Calculus II

Integrals of the form $\int \frac{a}{(bx+c)^n}\mathbf{d}x$

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Building block Ib

Building block Ib: $\int \frac{1}{x^n} \mathrm{d}x = \int x^{-n} \mathrm{d}x$, $n \neq 1$.

Example (Block Ib)

$$\int \frac{1}{x^n} \mathrm{d}x$$

Building block Ib

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Example (Block Ib)

$$\int \frac{1}{x^n} \mathrm{d}x = \int x^{-n} \mathrm{d}x = \textbf{?}$$

Building block Ib

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Example (Block Ib)

$$\int \frac{1}{x^n} \mathrm{d}x = \int x^{-n} \mathrm{d}x = \frac{x^{-n+1}}{-n+1} + C$$

Linear substitutions leading to building block Ib

Building block Ib: $\int \frac{1}{x^n}\mathrm{d}x = \int x^{-n}\mathrm{d}x = \frac{x^{-n+1}}{-n+1} + C,\ n \neq 1.$

Example

Integrate

$$\int \frac{1}{(3x+5)^3}\mathrm{d}x$$

Linear substitutions leading to building block Ib

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Example

Integrate

$$\int \frac{1}{(3x+5)^3}\mathrm{d}x = \int \frac{1}{(3x+5)^3}\frac{\mathrm{d}(3x)}{3}$$

Linear substitutions leading to building block Ib

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Example

Integrate

$$\begin{aligned}\int \frac{1}{(3x+5)^3}\mathrm{d}x &= \int \frac{1}{(3x+5)^3}\frac{\mathrm{d}(3x)}{3} \\ &= \int \frac{1}{(3x+5)^3}\frac{\mathrm{d}(3x+5)}{3}\end{aligned}$$

Linear substitutions leading to building block Ib

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Example

Integrate

$$\begin{aligned}
\int \frac{1}{(3x+5)^3}\mathrm{d}x &= \int \frac{1}{(3x+5)^3}\frac{\mathrm{d}(3x)}{3} \\
&= \int \frac{1}{(3x+5)^3}\frac{\mathrm{d}(3x+5)}{3} \qquad \Big| \text{ Set } u = 3x+5 \\
&= \int \frac{1}{u^3}\frac{\mathrm{d}u}{3}
\end{aligned}$$

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&= \int \frac{1}{u^3}\frac{\mathrm{d}u}{3} \\
&= \frac{1}{3}\int u^{-3}\mathrm{d}u = \frac{1}{3}\frac{u^{-2}}{(-2)} + C
\end{aligned}$$

Linear substitutions leading to building block Ib

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&= \int \frac{1}{u^3}\frac{\mathrm{d}u}{3} \\
&= \frac{1}{3}\int u^{-3}\mathrm{d}u = \frac{1}{3}\frac{u^{-2}}{(-2)} + C \\
&= -\frac{1}{6(3x+5)^2} + C \quad .
\end{aligned}$$

Linear substitutions leading to building block Ib

Building block Ib: $\int \frac{1}{x^n}\mathrm{d}x = \int x^{-n}\mathrm{d}x = \frac{x^{-n+1}}{-n+1} + C,\ n \neq 1.$

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Lin. subst. leading to building block Ib: general case

Building block Ib: $\int \frac{1}{x^n}\mathrm{d}x = \int x^{-n}\mathrm{d}x = \frac{x^{-n+1}}{-n+1} + C,\ n \neq 1.$

Example

Let $n \neq 1$. Integrate

$$\begin{aligned}
\int \frac{1}{(ax+b)^n}\mathrm{d}x &= \int \frac{1}{(ax+b)^n}\frac{\mathrm{d}(ax)}{a} \\
&= \int \frac{1}{(ax+b)^n}\frac{\mathrm{d}(ax+b)}{a} \qquad \Big| \text{ Set } u = ax+b \\
&= \int \frac{1}{u^3}\frac{\mathrm{d}u}{a} \\
&= \frac{1}{a}\int u^{-n}\mathrm{d}u = -\frac{1}{a}\frac{u^{-n+1}}{(n-1)} + C \\
&= -\frac{1}{a(n-1)(ax+b)^{n-1}} + C \quad .
\end{aligned}$$