Precalculus
Trigonometric equations and inequalities

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2019

Outline

1. Trigonometric equations and inequalities
   - The Equations $\sin x = A$, $\cos x = B$
   - Equations that reduce to $\sin x = A$, $\cos x = B$

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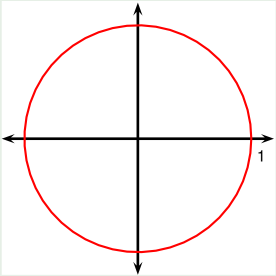
Trigonometric equations

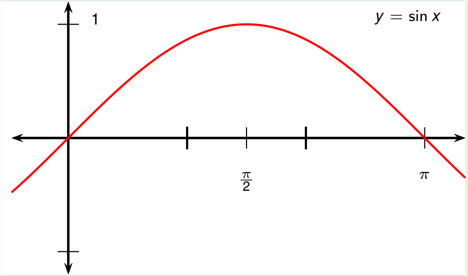
- Some problems will not ask you to prove a trigonometric identity, but rather to solve a trigonometric equation.
- Consider the problem of finding all values of x for which $\sin x = \sin(2x) = 2\sin x \cos x$.
- This is not a trigonometric identity - the two sides are different.
- However, there are values for x which the above equality holds.

Example

Find all solutions and then find those that lie between -360° and 360°.

$$\sin\theta = \frac{\sqrt{3}}{2}$$





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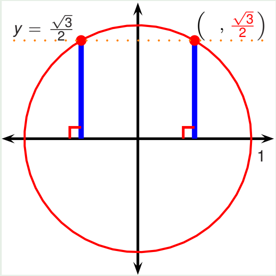

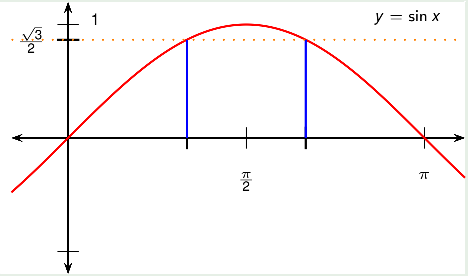

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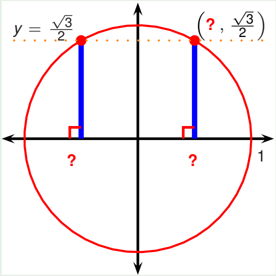

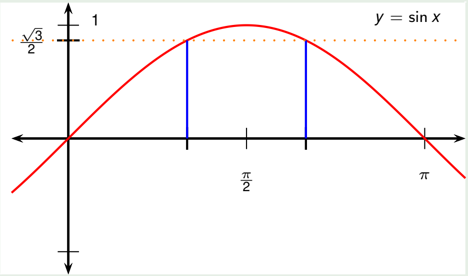

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Find all solutions and then find those that lie between $-360°$ and $360°$.

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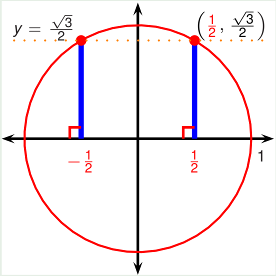
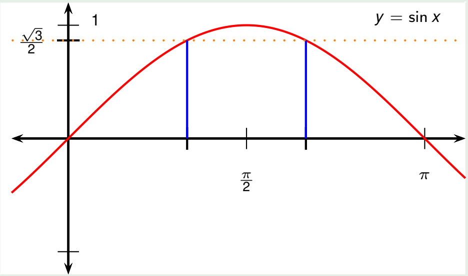
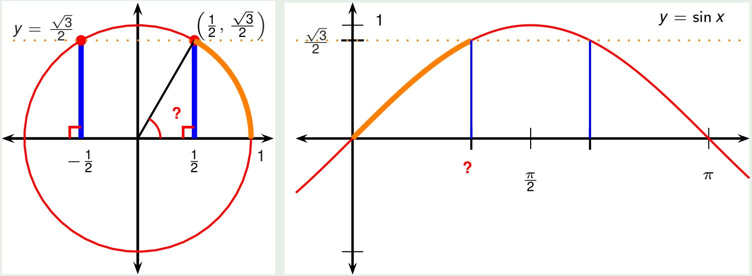




Example

Find all solutions and then find those that lie between -360° and 360°.

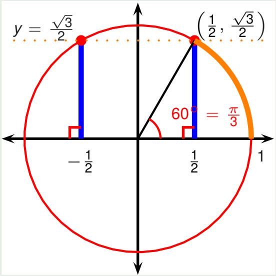
$$\begin{aligned}\sin\theta &= \frac{\sqrt{3}}{2}\\ \theta &= \textbf{?}\end{aligned}$$

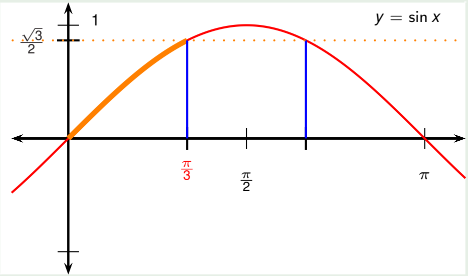


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned}\sin\theta &= \frac{\sqrt{3}}{2}\\ \theta &= 60^\circ\end{aligned}$$

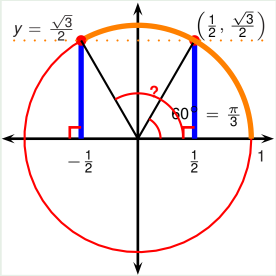


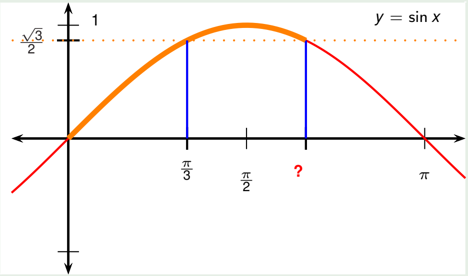


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned}\sin\theta &= \frac{\sqrt{3}}{2}\\ \theta &= 60^\circ\\ &\quad \textbf{or}\\ &\quad \textbf{?}\end{aligned}$$





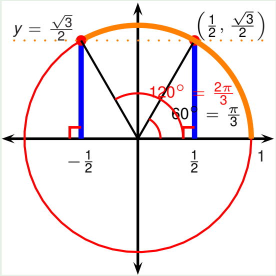
Example

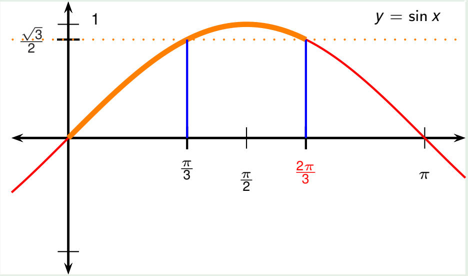
Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned} \sin\theta &= \frac{\sqrt{3}}{2} \\ \theta &= 60^\circ \end{aligned}$$

or

120°





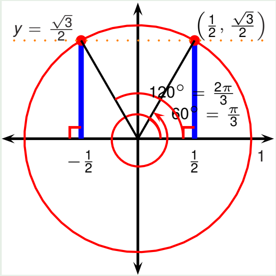
Example

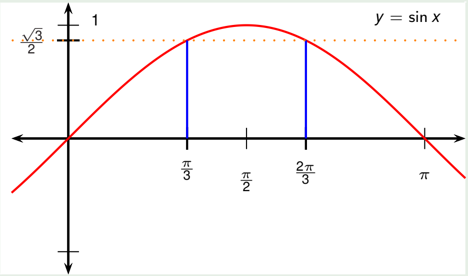
Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned} \sin\theta &= \frac{\sqrt{3}}{2} \\ \theta &= 60^\circ + k\cdot 360^\circ \end{aligned}$$

or

120°

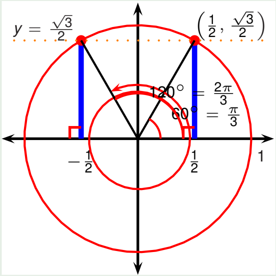


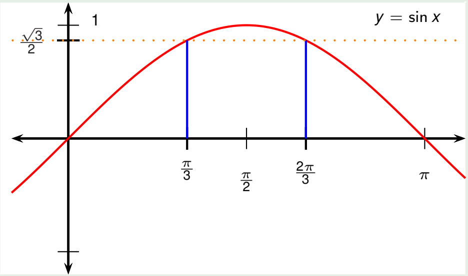


Example

Find all solutions and then find those that lie between $-360°$ and $360°$.

$$\begin{aligned} \sin\theta &= \frac{\sqrt{3}}{2} \\ \theta &= 60° + k \cdot 360° \\ &\quad \textbf{or} \\ &\quad 120° + k \cdot 360° \end{aligned}$$

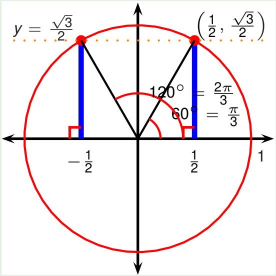


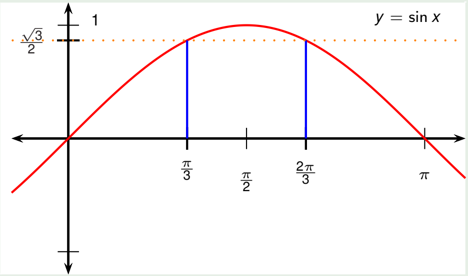


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned} \sin\theta &= \frac{\sqrt{3}}{2} \\ \theta &= 60^\circ + k\cdot 360^\circ = \quad \dots\ -660^\circ, \\ & \textbf{or} \qquad\qquad\qquad\quad \cdots \quad k{=}{-2} \\ & 120^\circ + k\cdot 360^\circ = \dots\ -600^\circ, \end{aligned}$$

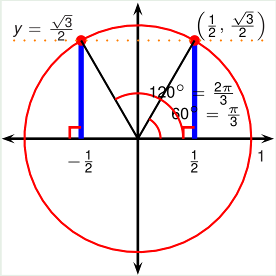


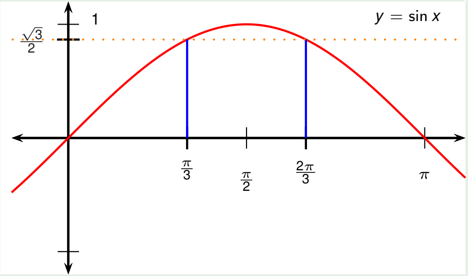


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned}
\sin\theta &= \frac{\sqrt{3}}{2} \\
\theta &= 60^\circ + {\color{red}k}\cdot 360^\circ = \quad \dots\ \underset{k=-2}{-660^\circ},\ {\color{red}\underset{k=-1}{-300^\circ},} \\
&\quad \textbf{or} \\
&\quad 120^\circ + {\color{red}k}\cdot 360^\circ = \dots\ -600^\circ,\ {\color{red}-240^\circ,}
\end{aligned}$$

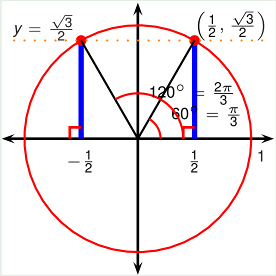


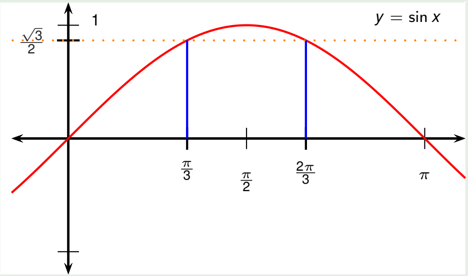


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned}
\sin\theta &= \frac{\sqrt{3}}{2}\\
\theta &= 60^\circ + k\cdot 360^\circ = \quad \dots\ -660^\circ,\ -300^\circ,\ 60^\circ,\\
&\quad\textbf{or} \qquad\qquad\qquad\qquad \cdots\quad k=-2 \quad k=-1 \quad k=0\\
&\quad 120^\circ + k\cdot 360^\circ = \dots\ -600^\circ,\ -240^\circ,\ 120^\circ,
\end{aligned}$$

