Precalculus
§ Text problems leading to polynomial systems

Todor Milev

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Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$
\begin{aligned}
x + y &= 25 && \text{|Solve for } y \\
y &= 25 - x \\
x^2 + y^2 &= 313 \\
x^2 + (25 - x)^2 &= 313 \\
x^2 + \left(25^2 - 2 \cdot 25 \cdot x + x^2\right) - 313 &= 0 && \left|(a-b)^2 = a^2 - 2ab + b^2\right. \\
2x^2 - 50x + 625 - 313 &= 0 \\
2x^2 - 50x + 312 &= 0 && \text{|Divide by 2} \\
x^2 - 25x + 156 &= 0
\end{aligned}
$$

$$
\begin{aligned}
x &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1} \\
&= \frac{25 \pm \sqrt{625 - 624}}{2} \\
&= \frac{25 \pm 1}{2} = \begin{cases} \frac{25+1}{2} = 13 \\ \frac{25-1}{2} = 12 \end{cases}
\end{aligned}
$$

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&= \frac{25 \pm 1}{2} = \begin{cases} \frac{25+1}{2} = 13 \\ \frac{25-1}{2} = 12 \end{cases} \\
y = 25 - x &= \begin{cases} 25 - 13 = 12 \\ 25 - 12 = 13 \end{cases}
\end{aligned}
$$

The two solution candidates are $x = 12, y = 13$ and $x = 13, y = 12$. Since $y \geq x$, one of the solutions needs to be discarded and our final answer is $x = 12, y = 13$.