

## Precalculus

# Definition of the trigonometric functions and basic computations

Todor Milev

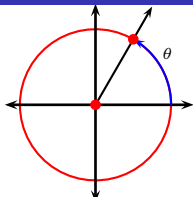
2019

# Outline

## 1 Trigonometry

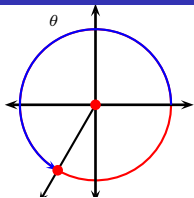
- Definition of the Trigonometric Functions
- Basic Computations with Trigonometric Functions
- Reference Angles
- Geometric Interpretation of the Trigonometric Functions
- Periodicity and Symmetries of the Trig Functions

# Definition of the trigonometric functions



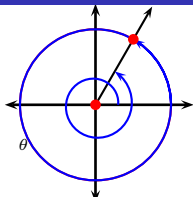
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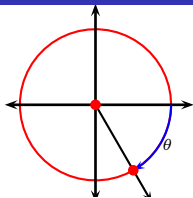
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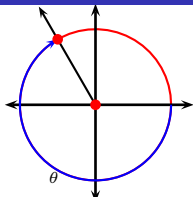
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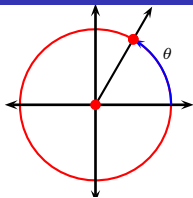
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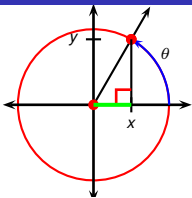
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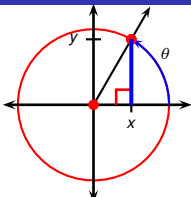
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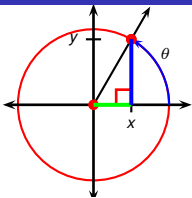
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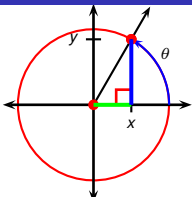
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## Definition (additional trigonometric functions)

The functions **tangent**, cotangent, secant and cosecant of the angle  $\theta$ , denoted by  $\tan \theta$ ,  $\cot \theta$ ,  $\sec \theta$ ,  $\csc \theta$ , are defined by

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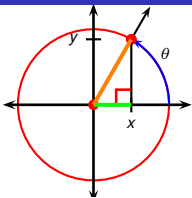
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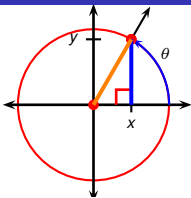
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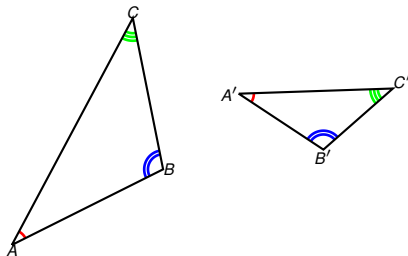
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## Definition (Similar triangles)

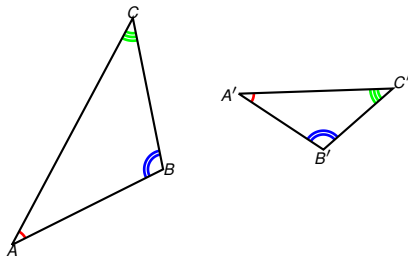
We say that a triangle  $\triangle ABC$  is similar to a triangle  $\triangle A'B'C'$  if the two triangles have equal angles.



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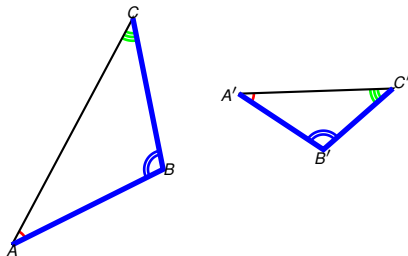




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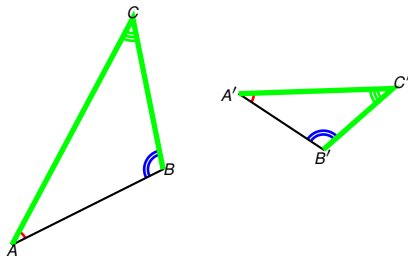
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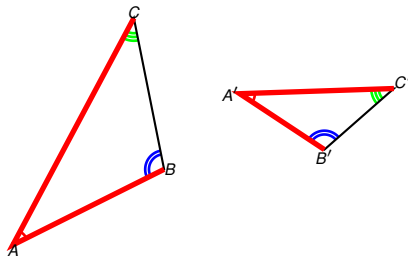
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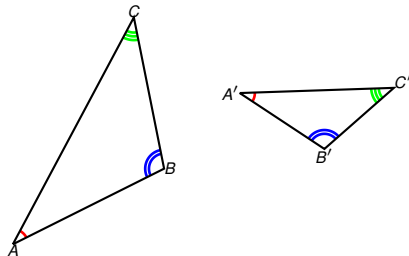


The following statement is proved in the subject of Euclidean (planar) geometry.

### Theorem (Similar triangles have equal side ratios)

*Let  $\triangle ABC$  and  $\triangle A'B'C'$  be two similar triangles. Then the ratios of the lengths of the sides of the two triangles are equal, that is*

$$\frac{|AB|}{|BC|} = \frac{|A'B'|}{|B'C'|} \quad \frac{|BC|}{|CA|} = \frac{|B'C'|}{|C'A'|} \quad \frac{|CA|}{|AB|} = \frac{|C'A'|}{|A'B'|}$$

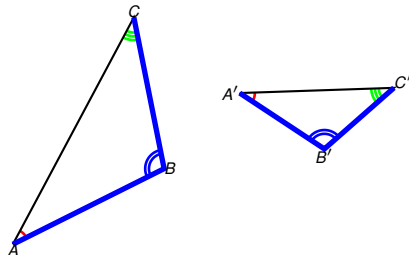


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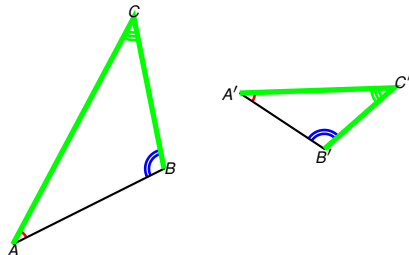


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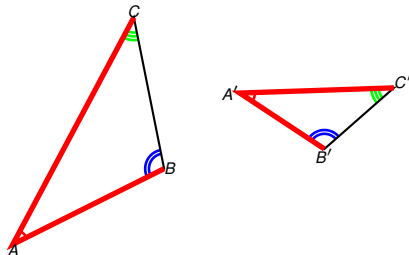


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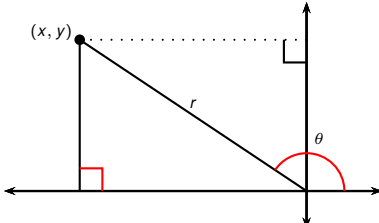
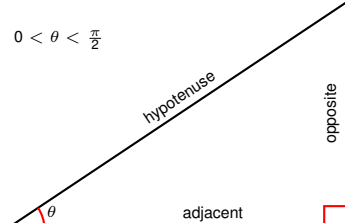
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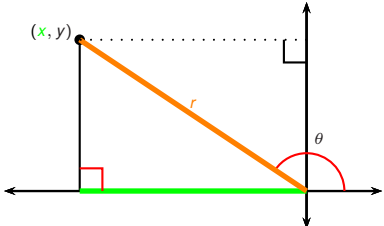
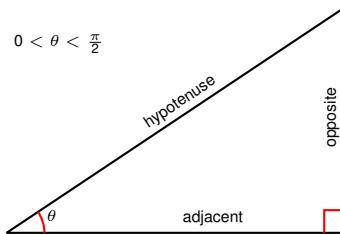
# Trigonometric Functions and Right Angle Triangles

	
$\cos \theta$ $\sin \theta$ $\tan \theta$	$\sec \theta$ $\csc \theta$ $\cot \theta$
All angles	Acute angles

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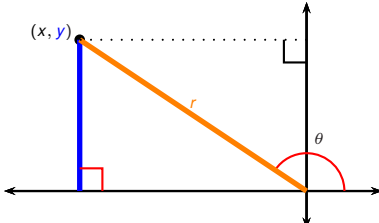
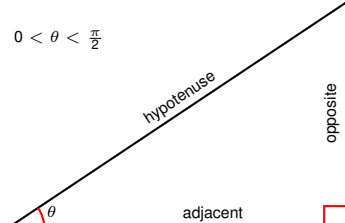


# Trigonometric Functions and Right Angle Triangles

 <p> <math>\cos \theta = \frac{x}{r}</math>      <math>\sec \theta</math>  <math>\sin \theta</math>      <math>\csc \theta</math>  <math>\tan \theta</math>      <math>\cot \theta</math> </p>	<p><math>0 &lt; \theta &lt; \frac{\pi}{2}</math></p>  <p> <math>\cos \theta</math>      <math>\sec \theta</math>  <math>\sin \theta</math>      <math>\csc \theta</math>  <math>\tan \theta</math>      <math>\cot \theta</math> </p>
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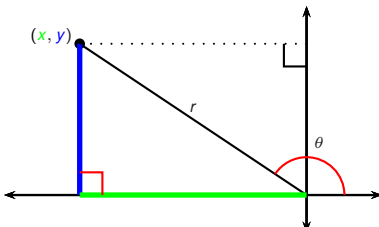
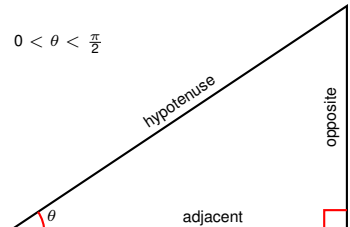
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- To do so we rescale by the distance  $r$  from the origin.

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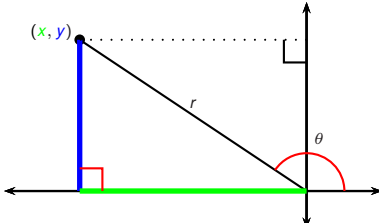
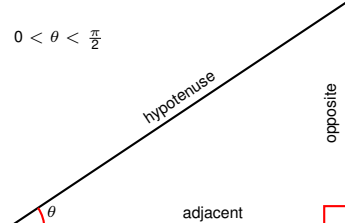
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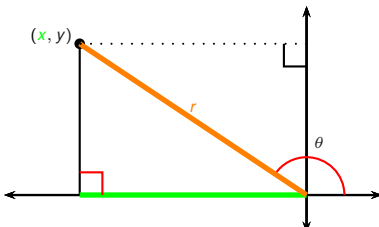
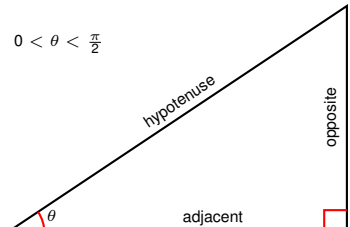
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 <p>Diagram showing an angle <math>\theta</math> in standard position. The terminal arm passes through point <math>(x, y)</math>. The distance from the origin to <math>(x, y)</math> is <math>r</math>. The x-axis is green, and the y-axis is blue. A right angle is shown at the origin between the axes.</p>	 <p>Diagram showing a right triangle for an acute angle <math>\theta</math>. The hypotenuse is the longest side, the adjacent side is next to <math>\theta</math>, and the opposite side is opposite <math>\theta</math>. A right angle is shown at the bottom right vertex.</p>
$\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$	$\sec \theta = \frac{r}{x}$ $\csc \theta = \frac{r}{y}$ $\cot \theta = \frac{x}{y}$
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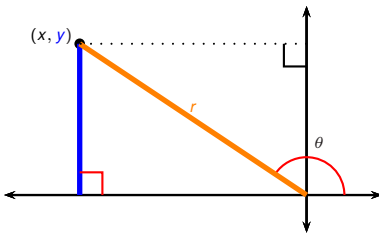
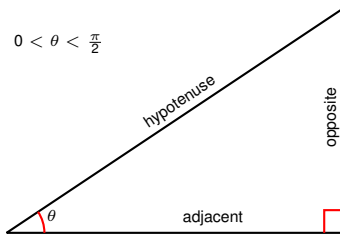
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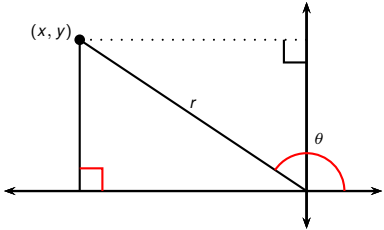
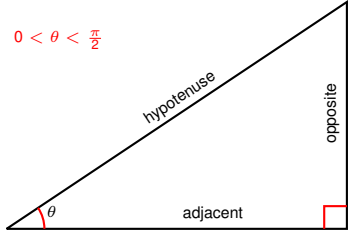
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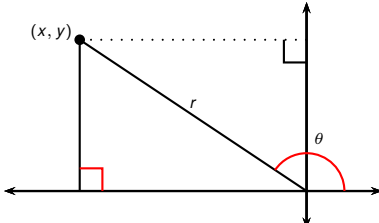
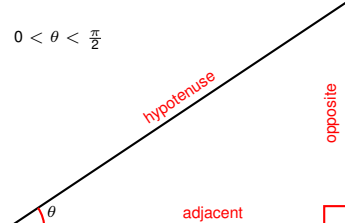
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- To do so we rescale by the distance  $r$  from the origin.
- The trig functions of **acute  $\theta$  (between 0 and  $\frac{\pi}{2}$ )** can be interpreted as ratios of sides of right angle triangle with angle  $\theta$ .

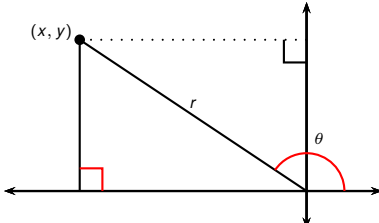
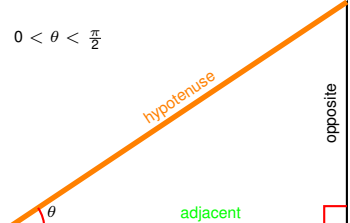
# Trigonometric Functions and Right Angle Triangles

	
$\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$	$\cos \theta$ $\sin \theta$ $\tan \theta$
$\sec \theta = \frac{r}{x}$ $\csc \theta = \frac{r}{y}$ $\cot \theta = \frac{x}{y}$	$\sec \theta$ $\csc \theta$ $\cot \theta$
All angles	Acute angles

- The trigonometric functions can be defined without requesting that the pt.  $(x, y)$  on the terminal arm of the angle lie on the unit circle.
- To do so we rescale by the distance  $r$  from the origin.
- The trig functions of acute  $\theta$  (between 0 and  $\frac{\pi}{2}$ ) can be interpreted as ratios of **sides of right angle triangle** with angle  $\theta$ .

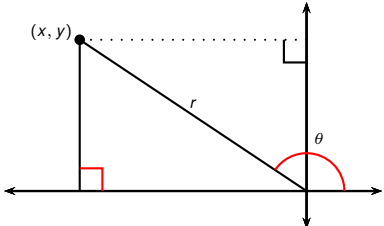
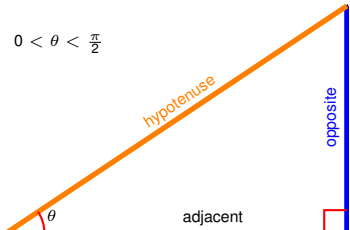


# Trigonometric Functions and Right Angle Triangles

	
$\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\sin \theta$ $\tan \theta$
$\sec \theta = \frac{r}{x}$ $\csc \theta = \frac{r}{y}$ $\cot \theta = \frac{x}{y}$	$\sec \theta$ $\csc \theta$ $\cot \theta$
All angles	Acute angles

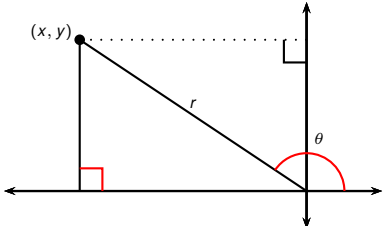
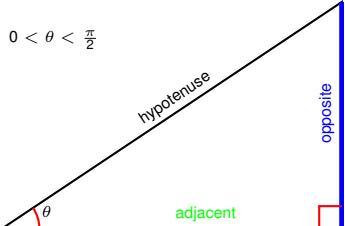
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# Trigonometric Functions and Right Angle Triangles

	
$\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ $\tan \theta$
$\sec \theta = \frac{r}{x}$ $\csc \theta = \frac{r}{y}$ $\cot \theta = \frac{x}{y}$	$\sec \theta$ $\csc \theta$ $\cot \theta$
All angles	Acute angles

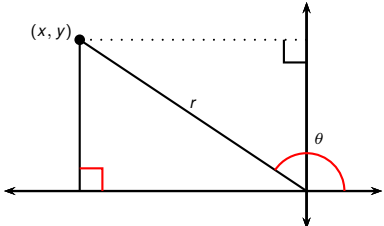
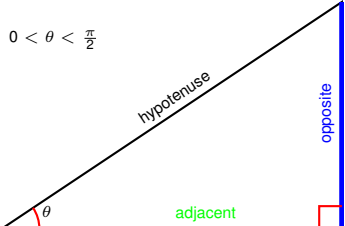
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# Trigonometric Functions and Right Angle Triangles

	
$\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ $\tan \theta = \frac{\text{opp}}{\text{adj}}$
All angles	Acute angles

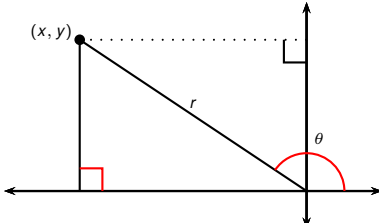
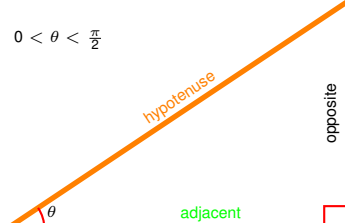
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# Trigonometric Functions and Right Angle Triangles

