Precalculus
Computing sine, cosine of a half-angle

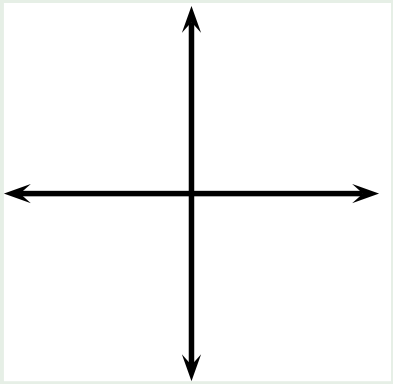
Todor Milev

2019

Example

Using radicals, find the exact value of the trigonometric expression.

$$\cos 105^\circ$$



Recall the half angle formula $\cos\alpha = \pm\sqrt{\frac{1+\cos(2\alpha)}{2}}$.

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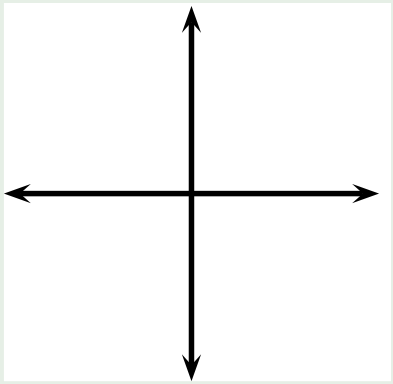

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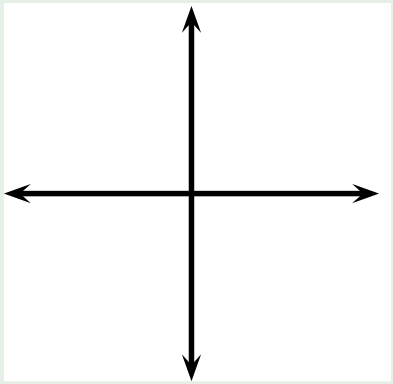

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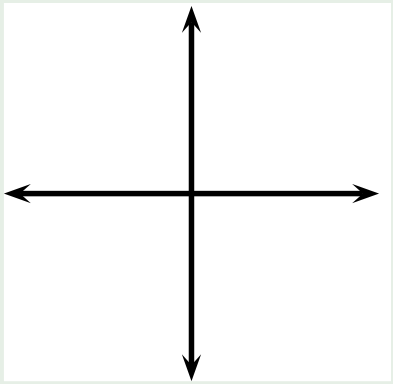

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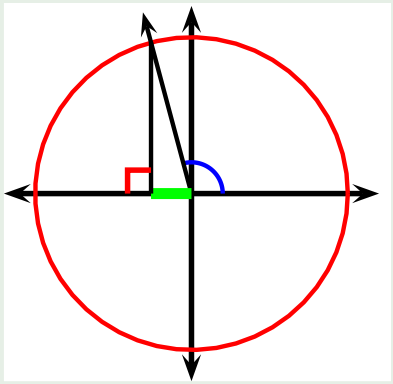

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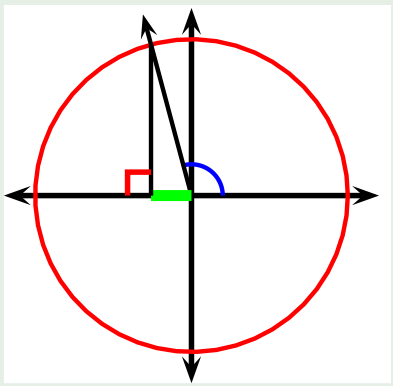
$$= -\sqrt{\frac{1+\cos(210^\circ)}{2}}$$



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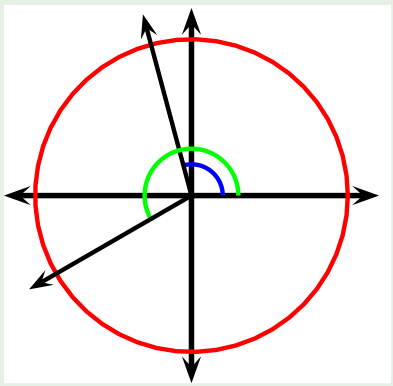

