Calculus II
Integrals of the form $\int \sin^n x \cos^m x dx$, both powers even

Todor Milev

2019

Example

$$\int_0^{\frac{\pi}{2}} \sin^2 x \mathrm{d}x$$

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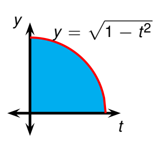

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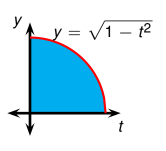

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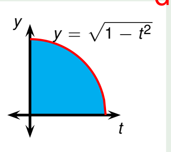

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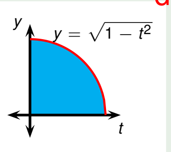

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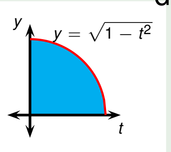

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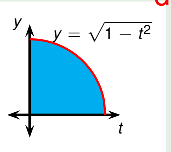

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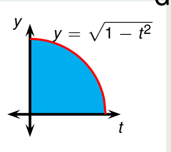

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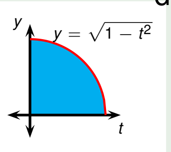

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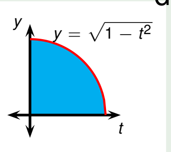

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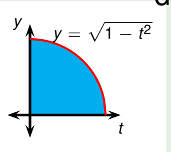

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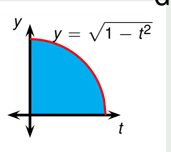

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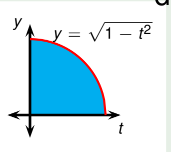

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