Calculus I
Trigonometry review

Todor Milev

2019

Outline

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Degrees and radians

- Degrees is a unit for measuring angles, denoted by $^\circ$.

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1^\circ &= \frac{\pi}{180} \text{ rad} \approx 0.017 \text{ rad.}
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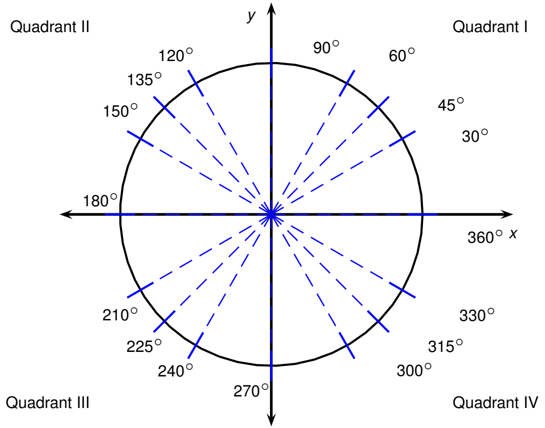
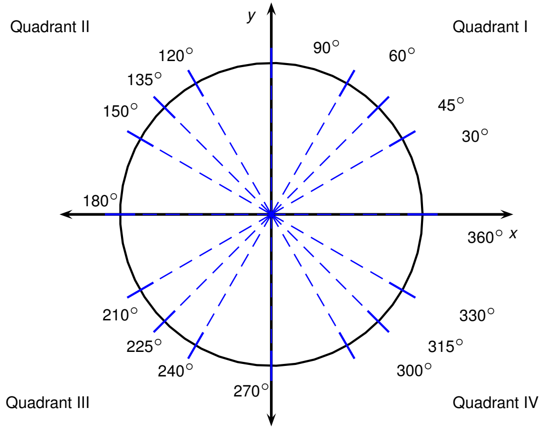
- In other words, a half-turn is measured by πrad or 180°.
- Degrees are useful because the most frequently encountered fractions of a half turn are measured by a whole number of degrees.
- If a measurement unit is not specified, it is implied to be radians. For example, in $\sin 5$, the number 5 stands for 5 radians.



The most frequently encountered angles are given in the table below.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	?										



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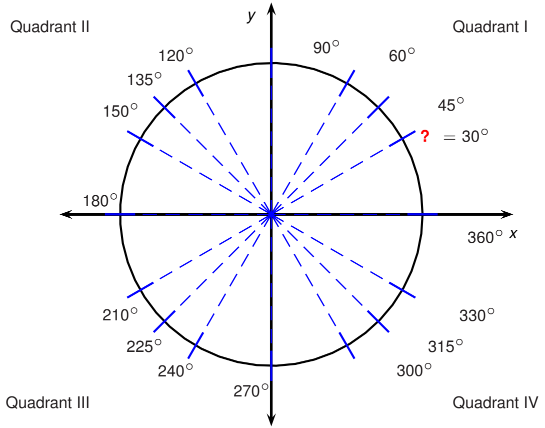

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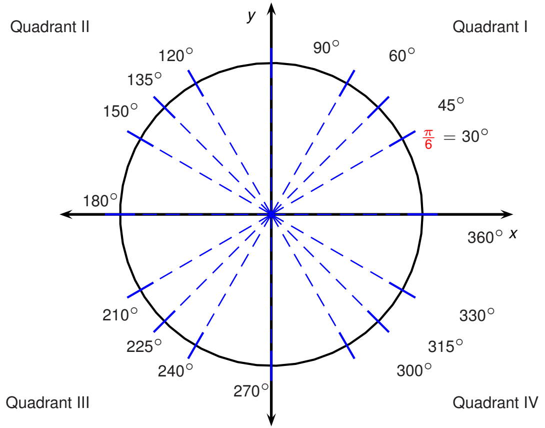

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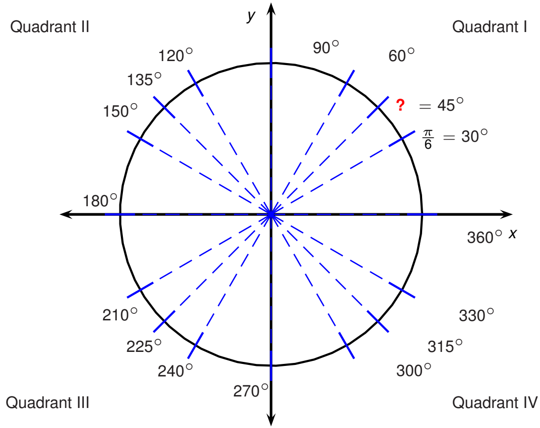

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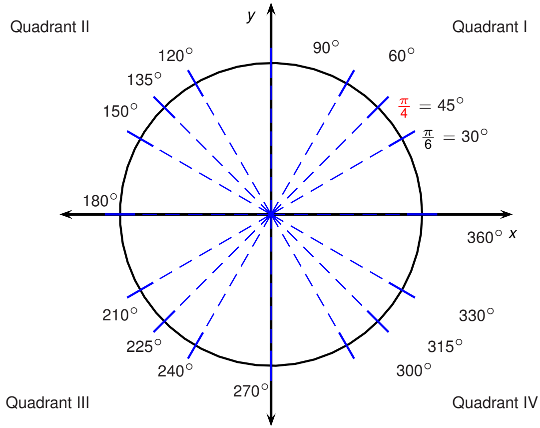

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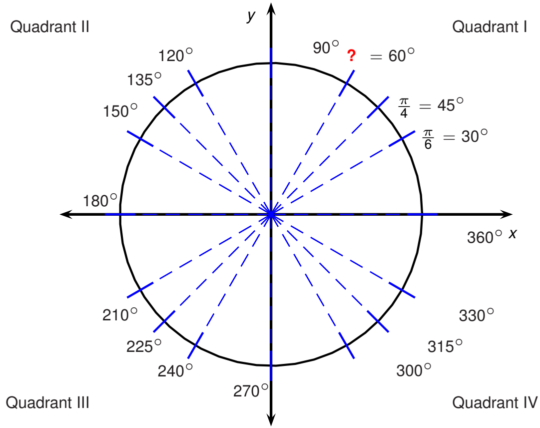

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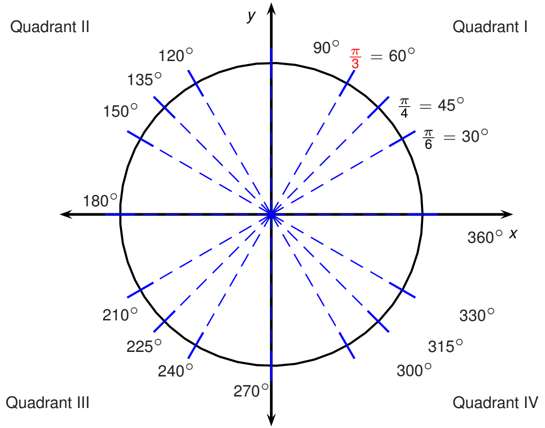

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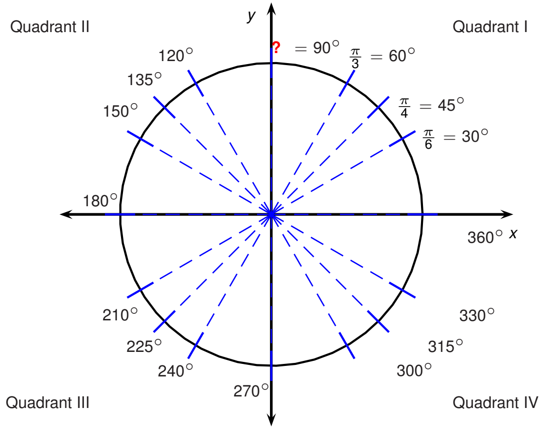

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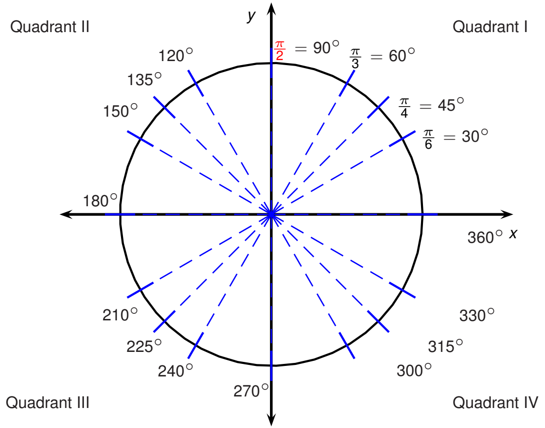

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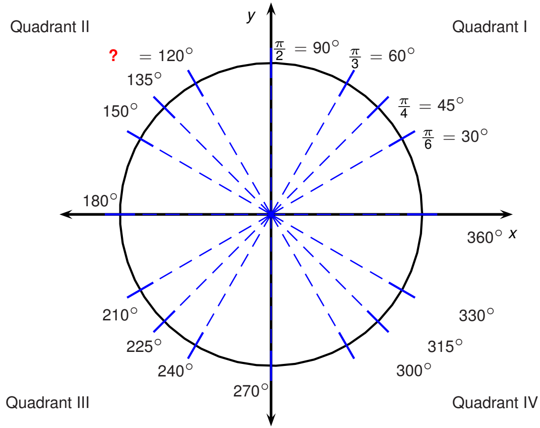

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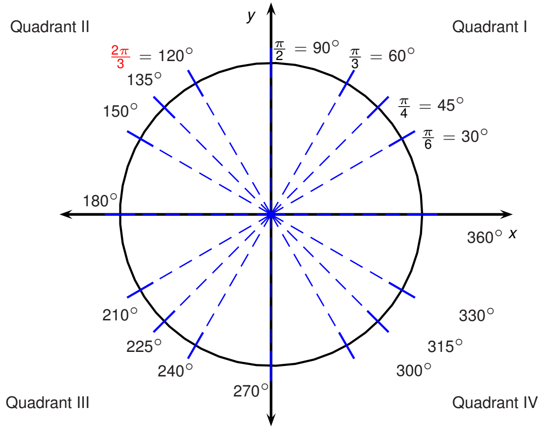

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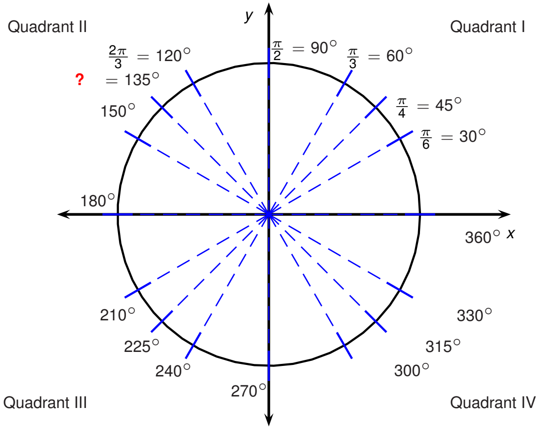

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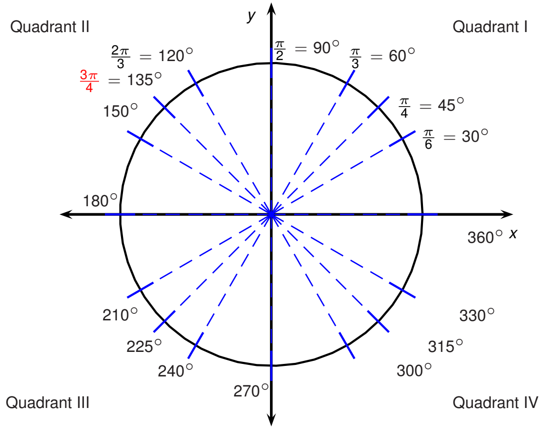

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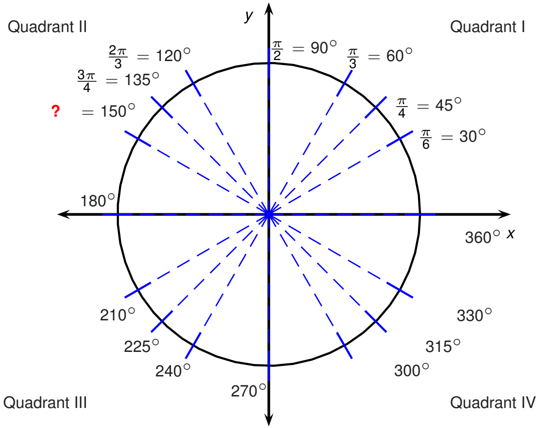

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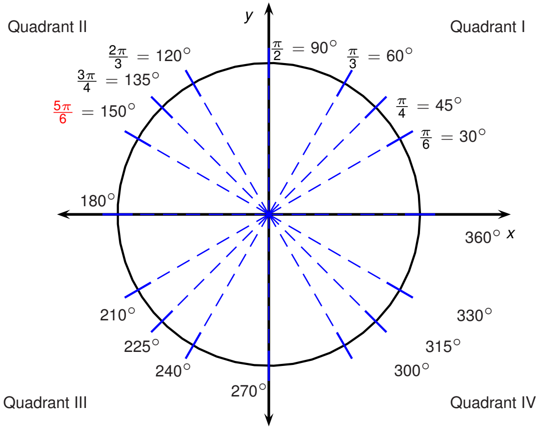

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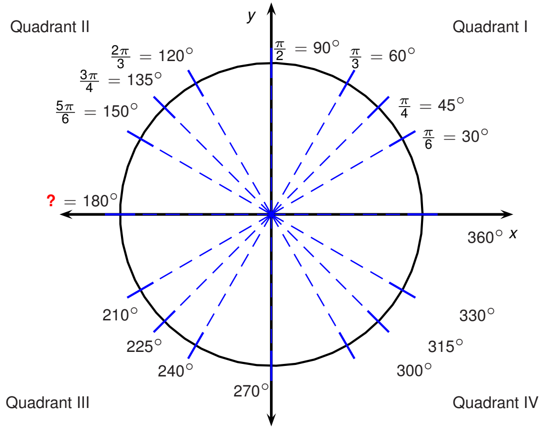

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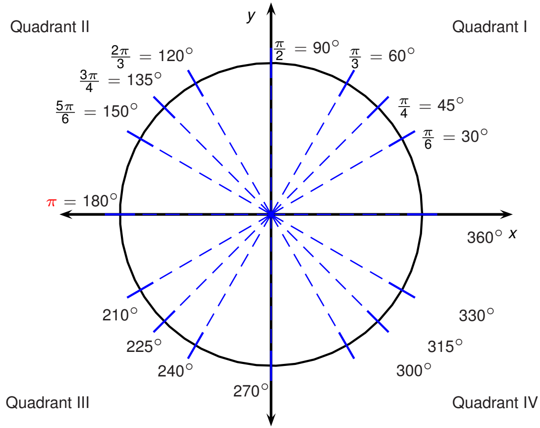

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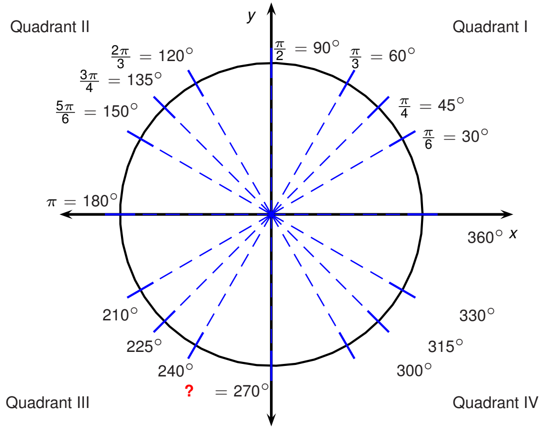

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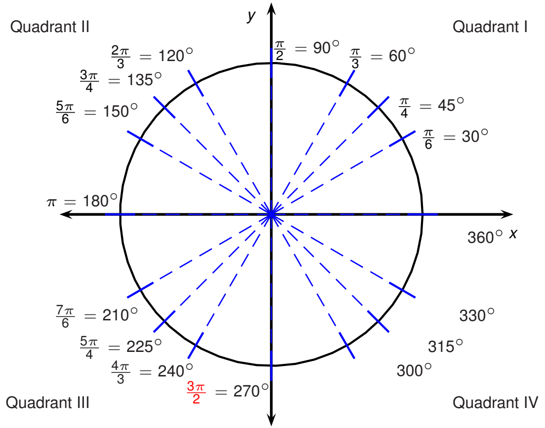

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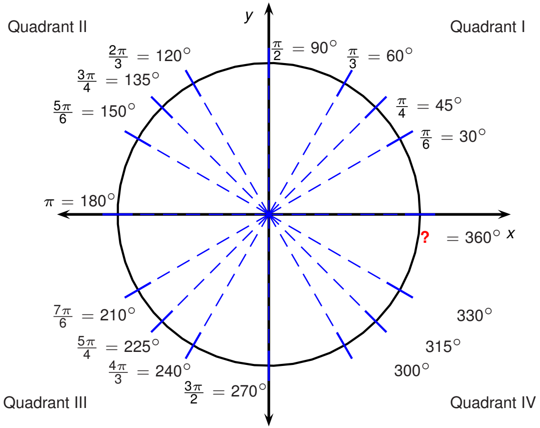

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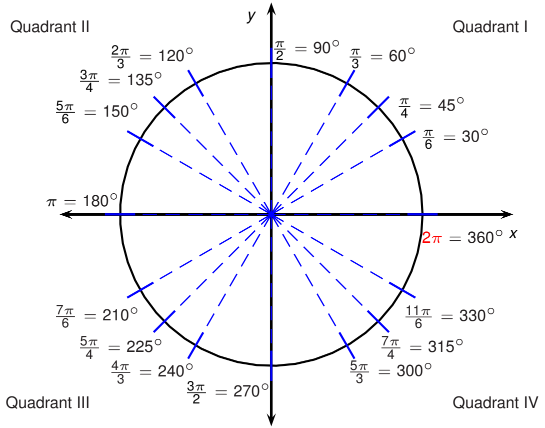

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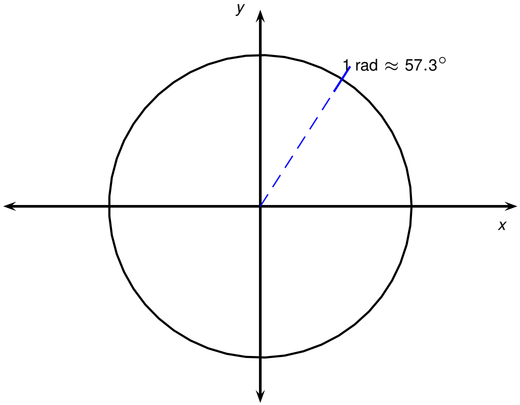
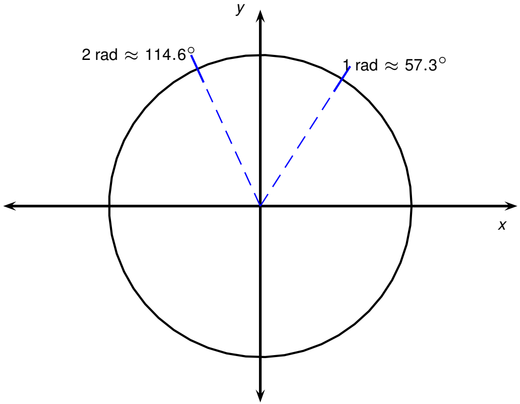


- Integer quantities of radians are not rational multiples of (the measure of) a half-turn and are not easy to compute with.



