Calculus I
Maxima and minima over closed intervals

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Outline

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Fermat's Theorem suggests that we should look at three types of points to find local maxima and minima:

1. Points c for which $f'(c) = 0$.
2. Points c for which $f'(c)$ doesn't exist.
3. Points c at ends of intervals where f is defined. Here, we need also that f be defined at c.

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Fermat's Theorem says that if f has a local maximum or minimum at c, and c is not an endpoint, then c is a critical number for f.

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- f isn't defined at $-\frac{2}{3}$. Therefore the critical numbers are 0 and $\frac{2}{3}$.

The Closed Interval Method

We know from the Extreme Value Theorem that a continuous function attains its maximum and minimum on a closed interval $[a, b]$. The maximum might occur at an endpoint. The minimum might occur at an endpoint.

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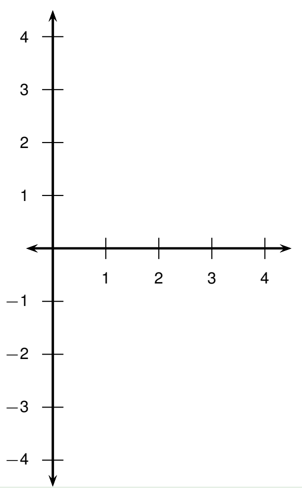
We know from the Extreme Value Theorem that a continuous function attains its maximum and minimum on a closed interval $[a, b]$. The maximum might occur at an endpoint. The minimum might occur at an endpoint.

To find the maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in $[a, b]$.
   - Find the values c with $f'(c) = 0$.
   - Find the values c where f' does not exist.
2. Find the values of f at the endpoints a and b.
3. The maximum of f is maximum of the preceding values; the minimum value is the minimum.

Example

Find the maximum and minimum values of the function
$f(x) = -x^3 + 2x^2 + 4x - 5$ on the interval $[1, 3]$.



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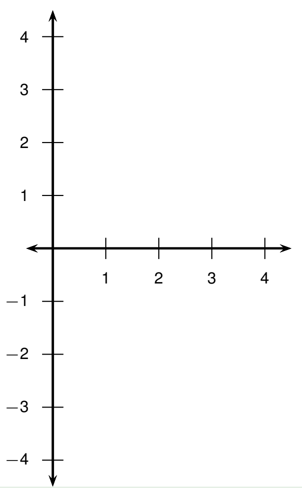

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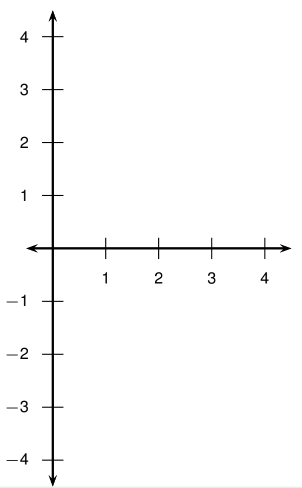

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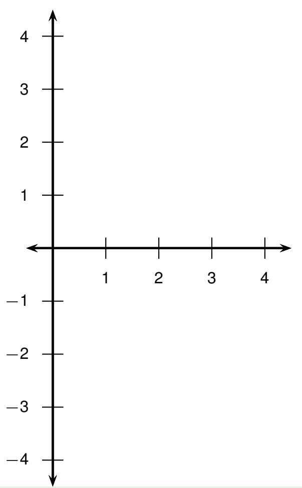

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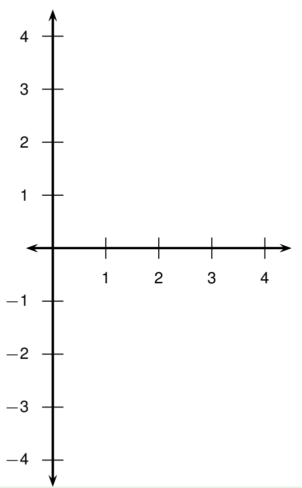

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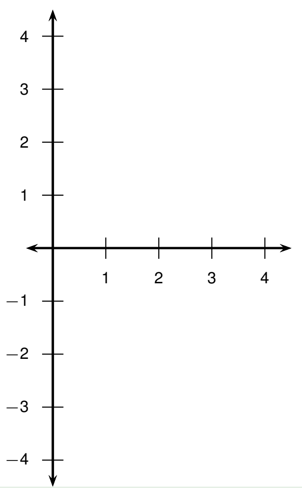

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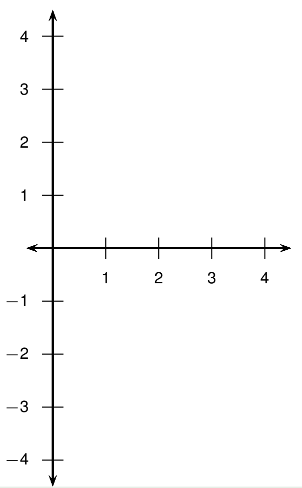

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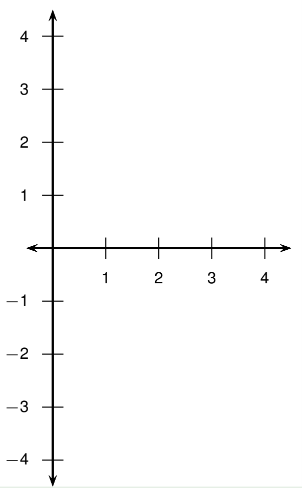

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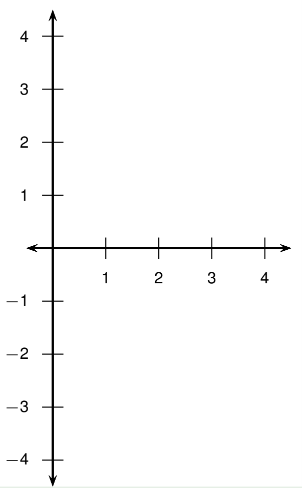

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2	
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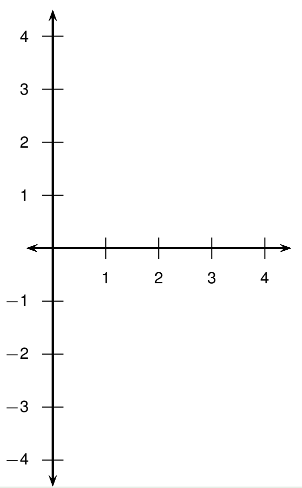

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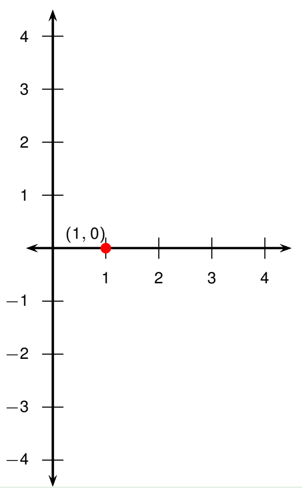

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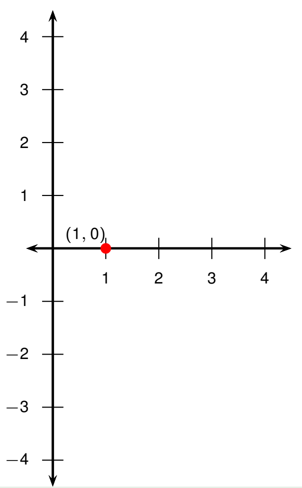

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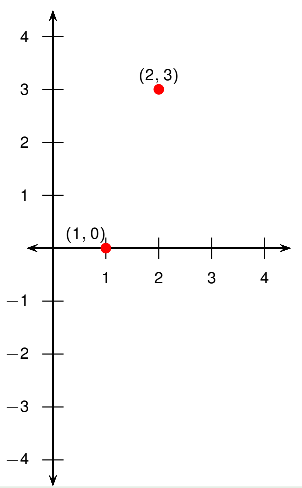

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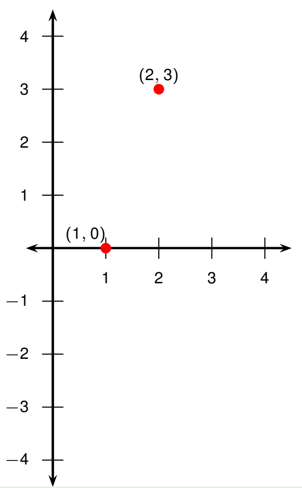

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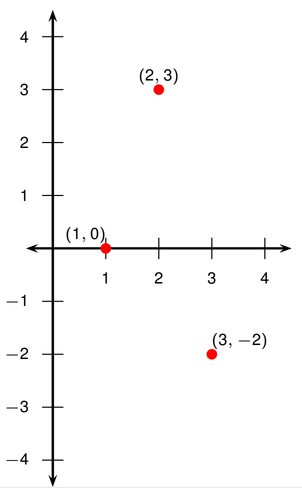

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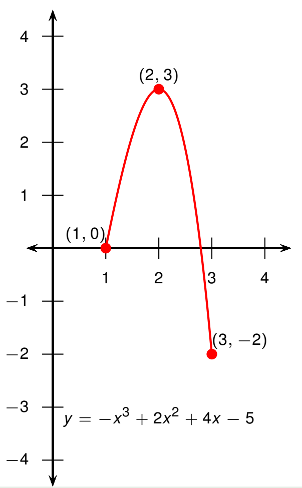

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If $f'(x) = 0$, $x = -\frac{2}{3}$ or 2.

