Precalculus
Quadratic trigonometric inequalities

Todor Milev

2019

Example

- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2\cos^2\theta + 2\cos\theta + 1 \leq \cos\theta + 2$ lying in $[-360^\circ, 360^\circ]$.

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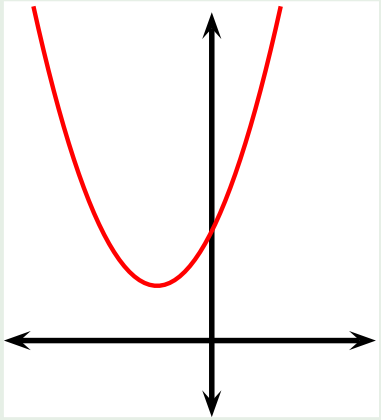

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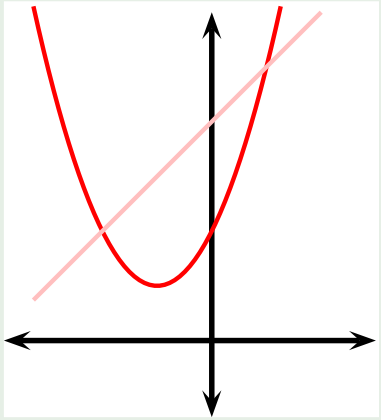

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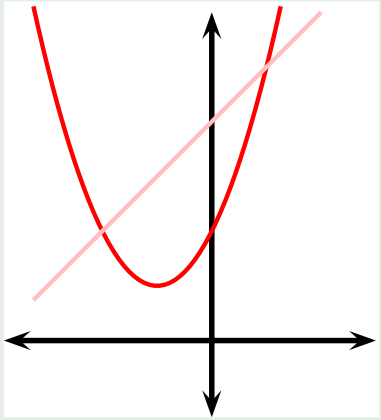

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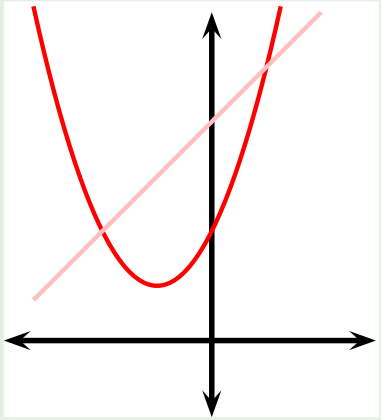

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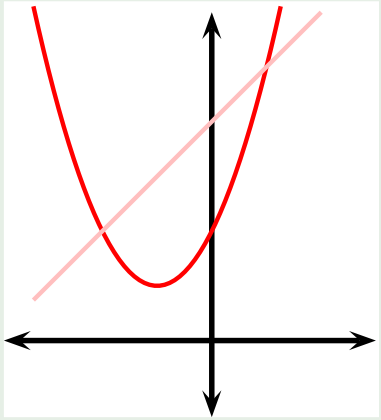

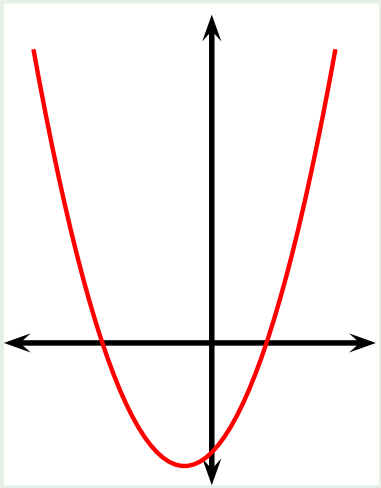

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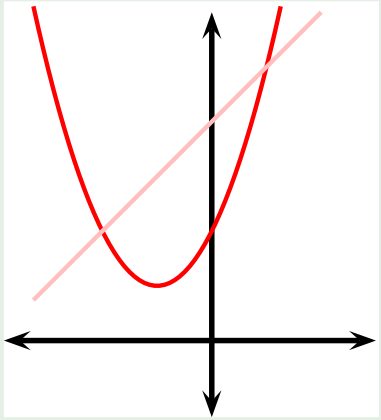

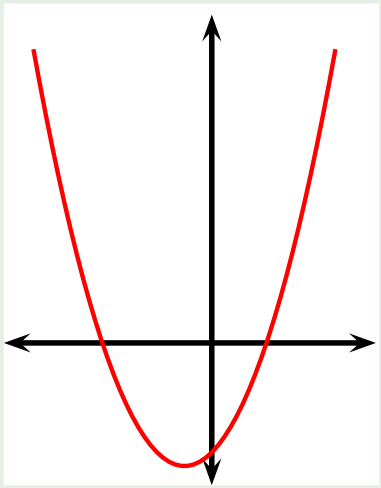

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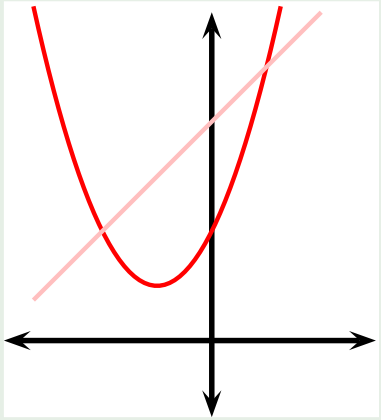

