Precalculus
Quadratic trigonometric inequalities

Todor Milev

2019

Example

- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2\cos^2\theta + 2\cos\theta + 1 \leq \cos\theta + 2$ lying in $[-360^\circ, 360^\circ]$.

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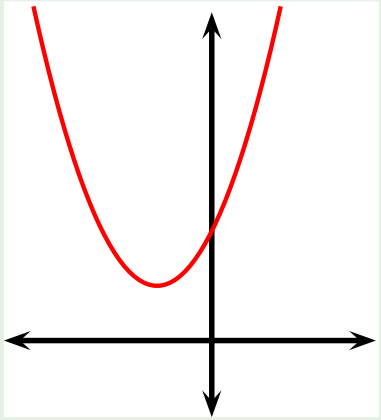

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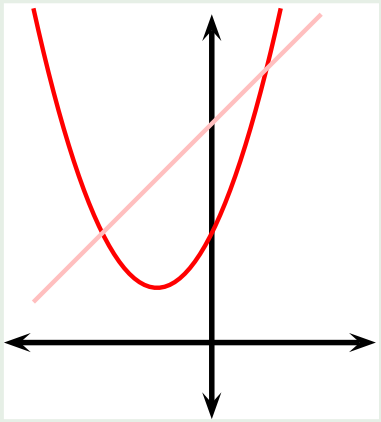

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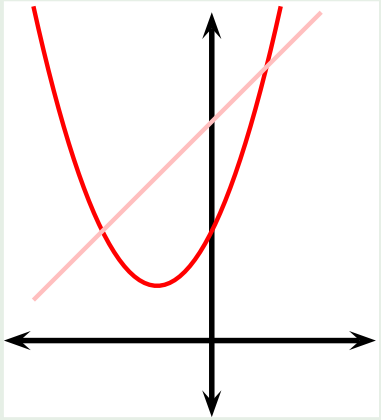

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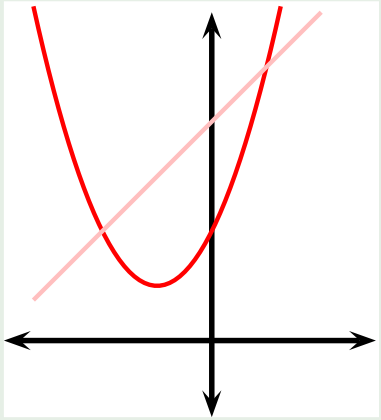

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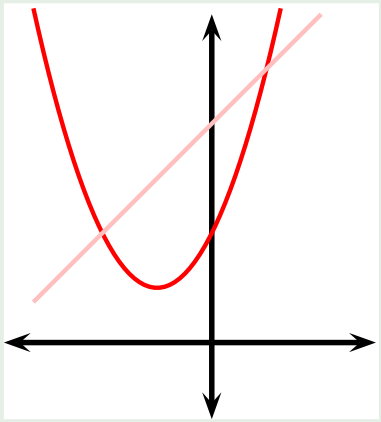

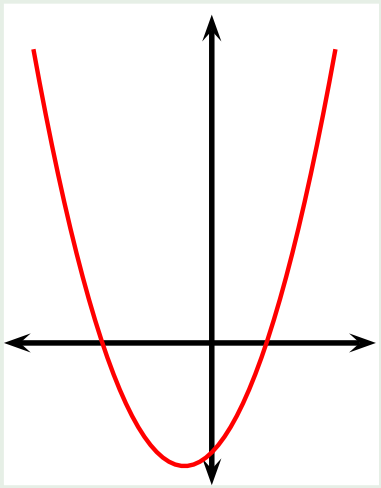

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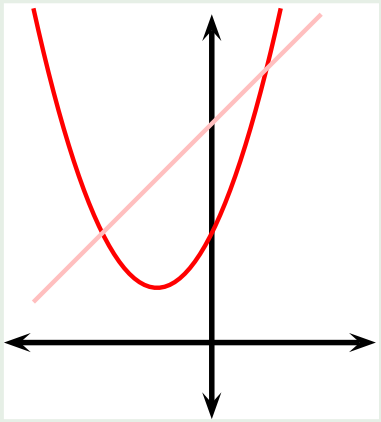

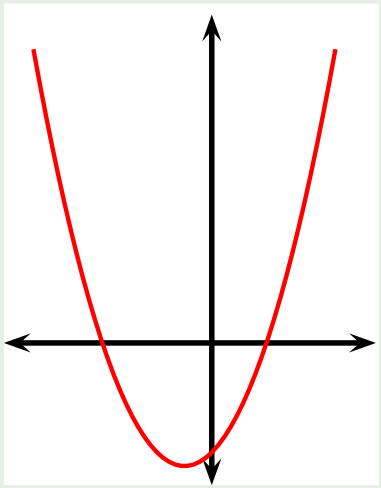

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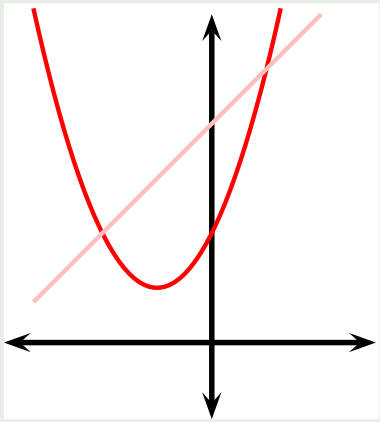

