Calculus I
The algebra behind derivatives

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2019

Outline

1 Understanding computations with derivatives

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We derived the first set of rules by directly computing limits. The second set of rules can be derived from the first set algebraically.

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Let *c* be a constant. Derive the constant multiple rule

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$$

Example

Let n be a positive integer. Derive the negative integer power rule

$$(x^{-n})' = \left(\frac{1}{x^n}\right)' = -nx^{-n-1} = -\frac{n}{x^{n+1}}$$

using the product rule, the constant derivative rule and the power rule for positive integers.

$$
\begin{aligned}
x^n x^{-n} &= 1 & &\Big|\ \tfrac{\mathrm{d}}{\mathrm{d}x} \\
(x^n x^{-n})' &= (1)' \\
(x^n)' x^{-n} + x^n (x^{-n})' &= 0 \\
nx^{n-1} x^{-n} + x^n (x^{-n})' &= 0 \\
\tfrac{n}{x} + x^n (x^{-n})' &= 0 \\
x^n (x^{-n})' &= -\tfrac{n}{x} & &\Big|\ \tfrac{1}{x^n} \\
(x^{-n})' &= -\tfrac{n}{x^{n+1}}
\end{aligned}
$$

Example

For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$

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For positive integer q, derive the power rule $\left(x^{\frac{1}{q}} \right)' = \frac{1}{q} x^{\frac{1}{q}-1}$

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Example

For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$

$$\left(x^{\frac{1}{q}}\right)^q = x \qquad\qquad \left|\; \frac{\mathrm{d}}{\mathrm{d}x}\right.$$

$$\left(\left(x^{\frac{1}{q}}\right)^q\right)' = (x)'$$

Example

For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$ using the rule $(x)' = 1$,

$$\left(x^{\frac{1}{q}}\right)^{q} = x \qquad\qquad \left|\ \frac{\mathrm{d}}{\mathrm{d}x}\right.$$
$$\left(\left(x^{\frac{1}{q}}\right)^{q}\right)' = 1$$

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$$\left(x^{\frac{1}{q}}\right)^q = x$$
$$\left(\left(x^{\frac{1}{q}}\right)^q\right)' = 1$$
$$\left(u^q\right)' = 1$$

$\frac{d}{dx}$

Set $u = x^{\frac{1}{q}}$

Example

For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$ using the
rule $(x)' = 1$, the chain rule

$$
\begin{aligned}
\left(x^{\frac{1}{q}}\right)^q &= x \\
\left(\left(x^{\frac{1}{q}}\right)^q\right)' &= 1 \\
(u^q)' &= 1 \\
\frac{\mathrm{d}}{\mathrm{d}u}(u^q)\, u' &= 1
\end{aligned}
$$

$\frac{\mathrm{d}}{\mathrm{d}x}$

Set $u = x^{\frac{1}{q}}$

Example

For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$ using the rule $(x)' = 1$, the chain rule and the integer power rule $\frac{\mathrm{d}}{\mathrm{d}u}(u^q) = qu^{q-1}$.

$$
\begin{aligned}
\left(x^{\frac{1}{q}}\right)^q &= x \\
\left(\left(x^{\frac{1}{q}}\right)^q\right)' &= 1 \\
(u^q)' &= 1 \\
\frac{\mathrm{d}}{\mathrm{d}u}(u^q)\, u' &= 1 \\
q(u)^{q-1}(u)' &= 1
\end{aligned}
$$

$\left|\begin{array}{l} \frac{\mathrm{d}}{\mathrm{d}x} \\[2mm] \text{Set } u = x^{\frac{1}{q}} \end{array}\right.$

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q(u)^{q-1}(u)' &= 1 \\
q(x^{\frac{1}{q}})^{q-1}\left(x^{\frac{1}{q}}\right)' &= 1
\end{aligned}
$$

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qx^{\frac{q-1}{q}}\left(x^{\frac{1}{q}}\right)' &= 1
\end{aligned}
$$

$\left|\begin{array}{l} \frac{d}{dx} \\[2mm] \text{Set } u = x^{\frac{1}{q}} \end{array}\right.$

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\end{aligned}
$$

$\left.\begin{array}{l} \frac{d}{dx} \\[2mm] \text{Set } u = x^{\frac{1}{q}} \end{array}\right.$

divide by $qx^{\frac{q-1}{q}}$

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\end{aligned}
$$

$\left.\begin{array}{l}\\\\\end{array}\right|$ $\frac{d}{dx}$

Set $u = x^{\frac{1}{q}}$

divide by $qx^{\frac{q-1}{q}}$

Example

For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$ using the rule $(x)' = 1$, the chain rule and the integer power rule $\frac{d}{du}(u^q) = qu^{q-1}$.

