Calculus II
Homework
Series basic facts

1. Find whether the series is convergent or divergent using an appropriate test. Some of the problems require the alternating series test. The test states the following.

   Alternating series test. Suppose $b_n \searrow 0$. Then $\sum(-1)^n b_n$ is convergent.

   Here, $b_n \searrow 0$ means the following.

   - The sequence of numbers b_n is decreasing.
   - The sequence decreases to 0, that is,
   $$\lim_{n\to\infty} b_n = 0 \quad .$$

   (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \ln n.$

   (b) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{\ln n}.$

   (c) $\displaystyle\sum_{n=2}^{\infty}\frac{n}{\ln n}$

   (d) $\displaystyle\sum_{n=2}^{\infty}\frac{\ln n}{n}$

2. Use the integral test, the comparison test or the limit comparison test to determine whether the series is convergent or divergent. Justify your answer.

   (a) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2n+1}.$

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2n^2+n^3}.$

   (c) $\displaystyle\sum_{n=1}^{\infty}\frac{n^2+3}{3n^5+n}$

   (d) $\displaystyle\sum_{n=0}^{\infty}\frac{1}{3^n+5}.$

   (e) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln n}$

   (f) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{(2n+1)\ln(n)}.$

   (g) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^2}$

   (h) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{(2n+1)(\ln(n))^2}.$

   (i) Determine all values of p, q r for which the series
   $$\sum_{n=30}^{\infty}\frac{1}{n^p(\ln n)^q(\ln(\ln n))^r}$$
   is convergent.