Precalculus
Quadratic trigonometric inequalities

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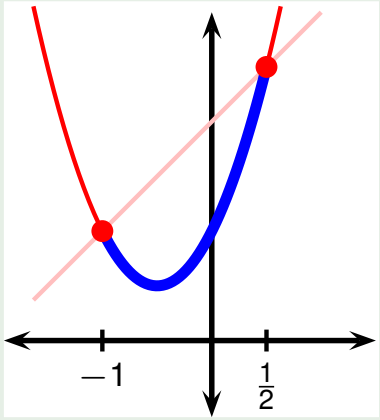
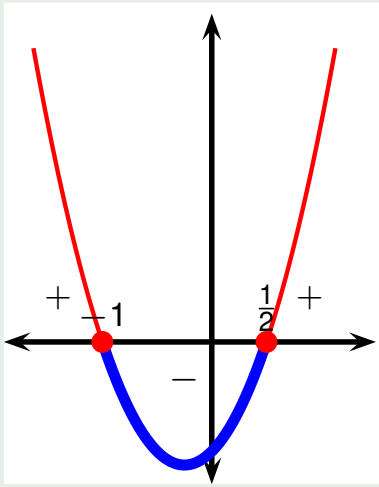
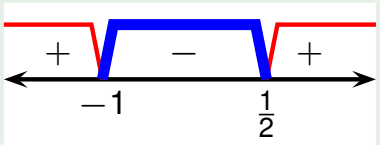
Example





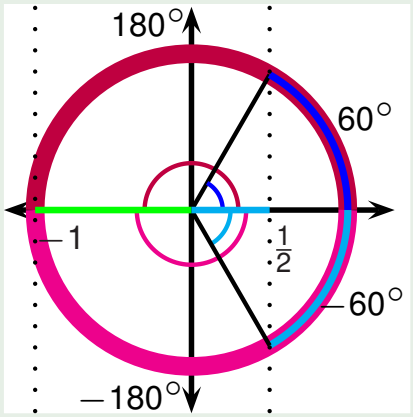


- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2\cos^2\theta + 2\cos\theta + 1 \leq \cos\theta + 2$ lying in $[-360°, 360°]$.

$$\begin{aligned} 2u^2 + 2u + 1 &\leq u + 2 \\ 2u^2 + u - 1 &\leq 0 \\ 2\left(u - \tfrac{1}{2}\right)(u + 1) &\leq 0 \\ u &\in \left[-1, \tfrac{1}{2}\right] \end{aligned}$$

$$\begin{aligned} 2\cos^2\theta + 2\cos\theta + 1 &\leq \cos\theta + 2 \quad \text{Set } \cos\theta = u \\ 2u^2 + 2u + 1 &\leq u + 2 \\ u &\in \left[-1, \tfrac{1}{2}\right] \quad \text{(solved above)} \\ \cos\theta &\in \left[-1, \tfrac{1}{2}\right] \\ -1 \leq \cos\theta &\leq \tfrac{1}{2} \end{aligned}$$

Example



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$$\theta \in [-180° + k360°, -60° + k360°] \cup [60° + k360°, 180° + k360°]$$

$$\begin{aligned} \theta \in \; & \cancel{\cup [-540°, -420°]} \cup [-300°, -180°] && k = -1 \\ & \cup [-180°, -60°] \cup [60°, 180°] && k = 0 \\ & \cup [180°, 300°] \cup \cancel{[420°, 540°]} && k = 1 \end{aligned}$$

$$\theta \in [-300°, -60°] \cup [60°, 300°]$$

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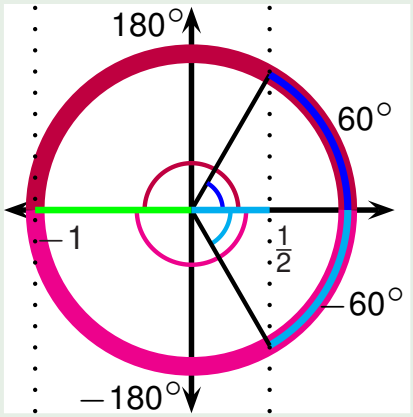

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