Precalculus
Homework
Exponential and logarithtmic models

1. Convert from degrees to radians.

   (a) $15°$.
   (b) $30°$.
   (c) $36°$.
   (d) $45°$.
   (e) $60°$.
   (f) $75°$.
   (g) $90°$.
   (h) $120°$.
   (i) $135°$.
   (j) $150°$.
   (k) $180°$.
   (l) $225°$.
   (m) $270°$.
   (n) $305°$.
   (o) $360°$.
   (p) $405°$.
   (q) $1200°$.
   (r) $-900°$.
   (s) $-2014°$.

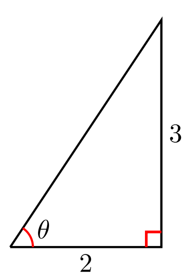
2. Convert from radians to degrees. The answer key has not been proofread, use with caution.

   (a) 4π.
   (b) $-\frac{7}{6}\pi$.
   (c) $\frac{7}{12}\pi$.
   (d) $\frac{4}{3}\pi$.
   (e) $-\frac{3}{8}\pi$.
   (f) 2014π.
   (g) 5.
   (h) -2014.

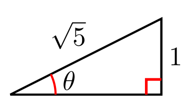
3. Find the indicated circle arc-length. The answer key has not been proofread, use with caution.

   (a) Circle of radius 3, arc of measure $36°$.
   (b) Circle of radius $\frac{1}{2}$, arc of measure $100°$.
   (c) Circle of radius 1, arc of measure 3 (radians).
   (d) Circle of radius 3, arc of measure $300°$.

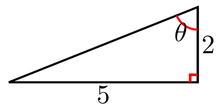
4. Find the 6 trigonometric functions of the indicated angle in the indicated right triangle.

   (a)

   (b)

(c) 

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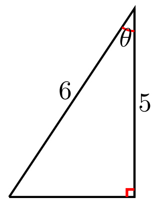


5. Find the exact value of the trigonometric function (using radicals).

   (a) $\cos 135°$.
   (b) $\sin 225°$.
   (c) $\cos 495°$.
   (d) $\sin 560°$.
   (e) $\sin\left(\frac{3\pi}{2}\right)$.
   (f) $\cos\left(\frac{11\pi}{6}\right)$.
   (g) $\sin\left(\frac{2015\pi}{3}\right)$.
   (h) $\cos\left(\frac{17\pi}{3}\right)$.

6. Find all solutions of the equation in the interval $[0, 2\pi)$. The answer key has not been proofread, use with caution.

   (a) $\sin x = -\frac{\sqrt{2}}{2}$.
   (b) $\cos x = \frac{\sqrt{3}}{2}$.
   (c) $\sin(3x) = \frac{1}{2}$.
   (d) $\cos(7x) = 0$.
   (e) $\cos\left(3x + \frac{\pi}{2}\right) = 0$.
   (f) $\sin\left(5x - \frac{\pi}{3}\right) = 0$.

7. Use the known values of $\sin 30°, \cos 30°, \sin 45°, \cos 45°, \sin 60°, \cos 60°, \dots$, the angle sum formulas and the cofunction identities to find an exact value (using radicals) for the trigonometric function.

   (a) The six trigonometric functions of $105° = 45° + 60°$:
   - $\sin(105°)$.
   - $\cos(105°)$. Should your answer be a positive or a negative number?
   - $\tan(105°)$.
   - $\cot(105°)$.
   - $\sec(105°)$.
   - $\csc(105°)$.

   (b) The six trigonometric functions of $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$:
   - $\sin\left(\frac{\pi}{12}\right)$.
   - $\cos\left(\frac{\pi}{12}\right)$. Should $\sin\left(\frac{\pi}{12}\right)$ be larger or smaller than $\cos\left(\frac{\pi}{12}\right)$?
   - $\tan\left(\frac{\pi}{12}\right)$.
   - $\cot\left(\frac{\pi}{12}\right)$.
   - $\sec\left(\frac{\pi}{12}\right)$.
   - $\csc\left(\frac{\pi}{12}\right)$.

