Calculus I
Derivative of a^x

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Example (Chain Rule, general exponential function)

Differentiate $\quad y = 2^x$.
$$y = \left(e^{\ln 2}\right)^x$$
$$y = e^{x \ln 2}.$$

$$\text{Let} \quad u =$$
$$\text{Then} \quad y =$$
$$\text{Chain Rule:} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x}$$
$$= (\quad)(\quad)$$
$$=$$
$$=$$
$$=$$

Example (Chain Rule, general exponential function)

Differentiate $y = a^x$.

$$y = \left(e^{\ln a}\right)^x$$

$$y = e^{x \ln a}.$$

$$\text{Let} \quad u = x \ln a.$$

$$\text{Then} \quad y = e^u.$$

$$\text{Chain Rule:} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x}$$

$$= (e^u)(\ln a)$$

$$= \left(e^{(x \ln a)}\right)(\ln a)$$

$$= \left(e^{\ln a}\right)^x (\ln a)$$

$$= a^x \ln a.$$

Theorem (The Derivative of a^x)

$$\frac{\mathrm{d}}{\mathrm{d}x}(a^x) = a^x \ln a.$$