

## Precalculus

# Factor cubic with one real root using its plot

Todor Milev

2019

## Example

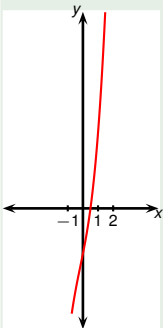
Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

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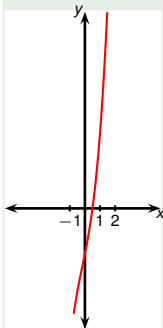


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We see only one root,  $x = ?$  .

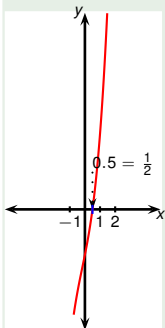


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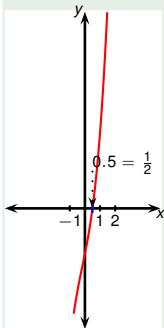


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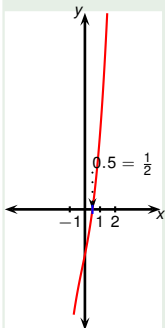
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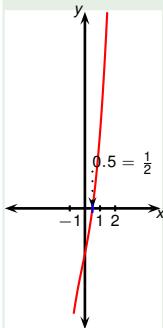
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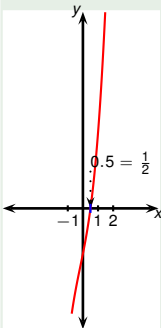
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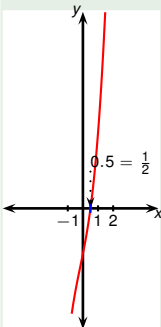
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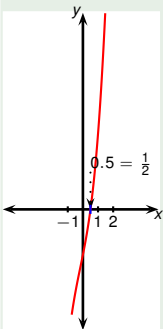


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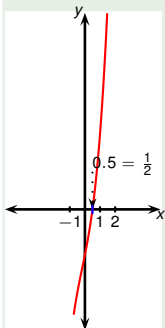
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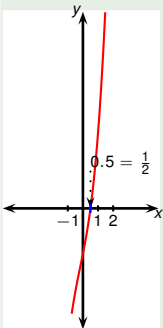
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$$x - \frac{1}{2} \overline{) \begin{array}{l} 2x^3 + x^2 + 5x - 3 \\ \phantom{2x^3 + } 2x^2 \phantom{+ 5x - 3} \\ \hline \phantom{2x^3 + } ? \phantom{+ 5x - 3} \\ \phantom{2x^3 + } ? \phantom{+ 5x - 3} \end{array}}$$

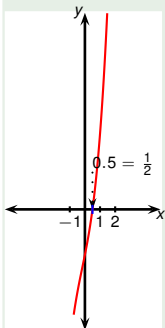
Multiply  $2x^2$  by divisor.

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$$\begin{array}{r}
 2x^2 \\
 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
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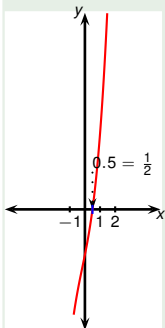
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 \phantom{x - \frac{1}{2}} \phantom{2x^3 -} \phantom{x^2 +} \phantom{5x -} \phantom{3} \phantom{0} \phantom{0} \phantom{0}
 \end{array}$$

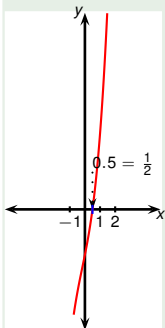
Subtract last two polynomials.

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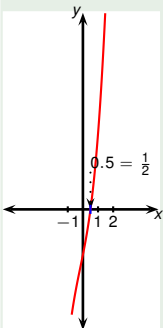


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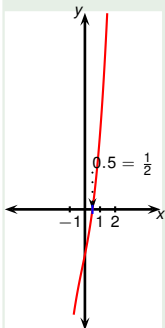
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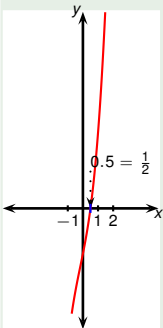
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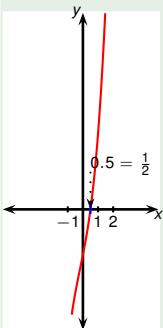
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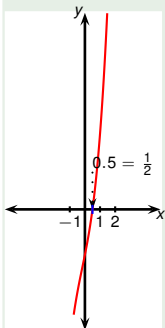
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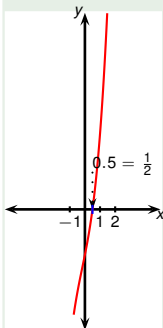
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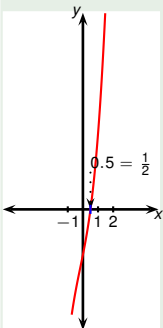
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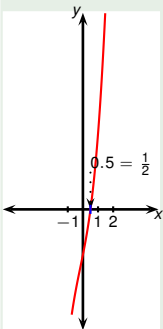
Divide  $6x$  by  $x$ .

