

# Precalculus

## Factor quadratic over the complex numbers

Todor Milev

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$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

$$\begin{cases} x_1 x_2 = \frac{c}{a} \\ x_1 + x_2 = -\frac{b}{a} \end{cases}$$

## Example

Factor the quadratic, using complex numbers if needed.

$$x^2 + x + 1 = \left( x - \left( \frac{-1 + \sqrt{3}i}{2} \right) \right) \left( x - \left( \frac{-1 - \sqrt{3}i}{2} \right) \right)$$

- The product of the two roots:  $x_1 x_2 = 1$ .
- Integer options:  $x_1 = 1, x_2 = 1$  and  $x_1 = -1, x_2 = -1$ .
- $\begin{aligned} (x - 1)(x - 1) &= (x - 1)^2 = x^2 - 2x + 1 \\ (x + 1)(x + 1) &= (x + 1)^2 = x^2 + 2x + 1 \end{aligned}$  both don't work.
- $\Rightarrow$  No easy factorization; must use quadratic formula.

$$\begin{aligned} x_1, x_2 &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} \\ &= \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2} \end{aligned}$$