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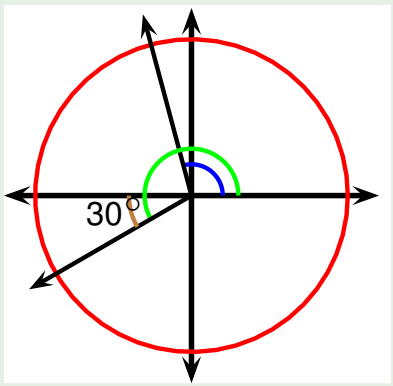

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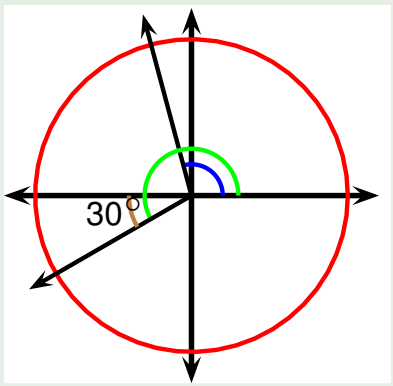

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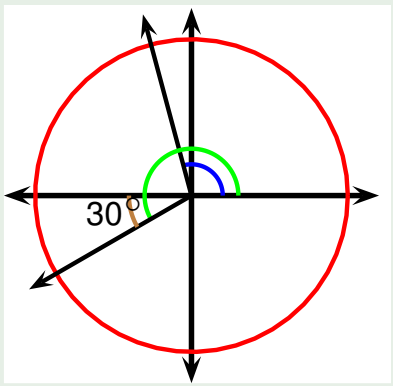

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$$= -\sqrt{\frac{1-?}{2}}$$

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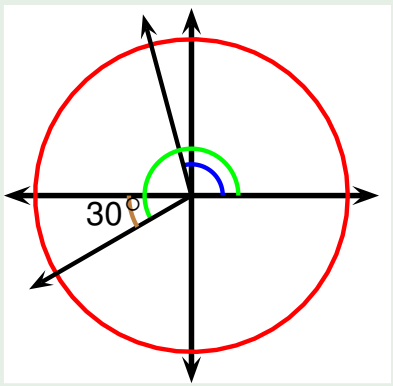

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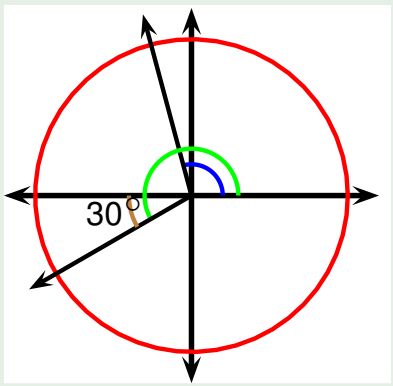

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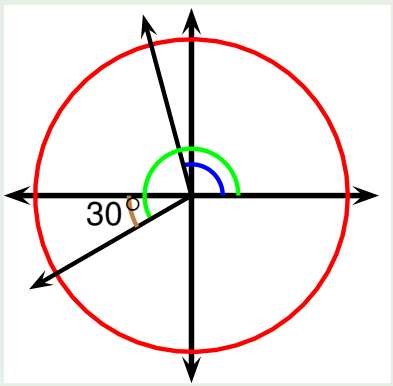

$$\begin{aligned}
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&= -\sqrt{\frac{1+\cos(210^\circ)}{2}} \\
&= -\sqrt{\frac{1-\cos(30^\circ)}{2}} \\
&= -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2-\sqrt{3}}{2\cdot 2}} \\
&= -\frac{\sqrt{2-\sqrt{3}}}{2}
\end{aligned}$$