Precalculus
Quadratic polynomials viewed as functions

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2019

Definition

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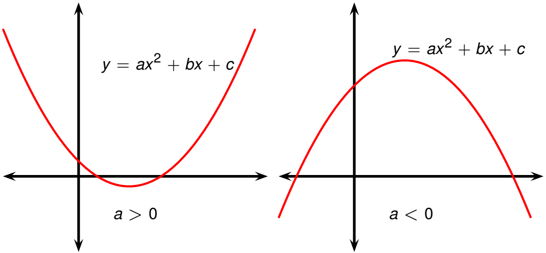
- The graph of a quadratic function is called a parabola.



Example (Completing the square)

Complete the square.

$$3x^2 - 5x + 1$$

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$$3x^2 - 5x + 1 \quad = \quad 3\left(x^2 - \textbf{?}\, x\right) + 1$$

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$$3x^2 - 5x + 1 \quad = \quad 3\left(x^2 - \frac{5}{3}x\right) + 1$$

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$$\begin{aligned}3x^2 - 5x + 1 &= 3\left(x^2 - \frac{5}{3}x\right) + 1\\ &= 3\left(x^2 - {\color{red}2}\cdot\frac{5}{{\color{red}2}\cdot 3}x\right) + 1\end{aligned}$$

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$$\begin{aligned}
3x^2 - 5x + 1 &= 3\left(x^2 - \frac{5}{3}x\right) + 1 \\
&= 3\left(x^2 - 2\cdot\frac{5}{\color{red}{2\cdot 3}}x\right) + 1 \\
&= 3\left(x^2 - 2\cdot\frac{5}{\color{red}{6}}x + \mathbf{?} \qquad - \mathbf{?} \qquad\right) + 1
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Add & subtract $\left(\quad\right)^2$

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&= a\left({\color{red}\left(x+\frac{b}{2a}\right)^2}-\frac{b^2}{4a^2}\right)+c
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use $(A+B)^2 = A^2+2AB+B^2$

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&= a\left(\left(x+{\color{red}\frac{b}{2a}}\right)^2-\frac{b^2}{4a^2}\right)+c
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&= a\left(x + \frac{b}{2a}\right)^2 - \cancel{a}\cdot\frac{b^2}{4a^{\cancel{2}}} + c \\
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Add & subtract $\left(\frac{b}{2a}\right)^2$ use $(A+B)^2 = A^2 + 2AB + B^2$

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$$\begin{array}{rcll} f(x) & = & a\left(x + \dfrac{b}{2a}\right)^2 + c - \dfrac{b^2}{4a} & \Bigg| \text{ complete the square} \\ & = & a\left(x - \left(-\dfrac{b}{2a}\right)\right)^2 - \dfrac{b^2 - 4ac}{4a} & \end{array}$$

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Definition (Discriminant of quadratic function)

The quantity $D = b^2 - 4ac$ is called the *discriminant* of the quadratic function $ax^2 + bx + c$.

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The expression $f(x) = a(x - h)^2 + k$, where $h = -\frac{b}{2a}$ and $k = -\frac{D}{4a} = -\frac{b^2 - 4ac}{4a}$ is called the standard form of $ax^2 + bx + c$.

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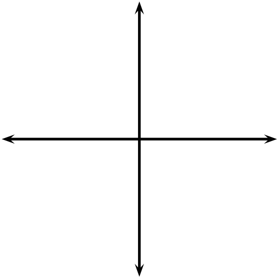

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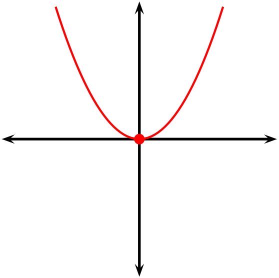

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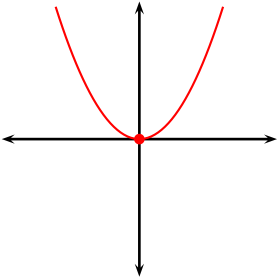

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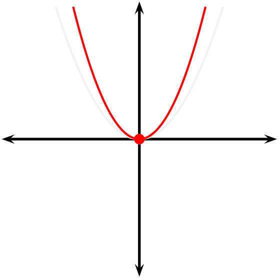

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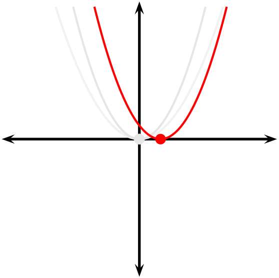

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  - $a(x-h)^2$ shifts $y = ax^2$ by h units right.

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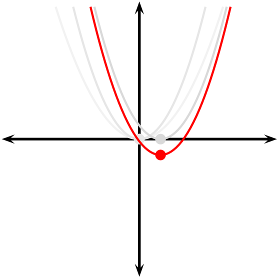

- The graph of $y = x^2$ is a parabola; its shape is assumed known.
- The standard form shows how the graph of an arbitrary quadratic is obtained from the graph of $y = x^2$:
  - ax^2 stretches $y = x^2$ by factor of a and possibly reflects across the x axis.
  - $a(x-h)^2$ shifts $y = ax^2$ by h units right.
  - $a(x-h)^2 + k$ shifts $y = a(x-h)^2 + k$ by k units up.

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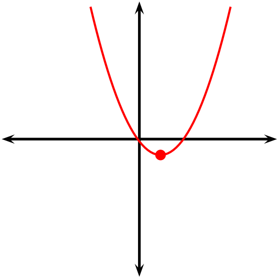

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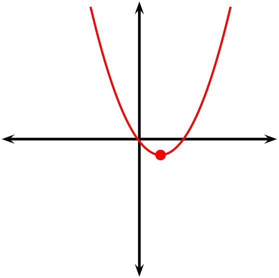

- The graph of a quadratic function is a parabola.
- When $a > 0$ the parabola opens upwards.

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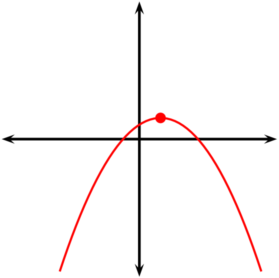

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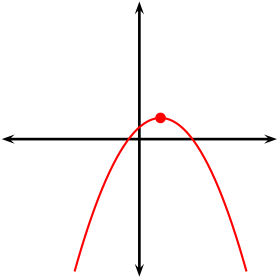

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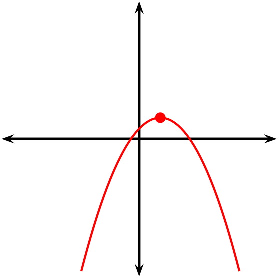

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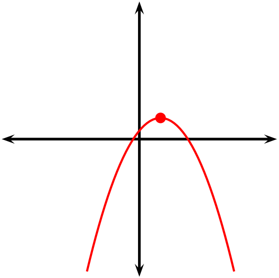

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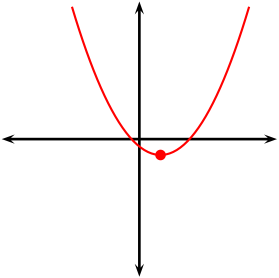

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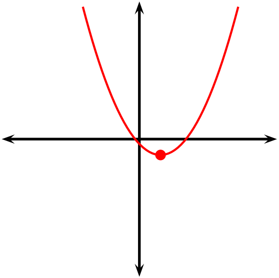

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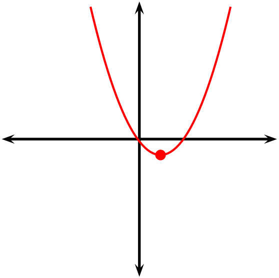

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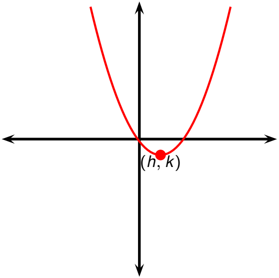

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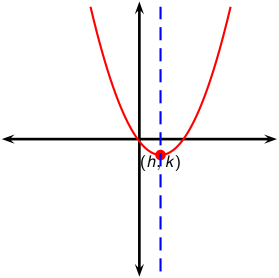

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- The point $(h, k) = \left(-\frac{b}{2a}, -\frac{D}{4a}\right)$ is called the vertex of the parabola.
- The parabola is symmetric with respect to the line $x = h = -\frac{b}{2a}$, i.e., the vertical line through its vertex.

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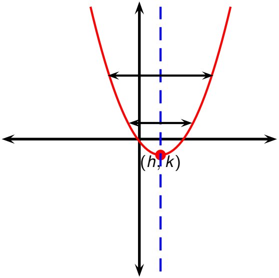

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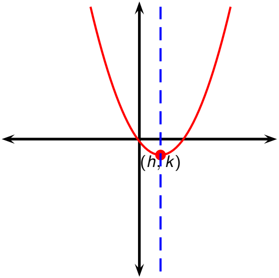

- When we change h and k we move the vertex of the parabola without change in steepness.

