Calculus I
Inverse functions

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Outline

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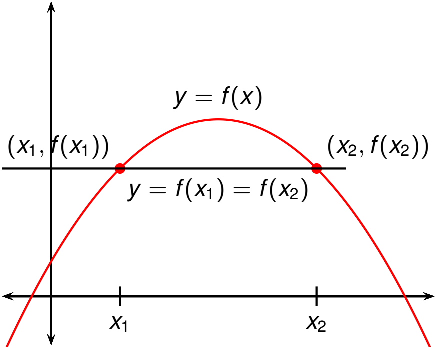
One-to-one Functions

Definition (One-to-one Function)

A function f is a one-to-one function if it never takes on the same value twice; that is,

$$f(x_1) \neq f(x_2) \text{ whenever } x_1 \neq x_2.$$

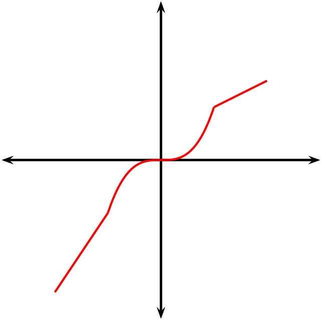


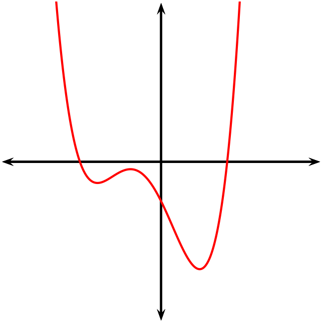
$\leftarrow$ This function is not one-to-one.

Question: How can we tell from the graph of a function whether it is one-to-one or not?
Answer: Use the horizontal line test.

The Horizontal Line Test.

A function is one-to-one if and only if no horizontal line intersects it more than once. □





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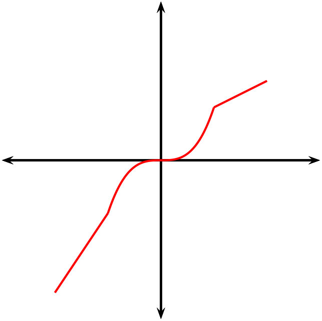

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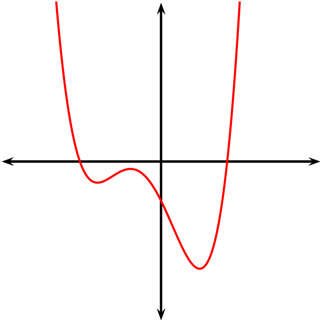

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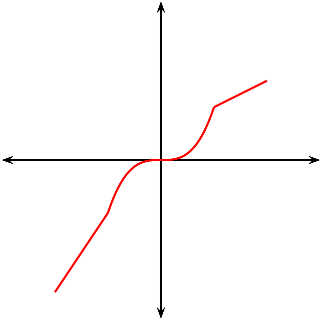

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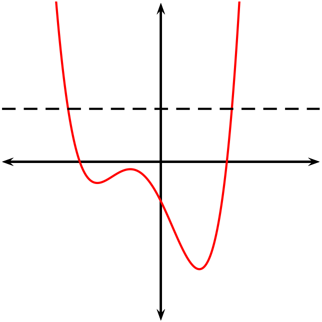

Not one-to-one

The Definition of the Inverse of f

Definition (f^{-1})

Let f be a one-to-one function with domain A and range B. Then the inverse of f is the function f^{-1} that has domain B and range A and is defined by

$$f^{-1}(y) = x \qquad \Leftrightarrow \qquad f(x) = y$$

for all y in B.

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Example ($f(x) = x^3$)

The inverse of $f(x) = x^3$ is $f^{-1}(x) = \sqrt[3]{x}$. This is because if $y = x^3$, then

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No one blamed English language of being logical.