$$
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\left(\left(x^{\frac{1}{q}}\right)^q\right)' &= 1 & & \text{Set } u = x^{\frac{1}{q}} \\
(u^q)' &= 1 & & \\
\frac{d}{du}(u^q)\, u' &= 1 & & \\
q(u)^{q-1}(u)' &= 1 & & \\
q(x^{\frac{1}{q}})^{q-1}\left(x^{\frac{1}{q}}\right)' &= 1 & & \\
qx^{\frac{q-1}{q}}\left(x^{\frac{1}{q}}\right)' &= 1 & & \text{divide by } qx^{\frac{q-1}{q}} \\
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\end{aligned}
$$

Example

Derive the quotient rules

$$
\begin{aligned}
\left(\frac{1}{g}\right)' &= -\frac{g'}{g^2} \\
\left(\frac{f}{g}\right)' &= \frac{f'g - fg'}{g^2}
\end{aligned}
$$

Example

Derive the quotient rules

$$\begin{array}{rcl} \left(\dfrac{1}{g}\right)' & = & -\dfrac{g'}{g^2} \\[2mm] \left(\dfrac{f}{g}\right)' & = & \dfrac{f'g - fg'}{g^2} \end{array}$$

$$\left(\frac{1}{g}\right)' =$$

Example

Derive the quotient rules

$$\left(\frac{1}{g}\right)' = -\frac{g'}{g^2}$$
$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

using the chain rule,

$$\left(\frac{1}{g}\right)' = \frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{1}{g}\right)g' =$$

Example

Derive the quotient rules

$$\left(\frac{1}{g}\right)' = -\frac{g'}{g^2}$$
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using the chain rule, the negative power rule

$$\left(\frac{1}{g}\right)' = \frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{1}{g}\right)g' = -\frac{1}{g^2}g'$$

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$$\left(\frac{f}{g}\right)'$$

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$$\left(\frac{1}{g}\right)' = \frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{1}{g}\right)g' = -\frac{1}{g^2}g' \qquad \text{as desired}$$

$$\left(\frac{f}{g}\right)' = \left(f\frac{1}{g}\right)' =$$

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Derive the quotient rules

$$\begin{aligned}
\left(\frac{1}{g}\right)' &= -\frac{g'}{g^2} \\
\left(\frac{f}{g}\right)' &= \frac{f'g - fg'}{g^2}
\end{aligned}$$

using the chain rule, the negative power rule and the product rule.

$$\left(\frac{1}{g}\right)' = \frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{1}{g}\right)g' = -\frac{1}{g^2}g' \qquad \text{as desired}$$

$$\left(\frac{f}{g}\right)' = \left(f\frac{1}{g}\right)' = f'\frac{1}{g} + f\left(\frac{1}{g}\right)' =$$

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Derive the quotient rules

$$\left(\frac{1}{g}\right)' = -\frac{g'}{g^2}$$
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$$= \frac{f'g - fg'}{g^2} \qquad \Big| \text{ as desired}$$

Example

Derive the exponent rule $(e^x)' = e^x$

Example

Derive the exponent rule $(e^x)' = e^x$ using the Calc II formula below,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots,$$

where $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$.

$$(e^x)' = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \right)'$$

Example

Derive the exponent rule $(e^x)' = e^x$ using the Calc II formula below, the infinite (both sides uniformly convergent) sum rule
$(f_1 + f_2 + f_3 + \dots)' = f_1' + f_2' + f_3' + \dots$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots,$$

where $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$.

$$
\begin{aligned}
(e^x)' &= \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \right)' \\
&= (1)' + (x)' + \frac{(x^2)'}{2!} + \frac{(x^3)'}{3!} + \dots + \frac{(x^n)'}{n!} + \dots
\end{aligned}
$$

Example

Derive the exponent rule $(e^x)' = e^x$ using the Calc II formula below, the infinite (both sides uniformly convergent) sum rule $(f_1 + f_2 + f_3 + \dots)' = f_1' + f_2' + f_3' + \dots$ and the power rule $(x^n)' = nx^{n-1}$.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots,$$

where $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$.

$$
\begin{aligned}
(e^x)' &= \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\right)' \\
&= (1)' + (x)' + \frac{(x^2)'}{2!} + \frac{(x^3)'}{3!} + \dots + \frac{(x^n)'}{n!} + \dots \\
&= 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \dots + \frac{nx^{n-1}}{n!} + \dots
\end{aligned}
$$

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Derive the exponent rule $(e^x)' = e^x$ using the Calc II formula below, the infinite (both sides uniformly convergent) sum rule $(f_1 + f_2 + f_3 + \dots)' = f_1' + f_2' + f_3' + \dots$ and the power rule $(x^n)' = nx^{n-1}$.

