Arithmetics
Division and fractions
calculator-algebra.org

Todor Milev

2019

Definition (Division, exact)

To divide a number p (**dividend**) by a number d (**divisor**) means to find a number q (**quotient**) so that

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The quotient of two integers equals the fraction formed by putting the dividend as the numerator and the divisor as the denominator.

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- Recall exact division: $$\begin{aligned} p' &= q' \cdot d' \\ q' &= \frac{p'}{d'} \end{aligned}$$
- Quotient may fail to reduce to an integer.
- What if we want an integer quotient?

Definition (Integer division with remainder)

To **divide** an integer $p > 0$ by an integer $d > 0$ **with remainder** $r \geq 0$ means to find the largest integer $q \geq 0$ and the smallest $0 \leq r$ so that:

$$p = q \cdot d + r$$

p is called the **dividend**, d is called the **divisor**, q is called the **quotient** and r is called the **remainder**.

Example

Divide 7 by 3 with remainder. $7 = 2 \cdot 3 + 1$.

- Differences between exact division integer division.
  - Integer division quotient is integer, exact division quotient is fraction.
  - Exact division: no notion of remainder.

- Recall integer division of p by d with remainder: $p = q \cdot d + r$.

- p may fail to be integer-divisible by d, then we have remainder.

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- Integer division of p by d answers the following question:

Question

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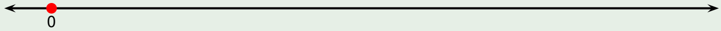

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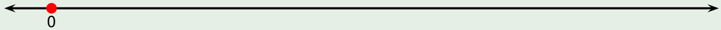

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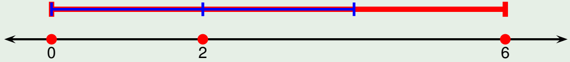

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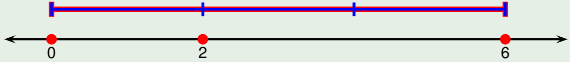

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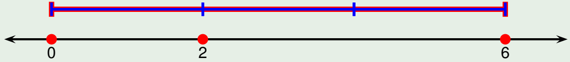

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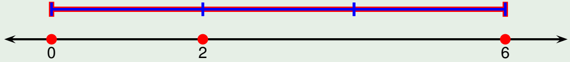

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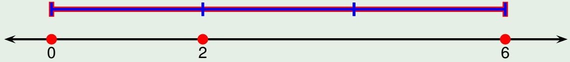

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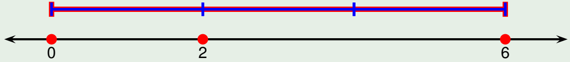

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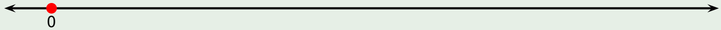

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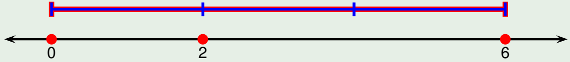

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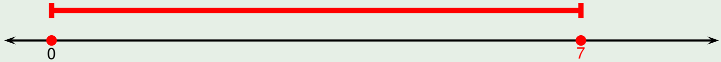

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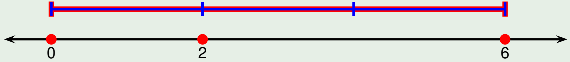

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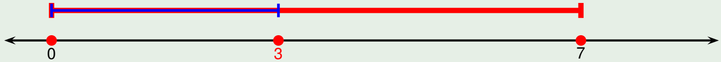

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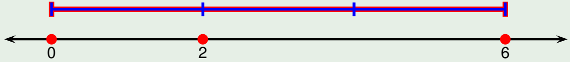

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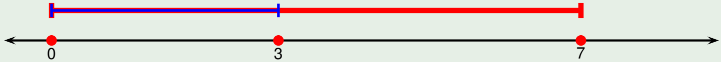

