Calculus I
Type 3: Exponent equation that reduces to quadratic

Todor Milev

2019

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

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Set $u = \mathbf{?}$.

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2u - 3 = 0 \quad &\text{or} \quad 2u + 1 = 0\\
u = \frac{3}{2} \quad &\text{or} \quad u = -\frac{1}{2}\\
2^x = \frac{3}{2} \quad &\text{or} \quad 2^x = -\frac{1}{2}\\
x = \log_2\left(\frac{3}{2}\right) \quad &\text{or} \quad \text{no real solution}
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x = \log_2\left(\frac{3}{2}\right) = \frac{\ln\left(\frac{3}{2}\right)}{\ln 2} \approx 0.58496 \quad &\text{or} \quad \text{no real solution}
\end{aligned}$$