

Calculus II

Integrals of the form $\int \frac{a}{bx + c} dx$

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Building block Ia

Building block Ia: $\int \frac{1}{x} dx$.

Example

Integrate building block Ia

$$\int \frac{1}{x} dx = \ln |x| + C$$

Linear substitutions leading to building block Ia

Building block Ia: $\int \frac{1}{x} dx = \ln |x| + C.$

Example

Integrate

$$\begin{aligned}
 \int \frac{1}{-4x+5} dx &= \int \frac{1}{(-4x+5)} \frac{d(-4x)}{(-4)} \\
 &= \int \frac{1}{(-4x+5)} \frac{d(-4x+5)}{(-4)} && \left| \text{Set } u = -4x+5 \right. \\
 &= \int \frac{1}{u} \frac{du}{(-4)} \\
 &= -\frac{1}{4} \int u^{-1} du = -\frac{1}{4} \ln |u| + C \\
 &= -\frac{1}{4} \ln |-4x+5| + C .
 \end{aligned}$$

Lin. subst. leading to building block 1a: general case

Building block 1a: $\int \frac{1}{x} dx = \ln |x| + C.$

Example

Integrate

$$\begin{aligned}
 \int \frac{1}{ax+b} dx &= \int \frac{1}{(ax+b)} \frac{d(ax)}{a} \\
 &= \int \frac{1}{(ax+b)} \frac{d(ax+b)}{a} && \left| \text{Set } u = ax+b \right. \\
 &= \int \frac{1}{u} \frac{du}{a} \\
 &= \frac{1}{a} \int u^{-1} du = \frac{1}{a} \ln |u| + C \\
 &= \frac{1}{a} \ln |ax+b| + C .
 \end{aligned}$$