

# Arithmetics

## Long division

[calculator-algebra.org](http://calculator-algebra.org)

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2019

- We present long integer division through examples.

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- We present each algorithm step by example.

# Introducing long integer division

Divide 13 by 4 with quotient and remainder. Use long integer division notation.

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$$\begin{array}{r} \boxed{13} \\ \hline 4 \end{array}$$

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Divide 13 by 4 with quotient and remainder. Use long integer division notation.

$$\begin{array}{r} 4 \sqrt{13} \\ \end{array}$$

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$$\begin{array}{r} ? \\ 4 \overline{)13} \end{array}$$

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$$\left[ \frac{13}{4} \right]$$

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$$\left[ \begin{array}{r} 13 \\ 4 \end{array} \right]$$

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$$\begin{array}{r} ? \\ 4 \overline{)13} \end{array}$$

$$\left[ \frac{13}{4} \right] = ?$$

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- Put result digit above the dividend in the position indicated.
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$$\begin{array}{r} 3 \\ 4 \overline{)13} \end{array}$$

$$\left[ \frac{13}{4} \right] = \left[ \frac{12}{4} + \frac{1}{4} \right] = \left[ 3 + \frac{1}{4} \right] = 3$$

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- Put result digit above the dividend in the position indicated.

$$\begin{array}{r} 3 \\ 4 \overline{)13} \end{array}$$

- Divide the two-digit integers by single-digit integers by guessing.

- Multiply quotient digit by divisor, put result under current dividend.

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Divide 13 by 4 with quotient and remainder. Use long integer division notation.

- Put result digit above the dividend in the position indicated.

$$\begin{array}{r} & 3 \\ ? \sqrt[4]{13} \\ & ? \end{array}$$

- Divide the two-digit integers by single-digit integers by guessing.
- Multiply quotient digit by divisor, put result under current dividend.

$$3 \cdot 4 = ?$$

# Introducing long integer division

Divide 13 by 4 with quotient and remainder. Use long integer division notation.

- Put result digit above the dividend in the position indicated.

$$\begin{array}{r} 3 \\ 4 \overline{)13} \\ 12 \\ \hline 2 \end{array}$$

- Divide the two-digit integers by single-digit integers by guessing.
- Multiply quotient digit by divisor, put result under current dividend.

$$3 \cdot 4 = 12$$

# Introducing long integer division

Divide 13 by 4 with quotient and remainder. Use long integer division notation.

- Put result digit above the dividend in the position indicated.

$$\begin{array}{r} 3 \\ 4 \overline{)13} \\ \textcolor{red}{?}2 \end{array}$$

- Divide the two-digit integers by single-digit integers by guessing.

- Multiply quotient digit by divisor, put result under current dividend.

$$3 \cdot 4 = \textcolor{red}{12}$$

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- Put result digit above the dividend in the position indicated.

$$\begin{array}{r} 3 \\ 4 \overline{)13} \\ -12 \\ \hline \end{array}$$

- Divide the two-digit integers by single-digit integers by guessing.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.

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Divide 13 by 4 with quotient and remainder. Use long integer division notation.

- Put result digit above the dividend in the position indicated.

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- Divide the two-digit integers by single-digit integers by guessing.
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- Subtract.

$$3 - 2 = ?$$

# Introducing long integer division

Divide 13 by 4 with quotient and remainder. Use long integer division notation.

- Put result digit above the dividend in the position indicated.

$$\begin{array}{r} 3 \\ 4 \overline{)13} \\ -12 \\ \hline 1 \end{array}$$

- Divide the two-digit integers by single-digit integers by guessing.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.

$$3 - 2 = 1$$

# Introducing long integer division

Divide 13 by 4 with quotient and remainder. Use long integer division notation.

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- Put result digit above the dividend in the position indicated.
- Divide the two-digit integers by single-digit integers by guessing.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.

$$\begin{aligned} 3 - 2 &= 1 \\ 1 - 1 &= ? \end{aligned}$$

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$$3 - 2 = 1$$

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- Final answer. **Dividend = 13**. Divisor = 4. Quotient = 3. Remainder = 1.

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- Final answer. **Dividend = 13**. Divisor = 4. Quotient = 3. Remainder = 1.
- Check.  $13 \stackrel{?}{=} 4 \cdot 3 + 1$

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- Final answer. Dividend = 13. **Divisor = 4.**  
Quotient = 3. Remainder = 1.
- Check.  $13 \stackrel{?}{=} 4 \cdot 3 + 1$

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- Final answer. Dividend = 13. Divisor = 4.  
**Quotient = 3. Remainder = 1.**
- Check.  $13 \stackrel{?}{=} 4 \cdot 3 + 1$

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Divide 13 by 4 with quotient and remainder. Use long integer division notation.

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- Put result digit above the dividend in the position indicated.
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- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Final answer. Dividend = 13. Divisor = 4. Quotient = 3. **Remainder = 1**.
- Check.  $13 \stackrel{?}{=} 4 \cdot 3 + 1$

- Division by single-digit divisor is different from division by multiple-digit divisors.
- We teach division by single-digit divisors first.
- We teach then extend to divisors with multiple digits.
- We point out all steps that are different on examples.

Divide 97 by 3 with quotient and remainder.

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$$3 \overline{)97}$$

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{|r} \hline 3 \\ \hline \end{array} \quad \begin{array}{r} 97 \\ \hline \end{array}$$

Divide 97 by 3 with quotient and remainder.

- Find shortest dividend start larger than divisor.

$$\begin{array}{|r} \hline 3 \\ \hline 97 \end{array}$$

Divide 97 by 3 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

$$\begin{array}{r} ? \\ 3 \overline{)97} \end{array}$$

Divide 97 by 3 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit

$$\begin{array}{r} ? \\ 3 \overline{)97} \end{array}$$

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} ? \\ 3 \overline{)97} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

$$\left| \begin{array}{r} 9 \\ \hline 3 \end{array} \right|$$

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} ? \\ 3 \overline{)97} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit **by divisor digit**. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

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Divide 97 by 3 with quotient and remainder.

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- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

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— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

$$\left[ \frac{9}{3} \right] = ?$$

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 97 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

$$\left\lfloor \frac{9}{3} \right\rfloor = \lfloor 3 \rfloor = 3$$

Divide 97 by 3 with quotient and remainder.

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- Multiply quotient digit by divisor, put result under current dividend.

Divide 97 by 3 with quotient and remainder.

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- Multiply quotient digit by divisor, put result under current dividend.

$$3 \cdot 3 = ?$$

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 3 \\ 3 \overline{)97} \\ 9 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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$$\begin{array}{r} 3 \\ \hline 3 | 97 \\ 9 \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend.

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 97 \\ -90 \\ \hline 7 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**

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$$7 - 0 = ?$$

Divide 97 by 3 with quotient and remainder.

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$$7 - 0 = 7$$

$$9 - 9 = ?$$

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$7 - 0 = 7$$

$$\textcolor{red}{9 - 9 = 0}$$

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- Find shortest dividend start larger than divisor.
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- Subtract.

$$7 - 0 = 7$$

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 3? \\ \hline 3 | \overline{97} \\ \hline 90 \\ \hline 7 \end{array}$$

- Find shortest dividend start larger than divisor.
- **Next quotient digit:** put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
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- Subtract.
- Repeat.

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 3? \\ \hline 3 | \overline{97} \\ \hline 90 \\ \hline 7 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

– Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{7}{3} \right] = ?$$

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

– Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{7}{3} \right\rfloor = \left\lfloor \frac{6}{3} + \frac{1}{3} \right\rfloor = \left\lfloor 2 + \frac{1}{3} \right\rfloor = 2$$

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.**
- Subtract.
- Repeat.

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ -6 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$2 \cdot 3 = 6$$

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ -6 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ -6 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.**
- Subtract.
- Repeat.

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ -6 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ -6 \\ \hline ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ -6 \\ \hline 1 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract.**
- **Repeat.**

$$7 - 6 = 1$$

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ -6 \\ \hline 1 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$7 - 6 = 1$$

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ -6 \\ \hline 1 \end{array}$$

- Find shortest dividend start larger than divisor.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -9 \\ \hline 7 \\ -6 \\ \hline 1 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ -6 \\ \hline 1 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
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- Repeat.

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- Final answer. **Dividend = 97**. Divisor = 3. Quotient = 32. Remainder = 1.

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ -6 \\ \hline 1 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
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- Final answer. Dividend = 97. Divisor = 3. Quotient = 32. Remainder = 1.

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- Subtract.
- Repeat.
- Final answer. Dividend = 97. Divisor = 3. Quotient = 32. **Remainder = 1.**

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ -6 \\ \hline 1 \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- Final answer. **Dividend = 97**. Divisor = 3. Quotient = 32. Remainder = 1.
- Check.  $97 \stackrel{?}{=} 3 \cdot 32 + 1$

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ -6 \\ \hline 1 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- Final answer. Dividend = 97. **Divisor = 3**. Quotient = 32. Remainder = 1.
- Check.  $97 \stackrel{?}{=} 3 \cdot 32 + 1$

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ -6 \\ \hline 1 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- Final answer. Dividend = 97. Divisor = 3. Quotient = 32. Remainder = 1.
- Check.  $97 \stackrel{?}{=} 3 \cdot 32 + 1$

Divide 97 by 3 with quotient and remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)97} \\ -90 \\ \hline 7 \\ -6 \\ \hline 1 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- Final answer. Dividend = 97. Divisor = 3. Quotient = 32. **Remainder = 1**.
- Check.  $97 \stackrel{?}{=} 3 \cdot 32 + 1$

Divide 476 by 9 with quotient and remainder.

Divide **476** by 9 with quotient and remainder.

$$\begin{array}{r} \boxed{476} \\ \hline 9 \end{array}$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} \boxed{9} \sqrt{476} \\ \end{array}$$

Divide 476 by 9 with quotient and remainder.

- Find shortest dividend start larger than divisor.

$$\begin{array}{r} \boxed{476} \\ \hline 9 \end{array}$$

Divide 476 by 9 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

$$\begin{array}{r} ? \\ 9 \overline{)476} \end{array}$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} ? \\ 9 \overline{)476} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} ? \\ 9 \overline{)476} \end{array}$$

- Find shortest dividend start larger than divisor.

- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

$$\left| \begin{array}{r} 47 \\ \hline 9 \end{array} \right|$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} ? \\ 9 \overline{)476} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits **by divisor digit**. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

$$\left| \begin{array}{r} 47 \\ \hline 9 \end{array} \right|$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} ? \\ 9 \overline{)476} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. **Round down if needed.**

$$\left| \begin{array}{r} 47 \\ \hline 9 \end{array} \right|$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} ? \\ 9 \overline{)476} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

$$\left\lfloor \frac{47}{9} \right\rfloor = ?$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 9 \overline{)476} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

$$\left\lfloor \frac{47}{9} \right\rfloor = \left\lfloor \frac{45}{9} + \frac{2}{9} \right\rfloor = \left\lfloor 5 + \frac{2}{9} \right\rfloor = 5$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 9 \overline{)476} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} & 5 \\ ? & \sqrt{ } \\ 9 & | 476 \\ & ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$5 \cdot 9 = ?$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} & 5 \\ 4 \sqrt{ } & 476 \\ 9 & \\ & 5 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$5 \cdot 9 = 45$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 4 \overline{)9 \ 476} \\ \underline{-36} \\ 116 \\ \underline{-90} \\ 26 \\ \underline{-27} \\ -1 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$5 \cdot 9 = 45$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 4 \overline{)476} \\ 45 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$5 \cdot 9 = 45$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 4 \overline{)9 \ 476} \\ 45 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$5 \cdot 9 = 45$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 9 \overline{)476} \\ 45 \\ \hline 16 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 9 \overline{)476} \\ 45 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 9 \overline{)476} \\ 45 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**

## Long division

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 9 \overline{)476} \\ -45 \\ \hline 26 \\ -27 \\ \hline -1 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 9 \overline{)476} \\ -450 \\ \hline ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$6 - 0 = ?$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 9 \overline{)476} \\ -450 \\ \hline 6 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$6 - 0 = 6$$

## Long division

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 9 \overline{)476} \\ -450 \\ \hline ?6 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$6 - 0 = 6$$

$$7 - 5 = ?$$

## Long division

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$6 - 0 = 6$$

$$7 - 5 = 2$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 6 - 0 &= 6 \\ 7 - 5 &= 2 \\ 4 - 4 &= ? \end{aligned}$$

## Long division

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.

- Subtract.

$$\begin{aligned} 6 - 0 &= 6 \\ 7 - 5 &= 2 \\ 4 - 4 &= 0 \end{aligned}$$

## Long division

Divide 476 by 9 with quotient and remainder.

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{array}{r} 6 - 0 = 6 \\ 7 - 5 = 2 \\ 4 - 4 = 0 \end{array}$$

Divide 476 by 9 with quotient and remainder.

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- Subtract.
- Repeat.

## Long division

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- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

## Long division

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5? \\ 9 \overline{)476} \\ -450 \\ \hline 26 \end{array}$$

- Find shortest dividend start larger than divisor.
- **Next quotient digit:** put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

– Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 5? \\ 9 \overline{)476} \\ -450 \\ \hline 26 \end{array}$$

- Find shortest dividend start larger than divisor.

- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

– Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.

- Subtract.

- Repeat.

$$\left[ \frac{26}{9} \right] = ?$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{26}{9} \right\rfloor = \left\lfloor \frac{18}{9} + \frac{8}{9} \right\rfloor = \left\lfloor 2 + \frac{8}{9} \right\rfloor = 2$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- **Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.**
- Subtract.
- Repeat.

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$2 \cdot 9 = 18$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- **Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.**
- Subtract.
- Repeat.

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

## Long division

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline -1 \\ -\quad 26 \\ -\quad 18 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.

- Subtract.
- Repeat.

$$\begin{array}{r} 6 - 8 = -2 = -10 + 8 \\ -1 + 2 - 1 = 0 \end{array}$$

## Long division

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{array}{r} 6 - 8 = -2 = -10 + 8 \\ -1 + 2 - 1 = 0 \end{array}$$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

## Long division

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- Final answer. **Dividend = 476**. Divisor = 9. Quotient = 52. Remainder = 8.

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- Final answer. Dividend = 476. **Divisor = 9**. Quotient = 52. Remainder = 8.

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- Final answer. Dividend = 476. Divisor = 9. Quotient = 52. Remainder = 8.

## Long division

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- Final answer. Dividend = 476. Divisor = 9. Quotient = 52. **Remainder = 8.**

## Long division

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- Final answer. **Dividend = 476**. Divisor = 9. Quotient = 52. Remainder = 8.
- Check.  $476 \stackrel{?}{=} 9 \cdot 52 + 8$

## Long division

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- Final answer. Dividend = 476. **Divisor = 9**. Quotient = 52. Remainder = 8.
- Check.  $476 \stackrel{?}{=} 9 \cdot 52 + 8$

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- Final answer. Dividend = 476. Divisor = 9. Quotient = 52. Remainder = 8.
- Check.  $476 \stackrel{?}{=} 9 \cdot 52 + 8$

## Long division

Divide 476 by 9 with quotient and remainder.

$$\begin{array}{r} 52 \\ 9 \overline{)476} \\ -450 \\ \hline 26 \\ -18 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- Final answer. Dividend = 476. Divisor = 9. Quotient = 52. **Remainder = 8.**
- Check.  $476 \stackrel{?}{=} 9 \cdot 52 + 8$

Divide 8375 by 4 with quotient and remainder.

Divide **8375** by 4 with quotient and remainder.

$$\begin{array}{r} \boxed{8375} \\ \hline 4 | \end{array}$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} \boxed{4} \quad 8375 \\ \end{array}$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.

$$\begin{array}{r} & \boxed{8375} \\ 4 | & \end{array}$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} ? \\ 4 \overline{)8375} \end{array}$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} ? \\ 4 \overline{)8375} \end{array}$$

— Current leading digit > divisor leading digit

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} ? \\ \hline 4 | \quad 8375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  **divide leading digit** by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

$$\left\lfloor \frac{8}{4} \right\rfloor$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} ? \\ \hline 4 | 8375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit **by divisor digit**. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

$$\left[ \begin{array}{r} 8 \\ \hline 4 \end{array} \right]$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} ? \\ \hline 4 | 8375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. **Round down if needed.**

$$\left[ \begin{array}{r} 8 \\ \hline 4 \end{array} \right]$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} ? \\ \hline 4 | 8375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

$$\left[ \frac{8}{4} \right] = ?$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ \hline 4 | 8375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

$$\left\lfloor \frac{8}{4} \right\rfloor = \lfloor 2 \rfloor = 2$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2 \\ \hline 4 | 8375 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ 4 \overline{)8375} \\ ? \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

$$2 \cdot 4 = ?$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ 4 \overline{)8375} \\ 8 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

$$2 \cdot 4 = 8$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2 \\ \hline 4 | 8375 \\ 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

$$2 \cdot 4 = 8$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2 \\ \hline 4 | 8375 \\ 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2 \\ \hline 4 | 8375 \\ 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2 \\ \hline 4 | 8375 \\ 8000 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ 4 \overline{)8375} \\ \underline{8000} \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ 4 \overline{)8375} \\ -8000 \\ \hline ? \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$5 - 0 = ?$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ 4 \overline{)8375} \\ -8000 \\ \hline 5 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$5 - 0 = 5$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ 4 \overline{)8375} \\ -8000 \\ \hline 35 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$5 - 0 = 5$$

$$7 - 0 = ?$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ 4 \overline{)8375} \\ -8000 \\ \hline 75 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$5 - 0 = 5$$

$$7 - 0 = 7$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 5 - 0 &= 5 \\ 7 - 0 &= 7 \\ 3 - 0 &= ? \end{aligned}$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 5 - 0 &= 5 \\ 7 - 0 &= 7 \\ 3 - 0 &= 3 \end{aligned}$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{array}{r} 5 - 0 = 5 \\ 7 - 0 = 7 \\ 3 - 0 = 3 \\ 8 - 8 = ? \end{array}$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{array}{r} 5 - 0 = 5 \\ 7 - 0 = 7 \\ 3 - 0 = 3 \\ 8 - 8 = 0 \end{array}$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 5 - 0 &= 5 \\ 7 - 0 &= 7 \\ 3 - 0 &= 3 \\ 8 - 8 &= 0 \end{aligned}$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ \hline 4 | \quad 8375 \\ \quad 8000 \\ \hline \quad 375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 5 - 0 &= 5 \\ 7 - 0 &= 7 \\ 3 - 0 &= 3 \\ 8 - 8 &= 0 \end{aligned}$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{array}{r} 5 - 0 = 5 \\ 7 - 0 = 7 \\ 3 - 0 = 3 \\ 8 - 8 = 0 \end{array}$$

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 2 \ ? \\ \hline 4 | 8375 \\ \quad - 8000 \\ \hline \quad \quad 375 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

## Long division

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 20? \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. **If this leaves gaps: fill with 0.**

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 20? \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 20? \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

- Repeat.

$$\left[ \frac{37}{4} \right]$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 20? \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits **by divisor digit**. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{37}{4} \right]$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 20? \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. **Round down if needed.**

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \begin{array}{r} 37 \\ \hline 4 \end{array} \right]$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 20? \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{37}{4} \right] = ?$$

## Long division

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209 \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{37}{4} \right\rfloor = \left\lfloor \frac{36}{4} + \frac{1}{4} \right\rfloor = \left\lfloor 9 + \frac{1}{4} \right\rfloor = 9$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209 \\ 3 \overline{)8375} \\ -8000 \\ \hline 375 \\ -36 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.

- Subtract.

- Repeat.

$$9 \cdot 4 = 36$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -36 \\ \hline 1 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -36 \\ \hline 9 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.

- Subtract.

- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
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- Subtract.
- Repeat.

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$$\begin{array}{r} 209 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.

- Subtract.

- Repeat.

$$\begin{aligned} 5 - 0 &= 5 \\ 7 - 6 &= 1 \\ 3 - 3 &= 0 \end{aligned}$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.

- Subtract.

- Repeat.

$$\begin{aligned} 5 - 0 &= 5 \\ 7 - 6 &= 1 \\ 3 - 3 &= 0 \end{aligned}$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
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- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209? \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \\ - 360 \\ \hline 15 \end{array}$$

- Find shortest dividend start larger than divisor.
- **Next quotient digit: put above last digit of that start.** If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209? \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \\ - 360 \\ \hline 15 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 209? \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{15}{4} \right] = ?$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2093 \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \\ - 360 \\ \hline 15 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{15}{4} \right\rfloor = \left\lfloor \frac{12}{4} + \frac{3}{4} \right\rfloor = \left\lfloor 3 + \frac{3}{4} \right\rfloor = 3$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2093 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \\ ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.

- Subtract.

- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2093 \\ 4 \overline{)8375} \\ 8000 \\ \hline 375 \\ -360 \\ \hline 15 \\ 12 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

- Repeat.

$$3 \cdot 4 = 12$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2093 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \\ -12 \\ \hline 3 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2093 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \\ -12 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2093 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \\ -12 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2093 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \\ -12 \\ \hline ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2093 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \\ -12 \\ \hline 3 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.

- Subtract.

- Repeat.

$$5 - 2 = 3$$

$$1 - 1 = 0$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2093 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \\ -12 \\ \hline 3 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

- Repeat.

$$5 - 2 = 3$$

$$1 - 1 = 0$$

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2093 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \\ -12 \\ \hline 3 \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2093 \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \\ - 360 \\ \hline 15 \\ - 12 \\ \hline 3 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2093 \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \\ - 360 \\ \hline 15 \\ - 12 \\ \hline 3 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit. Unlike division by multi-digit divisor, don't add one to divisor leading digit. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2093 \\ 4 \overline{)8375} \\ -8000 \\ \hline 375 \\ -360 \\ \hline 15 \\ -12 \\ \hline 3 \end{array}$$

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- Final answer. **Dividend = 8375**. Divisor = 4. Quotient = 2093. Remainder = 3.

Divide 8375 by 4 with quotient and remainder.

$$\begin{array}{r} 2093 \\ \hline 4 | 8375 \\ - 8000 \\ \hline 375 \\ - 360 \\ \hline 15 \\ - 12 \\ \hline 3 \end{array}$$

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- We introduce multiple-digit divisors.
- The multi-digit division algorithm is similar to the one-digit divisor, with some differences.
- The multi-digit division uses the leading digit of the divisor plus one, whereas the single-digit division uses leading digit as is.
- An extra step to collect the quotient digits is needed.
- In the division step, there are three major cases:
  - ① Leading digit of remaining dividend is larger than leading digit of divisor.
  - ② Leading digits of remaining dividend equals the divisor leading digit and the start of the dividend is larger than the divisor.
  - ③ The remaining case: either:
    - leading digit of remaining dividend equals the divisor leading digit, but the dividend start is smaller than the divisor
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# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

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$$\begin{array}{r} & \boxed{\phantom{0}} \\ 12 | & 2 \ 00 \end{array}$$

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Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} & \boxed{12} \\ & | \\ 2 & 00 \end{array}$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

- Find shortest dividend start larger than divisor.

$$\begin{array}{r} & \\ 12 & \overline{)2\ 00} \end{array}$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 12 | & 2 & 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 12 \overline{)2 \ 00} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 12 \overline{)2 \ 00} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  **divide leading digit** by divisor digit plus one. Round down if needed.

$$\left[ \frac{2}{1+1} \right]$$

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Divide 200 by 12 with quotient and remainder.

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- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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$$\left\lfloor \frac{2}{1+1} \right\rfloor$$

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$$\left[ \frac{2}{1+1} \right] = ?$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 12 | & 2 \ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

$$\left\lfloor \frac{2}{1+1} \right\rfloor = \left\lfloor \frac{2}{2} \right\rfloor = \lfloor 1 \rfloor = 1$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

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Divide 200 by 12 with quotient and remainder.

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- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 12 \overline{)2\ 00} \\ - \underline{1\ 20} \end{array}$$

- Find shortest dividend start larger than divisor.
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$$0 - 2 = ?$$

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$$\begin{aligned} 0 - 0 &= 0 \\ 0 - 2 &= -2 \end{aligned}$$

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# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline 80 \end{array}$$

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- Subtract.
- Repeat.

$$\left[ \frac{8}{1+1} \right] = ?$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 14 \\ 12 \overline{)2\ 00} \\ - \underline{1\ 2} \\ \phantom{1}20 \\ - \underline{8} \\ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{8}{1+1} \right\rfloor = \left\lfloor \frac{8}{2} \right\rfloor = \lfloor 4 \rfloor = 4$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline 80 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline 80 \\ ? \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$4 \cdot 2 = ?$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 14 \\ 12 \overline{)200} \\ -12 \\ \hline 80 \\ -80 \\ \hline 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$4 \cdot 2 = 8$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 14 \\ \hline 12 \overline{)2\ 00} \\ - \underline{1\ 2} \\ \hline 80 \\ \hline ?8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$4 \cdot 2 = 8$$

$$4 \cdot 1 = ?$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 14 \\ \hline 12 \overline{)2\ 00} \\ - \underline{1\ 2} \\ \hline 80 \\ \hline 48 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$4 \cdot 2 = 8$$

$$4 \cdot 1 = 4$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 14 \\ \hline 12 \overline{)2\ 00} \\ - \underline{1\ 2} \\ \hline 80 \\ - \underline{48} \\ \hline \end{array}$$

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$$4 \cdot 2 = 8$$

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$$\begin{array}{r} 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline 80 \\ 48 \end{array}$$

- Find shortest dividend start larger than divisor.
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  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 14 \\ \hline 12 | \overline{2 \ 00} \\ - \quad \underline{1 \ 20} \\ \hline - \quad \underline{\color{red}{80}} \\ - \quad \underline{\color{red}{48}} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 14 \\ \hline 12 | \overline{2 \ 00} \\ - \quad 1 \ 20 \\ \hline - \quad \quad \quad \color{red}{80} \\ - \quad \quad \quad \color{red}{48} \\ \hline \quad \quad \quad \color{red}{??} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline - \ 1\ 8 \\ - \ 48 \\ \hline 32 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 0 - 8 &= -8 = -10 + 2 \\ -1 + 8 - 4 &= 3 \end{aligned}$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline -\ 1\ 8 \\ - \ 48 \\ \hline 32 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
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- Subtract.
- Repeat.

$$\begin{aligned} 0 - 8 &= -8 = -10 + 2 \\ -1 + 8 - 4 &= 3 \end{aligned}$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 14 \\ \hline 12 | \overline{2 \ 00} \\ - \quad 1 \ 20 \\ \hline - \quad \quad 80 \\ - \quad \quad 48 \\ \hline \quad \quad \quad 32 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
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- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline - \ 80 \\ - \ 48 \\ \hline 32 \end{array}$$

- Find shortest dividend start larger than divisor.
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  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
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- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 14 \\ \hline 12 | \overline{2 \ 00} \\ - \ 1 \ 20 \\ \hline - \ 80 \\ - \ 48 \\ \hline 32 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} & ? \\ 14 & \\ \hline 12 & \left[ \begin{array}{r} 2 \ 00 \\ - 1 \ 20 \\ \hline 80 \\ - 48 \\ \hline 32 \end{array} \right] \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
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- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} ? \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline - \ 80 \\ - \ 48 \\ \hline 32 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{3}{1+1} \right] = ?$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \quad 1\ 20 \\ \hline - \quad \quad 80 \\ - \quad \quad 48 \\ \hline \quad \quad \quad 32 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{3}{1+1} \right] = \left[ \frac{3}{2} \right] = \left[ \frac{2}{2} + \frac{1}{2} \right] = \left[ 1 + \frac{1}{2} \right] = 1$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline - \ 80 \\ - \ 48 \\ \hline 32 \\ ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline - \ 80 \\ - \ 48 \\ \hline 32 \\ 12 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline \ 80 \\ - \ 48 \\ \hline \ 32 \\ - \ 12 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
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Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \underline{1\ 2} \\ \hline 80 \\ - \underline{48} \\ \hline 32 \\ - \underline{12} \\ \hline \end{array}$$

??

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline 80 \\ - \ 48 \\ \hline 32 \\ - \ 12 \\ \hline 20 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract.**
- **Repeat.**

$$\begin{aligned} 2 - 2 &= 0 \\ 3 - 1 &= 2 \end{aligned}$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline 80 \\ - \ 48 \\ \hline 32 \\ - \ 12 \\ \hline 20 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$2 - 2 = 0$$

$$3 - 1 = 2$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 14 \\ \hline 12 | & 2 \ 00 \\ - & 1 \ 20 \\ \hline & 80 \\ - & 48 \\ \hline & 32 \\ - & 12 \\ \hline & 20 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 14 \\ \hline 12 | & 2 \ 00 \\ - & 1 \ 20 \\ \hline & 80 \\ - & 48 \\ \hline & 32 \\ - & 12 \\ \hline & 20 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 14 \\ \hline 12 | & 2 \ 00 \\ - & 1 \ 20 \\ \hline & 80 \\ - & 48 \\ \hline & 32 \\ - & 12 \\ \hline & 20 \end{array}$$

- Find shortest dividend start larger than divisor.

- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.

- Subtract.

- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline \ 80 \\ - \ 48 \\ \hline \ 32 \\ - \ 12 \\ \hline \end{array}$$

20

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline 80 \\ - \ 48 \\ \hline 32 \\ - \ 12 \\ \hline 20 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{2}{1+1} \right] = ?$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline 1\ 20 \\ - \ 1\ 20 \\ \hline 0 \\ \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{2}{1+1} \right\rfloor = \left\lfloor \frac{2}{2} \right\rfloor = [1] = 1$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ \hline 14 \\ \hline 12 \left[ \begin{array}{r} 2 \ 00 \\ - 1 \ 20 \\ \hline 80 \\ - 48 \\ \hline 32 \\ - 12 \\ \hline 20 \\ ?? \end{array} \right] \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ \hline 14 \\ \hline 12 \left[ \begin{array}{r} 2 \ 00 \\ - 1 \ 20 \\ \hline 80 \\ - 48 \\ \hline 32 \\ - 12 \\ \hline 20 \\ \hline 12 \end{array} \right] \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline \ 80 \\ - \ 48 \\ \hline \ 32 \\ - \ 12 \\ \hline \ 20 \\ - \ 12 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
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# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline \ 80 \\ - \ 48 \\ \hline \ 32 \\ - \ 12 \\ \hline \ 20 \\ - \ 12 \\ \hline ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline \ 80 \\ - \ 48 \\ \hline \ 32 \\ - \ 12 \\ \hline -1 \\ - \ 20 \\ \hline -12 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract**.
- **Repeat**.

$$\begin{aligned} 0 - 2 &= -2 = -10 + 8 \\ -1 + 2 - 1 &= 0 \end{aligned}$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline \ 80 \\ - \ 48 \\ \hline \ 32 \\ - \ 12 \\ \hline \textcolor{red}{-1} \\ - \ 20 \\ \hline \ 12 \\ \hline \ 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 0 - 2 &= -2 = -10 + 8 \\ -1 + 2 - 1 &= 0 \end{aligned}$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline \ 80 \\ - \ 48 \\ \hline \ 32 \\ - \ 12 \\ \hline \ 20 \\ - \ 12 \\ \hline \ 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 14 \\ \hline 12 \overline{)2\ 00} \\ - \ 1\ 20 \\ \hline \ 80 \\ - \ 48 \\ \hline \ 32 \\ - \ 12 \\ \hline \ 20 \\ - \ 12 \\ \hline \ 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 1 \\ & 1 \\ 12 & \overline{)14} \\ - & 12 \\ \hline & 20 \\ - & 24 \\ \hline & 80 \\ - & 48 \\ \hline & 32 \\ - & 24 \\ \hline & 8 \\ \end{array}$$

- Find shortest dividend start larger than divisor. **If none, we're done.**
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ \hline & 14 \\ \hline 12 | & 2 \ 00 \\ - & 1 \ 20 \\ \hline & 80 \\ - & 48 \\ \hline & 32 \\ - & 12 \\ \hline & 20 \\ - & 12 \\ \hline & 8 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ \hline & 14 \\ - & 12 \\ \hline & 6 \\ 12 | & 2 \ 00 \\ - & 1 \ 20 \\ \hline & 80 \\ - & 48 \\ \hline & 32 \\ - & 12 \\ \hline & 20 \\ - & 12 \\ \hline & 8 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

$$4 + 1 + 1 = 6$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ \hline & 14 \\ - & 16 \\ \hline & 16 \\ 12 | & 2 \ 00 \\ - & 1 \ 20 \\ \hline & 80 \\ - & 48 \\ \hline & 32 \\ - & 12 \\ \hline & 20 \\ - & 12 \\ \hline & 8 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

$$\begin{aligned} 4 + 1 + 1 &= 6 \\ 1 &= 1 \end{aligned}$$

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ \hline & 14 \\ - & 12 \\ \hline & 16 \\ \boxed{12} & | \quad \textcolor{red}{2 \ 00} \\ - & 1 \ 20 \\ \hline & 80 \\ - & 48 \\ \hline & 32 \\ - & 12 \\ \hline & 20 \\ - & 12 \\ \hline & 8 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. **Dividend = 200**. Divisor = 12. Quotient = 16. Remainder = 8.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ \hline 14 & \\ \hline 16 & \\ \boxed{12} & \left[ \begin{array}{r} 2\ 00 \\ - 1\ 20 \\ \hline 80 \\ - 48 \\ \hline 32 \\ - 12 \\ \hline 20 \\ - 12 \\ \hline 8 \end{array} \right] \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 200. **Divisor = 12**. Quotient = 16. Remainder = 8.

# Divisor leading digit small relative to dividend

Divide 200 by 12 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ \hline & 14 \\ - & \underline{16} \\ \hline 12 | & 2 \ 00 \\ - & 1 \ 20 \\ \hline & 80 \\ - & 48 \\ \hline & 32 \\ - & 12 \\ \hline & 20 \\ - & 12 \\ \hline & 8 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 200. Divisor = 12. **Quotient = 16**. Remainder = 8.

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- Check.  $200 \stackrel{?}{=} 12 \cdot 16 + 8$

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# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

$$\begin{array}{r} \boxed{200} \\ \hline 77 \end{array}$$

# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

$$\begin{array}{r} \boxed{77} \\ \sqrt{200} \end{array}$$

# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

- Find shortest dividend start larger than divisor.

$$\begin{array}{r} 200 \\ \hline 77 \end{array}$$

# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

- Find shortest dividend start larger than divisor.

- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

$$\begin{array}{r} ? \\ 77 \overline{)200} \end{array}$$

# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits

$$\begin{array}{r} ? \\ \hline 77 \overline{)200} \end{array}$$

# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

- Find shortest dividend start larger than divisor.

- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one.

Round down if needed.

$$\begin{array}{r} ? \\ 77 \overline{)200} \end{array}$$

$$\left[ \frac{20}{7+1} \right]$$

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Round down if needed.

$$\begin{array}{r} ? \\ 77 \overline{)200} \end{array}$$

$$\left\lfloor \frac{20}{7+1} \right\rfloor$$

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Round down if needed.

$$\begin{array}{r} ? \\ \hline 77 \overline{)200} \end{array}$$

$$\left[ \begin{array}{r} 20 \\ \hline 7+1 \end{array} \right]$$

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- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one.

Round down if needed.

$$\begin{array}{r} ? \\ 77 \overline{)200} \end{array}$$

$$\left[ \frac{20}{7+1} \right] = ?$$

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Round down if needed.

$$\begin{array}{r} & 2 \\ 77 \sqrt{ } & 200 \end{array}$$

$$\left\lfloor \frac{20}{7+1} \right\rfloor = \left\lfloor \frac{20}{8} \right\rfloor = \left\lfloor \frac{16}{8} + \frac{4}{8} \right\rfloor = \left\lfloor 2 + \frac{4}{8} \right\rfloor = 2$$

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  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one.
- Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.**

# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

- Find shortest dividend start larger than divisor.

$$\begin{array}{r} & 2 \\ ? & \overline{)77} \\ 200 & \\ ? & \end{array}$$

- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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- Multiply quotient digit by divisor, put result under current dividend.

$$2 \cdot 7 = ?$$

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- Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{array}{r} & 2 \\ \overline{7} \Big) & 200 \\ & 4 \end{array}$$

$$2 \cdot 7 = 14$$

# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

- Find shortest dividend start larger than divisor.

