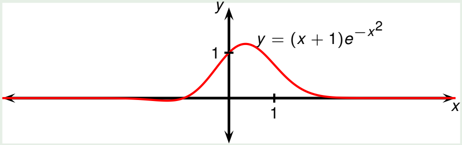
Calculus I
Miscellaneous problems, part 1

Todor Milev

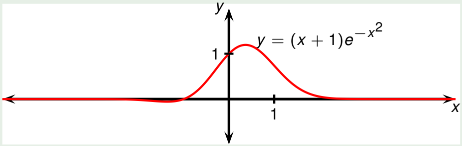
2019

Example



Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

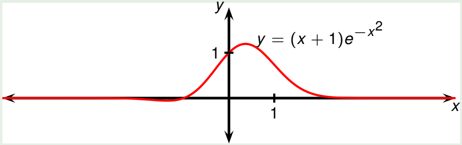
Example



Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right)$$

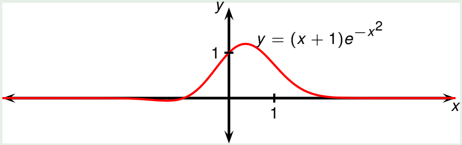
Example



Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = \frac{\mathrm{d}}{\mathrm{d}x}(x+1)e^{-x^2} + (x+1)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right)$$

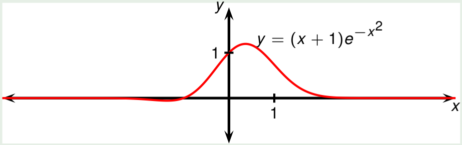
Example



Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = \frac{\mathrm{d}}{\mathrm{d}x}(x+1)e^{-x^2} + (x+1)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right)$$

Example



Find the value of x for which

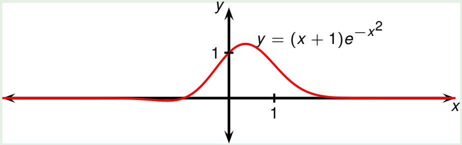
$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = \frac{\mathrm{d}}{\mathrm{d}x}(x+1)e^{-x^2} + (x+1)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right)$$

$$= \mathbf{?} \cdot e^{-x^2} + (x+1)\mathbf{?}$$

Example



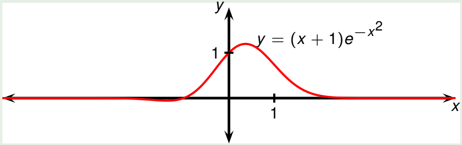
Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}(x+1)e^{-x^2} + (x+1)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right)\\ &= 1\cdot e^{-x^2} + (x+1)\mathbf{?}\end{aligned}$$

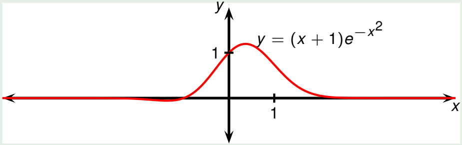
Example



Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}(x+1)e^{-x^2} + (x+1)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right)\\ &= 1\cdot e^{-x^2} + (x+1)\mathbf{?}\end{aligned}$$

Example



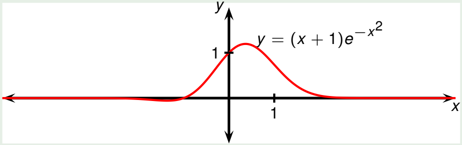
Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}(x+1)e^{-x^2} + (x+1)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right)\\ &= 1\cdot e^{-x^2} + (x+1)\,e^{-x^2}\left(-x^2\right)'\end{aligned}$$

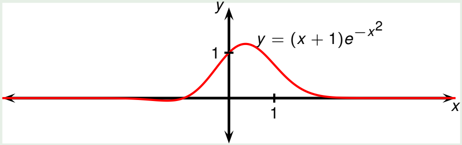
Example



Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}(x+1)e^{-x^2} + (x+1)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right)\\ &= 1\cdot e^{-x^2} + (x+1)\,e^{-x^2}\left(-x^2\right)'\\ &= 1\cdot e^{-x^2} + (x+1)e^{-x^2}(\textbf{?} \quad)\end{aligned}$$

Example



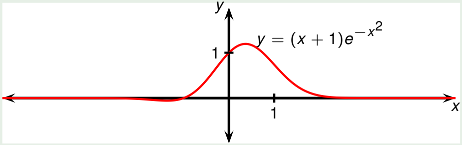
Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}(x+1)e^{-x^2} + (x+1)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right)\\
&= 1\cdot e^{-x^2} + (x+1)\,e^{-x^2}\left(-x^2\right)'\\
&= 1\cdot e^{-x^2} + (x+1)e^{-x^2}(-2x)
\end{aligned}$$

