# Precalculus
# Quadratic polynomials viewed as functions

Todor Milev

2019

## Definition

Let $a, b, c$ be real numbers with $a \neq 0$. The function

$$f(x) = ax^2 + bx + c$$

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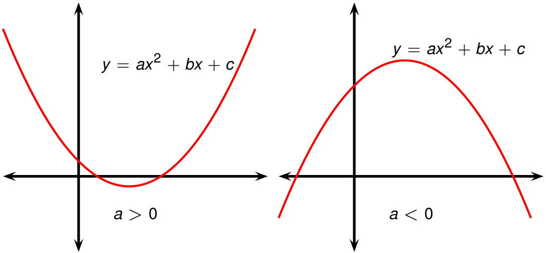
- The graph of a quadratic function is called a parabola.



## Example (Completing the square)

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&= a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right) + c
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Add & subtract $\left(\frac{b}{2a}\right)^2$

use $(A+B)^2 = A^2 + 2AB + B^2$

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The quantity $D = b^2 - 4ac$ is called the *discriminant* of the quadratic function $ax^2 + bx + c$.

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The expression $f(x) = a(x-h)^2 + k$, where $h = -\frac{b}{2a}$ and $k = -\frac{D}{4a} = -\frac{b^2-4ac}{4a}$ is called the standard form of $ax^2 + bx + c$.

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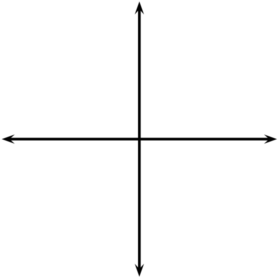

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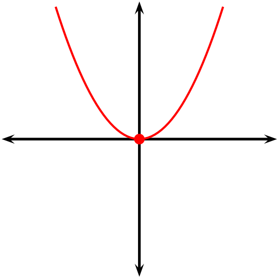

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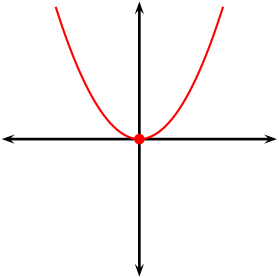

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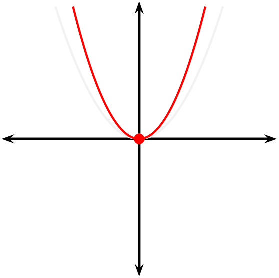

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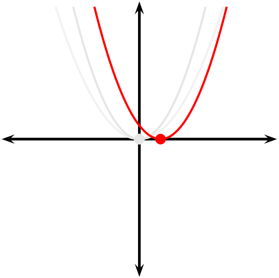

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  - $a(x-h)^2$ shifts $y = ax^2$ by $h$ units right.

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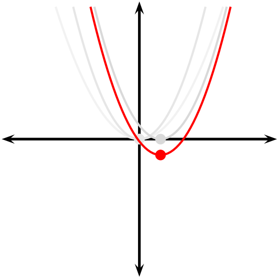

- The graph of $y = x^2$ is a parabola; its shape is assumed known.
- The standard form shows how the graph of an arbitrary quadratic is obtained from the graph of $y = x^2$:
  - $ax^2$ stretches $y = x^2$ by factor of $a$ and possibly reflects across the $x$ axis.
  - $a(x-h)^2$ shifts $y = ax^2$ by $h$ units right.
  - $a(x-h)^2 + k$ shifts $y = a(x-h)^2 + k$ by $k$ units up.

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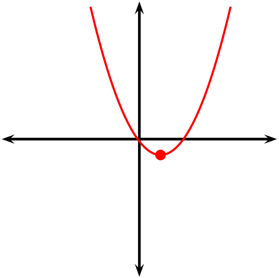

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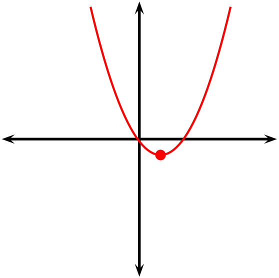

- The graph of a quadratic function is a parabola.
- When $a > 0$ the parabola opens upwards.

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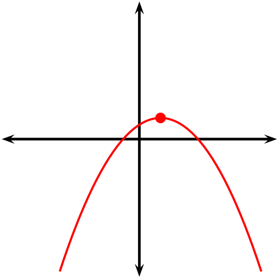

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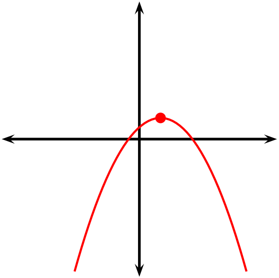

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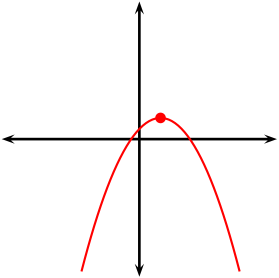

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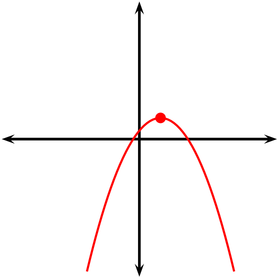

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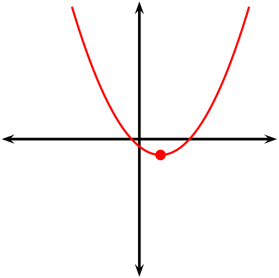

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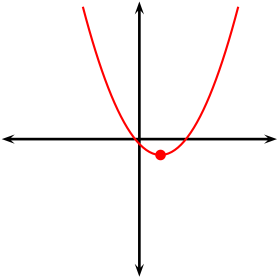

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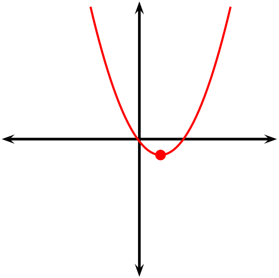

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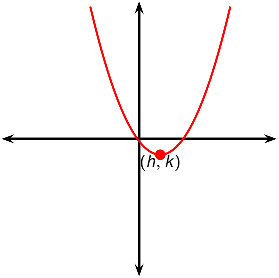


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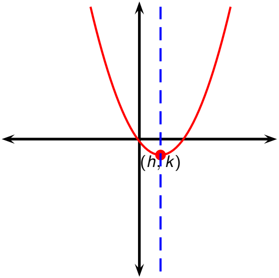


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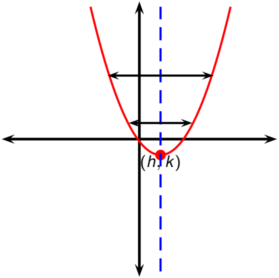


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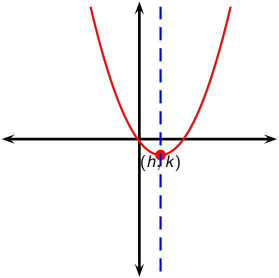


- When we change $h$ and $k$ we move the vertex of the parabola without change in steepness.

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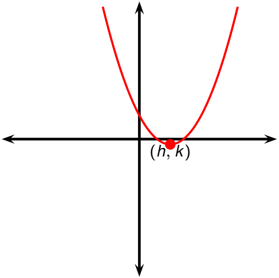


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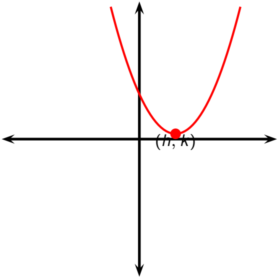


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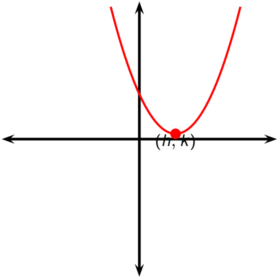
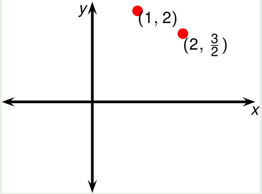


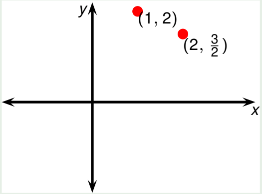

- When we change $h$ and $k$ we move the vertex of the parabola without change in steepness.
- Therefore when we change $b$ and $c$ we move the vertex of the parabola without change in steepness.

