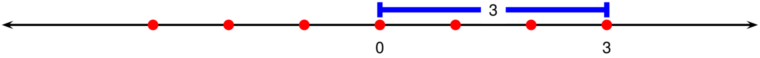
Arithmetics
Fraction basics
calculator-algebra.org

Todor Milev

2019

Division and the number line

- Numbers represent lengths by measuring distances.



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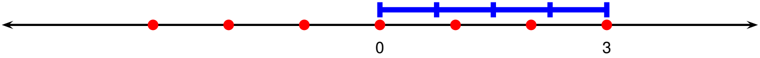

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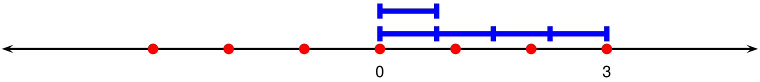

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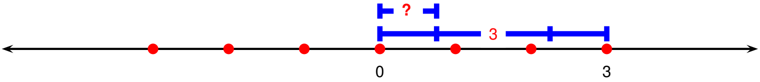

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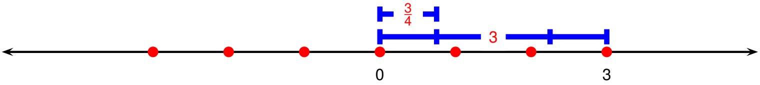

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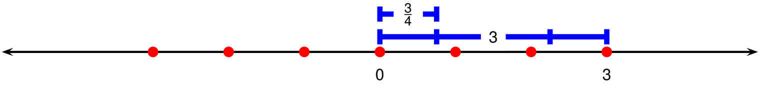

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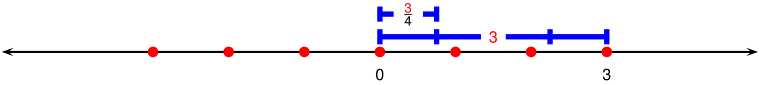

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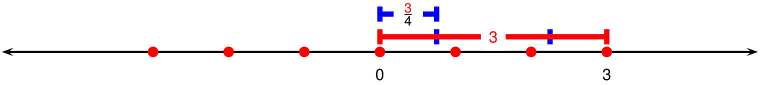

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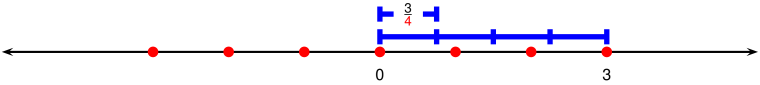

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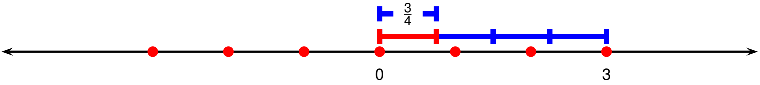

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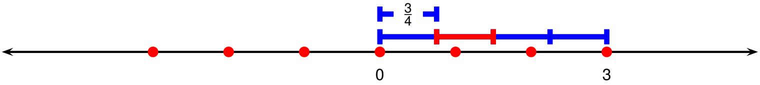

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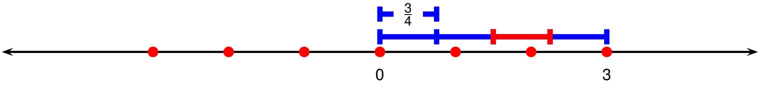

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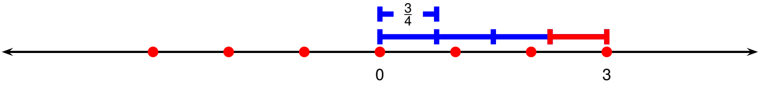

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For any number a, the following equality holds.

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Simplify.

$$\frac{5}{1} = \textbf{?}$$

$$\frac{-3}{1} = \textbf{?}$$

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Example (Reading fractions)

Read the fraction. Honor your English dialect naming convention, if different from the one given here.

- $\frac{1}{3}$ **?**
- $\frac{1}{2}$
- $\frac{1}{4}$
- $\frac{3}{4}$
- $\frac{6}{7}$
- $\frac{11}{6}$

Example (Reading fractions)

Read the fraction. Honor your English dialect naming convention, if different from the one given here.

- $\frac{1}{3}$ one third (can also say “a third”).
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- Most frequently, a fraction $\frac{a}{b}$ is read along the template “*a* *b*-th(s)”.
- Most important exceptions: $\dfrac{1}{2}$ is read as “one half” (or just “half”). $\dfrac{1}{4}$ is read as both “one fourth” and as ”one quarter”.

Observation

Fractions with same denominator are added by adding their numerators.

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

Example

$$\frac{2}{3} + \frac{5}{3}$$

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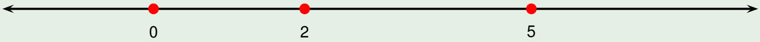

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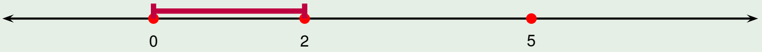

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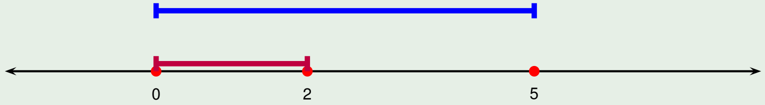

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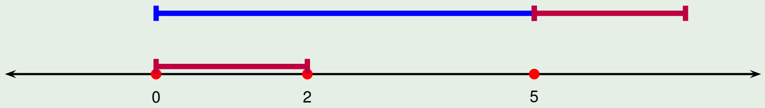

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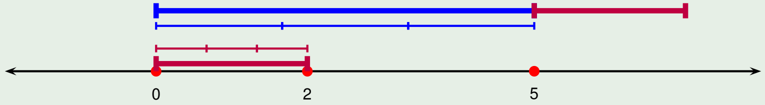

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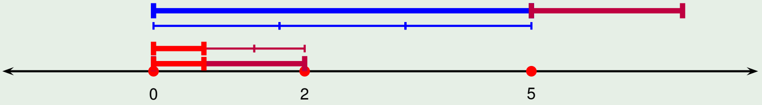

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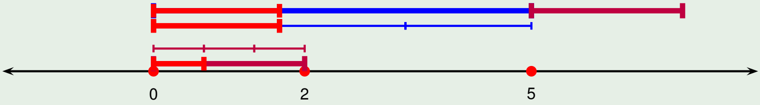

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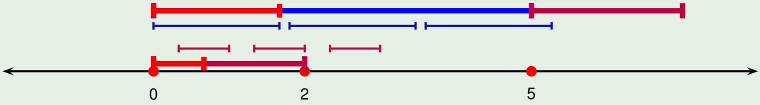