-Bjarne Stroustrup, creator of the programming language C++

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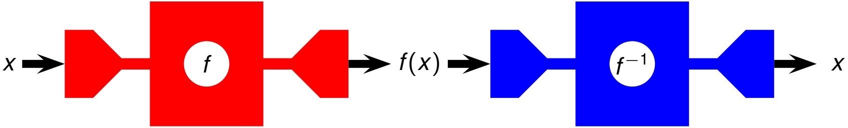

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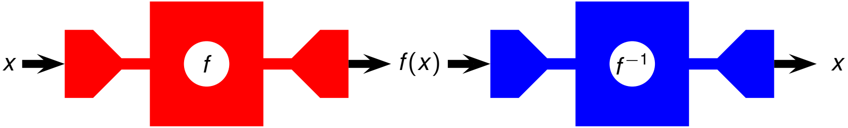

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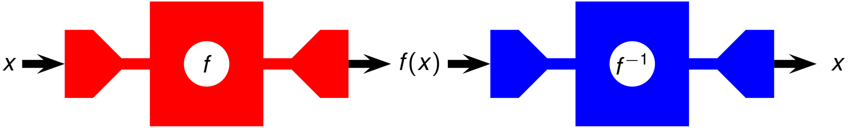

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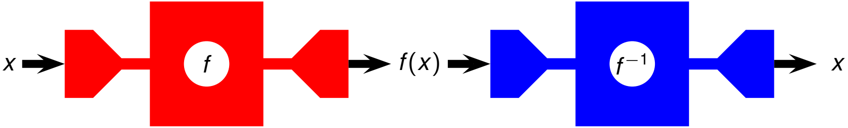

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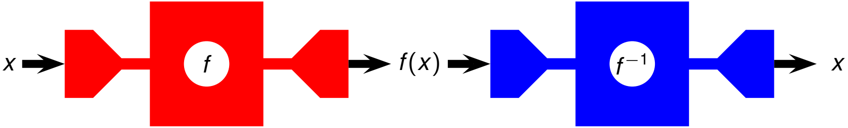

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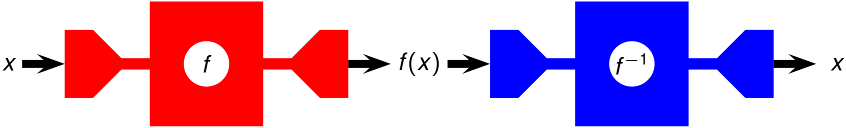

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If $f(x) = 2x + \sin 2x + e^{\frac{x}{2}}$, find $f^{-1}(1)$. You do not need to show that f has an inverse.

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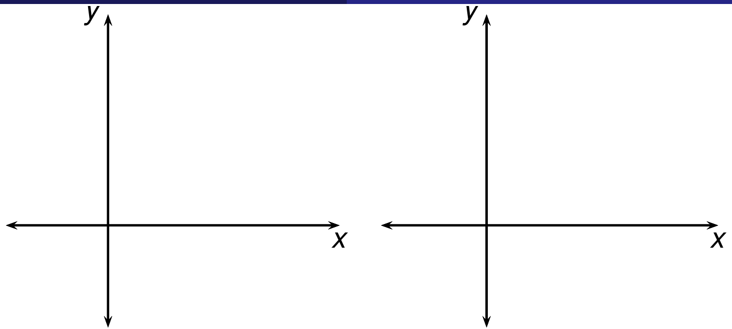
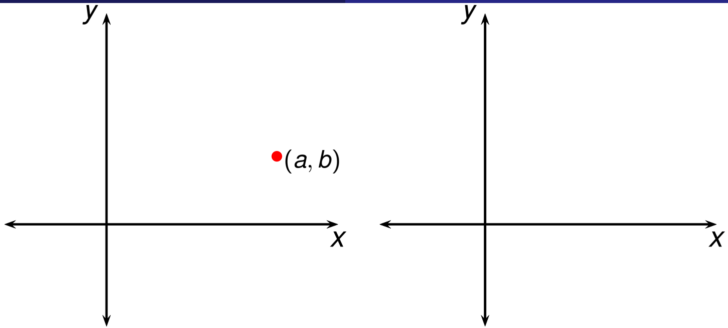


