

Precalculus

Trigonometry and triangles

Todor Milev

2019

Outline

1 Law of sines

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2 Law of cosines

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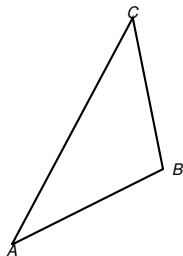
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$$\text{Triangle area} = \frac{1}{2} \text{base} \cdot \text{height}$$

Proposition (Triangle area)

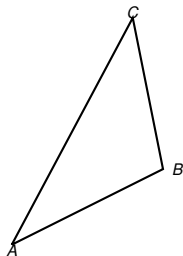
$$\text{Area}(\triangle ABC) = ?$$



$$\text{Triangle area} = \frac{1}{2} \text{base} \cdot \text{height}$$

Proposition (Triangle area)

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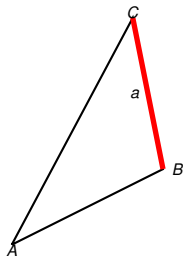


Triangle area = $\frac{1}{2}$ base \cdot height

Let $\triangle ABC$ have **side length a** and height length h_a , as indicated - **side a is opposite to vertex A** and h_a starts at A

Proposition (Triangle area)

$$\text{Area}(\triangle ABC) = \frac{1}{2} \text{height} \cdot \text{base} = \frac{1}{2} h_a a$$

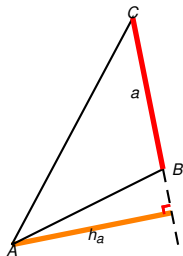


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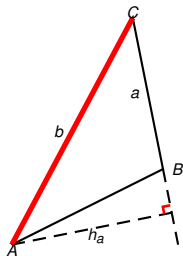


Triangle area = $\frac{1}{2}$ base \cdot height

Let $\triangle ABC$ have **side lengths** a, b and height lengths h_a, h_b , as indicated - side a is opposite to vertex A and h_a starts at A , and so on.

Proposition (Triangle area)

$$\text{Area}(\triangle ABC) = \frac{1}{2} \text{height} \cdot \text{base} = \frac{1}{2} h_a a = \frac{1}{2} h_b b .$$

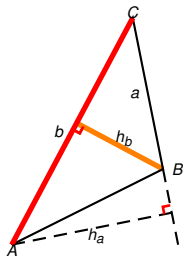


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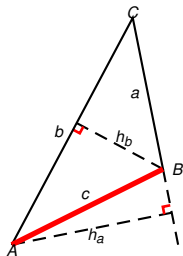


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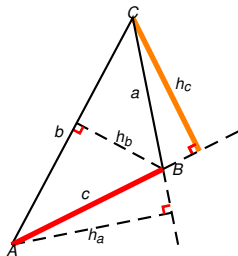


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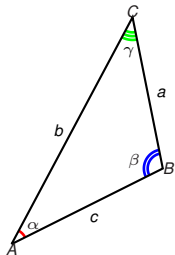
Triangle area from two sides and angle between them

Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ , as indicated: α is opposite to a , β is opposite to b , γ is opposite to c .

Proposition (\triangle area from two sides and angle between them)

The area of a triangle is half the product of the lengths of two of its sides times the sine of the angle between them. In other words,

$$\text{Area}(\triangle ABC) = \frac{ab \sin \gamma}{2} = \frac{bc \sin \alpha}{2} = \frac{ca \sin \beta}{2}$$



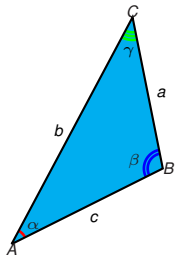
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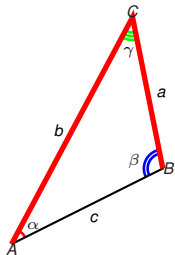
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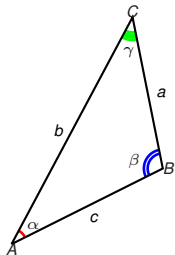
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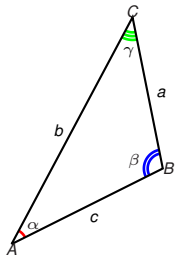
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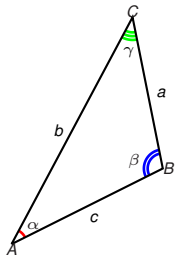
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Proof.

