

Precalculus

Factor cubic with one real root using its plot

Todor Milev

2019

Example

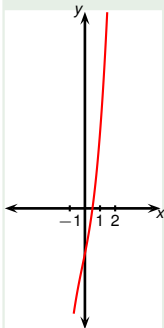
Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

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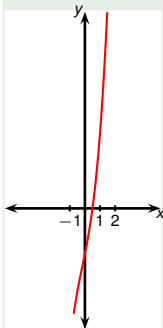


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We see only one root, $x = ?$.

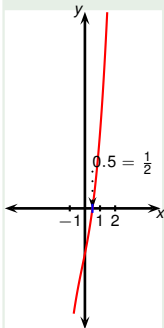


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We see only one root, $x = 0.5 = \frac{1}{2}$.

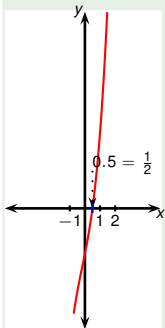


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Is there another root (far away from 0)?



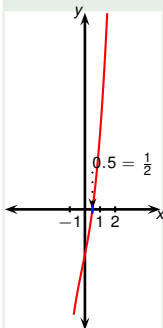
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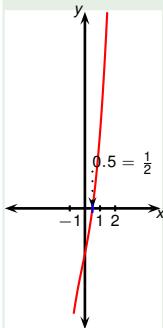
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$$x - \frac{1}{2} \quad \overline{2x^3 + x^2 + 5x - 3}$$



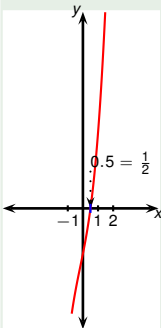
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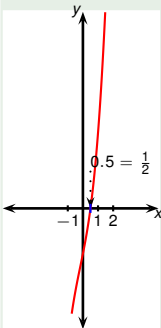
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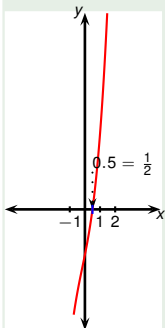


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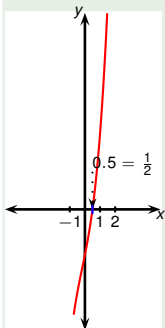
Divide $2x^3$ by x .

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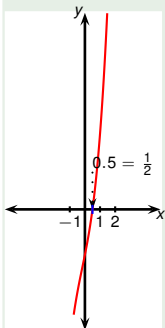
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$$\begin{array}{r}
 2x^2 \\
 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
 \underline{\phantom{x - \frac{1}{2}} ? ?}
 \end{array}$$

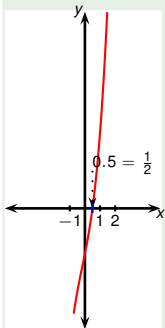
Multiply $2x^2$ by divisor.

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Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

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$$\begin{array}{r}
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 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
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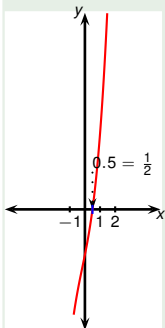
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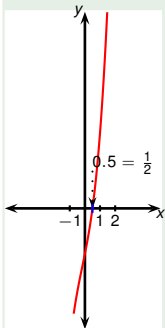
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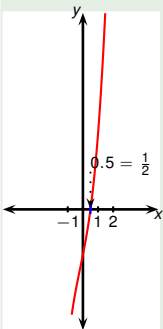
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 \hline
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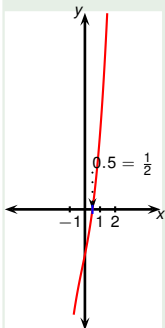
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$$\begin{array}{r}
 2x^2 + 2x \\
 \hline
 x - \frac{1}{2} \quad \overline{2x^3 + x^2 + 5x - 3} \\
 \quad \quad \quad \underline{2x^3 - x^2} \\
 \quad \quad \quad \quad \quad \quad 2x^2 + 5x - 3
 \end{array}$$

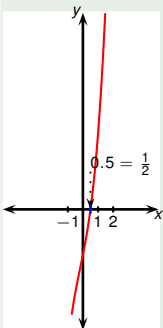
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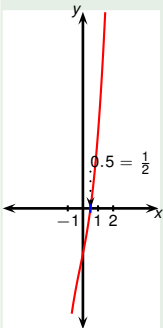
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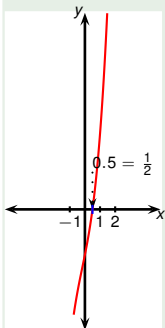
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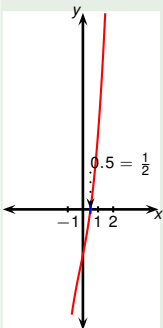
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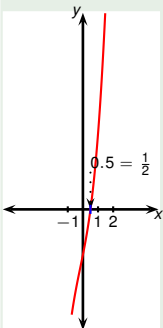
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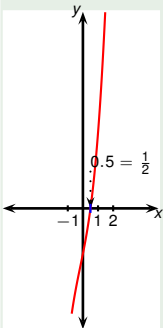
Divide $6x$ by x .

