

Precalculus

Equations formed by setting trigonometric sum equal to 0

Todor Milev

2019

Proposition (Product to sum formulas)

$$\begin{aligned}\sin \alpha \sin \beta &= \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta)) \\ \cos \alpha \cos \beta &= \frac{1}{2} (\cos(\alpha - \beta) + \cos(\alpha + \beta)) \\ \sin \alpha \cos \beta &= \frac{1}{2} (\sin(\alpha + \beta) + \sin(\alpha - \beta))\end{aligned}$$

Proof.



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Proof.

?

$$= \cos(\alpha + \beta)$$



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Proof.

$$\cos \alpha \cos \beta - \sin \alpha \sin \beta = \cos(\alpha + \beta)$$



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Proof.

$$\begin{aligned}\textcolor{red}{?} &= \textcolor{red}{\cos(\alpha - \beta)} \\ \cos \alpha \cos \beta - \sin \alpha \sin \beta &= \cos(\alpha + \beta)\end{aligned}$$



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Proof.

$$\begin{aligned}\cos \alpha \cos \beta + \sin \alpha \sin \beta &= \cos(\alpha - \beta) \\ \cos \alpha \cos \beta - \sin \alpha \sin \beta &= \cos(\alpha + \beta)\end{aligned}$$



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Proof.

$$\begin{aligned}+ \quad & \cos \alpha \cos \beta + \sin \alpha \sin \beta = \cos(\alpha - \beta) \\ & \cos \alpha \cos \beta - \sin \alpha \sin \beta = \cos(\alpha + \beta) \\ & \quad ? = \cos(\alpha - \beta) + \cos(\alpha + \beta)\end{aligned}$$



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$$\begin{aligned}+ \quad \cos \alpha \cos \beta + \cancel{\sin \alpha \sin \beta} &= \cos(\alpha - \beta) \\ \cos \alpha \cos \beta - \cancel{\sin \alpha \sin \beta} &= \cos(\alpha + \beta) \\ 2 \cos \alpha \cos \beta &= \cos(\alpha - \beta) + \cos(\alpha + \beta)\end{aligned}$$



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$$\sin \alpha \sin \beta = \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\cos \alpha \cos \beta = \frac{1}{2} (\cos(\alpha - \beta) + \cos(\alpha + \beta))$$

$$\sin \alpha \cos \beta = \frac{1}{2} (\sin(\alpha + \beta) + \sin(\alpha - \beta))$$

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$$\begin{aligned}
 + \quad & \cos \alpha \cos \beta + \cancel{\sin \alpha \sin \beta} = \cos(\alpha - \beta) \\
 & \cos \alpha \cos \beta - \cancel{\sin \alpha \sin \beta} = \cos(\alpha + \beta) \\
 & \quad \quad \quad 2 \cos \alpha \cos \beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)
 \end{aligned}$$



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$$\begin{array}{rcll} + & \cos \alpha \cos \beta + \cancel{\sin \alpha \sin \beta} & = & \cos(\alpha - \beta) \\ & \cos \alpha \cos \beta - \cancel{\sin \alpha \sin \beta} & = & \cos(\alpha + \beta) \\ & 2 \cos \alpha \cos \beta & = & \cos(\alpha - \beta) + \cos(\alpha + \beta) \\ \hline - & \cos \alpha \cos \beta + \sin \alpha \sin \beta & = & \cos(\alpha - \beta) \\ & \cos \alpha \cos \beta - \sin \alpha \sin \beta & = & \cos(\alpha + \beta) \\ & 2 \sin \alpha \sin \beta & = & \cos(\alpha - \beta) - \cos(\alpha + \beta) \end{array}$$



Proposition (Product to sum formulas)

$$\begin{aligned}\sin \alpha \sin \beta &= \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta)) \\ \cos \alpha \cos \beta &= \frac{1}{2} (\cos(\alpha - \beta) + \cos(\alpha + \beta)) \\ \sin \alpha \cos \beta &= \frac{1}{2} (\sin(\alpha + \beta) + \sin(\alpha - \beta))\end{aligned}$$

Proof.

