Calculus I
Homework
Fermat's Theorem and the Mean Value Theorem

1. Use the Intermediate Value theorem and the Mean Value Theorem/Rolle's Theorem to prove that the function has **exactly one** real root.

   (a) $f(x) = x^3 + 4x + 7$.

   (b) $f(x) = x^3 + x^2 + x + 1$.

   (c) $f(x) = \cos^3\left(\frac{x}{3}\right) + \sin x - 3x$.

2. Use the Intermediate Value theorem and the Mean Value Theorem/Rolle's Theorem to prove that the function has **exactly one** real root.

   (a) $x^5 + 7x = 2$.

   (b) $x^7 + x^5 + x^3 = 3$.

   (c) $2x - 1 = \sin x$.

   (d) $e^x + 2x = 3$.