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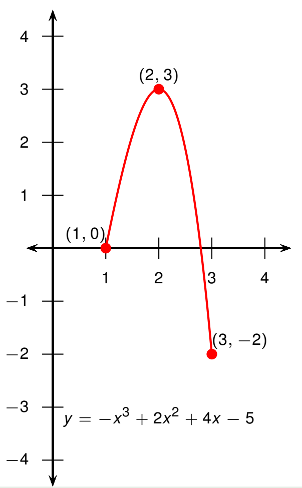

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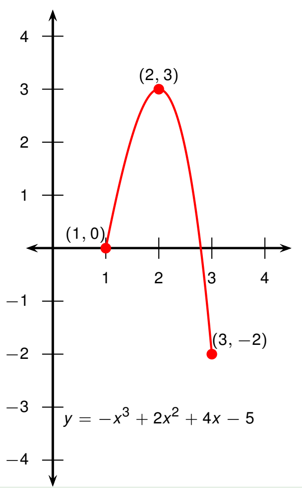

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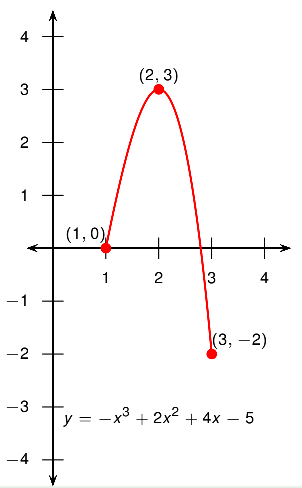

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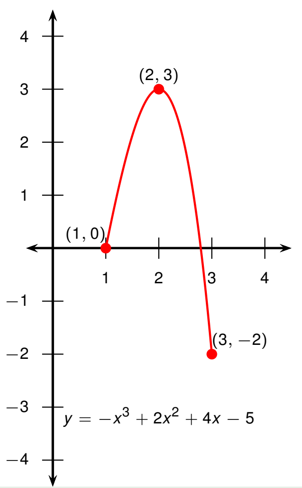

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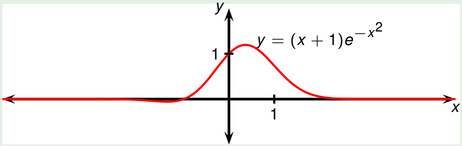

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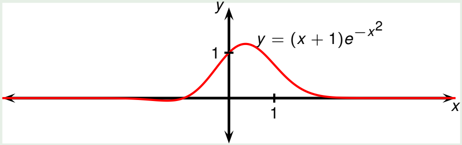

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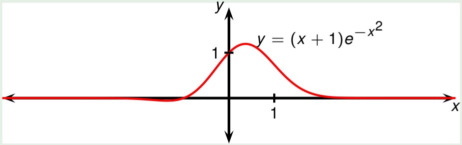

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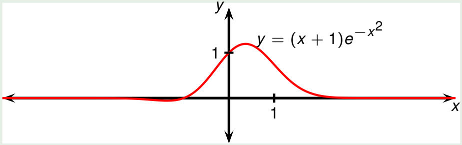

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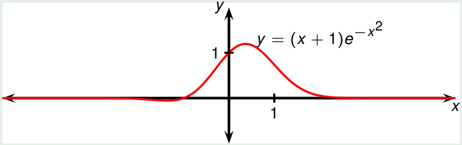

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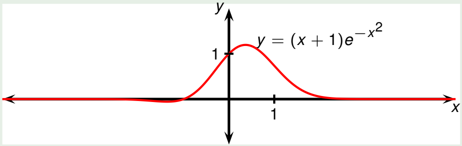

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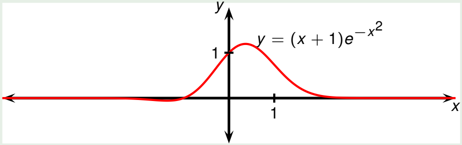

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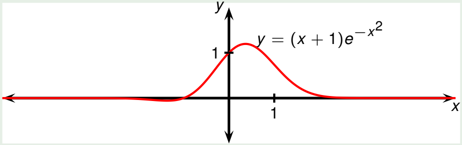

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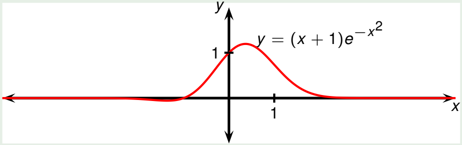

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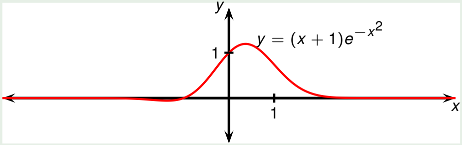

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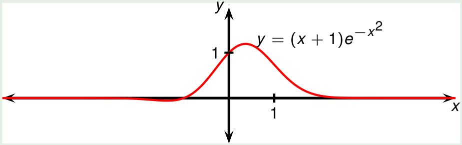

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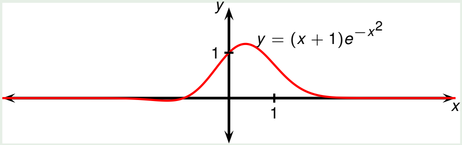

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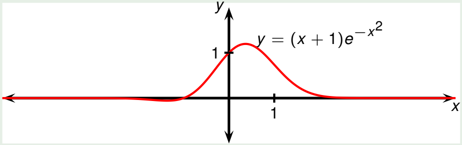

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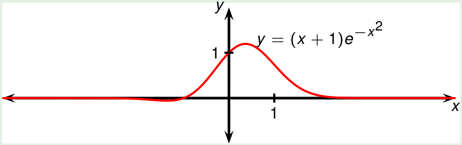

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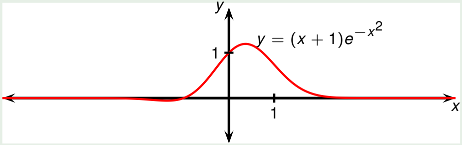

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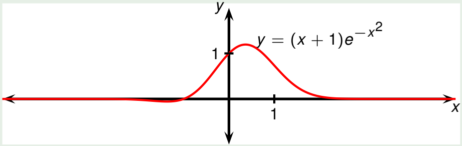

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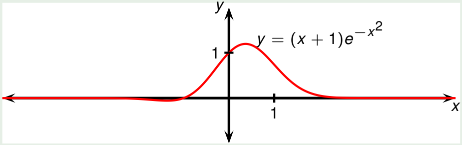

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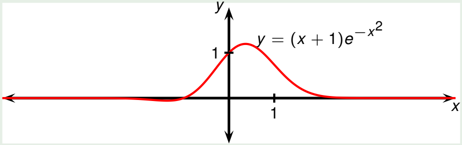

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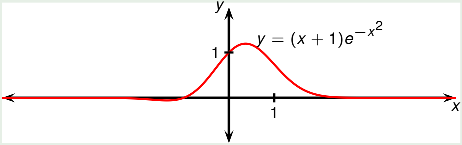

