Precalculus
Find extremum of quadratic, text problem.

Todor Milev

2019

Maximum or minimum value of a quadratic function

- Let $f(x) = ax^2 + bx + c$ - quadratic ($a \neq 0$).
- Let D be the discriminant $D = b^2 - 4ac$.

$$f(x) = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a} \quad \Big| \text{ complete the square}$$

Maximum or minimum value of a quadratic function

- Let $f(x) = ax^2 + bx + c$ - quadratic ($a \neq 0$).
- Let D be the discriminant $D = b^2 - 4ac$.

$$f(x) = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a} \quad \Bigg| \text{ complete the square}$$

- Therefore if $a > 0$ then $f(x) = a(\text{square}) - \dfrac{D}{4a}$.

Maximum or minimum value of a quadratic function

- Let $f(x) = ax^2 + bx + c$ - quadratic ($a \neq 0$).
- Let D be the discriminant $D = b^2 - 4ac$.

$$f(x) = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a} \quad \Big| \text{ complete the square}$$

- Therefore if $a > 0$ then $f(x) = a(\text{square}) - \frac{D}{4a} \geq -\frac{D}{4a}$.

Maximum or minimum value of a quadratic function

- Let $f(x) = ax^2 + bx + c$ - quadratic ($a \neq 0$).
- Let D be the discriminant $D = b^2 - 4ac$.

$$f(x) = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a} \quad \Bigg| \text{ complete the square}$$

- Therefore if $a > 0$ then $f(x) = a(\text{square}) - \dfrac{D}{4a} \geq -\dfrac{D}{4a}$.
- Similarly if $a < 0$ then $f(x) = a(\text{square}) - \dfrac{D}{4a} \leq -\dfrac{D}{4a}$.

Maximum or minimum value of a quadratic function

- Let $f(x) = ax^2 + bx + c$ - quadratic ($a \neq 0$).
- Let D be the discriminant $D = b^2 - 4ac$.

$$f(x) = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a} \quad \Big| \text{ complete the square}$$

- Therefore if $a > 0$ then $f(x) = a(\text{square}) - \frac{D}{4a} \geq -\frac{D}{4a}$.
- Similarly if $a < 0$ then $f(x) = a(\text{square}) - \frac{D}{4a} \leq -\frac{D}{4a}$.

Recall $f(x) = ax^2 + bx + c = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a}$.

Proposition

Let $f(x) = ax^2 + bx + c$, $a \neq 0$ and let $D = b^2 - 4ac$.

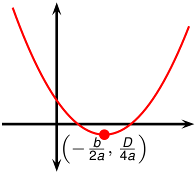
- *If $a > 0$ then $f(x)$ has no maximum and has minimum at $x = -\frac{b}{2a}$.*
- *If $a < 0$ then $f(x)$ has no minimum and has maximum at $x = -\frac{b}{2a}$.*
- *In both cases, the extremal value (either maximum or minimum) is $f\left(-\frac{b}{2a}\right) = -\frac{b^2-4ac}{4a} = -\frac{D}{4a}$.*

Recall $f(x) = ax^2 + bx + c = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a}$.

Proposition

Let $f(x) = ax^2 + bx + c$, $a \neq 0$ and let $D = b^2 - 4ac$.

- *If $a > 0$ then $f(x)$ has no maximum and has minimum at $x = -\frac{b}{2a}$.*
- *If $a < 0$ then $f(x)$ has no minimum and has maximum at $x = -\frac{b}{2a}$.*
- *In both cases, the extremal value (either maximum or minimum) is $f\left(-\frac{b}{2a}\right) = -\frac{b^2-4ac}{4a} = -\frac{D}{4a}$.*

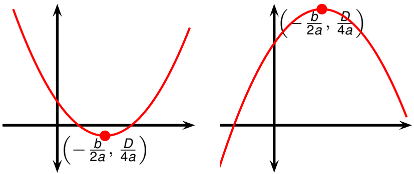


Recall $f(x) = ax^2 + bx + c = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a}$.

