Precalculus
Polynomial division and factorization of cubics with rational root

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Outline

1. Polynomial division

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2. Factoring cubics with rational root

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

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$$x - 1 \quad \overline{\left)\, x^3 + 2x^2 \qquad + 1 \right.}$$

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Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

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Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$x - 1 \overline{\smash{\big)}\ x^3 + 2x^2 \qquad + 1}^{\textbf{?}}$$

Divide x^3 by x.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 + 1}
\end{array}
$$

Divide x^3 by x.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 + 1} \\
\textbf{?} \quad \textbf{?}
\end{array}
$$

Multiply x^2 by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 \\
x - 1 \enclose{}{\overline{\smash{\big)}\, x^3 + 2x^2 + 1}} \\
\underline{x^3 - x^2 }
\end{array}
$$

Multiply x^2 by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 + 1} \\
\underline{x^3 - x^2 } \\
\textbf{?} \textbf{?}
\end{array}
$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 \\
x - 1 \enclose{verticalline}{} \overline{x^3 + 2x^2 + 1} \\
x^3 - x^2 \\
\hline
3x^2 + 1
\end{array}
$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 \quad \textbf{?} \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 \qquad + 1} \\
\underline{x^3 - x^2} \qquad\qquad \\
3x^2 \qquad + 1
\end{array}
$$

Divide $3x^2$ by x.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 + 3x \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 \quad\quad + 1} \\
\underline{x^3 - x^2 \quad\quad\quad\quad} \\
3x^2 \quad\quad + 1
\end{array}
$$

Divide $3x^2$ by x.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 + 3x \\
x - 1 \quad \overline{\smash{\big)}\, x^3 + 2x^2 \quad\quad + 1} \\
\underline{x^3 - x^2} \\
3x^2 \quad\quad + 1 \\
\underline{\mathbf{?} \quad\quad \mathbf{?}}
\end{array}
$$

Multiply $3x$ by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 + 3x \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 \qquad + 1} \\
\underline{x^3 - x^2 \qquad\qquad} \\
3x^2 \qquad + 1 \\
\underline{3x^2 - 3x \qquad}
\end{array}
$$

Multiply $3x$ by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 + 3x \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 + 1} \\
\underline{x^3 - x^2} \\
3x^2 + 1 \\
\underline{3x^2 - 3x} \\
??
\end{array}
$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 + 3x \\
x - 1 \enclose{} {\overline{\smash{\big)}\,x^3 + 2x^2 \qquad + 1}} \\
\underline{x^3 - x^2} \qquad\qquad \\
3x^2 \qquad + 1 \\
\underline{3x^2 - 3x} \\
3x + 1
\end{array}
$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 + 3x \quad \textbf{?} \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 + 1} \\
\underline{x^3 - x^2} \\
3x^2 + 1 \\
\underline{3x^2 - 3x} \\
3x + 1
\end{array}
$$

Divide $3x$ by x.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 + 3x + 3 \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 + 1} \\
\underline{x^3 - x^2 } \\
3x^2 + 1 \\
\underline{3x^2 - 3x } \\
3x + 1
\end{array}
$$

Divide $3x$ by x.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 + 3x + 3 \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 + 1} \\
\underline{x^3 - x^2} \\
3x^2 + 1 \\
\underline{3x^2 - 3x} \\
3x + 1 \\
\underline{ ? ?}
\end{array}
$$

Multiply 3 by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 + 3x + 3 \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 + 1} \\
\underline{x^3 - x^2} \\
3x^2 + 1 \\
\underline{3x^2 - 3x} \\
3x + 1 \\
\underline{3x - 3}
\end{array}
$$

Multiply 3 by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 + 3x + 3 \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 + 1} \\
\underline{x^3 - x^2} \\
3x^2 + 1 \\
\underline{3x^2 - 3x} \\
3x + 1 \\
\underline{3x - 3} \\
\mathbf{?}
\end{array}
$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 + 3x + 3 \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 \qquad + 1} \\
\underline{x^3 - x^2} \\
3x^2 \qquad + 1 \\
\underline{3x^2 - 3x} \\
3x + 1 \\
\underline{3x - 3} \\
4
\end{array}
$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
x^2 + 3x + 3 \\
x - 1 \overline{\smash{\big)} x^3 + 2x^2 \qquad + 1} \\
\underline{x^3 - x^2} \\
3x^2 \qquad + 1 \\
\underline{3x^2 - 3x} \\
3x + 1 \\
\underline{3x - 3} \\
4
\end{array}
$$

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
\textbf{Quotient:} \quad x^2 + 3x + 3 \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 \qquad + 1} \\
\underline{-\quad x^3 - x^2} \\
3x^2 \qquad + 1 \\
\underline{-\quad 3x^2 - 3x} \\
3x + 1 \\
\underline{-\quad 3x - 3} \\
4
\end{array}
$$

$$
\begin{aligned}
\textbf{(Dividend)} &= \textbf{(Quotient)} \cdot \textbf{(Divisor)} + \textbf{(Remainder)} \\
(x^3 + 2x^2 + 1) &= (x^2 + 3x + 3) \cdot (x - 1) + 4
\end{aligned}
$$

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r}
\textbf{Quotient:} \quad x^2 + 3x + 3 \\
x - 1 \enclose{longdiv}{x^3 + 2x^2 \quad\; + 1} \\
- \quad\; \underline{x^3 - x^2} \qquad\qquad \\
3x^2 \quad\; + 1 \\
- \qquad \underline{3x^2 - 3x} \qquad \\
3x + 1 \\
- \qquad\qquad \underline{3x - 3} \\
\textbf{Remainder:} \qquad 4
\end{array}
$$

$$
\begin{aligned}
\textbf{(Dividend)} &= \textbf{(Quotient)} \cdot \textbf{(Divisor)} + \textbf{(Remainder)} \\
(x^3 + 2x^2 + 1) &= (x^2 + 3x + 3) \cdot (x - 1) + 4
\end{aligned}
$$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$2x - 3 \enclose{longdiv}{6x^3 - 19x^2 + 17x - 3}$$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$2x - 3 \quad \overline{\smash{\big)}\, 6x^3 - 19x^2 + 17x - 3}$$