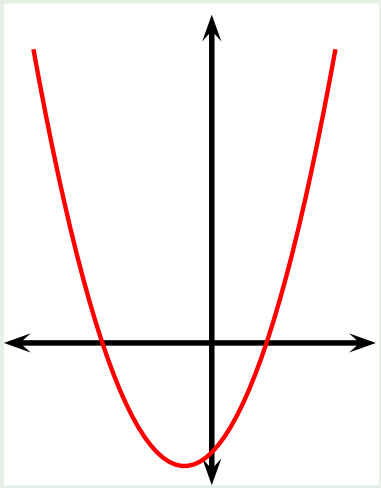

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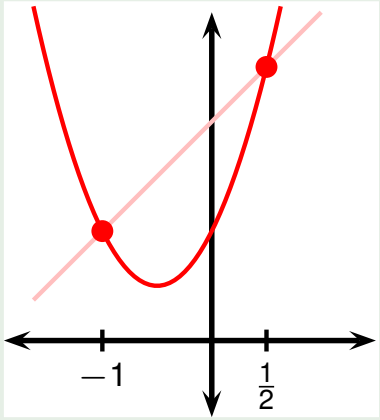

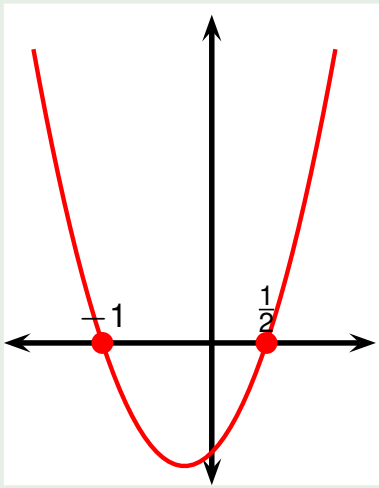

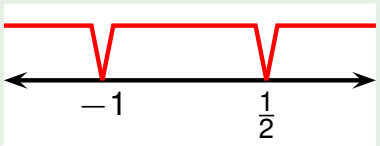

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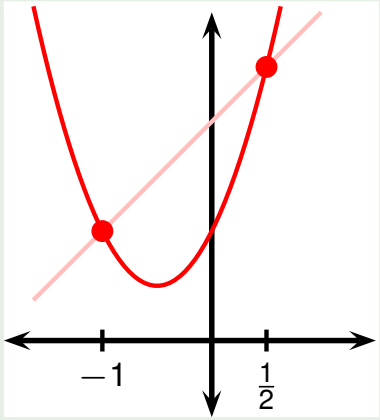

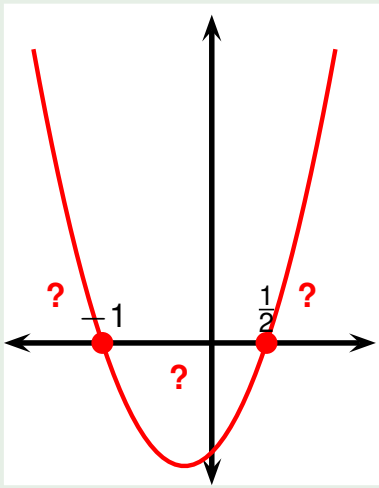

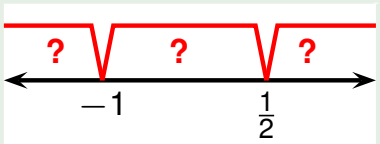

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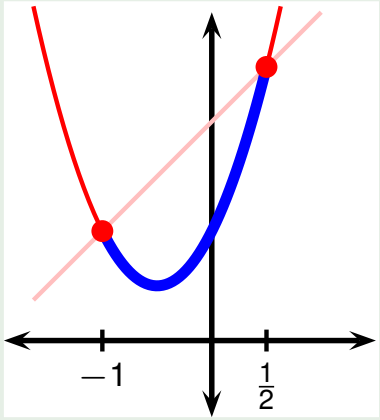

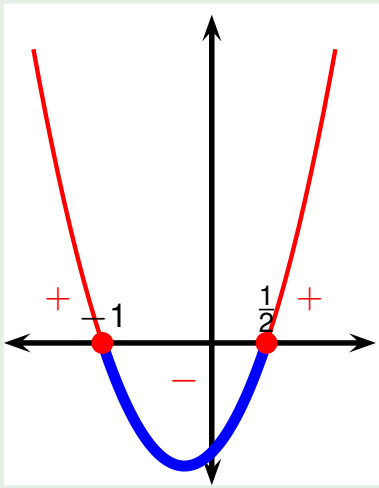

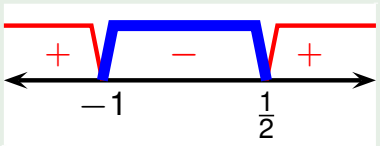

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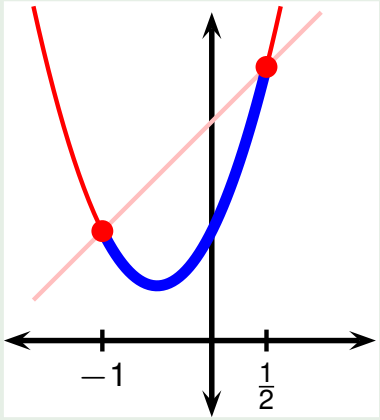