Proposition

Let $f(x) = ax^2 + bx + c$, $a \neq 0$ and let $D = b^2 - 4ac$.

- *If $a > 0$ then $f(x)$ has no maximum and has minimum at $x = -\frac{b}{2a}$.*
- *If $a < 0$ then $f(x)$ has no minimum and has maximum at $x = -\frac{b}{2a}$.*
- *In both cases, the extremal value (either maximum or minimum) is $f\left(-\frac{b}{2a}\right) = -\frac{b^2-4ac}{4a} = -\frac{D}{4a}$.*



Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.

Maximizing:

$$xz$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.

$$x + z = 12$$

Maximizing:

$$xz$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.

$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

Maximizing:

$$xz$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.

$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

Maximizing:

$$xz = x(12 - x)$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.

$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

Maximizing:

$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.

$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

Maximizing:

$$\begin{aligned} xz &= {\color{red}x}({\color{red}12} - x) \\ &= -x^2 + {\color{red}12x} \end{aligned}$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.

$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

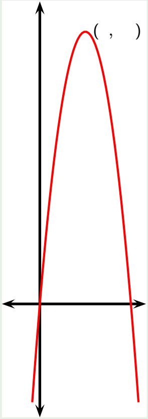
Maximizing:

$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

Parabola

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

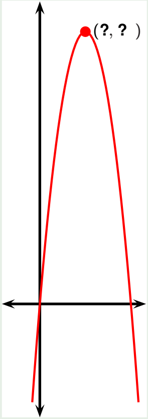
Maximizing:

$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

Parabola opens down

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

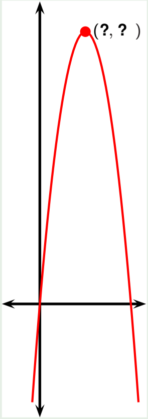
Maximizing:

$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

Maximizing:

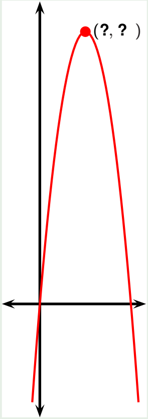
$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$x = \textbf{?}$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

Maximizing:

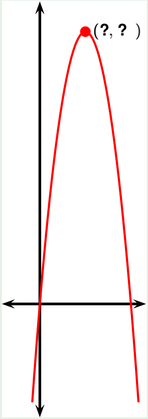
$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$x = -\frac{b}{2a}$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

Maximizing:

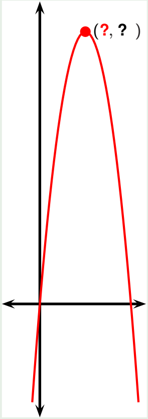
$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{12}{-2} \end{aligned}$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

Maximizing:

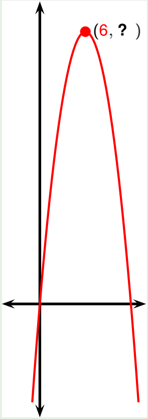
$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{12}{-2} \end{aligned}$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$
\begin{aligned}
x + z &= 12 \\
z &= 12 - x
\end{aligned}
$$

Maximizing:

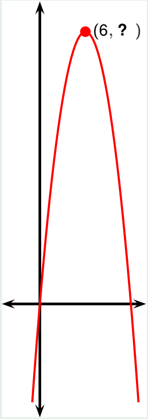
$$
\begin{aligned}
xz &= x(12 - x) \\
&= -x^2 + 12x
\end{aligned}
$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$
\begin{aligned}
x &= -\frac{b}{2a} \\
&= -\frac{12}{-2} = 6
\end{aligned}
$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

Maximizing:

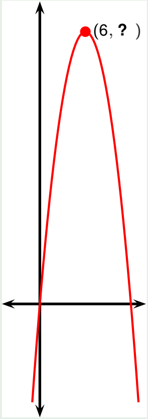
$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{12}{-2} = 6 \\ z &= 12 - x \end{aligned}$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

Maximizing:

