

Calculus I

Definite integral of $\frac{1}{(ax + b)^n}$

Todor Milev

2019

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = ?$.
- Then $du = ?$.

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = ?$.
- Then $du = ?$.

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = ?$.

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = ?$.

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = ?$.

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3} du$.

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3}du$.
- When $x = 1$, $u = ?$.
- When $x = 2$, $u = ?$.

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3}du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = ?$.

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3}du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = ?$.

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3} du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3}du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

$$\int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} = \int \quad \quad \quad$$

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3} du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

$$\int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} = \int \frac{1}{u^2}$$

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3}du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

$$\int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} = -\frac{1}{3} \int \frac{du}{u^2}$$

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3}du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

$$\int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} = -\frac{1}{3} \int_{u=-1}^{u=-4} \frac{du}{u^2}$$

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3} du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

$$\int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} = -\frac{1}{3} \int_{u=-1}^{u=-4} \frac{du}{u^2}$$

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3} du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

$$\int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} = -\frac{1}{3} \int_{u=-1}^{u=-4} \frac{du}{u^2} = -\frac{1}{3} \int_{-1}^{-4} u^{-2} du$$

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3} du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

$$\begin{aligned} \int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} &= -\frac{1}{3} \int_{u=-1}^{u=-4} \frac{du}{u^2} = -\frac{1}{3} \int_{-1}^{-4} u^{-2} du \\ &= -\frac{1}{3} \cdot \left[? \right]_{-1}^{-4} \end{aligned}$$

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3} du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

$$\begin{aligned} \int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} &= -\frac{1}{3} \int_{u=-1}^{u=-4} \frac{du}{u^2} = -\frac{1}{3} \int_{-1}^{-4} u^{-2} du \\ &= -\frac{1}{3} \cdot \left[-\frac{1}{u} \right]_{-1}^{-4} \end{aligned}$$

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3} du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

$$\begin{aligned} \int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} &= -\frac{1}{3} \int_{u=-1}^{u=-4} \frac{du}{u^2} = -\frac{1}{3} \int_{-1}^{-4} u^{-2} du \\ &= -\frac{1}{3} \cdot \left[-\frac{1}{u} \right]_{-1}^{-4} = \frac{1}{3} \left[\frac{1}{u} \right]_{-1}^{-4} \end{aligned}$$

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3}du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

$$\begin{aligned} \int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} &= -\frac{1}{3} \int_{u=-1}^{u=-4} \frac{du}{u^2} = -\frac{1}{3} \int_{-1}^{-4} u^{-2} du \\ &= -\frac{1}{3} \cdot \left[-\frac{1}{u} \right]_{-1}^{-4} = \frac{1}{3} \left[\frac{1}{u} \right]_{-1}^{-4} \\ &= \frac{1}{3} \left(\frac{1}{-4} - \frac{1}{-1} \right) \end{aligned}$$

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3} du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

$$\begin{aligned} \int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} &= -\frac{1}{3} \int_{u=-1}^{u=-4} \frac{du}{u^2} = -\frac{1}{3} \int_{-1}^{-4} u^{-2} du \\ &= -\frac{1}{3} \cdot \left[-\frac{1}{u} \right]_{-1}^{-4} = \frac{1}{3} \left[\frac{1}{u} \right]_{-1}^{-4} \\ &= \frac{1}{3} \left(\frac{1}{-4} - \frac{1}{-1} \right) \end{aligned}$$

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3}du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

$$\begin{aligned} \int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} &= -\frac{1}{3} \int_{u=-1}^{u=-4} \frac{du}{u^2} = -\frac{1}{3} \int_{-1}^{-4} u^{-2} du \\ &= -\frac{1}{3} \cdot \left[-\frac{1}{u} \right]_{-1}^{-4} = \frac{1}{3} \left[\frac{1}{u} \right]_{-1}^{-4} \\ &= \frac{1}{3} \left(\frac{1}{-4} - \frac{1}{-1} \right) = \frac{1}{3} \left(1 - \frac{1}{4} \right) \end{aligned}$$

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3 dx$.
- Therefore $dx = -\frac{1}{3}du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = -4$.

$$\begin{aligned} \int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} &= -\frac{1}{3} \int_{u=-1}^{u=-4} \frac{du}{u^2} = -\frac{1}{3} \int_{-1}^{-4} u^{-2} du \\ &= -\frac{1}{3} \cdot \left[-\frac{1}{u} \right]_{-1}^{-4} = \frac{1}{3} \left[\frac{1}{u} \right]_{-1}^{-4} \\ &= \frac{1}{3} \left(\frac{1}{-4} - \frac{1}{-1} \right) = \frac{1}{3} \left(1 - \frac{1}{4} \right) = \frac{1}{4}. \end{aligned}$$