$$\text{Area}(\triangle ABC) = \frac{\text{base} \cdot \text{height}}{2}$$



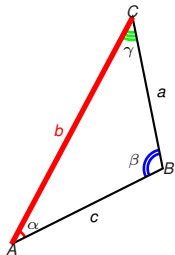
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Proof.

$$\text{Area}(\triangle ABC) = \frac{\text{base} \cdot \text{height}}{2} = \frac{bh_b}{2}$$



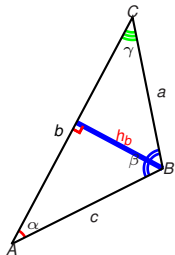
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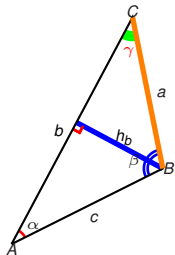
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Proof.

$$\begin{aligned} \text{Area}(\triangle ABC) &= \frac{\text{base} \cdot \text{height}}{2} = \frac{b h_b}{2} \\ &= \frac{b a \sin \gamma}{2}. \end{aligned}$$



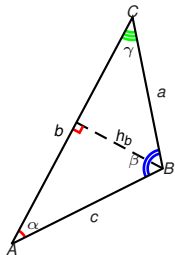
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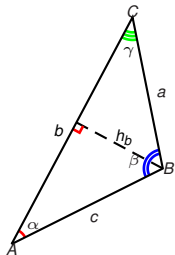
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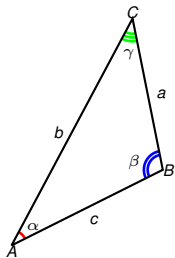
The proof of the other two cases is similar. □

Law of sines

Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ , as indicated: α is opposite to a , β is opposite to b , γ is opposite to c .

Proposition (Law of Sines)

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}.$$

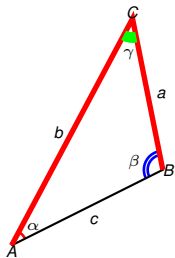


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Proof.

$$\text{Area}(\triangle ABC) = \frac{ab \sin \gamma}{2} = \frac{bc \sin \alpha}{2}$$

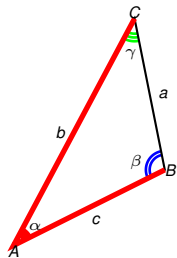


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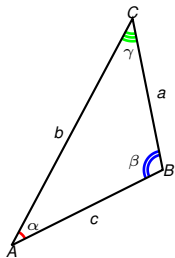


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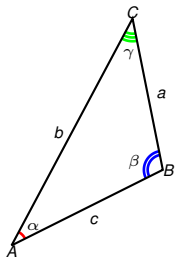


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Proof.

$$\text{Area}(\triangle ABC) = \frac{ab \sin \gamma}{2} = \frac{bc \sin \alpha}{2} \quad \left| \text{Div. by } \frac{b}{2} \right.$$

$$a \sin \gamma = c \sin \alpha$$

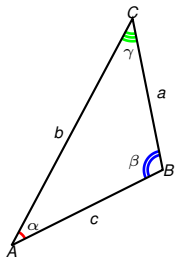


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$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}.$$



Proof.

$$\begin{aligned} \text{Area}(\triangle ABC) &= \frac{ab \sin \gamma}{2} = \frac{bc \sin \alpha}{2} & \Bigg| \text{ Div. by } \frac{b}{2} \\ & \frac{a \sin \gamma}{a} = \frac{c \sin \alpha}{c} \\ & \frac{\sin \gamma}{\sin \alpha} \end{aligned}$$

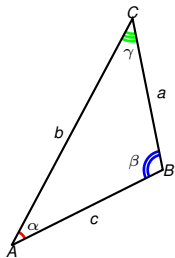


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Proof.

$$\begin{aligned} \text{Area}(\triangle ABC) &= \frac{ab \sin \gamma}{2} = \frac{bc \sin \alpha}{2} & \left| \text{Div. by } \frac{b}{2} \right. \\ & \quad a \sin \gamma = c \sin \alpha \\ & \quad \frac{a}{\sin \alpha} = \frac{c}{\sin \gamma}. \end{aligned}$$

