

Calculus I

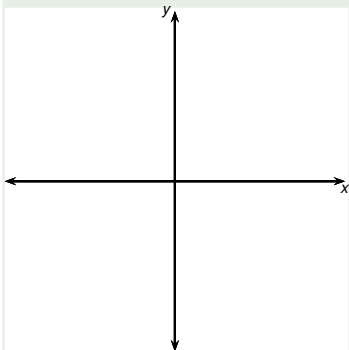
Rational function asymptotes, part 1

Todor Milev

2019

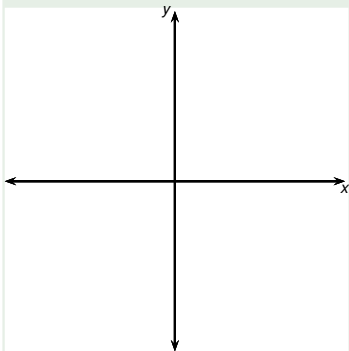
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Find the horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{3x^2+1}}{2x-3}$.



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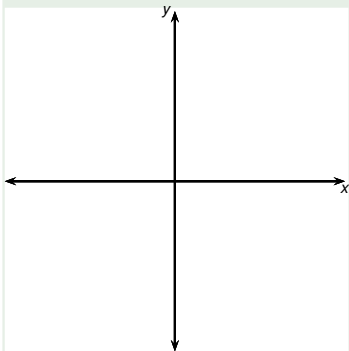


$$\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2+1}}{2x-3}$$

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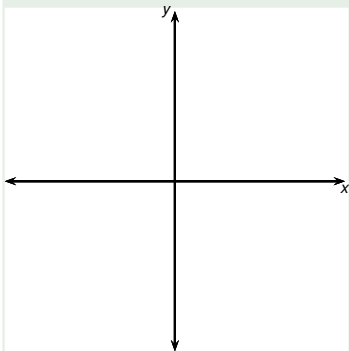


$$\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

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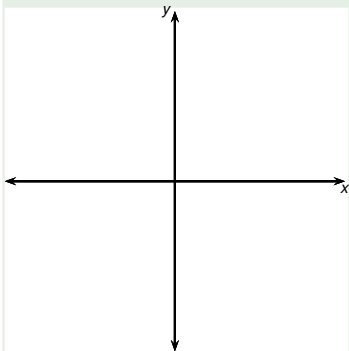
If $x > 0$ then $x = \sqrt{x^2}$.

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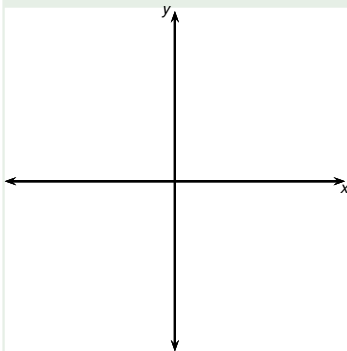
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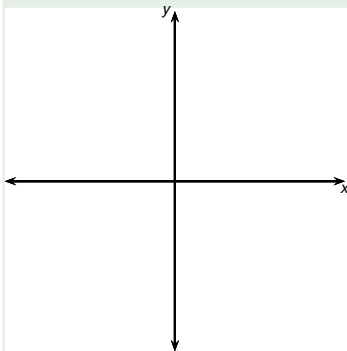
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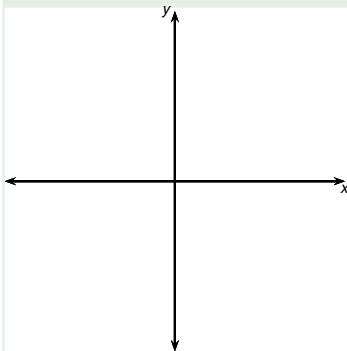
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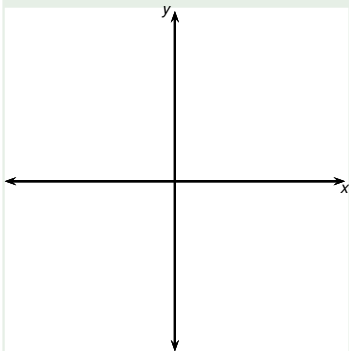
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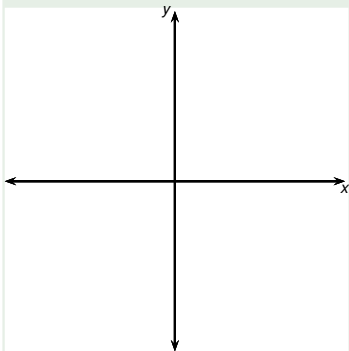
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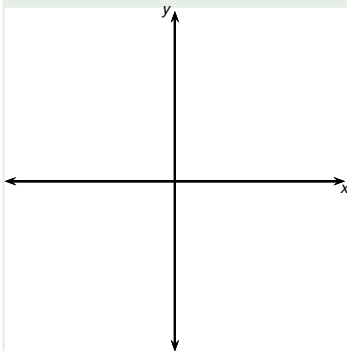


