

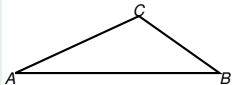
## Precalculus

# Solve triangle from two sides and an angle

Todor Milev

2019

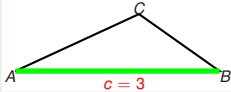
## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

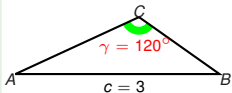
## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

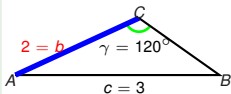
## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

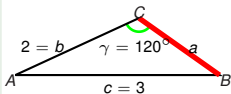
## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . **Another side of that triangle has length 2.**

- Find the length of the third side.
- Find the area of the triangle.

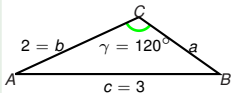
## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

## Example

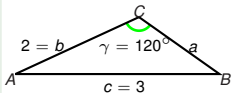


The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

| Law of cosines

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

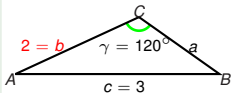
- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

| Law of cosines



## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

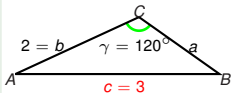
- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

| Law of cosines

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

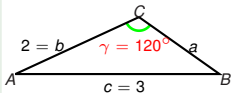
- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

| Law of cosines

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

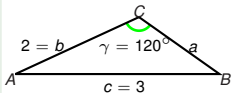
- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

| Law of cosines

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

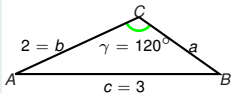
- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

Law of cosines  
Solve for  $a$  :

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

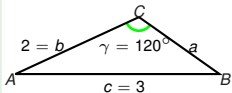
$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

$$a^2 - 4a \left( \quad \right) - 5 = 0$$

Law of cosines  
Solve for  $a$  :

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

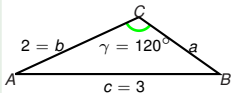
$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

$$a^2 - 4a \left( \quad \right) - 5 = 0$$

Law of cosines  
Solve for  $a$  :

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

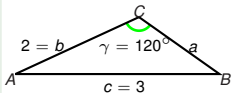
$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

$$a^2 - 4a \left( ? \right) - 5 = 0$$

Law of cosines  
Solve for  $a$  :

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

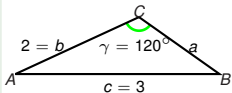
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

$$a^2 - 4a \left( -\frac{1}{2} \right) - 5 = 0$$

Law of cosines  
Solve for  $a$  :



## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

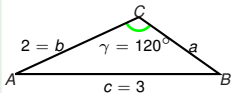
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

$$a^2 - 4a \left( -\frac{1}{2} \right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

Law of cosines  
Solve for  $a$  :

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

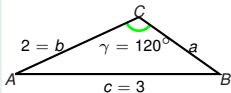
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

$$a^2 - 4a \left( -\frac{1}{2} \right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

Law of cosines  
Solve for  $a$  :

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

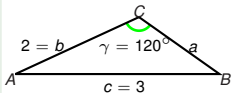
$$a^2 - 4a \left( -\frac{1}{2} \right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

Law of cosines  
Solve for  $a$  :

$$a = ?$$

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

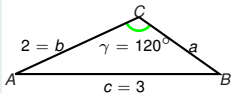
$$a^2 - 4a \left( -\frac{1}{2} \right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

Law of cosines  
Solve for  $a$  :

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

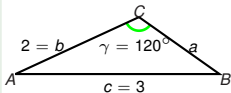
$$a^2 - 4a \left( -\frac{1}{2} \right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

Law of cosines  
Solve for  $a$  :

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

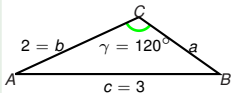
$$a^2 - 4a \left( -\frac{1}{2} \right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

Law of cosines  
Solve for  $a$  :

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

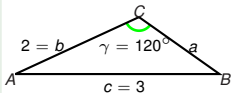
$$a^2 - 4a \left( -\frac{1}{2} \right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

Law of cosines  
Solve for  $a$  :

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

$$a^2 - 4a \left( -\frac{1}{2} \right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

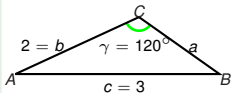
$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= \frac{-2 \pm \sqrt{24}}{2}$$

Law of cosines  
Solve for  $a$  :



## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

$$a^2 - 4a \left( -\frac{1}{2} \right) - 5 = 0$$

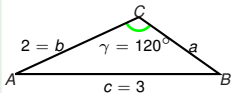
$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 + 2\sqrt{6}}{2}$$

