Calculus I
Type 5: Limits involving conjugate radicals.

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Example

$$\text{Find} \quad \lim_{t \to 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}$$

$$\text{Plug in 0:} \quad \frac{\sqrt{(0)^2 + 9} - 3}{(0)^2} = \frac{0}{0}$$

Zero over zero is undefined, so we can't use direct substitution.
Multiply top & bottom by (minus) the conjugate radical:

$$\lim_{t \to 0} \frac{\sqrt{t^2 + 9} - 3}{t^2} = \lim_{t \to 0} \frac{\sqrt{t^2 + 9} - 3}{t^2} \cdot \frac{\sqrt{t^2 + 9} + 3}{\sqrt{t^2 + 9} + 3}$$

$$= \lim_{t \to 0} \frac{(t^2 + 9) - 9}{t^2 \left(\sqrt{t^2 + 9} + 3 \right)} = \lim_{t \to 0} \frac{\cancel{t^2}}{\cancel{t^2}(\sqrt{t^2 + 9} + 3)}$$

$$= \lim_{t \to 0} \frac{1}{\sqrt{t^2 + 9} + 3}$$

$$\text{Plug in 0:} \quad =$$