

Calculus II

Integral of rational function with cubic denominator, part 3

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Example

Find $\int \frac{2x^2 - x + 4}{x^3 + 4x} dx$.

- $\deg(2x^2 - x + 4) < \deg(x^3 + 4x)$: don't divide.
- Factor denominator: $x^3 + 4x = x(x^2 + 4)$.

$$\frac{2x^2 - x + 4}{x(x^2 + 4)} = \frac{A}{x} + \frac{Bx + C}{(x^2 + 4)}$$

$$2x^2 - x + 4 = A(x^2 + 4) + (Bx + C)x$$

$$2x^2 - x + 4 = (A + B)x^2 + Cx + 4A$$

$$A = 1 \quad C = -1 \quad A + B = 2, \text{ therefore } B = 1$$

$$\begin{aligned} \int \frac{2x^2 - x + 4}{x(x^2 + 4)} dx &= \int \left(\frac{1}{x} + \frac{x - 1}{x^2 + 4} \right) dx \\ &= \int \frac{1}{x} dx + \int \frac{x}{x^2 + 4} dx - \int \frac{1}{x^2 + 4} dx \\ &= \ln|x| + \frac{1}{2} \ln(x^2 + 4) - \frac{1}{2} \arctan\left(\frac{x}{2}\right) + K \end{aligned}$$

$Q(x)$ contains quadratic factors, multiplicity 1

- Suppose $Q(x)$ contains quadratic factors $ax^2 + bx + c$ with where $b^2 - 4ac < 0$ (i.e., the factor is irreducible).
- Suppose none of the quadratic factors is repeated.
- The for each quadratic factor we need to add a partial fraction of the form

$$\frac{Ax + B}{ax^2 + bx + c}.$$