Calculus I
Homework
Limits involving infinity

1. Show the following limits do not exist and compute whether they evaluate to ∞, $-\infty$, or neither.

   (a) $\lim_{x\to 3^+} \dfrac{x^2+x-1}{x^2-2x-3}$.

   (b) $\lim_{x\to 3^-} \dfrac{x^2+x-1}{x^2-2x-3}$.

   (c) $\lim_{x\to 1^+} \dfrac{x^2+1}{\sqrt{x^2+3}-2}$.

   (d) $\lim_{x\to 1^-} \dfrac{x^2+1}{\sqrt{x^2+3}-2}$.

   (e) $\lim_{x\to 2^+} \dfrac{\sqrt{x^3-8}}{-x^2+x+2}$.

   (f) $\lim_{x\to -1^+} \dfrac{\sqrt[3]{x^2+2x+1}}{x^2-2x-3}$.

2. Find the limit or show that it does not exist. If the limit does not exist, indicate whether it is $\pm\infty$, or neither. The answer key has not been proofread, use with caution.

   (a) $\lim_{x\to\infty} \dfrac{x-2}{2x+1}$.

   (b) $\lim_{x\to\infty} \dfrac{1-x^2}{x^3-x-1}$.

   (c) $\lim_{x\to-\infty} \dfrac{x-2}{x^2+5}$.

   (d) $\lim_{x\to-\infty} \dfrac{3x^3+2}{2x^3-4x+5}$.

   (e) $\lim_{x\to\infty} \dfrac{\sqrt{x}+x^2}{\sqrt{x}-x^2}$.

   (f) $\lim_{x\to\infty} \dfrac{3-x\sqrt{x}}{2x^{\frac{3}{2}}-2}$.

   (g) $\lim_{x\to\infty} \dfrac{(2x^2+3)^2}{(x-1)^2(x^2+1)}$.

   (h) $\lim_{x\to\infty} \dfrac{x^2-3}{\sqrt{x^4+3}}$.

   (i) $\lim_{x\to-\infty} \dfrac{\sqrt{x^2+1}}{x+1}$.

   (j) $\lim_{x\to\infty} \dfrac{\sqrt{16x^6-3x}}{x^3+2}$.

   (k) $\lim_{x\to-\infty} \dfrac{\sqrt{16x^6-3x}}{x^3+2}$.

   (l) $\lim_{x\to\infty} \dfrac{\sqrt{3x^2+2x+1}}{x+1}$.

   (m) $\lim_{x\to\infty} \sqrt{4x^2+x}-2x$.

   (n) $\lim_{x\to-\infty} x+\sqrt{x^2+3x}$.

   (o) $\lim_{x\to\infty} \sqrt{x^2+2x}-\sqrt{x^2-2x}$.

   (p) $\lim_{x\to-\infty} \sqrt{x^2+x}-\sqrt{x^2-x}$.

   (q) $\lim_{x\to\infty} \sqrt{x^2+ax}-\sqrt{x^2+bx}$.

   (r) $\lim_{x\to\infty} \cos x$.

   (s) $\lim_{x\to\infty} \dfrac{x^4+x}{x^3-x+2}$.

   (t) $\lim_{x\to\infty} \sqrt{x^2+1}$.

   (u) $\lim_{x\to-\infty} (x^4+x^5)$.

   (v) $\lim_{x\to-\infty} \dfrac{\sqrt{1+x^6}}{1+x^2}$.

   (w) $\lim_{x\to\infty} (x-\sqrt{x})$.

   (x) $\lim_{x\to\infty} (x^2-x^3)$.

   (y) $\lim_{x\to\infty} x\sin x$.

   (z) $\lim_{x\to\infty} \sqrt{x}\sin x$.

3. Find the horizontal and vertical asymptotes of the graph of the function. If a graphing device is available, check your work by plotting the function.

   (a) $y = \dfrac{2x}{\sqrt{x^2+x+3}-3}$.

   (b) $y = \dfrac{3x^2}{\sqrt{x^2+2x+10}-5}$.

   (c) $y = \dfrac{3x+1}{x-2}$.

   (d) $y = \dfrac{x^2-1}{2x^2-3x-2}$.

   (e) $y = \dfrac{2x^2-2x-1}{x^2+x-2}$.

   (f) $f(x) = \dfrac{-5x^2-3x+5}{x^2-2x-3}$

   (g) $y = \dfrac{1+x^4}{x^2-x^4}$.

   (h) $y = \dfrac{x^3-x}{x^2-7x+6}$.

   (i) $y = \dfrac{x-9}{\sqrt{4x^2+3x+3}}$.

   (j) $y = \dfrac{\sqrt{x^2+1}-x}{x}$.

   (k) $f(x) = \dfrac{x}{\sqrt{x^2+3}-2x}$