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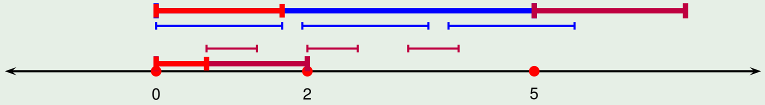

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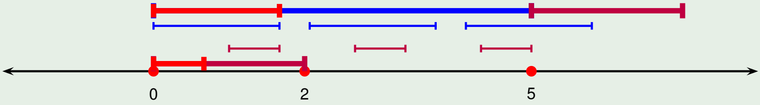

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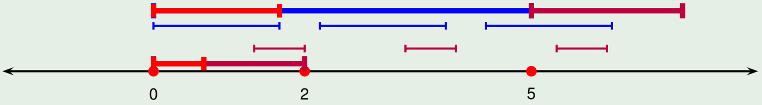

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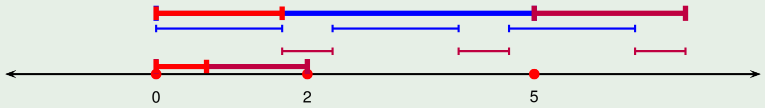

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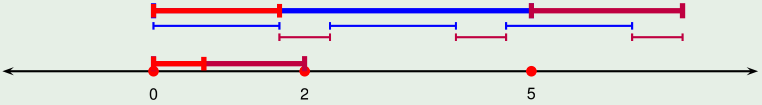

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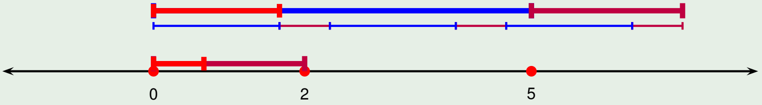

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Fractions with same denominator are added by adding their numerators.

$$\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}$$

A similar rule holds for subtraction.

Example

$$\begin{aligned} \frac{2}{3} + \frac{5}{3} &= \frac{2+5}{3} = \frac{7}{3} \\ \frac{4}{3} - \frac{1}{3} &= \frac{4-1}{3} = \frac{3}{3} \end{aligned}$$

Example (Add numbers with same denominator)

$$\frac{1}{4}+\frac{2}{4} =$$

$$\frac{7}{6}+\frac{10}{6} =$$

$$\frac{302}{111}+\frac{24}{111} =$$

$$\frac{3}{6}-\frac{2}{6} =$$

$$\frac{103}{101}-\frac{4}{101} =$$

$$\frac{3}{8}-\frac{6}{8} =$$

$$\frac{11}{13}-\frac{20}{13} =$$

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$$\frac{103}{101} - \frac{4}{101} =$$

$$\frac{3}{8} - \frac{6}{8} =$$

$$\frac{11}{13} - \frac{20}{13} =$$

Example (Add numbers with same denominator)

$$\frac{1}{4}+\frac{2}{4} = \frac{3}{4}$$

$$\frac{7}{6}+\frac{10}{6} = \textbf{?}$$

$$\frac{302}{111}+\frac{24}{111} =$$

$$\frac{3}{6}-\frac{2}{6} =$$

$$\frac{103}{101}-\frac{4}{101} =$$

$$\frac{3}{8}-\frac{6}{8} =$$

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Example (Add numbers with same denominator)

$$\frac{1}{4}+\frac{2}{4} = \frac{3}{4}$$

$$\frac{7}{6}+\frac{10}{6} = \frac{17}{6}$$

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$$\frac{3}{6}-\frac{2}{6} =$$

$$\frac{103}{101}-\frac{4}{101} =$$

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Example (Add numbers with same denominator)

$$
\begin{aligned}
\frac{1}{4}+\frac{2}{4} &= \frac{3}{4}\\
{\color{red}\frac{7}{6}+\frac{10}{6}} &= {\color{red}\frac{17}{6}}\\
\frac{302}{111}+\frac{24}{111} &=\\
\frac{3}{6}-\frac{2}{6} &=\\
\frac{103}{101}-\frac{4}{101} &=\\
\frac{3}{8}-\frac{6}{8} &=\\
\frac{11}{13}-\frac{20}{13} &=
\end{aligned}
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$$\frac{1}{4}+\frac{2}{4} = \frac{3}{4}$$

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\frac{103}{101}-\frac{4}{101} &= \frac{99}{101}\\
\frac{3}{8}-\frac{6}{8} &= \\
\frac{11}{13}-\frac{20}{13} &=
\end{aligned}
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\frac{3}{8}-\frac{6}{8} &= \textbf{?}\\
\frac{11}{13}-\frac{20}{13} &=
\end{aligned}$$

Example (Add numbers with same denominator)

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\begin{aligned}
\frac{1}{4}+\frac{2}{4} &= \frac{3}{4}\\
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\frac{3}{8}-\frac{6}{8} &= \frac{-3}{8}\\
\frac{11}{13}-\frac{20}{13} &=
\end{aligned}
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\frac{103}{101}-\frac{4}{101} &= \frac{99}{101}\\
\frac{3}{8}-\frac{6}{8} &= \frac{-3}{8} = -\frac{3}{8}\\
\frac{11}{13}-\frac{20}{13} &=
\end{aligned}
$$

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\frac{1}{4}+\frac{2}{4} &= \frac{3}{4}\\
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\frac{3}{6}-\frac{2}{6} &= \frac{1}{6}\\
\frac{103}{101}-\frac{4}{101} &= \frac{99}{101}\\
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\end{aligned}$$

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\frac{11}{13}-\frac{20}{13} &= \frac{-9}{13}
\end{aligned}$$

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\frac{11}{13}-\frac{20}{13} &= \frac{-9}{13}
\end{aligned}
$$

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\frac{103}{101}-\frac{4}{101} &= \frac{99}{101}\\
\frac{3}{8}-\frac{6}{8} &= \frac{-3}{8}=-\frac{3}{8}\\
\frac{11}{13}-\frac{20}{13} &= \frac{-9}{13}=-\frac{9}{13}
\end{aligned}
$$

Definition (Factor a number (properly))

To factor an integer a (properly) means to find integers $b > 1$ and $c > 1$ so that

$$a = \pm b \cdot c$$

The numbers b and c are called **factors.**

Example (Proper factorization)

$$\begin{aligned} 4 &= 2 \cdot 2 \\ 6 &= 3 \cdot 2 = 2 \cdot 3 \\ -8 &= -2 \cdot 4 = -4 \cdot 2 = -2 \cdot 2 \cdot 2 \end{aligned}$$

Example (Not a proper factorization)

$$\begin{aligned} -3 &= (-1) \cdot 3 \\ 4 &= 1 \cdot 4 \\ 1 &= 2 \cdot \frac{1}{2} \\ 1 &= 0.25 \cdot 4 \end{aligned}$$

Definition

An integer greater than one is prime if it cannot be factored properly.