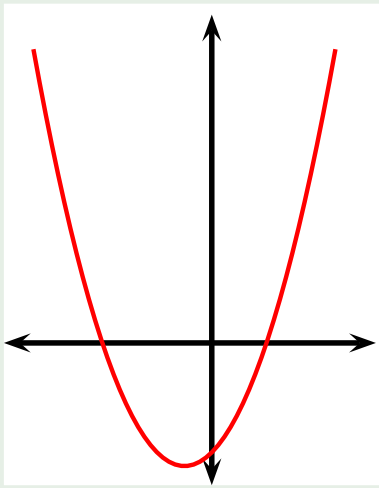

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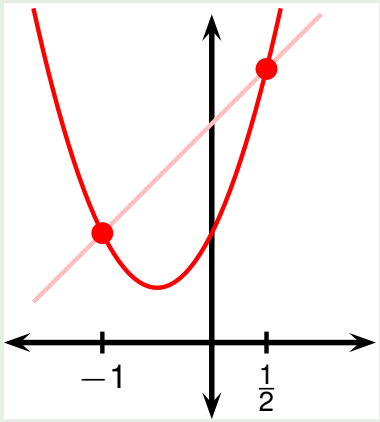

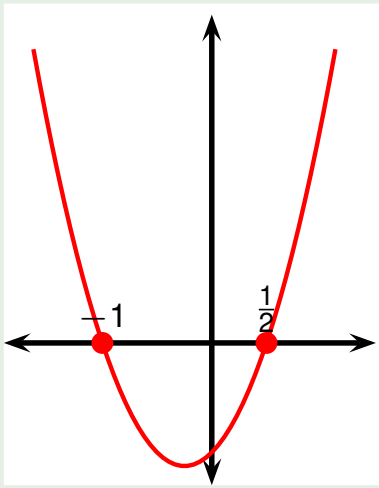

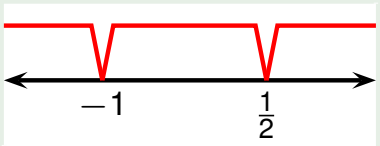

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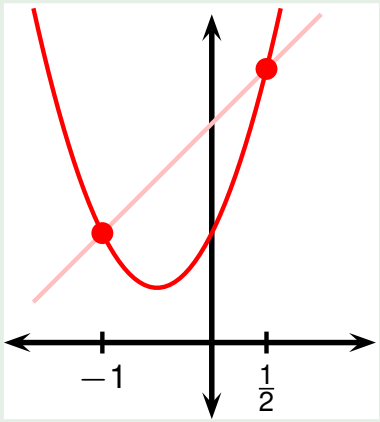

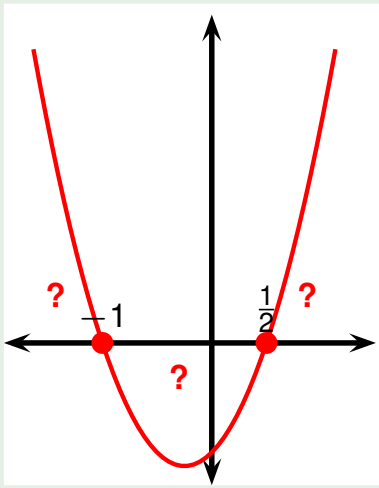

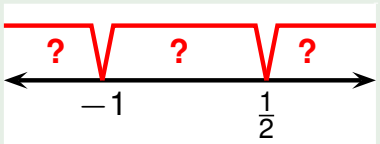

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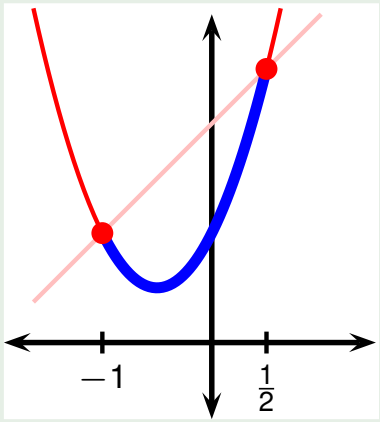

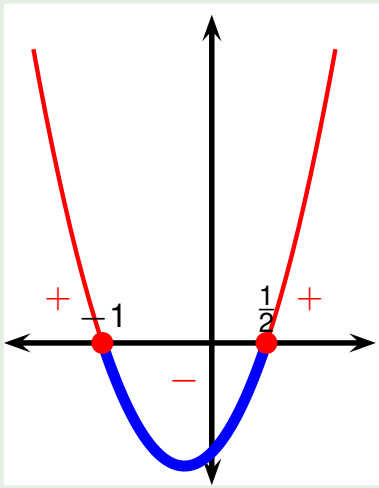

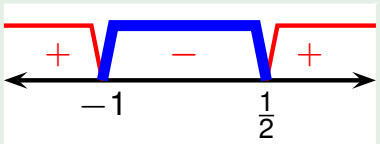

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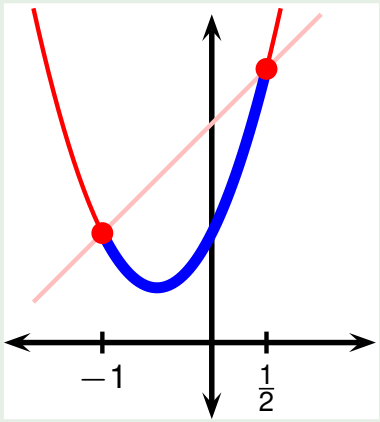

