Calculus I
Derivative of $(a(x))^{b(x)}$

Todor Milev

2019

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right)$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\mathbf{?}}\quad\right)^{\tan x}\right)$$

Convert base to $e^{?}$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln x}\right)^{\tan x}\right)$$

Convert base to $e^?$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln x}\right)^{\tan x}\right)$$
$$= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{(\ln x)\tan x}\right)$$

Convert base to $e^?$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \Big| \text{ Convert base to } e^{?} \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{(\ln x)\tan x}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) && \Big| \text{ Set } (\ln x)\tan x = u
\end{aligned}$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \text{Convert base to } e^{?} \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{(\ln x)\tan x}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) && \text{Set } (\ln x)\tan x = u \\
&= \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \text{Chain rule}
\end{aligned}$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \text{Convert base to } e^{?} \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{(\ln x)\tan x}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) && \text{Set } (\ln x)\tan x = u \\
&= \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \text{Chain rule} \\
&= \mathbf{?}\ \frac{\mathrm{d}}{\mathrm{d}x}\left((\ln x)\tan x\right)
\end{aligned}$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \text{Convert base to } e^{?} \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{(\ln x)\tan x}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) && \text{Set } (\ln x)\tan x = u \\
&= {\color{red}\frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)}\frac{\mathrm{d}u}{\mathrm{d}x} && \text{Chain rule} \\
&= {\color{red}\mathbf{?}}\ \frac{\mathrm{d}}{\mathrm{d}x}\left((\ln x)\tan x\right)
\end{aligned}$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \text{Convert base to } e^{?}\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{(\ln x)\tan x}\right)\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) && \text{Set } (\ln x)\tan x = u\\
&= \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \text{Chain rule}\\
&= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left((\ln x)\tan x\right)
\end{aligned}
$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \text{Convert base to } e^{?} \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{(\ln x)\tan x}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) && \text{Set } (\ln x)\tan x = u \\
&= \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \text{Chain rule} \\
&= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left((\ln x)\tan x\right) \\
&= e^{(\ln x)\tan x}\left((\ln x)'\tan x + (\ln x)(\tan x)'\right)
\end{aligned}$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$
\begin{aligned}
\frac{d}{dx}\left(x^{\tan x}\right) &= \frac{d}{dx}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \text{Convert base to } e^{?} \\
&= \frac{d}{dx}\left(e^{(\ln x)\tan x}\right) \\
&= \frac{d}{dx}\left(e^{u}\right) && \text{Set } (\ln x)\tan x = u \\
&= \frac{d}{du}\left(e^{u}\right)\frac{du}{dx} && \text{Chain rule} \\
&= e^{u}\frac{d}{dx}\left((\ln x)\tan x\right) \\
&= e^{(\ln x)\tan x}\left((\ln x)'\tan x + (\ln x)(\tan x)'\right) && \text{Prod. rule}
\end{aligned}
$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \text{Convert base to } e^{?} \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{(\ln x)\tan x}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) && \text{Set } (\ln x)\tan x = u \\
&= \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \text{Chain rule} \\
&= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left((\ln x)\tan x\right) \\
&= e^{(\ln x)\tan x}\left((\ln x)'\tan x + (\ln x)(\tan x)'\right) && \text{Prod. rule}
\end{aligned}$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \Big| \text{ Convert base to } e^{?} \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{(\ln x)\tan x}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) && \Big| \text{ Set } (\ln x)\tan x = u \\
&= \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \Big| \text{ Chain rule} \\
&= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left((\ln x)\tan x\right) \\
&= e^{(\ln x)\tan x}\left((\ln x)'\tan x + (\ln x)(\tan x)'\right) && \Big| \text{ Prod. rule}
\end{aligned}$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \text{Convert base to } e^{?} \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{(\ln x)\tan x}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) && \text{Set } (\ln x)\tan x = u \\
&= \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \text{Chain rule} \\
&= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left((\ln x)\tan x\right) \\
&= e^{(\ln x)\tan x}\left((\ln x)'\tan x + (\ln x)(\tan x)'\right) && \text{Prod. rule} \\
&= x^{\tan x}\left(\mathbf{?}\ \tan x + (\ln x)\mathbf{?}\qquad\right)
\end{aligned}
$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \text{Convert base to } e^{?} \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{(\ln x)\tan x}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) && \text{Set } (\ln x)\tan x = u \\
&= \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \text{Chain rule} \\
&= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left((\ln x)\tan x\right) \\
&= e^{(\ln x)\tan x}\left({\color{red}(\ln x)'}\tan x + (\ln x)(\tan x)'\right) && \text{Prod. rule} \\
&= x^{\tan x}\left({\color{red}\mathbf{?}}\ \tan x + (\ln x)\mathbf{?}\qquad\right)
\end{aligned}$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \Big| \text{ Convert base to } e^{?}\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{(\ln x)\tan x}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) && \Big| \text{ Set } (\ln x)\tan x = u\\
&= \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \Big| \text{ Chain rule}\\
&= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left((\ln x)\tan x\right) \\
&= e^{(\ln x)\tan x}\left((\ln x)'\tan x + (\ln x)(\tan x)'\right) && \Big| \text{ Prod. rule}\\
&= x^{\tan x}\left(\frac{1}{x}\tan x + (\ln x)\mathbf{?} \qquad\right)
\end{aligned}$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$\begin{aligned}
\frac{d}{dx}\left(x^{\tan x}\right) &= \frac{d}{dx}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \text{Convert base to } e^{?} \\
&= \frac{d}{dx}\left(e^{(\ln x)\tan x}\right) \\
&= \frac{d}{dx}\left(e^{u}\right) && \text{Set } (\ln x)\tan x = u \\
&= \frac{d}{du}\left(e^{u}\right)\frac{du}{dx} && \text{Chain rule} \\
&= e^{u}\frac{d}{dx}\left((\ln x)\tan x\right) \\
&= e^{(\ln x)\tan x}\left((\ln x)'\tan x + (\ln x)\,(\tan x)'\right) && \text{Prod. rule} \\
&= x^{\tan x}\left(\frac{1}{x}\tan x + (\ln x)\,\mathbf{?}\qquad\right)
\end{aligned}$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{\tan x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \text{Convert base to } e^{?}\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{(\ln x)\tan x}\right)\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) && \text{Set } (\ln x)\tan x = u\\
&= \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \text{Chain rule}\\
&= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left((\ln x)\tan x\right)\\
&= e^{(\ln x)\tan x}\left((\ln x)'\tan x + (\ln x)\,{\color{red}(\tan x)'}\right) && \text{Prod. rule}\\
&= x^{\tan x}\left(\frac{1}{x}\tan x + (\ln x)\,{\color{red}\sec^2 x}\right)
\end{aligned}
$$