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We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:



$$\begin{array}{r}
 2x^2 + 2x + 6 \\
 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
 \underline{2x^3 - x^2} \\
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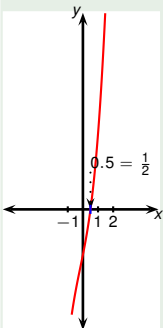
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 \phantom{x - \frac{1}{2}} 0
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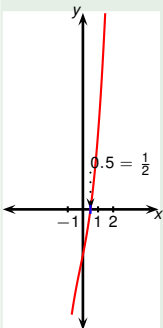
Multiply 6 by divisor.

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 6x - 3 \\
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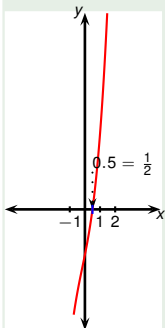
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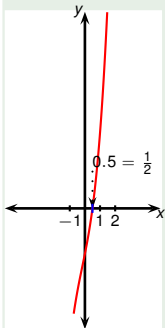
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 \underline{2x^3 - x^2} \\
 2x^2 + 5x - 3 \\
 \underline{2x^2 - x} \\
 6x - 3 \\
 \underline{6x - 3} \\
 0
 \end{array}$$

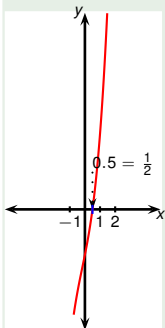
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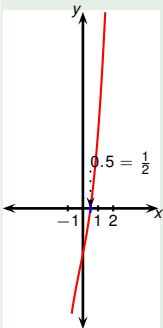
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 \underline{2x^2 - x} \\
 6x - 3 \\
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 0
 \end{array}$$

Example



Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2}) (2x^2 + 2x + 6) + 0 = 0$$

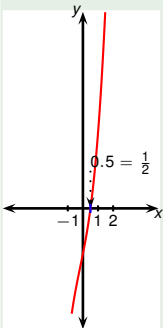
We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct?

Is there another root (far away from 0)? Factor:

Quotient: $2x^2 + 2x + 6$

$$\begin{array}{r}
 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
 \underline{2x^3 - x^2} \\
 2x^2 + 5x - 3 \\
 \underline{2x^2 - x} \\
 6x - 3 \\
 \underline{6x - 3} \\
 0
 \end{array}$$

Example



Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2})(2x^2 + 2x + 6) + 0 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct?

Is there another root (far away from 0)? Factor:

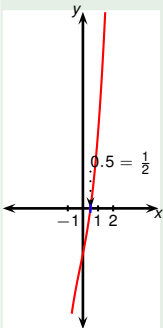
Quotient: $2x^2 + 2x + 6$

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 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
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 2x^2 + 5x - 3 \\
 \underline{2x^2 - x} \\
 6x - 3 \\
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 \end{array}$$

Remainder:

0

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 \underline{2x^2 - x} \\
 6x - 3 \\
 \underline{6x - 3} \\
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Remainder:

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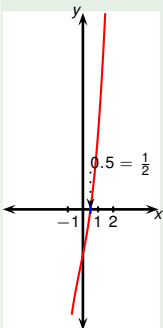
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$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2})(2x^2 + 2x + 6) = 0$$

$$x - \frac{1}{2} = 0 \quad \text{or} \quad x =$$



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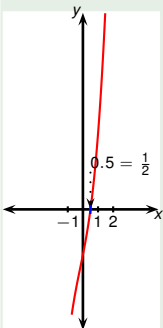
Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2})(2x^2 + 2x + 6) = 0$$

$$x - \frac{1}{2} = 0 \quad \text{or} \quad x =$$

$$x = \frac{1}{2}$$



We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)?

Example

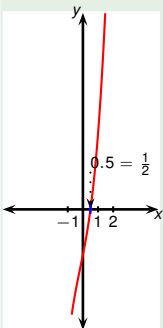
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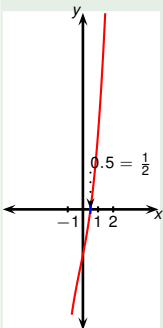
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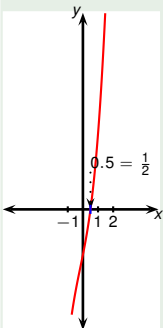
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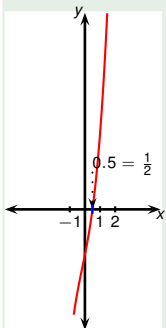
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no real solution

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