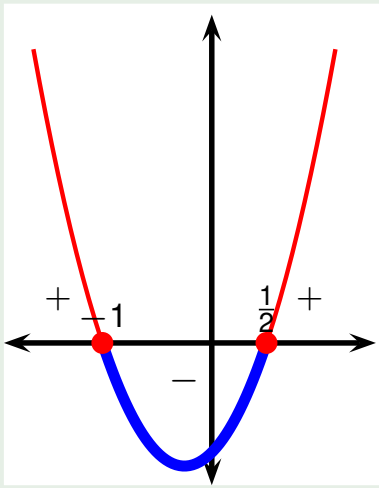

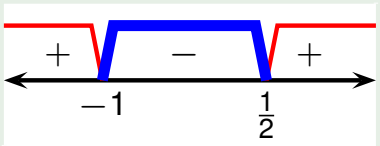

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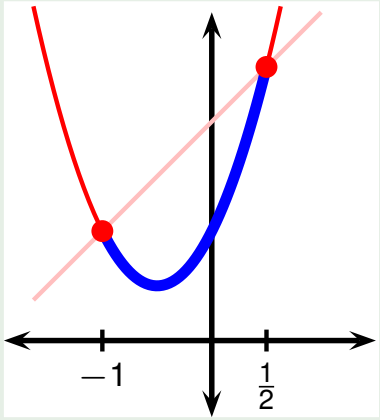

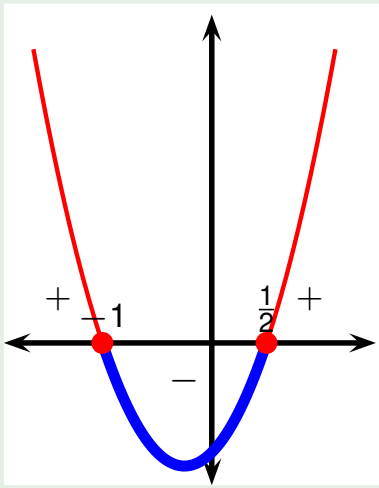

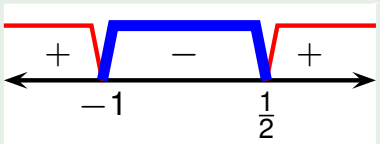

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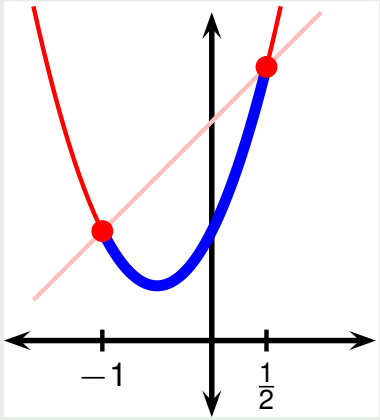

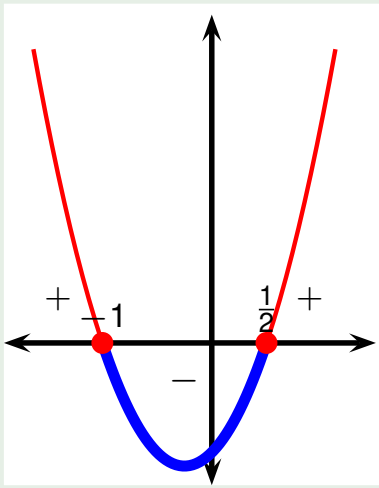

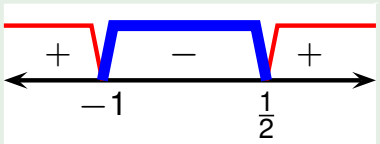

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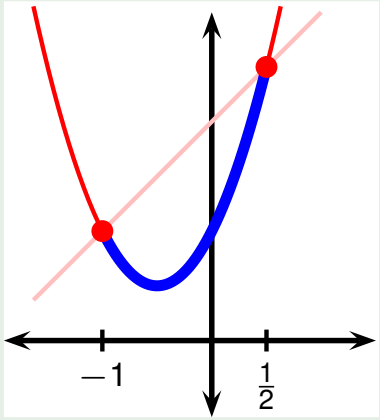

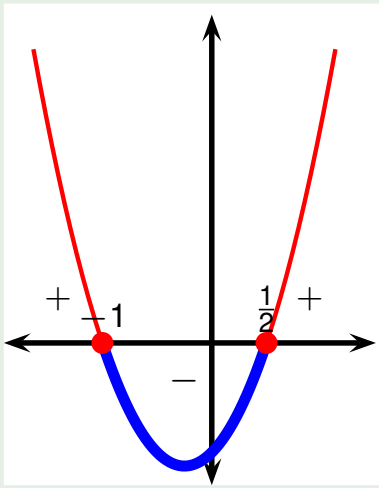

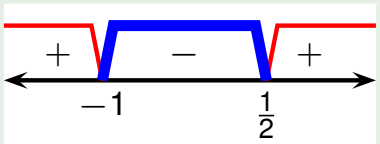

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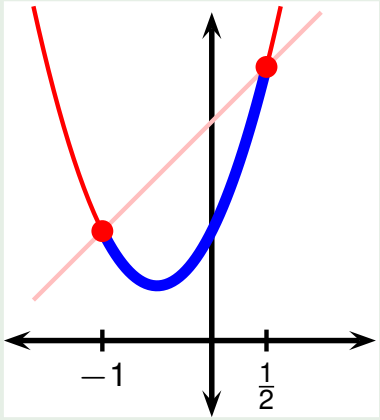

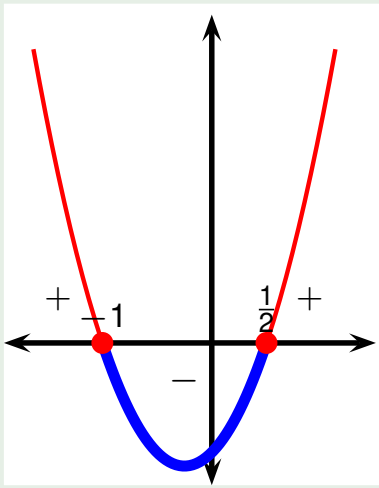

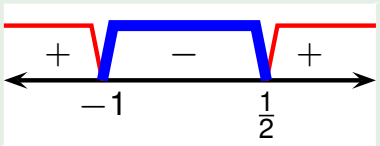

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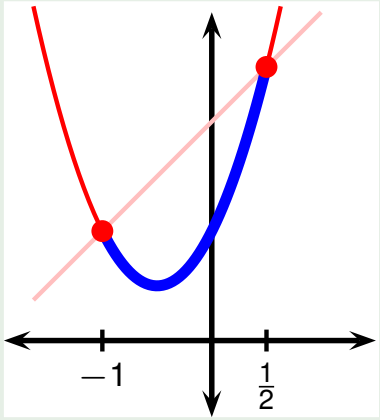

