Calculus I
Reference: Computing a Riemann Sum limit directly, part 1

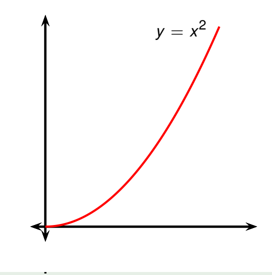
Todor Milev

2019

Example

For the region S underneath the parabola $y = x^2$ from 0 to 1, show that the area under the approximating rectangles approaches $\frac{1}{3}$, that is,

$$\lim_{n\to\infty} R_n = \frac{1}{3}.$$



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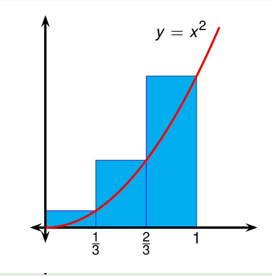

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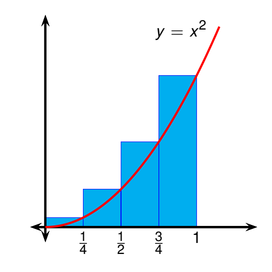

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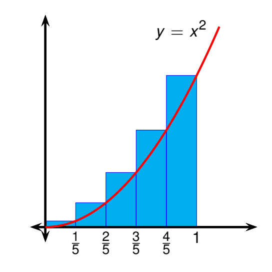

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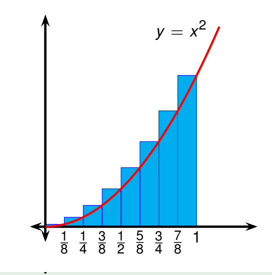

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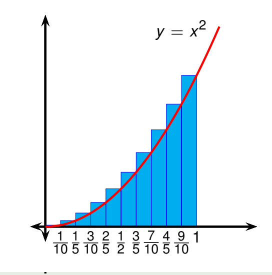

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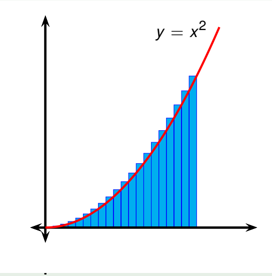

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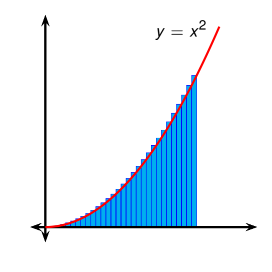

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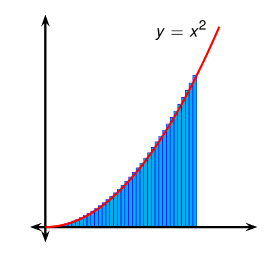

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- The heights are **?** , **?** , . . . , **?** .

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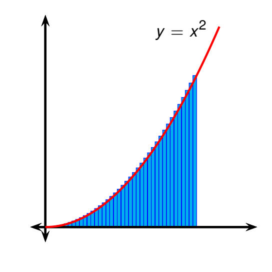

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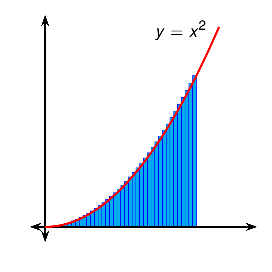

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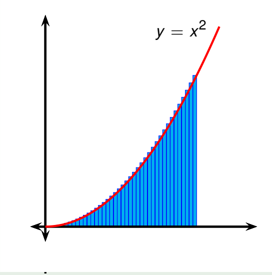

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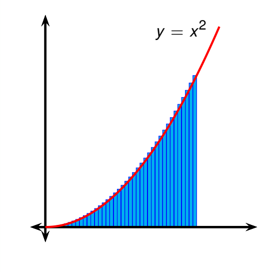

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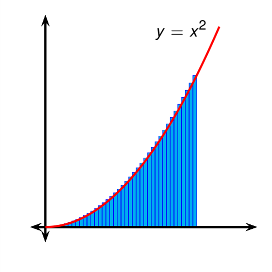

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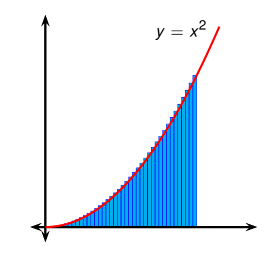

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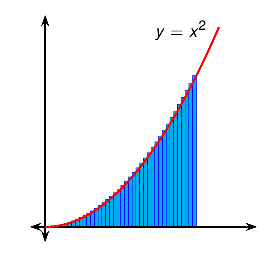

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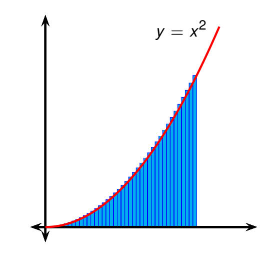

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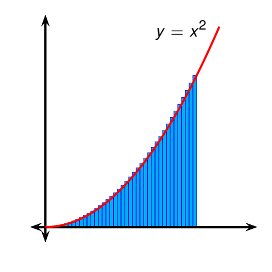

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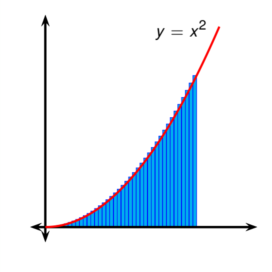

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