Precalculus
Solve triangle from side and two angles

Todor Milev

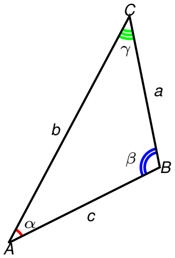
2019

Law of sines

Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ, as indicated: α is opposite to a, β is opposite to b, γ is opposite to c.

Proposition (Law of Sines)

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}.$$



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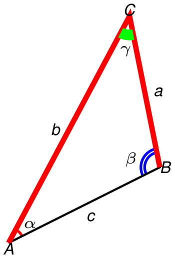

Proof.

$$\text{Area}(\triangle ABC) = \frac{ab\sin\gamma}{2} = \frac{bc\sin\alpha}{2}$$

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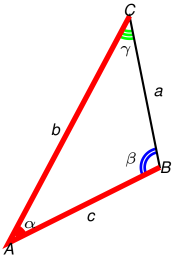

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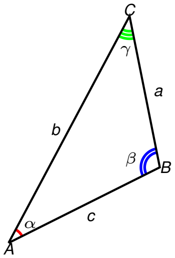

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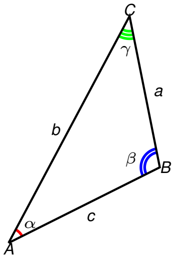

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$$\begin{aligned} \text{Area}(\triangle ABC) = \frac{ab\sin\gamma}{2} &= \frac{bc\sin\alpha}{2} && \Big| \text{ Div. by } \frac{b}{2} \\ a\sin\gamma &= c\sin\alpha \end{aligned}$$

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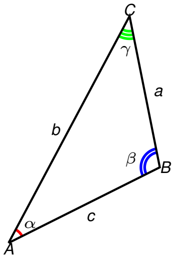

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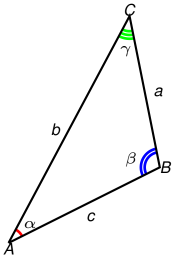

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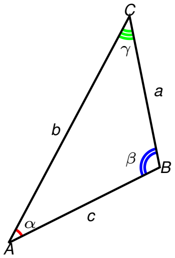

Proof.

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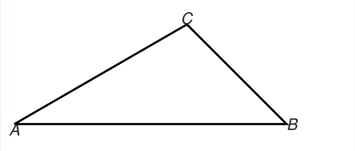
The remaining cases are similar. $\square$

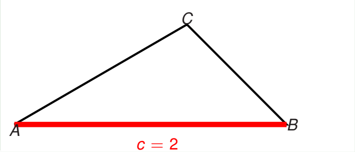
Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

Example



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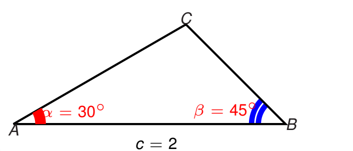
- Let the known side be $c = 2cm$.

Example



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- Find the other two sides of the triangle.
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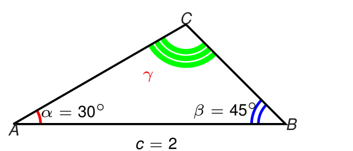
- Let the known side be $c = 2cm$.
- Let the known angles 30°, 45° be arranged as in the figure

Example

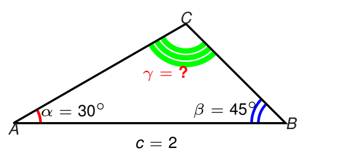


A triangle has a side of length $2cm$; the two angles adjacent to it are $30°$ and $45°$.

- Find the other two sides of the triangle.
- Find the area of the triangle.

- Let the known side be $c = 2cm$.
- Let the known angles $30°$, $45°$ be arranged as in the figure, and let the third angle be γ .

Example



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- Find the other two sides of the triangle.
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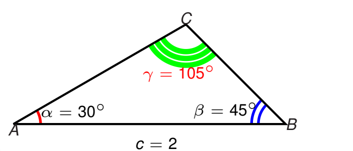
- Let the known side be $c = 2cm$.
- Let the known angles 30°, 45° be arranged as in the figure, and let the third angle be $\gamma = ?$.

Example



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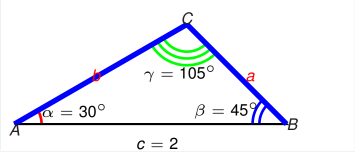
Example



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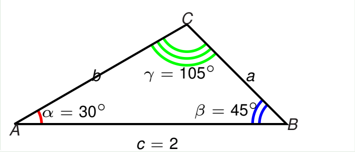
- Label the unknown sides a, b as indicated.

