

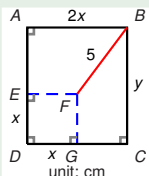
Precalculus

§ Geometric-text problems leading to polynomial systems, part 2

Todor Milev

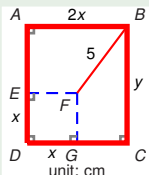
2019

Example



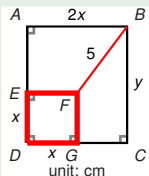
$ABCD$ is a rectangle. Points E , F , G and D form a square as indicated. Given: $|ED| = |DG| = x \text{ cm}$
 $|AB| = 2x \text{ cm}$, $|BC| = y \text{ cm}$, $|BF| = 5 \text{ cm}$,
 $Perimeter(ABCD) = 26 \text{ cm}$. Find x and y so that $y > x$.

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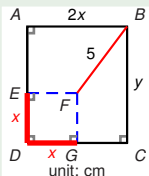
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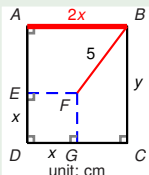
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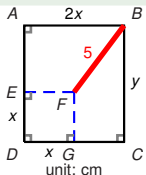
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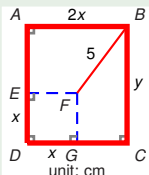
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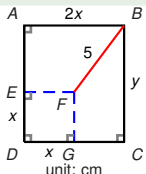
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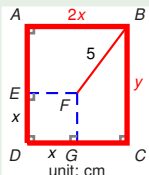
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$$2 \cdot (2x + y) = 26$$

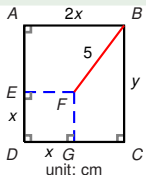
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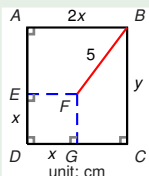
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$$\begin{aligned} 2 \cdot (2x + y) &= 26 && |\text{Div. by } 2 \\ 2x + y &= 13 \end{aligned}$$

Example



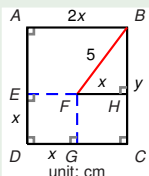
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$$2x + y = 13$$

$$y = 13 - 2x$$

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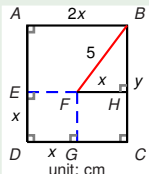
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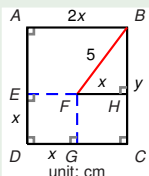
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$$|FH|^2 + |BH|^2 = |BF|^2$$

$$x^2 + (y - x)^2 = 5^2$$

$$x^2 + y^2 - 2xy + x^2 - 5^2 = 0$$

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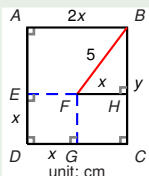
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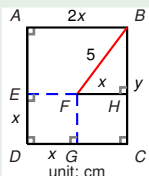
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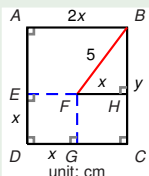
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$$2x^2 + 169 - 52x + 4x^2 - 26x + 4x^2 - 25 = 0$$

$$10x^2 - 78x + 144 = 0$$

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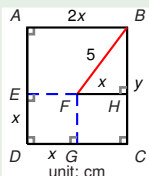
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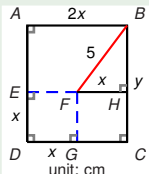
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$$\begin{aligned} x_1, x_2 &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-39) \pm \sqrt{39^2 - 4 \cdot 5 \cdot 72}}{2 \cdot 5} \end{aligned}$$

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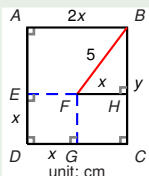
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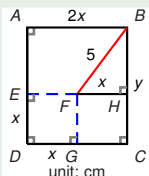
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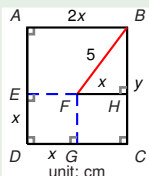
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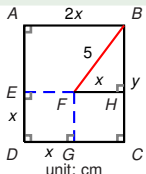
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$$x_1, x_2 = \frac{39 \pm 9}{10}$$

$$\text{Case 1. } x = \frac{39 + 9}{10} = \frac{48}{10} = 4.8$$

$$y = 13 - 2x = 13 - 2 \cdot 4.8 = 13 - 9.6$$

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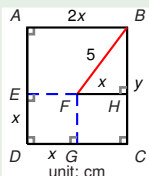
$$y = 13 - 2x = 13 - 2 \cdot 4.8 = 13 - 9.6 = 3.4 < 4.8 = x$$

Solution rejected as problem requires $y > x$.

$$\text{Case 2. } x = \frac{39 - 9}{10} = 3$$

$$y = 13 - 2 \cdot x = 13 - 2 \cdot 3$$

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