## Example




Write an equation of a parabola with vertex at $(1, 2)$ that passes through the point $\left(2, \frac{3}{2}\right)$.

## Example

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$$a(x-h)^2+k \quad = \quad y$$

Standard form

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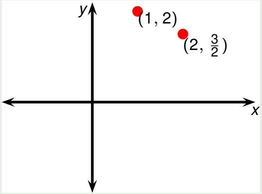


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Standard form

$$a(x - ?)^2 + ? = y$$

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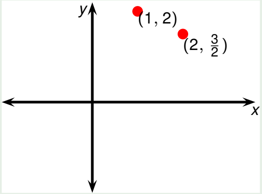


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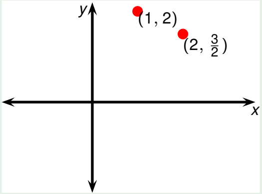
| | |
|---|---|
| $a(x - h)^2 + k = y$ | Standard form |
| $a(x - 1)^2 + 2 = y$ | Vertex at $(1, 2)$ |

## Example

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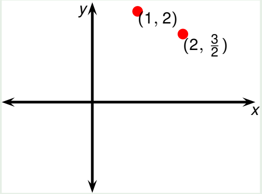


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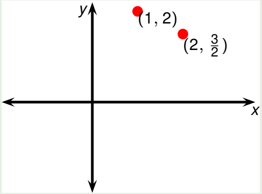


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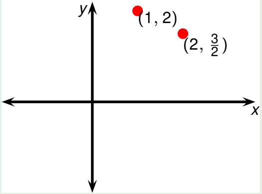
## Example

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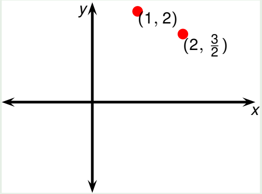
## Example

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Standard form

Vertex at $(1, 2)$

Passes through $\left(2, \frac{2}{3}\right)$

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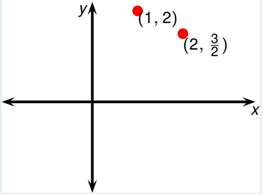


Write an equation of a parabola with vertex at $(1, 2)$ that passes through the point $\left(2, \frac{3}{2}\right)$.

$$\begin{aligned} a(x-h)^2+k &= y \\ a(x-1)^2+2 &= y \\ a\color{red}{(2-1)^2}+2 &= \frac{3}{2} \\ a &= \frac{3}{2}-2 \end{aligned}$$

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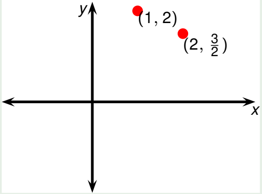


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$$\begin{aligned} a(x-h)^2 + k &= y \\ a(x-1)^2 + 2 &= y \\ a(2-1)^2 + {\color{red}2} &= \frac{3}{2} \\ a &= \frac{3}{2} {\color{red}- 2} \end{aligned}$$

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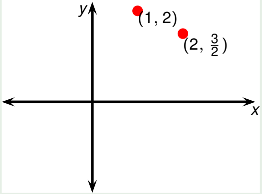


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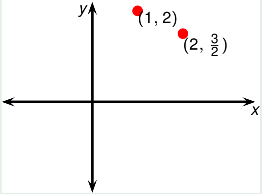


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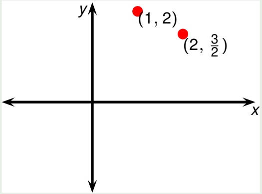


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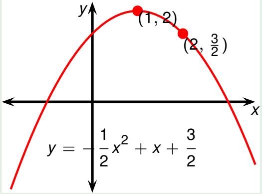


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y &= -\frac{1}{2}x^2 + x + \frac{3}{2} && \text{Alternative answer}
\end{aligned}
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## Problem (Quadratic equation formula)

*Solve the general quadratic equation*

$$ax^2 + bx + c = 0$$

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$$\begin{aligned} ax^2 + bx + c &= 0 \quad \Big| \textit{ complete the square} \\ a\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a} &= 0 \end{aligned}$$

## Problem (Quadratic equation formula)

*Solve the general quadratic equation*

$$\begin{array}{rcl|l} ax^2+bx+c &=& 0 & \text{complete the square} \\ a\left(x+\dfrac{b}{2a}\right)^2-\dfrac{D}{4a} &=& 0 & \text{where } D=b^2-4ac \end{array}$$

## Problem (Quadratic equation formula)

*Solve the general quadratic equation*

$$\begin{aligned} ax^2 + bx + c &= 0 \\ a\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a} &= 0 \\ a\left(\left(x + \frac{b}{2a}\right)^2 - \frac{D}{4a^2}\right) &= 0 \end{aligned}$$

*complete the square*

*where* $D = b^2 - 4ac$

## Problem (Quadratic equation formula)

*Solve the general quadratic equation*

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## Problem (Quadratic equation formula)

*Solve the general quadratic equation*

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x &= \frac{-b+\sqrt{D}}{2a}
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## Theorem

*The solutions of the quadratic equation*

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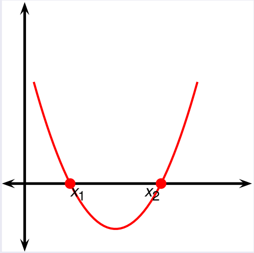


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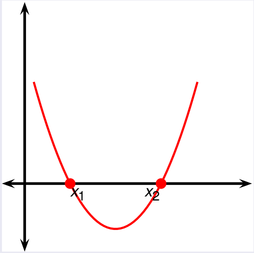


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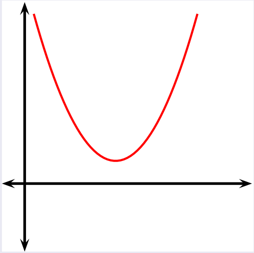

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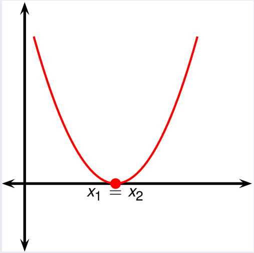
- If $D < 0$ then $\sqrt{D}$ is not a real $\Rightarrow$ quadratic has no real solutions.

## Theorem

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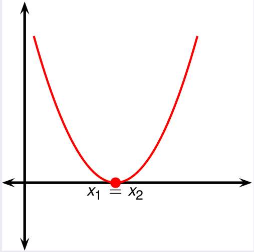


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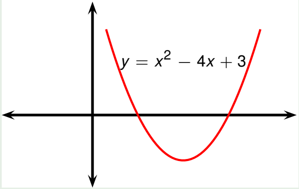
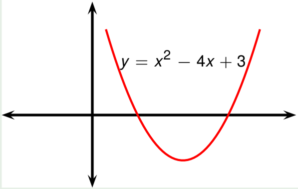
# Example




Find the $x$-intercepts of $x^2 - 4x + 3$.

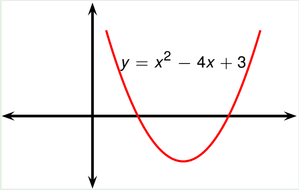
## Example




Find the $x$-intercepts of $x^2 - 4x + 3$.

