

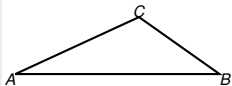
Precalculus

Solve triangle from two sides and an angle

Todor Milev

2019

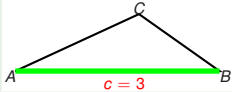
Example



The longest side of a triangle has length 3 and the angle opposite to it is 120° . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

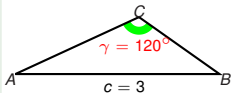
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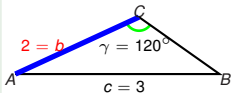
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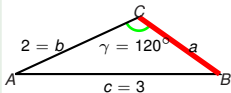
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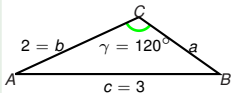
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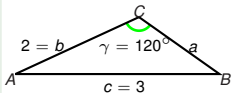
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| Law of cosines

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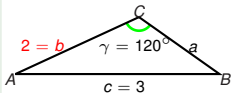
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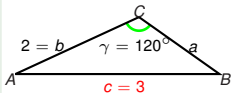
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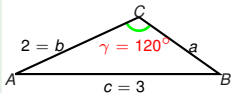
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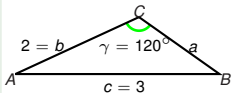
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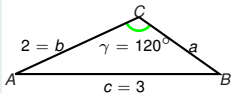
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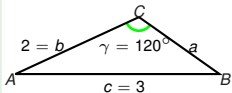
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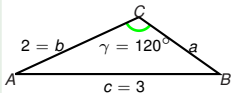
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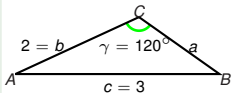
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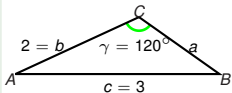
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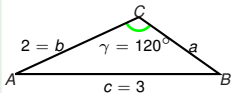
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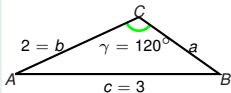
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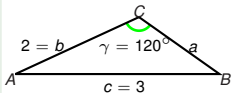
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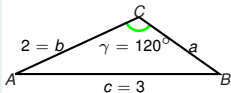
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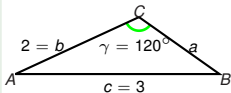
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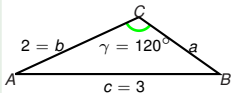
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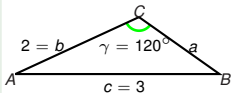
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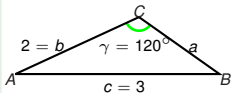
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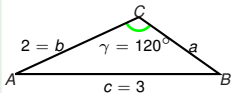
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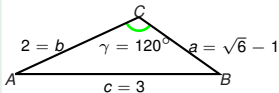
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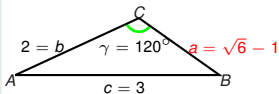
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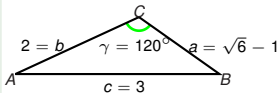
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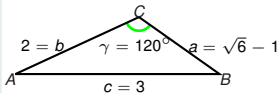
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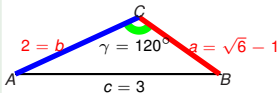
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Area = ?

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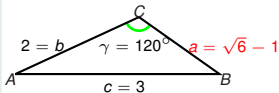
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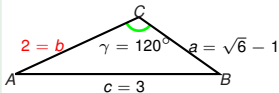
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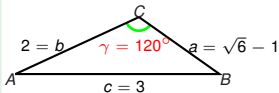
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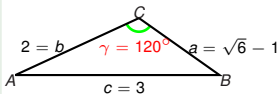
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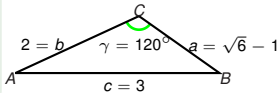
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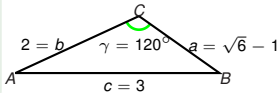
Law of cosines
Solve for a :

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$$= -1 + \sqrt{6}$$

$$\text{Area} = \frac{ab \sin \gamma}{2} = \frac{(\sqrt{6} - 1) \cancel{2} \sqrt{3}}{\cancel{2}} \frac{\sqrt{3}}{2}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120° . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

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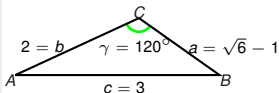
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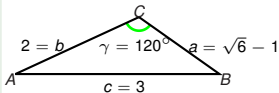
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