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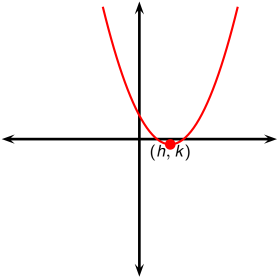

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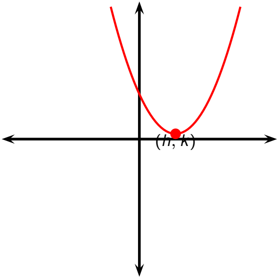

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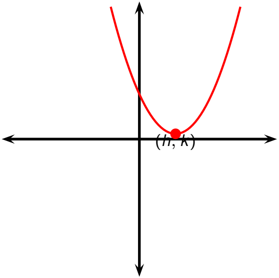
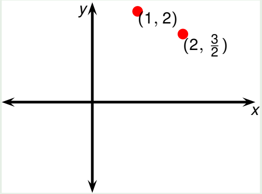


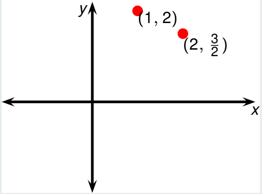
- When we change h and k we move the vertex of the parabola without change in steepness.
- Therefore when we change b and c we move the vertex of the parabola without change in steepness.

Example



Write an equation of a parabola with vertex at $(1, 2)$ that passes through the point $\left(2, \frac{3}{2}\right)$.

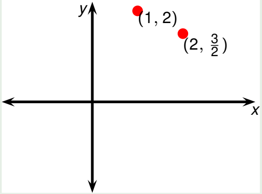
Example



Write an equation of a parabola with vertex at $(1, 2)$ that passes through the point $\left(2, \frac{3}{2}\right)$.

$a(x-h)^2+k \quad = \quad y$	Standard form

Example



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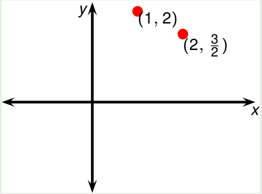
$$a(x - h)^2 + k = y$$

Standard form

$$a(x - ?)^2 + ? = y$$

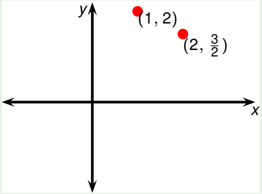
Example



Write an equation of a parabola with vertex at $(1, 2)$ that passes through the point $\left(2, \frac{3}{2}\right)$.

$a(x - h)^2 + k = y$	Standard form
$a(x - 1)^2 + 2 = y$	Vertex at $(1, 2)$

Example



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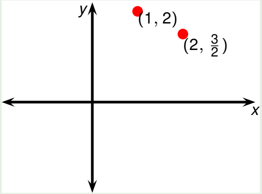
Vertex at $(1, 2)$

Example



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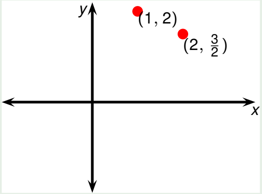

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$$
\begin{array}{rcl|l}
a(x-h)^2 + k & = & y & \text{Standard form} \\
a(x-1)^2 + 2 & = & y & \text{Vertex at } (1, 2) \\
a(2-1)^2 + 2 & = & \frac{3}{2} & \text{Passes through } \left(2, \frac{2}{3}\right)
\end{array}
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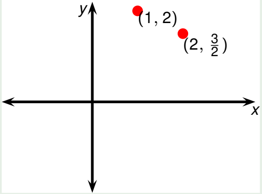
Example



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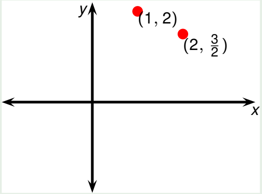
Example



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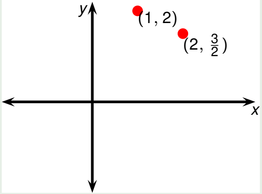


Write an equation of a parabola with vertex at $(1,2)$ that passes through the point $\left(2, \frac{3}{2}\right)$.

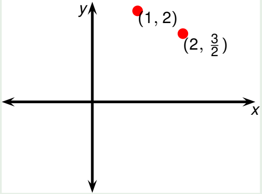
$$\begin{aligned} a(x-h)^2+k &= y \\ a(x-1)^2+2 &= y \\ a(2-1)^2+2 &= \frac{3}{2} \\ a &= \frac{3}{2}-2 \end{aligned}$$

Standard form

Vertex at $(1,2)$

Passes through $\left(2, \frac{2}{3}\right)$

Example



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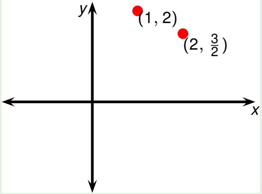
\begin{aligned}
a(x-h)^2+k &= y && \text{Standard form}\\
a(x-1)^2+2 &= y && \text{Vertex at } (1,2)\\
a(2-1)^2+{\color{red}2} &= \frac{3}{2} && \text{Passes through } \left(2, \tfrac{2}{3}\right)\\
a &= \frac{3}{2}{\color{red}-2}
\end{aligned}
$$

Example



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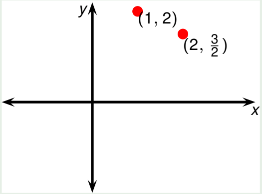
Standard form

Vertex at $(1,2)$

Passes through $\left(2, \frac{2}{3}\right)$

Example



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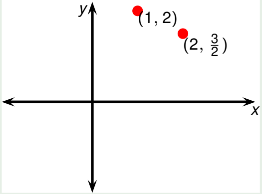
a & = & \frac{3}{2} - 2 = -\frac{1}{2} & \\
y & = & -\frac{1}{2}(x-1)^2+2 & \text{Final answer}
\end{array}
$$

Example



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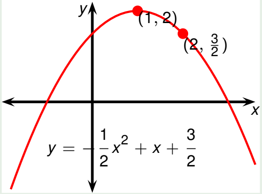

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$a(x-h)^2 + k = y$	Standard form
$a(x-1)^2 + 2 = y$	Vertex at $(1, 2)$
$a(2-1)^2 + 2 = \frac{3}{2}$	Passes through $\left(2, \frac{2}{3}\right)$
$a = \frac{3}{2} - 2 = -\frac{1}{2}$	
$y = -\frac{1}{2}(x-1)^2 + 2$	Final answer
$y = -\frac{1}{2}x^2 + x + \frac{3}{2}$	Alternative answer

Problem (Quadratic equation formula)

Solve the general quadratic equation

$$ax^2 + bx + c = 0$$

Problem (Quadratic equation formula)

Solve the general quadratic equation

$$\begin{aligned} ax^2+bx+c &= 0 \quad \Big| \textit{ complete the square} \\ a\left(x+\frac{b}{2a}\right)^2-\frac{D}{4a} &= 0 \end{aligned}$$

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Solve the general quadratic equation

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Problem (Quadratic equation formula)

Solve the general quadratic equation

$$\begin{array}{rcll} ax^2+bx+c &=& 0 & \Big| \ \textit{complete the square} \\ {\color{red}a}\left(x+\dfrac{b}{2a}\right)^2-\dfrac{D}{4{\color{red}a}} &=& 0 & \Big| \ \textit{where } D=b^2-4ac \\ {\color{red}a}{\color{red}\left(\color{black}\left(x+\dfrac{b}{2a}\right)^2-\dfrac{D}{4{\color{red}a^2}}\color{red}\right)} &=& 0 & \end{array}$$

Problem (Quadratic equation formula)

Solve the general quadratic equation

$$\begin{aligned} ax^2+bx+c &= 0 && \text{complete the square} \\ a\left(x+\frac{b}{2a}\right)^2-\frac{D}{4a} &= 0 && \text{where } D=b^2-4ac \\ a\left(\left(x+\frac{b}{2a}\right)^2-\frac{D}{4a^2}\right) &= 0 \\ a\left(\left(x+\frac{b}{2a}\right)^2-\left(\frac{\sqrt{D}}{2a}\right)^2\right) &= 0 \end{aligned}$$

Problem (Quadratic equation formula)

Solve the general quadratic equation

$$\begin{aligned} ax^2+bx+c &= 0 && \text{complete the square} \\ a\left(x+\frac{b}{2a}\right)^2-\frac{D}{4a} &= 0 && \text{where } D=b^2-4ac \\ a\left(\left(x+\frac{b}{2a}\right)^2-\frac{D}{4a^2}\right) &= 0 \\ a\left(\left(x+\frac{b}{2a}\right)^2-\left(\frac{\sqrt{D}}{2a}\right)^2\right) &= 0 \end{aligned}$$

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Solve the general quadratic equation