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

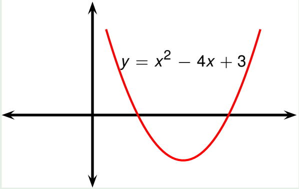
## Example




Find the $x$-intercepts of $x^2 - 4x + 3$.

$$
\begin{aligned}
x_1, x_2 &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}
\end{aligned}
$$

## Example

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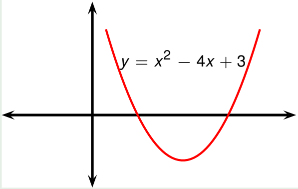


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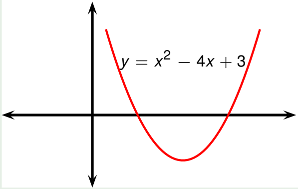


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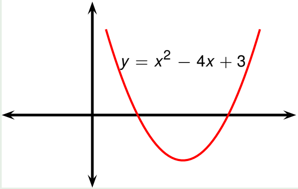


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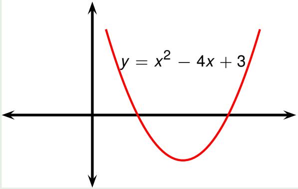


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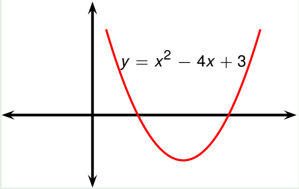


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&= \frac{4 \pm 2}{2} \\
&= \begin{cases} \dfrac{4 + 2}{2} \\ \dfrac{4 - 2}{2} \end{cases}
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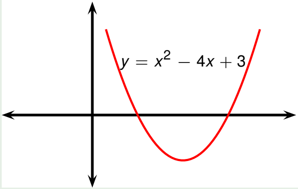
# Example

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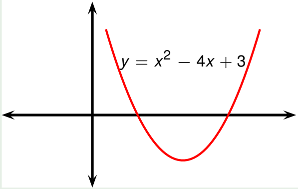
&= \begin{cases} \frac{4+2}{2} = \frac{6}{2} \\ \frac{4-2}{2} \end{cases}
\end{aligned}
$$

## Example




Find the $x$-intercepts of $x^2 - 4x + 3$.

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x_1, x_2 &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
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&= \frac{4 \pm \sqrt{4}}{2} \\
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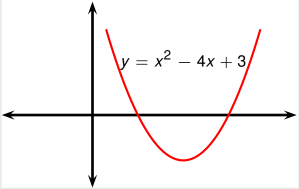


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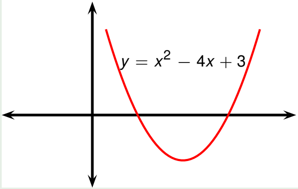


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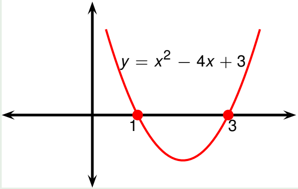


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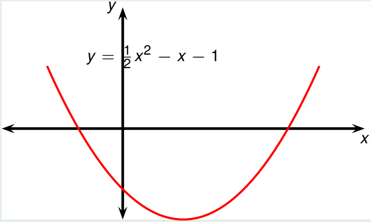


Find the $x$-intercepts of $\dfrac{x^2}{2} - x - 1$.

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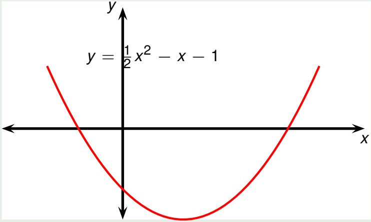


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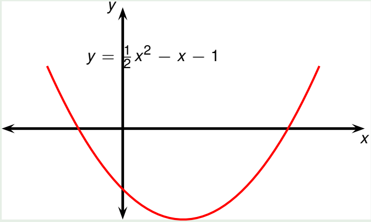


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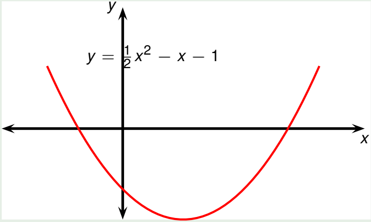


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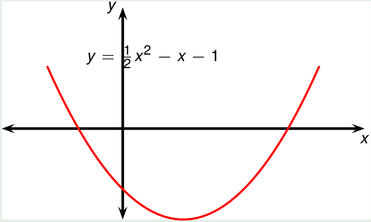


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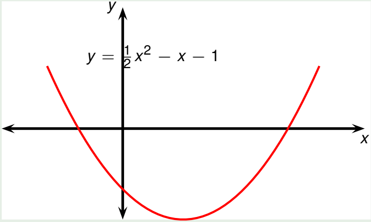


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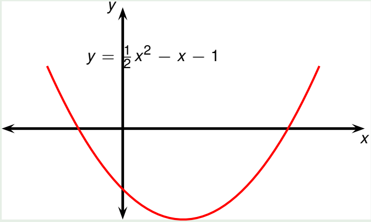


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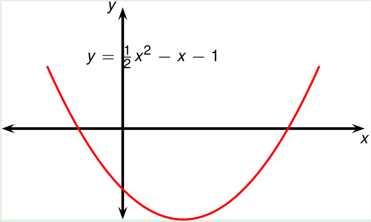


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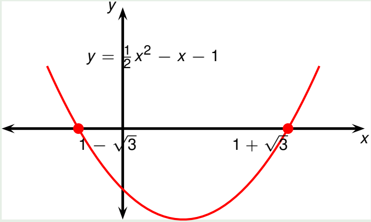


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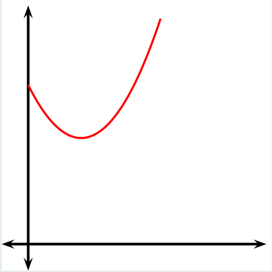

Find the $x$-intercepts of $x^2 - 2x + 3$.

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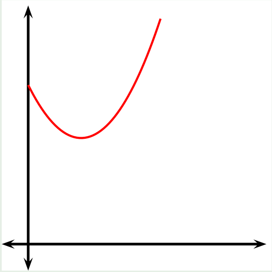

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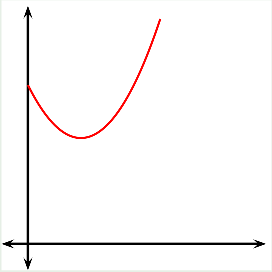

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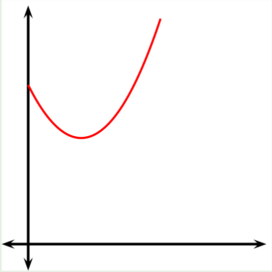

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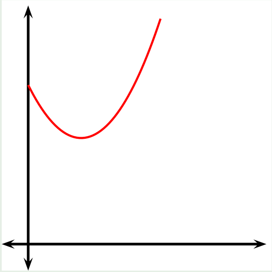

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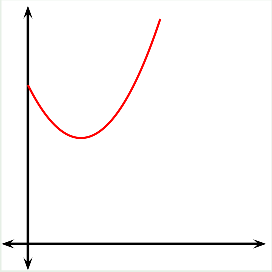

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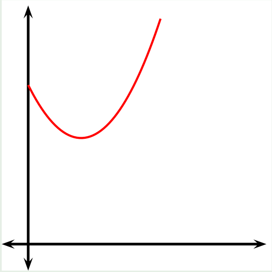

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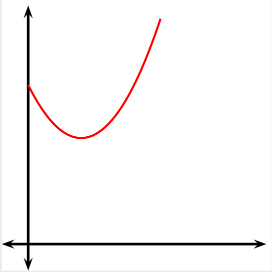

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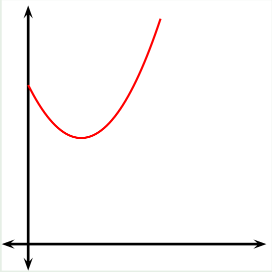

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no $x$ – intercepts

## Proposition

*Let $ax^2 + bx + c$, $a \neq 0$ be a quadratic with discriminant $D = b^2 - 4ac$ and roots $x_1$ and $x_2$. Then $D = a^2 (x_1 - x_2)^2$.*

## Proof.

$\square$

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$$a^2(x_1 - x_2)^2 = a^2 \left( \frac{-b + \sqrt{D}}{2a} - \frac{-b - \sqrt{D}}{2a} \right)$$

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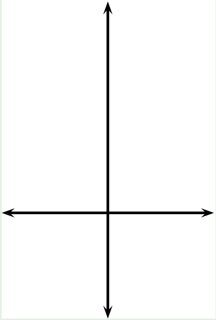
- Discriminant is zero $\Leftrightarrow$ the quadratic has non-distinct roots, hence the discriminant discriminates between the two roots.

