Calculus II
Trigonometry review

Todor Milev

2019

Outline

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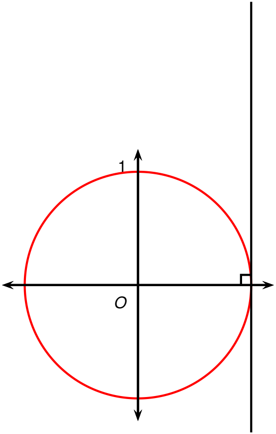
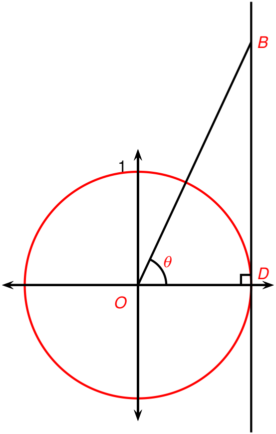
Geometric interpretation of all trigonometric functions



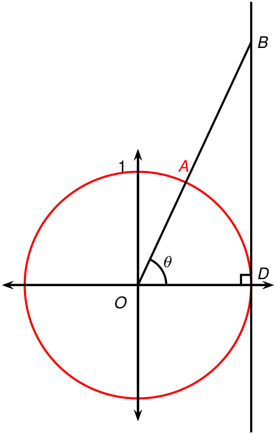
Fix unit circle, center O, coordinates $(0, 0)$.

Geometric interpretation of all trigonometric functions



Fix unit circle, center O, coordinates $(0, 0)$.
Let $\angle DOB = \theta$.

Geometric interpretation of all trigonometric functions



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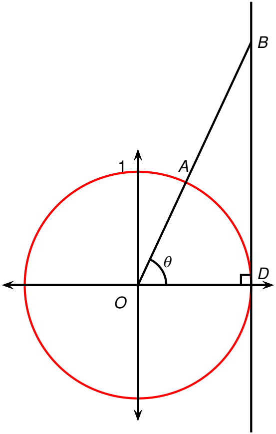
Geometric interpretation of all trigonometric functions



Fix unit circle, center O, coordinates $(0, 0)$. Let $\angle DOB = \theta$. Let OB intersect the circle at point A. Coordinates of A are $(\cos\theta, \sin\theta)$.

$\sin\theta$

$\cos\theta$

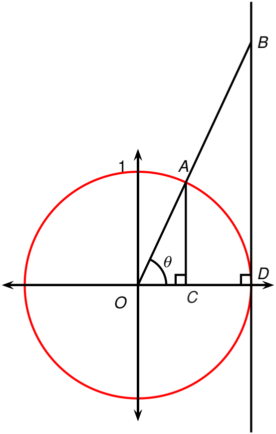
$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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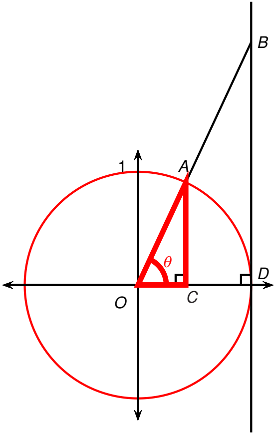

Fix unit circle, center O, coordinates $(0, 0)$. Let $\angle DOB = \theta$. Let OB intersect the circle at point A. Coordinates of A are $(\cos\theta, \sin\theta)$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$\cos\theta$

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$\sec\theta$

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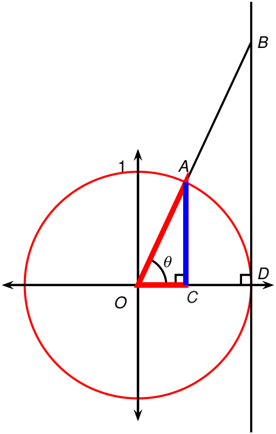
Geometric interpretation of all trigonometric functions



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$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|}$$

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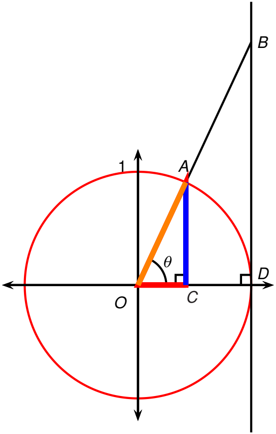
Geometric interpretation of all trigonometric functions



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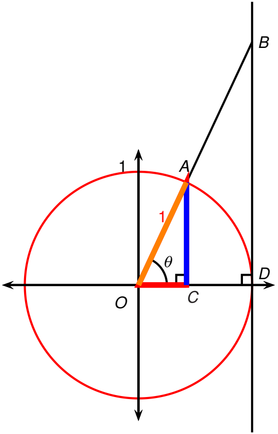

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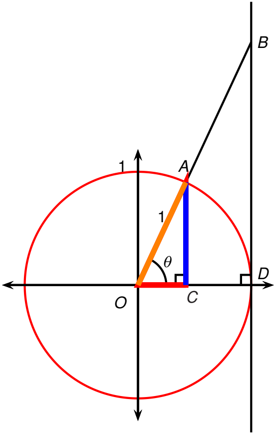

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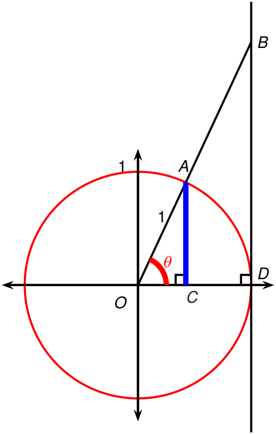

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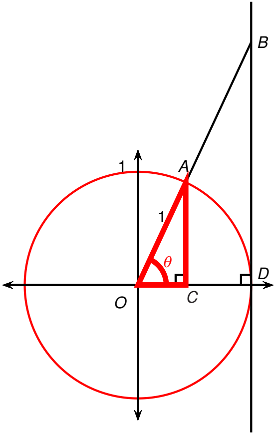

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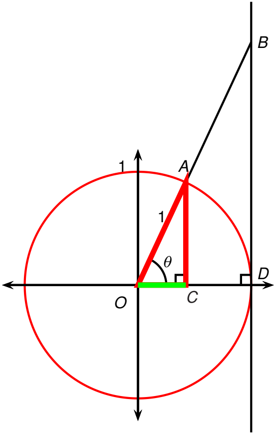

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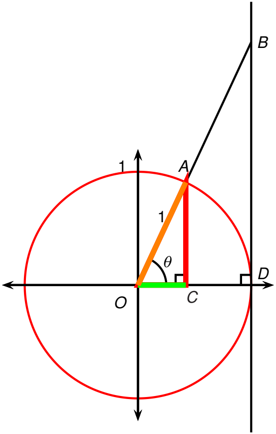

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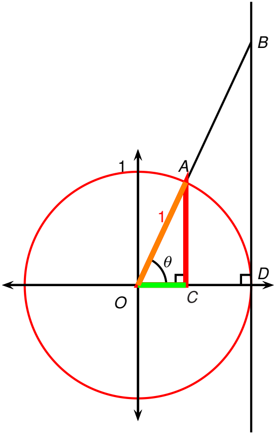

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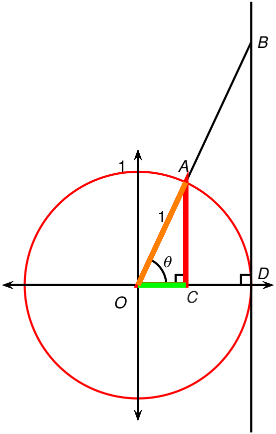

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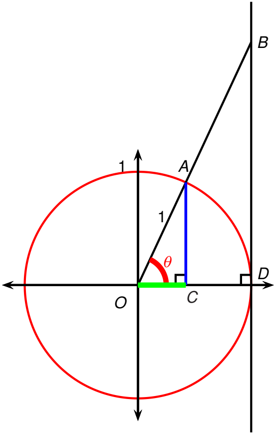

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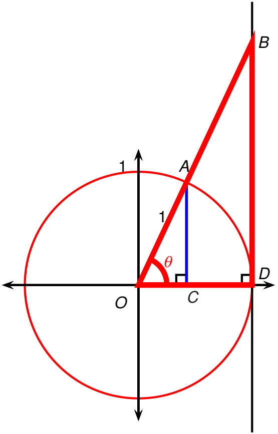

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$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

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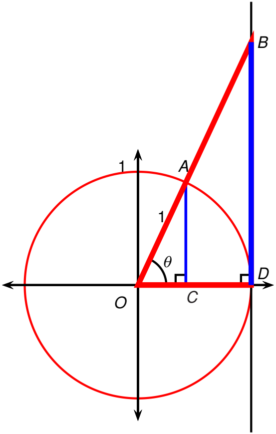

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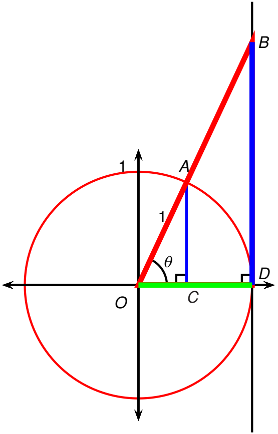
\tan\theta &= \frac{\text{opp}}{\text{adj}} = \frac{|BD|}{|OD|}
\end{aligned}$$

$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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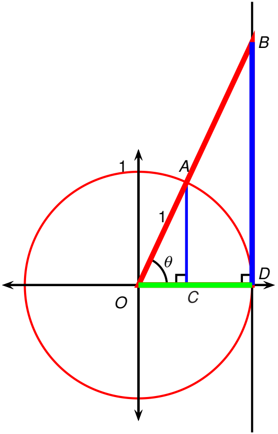

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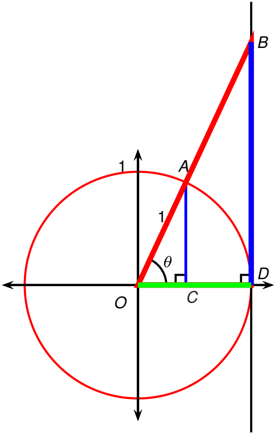
$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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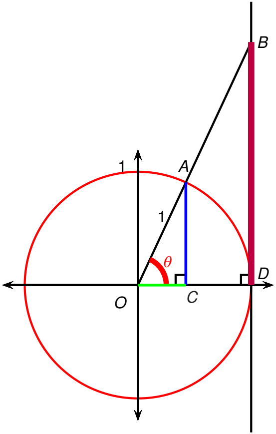

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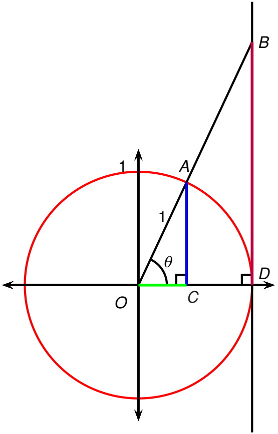

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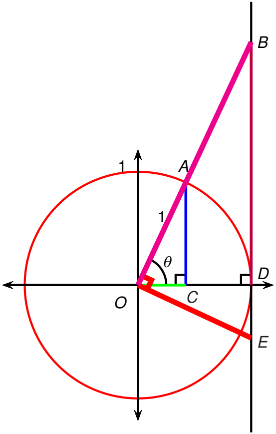

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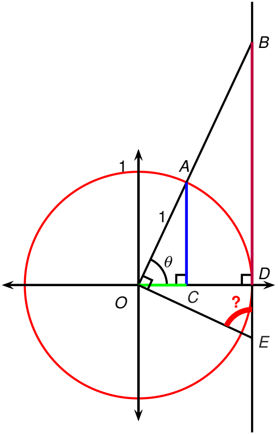

$\angle OED =$ **?**

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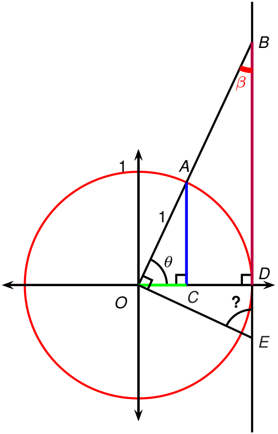

$\beta = \mathbf{?}$

$\angle OED = \mathbf{?}$

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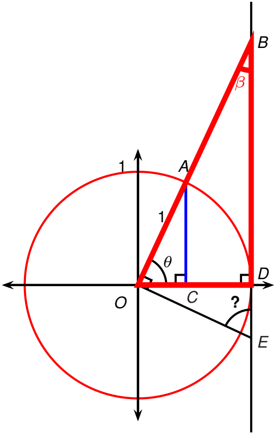

$\beta = 180° - 90° - \theta$

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\cot\theta &= \frac{\text{adj}}{\text{opp}}
\end{aligned}$$

$\sec\theta$

$\csc\theta$

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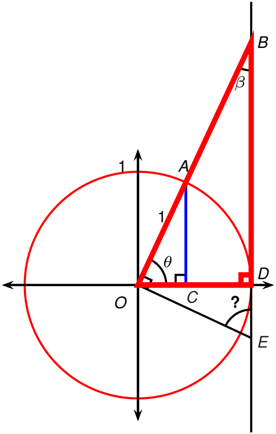

$\beta = 180^\circ - 90^\circ - \theta$

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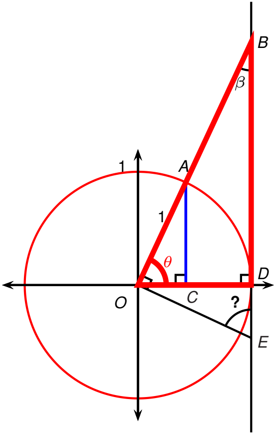

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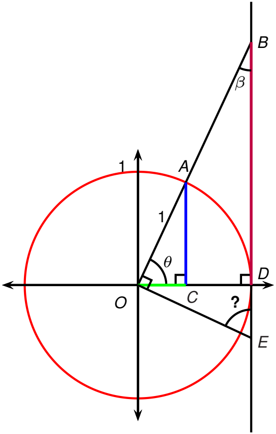

$$\beta = 180^\circ - 90^\circ - \theta$$
$$= 90^\circ - \theta$$
$$\angle OED = \textbf{?}$$

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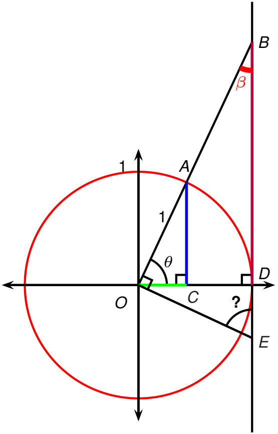
$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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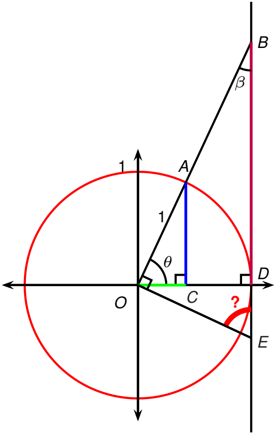
Geometric interpretation of all trigonometric functions