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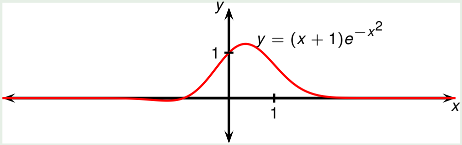

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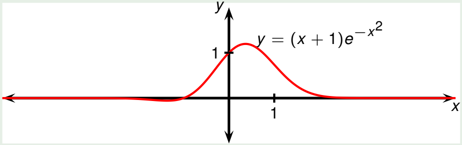

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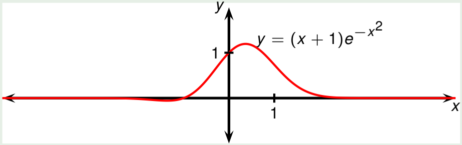

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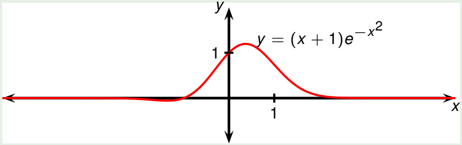

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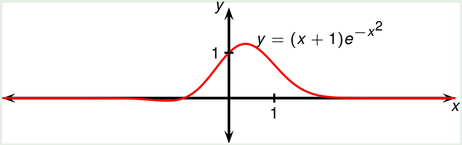

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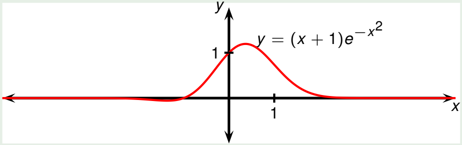

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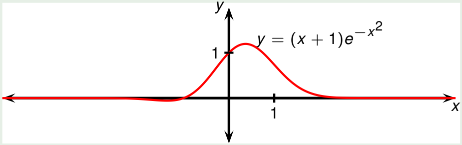

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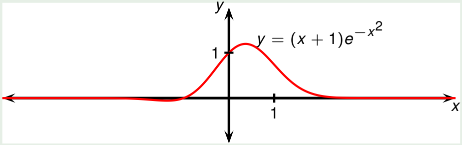

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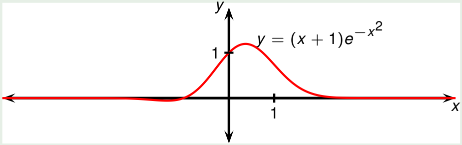

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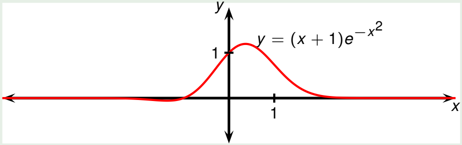

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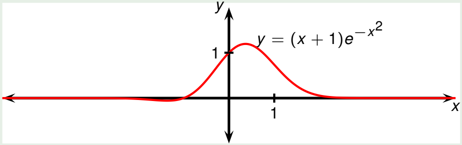

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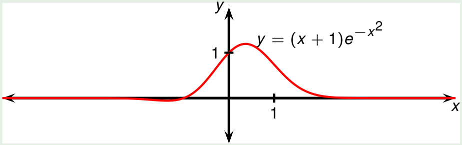

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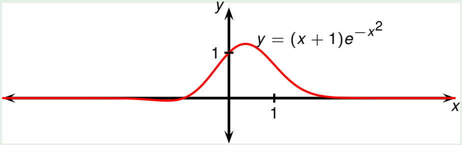

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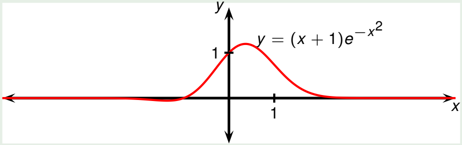

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x	$f(x)$
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5	

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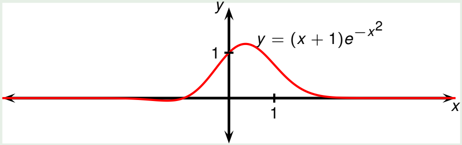

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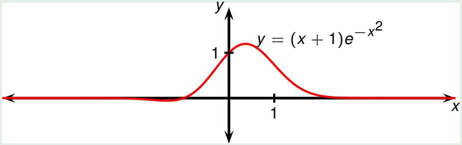

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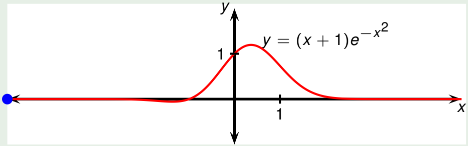

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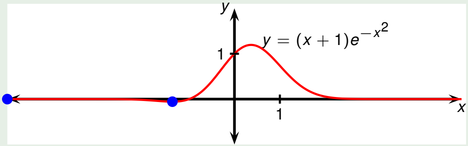

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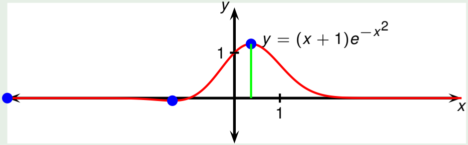

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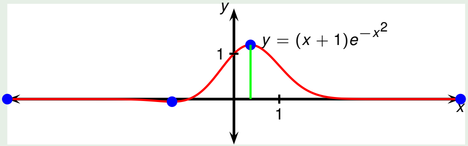

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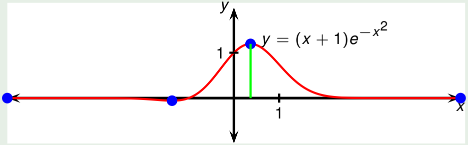

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Final answer:	$\frac{-1+\sqrt{3}}{2}$	positive, max from plot
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3. Use the closed interval method to find the maximum/minimum value of the desired quantity.

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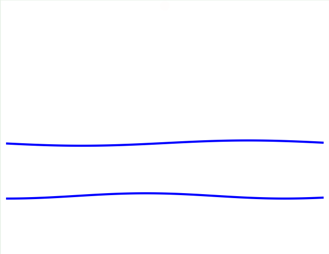
A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He doesn't need to put fencing along the river. What are the dimensions of the field with the largest area?



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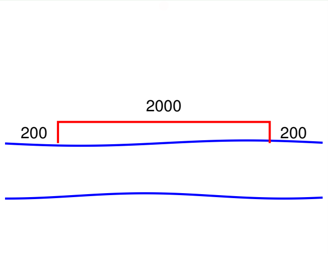

$\text{Area} = 200 \cdot 2000 = 400,000\text{ft}^2$

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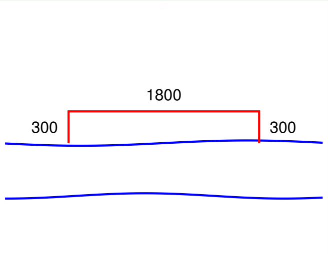

Area $= 300 \cdot 1800 = 540,000\text{ft}^2$

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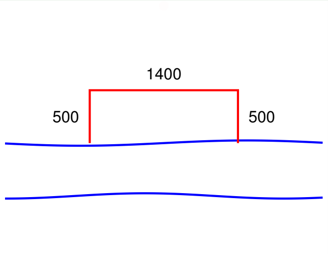

Area $= 500 \cdot 1400 = 700,000\text{ft}^2$

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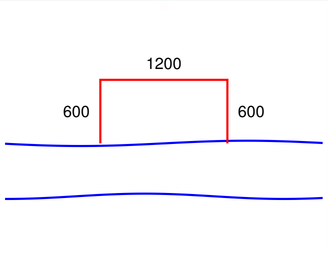

Area $= 600 \cdot 1200 = 720,000 \mathrm{ft}^2$

Example

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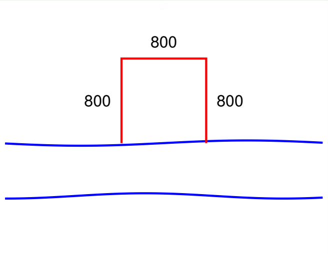

Area $= 800 \cdot 800 = 640,000\text{ft}^2$

Example

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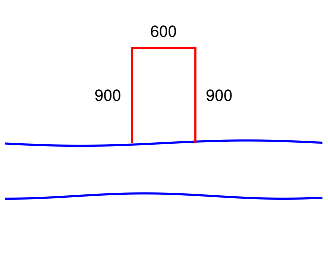

Area $= 900 \cdot 600 = 540,000\text{ft}^2$

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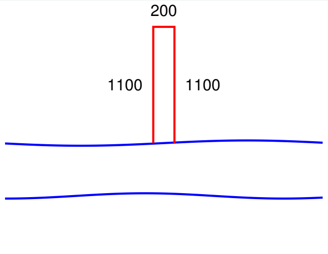

Area $= 1100 \cdot 200 = 220,000\text{ft}^2$

Example

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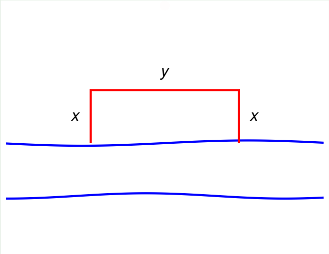

Let x and y denote the depth and width of the rectangle (in feet). Let A be its area.

