

Calculus II

Integrals of the form $\int \tan^m x \sec^n x dx$, n -positive and even

Todor Milev

2019

Example

$$\begin{aligned}
 \int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx \\
 &= \int \tan^8 x \sec^2 x d(\tan x) && \left. \begin{array}{l} \text{Can we rewrite} \\ \sec^2 x \text{ via } \tan x? \end{array} \right| \\
 &= \int \tan^8 x (1 + \tan^2 x) d(\tan x) && \left| \begin{array}{l} \text{Set } u = \tan x \end{array} \right. \\
 &= \int u^8 (1 + u^2) du \\
 &= \int (u^8 + u^{10}) du \\
 &= \frac{u^9}{9} + \frac{u^{11}}{11} + C \\
 &= \frac{\tan^9 x}{9} + \frac{\tan^{11} x}{11} + C .
 \end{aligned}$$