Precalculus
Polynomial systems basics

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Outline

1 Overview of polynomial systems

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1. Overview of polynomial systems

2. Ad hoc methods for solving polynomial systems

Systems of polynomial equations

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- Typical polynomial systems have more than one variable/equation:
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Here we have 2 variables (x, y), 2 equations.
- The number of variables and equations need not be equal:
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Here we have 4 variables (x, y, z, w), 3 equations.

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- Polynomial systems may have infinitely many solutions:

  $\left| \begin{array}{rcl} x & = & 0 \\ y + z & = & 1. \end{array} \right.$ If we set $x = 0$, $y = 1 - z$, we produce infinitely many solutions for every possible value of z.

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- Solving polynomial systems is an indispensable mathematical tool used in other branches of science and mathematics.
- Polynomial systems also have direct practical applications, for example kinematics - the configurations of a robotic arm can be parametrized with polynomials.

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- A system doable by hand would typically be solved easily using ad-hoc techniques.

Example

Solve the polynomial system. $\left|\begin{array}{rcl} x-4y &=& 5\\ y^2+xy &=& 10 \end{array}\right.$

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Solve for x in first eq-n.
Substitute x away

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$x=5+4y$

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$$\begin{aligned} x &= 5+4y\\ &= 5+4(-2)\end{aligned}$$

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$$x = 5 + 4y \qquad\qquad x = 5 + 4y$$

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$$\begin{array}{ll}
x = 5+4y & x = 5+4y\\
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Final answer: ${\color{red}x=-3, y=-2}$ or $x=9, y=1$.

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Check answer $y=1, x=9$:

$$\left|\begin{array}{r} x-4y\\ y^2+xy\end{array}\right.$$

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Final answer: $x=-3, y=-2$ or $x=9, y=1$.

Check answer $x=-3, y=-2$:

$$\left|\begin{array}{rclcl} x-4y &=& (-3)-4(-2) &=& 5\\ y^2+xy &=& (-2)^2+(-3)(-2) &=& 10\end{array}\right.$$

Check answer $y=1, x=9$:

$$\left|\begin{array}{rclcl} x-4y &=& 9-4\cdot 1 &=& 5\\ y^2+xy &=& 1^2+9\cdot 1 &=& 10.\end{array}\right.$$

Example

Solve the polynomial system. $\left|\begin{array}{rcl} x-4y & = & 5 \\ y^2+xy & = & 10 \end{array}\right.$

Final answer: $x=-3, y=-2$ or $x=9, y=1$.

Check answer $x=-3, y=-2$:

$$\left|\begin{array}{rclcl} x-4y & = & (-3)-4(-2) & = & 5 \\ y^2+xy & = & (-2)^2+(-3)(-2) & = & 10 \end{array}\right.$$

Check answer $y=1, x=9$:

$$\left|\begin{array}{rclcl} x-4y & = & 9-4\cdot 1 & = & 5 \\ y^2+xy & = & 1^2+9\cdot 1 & = & 10. \end{array}\right.$$

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

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$$x + y = 25$$

$$x^2 + y^2 = 313$$

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$\begin{aligned} {\color{red}x} + y &= 25 \qquad &|\text{Solve for } y \\ y &= 25 - {\color{red}x} \\ x^2 + y^2 &= 313 \end{aligned}$$

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$\begin{aligned} x + y &= 25 \qquad &&|\text{Solve for } y\\ y &= 25 - x \\ x^2 + y^2 &= 313 \end{aligned}$$

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$\begin{aligned} x + y &= 25 \qquad |\text{Solve for } y\\ y &= 25 - x\\ x^2 + y^2 &= 313\\ x^2 + (25 - x)^2 &= 313 \end{aligned}$$

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$\begin{aligned} x + y &= 25 && \Big| \text{Solve for } y \\ y &= 25 - x \\ x^2 + y^2 &= 313 \\ x^2 + (25 - x)^2 &= 313 \\ x^2 + \left(25^2 - 2 \cdot 25 \cdot x + x^2\right) - 313 &= 0 && \Big| (a-b)^2 = a^2 - 2ab + b^2 \end{aligned}$$

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$\begin{aligned} x + y &= 25 && \big| \text{Solve for } y \\ y &= 25 - x \\ x^2 + y^2 &= 313 \\ x^2 + (25 - x)^2 &= 313 \\ x^2 + \left(25^2 - 2 \cdot 25 \cdot x + x^2\right) - 313 &= 0 && \Big| (a-b)^2 = a^2 - 2ab + b^2 \end{aligned}$$

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The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$\begin{aligned} x + y &= 25 && \Big| \text{Solve for } y \\ y &= 25 - x \\ x^2 + y^2 &= 313 \\ x^2 + (25 - x)^2 &= {\color{red}313} \\ x^2 + \left(25^2 - 2 \cdot 25 \cdot x + x^2\right) - {\color{red}313} &= 0 && \Big| (a-b)^2 = a^2 - 2ab + b^2 \end{aligned}$$