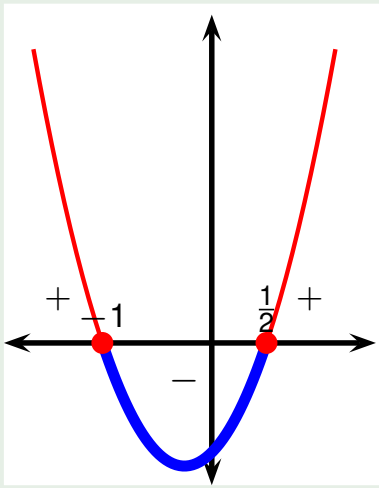

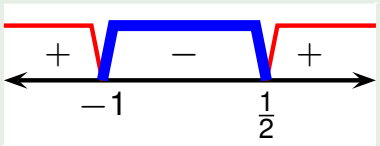

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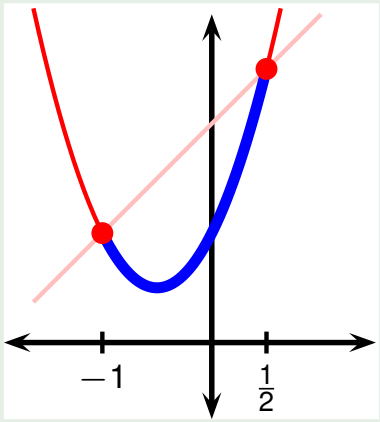

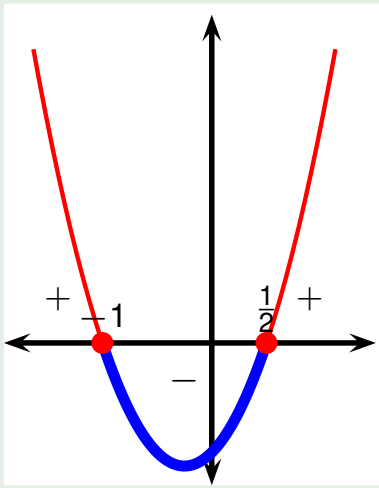

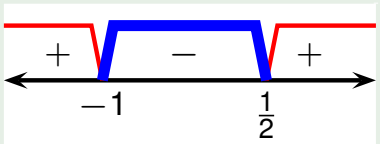

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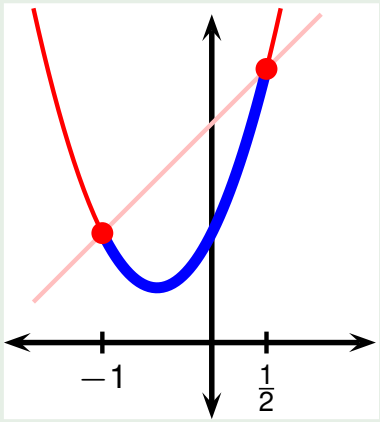

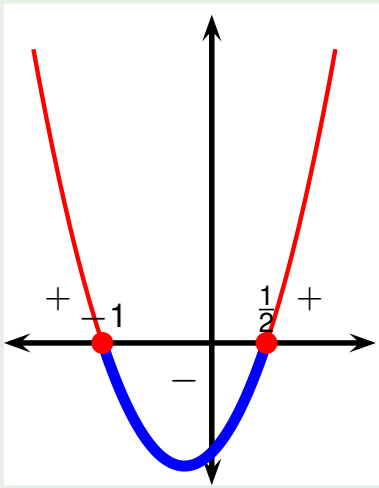

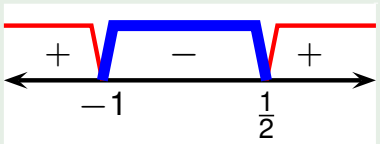

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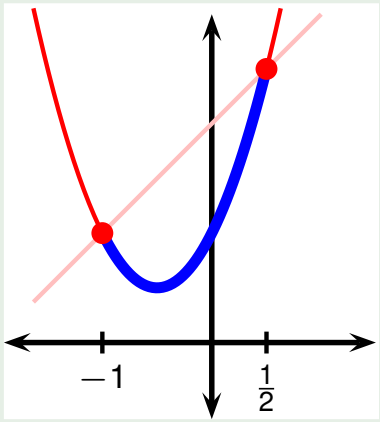

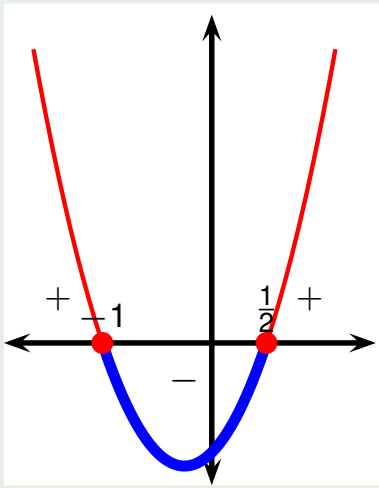

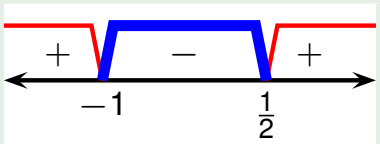

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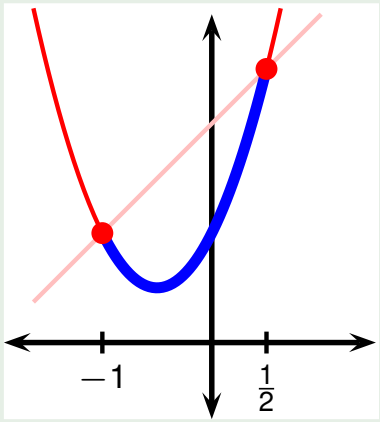

