Cryptography 101 Addition and subtraction in base 10 calculator-algebra.org

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Addition 2/13

• In what follows we show integer addition by example.

- In what follows we show integer addition by example.
- We show a series of examples that lead to a complete algorithm for addition.

Example

$$1+2 = 2+2 = 2+5 = 9+2 = 7+5 = 9+7 = 0+9 = 9+7$$

Example

$$1+2 = ?$$
 $2+2 =$
 $2+5 =$
 $9+2 =$
 $7+5 =$
 $9+7 =$
 $0+9 =$

Example

$$1+2 = 3$$
 $2+2 = 2+5 = 9+2 = 7+5 = 9+7 = 0+9 = 3$

Example

$$1+2 = 3$$

 $2+2 = ?$
 $2+5 = 9+2 = 7+5 = 9+7 = 0+9 = 9+7$

Example

$$1+2 = 3$$

 $2+2 = 4$
 $2+5 = 9+2 = 7+5 = 9+7 = 0+9 = 9+7$

Example

$$1+2 = 3$$

 $2+2 = 4$
 $2+5 = ?$
 $9+2 = 7+5 = 9+7 = 0+9 = 0$

Example

$$1+2 = 3$$

 $2+2 = 4$
 $2+5 = 7$
 $9+2 = 7$
 $7+5 = 9+7 = 1$
 $9+9 = 1$

Example

$$1+2 = 3$$

 $2+2 = 4$
 $2+5 = 7$
 $9+2 = ?$
 $7+5 = 9+7 = 0+9 = 9$

Example

$$1+2 = 3$$
 $2+2 = 4$
 $2+5 = 7$
 $9+2 = 11$
 $7+5 = 9+7 = 0+9 = 0$

Example

$$1+2 = 3$$
 $2+2 = 4$
 $2+5 = 7$
 $9+2 = 11$
 $7+5 = ?$
 $9+7 = 0+9 = 1$

Example

$$1+2 = 3$$
 $2+2 = 4$
 $2+5 = 7$
 $9+2 = 11$
 $7+5 = 12$
 $9+7 = 0+9 = 12$

Example

$$1+2 = 3$$
 $2+2 = 4$
 $2+5 = 7$
 $9+2 = 11$
 $7+5 = 12$
 $9+7 = ?$
 $0+9 =$

Example

$$1+2 = 3$$
 $2+2 = 4$
 $2+5 = 7$
 $9+2 = 11$
 $7+5 = 12$
 $9+7 = 16$
 $0+9 = 1$

Example

$$1+2 = 3$$
 $2+2 = 4$
 $2+5 = 7$
 $9+2 = 11$
 $7+5 = 12$
 $9+7 = 16$
 $0+9 = ?$

Example

$$1+2 = 3$$
 $2+2 = 4$
 $2+5 = 7$
 $9+2 = 11$
 $7+5 = 12$
 $9+7 = 16$
 $0+9 = 9$

$$\begin{array}{rrr}
 1 + 3 & = \\
 4 + 7 & = \\
 2 + 8 & = \\
 9 + 8 & = \\
 5 + 5 & = \\
 \end{array}$$

Add the one-digit numbers.

$$1+3 = 4+7 = 2+8 = 9+8 = 5+5 =$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
5										
6										
7										
8										
9										

Add the one-digit numbers.

$$1+3 = ?$$
 $4+7 =$
 $2+8 =$
 $9+8 =$
 $5+5 =$

+	0	1	2	3	4	5	6	7	8	9
0	H									
1	\vdash			?						
2	-			•						
3										
4										
5										
6										
7										
8										
9										

Add the one-digit numbers.

$$1+3 = 4$$
 $4+7 = 2+8 = 9+8 = 5+5 = 4$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2										
3										
4										
5										
6										
7										
8										
9										

Add the one-digit numbers.

$$1+3 = 4$$

 $4+7 = ?$
 $2+8 = 9+8 = 5+5 =$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2										
3										
4								?		
5										
6										
7										
8										
9										

Add the one-digit numbers.

