Precalculus
Compute trigonometric function of a complementary angle, part 1

Todor Milev

2019

Cofunction identities

Proposition (Cofunction identities)

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\begin{array}{rclrcl}
\sin\left(\frac{\pi}{2}-\alpha\right) & = & \cos\alpha & \sin\left(\frac{\pi}{2}+\alpha\right) & = & \cos\alpha\\
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- The proof each formula is broken into 4 cases depending on which quadrant contains α.

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- This makes a total of 4 formulas $\times 4$ cases per formula = 16 cases.

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- We show only a few of the cases.
- The proof provides intuition why the formulas are true.
- The Quadrant I part of the proof serves as a visual aid for memorization.
- There is an algebraically simpler (but theoretically advanced) way to prove the above identities through the angle sum f-las, derived in turn from Euler's formula (studied later/in another course).

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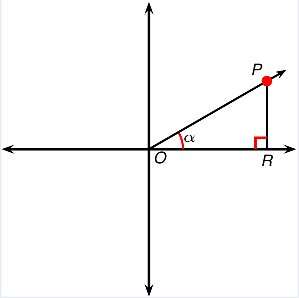
Part of Proof.



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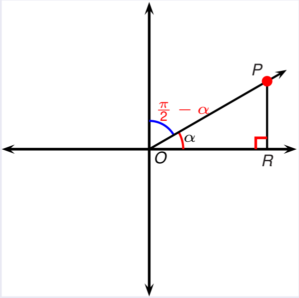

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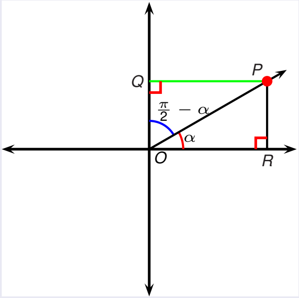

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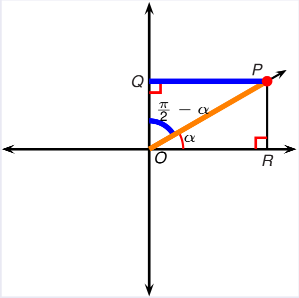

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$$\sin\left(\frac{\pi}{2}-\alpha\right) = \frac{|PQ|}{|OP|}$$

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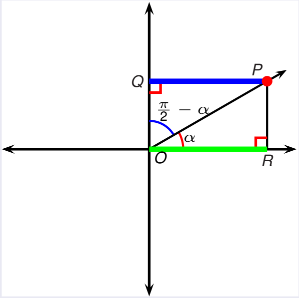

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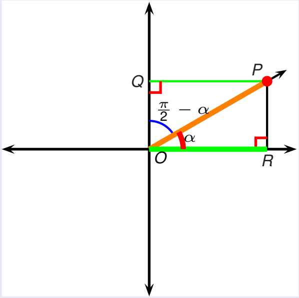

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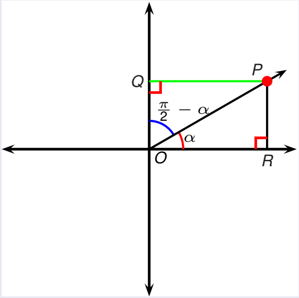

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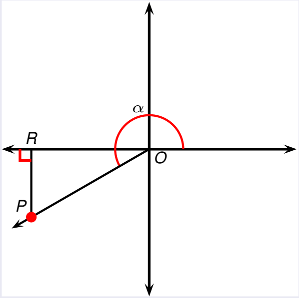

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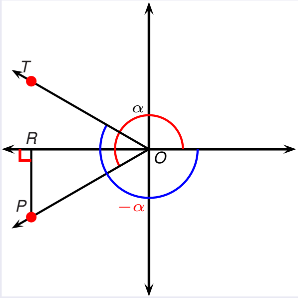

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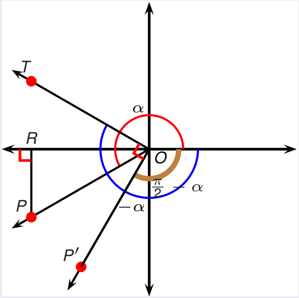

