# Calculus I
# Homework
# Limits

1. Evaluate the limits. Justify your computations.

   (a) $\lim_{x\to 2} 2x^2 - 3x - 6.$ answer: $-4$

   (b) $\lim_{x\to -1} \dfrac{x^4 - x}{x^2 + 2x + 3}.$ answer: 1

   (c) $\lim_{x\to -1} \dfrac{1}{x^2 - 3x + 2}.$ answer: $\frac{1}{6}$

   (d) $\lim_{x\to -2} \sqrt{x^4 + 16}.$ answer: $\sqrt{32}$

   (e) $\lim_{x\to 8} (1 + \sqrt[3]{x})(2 - x).$ answer: $-18$

2. Evaluate the limit if it exists.

   (a) $\lim_{x\to 2} \dfrac{x^2 - 5x + 6}{x - 2}.$ answer: $-1$

   (b) $\lim_{x\to 3} \dfrac{x^2 - 3x}{x^2 - 2x - 3}.$ answer: $\frac{3}{4}$

   (c) $\lim_{x\to -2} \dfrac{2x^2 + x - 6}{x^2 - 4}$ answer: $\frac{7}{4}$

   (d) $\lim_{x\to 2} \dfrac{x^2 - 5x - 6}{x - 2}.$ answer: DNE

   (e) $\lim_{x\to -1} \dfrac{x^2 - 3x}{x^2 - 2x - 3}.$ answer: DNE

   (f) $\lim_{x\to -2} \dfrac{x^2 - 4}{2x^2 + 5x + 2}.$ answer: $\frac{4}{3}$

   (g) $\lim_{x\to -1} \dfrac{2x^2 + 3x + 1}{3x^2 - 2x - 5}.$ answer: $\frac{1}{8}$

   (h) $\lim_{x\to -4} \dfrac{x^2 + 7x + 12}{x^2 + 6x + 8}.$ answer: $\frac{1}{2}$

   (i) $\lim_{h\to 0} \dfrac{(-3 + h)^2 - 9}{h}.$ answer: $-6$

   (j) $\lim_{h\to 0} \dfrac{(-2 + h)^3 + 8}{h}.$ answer: 12

   (k) $\lim_{x\to -3} \dfrac{x + 3}{x^3 + 27}.$ answer: $\frac{1}{27}$

   (l) $\lim_{x\to 1} \dfrac{x^4 - 1}{x^3 - 1}.$ answer: $\frac{4}{3}$

   (m) $\lim_{h\to 0} \dfrac{\sqrt{4 + h} - 2}{h}.$ answer: $\frac{1}{4}$

   (n) $\lim_{x\to 3} \dfrac{\sqrt{5x + 1} - 4}{x - 3}.$ answer: $\frac{5}{8}$

   (o) $\lim_{x\to -3} \dfrac{\sqrt{x^2 + 16} - 5}{x + 3}.$ answer: $-\frac{3}{5}$

   (p) $\lim_{x\to -3} \dfrac{\frac{1}{3} + \frac{1}{x}}{3 + x}.$ answer: $-\frac{1}{9}$

   (q) $\lim_{x\to -2} \dfrac{x^2 + 4x + 4}{x^4 - 16}.$ answer: 0

   (r) $\lim_{x\to 0} \dfrac{\sqrt{1 + x} - \sqrt{1 - x}}{x}.$ answer: 1

   (s) $\lim_{x\to 0} \left(\dfrac{1}{x} - \dfrac{1}{x^2 + x}\right).$ answer: 1

   (t) $\lim_{x\to 9} \dfrac{3 - \sqrt{x}}{9x - x^2}.$ answer: $\frac{1}{54}$

   (u) $\lim_{h\to 0} \dfrac{(2 + h)^{-1} - 2^{-1}}{h}.$ answer: $-\frac{1}{4}$

   (v) $\lim_{x\to 0} \left(\dfrac{1}{x\sqrt{1 + x}} - \dfrac{1}{x}\right).$ answer: $-\frac{1}{2}$

   (w) $\lim_{h\to 0} \dfrac{(x + h)^3 - x^3}{h}.$ answer: $3x^2$

   (x) $\lim_{h\to 0} \dfrac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}.$ answer: $-\frac{2}{x^3}$

   (y) $\lim_{h\to 0} \dfrac{\frac{1}{(2+h)^2} - \frac{1}{4}}{h}.$ answer: $-\frac{1}{4}$

(z) $\displaystyle\lim_{h\to 0}\frac{\frac{1}{(1+h)^2}-1}{h}$.

answer: $-2$

**Solution.** 2.a

$$\begin{array}{rcll}\displaystyle\lim_{x\to 2}\frac{x^2-5x+6}{x-2} &=& \displaystyle\lim_{x\to 2}\frac{(x-3)\cancel{(x-2)}}{\cancel{x-2}} & \Big|\ \text{factor and cancel}\\ &=& 2-3=-1 & \end{array}$$