Example

Differentiate $x^{\tan x}$, where $x > 0$.

$$
\begin{aligned}
\frac{d}{dx}\left(x^{\tan x}\right) &= \frac{d}{dx}\left(\left(e^{\ln x}\right)^{\tan x}\right) && \text{Convert base to } e^{?} \\
&= \frac{d}{dx}\left(e^{(\ln x)\tan x}\right) \\
&= \frac{d}{dx}\left(e^{u}\right) && \text{Set } (\ln x)\tan x = u \\
&= \frac{d}{du}\left(e^{u}\right)\frac{du}{dx} && \text{Chain rule} \\
&= e^{u}\frac{d}{dx}\left((\ln x)\tan x\right) \\
&= e^{(\ln x)\tan x}\left((\ln x)'\tan x + (\ln x)(\tan x)'\right) && \text{Prod. rule} \\
&= x^{\tan x}\left(\frac{1}{x}\tan x + (\ln x)\sec^2 x\right)
\end{aligned}
$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right)$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\mathbf{?}} \qquad \right)^{\ln x}\right) \quad \Bigg| \text{ Convert base to } e^{?}$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right)\quad\Bigg|\ \text{Convert base to } e^{?}$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) && \Big|\ \text{Convert base to } e^{?} \\ &= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\ln(3x+1)\ln x}\right) \end{aligned}$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\begin{aligned}
\frac{d}{dx}\left((3x+1)^{\ln x}\right) &= \frac{d}{dx}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) && \Big|\ \text{Convert base to } e^?\\
&= \frac{d}{dx}\left(e^{\color{red}{\ln(3x+1)\ln x}}\right)\\
&= \frac{d}{dx}\left(e^{\color{red}{u}}\right) && \Big|\ \color{red}{\text{Set } \ln(3x+1)\ln x = u}
\end{aligned}$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) \quad \Big| \text{ Convert base to } e^?$$

