Calculus II L'Hospital's rule

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2019

Outline

- Indeterminate Forms and L'Hospital's Rule
 - Indeterminate Products
 - Indeterminate Differences
 - Indeterminate Powers

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Find
$$\lim_{x\to 1} \frac{\ln x}{x-1}$$
.

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- We don't get any cancellation between top and bottom.
- We need new techniques.

Theorem (L'Hospital's Rule)

Suppose that f and g are differentiable and $g'(x) \neq 0$ on an open interval that contains a (except possibly at a). Suppose that

and $\lim_{x\to a} g(x) = 0$

or that
$$\lim_{x\to a} f(x) = \pm \infty$$
 and $\lim_{x\to a} g(x) = \pm \infty$

 $\lim_{x\to a} f(x) = 0$

(In other words, we have an indeterminate form of type 0/0 or ∞/∞ .) Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}.$$

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Example

Find
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Example

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Indeterminate Products

If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = \pm \infty$, then it isn't clear what $\lim_{x\to a} (fg)(x)$ will be.

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In such a case, write the product fg as a quotient:

$$fg = \frac{f}{1/g}$$
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$$fg = \frac{f}{1/g}$$
 or $fg = \frac{g}{1/f}$.

This converts the given limit into an indeterminate form of type 0/0 or ∞/∞ .

Evaluate $\lim_{x\to 0^+} x \ln x$.

- $\bullet \lim_{x\to 0^+} \ln x =$
- $\bullet \lim_{x\to 0^+} x = .$

- $\bullet \lim_{x\to 0^+} \ln x = ? \quad .$
- $\bullet \lim_{x\to 0^+} x = .$

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Example

- $\bullet \lim_{x\to 0^+} \ln x = -\infty.$
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$$= \lim_{x \to 0^{+}} \frac{\frac{1}{x}}{-\frac{1}{\sqrt{2}}} = \lim_{x \to 0^{+}} (-x)$$

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L'Hospital's rule

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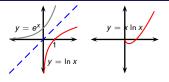
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Indeterminate Differences

If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$, then the limit

$$\lim_{x\to a}[f(x)-g(x)]$$

is called an indeterminate form of type $\infty - \infty$.

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is called an indeterminate form of type $\infty - \infty$.

To compute such a limit, try to convert it into a quotient (by using a common denominator, or by rationalizing, or by factoring out a common factor).

Evaluate $\lim_{x\to(\pi/2)^-}(\sec x - \tan x)$.

Evaluate $\lim_{x\to(\pi/2)^-} (\sec x - \tan x)$.

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- $\lim_{X\to(\pi/2)^-}\sec X=\infty$.
- $\lim_{X\to(\pi/2)^-}\tan x =$

Evaluate $\lim_{X\to(\pi/2)^-} (\sec X - \tan X)$.

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Indeterminate Powers

Several indeterminate forms arise from the limit $\lim_{x\to a} f(x)^{g(x)}$.

$$\lim_{x\to a} f(x) = 0$$
 and $\lim_{x\to a} g(x) = 0$ type 0^0

$$\lim_{x\to a} f(x) = \infty$$
 and $\lim_{x\to a} g(x) = 0$ type ∞^0

$$\lim_{x\to a} f(x) = 1$$
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These can all be solved either by taking the natural logarithm:

let
$$y = [f(x)]^{g(x)}$$
, then $\ln y = g(x) \ln f(x)$

or by writing the function as an exponential:

$$[f(x)]^{g(x)} = e^{g(x)\ln f(x)}.$$

Find $\lim_{x\to 0^+} x^x$.

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- Therefore

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$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x$$

$$\lim_{x \to \infty} \left(1 + \frac{k}{x} \right)^x = \lim_{x \to \infty} e^{\ln \left(1 + \frac{k}{x} \right)^x}$$

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 form "\frac{0}{0}", use L'Hospital
$$= \lim_{x \to \infty} \frac{1 + \frac{k}{x}}{1 + \frac{k}{x}} \left(1 + \frac{k}{x} \right)' - \frac{1}{x^2}$$

$$= \lim_{x \to \infty} \frac{k}{1 + \frac{k}{x}} = ?$$

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exponent= continuous f-n

L'Hospital's rule 2019

$$\lim_{x \to \infty} \left(1 + \frac{k}{x} \right)^x = \lim_{x \to \infty} e^{\ln(1 + \frac{k}{x})^x} \quad \text{exponent= continuous f-n}$$

$$= e^{\lim_{x \to \infty} \ln(1 + \frac{k}{x})^x} = e^k$$

$$\lim_{x \to \infty} \ln\left(1 + \frac{k}{x}\right)^x = \lim_{x \to \infty} x \ln\left(1 + \frac{k}{x}\right)$$

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Todor Milev

Example

$$\lim_{x \to \infty} \left(1 + \frac{k}{x} \right)^x = \lim_{x \to \infty} e^{\ln(1 + \frac{k}{x})^x}$$
 exponent= continuous formula to the continuous of the exponent of the continuous of the exponent of the exp

exponent= continuous f-n

limit computed below

L'Hospital's rule 2019