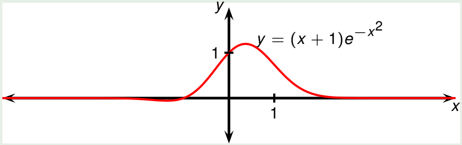
Example



Find the value of x for which
$$f(x)=(x+1)e^{-x^2}$$
attains its maximum in the interval $[-5,5]$. Use the given plot.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}(x+1)e^{-x^2}+(x+1)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right)\\ &= 1\cdot e^{-x^2}+(x+1)\,e^{-x^2}\left(-x^2\right)'\\ &= 1\cdot {\color{red}e^{-x^2}}+(x+1){\color{red}e^{-x^2}}(-2x)\\ &= (1+(x+1)(-2x))\,{\color{red}e^{-x^2}}\end{aligned}$$

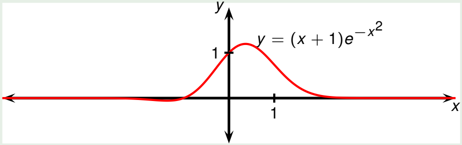
Example



Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}(x+1)e^{-x^2} + (x+1)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right)\\ &= 1\cdot e^{-x^2} + (x+1)\,e^{-x^2}\left(-x^2\right)'\\ &= 1\cdot e^{-x^2} + (x+1)e^{-x^2}(-2x)\\ &= (1+(x+1)(-2x))\,e^{-x^2}\end{aligned}$$

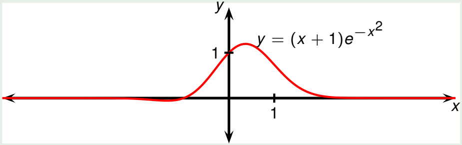
Example



Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}(x+1)e^{-x^2} + (x+1)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right)\\ &= 1\cdot e^{-x^2} + (x+1)\,e^{-x^2}\left(-x^2\right)'\\ &= 1\cdot e^{-x^2} + (x+1)e^{-x^2}(-2x)\\ &= (1+(x+1)(-2x))\,e^{-x^2} = \left(-2x^2-2x+1\right)e^{-x^2}\end{aligned}$$

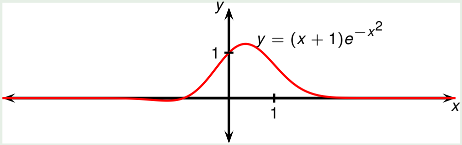
Example



Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5,5]$. Use the given plot.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}(x+1)e^{-x^2} + (x+1)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right)\\ &= 1\cdot e^{-x^2} + (x+1)\,e^{-x^2}\left(-x^2\right)'\\ &= 1\cdot e^{-x^2} + (x+1)e^{-x^2}(-2x)\\ &= (1+(x+1)(-2x))\,e^{-x^2} = \left(-2x^2-2x+1\right)e^{-x^2}\end{aligned}$$

Example



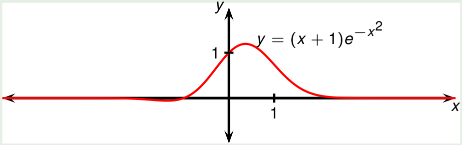
Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}(x+1)e^{-x^2} + (x+1)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right)\\
&= 1\cdot e^{-x^2} + (x+1)\,e^{-x^2}\left(-x^2\right)'\\
&= 1\cdot e^{-x^2} + (x+1)e^{-x^2}(-2x)\\
&= (1+(x+1)(-2x))\,e^{-x^2} = \left(-2x^2-2x+1\right)e^{-x^2}
\end{aligned}$$

