

Precalculus

Compute the trigonometric functions of an angle not in the first quadrant

Todor Milev

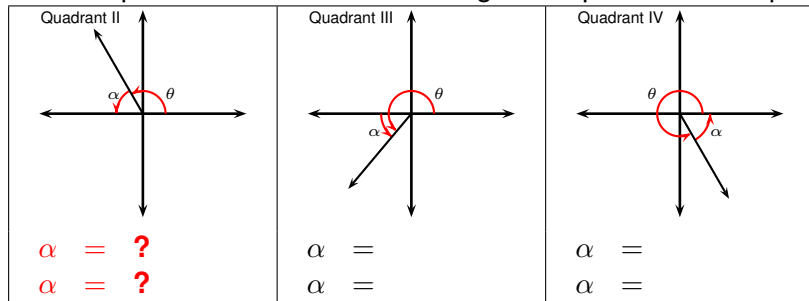
2019

To compute trigonometric functions from obtuse ($> 90^\circ$) or negative angles, we can use the following visual aid.

Definition (Reference Angle)

Let θ be an angle in standard position. Its reference angle is the acute positive angle formed by the terminal arm and the x axis.

The computation of the reference angle α depends on the quadrant.

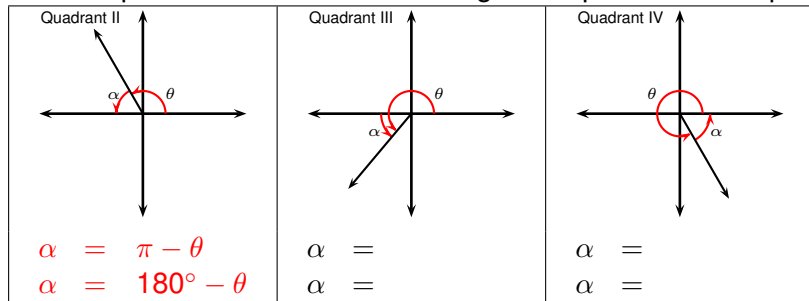


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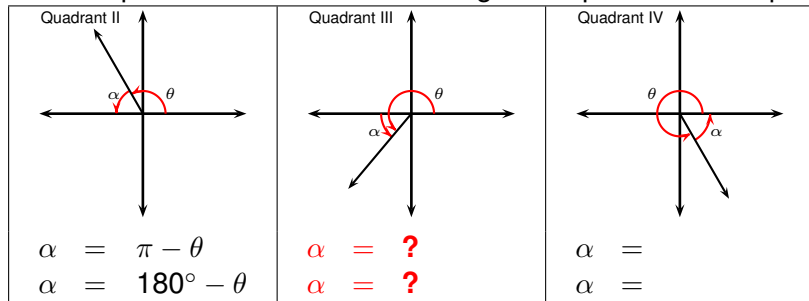


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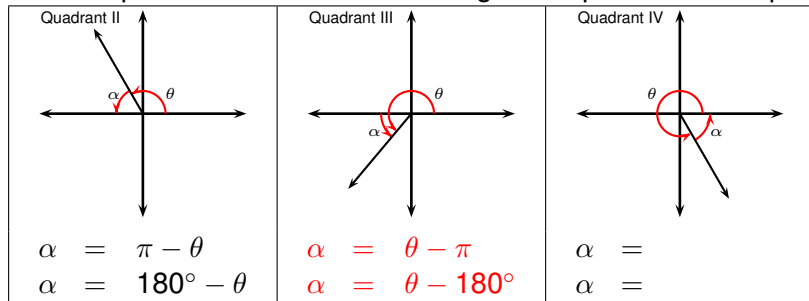


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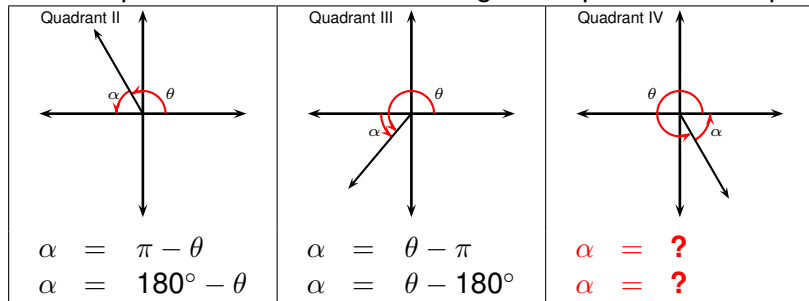