$$1+3 = 4$$
 $4+7 = 11$
 $2+8 = 9+8 = 5+5 =$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2										
3										
4								11		
5										
6										
7										
8										
9										

Add the one-digit numbers.

$$1+3 = 4$$
 $4+7 = 11$
 $2+8 = ?$
 $9+8 = 5+5 =$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2									?	
3										
4								11		
5										
6										
7										
8										
9										

Add the one-digit numbers.

$$1+3 = 4$$
 $4+7 = 11$
 $2+8 = 10$
 $9+8 = 5+5 =$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2									10	
3										
4								11		
5										
6										
7										
8										
9										

Add the one-digit numbers.

$$1+3 = 4$$
 $4+7 = 11$
 $2+8 = 10$
 $9+8 = ?$
 $5+5 =$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2									10	
3										
4								11		
5										
6										
7										
8										
9									?	

Add the one-digit numbers.

$$1+3 = 4$$
 $4+7 = 11$
 $2+8 = 10$
 $9+8 = 17$
 $5+5 =$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2									10	
3										
4								11		
5										
6										
7										
8										
9									17	

Add the one-digit numbers.

$$1+3 = 4$$
 $4+7 = 11$
 $2+8 = 10$
 $9+8 = 17$
 $5+5 = ?$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2									10	
3										
4								11		
5						?				
6										
7										
8										
9									17	

Add the one-digit numbers.

$$1+3 = 4$$
 $4+7 = 11$
 $2+8 = 10$
 $9+8 = 17$
 $5+5 = 10$

+	0	1	2	3	4	5	6	7	8	9
0										
1				4						
2									10	
3										
4								11		
5						10				
6										
7										
8										
9									17	

Add the one-digit numbers.

$$1+3 = 4$$
 $4+7 = 11$
 $2+8 = 10$
 $9+8 = 17$
 $5+5 = 10$

+	0	1	2	3	4	5	6	7	8	9
0	?	?	?	?	?	?	?	?	?	?
1	?	?	?	4	?	?	?	?	?	?
2	?	?	?	?	?	?	?	?	10	?
3	?	?	?	?	?	?	?	?	?	?
4	?	?	?	?	?	?	?	11	?	?
5	?	?	?	?	?	10	?	?	?	?
6	?	?	?	?	?	?	?	?	?	?
7	?	?	?	?	?	?	?	?	?	?
8	?	?	?	?	?	?	?	?	?	?
9	?	?	?	?	?	?	?	?	17	?

Add the one-digit numbers.

$$1+3 = 4$$
 $4+7 = 11$
 $2+8 = 10$
 $9+8 = 17$
 $5+5 = 10$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Add the one-digit numbers.

$$^{+}\frac{3}{5}$$

	_	-	_	_	-		_	_	_	_
+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
5										
6										
7										
8										
9										

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$3 + 5 = ?$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						?				
4										
5										
6										
7										
8										
9										

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$3 + 5 = 8$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7										
8										
9										

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$3 + 5 = 8$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7										
8										
9										

Add the one-digit numbers.

$$+\frac{3}{5}$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7										
8										
9										

Add the one-digit numbers.

$$+\frac{3}{5}$$

+	0	1	2	3	4	5	6	7	8	6
0										
1										
2										
3						8				
4										
5										
6										
7										
8										
9										

Add the one-digit numbers.

$$+\frac{3}{5}$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							?			
8										
9										

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$+\frac{1}{6}$$

$$7 + 6 = 13$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9										

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$+\frac{1}{6}$$

$$7 + 6 = 13$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9										

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$+ \frac{1}{6}$$

$$7 + 6 = 13$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9										

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$+\frac{1}{7}$$

+	0	1	2	3	4	5	6	7	8	9
0	П									
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9										

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$+\frac{1}{7}$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9										

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$+\frac{1}{6}$$

$$9 + 2 = ?$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9			?							

