

Precalculus

Homework

Trig cofunction identities and angle-sum formulas

1. Use the known values of $\sin 30^\circ$, $\cos 30^\circ$, $\sin 45^\circ$, $\cos 45^\circ$, $\sin 60^\circ$, $\cos 60^\circ$, \dots , the angle sum formulas and the cofunction identities to find an exact value (using radicals) for the trigonometric function.

(a) The six trigonometric functions of $105^\circ = 45^\circ + 60^\circ$:

• $\sin(105^\circ)$.

• $\cos(105^\circ)$. Should your answer be a positive or a negative number?

• $\tan(105^\circ)$.

• $\cot(105^\circ)$.

• $\sec(105^\circ)$.

• $\csc(105^\circ)$.

(b) The six trigonometric functions of $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$:

• $\sin\left(\frac{\pi}{12}\right)$.

• $\cos\left(\frac{\pi}{12}\right)$. Should $\sin\left(\frac{\pi}{12}\right)$ be larger or smaller than $\cos\left(\frac{\pi}{12}\right)$?

• $\tan\left(\frac{\pi}{12}\right)$.

• $\cot\left(\frac{\pi}{12}\right)$.

• $\sec\left(\frac{\pi}{12}\right)$.

• $\csc\left(\frac{\pi}{12}\right)$.

2. Simplify to a trigonometric function of the angle θ . The answer key has not been proofread, use with caution.

(a) $\sin\left(\frac{\pi}{2} - \theta\right)$.

(b) $\cos\left(\frac{13\pi}{2} - \theta\right)$.

(c) $\tan(\pi - \theta)$

(d) $\cot\left(\frac{3\pi}{2} - \theta\right)$

(e) $\csc\left(\frac{3\pi}{2} + \theta\right)$

3. Using the power-reducing formulas, rewrite the expression in terms of first powers of the cosines and sines of multiples of the angle θ .

(a) $\sin^4 \theta$.

(b) $\cos^4 \theta$.

(c) $\sin^6 \theta$.

(d) $\cos^6 \theta$.

$$\frac{91}{5} + (\theta 2) \cos \frac{2\pi}{5} + (\theta 4) \cos \frac{4\pi}{5} + (\theta 9) \cos \frac{6\pi}{5} = \theta 9 \cos \frac{8\pi}{5} \quad \text{ANSWER}$$

4. Use the sum-to-product formulas to find all solutions of the trigonometric equation in the interval $[0, 2\pi)$.

Please note that typing a query such as “solve(sin(x)+sin (3x)=0)” at www.wolframalpha.com will provide you with a correct answer and a function plot.

(a) $\sin(x) + \sin(3x) = 0$.

$$\frac{2}{\pi 5}, \frac{2}{\pi}, \frac{6}{\pi}, 0 = x \quad \text{ANSWER}$$

(b) $\cos(x) + \cos(-3x) = 0$.

$$\frac{\pi}{12}, \frac{2\pi}{5}, \frac{\pi}{12}, \pi, \frac{\pi}{12}, \frac{2\pi}{5}, \frac{\pi}{12} = x \quad \text{ANSWER}$$

(c) $\sin(x) - \sin(3x) = 0$.

$$\frac{\pi}{12}, \frac{\pi}{12}, \pi, \frac{\pi}{12}, \frac{\pi}{12}, 0 = x \quad \text{ANSWER}$$

(d) $\cos(2x) - \cos(3x) = 0$.

$$\frac{9}{\pi 8}, \frac{9}{\pi 9}, \frac{9}{\pi 4}, \pi, \frac{9}{\pi 4}, 0 = x \quad \text{ANSWER}$$