Calculus II
Review of integration basics

Todor Milev

2019

Outline

1. Integration, Review
    - The Evaluation Theorem (FTC part 2)

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   - The Evaluation Theorem (FTC part 2)

2. Integration Techniques from Calc I, Review
   - Differential Forms, Review

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Antiderivatives

Definition (Antiderivative)

A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

Theorem (The Evaluation Theorem (FTC part 2))

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a),$$

where F is any antiderivative of f.

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Theorem

Let f be a continuous function on $[a, b]$. Then f is integrable over $[a, b]$.

In other words, $\int_a^b f(x)dx$ exists for any continuous (over $[a, b]$) function f.

Theorem (The Evaluation Theorem (FTC part 2))

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Indefinite Integrals

- The Evaluation Theorem establishes a connection between antiderivatives and definite integrals.
- It says that $\int_a^b f(x)\mathrm{d}x$ equals $F(b) - F(a)$, where F is an antiderivative of f.
- We need convenient notation for writing antiderivatives.
- This is what the indefinite integral is.

Definition (Indefinite Integral)

The indefinite integral of f is another way of saying the antiderivative of f, and is written $\int f(x)\mathrm{d}x$. In other words,

$$\int f(x)\mathrm{d}x = F(x) \qquad \text{means} \qquad F'(x) = f(x).$$

Example

$$\int x^4 \mathrm{d}x = \textbf{?}$$

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$$\int x^4 \mathrm{d}x = \frac{x^5}{5}$$

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$$\int x^4 dx = \frac{x^5}{5} + C$$

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- We adopt the convention that the constant participating in an indefinite integral is only valid on one interval.
- $\int \frac{1}{x}\mathrm{d}x = \ln|x| + C$, and this is valid either on $(-\infty, 0)$ or $(0, \infty)$.

Differentials

- Recall $\Delta y, \Delta x$ stand for change of x, y. Recall: $\Delta y \approx \frac{\mathrm{d}y}{\mathrm{d}x}\Delta x$
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- Define the *differential* d and the *differential forms* $\mathrm{d}x$, $\mathrm{d}(f(x))$ by requesting that d and $\mathrm{d}x$ satisfy the transformation law

  $$\mathrm{d}(f(x)) = f'(x)\mathrm{d}x$$

  for any differentiable function $f(x)$. In abbreviated notation:

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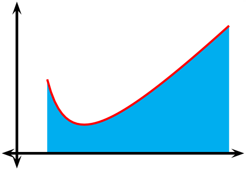

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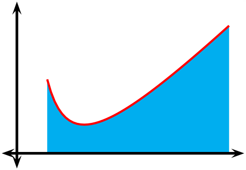

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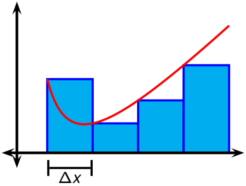

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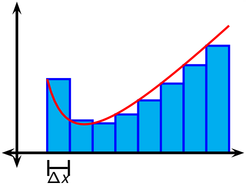

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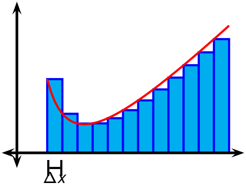

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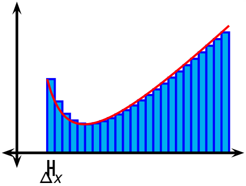

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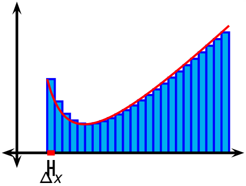

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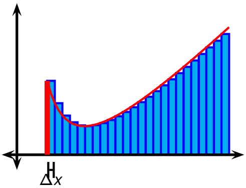

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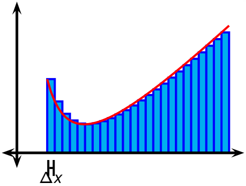

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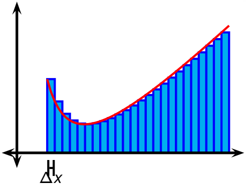

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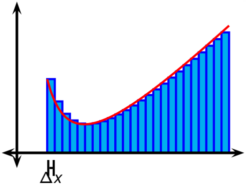

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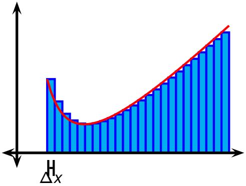

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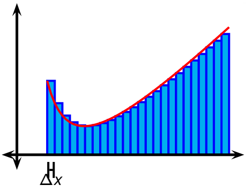

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- The rules for computing differential forms are a direct consequence of the corresponding derivative rules and the transformation law $\mathrm{d}(f(x)) = f'(x)\mathrm{d}x$.

