Precalculus Computing sine, cosine of a half-angle

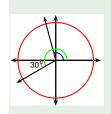
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Recall the half angle formula $\cos \alpha = \pm \sqrt{\frac{1 + \cos(2\alpha)}{2}}$.

Example

Using radicals, find the exact value of the trigonometric expression.



$$\begin{aligned} \cos 105^{\circ} &= \pm \sqrt{\frac{1 + \cos{(2 \cdot 105^{\circ})}}{2}} & \left| \cos 105^{\circ} < 0 \right. \\ &= -\sqrt{\frac{1 + \cos{(210^{\circ})}}{2}} \\ &= -\sqrt{\frac{1 - \cos{(30^{\circ})}}{2}} \\ &= -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2 - \sqrt{3}}{2 \cdot 2}} \\ &= -\frac{\sqrt{2 - \sqrt{3}}}{2} \end{aligned}$$