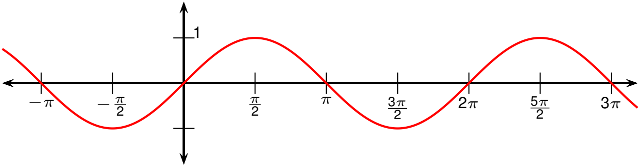
Precalculus
Graphs of trig functions; inverse trig

Todor Milev

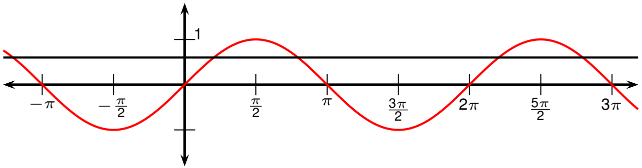
2019

Outline

Inverse Trigonometric Functions



Inverse Trigonometric Functions



- $\sin x$ isn't one-to-one.

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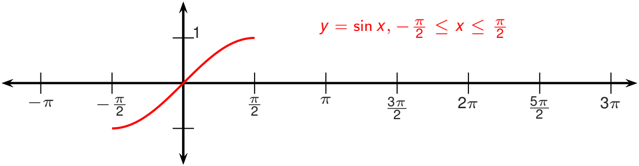

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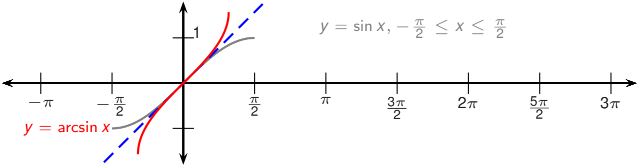

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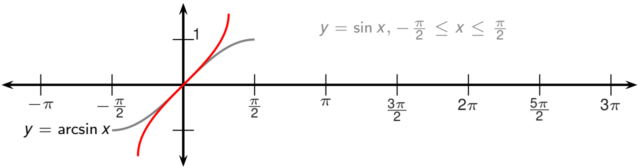

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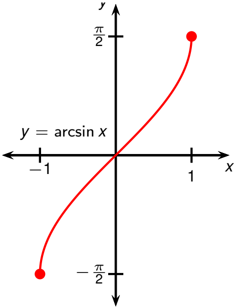

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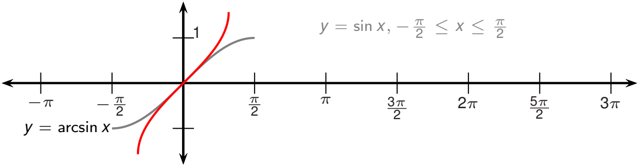

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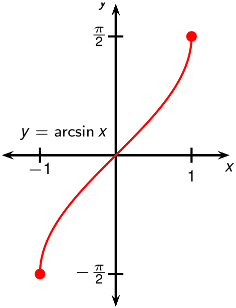
- It is if we restrict the domain to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
- Then it has an inverse function.
- We call it arcsin or $\sin^{-1}$.
- $\arcsin x = y \Leftrightarrow \sin y = x$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.



Example

Find $\arcsin\left(\frac{1}{2}\right)$.

Observation

- $\arcsin y =$ *the appropriate angle whose sine equals* y.

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- $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}.$

Observation

- $\arcsin y =$ *the appropriate angle whose sine equals* y.
- *Important: the output angle must lie in the interval* $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Example

$$\text{Find } \arcsin\left(\frac{1}{2}\right).$$

- $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.
- $-\frac{\pi}{2} \leq \frac{\pi}{6} \leq \frac{\pi}{2}$.

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- $\arcsin y =$ *the appropriate angle whose sine equals* y*.*
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Example

$$\text{Find } \arcsin\left(\frac{1}{2}\right).$$

- $\sin\left(\frac{\pi}{6}\right) = \dfrac{1}{2}$.
- $-\dfrac{\pi}{2} \leq \dfrac{\pi}{6} \leq \dfrac{\pi}{2}$.
- Therefore $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

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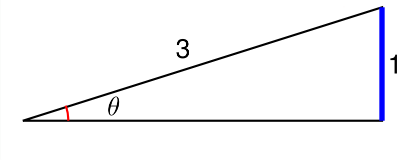

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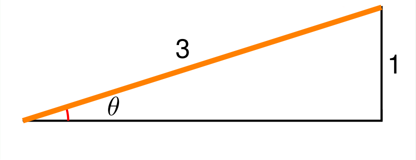

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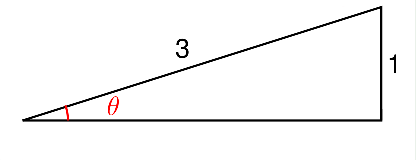

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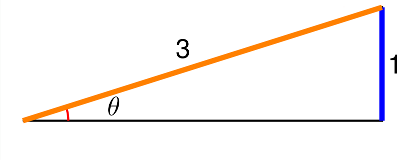

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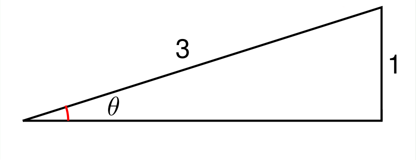

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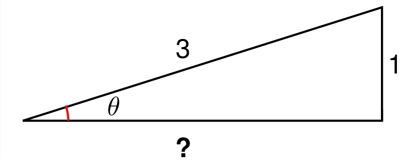
- Length of adjacent side $=$ **?**



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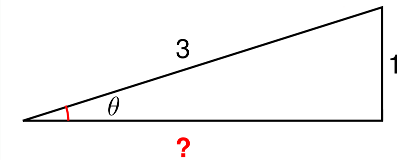
- Length of adjacent side $= \sqrt{3^2 - 1^2}$



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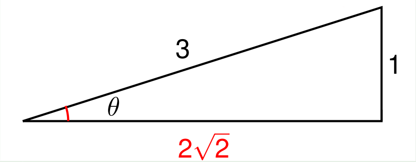
- Length of adjacent side $= \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$.



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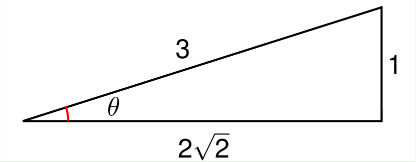

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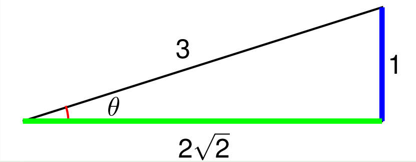
- Length of adjacent side $= \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$.
- Then $\tan\left(\arcsin\left(\frac{1}{3}\right)\right) = \frac{1}{2\sqrt{2}}$.



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- Therefore $\arcsin(\sin 1.5) = 1.5$.

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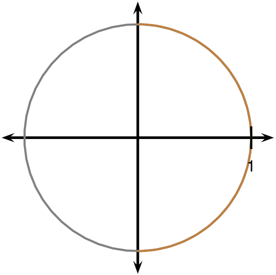
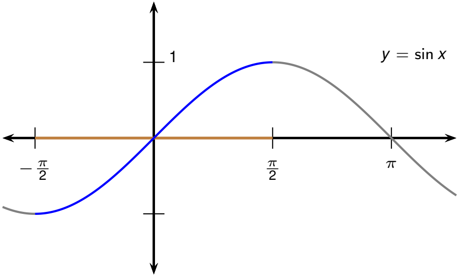
Find $\arcsin(\sin 2)$.