Area $= A = xy$

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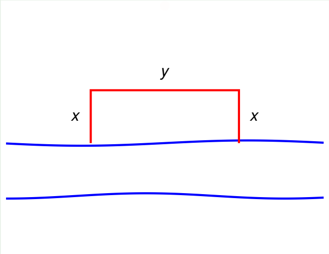

Let x and y denote the depth and width of the rectangle (in feet). Let A be its area.

$$2x + y = 2400$$

Area $= A = xy$

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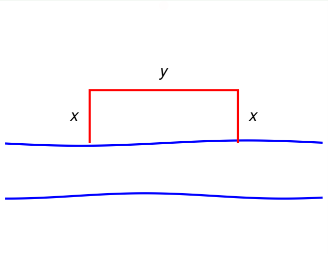

Let x and y denote the depth and width of the rectangle (in feet). Let A be its area.

$$2x + y = 2400$$
$$y = 2400 - 2x$$

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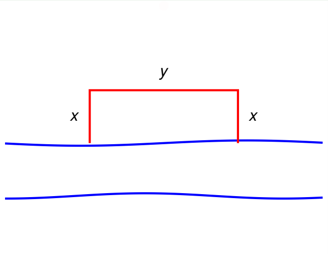

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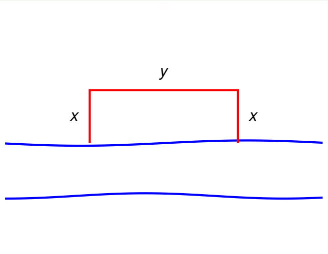

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Let x and y denote the depth and width of the rectangle (in feet). Let A be its area.

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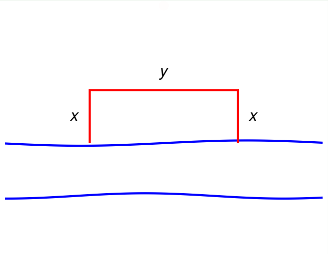

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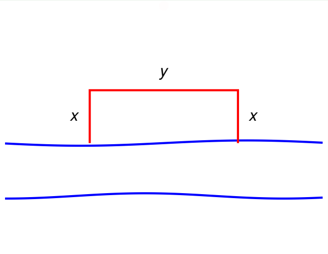

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Notice that $0 \leq x \leq 1200$.

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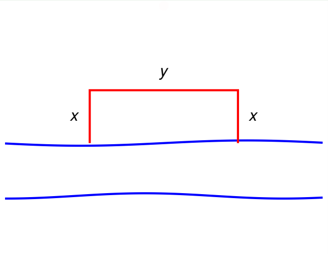

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$$A'(x) = \mathbf{?}$$

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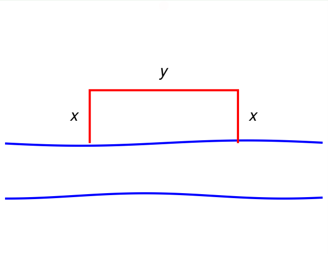

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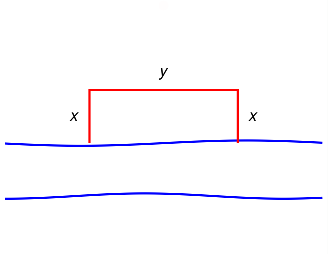

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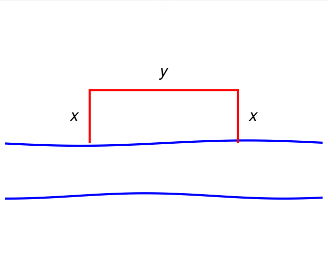

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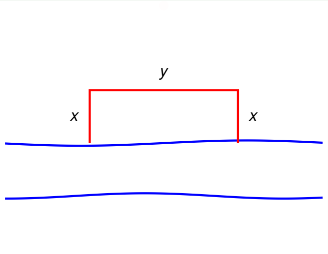

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x	$A(x)$
0	
600	
1200	

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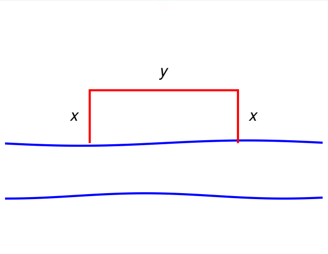

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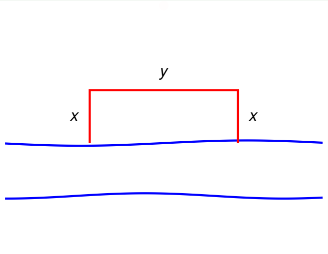

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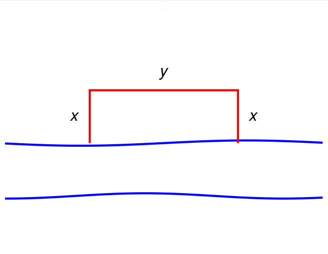

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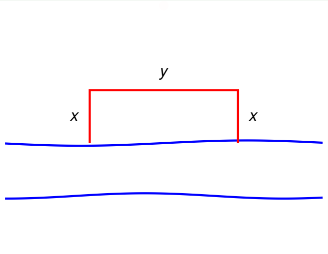

Area $= A = xy$

x	$A(x)$
0	0
600	720,000
1200	

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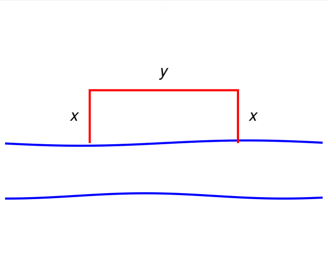

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x	$A(x)$
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600	$720,000$
1200	**?**

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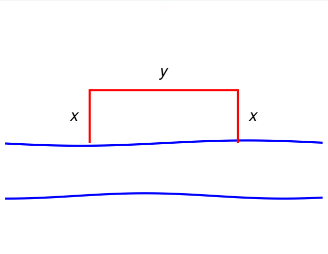

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x	$A(x)$
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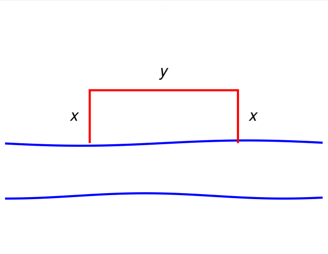

Area $= A = xy$

x	$A(x)$
0	0
600	720,000
1200	0

Let x and y denote the depth and width of the rectangle (in feet). Let A be its area.

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Notice that $0 \leq x \leq 1200$.
Maximize the function $A(x)$:

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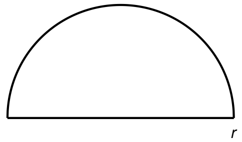
Critical number: $x = 600$.

Therefore the maximum area occurs when $x = 600$ft and $y = 1200$ft.

Example

Find the largest possible area of a rectangle inscribed in a semicircle of radius r.



