Precalculus
Factor cubic with one rational and two real roots using its plot

Todor Milev

2019

Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

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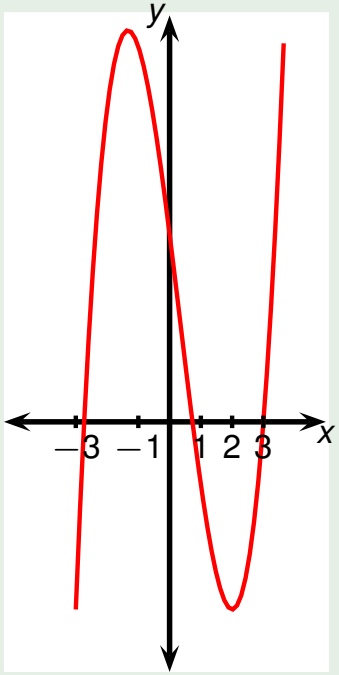

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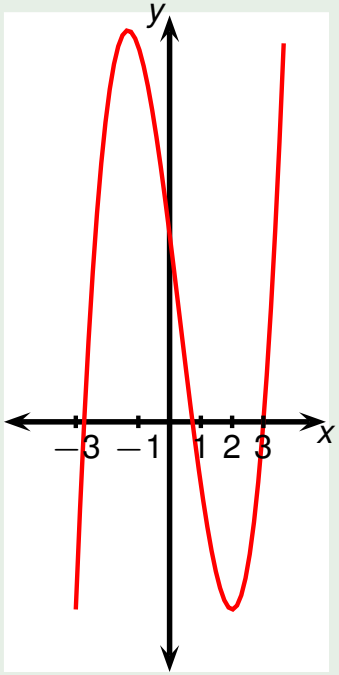

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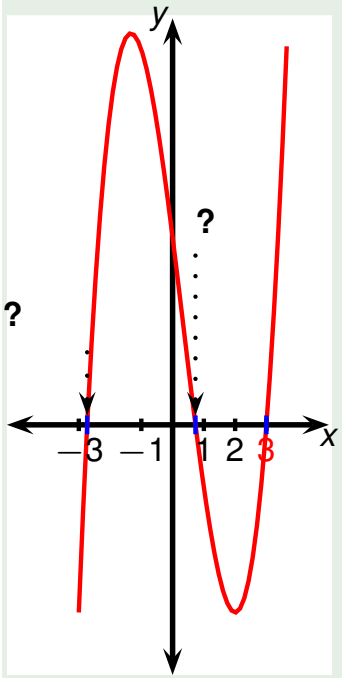

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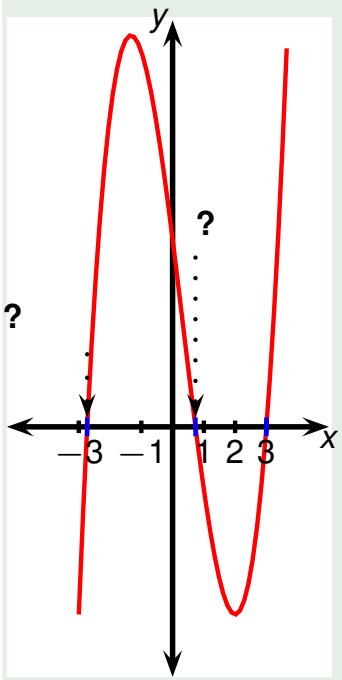

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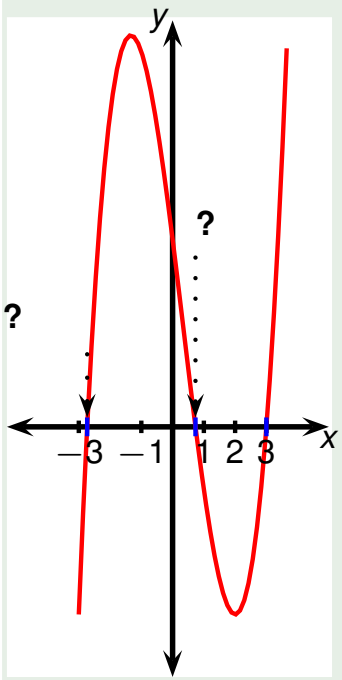