Example

	Prime?	Full factorization		Prime?	Full factorization
1			9		
2			10		
3			11		
4			12		
5			13		
6			14		
7			15		
8			16		

Definition

An integer greater than one is prime if it cannot be factored properly.

- 0 and 1 are not prime by definition.

Example

	Prime?	Full factorization		Prime?	Full factorization
1			9		
2			10		
3			11		
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An integer greater than one is prime if it cannot be factored properly.

- 0 and 1 are not prime by definition.
- Whether negatives can be prime is usually left undefined.

Example

	Prime?	Full factorization		Prime?	Full factorization
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    - Both options could be made to make sense.

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	Prime?	Full factorization		Prime?	Full factorization
1			9		
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  - To avoid confusion: avoid question of prime negative numbers.

Example

	Prime?	Full factorization		Prime?	Full factorization
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Example

	Prime?	Full factorization		Prime?	Full factorization
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- 0 and 1 are not prime by definition.
- Whether negatives can be prime is usually left undefined.
  - Both options could be made to make sense.
  - To avoid confusion: avoid question of prime negative numbers.

Example

	Prime?	Full factorization		Prime?	Full factorization
1	no	-	9	no	$3 \cdot 3$
2	yes	2	10	no	$2 \cdot 5$
3	yes	3	11	yes	11
4	no	$2 \cdot 2$	12	**?**	
5	yes	5	13		
6	no	$2 \cdot 3$	14		
7	yes	7	15		
8	no	$2 \cdot 2 \cdot 2$	16		

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Definition (Factor positive integer completely)

To factor a positive integer completely means to write it as a product of prime factors.

$$x = p_1 \cdot p_2 \cdots p_n$$

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$$\begin{aligned} 16 &= ? \\ 36 &= \end{aligned}$$

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Example

Factor the number completely. If applicable, show two answers: with and without exponent notation (x^3 vs $x \cdot x \cdot x$).

$$
\begin{aligned}
4 &= \textbf{?} \\
6 &= \\
7 &= \\
8 &= \\
9 &= \\
15 &= \\
24 &= \\
36 &= \\
52 &= \\
67 &= \\
91 &=
\end{aligned}
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Example

Factor the number completely. If applicable, show two answers: with and without exponent notation (x^3 vs $x \cdot x \cdot x$).

$$\begin{aligned}
4 &= 2 \cdot 2 = 2^2 \\
6 &= \\
7 &= \\
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$$
\begin{aligned}
4 &= 2 \cdot 2 = 2^2 \\
6 &= 2 \cdot 3 \text{ [exponent notation not needed]} \\
7 &= \\
8 &= \\
9 &= \\
15 &= \\
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Definition (Factor a number)

To factor an integer a means to find integers $b > 1$ and $c > 1$ so that

$$a = \pm b \cdot c$$

We say that b, c are **factors** of a.

Definition (Prime number)

A number is prime if it cannot be factored.

Definition (Complete factorization)

To find a complete factorization of an integer a means to find prime numbers $p_1 > 1, p_2 > 1, \ldots, p_n > 1$ with

$$a = \pm p_1 \cdot p_2 \cdots p_n$$

- Is the number 67414977753059 prime?

$$67414977753059 =$$

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$$\underbrace{67414977753059}_{\text{not prime}} =$$

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- In other words, there exist fast algorithms for knowing whether a proper factorization exists.
- However, even when we know x can be factored, as of 2019, there are no known fast computer algorithms for finding an actual factorization.
- In fact, the number above was generated by first making two large primes and then multiplying them.
- Each of the two known large primes above was in turn generated by trying large integers at random.

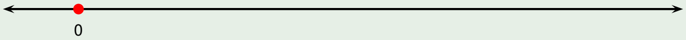
Observation

Fractions do not change when we multiply their numerator and denominator by the same number.

$$\frac{a}{b} = \frac{x \cdot a}{x \cdot b}$$

Example

$$\frac{2}{3}$$



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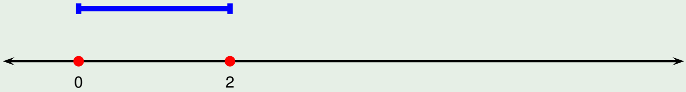

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$$\frac{a}{\color{red}{b}} = \frac{x \cdot a}{x \cdot b}$$

Example

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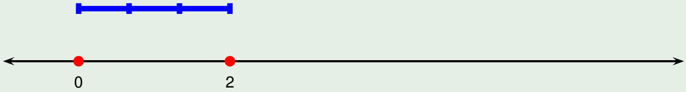

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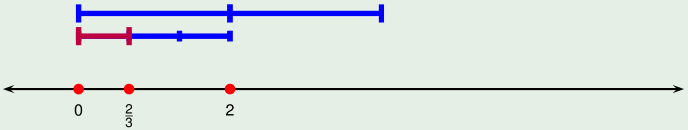

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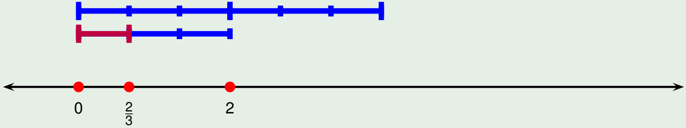

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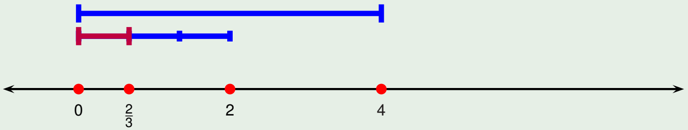
$$\frac{2}{3} = \frac{2 \cdot 2}{2 \cdot 3} = \frac{4}{6}$$



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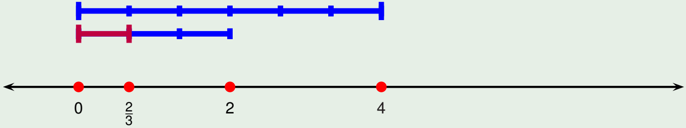

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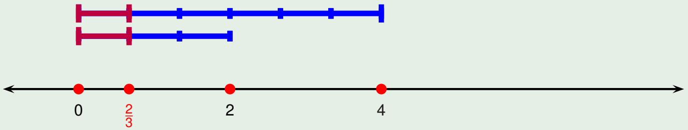

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Example

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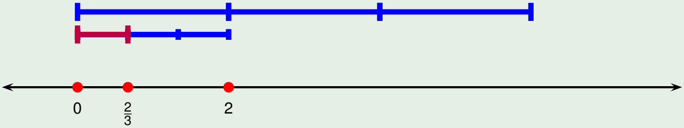

