

Precalculus

Equations involving logarithms and exponents

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Outline

1 Equations involving logarithms

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- 1 Equations involving logarithms
- 2 Equations involving exponents

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- 1 Equations involving logarithms
- 2 Equations involving exponents
- 3 Inverse function problems and exponents

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- 1 Equations involving logarithms
- 2 Equations involving exponents
- 3 Inverse function problems and exponents
- 4 Basic exponential inequalities

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Example

Solve the equation.

$$\log_3(2x^2 + 1) = 2$$

Example

Solve the equation.

$$\begin{aligned}\log_3(2x^2 + 1) &= 2 \\ \color{red}{3}^{\log_3(2x^2 + 1)} &= \color{red}{3}^2\end{aligned}$$

| Exponentiate base $\color{red}{3}$

Example

Solve the equation.

$$\begin{aligned}\log_3(2x^2 + 1) &= 2 \\ 3^{\log_3(2x^2 + 1)} &= 3^2 \\ 2x^2 + 1 &= 9\end{aligned}$$

| Exponentiate base 3

Example

Solve the equation.

$$\begin{aligned}\log_3(2x^2 + 1) &= 2 \\ 3^{\log_3(2x^2 + 1)} &= 3^2 \\ 2x^2 + 1 &= 9 \\ 2x^2 &= 8\end{aligned}$$

| Exponentiate base 3

Example

Solve the equation.

$$\log_3(2x^2 + 1) = 2$$

$$3^{\log_3(2x^2 + 1)} = 3^2$$

$$2x^2 + 1 = 9$$

$$2x^2 = 8$$

$$x^2 = \frac{8}{2} = 4$$

| Exponentiate base 3

Example

Solve the equation.

$$\begin{aligned}\log_3(2x^2 + 1) &= 2 & | \text{ Exponentiate base 3} \\ 3^{\log_3(2x^2 + 1)} &= 3^2 \\ 2x^2 + 1 &= 9 \\ 2x^2 &= 8 \\ x^2 &= \frac{8}{2} = 4 \\ x &= \pm\sqrt{4} = \pm 2\end{aligned}$$

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Solve the equation.

$$\begin{aligned}\log_3(2x^2 + 1) &= 2 && | \text{Exponentiate base 3} \\ 3^{\log_3(2x^2 + 1)} &= 3^2 \\ 2x^2 + 1 &= 9 \\ 2x^2 &= 8 \\ x^2 &= \frac{8}{2} = 4 \\ x &= \pm\sqrt{4} = \pm 2 \\ x = 2 \text{ or } x = -2 && | \text{final answer}\end{aligned}$$

The logarithmic property $\log_a(xy) = \log_a x + \log_a y$ holds only for positive x, y . Failure to check the positivity of x, y can result in extraneous (fake) solutions to logarithmic equations.

Example

Solve the equation.

$$\log_2(x + 2) + \log_2(x - 1) = 2$$

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Example

Solve the equation.

$$\log_2(x+2) + \log_2(x-1) = 2$$

$$\log_2((x+2)(x-1)) = 2$$

$$(x+2)(x-1) = 2^2$$

$$x^2 + x - 2 = 4$$

$$x^2 + x - 6 = 0$$

$$(x-2)(x+3) = 0$$

$$x = 2 \quad \text{or} \quad x = -3$$

$x = -3$ not a solution (outside of domain)

Domain: $x > 1$

Exponentiate base 2

Example (Solve exponential equation without logarithms)

Solve for t .

$$16^{4t} = 8^{t-2}$$

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Solve for t .

$$\begin{array}{rcl} 16^{4t} & = & 8^{t-2} \\ \text{Find a common base: } (?)^{4t} & = & (?)^{t-2} \end{array}$$

Example (Solve exponential equation without logarithms)

Solve for t .

Find a common base:

$$\begin{array}{rcl} 16^{4t} & = & 8^{t-2} \\ (2^4)^{4t} & = & (2^3)^{t-2} \end{array}$$

Example (Solve exponential equation without logarithms)

Solve for t .

Find a common base:

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Solve for t .

