

## Precalculus

# Factor cubic with one real root using its plot

Todor Milev

2019

## Example

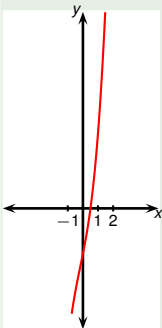
Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

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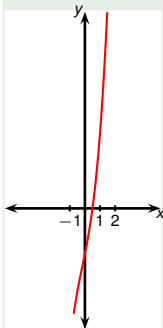


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We see only one root,  $x = ?$  .

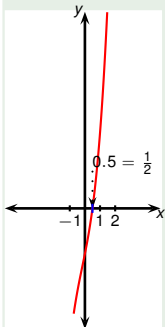


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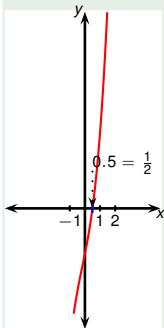


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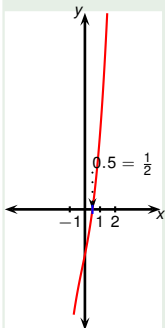
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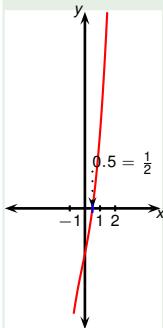
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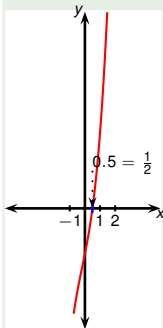
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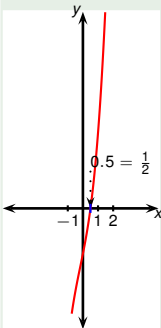
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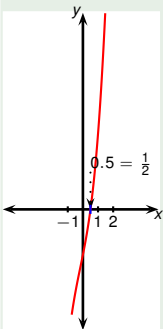


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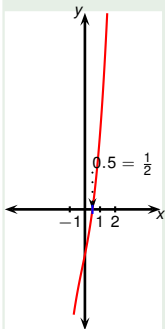
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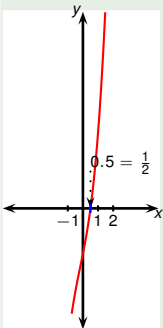
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$$x - \frac{1}{2} \overline{) \begin{array}{l} 2x^3 + x^2 + 5x - 3 \\ \phantom{2x^3 + } 2x^2 \phantom{+ 5x - 3} \\ \hline \phantom{2x^3 + } ? \phantom{+ 5x - 3} \\ \phantom{2x^3 + } ? \phantom{+ 5x - 3} \end{array}}$$

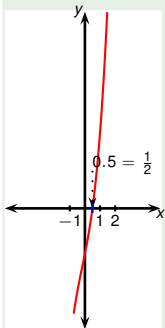
Multiply  $2x^2$  by divisor.

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$$\begin{array}{r}
 2x^2 \\
 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
 \underline{2x^3 - x^2} \phantom{+ 5x - 3}
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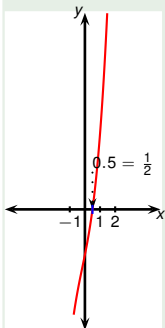
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 \phantom{x - \frac{1}{2}} \phantom{2x^2 +} 5x - 3 \phantom{=} \\
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 \end{array}$$

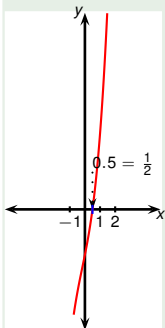
Subtract last two polynomials.

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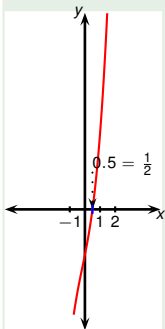


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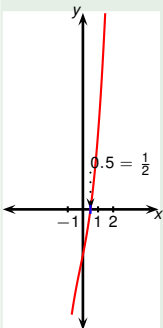
Divide  $2x^2$  by  $x$ .

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 2x^2 + 2x \\
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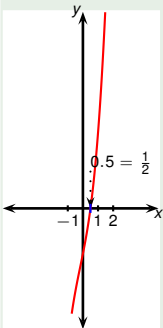
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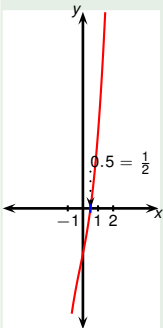
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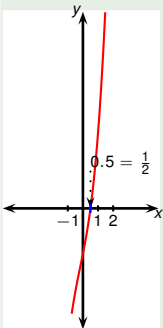
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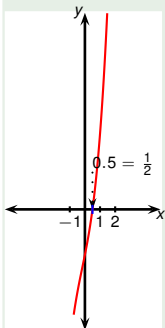
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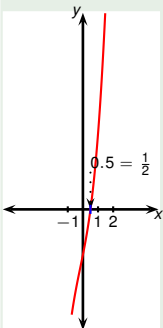
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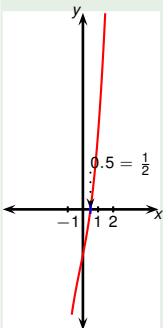
Divide  $6x$  by  $x$ .

