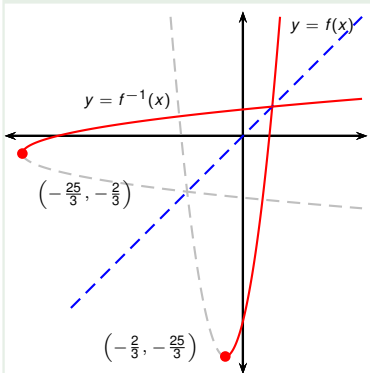# Calculus I
Inverse of a quadratic function

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Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.

$$3x^2 + 4x - 7 = y$$
$$3x^2 + 4x + (-7 - y) = 0$$

That's a quadratic equation in x. Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

$$= -\frac{2 \pm \sqrt{25 + 3y}}{3} = -\frac{2}{3} \pm \frac{\sqrt{25 + 3y}}{3}$$
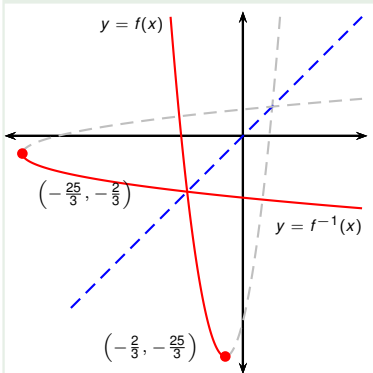
Final answer, relabelled:

$$f^{-1}(x) = -\frac{2}{3} + \frac{\sqrt{25 + 3x}}{3}$$

We are given $x \geq -\frac{2}{3}$, therefore
$$x = -\frac{2}{3} + \frac{\sqrt{25 + 3y}}{3} = f^{-1}(y).$$

Example (What if we change the problem to $x \leq -\frac{2}{3}$?)

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \leq -\frac{2}{3}$. Find $f^{-1}(x)$.

$$
\begin{aligned}
3x^2 + 4x - 7 &= y \\
3x^2 + 4x + (-7 - y) &= 0
\end{aligned}
$$

That's a quadratic equation in x. Solve:

$$
\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}
$$

$$
= -\frac{2 \pm \sqrt{25 + 3y}}{3} = -\frac{2}{3} \pm \frac{\sqrt{25 + 3y}}{3}
$$

Final answer, relabelled:

$$
f^{-1}(x) = -\frac{2}{3} - \frac{\sqrt{25 + 3x}}{3}
$$

We are given $x \leq -\frac{2}{3}$, therefore
$x = -\frac{2}{3} - \frac{\sqrt{25+3y}}{3} = f^{-1}(y)$.