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Example

Solve the equation.

$$2^{1-5x} = 12$$

Example

Solve the equation.

$$\begin{array}{rcl} 2^{1-5x} & = & 12 \\ \log_2(2^{1-5x}) & = & \log_2 12 \end{array} \quad \left| \text{ apply } \log_2 \right.$$

Example

Solve the equation.

$$\begin{array}{rcl} 2^{1-5x} & = & 12 \\ \log_2(2^{1-5x}) & = & \log_2 12 \\ 1 - 5x & = & \log_2 12 \end{array} \quad \left| \text{ apply } \log_2 \right.$$

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Solve the equation.

$$\begin{array}{rcl} 2^{1-5x} & = & 12 \\ \log_2(2^{1-5x}) & = & \log_2 12 \\ 1 - 5x & = & \log_2 12 = \log_2(4 \cdot 3) \end{array} \quad \left| \begin{array}{l} \text{apply } \log_2 \end{array} \right.$$

Example

Solve the equation.

$$\begin{aligned} 2^{1-5x} &= 12 && | \text{ apply } \log_2 \\ \log_2(2^{1-5x}) &= \log_2 12 \\ 1 - 5x &= \log_2 12 = \log_2(4 \cdot 3) \\ 1 - 5x &= \log_2 4 + \log_2 3 \end{aligned}$$

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 1 - 5x &= 2 + \log_2 3 \\
 5x &= 1 - (2 + \log_2 3) \\
 &= -1 \\
 x &= \underline{\hspace{2cm}}
 \end{aligned}$$

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 \textcolor{red}{5}x & = & 1 - (2 + \log_2 3) & \\
 & & - 1 - \log_2 3 & \\
 x & = & \frac{\quad}{\textcolor{red}{5}} &
 \end{array}$$

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Example

Solve the equation.

$$e^{x-3} = 2e^{2x-1}$$

Example

Solve the equation.

$$\begin{array}{l} e^{x-3} = 2e^{2x-1} \\ \hline \frac{e^{x-3}}{e^{2x-1}} = 2 \end{array} \quad \left| \text{Divide by } e^{2x-1} \right.$$

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Solve the equation.

$$e^{x-3} = 2e^{2x-1}$$

Divide by e^{2x-1}

$$\frac{e^{x-3}}{e^{2x-1}} = 2$$

$$e^{x-3-(2x-1)} = 2$$

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Apply \ln

$$-x - 2 = \ln 2$$

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$$-x - 2 = \ln 2$$

$$-x = \ln 2 + 2$$

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Apply \ln

$$-x - 2 = \ln 2$$

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$$x = -(\ln 2 + 2)$$

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$$\frac{e^{x-3}}{e^{2x-1}} = 2$$

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$$e^{-x-2} = 2 \quad \left| \text{Apply } \ln \right.$$

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$$x = -\ln 2 - 2 \quad \left| \text{Final answer} \right.$$

Example

Solve the equation.

$$e^{x-3} = 2e^{2x-1}$$

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Apply \ln

$$-x - 2 = \ln 2$$

$$-x = \ln 2 + 2$$

$$x = -(\ln 2 + 2)$$

$$x = -\ln 2 - 2$$

Final answer

$$x \approx -2.693$$

Calculator

Example

Solve.

$$3^{2x+5} = 5 \cdot 2^{-x+1}$$

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$$3^{2x+5} = 5 \cdot 2^{-x+1}$$

Common base

Example

Solve.

$$3^{2x+5} = 5 \cdot 2^{-x+1}$$

$$\left(2^{\log_2 3}\right)^{2x+5} = 5 \cdot 2^{-x+1}$$

Common base

$$a = b^{\log_b a}$$

Example

Solve.

$$3^{2x+5} = 5 \cdot 2^{-x+1}$$

$$\left(2^{\log_2 3}\right)^{2x+5} = 5 \cdot 2^{-x+1}$$

$$2^{(\log_2 3)(2x+5)} = 5 \cdot 2^{-x+1}$$

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$$(\log_2 3)(2x + 5) + x - 1 = \log_2 5$$

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Apply \log_2

Example

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$$(\log_2 3)(2x + 5) + x - 1 = \log_2 5$$

$$x(2\log_2 3 + 1) + 5\log_2 3 - 1 = \log_2 5$$

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$$a = b^{\log_b a}$$

Apply \log_2

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$$3^{2x+5} = 5 \cdot 2^{-x+1}$$

