Precalculus
Trigonometric equation that reduces to quadratic, masked by identity

Todor Milev

2019

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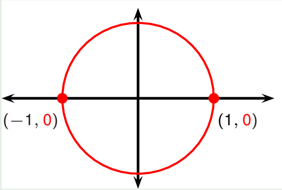

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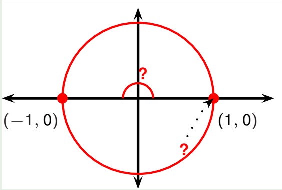

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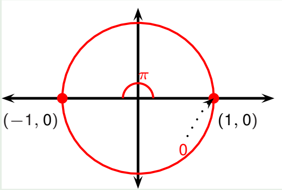

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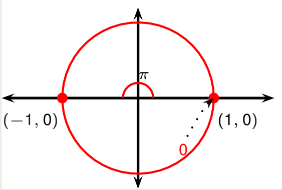

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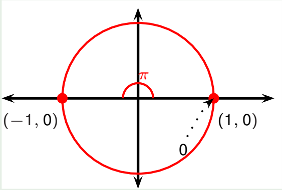

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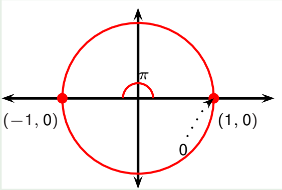

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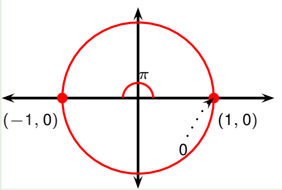

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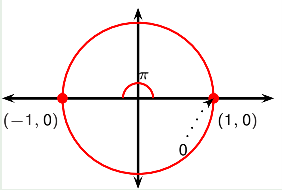

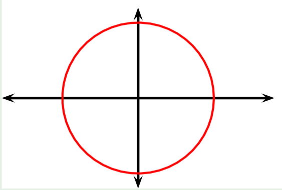

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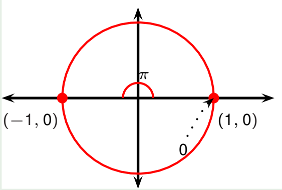

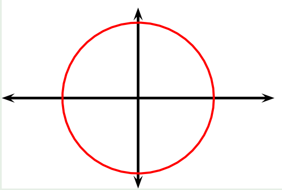

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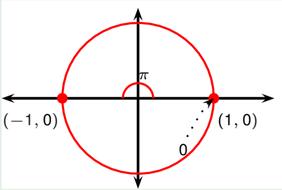

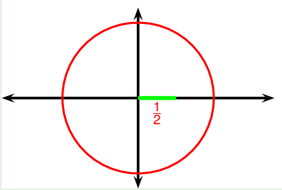

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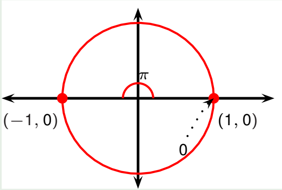

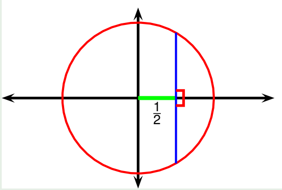

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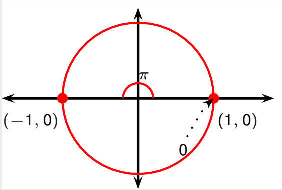

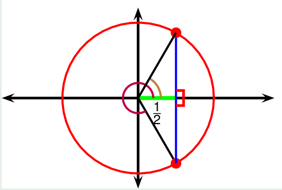

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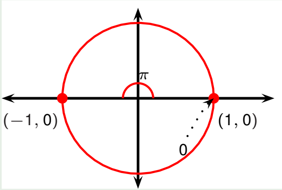

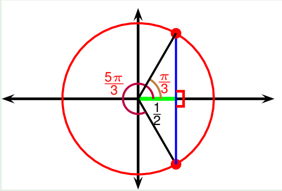

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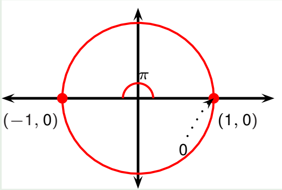