$$\begin{array}{rcll} + & \cos \alpha \cos \beta + \cancel{\sin \alpha \sin \beta} & = & \cos(\alpha - \beta) \\ & \cos \alpha \cos \beta - \cancel{\sin \alpha \sin \beta} & = & \cos(\alpha + \beta) \\ & 2 \cos \alpha \cos \beta & = & \cos(\alpha - \beta) + \cos(\alpha + \beta) \\ \hline - & \cos \alpha \cos \beta + \sin \alpha \sin \beta & = & \cos(\alpha - \beta) \\ & \cos \alpha \cos \beta - \sin \alpha \sin \beta & = & \cos(\alpha + \beta) \\ & 2 \sin \alpha \sin \beta & = & \cos(\alpha - \beta) - \cos(\alpha + \beta) \\ \hline + & \sin \alpha \cos \beta + \cos \alpha \sin \beta & = & \sin(\alpha + \beta) \\ & \sin \alpha \cos \beta - \cos \alpha \sin \beta & = & \sin(\alpha - \beta) \\ & 2 \sin \alpha \cos \beta & = & \sin(\alpha + \beta) + \sin(\alpha - \beta) \end{array}$$



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- Product to sum formulas are used when integrating (a topic to be studied later/in another course).

Proposition (Sum to product formulas)

$$\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$$

$$\sin \alpha - \sin \beta = 2 \sin \left(\frac{\alpha - \beta}{2} \right) \cos \left(\frac{\alpha + \beta}{2} \right)$$

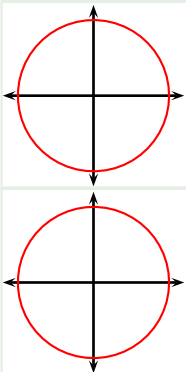
$$\cos \alpha + \cos \beta = 2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$$

$$\cos \alpha - \cos \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\alpha - \beta}{2} \right)$$

Example

Find all solutions in the interval $[0, 2\pi)$.

$$\sin(2x) + \sin(5x) = 0$$

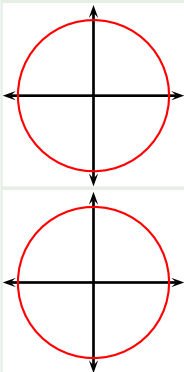


Recall the formula $\sin \alpha + \sin \beta = ?$

Example

Find all solutions in the interval $[0, 2\pi)$.

$$\sin(2x) + \sin(5x) = 0 \quad | \quad \text{use f-l-a}$$

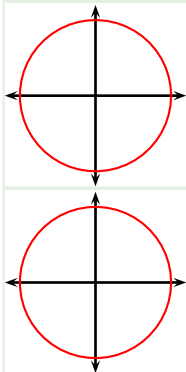


Recall the formula $\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$

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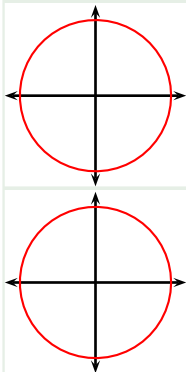
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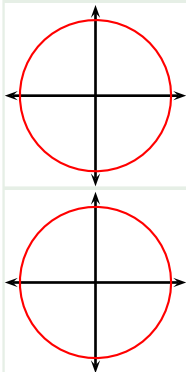
$$\sin(2x) + \sin(5x) = 0 \quad | \text{ use f-l-a}$$

$$2 \sin \left(\frac{2x + 5x}{2} \right) \cos \left(\frac{2x - 5x}{2} \right) = 0$$

Recall the formula $\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$

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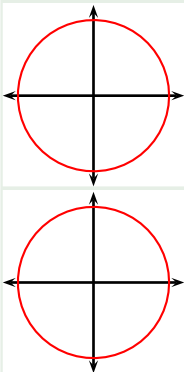
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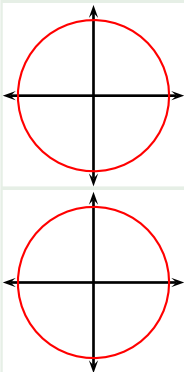


$$\begin{aligned} \sin(2x) + \sin(5x) &= 0 \quad | \text{ use f-l-a} \\ 2 \sin \left(\frac{2x + 5x}{2} \right) \cos \left(\frac{2x - 5x}{2} \right) &= 0 \\ 2 \sin \left(\frac{7}{2}x \right) \cos \left(-\frac{3}{2}x \right) &= 0 \end{aligned}$$

