

## Precalculus

# Exponent equation that reduces to quadratic, shifted exponents

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2019

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Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

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Set  $u = ?$  .

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Set  $u = 2^x$ . Then  $4^{x+1} = 4u^2$ ,  $2^{x+2} = ?$ .



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$$x = \log_2 \left( \frac{3}{2} \right) = \frac{\ln \left( \frac{3}{2} \right)}{\ln 2} \approx 0.58496 \quad \text{or} \quad \text{no real solution}$$