Precalculus
Trigonometric equations and inequalities

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2019

Outline

1. Trigonometric equations and inequalities
   - The Equations $\sin x = A$, $\cos x = B$
   - Equations that reduce to $\sin x = A$, $\cos x = B$

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2. Product-to-Sum Formulas

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1. Trigonometric equations and inequalities
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   - Equations that reduce to $\sin x = A$, $\cos x = B$
2. Product-to-Sum Formulas
3. Trigonometric inequalities

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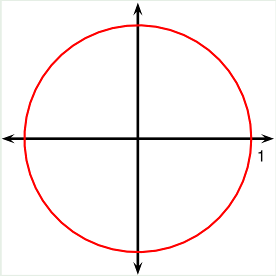
Trigonometric equations

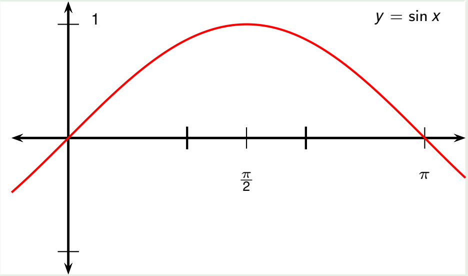
- Some problems will not ask you to prove a trigonometric identity, but rather to solve a trigonometric equation.
- Consider the problem of finding all values of x for which $\sin x = \sin(2x) = 2\sin x \cos x$.
- This is not a trigonometric identity - the two sides are different.
- However, there are values for x which the above equality holds.

Example

Find all solutions and then find those that lie between -360° and 360°.

$$\sin\theta = \frac{\sqrt{3}}{2}$$





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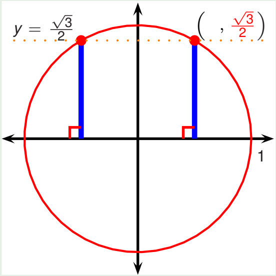

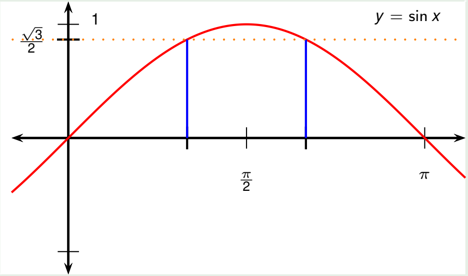

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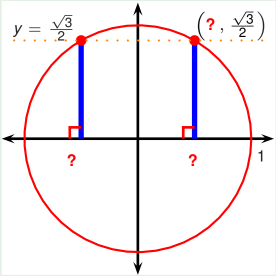

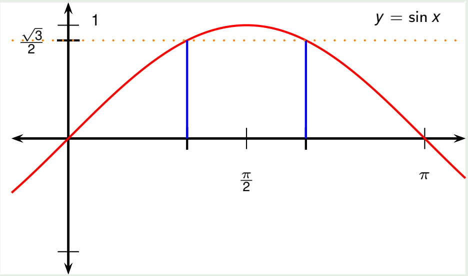

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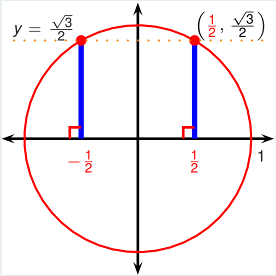
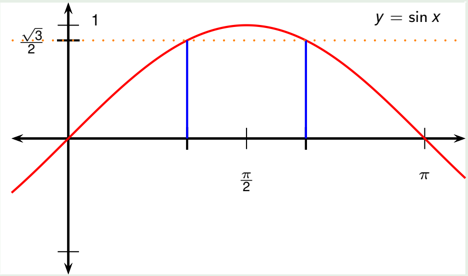
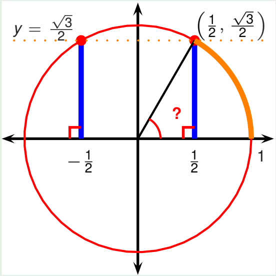


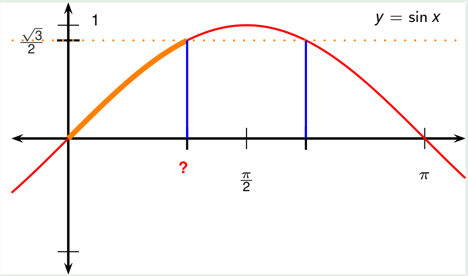


Example

Find all solutions and then find those that lie between $-360°$ and $360°$.

$$\sin\theta = \frac{\sqrt{3}}{2}$$
$$\theta = ?$$

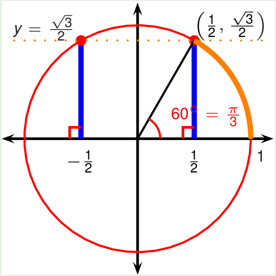


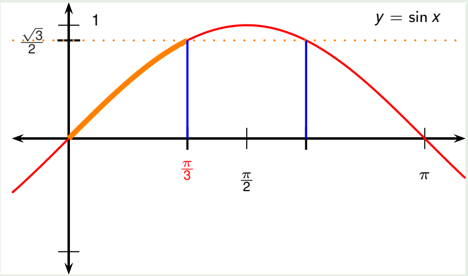


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned} \sin\theta &= \frac{\sqrt{3}}{2} \\ \theta &= 60^\circ \end{aligned}$$





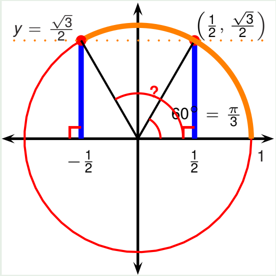
Example

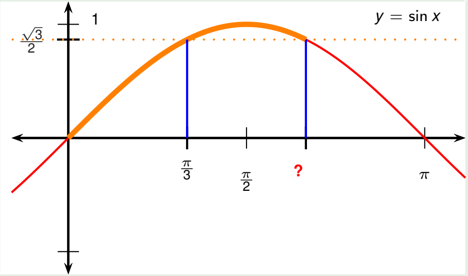
Find all solutions and then find those that lie between -360° and 360°.

$$\sin\theta = \frac{\sqrt{3}}{2}$$
$$\theta = 60^\circ$$

or

?

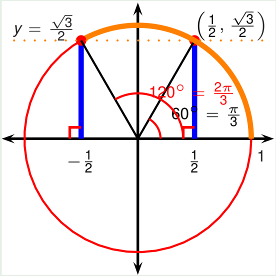


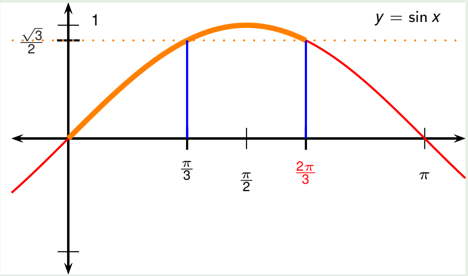


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned} \sin\theta &= \frac{\sqrt{3}}{2} \\ \theta &= 60^\circ \\ &\quad \textbf{or} \\ &\quad 120^\circ \end{aligned}$$

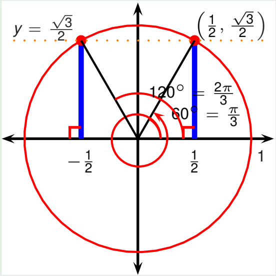


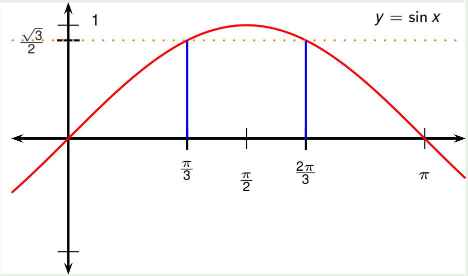


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned} \sin\theta &= \frac{\sqrt{3}}{2} \\ \theta &= 60^\circ + k\cdot 360^\circ \\ &\quad \textbf{or} \\ &\quad 120^\circ \end{aligned}$$





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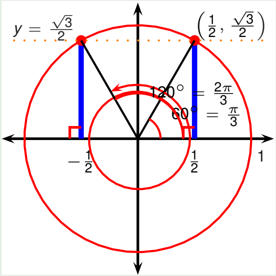
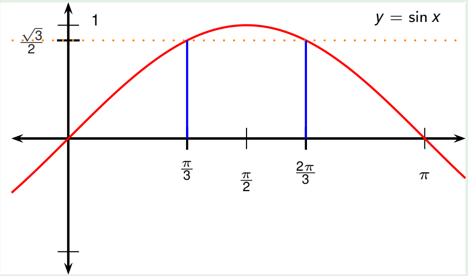
Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned} \sin\theta &= \frac{\sqrt{3}}{2} \\ \theta &= 60^\circ + k\cdot 360^\circ \\ &\textbf{or} \\ &120^\circ + k\cdot 360^\circ \end{aligned}$$





Example

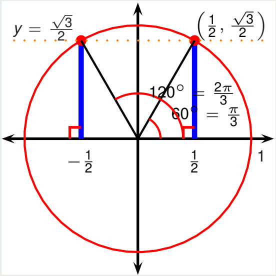
Find all solutions and then find those that lie between -360° and 360°.

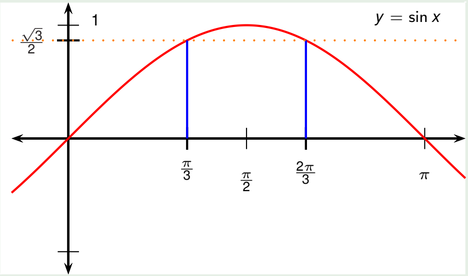
$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = 60^\circ + k\cdot 360^\circ = \quad \dots \quad -660^\circ,$$

or $\quad \dots \quad k=-2$

$$120^\circ + k\cdot 360^\circ = \dots \; -600^\circ,$$





Example

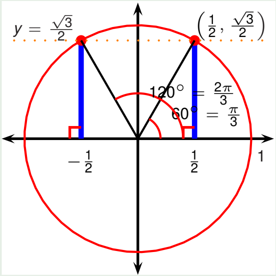
Find all solutions and then find those that lie between -360° and 360°.

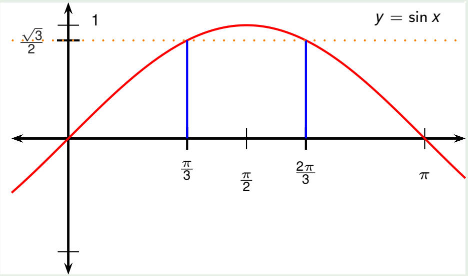
$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = 60^\circ + k\cdot 360^\circ = \dots\ -660^\circ,\ -300^\circ,$$

or $\dots$ $k=-2$ $k=-1$

$$120^\circ + k\cdot 360^\circ = \dots\ -600^\circ,\ -240^\circ,$$

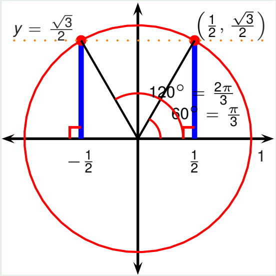


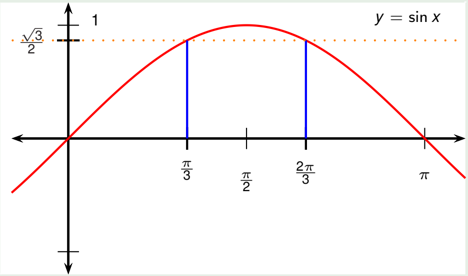


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned} \sin\theta &= \frac{\sqrt{3}}{2} \\ \theta &= 60^\circ + k\cdot 360^\circ = \dots\ -660^\circ,\ -300^\circ,\ 60^\circ, \\ & \quad \textbf{or} \qquad\qquad\qquad\qquad \dots\quad k{=}-2 \quad k{=}-1 \quad k{=}0 \\ & \quad 120^\circ + k\cdot 360^\circ = \dots\ -600^\circ,\ -240^\circ,\ 120^\circ, \end{aligned}$$





Example

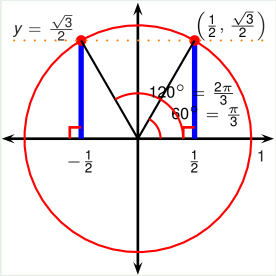
Find all solutions and then find those that lie between -360° and 360°.