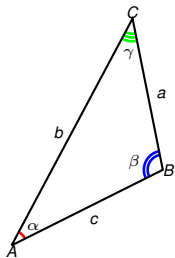


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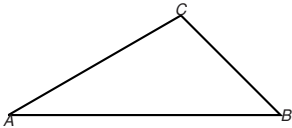


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$$\begin{aligned} \text{Area}(\triangle ABC) &= \frac{ab \sin \gamma}{2} = \frac{bc \sin \alpha}{2} & \left| \text{Div. by } \frac{b}{2} \right. \\ a \sin \gamma &= c \sin \alpha \\ \frac{a}{\sin \alpha} &= \frac{c}{\sin \gamma}. \end{aligned}$$

The remaining cases are similar. □

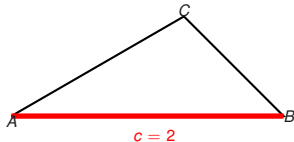
Example



A triangle has a side of length 2cm ; the two angles adjacent to it are 30° and 45° .

- Find the other two sides of the triangle.
- Find the area of the triangle.

Example

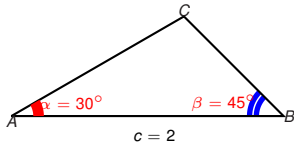


A triangle has a **side of length 2cm** ; the two angles adjacent to it are 30° and 45° .

- Find the other two sides of the triangle.
- Find the area of the triangle.

- Let the known side be $c = 2\text{cm}$.

Example

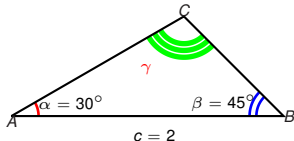


A triangle has a side of length 2cm ; the two angles adjacent to it are 30° and 45° .

- Find the other two sides of the triangle.
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- Let the known side be $c = 2\text{cm}$.
- Let the known angles 30° , 45° be arranged as in the figure

Example

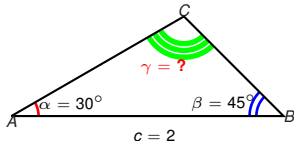


A triangle has a side of length 2cm ; the two angles adjacent to it are 30° and 45° .

- Find the other two sides of the triangle.
- Find the area of the triangle.

- Let the known side be $c = 2\text{cm}$.
- Let the known angles 30° , 45° be arranged as in the figure, and let the **third angle be γ**

Example

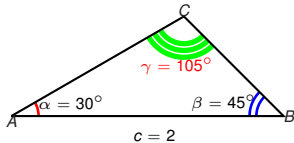


A triangle has a side of length 2cm ; the two angles adjacent to it are 30° and 45° .

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- Let the known side be $c = 2\text{cm}$.
- Let the known angles 30° , 45° be arranged as in the figure, and let the third angle be $\gamma = ?$

Example

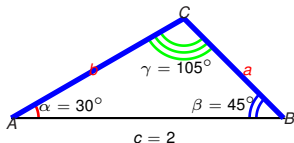


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- Let the known side be $c = 2\text{cm}$.
- Let the known angles 30° , 45° be arranged as in the figure, and let the third angle be $\gamma = 180^\circ - 30^\circ - 45^\circ = 180^\circ - 75^\circ = 105^\circ$.

Example

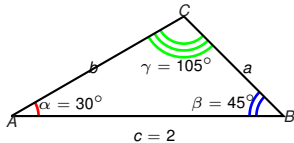


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- Let the known angles 30° , 45° be arranged as in the figure, and let the third angle be $\gamma = 180^\circ - 30^\circ - 45^\circ = 180^\circ - 75^\circ = 105^\circ$.
- Label the unknown sides a, b as indicated.

