

## Precalculus

# Complex multiplication involving real radicals

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## Example (Complex multiplication)

Multiply  $u = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$  by  $v = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ .

$$u \cdot v$$

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Multiply  $u = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$  by  $v = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ .

$$u \cdot v = \left( \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right) \cdot \left( \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \right)$$

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 &= \frac{2}{4} - \frac{2}{4}(-1) \\
 &= \frac{1}{2} + \frac{1}{2} \\
 &= 1
 \end{aligned}$$