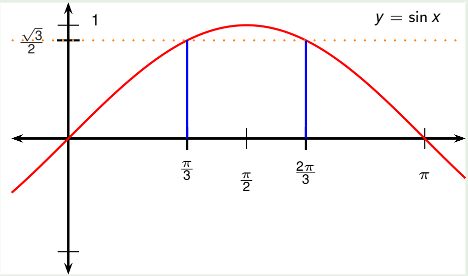
$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = 60^\circ + k\cdot 360^\circ = \dots\ -660^\circ,\ -300^\circ,\ 60^\circ,\ 420^\circ,\ \dots$$

or $\quad \cdots \quad k=-2 \quad k=-1 \quad k=0 \quad k=1 \quad \cdots$

$$120^\circ + k\cdot 360^\circ = \dots\ -600^\circ,\ -240^\circ,\ 120^\circ,\ 480^\circ,\ \dots$$





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or

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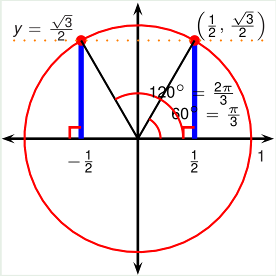

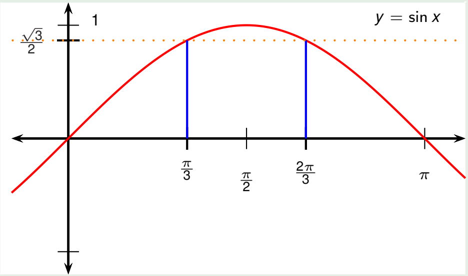

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$$
\begin{array}{rcl}
\sin\theta &=& \frac{\sqrt{3}}{2} \\
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&& \textbf{or} \qquad\qquad\qquad\qquad \cdots\ k{=}-2 \quad k{=}-1 \quad k{=}0 \quad k{=}1\ \cdots \\
&& 120^\circ + k\cdot 360^\circ = \dots\ -600^\circ,\ -240^\circ,\ 120^\circ,\ 480^\circ,\ \dots \\
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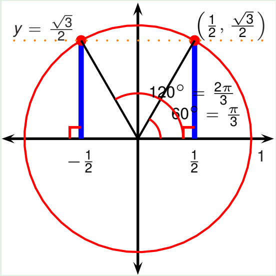
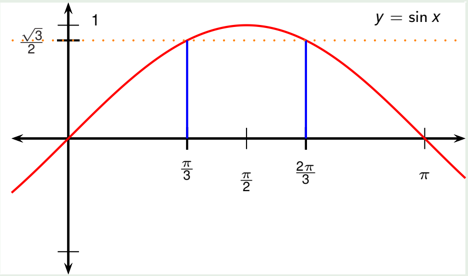
\theta &=& \dots\ -660^\circ,\ -300^\circ,\ 60^\circ,\ 420^\circ,\ \dots \\
&& \dots\ -600^\circ,\ -240^\circ,\ 120^\circ,\ 480^\circ,\ \dots
\end{array}
$$





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$$
\begin{array}{rclllllll}
\sin\theta &=& \frac{\sqrt{3}}{2} \\
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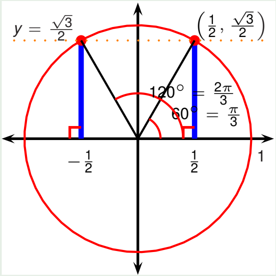
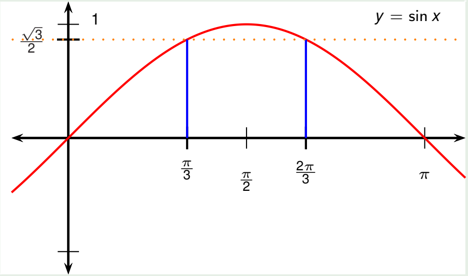
&& \textbf{or} & \cdots & k=-2 & k=-1 & k=0 & k=1 & \cdots \\
&& 120^\circ + k\cdot 360^\circ = & \dots & -600^\circ, & -240^\circ, & 120^\circ, & 480^\circ, & \dots \\
\hline
\theta &=& & \cancel{\dots} & \cancel{-660^\circ}, & -300^\circ, & 60^\circ, & \cancel{420^\circ}, & \cancel{\dots} \\
&& & \cancel{\dots} & \cancel{-600^\circ}, & -240^\circ, & 120^\circ, & \cancel{480^\circ}, & \cancel{\dots}
\end{array}
$$

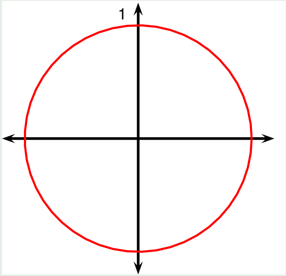


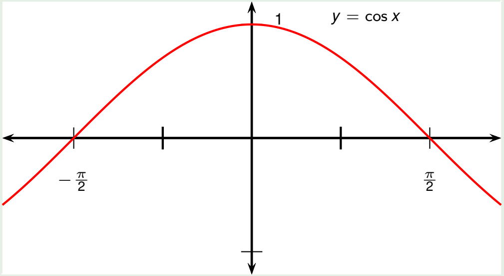


Example

Find all solutions and then find those that lie between -180° and 180°.

$$\cos\theta = \frac{\sqrt{2}}{2}$$





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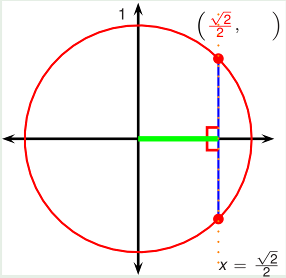

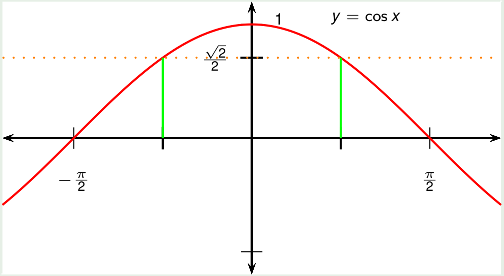

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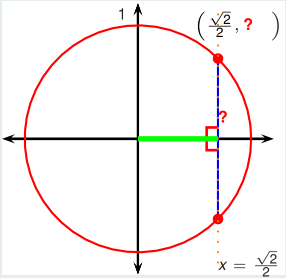

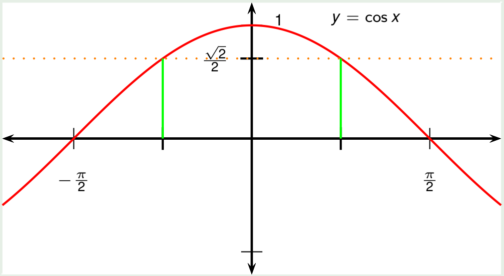

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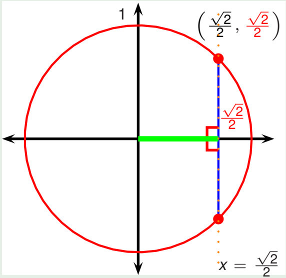
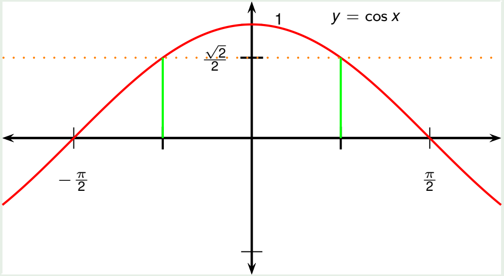
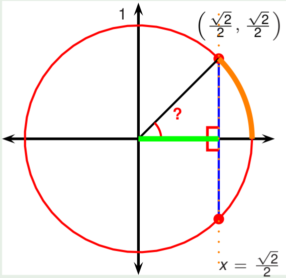


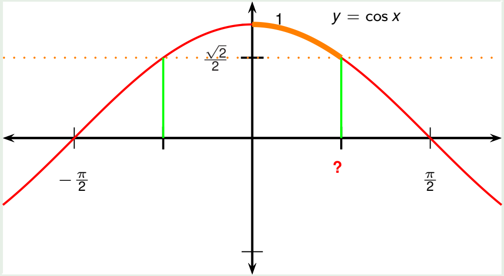


Example

Find all solutions and then find those that lie between $-180°$ and $180°$.

$$\cos\theta = \frac{\sqrt{2}}{2}$$
$$\theta = \textbf{?}$$

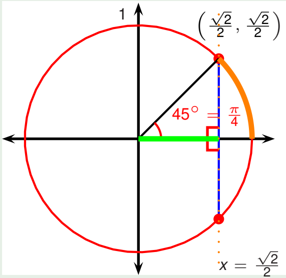


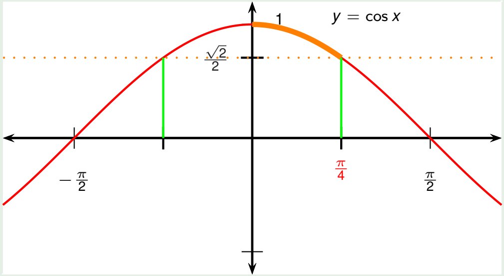


Example

Find all solutions and then find those that lie between -180° and 180°.

$$\cos\theta = \frac{\sqrt{2}}{2}$$
$$\theta = 45^\circ$$





Example

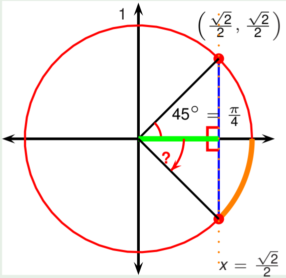
Find all solutions and then find those that lie between -180° and 180°.

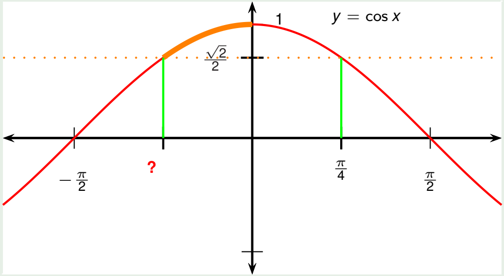
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ$$

or

?





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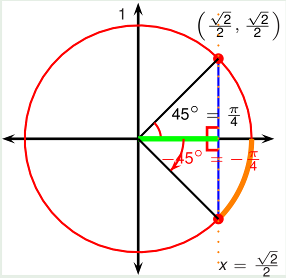
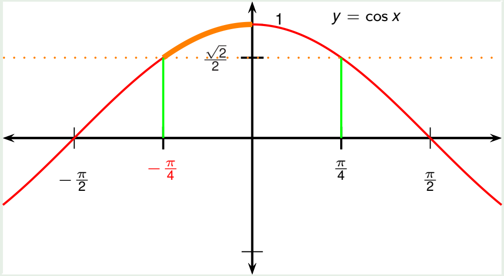
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ$$

or

-45°





Example

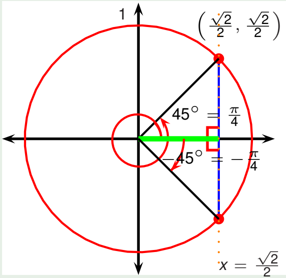
Find all solutions and then find those that lie between -180° and 180°.

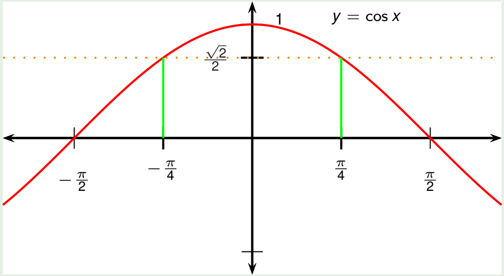
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ + k\cdot 360^\circ$$

or

$$-45^\circ$$





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$$\theta = 45^\circ + k\cdot 360^\circ$$

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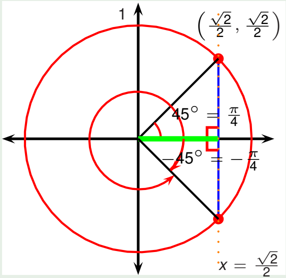
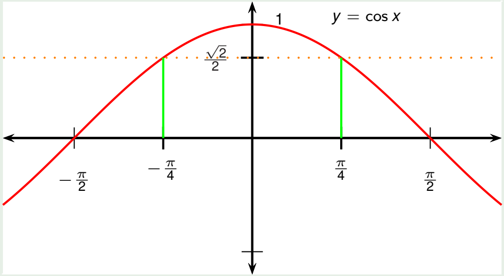




Example

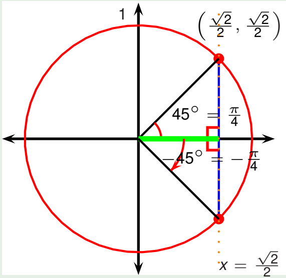
Find all solutions and then find those that lie between -180° and 180°.

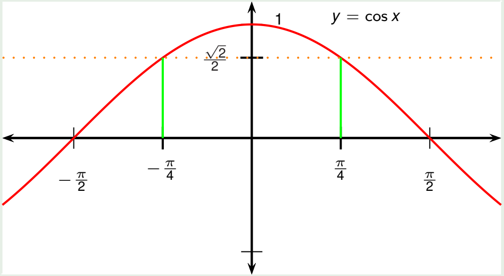
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ + k\cdot 360^\circ = \quad \dots \quad -675^\circ,$$

or $\qquad \dots \quad {\scriptstyle k=-2}$

$$-45^\circ + k\cdot 360^\circ = \dots \quad -765^\circ,$$





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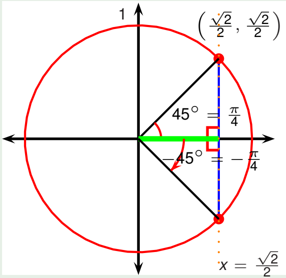
Find all solutions and then find those that lie between -180° and 180°.

