Calculus I
Reference: Computing a Riemann Sum limit directly, part 1

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Example

For the region S underneath the parabola $y = x^2$ from 0 to 1, show that the area under the approximating rectangles approaches $\frac{1}{3}$, that is,

$$\lim_{n \to \infty} R_n = \frac{1}{3}.$$

- Each rectangle has width $\frac{1}{n}$.
- The heights are $\left(\frac{1}{n}\right)^2$, $\left(\frac{2}{n}\right)^2$, ..., $\left(\frac{n}{n}\right)^2$.
- New formula:
- $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

$$R_n = \frac{1}{n}\left(\frac{1}{n}\right)^2 + \frac{1}{n}\left(\frac{2}{n}\right)^2 + \cdots + \frac{1}{n}\left(\frac{n}{n}\right)^2 = \frac{1}{n^3}\left(1^2 + 2^2 + \cdots + n^2\right)$$

$$\lim_{n \to \infty} R_n = \lim_{n \to \infty} \frac{1}{n^3}\frac{n(n+1)(2n+1)}{6} = \lim_{n \to \infty} \frac{1}{6}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) = \frac{1}{3}$$