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 &= \frac{\sqrt{? + ?}}{? - ?} \\
 \lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2+1}}{2x-3}
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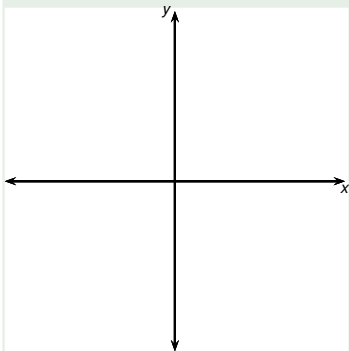


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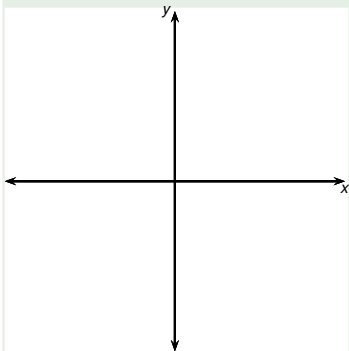


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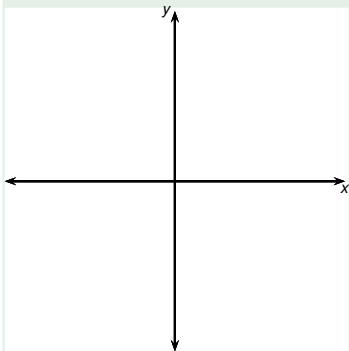


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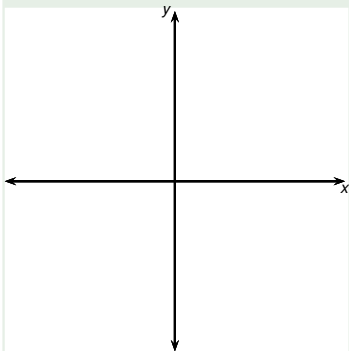


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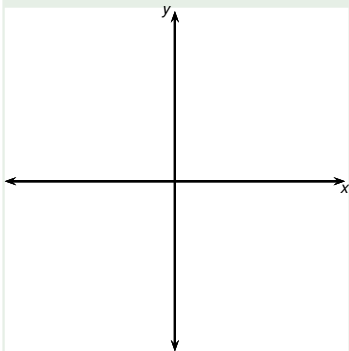


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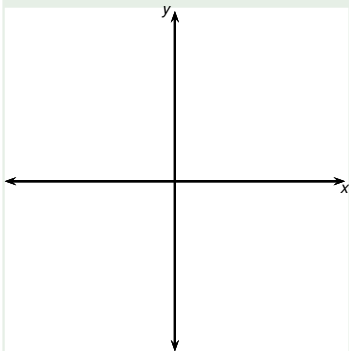


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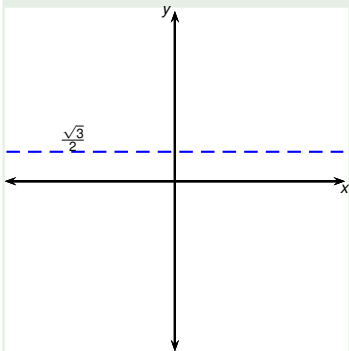


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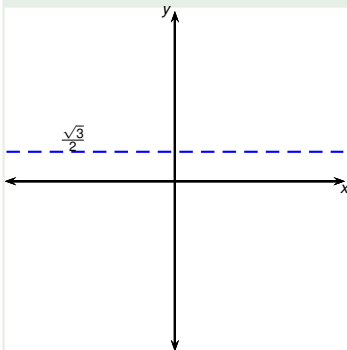


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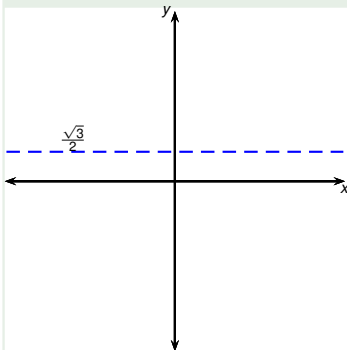