Recall the formula $\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$

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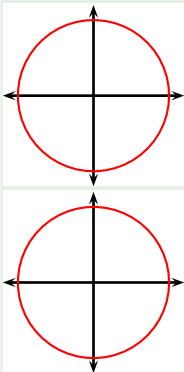


$$\begin{aligned} \sin(2x) + \sin(5x) &= 0 \quad | \text{ use f-l-a} \\ 2 \sin \left(\frac{2x + 5x}{2} \right) \cos \left(\frac{2x - 5x}{2} \right) &= 0 \\ 2 \sin \left(\frac{7}{2}x \right) \cos \left(-\frac{3}{2}x \right) &= 0 \quad | \begin{array}{l} \cos \\ \text{is even} \end{array} \\ 2 \sin \left(\frac{7}{2}x \right) \cos \left(\frac{3}{2}x \right) &= 0 \end{aligned}$$

Recall the formula $\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$

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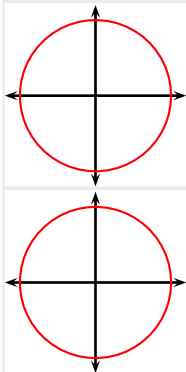


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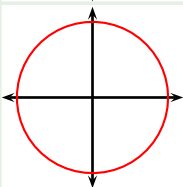
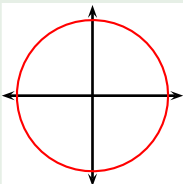
$$\sin(2x) + \sin(5x) = 0 \quad | \quad \text{use f-l-a}$$

$$2 \sin \left(\frac{7}{2}x \right) \cos \left(\frac{3}{2}x \right) = 0$$

Recall the formula $\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$

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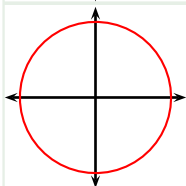
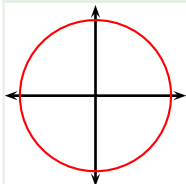
$$\cos \left(\frac{3}{2}x \right) = 0$$

or

Recall the formula $\sin \alpha + \sin \beta = 2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right)$

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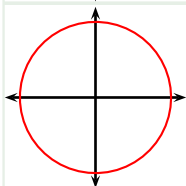
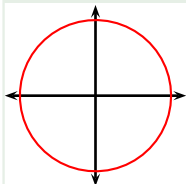
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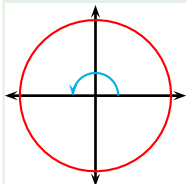
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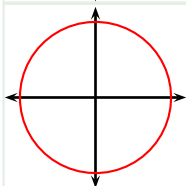


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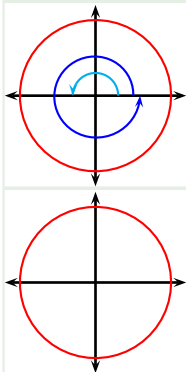
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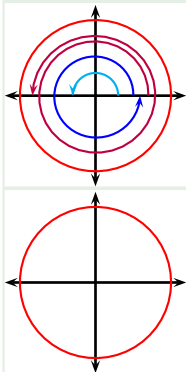
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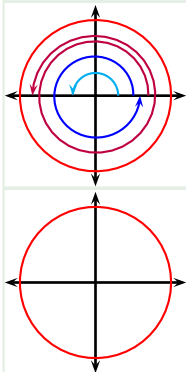
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Example

Find all solutions in the interval $[0, 2\pi)$.