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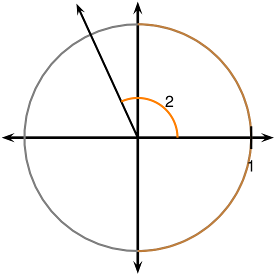

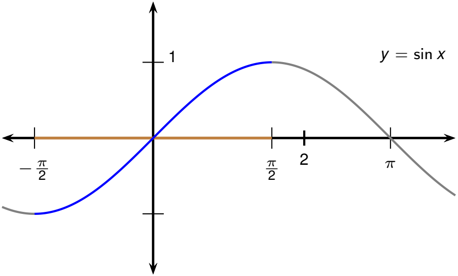

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- 2 is not between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.
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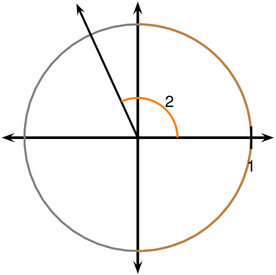

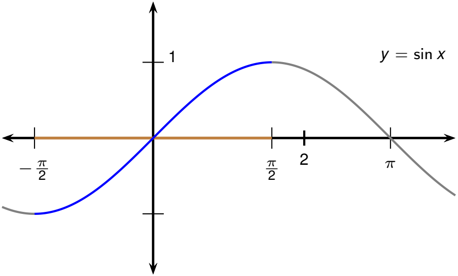

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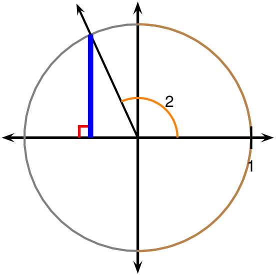

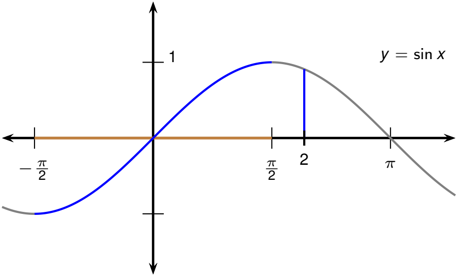

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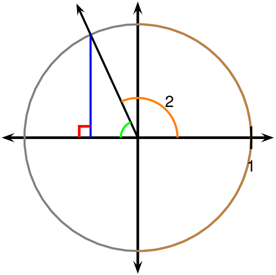

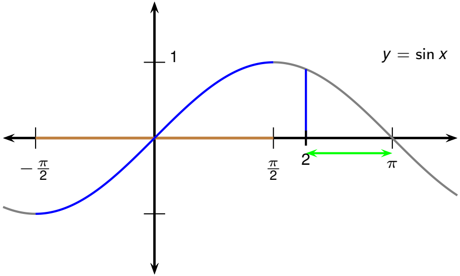

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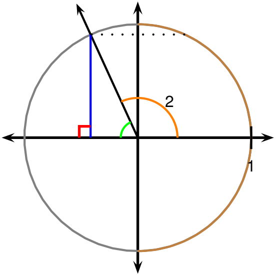

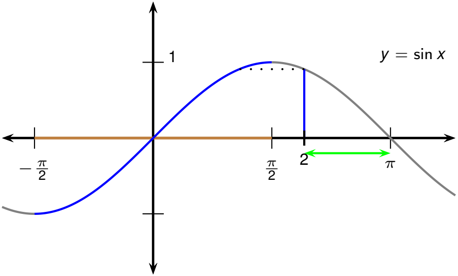

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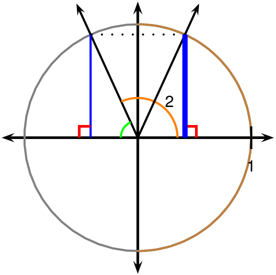

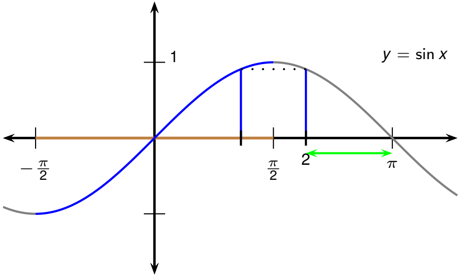

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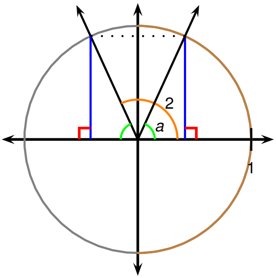

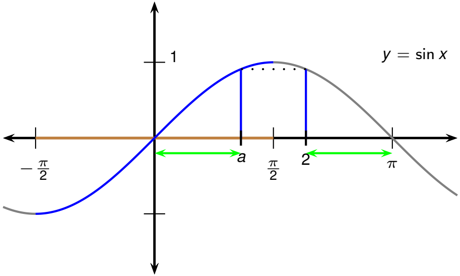

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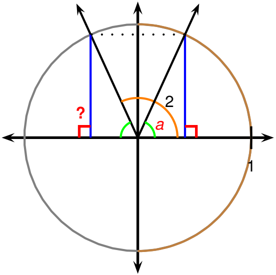
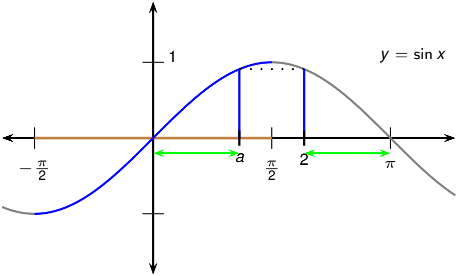
$$a = ?$$





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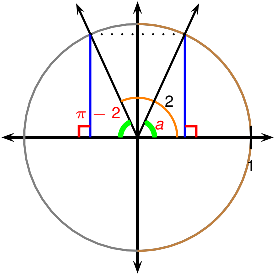

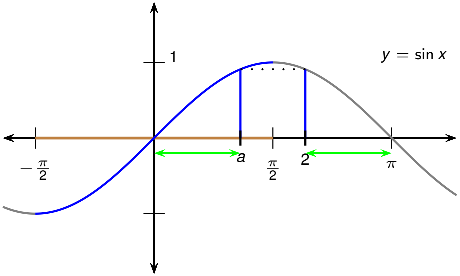

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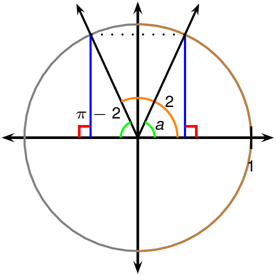
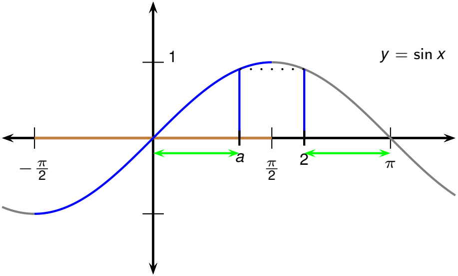
$$\begin{array}{rrcl} & a & = & \pi - 2. \\ \text{Therefore} & \arcsin(\sin 2) & = & \arcsin(\sin a) \end{array}$$





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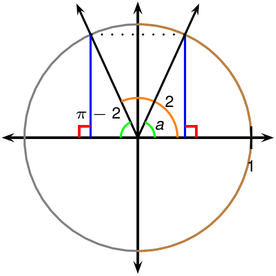

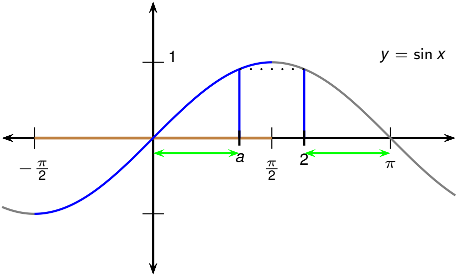

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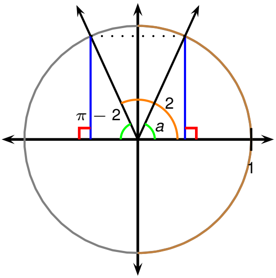
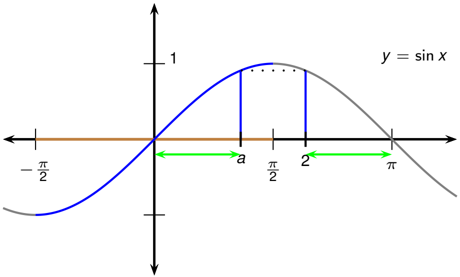
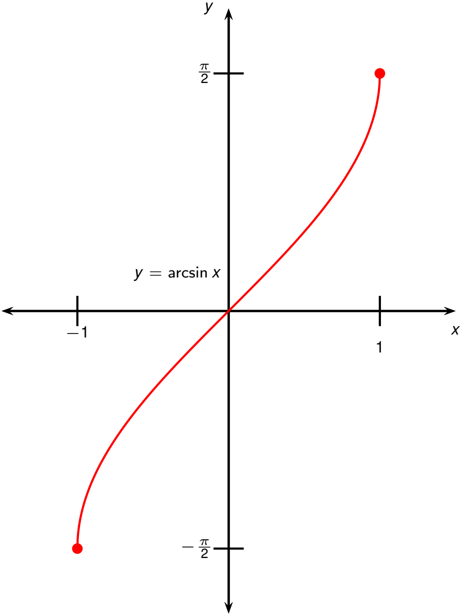