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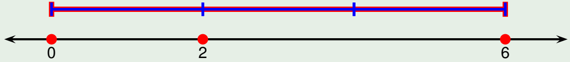

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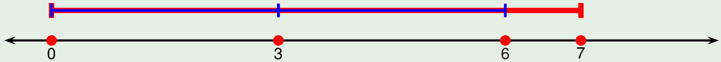

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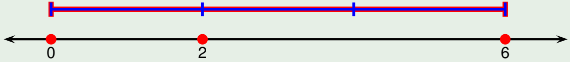

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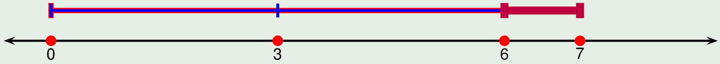

3 divides 7 non-exactly 2 times with remainder 1.

Example (Divisor 1-digit, quotient 1-digit)

Divide 7 by 2 with remainder. To solve, we need to answer: what is the largest integer which, when multiplied by 2, stays smaller than 7? Try:

$$\begin{aligned} 0\cdot 2 &= 0\\ 1\cdot 2 &= 2\\ 2\cdot 2 &= 4\\ 3\cdot 2 &= 6\\ 4\cdot 2 &= 8 > 7 \end{aligned}$$

$\Rightarrow 7 = 3\cdot 2 + x$. Solve: $x = 7 - 3\cdot 2 = 1$. Therefore $7 = 3\cdot 2 + 1$.

Observation (Question to answer when dividing with remainder)

What is the largest integer which, when multiplied by d, remains smaller than p?

- To answer this question, we guess quickly as shown above.
- Later on we learn to divide large numbers without guessing.
- However, we still need the guessing approach as a building block of the complete division algorithm.

Example (Integer division: 1-digit dividend, 1-digit divisor)

Divide with remainder:

$$\begin{array}{rrcl}
5 \text{ by } 2: & 5 & = & 2\cdot 2+1\\
3 \text{ by } 5: & 5 & = & 0\cdot 5+3\\
8 \text{ by } 4: & 5 & = & 2\cdot 4\\
9 \text{ by } 8: & 5 & = & 1\cdot 8+1\\
3 \text{ by } 1: & 3 & = & 3\cdot 1\\
9 \text{ by } 2: & 6 & = & 4\cdot 2+1
\end{array}$$

All quotients are known to be one-digit numbers.

Example (Integer division: 1-digit divisor, 1-digit quotient)

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\begin{array}{lrcl}
12 \text{ by } 6: & 12 & = & 2\cdot 6\\
14 \text{ by } 5: & 14 & = & 2\cdot 5+4\\
37 \text{ by } 5: & 37 & = & 7\cdot 5+2\\
40 \text{ by } 9: & 40 & = & 4\cdot 9+4\\
49 \text{ by } 7: & 49 & = & 7\cdot 7\\
57 \text{ by } 8: & 57 & = & 7\cdot 8+1\\
67 \text{ by } 7: & 67 & = & 9\cdot 7+4\\
82 \text{ by } 9: & 82 & = & 9\cdot 9+1
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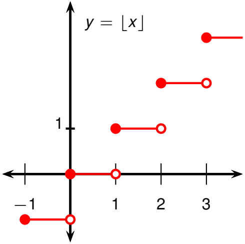
The *greatest integer function* $\lfloor x \rfloor$ is defined as the largest integer that is less than or equal to x.

In computer science this function is called the *floor* function, also the *round-down* function.



