

Precalculus

Polynomial division and factorization of cubics with rational root

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Outline

- 1 Polynomial division
- 2 Factoring cubics with rational root

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$\begin{array}{r}
 \text{Quotient:} \quad x^2 + 3x + 3 \\
 x - 1 \overline{) x^3 + 2x^2 + 1} \\
 \underline{x^3 - x^2} \\
 3x^2 \\
 \underline{3x^2 - 3x} \\
 3x + 1 \\
 \underline{3x - 3} \\
 4
 \end{array}$$

Remainder: 4

$$\begin{aligned}
 (\text{Dividend}) &= (\text{Quotient}) \cdot (\text{Divisor}) + (\text{Remainder}) \\
 (x^3 + 2x^2 + 1) &= (x^2 + 3x + 3) \cdot (x - 1) + 4
 \end{aligned}$$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$\begin{array}{r}
 \text{Quotient:} \quad 3x^2 - 5x + 1 \\
 2x - 3 \overline{) 6x^3 - 19x^2 + 17x - 3} \\
 \underline{6x^3 - 9x^2} \\
 -10x^2 + 17x - 3 \\
 \underline{-10x^2 + 15x} \\
 2x - 3 \\
 \underline{2x - 3} \\
 0
 \end{array}$$

Remainder: 0

$$(\text{Dividend}) = (\text{Quotient}) \cdot (\text{Divisor}) + (\text{Remainder})$$

$$(6x^3 - 19x^2 + 17x - 3) = (3x^2 - 5x + 1) \cdot (2x - 3)$$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$\begin{aligned}(6x^3 - 19x^2 + 17x - 3) &= (3x^2 - 5x + 1) \cdot (2x - 3) \\ &= 3 \left(x - \left(\frac{5+\sqrt{13}}{6} \right) \right) \left(x - \left(\frac{5-\sqrt{13}}{6} \right) \right) (2x - 3)\end{aligned}$$

No easy factorization of quadratic, so use formula:

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 3 \cdot 1}}{2 \cdot 3} = \frac{5 \pm \sqrt{13}}{6}$$

We are ready to solve the equation.

$$\begin{aligned}6x^3 - 19x^2 + 17x - 3 &= 0 \\ 3 \left(x - \left(\frac{5+\sqrt{13}}{6} \right) \right) \left(x - \left(\frac{5-\sqrt{13}}{6} \right) \right) (2x - 3) &= 0 \\ 2x - 3 = 0 \quad \text{or} \quad x = \left(\frac{5+\sqrt{13}}{6} \right) \quad \text{or} \quad x = \left(\frac{5-\sqrt{13}}{6} \right) \\ x &= \frac{3}{2}\end{aligned}$$

Example



Plot the left hand side of the equation with a graphing calculator. Solve the equation.

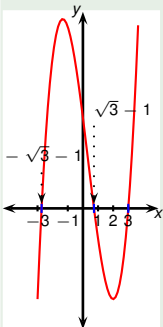
$$\begin{aligned}
 2x^3 + x^2 - 7x - 6 &= 0 \\
 (2x + 3)(x + 1)(x - 2) &= 0 \\
 x = -\frac{3}{2} \quad \text{or} \quad x = -1 \quad \text{or} \quad x = 2
 \end{aligned}$$

Make sure to practice with the graphing calculator you will on your exam(s). The graph appears to intersect the x axis at: $-1.5, -1, 2$. The left hand side should factor as:

$$\begin{aligned}
 2(x - (-1.5))(x - (-1))(x - 2) &= (2x + 3)(x + 1)(x - 2) \\
 &= (2x^2 + 5x + 3)(x - 2) = (2x^3 + 5x^2 + 3x) - (4x^2 + 10x + 6) \\
 &= 2x^3 + x^2 - 7x - 6
 \end{aligned}$$

Check work to make sure we guessed the roots correctly.

Example



Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$\begin{aligned}x^3 - x^2 - 8x + 6 &= 0 \\(x - 3)(x^2 + 2x - 2) &= 0\end{aligned}$$

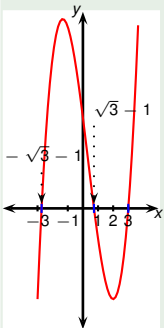
The graph appears to intersect the x axis at:
 $-\sqrt{3} - 1, \sqrt{3} - 1, 3$. What are the two roots besides 3?

Quotient:	$x^2 + 2x - 2$
$x - 3$	$\begin{array}{r} x^3 - x^2 - 8x + 6 \\ x^3 - 3x^2 \\ \hline 2x^2 - 8x + 6 \\ 2x^2 - 6x \\ \hline -2x + 6 \\ -2x + 6 \\ \hline 0 \end{array}$

Remainder:

0

Example



Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x = 3 \quad x = \frac{-2 \pm \sqrt{12}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}.$$

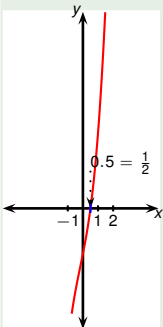
The graph appears to intersect the x axis at:

$-1 - \sqrt{3}$, $-1 + \sqrt{3}$, 3. What are the two roots besides 3?

Final answer:

$$x = 3 \quad \text{or} \quad x = -1 - \sqrt{3} \quad \text{or} \quad x = -1 + \sqrt{3}.$$

Example



Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2})(2x^2 + 2x + 6) + 0 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct?

Is there another root (far away from 0)? Factor:

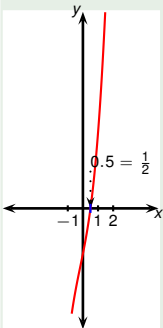
Quotient: $2x^2 + 2x + 6$

$$\begin{array}{r}
 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
 \underline{2x^3 - x^2} \\
 2x^2 + 5x - 3 \\
 \underline{2x^2 - x} \\
 6x - 3 \\
 \underline{6x - 3} \\
 0
 \end{array}$$

Remainder:

0

Example



Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2})(2x^2 + 2x + 6) + 0 = 0$$

$$x - \frac{1}{2} = 0 \quad \text{or} \quad x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2}$$

$$x = \frac{1}{2} \quad x = \frac{-2 \pm \sqrt{-44}}{2 \cdot 2}$$

no real solution

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)?