# Calculus II Power series expansion of the exponent

**Todor Miley** 

2019

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• To find the radius of convergence, let  $a_n = \frac{x^n}{n!}$ .

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- Therefore  $R = \infty$ .