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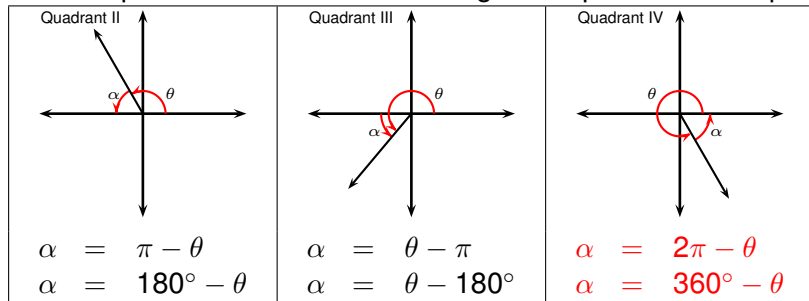


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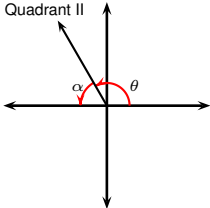
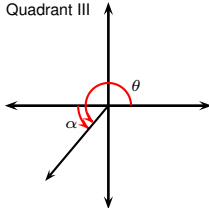
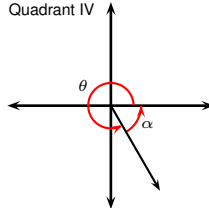


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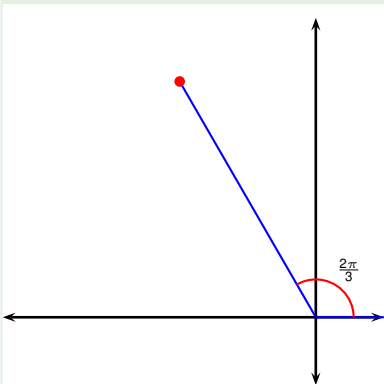
<p>Quadrant II</p>  $\alpha = \pi - \theta$ $\alpha = 180^\circ - \theta$	<p>Quadrant III</p>  $\alpha = \theta - \pi$ $\alpha = \theta - 180^\circ$	<p>Quadrant IV</p>  $\alpha = 2\pi - \theta$ $\alpha = 360^\circ - \theta$
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Observation

One can find the value of a trigonometric function of θ as follows.

- *Find the reference angle α associated to θ .*
- *Find the trig function of α .*
- *Use the quadrant in which θ lies to affix an appropriate sign to the function value.*

Example



Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

$$\sin\left(\frac{2\pi}{3}\right) =$$

$$\cos\left(\frac{2\pi}{3}\right) =$$

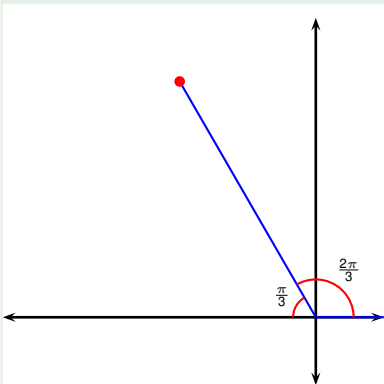
$$\tan\left(\frac{2\pi}{3}\right) =$$

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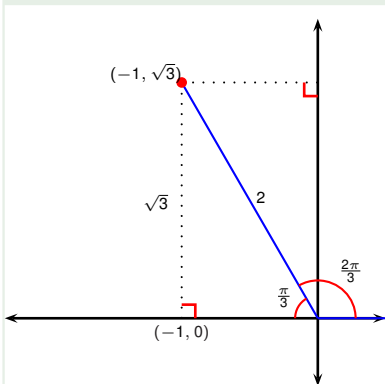
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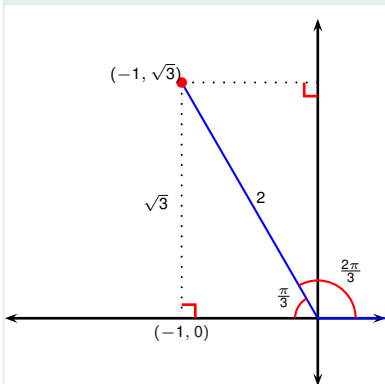
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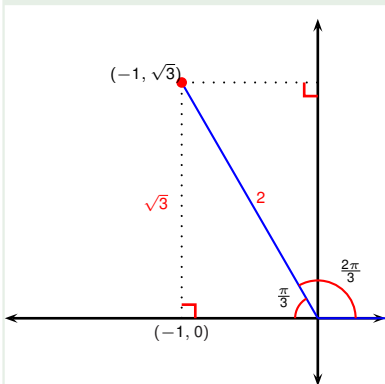
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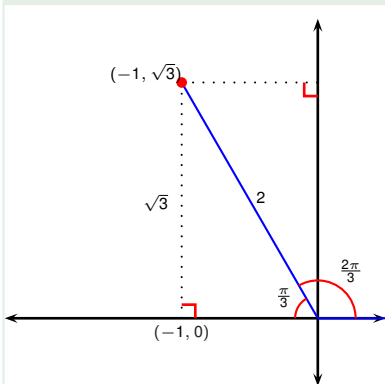
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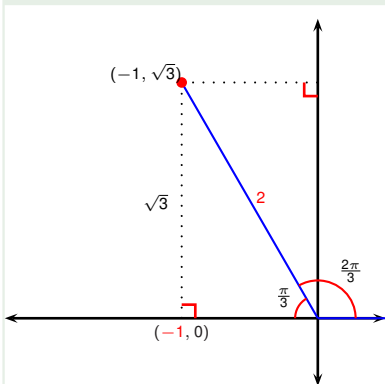
$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \cos\left(\frac{2\pi}{3}\right) = ?$$

