Precalculus
Compute the trigonometric functions of an angle not in the first quadrant

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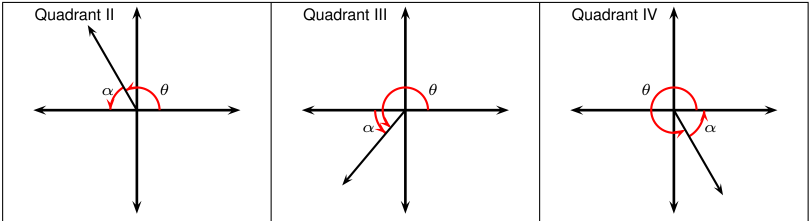
The computation of the reference angle α depends on the quadrant.



$$\begin{aligned}\alpha &= \pi - \theta\\ \alpha &= 180^\circ - \theta\end{aligned}$$

$$\begin{aligned}\alpha &= \theta - \pi\\ \alpha &= \theta - 180^\circ\end{aligned}$$

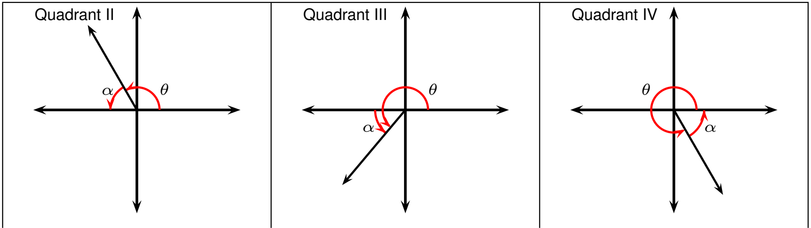
$$\begin{aligned}\alpha &= 2\pi - \theta\\ \alpha &= 360^\circ - \theta\end{aligned}$$

To compute trigonometric functions from obtuse ($> 90^\circ$) or negative angles, we can use the following visual aid.

Definition (Reference Angle)

Let θ be an angle in standard position. Its reference angle is the acute positive angle formed by the terminal arm and the x axis.

The computation of the reference angle α depends on the quadrant.



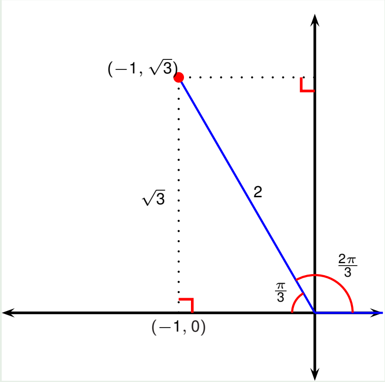
Quadrant II	Quadrant III	Quadrant IV
$\alpha = \pi - \theta$	$\alpha = \theta - \pi$	$\alpha = 2\pi - \theta$
$\alpha = 180^\circ - \theta$	$\alpha = \theta - 180^\circ$	$\alpha = 360^\circ - \theta$

Observation

One can find the value of a trigonometric function of θ as follows.

- *Find the reference angle α associated to θ.*
- *Find the trig function of α.*
- *Use the quadrant in which θ lies to affix an appropriate sign to the function value.*

Example



Find the exact values of the trigonometric functions of $\theta = \frac{2\pi}{3} = 120^\circ$.

$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} \quad \tan\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

$$\csc\left(\frac{2\pi}{3}\right) = \frac{2}{\sqrt{3}} \quad \sec\left(\frac{2\pi}{3}\right) = -\frac{2}{1} = -2 \quad \cot\left(\frac{2\pi}{3}\right) = -\frac{1}{\sqrt{3}}$$