Calculus I
Homework
Limits

1. Evaluate the difference quotient and simplify your answer.

(a) $\dfrac{f(2+h)-f(2)}{h}$, where $f(x)=x^2-x-1$.

answer: $h+3$

(b) $\dfrac{f(a+h)-f(a)}{h}$, where $f(x)=x^2$.

answer: $h+2a$

(c) $\dfrac{f(a+h)-f(a)}{h}$, where $f(x)=x^3$.

answer: h^2+3a^2+3ah

(d) $\dfrac{f(a+h)-f(a)}{h}$, where $f(x)=x^4$.

answer: $6a^2h+4ah^2+h^3+4a^3$

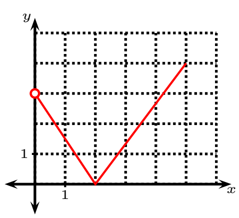
(e) $\dfrac{f(x)-f(a)}{x-a}$, where $f(x)=\frac{1}{x}$.

answer: $-\frac{1}{ax}$.

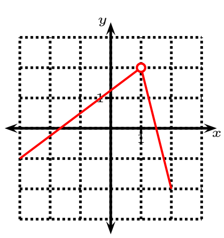
(f) $\dfrac{f(x)-f(1)}{x-1}$, where $f(x)=\frac{x-1}{x+1}$.

answer: $\frac{1}{x+1}$.

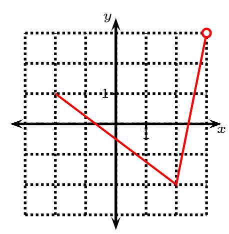
2. Write down a formula for a function whose graphs is given below. The graphs are up to scale. Please note that there is more than one way to write down a correct answer.

(a) 

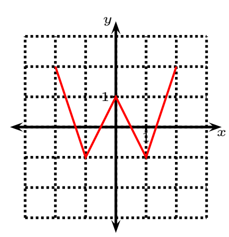
answer: $y = \begin{cases} -\frac{3}{2}x+3 & \text{if } 0<x\leq 2 \\ \frac{4}{3}x-\frac{8}{3} & \text{if } 2<x\leq 6 \end{cases}$

(b) 

answer: $y = \begin{cases} \frac{3}{4}x+\frac{5}{4} & \text{if } -3\leq x<1 \\ -4x+6 & \text{if } 1<x\leq 2 \end{cases}$

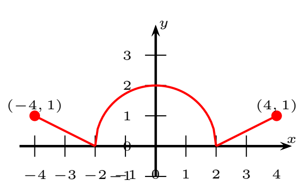
(c) 

answer: $y = \begin{cases} -\frac{3}{4}x-\frac{1}{2} & \text{if } -2\leq x<2 \\ 5x-12 & \text{if } 2\leq x<3 \end{cases}$

(d) 

answer: $y = \begin{cases} -3x-4 & \text{if } -2\leq x<-1 \\ 2x+1 & \text{if } -1\leq x<0 \\ -2x+1 & \text{if } 0\leq x<1 \\ 3x-4 & \text{if } 1\leq x\leq 2 \end{cases}$

3. Write down formulas for function whose graphs are as follows. The graphs are up to scale. All arcs are parts of circles.

(a) 

4. Evaluate the difference quotient and simplify your answer.

(a) $\dfrac{f(2+h)-f(2)}{h}$, where $f(x)=x^2-x-1$.

answer: $h+3$

(b) $\dfrac{f(a+h)-f(a)}{h}$, where $f(x)=x^2$.

answer: $h+2a$

(c) $\dfrac{f(a+h)-f(a)}{h}$, where $f(x)=x^3$.

answer: h^2+3a^2+3ah

(d) $\dfrac{f(a+h)-f(a)}{h}$, where $f(x)=x^4$.

answer: $6a^2h+4ah^2+h^3+4a^3$

(e) $\dfrac{f(x)-f(a)}{x-a}$, where $f(x)=\frac{1}{x}$.

answer: $-\frac{1}{ax}$.

(f) $\dfrac{f(x)-f(1)}{x-1}$, where $f(x)=\frac{x-1}{x+1}$.

answer: $\frac{1}{x+1}$.

