Calculus I
Definite integrals and areas between curves

Todor Milev

2019

Outline

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Symmetry

Theorem (Integrals of Symmetric Functions)

Suppose f is continuous on $[-a, a]$.

1. *If f is even (that is, $f(-x) = f(x)$), then* $\displaystyle\int_{-a}^{a} f(x)\,\mathrm{d}x = 2\int_{0}^{a} f(x)\,\mathrm{d}x$.
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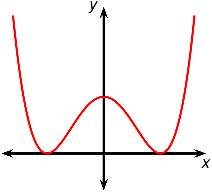

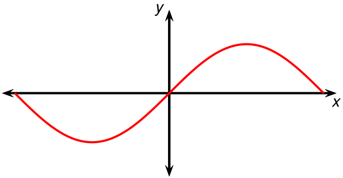

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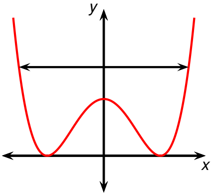

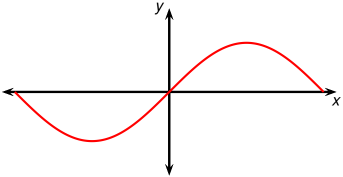

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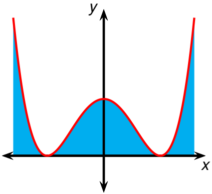

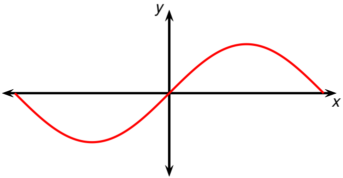

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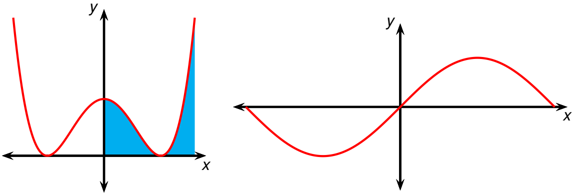

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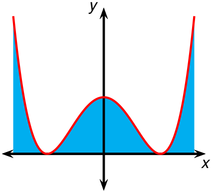

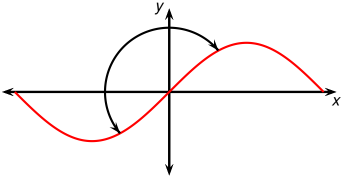

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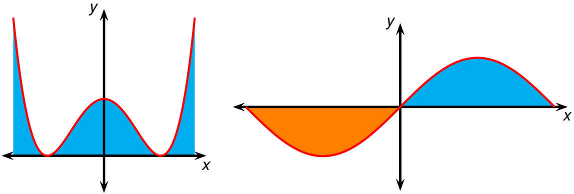

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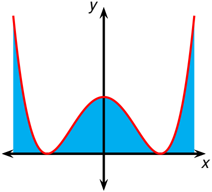

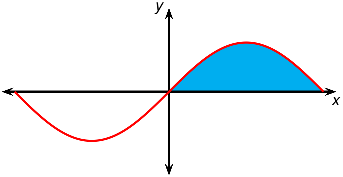

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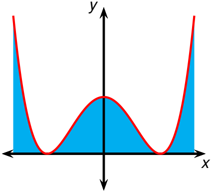

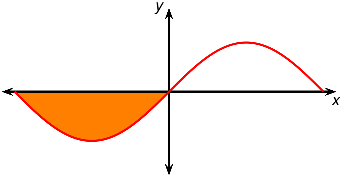

Example

Since $f(x) = x^6 + 1$ satisfies $f(-x) = f(x)$, it is even, and so

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$$\begin{aligned}\int_{-2}^{2} (x^6 + 1)\, \mathrm{d}x &= 2 \int_{0}^{2} (x^6 + 1)\, \mathrm{d}x \\ &= 2 \left[\frac{1}{7} x^7 + x \right]_0^2\end{aligned}$$

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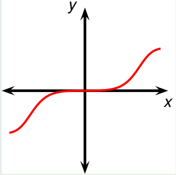
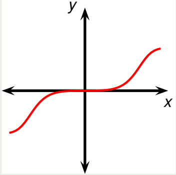
Example