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Apply \log_2

$$(\log_2 3)(2x + 5) + x - 1 = \log_2 5$$

$$x(2 \log_2 3 + 1) + 5 \log_2 3 = \log_2 5 + 1$$

$$x = \frac{\log_2 5 + 1 - 5 \log_2 3}{2 \log_2 3 + 1}$$

Example

Solve.

$$3^{2x+5} = 5 \cdot 2^{-x+1}$$

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$$(\log_2 3)(2x + 5) + x - 1 = \log_2 5$$

$$x(2 \log_2 3 + 1) + 5 \log_2 3 = \log_2 5 + 1$$

$$x = \frac{\log_2 5 + 1 - 5 \log_2 3}{2 \log_2 3 + 1}$$

$$x \approx -1.1038$$

Calculator

Example

Solve the equation.

$$e^{5-3x} = 10$$

Example

Solve the equation.

$$\begin{array}{rcl} e^{5-3x} & = & 10 \\ \ln(e^{5-3x}) & = & \ln 10 \end{array} \quad \text{apply } \ln$$

Example

Solve the equation.

$$\begin{array}{rcl} e^{5-3x} & = & 10 \\ \ln(e^{5-3x}) & = & \ln 10 \\ 5 - 3x & = & \ln 10 \end{array} \quad \text{apply } \ln$$

Example

Solve the equation.

$$\begin{aligned} e^{5-3x} &= 10 && \text{apply } \ln \\ \ln(e^{5-3x}) &= \ln 10 \\ \textcolor{red}{5} - 3x &= \ln 10 \\ 3x &= \textcolor{red}{5} - \ln 10 \end{aligned}$$

Example

Solve the equation.

$$\begin{array}{rcll} e^{5-3x} & = & 10 & \text{apply ln} \\ \ln(e^{5-3x}) & = & \ln 10 & \\ 5 - 3x & = & \ln 10 & \\ \textcolor{red}{3}x & = & 5 - \ln 10 & \\ x & = & \frac{5 - \ln 10}{\textcolor{red}{3}} & \end{array}$$

Example

Solve the equation.

$$\begin{aligned}e^{5-3x} &= 10 && \text{apply } \ln \\ \ln(e^{5-3x}) &= \ln 10 \\ 5 - 3x &= \ln 10 \\ 3x &= 5 - \ln 10 \\ x &= \frac{5 - \ln 10}{3} \\ \text{Calculator: } x &\approx 0.8991.\end{aligned}$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

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Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months.

When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

Solve for t :

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same **number of chickens** as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

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Solve for t : $c(t) = r(t)$

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Therefore the chicken and rabbit populations are equal after 3 years.

Example (Solving a quadratic exponential equation)

Solve for x .

$$9^x = 2 \cdot 3^x + 63$$

Example (Solving a quadratic exponential equation)

Solve for x .

$$\begin{aligned}9^x &= 2 \cdot 3^x + 63 \\9^x - 2 \cdot 3^x - 63 &= 0\end{aligned}$$

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$$? - 2u - 63 = 0$$

Substitute $u = 3^x$

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$$(\textcolor{red}{?})(\textcolor{red}{?}) = 0$$

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Therefore $x = 2$ is the solution.

Example

Solve the equation

$$e^{2x} - 3e^x - 4 = 0$$

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$$e^{2x} - 3e^x - 4 = 0$$

Set $e^x = u$.

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Set $e^x = u$. Then $e^{2x} = ?$.

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$$u^2 - 3u - 4 = 0$$

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Set $e^x = u$. Then $e^{2x} = u^2$.