Example



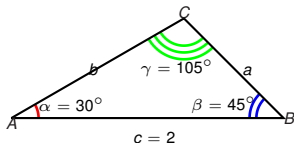
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- Find the other two sides of the triangle.
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$$\frac{a}{\sin \alpha} = \frac{c}{\sin \gamma}$$

| Law of sines

Example



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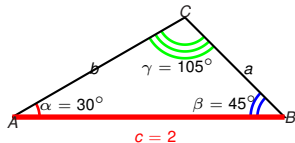
- Find the other two sides of the triangle.
- Find the area of the triangle.

$$\frac{a}{\sin \alpha} = \frac{c}{\sin \gamma}$$

$$a = \frac{c \sin \alpha}{\sin \gamma}$$

| Law of sines

Example



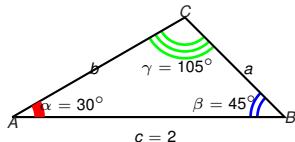
A triangle has a side of length 2cm ; the two angles adjacent to it are 30° and 45° .

- Find the other two sides of the triangle.
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$$\frac{a}{\sin \alpha} = \frac{c}{\sin \gamma} \quad | \text{Law of sines}$$

$$a = \frac{c \sin \alpha}{\sin \gamma} = \frac{2 \sin 30^\circ}{\sin 105^\circ}$$

Example



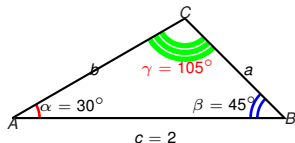
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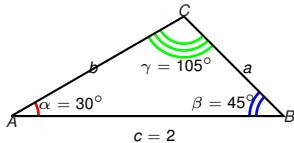
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Example



$\sin 105^\circ$

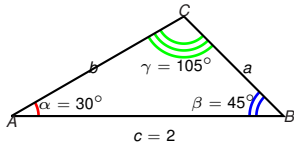
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Example



A triangle has a side of length 2cm ; the two angles adjacent to it are 30° and 45° .

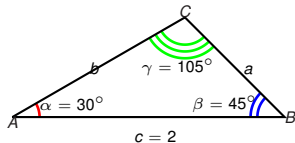
- Find the other two sides of the triangle.
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$$\sin 105^\circ = \sin(60^\circ + 45^\circ)$$

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A triangle has a side of length 2cm ; the two angles adjacent to it are 30° and 45° .

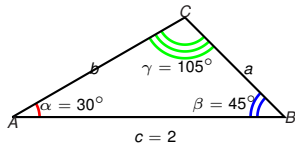
- Find the other two sides of the triangle.
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$$\sin 105^\circ = \sin(60^\circ + 45^\circ) = ?$$

$$\frac{a}{\sin \alpha} = \frac{c}{\sin \gamma} \quad | \text{Law of sines}$$

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Example



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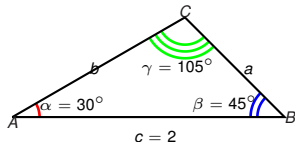
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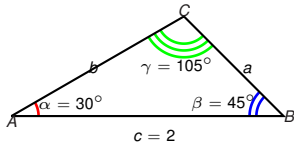
- Find the other two sides of the triangle.
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$$\begin{aligned}\sin 105^\circ &= \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ \\ &= \text{? ?} + \text{? ?}\end{aligned}$$

$$\frac{a}{\sin \alpha} = \frac{c}{\sin \gamma} \quad \text{| Law of sines}$$

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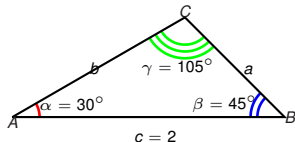
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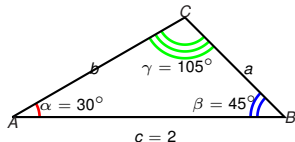
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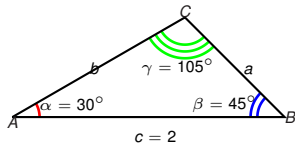
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$$\begin{aligned}\sin 105^\circ &= \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ \\ &= \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} + ??\end{aligned}$$

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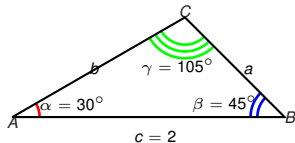
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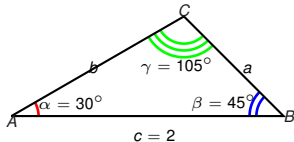
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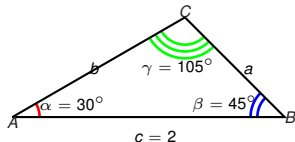
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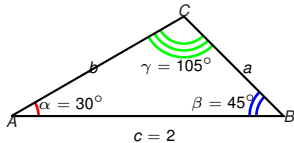
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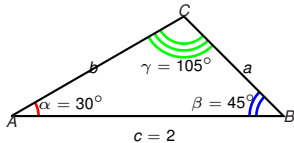
$$\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2} \frac{\sqrt{2}}{2} + \frac{1}{2} \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

