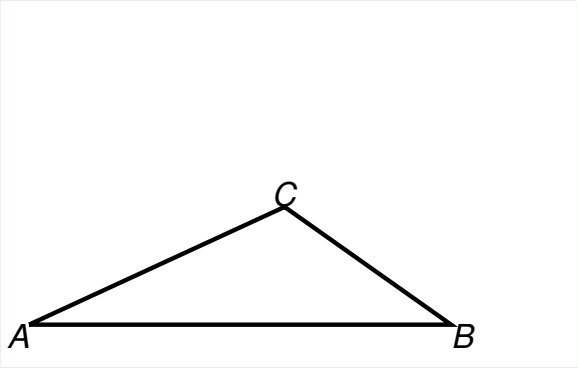
Precalculus
Solve triangle from two sides and an angle

Todor Milev

2019

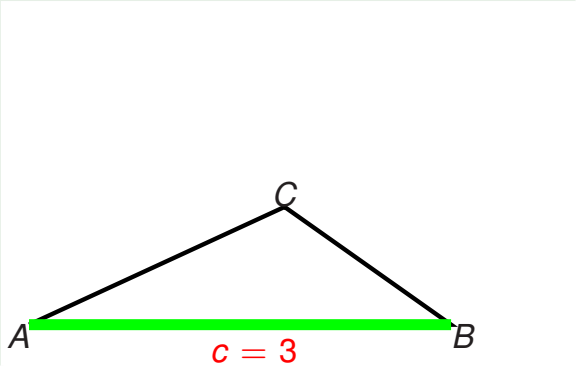
Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

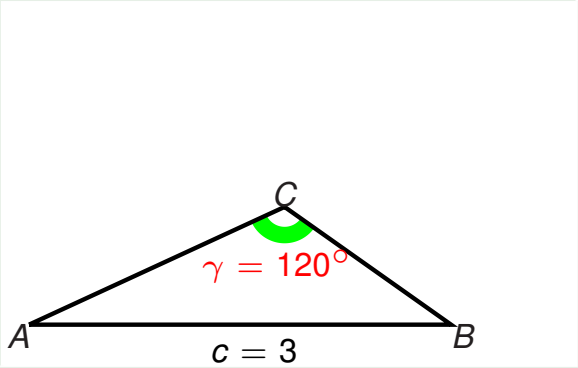
Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

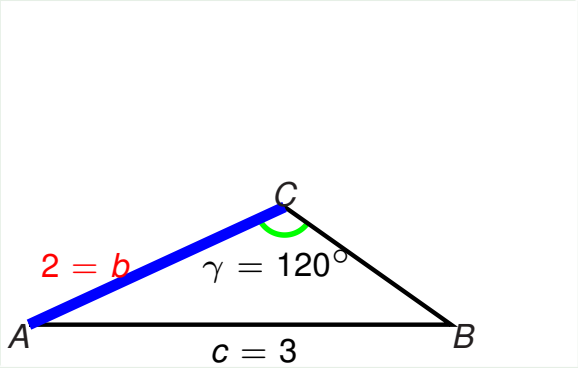
Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

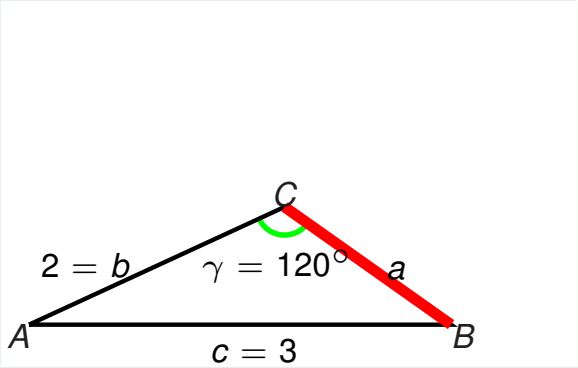
Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

