Calculus I
Derivative of $ax^3 + bx^2 + cx + d$

Todor Milev

2019

Theorem (The Sum Rule)

If f and g are both differentiable, then
$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)+g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}f(x) + \frac{\mathrm{d}}{\mathrm{d}x}g(x).$$

Proof.

$\square$

Theorem (The Sum Rule)

If f and g are both differentiable, then

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)+g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}f(x) + \frac{\mathrm{d}}{\mathrm{d}x}g(x).$$

Proof.

$$\text{Let} \quad F(x) = f(x) + g(x).$$

$\square$

Theorem (The Sum Rule)

If f and g are both differentiable, then

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)+g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}f(x) + \frac{\mathrm{d}}{\mathrm{d}x}g(x).$$

Proof.

$$\begin{aligned} \text{Let} \quad & F(x) = f(x) + g(x). \\ \text{Then} \quad & F'(x) = \lim_{h\to 0} \frac{F(x+h)-F(x)}{h} \end{aligned}$$

$\square$

Theorem (The Sum Rule)

If f and g are both differentiable, then

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)+g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}f(x) + \frac{\mathrm{d}}{\mathrm{d}x}g(x).$$

Proof.

$$\begin{aligned} \text{Let} \quad & F(x) = f(x) + g(x). \\ \text{Then} \quad & F'(x) = \lim_{h\to 0} \frac{F(x+h)-F(x)}{h} \\ & = \lim_{h\to 0} \frac{[f(x+h)+g(x+h)]-[f(x)+g(x)]}{h} \end{aligned}$$

□

Theorem (The Sum Rule)

If f and g are both differentiable, then

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)+g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}f(x) + \frac{\mathrm{d}}{\mathrm{d}x}g(x).$$

Proof.

$$\begin{aligned}
\text{Let} \quad F(x) &= f(x)+g(x). \\
\text{Then} \quad F'(x) &= \lim_{h\to 0}\frac{F(x+h)-F(x)}{h} \\
&= \lim_{h\to 0}\frac{[f(x+h)+g(x+h)]-[f(x)+g(x)]}{h} \\
&= \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}+\frac{g(x+h)-g(x)}{h}\right]
\end{aligned}$$

□

Theorem (The Sum Rule)

If f and g are both differentiable, then

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)+g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}f(x) + \frac{\mathrm{d}}{\mathrm{d}x}g(x).$$

Proof.

$$\begin{aligned}
\text{Let} \quad F(x) &= f(x) + g(x). \\
\text{Then} \quad F'(x) &= \lim_{h\to 0} \frac{F(x+h)-F(x)}{h} \\
&= \lim_{h\to 0} \frac{[f(x+h)+g(x+h)]-[f(x)+g(x)]}{h} \\
&= \lim_{h\to 0} \left[\frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h}\right]
\end{aligned}$$

$\square$

Theorem (The Sum Rule)

If f and g are both differentiable, then

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)+g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}f(x) + \frac{\mathrm{d}}{\mathrm{d}x}g(x).$$

Proof.

$$\begin{aligned}
\text{Let} \quad F(x) &= f(x) + g(x). \\
\text{Then} \quad F'(x) &= \lim_{h\to 0} \frac{F(x+h) - F(x)}{h} \\
&= \lim_{h\to 0} \frac{[f(x+h) + g(x+h)] - [f(x) + g(x)]}{h} \\
&= \lim_{h\to 0} \left[\frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h} \right]
\end{aligned}$$

$\square$

Theorem (The Sum Rule)

If f and g are both differentiable, then

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x)+g(x)] = \frac{\mathrm{d}}{\mathrm{d}x}f(x) + \frac{\mathrm{d}}{\mathrm{d}x}g(x).$$

Proof.

$$\begin{aligned}
\text{Let}\quad F(x) &= f(x)+g(x). \\
\text{Then}\quad F'(x) &= \lim_{h\to 0}\frac{F(x+h)-F(x)}{h} \\
&= \lim_{h\to 0}\frac{[f(x+h)+g(x+h)]-[f(x)+g(x)]}{h} \\
&= \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}+\frac{g(x+h)-g(x)}{h}\right] \\
\text{Limit Law 1:}\quad &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}
\end{aligned}$$

$\square$

Theorem (The Sum Rule)

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x).$$

Proof.

$$\begin{aligned}
\text{Let}\quad F(x) &= f(x)+g(x).\\
\text{Then}\quad F'(x) &= \lim_{h\to 0}\frac{F(x+h)-F(x)}{h}\\
&= \lim_{h\to 0}\frac{[f(x+h)+g(x+h)]-[f(x)+g(x)]}{h}\\
&= \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}+\frac{g(x+h)-g(x)}{h}\right]\\
\text{Limit Law 1:}\quad &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}\\
&= f'(x)+g'(x).
\end{aligned}$$

$\square$

Theorem (The Sum Rule)

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x).$$

Proof.

