PrecalculusPolynomial inequalities

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2019

Outline

Polynomial inequalities

Solve the inequality.

$$2x^2 + 3x - 5 \ge 0$$

Solve the inequality.

$$\begin{array}{ccc} 2x^2 + 3x - 5 & \geq & 0 \\ (? &)(? &) & \geq & 0 \end{array}$$

Solve the inequality.

$$\begin{array}{rcl} 2x^2 + 3x - 5 & \geq & 0 \\ (2x + 5)(x - 1) & \geq & 0 \end{array}$$

Solve the inequality.

$$2x^2 + 3x - 5 \ge 0$$
$$(2x + 5)(x - 1) \ge 0$$

Left hand side vanishes when $x = -\frac{5}{2}$ and when x = 1.



Solve the inequality.

$$\begin{array}{rcl} 2x^2 + 3x - 5 & \geq & 0 \\ (2x + 5)(x - 1) & \geq & 0 \end{array}$$

Left hand side vanishes when $x = -\frac{5}{2}$ and when x = 1.



Solve the inequality.

$$\begin{array}{rcl} 2x^2 + 3x - 5 & \geq & 0 \\ (2x + 5)(x - 1) & \geq & 0 \end{array}$$

$$\left(-\infty,-\frac{5}{2}\right),\left(-\frac{5}{2},1\right),\left(1,\infty\right).$$



	Interval	Factor signs	Final sign	
	$\left(-\infty,-\frac{5}{2}\right)$			
Ī				
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Interval	Factor signs	Final sign	
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Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$			
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Solve the inequality.

$$2x^2 + 3x - 5 \ge 0 (2x + 5)(x - 1) \ge 0$$

Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$			
$(-\frac{5}{2},1)$			
$(1,\infty)$	(?)(?)		

Solve the inequality.

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Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$			
$(-\frac{5}{2},1)$			
$(1,\infty)$	(+)(?)		

Solve the inequality.

$$2x^2 + 3x - 5 \ge 0 (2x + 5)(x - 1) \ge 0$$

$$(\infty, 2), (2,1), (1,\infty).$$



Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$			
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Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$			
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$\left(-\infty,-\frac{5}{2}\right)$			
$\left(-\frac{5}{2}, 1 \right)$	(+)(-)		
$(1,\infty)$	(+)(+)	+	

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$(-\frac{5}{2},1)$	(+)(-)	_	
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Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$	(-)(?)		
$(-\frac{5}{2},1)$	(+)(-)	_	
$(1,\infty)$	(+)(+)	+	

Solve the inequality.

$$2x^2 + 3x - 5 \ge 0 (2x + 5)(x - 1) \ge 0$$

$$\left(-\infty,-\frac{5}{2}\right),\left(-\frac{5}{2},1\right),\left(1,\infty\right).$$



Interval	Factor signs	Final sign	
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$\left(-\frac{5}{2},1\right)$	(+)(-)	-	
$(1,\infty)$	(+)(+)	+	

Solve the inequality.

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	Interval	Factor signs	Final sign	
ĺ	$\left(-\infty,-\frac{5}{2}\right)$	(-)(-)		
	$\left(-\frac{5}{2}, 1\right)$	(+)(-)	-	
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Interval	Factor signs	Final sign	
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$\left(-\frac{5}{2},1\right)$	(+)(-)	_	
$(1,\infty)$	(+)(+)	+	

Solve the inequality.

$$\begin{array}{rcl} 2x^2 + 3x - 5 & \geq & 0 \\ (2x + 5)(x - 1) & \geq & 0 \end{array}$$

$$(-\infty, -\frac{3}{2}), (-\frac{3}{2}, 1), (1, \infty).$$



Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$	(-)(-)	+	
$\left(-\frac{5}{2},1\right)$	(+)(-)	-	
$(1,\infty)$	(+)(+)	+	

Solve the inequality.

$$\begin{array}{rcl} 2x^2 + 3x - 5 & \geq & 0 \\ (2x + 5)(x - 1) & \geq & 0 \end{array}$$

Interval	Factor signs	Final sign	Sample pt	Value at sample pt
$\left(-\infty,-\frac{5}{2}\right)$	(-)(-)	+	-100	f(-100) > 0
$(-\frac{5}{2},1)$	(+)(-)	_	0	f(0) = -5 < 0
$(1,\infty)$	(+)(+)	+	100	f(100) > 0

Solve the inequality.