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a\left(\left(x+\frac{b}{2a}\right)^2-\frac{D}{4a^2}\right) &= 0\\
a\left(\left(\color{red}{x+\frac{b}{2a}}\right)^2-\left(\frac{\sqrt{D}}{2a}\right)^2\right) &= 0\\
a\left(\color{red}{x+\frac{b}{2a}}-\frac{\sqrt{D}}{2a}\right)\left(\color{red}{x+\frac{b}{2a}}+\frac{\sqrt{D}}{2a}\right) &= 0 && \text{use } \color{red}{A}^2-B^2 = (\color{red}{A}-B)(\color{red}{A}+B)
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x+\frac{b}{2a}-\frac{\sqrt{D}}{2a} = 0 \qquad \text{or} \qquad x+\frac{b}{2a}+\frac{\sqrt{D}}{2a} = 0
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a\left(x+\frac{b}{2a}-\frac{\sqrt{D}}{2a}\right)\left({\color{red}x+\frac{b}{2a}+\frac{\sqrt{D}}{2a}}\right) &{\color{red}=}\ {\color{red}0} && \text{use } A^2-B^2 = (A-B)(A+B)
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Solve the general quadratic equation

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Theorem

The solutions of the quadratic equation

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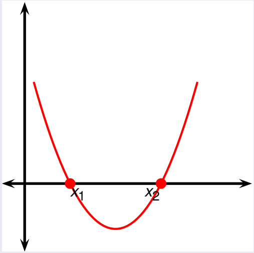

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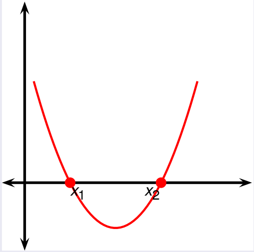

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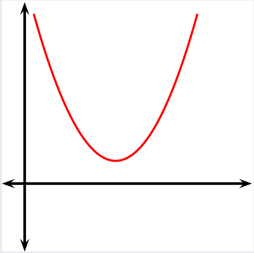

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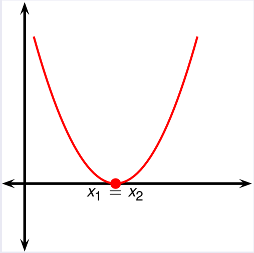
- If $D < 0$ then $\sqrt{D}$ is not a real $\Rightarrow$ quadratic has no real solutions.

Theorem



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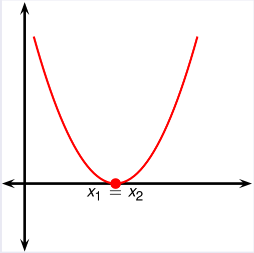

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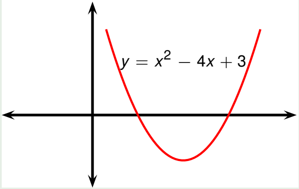
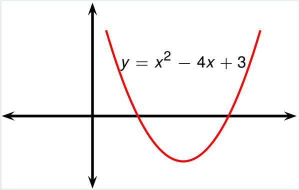
- If $D < 0$ then $\sqrt{D}$ is not a real $\Rightarrow$ quadratic has no real solutions.
- If $D = 0$ then $x_1 = x_2$, the equation has only one zero (with multiplicity two). The zero is located at the vertex of the parabola.

Example



Find the x-intercepts of $x^2 - 4x + 3$.

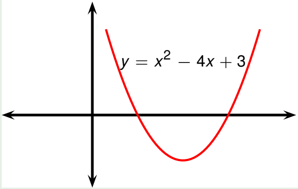
Example



Find the x-intercepts of $x^2 - 4x + 3$.

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

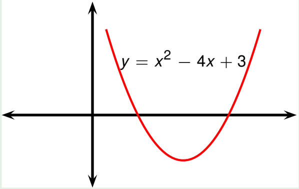
Example



Find the x-intercepts of $x^2 - 4x + 3$.

$$\begin{aligned} x_1, x_2 &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} \end{aligned}$$

Example



Find the x-intercepts of $x^2 - 4x + 3$.

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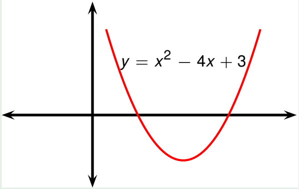

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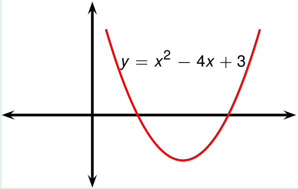

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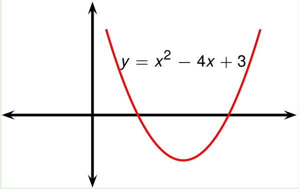
Example



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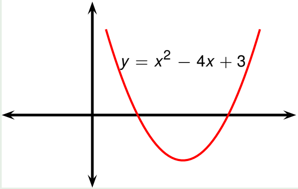

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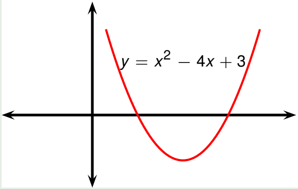

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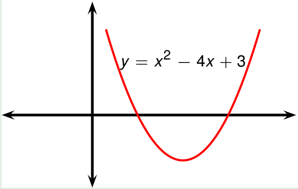
&= \begin{cases} \dfrac{4 + 2}{2} \\ \dfrac{4 - 2}{2} \end{cases}
\end{aligned}
$$

Example



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&= \frac{4 \pm 2}{2} \\
&= \begin{cases} \frac{4+2}{2} = \frac{6}{2} \\ \frac{4-2}{2} \end{cases}
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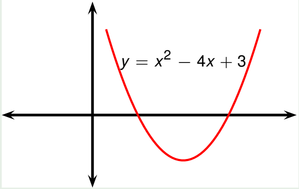
Example



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&= \begin{cases} \frac{4+2}{2} = \frac{6}{2} = 3 \\ \frac{4-2}{2} \end{cases}
\end{aligned}
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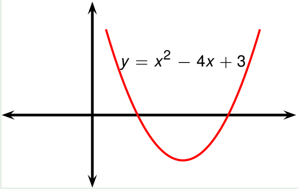
Example



Find the x-intercepts of $x^2 - 4x + 3$.

$$\begin{aligned}
x_1, x_2 &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} \\
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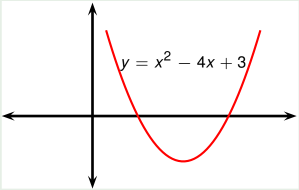

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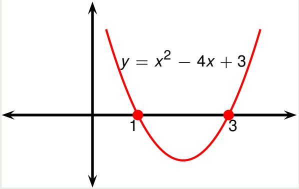

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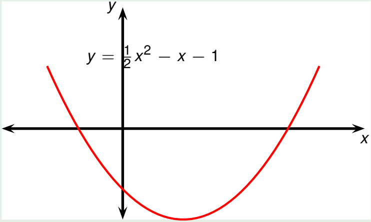
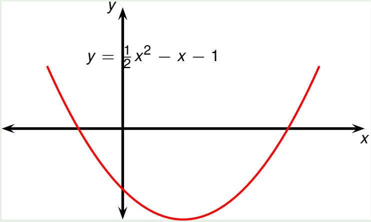


Find the x-intercepts of $\frac{x^2}{2} - x - 1$.

Example



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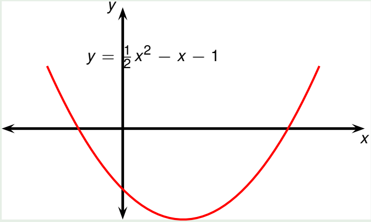

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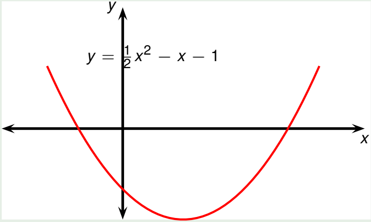

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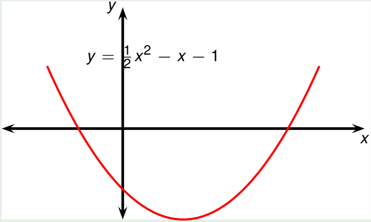

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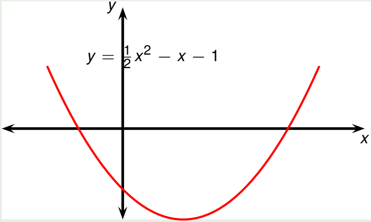

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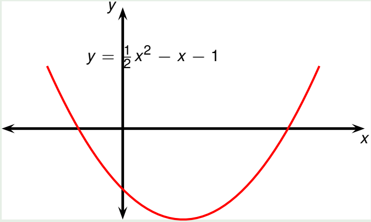

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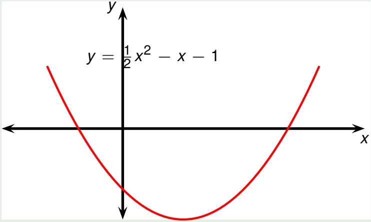