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- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{array}{r} & 2 \\ 77 & \overline{)200} \\ & 4 \end{array}$$

$$2 \cdot 7 = 14$$

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# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

- Find shortest dividend start larger than divisor.

$$\begin{array}{r} & 2 \\ \overline{77} \Big) & 200 \\ & ?4 \end{array}$$

- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one.
- Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{aligned} 2 \cdot 7 &= 14 \\ 2 \cdot 7 + 1 &= ? \end{aligned}$$

# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

- Find shortest dividend start larger than divisor.

$$\begin{array}{r} & 2 \\ \overline{77} \Big) & 200 \\ & 54 \end{array}$$

- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one.
- Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{aligned} 2 \cdot 7 &= 14 \\ 2 \cdot 7 + 1 &= 15 \end{aligned}$$

# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

- Find shortest dividend start larger than divisor.

$$\begin{array}{r} & 2 \\ \overline{77} \Big) & 200 \\ & \textcolor{red}{?} 54 \end{array}$$

- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one.
- Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{aligned} 2 \cdot 7 &= 14 \\ 2 \cdot 7 + 1 &= \textcolor{red}{15} \end{aligned}$$

# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

- Find shortest dividend start larger than divisor.

$$\begin{array}{r} & 2 \\ \overline{77} \Big) & 200 \\ & 154 \end{array}$$

- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one.
- Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{aligned} 2 \cdot 7 &= 14 \\ 2 \cdot 7 + 1 &= 15 \end{aligned}$$

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Divide 200 by 77 with quotient and remainder.

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Round down if needed.

- **Multiply quotient digit by divisor, put result under current dividend.**

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Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

- **Subtract.**

$$\begin{array}{r} & 2 \\ 77 \sqrt{)200} \\ -154 \\ \hline \end{array}$$

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$$\begin{array}{r} & 2 \\ 77 & \overline{)200} \\ -154 & \hline \end{array}$$

$$0 - 4 = ?$$

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$$\begin{array}{r} & 2 \\ 77 & \overline{)200} \\ -154 & \hline ? \end{array}$$

$$0 - 4 = -4 = -10 + ?$$

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- Subtract.

$$\begin{array}{r} & 2 \\ 77 & \overline{)200} \\ - & 154 \\ \hline & 6 \end{array}$$

$$0 - 4 = -4 = -10 + 6$$

# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

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$$\begin{aligned} 0 - 4 &= -4 = -10 + 6 \\ -1 + 0 - 5 &= ? \end{aligned}$$

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- Subtract.

$$\begin{array}{r} & 2 \\ 77 \sqrt{)200} & -1-1 \\ & \underline{-154} \\ & \underline{\underline{?6}} \end{array}$$

$$0 - 4 = -4 = -10 + 6$$

$$-1 + 0 - 5 = -6 = -10 + ?$$

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- Subtract.

$$\begin{array}{r} & 2 \\ 77 \sqrt{)200} & -1-1 \\ & \underline{-154} \\ & \underline{\underline{46}} \end{array}$$

$$0 - 4 = -4 = -10 + 6$$

$$-1 + 0 - 5 = -6 = -10 + 4$$

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- Subtract.

$$\begin{array}{r} & 2 \\ 77 & \overline{)200} \\ -154 & \hline 46 \end{array}$$

$$\begin{aligned}
 0 - 4 &= -4 = -10 + 6 \\
 -1 + 0 - 5 &= -6 = -10 + 4 \\
 -1 + 2 - 1 &= ?
 \end{aligned}$$

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$$0 - 4 = -4 = -10 + 6$$

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$$\textcolor{red}{-1 + 2 - 1 = 0}$$

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- Subtract.

- Repeat.

$$\begin{array}{r} & 2 \\ 77 \sqrt{)200} & -154 \\ \hline & 46 \end{array}$$

# Divisor leading digit large relative to dividend

Divide 200 by 77 with quotient and remainder.

$$\begin{array}{r} & 2 \\ 77 \sqrt{)200} \\ -154 \\ \hline 46 \end{array}$$

- Find shortest dividend start larger than divisor.  
**If none, we're done.**
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one.
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- Final answer. **Dividend = 200**. Divisor = 77.  
Quotient = 2. Remainder = 46.

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Divide 3600 by 33 with quotient and remainder.

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$$\begin{array}{r} \boxed{3600} \\ \hline 33 \end{array}$$

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Divide 3600 by 33 with quotient and remainder.

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— Equal leading digits;

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— Equal leading digits; **larger divisor start has same # of digits**

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$$\begin{array}{r} 1 \\ \hline 33 \overline{)3\ 6\ 00} \end{array}$$

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$$\begin{array}{r} & 1 \\ \overline{)33} & \quad 3 \ 6 \ 00 \\ & ? \ ? \end{array}$$

- Multiply quotient digit by divisor, put result under current dividend.

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1 0?

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$$\left[ \frac{30}{3+1} \right]$$

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$$\left[ \frac{30}{3+1} \right] = ?$$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \text{ } 0\textcolor{red}{7} \\ \hline 33 \overline{)3 \text{ } 6 \text{ } 00} \\ -3 \text{ } 3 \text{ } 00 \\ \hline 3 \text{ } 00 \end{array}$$

- Find shortest dividend start larger than divisor.
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- Subtract.
- Repeat.

$$\left\lfloor \frac{30}{3+1} \right\rfloor = \left\lfloor \frac{30}{4} \right\rfloor = \left\lfloor \frac{28}{4} + \frac{2}{4} \right\rfloor = \left\lfloor 7 + \frac{2}{4} \right\rfloor = 7$$

# Dividend and divisor leading digits equal

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- Find shortest dividend start larger than divisor.
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$$\begin{array}{r} 1 \ 07 \\ \hline 33 \overline{)3\ 6\ 00} \\ -3\ 3\ 00 \\ \hline 3\ 00 \\ \quad ? \end{array}$$

- Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
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$$7 \cdot 3 = ?$$

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$$7 \cdot 3 = 21$$

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# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \text{ } 0\textcolor{red}{7} \\ \hline 33 \overline{)3\text{ } 6\text{ } 00} \\ -3\text{ } 3\text{ } 00 \\ \hline 3\text{ } 00 \\ \quad \quad \quad \textcolor{red}{?}1 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{array}{r} 7 \cdot 3 = 21 \\ 7 \cdot 3 + 2 = ? \end{array}$$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \ 07 \\ \hline 33 \overline{)3\ 6\ 00} \\ -3\ 3\ 00 \\ \hline 3\ 00 \\ \hline 31 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
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- Repeat.

$$\begin{aligned} 7 \cdot 3 &= 21 \\ 7 \cdot 3 + 2 &= 23 \end{aligned}$$

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$$\begin{array}{r} 1 \ 07 \\ \hline 33 \overline{)3\ 6\ 00} \\ -3\ 3\ 00 \\ \hline 3\ 00 \\ \textcolor{red}{?}\ \textcolor{red}{31} \end{array}$$

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$$\begin{aligned} 7 \cdot 3 &= 21 \\ 7 \cdot 3 + 2 &= \textcolor{red}{23} \end{aligned}$$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \ 07 \\ \hline 22 \overline{)3\ 6\ 00} \\ 33 \overline{-} 3\ 3\ 00 \\ \hline 3\ 00 \\ 2\ 31 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

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# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \ 07 \\ \hline 33 \overline{)3\ 6\ 00} \\ -3\ 3\ 00 \\ \hline \end{array}$$

$$\begin{array}{r} 3\ 00 \\ 2\ 31 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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$$\begin{array}{r} 7 \cdot 3 = 21 \\ 7 \cdot 3 + 2 = 23 \\ \hline \end{array}$$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \ 07 \\ \hline 33 | \overline{3 \ 6 \ 00} \\ -3 \ 3 \ 00 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \ 00 \\ 2 \ 31 \\ \hline \end{array}$$

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$$\begin{array}{r} 1 \ 07 \\ \hline 33 \overline{)3\ 6\ 00} \\ -3\ 3\ 00 \\ \hline \end{array}$$

$\begin{array}{r} 3\ 00 \\ -2\ 31 \\ \hline \end{array}$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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- Repeat.

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# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r}
 1 \phantom{0}7 \\
 \hline
 33 \overline{)3 \phantom{0}6 \phantom{0}00} \\
 -3 \phantom{3}3 \phantom{0}00 \\
 \hline
 \phantom{-}1\phantom{1}1 \\
 -\phantom{1}3 \phantom{0}00 \\
 \hline
 \phantom{-}2\phantom{1}31 \\
 -\phantom{2}69 \\
 \hline
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 0 - 1 &= -1 = -10 + 9 \\
 -1 + 0 - 3 &= -4 = -10 + 6 \\
 -1 + 3 - 2 &= 0
 \end{aligned}$$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r}
 1 \ 07 \\
 \hline
 33 \overline{)3\ 6\ 00} \\
 -3\ 3\ 00 \\
 \hline
 \textcolor{red}{-1-1} \\
 -3\ 00 \\
 \hline
 2\ 31 \\
 \hline
 69
 \end{array}$$

- Find shortest dividend start larger than divisor.
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Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \ 07 \\ \hline 33 \overline{)3\ 6\ 00} \\ -3\ 3\ 00 \\ \hline -\ 3\ 00 \\ -\ 2\ 31 \\ \hline 69 \end{array}$$

- Find shortest dividend start larger than divisor.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
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# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \ 07 \\ \hline 33 | \overline{3 \ 6 \ 00} \\ -3 \ 3 \ 00 \\ \hline \end{array}$$
$$\begin{array}{r} 3 \ 00 \\ -2 \ 31 \\ \hline 69 \end{array}$$

- Find shortest dividend start larger than divisor.
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- Repeat.

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Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \ 07 \\ \hline 33 | \overline{3 \ 6 \ 00} \\ -3 \ 3 \ 00 \\ \hline \quad 3 \ 00 \\ -\quad 2 \ 31 \\ \hline \quad \quad 69 \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
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# Dividend and divisor leading digits equal

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$$\begin{array}{r} & \boxed{?} \\ 1 & 07 \\ \hline 33 | & 3 \ 6 \ 00 \\ & - 3 \ 3 \ 00 \\ \hline & \quad 3 \ 00 \\ & - \quad 2 \ 31 \\ \hline & \quad \quad 69 \end{array}$$

- Find shortest dividend start larger than divisor.
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  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} & ? \\ 1 & 07 \\ \hline 33 | & 3 & 6 & 00 \\ & -3 & 3 & 00 \\ \hline & 3 & 00 \\ & -2 & 31 \\ \hline & & 69 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - **Current leading digit > divisor leading digit**
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \quad 07 \\ \hline 33 \overline{)3 \quad 6 \quad 00} \\ -3 \quad 3 \quad 00 \\ \hline - \quad 3 \quad 00 \\ - \quad 2 \quad 31 \\ \hline \quad \quad \quad 69 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  **divide leading digit** by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{6}{3+1} \right]$$

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 \hline
 33 | & 3 6 00 \\
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 \hline
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 \hline
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 \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{6}{3+1} \right\rfloor = \left\lfloor \frac{6}{4} \right\rfloor = \left\lfloor \frac{4}{4} + \frac{2}{4} \right\rfloor = \left\lfloor 1 + \frac{2}{4} \right\rfloor = 1$$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 33) & \overline{3\ 6\ 00} \\ -3\ 3 & \\ \hline & 3\ 00 \\ -2\ 31 & \\ \hline & 69 \\ & ?? \end{array}$$

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- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
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- Subtract.
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# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 07 \\ \hline 33 \overline{)3\ 6\ 00} \\ -3\ 3\ 00 \\ \hline 3\ 00 \\ -2\ 31 \\ \hline 69 \\ -33 \\ \hline \end{array}$$

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# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 07 \\ \hline 33 \overline{)3\ 6\ 00} \\ -3\ 3\ 00 \\ \hline 3\ 00 \\ -2\ 31 \\ \hline 69 \\ -33 \\ \hline 36 \end{array}$$

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- **Subtract.**
- **Repeat.**

$$\begin{aligned} 9 &- 3 = 6 \\ 6 &- 3 = 3 \end{aligned}$$

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- Repeat.

$$\begin{aligned} 9 - 3 &= 6 \\ 6 - 3 &= 3 \end{aligned}$$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

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$$\begin{array}{r} & 1 \\ 33) & \overline{3\ 6\ 00} \\ -3\ 3 & \hline 3\ 0 \\ -2\ 1 & \hline 9 \\ -3\ 3 & \hline 36 \end{array}$$

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- Repeat.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \\ \hline 1 \ 07 \\ \hline \end{array}$$
$$\begin{array}{r} 3 \ 6 \ 00 \\ -3 \ 3 \ 00 \\ \hline \end{array}$$
$$\begin{array}{r} 3 \ 00 \\ -2 \ 31 \\ \hline 69 \\ -33 \\ \hline 36 \end{array}$$

- Find shortest dividend start larger than divisor.
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$$\begin{array}{r} & \textcolor{red}{?} \\ 33 | & 1 \quad 07 \\ & -3 \quad 6 \quad 00 \\ \hline & -3 \quad 3 \quad 00 \\ \hline & \quad 3 \quad 00 \\ & - \quad 2 \quad 31 \\ \hline & \quad \quad 69 \\ & - \quad \quad 33 \\ \hline & \quad \quad 36 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 33) & \overline{3\ 6\ 00} \\ -3\ 3 & \hline 3\ 0 \\ -2\ 1 & \hline 69 \\ -33 & \hline 36 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 33 | & 1 \\ & 1 \\ \hline & 07 \\ \\ 33 | & \boxed{3} & 6 & 00 \\ & -3 & 3 & 00 \\ \hline & 3 & 00 \\ & -2 & 31 \\ \hline & 69 \\ & -33 \\ \hline & 36 \\ & ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 33 | & 1 \\ & 1 \\ \hline & 07 \\ \\ \begin{array}{r} 3\ 6\ 00 \\ -3\ 3\ 00 \\ \hline 3\ 00 \\ -2\ 31 \\ \hline 69 \\ -33 \\ \hline 36 \\ -33 \\ \hline 3 \end{array} & \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ \hline 1 \ 07 \end{array}$$

$$\begin{array}{r} & 3 & 6 & 00 \\ 33 | & -3 & 3 & 00 \\ & \hline & 3 & 00 \\ & -2 & 31 \\ & \hline & 69 \\ & -33 \\ & \hline & 36 \\ & -33 \\ & \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ \hline 1 \ 07 \end{array}$$

$$\begin{array}{r} & 3 & 6 & 00 \\ 33 | & -3 & 3 & 00 \\ & \hline & 3 & 00 \\ & -2 & 31 \\ & \hline & 69 \\ & -33 \\ & \hline & 36 \\ & -33 \\ & \hline & ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ \hline 1 \ 07 \end{array}$$

$$\begin{array}{r} & 3 & 6 & 00 \\ 33 | & 3 & 3 & 00 \\ - & 3 & 00 \\ \hline & 6 & 9 \\ - & 6 & 9 \\ \hline & 3 & 6 \\ - & 3 & 3 \\ \hline & & 3 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract**.
- Repeat.

$$\begin{array}{r} 6 - 3 = 3 \\ 3 - 3 = 0 \end{array}$$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ \hline 1 \ 07 \end{array}$$

$$\begin{array}{r} & 3 & 6 & 00 \\ 33 | & \underline{-} & 3 & 3 & 00 \\ & 3 & 00 \\ - & \underline{-} & 2 & 31 \\ & 69 \\ - & \underline{-} & 33 \\ & 36 \\ - & \underline{-} & 33 \\ & 3 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 6 - 3 &= 3 \\ 3 - 3 &= 0 \end{aligned}$$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ \hline 1 \ 07 \end{array}$$

$$\begin{array}{r} & 3 & 6 & 00 \\ 33 | & -3 & 3 & 00 \\ & \hline & 3 & 00 \\ & -2 & 31 \\ & \hline & 69 \\ & -33 \\ & \hline & 36 \\ & -33 \\ & \hline & 3 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ \hline 1 \ 07 \end{array}$$

$$\begin{array}{r} & 3 & 6 & 00 \\ 33 | & 3 & 3 & 00 \\ - & 3 & 00 \\ \hline & 2 & 31 \\ - & 69 \\ \hline & 33 \\ - & 33 \\ \hline & 36 \\ - & 33 \\ \hline & 3 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ \hline 1 \ 07 \end{array}$$

$$\begin{array}{r} & 1 \\ 33 | & 3 \ 6 \ 00 \\ & -3 \ 3 \ 00 \\ \hline & 3 \ 00 \\ - & 2 \ 31 \\ \hline & 69 \\ - & 33 \\ \hline & 36 \\ - & 33 \\ \hline & 3 \end{array}$$

- Find shortest dividend start larger than divisor. **If none, we're done.**
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ \hline 1 & 07 \\ \hline \end{array}$$
$$\begin{array}{r} 3600 \\ - 33 \\ \hline 33 \\ - 231 \\ \hline 69 \\ - 33 \\ \hline 36 \\ - 33 \\ \hline 3 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- **If more than one quotient row, add them.**

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ \hline 1 & 07 \\ - & 9 \\ \hline \boxed{3} & 6 \quad 00 \\ 33 | & 3 \quad 3 \quad 00 \\ - & 3 \quad 00 \\ \hline & 69 \\ - & 33 \\ \hline & 36 \\ - & 33 \\ \hline & 3 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

$$7 + 1 + 1 = 9$$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 + & 1 \\
 \hline
 & 07 \\
 \hline
 & 09 \\
 \boxed{33} & \overline{)3\ 6\ 00} \\
 - & 3\ 3\ 00 \\
 \hline
 & 3\ 00 \\
 - & 2\ 31 \\
 \hline
 & 69 \\
 - & 33 \\
 \hline
 & 36 \\
 - & 33 \\
 \hline
 & 3
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

$$\begin{aligned}
 7 + 1 + 1 &= 9 \\
 0 &= 0
 \end{aligned}$$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & + & 1 \\
 & 1 & 07 \\
 \hline
 & 1 & 09 \\
 \boxed{33} & \overline{)3} & 6 & 00 \\
 & - & 3 & 3 & 00 \\
 \hline
 & & 3 & 00 \\
 & - & 2 & 31 \\
 \hline
 & & 69 \\
 & - & 33 \\
 \hline
 & & 36 \\
 & - & 33 \\
 \hline
 & & 3
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

$$\begin{aligned}
 7 + 1 + 1 &= 9 \\
 0 &= 0 \\
 1 &= 1
 \end{aligned}$$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ \hline & 1 07 \\ \hline & 1 09 \\ \hline \boxed{33} & \begin{array}{r} 3 \ 6 \ 00 \\ - 3 \ 3 \ 00 \\ \hline 3 \ 00 \\ - 2 \ 31 \\ \hline 69 \\ - 33 \\ \hline 36 \\ - 33 \\ \hline 3 \end{array} \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. **Dividend = 3600**. Divisor = 33. Quotient = 109. Remainder = 3.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ \hline 1 & 07 \\ \hline 1 & 09 \\ \hline \end{array}$$

**33** |  $\begin{array}{r} 3 & 6 & 00 \\ - 3 & 3 & 00 \\ \hline 3 & 00 \\ - 2 & 31 \\ \hline 69 \\ - 33 \\ \hline 36 \\ - 33 \\ \hline 3 \end{array}$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 3600. **Divisor = 33**. Quotient = 109. Remainder = 3.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ \hline & 1 07 \\ - & 1 09 \\ \hline & 0 \\ \overline{)33} & \overline{3\ 6\ 00} \\ - & 3\ 3\ 00 \\ \hline & 3\ 00 \\ - & 2\ 31 \\ \hline & 69 \\ - & 33 \\ \hline & 36 \\ - & 33 \\ \hline & 3 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 3600. Divisor = 33. Quotient = 109. Remainder = 3.

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ \hline & 1 07 \\ \hline & 1 09 \\ \hline \boxed{33} & 3 6 00 \\ - & 3 3 00 \\ \hline & 3 00 \\ - & 2 31 \\ \hline & 69 \\ - & 33 \\ \hline & 36 \\ - & 33 \\ \hline & 3 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 3600. Divisor = 33. Quotient = 109. **Remainder = 3.**

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 + & 1 \\
 \hline
 & 1 07 \\
 \hline
 & 1 09 \\
 \overline{33} & \overline{3 \ 6 \ 00} \\
 - & 3 \ 3 \ 00 \\
 \hline
 & 3 \ 00 \\
 - & 2 \ 31 \\
 \hline
 & 69 \\
 - & 33 \\
 \hline
 & 36 \\
 - & 33 \\
 \hline
 & 3
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. **Dividend = 3600**. Divisor = 33. Quotient = 109. Remainder = 3.
- Check.  $3600 \stackrel{?}{=} 33 \cdot 109 + 3$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ \hline 1 & 07 \\ \hline 1 & 09 \\ \hline \end{array}$$

**33** |  $\begin{array}{r} 3 & 6 & 00 \\ - 3 & 3 & 00 \\ \hline 3 & 00 \\ - 2 & 31 \\ \hline 69 \\ - 33 \\ \hline 36 \\ - 33 \\ \hline 3 \end{array}$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 3600. **Divisor = 33**. Quotient = 109. Remainder = 3.
- Check.  $3600 \stackrel{?}{=} 33 \cdot 109 + 3$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ \hline 1 & 07 \\ \hline 1 & 09 \\ \hline \end{array}$$

33 | 3 6 00

$$\begin{array}{r} - 3 3 00 \\ \hline 3 00 \\ - 2 31 \\ \hline 69 \\ - 33 \\ \hline 36 \\ - 33 \\ \hline 3 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 3600. Divisor = 33. Quotient = 109. Remainder = 3.
- Check.  $3600 \stackrel{?}{=} 33 \cdot 109 + 3$

# Dividend and divisor leading digits equal

Divide 3600 by 33 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 + & 1 \\
 \hline
 & 1 07 \\
 \hline
 & 1 09 \\
 \boxed{33} & \overline{)3\ 6\ 00} \\
 - & 3\ 3\ 00 \\
 \hline
 & 3\ 00 \\
 - & 2\ 31 \\
 \hline
 & 69 \\
 - & 33 \\
 \hline
 & 36 \\
 - & 33 \\
 \hline
 & 3
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 3600. Divisor = 33. Quotient = 109. **Remainder = 3.**
- Check.  $3600 \stackrel{?}{=} 33 \cdot 109 + 3$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & \boxed{4\ 9\ 2\ 6\ 19} \\ 46 & \end{array}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} \boxed{46} \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ \end{array}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

- Find shortest dividend start larger than divisor.

$$\begin{array}{r} & \boxed{4\ 9\ 2\ 6\ 19} \\ 46 & \end{array}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits;

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits; **larger divisor start has same # of digits**

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \end{array}$$

- Find shortest dividend start larger than divisor.
  - Next quotient digit: put above last digit of that start.
- Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ ?\ ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ 4\ 6 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ 4\ 6 \\ \hline 0\ 0\ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 \\ \hline 4\ 0 \\ -4\ 6 \\ \hline 0\ 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 \\ \hline 4\ 0 \\ -4\ 6 \\ \hline 0 \\ ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$9 - 0 = ?$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 \\ \hline 9 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$9 - 0 = 9$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 \\ \hline 4\ 0 \\ -4\ 6 \\ \hline 0\ 0 \\ \hline \end{array}$$

?9

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{array}{r} 9 - 0 = 9 \\ 1 - 0 = ? \end{array}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 \\ \hline 4\ 0 \\ -4\ 6 \\ \hline 0\ 0 \\ \hline \end{array}$$

19

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{array}{r} 9 - 0 = 9 \\ 1 - 0 = 1 \end{array}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6\ 0\ 0\ 00 \\ \hline ?\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 9 - 0 &= 9 \\ 1 - 0 &= 1 \\ 6 - 0 &= ? \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6\ 0\ 0\ 00 \\ \hline 6\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 9 - 0 &= 9 \\ 1 - 0 &= 1 \\ 6 - 0 &= 6 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 \\ \hline 0\ 0\ 0\ 0 \\ \hline ?\ 6\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 9 - 0 &= 9 \\ 1 - 0 &= 1 \\ 6 - 0 &= 6 \\ 2 - 0 &= ? \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 \\ \hline 2\ 6\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 9 - 0 &= 9 \\ 1 - 0 &= 1 \\ 6 - 0 &= 6 \\ 2 - 0 &= 2 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 \\ \hline 2\ 6\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 9 - 0 &= 9 \\ 1 - 0 &= 1 \\ 6 - 0 &= 6 \\ 2 - 0 &= 2 \\ 9 - 6 &= ? \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 \\ \hline 3\ 2\ 6\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 9 - 0 &= 9 \\ 1 - 0 &= 1 \\ 6 - 0 &= 6 \\ 2 - 0 &= 2 \\ \textcolor{red}{9 - 6} &= \textcolor{red}{3} \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 \\ \hline 3\ 2\ 6\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 9 - 0 &= 9 \\ 1 - 0 &= 1 \\ 6 - 0 &= 6 \\ 2 - 0 &= 2 \\ 9 - 6 &= 3 \\ 4 - 4 &= ? \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 \\ \hline 3\ 2\ 6\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 9 - 0 &= 9 \\ 1 - 0 &= 1 \\ 6 - 0 &= 6 \\ 2 - 0 &= 2 \\ 9 - 6 &= 3 \\ \textcolor{red}{4} - \textcolor{red}{4} &= \textcolor{red}{0} \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 \\ \hline 3\ 2\ 6\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 9 - 0 &= 9 \\ 1 - 0 &= 1 \\ 6 - 0 &= 6 \\ 2 - 0 &= 2 \\ 9 - 6 &= 3 \\ 4 - 4 &= 0 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 \\ \hline 3\ 2\ 6\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 9 - 0 &= 9 \\ 1 - 0 &= 1 \\ 6 - 0 &= 6 \\ 2 - 0 &= 2 \\ 9 - 6 &= 3 \\ 4 - 4 &= 0 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 \\ \hline 3\ 2\ 6\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{array}{r} 9 - 0 = 9 \\ 1 - 0 = 1 \\ 6 - 0 = 6 \\ 2 - 0 = 2 \\ 9 - 6 = 3 \\ 4 - 4 = 0 \end{array}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 \\ \hline 3\ 2\ 6\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 \\ \hline 3\ 2\ 6\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 \\ \hline 3\ 2\ 6 \\ -4\ 6 \\ \hline 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \quad ? \\ \overline{46} \overline{\left. \begin{array}{rrrrr} 4 & 9 & 2 & 6 & 19 \\ -4 & 6 & 0 & 0 & 00 \\ \hline 3 & 2 & 6 & 19 \end{array} \right)} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ ? \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \\ \hline 3 \ 2 \ 6 \ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. **If this leaves gaps: fill with 0.**

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ ? \\ 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \\ \hline 3 \ 2 \ 6 \ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.

— Larger divisor start has more digits

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ ? \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{32}{4+1} \right]$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ ? \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits **by divisor digit** plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left| \frac{32}{4+1} \right|$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ ? \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit **plus one**. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left| \frac{32}{4+1} \right|$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ ? \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. **Round down if needed.**

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \begin{array}{r} 32 \\ \hline 4+1 \end{array} \right]$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ ? \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{32}{4+1} \right] = ?$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{32}{4+1} \right] = \left[ \frac{32}{5} \right] = \left[ \frac{30}{5} + \frac{2}{5} \right] = \left[ 6 + \frac{2}{5} \right] = 6$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 \quad 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 \quad \quad \quad \quad \quad \\ \hline 3\ 2\ 6\ 19 \\ ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$6 \cdot 6 = ?$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ 46 \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 \\ \hline 3\ 2\ 6\ 19 \\ \quad \quad \quad 6 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$6 \cdot 6 = 36$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \\ \quad \quad \quad 6 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$6 \cdot 6 = 36$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 1 \ 9} \\ -4 \ 6 \ 0 \ 0 \ 0 \ 0 \\ \hline 3 \ 2 \ 6 \ 1 \ 9 \\ \quad 6 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$6 \cdot 6 = 36$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 \\ \hline 3\ 2\ 6\ 19 \\ ?\ 6 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 6 \cdot 6 &= 36 \\ 6 \cdot 4 + 3 &=? \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 1 \ 0 \ 6 \\
 \hline
 46 \overline{)4\ 9\ 2\ 6\ 19} \\
 -4\ 6 \\
 \hline
 3\ 2\ 6\ 19 \\
 -7\ 6 \\
 \hline
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 6 \cdot 6 &= 36 \\
 6 \cdot 4 + 3 &= 27
 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 23 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \\ \hline 3 \ 2 \ 6 \ 19 \\ ? \ 7 \ 6 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 6 \cdot 6 &= 36 \\ 6 \cdot 4 + 3 &= 27 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 23 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \\ \hline 3 \ 2 \ 6 \ 19 \\ -2 \ 7 \ 6 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 6 \cdot 6 &= 36 \\ 6 \cdot 4 + 3 &= 27 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 23 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \\ \hline 3 \ 2 \ 6 \ 19 \\ -2 \ 7 \ 6 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 6 \cdot 6 &= 36 \\ 6 \cdot 4 + 3 &= 27 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \\ -2 \ 7 \ 6 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 \quad 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \\ -2 \ 7 \ 6 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \\ 2 \ 7 \ 6 \ \textcolor{red}{00} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \\ -2 \ 7 \ 6 \ 00 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \\ -2 \ 7 \ 6 \ 00 \\ \hline ? \ ? \ ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 1 \ 0 \ 6 \\
 \hline
 46 \overline{)4 \ 9 \ 2 \ 6 \ 1 \ 9} \\
 -4 \ 6 \ 0 \ 0 \ 0 \ 0 \\
 \hline
 -1 \\
 3 \ 2 \ 6 \ 1 \ 9 \\
 -2 \ 7 \ 6 \ 0 \ 0 \\
 \hline
 5 \ 0 \ 1 \ 9
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract**.
- Repeat.

$$\begin{aligned}
 9 - 0 &= 9 \\
 1 - 0 &= 1 \\
 6 - 6 &= 0 \\
 2 - 7 &= -5 = -10 + 5 \\
 -1 + 3 - 2 &= 0
 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 1 \ 9} \\ -4 \ 6 \ 0 \ 0 \ 0 \ 0 \\ \hline -1 \\ \hline 3 \ 2 \ 6 \ 1 \ 9 \\ -2 \ 7 \ 6 \ 0 \ 0 \\ \hline \end{array}$$

5 0 19

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 9 - 0 &= 9 \\
 1 - 0 &= 1 \\
 6 - 6 &= 0 \\
 2 - 7 &= -5 = -10 + 5 \\
 -1 + 3 - 2 &= 0
 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \\ -2 \ 7 \ 6 \ 00 \\ \hline 5 \ 0 \ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 1 \ 9} \\ -4 \ 6 \ 0 \ 0 \ 0 \ 0 \\ \hline 3 \ 2 \ 6 \ 1 \ 9 \\ -2 \ 7 \ 6 \ 0 \ 0 \\ \hline 5 \ 0 \ 1 \ 9 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 0 \ 6 \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 1 \ 9} \\ -4 \ 6 \ 0 \ 0 \ 0 \ 0 \\ \hline 3 \ 2 \ 6 \ 1 \ 9 \\ -2 \ 7 \ 6 \ 0 \ 0 \\ \hline 5 \ 0 \ 1 \ 9 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \ 0 \ 6 \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \\ -2 \ 7 \ 6 \ 00 \\ \hline 5 \ 0 \ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- **Next quotient digit: put above last digit of that start.** If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \ 0 \ 6 \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline 3 \ 2 \ 6 \ 19 \\ -2 \ 7 \ 6 \ 00 \\ \hline 5 \ 0 \ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
    - Current leading digit  $>$  divisor leading digit
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 ? \\
 1 \ 0 \ 6 \\
 \hline
 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\
 -4 \ 6 \ 0 \ 0 \ 00 \\
 \hline
 -3 \ 2 \ 6 \ 19 \\
 -2 \ 7 \ 6 \ 00 \\
 \hline
 5 \ 0 \ 19
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  **divide leading digit** by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{5}{4+1} \right]$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 ? \\
 1 \ 0 \ 6 \\
 \hline
 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\
 -4 \ 6 \ 0 \ 0 \ 00 \\
 \hline
 - \ 3 \ 2 \ 6 \ 19 \\
 - \ 2 \ 7 \ 6 \ 00 \\
 \hline
 5 \ 0 \ 19
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit **by divisor digit** plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{5}{4+1} \right\rfloor$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 ? \\
 1 \ 0 \ 6 \\
 \hline
 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\
 -4 \ 6 \ 0 \ 0 \ 00 \\
 \hline
 -3 \ 2 \ 6 \ 19 \\
 -2 \ 7 \ 6 \ 00 \\
 \hline
 5 \ 0 \ 19
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit **plus one**. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{5}{4+1} \right\rfloor$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 ? \\
 1 \ 0 \ 6 \\
 \hline
 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\
 -4 \ 6 \ 0 \ 0 \ 00 \\
 \hline
 -3 \ 2 \ 6 \ 19 \\
 -2 \ 7 \ 6 \ 00 \\
 \hline
 5 \ 0 \ 19
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. **Round down if needed.**
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{5}{4+1} \right]$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & & ? \\
 & 1 & 0 & 6 \\
 \hline
 46 & \overline{)4} & 9 & 2 & 6 & 19 \\
 - & 4 & 6 & 0 & 0 & 00 \\
 \hline
 & 3 & 2 & 6 & 19 \\
 - & 2 & 7 & 6 & 00 \\
 \hline
 & & 5 & 0 & 19
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{5}{4+1} \right] = ??$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 1 & 0 & 6 \\
 \hline
 46 & \overline{)4\ 9\ 2\ 6\ 19} \\
 -4\ 6 & \overline{-\phantom{4}6\ 0\ 0\ 00} \\
 \hline
 & 3 & 2 & 6 & 19 \\
 - & 2 & 7 & 6 & 00 \\
 \hline
 & & 5 & 0 & 19
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{5}{4+1} \right\rfloor = \left\lfloor \frac{5}{5} \right\rfloor = [1] = 1$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 & \hline 3\ 2\ 6\ 1\ 9 \\ -2\ 7\ 6 & \hline 0\ 0 \\ 5\ 0\ 1\ 9 & \\ ?\ ? & \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 & \hline 3\ 2\ 6\ 1\ 9 \\ -2\ 7\ 6 & \hline 0\ 0 \\ 5\ 0 & \hline 4\ 6 \\ 5\ 0 & \hline 1\ 9 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 & \hline 4\ 0\ 0\ 0\ 0 \\ -4\ 6 & \hline 3\ 2\ 6\ 1\ 9 \\ -2\ 7\ 6 & \hline 0\ 0 \\ 4\ 6 & \hline 0\ 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 & \hline 0\ 6\ 0\ 0\ 0\ 0 \\ -4\ 6 & \hline 0\ 1\ 9 \\ -4\ 6 & \hline 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 & \hline 0\ 6\ 0\ 0\ 00 \\ -4\ 6 & \hline 0\ 19 \\ -4\ 6 & \hline 0 \\ ? & ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 1 & 0 & 6 \\
 \hline
 46 & \overline{)4\ 9\ 2\ 6\ 19} \\
 -4\ 6 & \overline{)0\ 0\ 0\ 00} \\
 \hline
 -3 & 2 & 6 & 19 \\
 -2 & 7 & 6 & 00 \\
 \hline
 -1 & & & \\
 5 & 0 & 19 \\
 -4 & 6 & 00 \\
 \hline
 & 4 & 19
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract.**
- Repeat.