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$+\frac{1}{6}$$

$$9 + 2 = 11$$

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9			11							

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$+\frac{1}{6}$$

$$9 + 2 = 11$$

	0	1	2	3	4	5	6	7	8	9
	U	ı		3	4	5	O	,	0	Э
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9			11							

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$+\frac{1}{7}$$

$$9 + 2 = 11$$

$\overline{}$	_		_						_	_
+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9			11							

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$+\frac{1}{6}$$

$$+\frac{9}{2}$$

+	0	1	2	3	4	5	6	7	8	9
۱	\vdash	<u>'</u>			'				Ŭ	
0										
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9			11							

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$+\frac{1}{6}$$

$$+\frac{9}{2}$$

+	0	1	2	3	4	5	6	7	8	9
0	Г									
1										
2										
3						8				
4										
5										
6										
7							13			
8										
9			11							

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$+\frac{1}{6}$$

$$+\frac{9}{2}$$

+	0	1	2	3	4	5	6	7	8	9
0	?	?	?	?	?	?	?	?	?	?
1	?	?	?	?	?	?	?	?	?	?
2	?	?	?	?	?	?	?	?	?	?
3	?	?	?	?	?	8	?	?	?	?
4	?	?	?	?	?	?	?	?	?	?
5	?	?	?	?	?	?	?	?	?	?
6	?	?	?	?	?	?	?	?	?	?
7	?	?	?	?	?	?	13	?	?	?
8	?	?	?	?	?	?	?	?	?	?
9	?	?	11	?	?	?	?	?	?	?

Add the one-digit numbers.

