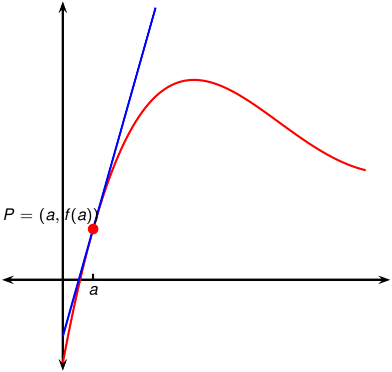
Calculus I
Reference: tangents to graphs of functions

Todor Milev

2019



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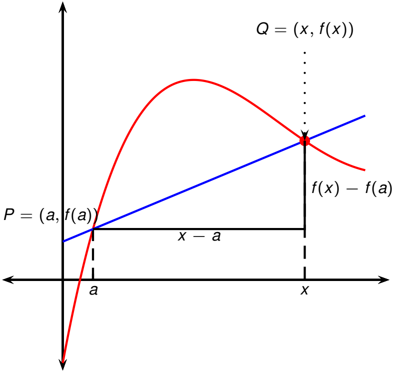

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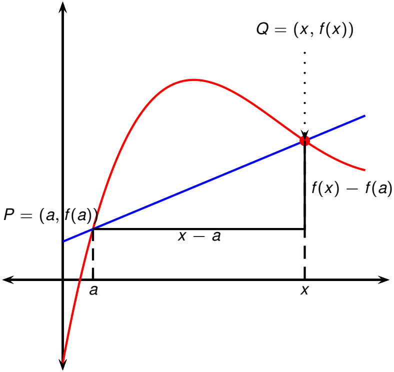

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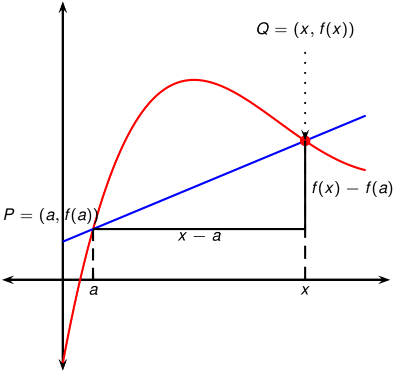

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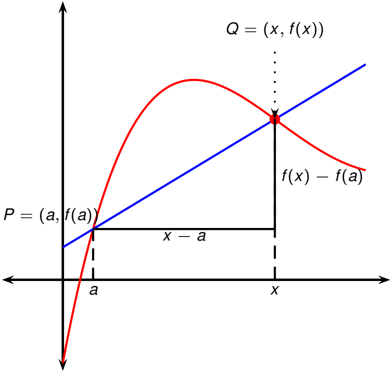

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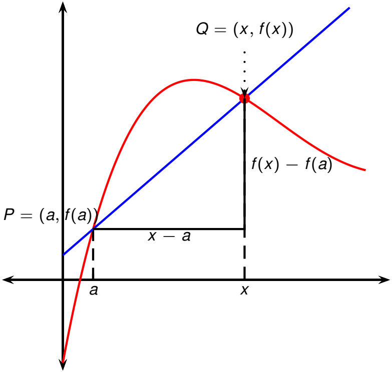

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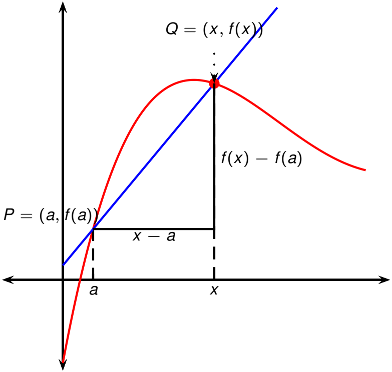

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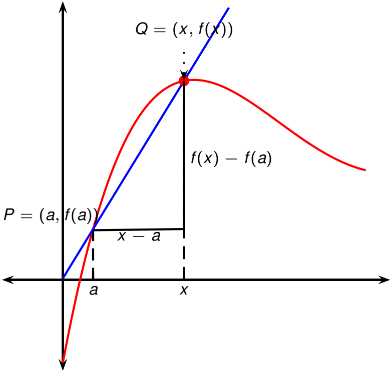