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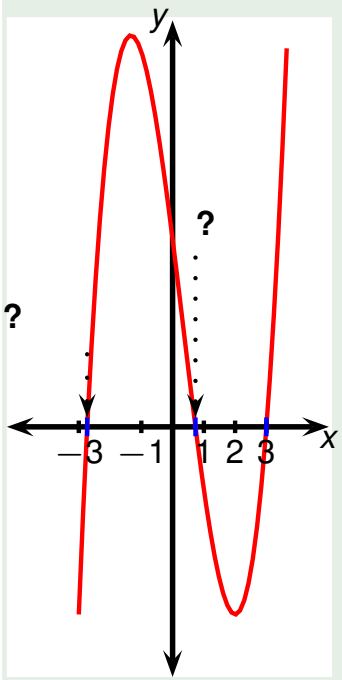

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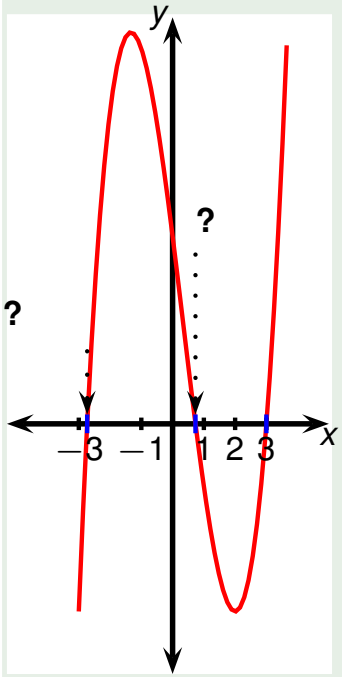

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Divide x^3 by x.

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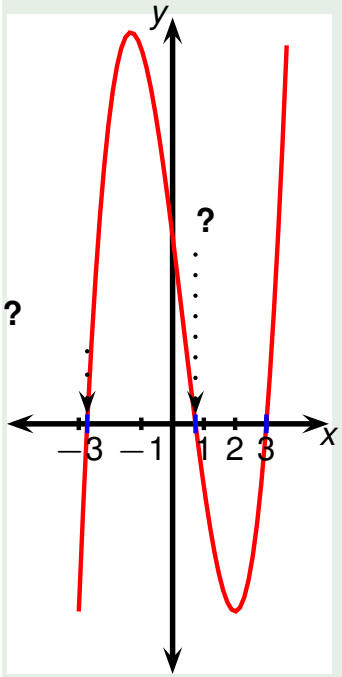

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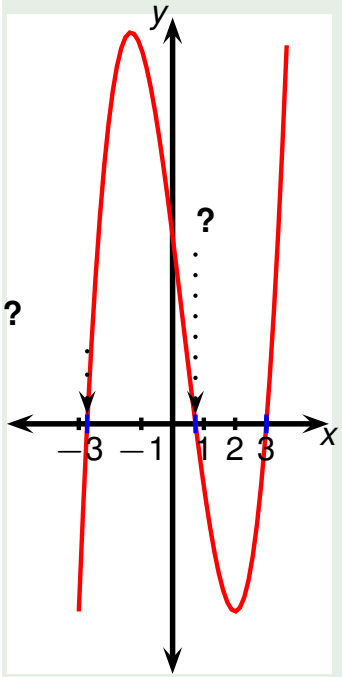

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Multiply x^2 by divisor.