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$$u^2 - 3u - 4 = 0$$

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$$u = 4$$

or

$$u = -1$$

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$$x = \ln 4$$

or

no real solution

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no real solution

$$x \approx 1.3863$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

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$$2^x = \frac{3}{2} \quad \text{or} \quad 2^x = -\frac{1}{2}$$

$$x = \log_2 \left(\frac{3}{2} \right)$$

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$$x = \log_2 \left(\frac{3}{2} \right) = \frac{\ln(?)}{\ln ?}$$

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$$2^x = \frac{3}{2} \quad \text{or} \quad 2^x = -\frac{1}{2}$$

$$x = \log_2 \left(\frac{3}{2} \right) = \frac{\ln \left(\frac{3}{2} \right)}{\ln 2} \approx 0.58496 \quad \text{or} \quad \text{no real solution}$$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$3^{2x} = 2 + 63 \cdot 3^{-2x}$$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

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> 2 terms \Rightarrow
transfer one side

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$$3^{2x} = u$$

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Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$3^{2x} = 2 + 63 \cdot 3^{-2x}$$

$$3^{2x} - 2 - 63 \cdot 3^{-2x} = 0$$

$$u - 2 - 63u^{-1} = 0$$

> 2 terms \Rightarrow
transfer one side

$$3^{2x} = u$$

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Multiply $\cdot u$

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$$(\text{?})(\text{?}) = 0$$

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$$(u - 9)(u + 7) = 0$$

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$$u - 9 = 0 \text{ or } u + 7 = 0$$

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$$\text{or} \quad u = -7$$

> 2 terms \Rightarrow

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$$(u - 9)(u + 7) = 0$$

$$u - 9 = 0 \quad \text{or} \quad u + 7 = 0$$

$$\text{or} \quad u = -7$$

or **no real solution**

> 2 terms \Rightarrow

transfer one side

$$3^{2x} = u$$

$$3^{-2x} = (3^{2x})^{-1} = u^{-1}$$

Multiply $\cdot u$

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> 2 terms \Rightarrow

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$$u = 9 \quad \text{or} \quad u = -7$$

$$3^{2x} = 9 \quad \text{or} \quad \text{no real solution}$$

> 2 terms \Rightarrow
transfer one side

$$3^{2x} = u$$

$$3^{-2x} = (3^{2x})^{-1} = u^{-1}$$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

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$$u - 9 = 0 \quad \text{or} \quad u + 7 = 0$$

$$u = 9 \quad \text{or} \quad u = -7$$

$$3^{2x} = 9 \quad \text{or} \quad \text{no real solution}$$

$$2x = \log_3 9$$

> 2 terms \Rightarrow

transfer one side

$$3^{2x} = u$$

$$3^{-2x} = (3^{2x})^{-1} = u^{-1}$$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$3^{2x} = 2 + 63 \cdot 3^{-2x}$$

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$$u - 9 = 0 \quad \text{or} \quad u + 7 = 0$$

$$u = 9 \quad \text{or} \quad u = -7$$

$$3^{2x} = 9 \quad \text{or} \quad \text{no real solution}$$

$$2x = \log_3 9$$

$$2x = ?$$

> 2 terms \Rightarrow

transfer one side

$$3^{2x} = u$$

$$3^{-2x} = (3^{2x})^{-1} = u^{-1}$$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$3^{2x} = 2 + 63 \cdot 3^{-2x}$$

$$3^{2x} - 2 - 63 \cdot 3^{-2x} = 0$$

$$u - 2 - 63u^{-1} = 0$$

$$u^2 - 2u - 63 = 0$$

$$(u - 9)(u + 7) = 0$$

$$u - 9 = 0 \quad \text{or} \quad u + 7 = 0$$

$$u = 9 \quad \text{or} \quad u = -7$$

$$3^{2x} = 9 \quad \text{or} \quad \text{no real solution}$$

$$2x = \log_3 9$$

$$2x = 2$$

> 2 terms \Rightarrow

transfer one side

$$3^{2x} = u$$

$$3^{-2x} = (3^{2x})^{-1} = u^{-1}$$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$3^{2x} = 2 + 63 \cdot 3^{-2x}$$

$$3^{2x} - 2 - 63 \cdot 3^{-2x} = 0$$

$$u - 2 - 63u^{-1} = 0$$

$$u^2 - 2u - 63 = 0$$

$$(u - 9)(u + 7) = 0$$

$$u - 9 = 0 \quad \text{or} \quad u + 7 = 0$$

$$u = 9 \quad \text{or} \quad u = -7$$

$$3^{2x} = 9 \quad \text{or} \quad \text{no real solution}$$

$$2x = \log_3 9$$

$$2x = 2$$

$$x = 1$$

> 2 terms \Rightarrow

transfer one side

$$3^{2x} = u$$

$$3^{-2x} = (3^{2x})^{-1} = u^{-1}$$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$3^{2x} = 2 + 63 \cdot 3^{-2x}$$

