

Calculus II

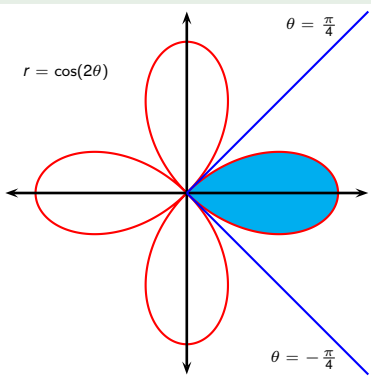
Area swept by one clover leaf $r = \sin(n\theta)$,
 $r = \cos(n\theta)$

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Example

Find the area enclosed by one loop of the four-leaved rose $r = \cos 2\theta$.



The region enclosed by the right loop corresponds to points whose θ polar coordinate lies in the interval $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$.

$$\begin{aligned}
 A &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2} r^2 d\theta \\
 &= \frac{1}{2} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2(2\theta) d\theta \\
 &= \int_0^{\frac{\pi}{4}} \cos^2(2\theta) d\theta \\
 &= \int_0^{\frac{\pi}{4}} \frac{1}{2} (1 + \cos(4\theta)) d\theta \\
 &= \frac{1}{2} \left[\theta + \frac{1}{4} \sin(4\theta) \right]_0^{\frac{\pi}{4}} \\
 &= \frac{\pi}{8}
 \end{aligned}$$