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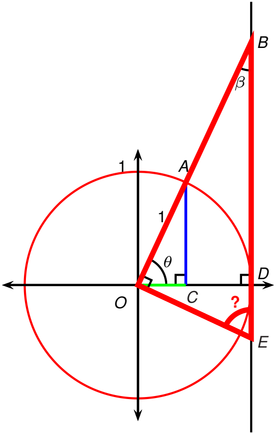
Geometric interpretation of all trigonometric functions



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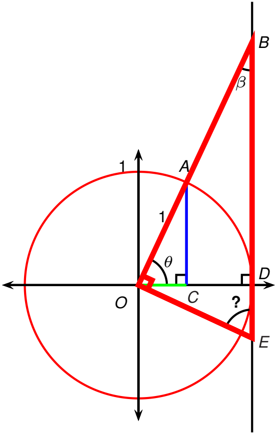
Geometric interpretation of all trigonometric functions



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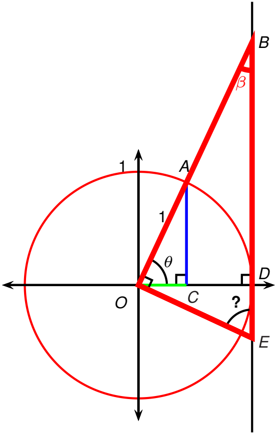

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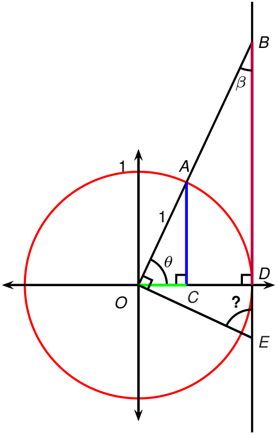
Geometric interpretation of all trigonometric functions



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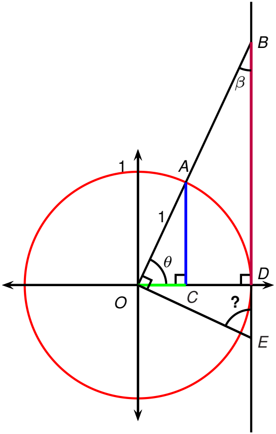
Geometric interpretation of all trigonometric functions



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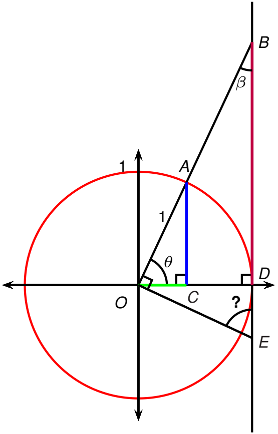
Geometric interpretation of all trigonometric functions



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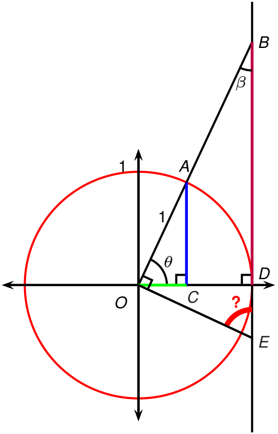
Geometric interpretation of all trigonometric functions



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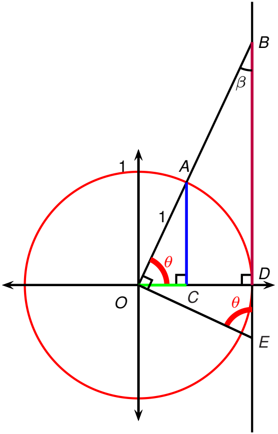
Geometric interpretation of all trigonometric functions



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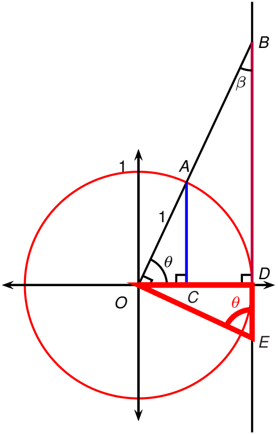
Geometric interpretation of all trigonometric functions



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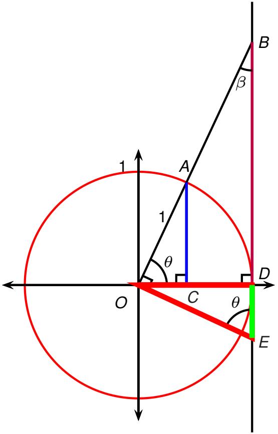

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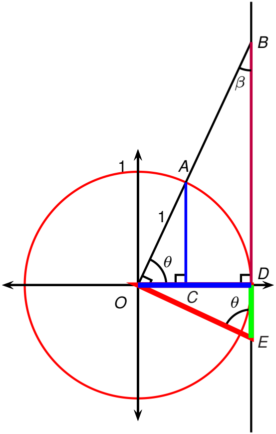
Geometric interpretation of all trigonometric functions



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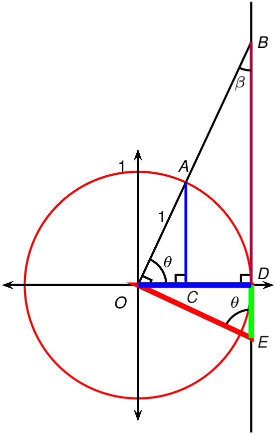
$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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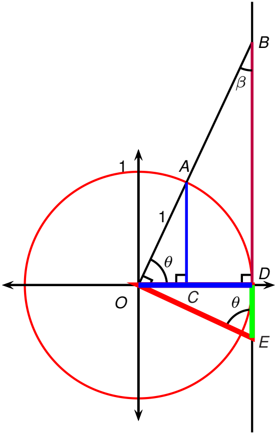
Geometric interpretation of all trigonometric functions



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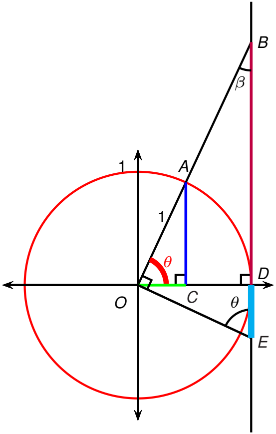
Geometric interpretation of all trigonometric functions



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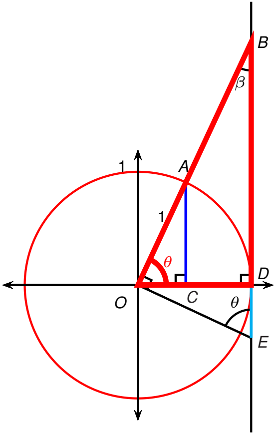
Geometric interpretation of all trigonometric functions



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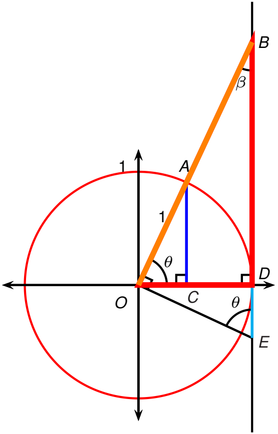
Geometric interpretation of all trigonometric functions



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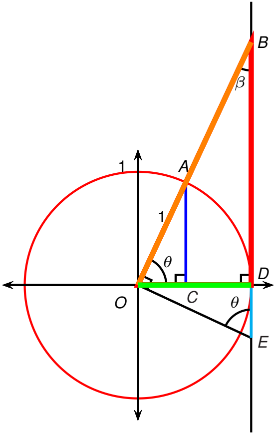
Geometric interpretation of all trigonometric functions



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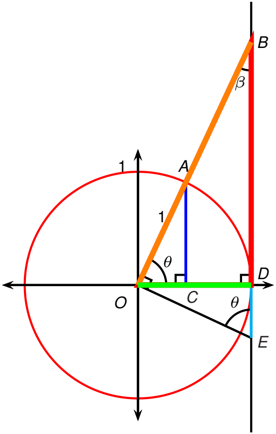
Geometric interpretation of all trigonometric functions



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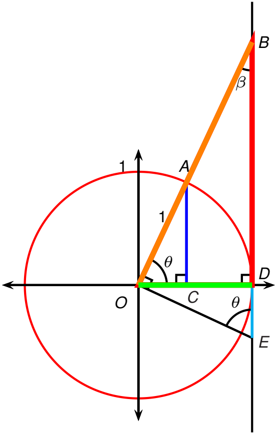
Geometric interpretation of all trigonometric functions



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Fix unit circle, center O, coordinates $(0, 0)$. Let $\angle DOB = \theta$. Let OB intersect the circle at point A. Coordinates of A are $(\cos\theta, \sin\theta)$.

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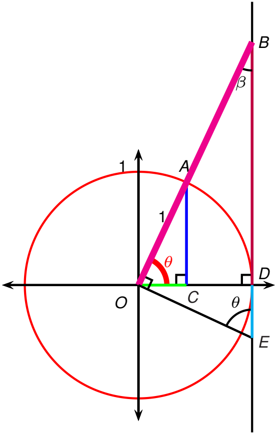

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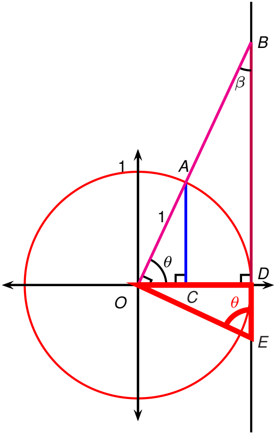

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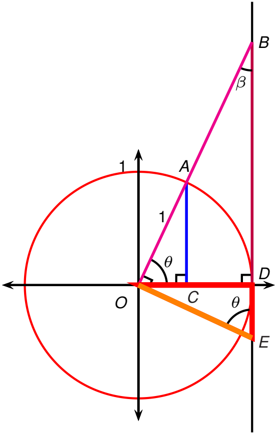

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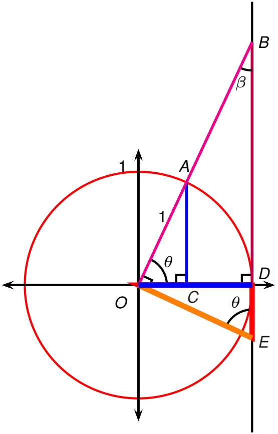

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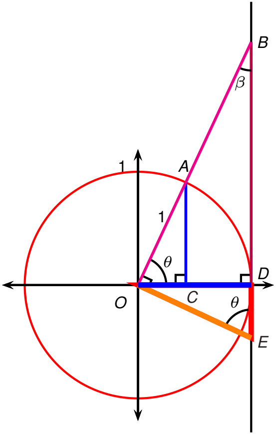

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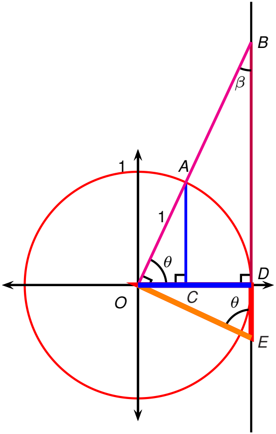

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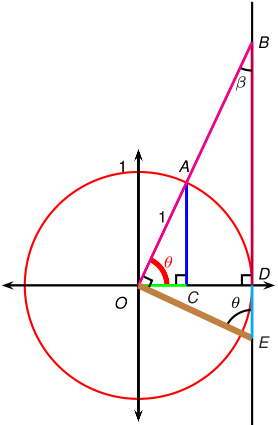

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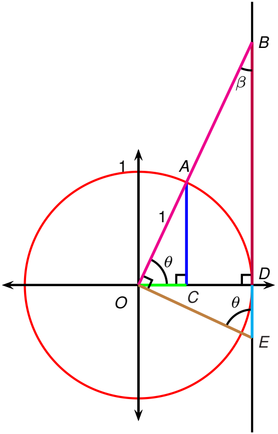

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Definition (Trigonometric Identity)

A trigonometric identity is an equality between the trigonometric functions in one or more variables that holds for all values of the involved variables in the domains of all of the expressions.

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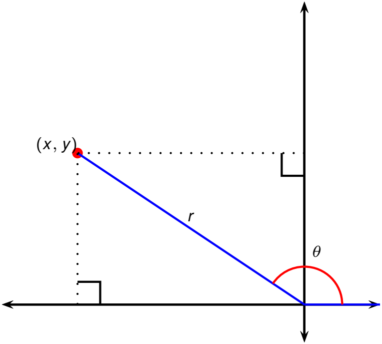
- By convention, when dealing with trigonometric identities we do not account for the domains of the involved expressions.
- For example, $\frac{\sin\theta}{\sin\theta} = 1$ is considered a valid trigonometric identity, although, when considered as a function, the left hand side is not defined for $\theta \neq 0$.



