Calculus II

Integrals of the form $\int \sin^n x \cos^m x \mathrm{d}x$, theory

Todor Milev

2019

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When n – odd:

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$= d(\sin x)$

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If both m, n- even, use $\left| \begin{array}{lcl} \sin^2 x & = & \frac{1-\cos(2x)}{2} \\ \cos^2 x & = & \frac{\cos(2x)+1}{2} \end{array} \right.$ and substitute $s = 2x$ to lower trig powers. Repeat above considerations.

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