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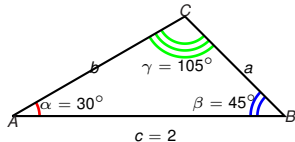
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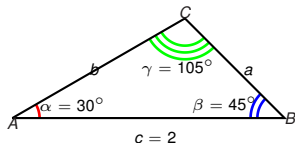
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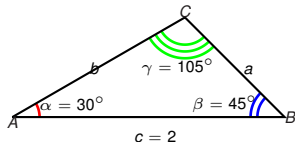
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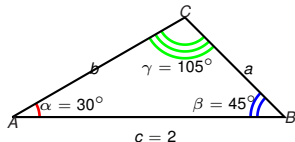
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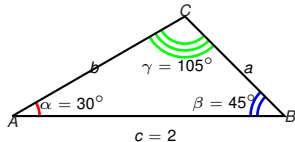
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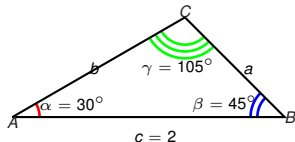
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Example



A triangle has a side of length 2cm ; the two angles adjacent to it are 30° and 45° .

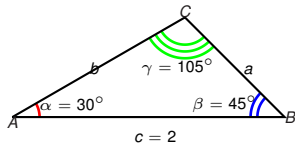
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Example



A triangle has a side of length 2cm ; the two angles adjacent to it are 30° and 45° .

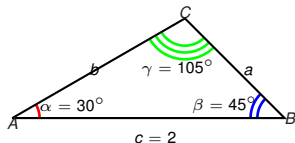
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Example



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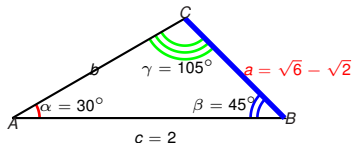
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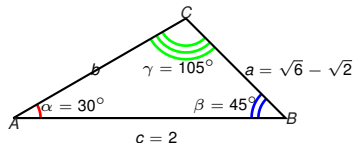
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Example



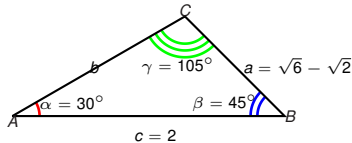
$$\frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

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| Law of sines

Example



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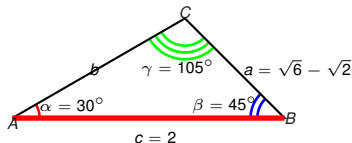
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| Law of sines

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$$b = \frac{c \sin \beta}{\sin \gamma}$$

Example



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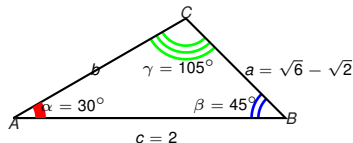
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| Law of sines

$$\frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

$$b = \frac{c \sin \beta}{\sin \gamma} = \frac{2 \sin 45^\circ}{\sin 105^\circ}$$

Example



A triangle has a side of length 2cm ; the two angles adjacent to it are 30° and 45° .

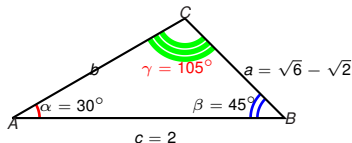
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$$\frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

$$b = \frac{c \sin \beta}{\sin \gamma} = \frac{2 \sin 45^\circ}{\sin 105^\circ}$$

