

# Calculus II

## Integration by parts

Todor Milev

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# Outline

## 1 Integration by Parts

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# Integration by Parts

Every differentiation rule

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Every **differentiation rule**

$$(uv)' = u'v + uv' \quad | \text{ Product Rule}$$

# Integration by Parts

Every differentiation rule corresponds to a **differential form rule**

$$\begin{aligned}(uv)' &= u'v + uv' \\ d(uv) &= vdu + u dv\end{aligned}$$

Product Rule  
Differential Prod. Rule

# Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

$(uv)' = u'v + uv'$	Product Rule
$d(uv) = vdu + u dv$	Differential Prod. Rule
$\int d(uv) = \int vdu + \int u dv$	integration of the above

# Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

$(uv)'$	$=$	$u'v + uv'$	Product Rule Differential Prod. Rule integration of the above
$d(uv)$	$=$	$vdu + u dv$	
$\int d(uv)$	$=$	$\int vdu + \int u dv$	
$uv$	$=$	$\int vdu + \int u dv$	



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$\textcolor{red}{uv} = \int vdu + \int u dv$	rearrange
$\int u dv = \textcolor{red}{uv} - \int vdu$	

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$\int udv = uv - \int vdu$	

We just proved the following.

## Proposition ((Rule of) Integration by Parts)

$$\int udv = uv - \int vdu$$

# Integration by parts: strategy for applying

Integration by parts:

$$\int u dv = uv - \int v du.$$

Generally: Choose  $u$  in this order: **LIPET**

# Integration by parts: strategy for applying

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Generally: Choose  $u$  in this order: **LIPET**

**Logs, Inverse trig, Polynomial, Exponential, Trig**

Integration by parts:  $\int u dv = uv - \int v du.$

### Example

$$\int x \sin x dx =$$

Integration by parts:  $\int u dv = uv - \int v du.$

### Example

$$\int x \sin x dx =$$

$$\sin x dx = d(?) \quad )$$



Integration by parts:  $\int u dv = uv - \int v du.$

### Example

$$\int x \sin x dx =$$

$$\sin x dx = d(-\cos x)$$

Integration by parts:  $\int u dv = uv - \int v du.$

### Example

$$\int x \sin x dx = \int x d(-\cos x) \quad \Bigg| \quad \sin x dx = d(-\cos x)$$

Integration by parts:  $\int u dv = uv - \int v du.$

### Example

$$\begin{aligned}\int x \sin x dx &= \int x d(-\cos x) & \left| \sin x dx = d(-\cos x) \right. \\ &= x(-\cos x) - \int (-\cos x) dx\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

### Example

$$\begin{aligned}\int x \sin x dx &= \int x d(-\cos x) & \left| \sin x dx = d(-\cos x) \right. \\ &= x(-\cos x) - \int (-\cos x) dx\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

### Example

$$\begin{aligned}\int x \sin x dx &= \int x d(-\cos x) && \left| \sin x dx = d(-\cos x) \right. \\ &= x(-\cos x) - \int (-\cos x) dx \\ &= -x \cos x + \int \cos x dx\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

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$$\begin{aligned}\int x \sin x dx &= \int x d(-\cos x) && \left| \sin x dx = d(-\cos x) \right. \\ &= x(-\cos x) - \int (-\cos x) dx \\ &= -x \cos x + \int \cos x dx \\ &= -x \cos x + ?\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

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$$\begin{aligned}\int x \sin x dx &= \int x d(-\cos x) && \left| \sin x dx = d(-\cos x) \right. \\ &= x(-\cos x) - \int (-\cos x) dx \\ &= -x \cos x + \int \cos x dx \\ &= -x \cos x + \sin x + C\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

### Example

$$\int \ln x dx =$$



Integration by parts:  $\int u dv = uv - \int v du.$

### Example

$$\int \ln x dx = (\ln x)x - \int x d(\ln x) \quad \left| \text{integrate by parts} \right.$$

Integration by parts:  $\int u dv = uv - \int v du.$

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### Example

$$\begin{aligned}\int \ln x dx &= (\ln x)x - \int x d(\ln x) && \left| \text{integrate by parts} \right. \\ &= x \ln x - \int x (\ln x)' dx\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

### Example

$$\begin{aligned}\int \ln x dx &= (\ln x)x - \int x d(\ln x) && \left| \text{integrate by parts} \right. \\ &= x \ln x - \int x (\ln x)' dx \\ &= x \ln x - \int x ? dx\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

### Example

$$\begin{aligned}\int \ln x dx &= (\ln x)x - \int x d(\ln x) && \left| \text{integrate by parts} \right. \\ &= x \ln x - \int x (\ln x)' dx \\ &= x \ln x - \int x \frac{1}{x} dx\end{aligned}$$