$$= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\ln(3x+1)\ln x}\right)$$

$$= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) = \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} \quad \Big| \text{ Set } \ln(3x+1)\ln x = u$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\begin{aligned}
\frac{d}{dx}\left((3x+1)^{\ln x}\right) &= \frac{d}{dx}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) && \Big| \text{ Convert base to } e^{?}\\
&= \frac{d}{dx}\left(e^{\ln(3x+1)\ln x}\right)\\
&= \frac{d}{dx}\left(e^{u}\right) = \frac{d}{du}\left(e^{u}\right)\frac{du}{dx} && \Big| \text{ Set } \ln(3x+1)\ln x = u\\
&= \mathbf{?}\ \frac{d}{dx}\left(\ln(3x+1)\ln x\right)
\end{aligned}$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) && \Big| \text{ Convert base to } e^{?} \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\ln(3x+1)\ln x}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) = \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \Big| \text{ Set } \ln(3x+1)\ln x = u \\
&= \textbf{?}\ \frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(3x+1)\ln x\right)
\end{aligned}$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) && \Big| \text{ Convert base to } e^{?}\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\ln(3x+1)\ln x}\right)\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) = \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \Big| \text{ Set } \ln(3x+1)\ln x = u\\
&= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(3x+1)\ln x\right)
\end{aligned}$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) && \Bigg| \text{ Convert base to } e^{?}\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\ln(3x+1)\ln x}\right)\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) = \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \Bigg| \text{ Set } \ln(3x+1)\ln x = u\\
&= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(3x+1)\ln x\right)\\
&= e^{\ln(3x+1)\ln x}\left((\ln(3x+1))'\ln x + \ln(3x+1)\left(\ln x\right)'\right)
\end{aligned}
$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) && \Big|\ \text{Convert base to } e^? \\ &= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\ln(3x+1)\ln x}\right) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) = \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \Big|\ \text{Set } \ln(3x+1)\ln x = u \\ &= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(3x+1)\ln x\right) \\ &= e^{\ln(3x+1)\ln x}\left(\left(\ln(3x+1)\right)'\ln x + \ln(3x+1)\left(\ln x\right)'\right)\end{aligned}$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) && \Big|\ \text{Convert base to } e^?\\ &= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\ln(3x+1)\ln x}\right)\\ &= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) = \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \Big|\ \text{Set } \ln(3x+1)\ln x = u\\ &= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(3x+1)\ln x\right)\\ &= e^{\ln(3x+1)\ln x}\left((\ln(3x+1))'\ln x + \ln(3x+1)(\ln x)'\right)\end{aligned}$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) && \Big| \text{ Convert base to } e^{?}\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\ln(3x+1)\ln x}\right)\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) = \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \Big| \text{ Set } \ln(3x+1)\ln x = u\\
&= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(3x+1)\ln x\right)\\
&= e^{\ln(3x+1)\ln x}\left((\ln(3x+1))'\ln x + \ln(3x+1)\left(\ln x\right)'\right)
\end{aligned}$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) && \Big|\ \text{Convert base to } e^{?} \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\ln(3x+1)\ln x}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) = \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \Big|\ \text{Set } \ln(3x+1)\ln x = u \\
&= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(3x+1)\ln x\right) \\
&= e^{\ln(3x+1)\ln x}\left((\ln(3x+1))'\ln x + \ln(3x+1)(\ln x)'\right) \\
&= (3x+1)^{\ln x}\left(\mathbf{?} \qquad \ln x + \ln(3x+1)\mathbf{?}\right)
\end{aligned}$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) \quad \Big| \text{ Convert base to } e^{?}$$

$$= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\ln(3x+1)\ln x}\right)$$

$$= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) = \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} \quad \Big| \text{ Set } \ln(3x+1)\ln x = u$$

$$= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(3x+1)\ln x\right)$$

$$= e^{\ln(3x+1)\ln x}\left((\ln(3x+1))'\ln x + \ln(3x+1)(\ln x)'\right)$$

$$= (3x+1)^{\ln x}\left(\mathbf{?} \qquad \ln x + \ln(3x+1)\mathbf{?}\right)$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right)$$