$$\begin{aligned}
 9 - 0 &= 9 \\
 1 - 0 &= 1 \\
 0 - 6 &= -6 = -10 + 4 \\
 -1 + 5 - 4 &= 0
 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 1 & 0 & 6 \\
 \hline
 46 & \overline{)4\ 9\ 2\ 6\ 19} \\
 -4\ 6 & \overline{)0\ 0\ 0\ 00} \\
 \hline
 -3 & 2 & 6 & 19 \\
 -2 & 7 & 6 & 00 \\
 \hline
 -1 & & & \\
 -5 & 0 & 19 \\
 -4 & 6 & 00 \\
 \hline
 & 4 & 19
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 9 - 0 &= 9 \\
 1 - 0 &= 1 \\
 0 - 6 &= -6 = -10 + 4 \\
 -1 + 5 - 4 &= 0
 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 & \hline 0\ 6\ 0\ 0\ 0\ 0 \\ -4\ 6 & \hline 0\ 1\ 9 \\ -4\ 6 & \hline 4\ 1\ 9 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 & \hline 0\ 6\ 0\ 0\ 00 \\ -4\ 6 & \hline 0\ 19 \\ -4\ 6 & \hline 4\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 & \hline 0\ 6\ 0\ 0\ 0\ 0 \\ -4\ 6 & \hline 0\ 1\ 9 \\ -4\ 6 & \hline 4\ 1\ 9 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 1 & 0 & 6 & ? \\ \hline 46 & \overline{4 & 9 & 2 & 6 & 19} \\ & -4 & 6 & 0 & 0 & 00 \\ \hline & 3 & 2 & 6 & 19 \\ & -2 & 7 & 6 & 00 \\ \hline & & 5 & 0 & 19 \\ & - & 4 & 6 & 00 \\ \hline & & & 4 & 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- **Next quotient digit: put above last digit of that start.** If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 & \hline 0\ 0\ 0\ 0 \\ -4\ 6 & \hline 0\ 0 \\ -4\ 6 & \hline 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. **If this leaves gaps: fill with 0.**
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 19} \\ -4\ 6 & \hline 3\ 2\ 6\ 19 \\ -2\ 7 & \hline 5\ 0\ 19 \\ -4\ 6 & \hline 4\ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 1 & 0 & 6 & 0 & ? \\
 \hline
 46 & \overline{)4} & 9 & 2 & 6 & 19 \\
 - & 4 & 6 & 0 & 0 & 00 \\
 \hline
 & 3 & 2 & 6 & 19 \\
 - & 2 & 7 & 6 & 00 \\
 \hline
 & 5 & 0 & 19 \\
 - & 4 & 6 & 00 \\
 \hline
 & & 4 & 19
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{41}{4+1} \right] = ?$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 1 & 0 & 6 & 0 & 8 \\
 \hline
 46 & \overline{)4} & 9 & 2 & 6 & 19 \\
 - & 4 & 6 & 0 & 0 & 00 \\
 \hline
 & 3 & 2 & 6 & 19 \\
 - & 2 & 7 & 6 & 00 \\
 \hline
 & 5 & 0 & 19 \\
 - & 4 & 6 & 00 \\
 \hline
 & & 4 & 19
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{41}{4+1} \right\rfloor = \left\lfloor \frac{41}{5} \right\rfloor = \left\lfloor \frac{40}{5} + \frac{1}{5} \right\rfloor = \left\lfloor 8 + \frac{1}{5} \right\rfloor = 8$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 & \hline 0\ 6\ 0\ 0\ 0\ 0 \\ -4\ 6 & \hline 0\ 0\ 1\ 9 \\ -4\ 6 & \hline 5\ 0\ 1\ 9 \\ -4\ 6 & \hline 1\ 9 \\ ? & ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 1 & 0 & 6 & 0 & 8 \\
 \hline
 34 & \overline{)4} & 9 & 2 & 6 & 19 \\
 46 & - & 4 & 6 & 0 & 0 & 00 \\
 \hline
 & 3 & 2 & 6 & 19 \\
 & - & 2 & 7 & 6 & 00 \\
 \hline
 & 5 & 0 & 19 \\
 & - & 4 & 6 & 00 \\
 \hline
 & 4 & 19 \\
 & 3 & 68
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 8 \cdot 6 &= 48 \\
 8 \cdot 4 + 4 &= 36
 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 & \hline 4\ 0\ 0\ 0\ 0 \\ -4\ 6 & \hline 0\ 0\ 0 \\ -4\ 6 & \hline 0\ 0 \\ 4 & 1\ 9 \\ -3 & \hline 6\ 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 & \hline 0\ 6\ 0\ 0\ 0\ 0 \\ -4\ 6 & \hline 0\ 0\ 1\ 9 \\ -4\ 6 & \hline 0\ 0\ 3\ 6 \\ & 3\ 6 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 1 & 0 & 6 & 0 & 8 \\ \hline 46 & \boxed{4} & 9 & 2 & 6 & 19 \\ & -4 & 6 & 0 & 0 & 00 \\ \hline & 3 & 2 & 6 & 19 \\ & -2 & 7 & 6 & 00 \\ \hline & 5 & 0 & 19 \\ & -4 & 6 & 00 \\ \hline & & 4 & 19 \\ & - & 3 & 68 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 1 & 0 & 6 & 0 & 8 \\
 \hline
 46 & \overline{4} & 9 & 2 & 6 & 19 \\
 - & 4 & 6 & 0 & 0 & 00 \\
 \hline
 & 3 & 2 & 6 & 19 \\
 - & 2 & 7 & 6 & 00 \\
 \hline
 & 5 & 0 & 19 \\
 - & 4 & 6 & 00 \\
 \hline
 & 4 & 19 \\
 - & 3 & 68 \\
 \hline
 & & ?? \\
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 1 & 0 & 6 & 0 & 8 \\
 \hline
 46 & \overline{4} & 9 & 2 & 6 & 19 \\
 - & 4 & 6 & 0 & 0 & 00 \\
 \hline
 & 3 & 2 & 6 & 19 \\
 - & 2 & 7 & 6 & 00 \\
 \hline
 & 5 & 0 & 19 \\
 - & 4 & 6 & 00 \\
 \hline
 & -1 \\
 - & 4 & 19 \\
 \hline
 & 3 & 68 \\
 \hline
 & 51
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract.**
- Repeat.

$$\begin{aligned}
 9 - 8 &= 1 \\
 1 - 6 &= -5 = -10 + 5 \\
 -1 + 4 - 3 &= 0
 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 1 & 0 & 6 & 0 & 8 \\
 \hline
 46 & \overline{4} & 9 & 2 & 6 & 19 \\
 - & 4 & 6 & 0 & 0 & 00 \\
 \hline
 & 3 & 2 & 6 & 19 \\
 - & 2 & 7 & 6 & 00 \\
 \hline
 & 5 & 0 & 19 \\
 - & 4 & 6 & 00 \\
 \hline
 & & \textcolor{red}{-1} \\
 - & & 4 & 19 \\
 \hline
 & & 3 & 68 \\
 \hline
 & & & 51
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 9 - 8 &= 1 \\
 1 - 6 &= -5 = -10 + 5 \\
 -1 + 4 - 3 &= 0
 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 & \hline 0\ 6\ 0\ 0\ 0\ 0 \\ -4\ 6 & \hline 0\ 0\ 0 \\ -4\ 6 & \hline 0\ 0\ 0 \\ -4\ 6 & \hline 0\ 0\ 0 \\ -3\ 6 & \hline 4\ 1\ 9 \\ -3\ 6 & \hline 3\ 6\ 8 \end{array}$$

51

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 & \hline 4\ 0\ 0\ 0\ 0 \\ -4\ 6 & \hline 3\ 2\ 6\ 1\ 9 \\ -2\ 7 & \hline 6\ 0\ 0 \\ -4\ 6 & \hline 5\ 0\ 1\ 9 \\ -4\ 6 & \hline 4\ 1\ 9 \\ -3\ 6 & \hline 5\ 6\ 8 \end{array}$$

51

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 46 & \overline{)4\ 9\ 2\ 6\ 1\ 9} \\ -4\ 6 & \hline 4\ 0\ 0\ 0\ 0 \\ -4\ 6 & \hline 3\ 2\ 6\ 1\ 9 \\ -2\ 7 & \hline 6\ 0\ 0 \\ -4\ 6 & \hline 5\ 0\ 1\ 9 \\ -4\ 6 & \hline 4\ 1\ 9 \\ -3\ 6 & \hline 3\ 6\ 8 \end{array}$$

51

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & ? \\
 1 & 0 & 6 & 08 \\
 \hline
 46 & \overline{4} & 9 & 2 & 6 & 19 \\
 & -4 & 6 & 0 & 0 & 00 \\
 \hline
 & 3 & 2 & 6 & 19 \\
 & -2 & 7 & 6 & 00 \\
 \hline
 & 5 & 0 & 19 \\
 & -4 & 6 & 00 \\
 \hline
 & 4 & 19 \\
 & -3 & 68
 \end{array}$$

51

- Find shortest dividend start larger than divisor.
- **Next quotient digit: put above last digit of that start.** If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \text{ } 0? \\ 1 \text{ } 0 \text{ } 6 \text{ } 08 \\ \hline 46 \overline{)4 \text{ } 9 \text{ } 2 \text{ } 6 \text{ } 19} \\ -4 \text{ } 6 \text{ } 0 \text{ } 0 \text{ } 00 \\ \hline -3 \text{ } 2 \text{ } 6 \text{ } 19 \\ -2 \text{ } 7 \text{ } 6 \text{ } 00 \\ \hline 5 \text{ } 0 \text{ } 19 \\ -4 \text{ } 6 \text{ } 00 \\ \hline 4 \text{ } 19 \\ -3 \text{ } 68 \\ \hline \end{array}$$

51

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. **If this leaves gaps: fill with 0.**
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & ? \\
 1 & 0 & 6 & 08 \\
 \hline
 46 & \overline{4} & 9 & 2 & 6 & 19 \\
 & -4 & 6 & 0 & 0 & 00 \\
 \hline
 & 3 & 2 & 6 & 19 \\
 & -2 & 7 & 6 & 00 \\
 \hline
 & 5 & 0 & 19 \\
 & -4 & 6 & 00 \\
 \hline
 & 4 & 19 \\
 & -3 & 68 \\
 \hline
 & & 51
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
    - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{5}{4+1} \right] = ?$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 1 & 0 & 6 & 0 & 8 \\
 \hline
 46 \overline{)4} & 9 & 2 & 6 & 19 \\
 - & 4 & 6 & 0 & 0 & 00 \\
 \hline
 & 3 & 2 & 6 & 19 \\
 - & 2 & 7 & 6 & 00 \\
 \hline
 & 5 & 0 & 19 \\
 - & 4 & 6 & 00 \\
 \hline
 & 4 & 19 \\
 - & 3 & 68 \\
 \hline
 & & 51
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{5}{4+1} \right\rfloor = \left\lfloor \frac{5}{5} \right\rfloor = \lfloor 1 \rfloor = 1$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 01 \\ 1 \ 0 \ 6 \ 08 \\ \hline \end{array}$$

$$\begin{array}{r} 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline - \ 3 \ 2 \ 6 \ 19 \\ - \ 2 \ 7 \ 6 \ 00 \\ \hline - \ 5 \ 0 \ 19 \\ - \ 4 \ 6 \ 00 \\ \hline - \ 4 \ 19 \\ - \ 3 \ 68 \\ \hline \end{array}$$

51  
??

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 01 \\ 1 \ 0 \ 6 \ 08 \\ \hline \end{array}$$

$$\begin{array}{r} 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline - \ 3 \ 2 \ 6 \ 19 \\ - \ 2 \ 7 \ 6 \ 00 \\ \hline - \ 5 \ 0 \ 19 \\ - \ 4 \ 6 \ 00 \\ \hline - \ 4 \ 19 \\ - \ 3 \ 68 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 01 \\ 1 \ 0 \ 6 \ 08 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \ 9 \ 2 \ 6 \ 19 \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \ 2 \ 6 \ 19 \\ -2 \ 7 \ 6 \ 00 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \ 0 \ 19 \\ -4 \ 6 \ 00 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \ 19 \\ -3 \ 68 \\ \hline \end{array}$$

$$\begin{array}{r} 51 \\ -46 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

## Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 0 \\
 & 1 & 0 & 6 & 08 \\
 \hline
 46 & \overline{)4} & 9 & 2 & 6 & 19 \\
 & -4 & 6 & 0 & 0 & 00 \\
 \hline
 & 3 & 2 & 6 & 19 \\
 & -2 & 7 & 6 & 00 \\
 \hline
 & 5 & 0 & 19 \\
 & -4 & 6 & 00 \\
 \hline
 & 4 & 19 \\
 & -3 & 68 \\
 \hline
 & 51 \\
 & -46 \\
 \hline
 & ?
 \end{array}$$

- Find shortest dividend start larger than divisor.
  - Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
    - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
    - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
    - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
  - Subtract.
  - Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 46 & \overline{)4} & 9 & 2 & 6 & 1 & 9 \\
 & -4 & 6 & 0 & 0 & 0 & 0 \\
 \hline
 & 3 & 2 & 6 & 1 & 9 \\
 & -2 & 7 & 6 & 0 & 0 \\
 \hline
 & 5 & 0 & 1 & 9 \\
 & -4 & 6 & 0 & 0 \\
 \hline
 & 4 & 1 & 9 \\
 & -3 & 6 & 8 \\
 \hline
 & & & -1 \\
 & - & 5 & 1 \\
 \hline
 & & & 4 & 6 \\
 \hline
 & & & & 5
 \end{array}$$

$$\begin{aligned}
 1 - 6 &= -5 = -10 + 5 \\
 -1 + 5 - 4 &= 0
 \end{aligned}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract.**
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 46 & \overline{)4} & 9 & 2 & 6 & 1 & 9 \\
 & -4 & 6 & 0 & 0 & 0 & 0 \\
 \hline
 & 3 & 2 & 6 & 1 & 9 \\
 & -2 & 7 & 6 & 0 & 0 \\
 \hline
 & 5 & 0 & 1 & 9 \\
 & -4 & 6 & 0 & 0 \\
 \hline
 & 4 & 1 & 9 \\
 & -3 & 6 & 8 \\
 \hline
 & & & -1 \\
 & - & 5 & 1 \\
 \hline
 & & & 4 & 6 \\
 \hline
 & & & & 5
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 1 - 6 &= -5 = -10 + 5 \\
 -1 + 5 - 4 &= 0
 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} 1 \ 01 \\ 1 \ 0 \ 6 \ 08 \\ \hline 46 \overline{)4 \ 9 \ 2 \ 6 \ 19} \\ -4 \ 6 \ 0 \ 0 \ 00 \\ \hline -3 \ 2 \ 6 \ 19 \\ -2 \ 7 \ 6 \ 00 \\ \hline 5 \ 0 \ 19 \\ -4 \ 6 \ 00 \\ \hline 4 \ 19 \\ -3 \ 68 \\ \hline 51 \\ -46 \\ \hline 5 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 1 & 0 & 6 & 0 & 8 \\
 \hline
 46 \overline{)4} & 9 & 2 & 6 & 19 \\
 - & 4 & 6 & 0 & 0 & 00 \\
 \hline
 & 3 & 2 & 6 & 19 \\
 - & 2 & 7 & 6 & 00 \\
 \hline
 & 5 & 0 & 19 \\
 - & 4 & 6 & 00 \\
 \hline
 & 4 & 19 \\
 - & 3 & 68 \\
 \hline
 & 51 \\
 - & 46 \\
 \hline
 & 5
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 46 & \overline{)4} & 9 & 2 & 6 & 19 \\
 & -4 & 6 & 0 & 0 & 00 \\
 \hline
 & 3 & 2 & 6 & 19 \\
 & -2 & 7 & 6 & 00 \\
 \hline
 & 5 & 0 & 19 \\
 & -4 & 6 & 00 \\
 \hline
 & 4 & 19 \\
 & -3 & 68 \\
 \hline
 & & 51 \\
 & - & 46 \\
 \hline
 & & 5
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 + & 1 & 0 & 6 & 0 & 8 \\
 \hline
 & 4 & 9 & 2 & 6 & 19 \\
 46 \overline{) } & - & 4 & 6 & 0 & 0 & 00 \\
 & 3 & 2 & 6 & 19 \\
 - & 2 & 7 & 6 & 00 \\
 \hline
 & 5 & 0 & 19 \\
 - & 4 & 6 & 00 \\
 \hline
 & 4 & 19 \\
 - & 3 & 68 \\
 \hline
 & 51 \\
 - & 46 \\
 \hline
 & 5
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 + & 1 & 0 & 6 & 0 & 8 \\
 \hline
 & & & & 9 \\
 46 \overline{)4} & 9 & 2 & 6 & 1 & 9 \\
 - & 4 & 6 & 0 & 0 & 0 & 0 \\
 \hline
 & 3 & 2 & 6 & 1 & 9 \\
 - & 2 & 7 & 6 & 0 & 0 \\
 \hline
 & 5 & 0 & 1 & 9 \\
 - & 4 & 6 & 0 & 0 \\
 \hline
 & 4 & 1 & 9 \\
 - & 3 & 6 & 8 \\
 \hline
 & 5 & 1 \\
 - & 4 & 6 \\
 \hline
 & 5
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

$$8 + 1 = 9$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 + & 1 & 0 & 6 & 0 & 8 \\
 \hline
 & & & 0 & 9
 \end{array}$$
  

$$\begin{array}{r}
 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\
 -4\ 6 \\
 \hline
 4\ 2\ 6\ 1\ 9 \\
 -2\ 7\ 6\ 0\ 0 \\
 \hline
 5\ 0\ 1\ 9 \\
 -4\ 6\ 0\ 0 \\
 \hline
 4\ 1\ 9 \\
 -3\ 6\ 8 \\
 \hline
 5\ 1 \\
 -4\ 6 \\
 \hline
 5
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

$$8 + 1 = 9$$

$$\textcolor{red}{0 + 0 = 0}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 + & 1 & 0 & 6 & 0 & 8 \\
 \hline
 & & 7 & 0 & 9
 \end{array}$$
  

$$\begin{array}{r}
 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\
 - 4\ 6 \\
 \hline
 4\ 0\ 0\ 0\ 0
 \end{array}$$
  

$$\begin{array}{r}
 - 3\ 2\ 6\ 1\ 9 \\
 - 2\ 7\ 6\ 0\ 0 \\
 \hline
 5\ 0\ 1\ 9
 \end{array}$$
  

$$\begin{array}{r}
 - 4\ 6\ 0\ 0 \\
 \hline
 5\ 0\ 1\ 9
 \end{array}$$
  

$$\begin{array}{r}
 - 4\ 1\ 9 \\
 - 3\ 6\ 8 \\
 \hline
 5\ 1
 \end{array}$$
  

$$\begin{array}{r}
 - 4\ 6 \\
 \hline
 5
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

$$8 + 1 = 9$$

$$0 + 0 = 0$$

$$\textcolor{red}{6 + 1 = 7}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 + & 1 & 0 & 6 & 0 & 8 \\
 \hline
 & 0 & 7 & 0 & 9 \\
 \\ 
 46 \overline{)4} & 9 & 2 & 6 & 1 & 9 \\
 - & 4 & 6 & 0 & 0 & 0 & 0 \\
 \hline
 & 3 & 2 & 6 & 1 & 9 \\
 - & 2 & 7 & 6 & 0 & 0 \\
 \hline
 & 5 & 0 & 1 & 9 \\
 - & 4 & 6 & 0 & 0 \\
 \hline
 & 4 & 1 & 9 \\
 - & 3 & 6 & 8 \\
 \hline
 & 5 & 1 \\
 - & 4 & 6 \\
 \hline
 & 5
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

$$\begin{aligned}
 8 + 1 &= 9 \\
 0 + 0 &= 0 \\
 6 + 1 &= 7 \\
 \textcolor{red}{0} &= 0
 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 + & 1 & 0 & 6 & 0 & 8 \\
 \hline
 & 1 & 0 & 7 & 0 & 9
 \end{array}$$
  

$$\begin{array}{r}
 46 \overline{)4} & 9 & 2 & 6 & 1 & 9 \\
 - & 4 & 6 & 0 & 0 & 0 & 0 \\
 \hline
 & 3 & 2 & 6 & 1 & 9 \\
 - & 2 & 7 & 6 & 0 & 0 \\
 \hline
 & 5 & 0 & 1 & 9 \\
 - & 4 & 6 & 0 & 0 \\
 \hline
 & 4 & 1 & 9 \\
 - & 3 & 6 & 8 \\
 \hline
 & 5 & 1 \\
 - & 4 & 6 \\
 \hline
 & 5
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

$$\begin{aligned}
 8 + 1 &= 9 \\
 0 + 0 &= 0 \\
 6 + 1 &= 7 \\
 0 &= 0 \\
 \textcolor{red}{1} &= \textcolor{red}{1}
 \end{aligned}$$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 + & 1 & 0 & 6 & 0 & 8 \\
 \hline
 & 1 & 0 & 7 & 0 & 9
 \end{array}$$
  

$$\begin{array}{r}
 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\
 -4\ 6 \\
 \hline
 4\ 2\ 6\ 1\ 9 \\
 -4\ 6\ 0\ 0\ 0\ 0 \\
 \hline
 3\ 2\ 6\ 1\ 9 \\
 -2\ 7\ 6\ 0\ 0 \\
 \hline
 5\ 0\ 1\ 9 \\
 -4\ 6\ 0\ 0 \\
 \hline
 4\ 1\ 9 \\
 -3\ 6\ 8 \\
 \hline
 5\ 1 \\
 -4\ 6 \\
 \hline
 5
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. **Dividend = 492619**. Divisor = 46. Quotient = 10709. Remainder = 5.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r} & 1 \ 0 1 \\ + & 1 \ 0 \ 6 \ 0 8 \\ \hline & 1 \ 0 \ 7 \ 0 9 \\ \boxed{46} & \overline{4 \ 9 \ 2 \ 6 \ 1 9} \\ - & 4 \ 6 \ 0 \ 0 \ 0 0 \\ \hline & 3 \ 2 \ 6 \ 1 9 \\ - & 2 \ 7 \ 6 \ 0 0 \\ \hline & 5 \ 0 \ 1 9 \\ - & 4 \ 6 \ 0 0 \\ \hline & 4 \ 1 9 \\ - & 3 \ 6 8 \\ \hline & 5 1 \\ - & 4 6 \\ \hline & 5 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 492619. **Divisor = 46**. Quotient = 10709. Remainder = 5.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 + & 1 & 0 & 6 & 0 & 8 \\
 \hline
 & 1 & 0 & 7 & 0 & 9
 \end{array}$$
  

$$\begin{array}{r}
 46 \overline{)4} & 9 & 2 & 6 & 1 & 9 \\
 - & 4 & 6 & 0 & 0 & 0 & 0 \\
 \hline
 & 3 & 2 & 6 & 1 & 9 \\
 - & 2 & 7 & 6 & 0 & 0 \\
 \hline
 & 5 & 0 & 1 & 9 \\
 - & 4 & 6 & 0 & 0 \\
 \hline
 & 4 & 1 & 9 \\
 - & 3 & 6 & 8 \\
 \hline
 & 5 & 1 \\
 - & 4 & 6 \\
 \hline
 & 5
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 492619. Divisor = 46. Quotient = 10709.  
Remainder = 5.

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 + & 1 & 0 & 6 & 0 & 8 \\
 \hline
 & 1 & 0 & 7 & 0 & 9
 \end{array}$$
  

$$\begin{array}{r}
 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\
 -4\ 6 \\
 \hline
 4\ 2\ 6\ 1\ 9 \\
 -4\ 6\ 0\ 0\ 0\ 0 \\
 \hline
 3\ 2\ 6\ 1\ 9 \\
 -2\ 7\ 6\ 0\ 0 \\
 \hline
 5\ 0\ 1\ 9 \\
 -4\ 6\ 0\ 0 \\
 \hline
 4\ 1\ 9 \\
 -3\ 6\ 8 \\
 \hline
 5\ 1 \\
 -4\ 6 \\
 \hline
 5
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 492619. Divisor = 46. Quotient = 10709.  
**Remainder = 5.**

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 + & 1 & 0 & 6 & 0 & 8 \\
 \hline
 & 1 & 0 & 7 & 0 & 9
 \end{array}$$
  

$$\begin{array}{r}
 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\
 -4\ 6 \\
 \hline
 4\ 2\ 6\ 1\ 9 \\
 -2\ 7\ 6\ 0 & 0 \\
 \hline
 5\ 0\ 1\ 9 \\
 -4\ 6\ 0 & 0 \\
 \hline
 4\ 1\ 9 \\
 -3\ 6\ 8 \\
 \hline
 5\ 1 \\
 -4\ 6 \\
 \hline
 5
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. **Dividend = 492619**. Divisor = 46. Quotient = 10709.  
Remainder = 5.
- Check. **492619**  $\stackrel{?}{=}$   $46 \cdot 10709 + 5$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 + & 1 & 0 & 6 & 0 & 8 \\
 \hline
 & 1 & 0 & 7 & 0 & 9
 \end{array}$$
  

$$\begin{array}{r}
 46 \overline{)4\ 9\ 2\ 6\ 1\ 9} \\
 -4\ 6 \\
 \hline
 4\ 2\ 6\ 1\ 9 \\
 -4\ 6\ 0\ 0\ 0\ 0 \\
 \hline
 3\ 2\ 6\ 1\ 9 \\
 -2\ 7\ 6\ 0\ 0 \\
 \hline
 5\ 0\ 1\ 9 \\
 -4\ 6\ 0\ 0 \\
 \hline
 4\ 1\ 9 \\
 -3\ 6\ 8 \\
 \hline
 5\ 1 \\
 -4\ 6 \\
 \hline
 5
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 492619. **Divisor = 46**. Quotient = 10709. Remainder = 5.
- Check.  $492619 = 46 \cdot 10709 + 5$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 + & 1 & 0 & 6 & 0 & 8 \\
 \hline
 & 1 & 0 & 7 & 0 & 9
 \end{array}$$
  

$$\begin{array}{r}
 46 \overline{)4} & 9 & 2 & 6 & 1 & 9 \\
 - & 4 & 6 & 0 & 0 & 0 & 0 \\
 \hline
 & 3 & 2 & 6 & 1 & 9 \\
 - & 2 & 7 & 6 & 0 & 0 \\
 \hline
 & 5 & 0 & 1 & 9 \\
 - & 4 & 6 & 0 & 0 \\
 \hline
 & 4 & 1 & 9 \\
 - & 3 & 6 & 8 \\
 \hline
 & 5 & 1 \\
 - & 4 & 6 \\
 \hline
 & 5
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 492619. Divisor = 46. Quotient = 10709.  
Remainder = 5.
- Check.  $492619 \stackrel{?}{=} 46 \cdot 10709 + 5$

# Full algorithm with all cases

Divide 492619 by 46 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 1 \\
 + & 1 & 0 & 6 & 0 & 8 \\
 \hline
 & 1 & 0 & 7 & 0 & 9
 \end{array}$$
  

$$\begin{array}{r}
 46 \overline{)4} & 9 & 2 & 6 & 1 & 9 \\
 - & 4 & 6 & 0 & 0 & 0 & 0 \\
 \hline
 & 3 & 2 & 6 & 1 & 9 \\
 - & 2 & 7 & 6 & 0 & 0 \\
 \hline
 & 5 & 0 & 1 & 9 \\
 - & 4 & 6 & 0 & 0 \\
 \hline
 & 4 & 1 & 9 \\
 - & 3 & 6 & 8 \\
 \hline
 & 5 & 1 \\
 - & 4 & 6 \\
 \hline
 & 5
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 492619. Divisor = 46. Quotient = 10709.  
**Remainder = 5.**
- Check.  $492619 \stackrel{?}{=} 46 \cdot 10709 + 5$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & \boxed{6\ 1\ 9\ 8\ 21} \\ 343 & \end{array}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & \boxed{ } \\ 343 & \overline{)6\ 1\ 9\ 8\ 21} \end{array}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.

$$\begin{array}{r} & \boxed{ } \\ 343 & \overline{)6\ 1\ 9\ 8\ 21} \end{array}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

?

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} & \boxed{6} & 1 & 9 & 8 & 21 \\ 343 & \overline{)619821} \end{array}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} ? \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \end{array}$$

— Current leading digit > divisor leading digit

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

?

$$\begin{array}{r} & \boxed{6} & 1 & 9 & 8 & 21 \\ 343 & \overline{)619821} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Current leading digit  $>$  divisor leading digit  $\Rightarrow$  **divide leading digit** by divisor digit plus one. Round down if needed.

$$\left\lfloor \frac{6}{3+1} \right\rfloor$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

?

$$\begin{array}{r} \boxed{6\ 1\ 9\ 8\ 21} \\ 343 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit **by divisor digit** plus one. Round down if needed.

$$\left\lfloor \frac{6}{3+1} \right\rfloor$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

?

$$\begin{array}{r} \boxed{6\ 1\ 9\ 8\ 21} \\ 343 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit **plus one**. Round down if needed.

$$\left\lfloor \frac{6}{3+1} \right\rfloor$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

?

$$\begin{array}{r} \boxed{6\ 1\ 9\ 8\ 21} \\ 343 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. **Round down if needed.**

$$\left[ \frac{6}{3+1} \right]$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} ? \\ \hline 343 \overline{)619821} \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

$$\left\lfloor \frac{6}{3+1} \right\rfloor = ?$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} & 1 \\ \hline 343 & \overline{)6\ 1\ 9\ 8\ 21} \end{array}$$

— Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

$$\left\lfloor \frac{6}{3+1} \right\rfloor = \left\lfloor \frac{6}{4} \right\rfloor = \left\lfloor \frac{4}{4} + \frac{2}{4} \right\rfloor = \left\lfloor 1 + \frac{2}{4} \right\rfloor = 1$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 343 \sqrt{619821} \\ & 619 \\ & 343 \\ & \underline{-} \\ & 278 \\ & 278 \\ & \underline{-} \\ & 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

## Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
  - Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 343 \\ \hline 6 & 1 & 9 & 8 & 21 \\ 3 & 4 & 3 \end{array}$$

- Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ \overline{)343} & 6 \ 1 \ 9 \ 8 \ 21 \\ & 3 \ 4 \ 3 \ 0 \ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 343 \overline{)619821} \\ - \underline{343} \\ 276 \\ - \underline{276} \\ 021 \\ - \underline{021} \\ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 2\ 1} \\ - \ 3\ 4\ 3\ 0\ 0\ 0 \\ \hline ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$1 - 0 = ?$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 1 \\ \hline 343 \overline{)619821} \\ - \underline{343} \\ 278 \\ - \underline{278} \\ 021 \\ - \underline{021} \\ 1 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$1 - 0 = 1$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 1 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4\ 3\ 0\ 00 \\ \hline ?1 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 1 - 0 &= 1 \\ 2 - 0 &= ? \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 343 \overline{)619821} \\ - \underline{343} \\ 278 \\ - \underline{278} \\ 021 \\ - \underline{021} \\ 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{array}{r} 1 - 0 = 1 \\ 2 - 0 = 2 \end{array}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 1 \\ \hline 343 \overline{)619821} \\ - \quad 343 \\ \hline 278 \quad 00 \\ - \quad 274 \\ \hline 42 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 1 - 0 &= 1 \\ 2 - 0 &= 2 \\ 8 - 0 &= ? \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 1 \\ \hline 343 \overline{)619821} \\ - \quad 343 \\ \hline 278 \quad 00 \\ - \quad 274 \\ \hline 421 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 1 - 0 &= 1 \\ 2 - 0 &= 2 \\ 8 - 0 &= 8 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 1 \\ \hline 343 \overline{)619821} \\ - \quad 343 \\ \hline 278 \quad 00 \\ - \quad 274 \\ \hline 421 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 1 - 0 &= 1 \\ 2 - 0 &= 2 \\ 8 - 0 &= 8 \\ 9 - 3 &= ? \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

1

$$\begin{array}{r} & 1 \\ \overline{)343} & \overline{6 \ 1 \ 9 \ 8 \ 21} \\ - & 3 \ 4 \\ & 3 \ 0 \ 0 0 \\ & \underline{6 \ 8 \ 21} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 1 - 0 &= 1 \\ 2 - 0 &= 2 \\ 8 - 0 &= 8 \\ 9 - 3 &= 6 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 1 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4 \\ \hline 6\ 8\ 21 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$1 - 0 = 1$$

$$2 - 0 = 2$$

$$8 - 0 = 8$$

$$9 - 3 = 6$$

$$1 - 4 = ?$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \\ \hline 343 \overline{)619821} \\ - \quad 34 \\ \hline 279 \\ - \quad 34 \\ \hline 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 1 - 0 &= 1 \\ 2 - 0 &= 2 \\ 8 - 0 &= 8 \\ 9 - 3 &= 6 \\ 1 - 4 &= -3 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 1 \\ \hline 343 \overline{)619821} \\ - \underline{343} \\ 278 \\ - \underline{278} \\ 021 \\ - \underline{021} \\ 000 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$1 - 0 = 1$$

$$2 - 0 = 2$$

$$8 - 0 = 8$$

$$9 - 3 = 6$$

$$1 - 4 = -3 = -10 + ?$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 1 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4 \\ \hline 7\ 6\ 8\ 21 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$1 - 0 = 1$$

$$2 - 0 = 2$$

$$8 - 0 = 8$$

$$9 - 3 = 6$$

$$1 - 4 = -3 = -10 + 7$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} & 1 \\ \overline{343} \left| \begin{array}{r} -1 \\ 6 \ 1 \ 9 \ 8 \ 21 \\ - \ 3 \ 4 \ 3 \ 0 \ 00 \\ \hline 7 \ 6 \ 8 \ 21 \end{array} \right. \end{array}$$

— Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$1 - 0 = 1$$

$$2 - 0 = 2$$

$$8 - 0 = 8$$

$$9 - 3 = 6$$

$$1 - 4 = -3 = -10 + 7$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} & 1 \\ \overline{343} \left| \begin{array}{r} -1 \\ 6 \ 1 \ 9 \ 8 \ 21 \\ - 3 \ 4 \ 3 \ 0 \ 00 \\ \hline 7 \ 6 \ 8 \ 21 \end{array} \right. \end{array}$$

- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$1 - 0 = 1$$

$$2 - 0 = 2$$

$$8 - 0 = 8$$

$$9 - 3 = 6$$

$$1 - 4 = -3 = -10 + 7$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 1 \\ \hline 343 \overline{)619821} \\ - \underline{343} \\ \hline 276821 \\ - \underline{2763} \\ \hline 500 \end{array}$$

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$1 - 0 = 1$$

$$2 - 0 = 2$$

$$8 - 0 = 8$$

$$9 - 3 = 6$$

$$1 - 4 = -3 = -10 + 7$$

$$-1 + 6 - 3 = ?$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r}
 & 1 \\
 \overline{343} \Big) \quad & -1 \\
 & \underline{-} \quad 6 \quad 1 \quad 9 \quad 8 \quad 21 \\
 & \quad 3 \quad 4 \quad 3 \quad 0 \quad 00 \\
 & \underline{\underline{-}} \quad 2 \quad 7 \quad 6 \quad 8 \quad 21
 \end{array}$$

- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$1 - 0 = 1$$

$$2 - 0 = 2$$

$$8 - 0 = 8$$

$$9 - 3 = 6$$

$$1 - 4 = -3 = -10 + 7$$

$$\textcolor{red}{-1 + 6 - 3 = 2}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} & 1 \\ \overline{343} \Big) & -1 \\ & 6 \ 1 \ 9 \ 8 \ 21 \\ & - 3 \ 4 \ 3 \ 0 \ 00 \\ \hline & 2 \ 7 \ 6 \ 8 \ 21 \end{array}$$

- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 1 - 0 &= 1 \\ 2 - 0 &= 2 \\ 8 - 0 &= 8 \\ 9 - 3 &= 6 \\ 1 - 4 &= -3 = -10 + 7 \\ -1 + 6 - 3 &= 2 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r}
 & 1 \\
 \overline{343} \Big) & -1 \\
 & 6 \ 1 \ 9 \ 8 \ 21 \\
 & - 3 \ 4 \ 3 \ 0 \ 00 \\
 \hline
 & 2 \ 7 \ 6 \ 8 \ 21
 \end{array}$$

- Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned}
 1 - 0 &= 1 \\
 2 - 0 &= 2 \\
 8 - 0 &= 8 \\
 9 - 3 &= 6 \\
 1 - 4 &= -3 = -10 + 7 \\
 -1 + 6 - 3 &= 2
 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r}
 & 1 \\
 \overline{343} \Big) \quad & -1 \\
 & 6 \ 1 \ 9 \ 8 \ 21 \\
 & - 3 \ 4 \ 3 \ 0 \ 00 \\
 \hline
 & 2 \ 7 \ 6 \ 8 \ 21
 \end{array}$$

- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$1 - 0 = 1$$

$$2 - 0 = 2$$

$$8 - 0 = 8$$

$$9 - 3 = 6$$

$$1 - 4 = -3 = -10 + 7$$

$$-1 + 6 - 3 = 2$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 1 \\ \hline 343 \overline{)619821} \\ - \quad 343 \\ \hline 276821 \\ - \quad 2743 \\ \hline 2500 \\ - \quad 2409 \\ \hline 101 \end{array}$$

- Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} & 1 \\ \overline{343} & \overline{619821} \\ - & 343 \\ \hline & 276821 \\ & \quad 274 \\ \hline & \quad 27 \\ & \quad 21 \\ \hline & \quad 00 \end{array}$$

- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} & 1 \\ \overline{343} & \overline{619821} \\ - & 343 \\ \hline & 276821 \\ - & 2768 \\ \hline & 21 \end{array}$$

— Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

## Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
  - Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 343 \\ \sqrt{619821} \\ - \end{array}$$

- Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
  - Subtract.
  - Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 ? \\ \hline 343 \overline{)619821} \\ - \underline{343} \\ 276 \\ - \underline{21} \\ 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 ? \\ \hline 343 \overline{)619821} \\ - \underline{343} \\ 276821 \\ - \underline{27} \\ 6821 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{27}{3+1} \right]$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

1 ?

$$\begin{array}{r} 343 \overline{)619821} \\ - \quad 343 \\ \hline 276821 \\ - \quad 274 \\ \hline 2721 \\ - \quad 274 \\ \hline 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits **by divisor digit** plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{27}{3+1} \right]$$

## Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
  - Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 343 \\ \hline - 619821 \\ - 343000 \\ \hline 276821 \end{array}$$

- Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
  - Subtract.
  - Repeat.

