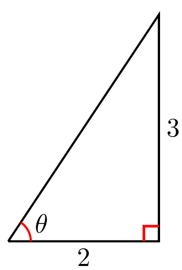
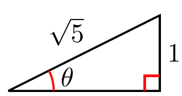
Precalculus
Homework
Definition of the trigonometric functions and basic computations

1. Find the 6 trigonometric functions of the indicated angle in the indicated right triangle.



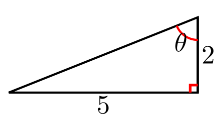
(a)

answer: $\sin\theta = \frac{3}{13}\sqrt{13}$, $\cos\theta = \frac{2}{13}\sqrt{13}$, $\tan\theta = \frac{3}{2}$, $\cot\theta = \frac{2}{3}$, $\sec\theta = \frac{\sqrt{13}}{2}$, $\csc\theta = \frac{\sqrt{13}}{3}$



(b)

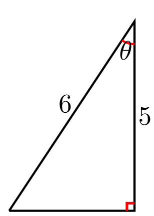
answer: $\sin\theta = \frac{\sqrt{5}}{5}$, $\cos\theta = \frac{2\sqrt{5}}{5}$, $\tan\theta = \frac{1}{2}$, $\cot\theta = 2$, $\sec\theta = \frac{\sqrt{5}}{2}$, $\csc\theta = \sqrt{5}$



(c)

answer: $\sin\theta = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29}$, $\cos\theta = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$, $\tan\theta = \frac{5}{2}$, $\cot\theta = \frac{2}{5}$, $\sec\theta = \frac{\sqrt{29}}{2}$, $\csc\theta = \frac{\sqrt{29}}{5}$

(d)



answer: $\sin\theta = \frac{\sqrt{11}}{6}$, $\cos\theta = \frac{5}{6}$, $\tan\theta = \frac{\sqrt{11}}{5}$, $\cot\theta = \frac{5}{\sqrt{11}} = \frac{5\sqrt{11}}{11}$, $\sec\theta = \frac{6}{5}$, $\csc\theta = \frac{6}{\sqrt{11}} = \frac{6\sqrt{11}}{11}$

2. Find the exact value of the trigonometric function (using radicals).

(a) $\cos 135°$.

answer:

(b) $\sin 225°$.

answer:

(c) $\cos 495°$.

answer:

(d) $\sin 560°$.

answer:

(e) $\sin\left(\frac{3\pi}{2}\right)$.

answer:

(f) $\cos\left(\frac{11\pi}{6}\right)$.

answer:

(g) $\sin\left(\frac{2015\pi}{3}\right)$.

answer:

(h) $\cos\left(\frac{17\pi}{3}\right)$.

answer:

3. Find all solutions of the equation in the interval $[0, 2\pi)$. The answer key has not been proofread, use with caution.

(a) $\sin x = -\frac{\sqrt{2}}{2}$.

answer: $x = \frac{5\pi}{4}, \frac{7\pi}{4}$

(b) $\cos x = \frac{\sqrt{3}}{2}$.

answer: $x = \frac{\pi}{6}, \frac{11\pi}{6}$

(c) $\sin(3x) = \frac{1}{2}$.

answer: $x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{16\pi}{9}$

(d) $\cos(7x) = 0$.

answer: $x = \frac{\pi}{14}, \frac{3\pi}{14}, \frac{5\pi}{14}, \frac{\pi}{2}, \frac{9\pi}{14}, \frac{11\pi}{14}, \frac{13\pi}{14}, \frac{15\pi}{14}, \frac{17\pi}{14}, \frac{19\pi}{14}, \frac{3\pi}{2}, \frac{23\pi}{14}, \frac{25\pi}{14}, \frac{27\pi}{14}$

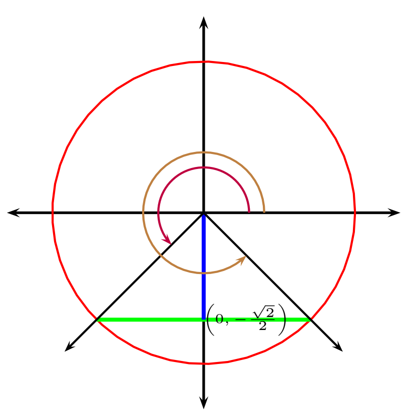
(e) $\cos\left(3x + \frac{\pi}{2}\right) = 0$.

answer: $x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

(f) $\sin\left(5x - \frac{\pi}{3}\right) = 0$.

answer: $x = \frac{\pi}{15}, \frac{4\pi}{15}, \frac{7\pi}{15}, \frac{2\pi}{3}, \frac{13\pi}{15}, \frac{16\pi}{15}, \frac{19\pi}{15}, \frac{22\pi}{15}, \frac{5\pi}{3}, \frac{28\pi}{15}$

Solution. 3.a



$$\sin x = -\frac{\sqrt{2}}{2}$$

Since $\sin x$ is negative it must be either in Quadrant III or IV. Therefore the angle x is coterminal either with $225° = \frac{5\pi}{4}$ (Quadrant III) or $315° = \frac{7\pi}{4}$ (Quadrant IV).

