Calculus I
Reference: The Evaluation Theorem (Fundamental Theorem of Calculus, part 2)

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Theorem (The Evaluation Theorem (FTC part 2))

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a),$$

where F is any antiderivative of f.

$$\int_a^b f(x)dx \text{ exists for any continuous (over } [a, b]) $$
function f.

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Theorem

Let f be a continuous function on $[a, b]$. Then f is integrable over $[a, b]$.

In other words, $\int_a^b f(x)dx$ exists for any continuous (over $[a, b]$) function f.

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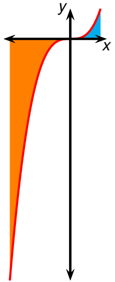
Example



Evaluate the integral $\displaystyle\int_{-2}^{1} x^3 \, \mathrm{d}x$.

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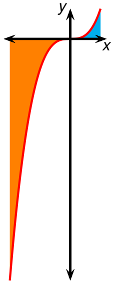

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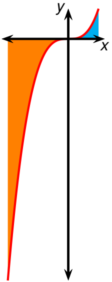

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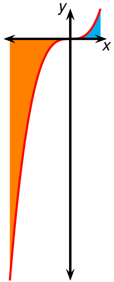

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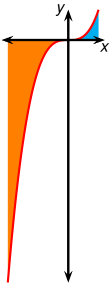

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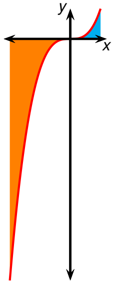

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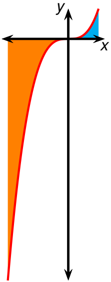

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