Calculus II
Integration by parts

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Outline

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Integration by Parts

Every differentiation rule

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$$(uv)' = u'v + uv' \qquad \Big| \text{ Product Rule}$$

Integration by Parts

Every differentiation rule corresponds to a differential form rule

$(uv)' = u'v + uv'$	Product Rule
$\mathrm{d}(uv) = v\mathrm{d}u + u\mathrm{d}v$	Differential Prod. Rule

Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

$(uv)' = u'v + uv'$	Product Rule
$\mathrm{d}(uv) = v\mathrm{d}u + u\mathrm{d}v$	Differential Prod. Rule
$\int \mathrm{d}(uv) = \int v\mathrm{d}u + \int u\mathrm{d}v$	integration of the above

Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

$$\begin{array}{rcl|l}
(uv)' &=& u'v + uv' & \text{Product Rule} \\
\mathrm{d}(uv) &=& v\mathrm{d}u + u\mathrm{d}v & \text{Differential Prod. Rule} \\
\int \mathrm{d}(uv) &=& \int v\mathrm{d}u + \int u\mathrm{d}v & \text{integration of the above} \\
uv &=& \int v\mathrm{d}u + \int u\mathrm{d}v &
\end{array}$$

Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

$$
\begin{array}{rcl|l}
(uv)' & = & u'v + uv' & \text{Product Rule} \\
\mathrm{d}(uv) & = & v\mathrm{d}u + u\mathrm{d}v & \text{Differential Prod. Rule} \\
\displaystyle\int \mathrm{d}(uv) & = & \displaystyle\int v\mathrm{d}u + \int u\mathrm{d}v & \text{integration of the above} \\
{\color{red}uv} & = & \displaystyle\int v\mathrm{d}u + \int u\mathrm{d}v & \text{rearrange} \\
\displaystyle\int u\mathrm{d}v & = & {\color{red}uv} - \displaystyle\int v\mathrm{d}u &
\end{array}
$$

Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

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\int \mathrm{d}(uv) & = & \int v\mathrm{d}u + \int u\mathrm{d}v & \text{integration of the above} \\
uv & = & \int v\mathrm{d}u + {\color{red}\int u\mathrm{d}v} & \text{rearrange} \\
{\color{red}\int u\mathrm{d}v} & = & uv - \int v\mathrm{d}u &
\end{array}
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Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

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\int \mathrm{d}(uv) & = & \int v\mathrm{d}u + \int u\mathrm{d}v & \text{integration of the above} \\
uv & = & {\color{red}\int v\mathrm{d}u} + \int u\mathrm{d}v & \text{rearrange} \\
\int u\mathrm{d}v & = & uv - {\color{red}\int v\mathrm{d}u} &
\end{array}
$$

Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

$$\begin{array}{rcl|l} (uv)' &=& u'v + uv' & \text{Product Rule} \\ \mathrm{d}(uv) &=& v\mathrm{d}u + u\mathrm{d}v & \text{Differential Prod. Rule} \\ \int \mathrm{d}(uv) &=& \int v\mathrm{d}u + \int u\mathrm{d}v & \text{integration of the above} \\ uv &=& \int v\mathrm{d}u + \int u\mathrm{d}v & \text{rearrange} \\ \int u\mathrm{d}v &=& uv - \int v\mathrm{d}u & \end{array}$$

We just proved the following.

Proposition ((Rule of) Integration by Parts)

$$\int u\mathrm{d}v = uv - \int v\mathrm{d}u$$

Integration by parts: strategy for applying

Integration by parts:

$$\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$$

Generally: Choose u in this order: **LIPET**

Integration by parts: strategy for applying

Integration by parts:

$$\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$$

Generally: Choose u in this order: **LIPET**

Logs, Inverse trig, Polynomial, Exponential, Trig

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\int x \sin x \mathrm{d}x =$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$\int x \sin x \mathrm{d}x =$ | $\sin x \mathrm{d}x = \mathrm{d}(\textbf{?} \quad)$

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x\sin x\mathrm{d}x = \qquad \Bigg| \quad \sin x\mathrm{d}x = \mathrm{d}(-\cos x)$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u$.

