

Precalculus

Trigonometric equation that reduces to quadratic, masked by identity

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Example

Find all values of θ in the interval $[0, 2\pi]$ such that $\sin \theta = \sin(2\theta)$.

$$\sin \theta = \sin(2\theta)$$

$$\sin \theta = 2 \sin \theta \cos \theta$$

$$0 = 2 \sin \theta \cos \theta - \sin \theta$$

$$0 = \sin \theta (2 \cos \theta - 1)$$

$$\sin \theta = 0$$

$$\theta = 0 + 2k\pi$$

$$\text{or } \pi + 2k\pi$$

$$\theta = 0 \text{ or } 2\pi \text{ or } \pi$$

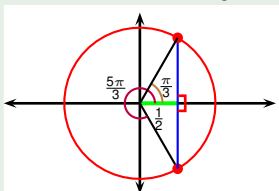
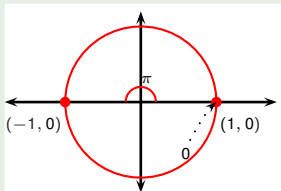
or

$$2 \cos \theta - 1 = 0$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3} + 2k\pi \text{ or } \frac{5\pi}{3} + 2k\pi$$

$$\theta = \frac{\pi}{3} \text{ or } \frac{5\pi}{3}$$



Example

Find all values of θ in the interval $\theta \in [0, 2\pi]$ for which

$$\cos(2\theta) = \cos \theta$$

$$\cos^2 \theta - \sin^2 \theta - \cos \theta = 0 \quad \left| \text{Express via } \cos \theta \right.$$

$$\cos^2 \theta - (1 - \cos^2 \theta) - \cos \theta = 0$$

$$2 \cos^2 \theta - \cos \theta - 1 = 0 \quad \left| \text{Set } \cos \theta = u \right.$$

$$2u^2 - u - 1 = 0$$

$$(u - 1)(2u + 1) = 0$$

$$u - 1 = 0$$

$$\cos \theta = 1$$

$$\theta = 0 + 2k\pi$$

$$\theta = 0 \text{ or } 2\pi$$

or

$$2u + 1 = 0$$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = \frac{2\pi}{3} + 2k\pi \text{ or } \frac{4\pi}{3} + 2k\pi$$

$$\theta = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}$$

