Calculus II

Integrals of the form $\int \frac{a}{bx^2 + c} \mathrm{d}x$, $c > 0$

Todor Milev

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Linear substitutions leading to blocks IIa and IIIa

Building block IIIa: $\int \frac{1}{u^2+1}\mathrm{d}u = \arctan u + C$.

Example

$$
\begin{aligned}
\int \frac{1}{x^2+2}\mathrm{d}x &= \int \frac{1}{2\left(\frac{1}{2}x^2+1\right)}\mathrm{d}x \\
&= \int \frac{1}{2\left(\left(\frac{x}{\sqrt{2}}\right)^2+1\right)}\sqrt{2}\mathrm{d}\left(\frac{x}{\sqrt{2}}\right) \\
&= \frac{1}{\sqrt{2}}\int \frac{1}{u^2+1}\mathrm{d}u \\
&= \frac{1}{\sqrt{2}}\arctan(u) + C \\
&= \frac{1}{\sqrt{2}}\arctan\left(\frac{x}{\sqrt{2}}\right) + C
\end{aligned}
$$

Use $2 = \left(\sqrt{2}\right)^2$

Set $\frac{x}{\sqrt{2}} = u$

Linear substitutions leading to blocks IIa and IIIa

Building block IIIa: $\int \frac{1}{x^2+1}\mathrm{d}x = \arctan x + C$. **Let** $a > 0$**.**

Example

$$
\begin{aligned}
\int \frac{1}{x^2+a}\mathrm{d}x &= \int \frac{1}{a\left(\frac{1}{a}x^2+1\right)}\mathrm{d}x \\
&= \int \frac{1}{a\left(\left(\frac{x}{\sqrt{a}}\right)^2+1\right)}\sqrt{a}\mathrm{d}\left(\frac{x}{\sqrt{a}}\right) \\
&= \frac{1}{\sqrt{a}}\int \frac{1}{u^2+1}\mathrm{d}u \\
&= \frac{1}{\sqrt{a}}\arctan(u) + C \\
&= \frac{1}{\sqrt{a}}\arctan\left(\frac{x}{\sqrt{a}}\right) + C
\end{aligned}
$$

Use $a = \left(\sqrt{a}\right)^2$

Set $u = \frac{x}{\sqrt{a}}$