

Calculus I

Homework

Logarithmic derivatives

1. Compute the derivative.

(a) $\ln(4x)$

(j) $\ln(-6x + 2)$

(b) $\ln(-13x)$

(k) $\ln\left(\frac{3x-2}{-2x+3}\right)$

(c) $\log_2(5x)$

(l) $\ln\left(\frac{5x-4}{-x-5}\right)$

(d) $\log_{10}(-3x)$

(m) $\ln\left(\frac{3x+1}{4x-5}\right)$

(e) $x^6 \ln(2x)$

(f) $x^4 \ln(2x)$

(g) $\ln(x^4)$

(n) $\ln(\cot x)$

(h) $(\ln(x))^4$

(o) $\ln(\sec(2x))$

(i) $\ln(7x+1)$

(p) $f(x) = \ln(\sec x) + \ln(\cot x)$.

Solution. 1.k

$$\begin{aligned}
 \frac{d}{dx} \left(\ln \left(\frac{3x-2}{-2x+3} \right) \right) &= \frac{d}{dx} (\ln(3x-2) - \ln(-2x+3)) && \left| \begin{array}{l} \text{logarithm properties} \\ \text{chain rule} \end{array} \right. \\
 &= \frac{(3x-2)'}{(3x-2)} - \frac{(-2x+3)'}{-2x+3} \\
 &= \frac{3x-2}{3} - \frac{-2x+3}{-2} \\
 &= \frac{3x-2}{3} - \frac{2x-3}{2} \\
 &= \frac{3x-2}{-5} - \frac{2x-3}{2} \\
 &= \frac{6x^2-13x+6}{-10} && \left| \begin{array}{l} \text{combine fractions (optional).} \end{array} \right.
 \end{aligned}$$

Solution. 1.m

$$\begin{aligned}
 \frac{d}{dx} \left(\ln \left(\frac{3x+1}{4x-5} \right) \right) &= \frac{d}{dx} (\ln(3x+1) - \ln(4x-5)) \\
 &= \frac{(3x+1)'}{3x+1} - \frac{(4x-5)'}{4x-5} \\
 &= \frac{3x+1}{-19} - \frac{4x-5}{4x-5} \\
 &= \frac{3x+1}{12x^2-11x-5} && \left| \begin{array}{l} \text{step optional.} \end{array} \right.
 \end{aligned}$$

Solution. 1.p

$$\begin{aligned}
 \frac{d}{dx}(\ln(\sec x) + \ln(\cot x)) &= \frac{d}{dx} \left(\ln \left(\frac{1}{\cos x} \right) + \ln \left(\frac{\cos x}{\sin x} \right) \right) \\
 &= \frac{d}{dx} (-\ln(\cos x) + \ln(\cos x) - \ln(\sin x)) \\
 &= -\frac{d}{dx}(\ln(\sin x)) \quad \left| \text{chain rule} \right. \\
 &= -\frac{1}{\sin x} \frac{d}{dx}(\sin x) \\
 &= -\frac{\cos x}{\sin x} \\
 &= -\cot x
 \end{aligned}$$

2. Differentiate.

(a) 10^{x^3} .

(b) $2^{\tan x}$.

(c) x^x .

(d) x^{x^x} .

(e) $(\sin x)^{\cos x}$.

(f) $(\ln x)^{\ln x}$.

3. Find the limit.

(a) $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x} \right)^x$.

(b) $\lim_{x \rightarrow 0} (1 - x)^{\frac{1}{x}}$.

(c) $\lim_{x \rightarrow \infty} \left(\frac{x}{x-5} \right)^x$.

(d) $\lim_{x \rightarrow \infty} \left(\frac{x}{x-2} \right)^{3x+2}$.