

Calculus II

Homework

Series absolute convergence, the ratio and root tests

1. Establish whether the series is convergent or divergent. Use the ratio or root tests. Show all your work. The answer key has not been proofread, use with caution.

(a) $\sum_{n=0}^{\infty} (-1)^n n^2 3^{-n}$

(b) $\sum_{n=1}^{\infty} \left(\frac{n+1}{4n} \right)^n$

(c) $\sum_{n=1}^{\infty} \left(\frac{4n+1}{n} \right)^n$

(d) $\sum_{n=1}^{\infty} \frac{n^n}{4^n n!}$

(e) $\sum_{n=1}^{\infty} \frac{(4n)^n}{n!}$

2. Except for $x = \pm e$, use the ratio test to determine all real values of x for which

$$\sum_{n=0}^{\infty} x^n \frac{n!}{n^n}$$

is convergent. You are expected to use in your solution the fact that

$$\lim_{x \rightarrow 0} \left(1 + \frac{x}{n} \right)^n = e^x \quad .$$