Calculus II
Basic and alternating series tests

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Alternating Series

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$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots = \sum_{n=1}^{\infty} \qquad -$$

$$-\frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \frac{4}{5} - \frac{5}{6} + \frac{6}{7} - \cdots = \sum_{n=1}^{\infty} \qquad \frac{\quad}{\quad}$$

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The nth term of an alternating series has the form

$$a_n = (-1)^{n-1} b_n \qquad \text{or} \qquad a_n = (-1)^n b_n$$

where b_n is positive.

Theorem (The Alternating Series Test)

If the alternating series

$$\sum_{n=1}^{\infty}(-1)^{n-1}b_n = b_1 - b_2 + b_3 - b_4 + b_5 - \cdots, \qquad b_n > 0$$

satisfies

1. $b_{n+1} \leq b_n$ *for all* n *and*
2. $\lim_{n\to\infty} b_n = 0$

then the series is convergent.

Example

The alternating harmonic series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$

satisfies

1. $b_{n+1} < b_n$ because $\frac{1}{n+1} < \frac{1}{n}$.
2. $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n}$

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2. $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{n} = 0$.

Therefore the series is convergent by the Alternating Series Test.

Example

The series $\sum_{n=1}^{\infty}(-1)^n \frac{3n}{4n-1}$ is alternating, but

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Therefore the series is divergent by the basic Divergence Test.