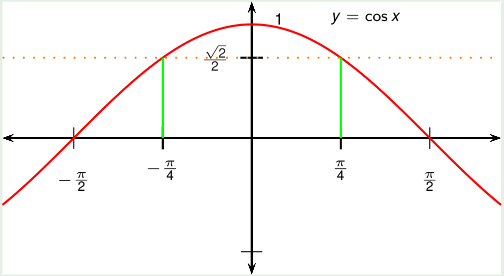
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ + {\color{red}k}\cdot 360^\circ = \quad \dots \quad \underset{k=-2}{-675^\circ}, \quad {\color{red}\underset{k=-1}{-315^\circ}},$$

or

$$-45^\circ + {\color{red}k}\cdot 360^\circ = \quad \dots \quad -765^\circ, \quad {\color{red}-405^\circ},$$





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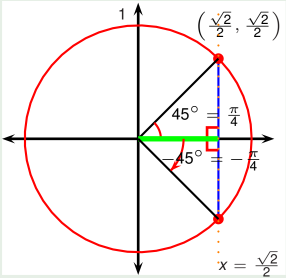
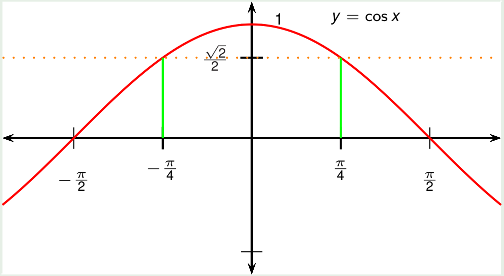
$$-45^\circ + k\cdot 360^\circ = \dots \; -765^\circ, \; -405^\circ, \; -45^\circ,$$





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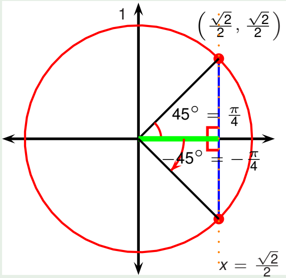

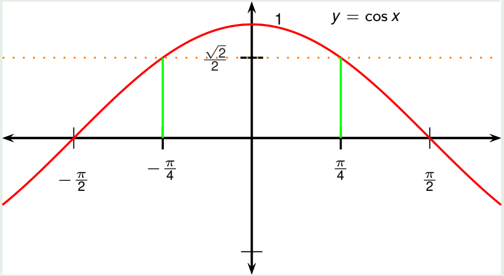

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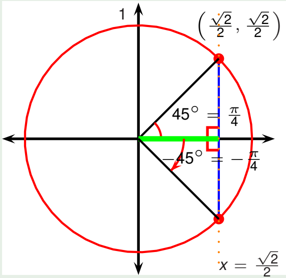

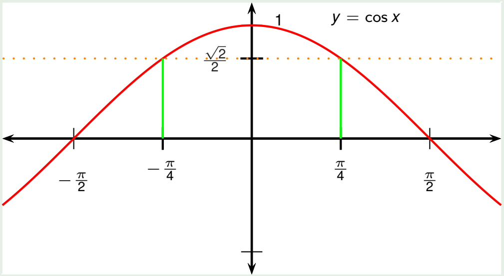

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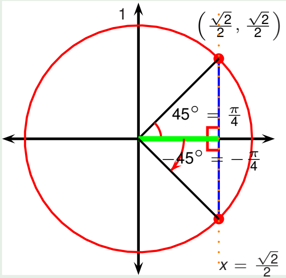

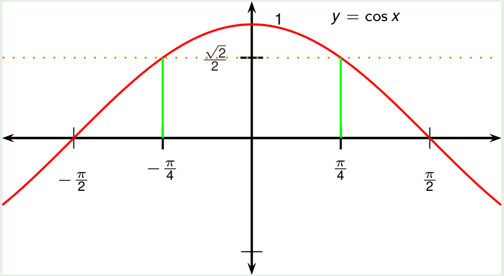

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\hline
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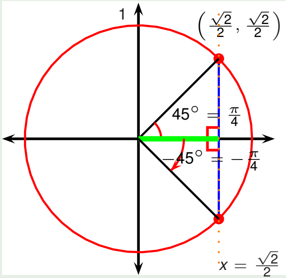
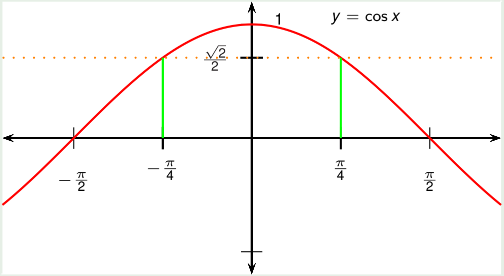
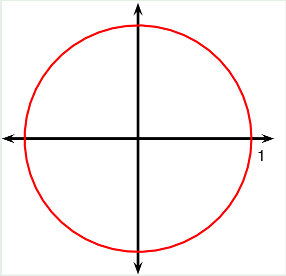


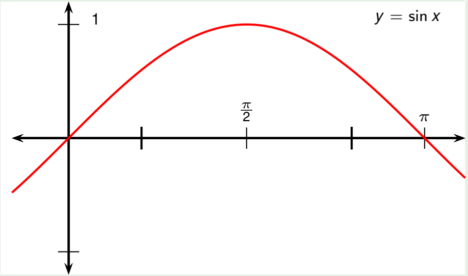


Example

Find all solutions of the equation.

$$\sin\theta = \frac{3}{5}$$





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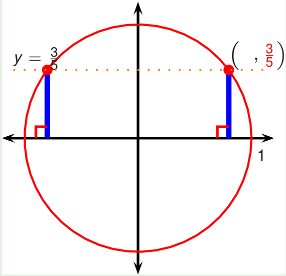

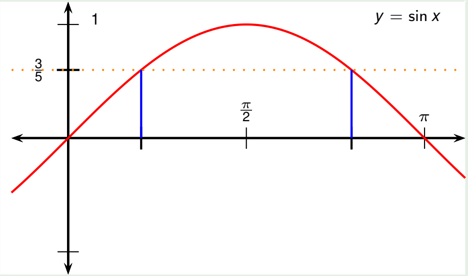

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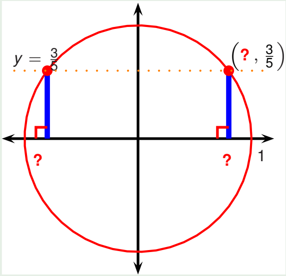

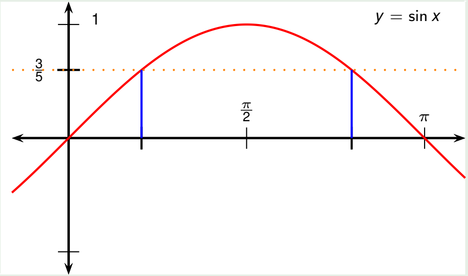

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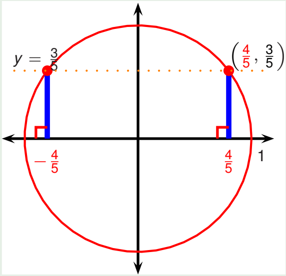

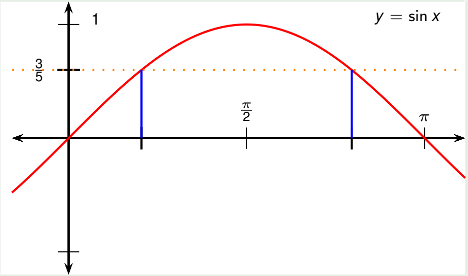

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$$\sin\theta = \frac{3}{5}$$

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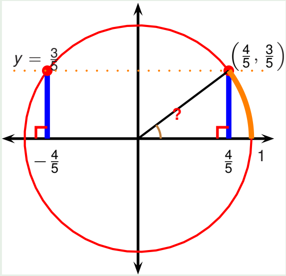

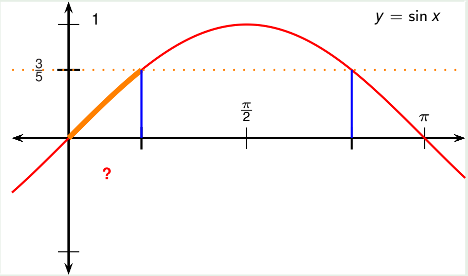

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$$\sin\theta = \frac{3}{5}$$

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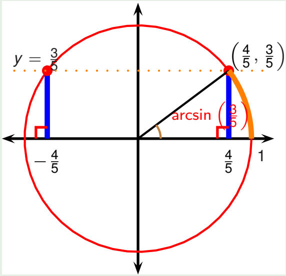
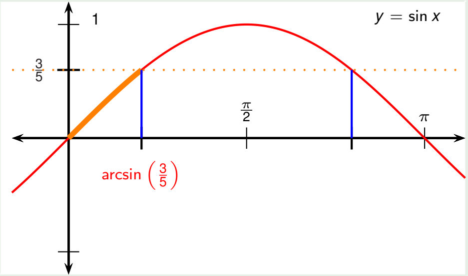
| arcsin implies radians





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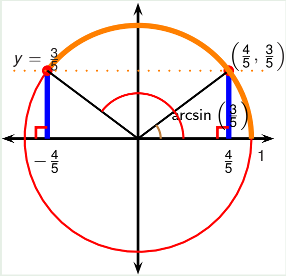

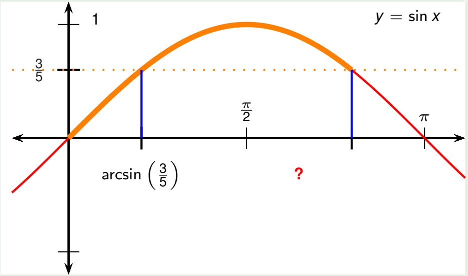

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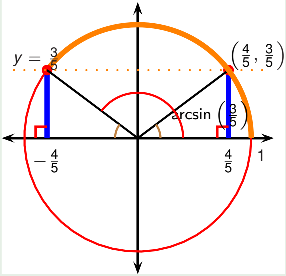

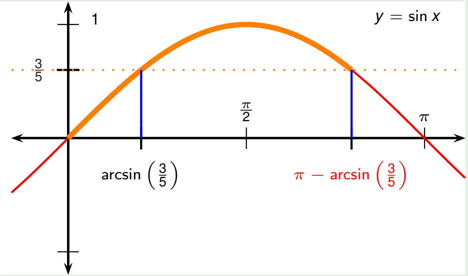

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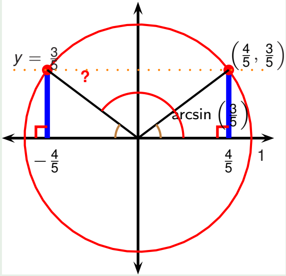
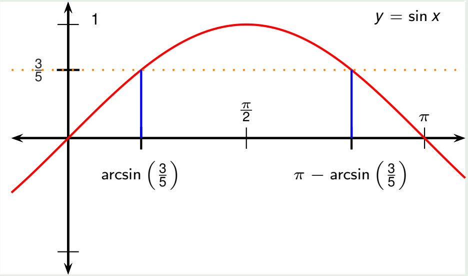




Example

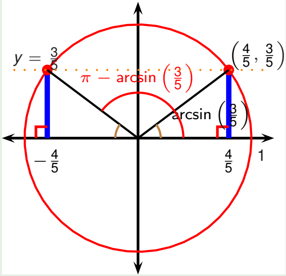
Find all solutions of the equation.

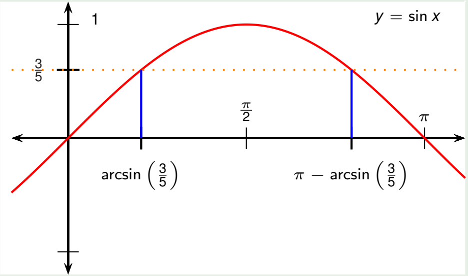
$$\sin\theta = \frac{3}{5}$$

$$\theta = \arcsin\left(\frac{3}{5}\right) \qquad \Big|\ \arcsin \text{ implies radians}$$

or

$$\pi - \arcsin\left(\frac{3}{5}\right)$$





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$$\sin\theta = \frac{3}{5}$$

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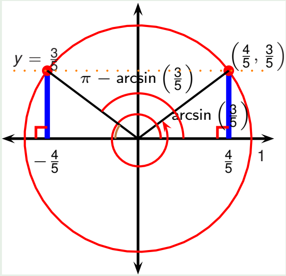

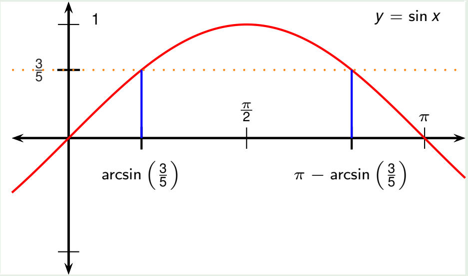

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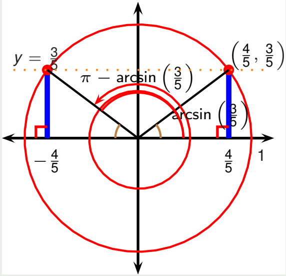

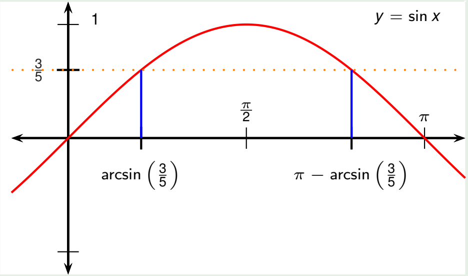

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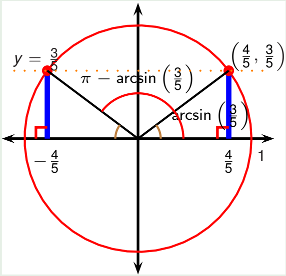

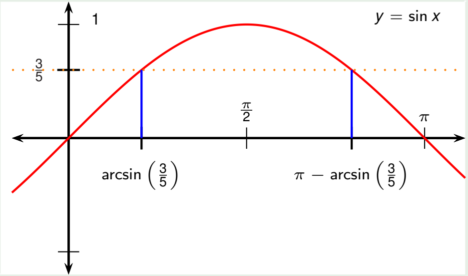

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Find all values of θ in the interval $[0, 2\pi]$ such that $\sin\theta = \sin(2\theta)$.