Example

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$$\sin x \mathrm{d}x = \mathrm{d}(-\cos x)$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\int x \sin x \mathrm{d}x = \int x \mathrm{d}(-\cos x) \qquad \Big| \quad \sin x \mathrm{d}x = \mathrm{d}(-\cos x)$$

$$= x(-\cos x) - \int (-\cos x) \mathrm{d}x$$

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x\sin x\mathrm{d}x = \int x\mathrm{d}(-\cos x) \qquad \Big| \; \sin x\mathrm{d}x = \mathrm{d}(-\cos x)$$
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Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int x\sin x\mathrm{d}x &= \int x\mathrm{d}(-\cos x) && \Big|\ \sin x\mathrm{d}x = \mathrm{d}(-\cos x)\\ &= x(-\cos x) - \int(-\cos x)\mathrm{d}x\\ &= -x\cos x + \int \cos x\mathrm{d}x\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u$.

Example

$$\begin{aligned}\int x \sin x \mathrm{d}x &= \int x \mathrm{d}(-\cos x) && \Big| \ \sin x \mathrm{d}x = \mathrm{d}(-\cos x)\\ &= x(-\cos x) - \int (-\cos x)\mathrm{d}x\\ &= -x\cos x + {\color{red}\int \cos x \mathrm{d}x}\\ &= -x\cos x + {\color{red}\mathbf{?}}\end{aligned}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u$.

Example

$$\begin{aligned}\int x \sin x \mathrm{d}x &= \int x \mathrm{d}(-\cos x) && \Big| \sin x \mathrm{d}x = \mathrm{d}(-\cos x)\\ &= x(-\cos x) - \int (-\cos x)\mathrm{d}x\\ &= -x\cos x + \int \cos x \mathrm{d}x\\ &= -x\cos x + \sin x + C\end{aligned}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\int \ln x \mathrm{d}x \quad =$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\int \ln x \mathrm{d}x = (\ln x)x - \int x \mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\int \ln x \mathrm{d}x = (\ln x)x - \int x \mathrm{d}(\ln x)$$ | integrate by parts

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\begin{aligned}\int \ln x \mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) && \text{integrate by parts} \\ &= x \ln x - \int x(\ln x)' \mathrm{d}x\end{aligned}$$

Integration by parts: $\int u dv = uv - \int v du.$

Example

$$\begin{aligned} \int \ln x dx &= (\ln x)x - \int x d(\ln x) && \Big| \text{ integrate by parts} \\ &= x \ln x - \int x {\color{red}(\ln x)'} dx \\ &= x \ln x - \int x \textbf{\color{red}?}\, dx \end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int \ln x \mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) && \Big| \text{ integrate by parts} \\
&= x\ln x - \int x(\ln x)'\mathrm{d}x \\
&= x\ln x - \int x\frac{1}{x}\mathrm{d}x
\end{aligned}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\begin{aligned} \int \ln x \mathrm{d}x &= (\ln x)x - \int x \mathrm{d}(\ln x) && \Big| \text{ integrate by parts} \\ &= x \ln x - \int x (\ln x)' \mathrm{d}x \\ &= x \ln x - \int x \frac{1}{x} \mathrm{d}x \\ &= x \ln x - \int \mathrm{d}x \end{aligned}$$

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

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\int \ln x\mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) && \Big| \text{ integrate by parts} \\
&= x\ln x - \int x(\ln x)'\mathrm{d}x \\
&= x\ln x - \int x\frac{1}{x}\mathrm{d}x \\
&= x\ln x - \int \mathrm{d}x \\
&= x\ln x - \textbf{?}
\end{aligned}$$