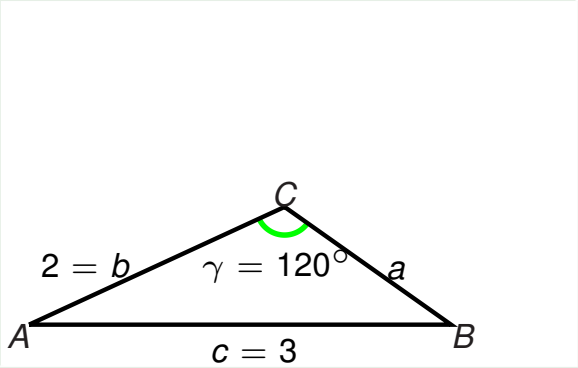
Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

Example



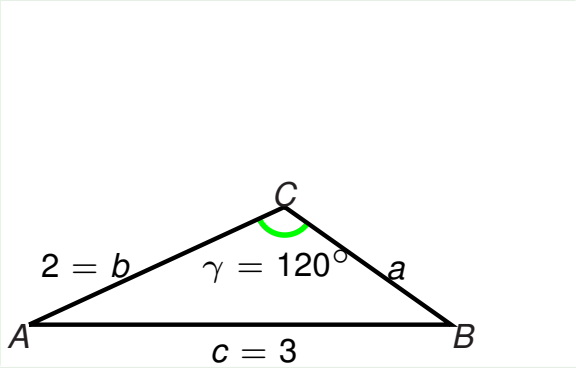
The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

?

Law of cosines

Example

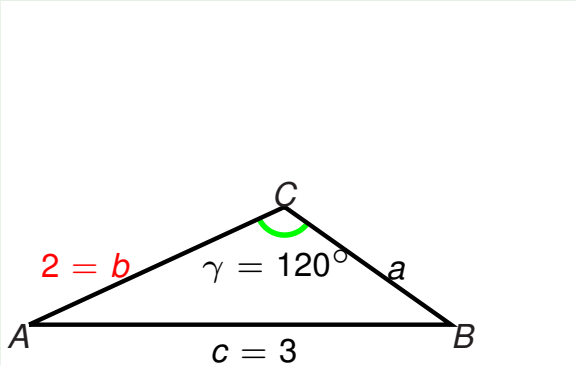


The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$a^2 + b^2 - 2ab\cos\gamma = c^2$ Law of cosines

Example



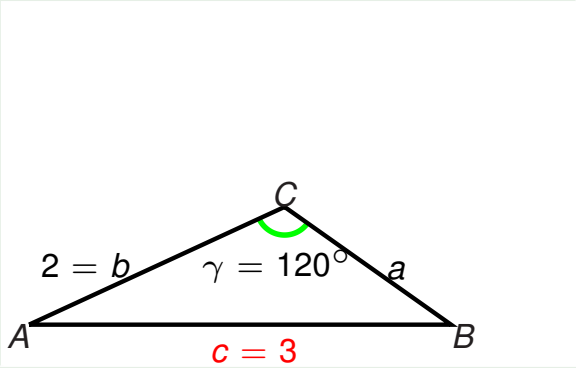
The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2$$ Law of cosines

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

Example



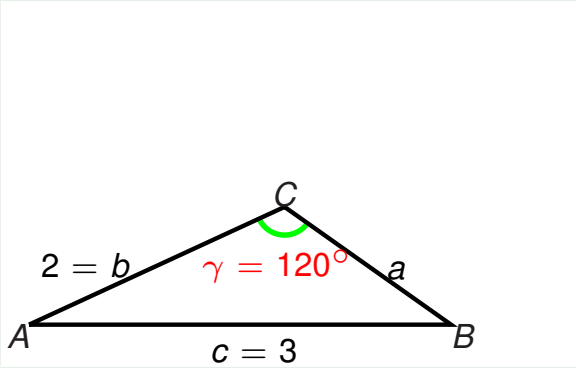
The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2$$ | Law of cosines