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$\begin{aligned} x + y &= 25 && \Big| \text{Solve for } y \\ y &= 25 - x \\ x^2 + y^2 &= 313 \\ x^2 + (25 - x)^2 &= 313 \\ {\color{red}x^2} + \left(25^2 - 2 \cdot 25 \cdot x + {\color{red}x^2}\right) - 313 &= 0 && \Big| (a-b)^2 = a^2 - 2ab + b^2 \\ {\color{red}2x^2} - 50x + 625 - 313 &= 0 \end{aligned}$$

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$\begin{aligned} x + y &= 25 & &|\text{Solve for } y \\ y &= 25 - x \\ x^2 + y^2 &= 313 \\ x^2 + (25 - x)^2 &= 313 \\ x^2 + \left(25^2 - {\color{red}2 \cdot 25} \cdot x + x^2\right) - 313 &= 0 & &\left|(a-b)^2 = a^2 - 2ab + b^2\right. \\ 2x^2 - {\color{red}50}x + 625 - 313 &= 0 \end{aligned}$$

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The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

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Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$\begin{aligned} x+y &= 25 && \text{|Solve for } y \\ y &= 25-x \\ x^2+y^2 &= 313 \\ x^2+(25-x)^2 &= 313 \\ x^2+\left(25^2-2\cdot 25\cdot x+x^2\right)-313 &= 0 && \Big|(a-b)^2=a^2-2ab+b^2 \\ 2x^2-50x+625-313 &= 0 \\ {\color{red}2}x^2-{\color{red}50}x+{\color{red}312} &= 0 && {\color{red}\text{|Divide by 2}} \\ x^2-{\color{red}25}x+{\color{red}156} &= 0 \end{aligned}$$

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

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x + y &= 25 && \text{|Solve for } y \\
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2x^2 - 50x + 625 - 313 &= 0 \\
2x^2 - 50x + 312 &= 0 && \text{|Divide by 2} \\
x^2 - 25x + 156 &= 0 \\
x &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1}
\end{aligned}
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2x^2 - 50x + 625 - 313 &= 0\\
2x^2 - 50x + 312 &= 0 && |\text{Divide by } 2\\
x^2 - 25x + {\color{red}156} &= 0\\
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2x^2 - 50x + 312 &= 0 & &|\text{Divide by } 2\\
x^2 - 25x + 156 &= 0\\
x &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1}\\
&= \frac{25 \pm \sqrt{625 - 624}}{2}
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2x^2 - 50x + 625 - 313 &= 0 \\
2x^2 - 50x + 312 &= 0 && \text{|Divide by 2} \\
x^2 - 25x + 156 &= 0 \\
x &= \frac{-(-25) \pm \sqrt{25^2 - {\color{red}4} \cdot 1 \cdot {\color{red}156}}}{2 \cdot 1} \\
&= \frac{25 \pm \sqrt{625 - {\color{red}624}}}{2}
\end{aligned}
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2x^2 - 50x + 625 - 313 &= 0\\
2x^2 - 50x + 312 &= 0 & &|\text{Divide by } 2\\
x^2 - 25x + 156 &= 0\\
x &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1}\\
&= \frac{25 \pm \sqrt{625 - 624}}{2}\\
&= \frac{25 \pm 1}{2}
\end{aligned}
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2x^2 - 50x + 625 - 313 &= 0\\
2x^2 - 50x + 312 &= 0 && \text{|Divide by 2}\\
x^2 - 25x + 156 &= 0\\
x &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1}\\
&= \frac{25 \pm \sqrt{625 - 624}}{2}\\
&= \frac{25 \pm 1}{2} = \begin{cases} \frac{25+1}{2} \\ \frac{25-1}{2} \end{cases}
\end{aligned}$$

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2x^2 - 50x + 625 - 313 &= 0\\
2x^2 - 50x + 312 &= 0 && |\text{Divide by } 2\\
x^2 - 25x + 156 &= 0\\
x &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1}\\
&= \frac{25 \pm \sqrt{625 - 624}}{2}\\
&= \frac{25 \pm 1}{2} = \begin{cases} \frac{25+1}{2} = 13 \\ \frac{25-1}{2} \end{cases}
\end{aligned}
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The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

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2x^2 - 50x + 625 - 313 &= 0 \\
2x^2 - 50x + 312 &= 0 && |\text{Divide by } 2 \\
x^2 - 25x + 156 &= 0
\end{aligned}$$

$$\begin{aligned}
x &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1} \\
&= \frac{25 \pm \sqrt{625 - 624}}{2} \\
&= \frac{25 \pm 1}{2} = \begin{cases} \frac{25+1}{2} = 13 \\ \frac{25-1}{2} = 12 \end{cases}
\end{aligned}$$