Convert base to e?

$$= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\ln(3x+1)\ln x}\right)$$

$$= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) = \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x}$$

Set $\ln(3x+1)\ln x = u$

$$= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(3x+1)\ln x\right)$$

$$= e^{\ln(3x+1)\ln x}\left((\ln(3x+1))'\ln x + \ln(3x+1)(\ln x)'\right)$$

$$= (3x+1)^{\ln x}\left(\frac{(3x+1)'}{3x+1}\ln x + \ln(3x+1)\textbf{?}\right)$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) && \Big| \text{ Convert base to } e^{?} \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\ln(3x+1)\ln x}\right) \\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) = \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \Big| \text{ Set } \ln(3x+1)\ln x = u \\
&= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(3x+1)\ln x\right) \\
&= e^{\ln(3x+1)\ln x}\left((\ln(3x+1))'\ln x + \ln(3x+1)\color{red}{(\ln x)'}\right) \\
&= (3x+1)^{\ln x}\left(\frac{(3x+1)'}{3x+1}\ln x + \ln(3x+1)\color{red}{\mathbf{?}}\right)
\end{aligned}
$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) && \Big| \text{ Convert base to } e^{?}\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\ln(3x+1)\ln x}\right)\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{u}\right) = \frac{\mathrm{d}}{\mathrm{d}u}\left(e^{u}\right)\frac{\mathrm{d}u}{\mathrm{d}x} && \Big| \text{ Set } \ln(3x+1)\ln x = u\\
&= e^{u}\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(3x+1)\ln x\right)\\
&= e^{\ln(3x+1)\ln x}\left((\ln(3x+1))'\ln x + \ln(3x+1)\,{\color{red}(\ln x)'}\right)\\
&= (3x+1)^{\ln x}\left(\frac{(3x+1)'}{3x+1}\ln x + \ln(3x+1)\,{\color{red}\frac{1}{x}}\right)
\end{aligned}$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{d}{dx}\left((3x+1)^{\ln x}\right) = \frac{d}{dx}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) \quad \Big| \text{ Convert base to } e^?$$

$$= \frac{d}{dx}\left(e^{\ln(3x+1)\ln x}\right)$$

$$= \frac{d}{dx}\left(e^{u}\right) = \frac{d}{du}\left(e^{u}\right)\frac{du}{dx} \quad \Big| \text{ Set } \ln(3x+1)\ln x = u$$

$$= e^{u}\frac{d}{dx}\left(\ln(3x+1)\ln x\right)$$

$$= e^{\ln(3x+1)\ln x}\left((\ln(3x+1))'\ln x + \ln(3x+1)(\ln x)'\right)$$

$$= (3x+1)^{\ln x}\left(\frac{(3x+1)'}{3x+1}\ln x + \ln(3x+1)\frac{1}{x}\right)$$

$$= (3x+1)^{\ln x}\left(\frac{3\ln x}{3x+1} + \ln(3x+1)\frac{1}{x}\right)$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$
\begin{aligned}
\frac{d}{dx}\left((3x+1)^{\ln x}\right) &= \frac{d}{dx}\left(\left(e^{\ln(3x+1)}\right)^{\ln x}\right) && \Big| \text{ Convert base to } e^? \\
&= \frac{d}{dx}\left(e^{\ln(3x+1)\ln x}\right) \\
&= \frac{d}{dx}\left(e^{u}\right) = \frac{d}{du}\left(e^{u}\right)\frac{du}{dx} && \Big| \text{ Set } \ln(3x+1)\ln x = u \\
&= e^{u}\frac{d}{dx}\left(\ln(3x+1)\ln x\right) \\
&= e^{\ln(3x+1)\ln x}\left((\ln(3x+1))'\ln x + \ln(3x+1)(\ln x)'\right) \\
&= (3x+1)^{\ln x}\left(\frac{(3x+1)'}{3x+1}\ln x + \ln(3x+1)\frac{1}{x}\right) \\
&= (3x+1)^{\ln x}\left(\frac{3\ln x}{3x+1} + \ln(3x+1)\frac{1}{x}\right)
\end{aligned}
$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) = (3x+1)^{\ln x}\left(\frac{3\ln x}{3x+1} + \ln(3x+1)\frac{1}{x}\right)$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) = (3x+1)^{\ln x}\left(\frac{3\ln x}{3x+1} + \ln(3x+1)\frac{1}{x}\right)$$

