Precalculus
Factor quadratic with irrational real roots

Todor Milev

2019

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

$$\left| \begin{array}{rcl} x_1 x_2 & = & \frac{c}{a} \\ x_1 + x_2 & = & -\frac{b}{a} \end{array} \right.$$

Example

Factor the quadratic.

$$x^2 + 3x + 1$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

$$\left|\begin{array}{rcl} x_1 x_2 & = & \frac{c}{a} \\ x_1 + x_2 & = & -\frac{b}{a} \end{array}\right.$$

Example

Factor the quadratic.

$$x^2 + 3x + 1 = \left(x + \mathbf{?} \qquad\qquad \right)\left(x + \mathbf{?} \qquad\qquad \right)$$

- The product of the two roots: $x_1 x_2 = 1$.

$$ax^2 + bx + c = a\left(x - x_1\right)\left(x - x_2\right),$$

$$\left|\begin{array}{rcl} x_1 x_2 & = & \frac{c}{a} \\ x_1 + x_2 & = & -\frac{b}{a} \end{array}\right.$$

Example

Factor the quadratic.

$$x^2 + 3x + 1 = \left(x + \textbf{?} \qquad\qquad\right)\left(x + \textbf{?} \qquad\qquad\right)$$

- The product of the two roots: $x_1 x_2 = 1$.
- Integer options: $x_1 = 1, x_2 = 1$ and $x_1 = -1, x_2 = -1$.

$$ax^2 + bx + c = a\left(x - x_1\right)\left(x - x_2\right),$$

$$\left|\begin{array}{rcl} x_1 x_2 & = & \frac{c}{a} \\ x_1 + x_2 & = & -\frac{b}{a} \end{array}\right.$$

Example

Factor the quadratic.

$$x^2 + 3x + 1 = \left(x + \textbf{?} \qquad\qquad \right)\left(x + \textbf{?} \qquad\qquad \right)$$

- The product of the two roots: $x_1 x_2 = 1$.
- Integer options: $x_1 = 1, x_2 = 1$ and $x_1 = -1, x_2 = -1$.
- $(x - 1)(x - 1)$
  $(x + 1)(x + 1)$

$$ax^2 + bx + c = a\,(x - x_1)\,(x - x_2)\,,$$

$$\left|\begin{array}{rcl} x_1 x_2 & = & \dfrac{c}{a} \\ x_1 + x_2 & = & -\dfrac{b}{a} \end{array}\right.$$

Example

Factor the quadratic.

$$x^2 + 3x + 1 = \left(x + \textbf{?} \qquad\qquad\right)\left(x + \textbf{?} \qquad\qquad\right)$$

- The product of the two roots: $x_1 x_2 = 1$.
- Integer options: $x_1 = 1, x_2 = 1$ and $x_1 = -1, x_2 = -1$.
- $(x - 1)(x - 1)$
  $(x + 1)(x + 1)$

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

$$\begin{array}{|rcl} x_1 x_2 & = & \frac{c}{a} \\ x_1 + x_2 & = & -\frac{b}{a} \end{array}$$

Example

Factor the quadratic.

$$x^2 + 3x + 1 = \left(x + \textbf{?} \qquad\qquad \right)\left(x + \textbf{?} \qquad\qquad \right)$$

- The product of the two roots: $x_1 x_2 = 1$.
- Integer options: $x_1 = 1, x_2 = 1$ and $x_1 = -1, x_2 = -1$.
- $\begin{array}{rcl} (x-1)(x-1) & = & (x-1)^2 = x^2 - 2x + 1 \\ (x+1)(x+1) & = & (x+1)^2 = x^2 + 2x + 1 \end{array}$ both don't work.

$$ax^2+bx+c=a(x-x_1)(x-x_2),$$

$$\left|\begin{array}{rcl} x_1x_2 &=& \frac{c}{a}\\ x_1+x_2 &=& -\frac{b}{a}\end{array}\right.$$

Example

Factor the quadratic.

$$x^2+3x+1=\left(x+\textbf{?}\qquad\qquad\right)\left(x+\textbf{?}\qquad\qquad\right)$$

- The product of the two roots: $x_1x_2=1$.
- Integer options: $x_1=1, x_2=1$ and $x_1=-1, x_2=-1$.
- $\begin{array}{rcl}(x-1)(x-1)&=&(x-1)^2=x^2-2x+1\\(x+1)(x+1)&=&(x+1)^2=x^2+2x+1\end{array}$ both don't work.
- $\Rightarrow$ No easy factorization; must use quadratic formula.

$$x_1,x_2 = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

$$\left|\begin{array}{rcl} x_1 x_2 & = & \frac{c}{a} \\ x_1 + x_2 & = & -\frac{b}{a} \end{array}\right.$$

Example

Factor the quadratic.

$$x^2 + 3x + 1 = \left(x + \textbf{?} \qquad\qquad\qquad \right)\left(x + \textbf{?} \qquad\qquad\qquad \right)$$

- The product of the two roots: $x_1 x_2 = 1$.
- Integer options: $x_1 = 1, x_2 = 1$ and $x_1 = -1, x_2 = -1$.
- $\begin{array}{rcl} (x-1)(x-1) & = & (x-1)^2 = x^2 - 2x + 1 \\ (x+1)(x+1) & = & (x+1)^2 = x^2 + 2x + 1 \end{array}$ both don't work.
- $\Rightarrow$ No easy factorization; must use quadratic formula.