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

Example



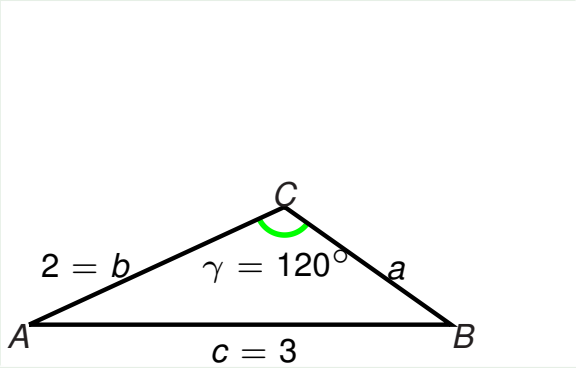
The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2$$ | Law of cosines

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

Example



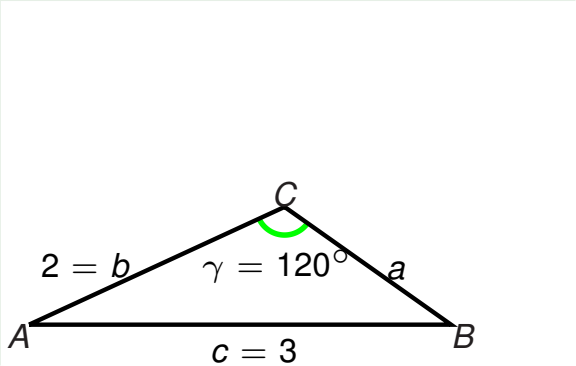
The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

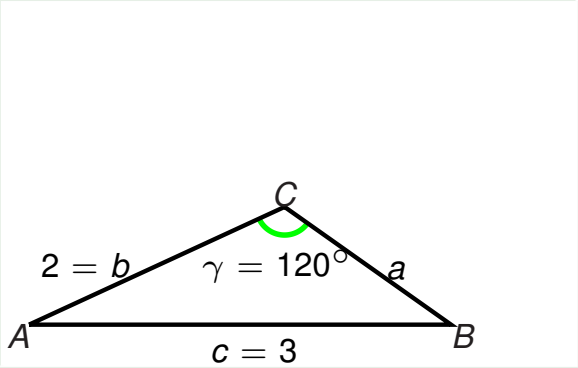
- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(\quad\right) - 5 = 0$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

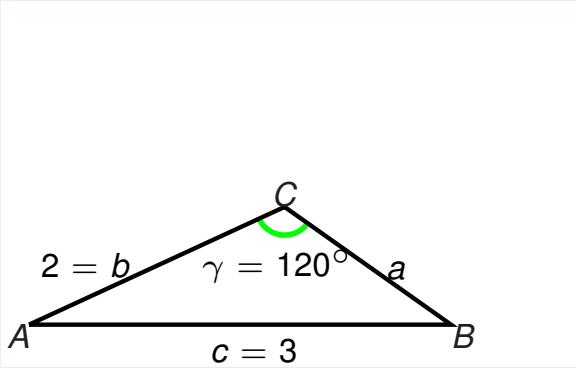
- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(\quad\right) - 5 = 0$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

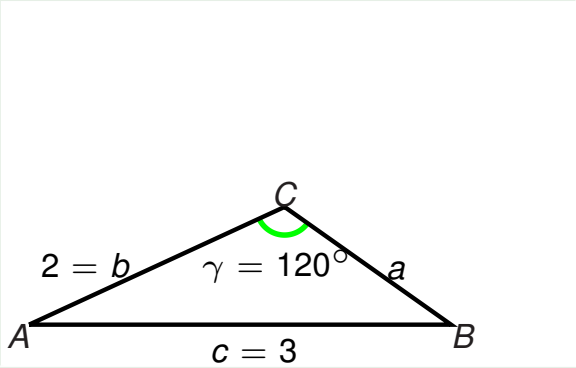
- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(\ ?\ \right) - 5 = 0$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