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$
\begin{aligned}
\int \ln x\mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) && \Big| \text{ integrate by parts} \\
&= x\ln x - \int x(\ln x)'\mathrm{d}x \\
&= x\ln x - \int x\frac{1}{x}\mathrm{d}x \\
&= x\ln x - \int \mathrm{d}x \\
&= x\ln x - x + C \quad .
\end{aligned}
$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int t^2 e^t \mathrm{d}t$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\int t^2 e^t \mathrm{d}t \quad = \quad \int t^2 \mathrm{d}\left(\mathbf{?} \right)$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\int t^2 e^t \mathrm{d}t = \int t^2 \mathrm{d}\left(e^t\right)$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\int t^2 e^t \mathrm{d}t = \int t^2 \mathrm{d}\left(e^t\right)$$

integrate by parts

$$= \textbf{?}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\begin{aligned} \int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \Big| \text{ integrate by parts} \\ &= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \end{aligned}$$

Integration by parts: $\int u \mathrm{d} v = uv - \int v \mathrm{d} u$.

Example

$$\begin{aligned} \int t^2 e^t \mathrm{d} t &= \int t^2 \mathrm{d}\left(e^t\right) \\ &= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \end{aligned}$$

integrate by parts

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\begin{aligned} \int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \Big|\ \text{integrate by parts} \\ &= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) \\ &= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\ &= t^2 e^t - \int e^t \mathbf{?}\, \mathrm{d}t\end{aligned}$$

| integrate by parts

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) \\ &= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\ &= t^2 e^t - \int e^t 2t\mathrm{d}t\end{aligned}$$

integrate by parts

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\begin{aligned} \int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) \\ &= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\ &= t^2 e^t - \int e^t 2t \mathrm{d}t \\ &= t^2 e^t - 2\int t \mathrm{d}(\mathbf{?}\ ) \end{aligned}$$

integrate by parts

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$
\begin{array}{rcl|l}
\displaystyle\int t^2 e^t \mathrm{d}t & = & \displaystyle\int t^2 \mathrm{d}\left(e^t\right) & \text{integrate by parts} \\
& = & \displaystyle t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) & \\
& = & \displaystyle t^2 e^t - \int e^t 2t \mathrm{d}t & \\
& = & \displaystyle t^2 e^t - 2\int t \mathrm{d}\left(e^t\right) &
\end{array}
$$

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$
\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \Big|\ \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
&= t^2 e^t - \int e^t\, 2t\mathrm{d}t \\
&= t^2 e^t - 2\int t\mathrm{d}(e^t) && \Big|\ \text{integrate by parts} \\
&= t^2 e^t - 2\,\mathbf{?}
\end{aligned}
$$

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$
\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
&= t^2 e^t - \int e^t\, 2t \mathrm{d}t \\
&= t^2 e^t - 2\int t \mathrm{d}(e^t) && \text{integrate by parts} \\
&= t^2 e^t - 2\left(te^t - \int e^t \mathrm{d}t\right)
\end{aligned}
$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \Big|\ \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
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&= t^2 e^t - 2\int t\mathrm{d}(e^t) && \Big|\ \text{integrate by parts} \\
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\end{aligned}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u$.

Example

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\begin{array}{rcll}
\int t^2 e^t \mathrm{d}t & = & \int t^2 \mathrm{d}\left(e^t\right) & \Big| \text{ integrate by parts} \\
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\end{array}
$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$
\begin{array}{rcll}
\int t^2 e^t \mathrm{d}t &=& \int t^2 \mathrm{d}\left(e^t\right) & \Big| \text{ integrate by parts} \\
&=& t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) & \\
&=& t^2 e^t - \int e^t 2t \mathrm{d}t & \\
&=& t^2 e^t - 2\int t \mathrm{d}(e^t) & \Big| \text{ integrate by parts} \\
&=& t^2 e^t - 2\left(te^t - \int e^t \mathrm{d}t\right) & \\
&=& t^2 e^t - 2te^t + 2e^t + C &
\end{array}
$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \Big| \text{ integrate by parts} \\
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&= t^2 e^t - 2te^t + 2e^t + C
\end{aligned}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\int e^x \sin x \mathrm{d}x \quad =$$

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int e^x \sin x \mathrm{d}x \quad = \quad \int \sin x \,\mathrm{d}\left(\textbf{?} \quad\right)$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u$.