Interchanging x and y suggests relation between the graphs of f^{-1} and f:



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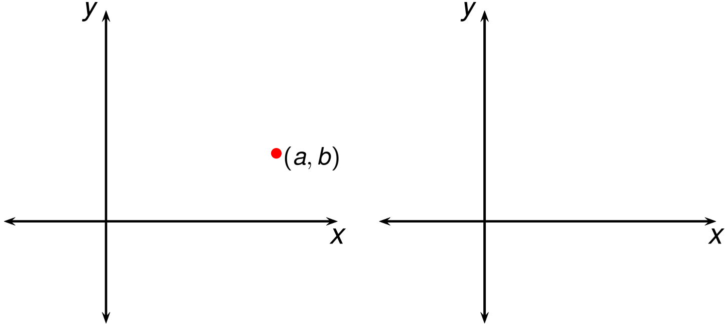

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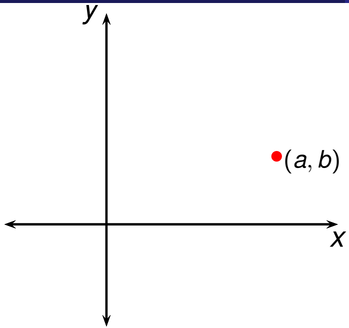

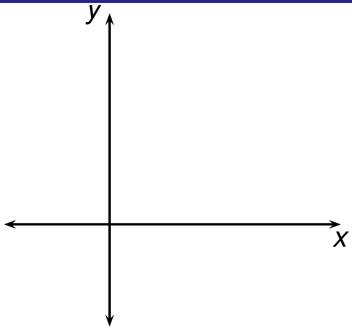

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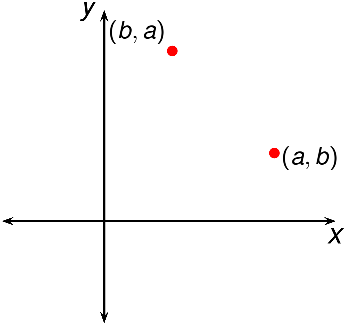

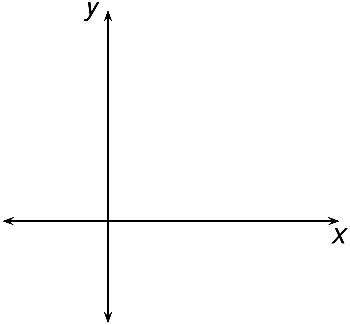

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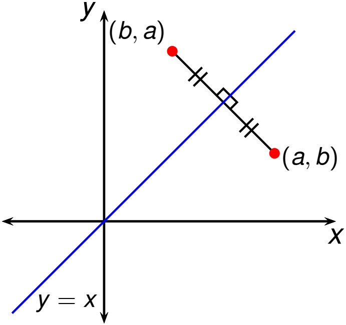
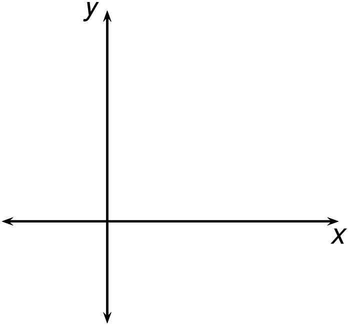
- Then (b, a) is on the graph of f^{-1}.





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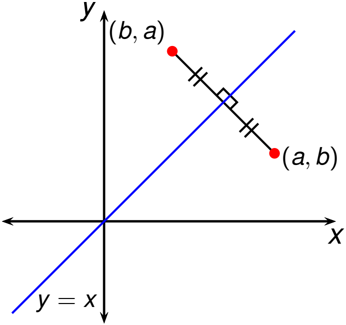
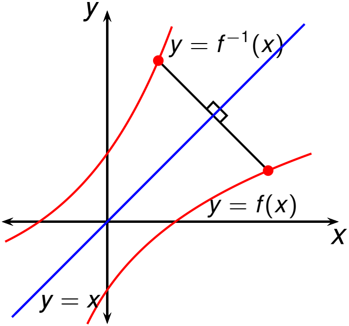
- (b, a) is the reflection of (a, b) in the line $y = x$.