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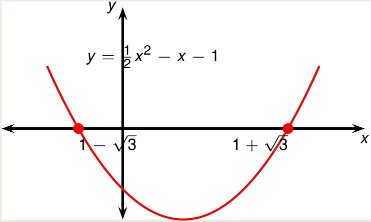

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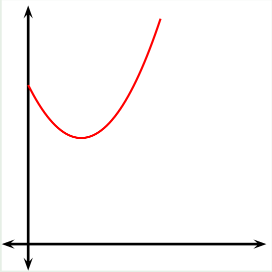

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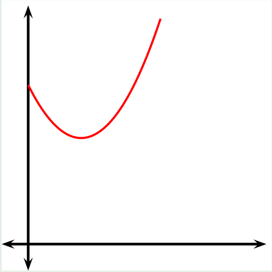

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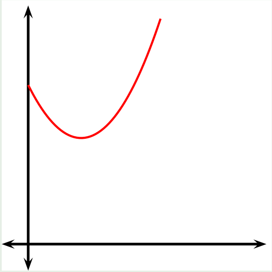

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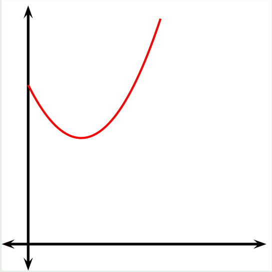

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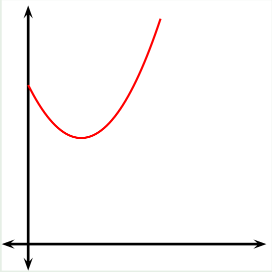

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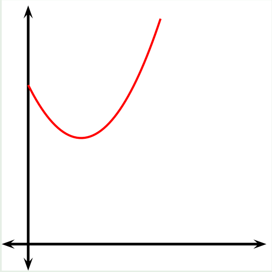

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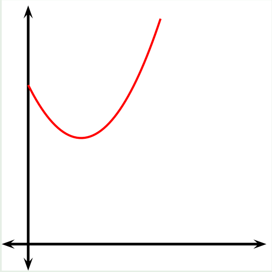

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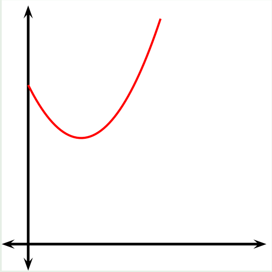

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no real solutions

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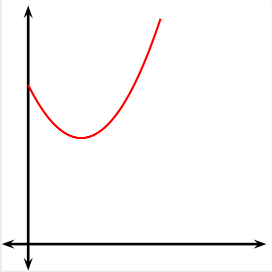

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Proposition

Let $ax^2 + bx + c$, $a \neq 0$ be a quadratic with discriminant $D = b^2 - 4ac$ and roots x_1 and x_2. Then $D = a^2 (x_1 - x_2)^2$.

Proof.

$\square$

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Proof.

$$a^2(x_1 - x_2)^2 = a^2 \left(\frac{-b + \sqrt{D}}{2a} - \frac{-b - \sqrt{D}}{2a} \right)$$

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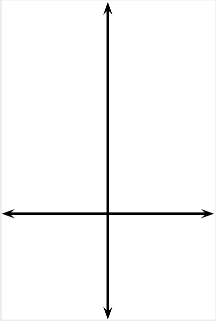
\end{aligned}$$

□

- Discriminant is zero $\Leftrightarrow$ the quadratic has non-distinct roots, hence the discriminant discriminates between the two roots.

Example

Find the values of the parameter k for which the equation $3x^2 - kx + 1$ has two real distinct roots.



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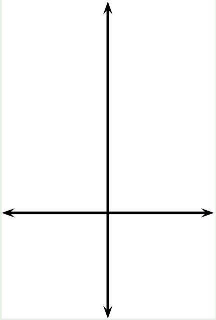
- Quadratic roots: $x_1, x_2 =$ **?** .



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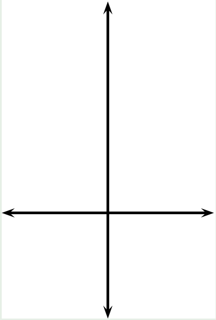

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- Quadratic roots: $x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- $\Rightarrow$ The roots x_1, x_2 are real and distinct when

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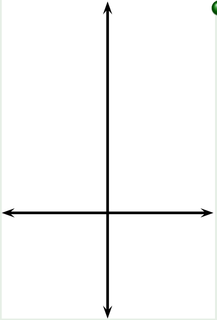

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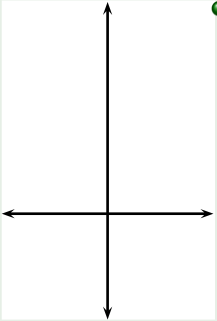

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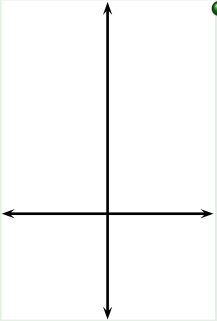

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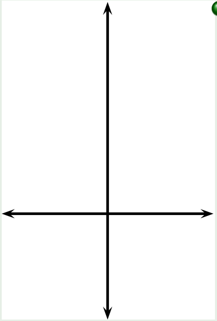

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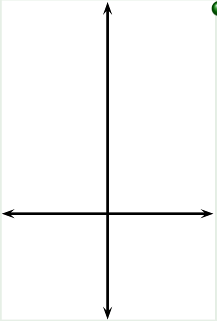

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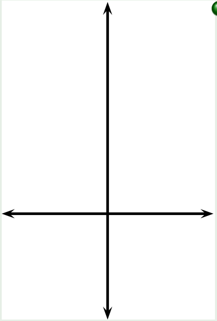

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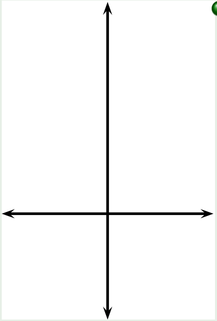

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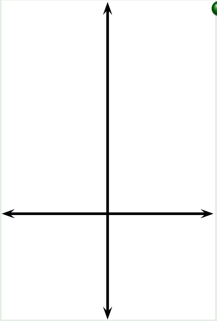

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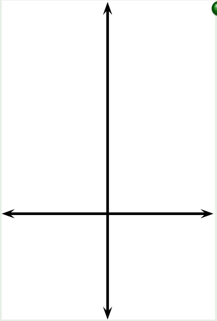

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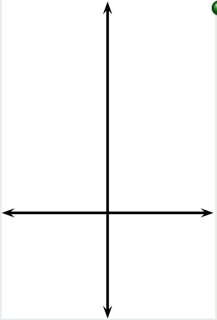

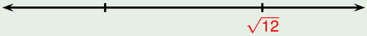

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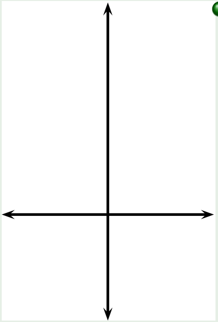

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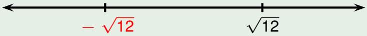

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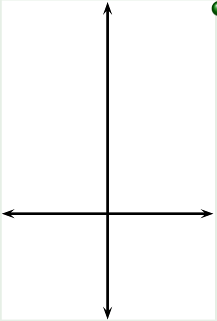

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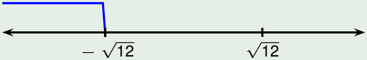

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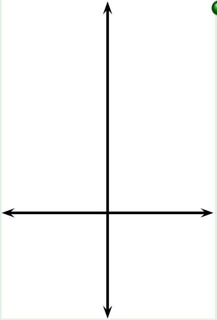

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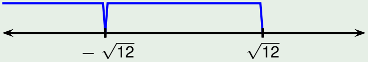

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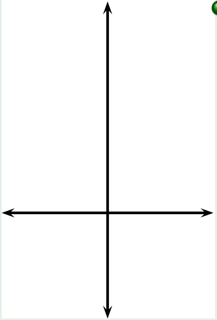

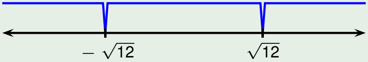

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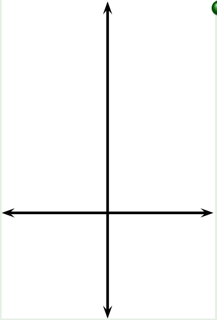

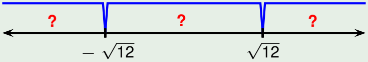

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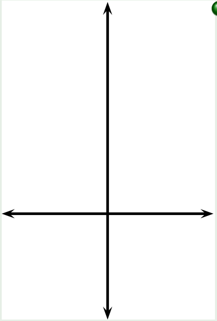

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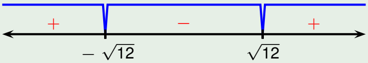

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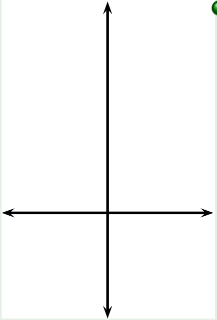