$$2x^{2} + 3x - 5 \ge 0$$

$$(2x + 5)(x - 1) \ge 0$$
 $x \in ?$

Interval	Factor signs	Final sign	Sample pt	Value at sample pt
$\left(-\infty,-\frac{5}{2}\right)$	(-)(-)	+	-100	f(-100) > 0
$(-\frac{5}{2},1)$	(+)(-)	_	0	f(0) = -5 < 0
$(1,\infty)$	(+)(+)	+	100	f(100) > 0

Solve the inequality.

$$2x^2 + 3x - 5 \geq 0$$

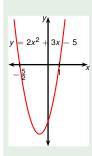
$$(2x + 5)(x - 1) \geq 0$$

$$x \in \left(-\infty, -\frac{5}{2}\right] \cup \left[1, \infty\right)$$

Left hand side vanishes when $x = -\frac{5}{2}$ and when x = 1. The two roots split the real line into three intervals: $(-\infty, -\frac{5}{2})$, $(-\frac{5}{2}, 1)$, $(1, \infty)$.

	Interval	Factor signs	Final sign	Sample pt	Value at sample pt
	$\left(-\infty,-\frac{5}{2}\right)$	(-)(-)	+	-100	f(-100) > 0
Ī	$(-\frac{5}{2},1)$	(+)(-)	_	0	f(0) = -5 < 0
Ī	$(1,\infty)$	(+)(+)	+	100	f(100) > 0

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Solve the inequality.

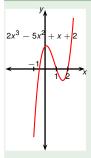
$$\begin{array}{ccc} 2x^2+3x-5 & \geq & 0 \\ (2x+5)(x-1) & \geq & 0 \\ x \in \left(-\infty, -\frac{5}{2}\right] \cup \left[1, \infty\right) \end{array}$$

$$\left(-\infty,-\frac{5}{2}\right),\left(-\frac{5}{2},1\right),\left(1,\infty\right).$$

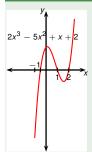


Interval	Factor signs	Final sign	Sample pt	Value at sample pt
$\left(-\infty,-\frac{5}{2}\right)$	(-)(-)	+	-100	f(-100) > 0
$(-\frac{5}{2},1)$	(+)(-)	_	0	f(0) = -5 < 0
$(1,\infty)$	(+)(+)	+	100	f(100) > 0

Plot the function
$$2x^3 - 5x^2 + x + 2$$
. Solve the inequality. $2x^3 - 5x^2 + x + 2 > 0$



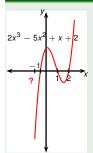
Plot the function
$$2x^3 - 5x^2 + x + 2$$
. Solve the inequality.
 $2x^3 - 5x^2 + x + 2 > 0$



Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality.

$$2x^3 - 5x^2 + x + 2 > 0$$

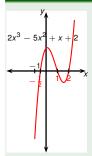
$$2x^3 - 5x^2 + x + 2 > 0$$
? $(x -)(x -)(x -) > 0$



Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality.

$$2x^3 - 5x^2 + x + 2 > 0$$

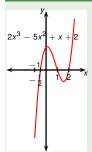
$$2x^3 - 5x^2 + x + 2 > 0$$
? $(x - ?)(x - ?)(x - ?) > 0$



Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality.

$$2x^3 - 5x^2 + x + 2 > 0$$

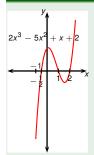
$$? \left(x - \left(-\frac{1}{2}\right)\right) \left(x - 1\right) \left(x - 2\right) > 0$$



Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality.

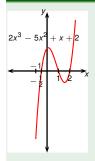
$$2x^3 - 5x^2 + x + 2 > 0$$

?
$$(x-(-\frac{1}{2}))(x-1)(x-2) > 0$$



Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality. $2x^3 - 5x^2 + x + 2 > 0$

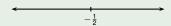
$$\frac{2}{2}(x-(-\frac{1}{2}))(x-1)(x-2) > 0$$

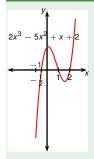


Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality. $2x^3 - 5x^2 + x + 2 > 0$

$$2(x-(-\frac{1}{2}))(x-1)(x-2) > 0$$

Left hand side vanishes when $x = -\frac{1}{2}$, when x = 1 and when x = 2.

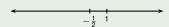


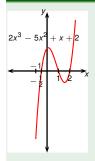


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Left hand side vanishes when $x = -\frac{1}{2}$, when x = 1 and when x = 2.

