

# Precalculus

## Homework

### Trig cofunction identities and angle-sum formulas

1. Use the known values of  $\sin 30^\circ$ ,  $\cos 30^\circ$ ,  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\dots$ , the angle sum formulas and the cofunction identities to find an exact value (using radicals) for the trigonometric function.

(a) The six trigonometric functions of  $105^\circ = 45^\circ + 60^\circ$ :

•  $\sin(105^\circ)$ .

•  $\cos(105^\circ)$ . Should your answer be a positive or a negative number?

•  $\tan(105^\circ)$ .

•  $\cot(105^\circ)$ .

•  $\sec(105^\circ)$ .

•  $\csc(105^\circ)$ .

(b) The six trigonometric functions of  $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ :

•  $\sin\left(\frac{\pi}{12}\right)$ .

•  $\cos\left(\frac{\pi}{12}\right)$ . Should  $\sin\left(\frac{\pi}{12}\right)$  be larger or smaller than  $\cos\left(\frac{\pi}{12}\right)$ ?

•  $\tan\left(\frac{\pi}{12}\right)$ .

•  $\cot\left(\frac{\pi}{12}\right)$ .

•  $\sec\left(\frac{\pi}{12}\right)$ .

•  $\csc\left(\frac{\pi}{12}\right)$ .

2. Simplify to a trigonometric function of the angle  $\theta$ . The answer key has not been proofread, use with caution.

(a)  $\sin\left(\frac{\pi}{2} - \theta\right)$ .

(b)  $\cos\left(\frac{13\pi}{2} - \theta\right)$ .

(c)  $\tan(\pi - \theta)$

(d)  $\cot\left(\frac{3\pi}{2} - \theta\right)$

(e)  $\csc\left(\frac{3\pi}{2} + \theta\right)$

3. Using the power-reducing formulas, rewrite the expression in terms of first powers of the cosines and sines of multiples of the angle  $\theta$ .

(a)  $\sin^4 \theta$ .

(b)  $\cos^4 \theta$ .

(c)  $\sin^6 \theta$ .

(d)  $\cos^6 \theta$ .

$$\frac{91}{5} + (\theta 2) \cos \frac{2\pi}{5} + (\theta 4) \cos \frac{4\pi}{5} + (\theta 9) \cos \frac{6\pi}{5} = \theta 9 \cos \frac{8\pi}{5}$$

4. Use the sum-to-product formulas to find all solutions of the trigonometric equation in the interval  $[0, 2\pi)$ .

Please note that typing a query such as “solve( sin(x)+sin (3x)=0)” at [www.wolframalpha.com](http://www.wolframalpha.com) will provide you with a correct answer and a function plot.

(a)  $\sin(x) + \sin(3x) = 0$ .

$$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 0 = x \text{ :ANSWER}$$

(b)  $\cos(x) + \cos(-3x) = 0$ .

$$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2}, \pi, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2}, \pi = x \text{ :ANSWER}$$

(c)  $\sin(x) - \sin(3x) = 0$ .

$$\frac{\pi}{2}, \frac{\pi}{2}, \pi, \frac{\pi}{2}, \frac{\pi}{2}, 0 = x \text{ :ANSWER}$$

(d)  $\cos(2x) - \cos(3x) = 0$ .

$$\frac{\pi}{8}, \frac{\pi}{8}, \frac{\pi}{8}, \frac{\pi}{8}, \pi, \frac{\pi}{8}, \pi, \frac{\pi}{8} = x \text{ :ANSWER}$$