Precalculus
Equations formed by setting trigonometric sum equal to 0

Todor Milev

2019

Proposition (Product to sum formulas)

$$
\begin{aligned}
\sin\alpha\sin\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)\\
\cos\alpha\cos\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)+\cos(\alpha+\beta)\right)\\
\sin\alpha\cos\beta &= \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)
\end{aligned}
$$

Proof.

$\square$

Proposition (Product to sum formulas)

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$$

Proof.

? $\qquad = \cos(\alpha+\beta)$

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Proposition (Product to sum formulas)

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Proof.

$$\cos\alpha\cos\beta-\sin\alpha\sin\beta = \cos(\alpha+\beta)$$

$\square$

Proposition (Product to sum formulas)

$$\begin{aligned}
\sin\alpha\sin\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)\\
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\end{aligned}$$

Proof.

$$\begin{aligned}
\textbf{?} &= \cos(\alpha-\beta)\\
\cos\alpha\cos\beta-\sin\alpha\sin\beta &= \cos(\alpha+\beta)
\end{aligned}$$

□

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Proof.

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\begin{aligned}
\cos\alpha\cos\beta+\sin\alpha\sin\beta &= \cos(\alpha-\beta)\\
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\cos\alpha\cos\beta+\cancel{\sin\alpha\sin\beta} &= \cos(\alpha-\beta)\\
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\end{aligned}$$

Proof.

$$\begin{aligned}
+\quad \cos\alpha\cos\beta + \cancel{\sin\alpha\sin\beta} &= \cos(\alpha-\beta)\\
\cos\alpha\cos\beta - \cancel{\sin\alpha\sin\beta} &= \cos(\alpha+\beta)\\
2\cos\alpha\cos\beta &= \cos(\alpha-\beta)+\cos(\alpha+\beta)
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Proof.

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\end{aligned}$$

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Proof.

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\end{aligned}
$$

Proof.

$$
\begin{array}{crcl}
+ & \cos\alpha\cos\beta + \cancel{\sin\alpha\sin\beta} &=& \cos(\alpha-\beta)\\
 & \cos\alpha\cos\beta - \cancel{\sin\alpha\sin\beta} &=& \cos(\alpha+\beta)\\
 & 2\cos\alpha\cos\beta &=& \cos(\alpha-\beta)+\cos(\alpha+\beta)\\
\hline
- & \cos\alpha\cos\beta + \sin\alpha\sin\beta &=& \cos(\alpha-\beta)\\
 & \cos\alpha\cos\beta - \sin\alpha\sin\beta &=& \cos(\alpha+\beta)\\
 & 2\sin\alpha\sin\beta &=& \cos(\alpha-\beta)-\cos(\alpha+\beta)
\end{array}
$$

$\square$

Proposition (Product to sum formulas)

$$\begin{aligned}
\sin\alpha\sin\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)\\
\cos\alpha\cos\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)+\cos(\alpha+\beta)\right)\\
\color{red}{\sin\alpha\cos\beta} &\color{red}{=} \color{red}{\frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)}
\end{aligned}$$

Proof.

$$\begin{array}{crcl}
+ & \cos\alpha\cos\beta+\cancel{\sin\alpha\sin\beta} &=& \cos(\alpha-\beta)\\
 & \cos\alpha\cos\beta-\cancel{\sin\alpha\sin\beta} &=& \cos(\alpha+\beta)\\
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\hline
\color{red}{+} & \sin\alpha\cos\beta+\cos\alpha\sin\beta &=& \color{red}{\sin(\alpha+\beta)}\\
 & \sin\alpha\cos\beta-\cos\alpha\sin\beta &=& \color{red}{\sin(\alpha-\beta)}\\
 & 2\sin\alpha\cos\beta &=& \sin(\alpha+\beta)+\sin(\alpha+\beta)
\end{array}$$

$\square$

Proposition (Product to sum formulas)

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\sin\alpha\sin\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)\\
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\sin\alpha\cos\beta &= \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)
\end{aligned}$$

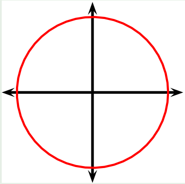
- Product to sum formulas are used when integrating (a topic to be studied later/in another course).

