

# Calculus I

**Reference: Indefinite integrals = antiderivatives**

Todor Milev

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# Indefinite Integrals

- The Evaluation Theorem establishes a connection between antiderivatives and definite integrals.
- It says that  $\int_a^b f(x)dx$  equals  $F(b) - F(a)$ , where  $F$  is an antiderivative of  $f$ .
- We need convenient notation for writing antiderivatives.
- This is what the indefinite integral is.

## Definition (Indefinite Integral)

The indefinite integral of  $f$  is another way of saying the antiderivative of  $f$ , and is written  $\int f(x)dx$ . In other words,

$$\int f(x)dx = F(x) \quad \text{means} \quad F'(x) = f(x).$$

## Example

$$\int x^4 dx = \frac{x^5}{5} + C$$

because

$$\frac{d}{dx} \left( \frac{x^5}{5} + C \right) = x^4.$$

- The indefinite integral represents a whole family of functions.
- Example: the general antiderivative of  $\frac{1}{x}$  is

$$F(x) = \begin{cases} \ln|x| + C_1 & \text{if } x > 0 \\ \ln|x| + C_2 & \text{if } x < 0 \end{cases}$$

- We adopt the convention that the constant participating in an indefinite integral is only valid on one interval.
- $\int \frac{1}{x} dx = \ln|x| + C$ , and this is valid either on  $(-\infty, 0)$  or  $(0, \infty)$ .