

## Precalculus

**Compute the trigonometric functions of an angle not in the first quadrant**

Todor Milev

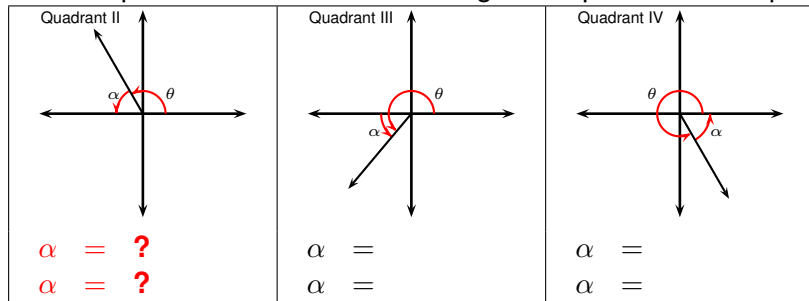
2019

To compute trigonometric functions from obtuse ( $> 90^\circ$ ) or negative angles, we can use the following visual aid.

### Definition (Reference Angle)

Let  $\theta$  be an angle in standard position. Its reference angle is the acute positive angle formed by the terminal arm and the x axis.

The computation of the reference angle  $\alpha$  depends on the quadrant.

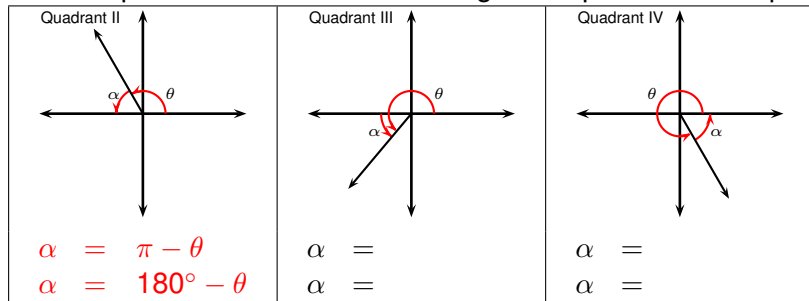


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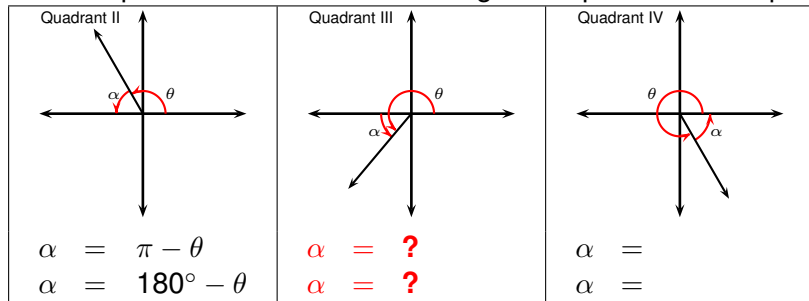


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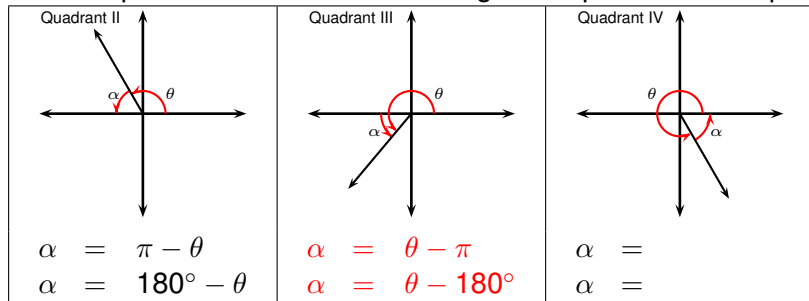


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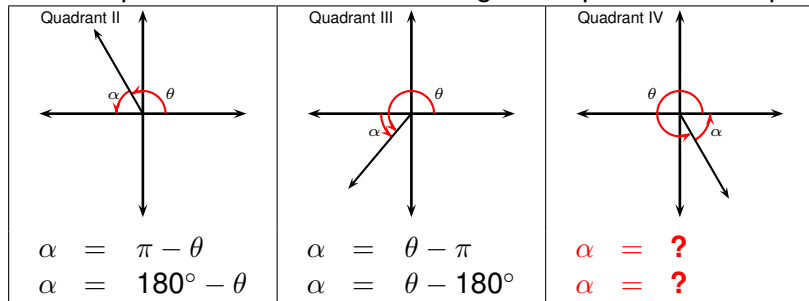


