

Calculus II
Integrals of the form $\int \tan^m x \sec^n x dx$,
 n -positive and even

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Example

$$\int \tan^8 x \sec^4 x dx$$

Example

$$\int \tan^8 x \sec^4 x dx = \int \tan^8 x \sec^2 x \sec^2 x dx$$

Example

$$\begin{aligned}\int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx \\ &= \int \tan^8 x \sec^2 x d(?)\end{aligned}$$

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$$\begin{aligned}\int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx \\ &= \int \tan^8 x \sec^2 x d(\tan x)\end{aligned}$$

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$$\int \tan^8 x \sec^4 x dx = \int \tan^8 x \sec^2 x \sec^2 x dx$$

$$= \int \tan^8 x \sec^2 x d(\tan x)$$

$$= \int \tan^8 x \left(? \right) d(\tan x)$$

Can we rewrite
 $\sec^2 x$ via $\tan x$?

Example

$$\int \tan^8 x \sec^4 x dx = \int \tan^8 x \sec^2 x \sec^2 x dx$$

$$= \int \tan^8 x \sec^2 x d(\tan x)$$

$$= \int \tan^8 x (1 + \tan^2 x) d(\tan x)$$

Can we rewrite
 $\sec^2 x$ via $\tan x$?

Example

$$\begin{aligned}
 \int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx \\
 &= \int \tan^8 x \sec^2 x d(\tan x) && \left| \begin{array}{l} \text{Can we rewrite} \\ \sec^2 x \text{ via } \tan x? \end{array} \right. \\
 &= \int \tan^8 x (1 + \tan^2 x) d(\tan x) && \left| \text{Set } u = \tan x \right. \\
 &= \int u^8 (1 + u^2) du
 \end{aligned}$$

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 \int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx \\
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 &= \int \tan^8 x (1 + \tan^2 x) d(\tan x) && \left| \text{Set } u = \tan x \right. \\
 &= \int u^8 (1 + u^2) du \\
 &= \int (u^8 + u^{10}) du
 \end{aligned}$$

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 &= \int u^8 (1 + u^2) du \\
 &= \int (u^8 + u^{10}) du \\
 &= ?
 \end{aligned}$$

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 \int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx \\
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 &= \int \tan^8 x (1 + \tan^2 x) d(\tan x) && \left| \text{Set } u = \tan x \right. \\
 &= \int u^8 (1 + u^2) du \\
 &= \int (u^8 + u^{10}) du \\
 &= \frac{u^9}{9} + \frac{u^{11}}{11} + C
 \end{aligned}$$

Example

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 \int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx \\
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 &= \int \tan^8 x (1 + \tan^2 x) d(\tan x) && \left. \begin{array}{l} \text{Set } u = \tan x \end{array} \right\} \\
 &= \int u^8 (1 + u^2) du \\
 &= \int (u^8 + u^{10}) du \\
 &= \frac{u^9}{9} + \frac{u^{11}}{11} + C \\
 &= \frac{\tan^9 x}{9} + \frac{\tan^{11} x}{11} + C .
 \end{aligned}$$