Calculus II
Add geometric progression, part 1

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2019

For $|r| < 1$, recall that the sum of a geometric series is

$$a + ar + ar^2 + ar^3 + \ldots = a\left(1 + r + r^2 + r^3 + \ldots\right) = \frac{a}{1-r}$$
$$\text{alternatively}$$
$$\sum_{n=1}^{\infty} ar^{n-1} = \sum_{m=0}^{\infty} ar^m = a\sum_{m=0}^{\infty} r^m = \frac{a}{1-r}$$

Example

Find the sum of the geometric series $\qquad -2 + \dfrac{6}{5} - \dfrac{18}{25} + \dfrac{54}{125} - \cdots$

- The first term is $a = -2$.
- The common ratio is $r = \dfrac{\frac{6}{5}}{-2} = -\dfrac{3}{5}$.
- Therefore the sum is

$$\sum_{n=1}^{\infty}(-2)\left(-\frac{3}{5}\right)^{n-1} = \frac{(-2)}{1-\left(-\frac{3}{5}\right)} = -\frac{2}{\frac{8}{5}} = -\frac{5}{4}$$