	
$\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ $\tan \theta = \frac{\text{opp}}{\text{adj}}$
$\sec \theta = \frac{r}{x}$ $\csc \theta = \frac{r}{y}$ $\cot \theta = \frac{x}{y}$	$\sec \theta = \frac{\text{hyp}}{\text{adj}}$ $\csc \theta = \frac{\text{hyp}}{\text{opp}}$ $\cot \theta = \frac{\text{adj}}{\text{opp}}$
All angles	Acute angles

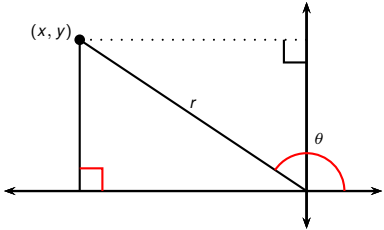
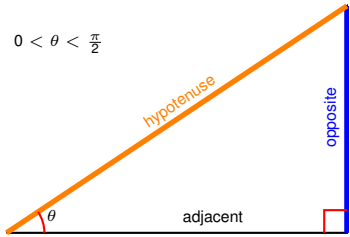
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# Trigonometric Functions and Right Angle Triangles

	
$\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ $\tan \theta = \frac{\text{opp}}{\text{adj}}$
$\sec \theta = \frac{r}{x}$ $\csc \theta = \frac{r}{y}$ $\cot \theta = \frac{x}{y}$	$\sec \theta = \frac{\text{hyp}}{\text{adj}}$ $\csc \theta = \frac{\text{hyp}}{\text{opp}}$ $\cot \theta = \frac{\text{adj}}{\text{opp}}$
All angles	Acute angles

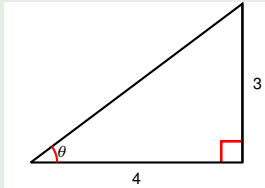
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# Trigonometric Functions and Right Angle Triangles

 $\cos \theta = \frac{x}{r} \qquad \sec \theta = \frac{r}{x}$ $\sin \theta = \frac{y}{r} \qquad \csc \theta = \frac{r}{y}$ $\tan \theta = \frac{y}{x} \qquad \cot \theta = \frac{x}{y}$ <p style="text-align: center;">All angles</p>	 $0 < \theta < \frac{\pi}{2}$ $\cos \theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec \theta = \frac{\text{hyp}}{\text{adj}}$ $\sin \theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc \theta = \frac{\text{hyp}}{\text{opp}}$ $\tan \theta = \frac{\text{opp}}{\text{adj}} \qquad \cot \theta = \frac{\text{adj}}{\text{opp}}$ <p style="text-align: center;">Acute angles</p>
--	--

- The trigonometric functions can be defined without requesting that the pt.  $(x, y)$  on the terminal arm of the angle lie on the unit circle.
- To do so we rescale by the distance  $r$  from the origin.
- The trig functions of acute  $\theta$  (between 0 and  $\frac{\pi}{2}$ ) can be interpreted as ratios of sides of right angle triangle with angle  $\theta$ .

## Example

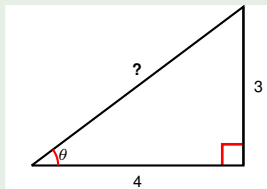


<sup>3</sup> Let the angle  $\theta$  be as indicated in the figure. Find the values of the six trigonometric functions of  $\theta$ .

$$\sin \theta = \quad \cos \theta = \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

## Example



<sup>3</sup> Let the angle  $\theta$  be as indicated in the figure. Find the values of the six trigonometric functions of  $\theta$ .

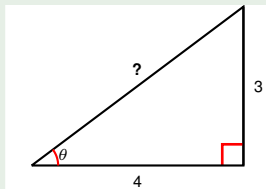
To find the trigonometric functions, we need to know the length of the hypotenuse.

$$\sin \theta = \quad \cos \theta = \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$



## Example



<sup>3</sup> Let the angle  $\theta$  be as indicated in the figure. Find the values of the six trigonometric functions of  $\theta$ .

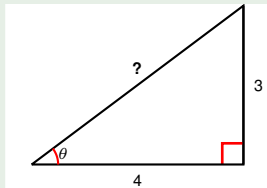
To find the trigonometric functions, we need to know the length of the hypotenuse.

hypotenuse = ?

$$\sin \theta = \quad \cos \theta = \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

## Example



<sup>3</sup> Let the angle  $\theta$  be as indicated in the figure. Find the values of the six trigonometric functions of  $\theta$ .

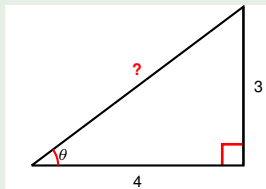
To find the trigonometric functions, we need to know the length of the hypotenuse.

$$\text{hypotenuse} = \sqrt{4^2 + 3^2}$$

$$\sin \theta = \quad \cos \theta = \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

## Example



<sup>3</sup> Let the angle  $\theta$  be as indicated in the figure. Find the values of the six trigonometric functions of  $\theta$ .

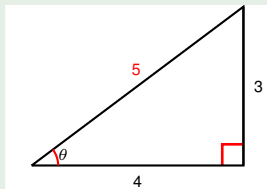
To find the trigonometric functions, we need to know the length of the hypotenuse.

$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25}$$

$$\sin \theta = \quad \cos \theta = \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

## Example



<sup>3</sup> Let the angle  $\theta$  be as indicated in the figure. Find the values of the six trigonometric functions of  $\theta$ .

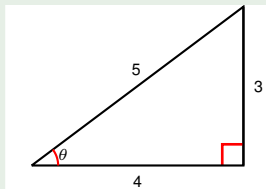
To find the trigonometric functions, we need to know the length of the hypotenuse.

$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

$$\sin \theta = \quad \cos \theta = \quad \tan \theta =$$

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## Example



<sup>3</sup> Let the angle  $\theta$  be as indicated in the figure. Find the values of the six trigonometric functions of  $\theta$ .

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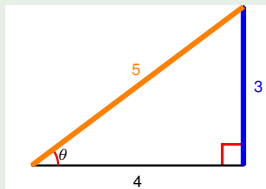
$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

Using the right angle triangle ratio interpretations of the trig functions, we can compute:

$$\sin \theta = ? \quad \cos \theta = \quad \tan \theta =$$

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## Example



Let the angle  $\theta$  be as indicated in the figure. Find the values of the six trigonometric functions of  $\theta$ .

To find the trigonometric functions, we need to know the length of the hypotenuse.

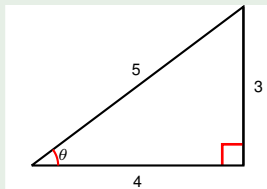
$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

Using the right angle triangle ratio interpretations of the trig functions, we can compute:

$$\sin \theta = \frac{3}{5} \quad \cos \theta = \quad \tan \theta =$$

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## Example



Let the angle  $\theta$  be as indicated in the figure. Find the values of the six trigonometric functions of  $\theta$ .

To find the trigonometric functions, we need to know the length of the hypotenuse.

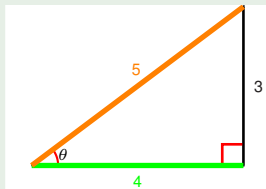
$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

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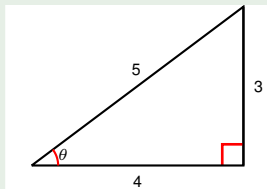
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$$\begin{array}{lll} \sin \theta = \frac{3}{5} & \cos \theta = \frac{4}{5} & \tan \theta = \\ \csc \theta = & \sec \theta = & \cot \theta = \end{array}$$



## Example



Let the angle  $\theta$  be as indicated in the figure. Find the values of the six trigonometric functions of  $\theta$ .

To find the trigonometric functions, we need to know the length of the hypotenuse.

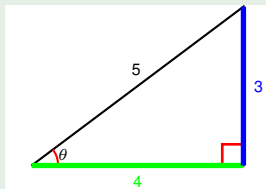
$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

Using the right angle triangle ratio interpretations of the trig functions, we can compute:

$$\sin \theta = \frac{3}{5} \quad \cos \theta = \frac{4}{5} \quad \tan \theta = ?$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

## Example



Let the angle  $\theta$  be as indicated in the figure. Find the values of the six trigonometric functions of  $\theta$ .

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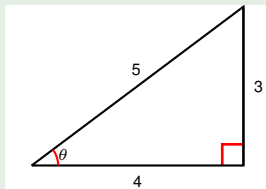
$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

Using the right angle triangle ratio interpretations of the trig functions, we can compute:

$$\sin \theta = \frac{3}{5} \quad \cos \theta = \frac{4}{5} \quad \tan \theta = \frac{3}{4}$$

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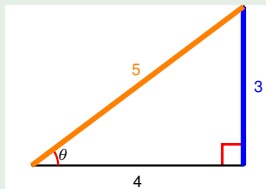
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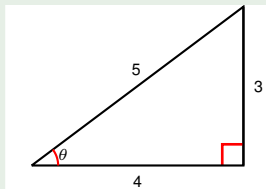
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Let the angle  $\theta$  be as indicated in the figure. Find the values of the six trigonometric functions of  $\theta$ .

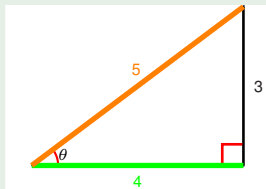
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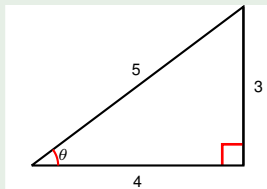
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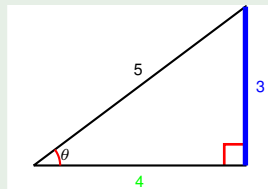
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## Example



Let the angle  $\theta$  be as indicated in the figure. Find the values of the six trigonometric functions of  $\theta$ .

To find the trigonometric functions, we need to know the length of the hypotenuse.

$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

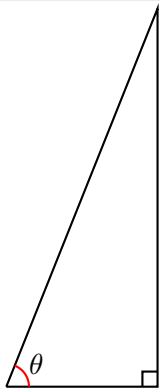
Using the right angle triangle ratio interpretations of the trig functions, we can compute:

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## Example

If  $\cos \theta = \frac{2}{5}$  and  $0 < \theta < \frac{\pi}{2}$ , find the other five trigonometric functions of  $\theta$ .



$$\sin \theta =$$

$$\tan \theta =$$

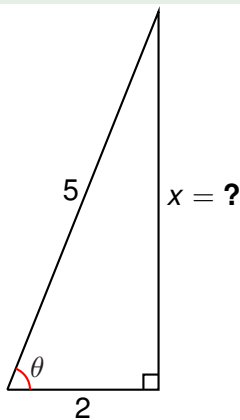
$$\csc \theta =$$

$$\sec \theta =$$

$$\cot \theta =$$

## Example

If  $\cos \theta = \frac{2}{5}$  and  $0 < \theta < \frac{\pi}{2}$ , find the other five trigonometric functions of  $\theta$ .



- Label the hypotenuse with length 5 and the adjacent side with length 2.

$$\sin \theta =$$

$$\tan \theta =$$

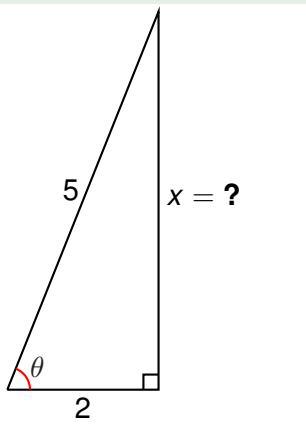
$$\csc \theta =$$

$$\sec \theta =$$

$$\cot \theta =$$

## Example

If  $\cos \theta = \frac{2}{5}$  and  $0 < \theta < \frac{\pi}{2}$ , find the other five trigonometric functions of  $\theta$ .



- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem:  $x^2 + 2^2 = 5^2$ .

$$\sin \theta =$$

$$\tan \theta =$$

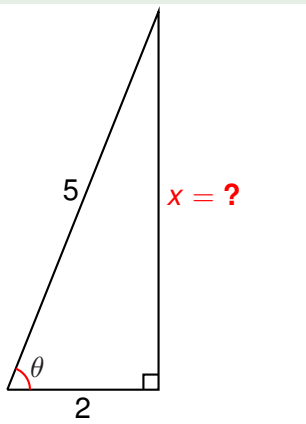
$$\csc \theta =$$

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## Example

If  $\cos \theta = \frac{2}{5}$  and  $0 < \theta < \frac{\pi}{2}$ , find the other five trigonometric functions of  $\theta$ .



- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem:  $x^2 + 2^2 = 5^2$ .
- Therefore  $x^2 = ?$ , so  $x = ?$ .

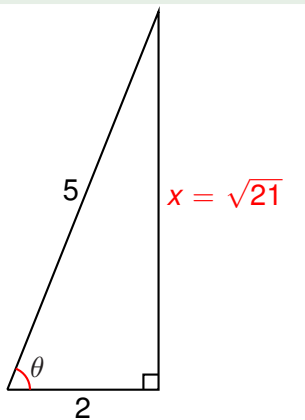
$$\sin \theta = \quad \quad \tan \theta =$$

$$\csc \theta = \quad \quad \sec \theta =$$

$$\cot \theta =$$

## Example

If  $\cos \theta = \frac{2}{5}$  and  $0 < \theta < \frac{\pi}{2}$ , find the other five trigonometric functions of  $\theta$ .



- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem:  $x^2 + 2^2 = 5^2$ .
- Therefore  $x^2 = 21$ , so  $x = \sqrt{21}$ .

$$\sin \theta =$$

$$\tan \theta =$$

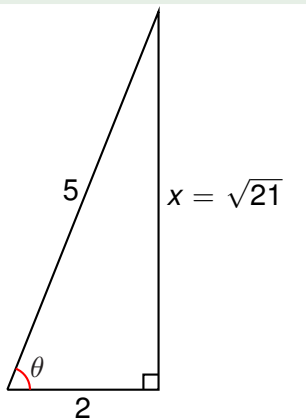
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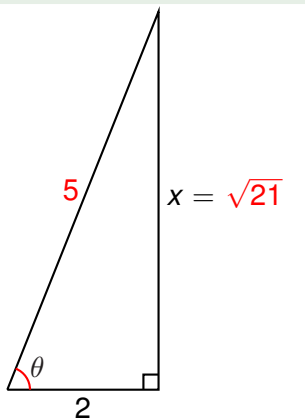
$$\sin \theta = ? \quad \tan \theta =$$

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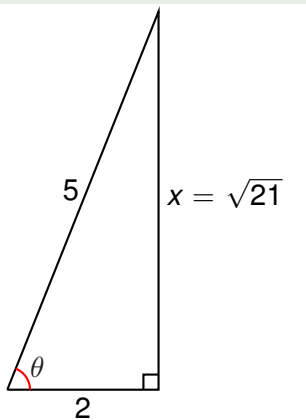
$$\sin \theta = \frac{\sqrt{21}}{5} \quad \tan \theta =$$

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$$\sin \theta = \frac{\sqrt{21}}{5} \quad \tan \theta = ?$$

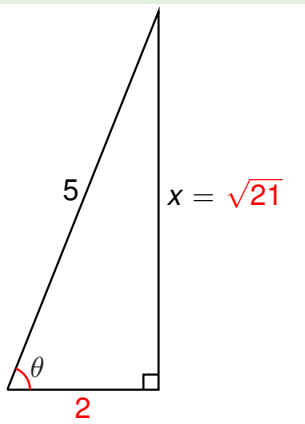
$$\csc \theta = \quad \sec \theta =$$

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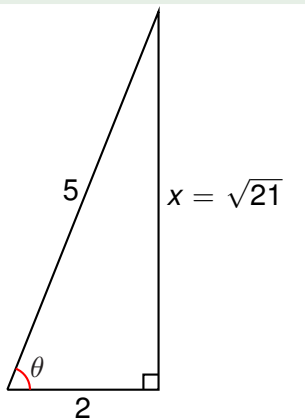
$$\sin \theta = \frac{\sqrt{21}}{5} \quad \tan \theta = \frac{\sqrt{21}}{2}$$

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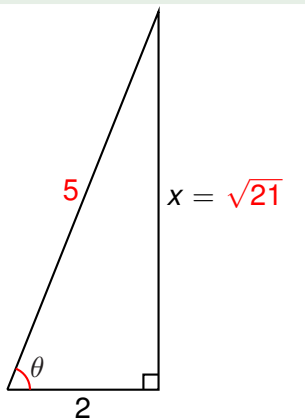
$$\sin \theta = \frac{\sqrt{21}}{5} \quad \tan \theta = \frac{\sqrt{21}}{2}$$

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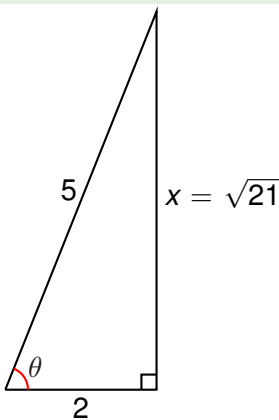
$$\sin \theta = \frac{\sqrt{21}}{5} \quad \tan \theta = \frac{\sqrt{21}}{2}$$

$$\csc \theta = \frac{5}{\sqrt{21}} \quad \sec \theta =$$

$$\cot \theta =$$

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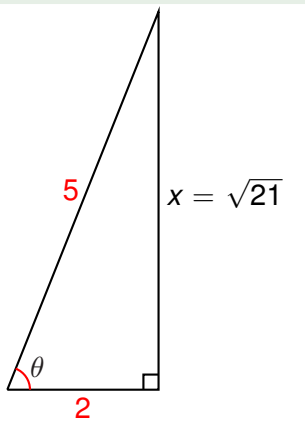
$$\sin \theta = \frac{\sqrt{21}}{5} \quad \tan \theta = \frac{\sqrt{21}}{2}$$

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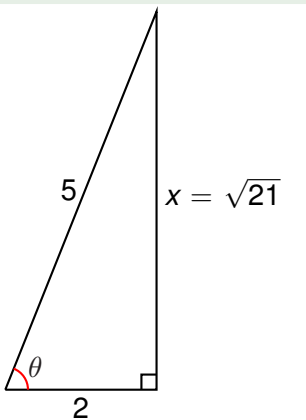
$$\sin \theta = \frac{\sqrt{21}}{5} \quad \tan \theta = \frac{\sqrt{21}}{2}$$

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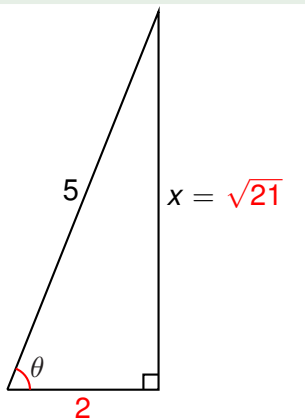
$$\sin \theta = \frac{\sqrt{21}}{5} \quad \tan \theta = \frac{\sqrt{21}}{2}$$

$$\csc \theta = \frac{5}{\sqrt{21}} \quad \sec \theta = \frac{5}{2}$$

$$\cot \theta = ?$$

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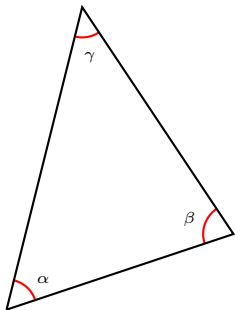


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- Pythagorean theorem:  $x^2 + 2^2 = 5^2$ .
- Therefore  $x^2 = 21$ , so  $x = \sqrt{21}$ .

$$\sin \theta = \frac{\sqrt{21}}{5} \quad \tan \theta = \frac{\sqrt{21}}{2}$$

$$\csc \theta = \frac{5}{\sqrt{21}} \quad \sec \theta = \frac{5}{2}$$

$$\cot \theta = \frac{2}{\sqrt{21}}$$



### Proposition

*The angles of every triangle sum up to  $\pi = 180^\circ$ .*

In other words, if  $\alpha, \beta, \gamma$  are the angles indicated in the figure, then we have:

$$\alpha + \beta + \gamma = 180^\circ.$$



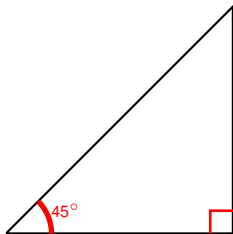
## Example

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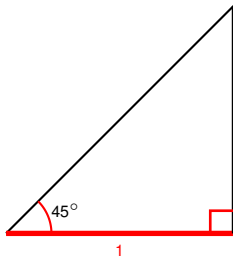
- Draw the  $45^\circ$  angle in right angle triangle,



## Example

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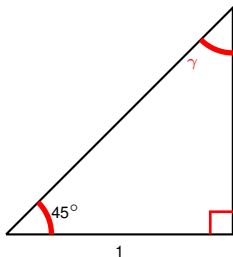
- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length **1**.