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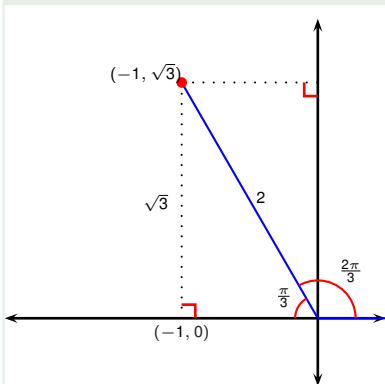
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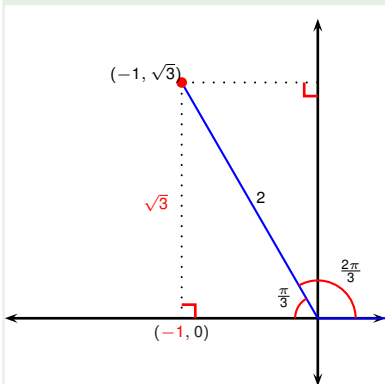
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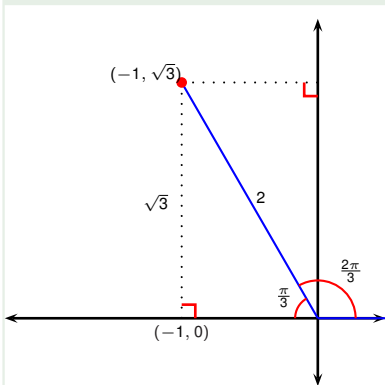
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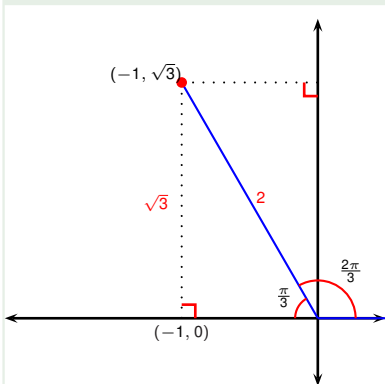
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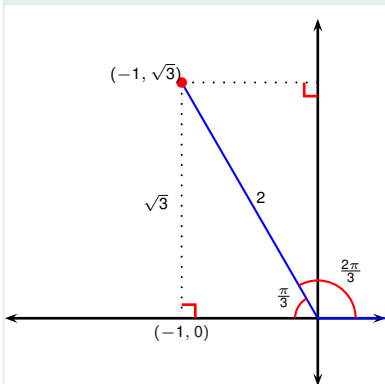
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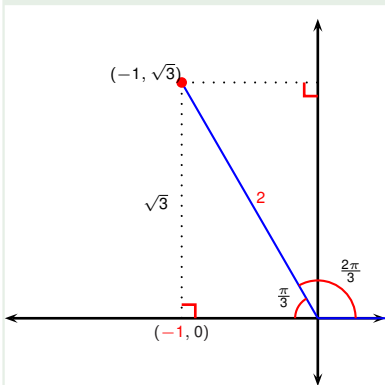
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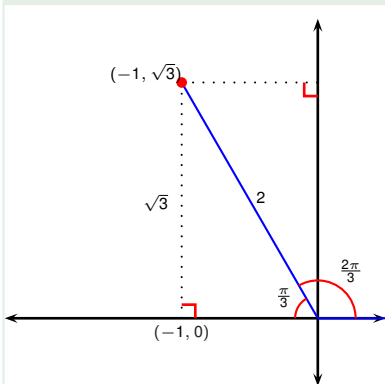


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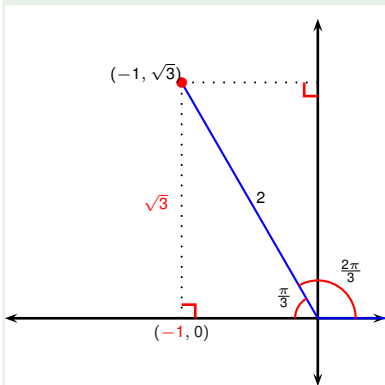


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