Calculus I
Derivative of a power of expression

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- In the example $y = \cos^3 x$, the outer function was a power function: $y = u^3$.
- The derivative was $\frac{dy}{dx} = 3u^2 \frac{du}{dx} = (3\cos^2 x)(-\sin x)$.
- We can generalize this:

Observation (The Power Rule Combined with the Chain Rule)

If n is any real number and $u = h(x)$ is differentiable, then

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

Alternatively,

$$\frac{d}{dx}[h(x)]^n = n[h(x)]^{n-1} \cdot h'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(u^n) = nu^{n-1}\frac{\mathrm{d}u}{\mathrm{d}x}$$

$$\begin{aligned}
(g(h(x)))' &= g'(h(x)) \cdot h'(x) && && \text{(notation 1)}\\
(g(u))' &= g'(u)u' && \text{where } u = h(x) && \text{(notation 2)}\\
\frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} && \text{where } y = g(u) && \text{(notation 3)}\quad .
\end{aligned}$$

Example (Chain Rule, Notation 3, Power Rule)

$$\begin{aligned}
\text{Differentiate} \quad y &= (x^3-1)^{100}.\\
\text{Let} \quad u &= \\
\text{Then} \quad y &= \\
\text{Chain Rule:} \quad \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x}\\
&= \left(\qquad\right)\left(\quad\right)\\
&=
\end{aligned}$$