Since $f(x) = \dfrac{\tan x - x}{1 - 2x^2 + 2x^4}$ satisfies $f(-x) = -f(x)$, it is odd, and so

$$\int_{-1}^{1} \frac{\tan x - x}{1 - 2x^2 + 2x^4} \mathrm{d}x =$$

Example



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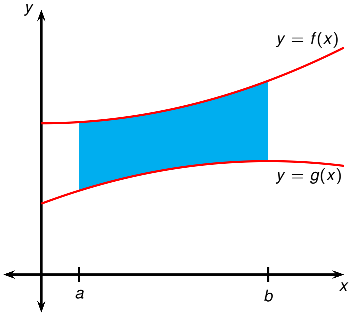
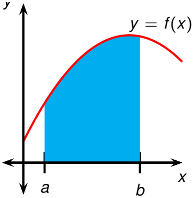
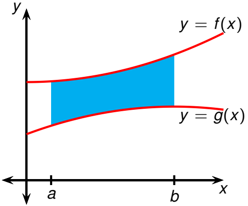
More About Areas

Suppose two curves, $y = f(x)$ and $y = g(x)$, are given.
How do we find the area bounded by those curves between the endpoints $x = a$ and $x = b$?



The Area Under a Curve	The Area Between Two Curves
	

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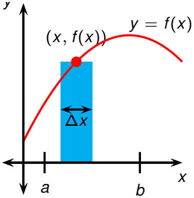
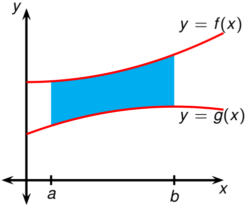


rectangle area = height·width

The Area Between Two Curves



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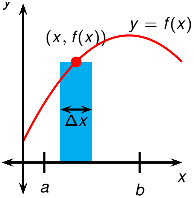

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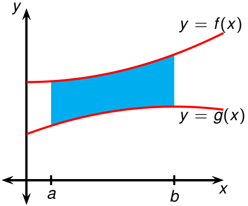

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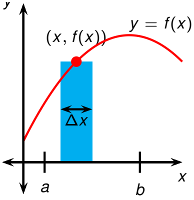

rectangle area $=$ height$\cdot\Delta x$

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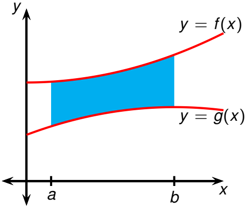

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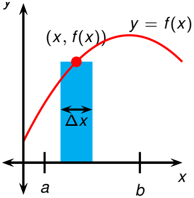

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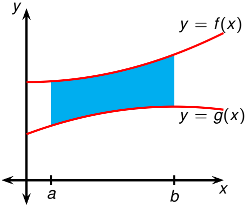

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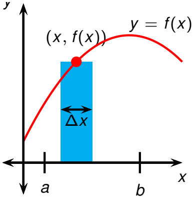

rectangle area $= f(x) \cdot \Delta x$

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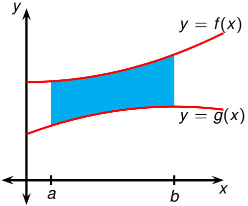

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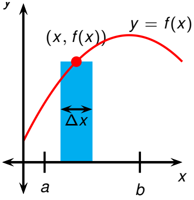

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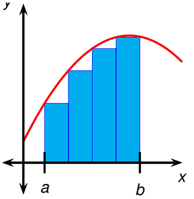

rectangles $= n = 4$

$A = \sum_{i=1}^{4} f(x_i)\Delta x$

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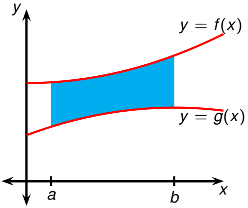

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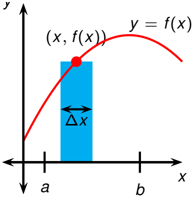

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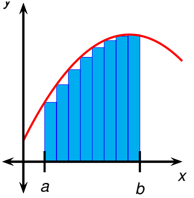

rectangles $= n = 8$

A = $\sum_{i=1}^{8} f(x_i)\Delta x$

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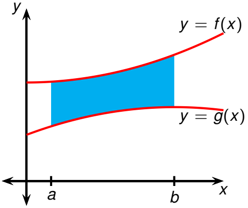

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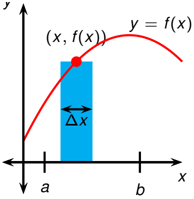

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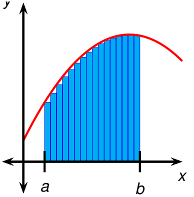

\# rectangles $= n = 16$

A = $\sum_{i=1}^{16} f(x_i)\Delta x$

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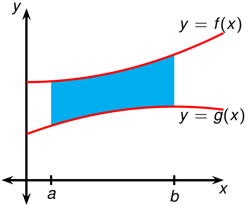