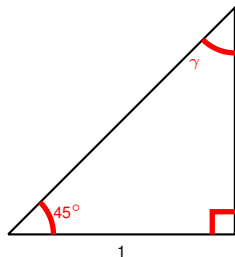
## Example

Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
- Let  $\gamma$  be the angle indicated on the plot.



## Example

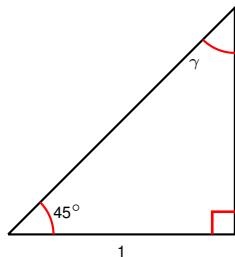


Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
- Let  $\gamma$  be the angle indicated on the plot.
- Angles in triangle sum to  $180^\circ$ :

$$45^\circ + 90^\circ + \gamma = 180^\circ$$

## Example

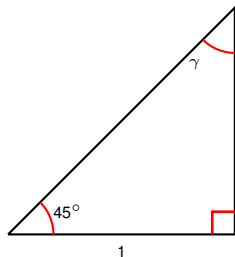


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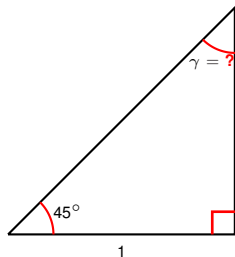


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- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
- Let  $\gamma$  be the angle indicated on the plot.
- Angles in triangle sum to  $180^\circ$ :

$$\begin{aligned} 45^\circ + 90^\circ + \gamma &= 180^\circ \\ \gamma &= 180^\circ - 90^\circ - 45^\circ \end{aligned}$$

## Example



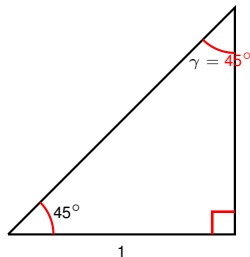
Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
- Let  $\gamma$  be the angle indicated on the plot.
- Angles in triangle sum to  $180^\circ$ :

$$\begin{aligned} 45^\circ + 90^\circ + \gamma &= 180^\circ \\ \gamma &= 180^\circ - 90^\circ - 45^\circ = ? \end{aligned}$$



## Example

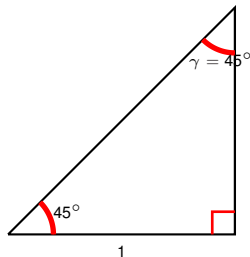


Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
- Let  $\gamma$  be the angle indicated on the plot.
- Angles in triangle sum to  $180^\circ$ :

$$\begin{aligned} 45^\circ + 90^\circ + \gamma &= 180^\circ \\ \gamma &= 180^\circ - 90^\circ - 45^\circ = 45^\circ. \end{aligned}$$

## Example



Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

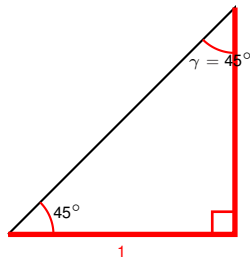
- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
- Let  $\gamma$  be the angle indicated on the plot.
- Angles in triangle sum to  $180^\circ$ :

$$45^\circ + 90^\circ + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 90^\circ - 45^\circ = 45^\circ.$$

- Triangle has two equal angles

## Example



Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

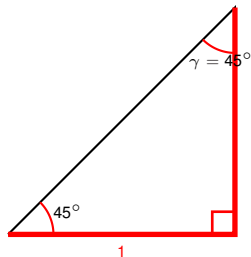
- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
- Let  $\gamma$  be the angle indicated on the plot.
- Angles in triangle sum to  $180^\circ$ :

$$45^\circ + 90^\circ + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 90^\circ - 45^\circ = 45^\circ.$$

- Triangle has two equal angles  $\Rightarrow$  is **isosceles**

## Example



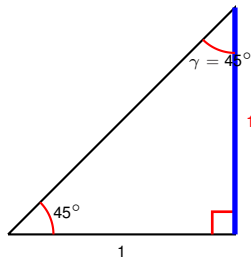
Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
- Let  $\gamma$  be the angle indicated on the plot.
- Angles in triangle sum to  $180^\circ$ :

$$\begin{aligned}45^\circ + 90^\circ + \gamma &= 180^\circ \\ \gamma &= 180^\circ - 90^\circ - 45^\circ = 45^\circ.\end{aligned}$$

- Triangle has two equal angles  $\Rightarrow$  is **isosceles (has two equal sides)**.

## Example



Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

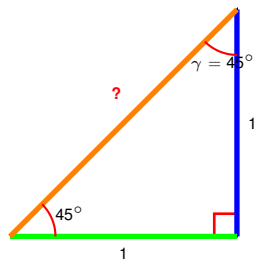
- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
- Let  $\gamma$  be the angle indicated on the plot.
- Angles in triangle sum to  $180^\circ$ :

$$45^\circ + 90^\circ + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 90^\circ - 45^\circ = 45^\circ.$$

- Triangle has two equal angles  $\Rightarrow$  is isosceles (has two equal sides).
- $\Rightarrow$  Opposite leg: length 1

## Example



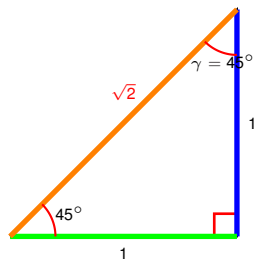
Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
- Let  $\gamma$  be the angle indicated on the plot.
- Angles in triangle sum to  $180^\circ$ :

$$\begin{aligned} 45^\circ + 90^\circ + \gamma &= 180^\circ \\ \gamma &= 180^\circ - 90^\circ - 45^\circ = 45^\circ. \end{aligned}$$

- Triangle has two equal angles  $\Rightarrow$  is isosceles (has two equal sides).
- $\Rightarrow$  Opposite leg: length 1  $\Rightarrow$  length(hyp) = ?

## Example



Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

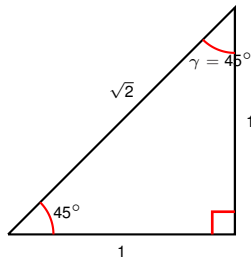
- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
- Let  $\gamma$  be the angle indicated on the plot.
- Angles in triangle sum to  $180^\circ$ :

$$45^\circ + 90^\circ + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 90^\circ - 45^\circ = 45^\circ.$$

- Triangle has two equal angles  $\Rightarrow$  is isosceles (has two equal sides).
- $\Rightarrow$  Opposite leg: length 1  $\Rightarrow$  length(hyp) =  $\sqrt{1^2 + 1^2} = \sqrt{2}$ .

## Example



Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
- Let  $\gamma$  be the angle indicated on the plot.
- Angles in triangle sum to  $180^\circ$ :

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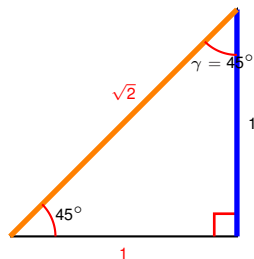
•  $\sin 45^\circ = ?$

$\cos 45^\circ = ?$

$\tan 45^\circ = ?$



## Example



Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
- Let  $\gamma$  be the angle indicated on the plot.
- Angles in triangle sum to  $180^\circ$ :

$$45^\circ + 90^\circ + \gamma = 180^\circ$$

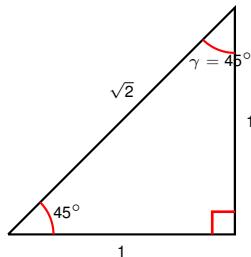
$$\gamma = 180^\circ - 90^\circ - 45^\circ = 45^\circ.$$

- Triangle has two equal angles  $\Rightarrow$  is isosceles (has two equal sides).
- $\Rightarrow$  Opposite leg: length 1  $\Rightarrow$  length(hyp) =  $\sqrt{1^2 + 1^2} = \sqrt{2}$ .

- $\sin 45^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{2}}{2}$        $\cos 45^\circ = ?$

$$\tan 45^\circ = ?$$

## Example



Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
- Let  $\gamma$  be the angle indicated on the plot.
- Angles in triangle sum to  $180^\circ$ :

$$45^\circ + 90^\circ + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 90^\circ - 45^\circ = 45^\circ.$$

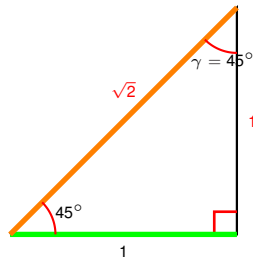
- Triangle has two equal angles  $\Rightarrow$  is isosceles (has two equal sides).
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- $\sin 45^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{2}}{2}$

$$\cos 45^\circ = ?$$

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## Example



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- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
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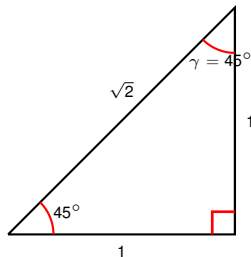
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$$\sin 45^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = ?$$

## Example



Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

- Draw the  $45^\circ$  angle in right angle triangle, adjacent side of length 1.
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- Angles in triangle sum to  $180^\circ$ :

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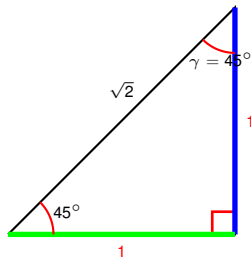
$$\gamma = 180^\circ - 90^\circ - 45^\circ = 45^\circ.$$

- Triangle has two equal angles  $\Rightarrow$  is isosceles (has two equal sides).
- $\Rightarrow$  Opposite leg: length 1  $\Rightarrow$  length(hyp) =  $\sqrt{1^2 + 1^2} = \sqrt{2}$ .

$$\sin 45^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{2}}{2} \qquad \cos 45^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = ?$$

## Example



Find the values of  $\sin 45^\circ$ ,  $\cos 45^\circ$ ,  $\tan 45^\circ$ .

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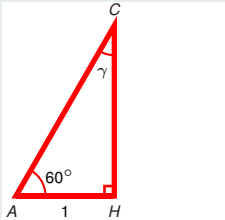
$$\sin 45^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{2}}{2} \qquad \cos 45^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{\text{opp}}{\text{adj}} = \frac{1}{1} = 1.$$

## Example

Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
 $\sin 30^\circ$ ,  $\cos 30^\circ$ ,  $\tan 30^\circ$ .

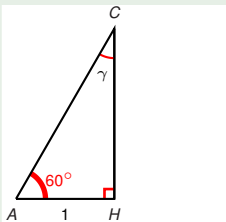
## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
 $\sin 30^\circ$ ,  $\cos 30^\circ$ ,  $\tan 30^\circ$ .

Construct a right angled  $\triangle AHC$  as indicated:

## Example

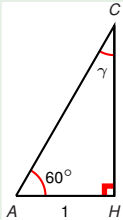


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Construct a right angled  $\triangle AHC$  as indicated: angles  
 $60^\circ$ ,  $90^\circ$ ,  $\gamma$ .



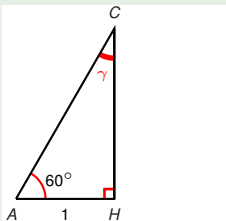
## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
 $\sin 30^\circ$ ,  $\cos 30^\circ$ ,  $\tan 30^\circ$ .

Construct a right angled  $\triangle AHC$  as indicated: angles  
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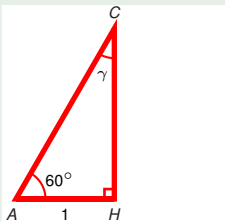
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Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
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## Example

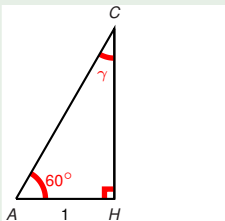


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Construct a right angled  $\triangle AHC$  as indicated: angles  
 $60^\circ$ ,  $90^\circ$ ,  $\gamma$ . Angles **in  $\triangle$**  sum to  $180^\circ$ :

$$60^\circ + 90^\circ + \gamma = 180^\circ$$

## Example

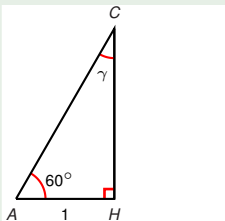


Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  $\sin 30^\circ$ ,  $\cos 30^\circ$ ,  $\tan 30^\circ$ .

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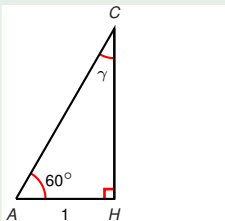
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$$60^\circ + 90^\circ + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 90^\circ - 60^\circ$$

## Example



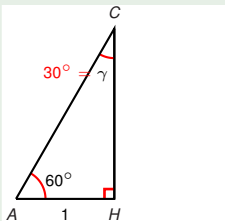
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$$60^\circ + 90^\circ + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 90^\circ - 60^\circ = ?$$

## Example



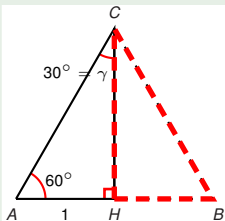
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$$\gamma = 180^\circ - 90^\circ - 60^\circ = 30^\circ.$$

## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
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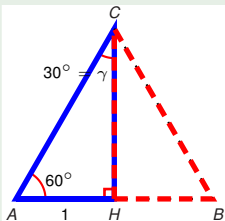
$$60^\circ + 90^\circ + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 90^\circ - 60^\circ = 30^\circ.$$

Construct  $\triangle HBC$  as indicated so that  $\triangle HBC \cong \triangle HAC$ .



## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
 $\sin 30^\circ$ ,  $\cos 30^\circ$ ,  $\tan 30^\circ$ .

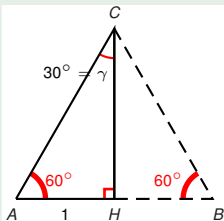
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 $60^\circ$ ,  $90^\circ$ ,  $\gamma$ . Angles in  $\triangle$  sum to  $180^\circ$ :

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## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
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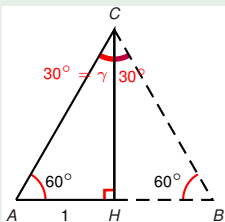
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 $60^\circ$ ,  $90^\circ$ ,  $\gamma$ . Angles in  $\triangle$  sum to  $180^\circ$ :

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## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
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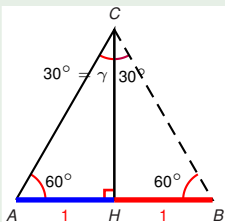
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## Example



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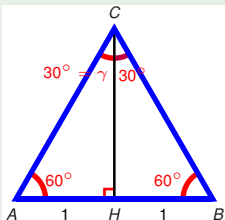
Construct a right angled  $\triangle AHC$  as indicated: angles  
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## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
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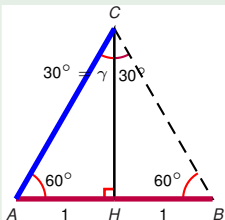
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Construct  $\triangle HBC$  as indicated so that  $\triangle HBC \cong \triangle HAC$ .  $\triangle ABC$  has  
 three equal angles ( $= 60^\circ$ )

## Example



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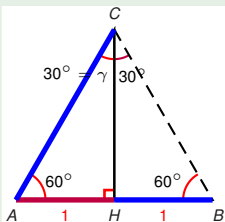
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$$|AC| = |AB|$$

## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
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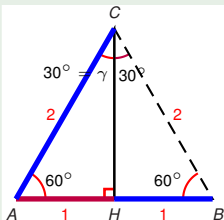
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 three equal angles ( $= 60^\circ$ )  $\Rightarrow$  its sides are of equal length. Therefore

$$|AC| = |AB| = 1 + 1$$

## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
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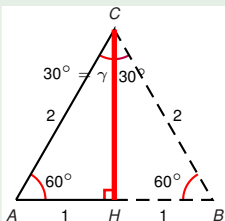
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Construct  $\triangle HBC$  as indicated so that  $\triangle HBC \cong \triangle HAC$ .  $\triangle ABC$  has  
 three equal angles ( $= 60^\circ$ )  $\Rightarrow$  its sides are of equal length. Therefore

$$|AC| = |AB| = 1 + 1 = 2$$



## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
 $\sin 30^\circ$ ,  $\cos 30^\circ$ ,  $\tan 30^\circ$ .

