Calculus I
§ Definite integrals of rational power monomials

Todor Milev

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Example

Evaluate: $\int_1^9 \frac{2t^3 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$

$$= \int_1^9 \left(2t + t^{\frac{1}{2}} - t^{-2}\right) \mathrm{d}t = \left[\int (2t + t^{\frac{1}{2}} - t^{-2}) \mathrm{d}t\right]_1^9$$

$$= \left[\int 2t\mathrm{d}t + \int t^{\frac{1}{2}} \mathrm{d}t - \int t^{-2} \mathrm{d}t\right]_1^9$$

$$= \left[t^2 + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{-1}}{-1}\right]_1^9 = \left[t^2 + \frac{2}{3}t^{\frac{3}{2}} + \frac{1}{t}\right]_1^9$$

$$= \left(9^2 + \frac{2}{3} \cdot 9^{\frac{3}{2}} + \frac{1}{9}\right) - \left(1^2 + \frac{2}{3} \cdot 1^{\frac{3}{2}} + \frac{1}{1}\right)$$

$$= 81 + 18 + \frac{1}{9} - 1 - \frac{2}{3} - 1 = \frac{868}{9}.$$