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 2x^2 + 2x + 6 \\
 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
 \underline{2x^3 - x^2} \phantom{- 3} \\
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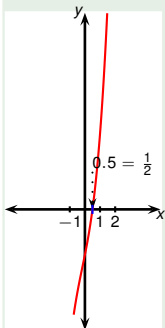


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 6x - 3 \\
 \underline{\phantom{6x} - 3} \\
 6x - 6 \\
 \underline{6x - 6} \\
 0
 \end{array}$$

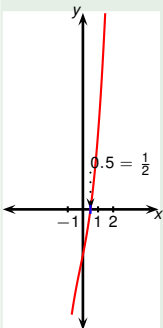
Multiply 6 by divisor.

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 \underline{\phantom{x - \frac{1}{2}} 6x - 3} \\
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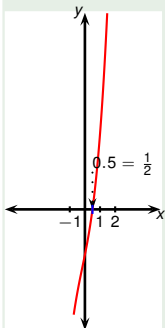
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 6x - 3 \\
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 ?
 \end{array}$$

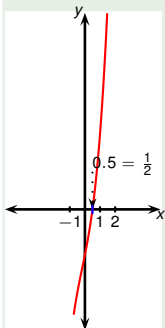
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 \phantom{x - \frac{1}{2}} \phantom{2x^2 + 5x - 3} \phantom{6x - 3} 0
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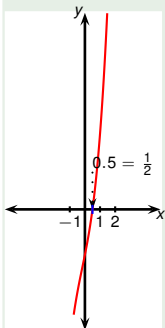
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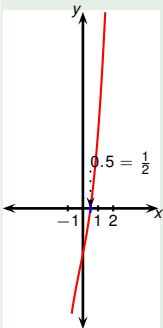
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 6x - 3 \\
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 0
 \end{array}$$

## Example



Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2}) (2x^2 + 2x + 6) + 0 = 0$$

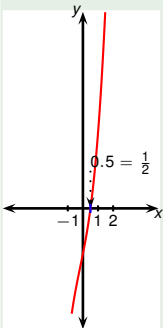
We see only one root,  $x = 0.5 = \frac{1}{2}$ . Is our guess correct?

Is there another root (far away from 0)? Factor:

**Quotient:**  $2x^2 + 2x + 6$

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$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2})(2x^2 + 2x + 6) + 0 = 0$$

We see only one root,  $x = 0.5 = \frac{1}{2}$ . Is our guess correct?

Is there another root (far away from 0)? Factor:

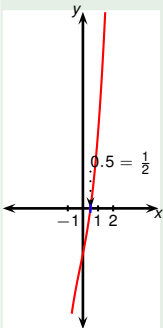
**Quotient:**  $2x^2 + 2x + 6$

$$\begin{array}{r}
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 \underline{2x^3 - x^2} \phantom{- 3} \\
 2x^2 + 5x - 3 \\
 \underline{2x^2 - x} \phantom{- 3} \\
 6x - 3 \\
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 \end{array}$$

**Remainder:**

0

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Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2})(2x^2 + 2x + 6) = 0$$

We see only one root,  $x = 0.5 = \frac{1}{2}$ . Is our guess correct?

Is there another root (far away from 0)? Factor:

**Quotient:**  $2x^2 + 2x + 6$

$$\begin{array}{r}
 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
 \underline{2x^3 - x^2} \phantom{- 3} \\
 2x^2 + 5x - 3 \\
 \underline{2x^2 - x} \phantom{- 3} \\
 6x - 3 \\
 \underline{6x - 3} \\
 0
 \end{array}$$

**Remainder:**

0



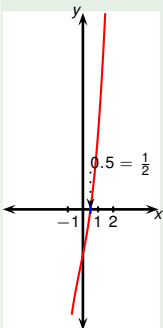
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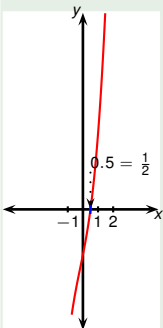
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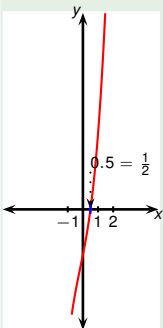
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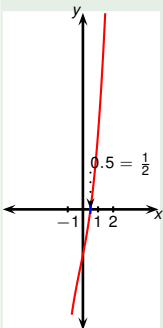
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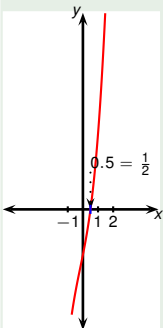
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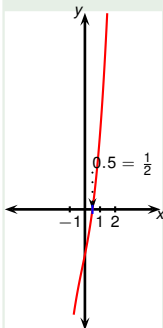
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no real solution

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