

Precalculus

Factor cubic with one rational and two real roots using its plot

Todor Milev

2019

Example

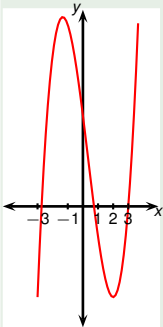
Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

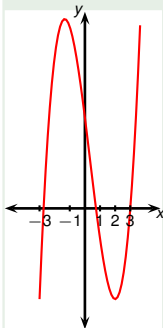


Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
?

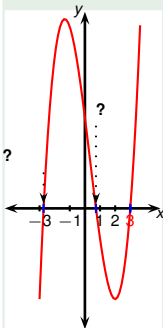


Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3.

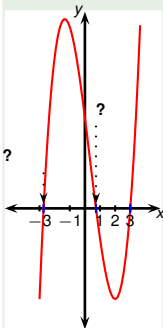


Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



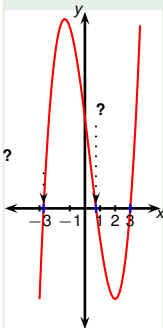
$$x - 3 \quad \overline{x^3 - x^2 - 8x + 6}$$

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



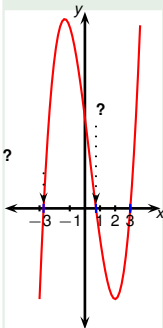
$$x - 3 \overline{) x^3 - x^2 - 8x + 6}$$

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



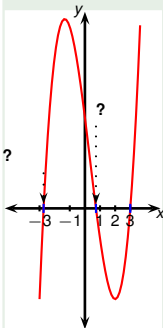
$$x - 3 \quad \overline{x^3 - x^2 - 8x + 6}$$

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$x - 3 \overline{) x^3 - x^2 - 8x + 6}$$

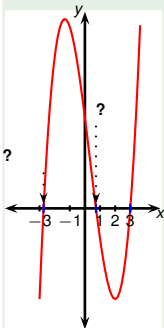
Divide x^3 by x .

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$x - 3 \overline{) \overset{x^2}{x^3 - x^2 - 8x + 6}}$$

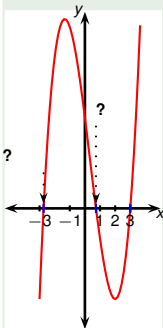
Divide x^3 by x .

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 \\
 x - 3 \overline{) x^3 - x^2 - 8x + 6} \\
 \underline{ x^2 } \\
 ?
 \end{array}$$

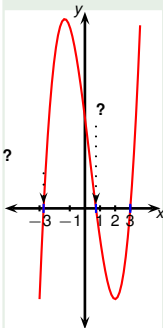
Multiply x^2 by divisor.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x - 3 \overline{) \begin{array}{l} x^3 - x^2 - 8x + 6 \\ x^3 - 3x^2 \end{array}} \\
 \hline
 \end{array}$$

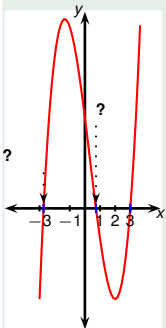
Multiply x^2 by divisor.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 \\
 x - 3 \overline{) x^3 - x^2 - 8x + 6} \\
 \underline{x^3 - 3x^2} \\
 2x^2 - 8x + 6 \\
 \underline{2x^2 - 6x} \\
 2x + 6 \\
 \underline{2x + 6} \\
 0
 \end{array}$$

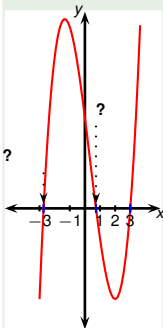
Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 \\
 x - 3 \overline{) x^3 - x^2 - 8x + 6} \\
 \underline{x^3 - 3x^2} \\
 2x^2 - 8x + 6
 \end{array}$$

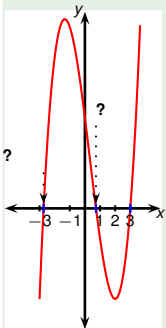
Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 \quad ? \\
 x - 3 \overline{) x^3 - x^2 - 8x + 6} \\
 \underline{x^3 - 3x^2} \\
 2x^2 - 8x + 6
 \end{array}$$

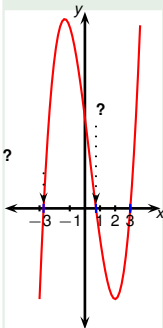
Divide $2x^2$ by x .