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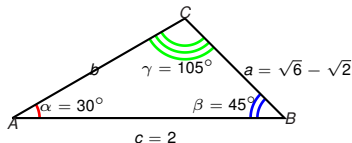
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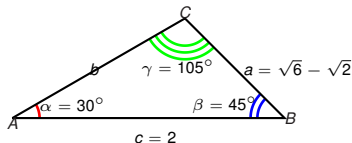
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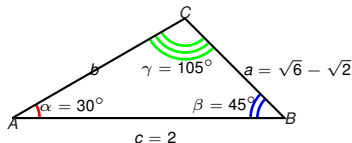
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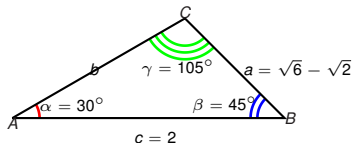
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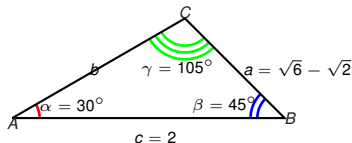
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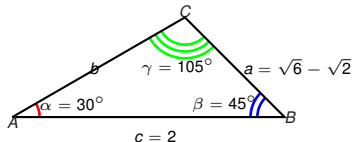
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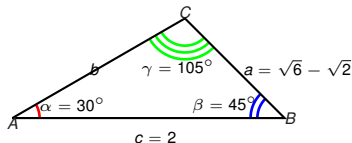
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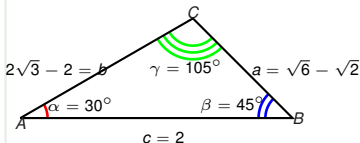
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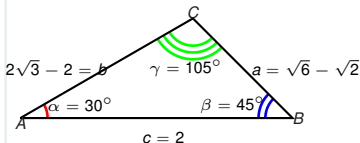
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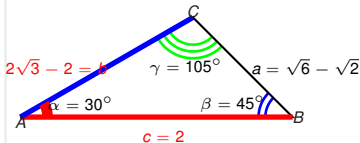
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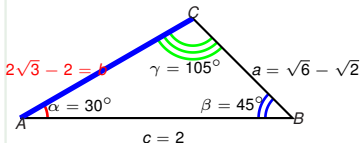
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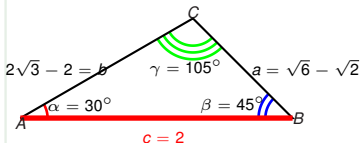
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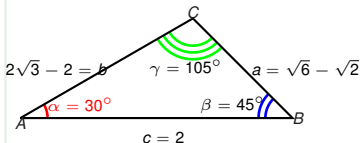
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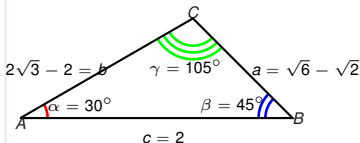
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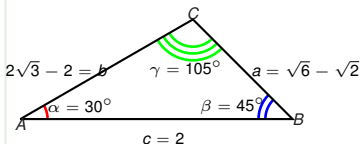
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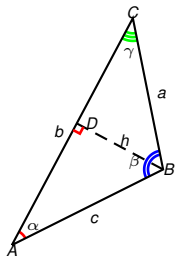
Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ , as indicated.

Proposition (Law of Cosines)

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

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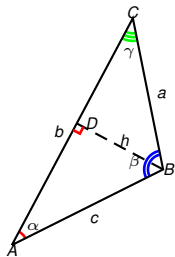
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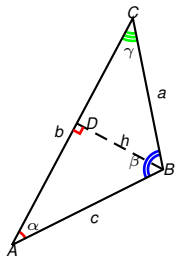
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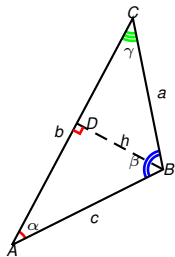
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Proof if $\gamma < 90^\circ$.

Drop a perpendicular h from B to AC .

$$|CD| = a \cos \gamma$$

$$h = a \sin \gamma$$

$$|AD| = b - |CD| = b - a \cos \gamma$$

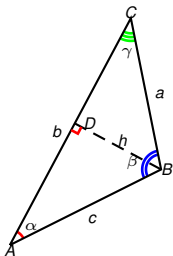
$$c^2 = |AD|^2 + h^2$$

$$= (b - a \cos \gamma)^2 + (a \sin \gamma)^2$$

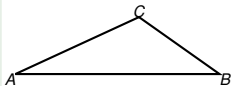
$$= b^2 - 2ab \cos \gamma + a^2 \cos^2 \gamma + a^2 \sin^2 \gamma$$

$$= b^2 - 2ab \cos \gamma + a^2.$$

Pyth. thm.
 $\triangle BDA$



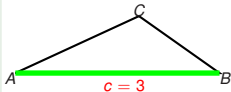
Example



The longest side of a triangle has length 3 and the angle opposite to it is 120° . Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

