

Precalculus

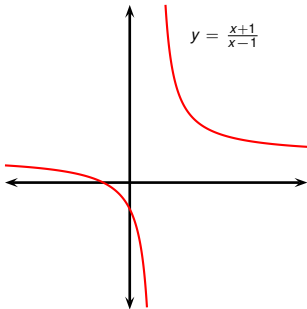
Inverse of fractional linear transformation

Todor Milev

2019

Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.

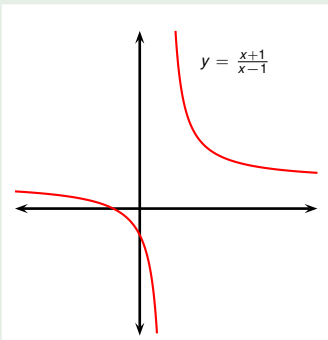


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We deal with domains and ranges later:

$$y = \frac{x+1}{x-1}$$

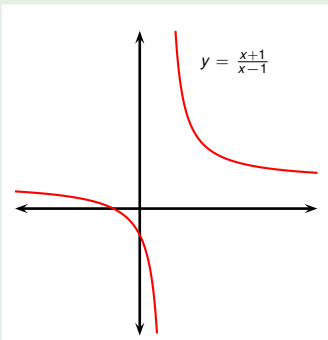


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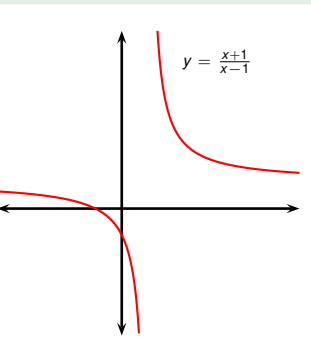
$$\begin{array}{rcl} y & = & \frac{x+1}{x-1} \\ y(x-1) & = & x+1 \end{array} \quad \left| \begin{array}{l} \text{mult. by } (x-1) \end{array} \right.$$



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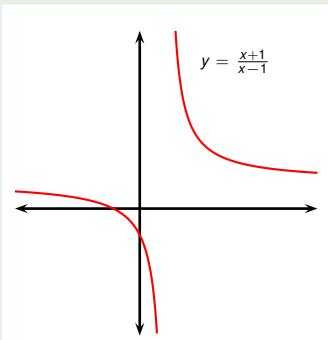


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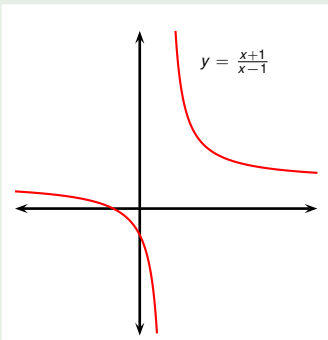


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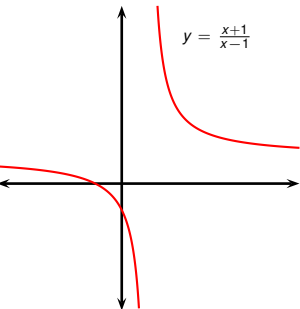


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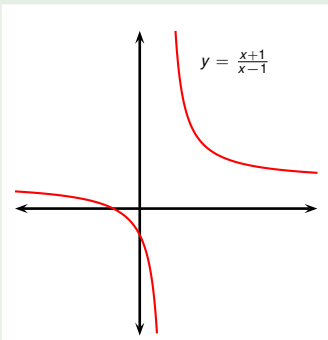


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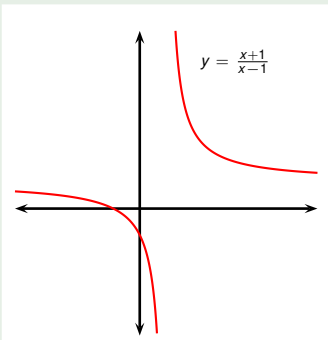


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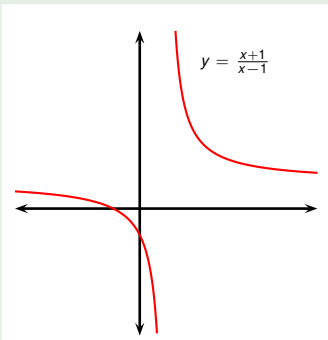


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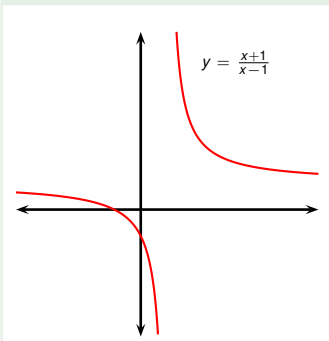


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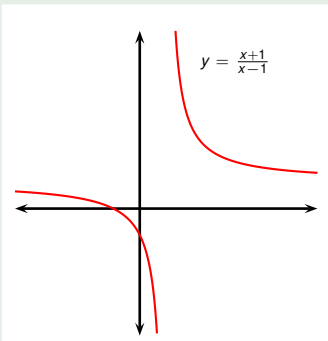
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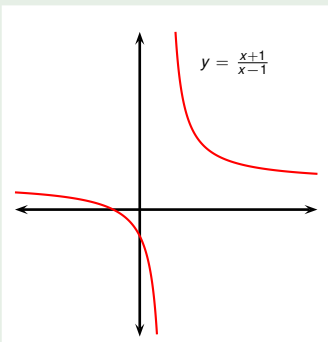
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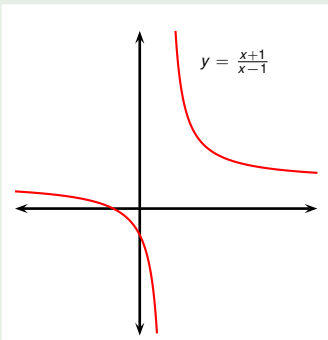
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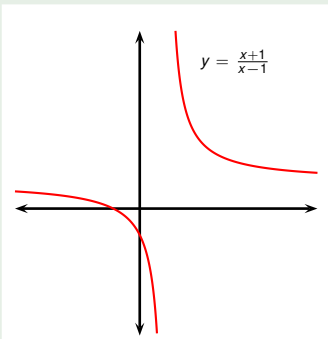
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Can a non-identity function be its own inverse?

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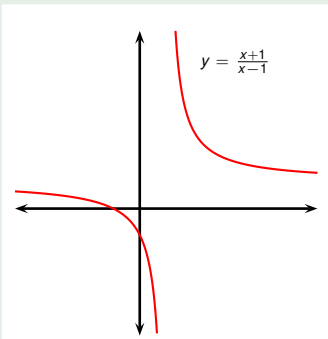
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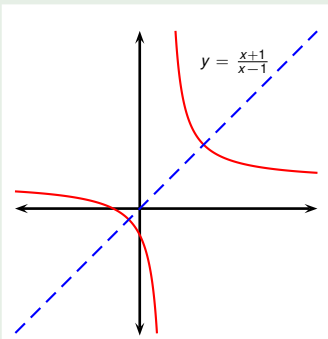
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Graph of f is symmetric across $y = x$.