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$$2x - 3 \quad \overline{\smash{\big)}\, 6x^3 - 19x^2 + 17x - 3}^{\textstyle \mathbf{?}}$$

Divide $6x^3$ by $2x$.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 \\
2x - 3 \enclose{longdiv}{6x^3 - 19x^2 + 17x - 3}
\end{array}
$$

Divide $6x^3$ by $2x$.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 \\
2x - 3 \overline{\smash{\big)}\, 6x^3 - 19x^2 + 17x - 3} \\
\mathbf{?} \qquad \mathbf{?}
\end{array}
$$

Multiply $3x^2$ by divisor.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 \\
2x - 3 \overline{\smash{\big)}\ 6x^3 - 19x^2 + 17x - 3} \\
6x^3 - 9x^2
\end{array}
$$

Multiply $3x^2$ by divisor.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 \\
2x - 3 \enclose{longdiv}{6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2 } \\
?\qquad ?\qquad ?
\end{array}
$$

Subtract last two polynomials.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 \\
2x - 3 \overline{\smash{)}6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2} \\
-10x^2 + 17x - 3
\end{array}
$$

Subtract last two polynomials.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 \quad \textbf{?} \\
2x - 3 \overline{\smash{\big)}\ 6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2} \\
-10x^2 + 17x - 3
\end{array}
$$

Divide $-10x^2$ by $2x$.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 - 5x \\
2x - 3 \overline{\smash{\big)}\, 6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2 } \\
-10x^2 + 17x - 3
\end{array}
$$

Divide $-10x^2$ by $2x$.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 - 5x \\
2x - 3 \overline{\smash{\big)}\ 6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2 } \\
-10x^2 + 17x - 3 \\
\underline{??}
\end{array}
$$

Multiply $-5x$ by divisor.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 - 5x \\
2x - 3 \enclose{longdiv}{6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2 } \\
-10x^2 + 17x - 3 \\
\underline{-10x^2 + 15x }
\end{array}
$$

Multiply $-5x$ by divisor.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 - 5x \\
2x - 3 \overline{\smash{\big)}\ 6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2} \\
-10x^2 + 17x - 3 \\
\underline{-10x^2 + 15x} \\
\textbf{?} \qquad \textbf{?}
\end{array}
$$

Subtract last two polynomials.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 - 5x \\
2x - 3 \enclose{longdiv}{6x^3 - 19x^2 + 17x - 3} \\
6x^3 - 9x^2 \\
\hline
-10x^2 + 17x - 3 \\
-10x^2 + 15x \\
\hline
2x - 3
\end{array}
$$

Subtract last two polynomials.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 - 5x \quad \textcolor{red}{\textbf{?}} \\
\textcolor{red}{2x} - 3 \enclose{longdiv}{6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2} \\
-10x^2 + 17x - 3 \\
\underline{-10x^2 + 15x} \\
\textcolor{red}{2x} - 3
\end{array}
$$

Divide $\textcolor{red}{2x}$ by $\textcolor{red}{2x}$.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 - 5x + 1 \\
2x - 3 \overline{\smash{)}6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2} \\
-10x^2 + 17x - 3 \\
\underline{-10x^2 + 15x} \\
2x - 3
\end{array}
$$

Divide $2x$ by $2x$.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 - 5x + 1 \\
2x - 3 \overline{\smash{\big)}\ 6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2} \\
-10x^2 + 17x - 3 \\
\underline{-10x^2 + 15x} \\
2x - 3 \\
\underline{\textbf{?} \qquad \textbf{?}}
\end{array}
$$

Multiply 1 by divisor.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 - 5x + 1 \\
2x - 3 \overline{\smash{\big)}\, 6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2} \\
-10x^2 + 17x - 3 \\
\underline{-10x^2 + 15x} \\
2x - 3 \\
\underline{2x - 3}
\end{array}
$$

Multiply 1 by divisor.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 - 5x + 1 \\
2x - 3 \enclose{longdiv}{6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2} \\
-10x^2 + 17x - 3 \\
\underline{-10x^2 + 15x} \\
2x - 3 \\
\underline{2x - 3} \\
?
\end{array}
$$

Subtract last two polynomials.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 - 5x + 1 \\
2x - 3 \overline{\smash{\big)}\ 6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2} \\
-10x^2 + 17x - 3 \\
\underline{-10x^2 + 15x} \\
2x - 3 \\
\underline{2x - 3} \\
0
\end{array}
$$

Subtract last two polynomials.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
3x^2 - 5x + 1 \\
2x - 3 \overline{\smash{\big)}\, 6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2} \\
-10x^2 + 17x - 3 \\
\underline{-10x^2 + 15x} \\
2x - 3 \\
\underline{2x - 3} \\
0
\end{array}
$$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
\textbf{Quotient:} \quad 3x^2 - 5x + 1 \\
2x - 3 \; \overline{\big)\; 6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2} \\
-10x^2 + 17x - 3 \\
\underline{-10x^2 + 15x} \\
2x - 3 \\
\underline{2x - 3} \\
0
\end{array}
$$

(Dividend)=**(Quotient)** $\cdot$ **(Divisor)** $+$ **(Remainder)**

$(6x^3 - 19x^2 + 17x - 3)$=$(3x^2 - 5x + 1) \cdot (2x - 3)$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
\textbf{Quotient:} \quad 3x^2 - 5x + 1 \\
2x - 3 \overline{\smash{\big)}\, 6x^3 - 19x^2 + 17x - 3} \\
\underline{\; 6x^3 - 9x^2} \\
-10x^2 + 17x - 3 \\
\underline{-10x^2 + 15x} \\
2x - 3 \\
\underline{2x - 3} \\
\textbf{Remainder:} \quad 0
\end{array}
$$