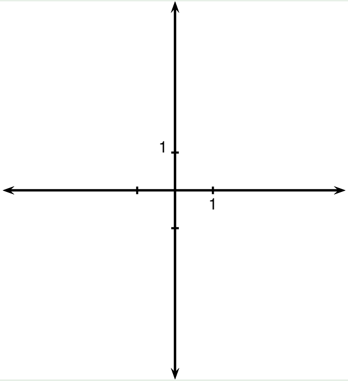




Interchanging x and y suggests relation between the graphs of f^{-1} and f:

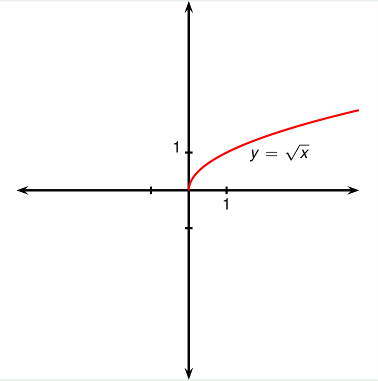
- Suppose (a, b) is on the graph of f.
- Then $f(a) = b$.
- Then $f^{-1}(b) = a$.
- Then (b, a) is on the graph of f^{-1}.
- (b, a) is the reflection of (a, b) in the line $y = x$.
- Thus the graph of f^{-1} is obtained by reflecting the graph of f in the line $y = x$.

Example



Sketch the graph of $f(x) = \sqrt{-x-1}$ and its inverse function.

Example



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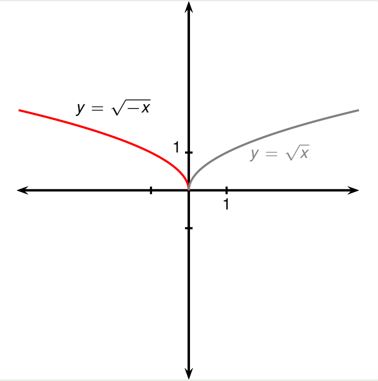
- Draw the graph of $y = \sqrt{x}$.

Example



Sketch the graph of $f(x) = \sqrt{-x-1}$ and its inverse function.

- Draw the graph of $y = \sqrt{x}$.
- $y = \sqrt{-x}$ is the reflection of $y = \sqrt{x}$ in the y-axis.

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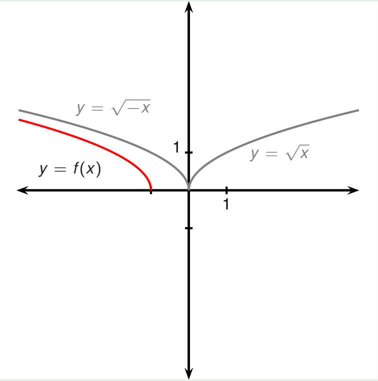

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- Draw the graph of $y = \sqrt{x}$.
- $y = \sqrt{-x}$ is the reflection of $y = \sqrt{x}$ in the y-axis.
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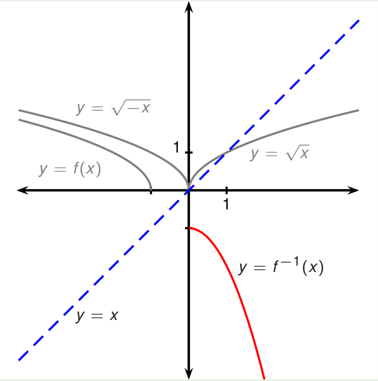