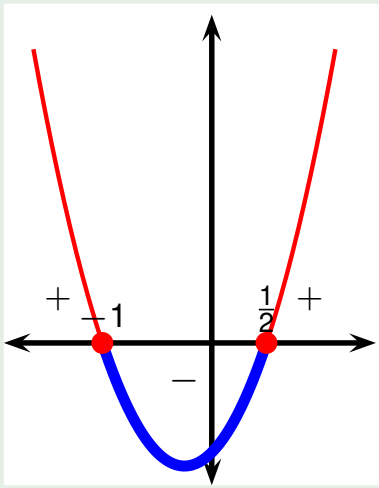

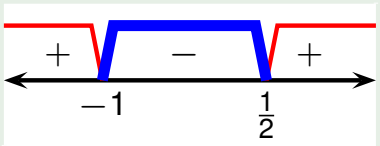

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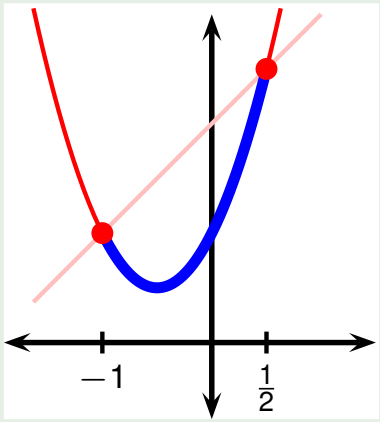

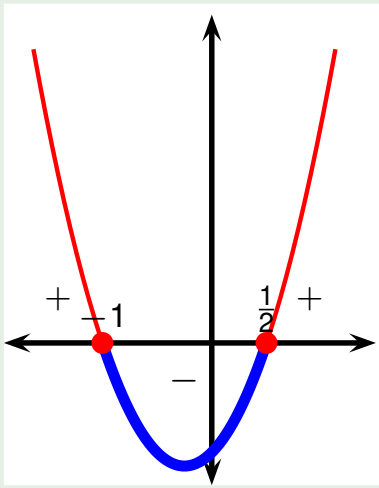

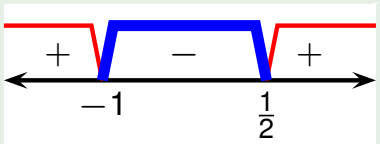

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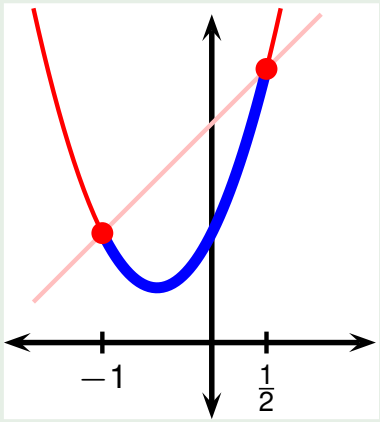
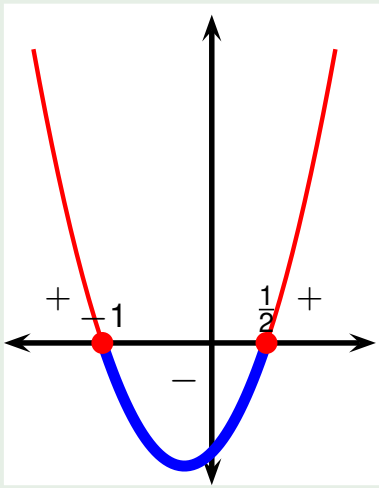
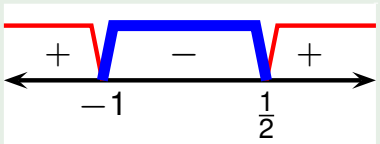
Example







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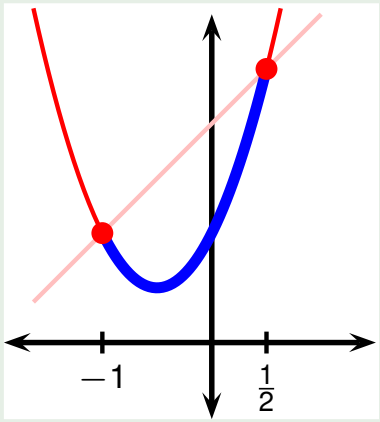

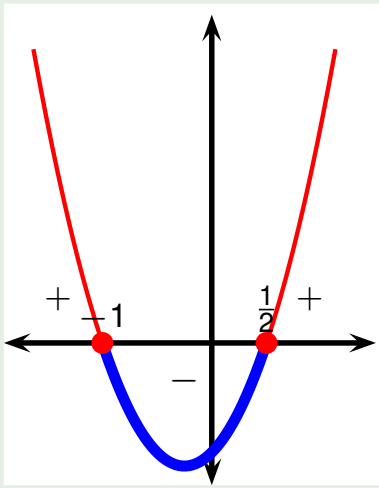

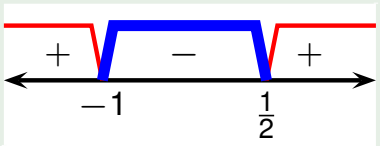

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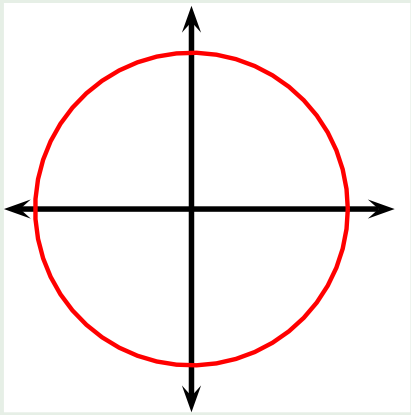