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$$\begin{aligned} \sin\theta &= \sin(2\theta) \\ \sin\theta &= \textbf{?} \end{aligned}$$

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Find all values of θ in the interval $[0, 2\pi]$ such that $\sin\theta = \sin(2\theta)$.

$$\begin{aligned}\sin\theta &= \sin(2\theta)\\ \sin\theta &= 2\sin\theta\cos\theta\end{aligned}$$

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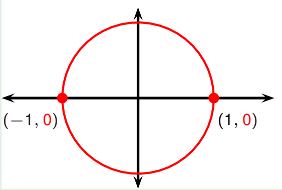

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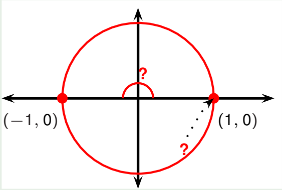

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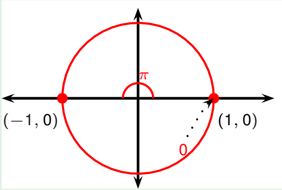

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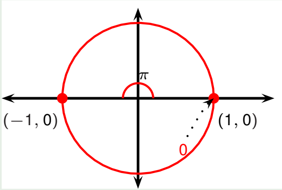

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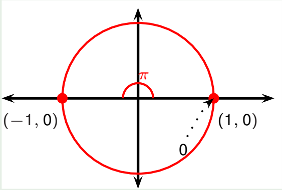

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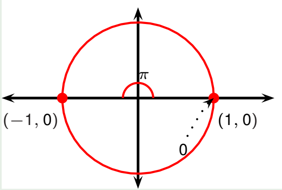

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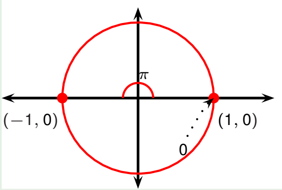

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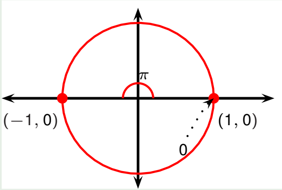

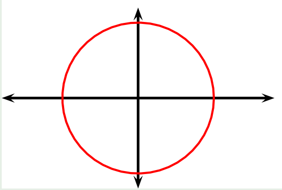

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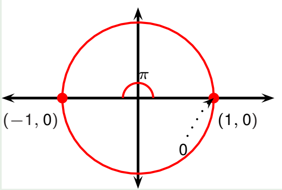

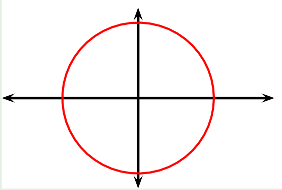

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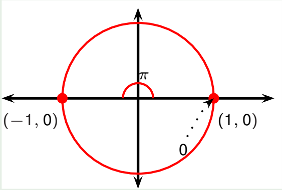

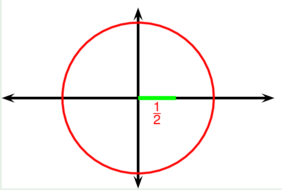

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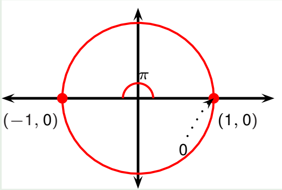

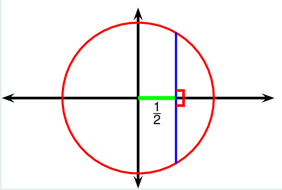

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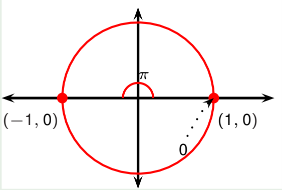

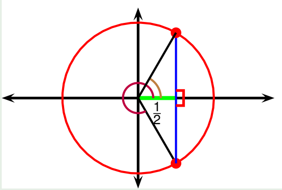

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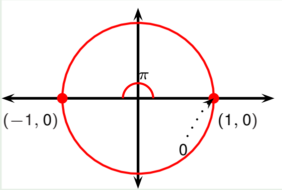

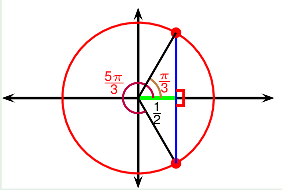

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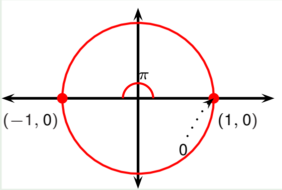

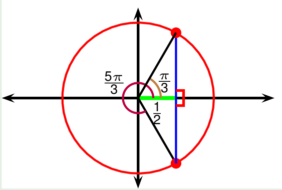

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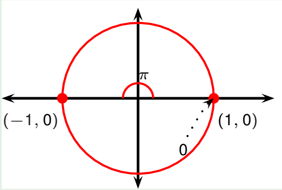

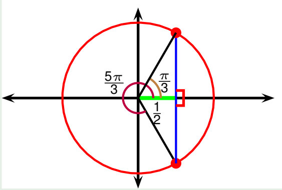

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Find all values of θ in the interval $\theta \in [0, 2\pi]$ for which

$$\cos(2\theta) = \cos\theta$$

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$$\begin{aligned} \cos(2\theta) &= \cos\theta \\ \cos^2\theta - \sin^2\theta - \cos\theta &= 0 \quad \Big| \text{ Express via } \cos\theta \\ \cos^2\theta - (\textbf{?} \qquad\qquad) - \cos\theta &= 0 \end{aligned}$$

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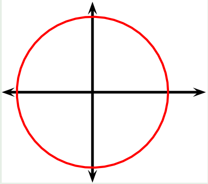

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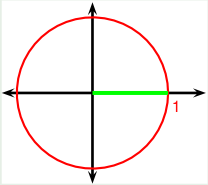

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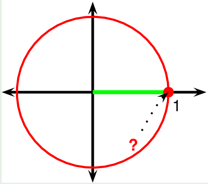

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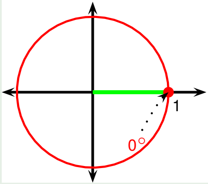

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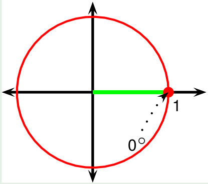

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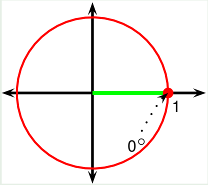
$$2u + 1 = 0$$



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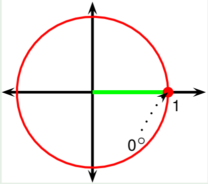

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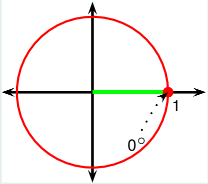

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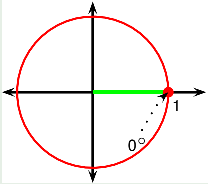

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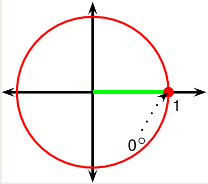

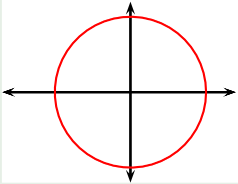

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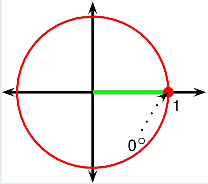

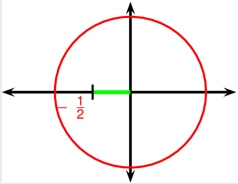

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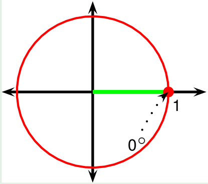

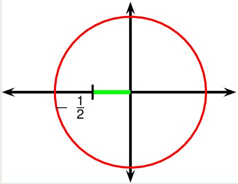

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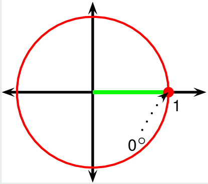

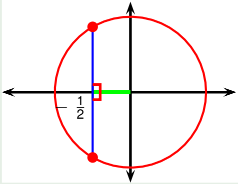

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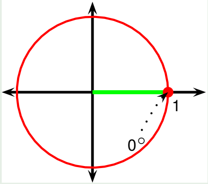

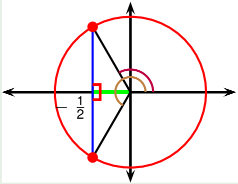

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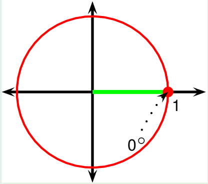

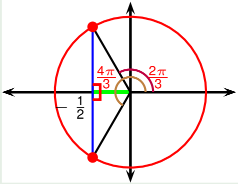

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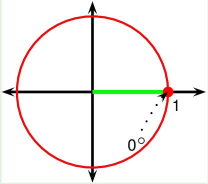

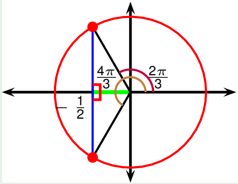

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$$\begin{array}{rcll} \cos(2\theta) &=& \cos\theta & \\ \cos^2\theta - \sin^2\theta - \cos\theta &=& 0 & \Big| \text{ Express via } \cos\theta \\ \cos^2\theta - (1-\cos^2\theta) - \cos\theta &=& 0 & \\ 2\cos^2\theta - \cos\theta - 1 &=& 0 & \Big| \text{ Set } \cos\theta = u \\ 2u^2 - u - 1 &=& 0 & \\ (u-1)(2u+1) &=& 0 & \end{array}$$

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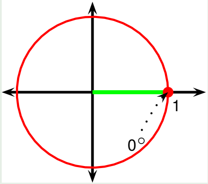

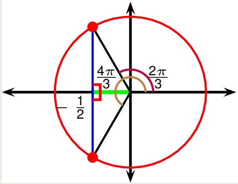

Strategy for solving trigonometric equations

- Suppose we want to solve an algebraic trigonometric equation.
- More precisely, the equation should be an algebraic expressions of the trigonometric functions of a single variable.
- Here is a general strategy for solving such a problem:
  - Using trig identities, rewrite in terms of $\sin x$ and $\cos x$ only.
  - Suppose $x \in [2n\pi, (2n+1)\pi]$.
    - Set $\sin x = \sqrt{1-\cos^2 x}$ (allowed due to restrictions on x).
    - Set $\cos x = u$. Solve the resulting algebraic equation for u.
    - For the found solutions for u, solve $\cos x = u$.
    - Check whether your solutions satisfy $x \in [2n\pi, (2n+1)\pi]$.
  - Suppose $x \in [(2n-1)\pi, 2n\pi]$.
    - Set $\sin x = -\sqrt{1-\cos^2 x}$ (allowed due to restrictions on x).
    - Set $\cos x = u$. Solve the resulting algebraic equation for u.
    - For the found solutions for u, solve $\cos x = u$.
    - Check whether your solutions satisfy $x \in [(2n-1)\pi, 2n\pi]$.
- A similar strategy exists for $u = \sin x$ instead of $u = \cos x$.
- Problems requiring full algorithm may be too hard for Calc exams.

Proposition (Product to sum formulas)

$$\begin{aligned}
\sin\alpha\sin\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)\\
\cos\alpha\cos\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)+\cos(\alpha+\beta)\right)\\
\sin\alpha\cos\beta &= \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)
\end{aligned}$$

Proof.