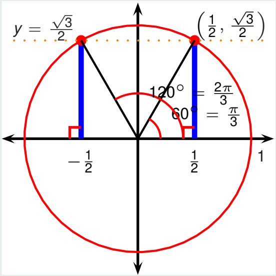


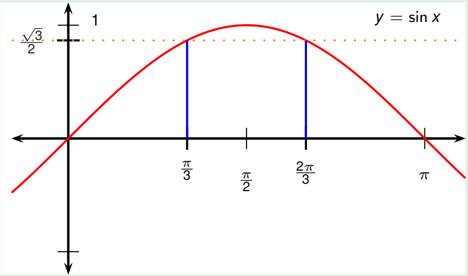


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{array}{rcl} \sin\theta &=& \frac{\sqrt{3}}{2} \\ \theta &=& 60^\circ + k\cdot 360^\circ = \quad \dots\ -660^\circ,\ -300^\circ,\ 60^\circ,\ 420^\circ,\ \dots \\ & & \textbf{or} \qquad\qquad\qquad\qquad \cdots \quad k{=}{-2} \quad k{=}{-1} \quad k{=}0 \quad k{=}1 \quad \cdots \\ & & 120^\circ + k\cdot 360^\circ = \dots\ -600^\circ,\ -240^\circ,\ 120^\circ,\ 480^\circ,\ \dots \end{array}$$





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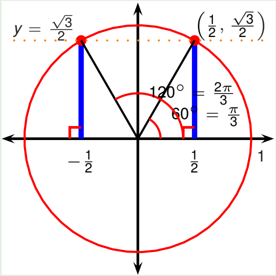
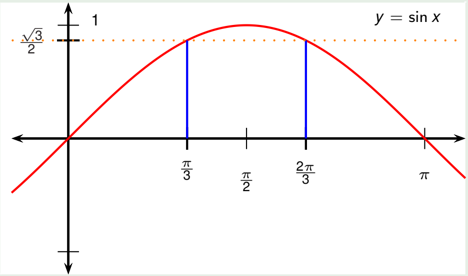
$$\begin{array}{rcl} \sin\theta &=& \dfrac{\sqrt{3}}{2} \\ \theta &=& 60^\circ + k\cdot 360^\circ = \underset{\cdots}{\dots}\ \underset{k=-2}{-660^\circ},\ \underset{k=-1}{-300^\circ},\ \underset{k=0}{60^\circ},\ \underset{k=1}{420^\circ},\ \underset{\cdots}{\dots} \\ && \textbf{or} \\ && 120^\circ + k\cdot 360^\circ = \dots\ -600^\circ,\ -240^\circ,\ 120^\circ,\ 480^\circ,\ \dots \end{array}$$





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&& \textbf{or} \qquad\qquad\qquad\quad \cdots\ \ k=-2\ \ \ k=-1\ \ \ k=0\ \ \ k=1\ \ \cdots\\
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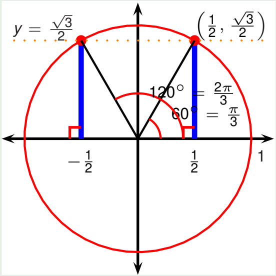
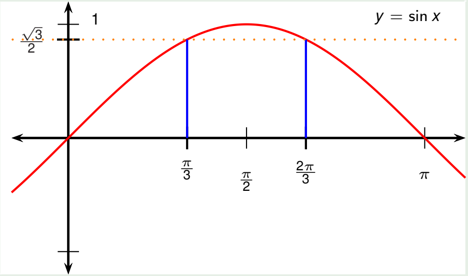
\end{array}$$





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$$
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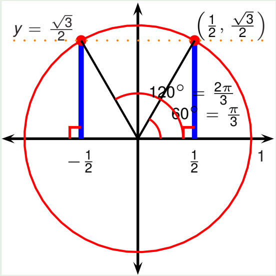
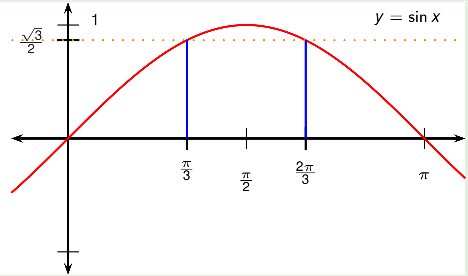
&& \textbf{or} & \quad\cdots\ \ k{=}-2\quad k{=}-1\quad k{=}0\quad k{=}1\ \ \cdots \\
&& 120^\circ + k\cdot 360^\circ = & \dots\ -600^\circ,\ -240^\circ,\ 120^\circ,\ 480^\circ,\ \dots \\
\hline
\theta &=& & \cancel{\dots}\ \cancel{-660^\circ},\ -300^\circ,\ 60^\circ,\ \cancel{420^\circ},\ \cancel{\dots} \\
&& & \cancel{\dots}\ \cancel{-600^\circ},\ -240^\circ,\ 120^\circ,\ \cancel{480^\circ},\ \cancel{\dots}
\end{array}
$$

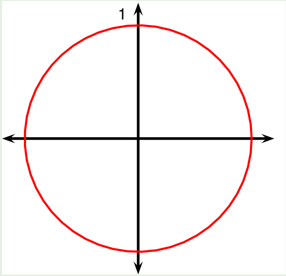


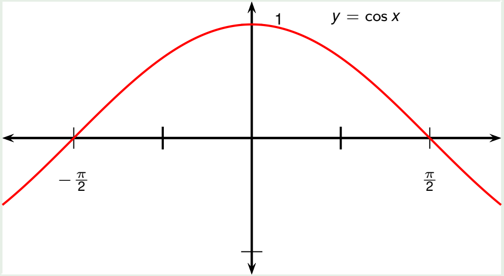


Example

Find all solutions and then find those that lie between -180° and 180°.

$$\cos\theta = \frac{\sqrt{2}}{2}$$





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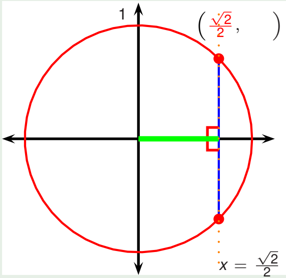

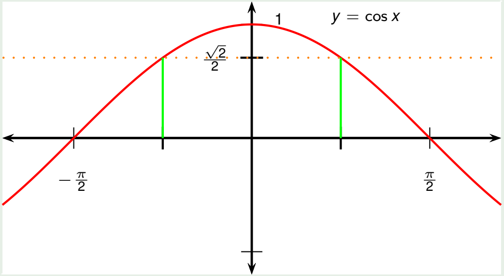

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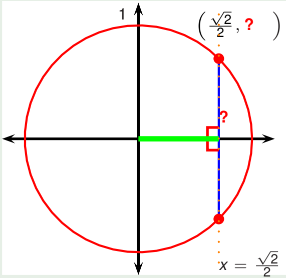

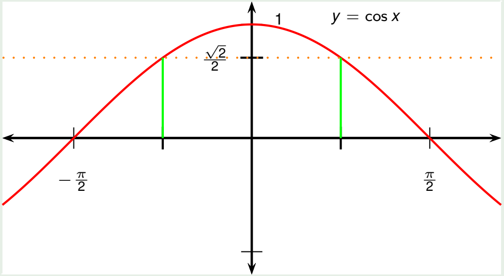

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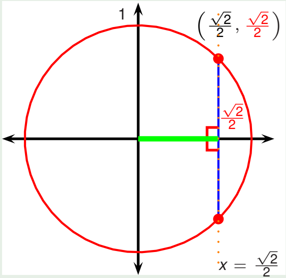
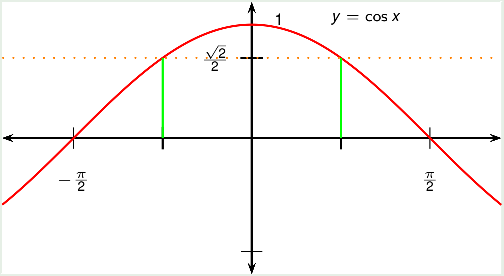
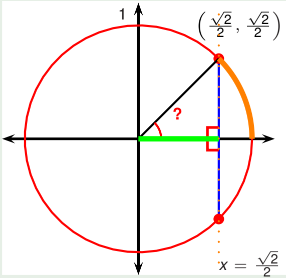


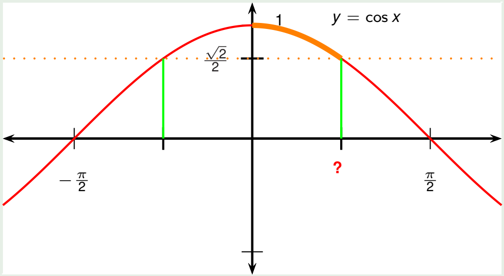


Example

Find all solutions and then find those that lie between -180° and 180°.

$$\cos\theta = \frac{\sqrt{2}}{2}$$
$$\theta = ?$$

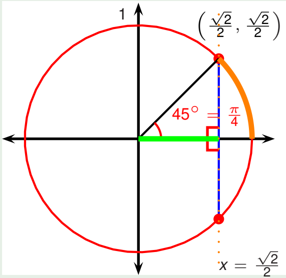


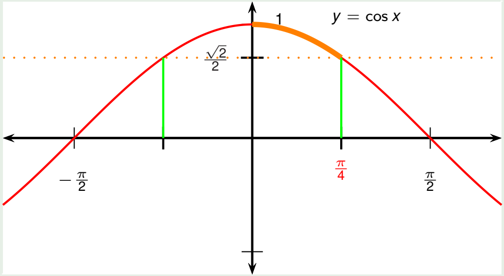


Example

Find all solutions and then find those that lie between -180° and 180°.

$$\cos\theta = \frac{\sqrt{2}}{2}$$
$$\theta = 45^\circ$$





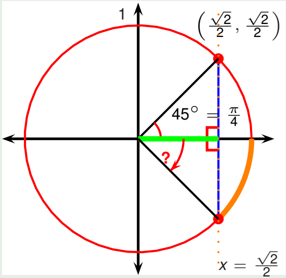
Example

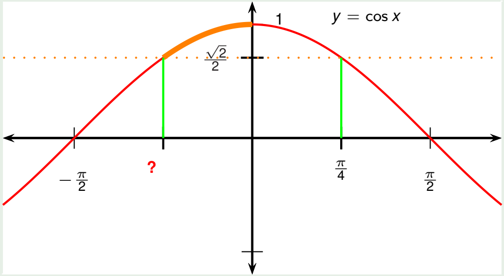
Find all solutions and then find those that lie between -180° and 180°.

$$\cos\theta = \frac{\sqrt{2}}{2}$$
$$\theta = 45^\circ$$

or

?





Example

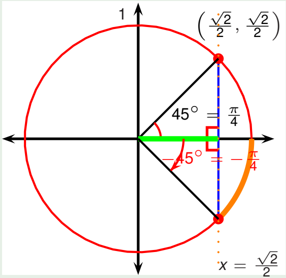
Find all solutions and then find those that lie between -180° and 180°.

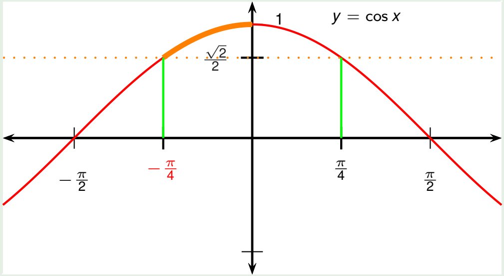
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ$$

or

-45°





Example

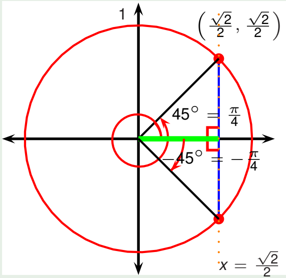
Find all solutions and then find those that lie between -180° and 180°.

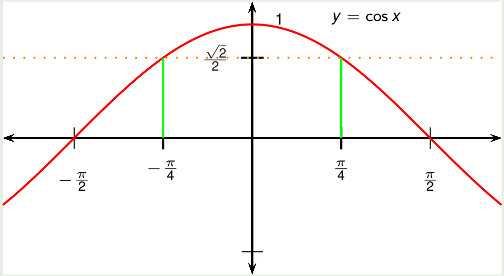
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ + k \cdot 360^\circ$$

or

$$-45^\circ$$





Example

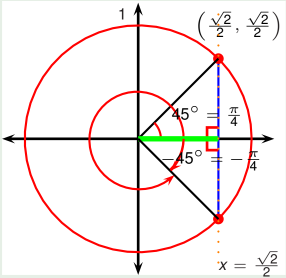
Find all solutions and then find those that lie between -180° and 180°.