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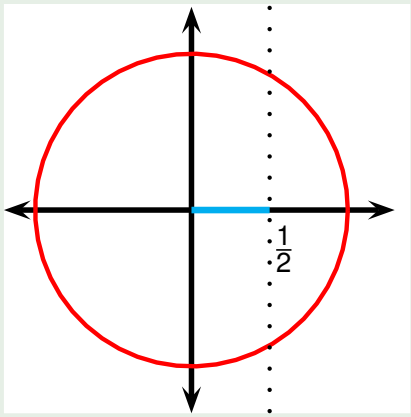

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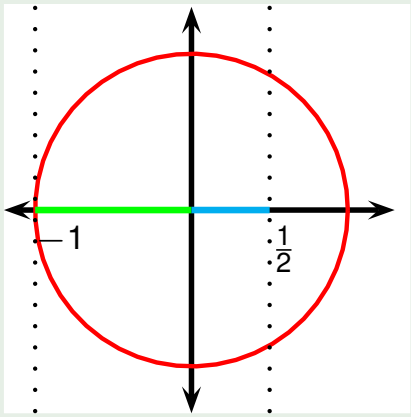

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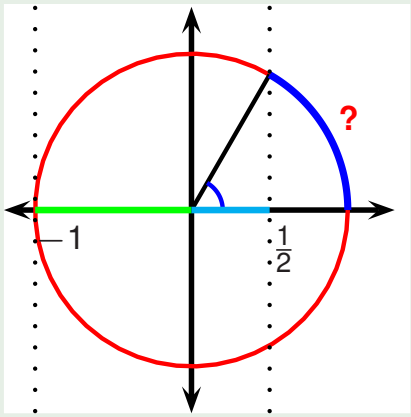

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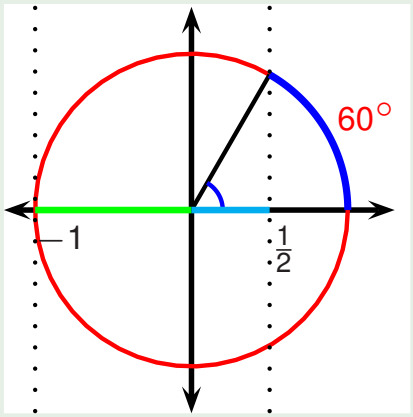

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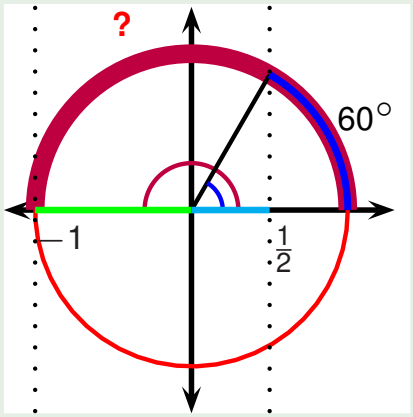
Example



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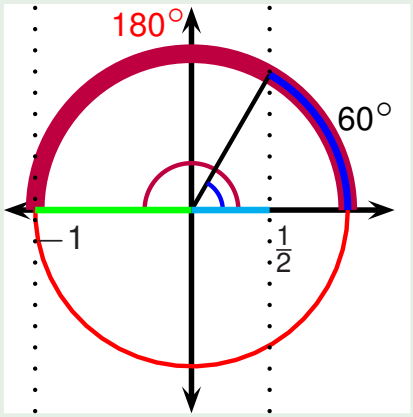
Example



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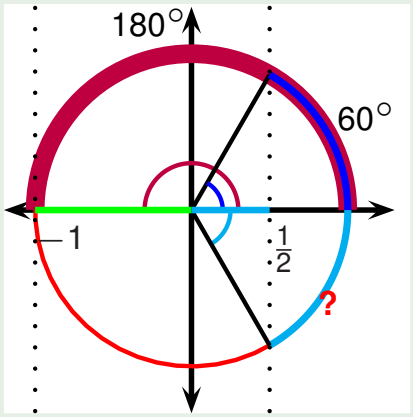
Example



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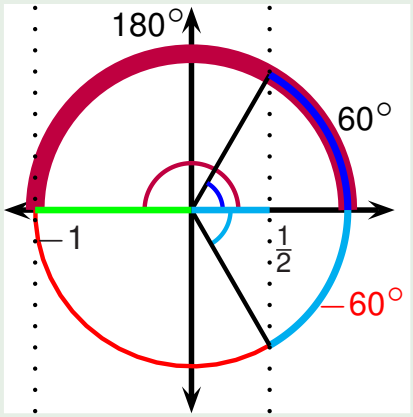
Example



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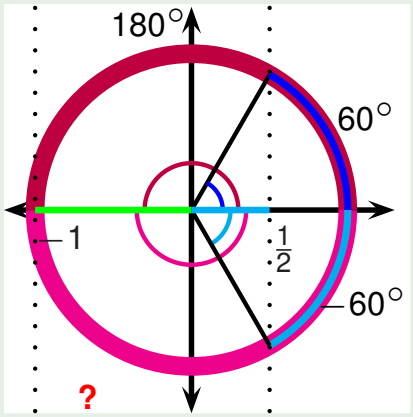
Example



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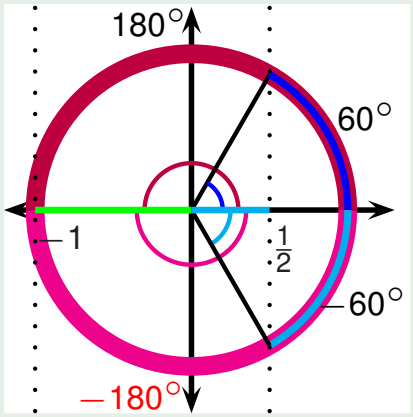
Example