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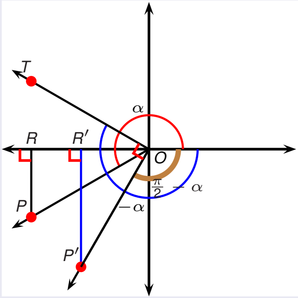

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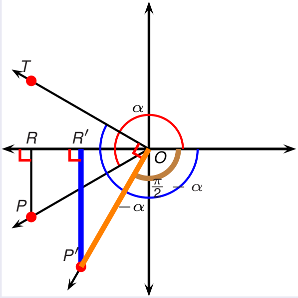

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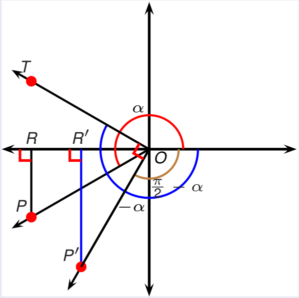

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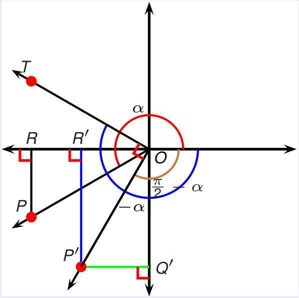

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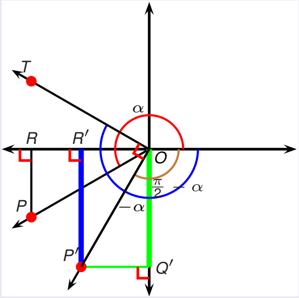

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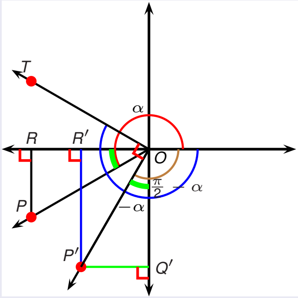

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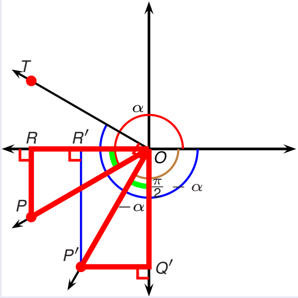

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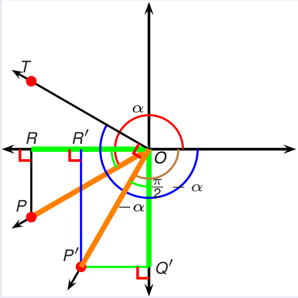

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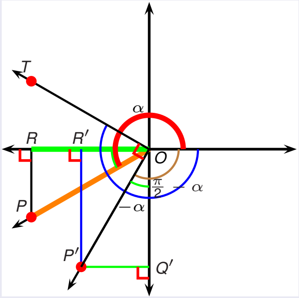

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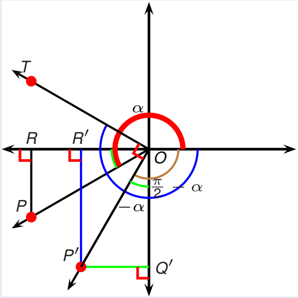

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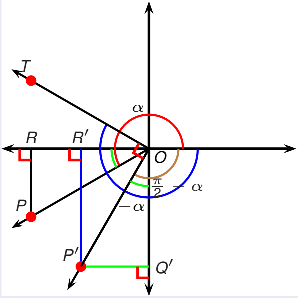

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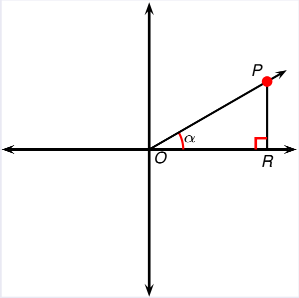

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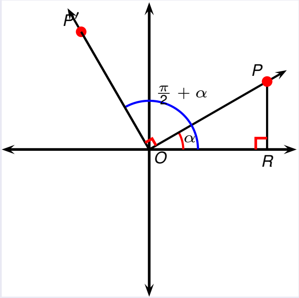
Part of Proof.



