Calculus II
Split partial fraction, general case

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Example

Write out the form of the partial fraction decomposition of

$$\frac{x^3+x^2+1}{x(x-1)(x^2+x+1)(x^2+1)^3}$$

$$= \frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+x+1} + \frac{Ex+F}{x^2+1} + \frac{Gx+H}{(x^2+1)^2} + \frac{Ix+J}{(x^2+1)^3}.$$

For example of this size it makes sense to use a computer algebra system; one such system easily produces the decomposition:

$$= \frac{-1}{x} + \frac{\frac{1}{8}}{x-1} + \frac{-x-1}{(x^2+x+1)} + \frac{\frac{15}{8}x-\frac{1}{8}}{(x^2+1)} + \frac{\frac{3}{4}x+\frac{3}{4}}{(x^2+1)^2} + \frac{-\frac{x}{2}+\frac{1}{2}}{(x^2+1)^3}.$$

$Q(x)$ has quadratic factors with multiplicity > 1

- Suppose $Q(x)$ has the factor $(ax^2 + bx + c)^r$, where $b^2 - 4ac < 0$ and $r > 1$.
- Then the partial fraction decomposition should include summands of the form

$$\frac{A_1 x + B_1}{ax^2 + bx + c} + \frac{A_2 x + B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{A_r x + B_r}{(ax^2 + bx + c)^r}$$