Example



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$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

Law of sines

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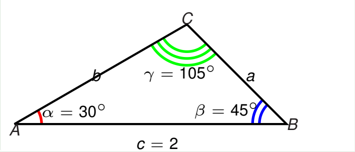

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$$a = \frac{c\sin\alpha}{\sin\gamma}$$

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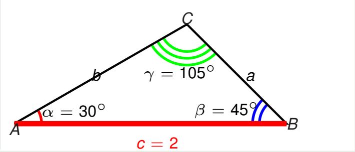

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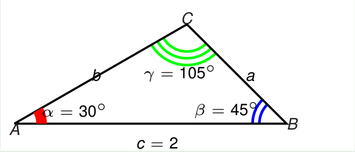
$$a = \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30°}{\sin 105°}$$

Example



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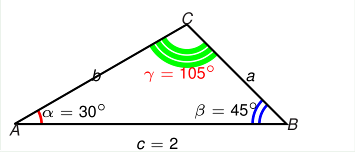

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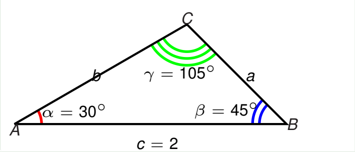

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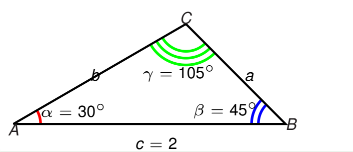

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$$\sin 105^\circ = \sin(60^\circ + 45^\circ)$$

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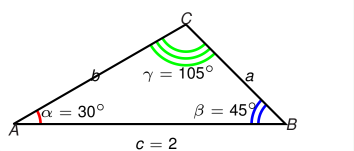

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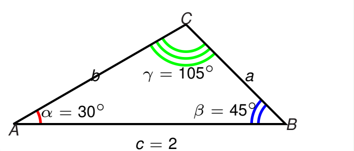

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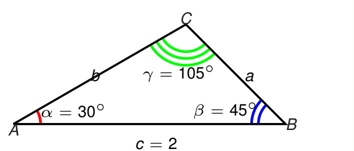

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&= ? \quad ? \quad + ?\ ? \\
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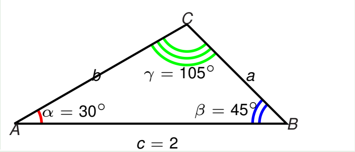

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\sin 105^\circ &= \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ \\
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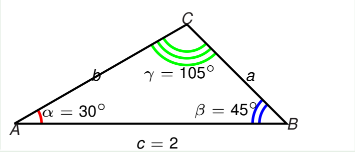

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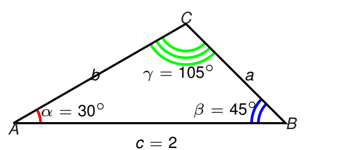

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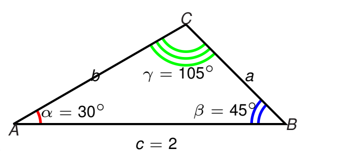

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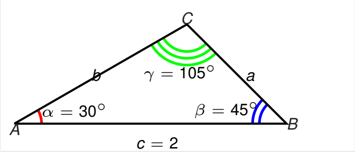

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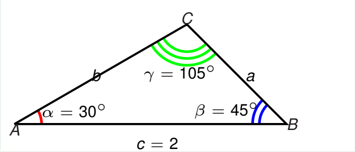

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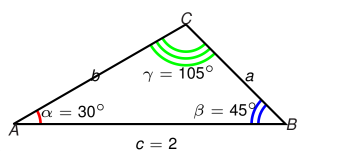

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\frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} && |\text{Law of sines} \\
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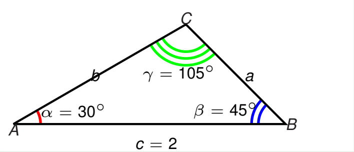

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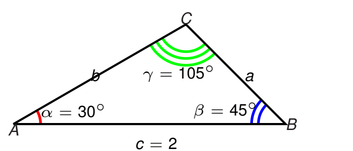
&= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4} \\
\frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} && |\text{Law of sines} \\
a &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ}
\end{aligned}$$

Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
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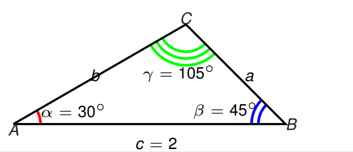

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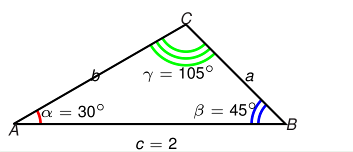
Example



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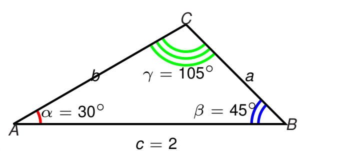

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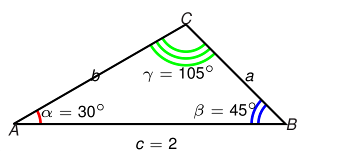

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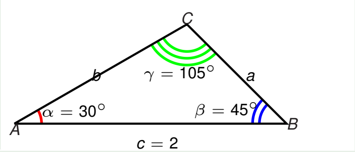
Example



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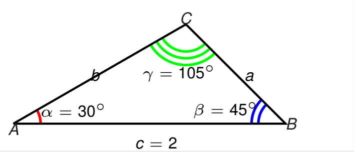
\frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} \qquad\qquad |\text{Law of sines} \\
a &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ} = \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{4}{(\sqrt{6}+\sqrt{2})}
\end{aligned}$$

Example



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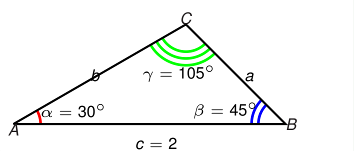
\end{aligned}
$$

Example



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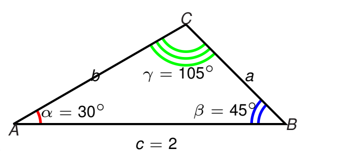
$$= \frac{4(\sqrt{6}-\sqrt{2})}{6-2}$$

Example



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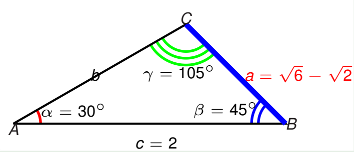
&= \frac{4(\sqrt{6}-\sqrt{2})}{6-2} = \sqrt{6}-\sqrt{2}
\end{aligned}
$$

Example



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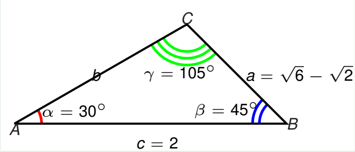
&= \frac{4(\sqrt{6}-\sqrt{2})}{6-2} {\color{red}= \sqrt{6}-\sqrt{2}}
\end{aligned}$$

Example



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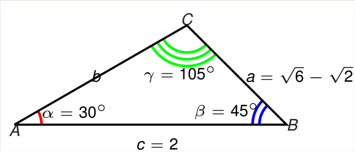
|Law of sines

Example



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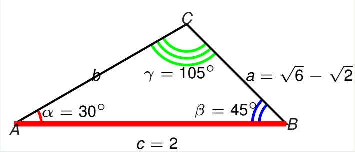
Example



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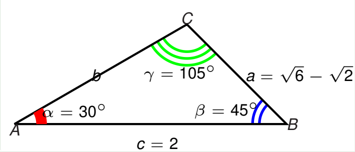
Example



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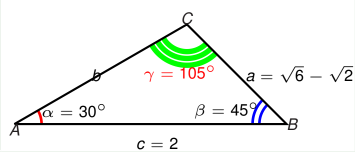

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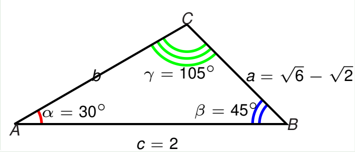
Example



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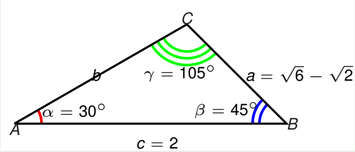
Example



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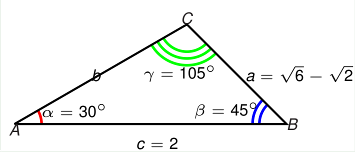

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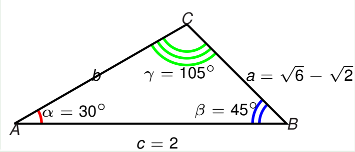

