Precalculus
Double/half angle formulas, theory

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Double angle formulas

Proposition (Double angle formulas)

$$\begin{aligned}
\sin(2\alpha) &= 2\sin\alpha\cos\alpha \\
\cos(2\alpha) &= \cos^2\alpha - \sin^2\alpha \\
&= 2\cos^2\alpha - 1 \\
&= 1 - 2\sin^2\alpha
\end{aligned}$$

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- The double angle formulas play a special role in integration.

Example

Derive the double-angle formulas.

$\sin(2\alpha) =$

$\cos(2\alpha) =$

Example

Derive the double-angle formulas.

$$
\begin{aligned}
\sin(2\alpha) &= \sin(\alpha+\alpha) \\
&= \sin\alpha\cos\alpha + \cos\alpha\sin\alpha \\
&= 2\sin\alpha\cos\alpha \\
\cos(2\alpha) &= \cos(\alpha+\alpha) \\
&= \cos\alpha\cos\alpha - \sin\alpha\sin\alpha \\
&= \cos^2\alpha - \sin^2\alpha \\
&= \cos^2\alpha - (1-\cos^2\alpha) \\
&= 2\cos^2\alpha - 1 \\
&= 1 - \sin^2\alpha - \sin^2\alpha \\
&= 1 - 2\sin^2\alpha
\end{aligned}
$$