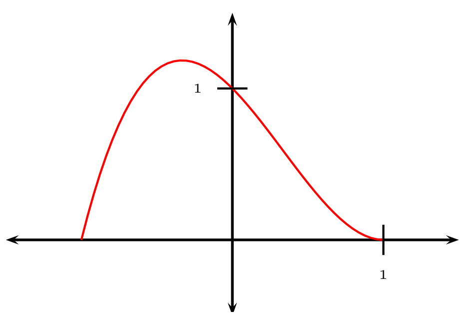
Calculus I
Homework
Maxima and minima over closed intervals

1. Find the critical points of the function. Identify whether those are local maxima, minima, or neither. The answer key has not been proofread, use with caution.

   (a) $f(x) = \dfrac{x}{1+x^2}$.
   (b) $f(x) = x^3 - x^2 - x - 1$.
   (c) $f(x) = 2x^3 - x^2 - 20x + 1$.
   (d) $f(x) = x + \dfrac{1}{x}$.
   (e) $f(x) = \dfrac{x - \frac{1}{2}}{x^2 - 2x + \frac{7}{4}}$.

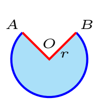
2. Find the maximum and minimum values of f on the given interval and the values of x for which they are attained.

   (a) $f(x) = 9 + 3x - x^2$, $x \in [0, 4]$.
   (b) $f(x) = 5 + 4x - 2x^3$, $x \in [-1, 1]$.
   (c) $f(x) = 2x^3 - x^2 - 20x + 1$, $x \in [-4, 3]$.
   (d) $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$, $x \in [-2, 3]$.
   (e) $f(x) = x^3 - x^2 - x + 1$, $x \in [-1, 1]$.

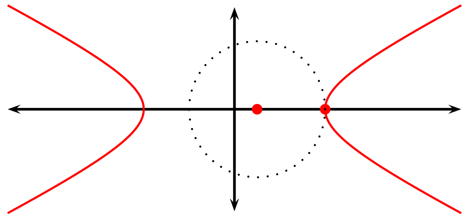


   (f) $f(x) = x^3 - x + 1$, $x \in [-2, 1]$.
   (g) $f(x) = (x^2 - 1)^3$, $x \in [-1, 2]$.
   (h) $f(x) = x + \dfrac{1}{x}$, $x \in [0.2, 4]$.
   (i) $f(x) = \dfrac{x}{x^2 - x + 1}$, $x \in [0, 3]$.
   (j) $f(t) = t\sqrt{4 - t^2}$, $x \in [-1, 2]$.
   (k) $f(t) = \sqrt[3]{t}(8 - t)$, $x \in [0, 8]$.
   (l) $f(t) = 2\cos t + \sin(2t)$, $x \in [0, \frac{\pi}{2}]$.
   (m) $f(t) = t + \cot\left(\dfrac{t}{2}\right)$, $x \in [\frac{\pi}{4}, \frac{7\pi}{4}]$.
   (n) $f(t) = t + \cot\left(\dfrac{t}{2}\right)$, $x \in [\frac{\pi}{4}, \frac{7\pi}{4}]$.
   (o) $f(x) = xe^{3x}$, $x \in \left[-3, \frac{1}{6}\right]$.
   (p) $f(x) = (x-2)(x+1)e^x$, $x \in [-5, 2]$.
   (q) $f(x) = (x+1)e^{-x^2}$, $x \in [-3, 3]$.
   (r) $f(x) = xe^{2x}$, $x \in \left[-2, \frac{1}{2}\right]$.

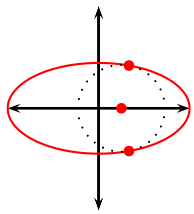
3. (a) Find the dimensions of a rectangle with area 1000 m^2 whose perimeter is as small as possible.
   (b) A box with an open top is to be constructed from a square piece of cardboard, 1m wide, by cutting out a square from each of the four corners and bending up the sides. Find the largest volume that such a box can have.
   (c) A right circular cylinder is inscribed in a sphere of radius r. Find the largest possible volume of such a cylinder.
   (d) A wedge of radius 2 (depicted below) is folded into a cone cup. The volume varies depending on the angle of the wedge. Find the maximal possible volume of the cone cup and the angle of the wedge for which this maximal volume is achieved.



4. (a) What is the x-coordinate of the point on the hyperbola $x^2 - 4y^2 = 16$ that is closest to the point $(1, 0)$?



(b) What is the x-coordinate of the point on the ellipse $x^2 + 4y^2 = 16$ closest to the point $(1, 0)$?



(c) A rectangular box with a square base is being built out of sheet metal. 2 pieces of sheet will be used for the bottom of the box, and a single piece of sheet metal for the 4 sides and the top of the box. What is the largest possible volume of the resulting box that can be obtained with $36m^2$ of metal sheet?

(d) Recall that the volume of a cylinder is computed as the product of the area of its base by its height. Recall also that the surface area of the wall of a cylinder is given by multiplying the perimeter of the base by the height of the cylinder.

A cylindrical container with an open top is being built from metal sheet. The total surface area of metal used must equal $10m^2$. Let r denote the radius of the base of the cylinder, and h its height. How should one choose h and r so as to get the maximal possible container volume? What will the resulting container volume be?