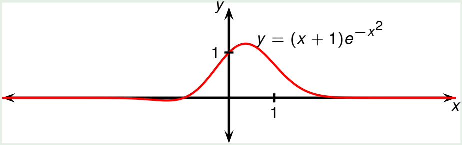
Example



Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

Example



Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

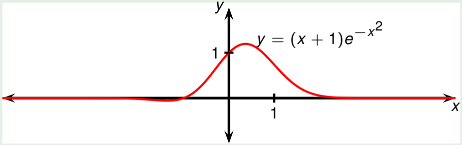
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1+(x+1)(-2x))\,e^{-x^2} = \left(-2x^2-2x+1\right)e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

$$(-2x^2-2x+1)e^{-x^2} = 0$$

Example



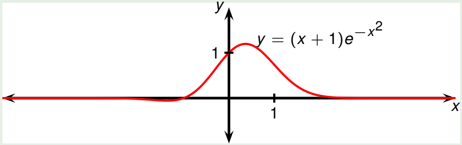
Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = \left(1+(x+1)(-2x)\right)e^{-x^2} = \left(-2x^2-2x+1\right)e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

$$(-2x^2-2x+1)e^{-x^2} = 0 \qquad \Big|\,\text{Div. by } e^{-x^2}$$

Example



Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

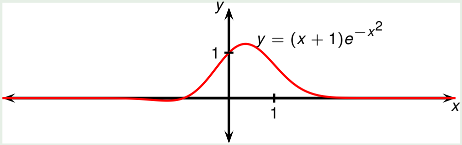
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1+(x+1)(-2x))\,e^{-x^2} = \left(-2x^2-2x+1\right)\,e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

$$(-2x^2-2x+1)e^{-x^2} = 0 \qquad \Big| \text{Div. by } e^{-x^2} \neq 0$$

Example



Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

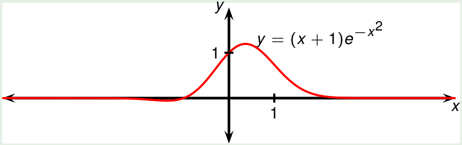
$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = \left(1 + (x+1)(-2x)\right)e^{-x^2} = \left(-2x^2 - 2x + 1\right)e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

$$(-2x^2 - 2x + 1)e^{-x^2} = 0 \qquad \Big| \text{Div. by } e^{-x^2} \neq 0$$

$$-2x^2 - 2x + 1 = 0$$

Example



Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

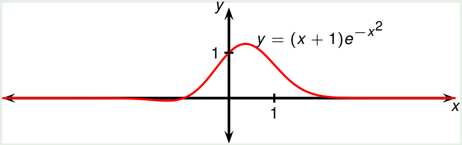
$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

$$\begin{aligned}(-2x^2 - 2x + 1)e^{-x^2} &= 0 && \Big| \text{Div. by } e^{-x^2} \neq 0\\ -2x^2 - 2x + 1 &= 0\end{aligned}$$

$$x_1, x_2 = \textbf{?}$$

Example



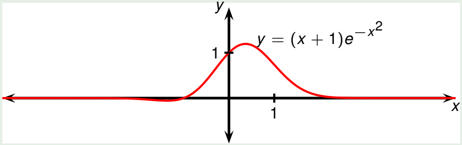
Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

$$(-2x^2 - 2x + 1)e^{-x^2} = 0 \qquad \Big| \text{Div. by } e^{-x^2} \neq 0$$
$$-2x^2 - 2x + 1 = 0$$
$$x_1, x_2 = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-2)\cdot 1}}{2(-2)}$$

Example



Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

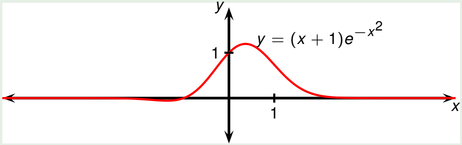
Find critical points: set $f'(x) = 0$ and solve for x:

$$(-2x^2 - 2x + 1)e^{-x^2} = 0 \qquad \Big| \text{Div. by } e^{-x^2} \neq 0$$

$$-2x^2 - 2x + 1 = 0$$

$$x_1, x_2 = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-2)\cdot 1}}{2(-2)}$$

Example



Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

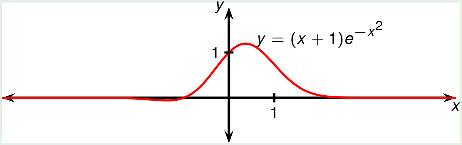
Find critical points: set $f'(x) = 0$ and solve for x:

$$(-2x^2 - 2x + 1)e^{-x^2} = 0 \qquad \Big| \text{Div. by } e^{-x^2} \neq 0$$

$$-2x^2 - 2x + 1 = 0$$

$$x_1, x_2 = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-2)\cdot 1}}{2(-2)}$$