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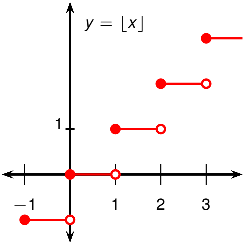

$$\lfloor 4 \rfloor = ?$$
$$\lfloor 4.8 \rfloor =$$
$$\lfloor 1.5 \rfloor =$$
$$\left\lfloor 1 + \frac{1}{2} \right\rfloor =$$
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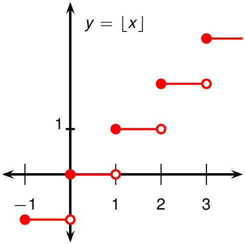

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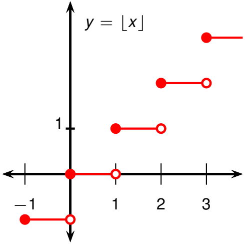

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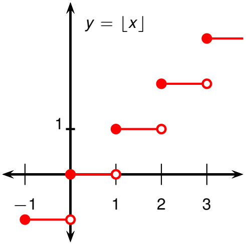

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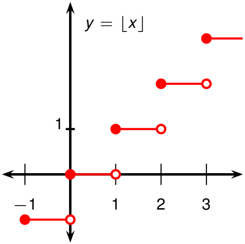

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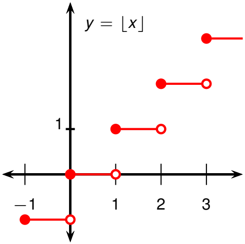

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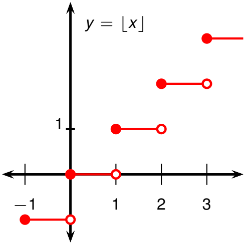

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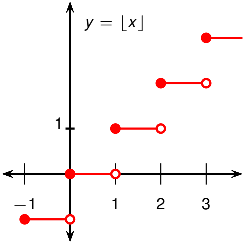

$$\left\lfloor 1 + \frac{1}{2} \right\rfloor = 1$$

$$\left\lfloor \frac{3}{2} \right\rfloor =$$

$$\lfloor \pi \rfloor =$$

Definition (Greatest Integer Function)

The *greatest integer function* $\lfloor x \rfloor$ is defined as the largest integer that is less than or equal to x.

In computer science this function is called the *floor* function, also the *round-down* function.

$$\lfloor 4 \rfloor = 4$$
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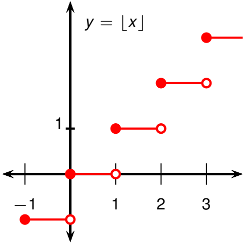

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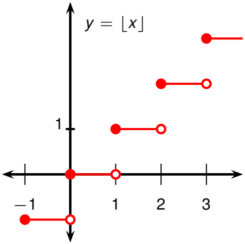

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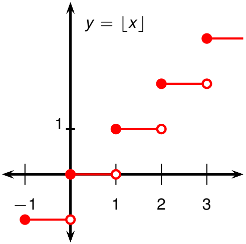

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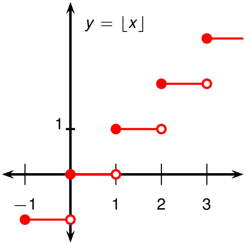

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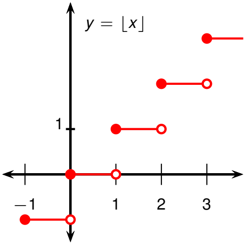

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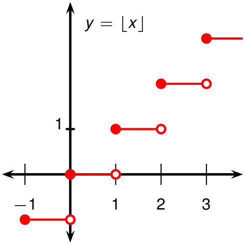

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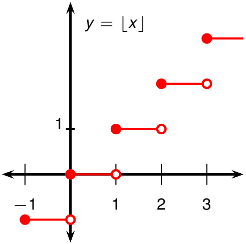

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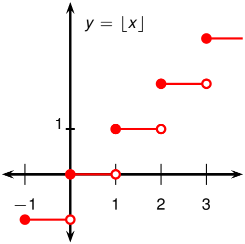

$$\left\lfloor 1+\frac{1}{2} \right\rfloor = 1$$

$$\left\lfloor \frac{3}{2} \right\rfloor = \left\lfloor \frac{2+1}{2} \right\rfloor = \left\lfloor \frac{2}{2}+\frac{1}{2} \right\rfloor = \left\lfloor 1+\frac{1}{2} \right\rfloor = 1$$

$$\lfloor \pi \rfloor = \lfloor 3.1415\ldots \rfloor = 3$$

Example

Compute the floor (round-down) function.

