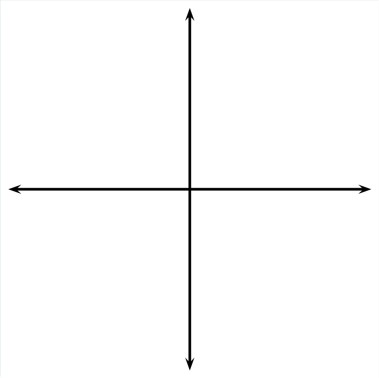
Calculus II
Area swept by one clover leaf $r = \sin(n\theta)$, $r = \cos(n\theta)$

Todor Milev

2019

Example

Find the area enclosed by one loop of the four-leaved rose $r = \cos 2\theta$.



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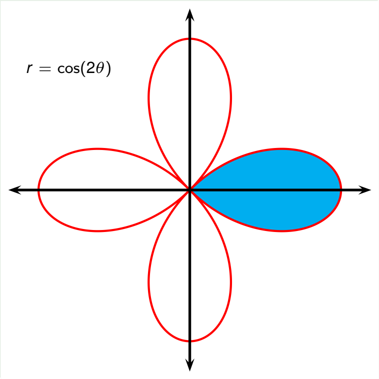

The region enclosed by the right loop corresponds to points whose θ polar coordinate lies in the interval

$$\leq \theta \leq \textbf{?} .$$

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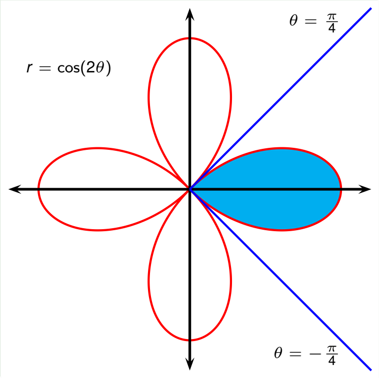

The region enclosed by the right loop corresponds to points whose θ polar coordinate lies in the interval $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$.

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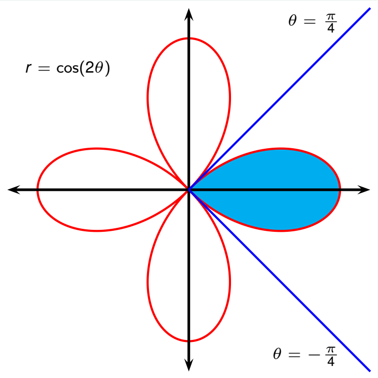

$$A = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2} r^2 \mathrm{d}\theta$$

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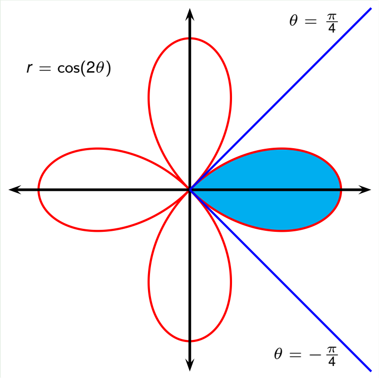

$$
\begin{aligned}
A &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2} r^2 \mathrm{d}\theta \\
&= \frac{1}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2(2\theta) \mathrm{d}\theta
\end{aligned}
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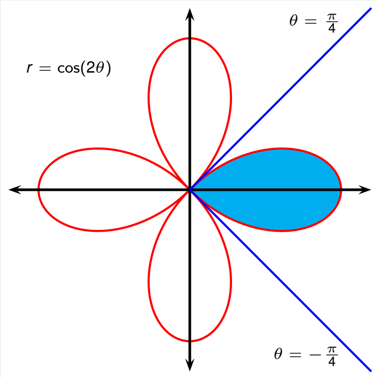

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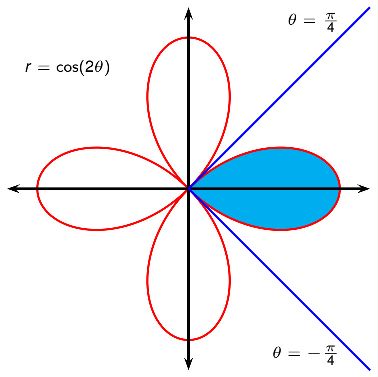

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&= \int_{0}^{\frac{\pi}{4}} \frac{1}{2}(1 + \cos(4\theta))\mathrm{d}\theta
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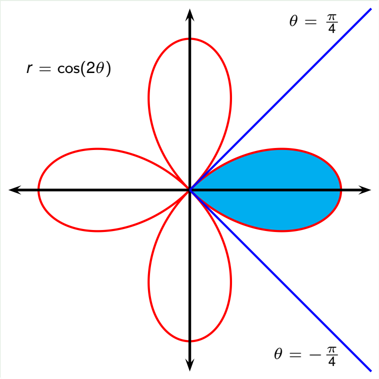

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&= \frac{1}{2} \left[\theta + \frac{1}{4} \sin(4\theta) \right]_{0}^{\frac{\pi}{4}}
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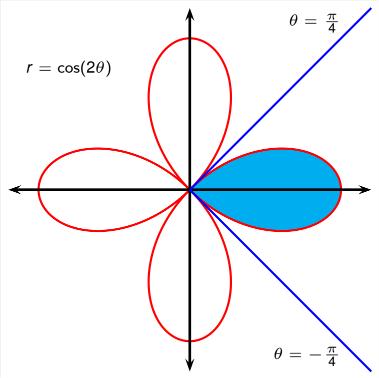

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&= \int_{0}^{\frac{\pi}{4}} \frac{1}{2}(1 + \cos(4\theta)) \mathrm{d}\theta \\
&= \frac{1}{2} \left[\theta + \frac{1}{4} \sin(4\theta) \right]_{0}^{\frac{\pi}{4}} \\
&= \frac{\pi}{8}
\end{aligned}
$$