

## Precalculus

### § Polynomial system that reduces to quadratic, part 1

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2019

## Example

Solve the polynomial system. 
$$\left| \begin{array}{rcl} x - 4y & = & 5 \\ y^2 + xy & = & 10 \end{array} \right.$$

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Solve the polynomial system. 
$$\begin{cases} x - 4y = 5 \\ y^2 + xy = 10 \end{cases}$$

$$x = 5 + 4y \quad \left| \text{Solve for } x \text{ in first eq-n.} \right.$$

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$$\begin{array}{rcl} x & = & 5 + 4y \\ y^2 + xy & = & 10 \end{array}$$

Solve for  $x$  in first eq-n.

## Example

Solve the polynomial system.  $\left| \begin{array}{l} x - 4y = 5 \\ y^2 + xy = 10 \end{array} \right.$

$$\begin{aligned} x &= 5 + 4y \\ y^2 + xy &= 10 \\ y^2 + (5 + 4y)y &= 10 \end{aligned}$$

Solve for  $x$  in first eq-n.

Substitute  $x$  away

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Solve the polynomial system.  $\left| \begin{array}{l} x - 4y = 5 \\ y^2 + xy = 10 \end{array} \right.$

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$$y^2 + xy = 10$$

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Divide by 5

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$$= 5 + 4 \cdot 1$$

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Final answer:  $x = -3, y = -2$  or  $x = 9, y = 1$ .

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Check answer  $x = -3, y = -2$ :

$$\left| \begin{array}{l} x - 4y \\ y^2 + xy \end{array} \right.$$

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$$\left| \begin{array}{rcl} x - 4y & = & 5 \\ y^2 + xy & = & 10 \end{array} \right.$$

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$$\left| \begin{array}{rcl} x - 4y & = & (-3) - 4(-2) = 5 \\ y^2 + xy & = & (-2)^2 + (-3)(-2) = 10 \end{array} \right.$$

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Solve the polynomial system. 
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Check answer  $y = 1, x = 9$ :

$$\left| \begin{array}{rcl} x - 4y & = & 9 - 4 \cdot 1 = 5 \\ y^2 + xy & = & 1^2 + 9 \cdot 1 = 10. \end{array} \right.$$

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$$\left| \begin{array}{rcl} x - 4y & = & 9 - 4 \cdot 1 = 5 \\ y^2 + xy & = & 1^2 + 9 \cdot 1 = 10. \end{array} \right.$$