Calculus I

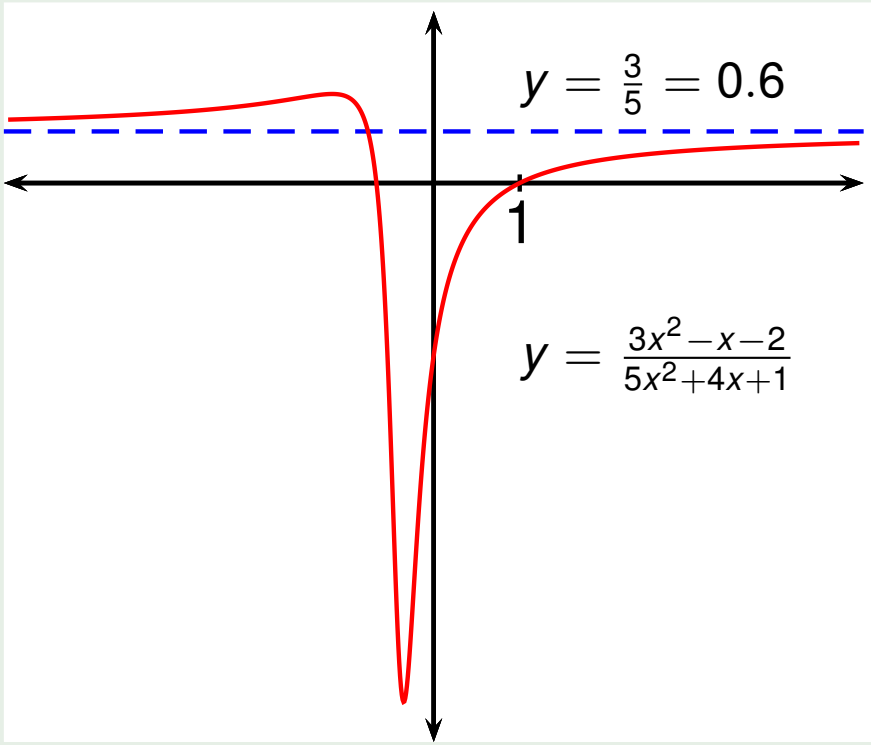
Type 1: $\lim_{x \to \infty}$ of rational function.

Todor Milev

2019

Example

Evaluate $\lim_{x\to\infty} \frac{3x^2-x-2}{5x^2+4x+1}$.



A similar calculation shows that the limit as $x \to -\infty$ is also $\frac{3}{5}$.

Standard approach: divide top and bottom by the highest power of x in the denominator.

$$\lim_{x\to\infty} \frac{(3x^2 - x - 2)}{(5x^2 + 4x + 1)} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{3 - \frac{1}{x} - \frac{2}{x^2}}{5 + \frac{4}{x} + \frac{1}{x^2}}$$

$$= \frac{\lim_{x\to\infty} 3 - \lim_{x\to\infty} \frac{1}{x} - 2 \lim_{x\to\infty} \frac{1}{x^2}}{\lim_{x\to\infty} 5 + 4 \lim_{x\to\infty} \frac{1}{x} + \lim_{x\to\infty} \frac{1}{x^2}}$$

$$= \frac{3 - 0 - 0}{5 + 0 + 0} = \frac{3}{5}$$