

Calculus II

Homework

Building block integrals

1. Integrate. Illustrate the steps of your solution.

(a) $\int \frac{1}{x+1} dx$

(b) $\int \frac{x-1}{x+1} dx$

(c) $\int \frac{1}{(x+1)^2} dx$

(d) $\int \frac{x}{(x+1)^2} dx$

(e) $\int \frac{1}{(2x+3)^2} dx$

(f) $\int \frac{x}{2x^2+3} dx$

(g) $\int \frac{1}{2x^2+3} dx$

(h) $\int \frac{x}{2x^2+x+1} dx$.

(i) $\int \frac{x}{2x^2+x+3} dx$

(j) $\int \frac{x}{x^2-x+3} dx$

(k) $\int \frac{1}{(x^2+1)^2} dx$

(l) $\int \frac{1}{(x^2+x+1)^2} dx$

(m) $\int \frac{1}{(x^2+1)^3} dx$

2. Let a, b, c, A, B be real numbers. Suppose in addition $a \neq 0$ and $b^2 - 4ac < 0$. Integrate

$$\int \frac{Ax+B}{ax^2+bx+c} dx \quad .$$

The purpose of this exercise is to produce a formula in form ready for implementation in a computer algebra system.

3. Let a, b, c, A, B be real numbers and let $n > 1$ be an integer. Suppose in addition $a \neq 0$ and $b^2 - 4ac < 0$. Let

$$J(n) = \int \frac{1}{\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)^n} dx \quad .$$

(a) Express the integral

$$\int \frac{Ax+B}{(ax^2+bx+c)^n} dx$$

via $J(n)$.

(b) Express $J(n)$ recursively via $J(n-1)$

The purpose of this exercise is to produce a formula in form ready for implementation in a computer algebra system.