

# Calculus II

## Complex multiplication

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2019

Let  $u = 2 + 3i$ ,  $v = 5 - 7i$ .

### Example (Addition)

$$u + v =$$

.

### Example (Subtraction)

$$u - v =$$

.

### Example (Multiplication)

$$u \cdot v =$$

Let  $u = 2 + 3i$ ,  $v = 5 - 7i$ .

### Example (Addition)

$$u + v = (2 + 3i) + (5 - 7i) = ?$$

### Example (Subtraction)

$$u - v =$$

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Let  $u = 2 + 3i$ ,  $v = 5 - 7i$ .

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$$u + v = (2 + 3i) + (5 - 7i) = (2 + 5) + (3 - 7)i = 7 - 4i.$$

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$$u - v = (2 + 3i) - (5 - 7i) = ?$$

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$$u - v = (2 + 3i) - (5 - 7i) = (2 - 5) + (3 - (-7))i = -3 + 10i.$$

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