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- $\csc\theta = \frac{1}{\sin\theta}$
- $\sec\theta = \frac{1}{\cos\theta}$
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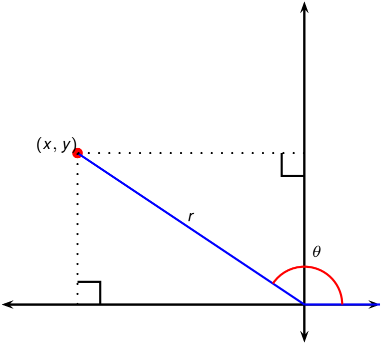

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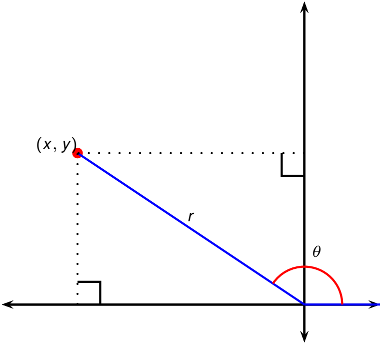

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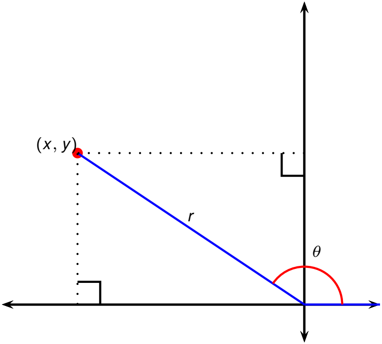
$$\sin^2\theta + \cos^2\theta$$



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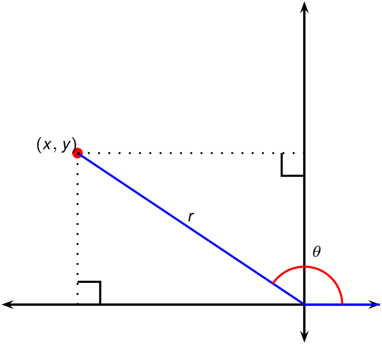

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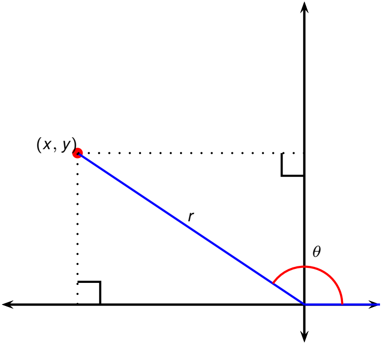

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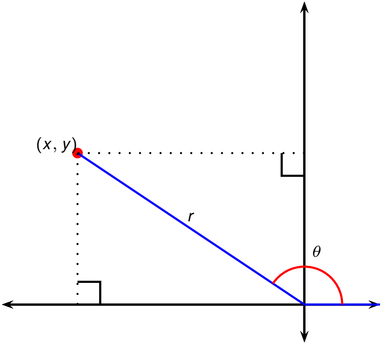

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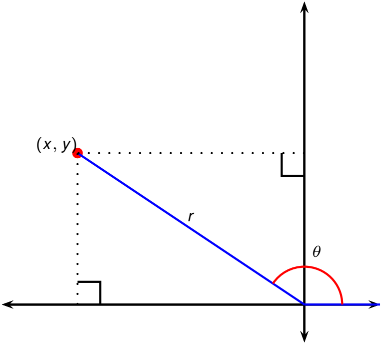

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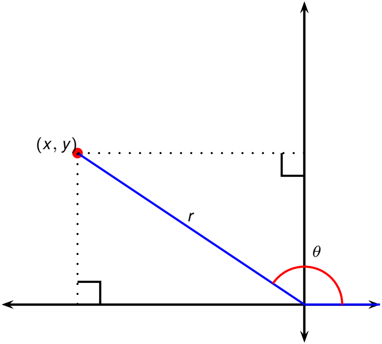
Therefore $\sin^2\theta + \cos^2\theta = 1$.



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Example ($\tan^2\theta + 1 = \sec^2\theta$)

Prove the identity
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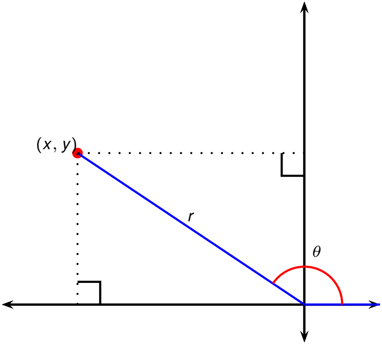

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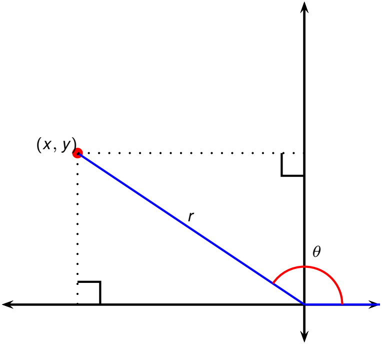
$$\sin^2\theta + \cos^2\theta \quad = \quad 1$$



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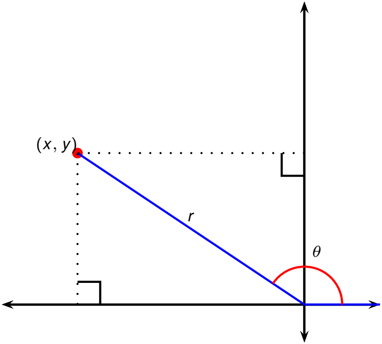
$$\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} &= \frac{1}{\cos^2\theta} \end{aligned}$$



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The remaining identities are consequences of the addition formulas:

$$\begin{aligned}
\sin(x+y) &= \sin x \cos y + \cos x \sin y \\
\cos(x+y) &= \cos x \cos y - \sin x \sin y
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Substitute $-y$ for y, and use the fact that $\sin(-y) = -\sin y$ and $\cos(-y) = \cos y$:

$$\begin{aligned}\sin(x-y) &= \sin x \cos y - \cos x \sin y\\ \cos(x-y) &= \cos x \cos y + \sin x \sin y\end{aligned}$$

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To get the double angle formulas, substitute x for y:

$$\begin{aligned}\sin(2x) &= 2\sin x\cos x\\ \cos(2x) &= \cos^2 x - \sin^2 x\end{aligned}$$

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$$\begin{aligned}\sin(2x) &= 2\sin x\cos x\\ \cos(2x) &= \cos^2 x - \sin^2 x\end{aligned}$$

Rewrite the second double angle formula in two ways, using $\cos^2 x = 1 - \sin^2 x$ and $\sin^2 x = 1 - \cos^2 x$:

$$\begin{aligned}\cos(2x) &= 2\cos^2 x - 1\\ \cos(2x) &= 1 - 2\sin^2 x\end{aligned}$$

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To get the half-angle formulas, solve these equations for $\cos^2 x$ and $\sin^2 x$ respectively.

$$\cos^2 x = \frac{1+\cos(2x)}{2}, \qquad \sin^2 x = \frac{1-\cos(2x)}{2}$$

The remaining identities are consequences of the addition formulas:

$$\begin{array}{rcccc}\sin(x+y) & = & \sin x \cos y & + & \cos x \sin y\\ \cos(x+y) & = & \cos x \cos y & - & \sin x \sin y\end{array}$$

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Divide the first equation by the second, and then cancel $\cos x\cos y$ from the top and bottom:

$$\tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y}$$

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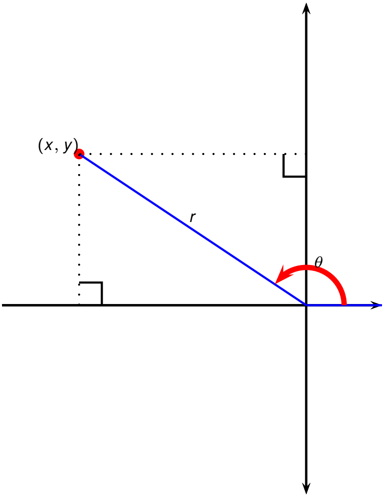
$$\tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y}$$

Do the same for the subtraction formulas:

$$\tan(x-y) = \frac{\tan x-\tan y}{1+\tan x\tan y}$$



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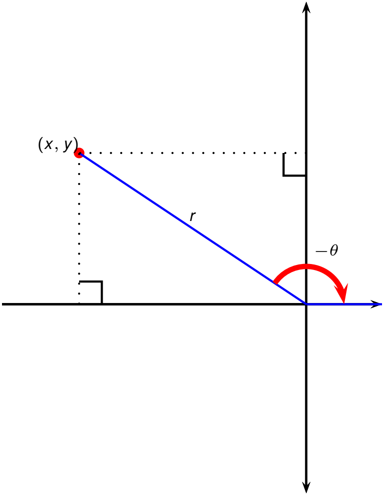

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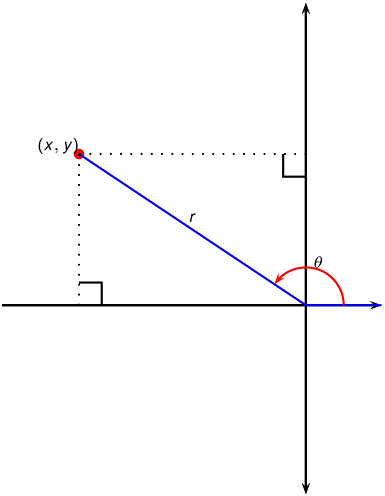

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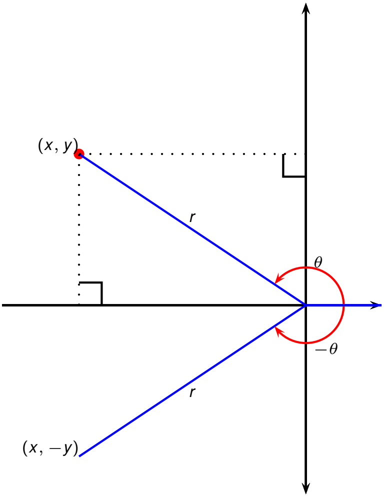

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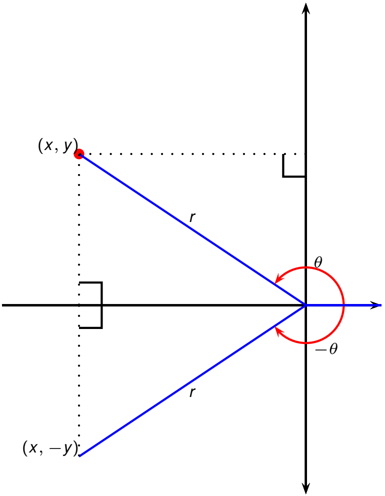

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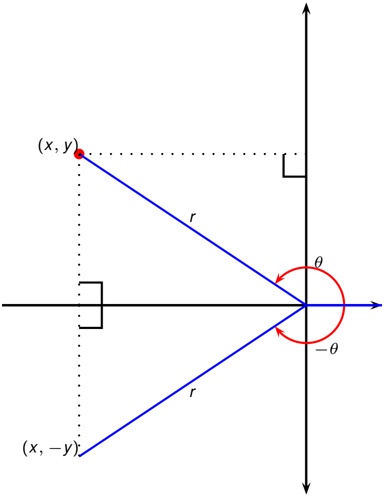

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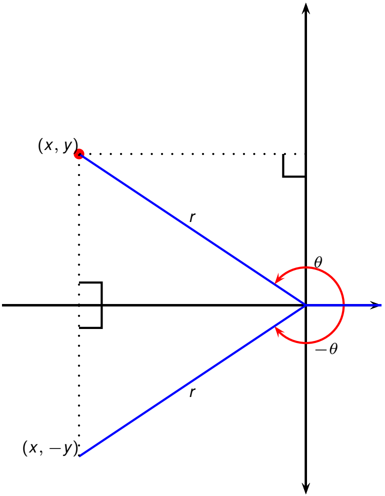

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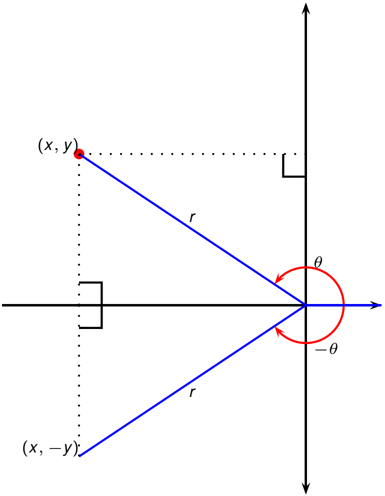

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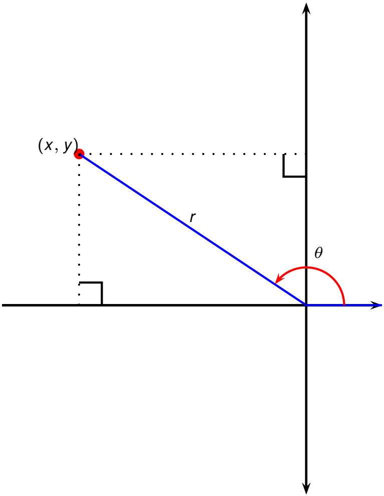

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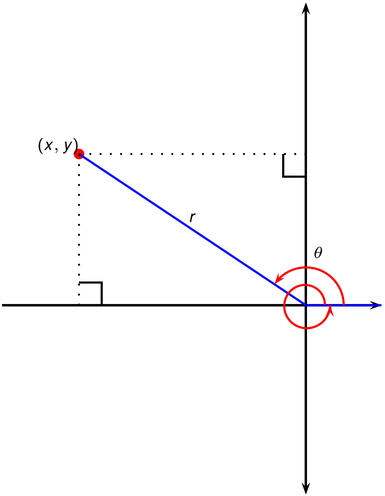

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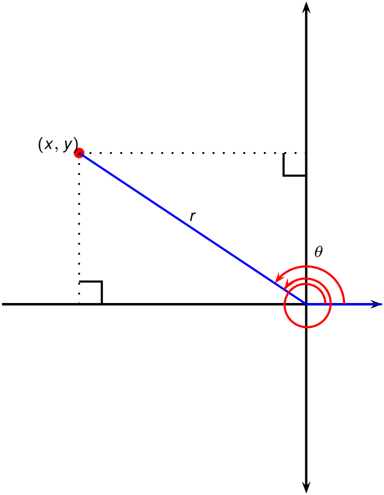
- 2π represents a full rotation.



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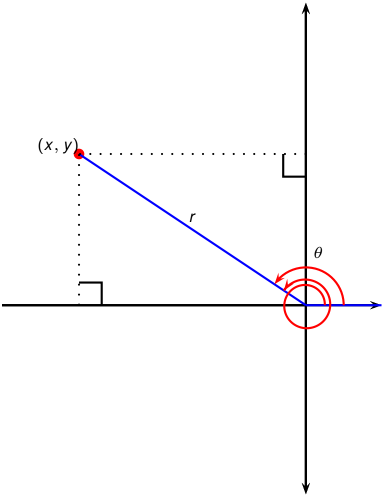

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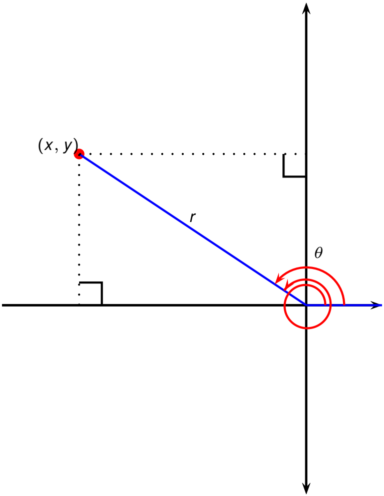

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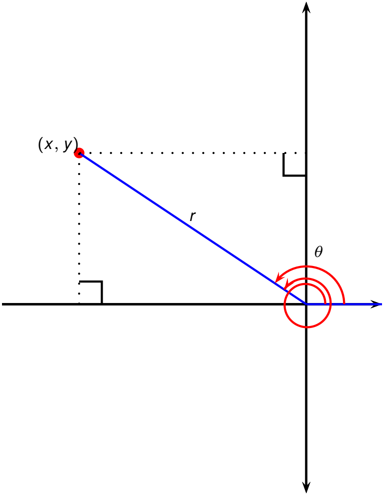

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- We say $\sin$ and $\cos$ are 2π-periodic.

Definition (Complex numbers)

The set of complex numbers $\mathbb{C}$ is defined as the set

$$\{a + bi | a, b - \text{real numbers}\},$$

where the number i is a number for which

$$i^2 = -1 \quad .$$

The number i is called the imaginary unit.