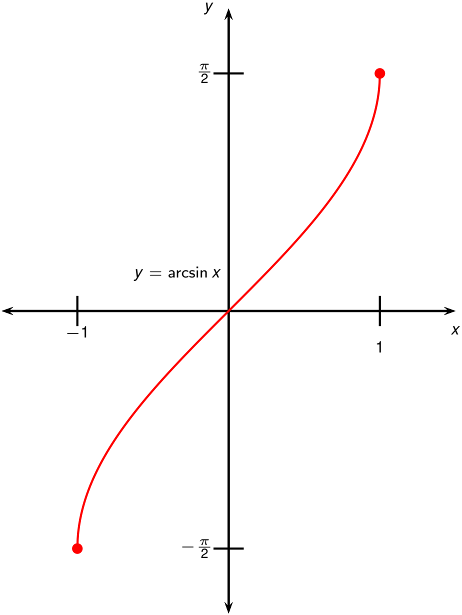


Important facts about arcsin:



1. Domain: **?**
2. Range: **?**
3. $\arcsin x = y \Leftrightarrow \sin y = x$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.
4. $\arcsin(\sin x) = x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.
5. $\sin(\arcsin x) = x$ for $-1 \leq x \leq 1$.
6. $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$.

Important facts about arcsin:



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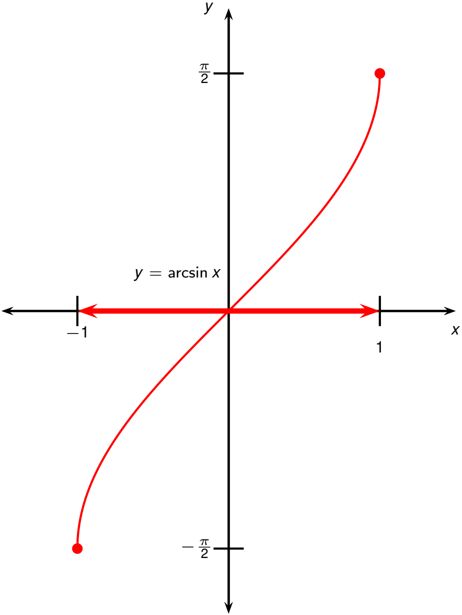

1. Domain: $[-1, 1]$.
2. Range: **?**
3. $\arcsin x = y \Leftrightarrow \sin y = x$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.
4. $\arcsin(\sin x) = x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.
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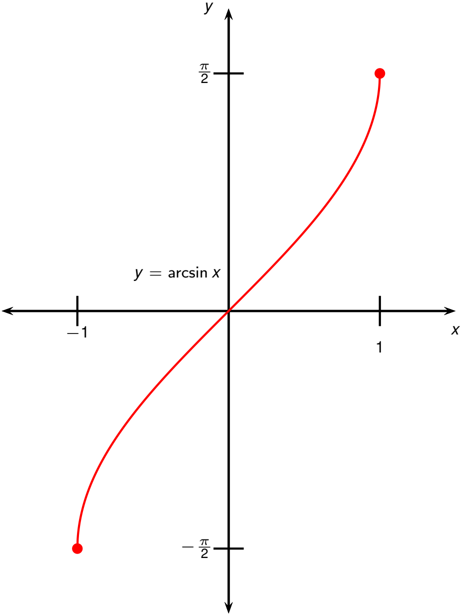

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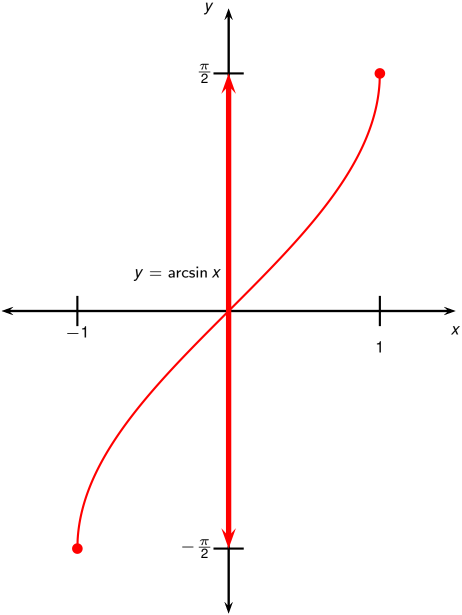
Important facts about arcsin:



1. Domain: $[-1, 1]$.
2. Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
3. $\arcsin x = y \Leftrightarrow \sin y = x$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.
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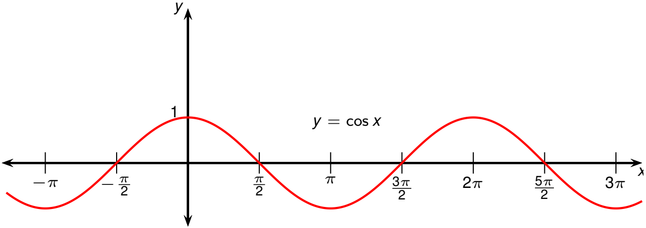
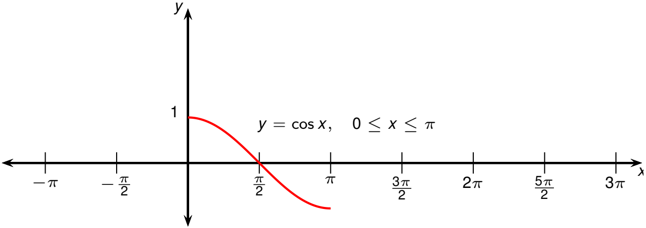
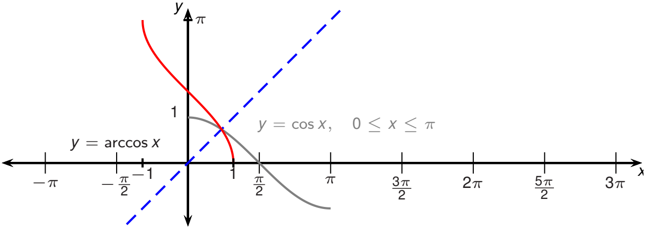


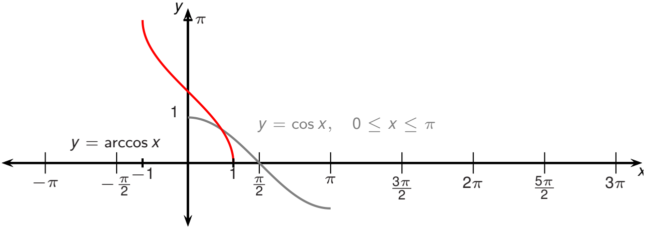
- Same for $\cos x$.



- Same for $\cos x$.
- Restrict the domain to $[0, \pi]$.



- Same for $\cos x$.
- Restrict the domain to $[0, \pi]$.
- The inverse is called $\arccos$ or $\cos^{-1}$.