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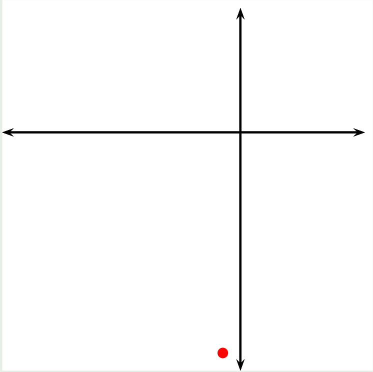
- $y = f(x) = \sqrt{-(x+1)} = \sqrt{-x-1}$ is the shift of $y = \sqrt{-x}$ one unit to the left.
- $y = f^{-1}(x)$ is the reflection of $y = f(x)$ across the line $y = x$.

Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



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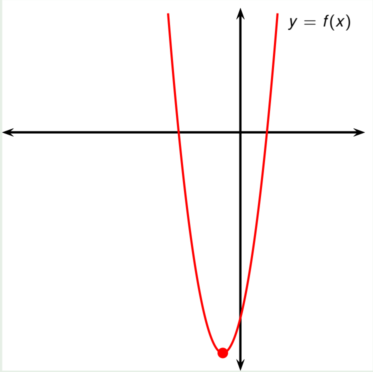

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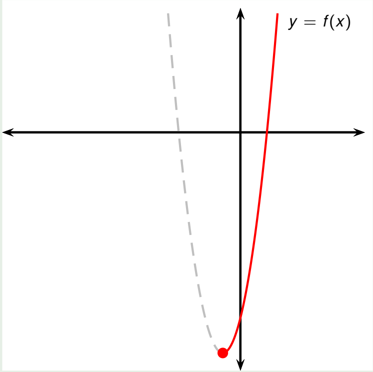

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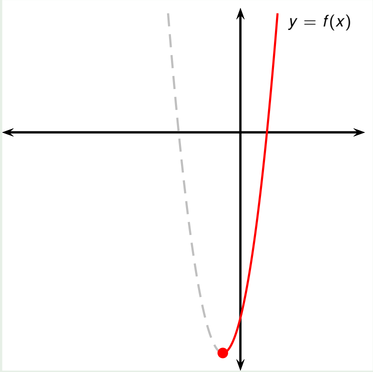

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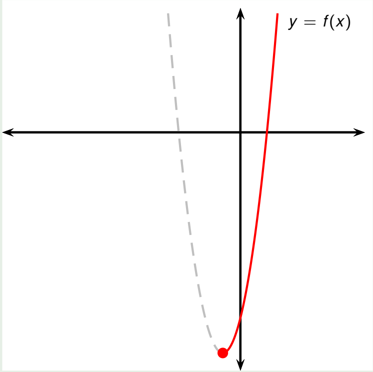

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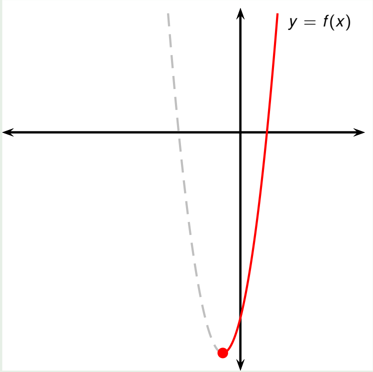

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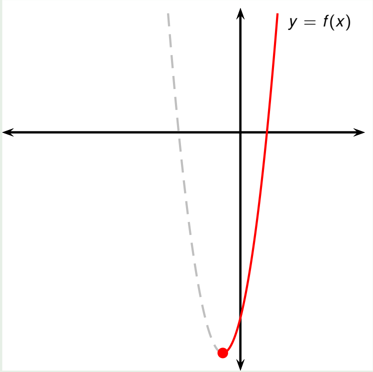

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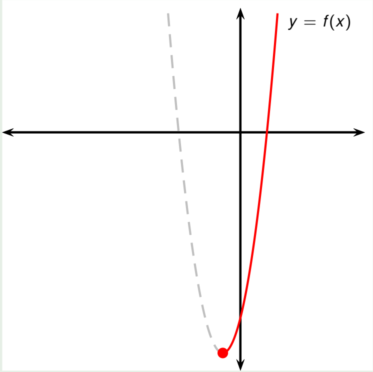