27  
3+1

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 ? \\ \hline 343 \overline{)619821} \\ - \quad 343 \\ \hline 276 \\ - \quad 276 \\ \hline 00 \\ - \quad 00 \\ \hline 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. **Round down if needed.**
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \begin{array}{r} 27 \\ 3+1 \end{array} \right]$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \ ? \\ 343 \overline{)619821} \\ - \underline{343} & \\ \hline 276821 \\ - \underline{276} & \\ \hline 821 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{27}{3+1} \right] = ?$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \text{ } 6 \\
 \hline
 343 & \overline{)6 \text{ } 1 \text{ } 9 \text{ } 8 \text{ } 21} \\
 - & 3 \text{ } 4 \text{ } 3 \text{ } 0 \text{ } 00 \\
 \hline
 & 2 \text{ } 7 \text{ } 6 \text{ } 8 \text{ } 21
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{27}{3+1} \right\rfloor = \left\lfloor \frac{27}{4} \right\rfloor = \left\lfloor \frac{24}{4} + \frac{3}{4} \right\rfloor = \left\lfloor 6 + \frac{3}{4} \right\rfloor = 6$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \text{ } 6 \\ \hline 343 \overline{)619821} \\ - \quad 343 \\ \hline 276 \\ - \quad 274 \\ \hline 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 1 \text{ } 6 \\ \hline 343 \overline{)619821} \\ - \quad 343 \\ \hline 276 \\ - \quad 276 \\ \hline 821 \\ - \quad 821 \\ \hline ? \end{array}$$

- Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$6 \cdot 3 = ?$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \ 6 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4 \\ \hline 2\ 7 \\ - \ 2\ 7 \\ \hline 0\ 0 \\ - \ 0\ 0 \\ \hline 2\ 1 \\ - \ 2\ 1 \\ \hline 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$6 \cdot 3 = 18$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \ 6 \\ 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4 \\ \hline 2\ 7 \\ - \ 2\ 7 \\ \hline 0\ 0 \\ - \ 0\ 0 \\ \hline 2\ 1 \\ - \ 2\ 1 \\ \hline 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$6 \cdot 3 = 18$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \ 6 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4 \\ \hline 2\ 7 \\ - \ 2\ 7 \\ \hline 0\ 0 \\ - \ 0\ 0 \\ \hline 2\ 1 \\ - \ 2\ 1 \\ \hline 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$6 \cdot 3 = 18$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \ 6 \\ \hline ?1 & \overline{)6 \ 1 \ 9 \ 8 \ 21} \\ 343 & - \ 3 \ 4 \ 3 \ 0 \ 00 \\ & \underline{-} \ 2 \ 7 \ 6 \ 8 \ 21 \\ & \quad \underline{\underline{?}} \ 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{array}{r} 6 \cdot 3 = 18 \\ 6 \cdot 4 + 1 = ? \end{array}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 21 \\ 343 \overline{)619821} \\ - \quad 343 \\ \hline 276 \\ - \quad 274 \\ \hline 21 \\ - \quad 21 \\ \hline 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{array}{r} 6 \cdot 3 = 18 \\ 6 \cdot 4 + 1 = 25 \end{array}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \ 6 \\ 21 \overline{)6\ 1\ 9\ 8\ 21} \\ 343 \overline{-} \\ \phantom{6}3\ 4\ 3\ 0\ 00 \\ \phantom{3}2\ 7\ 6\ 8\ 21 \\ \phantom{2}5\ 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 6 \cdot 3 &= 18 \\ 6 \cdot 4 + 1 &= 25 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 21 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4 \\ \hline 2\ 7 \\ - \ 2\ 7 \\ \hline 0\ 0 \\ - \ 0\ 0 \\ \hline 5\ 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 6 \cdot 3 &= 18 \\ 6 \cdot 4 + 1 &= 25 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

1 6

$$\begin{array}{r} \boxed{2}1 \\ 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4 \\ \hline 2\ 7 \\ \quad \boxed{5}\ 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

- Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 6 \cdot 3 &= 18 \\ 6 \cdot 4 + 1 &= 25 \\ 6 \cdot 3 + 2 &=? \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

1 6

$$\begin{array}{r} 221 \\ 343 \overline{)619821} \\ -343 \\ \hline 276 \\ -276 \\ \hline 821 \\ -686 \\ \hline 058 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 6 \cdot 3 &= 18 \\ 6 \cdot 4 + 1 &= 25 \\ 6 \cdot 3 + 2 &= 20 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

$$\begin{array}{r} 1 \ 6 \\ 221 \overline{)6\ 1\ 9\ 8\ 21} \\ 343 \quad - \ 3\ 4\ 3\ 0\ 00 \\ \hline 2\ 7\ 6\ 8\ 21 \\ ?\ 0\ 5\ 8 \end{array}$$

- Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 6 \cdot 3 &= 18 \\ 6 \cdot 4 + 1 &= 25 \\ 6 \cdot 3 + 2 &= 20 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

1 6

$$\begin{array}{r} 221 \\ 343 \overline{)619821} \\ - \quad 343 \\ \hline 276 \\ - \quad 276 \\ \hline 058 \end{array}$$

- Find shortest dividend start larger than divisor.
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  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 6 \cdot 3 &= 18 \\ 6 \cdot 4 + 1 &= 25 \\ 6 \cdot 3 + 2 &= 20 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

1 6

$$\begin{array}{r} \boxed{221} \\[-1ex] 343 \overline{)6\ 1\ 9\ 8\ 21} \\[-1ex] - \quad 3\ 4\ 3\ 0\ 00 \\ \hline 2\ 7\ 6\ 8\ 21 \\[-1ex] - \quad 2\ 0\ 5\ 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 6 \cdot 3 &= 18 \\ 6 \cdot 4 + 1 &= 25 \\ 6 \cdot 3 + 2 &= 20 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \ 6 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4\ 3 \\ \hline 2\ 7\ 6\ 8\ 21 \\ - \ 2\ 0\ 5\ 8 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \ 6 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4 \\ \hline 2\ 7 \\ - \ 2\ 7 \\ \hline 0\ 5 \\ - \ 0\ 5 \\ \hline 8 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \ 6 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4\ 3 \\ \hline 2\ 7\ 6\ 8\ 21 \\ - \ 2\ 0\ 5\ 8\ 00 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \ 6 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \underline{3\ 4} \quad 3\ 0\ 00 \\ \underline{-} \quad \underline{2\ 7\ 6\ 8\ 21} \\ \underline{\underline{2\ 0\ 5\ 8\ 00}} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \ 6 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4 \\ \hline 2\ 7 \\ - \ 2\ 7 \\ \hline 0\ 5 \\ - \ 0\ 5 \\ \hline 0\ 0 \\ \end{array}$$

? ? ? ??

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \ 6 \\
 \hline
 343 & \overline{)6\ 1\ 9\ 8\ 21} \\
 - & 3\ 4\ 3\ 0\ 00 \\
 \hline
 & 2\ 7\ 6\ 8\ 21 \\
 - & 2\ 0\ 5\ 8\ 00 \\
 \hline
 & 7\ 1\ 0\ 21
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract.**
- Repeat.

$$\begin{aligned}
 1 - 0 &= 1 \\
 2 - 0 &= 2 \\
 8 - 8 &= 0 \\
 6 - 5 &= 1 \\
 7 - 0 &= 7 \\
 2 - 2 &= 0
 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \ 6 \\
 \hline
 343 & \overline{)6\ 1\ 9\ 8\ 21} \\
 - & 3\ 4\ 3\ 0\ 00 \\
 \hline
 & 2\ 7\ 6\ 8\ 21 \\
 - & 2\ 0\ 5\ 8\ 00 \\
 \hline
 & 7\ 1\ 0\ 21
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 1 - 0 &= 1 \\
 2 - 0 &= 2 \\
 8 - 8 &= 0 \\
 6 - 5 &= 1 \\
 7 - 0 &= 7 \\
 2 - 2 &= 0
 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \ 6 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4 \\ \hline 2\ 7\ 6\ 8\ 21 \\ - \ 2\ 0\ 5\ 8\ 00 \\ \hline 7\ 1\ 0\ 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \ 6 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4 \\ \hline 2\ 7\ 6\ 8\ 21 \\ - \ 2\ 0\ 5\ 8\ 00 \\ \hline 7\ 1\ 0\ 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \ 6 \\ \hline 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4 \\ \hline 2\ 7\ 6\ 8\ 21 \\ - \ 2\ 0\ 5\ 8\ 00 \\ \hline 7\ 1\ 0\ 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & \text{?} \\ 343 & \overline{)619821} \\ - & 343 \\ \hline & 276 \\ - & 276 \\ \hline & 00 \\ & 71021 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 \text{?} \\
 1 \ 6 \\
 \hline
 343 \overline{)6\ 1\ 9\ 8\ 21} \\
 - \ 3\ 4 \\
 \hline
 \ 3\ 4\ 3\ 0\ 00 \\
 - \ 2\ 7 \\
 \hline
 \ 2\ 7\ 6\ 8\ 21 \\
 - \ 2\ 0\ 5 \\
 \hline
 \ 2\ 0\ 5\ 8\ 00
 \end{array}$$

7 1 0 21

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{7}{3+1} \right] = ?$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & 1 & 6 \\
 \hline
 343 & \overline{)619821} \\
 - & 343 & 000 \\
 \hline
 & 276821 \\
 - & 205800 \\
 \hline
 & 71021
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{7}{3+1} \right\rfloor = \left\lfloor \frac{7}{4} \right\rfloor = \left\lfloor \frac{4}{4} + \frac{3}{4} \right\rfloor = \left\lfloor 1 + \frac{3}{4} \right\rfloor = 1$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & & 1 \\ & 1 & 6 \\ \hline 343 & \overline{)619821} \\ - & 343 \\ \hline & 276 \\ - & 276 \\ \hline & 21 \\ ? & ? & ? \\ \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & & 1 \\ & 1 & 6 \\ \hline 343 & \overline{)6\ 1\ 9\ 8\ 21} \\ - & 3\ 4 & 3 & 0\ 00 \\ \hline & 2 & 7 & 6 & 8\ 21 \\ - & 2 & 0 & 5 & 8\ 00 \\ \hline & 7 & 1 & 0 & 21 \\ & 3 & 4 & 3 & \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 343 & \overline{)619821} \\ - & 343 \\ \hline & 276 \\ - & 276 \\ \hline & 00 \\ & 71021 \\ - & 343 \\ \hline & 343 \\ - & 343 \\ \hline & 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.

- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 343 & \overline{)619821} \\ - & 343 \\ \hline & 276 \\ - & 276 \\ \hline & 0 \\ - & 0 \\ \hline & 21 \\ - & 21 \\ \hline & 0 \end{array}$$

The quotient digits are 1, 6, 7, 1, 0, 21. The remainder is 0.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

## Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & & & 1 \\
 & & 1 & 6 \\
 & 1 & 6 & 8 & 21 \\
 343 | & \underline{-} & 6 & 1 & 9 & 8 & 21 \\
 & & 3 & 4 & 3 & 0 & 00 \\
 & & \underline{-} & 2 & 7 & 6 & 8 & 21 \\
 & & & 2 & 0 & 5 & 8 & 00 \\
 & & & - & 7 & 1 & 0 & 21 \\
 & & & & 3 & 4 & 3 & 00 \\
 & & & & \underline{-} & 7 & 1 & 0 & 21 \\
 & & & & & ? & ? & ? & ???
 \end{array}$$

- Find shortest dividend start larger than divisor.
  - Next quotient digit: put above last digit of that start.
    - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
    - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
  - Subtract.
  - Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 1 & 6 \\
 \hline
 343 & \overline{)619821} \\
 - & 343 \\
 \hline
 & 276821 \\
 - & 272 \\
 \hline
 & 4821 \\
 - & 343 \\
 \hline
 & 138 \\
 - & 138 \\
 \hline
 & 0
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract.**
- Repeat.

$$\begin{aligned}
 1 - 0 &= 1 \\
 2 - 0 &= 2 \\
 0 - 3 &= -3 = -10 + 7 \\
 -1 + 1 - 4 &= -4 = -10 + 6 \\
 -1 + 7 - 3 &= 3
 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 1 & 6 \\
 \hline
 343 & \overline{)6\ 1\ 9\ 8\ 21} \\
 - & 3\ 4\ 3 \\
 \hline
 & 2\ 7\ 6\ 8\ 21 \\
 - & 2\ 0\ 5\ 8\ 00 \\
 \hline
 & \underline{-1-1} \\
 & 7\ 1\ 0\ 21 \\
 - & 3\ 4\ 3\ 00 \\
 \hline
 & 3\ 6\ 7\ 21
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 1 - 0 &= 1 \\
 2 - 0 &= 2 \\
 0 - 3 &= -3 = -10 + 7 \\
 -1 + 1 - 4 &= -4 = -10 + 6 \\
 -1 + 7 - 3 &= 3
 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 343 & \overline{)619821} \\ - & 343 \\ \hline & 276 \\ - & 276 \\ \hline & 0 \\ - & 0 \\ \hline & 21 \\ - & 21 \\ \hline & 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 343 & \overline{)619821} \\ - & 343 \\ \hline & 276 \\ - & 276 \\ \hline & 0 \\ - & 0 \\ \hline & 21 \\ - & 21 \\ \hline & 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 343 & \overline{)619821} \\ - & 343 \\ \hline & 276 \\ - & 276 \\ \hline & 0 \\ - & 0 \\ \hline & 21 \\ - & 21 \\ \hline & 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} ? \\ \overline{1} \\ 1 \ 6 \\ \hline 343 \overline{)6 \ 1 \ 9 \ 8 \ 21} \\ - \ 3 \ 4 \ 3 \ 0 \ 00 \\ \hline \ 2 \ 7 \ 6 \ 8 \ 21 \\ - \ 2 \ 0 \ 5 \ 8 \ 00 \\ \hline \ 7 \ 1 \ 0 \ 21 \\ - \ 3 \ 4 \ 3 \ 00 \\ \hline \ 3 \ 6 \ 7 \ 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

## Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

- Find shortest dividend start larger than divisor.
  - Next quotient digit: put above last digit of that start.
    - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
    - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
    - Equal leading digits;
  - Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
  - Subtract.
  - Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} ? \\ \overline{1} \\ 1 \ 6 \\ \hline 343 \overline{)6 \ 1 \ 9 \ 8 \ 21} \\ - \ 3 \ 4 \ 3 \ 0 \ 00 \\ \hline 2 \ 7 \ 6 \ 8 \ 21 \\ - \ 2 \ 0 \ 5 \ 8 \ 00 \\ \hline 7 \ 1 \ 0 \ 21 \\ - \ 3 \ 4 \ 3 \ 00 \\ \hline 3 \ 6 \ 7 \ 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; **larger divisor start has same # of digits**
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 343 & \overline{)619821} \\ - & 343 \\ \hline & 276 \\ - & 276 \\ \hline & 0 \\ - & 0 \\ \hline & 21 \\ - & 21 \\ \hline & 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

## Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & & & & 1 \\
 & & & 1 \\
 & & 1 & 6 \\
 \hline
 343 & - & 6 & 1 & 9 & 8 & 21 \\
 & - & 3 & 4 & 3 & 0 & 00 \\
 \hline
 & - & 2 & 7 & 6 & 8 & 21 \\
 & - & 2 & 0 & 5 & 8 & 00 \\
 \hline
 & - & 7 & 1 & 0 & 21 \\
 & - & 3 & 4 & 3 & 00 \\
 \hline
 & & 3 & 6 & 7 & 21 \\
 & & ? & ? & ? & ?
 \end{array}$$

- Find shortest dividend start larger than divisor.
  - Next quotient digit: put above last digit of that start.
    - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
    - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
    - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
  - Subtract.
  - Repeat.

## Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & & & & 1 \\
 & & & & 1 \\
 & & & 1 & 6 \\
 \hline
 \\[-1.5ex]
 \boxed{\begin{array}{r}
 343 \\
 - 619821 \\
 \hline
 343000 \\
 - 276821 \\
 \hline
 205800 \\
 \\[-1.5ex]
 - 71021 \\
 \hline
 343000 \\
 - 343 \\
 \hline
 36721
 \end{array}}
 \end{array}$$

- Find shortest dividend start larger than divisor.
  - Next quotient digit: put above last digit of that start.
    - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
    - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
    - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - **Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.**
  - Subtract.
  - Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 343 & \overline{)619821} \\ - & 343 \\ \hline & 276821 \\ - & 276 \\ \hline & 205821 \\ - & 205 \\ \hline & 21 \\ - & 21 \\ \hline & 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**
- Subtract.
- Repeat.

## Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & & & & 1 \\
 & & & & 1 \\
 & & & 1 & 6 \\
 \hline
 343 & \overline{)619821} \\
 & - 343 \\
 \hline
 & 276821 \\
 & - 205800 \\
 \hline
 & & 71021 \\
 & - 343 \\
 \hline
 & & 36721 \\
 & - 343 \\
 \hline
 & & 000
 \end{array}$$

- Find shortest dividend start larger than divisor.
  - Next quotient digit: put above last digit of that start.
    - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
    - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
    - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
  - Subtract.
  - Repeat.

## Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & & & 1 \\
 & & & 1 \\
 & & 1 & 6 \\
 \hline
 343 & \overline{-} & 6 & 1 & 9 & 8 & 21 \\
 & - & 3 & 4 & 3 & 0 & 00 \\
 \hline
 & - & 2 & 7 & 6 & 8 & 21 \\
 & - & 2 & 0 & 5 & 8 & 00 \\
 \hline
 & - & 7 & 1 & 0 & 21 \\
 & - & 3 & 4 & 3 & 00 \\
 \hline
 & - & 3 & 6 & 7 & 21 \\
 & - & 3 & 4 & 3 & 00 \\
 \hline
 & & ? & ? & ???
 \end{array}$$

- Find shortest dividend start larger than divisor.
  - Next quotient digit: put above last digit of that start.
    - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
    - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
    - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
  - Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
  - Subtract.
  - Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \\
 343 & \overline{)6\ 1\ 9\ 8\ 21} \\
 - & 3\ 4\ 3\ 0\ 00 \\
 \hline
 & 2\ 7\ 6\ 8\ 21 \\
 - & 2\ 0\ 5\ 8\ 00 \\
 \hline
 & 7\ 1\ 0\ 21 \\
 - & 3\ 4\ 3\ 00 \\
 \hline
 & 3\ 6\ 7\ 21 \\
 - & 3\ 4\ 3\ 00 \\
 \hline
 & 2\ 4\ 21
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
- Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract.**
- Repeat.

$$\begin{aligned}
 1 - 0 &= 1 \\
 2 - 0 &= 2 \\
 7 - 3 &= 4 \\
 6 - 4 &= 2 \\
 3 - 3 &= 0
 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \\
 343 & \overline{)6\ 1\ 9\ 8\ 21} \\
 - & 3\ 4\ 3\ 0\ 00 \\
 \hline
 & 2\ 7\ 6\ 8\ 21 \\
 - & 2\ 0\ 5\ 8\ 00 \\
 \hline
 & 7\ 1\ 0\ 21 \\
 - & 3\ 4\ 3\ 00 \\
 \hline
 & 3\ 6\ 7\ 21 \\
 - & 3\ 4\ 3\ 00 \\
 \hline
 & 2\ 4\ 21
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
- Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 1 - 0 &= 1 \\
 2 - 0 &= 2 \\
 7 - 3 &= 4 \\
 6 - 4 &= 2 \\
 3 - 3 &= 0
 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 1 \\ 343 & \overline{)6\ 1\ 9\ 8\ 21} \\ - & 3\ 4\ 3 \\ \hline & 2\ 7\ 6\ 8\ 21 \\ - & 2\ 0\ 5\ 8\ 00 \\ \hline & 7\ 1\ 0\ 21 \\ - & 3\ 4\ 3\ 00 \\ \hline & 3\ 6\ 7\ 21 \\ - & 3\ 4\ 3\ 00 \\ \hline & 2\ 4\ 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 1 \\ 1 & 6 \\ \hline 343 & \left[ \begin{array}{r} 6 & 1 & 9 & 8 & 21 \\ - & 3 & 4 & 3 & 0 \\ \hline 2 & 7 & 6 & 8 & 21 \\ - & 2 & 0 & 5 & 8 \\ \hline 2 & 1 & 0 & 21 \\ - & 3 & 4 & 3 \\ \hline 3 & 6 & 7 & 21 \\ - & 3 & 4 & 3 \\ \hline 2 & 4 & 21 \end{array} \right] \\ & 2 & 4 & 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 1 \\ 343 & \overline{)6\ 1\ 9\ 8\ 21} \\ - & 3\ 4\ 3 \\ \hline & 2\ 7\ 6\ 8\ 21 \\ - & 2\ 0\ 5\ 8\ 00 \\ \hline & 7\ 1\ 0\ 21 \\ - & 3\ 4\ 3\ 00 \\ \hline & 3\ 6\ 7\ 21 \\ - & 3\ 4\ 3\ 00 \\ \hline & 2\ 4\ 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 1 \\ 343 & \overline{)6\ 1\ 9\ 8\ 21} \\ - & 3\ 4\ 3 \\ \hline & 2\ 7\ 6\ 8\ 21 \\ - & 2\ 0\ 5\ 8\ 00 \\ \hline & 7\ 1\ 0\ 21 \\ - & 3\ 4\ 3\ 00 \\ \hline & 3\ 6\ 7\ 21 \\ - & 3\ 4\ 3\ 00 \\ \hline & 2\ 4\ 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 1 \\ 1 & 6 \quad 0? \\ \hline 343 \overline{)619821} \\ - \quad 343 \\ \hline 276821 \\ - \quad 205800 \\ \hline 71021 \\ - \quad 34300 \\ \hline 36721 \\ - \quad 34300 \\ \hline 2421 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. **If this leaves gaps: fill with 0.**
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 1 \\ 1 & 6 \ 0? \\ \hline \end{array}$$

343

$$\begin{array}{r} 6 \ 1 \ 9 \ 8 \ 21 \\ - 3 \ 4 \ 3 \ 0 \ 00 \\ \hline 2 \ 7 \ 6 \ 8 \ 21 \\ - 2 \ 0 \ 5 \ 8 \ 00 \\ \hline 7 \ 1 \ 0 \ 21 \\ - 3 \ 4 \ 3 \ 00 \\ \hline 3 \ 6 \ 7 \ 21 \\ - 3 \ 4 \ 3 \ 00 \\ \hline 2 \ 4 \ 21 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \\
 343 & \overline{)6\ 1\ 9\ 8\ 21} \\
 - & 3\ 4\ 3 \\
 \hline
 & 2\ 7\ 6\ 8\ 21 \\
 - & 2\ 0\ 5\ 8\ 00 \\
 \hline
 & 7\ 1\ 0\ 21 \\
 - & 3\ 4\ 3\ 00 \\
 \hline
 & 3\ 6\ 7\ 21 \\
 - & 3\ 4\ 3\ 00 \\
 \hline
 & 2\ 4\ 21
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{24}{3+1} \right] = ?$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \\
 343 & \overline{)6\ 1\ 9\ 8\ 21} \\
 - & 3\ 4\ 3\ 0\ 00 \\
 \hline
 & 2\ 7\ 6\ 8\ 21 \\
 - & 2\ 0\ 5\ 8\ 00 \\
 \hline
 & 7\ 1\ 0\ 21 \\
 - & 3\ 4\ 3\ 00 \\
 \hline
 & 3\ 6\ 7\ 21 \\
 - & 3\ 4\ 3\ 00 \\
 \hline
 & 2\ 4\ 21
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{24}{3+1} \right\rfloor = \left\lfloor \frac{24}{4} \right\rfloor = \lfloor 6 \rfloor = 6$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \\
 343 & \overline{)6\ 1\ 9\ 8\ 21} \\
 - & 3\ 4\ 3\ 0\ 00 \\
 \hline
 & 2\ 7\ 6\ 8\ 21 \\
 - & 2\ 0\ 5\ 8\ 00 \\
 \hline
 & 7\ 1\ 0\ 21 \\
 - & 3\ 4\ 3\ 00 \\
 \hline
 & 3\ 6\ 7\ 21 \\
 - & 3\ 4\ 3\ 00 \\
 \hline
 & 2\ 4\ 21 \\
 & \color{red}?\ ?\ ??
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \\
 1 & 6 & 06 \\
 \hline
 221 & \left[ \begin{array}{r} 6 & 1 & 9 & 8 & 21 \\ - 343 & \hline & 3 & 4 & 3 & 0 & 00 \\ - 276 & \hline & 2 & 7 & 6 & 8 & 21 \\ - 205 & \hline & 2 & 0 & 5 & 8 & 00 \\ \hline & 7 & 1 & 0 & 21 \\ - 343 & \hline & 3 & 6 & 7 & 21 \\ - 343 & \hline & 2 & 0 & 58 \end{array} \right] \\
 343 & \hline
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 6 \cdot 3 &= 18 \\
 6 \cdot 4 + 1 &= 25 \\
 6 \cdot 3 + 2 &= 20
 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 1 \\ 1 & 6 \ 06 \\ \hline \end{array}$$
$$\begin{array}{r} 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4\ 3 \\ \hline \ 2\ 7\ 6\ 8\ 21 \\ - \ 2\ 0\ 5\ 8\ 00 \\ \hline \ 7\ 1\ 0\ 21 \\ - \ 3\ 4\ 3\ 00 \\ \hline \ 3\ 6\ 7\ 21 \\ - \ 3\ 4\ 3\ 00 \\ \hline \ 2\ 4\ 21 \\ - \ 2\ 0\ 58 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 1 \\ 343 & \overline{)6\ 1\ 9\ 8\ 21} \\ - & 3\ 4\ 3 \\ \hline & 2\ 7\ 6\ 8\ 21 \\ - & 2\ 0\ 5\ 8\ 00 \\ \hline & 7\ 1\ 0\ 21 \\ - & 3\ 4\ 3\ 00 \\ \hline & 3\ 6\ 7\ 21 \\ - & 3\ 4\ 3\ 00 \\ \hline & 2\ 4\ 21 \\ & 2\ 0\ 58 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 1 \\ 1 & 6 \ 06 \\ \hline \end{array}$$
$$\begin{array}{r} 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4\ 3 \\ \hline 2\ 7\ 6\ 8\ 21 \\ - \ 2\ 0\ 5\ 8\ 00 \\ \hline 7\ 1\ 0\ 21 \\ - \ 3\ 4\ 3\ 00 \\ \hline 3\ 6\ 7\ 21 \\ - \ 3\ 4\ 3\ 00 \\ \hline 2\ 4\ 21 \\ - \ 2\ 0\ 58 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \\
 1 & 6 & 06 \\
 \hline
 343 \overline{)619821} & & \\
 - & 6 & 1 & 9 & 8 & 21 \\
 & 3 & 4 & 3 & 0 & 00 \\
 \hline
 & 2 & 7 & 6 & 8 & 21 \\
 - & 2 & 0 & 5 & 8 & 00 \\
 \hline
 & 7 & 1 & 0 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 3 & 6 & 7 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 2 & 4 & 21 \\
 - & 2 & 0 & 58 \\
 \hline
 & & ? & ?? \\
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \\
 1 & 6 \quad 06 \\
 \hline
 343 \overline{)6\ 1\ 9\ 8\ 21} \\
 - \quad 3\ 4\ 3\ 0\ 00 \\
 \hline
 & 2\ 7\ 6\ 8\ 21 \\
 - \quad 2\ 0\ 5\ 8\ 00 \\
 \hline
 & 7\ 1\ 0\ 21 \\
 - \quad 3\ 4\ 3\ 00 \\
 \hline
 & 3\ 6\ 7\ 21 \\
 - \quad 3\ 4\ 3\ 00 \\
 \hline
 & \color{red}{-1-1} \\
 - \quad & 2\ 4\ 21 \\
 \hline
 & 2\ 0\ 58 \\
 \hline
 & 3\ 63
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract.**
- Repeat.

$$\begin{aligned}
 1 - 8 &= -7 = -10 + 3 \\
 -1 + 2 - 5 &= -4 = -10 + 6 \\
 -1 + 4 - 0 &= 3 \\
 2 - 2 &= 0
 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \\
 1 & 6 \quad 06 \\
 \hline
 343 \overline{)6\ 1\ 9\ 8\ 21} \\
 - \quad 3\ 4\ 3\ 0\ 00 \\
 \hline
 & 2\ 7\ 6\ 8\ 21 \\
 - \quad 2\ 0\ 5\ 8\ 00 \\
 \hline
 & 7\ 1\ 0\ 21 \\
 - \quad 3\ 4\ 3\ 00 \\
 \hline
 & 3\ 6\ 7\ 21 \\
 - \quad 3\ 4\ 3\ 00 \\
 \hline
 & \textcolor{red}{-1-1} \\
 - \quad 2\ 4\ 21 \\
 \hline
 & 2\ 0\ 58 \\
 \hline
 & 3\ 63
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 1 - 8 &= -7 = -10 + 3 \\
 -1 + 2 - 5 &= -4 = -10 + 6 \\
 -1 + 4 - 0 &= 3 \\
 2 - 2 &= 0
 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 1 \\ 1 & 6 \ 06 \\ \hline \end{array}$$
$$\begin{array}{r} 343 \overline{)6\ 1\ 9\ 8\ 21} \\ - \ 3\ 4\ 3 \\ \hline 2\ 7\ 6\ 8\ 21 \\ - \ 2\ 0\ 5\ 8\ 00 \\ \hline 7\ 1\ 0\ 21 \\ - \ 3\ 4\ 3\ 00 \\ \hline 3\ 6\ 7\ 21 \\ - \ 3\ 4\ 3\ 00 \\ \hline 2\ 4\ 21 \\ - \ 2\ 0\ 58 \\ \hline 3\ 63 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 1 \\ 343 & \overline{)6\ 1\ 9\ 8\ 21} \\ - & 3\ 4\ 3 \\ \hline & 2\ 7\ 6\ 8\ 21 \\ - & 2\ 0\ 5\ 8\ 00 \\ \hline & 7\ 1\ 0\ 21 \\ - & 3\ 4\ 3\ 00 \\ \hline & 3\ 6\ 7\ 21 \\ - & 3\ 4\ 3\ 00 \\ \hline & 2\ 4\ 21 \\ - & 2\ 0\ 58 \\ \hline & 3\ 63 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \\
 1 & 6 \quad 06 \\
 \hline
 343 \overline{)6\ 1\ 9\ 8\ 21} \\
 - \quad 3\ 4\ 3\ 0\ 00 \\
 \hline
 & 2\ 7\ 6\ 8\ 21 \\
 - \quad 2\ 0\ 5\ 8\ 00 \\
 \hline
 & 7\ 1\ 0\ 21 \\
 - \quad 3\ 4\ 3\ 00 \\
 \hline
 & 3\ 6\ 7\ 21 \\
 - \quad 3\ 4\ 3\ 00 \\
 \hline
 & 2\ 4\ 21 \\
 - \quad 2\ 0\ 58 \\
 \hline
 & 3\ 63
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \quad ? \\
 1 \quad 6 \quad 06 \\
 \hline
 343 \overline{)6 \quad 1 \quad 9 \quad 8 \quad 21} \\
 - \quad 3 \quad 4 \quad 3 \quad 0 \quad 00 \\
 \hline
 & 2 \quad 7 \quad 6 \quad 8 \quad 21 \\
 - \quad 2 \quad 0 \quad 5 \quad 8 \quad 00 \\
 \hline
 & 7 \quad 1 \quad 0 \quad 21 \\
 - \quad 3 \quad 4 \quad 3 \quad 00 \\
 \hline
 & 3 \quad 6 \quad 7 \quad 21 \\
 - \quad 3 \quad 4 \quad 3 \quad 00 \\
 \hline
 & 2 \quad 4 \quad 21 \\
 - \quad 2 \quad 0 \quad 58 \\
 \hline
 & 3 \quad 63
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 343 & \overline{)619821} \\ - & 343 \\ \hline & 276821 \\ - & 276 \\ \hline & 205821 \\ - & 205 \\ \hline & 21 \\ - & 20 \\ \hline & 1 \\ - & 0 \\ \hline & 21 \\ - & 20 \\ \hline & 1 \\ - & 0 \\ \hline & 58 \\ - & 56 \\ \hline & 21 \\ - & 20 \\ \hline & 1 \\ - & 0 \\ \hline & 63 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. **If this leaves gaps: fill with 0.**
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \text{ } 0\text{?} \\
 1 \text{ } 6 \text{ } 06 \\
 \hline
 343 \overline{)6 \text{ } 1 \text{ } 9 \text{ } 8 \text{ } 21} \\
 - \quad 3 \text{ } 4 \text{ } 3 \text{ } 0 \text{ } 00 \\
 \hline
 & 2 \text{ } 7 \text{ } 6 \text{ } 8 \text{ } 21 \\
 - \quad 2 \text{ } 0 \text{ } 5 \text{ } 8 \text{ } 00 \\
 \hline
 & 7 \text{ } 1 \text{ } 0 \text{ } 21 \\
 - \quad 3 \text{ } 4 \text{ } 3 \text{ } 00 \\
 \hline
 & 3 \text{ } 6 \text{ } 7 \text{ } 21 \\
 - \quad 3 \text{ } 4 \text{ } 3 \text{ } 00 \\
 \hline
 & 2 \text{ } 4 \text{ } 21 \\
 - \quad 2 \text{ } 0 \text{ } 58 \\
 \hline
 & 3 \text{ } 63
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 01 \\ 1 \ 6 \ 06 \\ \hline \\ 343 \overline{)6 \ 1 \ 9 \ 8 \ 21} \\ - \ 3 \ 4 \ 3 \ 0 \ 00 \\ \hline \ 2 \ 7 \ 6 \ 8 \ 21 \\ - \ 2 \ 0 \ 5 \ 8 \ 00 \\ \hline \\ - \ 7 \ 1 \ 0 \ 21 \\ - \ 3 \ 4 \ 3 \ 00 \\ \hline \ 3 \ 6 \ 7 \ 21 \\ - \ 3 \ 4 \ 3 \ 00 \\ \hline \\ - \ 2 \ 4 \ 21 \\ - \ 2 \ 0 \ 58 \\ \hline \ 3 \ 63 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
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  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \text{ } 01 \\
 1 \text{ } 6 \text{ } 06 \\
 \hline
 343 \overline{)6 \text{ } 1 \text{ } 9 \text{ } 8 \text{ } 21} \\
 - \quad 3 \text{ } 4 \text{ } 3 \text{ } 0 \text{ } 00 \\
 \hline
 & 2 \text{ } 7 \text{ } 6 \text{ } 8 \text{ } 21 \\
 - \quad 2 \text{ } 0 \text{ } 5 \text{ } 8 \text{ } 00 \\
 \hline
 & 7 \text{ } 1 \text{ } 0 \text{ } 21 \\
 - \quad 3 \text{ } 4 \text{ } 3 \text{ } 00 \\
 \hline
 & 3 \text{ } 6 \text{ } 7 \text{ } 21 \\
 - \quad 3 \text{ } 4 \text{ } 3 \text{ } 00 \\
 \hline
 & 2 \text{ } 4 \text{ } 21 \\
 - \quad 2 \text{ } 0 \text{ } 58 \\
 \hline
 & 3 \text{ } 63 \\
 \text{? } \text{??}
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 343 & \overline{)619821} \\ - & 343 \\ \hline & 276821 \\ - & 276 \\ \hline & 21 \\ - & 21 \\ \hline & 0 \\ - & 0 \\ \hline & 21 \\ - & 21 \\ \hline & 0 \\ - & 0 \\ \hline & 21 \\ - & 21 \\ \hline & 0 \\ - & 0 \\ \hline & 63 \\ - & 63 \\ \hline & 0 \\ - & 0 \\ \hline & 43 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 343 & \overline{)619821} \\ - & 343 \\ \hline & 276821 \\ - & 276 \\ \hline & 205800 \\ - & 205 \\ \hline & 21 \\ - & 34 \\ \hline & 36721 \\ - & 34 \\ \hline & 2421 \\ - & 20 \\ \hline & 421 \\ - & 34 \\ \hline & 363 \\ - & 34 \\ \hline & 43 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \ 01 \\
 1 \ 6 \ 06 & \overline{-} \\
 343 \overline{)6 \ 1 \ 9 \ 8 \ 21} \\
 - 3 \ 4 \ 3 \ 0 \ 00 \\
 \hline
 2 \ 7 \ 6 \ 8 \ 21 \\
 - 2 \ 0 \ 5 \ 8 \ 00 \\
 \hline
 & 7 \ 1 \ 0 \ 21 \\
 - 3 \ 4 \ 3 \ 00 \\
 \hline
 3 \ 6 \ 7 \ 21 \\
 - 3 \ 4 \ 3 \ 00 \\
 \hline
 & 2 \ 4 \ 21 \\
 - 2 \ 0 \ 58 \\
 \hline
 & 3 \ 63 \\
 - 3 \ 43 \\
 \hline
 & ?? \\
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \quad 01 \\
 1 \quad 6 \quad 06 \\
 \hline
 343 \overline{)6 \quad 1 \quad 9 \quad 8 \quad 21} \\
 - \quad 3 \quad 4 \quad 3 \quad 0 \quad 00 \\
 \hline
 & 2 \quad 7 \quad 6 \quad 8 \quad 21 \\
 - \quad 2 \quad 0 \quad 5 \quad 8 \quad 00 \\
 \hline
 & 7 \quad 1 \quad 0 \quad 21 \\
 - \quad 3 \quad 4 \quad 3 \quad 00 \\
 \hline
 & 3 \quad 6 \quad 7 \quad 21 \\
 - \quad 3 \quad 4 \quad 3 \quad 00 \\
 \hline
 & 2 \quad 4 \quad 21 \\
 - \quad 2 \quad 0 \quad 58 \\
 \hline
 & 3 \quad 63 \\
 - \quad 3 \quad 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract.**
- Repeat.