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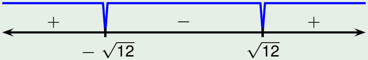

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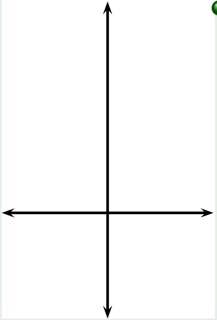

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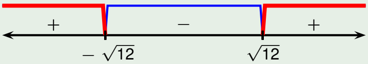

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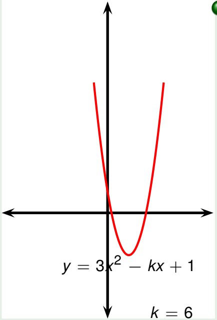

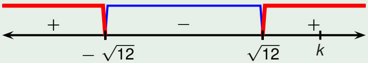

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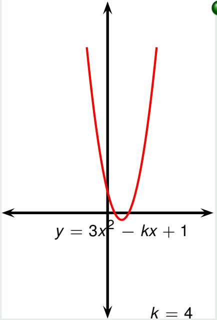

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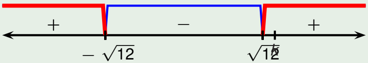

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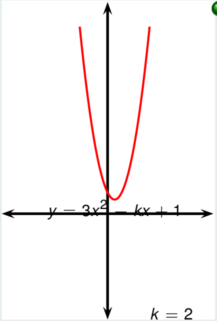

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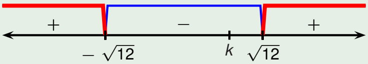

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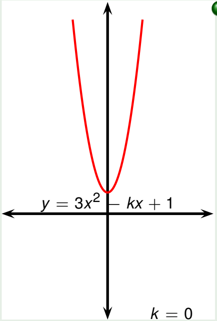

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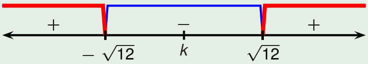

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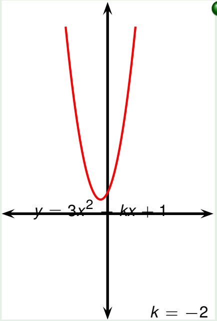

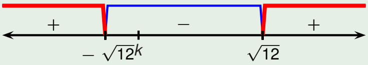

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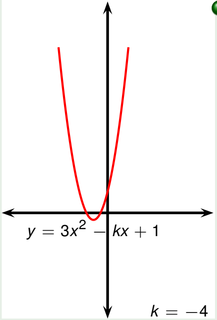

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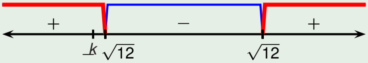

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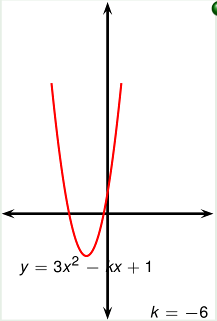

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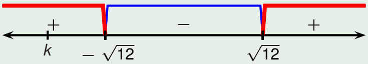

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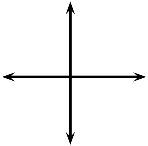

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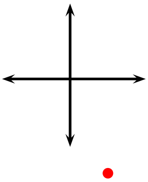

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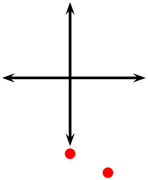

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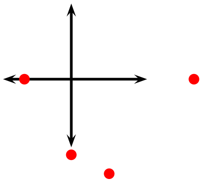
- Find the vertex of the parabola.
- Find the y intercept.
- Find the x intercept(s) if any.



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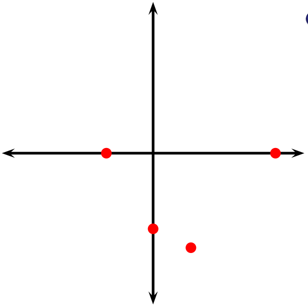

- Find the vertex of the parabola.
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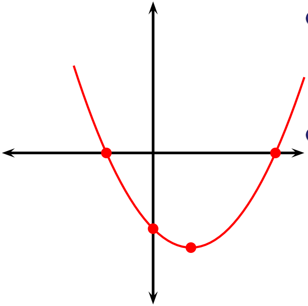

- Find the vertex of the parabola.
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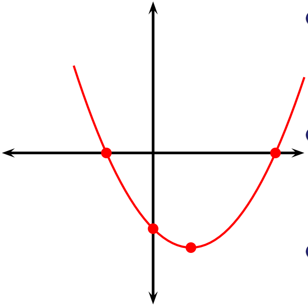

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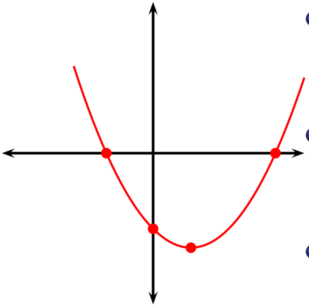

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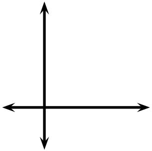
- If $a > 0$ your parabola should open upwards, if $a < 0$ your parabola should open downwards.
- For $|a| > 1$ we should aim to draw the graph steeper than $a = x^2$, for $|a| < 1$ we should aim to draw the graph flatter than $a = x^2$.

Example



Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

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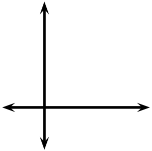

Plot roughly by hand the graph of
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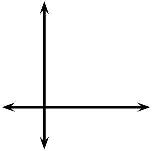
- The vertex of the parabola is given by:

$$x = ?$$

$$y = ?$$

Example



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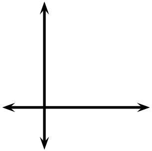
- The vertex of the parabola is given by:

$$x = -\frac{b}{2a} = -\frac{7}{2\left(-\frac{2}{3}\right)}$$

$$y = \textbf{?}$$

Example



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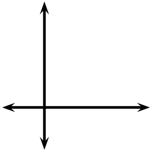

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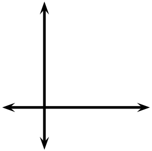

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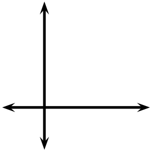

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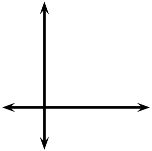

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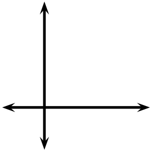

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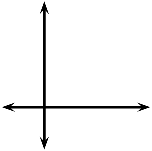

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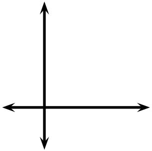

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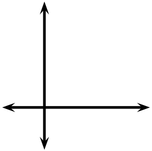

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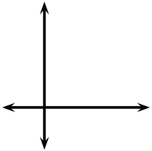

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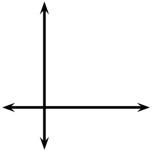

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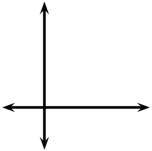

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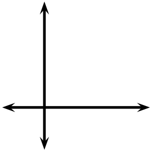

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&= -\frac{7^2 - {\color{red}4}\left(-\frac{{\color{red}2}}{\cancel{3}}\right)\cancel{3}}{4\left(-\frac{2}{3}\right)} = \frac{49+{\color{red}8}}{\frac{8}{3}}
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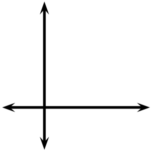
Example



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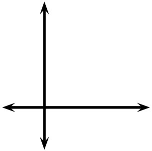

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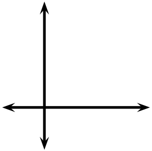

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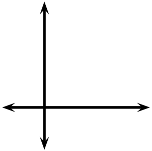

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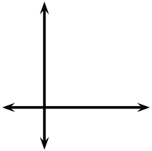
Example



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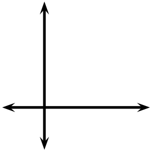

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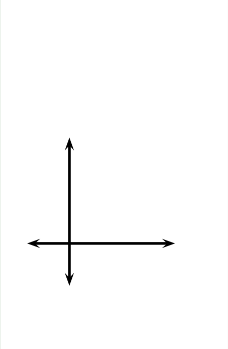
Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$

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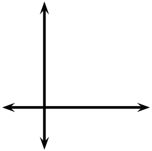

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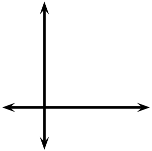
- The y-intercept is $f(0) = \textbf{?}$.

Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$

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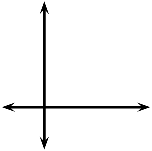

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- The x intercepts are given by the solutions of
$$-\frac{2}{3}x^2 + 7x + 3 = 0$$

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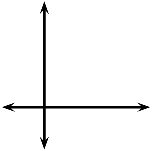


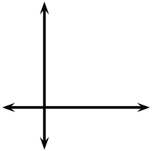
Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The x intercepts are given by the solutions of

$$\begin{aligned} -\frac{2}{3}x^2 + 7x + 3 &= 0 \quad \Big| \cdot 3 \\ -2x^2 + 21x + 9 &= 0 \end{aligned}$$

Example



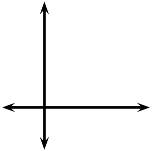
Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The x intercepts are given by the solutions of

$$
\begin{aligned}
-\frac{2}{3}x^2 + 7x + 3 &= 0 && \Big|\ \cdot 3 \\
-2x^2 + 21x + 9 &= 0 \\
x &= \textbf{?}
\end{aligned}
$$

Example



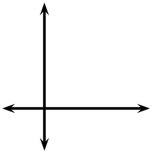
Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The x intercepts are given by the solutions of

$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$
$$-2x^2 + 21x + 9 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example



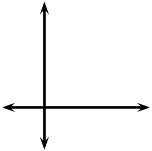
Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- The x intercepts are given by the solutions of

$$\begin{aligned} -\frac{2}{3}x^2 + 7x + 3 &= 0 \qquad \Big| \cdot 3 \\ -2x^2 + 21x + 9 &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)} \end{aligned}$$

Example



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y-intercept at $y = 3$

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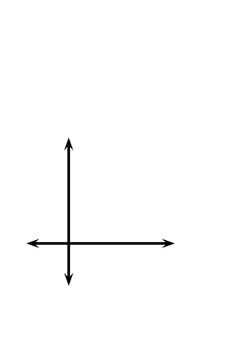
$$\begin{aligned} -\frac{2}{3}x^2 + 7x + 3 &= 0 \quad \Big| \cdot 3 \\ -2x^2 + 21x + 9 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-21 \pm \sqrt{21^2 - 4 \cdot (-2) \cdot 9}}{2 \cdot (-2)} \end{aligned}$$

Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$

Plot roughly by hand the graph of
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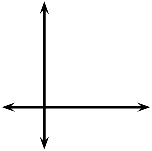
$$-\frac{2}{3}x^2 + 7x + 3 = 0 \quad \Big| \cdot 3$$
$$-2x^2 + 21x + 9 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)}$$

Example



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Plot roughly by hand the graph of
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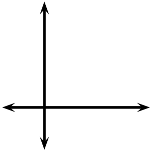
- The x intercepts are given by the solutions of

$$
\begin{aligned}
-\frac{2}{3}x^2 + 7x + 3 &= 0 \qquad \Big| \cdot 3\\
-2x^2 + 21x + 9 &= 0\\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\
&= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)}\\
&= \frac{-21 \pm \sqrt{441 + 72}}{-4}
\end{aligned}
$$

Example



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$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$
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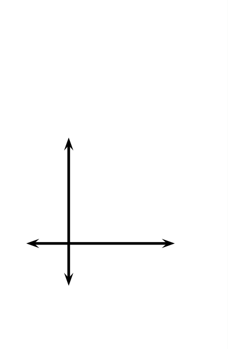
Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
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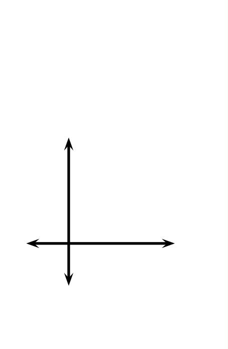
$$\begin{aligned} -\frac{2}{3}x^2 + 7x + 3 &= 0 & \Big| \cdot 3 \\ -2x^2 + 21x + 9 &= 0 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-21 \pm \sqrt{21^2 - 4 \cdot (-2) \cdot 9}}{2 \cdot (-2)} \\ &= \frac{-21 \pm \sqrt{441 + 72}}{-4} \end{aligned}$$

Example



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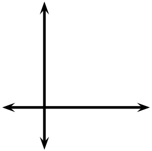
- The x intercepts are given by the solutions of

$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$
$$-2x^2 + 21x + 9 = 0$$
$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)} \\ &= \frac{-21 \pm \sqrt{441 + 72}}{-4} \\ &= \frac{21 \mp \sqrt{513}}{4} \end{aligned}$$

Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

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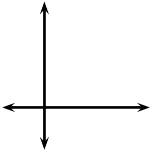
$$-2x^2 + 21x + 9 = 0$$
$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)} \\ &= \frac{-21 \pm \sqrt{441 + 72}}{-4} \\ &= \frac{21 \mp \sqrt{513}}{4} \end{aligned}$$

Example



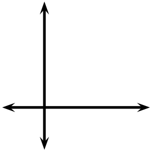
Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3.$

- The x intercepts are given by the solutions of

$$
\begin{aligned}
-\frac{2}{3}x^2 + 7x + 3 &= 0 \qquad \Big| \cdot 3 \\
-2x^2 + 21x + 9 &= 0 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)} \\
&= \frac{-21 \pm \sqrt{441 + 72}}{-4} \\
&= \frac{21 \mp \sqrt{513}}{4} \\
&= \frac{21 \mp \sqrt{9\cdot 57}}{4}
\end{aligned}
$$

Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$

Plot roughly by hand the graph of
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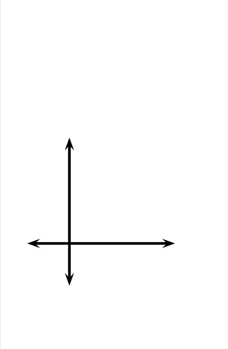
\begin{aligned}
-\frac{2}{3}x^2 + 7x + 3 &= 0 \qquad \Big| \cdot 3\\
-2x^2 + 21x + 9 &= 0
\end{aligned}
$$

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\
&= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)}\\
&= \frac{-21 \pm \sqrt{441 + 72}}{-4}\\
&= \frac{21 \mp \sqrt{513}}{4}\\
&= \frac{21 \mp \sqrt{9\cdot 57}}{4}\\
&= \frac{21 \mp \sqrt{9}\sqrt{57}}{4}
\end{aligned}
$$

Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

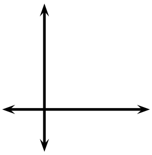
- The x intercepts are given by the solutions of

$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$

$$-2x^2 + 21x + 9 = 0$$

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-21 \pm \sqrt{21^2 - 4 \cdot (-2) \cdot 9}}{2 \cdot (-2)} \\
&= \frac{-21 \pm \sqrt{441 + 72}}{-4} \\
&= \frac{21 \mp \sqrt{513}}{4} \\
&= \frac{21 \mp \sqrt{9 \cdot 57}}{4} \\
&= \frac{21 \mp \sqrt{9}\sqrt{57}}{4} \\
&= \frac{21 \mp 3\sqrt{57}}{4}
\end{aligned}$$

Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$
x-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

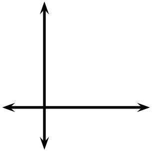
- The x intercepts are given by the solutions of

$$-\frac{2}{3}x^2 + 7x + 3 = 0 \qquad \Big| \cdot 3$$

$$-2x^2 + 21x + 9 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-21 \pm \sqrt{21^2 - 4\cdot(-2)\cdot 9}}{2\cdot(-2)} \\ &= \frac{-21 \pm \sqrt{441 + 72}}{-4} \\ &= \frac{21 \mp \sqrt{513}}{4} \\ &= \frac{21 \mp \sqrt{9\cdot 57}}{4} \\ &= \frac{21 \mp \sqrt{9}\sqrt{57}}{4} \\ &= \frac{21 \mp 3\sqrt{57}}{4} \end{aligned}$$

Example

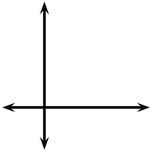


Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$
x-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4}$

Example

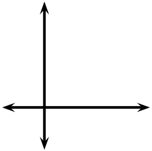


Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$
x-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.

Example

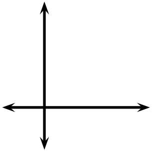


Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$
x-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.
  - $\frac{171}{8}$ is between the integers **?**

Example

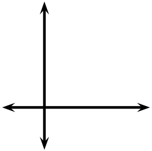


Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$
x-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.
  - $\frac{171}{8}$ is between the integers 21 and 22.

Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$
x-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
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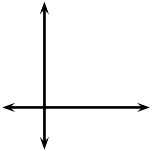
- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.
  - $\frac{171}{8}$ is between the integers 21 and 22.
  - $\frac{21+3\sqrt{57}}{4}$ is close to $\frac{21+3\sqrt{64}}{4}$

Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$
x-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
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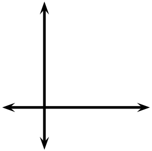
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.
  - $\frac{171}{8}$ is between the integers 21 and 22.
  - $\frac{21+3\sqrt{57}}{4}$ is close to $\frac{21+3\sqrt{64}}{4} = \frac{21+24}{4}$

Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$
x-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of
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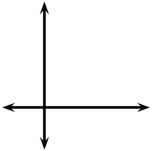
- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.
  - $\frac{171}{8}$ is between the integers 21 and 22.
  - $\frac{21+3\sqrt{57}}{4}$ is close to $\frac{21+3\sqrt{64}}{4} = \frac{21+24}{4} = \frac{45}{4}$

Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
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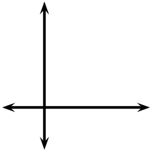
Plot roughly by hand the graph of
$f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.
  - $\frac{171}{8}$ is between the integers 21 and 22.
  - $\frac{21+3\sqrt{57}}{4}$ is close to $\frac{21+3\sqrt{64}}{4} = \frac{21+24}{4} = \frac{45}{4}$ which is close to $\frac{44}{4} = 11$.