Example



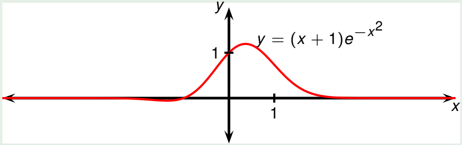
Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

$$\begin{aligned}(-2x^2 - 2x + 1)e^{-x^2} &= 0 && \Big|\text{Div. by } e^{-x^2} \neq 0\\ -2x^2 - 2x + 1 &= 0\\ x_1, x_2 &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-2)\cdot 1}}{2(-2)}\end{aligned}$$

Example



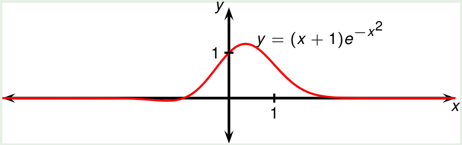
Find the value of x for which
$$f(x)=(x+1)e^{-x^2}$$
attains its maximum in the interval $[-5,5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right)=(1+(x+1)(-2x))\,e^{-x^2}=\left(-2x^2-2x+1\right)e^{-x^2}$$

Find critical points: set $f'(x)=0$ and solve for x:

$$\begin{aligned}(-2x^2-2x+1)e^{-x^2}&=0 && \Big|\text{Div. by } e^{-x^2}\neq 0\\ -2x^2-2x+1&=0\\ x_1,x_2&=\frac{-(-2)\pm\sqrt{(-2)^2-4(-2)\cdot 1}}{2(-2)}\\ &=\frac{2\pm\sqrt{12}}{-4}\end{aligned}$$

Example



Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

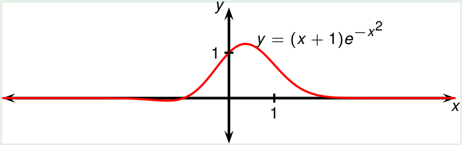
$$(-2x^2 - 2x + 1)e^{-x^2} = 0 \qquad \Big| \text{Div. by } e^{-x^2} \neq 0$$

$$-2x^2 - 2x + 1 = 0$$

$$x_1, x_2 = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-2)\cdot 1}}{2(-2)}$$

$$= \frac{2 \pm \sqrt{12}}{-4}$$

Example



Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

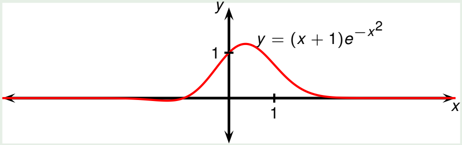
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = \left(1 + (x+1)(-2x)\right)e^{-x^2} = \left(-2x^2 - 2x + 1\right)e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

$$\begin{aligned}
(-2x^2 - 2x + 1)e^{-x^2} &= 0 && \Big| \text{Div. by } e^{-x^2} \neq 0 \\
-2x^2 - 2x + 1 &= 0 \\
x_1, x_2 &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-2)\cdot 1}}{2(-2)} \\
&= \frac{2 \pm \sqrt{12}}{-4} = \frac{-2 \mp 2\sqrt{3}}{4}
\end{aligned}$$

Example



Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

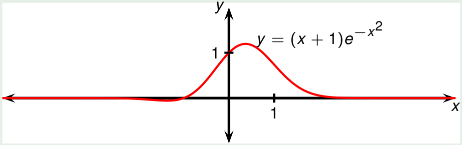
$$(-2x^2 - 2x + 1)e^{-x^2} = 0 \qquad \Big| \text{Div. by } e^{-x^2} \neq 0$$

$$-2x^2 - 2x + 1 = 0$$

$$x_1, x_2 = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-2)\cdot 1}}{2(-2)}$$

$$= \frac{2 \pm \sqrt{12}}{-4} = \frac{-2 \mp 2\sqrt{3}}{4}$$

Example



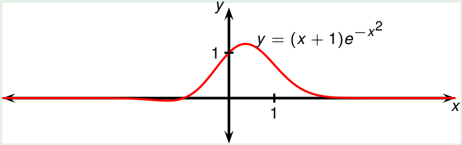
Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1+(x+1)(-2x))\,e^{-x^2} = \left(-2x^2-2x+1\right)\,e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

$$(-2x^2-2x+1)e^{-x^2} = 0 \qquad \Big|\text{Div. by } e^{-x^2} \neq 0$$
$$-2x^2-2x+1 = 0$$
$$x_1, x_2 = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-2)\cdot 1}}{2(-2)}$$
$$= \frac{2 \pm \sqrt{12}}{-4} = \frac{-2 \mp 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