$$3^{2x} - 2 - 63 \cdot 3^{-2x} = 0$$

$$u - 2 - 63u^{-1} = 0$$

$$u^2 - 2u - 63 = 0$$

$$(u - 9)(u + 7) = 0$$

$$u - 9 = 0 \quad \text{or} \quad u + 7 = 0$$

$$u = 9 \quad \text{or} \quad u = -7$$

$$3^{2x} = 9 \quad \text{or} \quad \text{no real solution}$$

$$2x = \log_3 9$$

$$2x = 2$$

$$x = 1$$

> 2 terms \Rightarrow

transfer one side

$$3^{2x} = u$$

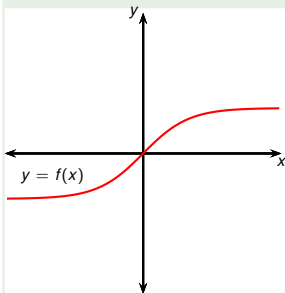
$$3^{-2x} = (3^{2x})^{-1} = u^{-1}$$

Multiply $\cdot u$

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

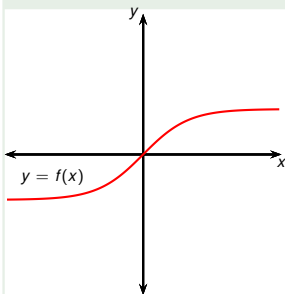


Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$



Example

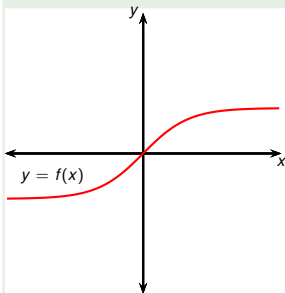
Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{(u - ?)}{(u + ?)} = y$$

Set $u = e^x$



Example

Find $f^{-1}(x)$ for

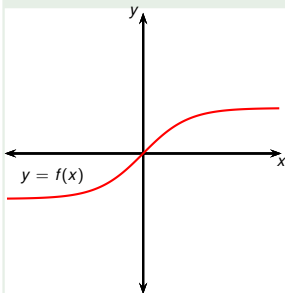
$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{(u - ?)}{(u + ?)} = y$$

Set $u = e^x$

$$e^{-x} = ?$$



Example

Find $f^{-1}(x)$ for

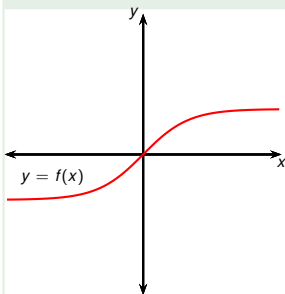
$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{(u - \frac{1}{u})}{(u + \frac{1}{u})} = y$$

Set $u = e^x$

$$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$$



Example

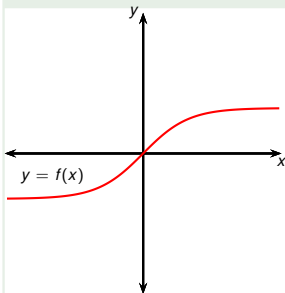
Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{(u - \frac{1}{u}) \textcolor{red}{u}}{(u + \frac{1}{u}) \textcolor{red}{u}} = y$$

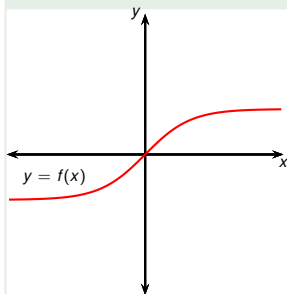
$$\begin{array}{l} \text{Set } u = e^x \\ e^{-x} = \frac{1}{e^x} = \frac{1}{u} \end{array}$$



Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{(u - \frac{1}{u})u}{(u + \frac{1}{u})u} = y$$