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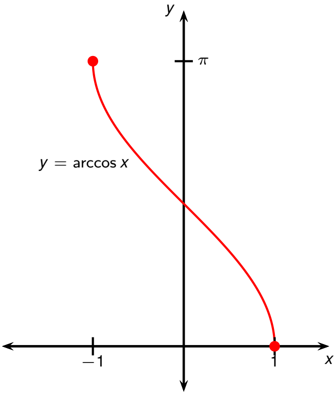
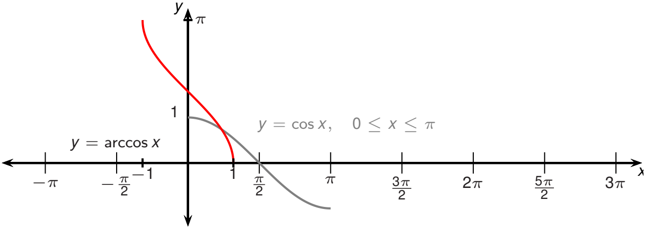
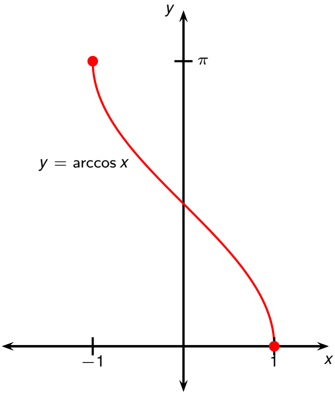
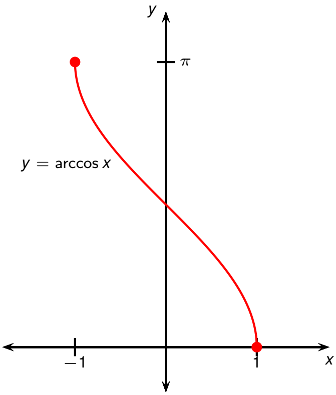




- Same for $\cos x$.
- Restrict the domain to $[0, \pi]$.
- The inverse is called $\arccos$ or $\cos^{-1}$.
- $\arccos(x) = y \Leftrightarrow \cos y = x$ and $0 \leq y \leq \pi$.

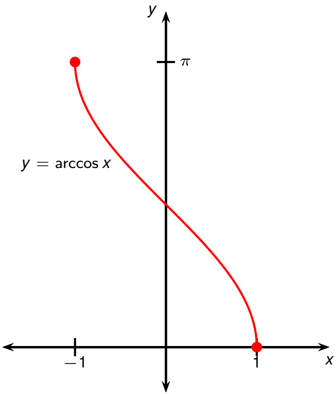


Important facts about arccos:



1. Domain:
2. Range:
3. $\arccos x = y \Leftrightarrow \cos y = x$ and $0 \leq y \leq \pi$.
4. $\arccos(\cos x) = x$ for $0 \leq x \leq \pi$.
5. $\cos(\arccos x) = x$ for $-1 \leq x \leq 1$.
6. $\dfrac{d}{dx}(\arccos x) = -\dfrac{1}{\sqrt{1-x^2}}$.

Important facts about arccos:



1. Domain: **?**
2. Range:
3. $\arccos x = y \Leftrightarrow \cos y = x$ and $0 \leq y \leq \pi$.
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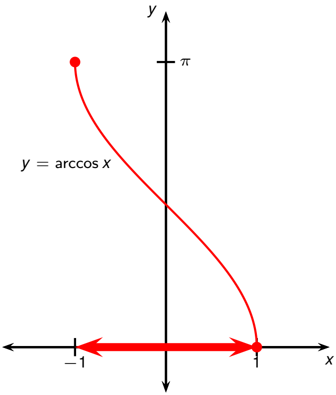

1. Domain: $[-1, 1]$.
2. Range:
3. $\arccos x = y \Leftrightarrow \cos y = x$ and $0 \leq y \leq \pi$.
4. $\arccos(\cos x) = x$ for $0 \leq x \leq \pi$.
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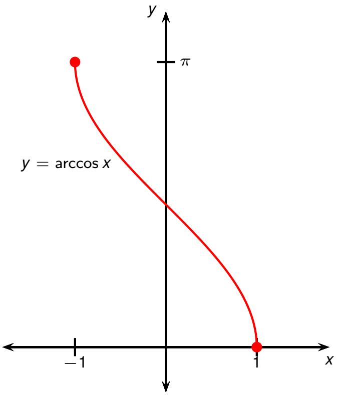
Important facts about arccos:



1. Domain: $[-1, 1]$.
2. Range: **?**
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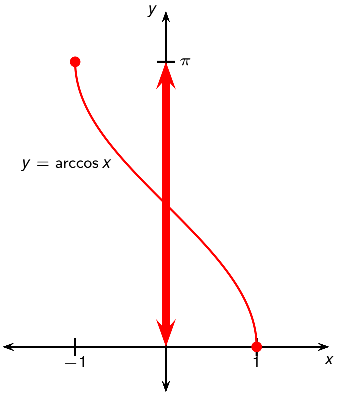

1. Domain: $[-1, 1]$.
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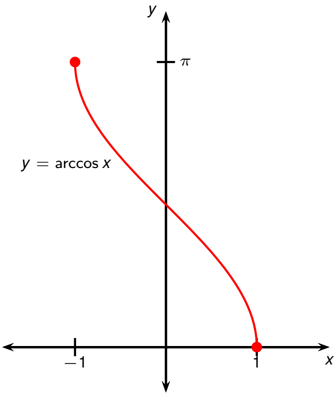

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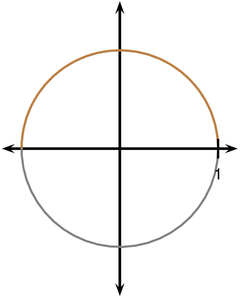
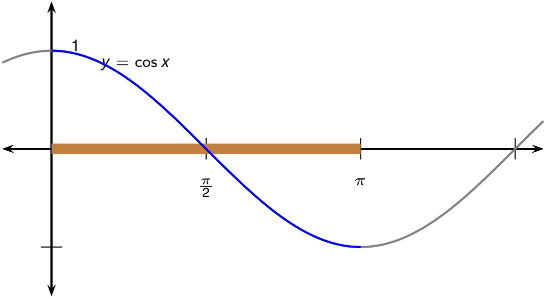
6. $\displaystyle \frac{\mathrm{d}}{\mathrm{d}x}(\arccos x) = -\frac{1}{\sqrt{1 - x^2}}$.
(The proof is similar to the proof of the formula for the derivative of $\arcsin x$.)

Example

Find $\arccos(\cos 4)$.

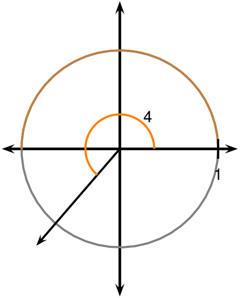


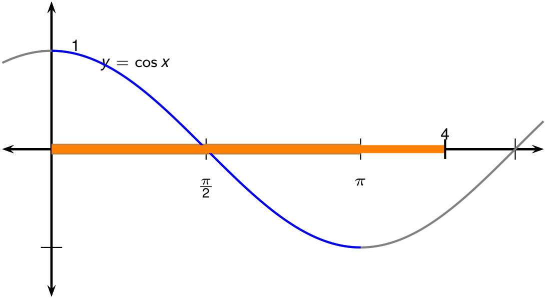


Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.

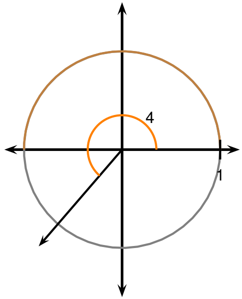


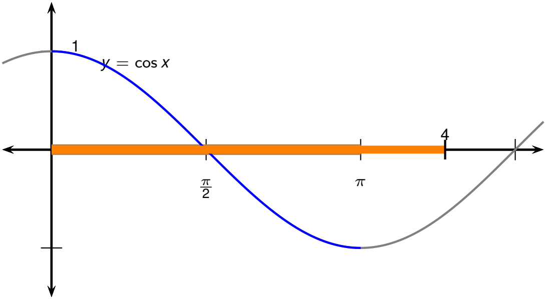


Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.