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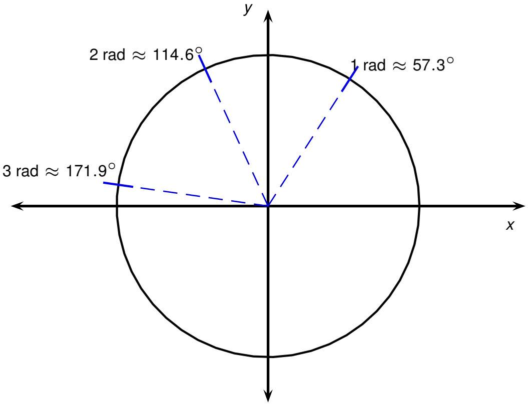

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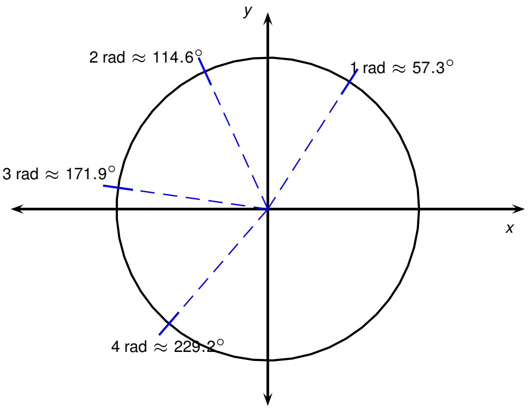

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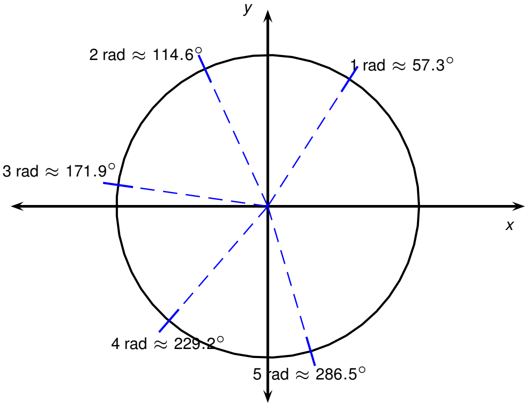

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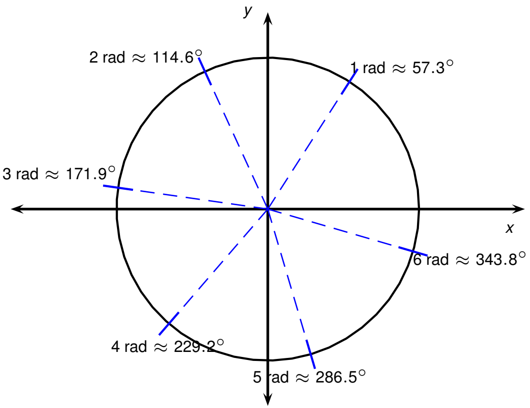

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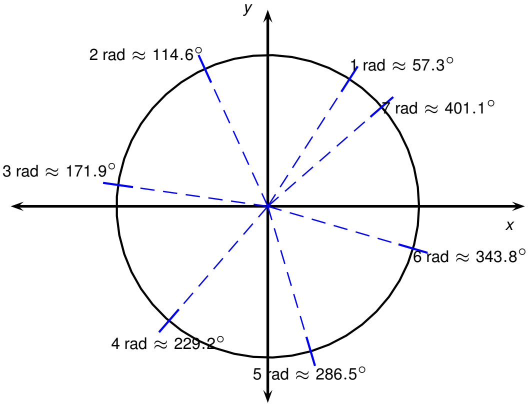

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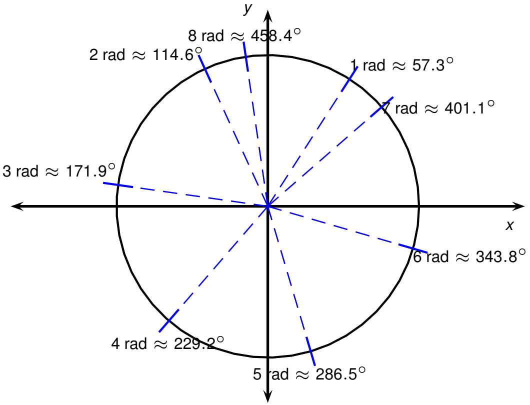

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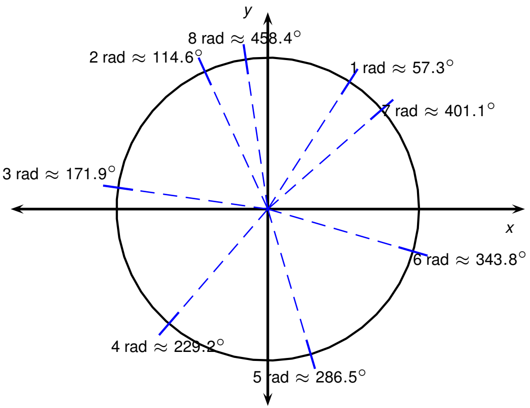
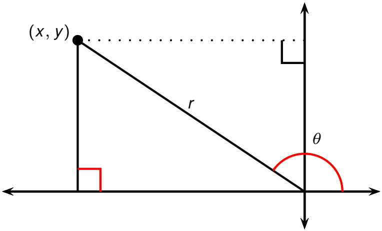
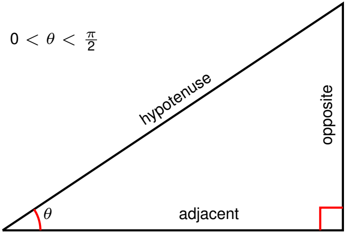


- Integer quantities of radians are not rational multiples of (the measure of) a half-turn and are not easy to compute with.
- For example to determine in which quadrant is an angle of k radians located one needs to know the numerical value of $\frac{k}{\pi}$, which requires knowledge of π with great numerical accuracy.

Trigonometric Functions and Right Angle Triangles





$\cos\theta$	$\sec\theta$	$\cos\theta$	$\sec\theta$
$\sin\theta$	$\csc\theta$	$\sin\theta$	$\csc\theta$
$\tan\theta$	$\cot\theta$	$\tan\theta$	$\cot\theta$
All angles		Acute angles	

- The trigonometric functions can be defined without requesting that the pt. (x, y) on the terminal arm of the angle lie on the unit circle.

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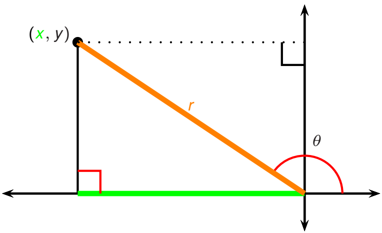

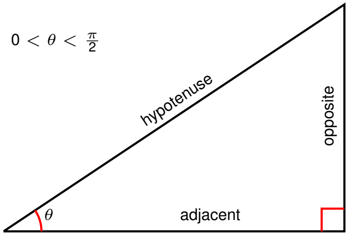

$\cos\theta = \frac{x}{r}$	$\sec\theta$	$\cos\theta$	$\sec\theta$
$\sin\theta$	$\csc\theta$	$\sin\theta$	$\csc\theta$
$\tan\theta$	$\cot\theta$	$\tan\theta$	$\cot\theta$
All angles		Acute angles	

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- To do so we rescale by the distance r from the origin.

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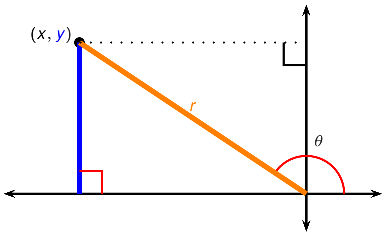

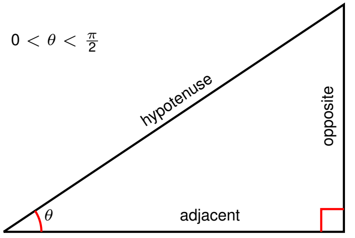

$\cos\theta = \frac{x}{r}$	$\sec\theta$	$\cos\theta$	$\sec\theta$
$\sin\theta = \frac{y}{r}$	$\csc\theta$	$\sin\theta$	$\csc\theta$
$\tan\theta$	$\cot\theta$	$\tan\theta$	$\cot\theta$
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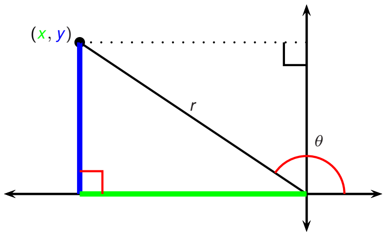

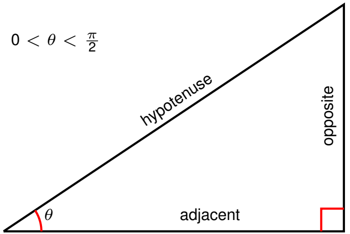

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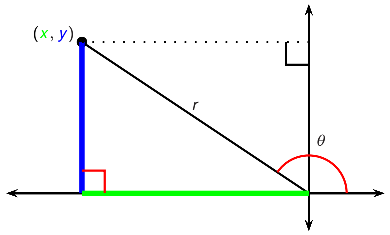
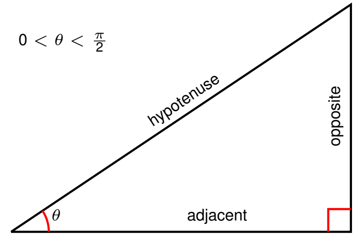
Trigonometric Functions and Right Angle Triangles





$\cos\theta = \frac{x}{r}$	$\sec\theta$	$\cos\theta$	$\sec\theta$
$\sin\theta = \frac{y}{r}$	$\csc\theta$	$\sin\theta$	$\csc\theta$
$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$	$\tan\theta$	$\cot\theta$
All angles		Acute angles	

- The trigonometric functions can be defined without requesting that the pt. (x, y) on the terminal arm of the angle lie on the unit circle.
- To do so we rescale by the distance r from the origin.

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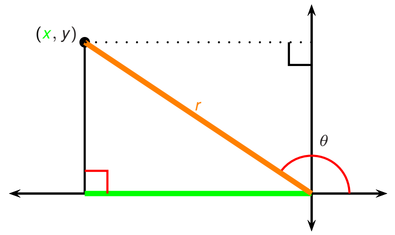

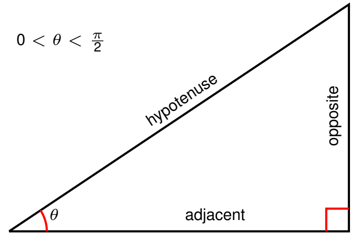

$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$\cos\theta$	$\sec\theta$
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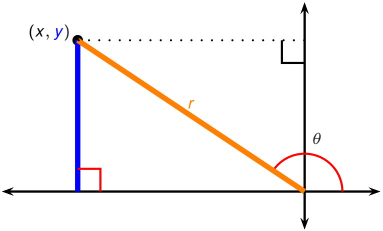

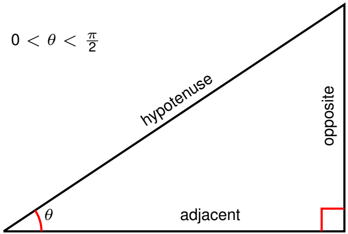

$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$\cos\theta$	$\sec\theta$
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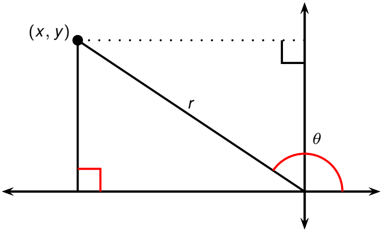
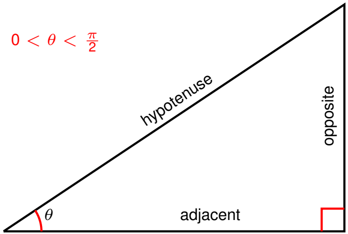
All angles		Acute angles	
			
$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$\cos\theta$	$\sec\theta$
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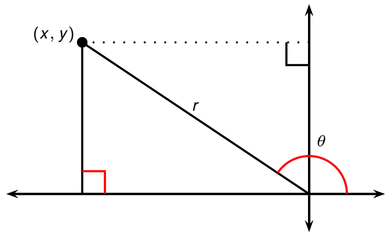
Trigonometric Functions and Right Angle Triangles



$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$
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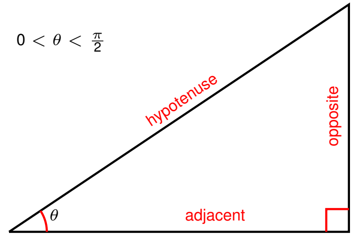
All angles



$\cos\theta$ $\qquad$ $\sec\theta$

$\sin\theta$ $\qquad$ $\csc\theta$

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Acute angles

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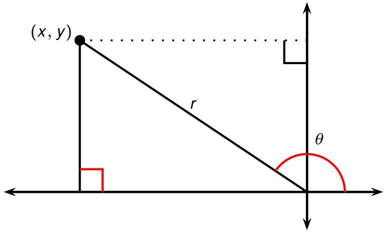

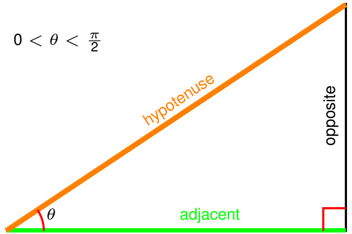

$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\sec\theta$
$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$	$\sin\theta$	$\csc\theta$
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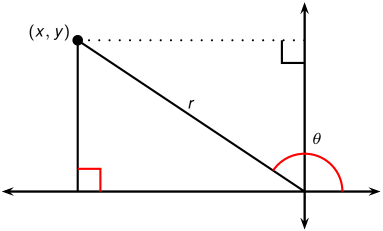

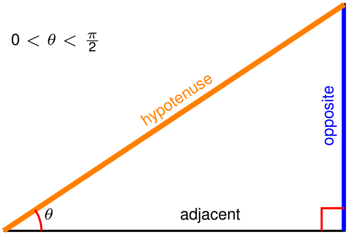

$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\sec\theta$
$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$	$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	$\csc\theta$
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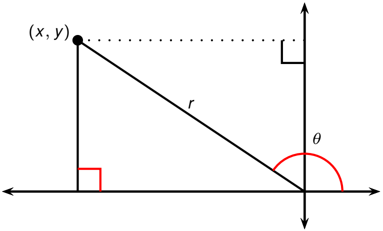

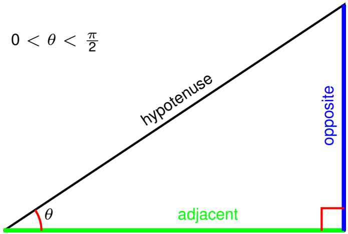

$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\sec\theta$
$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$	$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	$\csc\theta$
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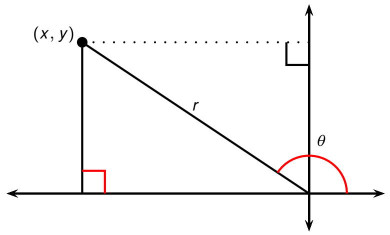

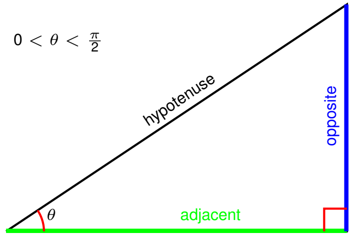

$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\sec\theta$
$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$	$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	$\csc\theta$
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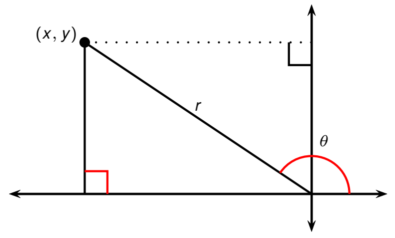

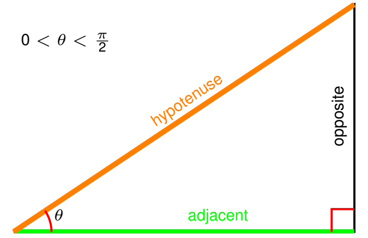

$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\sec\theta = \frac{\text{hyp}}{\text{adj}}$
$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$	$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	$\csc\theta$
$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$	$\tan\theta = \frac{\text{opp}}{\text{adj}}$	$\cot\theta = \frac{\text{adj}}{\text{opp}}$
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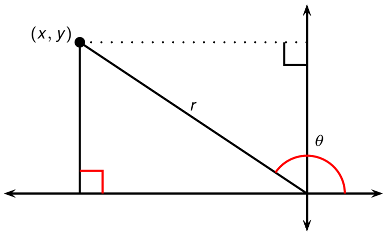

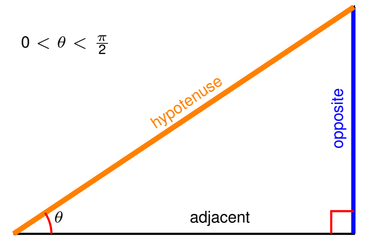

$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\sec\theta = \frac{\text{hyp}}{\text{adj}}$
$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$	$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	$\csc\theta = \frac{\text{hyp}}{\text{opp}}$
$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$	$\tan\theta = \frac{\text{opp}}{\text{adj}}$	$\cot\theta = \frac{\text{adj}}{\text{opp}}$
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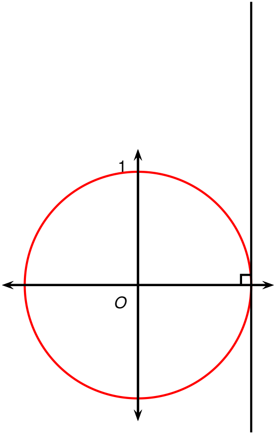
Geometric interpretation of all trigonometric functions



Fix unit circle, center O, coordinates $(0, 0)$.

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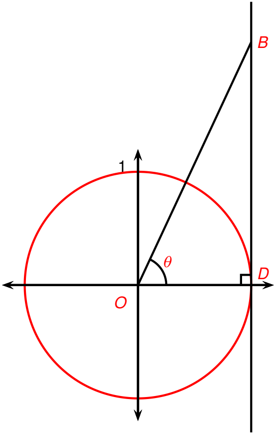
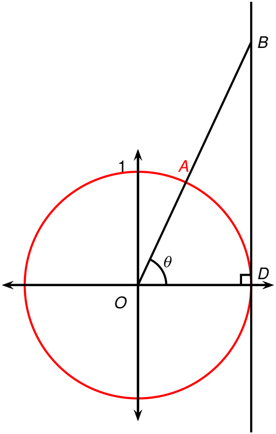


Fix unit circle, center O, coordinates $(0, 0)$.
Let $\angle DOB = \theta$.

Geometric interpretation of all trigonometric functions



Fix unit circle, center O, coordinates $(0, 0)$. Let $\angle DOB = \theta$. Let OB intersect the circle at point A.

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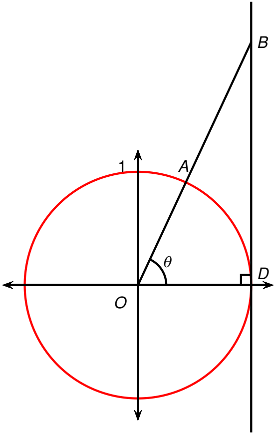

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$\sin\theta$

$\cos\theta$

$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

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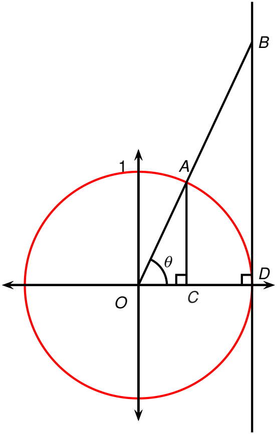

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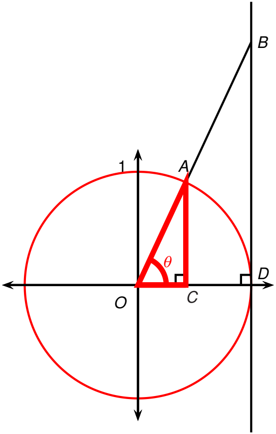

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$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$\cos\theta$

$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

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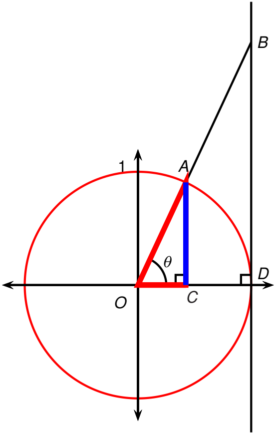

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$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|}$$

$\cos\theta$

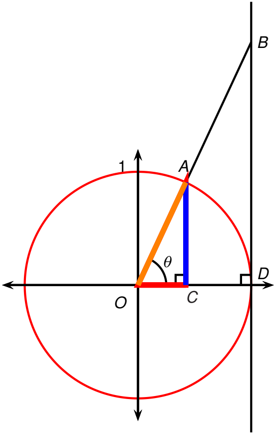
$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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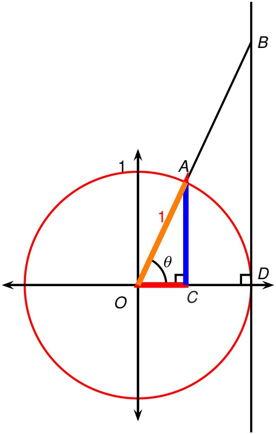

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$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|} = \frac{|AC|}{1}$$

$\cos\theta$

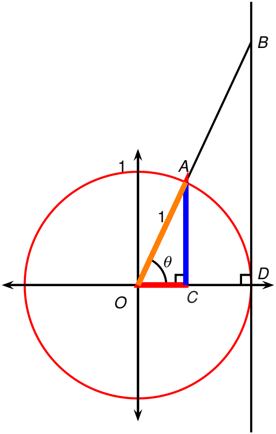
$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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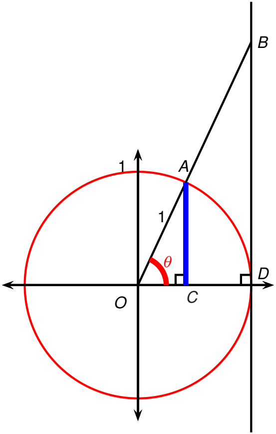

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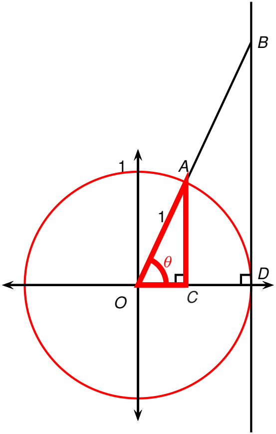

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$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

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$\cot\theta$

$\sec\theta$

$\csc\theta$

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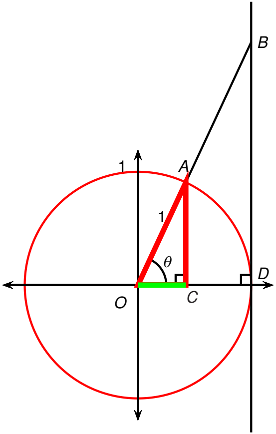

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$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|} = \frac{|AC|}{1} = |AC|$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{|OC|}{|OA|}$$

$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

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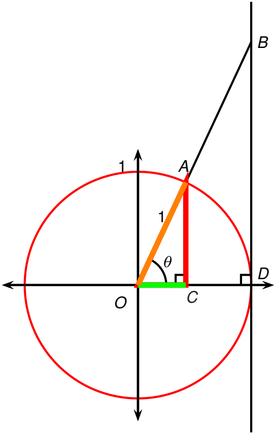

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$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{|OC|}{|OA|}$$

$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

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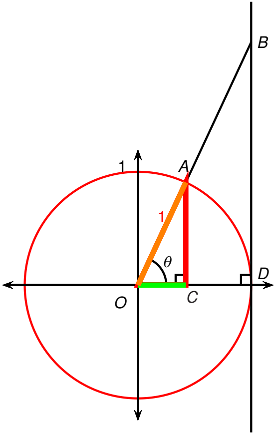

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$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{|OC|}{|OA|} = \frac{|OC|}{1}$$

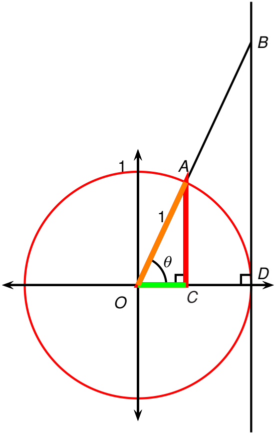
$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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$\sec\theta$