(Dividend)=**(Quotient)** $\cdot$ **(Divisor)** $+$ **(Remainder)**

$(6x^3 - 19x^2 + 17x - 3)$=$(3x^2 - 5x + 1) \cdot (2x - 3)$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r}
\textbf{Quotient:} \quad 3x^2 - 5x + 1 \\
2x - 3 \enclose{longdiv}{6x^3 - 19x^2 + 17x - 3} \\
\underline{6x^3 - 9x^2} \\
-10x^2 + 17x - 3 \\
\underline{-10x^2 + 15x} \\
2x - 3 \\
\underline{2x - 3} \\
\textbf{Remainder:} \quad 0
\end{array}
$$

(Dividend)=**(Quotient)** · **(Divisor)** + **(Remainder)**

$(6x^3 - 19x^2 + 17x - 3)$=$(3x^2 - 5x + 1) \cdot (2x - 3)$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$(6x^3 - 19x^2 + 17x - 3) = (3x^2 - 5x + 1) \cdot (2x - 3)$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$(6x^3 - 19x^2 + 17x - 3) = (3x^2 - 5x + 1) \cdot (2x - 3)$$
$$= 3 \left(x - \textbf{?} \right) \left(x - \textbf{?} \right) (2x - 3)$$

No easy factorization of quadratic, so use formula:

$x_1, x_2 = \textbf{?}$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$(6x^3 - 19x^2 + 17x - 3) = (3x^2 - 5x + 1) \cdot (2x - 3)$$
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Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$2x^3 + x^2 - 7x - 6 = 0$$

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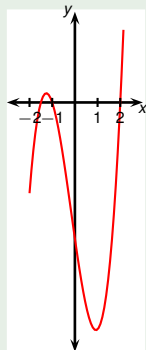
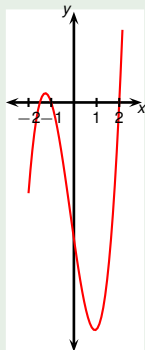Plot the left hand side of the equation with a graphing calculator. Solve the equation.

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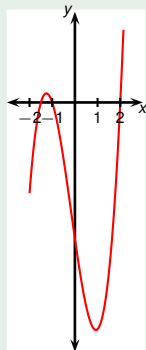Plot the left hand side of the equation with a graphing calculator. Solve the equation.

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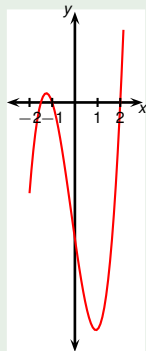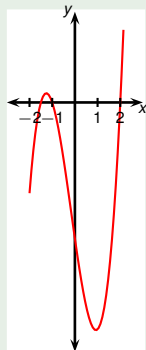
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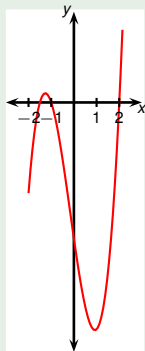Plot the left hand side of the equation with a graphing calculator. Solve the equation.

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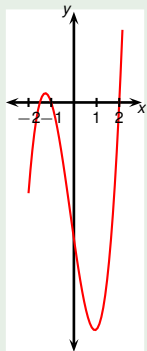Plot the left hand side of the equation with a graphing calculator. Solve the equation.

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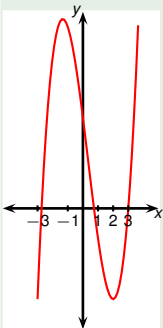Plot the left hand side of the equation with a graphing calculator. Solve the equation.

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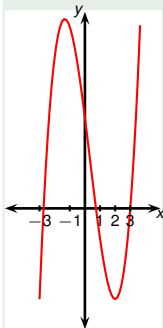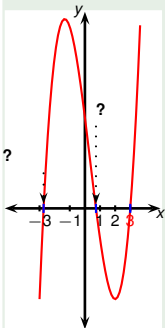
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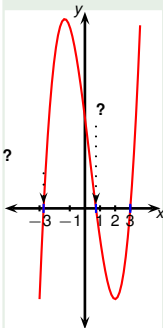
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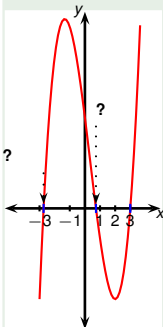$$= 2x^3 + x^2 - 7x - 6$$

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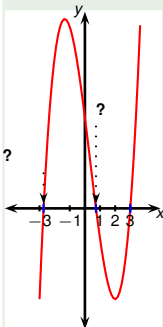$$= 2x^3 + x^2 - 7x - 6$$

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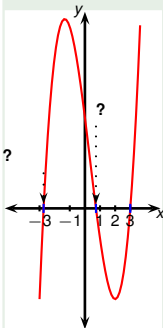
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\begin{aligned}
2x^3 + x^2 - 7x - 6 &= 0 \\
(2x+3)(x+1)(x-2) &= 0 \\
x = -\tfrac{3}{2} \quad \textbf{or} \quad x = -1 \quad &\textbf{or} \quad x = 2
\end{aligned}
$$

Make sure to practice with the graphing calculator you will on your exam(s). The graph appears to intersect the x axis at: $-1.5, -1, 2$. The left hand side should factor as:

$$
2(x - (-1.5))(x - (-1))(x - 2) = (2x+3)(x+1)(x-2)
$$

$$
= (2x^2 + 5x + 3)(x - 2) = (2x^3 + 5x^2 + 3x) - (4x^2 + 10x + 6)
$$

$$
= 2x^3 + x^2 - 7x - 6
$$

Check work to make sure we guessed the roots correctly.

Example

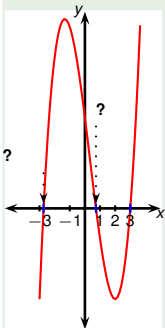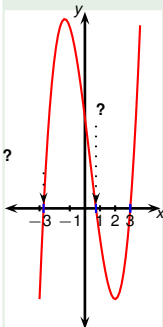Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 = 0$$

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
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Example

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$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
?