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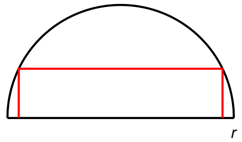

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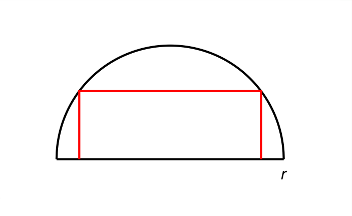

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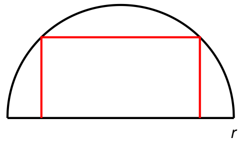

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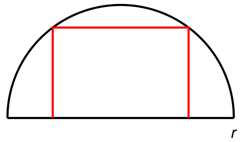

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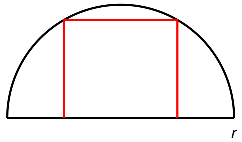

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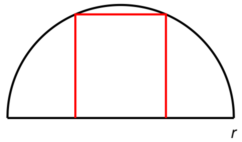

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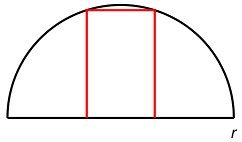

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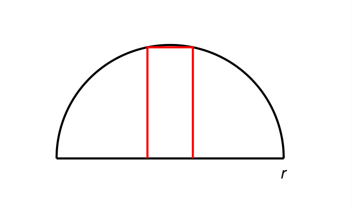

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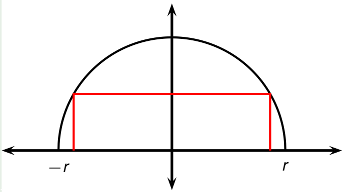

Let the semicircle have center at the origin.

Example

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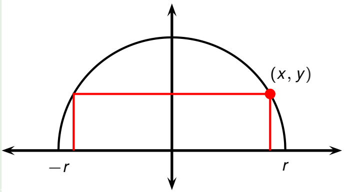

Let the semicircle have center at the origin. Let (x, y) -coord. of top right corner of rectangle.

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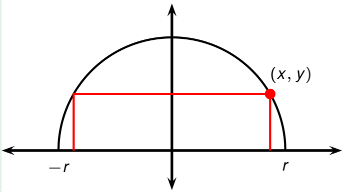
Find the largest possible area of a rectangle inscribed in a semicircle of radius r.



Let the semicircle have center at the origin. Let (x, y) -coord. of top right corner of rectangle. Let A be its area.

$A =$ base $\cdot$ height

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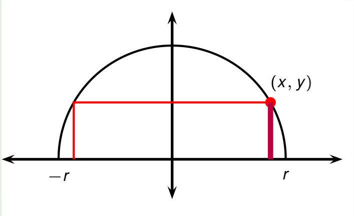

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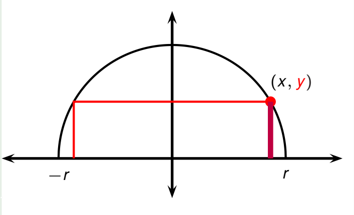
A = base $\cdot$ height

$= \mathbf{?} \quad \cdot \mathbf{?}$

Example

Find the largest possible area of a rectangle inscribed in a semicircle of radius r.



Let the semicircle have center at the origin. Let (x, y) -coord. of top right corner of rectangle. Let A be its area.

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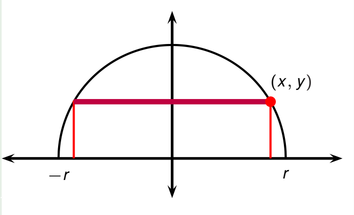

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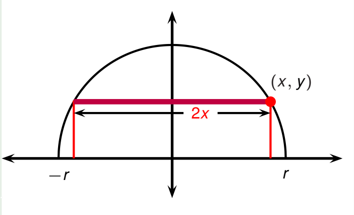

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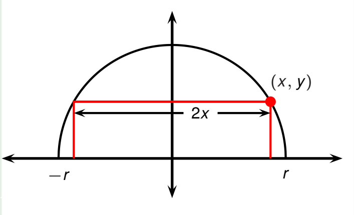

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To eliminate y, use that (x, y) lies on the semicircle.

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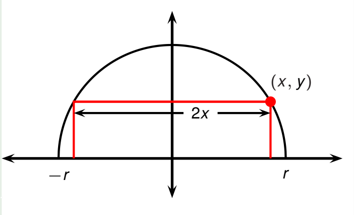

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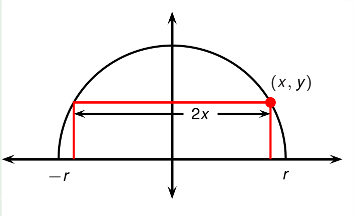

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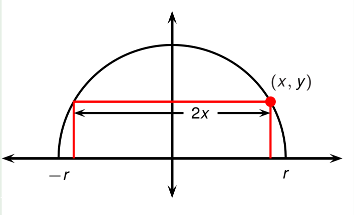

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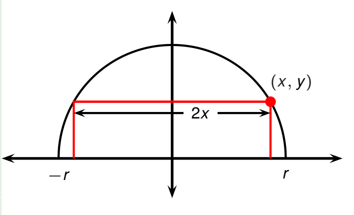

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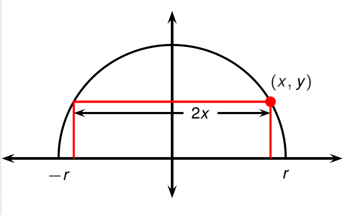

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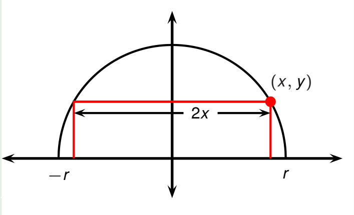

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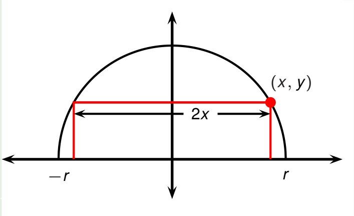

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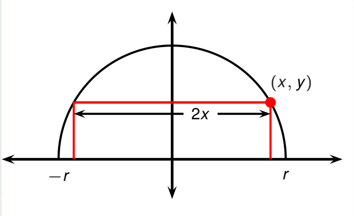

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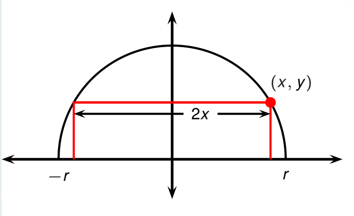

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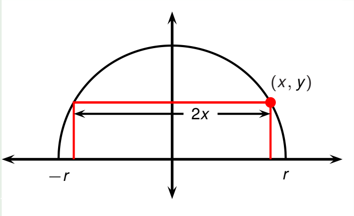

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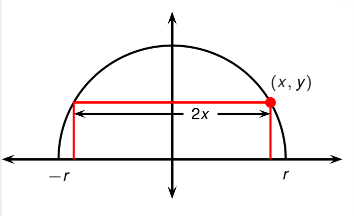

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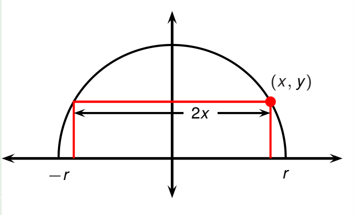

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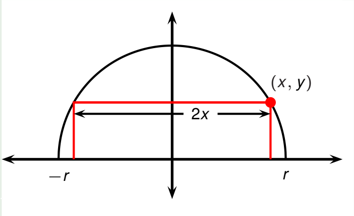

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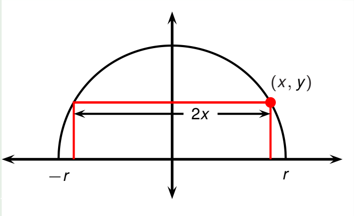

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Critical numbers: $x = \frac{r}{\sqrt{2}}$ and r.

