Calculus II
Homework
Series basics

1. Express the infinite decimal number as a rational number.

   (a) $0.\overline{9} = 0.99999\ldots$
   (b) $1.\overline{6} = 1.6666\ldots$
   (c) $1.\overline{3} = 1.3333\ldots$
   (d) $1.\overline{19} = 1.191919\ldots.$
   (e) $0.\overline{09} = 0.0909090909\ldots.$
   (f) $2.\overline{16} = 2.16161616\ldots$
   (g) $2014.\overline{2014} = 2014.2014201420142014\ldots$

2. Express the sum of the series as a rational number.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + 3^n}{5^n}$
   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n + 5^n}{10^n}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n - 3^n}{7^n}$
   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+1} + 7^{n-1}}{21^n}$
   (e) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n+1} + (-3)^{n-1}}{5^n}$

3. Sum the telescoping series (a sum is "telescoping" if it can be broken into summands so that consecutive terms cancel).

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{-6}{9n^2 + 3n - 2}$.
   (b) $\displaystyle\sum_{n=3}^{\infty} \frac{3}{n^2 - 3n + 2}$.
   (c) $\displaystyle\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)$. (Hint: Use the properties of the logarithm to aim for a telescoping series).

4. Use partial fractions to sum the telescoping series (a sum is "telescoping" if it can be broken into summands so that consecutive terms cancel).

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$
   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{2n + 1}{n^4 + 2n^3 - n^2 - 2n}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n^4 - 3n^2 + 1}$
   (d) $\displaystyle\sum_{n=3}^{\infty} \frac{n^2 + n + 2}{n^4 - 5n^2 + 4}$