$$\begin{aligned}
 3 - 3 &= 0 \\
 6 - 4 &= 2 \\
 3 - 3 &= 0
 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \quad 01 \\
 1 \quad 6 \quad 06 \\
 \hline
 343 \overline{)6 \quad 1 \quad 9 \quad 8 \quad 21} \\
 - \quad 3 \quad 4 \quad 3 \quad 0 \quad 00 \\
 \hline
 & 2 \quad 7 \quad 6 \quad 8 \quad 21 \\
 - \quad 2 \quad 0 \quad 5 \quad 8 \quad 00 \\
 \hline
 & 7 \quad 1 \quad 0 \quad 21 \\
 - \quad 3 \quad 4 \quad 3 \quad 00 \\
 \hline
 & 3 \quad 6 \quad 7 \quad 21 \\
 - \quad 3 \quad 4 \quad 3 \quad 00 \\
 \hline
 & 2 \quad 4 \quad 21 \\
 - \quad 2 \quad 0 \quad 58 \\
 \hline
 & 3 \quad 63 \\
 - \quad 3 \quad 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$3 - 3 = 0$$

$$6 - 4 = 2$$

$$3 - 3 = 0$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \quad 01 \\
 1 \quad 6 \quad 06 \\
 \hline
 343 \overline{)6 \quad 1 \quad 9 \quad 8 \quad 21} \\
 - \quad 3 \quad 4 \quad 3 \quad 0 \quad 00 \\
 \hline
 & 2 \quad 7 \quad 6 \quad 8 \quad 21 \\
 - \quad 2 \quad 0 \quad 5 \quad 8 \quad 00 \\
 \hline
 & 7 \quad 1 \quad 0 \quad 21 \\
 - \quad 3 \quad 4 \quad 3 \quad 00 \\
 \hline
 & 3 \quad 6 \quad 7 \quad 21 \\
 - \quad 3 \quad 4 \quad 3 \quad 00 \\
 \hline
 & 2 \quad 4 \quad 21 \\
 - \quad 2 \quad 0 \quad 58 \\
 \hline
 & 3 \quad 63 \\
 - \quad 3 \quad 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \quad 01 \\
 1 \quad 6 \quad 06 \\
 \hline
 343 \overline{)6 \quad 1 \quad 9 \quad 8 \quad 21} \\
 - \quad 3 \quad 4 \quad 3 \quad 0 \quad 00 \\
 \hline
 & 2 \quad 7 \quad 6 \quad 8 \quad 21 \\
 - \quad 2 \quad 0 \quad 5 \quad 8 \quad 00 \\
 \hline
 & 7 \quad 1 \quad 0 \quad 21 \\
 - \quad 3 \quad 4 \quad 3 \quad 00 \\
 \hline
 & 3 \quad 6 \quad 7 \quad 21 \\
 - \quad 3 \quad 4 \quad 3 \quad 00 \\
 \hline
 & 2 \quad 4 \quad 21 \\
 - \quad 2 \quad 0 \quad 58 \\
 \hline
 & 3 \quad 63 \\
 - \quad 3 \quad 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 01 \\ 1 \ 6 \ 06 \end{array}$$

$$\begin{array}{r} & 6 & 1 & 9 & 8 & 21 \\ 343 & - & 3 & 4 & 3 & 0 \ 00 \\ & \underline{-} & 2 & 7 & 6 & 8 \ 21 \\ & - & 2 & 0 & 5 & 8 \ 00 \end{array}$$

$$\begin{array}{r} & 7 & 1 & 0 & 21 \\ - & 3 & 4 & 3 & 00 \\ \hline & 3 & 6 & 7 & 21 \\ - & 3 & 4 & 3 & 00 \\ \hline \end{array}$$

$$\begin{array}{r} & 2 & 4 & 21 \\ - & 2 & 0 & 58 \\ \hline & 3 & 63 \\ - & 3 & 43 \\ \hline & 20 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 1 \text{ } 00 \\
 1 \text{ } 01 \\
 1 \text{ } 6 \text{ } 06
 \end{array}$$

$$\begin{array}{r}
 343 \overline{)6 \text{ } 1 \text{ } 9 \text{ } 8 \text{ } 21} \\
 - \text{ } 3 \text{ } 4 \text{ } 3 \text{ } 0 \text{ } 00 \\
 \hline
 2 \text{ } 7 \text{ } 6 \text{ } 8 \text{ } 21 \\
 - \text{ } 2 \text{ } 0 \text{ } 5 \text{ } 8 \text{ } 00
 \end{array}$$

$$\begin{array}{r}
 7 \text{ } 1 \text{ } 0 \text{ } 21 \\
 - \text{ } 3 \text{ } 4 \text{ } 3 \text{ } 00 \\
 \hline
 3 \text{ } 6 \text{ } 7 \text{ } 21 \\
 - \text{ } 3 \text{ } 4 \text{ } 3 \text{ } 00
 \end{array}$$
  

$$\begin{array}{r}
 2 \text{ } 4 \text{ } 21 \\
 - \text{ } 2 \text{ } 0 \text{ } 58 \\
 \hline
 3 \text{ } 63 \\
 - \text{ } 3 \text{ } 43 \\
 \hline
 20
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- **If any, fill gaps in quotients with zeroes.**

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \text{ } 00 \\
 + & 1 \text{ } 01 \\
 \hline
 & 1 \text{ } 6 \text{ } 06 \\
 \\ 
 \boxed{343} & \overline{)6 \text{ } 1 \text{ } 9 \text{ } 8 \text{ } 21} \\
 - & 3 \text{ } 4 \text{ } 3 \text{ } 0 \text{ } 00 \\
 \hline
 & 2 \text{ } 7 \text{ } 6 \text{ } 8 \text{ } 21 \\
 - & 2 \text{ } 0 \text{ } 5 \text{ } 8 \text{ } 00 \\
 \hline
 & 7 \text{ } 1 \text{ } 0 \text{ } 21 \\
 - & 3 \text{ } 4 \text{ } 3 \text{ } 00 \\
 \hline
 & 3 \text{ } 6 \text{ } 7 \text{ } 21 \\
 - & 3 \text{ } 4 \text{ } 3 \text{ } 00 \\
 \hline
 & 2 \text{ } 4 \text{ } 21 \\
 - & 2 \text{ } 0 \text{ } 58 \\
 \hline
 & 3 \text{ } 63 \\
 - & 3 \text{ } 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If any, fill gaps in quotients with zeroes.
- **If more than one quotient row, add them.**

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \text{ } 00 \\
 + & 1 \text{ } 01 \\
 \hline
 & 1 \text{ } 6 \text{ } 06 \\
 \hline
 & 7 \\
 343 \overline{)6 \text{ } 1 \text{ } 9 \text{ } 8 \text{ } 21} & \\
 - & 3 \text{ } 4 \text{ } 3 \text{ } 0 \text{ } 00 \\
 \hline
 & 2 \text{ } 7 \text{ } 6 \text{ } 8 \text{ } 21 \\
 - & 2 \text{ } 0 \text{ } 5 \text{ } 8 \text{ } 00 \\
 \hline
 & 7 \text{ } 1 \text{ } 0 \text{ } 21 \\
 - & 3 \text{ } 4 \text{ } 3 \text{ } 00 \\
 \hline
 & 3 \text{ } 6 \text{ } 7 \text{ } 21 \\
 - & 3 \text{ } 4 \text{ } 3 \text{ } 00 \\
 \hline
 & 2 \text{ } 4 \text{ } 21 \\
 - & 2 \text{ } 0 \text{ } 58 \\
 \hline
 & 3 \text{ } 63 \\
 - & 3 \text{ } 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If any, fill gaps in quotients with zeroes.
- If more than one quotient row, add them.

$$6 + 1 + 0 = 7$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 \text{ } 00 \\
 + & 1 \text{ } 01 \\
 \hline
 & 1 \text{ } 6 \text{ } 06 \\
 \hline
 & 07 \\
 \\ 
 343 \overline{)6 \text{ } 1 \text{ } 9 \text{ } 8 \text{ } 21} \\
 - & 3 \text{ } 4 \text{ } 3 \text{ } 0 \text{ } 00 \\
 \hline
 & 2 \text{ } 7 \text{ } 6 \text{ } 8 \text{ } 21 \\
 - & 2 \text{ } 0 \text{ } 5 \text{ } 8 \text{ } 00 \\
 \hline
 & 7 \text{ } 1 \text{ } 0 \text{ } 21 \\
 - & 3 \text{ } 4 \text{ } 3 \text{ } 00 \\
 \hline
 & 3 \text{ } 6 \text{ } 7 \text{ } 21 \\
 - & 3 \text{ } 4 \text{ } 3 \text{ } 00 \\
 \hline
 & 2 \text{ } 4 \text{ } 21 \\
 - & 2 \text{ } 0 \text{ } 58 \\
 \hline
 & 3 \text{ } 63 \\
 - & 3 \text{ } 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If any, fill gaps in quotients with zeroes.
- If more than one quotient row, add them.

$$\begin{aligned}
 6 + 1 + 0 &= 7 \\
 \textcolor{red}{0 + 0 + 0} &= \textcolor{red}{0}
 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 00 \\
 + & 1 & 01 \\
 & 1 & 6 & 06 \\
 \hline
 & 8 & 07 \\
 \\ 
 343 \overline{)6} & 1 & 9 & 8 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 2 & 7 & 6 & 8 & 21 \\
 - & 2 & 0 & 5 & 8 & 00 \\
 \hline
 & 7 & 1 & 0 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 3 & 6 & 7 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 2 & 4 & 21 \\
 - & 2 & 0 & 58 \\
 \hline
 & 3 & 63 \\
 - & 3 & 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If any, fill gaps in quotients with zeroes.
- If more than one quotient row, add them.

$$\begin{aligned}
 6 + 1 + 0 &= 7 \\
 0 + 0 + 0 &= 0 \\
 \textcolor{red}{6 + 1 + 1} &= \textcolor{red}{8}
 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 00 \\
 + & & 1 & 01 \\
 & 1 & 6 & 06 \\
 \hline
 & 1 & 8 & 07 \\
 \\ 
 343 \overline{)6} & 1 & 9 & 8 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 2 & 7 & 6 & 8 & 21 \\
 - & 2 & 0 & 5 & 8 & 00 \\
 \hline
 & 7 & 1 & 0 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 3 & 6 & 7 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 2 & 4 & 21 \\
 - & 2 & 0 & 58 \\
 \hline
 & 3 & 63 \\
 - & 3 & 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If any, fill gaps in quotients with zeroes.
- If more than one quotient row, add them.

$$\begin{aligned}
 6 + 1 + 0 &= 7 \\
 0 + 0 + 0 &= 0 \\
 6 + 1 + 1 &= 8 \\
 \textcolor{red}{1} &= 1
 \end{aligned}$$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 0 \\
 + & & 1 & 0 & 1 \\
 & 1 & 6 & 0 & 6 \\
 \hline
 & 1 & 8 & 0 & 7 \\
 \\ 
 343 \overline{)6\ 1\ 9\ 8\ 2\ 1} \\
 - & 3 & 4 & 3 & 0 & 00 \\
 \hline
 & 2 & 7 & 6 & 8 & 21 \\
 - & 2 & 0 & 5 & 8 & 00 \\
 \hline
 & 7 & 1 & 0 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 3 & 6 & 7 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 2 & 4 & 21 \\
 - & 2 & 0 & 58 \\
 \hline
 & 3 & 63 \\
 - & 3 & 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If any, fill gaps in quotients with zeroes.
- If more than one quotient row, add them.
- Final answer. **Dividend = 619821**. Divisor = 343. Quotient = 1807. Remainder = 20.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 0 \\
 + & & 1 & 0 & 1 \\
 & 1 & 6 & 0 & 6 \\
 \hline
 & 1 & 8 & 0 & 7 \\
 \\ 
 \boxed{343} & 6 & 1 & 9 & 8 & 21 \\
 - & 3 & 4 & 3 & 0 & 00 \\
 \hline
 & 2 & 7 & 6 & 8 & 21 \\
 - & 2 & 0 & 5 & 8 & 00 \\
 \hline
 & 7 & 1 & 0 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 3 & 6 & 7 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 2 & 4 & 21 \\
 - & 2 & 0 & 58 \\
 \hline
 & 3 & 63 \\
 - & 3 & 43 \\
 \hline
 & 20
 \end{array}$$

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  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
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- Subtract.
- Repeat.
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  - If more than one quotient row, add them.
- Final answer. Dividend = 619821. **Divisor = 343**. Quotient = 1807. Remainder = 20.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 0 \\
 + & & 1 & 0 & 1 \\
 & 1 & 6 & 0 & 6 \\
 \hline
 & 1 & 8 & 0 & 7 \\
 \\ 
 343 \overline{)6} & 1 & 9 & 8 & 21 \\
 - & 3 & 4 & 3 & 0 & 00 \\
 \hline
 & 2 & 7 & 6 & 8 & 21 \\
 - & 2 & 0 & 5 & 8 & 00 \\
 \hline
 & 7 & 1 & 0 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 3 & 6 & 7 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 2 & 4 & 21 \\
 - & 2 & 0 & 58 \\
 \hline
 & 3 & 63 \\
 - & 3 & 43 \\
 \hline
 & 20
 \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
  - If any, fill gaps in quotients with zeroes.
  - If more than one quotient row, add them.
- Final answer. Dividend = 619821. Divisor = 343. **Quotient = 1807**. Remainder = 20.

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 0 \\
 + & & 1 & 0 & 1 \\
 & 1 & 6 & 0 & 6 \\
 \hline
 & 1 & 8 & 0 & 7 \\
 \\ 
 343 \overline{)6} & 1 & 9 & 8 & 21 \\
 - & 3 & 4 & 3 & 0 & 00 \\
 \hline
 & 2 & 7 & 6 & 8 & 21 \\
 - & 2 & 0 & 5 & 8 & 00 \\
 \hline
 & 7 & 1 & 0 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 3 & 6 & 7 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 2 & 4 & 21 \\
 - & 2 & 0 & 58 \\
 \hline
 & 3 & 63 \\
 - & 3 & 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
  - If any, fill gaps in quotients with zeroes.
  - If more than one quotient row, add them.
- Final answer. Dividend = 619821. Divisor = 343. Quotient = 1807.  
**Remainder = 20.**

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 0 \\
 + & & 1 & 0 & 1 \\
 & 1 & 6 & 0 & 6 \\
 \hline
 & 1 & 8 & 0 & 7 \\
 \\ 
 343 \overline{)6\ 1\ 9\ 8\ 2\ 1} \\
 - & 3 & 4 & 3 & 0 & 00 \\
 \hline
 & 2 & 7 & 6 & 8 & 21 \\
 - & 2 & 0 & 5 & 8 & 00 \\
 \hline
 & 7 & 1 & 0 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 3 & 6 & 7 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 2 & 4 & 21 \\
 - & 2 & 0 & 58 \\
 \hline
 & 3 & 63 \\
 - & 3 & 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If any, fill gaps in quotients with zeroes.
- If more than one quotient row, add them.
- Final answer. **Dividend = 619821**. Divisor = 343. Quotient = 1807. Remainder = 20.
- Check.  $619821 \stackrel{?}{=} 343 \cdot 1807 + 20$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 0 \\
 + & & 1 & 0 & 1 \\
 & 1 & 6 & 0 & 6 \\
 \hline
 & 1 & 8 & 0 & 7 \\
 \\ 
 \boxed{343} & 6 & 1 & 9 & 8 & 21 \\
 - & 3 & 4 & 3 & 0 & 00 \\
 \hline
 & 2 & 7 & 6 & 8 & 21 \\
 - & 2 & 0 & 5 & 8 & 00 \\
 \hline
 & 7 & 1 & 0 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 3 & 6 & 7 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 2 & 4 & 21 \\
 - & 2 & 0 & 58 \\
 \hline
 & 3 & 63 \\
 - & 3 & 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If any, fill gaps in quotients with zeroes.
- If more than one quotient row, add them.
- Final answer. Dividend = 619821. **Divisor = 343**. Quotient = 1807. Remainder = 20.
- Check.  $619821 \stackrel{?}{=} 343 \cdot 1807 + 20$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 0 & 0 \\
 + & & 1 & 0 & 1 \\
 & 1 & 6 & 0 & 6 \\
 \hline
 & 1 & 8 & 0 & 7 \\
 \\ 
 343 \overline{)6} & 1 & 9 & 8 & 21 \\
 - & 3 & 4 & 3 & 0 & 00 \\
 \hline
 & 2 & 7 & 6 & 8 & 21 \\
 - & 2 & 0 & 5 & 8 & 00 \\
 \hline
 & 7 & 1 & 0 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 3 & 6 & 7 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 2 & 4 & 21 \\
 - & 2 & 0 & 58 \\
 \hline
 & 3 & 63 \\
 - & 3 & 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If any, fill gaps in quotients with zeroes.
- If more than one quotient row, add them.
- Final answer. Dividend = 619821. Divisor = 343. Quotient = 1807. Remainder = 20.
- Check.  $619821 \stackrel{?}{=} 343 \cdot 1807 + 20$

# Full algorithm: a large example

Divide 619821 by 343 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 00 \\
 + & 1 & 01 \\
 & 1 & 6 & 06 \\
 \hline
 & 1 & 8 & 07 \\
 \\ 
 343 \overline{)6} & 1 & 9 & 8 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 2 & 7 & 6 & 8 & 21 \\
 - & 2 & 0 & 5 & 8 & 00 \\
 \hline
 & 7 & 1 & 0 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 3 & 6 & 7 & 21 \\
 - & 3 & 4 & 3 & 00 \\
 \hline
 & 2 & 4 & 21 \\
 - & 2 & 0 & 58 \\
 \hline
 & 3 & 63 \\
 - & 3 & 43 \\
 \hline
 & 20
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
  - If any, fill gaps in quotients with zeroes.
  - If more than one quotient row, add them.
- Final answer. Dividend = 619821. Divisor = 343. Quotient = 1807.  
**Remainder = 20.**
- Check.  $619821 \stackrel{?}{=} 343 \cdot 1807 + 20$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & \boxed{1\ 0\ 0\ 0\ 0\ 00} \\ 2019 & \end{array}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & \boxed{ } \\ 2019 & | 1 \ 0 \ 0 \ 0 \ 0 \ 00 \end{array}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & \boxed{ } \\ 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 00} \end{array}$$

- Find shortest dividend start larger than divisor.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 2019 \quad 1 \ 0 \ 0 \ 0 \ 0 \ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 2019 \quad 1 \ 0 \ 0 \ 0 \ 0 \ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 00} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

$$\left[ \frac{10}{2+1} \right]$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 2019 \quad 1 \ 0 \ 0 \ 0 \ 0 \ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits **by divisor digit** plus one. Round down if needed.

$$\left[ \frac{10}{2+1} \right]$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 2019 \quad 1 \ 0 \ 0 \ 0 \ 0 \ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit **plus one**. Round down if needed.

$$\left[ \frac{10}{2+1} \right]$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 2019 \quad 1 \ 0 \ 0 \ 0 \ 0 \ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. **Round down if needed.**

$$\left[ \frac{10}{2+1} \right]$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 00} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

$$\left[ \frac{10}{2+1} \right] = ?$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 3 \\ \hline 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 00} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

$$\left[ \frac{10}{2+1} \right] = \left[ \frac{10}{3} \right] = \left[ \frac{9}{3} + \frac{1}{3} \right] = \left[ 3 + \frac{1}{3} \right] = 3$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 3 \\ \hline 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 00} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

## Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

3  
?

- Find shortest dividend start larger than divisor.
  - Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
    - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Multiply quotient digit by divisor, put result under current dividend.

$$3 \cdot 9 = ?$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 3 \\ \overline{2} & \sqrt{1\ 0\ 0\ 0\ 0\ 00} \\ 2019 & \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$3 \cdot 9 = 27$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ 2 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\ \underline{-7} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$3 \cdot 9 = 27$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 3 \\ \overline{2} & \quad | \\ 2019 & \quad | \\ 1 & 0 & 0 & 0 & 0 & 00 \\ & 7 \\ & \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$3 \cdot 9 = 27$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 3 \\ \overline{2} & \quad | \\ 2019 & \quad | \\ 1 & 0 & 0 & 0 & 0 & 00 \\ ? & 7 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{aligned} 3 \cdot 9 &= 27 \\ 3 \cdot 1 + 2 &=? \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 3 \\ \overline{2} & \quad | \\ 2019 & \quad | \\ 1 & 0 & 0 & 0 & 0 & 00 \\ & 5 & 7 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{array}{r} 3 \cdot 9 = 27 \\ 3 \cdot 1 + 2 = 5 \end{array}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 3 \\ \overline{2} & \sqrt{2019 \overline{)1000000}} \\ 2019 & \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{array}{r} 3 \cdot 9 = 27 \\ 3 \cdot 1 + 2 = 5 \\ 3 \cdot 0 = ? \end{array}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 3 \\ \overline{2} & \sqrt{2019 \overline{)1000000}} \\ & 057 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{array}{r} 3 \cdot 9 = 27 \\ 3 \cdot 1 + 2 = 5 \\ 3 \cdot 0 = 0 \end{array}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 3 \\ \overline{2} & \quad | \\ 2019 & \quad | \\ 1 & 0 & 0 & 0 & 0 & 00 \\ ? & 0 & 5 & 7 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{aligned} 3 \cdot 9 &= 27 \\ 3 \cdot 1 + 2 &= 5 \\ 3 \cdot 0 &= 0 \\ 3 \cdot 2 &= ? \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 3 \\ \overline{2} & \sqrt{2019 \overline{)1000000}} \\ & 6 \\ & \underline{60} \\ & 57 \\ & \underline{54} \\ & 30 \\ & \underline{30} \\ & 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{aligned} 3 \cdot 9 &= 27 \\ 3 \cdot 1 + 2 &= 5 \\ 3 \cdot 0 &= 0 \\ 3 \cdot 2 &= 6 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 3 \\ \overline{2} & \quad | \\ 2019 & \quad | \\ 1 & 0 & 0 & 0 & 0 & 00 \\ & 6 & 0 & 5 & 7 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{aligned} 3 \cdot 9 &= 27 \\ 3 \cdot 1 + 2 &= 5 \\ 3 \cdot 0 &= 0 \\ 3 \cdot 2 &= 6 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 00} \\ 6\ 0\ 5\ 7 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 3 \\ \overline{)2019} & 1 \ 0 \ 0 \ 0 \ 0 \ 00 \\ & 6 \ 0 \ 5 \ 7 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 00} \\ 6\ 0\ 5\ 7\ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 00} \\ - \underline{6\ 0\ 5\ 7\ 00} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0} \\ - \ 6\ 0\ 5\ 7\ 00 \\ \hline \quad \quad \quad ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = ?$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1000000} \\ - \quad 6 \\ \hline 40 \\ - \quad 36 \\ \hline 40 \\ - \quad 36 \\ \hline 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = 0$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\ - \ 6\ 0\ 5\ 7\ 00 \\ \hline ?0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{array}{r} 0 - 0 = 0 \\ 0 - 0 = ? \end{array}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0} \\ - \ 6\ 0\ 5\ 7\ 00 \\ \hline 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{array}{r} 0 - 0 = 0 \\ 0 - 0 = 0 \end{array}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0} \\ - \ 1\ 0 \\ \hline \ 6\ 0\ 5\ 7\ 00 \\ \quad \quad \quad 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 0 - 0 &= 0 \\ 0 - 0 &= 0 \\ 0 - 7 &= ? \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0} \\ - \ 6\ 0\ 5\ 7\ 00 \\ \hline 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 0 - 0 &= 0 \\ 0 - 0 &= 0 \\ 0 - 7 &= -7 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 3 \\
 & \overline{-1} \\
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0\ 00} \\
 - & 6\ 0\ 5\ 7\ 00 \\
 & \textcolor{red}{?}\ 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned}
 0 - 0 &= 0 \\
 0 - 0 &= 0 \\
 0 - 7 &= \textcolor{red}{-7 = -10 + ?}
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 00} \\ - 6\ 0\ 5\ 7\ 00 \\ \hline 3\ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = 0$$

$$0 - 0 = 0$$

$$0 - 7 = -7 = -10 + 3$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 3 \\ \overline{)2019} & \begin{array}{r} -1 \\ -1 \\ \hline 605700 \\ -300 \\ \hline 300 \end{array} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 0 - 0 &= 0 \\ 0 - 0 &= 0 \\ 0 - 7 &= -7 = -10 + 3 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 3 \\
 \overline{2019} \overline{-} & 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 00 \\
 & - 6 \ 0 \ 5 \ 7 \ 00 \\
 \hline
 & 3 \ 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = 0$$

$$0 - 0 = 0$$

$$0 - 7 = -7 = -10 + 3$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 3 \\
 \overline{2019} \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 0\ 0 \\
 & \underline{-} \\
 & 6\ 0\ 5\ 7\ 00 \\
 & \underline{-} \\
 & 3\ 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = 0$$

$$0 - 0 = 0$$

$$0 - 7 = -7 = -10 + 3$$

$$\textcolor{red}{-1 + 0 - 5 = ?}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 3 \\
 \overline{2019} \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 0\ 0 \\
 & \underline{-} \\
 & 6\ 0\ 5\ 7\ 00 \\
 & 3\ 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned}
 0 - 0 &= 0 \\
 0 - 0 &= 0 \\
 0 - 7 &= -7 = -10 + 3 \\
 -1 + 0 - 5 &= -6
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 3 \\
 & \overline{-1\phantom{1}} \\
 2019 & \overline{)1\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}0} \\
 - & 1\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}0 \\
 \hline
 & 6\phantom{0}0\phantom{0}5\phantom{0}7\phantom{0}00 \\
 & \overline{? \phantom{1}3\phantom{0}00}
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = 0$$

$$0 - 0 = 0$$

$$0 - 7 = -7 = -10 + 3$$

$$-1 + 0 - 5 = -6 = -10 + ?$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 3 \\
 & \overline{-1\phantom{1}} \\
 2019 & \overline{)1\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}0} \\
 - & 1\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}0 \\
 & \overline{6\phantom{0}0\phantom{0}5\phantom{0}7\phantom{0}00} \\
 & \phantom{1}4\phantom{0}3\phantom{0}00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned}
 0 - 0 &= 0 \\
 0 - 0 &= 0 \\
 0 - 7 &= -7 = -10 + 3 \\
 -1 + 0 - 5 &= -6 = -10 + 4
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 3 \\
 \overline{2019} \overline{)1\ 0\ 0\ 0\ 0\ 0\ 00} \\
 -1\ 0 \\
 \hline
 & 0\ 5\ 7\ 00 \\
 & 4\ 0\ 0 \\
 \hline
 & 3\ 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned}
 0 - 0 &= 0 \\
 0 - 0 &= 0 \\
 0 - 7 &= -7 = -10 + 3 \\
 -1 + 0 - 5 &= -6 = -10 + 4
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 3 \\
 & \overline{-1\phantom{1}} \\
 2019 & \overline{)1\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}0} \\
 - & 1\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}0 \\
 & \underline{-\phantom{1}6\phantom{0}0\phantom{0}5\phantom{0}7\phantom{0}00} \\
 & 4\phantom{1}3\phantom{0}00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = 0$$

$$0 - 0 = 0$$

$$0 - 7 = -7 = -10 + 3$$

$$-1 + 0 - 5 = -6 = -10 + 4$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 3 \\
 & \overline{-1\phantom{1}} \\
 2019 & \overline{)1\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}00} \\
 - & 1\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}00 \\
 & \underline{-\phantom{1}6\phantom{0}0\phantom{0}5\phantom{0}7\phantom{0}00} \\
 & 4\phantom{1}3\phantom{0}00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned}
 0 - 0 &= 0 \\
 0 - 0 &= 0 \\
 0 - 7 &= -7 = -10 + 3 \\
 -1 + 0 - 5 &= -6 = -10 + 4
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 3 \\
 & \overline{-1\phantom{1}} \\
 2019 & \overline{)1\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}00} \\
 - & 1\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}00 \\
 & \underline{-\phantom{1}6\phantom{0}0\phantom{0}5\phantom{0}7\phantom{0}00} \\
 & 4\phantom{1}3\phantom{0}00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = 0$$

$$0 - 0 = 0$$

$$0 - 7 = -7 = -10 + 3$$

$$-1 + 0 - 5 = -6 = -10 + 4$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 3 \\
 & -1 & -1 & -1 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0\ 00} \\
 - & 1\ 0 & 0 & 5 & 7 & 00 \\
 \hline
 & ? & 4 & 3 & 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = 0$$

$$0 - 0 = 0$$

$$0 - 7 = -7 = -10 + 3$$

$$-1 + 0 - 5 = -6 = -10 + 4$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 3 \\
 & -1 & -1 & -1 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0 & 0 & 5 & 7 & 0 0 \\
 \hline
 & 9 & 4 & 3 & 0 0
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = 0$$

$$0 - 0 = 0$$

$$0 - 7 = -7 = -10 + 3$$

$$-1 + 0 - 5 = -6 = -10 + 4$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1000000} \\ -10 \\ \hline 100 \\ -60 \\ \hline 40 \\ -40 \\ \hline 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 0 - 0 &= 0 \\ 0 - 0 &= 0 \\ 0 - 7 &= -7 = -10 + 3 \\ -1 + 0 - 5 &= -6 = -10 + 4 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 3 \\
 \overline{2019} \overline{\overline{-1-1-1}} \\
 - 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 00 \\
 \underline{-} \quad 6 \quad 0 \quad 5 \quad 7 \quad 00 \\
 \hline
 & 9 \quad 4 \quad 3 \quad 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = 0$$

$$0 - 0 = 0$$

$$0 - 7 = -7 = -10 + 3$$

$$-1 + 0 - 5 = -6 = -10 + 4$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1000000} \\ -10 \\ \hline 10 \\ -6 \\ \hline 40 \\ -40 \\ \hline 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = 0$$

$$0 - 0 = 0$$

$$0 - 7 = -7 = -10 + 3$$

$$-1 + 0 - 5 = -6 = -10 + 4$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 3 \\
 & \overline{-1-1-1} \\
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0 \\
 & \underline{-\ 6\ 0} \\
 & 4\ 3\ 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = 0$$

$$0 - 0 = 0$$

$$0 - 7 = -7 = -10 + 3$$

$$-1 + 0 - 5 = -6 = -10 + 4$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1000000} \\ -10 \\ \hline 100 \\ -60 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 0 - 0 &= 0 \\ 0 - 0 &= 0 \\ 0 - 7 &= -7 = -10 + 3 \\ -1 + 0 - 5 &= -6 = -10 + 4 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 3 \\
 \overline{2019} \overline{\overline{-1-1-1-1}} \\
 -1\ 0\ 0\ 0\ 0\ 0\ 00 \\
 \hline
 -6\ 0\ 5\ 7\ 00 \\
 \hline
 3\ 9\ 4\ 3\ 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned}
 0 - 0 &= 0 \\
 0 - 0 &= 0 \\
 0 - 7 &= -7 = -10 + 3 \\
 -1 + 0 - 5 &= -6 = -10 + 4
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1000000} \\ -10 \\ \hline 100 \\ -60 \\ \hline 40 \\ -40 \\ \hline 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = 0$$

$$0 - 0 = 0$$

$$0 - 7 = -7 = -10 + 3$$

$$-1 + 0 - 5 = -6 = -10 + 4$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1000000} \\ -10 \\ \hline 100 \\ -60 \\ \hline 40 \\ -40 \\ \hline 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned} 0 - 0 &= 0 \\ 0 - 0 &= 0 \\ 0 - 7 &= -7 = -10 + 3 \\ -1 + 0 - 5 &= -6 = -10 + 4 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1000000} \\ -10 \\ \hline 100 \\ -60 \\ \hline 40 \\ -40 \\ \hline 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$0 - 0 = 0$$

$$0 - 0 = 0$$

$$0 - 7 = -7 = -10 + 3$$

$$-1 + 0 - 5 = -6 = -10 + 4$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 3 \\
 \overline{2019} \overline{)1000000} \\
 -10 \\
 \hline
 100 \\
 -60 \\
 \hline
 40 \\
 -40 \\
 \hline
 00
 \end{array}$$

The quotient is 3. The remainder is 00.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{array}{r}
 0 - 0 = 0 \\
 0 - 0 = 0 \\
 0 - 7 = -7 = -10 + 3 \\
 -1 + 0 - 5 = -6 = -10 + 4
 \end{array}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 3 \\
 \overline{2019} \overline{\overline{-1-1-1-1}} \\
 - 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 00 \\
 \hline
 & 6 \quad 0 \quad 5 \quad 7 \quad 00 \\
 & 3 \quad 9 \quad 4 \quad 3 \quad 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

$$\begin{aligned}
 0 - 0 &= 0 \\
 0 - 0 &= 0 \\
 0 - 7 &= -7 = -10 + 3 \\
 -1 + 0 - 5 &= -6 = -10 + 4
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0} \\ - \ 6\ 0\ 5\ 7\ 00 \\ \hline 3\ 9\ 4\ 3\ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0} \\ - 1\ 6 \\ \hline 3\ 9\ 4\ 3\ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 3 \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0} \\ - \ 6\ 0\ 5\ 7\ 00 \\ \hline 3\ 9\ 4\ 3\ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & \text{?} \\ 3 & \\ \hline 2019 & \overline{-\phantom{0}1\phantom{0}0\phantom{0}0\phantom{0}0\phantom{0}00} \\ & -\phantom{0}6\phantom{0}0\phantom{0}5\phantom{0}7\phantom{0}00 \\ & \phantom{-}3\phantom{0}9\phantom{0}4\phantom{0}3\phantom{0}00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} ? \\ 3 \\ \hline 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 00} \\ - \ 1\ 6 \\ \hline 3\ 9\ 4\ 3\ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit > divisor leading digit

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 ? \\
 3 \\
 \hline
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \ 6\ 0\ 5\ 7\ 0\ 0 \\
 \hline
 3\ 9\ 4\ 3\ 0\ 0
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  **divide leading digit** by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{3}{2+1} \right]$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 ? \\
 3 \\
 \hline
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - \ 6\ 0\ 5\ 7\ 00 \\
 \hline
 3\ 9\ 4\ 3\ 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit **by divisor digit** plus one. Round down if needed.
  
