# Precalculus
# Solve triangle from two sides and an angle

Todor Milev

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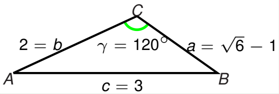
## Example




The longest side of a triangle has length 3 and the angle opposite to it is $120^\circ$. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$
\begin{aligned}
a^2+b^2-2ab\cos\gamma &= c^2 && \text{Law of cosines}\\
a^2+2^2-2a\cdot 2\cdot\cos 120^\circ &= 3^2 && \text{Solve for } a:\\
a^2-4a\left(-\frac{1}{2}\right)-5 &= 0\\
a^2+2a-5 &= 0\\
a &= \frac{-2\pm\sqrt{2^2-4\cdot(-5)\cdot 1}}{2\cdot 1}\\
&= \frac{-2\pm\sqrt{24}}{2} = \frac{-2+2\sqrt{6}}{2} && a>0\\
&= -1+\sqrt{6}
\end{aligned}
$$

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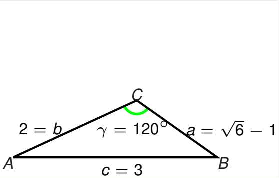


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$$\begin{aligned} a &= \frac{-2\pm\sqrt{2^2-4\cdot(-5)\cdot 1}}{2\cdot 1}\\ &= -1+\sqrt{6}\end{aligned}$$

$$\begin{aligned} \textbf{Area} &= \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6}-1\right)\cancel{2}}{\cancel{2}}\frac{\sqrt{3}}{2}\\ &= \frac{3\sqrt{2}-\sqrt{3}}{2}\end{aligned}$$