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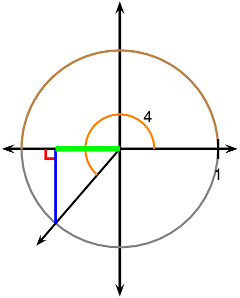
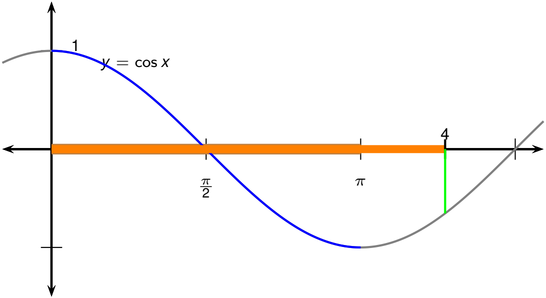
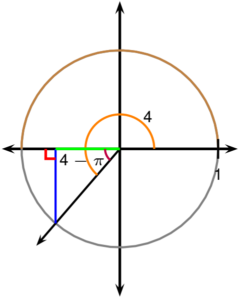


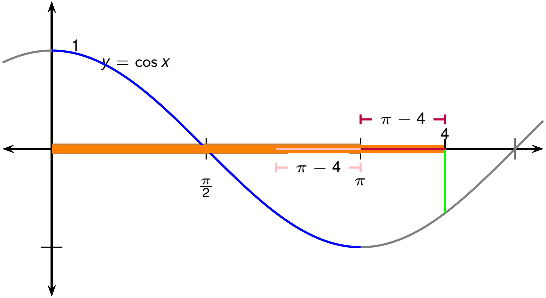


Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

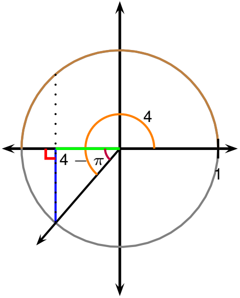


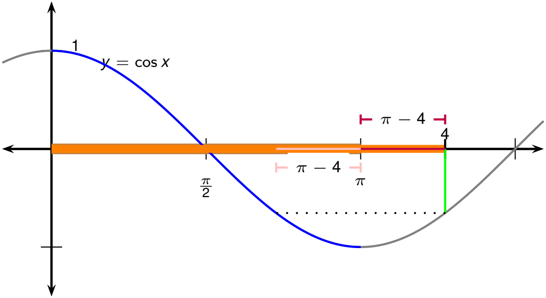


Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.
- We need the angle *a* between 0 and π for which $\cos 4 = \cos a$.





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- 4 is not between 0 and π.
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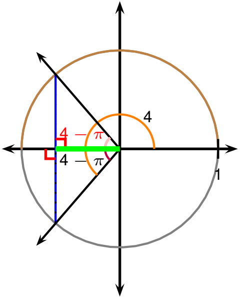

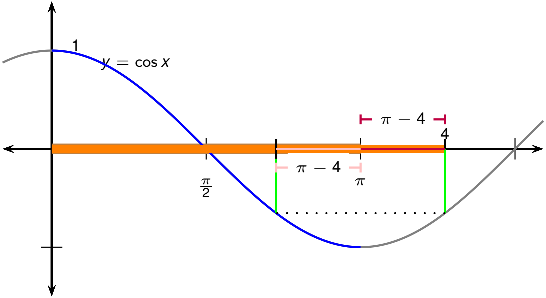

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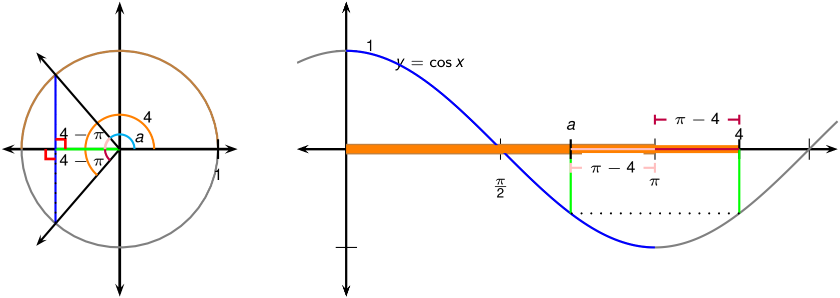
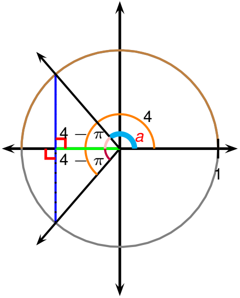


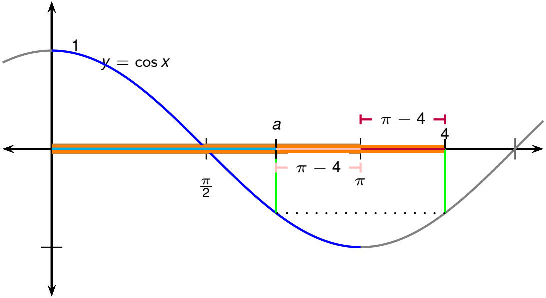
Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$a = \textbf{?}$$



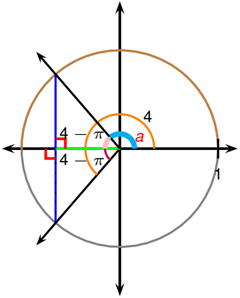


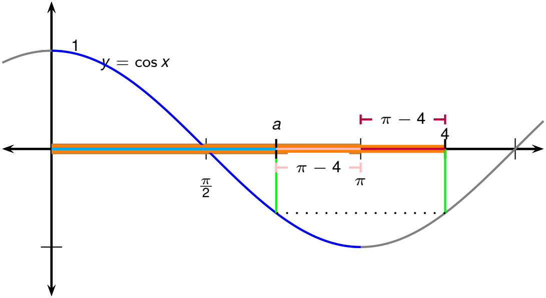
Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$a = \pi - (4 - \pi)$$



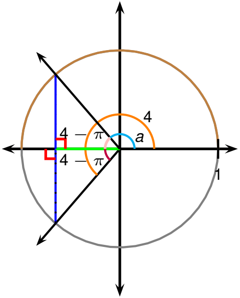


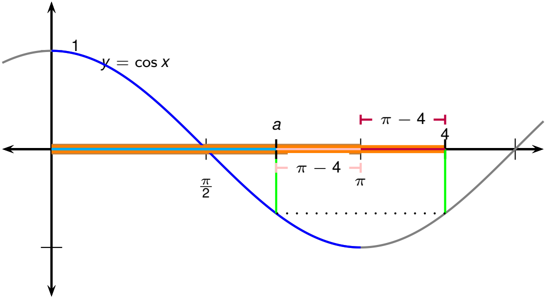
Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$a = \pi - (4 - \pi) = 2\pi - 4$$



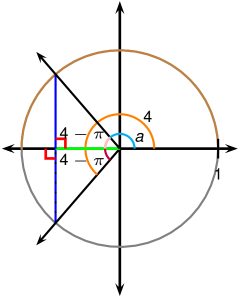


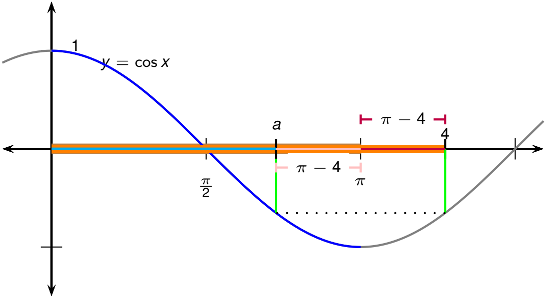
Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$\begin{aligned} a &= \pi - (4-\pi) = 2\pi - 4 \\ \text{Therefore} \quad \arccos(\cos 4) &= \arccos(\cos a) \end{aligned}$$



