Precalculus
§ Polynomial system that reduces to quadratic, part 1

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Example

Solve the polynomial system. $\left| \begin{array}{rcl} x - 4y & = & 5 \\ y^2 + xy & = & 10 \end{array} \right.$

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$x = 5 + 4y$ | Solve for x in first eq-n.

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Solve for x in first eq-n.
Substitute x away

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$$\begin{aligned} x &= 5+4y \\ &= 5+4(-2) \end{aligned}$$

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$$\begin{array}{ll} x=5+4y & x=5+4y \\ \quad =5+4(-2)=-3 & \quad =5+4\cdot 1 \end{array}$$

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$$\begin{array}{ll} x=5+4y & x=5+4y \\ =5+4(-2)=-3 & =5+4\cdot 1=9 \end{array}$$

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$$\begin{array}{ll} x=5+4y & x=5+4y \\ =5+4(-2)=-3 & =5+4\cdot 1=9 \end{array}$$

Final answer: $x=-3, y=-2$ or $x=9, y=1$.

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Check answer $x = -3, y = -2$:

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