$$\begin{aligned}
\left\lfloor \frac{1}{3} \right\rfloor &= \textbf{?} \\
\left\lfloor \frac{6}{7} \right\rfloor &= \\
\left\lfloor 2 + \frac{3}{11} \right\rfloor &= \\
\left\lfloor 10 + \frac{1}{7} \right\rfloor &=
\end{aligned}$$

Example

Compute the floor (round-down) function.

$$\left\lfloor \frac{1}{3} \right\rfloor = 0$$

$$\left\lfloor \frac{6}{7} \right\rfloor =$$

$$\left\lfloor 2 + \frac{3}{11} \right\rfloor =$$

$$\left\lfloor 10 + \frac{1}{7} \right\rfloor =$$

Example

Compute the floor (round-down) function.

$$\begin{aligned}
\left\lfloor \frac{1}{3} \right\rfloor &= 0 \\
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Observation

$$\left\lfloor \frac{p}{q} \right\rfloor = 0 \qquad \textit{whenever } \ 0 \leq p < q$$

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Compute the floor (round-down) function.

$$\begin{aligned}
\left\lfloor \frac{1}{3} \right\rfloor &= 0\\
\left\lfloor \frac{6}{7} \right\rfloor &=\\
\left\lfloor 2+\frac{3}{11} \right\rfloor &=\\
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$$\lfloor n+x \rfloor = n + \lfloor x \rfloor \qquad \textit{whenever } n \textit{ is integer}$$

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$$\begin{aligned}
\left\lfloor \frac{1}{3} \right\rfloor &= 0 \\
\left\lfloor \frac{6}{7} \right\rfloor &= 0 \\
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$$\left\lfloor 10 + \frac{1}{7} \right\rfloor = \mathbf{?}$$

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\left\lfloor 10+\frac{1}{7} \right\rfloor &= 10+\left\lfloor \frac{1}{7} \right\rfloor = 10
\end{aligned}$$

Observation

$$\left\lfloor \frac{p}{q} \right\rfloor = 0 \qquad \textit{whenever } 0 \leq p < q$$

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Example

Compute the floor (round-down) of $\frac{8}{3}$.

$$\left\lfloor \frac{8}{3} \right\rfloor =$$

Observation

The floor (round-down) of $\frac{p}{q}$ is computed as

$$\left\lfloor \frac{p}{d} \right\rfloor = q,$$

where q is the the quotient obtained by integer division of p by d.

Example

Compute the floor (round-down) of $\frac{8}{3}$.

$$\left\lfloor \frac{8}{3} \right\rfloor =$$

Divide 8 by 3 with remainder.

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The floor (round-down) of $\frac{p}{q}$ is computed as

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Divide 8 by 3 with remainder. Try:

$$0 \cdot 3 = 0$$

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Divide 8 by 3 with remainder. Try:

$$\begin{array}{rcl} 0 \cdot 3 & = & 0 \\ \color{red}{1 \cdot 3} & \color{red}{=} & \color{red}{3} \end{array}$$

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$$\begin{array}{rcl} 0 \cdot 3 & = & 0 \\ 1 \cdot 3 & = & 3 \\ 2 \cdot 3 & = & 6 \end{array}$$

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$$\left\lfloor \frac{8}{3} \right\rfloor =$$

Divide 8 by 3 with remainder. Try:

$$\begin{array}{rcl} 0 \cdot 3 &=& 0 \\ 1 \cdot 3 &=& 3 \\ 2 \cdot 3 &=& 6 \\ \color{red}{3 \cdot 3} &\color{red}{=}& \color{red}{9} \end{array}$$

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The floor (round-down) of $\frac{p}{q}$ is computed as

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Compute the floor (round-down) of $\frac{8}{3}$.