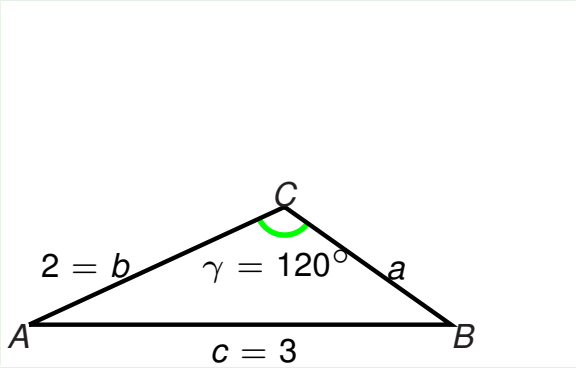
- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(-\frac{1}{2}\right) - 5 = 0$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

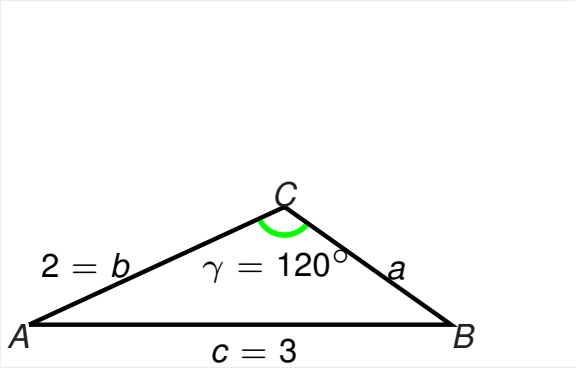
$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(-\frac{1}{2}\right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

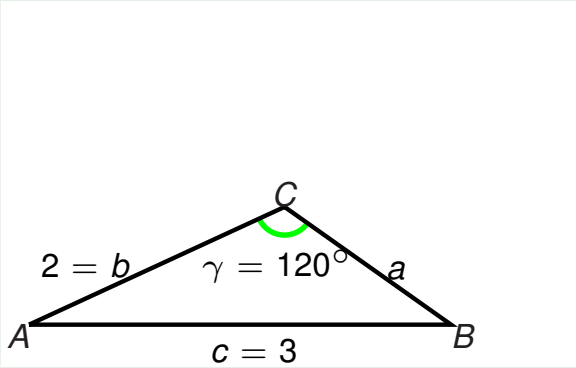
$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(-\frac{1}{2}\right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

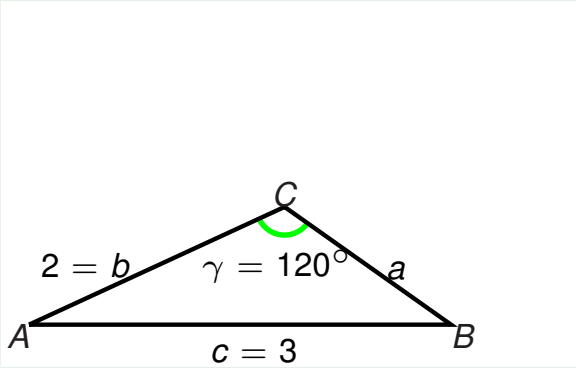
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(-\frac{1}{2}\right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

$a =$ **?**

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

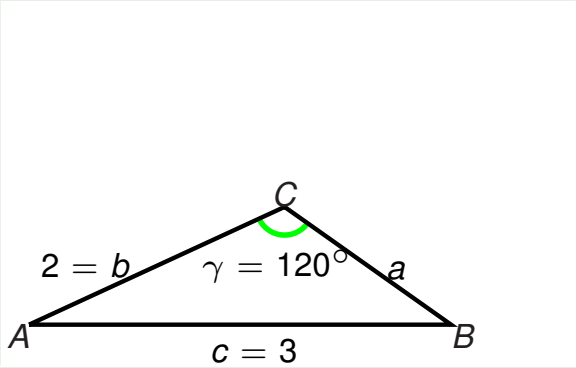
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(-\frac{1}{2}\right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

