

## Precalculus

# Trigonometric equation that reduces to quadratic, masked by identity

Todor Milev

2019

## Example

Find all values of  $\theta$  in the interval  $[0, 2\pi]$  such that  $\sin \theta = \sin(2\theta)$ .

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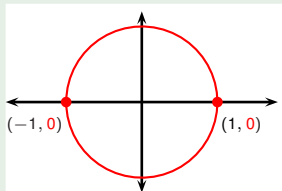
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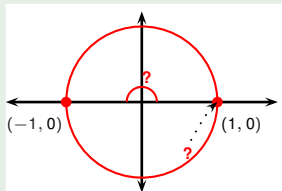
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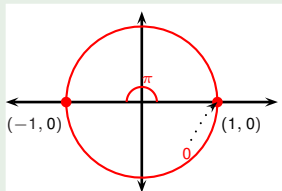
$$\sin \theta = 0$$

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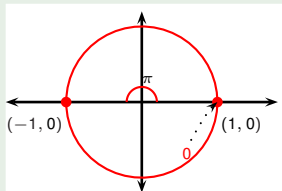
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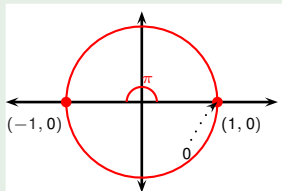
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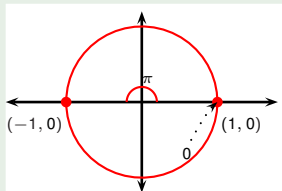
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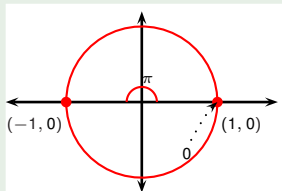
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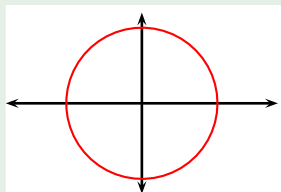
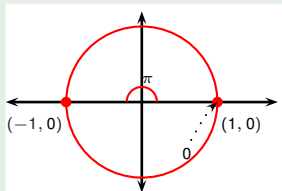
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$$2 \cos \theta - 1 = 0$$

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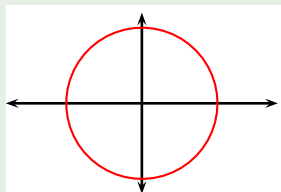
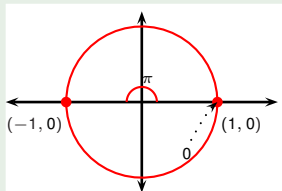
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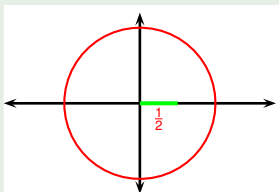
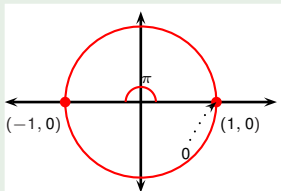
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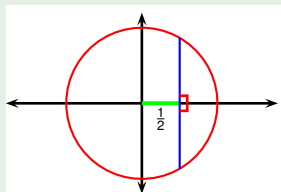
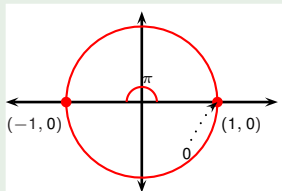
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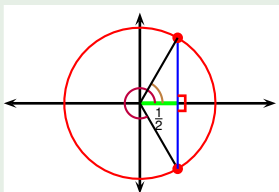
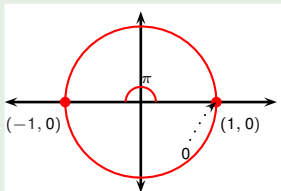
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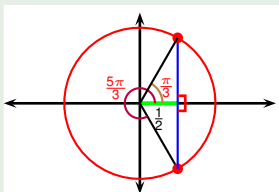
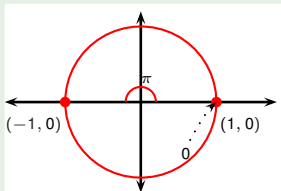
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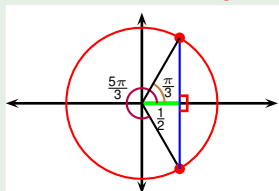
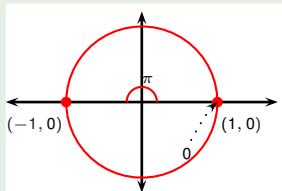
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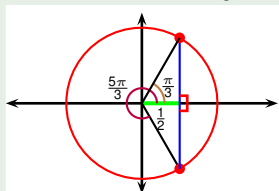
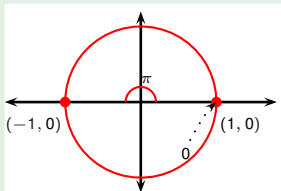
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$$\cos(2\theta) = \cos \theta$$