Example

$$\int e^x \sin x \mathrm{d}x = \int \sin x \, \mathrm{d}\left(e^x\right)$$

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int e^x \sin x \mathrm{d}x &= \int \sin x \,\mathrm{d}\left(e^x\right)\\ &= ?\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int e^x \sin x\mathrm{d}x &= \int \sin x \,\mathrm{d}\left(e^x\right)\\ &= (\sin x)e^x - \int e^x \mathrm{d}(\sin x)\end{aligned}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u$.

Example

$$\begin{aligned} \int e^x \sin x \mathrm{d}x &= \int \sin x \,\mathrm{d}\left(e^x\right) \\ &= (\sin x) e^x - \int e^x \mathrm{d}(\sin x) \end{aligned}$$

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Integration by parts: $\int u \mathrm{d} v = uv - \int v \mathrm{d} u.$

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Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u$.

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\int e^x \sin x \mathrm{d}x &= \int \sin x \,\mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x
\end{aligned}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

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\int e^x \sin x \mathrm{d}x &= \int \sin x \,\mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(\textbf{?} \quad\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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$$\begin{aligned}\int e^x \sin x\mathrm{d}x &= \int \sin x\,\mathrm{d}\left(e^x\right)\\ &= (\sin x)e^x - \int e^x\mathrm{d}(\sin x)\\ &= e^x\sin x - \int e^x \cos x\mathrm{d}x\\ &= e^x\sin x - \int \cos x\mathrm{d}\left(e^x\right)\end{aligned}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u$.

Example

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\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \,\mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(e^x\right) \\
&= e^x \sin x - \left(\textbf{?} \qquad\qquad\qquad \right)
\end{aligned}
$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

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\int e^x \sin x \mathrm{d}x &= \int \sin x \,\mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(e^x\right) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x) \right)
\end{aligned}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

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\int e^x \sin x \mathrm{d}x &= \int \sin x \,\mathrm{d}\left(e^x\right) \\
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\end{aligned}$$

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\int e^x \sin x \mathrm{d}x &= \int \sin x \,\mathrm{d}\left(e^x\right) \\
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&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x)\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$
\begin{aligned}
\int e^x \sin x\mathrm{d}x &= \int \sin x \,\mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x\mathrm{d}x \\
&= e^x \sin x - \int \cos x\mathrm{d}\left(e^x\right) \\
&= e^x \sin x {\color{red}-} \left((\cos x)e^x {\color{red}-} \int e^x \mathrm{d}(\cos x)\right) \\
&= e^x \sin x - \cos x e^x {\color{red}+} \int e^x \mathbf{?}
\end{aligned}
$$

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$
\begin{aligned}
\int e^x \sin x\mathrm{d}x &= \int \sin x \,\mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x\mathrm{d}x \\
&= e^x \sin x - \int \cos x\mathrm{d}\left(e^x\right) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x)\right) \\
&= e^x \sin x - \cos x e^x + \int e^x \mathbf{?}
\end{aligned}
$$

Integration by parts: $\int u \mathrm{d} v = uv - \int v \mathrm{d} u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \,\mathrm{d}\left(e^x\right) \\
&= (\sin x) e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(e^x\right) \\
&= e^x \sin x - \left((\cos x) e^x - \int e^x \mathrm{d}(\cos x) \right) \\
&= e^x \sin x - \cos x e^x + \int e^x (-\sin x) \mathrm{d}x
\end{aligned}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$
\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \,\mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(e^x\right) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x)\right) \\
&= e^x \sin x - \cos x e^x + \int e^x (- \sin x) \mathrm{d}x \\
&= e^x \sin x - \cos x e^x - \int e^x \sin x \mathrm{d}x
\end{aligned}
$$

Integration by parts: $\int u dv = uv - \int v du$.

