Calculus II
Comparison and limit-comparison tests, part 3

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Example

Test the series $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{7+n^5}}$ for convergence or divergence.

- The dominant part of the numerator is $2n^2$ and the dominant part of the denominator is $\sqrt{n^5} = n^{5/2}$.

$$\begin{aligned} a_n &= \frac{2n^2+3n}{\sqrt{7+n^5}}, \qquad b_n = \frac{2n^2}{n^{5/2}} = \frac{2}{n^{1/2}} \\ \lim_{n\to\infty} \frac{a_n}{b_n} &= \lim_{n\to\infty} \frac{2n^2+3n}{\sqrt{7+n^5}} \cdot \frac{n^{1/2}}{2} = \lim_{n\to\infty} \frac{2n^{5/2}+3n^{3/2}}{2\sqrt{7+n^5}} \frac{\frac{1}{n^{5/2}}}{\frac{1}{n^{5/2}}} \\ &= \lim_{n\to\infty} \frac{2+\frac{3}{n}}{2\sqrt{\frac{7}{n^5}+1}} = 1 > 0 \end{aligned}$$

- $\sum \frac{2}{n^{\frac{1}{2}}}$ is a constant multiple of a p-series with $p = \frac{1}{2}$.
- Therefore $\sum \frac{2}{n^{\frac{1}{2}}}$ is divergent, and so is $\sum \frac{2n^2+3n}{\sqrt{7+n^5}}$.