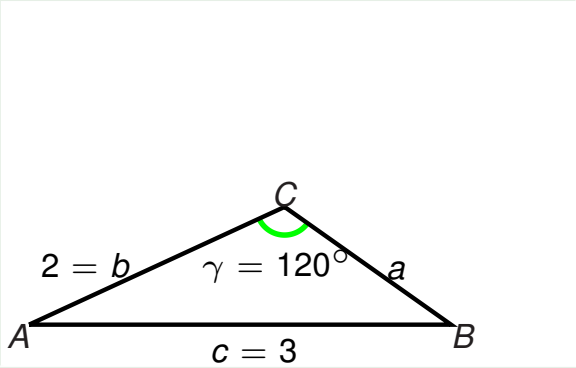
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(-\frac{1}{2}\right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

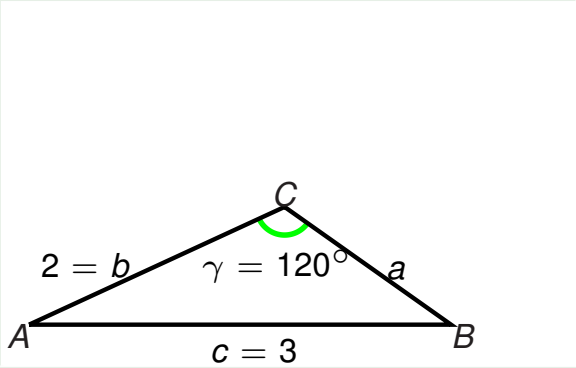
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(-\frac{1}{2}\right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

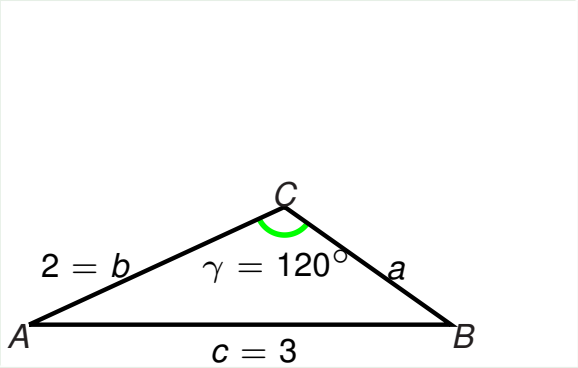
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(-\frac{1}{2}\right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

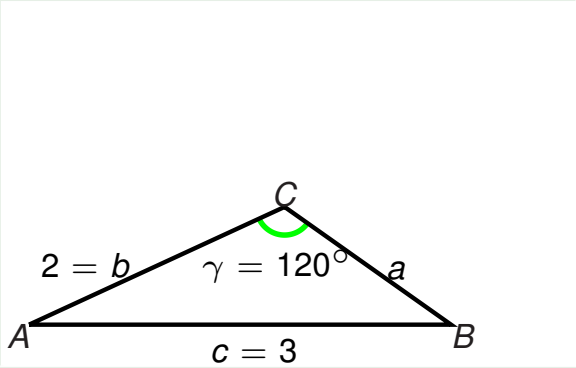
$$a^2 - 4a\left(-\frac{1}{2}\right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= \frac{-2 \pm \sqrt{24}}{2}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

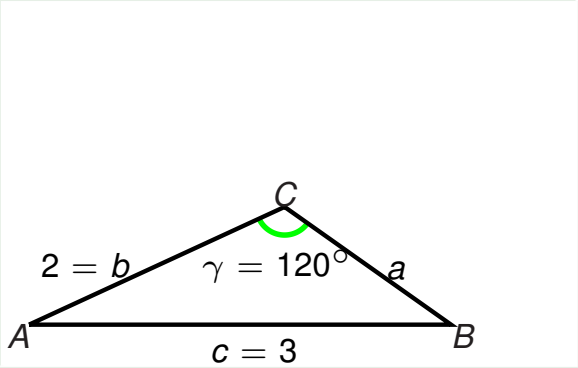
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(-\frac{1}{2}\right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1} = \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 + 2\sqrt{6}}{2}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

