Precalculus
Trickier logarithmic equations involving quadratics

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- Recall $\lg = \log_{10}$.
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$$\begin{aligned} \lg(x+2)+\lg(x-1) &= 1 \\ \lg\left((x+2)(x-1)\right) &= 1 \qquad \text{Exp. base 10} \\ (x+2)(x-1) &= 10^1 \end{aligned}$$

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Domain: $x > 1$

Exp. base 10

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- $\log_a(st) = \log_a(s) + \log_a(t)$, rule does not hold for negative s, t.
- Check whether answers are in domain of original expression: $\lg(t)$ is not a real number for $t < 0$.