Calculus II
Integrals of the form $\int \sin^n x \cos^m x \mathrm{d}x$, theory

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When n – odd:

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When m – odd:

$$\int \sin^m x \cos^n x dx = \int \sin^m x \cos^{n-1} x d(\sin x)$$

When n – odd:
$\cos x dx$
$= d(\sin x)$

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$$= \int \sin^m x \left(1 - \sin^2 x\right)^{\frac{n-1}{2}} d(\sin x)$$

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When m – odd:

$\int \sin^m x \cos^n x \mathrm{d}x = \int \sin^m x \cos^{n-1} x \mathrm{d}(\sin x)$	When n – odd: $\cos x \mathrm{d}x = \mathrm{d}(\sin x)$
$= \int \sin^m x \left(1 - \sin^2 x\right)^{\frac{n-1}{2}} \mathrm{d}(\sin x)$	Express $\cos x$ via $\sin x$
$= \int u^m \left(1 - u^2\right)^{\frac{n-1}{2}} \mathrm{d}u$	Set $\sin x = u$
$\int \sin^m x \cos^n x \mathrm{d}x = \int \sin^{m-1} x \cos^n x \mathrm{d}(-\cos x)$	When m – odd: $\sin x \mathrm{d}x = \mathrm{d}(-\cos x)$

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$$= -\int \left(1 - \cos^2 x\right)^{\frac{m-1}{2}} \cos^n x d(\cos x)$$

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When n – odd: $\cos x dx = d(\sin x)$

Express $\cos x$ via $\sin x$

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When m – odd: $\sin x dx = d(-\cos x)$

Express $\cos x$ via $\sin x$

$$\begin{aligned}\int \sin^m x \cos^n x dx &= \int \sin^m x \cos^{n-1} x d(\sin x)\\ &= \int \sin^m x\left(1-\sin^2 x\right)^{\frac{n-1}{2}} d(\sin x)\\ &= \int u^m \left(1-u^2\right)^{\frac{n-1}{2}} du\end{aligned}$$	When n – odd: $\cos x dx = d(\sin x)$ Express $\cos x$ via $\sin x$. Set $\sin x = u$
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If both m, n- even,

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When $n-$ odd: $\cos x dx = d(\sin x)$

Express $\cos x$ via $\sin x$

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When $m-$ odd: $\sin x dx = d(-\cos x)$

Express $\cos x$ via $\sin x$

Set $\cos x = u$

If both m, n- even, use $\left|\begin{array}{lcl} \sin^2 x & = & \frac{1-\cos(2x)}{2} \\ \cos^2 x & = & \frac{\cos(2x)+1}{2} \end{array}\right.$ and substitute $s = 2x$ to lower trig powers. Repeat above considerations.

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