

Precalculus

Factor cubic with one real root using its plot

Todor Milev

2019

Example

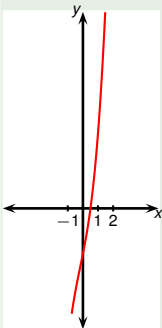
Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

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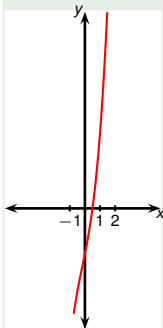


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We see only one root, $x = ?$.

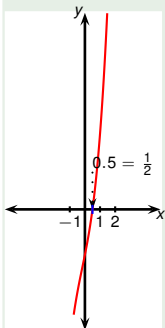


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We see only one root, $x = 0.5 = \frac{1}{2}$.

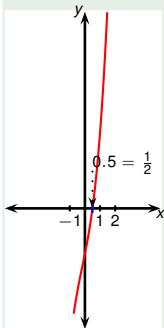


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We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct?
Is there another root (far away from 0)?



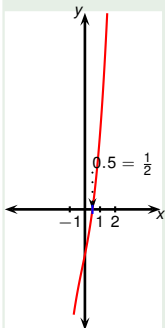
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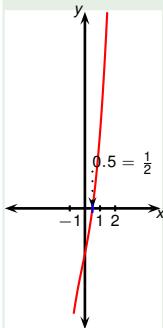
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$$x - \frac{1}{2} \quad \overline{2x^3 + x^2 + 5x - 3}$$



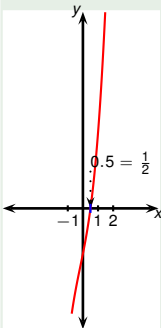
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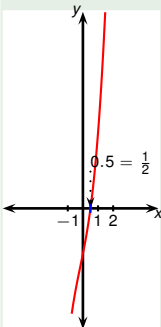
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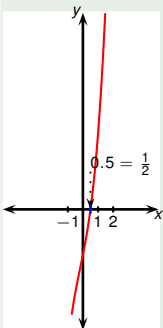


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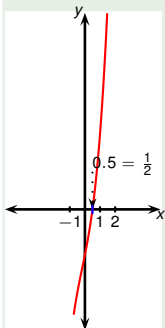
Divide $2x^3$ by x .

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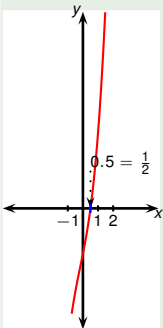
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$$x - \frac{1}{2} \overline{) \begin{array}{l} 2x^3 + x^2 + 5x - 3 \\ 2x^2 \\ \hline ? \\ ? \end{array}}$$

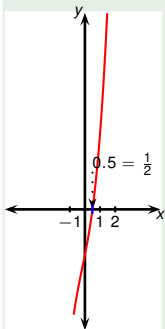
Multiply $2x^2$ by divisor.

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 2x^2 \\
 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
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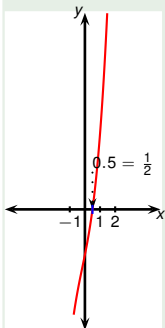
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 x - \frac{1}{2} \quad \overline{2x^3 + x^2 + 5x - 3} \\
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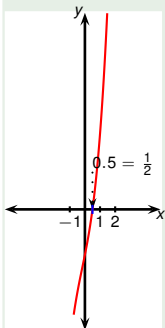
Subtract last two polynomials.

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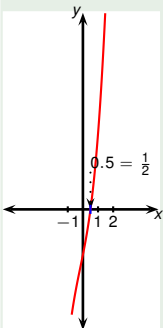
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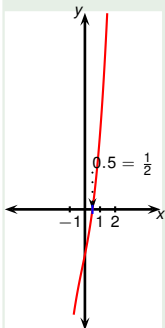
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 2x^2 + 2x \\
 \hline
 x - \frac{1}{2} \quad \overline{2x^3 + x^2 + 5x - 3} \\
 \quad \quad \quad \underline{2x^3 - x^2} \\
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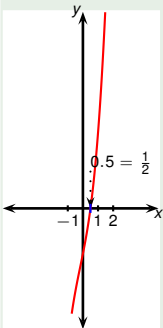
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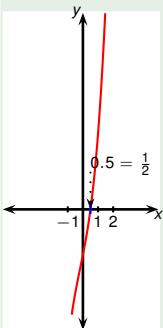
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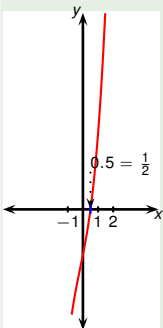
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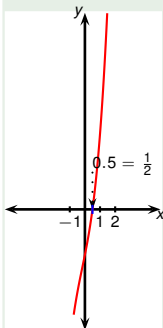
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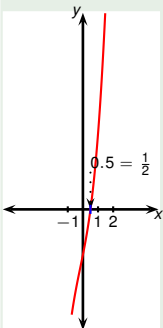
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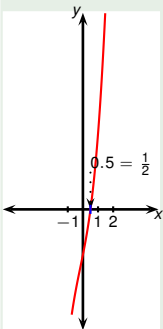
Divide $6x$ by x .

