Precalculus
Use polynomial division with remainder 0 to factor a polynomial

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Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{lr}
\textbf{Quotient:} & 3x^2 - 5x + 1 \\
2x - 3 & \overline{)\,6x^3 - 19x^2 + 17x - 3} \\
- & 6x^3 - 9x^2 \\
\hline
 & -10x^2 + 17x - 3 \\
- & -10x^2 + 15x \\
\hline
 & 2x - 3 \\
- & 2x - 3 \\
\hline
\textbf{Remainder:} & 0
\end{array}
$$

(Dividend)$=$(Quotient) $\cdot$ (Divisor) $+$ (Remainder)

$$(6x^3 - 19x^2 + 17x - 3) = (3x^2 - 5x + 1) \cdot (2x - 3)$$

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$$\begin{aligned}(6x^3 - 19x^2 + 17x - 3) &= (3x^2 - 5x + 1) \cdot (2x - 3) \\ &= 3\left(x - \left(\frac{5+\sqrt{13}}{6}\right)\right)\left(x - \left(\frac{5-\sqrt{13}}{6}\right)\right)(2x - 3)\end{aligned}$$

No easy factorization of quadratic, so use formula:

$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 3 \cdot 1}}{2 \cdot 3} = \frac{5 \pm \sqrt{13}}{6}$ We are ready to solve the equation.

$$\begin{aligned}6x^3 - 19x^2 + 17x - 3 &= 0 \\ 3\left(x - \left(\frac{5+\sqrt{13}}{6}\right)\right)\left(x - \left(\frac{5-\sqrt{13}}{6}\right)\right)(2x - 3) &= 0\end{aligned}$$

$$\begin{aligned}2x - 3 &= 0 \quad \textbf{or} \quad x = \left(\frac{5+\sqrt{13}}{6}\right) \quad \textbf{or} \quad x = \left(\frac{5-\sqrt{13}}{6}\right) \\ x &= \frac{3}{2}\end{aligned}$$