Example

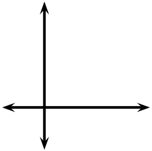


Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$
x-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of $f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.
  - $\frac{171}{8}$ is between the integers 21 and 22.
  - $\frac{21+3\sqrt{57}}{4}$ is close to $\frac{21+3\sqrt{64}}{4} = \frac{21+24}{4} = \frac{45}{4}$ which is close to $\frac{44}{4} = 11$.
  - $\frac{21-3\sqrt{57}}{4}$ is close to $\frac{21-3\sqrt{64}}{4}$

Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
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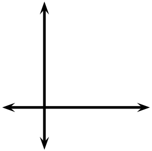
x-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
$x = \frac{21+3\sqrt{57}}{4}$.

Plot roughly by hand the graph of $f(x) = -\frac{2}{3}x^2 + 7x + 3$.

- Select scale to fit the picture:
  - $\frac{21}{4}$ is close to $\frac{20}{4} = 5$.
  - $\frac{171}{8}$ is between the integers 21 and 22.
  - $\frac{21+3\sqrt{57}}{4}$ is close to $\frac{21+3\sqrt{64}}{4} = \frac{21+24}{4} = \frac{45}{4}$ which is close to $\frac{44}{4} = 11$.
  - $\frac{21-3\sqrt{57}}{4}$ is close to $\frac{21-3\sqrt{64}}{4} = \frac{21-24}{4}$

Example



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y-intercept at $y = 3$
x-intercepts at
$x = \frac{21-3\sqrt{57}}{4}$,
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Plot roughly by hand the graph of $f(x) = -\frac{2}{3}x^2 + 7x + 3$.

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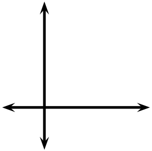
Example



Vertex at: $\left(\frac{21}{4}, \frac{171}{8}\right)$
y-intercept at $y = 3$
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$x = \frac{21-3\sqrt{57}}{4}$,
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Plot roughly by hand the graph of
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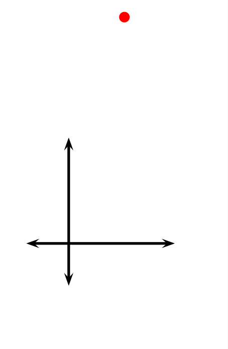

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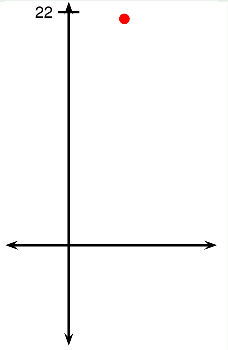

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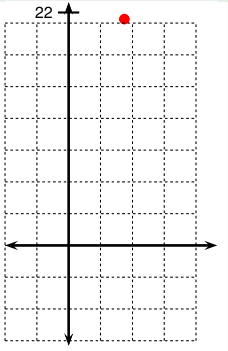

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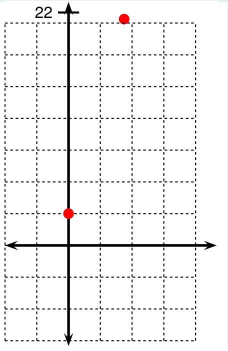

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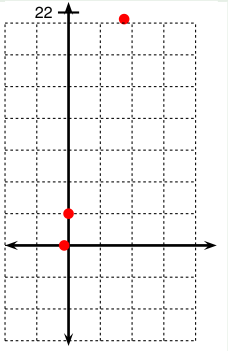

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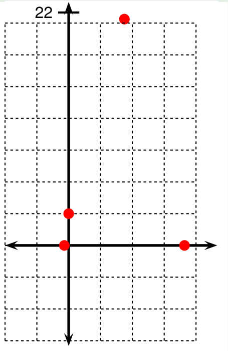

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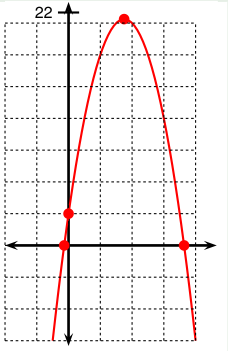

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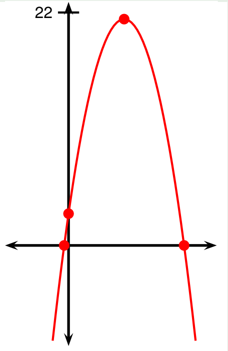

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Example

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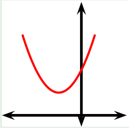
Example



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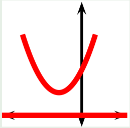
- Leading coefficient is positive $\Rightarrow$ graph opens up

Example



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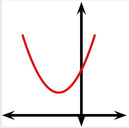
- Leading coefficient is positive $\Rightarrow$ graph opens up $\Rightarrow$ is above x axis if it does not intersect it

Example



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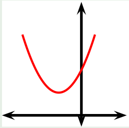

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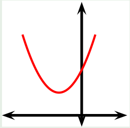
- The roots of a quadratic are $x_1, x_2 =$ **?**

Example



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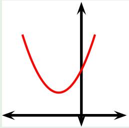

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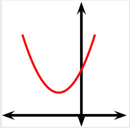
$$b^2 - 4ac < 0$$

Example



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$$\begin{aligned} b^2 - 4ac &< 0 \\ (k+1)^2 - 4 \cdot 1 \cdot 2k &< 0 \end{aligned}$$

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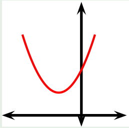

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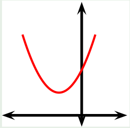

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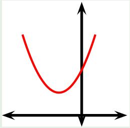

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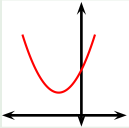

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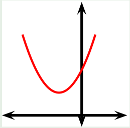

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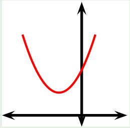

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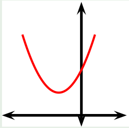

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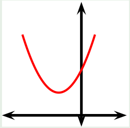

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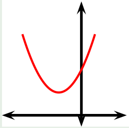

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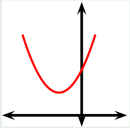

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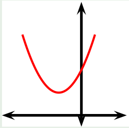

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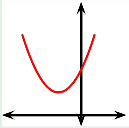

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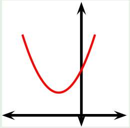

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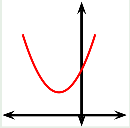

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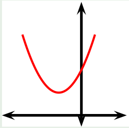

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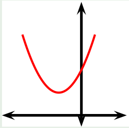

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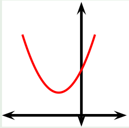

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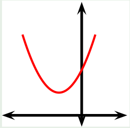

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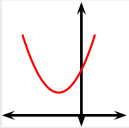

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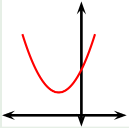

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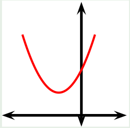

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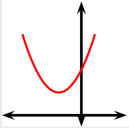

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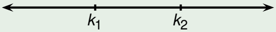

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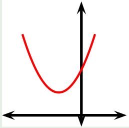

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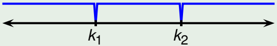

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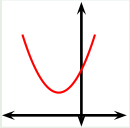

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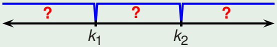

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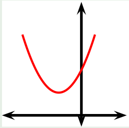

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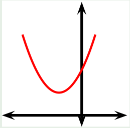

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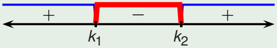

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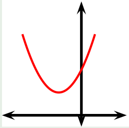

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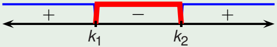
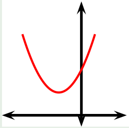


Example



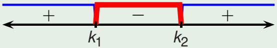
Find the values of the parameter k for which $x^2 + (k+1)x + 2k > 0$ holds for all real x.

- In order for the quadratic to be positive, its graph must lie entirely above the x axis.