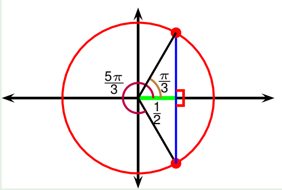

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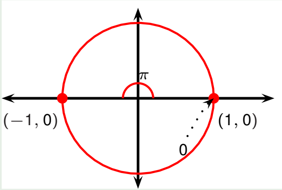

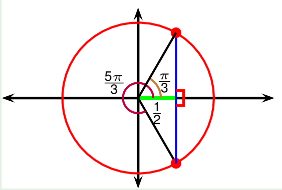

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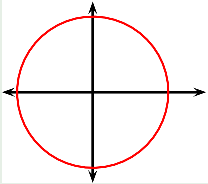

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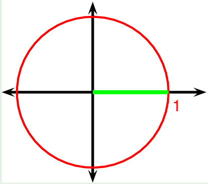

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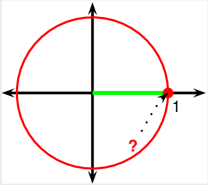

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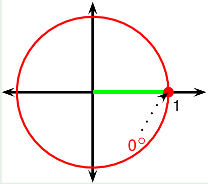

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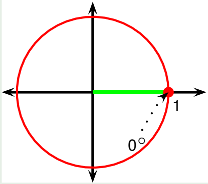

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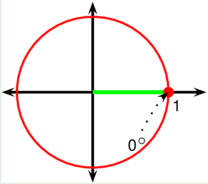

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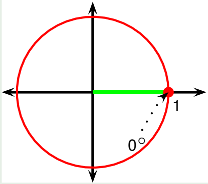

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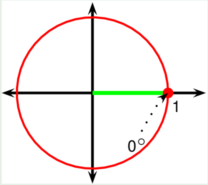

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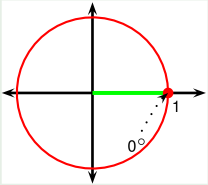

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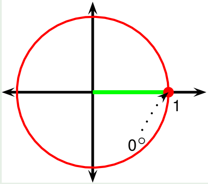

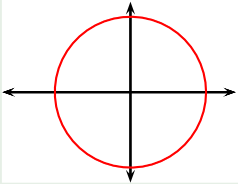

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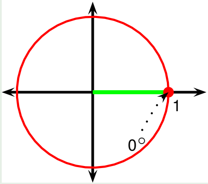

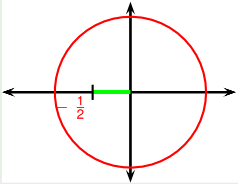

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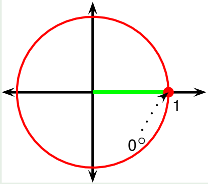

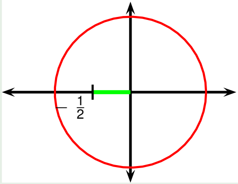

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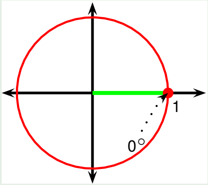

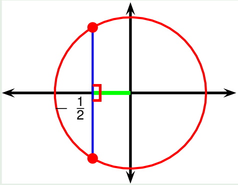

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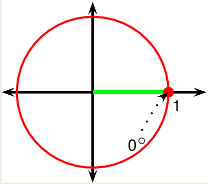

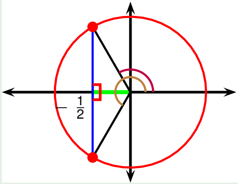

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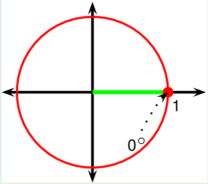

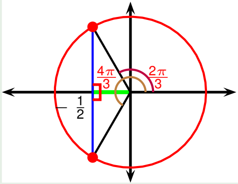

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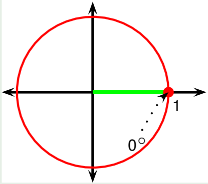

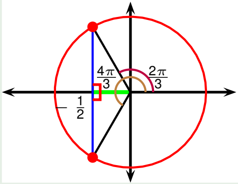

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