

Calculus II

Integrals of the form $\int \tan^m x \sec^n x dx$, $n, m > 0$,
 m -odd

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Example

$$\begin{aligned}
 \int \tan^5 x \sec^9 x dx &= \int \tan^4 x \sec^8 x \tan x \sec x dx \\
 &= \int \tan^4 x \sec^8 x d(\sec x) && \left| \begin{array}{l} \text{Can we rewrite} \\ \tan^4 x \text{ via } \sec x? \end{array} \right. \\
 &= \int (\tan^2 x)^2 \sec^8 x d(\sec x) \\
 &= \int (\sec^2 x - 1)^2 \sec^8 x d(\sec x) && \left| \begin{array}{l} \text{Set } u = \sec x \end{array} \right. \\
 &= \int (1 - u^2)^2 u^8 du \\
 &= \int (1 - 2u^2 + u^4) u^8 du \\
 &= \int (u^8 - 2u^{10} + u^{12}) du \\
 &= \frac{u^9}{9} - 2\frac{u^{11}}{11} + \frac{u^{13}}{13} + C \\
 &= \frac{\sec^9 x}{9} - 2\frac{\sec^{11} x}{11} + \frac{\sec^{13} x}{13} + C .
 \end{aligned}$$