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Divide 8 by 3 with remainder. Try:

$$\begin{aligned} 0\cdot 3 &= 0\\ 1\cdot 3 &= 3\\ 2\cdot 3 &= 6\\ 3\cdot 3 &= 9 > 8 \end{aligned}$$

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Example

Compute the floor (round-down) of $\frac{8}{3}$.

$$\left\lfloor \frac{8}{3} \right\rfloor = \left\lfloor \frac{2 \cdot 3 + \mathbf{?}}{3} \right\rfloor$$

Divide 8 by 3 with remainder. Try:

$$\begin{array}{rcl} 0 \cdot 3 & = & 0 \\ 1 \cdot 3 & = & 3 \\ 2 \cdot 3 & = & 6 \\ 3 \cdot 3 & = & 9 > 8 \end{array}$$

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Example

Compute the floor (round-down) of $\frac{8}{3}$.

$$\left\lfloor \frac{8}{3} \right\rfloor = \left\lfloor \frac{2 \cdot 3 + 2}{3} \right\rfloor$$

Divide 8 by 3 with remainder. Try:

$$\begin{array}{rcl} 0 \cdot 3 &=& 0 \\ 1 \cdot 3 &=& 3 \\ 2 \cdot 3 &=& 6 \\ 3 \cdot 3 &=& 9 > 8 \end{array}$$

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Example

Compute the floor (round-down) of $\frac{8}{3}$.

$$\left\lfloor \frac{8}{3} \right\rfloor = \left\lfloor \frac{2\cdot 3 + 2}{3} \right\rfloor$$

$$= \left\lfloor \frac{2\cdot 3}{3} + \frac{2}{3} \right\rfloor$$

Divide 8 by 3 with remainder. Try:

$$\begin{aligned} 0\cdot 3 &= 0 \\ 1\cdot 3 &= 3 \\ 2\cdot 3 &= 6 \\ 3\cdot 3 &= 9 > 8 \end{aligned}$$

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Example

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$$
\begin{aligned}
\left\lfloor \frac{8}{3} \right\rfloor &= \left\lfloor \frac{2\cdot 3+2}{3} \right\rfloor \\
&= \left\lfloor \frac{2\cdot \cancel{3}}{\cancel{3}} + \frac{2}{3} \right\rfloor \\
&= \left\lfloor 2+\frac{2}{3} \right\rfloor
\end{aligned}
$$

Divide 8 by 3 with remainder. Try:

$$
\begin{aligned}
0\cdot 3 &= 0 \\
1\cdot 3 &= 3 \\
2\cdot 3 &= 6 \\
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\end{aligned}
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$$= \left\lfloor \frac{2\cdot \cancel{3}}{\cancel{3}} + \frac{2}{3} \right\rfloor$$

$$= \left\lfloor 2+\frac{2}{3} \right\rfloor = 2$$

because $2 \leq 2+\frac{2}{3} < 3$

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Divide 8 by 3 with remainder. Try:

$$\begin{array}{rcl} 0\cdot 3 &=& 0\\ 1\cdot 3 &=& 3\\ 2\cdot 3 &=& 6\\ 3\cdot 3 &=& 9>8 \end{array}$$

$$= \left\lfloor \frac{2\cdot \cancel{3}}{\cancel{3}} + \frac{2}{3} \right\rfloor$$

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Compute the floor (round-down) of $\frac{8}{3}$.

$$\left\lfloor \frac{8}{3} \right\rfloor = \left\lfloor \frac{2\cdot 3 + 2}{3} \right\rfloor = \left\lfloor \frac{2\cdot \cancel{3}}{\cancel{3}} + \frac{2}{3} \right\rfloor = \left\lfloor {\color{red}2 + \frac{2}{3}} \right\rfloor = 2$$

Divide 8 by 3 with remainder. Try:

$$\begin{aligned} 0\cdot 3 &= 0\\ 1\cdot 3 &= 3\\ 2\cdot 3 &= 6\\ 3\cdot 3 &= 9 > 8 \end{aligned}$$

because $2 \leq {\color{red}2 + \frac{2}{3}} < 3$

Observation

The floor (round-down) of $\frac{p}{q}$ is computed as

$$\left\lfloor \frac{p}{d} \right\rfloor = q,$$

where q is the the quotient obtained by integer division of p by d.