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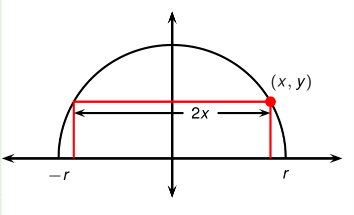

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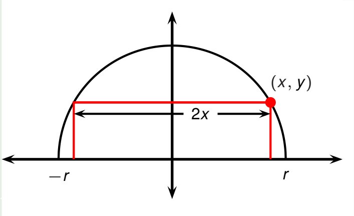

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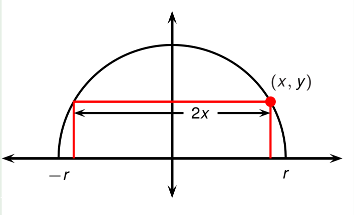

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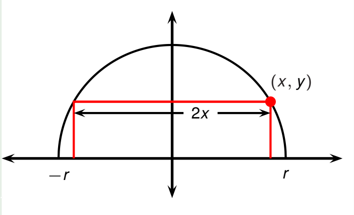

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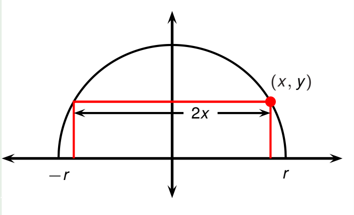

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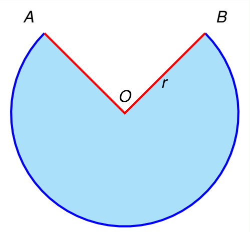
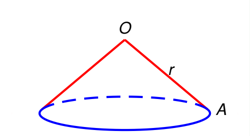
We have $0 \leq x \leq r$ and so the critical numbers together with the endpoints are $x = 0, \frac{r}{\sqrt{2}}, r$. Since $A(0) = 0 = A(r)$, the max is achieved at $x = y = \frac{r}{\sqrt{2}}$. The max area is $A(\frac{r}{\sqrt{2}}) = 2\frac{r}{\sqrt{2}}\sqrt{r^2 - \frac{r^2}{2}} = r^2$.

Example

A cone is folded from a wedge-shaped profile of radius r. Find the maximal possible volume V of such a cone.





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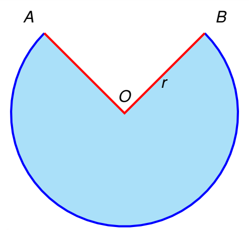

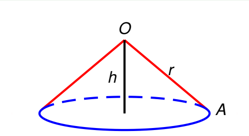

Set h - cone height,

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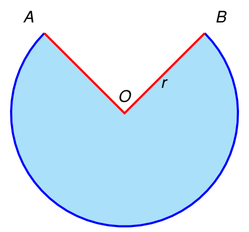

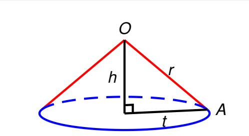

Set h - cone height, t - cone radius.

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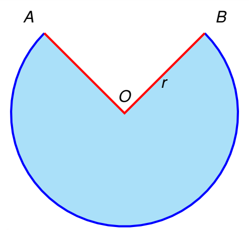

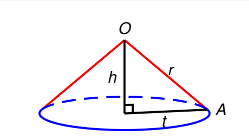

Set h - cone height, t - cone radius. Then

$V =$.

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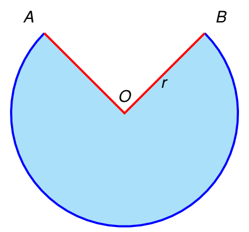

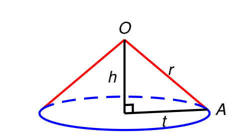

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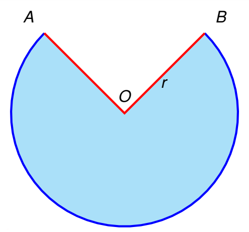

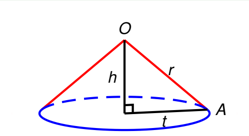

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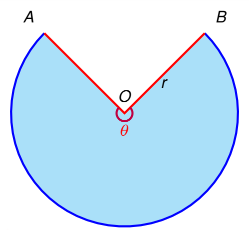

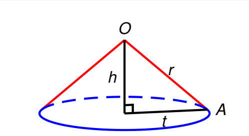

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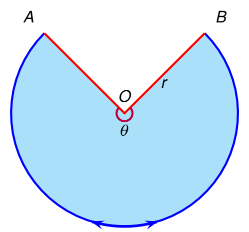

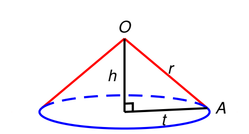

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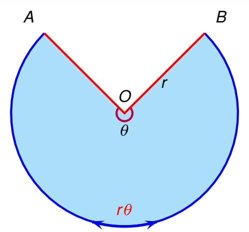

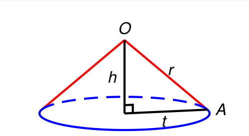

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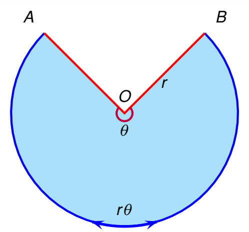

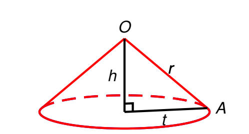

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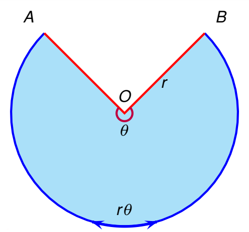

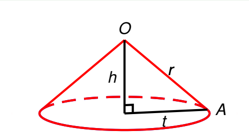

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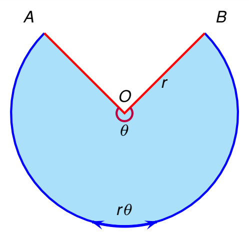

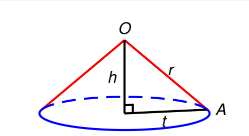

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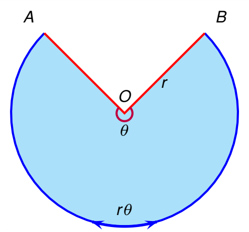

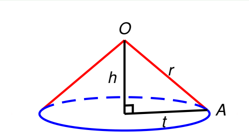

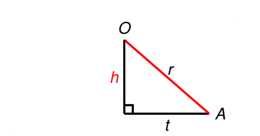

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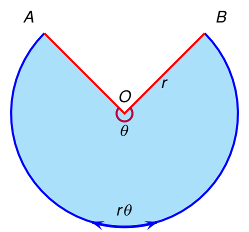

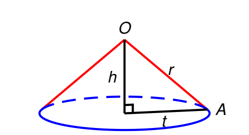

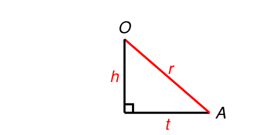

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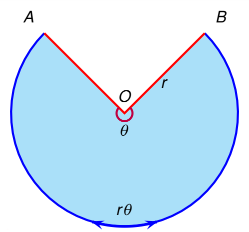

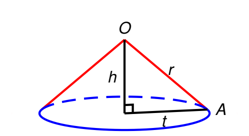

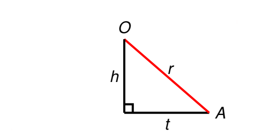

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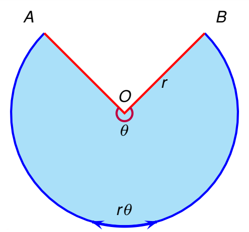

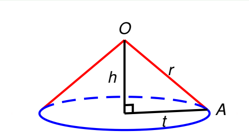

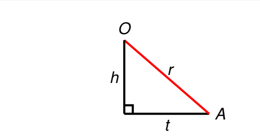

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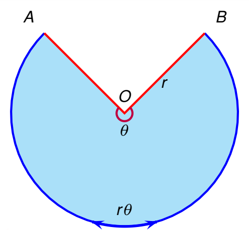

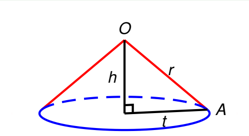

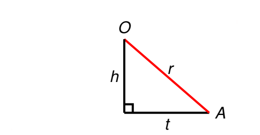

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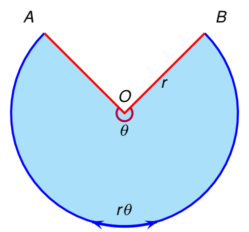

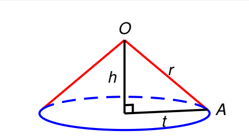

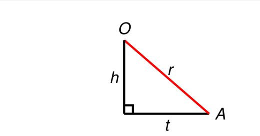

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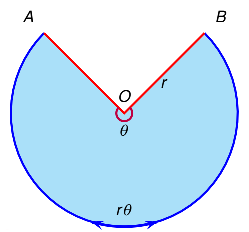

