Arithmetics
Subtraction base 10
calculator-algebra.org

Todor Milev

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Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$
\begin{array}{rcl|l}
5 - 3 & = & 2 & \text{because } 3 + 2 = 5 \\
4 - 0 & = & 4 & \text{because } 0 + 4 = 4 \\
7 - 4 & = & 3 & \text{because } 4 + 3 = 7 \\
8 - 2 & = & 6 & \text{because } 2 + 6 = 8 \\
9 - 7 & = & 2 & \text{because } 7 + 2 = 9
\end{array}
$$

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$\begin{array}{rcl}
6 - 1 & = & 5 \\
9 - 5 & = & 4 \\
8 - 2 & = & 6
\end{array}$$
because $1 + 5 = 6$
because $5 + 4 = 9$
because $2 + 6 = 8$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

• To do one-digit subtraction: guess from addition table.

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$9 - 2 = 7$$
$$8 - 4 = 4$$
$$7 - 7 = 0$$

$$
\begin{array}{r}
9 \\
- \quad 2 \\
\hline
7
\end{array}
\qquad
\begin{array}{r}
8 \\
- \quad 4 \\
\hline
4
\end{array}
\qquad
\begin{array}{r}
7 \\
- \quad 7 \\
\hline
0
\end{array}
$$

$+$	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

• Subtraction can also be written in columns.

Example (One digit subtraction, result > 0)

Subtract the one-digit numbers.

$$11 - 3 = 8$$
$$10 - 5 = 5$$
$$18 - 9 = 9$$

$$
\begin{array}{r}
11 \\
-3 \\
\hline
8
\end{array}
\qquad
\begin{array}{r}
10 \\
-5 \\
\hline
5
\end{array}
\qquad
\begin{array}{r}
18 \\
-9 \\
\hline
9
\end{array}
$$

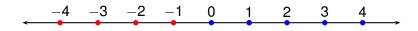| +	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

• Addition table: can be used for subtraction from small two-digit numbers.

Negative integers

- The negative integers are the numbers:

$$\ldots, -6, -5, -4, -3, -2, -1$$

- Written as the minus sign $-$ followed by a (positive) number.
- Negatives are to the left of 0 on the number line.

- -1 is as far away from 0 as 1 is.
- -2 is as far away from 0 as 2 is.
- ... and so on.

Negative sign as a function

- The negative sign can be regarded as a function/operator:

Rule

$$-(a) = -a \qquad \text{if } a > 0$$

Example

$$-(5) = -5$$

- On the left, $-$ is regarded as a function that takes as input a positive number and produces an output that is a negative number.
- On the right, $-$ is regarded as a part of the notation for negative numbers.

Absolute value (magnitude) of a number

Definition (Magnitude of a number)

The magnitude or absolute value $|x|$ of a number x is defined as:

- The number itself, if the number is non-negative.

$$|a| = a, \qquad \text{if } a \text{ is non-negative.}$$

- The number with negative sign removed, if the number is negative.

$$|-a| = a, \qquad \text{if } a \text{ is negative.}$$

Example

$$
\begin{aligned}
|4| &= 4 \\
|-5| &= 5 \\
|0| &= 0
\end{aligned}
$$

Negative of a negative

Rule

The negative of a negative of a number is the number itself.

$$-(-a) = a$$

- Parenthesis are necessary when using multiple negative signs.

  Incorrect   Correct
  $$-\!\!-\!\!a \qquad -(-a)$$
- The rule is independent of whether *a* is positive or negative.
- Can be applied consecutively for more than 2 negative signs.

Example

$$
\begin{aligned}
-(-5) &= 5 \\
-(-(-7)) &= -(7) = -7 \\
-(-(-(-1))) &= -(-1) = 1
\end{aligned}
$$

Sum with a negative

Rule

Subtracting a number is the same as adding its negative.

$$\begin{aligned} a + (-b) &= a - b \\ -a + b &= b - a \end{aligned}$$

Example

$$\begin{aligned} 5 + (-3) &= 5 - 3 &= 2 \\ 10 + (-5) &= 10 - 5 &= 5 \\ -1 + 8 &= 8 - 1 &= 7 \\ -8 + 14 &= 14 - 8 &= 6 \end{aligned}$$

Negative of a sum

Rule

The sum of negatives is the negative of the sum.

$$-a - b = -(a + b)$$

The difference of two numbers is minus the opposite difference.

$$a - b = -(b - a)$$

Example

$$
\begin{aligned}
-5 - 7 &= -(5 + 7) &&= -12 \\
-7 + (-8) &= -7 - 8 &= -(7 + 8) &= -15 \\
5 - 9 &= -(9 - 5) &&= -4 \\
6 - 11 &= -(11 - 6) &&= -5 \\
-9 + 3 &= 3 - 9 &= -(9 - 3) &= -6
\end{aligned}
$$

Summary of algebra rules involving subtraction

Rule

$$
\begin{aligned}
-(-a) &= a \\
a + (-b) &= a - b \\
-a + b &= b - a \\
-a - b &= -(a + b) \\
a - b &= -(b - a)
\end{aligned}
$$

Example

Find the number x so that: $5 + x = 9$. Solution:

$5 + x$	$=$	9	transfer 5 to the right hand side
x	$=$	$9 - 5$	When transferred, 5 acquires negative sign
x	$=$	4	

Observation

At the price of a negative sign, one is allowed to transfer summands from one side of an equation to the other.