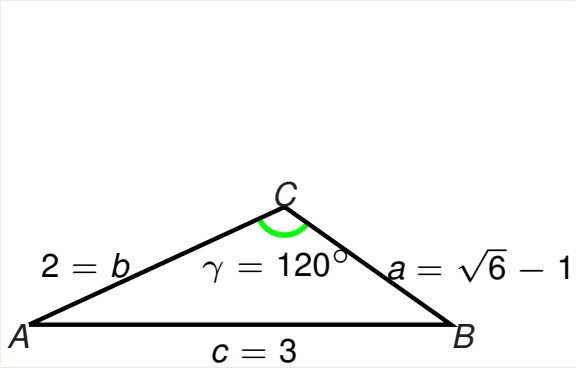
$$a^2 - 4a\left(-\frac{1}{2}\right) - 5 = 0$$

$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 + 2\sqrt{6}}{2} \qquad a > 0$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(-\frac{1}{2}\right) - 5 = 0$$

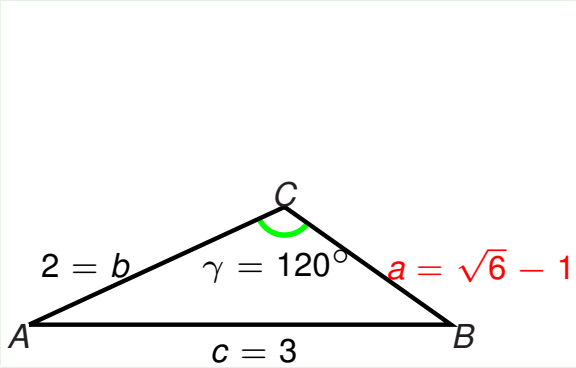
$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 + 2\sqrt{6}}{2} \qquad a > 0$$

$$= -1 + \sqrt{6}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a^2 - 4a\left(-\frac{1}{2}\right) - 5 = 0$$

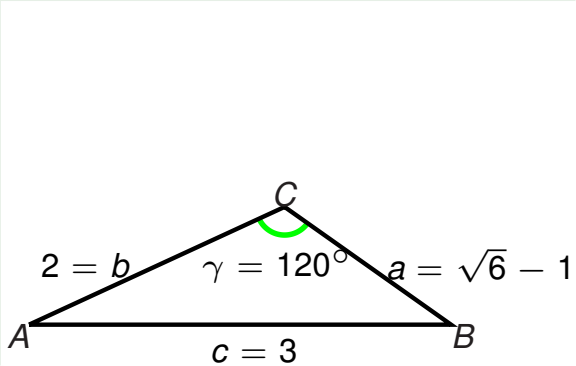
$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 + 2\sqrt{6}}{2} \qquad a > 0$$

$$= -1 + \sqrt{6}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

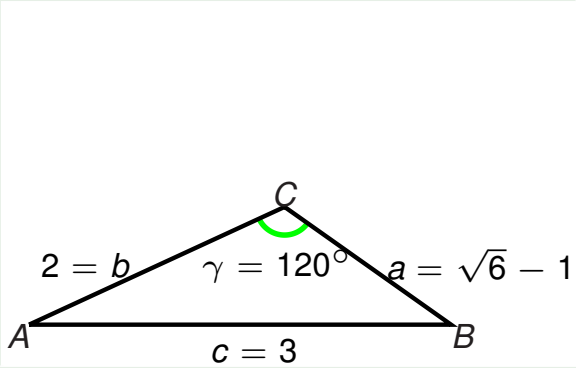
$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

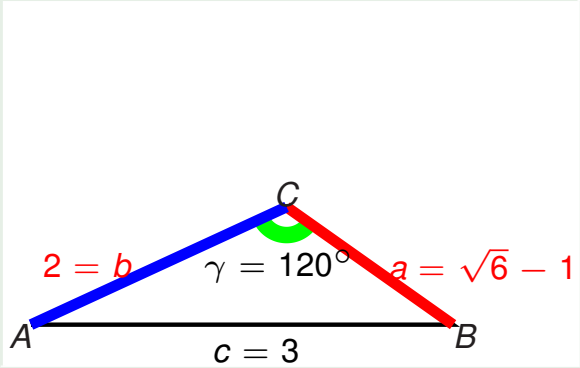
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

Area = ?

