

Calculus II

Integrals of the form $\int \frac{Ax + B}{(ax^2 + bx + c)^n} dx$,
denominator has no real roots

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2019

Building block IIIb: example illustrating main idea

Example

Integrate $\int \frac{dx}{(x^2+1)^2}$. We start with an already known integral:

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$$\arctan x + C = \int \frac{1}{x^2 + 1} dx$$

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Rearrange terms

$$2 \int \frac{dx}{(1 + x^2)^2} = \left(\frac{x}{x^2 + 1} + \arctan x \right) + L \quad .$$

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Rearrange terms and divide by 2 to get the desired integral:

$$\int \frac{dx}{(1+x^2)^2} = \frac{1}{2} \left(\frac{x}{x^2+1} + \arctan x \right) + K .$$

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$$\int \frac{1}{(x^2 + 1)} dx = \arctan x + C \quad .$$

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$$J(n) = \int \frac{1}{(x^2 + 1)^n} dx$$

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- Set $J(n) = \int \frac{1}{(x^2+1)^n} dx$. We are looking for a formula for $J(n)$.

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- In this way we end up expressing $J(n)$ via $J(n-1)$.
- We work our way from $J(n)$ to $J(n-1)$, from $J(n-1)$ to $J(n-2)$, and so on, until we get to $J(1)$.

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Rearrange to get:

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$$J(n-1) = \frac{x}{(x^2+1)^{n-1}} + 2(n-1)J(n-1) - 2(n-1)J(n) \quad .$$

Rearrange to get:

$$\begin{aligned} 2(n-1)J(n) &= \frac{x}{(x^2+1)^{n-1}} + (2n-3)J(n-1) \\ J(n) &= \frac{x}{(2n-2)(x^2+1)^{n-1}} + \frac{2n-3}{2n-2}J(n-1) \quad . \end{aligned}$$

In this way we expressed $J(n)$ using $J(n-1)$. We apply the above formula consecutively:

$$J(n) = \frac{x}{(2n-2)(x^2+1)^{n-1}} + \frac{2n-3}{2n-2} \left(\frac{x}{(2n-4)(x^2+1)^{n-2}} + \frac{2n-5}{2n-4} J(n-2) \right) = \dots$$

and so on.

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and so on. The above can be used to write a formula for the final result, but that is as complicated as the process above.