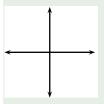
Precalculus Computing sine, cosine of a half-angle

Todor Miley

2019

Using radicals, find the exact value of the trigonometric expression.

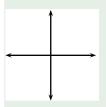
 $\cos 105^{\circ}$



Example

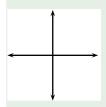
Using radicals, find the exact value of the trigonometric expression.

 $\cos 105^{\circ}$



Recall the half angle formula
$$\cos \alpha = \pm \sqrt{\frac{1 + \cos(2\alpha)}{2}}$$
.

$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos (2 \cdot 105^{\circ})}{2}}$$

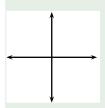


Example

Using radicals, find the exact value of the trigonometric expression.

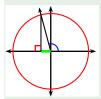
$$\cos 105^\circ = \pm \sqrt{\frac{1 + \cos \left(2 \cdot 105^\circ\right)}{2}}$$

cos 105°? 0

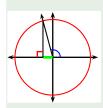


Example

$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos (2 \cdot 105^{\circ})}{2}}$$
 $\cos 105^{\circ} < 0$



Recall the half angle formula
$$\cos \alpha = \pm \sqrt{\frac{1 + \cos(2\alpha)}{2}}$$
.

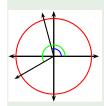


$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos (2 \cdot 105^{\circ})}{2}}$$
$$= -\sqrt{\frac{1 + \cos (210^{\circ})}{2}}$$



Recall the half angle formula
$$\cos \alpha = \pm \sqrt{\frac{1 + \cos(2\alpha)}{2}}$$
.

Using radicals, find the exact value of the trigonometric expression.



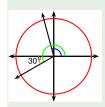
$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos (2 \cdot 105^{\circ})}{2}}$$
$$= -\sqrt{\frac{1 + \cos (210^{\circ})}{2}}$$

cos 105° <0

 $\cos 105^{\circ} < 0$

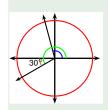
Recall the half angle formula
$$\cos \alpha = \pm \sqrt{\frac{1 + \cos(2\alpha)}{2}}$$
.

Example



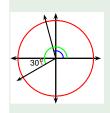
$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos (2 \cdot 105^{\circ})}{2}} \\ = -\sqrt{\frac{1 + \cos (210^{\circ})}{2}}$$

Recall the half angle formula
$$\cos \alpha = \pm \sqrt{\frac{1 + \cos(2\alpha)}{2}}$$
.



$$\begin{aligned} \cos 105^\circ &= \pm \sqrt{\frac{1 + \cos \left(2 \cdot 105^\circ\right)}{2}} & & \left| \cos 105^\circ < 0 \right. \\ &= -\sqrt{\frac{1 + \cos \left(210^\circ\right)}{2}} \\ &= -\sqrt{\frac{1 - \cos \left(30^\circ\right)}{2}} \end{aligned}$$

Example



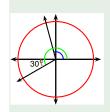
$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos(2 \cdot 105^{\circ})}{2}} \qquad \left| \cos 105^{\circ} < 0 \right|$$

$$= -\sqrt{\frac{1 + \cos(210^{\circ})}{2}}$$

$$= -\sqrt{\frac{1 - \cos(30^{\circ})}{2}}$$

$$= -\sqrt{\frac{1 - ?}{2}}$$

Example



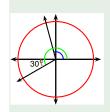
$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos(2 \cdot 105^{\circ})}{2}} \qquad \left| \cos 105^{\circ} < 0 \right|$$

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$$= -\sqrt{\frac{1 - \cos(30^{\circ})}{2}}$$

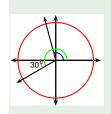
$$= -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$$

Example



$$\begin{split} \cos 105^\circ &= \pm \sqrt{\frac{1 + \cos \left(2 \cdot 105^\circ\right)}{2}} \\ &= -\sqrt{\frac{1 + \cos \left(210^\circ\right)}{2}} \\ &= -\sqrt{\frac{1 - \cos \left(30^\circ\right)}{2}} \\ &= -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2 - \sqrt{3}}{2 \cdot 2}} \end{split}$$

Example



$$\cos 105^{\circ} = \pm \sqrt{\frac{1 + \cos(2 \cdot 105^{\circ})}{2}} \quad \left| \cos 105^{\circ} < 0 \right|$$

$$= -\sqrt{\frac{1 + \cos(210^{\circ})}{2}}$$

$$= -\sqrt{\frac{1 - \cos(30^{\circ})}{2}}$$

$$= -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2 - \sqrt{3}}{2 \cdot 2}}$$

$$= -\frac{\sqrt{2 - \sqrt{3}}}{2}$$