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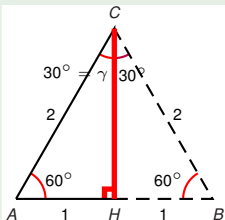
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$$|AC| = |AB| = 1 + 1 = 2$$

$$|CH| = ?$$

## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
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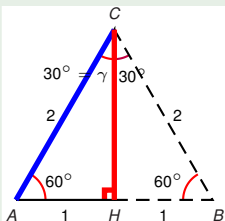
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$$|AC| = |AB| = 1 + 1 = 2$$

$$|CH| = \sqrt{|AC|^2 - |AH|^2} \quad \bigg| \quad \text{Pythagorean theorem}$$

## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
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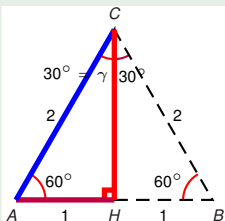
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## Example



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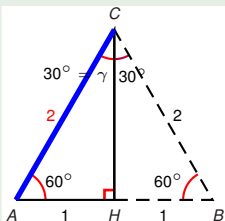
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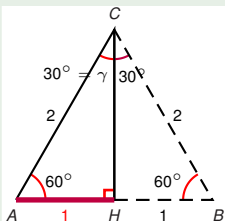
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$$|CH| = \sqrt{|AC|^2 - |AH|^2} \quad | \text{ Pythagorean theorem}$$

$$= \sqrt{2^2 - 1^2}$$

## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
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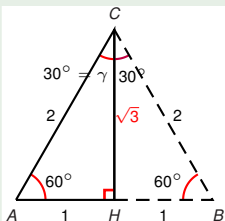
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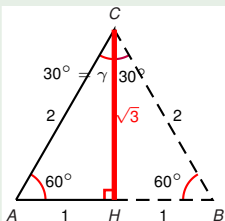
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$$= \sqrt{2^2 - 1^2} = \sqrt{3}$$

## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
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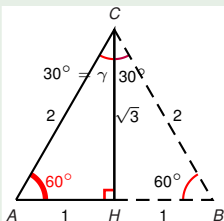
$$|AC| = |AB| = 1 + 1 = 2$$

$$|CH| = \sqrt{|AC|^2 - |AH|^2} \quad | \text{ Pythagorean theorem}$$

$$= \sqrt{2^2 - 1^2} = \sqrt{3}$$



## Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
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$$|AC| = |AB| = 1 + 1 = 2$$

$$|CH| = \sqrt{|AC|^2 - |AH|^2} \quad \left| \text{Pythagorean theorem} \right.$$

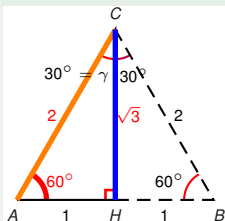
$$= \sqrt{2^2 - 1^2} = \sqrt{3}$$

$$\sin 60^\circ = ?$$

$$\cos 60^\circ = ?$$

$$\tan 60^\circ = ?$$

# Example



Find the values of  $\sin 60^\circ$ ,  $\cos 60^\circ$ ,  $\tan 60^\circ$ ,  
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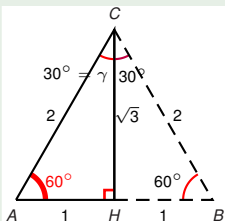
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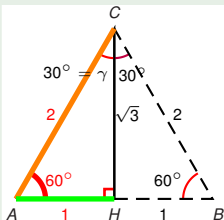
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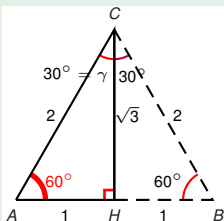
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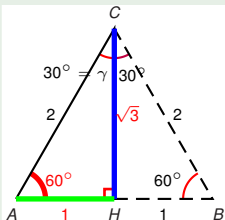
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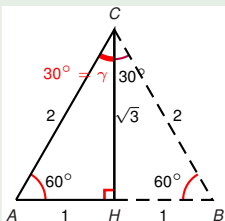
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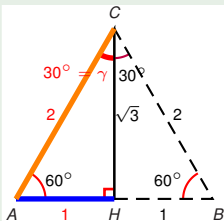
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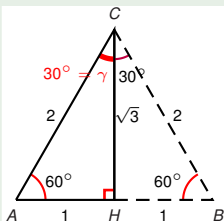
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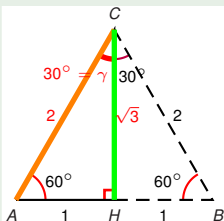
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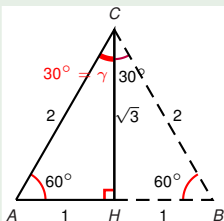
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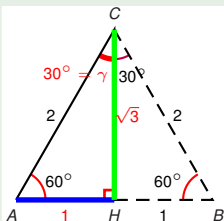
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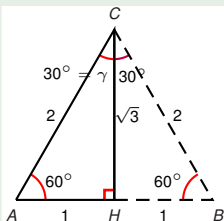
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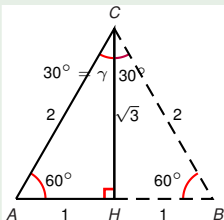
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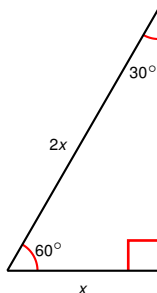
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## Observation

- *If the hypotenuse of a right angle triangle is twice larger than one of the sides, then the angle opposite to that side is  $30^\circ$ .*
- *Conversely, in a right angle triangle with angle  $30^\circ$ , the hypotenuse is twice longer than the shorter of the two legs.*



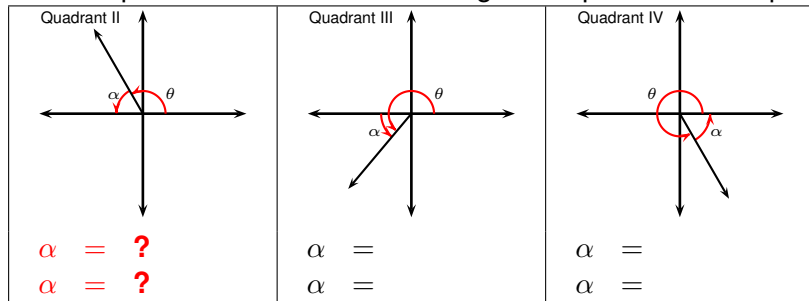
To compute trigonometric functions from obtuse ( $> 90^\circ$ ) or negative angles, we can use the following visual aid.

### Definition (Reference Angle)

Let  $\theta$  be an angle in standard position. Its reference angle is the acute positive angle formed by the terminal arm and the x axis.



The computation of the reference angle  $\alpha$  depends on the quadrant.

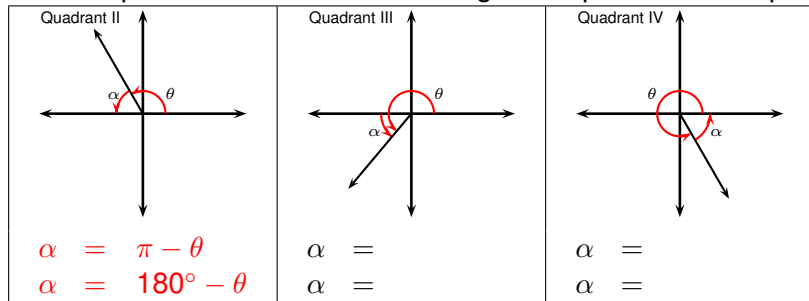


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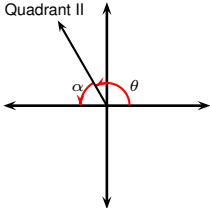
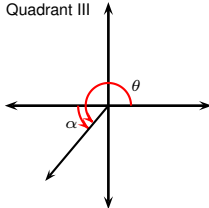
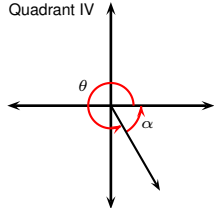


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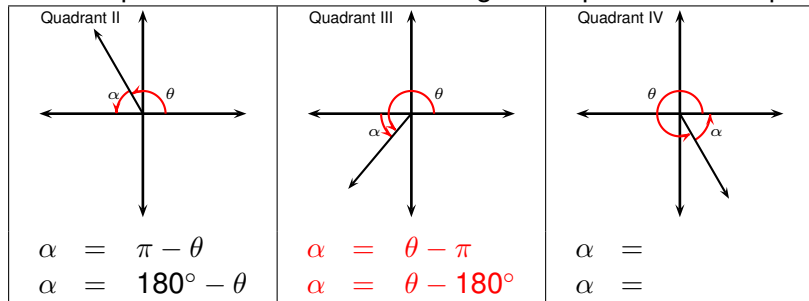
<p>Quadrant II</p>  <p> <math>\alpha = \pi - \theta</math>  <math>\alpha = 180^\circ - \theta</math> </p>	<p>Quadrant III</p>  <p> <math>\alpha = ?</math>  <math>\alpha = ?</math> </p>	<p>Quadrant IV</p>  <p> <math>\alpha =</math>  <math>\alpha =</math> </p>
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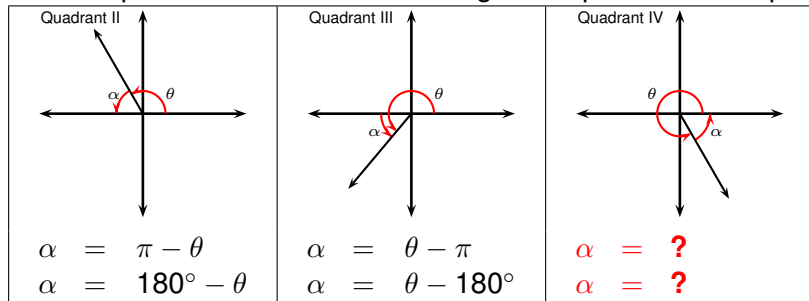


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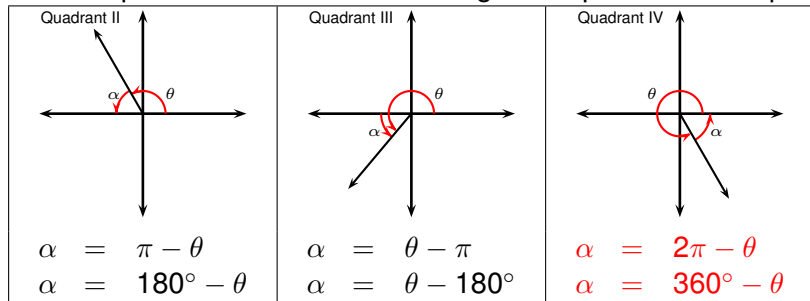


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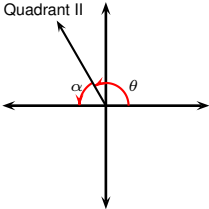
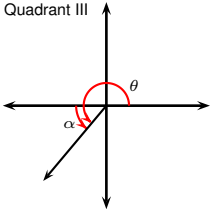
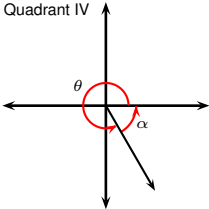


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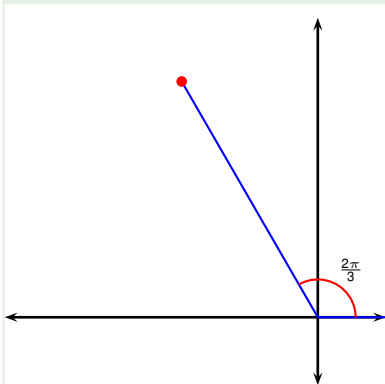
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## Observation

*One can find the value of a trigonometric function of  $\theta$  as follows.*

- *Find the reference angle  $\alpha$  associated to  $\theta$ .*
- *Find the trig function of  $\alpha$ .*
- *Use the quadrant in which  $\theta$  lies to affix an appropriate sign to the function value.*

## Example



Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

$$\sin\left(\frac{2\pi}{3}\right) =$$

$$\cos\left(\frac{2\pi}{3}\right) =$$

$$\tan\left(\frac{2\pi}{3}\right) =$$

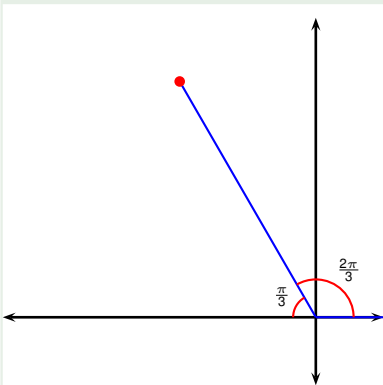
$$\csc\left(\frac{2\pi}{3}\right) =$$

$$\sec\left(\frac{2\pi}{3}\right) =$$

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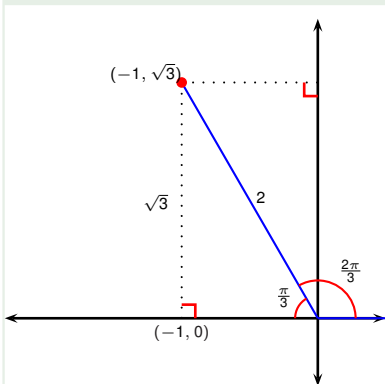
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$$\csc\left(\frac{2\pi}{3}\right) =$$

$$\sec\left(\frac{2\pi}{3}\right) =$$

$$\cot\left(\frac{2\pi}{3}\right) =$$

# Example



Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

$$\sin\left(\frac{2\pi}{3}\right) =$$

$$\cos\left(\frac{2\pi}{3}\right) =$$

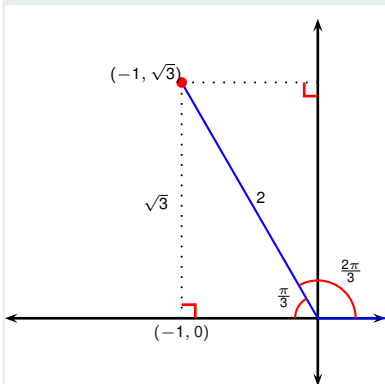
$$\tan\left(\frac{2\pi}{3}\right) =$$

$$\csc\left(\frac{2\pi}{3}\right) =$$

$$\sec\left(\frac{2\pi}{3}\right) =$$

$$\cot\left(\frac{2\pi}{3}\right) =$$

# Example



Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

$$\sin\left(\frac{2\pi}{3}\right) = ?$$

$$\csc\left(\frac{2\pi}{3}\right) =$$

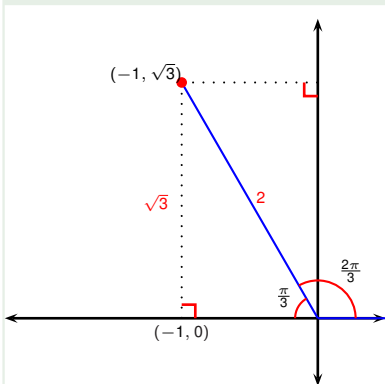
$$\cos\left(\frac{2\pi}{3}\right) =$$

$$\sec\left(\frac{2\pi}{3}\right) =$$

$$\tan\left(\frac{2\pi}{3}\right) =$$

$$\cot\left(\frac{2\pi}{3}\right) =$$

# Example



Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\csc\left(\frac{2\pi}{3}\right) =$$

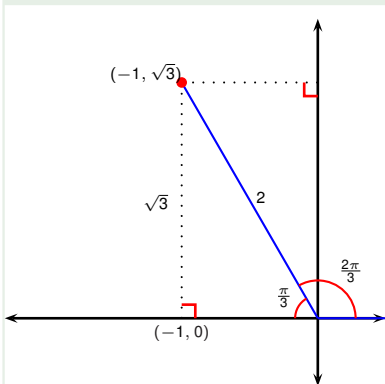
$$\cos\left(\frac{2\pi}{3}\right) =$$

$$\sec\left(\frac{2\pi}{3}\right) =$$

$$\tan\left(\frac{2\pi}{3}\right) =$$

$$\cot\left(\frac{2\pi}{3}\right) =$$

# Example



Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

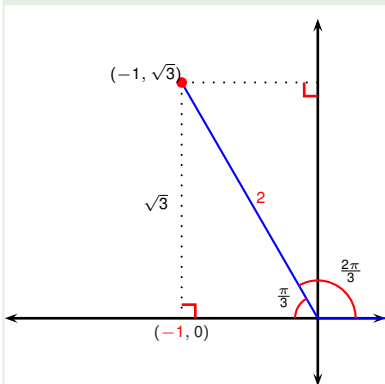
$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \cos\left(\frac{2\pi}{3}\right) = ?$$

$$\csc\left(\frac{2\pi}{3}\right) = \quad \sec\left(\frac{2\pi}{3}\right) =$$

$$\tan\left(\frac{2\pi}{3}\right) =$$

$$\cot\left(\frac{2\pi}{3}\right) =$$

# Example



Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

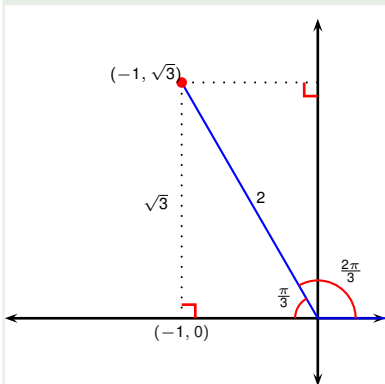
$$\tan\left(\frac{2\pi}{3}\right) =$$

$$\csc\left(\frac{2\pi}{3}\right) =$$

$$\sec\left(\frac{2\pi}{3}\right) =$$

$$\cot\left(\frac{2\pi}{3}\right) =$$

# Example



Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

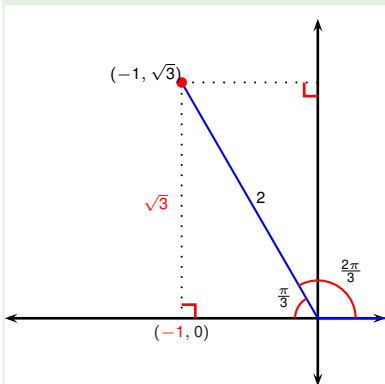
$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$\csc\left(\frac{2\pi}{3}\right) = \quad \sec\left(\frac{2\pi}{3}\right) =$$

$$\tan\left(\frac{2\pi}{3}\right) = ?$$

$$\cot\left(\frac{2\pi}{3}\right) =$$

# Example



Find the exact values of the trigonometric functions of

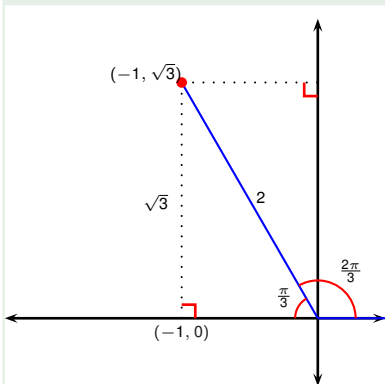
$$\theta = \frac{2\pi}{3} = 120^\circ.$$

$$\begin{aligned} \sin\left(\frac{2\pi}{3}\right) &= \frac{\sqrt{3}}{2} & \cos\left(\frac{2\pi}{3}\right) &= -\frac{1}{2} \\ \csc\left(\frac{2\pi}{3}\right) &= & \sec\left(\frac{2\pi}{3}\right) &= \end{aligned}$$

$$\begin{aligned} \tan\left(\frac{2\pi}{3}\right) &= \frac{\sqrt{3}}{-1} = -\sqrt{3} \\ \cot\left(\frac{2\pi}{3}\right) &= \end{aligned}$$