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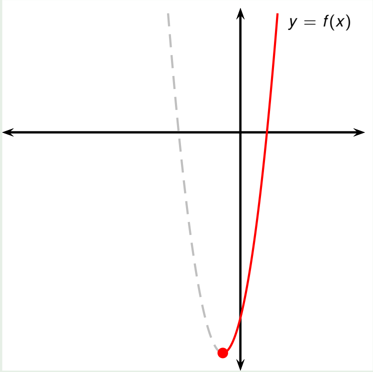

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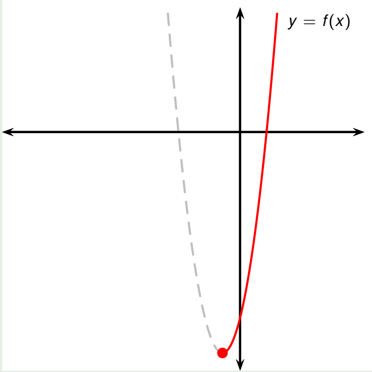

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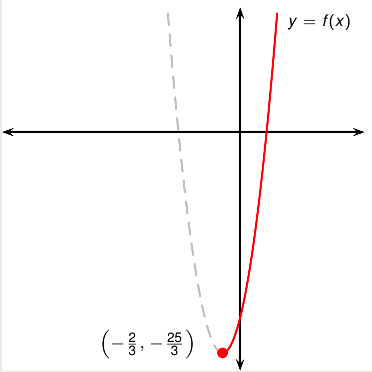

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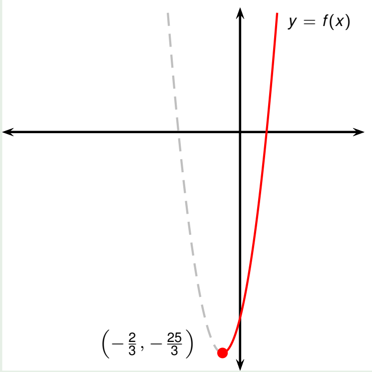

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answer

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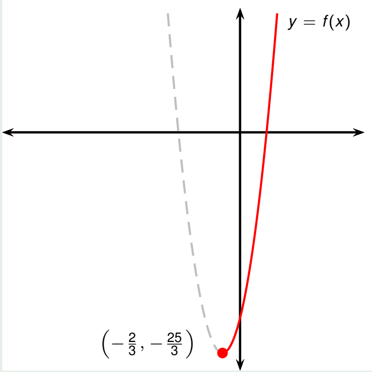

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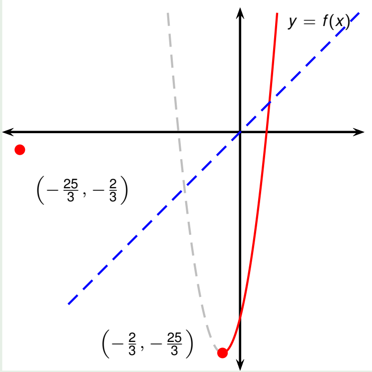

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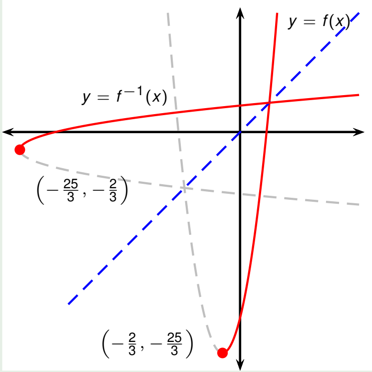

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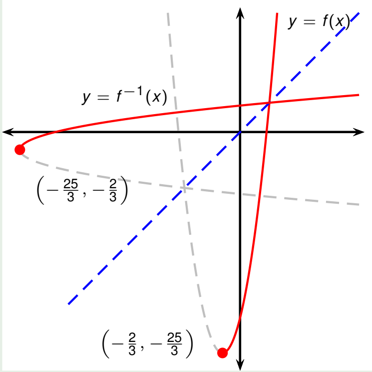