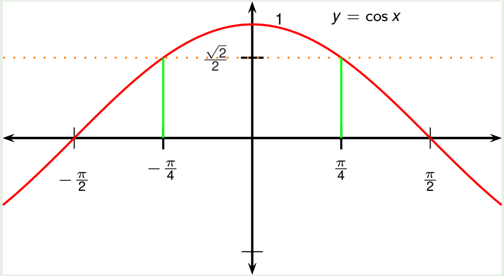
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ + k\cdot 360^\circ$$

or

$$-45^\circ + k\cdot 360^\circ$$





Example

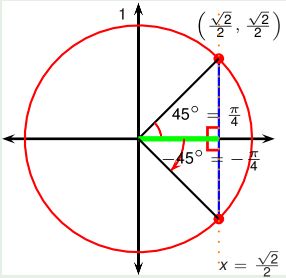
Find all solutions and then find those that lie between -180° and 180°.

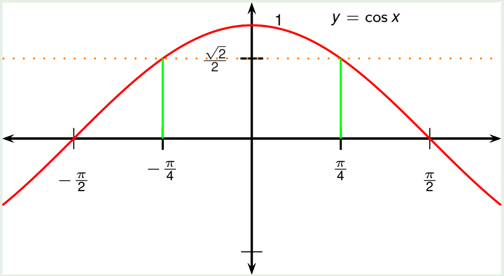
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ + k\cdot 360^\circ = \quad \ldots \quad -675^\circ,$$

or $\quad \ldots \quad k=-2$

$$-45^\circ + k\cdot 360^\circ = \ldots \quad -765^\circ,$$





Example

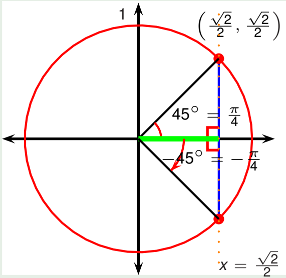
Find all solutions and then find those that lie between -180° and 180°.

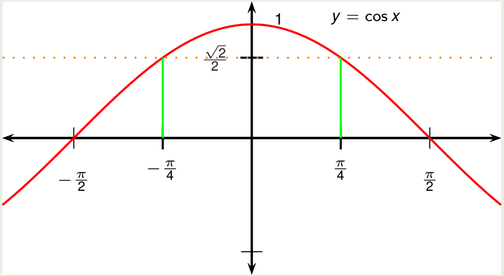
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ + k\cdot 360^\circ = \quad \dots \; \underset{k=-2}{-675^\circ}, \; \underset{k=-1}{-315^\circ},$$

or

$$-45^\circ + k\cdot 360^\circ = \quad \dots \; -765^\circ, \; -405^\circ,$$





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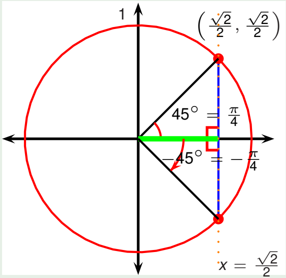

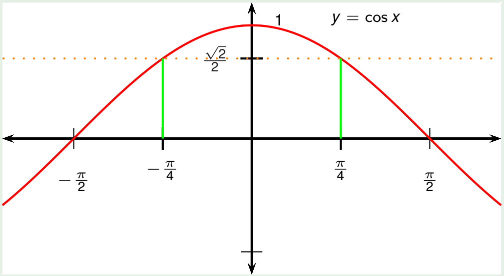

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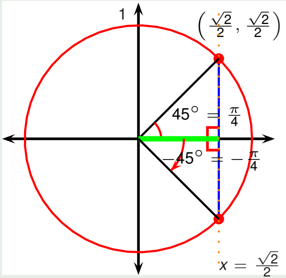

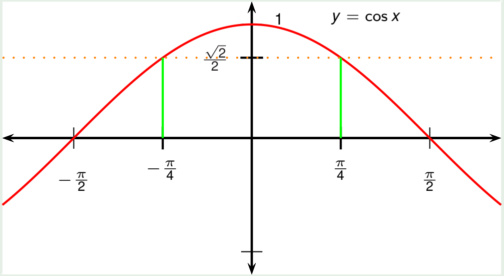

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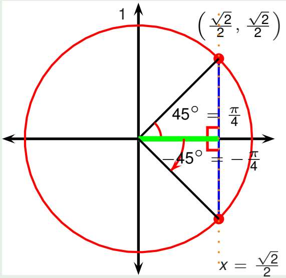

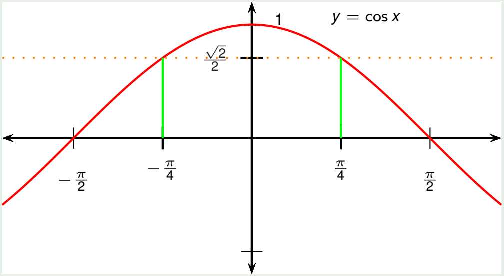

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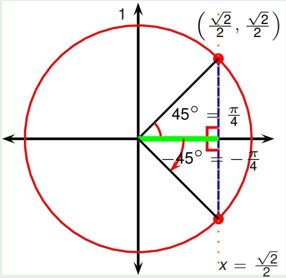

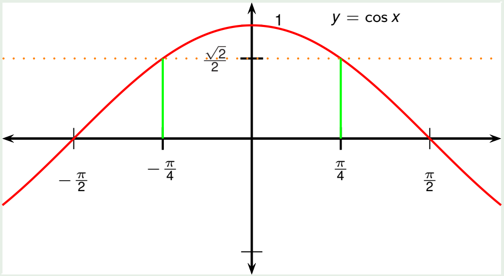

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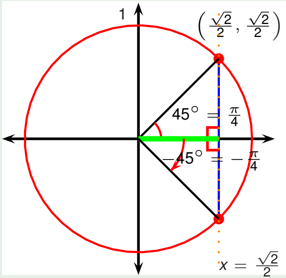
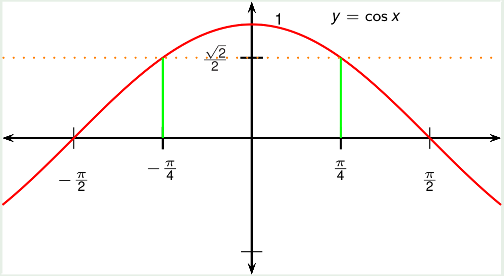
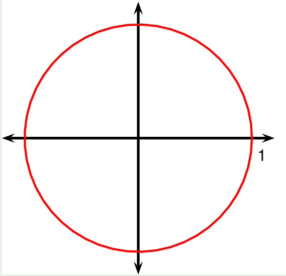


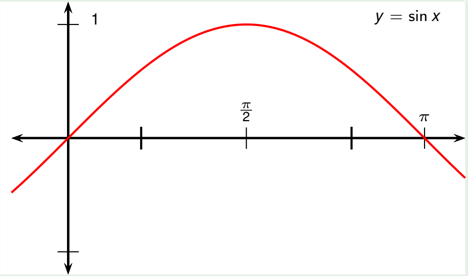


Example

Find all solutions of the equation.

$$\sin\theta = \frac{3}{5}$$





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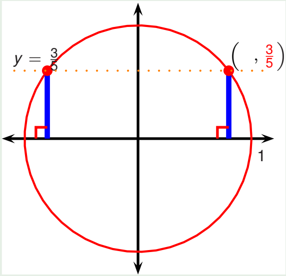

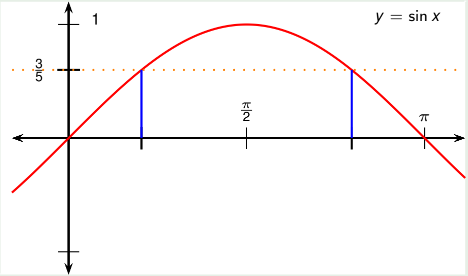

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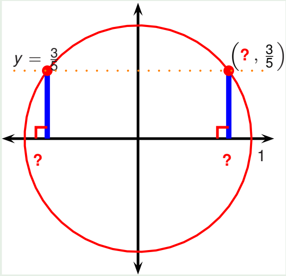

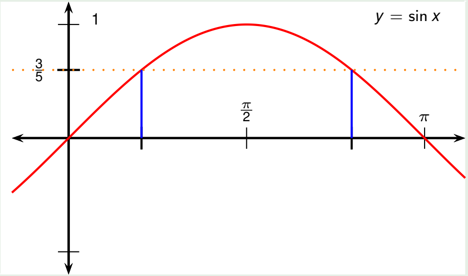

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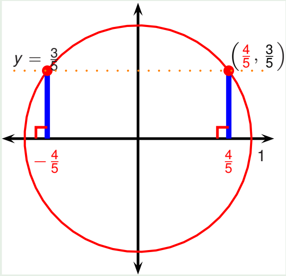

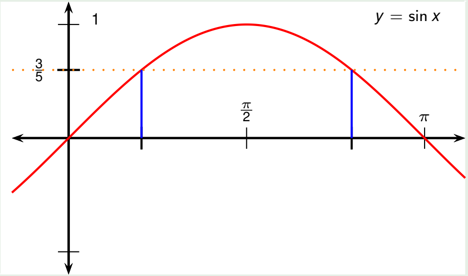

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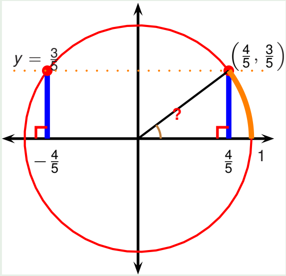
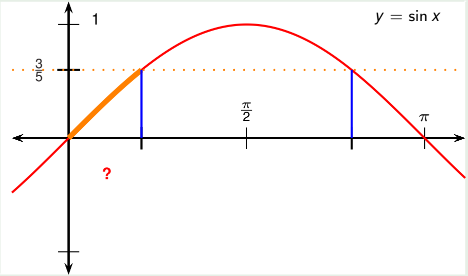
$$\sin\theta = \frac{3}{5}$$

$$\theta = ?$$





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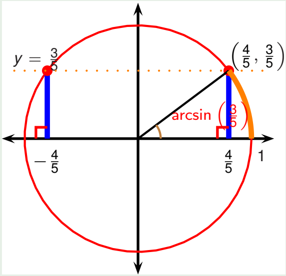
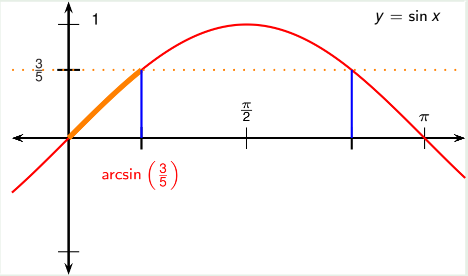
$$\sin\theta = \frac{3}{5}$$

$$\theta = \arcsin\left(\frac{3}{5}\right)$$

| arcsin implies radians





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$$\sin\theta = \frac{3}{5}$$

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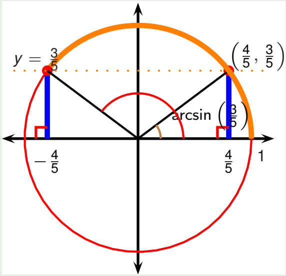

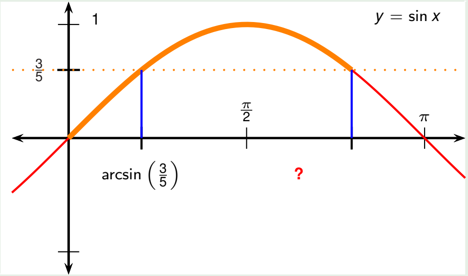

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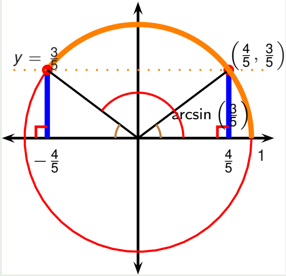

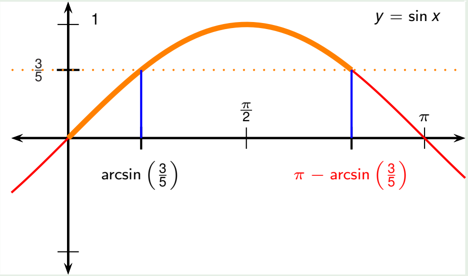

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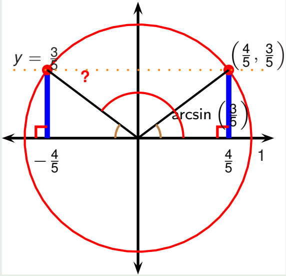

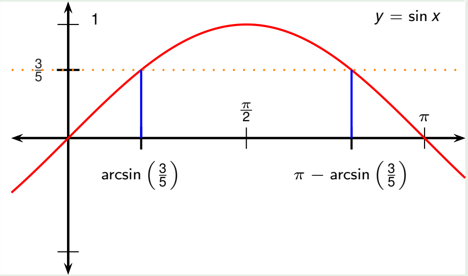

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$$\sin\theta = \frac{3}{5}$$