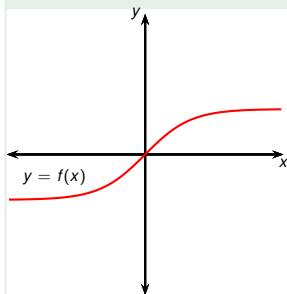
$$\frac{u^2 - 1}{u^2 + 1} = y$$

$$\begin{aligned} \text{Set } u &= e^x \\ e^{-x} &= \frac{1}{e^x} = \frac{1}{u} \end{aligned}$$

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

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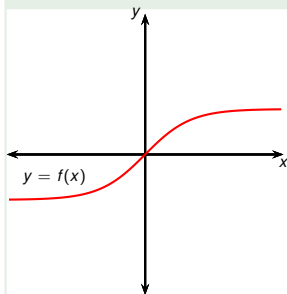
$$u^2 - 1 = y(u^2 + 1)$$

$$\begin{aligned} \text{Set } u &= e^x \\ e^{-x} &= \frac{1}{e^x} = \frac{1}{u} \end{aligned}$$

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

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$$\frac{u^2 - 1}{u^2 + 1} = y$$

$$u^2 - 1 = y(u^2 + 1)$$

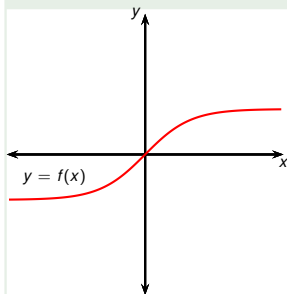
$$u^2(1 - y) = 1 + y$$

$$\begin{aligned} \text{Set } u &= e^x \\ e^{-x} &= \frac{1}{e^x} = \frac{1}{u} \end{aligned}$$

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



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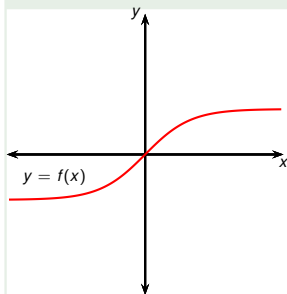
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$$u^2 - 1 = y(u^2 + 1)$$

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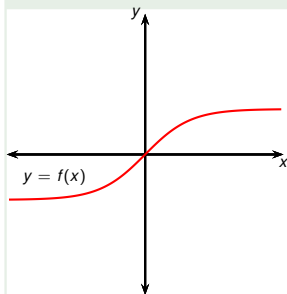
$$u^2 = \frac{1 + y}{1 - y}$$

$$\begin{aligned} \text{Set } u &= e^x \\ e^{-x} &= \frac{1}{e^x} = \frac{1}{u} \end{aligned}$$

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{(u - \frac{1}{u})u}{(u + \frac{1}{u})u} = y$$

$$\frac{u^2 - 1}{u^2 + 1} = y$$

$$u^2 - 1 = y(u^2 + 1)$$

$$u^2(1 - y) = 1 + y$$

$$u^2 = \frac{1 + y}{1 - y}$$

$$(e^x)^2 = \frac{1 + y}{1 - y}$$

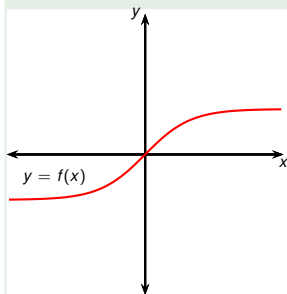
Set $u = e^x$

$$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$$

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



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$$\frac{u^2 - 1}{u^2 + 1} = y$$

$$u^2 - 1 = y(u^2 + 1)$$

$$u^2(1 - y) = 1 + y$$

$$u^2 = \frac{1 + y}{1 - y}$$

$$(e^x)^2 = \frac{1 + y}{1 - y}$$

$$e^{2x} = \frac{1 + y}{1 - y}$$

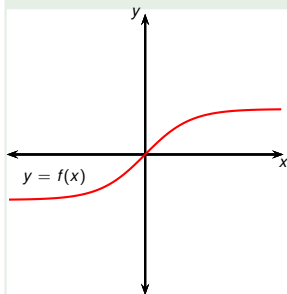
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Example