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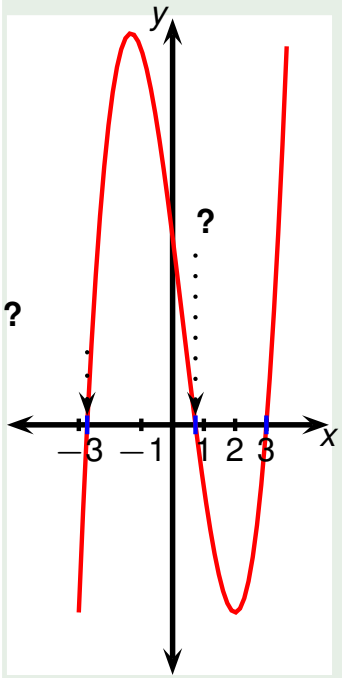

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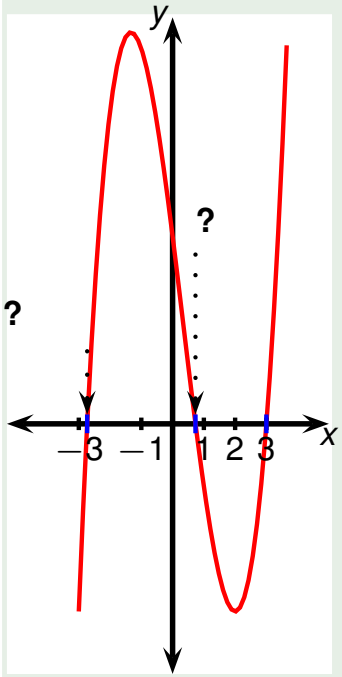

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Subtract last two polynomials.

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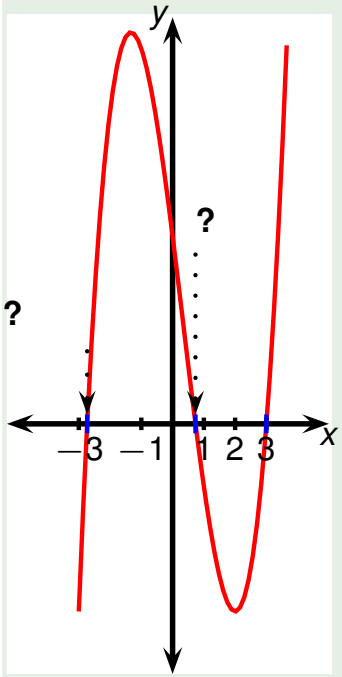

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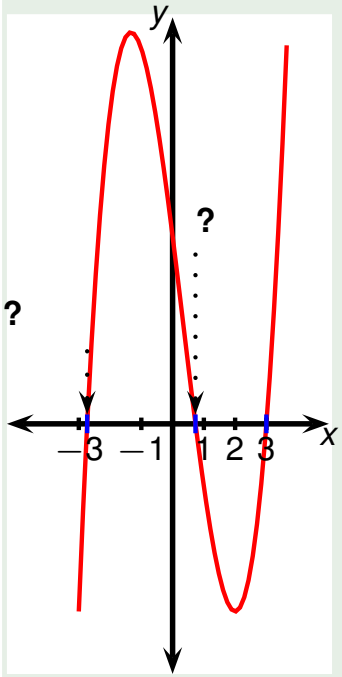

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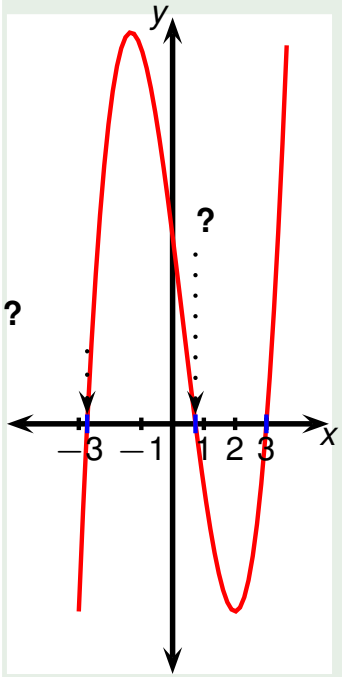

