt{6}$$

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## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

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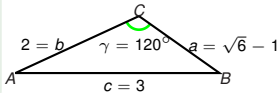
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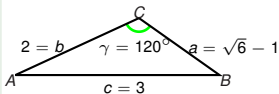
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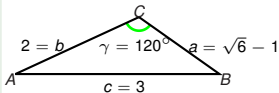
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