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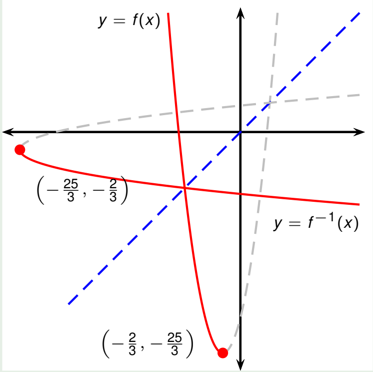

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Final answer, relabelled:

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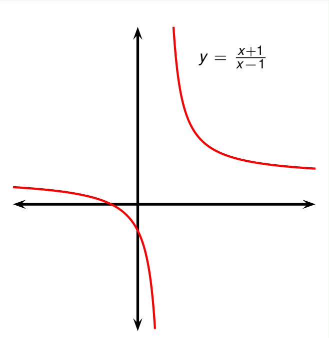
Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.



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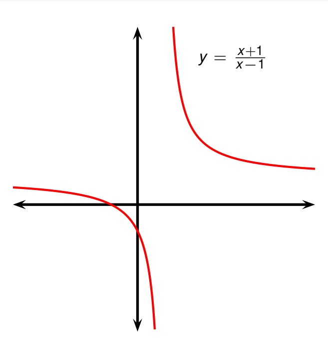

We deal with domains and ranges later:

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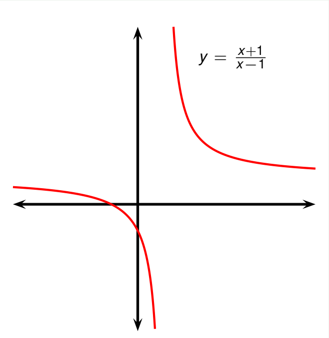

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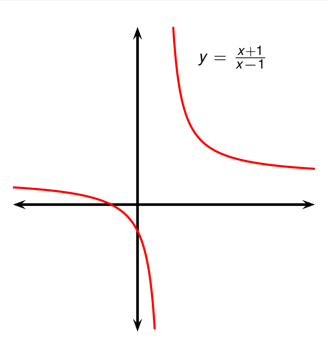

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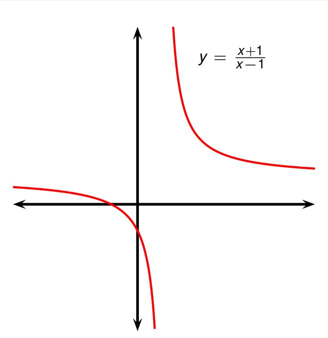

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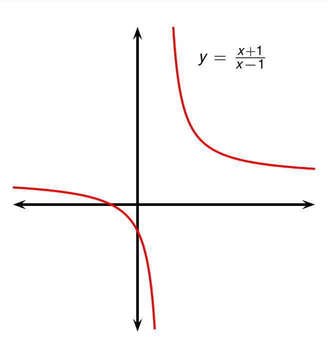

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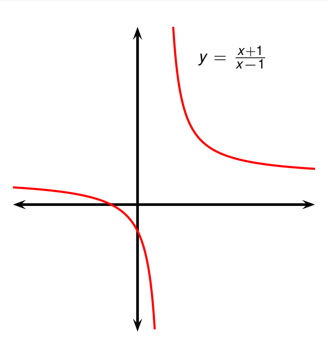

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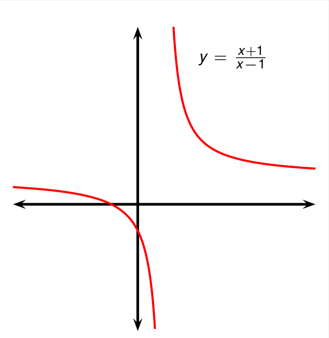