## Example

Find the values of the parameter $k$ for which the equation $3x^2 - kx + 1$ has two real distinct roots.



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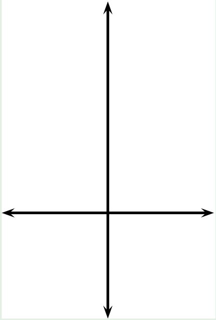

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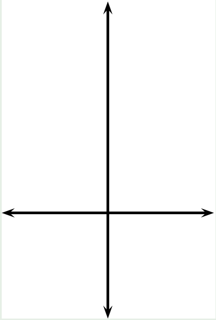

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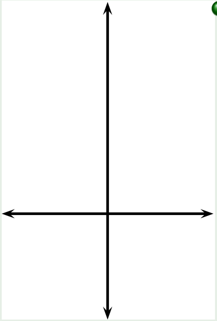
- $\Rightarrow$ The roots $x_1, x_2$ are real and distinct when
$$b^2 - 4ac > 0$$



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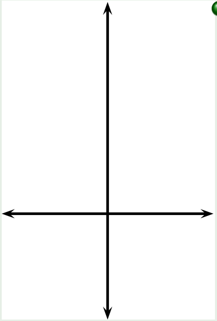

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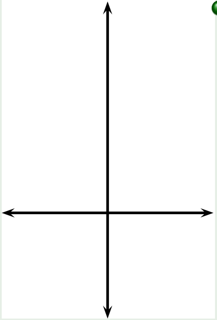

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$$b^2 - 4ac > 0$$
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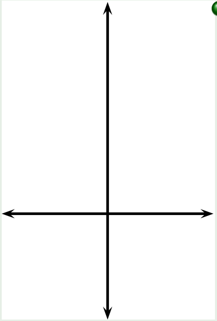

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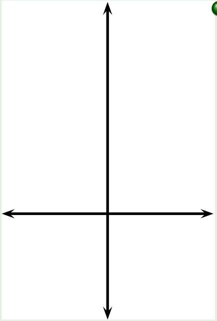

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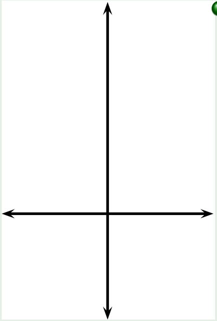

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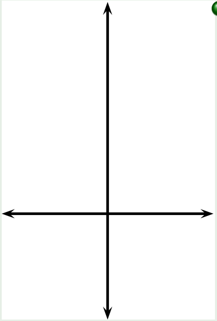

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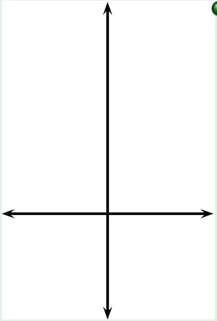

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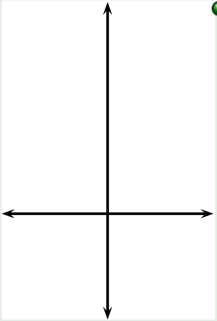

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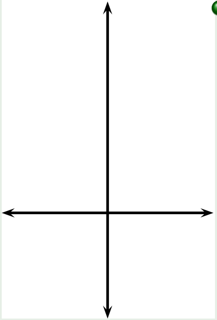

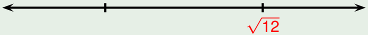

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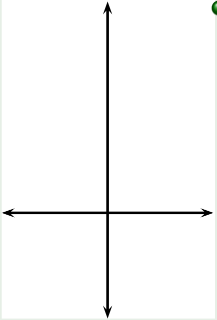

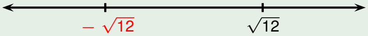

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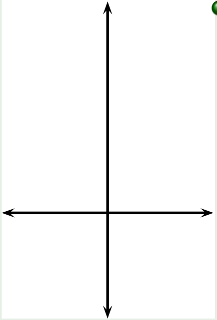

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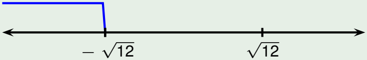

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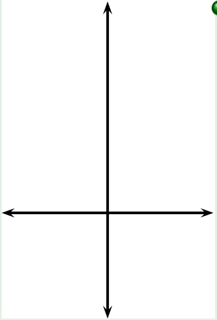

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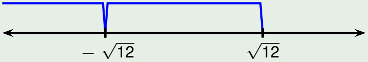

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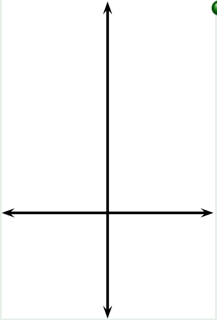

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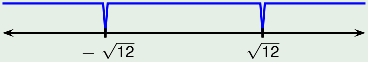

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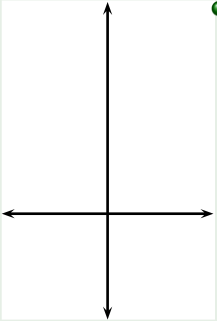

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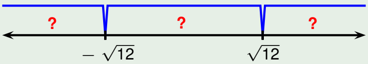

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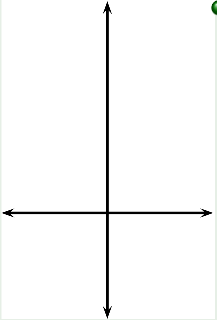

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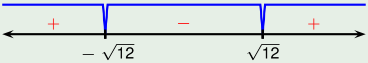

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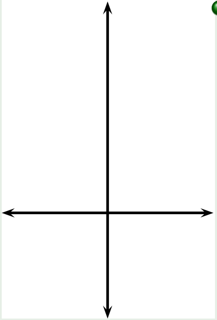

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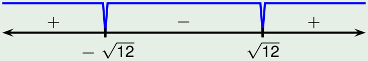

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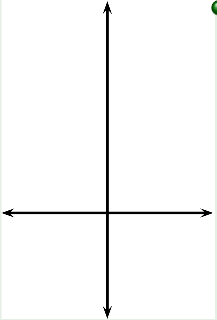

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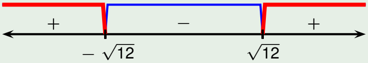

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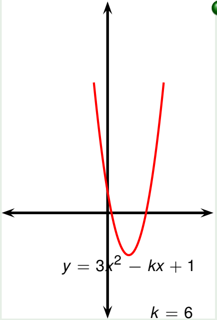


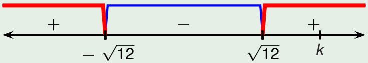

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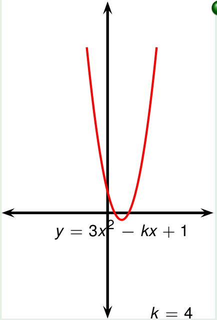


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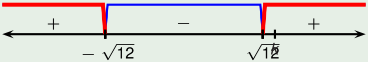

