

Arithmetics
Subtraction base 10
calculator-algebra.org

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Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$5 - 3 = 2$	because $3 + 2 = 5$
$4 - 0 = 4$	because $0 + 4 = 4$
$7 - 4 = 3$	because $4 + 3 = 7$
$8 - 2 = 6$	because $2 + 6 = 8$
$9 - 7 = 2$	because $7 + 2 = 9$

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$6 - 1 = 5$	because $1 + 5 = 6$
$9 - 5 = 4$	because $5 + 4 = 9$
$8 - 2 = 6$	because $2 + 6 = 8$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

- To do one-digit subtraction: guess from addition table.

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$9 - 2 = 7$$

$$8 - 4 = 4$$

$$7 - 7 = 0$$

$$\begin{array}{r} 9 \\ - 2 \\ \hline 7 \end{array}$$

$$\begin{array}{r} 8 \\ - 4 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 7 \\ - 7 \\ \hline 0 \end{array}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

- Subtraction can also be written in columns.

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$11 - 3 = 8$$

$$10 - 5 = 5$$

$$18 - 9 = 9$$

$$\begin{array}{r} 11 \\ - 3 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 10 \\ - 5 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 18 \\ - 9 \\ \hline 9 \end{array}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

- Addition table: can be used for subtraction from small two-digit numbers.

Negative integers

- The negative integers are the numbers:

$\dots, -6, -5, -4, -3, -2, -1$

- Written as the minus sign $-$ followed by a (positive) number.
- Negatives are to the left of 0 on the number line.



- -1 is as far away from 0 as 1 is.
- -2 is as far away from 0 as 2 is.
- \dots and so on.

Negative sign as a function

- The negative sign can be regarded as a function/operator:

Rule

$$-(a) = -a \quad \text{if } a > 0$$

Example

$$-(5) = -5$$

- On the left, $-$ is regarded as a function that takes as input a positive number and produces an output that is a negative number.
- On the right, $-$ is regarded as a part of the notation for negative numbers.

Absolute value (magnitude) of a number

Definition (Magnitude of a number)

The magnitude or absolute value $|x|$ of a number x is defined as:

- The number itself, if the number is non-negative.

$$|a| = a, \quad \text{if } a \text{ is non-negative.}$$

- The number with negative sign removed, if the number is negative.

$$|-a| = a, \quad \text{if } a \text{ is negative.}$$

Example

$$\begin{array}{rcl} |4| & = & 4 \\ |-5| & = & 5 \\ |0| & = & 0 \end{array}$$

Negative of a negative

Rule

The negative of a negative of a number is the number itself.

$$-(-a) = a$$

- Parenthesis are necessary when using multiple negative signs.

Incorrect Correct

$$\cancel{> a} \quad -(-a)$$

- The rule is independent of whether a is positive or negative.
- Can be applied consecutively for more than 2 negative signs.

Example

$$\begin{aligned} -(-5) &= 5 \\ -(-(-7)) &= -(7) = -7 \\ -(-(-(-1))) &= -(-1) = 1 \end{aligned}$$

Sum with a negative

Rule

Subtracting a number is the same as adding its negative.

$$a + (-b) = a - b$$

$$-a + b = b - a$$

Example

$$5 + (-3) = 5 - 3 = 2$$

$$10 + (-5) = 10 - 5 = 5$$

$$-1 + 8 = 8 - 1 = 7$$

$$-8 + 14 = 14 - 8 = 6$$

Negative of a sum

Rule

The sum of negatives is the negative of the sum.

$$-a - b = -(a + b)$$

The difference of two numbers is minus the opposite difference.

$$a - b = -(b - a)$$

Example

$$\begin{array}{rclcl} -5 - 7 & = & -(5 + 7) & = & -12 \\ -7 + (-8) & = & -7 - 8 & = & -(7 + 8) = -15 \\ 5 - 9 & = & -(9 - 5) & = & -4 \\ 6 - 11 & = & -(11 - 6) & = & -5 \\ -9 + 3 & = & 3 - 9 & = & -(9 - 3) = -6 \end{array}$$

Summary of algebra rules involving subtraction

Rule

$$\begin{aligned} -(-a) &= a \\ a + (-b) &= a - b \\ -a + b &= b - a \\ -a - b &= -(a + b) \\ a - b &= -(b - a) \end{aligned}$$

