Calculus I
Definite integrals of rational power monomials

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Example

Evaluate: $$\int_1^9 \frac{2t^3 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$$

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$$= \left(9^2 + \frac{2}{3}\cdot 9^{\frac{3}{2}} + \frac{1}{9}\right) - \left(1^2 + \frac{2}{3}\cdot 1^{\frac{3}{2}} + \frac{1}{1}\right)$$

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$$= 81 + 18 + \frac{1}{9} - 1 - \frac{2}{3} - 1$$

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$$= 81+18+\frac{1}{9}-1-\frac{2}{3}-1=\frac{868}{9}.$$