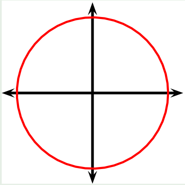
Proposition (Sum to product formulas)

$$
\begin{aligned}
\sin\alpha + \sin\beta &= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\\
\sin\alpha - \sin\beta &= 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)\\
\cos\alpha + \cos\beta &= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\\
\cos\alpha - \cos\beta &= 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)
\end{aligned}
$$

Example

Find all solutions in the interval $[0, 2\pi)$.



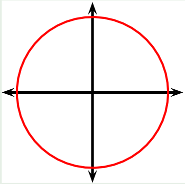


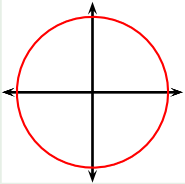
$$\sin(2x) + \sin(5x) = 0$$

Recall the formula $\sin\alpha + \sin\beta = \textbf{?}$

Example

Find all solutions in the interval $[0, 2\pi)$.



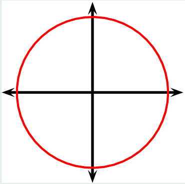


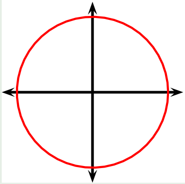
$\sin(2x) + \sin(5x) = 0$ | use f-la

Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

Example

Find all solutions in the interval $[0, 2\pi)$.





$$\sin(2x) + \sin(5x) = 0 \;\Big|\; \text{use f-la}$$

Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

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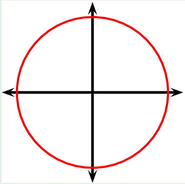

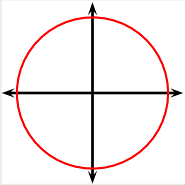

$$\sin(2x) + \sin(5x) = 0 \quad \Big|\ \text{use f-la}$$

$$2\sin\left(\frac{2x+5x}{2}\right)\cos\left(\frac{2x-5x}{2}\right) = 0$$

Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

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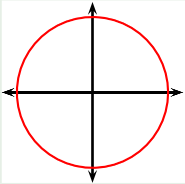

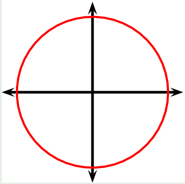

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$$2\sin\left(\frac{2x+5x}{2}\right)\cos\left(\frac{2x-5x}{2}\right) = 0$$

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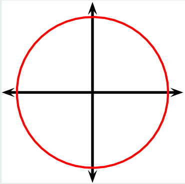
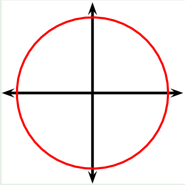




$$\sin(2x) + \sin(5x) = 0 \quad \Big| \text{ use f-la}$$

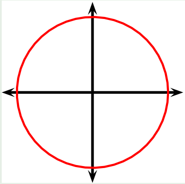
$$2\sin\left(\frac{2x+5x}{2}\right)\cos\left(\frac{2x-5x}{2}\right) = 0$$

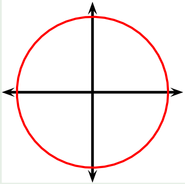
$$2\sin\left(\frac{7}{2}x\right)\cos\left(-\frac{3}{2}x\right) = 0$$

Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

Example

Find all solutions in the interval $[0, 2\pi)$.



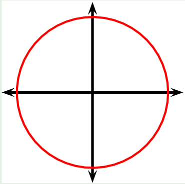


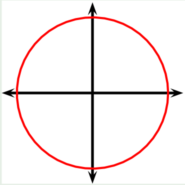
$$
\begin{aligned}
\sin(2x) + \sin(5x) &= 0 && \text{use f-la}\\
2\sin\left(\frac{2x+5x}{2}\right)\cos\left(\frac{2x-5x}{2}\right) &= 0\\
2\sin\left(\frac{7}{2}x\right)\cos\left(-\frac{3}{2}x\right) &= 0 && \text{cos is even}\\
2\sin\left(\frac{7}{2}x\right)\cos\left(\frac{3}{2}x\right) &= 0
\end{aligned}
$$

Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

Example

Find all solutions in the interval $[0, 2\pi)$.



