

# Precalculus

## Law of cosines theory

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Let  $\triangle ABC$  have sides lengths  $a, b, c$  angles  $\alpha, \beta, \gamma$ , as indicated.

## Proposition (Law of Cosines)

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = c^2 + a^2 - 2ca \cos \beta$$

Proof if  $\gamma < 90^\circ$ .

Drop a perpendicular  $h$  from  $B$  to  $AC$ .

$$|CD| = a \cos \gamma$$

$$h = a \sin \gamma$$

$$|AD| = b - |CD| = b - a \cos \gamma$$

$$c^2 = |AD|^2 + h^2$$

$$= (b - a \cos \gamma)^2 + (a \sin \gamma)^2$$

$$= b^2 - 2ab \cos \gamma + a^2 \cos^2 \gamma + a^2 \sin^2 \gamma$$

$$= b^2 - 2ab \cos \gamma + a^2.$$

Pyth. thm.  
 $\triangle BDA$