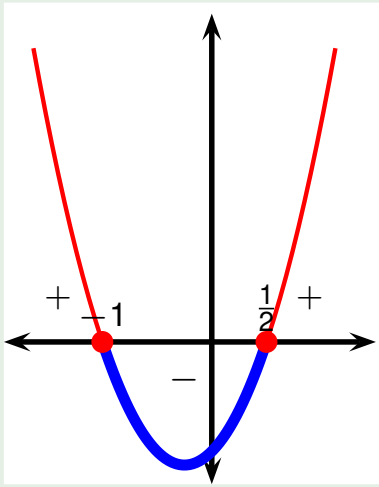

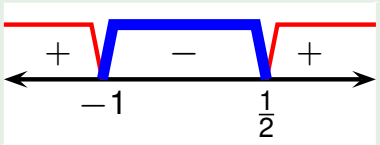

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2u^2 + 2u + 1 &\leq u + 2 \\
2u^2 + u - 1 &\leq 0 \\
2\left(u - \tfrac{1}{2}\right)(u + 1) &\leq 0 \\
u &\in \left[-1, \tfrac{1}{2}\right]
\end{aligned}
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$$
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2\cos^2\theta + 2\cos\theta + 1 &\leq \cos\theta + 2 \quad \text{Set } \cos\theta = u \\
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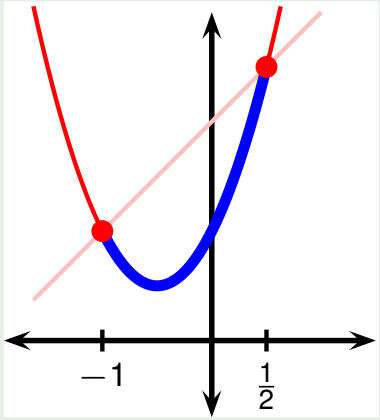
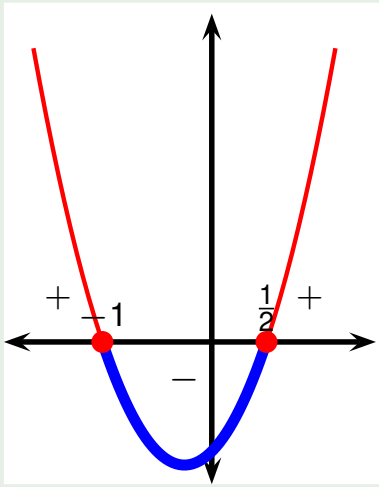
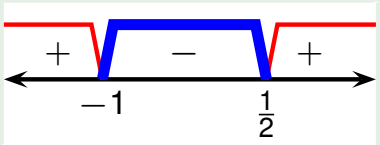
Example







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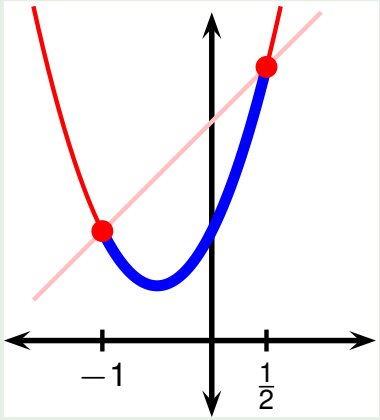

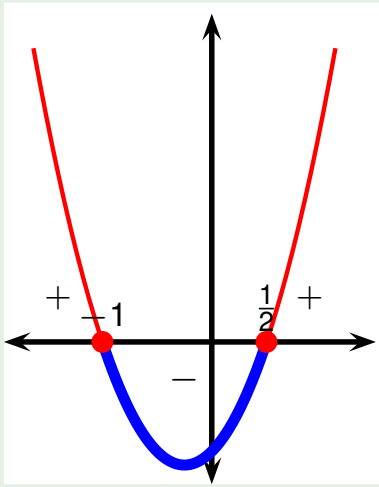

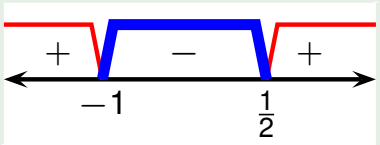

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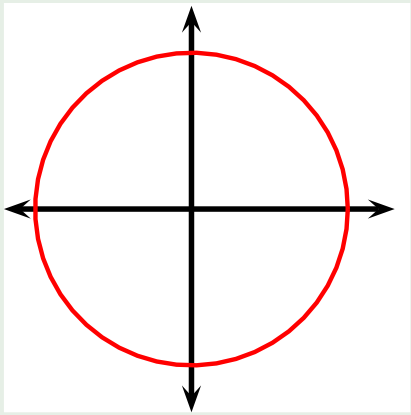

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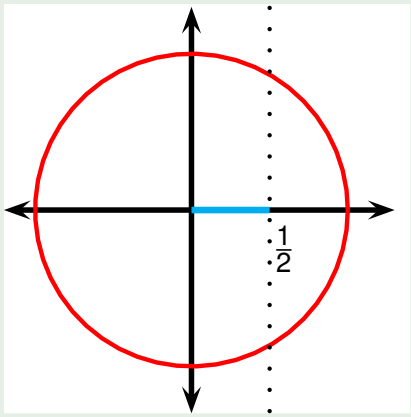
Example