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$\frac{n}{n!} =$

$$
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Example

Derive the exponent rule $(e^x)' = e^x$ using the Calc II formula below, the infinite (both sides uniformly convergent) sum rule $(f_1 + f_2 + f_3 + \dots)' = f_1' + f_2' + f_3' + \dots$ and the power rule $(x^n)' = nx^{n-1}$.

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&= 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^{n-1}}{(n-1)!} + \dots =
\end{aligned}$$

Example

Derive the exponent rule $(e^x)' = e^x$ using the Calc II formula below, the infinite (both sides uniformly convergent) sum rule $(f_1 + f_2 + f_3 + \dots)' = f_1' + f_2' + f_3' + \dots$ and the power rule $(x^n)' = nx^{n-1}$.

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\begin{aligned}
(e^x)' &= \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\right)' \\
&= (1)' + (x)' + \frac{(x^2)'}{2!} + \frac{(x^3)'}{3!} + \dots + \frac{(x^n)'}{n!} + \dots \\
&= 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \dots + \frac{nx^{n-1}}{n!} + \dots \\
&= 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^{n-1}}{(n-1)!} + \dots = e^x
\end{aligned}
$$

as desired.

Example

Derive the logarithm derivative rules

$$
\begin{array}{rcl}
(\ln x)' &=& \frac{1}{x} \\
(\log_a x)' &=& \frac{1}{x \ln a}
\end{array}
$$

Example

Derive the logarithm derivative rules

$$\begin{array}{rcl}
(\ln x)' & = & \frac{1}{x} \\
(\log_a x)' & = & \frac{1}{x \ln a}
\end{array}$$

$$e^{\ln x} = x$$

Example

Derive the logarithm derivative rules

$$\begin{array}{rcl}
(\ln x)' &=& \frac{1}{x} \\
(\log_a x)' &=& \frac{1}{x \ln a}
\end{array}$$

$$\begin{array}{rcl}
e^{\ln x} &=& x \\
e^u &=& x
\end{array} \qquad \Big|\ \text{set}\quad u = \ln x$$

Example

Derive the logarithm derivative rules

$$\begin{array}{rcl} (\ln x)' &=& \frac{1}{x} \\ (\log_a x)' &=& \frac{1}{x \ln a} \end{array}$$

$$\begin{array}{rcl} e^{\ln x} &=& x \\ e^{u} &=& x \end{array} \qquad\qquad \left| \begin{array}{l} \text{set} \quad u = \ln x \\ \frac{\mathrm{d}}{\mathrm{d}x} \end{array} \right.$$

Example

Derive the logarithm derivative rules

$$\begin{aligned} (\ln x)' &= \frac{1}{x} \\ (\log_a x)' &= \frac{1}{x \ln a} \end{aligned}$$

using the chain rule,

$$\begin{aligned} e^{\ln x} &= x \\ e^u &= x \\ \frac{\mathrm{d}}{\mathrm{d}u}(e^u)u' &= (x)' \end{aligned} \qquad\qquad \left| \begin{array}{l} \text{set} \quad u = \ln x \\ \frac{\mathrm{d}}{\mathrm{d}x} \end{array} \right.$$

Example

Derive the logarithm derivative rules

$$
\begin{aligned}
(\ln x)' &= \tfrac{1}{x} \\
(\log_a x)' &= \tfrac{1}{x \ln a}
\end{aligned}
$$

using the chain rule, the exponent derivative rule $(e^x)' = e^x$,

$$
\begin{aligned}
e^{\ln x} &= x \\
e^u &= x \\
\tfrac{\mathrm{d}}{\mathrm{d}u}(e^u)u' &= (x)' \\
e^u u' &= 1
\end{aligned}
\qquad
\left|
\begin{array}{l}
\text{set} \quad u = \ln x \\
\tfrac{\mathrm{d}}{\mathrm{d}x}
\end{array}
\right.
$$

Example

Derive the logarithm derivative rules

$$
\begin{array}{rcl}
(\ln x)' &=& \frac{1}{x} \\
(\log_a x)' &=& \frac{1}{x \ln a}
\end{array}
$$

using the chain rule, the exponent derivative rule $(e^x)' = e^x$, the rule $(x)' = 1$

$$
\begin{array}{rcl}
e^{\ln x} &=& x \\
e^u &=& x \\
\frac{\mathrm{d}}{\mathrm{d}u}(e^u)u' &=& (x)' \\
e^u u' &=& 1
\end{array}
\qquad
\left|
\begin{array}{l}
\text{set} \quad u = \ln x \\
\frac{\mathrm{d}}{\mathrm{d}x}
\end{array}
\right.
$$