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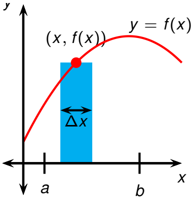

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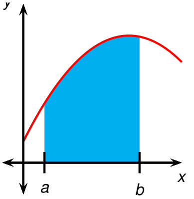

\# rectangles $= n \to \infty$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

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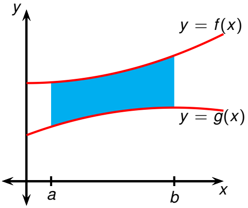

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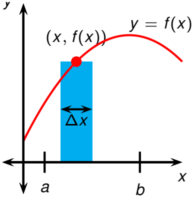

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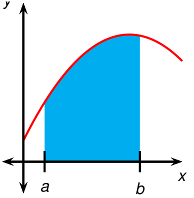

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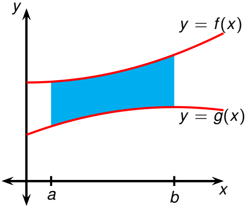

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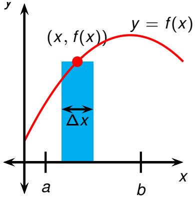

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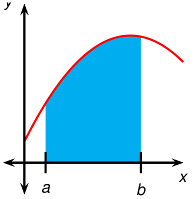

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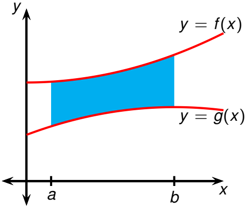

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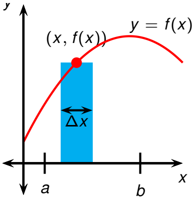

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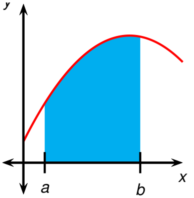

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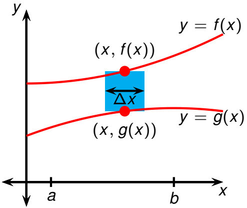

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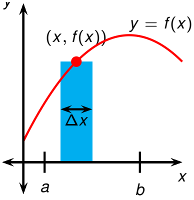

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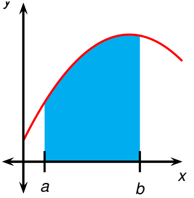

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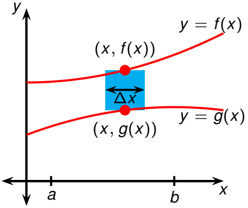
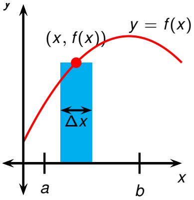
$$A = \int_a^b f(x) dx$$