Theorem

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((a(x))^{b(x)}\right) = (a(x))^{b(x)}\left(\frac{a'(x)}{a(x)}b(x) + \ln(a(x))b'(x)\right), \quad a(x)>0$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{d}{dx}\left((3x+1)^{\ln x}\right) = (3x+1)^{\ln x}\left(\frac{3\ln x}{3x+1} + \ln(3x+1)\frac{1}{x}\right)$$

Theorem

$$\frac{d}{dx}\left((a(x))^{b(x)}\right) = (a(x))^{b(x)}\left(\frac{a'(x)}{a(x)}b(x) + \ln(a(x))b'(x)\right), \quad a(x)>0$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) = (3x+1)^{\ln x}\left(\frac{3\ln x}{3x+1} + \ln(3x+1)\frac{1}{x}\right)$$

Theorem

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((a(x))^{b(x)}\right) = (a(x))^{b(x)}\left(\frac{a'(x)}{a(x)}b(x) + \ln(a(x))b'(x)\right), \quad a(x) > 0$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{d}{dx}\left((3x+1)^{\ln x}\right) = (3x+1)^{\ln x}\left(\frac{3\ln x}{3x+1} + \ln(3x+1)\frac{1}{x}\right)$$

Theorem

$$\frac{d}{dx}\left((a(x))^{b(x)}\right) = (a(x))^{b(x)}\left(\frac{a'(x)}{a(x)}b(x) + \ln(a(x))b'(x)\right), \quad a(x) > 0$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) = (3x+1)^{\ln x}\left(\frac{3\ln x}{3x+1} + \ln(3x+1)\frac{1}{x}\right)$$

Theorem

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((a(x))^{b(x)}\right) = (a(x))^{b(x)}\left(\frac{a'(x)}{a(x)}b(x) + \ln(a(x))b'(x)\right), \quad a(x)>0$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) = (3x+1)^{\ln x}\left(\frac{3\ln x}{3x+1} + \ln(3x+1)\frac{1}{x}\right)$$

Theorem

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((a(x))^{b(x)}\right) = (a(x))^{b(x)}\left(\frac{a'(x)}{a(x)}b(x) + \ln(a(x))b'(x)\right), \quad a(x)>0$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) = (3x+1)^{\ln x}\left(\frac{3\ln x}{3x+1} + \ln(3x+1)\frac{1}{x}\right)$$

Theorem

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((a(x))^{b(x)}\right) = (a(x))^{b(x)}\left(\frac{a'(x)}{a(x)}b(x) + \ln(a(x))b'(x)\right), \quad a(x)>0$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) = (3x+1)^{\ln x}\left(\frac{3\ln x}{3x+1} + \ln(3x+1)\frac{1}{x}\right)$$

Theorem

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((a(x))^{b(x)}\right) = (a(x))^{b(x)}\left(\frac{a'(x)}{a(x)}b(x) + \ln(a(x))b'(x)\right), \quad a(x)>0$$

Example

Differentiate $(3x+1)^{\ln x}$, where $3x+1>0$.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((3x+1)^{\ln x}\right) = (3x+1)^{\ln x}\left(\frac{3\ln x}{3x+1} + \ln(3x+1)\frac{1}{x}\right)$$

Theorem

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((a(x))^{b(x)}\right) = (a(x))^{b(x)}\left(\frac{a'(x)}{a(x)}b(x) + \ln(a(x))b'(x)\right), \quad a(x)>0$$