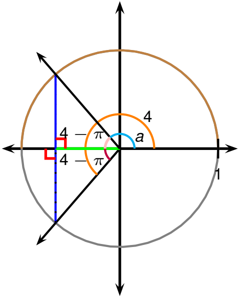


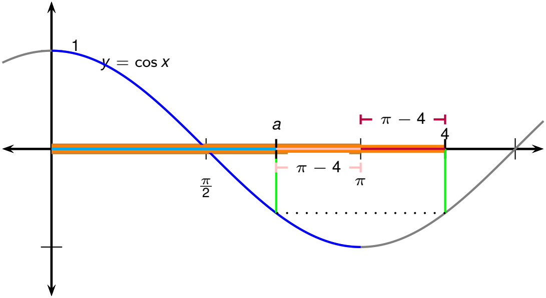
Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$
\begin{aligned}
& & {\color{red}a} &= \pi - (4 - \pi) = 2\pi - 4 \\
&\text{Therefore} & \arccos(\cos 4) &= \arccos(\cos a) \\
& & &= {\color{red}a}
\end{aligned}
$$



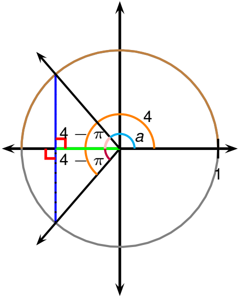


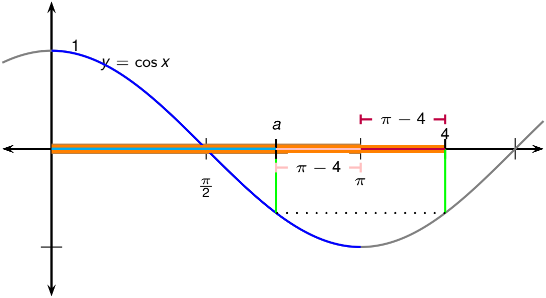
Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$
\begin{array}{rrcl}
 & a & = & \pi - (4 - \pi) = 2\pi - 4 \\
\text{Therefore} & \arccos(\cos 4) & = & \arccos(\cos a) \\
 & & = & a = 2\pi - 4.
\end{array}
$$

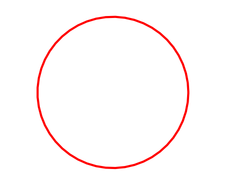




Example

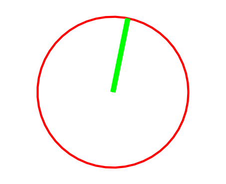
The horizon at sea is viewed at from a ship height of 10m. How far does the ship need to travel to reach the horizon just observed?

Example



The horizon at sea is viewed at from a ship height of 10m. How far does the ship need to travel to reach the horizon just observed? Assume earth is round with radius 6371 km and that the ship sails along the shortest curved path.

Example



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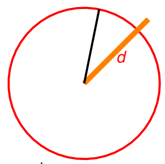
Example

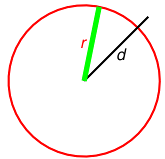


not to scale

The horizon at sea is viewed at from a ship height of 10m. How far does the ship need to travel to reach the horizon just observed? Assume earth is round with radius 6371 km and that the ship sails along the shortest curved path.

- Let d be the distance from eyes of seaman to the center of earth.

Example

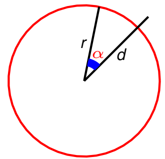


not to scale

The horizon at sea is viewed at from a ship height of 10m. How far does the ship need to travel to reach the horizon just observed? Assume earth is round with radius 6371 km and that the ship sails along the shortest curved path.

- Let d be the distance from eyes of seaman to the center of earth.
- Let r be the radius of earth.

Example



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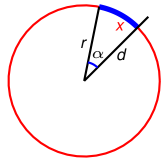
- Let r be the radius of earth. Let α be the indicated angle.

Example



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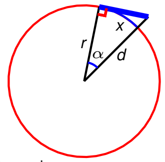
- Let the distance to the horizon be x.

Example



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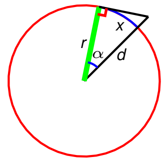

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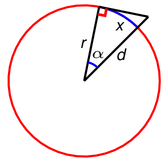
$r = 6371$km

Example



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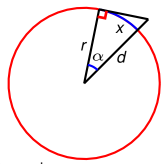
Example



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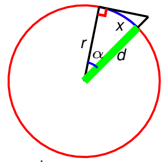
$r = 6371\text{km}$

$d = 6371\text{km} + 0.01\text{km}$

Example



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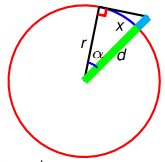

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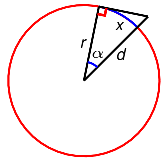
Example



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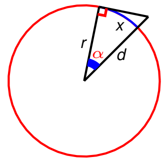
r=6371km

d=6371km + 0.01km = 6371.01km

Example



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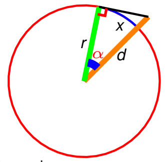
r=6371km
d=6371km + 0.01km = 6371.01km

$\cos\alpha$=**?**

Example



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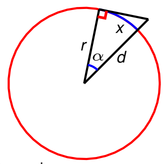
$r = 6371\text{km}$

$d = 6371\text{km} + 0.01\text{km} = 6371.01\text{km}$

$\cos\alpha = \frac{r}{d}$

Example



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r &= 6371\text{km}\\
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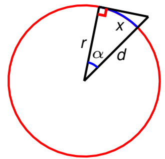
\cos\alpha &= \frac{r}{d}\\
\alpha &= \arccos\left(\frac{r}{d}\right)
\end{aligned}
$$

Example



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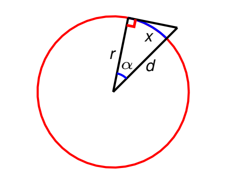
$$x = \textbf{?}$$

Example



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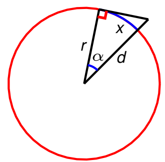
$$\alpha = \arccos\left(\frac{r}{d}\right)$$

$$x = r\alpha$$

Example



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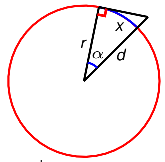
Example



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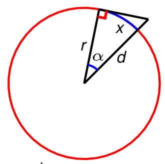
Example



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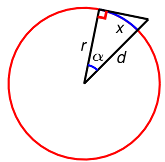

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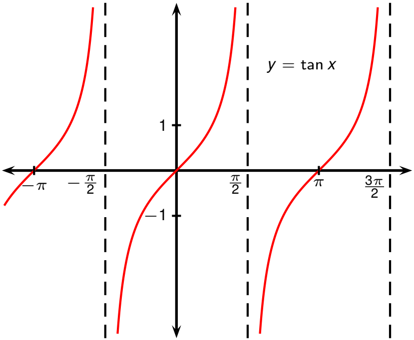
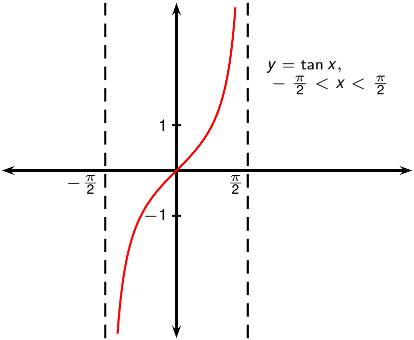
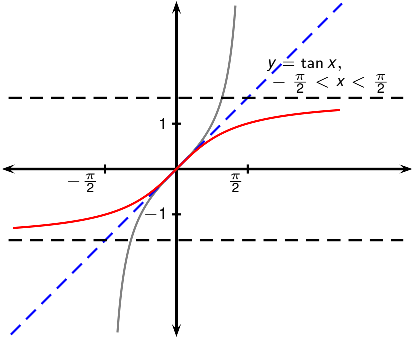
$$\begin{aligned} r &= 6371\text{km} \\ d &= 6371\text{km} + 0.01\text{km} = 6371.01\text{km} \\ \cos\alpha &= \frac{r}{d} \\ \alpha &= \arccos\left(\frac{r}{d}\right) \\ x &= r\alpha = r\arccos\left(\frac{r}{d}\right) = 6371\text{km}\arccos\left(\frac{6371\text{km}}{6371.01\text{km}}\right) \approx 11.29\text{km} \end{aligned}$$



- $\tan x$ isn't one-to-one.