# Example



Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

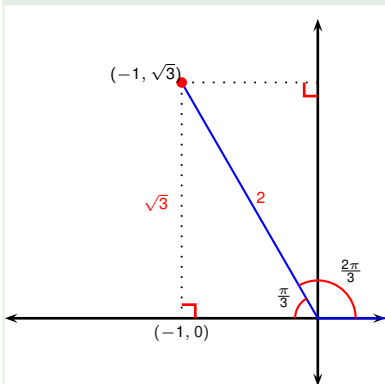
$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$\tan\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

$$\text{csc}\left(\frac{2\pi}{3}\right) = ? \quad \sec\left(\frac{2\pi}{3}\right) =$$

$$\cot\left(\frac{2\pi}{3}\right) =$$

# Example



Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

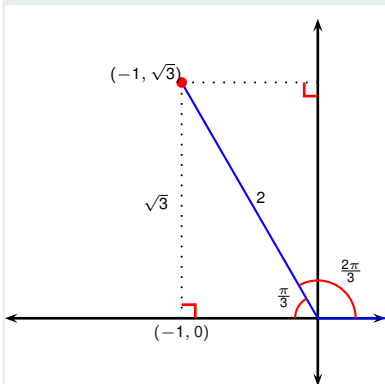
$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$\text{csc}\left(\frac{2\pi}{3}\right) = \frac{2}{\sqrt{3}} \quad \sec\left(\frac{2\pi}{3}\right) =$$

$$\tan\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

$$\cot\left(\frac{2\pi}{3}\right) =$$

## Example



Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

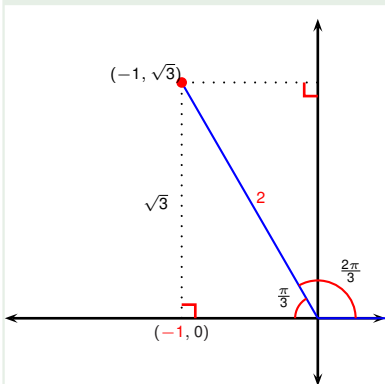
$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$\csc\left(\frac{2\pi}{3}\right) = \frac{2}{\sqrt{3}} \quad \sec\left(\frac{2\pi}{3}\right) = ?$$

$$\tan\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

$$\cot\left(\frac{2\pi}{3}\right) =$$

# Example

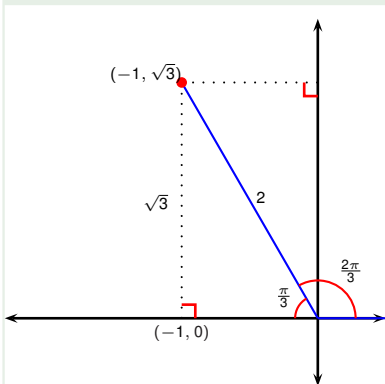


Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

$$\begin{aligned} \sin\left(\frac{2\pi}{3}\right) &= \frac{\sqrt{3}}{2} & \cos\left(\frac{2\pi}{3}\right) &= -\frac{1}{2} & \tan\left(\frac{2\pi}{3}\right) &= \frac{\sqrt{3}}{-1} = -\sqrt{3} \\ \csc\left(\frac{2\pi}{3}\right) &= \frac{2}{\sqrt{3}} & \sec\left(\frac{2\pi}{3}\right) &= -\frac{2}{1} = -2 & \cot\left(\frac{2\pi}{3}\right) &= \end{aligned}$$

# Example

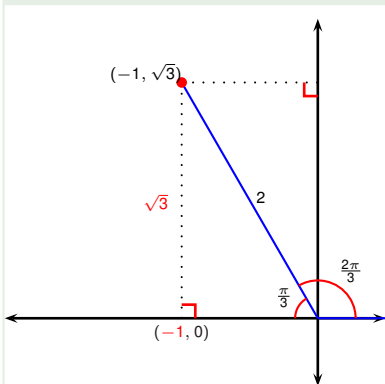


Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

$$\begin{aligned} \sin\left(\frac{2\pi}{3}\right) &= \frac{\sqrt{3}}{2} & \cos\left(\frac{2\pi}{3}\right) &= -\frac{1}{2} & \tan\left(\frac{2\pi}{3}\right) &= \frac{\sqrt{3}}{-1} = -\sqrt{3} \\ \csc\left(\frac{2\pi}{3}\right) &= \frac{2}{\sqrt{3}} & \sec\left(\frac{2\pi}{3}\right) &= -\frac{2}{1} = -2 & \cot\left(\frac{2\pi}{3}\right) &= ? \end{aligned}$$

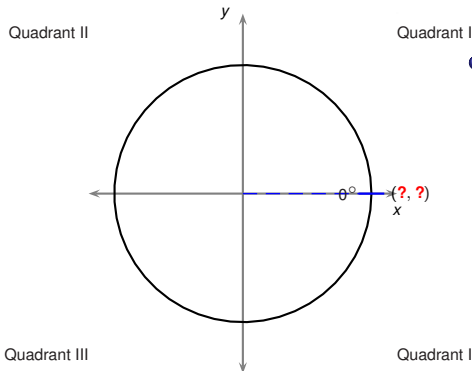
# Example



Find the exact values of the trigonometric functions of

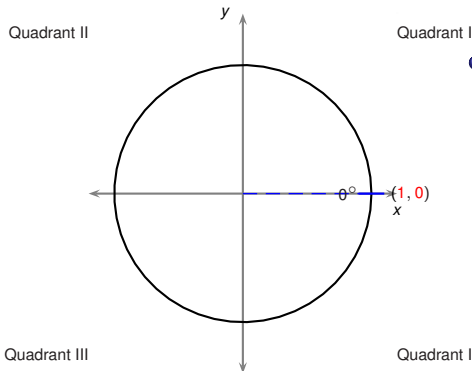
$$\theta = \frac{2\pi}{3} = 120^\circ.$$

$$\begin{aligned} \sin\left(\frac{2\pi}{3}\right) &= \frac{\sqrt{3}}{2} & \cos\left(\frac{2\pi}{3}\right) &= -\frac{1}{2} & \tan\left(\frac{2\pi}{3}\right) &= \frac{\sqrt{3}}{-1} = -\sqrt{3} \\ \csc\left(\frac{2\pi}{3}\right) &= \frac{2}{\sqrt{3}} & \sec\left(\frac{2\pi}{3}\right) &= -\frac{2}{1} = -2 & \cot\left(\frac{2\pi}{3}\right) &= -\frac{1}{\sqrt{3}} \end{aligned}$$



- One only needs to memorize sines and cosines in Quadrant I and on the axes.

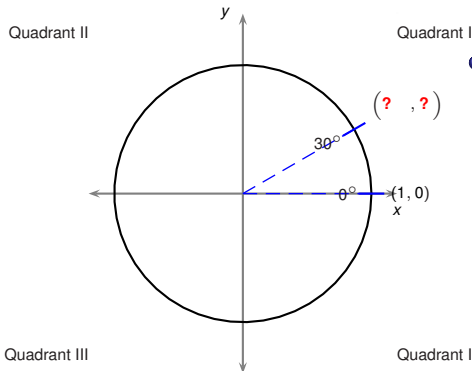
Deg.	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$	$270^\circ$	$360^\circ$
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	?										
cos	?										



- One only needs to memorize sines and cosines in Quadrant I and on the axes.

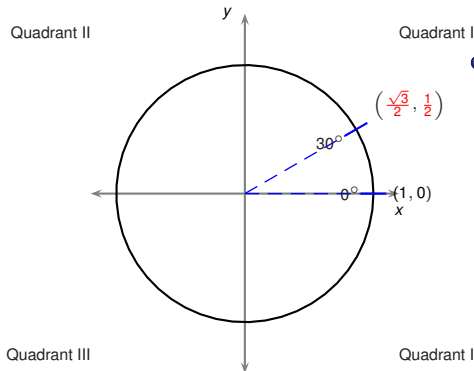
Deg.	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$	$270^\circ$	$360^\circ$
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0										
cos	1										





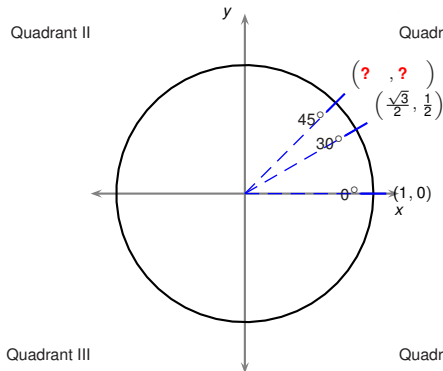
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	?									
cos	1	?									



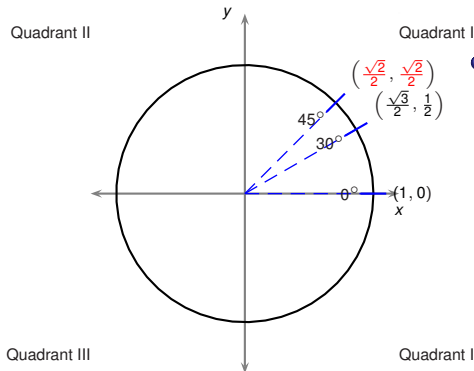
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$									
cos	1	$\frac{\sqrt{3}}{2}$									



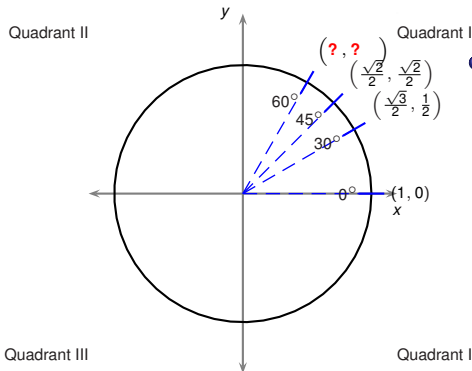
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	?								
cos	1	$\frac{\sqrt{3}}{2}$	?								



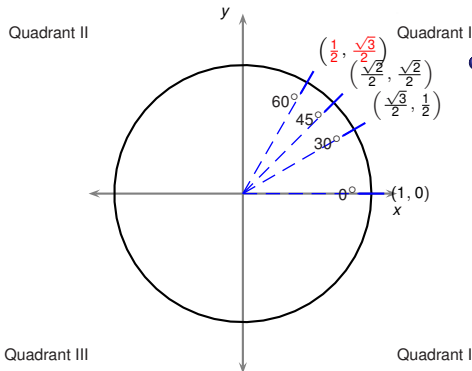
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$								
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$								



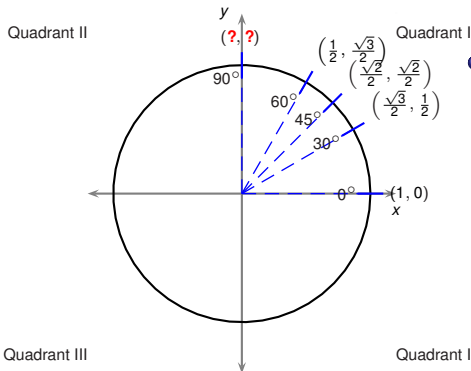
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	?							
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	?							



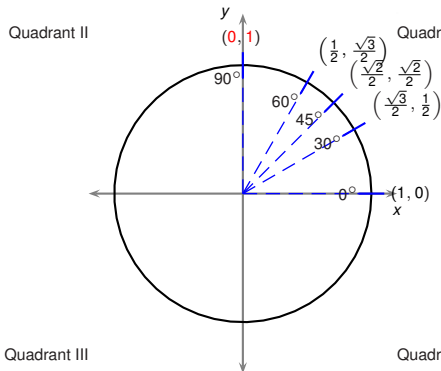
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$							
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$							



- One only needs to memorize sines and cosines in Quadrant I and on the axes.

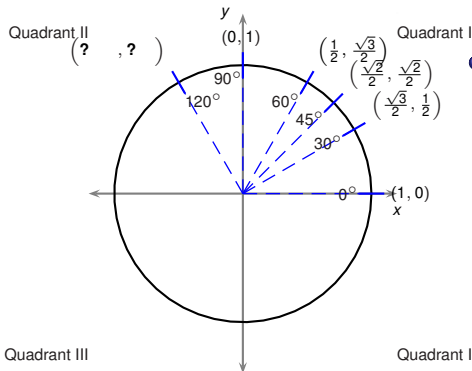
Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	?						
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	?						



- One only needs to memorize sines and cosines in Quadrant I and on the axes.

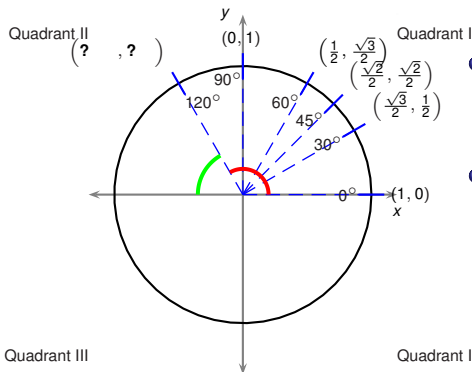
Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1						
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0						





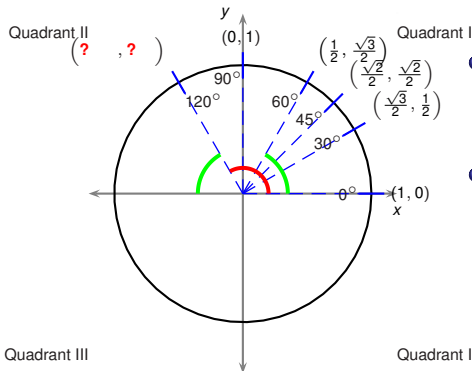
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	?					
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	?					



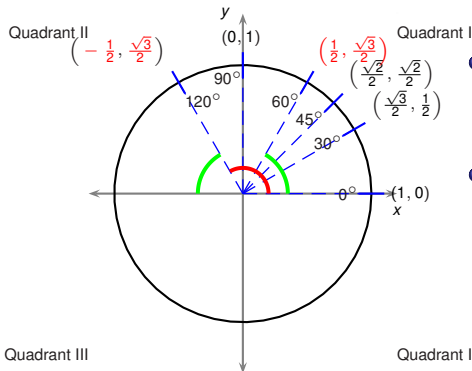
- One only needs to memorize sines and cosines in Quadrant I and on the axes.
- The remaining sines and cosines are extracted by
  - taking the sine/cosine of **the reference angle**

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	?					
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	?					



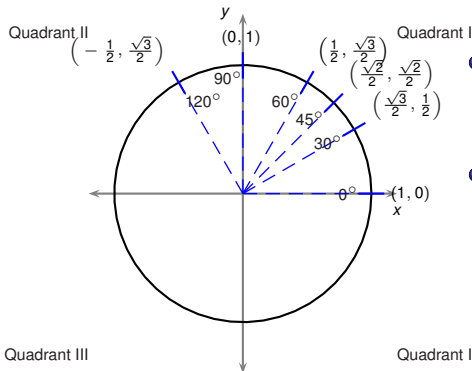
- One only needs to memorize sines and cosines in Quadrant I and on the axes.
- The remaining sines and cosines are extracted by
  - taking the sine/cosine of **the reference angle**

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	?					
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	?					