$\square$

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? $= \cos(\alpha+\beta)$

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$$\cos\alpha\cos\beta-\sin\alpha\sin\beta = \cos(\alpha+\beta)$$

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$$\begin{aligned}
\cos\alpha\cos\beta+\sin\alpha\sin\beta &= \cos(\alpha-\beta)\\
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□

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$$
\begin{aligned}
+\quad \cos\alpha\cos\beta + \cancel{\sin\alpha\sin\beta} &= \cos(\alpha-\beta)\\
\cos\alpha\cos\beta - \cancel{\sin\alpha\sin\beta} &= \cos(\alpha+\beta)\\
2\cos\alpha\cos\beta &= \cos(\alpha-\beta)+\cos(\alpha+\beta)
\end{aligned}
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$$

$\square$

Proposition (Product to sum formulas)

$$
\begin{aligned}
\sin\alpha\sin\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)\\
\cos\alpha\cos\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)+\cos(\alpha+\beta)\right)\\
\sin\alpha\cos\beta &= \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)
\end{aligned}
$$

Proof.

$$
\begin{array}{lrcl}
 & \cos\alpha\cos\beta+\cancel{\sin\alpha\sin\beta} &=& \cos(\alpha-\beta)\\
+ & \cos\alpha\cos\beta-\cancel{\sin\alpha\sin\beta} &=& \cos(\alpha+\beta)\\
 & 2\cos\alpha\cos\beta &=& \cos(\alpha-\beta)+\cos(\alpha+\beta)
\end{array}
$$

□

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$\square$

Proposition (Product to sum formulas)

$$
\begin{aligned}
\sin\alpha \sin\beta &= \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)\\
\cos\alpha \cos\beta &= \frac{1}{2}\left(\cos(\alpha-\beta) + \cos(\alpha+\beta)\right)\\
\sin\alpha \cos\beta &= \frac{1}{2}\left(\sin(\alpha+\beta) + \sin(\alpha-\beta)\right)
\end{aligned}
$$

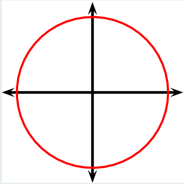
- Product to sum formulas are used when integrating (a topic to be studied later/in another course).

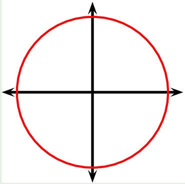
Proposition (Sum to product formulas)

$$\begin{aligned}
\sin\alpha + \sin\beta &= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\\
\sin\alpha - \sin\beta &= 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)\\
\cos\alpha + \cos\beta &= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\\
\cos\alpha - \cos\beta &= 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)
\end{aligned}$$

Example

Find all solutions in the interval $[0, 2\pi)$.





$$\sin(2x) + \sin(5x) = 0$$

Recall the formula $\sin\alpha + \sin\beta = \textbf{?}$

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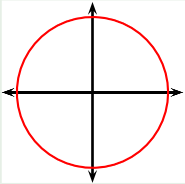

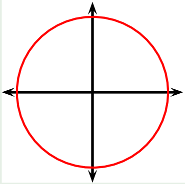

$\sin(2x) + \sin(5x) = 0 \;\Big|\;$ use f-la

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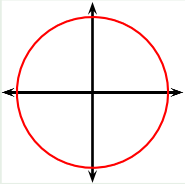

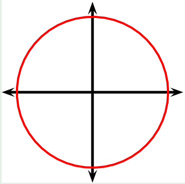

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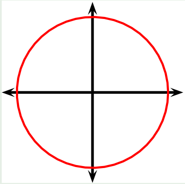

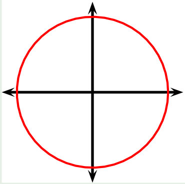

$$\sin(2x) + \sin(5x) = 0 \quad \Big| \text{ use f-la}$$

$$2\sin\left(\frac{2x+5x}{2}\right)\cos\left(\frac{2x-5x}{2}\right) = 0$$

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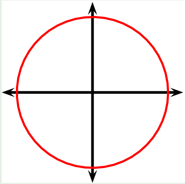

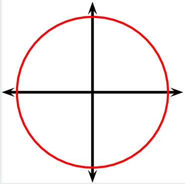

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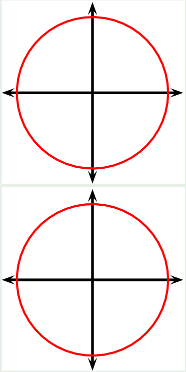

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$$2\sin\left(\frac{7}{2}x\right)\cos\left(-\frac{3}{2}x\right) = 0$$

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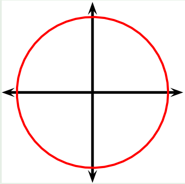

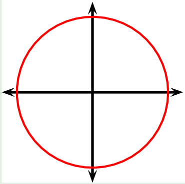

$$\sin(2x) + \sin(5x) = 0 \quad | \text{ use f-la}$$
$$2\sin\left(\frac{2x+5x}{2}\right)\cos\left(\frac{2x-5x}{2}\right) = 0$$
$$2\sin\left(\frac{7}{2}x\right)\cos\left(-\frac{3}{2}x\right) = 0 \quad | \text{ cos is even}$$
$$2\sin\left(\frac{7}{2}x\right)\cos\left(\frac{3}{2}x\right) = 0$$

Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

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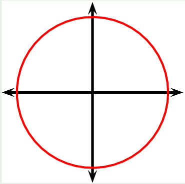

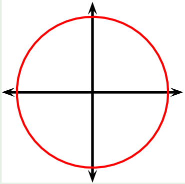

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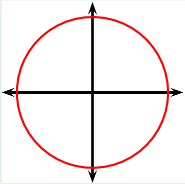

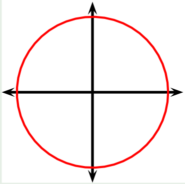

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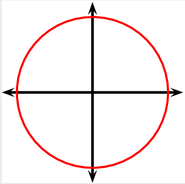
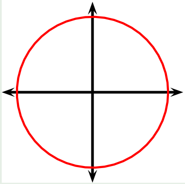


$\sin\left(\frac{7}{2}x\right) = 0$

or



$\cos\left(\frac{3}{2}x\right) = 0$

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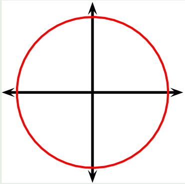

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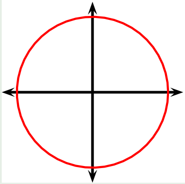

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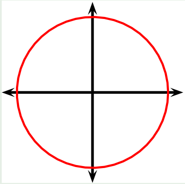

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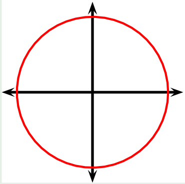

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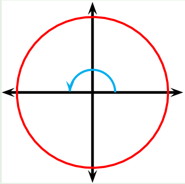

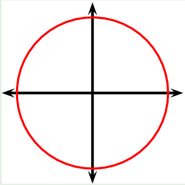

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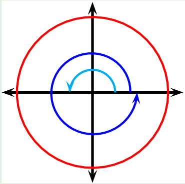

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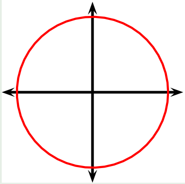

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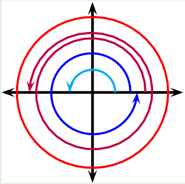

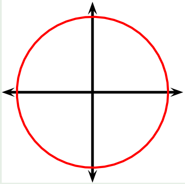

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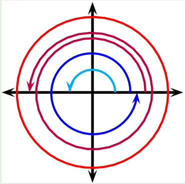
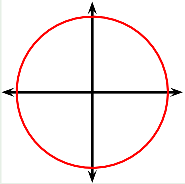


$\sin\left(\frac{7}{2}x\right) = 0$

$\frac{7}{2}x = k\pi$ $\quad$ k – integer

or



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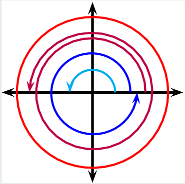

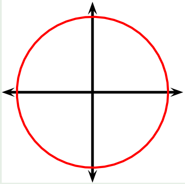

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$$\tfrac{7}{2}x = k\pi \qquad k - \text{integer}$$

$$x = \tfrac{2k\pi}{7}$$

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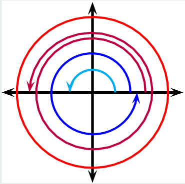

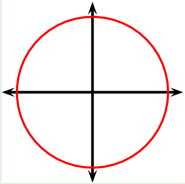

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$$x = \ldots, \frac{-2\pi}{7}, 0, \frac{2\pi}{7}, \frac{4\pi}{7}, \frac{6\pi}{7}, \frac{8\pi}{7}, \frac{10\pi}{7}, \frac{12\pi}{7}, \frac{14\pi}{7}, \ldots$$

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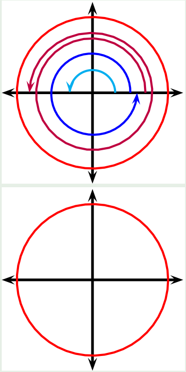

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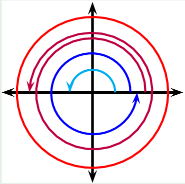

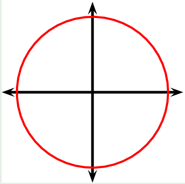

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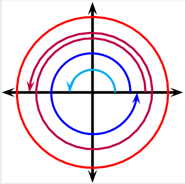

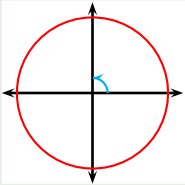

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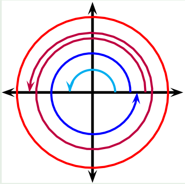

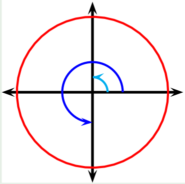

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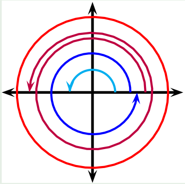

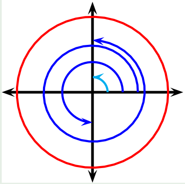

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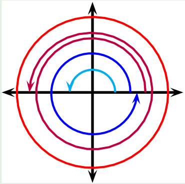

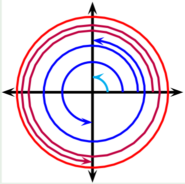

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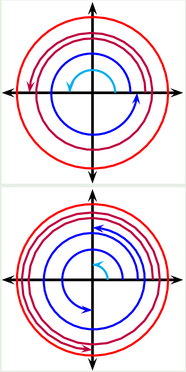

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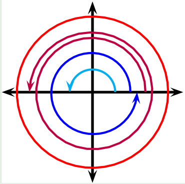

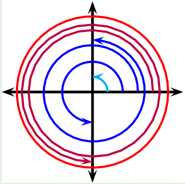

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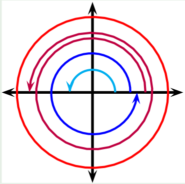

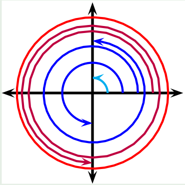

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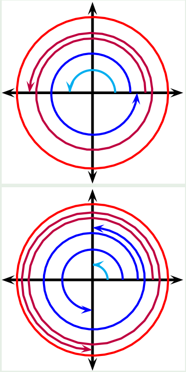

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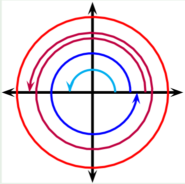

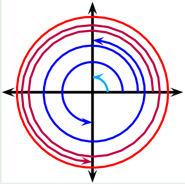

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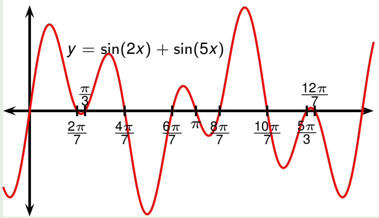

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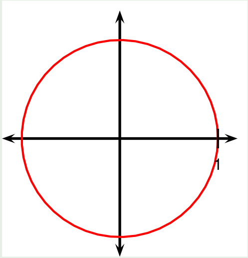
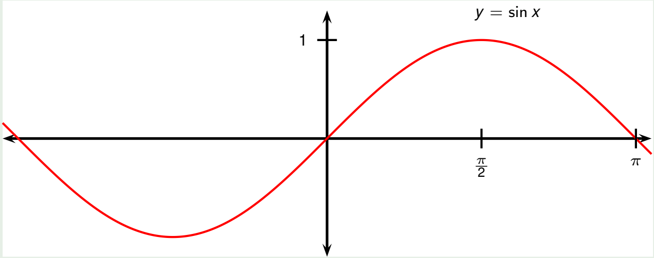
Solve. Among your solutions, find those between -360° and 450°.