The Area Between Two Curves

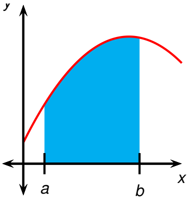


rectangle area $=$ height$\cdot$width

The Area Under a Curve



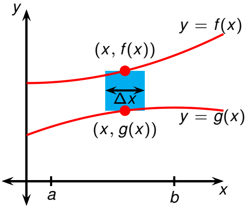
rectangle area $= f(x) \cdot \Delta x$



rectangles $= n \to \infty$

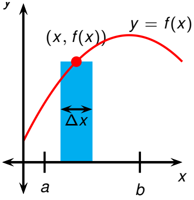
$$A = \int_a^b f(x) dx$$

The Area Between Two Curves

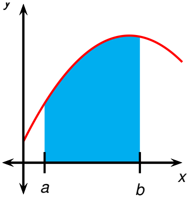


rectangle area $=$ height$\cdot \Delta x$

The Area Under a Curve



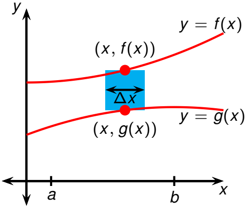
rectangle area $= f(x) \cdot \Delta x$



rectangles $= n \to \infty$

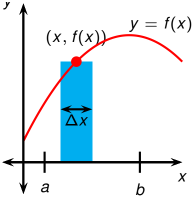
$$A = \int_a^b f(x) dx$$

The Area Between Two Curves

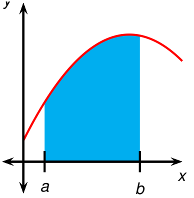


rectangle area $=$ height$\cdot \Delta x$

The Area Under a Curve



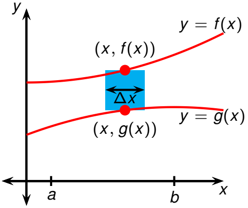
rectangle area $= f(x) \cdot \Delta x$



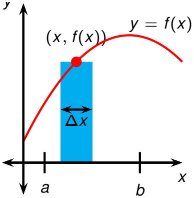
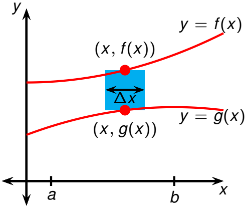
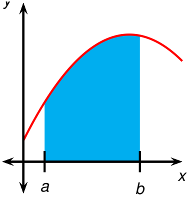
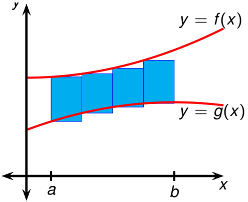
rectangles $= n \to \infty$

$$A = \int_a^b f(x) dx$$

The Area Between Two Curves

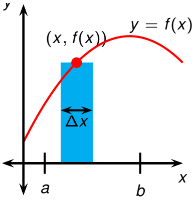


rectangle area $= (f(x) - g(x)) \cdot \Delta x$

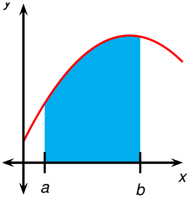
The Area Under a Curve	The Area Between Two Curves
	
rectangle area $= f(x) \cdot \Delta x$	rectangle area $= (f(x) - g(x)) \cdot \Delta x$
	

# rectangles $= n \to \infty$ 	
A = $\int_a^b f(x) dx$
	# rectangles $= n = 4$
	A = $\sum_{i=1}^{4} (f(x_i) - g(x_i)) \Delta x$

The Area Under a Curve



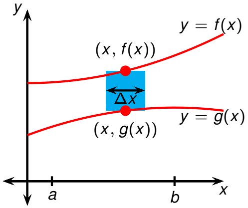
rectangle area $= f(x) \cdot \Delta x$



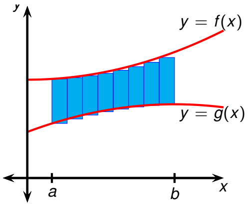
rectangles $= n \to \infty$

A = $\int_a^b f(x) dx$

The Area Between Two Curves



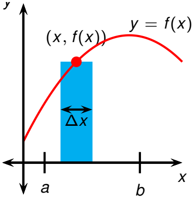
rectangle area $= (f(x) - g(x)) \cdot \Delta x$



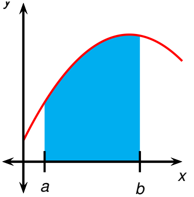
rectangles $= n = 8$

A = $\sum_{i=1}^{8} (f(x_i) - g(x_i)) \Delta x$

The Area Under a Curve



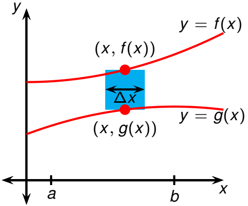
rectangle area $= f(x) \cdot \Delta x$



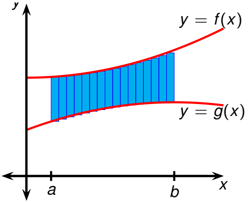
rectangles $= n \to \infty$

$$A = \int_a^b f(x) dx$$

The Area Between Two Curves



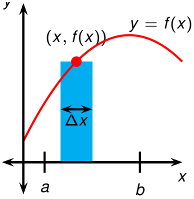
rectangle area $= (f(x) - g(x)) \cdot \Delta x$



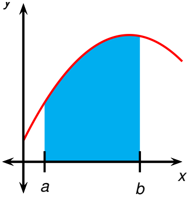
rectangles $= n = 16$

$$A = \sum_{i=1}^{16} (f(x_i) - g(x_i)) \Delta x$$

The Area Under a Curve



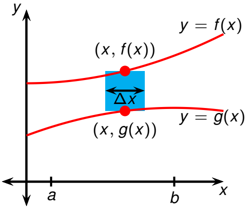
rectangle area $= f(x) \cdot \Delta x$



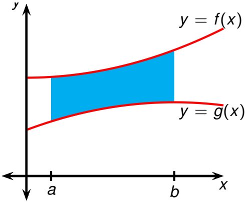
rectangles $= n \to \infty$

$$A = \int_a^b f(x) dx$$

The Area Between Two Curves



rectangle area $= (f(x) - g(x)) \cdot \Delta x$



rectangles $= n \to \infty$

$$A = \int_a^b [f(x) - g(x)] dx$$

Definition (The Area Between Two Curves)

