

Precalculus

Exponent equation that reduces to quadratic, shifted exponents

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2019

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Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

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Set $u = ?$.

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