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$$u^2 - 1 = y(u^2 + 1)$$

$$u^2(1 - y) = 1 + y$$

$$u^2 = \frac{1 + y}{1 - y}$$

$$(e^x)^2 = \frac{1 + y}{1 - y}$$

$$e^{2x} = \frac{1 + y}{1 - y}$$

$$2x = \ln \left(\frac{1 + y}{1 - y} \right)$$

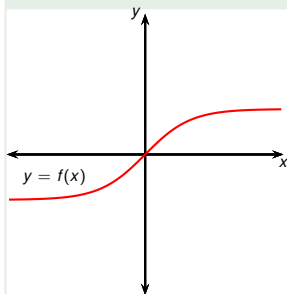
$$\begin{aligned} \text{Set } u &= e^x \\ e^{-x} &= \frac{1}{e^x} = \frac{1}{u} \end{aligned}$$

Take \ln

Example

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$$u^2 - 1 = y(u^2 + 1)$$

$$u^2(1 - y) = 1 + y$$

$$u^2 = \frac{1 + y}{1 - y}$$

$$(e^x)^2 = \frac{1 + y}{1 - y}$$

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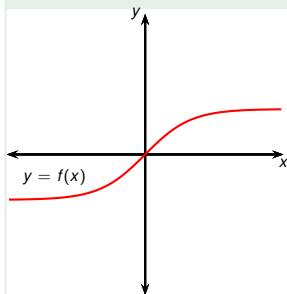
$$\begin{aligned} \text{Set } u &= e^x \\ e^{-x} &= \frac{1}{e^x} = \frac{1}{u} \end{aligned}$$

Take \ln

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



answer

$$f^{-1}(y) = \frac{1}{2} \ln \left(\frac{1+y}{1-y} \right)$$

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{(u - \frac{1}{u})u}{(u + \frac{1}{u})u} = y$$

$$\frac{u^2 - 1}{u^2 + 1} = y$$

$$u^2 - 1 = y(u^2 + 1)$$

$$u^2(1 - y) = 1 + y$$

$$u^2 = \frac{1+y}{1-y}$$

$$(e^x)^2 = \frac{1+y}{1-y}$$

$$e^{2x} = \frac{1+y}{1-y}$$

$$x = \frac{1}{2} \ln \left(\frac{1+y}{1-y} \right)$$

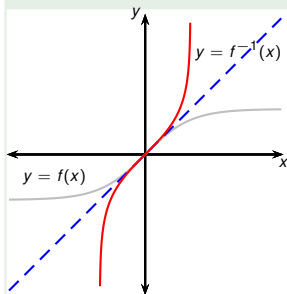
$$\begin{aligned} \text{Set } u &= e^x \\ e^{-x} &= \frac{1}{e^x} = \frac{1}{u} \end{aligned}$$

Take \ln

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



Final answer, **relabelled**:

$$f^{-1}(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$$

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{(u - \frac{1}{u})u}{(u + \frac{1}{u})u} = y$$

$$\frac{u^2 - 1}{u^2 + 1} = y$$

$$u^2 - 1 = y(u^2 + 1)$$

$$u^2(1 - y) = 1 + y$$

$$u^2 = \frac{1 + y}{1 - y}$$

$$(e^x)^2 = \frac{1 + y}{1 - y}$$

$$e^{2x} = \frac{1 + y}{1 - y}$$

$$x = \frac{1}{2} \ln \left(\frac{1 + y}{1 - y} \right)$$

$$\begin{aligned} \text{Set } u &= e^x \\ e^{-x} &= \frac{1}{e^x} = \frac{1}{u} \end{aligned}$$

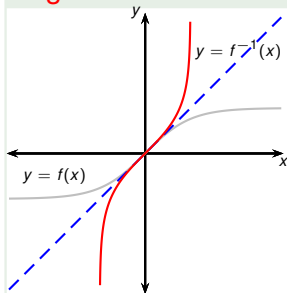
Take \ln

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

$f = \tanh =$ **hyperbolic tangent function.**



Final answer, relabeled:

$$f^{-1}(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$$

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{(u - \frac{1}{u})u}{(u + \frac{1}{u})u} = y$$

$$\frac{u^2 - 1}{u^2 + 1} = y$$

$$u^2 - 1 = y(u^2 + 1)$$

$$u^2(1 - y) = 1 + y$$

$$u^2 = \frac{1 + y}{1 - y}$$

$$(e^x)^2 = \frac{1 + y}{1 - y}$$

$$e^{2x} = \frac{1 + y}{1 - y}$$

$$x = \frac{1}{2} \ln \left(\frac{1 + y}{1 - y} \right)$$

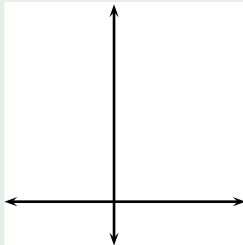
$$\begin{aligned} \text{Set } u &= e^x \\ e^{-x} &= \frac{1}{e^x} = \frac{1}{u} \end{aligned}$$