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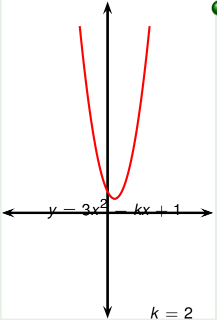


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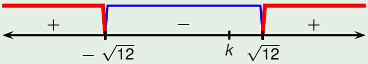

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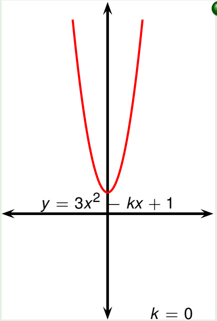


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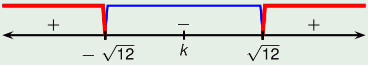

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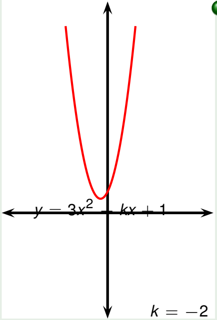


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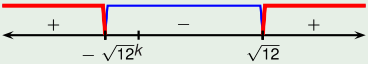

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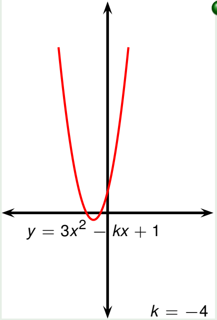


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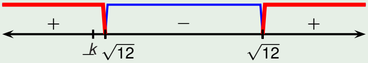

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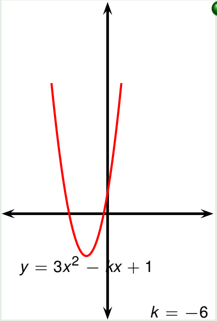


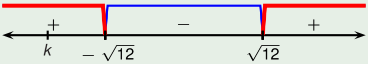

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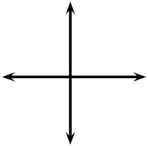

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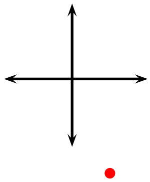
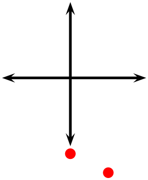


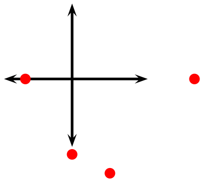
To plot a parabola by hand roughly, we need to do the following.

- Find the vertex of the parabola.
- Find the $y$ intercept.



To plot a parabola by hand roughly, we need to do the following.

- Find the vertex of the parabola.
- Find the $y$ intercept.
- Find the $x$ intercept(s) if any.



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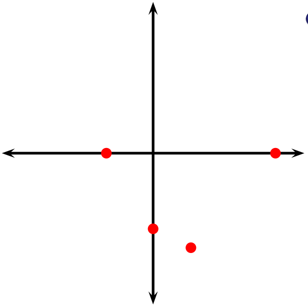

- Find the vertex of the parabola.
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- Select (or re-select) axes scale so all important points found in the preceding items fit in the plot.

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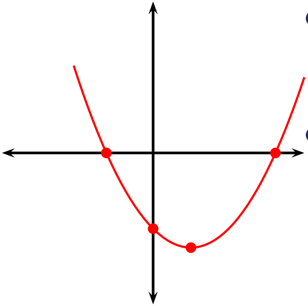

- Find the vertex of the parabola.
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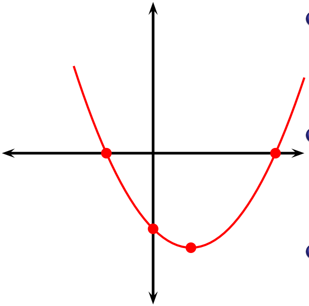

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- If $a > 0$ your parabola should open upwards, if $a < 0$ your parabola should open downwards.

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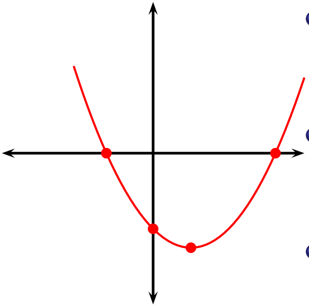

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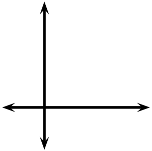
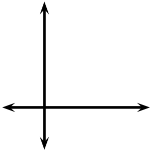
- If $a > 0$ your parabola should open upwards, if $a < 0$ your parabola should open downwards.
- For $|a| > 1$ we should aim to draw the graph steeper than $a = x^2$, for $|a| < 1$ we should aim to draw the graph flatter than $a = x^2$.

## Example



Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

## Example



Plot roughly by hand the graph of
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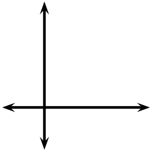
- The vertex of the parabola is given by:

$$
\begin{aligned}
x &= \textcolor{red}{\textbf{?}} \\
y &= \textbf{?}
\end{aligned}
$$

## Example



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- The vertex of the parabola is given by:

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\begin{aligned}
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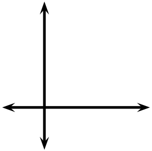
## Example



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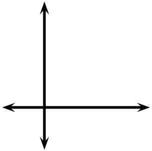

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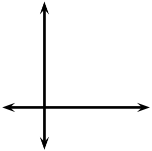

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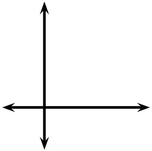

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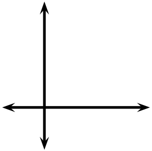

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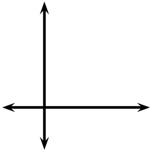

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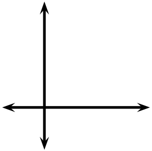

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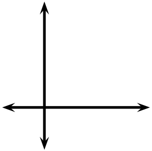
## Example



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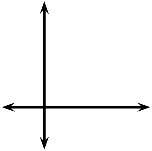

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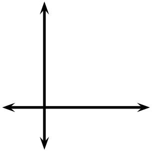

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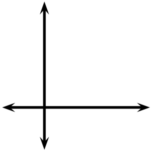

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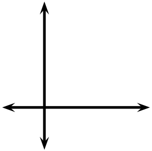

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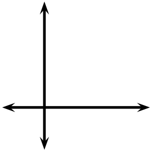

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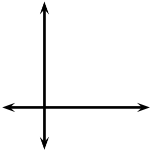

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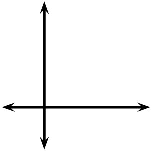

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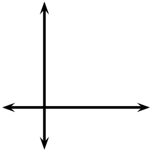

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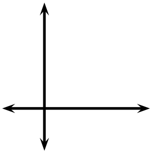

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&= \frac{3\cdot 57}{8}
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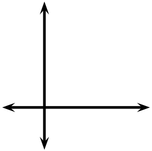
## Example



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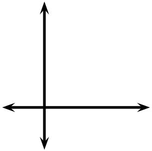

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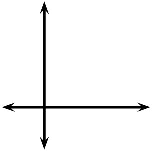
## Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$

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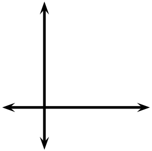

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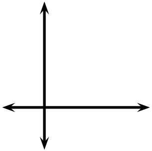
x &= -\frac{b}{2a} = -\frac{7}{2\left(-\frac{2}{3}\right)} = \frac{21}{4} \\
y &= f\left(-\frac{b}{2a}\right) = -\frac{D}{4a} = -\frac{\left(b^2 - 4ac\right)}{4a} \\
&= -\frac{7^2 - 4\left(-\frac{2}{\cancel{3}}\right)\cancel{3}}{4\left(-\frac{2}{3}\right)} = \frac{49 + 8}{\frac{8}{3}} \\
&= \frac{3 \cdot 57}{8} = \frac{171}{8}.
\end{aligned}
$$

- The $y$-intercept is $f(0) = \mathbf{?}$.

## Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

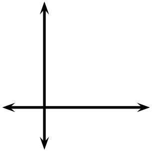
Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The vertex of the parabola is given by:

$$
\begin{aligned}
x &= -\frac{b}{2a} = -\frac{7}{2\left(-\frac{2}{3}\right)} = \frac{21}{4}\\
y &= f\left(-\frac{b}{2a}\right) = -\frac{D}{4a} = -\frac{(b^2-4ac)}{4a}\\
&= -\frac{7^2 - 4\left(-\frac{2}{\cancel{3}}\right)\cancel{3}}{4\left(-\frac{2}{3}\right)} = \frac{49+8}{\frac{8}{3}}\\
&= \frac{3\cdot 57}{8} = \frac{171}{8}.
\end{aligned}
$$

- The $y$-intercept is $f(0) = 3$.

## Example

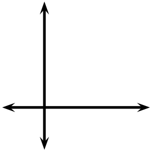


Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The $x$ intercepts are given by the solutions of
$$-\frac{2}{3}x^2 + 7x + 3 = 0$$

## Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$

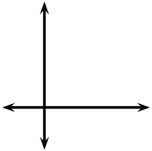
$y$-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The $x$ intercepts are given by the solutions of

$$\begin{aligned} -\frac{2}{3}x^2 + 7x + 3 &= 0 \qquad \Big|\ \cdot 3 \\ -2x^2 + 21x + 9 &= 0 \end{aligned}$$

## Example



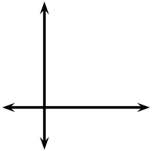
Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The $x$ intercepts are given by the solutions of

$$\begin{aligned} -\frac{2}{3}x^2 + 7x + 3 &= 0 && \Big| \cdot 3 \\ -2x^2 + 21x + 9 &= 0 \\ x &= ? \end{aligned}$$

## Example



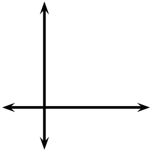
Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The $x$ intercepts are given by the solutions of

$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$
$$-2x^2 + 21x + 9 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## Example



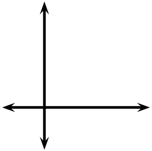
Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The $x$ intercepts are given by the solutions of

$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$
$$-2x^2 + 21x + 9 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)}$$

## Example



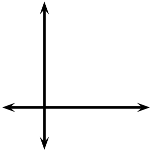
Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The $x$ intercepts are given by the solutions of

$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$
$$-2x^2 + 21x + 9 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-21 \pm \sqrt{21^2 - 4 \cdot (-2) \cdot 9}}{2 \cdot (-2)}$$

## Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The $x$ intercepts are given by the solutions of

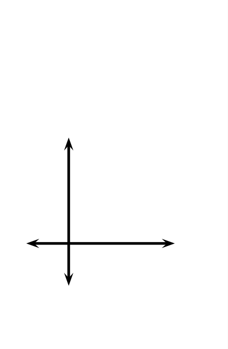
$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$

$$-2x^2 + 21x + 9 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)}$$

## Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

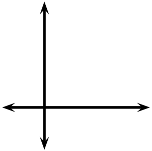
Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3.$

- The $x$ intercepts are given by the solutions of

$$\begin{aligned} -\frac{2}{3}x^2 + 7x + 3 &= 0 & \Big| \cdot 3 \\ -2x^2 + 21x + 9 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-21 \pm \sqrt{21^2 - 4 \cdot (-2) \cdot 9}}{2 \cdot (-2)} \\ &= \frac{-21 \pm \sqrt{441 + 72}}{-4} \end{aligned}$$

## Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

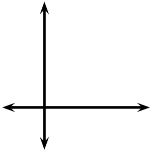
- The $x$ intercepts are given by the solutions of

$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$

$$-2x^2 + 21x + 9 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-21 \pm \sqrt{21^2 - 4 \cdot (-2) \cdot 9}}{2 \cdot (-2)} \\ &= \frac{-21 \pm \sqrt{441 + 72}}{-4} \end{aligned}$$

## Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

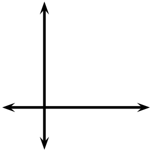
- The $x$ intercepts are given by the solutions of

$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$

$$-2x^2 + 21x + 9 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-21 \pm \sqrt{21^2 - 4 \cdot (-2) \cdot 9}}{2 \cdot (-2)} \\ &= \frac{-21 \pm \sqrt{441 + 72}}{-4} \end{aligned}$$

## Example



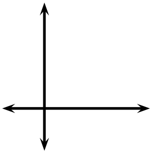
Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The $x$ intercepts are given by the solutions of

$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$
$$-2x^2 + 21x + 9 = 0$$
$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)} \\ &= \frac{-21 \pm \sqrt{441 + 72}}{-4} \\ &= \frac{21 \mp \sqrt{513}}{4} \end{aligned}$$

## Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

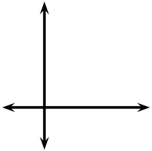
- The $x$ intercepts are given by the solutions of

$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$

$$-2x^2 + 21x + 9 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-21 \pm \sqrt{21^2 - 4 \cdot (-2) \cdot 9}}{2 \cdot (-2)} \\ &= \frac{-21 \pm \sqrt{441 + 72}}{-4} \\ &= \frac{21 \mp \sqrt{513}}{4} \end{aligned}$$

## Example



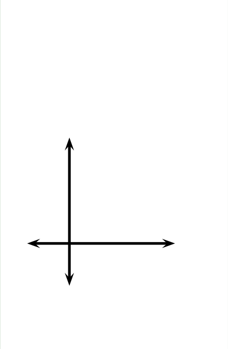
Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The $x$ intercepts are given by the solutions of

$$
\begin{aligned}
-\frac{2}{3}x^2 + 7x + 3 &= 0 \qquad \Big| \cdot 3\\
-2x^2 + 21x + 9 &= 0\\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\
&= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)}\\
&= \frac{-21 \pm \sqrt{441 + 72}}{-4}\\
&= \frac{21 \mp \sqrt{513}}{4}\\
&= \frac{21 \mp \sqrt{9\cdot 57}}{4}
\end{aligned}
$$

## Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3.$

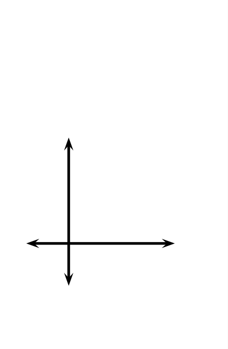
- The $x$ intercepts are given by the solutions of

$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$

$$-2x^2 + 21x + 9 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)} \\ &= \frac{-21 \pm \sqrt{441 + 72}}{-4} \\ &= \frac{21 \mp \sqrt{513}}{4} \\ &= \frac{21 \mp \sqrt{9\cdot 57}}{4} \\ &= \frac{21 \mp \sqrt{9}\sqrt{57}}{4} \end{aligned}$$

## Example



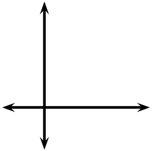
Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The $x$ intercepts are given by the solutions of

$$
\begin{aligned}
-\frac{2}{3}x^2 + 7x + 3 &= 0 \qquad \Big| \cdot 3\\
-2x^2 + 21x + 9 &= 0\\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\
&= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)}\\
&= \frac{-21 \pm \sqrt{441 + 72}}{-4}\\
&= \frac{21 \mp \sqrt{513}}{4}\\
&= \frac{21 \mp \sqrt{9\cdot 57}}{4}\\
&= \frac{21 \mp \sqrt{9}\sqrt{57}}{4}\\
&= \frac{21 \mp 3\sqrt{57}}{4}
\end{aligned}
$$

## Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$
$x$-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

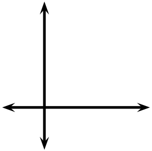
Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The $x$ intercepts are given by the solutions of

$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$
$$-2x^2 + 21x + 9 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)} \\ &= \frac{-21 \pm \sqrt{441+72}}{-4} \\ &= \frac{21 \mp \sqrt{513}}{4} \\ &= \frac{21 \mp \sqrt{9\cdot 57}}{4} \\ &= \frac{21 \mp \sqrt{9}\sqrt{57}}{4} \\ &= \frac{21 \mp 3\sqrt{57}}{4} \end{aligned}$$

## Example

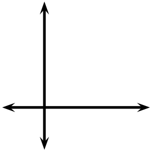


Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$
$x$-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4}$

## Example

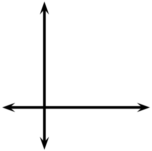


Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$
$x$-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.