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 2x^2 + 2x + 6 \\
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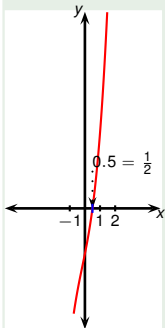
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 6x - 3 \\
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 6x - 6 \\
 \underline{6x - 6} \\
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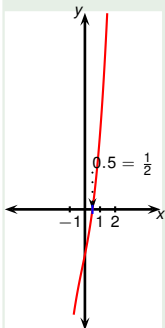
Multiply 6 by divisor.

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 6x - 3 \\
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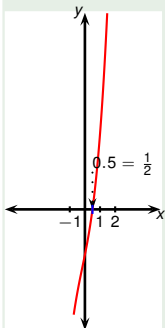
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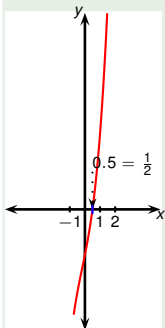
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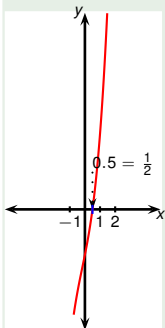
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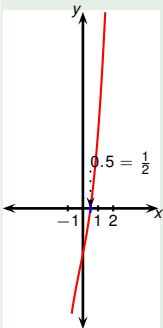
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Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2}) (2x^2 + 2x + 6) + 0 = 0$$

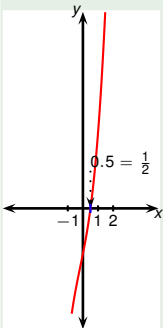
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Is there another root (far away from 0)? Factor:

Quotient: $2x^2 + 2x + 6$

$$\begin{array}{r}
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 2x^2 + 5x - 3 \\
 \underline{2x^2 - x} \\
 6x - 3 \\
 \underline{6x - 3} \\
 0
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Example



Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2})(2x^2 + 2x + 6) + 0 = 0$$

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Is there another root (far away from 0)? Factor:

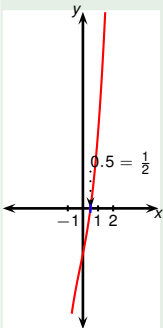
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 x - \frac{1}{2} \overline{) 2x^3 + x^2 + 5x - 3} \\
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 2x^2 + 5x - 3 \\
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 6x - 3 \\
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Remainder:

0

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 \underline{2x^2 - x} \\
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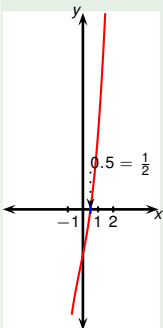
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$$(x - \frac{1}{2})(2x^2 + 2x + 6) = 0$$

$$x - \frac{1}{2} = 0 \quad \text{or} \quad x =$$



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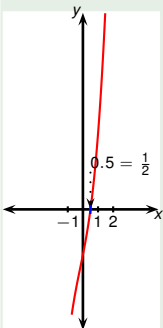
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$$x = \frac{1}{2}$$



We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)?

Example

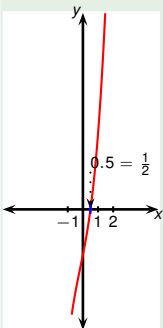
Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2})(2x^2 + 2x + 6) = 0$$

$$x - \frac{1}{2} = 0 \quad \text{or} \quad x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2}$$

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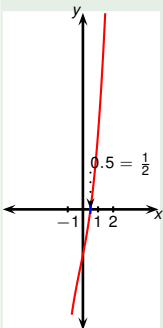
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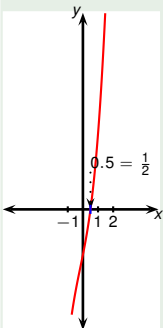
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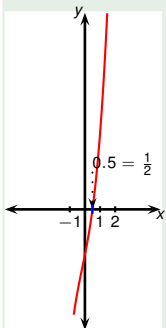
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$$x = \frac{1}{2} \quad x = \frac{-2 \pm \sqrt{-44}}{2 \cdot 2}$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)?



Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

$$(x - \frac{1}{2})(2x^2 + 2x + 6) = 0$$

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$$x = \frac{1}{2} \quad x = \frac{-2 \pm \sqrt{-44}}{2 \cdot 2}$$

no real solution

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)?

