

Calculus II

Integrals of the form $\int \frac{a}{(bx + c)^n} dx$

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Building block Ib

Building block Ib: $\int \frac{1}{x^n} dx = \int x^{-n} dx, n \neq 1.$

Example (Block Ib)

$$\int \frac{1}{x^n} dx$$

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Example (Block Ib)

$$\int \frac{1}{x^n} dx = \int x^{-n} dx = ?$$

Building block Ib

Building block Ib: $\int \frac{1}{x^n} dx = \int x^{-n} dx, n \neq 1.$

Example (Block Ib)

$$\int \frac{1}{x^n} dx = \int x^{-n} dx = \frac{x^{-n+1}}{-n+1} + C$$

Linear substitutions leading to building block Ib

Building block Ib: $\int \frac{1}{x^n} dx = \int x^{-n} dx = \frac{x^{-n+1}}{-n+1} + C, n \neq 1.$

Example

Integrate

$$\int \frac{1}{(3x+5)^3} dx$$

Linear substitutions leading to building block Ib

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Example

Integrate

$$\int \frac{1}{(3x+5)^3} dx = \int \frac{1}{(3x+5)^3} \frac{d(\textcolor{red}{3}x)}{\textcolor{red}{3}}$$

Linear substitutions leading to building block Ib

Building block Ib: $\int \frac{1}{x^n} dx = \int x^{-n} dx = \frac{x^{-n+1}}{-n+1} + C, n \neq 1.$

Example

Integrate

$$\begin{aligned} \int \frac{1}{(3x+5)^3} dx &= \int \frac{1}{(3x+5)^3} \frac{\overset{\text{red}}{d(3x)}}{3} \\ &= \int \frac{1}{(3x+5)^3} \frac{\overset{\text{red}}{d(3x+5)}}{3} \end{aligned}$$

Linear substitutions leading to building block Ib

Building block Ib: $\int \frac{1}{x^n} dx = \int x^{-n} dx = \frac{x^{-n+1}}{-n+1} + C, n \neq 1.$

Example

Integrate

$$\begin{aligned}
 \int \frac{1}{(3x+5)^3} dx &= \int \frac{1}{(3x+5)^3} \frac{d(3x)}{3} \\
 &= \int \frac{1}{(\textcolor{red}{3x+5})^3} \frac{d(\textcolor{red}{3x+5})}{3} && \left| \text{Set } \textcolor{red}{u} = \textcolor{red}{3x+5} \right. \\
 &= \int \frac{1}{\textcolor{red}{u}^3} \frac{d\textcolor{red}{u}}{3}
 \end{aligned}$$

Linear substitutions leading to building block Ib

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 &= \int \frac{1}{(3x+5)^3} \frac{d(3x+5)}{3} && \left| \text{Set } u = 3x+5 \right. \\
 &= \int \frac{1}{u^3} \frac{du}{3} \\
 &= \frac{1}{3} \int u^{-3} du
 \end{aligned}$$

Linear substitutions leading to building block Ib

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 &= \int \frac{1}{(3x+5)^3} \frac{d(3x+5)}{3} && \left| \text{Set } u = 3x + 5 \right. \\
 &= \int \frac{1}{u^3} \frac{du}{3} \\
 &= \frac{1}{3} \int u^{-3} du = \frac{1}{3} \frac{u^{-2}}{(-2)} + C
 \end{aligned}$$

Linear substitutions leading to building block Ib

Building block Ib: $\int \frac{1}{x^n} dx = \int x^{-n} dx = \frac{x^{-n+1}}{-n+1} + C, n \neq 1.$

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 &= \int \frac{1}{u^3} \frac{du}{3} \\
 &= \frac{1}{3} \int u^{-3} du = \frac{1}{3} \frac{u^{-2}}{(-2)} + C \\
 &= -\frac{1}{6(3x+5)^2} + C.
 \end{aligned}$$

Linear substitutions leading to building block Ib

Building block Ib: $\int \frac{1}{x^n} dx = \int x^{-n} dx = \frac{x^{-n+1}}{-n+1} + C, n \neq 1.$

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Integrate

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 &= \int \frac{1}{u^3} \frac{du}{3} \\
 &= \frac{1}{3} \int u^{-3} du = \frac{1}{3} \frac{u^{-2}}{(-2)} + C \\
 &= -\frac{1}{6(3x+5)^2} + C.
 \end{aligned}$$

Lin. subst. leading to building block Ib: general case

Building block Ib: $\int \frac{1}{x^n} dx = \int x^{-n} dx = \frac{x^{-n+1}}{-n+1} + C, n \neq 1.$

Example

Let $n \neq 1$. Integrate

$$\begin{aligned}
 \int \frac{1}{(ax+b)^n} dx &= \int \frac{1}{(ax+b)^n} \frac{d(ax)}{a} \\
 &= \int \frac{1}{(ax+b)^n} \frac{d(ax+b)}{a} && \left| \text{Set } u = ax+b \right. \\
 &= \int \frac{1}{u^n} \frac{du}{a} \\
 &= \frac{1}{a} \int u^{-n} du = -\frac{1}{a} \frac{u^{-n+1}}{n-1} + C \\
 &= -\frac{1}{a(n-1)(ax+b)^{n-1}} + C.
 \end{aligned}$$