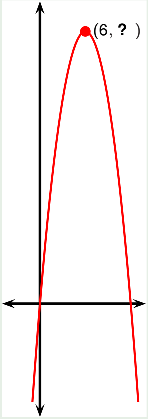
$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$\begin{aligned} {\color{red}x} &= -\frac{b}{2a} \\ &= -\frac{12}{-2} {\color{red}= 6} \\ z &= 12 - {\color{red}x} = 12 - {\color{red}6} \end{aligned}$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

Maximizing:

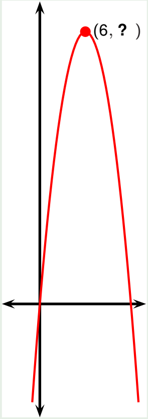
$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{12}{-2} = 6 \\ z &= 12 - x = 12 - 6 = 6 \end{aligned}$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$x + z = 12$$
$$z = 12 - x$$

Maximizing:

$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

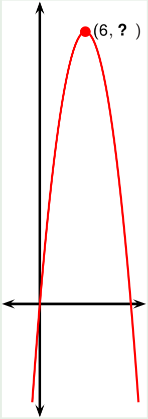
Parabola opens down $\Rightarrow$ has maximum, attained at:

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{12}{-2} = 6 \\ z &= 12 - x = 12 - 6 = 6 \end{aligned}$$

Max. product $= xz$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$
\begin{aligned}
x + z &= 12 \\
z &= 12 - x
\end{aligned}
$$

Maximizing:

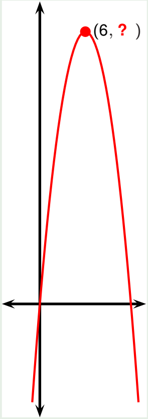
$$
\begin{aligned}
xz &= x(12 - x) \\
&= -x^2 + 12x
\end{aligned}
$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$
\begin{aligned}
x &= -\frac{b}{2a} \\
&= -\frac{12}{-2} = 6 \\
z &= 12 - x = 12 - 6 = 6 \\
\text{Max. product} &= xz = 6 \cdot 6
\end{aligned}
$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$\begin{aligned} x+z &= 12 \\ z &= 12-x \end{aligned}$$

Maximizing:

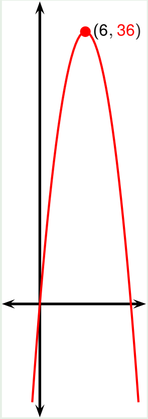
$$\begin{aligned} xz &= x(12-x) \\ &= -x^2+12x \end{aligned}$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{12}{-2} = 6 \\ z &= 12-x = 12-6 = 6 \\ \text{Max. product} &= xz = 6 \cdot 6 \end{aligned}$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$
\begin{aligned}
x + z &= 12\\
z &= 12 - x
\end{aligned}
$$

Maximizing:

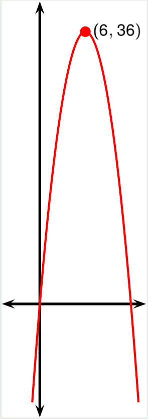
$$
\begin{aligned}
xz &= x(12 - x)\\
&= -x^2 + 12x
\end{aligned}
$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$
\begin{aligned}
x &= -\frac{b}{2a}\\
&= -\frac{12}{-2} = 6\\
z &= 12 - x = 12 - 6 = 6\\
\text{Max. product} &= xz = 6 \cdot 6 = 36.
\end{aligned}
$$

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.



$$
\begin{aligned}
x + z &= 12\\
z &= 12 - x
\end{aligned}
$$

Maximizing:

$$
\begin{aligned}
xz &= x(12 - x)\\
&= -x^2 + 12x
\end{aligned}
$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$
\begin{aligned}
x &= -\frac{b}{2a}\\
&= -\frac{12}{-2} = 6\\
z &= 12 - x = 12 - 6 = 6\\
\text{Max. product} &= xz = 6 \cdot 6 = 36.
\end{aligned}
$$