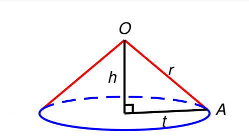

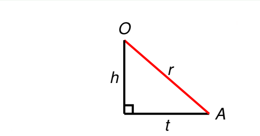

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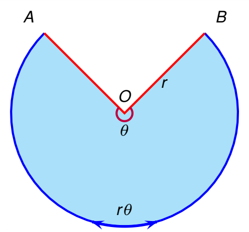

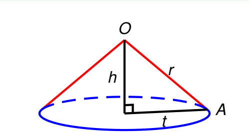

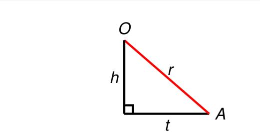

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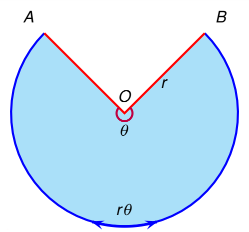

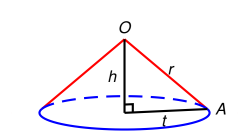

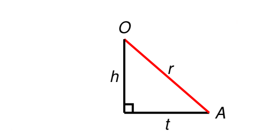

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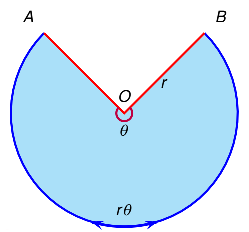

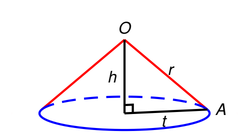

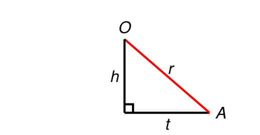

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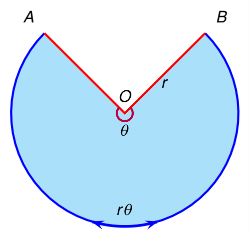

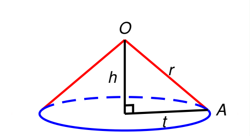

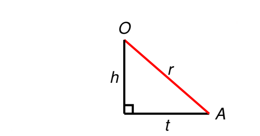

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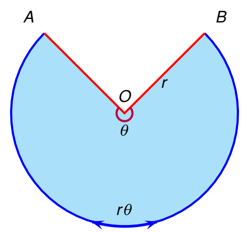

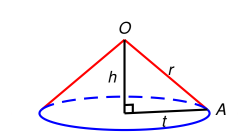

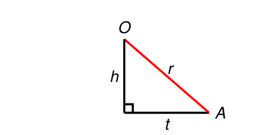

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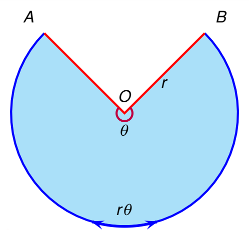
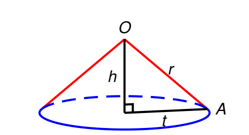
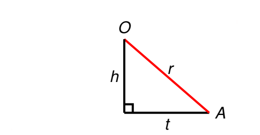
as function of θ (using the closed interval method). We need to find the critical points of V, i.e., the values of θ for which $\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0$ and the values of θ for which $\frac{\mathrm{d}V}{\mathrm{d}\theta}$ is not defined.

Example

A cone is folded from a wedge-shaped profile of radius r. Find the maximal possible volume V of such a cone.







$$V = \frac{r^3}{24\pi^2}\theta^2\sqrt{4\pi^2-\theta^2}, \qquad 0 \leq \theta \leq 2\pi$$

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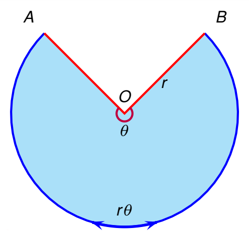

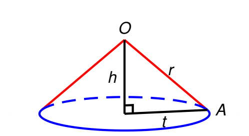

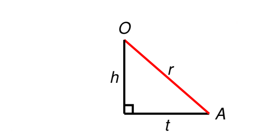

$$V = \frac{r^3}{24\pi^2}\theta^2\sqrt{4\pi^2-\theta^2}, \qquad 0 \leq \theta \leq 2\pi$$

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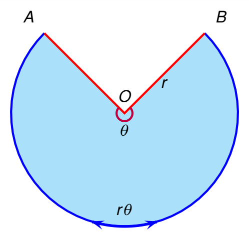

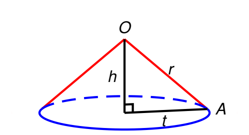

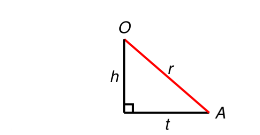

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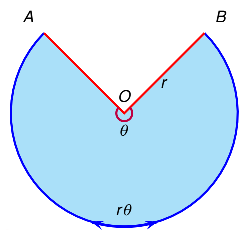

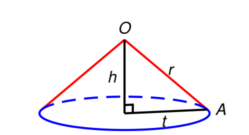

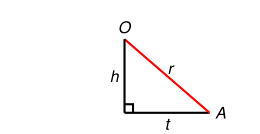

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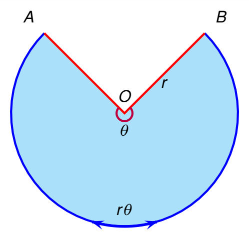

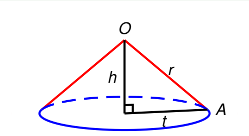

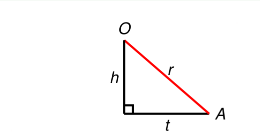

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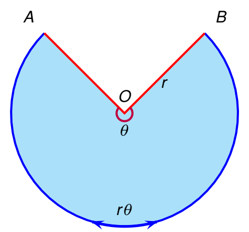

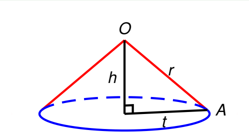

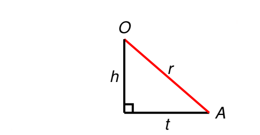

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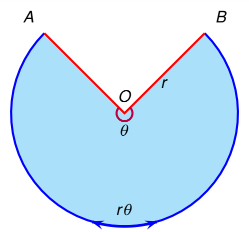

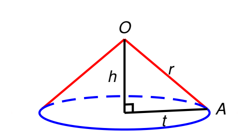

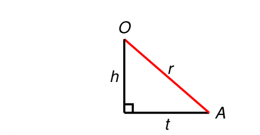

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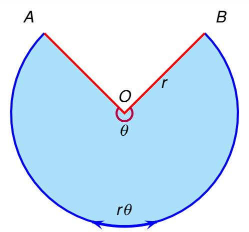

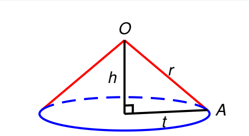

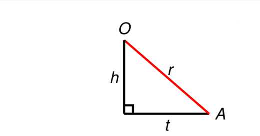

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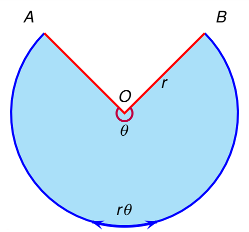

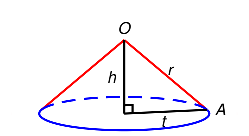

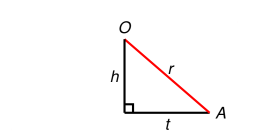

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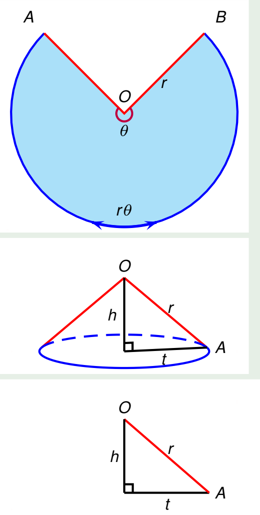

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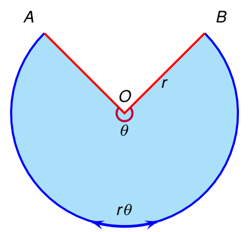

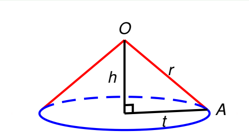

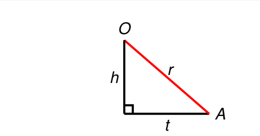

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Example

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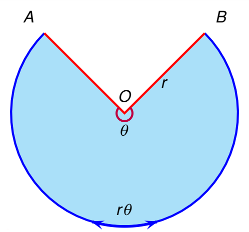

