

Precalculus

Express $\sin(kx)$, $\cos(kx)$ **via** $\sin x$, $\cos x$

Todor Milev

2019

Example

Express $\sin(3x)$ and $\cos(3x)$ via $\cos x$ and $\sin x$.

$$\sin(3x)$$

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$$\sin(3x) = \sin(x + 2x)$$

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Recall the formulas $\sin(\alpha + \beta) = ?$
 $\cos(\alpha + \beta) = ?$

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