Example



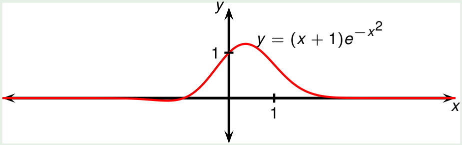
Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

$$(-2x^2 - 2x + 1)e^{-x^2} = 0 \qquad \Big| \text{Div. by } e^{-x^2} \neq 0$$
$$-2x^2 - 2x + 1 = 0$$
$$x_1, x_2 = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-2)\cdot 1}}{2(-2)}$$
$$= \frac{2 \pm \sqrt{12}}{-4} = \frac{-2 \mp 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

Example



Find the value of x for which

$$f(x)=(x+1)e^{-x^2}$$

attains its maximum in the interval $[-5,5]$. Use the given plot.

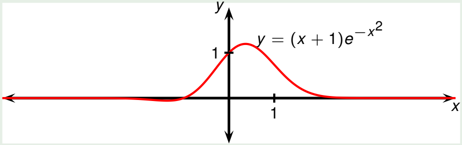
$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right)=(1+(x+1)(-2x))\,e^{-x^2}=\left(-2x^2-2x+1\right)\,e^{-x^2}$$

Find critical points: set $f'(x)=0$ and solve for x:

$$(-2x^2-2x+1)e^{-x^2}=0 \qquad \Big| \text{Div. by } e^{-x^2}\neq 0$$

$$x_1,x_2=\frac{2\pm\sqrt{12}}{-4}=\frac{-2\mp 2\sqrt{3}}{4}=\frac{-1\pm\sqrt{3}}{2}$$

Example



Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

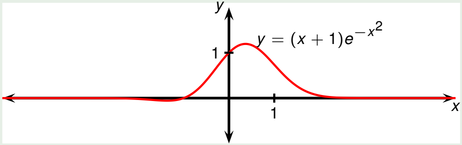
$$(-2x^2 - 2x + 1)e^{-x^2} = 0 \qquad \Big| \text{Div. by } e^{-x^2} \neq 0$$

$$x_1, x_2 = \frac{2 \pm \sqrt{12}}{-4} = \frac{-2 \mp 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

Compare the values of f at the endpoints and the critical points:

x	$f(x)$
-5	
$\frac{-1-\sqrt{3}}{2}$	
$\frac{-1+\sqrt{3}}{2}$	
5	

Example



Find the value of x for which
$$f(x) = (x+1)e^{-x^2}$$
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{d}{dx}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

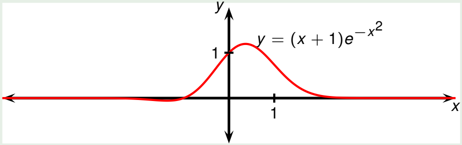
$$(-2x^2 - 2x + 1)e^{-x^2} = 0 \qquad \Big| \text{Div. by } e^{-x^2} \neq 0$$

$$x_1, x_2 = \frac{2 \pm \sqrt{12}}{-4} = \frac{-2 \mp 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

Compare the values of f at the endpoints and the critical points:

x	$f(x)$
-5	
$\frac{-1-\sqrt{3}}{2}$	
$\frac{-1+\sqrt{3}}{2}$	
5	

Example



Find the value of x for which
$$f(x)=(x+1)e^{-x^2}$$
attains its maximum in the interval $[-5,5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right)=(1+(x+1)(-2x))\,e^{-x^2}=\left(-2x^2-2x+1\right)e^{-x^2}$$

Find critical points: set $f'(x)=0$ and solve for x:

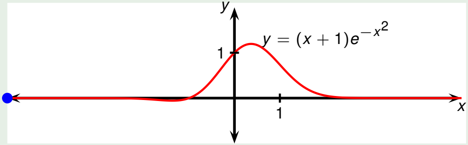
$(-2x^2-2x+1)e^{-x^2}=0$ $\qquad$ |Div. by $e^{-x^2}\neq 0$

$$x_1,x_2=\frac{2\pm\sqrt{12}}{-4}=\frac{-2\mp 2\sqrt{3}}{4}=\frac{-1\pm\sqrt{3}}{2}$$