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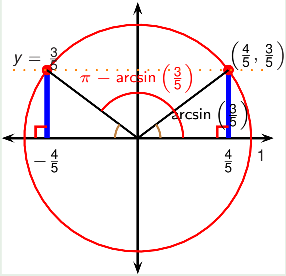

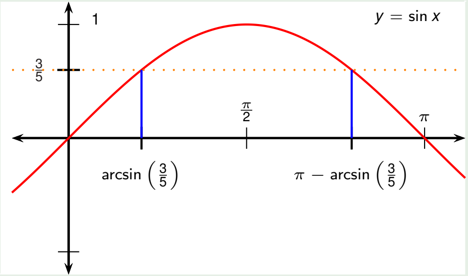

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$$\sin\theta = \frac{3}{5}$$

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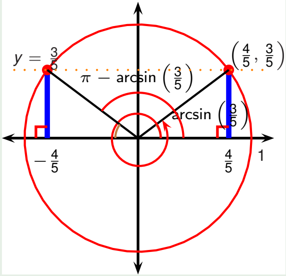

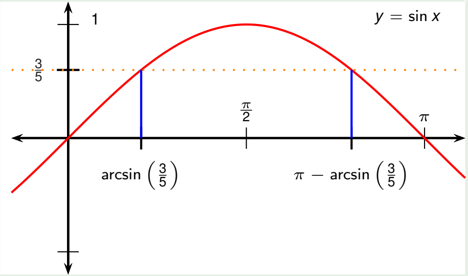

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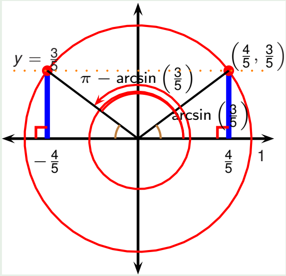

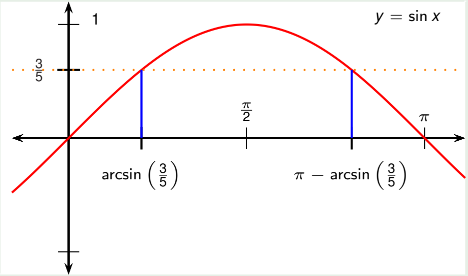

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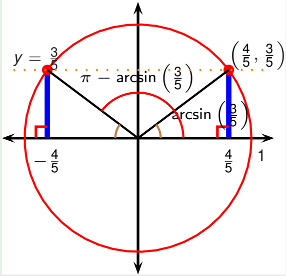

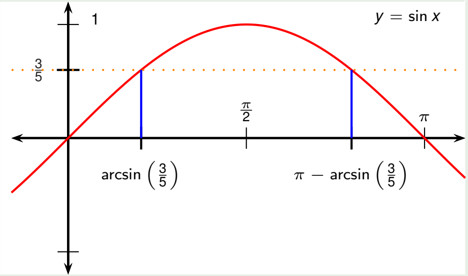

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Find all values of θ in the interval $[0, 2\pi]$ such that $\sin\theta = \sin(2\theta)$.

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$$\begin{aligned} \sin\theta &= \sin(2\theta) \\ \sin\theta &= \textbf{?} \end{aligned}$$

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Find all values of θ in the interval $[0, 2\pi]$ such that $\sin\theta = \sin(2\theta)$.

$$\begin{aligned}\sin\theta &= \sin(2\theta)\\ \sin\theta &= 2\sin\theta\cos\theta\end{aligned}$$

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$$\begin{aligned}
\sin\theta &= \sin(2\theta)\\
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\end{aligned}$$

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Find all values of θ in the interval $[0, 2\pi]$ such that $\sin\theta = \sin(2\theta)$.

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$$\color{red}{\sin\theta} = 0 \qquad\qquad 2\cos\theta - 1 = 0$$

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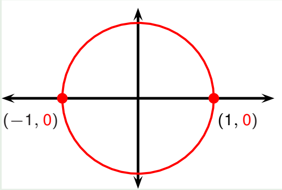
$\sin\theta = 0$ or $2\cos\theta - 1 = 0$



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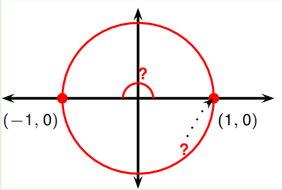

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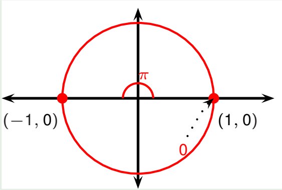

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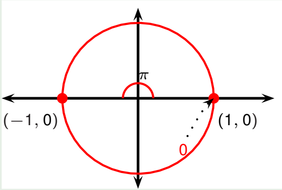

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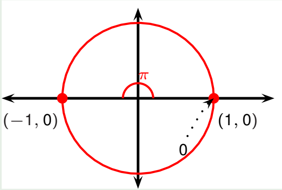

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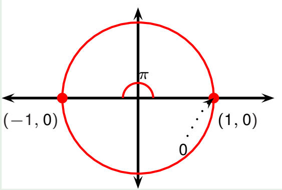

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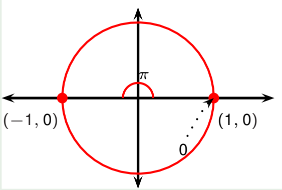

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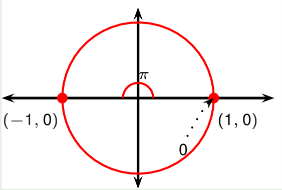

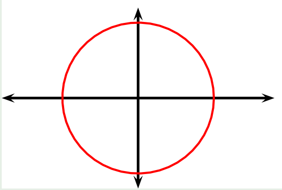

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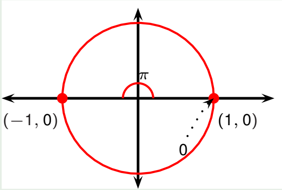

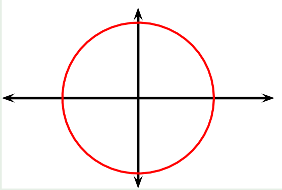

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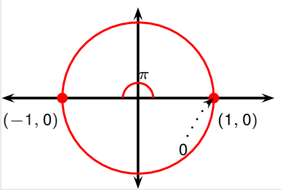

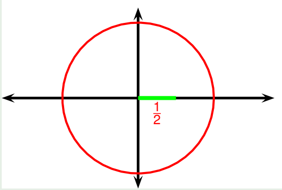

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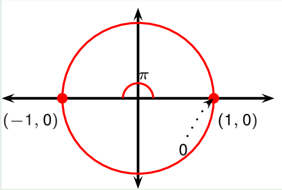

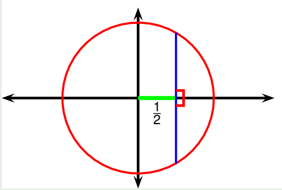

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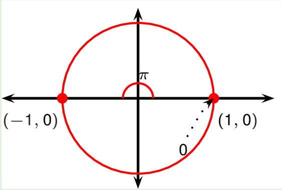

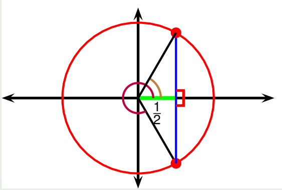

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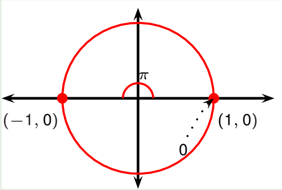

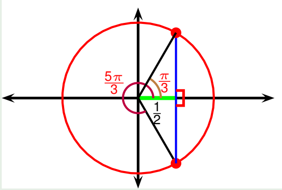

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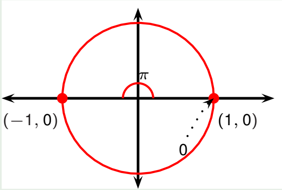

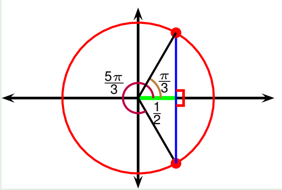

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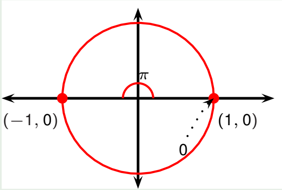

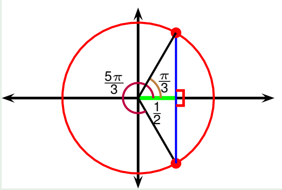

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Find all values of θ in the interval $\theta \in [0, 2\pi]$ for which

$$\cos(2\theta) = \cos\theta$$

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Find all values of θ in the interval $\theta \in [0, 2\pi]$ for which

$$\begin{array}{lrcl} & \cos(2\theta) & = & \cos\theta \\ \textbf{?} & -\cos\theta & = & 0 \end{array}$$

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\cos(2\theta) &=& \cos\theta & \\
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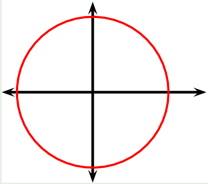

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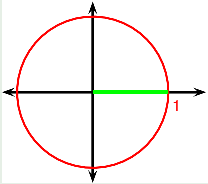

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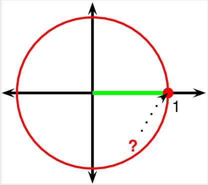

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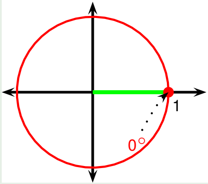

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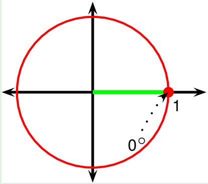

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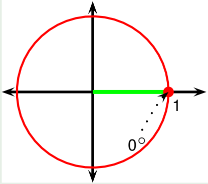

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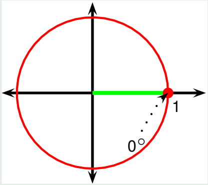

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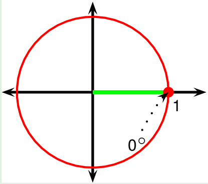

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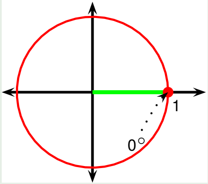

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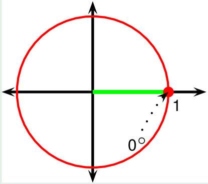

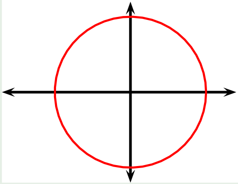

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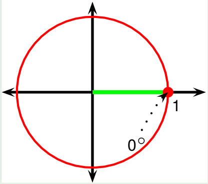

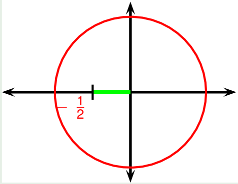

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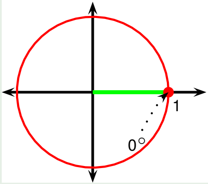

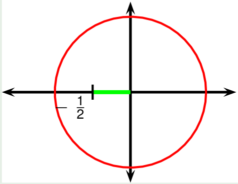

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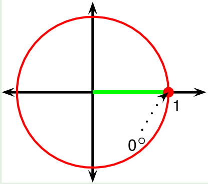

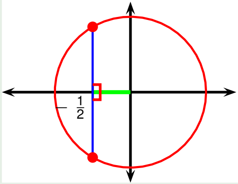

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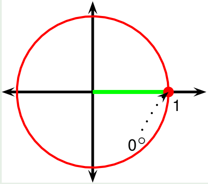

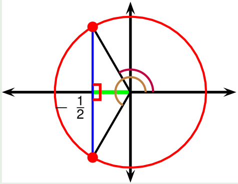

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2u + 1 &=& 0 \\
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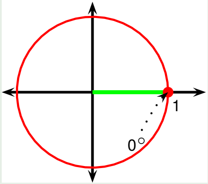

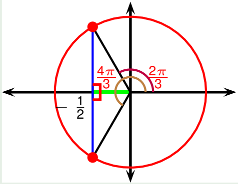

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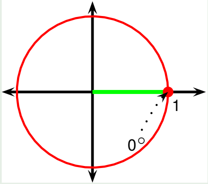

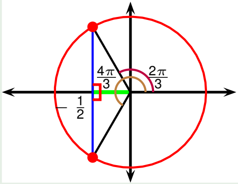

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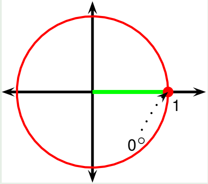

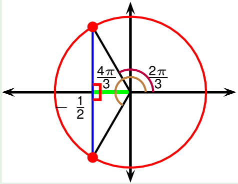

Strategy for solving trigonometric equations

- Suppose we want to solve an algebraic trigonometric equation.
- More precisely, the equation should be an algebraic expressions of the trigonometric functions of a single variable.
- Here is a general strategy for solving such a problem:
  - Using trig identities, rewrite in terms of $\sin x$ and $\cos x$ only.
  - Suppose $x \in [2n\pi, (2n+1)\pi]$.
    - Set $\sin x = \sqrt{1 - \cos^2 x}$ (allowed due to restrictions on x).
    - Set $\cos x = u$. Solve the resulting algebraic equation for u.
    - For the found solutions for u, solve $\cos x = u$.
    - Check whether your solutions satisfy $x \in [2n\pi, (2n+1)\pi]$.
  - Suppose $x \in [(2n-1)\pi, 2n\pi]$.
    - Set $\sin x = -\sqrt{1 - \cos^2 x}$ (allowed due to restrictions on x).
    - Set $\cos x = u$. Solve the resulting algebraic equation for u.
    - For the found solutions for u, solve $\cos x = u$.
    - Check whether your solutions satisfy $x \in [(2n-1)\pi, 2n\pi]$.
- A similar strategy exists for $u = \sin x$ instead of $u = \cos x$.
- Problems requiring full algorithm may be too hard for Calc exams.