Rule name: product rule.

Differential rule

Derivative rule

$(fg)' = f'g + fg'$

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Differential rule

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Derivative rule

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Let c be a constant. Rule name: constant derivative rule.

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
	$(c)' = 0$

Let c be a constant. Rule name: constant derivative rule.

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$

Let c be a constant. Rule name:

Differential rule	Derivative rule
$d(fg) = \quad g df + \quad f dg$	$(fg)' = f'g + fg'$
$dc = 0$	$(c)' = 0$
	$(cf)' = cf'$

Let c be a constant. Rule name:

Differential rule	Derivative rule
$d(fg) = \quad g df + \quad f dg$	$(fg)' = f'g + fg'$
$dc = 0$	$(c)' = 0$
$d(cf) = c \quad df$	$(cf)' = cf'$

Let c be a constant. Rule name: sum rule.

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$
$\mathrm{d}(cf) = c \quad \mathrm{d}f$	$(cf)' = cf'$
	$(f+g)' = f' + g'$

Let c be a constant. Rule name: sum rule.

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$
$\mathrm{d}(cf) = c \quad \mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \quad \mathrm{d}f + \quad \mathrm{d}g$	$(f+g)' = f' + g'$

Let c be a constant. Rule name: chain rule.

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$
$\mathrm{d}(cf) = c \quad \mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \quad \mathrm{d}f + \quad \mathrm{d}g$	$(f+g)' = f' + g'$
	$(f(g(x)))' = f'(g(x))g'(x)$

Let c be a constant. Rule name: chain rule.

Differential rule	Derivative rule
$d(fg) = \quad g df + \quad f dg$	$(fg)' = f'g + fg'$
$dc = 0$	$(c)' = 0$
$d(cf) = c \quad df$	$(cf)' = cf'$
$d(f+g) = \quad df + \quad dg$	$(f+g)' = f' + g'$
$df(g(x)) = \quad f'(g(x))dg(x)$	
$= \quad f'(g(x))g'(x)dx$	$(f(g(x)))' = f'(g(x))g'(x)$
$df(g) \quad = \quad f'(g)dg$	

Let c be a constant. Rule name: power rule.

Differential rule	Derivative rule
$\mathrm{d}(fg) = g\mathrm{d}f + f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$
$\mathrm{d}(cf) = c\,\mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \mathrm{d}f + \mathrm{d}g$	$(f+g)' = f' + g'$
$\mathrm{d}f(g(x)) = f'(g(x))\mathrm{d}g(x)$	
$= f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) = f'(g)\mathrm{d}g$	
	$(x^n)' = nx^{n-1}$

Let c be a constant. Rule name: power rule.

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$
$\mathrm{d}(cf) = c \quad \mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \quad \mathrm{d}f + \quad \mathrm{d}g$	$(f+g)' = f' + g'$
$\mathrm{d}f(g(x)) = \quad f'(g(x))\mathrm{d}g(x)$	
$= \quad f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) \quad = \quad f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$

Let c be a constant. Rule name: exponent derivative rule.

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$
$\mathrm{d}(cf) = c \quad \mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \quad \mathrm{d}f + \quad \mathrm{d}g$	$(f+g)' = f' + g'$
$\mathrm{d}f(g(x)) = \quad f'(g(x))\mathrm{d}g(x)$	
$= \quad f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) \quad = \quad f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
	$(e^x)' = e^x$

Let c be a constant. Rule name: exponent derivative rule.