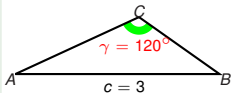
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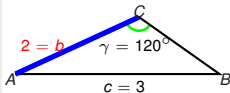
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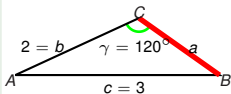
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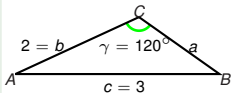
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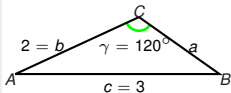


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| Law of cosines

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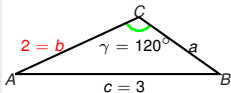
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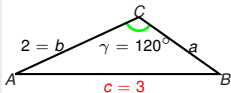
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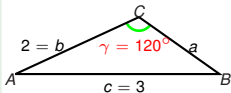
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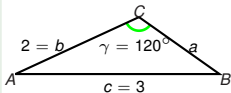
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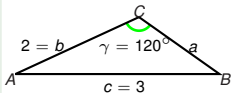
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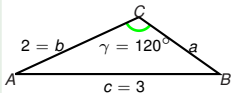
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$$a^2 - 4a \left(\quad \right) - 5 = 0$$

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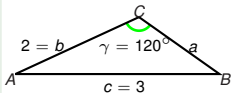
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Law of cosines
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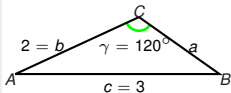
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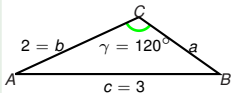
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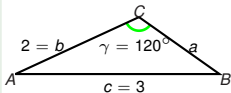
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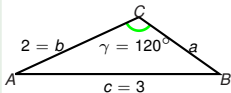
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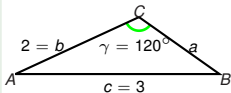
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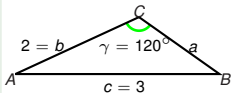
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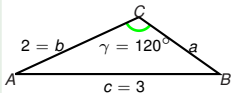
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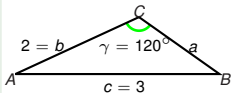
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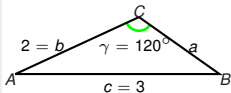
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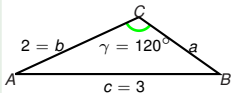
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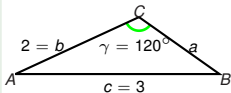
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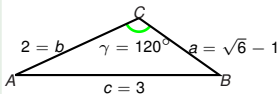
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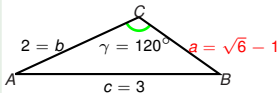
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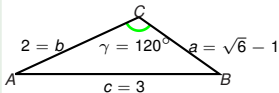
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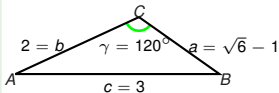
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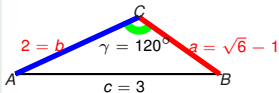
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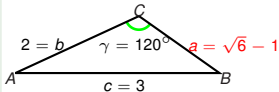
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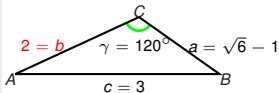
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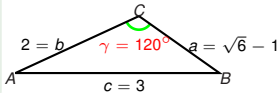
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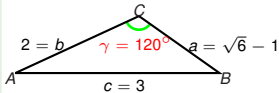
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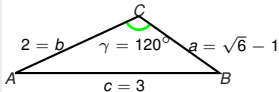
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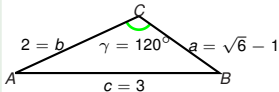
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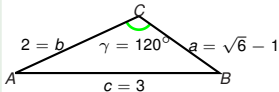
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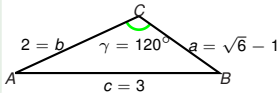
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