$$\begin{aligned}
\text{Let} \quad F(x) &= f(x) + g(x). \\
\text{Then} \quad F'(x) &= \lim_{h\to 0} \frac{F(x+h)-F(x)}{h} \\
&= \lim_{h\to 0} \frac{[f(x+h)+g(x+h)] - [f(x)+g(x)]}{h} \\
&= \lim_{h\to 0} \left[\frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h} \right] \\
\text{Limit Law 1:} \quad &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} \\
&= f'(x) + g'(x).
\end{aligned}$$

$\square$

The Sum Rule can be extended to any number of summands. For instance, using the theorem twice, we get

$$(f+g+h)' = [(f+g)+h]' = (f+g)'+h' = f'+g'+h'.$$

The Sum Rule can be extended to any number of summands. For instance, using the theorem twice, we get

$$(f+g+h)' = [(f+g)+h]' = (f+g)' + h' = f' + g' + h'.$$

By writing $f - g$ as $f + (-1)g$ and applying the Sum Rule and the Constant Multiple Rule, we get

Theorem (The Difference Rule)

If f and g are both differentiable, then

$$\frac{\mathrm{d}}{\mathrm{d}x}[f(x) - g(x)] = \frac{\mathrm{d}}{\mathrm{d}x} f(x) - \frac{\mathrm{d}}{\mathrm{d}x} g(x).$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

If $\quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \dfrac{x}{8} - 5,$

Then $\quad \dfrac{\mathrm{d}y}{\mathrm{d}x} =$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

$$\text{If} \quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$$

$$\text{Then} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right)$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

$$\text{If} \quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$$

$$\begin{aligned} \text{Then} \quad \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}}{\mathrm{d}x}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right) \\ &= \frac{\mathrm{d}}{\mathrm{d}x}\left(x^{16}\right) + \frac{\mathrm{d}}{\mathrm{d}x}\left(2\sqrt{3}x^7\right) - \frac{\mathrm{d}}{\mathrm{d}x}\left(4x^3\right) + \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x}{8}\right) - \frac{\mathrm{d}}{\mathrm{d}x}(5) \end{aligned}$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

$$\text{If} \quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$$

$$\begin{aligned}\text{Then} \quad \frac{dy}{dx} &= \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right) \\ &= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5) \\ &= \frac{d}{dx}\left(x^{16}\right) + 2\sqrt{3}\frac{d}{dx}\left(x^7\right) - 4\frac{d}{dx}\left(x^3\right) + \frac{1}{8}\frac{d}{dx}(x) - \frac{d}{dx}(5)\end{aligned}$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

$$\text{If} \quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$$

$$\begin{aligned}
\text{Then} \quad \frac{dy}{dx} &= \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right) \\
&= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5) \\
&= \frac{d}{dx}\left(x^{16}\right) + 2\sqrt{3}\frac{d}{dx}\left(x^7\right) - 4\frac{d}{dx}\left(x^3\right) + \frac{1}{8}\frac{d}{dx}(x) - \frac{d}{dx}(5) \\
&= (\textbf{?} \qquad) + 2\sqrt{3}\left(\textbf{?} \quad \right) - 4\left(\textbf{?} \quad \right) + \frac{1}{8}(\textbf{?}) - (\textbf{?})
\end{aligned}$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

$$\text{If} \quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$$

$$\begin{aligned}
\text{Then} \quad \frac{dy}{dx} &= \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right) \\
&= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5) \\
&= \frac{d}{dx}\left(x^{16}\right) + 2\sqrt{3}\frac{d}{dx}\left(x^7\right) - 4\frac{d}{dx}\left(x^3\right) + \frac{1}{8}\frac{d}{dx}(x) - \frac{d}{dx}(5) \\
&= \left(16x^{15}\right) + 2\sqrt{3}\left(\mathbf{?} \quad\right) - 4\left(\mathbf{?} \quad\right) + \frac{1}{8}(\mathbf{?}) - (\mathbf{?})
\end{aligned}$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

If $y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$

Then
$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right)\\ &= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5)\\ &= \frac{d}{dx}\left(x^{16}\right) + 2\sqrt{3}\frac{d}{dx}\left(x^7\right) - 4\frac{d}{dx}\left(x^3\right) + \frac{1}{8}\frac{d}{dx}(x) - \frac{d}{dx}(5)\\ &= (16x^{15}) + 2\sqrt{3}\left(\mathbf{?}\quad\right) - 4\left(\mathbf{?}\quad\right) + \frac{1}{8}(\mathbf{?}) - (\mathbf{?})\end{aligned}$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

$$\text{If} \quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$$

$$\begin{aligned}
\text{Then} \quad \frac{dy}{dx} &= \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right) \\
&= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5) \\
&= \frac{d}{dx}\left(x^{16}\right) + 2\sqrt{3}\frac{d}{dx}\left(x^7\right) - 4\frac{d}{dx}\left(x^3\right) + \frac{1}{8}\frac{d}{dx}(x) - \frac{d}{dx}(5) \\
&= (16x^{15}) + 2\sqrt{3}\left(7x^6\right) - 4\left(\mathbf{?} \quad \right) + \frac{1}{8}(\mathbf{?}) - (\mathbf{?})
\end{aligned}$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