- $\tan x$ isn't one-to-one.
- Restrict the domain to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.



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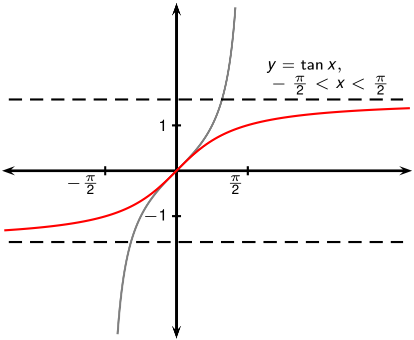

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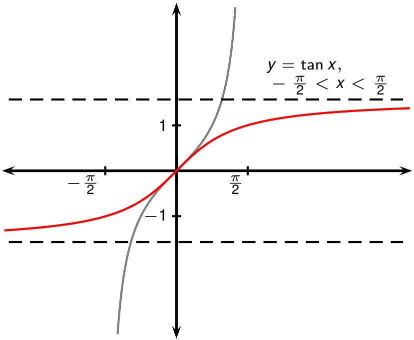

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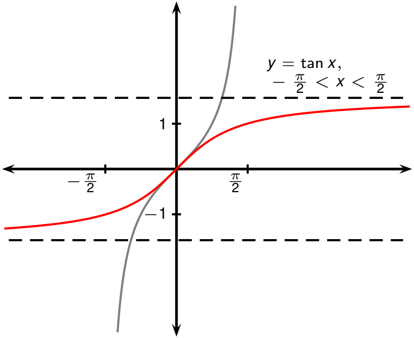
- Domain of $\arctan$: **?**
- Range of $\arctan$:



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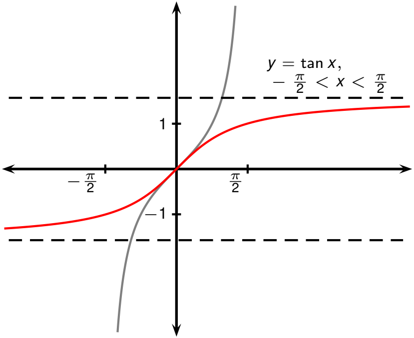

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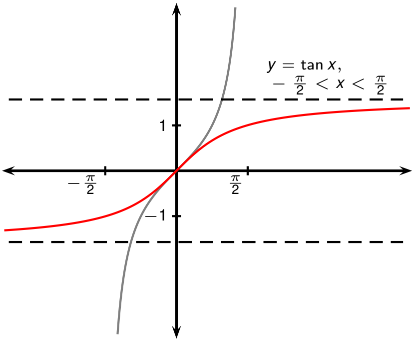

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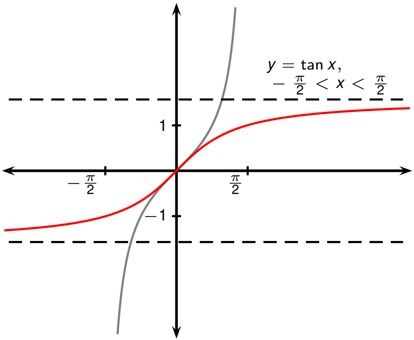

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- $\lim_{x\to\infty} \arctan x =$ **?**
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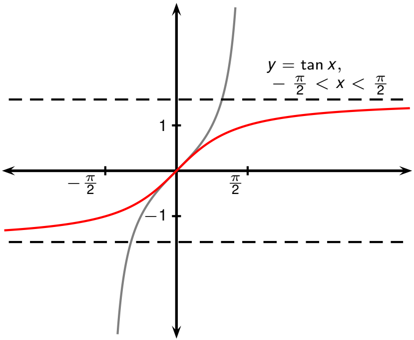

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- Domain of $\arctan$: $(-\infty, \infty)$.
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- $\lim\limits_{x\to\infty} \arctan x = \dfrac{\pi}{2}$.
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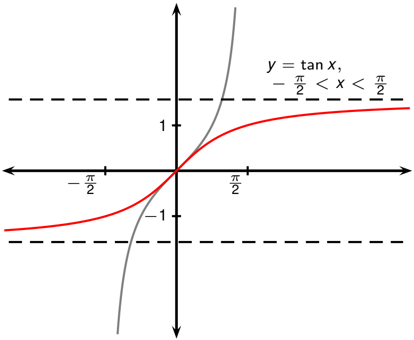

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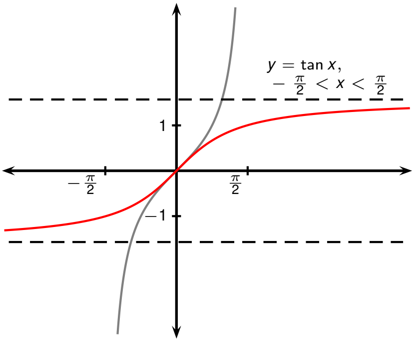

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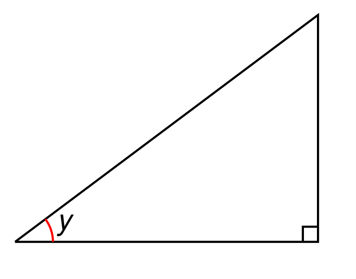
- $\lim_{x\to\infty} \arctan x = \frac{\pi}{2}$.
- $\lim_{x\to-\infty} \arctan x = -\frac{\pi}{2}$.

Example

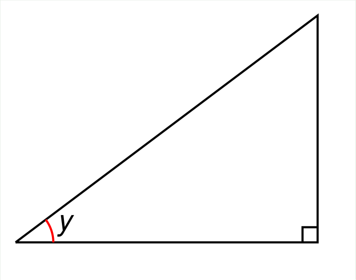
Simplify the expression $\cos(\arctan x)$.



Example

Simplify the expression $\cos(\arctan x)$.

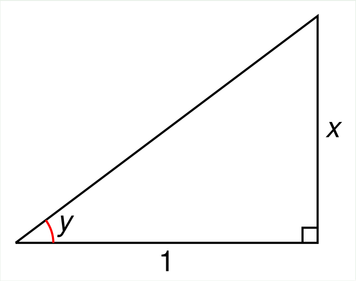
- Let $y = \arctan x$, so $\tan y = x$.



Example

Simplify the expression $\cos(\arctan x)$.

- Let $y = \arctan x$, so $\tan y = x$.
- Draw a right triangle with opposite x and adjacent 1.



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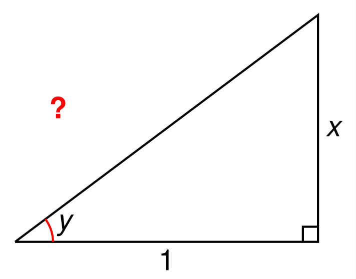
- Let $y = \arctan x$, so $\tan y = x$.
- Draw a right triangle with opposite x and adjacent 1.
- Length of hypotenuse $=$ **?**



Example

Simplify the expression $\cos(\arctan x)$.

- Let $y = \arctan x$, so $\tan y = x$.
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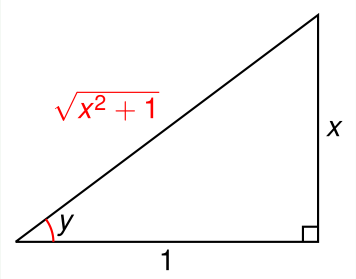
- Length of hypotenuse $= \sqrt{1^2 + x^2}$.