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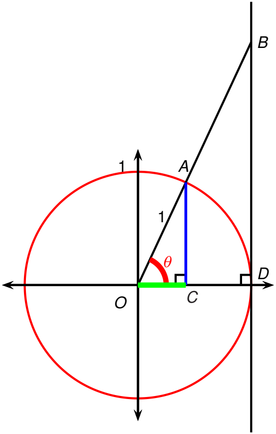
Geometric interpretation of all trigonometric functions



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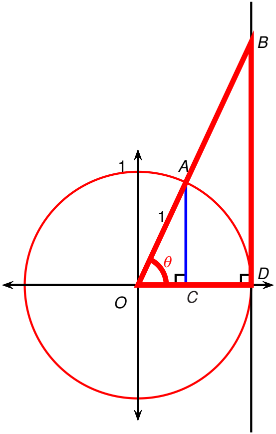

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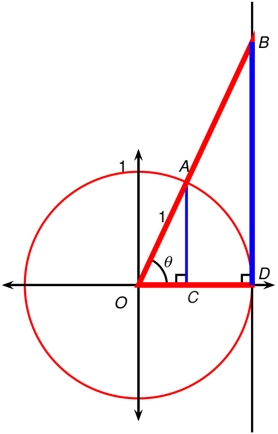

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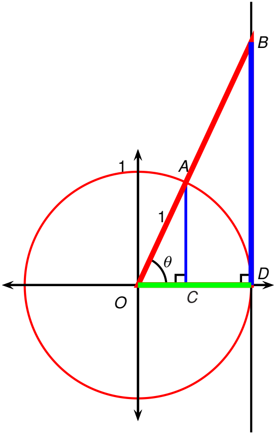

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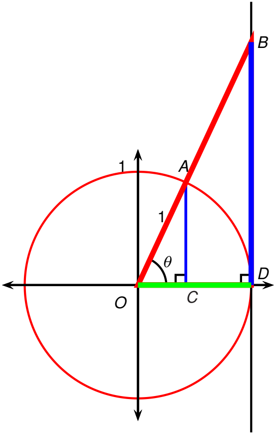

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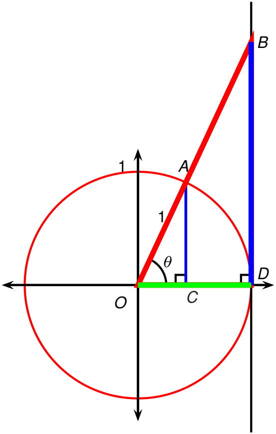

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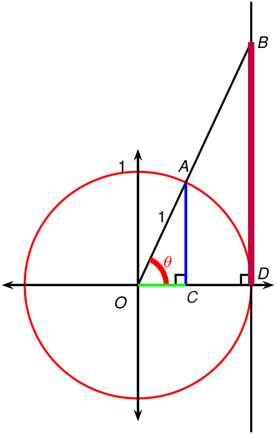

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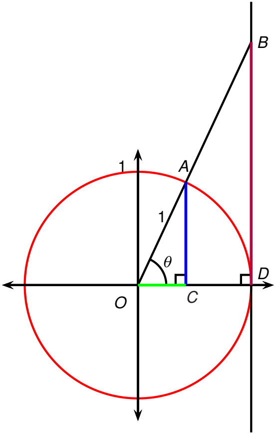

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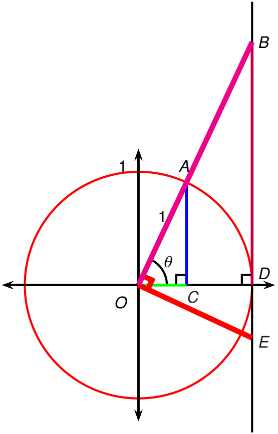

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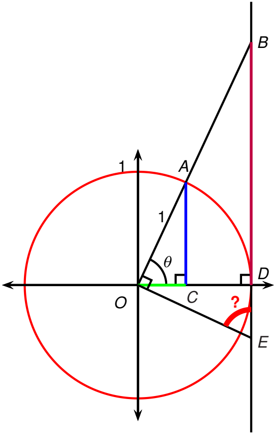

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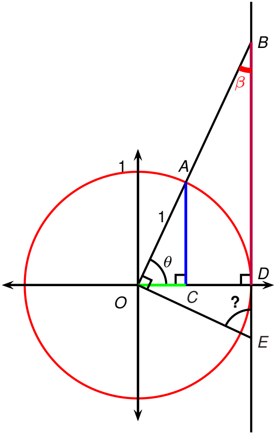
Geometric interpretation of all trigonometric functions



$\beta = \mathbf{?}$

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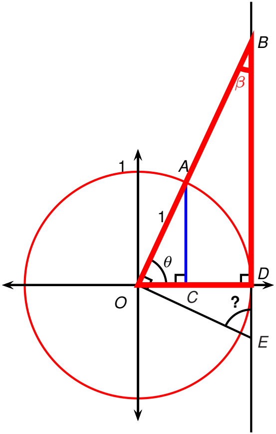
Geometric interpretation of all trigonometric functions



$\beta = 180° - 90° - \theta$

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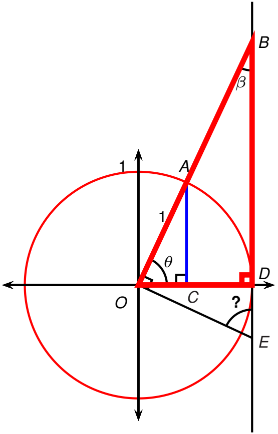
Geometric interpretation of all trigonometric functions



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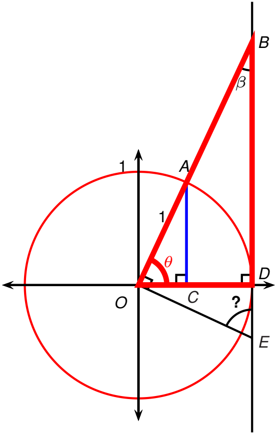

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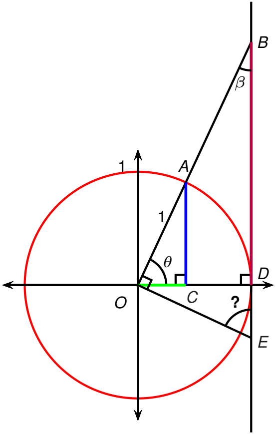
Geometric interpretation of all trigonometric functions



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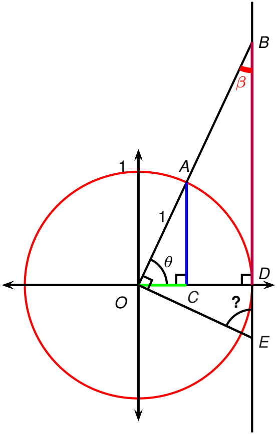

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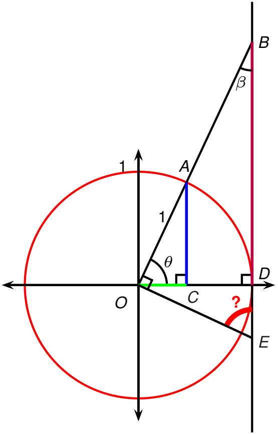

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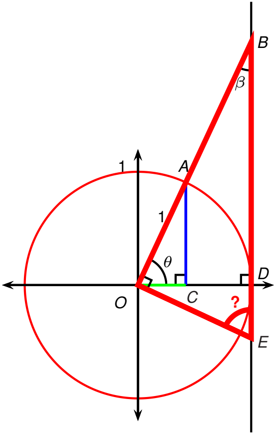

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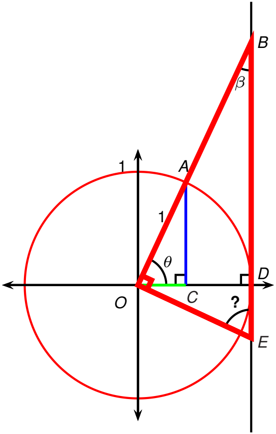

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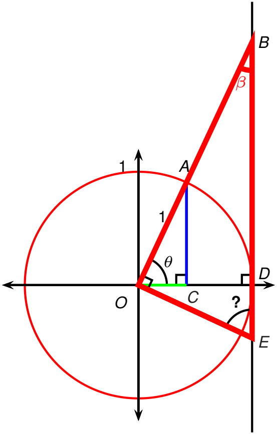

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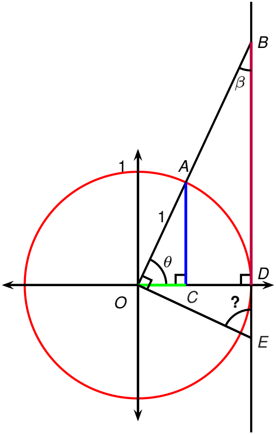

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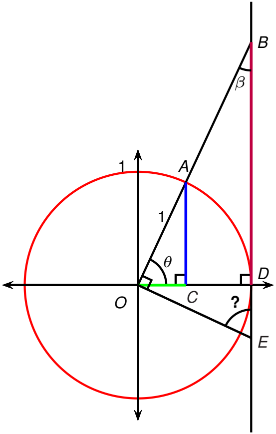

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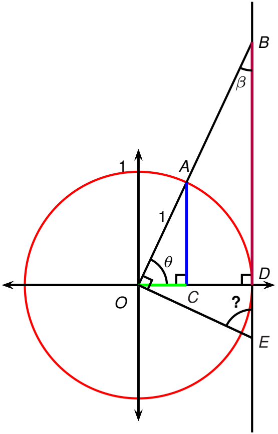

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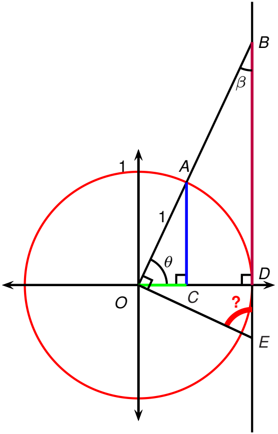

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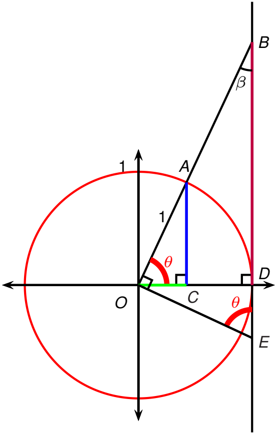
Geometric interpretation of all trigonometric functions



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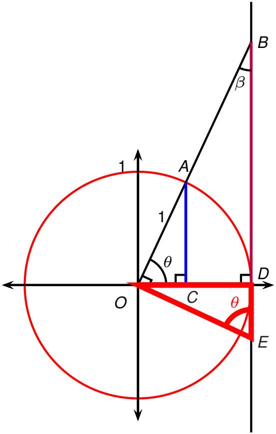

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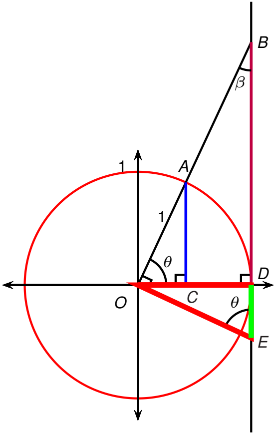

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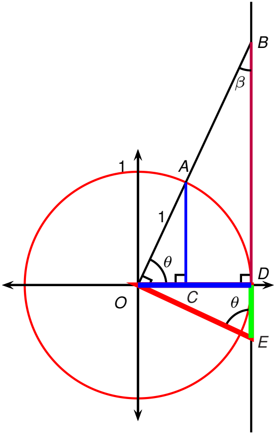
Geometric interpretation of all trigonometric functions



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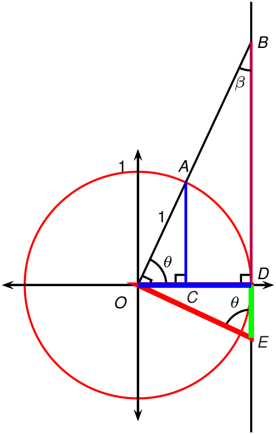
Geometric interpretation of all trigonometric functions



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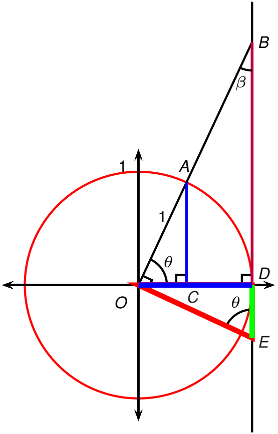

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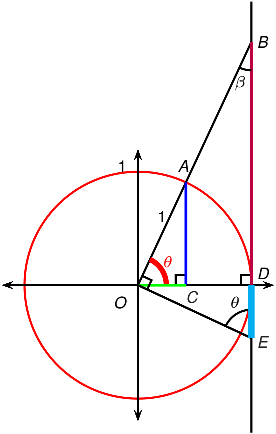

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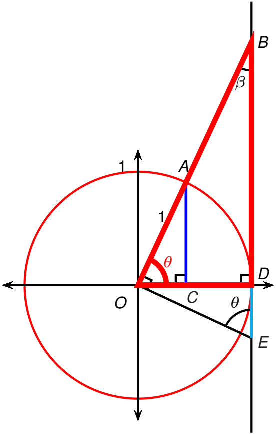
Geometric interpretation of all trigonometric functions



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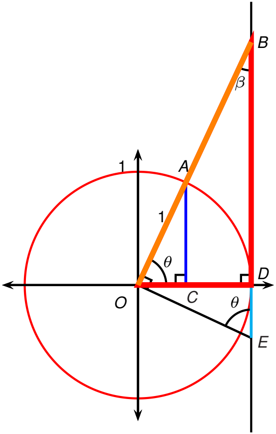
Geometric interpretation of all trigonometric functions



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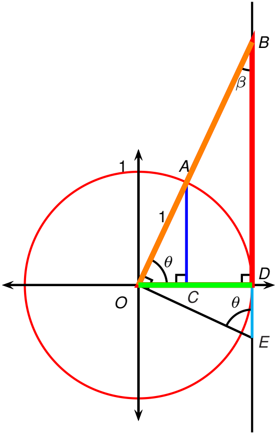
Geometric interpretation of all trigonometric functions



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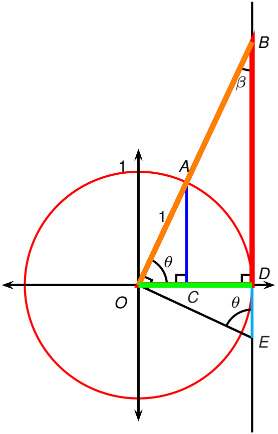

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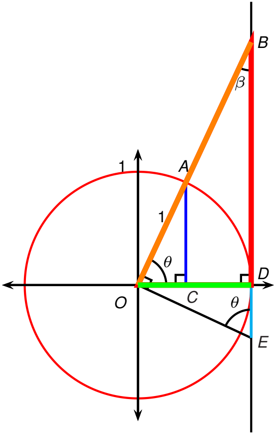
Geometric interpretation of all trigonometric functions



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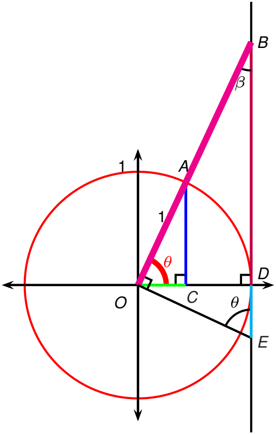
Geometric interpretation of all trigonometric functions



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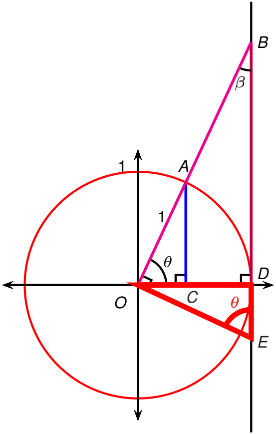
Geometric interpretation of all trigonometric functions



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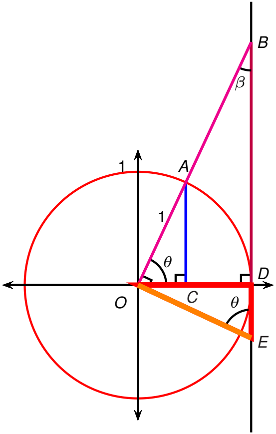
Geometric interpretation of all trigonometric functions



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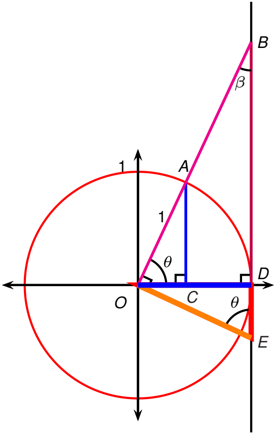
Geometric interpretation of all trigonometric functions



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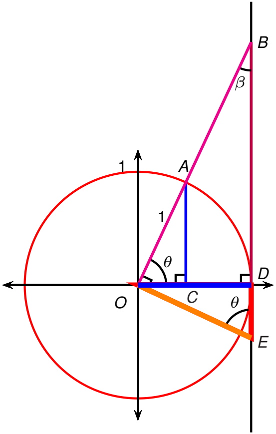
Geometric interpretation of all trigonometric functions



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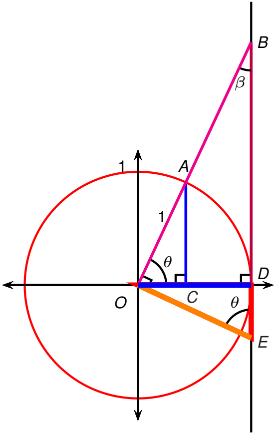
Geometric interpretation of all trigonometric functions



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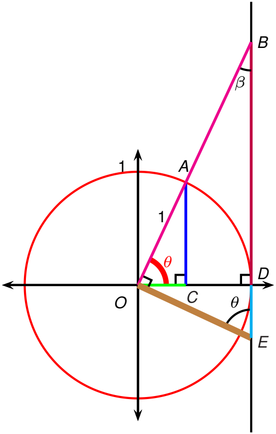

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$$\angle OED = 180^\circ - 90^\circ - \beta$$
$$= 90^\circ - (90^\circ - \theta)$$
$$= \theta$$

Fix unit circle, center O, coordinates $(0,0)$. Let $\angle DOB = \theta$. Let OB intersect the circle at point A. Coordinates of A are $(\cos\theta, \sin\theta)$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|} = \frac{|AC|}{1} = |AC|$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{|OC|}{|OA|} = \frac{|OC|}{1} = |OC|$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{|BD|}{|OD|} = \frac{|BD|}{1} = |BD|$$
$$\cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{|DE|}{|OD|} = \frac{|DE|}{1} = |DE|$$
$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{|OB|}{|OD|} = \frac{|OB|}{1} = |OB|$$
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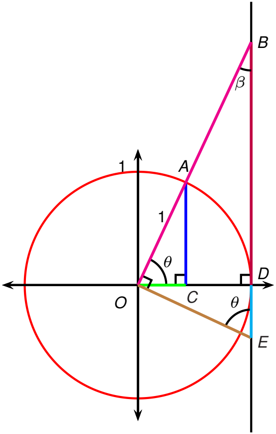
Geometric interpretation of all trigonometric functions



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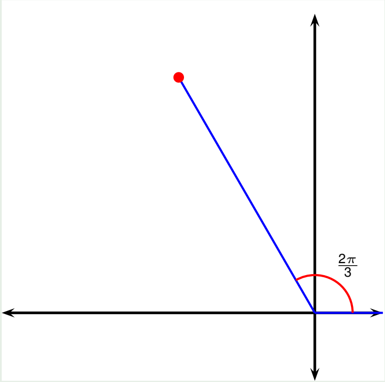
Example



Find the exact values of the trigonometric functions of $\theta = \dfrac{2\pi}{3} = 120^\circ$.

$$\sin\left(\frac{2\pi}{3}\right) = \qquad \cos\left(\frac{2\pi}{3}\right) = \qquad \tan\left(\frac{2\pi}{3}\right) =$$

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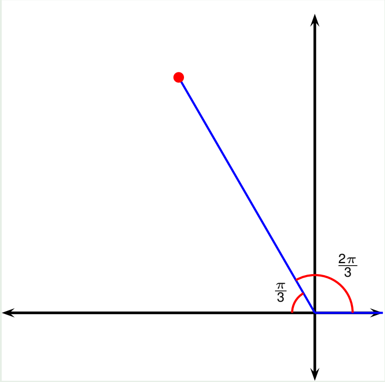
Example



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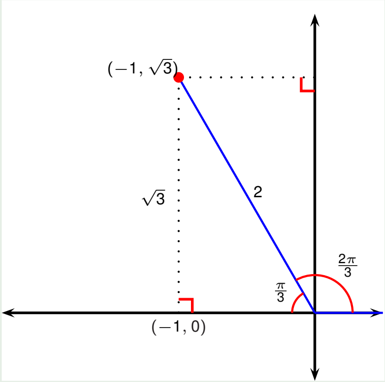
Example



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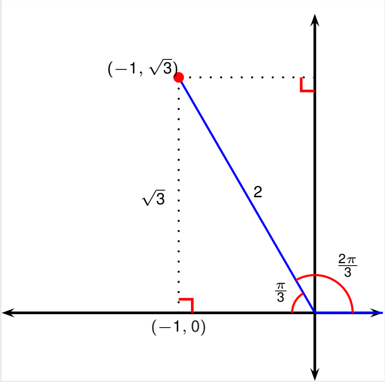
Example



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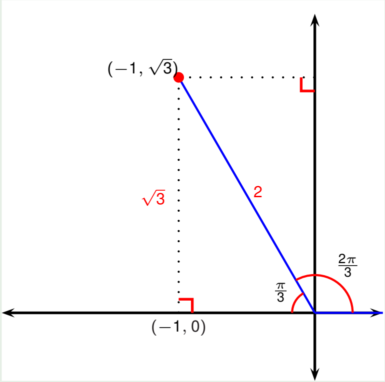
Example



Find the exact values of the trigonometric functions of $\theta = \frac{2\pi}{3} = 120^\circ$.

