Calculus II
Convergence of sequences related to the number e as a limit, part 1

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Theorem

If $\lim_{n\to\infty} a_n = L$ *and the function* f *is continuous at* L*, then*

$$\lim_{n\to\infty} f(a_n) = f(L)$$

Example

$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} e^{\ln\left(1+\frac{k}{x}\right)^x} \quad \Big| \text{ exponent= continuous f-n}$$

$$= e^{\lim\limits_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x} = e^k \quad \Big| \text{ limit computed below}$$

$$\lim_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} x\ln\left(1+\frac{k}{x}\right)$$

$$= \lim_{x\to\infty}\frac{\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(1+\frac{k}{x}\right)\right)}{\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{x}\right)} \quad \Big| \text{ form } \text{“}\tfrac{0}{0}\text{”, use L'Hospital}$$

$$= \lim_{x\to\infty}\frac{\frac{1}{1+\frac{k}{x}}\left(1+\frac{k}{x}\right)'}{-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty}\frac{\frac{1}{1+\frac{k}{x}}\left(-\frac{k}{x^2}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x\to\infty}\frac{k}{1+\frac{k}{x}} = k$$