Example

Find the number x so that: $5 + x = 9$. Solution:

$$\begin{array}{rcl|l} 5 + x & = & 9 & \text{transfer 5 to the right hand side} \\ x & = & 9 - 5 & \text{When transferred, 5 acquires negative sign} \\ x & = & 4 & \end{array}$$

Observation

At the price of a negative sign, one is allowed to transfer summands from one side of an equation to the other.

$$\begin{array}{rcl|l} a + b & = & c & \text{transfer } a \text{ to the right hand side} \\ b & = & c - a & \end{array}$$

Example

Solve the equation.

$$8 + x = 9 \quad \text{Answer: } x = 1$$

$$3 + t = 11 \quad \text{Answer: } t = 8$$

$$5 + a = 10 \quad \text{Answer: } a = 5$$

$$8 + s = 16 \quad \text{Answer: } s = 8$$

Example

Solve the equation.

$$7 + x = 2$$

Solution.

$$7 + x = 2$$

Transfer 7 to the other side

$$x = 2 - 7 = -5$$

Example

Solve the equation.

$$3 + x = 7 \quad \text{Answer: } x = 4$$

$$1 + a = 10 \quad \text{Answer: } a = 9$$

$$3 + x = 1 \quad \text{Answer: } x = -2$$

$$5 + x = 0 \quad \text{Answer: } x = -5$$

$$9 + a = 15 \quad \text{Answer: } a = 6$$

$$4 + z = 13 \quad \text{Answer: } z = 9$$

$$9 + x = 8 \quad \text{Answer: } x = -1$$

$$9 + x = 1 \quad \text{Answer: } x = -8$$

Example

Find w, x, y, z so as to satisfy each equality below.

$$-4 = -10 + w$$

Solution

$$-4 = (-10 + 10) - 4 = -10 + (10 - 4) = -10 + 6 \Rightarrow w = 6$$

$$-2 = -10 + x$$

Solution

$$-2 = (-10 + 10) - 2 = -10 + (10 - 2) = -10 + 8 \Rightarrow x = 8$$

$$-1 = -10 + y$$

Solution

$$-1 = (-10 + 10) - 1 = -10 + (10 - 1) = -10 + 9 \Rightarrow y = 9$$

$$-9 = -10 + z$$

Solution

$$-9 = (-10 + 10) - 9 = -10 + (10 - 9) = -10 + 1 \Rightarrow z = 1$$

Subtract 5 from 71.

$$\begin{array}{r}
 -1 \\
 71 \\
 -5 \\
 \hline
 66
 \end{array}$$

- Ensure summand $>$ subtracand.

$$\begin{aligned}
 1 - 5 &= -4 = -10 + 6 \\
 -1 + 7 - 0 &= 6
 \end{aligned}$$

When the subtracand is larger than the summand, we first use the rule $a - b = -(b - a)$.

Compute $447 - 509 = -(509 - 447)$.

$$\begin{array}{r}
 -1 \\
 5 9 \\
 - 4 7 \\
 \hline
 0 2
 \end{array}$$

- Ensure summand $>$ subtracand.
- Remove leading zeroes.

$$\begin{aligned}
 9 - 7 &= 2 \\
 0 - 4 &= -4 = -10 + 6 \\
 -1 + 5 - 4 &= 0
 \end{aligned}$$

Subtract 1234567 from 20182019.

$$\begin{array}{r}
 -1 -1 -1 -1 -1 \\
 2 0 1 8 2 0 1 9 \\
 1 2 3 4 5 6 7 \\
 \hline
 1 8 9 4 7 4 5 2
 \end{array}$$

- Ensure summand $>$ subtracand.

$$\begin{aligned}
 9 - 7 &= 2 \\
 1 - 6 &= -5 = -10 + 5 \\
 -1 + 0 - 5 &= -6 = -10 + 4 \\
 -1 + 2 - 4 &= -3 = -10 + 7 \\
 -1 + 8 - 3 &= 4 \\
 1 - 2 &= -1 = -10 + 9 \\
 -1 + 0 - 1 &= -2 = -10 + 8 \\
 -1 + 2 - 0 &= 1
 \end{aligned}$$