Calculus I
Linearize a given function, part 1

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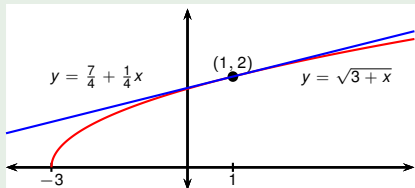
Example

Find the linearization of the function $f(x) = \sqrt{x + 3}$ at $a = 1$ and use it to approximate the numbers $\sqrt{3.98}$ and $\sqrt{4.05}$. Are these approximations overestimates or underestimates?

- $f'(x) = \frac{1}{2\sqrt{x+3}}$.
- $f(1) = \sqrt{1 + 3} = 2$.
- $f'(1) = \frac{1}{2\sqrt{1+3}} = \frac{1}{4}$.
- Linearization:
$$L(x) = 2 + \frac{1}{4}(x - 1)$$
$$= \frac{7}{4} + \frac{x}{4}$$

The graph of the linearization is above the curve, so these are overestimates.

- $\sqrt{3.98} = f(0.98) \approx \frac{7}{4} + \frac{0.98}{4} = 1.995$.
- $\sqrt{4.05} = f(1.05) \approx \frac{7}{4} + \frac{1.05}{4} = 2.0125$.