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Euler's Formula

Theorem (Euler's Formula)

$$e^{ix} = \cos x + i \sin x,$$

where $e \approx 2.71828$ *is Euler's/Napier's constant .*

Proof.

Recall $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n$. Borrow from Calc II the f-las:

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$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \dots$$

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$$\begin{array}{rcccccccc}
\sin x &=& & x & & -\frac{x^3}{3!} & & +\frac{x^5}{5!} & -\dots \\
\cos x &=& 1 & & -\frac{x^2}{2!} & & +\frac{x^4}{4!} & & +\dots \\
\hline
e^z &=& 1 & +z & +\frac{z^2}{2!} & +\frac{z^3}{3!} & +\frac{z^4}{4!} & +\frac{z^5}{5!} & +\dots
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Rearrange.

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Rearrange. Plug-in $z = ix$.

$\square$

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e^{ix} & = & 1 & +ix & \color{red}{-}\frac{\color{red}{x^2}}{2!} & \color{red}{-i}\frac{\color{red}{x^3}}{3!} & +\frac{\color{red}{x^4}}{4!} & +\color{red}{i}\frac{\color{red}{x^5}}{5!} & \color{red}{-}\dots
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Rearrange. Plug-in $z = ix$. Use $i^2 = -1$.

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$$\begin{array}{rcllllllll}
i \sin x & = & & ix & & -i\frac{x^3}{3!} & & +i\frac{x^5}{5!} & - \dots \\
\cos x & = & 1 & & -\frac{x^2}{2!} & & +\frac{x^4}{4!} & & + \dots \\
\hline
e^{ix} & = & 1 & +ix & -\frac{x^2}{2!} & -i\frac{x^3}{3!} & +\frac{x^4}{4!} & +i\frac{x^5}{5!} & - \dots
\end{array}$$

Rearrange. Plug-in $z = ix$. Use $i^2 = -1$. Multiply $\sin x$ by i.

□

Euler's Formula

Theorem (Euler's Formula)

$$e^{ix} = \cos x + i \sin x,$$

where $e \approx 2.71828$ is Euler's/Napier's constant .

Proof.

Recall $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n$. Borrow from Calc II the f-las:

$$\begin{array}{rcccccccc}
i \sin x & = & & ix & & -i\frac{x^3}{3!} & & +i\frac{x^5}{5!} & - \dots \\
\cos x & = & 1 & & -\frac{x^2}{2!} & & +\frac{x^4}{4!} & & + \dots \\
\hline
e^{ix} & = & 1 & +ix & -\frac{x^2}{2!} & -i\frac{x^3}{3!} & +\frac{x^4}{4!} & +i\frac{x^5}{5!} & - \dots
\end{array}$$

Rearrange. Plug-in $z = ix$. Use $i^2 = -1$. Multiply $\sin x$ by i. Add to get $e^{ix} = \cos x + i \sin x$. $\square$

Trigonometric Identities Revisited

- $e^{ix} = \cos x + i \sin x$ (Euler's Formula).
- $e^{ix} e^{iy} = e^{ix+iy} = e^{i(x+y)}$ (exponentiation rule: valid for $\mathbb{C}$).
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All trigonometric formulas can be easily derived using the above formulas.

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Example

$$\begin{aligned} \sin(x+y) &= \sin x \cos y + \sin y \cos x \\ \cos(x+y) &= \cos x \cos y - \sin x \sin y \end{aligned} \quad .$$

Proof.

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Compare coefficient in front of i and remaining terms to get the desired equalities. □

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Compare coefficient in front of i and

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Example

Express $\sin(3x)$ and $\cos(3x)$ via $\cos x$ and $\sin x$.

- Recall Euler's formula: $e^{i\alpha} = \cos\alpha + i\sin\alpha$.

Example

Express $\sin(3x)$ and $\cos(3x)$ via $\cos x$ and $\sin x$.

$$\begin{array}{ll} \cos(3x) + i\sin(3x) & \Big| \text{ Euler's f-la} \\ = e^{3ix} & \end{array}$$

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- Recall Euler's formula: $e^{i\alpha} = \cos\alpha + i\sin\alpha$.
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Example

Express $\sin(3x)$ and $\cos(3x)$ via $\cos x$ and $\sin x$.

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= (\cos^3 x - 3\cos x\sin^2 x) + i(3\cos^2 x\sin x - \sin^3 x) &
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&= \cos^3 x + 3\cos^2 x(i\sin x) + 3\cos x(i\sin x)^2 + (i\sin x)^3\\
&= \cos^3 x + 3i\cos^2 x\sin x + 3i^2\cos x\sin^2 x + i^3\sin^3 x\\
&= \cos^3 x + 3i\cos^2 x\sin x - 3\cos x\sin^2 x - i\sin^3 x && \Big| \text{ Use } i^2 = -1\\
&= (\cos^3 x - 3\cos x\sin^2 x) + i(3\cos^2 x\sin x - \sin^3 x)
\end{aligned}$$

- Recall Euler's formula: $e^{i\alpha} = \cos\alpha + i\sin\alpha$.
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= (\cos^3 x - 3\cos x\sin^2 x) + i(3\cos^2 x\sin x - \sin^3 x) &
\end{array}$$

The real parts of the starting and final expression must be equal; therefore:

$$\cos(3x) = \cos^3 x - 3\cos x\sin^2 x$$

- Recall Euler's formula: $e^{i\alpha} = \cos\alpha + i\sin\alpha$.
- Recall the formula: $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

Example

Express $\sin(3x)$ and $\cos(3x)$ via $\cos x$ and $\sin x$.

$$
\begin{aligned}
& \cos(3x) + i\sin(3x) && \Big| \text{ Euler's f-la} \\
&= e^{3ix} \\
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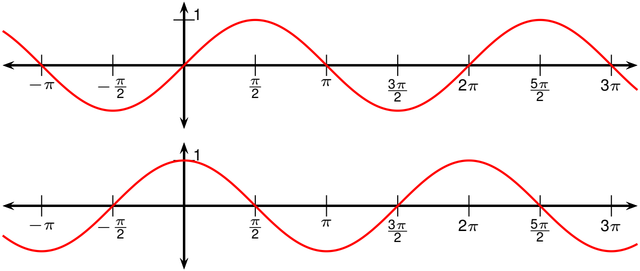
&= \cos^3 x + 3i\cos^2 x\sin x - 3\cos x\sin^2 x - i\sin^3 x && \Big| \text{ Use } i^2 = -1 \\
&= (\cos^3 x - 3\cos x\sin^2 x) + i(3\cos^2 x\sin x - \sin^3 x)
\end{aligned}
$$

The real parts of the starting and final expression must be equal; likewise the imaginary parts must be equal; therefore:

$$
\begin{aligned}
\cos(3x) &= \cos^3 x - 3\cos x\sin^2 x \\
\sin(3x) &= 3\cos^2 x\sin x - \sin^3 x
\end{aligned}
$$

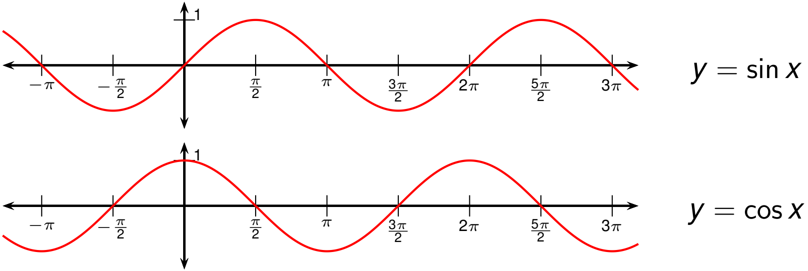
Graphs of the Trigonometric Functions



$y = \sin x$

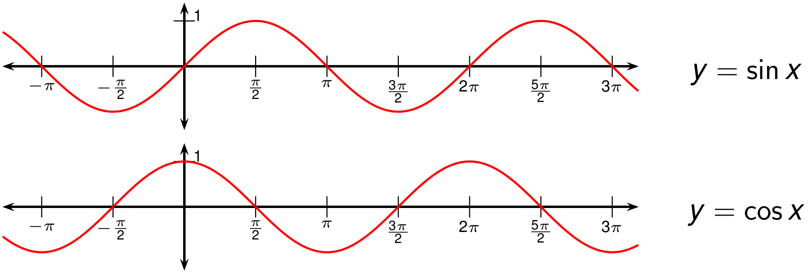
$y = \cos x$

Graphs of the Trigonometric Functions



- $\sin x$ has zeroes at $n\pi$ for all integers n.

Graphs of the Trigonometric Functions



- $\sin x$ has zeroes at $n\pi$ for all integers n.
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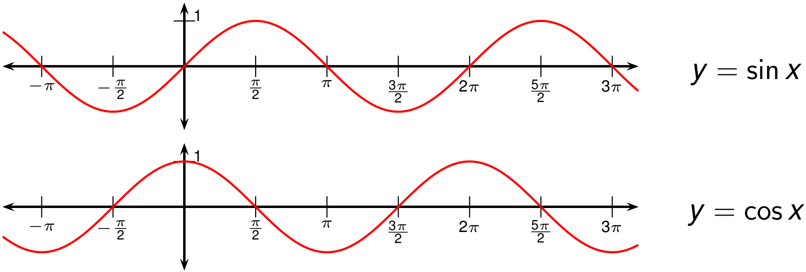
Graphs of the Trigonometric Functions



- $\sin x$ has zeroes at $n\pi$ for all integers n.
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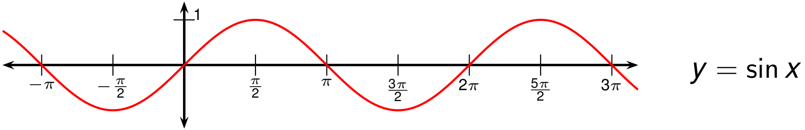
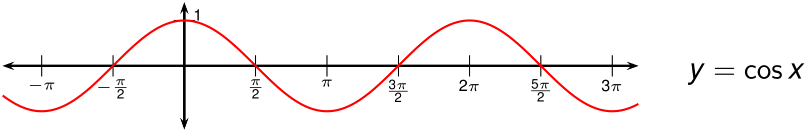
- $-1 \leq \sin x \leq 1$.

Graphs of the Trigonometric Functions





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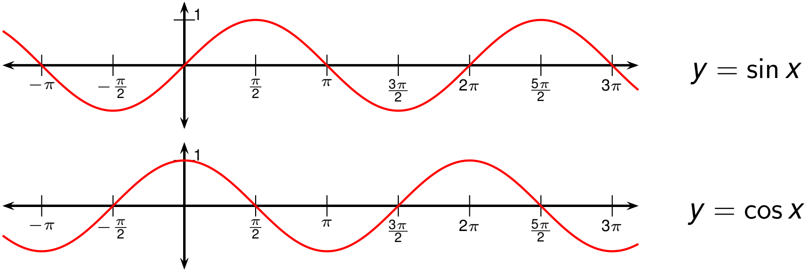

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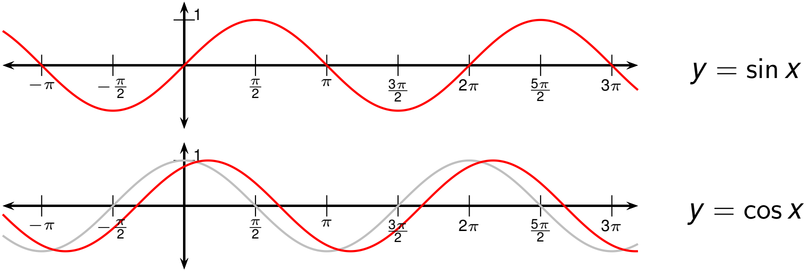
- If we translate the graph of $\cos x$ by $\frac{\pi}{2}$ units to the right

Graphs of the Trigonometric Functions



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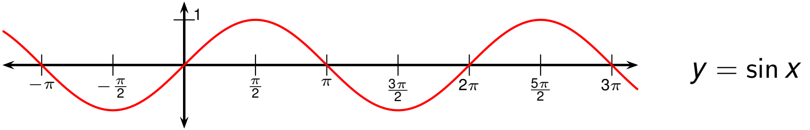

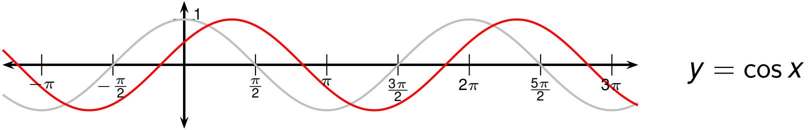

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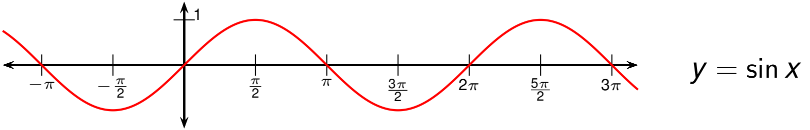

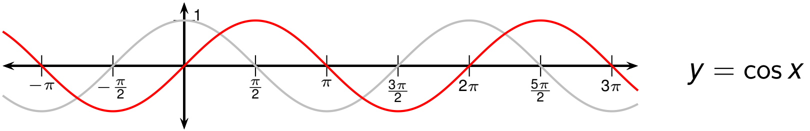

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- If we translate the graph of $\cos x$ by $\frac{\pi}{2}$ units to the right we get the graph of $\sin x$.

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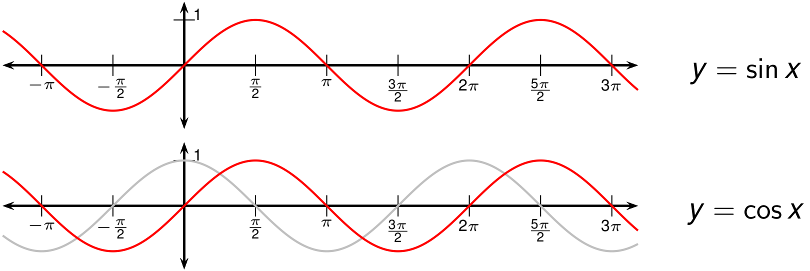

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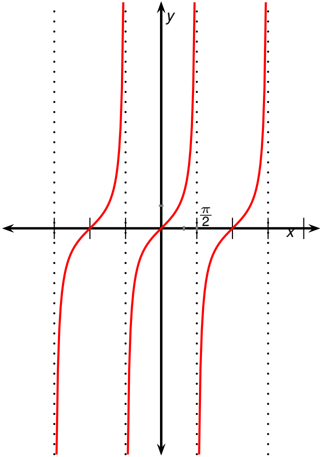
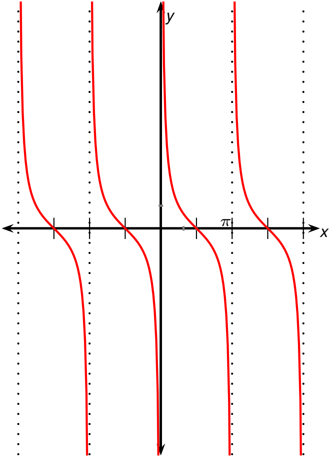
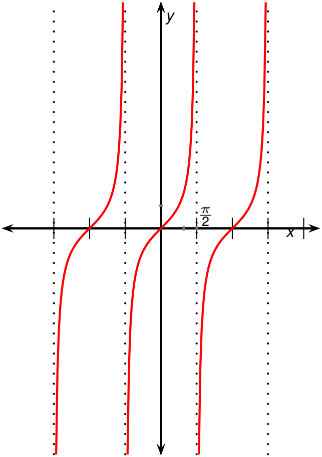
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.
- $-1 \leq \sin x \leq 1$.
- $-1 \leq \cos x \leq 1$.
- If we translate the graph of $\cos x$ by $\frac{\pi}{2}$ units to the right we get the graph of $\sin x$. This is a consequence of $\cos\left(x - \frac{\pi}{2}\right) = \sin x$.

