Calculus II

Integrals of the form $\int \frac{a}{bx+c}\mathbf{d}x$

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Building block Ia

Building block Ia: $\int \frac{1}{x} \mathrm{d}x.$

Example

Integrate building block Ia

$$\int \frac{1}{x} \mathrm{d}x$$

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Integrate building block Ia

$$\int \frac{1}{x} \mathrm{d}x = \textbf{?}$$

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Example

Integrate building block Ia

$$\int \frac{1}{x} \mathrm{d}x = \ln |x| + C$$

Linear substitutions leading to building block Ia

Building block Ia: $\int \frac{1}{x}\mathrm{d}x = \ln|x| + C.$

Example

Integrate

$$\int \frac{1}{-4x+5}\mathrm{d}x$$

Linear substitutions leading to building block Ia

Building block Ia: $\int \frac{1}{x}\mathrm{d}x = \ln|x| + C.$

Example

Integrate

$$\int \frac{1}{-4x+5}\mathrm{d}x = \int \frac{1}{(-4x+5)}\frac{\mathrm{d}(-4x)}{(-4)}$$

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Example

Integrate

$$\begin{aligned}\int \frac{1}{-4x+5}\mathrm{d}x &= \int \frac{1}{(-4x+5)}\frac{\mathrm{d}(-4x)}{(-4)} \\ &= \int \frac{1}{(-4x+5)}\frac{\mathrm{d}(-4x+5)}{(-4)} \qquad \Big| \text{ Set } u = -4x+5 \\ &= \int \frac{1}{u}\frac{\mathrm{d}u}{(-4)}\end{aligned}$$

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&= \int \frac{1}{(-4x+5)}\frac{\mathrm{d}(-4x+5)}{(-4)} \qquad \Big| \text{ Set } u = -4x+5 \\
&= \int \frac{1}{u}\frac{\mathrm{d}u}{(-4)} \\
&= -\frac{1}{4}\int u^{-1}\mathrm{d}u
\end{aligned}$$

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&= -\frac{1}{4}\int u^{-1}\mathrm{d}u = -\frac{1}{4}\ln|u| + C \\
&= -\frac{1}{4}\ln|-4x+5| + C \quad .
\end{aligned}$$

Linear substitutions leading to building block Ia

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&= -\frac{1}{4}\ln|-4x+5| + C \quad .
\end{aligned}$$

Lin. subst. leading to building block Ia: general case

Building block Ia: $\int \frac{1}{x}\mathrm{d}x = \ln|x| + C.$

Example

Integrate

$$\begin{aligned}
\int \frac{1}{ax+b}\mathrm{d}x &= \int \frac{1}{(\quad ax+b)}\frac{\mathrm{d}(\quad ax)}{a} \\
&= \int \frac{1}{(\quad ax+b)}\frac{\mathrm{d}(\quad ax+b)}{a} \qquad \Big| \text{ Set } u = ax+b \\
&= \int \frac{1}{u}\frac{\mathrm{d}u}{a} \\
&= \frac{1}{a}\int u^{-1}\mathrm{d}u = \frac{1}{a}\ln|u| + C \\
&= \frac{1}{a}\ln|ax+b| + C \quad .
\end{aligned}$$