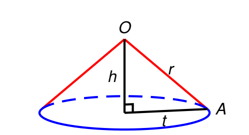

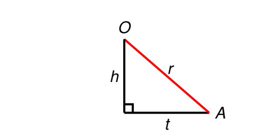

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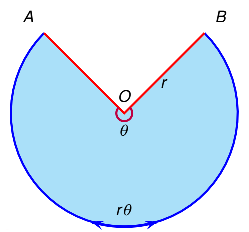

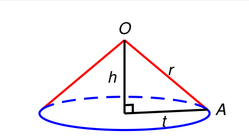

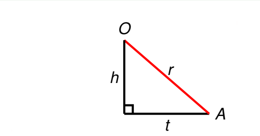

$$V = \frac{r^3}{24\pi^2}\theta^2\sqrt{4\pi^2-\theta^2}, \qquad 0 \leq \theta \leq 2\pi$$

$$\frac{dV}{d\theta} = \left(\frac{r^3}{24\pi^2}\right)\frac{8\theta\pi^2 - 3\theta^3}{\sqrt{4\pi^2-\theta^2}}$$

We have that $\frac{dV}{d\theta} = 0$ when

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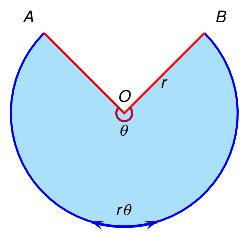

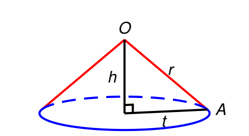

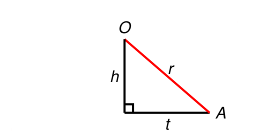

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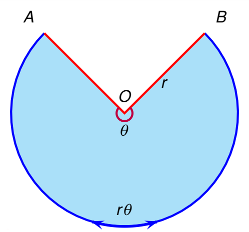

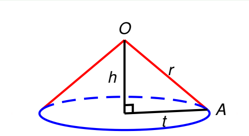

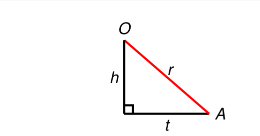

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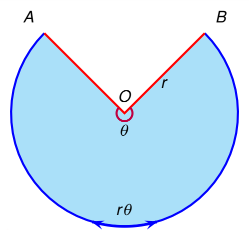

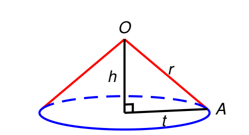

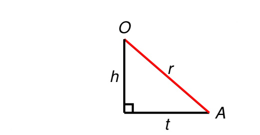

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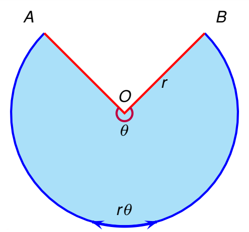

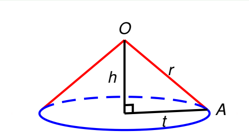

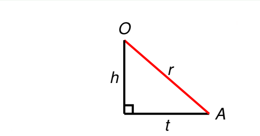

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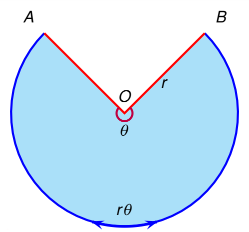

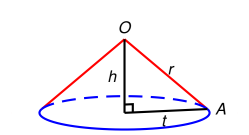

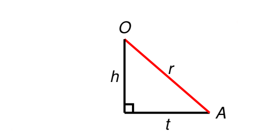

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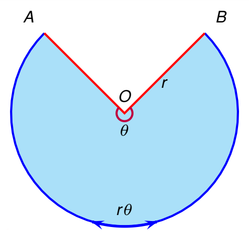

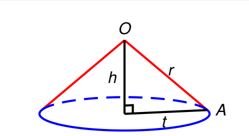

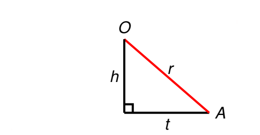

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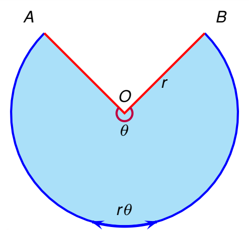

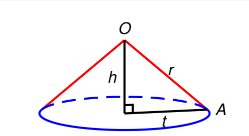

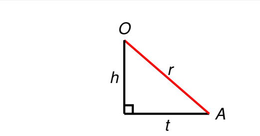

$$\begin{aligned} V &= \frac{r^3}{24\pi^2}\theta^2\sqrt{4\pi^2-\theta^2}, \qquad {\color{red}0\leq\theta\leq 2\pi}\\ \frac{\mathrm{d}V}{\mathrm{d}\theta} &= \left(\frac{r^3}{24\pi^2}\right)\frac{8\theta\pi^2-3\theta^3}{\sqrt{4\pi^2-\theta^2}}\end{aligned}$$

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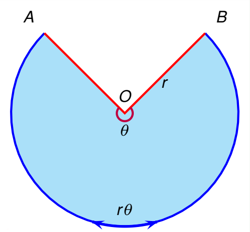

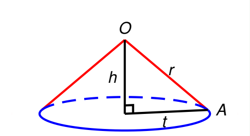

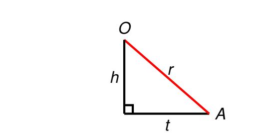

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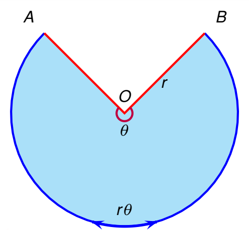

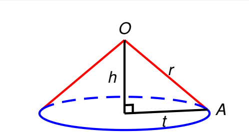

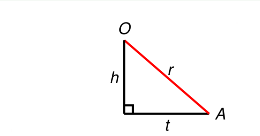

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Finally, the answer to the problem is

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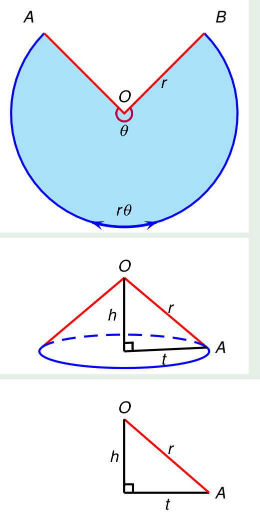

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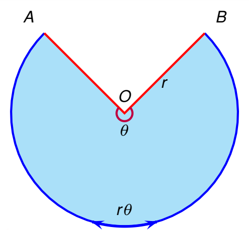

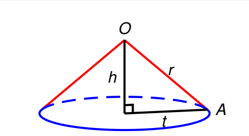

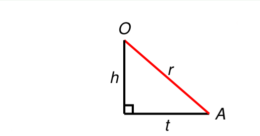

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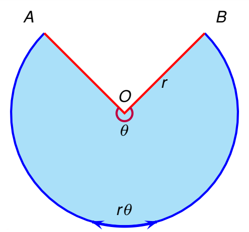

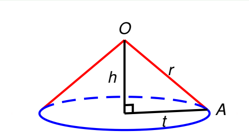

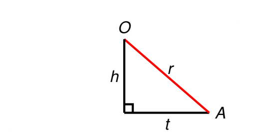

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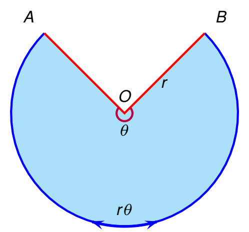

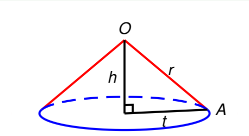

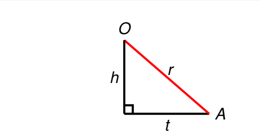

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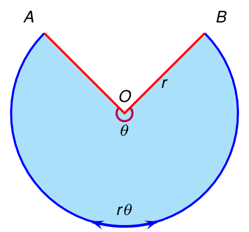

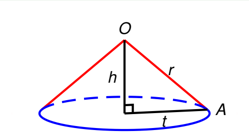

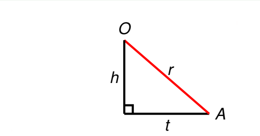

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