Compare the values of f at the endpoints and the critical points:

x	$f(x)$
-5	
$\frac{-1-\sqrt{3}}{2}$	
$\frac{-1+\sqrt{3}}{2}$	
5	

Example



Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

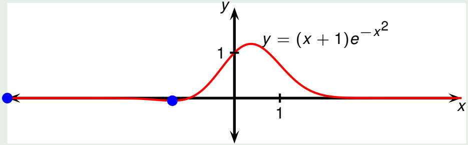
$$(-2x^2 - 2x + 1)e^{-x^2} = 0 \qquad \Big| \text{Div. by } e^{-x^2} \neq 0$$

$$x_1, x_2 = \frac{2 \pm \sqrt{12}}{-4} = \frac{-2 \mp 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

Compare the values of f at the endpoints and the critical points:

x	$f(x)$
-5	close to 0 from plot
$\frac{-1-\sqrt{3}}{2}$	
$\frac{-1+\sqrt{3}}{2}$	
5	

Example



Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

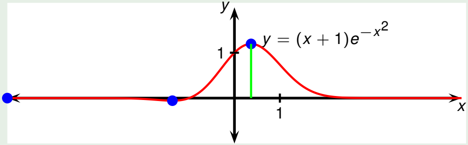
$$(-2x^2 - 2x + 1)e^{-x^2} = 0 \qquad \Big|\text{Div. by } e^{-x^2} \neq 0$$

$$x_1, x_2 = \frac{2 \pm \sqrt{12}}{-4} = \frac{-2 \mp 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

Compare the values of f at the endpoints and the critical points:

x	$f(x)$
-5	close to 0 from plot
$\frac{-1-\sqrt{3}}{2}$	negative, min from plot
$\frac{-1+\sqrt{3}}{2}$	
5	

Example



Find the value of x for which
$$f(x)=(x+1)e^{-x^2}$$
attains its maximum in the interval $[-5,5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right)=(1+(x+1)(-2x))\,e^{-x^2}=\left(-2x^2-2x+1\right)\,e^{-x^2}$$

Find critical points: set $f'(x)=0$ and solve for x:

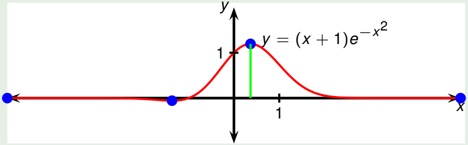
$$(-2x^2-2x+1)e^{-x^2}=0 \qquad \Big| \text{Div. by } e^{-x^2}\neq 0$$

$$x_1,x_2=\frac{2\pm\sqrt{12}}{-4}=\frac{-2\mp 2\sqrt{3}}{4}=\frac{-1\pm\sqrt{3}}{2}$$

Compare the values of f at the endpoints and the critical points:

x	$f(x)$
-5	close to 0 from plot
$\frac{-1-\sqrt{3}}{2}$	negative, min from plot
$\frac{-1+\sqrt{3}}{2}$	positive, max from plot
5	

Example



Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\,e^{-x^2} = \left(-2x^2 - 2x + 1\right)\,e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

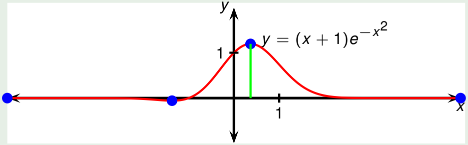
$(-2x^2 - 2x + 1)e^{-x^2} = 0$ | Div. by $e^{-x^2} \neq 0$

$$x_1, x_2 = \frac{2 \pm \sqrt{12}}{-4} = \frac{-2 \mp 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

Compare the values of f at the endpoints and the critical points:

x	$f(x)$
-5	close to 0 from plot
$\frac{-1-\sqrt{3}}{2}$	negative, min from plot
$\frac{-1+\sqrt{3}}{2}$	positive, max from plot
5	close to 0 from plot

Example



Find the value of x for which
$$f(x)=(x+1)e^{-x^2}$$
attains its maximum in the interval $[-5,5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right)=(1+(x+1)(-2x))\,e^{-x^2}=\left(-2x^2-2x+1\right)\,e^{-x^2}$$

Find critical points: set $f'(x)=0$ and solve for x:

$$(-2x^2-2x+1)e^{-x^2}=0 \qquad \Big| \text{Div. by } e^{-x^2}\neq 0$$

$$x_1,x_2=\frac{2\pm\sqrt{12}}{-4}=\frac{-2\mp2\sqrt{3}}{4}=\frac{-1\pm\sqrt{3}}{2}$$

Compare the values of f at the endpoints and the critical points:

x	$f(x)$
-5	close to 0 from plot
$\frac{-1-\sqrt{3}}{2}$	negative, min from plot
Final answer: $\frac{-1+\sqrt{3}}{2}$	positive, max from plot
5	close to 0 from plot