Calculus I
Implicit derivatives involving trigonometry, part 1

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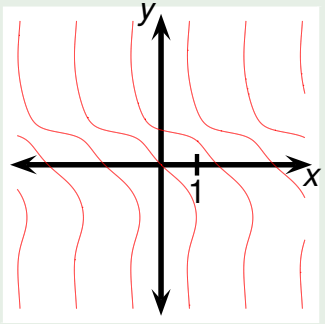
Example



Find y' as an expression of x and y.

$$\sin(2(x+y)) = y^2\cos(2x).$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sin(2(x+y))) = \frac{\mathrm{d}}{\mathrm{d}x}(y^2\cos(2x))$$

$$\cos(2(x+y))\frac{\mathrm{d}}{\mathrm{d}x}(2(x+y)) = \frac{\mathrm{d}}{\mathrm{d}x}(y^2)\cos(2x) + (y^2)\frac{\mathrm{d}}{\mathrm{d}x}(\cos(2x))$$

$$\cos(2(x+y))\left(2+2y'\right) = 2yy'\cos(2x) + y^2(-\sin(2x))\frac{\mathrm{d}}{\mathrm{d}x}(2x)$$

$$2\cos(2(x+y))\left(1+y'\right) = 2yy'\cos(2x) - y^2\sin(2x)2$$

$$\cos(2(x+y)) + y'\cos(2(x+y)) = yy'\cos(2x) - y^2\sin(2x)$$

$$y'\cos(2(x+y)) - yy'\cos(2x) = -\cos(2(x+y)) - y^2\sin(2x)$$

$$y'(\cos(2(x+y)) - y\cos(2x)) = -\cos(2(x+y)) - y^2\sin(2x)$$

$$y' = \frac{-\cos(2(x+y)) - y^2\sin(2x)}{\cos(2(x+y)) - y\cos(2x)}.$$