8. Simplify to a trigonometric function of the angle θ. The answer key has not been proofread, use with caution.

   (a) $\sin\left(\frac{\pi}{2} - \theta\right)$.

(b) $\cos\left(\frac{13\pi}{2}-\theta\right)$.

(c) $\tan(\pi-\theta)$

(d) $\cot\left(\frac{3\pi}{2}-\theta\right)$

(e) $\csc\left(\frac{3\pi}{2}+\theta\right)$

9. Using the power-reducing formulas, rewrite the expression in terms of first powers of the cosines and sines of multiples of the angle θ.

   (a) $\sin^4\theta$.

   (b) $\cos^4\theta$.

   (c) $\sin^6\theta$.

   (d) $\cos^6\theta$.

10. Use the sum-to-product formulas to find all solutions of the trigonometric equation in the interval $[0, 2\pi)$.

    Please note that typing a query such as "solve(sin(x)+sin (3x)=0)" at `www.wolframalpha.com` will provide you with a correct answer and a function plot.

    (a) $\sin(x)+\sin(3x)=0$.

    (b) $\cos(x)+\cos(-3x)=0$.

    (c) $\sin(x)-\sin(3x)=0$.

    (d) $\cos(2x)-\cos(3x)=0$.

11. Find the inverse function. You are asked to do the algebra only; you are not asked to determine the domain or range of the function or its inverse.

    (a) $f(x)=3x^2+4x-7$, where $x\geq-\frac{2}{3}$.

    (b) $f(x)=2x^2+3x-5$, where $x\geq-\frac{3}{4}$.

    (c) $f(x)=\dfrac{2x+5}{x-4}$, where $x\neq 4$.

    (d) $f(x)=\dfrac{3x+5}{2x-4}$, where $x\neq 2$.

    (e) $f(x)=\dfrac{5x+6}{4x+5}$, where $x\neq-\frac{5}{4}$.

    (f) $f(x)=\dfrac{2x-3}{-3x+4}$, where $x\neq\frac{4}{3}$..

12. Find the inverse function and its domain.

    (a) $y=\ln(x+3)$.

    (b) $y=4\ln(x-3)-4$.

    (c) $y=2\ln(-2x+4)+1$

    (d) $f(x)=e^{x^3}$.

    (e) $y=(\ln x)^2$, $x\geq 1$.

    (f) $y=\dfrac{e^x}{1+2e^x}$.

    (g) $f(x)=2^{2x}+2^x-2$.

13. Find each of the following values. Express your answers precisely, not as decimals.

    (a) $\arcsin(\sin 4)$.

    (b) $\arcsin(\sin 0.5)$.

    (c) $\arcsin(\cos 120^\circ)$.

    (d) $\arccos(\cos(3))$.

    (e) $\arccos(\cos(-2))$.

    (f) $\arccos(\sin(-4))$.

    (g) $\arctan(\tan 5)$.

14. Express as the following as an algebraic expression of x. In other words, "get rid" of the trigonometric and inverse trigonometric expressions.

(a) $\cos^2(\arctan x)$.

(b) $-\sin^2(\operatorname{arccot} x)$.

(c) $\dfrac{1}{\cos(\arcsin x)}$.

(d) $-\dfrac{1}{\sin(\arccos x)}$.

15. Let $x \in (0,1)$. Express the following using x and $\sqrt{1-x^2}$.

    (a) $\sin(\arcsin(x))$.

    (b) $\sin(2\arcsin(x))$.

    (c) $\sin(3\arcsin(x))$.

    (d) $\sin(\arccos(x))$.

    (e) $\sin(2\arccos(x))$.

    (f) $\sin(3\arccos(x))$.

    (g) $\cos(2\arcsin(x))$.

    (h) $\cos(3\arccos(x))$.

16. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation.

    (a) $2\cos x - 1 = 0$.

    (b) $\sin(2x) = \cos x$.

    (c) $\sqrt{3}\sin x = \sin(2x)$. .

    (d) $2\sin^2 x = 1$.

    (e) $2 + \cos(2x) = 3\cos x$.

    (f) $2\cos x + \sin(2x) = 0$.

    (g) $2\cos^2 x - (1+\sqrt{2})\cos x + \dfrac{\sqrt{2}}{2} = 0$.
    .

    (h) $|\tan x| = 1$.

    (i) $3\cot^2 x = 1$.

    (j) $\sin x = \tan x$.

17. Express each of the following as a single power.

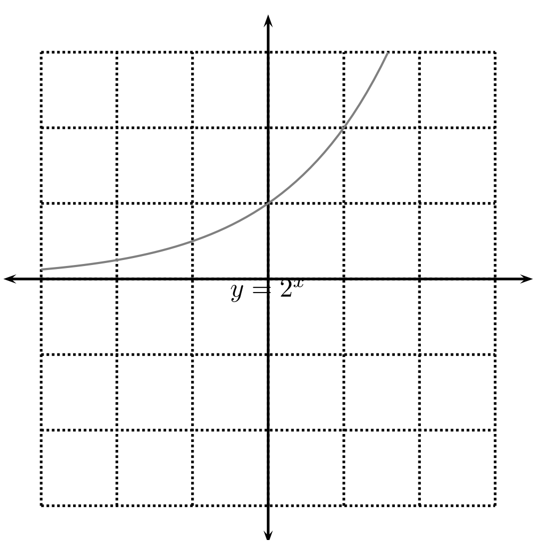
    (a) $\dfrac{2^5 \cdot 2^7}{2\sqrt{2}}$

    (b) $\dfrac{3^2 \cdot 3^{-1}}{3^3 \cdot \sqrt{3^3}}$

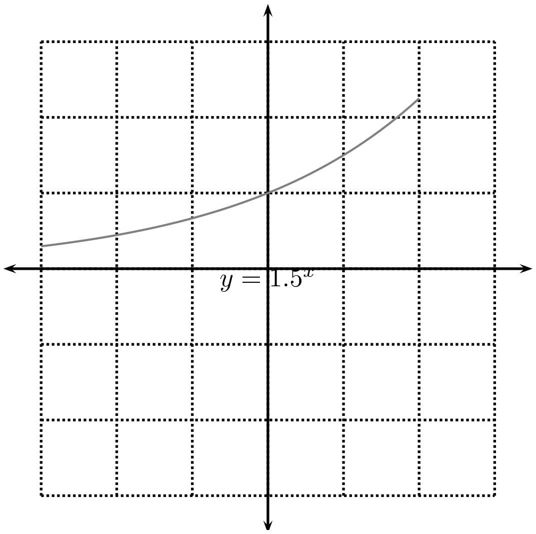
    (c) $\dfrac{\pi^3}{\pi^{-1}\sqrt{\pi^5}}$

18. Sketch by hand approximately the given function. The function is obtained by transforming linearly the graph of a known function. The known function has been sketched for you by computer.

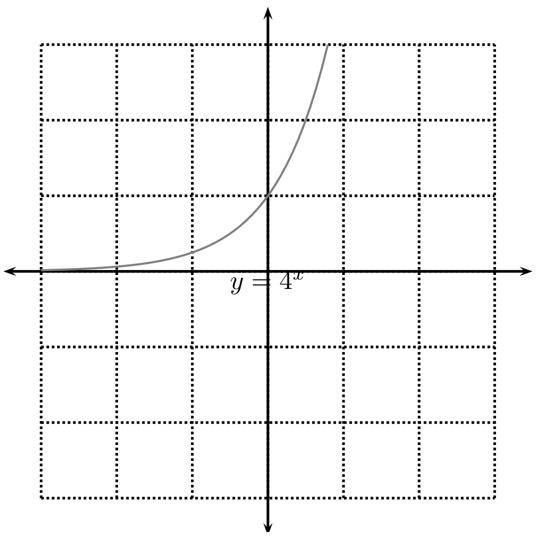
    (a) $f(x) = 2^{x+1} - 1$.



