

# Calculus I

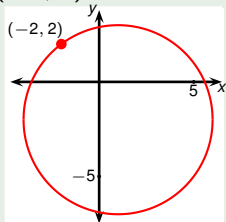
## Tangent to implicit curve, part 2

Todor Milev

2019

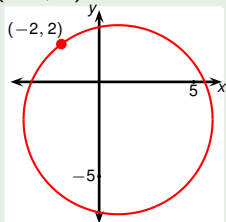
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Find an equation of the tangent line to  $(x - 1)^2 + (y + 2)^2 = 25$  at  $(-2, 2)$ .



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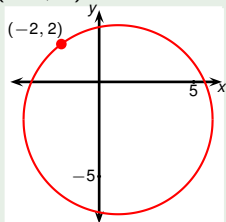
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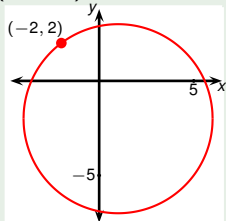
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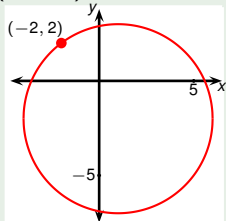
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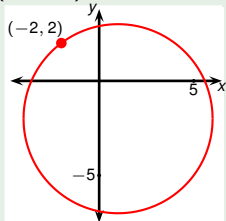
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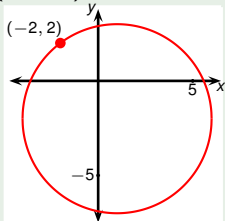
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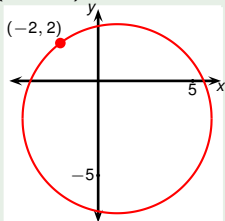
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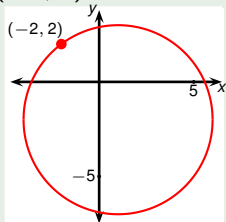
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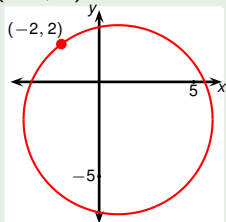
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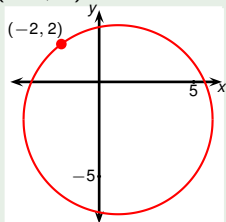


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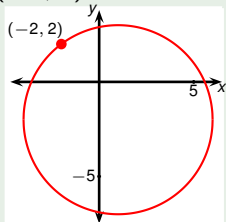


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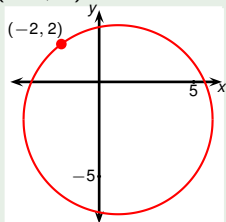


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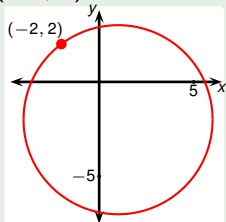
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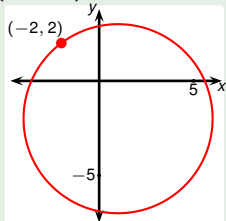
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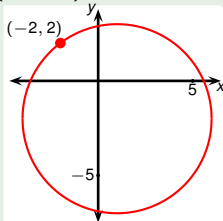
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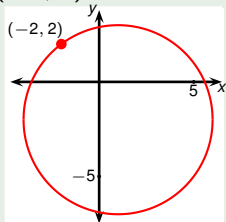
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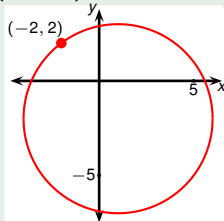
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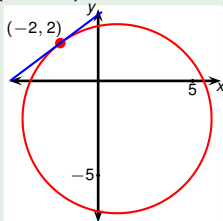
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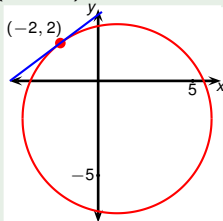
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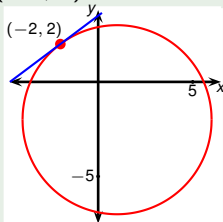
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