Precalculus

Express $\sin(kx)$, $\cos(kx)$ **via** $\sin x$, $\cos x$

Todor Milev

2019

$$\sin(3x) = \sin(x + 2x)$$

Recall the formulas
$$\frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = ?$$

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$$\frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos(\alpha + \beta)}$$

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$$\sin(3x) = \sin(x + 2x)$$

$$= \sin x \cos(2x) + \cos x \sin(2x)$$

Recall the formulas $\frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\cos \alpha \cos \beta - \sin \alpha \sin \beta}.$

Example

$$\sin(3x) = \sin(x + \frac{2x}{2x})$$

$$= \sin x \cos(\frac{2x}{2x}) + \cos x \sin(\frac{2x}{2x})$$

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Example

Express sin(3x) and cos(3x) via cos x and sin x.

$$\sin(3x) = \sin(x+2x)$$

$$= \sin x \cos(2x) + \cos x \sin(2x)$$

$$= \sin x ?$$

cos(3x)

Recall the formulas
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Example

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