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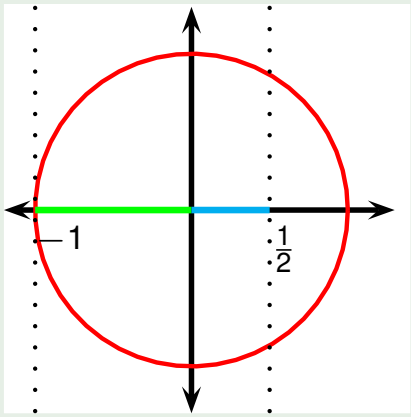

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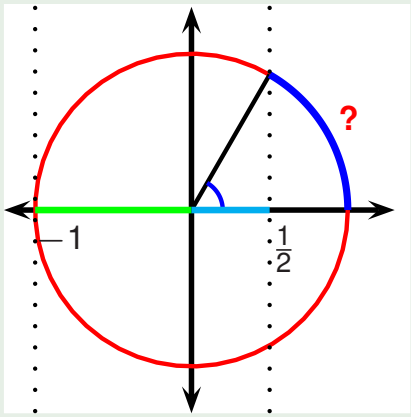

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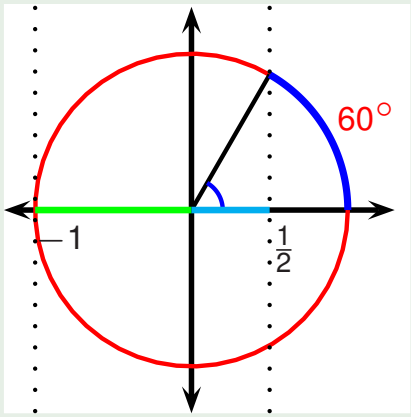
Example



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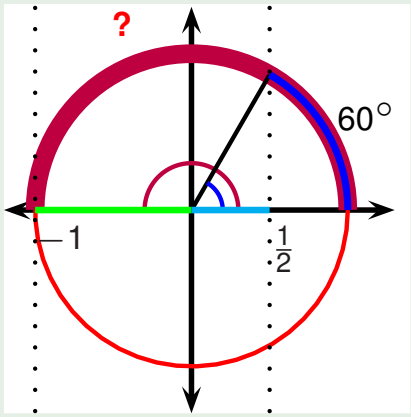
$\theta \in$ $[60^\circ$, **?** $]$

Example



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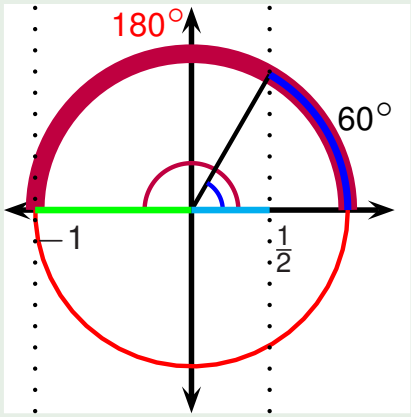
$\theta \in$ [60° , **?**]

Example



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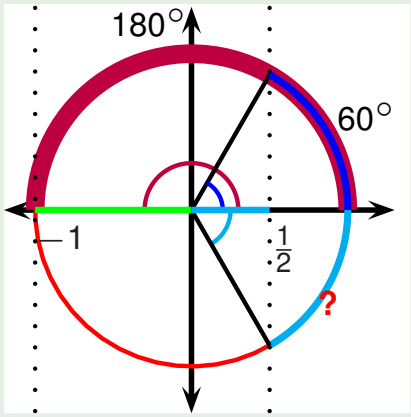
$\theta \in$ [60° , 180°]

Example



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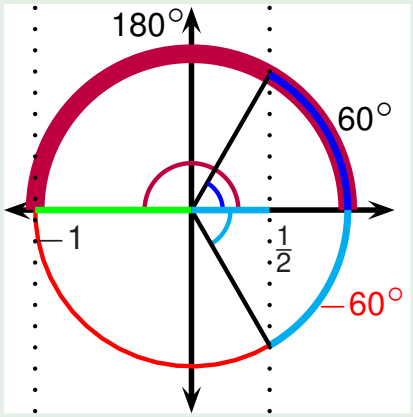
Example



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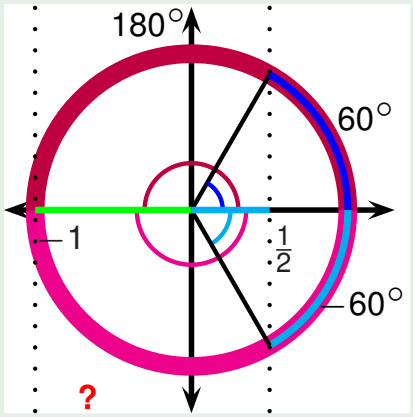
Example



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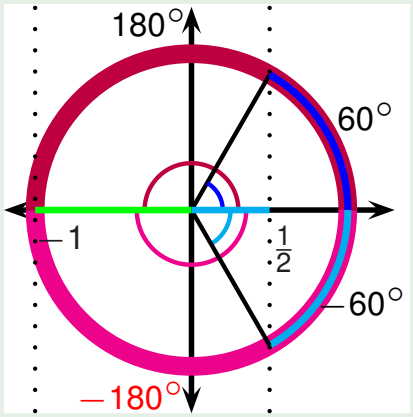
Example