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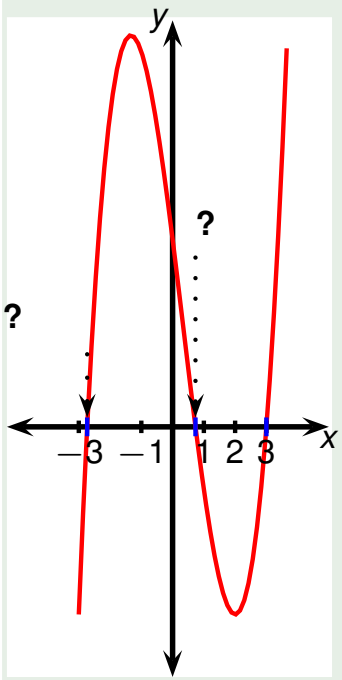

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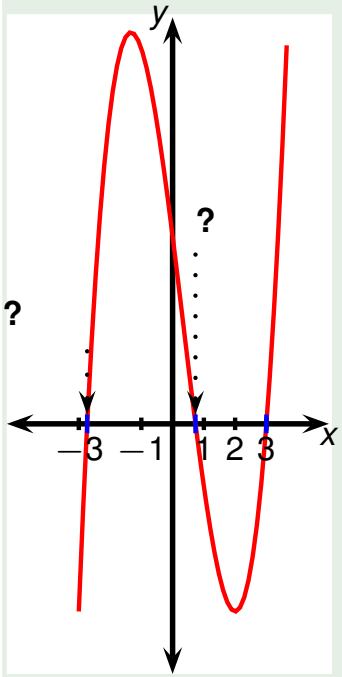

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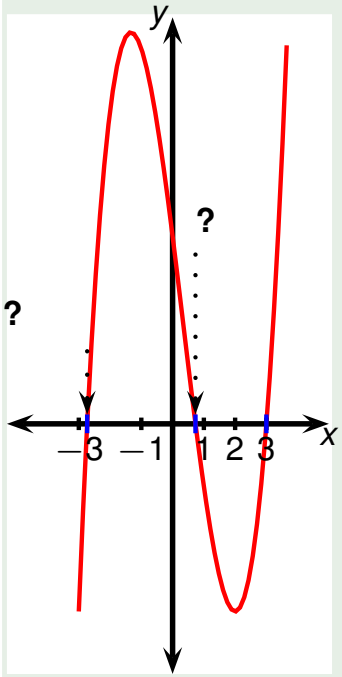

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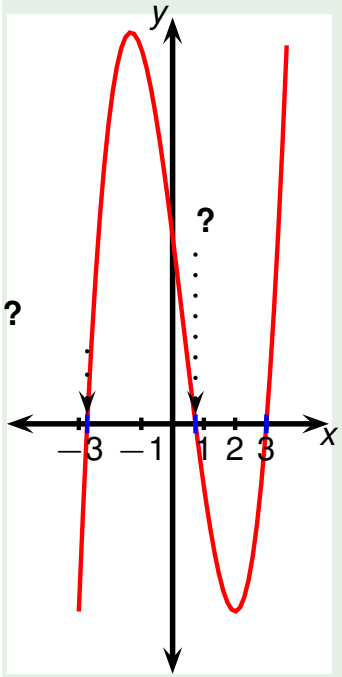

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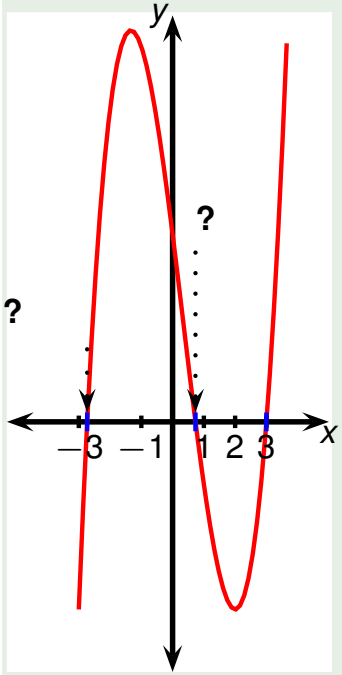