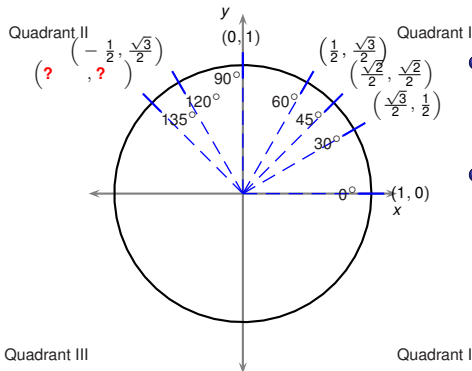
- One only needs to memorize sines and cosines in Quadrant I and on the axes.
- The remaining sines and cosines are extracted by
  - taking the **sine/cosine of the reference angle**

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$					
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$					



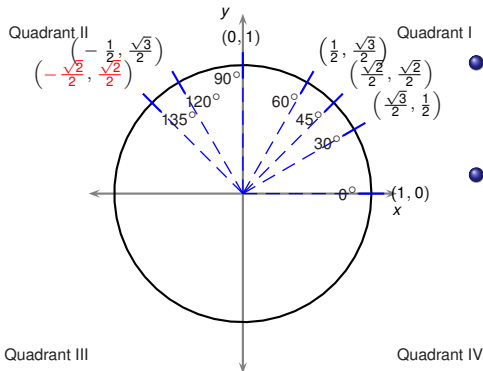
- One only needs to memorize sines and cosines in Quadrant I and on the axes.
- The remaining sines and cosines are extracted by
  - taking the sine/cosine of the reference angle
  - and **adjusting the sign according to the quadrant.**

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$					
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$					



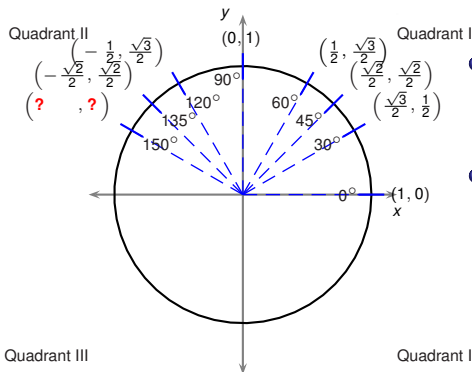
- One only needs to memorize sines and cosines in Quadrant I and on the axes.
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  - taking the sine/cosine of the reference angle
  - and adjusting the sign according to the quadrant.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	?				
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	?				



- One only needs to memorize sines and cosines in Quadrant I and on the axes.
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  - taking the sine/cosine of the reference angle
  - and adjusting the sign according to the quadrant.

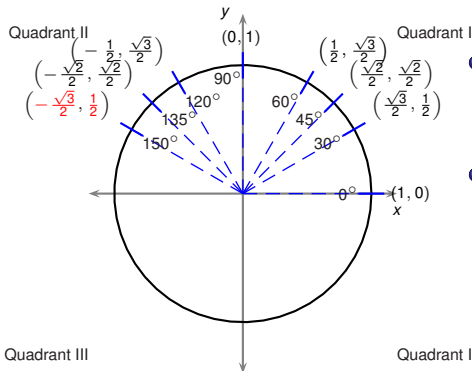
Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$				
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$				



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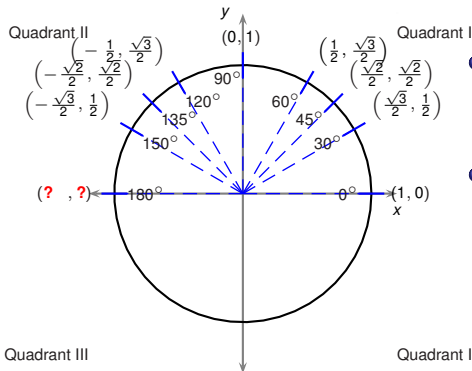
Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	?			
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	?			





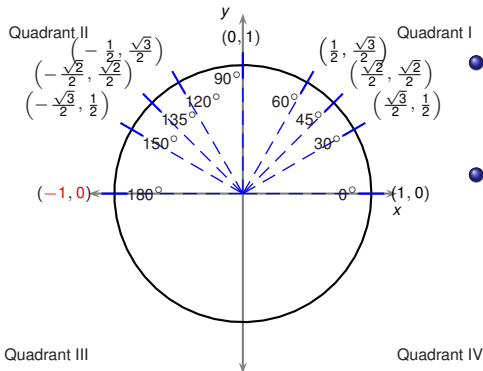
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Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$			
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$			



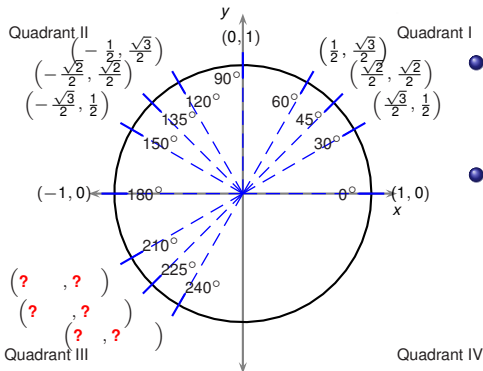
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Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	?		
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	?		



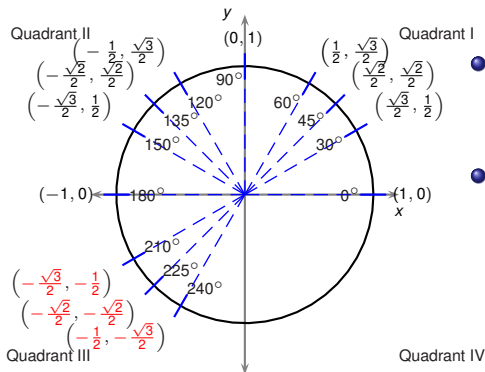
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Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0		
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1		



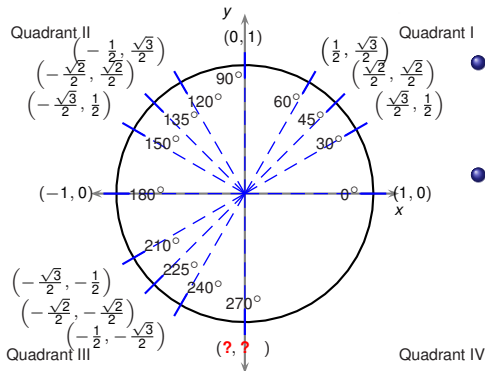
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sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0		
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1		



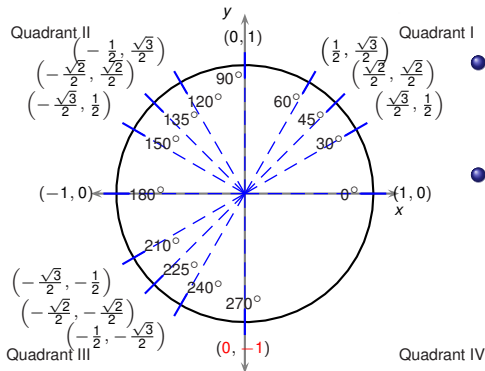
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Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0		
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1		



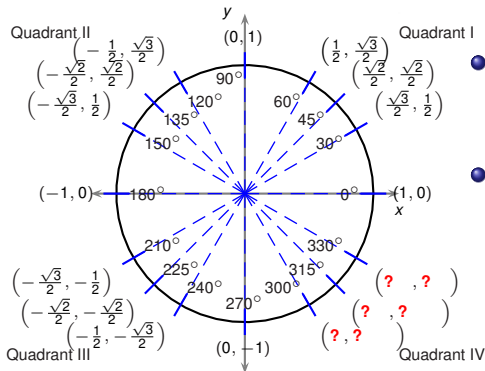
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Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	?	
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	?	



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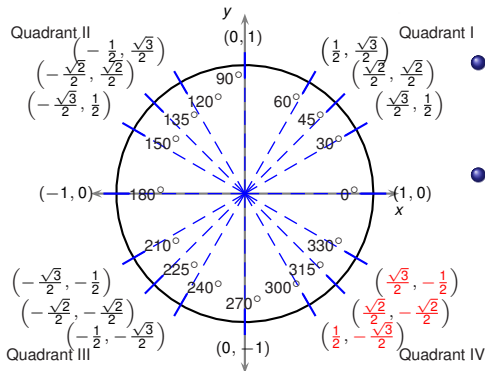
Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	



- One only needs to memorize sines and cosines in Quadrant I and on the axes.
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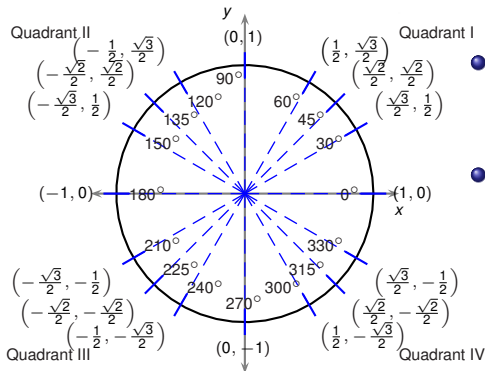
Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	





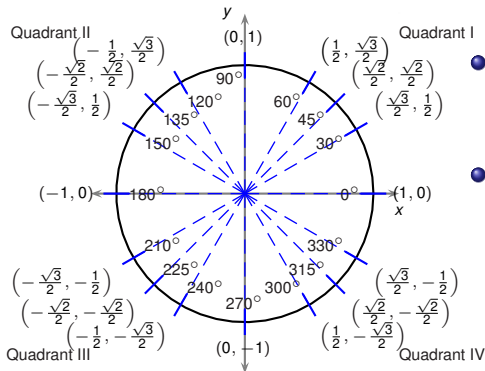
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Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	



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  - taking the sine/cosine of the reference angle
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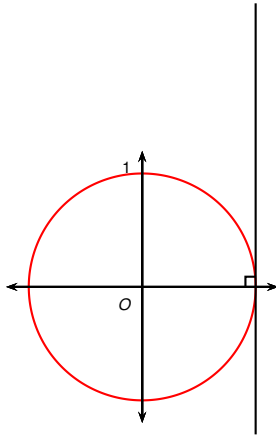
Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	?
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	?



- One only needs to memorize sines and cosines in Quadrant I and on the axes.
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  - taking the sine/cosine of the reference angle
  - and adjusting the sign according to the quadrant.

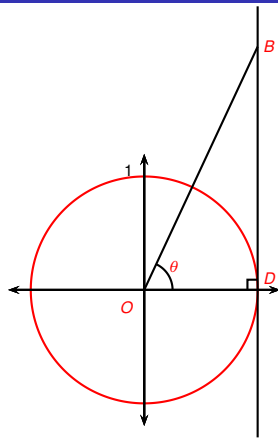
Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$	$\frac{3\pi}{2}$	$2\pi$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	1

# Geometric interpretation of all trigonometric functions



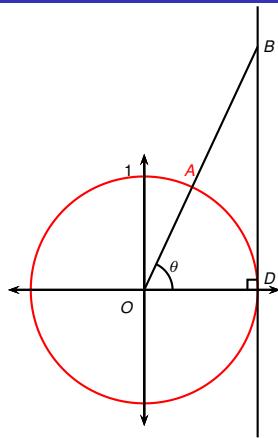
Fix unit circle, center  $O$ , coordinates  $(0, 0)$ .

# Geometric interpretation of all trigonometric functions



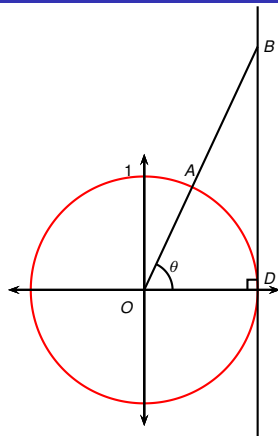
Fix unit circle, center  $O$ , coordinates  $(0, 0)$ .  
Let  $\angle DOB = \theta$ .

# Geometric interpretation of all trigonometric functions



Fix unit circle, center  $O$ , coordinates  $(0, 0)$ .  
Let  $\angle DOB = \theta$ . Let  $OB$  intersect the circle at point  $A$ .

# Geometric interpretation of all trigonometric functions



Fix unit circle, center  $O$ , coordinates  $(0, 0)$ .  
Let  $\angle DOB = \theta$ . Let  $OB$  intersect the circle at point  $A$ . Coordinates of  $A$  are  $(\cos \theta, \sin \theta)$ .

$$\sin \theta$$

$$\cos \theta$$

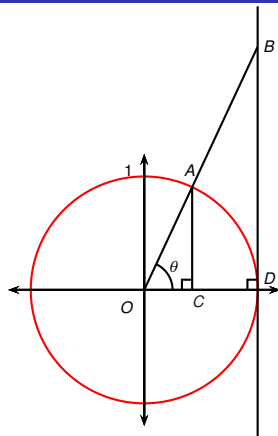
$$\tan \theta$$

$$\cot \theta$$

$$\sec \theta$$

$$\csc \theta$$

# Geometric interpretation of all trigonometric functions



Fix unit circle, center  $O$ , coordinates  $(0, 0)$ .  
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$$\sin \theta$$

$$\cos \theta$$

$$\tan \theta$$

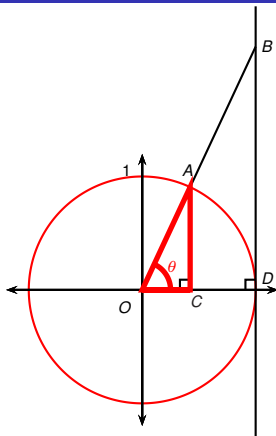
$$\cot \theta$$

$$\sec \theta$$

$$\csc \theta$$



# Geometric interpretation of all trigonometric functions



Fix unit circle, center  $O$ , coordinates  $(0, 0)$ .  
 Let  $\angle DOB = \theta$ . Let  $OB$  intersect the circle at point  $A$ . Coordinates of  $A$  are  $(\cos \theta, \sin \theta)$ .

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta$$

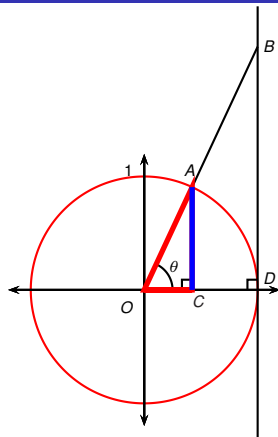
$$\tan \theta$$

$$\cot \theta$$

$$\sec \theta$$

$$\csc \theta$$

# Geometric interpretation of all trigonometric functions



Fix unit circle, center  $O$ , coordinates  $(0, 0)$ .  
 Let  $\angle DOB = \theta$ . Let  $OB$  intersect the circle at point  $A$ . Coordinates of  $A$  are  $(\cos \theta, \sin \theta)$ .

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|}$$

$$\cos \theta$$

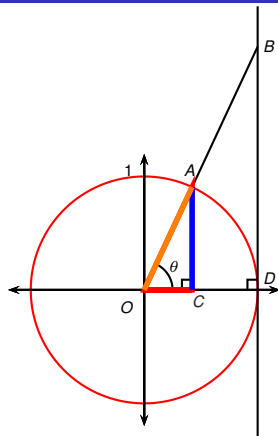
$$\tan \theta$$

$$\cot \theta$$

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$$\csc \theta$$

# Geometric interpretation of all trigonometric functions



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$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|}$$

$$\cos \theta$$

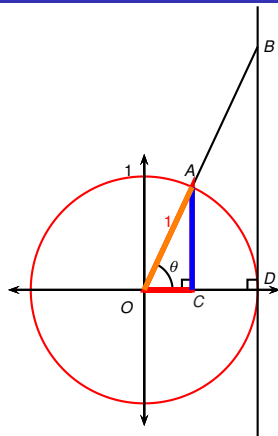
$$\tan \theta$$

$$\cot \theta$$

$$\sec \theta$$

$$\csc \theta$$

# Geometric interpretation of all trigonometric functions



Fix unit circle, center  $O$ , coordinates  $(0, 0)$ .  
 Let  $\angle DOB = \theta$ . Let  $OB$  intersect the circle at point  $A$ . Coordinates of  $A$  are  $(\cos \theta, \sin \theta)$ .

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|} = \frac{|AC|}{1}$$

$$\cos \theta$$

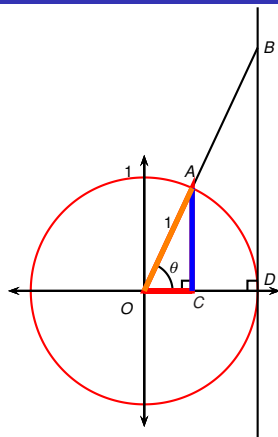
$$\tan \theta$$

$$\cot \theta$$

$$\sec \theta$$

$$\csc \theta$$

# Geometric interpretation of all trigonometric functions



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$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|} = \frac{|AC|}{1} = |AC|$$

$$\cos \theta$$

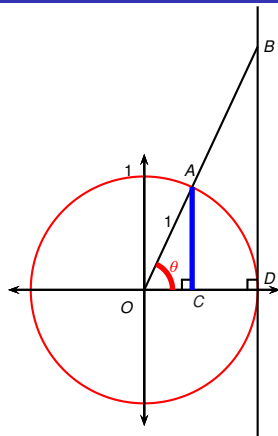
$$\tan \theta$$

$$\cot \theta$$

$$\sec \theta$$

$$\csc \theta$$

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$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|} = \frac{|AC|}{1} = |AC|$$

$$\cos \theta$$

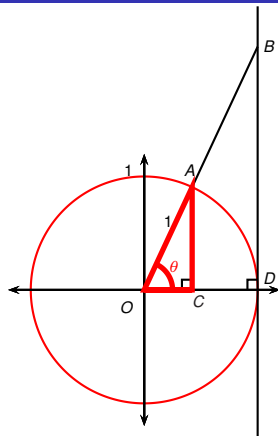
$$\tan \theta$$

$$\cot \theta$$

$$\sec \theta$$

$$\csc \theta$$

# Geometric interpretation of all trigonometric functions



Fix unit circle, center  $O$ , coordinates  $(0, 0)$ .  
 Let  $\angle DOB = \theta$ . Let  $OB$  intersect the circle at point  $A$ . Coordinates of  $A$  are  $(\cos \theta, \sin \theta)$ .

