Precalculus
Find extremum of quadratic, text problem.

Todor Milev

2019

Maximum or minimum value of a quadratic function

- Let $f(x) = ax^2 + bx + c$ - quadratic ($a \neq 0$).
- Let D be the discriminant $D = b^2 - 4ac$.

$$f(x) = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a} \quad \Bigg| \text{ complete the square}$$

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Recall $f(x) = ax^2 + bx + c = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a}$.

Proposition

Let $f(x) = ax^2 + bx + c$, $a \neq 0$ and let $D = b^2 - 4ac$.

- *If $a > 0$ then $f(x)$ has no maximum and has minimum at $x = -\frac{b}{2a}$.*
- *If $a < 0$ then $f(x)$ has no minimum and has maximum at $x = -\frac{b}{2a}$.*
- *In both cases, the extremal value (either maximum or minimum) is $f\left(-\frac{b}{2a}\right) = -\frac{b^2-4ac}{4a} = -\frac{D}{4a}$.*

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Let $f(x) = ax^2 + bx + c$, $a \neq 0$ and let $D = b^2 - 4ac$.

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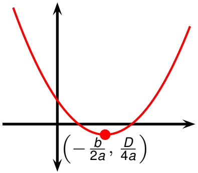

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Let $f(x) = ax^2 + bx + c$, $a \neq 0$ and let $D = b^2 - 4ac$.

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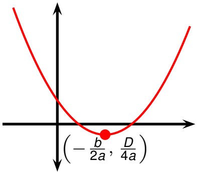

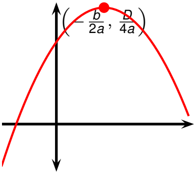

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.

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Maximizing:

$$xz$$

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$$x + z = 12$$

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Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.

$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

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$$xz = x(12 - x)$$

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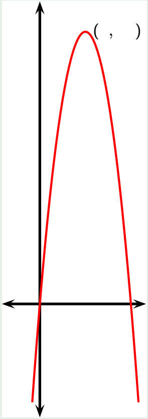

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Parabola opens down

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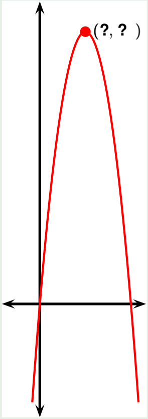

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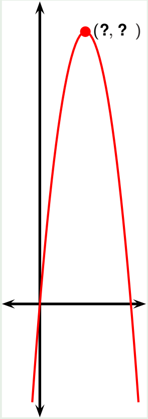

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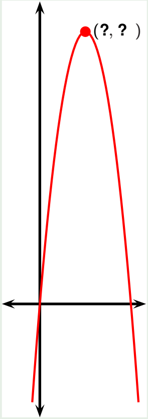

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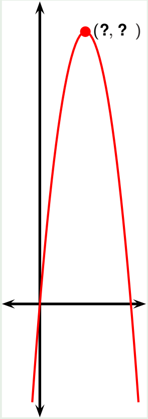

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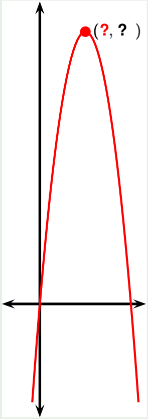

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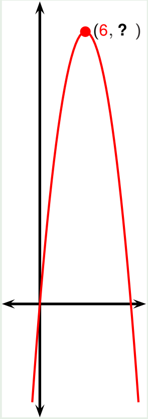

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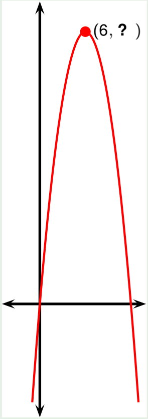

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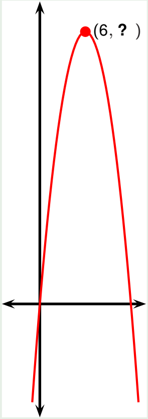

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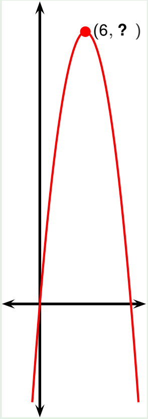

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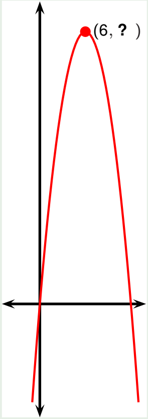

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Max. product $= xz$

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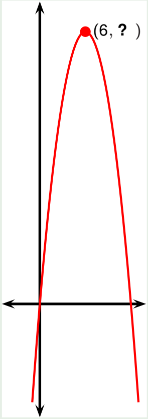

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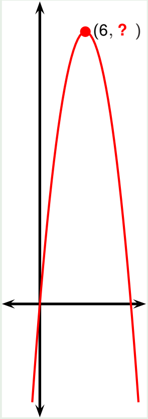

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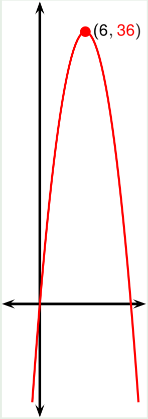

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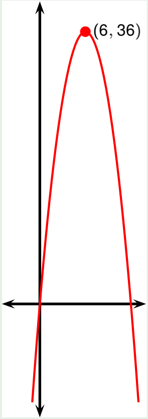

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