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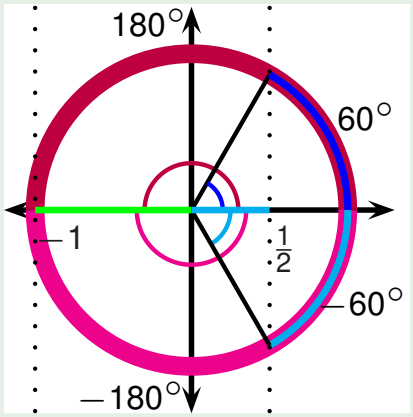
Example



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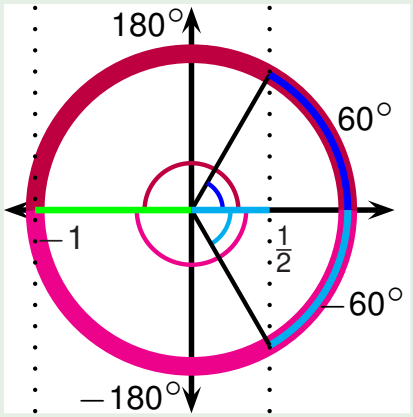

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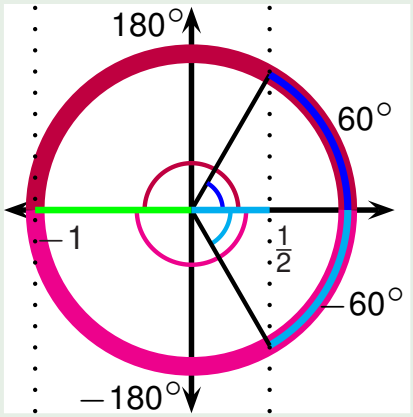

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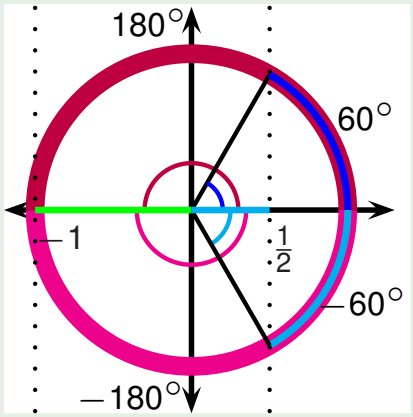

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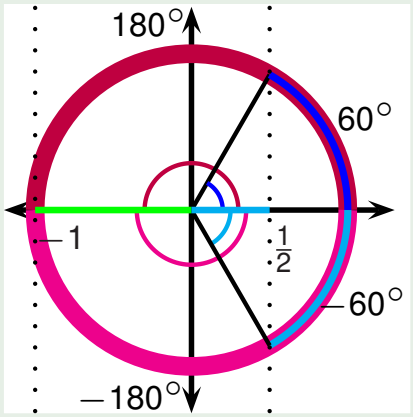

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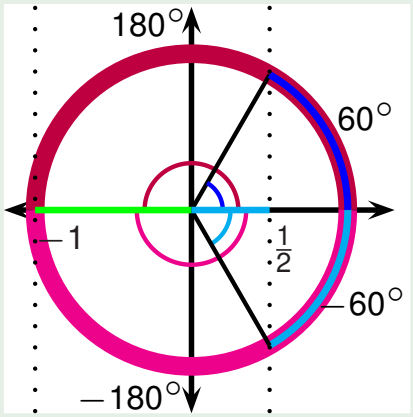

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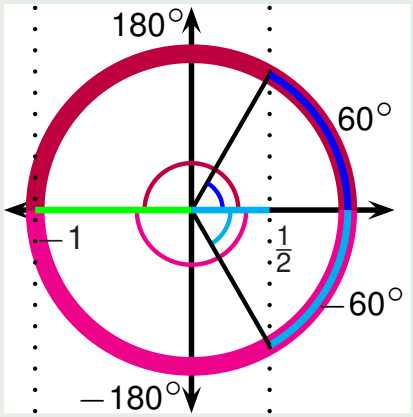
Example



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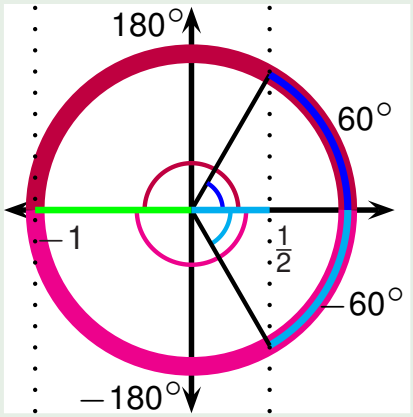

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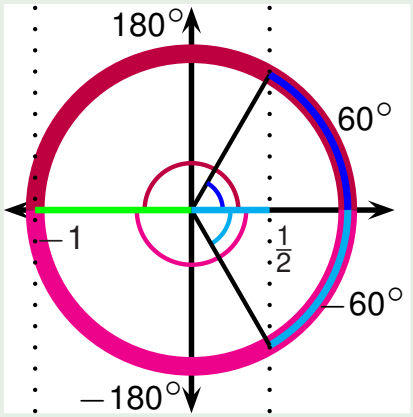