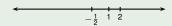


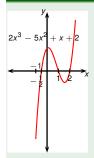


Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality. $2x^3 - 5x^2 + x + 2 > 0$

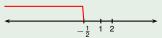
$$2(x-(-\frac{1}{2}))(x-1)(x-\frac{2}{2}) > 0$$

Left hand side vanishes when $x = -\frac{1}{2}$, when x = 1 and when x = 2.

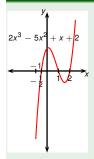




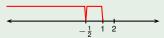
Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality. $2x^3 - 5x^2 + x + 2 > 0$ $2(x - (-\frac{1}{2}))(x - 1)(x - 2) > 0$



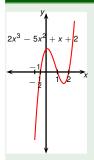
Interval	Factor signs	Final sign from plot
$\left(-\infty,-\frac{1}{2}\right)$		



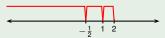
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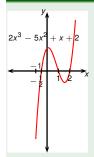
Interval	Factor signs	Final sign from plot
$ \begin{pmatrix} (-\infty, -\frac{1}{2}) \\ (-\frac{1}{2}, 1) \end{pmatrix} $		



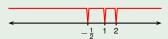
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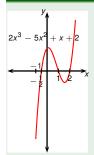
Interval	Factor signs	Final sign from plot
$ \begin{pmatrix} (-\infty, -\frac{1}{2}) \\ (-\frac{1}{2}, 1) \\ (1, 2) \end{pmatrix} $		



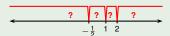
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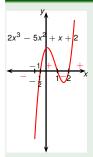
Interval	Factor signs	Final sign from plot
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(1,2)		
$(2,\infty)$		



Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality. $2x^3 - 5x^2 + x + 2 > 0$ $2(x - (-\frac{1}{2}))(x - 1)(x - 2) > 0$



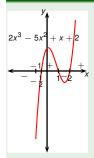
Interval	Factor signs	Final sign from plot
$\left(-\infty,-\frac{1}{2}\right)$?	?
$\left(-\frac{1}{2},1\right)$?	?
(1,2)	?	?
$(2,\infty)$?	?



Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality. $2x^3 - 5x^2 + x + 2 > 0$ $2(x - (-\frac{1}{2}))(x - 1)(x - 2) > 0$



Interval	Factor signs	Final sign from plot
$\left(-\infty,-\frac{1}{2}\right)$	(-)(-)(-)	_
$\left(-\frac{1}{2},1\right)$	(+)(-)(-)	+
(1,2)	(+)(+)(-)	_
$(2,\infty)$	(+)(+)(+)	+



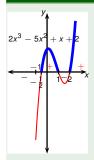
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Left hand side vanishes when $x = -\frac{1}{2}$, when x = 1 and when x = 2. The two roots split the real line into four intervals: $(-\infty, -\frac{1}{2})$, $(-\frac{1}{2}, 1)$, (1, 2), $(2, \infty)$.



x ∈ ?

Interval	Factor signs	Final sign from plot
$\left(-\infty,-\frac{1}{2}\right)$	(-)(-)(-)	_
$\left(-\frac{1}{2},1\right)^{-1}$	(+)(-)(-)	+
(1,2)	(+)(+)(-)	_
$(2,\infty)$	(+)(+)(+)	+



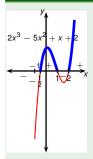
Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality. $2x^3 - 5x^2 + x + 2 > 0$

$$2(x - (-\frac{1}{2}))(x - 1)(x - 2) > 0$$

$$x \in (-\frac{1}{2}, 1) \cup (2, \infty)$$



Interval	Factor signs	Final sign from plot
$\left(-\infty,-\frac{1}{2}\right)$	(-)(-)(-)	_
$\left(-\frac{1}{2},1\right)^{-1}$	(+)(-)(-)	+
(1,2)	(+)(+)(-)	_
$(2,\infty)$	(+)(+)(+)	+



Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality. $2x^3 - 5x^2 + x + 2 > 0$

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 $x \in (-\frac{1}{2}, 1) \cup (2, \infty)$



Interval	Factor signs	Final sign from plot
$\left(-\infty,-\frac{1}{2}\right)$	(-)(-)(-)	_
$\left(-\frac{1}{2},1\right)$	(+)(-)(-)	+
(1,2)	(+)(+)(-)	_
$(2,\infty)$	(+)(+)(+)	+