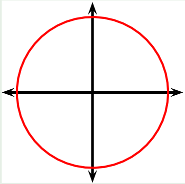


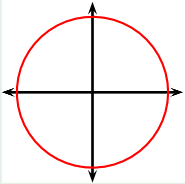
$$\begin{aligned} \sin(2x)+\sin(5x) &= 0 && \Big|\ \text{use f-la}\\ 2\sin\left(\frac{2x+5x}{2}\right)\cos\left(\frac{2x-5x}{2}\right) &= 0 \\ 2\sin\left(\frac{7}{2}x\right)\cos\left(-\frac{3}{2}x\right) &= 0 && \Big|\ \begin{array}{l}\cos\\ \text{is even}\end{array}\\ {\color{red}2\sin\left(\frac{7}{2}x\right)\cos\left(\frac{3}{2}x\right)} &{\color{red}= 0} \end{aligned}$$

Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

Example

Find all solutions in the interval $[0, 2\pi)$.





$$\sin(2x) + \sin(5x) = 0 \quad \Big| \text{ use f-la}$$

$$2\sin\left(\frac{7}{2}x\right)\cos\left(\frac{3}{2}x\right) = 0$$

Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

Example

Find all solutions in the interval $[0, 2\pi)$.

$$\sin(2x) + \sin(5x) = 0 \quad \Big| \text{ use f-la}$$

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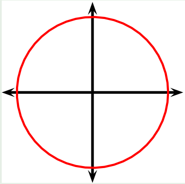
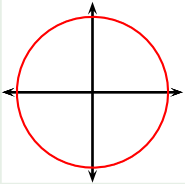


$\sin\left(\frac{7}{2}x\right) = 0$

or



$\cos\left(\frac{3}{2}x\right) = 0$

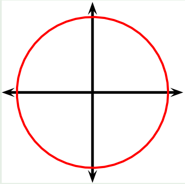
Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

Example

Find all solutions in the interval $[0, 2\pi)$.

$$\sin(2x) + \sin(5x) = 0 \quad \Big| \text{ use f-la}$$

$$2\sin\left(\frac{7}{2}x\right)\cos\left(\frac{3}{2}x\right) = 0$$



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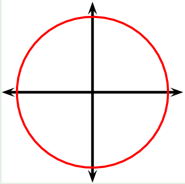


$\cos\left(\frac{3}{2}x\right) = 0$

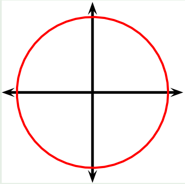
Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

Example

Find all solutions in the interval $[0, 2\pi)$.

$$\sin(2x) + \sin(5x) = 0 \quad \Big| \text{ use f-la}$$

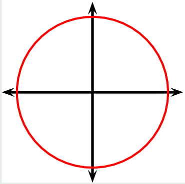
$$2\sin\left(\frac{7}{2}x\right)\cos\left(\frac{3}{2}x\right) = 0$$



$\sin\left(\frac{7}{2}x\right) = 0$

$\frac{7}{2}x = \textbf{?}$

or



$\cos\left(\frac{3}{2}x\right) = 0$

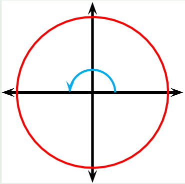
Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

Example

Find all solutions in the interval $[0, 2\pi)$.

$$\sin(2x) + \sin(5x) = 0 \quad \Big| \text{ use f-la}$$

$$2\sin\left(\frac{7}{2}x\right)\cos\left(\frac{3}{2}x\right) = 0$$



$\sin\left(\frac{7}{2}x\right) = 0$

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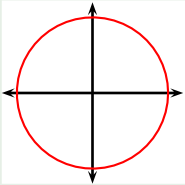

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Find all solutions in the interval $[0, 2\pi)$.

