

Precalculus

Degree lowering formulas

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Proposition (Power-Reducing Formulas)

$$\sin^2 \alpha = \frac{1 - \cos(2\alpha)}{2} \quad \cos^2 \alpha = \frac{1 + \cos(2\alpha)}{2}$$

Proof.

$$\begin{aligned} \cos(2\alpha) &= 1 - 2\sin^2 \alpha & \cos(2\alpha) &= 2\cos^2 \alpha - 1 \\ 2\sin^2 \alpha &= 1 - \cos(2\alpha) & 2\cos^2 \alpha &= 1 + \cos(2\alpha) \\ \sin^2 \alpha &= \frac{1 - \cos(2\alpha)}{2} & \cos^2 \alpha &= \frac{1 + \cos(2\alpha)}{2} \end{aligned}$$



Corollary

$$\sin \alpha = \pm \sqrt{\frac{1 - \cos(2\alpha)}{2}} \quad \cos \alpha = \pm \sqrt{\frac{1 + \cos(2\alpha)}{2}}$$

Corollary (Half-Angle Formulas)

$$\sin\left(\frac{\beta}{2}\right) = \pm \sqrt{\frac{1 - \cos \beta}{2}} \quad \cos\left(\frac{\beta}{2}\right) = \pm \sqrt{\frac{1 + \cos \beta}{2}}$$

Proposition (Power-Reducing Formulas)

$$\sin^2 \alpha = \frac{1 - \cos(2\alpha)}{2} \quad \cos^2 \alpha = \frac{1 + \cos(2\alpha)}{2}$$

- The power reducing formulas are used to express $\sin^k \alpha$ and $\cos^k \alpha$ via lower powers of the sin and cos functions (applied to angles other than α).
- This technique will play a key role in integration (studied later/in another course).

Recall the formulas: $\sin^2 \beta = \frac{1 - \cos(2\beta)}{2}$, $\cos^2 \beta = \frac{\cos(2\beta) + 1}{2}$.

Example

Rewrite $\sin^4 \alpha$ in terms of first powers of the cosines and sines of multiples of the angle α .

$$\begin{aligned}
 \sin^4 \alpha &= (\sin^2 \alpha)^2 \\
 &= \left(\frac{1 - \cos(2\alpha)}{2} \right)^2 \\
 &= \frac{1}{4} (1 - 2\cos(2\alpha) + \cos^2(2\alpha)) \\
 &= \frac{1}{4} \left(1 - 2\cos(2\alpha) + \frac{\cos(2 \cdot 2\alpha) + 1}{2} \right) \\
 &= \frac{1}{4} \left(1 - 2\cos(2\alpha) + \frac{\cos(2 \cdot 2\alpha)}{2} + \frac{1}{2} \right) \\
 &= \frac{1}{4} \left(\frac{3}{2} - 2\cos(2\alpha) + \frac{\cos(4\alpha)}{2} \right) \\
 &= \frac{1}{8} (3 - 4\cos(2\alpha) + \cos(4\alpha))
 \end{aligned}$$