Differential rule	Derivative rule
$d(fg) = \quad g df + \quad f dg$	$(fg)' = f'g + fg'$
$dc = 0$	$(c)' = 0$
$d(cf) = c \quad df$	$(cf)' = cf'$
$d(f+g) = \quad df + \quad dg$	$(f+g)' = f' + g'$
$df(g(x)) = \quad f'(g(x))dg(x)$	
$= \quad f'(g(x))g'(x)dx$	$(f(g(x)))' = f'(g(x))g'(x)$
$df(g) \quad = \quad f'(g)dg$	
$dx^n = nx^{n-1}dx$	$(x^n)' = nx^{n-1}$
$de^x = e^x dx$	$(e^x)' = e^x$

Let c be a constant. Rule name:

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$
$\mathrm{d}(cf) = c \quad \mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \quad \mathrm{d}f + \quad \mathrm{d}g$	$(f+g)' = f' + g'$
$\mathrm{d}f(g(x)) = \quad f'(g(x))\mathrm{d}g(x)$	
$= \quad f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) \quad = \quad f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
	$(\sin x)' = \cos x$

Let c be a constant. Rule name:

Differential rule	Derivative rule
$\mathrm{d}(fg) = g\mathrm{d}f + f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$
$\mathrm{d}(cf) = c\,\mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \mathrm{d}f + \mathrm{d}g$	$(f+g)' = f' + g'$
$\mathrm{d}f(g(x)) = f'(g(x))\mathrm{d}g(x)$	
$= f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) = f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$

Let c be a constant. Rule name:

Differential rule	Derivative rule
$\mathrm{d}(fg) = g\mathrm{d}f + f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$
$\mathrm{d}(cf) = c\,\mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \mathrm{d}f + \mathrm{d}g$	$(f+g)' = f' + g'$
$\mathrm{d}f(g(x)) = f'(g(x))\mathrm{d}g(x)$	
$= f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) = f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$
	$(\cos x)' = -\sin x$

Let c be a constant. Rule name:

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$
$\mathrm{d}(cf) = c \quad \mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \quad \mathrm{d}f + \quad \mathrm{d}g$	$(f+g)' = f' + g'$
$\mathrm{d}f(g(x)) = \quad f'(g(x))\mathrm{d}g(x)$	
$= \quad f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) = \quad f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$
$\mathrm{d}\cos x = -\sin x\mathrm{d}x$	$(\cos x)' = -\sin x$

Let c be a constant. Rule name:

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$
$\mathrm{d}(cf) = c \quad \mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \quad \mathrm{d}f + \quad \mathrm{d}g$	$(f+g)' = f' + g'$
$\mathrm{d}f(g(x)) = \quad f'(g(x))\mathrm{d}g(x)$	
$= \quad f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) \quad = \quad f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$
$\mathrm{d}\cos x = -\sin x\mathrm{d}x$	$(\cos x)' = -\sin x$
	$(\ln x)' = \dfrac{1}{x}$

Let c be a constant. Rule name:

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$
$\mathrm{d}(cf) = c \quad \mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \quad \mathrm{d}f + \quad \mathrm{d}g$	$(f+g)' = f' + g'$
$\mathrm{d}f(g(x)) = \quad f'(g(x))\mathrm{d}g(x)$	
$\qquad = \quad f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) \quad = \quad f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$
$\mathrm{d}\cos x = -\sin x\mathrm{d}x$	$(\cos x)' = -\sin x$
$\mathrm{d}\ln x = \frac{1}{x}\mathrm{d}x$	$(\ln x)' = \frac{1}{x}$

Let c be a constant. Rule name:
Corresponding integration rules.

Integration rule	Derivative rule
$\int \mathrm{d}(fg) = \int g\mathrm{d}f + \int f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\int \mathrm{d}c = 0$	$(c)' = 0$
$\int \mathrm{d}(cf) = c\int \mathrm{d}f$	$(cf)' = cf'$
$\int \mathrm{d}(f+g) = \int \mathrm{d}f + \int \mathrm{d}g$	$(f+g)' = f' + g'$
$\int \mathrm{d}f(g(x)) = \int f'(g(x))\mathrm{d}g(x)$	
$= \int f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\int \mathrm{d}f(g) = \int f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$
$\mathrm{d}\cos x = -\sin x\mathrm{d}x$	$(\cos x)' = -\sin x$
$\mathrm{d}\ln x = \frac{1}{x}\mathrm{d}x$	$(\ln x)' = \frac{1}{x}$

Let c be a constant. Rule name: Integration by parts.
Corresponding integration rules.