5. Find the implied domain of the function.

(a) $f(x)=\dfrac{x+4}{x^2-4}$.

answer: $x\neq\pm2$, alternatively: $x\in(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

(b) $f(x)=\dfrac{2x^3-5}{x^2+5x+6}$.

answer: $x\neq-2,-3$, alternatively: $x\in(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$

(c) $f(t)=\sqrt[3]{3t-1}$.

answer: $x\in\mathbb{R}$ (the domain is all real numbers)

(d) $g(t)=\sqrt{5-t}-\sqrt{1+t}$.

answer: $x\in[-1,5]$.

(e) $h(x)=\dfrac{1}{\sqrt[6]{x^2-7x}}$.

answer: $x\in(-\infty,0)\cup(7,\infty)$.

(f) $f(u)=\dfrac{u+1}{1+\frac{1}{u+1}}$.

answer: $u\neq-1,-2$ or $u\in(-\infty,-2)\cup(-2,-1)\cup(-1,\infty)$..

(g) $F(x)=\sqrt{10-\sqrt{x}}$.

answer: $x\in[0,100]$

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(f) $f(u)=\dfrac{u+1}{1+\frac{1}{u+1}}$.

answer: $u\neq-1,-2$ or $u\in(-\infty,-2)\cup(-2,-1)\cup(-1,\infty)$..

(g) $F(x)=\sqrt{10-\sqrt{x}}$.

answer: $x\in[0,100]$

7. Compute the composite functions $(f\circ g)(x)$, $(g\circ f)(x)$. Simplify your answer to a single fraction. Find the domain of the composite function.

(a) $f(x)=\dfrac{x+2}{x-2}, g(x)=\dfrac{x-1}{x+2}$.

answer: $(f\circ g)(x)=\frac{3+3x}{-5-x}$ $\quad x\neq-2,-5$
$(g\circ f)(x)=\frac{4}{-2+3x}$ $\quad x\neq2,\frac{2}{3}$

(b) $f(x)=\dfrac{x+1}{3x-2}, g(x)=\dfrac{x-2}{x-1}$.

answer: $(f\circ g)(x)=\frac{-3+2x}{-4+x}$ $\quad x\neq4,1$
$(g\circ f)(x)=\frac{5-5x}{3-2x}$ $\quad x\neq\frac{2}{3},\frac{3}{2}$

(c) $f(x)=\dfrac{2x+1}{3x-1}, g(x)=\dfrac{x-2}{2x-1}$.

answer: $(f\circ g)(x)=\frac{-5+4x}{-5+x}$ $\quad x\neq5,\frac{1}{2}$
$(g\circ f)(x)=\frac{3-4x}{3+x}$ $\quad x\neq-3,\frac{1}{3}$

(d) $f(x)=\dfrac{x+1}{x-2}, g(x)=\dfrac{x+2}{2x-1}$.

answer: $(f\circ g)(x)=\frac{1+3x}{4-3x}$ $\quad x\neq\frac{4}{3},\frac{1}{2}$
$(g\circ f)(x)=\frac{-3+3x}{4+x}$ $\quad x\neq-4,2$

(e) $f(x)=\dfrac{5x+1}{4x-1}, g(x)=\dfrac{4x-1}{3x+1}$.

answer: $(f\circ g)(x)=\frac{-4+23x}{-5+13x}$ $\quad x\neq\frac{5}{13},-\frac{1}{3}$
$(g\circ f)(x)=\frac{5+16x}{2+19x}$ $\quad x\neq-\frac{2}{19},\frac{1}{4}$

(f) $f(x) = \dfrac{3x-5}{x-2}$, $g(x) = \dfrac{x-2}{x-4}$.

answer: $(f\circ g)(x) = \frac{-2x+14}{-x+6}$ $\quad x \neq 6, 4$
$(g\circ f)(x) = \frac{x-1}{-x+3}$ $\quad x \neq 3, 2$

(g) $f(x) = \dfrac{x-3}{x+2}$, $g(y) = \dfrac{y+3}{y-4}$.

answer: $(f\circ g)(x) = \frac{-2x+15}{3x-5}$ $\quad x \neq \frac{5}{3}, 4$
$(g\circ f)(x) = \frac{4x+3}{-3x-11}$ $\quad x \neq -\frac{11}{3}, -2$

8. Find the functions $f\circ g$, $g\circ f$, $f\circ f$ and $g\circ g$ and their implied domains. The answer key has not been proofread, use with caution.

