Calculus II

Integrals of the form $\int \sin^n x \cos^m x dx$, both powers even

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Example

$$\int_0^{\frac{\pi}{2}} \sin^2 x \mathrm{d}x = \int_0^{\frac{\pi}{2}} \left(\frac{1-\cos(2x)}{2}\right) \mathrm{d}x$$

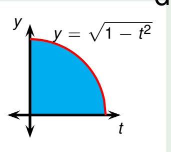
express $\sin^2 x$ via $\cos(2x)$

$$= \left[\frac{x}{2} - \frac{\sin(2x)}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \left(\frac{\pi}{4} - \frac{\sin\pi}{4}\right) - \left(0 - \frac{\sin 0}{4}\right) = \frac{\pi}{4}.$$

Example

Set $t = \cos x$, $x \in \left[0, \frac{\pi}{2}\right] \Rightarrow \sin x \geq 0$. Then $\mathrm{d}t = \mathrm{d}(\cos x) = -\sin x \mathrm{d}x$.



$$\int_{t=0}^{t=1} \sqrt{1-t^2}\mathrm{d}t = -\int_{x=\frac{\pi}{2}}^{x=0} \sqrt{1-\cos^2 x} \sin x \mathrm{d}x$$

$$= \int_{x=0}^{x=\frac{\pi}{2}} \sqrt{\sin^2 x} \sin x \mathrm{d}x$$

$$= \int_0^{\frac{\pi}{2}} \sin^2 x \mathrm{d}x = \frac{\pi}{4} \quad .$$