- Leading coefficient is positive $\Rightarrow$ graph opens up $\Rightarrow$ is above x axis if it does not intersect it $\Rightarrow$ the quadratic has no real solutions.
- The roots of a quadratic are $x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$
\begin{aligned}
b^2 - 4ac &< 0\\
(k+1)^2 - 4 \cdot 1 \cdot 2k &< 0\\
(k-k_1)(k-k_2) &\color{red}{<} 0
\end{aligned}
$$

$$
\begin{aligned}
k_1, k_2 &= \frac{2 \cdot 3 \pm \sqrt{4}\sqrt{8}}{2} = \frac{\cancel{2}\left(3 \pm \sqrt{8}\right)}{\cancel{2}} = 3 \pm \sqrt{8}\\
k &\in \color{red}{(}k_1, k_2\color{red}{)} = \color{red}{\left(}3 - \sqrt{8}, 3 + \sqrt{8}\color{red}{\right)}
\end{aligned}
$$



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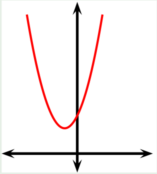
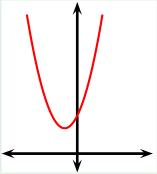


Find the minimum point on the curve $y = 3x^2 + 2x + 1$ by completing the square.

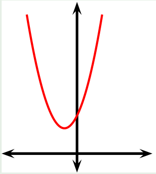
Example



Find the minimum point on the curve
$y = 3x^2 + 2x + 1$ by completing the square.

$$3x^2 + 2x + 1 \quad = \quad 3\left(x^2 + 2 \cdot \frac{1}{3}x + \mathbf{?} \; - \mathbf{?} \;\right) + 1$$

Example



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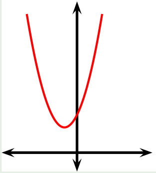

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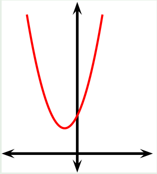
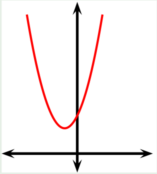


Find the minimum point on the curve $y = 3x^2 + 2x + 1$ by completing the square.

$$3x^2 + 2x + 1 = 3\left(x^2 + 2 \cdot \frac{1}{3}x + \frac{1}{9} - \frac{1}{9}\right) + 1$$

Example



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$$\begin{aligned} 3x^2 + 2x + 1 &= 3\left(x^2 + 2\cdot\frac{1}{3}x + \frac{1}{9} - \frac{1}{9}\right) + 1 \\ &= 3\left(\left(x + \frac{1}{3}\right)^2 - \frac{1}{9}\right) + 1 \end{aligned}$$

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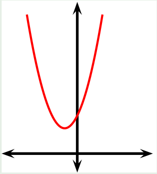

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&= 3\left(x + \frac{1}{3}\right)^2 {\color{red}-\frac{1}{3}} + 1
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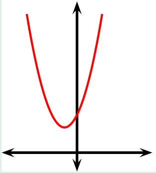

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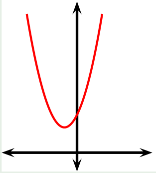

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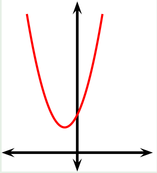

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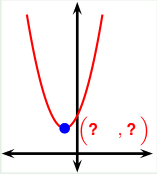

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\text{Minimum point} &= \left(\mathbf{?}\quad, \mathbf{?}\right)
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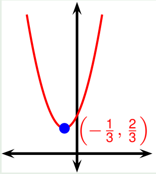

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Maximum or minimum value of a quadratic function

- Let $f(x) = ax^2 + bx + c$ - quadratic ($a \neq 0$).
- Let D be the discriminant $D = b^2 - 4ac$.

$$f(x) = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a} \quad \Bigg| \text{ complete the square}$$

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Recall $f(x) = ax^2 + bx + c = a\left(x - \left(-\frac{b}{2a}\right)\right)^2 - \frac{D}{4a}$.

Proposition

Let $f(x) = ax^2 + bx + c$, $a \neq 0$ and let $D = b^2 - 4ac$.

- *If $a > 0$ then $f(x)$ has no maximum and has minimum at $x = -\frac{b}{2a}$.*
- *If $a < 0$ then $f(x)$ has no minimum and has maximum at $x = -\frac{b}{2a}$.*
- *In both cases, the extremal value (either maximum or minimum) is $f\left(-\frac{b}{2a}\right) = -\frac{b^2-4ac}{4a} = -\frac{D}{4a}$.*

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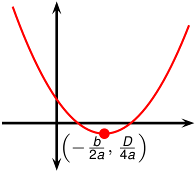

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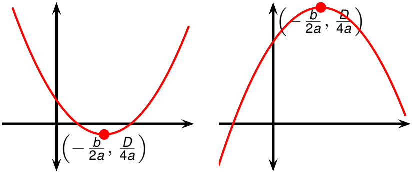

Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.

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Maximizing:

$$xz$$

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$$x + z = 12$$

Maximizing:

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Example

Let x, z be two numbers that add to 12. Choose x and z so that the product $x \cdot z$ is maximal.

$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

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Maximizing:

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Parabola

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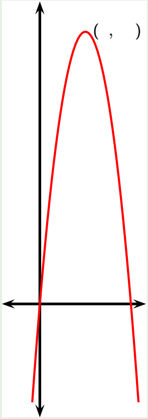

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Parabola opens down

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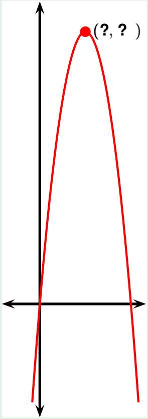

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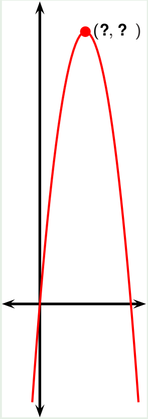

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$$x = ?$$

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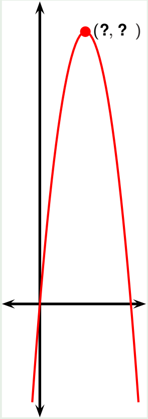

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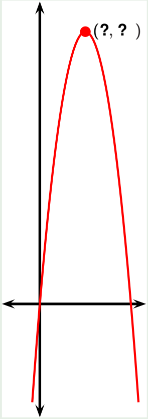

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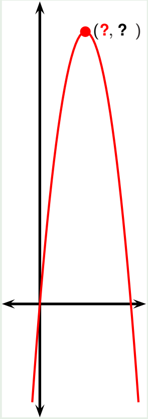

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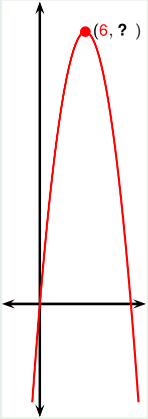

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$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{12}{-2} = 6 \end{aligned}$$

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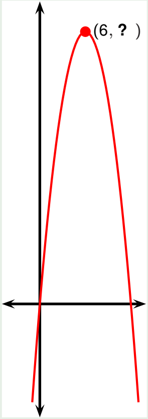

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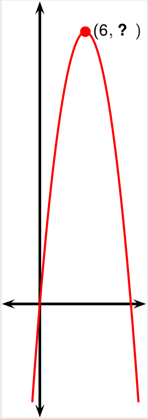

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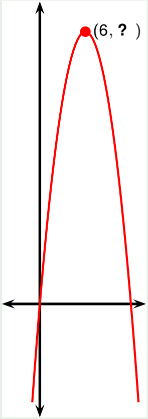

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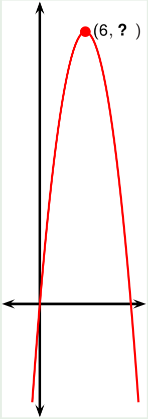

$$\begin{aligned} x + z &= 12 \\ z &= 12 - x \end{aligned}$$

Maximizing:

$$\begin{aligned} xz &= x(12 - x) \\ &= -x^2 + 12x \end{aligned}$$

Parabola opens down $\Rightarrow$ has maximum, attained at:

$$\begin{aligned} x &= -\frac{b}{2a} \\ &= -\frac{12}{-2} = 6 \\ z &= 12 - x = 12 - 6 = 6 \end{aligned}$$

Max. product $= xz$

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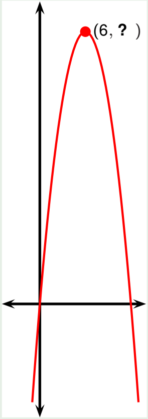

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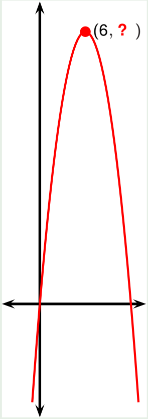

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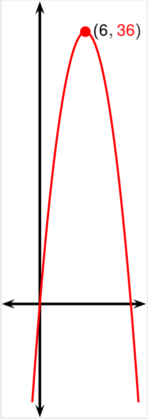

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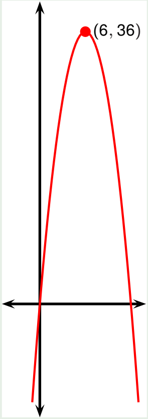

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