Calculus II

Simplify sin(k arcsin x), cos(k arcsin x), sin(k arccos x), cos(k arccos x)

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2019

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

sin(2 arccos(x))

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$.

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sin(2 arccos(x))

$$sin(2 \frac{arccos(x)}{arccos(x)}) = sin(2y)$$

Set
$$y = \arccos x$$

$$\sin(2\arccos(x)) = \frac{\sin(2y)}{2}$$
= ?

Set
$$y = \arccos x$$

Express via $\sin y, \cos y$

$$sin(2 arccos(x)) = sin(2y)$$

$$= 2 cos y sin y$$

Set
$$y = \arccos x$$

Express via $\sin y, \cos y$

$$\sin(2\arccos(x)) = \sin(2y)$$

$$= 2\cos y \sin y$$

$$= 2\cos y \left(\pm\sqrt{1-\cos^2 y}\right)$$
Set $y = \arccos x$
Express via $\sin y$, $\cos y$
Express $\sin y$ via $\cos y$

$$\sin(2 \arccos(x)) = \sin(2y)$$

$$= 2 \cos y \sin y$$

$$= 2 \cos y \left(\pm \sqrt{1 - \cos^2 y}\right)$$

$$= 2 \cos y \sqrt{1 - \cos^2 y}$$

Set
$$y = \arccos x$$

Express via $\sin y$, $\cos y$
Express $\sin y$ via $\cos y$
 $\sin y > 0$ because $0 < y < \pi$

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$$= 2 \cos y \left(\pm \sqrt{1 - \cos^2 y}\right)$$

$$= 2 \cos y \sqrt{1 - \cos^2 y}$$

$$= 2x\sqrt{1 - x^2}$$

Set
$$y = \arccos x$$

Express via $\sin y$, $\cos y$
Express $\sin y$ via $\cos y$
 $\sin y > 0$ because
 $0 \le y \le \pi$
use $x = \cos y$

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

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$$y = \arccos x$$

Express via $\sin y$, $\cos y$
Express $\sin y$ via $\cos y$
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Rewrite $cos(3 \operatorname{arccos}(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

cos(3 arccos(x))

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cos(3 arccos(x))

$$cos(3 \operatorname{arccos}(x)) = cos(3y)$$

$$y = \arccos x$$

$$\cos(3\arccos(x)) = \cos(3y) = \cos(2y + y)$$
 | $y = \arccos x$

$$cos(3 \operatorname{arccos}(x)) = cos(3y) = \frac{cos(2y + y)}{=?}$$
 | $y = \operatorname{arccos} x$ | Angle sum f-la

$$cos(3 \operatorname{arccos}(x)) = cos(3y) = \frac{cos(2y + y)}{= cos(2y) \cos y - \sin(2y) \sin y}$$
 $y = \operatorname{arccos} x$
Angle sum f-la

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\cos(3\arccos(x)) = \cos(3y) = \cos(2y + y)$$

$$= \cos(2y)\cos y - \sin(2y)\sin y$$

$$= (?) \cos y$$

$$-? \sin y$$

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$$= (\cos^2 y - \sin^2 y)\cos y$$

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$$- 2\sin y\cos y\sin y$$

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$$= (\cos^2 y - \sin^2 y)\cos y$$

$$- 2\sin y\cos y\sin y$$

$$= \cos^3 y - \sin^2 y\cos y - 2\sin^2 y\cos y$$

$$= \cos^3 y - \sin^2 y\cos y - 2\sin^2 y\cos y$$

$$y = \arccos x$$
Angle sum f-la
Express via
$$\sin y, \cos y$$

$$\cos(3\arccos(x)) = \cos(3y) = \cos(2y + y)$$

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$$= \cos^3 y - 3\sin^2 y\cos y$$

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$$= \cos^3 y - \sin^2 y\cos y - 2\sin^2 y\cos y$$

$$= \cos^3 y - 3\sin^2 y\cos y$$

$$= \cos^3 y - 3\sin^2 y\cos y$$

$$= \cos^3 y - 3(?)$$

$$\cos y$$

$$y = \arccos x$$
Angle sum f-late Express via
$$\sin y, \cos y$$

$$= \cos y$$
Express $\sin y$ via $\cos y$

Angle sum f-la

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the cos function.

$$\cos(3\arccos(x)) = \cos(3y) = \cos(2y + y)$$

$$= \cos(2y)\cos y - \sin(2y)\sin y$$

$$= (\cos^2 y - \sin^2 y)\cos y$$

$$- 2\sin y\cos y\sin y$$

$$= \cos^3 y - \sin^2 y\cos y - 2\sin^2 y\cos y$$

$$= \cos^3 y - 3\sin^2 y\cos y$$

$$= \cos^3 y - 3(1 - \cos^2 y)\cos y$$

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Angle sum f-la

$$\cos(3\arccos(x)) = \cos(3y) = \cos(2y + y)$$

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$$= \cos^3 y - 3(1 - \cos^2 y)\cos y$$

$$= 4\cos^3 y - 3\cos y$$

$$= \cos^3 y - 3\cos y$$

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$$\cos(3\arccos(x)) = \cos(3y) = \cos(2y + y)$$

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$$y = \arccos x$$
Angle sum f-la
Express via
$$\sin y, \cos y$$
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$$\cos(3\arccos(x)) = \cos(3y) = \cos(2y + y)$$

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Angle sum f-la
$$= (\cos^3 y - \sin^2 y)\cos y$$

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$$= \cos^3 y - 3\sin^2 y\cos y$$

$$= \cos^3 y - 3\cos^2 y\cos y$$

$$= 4\cos^3 y - 3\cos y$$

$$= 4x^3 - 3x$$

$$x = \cos y$$

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$$= \cos(2y)\cos y - \sin(2y)\sin y$$

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$$= (\cos^3 y - \sin^2 y)\cos y$$

$$= \cos^3 y - \sin^2 y\cos y - 2\sin^2 y\cos y$$

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