Calculus I

$$\int x^3\sqrt{ax^2+b}\,\mathbf{d}x$$

Todor Milev

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Example (Substitution Rule, more factors)

Evaluate $\displaystyle\int 3x^5\sqrt{1+x^3}\mathrm{d}x$.

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Let $u = \textbf{?}$

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Let $u = 1 + x^3$.

Example (Substitution Rule, more factors)

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$$\text{Let } u = 1 + x^3.$$

$$\text{Then } \mathrm{d}u = \mathbf{?}$$

Example (Substitution Rule, more factors)

Evaluate $\displaystyle\int 3x^5\sqrt{1+x^3}\mathrm{d}x$.

$$\text{Let } u = 1 + x^3.$$

$$\text{Then } \mathrm{d}u = 3x^2\mathrm{d}x.$$

Example (Substitution Rule, more factors)

Evaluate $\displaystyle\int 3x^5\sqrt{1+x^3}\mathrm{d}x = \int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x.$

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$$x^3 = u - 1.$$

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Evaluate $\displaystyle\int 3x^5\sqrt{1+x^3}\mathrm{d}x = \int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x.$

$$\begin{aligned}
\text{Let } u &= 1+x^3.\\
\text{Then } \mathrm{d}u &= 3x^2\mathrm{d}x.\\
x^3 &= u-1.\\
\int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x &= \int (u-1)\sqrt{u}\,\mathrm{d}u\\
&= \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u
\end{aligned}$$

Example (Substitution Rule, more factors)

Evaluate $\int 3x^5\sqrt{1+x^3}\mathrm{d}x = \int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x.$

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x^3 &= u-1.\\
\int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x &= \int (u-{\color{red}1}){\color{red}\sqrt{u}}\,\mathrm{d}u\\
&= \int \left(u^{\frac{3}{2}} - {\color{red}u^{\frac{1}{2}}}\right)\mathrm{d}u
\end{aligned}$$

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&= \left(\mathbf{?} \quad - \mathbf{?} \quad\right)
\end{aligned}$$

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&= \left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \mathbf{?}\quad\right)
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\int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x &= \int (u-1)\sqrt{u}\,\mathrm{d}u\\
&= \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u\\
&= \left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right)
\end{aligned}$$

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$$\text{Evaluate } \int 3x^5\sqrt{1+x^3}\mathrm{d}x = \int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x.$$

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&= \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u\\
&= \left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right) + C
\end{aligned}$$

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$$\text{Evaluate } \int 3x^5\sqrt{1+x^3}\mathrm{d}x = \int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x.$$

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\int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x &= \int (u-1)\sqrt{u}\,\mathrm{d}u\\
&= \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u\\
&= \left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right) + C\\
&= \frac{2}{5}\left(1+x^3\right)^{\frac{5}{2}} - \frac{2}{3}\left(1+x^3\right)^{\frac{3}{2}} + C.
\end{aligned}$$