Example

Derive the logarithm derivative rules

$$
\begin{array}{rcl}
(\ln x)' & = & \frac{1}{x} \\
(\log_a x)' & = & \frac{1}{x \ln a}
\end{array}
$$

using the chain rule, the exponent derivative rule $(e^x)' = e^x$, the rule $(x)' = 1$

$$
\begin{array}{rcl}
e^{\ln x} & = & x \\
e^u & = & x \\
\frac{\mathrm{d}}{\mathrm{d}u}(e^u)u' & = & (x)' \\
e^u u' & = & 1 \\
e^{\ln x}(\ln x)' & = & 1
\end{array}
$$

$$
\left|
\begin{array}{l}
\text{set} \quad u = \ln x \\
\frac{\mathrm{d}}{\mathrm{d}x}
\end{array}
\right.
$$

Example

Derive the logarithm derivative rules

$$\begin{array}{rcl} (\ln x)' & = & \frac{1}{x} \\ (\log_a x)' & = & \frac{1}{x \ln a} \end{array}$$

using the chain rule, the exponent derivative rule $(e^x)' = e^x$, the rule $(x)' = 1$

$$\begin{array}{rcl} e^{\ln x} & = & x \\ e^u & = & x \\ \frac{\mathrm{d}}{\mathrm{d}u}(e^u)u' & = & (x)' \\ e^u u' & = & 1 \\ \textcolor{red}{e^{\ln x}}(\ln x)' & = & 1 \\ \textcolor{red}{x}(\ln x)' & = & 1 \end{array} \qquad \left| \begin{array}{l} \text{set} \quad u = \ln x \\ \frac{\mathrm{d}}{\mathrm{d}x} \end{array} \right.$$

Example

Derive the logarithm derivative rules

$$\begin{array}{rcl} (\ln x)' & = & \frac{1}{x} \\ (\log_a x)' & = & \frac{1}{x \ln a} \end{array}$$

using the chain rule, the exponent derivative rule $(e^x)' = e^x$, the rule $(x)' = 1$

$$\begin{array}{rcl} e^{\ln x} & = & x \\ e^u & = & x \\ \frac{\mathrm{d}}{\mathrm{d}u}(e^u)u' & = & (x)' \\ e^u u' & = & 1 \\ e^{\ln x}(\ln x)' & = & 1 \\ x(\ln x)' & = & 1 \\ (\ln x)' & = & \frac{1}{x} \end{array} \qquad \begin{array}{l} \left| \begin{array}{l} \text{set} \quad u = \ln x \\ \frac{\mathrm{d}}{\mathrm{d}x} \end{array} \right. \\ \\ \\ \\ \left| \; \cdot \frac{1}{x} \right. \end{array}$$

Example

Derive the logarithm derivative rules

$$
\begin{aligned}
(\ln x)' &= \frac{1}{x} \\
(\log_a x)' &= \frac{1}{x \ln a}
\end{aligned}
$$

using the chain rule, the exponent derivative rule $(e^x)' = e^x$, the rule $(x)' = 1$

$$
\begin{array}{rcl|l}
e^{\ln x} &=& x & \text{set} \quad u = \ln x \\
e^u &=& x & \frac{\mathrm{d}}{\mathrm{d}x} \\
\frac{\mathrm{d}}{\mathrm{d}u}(e^u)u' &=& (x)' & \\
e^u u' &=& 1 & \\
e^{\ln x}(\ln x)' &=& 1 & \\
x(\ln x)' &=& 1 & \cdot\frac{1}{x} \\
(\ln x)' &=& \dfrac{1}{x} & \text{as desired}
\end{array}
$$

Example

Derive the logarithm derivative rules

$$
\begin{array}{rcl}
(\ln x)' & = & \frac{1}{x} \\
(\log_a x)' & = & \frac{1}{x \ln a}
\end{array}
$$

using the chain rule, the exponent derivative rule $(e^x)' = e^x$, the rule $(x)' = 1$

$$
\begin{array}{rcl}
e^{\ln x} & = & x \\
e^u & = & x \\
\frac{\mathrm{d}}{\mathrm{d}u}(e^u)u' & = & (x)' \\
e^u u' & = & 1 \\
e^{\ln x}(\ln x)' & = & 1 \\
x(\ln x)' & = & 1 \\
(\ln x)' & = & \frac{1}{x} \\
\hline
(\log_a x)' & = &
\end{array}
$$