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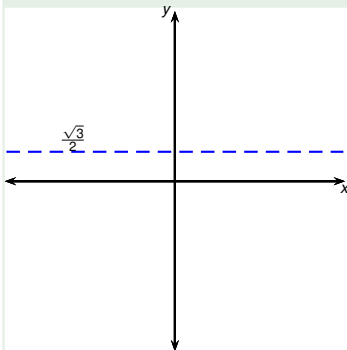


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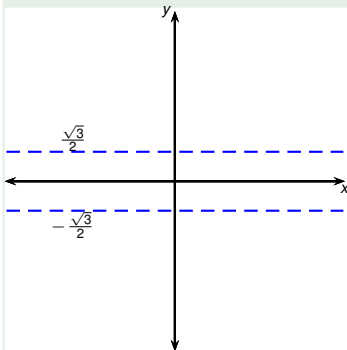
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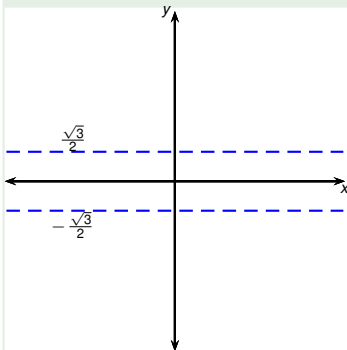
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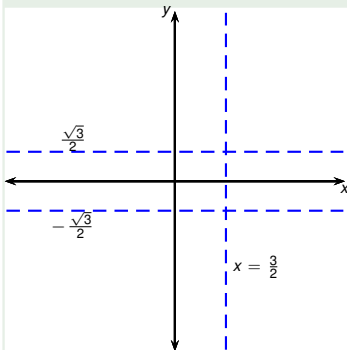
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Vertical Asymptote:

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 &= \lim_{x \rightarrow \infty} \frac{\sqrt{3 + \frac{1}{x^2}}}{2 - \frac{3}{x}} = \frac{\sqrt{\lim_{x \rightarrow \infty} 3 + \lim_{x \rightarrow \infty} \frac{1}{x^2}}}{\lim_{x \rightarrow \infty} 2 - 3 \lim_{x \rightarrow \infty} \frac{1}{x}} \\
 &= \frac{\sqrt{3+0}}{2-0} = \frac{\sqrt{3}}{2} \\
 \lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} &= \lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{-1}{\frac{1}{\sqrt{x^2}}} \\
 &= \lim_{x \rightarrow -\infty} -\frac{\sqrt{3 + \frac{1}{x^2}}}{2 - \frac{3}{x}} = -\frac{\sqrt{3}}{2}
 \end{aligned}$$

Example

Find the horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{3x^2+1}}{2x-3}$.



If $x > 0$ then $x = \sqrt{x^2}$.
If $x < 0$ then $x = -\sqrt{x^2}$.

Vertical Asymptote:

$$x = \frac{3}{2}.$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{1}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\sqrt{x^2}}{\frac{1}{x}}$$

$$= \lim_{x \rightarrow \infty} \frac{\sqrt{3 + \frac{1}{x^2}}}{2 - \frac{3}{x}} = \frac{\sqrt{\lim_{x \rightarrow \infty} 3 + \lim_{x \rightarrow \infty} \frac{1}{x^2}}}{\lim_{x \rightarrow \infty} 2 - 3 \lim_{x \rightarrow \infty} \frac{1}{x}}$$

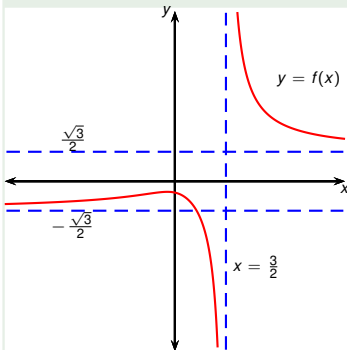
$$= \frac{\sqrt{3+0}}{2-0} = \frac{\sqrt{3}}{2}$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{1}{\frac{1}{x}} = \lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{-\frac{1}{\sqrt{x^2}}}{\frac{1}{x}}$$

$$= \lim_{x \rightarrow -\infty} -\frac{\sqrt{3 + \frac{1}{x^2}}}{2 - \frac{3}{x}} = -\frac{\sqrt{3}}{2}$$

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