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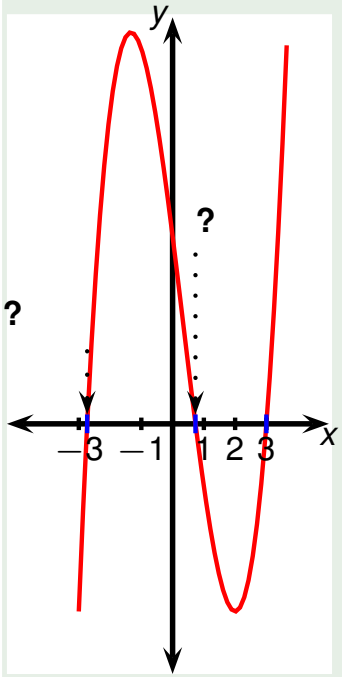

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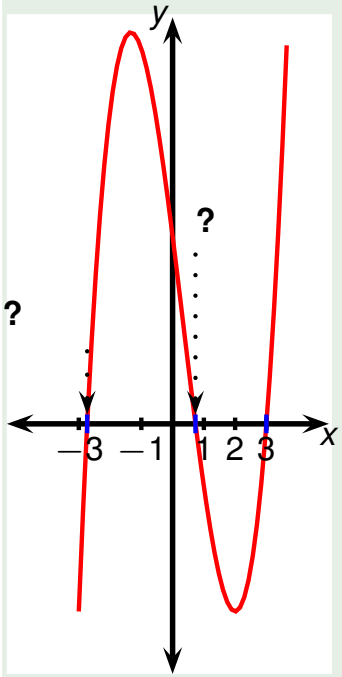

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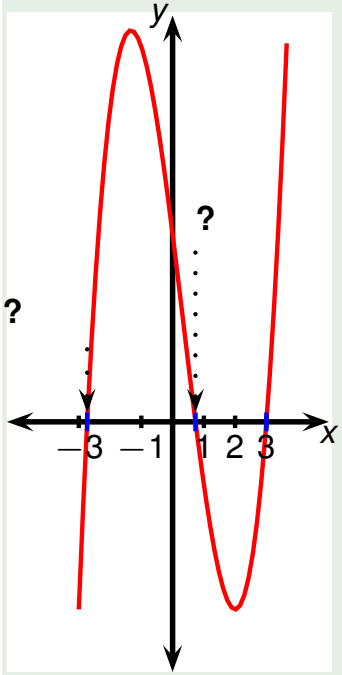

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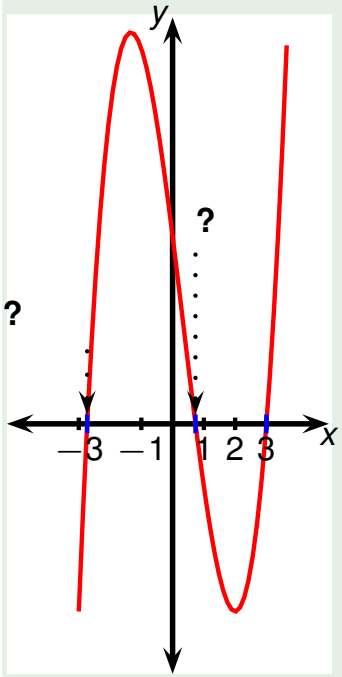

