

Calculus II

Definition of complex numbers

Todor Milev

2019

Definition (Complex numbers)

The set of complex numbers \mathbb{C} is defined as the set

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where the number i is a number for which

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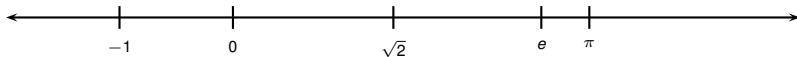
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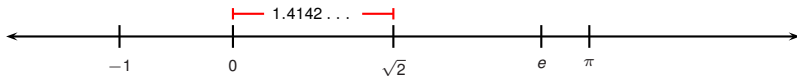
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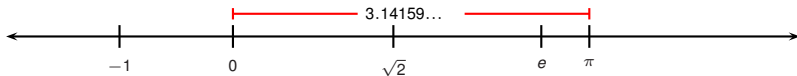
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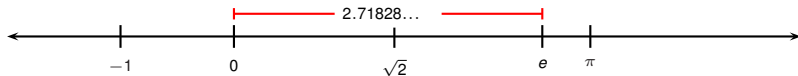
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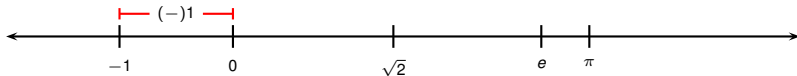
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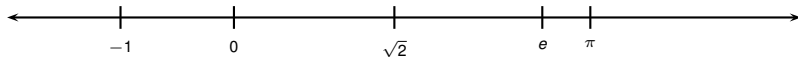
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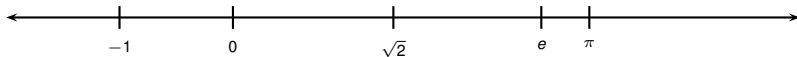
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- Geometric interpretation of complex numbers: beyond our scope.