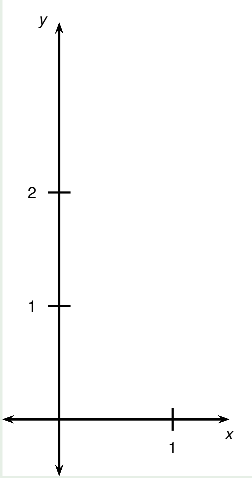
The area between two curves $y = f(x)$ and $y = g(x)$ bounded by the endpoints $x = a$ and $x = b$ is

$$\int_a^b |f(x) - g(x)| \mathrm{d}x.$$

Note that we use the absolute value, because in general we don't know which curve is above the other.

Example

Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



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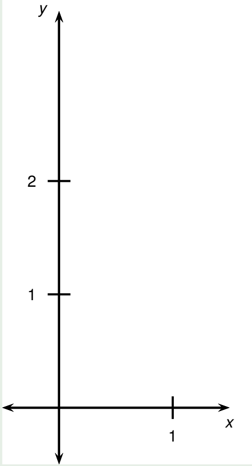
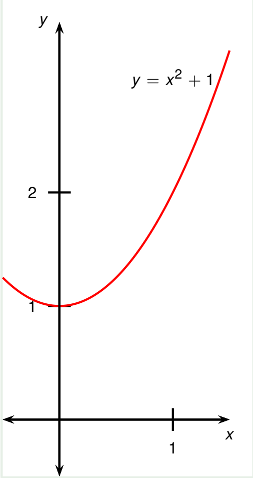


1. Graph the functions.

Example

Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



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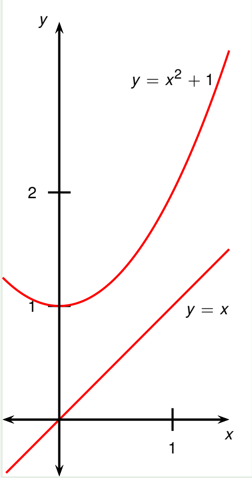

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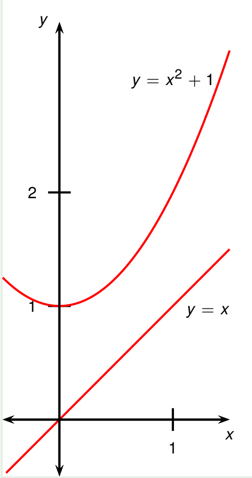
Example

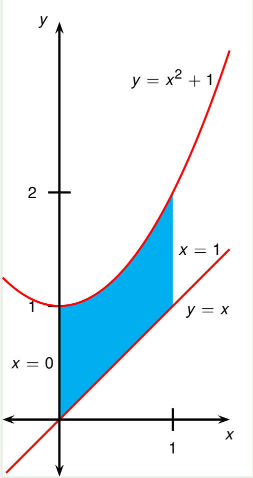
Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



1. Graph the functions.
2. Identify the region.

Example

Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



1. Graph the functions.
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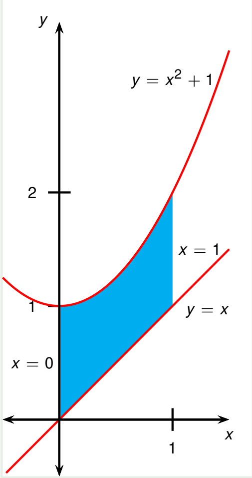
Example

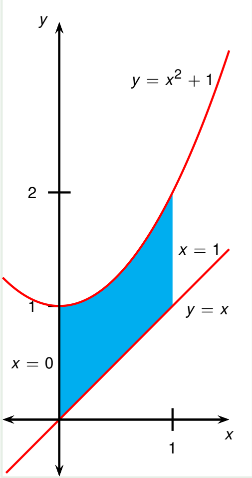
Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



1. Graph the functions.
2. Identify the region.
3. Integrate.

Example

Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.

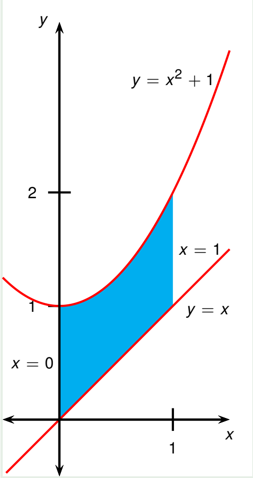


1. Graph the functions.
2. Identify the region.
3. Integrate.

$$
\begin{aligned}
A &= \int_0^1 |(x^2 + 1) - x| \mathrm{d}x \\
&= \int_0^1 (x^2 - x + 1) \mathrm{d}x
\end{aligned}
$$

Example

Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.