$\frac{1}{2} \leq \sin\theta < \frac{\sqrt{3}}{2}$





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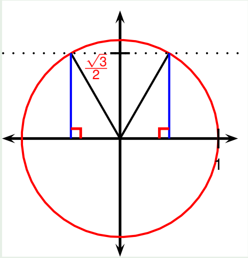

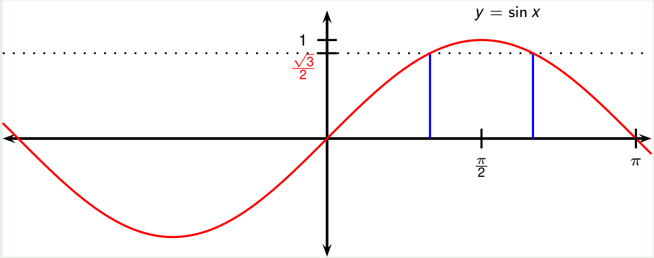

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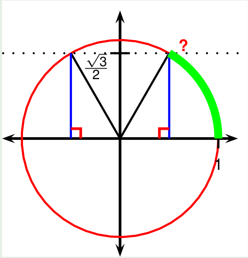

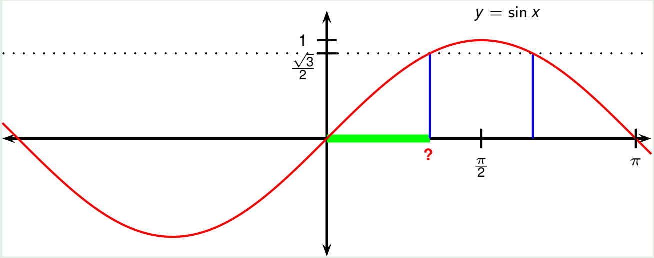

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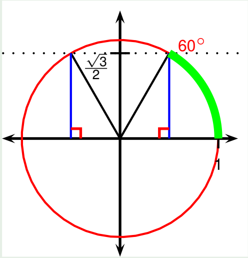

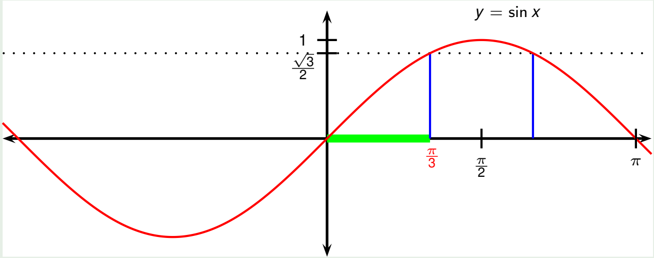

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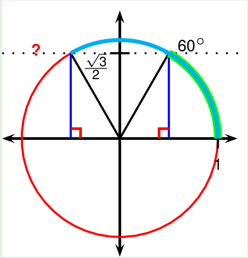

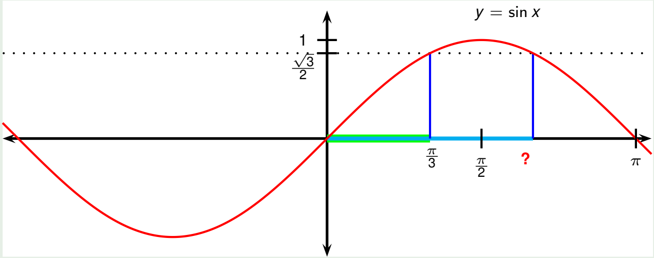

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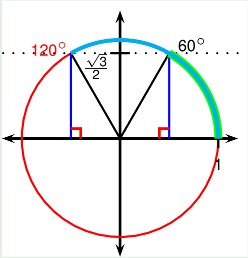

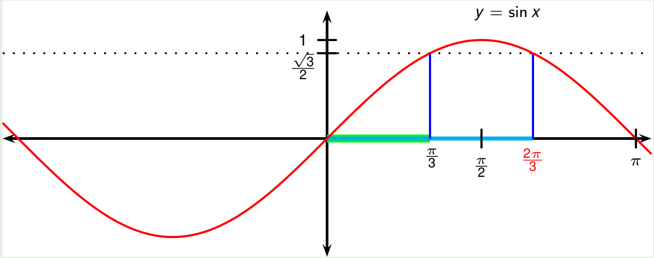

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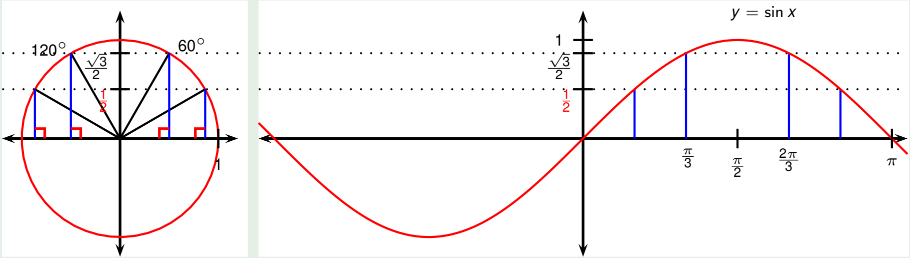

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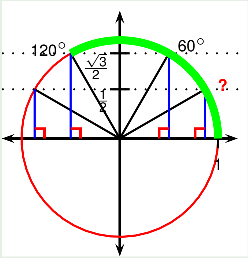

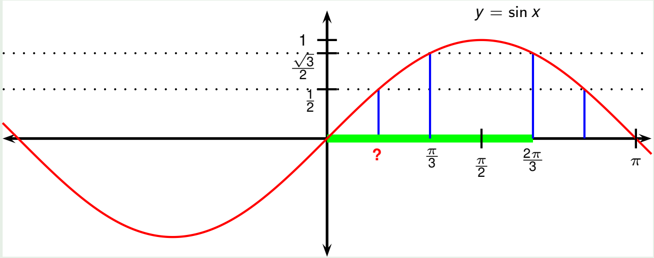

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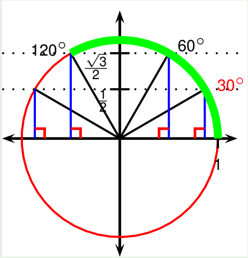

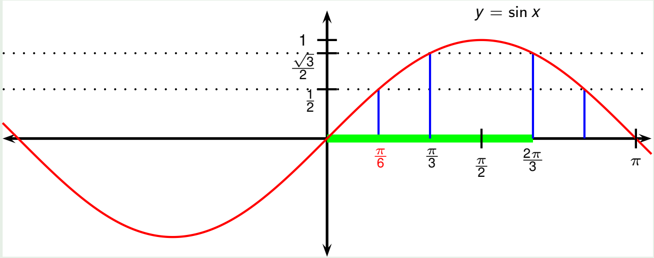

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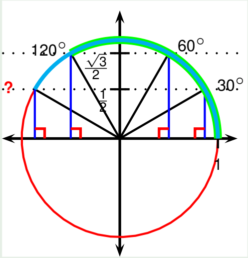
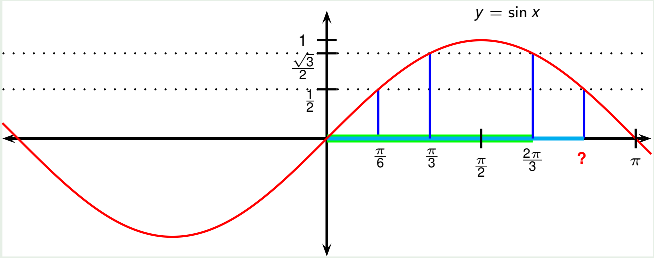
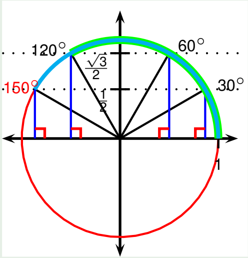


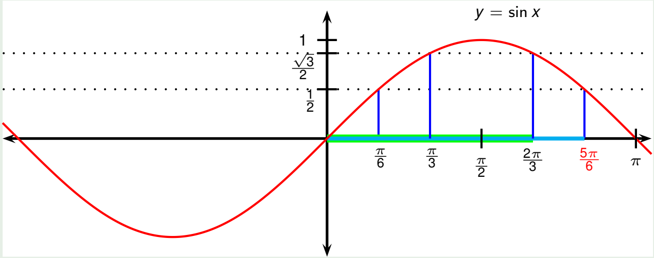


Example

Solve. Among your solutions, find those between -360° and 450°.

$\frac{1}{2} \leq \sin\theta < \frac{\sqrt{3}}{2}$



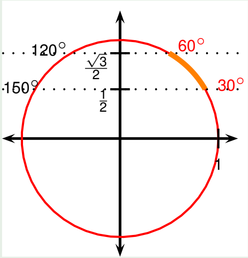


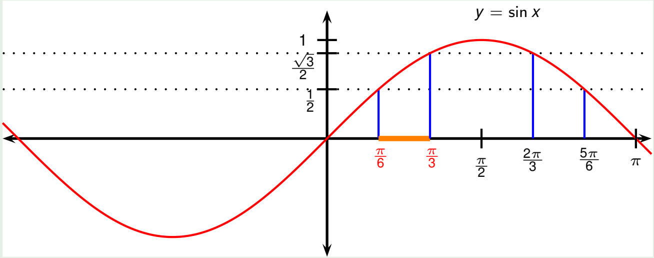
Example

Solve. Among your solutions, find those between -360° and 450°.

$\frac{1}{2} \leq \sin\theta < \frac{\sqrt{3}}{2}$

$x \in [30^\circ \quad , 60^\circ \quad)$





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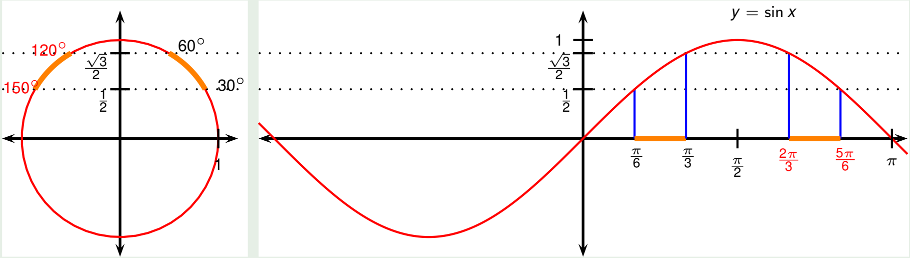
$x \in [30^\circ \quad , 60^\circ \quad) \cup (120^\circ \quad , 150^\circ \quad]$



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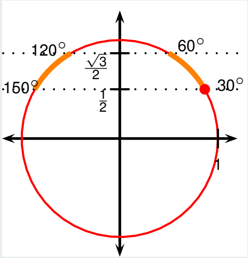

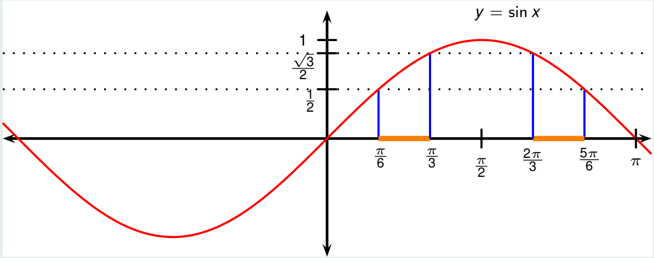

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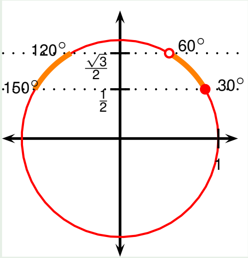

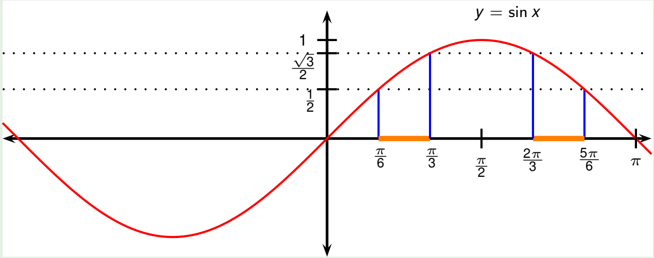

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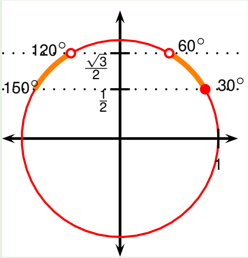

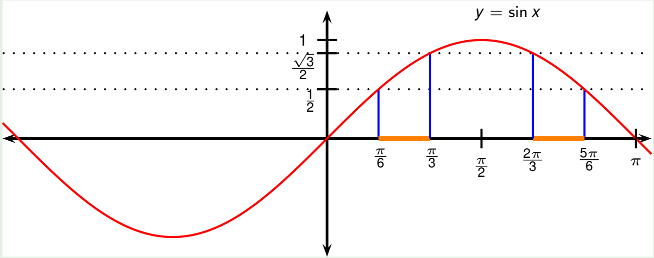

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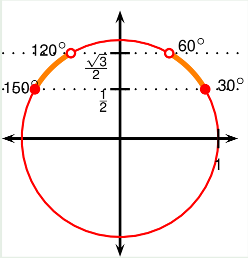
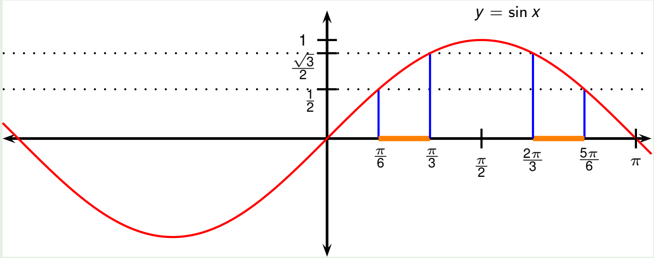


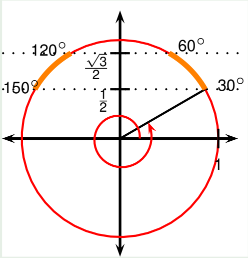


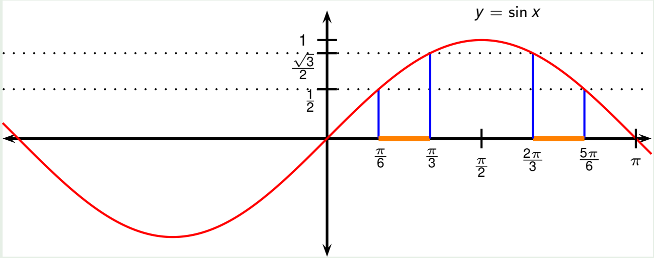
Example

Solve. Among your solutions, find those between -360° and 450°.