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
?   , **?**   , 3.
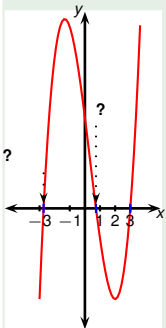
Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
? , **?** , 3. What are the two roots besides 3?

$$x - 3 \quad \overline{\smash{)}x^3 - x^2 - 8x + 6}$$
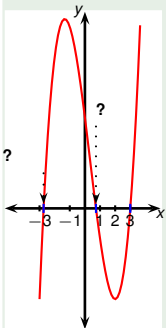
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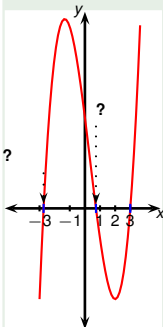
$$x - 3 \quad \overline{\smash{\big)}\,x^3 - x^2 - 8x + 6}$$

Example

Plot the left hand side of the equation with a graphing
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$$x^3 - x^2 - 8x + 6 \ = \ 0$$

The graph appears to intersect the x axis at:
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$$x - 3 \quad \overline{\smash{\big)}\ x^3 - x^2 - 8x + 6}$$

Divide x^3 by x.
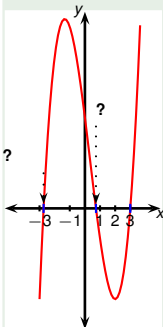
Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 \;=\; 0$$

The graph appears to intersect the x axis at:
?   , **?**   , 3. What are the two roots besides 3?

$$
\begin{array}{r}
x^2 \\
x - 3 \enclose{longdiv}{x^3 - x^2 - 8x + 6}
\end{array}
$$

Divide x^3 by x.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
? , **?** , 3. What are the two roots besides 3?

$$
\begin{array}{r}
x^2 \\
x - 3 \overline{\smash{\big)}\, x^3 - x^2 - 8x + 6} \\
\textbf{?} \quad \textbf{?}
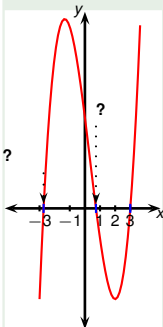\end{array}
$$

Multiply x^2 by divisor.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
?   , **?**   , 3. What are the two roots besides 3?

$$
\begin{array}{r}
x^2 \qquad\qquad\quad \\
x - 3 \overline{\smash{\big)}\, x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \qquad\qquad\;
\end{array}
$$

Multiply x^2 by divisor.
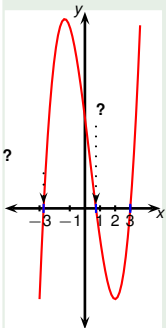
Example

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$$
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x - 3 \overline{\big)\, x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \\
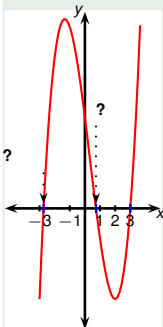\textbf{?} \quad \textbf{?} \quad \textbf{?}
\end{array}
$$

Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing
calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
?   , **?**   , 3. What are the two roots besides 3?

$$
\begin{array}{r}
x^2 \\
x - 3 \enclose{longdiv}{x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \\
2x^2 - 8x + 6
\end{array}
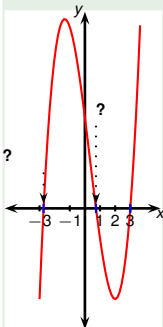$$

Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing
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$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
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$$
\begin{array}{r}
x^2 \quad \textbf{?} \\
x - 3 \enclose{longdiv}{x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \\
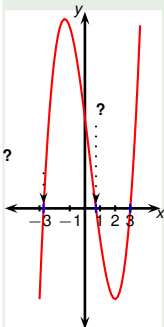2x^2 - 8x + 6
\end{array}
$$

Divide $2x^2$ by x.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
?   , **?**   , 3. What are the two roots besides 3?

$$
\begin{array}{r}
x^2 + 2x \\
x - 3 \overline{\smash{\big)}\, x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \\
2x^2 - 8x + 6
\end{array}
$$

Divide $2x^2$ by x.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 = 0$$

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$$
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\underline{x^3 - 3x^2} \\
2x^2 - 8x + 6 \\
\underline{\textbf{?}\quad\textbf{?}}
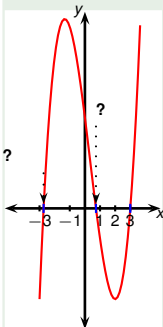\end{array}
$$

Multiply $2x$ by divisor.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
?   , **?**   , 3. What are the two roots besides 3?

$$
\begin{array}{r}
x^2 + 2x \\
x - 3 \overline{\smash{\big)}\ x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \\
2x^2 - 8x + 6 \\
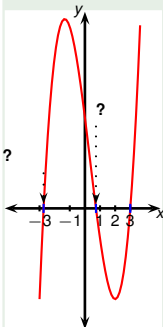\underline{2x^2 - 6x}
\end{array}
$$

Multiply $2x$ by divisor.

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Plot the left hand side of the equation with a graphing calculator. Solve the equation.
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The graph appears to intersect the x axis at:
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$$
\begin{array}{r}
x^2 + 2x \\
x - 3 \overline{\smash{\big)}\ x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \\
2x^2 - 8x + 6 \\
\underline{2x^2 - 6x} \\
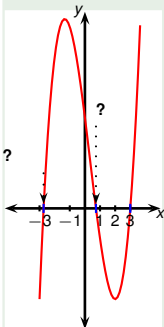\textbf{?} \quad \textbf{?}
\end{array}
$$

Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
?   , **?**   , 3. What are the two roots besides 3?

$$
\begin{array}{r}
x^2 + 2x \\
x - 3 \enclose{longdiv}{x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \\
2x^2 - 8x + 6 \\
\underline{2x^2 - 6x} \\
-2x + 6
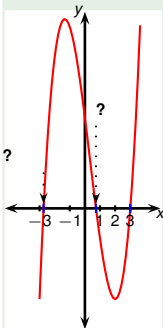\end{array}
$$