Proposition (Product to sum formulas)

$$
\begin{aligned}
\sin\alpha\sin\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)\\
\cos\alpha\cos\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)+\cos(\alpha+\beta)\right)\\
\sin\alpha\cos\beta &= \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)
\end{aligned}
$$

Proof.

$\square$

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Proof.

?

$$= \cos(\alpha+\beta)$$

$\square$

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Proof.

$$\cos\alpha\cos\beta-\sin\alpha\sin\beta = \cos(\alpha+\beta)$$

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\begin{aligned}
\cos\alpha\cos\beta+\sin\alpha\sin\beta &= \cos(\alpha-\beta)\\
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\end{array}$$

$\square$

Proposition (Product to sum formulas)

$$\begin{aligned}
\sin\alpha\sin\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)\\
\cos\alpha\cos\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)+\cos(\alpha+\beta)\right)\\
\sin\alpha\cos\beta &= \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)
\end{aligned}$$

Proof.

$$\begin{array}{crcl}
+ & \cos\alpha\cos\beta+\cancel{\sin\alpha\sin\beta} &=& \cos(\alpha-\beta)\\
 & \cos\alpha\cos\beta-\cancel{\sin\alpha\sin\beta} &=& \cos(\alpha+\beta)\\
 & 2\cos\alpha\cos\beta &=& \cos(\alpha-\beta)+\cos(\alpha+\beta)\\
\hline
- & \cos\alpha\cos\beta+\sin\alpha\sin\beta &=& \cos(\alpha-\beta)\\
 & \cos\alpha\cos\beta-\sin\alpha\sin\beta &=& \cos(\alpha+\beta)\\
 & 2\sin\alpha\sin\beta &=& \cos(\alpha-\beta)-\cos(\alpha+\beta)
\end{array}$$

$\square$

Proposition (Product to sum formulas)

$$\begin{aligned}\sin\alpha\sin\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)\\ \cos\alpha\cos\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)+\cos(\alpha+\beta)\right)\\ \sin\alpha\cos\beta &= \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)\end{aligned}$$

Proof.

$$\begin{array}{crcl} + & \cos\alpha\cos\beta + \cancel{\sin\alpha\sin\beta} &=& \cos(\alpha-\beta)\\ & \cos\alpha\cos\beta - \cancel{\sin\alpha\sin\beta} &=& \cos(\alpha+\beta)\\ & 2\cos\alpha\cos\beta &=& \cos(\alpha-\beta)+\cos(\alpha+\beta)\\ \hline - & \cos\alpha\cos\beta + \sin\alpha\sin\beta &=& \cos(\alpha-\beta)\\ & \cos\alpha\cos\beta - \sin\alpha\sin\beta &=& \cos(\alpha+\beta)\\ & 2\sin\alpha\sin\beta &=& \cos(\alpha-\beta)-\cos(\alpha+\beta)\\ \hline + & \sin\alpha\cos\beta + \cos\alpha\sin\beta &=& \sin(\alpha+\beta)\\ & \sin\alpha\cos\beta - \cos\alpha\sin\beta &=& \sin(\alpha-\beta)\\ & 2\sin\alpha\cos\beta &=& \sin(\alpha+\beta)+\sin(\alpha+\beta)\end{array}$$

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Proposition (Product to sum formulas)

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\sin\alpha\sin\beta &= \frac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta))\\
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\end{aligned}$$

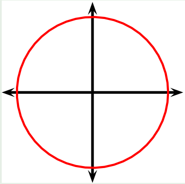
- Product to sum formulas are used when integrating (a topic to be studied later/in another course).

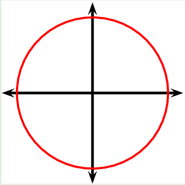
Proposition (Sum to product formulas)

$$\begin{aligned}
\sin\alpha + \sin\beta &= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\\
\sin\alpha - \sin\beta &= 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)\\
\cos\alpha + \cos\beta &= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\\
\cos\alpha - \cos\beta &= 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)
\end{aligned}$$

Example

Find all solutions in the interval $[0, 2\pi)$.





$$\sin(2x) + \sin(5x) = 0$$

Recall the formula $\sin\alpha + \sin\beta = $ **?**

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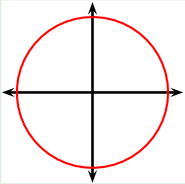

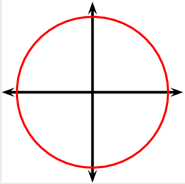

$\sin(2x) + \sin(5x) = 0$ | use f-la

Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

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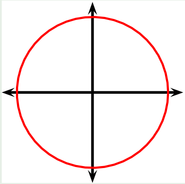

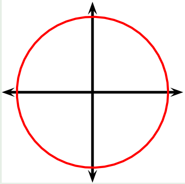

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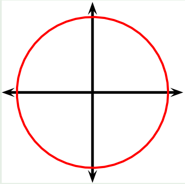

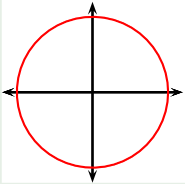

$$\sin(2x) + \sin(5x) = 0 \quad \Big| \text{ use f-la}$$

$$2\sin\left(\frac{2x+5x}{2}\right)\cos\left(\frac{2x-5x}{2}\right) = 0$$

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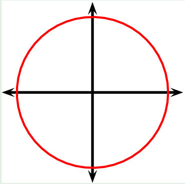

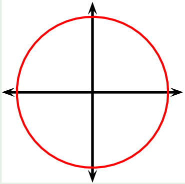

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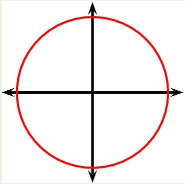

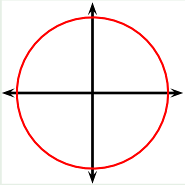

$$\sin(2x) + \sin(5x) = 0 \quad \Big| \text{ use f-la}$$
$$2\sin\left(\frac{2x+5x}{2}\right)\cos\left(\frac{2x-5x}{2}\right) = 0$$
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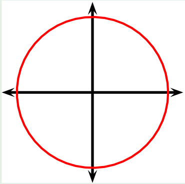

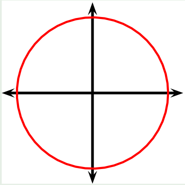

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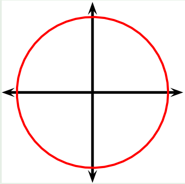

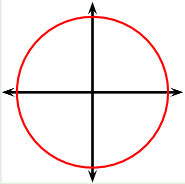

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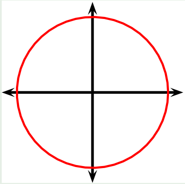

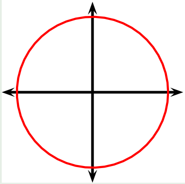

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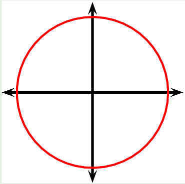
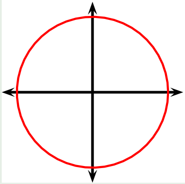


$\sin\left(\frac{7}{2}x\right) = 0$

or



$\cos\left(\frac{3}{2}x\right) = 0$

Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

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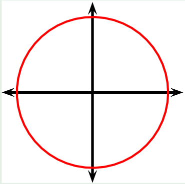

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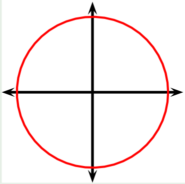

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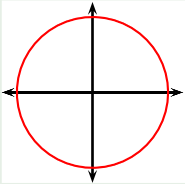

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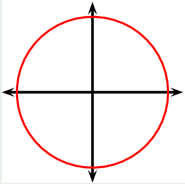

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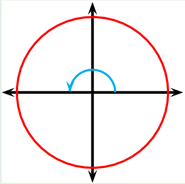

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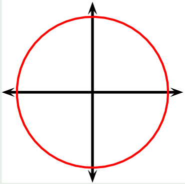

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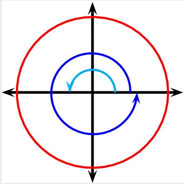

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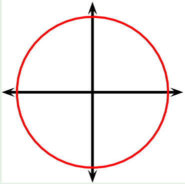

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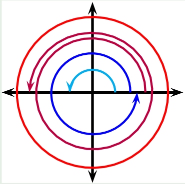

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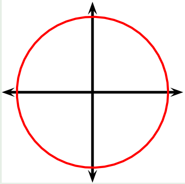

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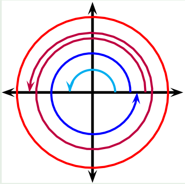

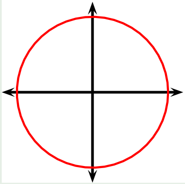

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$\sin\left(\frac{7}{2}x\right) = 0$

$\frac{7}{2}x = k\pi$ $\qquad k$ – integer

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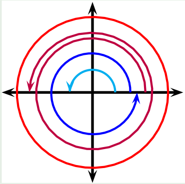

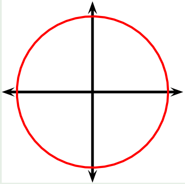

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$$x = \frac{2k\pi}{7}$$

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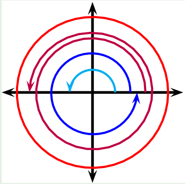

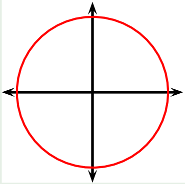

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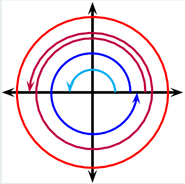

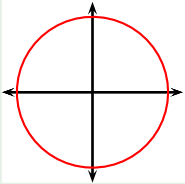

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$$\cos\left(\tfrac{3}{2}x\right) = 0$$

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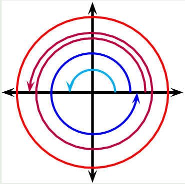
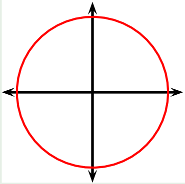
Example

Find all solutions in the interval $[0, 2\pi)$.





