Precalculus
A useful inverse hyperbolic function identity

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The inverse hyperbolic function $\operatorname{arcsinh} = \ln\left(x + \sqrt{1 + x^2}\right)$ is used when studying hyperbolas (types of curves in the plane).

Example

Demonstrate that $-\ln\left(\sqrt{1 + x^2} - x\right) = \ln\left(x + \sqrt{1 + x^2}\right)$.

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$$
\begin{aligned}
-\ln\left(\sqrt{1+x^2} - x\right) &= \ln\left(\frac{1}{\sqrt{x^2+1} - x}\right) && \Big| \text{ rationalize} \\
&= \ln\left(\frac{\left(\sqrt{x^2+1} + x\right)}{\left(\sqrt{x^2+1} - x\right)\left(\sqrt{x^2+1} + x\right)}\right) \\
&= \ln\left(\frac{\sqrt{x^2+1} + x}{x^2 + 1 - x^2}\right) \\
&= \ln\left(\sqrt{x^2+1} + x\right) \quad .
\end{aligned}
$$