Precalculus
Polynomial inequalities

Todor Milev

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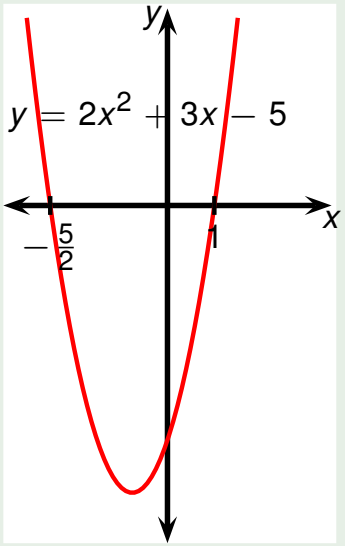
Outline

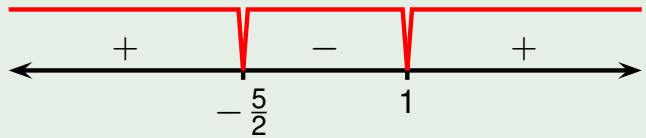
Example



Solve the inequality.

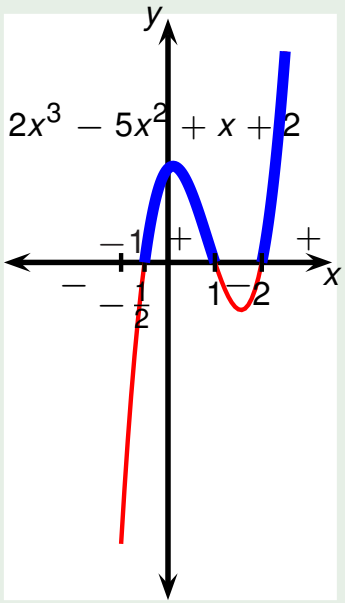
$$\begin{aligned} 2x^2 + 3x - 5 &\geq 0 \\ (2x+5)(x-1) &\geq 0 \\ x \in \left(-\infty, -\tfrac{5}{2}\right] \cup [1, \infty) & \end{aligned}$$

Left hand side vanishes when $x = -\frac{5}{2}$ and when $x = 1$.
The two roots split the real line into three intervals:
$\left(-\infty, -\frac{5}{2}\right), \left(-\frac{5}{2}, 1\right), (1, \infty)$.



Interval	Factor signs	Final sign	Sample pt	Value at sample pt
$\left(-\infty, -\frac{5}{2}\right)$	$(-)(-)$	$+$	-100	$f(-100) > 0$
$\left(-\frac{5}{2}, 1\right)$	$(+)(-)$	$-$	0	$f(0) = -5 < 0$
$(1, \infty)$	$(+)(+)$	$+$	100	$f(100) > 0$

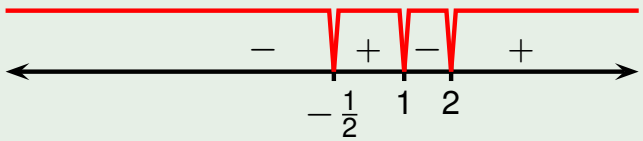
Example



Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality.

$$\begin{aligned} 2x^3 - 5x^2 + x + 2 &> 0 \\ 2\left(x - \left(-\tfrac{1}{2}\right)\right)(x-1)(x-2) &> 0 \\ x \in \left(-\tfrac{1}{2}, 1\right) \cup (2, \infty) & \end{aligned}$$

Left hand side vanishes when $x = -\frac{1}{2}$, when $x = 1$ and when $x = 2$. The two roots split the real line into four intervals: $\left(-\infty, -\frac{1}{2}\right), \left(-\frac{1}{2}, 1\right), (1, 2), (2, \infty)$.



Interval	Factor signs	Final sign from plot
$\left(-\infty, -\frac{1}{2}\right)$	$(-)(-)(-)$	$-$
$\left(-\frac{1}{2}, 1\right)$	$(+)(-)(-)$	$+$
$(1, 2)$	$(+)(+)(-)$	$-$
$(2, \infty)$	$(+)(+)(+)$	$+$