Precalculus
Tangent angle sum formula

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2019

Example

Prove the angle sum formula $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$.

$$
\begin{aligned}
\tan(\alpha + \beta) &= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} \\
&= \frac{(\sin \alpha \cos \beta + \cos \alpha \sin \beta) \frac{1}{\cos \alpha \cos \beta}}{(\cos \alpha \cos \beta - \sin \alpha \sin \beta) \frac{1}{\cos \alpha \cos \beta}} \\
&= \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} + \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} - \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}} \\
&= \frac{\frac{\sin \alpha}{\cos \alpha} + \frac{\sin \beta}{\cos \beta}}{1 - \frac{\sin \alpha}{\cos \alpha} \cdot \frac{\sin \beta}{\cos \beta}} \\
&= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}
\end{aligned}
$$