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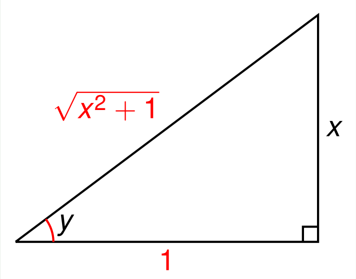

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- Let $y = \arctan x$, so $\tan y = x$.
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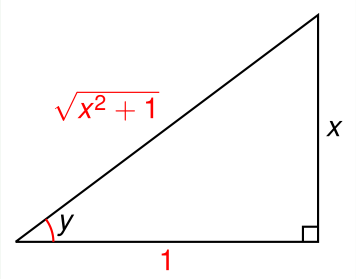

The remaining inverse trigonometric functions aren't used as often:

$$\begin{array}{llcllll}
y = \operatorname{arccsc} x & (|x| \geq 1) & \Leftrightarrow & \csc y = x & \text{and} & y \in \left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right] \\
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We will however make use of $\operatorname{arcsec} x$: we discuss in detail its domain.

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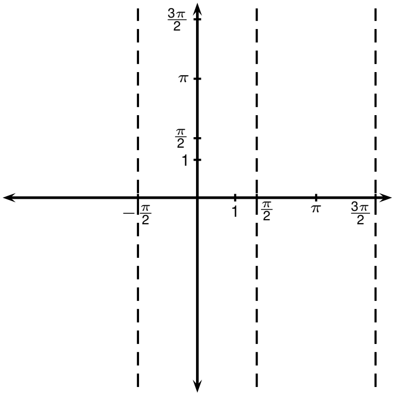

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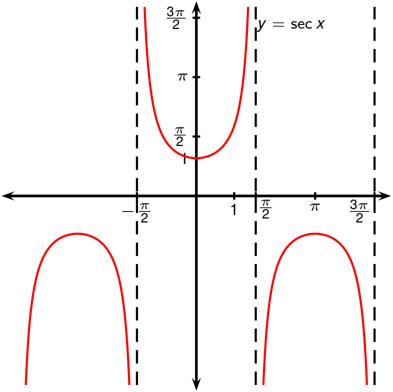

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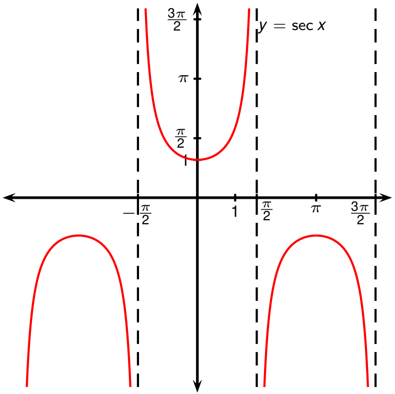

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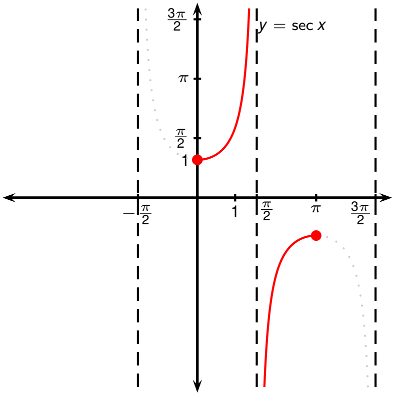

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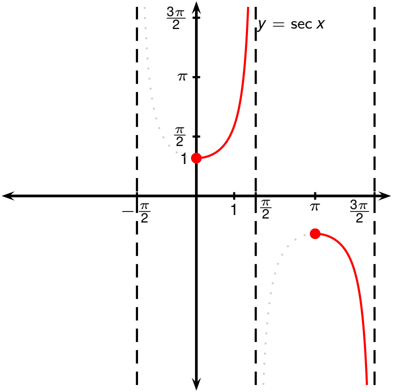

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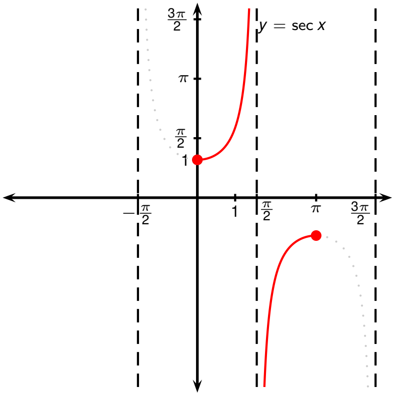

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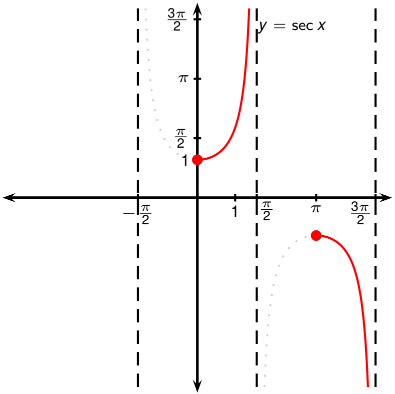

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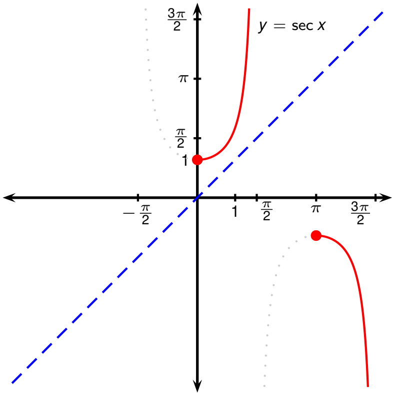

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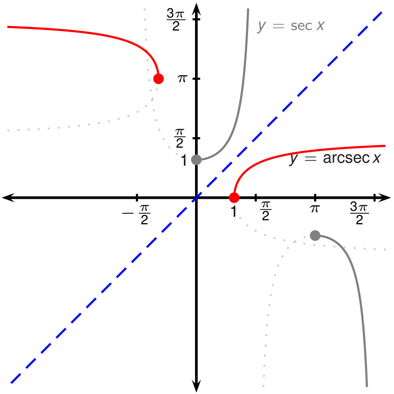

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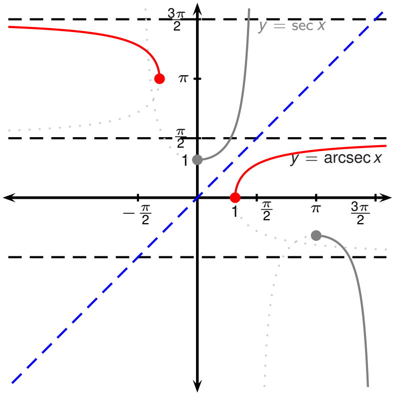

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Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

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Set $y = \arccos x$
Express via $\sin y, \cos y$

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$\sin(2\arccos(x)) = \sin(2y)$	Set $y = \arccos x$
$= 2\cos y \sin y$	Express via $\sin y, \cos y$
$= 2\cos y\left(\pm\sqrt{1-\cos^2 y}\right)$	Express $\sin y$ via $\cos y$
$= 2\cos y\sqrt{1-\cos^2 y}$	$\sin y > 0$ because $0 \leq y \leq \pi$
$= 2x\sqrt{1-x^2}$	use $x = \cos y$

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$$\cos(3\arccos(x)) = \cos(3y) \qquad \Big| \; y = \arccos x$$

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Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.
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$$\begin{aligned} \cos(3\arccos(x)) &= \cos(3y) = \cos(2y+y) && \Big| \; y = \arccos x \\ &= \cos(2y)\cos y - \sin(2y)\sin y && \Big| \; \text{Angle sum f-la} \end{aligned}$$

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= & \cos(2y)\cos y - \sin(2y)\sin y & \text{Angle sum f-la} \\
= & (\cos^2 y - \sin^2 y)\cos y & \text{Express via} \\
 & -2\sin y \cos y \sin y & \sin y, \cos y \\
= & \cos^3 y - \sin^2 y \cos y - 2\sin^2 y \cos y &
\end{array}$$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

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