

Precalculus

Express $\sin(kx)$, $\cos(kx)$ **via** $\sin x$, $\cos x$

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Recall the formulas

$$\begin{aligned}\sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta.\end{aligned}$$

Example

Express $\sin(3x)$ and $\cos(3x)$ via $\cos x$ and $\sin x$.

$$\begin{aligned}\sin(3x) &= \sin(x + 2x) \\ &= \sin x \cos(2x) + \cos x \sin(2x) \\ &= \sin x (\cos^2 x - \sin^2 x) + \cos x (2 \sin x \cos x) \\ &= \sin x \cos^2 x - \sin^3 x + 2 \sin x \cos^2 x \\ &= 3 \sin x \cos^2 x - \sin^3 x \\ \cos(3x) &= \cos(x + 2x) \\ &= \cos x \cos(2x) - \sin x \sin(2x) \\ &= \cos x (\cos^2 x - \sin^2 x) - \sin x (2 \sin x \cos x) \\ &= \cos^3 x - \cos x \sin^2 x - 2 \cos x \sin^2 x \\ &= \cos^3 x - 3 \cos x \sin^2 x\end{aligned}$$