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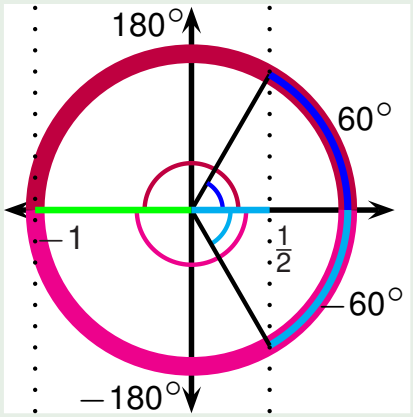
Example



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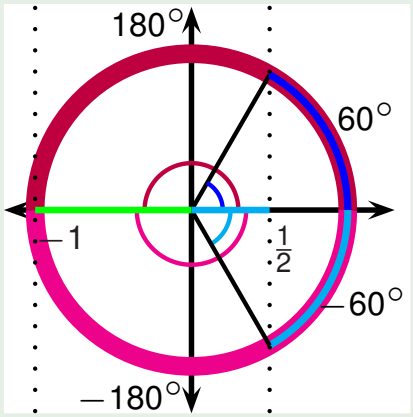

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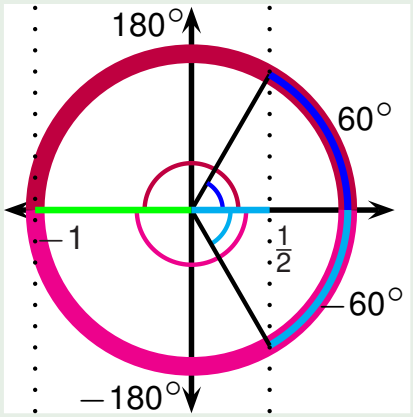

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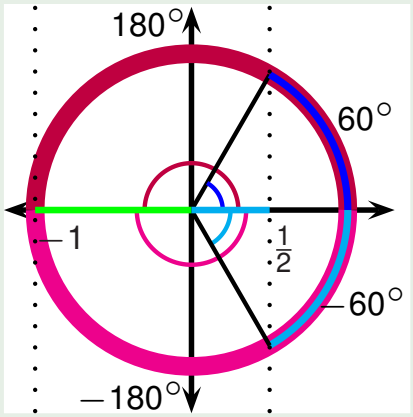
Example



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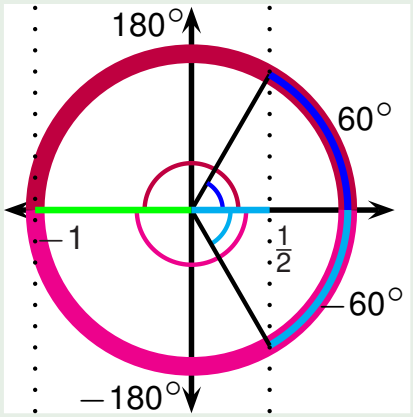

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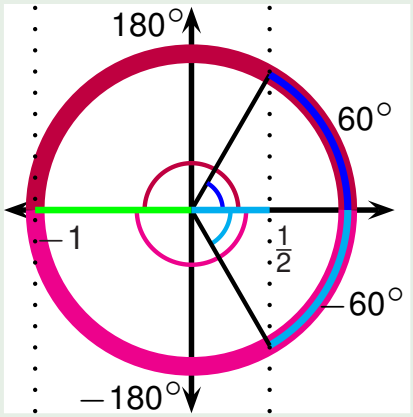

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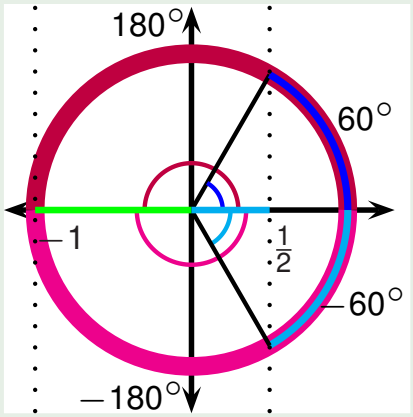
Example



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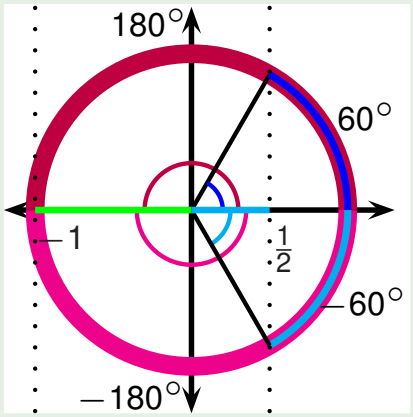
Example



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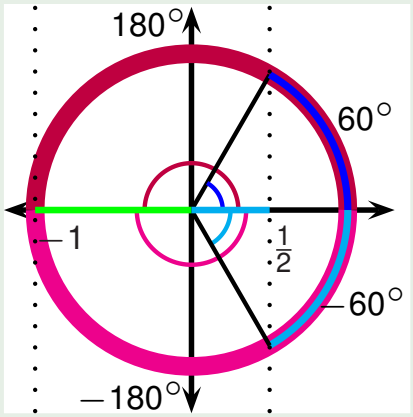
Example



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$$\begin{aligned} \theta \in \; & \cancel{\cup\, [-540°, -420°]} \cup [-300°, -180°] && k = -1 \\ & \cup [-180°, -60°] \cup [60°, 180°] && k = 0 \\ & \cup [180°, 300°] \cup \cancel{[420°, 540°]} && k = 1 \end{aligned}$$

