

Precalculus

Double/half angle formulas, theory

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Double angle formulas

Proposition (Double angle formulas)

$$\begin{aligned}\sin(2\alpha) &= 2 \sin \alpha \cos \alpha \\ \cos(2\alpha) &= \cos^2 \alpha - \sin^2 \alpha \\ &= 2 \cos^2 \alpha - 1 \\ &= 1 - 2 \sin^2 \alpha\end{aligned}$$

- The double angle formulas play a special role in integration.

Example

Derive the double-angle formulas.

$$\begin{aligned}\sin(2\alpha) &= \sin(\alpha + \alpha) \\ &= \sin \alpha \cos \alpha + \cos \alpha \sin \alpha \\ &= 2 \sin \alpha \cos \alpha\end{aligned}$$

$$\begin{aligned}\cos(2\alpha) &= \cos(\alpha + \alpha) \\ &= \cos \alpha \cos \alpha - \sin \alpha \sin \alpha \\ &= \cos^2 \alpha - \sin^2 \alpha \\ &= \cos^2 \alpha - (1 - \cos^2 \alpha) \\ &= 2 \cos^2 \alpha - 1 \\ &= 1 - \sin^2 \alpha - \sin^2 \alpha \\ &= 1 - 2 \sin^2 \alpha\end{aligned}$$