$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \qquad \cos\left(\frac{2\pi}{3}\right) = \qquad \tan\left(\frac{2\pi}{3}\right) =$$

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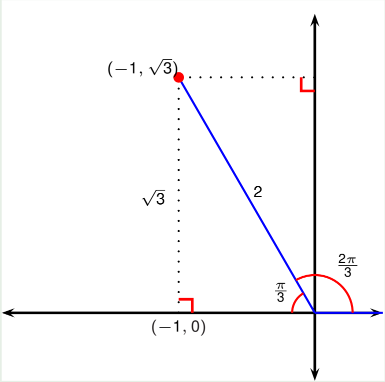

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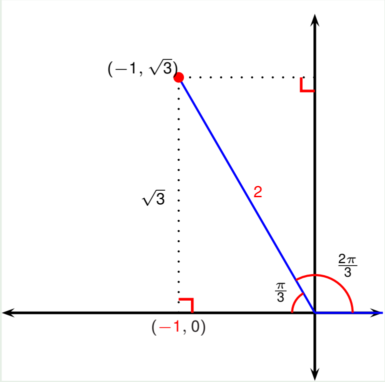

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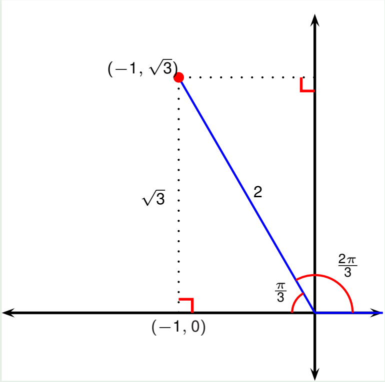
Example



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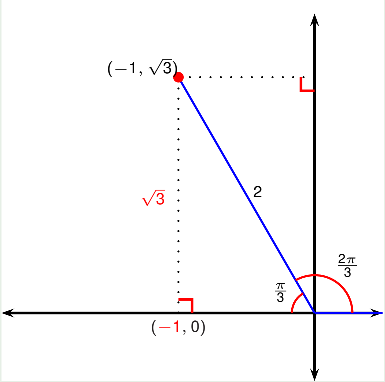

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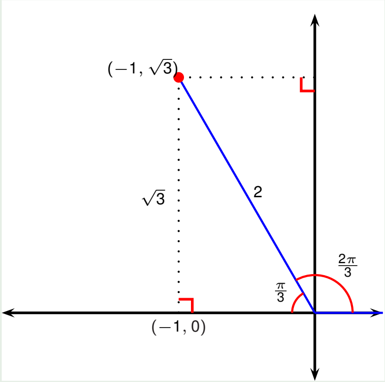
Example



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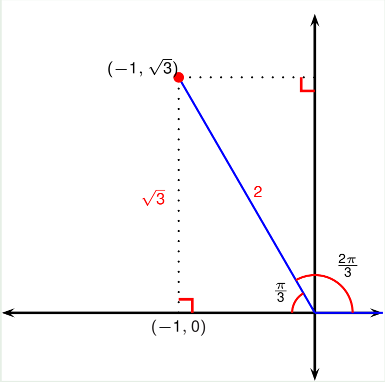
Example



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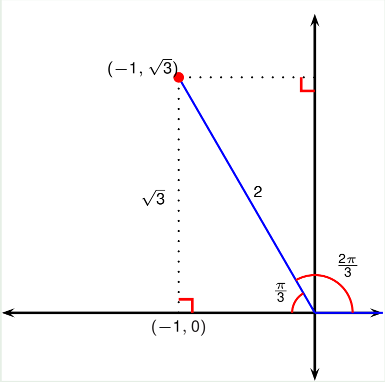
$$\csc\left(\frac{2\pi}{3}\right) = \frac{2}{\sqrt{3}} \quad \sec\left(\frac{2\pi}{3}\right) = \quad \cot\left(\frac{2\pi}{3}\right) =$$

Example



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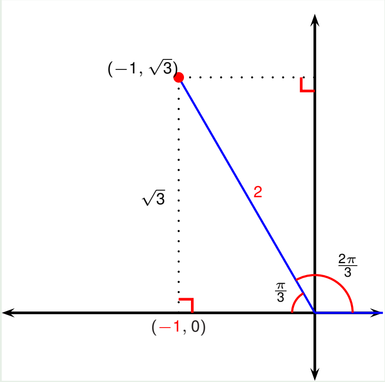

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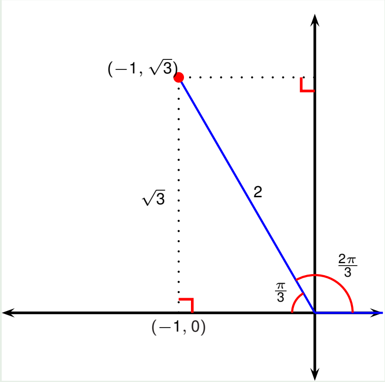
$$\csc\left(\frac{2\pi}{3}\right) = \frac{2}{\sqrt{3}} \quad \sec\left(\frac{2\pi}{3}\right) = -\frac{2}{1} = -2 \quad \cot\left(\frac{2\pi}{3}\right) =$$

Example



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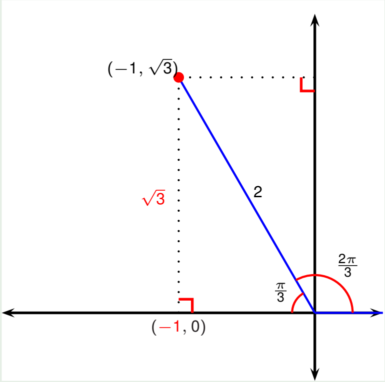
Example



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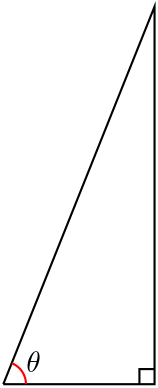
$$\csc\left(\frac{2\pi}{3}\right) = \frac{2}{\sqrt{3}} \quad \sec\left(\frac{2\pi}{3}\right) = -\frac{2}{1} = -2 \quad \cot\left(\frac{2\pi}{3}\right) = -\frac{1}{\sqrt{3}}$$

Example

If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



$\sin\theta =$ $\tan\theta =$

$\csc\theta =$ $\sec\theta =$

$\cot\theta =$

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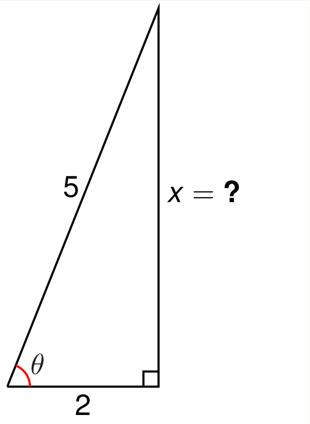


- Label the hypotenuse with length 5 and the adjacent side with length 2.

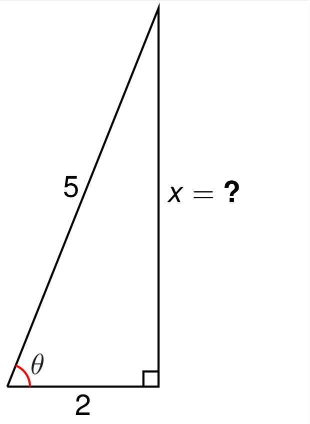
$\sin\theta =$ $\qquad$ $\tan\theta =$

$\csc\theta =$ $\qquad$ $\sec\theta =$

$\cot\theta =$

Example

If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem: $x^2 + 2^2 = 5^2$.

$\sin\theta =$ $\tan\theta =$

$\csc\theta =$ $\sec\theta =$

$\cot\theta =$

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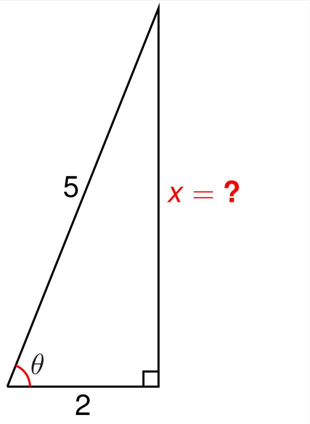
If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem: $x^2 + 2^2 = 5^2$.
- Therefore $x^2 = ?$, so $x = ?$.

$\sin\theta =$ $\tan\theta =$

$\csc\theta =$ $\sec\theta =$

$\cot\theta =$

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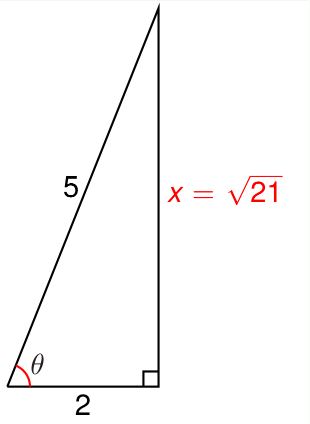
If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem: $x^2 + 2^2 = 5^2$.
- Therefore $x^2 = 21$, so $x = \sqrt{21}$.

$\sin\theta =$ $\qquad$ $\tan\theta =$

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$\cot\theta =$

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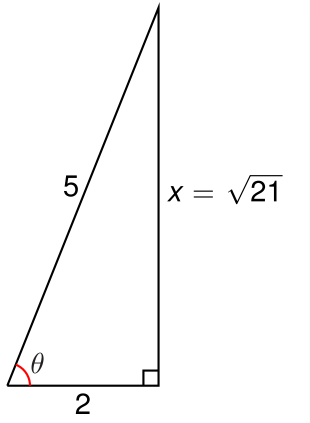
If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



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$\sin\theta = \textbf{?}$ $\tan\theta =$

$\csc\theta =$ $\sec\theta =$

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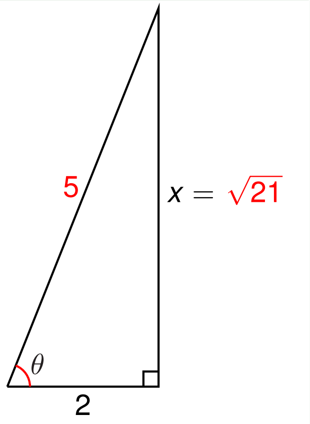
Example

If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



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$\sin\theta = \frac{\sqrt{21}}{5}$ $\quad$ $\tan\theta =$

$\csc\theta =$ $\quad$ $\sec\theta =$

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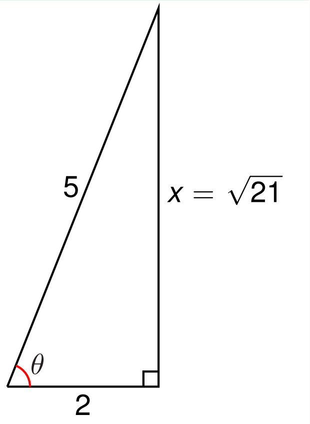
Example

If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



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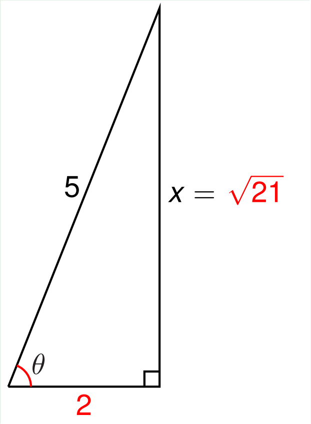

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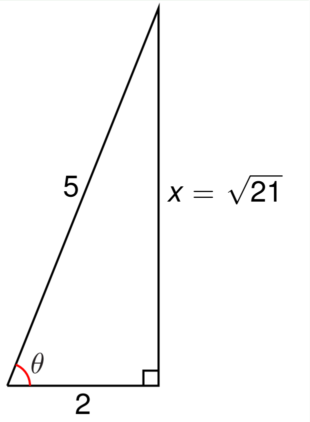
If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



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$$\sin\theta = \frac{\sqrt{21}}{5} \quad \tan\theta = \frac{\sqrt{21}}{2}$$

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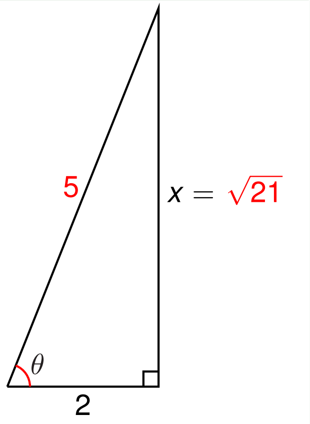

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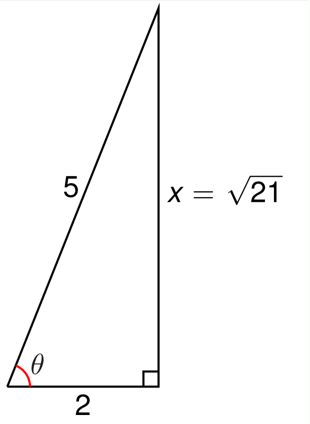
$$\sin\theta = \frac{\sqrt{21}}{5} \qquad \tan\theta = \frac{\sqrt{21}}{2}$$

$$\csc\theta = \frac{5}{\sqrt{21}} \qquad \sec\theta =$$

$$\cot\theta =$$

Example

If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



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$$\cot\theta =$$

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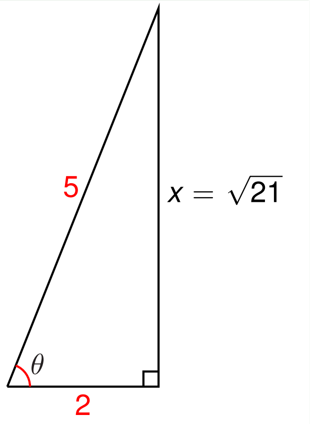

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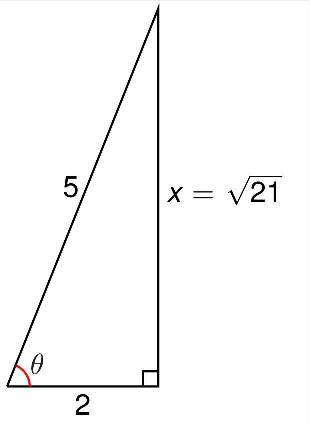
$$\cot\theta =$$

Example

If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem: $x^2 + 2^2 = 5^2$.
- Therefore $x^2 = 21$, so $x = \sqrt{21}$.

$$\sin\theta = \frac{\sqrt{21}}{5} \quad \tan\theta = \frac{\sqrt{21}}{2}$$

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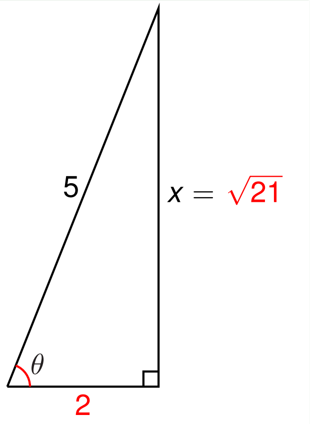

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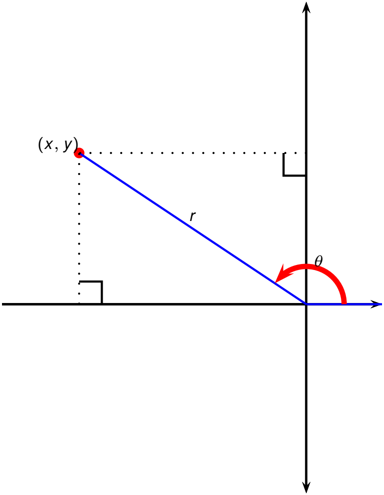

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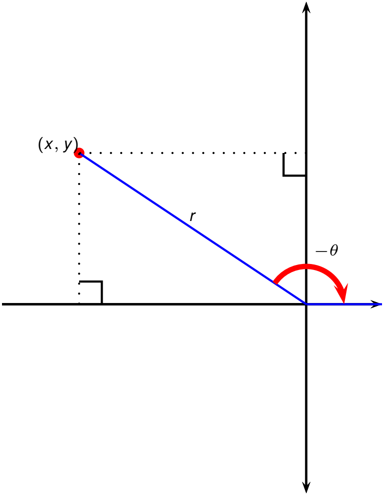

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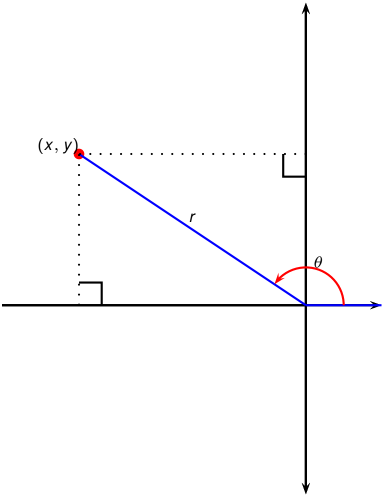

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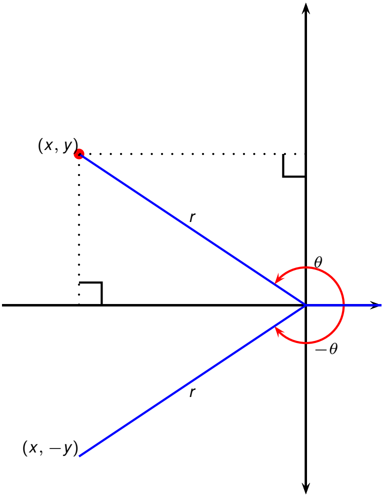

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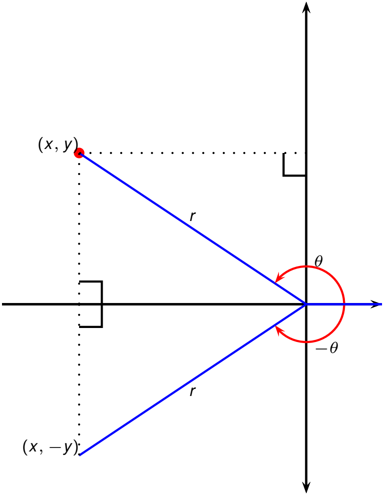

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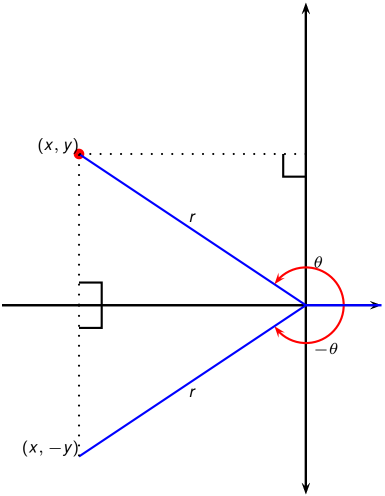

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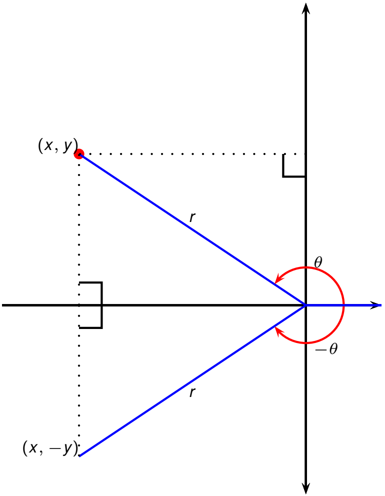

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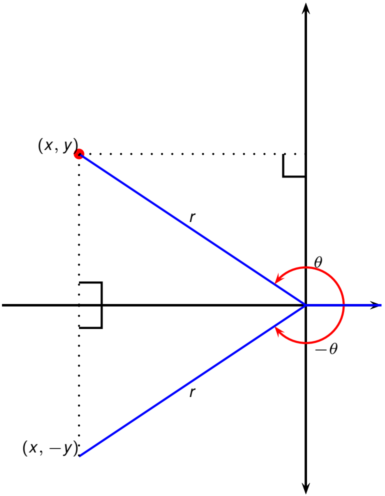

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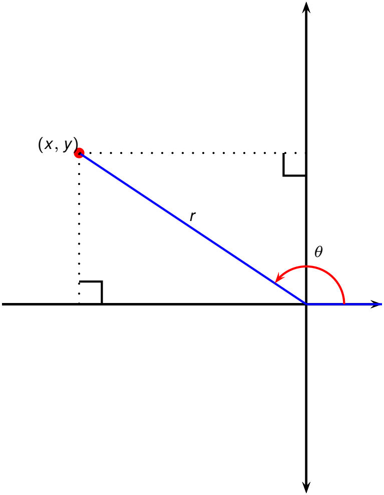
- cos is an even function.