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{3}{2+1} \right\rfloor$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 ? \\
 3 \\
 \hline
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \ 6\ 0\ 5\ 7\ 0\ 0 \\
 \hline
 3\ 9\ 4\ 3\ 0\ 0
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit **plus one**. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{3}{2+1} \right]$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 ? \\
 3 \\
 \hline
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - 1\ 0 \\
 \hline
 6\ 0\ 5\ 7\ 00 \\
 - 3\ 9\ 4\ 3\ 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. **Round down if needed.**
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{3}{2+1} \right]$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 \text{?} \\
 3 \\
 \hline
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \ 6\ 0\ 5\ 7\ 0\ 0 \\
 \hline
 3\ 9\ 4\ 3\ 0\ 0
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{3}{2+1} \right] = \text{?}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 3 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 2019 & -\ 1\ 0\ 5 \\
 & \hline
 & 6\ 0\ 5\ 7\ 00 \\
 & -\ 6\ 0\ 5 \\
 & \hline
 & 0\ 0\ 0\ 0\ 00 \\
 & -\ 0\ 0\ 0\ 0\ 00 \\
 & \hline
 & 0\ 0\ 0\ 0\ 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{3}{2+1} \right] = \left[ \frac{3}{3} \right] = [1] = 1$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 3 & \overline{)1\ 0\ 0\ 0\ 0\ 0\ 00} \\ - & 6\ 0\ 5\ 7\ 00 \\ \hline & 3\ 9\ 4\ 3\ 00 \\ & \textcolor{red}{?}\ \textcolor{red}{?}\ \textcolor{red}{?}\ \textcolor{red}{?} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 3 & \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\ - & 6\ 0\ 5\ 7\ 0\ 0 \\ \hline & 3\ 9\ 4\ 3\ 0\ 0 \\ & 2\ 0\ 1\ 9 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\ - & 6\ 0\ 5\ 7\ 00 \\ \hline & 3\ 9\ 4\ 3\ 00 \\ & 2\ 0\ 1\ 9\ \textcolor{red}{00} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 3 & \overline{)1\ 0\ 0\ 0\ 0\ 0\ 00} \\ - & 6\ 0\ 5\ 7\ 00 \\ \hline & 3\ 9\ 4\ 3\ 00 \\ - & 2\ 0\ 1\ 9\ 00 \\ \hline & & & & & & \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 3 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\ - & 6\ 0\ 5\ 7\ 00 \\ \hline & 3\ 9\ 4\ 3\ 00 \\ - & 2\ 0\ 1\ 9\ 00 \\ \hline & ?\ ?\ ?\ ?\ ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 3 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0\ 00} \\
 - & 1\ 0 \\
 \hline
 & 6\ 0\ 5\ 7\ 00 \\
 - & \quad \quad \quad \overline{-1} \\
 & 3\ 9\ 4\ 3\ 00 \\
 - & \quad \quad \quad 2\ 0\ 1\ 9\ 00 \\
 \hline
 & \quad \quad \quad 1\ 9\ 2\ 4\ 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract.**
- Repeat.

$$\begin{array}{r}
 0 - 0 = 0 \\
 0 - 0 = 0 \\
 3 - 9 = -6 = -10 + 4 \\
 -1 + 4 - 1 = 2
 \end{array}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 3 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0\ 00} \\
 - & 6\ 0\ 5\ 7\ 00 \\
 \hline
 & \quad \textcolor{red}{-1} \\
 - & 3\ 9\ 4\ 3\ 00 \\
 & 2\ 0\ 1\ 9\ 00 \\
 \hline
 & 1\ 9\ 2\ 4\ 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 0 - 0 &= 0 \\
 0 - 0 &= 0 \\
 3 - 9 &= -6 = -10 + 4 \\
 -1 + 4 - 1 &= 2
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 3 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\ - & 6\ 0\ 5\ 7\ 00 \\ \hline & 3\ 9\ 4\ 3\ 00 \\ - & 2\ 0\ 1\ 9\ 00 \\ \hline & 1\ 9\ 2\ 4\ 00 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ \hline 2019 & \overline{-} \begin{array}{r} 1 \ 0 \ 0 \ 0 \ 0 \ 0 \\ - \ 6 \ 0 \ 5 \ 7 \ 00 \\ \hline - \ 3 \ 9 \ 4 \ 3 \ 00 \\ - \ 2 \ 0 \ 1 \ 9 \ 00 \\ \hline 1 \ 9 \ 2 \ 4 \ 00 \end{array} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ \hline 2019 & \overline{-} \begin{array}{r} 1 \ 0 \ 0 \ 0 \ 0 \ 0 \\ - \ 6 \ 0 \ 5 \ 7 \ 00 \\ \hline - \ 3 \ 9 \ 4 \ 3 \ 00 \\ - \ 2 \ 0 \ 1 \ 9 \ 00 \\ \hline 1 \ 9 \ 2 \ 4 \ 00 \end{array} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 3 & \boxed{?} \\ \hline 2019 & \left[ \begin{array}{r} 1 & 0 & 0 & 0 & 0 & 0 \\ - & 6 & 0 & 5 & 7 & 00 \\ \hline & 3 & 9 & 4 & 3 & 00 \\ - & 2 & 0 & 1 & 9 & 00 \\ \hline & 1 & 9 & 2 & 4 & 00 \end{array} \right] \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 3 & \boxed{?} \\ \hline 2019 & \left[ \begin{array}{r} 1 & 0 & 0 & 0 & 0 & 00 \\ - & 6 & 0 & 5 & 7 & 00 \\ \hline - & 3 & 9 & 4 & 3 & 00 \\ - & 2 & 0 & 1 & 9 & 00 \\ \hline 1 & 9 & 2 & 4 & 00 \end{array} \right] \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 3 & ? \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 5\ 7\ 00 \\
 \hline
 & 3\ 9\ 4\ 3\ 00 \\
 - & 2\ 0\ 1\ 9\ 00 \\
 \hline
 & 1\ 9\ 2\ 4\ 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{19}{2+1} \right] = ?$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 3 \text{ } 6 \\
 \hline
 2019 & \overline{)1 \text{ } 0 \text{ } 0 \text{ } 0 \text{ } 0 \text{ } 0} \\
 - & 1 \text{ } 0 \text{ } 5 \text{ } 7 \text{ } 00 \\
 \hline
 - & 3 \text{ } 9 \text{ } 4 \text{ } 3 \text{ } 00 \\
 - & 2 \text{ } 0 \text{ } 1 \text{ } 9 \text{ } 00 \\
 \hline
 & 1 \text{ } 9 \text{ } 2 \text{ } 4 \text{ } 00
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{19}{2+1} \right] = \left[ \frac{19}{3} \right] = \left[ \frac{18}{3} + \frac{1}{3} \right] = \left[ 6 + \frac{1}{3} \right] = 6$$

## Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

- Find shortest dividend start larger than divisor.
  - Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
    - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
    - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
  - Subtract.
  - Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 3 \text{ } 6 \\
 \hline
 1 \text{ } 15 & \left[ \begin{array}{r} 1 \text{ } 0 \text{ } 0 \text{ } 0 \text{ } 0 \\ - 6 \text{ } 0 \text{ } 5 \text{ } 7 \text{ } 0 \end{array} \right] \\
 2019 & \left[ \begin{array}{r} 3 \text{ } 9 \text{ } 4 \text{ } 3 \text{ } 0 \text{ } 0 \\ - 2 \text{ } 0 \text{ } 1 \text{ } 9 \text{ } 0 \end{array} \right] \\
 \hline
 & 1 \text{ } 9 \text{ } 2 \text{ } 4 \text{ } 0 \text{ } 0 \\
 & \text{1 } \text{2 } \text{1 } \text{1 } \text{4}
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 6 \cdot 9 &= 54 \\
 6 \cdot 1 + 5 &= 11 \\
 6 \cdot 0 + 1 &= 1 \\
 6 \cdot 2 &= 12
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 3 & 6 \\ \hline 2019 & \left[ \begin{array}{r} 1 & 0 & 0 & 0 & 0 & 0 \\ - & 6 & 0 & 5 & 7 & 00 \\ \hline & 3 & 9 & 4 & 3 & 00 \\ - & 2 & 0 & 1 & 9 & 00 \\ \hline & 1 & 9 & 2 & 4 & 00 \\ & 1 & 2 & 1 & 1 & 4 \end{array} \right] \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\ - & 1\ 6 \\ \hline & 4\ 0 \\ - & 3\ 9 \\ \hline & 1\ 1 \\ - & 1\ 9 \\ \hline & 0\ 0 \\ - & 1\ 9 \\ \hline & 1\ 1 \\ - & 1\ 9 \\ \hline & 0\ 0 \\ - & 1\ 9 \\ \hline & 1\ 1 \\ - & 1\ 9 \\ \hline & 0\ 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 3 & 6 \\ \hline 2019 & \left[ \begin{array}{r} 1 & 0 & 0 & 0 & 0 & 0 \\ - & 6 & 0 & 5 & 7 & 00 \\ \hline & 3 & 9 & 4 & 3 & 00 \\ - & 2 & 0 & 1 & 9 & 00 \\ \hline & 1 & 9 & 2 & 4 & 00 \\ & 1 & 2 & 1 & 1 & 40 \end{array} \right] \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 3 & 6 \\ \hline 2019 & \left[ \begin{array}{r} 1 & 0 & 0 & 0 & 0 & 00 \\ - & 6 & 0 & 5 & 7 & 00 \\ \hline - & 3 & 9 & 4 & 3 & 00 \\ - & 2 & 0 & 1 & 9 & 00 \\ \hline - & 1 & 9 & 2 & 4 & 00 \\ - & 1 & 2 & 1 & 1 & 40 \\ \hline \end{array} \right] \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 3 & 6 \\ \hline 2019 & \left[ \begin{array}{r} 1 & 0 & 0 & 0 & 0 & 00 \\ - & 6 & 0 & 5 & 7 & 00 \\ \hline - & 3 & 9 & 4 & 3 & 00 \\ - & 2 & 0 & 1 & 9 & 00 \\ \hline - & 1 & 9 & 2 & 4 & 00 \\ - & 1 & 2 & 1 & 1 & 40 \\ \hline & & & & & \end{array} \right] \\ & \quad ? \quad ? \quad ? \quad ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 3 \ 6 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 5\ 7\ 0\ 0 \\
 \hline
 & 3\ 9\ 4\ 3\ 0\ 0 \\
 - & 2\ 0\ 1\ 9\ 0\ 0 \\
 \hline
 & \quad \quad \quad \textcolor{red}{-1} \\
 - & 1\ 9\ 2\ 4\ 0\ 0 \\
 \hline
 & 1\ 2\ 1\ 1\ 4\ 0 \\
 \hline
 & 7\ 1\ 2\ 6\ 0
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract.**
- Repeat.

$$\begin{aligned}
 0 - 0 &= 0 \\
 0 - 4 &= -4 = -10 + 6 \\
 -1 + 4 - 1 &= 2 \\
 2 - 1 &= 1
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 3 \ 6 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 5\ 7\ 0\ 0 \\
 \hline
 & 3\ 9\ 4\ 3\ 0\ 0 \\
 - & 2\ 0\ 1\ 9\ 0\ 0 \\
 \hline
 & \quad \textcolor{red}{-1} \\
 - & 1\ 9\ 2\ 4\ 0\ 0 \\
 \hline
 & 1\ 2\ 1\ 1\ 4\ 0 \\
 \hline
 & 7\ 1\ 2\ 6\ 0
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 0 - 0 &= 0 \\
 0 - 4 &= -4 = -10 + 6 \\
 -1 + 4 - 1 &= 2 \\
 2 - 1 &= 1
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 3 & 6 \\ \hline 2019 & \left[ \begin{array}{r} 1 & 0 & 0 & 0 & 0 & 00 \\ - & 6 & 0 & 5 & 7 & 00 \\ \hline & 3 & 9 & 4 & 3 & 00 \\ - & 2 & 0 & 1 & 9 & 00 \\ \hline & 1 & 9 & 2 & 4 & 00 \\ - & 1 & 2 & 1 & 1 & 40 \\ \hline & & 7 & 1 & 2 & 60 \end{array} \right] \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 3 & 6 \\ \hline 2019 & \left[ \begin{array}{r} 1 & 0 & 0 & 0 & 0 & 0 \\ - & 6 & 0 & 5 & 7 & 00 \\ \hline & 3 & 9 & 4 & 3 & 00 \\ - & 2 & 0 & 1 & 9 & 00 \\ \hline & 1 & 9 & 2 & 4 & 00 \\ - & 1 & 2 & 1 & 1 & 40 \\ \hline & & 7 & 1 & 2 & 60 \end{array} \right] \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \\ 3 & 6 \\ \hline 2019 & \left[ \begin{array}{r} 1 & 0 & 0 & 0 & 0 & 00 \\ - & 6 & 0 & 5 & 7 & 00 \\ \hline & 3 & 9 & 4 & 3 & 00 \\ - & 2 & 0 & 1 & 9 & 00 \\ \hline & 1 & 9 & 2 & 4 & 00 \\ - & 1 & 2 & 1 & 1 & 40 \\ \hline & & 7 & 1 & 2 & 60 \end{array} \right] \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 & ? \\ & \underline{3} & 6 \\ 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\ - & 1\ 0\ 5\ 7\ 00 \\ \hline & 3\ 9\ 4\ 3\ 00 \\ - & 2\ 0\ 1\ 9\ 00 \\ \hline & 1\ 9\ 2\ 4\ 00 \\ - & 1\ 2\ 1\ 1\ 40 \\ \hline & & 7 & 1 & 2 & 60 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 & ? \\
 & 3 & 6 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 5\ 7\ 00 \\
 \hline
 & 3\ 9\ 4\ 3\ 00 \\
 - & 2\ 0\ 1\ 9\ 00 \\
 \hline
 & 1\ 9\ 2\ 4\ 00 \\
 - & 1\ 2\ 1\ 1\ 40 \\
 \hline
 & 7\ 1\ 2\ 60
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{7}{2+1} \right] = ?$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 2 \\
 & 3 & 6 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 5\ 7\ 00 \\
 \hline
 & 3 & 9 & 4 & 3 & 00 \\
 - & 2 & 0 & 1 & 9 & 00 \\
 \hline
 & 1 & 9 & 2 & 4 & 00 \\
 - & 1 & 2 & 1 & 1 & 40 \\
 \hline
 & 7 & 1 & 2 & 60
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{7}{2+1} \right] = \left[ \frac{7}{3} \right] = \left[ \frac{6}{3} + \frac{1}{3} \right] = \left[ 2 + \frac{1}{3} \right] = 2$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 \textcolor{red}{2} \\
 3 \textcolor{red}{6} & \overline{)1000000} \\
 - & 605700 \\
 \hline
 - & 394300 \\
 - & 201900 \\
 \hline
 - & 192400 \\
 - & 121140 \\
 \hline
 & 71260 \\
 & \textcolor{red}{? ? ? ?}
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  
- **Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 2 \\
 2019 & \overline{)1000000} \\
 - & 1 & 0 & 0 & 0 & 0 & 00 \\
 & 6 & 0 & 5 & 7 & 00 \\
 - & 3 & 9 & 4 & 3 & 00 \\
 & 2 & 0 & 1 & 9 & 00 \\
 \hline
 & 1 & 9 & 2 & 4 & 00 \\
 - & 1 & 2 & 1 & 1 & 40 \\
 \hline
 & 7 & 1 & 2 & 60 \\
 & 4 & 0 & 3 & 8
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 2 \cdot 9 &= 18 \\
 2 \cdot 1 + 1 &= 3 \\
 2 \cdot 0 &= 0 \\
 2 \cdot 2 &= 4
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \ 2 \\ & \underline{3 \ 6} \\ 2019 & \overline{)1 \ 0 \ 0 \ 0 \ 0 \ 0} \\ - & \underline{6 \ 0 \ 5 \ 7 \ 0 \ 0} \\ & \underline{-} \\ & 3 \ 9 \ 4 \ 3 \ 0 \ 0 \\ - & \underline{2 \ 0 \ 1 \ 9 \ 0 \ 0} \\ & \underline{-} \\ & 1 \ 9 \ 2 \ 4 \ 0 \ 0 \\ - & \underline{1 \ 2 \ 1 \ 1 \ 4 \ 0} \\ & \underline{-} \\ & 7 \ 1 \ 2 \ 6 \ 0 \\ & \underline{4 \ 0 \ 3 \ 8} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \textcolor{red}{2} \\ 3 & 6 \\ \hline 2019 & \left[ \begin{array}{r} 1 \ 0 \ 0 \ 0 \ 0 \ 0 \\ - \ 6 \ 0 \ 5 \ 7 \ 0 0 \\ \hline 3 \ 9 \ 4 \ 3 \ 0 0 \\ - \ 2 \ 0 \ 1 \ 9 \ 0 0 \\ \hline 1 \ 9 \ 2 \ 4 \ 0 0 \\ - \ 1 \ 2 \ 1 \ 1 \ 4 0 \\ \hline 7 \ 1 \ 2 \ 6 0 \\ - \ 4 \ 0 \ 3 \ 8 \end{array} \right] \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 \ 2 \\ & \underline{3 \ 6} \\ 2019 | & \begin{array}{r} 1 \ 0 \ 0 \ 0 \ 0 \ 0 \\ - 6 \ 0 \ 5 \ 7 \ 0 \ 0 \\ \hline 3 \ 9 \ 4 \ 3 \ 0 \ 0 \\ - 2 \ 0 \ 1 \ 9 \ 0 \ 0 \\ \hline 1 \ 9 \ 2 \ 4 \ 0 \ 0 \\ - 1 \ 2 \ 1 \ 1 \ 4 \ 0 \\ \hline 7 \ 1 \ 2 \ 6 \ 0 \\ - 4 \ 0 \ 3 \ 8 \ 0 \end{array} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 & 2 \\ & 3 & 6 \\ \hline 2019 & \overline{-} & 1 & 0 & 0 & 0 & 0 & 00 \\ & - & 6 & 0 & 5 & 7 & 00 \\ \hline & - & 3 & 9 & 4 & 3 & 00 \\ & - & 2 & 0 & 1 & 9 & 00 \\ \hline & - & 1 & 9 & 2 & 4 & 00 \\ & - & 1 & 2 & 1 & 1 & 40 \\ \hline & & 7 & 1 & 2 & 60 \\ & & - & 4 & 0 & 3 & 80 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 2 \\
 & 3 & 6 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 5\ 7\ 00 \\
 \hline
 & 3 & 9 & 4 & 3 & 00 \\
 - & 2 & 0 & 1 & 9 & 00 \\
 \hline
 & 1 & 9 & 2 & 4 & 00 \\
 - & 1 & 2 & 1 & 1 & 40 \\
 \hline
 & & 7 & 1 & 2 & 60 \\
 - & & 4 & 0 & 3 & 80 \\
 \hline
 & & ? & ? & ? & ?
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 2 \\
 & 3 & 6 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 5\ 7\ 00 \\
 \hline
 & 3 & 9 & 4 & 3 & 00 \\
 - & 2 & 0 & 1 & 9 & 00 \\
 \hline
 & 1 & 9 & 2 & 4 & 00 \\
 - & 1 & 2 & 1 & 1 & 40 \\
 \hline
 & & & \textcolor{red}{-1-1} \\
 & & & 7 & 1 & 2 & 60 \\
 - & & & 4 & 0 & 3 & 80 \\
 \hline
 & 3 & 0 & 8 & 80
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract.**
- Repeat.

$$0 - 0 = 0$$

$$6 - 8 = -2 = -10 + 8$$

$$-1 + 2 - 3 = -2 = -10 + 8$$

$$-1 + 1 - 0 = 0$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 2 \\
 & 3 & 6 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 5\ 7\ 00 \\
 \hline
 & 3 & 9 & 4 & 3 & 00 \\
 - & 2 & 0 & 1 & 9 & 00 \\
 \hline
 & 1 & 9 & 2 & 4 & 00 \\
 - & 1 & 2 & 1 & 1 & 40 \\
 \hline
 & & & \textcolor{red}{-1-1} \\
 & & & 7 & 1 & 2 & 60 \\
 - & & & 4 & 0 & 3 & 80 \\
 \hline
 & & 3 & 0 & 8 & 80
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 0 - 0 &= 0 \\
 6 - 8 &= -2 = -10 + 8 \\
 -1 + 2 - 3 &= -2 = -10 + 8 \\
 -1 + 1 - 0 &= 0
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \ 2 \\ 3 \ 6 \end{array}$$

$$\begin{array}{r} & 1 & 2 & 0 & 0 & 0 & 0 \\ 2019 & \overline{-} & 1 & 0 & 0 & 0 & 0 \\ & - & 6 & 0 & 5 & 7 & 00 \\ & & \underline{-} & & & & \\ & & 3 & 9 & 4 & 3 & 00 \\ & - & 2 & 0 & 1 & 9 & 00 \\ & & \underline{-} & & & & \\ & & 1 & 9 & 2 & 4 & 00 \\ & - & 1 & 2 & 1 & 1 & 40 \\ & & \underline{-} & & & & \\ & & 7 & 1 & 2 & 60 \\ & - & 4 & 0 & 3 & 80 \\ & & \underline{-} & & & & \\ & & 3 & 0 & 8 & 80 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 & 2 \\
 & - & 3 & 6 \\
 \hline
 2019 & | & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & - & 6 & 0 & 5 & 7 & 0 & 0 \\
 \hline
 & - & 3 & 9 & 4 & 3 & 0 & 0 \\
 & - & 2 & 0 & 1 & 9 & 0 & 0 \\
 \hline
 & - & 1 & 9 & 2 & 4 & 0 & 0 \\
 & - & 1 & 2 & 1 & 1 & 4 & 0 \\
 \hline
 & - & 7 & 1 & 2 & 6 & 0 & 0 \\
 & - & 4 & 0 & 3 & 8 & 0 & 0 \\
 \hline
 & & 3 & 0 & 8 & 8 & 0 & 0
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} & 1 & 2 \\ & \underline{3} & \underline{6} \\ 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\ - & 1\ 0 & 5\ 7\ 00 \\ \hline & 3 & 9\ 4\ 3\ 00 \\ - & 2 & 0\ 1\ 9\ 00 \\ \hline & 1 & 9\ 2\ 4\ 00 \\ - & 1 & 2\ 1\ 1\ 40 \\ \hline & 7 & 1\ 2\ 60 \\ - & 4 & 0\ 3\ 80 \\ \hline & 3 & 0\ 8\ 80 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 \text{?} \\
 \begin{array}{r}
 1 \quad 2 \\
 3 \quad 6
 \end{array} \\
 \hline
 2019 \overline{)1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0} \\
 - \quad 6 \quad 0 \quad 5 \quad 7 \quad 00 \\
 \hline
 - \quad 3 \quad 9 \quad 4 \quad 3 \quad 00 \\
 - \quad 2 \quad 0 \quad 1 \quad 9 \quad 00 \\
 \hline
 - \quad 1 \quad 9 \quad 2 \quad 4 \quad 00 \\
 - \quad 1 \quad 2 \quad 1 \quad 1 \quad 40 \\
 \hline
 - \quad 7 \quad 1 \quad 2 \quad 60 \\
 - \quad 4 \quad 0 \quad 3 \quad 80 \\
 \hline
 \color{red}{3 \quad 0 \quad 8 \quad 80}
 \end{array}$$

- Find shortest dividend start larger than divisor.
- **Next quotient digit:** put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 \text{?} \\
 \begin{array}{r}
 1 \quad 2 \\
 3 \quad 6
 \end{array} \\
 \hline
 2019 \overline{)1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0} \\
 - \quad 6 \quad 0 \quad 5 \quad 7 \quad 0 \quad 0 \\
 \hline
 - \quad 3 \quad 9 \quad 4 \quad 3 \quad 0 \quad 0 \\
 - \quad 2 \quad 0 \quad 1 \quad 9 \quad 0 \quad 0 \\
 \hline
 - \quad 1 \quad 9 \quad 2 \quad 4 \quad 0 \quad 0 \\
 - \quad 1 \quad 2 \quad 1 \quad 1 \quad 4 \quad 0 \\
 \hline
 & \quad 7 \quad 1 \quad 2 \quad 6 \quad 0 \\
 - \quad & \quad 4 \quad 0 \quad 3 \quad 8 \quad 0 \\
 \hline
 & \quad 3 \quad 0 \quad 8 \quad 8 \quad 0
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
    - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{3}{2+1} \right\rfloor = ?$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & 1 & 2 \\
 & 3 & 6 \\
 \hline
 2019 & | & 1 & 0 & 0 & 0 & 0 & 0 \\
 & - & 6 & 0 & 5 & 7 & 0 & 0 \\
 \hline
 & - & 3 & 9 & 4 & 3 & 0 & 0 \\
 & - & 2 & 0 & 1 & 9 & 0 & 0 \\
 \hline
 & - & 1 & 9 & 2 & 4 & 0 & 0 \\
 & - & 1 & 2 & 1 & 1 & 4 & 0 \\
 \hline
 & - & 7 & 1 & 2 & 6 & 0 & 0 \\
 & - & 4 & 0 & 3 & 8 & 0 & 0 \\
 \hline
 & & 3 & 0 & 8 & 8 & 0 & 0
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
    - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{3}{2+1} \right\rfloor = \left\lfloor \frac{3}{3} \right\rfloor = [1] = 1$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & 1 & 2 \\
 2019 & \overline{-} & 1 & 0 & 0 & 0 & 0 & 0 \\
 & - & 6 & 0 & 5 & 7 & 0 & 0 \\
 & - & 3 & 9 & 4 & 3 & 0 & 0 \\
 & - & 2 & 0 & 1 & 9 & 0 & 0 \\
 \hline
 & - & 1 & 9 & 2 & 4 & 0 & 0 \\
 & - & 1 & 2 & 1 & 1 & 4 & 0 \\
 \hline
 & - & 7 & 1 & 2 & 6 & 0 & 0 \\
 & - & 4 & 0 & 3 & 8 & 0 & 0 \\
 \hline
 & & 3 & 0 & 8 & 8 & 0 & 0 \\
 & & ? & ? & ? & ? & ? & ?
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & 1 & 2 \\
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 5\ 7\ 0\ 0 \\
 \hline
 & 3 & 9 & 4 & 3 & 0\ 0 \\
 - & 2 & 0 & 1 & 9 & 0\ 0 \\
 \hline
 & 1 & 9 & 2 & 4 & 0\ 0 \\
 - & 1 & 2 & 1 & 1 & 4\ 0 \\
 \hline
 & 7 & 1 & 2 & 6\ 0 \\
 - & 4 & 0 & 3 & 8\ 0 \\
 \hline
 & 3 & 0 & 8 & 8\ 0 \\
 & 2 & 0 & 1 & 9
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & 1 & 2 \\
 & 3 & 6 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & \underline{1\ 0\ 5\ 7\ 0\ 0} \\
 & 3\ 9\ 4\ 3\ 0\ 0 \\
 - & \underline{2\ 0\ 1\ 9\ 0\ 0} \\
 & 1\ 9\ 2\ 4\ 0\ 0 \\
 - & \underline{1\ 2\ 1\ 1\ 4\ 0} \\
 & 7\ 1\ 2\ 6\ 0 \\
 - & \underline{4\ 0\ 3\ 8\ 0} \\
 & 3\ 0\ 8\ 8\ 0 \\
 & \quad 2\ 0\ 1\ 9\ 0
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. **Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 6 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 6\ 0\ 5\ 7\ 0\ 0 \\
 \hline
 - \quad 3\ 9\ 4\ 3\ 0\ 0 \\
 - \quad 2\ 0\ 1\ 9\ 0\ 0 \\
 \hline
 - \quad 1\ 9\ 2\ 4\ 0\ 0 \\
 - \quad 1\ 2\ 1\ 1\ 4\ 0 \\
 \hline
 & \quad 7\ 1\ 2\ 6\ 0 \\
 - & \quad 4\ 0\ 3\ 8\ 0 \\
 \hline
 & \quad 3\ 0\ 8\ 8\ 0 \\
 - & \quad 2\ 0\ 1\ 9\ 0 \\
 \hline
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 6 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 6\ 0\ 5\ 7\ 0\ 0 \\
 \hline
 - \quad 3\ 9\ 4\ 3\ 0\ 0 \\
 - \quad 2\ 0\ 1\ 9\ 0\ 0 \\
 \hline
 - \quad 1\ 9\ 2\ 4\ 0\ 0 \\
 - \quad 1\ 2\ 1\ 1\ 4\ 0 \\
 \hline
 & \quad 7\ 1\ 2\ 6\ 0 \\
 - & \quad 4\ 0\ 3\ 8\ 0 \\
 \hline
 - & \quad 3\ 0\ 8\ 8\ 0 \\
 - & \quad 2\ 0\ 1\ 9\ 0 \\
 \hline
 & \quad ?\ ?\ ?\ ??
 \end{array}$$

- Find shortest dividend start larger than divisor.
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  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & 1 & 2 \\
 & 3 & 6 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 5\ 7\ 00 \\
 \hline
 & 3\ 9\ 4\ 3\ 00 \\
 - & 2\ 0\ 1\ 9\ 00 \\
 \hline
 & 1\ 9\ 2\ 4\ 00 \\
 - & 1\ 2\ 1\ 1\ 40 \\
 \hline
 & 7\ 1\ 2\ 60 \\
 - & 4\ 0\ 3\ 80 \\
 \hline
 & \textcolor{red}{-1} \\
 - & 3\ 0\ 8\ 80 \\
 \hline
 & 2\ 0\ 1\ 90 \\
 \hline
 & \textcolor{red}{1\ 0\ 6\ 90}
 \end{array}$$

- Find shortest dividend start larger than divisor.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 0 - 0 &= 0 \\
 8 - 9 &= -1 = -10 + 9 \\
 -1 + 8 - 1 &= 6 \\
 0 - 0 &= 0
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & 1 & 2 \\
 & 3 & 6 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & \underline{6\ 0\ 5\ 7\ 00} \\
 \hline
 & 3 & 9 & 4 & 3 & 00 \\
 - & \underline{2\ 0\ 1\ 9\ 00} \\
 \hline
 & 1 & 9 & 2 & 4 & 00 \\
 - & \underline{1\ 2\ 1\ 1\ 40} \\
 \hline
 & & 7 & 1 & 2 & 60 \\
 - & & \underline{4\ 0\ 3\ 80} \\
 \hline
 & & & \textcolor{red}{-1} \\
 - & & \underline{3\ 0\ 8\ 80} \\
 \hline
 & & 2 & 0 & 1 & 90 \\
 \hline
 & & 1 & 0 & 6 & 90
 \end{array}$$

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# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & 1 & 2 \\
 & 3 & 6 \\
 \hline
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 5\ 7\ 00 \\
 \hline
 & 3\ 9\ 4\ 3\ 00 \\
 - & 2\ 0\ 1\ 9\ 00 \\
 \hline
 & 1\ 9\ 2\ 4\ 00 \\
 - & 1\ 2\ 1\ 1\ 40 \\
 \hline
 & 7\ 1\ 2\ 60 \\
 - & 4\ 0\ 3\ 80 \\
 \hline
 & 3\ 0\ 8\ 80 \\
 - & 2\ 0\ 1\ 90 \\
 \hline
 & 1\ 0\ 6\ 90
 \end{array}$$

- Find shortest dividend start larger than divisor.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & 1 & 2 \\
 & 3 & 6 \\
 \\ 
 \boxed{2019} & \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0 & 0 \\
 & 6 & 0 & 5 & 7 & 0 & 0 \\
 \hline
 & 3 & 9 & 4 & 3 & 0 & 0 \\
 - & 2 & 0 & 1 & 9 & 0 & 0 \\
 \hline
 & 1 & 9 & 2 & 4 & 0 & 0 \\
 - & 1 & 2 & 1 & 1 & 4 & 0 \\
 \hline
 & & 7 & 1 & 2 & 6 & 0 \\
 - & & 4 & 0 & 3 & 8 & 0 \\
 \hline
 & & 3 & 0 & 8 & 8 & 0 \\
 - & & 2 & 0 & 1 & 9 & 0 \\
 \hline
 & & & & & & 1 & 0 & 6 & 9 & 0
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
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- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & 1 & 2 \\
 & 3 & 6 \\
 \hline
 2019 & | & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 & - & 6 & 0 & 5 & 7 & 0 & 0 \\
 \hline
 & - & 3 & 9 & 4 & 3 & 0 & 0 \\
 & - & 2 & 0 & 1 & 9 & 0 & 0 \\
 \hline
 & - & 1 & 9 & 2 & 4 & 0 & 0 \\
 & - & 1 & 2 & 1 & 1 & 4 & 0 \\
 \hline
 & & 7 & 1 & 2 & 6 & 0 & 0 \\
 & - & 4 & 0 & 3 & 8 & 0 & 0 \\
 \hline
 & - & 3 & 0 & 8 & 8 & 0 & 0 \\
 & - & 2 & 0 & 1 & 9 & 0 & 0 \\
 \hline
 & & & & & 1 & 0 & 6 & 9 & 0
 \end{array}$$

- Find shortest dividend start larger than divisor.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 6? \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 6\ 0\ 5\ 7\ 00 \\
 \hline
 - \quad 3\ 9\ 4\ 3\ 00 \\
 - \quad 2\ 0\ 1\ 9\ 00 \\
 \hline
 - \quad 1\ 9\ 2\ 4\ 00 \\
 - \quad 1\ 2\ 1\ 1\ 40 \\
 \hline
 & \quad 7\ 1\ 2\ 60 \\
 - & \quad 4\ 0\ 3\ 80 \\
 \hline
 & \quad 3\ 0\ 8\ 80 \\
 - & \quad 2\ 0\ 1\ 90 \\
 \hline
 & \quad 1\ 0\ 6\ 90
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 6? \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 1\ 0\ 0\ 0\ 0\ 0 \\
 \hline
 \quad 6\ 0\ 5\ 7\ 00 \\
 - \quad 3\ 9\ 4\ 3\ 00 \\
 \hline
 \quad 2\ 0\ 1\ 9\ 00 \\
 - \quad 1\ 9\ 2\ 4\ 00 \\
 \hline
 \quad 1\ 2\ 1\ 1\ 40 \\
 - \quad 7\ 1\ 2\ 60 \\
 \hline
 \quad 4\ 0\ 3\ 80 \\
 - \quad 3\ 0\ 8\ 80 \\
 \hline
 \quad 2\ 0\ 1\ 90 \\
 \hline
 \quad 1\ 0\ 6\ 90
 \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 6? \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 6\ 0\ 5\ 7\ 00 \\
 \hline
 - \quad 3\ 9\ 4\ 3\ 00 \\
 - \quad 2\ 0\ 1\ 9\ 00 \\
 \hline
 - \quad 1\ 9\ 2\ 4\ 00 \\
 - \quad 1\ 2\ 1\ 1\ 40 \\
 \hline
 & \quad 7\ 1\ 2\ 60 \\
 - & \quad 4\ 0\ 3\ 80 \\
 \hline
 & \quad 3\ 0\ 8\ 80 \\
 - & \quad 2\ 0\ 1\ 90 \\
 \hline
 & \quad 1\ 0\ 6\ 90
 \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{10}{2+1} \right] = ?$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r} & 1 & 0 & 0 & 0 & 0 & 0 \\ 2019 & \overline{-} & 1 & 0 & 0 & 0 & 0 \\ & - & 6 & 0 & 5 & 7 & 00 \\ & & \underline{-} & & & & \\ & & 3 & 9 & 4 & 3 & 00 \\ & - & 2 & 0 & 1 & 9 & 00 \\ & & \underline{-} & & & & \\ & & 1 & 9 & 2 & 4 & 00 \\ & - & 1 & 2 & 1 & 1 & 40 \\ & & \underline{-} & & & & \\ & & 7 & 1 & 2 & 60 \\ & - & 4 & 0 & 3 & 80 \\ & & \underline{-} & & & & \\ & & 3 & 0 & 8 & 80 \\ & - & 2 & 0 & 1 & 90 \\ & & \underline{-} & & & & \\ & & 1 & 0 & 6 & 90 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left\lfloor \frac{10}{2+1} \right\rfloor = \left\lfloor \frac{10}{3} \right\rfloor = \left\lfloor \frac{9}{3} + \frac{1}{3} \right\rfloor = \left\lfloor 3 + \frac{1}{3} \right\rfloor = 3$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 1\ 0\ 5\ 7\ 00 \\
 \hline
 - \quad 3\ 9\ 4\ 3\ 00 \\
 - \quad 2\ 0\ 1\ 9\ 00 \\
 \hline
 - \quad 1\ 9\ 2\ 4\ 00 \\
 - \quad 1\ 2\ 1\ 1\ 40 \\
 \hline
 & \quad 7\ 1\ 2\ 60 \\
 - & \quad 4\ 0\ 3\ 80 \\
 \hline
 & \quad 3\ 0\ 8\ 80 \\
 - & \quad 2\ 0\ 1\ 90 \\
 \hline
 & \quad 1\ 0\ 6\ 90 \\
 & \quad ?\ ?\ ??
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  
- **Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & 1 & 2 \\
 3 & 63 \\
 \hline
 \end{array}$$
  

$$\begin{array}{r}
 2 \\
 \hline
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 6\ 0\ 5\ 7\ 00 \\
 \hline
 - \quad 3\ 9\ 4\ 3\ 00 \\
 - \quad 2\ 0\ 1\ 9\ 00 \\
 \hline
 - \quad 1\ 9\ 2\ 4\ 00 \\
 - \quad 1\ 2\ 1\ 1\ 40 \\
 \hline
 & \quad 7\ 1\ 2\ 60 \\
 - \quad 4\ 0\ 3\ 80 \\
 \hline
 - \quad 3\ 0\ 8\ 80 \\
 - \quad 2\ 0\ 1\ 90 \\
 \hline
 & \quad 1\ 0\ 6\ 90 \\
 & \quad 6\ 0\ 57
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 3 \cdot 9 &= 27 \\
 3 \cdot 1 + 2 &= 5 \\
 3 \cdot 0 &= 0 \\
 3 \cdot 2 &= 6
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 6\ 0\ 5\ 7\ 00 \\
 \hline
 - \quad 3\ 9\ 4\ 3\ 00 \\
 - \quad 2\ 0\ 1\ 9\ 00 \\
 \hline
 - \quad 1\ 9\ 2\ 4\ 00 \\
 - \quad 1\ 2\ 1\ 1\ 40 \\
 \hline
 & \quad 7\ 1\ 2\ 60 \\
 - & \quad 4\ 0\ 3\ 80 \\
 \hline
 - & \quad 3\ 0\ 8\ 80 \\
 - & \quad 2\ 0\ 1\ 90 \\
 \hline
 & \quad 1\ 0\ 6\ 90 \\
 & \quad 6\ 0\ 57
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 1\ 0\ 5\ 7\ 0\ 0 \\
 \hline
 - \quad 3\ 9\ 4\ 3\ 0\ 0 \\
 - \quad 2\ 0\ 1\ 9\ 0\ 0 \\
 \hline
 - \quad 1\ 9\ 2\ 4\ 0\ 0 \\
 - \quad 1\ 2\ 1\ 1\ 4\ 0 \\
 \hline
 & \quad 7\ 1\ 2\ 6\ 0 \\
 - & \quad 4\ 0\ 3\ 8\ 0 \\
 \hline
 & \quad 3\ 0\ 8\ 8\ 0 \\
 - & \quad 2\ 0\ 1\ 9\ 0 \\
 \hline
 & \quad 1\ 0\ 6\ 9\ 0 \\
 & \quad 6\ 0\ 5\ 7
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  
- **Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 6\ 0\ 5\ 7\ 00 \\
 \hline
 - \quad 3\ 9\ 4\ 3\ 00 \\
 - \quad 2\ 0\ 1\ 9\ 00 \\
 \hline
 - \quad 1\ 9\ 2\ 4\ 00 \\
 - \quad 1\ 2\ 1\ 1\ 40 \\
 \hline
 & \quad 7\ 1\ 2\ 60 \\
 - & \quad 4\ 0\ 3\ 80 \\
 \hline
 - & \quad 3\ 0\ 8\ 80 \\
 - & \quad 2\ 0\ 1\ 90 \\
 \hline
 & \quad 1\ 0\ 6\ 90 \\
 - & \quad 6\ 0\ 57 \\
 \hline
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r} & 1 & 0 & 0 & 0 & 0 & 0 \\ 2019 & \overline{-} & 1 & 0 & 0 & 0 & 0 \\ & - & 6 & 0 & 5 & 7 & 00 \\ & & \underline{-} & & & & \\ & & 3 & 9 & 4 & 3 & 00 \\ & - & 2 & 0 & 1 & 9 & 00 \\ & & \underline{-} & & & & \\ & & 1 & 9 & 2 & 4 & 00 \\ & - & 1 & 2 & 1 & 1 & 40 \\ & & \underline{-} & & & & \\ & & 7 & 1 & 2 & 60 \\ & - & 4 & 0 & 3 & 80 \\ & & \underline{-} & & & & \\ & & 3 & 0 & 8 & 80 \\ & - & 2 & 0 & 1 & 90 \\ & & \underline{-} & & & & \\ & & 1 & 0 & 6 & 90 \\ & - & 6 & 0 & 57 \\ & & \underline{-} & & & & \\ & & ? & ? & ? ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r} & 1 & 0 & 0 & 0 & 0 & 0 \\ 2019 & \overline{-} & 1 & 0 & 0 & 0 & 0 \\ & - & 6 & 0 & 5 & 7 & 00 \\ & & \underline{-} & & & & \\ & & 3 & 9 & 4 & 3 & 00 \\ & - & 2 & 0 & 1 & 9 & 00 \\ & & \underline{-} & & & & \\ & & 1 & 9 & 2 & 4 & 00 \\ & - & 1 & 2 & 1 & 1 & 40 \\ & & \underline{-} & & & & \\ & & 7 & 1 & 2 & 60 \\ & - & 4 & 0 & 3 & 80 \\ & & \underline{-} & & & \\ & & 3 & 0 & 8 & 80 \\ & - & 2 & 0 & 1 & 90 \\ & & \underline{-} & & & \\ & & 1 & 0 & 6 & 90 \\ & - & 6 & 0 & 57 \\ & & \underline{-} & & & \\ & & 4 & 6 & 33 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract**.
- Repeat.