Take \ln

Example

Solve the inequality.

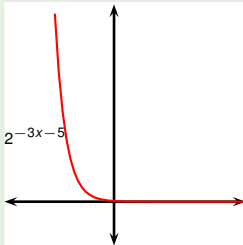
$$2^{-3x-5} < 7$$



Example

Solve the inequality.

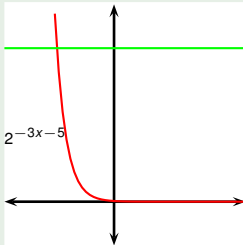
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Example

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$$2^{-3x-5} < 7$$



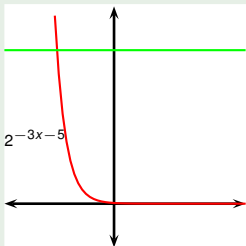
Example

Solve the inequality.

$$2^{-3x-5} < 7$$

$$\log_2 2^{-3x-5} < \log_2 7$$

Logarithms preserve
inequalities: apply \log_2



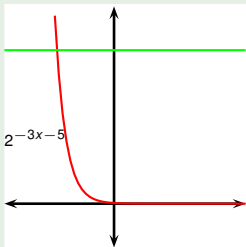
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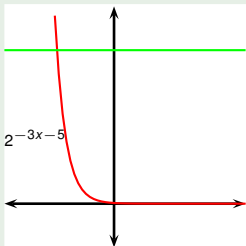
Example

Solve the inequality.

$$2^{-3x-5} < 7$$

$$\begin{aligned} \log_2 2^{-3x-5} &< \log_2 7 \\ -3x - 5 &< \log_2 7 \end{aligned}$$

Logarithms preserve
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Example

Solve the inequality.

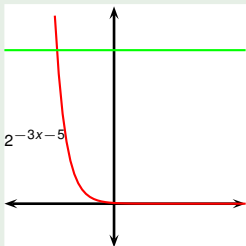
$$2^{-3x-5} < 7$$

$$\log_2 2^{-3x-5} < \log_2 7$$

$$-3x - 5 < \log_2 7$$

$$-3x < \log_2 7 + 5$$

Logarithms preserve
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Example

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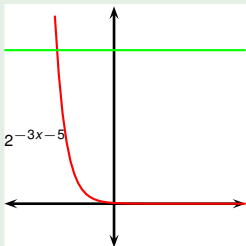
$$-3x - 5 < \log_2 7$$

$$-3x < \log_2 7 + 5$$

$$x > -\frac{\log_2 7 + 5}{3}$$

Logarithms preserve inequalities: apply \log_2

Division by negative number flips inequalities



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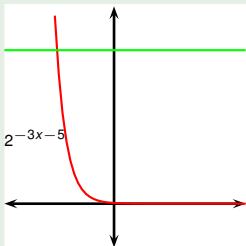
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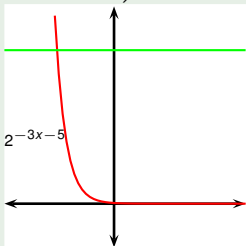
$$-3x < \log_2 7 + 5$$

$$x > -\frac{\log_2 7 + 5}{3}$$

$$x \in \left(-\frac{5 + \log_2 7}{3}, \infty \right)$$

Logarithms preserve inequalities: apply \log_2

Division by negative number flips inequalities



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$$-3x - 5 < \log_2 7$$

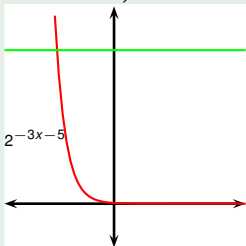
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