Precalculus
Trigonometric functions with arguments translated by a multiple of $\frac{\pi}{2}$

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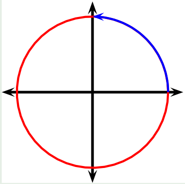
Example

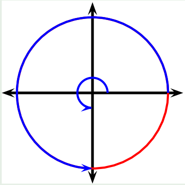
Use the angle sum/difference formulas to simplify.



$$
\begin{aligned}
\cos\left(\frac{\pi}{2} - x\right) &= \cos\left(\frac{\pi}{2}\right)\cos x + \sin\left(\frac{\pi}{2}\right)\sin x \\
&= 0 \cdot \cos(x) + 1 \cdot \sin x \\
&= \sin x
\end{aligned}
$$

Example

Use the angle sum/difference formulas to simplify.



$$
\begin{aligned}
\cot\left(\frac{3\pi}{2}+x\right) &= \frac{\cos\left(\frac{3\pi}{2}+x\right)}{\sin\left(\frac{3\pi}{2}+x\right)}\\
&= \frac{\cos\left(\frac{3\pi}{2}\right)\cos x - \sin\left(\frac{3\pi}{2}\right)\sin x}{\sin\left(\frac{3\pi}{2}\right)\cos x + \cos\left(\frac{3\pi}{2}\right)\sin x}\\
&= \frac{0\cdot\cos x - (-1)\sin x}{(-1)\cos x + 0\cdot\sin x}\\
&= \frac{\sin x}{-\cos x} = -\frac{\sin x}{\cos x}\\
&= -\tan x
\end{aligned}
$$

Example

Show that $\tan(\pi + x) = \tan x$ using the angle sum formulas.

$$\begin{aligned}
\tan(\pi + x) &= \frac{\sin(\pi + x)}{\cos(\pi + x)} \\
&= \frac{\sin\pi\cos x + \cos\pi\sin x}{\cos\pi\cos x - \sin\pi\sin x} \\
&= \frac{0\cdot\cos x + (-1)\cdot\sin x}{(-1)\cdot\cos x - 0\cdot\sin x} \\
&= \frac{-\sin x}{-\cos x} \\
&= \frac{\sin x}{\cos x} \\
&= \tan x,
\end{aligned}$$

as desired.

Proposition ($\tan, \cot$ are π-periodic)

The tangent and cotangent functions are π-periodic, in other words,

$$\begin{aligned} \tan(\theta + \pi) &= \tan \theta \\ \cot(\theta + \pi) &= \cot \theta \end{aligned}$$