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|} = \frac{|AC|}{1} = |AC|$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

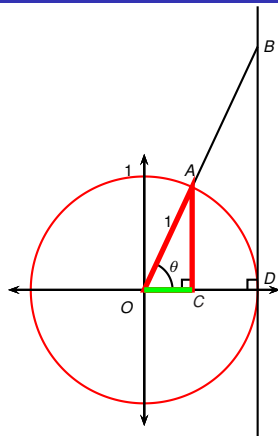
$$\tan \theta$$

$$\cot \theta$$

$$\sec \theta$$

$$\csc \theta$$

# Geometric interpretation of all trigonometric functions



Fix unit circle, center  $O$ , coordinates  $(0, 0)$ .  
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$$\tan \theta$$

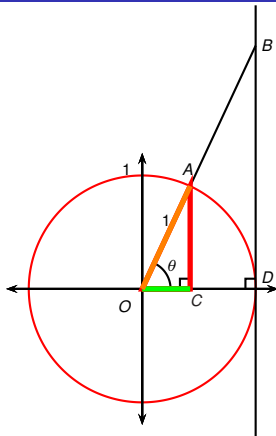
$$\cot \theta$$

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# Geometric interpretation of all trigonometric functions



Fix unit circle, center  $O$ , coordinates  $(0, 0)$ .  
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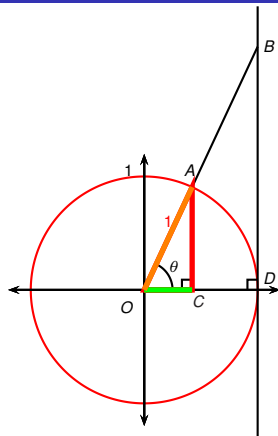
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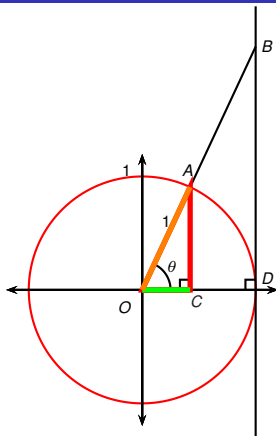
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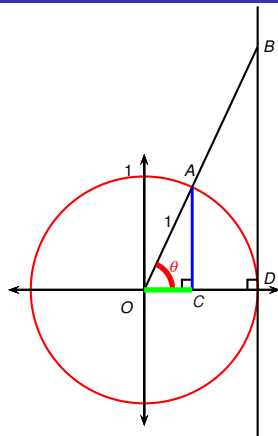
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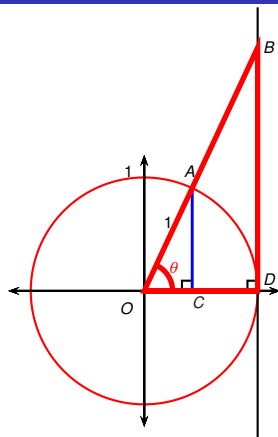
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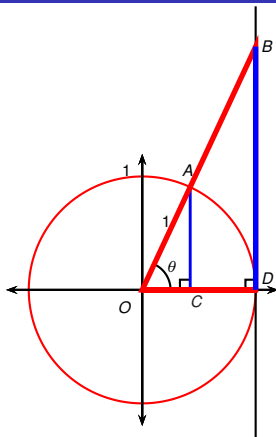
$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

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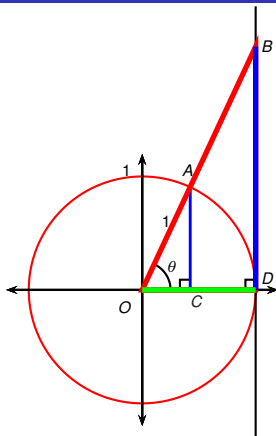
$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{|BD|}{|OD|}$$

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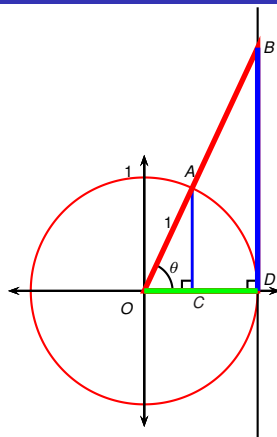
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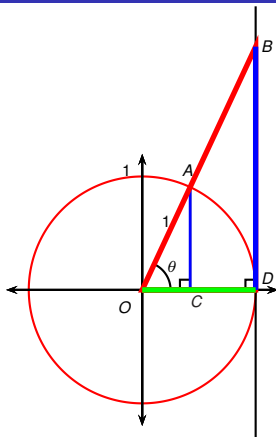
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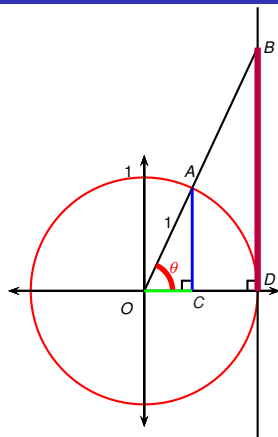
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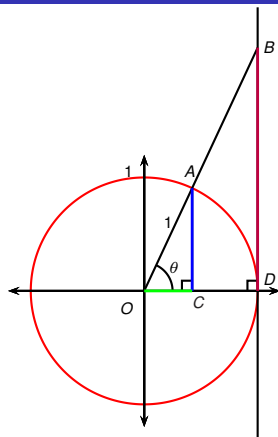
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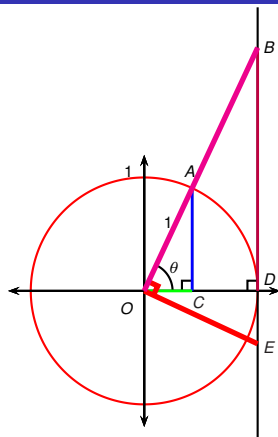
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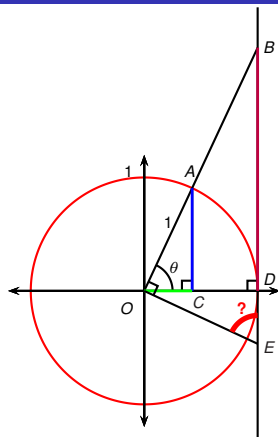
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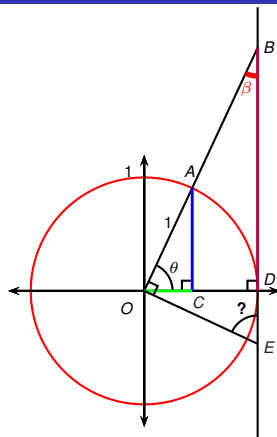
$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$

$$\sec \theta$$

$$\csc \theta$$

$$\angle OED = ?$$

# Geometric interpretation of all trigonometric functions



$$\beta = ?$$

$$\angle OED = ?$$

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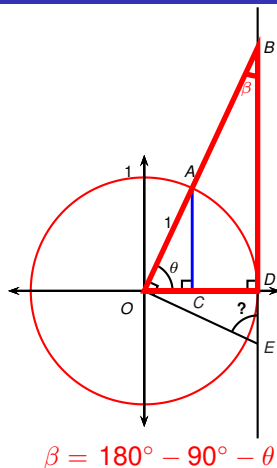
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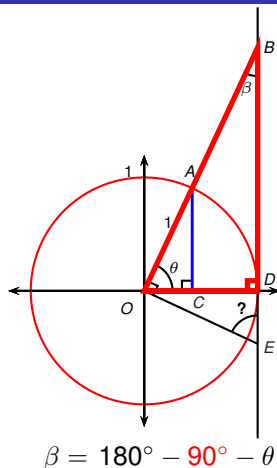
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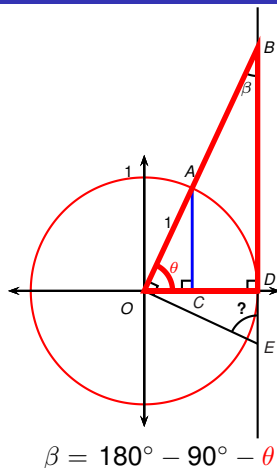
$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$

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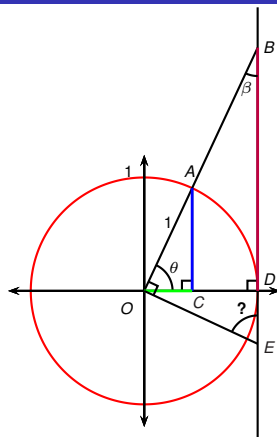
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# Geometric interpretation of all trigonometric functions



$$\begin{aligned}\beta &= 180^\circ - 90^\circ - \theta \\ &= 90^\circ - \theta\end{aligned}$$

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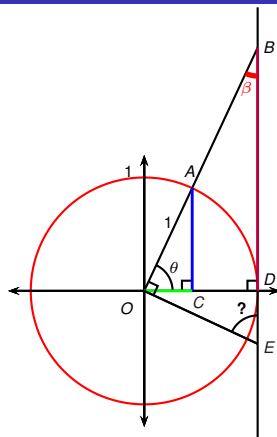
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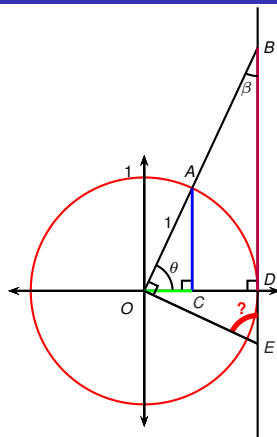
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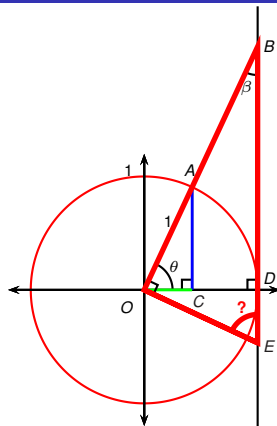
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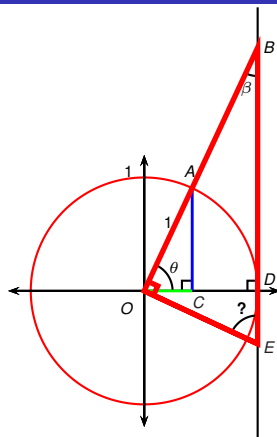
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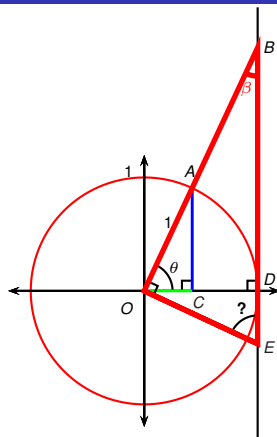
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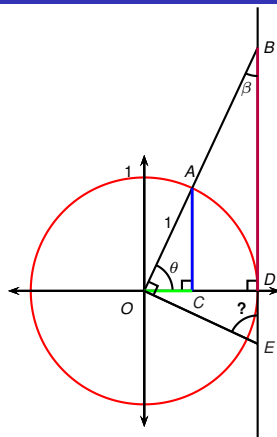
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$$\begin{aligned}\angle OED &= 180^\circ - 90^\circ - \beta \\ &= 90^\circ - (90^\circ - \theta)\end{aligned}$$

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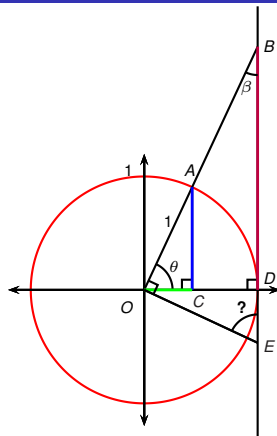
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# Geometric interpretation of all trigonometric functions



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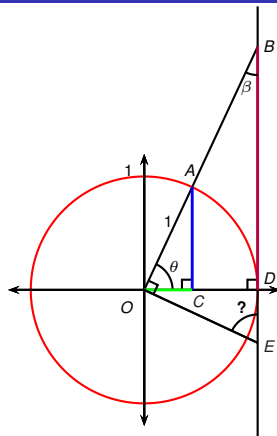
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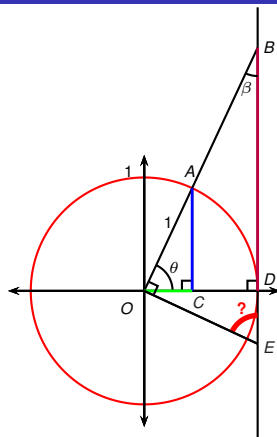
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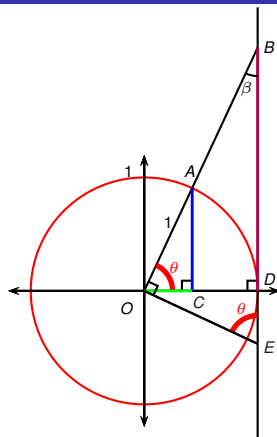
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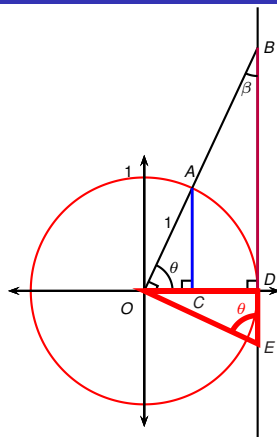
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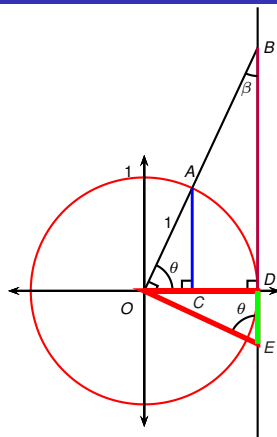
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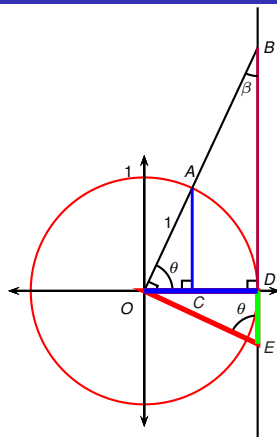
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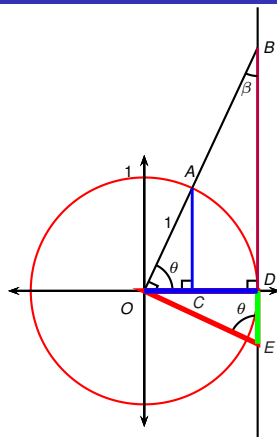
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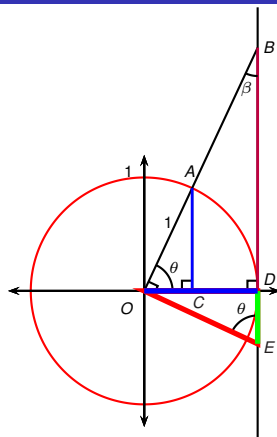
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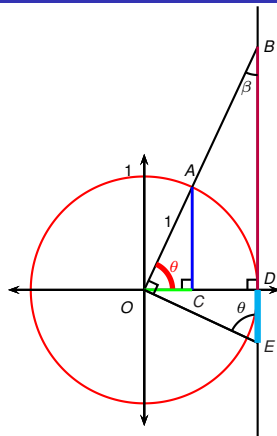
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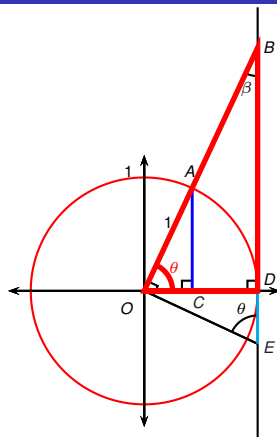
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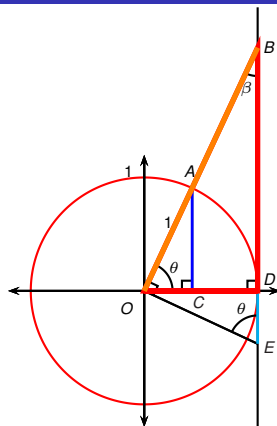
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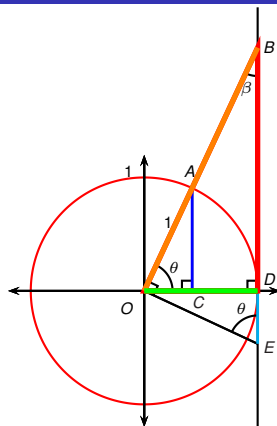
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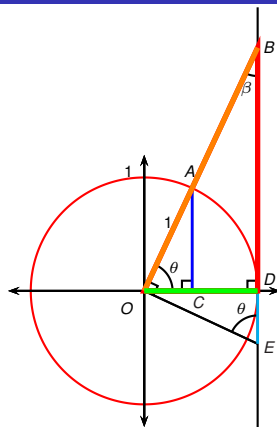
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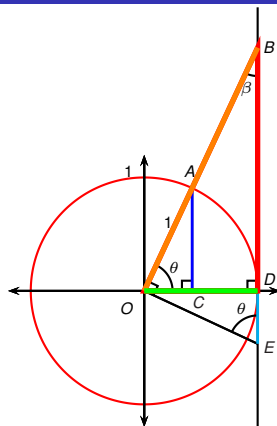
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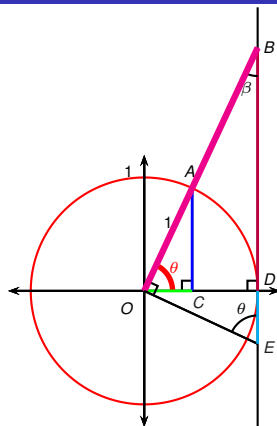
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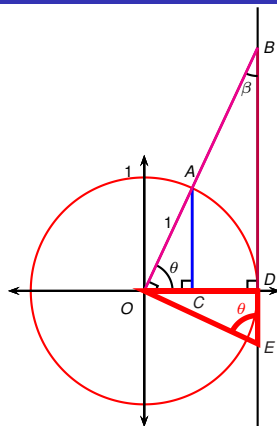
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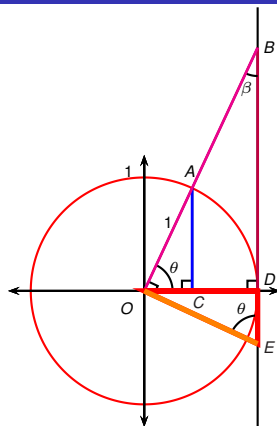
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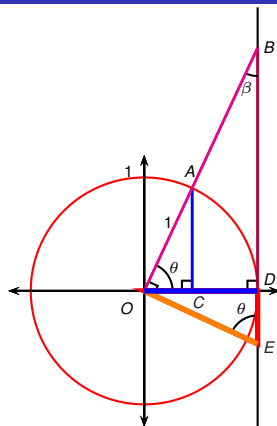
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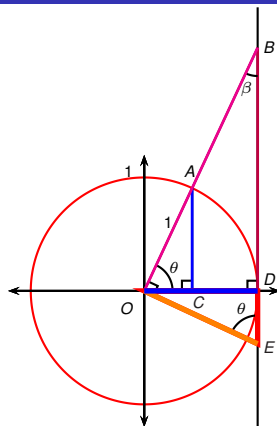
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$$\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{|OE|}{|DO|}$$

