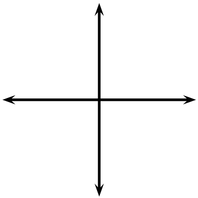
Calculus I
Plotting curves defined by implicit equations

Todor Milev

2019

(Elementary Computer algorithm for sketching graphs)

Let H-continuous; is there simple algorithm to sketch $H(x, y) = 0$?



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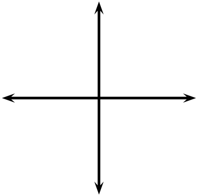

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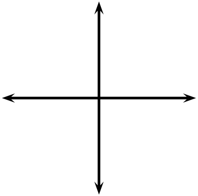

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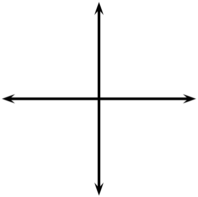

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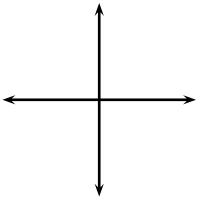

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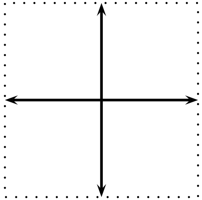

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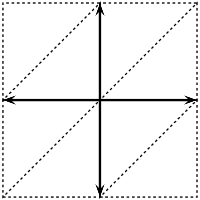

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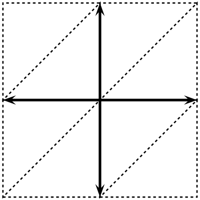

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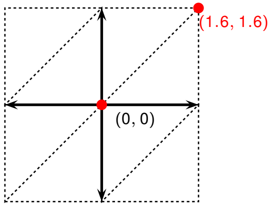

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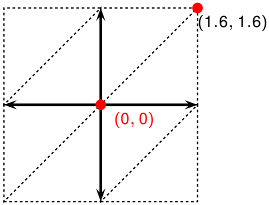

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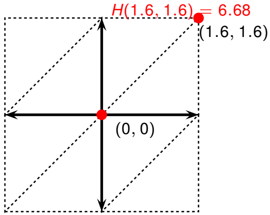

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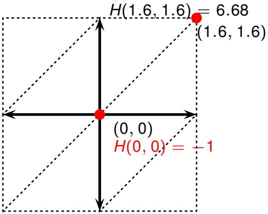

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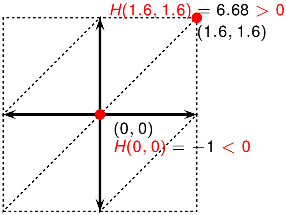

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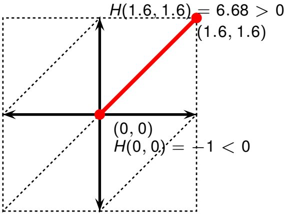

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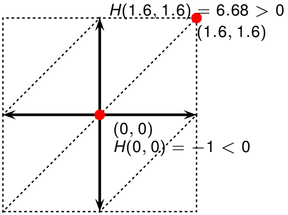

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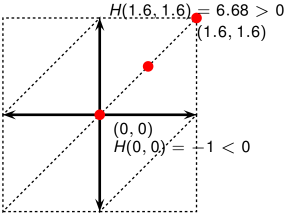

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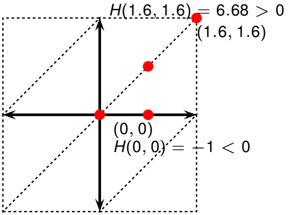

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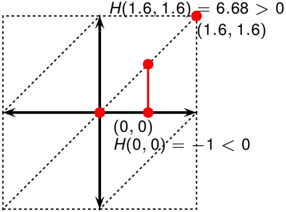

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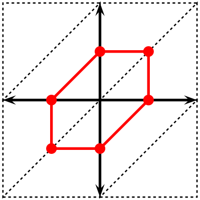

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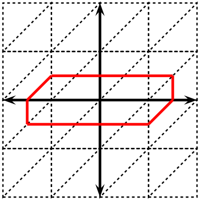

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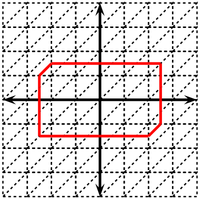

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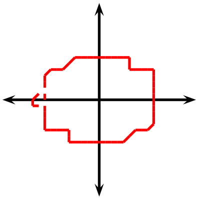

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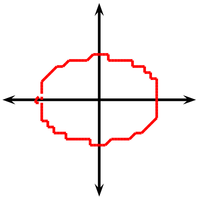

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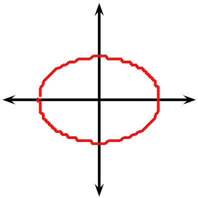

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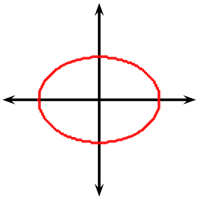

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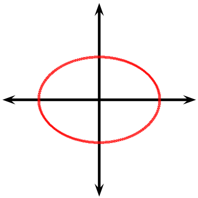

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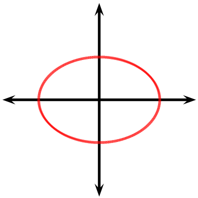

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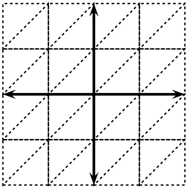

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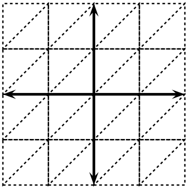

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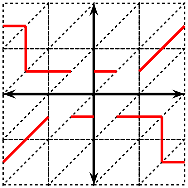

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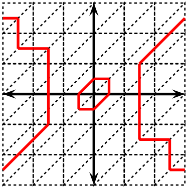

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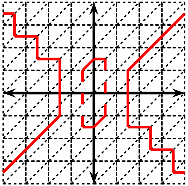

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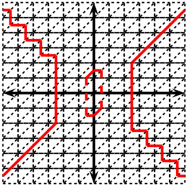

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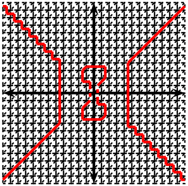

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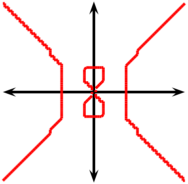

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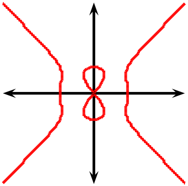

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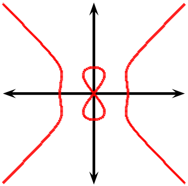

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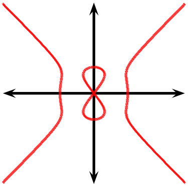

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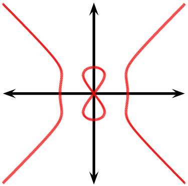

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