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The graph appears to intersect the x axis at:
?   , **?**   , 3. What are the two roots besides 3?

$$
\begin{array}{r}
x^2 + 2x \quad \textbf{?} \\
x - 3 \overline{\smash{\big)}\ x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \\
2x^2 - 8x + 6 \\
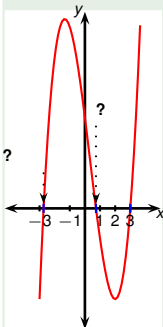\underline{2x^2 - 6x} \\
-2x + 6
\end{array}
$$

Divide $-2x$ by x.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
?   , **?**   , 3. What are the two roots besides 3?

$$
\begin{array}{r}
x^2 + 2x - 2 \\
x - 3 \overline{\smash{\big)}\ x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \\
2x^2 - 8x + 6 \\
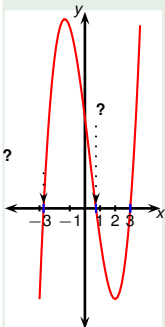\underline{2x^2 - 6x} \\
-2x + 6
\end{array}
$$

Divide $-2x$ by x.

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Plot the left hand side of the equation with a graphing calculator. Solve the equation.
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The graph appears to intersect the x axis at:
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$$
\begin{array}{r}
x^2 + 2x - 2 \\
x - 3 \overline{\smash{\big)}\ x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \\
2x^2 - 8x + 6 \\
\underline{2x^2 - 6x} \\
-2x + 6 \\
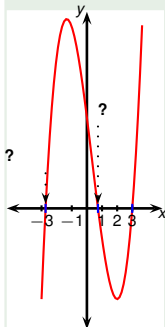\underline{\mathbf{?} \quad \mathbf{?}}
\end{array}
$$

Multiply -2 by divisor.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
? , **?** , 3. What are the two roots besides 3?

$$
\begin{array}{r}
x^2 + 2x - 2 \\
x - 3 \overline{\smash{\big)}\ x^3 - x^2 - 8x + 6} \\
x^3 - 3x^2 \\
\hline
2x^2 - 8x + 6 \\
2x^2 - 6x \\
\hline
-2x + 6 \\
-2x + 6 \\
\hline
\end{array}
$$

Multiply -2 by divisor.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 \ = \ 0$$

The graph appears to intersect the x axis at:
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$$
\begin{array}{r}
x^2 + 2x - 2 \\
x - 3 \overline{\smash{\big)}\ x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \\
2x^2 - 8x + 6 \\
\underline{2x^2 - 6x} \\
-2x + 6 \\
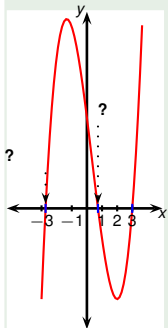\underline{-2x + 6} \\
?
\end{array}
$$

Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
?   , **?**   , 3. What are the two roots besides 3?

$$
\begin{array}{r}
x^2 + 2x - 2 \\
x - 3 \overline{\smash{)}\ x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \\
2x^2 - 8x + 6 \\
\underline{2x^2 - 6x} \\
-2x + 6 \\
\underline{-2x + 6} \\
0
\end{array}
$$

Subtract last two polynomials.

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x^2 + 2x - 2 \\
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\underline{x^3 - 3x^2} \\
2x^2 - 8x + 6 \\
\underline{2x^2 - 6x} \\
-2x + 6 \\
\underline{-2x + 6} \\
0
\end{array}
$$

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$
$$(x - 3)(x^2 + 2x - 2) + 0 = 0$$

The graph appears to intersect the x axis at:
?   , **?**   , 3. What are the two roots besides 3?

Quotient: $x^2 + 2x - 2$

$$
\begin{array}{r|l}
x - 3 & x^3 - x^2 - 8x + 6 \\
      & x^3 - 3x^2 \\
\hline
      & \quad 2x^2 - 8x + 6 \\
      & \quad 2x^2 - 6x \\
\hline
      & \qquad -2x + 6 \\
      & \qquad -2x + 6 \\
\hline
      & \qquad\qquad 0
\end{array}
$$

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$
$$(x - 3)(x^2 + 2x - 2) + 0 = 0$$

The graph appears to intersect the x axis at:
? , **?** , 3. What are the two roots besides 3?

$$
\begin{array}{r}
\textbf{Quotient:} \quad x^2 + 2x - 2 \\
x - 3 \enclose{longdiv}{x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \\
2x^2 - 8x + 6 \\
\underline{2x^2 - 6x} \\
-2x + 6 \\
\underline{-2x + 6} \\
\textbf{Remainder:} \quad 0
\end{array}
$$

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

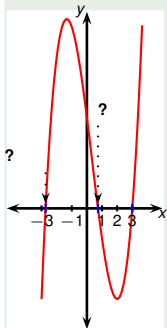$$x^3 - x^2 - 8x + 6 = 0$$
$$(x - 3)(x^2 + 2x - 2) = 0$$

The graph appears to intersect the x axis at:
?   , **?**   , 3. What are the two roots besides 3?

Quotient: $\quad x^2 + 2x - 2$

$$
\begin{array}{r}
x - 3 \enclose{longdiv}{x^3 - x^2 - 8x + 6} \\
\underline{x^3 - 3x^2} \\
2x^2 - 8x + 6 \\
\underline{2x^2 - 6x} \\
-2x + 6 \\
\underline{-2x + 6} \\
\end{array}
$$

Remainder: $\qquad 0$

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$
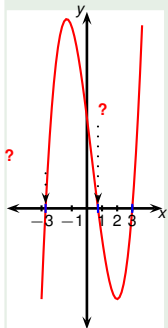$$(x - 3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \textbf{or} \quad x=$$

The graph appears to intersect the x axis at:
?   , **?**   , 3. What are the two roots besides 3?

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$
$$(x-3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \textbf{or} \quad x =$$

$$x = 3$$

The graph appears to intersect the x axis at:
? , **?** , 3. What are the two roots besides 3?

