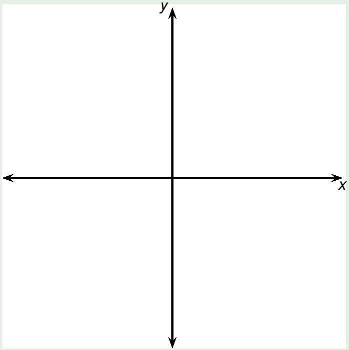
Calculus I
Rational function asymptotes, part 1

Todor Milev

2019

Example

Find the horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{3x^2+1}}{2x-3}$.



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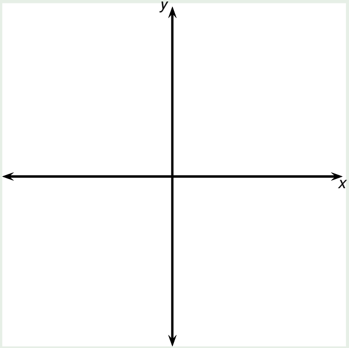

$$\lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-3}$$

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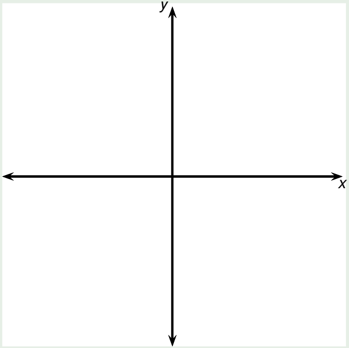

$$\lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

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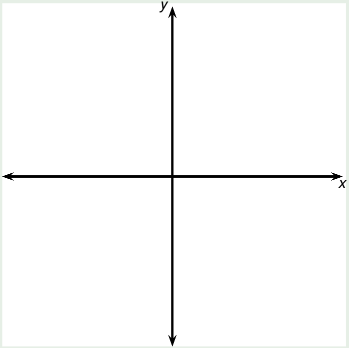

$$\lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{x}}$$

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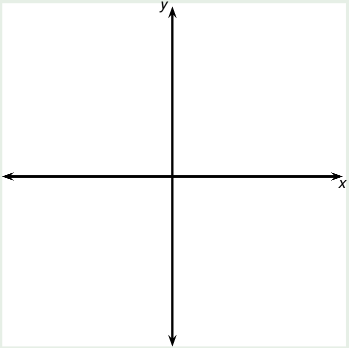

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$$= \lim_{x\to\infty} \frac{\sqrt{\textbf{?}}}{\textbf{?}}$$

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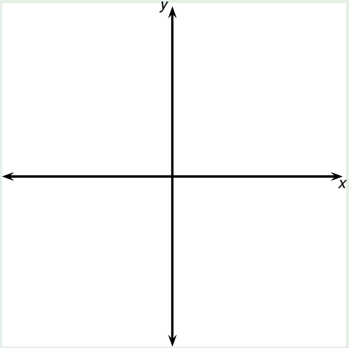

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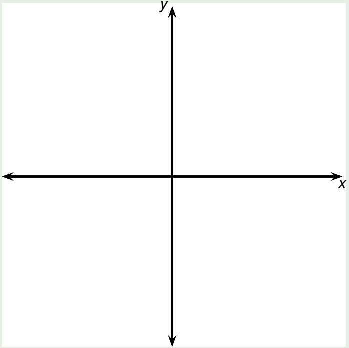

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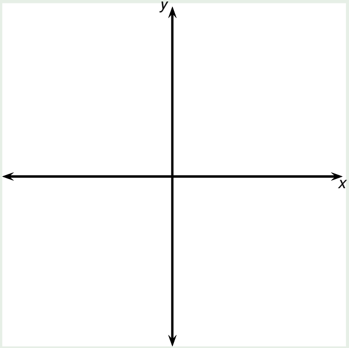

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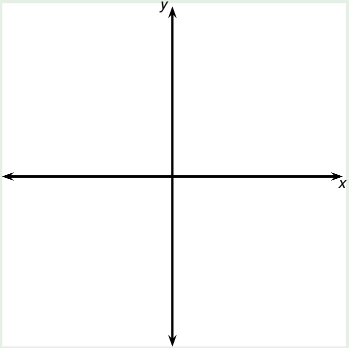

