Calculus I
Miscellaneous problems, part 1

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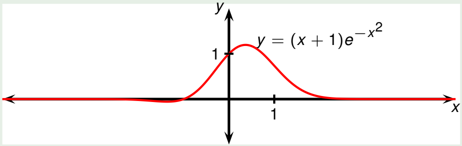
Example



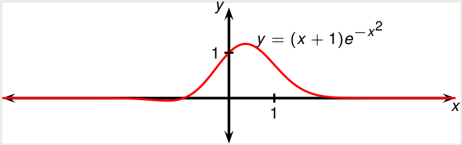
Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}(x+1)e^{-x^2} + (x+1)\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right)\\
&= 1\cdot e^{-x^2} + (x+1)\,e^{-x^2}\left(-x^2\right)'\\
&= 1\cdot e^{-x^2} + (x+1)e^{-x^2}(-2x)\\
&= (1+(x+1)(-2x))\,e^{-x^2} = \left(-2x^2-2x+1\right)e^{-x^2}
\end{aligned}$$

Example



Find the value of x for which

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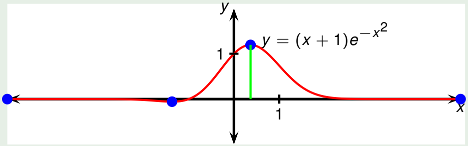
attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

$$\begin{aligned} (-2x^2 - 2x + 1)e^{-x^2} &= 0 && \Big| \text{Div. by } e^{-x^2} \neq 0 \\ -2x^2 - 2x + 1 &= 0 \\ x_1, x_2 &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-2)\cdot 1}}{2(-2)} \\ &= \frac{2 \pm \sqrt{12}}{-4} = \frac{-2 \mp 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2} \end{aligned}$$

Example



Find the value of x for which

$$f(x) = (x+1)e^{-x^2}$$

attains its maximum in the interval $[-5, 5]$. Use the given plot.

$$\frac{\mathrm{d}}{\mathrm{d}x}\left((x+1)e^{-x^2}\right) = (1 + (x+1)(-2x))\, e^{-x^2} = \left(-2x^2 - 2x + 1\right) e^{-x^2}$$

Find critical points: set $f'(x) = 0$ and solve for x:

$$(-2x^2 - 2x + 1)e^{-x^2} = 0 \qquad \Big| \text{Div. by } e^{-x^2} \neq 0$$

$$x_1, x_2 = \frac{2 \pm \sqrt{12}}{-4} = \frac{-2 \mp 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

Compare the values of f at the endpoints and the critical points:

x	$f(x)$
-5	close to 0 from plot
$\frac{-1-\sqrt{3}}{2}$	negative, min from plot
Final answer: $\frac{-1+\sqrt{3}}{2}$	positive, max from plot
5	close to 0 from plot