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 + 2x \\
 \hline
 x^3 - x^2 - 8x + 6 \\
 \underline{x^3 - 3x^2} \\
 2x^2 - 8x + 6
 \end{array}$$

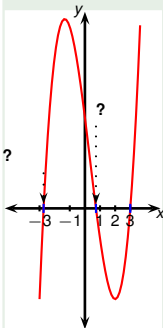
Divide $2x^2$ by x .

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 + 2x \\
 \hline
 x^3 - x^2 - 8x + 6 \\
 \underline{x^3 - 3x^2} \\
 2x^2 - 8x + 6 \\
 \underline{ } \\

 \end{array}$$

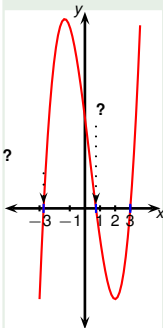
Multiply $2x$ by divisor.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 + 2x \\
 \overline{x^3 - x^2 - 8x + 6} \\
 x^3 - 3x^2 \\
 \hline
 2x^2 - 8x + 6 \\
 2x^2 - 6x \\
 \hline
 \end{array}$$

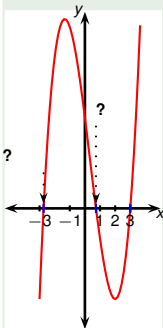
Multiply $2x$ by divisor.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 + 2x \\
 \overline{x^3 - x^2 - 8x + 6} \\
 x^3 - 3x^2 \\
 \hline
 2x^2 - 8x + 6 \\
 2x^2 - 6x \\
 \hline
 - 2x + 6 \\
 - 2x + 6 \\
 \hline
 0
 \end{array}$$

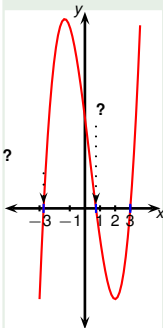
Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 + 2x \\
 \overline{x^3 - x^2 - 8x + 6} \\
 x^3 - 3x^2 \\
 \hline
 2x^2 - 8x + 6 \\
 2x^2 - 6x \\
 \hline
 -2x + 6
 \end{array}$$

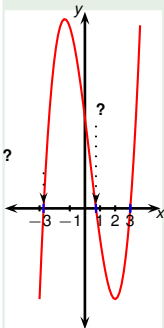
Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 + 2x \quad ? \\
 \hline
 x^3 - x^2 - 8x + 6 \\
 \underline{x^3 - 3x^2} \\
 2x^2 - 8x + 6 \\
 \underline{2x^2 - 6x} \\
 -2x + 6
 \end{array}$$

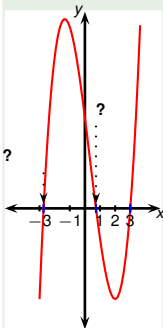
Divide $-2x$ by x .

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 + 2x - 2 \\
 x - 3 \overline{) x^3 - x^2 - 8x + 6} \\
 \underline{x^3 - 3x^2} \\
 2x^2 - 8x + 6 \\
 \underline{2x^2 - 6x} \\
 -2x + 6
 \end{array}$$

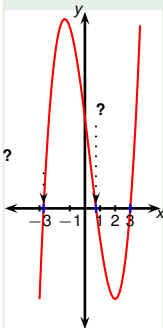
Divide $-2x$ by x .