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Integration by parts:  $\int u dv = uv - \int v du.$

### Example

$$\begin{aligned}\int \ln x dx &= (\ln x)x - \int x d(\ln x) && \left| \text{integrate by parts} \right. \\ &= x \ln x - \int x (\ln x)' dx \\ &= x \ln x - \int x \frac{1}{x} dx \\ &= x \ln x - \int dx \\ &= x \ln x - x + C.\end{aligned}$$



Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\int t^2 e^t dt$$

Integration by parts:  $\int u dv = uv - \int v du.$

### Example

$$\int t^2 e^t dt = \int t^2 d(?)$$

Integration by parts:  $\int u dv = uv - \int v du.$

### Example

$$\int t^2 e^t dt = \int t^2 d(e^t)$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned} \int t^2 e^t dt &= \int t^2 d(e^t) && \left| \text{integrate by parts} \right. \\ &= ? \end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}\int t^2 e^t dt &= \int t^2 d(e^t) && \left| \text{integrate by parts} \right. \\ &= t^2 e^t - \int e^t d(t^2)\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}\int t^2 e^t dt &= \int t^2 d(e^t) && \left| \text{integrate by parts} \right. \\ &= t^2 e^t - \int e^t d(t^2)\end{aligned}$$

Integration by parts:  $\int u d\mathbf{v} = u\mathbf{v} - \int \mathbf{v} du.$

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Integration by parts:  $\int u dv = uv - \int v du.$

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$$\begin{aligned}\int t^2 e^t dt &= \int t^2 d(e^t) && \left| \text{integrate by parts} \right. \\ &= t^2 e^t - \int e^t d(t^2) \\ &= t^2 e^t - \int e^t \textcolor{red}{?} dt\end{aligned}$$



Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}\int t^2 e^t dt &= \int t^2 d(e^t) && \left| \text{integrate by parts} \right. \\ &= t^2 e^t - \int e^t d(t^2) \\ &= t^2 e^t - \int e^t 2t dt\end{aligned}$$

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Integration by parts:  $\int u dv = uv - \int v du.$

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Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}\int t^2 e^t dt &= \int t^2 d(e^t) && \left| \text{integrate by parts} \right. \\ &= t^2 e^t - \int e^t d(t^2) \\ &= t^2 e^t - \int e^t 2t dt \\ &= t^2 e^t - 2 \int t d(e^t) && \left| \text{integrate by parts} \right. \\ &= t^2 e^t - 2 \left( t e^t - \int e^t dt \right)\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du$ .

## Example

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Integration by parts:  $\int u d\mathbf{v} = u\mathbf{v} - \int \mathbf{v} du.$

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Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\int e^x \sin x dx =$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

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## Example

$$\int e^x \sin x dx = \int \sin x d(e^x)$$

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## Example

$$\begin{aligned}\int e^x \sin x dx &= \int \sin x d(e^x) \\ &= ?\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}\int e^x \sin x dx &= \int \sin x d(e^x) \\ &= (\sin x)e^x - \int e^x d(\sin x)\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

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Integration by parts:  $\int u d\mathbf{v} = u\mathbf{v} - \int \mathbf{v} du.$

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Integration by parts:  $\int u dv = uv - \int v du.$

