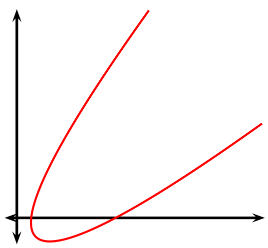
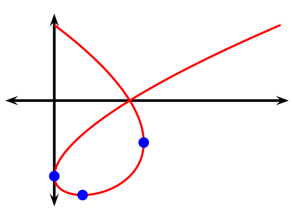
Calculus II
Homework
Tangents and curve length

1. Find the values of the parameter t for which the curve has horizontal and vertical tangents.

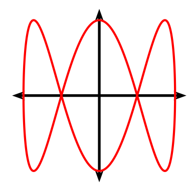
   (a) $x = t^2 - t + 1, y = t^2 + t - 1$

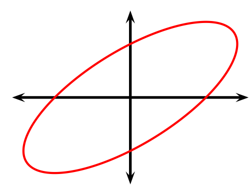
   (b) $x = t^3 - t^2 - t + 1, y = t^2 - t - 1.$

   (c) $x = \cos(t), y = \sin(3t)$

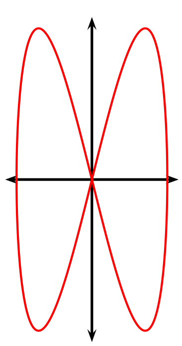
   

   (d) $x = \cos(t) + \sin(t)$, $y = \sin(t)$.

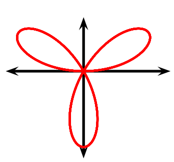
   

2. Show that the parametric curve has multiple tangents at the point and find their slopes.

   (a) $x = \cos t, y = 2\sin(2t)$, two tangents at $(x, y) = (0, 0)$.

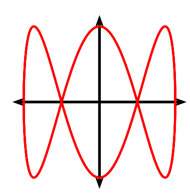
   

   (b) $x = \cos t \sin(3t)$, $y = \sin(t)\sin(3t)$, six tangents at

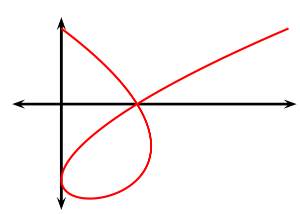
   $(x, y) = (0, 0)$.

   (c) $x = \cos t, y = \sin(3t)$, find the two points at which the curve has double tangent and find the slopes of both pairs

   of tangents.

   (d) $x = t^3 - t^2 - t + 1, y = t^2 - t - 1$, find a point where the curve has double tangent and find the slopes of the tangents.

3. Find the length of the curve.

   (a) $y = x^2, x \in [1, 2]$.

(b) $y=\sqrt{x}$, $x\in[1,2]$.

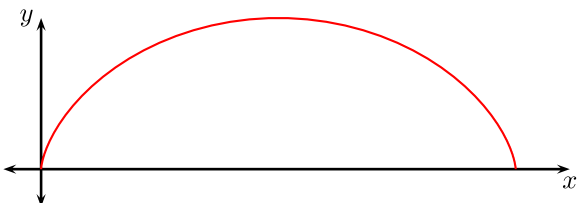
(c) $x=\sqrt{t}-2t$ and $y=\dfrac{8}{3}t^{\frac{3}{4}}$ from $t=1$ to $t=4$.

(d) $\gamma:\left|\begin{array}{lcl} x(t) & = & \frac{1}{t}+\frac{t^3}{3} \\ y(t) & = & 2t \end{array}\right., t\in[1,2]$.

(e) $\gamma:\left|\begin{array}{lcl} x(t) & = & \frac{1}{t}+t \\ y(t) & = & 2\ln t \end{array}\right., t\in[1,2]$.

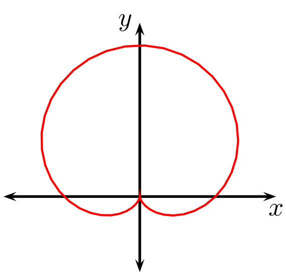
(f) One arch of the cycloid

$$\gamma:\left|\begin{array}{lcl} x(t) & = & t-\sin t \\ y(t) & = & 1-\cos t \end{array}\right., t\in[0,2\pi]$$



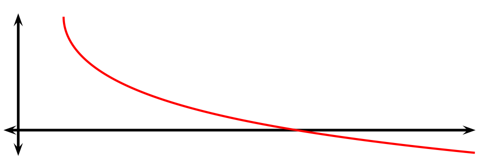
(g) The cardioid

$$\gamma:\left|\begin{array}{lcl} x(t) & = & (1+\sin t)\cos t \\ y(t) & = & (1+\sin t)\sin t \end{array}\right., t\in[0,2\pi]$$



4. Set up an integral that expresses the length of the curve and find the length of the curve.

(a) $\left|\begin{array}{lcl} x(t) & = & e^t+e^{-t} \\ y(t) & = & 5-2t \end{array}\right., t\in[0,3]$



(b) $\left|\begin{array}{lcl} x(t) & = & \sin t+\cos t \\ y(t) & = & \sin t-\cos t \end{array}\right., t\in[0,\pi]$