$$0 - 7 = -7 = -10 + 3$$

$$-1 + 9 - 5 = 3$$

$$6 - 0 = 6$$

$$0 - 6 = -6 = -10 + 4$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r} & 1 & 0 & 0 & 0 & 0 & 0 \\ 2019 & \overline{-} & 1 & 0 & 0 & 0 & 0 \\ & - & 6 & 0 & 5 & 7 & 00 \\ & & \underline{-} & 3 & 9 & 4 & 3 & 00 \\ & & - & 2 & 0 & 1 & 9 & 00 \\ & & & \underline{\underline{-}} & 1 & 9 & 2 & 4 & 00 \\ & & & - & 1 & 2 & 1 & 1 & 40 \\ & & & & \underline{\underline{-}} & 7 & 1 & 2 & 60 \\ & & & & - & 4 & 0 & 3 & 80 \\ & & & & & \underline{\underline{-}} & 3 & 0 & 8 & 80 \\ & & & & - & 2 & 0 & 1 & 90 \\ & & & & & \underline{\underline{-}} & \color{red}{-1} & \color{red}{-1} & & \\ & & & & - & 1 & 0 & 6 & 90 \\ & & & & - & 6 & 0 & 57 \\ & & & & & \underline{\underline{-}} & 4 & 6 & 33 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
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- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned} 0 - 7 &= -7 = -10 + 3 \\ -1 + 9 - 5 &= 3 \\ 6 - 0 &= 6 \\ 0 - 6 &= -6 = -10 + 4 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & 1 & 2 \\
 3 & 63 \\
 \\ 
 \boxed{2019} & \overline{-} & 1 & 0 & 0 & 0 & 0 & 0 \\
 & - & 6 & 0 & 5 & 7 & 00 \\
 & & \overline{-} & 3 & 9 & 4 & 3 & 00 \\
 & - & 2 & 0 & 1 & 9 & 00 \\
 & & \overline{-} & 1 & 9 & 2 & 4 & 00 \\
 & - & 1 & 2 & 1 & 1 & 40 \\
 & & \overline{-} & 7 & 1 & 2 & 60 \\
 & - & 4 & 0 & 3 & 80 \\
 & & \overline{-} & 3 & 0 & 8 & 80 \\
 & - & 2 & 0 & 1 & 90 \\
 & & \overline{-} & 1 & 0 & 6 & 90 \\
 & - & 6 & 0 & 57 \\
 & & \overline{-} & 4 & 6 & 33
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 1\ 0\ 0\ 0\ 0\ 0 \\
 \hline
 \quad 6\ 0\ 5\ 7\ 00 \\
 - \quad 3\ 9\ 4\ 3\ 00 \\
 \hline
 \quad 2\ 0\ 1\ 9\ 00 \\
 - \quad 1\ 9\ 2\ 4\ 00 \\
 \hline
 \quad 1\ 2\ 1\ 1\ 40 \\
 - \quad 7\ 1\ 2\ 60 \\
 \hline
 \quad 4\ 0\ 3\ 80 \\
 - \quad 3\ 0\ 8\ 80 \\
 \hline
 \quad 2\ 0\ 1\ 90 \\
 - \quad 1\ 0\ 6\ 90 \\
 \hline
 \quad 6\ 0\ 57 \\
 - \quad 4\ 6\ 33
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 1\ 0\ 0\ 0\ 0\ 0 \\
 \hline
 \quad 6\ 0\ 5\ 7\ 00 \\
 - \quad 3\ 9\ 4\ 3\ 00 \\
 \hline
 \quad 2\ 0\ 1\ 9\ 00 \\
 - \quad 1\ 9\ 2\ 4\ 00 \\
 \hline
 \quad 1\ 2\ 1\ 1\ 40 \\
 - \quad 7\ 1\ 2\ 60 \\
 \hline
 \quad 4\ 0\ 3\ 80 \\
 - \quad 3\ 0\ 8\ 80 \\
 \hline
 \quad 2\ 0\ 1\ 90 \\
 - \quad 1\ 0\ 6\ 90 \\
 \hline
 \quad 6\ 0\ 57 \\
 - \quad 4\ 6\ 33
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 ? \\ 3 \ 63 \end{array}$$

$$\begin{array}{r} & 1 & 0 & 0 & 0 & 0 & 0 \\ 2019 & \overline{-} & 1 & 0 & 0 & 0 & 0 \\ & - & 6 & 0 & 5 & 7 & 00 \\ & & \underline{-} & & & & \\ & & 3 & 9 & 4 & 3 & 00 \\ & - & 2 & 0 & 1 & 9 & 00 \\ & & \underline{-} & & & & \\ & & 1 & 9 & 2 & 4 & 00 \\ & - & 1 & 2 & 1 & 1 & 40 \\ & & \underline{-} & & & & \\ & & 7 & 1 & 2 & 60 \\ & - & 4 & 0 & 3 & 80 \\ & & \underline{-} & & & & \\ & & 3 & 0 & 8 & 80 \\ & - & 2 & 0 & 1 & 90 \\ & & \underline{-} & & & & \\ & & 1 & 0 & 6 & 90 \\ & - & 6 & 0 & 57 \\ & & \underline{-} & & & & \\ & & 4 & 6 & 33 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2 ? \\ 3 \ 63 \end{array}$$

$$\begin{array}{r} & 1 & 0 & 0 & 0 & 0 & 0 \\ 2019 & \overline{-} & 6 & 0 & 5 & 7 & 00 \\ & - & 3 & 9 & 4 & 3 & 00 \\ & - & 2 & 0 & 1 & 9 & 00 \\ & \hline & 1 & 9 & 2 & 4 & 00 \\ & - & 1 & 2 & 1 & 1 & 40 \\ & \hline & & 7 & 1 & 2 & 60 \\ & - & & 4 & 0 & 3 & 80 \\ & \hline & & 3 & 0 & 8 & 80 \\ & - & & 2 & 0 & 1 & 90 \\ & \hline & & 1 & 0 & 6 & 90 \\ & - & & 6 & 0 & 57 \\ & \hline & & 4 & 6 & 33 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
    - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{4}{2+1} \right] = ?$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2\textcolor{red}{1} \\ 3 \ 63 \end{array}$$

$$\begin{array}{r} 2019 \overline{)1 \ 0 \ 0 \ 0 \ 0 \ 0} \\ - \quad 6 \ 0 \ 5 \ 7 \ 00 \\ \hline - \quad 3 \ 9 \ 4 \ 3 \ 00 \\ - \quad 2 \ 0 \ 1 \ 9 \ 00 \\ \hline - \quad 1 \ 9 \ 2 \ 4 \ 00 \\ - \quad 1 \ 2 \ 1 \ 1 \ 40 \\ \hline - \quad \quad 7 \ 1 \ 2 \ 60 \\ - \quad \quad 4 \ 0 \ 3 \ 80 \\ \hline - \quad \quad 3 \ 0 \ 8 \ 80 \\ - \quad \quad 2 \ 0 \ 1 \ 90 \\ \hline - \quad \quad 1 \ 0 \ 6 \ 90 \\ - \quad \quad 6 \ 0 \ 57 \\ \hline \textcolor{red}{4} \ 6 \ 33 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
    - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\left[ \frac{4}{2+1} \right] = \left[ \frac{4}{3} \right] = \left[ \frac{3}{3} + \frac{1}{3} \right] = \left[ 1 + \frac{1}{3} \right] = 1$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2\textcolor{red}{1} \\ 3 \ 63 \end{array}$$

$$\begin{array}{r} 2019 \longdiv{1 \ 0 \ 0 \ 0 \ 0 \ 0} \\ - \ 1 \ 0 \ 5 \ 7 \ 0 0 \\ \hline - \ 3 \ 9 \ 4 \ 3 \ 0 0 \\ - \ 2 \ 0 \ 1 \ 9 \ 0 0 \\ \hline - \ 1 \ 9 \ 2 \ 4 \ 0 0 \\ - \ 1 \ 2 \ 1 \ 1 \ 4 0 \\ \hline - \ 7 \ 1 \ 2 \ 6 0 \\ - \ 4 \ 0 \ 3 \ 8 0 \\ \hline - \ 3 \ 0 \ 8 \ 8 0 \\ - \ 2 \ 0 \ 1 \ 9 0 \\ \hline - \ 1 \ 0 \ 6 \ 9 0 \\ - \ 6 \ 0 \ 5 7 \\ \hline 4 \ 6 \ 33 \\ \textcolor{red}{?} \ ? \ ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  
- **Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 2\textcolor{red}{1} \\ 3 \ 63 \end{array}$$

$$\begin{array}{r} 2019 \overline{)1 \ 0 \ 0 \ 0 \ 0 \ 0} \\ - \ 1 \ 0 \ 5 \ 7 \ 0 0 \\ \hline - \ 3 \ 9 \ 4 \ 3 \ 0 0 \\ - \ 2 \ 0 \ 1 \ 9 \ 0 0 \\ \hline - \ 1 \ 9 \ 2 \ 4 \ 0 0 \\ - \ 1 \ 2 \ 1 \ 1 \ 4 0 \\ \hline - \ 7 \ 1 \ 2 \ 6 0 \\ - \ 4 \ 0 \ 3 \ 8 0 \\ \hline - \ 3 \ 0 \ 8 \ 8 0 \\ - \ 2 \ 0 \ 1 \ 9 0 \\ \hline - \ 1 \ 0 \ 6 \ 9 0 \\ - \ 6 \ 0 \ 5 7 \\ \hline 4 \ 6 \ 33 \\ 2 \ 0 \ 19 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- **Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.**
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & 1 & 21 \\
 3 & 63 \\
 \\ 
 \boxed{2019} & \overline{-} & 1 & 0 & 0 & 0 & 0 & 0 \\
 & - & 6 & 0 & 5 & 7 & 00 \\
 & & \overline{-} & 3 & 9 & 4 & 3 & 00 \\
 & - & 2 & 0 & 1 & 9 & 00 \\
 & & \overline{-} & 1 & 9 & 2 & 4 & 00 \\
 & - & 1 & 2 & 1 & 1 & 40 \\
 & & \overline{-} & 7 & 1 & 2 & 60 \\
 & - & 4 & 0 & 3 & 80 \\
 & & \overline{-} & 3 & 0 & 8 & 80 \\
 & - & 2 & 0 & 1 & 90 \\
 & & \overline{-} & 1 & 0 & 6 & 90 \\
 & - & 6 & 0 & 57 \\
 & & \overline{-} & 4 & 6 & 33 \\
 & - & 2 & 0 & 19 \\
 & & \overline{-} & & & & 
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 21 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 6\ 0\ 5\ 7\ 00 \\
 \hline
 - \quad 3\ 9\ 4\ 3\ 00 \\
 - \quad 2\ 0\ 1\ 9\ 00 \\
 \hline
 - \quad 1\ 9\ 2\ 4\ 00 \\
 - \quad 1\ 2\ 1\ 1\ 40 \\
 \hline
 & \quad 7\ 1\ 2\ 60 \\
 - & \quad 4\ 0\ 3\ 80 \\
 \hline
 & \quad 3\ 0\ 8\ 80 \\
 - & \quad 2\ 0\ 1\ 90 \\
 \hline
 & \quad 1\ 0\ 6\ 90 \\
 - & \quad 6\ 0\ 57 \\
 \hline
 & \quad 4\ 6\ 33 \\
 - & \quad 2\ 0\ 19 \\
 \hline
 & \quad ?\ ?\ ?
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

## Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

1  
21  
63

2019	$\begin{array}{r} 1 \ 0 \ 0 \ 0 \ 0 \ 0 \\ - 6 \ 0 \ 5 \ 7 \ 0 \ 0 \\ \hline \end{array}$
	$\begin{array}{r} 3 \ 9 \ 4 \ 3 \ 0 \ 0 \\ - 2 \ 0 \ 1 \ 9 \ 0 \ 0 \\ \hline \end{array}$
	$\begin{array}{r} 1 \ 9 \ 2 \ 4 \ 0 \ 0 \\ - 1 \ 2 \ 1 \ 1 \ 4 \ 0 \\ \hline \end{array}$
	$\begin{array}{r} 7 \ 1 \ 2 \ 6 \ 0 \ 0 \\ - 4 \ 0 \ 3 \ 8 \ 0 \ 0 \\ \hline \end{array}$
	$\begin{array}{r} 3 \ 0 \ 8 \ 8 \ 0 \ 0 \\ - 2 \ 0 \ 1 \ 9 \ 0 \ 0 \\ \hline \end{array}$
	$\begin{array}{r} 1 \ 0 \ 6 \ 9 \ 0 \ 0 \\ - 6 \ 0 \ 5 \ 7 \ 0 \ 0 \\ \hline \end{array}$
	$\begin{array}{r} 4 \ 6 \ 3 \ 3 \ 0 \ 0 \\ - 2 \ 0 \ 1 \ 9 \ 0 \ 0 \\ \hline \end{array}$
	$2 \ 6 \ 14 \ 0 \ 0 \ 0$

- Find shortest dividend start larger than divisor.
  - Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
    - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
    - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
  - **Subtract.**
  - **Repeat.**

$$\begin{aligned}3 - 9 &= -6 = -10 + 4 \\3 - 1 &= 1 \\6 - 0 &= 6 \\4 - 2 &= 2\end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 1 \\
 & 1 \ 21 \\
 & 3 \ 63 \\
 \\ 
 \boxed{2019} & \left[ \begin{array}{r}
 1 \ 0 \ 0 \ 0 \ 0 \ 0 \\
 - 6 \ 0 \ 5 \ 7 \ 00 \\
 \hline
 3 \ 9 \ 4 \ 3 \ 00 \\
 - 2 \ 0 \ 1 \ 9 \ 00 \\
 \hline
 1 \ 9 \ 2 \ 4 \ 00 \\
 - 1 \ 2 \ 1 \ 1 \ 40 \\
 \hline
 7 \ 1 \ 2 \ 60 \\
 - 4 \ 0 \ 3 \ 80 \\
 \hline
 3 \ 0 \ 8 \ 80 \\
 - 2 \ 0 \ 1 \ 90 \\
 \hline
 1 \ 0 \ 6 \ 90 \\
 - 6 \ 0 \ 57 \\
 \hline
 \color{red}{-1} \\
 - 4 \ 6 \ 33 \\
 \hline
 2 \ 0 \ 19 \\
 \hline
 2 \ 6 \ 14
 \end{array} \right]
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 3 - 9 &= -6 = -10 + 4 \\
 -1 + 3 - 1 &= 1 \\
 6 - 0 &= 6 \\
 4 - 2 &= 2
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 \\
 & 1 & 21 \\
 3 & 63 \\
 \hline
 2019 & \left[ \begin{array}{r} 1 & 0 & 0 & 0 & 0 & 0 \\ - & 6 & 0 & 5 & 7 & 00 \\ \hline 3 & 9 & 4 & 3 & 00 \\ - & 2 & 0 & 1 & 9 & 00 \\ \hline 1 & 9 & 2 & 4 & 00 \\ - & 1 & 2 & 1 & 1 & 40 \\ \hline 7 & 1 & 2 & 60 \\ - & 4 & 0 & 3 & 80 \\ \hline 3 & 0 & 8 & 80 \\ - & 2 & 0 & 1 & 90 \\ \hline 1 & 0 & 6 & 90 \\ - & 6 & 0 & 57 \\ \hline 4 & 6 & 33 \\ - & 2 & 0 & 19 \\ \hline 2 & 6 & 14
 \end{array} \right]
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 21 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 1\ 0\ 0\ 0\ 0\ 0 \\
 \hline
 \quad 6\ 0\ 5\ 7\ 00 \\
 - \quad 3\ 9\ 4\ 3\ 00 \\
 \hline
 \quad 2\ 0\ 1\ 9\ 00 \\
 \hline
 \quad 1\ 9\ 2\ 4\ 00 \\
 - \quad 1\ 2\ 1\ 1\ 40 \\
 \hline
 \quad 7\ 1\ 2\ 60 \\
 - \quad 4\ 0\ 3\ 80 \\
 \hline
 \quad 3\ 0\ 8\ 80 \\
 - \quad 2\ 0\ 1\ 90 \\
 \hline
 \quad 1\ 0\ 6\ 90 \\
 - \quad 6\ 0\ 57 \\
 \hline
 \quad 4\ 6\ 33 \\
 - \quad 2\ 0\ 19 \\
 \hline
 \quad 2\ 6\ 14
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \ 21 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 1\ 0\ 0\ 0\ 0\ 0\ 0 \\
 \hline
 \quad 6\ 0\ 5\ 7\ 0\ 0 \\
 - \quad 3\ 9\ 4\ 3\ 0\ 0 \\
 \hline
 \quad 2\ 0\ 1\ 9\ 0\ 0 \\
 \hline
 \quad 1\ 9\ 2\ 4\ 0\ 0 \\
 - \quad 1\ 2\ 1\ 1\ 4\ 0 \\
 \hline
 \quad 7\ 1\ 2\ 6\ 0 \\
 - \quad 4\ 0\ 3\ 8\ 0 \\
 \hline
 \quad 3\ 0\ 8\ 8\ 0 \\
 - \quad 2\ 0\ 1\ 9\ 0 \\
 \hline
 \quad 1\ 0\ 6\ 9\ 0 \\
 - \quad 6\ 0\ 5\ 7 \\
 \hline
 \quad 4\ 6\ 3\ 3 \\
 - \quad 2\ 0\ 1\ 9 \\
 \hline
 \quad 2\ 6\ 14
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 & ? \\
 & 1 & 2 & 1 \\
 & 3 & 6 & 3 \\
 \\ 
 \boxed{2019} & \overline{)1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0} \\
 - & 1 \ 0 \ 5 \ 7 \ 0 & 0 \\
 \hline
 & 3 & 9 & 4 & 3 & 0 & 0 \\
 - & 2 & 0 & 1 & 9 & 0 & 0 \\
 \hline
 & 1 & 9 & 2 & 4 & 0 & 0 \\
 - & 1 & 2 & 1 & 1 & 4 & 0 \\
 \hline
 & & 7 & 1 & 2 & 6 & 0 \\
 - & & 4 & 0 & 3 & 8 & 0 \\
 \hline
 & & 3 & 0 & 8 & 8 & 0 \\
 - & & 2 & 0 & 1 & 9 & 0 \\
 \hline
 & & 1 & 0 & 6 & 9 & 0 \\
 - & & 6 & 0 & 5 & 7 & 0 \\
 \hline
 & & 4 & 6 & 3 & 3 & 0 \\
 - & & 2 & 0 & 1 & 9 & 0 \\
 \hline
 & & 2 & 6 & 1 & 4 & 0 \\
 & & & & & & 0 \\
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 & ? \\
 & 1 & 2 & 1 \\
 & 3 & 6 & 3 \\
 \\ 
 \boxed{2019} & \overline{)1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0} \\
 - & 1 \ 0 \ 5 \ 7 \ 0 & 0 \\
 \hline
 & 3 \ 9 \ 4 \ 3 \ 0 & 0 \\
 - & 2 \ 0 \ 1 \ 9 \ 0 & 0 \\
 \hline
 & 1 \ 9 \ 2 \ 4 \ 0 & 0 \\
 - & 1 \ 2 \ 1 \ 1 & 40 \\
 \hline
 & 7 \ 1 \ 2 \ 6 & 0 \\
 - & 4 \ 0 \ 3 \ 8 & 0 \\
 \hline
 & 3 \ 0 \ 8 \ 8 & 0 \\
 - & 2 \ 0 \ 1 \ 9 & 0 \\
 \hline
 & 1 \ 0 \ 6 \ 9 & 0 \\
 - & 6 \ 0 \ 5 & 7 \\
 \hline
 & 4 \ 6 \ 3 & 3 \\
 - & 2 \ 0 \ 1 & 9 \\
 \hline
 & 2 \ 6 \ 1 & 4
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits;
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 & ? \\
 & 1 & 2 & 1 \\
 & 3 & 6 & 3 \\
 \\ 
 \boxed{2019} & \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0 & 0 & 0 & 0 & 0 & 0 \\
 & - & 6 & 0 & 5 & 7 & 0 0 \\
 & & 3 & 9 & 4 & 3 & 0 0 \\
 - & & 2 & 0 & 1 & 9 & 0 0 \\
 & & \hline & & & & \\
 & & 1 & 9 & 2 & 4 & 0 0 \\
 - & & 1 & 2 & 1 & 1 & 4 0 \\
 & & \hline & & & & \\
 & & 7 & 1 & 2 & 6 0 \\
 - & & 4 & 0 & 3 & 8 0 \\
 & & \hline & & & \\
 & & 3 & 0 & 8 & 8 0 \\
 - & & 2 & 0 & 1 & 9 0 \\
 & & \hline & & & \\
 & & 1 & 0 & 6 & 9 0 \\
 - & & 6 & 0 & 5 7 \\
 & & \hline & & & \\
 & & 4 & 6 & 3 3 \\
 - & & 2 & 0 & 1 9 \\
 & & \hline & & & \\
 & & 2 & 6 & 1 4
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; **larger divisor start has same # of digits**
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 11 \\
 & 1 & 21 \\
 & 3 & 63 \\
 \\ 
 \boxed{2019} & \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0 & 0\ 0\ 0\ 0 \\
 & 6\ 0 & 5\ 7\ 0\ 0 \\
 \\ 
 - & 3\ 9 & 4\ 3\ 0\ 0 \\
 & 2\ 0 & 1\ 9\ 0\ 0 \\
 \hline & 1\ 9 & 2\ 4\ 0\ 0 \\
 - & 1\ 2 & 1\ 1\ 4\ 0 \\
 \hline & 7\ 1 & 2\ 6\ 0 \\
 - & 4\ 0 & 3\ 8\ 0 \\
 \hline & 3\ 0 & 8\ 8\ 0 \\
 - & 2\ 0 & 1\ 9\ 0 \\
 \hline & 1\ 0 & 6\ 9\ 0 \\
 - & 6\ 0 & 5\ 7 \\
 \hline & 4\ 6 & 3\ 3 \\
 - & 2\ 0 & 1\ 9 \\
 \hline & 2\ 6 & 14
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 11 \\ 1 \quad 21 \\ 3 \quad 63 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1000000} \\
 - \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 00 \\
 \underline{-} \quad 6 \quad 0 \quad 5 \quad 7 \quad 00 \\
 - \quad 3 \quad 9 \quad 4 \quad 3 \quad 00 \\
 \underline{-} \quad 2 \quad 0 \quad 1 \quad 9 \quad 00 \\
 \underline{\underline{-}} \quad 1 \quad 9 \quad 2 \quad 4 \quad 00 \\
 \underline{-} \quad 1 \quad 2 \quad 1 \quad 1 \quad 40 \\
 \underline{\underline{-}} \quad 7 \quad 1 \quad 2 \quad 60 \\
 \underline{-} \quad 4 \quad 0 \quad 3 \quad 80 \\
 \underline{\underline{-}} \quad 3 \quad 0 \quad 8 \quad 80 \\
 \underline{-} \quad 2 \quad 0 \quad 1 \quad 90 \\
 \underline{\underline{-}} \quad 1 \quad 0 \quad 6 \quad 90 \\
 \underline{-} \quad 6 \quad 0 \quad 57 \\
 \underline{\underline{-}} \quad 4 \quad 6 \quad 33 \\
 \underline{-} \quad 2 \quad 0 \quad 19 \\
 \underline{\underline{-}} \quad 2 \quad 6 \quad 14 \\
 \textcolor{red}{?} \quad \textcolor{red}{?} \quad \textcolor{red}{??}
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 11 \\ 1 \ 21 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 1\ 0\ 0\ 0\ 0\ 0\ 0 \\
 \hline
 \quad 6\ 0\ 5\ 7\ 0\ 0 \\
 - \quad 3\ 9\ 4\ 3\ 0\ 0 \\
 \hline
 \quad 2\ 0\ 1\ 9\ 0\ 0 \\
 \hline
 \quad 1\ 9\ 2\ 4\ 0\ 0 \\
 - \quad 1\ 2\ 1\ 1\ 4\ 0 \\
 \hline
 \quad 7\ 1\ 2\ 6\ 0 \\
 - \quad 4\ 0\ 3\ 8\ 0 \\
 \hline
 \quad 3\ 0\ 8\ 8\ 0 \\
 - \quad 2\ 0\ 1\ 9\ 0 \\
 \hline
 \quad 1\ 0\ 6\ 9\ 0 \\
 - \quad 6\ 0\ 5\ 7 \\
 \hline
 \quad 4\ 6\ 3\ 3 \\
 - \quad 2\ 0\ 1\ 9 \\
 \hline
 \quad 2\ 6\ 14 \\
 \textcolor{red}{-} \quad \textcolor{red}{2}\ \textcolor{red}{0}\ \textcolor{red}{19}
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- **Multiply quotient digit by divisor, put result under current dividend.** Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 11 \\
 & 1 \ 21 \\
 & 3 \ 63 \\
 \\ 
 \boxed{2019} & \left[ \begin{array}{r} 1 \ 0 \ 0 \ 0 \ 0 \ 0 \\ - 6 \ 0 \ 5 \ 7 \ 00 \\ \hline 3 \ 9 \ 4 \ 3 \ 00 \\ - 2 \ 0 \ 1 \ 9 \ 00 \\ \hline 1 \ 9 \ 2 \ 4 \ 00 \\ - 1 \ 2 \ 1 \ 1 \ 40 \\ \hline 7 \ 1 \ 2 \ 60 \\ - 4 \ 0 \ 3 \ 80 \\ \hline 3 \ 0 \ 8 \ 80 \\ - 2 \ 0 \ 1 \ 90 \\ \hline 1 \ 0 \ 6 \ 90 \\ - 6 \ 0 \ 57 \\ \hline 4 \ 6 \ 33 \\ - 2 \ 0 \ 19 \\ \hline 2 \ 6 \ 14 \\ - 2 \ 0 \ 19 \\ \hline \end{array} \right]
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 11 \\
 & 1 \ 21 \\
 & 3 \ 63 \\
 \\ 
 \boxed{2019} & \left[ \begin{array}{r} 1 \ 0 \ 0 \ 0 \ 0 \ 0 \\ - 6 \ 0 \ 5 \ 7 \ 00 \\ \hline 3 \ 9 \ 4 \ 3 \ 00 \\ - 2 \ 0 \ 1 \ 9 \ 00 \\ \hline 1 \ 9 \ 2 \ 4 \ 00 \\ - 1 \ 2 \ 1 \ 1 \ 40 \\ \hline 7 \ 1 \ 2 \ 60 \\ - 4 \ 0 \ 3 \ 80 \\ \hline 3 \ 0 \ 8 \ 80 \\ - 2 \ 0 \ 1 \ 90 \\ \hline 1 \ 0 \ 6 \ 90 \\ - 6 \ 0 \ 57 \\ \hline 4 \ 6 \ 33 \\ - 2 \ 0 \ 19 \\ \hline 2 \ 6 \ 14 \\ - 2 \ 0 \ 19 \\ \hline ? \ ??
 \end{array} \right]
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 11 \\
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 0\ 0\ 0\ 0 \\
 \hline
 & 6\ 0\ 5\ 7\ 00 \\
 - & 3\ 9\ 4\ 3\ 00 \\
 \hline
 & 2\ 0\ 1\ 9\ 00 \\
 - & 1\ 9\ 2\ 4\ 00 \\
 \hline
 & 1\ 2\ 1\ 1\ 40 \\
 - & 7\ 1\ 2\ 60 \\
 \hline
 & 4\ 0\ 3\ 80 \\
 - & 3\ 0\ 8\ 80 \\
 \hline
 & 2\ 0\ 1\ 90 \\
 - & 1\ 0\ 6\ 90 \\
 \hline
 & 6\ 0\ 57 \\
 - & 4\ 6\ 33 \\
 \hline
 & 2\ 0\ 19 \\
 - & \underline{\underline{-1-1}} \\
 & 2\ 6\ 14 \\
 - & 2\ 0\ 19 \\
 \hline
 & 5\ 95
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- **Subtract.**
- Repeat.

$$\begin{aligned}
 4 - 9 &= -5 = -10 + 5 \\
 -1 + 1 - 1 &= -1 = -10 + 9 \\
 -1 + 6 - 0 &= 5 \\
 2 - 2 &= 0
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 11 \\
 2019 & \overline{)1\ 0\ 0\ 0\ 0\ 0} \\
 - & 1\ 0\ 5\ 7\ 0\ 0 \\
 \hline
 & 3\ 9\ 4\ 3\ 0\ 0 \\
 - & 2\ 0\ 1\ 9\ 0\ 0 \\
 \hline
 & 1\ 9\ 2\ 4\ 0\ 0 \\
 - & 1\ 2\ 1\ 1\ 4\ 0 \\
 \hline
 & 7\ 1\ 2\ 6\ 0 \\
 - & 4\ 0\ 3\ 8\ 0 \\
 \hline
 & 3\ 0\ 8\ 8\ 0 \\
 - & 2\ 0\ 1\ 9\ 0 \\
 \hline
 & 1\ 0\ 6\ 9\ 0 \\
 - & 6\ 0\ 5\ 7 \\
 \hline
 & 4\ 6\ 3\ 3 \\
 - & 2\ 0\ 1\ 9 \\
 \hline
 & \underline{\underline{-1-1}} \\
 & 2\ 6\ 1\ 4 \\
 - & 2\ 0\ 1\ 9 \\
 \hline
 & 5\ 9\ 5
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

$$\begin{aligned}
 4 - 9 &= -5 = -10 + 5 \\
 -1 + 1 - 1 &= -1 = -10 + 9 \\
 -1 + 6 - 0 &= 5 \\
 2 - 2 &= 0
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & 11 \\
 & 1 \ 21 \\
 & 3 \ 63 \\
 \\ 
 \boxed{2019} & \left[ \begin{array}{r}
 1 \ 0 \ 0 \ 0 \ 0 \ 0 \\
 - 6 \ 0 \ 5 \ 7 \ 00 \\
 \hline
 3 \ 9 \ 4 \ 3 \ 00 \\
 - 2 \ 0 \ 1 \ 9 \ 00 \\
 \hline
 1 \ 9 \ 2 \ 4 \ 00 \\
 - 1 \ 2 \ 1 \ 1 \ 40 \\
 \hline
 7 \ 1 \ 2 \ 60 \\
 - 4 \ 0 \ 3 \ 80 \\
 \hline
 3 \ 0 \ 8 \ 80 \\
 - 2 \ 0 \ 1 \ 90 \\
 \hline
 1 \ 0 \ 6 \ 90 \\
 - 6 \ 0 \ 57 \\
 \hline
 4 \ 6 \ 33 \\
 - 2 \ 0 \ 19 \\
 \hline
 2 \ 6 \ 14 \\
 - 2 \ 0 \ 19 \\
 \hline
 5 \ 95
 \end{array} \right]
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 11 \\ 1 \quad 21 \\ 3 \quad 63 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 1\ 0\ 0\ 0\ 0\ 0\ 0 \\
 \hline
 \quad 6\ 0\ 5\ 7\ 0\ 0 \\
 - \quad 3\ 9\ 4\ 3\ 0\ 0 \\
 \hline
 \quad 2\ 0\ 1\ 9\ 0\ 0 \\
 \hline
 \quad 1\ 9\ 2\ 4\ 0\ 0 \\
 - \quad 1\ 2\ 1\ 1\ 4\ 0 \\
 \hline
 \quad 7\ 1\ 2\ 6\ 0 \\
 - \quad 4\ 0\ 3\ 8\ 0 \\
 \hline
 \quad 3\ 0\ 8\ 8\ 0 \\
 - \quad 2\ 0\ 1\ 9\ 0 \\
 \hline
 \quad 1\ 0\ 6\ 9\ 0 \\
 - \quad 6\ 0\ 5\ 7 \\
 \hline
 \quad 4\ 6\ 3\ 3 \\
 - \quad 2\ 0\ 1\ 9 \\
 \hline
 \quad 2\ 6\ 1\ 4 \\
 - \quad 2\ 0\ 1\ 9 \\
 \hline
 \quad 5\ 9\ 5
 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r} 11 \\ 1 \ 21 \\ 3 \ 63 \end{array}$$

$$\begin{array}{r}
 2019 \overline{)1\ 0\ 0\ 0\ 0\ 0\ 0} \\
 - \quad 6\ 0\ 5\ 7\ 00 \\
 \hline
 - \quad 3\ 9\ 4\ 3\ 00 \\
 - \quad 2\ 0\ 1\ 9\ 00 \\
 \hline
 - \quad 1\ 9\ 2\ 4\ 00 \\
 - \quad 1\ 2\ 1\ 1\ 40 \\
 \hline
 - \quad 7\ 1\ 2\ 60 \\
 - \quad 4\ 0\ 3\ 80 \\
 \hline
 - \quad 3\ 0\ 8\ 80 \\
 - \quad 2\ 0\ 1\ 90 \\
 \hline
 - \quad 1\ 0\ 6\ 90 \\
 - \quad 6\ 0\ 57 \\
 \hline
 - \quad 4\ 6\ 33 \\
 - \quad 2\ 0\ 19 \\
 \hline
 - \quad 2\ 6\ 14 \\
 - \quad 2\ 0\ 19 \\
 \hline
 & \quad 5\ 95
 \end{array}$$