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Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}\int e^x \sin x dx &= \int \sin x d(e^x) \\&= (\sin x)e^x - \int e^x d(\sin x) \\&= e^x \sin x - \int e^x \cos x dx \\&= e^x \sin x - \int \cos x d(e^x) \\&= e^x \sin x - \left( (\cos x)e^x - \int e^x d(\cos x) \right) \\&= e^x \sin x - \cos x e^x + \int e^x\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}\int e^x \sin x dx &= \int \sin x d(e^x) \\&= (\sin x)e^x - \int e^x d(\sin x) \\&= e^x \sin x - \int e^x \cos x dx \\&= e^x \sin x - \int \cos x d(e^x) \\&= e^x \sin x - \left( (\cos x)e^x - \int e^x d(\cos x) \right) \\&= e^x \sin x - \cos x e^x + \int e^x ?\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}\int e^x \sin x dx &= \int \sin x d(e^x) \\&= (\sin x)e^x - \int e^x d(\sin x) \\&= e^x \sin x - \int e^x \cos x dx \\&= e^x \sin x - \int \cos x d(e^x) \\&= e^x \sin x - \left( (\cos x)e^x - \int e^x d(\cos x) \right) \\&= e^x \sin x - \cos x e^x + \int e^x (-\sin x) dx\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}
 \int e^x \sin x dx &= \int \sin x d(e^x) \\
 &= (\sin x)e^x - \int e^x d(\sin x) \\
 &= e^x \sin x - \int e^x \cos x dx \\
 &= e^x \sin x - \int \cos x d(e^x) \\
 &= e^x \sin x - \left( (\cos x)e^x - \int e^x d(\cos x) \right) \\
 &= e^x \sin x - \cos x e^x + \int e^x (-\sin x) dx \\
 &= e^x \sin x - \cos x e^x - \int e^x \sin x dx
 \end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}\int e^x \sin x dx &= \int \sin x d(e^x) \\&= (\sin x)e^x - \int e^x d(\sin x) \\&= e^x \sin x - \int e^x \cos x dx \\&= e^x \sin x - \int \cos x d(e^x) \\&= e^x \sin x - \left( (\cos x)e^x - \int e^x d(\cos x) \right) \\&= e^x \sin x - \cos x e^x + \int e^x (-\sin x) dx \\&= e^x \sin x - \cos x e^x - \int e^x \sin x dx\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\int e^x \sin x dx = e^x \sin x - \cos x e^x - \int e^x \sin x dx$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}\int e^x \sin x dx &= e^x \sin x - \cos x e^x - \int e^x \sin x dx \\ 2 \int e^x \sin x dx &= e^x \sin x - \cos x e^x\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\int e^x \sin x dx = e^x \sin x - \cos x e^x - \int e^x \sin x dx$$

$$2 \int e^x \sin x dx = e^x \sin x - \cos x e^x$$

$$\int e^x \sin x dx = \frac{1}{2} (e^x \sin x - \cos x e^x)$$



Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}\int e^x \sin x dx &= e^x \sin x - \cos x e^x - \int e^x \sin x dx \\ 2 \int e^x \sin x dx &= e^x \sin x - \cos x e^x \\ \int e^x \sin x dx &= \frac{1}{2} (e^x \sin x - \cos x e^x) + C\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\int_0^1 \arctan x dx =$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\int_0^1 \arctan x dx = ?$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\int_0^1 \arctan x dx = [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x d(\arctan x)$$

Integration by parts:  $\int u dv = uv - \int v du$ .

## Example

$$\int_0^1 \arctan x dx = [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x d(\arctan x)$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\int_0^1 \arctan x dx = [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x d(\arctan x)$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}\int_0^1 \arctan x dx &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x d(\arctan x) \\ &= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x?\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}\int_0^1 \arctan x dx &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x d(\arctan x) \\ &= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x?\end{aligned}$$



Integration by parts:  $\int u dv = uv - \int v du.$

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Integration by parts:  $\int u dv = uv - \int v du.$

## Example

$$\begin{aligned}\int_0^1 \arctan x dx &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x d(\arctan x) \\ &= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x \frac{1}{1+x^2} dx\end{aligned}$$

Integration by parts:  $\int u dv = uv - \int v du.$

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Integration by parts:  $\int u dv = uv - \int v du.$

## Example

Set  $w = 1 + x^2$ .

$$\begin{aligned}\int_0^1 \arctan x dx &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x d(\arctan x) \\&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x \frac{1}{1+x^2} dx \\&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2} d\left(\frac{x^2}{2}\right) \\&= \frac{\pi}{4} - \frac{1}{2} \int_{x=0}^{x=1} \frac{1}{1+x^2} d(1+x^2)\end{aligned}$$

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 &= \frac{\pi}{4} - \frac{1}{2} \int_{x=0}^{x=1} \frac{1}{1+x^2} d(1+x^2) \\
 &= \frac{\pi}{4} - \frac{1}{2} \int_{x=0}^{x=1} \frac{1}{w} dw
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 &= \frac{\pi}{4} - \frac{1}{2} \int_{x=0}^{x=1} \frac{1}{w} dw = \frac{\pi}{4} - \frac{1}{2} [\ln |w|]_{x=0}^{x=1}
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 &= \frac{\pi}{4} - \frac{1}{2} \left[ \ln(1+x^2) \right]_{x=0}^{x=1}
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 &= \frac{\pi}{4} - \frac{1}{2} \left[ \ln(1+x^2) \right]_{x=0}^{x=1} \\
 &= \frac{\pi}{4} - \frac{1}{2} (\ln 2 - \ln 1)
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 &= \frac{\pi}{4} - \frac{1}{2} \left[ \ln(1+x^2) \right]_{x=0}^{x=1} \\
 &= \frac{\pi}{4} - \frac{1}{2} (\ln 2 - \ln 1) = \frac{\pi}{4} - \frac{1}{2} \ln 2.
 \end{aligned}$$