Calculus II

Power series expansion of rational functions with linear denominator, part 1

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Representations of Functions as Power Series

$$g(x) = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \qquad \text{for } |x| < 1$$

- This is a geometric series with $a = 1$ and $r = x$.
- It is convergent if $|x| < 1$ and divergent otherwise.
- If convergent, the sum is $\dfrac{1}{1-x}$.
- The domain of $g(x)$ is $|x| < 1$.
- The domain of $f(x) = \dfrac{1}{1-x}$ is $x \neq 1$.
- In this way $g(x) = \sum_{n=0}^{\infty} x^n$ is a new way to compute/expresses the function $f(x) = \dfrac{1}{1-x}$ for $|x| < 1$.
- Except for their domains, the functions $g(x)$ and $f(x)$ coincide.