Integration rule	Derivative rule
$\int \mathrm{d}(fg) = \int g\mathrm{d}f + \int f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\int \mathrm{d}c = 0$	$(c)' = 0$
$\int \mathrm{d}(cf) = c\int \mathrm{d}f$	$(cf)' = cf'$
$\int \mathrm{d}(f+g) = \int \mathrm{d}f + \int \mathrm{d}g$	$(f+g)' = f' + g'$
$\int \mathrm{d}f(g(x)) = \int f'(g(x))\mathrm{d}g(x)$	
$= \int f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\int \mathrm{d}f(g) = \int f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$
$\mathrm{d}\cos x = -\sin x\mathrm{d}x$	$(\cos x)' = -\sin x$
$\mathrm{d}\ln x = \frac{1}{x}\mathrm{d}x$	$(\ln x)' = \frac{1}{x}$

Let c be a constant. Rule name: Integration is linear.
Corresponding integration rules.

Integration rule	Derivative rule
$\int d(fg) = \int g df + \int f dg$	$(fg)' = f'g + fg'$
$\int dc = 0$	$(c)' = 0$
$\int d(cf) = c \int df$	$(cf)' = cf'$
$\int d(f+g) = \int df + \int dg$	$(f+g)' = f' + g'$
$\int df(g(x)) = \int f'(g(x)) dg(x)$	
$= \int f'(g(x)) g'(x) dx$	$(f(g(x)))' = f'(g(x))g'(x)$
$\int df(g) = \int f'(g) dg$	
$dx^n = nx^{n-1}dx$	$(x^n)' = nx^{n-1}$
$de^x = e^x dx$	$(e^x)' = e^x$
$d\sin x = \cos x dx$	$(\sin x)' = \cos x$
$d\cos x = -\sin x dx$	$(\cos x)' = -\sin x$
$d\ln x = \frac{1}{x} dx$	$(\ln x)' = \frac{1}{x}$

Let c be a constant. Rule name: Substitution rule.
Corresponding integration rules.

Integration rule	Derivative rule
$\int \mathrm{d}(fg) = \int g\mathrm{d}f + \int f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\int \mathrm{d}c = 0$	$(c)' = 0$
$\int \mathrm{d}(cf) = c\int \mathrm{d}f$	$(cf)' = cf'$
$\int \mathrm{d}(f+g) = \int \mathrm{d}f + \int \mathrm{d}g$	$(f+g)' = f' + g'$
$\int \mathrm{d}f(g(x)) = \int f'(g(x))\mathrm{d}g(x)$	
$= \int f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\int \mathrm{d}f(g) = \int f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$
$\mathrm{d}\cos x = -\sin x\mathrm{d}x$	$(\cos x)' = -\sin x$
$\mathrm{d}\ln x = \frac{1}{x}\mathrm{d}x$	$(\ln x)' = \frac{1}{x}$

Let c be a constant. Rule name:

Corresponding integration rules. Integration rules justified via the Fundamental Theorem of Calculus

Integration rule	Derivative rule
$\int \mathrm{d}(fg) = \int g\mathrm{d}f + \int f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\int \mathrm{d}c = 0$	$(c)' = 0$
$\int \mathrm{d}(cf) = c\int \mathrm{d}f$	$(cf)' = cf'$
$\int \mathrm{d}(f+g) = \int \mathrm{d}f + \int \mathrm{d}g$	$(f+g)' = f' + g'$
$\int \mathrm{d}f(g(x)) = \int f'(g(x))\mathrm{d}g(x)$	
$= \int f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\int \mathrm{d}f(g) = \int f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$
$\mathrm{d}\cos x = -\sin x\mathrm{d}x$	$(\cos x)' = -\sin x$
$\mathrm{d}\ln x = \frac{1}{x}\mathrm{d}x$	$(\ln x)' = \frac{1}{x}$

We recall from previous slides that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\ln|x|) = \frac{1}{x}.$$

This formula has a special application to integration:

Theorem (The Integral of $1/x$)

$$\int \frac{1}{x}\mathrm{d}x = \ln|x| + C.$$

We recall from previous slides that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\ln|x|) = \frac{1}{x}.$$

This formula has a special application to integration:

Theorem (The Integral of $1/x$)

$$\int \frac{1}{x}\mathrm{d}x = \ln|x| + C.$$

This fills in the gap in the rule for integrating power functions:

$$\int x^n \mathrm{d}x = \frac{x^{n+1}}{n+1} + C, \qquad n \neq -1.$$

Now we know the formula for $n = -1$ too.