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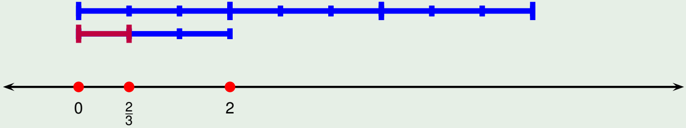

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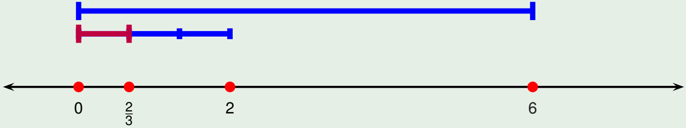

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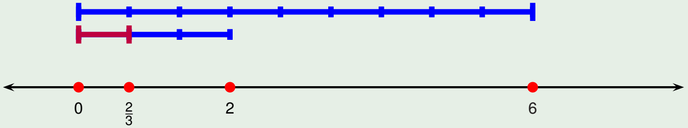
$$\begin{aligned}\frac{2}{3} &= \frac{2\cdot 2}{2\cdot 3} = \frac{4}{6}\\ &= \frac{3\cdot 2}{3\cdot 3} = \frac{6}{9}\end{aligned}$$



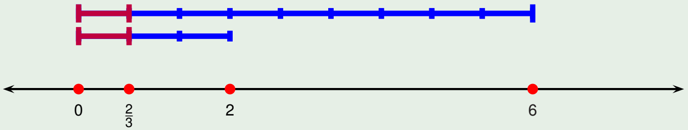
Observation

Fractions do not change when we multiply their numerator and denominator by the same number.

$$\frac{a}{b} = \frac{x \cdot a}{x \cdot b}$$

Example

$$\begin{aligned} \frac{2}{3} &= \frac{2\cdot 2}{2\cdot 3} = \frac{4}{6} \\ &= \frac{3\cdot 2}{3\cdot 3} = \frac{6}{9} \end{aligned}$$



Definition (Reduce a fraction)

A positive fraction $\frac{a}{b}$ is a reduction of a positive fraction $\frac{A}{B}$ when

$$\frac{A}{B} = \frac{a}{b}$$

and $\frac{a}{b}$ has smaller numerator and denominator, i.e., $A > a$ and $B > b$.

Recall that $\frac{x \cdot a}{x \cdot b} = \frac{a}{b}$.

Example

Reduce the fraction.

$$\begin{aligned} \frac{4}{6} &= \frac{2 \cdot 2}{2 \cdot 3} = \frac{2}{3} \\ \frac{2}{4} &= \frac{2 \cdot 1}{2 \cdot 2} = \frac{1}{2} \\ \frac{3}{9} &= \frac{3 \cdot 1}{3 \cdot 3} = \frac{1}{3} \end{aligned}$$

- To reduce a fraction, we use the rule:

$$\frac{x \cdot a}{x \cdot b} = \frac{a}{b}.$$

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- The uses of the cancel notation will become apparent in examples.

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$$\frac{x\cdot a}{x\cdot b}=\frac{\cancel{x}\cdot a}{\cancel{x}\cdot b}=\frac{a}{b}.$$

- To reduce excessive copying: use cancel notation.
- The uses of the cancel notation will become apparent in examples.
- Rules.
  - Use a single slanted line.
  - Unless circumstances dictate otherwise, slant your line from lower left corner to top right corner.
  - Do not use crosses, smudges, or any other notation that obscures the expression below the cancel line.

Example

Simplify (reduce) the fraction.

$$\frac{9}{12} =$$

$$\frac{14}{6} =$$

$$\frac{6}{2} =$$

$$\frac{6}{5} =$$

$$\frac{10}{15} =$$

$$\frac{8}{6} =$$

Example

Simplify (reduce) the fraction.

$$\frac{9}{12} =$$

$$\frac{14}{6} =$$

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Example

Simplify (reduce) the fraction.

$$\frac{9}{12} = \frac{3\cdot 3}{3\cdot 4}$$

$$\frac{14}{6} =$$

$$\frac{6}{2} =$$

$$\frac{6}{5} =$$

$$\frac{10}{15} =$$

$$\frac{8}{6} =$$

Example

Simplify (reduce) the fraction.

$$\frac{9}{\color{red}{12}} = \frac{3\cdot 3}{\color{red}{3\cdot 4}}$$

$$\frac{14}{6} =$$

$$\frac{6}{2} =$$

$$\frac{6}{5} =$$

$$\frac{10}{15} =$$

$$\frac{8}{6} =$$

Example

Simplify (reduce) the fraction.

$$\frac{9}{12} = \frac{\cancel{3}\cdot 3}{\cancel{3}\cdot 4} = \frac{3}{4}$$

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$$\frac{6}{5} =$$

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$$\frac{8}{6} =$$

Example

Simplify (reduce) the fraction.

$$\frac{9}{12} = \frac{\cancel{3}\cdot 3}{\cancel{3}\cdot 4} = \frac{3}{4}$$

$$\frac{14}{6} = \frac{2\cdot 7}{2\cdot 3}$$

$$\frac{6}{2} =$$

$$\frac{6}{5} =$$

$$\frac{10}{15} =$$

$$\frac{8}{6} =$$

Example

Simplify (reduce) the fraction.

$$\frac{9}{12} = \frac{\cancel{3}\cdot 3}{\cancel{3}\cdot 4} = \frac{3}{4}$$

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Simplify (reduce) the fraction.

$$
\begin{aligned}
\frac{9}{12} &= \frac{\cancel{3}\cdot 3}{\cancel{3}\cdot 4} = \frac{3}{4}\\
\frac{14}{6} &= \frac{\cancel{2}\cdot 7}{\cancel{2}\cdot 3} = \frac{7}{3}\\
\frac{6}{2} &= \\
\frac{6}{5} &= \\
\frac{10}{15} &= \\
\frac{8}{6} &=
\end{aligned}
$$

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Simplify (reduce) the fraction.

$$\frac{9}{12} = \frac{\cancel{3}\cdot 3}{\cancel{3}\cdot 4} = \frac{3}{4}$$

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$$\frac{6}{2} = \frac{2\cdot 3}{2\cdot 1}$$

$$\frac{6}{5} =$$

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$$\frac{6}{\color{red}{2}} = \frac{2\cdot 3}{\color{red}{2\cdot 1}}$$

$$\frac{6}{5} =$$

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\frac{6}{5} &=\\
\frac{10}{15} &=\\
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$$\frac{6}{5} = \textbf{?}$$

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\frac{6}{5} &= \text{already simplified: } \frac{6}{5} = \frac{3\cdot 2}{5}, \text{ no primes to cancel}\\
\frac{10}{15} &=\\
\frac{8}{6} &=
\end{aligned}$$

Lemma

If there are no common prime factors between the numerator and the denominator, the fraction is reduced.