**Solution.** 2.c

$$\begin{array}{rcll}\displaystyle\lim_{x\to -2}\frac{2x^2+x-6}{x^2-4} &=& \displaystyle\lim_{x\to -2}\frac{(2x-3)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}} & \Big|\ \text{factor and cancel}\\ &=& \displaystyle\frac{(2(-2)-3)}{-2-2} & \Big|\ \text{substitute}\\ &=& \displaystyle\frac{7}{4} & \end{array}$$

**Solution.** 2.f

$$\begin{array}{rcll}\displaystyle\lim_{x\to 2}\frac{x^2-4}{2x^2+5x+2} &=& \displaystyle\lim_{x\to -2}\frac{(x-2)\cancel{(x+2)}}{(2x+1)\cancel{(x+2)}} & \Big|\ \text{factor and cancel}\\ &=& \displaystyle\frac{(-2)-2}{2(-2)+1}=\frac{4}{3}. & \end{array}$$

**Solution.** 2.g

$$\begin{array}{rcll}\displaystyle\lim_{x\to -1}\frac{2x^2+3x+1}{3x^2-2x-5} &=& \displaystyle\lim_{x\to -1}\frac{(2x+1)\cancel{(x+1)}}{(3x-5)\cancel{(x+1)}} & \Big|\ \text{factor and cancel}\\ &=& \displaystyle\frac{2(-1)+1}{3(-1)-5}=\frac{1}{8}. & \end{array}$$

**Solution.** 2.h.

$$\begin{array}{rcll}\displaystyle\lim_{x\to -4}\frac{x^2+7x+12}{x^2+6x+8} &=& \displaystyle\lim_{x\to -4}\frac{(x+3)\cancel{(x+4)}}{(x+2)\cancel{(x+4)}} & \Big|\ \text{factor}\\ &=& \displaystyle\frac{-4+3}{-4+2}=-\frac{1}{2}. & \end{array}$$

**Solution.** 2.x

$$\begin{array}{rcl}\displaystyle\lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} &=& \displaystyle\lim_{h\to 0}\frac{x^2-(x+h)^2}{hx^2(x+h)^2}=\lim_{h\to 0}\frac{x^2-(x^2+2xh+h^2)}{hx^2(x+h)^2}\\ &=& \displaystyle\lim_{h\to 0}\frac{\cancel{h}(-2x+h)}{\cancel{h}x^2(x+h)^2}=\frac{-2x+0}{x^2(x+0)^2}=-\frac{2}{x^3}. \end{array}$$

**Solution.** 2.y.

**Variant I.**

$$\begin{array}{rcll}\displaystyle\lim_{h\to 0}\frac{\frac{1}{(2+h)^2}-\frac{1}{4}}{h} &=& \displaystyle\lim_{h\to 0}\frac{\frac{4-(2+h)^2}{4(2+h)^2}}{h} & \\ &=& \displaystyle\lim_{h\to 0}\frac{4-(4+4h+h^2)}{4h(2+h)^2} & \\ &=& \displaystyle\lim_{h\to 0}\frac{-4h-h^2}{4h(2+h)^2} & \\ &=& \displaystyle\lim_{h\to 0}\frac{\cancel{h}(-4-h)}{4\cancel{h}(2+h)^2} & \Big|\ \text{substitute } h=0\\ &=& \displaystyle\frac{-4-0}{4(2+0)^2} & \\ &=& \displaystyle-\frac{1}{4} & \end{array}$$

**Variant II.**

$$\lim_{h\to 0}\frac{\frac{1}{(2+h)^2}-\frac{1}{4}}{h} = \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{x^2}\right)_{|x=2} = \left(\frac{-2}{x^3}\right)_{|x=2} = -\frac{1}{4}$$

**Solution.** 2.z.

**Variant I.**

$$\begin{aligned}\lim_{h\to 0}\frac{\frac{1}{(1+h)^2}-1}{h} &= \lim_{h\to 0}\frac{\frac{1-(1+h)^2}{(1+h)^2}}{h}\\ &= \lim_{h\to 0}\frac{1-(1+2h+h^2)}{h(1+h)^2}\\ &= \lim_{h\to 0}\frac{-2h-h^2}{h(1+h)^2}\\ &= \lim_{h\to 0}\frac{\cancel{h}(-2-h)}{\cancel{h}(1+h)^2} && \Big|\ \text{substitute } h=0\\ &= \frac{-2-0}{(1+0)^2}\\ &= -2.\end{aligned}$$

**Variant II.**

$$\begin{aligned}\lim_{h\to 0}\frac{\frac{1}{(1+h)^2}-1}{h} &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{x^2}\right)_{|x=1} && \Big|\ \text{derivative definition}\\ &= \left(\frac{-2}{x^3}\right)_{|x=1}\\ &= -2.\end{aligned}$$