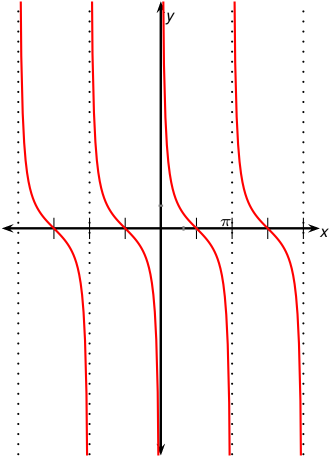


$y = \tan x$



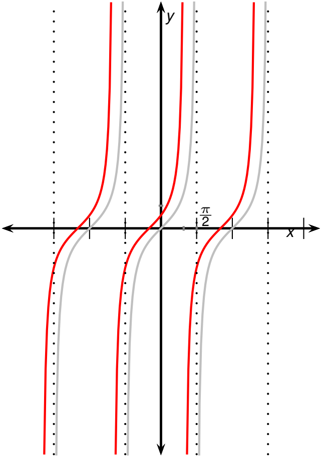
$y = \cot x$

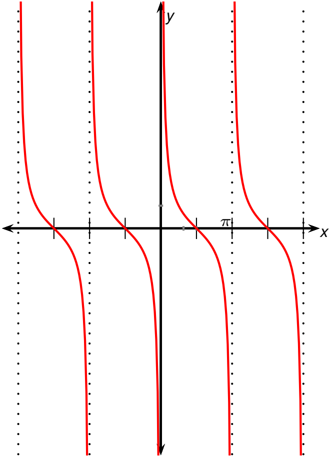




$y = \tan x$ $y = \cot x$

If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis





$y = \tan x$ $\qquad\qquad$ $y = \cot x$

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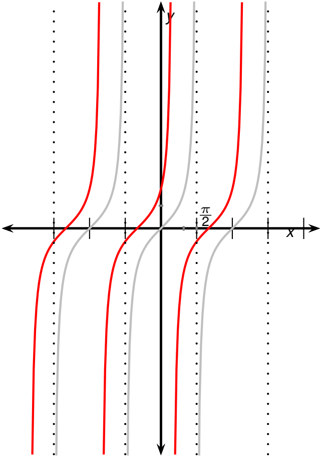

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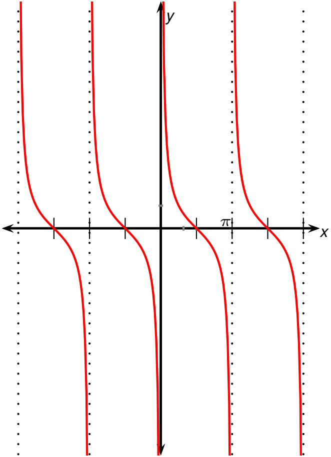

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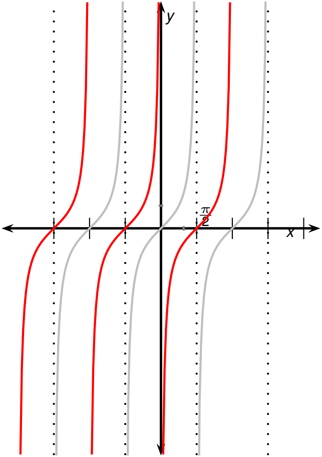
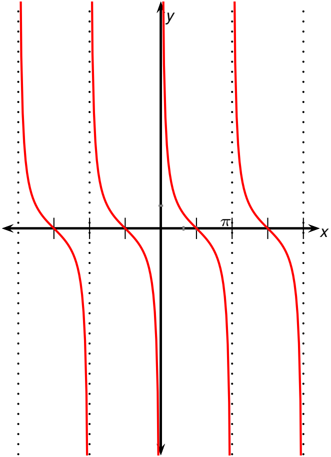


$y = \tan x$



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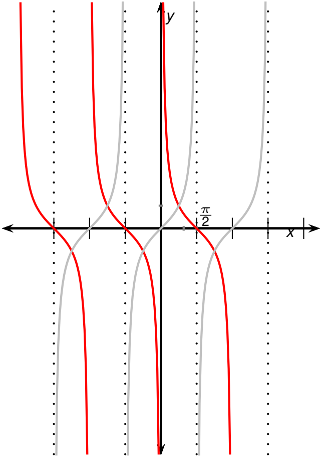
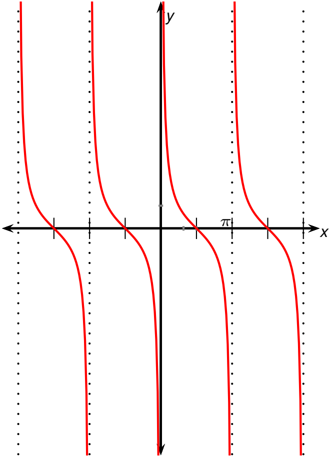
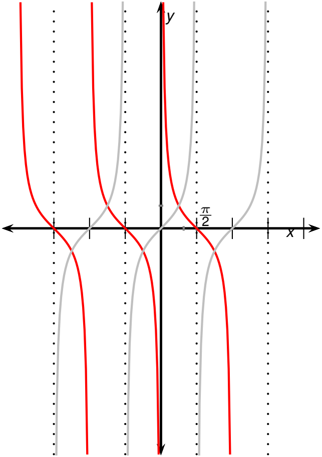
If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis



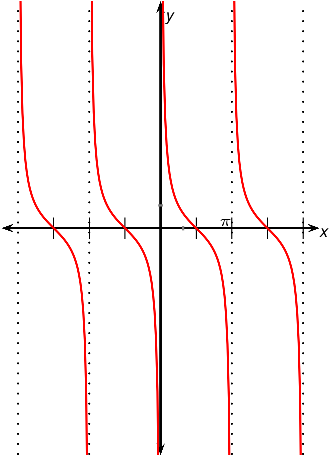


$y = \tan x$ $\qquad$ $y = \cot x$

If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis, we get the graph of $\cot x$.

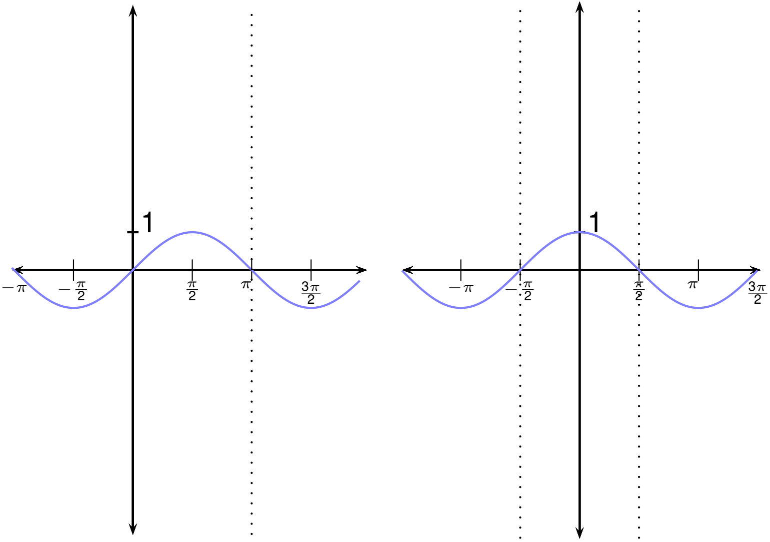


$y = \tan x$

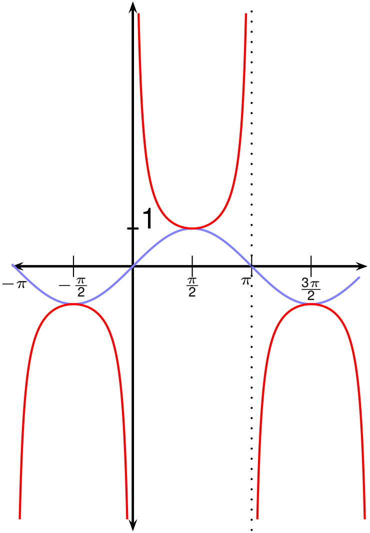


$y = \cot x$

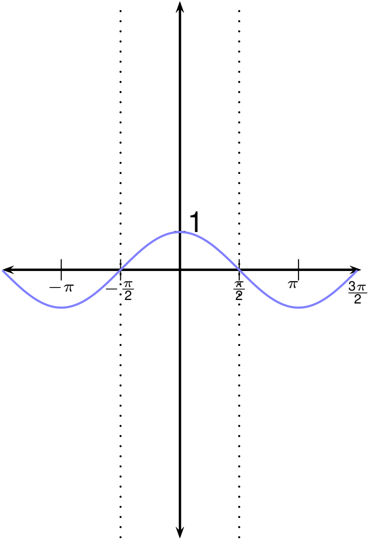
If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis, we get the graph of $\cot x$. This follows from $\tan\left(x \pm \frac{\pi}{2}\right) = -\cot x$.



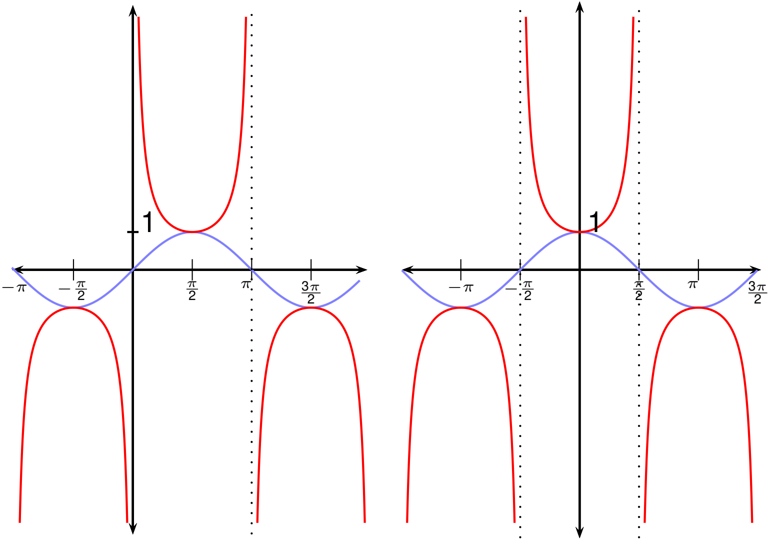
$y = \csc x$ $\qquad$ $y = \sec x$


1
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$

$y = \csc x$

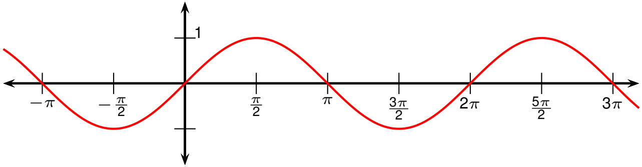

1
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$

$y = \sec x$

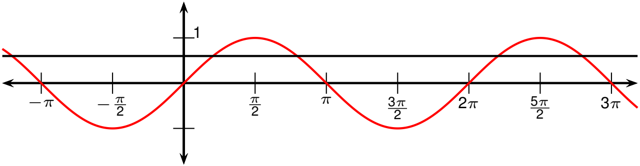


$y = \csc x$ $\qquad$ $y = \sec x$

Inverse Trigonometric Functions



Inverse Trigonometric Functions



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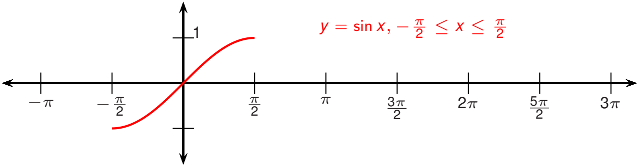

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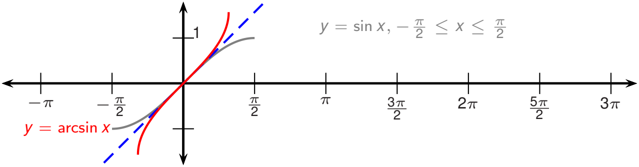

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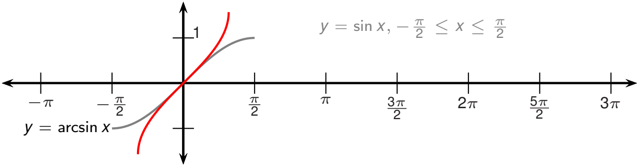

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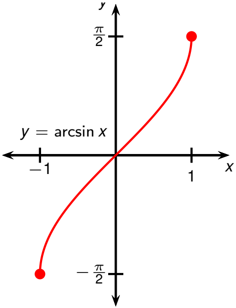

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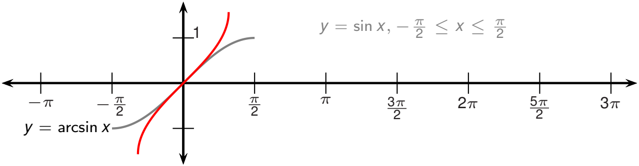

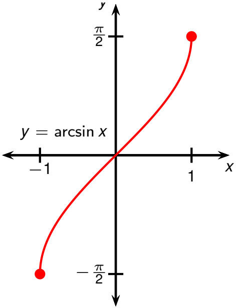

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- It is if we restrict the domain to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
- Then it has an inverse function.
- We call it arcsin or $\sin^{-1}$.
- $\arcsin x = y \Leftrightarrow \sin y = x$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

Example

Find $\arcsin\left(\frac{1}{2}\right)$.

Observation

- $\arcsin y =$ *the appropriate angle whose sine equals y.*

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- $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.

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- *Important: the output angle must lie in the interval* $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

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$$\text{Find } \arcsin\left(\frac{1}{2}\right).$$

- $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.
- $-\frac{\pi}{2} \leq \frac{\pi}{6} \leq \frac{\pi}{2}$.