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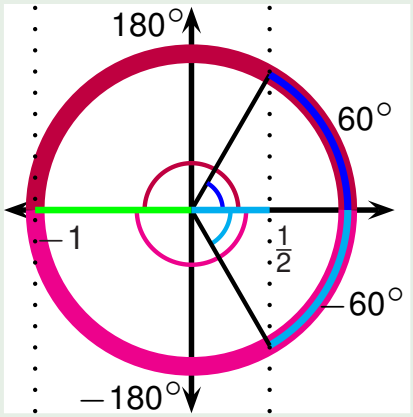
Example



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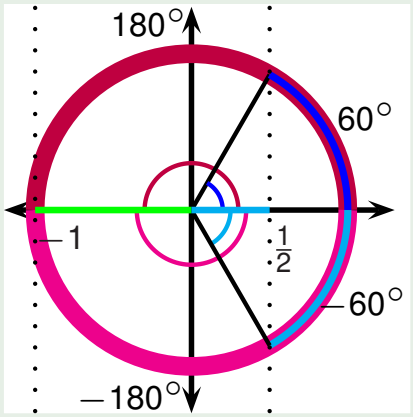

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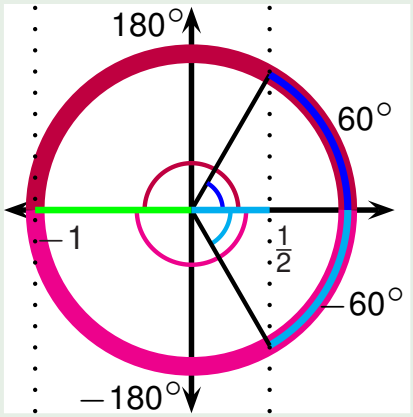
Example



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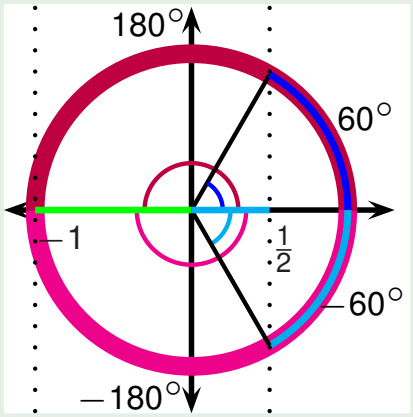

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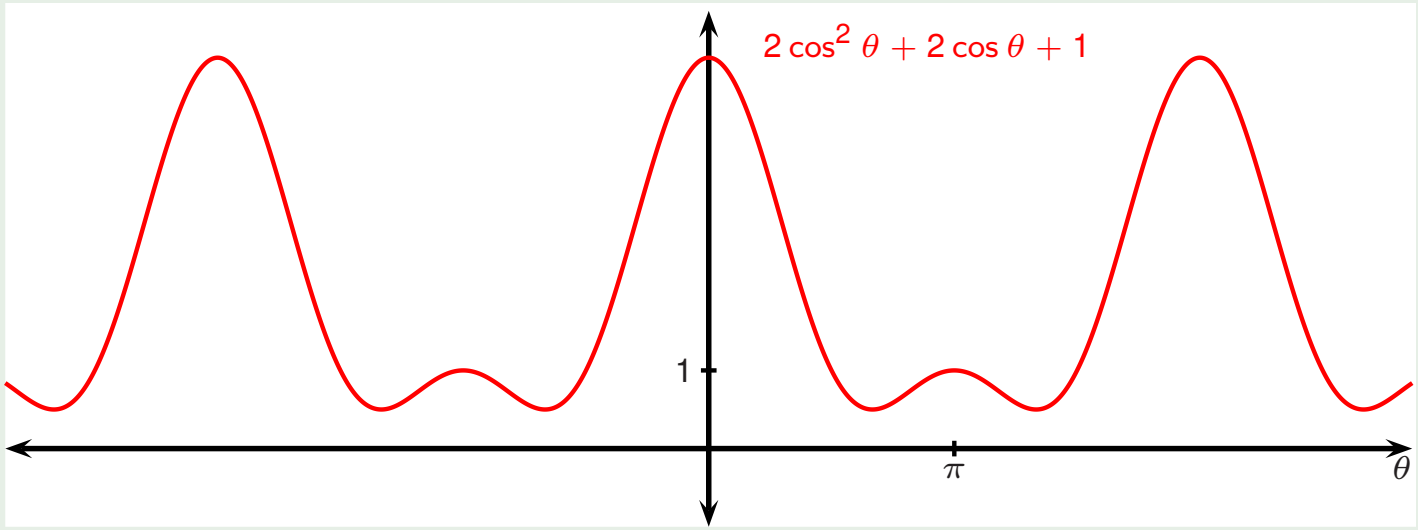

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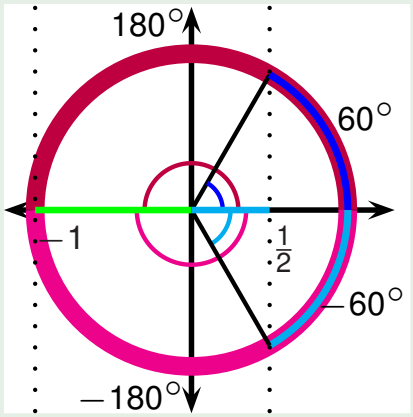

