

# Precalculus

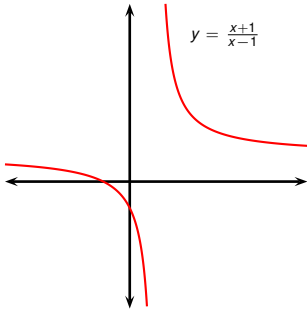
## Inverse of fractional linear transformation

Todor Milev

2019

## Example

Find  $f^{-1}(x)$  where  $f(x) = \frac{x+1}{x-1}$ .

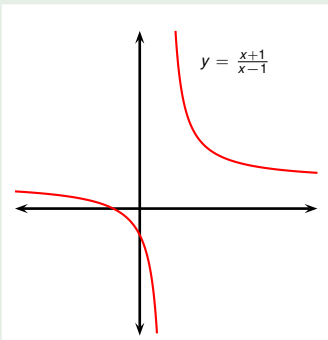


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Find  $f^{-1}(x)$  where  $f(x) = \frac{x+1}{x-1}$ .

We deal with domains and ranges later:

$$y = \frac{x+1}{x-1}$$

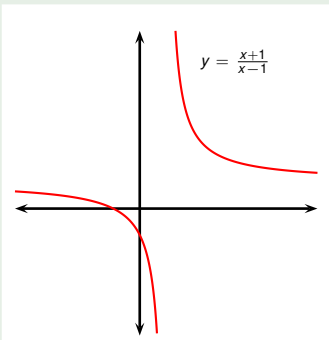


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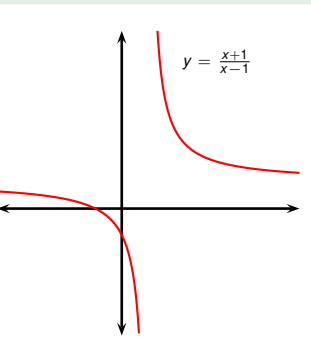
$$\begin{array}{lcl} y & = & \frac{x+1}{x-1} \\ y(x-1) & = & x+1 \end{array} \quad \left| \begin{array}{l} \text{mult. by } (x-1) \end{array} \right.$$



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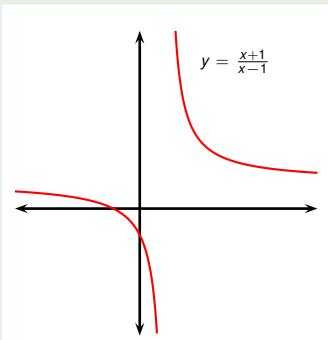


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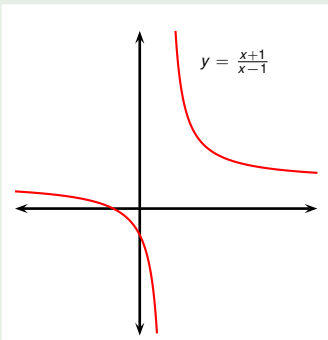


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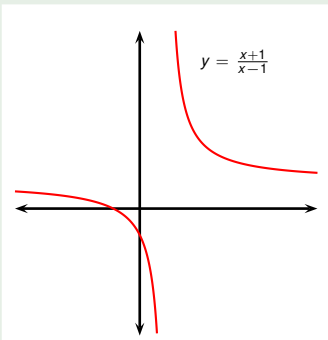


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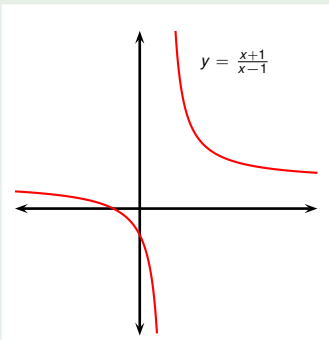
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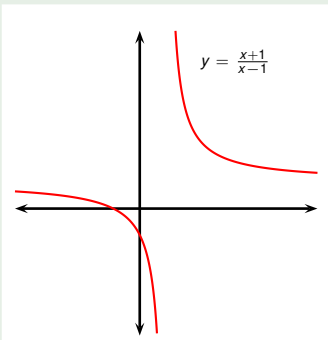


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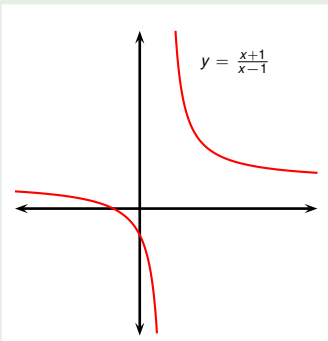


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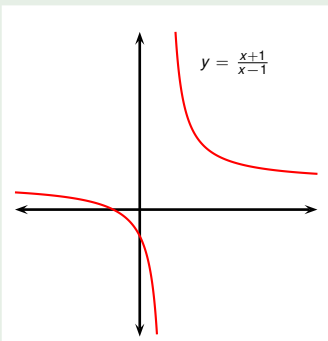


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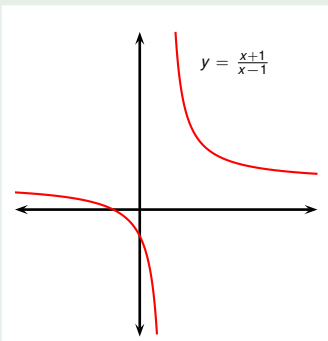
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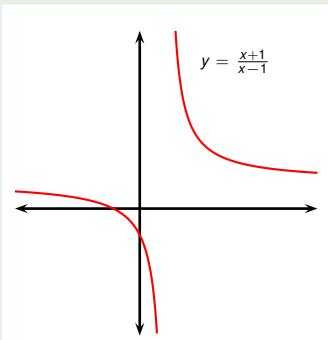
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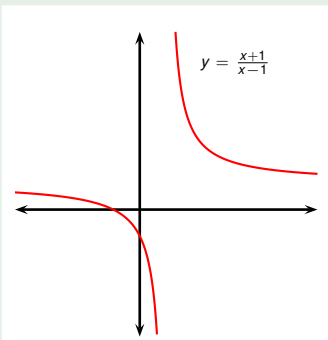
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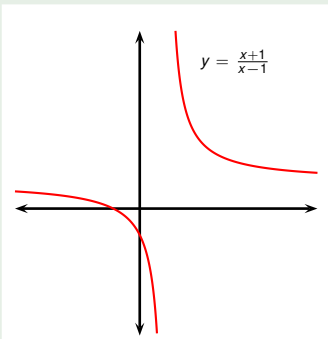
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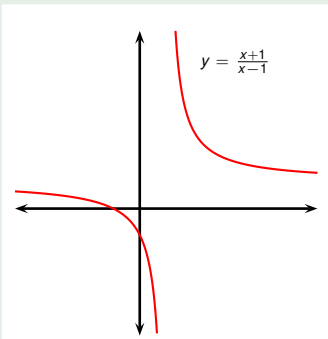
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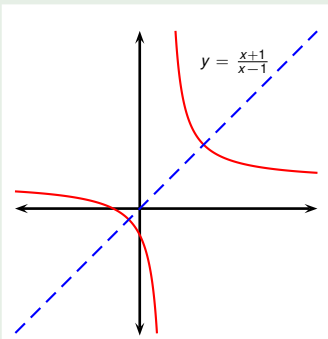
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**Graph of  $f$  is symmetric across  $y = x$ .**