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Example

$$\text{Find } \arcsin\left(\frac{1}{2}\right).$$

- $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.
- $-\frac{\pi}{2} \leq \frac{\pi}{6} \leq \frac{\pi}{2}$.
- Therefore $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

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Find $\tan\left(\arcsin\left(\frac{1}{3}\right)\right)$.

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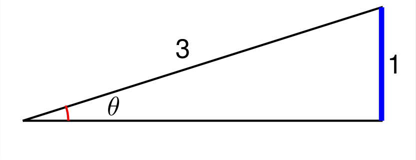

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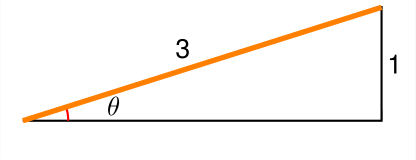

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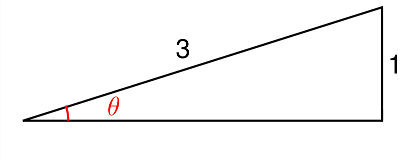

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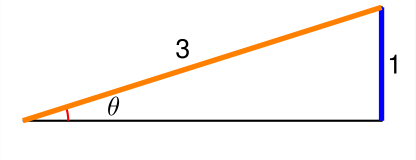

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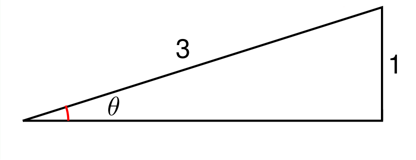

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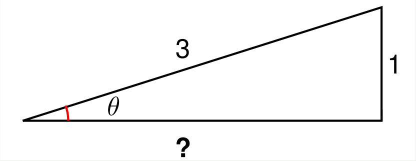
- Length of adjacent side $=$ **?**



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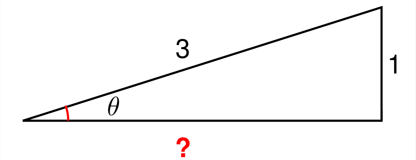
- Length of adjacent side $= \sqrt{3^2 - 1^2}$



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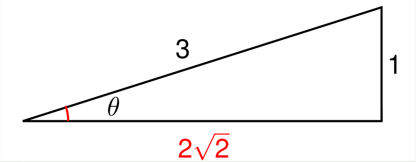
- Length of adjacent side $= \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$.



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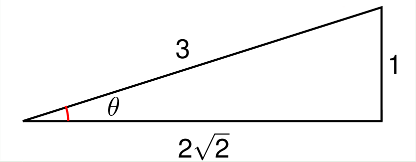
- Then $\tan\left(\arcsin\left(\frac{1}{3}\right)\right) = \textbf{?}$



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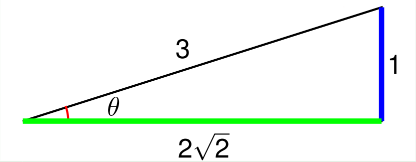
- Length of adjacent side $= \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$.
- Then $\tan\left(\arcsin\left(\frac{1}{3}\right)\right) = \frac{1}{2\sqrt{2}}$.



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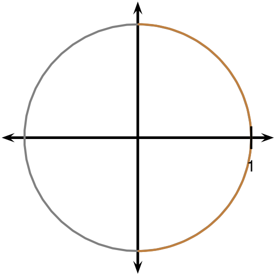
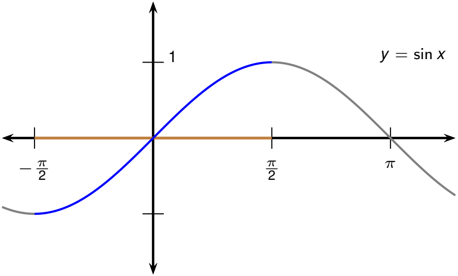
- $\frac{\pi}{2} \approx 1.57$.
- Therefore $-\frac{\pi}{2} \leq 1.5 \leq \frac{\pi}{2}$.
- Therefore $\arcsin(\sin 1.5) = 1.5$.

Example

Find $\arcsin(\sin 2)$.





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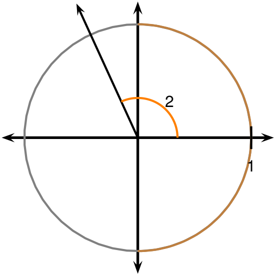

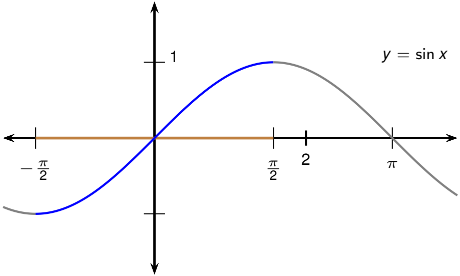

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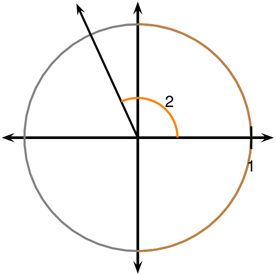

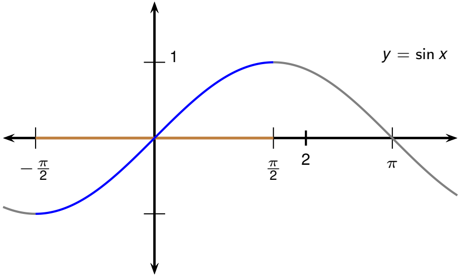

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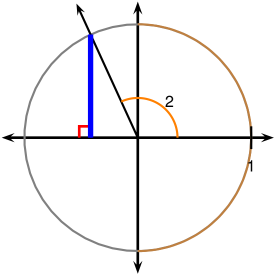

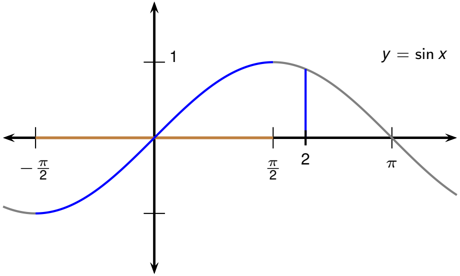

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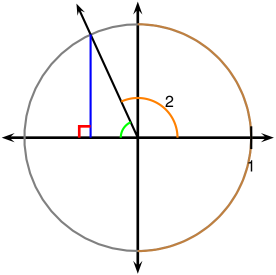

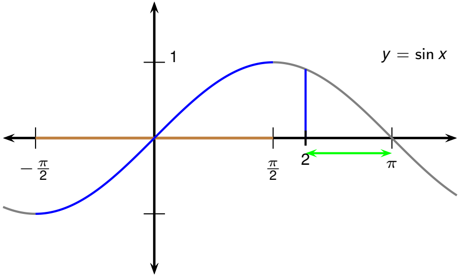

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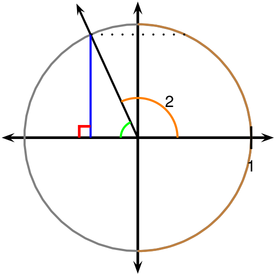

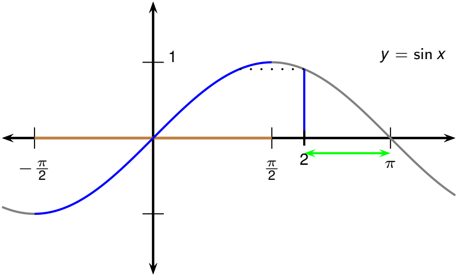

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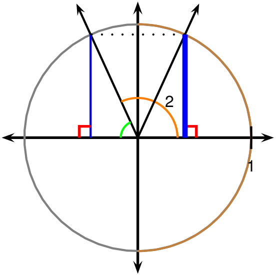

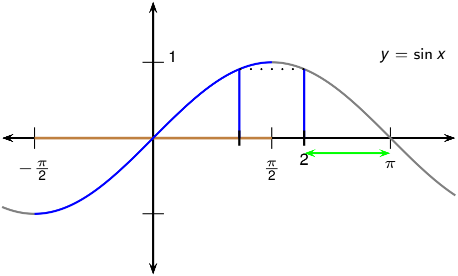

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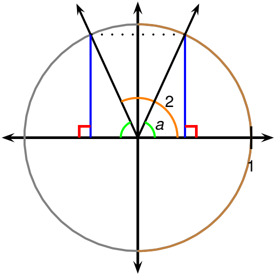

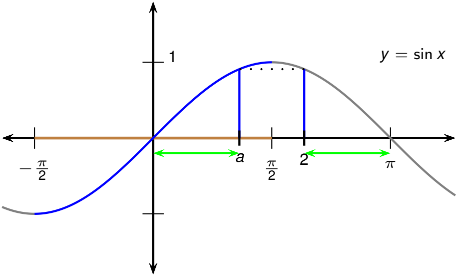

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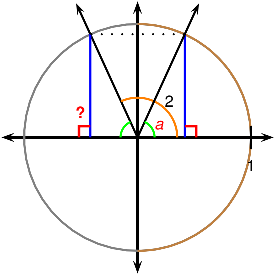
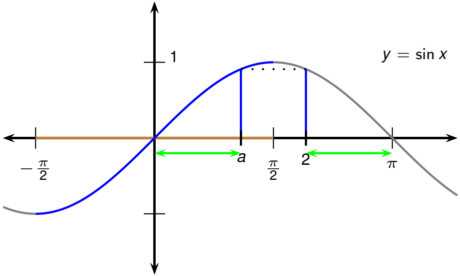
$$a \quad = \quad \textbf{?}$$





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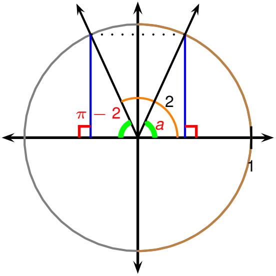
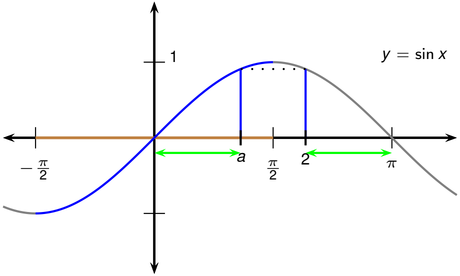
$$a = \pi - 2.$$





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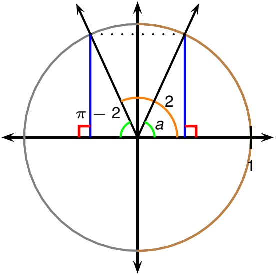

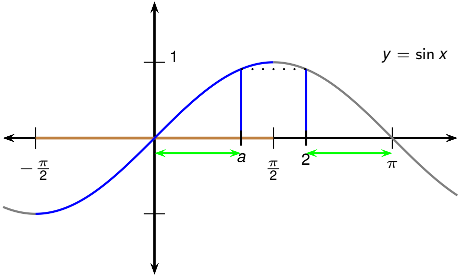

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$$
\begin{array}{rrcl}
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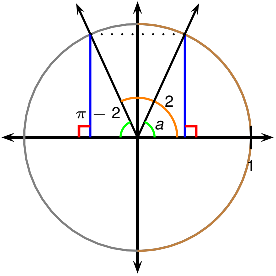
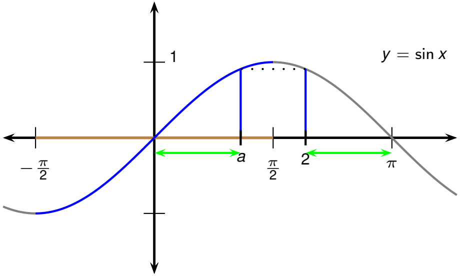
 & & = & a
\end{array}
$$





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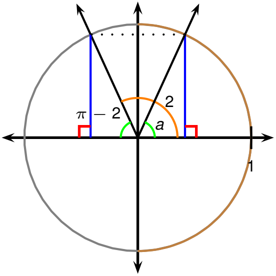

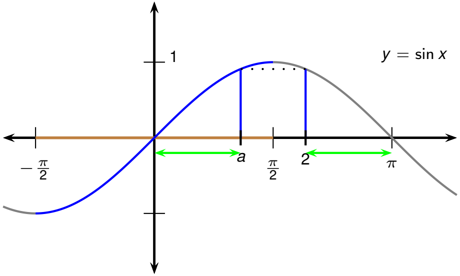

Theorem (The Derivative of $\arcsin x$)

$$\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}, \qquad -1 < x < 1.$$

Proof.

$\square$

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$$\text{Let} \quad y = \arcsin x.$$

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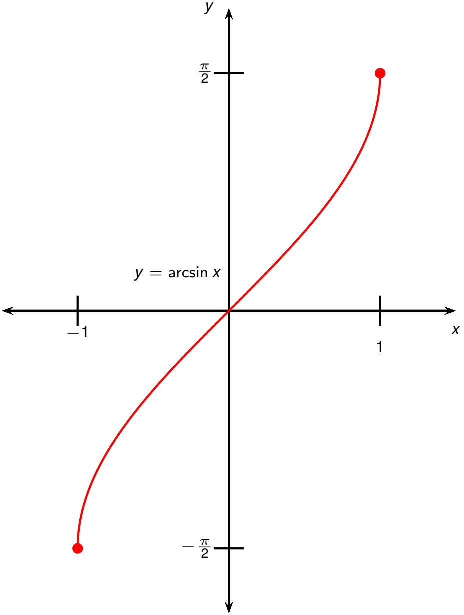
\end{aligned}$$

$\square$

Important facts about arcsin:



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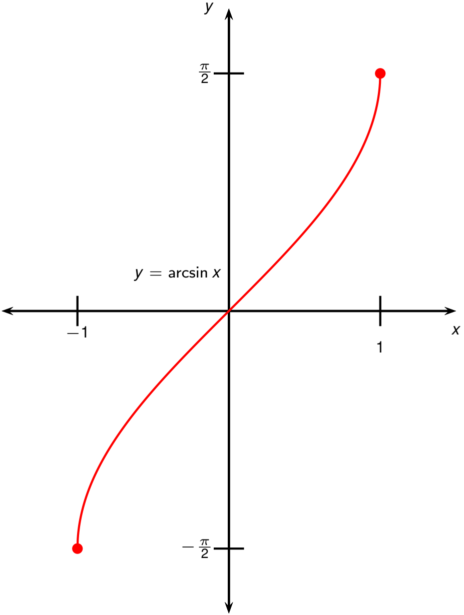

