Calculus II
Add geometric progression, part 1

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Example

Find the sum of the geometric series $$-2+\frac{6}{5}-\frac{18}{25}+\frac{54}{125}-\cdots$$

For $|r| < 1$, recall that the sum of a geometric series is

$$a + ar + ar^2 + ar^3 + \dots$$

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Find the sum of the geometric series $-2 + \frac{6}{5} - \frac{18}{25} + \frac{54}{125} - \cdots$

- The first term is $a = -2$.

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- The first term is $a = -2$.
- The common ratio is $r =$ **?** .

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Example

Find the sum of the geometric series $-2 + \frac{6}{5} - \frac{18}{25} + \frac{54}{125} - \cdots$

- The first term is $a = -2$.
- The common ratio is $r = \frac{\frac{6}{5}}{-2} = -\frac{3}{5}$.

For $|r| < 1$, recall that the sum of a geometric series is

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- Therefore the sum is

$$\sum_{n=1}^{\infty}(-2)\left(-\frac{3}{5}\right)^{n-1} = \frac{(-2)}{1-\left(-\frac{3}{5}\right)}$$

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