Precalculus
Law of cosines theory

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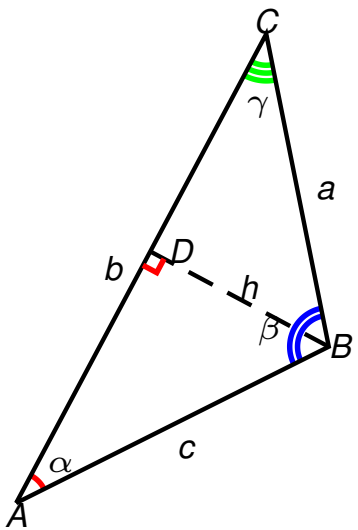
2019

Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ, as indicated.

Proposition (Law of Cosines)

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos\gamma \\ a^2 &= b^2 + c^2 - 2bc\cos\alpha \\ b^2 &= c^2 + a^2 - 2ca\cos\beta \end{aligned}$$



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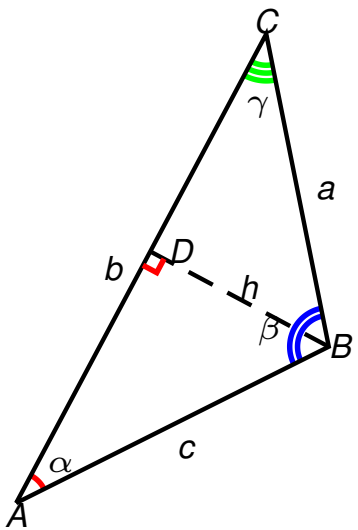

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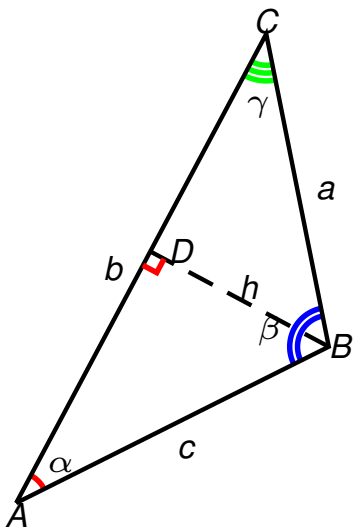

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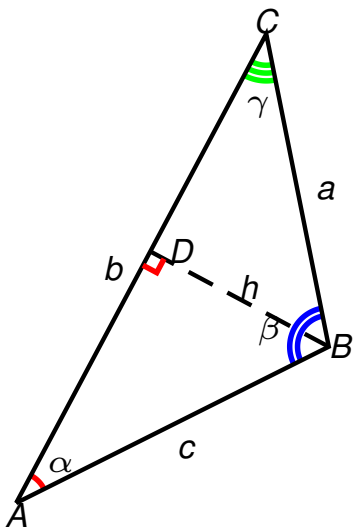

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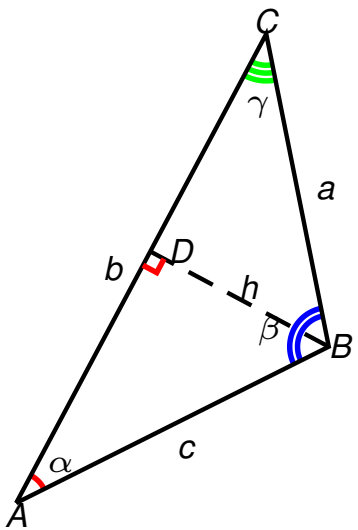

Proof if $\gamma < 90°$.

Drop a perpendicular h from B to AC.

$$\begin{aligned} |CD| &= a\cos\gamma \\ h &= a\sin\gamma \\ |AD| &= b - |CD| = b - a\cos\gamma \\ c^2 &= |AD|^2 + h^2 \qquad \text{Pyth. thm. } \triangle BDA \\ &= (b - a\cos\gamma)^2 + (a\sin\gamma)^2 \\ &= b^2 - 2ab\cos\gamma + a^2\cos^2\gamma + a^2\sin^2\gamma \\ &= b^2 - 2ab\cos\gamma + a^2. \end{aligned}$$

□