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

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x + y &= 25 && \text{|Solve for } y\\
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2x^2 - 50x + 312 &= 0 && \text{|Divide by 2}\\
x^2 - 25x + 156 &= 0\\
x &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1}\\
&= \frac{25 \pm \sqrt{625 - 624}}{2}\\
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Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

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x + y &= 25 \qquad |\text{Solve for } y\\
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$$\begin{aligned}
x + y &= 25 \qquad |\text{Solve for } y\\
y &= 25 - x\\
x^2 + y^2 &= 313\\
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&= \frac{25 \pm \sqrt{625 - 624}}{2}\\
&= \frac{25 \pm 1}{2} = \begin{cases} \frac{25+1}{2} = 13\\ \frac{25-1}{2} = 12\end{cases}\\
y &= 25 - x
\end{aligned}$$

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$\begin{aligned}
x + y &= 25 \qquad |\text{Solve for } y\\
y &= 25 - x\\
x^2 + y^2 &= 313\\
x^2 + (25-x)^2 &= 313\\
x &= \frac{-(-25) \pm \sqrt{25^2 - 4\cdot 1 \cdot 156}}{2\cdot 1}\\
&= \frac{25 \pm \sqrt{625 - 624}}{2}\\
&= \frac{25 \pm 1}{2} = \begin{cases} \frac{25+1}{2} = 13\\ \frac{25-1}{2} = 12\end{cases}\\
y = 25 - x &= \begin{cases} 25 - 13\\ 25 - 12\end{cases}
\end{aligned}$$

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

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\begin{aligned}
x + y &= 25 \qquad |\text{Solve for } y\\
y &= 25 - x\\
x^2 + y^2 &= 313\\
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&= \frac{25 \pm 1}{2} = \begin{cases} \frac{25+1}{2} = 13\\ \frac{25-1}{2} = 12 \end{cases}\\
y = 25 - x &= \begin{cases} 25 - 13 = 12\\ 25 - 12 = 13 \end{cases}
\end{aligned}
$$

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

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\begin{aligned}
x + y &= 25 \qquad |\text{Solve for } y\\
y &= 25 - x\\
x^2 + y^2 &= 313\\
x^2 + (25 - x)^2 &= 313\\
{\color{red}x} &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1}\\
&= \frac{25 \pm \sqrt{625 - 624}}{2}\\
&= \frac{25 \pm 1}{2} = \begin{cases} \frac{25+1}{2} = 13\\ \frac{25-1}{2} {\color{red}= 12} \end{cases}\\
{\color{red}y} = 25 - x &= \begin{cases} 25 - 13 = 12\\ 25 - {\color{red}12} = 13 \end{cases}
\end{aligned}
$$

The two solution candidates are ${\color{red}x = 12, y = 13}$ and $x = 13, y = 12$.

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$\begin{aligned}
x + y &= 25 \qquad |\text{Solve for } y\\
y &= 25 - x\\
x^2 + y^2 &= 313\\
x^2 + (25 - x)^2 &= 313\\
{\color{red}x} &= \frac{-(-25) \pm \sqrt{25^2 - 4 \cdot 1 \cdot 156}}{2 \cdot 1}\\
&= \frac{25 \pm \sqrt{625 - 624}}{2}\\
&= \frac{25 \pm 1}{2} = \begin{cases} \frac{25+1}{2} {\color{red}= 13}\\ \frac{25-1}{2} = 12 \end{cases}\\
{\color{red}y} &= 25 - x = \begin{cases} 25 - 13 = 12\\ 25 - 12 {\color{red}= 13} \end{cases}
\end{aligned}$$

The two solution candidates are $x = 12, y = 13$ and ${\color{red}x = 13, y = 12}$.

Example

The sum of two numbers x and y is 25 and the sum of their squares is 313. Given that $y \geq x$, find x and y.

$$\begin{aligned} x + y &= 25 \qquad |\text{Solve for } y\\ y &= 25 - x\\ x^2 + y^2 &= 313\\ x^2 + (25-x)^2 &= 313\\ x &= \frac{-(-25) \pm \sqrt{25^2 - 4\cdot 1\cdot 156}}{2\cdot 1}\\ &= \frac{25 \pm \sqrt{625-624}}{2}\\ &= \frac{25\pm 1}{2} = \begin{cases} \frac{25+1}{2} = 13\\ \frac{25-1}{2} = 12 \end{cases}\\ y = 25 - x &= \begin{cases} 25 - 13 = 12\\ 25-12 = 13\end{cases} \end{aligned}$$

The two solution candidates are $x = 12, y = 13$ and $x = 13, y = 12$. Since $y \geq x$, one of the solutions needs to be discarded and our final answer is $x = 12, y = 13$.