$$\text{If}\quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$$

$$\begin{aligned}
\text{Then}\quad \frac{dy}{dx} &= \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right)\\
&= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5)\\
&= \frac{d}{dx}\left(x^{16}\right) + 2\sqrt{3}\frac{d}{dx}\left(x^7\right) - 4\frac{d}{dx}\left(x^3\right) + \frac{1}{8}\frac{d}{dx}(x) - \frac{d}{dx}(5)\\
&= \left(16x^{15}\right) + 2\sqrt{3}\left(7x^6\right) - 4\left(\mathbf{?}\quad\right) + \frac{1}{8}(\mathbf{?}) - (\mathbf{?})
\end{aligned}$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

$$\text{If}\quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$$

$$\begin{aligned}
\text{Then}\quad \frac{dy}{dx} &= \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right)\\
&= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5)\\
&= \frac{d}{dx}\left(x^{16}\right) + 2\sqrt{3}\frac{d}{dx}\left(x^7\right) - 4\frac{d}{dx}\left(x^3\right) + \frac{1}{8}\frac{d}{dx}(x) - \frac{d}{dx}(5)\\
&= \left(16x^{15}\right) + 2\sqrt{3}\left(7x^6\right) - 4\left(3x^2\right) + \frac{1}{8}(\mathbf{?}) - (\mathbf{?})
\end{aligned}$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

$$\text{If} \quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$$

$$\begin{aligned}
\text{Then} \quad \frac{dy}{dx} &= \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right) \\
&= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5) \\
&= \frac{d}{dx}\left(x^{16}\right) + 2\sqrt{3}\frac{d}{dx}\left(x^7\right) - 4\frac{d}{dx}\left(x^3\right) + \frac{1}{8}\frac{d}{dx}(x) - \frac{d}{dx}(5) \\
&= \left(16x^{15}\right) + 2\sqrt{3}\left(7x^6\right) - 4\left(3x^2\right) + \frac{1}{8}(\mathbf{?}) - (\mathbf{?})
\end{aligned}$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

$$\text{If}\quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$$

$$\begin{aligned}
\text{Then}\quad \frac{\mathrm{d}y}{\mathrm{d}x} &= \frac{\mathrm{d}}{\mathrm{d}x}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right)\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(x^{16}\right) + \frac{\mathrm{d}}{\mathrm{d}x}\left(2\sqrt{3}x^7\right) - \frac{\mathrm{d}}{\mathrm{d}x}\left(4x^3\right) + \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x}{8}\right) - \frac{\mathrm{d}}{\mathrm{d}x}(5)\\
&= \frac{\mathrm{d}}{\mathrm{d}x}\left(x^{16}\right) + 2\sqrt{3}\frac{\mathrm{d}}{\mathrm{d}x}\left(x^7\right) - 4\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3\right) + \frac{1}{8}\frac{\mathrm{d}}{\mathrm{d}x}(x) - \frac{\mathrm{d}}{\mathrm{d}x}(5)\\
&= (16x^{15}) + 2\sqrt{3}\left(7x^6\right) - 4\left(3x^2\right) + \frac{1}{8}(1) - (\mathbf{?})
\end{aligned}$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

$$\text{If}\quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$$

$$\begin{aligned}
\text{Then}\quad \frac{dy}{dx} &= \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right)\\
&= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5)\\
&= \frac{d}{dx}\left(x^{16}\right) + 2\sqrt{3}\frac{d}{dx}\left(x^7\right) - 4\frac{d}{dx}\left(x^3\right) + \frac{1}{8}\frac{d}{dx}(x) - \frac{d}{dx}(5)\\
&= \left(16x^{15}\right) + 2\sqrt{3}\left(7x^6\right) - 4\left(3x^2\right) + \frac{1}{8}(1) - (\mathbf{?})
\end{aligned}$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

$$\text{If} \quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$$

$$\begin{aligned}\text{Then} \quad \frac{dy}{dx} &= \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right)\\ &= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5)\\ &= \frac{d}{dx}\left(x^{16}\right) + 2\sqrt{3}\frac{d}{dx}\left(x^7\right) - 4\frac{d}{dx}\left(x^3\right) + \frac{1}{8}\frac{d}{dx}(x) - \frac{d}{dx}(5)\\ &= (16x^{15}) + 2\sqrt{3}\left(7x^6\right) - 4\left(3x^2\right) + \frac{1}{8}(1) - (0)\end{aligned}$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

$$\text{If}\quad y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$$

$$\begin{aligned}
\text{Then}\quad \frac{dy}{dx} &= \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right) \\
&= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5) \\
&= \frac{d}{dx}\left(x^{16}\right) + 2\sqrt{3}\frac{d}{dx}\left(x^7\right) - 4\frac{d}{dx}\left(x^3\right) + \frac{1}{8}\frac{d}{dx}(x) - \frac{d}{dx}(5) \\
&= (16x^{15}) + 2\sqrt{3}\left(7x^6\right) - 4\left(3x^2\right) + \frac{1}{8}(1) - (0) \\
&= 16x^{15} + 14\sqrt{3}x^6 - 12x^2 + \frac{1}{8}.
\end{aligned}$$