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Lemma

If there are no common prime factors between the numerator and the denominator, the fraction is reduced.

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- That fraction simplification GCD algorithm is well-suited for and very fast on a computer.
- This algorithm is also practical for use by hand.
- However on small examples the factorization-cancellation guess-work technique shown in examples is faster for a human.

Observation

$$x \cdot \frac{a}{b} = \frac{x \cdot a}{b}$$

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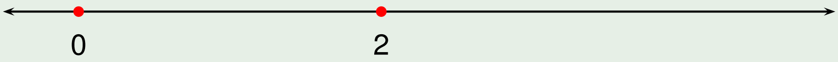
Example

$$2 \cdot \frac{2}{3} = \frac{4}{3}$$



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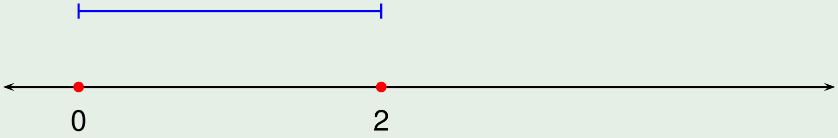

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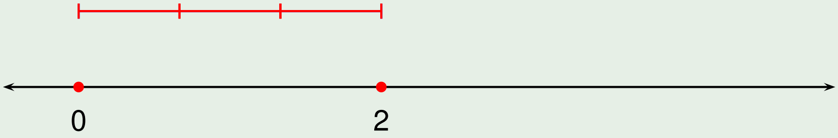

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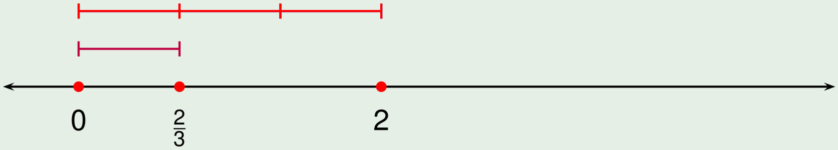

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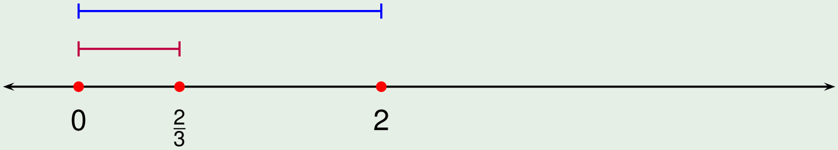

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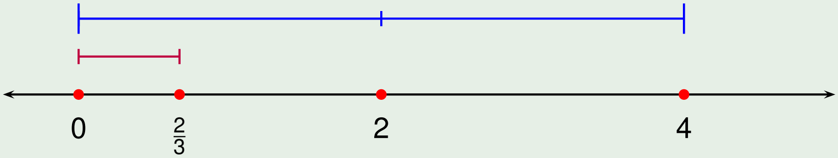

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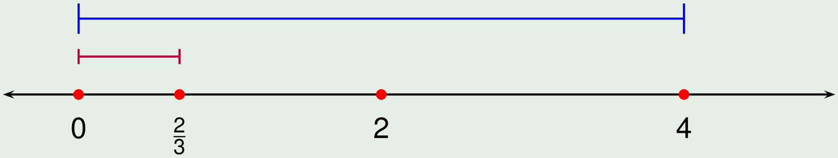


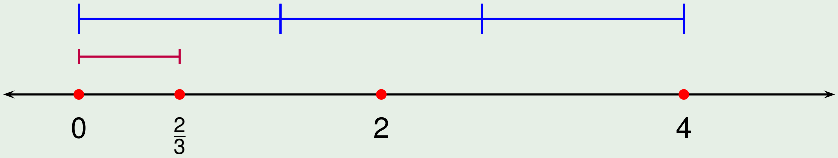
Observation

$$x \cdot \frac{a}{b} = \frac{x \cdot a}{b}$$

Multiplying a fraction by a number is equivalent to multiplying its numerator by that number.

Example

$$2 \cdot \frac{2}{3} = \frac{4}{3}$$



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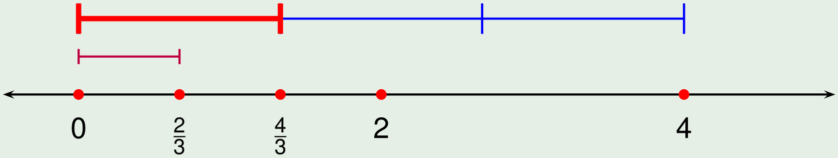

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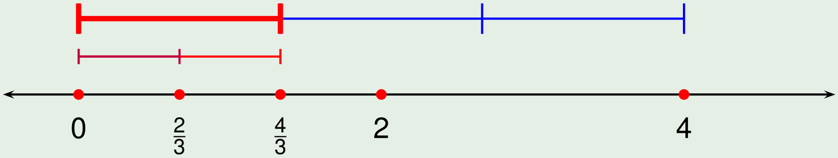

Example

Simplify the expression to a single reduced fraction.

$$2 \cdot \frac{2}{3} =$$
$$3 \cdot \frac{2}{15} =$$
$$3 \cdot \frac{1}{3} =$$
$$7 \cdot \frac{3}{21} =$$

$$6 \cdot \frac{2}{15} =$$
$$4 \cdot \frac{5}{18} =$$

Example

Simplify the expression to a single reduced fraction.

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$$4 \cdot \frac{5}{18} =$$

Example

Simplify the expression to a single reduced fraction.

$$\begin{aligned}
2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} \\
3 \cdot \frac{2}{15} &= \\
3 \cdot \frac{1}{3} &= \\
7 \cdot \frac{3}{21} &= \\
\\
6 \cdot \frac{2}{15} &= \\
4 \cdot \frac{5}{18} &=
\end{aligned}$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} = \frac{4}{3} \\
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7 \cdot \frac{3}{21} &= \\
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6 \cdot \frac{2}{15} &= \\
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\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

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2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} = \frac{4}{3} \\
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6 \cdot \frac{2}{15} &= \\
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\end{aligned}
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Example