Example

$$\begin{aligned}
\int e^x \sin x dx &= \int \sin x \, d\left(e^x\right) \\
&= (\sin x)e^x - \int e^x d(\sin x) \\
&= e^x \sin x - \int e^x \cos x dx \\
&= e^x \sin x - \int \cos x d\left(e^x\right) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x d(\cos x)\right) \\
&= e^x \sin x - \cos x e^x + \int e^x (-\sin x) dx \\
&= e^x \sin x - \cos x e^x - \int e^x \sin x dx
\end{aligned}$$

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int e^x \sin x\mathrm{d}x = e^x \sin x - \cos x e^x - \int e^x \sin x\mathrm{d}x$$

Integration by parts: $\int u dv = uv - \int v du.$

Example

$$\begin{aligned} \int e^x \sin x dx &= e^x \sin x - \cos x e^x - \int e^x \sin x dx \\ 2\int e^x \sin x dx &= e^x \sin x - \cos x e^x \end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= e^x \sin x - \cos x e^x - \int e^x \sin x \mathrm{d}x \\
{\color{red}2} \int e^x \sin x \mathrm{d}x &= e^x \sin x - \cos x e^x \\
\int e^x \sin x \mathrm{d}x &= \frac{1}{\color{red}2}\left(e^x \sin x - \cos x e^x\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x\mathrm{d}x &= e^x \sin x - \cos x e^x - \int e^x \sin x\mathrm{d}x \\
2\int e^x \sin x\mathrm{d}x &= e^x \sin x - \cos x e^x \\
\int e^x \sin x\mathrm{d}x &= \frac{1}{2}\left(e^x \sin x - \cos x e^x\right) + C
\end{aligned}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\int_0^1 \arctan x \mathrm{d}x \quad =$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int_0^1 \arctan x\mathrm{d}x \quad = \quad \textbf{?}$$

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int_0^1 \arctan x \mathrm{d}x = [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right)$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\int_0^1 \arctan x \mathrm{d}x = [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x \mathrm{d}(\arctan x)$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

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Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int_0^1 \arctan x \mathrm{d}x &= \left[(\arctan x)x\right]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right)\\ &= 1\cdot \arctan 1 - 0\cdot \arctan 0 - \int_{x=0}^{x=1} x\mathbf{?}\end{aligned}$$

Integration by parts: $\int u dv = uv - \int v du.$

Example

$$\begin{aligned}\int_0^1 \arctan x dx &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x d(\arctan x)\\ &= 1\cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x\mathbf{?}\end{aligned}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u$.

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Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

Example

$$\begin{aligned}
\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x \mathrm{d}(\arctan x) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x \frac{1}{1+x^2} \mathrm{d}x \\
&= \mathbf{?} - \int_{x=0}^{x=1} \frac{1}{1+x^2} \mathrm{d}\left(\mathbf{?} \quad \right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int_0^1 \arctan x \mathrm{d}x &= \left[(\arctan x)x\right]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right)\\ &= 1\cdot \arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\frac{1}{1+x^2}\mathrm{d}x\\ &= \frac{\pi}{4} - \int_{x=0}^{x=1}\frac{1}{1+x^2}\mathrm{d}\left(\mathbf{?}\quad\right)\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\ &= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\frac{1}{1+x^2}\mathrm{d}x \\ &= \frac{\pi}{4} - \int_{x=0}^{x=1}\frac{1}{1+x^2}\mathrm{d}\left(\mathbf{?}\quad\right)\end{aligned}$$

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\int_0^1 \arctan x \mathrm{d}x &= \left[(\arctan x)x\right]_{x=0}^{x=1} - \int_0^1 x \mathrm{d}\left(\arctan x\right) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x \frac{1}{1+x^2} \mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2} \mathrm{d}\left(\frac{x^2}{2}\right)
\end{aligned}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u$.

