

Precalculus

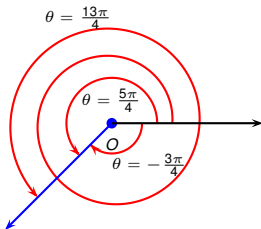
Find all angles coterminal to a given one

Todor Milev

2019

Definition (Coterminal Angles)

Two angles (angle measures) are called coterminal if the corresponding geometric angles have the same initial and terminal sides.



Observation

The set of angles coterminal with α consists of the angles $\alpha + 2k\pi$, where k runs over the set of integers. In other words, the angles coterminal with α are the angles:

$$\dots, \alpha - 6\pi, \alpha - 4\pi, \alpha - 2\pi, \alpha, \alpha + 2\pi, \alpha + 4\pi, \alpha + 6\pi, \dots$$

Example

- Find all angles that are coterminal to $\frac{\pi}{4}$.
- Find all angles in the interval $[-2\pi, \pi]$ that are coterminal to $\frac{\pi}{4}$.

By theory, the angles coterminal with $\frac{\pi}{4}$ are all angles of the form

$$\frac{\pi}{4} + 2k\pi.$$

To find which among the angles $\frac{\pi}{4} + 2k\pi$ lie in the interval $[-2\pi, \pi]$, we write them as an infinite list (we indicate the unboundedness of the list by ellipsis dots) and cross out the angles that lie outside of the desired interval.

$$\cancel{\dots}, \cancel{\frac{\pi}{4} - 4\pi}, \cancel{\frac{\pi}{4} - 2\pi}, \frac{\pi}{4}, \cancel{\frac{\pi}{4} + 2\pi}, \cancel{\frac{\pi}{4} + 4\pi}, \dots$$

Our final answer is $-\frac{7\pi}{4}, \frac{\pi}{4}$