

Calculus I

Type 1: Composing fractional linear transformations

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2019

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x - 1}{x + 2}$$

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$$(f \circ f)(x) = f(f(x)) = f\left(\frac{2x-1}{x+2}\right)$$

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$$= \frac{3x-4}{4x+3}$$

$$x \neq ?$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

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$$(g \circ f)(x) = \frac{7x+4}{3x-19}$$

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$$(g \circ g)(x) = \frac{19x-15}{-25x+64}$$

$$x \neq \frac{7}{5}, \frac{64}{25}$$