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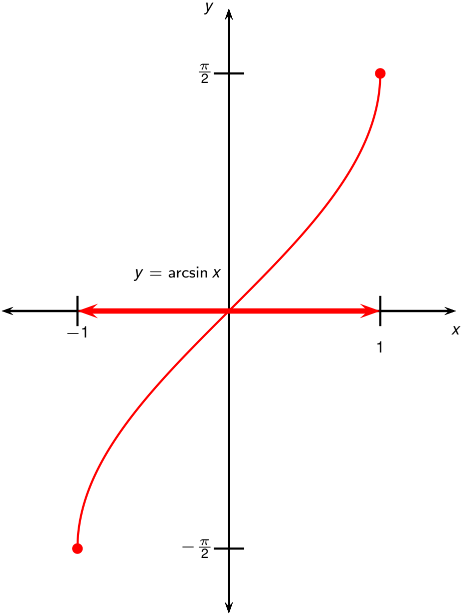

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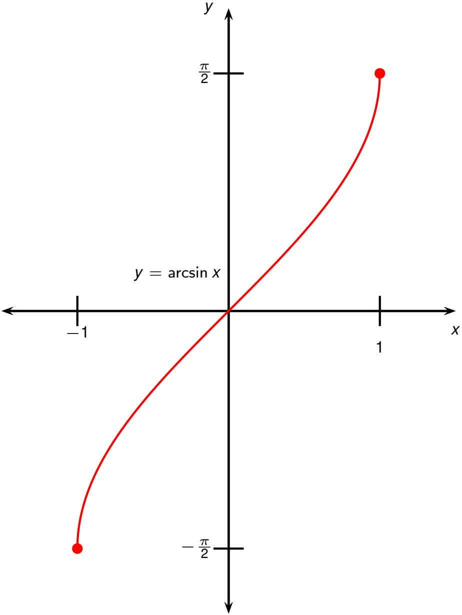

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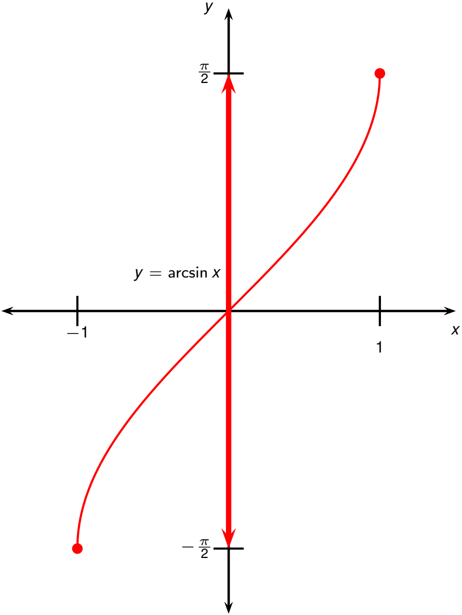

1. Domain: $[-1, 1]$.
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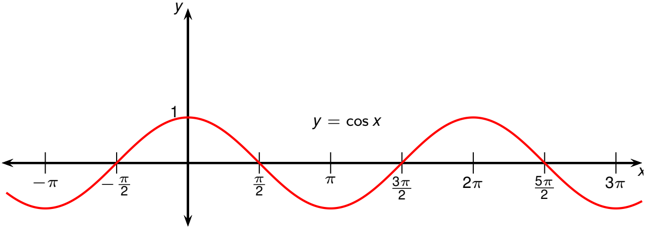
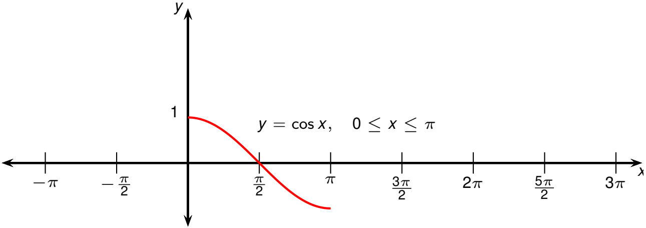
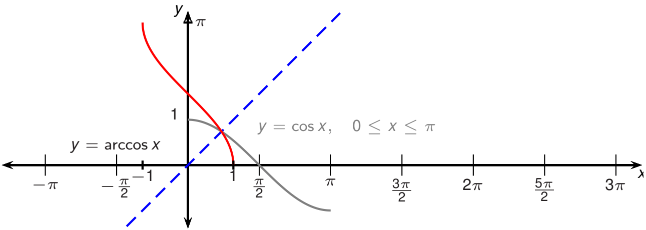


- Same for $\cos x$.



- Same for $\cos x$.
- Restrict the domain to $[0, \pi]$.



- Same for $\cos x$.
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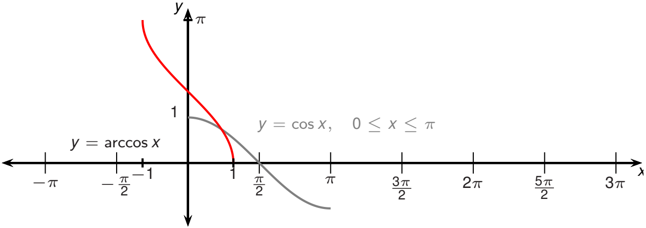

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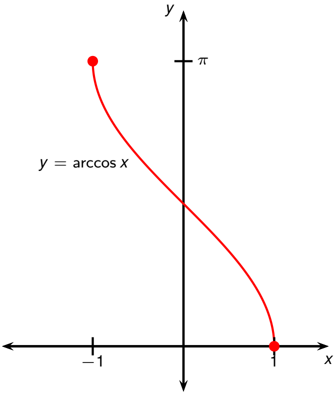
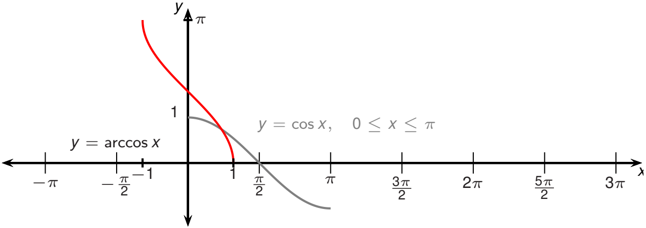
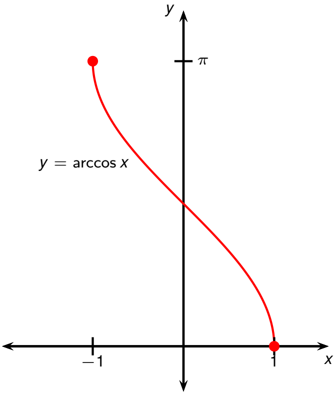
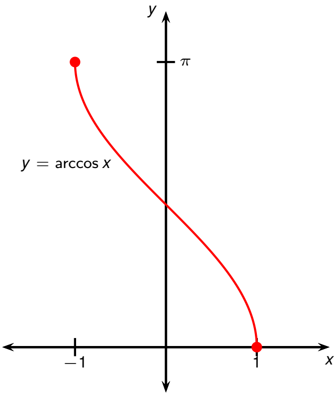




- Same for $\cos x$.
- Restrict the domain to $[0, \pi]$.
- The inverse is called arccos or $\cos^{-1}$.
- $\arccos(x) = y \Leftrightarrow \cos y = x$ and $0 \leq y \leq \pi$.



Important facts about arccos:



1. Domain:
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3. $\arccos x = y \Leftrightarrow \cos y = x$ and $0 \leq y \leq \pi$.
4. $\arccos(\cos x) = x$ for $0 \leq x \leq \pi$.
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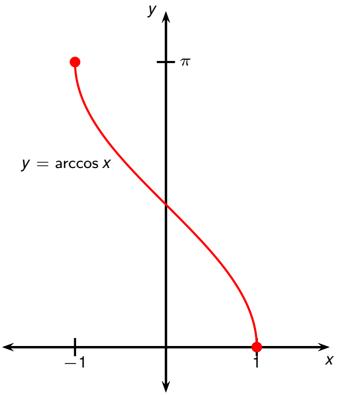

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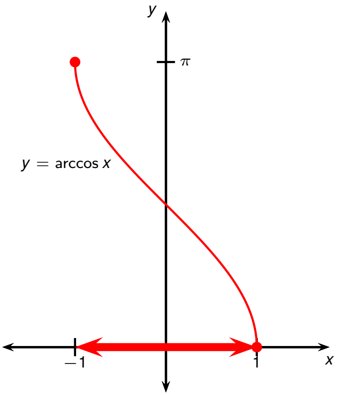

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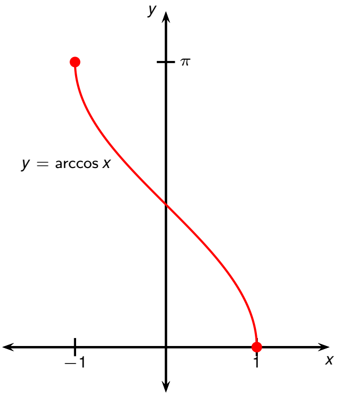

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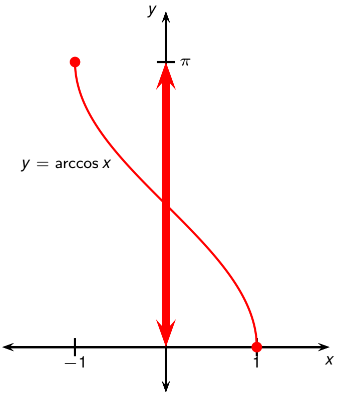

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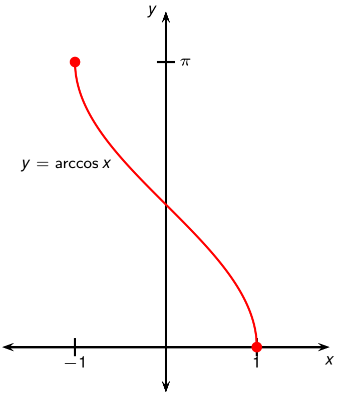

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6. $\frac{\mathrm{d}}{\mathrm{d}x}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$. (The proof is similar to the proof of the formula for the derivative of $\arcsin x$.)

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

$$\sin(2\arccos(x))$$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.
To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$.

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$\sin(2\arccos(x)) = \sin(2y)$ | Set $y = \arccos x$

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$$\begin{aligned}\sin(2\arccos(x)) &= \sin(2y)\\ &= \textbf{?}\end{aligned}$$

Set $y = \arccos x$
Express via $\sin y, \cos y$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\begin{aligned} \sin(2\arccos(x)) &= \sin(2y) \\ &= 2\cos y \sin y \end{aligned}$$

Set $y = \arccos x$
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Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$
\begin{array}{rll|l}
\sin(2\arccos(x)) &= \sin(2y) & & \text{Set } y = \arccos x \\
&= 2\cos y \sin y & & \text{Express via } \sin y, \cos y \\
&= 2\cos y \left(\pm\sqrt{1-\cos^2 y}\right) & & \text{Express } \sin y \text{ via } \cos y
\end{array}
$$

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Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

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&= 2\cos y\sqrt{1-\cos^2 y} && \sin y > 0 \text{ because } 0 \leq y \leq \pi
\end{aligned}
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$\sin(2\arccos(x)) = \sin(2y)$	Set $y = \arccos x$
$= 2\cos y \sin y$	Express via $\sin y, \cos y$
$= 2\cos y\left(\pm\sqrt{1-\cos^2 y}\right)$	Express $\sin y$ via $\cos y$
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Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

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$$\cos(3\arccos(x)) = \cos(3y) \qquad \Big| \; y = \arccos x$$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\cos(3\arccos(x)) = \cos(3y) = \cos(2y + y) \qquad \Big|\ y = \arccos x$$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the cos function.

$$\begin{aligned} \cos(3\arccos(x)) &= \cos(3y) = \cos(2y + y) \\ &= \mathbf{?} \end{aligned} \qquad \left| \begin{array}{l} y = \arccos x \\ \text{Angle sum f-la} \end{array} \right.$$

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Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\begin{aligned} \cos(3\arccos(x)) &= \cos(3y) = \cos(2y+y) && \Big| \; y = \arccos x \\ &= \cos(2y)\cos y - \sin(2y)\sin y && \Big| \; \text{Angle sum f-la} \end{aligned}$$

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Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\begin{aligned}
\cos(3\arccos(x)) &= \cos(3y) = \cos(2y+y) && \Big|\ y = \arccos x \\
&= \color{red}{\cos(2y)}\color{black}{\cos y - \sin(2y)\sin y} && \Big|\ \text{Angle sum f-la} \\
&= (\color{red}{\mathbf{?}}\color{black}{\qquad\qquad\qquad)\cos y} && \Big|\ \text{Express via} \\
&\quad -\mathbf{?}\qquad\qquad\qquad \sin y && \Big|\ \sin y, \cos y
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Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

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\cos(3\arccos(x)) &= \cos(3y) = \cos(2y+y) && \Big|\; y = \arccos x \\
&= \cos(2y)\cos y - \sin(2y)\sin y && \Big|\; \text{Angle sum f-la} \\
&= (\cos^2 y - \sin^2 y)\cos y && \Big|\; \text{Express via} \\
&\quad -\mathbf{?} \qquad\qquad \sin y && \Big|\; \sin y, \cos y
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Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

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= & \cos^3 y - 3\sin^2 y\cos y &
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&= 4\cos^3 y - 3\cos y
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& = 4x^3 - 3x & x = \cos y
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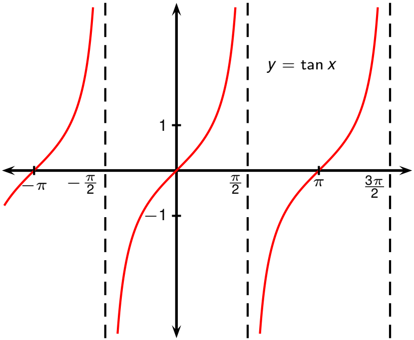
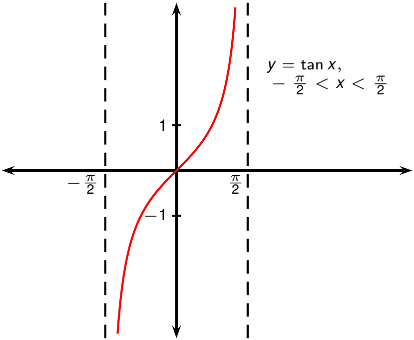


- $\tan x$ isn't one-to-one.



