Precalculus
Factor quadratic over the complex numbers

Todor Milev

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$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

$$\left| \begin{array}{rcl} x_1 x_2 & = & \frac{c}{a} \\ x_1 + x_2 & = & -\frac{b}{a} \end{array} \right.$$

Example

Factor the quadratic, using complex numbers if needed.

$$x^2 + x + 1 = \left(x - \left(\frac{-1+\sqrt{3}i}{2}\right)\right)\left(x - \left(\frac{-1-\sqrt{3}i}{2}\right)\right)$$

- The product of the two roots: $x_1 x_2 = 1$.
- Integer options: $x_1 = 1, x_2 = 1$ and $x_1 = -1, x_2 = -1$.
- $\begin{array}{rcl} (x-1)(x-1) & = & (x-1)^2 = x^2 - 2x + 1 \\ (x+1)(x+1) & = & (x+1)^2 = x^2 + 2x + 1 \end{array}$ both don't work.
- $\Rightarrow$ No easy factorization; must use quadratic formula.

$$\begin{aligned} x_1, x_2 &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} \\ &= \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2} \end{aligned}$$