

Precalculus

Factor quadratic with irrational real roots

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2019

$$ax^2 + bx + c = a(x - x_1)(x - x_2),$$

$$\left| \begin{array}{rcl} x_1 x_2 & = & \frac{c}{a} \\ x_1 + x_2 & = & -\frac{b}{a} \end{array} \right.$$

Example

Factor the quadratic.

$$x^2 + 3x + 1$$

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$$x^2 + 3x + 1 = (x + ?) (x + ?)$$

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$$x^2 + 3x + 1 = \left(x - \left(\frac{-3 + \sqrt{5}}{2} \right) \right) \left(x - \left(\frac{-3 - \sqrt{5}}{2} \right) \right)$$

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