

Calculus II

Integrals with irreducible quadratic denominator

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Building blocks IIa and IIIa

Building block IIa: $\int \frac{x}{1+x^2} dx$. Building block IIIa: $\int \frac{1}{1+x^2} dx$.

Example (Block IIa)

$$\begin{aligned}
 \int \frac{x}{1+x^2} dx &= \int \frac{1}{(1+x^2)} \frac{d(x^2)}{2} \\
 &= \int \frac{1}{1+x^2} \frac{d(1+x^2)}{2} && \left| \text{Set } u = 1+x^2 \right. \\
 &= \int \frac{1}{u} \frac{du}{2} \\
 &= \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln (1+x^2) + C .
 \end{aligned}$$

Example (Block IIIa)

$$\int \frac{1}{1+x^2} dx = \arctan x + C$$