

Calculus II

Integrals of the form $\int \frac{ax}{(bx^2 + c)^n} dx$

Todor Milev

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Building blocks IIa and IIb

We solve building block IIb. For completeness, we solve block IIa again as well.

Example

$$\int \frac{x}{(x^2 + 1)^n} dx$$

Building blocks IIa and IIb

We solve building block IIb. For completeness, we solve block IIa again as well.

Example

$$\int \frac{x}{(x^2 + 1)^n} dx = \int \frac{1}{(x^2 + 1)^n} \frac{d(x^2 + 1)}{2}$$

Building blocks IIa and IIb

We solve building block IIb. For completeness, we solve block IIa again as well.

Example

$$\begin{aligned}\int \frac{x}{(x^2 + 1)^n} dx &= \int \frac{1}{(x^2 + 1)^n} \frac{d(x^2 + 1)}{2} \\ &= \frac{1}{2} \int u^{-n} du\end{aligned}$$

where we used the substitution $u = x^2 + 1$.

Building blocks IIa and IIb

We solve building block IIb. For completeness, we solve block IIa again as well.

Example

$$\begin{aligned}
 \int \frac{x}{(x^2 + 1)^n} dx &= \int \frac{1}{(x^2 + 1)^n} \frac{d(x^2 + 1)}{2} \\
 &= \frac{1}{2} \int u^{-n} du \\
 &= \begin{cases} ? & \text{if } n = 1 \\ & \text{if } n \neq 1 \end{cases},
 \end{aligned}$$

where we used the substitution $u = x^2 + 1$.

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 \int \frac{x}{(x^2 + 1)^n} dx &= \int \frac{1}{(x^2 + 1)^n} \frac{d(x^2 + 1)}{2} \\
 &= \frac{1}{2} \int u^{-n} du \\
 &= \begin{cases} \frac{1}{2} \ln(x^2 + 1) + C & \text{if } n = 1 \\ & \text{if } n \neq 1 \end{cases},
 \end{aligned}$$

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 \int \frac{x}{(x^2 + 1)^n} dx &= \int \frac{1}{(x^2 + 1)^n} \frac{d(x^2 + 1)}{2} \\
 &= \frac{1}{2} \int u^{-n} du \\
 &= \begin{cases} \frac{1}{2} \ln(x^2 + 1) + C & \text{if } n = 1 \\ \frac{1}{2} \frac{(x^2 + 1)^{-n+1}}{(-n+1)} + C & \text{if } n \neq 1 \end{cases},
 \end{aligned}$$

where we used the substitution $u = x^2 + 1$.