

# Precalculus

## Interval notation, the $\in$ and the $\cup$ symbols

Todor Milev

2019

# Outline

## 1 Interval notation

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- 2 Inequalities and interval notation

# Review of standard interval notation

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  - the “belongs to”  $\in$  sign;
  - the **union**  $\cup$  sign.

Let  $a < b$  be two real numbers.

### Definition (Interval notation)

The set (interval) of all real numbers from  $a$  to  $b$  is denoted as follows.


Notation	Endpoints	Picture
$[a, b]$		
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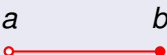
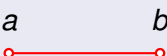
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Let  $a$  be a number.

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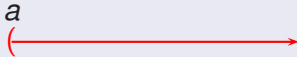
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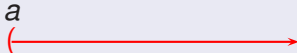
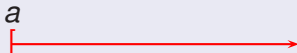
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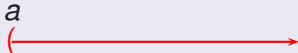


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Write the set of numbers  $x$  satisfying  $0 \leq x \leq 2$  in interval notation.

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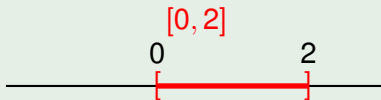
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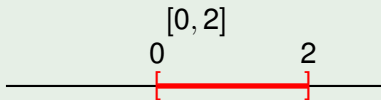
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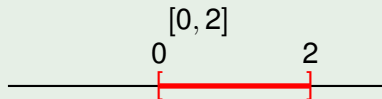
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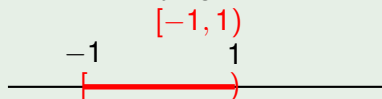
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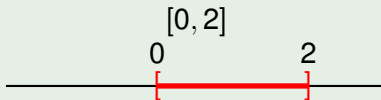


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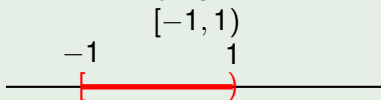
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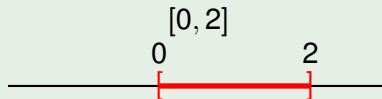
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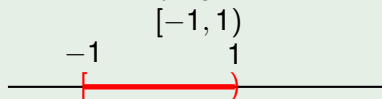
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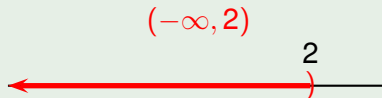
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- The union of  $A$  and  $B$  is the set consisting of the elements in  $A$  and the elements in  $B$ , without additional elements.
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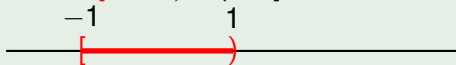
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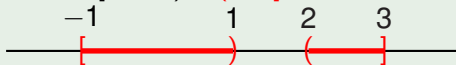
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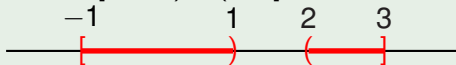
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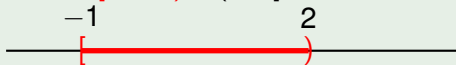
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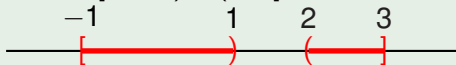
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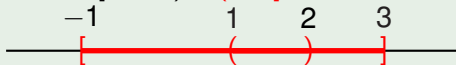
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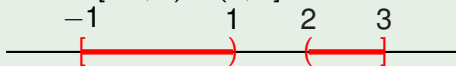
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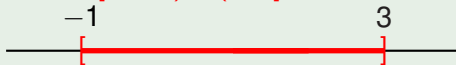
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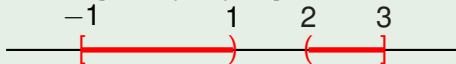
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- To draw the points of a union draw both on top of one another.



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