- Find shortest dividend start larger than divisor. **If none, we're done.**
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- **Repeat.**

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 11 \\
 & + & \\
 & & 1 \ 21 \\
 & & 3 \ 63 \\
 \hline
 \\[-1.5ex]
 2019 & \left[ \begin{array}{r} 1 \ 0 \ 0 \ 0 \ 0 \ 0 \\ - 6 \ 0 \ 5 \ 7 \ 0 0 \\ \hline 3 \ 9 \ 4 \ 3 \ 0 0 \\ - 2 \ 0 \ 1 \ 9 \ 0 0 \\ \hline 1 \ 9 \ 2 \ 4 \ 0 0 \\ - 1 \ 2 \ 1 \ 1 \ 4 0 \\ \hline 7 \ 1 \ 2 \ 6 0 \\ - 4 \ 0 \ 3 \ 8 0 \\ \hline 3 \ 0 \ 8 \ 8 0 \\ - 2 \ 0 \ 1 \ 9 0 \\ \hline 1 \ 0 \ 6 \ 9 0 \\ - 6 \ 0 \ 5 7 \\ \hline 4 \ 6 \ 3 3 \\ - 2 \ 0 \ 1 9 \\ \hline 2 \ 6 \ 1 4 \\ - 2 \ 0 \ 1 9 \\ \hline 5 \ 9 5 \end{array} \right] \\
 & - & \\
 & & \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- **If more than one quotient row, add them.**

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 1 & 1 \\
 & + & 1 & 2 & 1 \\
 & & 3 & 6 & 3 \\
 \hline
 & & & & 5
 \end{array}$$
  

$$\begin{array}{r}
 2019 \quad | \quad 1 & 0 & 0 & 0 & 0 & 0 \\
 - & 6 & 0 & 5 & 7 & 00 \\
 \hline
 & 3 & 9 & 4 & 3 & 00 \\
 - & 2 & 0 & 1 & 9 & 00 \\
 \hline
 & 1 & 9 & 2 & 4 & 00 \\
 - & 1 & 2 & 1 & 1 & 40 \\
 \hline
 & 7 & 1 & 2 & 60 \\
 - & 4 & 0 & 3 & 80 \\
 \hline
 & 3 & 0 & 8 & 80 \\
 - & 2 & 0 & 1 & 90 \\
 \hline
 & 1 & 0 & 6 & 90 \\
 - & 6 & 0 & 57 \\
 \hline
 & 4 & 6 & 33 \\
 - & 2 & 0 & 19 \\
 \hline
 & 2 & 6 & 14 \\
 - & 2 & 0 & 19 \\
 \hline
 & & 5 & 95
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

$$3 + 1 + 1 = 5$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 11 \\
 + & & 1 \quad 21 \\
 & 3 & 63 \\
 \hline
 & & 95 \\
 \\ 
 2019 & \overline{)1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0} \\
 - & 6 \quad 0 \quad 5 \quad 7 \quad 00 \\
 \hline
 & 3 \quad 9 \quad 4 \quad 3 \quad 00 \\
 - & 2 \quad 0 \quad 1 \quad 9 \quad 00 \\
 \hline
 & 1 \quad 9 \quad 2 \quad 4 \quad 00 \\
 - & 1 \quad 2 \quad 1 \quad 1 \quad 40 \\
 \hline
 & 7 \quad 1 \quad 2 \quad 60 \\
 - & 4 \quad 0 \quad 3 \quad 80 \\
 \hline
 & 3 \quad 0 \quad 8 \quad 80 \\
 - & 2 \quad 0 \quad 1 \quad 90 \\
 \hline
 & 1 \quad 0 \quad 6 \quad 90 \\
 - & 6 \quad 0 \quad 57 \\
 \hline
 & 4 \quad 6 \quad 33 \\
 - & 2 \quad 0 \quad 19 \\
 \hline
 & 2 \quad 6 \quad 14 \\
 - & 2 \quad 0 \quad 19 \\
 \hline
 & 5 \quad 95
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

$$3 + 1 + 1 = 5$$

$$\textcolor{red}{6 + 2 + 1 = 9}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 11 \\
 + & & 1 \quad 21 \\
 & 3 & 63 \\
 \hline
 & 4 & 95 \\
 \\ 
 2019 & \overline{)1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0} \\
 - & 6 \quad 0 \quad 5 \quad 7 \quad 00 \\
 \hline
 & 3 \quad 9 \quad 4 \quad 3 \quad 00 \\
 - & 2 \quad 0 \quad 1 \quad 9 \quad 00 \\
 \hline
 & 1 \quad 9 \quad 2 \quad 4 \quad 00 \\
 - & 1 \quad 2 \quad 1 \quad 1 \quad 40 \\
 \hline
 & 7 \quad 1 \quad 2 \quad 60 \\
 - & 4 \quad 0 \quad 3 \quad 80 \\
 \hline
 & 3 \quad 0 \quad 8 \quad 80 \\
 - & 2 \quad 0 \quad 1 \quad 90 \\
 \hline
 & 1 \quad 0 \quad 6 \quad 90 \\
 - & 6 \quad 0 \quad 57 \\
 \hline
 & 4 \quad 6 \quad 33 \\
 - & 2 \quad 0 \quad 19 \\
 \hline
 & 2 \quad 6 \quad 14 \\
 - & 2 \quad 0 \quad 19 \\
 \hline
 & 5 \quad 95
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

$$\begin{aligned}
 3 + 1 + 1 &= 5 \\
 6 + 2 + 1 &= 9 \\
 \textcolor{red}{3 + 1} &= 4
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 11 \\
 + & & 1 & 21 \\
 & 3 & 63 \\
 \hline
 & 4 & 95 \\
 \\ 
 2019 & \overline{)1 & 0 & 0 & 0 & 0 & 0} \\
 - & 6 & 0 & 5 & 7 & 00 \\
 \hline
 - & 3 & 9 & 4 & 3 & 00 \\
 - & 2 & 0 & 1 & 9 & 00 \\
 \hline
 - & 1 & 9 & 2 & 4 & 00 \\
 - & 1 & 2 & 1 & 1 & 40 \\
 \hline
 & 7 & 1 & 2 & 60 \\
 - & 4 & 0 & 3 & 80 \\
 \hline
 - & 3 & 0 & 8 & 80 \\
 - & 2 & 0 & 1 & 90 \\
 \hline
 - & 1 & 0 & 6 & 90 \\
 - & 6 & 0 & 57 \\
 \hline
 & 4 & 6 & 33 \\
 - & 2 & 0 & 19 \\
 \hline
 & 2 & 6 & 14 \\
 - & 2 & 0 & 19 \\
 \hline
 & 5 & 95
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. **Dividend = 1000000**. Divisor = 2019. Quotient = 495. Remainder = 595.

$$\begin{aligned}
 3 + 1 + 1 &= 5 \\
 6 + 2 + 1 &= 9 \\
 3 + 1 &= 4
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 11 \\
 + & & 1 & 21 \\
 & 3 & 63 \\
 \hline
 & 4 & 95 \\
 \\ 
 \boxed{2019} & - & 1 & 0 & 0 & 0 & 0 & 00 \\
 & - & 6 & 0 & 5 & 7 & 00 \\
 \hline
 & - & 3 & 9 & 4 & 3 & 00 \\
 & - & 2 & 0 & 1 & 9 & 00 \\
 \hline
 & - & 1 & 9 & 2 & 4 & 00 \\
 & - & 1 & 2 & 1 & 1 & 40 \\
 \hline
 & & 7 & 1 & 2 & 60 \\
 & - & 4 & 0 & 3 & 80 \\
 \hline
 & - & 3 & 0 & 8 & 80 \\
 & - & 2 & 0 & 1 & 90 \\
 \hline
 & - & 1 & 0 & 6 & 90 \\
 & - & 6 & 0 & 57 \\
 \hline
 & & 4 & 6 & 33 \\
 & - & 2 & 0 & 19 \\
 \hline
 & - & 2 & 6 & 14 \\
 & - & 2 & 0 & 19 \\
 \hline
 & & 5 & 95
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 1000000. **Divisor = 2019**. Quotient = 495. Remainder = 595.

$$\begin{aligned}
 3 + 1 + 1 &= 5 \\
 6 + 2 + 1 &= 9 \\
 3 + 1 &= 4
 \end{aligned}$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 11 \\
 + & & 1 \ 21 \\
 & 3 & 63 \\
 \hline
 & 4 & 95 \\
 \\ 
 \boxed{2019} & 1 & 0 & 0 & 0 & 0 & 00 \\
 - & 6 & 0 & 5 & 7 & 00 \\
 \hline
 - & 3 & 9 & 4 & 3 & 00 \\
 - & 2 & 0 & 1 & 9 & 00 \\
 \hline
 - & 1 & 9 & 2 & 4 & 00 \\
 - & 1 & 2 & 1 & 1 & 40 \\
 \hline
 & 7 & 1 & 2 & 60 \\
 - & 4 & 0 & 3 & 80 \\
 \hline
 - & 3 & 0 & 8 & 80 \\
 - & 2 & 0 & 1 & 90 \\
 \hline
 - & 1 & 0 & 6 & 90 \\
 - & 6 & 0 & 57 \\
 \hline
 & 4 & 6 & 33 \\
 - & 2 & 0 & 19 \\
 \hline
 & 2 & 6 & 14 \\
 - & 2 & 0 & 19 \\
 \hline
 & 5 & 95
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
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  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend. Fill gaps with zeroes.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 1000000. Divisor = 2019. **Quotient = 495.** Remainder = 595.

$$3 + 1 + 1 = 5$$

$$6 + 2 + 1 = 9$$

$$3 + 1 = 4$$

# Full algorithm: a large example

Divide 1000000 by 2019 with quotient and remainder.

$$\begin{array}{r}
 & & 11 \\
 + & & 1 & 21 \\
 & 3 & 63 \\
 \hline
 & 4 & 95 \\
 \\ 
 \boxed{2019} & 1 & 0 & 0 & 0 & 0 & 00 \\
 - & 6 & 0 & 5 & 7 & 00 \\
 \hline
 - & 3 & 9 & 4 & 3 & 00 \\
 - & 2 & 0 & 1 & 9 & 00 \\
 \hline
 - & 1 & 9 & 2 & 4 & 00 \\
 - & 1 & 2 & 1 & 1 & 40 \\
 \hline
 & 7 & 1 & 2 & 60 \\
 - & 4 & 0 & 3 & 80 \\
 \hline
 - & 3 & 0 & 8 & 80 \\
 - & 2 & 0 & 1 & 90 \\
 \hline
 - & 1 & 0 & 6 & 90 \\
 - & 6 & 0 & 57 \\
 \hline
 & 4 & 6 & 33 \\
 - & 2 & 0 & 19 \\
 \hline
 & 2 & 6 & 14 \\
 - & 2 & 0 & 19 \\
 \hline
 & 5 & 95
 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
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- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 1000000. Divisor = 2019. Quotient = 495.  
Remainder = 595.

$$\begin{aligned}
 3 + 1 + 1 &= 5 \\
 6 + 2 + 1 &= 9 \\
 3 + 1 &= 4
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Divide 1000000 by 2019 with quotient and remainder.

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- If more than one quotient row, add them.
- Final answer. **Dividend = 1000000**. Divisor = 2019. Quotient = 495. Remainder = 595.
- Check.  $1000000 \stackrel{?}{=} 2019 \cdot 495 + 595$

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 & - & 2 & 0 & 1 & 9 & 0 & 0 \\
 \hline
 & - & 1 & 9 & 2 & 4 & 0 & 0 \\
 & - & 1 & 2 & 1 & 1 & 4 & 0 \\
 \hline
 & - & 7 & 1 & 2 & 6 & 0 & 0 \\
 & - & 4 & 0 & 3 & 8 & 0 & 0 \\
 & - & 3 & 0 & 8 & 8 & 0 & 0 \\
 & - & 2 & 0 & 1 & 9 & 0 & 0 \\
 \hline
 & - & 1 & 0 & 6 & 9 & 0 & 0 \\
 & - & 6 & 0 & 5 & 7 & 0 & 0 \\
 \hline
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 & - & 2 & 0 & 1 & 9 & 0 & 0 \\
 \hline
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 \hline
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 & - & 1 & 9 & 2 & 4 & 0 & 0 \\
 & - & 1 & 2 & 1 & 1 & 4 & 0 \\
 & - & 7 & 1 & 2 & 6 & 0 & 0 \\
 & - & 4 & 0 & 3 & 8 & 0 & 0 \\
 & - & 3 & 0 & 8 & 8 & 0 & 0 \\
 & - & 2 & 0 & 1 & 9 & 0 & 0 \\
 & - & 1 & 0 & 6 & 9 & 0 & 0 \\
 & - & 6 & 0 & 5 & 7 & 0 & 0 \\
 & - & 4 & 6 & 3 & 3 & 0 & 0 \\
 & - & 2 & 0 & 1 & 9 & 0 & 0 \\
 & - & 2 & 6 & 1 & 4 & 0 & 0 \\
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 3 + 1 + 1 &= 5 \\
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Divide 1000000 by 2019 with quotient and remainder.

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 \end{array}$$

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- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 1000000. Divisor = 2019. Quotient = 495.  
**Remainder = 595.**
- Check.  $1000000 \stackrel{?}{=} 2019 \cdot 495 + \textcolor{red}{595}$

$$\begin{aligned}
 3 + 1 + 1 &= 5 \\
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# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$11 \overline{)99}$$

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Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 11 \overline{)99} \end{array}$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

- Find shortest dividend start larger than divisor.

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# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

?

$$11 \overline{)99}$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit

$$\begin{array}{r} ? \\ 11 \overline{)99} \end{array}$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} ? \\ 11 \overline{)99} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

$$\left[ \frac{9}{1+1} \right]$$

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Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} ? \\ 11 \overline{)99} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit **by divisor digit** plus one. Round down if needed.

$$\left\lfloor \frac{9}{1+1} \right\rfloor$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} ? \\ 11 \overline{)99} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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Divide 99 by 11 with quotient and remainder.

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- Find shortest dividend start larger than divisor.
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$$\left[ \frac{9}{1+1} \right]$$

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  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

$$\left[ \frac{9}{1+1} \right] =$$

?

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

$$\begin{array}{r} 4 \\ 11 \overline{)99} \end{array}$$

- Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

$$\left\lfloor \frac{9}{1+1} \right\rfloor = \left\lfloor \frac{9}{2} \right\rfloor = \left\lfloor \frac{8}{2} + \frac{1}{2} \right\rfloor = \left\lfloor 4 + \frac{1}{2} \right\rfloor = 4$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 4 \\ 11 \overline{)99} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 4 \\ 11 \overline{)99} \\ ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$4 \cdot 1 = ?$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 4 \\ 11 \overline{)99} \\ 4 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$4 \cdot 1 = 4$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 4 \\ 11 \overline{)99} \\ -9 \\ \hline 9 \\ -8 \\ \hline 1 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{array}{r} 4 \cdot 1 = 4 \\ 4 \cdot 1 = ? \end{array}$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 4 \\ 11 \overline{)99} \\ 44 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{array}{r} 4 \cdot 1 = 4 \\ 4 \cdot 1 = 4 \end{array}$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 4 \\ 11 \overline{)99} \\ 44 \end{array}$$

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- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$\begin{array}{r} 4 \cdot 1 = 4 \\ 4 \cdot 1 = 4 \end{array}$$

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Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 4 \\ 11 \overline{)99} \\ 44 \end{array}$$

- Find shortest dividend start larger than divisor.
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- Multiply quotient digit by divisor, put result under current dividend.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 4 \\ 11 \overline{)99} \\ \underline{-44} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 4 \\ 11 \overline{)99} \\ -44 \\ \hline ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.

$$9 - 4 = ?$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 4 \\ 11 \overline{)99} \\ -44 \\ \hline 5 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.

$$9 - 4 = 5$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 4 \\ 11 \overline{)99} \\ -44 \\ \hline 5 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.

$$\begin{array}{r} 9 - 4 = 5 \\ 9 - 4 = ? \end{array}$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 4 \\ 11 \overline{)99} \\ -44 \\ \hline 55 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.

$$\begin{array}{r} 9 - 4 = 5 \\ \textcolor{red}{9 - 4 = 5} \end{array}$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 4 \\ 11 \overline{)99} \\ -44 \\ \hline 55 \end{array}$$

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Divide 99 by 11 with quotient and remainder.

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$$\begin{array}{r} 9 - 4 = 5 \\ 9 - 4 = 5 \end{array}$$

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  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\left[ \frac{5}{1+1} \right] = ?$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\left\lfloor \frac{5}{1+1} \right\rfloor = \left\lfloor \frac{5}{2} \right\rfloor = \left\lfloor \frac{4}{2} + \frac{1}{2} \right\rfloor = \left\lfloor 2 + \frac{1}{2} \right\rfloor = 2$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\begin{array}{r} 2 \cdot 1 = 2 \\ 2 \cdot 1 = 2 \end{array}$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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$$\begin{array}{r} 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline ?? \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\begin{array}{r} 5 - 2 = 3 \\ 5 - 2 = 3 \end{array}$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \end{array}$$

- Find shortest dividend start larger than divisor.
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- Subtract.
- Repeat.

$$\left[ \frac{3}{1+1} \right] = ?$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

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- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\left\lfloor \frac{3}{1+1} \right\rfloor = \left\lfloor \frac{3}{2} \right\rfloor = \left\lfloor \frac{2}{2} + \frac{1}{2} \right\rfloor = \left\lfloor 1 + \frac{1}{2} \right\rfloor = 1$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 1 \\ 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ \textcolor{red}{??} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 1 \\ 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline \end{array}$$

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Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 1 \\ 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \end{array}$$

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- Subtract.
- Repeat.

$$\left[ \frac{2}{1+1} \right] = ?$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\left\lfloor \frac{2}{1+1} \right\rfloor = \left\lfloor \frac{2}{2} \right\rfloor = [1] = 1$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ \textcolor{red}{??} \end{array}$$

- Find shortest dividend start larger than divisor.
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- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

1  
1  
2  
4

$$\begin{array}{r} 99 \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ -11 \\ \hline ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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Divide 99 by 11 with quotient and remainder.

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- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\begin{array}{r} 2 - 1 = 1 \\ 2 - 1 = 1 \end{array}$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ -11 \\ \hline 11 \end{array}$$

- Find shortest dividend start larger than divisor.
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Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \\ 1 \\ 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ -11 \\ \hline 11 \end{array}$$

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- Repeat.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \\ 1 \\ 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ -11 \\ \hline 11 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
    - Equal leading digits;
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \\ 1 \\ 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ -11 \\ \hline 11 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; **larger divisor start has same # of digits**
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 1 \\ 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ -11 \\ \hline 11 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 1 \\ 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ -11 \\ \hline 11 \\ -11 \\ \hline ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
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# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 1 \\ 2 \\ 4 \\ \hline 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ -11 \\ \hline 11 \\ -11 \\ \hline 11 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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- Subtract.
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# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

1  
1  
1  
2  
4

$$\begin{array}{r} & 99 \\ 11 | & \overline{44} \\ & -55 \\ & \overline{22} \\ & -33 \\ & \overline{11} \\ & -22 \\ & \overline{11} \\ & -11 \\ & \overline{11} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
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- Subtract.
- Repeat.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

1  
1  
1  
2  
4

$$\begin{array}{r} 99 \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ -11 \\ \hline 11 \\ -11 \\ \hline ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
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- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

1  
1  
1  
2  
4

$$\begin{array}{r} 99 \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ -11 \\ \hline 11 \\ -11 \\ \hline 0 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit > divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
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- Subtract.
- Repeat.

$$1 - 1 = 0$$

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# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

1  
1  
1  
2  
4

$$\begin{array}{r} 99 \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ -11 \\ \hline 11 \\ -11 \\ \hline 0 \end{array}$$

- Find shortest dividend start larger than divisor.
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# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

1  
1  
1  
2  
4

$$\begin{array}{r} 99 \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ -11 \\ \hline 11 \\ -11 \\ \hline 0 \end{array}$$

- Find shortest dividend start larger than divisor.
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1  
1  
1  
2  
4

$$\begin{array}{r} 99 \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ -11 \\ \hline 11 \\ -11 \\ \hline 0 \end{array}$$

- Find shortest dividend start larger than divisor.
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1  
1  
1  
2  
4

$$\begin{array}{r} & 99 \\ 11 & \overline{)99} \\ & -44 \\ & \overline{55} \\ & -22 \\ & \overline{33} \\ & -11 \\ & \overline{22} \\ & -11 \\ & \overline{11} \\ & -11 \\ & \overline{0} \end{array}$$

- Find shortest dividend start larger than divisor. **If none, we're done.**
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
  - Equal leading digits; larger divisor start has same # of digits  $\Rightarrow$  set quotient digit to 1.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} & \overset{1}{\cancel{}} \\ + & \overset{1}{\cancel{}} \\ & \overset{1}{\cancel{}} \\ & \overset{2}{\cancel{}} \\ & \overset{4}{\cancel{}} \\ \hline 11 & \overline{)99} \\ & -44 \\ \hline & 55 \\ & -22 \\ \hline & 33 \\ & -11 \\ \hline & 22 \\ & -11 \\ \hline & 11 \\ & -11 \\ \hline & 0 \end{array}$$

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- **If more than one quotient row, add them.**

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ & 1 \\ & 2 \\ & 4 \\ \hline & 9 \\ 11 | & 99 \\ - & 44 \\ \hline & 55 \\ - & 22 \\ \hline & 33 \\ - & 11 \\ \hline & 22 \\ - & 11 \\ \hline & 11 \\ - & 11 \\ \hline & 0 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
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- Repeat.
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$$4 + 2 + 1 + 1 + 1 = 9$$

# Example: many quotient rows

Divide 99 by 11 with quotient and remainder.

$$\begin{array}{r} 1 \\ + 1 \\ 1 \\ 2 \\ 4 \\ \hline 9 \\ 11 \overline{)99} \\ -44 \\ \hline 55 \\ -22 \\ \hline 33 \\ -11 \\ \hline 22 \\ -11 \\ \hline 11 \\ -11 \\ \hline 0 \end{array}$$

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- Repeat.
- If more than one quotient row, add them.
- Final answer. **Dividend = 99**. Divisor = 11. Quotient = 9. Remainder = 0.

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- Check.  $99 \stackrel{?}{=} 11 \cdot 9 + 0$

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- Check.  $99 \stackrel{?}{=} 11 \cdot 9 + 0$

# Division algorithm: how many quotient rows?

- When dividing with remainder, we sometimes have to collect multiple quotient rows.
- In the example of dividing 99 by 11, we saw had to collect the quotients 4, 2, 1, 1, 1 from 5 different rows.
- Can it get any worse? No, example shows worst case scenario.

## Lemma

- With 10 digits, there are at most 5 quotient rows during division.*
  - With  $N$  digits, there are at most  $\lfloor \log_2 N \rfloor + 2$  quotient rows during division.*
- 
- We study counting systems that do not use 10 digits later.
  - Reader not familiar with logarithms may ignore the second part.
  - The lemma follows from the fact that each quotient digit that appears higher in the same column is at most half of the one below it, except possibly the highest two digits.

# The Knuth optimization, part 1

- When dividing with remainder, we sometimes have to collect multiple quotient rows.
- How much of a slow down does this cause?
- In the example of dividing 99 by 11, we saw had to collect the quotients 4, 2, 1, 1, 1 from 5 different rows.
- Can it get any worse? No, the example shows the worst case scenario.
- In fact, one can prove the following.

## Lemma

- *Using 10 digits, there are at most 5 quotient rows in the division algorithm.*
- *Using  $N$  digits, there are at most  $\lfloor \log_2 N \rfloor + 2$  quotient rows in the division algorithm.*
- Later on we study computations using different digit systems.

Divide 469 by 51 with quotient and remainder.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} \boxed{ } \\ 51 | \quad 469 \end{array}$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 469 \\ \hline 51 | \end{array}$$

Divide 469 by 51 with quotient and remainder.

- Find shortest dividend start larger than divisor.

$$\begin{array}{r} & \boxed{ } \\ 51 | & 469 \end{array}$$

Divide 469 by 51 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

$$\begin{array}{r} ? \\ \hline 51 \quad | \quad 469 \end{array}$$

Divide 469 by 51 with quotient and remainder.

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits

$$\begin{array}{r} ? \\ 51 \overline{)469} \end{array}$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} ? \\ 51 \overline{)469} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

$$\left[ \frac{46}{5+1} \right]$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 51 \overline{)469} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits **by divisor digit** plus one. Round down if needed.

$$\left\lfloor \frac{46}{5+1} \right\rfloor$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 51 \quad | \quad 469 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit **plus one**. Round down if needed.

$$\left[ \begin{array}{r} 46 \\ 5+1 \end{array} \right]$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} ? \\ 51 \overline{)469} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. **Round down if needed.**

$$\left[ \begin{array}{r} 46 \\ 5+1 \end{array} \right]$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} ? \\ \hline 51 \quad | \quad 469 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

$$\left\lfloor \frac{46}{5+1} \right\rfloor = ?$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 7 \\ 51 | & 469 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

$$\left\lfloor \frac{46}{5+1} \right\rfloor = \left\lfloor \frac{46}{6} \right\rfloor = \left\lfloor \frac{42}{6} + \frac{4}{6} \right\rfloor = \left\lfloor 7 + \frac{4}{6} \right\rfloor = 7$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 7 \\ 51 \overline{)469} \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

Divide 469 by 51 with quotient and remainder.

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- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

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$$7 \cdot 1 = ?$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 7 \\ 51 | & 469 \\ & 7 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

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- Multiply quotient digit by divisor, put result under current dividend.

$$7 \cdot 1 = 7$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 7 \\ ? \sqrt{ } & 469 \\ 51 | & ?7 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

$$7 \cdot 1 = 7$$

$$7 \cdot 5 = ?$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 7 \\ 3 \overline{)5} & \quad | \\ 469 & \\ 57 & \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

$$7 \cdot 1 = 7$$

$$7 \cdot 5 = 35$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 7 \\ 3 \overline{)469} \\ 25 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$7 \cdot 1 = 7$$

$$7 \cdot 5 = 35$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 7 \\ 3 \overline{)469} \\ 357 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

$$7 \cdot 1 = 7$$

$$7 \cdot 5 = 35$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 7 \\ 3 \overline{)469} \\ 51 \\ \hline 357 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

$$7 \cdot 1 = 7$$

$$7 \cdot 5 = 35$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 7 \\ \hline 51 | 469 \\ 357 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
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Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 7 \\ \hline 51 \overline{)469} \\ 357 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 7 \\ 51 \overline{)469} \\ -357 \\ \hline 12 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 7 \\ 51 | & \overline{469} \\ & - 357 \\ & \hline & ? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.

$$9 - 7 = ?$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 7 \\ 51 \overline{)469} \\ -357 \\ \hline 2 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.

$$9 - 7 = 2$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 7 \\ 51 \overline{)469} \\ - \underline{357} \\ \hline 2 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.

$$\begin{aligned} 9 - 7 &= 2 \\ 6 - 5 &= ? \end{aligned}$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 7 \\ 51 \overline{)469} \\ - \underline{357} \\ 12 \end{array}$$

- Find shortest dividend start larger than divisor.
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- Subtract.

$$\begin{array}{r} 9 - 7 = 2 \\ 6 - 5 = 1 \end{array}$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 7 \\ \hline 51 \overline{)469} \\ -357 \\ \hline 12 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.

$$9 - 7 = 2$$

$$6 - 5 = 1$$

$$4 - 3 = ?$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 7 \\ 51 \overline{)469} \\ -357 \\ \hline 112 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.

$$9 - 7 = 2$$

$$6 - 5 = 1$$

$$4 - 3 = 1$$

Divide 469 by 51 with quotient and remainder.

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$$9 - 7 = 2$$

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Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 7 \\ \hline 51 \overline{-} \quad 469 \\ \quad \quad \underline{357} \\ \quad \quad 112 \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\left\lfloor \frac{11}{5+1} \right\rfloor = ?$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 7 \\ 51 | & \overline{469} \\ - & 357 \\ \hline & 112 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\left\lfloor \frac{11}{5+1} \right\rfloor = \left\lfloor \frac{11}{6} \right\rfloor = \left\lfloor \frac{6}{6} + \frac{5}{6} \right\rfloor = \left\lfloor 1 + \frac{5}{6} \right\rfloor = 1$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 7 \\ \hline 51 \overline{)469} \\ -357 \\ \hline 112 \\ ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 7 \\ \hline 51 \overline{)469} \\ -357 \\ \hline 112 \\ -51 \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.

- Subtract.
- Repeat.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 7 \\ \hline 51 \overline{)469} \\ -357 \\ \hline 112 \\ -\underline{51} \\ \hline \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 7 \\ \hline 51 \overline{)469} \\ -357 \\ \hline \color{red}{-1} \\ -51 \\ \hline 61 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\begin{aligned} 2 - 1 &= 1 \\ 1 - 5 &= -4 = -10 + 6 \end{aligned}$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 7 \\ \hline 51 \overline{)469} \\ -357 \\ \hline \underline{-1} \\ -51 \\ \hline 61 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\begin{aligned} 2 - 1 &= 1 \\ 1 - 5 &= -4 = -10 + 6 \end{aligned}$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 7 \\ \hline 51 \overline{)469} \\ -357 \\ \hline 112 \\ -51 \\ \hline 61 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 7 \\ \hline 51 \overline{)469} \\ -357 \\ \hline 112 \\ -51 \\ \hline 61 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 7 \\ \hline 51 \overline{)469} \\ -357 \\ \hline 112 \\ -51 \\ \hline 61 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.

— Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.

- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline 61 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline 61 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - **Current leading digit > divisor leading digit**
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline 61 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  **divide leading digit** by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\left[ \frac{6}{5+1} \right]$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline 61 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit **by divisor digit** plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\left\lfloor \frac{6}{5+1} \right\rfloor$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline 61 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit **plus one**. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\left[ \frac{6}{5+1} \right]$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline 61 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. **Round down if needed.**
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\left[ \begin{array}{r} 6 \\ \hline 5+1 \end{array} \right]$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} ? \\ 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline 61 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\left[ \frac{6}{5+1} \right] = ?$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline 61 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\left\lfloor \frac{6}{5+1} \right\rfloor = \left\lfloor \frac{6}{6} \right\rfloor = \lfloor 1 \rfloor = 1$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline 61 \\ ?? \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- **Multiply quotient digit by divisor, put result under current dividend.**
- Subtract.
- Repeat.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline 61 \\ - 51 \\ \hline \end{array}$$

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- **Multiply quotient digit by divisor, put result under current dividend.**
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Divide 469 by 51 with quotient and remainder.

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Divide 469 by 51 with quotient and remainder.

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- Repeat.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline 61 \\ - 51 \\ \hline 10 \end{array}$$

- Find shortest dividend start larger than divisor.
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  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
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- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\begin{aligned} 1 - 1 &= 0 \\ 6 - 5 &= 1 \end{aligned}$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline 61 \\ - 51 \\ \hline 10 \end{array}$$

- Find shortest dividend start larger than divisor.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

$$\begin{aligned} 1 - 1 &= 0 \\ 6 - 5 &= 1 \end{aligned}$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline 61 \\ - 51 \\ \hline 10 \end{array}$$

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- Repeat.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} 1 \\ 1 \\ 7 \\ \hline 51 \overline{)469} \\ - 357 \\ \hline 112 \\ - 51 \\ \hline 61 \\ - 51 \\ \hline 10 \end{array}$$

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  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 1 \\ & 1 \\ & 7 \\ \hline 51 | & 469 \\ & - 357 \\ \hline & 112 \\ & - 51 \\ \hline & 61 \\ & - 51 \\ \hline & 10 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ & 7 \\ \hline & 469 \\ 51 | & - 357 \\ \hline & 112 \\ - & 51 \\ \hline & 61 \\ - & 51 \\ \hline & 10 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.
- **If more than one quotient row, add them.**

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ & 7 \\ \hline & 9 \\ 51 | & 469 \\ - & 357 \\ \hline & 112 \\ - & 51 \\ \hline & 61 \\ - & 51 \\ \hline & 10 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.
- If more than one quotient row, add them.

$$7 + 1 + 1 = 9$$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ & 7 \\ \hline & 9 \\ 51 | & \overline{469} \\ - & 357 \\ \hline & 112 \\ - & 51 \\ \hline & 61 \\ - & 51 \\ \hline & 10 \end{array}$$

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  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. **Dividend = 469**. Divisor = 51. Quotient = 9. Remainder = 10.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ & 7 \\ \hline & 9 \\ 51 | & 469 \\ - & 357 \\ \hline & 112 \\ - & 51 \\ \hline & 61 \\ - & 51 \\ \hline & 10 \end{array}$$

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- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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- Final answer. Dividend = 469. **Divisor = 51**. Quotient = 9. Remainder = 10.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ & 7 \\ \hline & 9 \\ 51 | & 469 \\ - & 357 \\ \hline & 112 \\ - & 51 \\ \hline & 61 \\ - & 51 \\ \hline & 10 \end{array}$$

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- Final answer. Dividend = 469. Divisor = 51. **Quotient = 9**. Remainder = 10.

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ & 7 \\ \hline & 9 \\ 51 | & 469 \\ - & 357 \\ \hline & 112 \\ - & 51 \\ \hline & 61 \\ - & 51 \\ \hline & 10 \end{array}$$

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- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 469. Divisor = 51. Quotient = 9. **Remainder = 10.**

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ & 7 \\ \hline & 9 \\ 51 | & \overline{469} \\ - & 357 \\ \hline & 112 \\ - & 51 \\ \hline & 61 \\ - & 51 \\ \hline & 10 \end{array}$$

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- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. **Dividend = 469**. Divisor = 51. Quotient = 9. Remainder = 10.
- Check.  $469 \stackrel{?}{=} 51 \cdot 9 + 10$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ & 7 \\ \hline & 9 \\ 51 | & 469 \\ - & 357 \\ \hline & 112 \\ - & 51 \\ \hline & 61 \\ - & 51 \\ \hline & 10 \end{array}$$

- Find shortest dividend start larger than divisor. If none, we're done.
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  - Larger divisor start has more digits  $\Rightarrow$  divide leading two digits by divisor digit plus one. Round down if needed.
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- Check.  $469 \stackrel{?}{=} 51 \cdot 9 + 10$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ & 7 \\ \hline & 9 \\ 51 | & 469 \\ - & 357 \\ \hline & 112 \\ - & 51 \\ \hline & 61 \\ - & 51 \\ \hline & 10 \end{array}$$

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- Final answer. Dividend = 469. Divisor = 51. Quotient = 9. Remainder = 10.
- Check.  $469 \stackrel{?}{=} 51 \cdot 9 + 10$

Divide 469 by 51 with quotient and remainder.

$$\begin{array}{r} & 1 \\ + & 1 \\ & 7 \\ \hline & 9 \\ 51 | & 469 \\ - & 357 \\ \hline & 112 \\ - & 51 \\ \hline & 61 \\ - & 51 \\ \hline & 10 \end{array}$$

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- Next quotient digit: put above last digit of that start. If this leaves gaps: fill with 0.
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  - Current leading digit  $>$  divisor leading digit  $\Rightarrow$  divide leading digit by divisor digit plus one. Round down if needed.
- Multiply quotient digit by divisor, put result under current dividend.
- Subtract.
- Repeat.
- If more than one quotient row, add them.
- Final answer. Dividend = 469. Divisor = 51. Quotient = 9. **Remainder = 10**.
- Check.  $469 \stackrel{?}{=} 51 \cdot 9 + 10$

# The Knuth optimization

- Let the leading digit of the divisor be  $q$ .
- Observation: when the leading divisor digit  $q$  is large, there are fewer quotient rows.
- Observation: if we multiply the divisor and the dividend by a number  $s$ , this doesn't change the quotient and multiplies the remainder by the same factor  $s$ .
- Donald Knuth suggests the following long division optimization.
- Before division, multiply dividend & divisor by one-digit  $q$ .
- Choose  $q$  to make the divisor leading digit as large as possible.
- More precisely, for divisor leading digit  $d$ , choose  $q$  to be

$$q = \left\lfloor \frac{10}{d+1} \right\rfloor$$

- Divide integers re-scaled by  $q$  in the usual way.
- The new quotient coincides with that of the original problem; the original remainder is obtained by dividing the new one by  $q$ .

# The Knuth optimization in other bases

- The Knuth optimization is intended for large examples and computations by computer.
- Thus, the Knuth optimization is not very beneficial when computing by hand.
- When using non-decimal counting systems with more than 10 digits on a computer, the Knuth optimization yields significant benefits.
- Important cases for the Knuth optimization would be the use of  $2^8 = 256$ ,  $2^{16} = 65536$ ,  $2^{32} = 4294967296$  digits, as they are easily available on most modern computers.