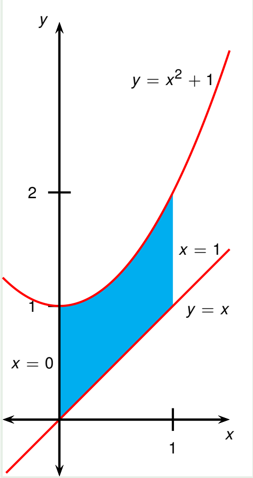


1. Graph the functions.
2. Identify the region.
3. Integrate.

$$A = \int_0^1 |(x^2+1) - x| \mathrm{d}x$$
$$= \int_0^1 (x^2 - x + 1) \mathrm{d}x$$
$$= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^1$$

Example

Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



1. Graph the functions.
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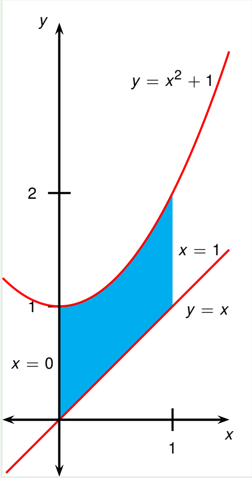
A &= \int_0^1 |(x^2+1) - x| \mathrm{d}x \\
&= \int_0^1 (x^2 - x + 1) \mathrm{d}x \\
&= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^1 \\
&= \frac{1}{3} - \frac{1}{2} + 1
\end{aligned}
$$

Example

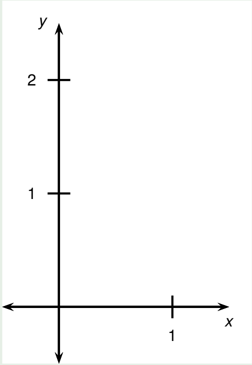
Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



1. Graph the functions.
2. Identify the region.
3. Integrate.

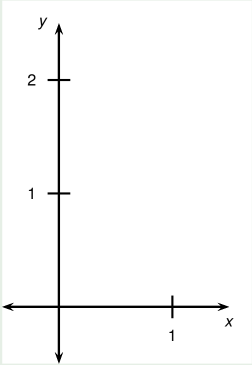
$$
\begin{aligned}
A &= \int_0^1 |(x^2+1) - x| \mathrm{d}x \\
&= \int_0^1 (x^2 - x + 1) \mathrm{d}x \\
&= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^1 \\
&= \frac{1}{3} - \frac{1}{2} + 1 = \frac{5}{6}.
\end{aligned}
$$

Example



Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

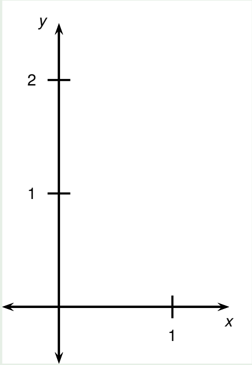
Example



Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

1. Find the point of intersection.

Example

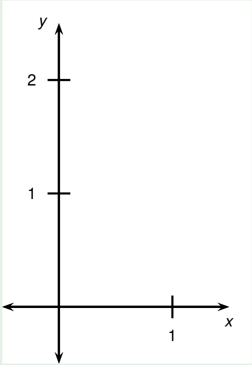


1. Find the point of intersection.

Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

$$x^2 = 2x - x^2$$

Example



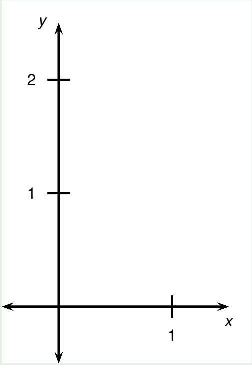
Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

$$x^2 = 2x - x^2$$

$$0 = 2x - 2x^2$$

1. Find the point of intersection.

Example



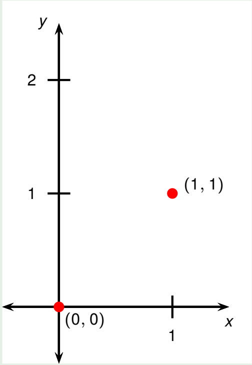
1. Find the point of intersection.

Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

$$x^2 = 2x - x^2$$

$$0 = 2x - 2x^2 = 2x(1 - x)$$

Example



1. Find the point of intersection.

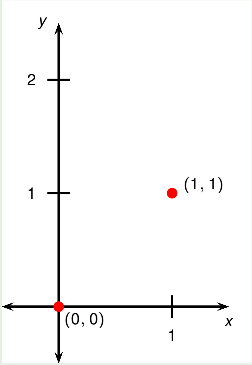
Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

$$x^2 = 2x - x^2$$

$$0 = 2x - 2x^2 = 2x(1 - x)$$

$$x = 0 \text{ or } 1.$$

Example



1. Find the point of intersection.
2. Graph the functions.

Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

$$x^2 = 2x - x^2$$

$$0 = 2x - 2x^2 = 2x(1 - x)$$

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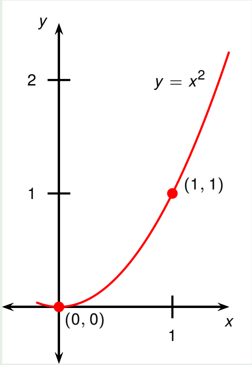
Example



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$$x^2 = 2x - x^2$$
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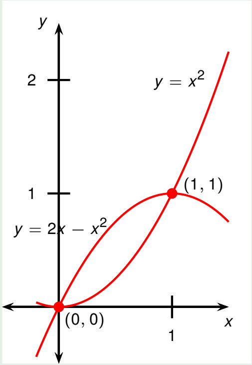
Example



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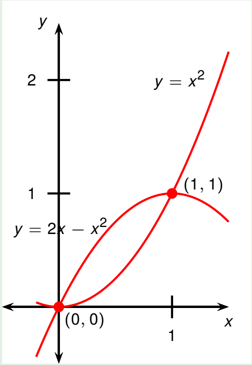
Example



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1. Find the point of intersection.
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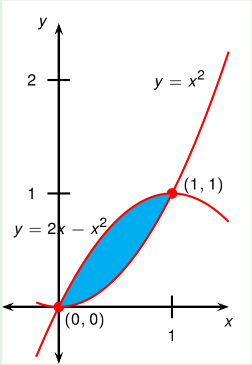
3. Identify the region.

Example



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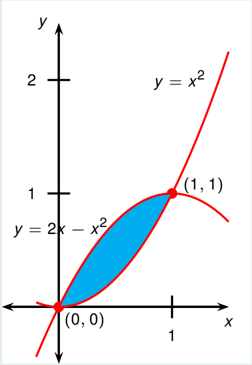
$$x^2 = 2x - x^2$$

$$0 = 2x - 2x^2 = 2x(1 - x)$$

$$x = 0 \text{ or } 1.$$

Example



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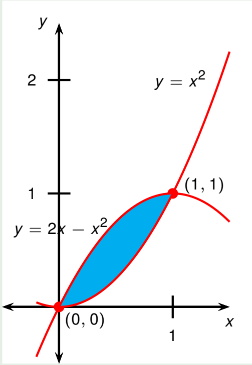
$$0 = 2x - 2x^2 = 2x(1 - x)$$

$$x = 0 \text{ or } 1.$$

1. Find the point of intersection.
2. Graph the functions.
3. Identify the region.
4. Integrate.

Example



1. Find the point of intersection.
2. Graph the functions.
3. Identify the region.
4. Integrate.

Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

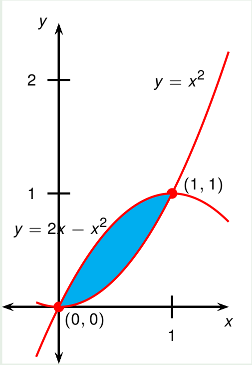
$$x^2 = 2x - x^2$$

$$0 = 2x - 2x^2 = 2x(1 - x)$$

$$x = 0 \text{ or } 1.$$

$$A = \int_0^1 (2x - 2x^2)\mathrm{d}x$$

Example



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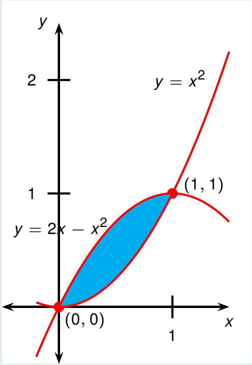
Example



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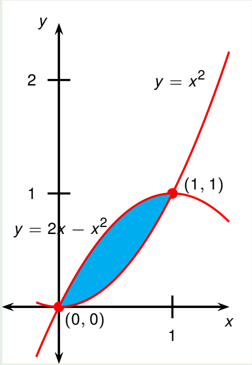

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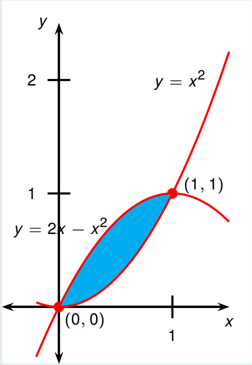