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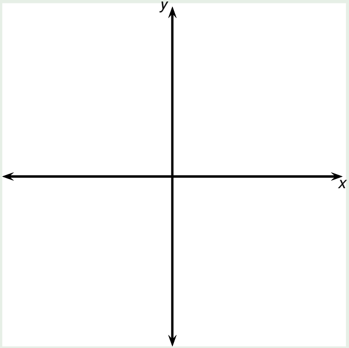

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$$= \frac{\sqrt{\mathbf{?} + \mathbf{?}}}{\mathbf{?} - \mathbf{?}}$$

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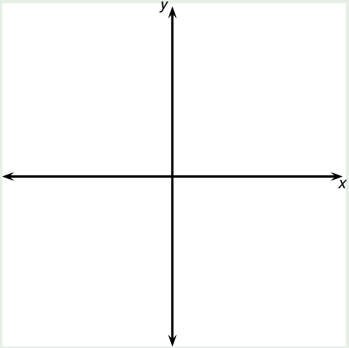

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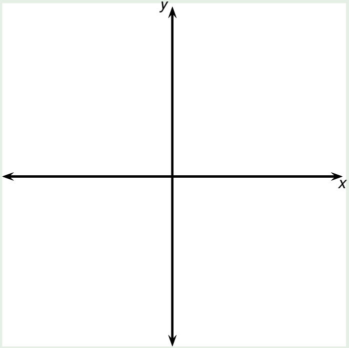

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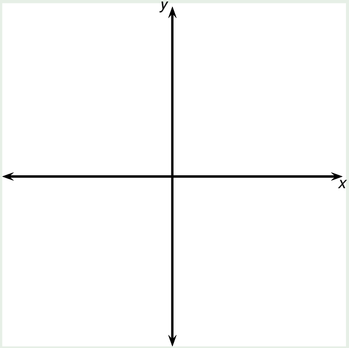

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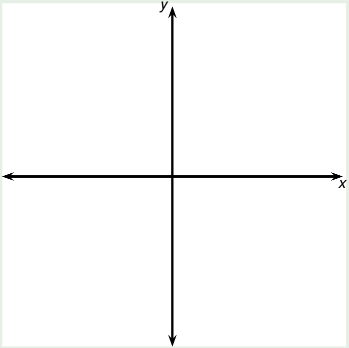

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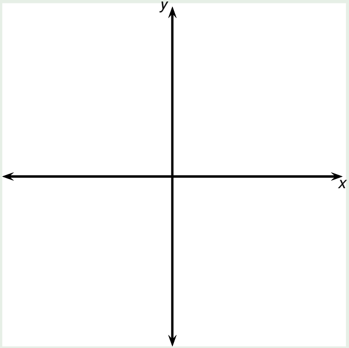

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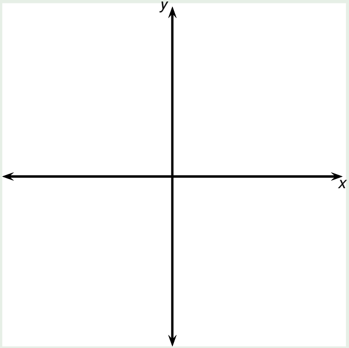

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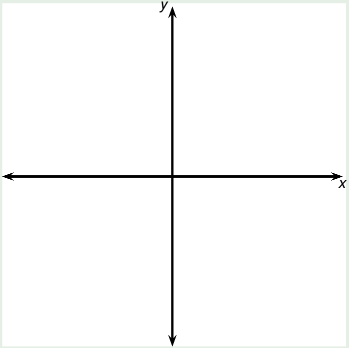

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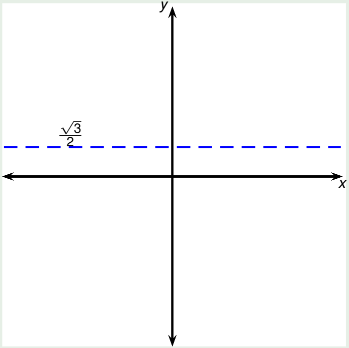