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

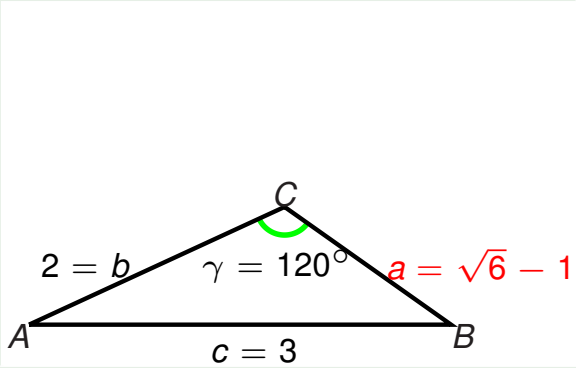
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\textbf{Area} = \frac{ab\sin\gamma}{2}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

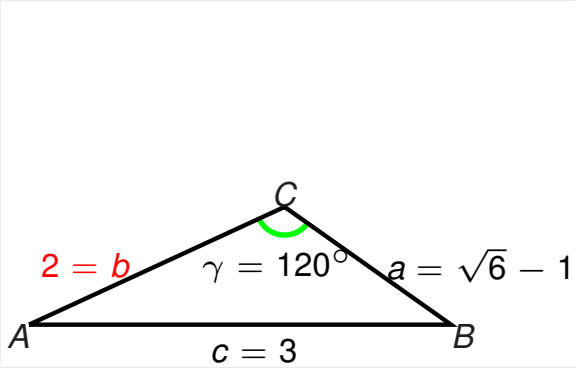
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\textbf{Area} = \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6}-1\right)2}{2}\textbf{?}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

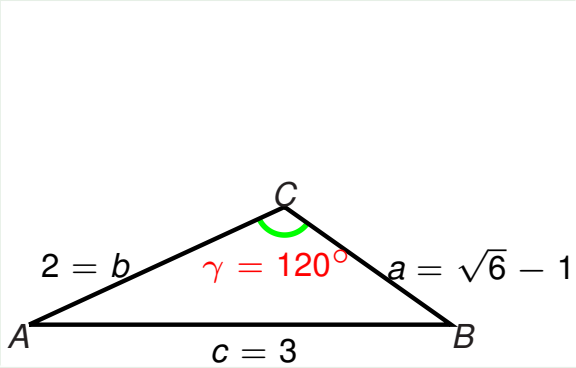
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\textbf{Area} = \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6} - 1\right) 2}{2}\textbf{?}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

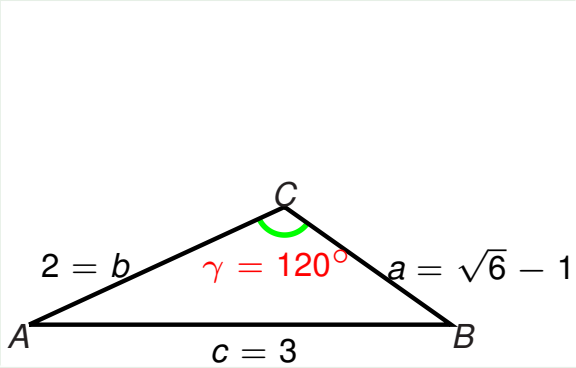
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\textbf{Area} = \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6}-1\right)2}{2}\,?$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

