

# Precalculus

## Quadratic inequality part 1

Todor Milev

2019

## Example

Solve the inequality.

$$2x^2 + 3x - 5 \geq 0$$

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$$\begin{aligned} 2x^2 + 3x - 5 &\geq 0 \\ (?)(?) &\geq 0 \end{aligned}$$

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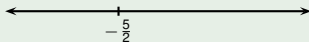
$$\begin{aligned} 2x^2 + 3x - 5 &\geq 0 \\ (2x + 5)(x - 1) &\geq 0 \end{aligned}$$

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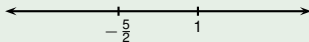


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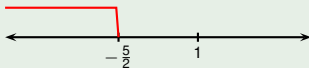


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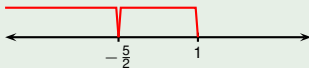
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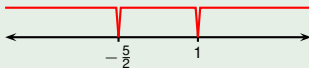


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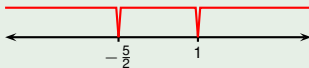
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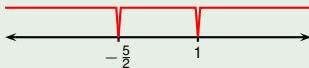
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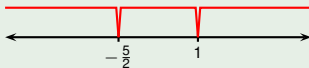
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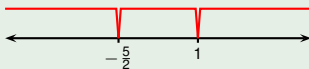
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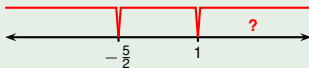
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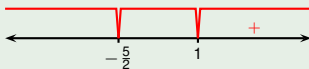
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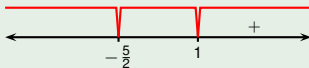
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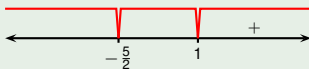
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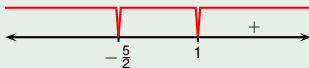
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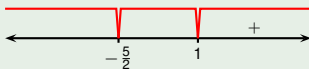
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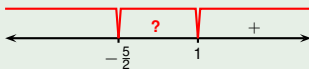
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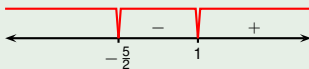
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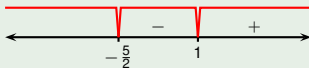
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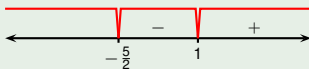
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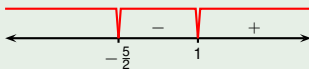
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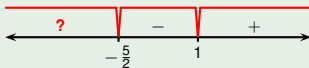
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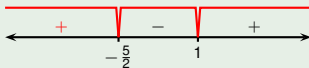
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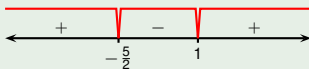
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$(-\frac{5}{2}, 1)$	$(+)(-)$	-	0	$f(0) = -5 < 0$
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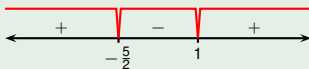
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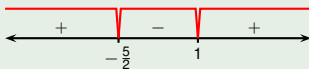
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 (2x + 5)(x - 1) &\geq 0 \\
 x &\in \left(-\infty, -\frac{5}{2}\right] \cup [1, \infty)
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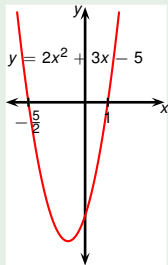
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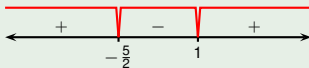


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