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 + 2x - 2 \\
 x - 3 \overline{) x^3 - x^2 - 8x + 6} \\
 \underline{x^3 - 3x^2} \\
 2x^2 - 8x + 6 \\
 \underline{2x^2 - 6x} \\
 -2x + 6 \\
 \underline{ 2x - 6} \\
 12
 \end{array}$$

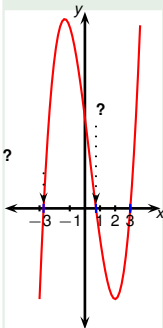
Multiply -2 by divisor.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 + 2x - 2 \\
 x - 3 \overline{) x^3 - x^2 - 8x + 6} \\
 \underline{x^3 - 3x^2} \\
 2x^2 - 8x + 6 \\
 \underline{2x^2 - 6x} \\
 -2x + 6 \\
 \underline{-2x + 6} \\
 0
 \end{array}$$

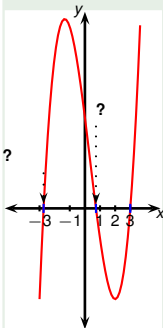
Multiply -2 by divisor.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 + 2x - 2 \\
 x - 3 \overline{) x^3 - x^2 - 8x + 6} \\
 \underline{x^3 - 3x^2} \\
 2x^2 - 8x + 6 \\
 \underline{2x^2 - 6x} \\
 -2x + 6 \\
 \underline{-2x + 6} \\
 ?
 \end{array}$$

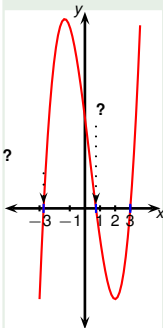
Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 + 2x - 2 \\
 x - 3 \overline{) x^3 - x^2 - 8x + 6} \\
 \underline{x^3 - 3x^2} \\
 2x^2 - 8x + 6 \\
 \underline{2x^2 - 6x} \\
 -2x + 6 \\
 \underline{-2x + 6} \\
 0
 \end{array}$$

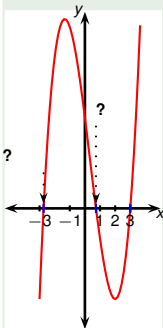
Subtract last two polynomials.

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



$$\begin{array}{r}
 x^2 + 2x - 2 \\
 x - 3 \overline{) x^3 - x^2 - 8x + 6} \\
 \underline{x^3 - 3x^2} \\
 2x^2 - 8x + 6 \\
 \underline{2x^2 - 6x} \\
 -2x + 6 \\
 \underline{-2x + 6} \\
 0
 \end{array}$$

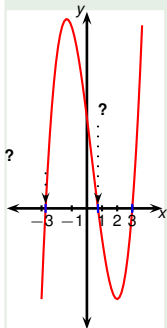
Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) + 0 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



Quotient: $x^2 + 2x - 2$

$$\begin{array}{r}
 x - 3 \overline{) x^3 - x^2 - 8x + 6} \\
 \underline{x^3 - 3x^2} \\
 2x^2 - 8x + 6 \\
 \underline{2x^2 - 6x} \\
 -2x + 6 \\
 \underline{-2x + 6} \\
 0
 \end{array}$$

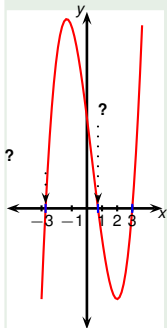
Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) + 0 = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



Quotient:	$x^2 + 2x - 2$
$x - 3$	$\begin{array}{r} x^3 - x^2 - 8x + 6 \\ x^3 - 3x^2 \\ \hline 2x^2 - 8x + 6 \\ 2x^2 - 6x \\ \hline -2x + 6 \\ -2x + 6 \\ \hline 0 \end{array}$
Remainder:	0