Law of cosines  
Solve for  $a$  :

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

Law of cosines  
Solve for  $a$  :

$$a^2 - 4a \left( -\frac{1}{2} \right) - 5 = 0$$

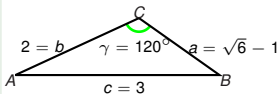
$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 + 2\sqrt{6}}{2}$$

$a > 0$

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

$$a^2 - 4a \left( -\frac{1}{2} \right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

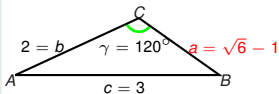
$$= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 + 2\sqrt{6}}{2}$$

$$= -1 + \sqrt{6}$$

Law of cosines  
Solve for  $a$ :

$$a > 0$$

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

$$a^2 - 4a \left( -\frac{1}{2} \right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

Law of cosines  
Solve for  $a$  :

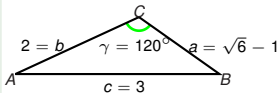
$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 + 2\sqrt{6}}{2}$$

$$= -1 + \sqrt{6}$$

$a > 0$

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

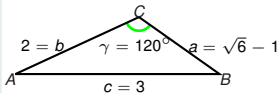
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

Law of cosines  
Solve for  $a$  :

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

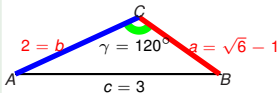
Law of cosines  
Solve for  $a$  :

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

**Area = ?**

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

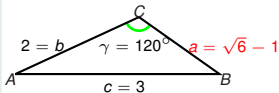
Law of cosines  
Solve for  $a$  :

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\text{Area} = \frac{ab \sin \gamma}{2}$$

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

Law of cosines  
Solve for  $a$ :

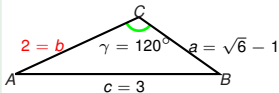
$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\text{Area} = \frac{ab \sin \gamma}{2} = \frac{(\sqrt{6} - 1) \cdot 2}{2} ?$$



## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

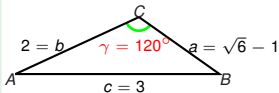
Law of cosines  
Solve for  $a$  :

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\text{Area} = \frac{ab \sin \gamma}{2} = \frac{(\sqrt{6} - 1) \cdot 2}{2} ?$$

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

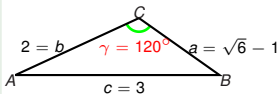
Law of cosines  
Solve for  $a$  :

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\text{Area} = \frac{ab \sin \gamma}{2} = \frac{(\sqrt{6} - 1) 2}{2} ?$$

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

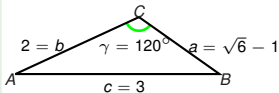
Law of cosines  
Solve for  $a$  :

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\text{Area} = \frac{ab \sin \gamma}{2} = \frac{(\sqrt{6} - 1) \cdot 2 \cdot \frac{\sqrt{3}}{2}}{2}$$

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

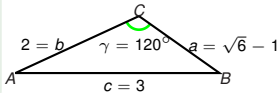
Law of cosines  
Solve for  $a$ :

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\text{Area} = \frac{ab \sin \gamma}{2} = \frac{(\sqrt{6} - 1) \cancel{2} \sqrt{3}}{\cancel{2}} \frac{\sqrt{3}}{2}$$

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

Law of cosines  
Solve for  $a$ :

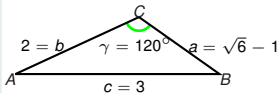
$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\text{Area} = \frac{ab \sin \gamma}{2} = \frac{(\sqrt{6} - 1) \cancel{2} \sqrt{3}}{\cancel{2} \cdot 2}$$

$$= \frac{3\sqrt{2} - \sqrt{3}}{2}$$

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

Law of cosines  
Solve for  $a$ :

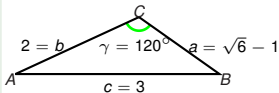
$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\text{Area} = \frac{ab \sin \gamma}{2} = \frac{(\sqrt{6} - 1) \cancel{2} \sqrt{3}}{\cancel{2} \cdot 2}$$

$$= \frac{3\sqrt{2} - \sqrt{3}}{2}$$

## Example



The longest side of a triangle has length 3 and the angle opposite to it is  $120^\circ$ . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

Law of cosines  
Solve for  $a$  :

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\text{Area} = \frac{ab \sin \gamma}{2} = \frac{(\sqrt{6} - 1) \cancel{2} \sqrt{3}}{\cancel{2} \cdot 2}$$

$$= \frac{3\sqrt{2} - \sqrt{3}}{2}$$