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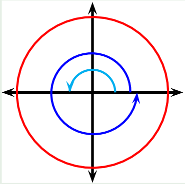

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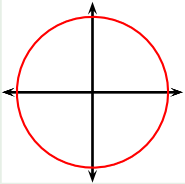

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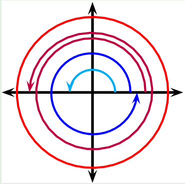

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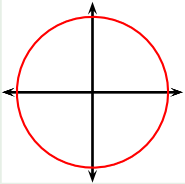

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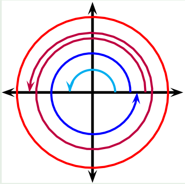

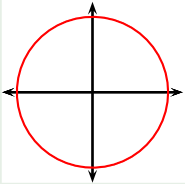

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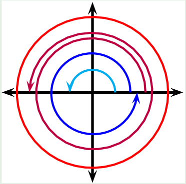

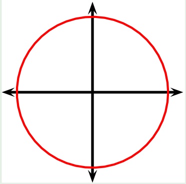

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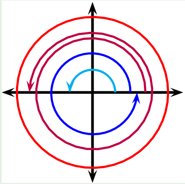

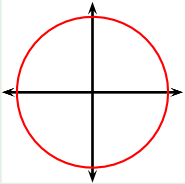

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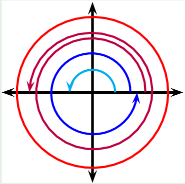

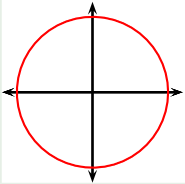

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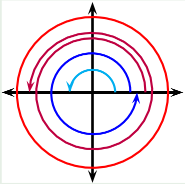

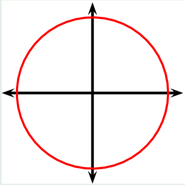

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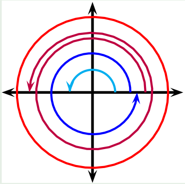

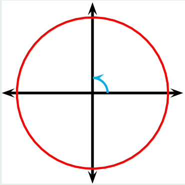

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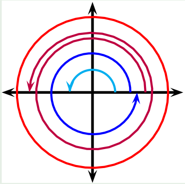

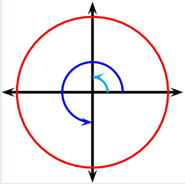

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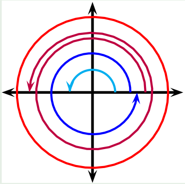

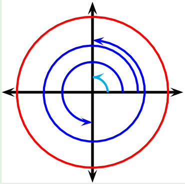

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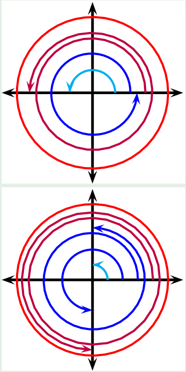

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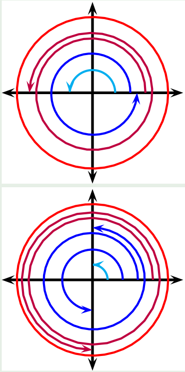

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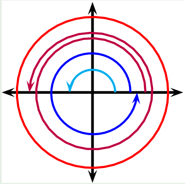

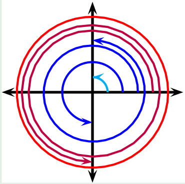

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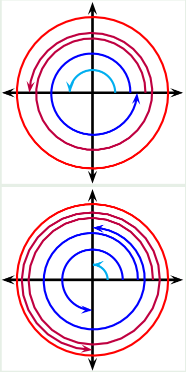

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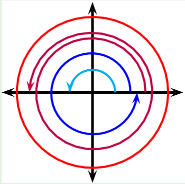

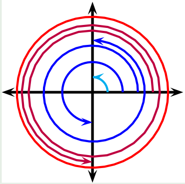

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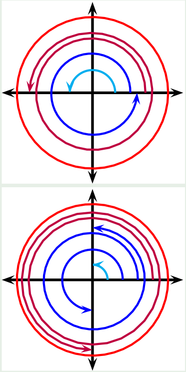

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