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$
$$(x - 3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \textbf{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$
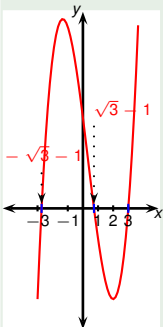
$$x = 3$$

The graph appears to intersect the x axis at:
? , **?** , 3. What are the two roots besides 3?

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Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$
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The graph appears to intersect the x axis at:
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Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.
$$x^3 - x^2 - 8x + 6 \; = \; 0$$
$$(x-3)(x^2 + 2x - 2) \qquad = \; 0$$

$$x - 3 = 0 \qquad \textbf{or} \qquad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x = 3$$

The graph appears to intersect the x axis at:
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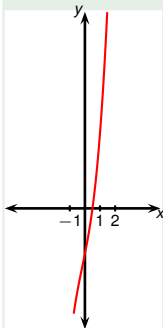
Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$
$$(x - 3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \textbf{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x = 3 \qquad\qquad x = \frac{-2 \pm \sqrt{12}}{2}$$

The graph appears to intersect the x axis at:
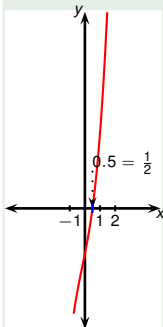**?** , **?** , 3. What are the two roots besides 3?
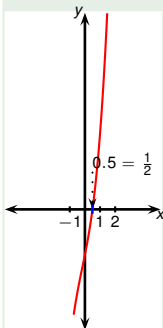
Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$
$$(x - 3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \textbf{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x = 3 \qquad\qquad x = \frac{-2 \pm \sqrt{12}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{3}}{2} \qquad\qquad .$$

The graph appears to intersect the x axis at:
? , **?** , 3. What are the two roots besides 3?
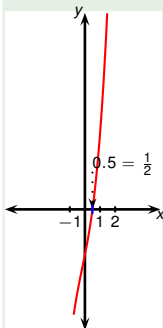
Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$
$$(x - 3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \textbf{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x = 3 \qquad \qquad x = \frac{-2 \pm \sqrt{12}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}.$$

The graph appears to intersect the x axis at:
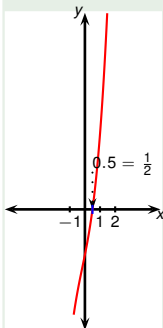**?**   , **?**   , 3. What are the two roots besides 3?

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$
$$(x - 3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \textbf{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x = 3 \qquad\qquad x = \frac{-2 \pm \sqrt{12}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}.$$

The graph appears to intersect the x axis at:
$-\sqrt{3} - 1, \ \sqrt{3} - 1, 3$. What are the two roots besides 3?

Final answer:

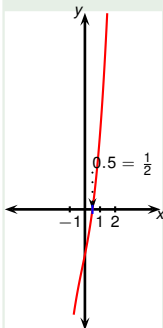$$x = 3 \quad \textbf{or} \quad x = -1 - \sqrt{3} \quad \textbf{or} \quad x = -1 + \sqrt{3}.$$

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
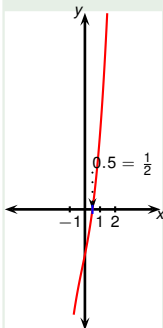$$2x^3 + x^2 + 5x - 3 = 0$$
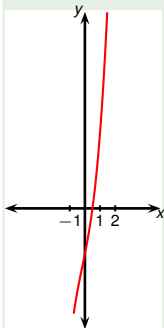
Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = $ **?**   .

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

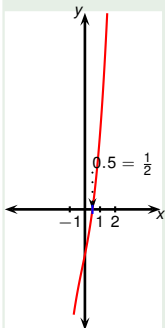We see only one root, $x = 0.5 = \frac{1}{2}$.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)?

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct?
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Example

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We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:
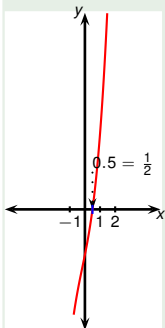
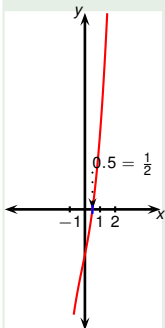$$x - \frac{1}{2} \quad \overline{\left\vert 2x^3 + x^2 + 5x - 3 \right.}$$

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$x - \tfrac{1}{2} \quad \overline{\left.2x^3 + x^2 + 5x - 3\right.}$$
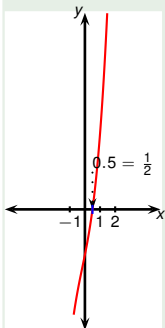
Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

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Example

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$$x - \frac{1}{2} \quad \overline{\left| 2x^3 + x^2 + 5x - 3 \right.}$$

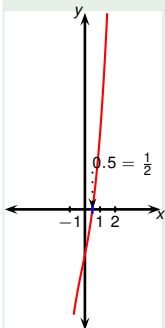Divide $2x^3$ by x.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$x - \tfrac{1}{2} \quad \overline{\big)\, 2x^3 + x^2 + 5x - 3}$$
$$2x^2$$

Divide $2x^3$ by x.
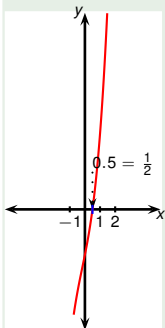
Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$x - \tfrac{1}{2} \enclose{longdiv}{2x^3 + x^2 + 5x - 3}$$

$$2x^2$$

?   **?**

Multiply $2x^2$ by divisor.

