

Precalculus

Trigonometric functions with arguments translated by a multiple of $\frac{\pi}{2}$

Todor Milev

2019

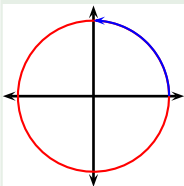
Example

Use the angle sum/difference formulas to simplify.

$$\cos\left(\frac{\pi}{2} - x\right)$$

Example

Use the angle sum/difference formulas to simplify.



$$\begin{aligned}\cos\left(\frac{\pi}{2} - x\right) &= \cos\left(\frac{\pi}{2}\right)\cos x + \sin\left(\frac{\pi}{2}\right)\sin x \\ &= 0 \cdot \cos(x) + 1 \cdot \sin x \\ &= \sin x\end{aligned}$$

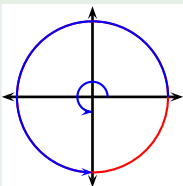
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Use the angle sum/difference formulas to simplify.

$$\cot \left(\frac{3\pi}{2} + x \right)$$

Example

Use the angle sum/difference formulas to simplify.



$$\begin{aligned}
 \cot \left(\frac{3\pi}{2} + x \right) &= \frac{\cos \left(\frac{3\pi}{2} + x \right)}{\sin \left(\frac{3\pi}{2} + x \right)} \\
 &= \frac{\cos \left(\frac{3\pi}{2} \right) \cos x - \sin \left(\frac{3\pi}{2} \right) \sin x}{\sin \left(\frac{3\pi}{2} \right) \cos x + \cos \left(\frac{3\pi}{2} \right) \sin x} \\
 &= \frac{0 \cdot \cos x - (-1) \sin x}{(-1) \cos x + 0 \cdot \sin x} \\
 &= \frac{-\cos x}{-\sin x} = -\frac{\sin x}{\cos x} \\
 &= -\tan x
 \end{aligned}$$

Example

Show that $\tan(\pi + x) = \tan x$ using the angle sum formulas.

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$$\begin{aligned}\tan(\pi + x) &= \frac{\sin(\pi + x)}{\cos(\pi + x)} \\ &= \frac{\sin \pi \cos x + \cos \pi \sin x}{\cos \pi \cos x - \sin \pi \sin x}\end{aligned}$$

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 &= \frac{\sin \pi \cos x + \cos \pi \sin x}{\cos \pi \cos x - \sin \pi \sin x} \\
 &= \frac{? \cdot \cos x + ? \cdot \sin x}{? \cdot \cos x - ? \cdot \sin x}
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as desired.

Proposition (\tan, \cot are π -periodic)

The tangent and cotangent functions are π -periodic, in other words,

$$\tan(\theta + \pi) = \tan \theta$$

$$\cot(\theta + \pi) = \cot \theta$$