

# Precalculus

## Homework

### Graphs of trig functions; inverse trig

1. Convert from degrees to radians.

- |                  |                   |                     |
|------------------|-------------------|---------------------|
| (a) $15^\circ$ . | (h) $120^\circ$ . | (o) $360^\circ$ .   |
| (b) $30^\circ$ . | (i) $135^\circ$ . | (p) $405^\circ$ .   |
| (c) $36^\circ$ . | (j) $150^\circ$ . | (q) $1200^\circ$ .  |
| (d) $45^\circ$ . | (k) $180^\circ$ . | (r) $-900^\circ$ .  |
| (e) $60^\circ$ . | (l) $225^\circ$ . | (s) $-2014^\circ$ . |
| (f) $75^\circ$ . | (m) $270^\circ$ . |                     |
| (g) $90^\circ$ . | (n) $305^\circ$ . |                     |

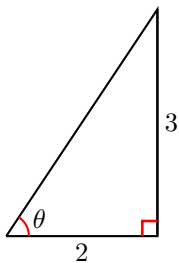
2. Convert from radians to degrees. The answer key has not been proofread, use with caution.

- |                         |                         |               |
|-------------------------|-------------------------|---------------|
| (a) $4\pi$ .            | (d) $\frac{4}{3}\pi$ .  | (g) 5.        |
| (b) $-\frac{7}{6}\pi$ . | (e) $-\frac{3}{8}\pi$ . | (h) $-2014$ . |
| (c) $\frac{7}{12}\pi$ . | (f) $2014\pi$ .         |               |

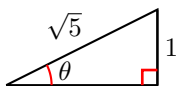
3. Find the indicated circle arc-length. The answer key has not been proofread, use with caution.

- Circle of radius 3, arc of measure  $36^\circ$ .
- Circle of radius  $\frac{1}{2}$ , arc of measure  $100^\circ$ .
- Circle of radius 1, arc of measure 3 (radians).
- Circle of radius 3, arc of measure  $300^\circ$ .

4. Find the 6 trigonometric functions of the indicated angle in the indicated right triangle.



(a)



(b)

- (c) 
- (d)



5. Find the exact value of the trigonometric function (using radicals).

- (a)  $\cos 135^\circ$ .  
 (b)  $\sin 225^\circ$ .  
 (c)  $\cos 495^\circ$ .  
 (d)  $\sin 560^\circ$ .  
 (e)  $\sin\left(\frac{3\pi}{2}\right)$ .  
 (f)  $\cos\left(\frac{11\pi}{6}\right)$ .  
 (g)  $\sin\left(\frac{2015\pi}{3}\right)$ .  
 (h)  $\cos\left(\frac{17\pi}{3}\right)$ .

6. Find all solutions of the equation in the interval  $[0, 2\pi)$ . The answer key has not been proofread, use with caution.

- (a)  $\sin x = -\frac{\sqrt{2}}{2}$ .  
 (b)  $\cos x = \frac{\sqrt{3}}{2}$ .  
 (c)  $\sin(3x) = \frac{1}{2}$ .  
 (d)  $\cos(7x) = 0$ .  
 (e)  $\cos\left(3x + \frac{\pi}{2}\right) = 0$ .  
 (f)  $\sin\left(5x - \frac{\pi}{3}\right) = 0$ .

7. Use the known values of  $\sin 30^\circ$ ,  $\cos 30^\circ$ ,  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\dots$ , the angle sum formulas and the cofunction identities to find an exact value (using radicals) for the trigonometric function.

(a) The six trigonometric functions of  $105^\circ = 45^\circ + 60^\circ$ :

- $\sin(105^\circ)$ .
- $\cos(105^\circ)$ . Should your answer be a positive or a negative number?
- $\tan(105^\circ)$ .
- $\cot(105^\circ)$ .
- $\sec(105^\circ)$ .
- $\csc(105^\circ)$ .

(b) The six trigonometric functions of  $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ :

- $\sin\left(\frac{\pi}{12}\right)$ .
- $\cos\left(\frac{\pi}{12}\right)$ . Should  $\sin\left(\frac{\pi}{12}\right)$  be larger or smaller than  $\cos\left(\frac{\pi}{12}\right)$ ?
- $\tan\left(\frac{\pi}{12}\right)$ .
- $\cot\left(\frac{\pi}{12}\right)$ .
- $\sec\left(\frac{\pi}{12}\right)$ .
- $\csc\left(\frac{\pi}{12}\right)$ .

