Precalculus

Express $\sin(kx), \cos(kx)$ via $\sin x, \cos x$

Todor Milev

2019

Example

Express $\sin(3x)$ and $\cos(3x)$ via $\cos x$ and $\sin x$.

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$$\sin(3x) = \sin(x + 2x)$$

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Recall the formulas $\begin{aligned} \sin(\alpha+\beta) &= \textbf{?} \\ \cos(\alpha+\beta) &= \textbf{?} \end{aligned}$.

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