Calculus I
Writing a Riemann sum, part 1

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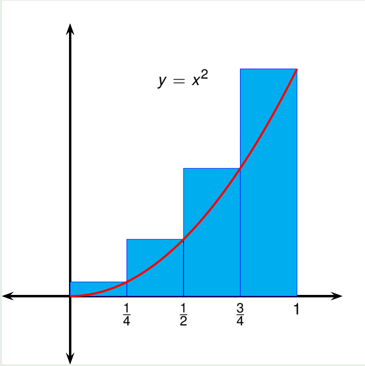
Example

Find the sum of the areas of the four approximating rectangles obtained using right endpoints.

- Let R_4 denote the sum of the areas of the rectangles.
- Each rectangle has width $\frac{1}{4}$.
- The heights are $\left(\frac{1}{4}\right)^2$, $\left(\frac{1}{2}\right)^2$, $\left(\frac{3}{4}\right)^2$, and 1^2.



$$R_4 = \frac{1}{4}\cdot\left(\frac{1}{4}\right)^2 + \frac{1}{4}\cdot\left(\frac{1}{2}\right)^2 + \frac{1}{4}\cdot\left(\frac{3}{4}\right)^2 + \frac{1}{4}\cdot(1)^2 = \frac{15}{32} = 0.46875$$

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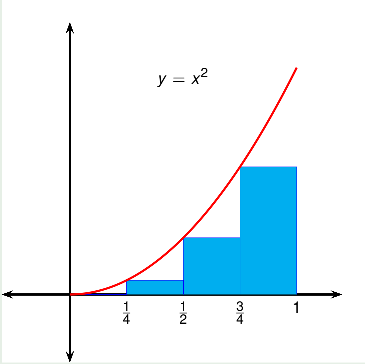
- A similar calculation works for L_4, the sum of the areas of the left endpoint rectangles.



$$R_4 = \frac{1}{4}\cdot\left(\frac{1}{4}\right)^2 + \frac{1}{4}\cdot\left(\frac{1}{2}\right)^2 + \frac{1}{4}\cdot\left(\frac{3}{4}\right)^2 + \frac{1}{4}\cdot(1)^2 = \frac{15}{32} = 0.46875$$

$$L_4 = \frac{1}{4}\cdot(0)^2 + \frac{1}{4}\cdot\left(\frac{1}{4}\right)^2 + \frac{1}{4}\cdot\left(\frac{1}{2}\right)^2 + \frac{1}{4}\cdot\left(\frac{3}{4}\right)^2 = \frac{7}{32} = 0.21875$$