Precalculus
Inverse of fractional linear transformation

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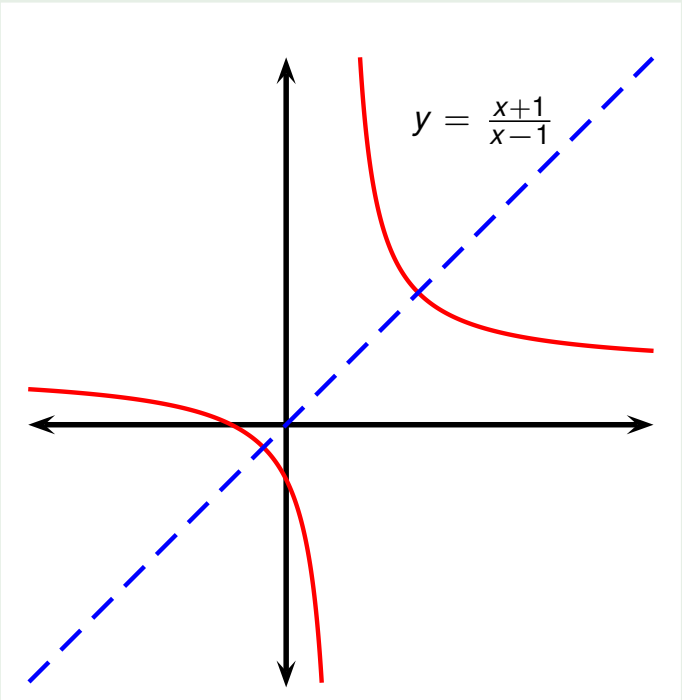
Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.



Answer: $f^{-1}(x) = \frac{x+1}{x-1}$, $x \neq 1$.

We deal with domains and ranges later:

$$\begin{aligned} y &= \frac{x+1}{x-1} && \text{mult. by } (x-1) \\ y(x-1) &= x+1 \\ x(y-1) &= y+1 && \text{div. by } (y-1) \\ f^{-1}(y) = x &= \frac{y+1}{y-1} && \text{relabel } x, y \\ f^{-1}(x) &= \frac{x+1}{x-1} \end{aligned}$$

We divided by $y - 1$ so $y \neq 1$. Therefore the domain of f^{-1} is all real numbers except 1.

Can a non-identity function be its own inverse? Yes, f is.
What does it mean for f to be its own inverse?
Graph of f is symmetric across $y = x$.