Calculus II
Integrals with irreducible quadratic denomonimator

Todor Milev

2019

Building blocks IIa and IIIa

Building block IIa: $\int \frac{x}{1+x^2}\mathrm{d}x$. Building block IIIa: $\int \frac{1}{1+x^2}\mathrm{d}x$.

Example (Block IIa)

$$\int \frac{x}{1+x^2}\mathrm{d}x$$

Building blocks IIa and IIIa

Building block IIa: $\int \frac{x}{1+x^2}\mathrm{d}x$. Building block IIIa: $\int \frac{1}{1+x^2}\mathrm{d}x$.

Example (Block IIa)

$$\int \frac{x}{1+x^2}\mathrm{d}x = \int \frac{1}{(1+x^2)}\frac{\mathrm{d}(x^2)}{2}$$

Building blocks IIa and IIIa

Building block IIa: $\int \frac{x}{1+x^2}\mathrm{d}x$. Building block IIIa: $\int \frac{1}{1+x^2}\mathrm{d}x$.

Example (Block IIa)

$$\begin{aligned}\int \frac{x}{1+x^2}\mathrm{d}x &= \int \frac{1}{(1+x^2)}\frac{\mathrm{d}(x^2)}{2} \\ &= \int \frac{1}{1+x^2}\frac{\mathrm{d}(1+x^2)}{2}\end{aligned}$$

Building blocks IIa and IIIa

Building block IIa: $\int \frac{x}{1+x^2}\mathrm{d}x$. Building block IIIa: $\int \frac{1}{1+x^2}\mathrm{d}x$.

Example (Block IIa)

$$\begin{aligned}\int \frac{x}{1+x^2}\mathrm{d}x &= \int \frac{1}{(1+x^2)}\frac{\mathrm{d}(x^2)}{2} \\ &= \int \frac{1}{1+x^2}\frac{\mathrm{d}(1+x^2)}{2} \\ &= \int \frac{1}{u}\frac{\mathrm{d}u}{2}\end{aligned}$$

Set $u = 1 + x^2$

Building blocks IIa and IIIa

Building block IIa: $\int \frac{x}{1+x^2}\mathrm{d}x$. Building block IIIa: $\int \frac{1}{1+x^2}\mathrm{d}x$.

Example (Block IIa)

$$\begin{aligned}
\int \frac{x}{1+x^2}\mathrm{d}x &= \int \frac{1}{(1+x^2)}\frac{\mathrm{d}(x^2)}{2} \\
&= \int \frac{1}{1+x^2}\frac{\mathrm{d}(1+x^2)}{2} \qquad \Big| \text{ Set } u = 1+x^2 \\
&= \int \frac{1}{u}\frac{\mathrm{d}u}{2} \\
&= \frac{1}{2}\ln|u| + C
\end{aligned}$$

Building blocks IIa and IIIa

Building block IIa: $\int \frac{x}{1+x^2}\mathrm{d}x$. Building block IIIa: $\int \frac{1}{1+x^2}\mathrm{d}x$.

Example (Block IIa)

$$\begin{aligned}
\int \frac{x}{1+x^2}\mathrm{d}x &= \int \frac{1}{(1+x^2)}\frac{\mathrm{d}(x^2)}{2} \\
&= \int \frac{1}{1+x^2}\frac{\mathrm{d}(1+x^2)}{2} \qquad \Big| \text{ Set } u = 1+x^2 \\
&= \int \frac{1}{u}\frac{\mathrm{d}u}{2} \\
&= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln\left(1+x^2\right) + C \quad .
\end{aligned}$$

Building blocks IIa and IIIa

Building block IIa: $\int \frac{x}{1+x^2}\mathrm{d}x$. Building block IIIa: $\int \frac{1}{1+x^2}\mathrm{d}x$.

Example (Block IIa)

$$\begin{aligned}\int \frac{x}{1+x^2}\mathrm{d}x &= \int \frac{1}{(1+x^2)}\frac{\mathrm{d}(x^2)}{2} \\ &= \int \frac{1}{1+x^2}\frac{\mathrm{d}(1+x^2)}{2} \qquad \Big| \text{ Set } u = 1+x^2 \\ &= \int \frac{1}{u}\frac{\mathrm{d}u}{2} \\ &= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln\left(1+x^2\right) + C \quad .\end{aligned}$$

Example (Block IIIa)

$$\int \frac{1}{1+x^2}\mathrm{d}x$$

Building blocks IIa and IIIa

Building block IIa: $\int \frac{x}{1+x^2}\mathrm{d}x$. Building block IIIa: $\int \frac{1}{1+x^2}\mathrm{d}x$.

Example (Block IIa)

$$\begin{aligned}\int \frac{x}{1+x^2}\mathrm{d}x &= \int \frac{1}{(1+x^2)}\frac{\mathrm{d}(x^2)}{2} \\ &= \int \frac{1}{1+x^2}\frac{\mathrm{d}(1+x^2)}{2} \qquad \Big| \text{ Set } u = 1+x^2 \\ &= \int \frac{1}{u}\frac{\mathrm{d}u}{2} \\ &= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln\left(1+x^2\right) + C \quad .\end{aligned}$$

Example (Block IIIa)

$$\int \frac{1}{1+x^2}\mathrm{d}x = \textbf{?}$$

Building blocks IIa and IIIa

Building block IIa: $\int \frac{x}{1+x^2}\mathrm{d}x$. Building block IIIa: $\int \frac{1}{1+x^2}\mathrm{d}x$.

Example (Block IIa)

$$\begin{aligned}\int \frac{x}{1+x^2}\mathrm{d}x &= \int \frac{1}{(1+x^2)}\frac{\mathrm{d}(x^2)}{2} \\ &= \int \frac{1}{1+x^2}\frac{\mathrm{d}(1+x^2)}{2} \qquad \Big| \text{ Set } u = 1+x^2 \\ &= \int \frac{1}{u}\frac{\mathrm{d}u}{2} \\ &= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln\left(1+x^2\right) + C \quad .\end{aligned}$$

Example (Block IIIa)

$$\int \frac{1}{1+x^2}\mathrm{d}x = \arctan x + C$$