Calculus II
Guess sequence formula, part 1

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2019

Example (Sequences via f-las: guess f-la from terms)

Find a formula for the general term a_n of the sequence

$$\left(0, \frac{1}{4}, -\frac{2}{8}, \frac{3}{16}, -\frac{4}{32}, \frac{5}{64}, \dots\right)$$

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- We take this into account by multiplying by $(-1)^n$.

$$(-1)^n \frac{n-1}{2^n}$$

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$$a_n = (-1)^n \frac{n-1}{2^n}$$