$$\sin \left(\frac{7}{2}x \right) = 0$$

$$\frac{7}{2}x = k\pi$$

$$\sin(2x) + \sin(5x) = 0 \quad | \text{ use f-l-a}$$

$$2 \sin \left(\frac{7}{2}x \right) \cos \left(\frac{3}{2}x \right) = 0$$

k – integer

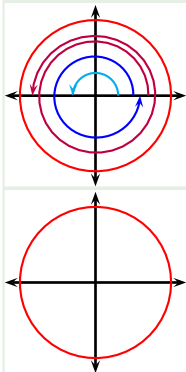
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Example

Find all solutions in the interval $[0, 2\pi)$.



$$\begin{aligned}\sin \left(\frac{7}{2}x \right) &= 0 \\ \frac{7}{2}x &= k\pi \\ x &= \frac{2k\pi}{7}\end{aligned}$$

$$\begin{aligned}\sin(2x) + \sin(5x) &= 0 \quad | \text{ use f-l-a} \\ 2 \sin \left(\frac{7}{2}x \right) \cos \left(\frac{3}{2}x \right) &= 0\end{aligned}$$

k – integer

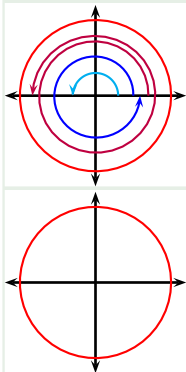
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$$\sin(2x) + \sin(5x) = 0 \quad | \text{ use f-l-a}$$

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k – integer

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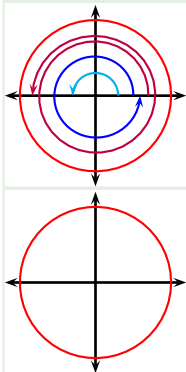
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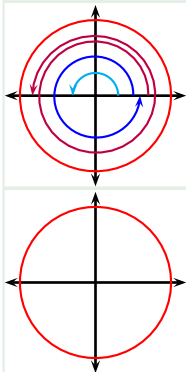
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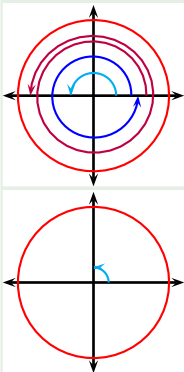
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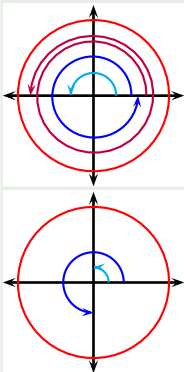
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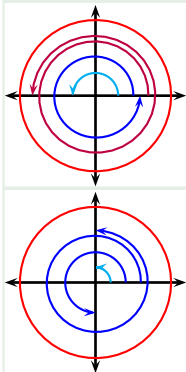
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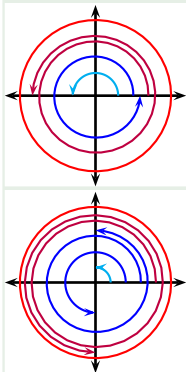
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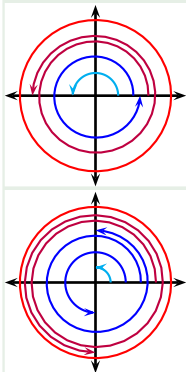
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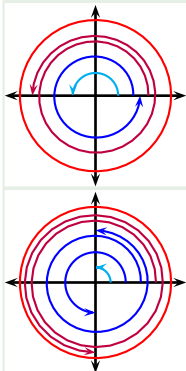
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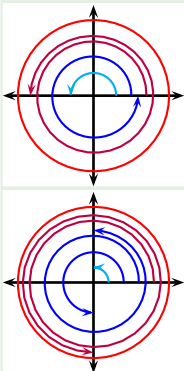
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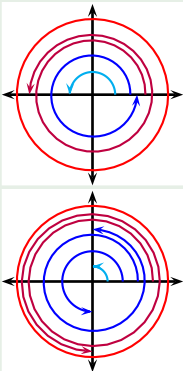
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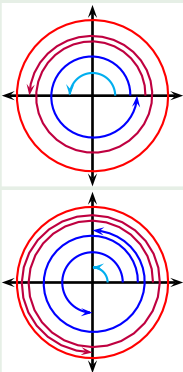
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