Calculus I
Derivative of a power of expression

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- We can generalize this:

Observation (The Power Rule Combined with the Chain Rule)

If n is any real number and $u = h(x)$ is differentiable, then

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$

Alternatively,

$$\frac{d}{dx}[h(x)]^n = n[h(x)]^{n-1} \cdot h'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(u^n) = nu^{n-1}\frac{\mathrm{d}u}{\mathrm{d}x}$$

$$(g(h(x)))' = g'(h(x)) \cdot h'(x) \qquad \text{(notation 1)}$$

$$(g(u))' = g'(u)u' \qquad \text{where } u = h(x) \qquad \text{(notation 2)}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} \qquad \text{where } y = g(u) \qquad \text{(notation 3)} \quad .$$

Example (Chain Rule, Notation 3, Power Rule)

Differentiate $y = (x^3 - 1)^{100}$.

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\text{Differentiate} \quad y &= (x^3 - 1)^{100}. \\
\text{Let} \quad u &= \textbf{?} \\
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&= 300x^2(x^3-1)^{99}.
\end{aligned}$$