

# Precalculus

## Interval notation, the $\in$ and the $\cup$ symbols

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# Outline

- 1 Interval notation
- 2 Inequalities and interval notation

# Review of standard interval notation

- If you are familiar with all aspects of the following notation:

$$x \in (-\infty, 7) \cup (9, 12],$$

feel free skip the next slides.

- In particular, in the next two slides we cover:
  - the open/closed/semi-closed interval notation;
  - the “belongs to”  $\in$  sign;
  - the union  $\cup$  sign.

Let  $a < b$  be two real numbers.

### Definition (Interval notation)

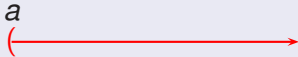
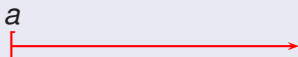
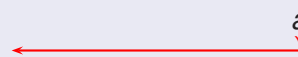

The set (interval) of all real numbers from  $a$  to  $b$  is denoted as follows.

Notation	Endpoints	Picture
$[a, b]$	including both $a$ and $b$	$a$ $b$ $a$ $b$ $\bullet$ ————— $\bullet$ or $[$ ————— $]$
$[a, b)$	including $a$ but not $b$	$a$ $b$ $a$ $b$ $\bullet$ ————— $\circ$ or $[$ ————— $)$
$(a, b]$	including $b$ but not $a$	$a$ $b$ $a$ $b$ $\circ$ ————— $\bullet$ or $($ ————— $]$
$(a, b)$	including neither $a$ nor $b$	$a$ $b$ $a$ $b$ $\circ$ ————— $\circ$ or $($ ————— $)$

Let  $a$  be a number.

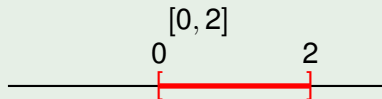
### Definition (Infinite intervals)

The set of all numbers greater than/smaller than  $a$  is denoted with the help of the  $\infty$  symbol.

Notation	Endpoint	Picture
$(a, \infty)$	pts. to the right of $a$ excluding $a$	
$[a, \infty)$	pts. to the right of $a$ including $a$	
$(-\infty, a)$	pts. to the left of $a$ excluding $a$	
$(-\infty, a]$	pts. to the left of $a$ including $a$	

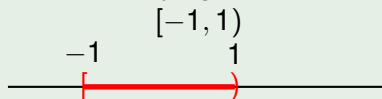
## Example

Write the set of numbers  $x$  satisfying  $0 \leq x \leq 2$  in interval notation.



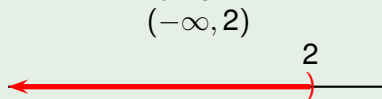
## Example

Write the set of numbers  $x$  satisfying  $-1 \leq x < 1$  in interval notation.



## Example

Write the set of numbers  $x$  satisfying  $x < 2$  in interval notation.



## Definition

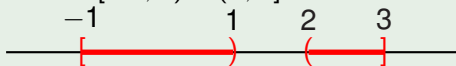
Let  $A$  and  $B$  be sets.

- The union of  $A$  and  $B$  is the set consisting of the elements in  $A$  and the elements in  $B$ , without additional elements.
- The union of  $A$  and  $B$  is denoted by

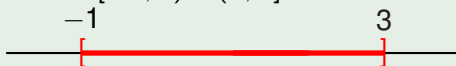
$$A \cup B$$

## Example

Plot the points in the set  $[-1, 1) \cup (2, 3]$ .



Plot the points in the set  $[-1, 2) \cup (1, 3]$ .



- To draw the points of a union draw both on top of one another.

## Definition

Let  $A$  be a set. The notation

$$x \in A$$

is read as

- $x$  belongs to  $A$  or
- $x$  is an element of  $A$ .

## Example

Express the statement  $-1 < x \leq 2$  using the  $\in$  symbol and the interval notation.

$$x \in (-1, 2]$$

Express the statement  $x < 0$  or  $1 \leq x < 2$  using the  $\in$  symbol and the interval notation.

$$x \in (-\infty, 0) \cup [1, 2)$$