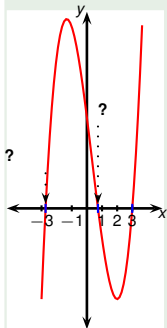
Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) = 0$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



Quotient:	$x^2 + 2x - 2$
$x - 3$	$\begin{array}{r} \overline{) x^3 - x^2 - 8x + 6} \\ \underline{x^3 - 3x^2} \\ 2x^2 - 8x + 6 \\ \underline{2x^2 - 6x} \\ -2x + 6 \\ \underline{-2x + 6} \\ 0 \end{array}$
Remainder:	0

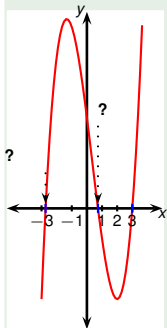
Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x =$$



The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

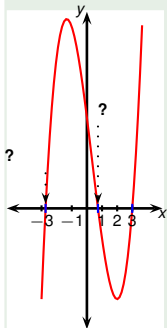
$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x =$$

$$x = 3$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

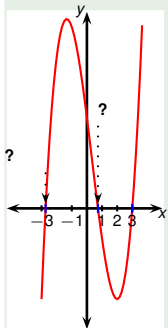
$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

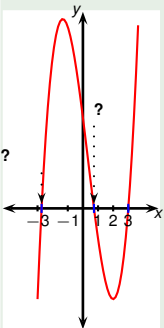
$$x = 3$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?



Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.



$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) = 0$$

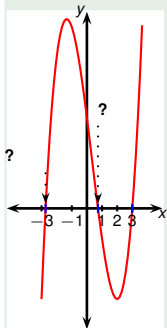
$$x - 3 = 0 \quad \text{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x = 3$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.



$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) = 0$$

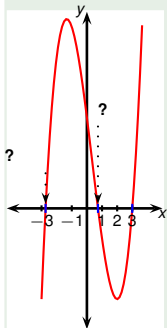
$$x - 3 = 0 \quad \text{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x = 3$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.



$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) = 0$$

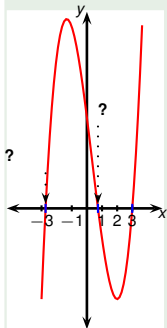
$$x - 3 = 0 \quad \text{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x = 3 \quad x = \frac{-2 \pm \sqrt{12}}{2}$$

The graph appears to intersect the x axis at:
 ? , ? , 3. What are the two roots besides 3?

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.



$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x = 3 \quad x = \frac{-2 \pm \sqrt{12}}{2}$$

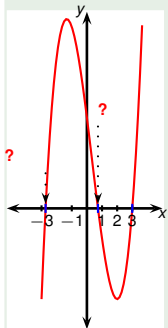
$$x = \frac{-2 \pm 2\sqrt{3}}{2}$$

The graph appears to intersect the x axis at:

?, ?, 3. What are the two roots besides 3?

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.



$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x = 3 \quad x = \frac{-2 \pm \sqrt{12}}{2}$$

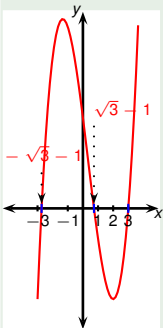
$$x = \frac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}.$$

The graph appears to intersect the x axis at:

?, ?, 3. What are the two roots besides 3?

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.



$$x^3 - x^2 - 8x + 6 = 0$$

$$(x - 3)(x^2 + 2x - 2) = 0$$

$$x - 3 = 0 \quad \text{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$x = 3$$

$$x = \frac{-2 \pm \sqrt{12}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}.$$

The graph appears to intersect the x axis at:

$-1 - \sqrt{3}$, $-1 + \sqrt{3}$, 3. What are the two roots besides 3?

Final answer:

$$x = 3 \quad \text{or} \quad x = -1 - \sqrt{3} \quad \text{or} \quad x = -1 + \sqrt{3}.$$