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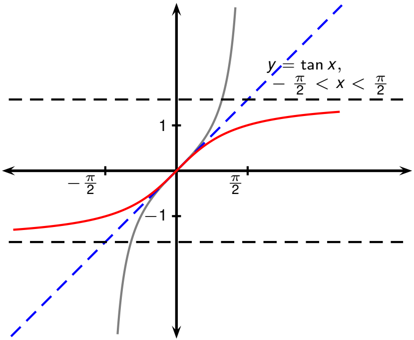

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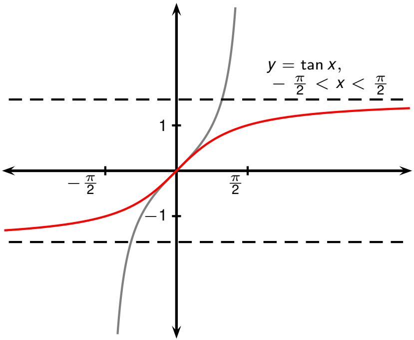

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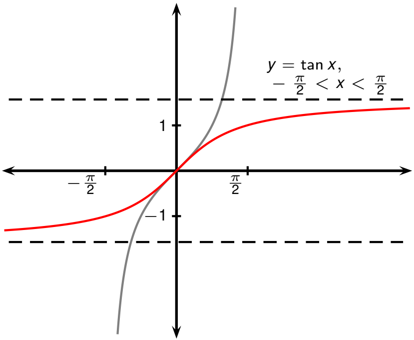

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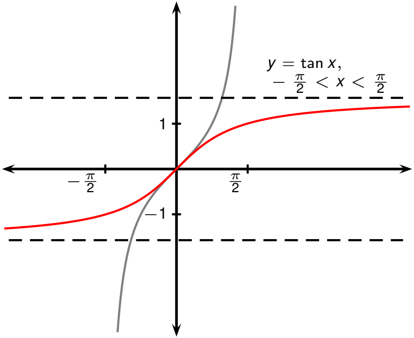
- Range of $\arctan$:



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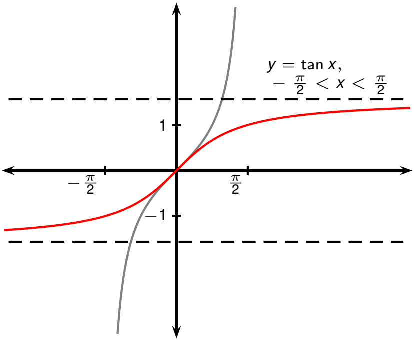

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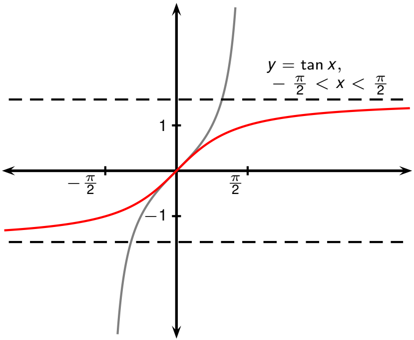

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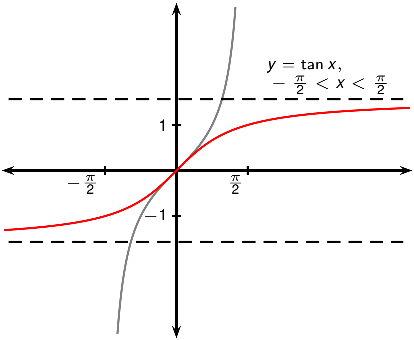

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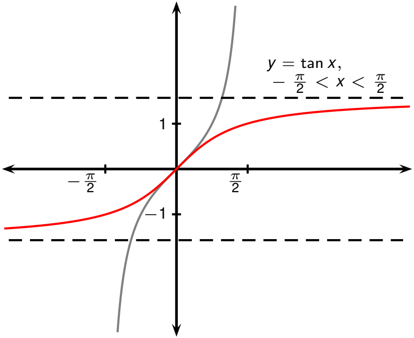

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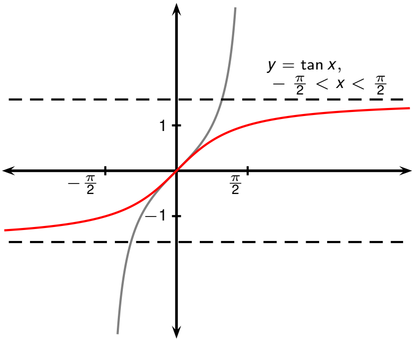

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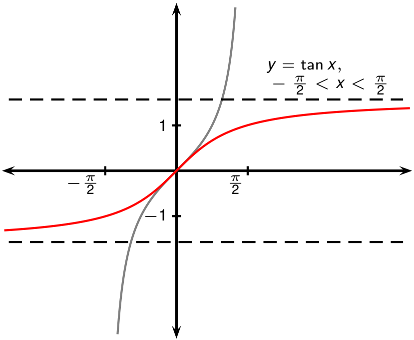

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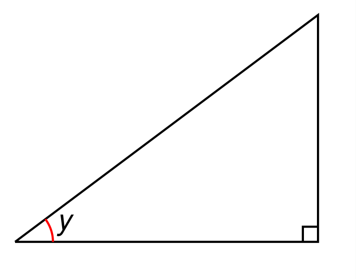
Example

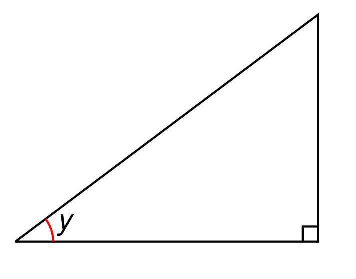
Simplify the expression $\cos(\arctan x)$.



Example

Simplify the expression $\cos(\arctan x)$.

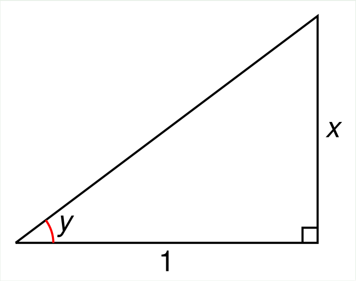
- Let $y = \arctan x$, so $\tan y = x$.



Example

Simplify the expression $\cos(\arctan x)$.

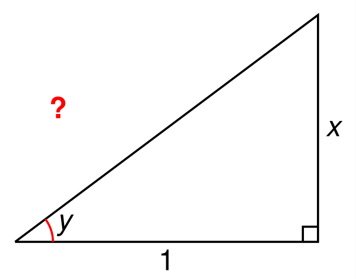
- Let $y = \arctan x$, so $\tan y = x$.
- Draw a right triangle with opposite x and adjacent 1.



Example

Simplify the expression $\cos(\arctan x)$.

- Let $y = \arctan x$, so $\tan y = x$.
- Draw a right triangle with opposite x and adjacent 1.
- Length of hypotenuse $=$ **?**



Example

Simplify the expression $\cos(\arctan x)$.

- Let $y = \arctan x$, so $\tan y = x$.
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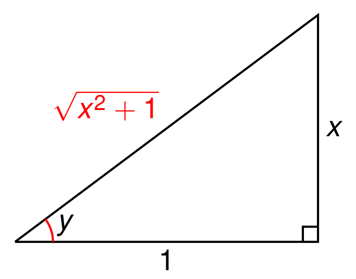
- Length of hypotenuse $= \sqrt{1^2 + x^2}$.



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- Let $y = \arctan x$, so $\tan y = x$.
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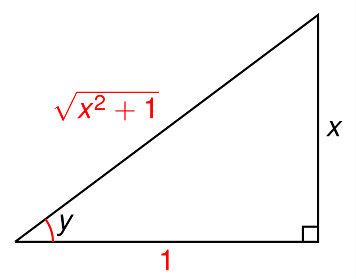
- Then $\cos(\arctan x) = \textbf{?}$



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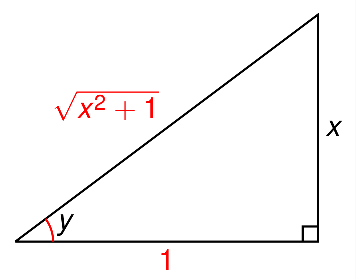

Example

Evaluate

$$\lim_{x \to 2^+} \arctan\left(\frac{1}{x-2}\right).$$

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$$\lim_{x \to 2^+} \arctan\left(\frac{1}{x-2}\right) = \textbf{?}$$

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Therefore

$$\lim_{x\to 2^+} \arctan\left(\frac{1}{x-2}\right) = \frac{\pi}{2}.$$

Theorem (The Derivative of $\arctan x$)

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}.$$

Proof.

$\square$

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$$\frac{\mathrm{d}}{\mathrm{d}x}(\arctan x) = \frac{1}{1+x^2}.$$

Proof.

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$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}.$$

Proof.

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The remaining inverse trigonometric functions aren't used as often:

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y = \operatorname{arccsc} x & (|x| \geq 1) & \Leftrightarrow & \csc y = x & \text{and} \quad y \in \left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right] \\
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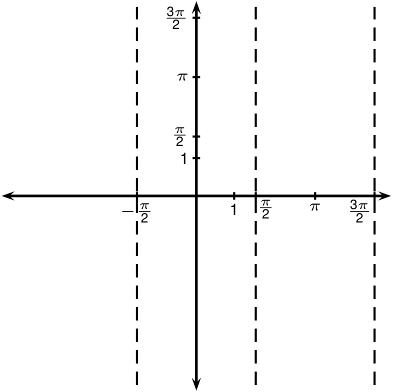

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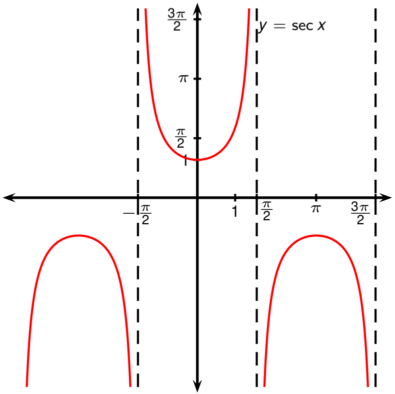

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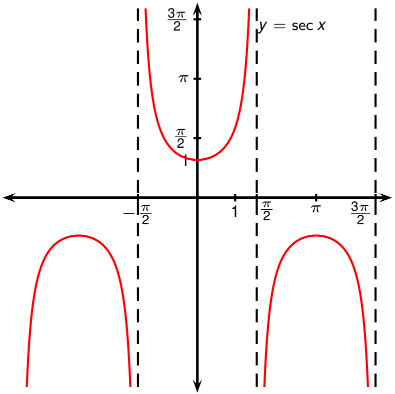

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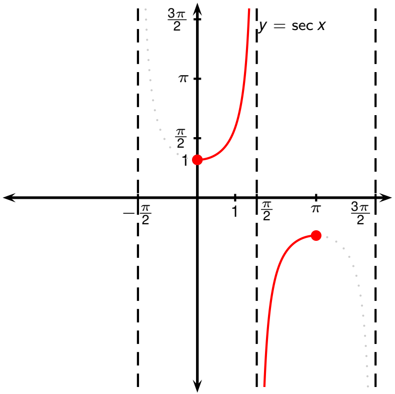

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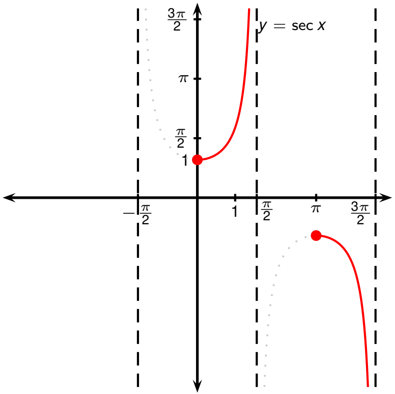

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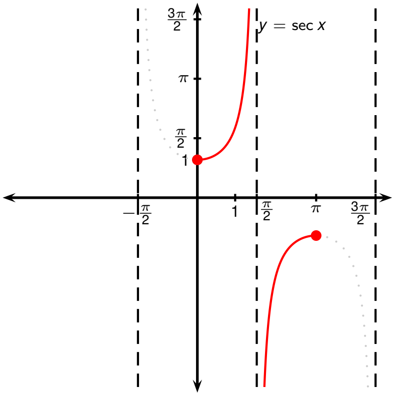

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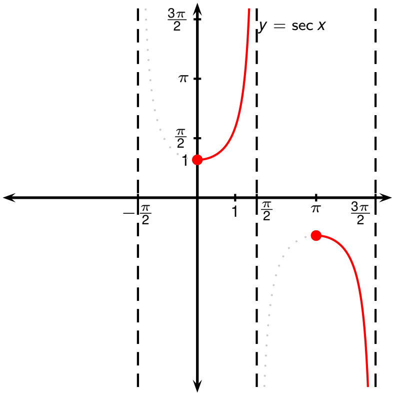

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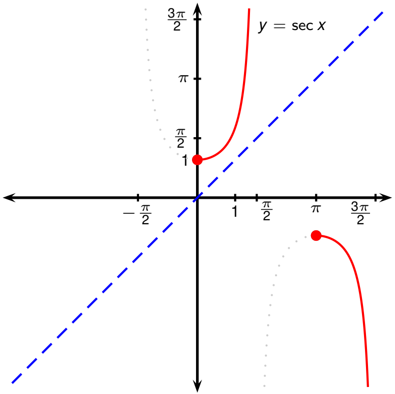

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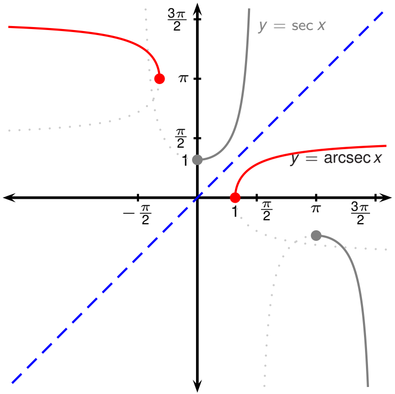

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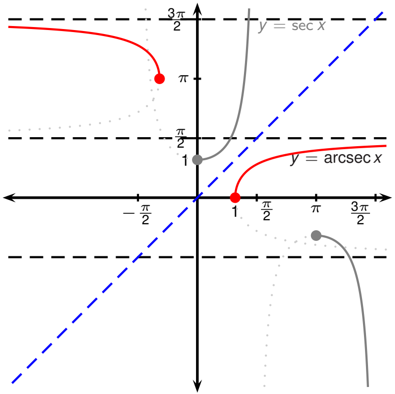

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Table of derivatives of inverse trigonometric functions:

$$\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{arccsc} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\arccos x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{arcsec} x) = \frac{1}{x\sqrt{x^2-1}}$$

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Differentiate $\quad y = \dfrac{1}{\arcsin x}.$

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$$\text{Chain Rule:} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x}$$

Example (Chain Rule)

Differentiate $y = \dfrac{1}{\arcsin x}$.

Let $u = \arcsin x$.

Then $y = u^{-1}$.

Chain Rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

$$= \left(\textbf{?} \qquad \right)\left(\qquad\qquad \right)$$

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$$= -\frac{1}{(\arcsin x)^2\sqrt{1-x^2}}.$$

All of the inverse trigonometric derivatives also give rise to integration formulas. These two are the most important:

$$\int \frac{1}{\sqrt{1-x^2}} \mathrm{d}x = \arcsin x + C.$$

$$\int \frac{1}{x^2+1} \mathrm{d}x = \arctan x + C.$$