$\left.\begin{array}{l} \text{set} \quad u = \ln x \\ \frac{\mathrm{d}}{\mathrm{d}x} \end{array}\right.$

$\cdot \frac{1}{x}$

as desired

Example

Derive the logarithm derivative rules

$$
\begin{array}{rcl}
(\ln x)' & = & \frac{1}{x} \\
(\log_a x)' & = & \frac{1}{x \ln a}
\end{array}
$$

using the chain rule, the exponent derivative rule $(e^x)' = e^x$, the rule $(x)' = 1$

$$
\begin{array}{rcl}
e^{\ln x} & = & x \\
e^{u} & = & x \\
\frac{d}{du}(e^u)u' & = & (x)' \\
e^u u' & = & 1 \\
e^{\ln x}(\ln x)' & = & 1 \\
x(\ln x)' & = & 1 \\
(\ln x)' & = & \frac{1}{x} \\
\hline
(\log_a x)' & = & \left(\frac{\ln x}{\ln a}\right)' =
\end{array}
$$

$\left|\begin{array}{l} \text{set} \quad u = \ln x \\ \frac{d}{dx} \end{array}\right.$

$\cdot \frac{1}{x}$

as desired

Example

Derive the logarithm derivative rules

$$\begin{array}{rcl} (\ln x)' & = & \frac{1}{x} \\ (\log_a x)' & = & \frac{1}{x \ln a} \end{array}$$

using the chain rule, the exponent derivative rule $(e^x)' = e^x$, the rule $(x)' = 1$ and the constant multiple rule $(cf)' = cf'$.

$$\begin{array}{rcl|l} e^{\ln x} & = & x & \text{set} \quad u = \ln x \\ e^u & = & x & \frac{d}{dx} \\ \frac{d}{du}(e^u)u' & = & (x)' & \\ e^u u' & = & 1 & \\ e^{\ln x}(\ln x)' & = & 1 & \\ x(\ln x)' & = & 1 & \cdot \frac{1}{x} \\ (\ln x)' & = & \dfrac{1}{x} & \text{as desired} \\ \hline (\log_a x)' & = & \left(\dfrac{\ln x}{\ln a}\right)' = \dfrac{(\ln x)'}{\ln a} = \dfrac{1}{x \ln a} & \end{array}$$

Example

Derive the logarithm derivative rules

$$\begin{aligned}
(\ln x)' &= \frac{1}{x} \\
(\log_a x)' &= \frac{1}{x \ln a}
\end{aligned}$$

using the chain rule, the exponent derivative rule $(e^x)' = e^x$, the rule $(x)' = 1$ and the constant multiple rule $(cf)' = cf'$.

$$\begin{array}{rcl|l}
e^{\ln x} &=& x & \text{set} \quad u = \ln x \\
e^u &=& x & \frac{d}{dx} \\
\frac{d}{du}(e^u)u' &=& (x)' & \\
e^u u' &=& 1 & \\
e^{\ln x}(\ln x)' &=& 1 & \\
x(\ln x)' &=& 1 & \cdot \frac{1}{x} \\
(\ln x)' &=& \dfrac{1}{x} & \text{as desired} \\
\hline
(\log_a x)' &=& \left(\frac{\ln x}{\ln a}\right)' = \frac{(\ln x)'}{\ln a} = \frac{1}{x \ln a} &
\end{array}$$

Example

Derive the logarithm derivative rules

$$\begin{array}{rcl} (\ln x)' &=& \frac{1}{x} \\ (\log_a x)' &=& \frac{1}{x \ln a} \end{array}$$

using the chain rule, the exponent derivative rule $(e^x)' = e^x$, the rule $(x)' = 1$ and the constant multiple rule $(cf)' = cf'$.

$$
\begin{array}{rcl|l}
e^{\ln x} &=& x & \text{set} \quad u = \ln x \\
e^u &=& x & \frac{d}{dx} \\
\frac{d}{du}(e^u)u' &=& (x)' & \\
e^u u' &=& 1 & \\
e^{\ln x}(\ln x)' &=& 1 & \\
x(\ln x)' &=& 1 & \cdot \frac{1}{x} \\
(\ln x)' &=& \frac{1}{x} & \text{as desired} \\
\hline
(\log_a x)' &=& \left(\frac{\ln x}{\ln a}\right)' = \frac{(\ln x)'}{\ln a} = \frac{1}{x \ln a} & \text{as desired}
\end{array}
$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

$$(x^r)' =$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

$$(x^r)' \;=\; \left((e^{\ln x})^r\right)' =$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

$$(x^r)' = \left((e^{\ln x})^r\right)' = \left(e^{r \ln x}\right)'$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

