

# Calculus I

## § Definite integrals of rational power monomials

Todor Milev

2019

## Example

Evaluate:  $\int_1^9 \frac{2t^3 + t^2\sqrt{t} - 1}{t^2} dt$

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$$= \int_1^9 \left( 2t + t^{\frac{1}{2}} - t^{-2} \right) dt$$

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$$= \int_1^9 (2t + t^{\frac{1}{2}} - t^{-2}) dt = \left[ \int (2t + t^{\frac{1}{2}} - t^{-2}) dt \right]_1^9$$

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Evaluate:  $\int_1^9 \frac{2t^3 + t^2\sqrt{t} - 1}{t^2} dt$

$$\begin{aligned} &= \int_1^9 (2t + t^{\frac{1}{2}} - t^{-2}) dt = \left[ \int (2t + t^{\frac{1}{2}} - t^{-2}) dt \right]_1^9 \\ &= \left[ \int 2t dt + \int t^{\frac{1}{2}} dt - \int t^{-2} dt \right]_1^9 \end{aligned}$$

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 \text{Evaluate: } & \int_1^9 \frac{2t^3 + t^2\sqrt{t} - 1}{t^2} dt \\
 &= \int_1^9 \left( \color{red}{2t} + t^{\frac{1}{2}} - t^{-2} \right) \color{red}{dt} = \left[ \int (2t + t^{\frac{1}{2}} - t^{-2}) dt \right]_1^9 \\
 &= \left[ \int \color{red}{2t} dt + \int t^{\frac{1}{2}} dt - \int t^{-2} dt \right]_1^9 \\
 &= \left[ \color{red}{?} + ? - ? \right]_1^9
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 &= \left( 9^2 + \frac{2}{3} \cdot 9^{\frac{3}{2}} + \frac{1}{9} \right) - \left( 1^2 + \frac{2}{3} \cdot 1^{\frac{3}{2}} + \frac{1}{1} \right)
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 &= 81 + 18 + \frac{1}{9} - 1 - \frac{2}{3} - 1
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 &= 81 + 18 + \frac{1}{9} - 1 - \frac{2}{3} - 1 = \frac{868}{9}.
 \end{aligned}$$