(a) $f(x) = x^2+1$, $g(x) = x+1$.

answer: Domain, all 4 cases: $x \in \mathbb{R}$ (all reals)
in some order: $(1+x)^2+1$, $(x)^2+2$, $((x)^2+1)^2+1$, $2+x$

(b) $f(x) = \sqrt{x+1}$, $g(x) = x+1$.

answer: Domain of $f\circ g$ is $x \geq -2$. Domain of $g\circ f$ is $x \geq -1$
Domain of $f\circ f$ is $x \geq -1$. Domain of $g\circ g$ is all reals ($x \in \mathbb{R}$).
in some order: $\sqrt{2+x}$, $1+\sqrt{1+x}$, $\sqrt{1+\sqrt{1+x}}$, $2+x$

(c) $f(x) = 2x$, $g(x) = \tan x$.

In this subproblem, you are not required to find the domain.

answer: Domain $f\circ f$: all reals ($x\in\mathbb{R}$). Domain $g\circ f$: $x \neq (2k+1)\frac{\pi}{2}$ for all $k \in \mathbb{Z}$
Domain $f\circ g$: $x \neq (4k+1)\frac{\pi}{4}$, $x \neq (4k+3)\frac{\pi}{4}$ for all $k \in \mathbb{Z}$
Domain $g\circ g$: $x \neq (2k+1)\frac{\pi}{2}$ and $x \neq k\pi + \arctan\left(\frac{\pi}{2}\right)$ for all $k\in\mathbb{Z}$
in some order: $2\tan x$, $\tan(2x)$, $4x$, $\tan(\tan x)$

(d) $f(x) = \dfrac{x+1}{x-1}$, $g(x) = \dfrac{x-1}{x+1}$.

answer: Domain $f\circ f$: $x \neq 1$. Domain $g\circ g$: $x\neq 0, x \neq -1$
Domain $f\circ g$: $x \neq -1$. Domain $g\circ f$: $x \neq 0, x\neq 1$
in some order: $-x$, $\frac{1}{x}$, x, $-\frac{1}{x}$

9. Convert from degrees to radians.

(a) 15°. answer: $\frac{\pi}{12} \approx 0.261799388$

(b) 30°. answer: $\frac{\pi}{6} \approx 0.523598776$

(c) 36°. answer: $\frac{\pi}{5} \approx 0.628318531$

(d) 45°. answer: $\frac{\pi}{4} \approx 0.785398163$

(e) 60°. answer: $\frac{\pi}{3} \approx 1.047197551$

(f) 75°. answer: $\frac{5\pi}{12} \approx 1.308997$

(g) 90°. answer: $\frac{\pi}{2}$

(h) 120°. answer: $\frac{2\pi}{3}$

(i) 135°. answer: $\frac{3\pi}{4}$

(j) 150°. answer: $\frac{5\pi}{6}$

(k) 180°. answer: π

(l) 225°. answer: $\frac{5\pi}{4}$

(m) 270°. answer: $\frac{3\pi}{2}$

(n) 305°. answer: $\frac{61\pi}{36} \approx 5.323254$

(o) 360°. answer: 2π

(p) 405°. answer: $\frac{9\pi}{4}$

(q) 1200°. answer: $\frac{20\pi}{3}$

(r) -900°. answer: -5π

(s) -2014°. answer: $-\frac{1007}{90}\pi \approx -35.150931$

10. Convert from radians to degrees. The answer key has not been proofread, use with caution.

(a) 4π. answer: 720°

(b) $-\frac{7}{6}\pi$. answer: -210°

(c) $\frac{7}{12}\pi$. answer: 105°

(d) $\frac{4}{3}\pi$. answer: 240°

(e) $-\frac{3}{8}\pi$. answer: -67.5°

(f) 2014π. answer: 362520°

(g) 5. answer: $\left(\frac{900}{\pi}\right)^\circ \approx 286^\circ$

(h) -2014. answer: -362520°

11. Prove the trigonometry identities.

(a) $\sin\theta\cot\theta = \cos\theta$.

(b) $(\sin\theta+\cos\theta)^2 = 1+\sin(2\theta)$.

(c) $\sec\theta - \cos\theta = \tan\theta\sin\theta$.

(d) $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$.

(e) $\cot^2\theta + \sec^2\theta = \tan^2\theta + \csc^2\theta$.