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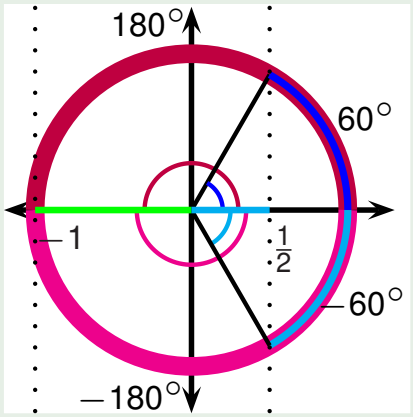
$$\begin{aligned} \theta \in \; & \cancel{\cup [-540°, -420°]} \cup [-300°, -180°] && k = -1 \\ & \cup [-180°, -60°] \cup [60°, 180°] && k = 0 \\ & \cup [180°, 300°] \cup \cancel{[420°, 540°]} && k = 1 \end{aligned}$$

$$\theta \in [-300°, -60°] \cup [60°, 300°]$$

Example



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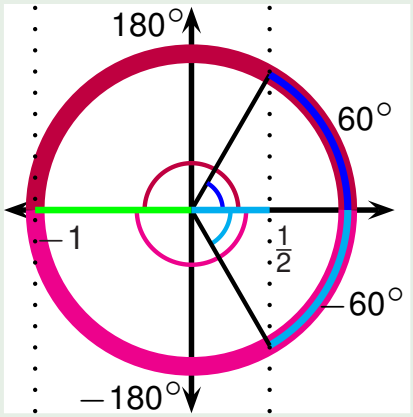

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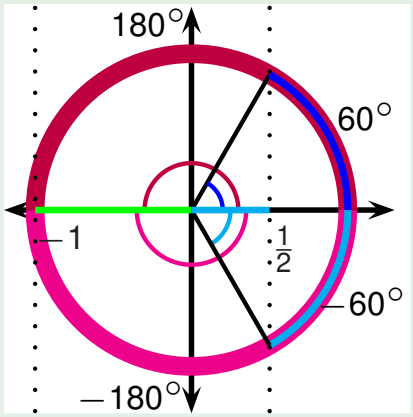
Example



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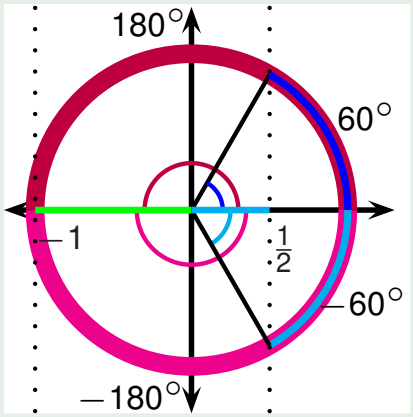
Example



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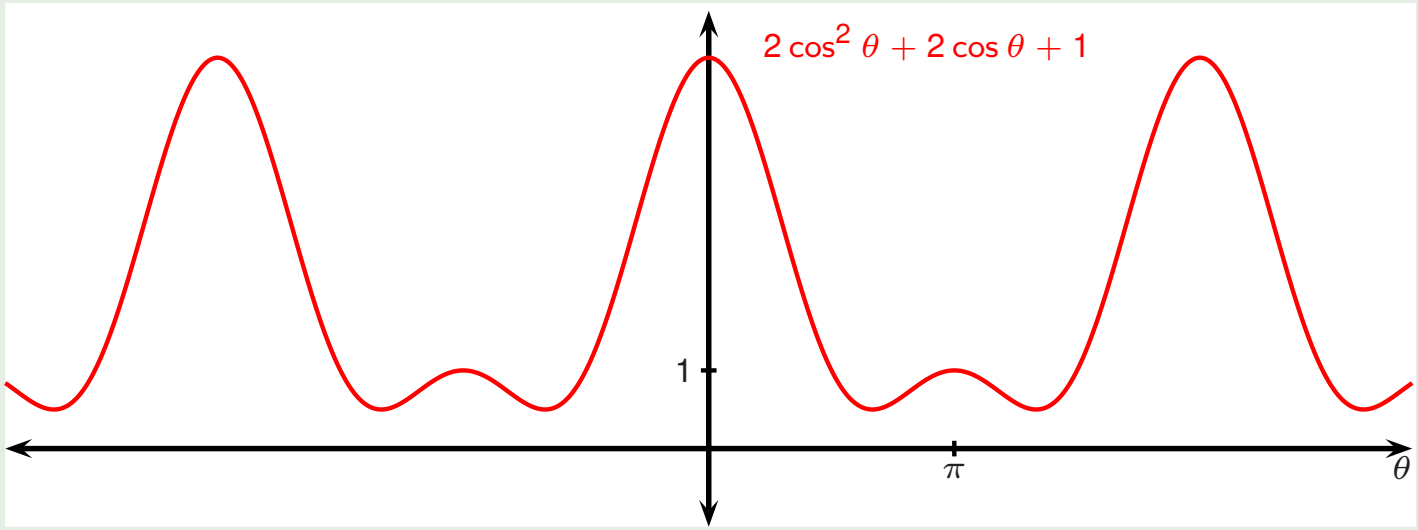

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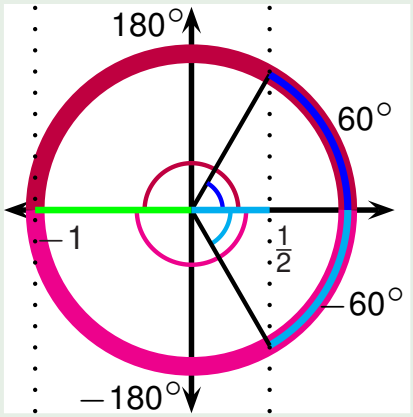

