

Precalculus

Complex multiplication involving real radicals

Todor Milev

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Example (Complex multiplication)

Multiply $u = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ by $v = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$.

$$u \cdot v$$

Example (Complex multiplication)

Multiply $u = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ by $v = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$.

$$u \cdot v = \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right) \cdot \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \right)$$

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$$\begin{aligned} u \cdot v &= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right) \cdot \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \right) \\ &= \frac{\sqrt{2}^2}{2^2} - \frac{\sqrt{2}^2}{2^2}i + \frac{\sqrt{2}^2}{2^2}i - \frac{\sqrt{2}^2}{2^2}i^2 \end{aligned}$$

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Multiply $u = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ by $v = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$.

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 &= \frac{2}{4} - \frac{2}{4}(-1)
 \end{aligned}$$

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 &= \frac{2}{4} - \frac{2}{4}(-1) \\
 &= \frac{1}{2} + \frac{1}{2} \\
 &= 1
 \end{aligned}$$