(f) $2\csc(2\theta) = \sec\theta\csc\theta$.

(g) $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$.

(h) $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} = 2\sec^2\theta$.

(i) $\tan\alpha + \tan\beta = \dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$.

(j) $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$.

(k) $\sin(3\theta) + \sin\theta = 2\sin(2\theta)\cos\theta$.

(l) $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$.

(m) $1 + \tan^2\theta = \sec^2\theta$.

(n) $1 + \csc^2\theta = \cot^2\theta$.

(o) $2\cos^2(2x) = 2\sin^4\theta + 2\cos^4\theta - \sin^2(2\theta)$.

(p) $\dfrac{1+\tan\left(\frac{\theta}{2}\right)}{1-\tan\left(\frac{\theta}{2}\right)} = \tan\theta + \sec\theta$.

12. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation.

(a) $2\cos x - 1 = 0$.

answer: $x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$

(b) $\sin(2x) = \cos x$.

answer: $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$, $x = \frac{\pi}{6}$, or $x = \frac{5\pi}{6}$

(c) $\sqrt{3}\sin x = \sin(2x)$.

answer: $x = \frac{\pi}{6}, \frac{11\pi}{6}, 0, \pi, 2\pi$

(d) $2\sin^2 x = 1$.

answer: $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$, $x = \frac{5\pi}{4}$, or $x = \frac{7\pi}{4}$

(e) $2 + \cos(2x) = 3\cos x$.

answer: $x = 0$, $x = 2\pi$, $x = \frac{\pi}{3}$, or $x = \frac{5\pi}{3}$

(f) $2\cos x + \sin(2x) = 0$.

answer: $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$

(g) $2\cos^2 x - (1+\sqrt{2})\cos x + \dfrac{\sqrt{2}}{2} = 0$.

answer: $x = \frac{\pi}{4}, \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{4}$

(h) $|\tan x| = 1$.

answer: $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$, $x = \frac{5\pi}{4}$, or $x = \frac{7\pi}{4}$

(i) $3\cot^2 x = 1$.

answer: $x = \frac{\pi}{3}$, $x = \frac{2\pi}{3}$, $x = \frac{4\pi}{3}$, or $x = \frac{5\pi}{3}$

(j) $\sin x = \tan x$.

answer: $x = 0, \pi$, or $x = 2\pi$

Solution. 12.g Set $\cos x = u$. Then

$$2\cos^2 x - (1+\sqrt{2})\cos x + \frac{\sqrt{2}}{2} = 0$$

becomes

$$2u^2 - (1+\sqrt{2})u + \frac{\sqrt{2}}{2} = 0.$$

This is a quadratic equation in u and therefore has solutions

$$\begin{aligned} u_1, u_2 &= \frac{1+\sqrt{2} \pm \sqrt{(1+\sqrt{2})^2 - 4\sqrt{2}}}{4} \\ &= \frac{1+\sqrt{2} \pm \sqrt{1 - 2\sqrt{2} + 2}}{4} \\ &= \frac{1+\sqrt{2} \pm \sqrt{(1-\sqrt{2})^2}}{4} \\ &= \frac{1+\sqrt{2} \pm (1-\sqrt{2})}{4} = \begin{cases} \frac{1}{2} & \text{or} \\ \frac{\sqrt{2}}{2} \end{cases} \end{aligned}$$

Therefore $u = \cos x = \frac{1}{2}$ or $u = \cos x = \frac{\sqrt{2}}{2}$, and, as x is in the interval $[0, 2\pi]$, we get $x = \frac{\pi}{3}, \frac{5\pi}{3}$ (for $\cos x = \frac{1}{2}$) or $x = \frac{\pi}{4}, \frac{7\pi}{4}$ (for $\cos x = \frac{\sqrt{2}}{2}$).