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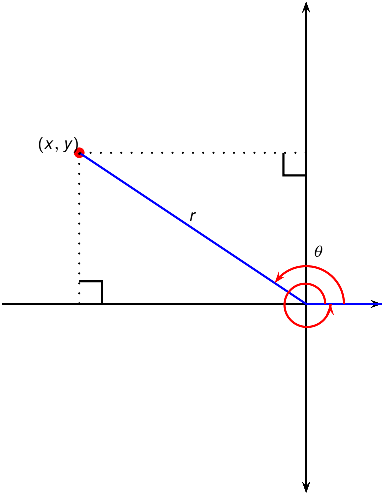

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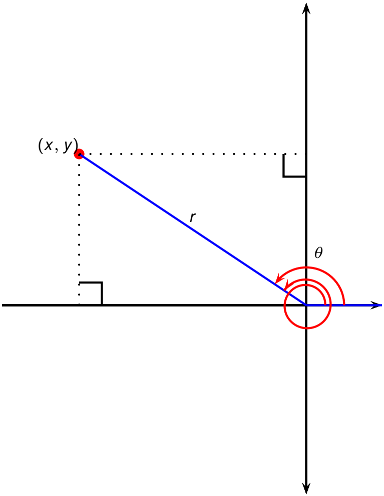

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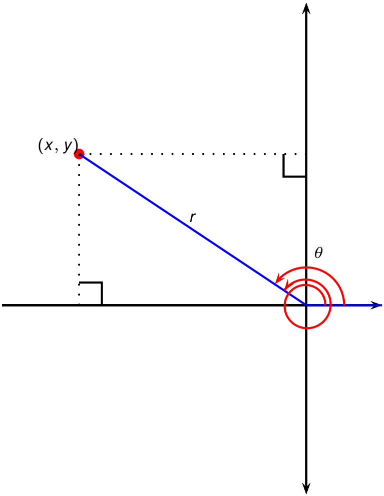

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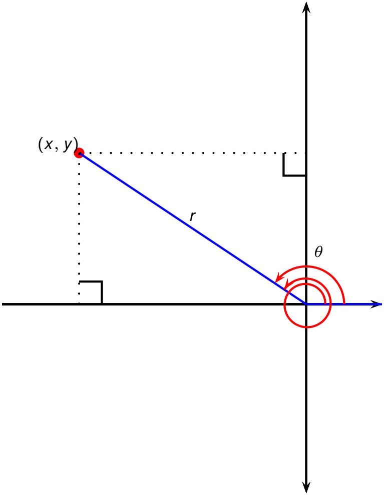

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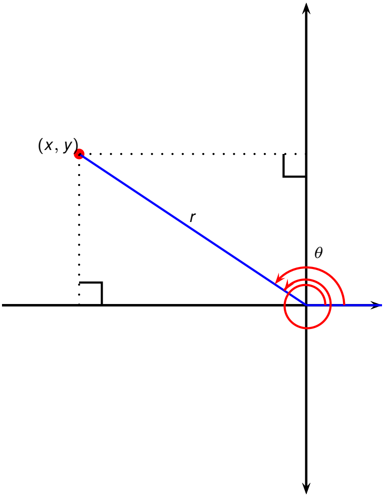

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- We say $\sin$ and $\cos$ are 2π-periodic.

Trigonometric Identities

Definition (Trigonometric Identity)

A trigonometric identity is an equality between the trigonometric functions in one or more variables that holds for all values of the involved variables in the domains of all of the expressions.

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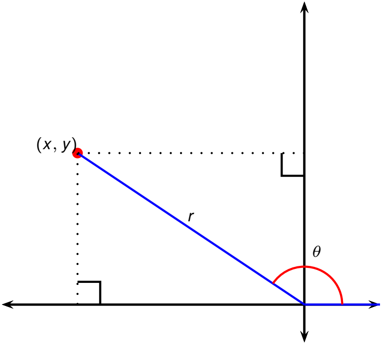
- By convention, when dealing with trigonometric identities we do not account for the domains of the involved expressions.
- For example, $\frac{\sin\theta}{\sin\theta} = 1$ is considered a valid trigonometric identity, although, when considered as a function, the left hand side is not defined for $\theta \neq 0$.



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- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
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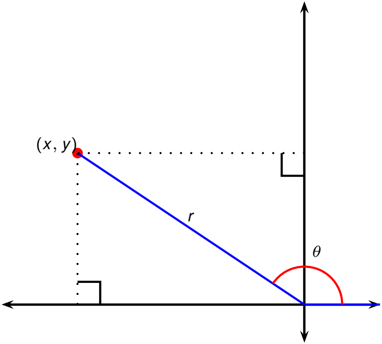

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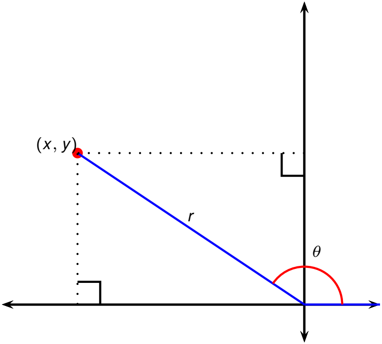

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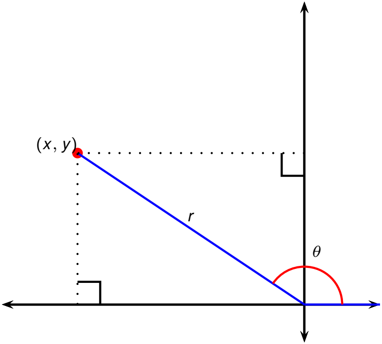
$$\sin^2\theta + \cos^2\theta$$



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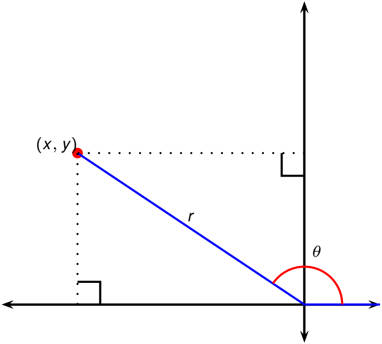

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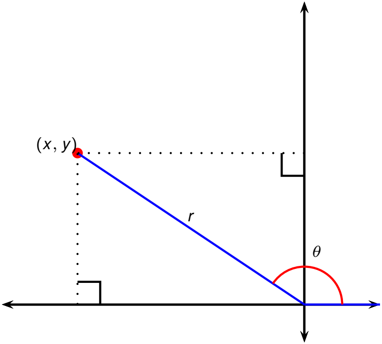

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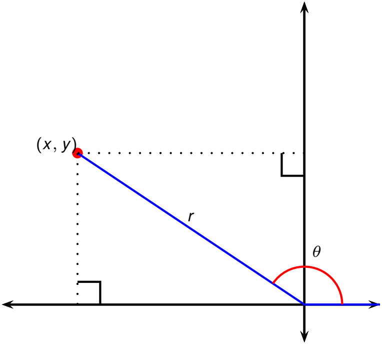

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\begin{array}{ll}
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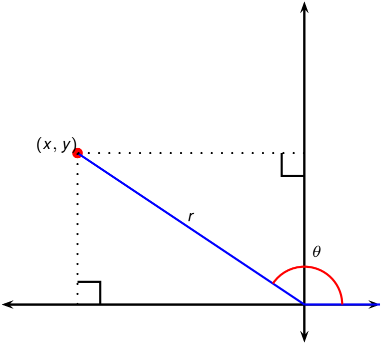
\end{array}
$$

$$
\begin{aligned}
& \sin^2\theta + \cos^2\theta \\
= \quad & \frac{y^2}{r^2} + \frac{x^2}{r^2} \\
= \quad & \frac{y^2 + x^2}{r^2} \\
= \quad & \frac{r^2}{r^2} \\
= \quad & 1
\end{aligned}
$$



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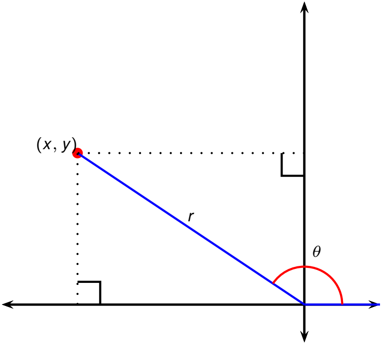
Therefore $\sin^2\theta + \cos^2\theta = 1$.



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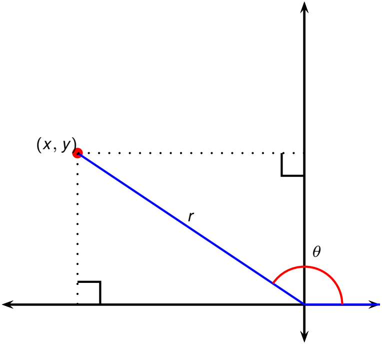
Example ($\tan^2\theta + 1 = \sec^2\theta$)

Prove the identity
$\tan^2\theta + 1 = \sec^2\theta$.



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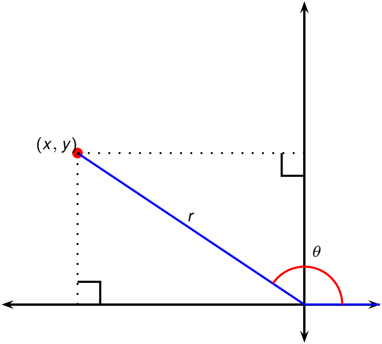
$$\sin^2\theta + \cos^2\theta = 1$$



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Prove the identity
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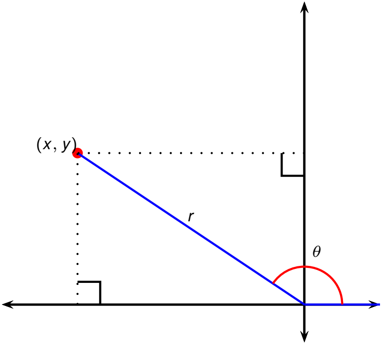
$$\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} &= \frac{1}{\cos^2\theta} \end{aligned}$$



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Prove the identity
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The remaining identities are consequences of the addition formulas:

$$
\begin{aligned}
\sin(x+y) &= \sin x \cos y + \cos x \sin y \\
\cos(x+y) &= \cos x \cos y - \sin x \sin y
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$$

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Substitute $-y$ for y, and use the fact that $\sin(-y) = -\sin y$ and $\cos(-y) = \cos y$:

$$\begin{aligned}
\sin(x-y) &= \sin x\cos y - \cos x\sin y\\
\cos(x-y) &= \cos x\cos y + \sin x\sin y
\end{aligned}$$

The remaining identities are consequences of the addition formulas:

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To get the double angle formulas, substitute x for y:

$$\begin{aligned} \sin(2x) &= 2 \sin x \cos x \\ \cos(2x) &= \cos^2 x - \sin^2 x \end{aligned}$$

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To get the double angle formulas, substitute x for y:

$$\begin{aligned}\sin(2x) &= 2\sin x \cos x\\ \cos(2x) &= \cos^2 x - \sin^2 x\end{aligned}$$

Rewrite the second double angle formula in two ways, using $\cos^2 x = 1 - \sin^2 x$ and $\sin^2 x = 1 - \cos^2 x$:

$$\begin{aligned}\cos(2x) &= 2\cos^2 x - 1\\ \cos(2x) &= 1 - 2\sin^2 x\end{aligned}$$

The remaining identities are consequences of the addition formulas:

$$\begin{aligned}\sin(x+y) &= \sin x\cos y + \cos x\sin y\\ \cos(x+y) &= \cos x\cos y - \sin x\sin y\end{aligned}$$

To get the double angle formulas, substitute x for y:

$$\begin{aligned}\sin(2x) &= 2\sin x\cos x\\ \cos(2x) &= \cos^2 x - \sin^2 x\end{aligned}$$

Rewrite the second double angle formula in two ways, using $\cos^2 x = 1-\sin^2 x$ and $\sin^2 x = 1-\cos^2 x$:

$$\begin{aligned}\cos(2x) &= 2\cos^2 x - 1\\ \cos(2x) &= 1 - 2\sin^2 x\end{aligned}$$

To get the half-angle formulas, solve these equations for $\cos^2 x$ and $\sin^2 x$ respectively.

$$\cos^2 x = \frac{1+\cos(2x)}{2}, \qquad \sin^2 x = \frac{1-\cos(2x)}{2}$$

The remaining identities are consequences of the addition formulas:

$$\begin{aligned}
\sin(x+y) &= \sin x \cos y + \cos x \sin y \\
\cos(x+y) &= \cos x \cos y - \sin x \sin y
\end{aligned}$$

The remaining identities are consequences of the addition formulas:

$$\begin{array}{rcccc} \sin(x+y) & = & \sin x \cos y & + & \cos x \sin y \\ \cos(x+y) & = & \cos x \cos y & - & \sin x \sin y \end{array}$$

Divide the first equation by the second, and then cancel $\cos x \cos y$ from the top and bottom:

$$\tan(x+y) \quad = \quad \tfrac{\tan x + \tan y}{1 - \tan x \tan y}$$

The remaining identities are consequences of the addition formulas:

$$\begin{aligned}\sin(x+y) &= \sin x \cos y + \cos x \sin y \\ \cos(x+y) &= \cos x \cos y - \sin x \sin y\end{aligned}$$

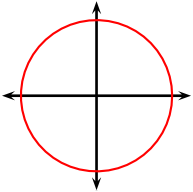
Divide the first equation by the second, and then cancel $\cos x \cos y$ from the top and bottom:

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

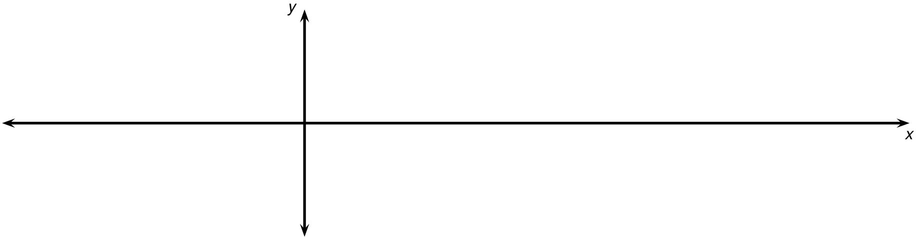
Do the same for the subtraction formulas:

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

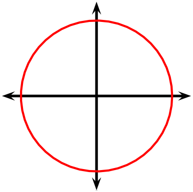
Graph of $\sin x$



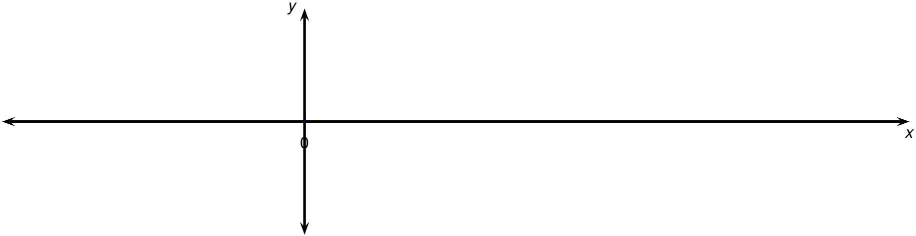
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	**?**								



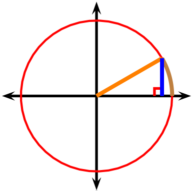
Graph of $\sin x$



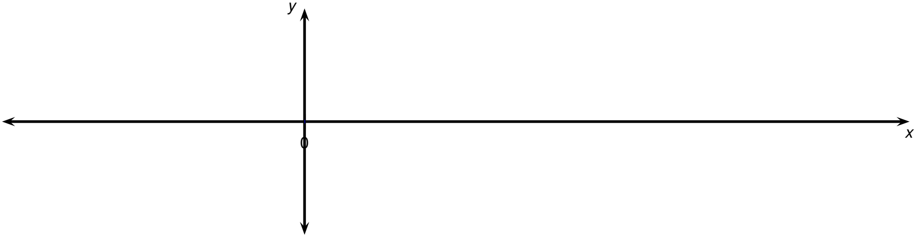
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0								



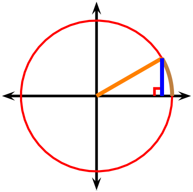
Graph of $\sin x$



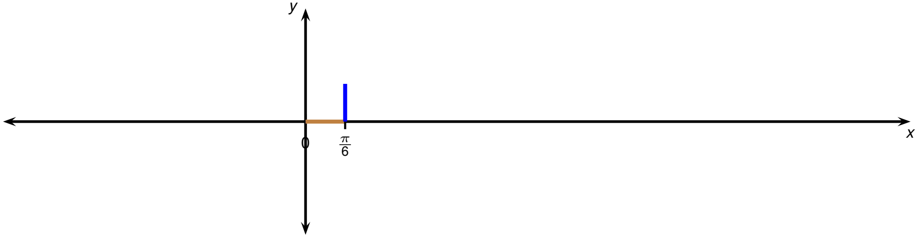
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	**?**							



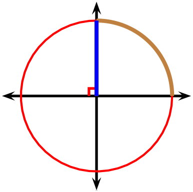
Graph of $\sin x$



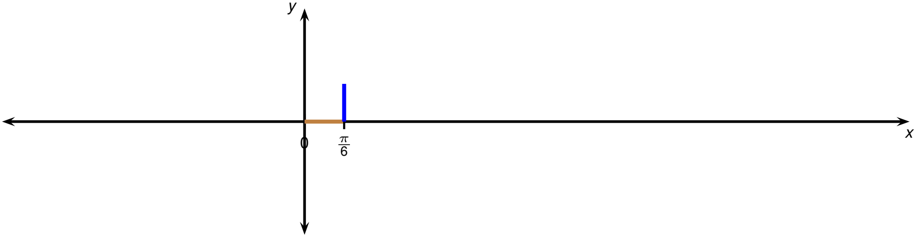
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$							



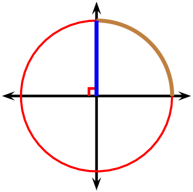
Graph of sin x



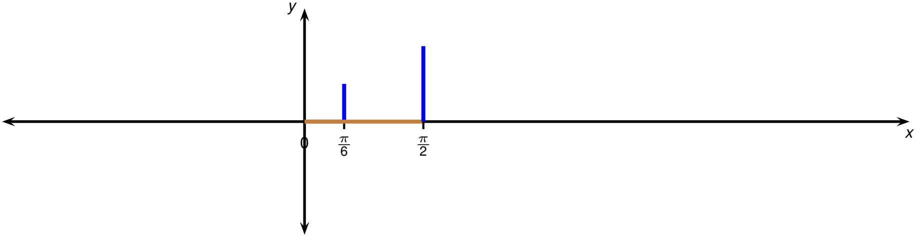
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	**?**						



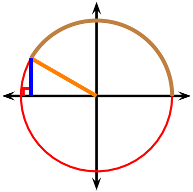
Graph of $\sin x$



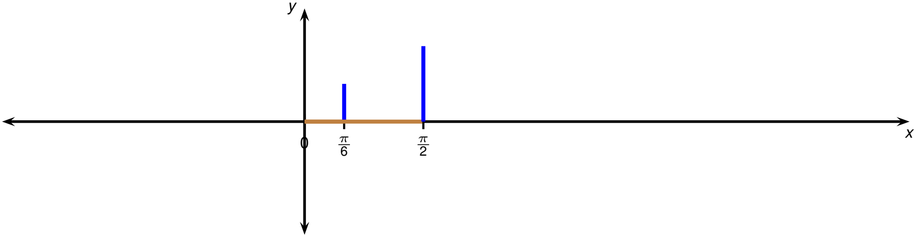
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1						



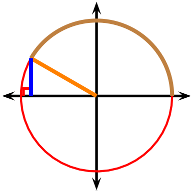
Graph of $\sin x$



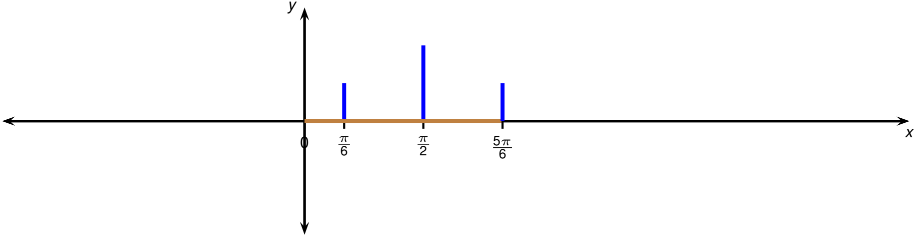
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	**?**					



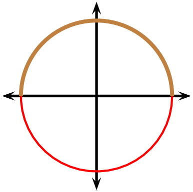
Graph of $\sin x$



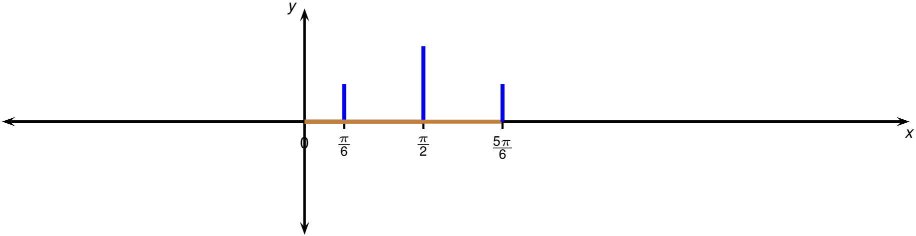
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$					



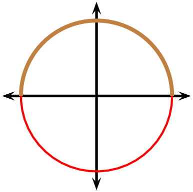
Graph of $\sin x$



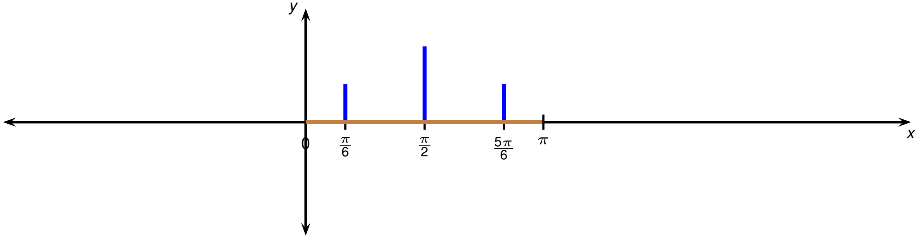
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	**?**				



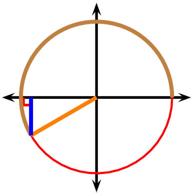
Graph of $\sin x$



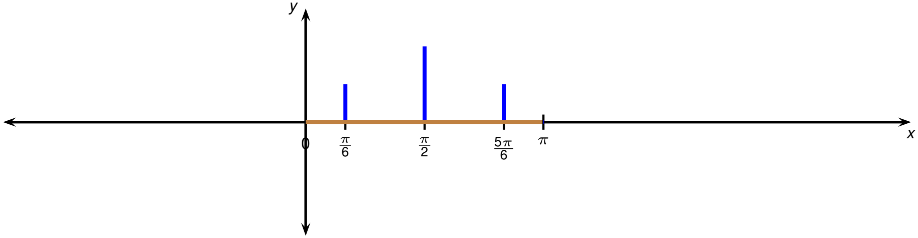
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0				



