Precalculus

Use polynomial division with remainder 0 to factor a polynomial

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Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by 2x - 3 using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

Quotient:
$$3x^2 - 5x + 1$$

$$2x - 3 = 6x^3 - 19x^2 + 17x - 3$$

$$- 6x^3 - 9x^2$$

$$- 10x^2 + 17x - 3$$

$$- 10x^2 + 15x$$

$$- 2x - 3$$
Remainder: 0

Remainder:

(Dividend)=(Quotient) · (Divisor) + (Remainder)

$$(6x^3 - 19x^2 + 17x - 3) = (3x^2 - 5x + 1) \cdot (2x - 3)$$

Example

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$$(6x^{3} - 19x^{2} + 17x - 3) = (3x^{2} - 5x + 1) \cdot (2x - 3)$$

$$= 3\left(x - \left(\frac{5 + \sqrt{13}}{6}\right)\right)\left(x - \left(\frac{5 - \sqrt{13}}{6}\right)\right)(2x - 3)$$

No easy factorization of quadratic, so use formula:

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 3 \cdot 1}}{2 \cdot 3} = \frac{5 \pm \sqrt{13}}{6}$$
 We are ready to solve the equation.

$$6x^{3} - 19x^{2} + 17x - 3 = 0$$

$$3\left(x - \left(\frac{5 + \sqrt{13}}{6}\right)\right)\left(x - \left(\frac{5 - \sqrt{13}}{6}\right)\right)(2x - 3) = 0$$

$$2x - 3 = 0 \quad \text{or} \quad x = \left(\frac{5 + \sqrt{13}}{6}\right) \quad \text{or} \quad x = \left(\frac{5 - \sqrt{13}}{6}\right)$$

$$x = \frac{3}{2}$$