Precalculus
Factor cubic with one rational and two real roots using its plot

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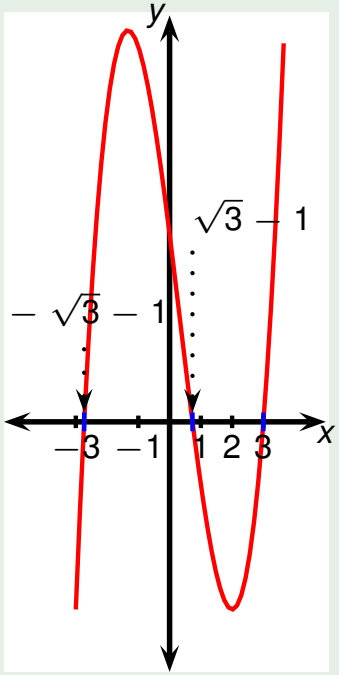
Example



Plot the left hand side of the equation with a graphing calculator. Solve the equation.

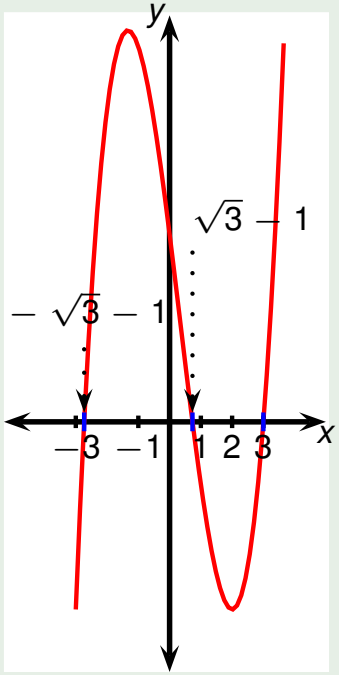
$$\begin{aligned} x^3 - x^2 - 8x + 6 &= 0 \\ (x-3)(x^2+2x-2) &= 0 \end{aligned}$$

The graph appears to intersect the x axis at: $-\sqrt{3}-1, \sqrt{3}-1, 3$. What are the two roots besides 3?

Quotient: $x^2 + 2x - 2$

$$\begin{array}{r|l} x-3 & x^3 - x^2 - 8x + 6 \\ - & x^3 - 3x^2 \\ \hline & 2x^2 - 8x + 6 \\ - & 2x^2 - 6x \\ \hline & -2x + 6 \\ - & -2x + 6 \\ \hline \textbf{Remainder:} & 0 \end{array}$$

Example



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$$\begin{aligned} x^3 - x^2 - 8x + 6 &= 0 \\ (x-3)(x^2+2x-2) &= 0 \end{aligned}$$

$x - 3 = 0$ **or** $x = \dfrac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$

$x = 3$ $\qquad x = \dfrac{-2 \pm \sqrt{12}}{2}$

$$x = \frac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}.$$

The graph appears to intersect the x axis at: $-\sqrt{3}-1, \sqrt{3}-1, 3$. What are the two roots besides 3?

Final answer:

$x = 3$ **or** $x = -1 - \sqrt{3}$ **or** $x = -1 + \sqrt{3}$.