Calculus II
Ratio test basic

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Example

Test the series $\sum_{n=1}^{\infty}(-1)^n \frac{n^3}{3^n}$ for absolute convergence.

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Therefore the series is by the Ratio Test.

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&= \frac{1}{3}\left(\frac{n+1}{n}\right)^3 \\
&= \frac{1}{3}\left(1+\frac{1}{n}\right)^3 \\
&\to \frac{1}{3} < 1
\end{aligned}$$

Therefore the series is absolutely convergent by the Ratio Test.