Example

$$\begin{aligned}
\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x \mathrm{d}\left(\arctan x\right) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x \frac{1}{1+x^2} \mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2} \mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2} \int_{x=0}^{x=1} \frac{1}{1+x^2} \mathrm{d}(\quad x^2)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

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\int_0^1 \arctan x \mathrm{d}x &= \left[(\arctan x)x\right]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

Set $w = 1 + x^2$.

$$\begin{aligned}
\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2)
\end{aligned}$$

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&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2} \mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2} \mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{w} \mathrm{d}w
\end{aligned}$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u.$

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\int_0^1 \arctan x \mathrm{d}x &= \left[(\arctan x)x\right]_{x=0}^{x=1} - \int_0^1 x \mathrm{d}\left(\arctan x\right) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x \frac{1}{1+x^2} \mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2} \mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2} \int_{x=0}^{x=1} \frac{1}{1+x^2} \mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2} \int_{x=0}^{x=1} \frac{1}{w} \mathrm{d}w = \frac{\pi}{4} - \frac{1}{2} \left[\ln |w|\right]_{x=0}^{x=1}
\end{aligned}$$

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

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$$
\begin{aligned}
\int_0^1 \arctan x \mathrm{d}x &= \left[(\arctan x)x\right]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
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&= \frac{\pi}{4} - \int_{x=0}^{x=1}\frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1}\frac{1}{1+x^2}\mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1}\frac{1}{w}\mathrm{d}w = \frac{\pi}{4} - \frac{1}{2}\left[\ln|w|\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}\left[\ln\left(1+x^2\right)\right]_{x=0}^{x=1}
\end{aligned}
$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u$.

Example

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\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right)\\
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&= \frac{\pi}{4} - \int_{x=0}^{x=1}\frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right)\\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1}\frac{1}{1+x^2}\mathrm{d}(1+x^2)\\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1}\frac{1}{w}\mathrm{d}w = \frac{\pi}{4} - \frac{1}{2}\left[\ln|w|\right]_{x=0}^{x=1}\\
&= \frac{\pi}{4} - \frac{1}{2}\left[\ln\left(1+x^2\right)\right]_{x=0}^{x=1}\\
&= \frac{\pi}{4} - \frac{1}{2}\left(\ln 2 - \ln 1\right)
\end{aligned}$$

Integration by parts: $\displaystyle\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

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$$
\begin{aligned}
\int_0^1 \arctan x\mathrm{d}x &= \left[(\arctan x)x\right]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right)\\
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&= \frac{\pi}{4} - \int_{x=0}^{x=1}\frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right)\\
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&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1}\frac{1}{w}\mathrm{d}w = \frac{\pi}{4} - \frac{1}{2}\left[\ln|w|\right]_{x=0}^{x=1}\\
&= \frac{\pi}{4} - \frac{1}{2}\left[\ln\left(1+x^2\right)\right]_{x=0}^{x=1}\\
&= \frac{\pi}{4} - \frac{1}{2}\left(\ln 2 - \ln 1\right)
\end{aligned}
$$

Integration by parts: $\int u \mathrm{d}v = uv - \int v \mathrm{d}u$.

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&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x \frac{1}{1+x^2} \mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2} \mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2} \int_{x=0}^{x=1} \frac{1}{1+x^2} \mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2} \int_{x=0}^{x=1} \frac{1}{w} \mathrm{d}w = \frac{\pi}{4} - \frac{1}{2} \left[\ln |w|\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2} \left[\ln\left(1+x^2\right)\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2} \left(\ln 2 - \ln 1\right) = \frac{\pi}{4} - \frac{1}{2} \ln 2 \quad .
\end{aligned}$$