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$$\begin{array}{r}
 2x^2 + 2x + 6 \\
 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
 \underline{2x^3 - x^2} \phantom{- 3} \\
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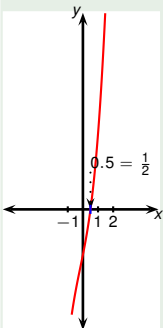


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 6x - 3 \\
 \underline{\phantom{6x} 6x - 3} \\
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 \end{array}$$

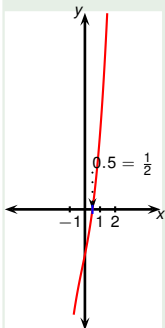
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 6x - 3 \\
 \underline{\phantom{6x} - 6x + 3} \\
 0
 \end{array}$$

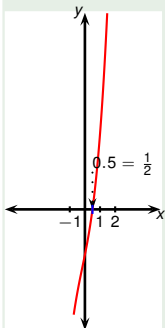
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 \phantom{x - \frac{1}{2}} \phantom{2x^2 + 5x - 3} \phantom{6x - 3} ?
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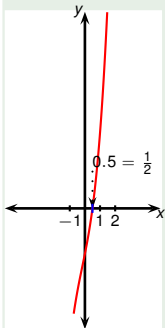
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 6x - 3 \\
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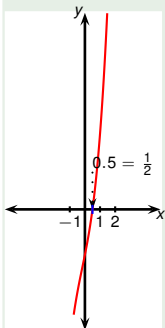
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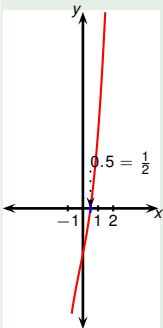
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Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2}) (2x^2 + 2x + 6) + 0 = 0$$

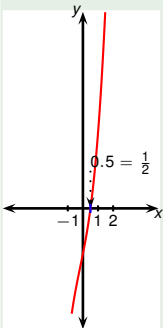
We see only one root,  $x = 0.5 = \frac{1}{2}$ . Is our guess correct?

Is there another root (far away from 0)? Factor:

**Quotient:**  $2x^2 + 2x + 6$

$$\begin{array}{r}
 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
 \underline{2x^3 - x^2} \phantom{- 3} \\
 2x^2 + 5x - 3 \\
 \underline{2x^2 - x} \phantom{- 3} \\
 6x - 3 \\
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 0
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Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2})(2x^2 + 2x + 6) + 0 = 0$$

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Is there another root (far away from 0)? Factor:

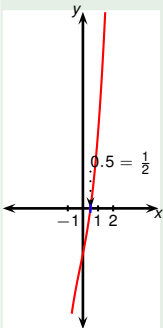
**Quotient:**  $2x^2 + 2x + 6$

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 \underline{2x^3 - x^2} \phantom{- 3} \\
 2x^2 + 5x - 3 \\
 \underline{2x^2 - x} \phantom{- 3} \\
 6x - 3 \\
 \underline{6x - 3} \\
 0
 \end{array}$$

**Remainder:**

0

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Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2})(2x^2 + 2x + 6) = 0$$

We see only one root,  $x = 0.5 = \frac{1}{2}$ . Is our guess correct?

Is there another root (far away from 0)? Factor:

**Quotient:**  $2x^2 + 2x + 6$

$$\begin{array}{r}
 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
 \underline{2x^3 - x^2} \phantom{- 3} \\
 2x^2 + 5x - 3 \\
 \underline{2x^2 - x} \phantom{- 3} \\
 6x - 3 \\
 \underline{6x - 3} \\
 0
 \end{array}$$

**Remainder:**

0



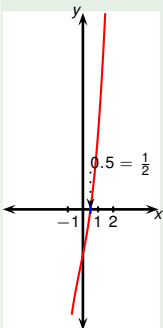
## Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2})(2x^2 + 2x + 6) = 0$$

$$x - \frac{1}{2} = 0 \quad \text{or} \quad x =$$



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## Example

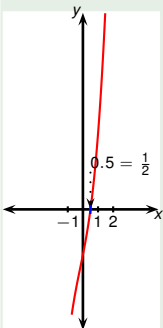
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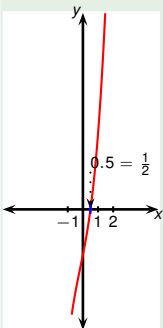
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$$x - \frac{1}{2} = 0 \quad \text{or} \quad x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2}$$

$$x = \frac{1}{2}$$



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## Example

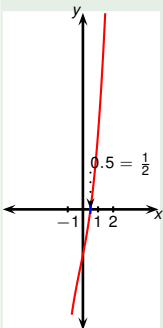
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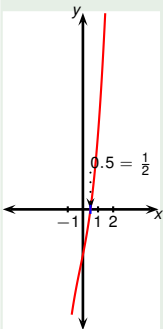
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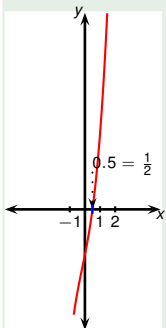
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$$x = \frac{1}{2} \quad x = \frac{-2 \pm \sqrt{-44}}{2 \cdot 2}$$

We see only one root,  $x = 0.5 = \frac{1}{2}$ . Is our guess correct? Is there another root (far away from 0)?



## Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2})(2x^2 + 2x + 6) = 0$$

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$$x = \frac{1}{2} \quad x = \frac{-2 \pm \sqrt{-44}}{2 \cdot 2}$$

no real solution

We see only one root,  $x = 0.5 = \frac{1}{2}$ . Is our guess correct? Is there another root (far away from 0)?