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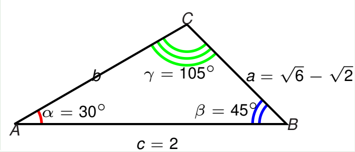
Example



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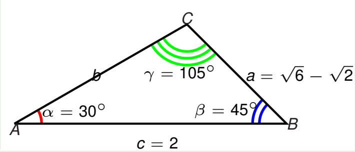
Example



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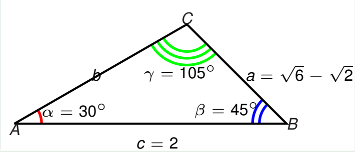
Example



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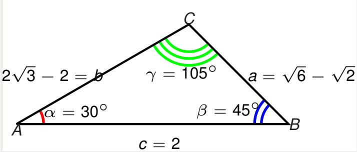
$$= \frac{\cancel{4}\sqrt{2}(\sqrt{6}-\sqrt{2})}{\cancel{4}}$$

Example



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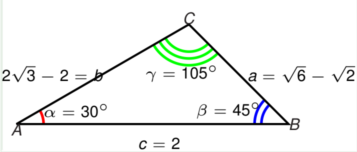
b &= \frac{c\sin\beta}{\sin\gamma} = \frac{2\sin 45^\circ}{\sin 105^\circ} = \frac{\cancel{2}\frac{\sqrt{2}}{\cancel{2}}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{4\sqrt{2}\left(\sqrt{6}-\sqrt{2}\right)}{\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}-\sqrt{2}\right)}\\
&= \frac{\cancel{4}\sqrt{2}(\sqrt{6}-\sqrt{2})}{\cancel{4}} = 2\sqrt{3}-2
\end{aligned}$$

Example



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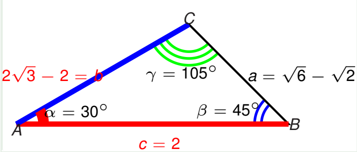
Example



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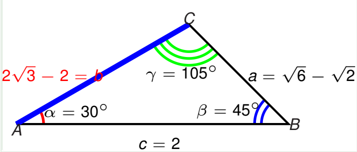
$$= \frac{\cancel{4}\sqrt{2}(\sqrt{6}-\sqrt{2})}{\cancel{4}} = 2\sqrt{3}-2$$

$$\text{Area} = \frac{bc\sin\alpha}{2}$$

Example



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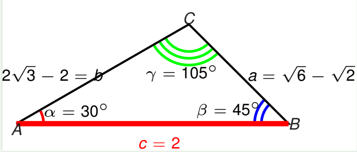
$$b = \frac{c\sin\beta}{\sin\gamma} = \frac{2\sin 45^\circ}{\sin 105^\circ} = \frac{\cancel{2}\frac{\sqrt{2}}{\cancel{2}}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{4\sqrt{2}\left(\sqrt{6}-\sqrt{2}\right)}{\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}-\sqrt{2}\right)}$$

$$= \frac{\cancel{4}\sqrt{2}(\sqrt{6}-\sqrt{2})}{\cancel{4}} = 2\sqrt{3}-2$$

$$\text{Area} = \frac{{\color{red}b}c\sin\alpha}{2} = \frac{({\color{red}2\sqrt{3}-2})2\frac{1}{2}}{2}$$

Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \qquad |\text{Law of sines}$$

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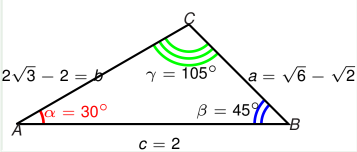
Example



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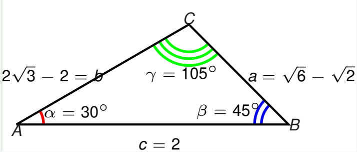
Example



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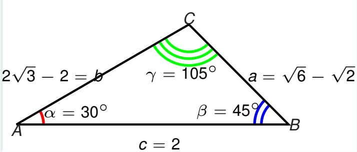
Example



A triangle has a side of length 2*cm*; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \qquad |\text{Law of sines}$$

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$$\text{Area} = \frac{bc\sin\alpha}{2} = \frac{(2\sqrt{3}-2)\cancel{2}\frac{1}{\cancel{2}}}{2} = \sqrt{3}-1 \quad \text{cm}^2$$