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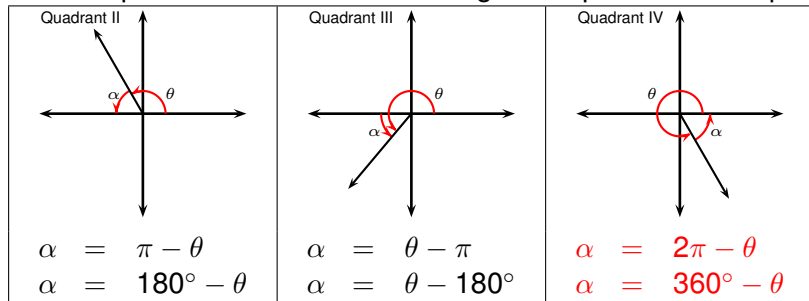


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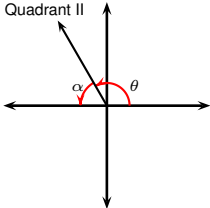
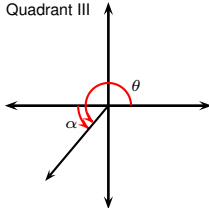
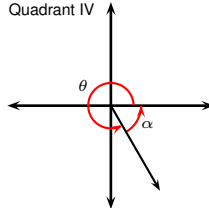
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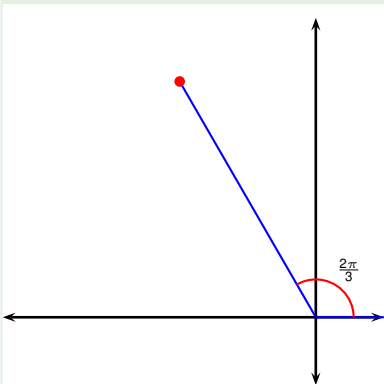
<p>Quadrant II</p>  $\alpha = \pi - \theta$ $\alpha = 180^\circ - \theta$	<p>Quadrant III</p>  $\alpha = \theta - \pi$ $\alpha = \theta - 180^\circ$	<p>Quadrant IV</p>  $\alpha = 2\pi - \theta$ $\alpha = 360^\circ - \theta$
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## Observation

*One can find the value of a trigonometric function of  $\theta$  as follows.*

- *Find the reference angle  $\alpha$  associated to  $\theta$ .*
- *Find the trig function of  $\alpha$ .*
- *Use the quadrant in which  $\theta$  lies to affix an appropriate sign to the function value.*

## Example



Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

$$\sin\left(\frac{2\pi}{3}\right) =$$

$$\cos\left(\frac{2\pi}{3}\right) =$$

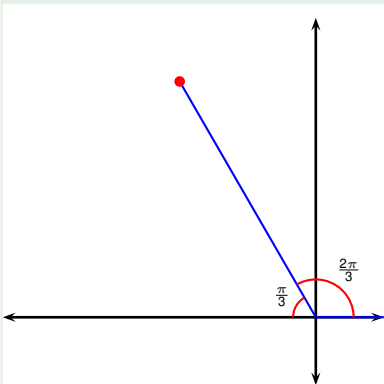
$$\tan\left(\frac{2\pi}{3}\right) =$$

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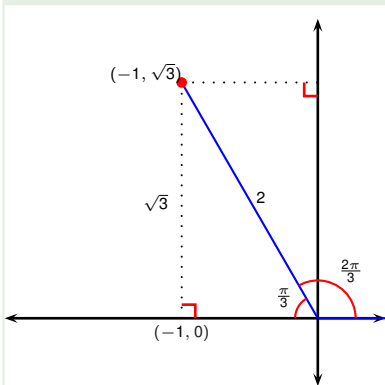
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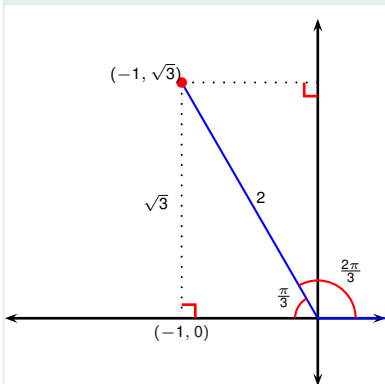
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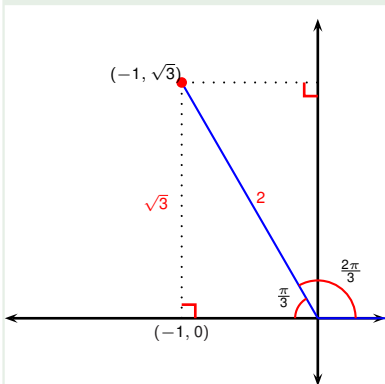
$$\cos\left(\frac{2\pi}{3}\right) =$$