$$(x^r)' = \left((e^{\ln x})^r\right)' = \left(e^{r \ln x}\right)' \qquad \text{Set } u = r \ln x$$

$$= (e^u)' =$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

using the chain rule,

$$
\begin{aligned}
(x^r)' &= \left((e^{\ln x})^r\right)' = \left(e^{r\ln x}\right)' & &\left|\ \text{Set } u = r\ln x\right. \\[2ex]
&= (e^u)' = \frac{\mathrm{d}}{\mathrm{d}u}(e^u)\, u' =
\end{aligned}
$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

using the chain rule, the the rule $(e^x)' = e^x$,

$$(x^r)' = \left((e^{\ln x})^r\right)' = \left(e^{r\ln x}\right)' \qquad \Big| \text{ Set } u = r\ln x$$

$$= (e^u)' = \frac{d}{du}(e^u)\, u' = e^u u' =$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

using the chain rule, the the rule $(e^x)' = e^x$,

$$
\begin{aligned}
(x^r)' &= \left((e^{\ln x})^r\right)' = \left(e^{r\ln x}\right)' && \text{Set } u = r\ln x \\[2mm]
&= (e^u)' = \frac{\mathrm{d}}{\mathrm{d}u}(e^u)\, u' = e^u u' = \\[2mm]
&= e^{r\ln x}(r\ln x)' =
\end{aligned}
$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

using the chain rule, the the rule $(e^x)' = e^x$,

$$
\begin{aligned}
(x^r)' &= \left((e^{\ln x})^r\right)' = \left(e^{r \ln x}\right)' & \quad \text{Set } u = r \ln x \\[2mm]
&= (e^u)' = \frac{\mathrm{d}}{\mathrm{d}u}(e^u)\, u' = e^u u' = \\[2mm]
&= e^{r \ln x}(r \ln x)' = \left(e^{\ln x}\right)^r r(\ln x)'
\end{aligned}
$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

using the chain rule, the the rule $(e^x)' = e^x$, the constant multiple derivative rule

$$
\begin{aligned}
(x^r)' &= \left((e^{\ln x})^r\right)' = \left(e^{r\ln x}\right)' & \text{Set } u = r\ln x \\[2mm]
&= (e^u)' = \frac{d}{du}(e^u)\, u' = e^u u' = \\[2mm]
&= e^{r\ln x}(r\ln x)' = \left(e^{\ln x}\right)^r r(\ln x)'
\end{aligned}
$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

using the chain rule, the the rule $(e^x)' = e^x$, the constant multiple derivative rule

$$
\begin{aligned}
(x^r)' &= \left((e^{\ln x})^r\right)' = \left(e^{r\ln x}\right)' & & \text{Set } u = r\ln x \\
&= (e^u)' = \frac{d}{du}(e^u)\, u' = e^u u' = \\
&= e^{r\ln x}(r\ln x)' = \left(e^{\ln x}\right)^r r(\ln x)' \\
&= x^r r\frac{1}{x} =
\end{aligned}
$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

using the chain rule, the the rule $(e^x)' = e^x$, the constant multiple derivative rule and the logarithm derivative rule $(\ln x)' = \frac{1}{x}$.

$$
\begin{aligned}
(x^r)' &= \left((e^{\ln x})^r \right)' = \left(e^{r \ln x} \right)' & \quad \left| \; \text{Set } u = r \ln x \right. \\[2mm]
&= (e^u)' = \frac{d}{du}(e^u)\, u' = e^u u' = \\[2mm]
&= e^{r \ln x}(r \ln x)' = \left(e^{\ln x} \right)^r r(\ln x)' \\[2mm]
&= x^r r \frac{1}{x} =
\end{aligned}
$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

using the chain rule, the the rule $(e^x)' = e^x$, the constant multiple derivative rule and the logarithm derivative rule $(\ln x)' = \frac{1}{x}$.

$$
\begin{aligned}
(x^r)' &= \left((e^{\ln x})^r \right)' = \left(e^{r \ln x} \right)' & \quad \left| \text{ Set } u = r \ln x \right. \\
&= (e^u)' = \frac{d}{du}(e^u)\, u' = e^u u' = \\
&= e^{r \ln x}(r \ln x)' = \left(e^{\ln x} \right)^r r(\ln x)' \\
&= x^r r \frac{1}{x} = rx^{r-1}
\end{aligned}
$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

using the chain rule, the the rule $(e^x)' = e^x$, the constant multiple derivative rule and the logarithm derivative rule $(\ln x)' = \frac{1}{x}$.