$\frac{1}{2} \leq \sin\theta < \frac{\sqrt{3}}{2}$

$x \in [30^\circ + k360^\circ, 60^\circ + k360^\circ) \cup (120^\circ + k360^\circ, 150^\circ + k360^\circ]$





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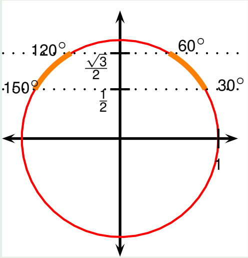
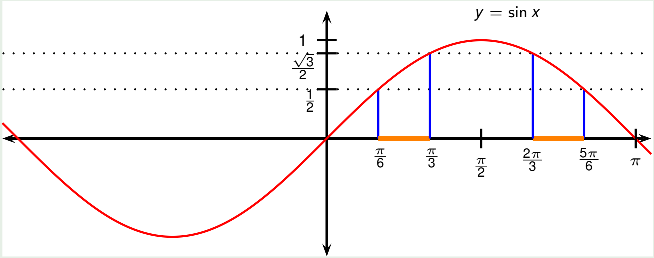
$x \in \quad [30^\circ, 60^\circ) \cup (120^\circ, 150^\circ] \quad \Big| \; k = 0$





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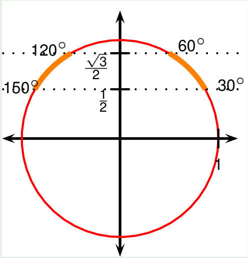
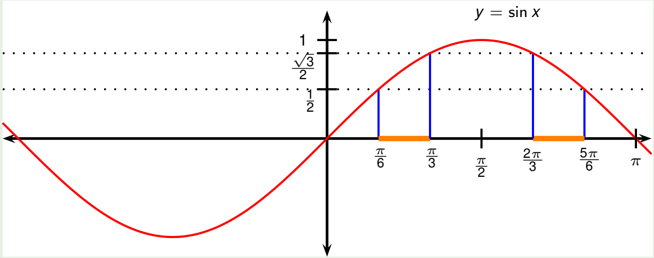
$$x \in \begin{array}{r} [30^\circ, 60^\circ) \cup (120^\circ, 150^\circ] \\ \cup [390^\circ, 420^\circ) \cup (480^\circ, 510^\circ] \end{array} \quad \left| \begin{array}{l} k = 0 \\ k = 1 \end{array} \right.$$





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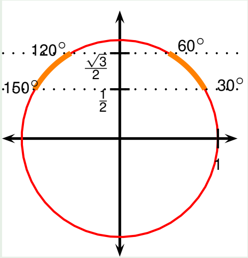

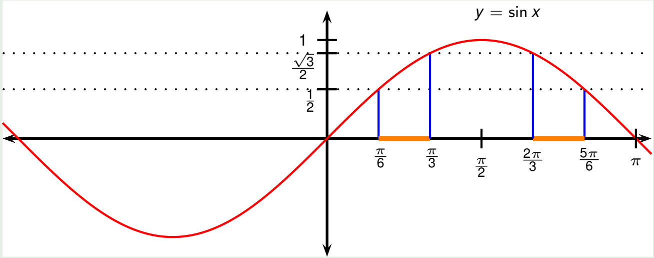

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Solve. Among your solutions, find those between $-360°$ and $450°$.

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$x \in [30° + k360°, 60° + k360°) \cup (120° + k360°, 150° + k360°]$

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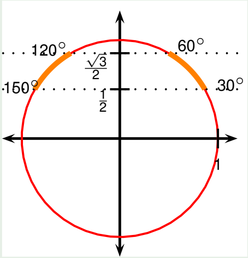

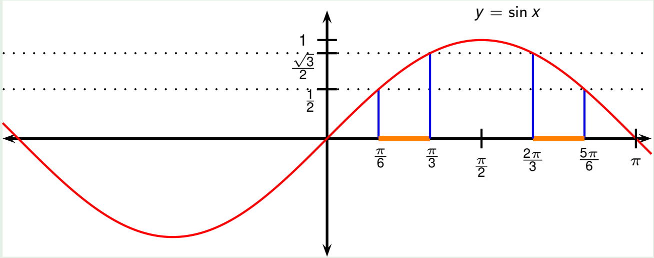

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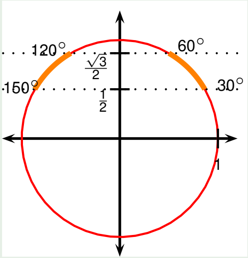

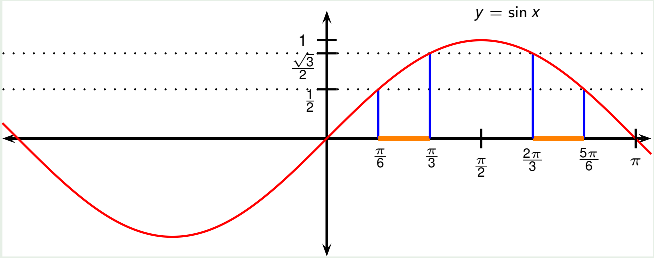

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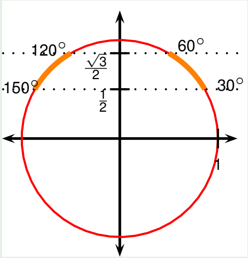

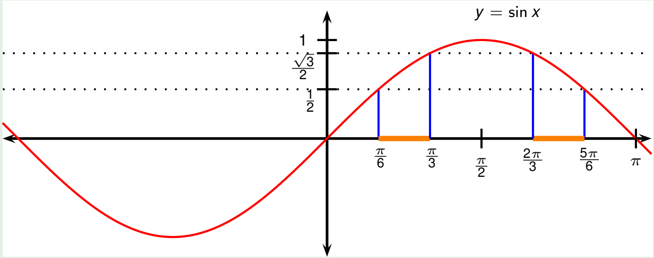

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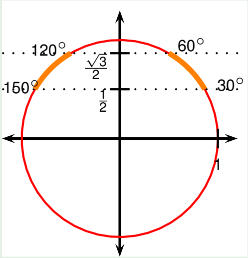

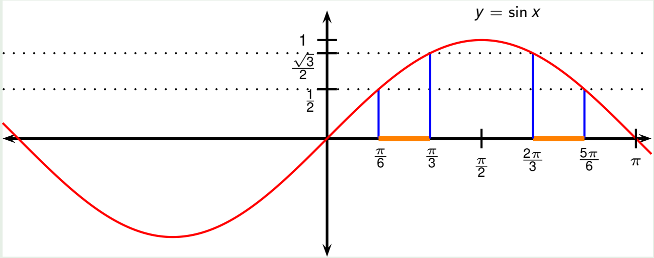

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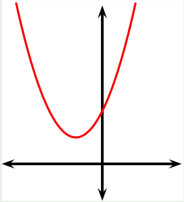
In radians:

$x \in \left[-\frac{11\pi}{6}, -\frac{5\pi}{3}\right) \cup \left[-\frac{4\pi}{3}, -\frac{7\pi}{6}\right) \cup \left[\frac{\pi}{6}, \frac{\pi}{3}\right) \cup \left[\frac{2\pi}{3}, \frac{5\pi}{6}\right) \cup \left[\frac{13\pi}{6}, \frac{7\pi}{3}\right)$

Example

- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2\cos^2\theta + 2\cos\theta + 1 \leq \cos\theta + 2$ lying in $[-360^\circ, 360^\circ]$.

Example



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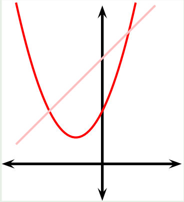

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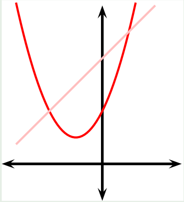

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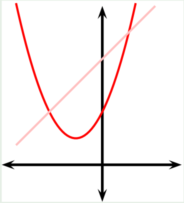
$$\begin{aligned} 2u^2 + 2u + 1 &\leq u + 2 \\ 2u^2 + u - 1 &\leq 0 \end{aligned}$$

Example



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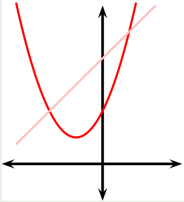
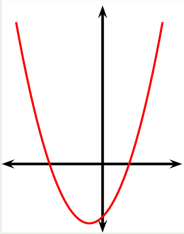
$$\begin{array}{rcl} 2u^2 + 2u + {\color{red}1} & \leq & u + {\color{red}2} \\ 2u^2 + u {\color{red}- 1} & \leq & 0 \end{array}$$

Example





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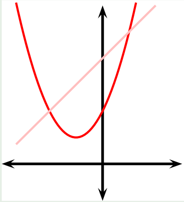

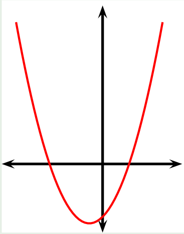

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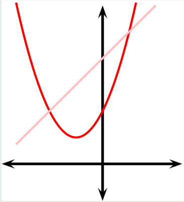
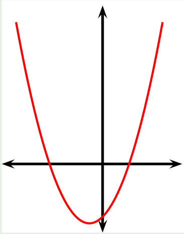
Example





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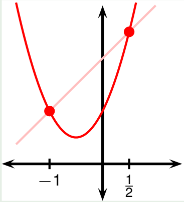
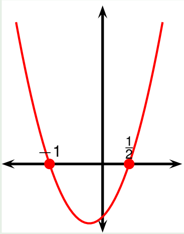
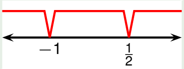
$$
\begin{array}{rcl}
2u^2 + 2u + 1 & \leq & u + 2 \\
2u^2 + u - 1 & \leq & 0 \\
2\left(u - \frac{1}{2}\right)\left(u + 1\right) & \leq & 0
\end{array}
$$

Example







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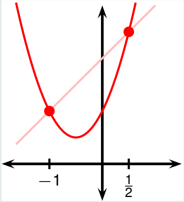
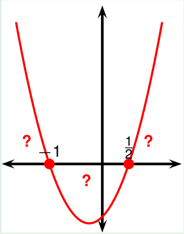
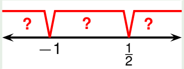
$$\begin{array}{rcl} 2u^2 + 2u + 1 & \leq & u + 2 \\ 2u^2 + u - 1 & \leq & 0 \\ 2\left(u - \frac{1}{2}\right)\left(u + 1\right) & \leq & 0 \end{array}$$

Example







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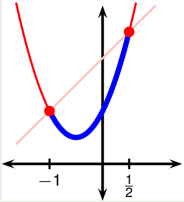

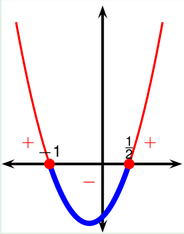

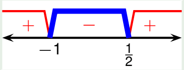

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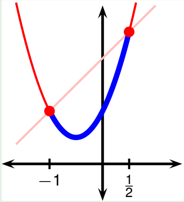

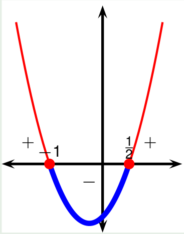

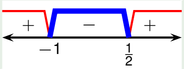

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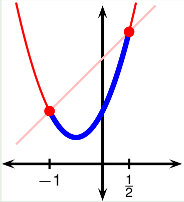

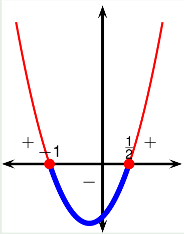

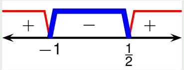

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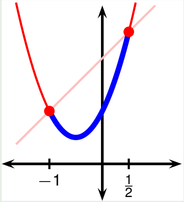

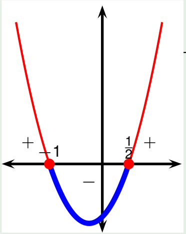

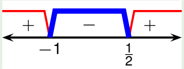

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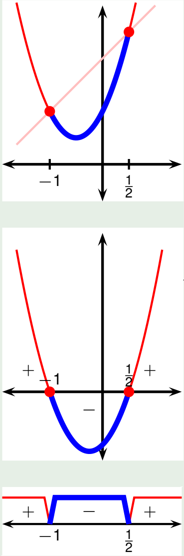
Example



