

Precalculus

Homework

Trigonometric equations and inequalities

1. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation.

(a) $2 \cos x - 1 = 0$.

$$\frac{x}{\pi} = x \text{ to } \frac{x}{\pi} = x \text{ : ANSWER}$$

(b) $\sin(2x) = \cos x$.

$$\frac{9}{\pi} = x \text{ to } \frac{9}{\pi} = x \cdot \frac{\pi}{\pi} = x \cdot \frac{\pi}{\pi} = x \text{ : ANSWER}$$

(c) $\sqrt{3} \sin x = \sin(2x)$.

$$u \text{ to } u \cdot \frac{9}{\pi} = x \text{ : ANSWER}$$

(d) $2 \sin^2 x = 1$.

$$\frac{\pi}{\pi} = x \text{ to } \frac{\pi}{\pi} = x \cdot \frac{\pi}{\pi} = x \cdot \frac{\pi}{\pi} = x \text{ : ANSWER}$$

(e) $2 + \cos(2x) = 3 \cos x$.

$$\frac{x}{\pi} = x \text{ to } \frac{x}{\pi} = x \cdot \frac{\pi}{\pi} = x \cdot \frac{\pi}{\pi} = x \text{ : ANSWER}$$

(f) $2 \cos x + \sin(2x) = 0$.

$$\frac{x}{\pi} = x \cdot \frac{\pi}{\pi} = x \text{ : ANSWER}$$

(g) $2 \cos^2 x - (1 + \sqrt{2}) \cos x + \frac{\sqrt{2}}{2} = 0$.

$$\frac{\pi}{\pi} = x \text{ to } \frac{\pi}{\pi} = x \cdot \frac{\pi}{\pi} = x \cdot \frac{\pi}{\pi} = x \text{ : ANSWER}$$

(h) $|\tan x| = 1$.

$$\frac{\pi}{\pi} = x \text{ to } \frac{\pi}{\pi} = x \cdot \frac{\pi}{\pi} = x \cdot \frac{\pi}{\pi} = x \text{ : ANSWER}$$

(i) $3 \cot^2 x = 1$.

$$\frac{x}{\pi} = x \text{ to } \frac{x}{\pi} = x \cdot \frac{\pi}{\pi} = x \cdot \frac{\pi}{\pi} = x \text{ : ANSWER}$$

(j) $\sin x = \tan x$.

$$u \text{ to } u \cdot \frac{\pi}{\pi} = x \cdot \frac{\pi}{\pi} = x \cdot \frac{\pi}{\pi} = x \text{ : ANSWER}$$

Solution. 1.g Set $\cos x = u$. Then

$$2 \cos^2 x - (1 + \sqrt{2}) \cos x + \frac{\sqrt{2}}{2} = 0$$

becomes

$$2u^2 - (1 + \sqrt{2})u + \frac{\sqrt{2}}{2} = 0.$$

This is a quadratic equation in u and therefore has solutions

$$\begin{aligned} u_1, u_2 &= \frac{1 + \sqrt{2} \pm \sqrt{(1 + \sqrt{2})^2 - 4\sqrt{2}}}{4} \\ &= \frac{1 + \sqrt{2} \pm \sqrt{1 - 2\sqrt{2} + 2}}{4} \\ &= \frac{1 + \sqrt{2} \pm \sqrt{(1 - \sqrt{2})^2}}{4} \\ &= \frac{1 + \sqrt{2} \pm (1 - \sqrt{2})}{4} = \left\{ \frac{1}{2} \quad \text{or} \quad \frac{\sqrt{2}}{2} \right\} \end{aligned}$$

Therefore $u = \cos x = \frac{1}{2}$ or $u = \cos x = \frac{\sqrt{2}}{2}$, and, as x is in the interval $[0, 2\pi]$, we get $x = \frac{\pi}{3}, \frac{5\pi}{3}$ (for $\cos x = \frac{1}{2}$) or $x = \frac{\pi}{4}, \frac{7\pi}{4}$ (for $\cos x = \frac{\sqrt{2}}{2}$).