Calculus II

Integrals of the form $\int x \sin(mx)\mathbf{d}x$

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2019

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x \sin x \mathrm{d}x =$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$\int x \sin x \mathrm{d}x =$ | $\sin x \mathrm{d}x = \mathrm{d}(\mathbf{?} \qquad)$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$\int x \sin x \mathrm{d}x =$ | $\sin x \mathrm{d}x = \mathrm{d}(-\cos x)$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x \sin x \mathrm{d}x = \int x \mathrm{d}(-\cos x) \quad \Big| \quad \sin x \mathrm{d}x = \mathrm{d}(-\cos x)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x\sin x\mathrm{d}x = \int x\mathrm{d}(-\cos x) \qquad \Big| \ \sin x\mathrm{d}x = \mathrm{d}(-\cos x)$$
$$= x(-\cos x) - \int(-\cos x)\mathrm{d}x$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x\sin x\mathrm{d}x = \int x\mathrm{d}(-\cos x) \qquad \Big| \ \sin x\mathrm{d}x = \mathrm{d}(-\cos x)$$
$$= x(-\cos x) - \int (-\cos x)\mathrm{d}x$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x\sin x\mathrm{d}x = \int x\mathrm{d}(-\cos x) \qquad \Big| \ \sin x\mathrm{d}x = \mathrm{d}(-\cos x)$$

$$= x(-\cos x) - \int (-\cos x)\mathrm{d}x$$

$$= -x\cos x + \int \cos x\mathrm{d}x$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x\sin x\mathrm{d}x = \int x\mathrm{d}(-\cos x) \quad \Big| \quad \sin x\mathrm{d}x = \mathrm{d}(-\cos x)$$

$$= x(-\cos x) - \int (-\cos x)\mathrm{d}x$$

$$= -x\cos x + \int \cos x\mathrm{d}x$$

$$= -x\cos x + \textbf{?}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int x\sin x\mathrm{d}x &= \int x\mathrm{d}(-\cos x) \qquad \Big| \ \sin x\mathrm{d}x = \mathrm{d}(-\cos x)\\ &= x(-\cos x) - \int(-\cos x)\mathrm{d}x\\ &= -x\cos x + \int \cos x\mathrm{d}x\\ &= -x\cos x + \sin x + C\end{aligned}$$