

Precalculus

Homework

Equations involving logarithms and exponents

1. Solve each equation for x . If available, use a calculator to give an (\approx) answer in decimal notation. If available, use a calculator to verify your approximate solutions.

(a) $e^{7-4x} = 7$.

(j) $\ln(\ln x) = 1$.

(b) $\ln(2x - 9) = 2$.

(k) $e^{e^x} = 10$.

(c) $\ln(x^2 - 2) = 3$.

(l) $\ln(2x + 1) = 3 - \ln x$.

(d) $2^{x-3} = 5$.

(m) $e^{2x} - 4e^x + 3 = 0$.

(e) $\ln x + \ln(x - 1) = 1$.

(n) $e^{4x} + 3e^{2x} - 4 = 0$.

(f) $e^{2x+1} = t$.

(o) $e^{2x} - e^x - 6 = 0$.

(g) $\log_2(mx) = c$.

(p) $4^{3x} - 2^{3x+2} - 5 = 0$.

(h) $e - e^{-2x} = 1$.

(q) $3 \cdot 2^x + 2\left(\frac{1}{2}\right)^{x-1} - 7 = 0$.

(i) $8(1 + e^{-x})^{-1} = 3$.

Solution. 1.d

| | |
|-----------------------------|---|
| $2^{x-3} = 5$ | take \log_2 add 3 to both sides answer is complete optional step: convert to \ln calculator |
| $x - 3 = \log_2(5)$ | |
| $x = \log_2(5) + 3$ | |
| $= \frac{\ln 5}{\ln 2} + 3$ | |
| ≈ 5.321928095 | |

Solution. 1.h

| | |
|-------------------------------|-------------|
| $e - e^{-2x} = 1$ | apply \ln |
| $e^{-2x} = e - 1$ | |
| $\ln e^{-2x} = \ln(e - 1)$ | |
| $-2x = \ln(e - 1)$ | |
| $x = -\frac{1}{2} \ln(e - 1)$ | calculator |
| ≈ -0.270662427 | |

Solution. 1.e

$$\begin{aligned}
 \ln x + \ln(x-1) &= 1 \\
 \ln(x^2 - x) &= 1 \\
 e^{\ln(x^2 - x)} &= e^1 \\
 x^2 - x &= e \\
 x^2 - x - e &= 0 \\
 \text{Quadratic formula: } x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-e)}}{2(1)} \\
 &= \frac{1 \pm \sqrt{1+4e}}{2}.
 \end{aligned}$$

However $\frac{1-\sqrt{1+4e}}{2}$ is negative, so $\ln\left(\frac{1-\sqrt{1+4e}}{2}\right)$ is undefined. Hence the only solution is $x = \frac{1+\sqrt{1+4e}}{2} \approx 2.2229$.

Solution. 1.p

$$\begin{aligned}
 4^{3x} - 2^{3x+2} - 5 &= 0 \\
 4^{3x} - 4 \cdot 2^{3x} - 5 &= 0 & \left| \begin{array}{l} \text{Set } 2^{3x} = u \\ 4^{3x} = u^2 \end{array} \right. \\
 u^2 - 4u - 5 &= 0 \\
 (u-5)(u+1) &= 0 \\
 u = 5 &\text{ or } u = -1 \\
 2^{3x} = 5 & \quad 2^{3x} = -1 \\
 3x = \log_2(5) & \quad \text{no real solution} \\
 x = \frac{\log_2 5}{3} \\
 \text{Calculator: } x &\approx 0.773976
 \end{aligned}$$

Solution. 1.q

$$\begin{aligned}
 3 \cdot 2^x + 2 \left(\frac{1}{2}\right)^{x-1} - 7 &= 0 \\
 3 \cdot 2^x + 2 \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{-1} - 7 &= 0 \\
 3 \cdot 2^x + 4 \left(\frac{1}{2}\right)^x - 7 &= 0 & \left| \begin{array}{l} \text{Set } 2^x = u \\ \text{Multiply by } u \end{array} \right. \\
 3u + \frac{4}{u} - 7 &= 0 \\
 3u^2 - 7u + 4 &= 0 \\
 (u-1)(3u-4) &= 0 \\
 u = 1 &\text{ or } 3u - 4 = 0 \\
 2^x = 1 & \quad u = \frac{4}{3} \\
 x = 0 & \quad 2^x = \frac{4}{3} \\
 & \quad x = \log_2 \frac{4}{3} = \log_2 4 - \log_2 3 \\
 & \quad x = 2 - \log_2 3 \\
 \text{Calculator: } &x \approx 0.415037
 \end{aligned}$$