$$+\frac{3}{5}$$

$$+\frac{1}{6}$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

$$+\frac{23}{34}$$

$$+\frac{23}{34}$$

$$3 + 4 = ?$$

$$+\frac{23}{34}$$

$$3 + 4 = 7$$

$$+\frac{23}{34}$$

$$3 + 4 = 7$$

$$+\frac{23}{34}$$

$$2 + 3 = ?$$

$$+\frac{23}{34}$$

$$2 + 3 = 5$$

$$+\frac{23}{34}$$
 $\frac{57}{57}$

$$2 + 3 = 5$$

$$+\frac{23}{34} \\ \overline{57}$$



$$+\frac{67}{8}$$

$$+\frac{67}{8}$$

$$7 + 8 = ?$$

$$+\frac{67}{8}$$

$$7 + 8 = 15$$

$$+\frac{67}{8}$$

$$7 + 8 = 15$$

$$+\frac{67}{8}$$

$$7 + 8 = 15$$

$$+\frac{67}{8}$$

$$1 + 6 + = ?$$

$$+\frac{67}{8}$$

$$1 + 6 += 7$$

$$+\frac{67}{8}$$

$$1 + 6 += 7$$





$$+ \underbrace{\begin{array}{c} 67 \\ 962 \end{array}}$$

$$7 + 2 = ?$$

$$+\frac{67}{962}$$

$$7 + 2 = 9$$

$$7 + 2 = 9$$

$$6 + 6 = ?$$

$$6 + 6 = 12$$

$$+\frac{67}{962}$$

$$6 + 6 = 12$$

$$+\frac{67}{962}$$

$$6 + 6 = 12$$

$$1 + 9 = ?$$

$$+\frac{67}{962}$$

$$1 + 9 = 10$$

$$+\frac{67}{962}$$

$$1 + 9 = 10$$

$$+\frac{67}{962}$$

$$1 + 9 = 10$$

$$+\frac{67}{962}$$

$$+\frac{{}^{1} {}^{1} {}^{0} {}^{$$

$$+ \frac{35461}{68072}$$

$$1 + 2 = ?$$

$$+ \frac{35461}{68072}$$

$$1 + 2 = 3$$

$$+\frac{35461}{68072}$$

$$1 + 2 = 3$$

$$6 + 7 = ?$$

$$+\frac{35461}{68072}$$

$$6 + 7 = 13$$

$$+\frac{35461}{68072}$$

$$6 + 7 = 13$$

$$+ \frac{35461}{68072}$$

$$6 + 7 = 13$$

$$+\frac{35461}{68072}$$
?33

$$1+4+0=?$$

$$+ \frac{35461}{68072} \\ - \frac{533}{533}$$

$$1 + 4 + 0 = 5$$

$$+\frac{35461}{68072}$$

$$-\frac{533}{533}$$

$$1+4+0=5$$

$$+\frac{\overset{?}{35461}}{\overset{68072}{2533}}$$

$$+\frac{35461}{68072}$$

$$-3533$$

$$5 + 8 = 13$$

$$+\frac{35461}{68072}$$

$$-\frac{3533}{3533}$$

$$5 + 8 = 13$$

$$+\frac{35461}{68072}$$

$$-3533$$

$$5 + 8 = 13$$

$$+\frac{35461}{68072}$$

$$-\frac{3533}{23533}$$

$$1 + 3 + 6 = ?$$

$$+\frac{{35461 \atop 35461 \atop 68072 \atop 03533}}$$

$$1 + 3 + 6 = 10$$

$$+\frac{35461}{68072}$$

$$-03533$$

$$1 + 3 + 6 = 10$$

$$+\frac{35461}{68072}$$

$$-03533$$

$$1 + 3 + 6 = 10$$

$$+\frac{35461}{68072}$$

$$+\frac{3533}{103533}$$



Addition 10/13

We covered addition by example; algorithm follows. Feel free to skip.

Algorithm (Addition base 10)

- 1. Set maxNumberOfDigits to the larger number of digits.
- 2. For each digit position i, starting at position 0:
- 2.1. Let topDigit and bottomDigit be the two digits in ith position. If smaller number has no digit at the position, set its digit to 0.
- 2.2. Set digitSum to topDigit + bottomDigit.
- 2.3. If digitSum \geq 10, set resultDigit = digitSum 10 and carryOver = 1.
 - Else digitSum < 10, so set resultDigit = digitSum and carryOver = 0.
- 2.4. Set the result's ith digit to resultDigit.
 - 3. If after last step carryOver is 1, set 1 as the result's (maxNumberOfDigits + 1)th digit.

Subtraction 11/13

• In what follows we show subtraction by example.

- In what follows we show subtraction by example.
- We show a series of example that lead to a complete algorithm for subtraction.

Example (One digit subtraction, result > 0)

$$5 - 3 =$$

$$4 - 0 =$$

$$7 - 4 =$$

$$8 - 2 =$$

$$9 - 7 =$$

Example (One digit subtraction, result > 0)

$$5-3 = ?$$
 | because $3+?=5$
 $4-0 = 7-4 = 8-2 = 9-7 = 9$

Example (One digit subtraction, result > 0)

$$5-3 = 2$$
 | because $3+2=5$

$$4 - 0 =$$

$$7 - 4 =$$

$$8 - 2 =$$

$$9 - 7 =$$

Example (One digit subtraction, result > 0)

$$5-3 = 2$$
 | because $3+2=5$
 $4-0 = ?$ | because $0+?=4$
 $7-4 = 8-2 = 9-7 = 9$

Example (One digit subtraction, result > 0)

$$5-3 = 2$$
 | because $3+2=5$
 $4-0 = 4$ | because $0+4=4$
 $7-4 = 8-2 = 9-7 = 9$

Example (One digit subtraction, result > 0)

$$5-3 = 2$$
 | because $3+2=5$
 $4-0 = 4$ | because $0+4=4$
 $7-4 = ?$ | because $4+?=7$
 $8-2 = 9-7 =$

Example (One digit subtraction, result > 0)

$$5-3 = 2$$
 | because $3+2=5$
 $4-0 = 4$ | because $0+4=4$
 $7-4 = 3$ | because $4+3=7$
 $8-2 = 9-7 = 9$

Example (One digit subtraction, result > 0)

$$5-3 = 2$$
 | because $3+2=5$
 $4-0 = 4$ | because $0+4=4$
 $7-4 = 3$ | because $4+3=7$
 $8-2 = ?$ | because $2+?=8$
 $9-7 =$

Example (One digit subtraction, result > 0)

$$5-3 = 2$$
 | because $3+2=5$
 $4-0 = 4$ | because $0+4=4$
 $7-4 = 3$ | because $4+3=7$
 $8-2 = 6$ | because $2+6=8$
 $9-7 = 6$