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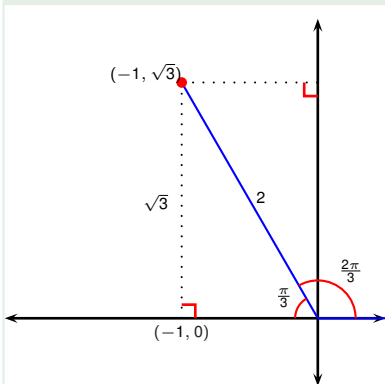
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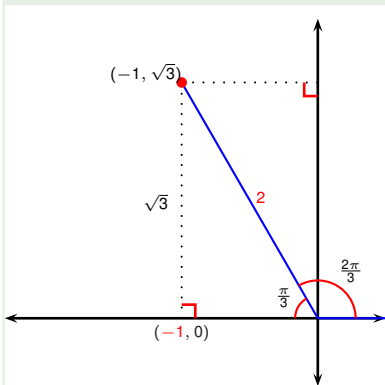
$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \cos\left(\frac{2\pi}{3}\right) = ?$$

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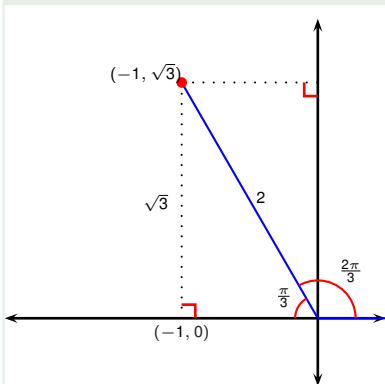
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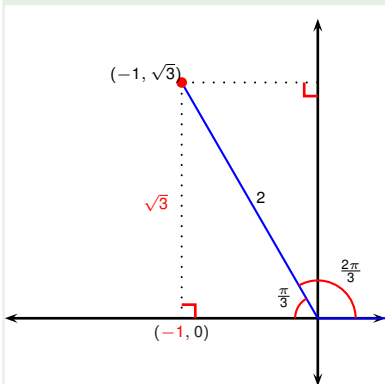
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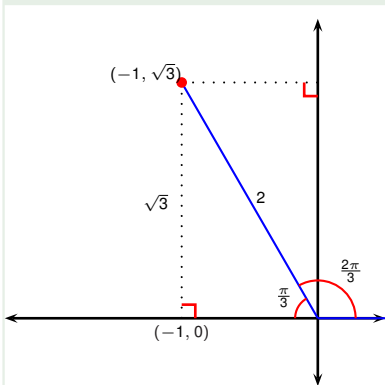
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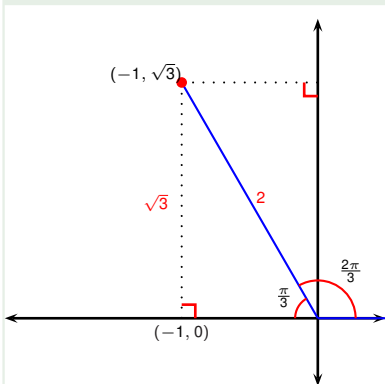
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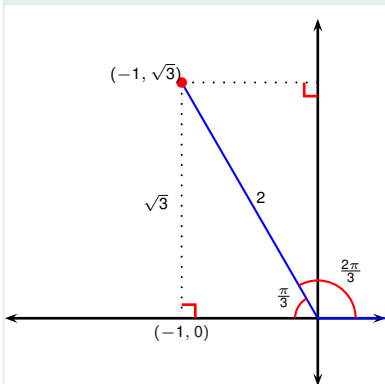
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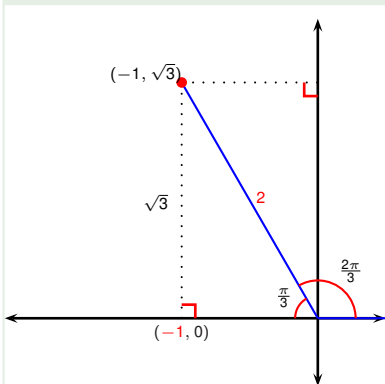
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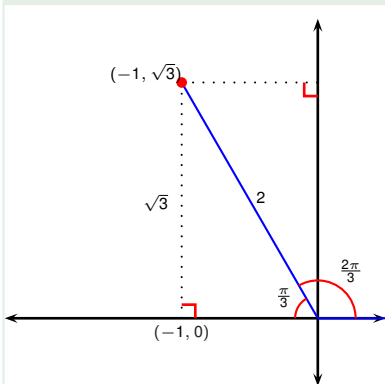


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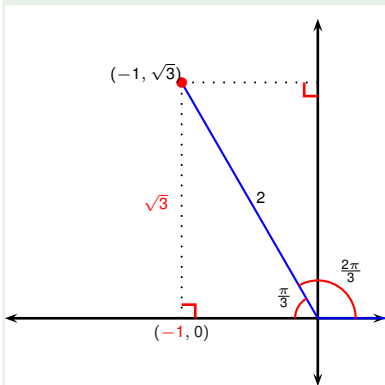


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