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$$(u - 1)(2u + 1) = 0$$

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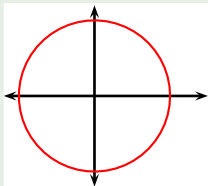
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$$\cos \theta = 1$$

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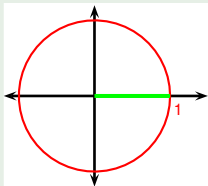
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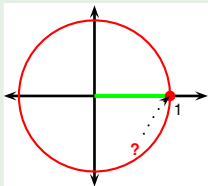
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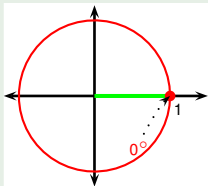
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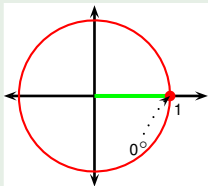
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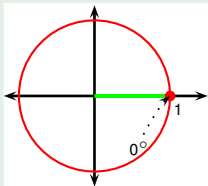
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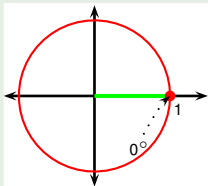
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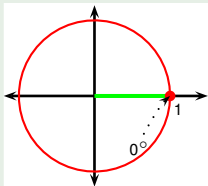
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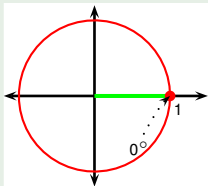
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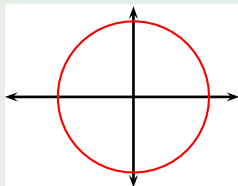
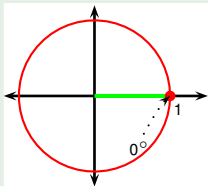
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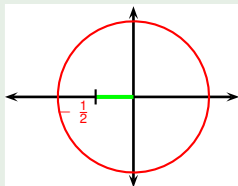
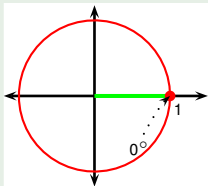
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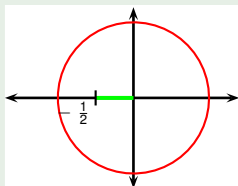
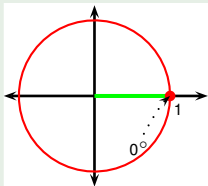
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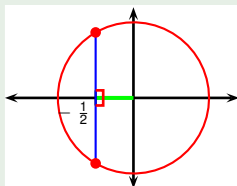
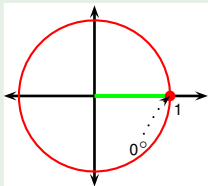
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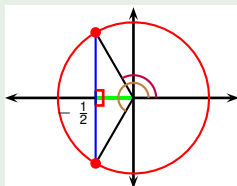
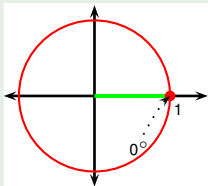
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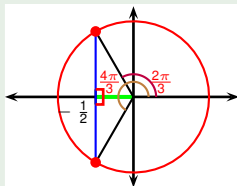
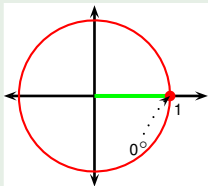
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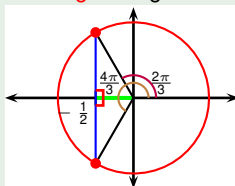
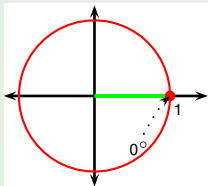
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