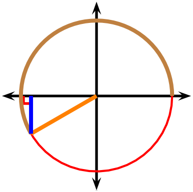
Graph of $\sin x$



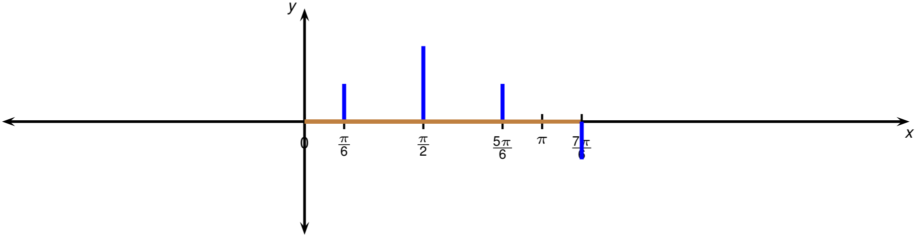
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	**?**			



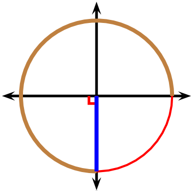
Graph of $\sin x$



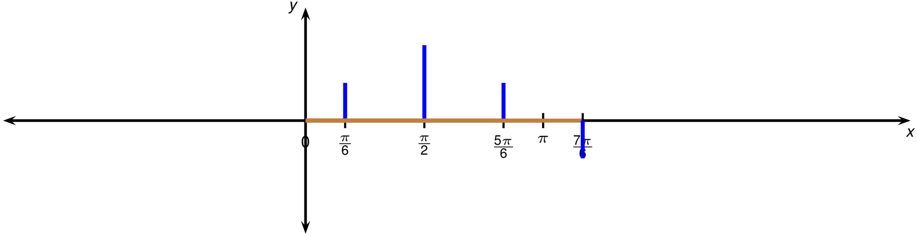
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$			



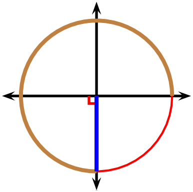
Graph of $\sin x$



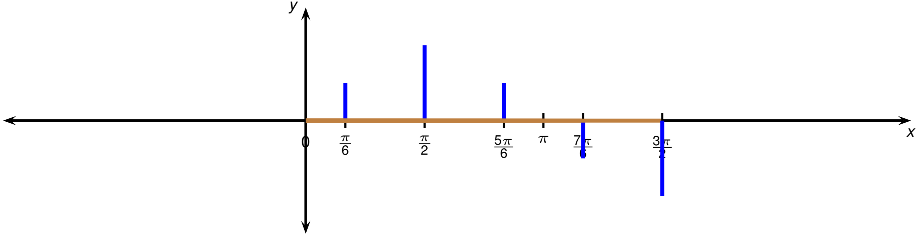
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	**?**		



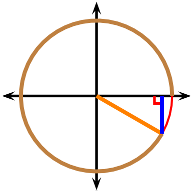
Graph of $\sin x$



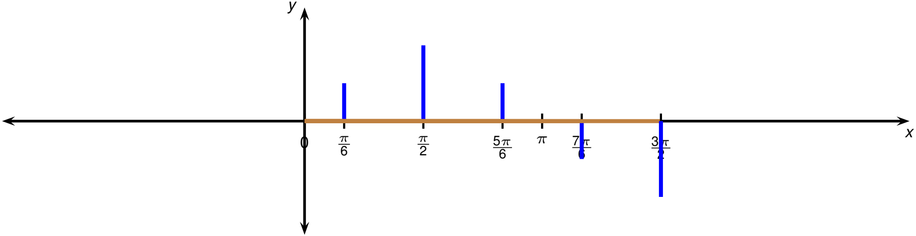
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1		



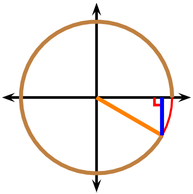
Graph of $\sin x$



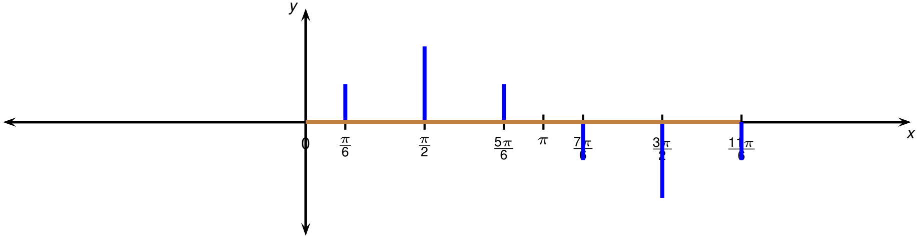
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	**?**	



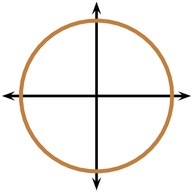
Graph of $\sin x$



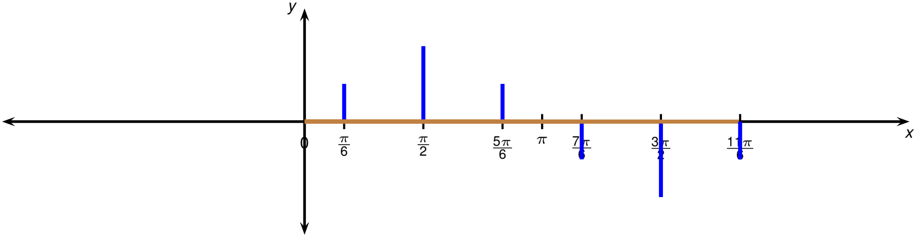
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	



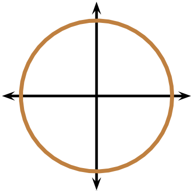
Graph of $\sin x$



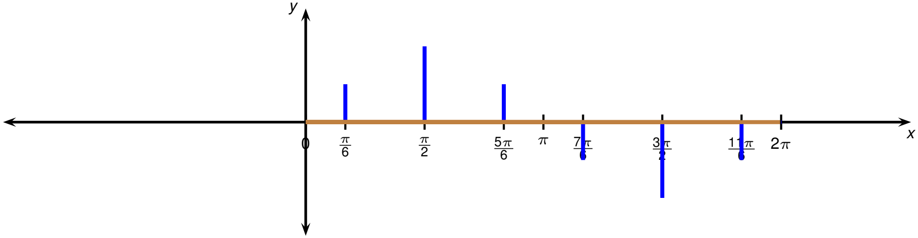
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	**?**



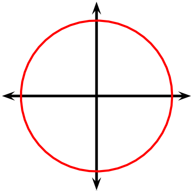
Graph of $\sin x$



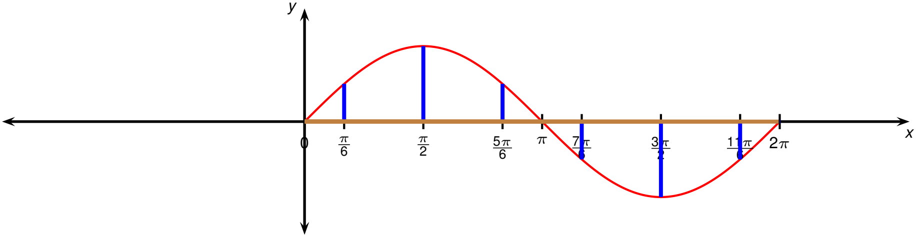
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0



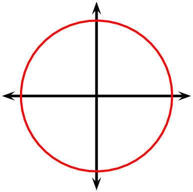
Graph of $\sin x$



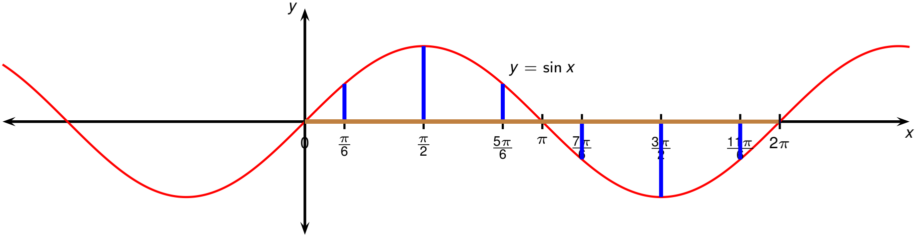
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0



Graph of $\sin x$

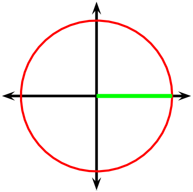


x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0

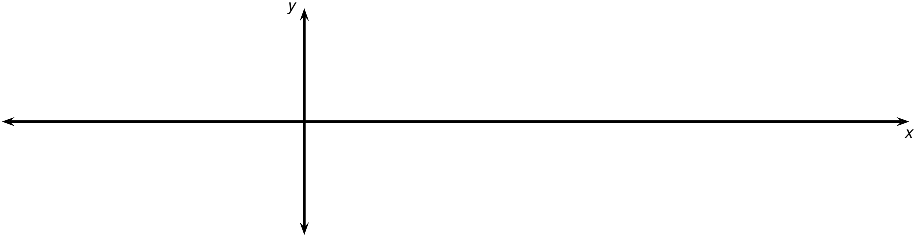


The graph of $\sin x$ is 2π-periodic so the rest of the graph can be inferred from the interval $[0, 2\pi]$.

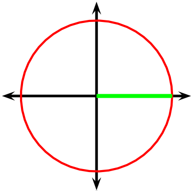
Graph of $\cos x$



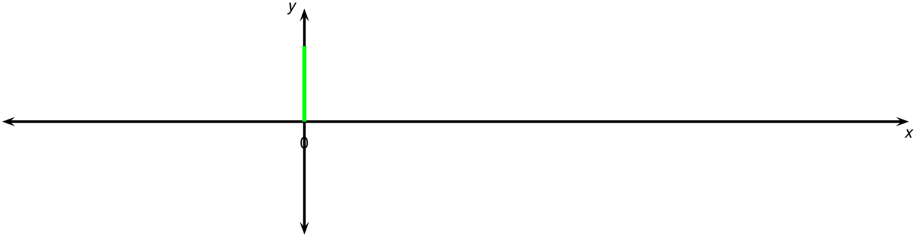
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	**?**								



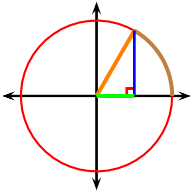
Graph of $\cos x$



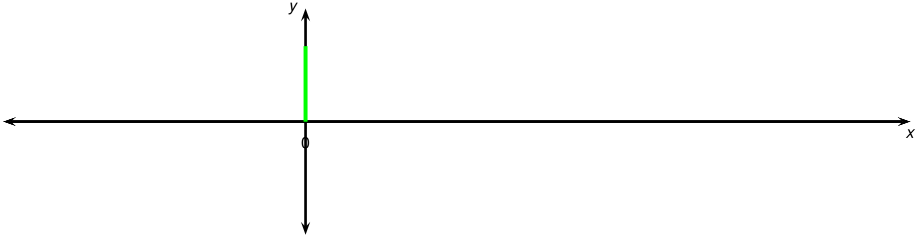
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1								



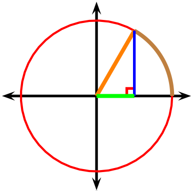
Graph of $\cos x$



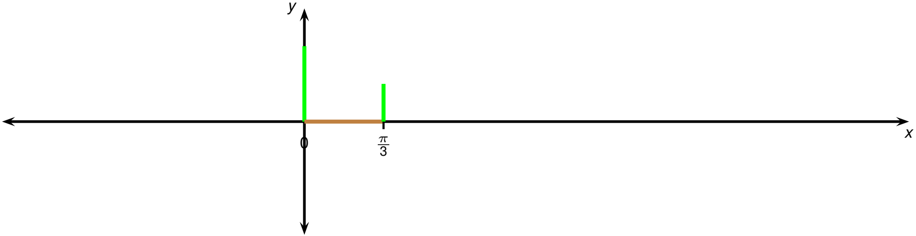
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	**?**							



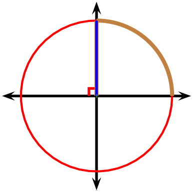
Graph of $\cos x$



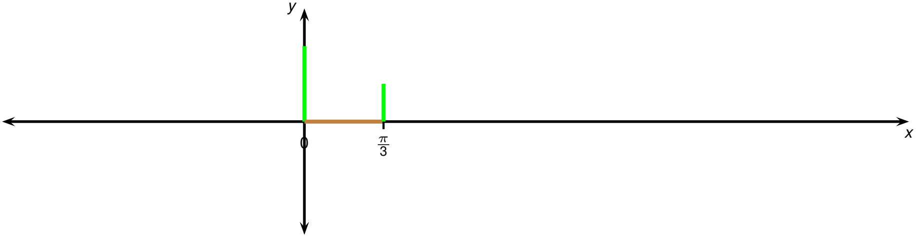
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$							



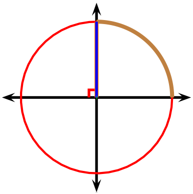
Graph of $\cos x$



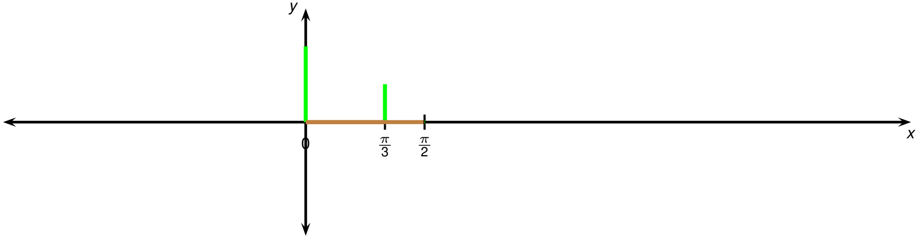
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	?						



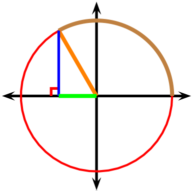
Graph of $\cos x$



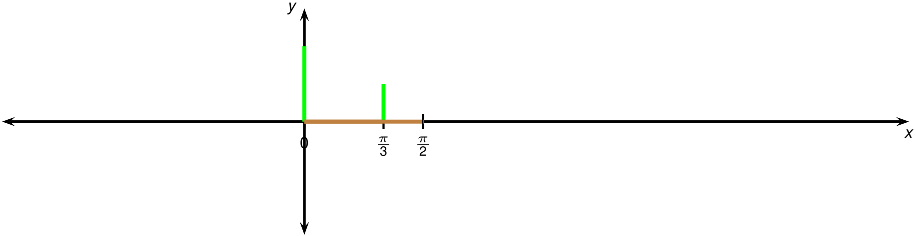
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0						



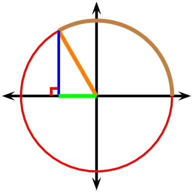
Graph of $\cos x$



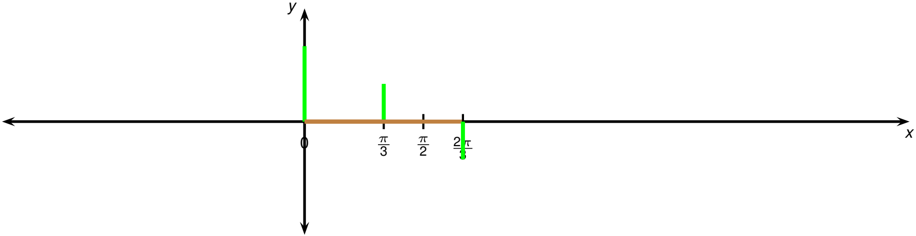
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	**?**					



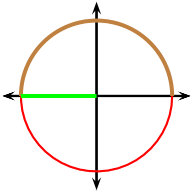
Graph of $\cos x$



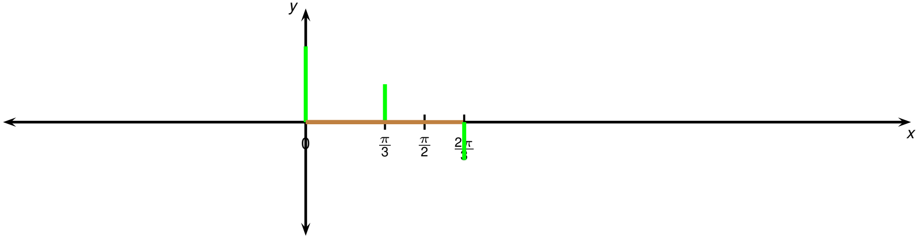
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$					



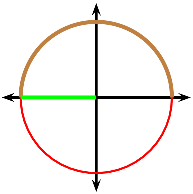
Graph of $\cos x$



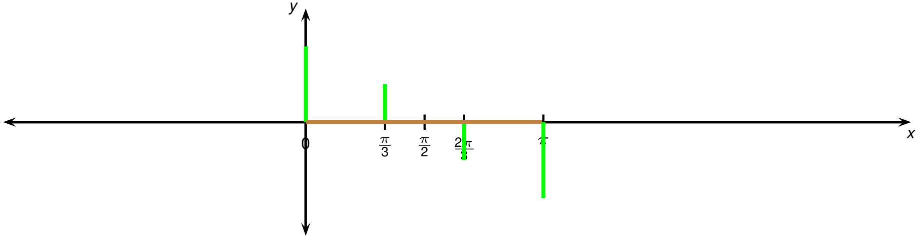
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	**?**				



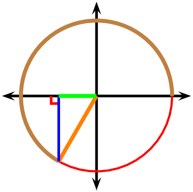
Graph of $\cos x$



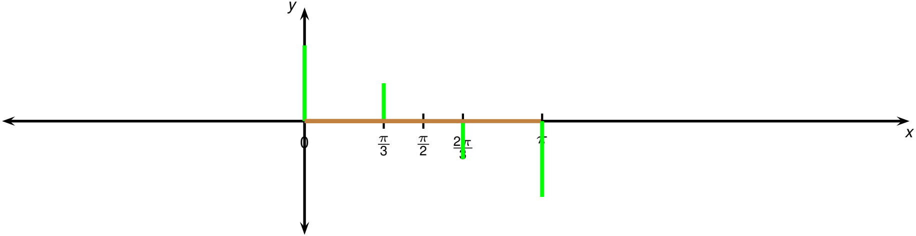
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1				



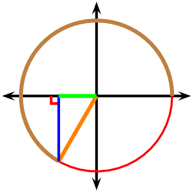
Graph of $\cos x$



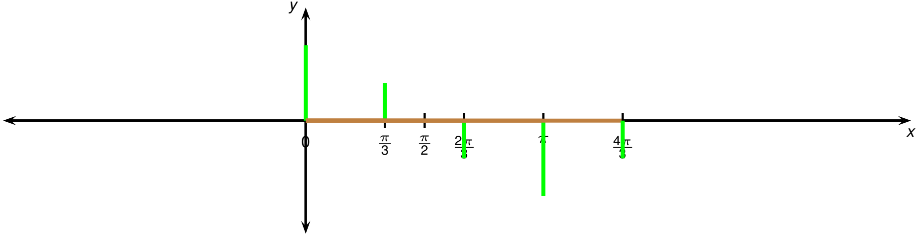
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	**?**			



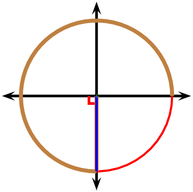
Graph of $\cos x$



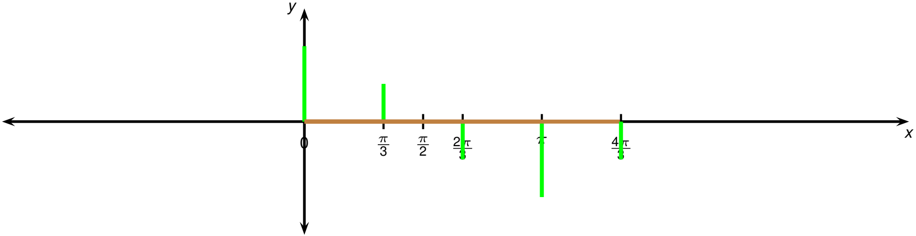
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$			



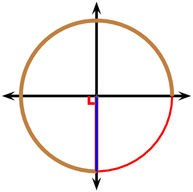
Graph of $\cos x$



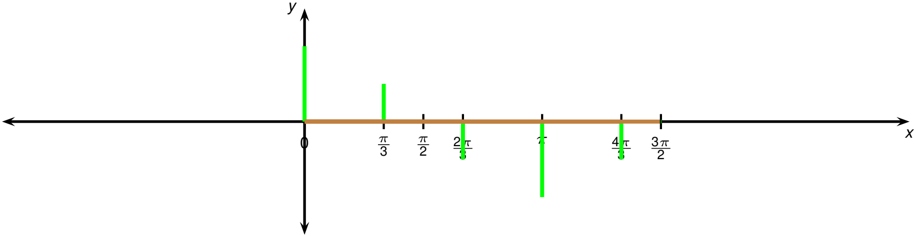
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	**?**		



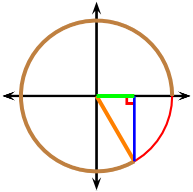
Graph of $\cos x$



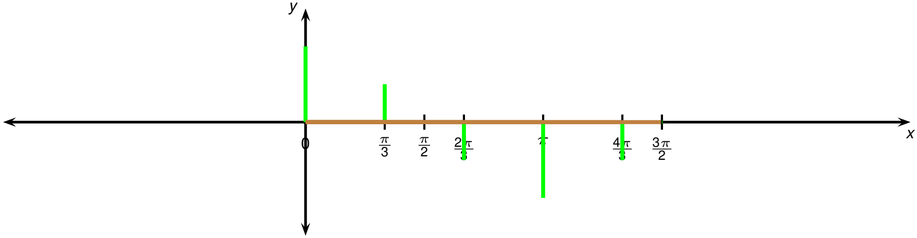
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0		