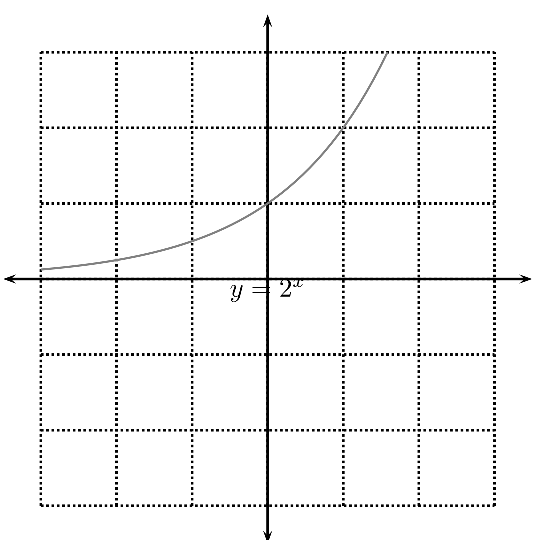
(b) $f(x) = 1.5^{x-2} + 2.$



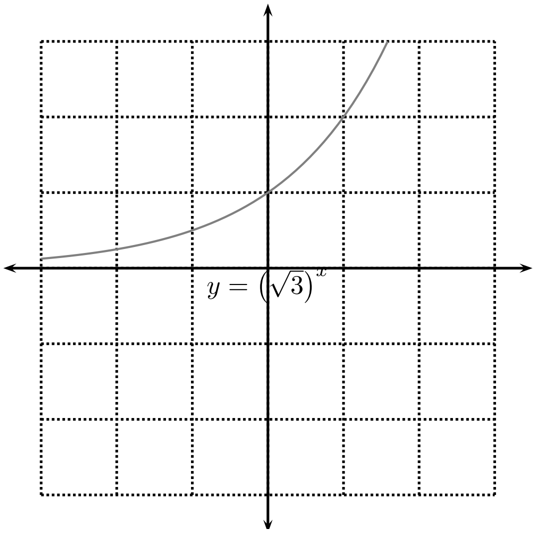
(c) $f(x) = 2^{2x-5}.$



(d) $f(x) = \dfrac{1}{2^{x-1}} + 1.$



(e) $f(x) = \dfrac{1}{3^{\frac{1}{2}x+1}} - 1$.



19. (a) A sum is held under a yearly compound interest of 1%. Make an approximation by hand (no calculators allowed) by what factor will have the money increased after 200 years. Can you do the computation in your head?

    (b) Decide, without using a calculator, which is more profitable: earning a yearly compound interest of 2% for 150 years or earning yearly simple interest of 11% for 150 years?

20. Use the definition of a logarithm to evaluate each of the following without using a calculator. The answer key has not been proofread, use with caution.

    (a) $\log_2 16$.
    (b) $\log_3 \left(\dfrac{1}{9}\right)$.
    (c) $\log_{10} 1000$.
    (d) $\log_6 36^{-\frac{2}{3}}$.
    (e) $\log_2(8\sqrt{2})$.
    (f) $\log_{\frac{1}{2}}(4)$.
    (g) $\log_{\frac{1}{9}}(\sqrt{3})$.