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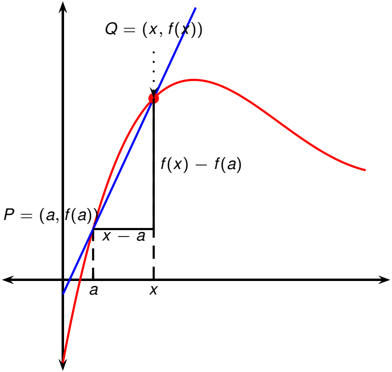

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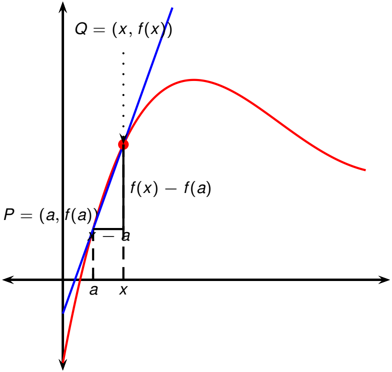

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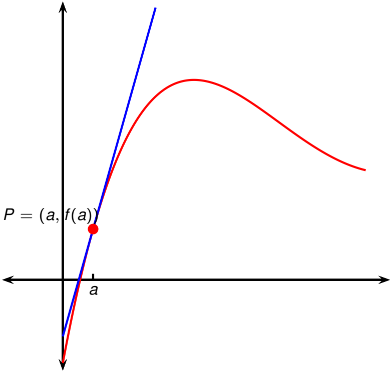

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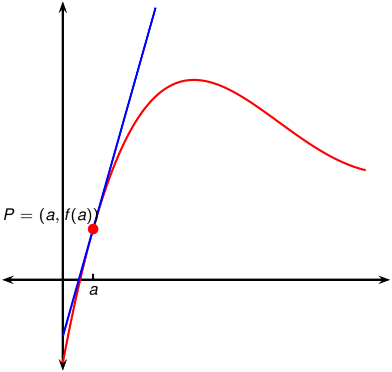
Definition (Non-vertical tangent line)

Let $P = (a, f(a))$. Suppose the limit $m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ exists. Define the tangent to $y = f(x)$ at P to be the line passing through P with slope m



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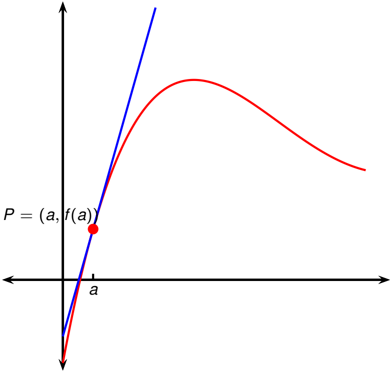

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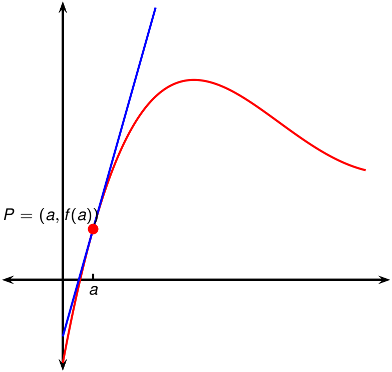

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Note. Even if the limit does not exist a reasonable notion of a tangent line may still exist.

Derivatives

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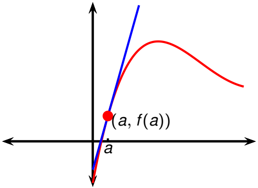
The derivative of a function f at a number a, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if the limit exists.



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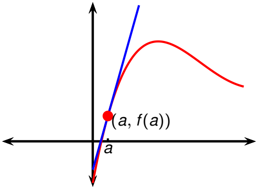

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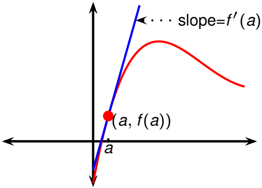

- The two alternative formulas result in equivalent definitions.
- Equivalent formulation. The derivative $f'(a)$ is the slope of the tangent line to $y = f(x)$ at $(a, f(a))$, provided that tangent line exists and is non-vertical.