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$ax^2+bx+c=a\left(x-x_1\right)\left(x-x_2\right),\qquad \left|\begin{array}{rcl} x_1x_2 &=& \frac{c}{a}\\ x_1+x_2 &=& -\frac{b}{a}\end{array}\right.$$

Example

Factor the quadratic.

$$x^2+{\color{red}3}x+1=\Big(x+\textbf{?}\qquad\qquad\Big)\Big(x+\textbf{?}\qquad\qquad\Big)$$

- The product of the two roots: $x_1x_2=1$.
- Integer options: $x_1=1, x_2=1$ and $x_1=-1, x_2=-1$.
- $\begin{array}{rcl}(x-1)(x-1) &=& (x-1)^2=x^2-2x+1\\ (x+1)(x+1) &=& (x+1)^2=x^2+2x+1\end{array}$ both don't work.
- $\Rightarrow$ No easy factorization; must use quadratic formula.

$$x_1,x_2 = \frac{-{\color{red}b}\pm\sqrt{{\color{red}b}^2-4ac}}{2a}=\frac{-{\color{red}3}\pm\sqrt{{\color{red}3}^2-4\cdot 1\cdot 1}}{2\cdot 1}$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

$$\begin{array}{rcl} x_1 x_2 & = & \frac{c}{a} \\ x_1 + x_2 & = & -\frac{b}{a} \end{array}$$

Example

Factor the quadratic.

$$x^2 + 3x + 1 = \left(x + \textbf{?} \qquad\qquad\right)\left(x + \textbf{?} \qquad\qquad\right)$$

- The product of the two roots: $x_1 x_2 = 1$.
- Integer options: $x_1 = 1, x_2 = 1$ and $x_1 = -1, x_2 = -1$.
- $\begin{array}{rcl} (x-1)(x-1) & = & (x-1)^2 = x^2 - 2x + 1 \\ (x+1)(x+1) & = & (x+1)^2 = x^2 + 2x + 1 \end{array}$ both don't work.
- $\Rightarrow$ No easy factorization; must use quadratic formula.

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$$

$$ax^2 + bx + c = a\left(x - x_1\right)\left(x - x_2\right),$$

$$\begin{array}{rcl} x_1 x_2 & = & \frac{c}{a} \\ x_1 + x_2 & = & -\frac{b}{a} \end{array}$$

Example

Factor the quadratic.

$$x^2 + 3x + 1 = \Big(x + \textbf{?} \qquad\qquad\qquad \Big)\Big(x + \textbf{?} \qquad\qquad\qquad \Big)$$

- The product of the two roots: $x_1 x_2 = 1$.
- Integer options: $x_1 = 1, x_2 = 1$ and $x_1 = -1, x_2 = -1$.
- $\begin{array}{rcl} (x-1)(x-1) & = & (x-1)^2 = x^2 - 2x + 1 \\ (x+1)(x+1) & = & (x+1)^2 = x^2 + 2x + 1 \end{array}$ both don't work.
- $\Rightarrow$ No easy factorization; must use quadratic formula.

$$\begin{array}{rcl} x_1, x_2 & = & \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} \\ & = & \frac{-3 \pm \sqrt{5}}{2} \end{array}$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

$$\left|\begin{array}{rcl} x_1 x_2 &=& \frac{c}{a} \\ x_1 + x_2 &=& -\frac{b}{a} \end{array}\right.$$

Example

Factor the quadratic.

$$x^2 + 3x + 1 = \left(x + \mathbf{?} \qquad\qquad \right)\left(x + \mathbf{?} \qquad\qquad \right)$$

- The product of the two roots: $x_1 x_2 = 1$.
- Integer options: $x_1 = 1, x_2 = 1$ and $x_1 = -1, x_2 = -1$.
- $\begin{array}{rcl} (x-1)(x-1) &=& (x-1)^2 = x^2 - 2x + 1 \\ (x+1)(x+1) &=& (x+1)^2 = x^2 + 2x + 1 \end{array}$ both don't work.
- $\Rightarrow$ No easy factorization; must use quadratic formula.

$$\begin{array}{rcl} x_1, x_2 &=& \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} \\ &=& \frac{-3 \pm \sqrt{5}}{2} \end{array}$$

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

$$\begin{array}{rcl} x_1 x_2 & = & \frac{c}{a} \\ x_1 + x_2 & = & -\frac{b}{a} \end{array}$$

Example

Factor the quadratic.

$$x^2 + 3x + 1 = \left(x - \left(\frac{-3+\sqrt{5}}{2}\right)\right)\left(x - \left(\frac{-3-\sqrt{5}}{2}\right)\right)$$

- The product of the two roots: $x_1 x_2 = 1$.
- Integer options: $x_1 = 1, x_2 = 1$ and $x_1 = -1, x_2 = -1$.
- $\begin{array}{rcl} (x-1)(x-1) & = & (x-1)^2 = x^2 - 2x + 1 \\ (x+1)(x+1) & = & (x+1)^2 = x^2 + 2x + 1 \end{array}$ both don't work.
- $\Rightarrow$ No easy factorization; must use quadratic formula.

$$\begin{array}{rcl} x_1, x_2 & = & \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} \\ & = & \frac{-3 \pm \sqrt{5}}{2} \end{array}$$