Case 1. x is coterminal with $225° = \frac{5\pi}{4}$. We can compute

$$\begin{array}{rcl|l} x &=& \dfrac{5\pi}{4} + 2k\pi & k \text{ is any integer} \\ x &=& \dfrac{5\pi}{4} + \dfrac{8k\pi}{4} \\ x &=& \dfrac{5\pi + 8k\pi}{4} \\ x &=& \dfrac{\pi(5+8k)}{4} \end{array}$$

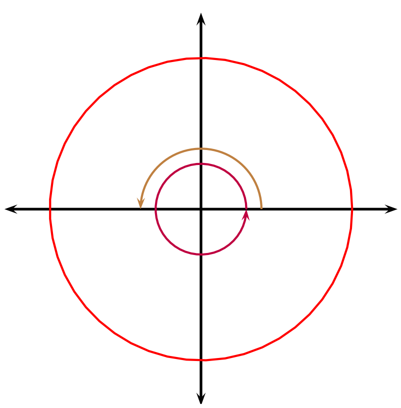
We are looking for solutions in the interval $[0, 2\pi)$ and so we must discard those values of the integer k for which $\frac{\pi(7+8k)}{4}$ is negative or is greater than or equal to 2π. Therefore the only solution in this case is $x = \frac{5\pi}{4}$.

Case 2.

$$\begin{array}{rcl} x &=& \dfrac{7\pi}{4} + 2k\pi \\ x &=& \dfrac{7\pi}{4} + \dfrac{8k\pi}{4} \\ x &=& \dfrac{7\pi + 8k\pi}{4} \\ x &=& \dfrac{\pi(7+8k)}{4} \end{array}$$

We are looking for solutions in the interval $[0, 2\pi)$ and so we must discard those values of the integer k for which $\frac{\pi(7+8k)}{4}$ is negative or is greater than or equal to 2π. Therefore the only solution in this case is $x = \frac{7\pi}{4}$.

Solution. 3.f



$$\sin\left(5x - \frac{\pi}{3}\right) = 0$$

Since $\sin 0 = 0$ and $\sin 180° = \sin\pi = 0$, the angle $5x - \frac{\pi}{3}$ must be coterminal with 0 or π.

Case 1. $5x - \frac{\pi}{3}$ is coterminal with 0. We compute

$$
\begin{aligned}
5x - \frac{\pi}{3} &= 0 + 2k\pi \\
5x &= \frac{\pi}{3} + 2k\pi \\
x &= \frac{\frac{\pi}{3} + 2k\pi}{5} \\
x &= \frac{\frac{\pi}{3} + \frac{6k\pi}{3}}{5} \\
x &= \frac{\frac{\pi + 6k\pi}{3}}{5} \\
x &= \frac{\pi + 6k\pi}{15} \\
x &= \frac{\pi(1 + 6k)}{15} \\
x &= \cancel{\ldots}, \frac{\pi[1+6(0)]}{15}, \frac{\pi[1+6(1)]}{15}, \frac{\pi[1+6(2)]}{15}, \frac{\pi(1+12)}{15}, \frac{\pi[1+6(3)]}{15}, \frac{\pi[1+6(4)]}{15}, \cancel{\ldots} \\
x &= \frac{\pi}{15}, \frac{7\pi}{15}, \frac{13\pi}{15}, \frac{19\pi}{15}, \frac{25\pi}{15}.
\end{aligned}
$$

Discard other values of k as they yield angles outside of $[0, 2\pi)$

Case 2.

$$
\begin{aligned}
5x - \frac{\pi}{3} &= \pi + 2k\pi \\
5x &= \pi + \frac{\pi}{3} + 2k\pi \\
5x &= \frac{4\pi}{3} + 2k\pi \\
x &= \frac{\frac{4\pi}{3} + 2k\pi}{5} \\
x &= \frac{\frac{4\pi}{3} + \frac{6k\pi}{3}}{5} \\
x &= \frac{\frac{4\pi + 6k\pi}{3}}{5} \\
x &= \frac{4\pi + 6k\pi}{15} \\
x &= \frac{2\pi(2 + 3k)}{15} \\
x &= \cancel{\ldots}, \frac{2\pi[2+3(0)]}{15}, \frac{2\pi[2+3(1)]}{15}, \frac{2\pi[2+3(2)]}{15}, \frac{2\pi[2+3(3)]}{15}, \frac{2\pi[2+3(4)]}{15}, \cancel{\ldots} \\
x &= \frac{4\pi}{15}, \frac{10\pi}{15}, \frac{16\pi}{15}, \frac{22\pi}{15}, \frac{28\pi}{15}.
\end{aligned}
$$

Discard other values of k as they yield angles outside of $[0, 2\pi)$

Our final answer (combined from the two cases) is $x = \frac{\pi}{15}, \frac{4\pi}{15}, \frac{7\pi}{15}, \frac{2\pi}{3}, \frac{13\pi}{15}, \frac{16\pi}{15}, \frac{19\pi}{15}, \frac{22\pi}{15}, \frac{5\pi}{3}$ or $\frac{28\pi}{15}$.