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\\
6 \cdot \frac{2}{15} &= \\
4 \cdot \frac{5}{18} &=
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} = \frac{4}{3} \\
{\color{red}3} \cdot \frac{2}{15} &= \frac{{\color{red}3} \cdot 2}{\textbf{?}} \\
3 \cdot \frac{1}{3} &= \\
7 \cdot \frac{3}{21} &= \\
\\
6 \cdot \frac{2}{15} &= \\
4 \cdot \frac{5}{18} &=
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

$$\begin{aligned}
2\cdot\frac{2}{3} &= \frac{2\cdot 2}{3} = \frac{4}{3}\\
3\cdot\frac{2}{\color{red}{15}} &= \frac{3\cdot 2}{\color{red}{\mathbf{?}}}\\
3\cdot\frac{1}{3} &=\\
7\cdot\frac{3}{21} &=\\
\\
6\cdot\frac{2}{15} &=\\
4\cdot\frac{5}{18} &=
\end{aligned}$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2\cdot\frac{2}{3} &= \frac{2\cdot 2}{3}=\frac{4}{3}\\
3\cdot\frac{2}{\color{red}{15}} &= \frac{3\cdot 2}{\color{red}{3\cdot 5}}\\
3\cdot\frac{1}{3} &=\\
7\cdot\frac{3}{21} &=\\
\\
6\cdot\frac{2}{15} &=\\
4\cdot\frac{5}{18} &=
\end{aligned}
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Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} = \frac{4}{3} \\
3 \cdot \frac{2}{15} &= \frac{\cancel{3} \cdot 2}{\cancel{3} \cdot 5} = \frac{2}{5} \\
3 \cdot \frac{1}{3} &= \\
7 \cdot \frac{3}{21} &=
\end{aligned}
$$

$$
\begin{aligned}
6 \cdot \frac{2}{15} &= \\
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Simplify the expression to a single reduced fraction.

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2\cdot\frac{2}{3} &= \frac{2\cdot 2}{3} = \frac{4}{3}\\
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3\cdot\frac{1}{3} &= \\
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6\cdot\frac{2}{15} &= \\
4\cdot\frac{5}{18} &=
\end{aligned}
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Simplify the expression to a single reduced fraction.

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2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} = \frac{4}{3} \\
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3 \cdot \frac{1}{3} &= \\
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\end{aligned}
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3 \cdot \frac{1}{3} &= \frac{3}{3} \\
7 \cdot \frac{3}{21} &= \\
\\
6 \cdot \frac{2}{15} &= \\
4 \cdot \frac{5}{18} &=
\end{aligned}
$$

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Simplify the expression to a single reduced fraction.

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\begin{aligned}
2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} = \frac{4}{3} \\
3 \cdot \frac{2}{15} &= \frac{\cancel{3} \cdot 2}{\cancel{3} \cdot 5} = \frac{2}{5} \\
3 \cdot \frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}} = 1 \\
7 \cdot \frac{3}{21} &= \\
\\
6 \cdot \frac{2}{15} &= \\
4 \cdot \frac{5}{18} &=
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

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\begin{aligned}
2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} = \frac{4}{3} \\
3 \cdot \frac{2}{15} &= \frac{\cancel{3} \cdot 2}{\cancel{3} \cdot 5} = \frac{2}{5} \\
3 \cdot \frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}} = 1 \\
7 \cdot \frac{3}{21} &= \\
\\
6 \cdot \frac{2}{15} &= \\
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3 \cdot \frac{2}{15} &= \frac{\cancel{3} \cdot 2}{\cancel{3} \cdot 5} = \frac{2}{5} \\
3 \cdot \frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}} = 1 \\
7 \cdot \frac{3}{21} &= \\
\\
6 \cdot \frac{2}{15} &= \\
4 \cdot \frac{5}{18} &=
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} = \frac{4}{3} \\
3 \cdot \frac{2}{15} &= \frac{\cancel{3} \cdot 2}{\cancel{3} \cdot 5} = \frac{2}{5} \\
3 \cdot \frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}} = 1 \\
{\color{red}7} \cdot \frac{3}{21} &= \frac{{\color{red}7} \cdot 3}{21}
\end{aligned}
$$

$$
\begin{aligned}
6 \cdot \frac{2}{15} &= \\
4 \cdot \frac{5}{18} &=
\end{aligned}
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Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} = \frac{4}{3}\\
3 \cdot \frac{2}{15} &= \frac{\cancel{3} \cdot 2}{\cancel{3} \cdot 5} = \frac{2}{5}\\
3 \cdot \frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}} = 1\\
7 \cdot \frac{3}{21} &= \frac{{\color{red}7 \cdot 3}}{21} = \frac{{\color{red}21}}{21}
\end{aligned}
$$

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\begin{aligned}
6 \cdot \frac{2}{15} &= \\
4 \cdot \frac{5}{18} &=
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3 \cdot \frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}} = 1 \\
7 \cdot \frac{3}{21} &= \frac{7 \cdot 3}{21} = \frac{\cancel{21}}{\cancel{21}} = 1
\end{aligned}
$$

$$
\begin{aligned}
6 \cdot \frac{2}{15} &= \\
4 \cdot \frac{5}{18} &=
\end{aligned}
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Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2\cdot\frac{2}{3} &= \frac{2\cdot 2}{3} = \frac{4}{3}\\
3\cdot\frac{2}{15} &= \frac{\cancel{3}\cdot 2}{\cancel{3}\cdot 5} = \frac{2}{5}\\
3\cdot\frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}} = 1\\
{\color{red}7\cdot\frac{3}{21}} &{\color{red}=} \frac{7\cdot 3}{21} = \frac{\cancel{21}}{\cancel{21}} = {\color{red}1}
\end{aligned}
$$

$$
\begin{aligned}
6\cdot\frac{2}{15} &=\\
4\cdot\frac{5}{18} &=
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} = \frac{4}{3} \\
3 \cdot \frac{2}{15} &= \frac{\cancel{3} \cdot 2}{\cancel{3} \cdot 5} = \frac{2}{5} \\
3 \cdot \frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}} = 1 \\
7 \cdot \frac{3}{\color{red}{21}} &= \frac{7 \cdot 3}{21} = \frac{\cancel{21}}{\cancel{21}} = 1 \\
\text{[alternatively]} &= 7 \cdot \frac{3}{\color{red}{3 \cdot 7}} \\
6 \cdot \frac{2}{15} &= \\
4 \cdot \frac{5}{18} &=
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

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\begin{aligned}
2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} = \frac{4}{3} \\
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7 \cdot \frac{3}{21} &= \frac{7 \cdot 3}{21} = \frac{\cancel{21}}{\cancel{21}} = 1 \\
\text{[alternatively]} &= 7 \cdot \frac{\cancel{3}}{\cancel{3} \cdot 7} = 7 \cdot \frac{1}{7} \\
6 \cdot \frac{2}{15} &= \\
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Simplify the expression to a single reduced fraction.

