Calculus II
Power series expansion of rational functions with linear denominator, part 1

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Representations of Functions as Power Series

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- It is convergent if **?** and divergent otherwise.

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- In this way $g(x) = \sum\limits_{n=0}^{\infty} x^n$ is a new way to compute/expresses the function $f(x) = \frac{1}{1-x}$ for $|x| < 1$.
- Except for their domains, the functions $g(x)$ and $f(x)$ coincide.