## Example

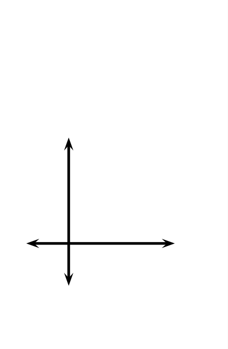


Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$
$x$-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.
  - $\frac{171}{8}$ is between the integers **?**

## Example

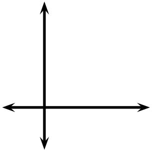


Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$
$x$-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.
  - $\frac{171}{8}$ is between the integers 21 and 22.

## Example

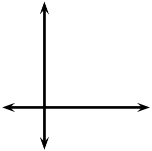


Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$
$x$-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.
  - $\frac{171}{8}$ is between the integers 21 and 22.
  - $\frac{21+3\sqrt{57}}{4}$ is close to $\frac{21+3\sqrt{64}}{4}$

## Example

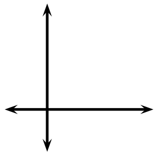


Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$
$x$-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.
  - $\frac{171}{8}$ is between the integers 21 and 22.
  - $\frac{21+3\sqrt{57}}{4}$ is close to $\frac{21+3\sqrt{64}}{4} = \frac{21+24}{4}$

## Example

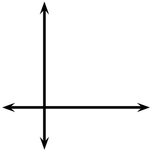


Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$
$x$-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of $f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.
  - $\frac{171}{8}$ is between the integers 21 and 22.
  - $\frac{21+3\sqrt{57}}{4}$ is close to $\frac{21+3\sqrt{64}}{4} = \frac{21+24}{4} = \frac{45}{4}$

## Example

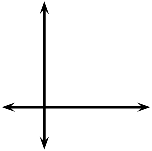


Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$
$x$-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.
  - $\frac{171}{8}$ is between the integers 21 and 22.
  - $\frac{21+3\sqrt{57}}{4}$ is close to $\frac{21+3\sqrt{64}}{4} = \frac{21+24}{4} = \frac{45}{4}$ which is close to $\frac{44}{4} = 11$.

## Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
$y$-intercept at $y = 3$
$x$-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

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  - $\frac{21-3\sqrt{57}}{4}$ is close to $\frac{21-3\sqrt{64}}{4}$

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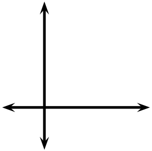

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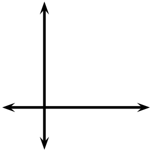

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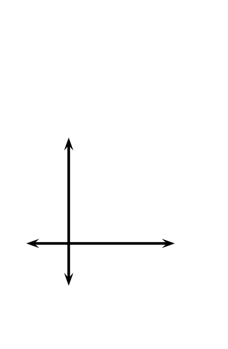

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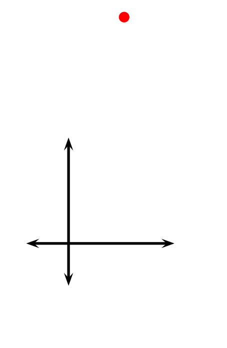

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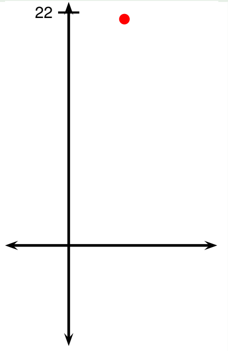


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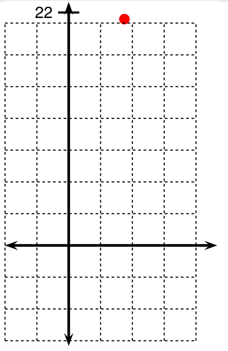


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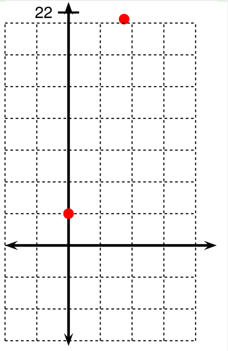


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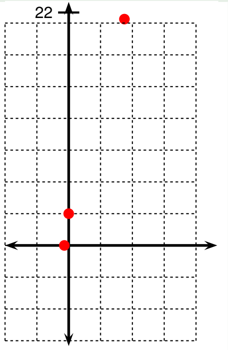


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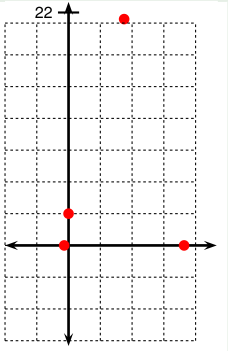


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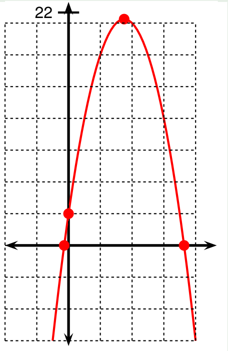


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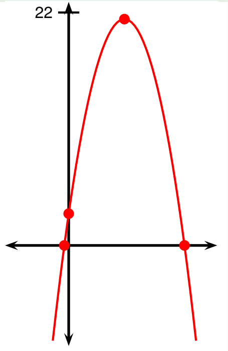


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## Example

Find the values of the parameter $k$ for which $x^2 + (k+1)x + 2k > 0$ holds for all real $x$.

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- Leading coefficient is positive

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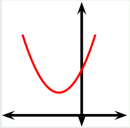

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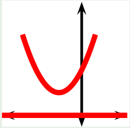
## Example



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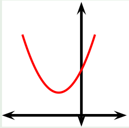

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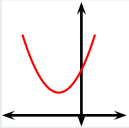

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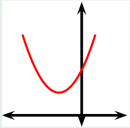

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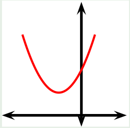

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$$b^2 - 4ac \quad < \quad 0$$

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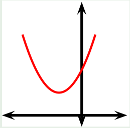

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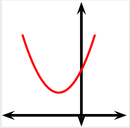

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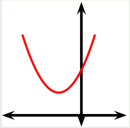

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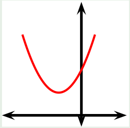

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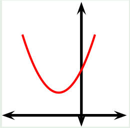

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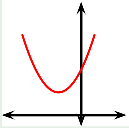

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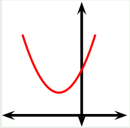

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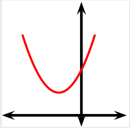

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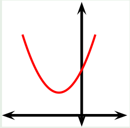

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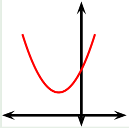

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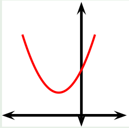

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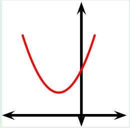

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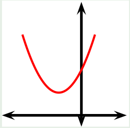

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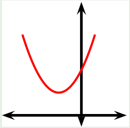

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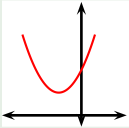

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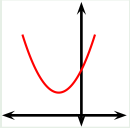

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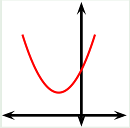

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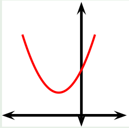

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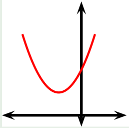

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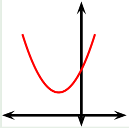

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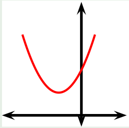

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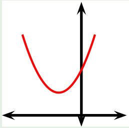

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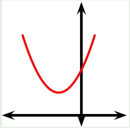

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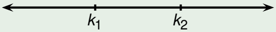

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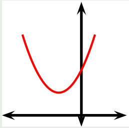

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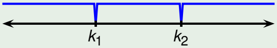

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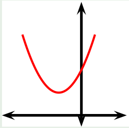

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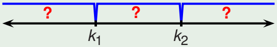

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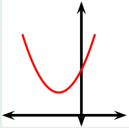

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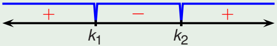
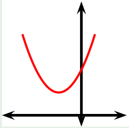
$$\color{red}{k \in \mathbf{?}}$$



# Example



Find the values of the parameter $k$ for which $x^2 + (k+1)x + 2k > 0$ holds for all real $x$.