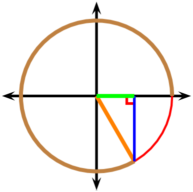
Graph of $\cos x$



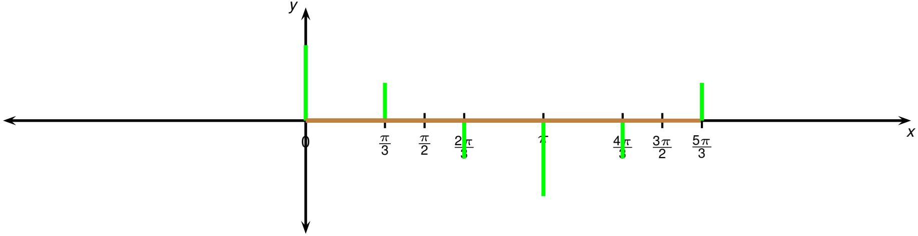
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	**?**	



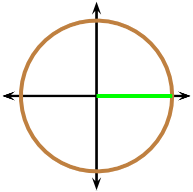
Graph of $\cos x$



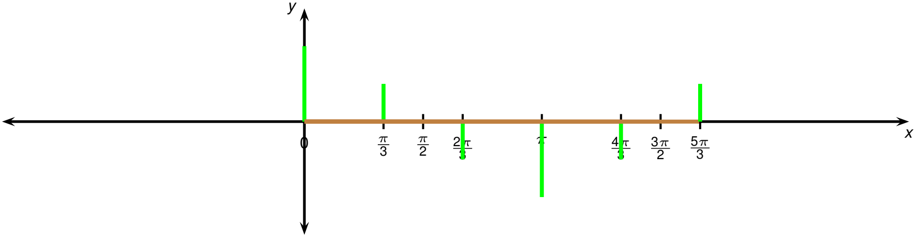
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	



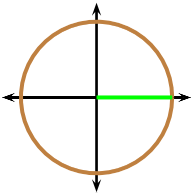
Graph of $\cos x$



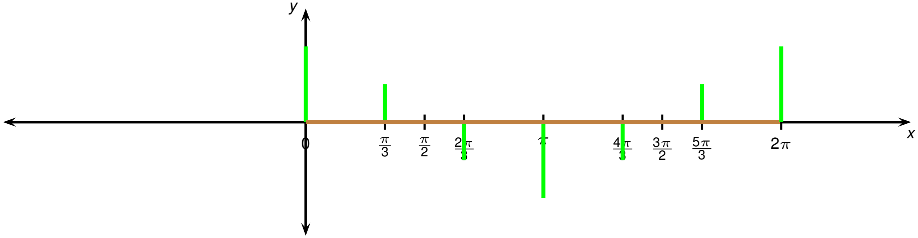
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	**?**



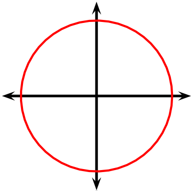
Graph of $\cos x$



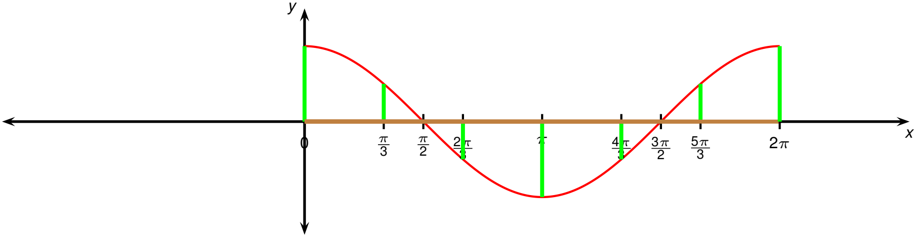
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0



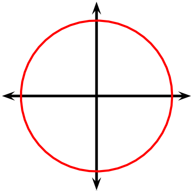
Graph of $\cos x$



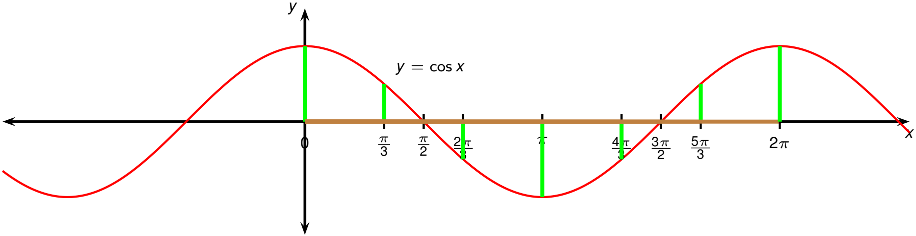
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0



Graph of $\cos x$

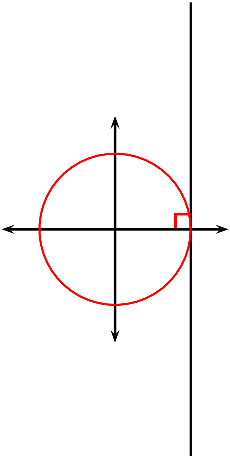


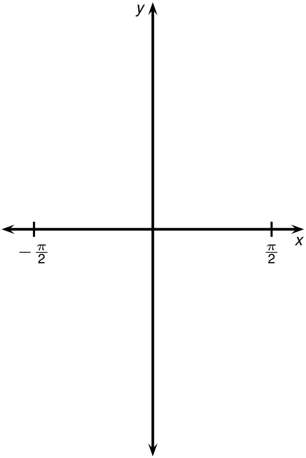
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0



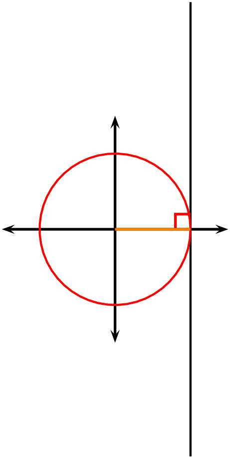
The graph of $\cos x$ is 2π-periodic so the rest of the graph can be inferred from the interval $[0, 2\pi]$.

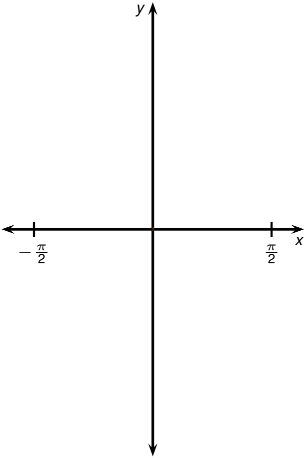
Graph of $\tan x$



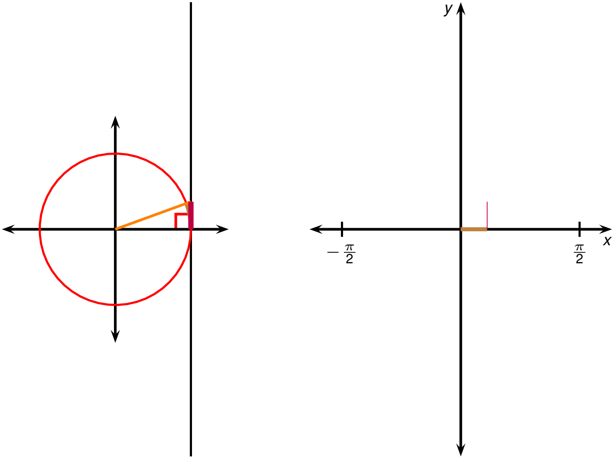


Graph of $\tan x$

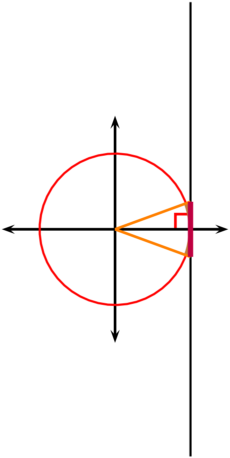


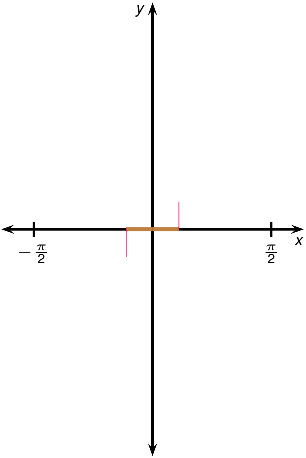


Graph of $\tan x$

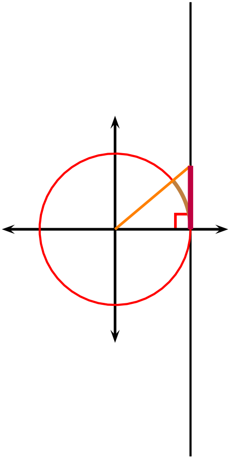


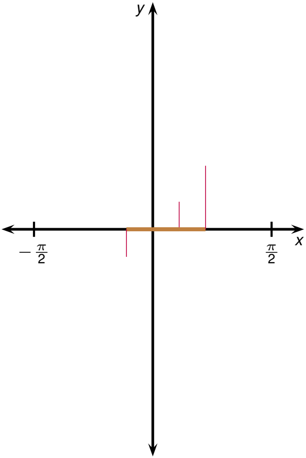
Graph of $\tan x$



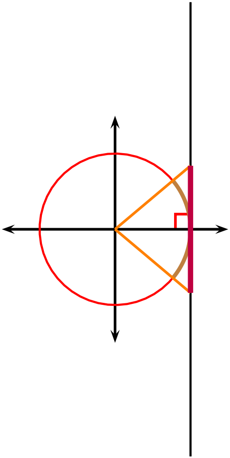


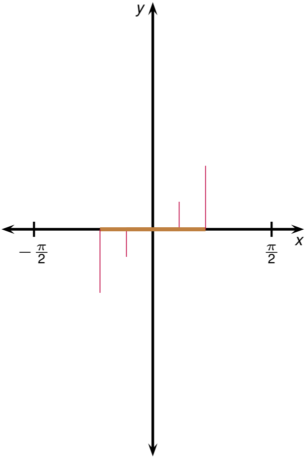
Graph of $\tan x$



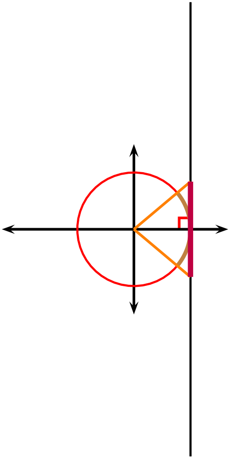


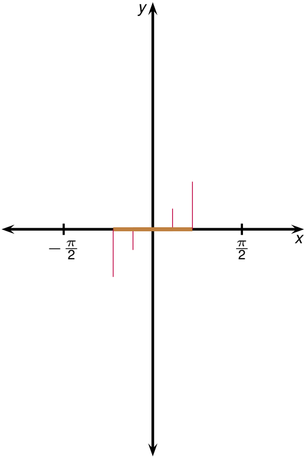
Graph of $\tan x$



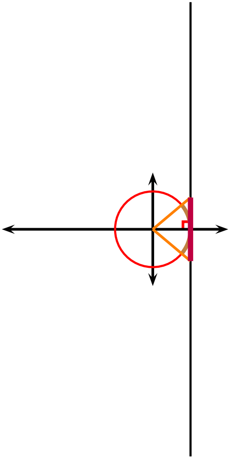


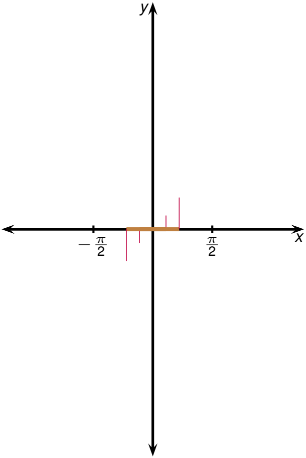
Graph of $\tan x$



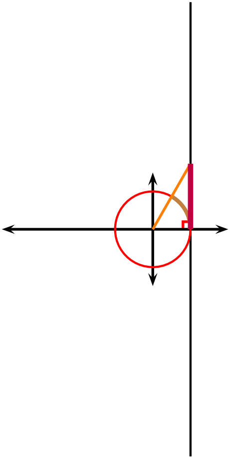


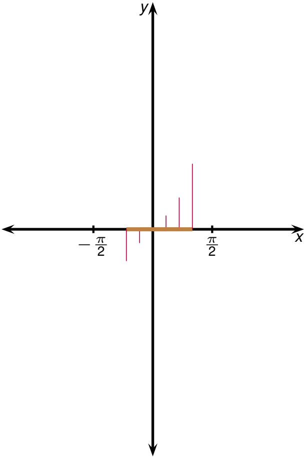
Graph of $\tan x$



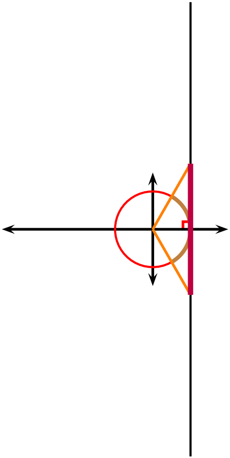


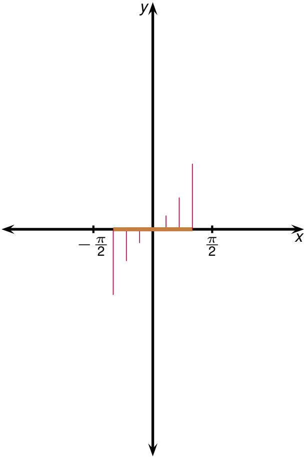
Graph of $\tan x$



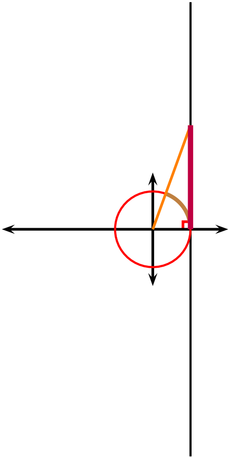


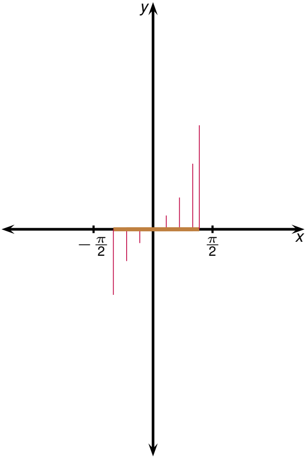
Graph of $\tan x$



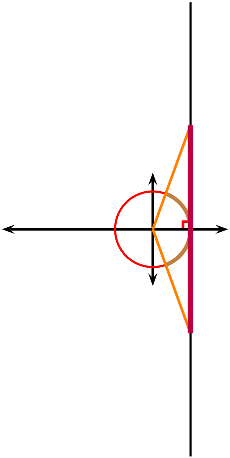


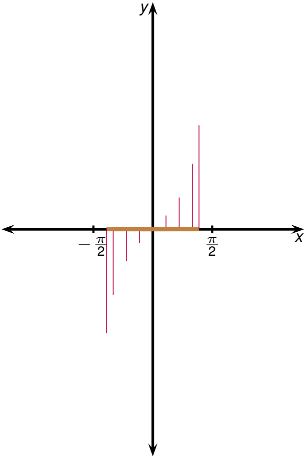
Graph of $\tan x$



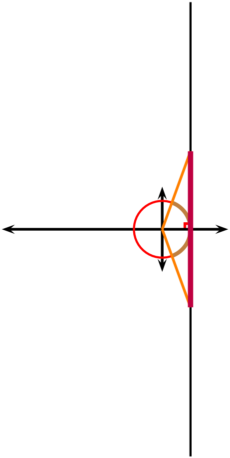


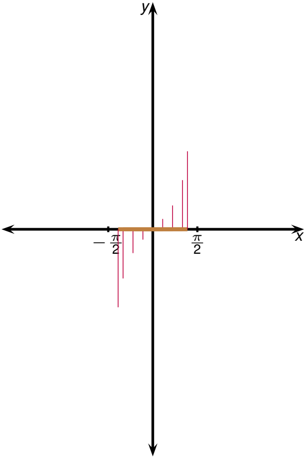
Graph of $\tan x$



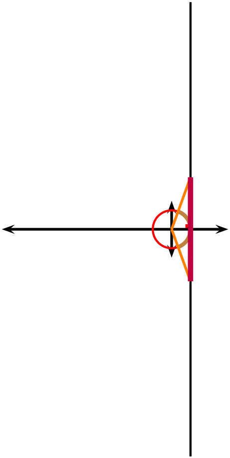


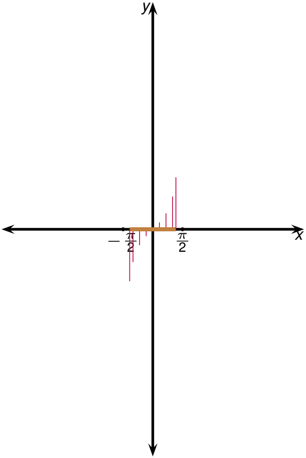
Graph of $\tan x$



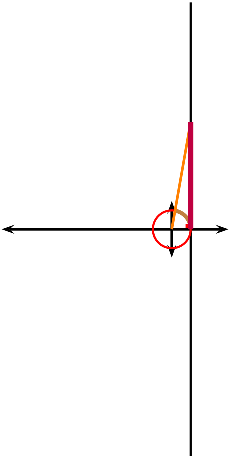


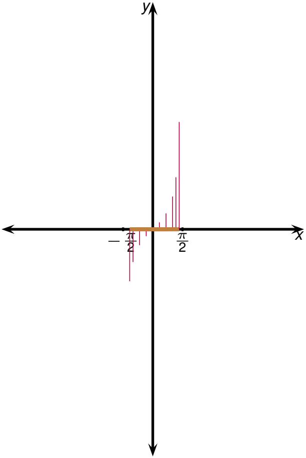
Graph of $\tan x$



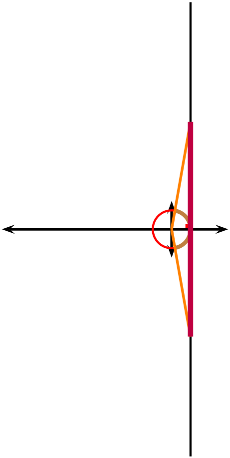


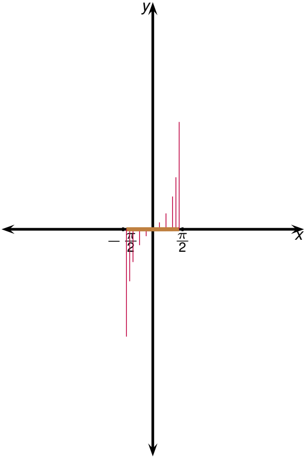
Graph of $\tan x$





Graph of $\tan x$





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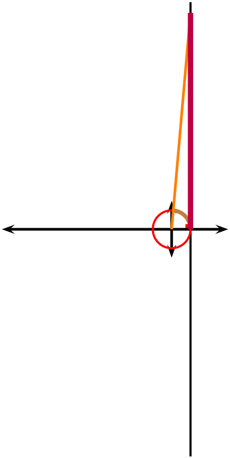

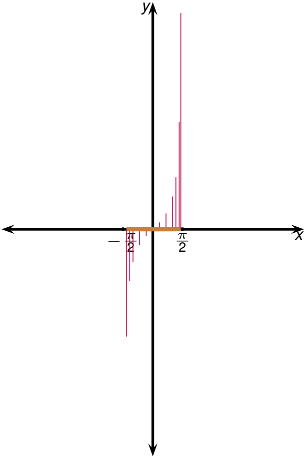

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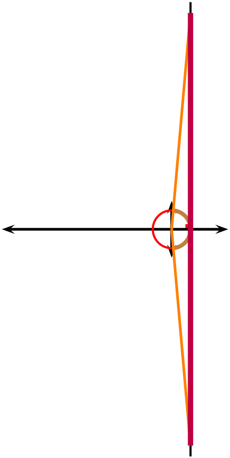

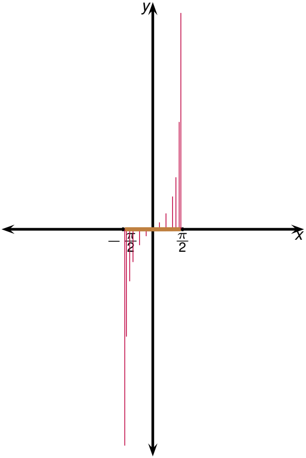

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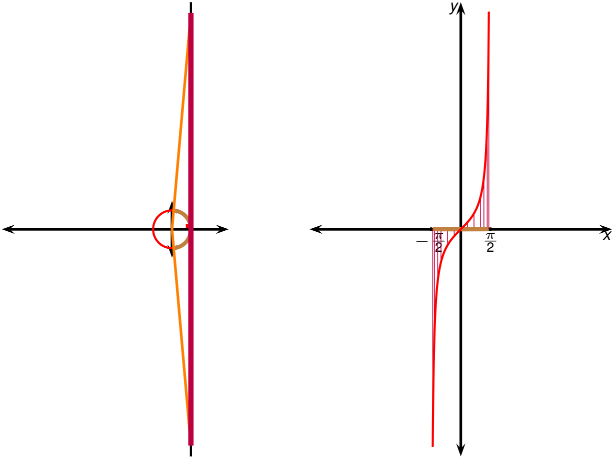

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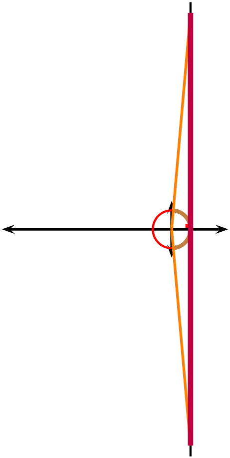

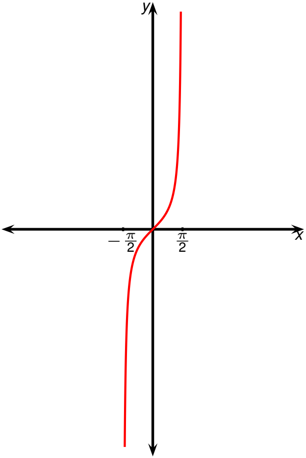

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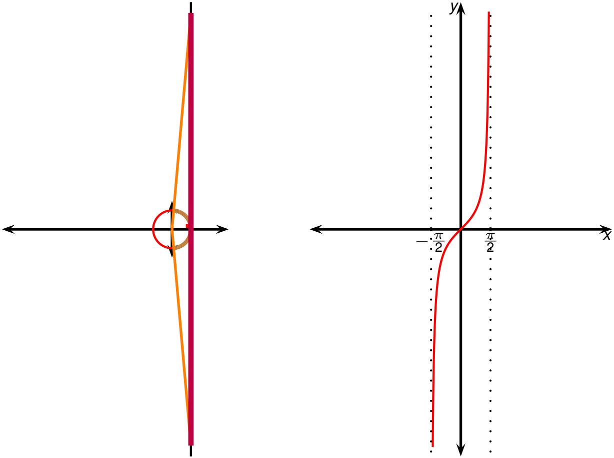
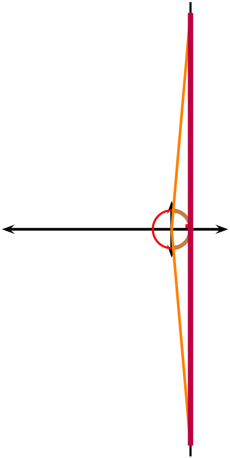
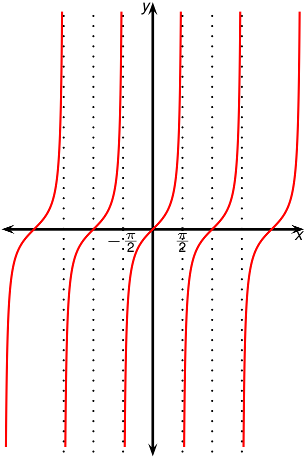


Near $\pm\frac{\pi}{2}$ the graph of $\tan x$ approaches $\pm\infty$.