Example

Compute the floor (round-down) of $\frac{8}{3}$.

$$\left\lfloor \frac{8}{3} \right\rfloor = \left\lfloor \frac{2\cdot 3+2}{3} \right\rfloor$$

Divide 8 by 3 with remainder. Try:

$$\begin{array}{rcl} 0\cdot 3 &=& 0 \\ 1\cdot 3 &=& 3 \\ 2\cdot 3 &=& 6 \\ 3\cdot 3 &=& 9 > 8 \end{array}$$

$$= \left\lfloor \frac{2\cdot \cancel{3}}{\cancel{3}} + \frac{2}{3} \right\rfloor$$

$$= \left\lfloor 2 + \frac{2}{3} \right\rfloor = 2$$

because $2 \leq 2 + \frac{2}{3} < 3$

Observation

The floor (round-down) of $\frac{p}{q}$ is computed as

$$\left\lfloor \frac{p}{d} \right\rfloor = q,$$

where q is the the quotient obtained by integer division of p by d.

Example

Compute the floor (round-down) function.

$$\left\lfloor \frac{4}{3} \right\rfloor = \textbf{?}$$
$$\left\lfloor \frac{15}{2} \right\rfloor =$$
$$\left\lfloor \frac{24}{4} \right\rfloor =$$
$$\left\lfloor \frac{43}{5} \right\rfloor =$$
$$\left\lfloor \frac{56}{7} \right\rfloor =$$
$$\left\lfloor \frac{79}{8} \right\rfloor =$$
$$\left\lfloor \frac{80}{9} \right\rfloor =$$

Example

Compute the floor (round-down) function.

$$\left\lfloor \frac{4}{3} \right\rfloor = \left\lfloor \frac{1\cdot 3+1}{3} \right\rfloor = \left\lfloor \frac{1\cdot 3}{3} + \frac{1}{3} \right\rfloor = \left\lfloor 1 + \frac{1}{3} \right\rfloor = 1$$

$$\left\lfloor \frac{15}{2} \right\rfloor =$$

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Example

Compute the floor (round-down) function.

$$
\begin{aligned}
\left\lfloor \frac{4}{3} \right\rfloor &= \left\lfloor \frac{1\cdot 3+1}{3} \right\rfloor = \left\lfloor \frac{1\cdot 3}{3} + \frac{1}{3} \right\rfloor = \left\lfloor 1 + \frac{1}{3} \right\rfloor = 1 \\
\left\lfloor \frac{15}{2} \right\rfloor &= \textbf{?} \\
\left\lfloor \frac{24}{4} \right\rfloor &= \\
\left\lfloor \frac{43}{5} \right\rfloor &= \\
\left\lfloor \frac{56}{7} \right\rfloor &= \\
\left\lfloor \frac{79}{8} \right\rfloor &= \\
\left\lfloor \frac{80}{9} \right\rfloor &=
\end{aligned}
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Example

Compute the floor (round-down) function.

