

# Precalculus

## Inverse of fractional linear transformation

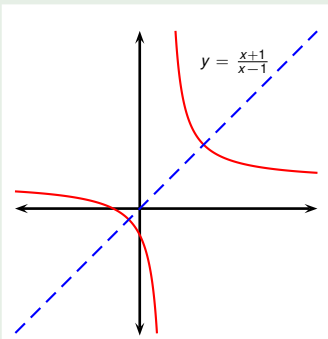
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## Example

Find  $f^{-1}(x)$  where  $f(x) = \frac{x+1}{x-1}$ .

We deal with domains and ranges later:



Answer:  $f^{-1}(x) = \frac{x+1}{x-1}$ ,  
 $x \neq 1$ .

$$\begin{array}{rcll}
 y & = & \frac{x+1}{x-1} & \left| \begin{array}{l} \text{mult. by } (x-1) \\ \text{div. by } (y-1) \\ \text{relabel } x, y \end{array} \right. \\
 y(x-1) & = & x+1 & \\
 x(y-1) & = & y+1 & \\
 f^{-1}(y) = x & = & \frac{y+1}{y-1} & \\
 f^{-1}(x) & = & \frac{x+1}{x-1} & 
 \end{array}$$

We divided by  $y-1$  so  $y \neq 1$ . Therefore the domain of  $f^{-1}$  is all real numbers except 1.

Can a non-identity function be its own inverse? Yes,  $f$  is.

What does it mean for  $f$  to be its own inverse?  
 Graph of  $f$  is symmetric across  $y = x$ .