Precalculus
§ Polynomial system that reduces to quadratic, part 1

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Example

Solve the polynomial system. $\left|\begin{array}{rcl} x-4y &=& 5 \\ y^2+xy &=& 10 \end{array}\right.$

$$\begin{array}{rcll} x &=& 5+4y & \left| \text{ Solve for } x \text{ in first eq-n.} \right. \\ y^2+xy &=& 10 & \left| \text{ Substitute } x \text{ away} \right. \\ y^2+(5+4y)y &=& 10 & \\ y^2+5y+4y^2-10 &=& 0 & \\ 5y^2+5y-10 &=& 0 & \left| \text{ Divide by 5} \right. \\ y^2+y-2 &=& 0 & \\ (y+2)(y-1) &=& 0 & \\ y=-2 & \text{or} & y=1 & \end{array}$$

$$\begin{array}{ll} x=5+4y & x=5+4y \\ =5+4(-2)=-3 & =5+4\cdot 1=9 \end{array}$$

Final answer: $x=-3, y=-2$ or $x=9, y=1$.

Example

Solve the polynomial system. $\left| \begin{array}{rcl} x - 4y & = & 5 \\ y^2 + xy & = & 10 \end{array} \right.$

Final answer: $x = -3, y = -2$ or $x = 9, y = 1$.

Check answer $x = -3, y = -2$:

$$\left| \begin{array}{rcll} x - 4y & = & (-3) - 4(-2) & = & 5 \\ y^2 + xy & = & (-2)^2 + (-3)(-2) & = & 10 \end{array} \right.$$

Check answer $y = 1, x = 9$:

$$\left| \begin{array}{rcll} x - 4y & = & 9 - 4 \cdot 1 & = & 5 \\ y^2 + xy & = & 1^2 + 9 \cdot 1 & = & 10. \end{array} \right.$$