$a + b$	$=$	c	*transfer a to the right hand side*
b	$=$	$c - a$	

Example

Solve the equation.

$8 + x$	$=$	9	Answer: $x = 1$
$3 + t$	$=$	11	Answer: $t = 8$
$5 + a$	$=$	10	Answer: $a = 5$
$8 + s$	$=$	16	Answer: $s = 8$

Example

Solve the equation.

$$7 + x = 2$$

Solution.

$$
\begin{array}{rcll}
7 + x &=& 2 & \text{Transfer 7 to the other side} \\
x &=& 2 - 7 = -5 &
\end{array}
$$

Example

Solve the equation.

$$
\begin{aligned}
3 + x &= 7 \quad \text{Answer: } x = 4 \\
1 + a &= 10 \quad \text{Answer: } a = 9 \\
3 + x &= 1 \quad \text{Answer: } x = -2 \\
5 + x &= 0 \quad \text{Answer: } x = -5 \\
9 + a &= 15 \quad \text{Answer: } a = 6 \\
4 + z &= 13 \quad \text{Answer: } z = 9 \\
9 + x &= 8 \quad \text{Answer: } x = -1 \\
9 + x &= 1 \quad \text{Answer: } x = -8
\end{aligned}
$$

Example

Find w, x, y, z so as to satisfy each equality below.

$$-4 \;=\; -10 + w$$

Solution

$$-4 \;=\; (-10 + 10) - 4 = -10 + (10 - 4) = -10 + 6 \;\Rightarrow\; w = 6$$

$$-2 \;=\; -10 + x$$

Solution

$$-2 \;=\; (-10 + 10) - 2 = -10 + (10 - 2) = -10 + 8 \;\Rightarrow\; x = 8$$

$$-1 \;=\; -10 + y$$

Solution

$$-1 \;=\; (-10 + 10) - 1 = -10 + (10 - 1) = -10 + 9 \;\Rightarrow\; y = 9$$

$$-9 \;=\; -10 + z$$

Solution

$$-9 \;=\; (-10 + 10) - 9 = -10 + (10 - 9) = -10 + 1 \;\Rightarrow\; z = 1$$

Subtract 5 from 71.

$$
\begin{array}{r}
-1 \\
7\ \ 1 \\
-\ \ 5 \\
\hline
6\ \ 6
\end{array}
$$

• Ensure summand $>$ subtracand.

$$1 - 5 = -4 = -10 + 6$$
$$-1 + 7 - 0 = 6$$

Subtract 108 from 111.

$$
\begin{array}{r}
-1 \\
1 \quad 1 \quad 1 \\
- \quad 1 \quad 0 \quad 8 \\
\hline
0 \quad 0 \quad 3
\end{array}
$$

• Ensure summand $>$ subtracand.
• Remove leading zeroes.

$$
\begin{aligned}
1 - 8 &= -7 = -10 + 3 \\
-1 + 1 - 0 &= 0 \\
1 - 1 &= 0
\end{aligned}
$$

When the subtracand is larger than the summand, we first use the rule
$a - b = -(b - a)$.
Compute $447 - 509 = -(509 - 447)$.

$$
\begin{array}{r}
{-1} \\
-\quad\begin{array}{ccc} 5 & 0 & 9 \\ 4 & 4 & 7 \end{array} \\
\hline
\begin{array}{ccc} 0 & 6 & 2 \end{array}
\end{array}
$$

- Ensure summand $>$ subtracand.
- Remove leading zeroes.

$$
\begin{aligned}
9 - 7 &= 2 \\
0 - 4 &= -4 = -10 + 6 \\
-1 + 5 - 4 &= 0
\end{aligned}
$$

Subtract 1234567 from 20182019.

$$
\begin{array}{r}
{-1}\ \ {-1}{-1}\ \ {-1}\ \ {-1} \\
\underline{-\ \ \begin{array}{ccccccc} 2 & 0 & 1 & 8 & 2 & 0 & 1 & 9 \\ & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array}} \\
\begin{array}{ccccccc} 1 & 8 & 9 & 4 & 7 & 4 & 5 & 2 \end{array}
\end{array}
$$

• Ensure summand
$>$ subtracand.

$$
\begin{aligned}
9 - 7 &= 2 \\
1 - 6 &= -5 = -10 + 5 \\
-1 + 0 - 5 &= -6 = -10 + 4 \\
-1 + 2 - 4 &= -3 = -10 + 7 \\
-1 + 8 - 3 &= 4 \\
1 - 2 &= -1 = -10 + 9 \\
-1 + 0 - 1 &= -2 = -10 + 8 \\
-1 + 2 - 0 &= 1
\end{aligned}
$$