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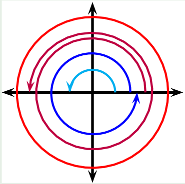

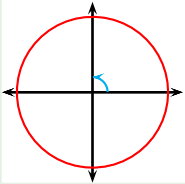

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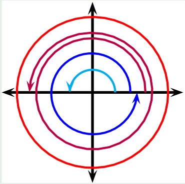

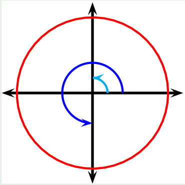

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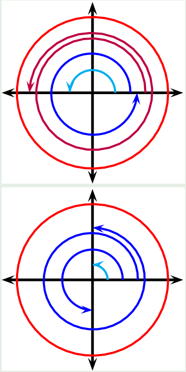

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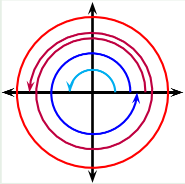
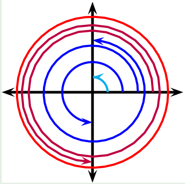

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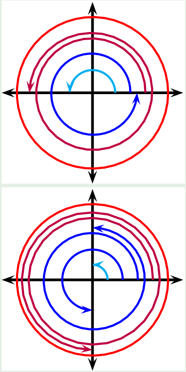

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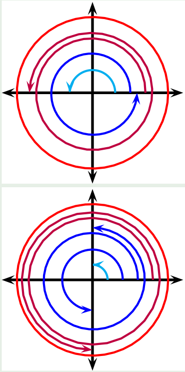

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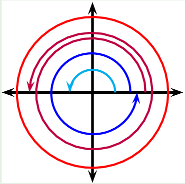

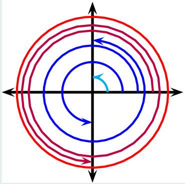

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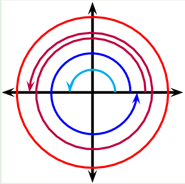

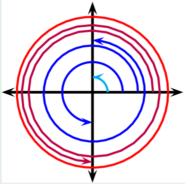

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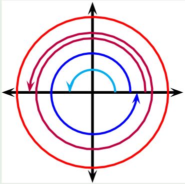

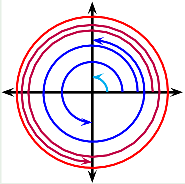

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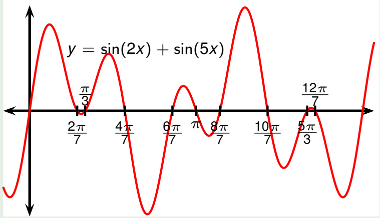
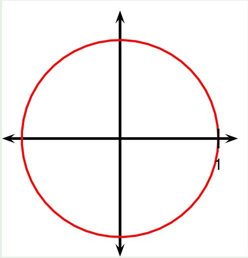
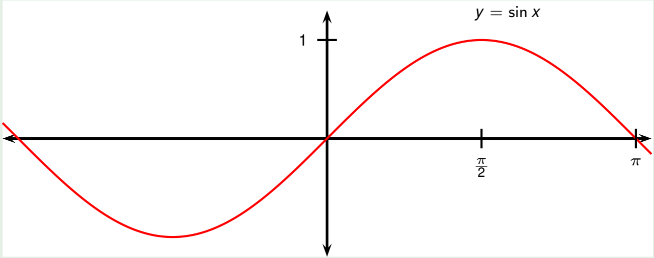


Example

Solve. Among your solutions, find those between -360° and 450°.

$\frac{1}{2} \leq \sin\theta < \frac{\sqrt{3}}{2}$





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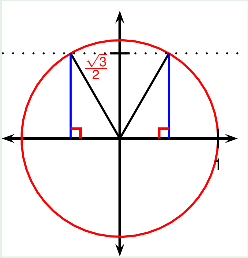

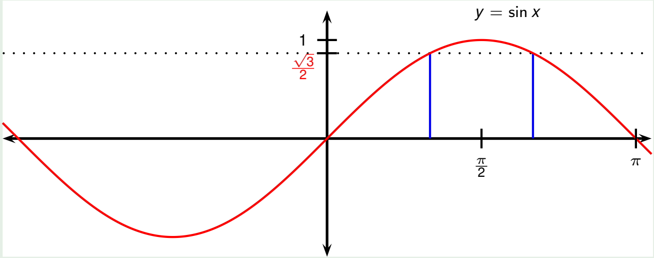

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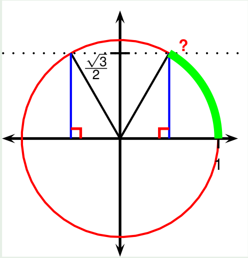

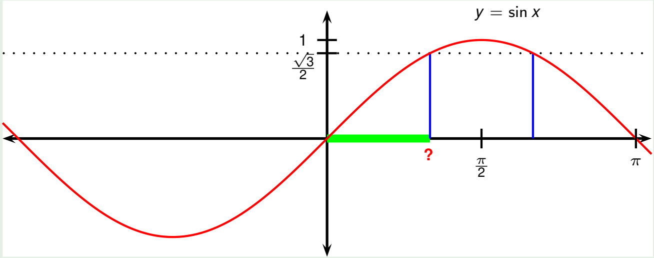

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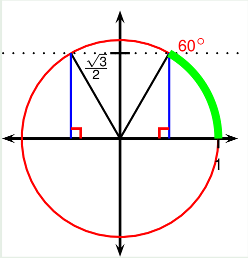

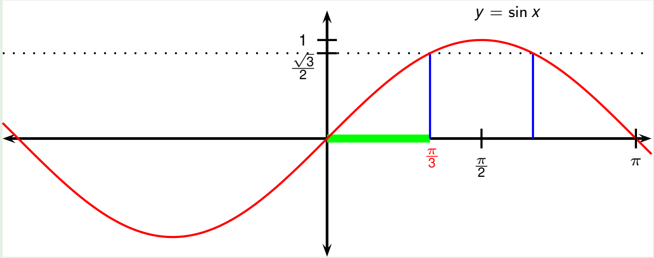

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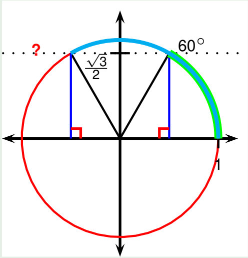

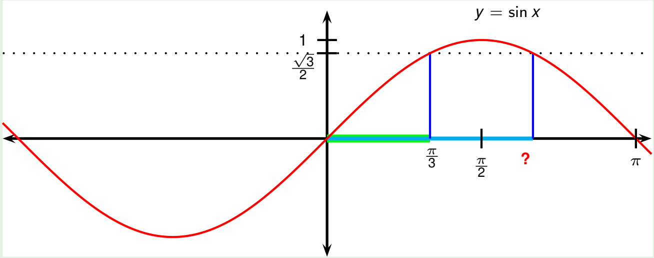

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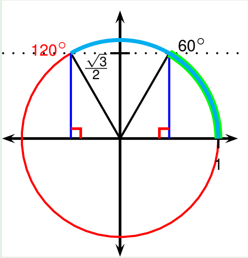

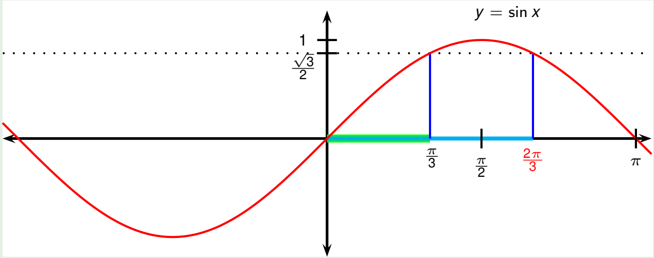

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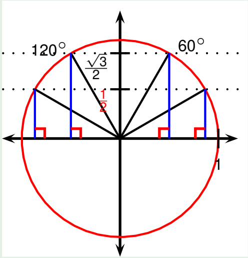

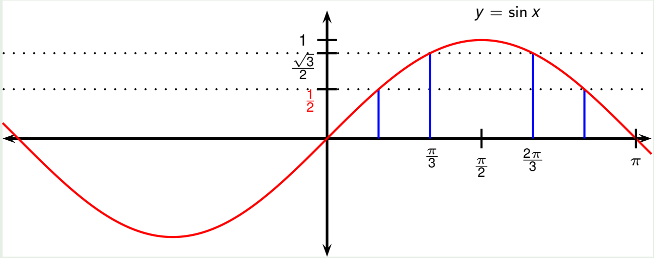

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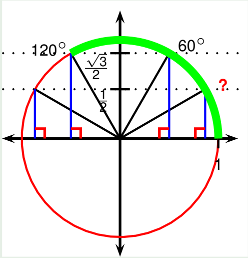

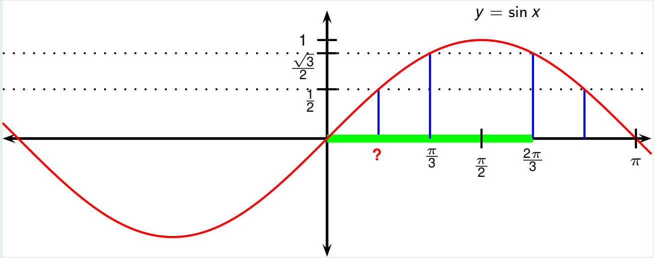

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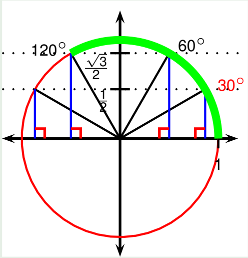

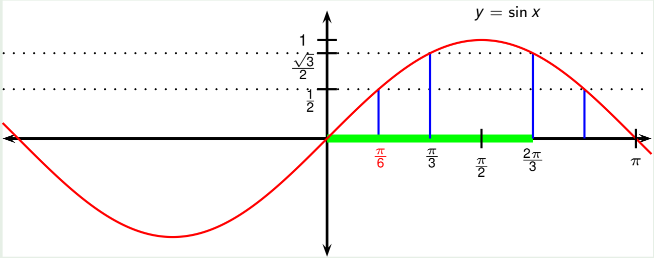

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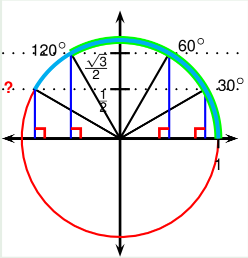

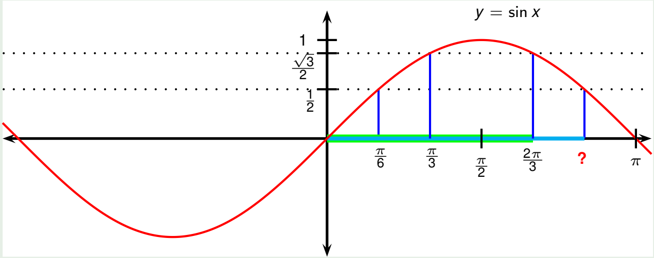

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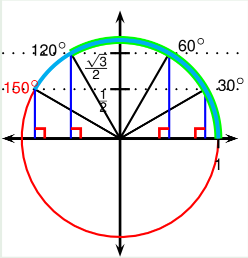
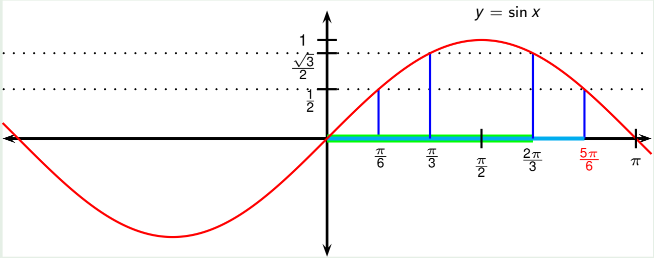


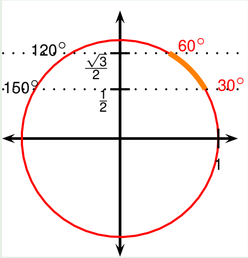


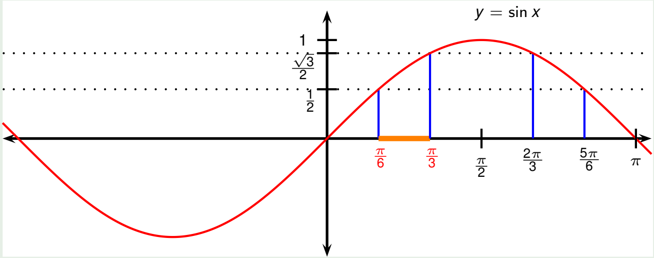
Example

Solve. Among your solutions, find those between -360° and 450°.