8. Simplify to a trigonometric function of the angle  $\theta$ . The answer key has not been proofread, use with caution.

- (a)  $\sin\left(\frac{\pi}{2} - \theta\right)$ .

- (b)  $\cos\left(\frac{13\pi}{2} - \theta\right)$ .
- (c)  $\tan(\pi - \theta)$
- (d)  $\cot\left(\frac{3\pi}{2} - \theta\right)$
- (e)  $\csc\left(\frac{3\pi}{2} + \theta\right)$

9. Using the power-reducing formulas, rewrite the expression in terms of first powers of the cosines and sines of multiples of the angle  $\theta$ .

- (a)  $\sin^4 \theta$ .
- (b)  $\cos^4 \theta$ .
- (c)  $\sin^6 \theta$ .
- (d)  $\cos^6 \theta$ .

10. Use the sum-to-product formulas to find all solutions of the trigonometric equation in the interval  $[0, 2\pi)$ .

Please note that typing a query such as “solve( sin(x)+sin (3x)=0)” at [www.wolframalpha.com](http://www.wolframalpha.com) will provide you with a correct answer and a function plot.

- (a)  $\sin(x) + \sin(3x) = 0$ .
- (b)  $\cos(x) + \cos(-3x) = 0$ .
- (c)  $\sin(x) - \sin(3x) = 0$ .
- (d)  $\cos(2x) - \cos(3x) = 0$ .

11. Find the inverse function. You are asked to do the algebra only; you are not asked to determine the domain or range of the function or its inverse.

- (a)  $f(x) = 3x^2 + 4x - 7$ , where  $x \geq -\frac{2}{3}$ .
- (b)  $f(x) = 2x^2 + 3x - 5$ , where  $x \geq -\frac{3}{4}$ .
- (c)  $f(x) = \frac{2x+5}{x-4}$ , where  $x \neq 4$ .
- (d)  $f(x) = \frac{3x+5}{2x-4}$ , where  $x \neq 2$ .
- (e)  $f(x) = \frac{5x+6}{4x+5}$ , where  $x \neq -\frac{5}{4}$ .
- (f)  $f(x) = \frac{2x-3}{-3x+4}$ , where  $x \neq \frac{4}{3}$ .

12. Find the inverse function and its domain.

- (a)  $y = \ln(x+3)$ .
- (b)  $y = 4 \ln(x-3) - 4$ .
- (c)  $y = 2 \ln(-2x+4) + 1$
- (d)  $f(x) = e^{x^3}$ .
- (e)  $y = (\ln x)^2, x \geq 1$ .
- (f)  $y = \frac{e^x}{1+2e^x}$ .
- (g)  $f(x) = 2^{2x} + 2^x - 2$ .

13. Find each of the following values. Express your answers precisely, not as decimals.

- (a)  $\arcsin(\sin 4)$ .
- (b)  $\arcsin(\sin 0.5)$ .
- (c)  $\arcsin(\cos 120^\circ)$ .
- (d)  $\arccos(\cos(3))$ .
- (e)  $\arccos(\cos(-2))$ .
- (f)  $\arccos(\sin(-4))$ .
- (g)  $\arctan(\tan 5)$ .

14. Express as the following as an algebraic expression of  $x$ . In other words, “get rid” of the trigonometric and inverse trigonometric expressions.

(a)  $\cos^2(\arctan x)$ .

(b)  $-\sin^2(\operatorname{arccot} x)$ .

(c)  $\frac{1}{\cos(\arcsin x)}$ .

(d)  $-\frac{1}{\sin(\arccos x)}$ .

15. Let  $x \in (0, 1)$ . Express the following using  $x$  and  $\sqrt{1 - x^2}$ .

(a)  $\sin(\arcsin(x))$ .

(b)  $\sin(2 \arcsin(x))$ .

(c)  $\sin(3 \arcsin(x))$ .

(d)  $\sin(\arccos(x))$ .

(e)  $\sin(2 \arccos(x))$ .

(f)  $\sin(3 \arccos(x))$ .

(g)  $\cos(2 \arcsin(x))$ .

(h)  $\cos(3 \arccos(x))$ .