

Calculus I

Derivatives involving trigonometry and exponents

Todor Milev

2019

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Product Rule:

$$y' = \frac{d}{d\theta} (\theta e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} \frac{d}{d\theta} (\tan \theta + \sec \theta)$$

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Product Rule:

$$y' = \frac{d}{d\theta} (\theta e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} \frac{d}{d\theta} (\tan \theta + \sec \theta)$$

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Product Rule:

$$y' = \frac{d}{d\theta} (\theta e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} \frac{d}{d\theta} (\tan \theta + \sec \theta)$$

Product Rule:

$$= \left(? \right) (\tan \theta + \sec \theta) + \theta e^{\theta} \left(? \right)$$

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Product Rule:

$$y' = \frac{d}{d\theta} (\theta e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} \frac{d}{d\theta} (\tan \theta + \sec \theta)$$

Product Rule:

$$= \left(\theta \frac{d}{d\theta} (e^{\theta}) + \frac{d}{d\theta} (\theta) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (?)$$

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Product Rule:

$$y' = \frac{d}{d\theta} (\theta e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} \frac{d}{d\theta} (\tan \theta + \sec \theta)$$

Product Rule:

$$= \left(\theta \frac{d}{d\theta} (e^{\theta}) + \frac{d}{d\theta} (\theta) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (?)$$

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Product Rule:

$$y' = \frac{d}{d\theta} (\theta e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} \frac{d}{d\theta} (\tan \theta + \sec \theta)$$

Product Rule:

$$= \left(\theta \frac{d}{d\theta} (e^{\theta}) + \frac{d}{d\theta} (\theta) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta)$$

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Product Rule:

$$y' = \frac{d}{d\theta} (\theta e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} \frac{d}{d\theta} (\tan \theta + \sec \theta)$$

Product Rule:

$$= \left(\theta \frac{d}{d\theta} (e^{\theta}) + \frac{d}{d\theta} (\theta) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta)$$

$$= \left(\theta (?) + (?) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta)$$

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Product Rule:

$$y' = \frac{d}{d\theta} (\theta e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} \frac{d}{d\theta} (\tan \theta + \sec \theta)$$

Product Rule:

$$\begin{aligned} &= \left(\theta \frac{d}{d\theta} (e^{\theta}) + \frac{d}{d\theta} (\theta) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta) \\ &= \left(\theta (e^{\theta}) + (?) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta) \end{aligned}$$

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Product Rule:

$$y' = \frac{d}{d\theta} (\theta e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} \frac{d}{d\theta} (\tan \theta + \sec \theta)$$

Product Rule:

$$\begin{aligned} &= \left(\theta \frac{d}{d\theta} (e^{\theta}) + \frac{d}{d\theta} (\theta) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta) \\ &= \left(\theta (e^{\theta}) + (?) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta) \end{aligned}$$

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Product Rule:

$$y' = \frac{d}{d\theta} (\theta e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} \frac{d}{d\theta} (\tan \theta + \sec \theta)$$

Product Rule:

$$\begin{aligned} &= \left(\theta \frac{d}{d\theta} (e^{\theta}) + \frac{d}{d\theta} (\theta) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta) \\ &= \left(\theta (e^{\theta}) + (1) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta) \end{aligned}$$

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Product Rule:

$$y' = \frac{d}{d\theta} (\theta e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} \frac{d}{d\theta} (\tan \theta + \sec \theta)$$

Product Rule:

$$\begin{aligned} &= \left(\theta \frac{d}{d\theta} (e^{\theta}) + \frac{d}{d\theta} (\theta) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta) \\ &= \left(\theta (e^{\theta}) + (1) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta) \\ &= \theta e^{\theta} \sec \theta (\sec \theta + \tan \theta) + e^{\theta} (\theta + 1) (\tan \theta + \sec \theta) \end{aligned}$$

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Product Rule:

$$y' = \frac{d}{d\theta} (\theta e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} \frac{d}{d\theta} (\tan \theta + \sec \theta)$$

Product Rule:

$$\begin{aligned} &= \left(\theta \frac{d}{d\theta} (e^{\theta}) + \frac{d}{d\theta} (\theta) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta) \\ &= \left(\theta (e^{\theta}) + (1) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta) \\ &= \theta e^{\theta} \sec \theta (\sec \theta + \tan \theta) + e^{\theta} (\theta + 1) (\tan \theta + \sec \theta) \end{aligned}$$

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Product Rule:

$$y' = \frac{d}{d\theta} (\theta e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} \frac{d}{d\theta} (\tan \theta + \sec \theta)$$

Product Rule:

$$\begin{aligned} &= \left(\theta \frac{d}{d\theta} (e^{\theta}) + \frac{d}{d\theta} (\theta) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta) \\ &= (\theta (e^{\theta}) + (1) e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta) \\ &= \theta e^{\theta} \sec \theta (\sec \theta + \tan \theta) + e^{\theta} (\theta + 1) (\tan \theta + \sec \theta) \\ &= (\theta \sec \theta + \theta + 1) e^{\theta} (\tan \theta + \sec \theta). \end{aligned}$$

Example (Using the Product Rule twice)

Differentiate:

$$y = \theta e^{\theta} (\tan \theta + \sec \theta).$$

Product Rule:

$$y' = \frac{d}{d\theta} (\theta e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} \frac{d}{d\theta} (\tan \theta + \sec \theta)$$

Product Rule:

$$\begin{aligned} &= \left(\theta \frac{d}{d\theta} (e^{\theta}) + \frac{d}{d\theta} (\theta) e^{\theta} \right) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta) \\ &= (\theta (e^{\theta}) + (1) e^{\theta}) (\tan \theta + \sec \theta) + \theta e^{\theta} (\sec^2 \theta + \tan \theta \sec \theta) \\ &= \theta e^{\theta} \sec \theta (\sec \theta + \tan \theta) + e^{\theta} (\theta + 1) (\tan \theta + \sec \theta) \\ &= (\theta \sec \theta + \theta + 1) e^{\theta} (\tan \theta + \sec \theta). \end{aligned}$$