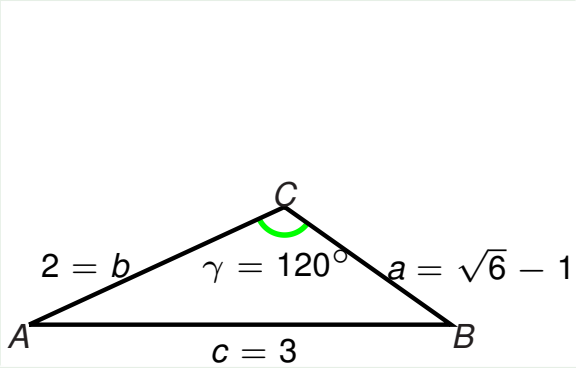
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\textbf{Area} = \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6}-1\right)2}{2}\,\frac{\sqrt{3}}{2}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

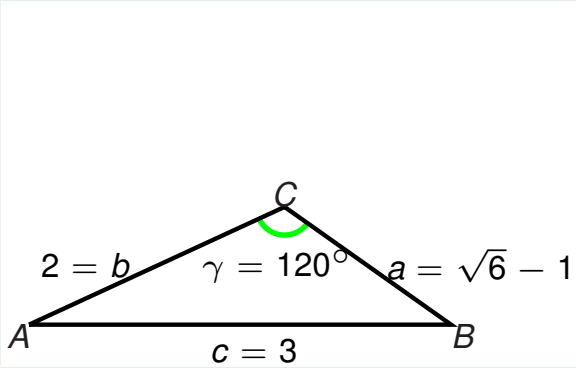
$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\textbf{Area} = \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6}-1\right)\cancel{2}}{\cancel{2}}\frac{\sqrt{3}}{2}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

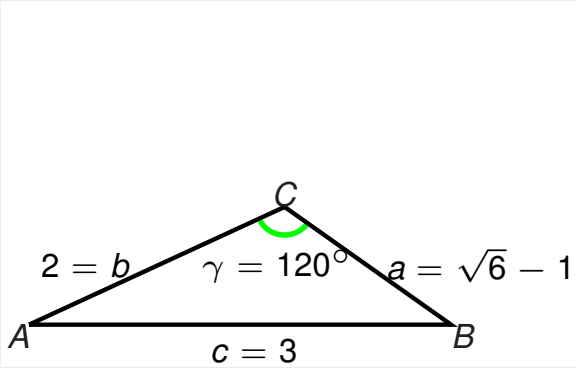
$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\textbf{Area} = \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6} - 1\right)\cancel{2}}{\cancel{2}} \frac{\sqrt{3}}{2}$$

$$= \frac{3\sqrt{2} - \sqrt{3}}{2}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

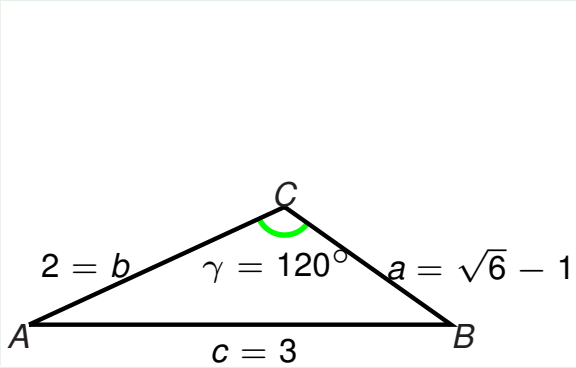
$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\textbf{Area} = \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6} - 1\right)\cancel{2}}{\cancel{2}} \frac{\sqrt{3}}{2}$$

$$= \frac{3\sqrt{2} - \sqrt{3}}{2}$$

Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

$$a^2 + b^2 - 2ab\cos\gamma = c^2 \qquad \text{Law of cosines}$$

$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2 \qquad \text{Solve for } a:$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

$$= -1 + \sqrt{6}$$

$$\textbf{Area} = \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6}-1\right)\cancel{2}}{\cancel{2}}\,\frac{\sqrt{3}}{2}$$

$$= \frac{3\sqrt{2} - \sqrt{3}}{2}$$