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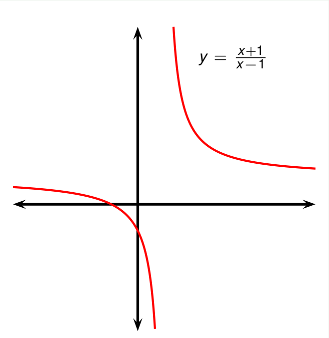

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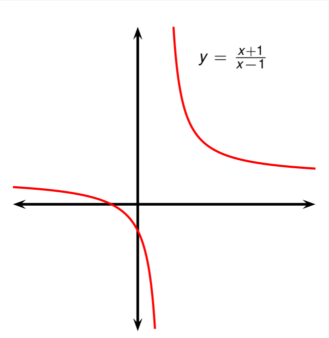

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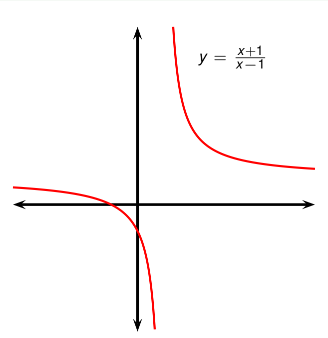
Example

Find $f^{-1}(x)$ where $f(x) = \frac{x+1}{x-1}$.



Answer: $f^{-1}(x) = \frac{x+1}{x-1}$

We deal with domains and ranges later:

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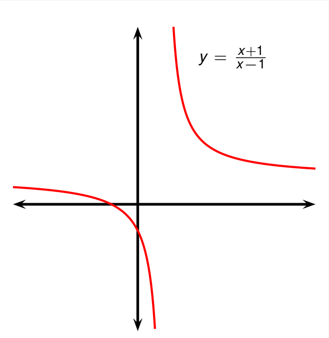

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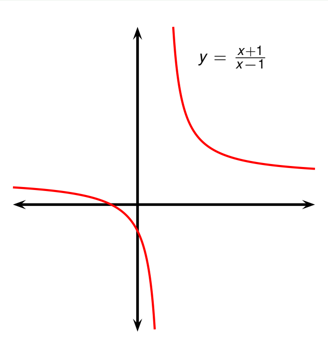

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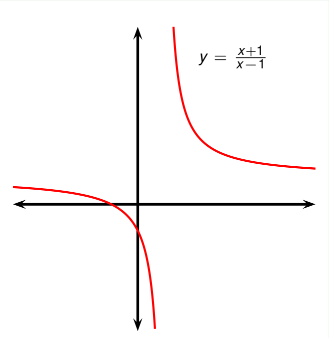

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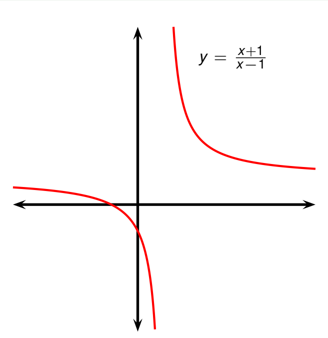

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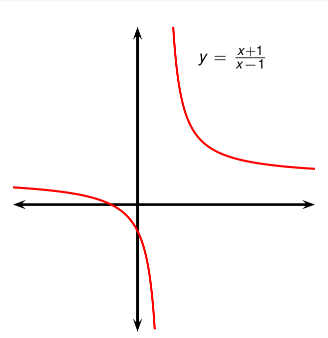

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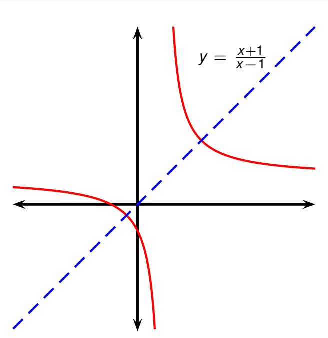

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What does it mean for f to be its own inverse?
Graph of f is symmetric across $y = x$.