21. Find the exact value of each expression.

    (a) $\log_5 125$.
    (b) $\log_3 \dfrac{1}{27}$.
    (c) $\ln \left(\dfrac{1}{e}\right)$.
    (d) $\log_{10} \sqrt{10}$.
    (e) $e^{\ln 4.5}$.
    (f) $\log_{10} 0.0001$.
    (g) $\log_{1.5} 2.25$.
    (h) $\log_5 4 - \log_5 500$.
    (i) $\log_2 6 - \log_2 15 + \log_2 20$.
    (j) $\log_3 100 - \log_3 18 - \log_3 50$.
    (k) $e^{-2\ln 5}$.
    (l) $\ln \left(\ln e^{e^{10}}\right)$.
    (m) $\log_7 \left(\dfrac{49^x}{343^y}\right)$

22. Using only the $\ln$ operation of your calculator compute the indicated logarithm. Confirm your computation numerically by exponentiation.

    (a) $\log_5(13)$.
    (b) $\log_{12}(9)$.
    (c) $\log_{13}(101)$.
    (d) $\log_{10}(2015)$.

23. Express each of the following as a single logarithm. If possible, compute the logarithm without using a calculator. The answer key has not been proofread, use with caution.

    (a) $\ln 4 + \ln 6 - \ln 5$.
    (b) $2\ln 2 - 3\ln 3 + 4\ln 4$.

(c) $\ln 36 - 2\ln 3 - 3\ln 2$.

(d) $\log_2(24) - \log_4 9$.

(e) $\log_7(24) + \log_{\frac{1}{7}} 3 - \log_{49}(64)$.

(f) $\log_3(24) + \log_3\left(\frac{3}{8}\right)$.

24. Demonstrate the identity(s).

(a) $-\ln\left(\sqrt{1+x^2} - x\right) = \ln\left(x + \sqrt{1+x^2}\right)$

25. Solve each equation for x. If available, use a calculator to give an ($\approx$) answer in decimal notation. If available, use a calculator to verify your approximate solutions.

(a) $e^{7-4x} = 7$.

(b) $\ln(2x-9) = 2$.

(c) $\ln(x^2-2) = 3$.

(d) $2^{x-3} = 5$.

(e) $\ln x + \ln(x-1) = 1$.

(f) $e^{2x+1} = t$.

(g) $\log_2(mx) = c$.

(h) $e - e^{-2x} = 1$.

(i) $8(1+e^{-x})^{-1} = 3$.

(j) $\ln(\ln x) = 1$.

(k) $e^{e^x} = 10$.

(l) $\ln(2x+1) = 3 - \ln x$.

(m) $e^{2x} - 4e^x + 3 = 0$.

(n) $e^{4x} + 3e^{2x} - 4 = 0$.

(o) $e^{2x} - e^x - 6 = 0$.

(p) $4^{3x} - 2^{3x+2} - 5 = 0$.

(q) $3 \cdot 2^x + 2\left(\frac{1}{2}\right)^{x-1} - 7 = 0$.

26. 1 day after the start of hypothetical experiment a population of fruit flies was measured to have 110 individuals. 3 days after the start there were 190 flies. Write down an exponential growth law that fits this data. According to the model, how may fruit were there at the start of the experiment? After 5 days? The answer key has not been proofread, use with caution.

27. In a hypothetical experiment, the number of E. Coli bacteria cells is modeled with a logistic curve $E(t) = \dfrac{2.8 \times 10^{11}}{1 + (3.5 \times 10^9)e^{-1.2t}}$, where t measures time in hours since the start of the experiment.

- According to the model, approximately how many cells were there at the start of the experiment?
- According to the model, how many hours are needed for the number of cells to be approximately 10^{10}?

The answer key has not been proofread, use with caution.

28. The Richter magnitude M_L of an earthquake is determined from the logarithm of the amplitude A of waves recorded by seismographs (with adjustment to compensate for the distance between the measuring station and the estimated epicenter of the earthquake). The formula is

$$M_L = \log_{10} A - J_0(\delta),$$

where $J_0(\delta)$ depends on the distance δ from the epicenter. Compare the amplitudes A_1 and A_2 of the seismographic waves of two hypothetical earthquakes of magnitudes 5 and 7.2 with the same epicenter.