Calculus II
Integrals of the form $\displaystyle\int \frac{Ax+B}{(ax^2+bx+c)^n}\mathbf{d}x$, denominator has no real roots

Todor Milev

2019

Building block IIIb: example illustrating main idea

Example

Integrate $\int \frac{dx}{(x^2+1)^2}$. We start with an already known integral:

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$$\arctan x + C = \int \frac{1}{x^2+1} dx$$

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Integrate $\int \frac{\mathrm{d}x}{(x^2+1)^2}$. We start with an already known integral:

$$\begin{aligned} \arctan x + C &= \int \frac{1}{x^2+1} \mathrm{d}x \\ &= \frac{1}{x^2+1} x - \int x \mathrm{d}\left(\frac{1}{x^2+1}\right) \end{aligned}$$

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Rearrange terms

$$2\int \frac{\mathrm{d}x}{(1+x^2)^2} = \left(\frac{x}{x^2+1} + \arctan x\right) + L \qquad .$$

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$$\arctan x + C \quad = \quad \frac{x}{x^2+1} + 2\arctan x - 2\int \frac{dx}{(x^2+1)^2}$$

Rearrange terms and divide by 2 to get the desired integral:

$$\int \frac{dx}{(1+x^2)^2} = \frac{1}{2}\left(\frac{x}{x^2+1} + \arctan x\right) + \quad K \quad .$$

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$$\int \frac{1}{(x^2+1)} \mathrm{d}x = \arctan x + C \quad .$$

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$$J(n) = \int \frac{1}{(x^2+1)^n}\mathrm{d}x$$

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- Set $J(n) = \int \frac{1}{(x^2+1)^n}\mathrm{d}x$. We are looking for a formula for $J(n)$.

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- Building block IIIa:

$$J(1) = \int \frac{1}{(x^2+1)} \mathrm{d}x = \arctan x + C \quad .$$

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- Building block IIIa:

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- In this way we end up expressing $J(n)$ via $J(n-1)$.
- We work our way from $J(n)$ to $J(n-1)$, from $J(n-1)$ to $J(n-2)$, and so on, until we get to $J(1)$.

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Rearrange to get:

$$2(n-1)J(n) = \frac{x}{(x^2+1)^{n-1}} + (2n-3)J(n-1)$$

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In this way we expressed $J(n)$ using $J(n-1)$.

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