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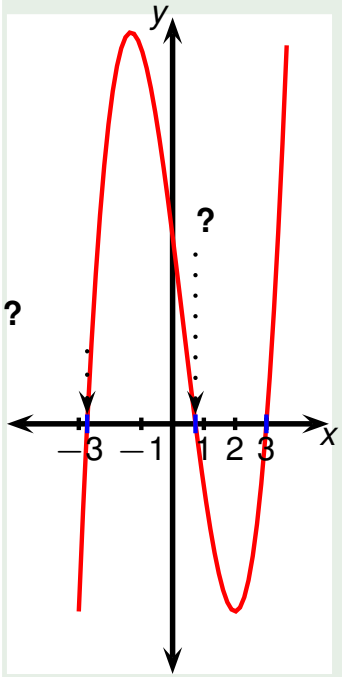

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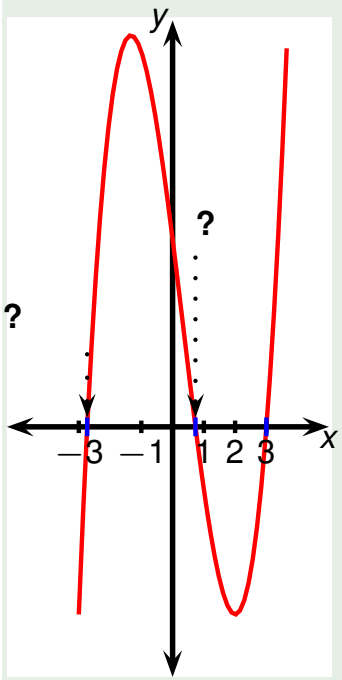

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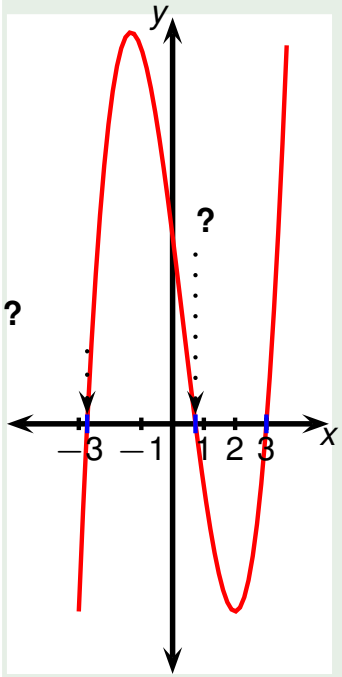

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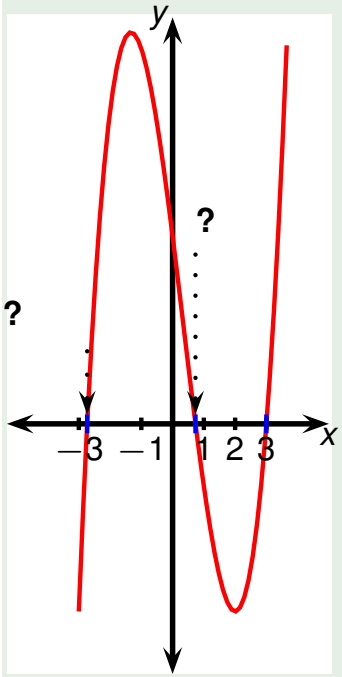

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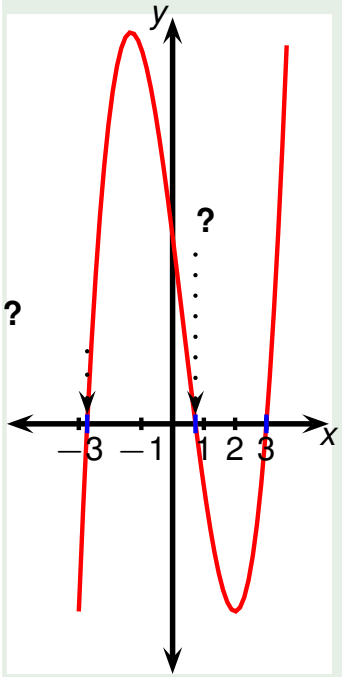

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$\begin{aligned} x^3 - x^2 - 8x + 6 &= 0 \\ (x-3)(x^2 + 2x - 2) &= 0 \end{aligned}$$

The graph appears to intersect the x axis at:
? , **?** , 3. What are the two roots besides 3?