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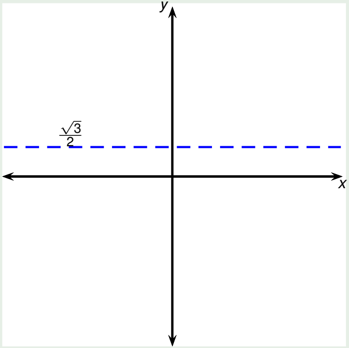

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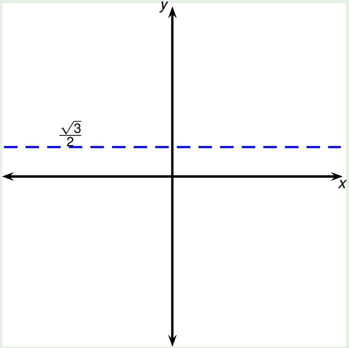

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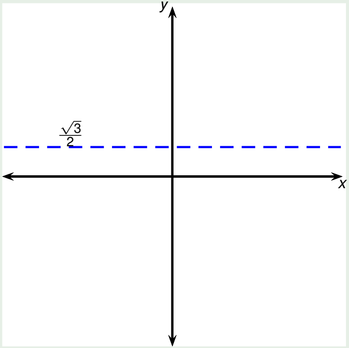

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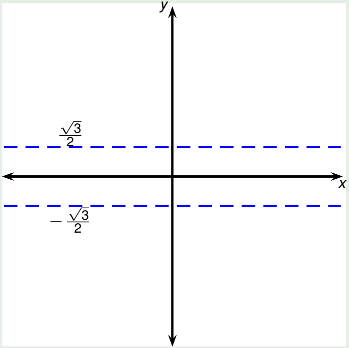

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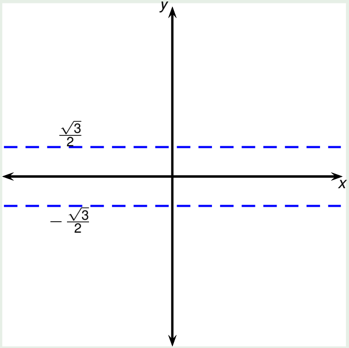
Example

Find the horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{3x^2+1}}{2x-3}$.



$$\lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{3+\frac{1}{x^2}}}{2-\frac{3}{x}} = \frac{\sqrt{\lim\limits_{x\to\infty} 3 + \lim\limits_{x\to\infty} \frac{1}{x^2}}}{\lim\limits_{x\to\infty} 2 - 3 \lim\limits_{x\to\infty} \frac{1}{x}}$$

$$= \frac{\sqrt{3+0}}{2-0} = \frac{\sqrt{3}}{2}$$

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If $x > 0$ then $x = \sqrt{x^2}$.
If $x < 0$ then $x = -\sqrt{x^2}$.

Vertical Asymptote:

Example

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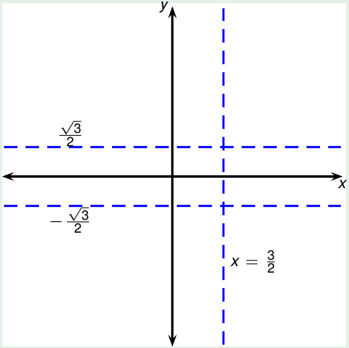

If $x > 0$ then $x = \sqrt{x^2}$.
If $x < 0$ then $x = -\sqrt{x^2}$.
Vertical Asymptote:
$x = \frac{3}{2}$.

$$\lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{3+\frac{1}{x^2}}}{2-\frac{3}{x}} = \frac{\sqrt{\lim\limits_{x\to\infty} 3 + \lim\limits_{x\to\infty} \frac{1}{x^2}}}{\lim\limits_{x\to\infty} 2 - 3 \lim\limits_{x\to\infty} \frac{1}{x}}$$

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Example

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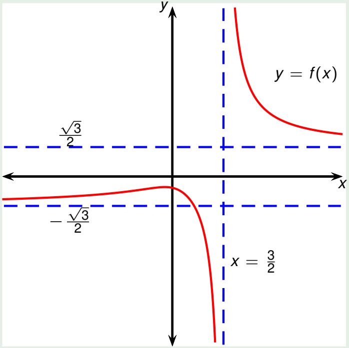

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