

Precalculus

A useful inverse hyperbolic function identity

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The inverse hyperbolic function $\operatorname{arcsinh} = \ln \left(x + \sqrt{1 + x^2} \right)$ is used when studying hyperbolas (types of curves in the plane).

Example

Demonstrate that $-\ln \left(\sqrt{1 + x^2} - x \right) = \ln \left(x + \sqrt{1 + x^2} \right)$.

$$\begin{aligned}
 -\ln \left(\sqrt{1 + x^2} - x \right) &= \ln \left(\frac{1}{\sqrt{x^2 + 1} - x} \right) && \left| \begin{array}{l} \text{rationalize} \end{array} \right. \\
 &= \ln \left(\frac{\left(\sqrt{x^2 + 1} + x \right)}{\left(\sqrt{x^2 + 1} - x \right) \left(\sqrt{x^2 + 1} + x \right)} \right) \\
 &= \ln \left(\frac{\sqrt{x^2 + 1} + x}{x^2 + 1 - x^2} \right) \\
 &= \ln \left(\sqrt{x^2 + 1} + x \right) .
 \end{aligned}$$