Precalculus
Quadratic inequality part 1

Todor Milev

2019

Example

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$$2x^2 + 3x - 5 \geq 0$$

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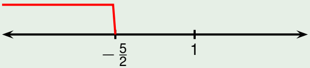

Interval	Factor signs	Final sign	
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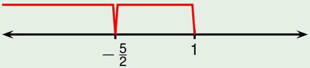

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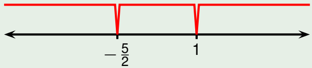

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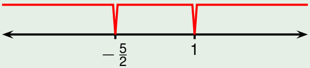

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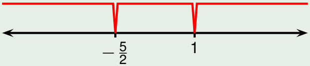

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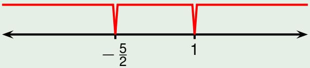

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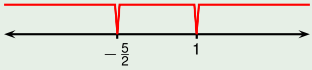

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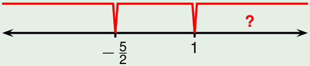

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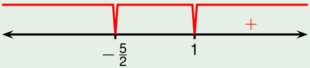

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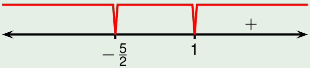

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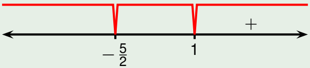

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${\color{red}\left(-\frac{5}{2}, 1\right)}$	$({\color{red}+})(\textbf{?}\ \ )$		
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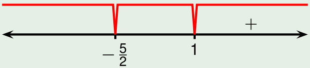

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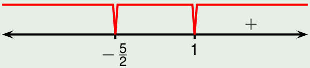

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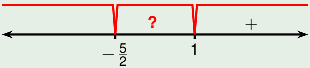

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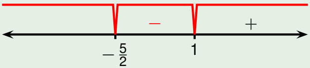

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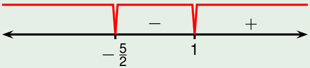

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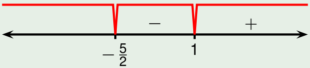

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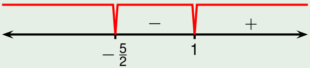

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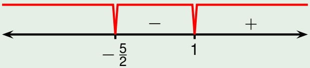

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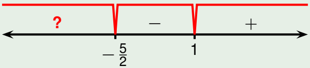

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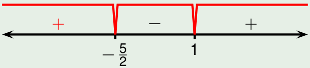

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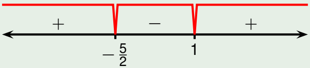

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$\left(-\frac{5}{2},1\right)$	$(+)(-)$	$-$	0	$f(0)=-5<0$
$(1,\infty)$	$(+)(+)$	$+$	100	$f(100)>0$

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$x \in$ **?**

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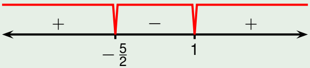

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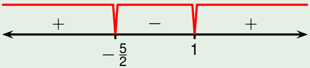

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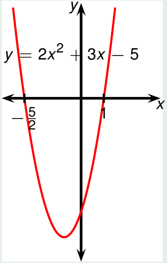

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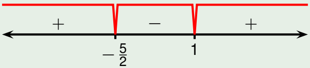

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$\left(-\frac{5}{2}, 1\right)$	$(+)(-)$	$-$	0	$f(0) = -5 < 0$
$(1, \infty)$	$(+)(+)$	$+$	100	$f(100) > 0$