Quotient: $x^2 + 2x - 2$

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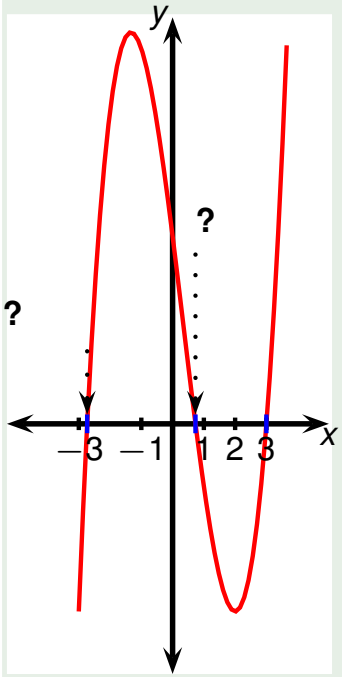

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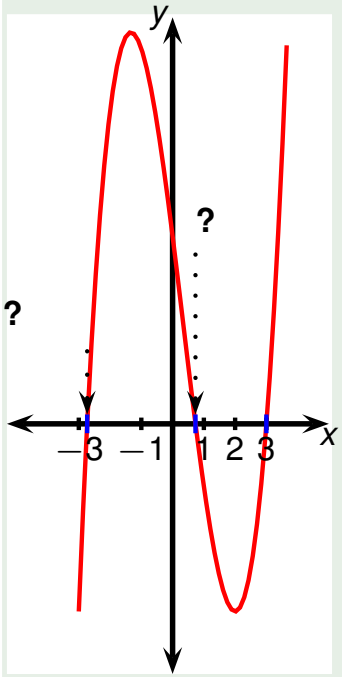

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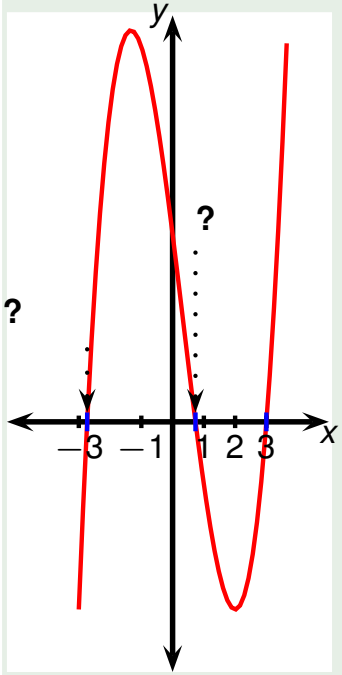

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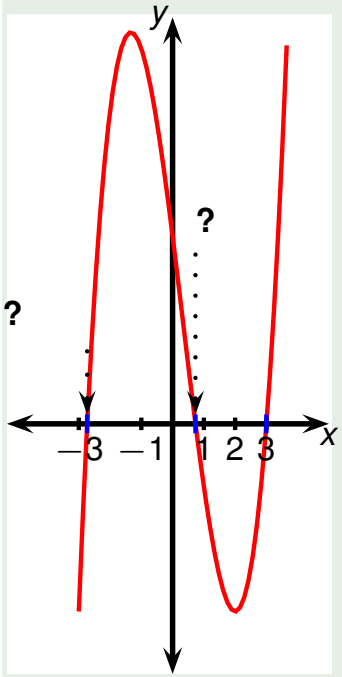

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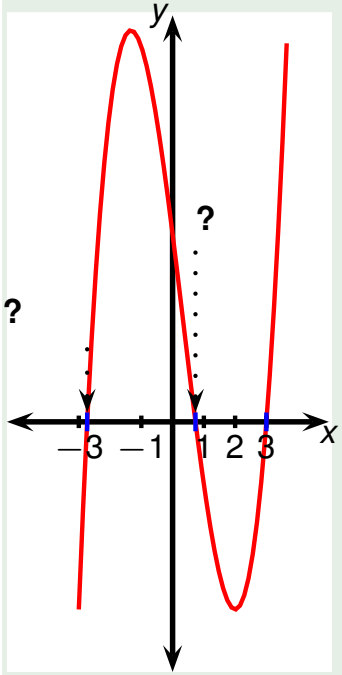