$\frac{1}{2} \leq \sin\theta < \frac{\sqrt{3}}{2}$

$x \in [30^\circ \quad , 60^\circ \quad)$





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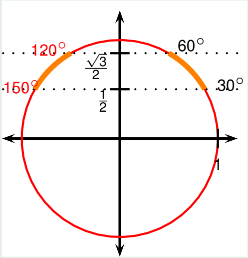
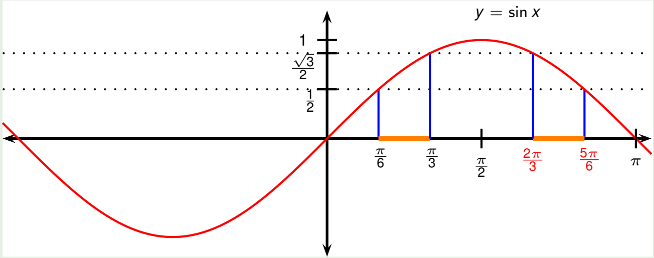
$x \in [30^\circ \quad , 60^\circ \quad) \cup ($ 120° $,$ 150° $]$





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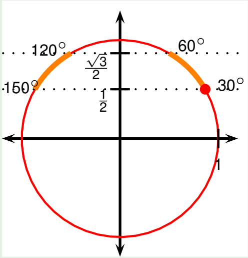

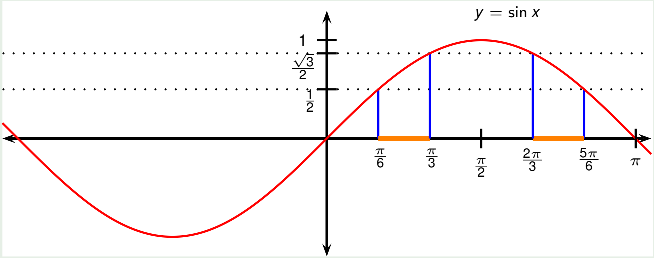

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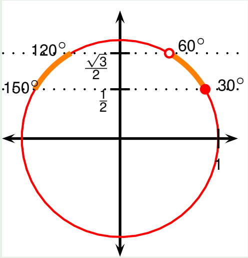

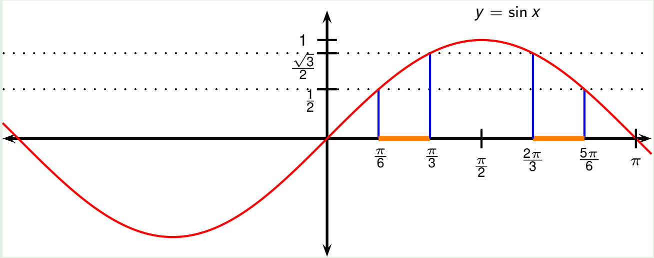

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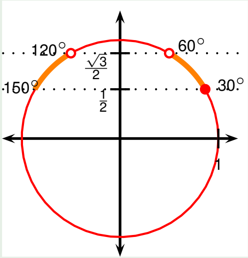

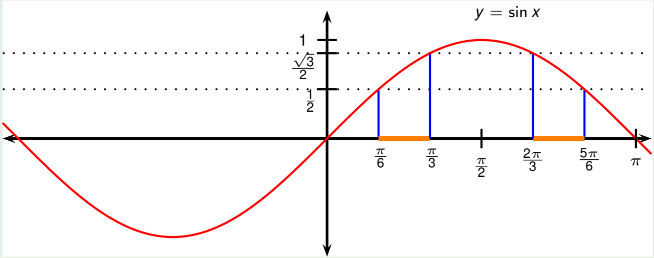

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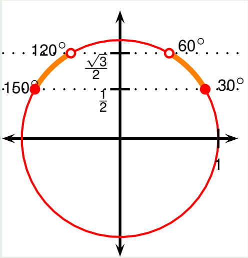

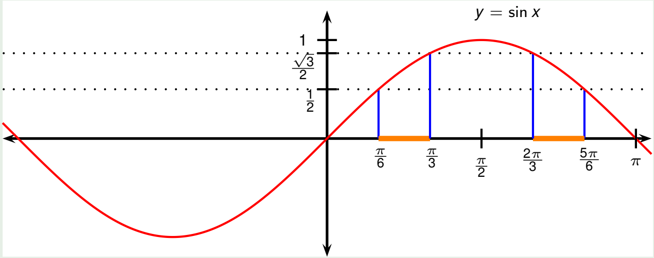

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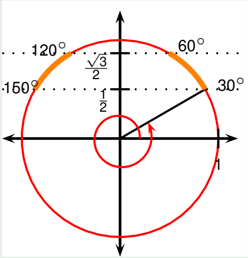

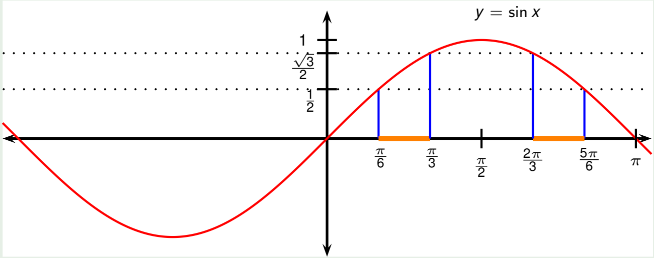

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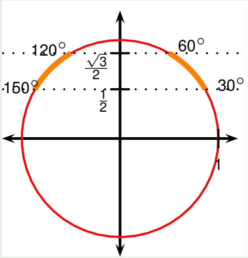

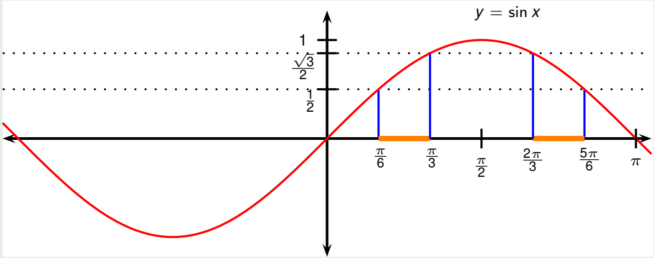

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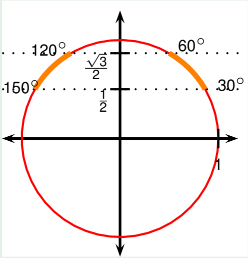
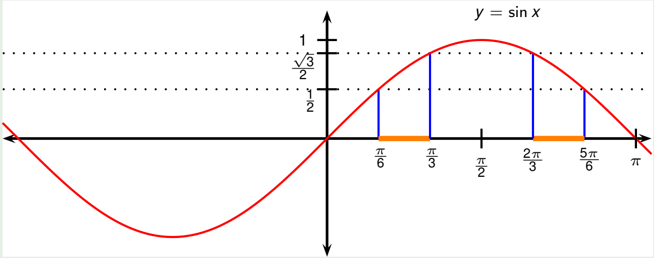
\end{array}
$$





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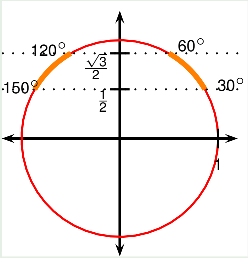

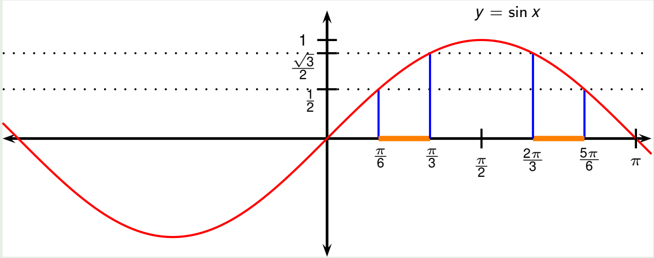

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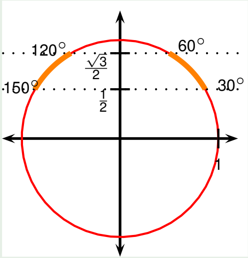

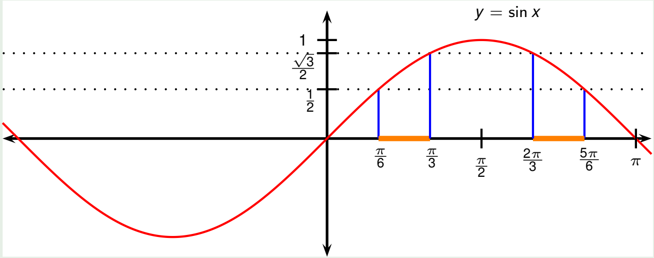

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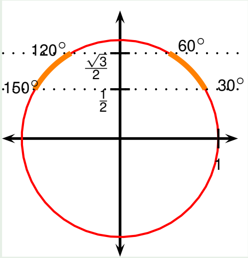

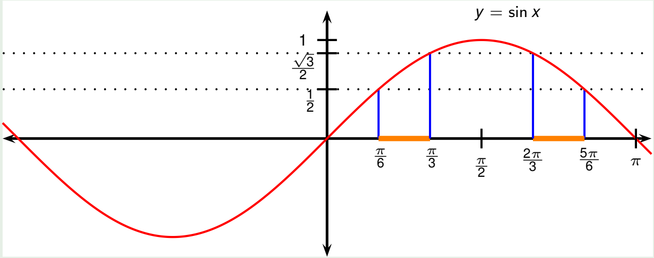

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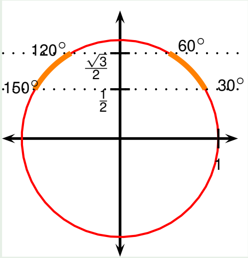

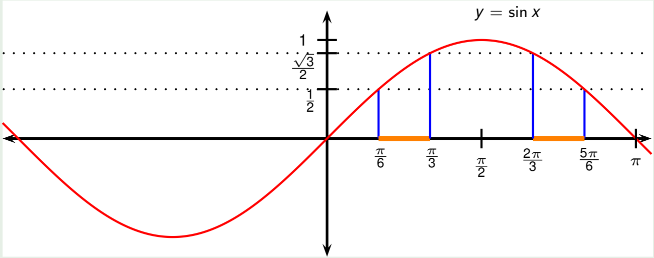

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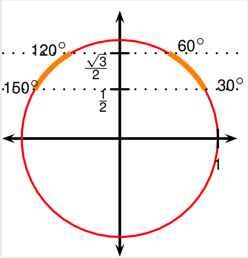

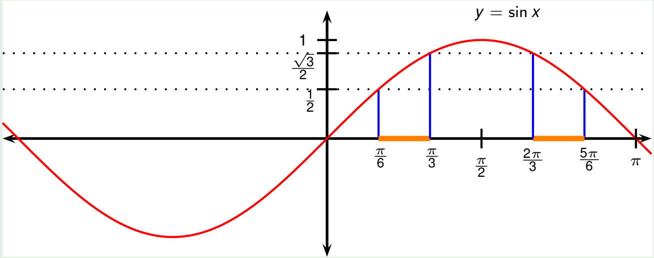

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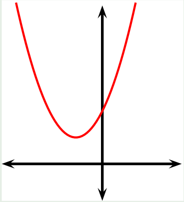
In radians:

$$x \in \left[-\frac{11\pi}{6}, -\frac{5\pi}{3}\right) \cup \left[-\frac{4\pi}{3}, -\frac{7\pi}{6}\right) \cup \left[\frac{\pi}{6}, \frac{\pi}{3}\right) \cup \left[\frac{2\pi}{3}, \frac{5\pi}{6}\right) \cup \left[\frac{13\pi}{6}, \frac{7\pi}{3}\right)$$

Example

- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2\cos^2\theta + 2\cos\theta + 1 \leq \cos\theta + 2$ lying in $[-360^\circ, 360^\circ]$.

