

# Precalculus

## Factor quadratic over the complex numbers

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2019

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

$$\left| \begin{array}{rcl} x_1 x_2 & = & \frac{c}{a} \\ x_1 + x_2 & = & -\frac{b}{a} \end{array} \right.$$

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Factor the quadratic, using complex numbers if needed.

$$x^2 + x + 1$$

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$$x^2 + x + 1 = \left( x - \left( \frac{-1 + \sqrt{3}i}{2} \right) \right) \left( x - \left( \frac{-1 - \sqrt{3}i}{2} \right) \right)$$

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