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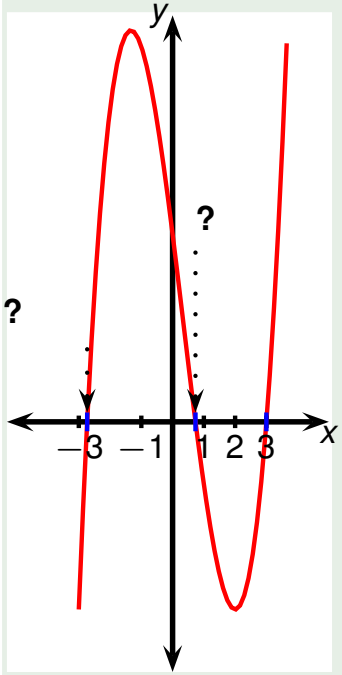

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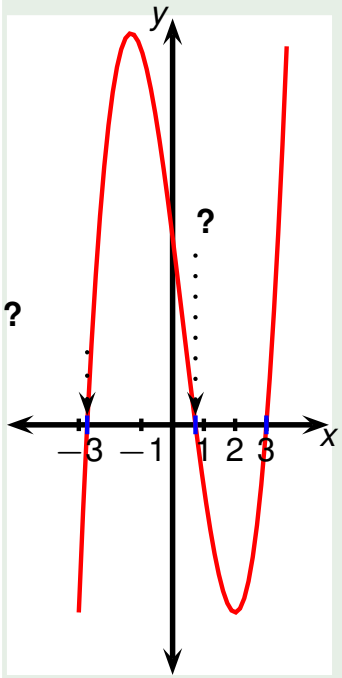

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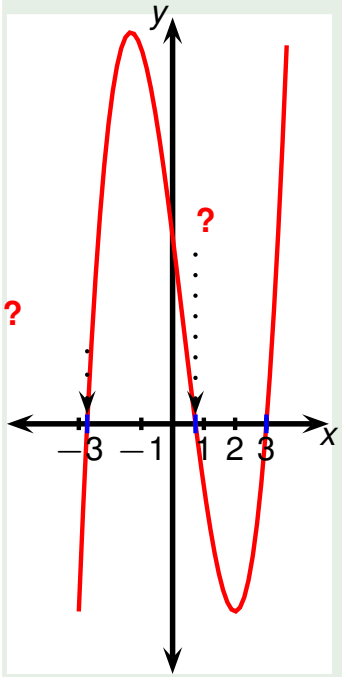

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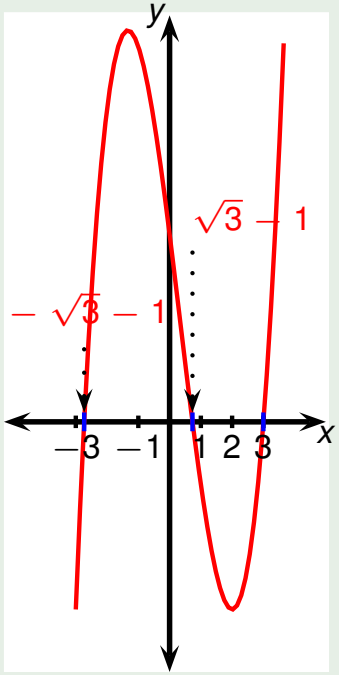

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The graph appears to intersect the x axis at: $-\sqrt{3}-1, \sqrt{3}-1, 3$. What are the two roots besides 3?

Final answer:

$x = 3$ **or** $x = -1 - \sqrt{3}$ **or** $x = -1 + \sqrt{3}$.