$$
\begin{aligned}
(x^r)' &= \left((e^{\ln x})^r\right)' = \left(e^{r\ln x}\right)' & \Big| \text{ Set } u = r\ln x \\[2mm]
&= (e^u)' = \frac{d}{du}(e^u)\, u' = e^u u' = \\[2mm]
&= e^{r\ln x}(r\ln x)' = \left(e^{\ln x}\right)^r r(\ln x)' \\[2mm]
&= x^r r\tfrac{1}{x} = rx^{r-1} & \Big| \text{ as desired}
\end{aligned}
$$

Example

Derive the sine and cosine rules

$$\begin{array}{rcl} (\sin x)' & = & \cos x \\ (\cos x)' & = & -\sin x \end{array}$$

Example

Derive the sine and cosine rules

$$(\sin x)' = \cos x$$
$$(\cos x)' = -\sin x$$

using Euler's formula,

$$e^{ix} = \cos x + i \sin x$$

Example

Derive the sine and cosine rules

$$(\sin x)' = \cos x$$
$$(\cos x)' = -\sin x$$

using Euler's formula,

$$
\begin{array}{rcl}
e^{ix} & = & \cos x + i \sin x \\
\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{ix}\right) & = & \frac{\mathrm{d}}{\mathrm{d}x}\left(\cos x + i \sin x\right)
\end{array}
\quad \Big| \; \frac{\mathrm{d}}{\mathrm{d}x}
$$

Example

Derive the sine and cosine rules

$$
\begin{aligned}
(\sin x)' &= \cos x \\
(\cos x)' &= -\sin x
\end{aligned}
$$

using Euler's formula, the exponent derivative rule, the chain rule,

$$
\begin{aligned}
e^{ix} &= \cos x + i \sin x & \Big| \ \tfrac{\mathrm{d}}{\mathrm{d}x} \\
\tfrac{\mathrm{d}}{\mathrm{d}x}\left(e^{ix}\right) &= \tfrac{\mathrm{d}}{\mathrm{d}x}\left(\cos x + i \sin x\right) \\
e^{ix}(ix)' &=
\end{aligned}
$$

Example

Derive the sine and cosine rules

$$
\begin{array}{rcl}
(\sin x)' &=& \cos x \\
(\cos x)' &=& -\sin x
\end{array}
$$

using Euler's formula, the exponent derivative rule, the chain rule,
Assume all rules are valid over

the complex numbers $\mathbb{C}$.

$$
\begin{array}{rcl}
e^{ix} &=& \cos x + i \sin x \\
\frac{d}{dx}\left(e^{ix}\right) &=& \frac{d}{dx}\left(\cos x + i \sin x\right) \\
e^{ix}(ix)' &=&
\end{array} \quad \Big| \ \frac{d}{dx}
$$

Example

Derive the sine and cosine rules

$$
\begin{array}{rcl}
(\sin x)' & = & \cos x \\
(\cos x)' & = & -\sin x
\end{array}
$$

using Euler's formula, the exponent derivative rule, the chain rule, the sum rule and the constant multiple rule. Assume all rules are valid over the complex numbers $\mathbb{C}$.

$$
\begin{array}{rcll}
e^{ix} & = & \cos x + i\sin x & \Big|\ \frac{\mathrm{d}}{\mathrm{d}x} \\
\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{ix}\right) & = & \frac{\mathrm{d}}{\mathrm{d}x}\left(\cos x + i\sin x\right) & \\
e^{ix}(ix)' & = & (\cos x)' + i(\sin x)' &
\end{array}
$$

Example

Derive the sine and cosine rules

$$
\begin{array}{rcl}
(\sin x)' & = & \cos x \\
(\cos x)' & = & -\sin x
\end{array}
$$

using Euler's formula, the exponent derivative rule, the chain rule, the sum rule and the constant multiple rule. Assume all rules are valid over the complex numbers $\mathbb{C}$.

$$
\begin{array}{rcll}
e^{ix} & = & \cos x + i\sin x & \Big|\ \frac{\mathrm{d}}{\mathrm{d}x} \\
\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{ix}\right) & = & \frac{\mathrm{d}}{\mathrm{d}x}\left(\cos x + i\sin x\right) & \\
e^{ix}(ix)' & = & (\cos x)' + i(\sin x)' & \\
ie^{ix} & = & (\cos x)' + i(\sin x)' &
\end{array}
$$

Example

Derive the sine and cosine rules

$$\begin{aligned}
(\sin x)' &= \cos x \\
(\cos x)' &= -\sin x
\end{aligned}$$

using Euler's formula, the exponent derivative rule, the chain rule, the sum rule and the constant multiple rule. Assume all rules are valid over the complex numbers $\mathbb{C}$.

