

# Calculus II

## Integrals of the form $\int \sin(nx) \cos(mx) dx$

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To evaluate integrals of the form

①  $\int \sin(mx) \cos(nx) dx$

②  $\int \sin(mx) \sin(nx) dx$

③  $\int \cos(mx) \cos(nx) dx$

use the corresponding identity:

①  $\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$

②  $\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$

③  $\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$

## Example

$$\int \sin(4x) \cos(5x) dx$$

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$$\int \sin(4x) \cos(5x) dx = \int \frac{1}{2} [\sin(4x - 5x) + \sin(4x + 5x)] dx$$

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$$\begin{aligned}\int \sin(4x) \cos(5x) dx &= \int \frac{1}{2} [\sin(4x - 5x) + \sin(4x + 5x)] dx \\ &= \frac{1}{2} \int (\sin(-x) + \sin(9x)) dx\end{aligned}$$

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$$\begin{aligned}\int \sin(4x) \cos(5x) dx &= \int \frac{1}{2} [\sin(4x - 5x) + \sin(4x + 5x)] dx \\ &= \frac{1}{2} \int (\sin(-x) + \sin(9x)) dx \\ &= \frac{1}{2} \int (-\sin x + \sin(9x)) dx\end{aligned}$$

## Example

$$\begin{aligned}\int \sin(4x) \cos(5x) dx &= \int \frac{1}{2} [\sin(4x - 5x) + \sin(4x + 5x)] dx \\ &= \frac{1}{2} \int (\sin(-x) + \sin(9x)) dx \\ &= \frac{1}{2} \int (-\sin x + \sin(9x)) dx \\ &= \frac{1}{2} \left( \cos x - \frac{1}{9} \cos(9x) \right) + C\end{aligned}$$