- In order for the quadratic to be positive, its graph must lie entirely above the $x$ axis.

- Leading coefficient is positive $\Rightarrow$ graph opens up $\Rightarrow$ is above $x$ axis if it does not intersect it $\Rightarrow$ the quadratic has no real solutions.
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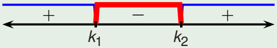

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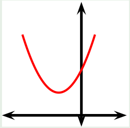

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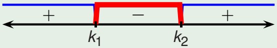

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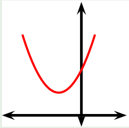

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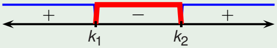

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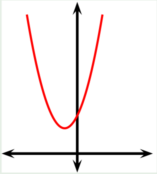

Find the minimum point on the curve
$y = 3x^2 + 2x + 1$ by completing the square.

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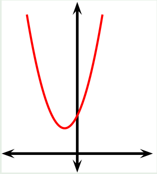

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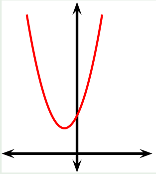

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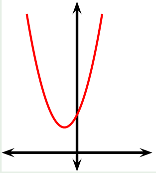

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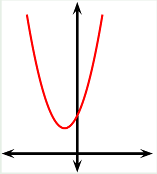

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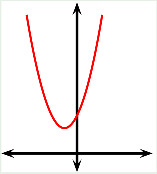

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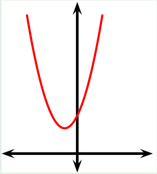

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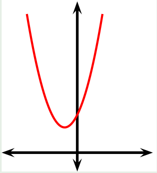

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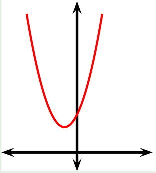

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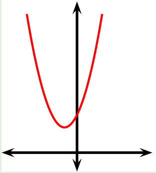

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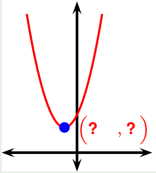


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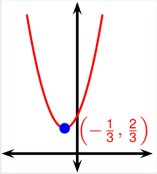


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# Maximum or minimum value of a quadratic function

- Let $f(x) = ax^2 + bx + c$ - quadratic ($a \neq 0$).
- Let $D$ be the discriminant $D = b^2 - 4ac$.

$$f(x) = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a} \quad \Big| \text{ complete the square}$$

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Recall $f(x) = ax^2 + bx + c = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a}$.

## Proposition

*Let $f(x) = ax^2 + bx + c$, $a \neq 0$ and let $D = b^2 - 4ac$.*

- *If $a > 0$ then $f(x)$ has no maximum and has minimum at $x = -\frac{b}{2a}$.*
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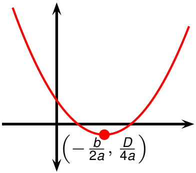

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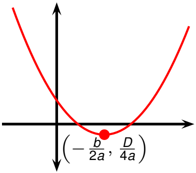


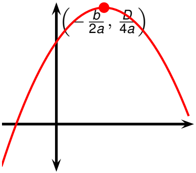

## Example

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$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

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Parabola

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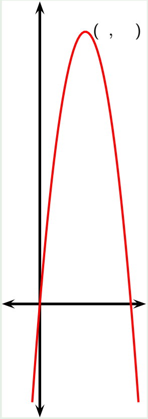


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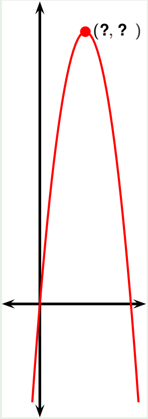


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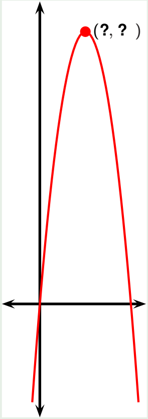


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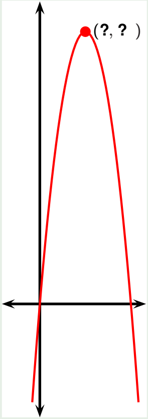


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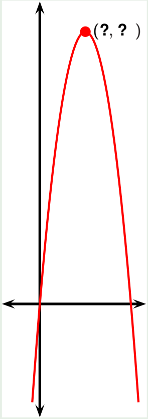


$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

Maximizing:

$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{12}{-2} \end{aligned}$$

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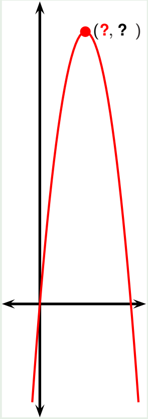


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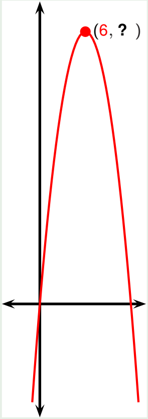


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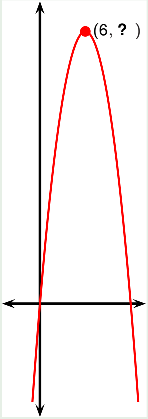


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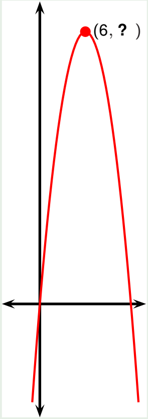


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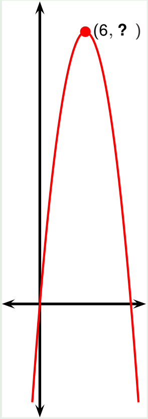


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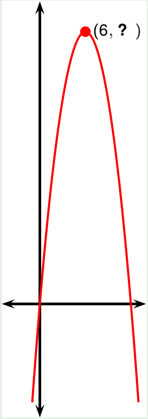


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Max. product $= xz$

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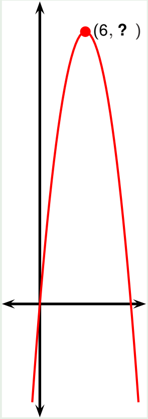


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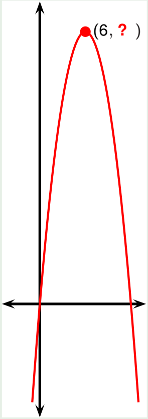


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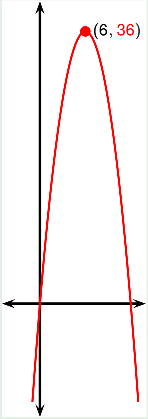


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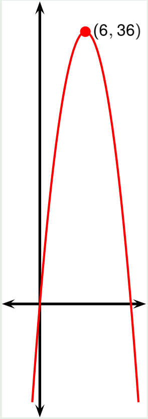


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