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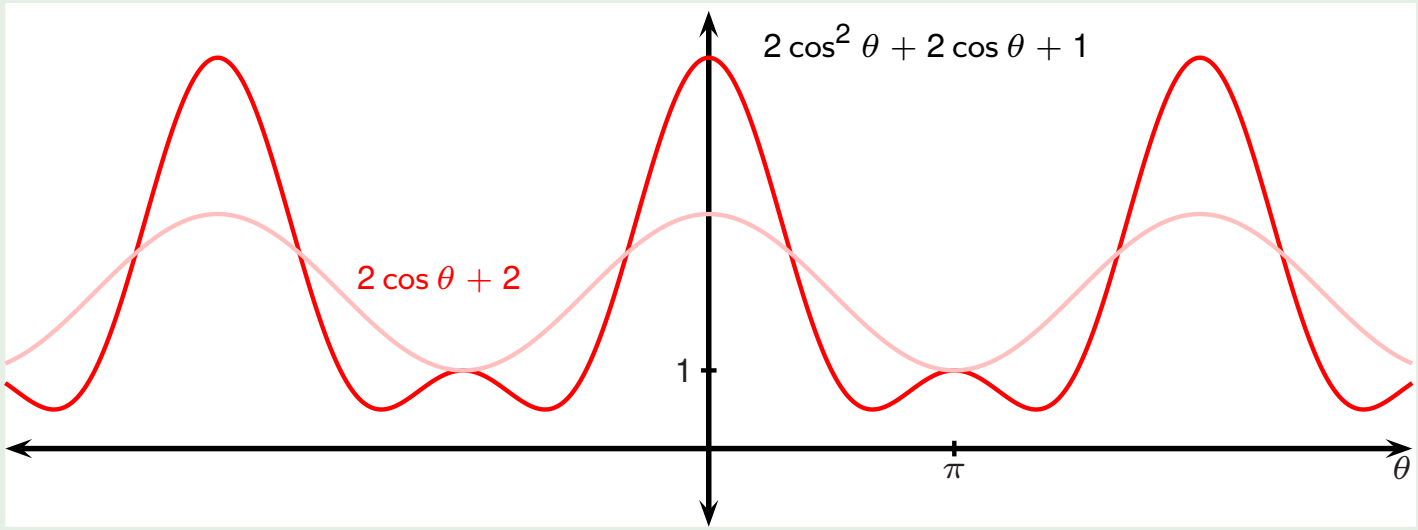

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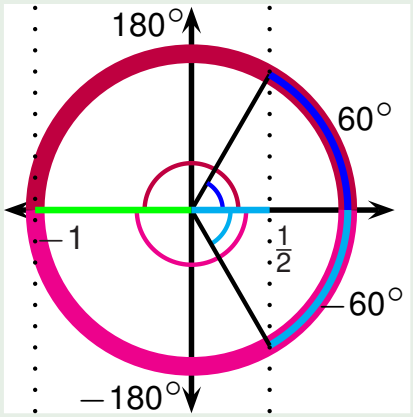

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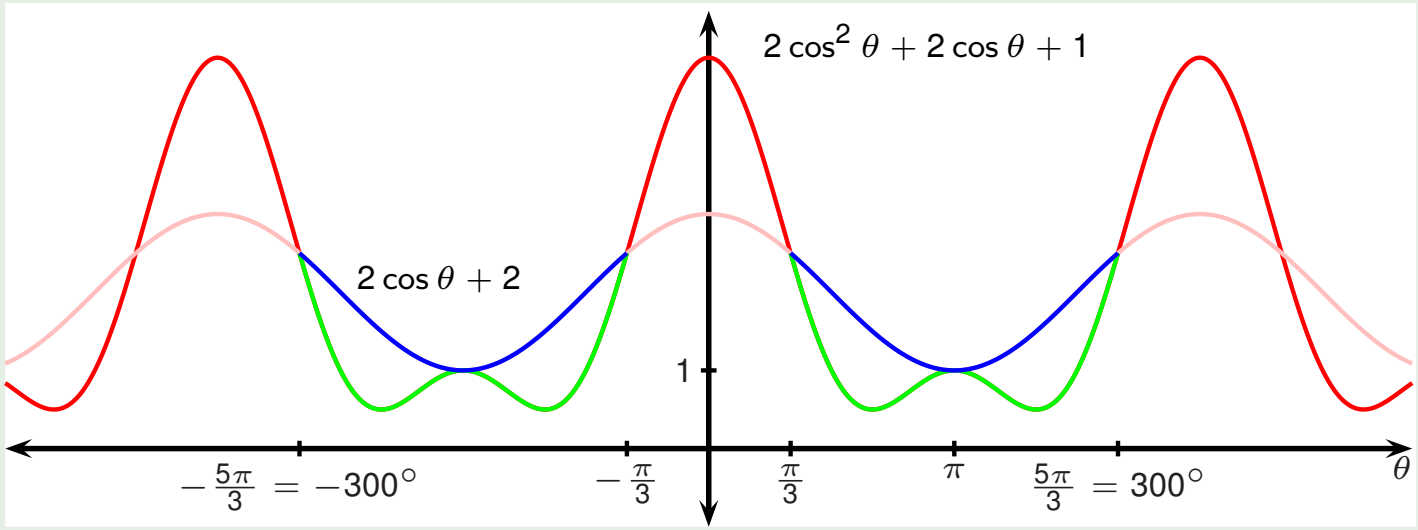
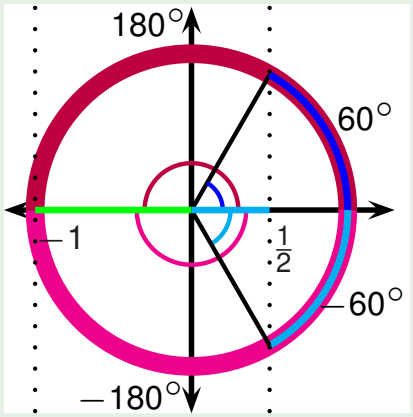


Example



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