Calculus II
Integrals of the form $\int \tan^m x \sec^n x \mathrm{d}x$, n-positive and even

Todor Milev

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Example

$$\int \tan^8 x \sec^4 x \mathrm{d}x$$

Example

$$\int \tan^8 x \sec^4 x dx = \int \tan^8 x \sec^2 x \sec^2 x dx$$

Example

$$\begin{aligned}\int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx\\ &= \int \tan^8 x \sec^2 x d\left(? \quad\right)\end{aligned}$$

Example

$$\int \tan^8 x \sec^4 x dx = \int \tan^8 x \sec^2 x \sec^2 x dx$$
$$= \int \tan^8 x \sec^2 x d(\tan x)$$

Example

$$\begin{aligned}\int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx \\ &= \int \tan^8 x \sec^2 x d(\tan x) \\ &= \int \tan^8 x \left(? \qquad\qquad \right) d(\tan x)\end{aligned}$$

Can we rewrite $\sec^2 x$ via $\tan x$?

Example

$$\begin{aligned}\int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx \\ &= \int \tan^8 x \sec^2 x d(\tan x) \\ &= \int \tan^8 x \left(1 + \tan^2 x\right) d(\tan x)\end{aligned}$$

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Can we rewrite $\sec^2 x$ via $\tan x$?

Set $u = \tan x$

Example

$$
\begin{aligned}
\int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx \\
&= \int \tan^8 x \sec^2 x d(\tan x) \\
&= \int \tan^8 x \left(1+\tan^2 x\right) d(\tan x) \\
&= \int u^8 \left(1+u^2\right) du \\
&= \int \left(u^8+u^{10}\right) du
\end{aligned}
$$

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Set $u = \tan x$

Example

$$\begin{aligned}
\int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx \\
&= \int \tan^8 x \sec^2 x d(\tan x) \\
&= \int \tan^8 x \left(1 + \tan^2 x\right) d(\tan x) \\
&= \int u^8 \left(1 + u^2\right) du \\
&= \int \left(u^8 + u^{10}\right) du \\
&= \mathbf{?}
\end{aligned}$$

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Set $u = \tan x$

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$$\begin{aligned}
\int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx \\
&= \int \tan^8 x \sec^2 x d(\tan x) \\
&= \int \tan^8 x \left(1 + \tan^2 x\right) d(\tan x) \\
&= \int u^8 \left(1 + u^2\right) du \\
&= \int \left(u^8 + u^{10}\right) du \\
&= \frac{u^9}{9} + \frac{u^{11}}{11} + C
\end{aligned}$$

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Set $u = \tan x$

Example

$$\begin{aligned}\int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx \\ &= \int \tan^8 x \sec^2 x d(\tan x) \\ &= \int \tan^8 x \left(1+\tan^2 x\right) d(\tan x) \\ &= \int u^8 \left(1+u^2\right) du \\ &= \int \left(u^8+u^{10}\right) du \\ &= \frac{u^9}{9}+\frac{u^{11}}{11}+C\end{aligned}$$

Can we rewrite $\sec^2 x$ via $\tan x$?

Set $u = \tan x$

Example

$$\int \tan^8 x \sec^4 x dx = \int \tan^8 x \sec^2 x \sec^2 x dx$$
$$= \int \tan^8 x \sec^2 x d(\tan x)$$
$$= \int \tan^8 x \left(1 + \tan^2 x\right) d(\tan x)$$
$$= \int u^8 \left(1 + u^2\right) du$$
$$= \int \left(u^8 + u^{10}\right) du$$
$$= \frac{u^9}{9} + \frac{u^{11}}{11} + C$$

Can we rewrite $\sec^2 x$ via $\tan x$?

Set $u = \tan x$

Example

$$\begin{aligned}
\int \tan^8 x \sec^4 x dx &= \int \tan^8 x \sec^2 x \sec^2 x dx \\
&= \int \tan^8 x \sec^2 x d\left(\tan x\right) \\
&= \int \tan^8 x \left(1+\tan^2 x\right) d(\tan x) \\
&= \int u^8 \left(1+u^2\right) du \\
&= \int \left(u^8+u^{10}\right) du \\
&= \frac{u^9}{9}+\frac{u^{11}}{11}+C \\
&= \frac{\tan^9 x}{9}+\frac{\tan^{11} x}{11}+C \quad .
\end{aligned}$$

Can we rewrite $\sec^2 x$ via $\tan x$?

Set $u = \tan x$