Todor Milev   Polynomial division and factorization of cu . . .   2019
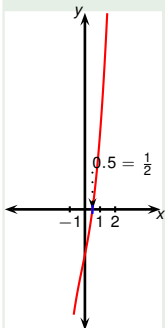
Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 \\
x - \tfrac{1}{2} \enclose{longdiv}{2x^3 + x^2 + 5x - 3} \\
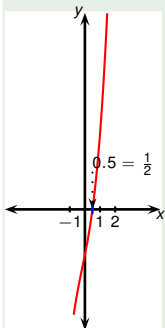\underline{2x^3 - x^2}
\end{array}
$$

Multiply $2x^2$ by divisor.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 \\
x - \tfrac{1}{2} \enclose{longdiv}{2x^3 + x^2 + 5x - 3} \\
\underline{2x^3 - x^2} \\
?\quad ?\quad ?
\end{array}
$$

Subtract last two polynomials.
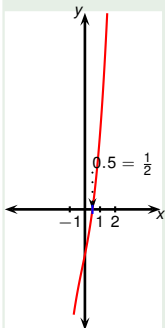
Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 \\
x - \frac{1}{2} \enclose{longdiv}{2x^3 + x^2 + 5x - 3} \\
\underline{2x^3 - x^2} \\
2x^2 + 5x - 3
\end{array}
$$

Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 \;=\; 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 \quad \textbf{?} \\
x - \tfrac{1}{2} \enclose{longdiv}{2x^3 + x^2 + 5x - 3} \\
\underline{2x^3 - x^2} \\
2x^2 + 5x - 3
\end{array}
$$

Divide $2x^2$ by x.
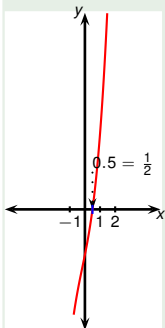
Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 + 2x \\
x - \tfrac{1}{2} \enclose{longdiv}{2x^3 + x^2 + 5x - 3} \\
2x^3 - x^2 \\
\hline
2x^2 + 5x - 3
\end{array}
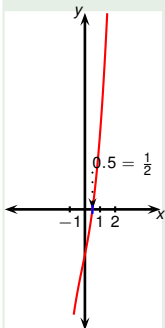$$

Divide $2x^2$ by x.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 + 2x \\
x - \frac{1}{2} \overline{\big)\ 2x^3 + x^2 + 5x - 3} \\
\underline{2x^3 - x^2} \\
2x^2 + 5x - 3 \\
\underline{\mathbf{?}\qquad\mathbf{?}}
\end{array}
$$

Multiply $2x$ by divisor.
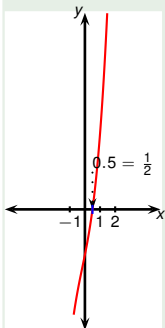
Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 + 2x \\
x - \frac{1}{2} \overline{\smash{\big)}\ 2x^3 + x^2 + 5x - 3} \\
\underline{2x^3 - x^2} \\
2x^2 + 5x - 3 \\
\underline{2x^2 - x}
\end{array}
$$

Multiply $2x$ by divisor.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
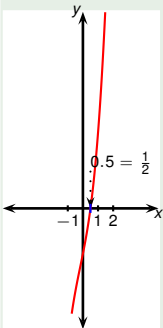$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 + 2x \\
x - \tfrac{1}{2} \overline{\big)\, 2x^3 + x^2 + 5x - 3} \\
\underline{2x^3 - x^2} \\
2x^2 + 5x - 3 \\
\underline{2x^2 - x} \\
\textbf{?} \qquad \textbf{?}
\end{array}
$$

Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 + 2x \\
x - \tfrac{1}{2} \overline{\smash{\big)}\ 2x^3 + x^2 + 5x - 3} \\
\underline{2x^3 - x^2} \\
2x^2 + 5x - 3 \\
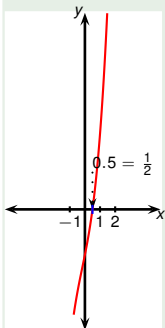\underline{2x^2 - x} \\
6x - 3
\end{array}
$$

Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 + 2x \quad \textbf{?} \\
x - \frac{1}{2} \overline{\smash{\big)}\, 2x^3 + x^2 + 5x - 3} \\
\underline{2x^3 - x^2 } \\
2x^2 + 5x - 3 \\
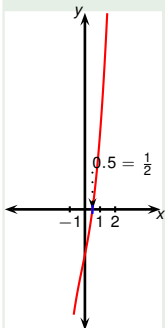\underline{2x^2 - x } \\
6x - 3
\end{array}
$$

Divide $6x$ by x.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 + 2x + 6 \\
x - \frac{1}{2} \overline{\smash{\big)}\ 2x^3 + x^2 + 5x - 3} \\
\underline{2x^3 - x^2} \\
2x^2 + 5x - 3 \\
\underline{2x^2 - x} \\
6x - 3
\end{array}
$$

Divide $6x$ by x.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 \; = \; 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 + 2x + 6 \\
x - \tfrac{1}{2} \; \overline{\big)\; 2x^3 + x^2 + 5x - 3} \\
\underline{-\quad 2x^3 - x^2 \qquad\qquad} \\
2x^2 + 5x - 3 \\
\underline{-\qquad 2x^2 - x \qquad} \\
6x - 3 \\
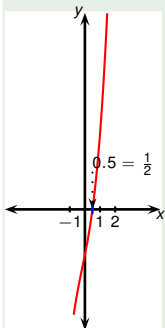\underline{\textbf{?} \qquad \textbf{?}}
\end{array}
$$

Multiply 6 by divisor.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 + 2x + 6 \\
x - \tfrac{1}{2} \overline{\smash{\big)}\ 2x^3 + x^2 + 5x - 3} \\
\underline{2x^3 - x^2} \\
2x^2 + 5x - 3 \\
\underline{2x^2 - x} \\
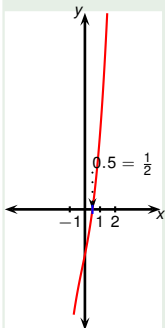6x - 3 \\
\underline{6x - 3}
\end{array}
$$

Multiply 6 by divisor.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 + 2x + 6 \\
x - \frac{1}{2} \overline{\smash{\big)}\ 2x^3 + x^2 + 5x - 3} \\
\underline{2x^3 - x^2} \\
2x^2 + 5x - 3 \\
\underline{2x^2 - x} \\
6x - 3 \\
\underline{6x - 3} \\
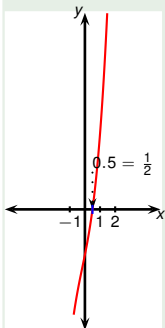\mathbf{?}
\end{array}
$$

Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

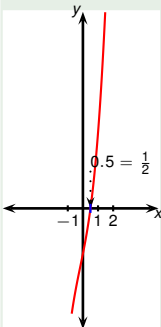We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 + 2x + 6 \\
x - \tfrac{1}{2} \overline{\smash{\big)}\ 2x^3 + x^2 + 5x - 3} \\
\underline{2x^3 - x^2} \\
2x^2 + 5x - 3 \\
\underline{2x^2 - x} \\
6x - 3 \\
\underline{6x - 3} \\
0
\end{array}
$$

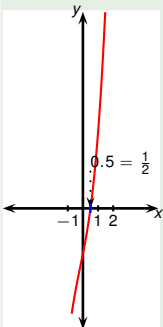Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$
\begin{array}{r}
2x^2 + 2x + 6 \\
x - \frac{1}{2} \overline{\smash{\big)}\ 2x^3 + x^2 + 5x - 3} \\
\underline{2x^3 - x^2} \\
2x^2 + 5x - 3 \\
\underline{2x^2 - x} \\
6x - 3 \\
\underline{6x - 3} \\
0
\end{array}
$$

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$
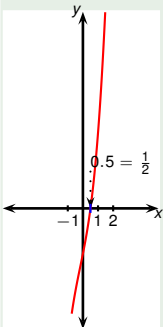$$\left(x - \tfrac{1}{2}\right)\left(2x^2 + 2x + 6\right) + 0 = 0$$

We see only one root, $x = 0.5 = \tfrac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

Quotient: $\quad 2x^2 + 2x + 6$

$$
\begin{array}{r}
x - \tfrac{1}{2} \\
- \\
\\
- \\
\\
- \\
\\
\end{array}
\begin{array}{l}
\overline{\left| 2x^3 + x^2 + 5x - 3 \right.} \\
2x^3 - x^2 \\
\overline{\ 2x^2 + 5x - 3} \\
2x^2 - x \\
\overline{\ 6x - 3} \\
6x - 3 \\
\overline{\ 0}
\end{array}
$$

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$
$$\left(x - \tfrac{1}{2}\right)\left(2x^2 + 2x + 6\right) + 0 = 0$$

We see only one root, $x = 0.5 = \tfrac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:
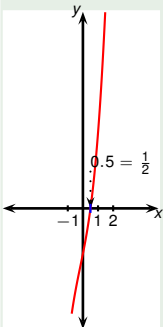
$$
\begin{array}{r}
\textbf{Quotient:} \quad 2x^2 + 2x + 6 \\
x - \tfrac{1}{2} \enclose{longdiv}{2x^3 + x^2 + 5x - 3} \\
\underline{2x^3 - x^2} \\
2x^2 + 5x - 3 \\
\underline{2x^2 - x} \\
6x - 3 \\
\underline{6x - 3} \\
\textbf{Remainder:} \quad 0
\end{array}
$$

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$
$$\left(x - \tfrac{1}{2}\right)\left(2x^2 + 2x + 6\right) = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

Quotient: $\quad 2x^2 + 2x + 6$

$$
x - \tfrac{1}{2} \overline{\left|2x^3 + x^2 + 5x - 3\right.}
$$

$$-\quad \underline{2x^3 - x^2}$$

$$2x^2 + 5x - 3$$

$$-\quad \underline{2x^2 - x}$$

$$6x - 3$$

$$-\quad \underline{6x - 3}$$
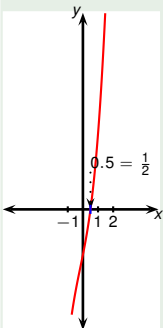
Remainder: $\qquad\qquad 0$

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$
$$\left(x - \tfrac{1}{2}\right)\left(2x^2 + 2x + 6\right) = 0$$

$$x - \tfrac{1}{2} = 0 \quad \textbf{or} \quad x =$$

We see only one root, $x = 0.5 = \tfrac{1}{2}$. Is our guess correct? Is there another root (far away from 0)?
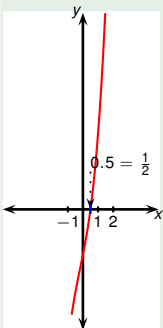
Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$
$$\left(x - \tfrac{1}{2}\right)\left(2x^2 + 2x + 6\right) = 0$$

$$x - \tfrac{1}{2} = 0 \quad \textbf{or} \quad x =$$

$$x = \tfrac{1}{2}$$

We see only one root, $x = 0.5 = \tfrac{1}{2}$. Is our guess correct? Is there another root (far away from 0)?

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.
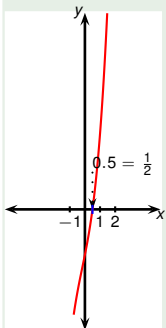
$$2x^3 + x^2 + 5x - 3 = 0$$
$$\left(x - \tfrac{1}{2}\right)\left(2x^2 + 2x + 6\right) = 0$$

$$x - \tfrac{1}{2} = 0 \quad \textbf{or} \quad x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2}$$

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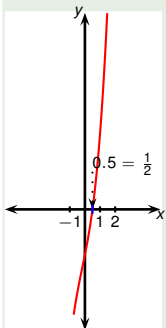
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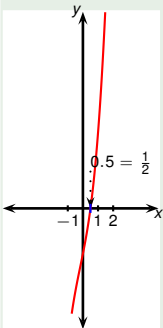$$x = \tfrac{1}{2}$$

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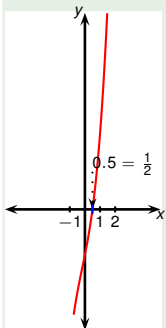
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no real solution

We see only one root, $x = 0.5 = \tfrac{1}{2}$. Is our guess correct? Is there another root (far away from 0)?