

# Calculus II

## Homework

### Trigonometry review

1. Let  $x \in (0, 1)$ . Express the following using  $x$  and  $\sqrt{1 - x^2}$ .

- |                            |                            |
|----------------------------|----------------------------|
| (a) $\sin(\arcsin(x))$ .   | (e) $\sin(2 \arccos(x))$ . |
| (b) $\sin(2 \arcsin(x))$ . | (f) $\sin(3 \arccos(x))$ . |
| (c) $\sin(3 \arcsin(x))$ . | (g) $\cos(2 \arcsin(x))$ . |
| (d) $\sin(\arccos(x))$ .   | (h) $\cos(3 \arccos(x))$ . |

2. Express as the following as an algebraic expression of  $x$ . In other words, “get rid” of the trigonometric and inverse trigonometric expressions.

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|--|------------------------------------|
| (a) $\cos^2(\arctan x)$ .                | (c) $\frac{1}{\cos(\arcsin x)}$ .  |
| (b) $-\sin^2(\operatorname{arccot} x)$ . | (d) $-\frac{1}{\sin(\arccos x)}$ . |

3. Rewrite as a rational function of  $t$ . This problem will be later used to derive the Euler substitutions (an important technique for integrating).

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|---------------------------|---|
| (a) $\cos(2 \arctan t)$ . | (g) $\cos(2 \operatorname{arccot} t)$ . |
| (b) $\sin(2 \arctan t)$ . | (h) $\sin(2 \operatorname{arccot} t)$ . |
| (c) $\tan(2 \arctan t)$ . | (i) $\tan(2 \operatorname{arccot} t)$ . |
| (d) $\cot(2 \arctan t)$ . | (j) $\cot(2 \operatorname{arccot} t)$ . |
| (e) $\csc(2 \arctan t)$ . | (k) $\csc(2 \operatorname{arccot} t)$ . |
| (f) $\sec(2 \arctan t)$ . | (l) $\sec(2 \operatorname{arccot} t)$ . |

4. Compute the derivative (derive the formula).

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|------------------------------------|---|
| (a) $(\arctan x)'$ .               | (d) $(\arccos x)'$ .  |
| (b) $(\operatorname{arccot} x)'$ . | (e) Let $\operatorname{arcsec}$ denote the inverse of the secant function. Compute $(\operatorname{arcsec} x)'$ . |
| (c) $(\arcsin x)'$ .               |   |

5. (a) Let  $a + b \neq k\pi$ ,  $a \neq k\pi + \frac{\pi}{2}$  and  $b \neq k\pi + \frac{\pi}{2}$  for any  $k \in \mathbb{Z}$  (integers). Prove that

$$\frac{\tan a + \tan b}{1 - \tan a \tan b} = \tan(a + b) \quad .$$

(b) Let  $x$  and  $y$  be real. Prove that, for  $xy \neq 1$ , we have

$$\arctan x + \arctan y = \arctan \left( \frac{x + y}{1 - xy} \right)$$

if the left hand side lies between  $(-\frac{\pi}{2}, \frac{\pi}{2})$ .