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7 \cdot \frac{3}{21} &= \frac{7 \cdot 3}{21} = \frac{\cancel{21}}{\cancel{21}} = 1 \\
\text{[alternatively]} &= 7 \cdot \frac{\cancel{3}}{\cancel{3} \cdot 7} = {\color{red}7} \cdot \frac{1}{7} = \frac{{\color{red}7}}{7} \\
6 \cdot \frac{2}{15} &= \\
4 \cdot \frac{5}{18} &=
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{array}{rcl}
2 \cdot \frac{2}{3} & = & \frac{2 \cdot 2}{3} = \frac{4}{3} \\
3 \cdot \frac{2}{15} & = & \frac{\cancel{3} \cdot 2}{\cancel{3} \cdot 5} = \frac{2}{5} \\
3 \cdot \frac{1}{3} & = & \frac{\cancel{3}}{\cancel{3}} = 1 \\
7 \cdot \frac{3}{21} & = & \frac{7 \cdot 3}{21} = \frac{\cancel{21}}{\cancel{21}} = 1 \\
\text{[alternatively]} & = & 7 \cdot \frac{\cancel{3}}{\cancel{3} \cdot 7} = 7 \cdot \frac{1}{7} = \frac{\cancel{7}}{\cancel{7}} = 1 \\
6 \cdot \frac{2}{15} & = & \\
4 \cdot \frac{5}{18} & = &
\end{array}
$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2\cdot\frac{2}{3} &= \frac{2\cdot 2}{3}=\frac{4}{3}\\
3\cdot\frac{2}{15} &= \frac{\cancel{3}\cdot 2}{\cancel{3}\cdot 5}=\frac{2}{5}\\
3\cdot\frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}}=1\\
{\color{red}7\cdot\frac{3}{21}} &{\color{red}=} \frac{7\cdot 3}{21}=\frac{\cancel{21}}{\cancel{21}}=1\\
\text{[alternatively]} &= 7\cdot\frac{\cancel{3}}{\cancel{3}\cdot 7}=7\cdot\frac{1}{7}=\frac{\cancel{7}}{\cancel{7}}={\color{red}1}\\
6\cdot\frac{2}{15} &=\\
4\cdot\frac{5}{18} &=
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2\cdot\frac{2}{3} &= \frac{2\cdot 2}{3}=\frac{4}{3}\\
3\cdot\frac{2}{15} &= \frac{\cancel{3}\cdot 2}{\cancel{3}\cdot 5}=\frac{2}{5}\\
3\cdot\frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}}=1\\
7\cdot\frac{3}{21} &= \frac{7\cdot 3}{21}=\frac{\cancel{21}}{\cancel{21}}=1\\
\text{[alternatively]} &= 7\cdot\frac{\cancel{3}}{\cancel{3}\cdot 7}=7\cdot\frac{1}{7}=\frac{\cancel{7}}{\cancel{7}}=1\\
6\cdot\frac{2}{15} &= \\
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\end{aligned}
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Simplify the expression to a single reduced fraction.

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\begin{aligned}
2\cdot\frac{2}{3} &= \frac{2\cdot 2}{3}=\frac{4}{3}\\
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3\cdot\frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}}=1\\
7\cdot\frac{3}{21} &= \frac{7\cdot 3}{21}=\frac{\cancel{21}}{\cancel{21}}=1\\
\text{[alternatively]} &= 7\cdot\frac{\cancel{3}}{\cancel{3}\cdot 7}=7\cdot\frac{1}{7}=\frac{\cancel{7}}{\cancel{7}}=1\\
6\cdot\frac{2}{15} &= \frac{6\cdot 2}{15}\\
4\cdot\frac{5}{18} &=
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2\cdot\frac{2}{3} &= \frac{2\cdot 2}{3}=\frac{4}{3}\\
3\cdot\frac{2}{15} &= \frac{\cancel{3}\cdot 2}{\cancel{3}\cdot 5}=\frac{2}{5}\\
3\cdot\frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}}=1\\
7\cdot\frac{3}{21} &= \frac{7\cdot 3}{21}=\frac{\cancel{21}}{\cancel{21}}=1\\
\text{[alternatively]} &= 7\cdot\frac{\cancel{3}}{\cancel{3}\cdot 7}=7\cdot\frac{1}{7}=\frac{\cancel{7}}{\cancel{7}}=1\\
6\cdot\frac{2}{15} &= \frac{{\color{red}6}\cdot 2}{15}=\frac{{\color{red}2\cdot 3}\cdot 2}{3\cdot 5}\\
4\cdot\frac{5}{18} &=
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} = \frac{4}{3} \\
3 \cdot \frac{2}{15} &= \frac{\cancel{3} \cdot 2}{\cancel{3} \cdot 5} = \frac{2}{5} \\
3 \cdot \frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}} = 1 \\
7 \cdot \frac{3}{21} &= \frac{7 \cdot 3}{21} = \frac{\cancel{21}}{\cancel{21}} = 1 \\
\text{[alternatively]} &= 7 \cdot \frac{\cancel{3}}{\cancel{3} \cdot 7} = 7 \cdot \frac{1}{7} = \frac{\cancel{7}}{\cancel{7}} = 1 \\
6 \cdot \frac{2}{15} &= \frac{6 \cdot 2}{15} = \frac{2 \cdot \cancel{3} \cdot 2}{\cancel{3} \cdot 5} = \frac{4}{5} \\
4 \cdot \frac{5}{18} &=
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} = \frac{4}{3} \\
3 \cdot \frac{2}{15} &= \frac{\cancel{3} \cdot 2}{\cancel{3} \cdot 5} = \frac{2}{5} \\
3 \cdot \frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}} = 1 \\
7 \cdot \frac{3}{21} &= \frac{7 \cdot 3}{21} = \frac{\cancel{21}}{\cancel{21}} = 1 \\
\text{[alternatively]} &= 7 \cdot \frac{\cancel{3}}{\cancel{3} \cdot 7} = 7 \cdot \frac{1}{7} = \frac{\cancel{7}}{\cancel{7}} = 1 \\
6 \cdot \frac{2}{15} &= \frac{6 \cdot 2}{15} = \frac{2 \cdot \cancel{3} \cdot 2}{\cancel{3} \cdot 5} = \frac{4}{5} \\
4 \cdot \frac{5}{18} &=
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