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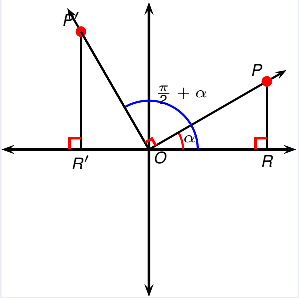

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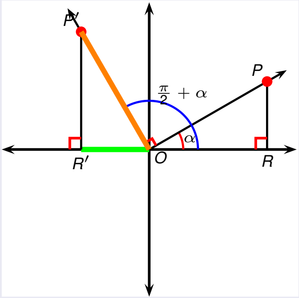

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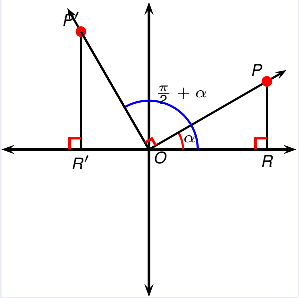

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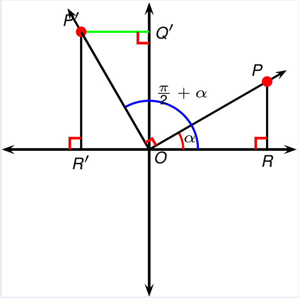

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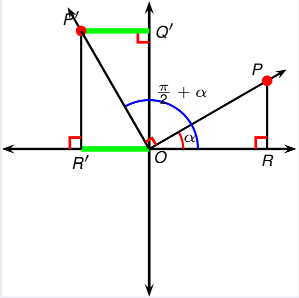

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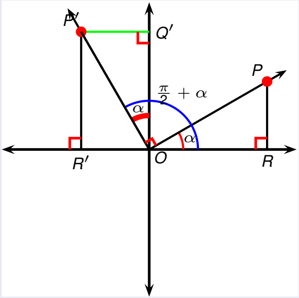

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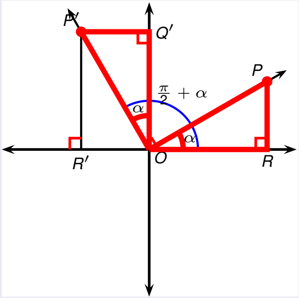

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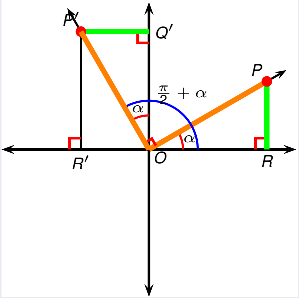

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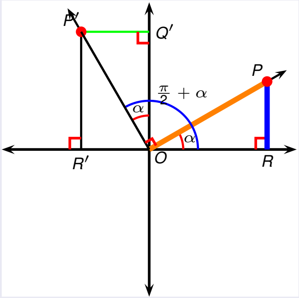

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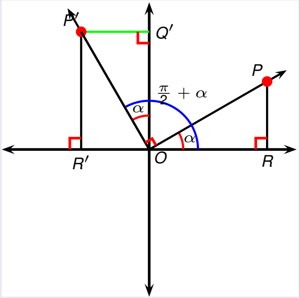

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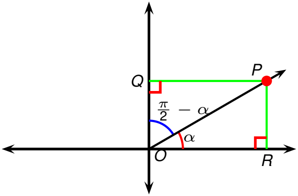
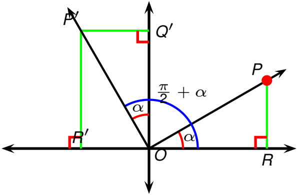
To memorize the cofunction identities it suffices to memorize the Quadrant I case via the two diagrams below.



$$\begin{aligned} \sin\left(\frac{\pi}{2}-\alpha\right) &= \frac{|PQ|}{|OP|} \\ \cos\left(\frac{\pi}{2}-\alpha\right) &= \frac{|OQ|}{|OP|} \end{aligned}$$



$$\begin{aligned} \sin\left(\frac{\pi}{2}+\alpha\right) &= \frac{|OQ'|}{|OP|} \\ \cos\left(\frac{\pi}{2}+\alpha\right) &= -\frac{|PQ'|}{|OP'|} = \frac{|PR|}{|OP|} \end{aligned}$$