# Geometric interpretation of all trigonometric functions



$$\begin{aligned}\beta &= 180^\circ - 90^\circ - \theta \\ &= 90^\circ - \theta\end{aligned}$$

$$\begin{aligned}\angle OED &= 180^\circ - 90^\circ - \beta \\ &= 90^\circ - (90^\circ - \theta) \\ &= \theta\end{aligned}$$

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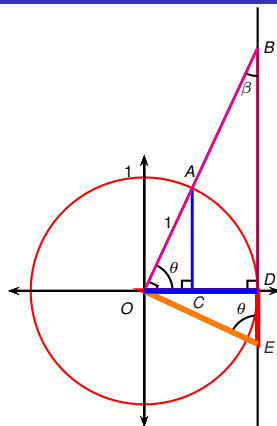
$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{|BD|}{|OD|} = \frac{|BD|}{1} = |BD|$$

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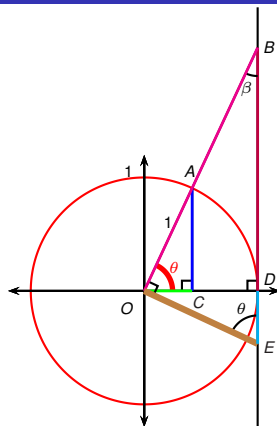
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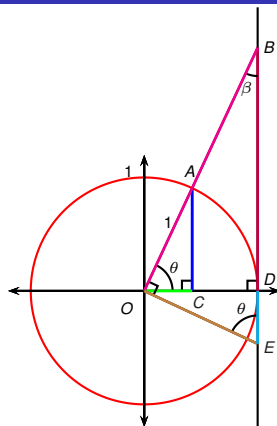
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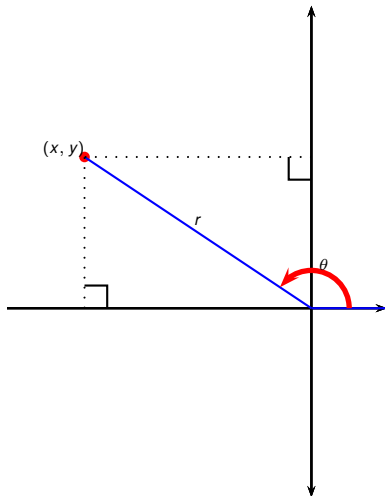
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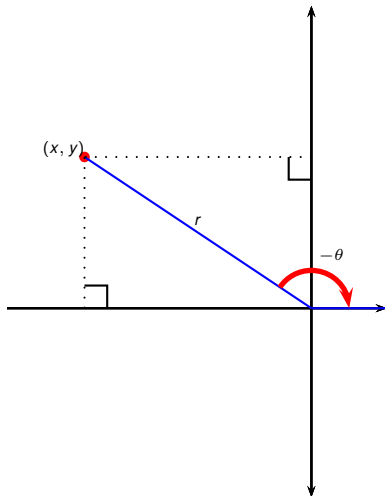
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- Positive angles are obtained by rotating counterclockwise.

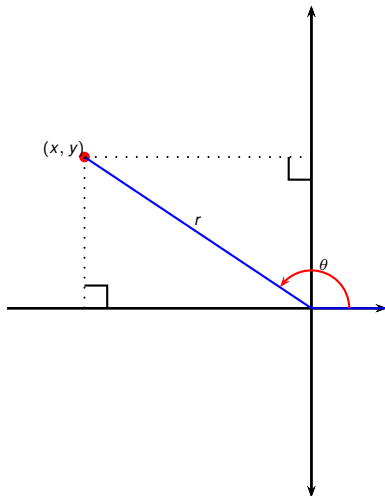
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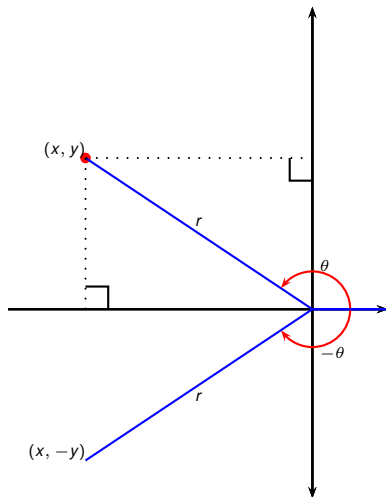
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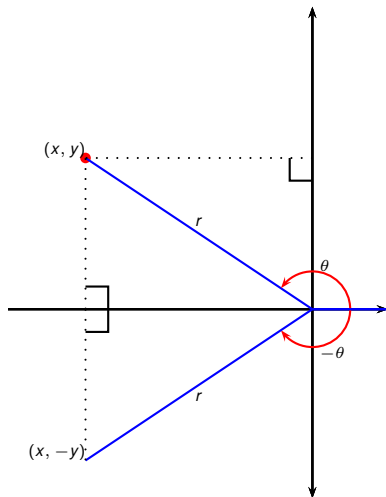
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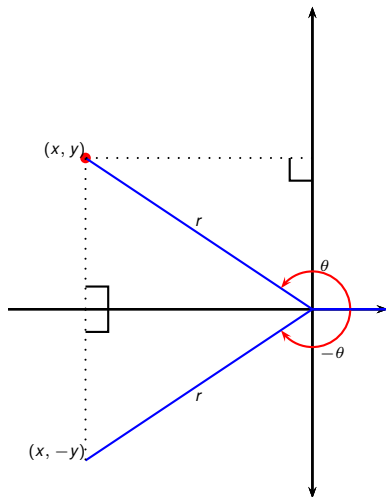
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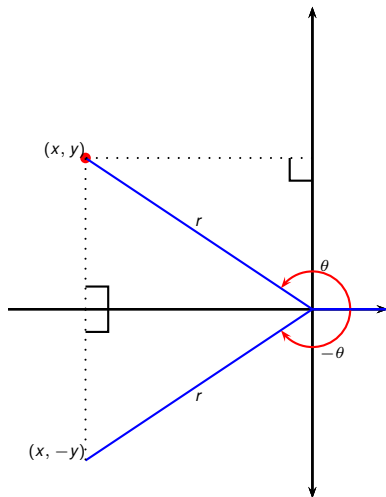
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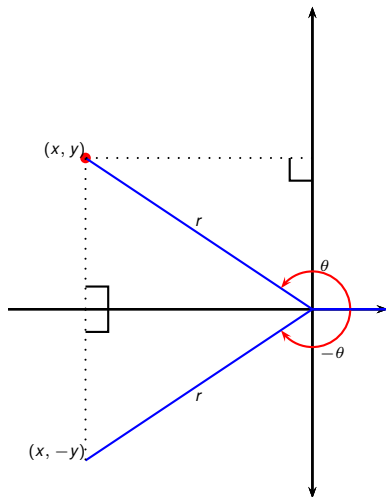
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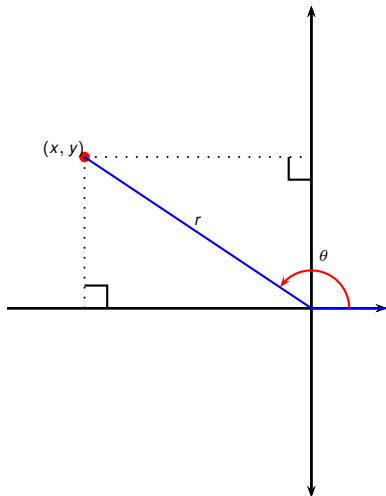
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- $\sin$  is an **odd function**.



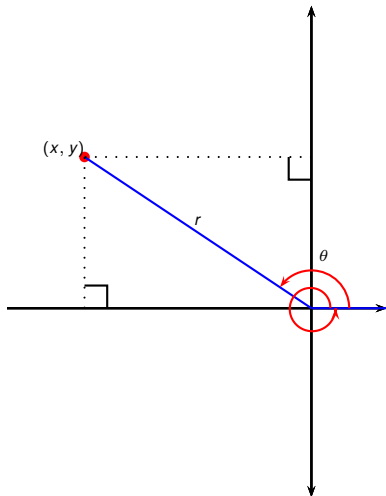
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- $\sin$  is an odd function.
- $\cos$  is an even function.



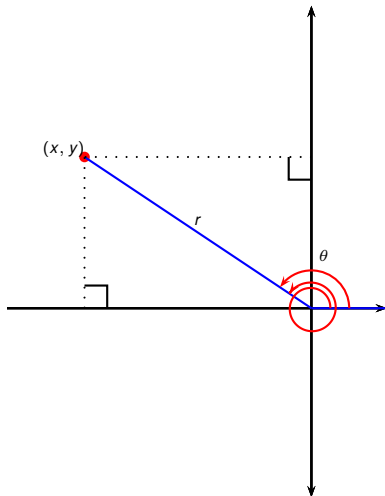
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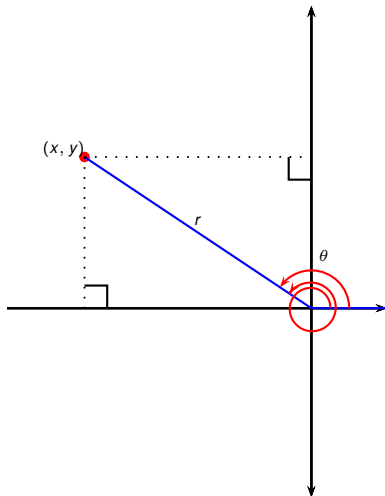
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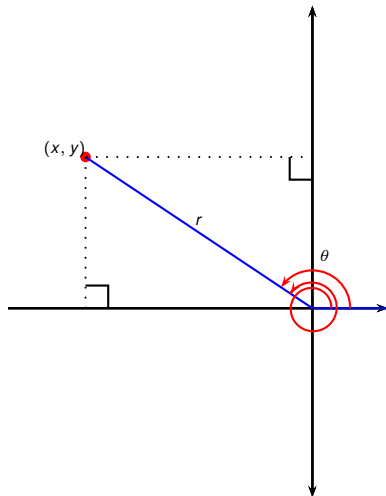
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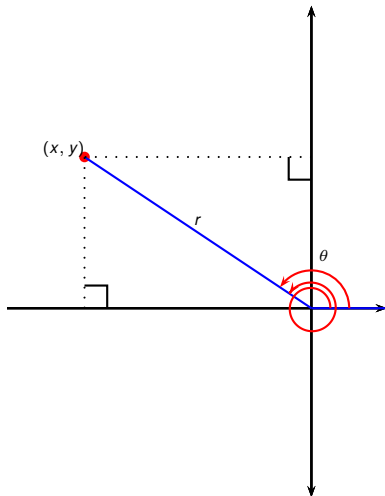
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- We say  $\sin$  and  $\cos$  are  $2\pi$ -periodic.

$$\begin{aligned} \sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y} \end{aligned}$$

# Trigonometric Identities

## Definition (Trigonometric Identity)

A trigonometric identity is an equality between the trigonometric functions in one or more variables that holds for all values of the involved variables in the domains of all of the expressions.

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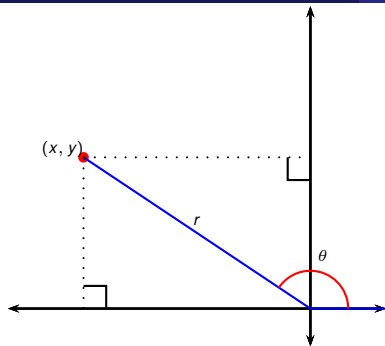
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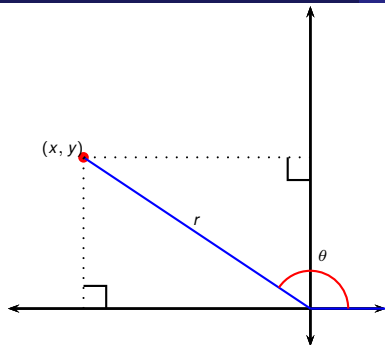
- By convention, when dealing with trigonometric identities we do not account for the domains of the involved expressions.
- For example,  $\frac{\sin \theta}{\sin \theta} = 1$  is considered a valid trigonometric identity, although, when considered as a function, the left hand side is not defined for  $\theta \neq 0$ .



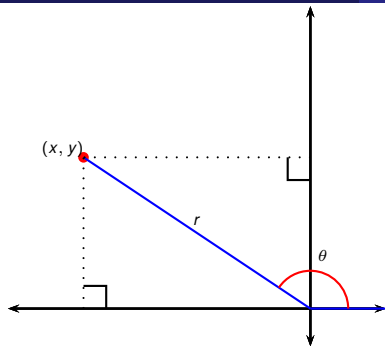


$$\begin{aligned}\sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$

- $\csc \theta = \frac{1}{\sin \theta}$
- $\sec \theta = \frac{1}{\cos \theta}$
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- $\tan \theta = \frac{\sin \theta}{\cos \theta}$
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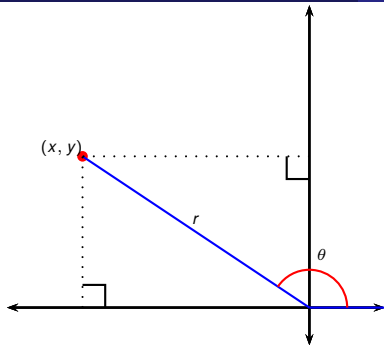


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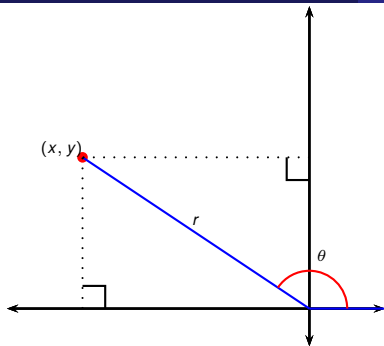
$$\sin^2 \theta + \cos^2 \theta$$

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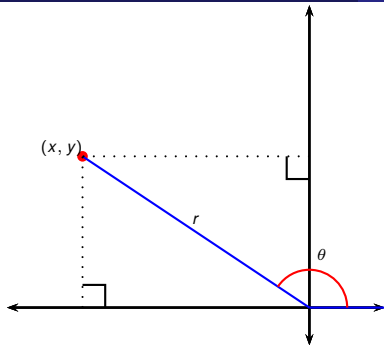
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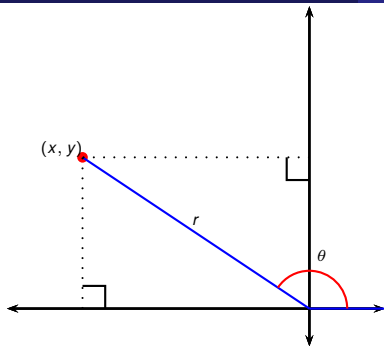
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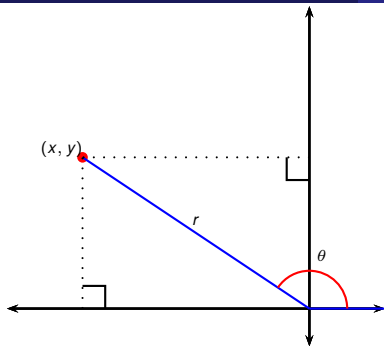
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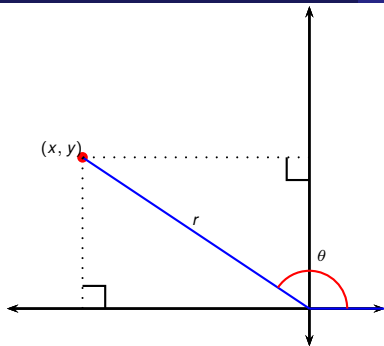


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$$\begin{aligned}& \sin^2 \theta + \cos^2 \theta \\ &= \frac{y^2}{r^2} + \frac{x^2}{r^2} \\ &= \frac{y^2 + x^2}{r^2} \\ &= \frac{r^2}{r^2} \\ &= 1\end{aligned}$$

Therefore  $\sin^2 \theta + \cos^2 \theta = 1$ .



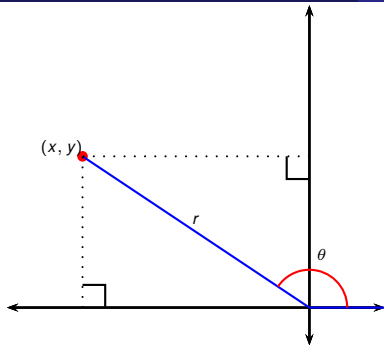


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### Example ( $\tan^2 \theta + 1 = \sec^2 \theta$ )

Prove the identity

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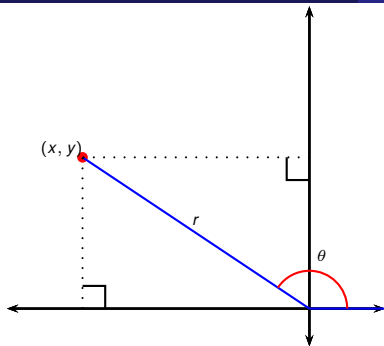
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Prove the identity

$$\tan^2 \theta + 1 = \sec^2 \theta.$$

$$\sin^2 \theta + \cos^2 \theta = 1$$



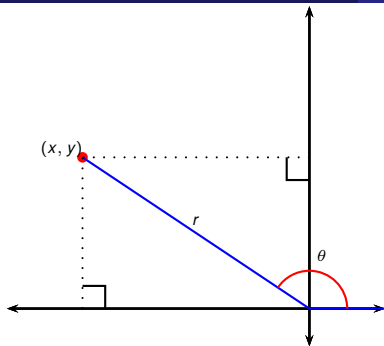
$$\begin{aligned}\sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$

### Example ( $\tan^2 \theta + 1 = \sec^2 \theta$ )

Prove the identity

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$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ \frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} &= \frac{1}{\cos^2 \theta}\end{aligned}$$



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