Example



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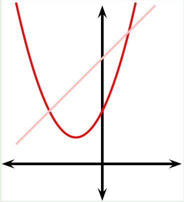

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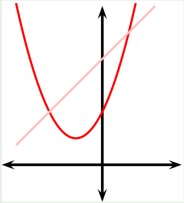

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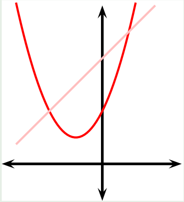

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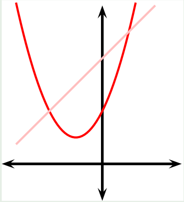

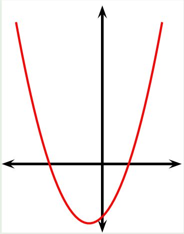

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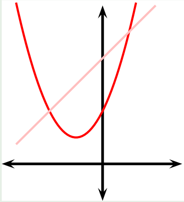

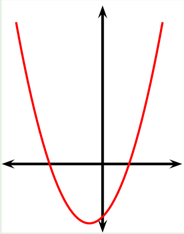

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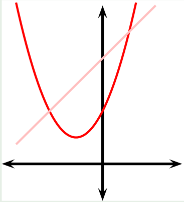

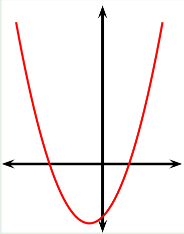

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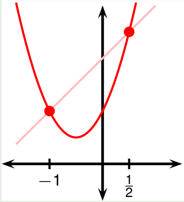
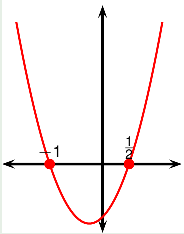
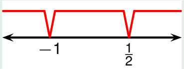
Example







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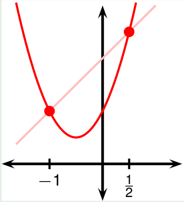

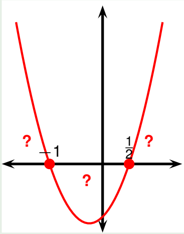

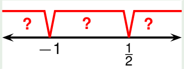

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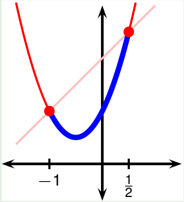
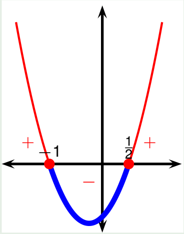
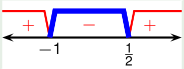
Example







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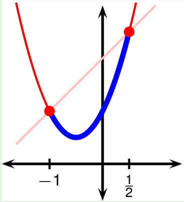

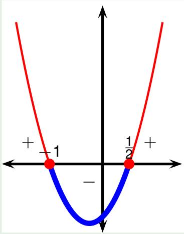

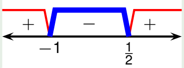

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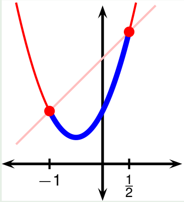

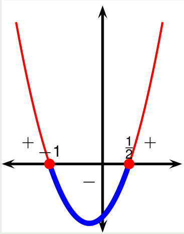

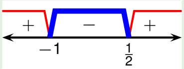

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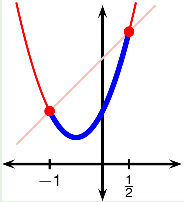

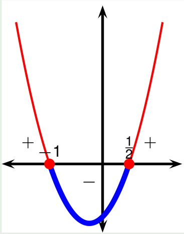

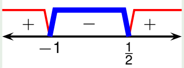

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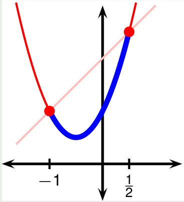

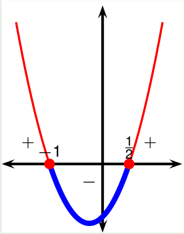

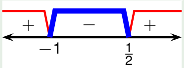

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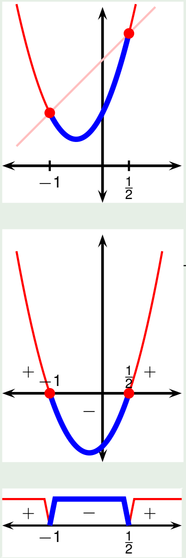

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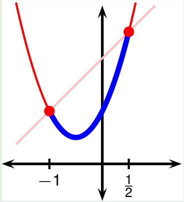

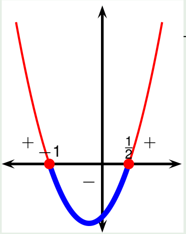

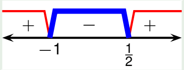

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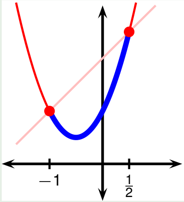

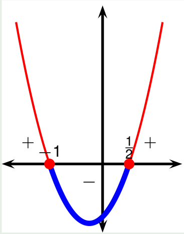

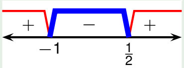

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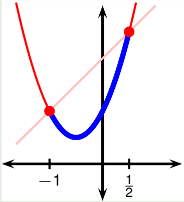

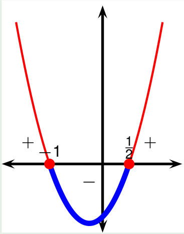

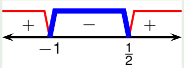

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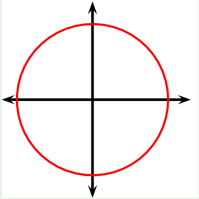

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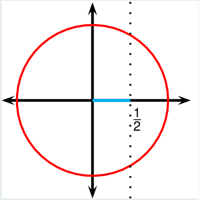
Example



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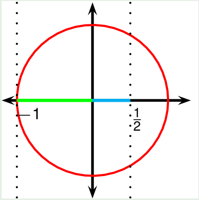

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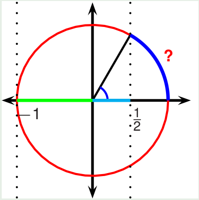

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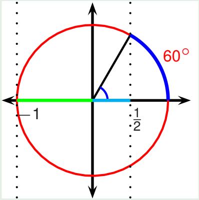

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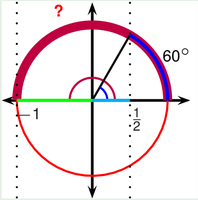
Example



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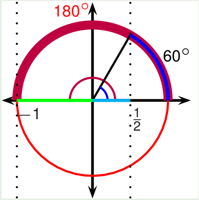

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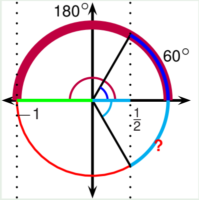
$\theta \in \quad [60° \quad , 180° \quad]$

Example



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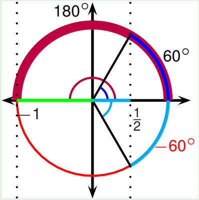
Example



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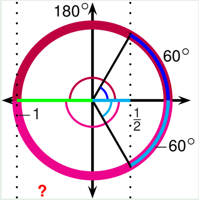
Example



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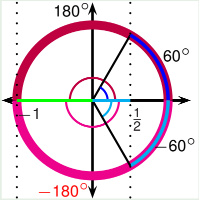
Example



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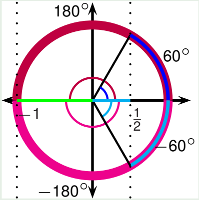
$$\theta \in [-180°, -60°] \cup [60°, 180°]$$

Example



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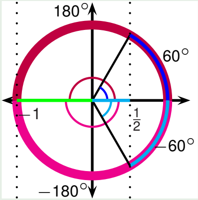
$$\theta \in [-180° + k360°, -60° + k360°] \cup [60° + k360°, 180° + k360°]$$

Example



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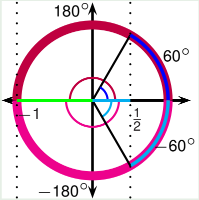
$$\theta \in [-180^\circ + k360^\circ, -60^\circ + k360^\circ] \cup [60^\circ + k360^\circ, 180^\circ + k360^\circ]$$

Example



- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
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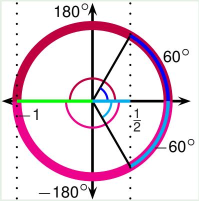
$$\theta \in [-180° + k360°, -60° + k360°] \cup [60° + k360°, 180° + k360°]$$

$$\theta \in [-180°, -60°] \cup [60°, 180°] \qquad k = 0$$

Example



- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2\cos^2\theta + 2\cos\theta + 1 \leq \cos\theta + 2$ lying in $[-360°, 360°]$.

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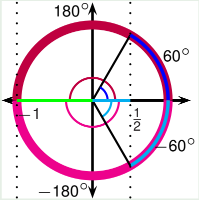
$$\begin{aligned} \theta \in \quad & [-180°, -60°] \cup [60°, 180°] && k = 0 \\ & \cup [180°, 300°] \cup [420°, 540°] && k = 1 \end{aligned}$$

Example



- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2\cos^2\theta + 2\cos\theta + 1 \leq \cos\theta + 2$ lying in $[-360°, 360°]$.

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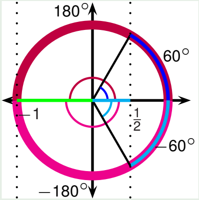
$$\begin{array}{lll} \theta \in & [-180°, -60°] \cup [60°, 180°] & k = 0 \\ & \cup [180°, 300°] \cup [420°, 540°] & k = 1 \\ & \ldots & \end{array}$$

Example



- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
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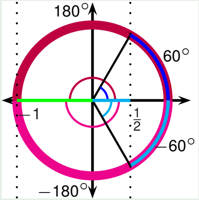
$$\theta \in [-180°+k360°, -60°+k360°] \cup [60°+k360°, 180°+k360°]$$

$$\theta \in \begin{array}{rl} [-540°, -420°] \cup [-300°, -180°] & k=-1 \\ \cup [-180°, -60°] \cup [60°, 180°] & k=0 \\ \cup [180°, 300°] \cup [420°, 540°] & k=1 \\ \dots & \end{array}$$

Example



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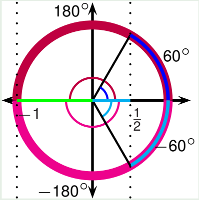
$$\theta \in \begin{array}{rcll} & \dots & & \\ \cup [-540^\circ, -420^\circ] & \cup & [-300^\circ, -180^\circ] & k = -1 \\ \cup [-180^\circ, -60^\circ] & \cup & [60^\circ, 180^\circ] & k = 0 \\ \cup [180^\circ, 300^\circ] & \cup & [420^\circ, 540^\circ] & k = 1 \\ & \dots & & \end{array}$$

Example



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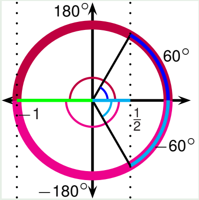
Example



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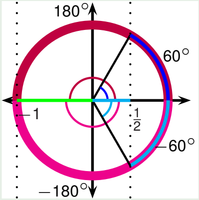
$$\theta \in [-300°, -60°] \cup [60°, 300°]$$

Example



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$$\cdots$$

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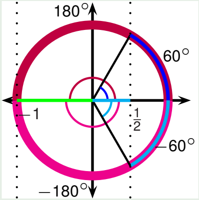
Example



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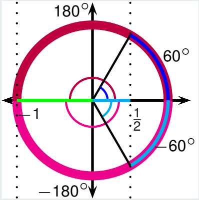
$$\color{red}{\theta \in [-300°, -60°] \cup [60°, 300°]}$$

Example



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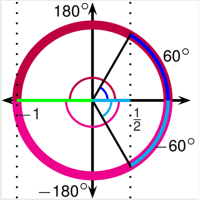
Example



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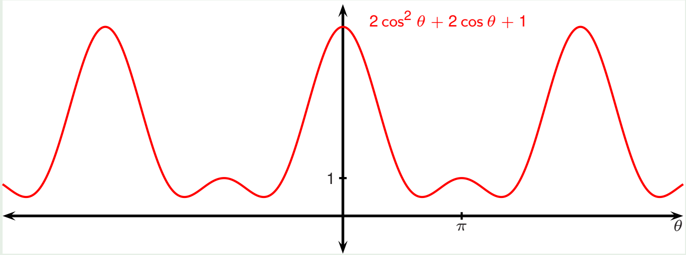
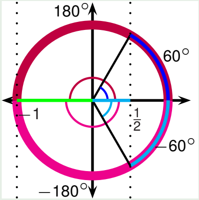


Example



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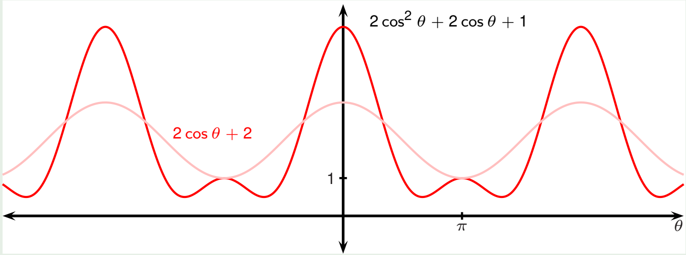
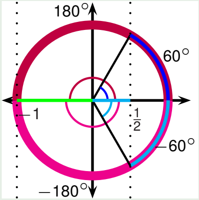


Example



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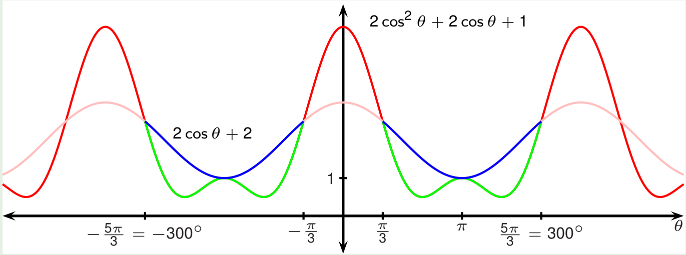
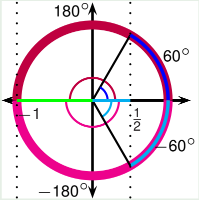


Example



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