Calculus II
Power series expansion related to geometric series

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Recall the geometric series formula:

$$\frac{1}{1-y} = \sum_{n=0}^{\infty} y^n = 1 + y + y^2 + y^3 + \dots \qquad \begin{array}{l}\text{if \& only if}\\ |y| < 1\end{array} \quad .$$

Example

Write $\frac{1}{1+x^2}$ as a power series and find the interval of convergence.

$$\begin{aligned}\frac{1}{1+x^2} &= \frac{1}{1-\left(-x^2\right)} = \sum_{n=0}^{\infty}\left(-x^2\right)^n \quad \left| \begin{array}{l}\text{if \& only if}\\ |-x^2| < 1\end{array}\right. \\ &= 1 + (-x^2) + (-x^2)^2 + (-x^2)^3 + \dots \\ &= 1 \quad - x^2 + x^4 - x^6 + \dots \\ &= \sum_{n=0}^{\infty} (-1)^n x^{2n}\end{aligned}$$

- This converges if and only if $\left| \begin{array}{rcl} |-x^2| & < & 1 \\ |x| & < & 1 \end{array}\right. .$
- Therefore the interval of convergence is $x \in (-1, 1)$.