Precalculus
Homework
Logarithms basics

1. Use the definition of a logarithm to evaluate each of the following without using a calculator. The answer key has not been proofread, use with caution.

   (a) $\log_2 16$.

   (b) $\log_3\left(\frac{1}{9}\right)$.

   (c) $\log_{10} 1000$.

   (d) $\log_6 36^{-\frac{2}{3}}$.

   (e) $\log_2(8\sqrt{2})$.

   (f) $\log_{\frac{1}{2}}(4)$.

   (g) $\log_{\frac{1}{9}}(\sqrt{3})$.

2. Find the exact value of each expression.

   (a) $\log_5 125$.

   (b) $\log_3 \frac{1}{27}$.

   (c) $\ln\left(\frac{1}{e}\right)$.

   (d) $\log_{10}\sqrt{10}$.

   (e) $e^{\ln 4.5}$.

   (f) $\log_{10} 0.0001$.

   (g) $\log_{1.5} 2.25$.

   (h) $\log_5 4 - \log_5 500$.

   (i) $\log_2 6 - \log_2 15 + \log_2 20$.

   (j) $\log_3 100 - \log_3 18 - \log_3 50$.

   (k) $e^{-2\ln 5}$.

   (l) $\ln\left(\ln e^{e^{10}}\right)$.

   (m) $\log_7\left(\frac{49^x}{343^y}\right)$

3. Using only the $\ln$ operation of your calculator compute the indicated logarithm. Confirm your computation numerically by exponentiation.

   (a) $\log_5(13)$.

   (b) $\log_{12}(9)$.

   (c) $\log_{13}(101)$.

   (d) $\log_{10}(2015)$.

4. Express each of the following as a single logarithm. If possible, compute the logarithm without using a calculator. The answer key has not been proofread, use with caution.

   (a) $\ln 4 + \ln 6 - \ln 5$.

   (b) $2\ln 2 - 3\ln 3 + 4\ln 4$.

   (c) $\ln 36 - 2\ln 3 - 3\ln 2$.

   (d) $\log_2(24) - \log_4 9$.

   (e) $\log_7(24) + \log_{\frac{1}{7}} 3 - \log_{49}(64)$.

   (f) $\log_3(24) + \log_3\left(\frac{3}{8}\right)$.

5. Demonstrate the identity(s).

   (a) $-\ln\left(\sqrt{1+x^2} - x\right) = \ln\left(x + \sqrt{1+x^2}\right)$