

Precalculus

Factoring quadratic polynomials

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Outline

1 Factoring quadratics

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2 Vietas' formulas

Definition ((Partial) Factorization)

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Example

$$\begin{aligned}x^2 - 1 &= (x - 1)(x + 1) \\ x^3 + 8 &= (x + 2)(x^2 - 2x + 4)\end{aligned}$$

Theorem

The quadratic $ax^2 + bx + c$ factors as follows.

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

where x_1 and x_2 are the roots of the quadratic, given by:

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Let $ax^2 + bx + c$ be a quadratic functions with zeros x_1 and x_2 . Then:

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Let $ax^2 + bx + c$ be a quadratic functions with zeros x_1 and x_2 . Then:

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The last two formulas are called Vieta's formulas (after François Viète (1540-1603), Latinized name: Franciscus Vieta).

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

$$x_1 + x_2 = -\frac{b}{a}$$

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Vieta's formulas

Example

Factor the quadratic.

$$x^2 + 5x + 6$$

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Example

Factor the quadratic.

$$x^2 + 5x + 6 = (x + ?)(x + ?)$$

- The product of the two roots: $x_1 x_2 = 6$.

$$ax^2 + bx + c = a(x - x_1)(x - x_2)$$

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Example

Factor the quadratic.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

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$$x^2 + 3x + 1 = \left(x - \left(\frac{-3 + \sqrt{5}}{2} \right) \right) \left(x - \left(\frac{-3 - \sqrt{5}}{2} \right) \right)$$

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Factor the quadratic, using complex numbers if needed.

$$x^2 + x + 1 = (x + ?)(x + ?)$$

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Example

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$$x^2 + x + 1 = \left(x - \left(\frac{-1 + \sqrt{3}i}{2} \right) \right) \left(x - \left(\frac{-1 - \sqrt{3}i}{2} \right) \right)$$

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