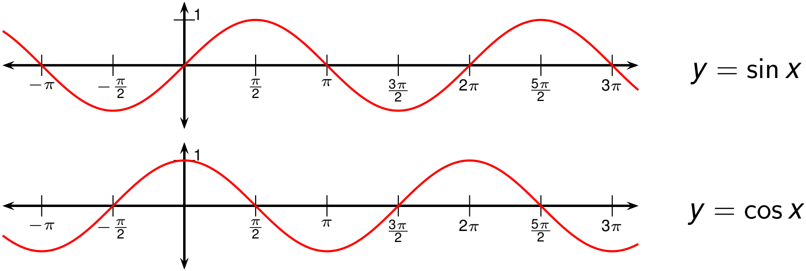
Graph of $\tan x$



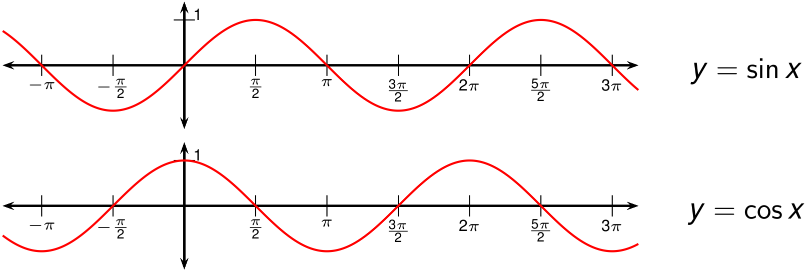


Near $\pm\frac{\pi}{2}$ the graph of $\tan x$ approaches $\pm\infty$. The graph of $\tan x$ is π-periodic so the rest of the graph can be inferred from the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Graphs of the Trigonometric Functions

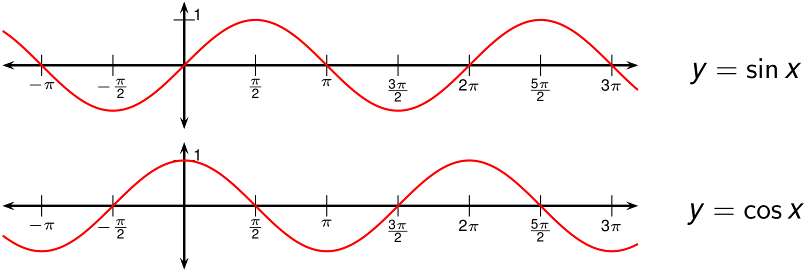


Graphs of the Trigonometric Functions



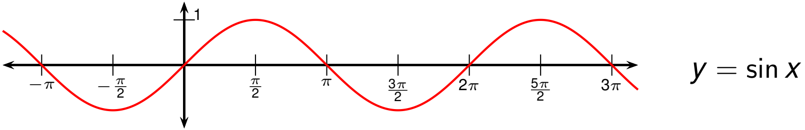
- $\sin x$ has zeroes at $n\pi$ for all integers n.

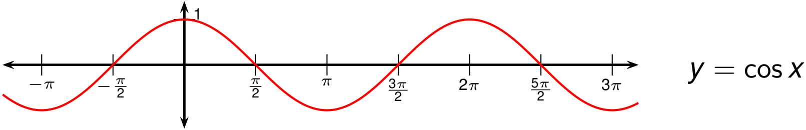
Graphs of the Trigonometric Functions



- $\sin x$ has zeroes at $n\pi$ for all integers n.
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.

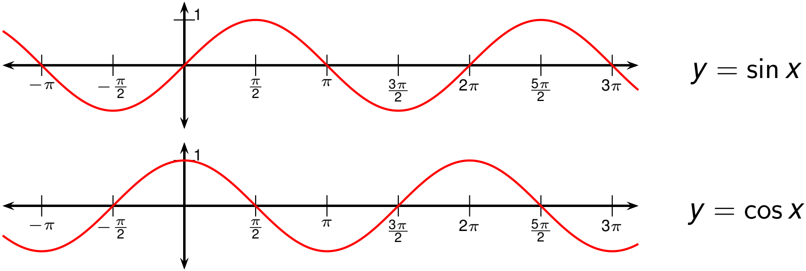
Graphs of the Trigonometric Functions





- $\sin x$ has zeroes at $n\pi$ for all integers n.
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.
- $-1 \leq \sin x \leq 1$.

Graphs of the Trigonometric Functions



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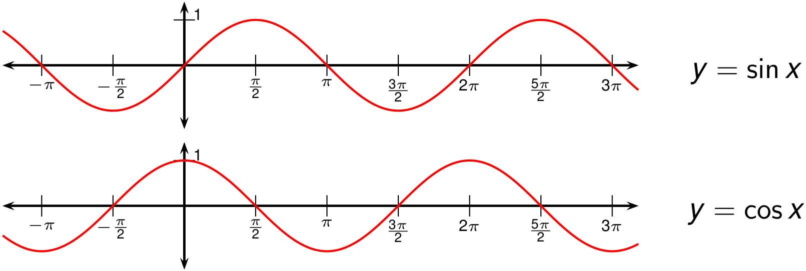

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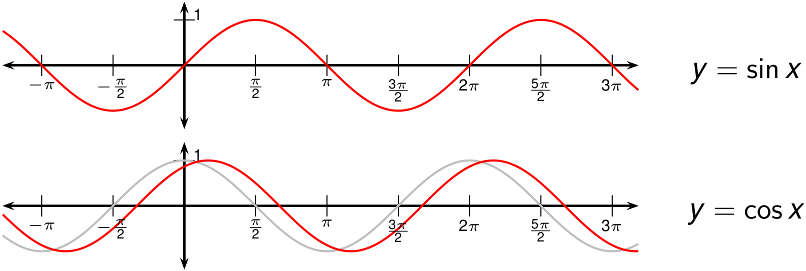
- If we translate the graph of $\cos x$ by $\frac{\pi}{2}$ units to the right

Graphs of the Trigonometric Functions



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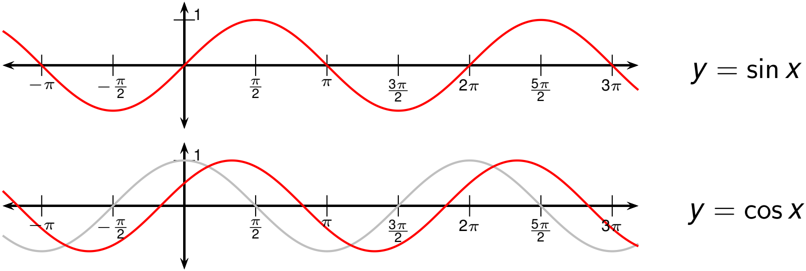

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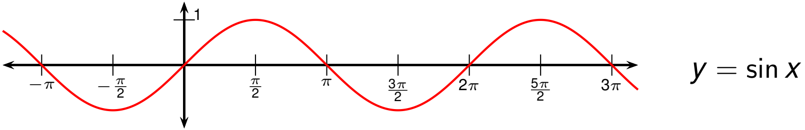
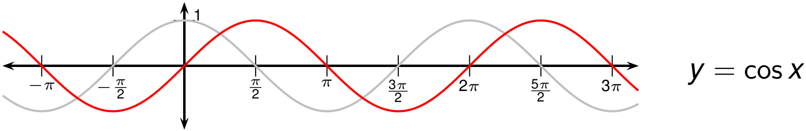
Graphs of the Trigonometric Functions





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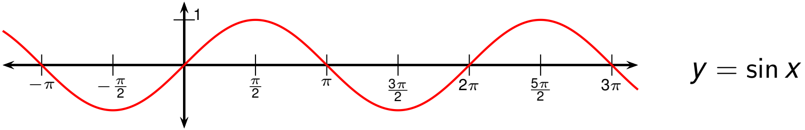
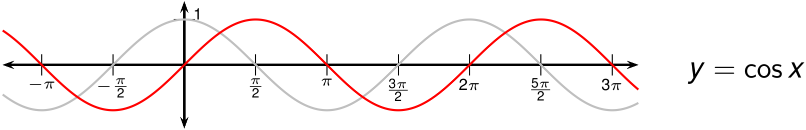
Graphs of the Trigonometric Functions





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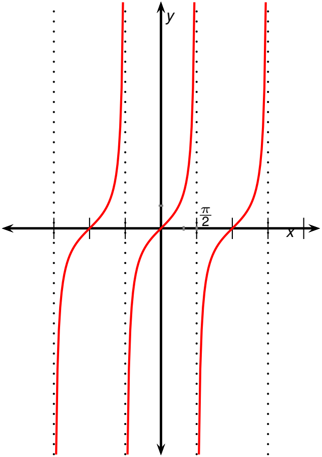
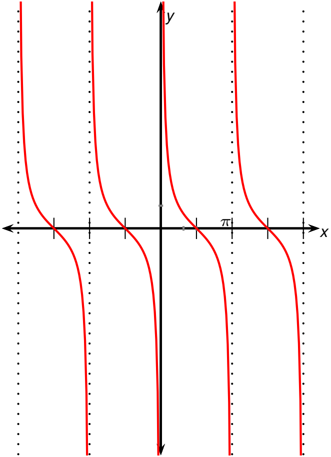
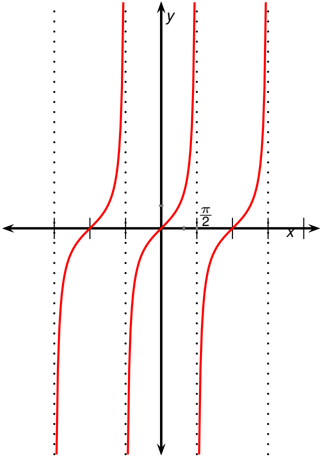
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.
- $-1 \leq \sin x \leq 1$.
- $-1 \leq \cos x \leq 1$.
- If we translate the graph of $\cos x$ by $\frac{\pi}{2}$ units to the right we get the graph of $\sin x$. This is a consequence of $\cos\left(x - \frac{\pi}{2}\right) = \sin x$.



$y = \tan x$



$y = \cot x$



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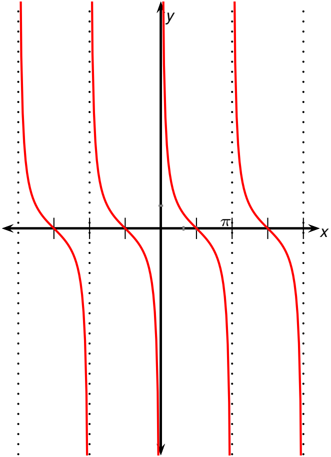
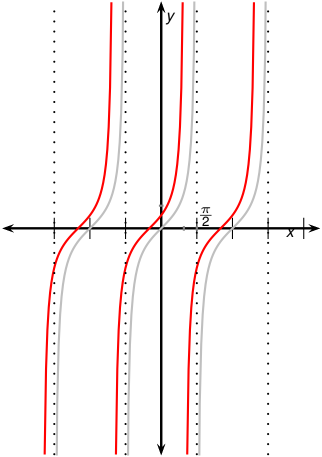


$y = \cot x$

If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis



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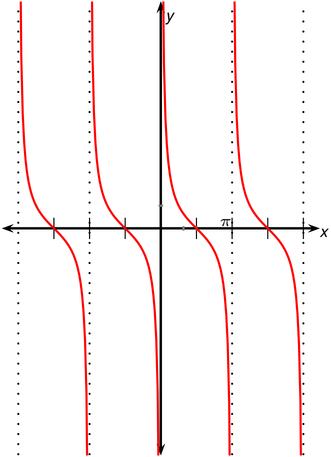

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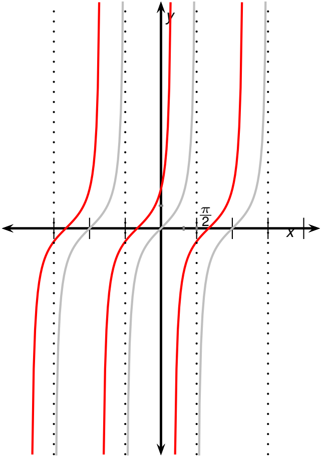

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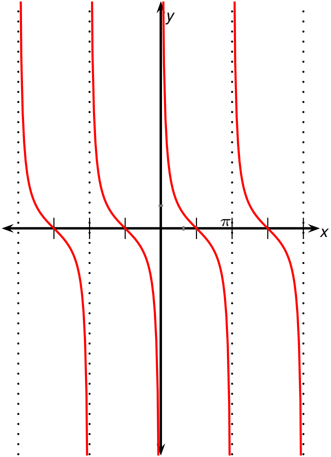

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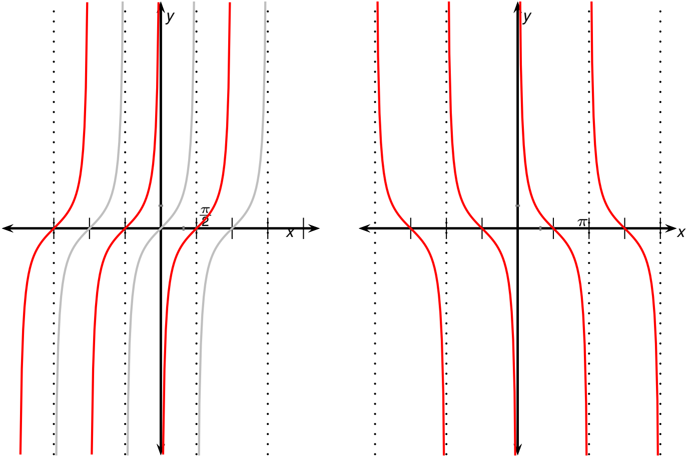
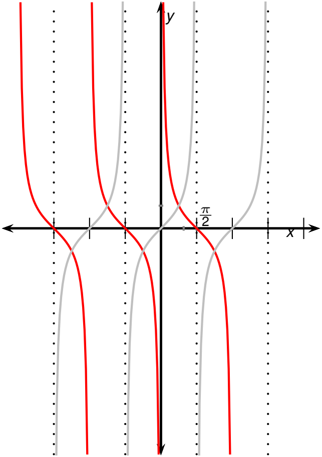
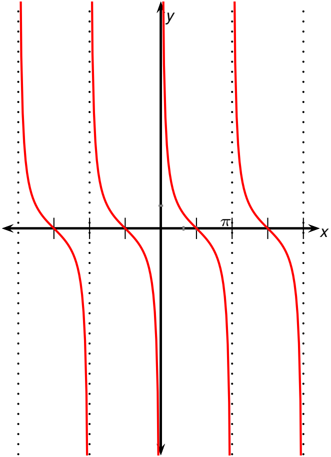


$y = \tan x$ $\qquad$ $y = \cot x$

If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis

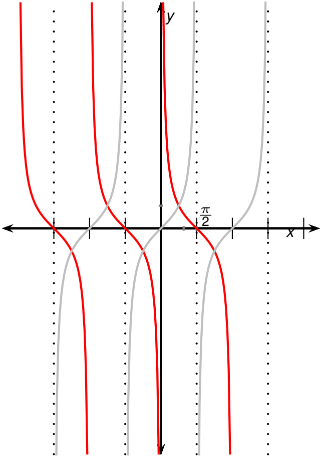


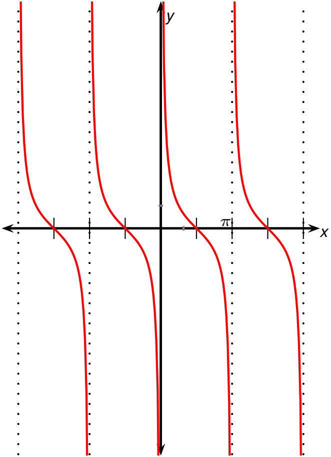
$y = \tan x$



$y = \cot x$

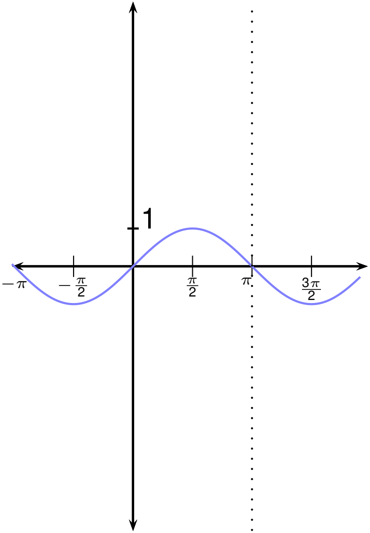
If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis, we get the graph of $\cot x$.



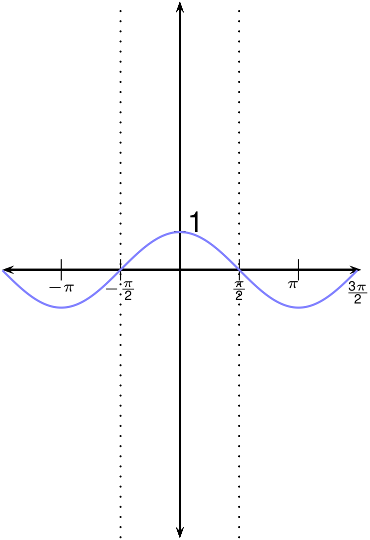


$y = \tan x$ $\qquad$ $y = \cot x$

If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis, we get the graph of $\cot x$. This follows from $\tan\left(x \pm \frac{\pi}{2}\right) = -\cot x$.


1
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$

$y = \csc x$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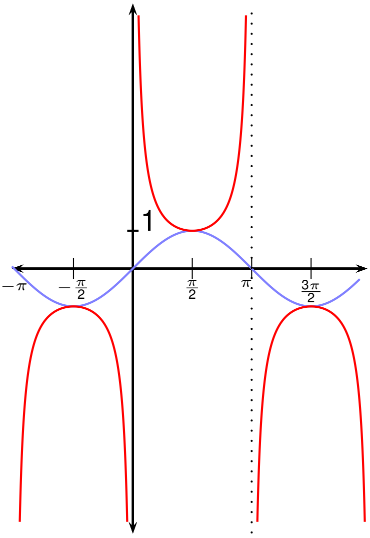
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$

$y = \sec x$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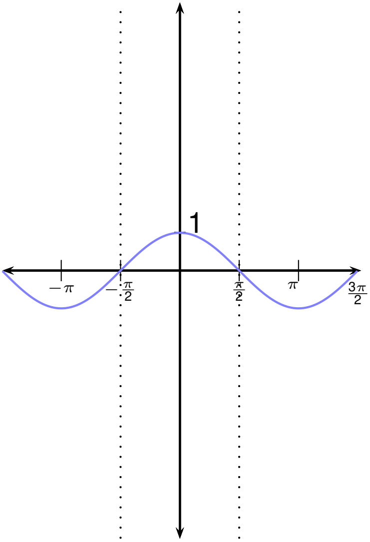
$y = \csc x$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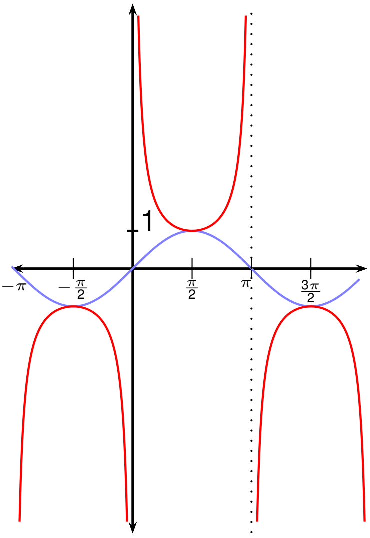
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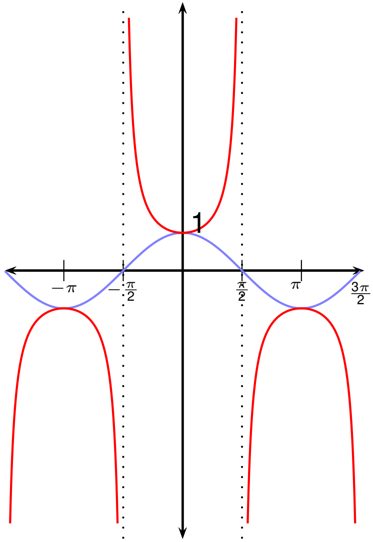
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