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$$\left\lfloor \frac{15}{2} \right\rfloor = \left\lfloor \frac{7\cdot 2+1}{2} \right\rfloor = \left\lfloor \frac{7\cdot 2}{2} + \frac{1}{2} \right\rfloor = \left\lfloor 7 + \frac{1}{2} \right\rfloor = 7$$

$$\left\lfloor \frac{24}{4} \right\rfloor =$$

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\left\lfloor \frac{15}{2} \right\rfloor &= \left\lfloor \frac{7\cdot 2+1}{2} \right\rfloor = \left\lfloor \frac{7\cdot 2}{2} + \frac{1}{2} \right\rfloor = \left\lfloor 7 + \frac{1}{2} \right\rfloor = 7\\
\left\lfloor \frac{24}{4} \right\rfloor &= \left\lfloor \frac{6\cdot 4}{4} \right\rfloor = \lfloor 6 \rfloor = 6\\
\left\lfloor \frac{43}{5} \right\rfloor &=\\
\left\lfloor \frac{56}{7} \right\rfloor &=\\
\left\lfloor \frac{79}{8} \right\rfloor &=\\
\left\lfloor \frac{80}{9} \right\rfloor &=
\end{aligned}
$$

Example

Compute the floor (round-down) function.

$$\left\lfloor \frac{4}{3} \right\rfloor = \left\lfloor \frac{1\cdot 3+1}{3} \right\rfloor = \left\lfloor \frac{1\cdot 3}{3}+\frac{1}{3} \right\rfloor = \left\lfloor 1+\frac{1}{3} \right\rfloor = 1$$

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Example

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Example

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\left\lfloor \frac{4}{3} \right\rfloor &= \left\lfloor \frac{1\cdot 3+1}{3} \right\rfloor = \left\lfloor \frac{1\cdot 3}{3} + \frac{1}{3} \right\rfloor = \left\lfloor 1 + \frac{1}{3} \right\rfloor = 1\\
\left\lfloor \frac{15}{2} \right\rfloor &= \left\lfloor \frac{7\cdot 2+1}{2} \right\rfloor = \left\lfloor \frac{7\cdot 2}{2} + \frac{1}{2} \right\rfloor = \left\lfloor 7 + \frac{1}{2} \right\rfloor = 7\\
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\left\lfloor \frac{43}{5} \right\rfloor &= \left\lfloor \frac{8\cdot 5+3}{5} \right\rfloor = \left\lfloor \frac{8\cdot 5}{5} + \frac{3}{5} \right\rfloor = \left\lfloor 8 + \frac{3}{5} \right\rfloor = 8\\
\left\lfloor \frac{56}{7} \right\rfloor &= \left\lfloor \frac{8\cdot 7}{7} \right\rfloor = \lfloor 8 \rfloor = 8\\
\left\lfloor \frac{79}{8} \right\rfloor &=\\
\left\lfloor \frac{80}{9} \right\rfloor &=
\end{aligned}
$$

Example

Compute the floor (round-down) function.

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Example

Compute the floor (round-down) function.

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Example

Compute the floor (round-down) function.

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\left\lfloor \frac{4}{3} \right\rfloor &= \left\lfloor \frac{1\cdot 3+1}{3} \right\rfloor = \left\lfloor \frac{1\cdot 3}{3} + \frac{1}{3} \right\rfloor = \left\lfloor 1 + \frac{1}{3} \right\rfloor = 1\\
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\left\lfloor \frac{80}{9} \right\rfloor &= \textbf{?}
\end{aligned}$$

Example

Compute the floor (round-down) function.

$$
\begin{aligned}
\left\lfloor\frac{4}{3}\right\rfloor &= \left\lfloor\frac{1\cdot 3+1}{3}\right\rfloor = \left\lfloor\frac{1\cdot 3}{3}+\frac{1}{3}\right\rfloor = \left\lfloor 1+\frac{1}{3}\right\rfloor = 1\\
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\left\lfloor\frac{80}{9}\right\rfloor &= \left\lfloor\frac{8\cdot 9+8}{9}\right\rfloor = \left\lfloor\frac{8\cdot 9}{9}+\frac{8}{9}\right\rfloor = \left\lfloor 8+\frac{8}{9}\right\rfloor = 8
\end{aligned}
$$