13. Evaluate the limits. Justify your computations.

(a) $\lim\limits_{x\to 2} 2x^2 - 3x - 6$.

answer: -4

(b) $\lim\limits_{x\to -1} \dfrac{x^4 - x}{x^2 + 2x + 3}$.

answer: 1

(c) $\lim\limits_{x\to -1} \dfrac{1}{x^2 - 3x + 2}$.

answer: $\frac{1}{6}$

(d) $\lim\limits_{x\to -2} \sqrt{x^4 + 16}$.

answer: $\sqrt{32}$

(e) $\lim\limits_{x\to 8} (1 + \sqrt[3]{x})(2 - x)$.

answer: -18

14. Evaluate the limit if it exists.

(a) $\lim\limits_{x\to 2} \dfrac{x^2 - 5x + 6}{x - 2}$.

answer: -1

(b) $\lim\limits_{x\to 3} \dfrac{x^2 - 3x}{x^2 - 2x - 3}$.

answer: $\frac{3}{4}$

(c) $\lim\limits_{x\to -2} \dfrac{2x^2 + x - 6}{x^2 - 4}$

answer: $\frac{7}{4}$

(d) $\lim\limits_{x\to 2} \dfrac{x^2 - 5x - 6}{x - 2}$.

answer: DNE

(e) $\lim\limits_{x\to -1} \dfrac{x^2 - 3x}{x^2 - 2x - 3}$.

answer: DNE

(f) $\lim\limits_{x\to -2} \dfrac{x^2-4}{2x^2+5x+2}$. answer: $\frac{4}{3}$

(g) $\lim\limits_{x\to -1} \dfrac{2x^2+3x+1}{3x^2-2x-5}$. answer: $\frac{1}{8}$

(h) $\lim\limits_{x\to -4} \dfrac{x^2+7x+12}{x^2+6x+8}$. answer: $\frac{1}{2}$

(i) $\lim\limits_{h\to 0} \dfrac{(-3+h)^2-9}{h}$. answer: -6

(j) $\lim\limits_{h\to 0} \dfrac{(-2+h)^3+8}{h}$. answer: 12

(k) $\lim\limits_{x\to -3} \dfrac{x+3}{x^3+27}$. answer: $\frac{1}{27}$

(l) $\lim\limits_{x\to 1} \dfrac{x^4-1}{x^3-1}$. answer: $\frac{4}{3}$

(m) $\lim\limits_{h\to 0} \dfrac{\sqrt{4+h}-2}{h}$. answer: $\frac{1}{4}$

(n) $\lim\limits_{x\to 3} \dfrac{\sqrt{5x+1}-4}{x-3}$. answer: $\frac{5}{8}$

(o) $\lim\limits_{x\to -3} \dfrac{\sqrt{x^2+16}-5}{x+3}$. answer: $-\frac{3}{5}$

(p) $\lim\limits_{x\to -3} \dfrac{\frac{1}{3}+\frac{1}{x}}{3+x}$. answer: $-\frac{1}{9}$

(q) $\lim\limits_{x\to -2} \dfrac{x^2+4x+4}{x^4-16}$. answer: 0

(r) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$. answer: 1

(s) $\lim\limits_{x\to 0} \left(\dfrac{1}{x}-\dfrac{1}{x^2+x}\right)$. answer: 1

(t) $\lim\limits_{x\to 9} \dfrac{3-\sqrt{x}}{9x-x^2}$. answer: $\frac{1}{54}$

(u) $\lim\limits_{h\to 0} \dfrac{(2+h)^{-1}-2^{-1}}{h}$. answer: $-\frac{1}{4}$

(v) $\lim\limits_{x\to 0} \left(\dfrac{1}{x\sqrt{1+x}}-\dfrac{1}{x}\right)$. answer: $-\frac{1}{2}$

(w) $\lim\limits_{h\to 0} \dfrac{(x+h)^3-x^3}{h}$. answer: $3x^2$

(x) $\lim\limits_{h\to 0} \dfrac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$. answer: $-\frac{2}{x^3}$

(y) $\lim\limits_{h\to 0} \dfrac{\frac{1}{(2+h)^2}-\frac{1}{4}}{h}$. answer: $-\frac{1}{4}$

(z) $\lim\limits_{h\to 0} \dfrac{\frac{1}{(1+h)^2}-1}{h}$. answer: -2

Solution. 14.a

$$\begin{array}{rcl|l} \lim\limits_{x\to 2}\dfrac{x^2-5x+6}{x-2} &=& \lim\limits_{x\to 2}\dfrac{(x-3)\cancel{(x-2)}}{\cancel{x-2}} & \text{factor and cancel}\\ &=& 2-3=-1 & \end{array}$$

