

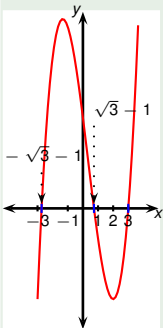
Precalculus

Factor cubic with one rational and two real roots using its plot

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Example



Plot the left hand side of the equation with a graphing calculator. Solve the equation.

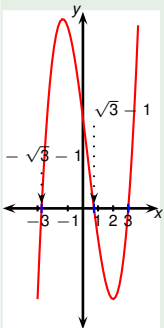
$$\begin{aligned}x^3 - x^2 - 8x + 6 &= 0 \\(x - 3)(x^2 + 2x - 2) &= 0\end{aligned}$$

The graph appears to intersect the x axis at:
 $-\sqrt{3} - 1, \sqrt{3} - 1, 3$. What are the two roots besides 3?

$$\begin{array}{r} \text{Quotient:} \quad x^2 + 2x - 2 \\ x - 3 \overline{) x^3 - x^2 - 8x + 6} \\ \underline{x^3 - 3x^2} \\ 2x^2 - 8x + 6 \\ \underline{2x^2 - 6x} \\ \underline{-2x + 6} \\ \underline{-2x + 6} \\ 0 \end{array}$$

Remainder: 0

Example



Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x = 3 \quad x = \frac{-2 \pm \sqrt{12}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}.$$

The graph appears to intersect the x axis at:

$- \sqrt{3} - 1$, $\sqrt{3} - 1$, 3. What are the two roots besides 3?

Final answer:

$$x = 3 \quad \text{or} \quad x = -1 - \sqrt{3} \quad \text{or} \quad x = -1 + \sqrt{3}.$$