

Calculus II

Integrals of the form $\int \sin^n x \cos^m x dx$, at least
one power is odd

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Example

$$\begin{aligned}
 \int \sin^3 x dx &= \int \sin^2 x \sin x dx \\
 &= \int \sin^2 x d(-\cos x) \\
 &= \int (-1) (1 - \cos^2 x) d(\cos x) \\
 &= \int (\cos^2 x - 1) d(\cos x) \\
 &= \int (u^2 - 1) du \\
 &= \frac{u^3}{3} - u + C \\
 &= \frac{1}{3} \cos^3 x - \cos x + C .
 \end{aligned}$$

Can we rewrite
 $\sin^2 x$ via $\cos x$?

Set $u = \cos x$

Example

$$\begin{aligned}
 \int \cos^5 x \sin^2 x dx &= \int \cos^4 x \sin^2 x \cos x dx \\
 &= \int \cos^4 x \sin^2 x d(\sin x) && \left| \begin{array}{l} \text{Can we rewrite} \\ \cos^4 x \text{ via } \sin x? \end{array} \right. \\
 &= \int (\cos^2 x)^2 \sin^2 x d(\sin x) \\
 &= \int (1 - \sin^2 x)^2 \sin^2 x d(\sin x) && \left| \begin{array}{l} \text{Set } u = \sin x \end{array} \right. \\
 &= \int (1 - u^2)^2 u^2 du \\
 &= \int (1 - 2u^2 + u^4) u^2 du \\
 &= \int (u^2 - 2u^4 + u^6) du \\
 &= \frac{u^3}{3} - 2\frac{u^5}{5} + \frac{u^7}{7} + C \\
 &= \frac{\sin^3 x}{3} - 2\frac{\sin^5 x}{5} + \frac{\sin^7 x}{7} + C .
 \end{aligned}$$