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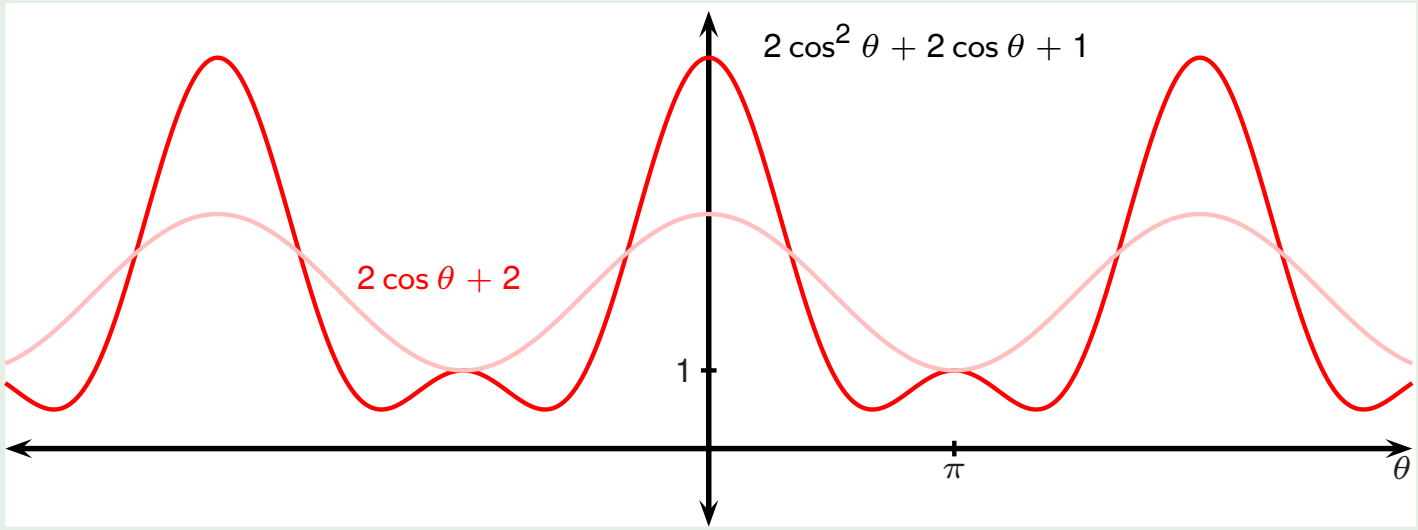

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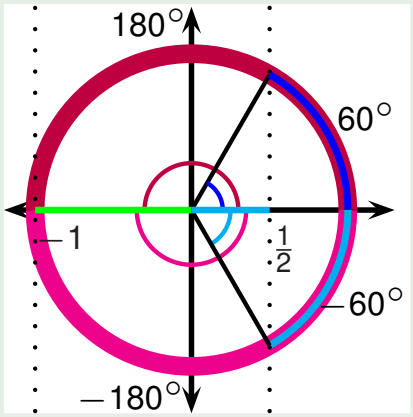

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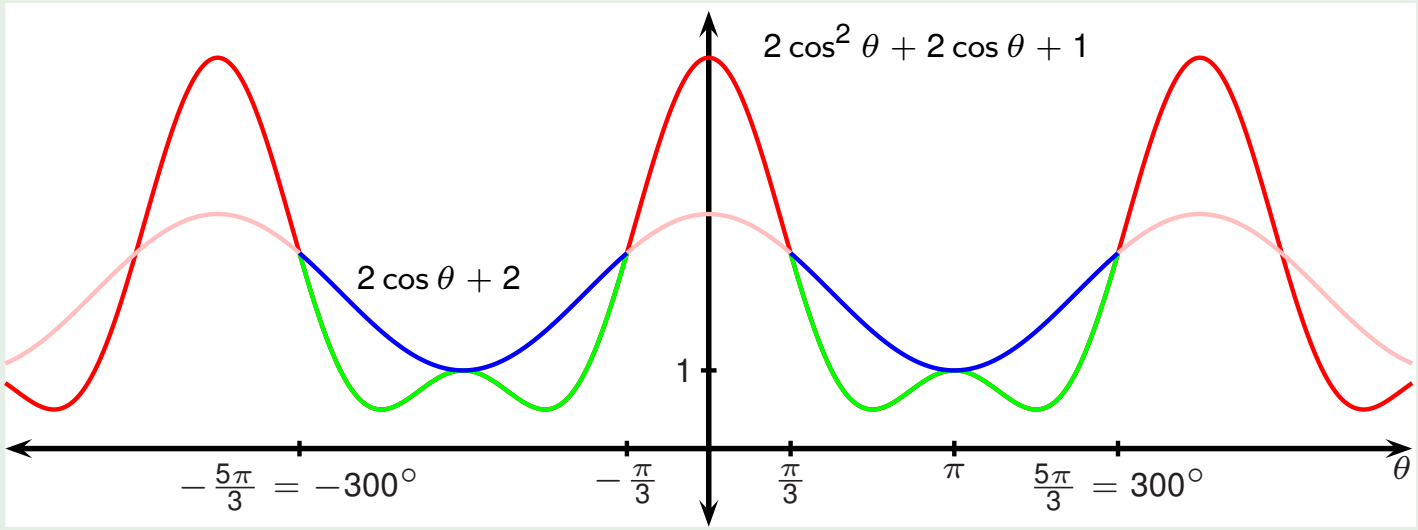

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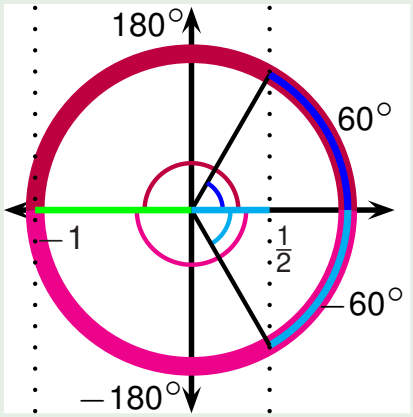

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