

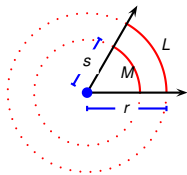
## Precalculus

# Find circle arclength from radius and angle

Todor Milev

2019

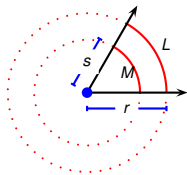
# Arc-length of a circle arc



## Proposition

*Let two circles have common center and radii  $s$  and  $r$ . Suppose an arbitrary geometric angle with vertex at the common center of the circles cuts off short arcs of length  $M$  and  $L$ . Then  $\frac{s}{r} = \frac{L}{M}$ .*

# Arc-length of a circle arc

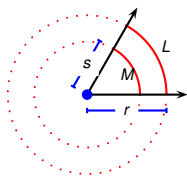


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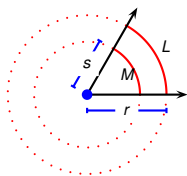
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$$\frac{s}{r} = \frac{M}{L}$$

$$\frac{1}{r} = \frac{\alpha}{L}$$

Choose  $s = 1$ , relabel  $M = \alpha$

# Arc-length of a circle arc



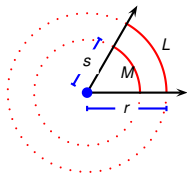
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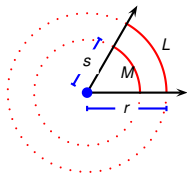
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$$\begin{aligned} \frac{s}{r} &= \frac{M}{L} \\ \frac{1}{r} &= \frac{\alpha}{L} \\ \frac{\textcolor{red}{L}}{r} &= \alpha r \end{aligned}$$

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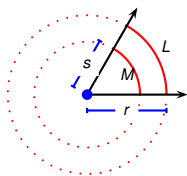
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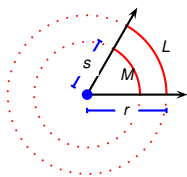
$$\begin{aligned}\frac{s}{r} &= \frac{M}{L} \\ \frac{1}{r} &= \frac{\alpha}{L} \\ L &= \alpha r\end{aligned}$$

Choose  $s = 1$ , relabel  $M = \alpha$

The angle-measure of a geometric angle is the arc-length cut off from a radius 1 circle, therefore we get the following.



# Arc-length of a circle arc



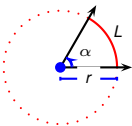
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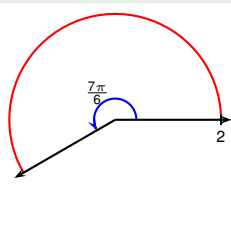
The angle-measure of a geometric angle is the arc-length cut off from a radius 1 circle, therefore we get the following.



## Corollary

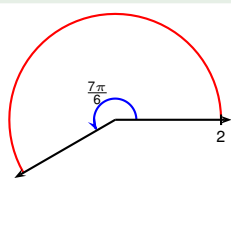
*The arc-length cut off by an angle with measure  $\alpha$  from a circle of radius  $r$  equals  $\alpha r$ .*

## Example



Find the length of an arc of a circle of radius 2 cut off by an angle of measure  $\frac{7\pi}{6}$  ( $= 210^\circ$ ).

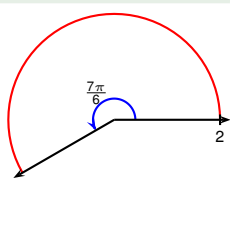
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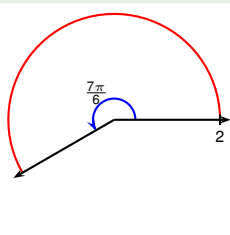
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Find the length of an arc of a circle of radius 2 cut off by an **angle of measure**  $\frac{7\pi}{6}$  ( $= 210^\circ$ ).

$$\text{arc-length} = \alpha r = \frac{7\pi}{6} \cdot 2$$

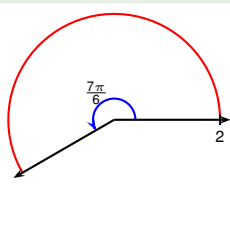
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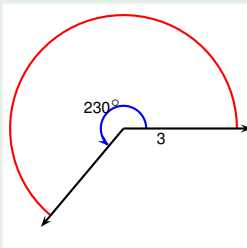
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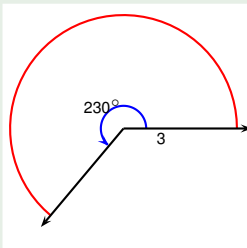
$$\text{arc-length} = \alpha r = \frac{7\pi}{6} \cdot 2 = \frac{7\pi}{3} \approx 7.33038 \text{ (units)}$$

## Example



Find the length of an arc of a circle of radius 3 cut off by an angle of measure  $230^\circ$ .

## Example

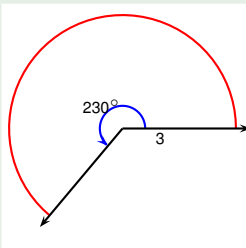


Find the length of an arc of a circle of radius 3 cut off by an angle of measure  $230^\circ$ .

$$\text{arc-length} = ar$$



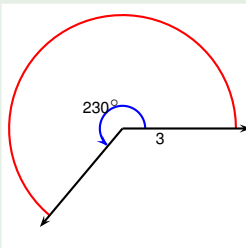
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Find the length of an arc of a circle of **radius 3** cut off by an angle of measure  $230^\circ$ .

$$\text{arc-length} = \alpha r = ? \cdot 3$$

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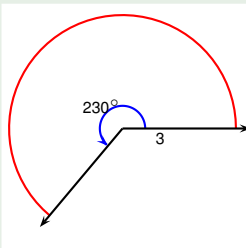


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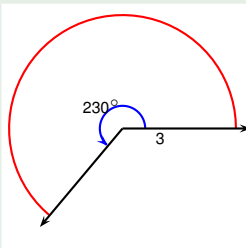
$$\alpha = 230^\circ$$

$$= ?$$

Convert to radians

$$\text{arc-length} = \alpha r = ? \cdot 3$$

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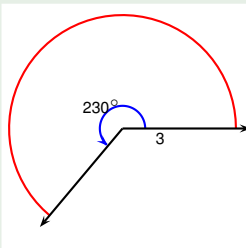
$$\alpha = 230^\circ$$

$$= 230^\circ \frac{\pi \text{ rad}}{180^\circ}$$

Convert to radians

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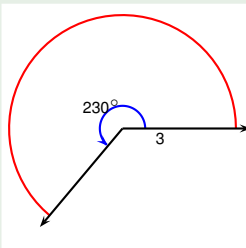
Find the length of an arc of a circle of radius 3 cut off by an angle of measure  $230^\circ$ .

$$\begin{aligned}\alpha &= 230^\circ \\ &= 230^\circ \frac{\pi \text{ rad}}{180^\circ} = \frac{23}{18} \pi \text{ rad}\end{aligned}$$

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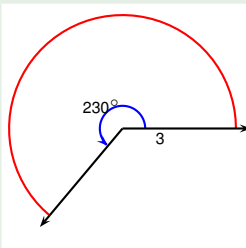
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Convert to radians

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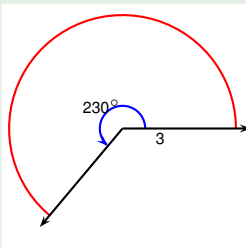
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Convert to radians

$$\text{arc-length} = \alpha r = \frac{23\pi}{18} \cdot 3 = \frac{23\pi}{6} \approx 12.043$$