Solution. 14.c

$$\begin{array}{rcl|l} \lim\limits_{x\to -2}\dfrac{2x^2+x-6}{x^2-4} &=& \lim\limits_{x\to -2}\dfrac{(2x-3)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}} & \text{factor and cancel}\\ &=& \dfrac{(2(-2)-3)}{-2-2} & \text{substitute}\\ &=& \dfrac{7}{4} & \end{array}$$

Solution. 14.f

$$\begin{array}{rcl|l} \lim\limits_{x\to 2}\dfrac{x^2-4}{2x^2+5x+2} &=& \lim\limits_{x\to -2}\dfrac{(x-2)\cancel{(x+2)}}{(2x+1)\cancel{(x+2)}} & \text{factor and cancel}\\ &=& \dfrac{(-2)-2}{2(-2)+1}=\dfrac{4}{3}. & \end{array}$$

Solution. 14.g

$$\begin{array}{rcl|l} \lim\limits_{x\to -1}\dfrac{2x^2+3x+1}{3x^2-2x-5} &=& \lim\limits_{x\to -1}\dfrac{(2x+1)\cancel{(x+1)}}{(3x-5)\cancel{(x+1)}} & \text{factor and cancel}\\ &=& \dfrac{2(-1)+1}{3(-1)-5}=\dfrac{1}{8}. & \end{array}$$

Solution. 14.h.

$$\begin{aligned}\lim_{x\to -4}\frac{x^2+7x+12}{x^2+6x+8} &= \lim_{x\to -4}\frac{(x+3)\cancel{(x+4)}}{(x+2)\cancel{(x+4)}} && \Big| \text{ factor}\\ &= \frac{-4+3}{-4+2} = -\frac{1}{2}.\end{aligned}$$

Solution. 14.x

$$\begin{aligned}\lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} &= \lim_{h\to 0}\frac{x^2-(x+h)^2}{hx^2(x+h)^2} = \lim_{h\to 0}\frac{x^2-(x^2+2xh+h^2)}{hx^2(x+h)^2}\\ &= \lim_{h\to 0}\frac{\cancel{h}(-2x+h)}{\cancel{h}x^2(x+h)^2} = \frac{-2x+0}{x^2(x+0)^2} = -\frac{2}{x^3}.\end{aligned}$$

Solution. 14.y.

Variant I.

$$\begin{aligned}\lim_{h\to 0}\frac{\frac{1}{(2+h)^2}-\frac{1}{4}}{h} &= \lim_{h\to 0}\frac{\frac{4-(2+h)^2}{4(2+h)^2}}{h}\\ &= \lim_{h\to 0}\frac{4-(4+4h+h^2)}{4h(2+h)^2}\\ &= \lim_{h\to 0}\frac{-4h-h^2}{4h(2+h)^2}\\ &= \lim_{h\to 0}\frac{\cancel{h}(-4-h)}{4\cancel{h}(2+h)^2} && \Big| \text{ substitute } h=0\\ &= \frac{-4-0}{4(2+0)^2}\\ &= -\frac{1}{4}\end{aligned}$$

Variant II.

$$\begin{aligned}\lim_{h\to 0}\frac{\frac{1}{(2+h)^2}-\frac{1}{4}}{h} &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{x^2}\right)_{|x=2}\\ &= \left(\frac{-2}{x^3}\right)_{|x=2}\\ &= -\frac{1}{4}\end{aligned}$$

Solution. 14.z.

Variant I.

$$\begin{aligned}\lim_{h\to 0}\frac{\frac{1}{(1+h)^2}-1}{h} &= \lim_{h\to 0}\frac{\frac{1-(1+h)^2}{(1+h)^2}}{h}\\ &= \lim_{h\to 0}\frac{1-(1+2h+h^2)}{h(1+h)^2}\\ &= \lim_{h\to 0}\frac{-2h-h^2}{h(1+h)^2}\\ &= \lim_{h\to 0}\frac{\cancel{h}(-2-h)}{\cancel{h}(1+h)^2} && \Big| \text{ substitute } h=0\\ &= \frac{-2-0}{(1+0)^2}\\ &= -2.\end{aligned}$$

Variant II.

$$\begin{aligned}\lim_{h\to 0}\frac{\frac{1}{(1+h)^2}-1}{h} &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{x^2}\right)_{|x=1} && \Big| \text{ derivative definition}\\ &= \left(\frac{-2}{x^3}\right)_{|x=1}\\ &= -2.\end{aligned}$$