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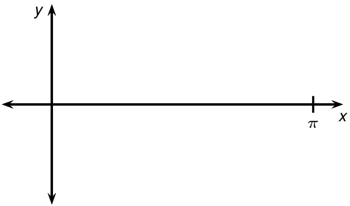
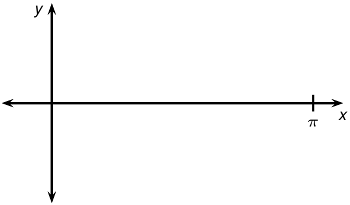
Example



Find the area of the region enclosed by the curves $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \pi/2$.

Example



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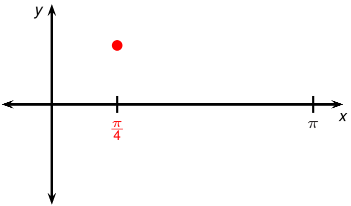

Find the area of the region enclosed by the curves $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \pi/2$.

The only point of intersection in the interval $[0, \pi/2]$ is $(\pi/4, 1/\sqrt{2})$.

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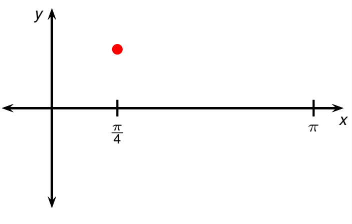
Example



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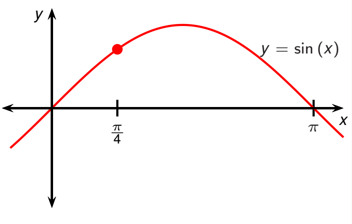
1. Find the point of intersection.
2. Graph the functions.

Example



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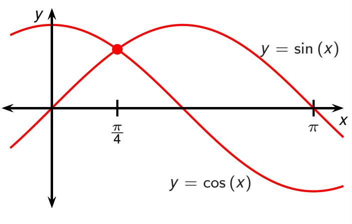

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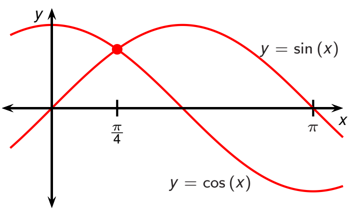

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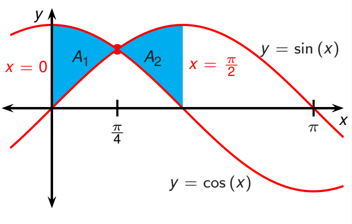

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$A = A_1 + A_2$

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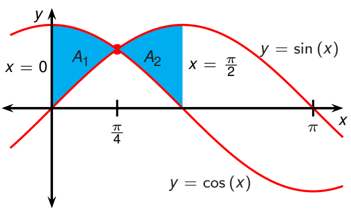
2. Graph the functions.
3. Identify the region.

Example



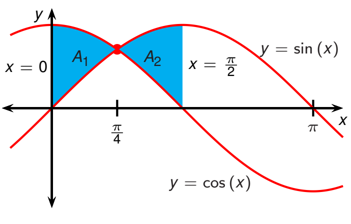
1. Find the point of intersection.
2. Graph the functions.
3. Identify the region.
4. Integrate.

Find the area of the region enclosed by the curves $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \pi/2$.

The only point of intersection in the interval $[0, \pi/2]$ is $(\pi/4, 1/\sqrt{2})$.

$$\begin{aligned} A &= A_1 + A_2 \\ &= \int_0^{\pi/4} (\cos x - \sin x) dx \\ &\quad + \int_{\pi/4}^{\pi/2} (\sin x - \cos x) dx \end{aligned}$$

Example



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$$
\begin{aligned}
A &= A_1 + A_2 \\
&= \int_0^{\pi/4} (\cos x - \sin x) dx \\
&\quad + \int_{\pi/4}^{\pi/2} (\sin x - \cos x) dx \\
&= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}
\end{aligned}
$$

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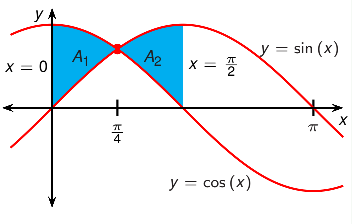

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&\quad + \int_{\pi/4}^{\pi/2} (\sin x - \cos x) dx \\
&= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2} \\
&= 2\sqrt{2} - 2.
\end{aligned}
$$