Example (One digit subtraction, result > 0)

$$5-3 = 2$$
 | because $3+2=5$
 $4-0 = 4$ | because $0+4=4$
 $7-4 = 3$ | because $4+3=7$
 $8-2 = 6$ | because $2+6=8$
 $9-7 = ?$ | because $7+?=9$

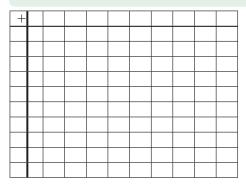
Example (One digit subtraction, result > 0)

$$5-3 = 2$$
 | because $3+2=5$
 $4-0 = 4$ | because $0+4=4$
 $7-4 = 3$ | because $4+3=7$
 $8-2 = 6$ | because $2+6=8$
 $9-7 = 2$ | because $7+2=9$

Subtract the one-digit numbers.

$$6 - 1 =$$

$$9-5 = 8-2 =$$

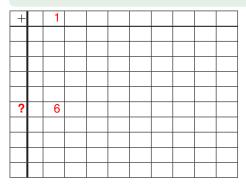


Subtract the one-digit numbers.

$$6-1 = ? | bec$$

$$6-1 = ?$$
 | because $1+?=6$

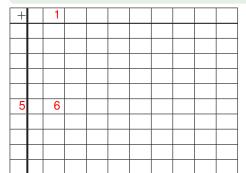
$$9-5 = 8-2 =$$



Subtract the one-digit numbers.

$$6-1 = 5 \mid \text{because } 1+5=6$$

$$9-5 = 8-2 =$$

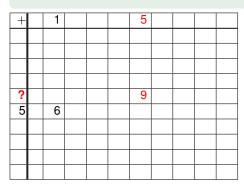


Subtract the one-digit numbers.

$$6-1 = 5$$

$$6-1 = 5$$
 | because $1+5=6$
 $9-5 = ?$ | because $5+?=9$

$$8 - 2 =$$

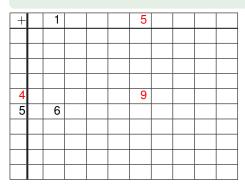


Subtract the one-digit numbers.

$$6-1 = 5$$
 | because $1+5=6$
 $9-5 = 4$ | because $5+4=9$

because
$$1+5=6$$

$$8 - 2 =$$



Subtract the one-digit numbers.

$$6-1 = 5$$

 $9-5 = 4$

$$6-1 = 5$$
 | because $1+5=6$
 $9-5 = 4$ | because $5+4=9$
 $8-2 = ?$ | because $2+?=8$

+	1	2		5		
4				9		
5 ?	6					
?		8				

Subtract the one-digit numbers.

$$6-1 = 5$$

 $9-5 = 4$

$$6-1 = 5$$
 | because $1+5=6$
 $9-5 = 4$ | because $5+4=9$
 $8-2 = 6$ | because $2+6=8$

+	1	2		5		
4				9		
5 6	6					
6		8				

Subtract the one-digit numbers.

$$6-1 = 5$$
 | be $9-5 = 4$ | be $8-2 = 6$ | be

because
$$1 + 5 = 6$$

because $5 + 4 = 9$
because $2 + 6 = 8$

+	?	1	2	?	?	5	?	?	?	?
?	?	?	?	?	?	?	?	?	?	?
?	?	?	?	?	?	?	?	?	?	?
?	?	?	?	?	?	?	?	?	?	?
?	?	?	?	?	?	?	?	?	?	?
4	?	?	?	?	?	9	?	?	?	?
5	?	6	?	?	?	?	?	?	?	?
6	?	?	8	?	?	?	?	?	?	?
?	?	?	?	?	?	?	?	?	?	?
?	?	?	?	?	?	?	?	?	?	?
?	?	?	?	?	?	?	?	?	?	?

Subtract the one-digit numbers.

$$6-1 = 5$$
 | because $1+5=6$
 $9-5 = 4$ | because $5+4=9$
 $8-2 = 6$ | because $2+6=8$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18