$$\begin{aligned}
e^{ix} &= \cos x + i\sin x & \Big|\ \tfrac{\mathrm{d}}{\mathrm{d}x} \\
\tfrac{\mathrm{d}}{\mathrm{d}x}\left(e^{ix}\right) &= \tfrac{\mathrm{d}}{\mathrm{d}x}\left(\cos x + i\sin x\right) \\
e^{ix}(ix)' &= (\cos x)' + i(\sin x)' \\
ie^{ix} &= (\cos x)' + i(\sin x)' \\
i(\cos x + i\sin x) &= (\cos x)' + i(\sin x)'
\end{aligned}$$

Example

Derive the sine and cosine rules

$$
\begin{array}{rcl}
(\sin x)' & = & \cos x \\
(\cos x)' & = & -\sin x
\end{array}
$$

using Euler's formula, the exponent derivative rule, the chain rule, the sum rule and the constant multiple rule. Assume all rules are valid over the complex numbers $\mathbb{C}$.

$$
\begin{array}{rcll}
e^{ix} & = & \cos x + i \sin x & \Big| \ \frac{d}{dx} \\
\frac{d}{dx}\left(e^{ix}\right) & = & \frac{d}{dx}\left(\cos x + i \sin x\right) & \\
e^{ix}(ix)' & = & (\cos x)' + i(\sin x)' & \\
ie^{ix} & = & (\cos x)' + i(\sin x)' & \\
i^2 \sin x + i \cos x = i(\cos x + i \sin x) & = & (\cos x)' + i(\sin x)' &
\end{array}
$$

Example

Derive the sine and cosine rules

$$\begin{aligned}(\sin x)' &= \cos x \\ (\cos x)' &= -\sin x\end{aligned}$$

using Euler's formula, the exponent derivative rule, the chain rule, the sum rule and the constant multiple rule. Assume all rules are valid over the complex numbers $\mathbb{C}$.

$$\begin{aligned}e^{ix} &= \cos x + i\sin x & \Big|\ \tfrac{\mathrm{d}}{\mathrm{d}x} \\ \tfrac{\mathrm{d}}{\mathrm{d}x}\left(e^{ix}\right) &= \tfrac{\mathrm{d}}{\mathrm{d}x}\left(\cos x + i\sin x\right) \\ e^{ix}(ix)' &= (\cos x)' + i(\sin x)' \\ ie^{ix} &= (\cos x)' + i(\sin x)' \\ i^2\sin x + i\cos x = i(\cos x + i\sin x) &= (\cos x)' + i(\sin x)' \\ -\sin x + i\cos x &= (\cos x)' + i(\sin x)'\end{aligned}$$

Example

Derive the sine and cosine rules

$$
\begin{aligned}
(\sin x)' &= \cos x \\
(\cos x)' &= -\sin x
\end{aligned}
$$

using Euler's formula, the exponent derivative rule, the chain rule, the sum rule and the constant multiple rule. Assume all rules are valid over the complex numbers $\mathbb{C}$.

$$
\begin{aligned}
e^{ix} &= \cos x + i \sin x & \Big| \ \tfrac{\mathrm{d}}{\mathrm{d}x} \\
\tfrac{\mathrm{d}}{\mathrm{d}x}(e^{ix}) &= \tfrac{\mathrm{d}}{\mathrm{d}x}(\cos x + i \sin x) \\
e^{ix}(ix)' &= (\cos x)' + i(\sin x)' \\
ie^{ix} &= (\cos x)' + i(\sin x)' \\
i^2 \sin x + i \cos x = i(\cos x + i \sin x) &= (\cos x)' + i(\sin x)' \\
-\sin x + i \cos x &= (\cos x)' + i(\sin x)'
\end{aligned}
$$

Compare real part   to get the desired equalities.

Example

Derive the sine and cosine rules

$$(\sin x)' = \cos x$$
$$(\cos x)' = -\sin x$$

using Euler's formula, the exponent derivative rule, the chain rule, the sum rule and the constant multiple rule. Assume all rules are valid over the complex numbers $\mathbb{C}$.

$$
\begin{aligned}
e^{ix} &= \cos x + i\sin x & \Big| \ \tfrac{d}{dx} \\
\tfrac{d}{dx}(e^{ix}) &= \tfrac{d}{dx}(\cos x + i\sin x) \\
e^{ix}(ix)' &= (\cos x)' + i(\sin x)' \\
ie^{ix} &= (\cos x)' + i(\sin x)' \\
i^2\sin x + i\cos x = i(\cos x + i\sin x) &= (\cos x)' + i(\sin x)' \\
-\sin x + i\cos x &= (\cos x)' + i(\sin x)'
\end{aligned}
$$

Compare real part and coefficients of i to get the desired equalities.