$$\begin{aligned}
2\cdot\frac{2}{3} &= \frac{2\cdot 2}{3}=\frac{4}{3}\\
3\cdot\frac{2}{15} &= \frac{\cancel{3}\cdot 2}{\cancel{3}\cdot 5}=\frac{2}{5}\\
3\cdot\frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}}=1\\
7\cdot\frac{3}{21} &= \frac{7\cdot 3}{21}=\frac{\cancel{21}}{\cancel{21}}=1\\
\text{[alternatively]} &= 7\cdot\frac{\cancel{3}}{\cancel{3}\cdot 7}=7\cdot\frac{1}{7}=\frac{\cancel{7}}{\cancel{7}}=1\\
{\color{red}6\cdot\frac{2}{15}} &{\color{red}=} \frac{6\cdot 2}{15}=\frac{2\cdot\cancel{3}\cdot 2}{\cancel{3}\cdot 5}={\color{red}\frac{4}{5}}\\
4\cdot\frac{5}{18} &=
\end{aligned}$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2\cdot\frac{2}{3} &= \frac{2\cdot 2}{3} = \frac{4}{3}\\
3\cdot\frac{2}{15} &= \frac{\cancel{3}\cdot 2}{\cancel{3}\cdot 5} = \frac{2}{5}\\
3\cdot\frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}} = 1\\
7\cdot\frac{3}{21} &= \frac{7\cdot 3}{21} = \frac{\cancel{21}}{\cancel{21}} = 1\\
\text{[alternatively]} &= 7\cdot\frac{\cancel{3}}{\cancel{3}\cdot 7} = 7\cdot\frac{1}{7} = \frac{\cancel{7}}{\cancel{7}} = 1\\
6\cdot\frac{2}{15} &= \frac{6\cdot 2}{15} = \frac{2\cdot\cancel{3}\cdot 2}{\cancel{3}\cdot 5} = \frac{4}{5}\\
4\cdot\frac{5}{18} &=
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2\cdot\frac{2}{3} &= \frac{2\cdot 2}{3}=\frac{4}{3}\\
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3\cdot\frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}}=1\\
7\cdot\frac{3}{21} &= \frac{7\cdot 3}{21}=\frac{\cancel{21}}{\cancel{21}}=1\\
\text{[alternatively]} &= 7\cdot\frac{\cancel{3}}{\cancel{3}\cdot 7}=7\cdot\frac{1}{7}=\frac{\cancel{7}}{\cancel{7}}=1\\
6\cdot\frac{2}{15} &= \frac{6\cdot 2}{15}=\frac{2\cdot\cancel{3}\cdot 2}{\cancel{3}\cdot 5}=\frac{4}{5}\\
{\color{red}4}\cdot\frac{5}{18} &= \frac{{\color{red}4}\cdot 5}{\textbf{?}}
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{array}{rcl}
2\cdot\frac{2}{3} & = & \frac{2\cdot 2}{3}=\frac{4}{3}\\
3\cdot\frac{2}{15} & = & \frac{\cancel{3}\cdot 2}{\cancel{3}\cdot 5}=\frac{2}{5}\\
3\cdot\frac{1}{3} & = & \frac{\cancel{3}}{\cancel{3}}=1\\
7\cdot\frac{3}{21} & = & \frac{7\cdot 3}{21}=\frac{\cancel{21}}{\cancel{21}}=1\\
\text{[alternatively]} & = & 7\cdot\frac{\cancel{3}}{\cancel{3}\cdot 7}=7\cdot\frac{1}{7}=\frac{\cancel{7}}{\cancel{7}}=1\\
6\cdot\frac{2}{15} & = & \frac{6\cdot 2}{15}=\frac{2\cdot\cancel{3}\cdot 2}{\cancel{3}\cdot 5}=\frac{4}{5}\\
4\cdot\frac{5}{18} & = & \frac{4\cdot 5}{?}
\end{array}
$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2 \cdot \frac{2}{3} &= \frac{2 \cdot 2}{3} = \frac{4}{3} \\
3 \cdot \frac{2}{15} &= \frac{\cancel{3} \cdot 2}{\cancel{3} \cdot 5} = \frac{2}{5} \\
3 \cdot \frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}} = 1 \\
7 \cdot \frac{3}{21} &= \frac{7 \cdot 3}{21} = \frac{\cancel{21}}{\cancel{21}} = 1 \\
\text{[alternatively]} &= 7 \cdot \frac{\cancel{3}}{\cancel{3} \cdot 7} = 7 \cdot \frac{1}{7} = \frac{\cancel{7}}{\cancel{7}} = 1 \\
6 \cdot \frac{2}{15} &= \frac{6 \cdot 2}{15} = \frac{2 \cdot \cancel{3} \cdot 2}{\cancel{3} \cdot 5} = \frac{4}{5} \\
4 \cdot \frac{5}{18} &= \frac{4 \cdot 5}{3 \cdot 6}
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2\cdot\frac{2}{3} &= \frac{2\cdot 2}{3}=\frac{4}{3}\\
3\cdot\frac{2}{15} &= \frac{\cancel{3}\cdot 2}{\cancel{3}\cdot 5}=\frac{2}{5}\\
3\cdot\frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}}=1\\
7\cdot\frac{3}{21} &= \frac{7\cdot 3}{21}=\frac{\cancel{21}}{\cancel{21}}=1\\
\text{[alternatively]} &= 7\cdot\frac{\cancel{3}}{\cancel{3}\cdot 7}=7\cdot\frac{1}{7}=\frac{\cancel{7}}{\cancel{7}}=1\\
6\cdot\frac{2}{15} &= \frac{6\cdot 2}{15}=\frac{2\cdot\cancel{3}\cdot 2}{\cancel{3}\cdot 5}=\frac{4}{5}\\
4\cdot\frac{5}{18} &= \frac{4\cdot 5}{3\cdot 6}=\frac{2\cdot 2\cdot 5}{3\cdot 2\cdot 3}
\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

$$
\begin{aligned}
2\cdot\frac{2}{3} &= \frac{2\cdot 2}{3} = \frac{4}{3}\\
3\cdot\frac{2}{15} &= \frac{\cancel{3}\cdot 2}{\cancel{3}\cdot 5} = \frac{2}{5}\\
3\cdot\frac{1}{3} &= \frac{\cancel{3}}{\cancel{3}} = 1\\
7\cdot\frac{3}{21} &= \frac{7\cdot 3}{21} = \frac{\cancel{21}}{\cancel{21}} = 1\\
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6\cdot\frac{2}{15} &= \frac{6\cdot 2}{15} = \frac{2\cdot\cancel{3}\cdot 2}{\cancel{3}\cdot 5} = \frac{4}{5}\\
4\cdot\frac{5}{18} &= \frac{{\color{red}4}\cdot 5}{3\cdot 6} = \frac{{\color{red}2\cdot 2}\cdot 5}{3\cdot 2\cdot 3}
\end{aligned}
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4 \cdot \frac{5}{18} &= \frac{4 \cdot 5}{3 \cdot 6} = \frac{2 \cdot \cancel{2} \cdot 5}{3 \cdot \cancel{2} \cdot 3} = \frac{2 \cdot 5}{3 \cdot 3}
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\end{aligned}
$$

Example

Simplify the expression to a single reduced fraction.

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2\cdot\frac{2}{3} &= \frac{2\cdot 2}{3}=\frac{4}{3}\\
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\end{aligned}$$

Example

Simplify the expression to a single reduced fraction.

$$
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\end{aligned}
$$