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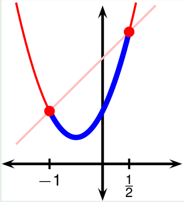
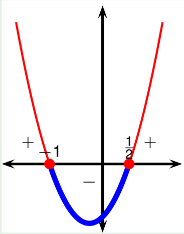
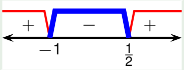
Example







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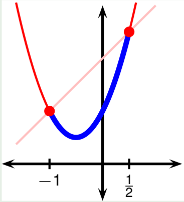
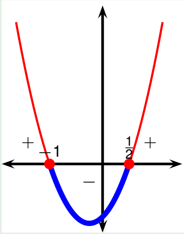
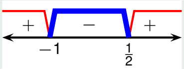
$$\begin{aligned} 2\cos^2\theta + 2\cos\theta + 1 &\leq \cos\theta + 2 && \text{Set } \color{red}{\cos\theta = u} \\ 2u^2 + 2u + 1 &\leq u + 2 \\ \color{red}{u} &\in \left[-1, \tfrac{1}{2}\right] && \text{(solved above)} \\ \color{red}{\cos\theta} &\in \left[-1, \tfrac{1}{2}\right] \end{aligned}$$

Example







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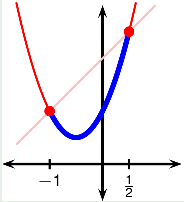
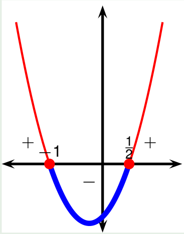
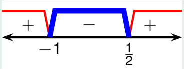
Example







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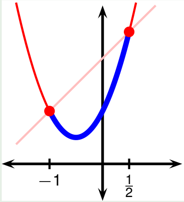

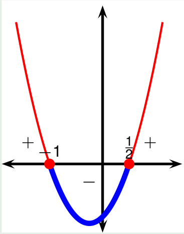

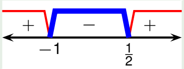

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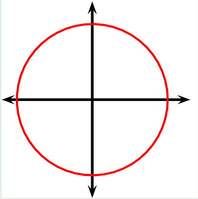

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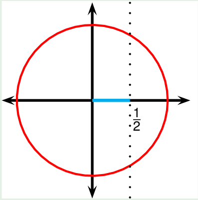
Example



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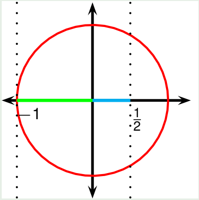

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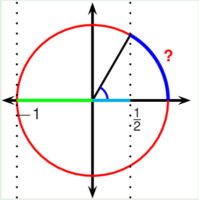

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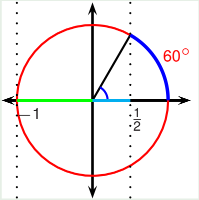

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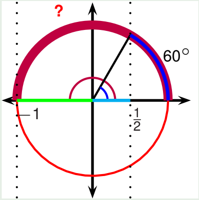
$\theta \in \quad [60° \quad , \mathbf{?} \quad]$

Example



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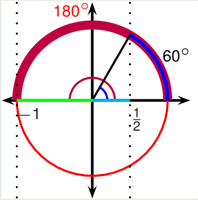
Example



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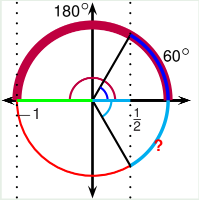
$\theta \in$ $[60°$, $180°$ $]$

Example



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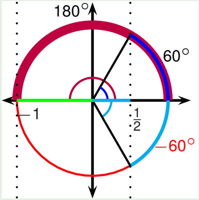
$\theta \in [$**?** , **?** $] \cup [60^\circ , 180^\circ]$

Example



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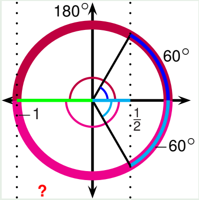
$\theta \in [\mathbf{?} \qquad , -60° \qquad] \cup [60° \qquad , 180° \qquad]$

Example



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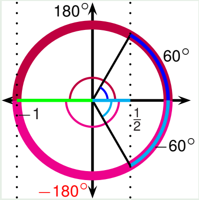
Example



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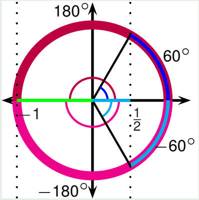
Example



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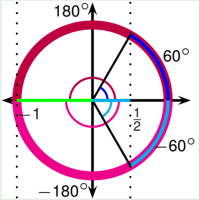
$$\theta \in [-180° + k360°, -60° + k360°] \cup [60° + k360°, 180° + k360°]$$

Example



- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2\cos^2\theta + 2\cos\theta + 1 \leq \cos\theta + 2$ lying in $[-360°, 360°]$.

$$\begin{aligned} \cos\theta &\in \left[-1, \tfrac{1}{2}\right] \\ -1 \leq \cos\theta &\leq \frac{1}{2} \end{aligned}$$

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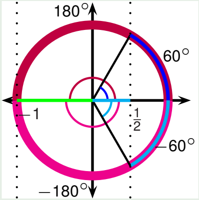
Example



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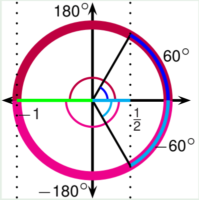
$$\theta \in [-180°, -60°] \cup [60°, 180°] \qquad k = 0$$

Example



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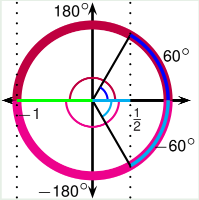
$$\begin{array}{lll} \theta \in & [-180°, -60°] \cup [60°, 180°] & k = 0 \\ & \cup [180°, 300°] \cup [420°, 540°] & k = 1 \end{array}$$

Example



- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2\cos^2\theta + 2\cos\theta + 1 \leq \cos\theta + 2$ lying in $[-360^\circ, 360^\circ]$.

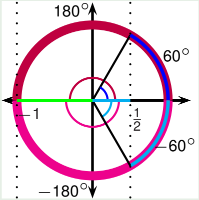
$$\begin{aligned} \cos\theta &\in \left[-1, \tfrac{1}{2}\right] \\ -1 \leq \cos\theta &\leq \tfrac{1}{2} \end{aligned}$$

$$\theta \in [-180^\circ + k360^\circ, -60^\circ + k360^\circ] \cup [60^\circ + k360^\circ, 180^\circ + k360^\circ]$$

$$\begin{aligned} \theta \in \quad & [-180^\circ, -60^\circ] \cup [60^\circ, 180^\circ] && k = 0 \\ & \cup [180^\circ, 300^\circ] \cup [420^\circ, 540^\circ] && k = 1 \end{aligned}$$

$\dots$

Example



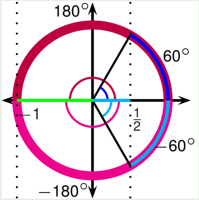
- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2\cos^2\theta + 2\cos\theta + 1 \leq \cos\theta + 2$ lying in $[-360^\circ, 360^\circ]$.

$$\begin{array}{rcl} \cos\theta & \in & \left[-1, \frac{1}{2}\right] \\ -1 \leq \cos\theta & \leq & \frac{1}{2} \end{array}$$

$$\theta \in [-180^\circ + k360^\circ, -60^\circ + k360^\circ] \cup [60^\circ + k360^\circ, 180^\circ + k360^\circ]$$

$$\theta \in \begin{array}{rll} [-540^\circ, -420^\circ] \cup [-300^\circ, -180^\circ] & & k = -1 \\ \cup [-180^\circ, -60^\circ] \cup [60^\circ, 180^\circ] & & k = 0 \\ \cup [180^\circ, 300^\circ] \cup [420^\circ, 540^\circ] & & k = 1 \\ \ldots & & \end{array}$$

Example



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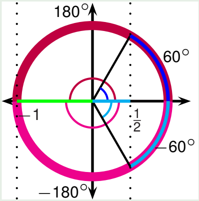
$$\theta \in [-180°+k360°, -60°+k360°] \cup [60°+k360°, 180°+k360°]$$

$$\theta \in \begin{array}{lr} \dots & \\ \cup[-540°, -420°] \cup [-300°, -180°] & k=-1 \\ \cup[-180°, -60°] \cup [60°, 180°] & k=0 \\ \cup[180°, 300°] \cup [420°, 540°] & k=1 \\ \dots & \end{array}$$

Example



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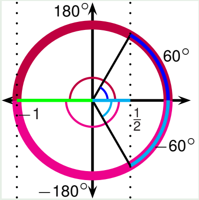
$$\begin{array}{lrl} & \cdots & \\ & \cup \cancel{[-540^\circ, -420^\circ]} \cup [-300^\circ, -180^\circ] & k = -1 \\ \theta \in & \cup [-180^\circ, -60^\circ] \cup [60^\circ, 180^\circ] & k = 0 \\ & \cup [180^\circ, 300^\circ] \cup \cancel{[420^\circ, 540^\circ]} & k = 1 \\ & \cdots & \end{array}$$

Example



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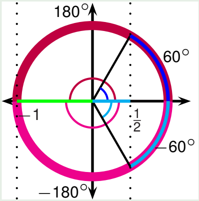
$$\theta \in \begin{array}{rclr} & \cancel{\dots} & & \\ \cup\ \cancel{[-540^\circ, -420^\circ]} & \cup & [-300^\circ, -180^\circ] & k = -1 \\ \cup\ [-180^\circ, -60^\circ] & \cup & [60^\circ, 180^\circ] & k = 0 \\ \cup\ [180^\circ, 300^\circ] & \cup & \cancel{[420^\circ, 540^\circ]} & k = 1 \\ & \cancel{\dots} & & \end{array}$$

$$\theta \in [-300^\circ, -60^\circ] \cup [60^\circ, 300^\circ]$$

Example



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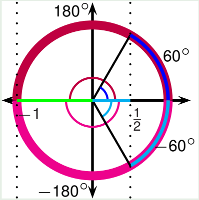
$$\theta \in [-300^\circ, -60^\circ] \cup \color{red}{[60^\circ, 300^\circ]}$$

Example



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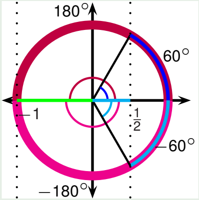
Example



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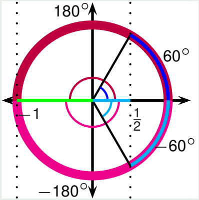
Example



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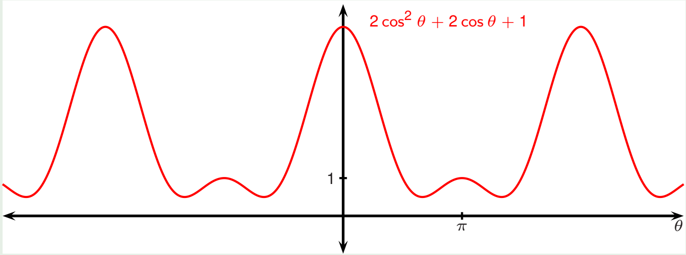
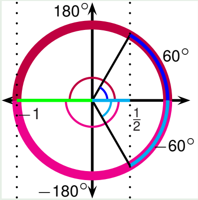


Example



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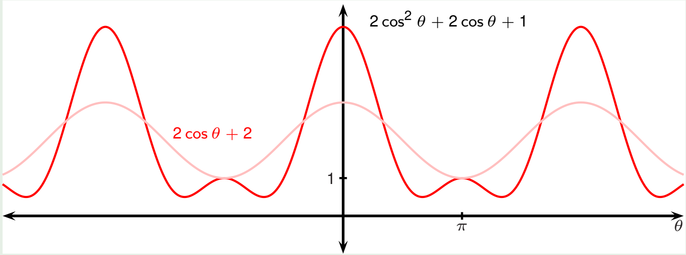
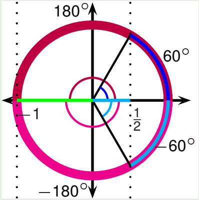


Example



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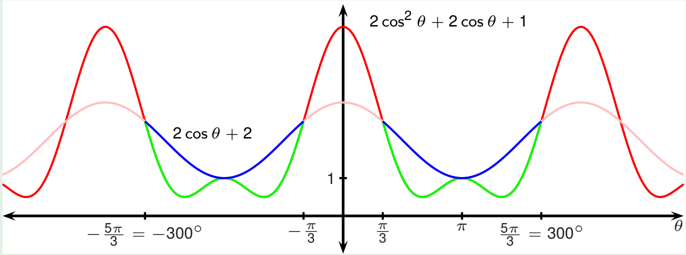
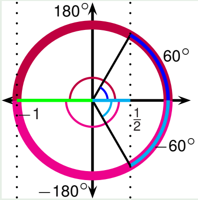


Example



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