Calculus II
Power series expansion of logarithms, part 1

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2019

Example

Find a power series for $\ln(1-x)$ and state its radius of convergence.

$$\begin{aligned}
\ln(1-x) &= \int \mathrm{d}(\ln(1-x)) = \int (\ln(1-x))' \mathrm{d}x \quad \Big| \text{ up to const.} \\
&= \int \left(-\frac{1}{1-x}\right) \mathrm{d}x \\
&= -\int \left(1 + x + x^2 + x^3 + \cdots\right) \mathrm{d}x \quad \Big| \text{ for } |x| < 1 \\
&= -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots\right) + C \\
&= C - \sum_{n=1}^{\infty} \frac{x^n}{n}
\end{aligned}$$

- To find C, plug in $x = 0$: $C = 0$.
- Therefore the theorem on integrating power series implies that $\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}$, for $|x| < 1$.
- By the same theorem, the radius of convergence remains $R = 1$.