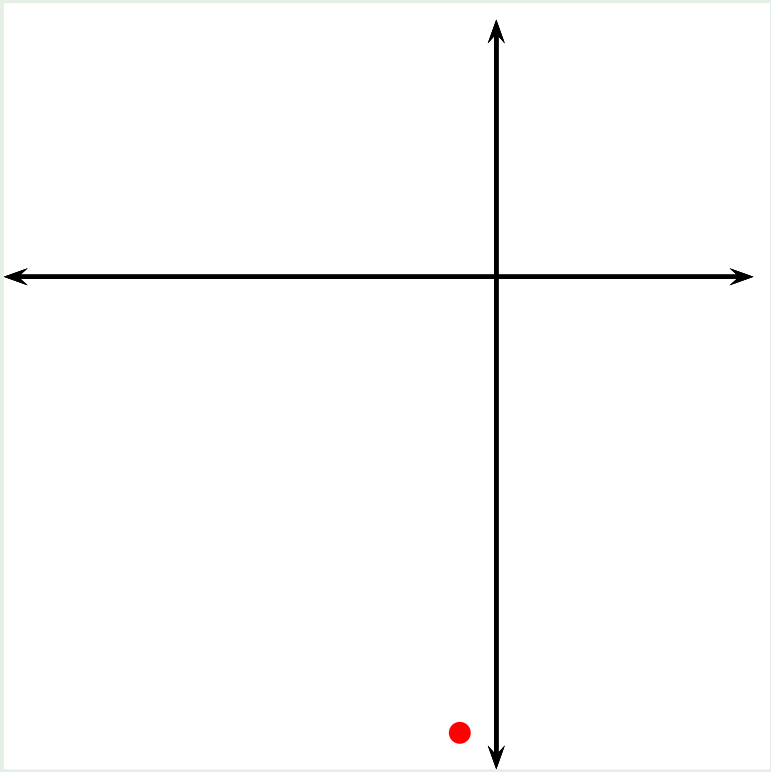
Calculus I
Inverse of a quadratic function

Todor Milev

2019

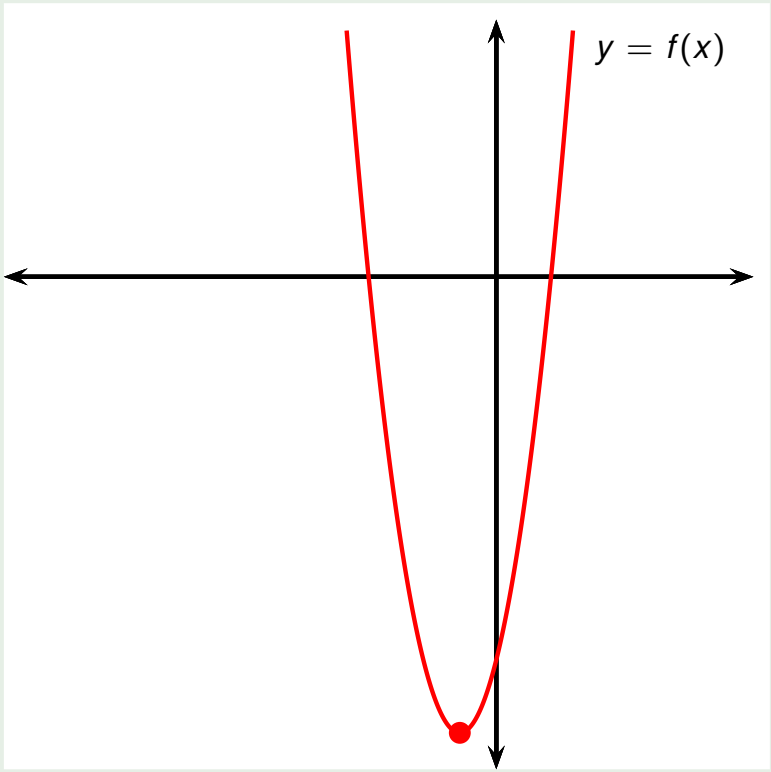
Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



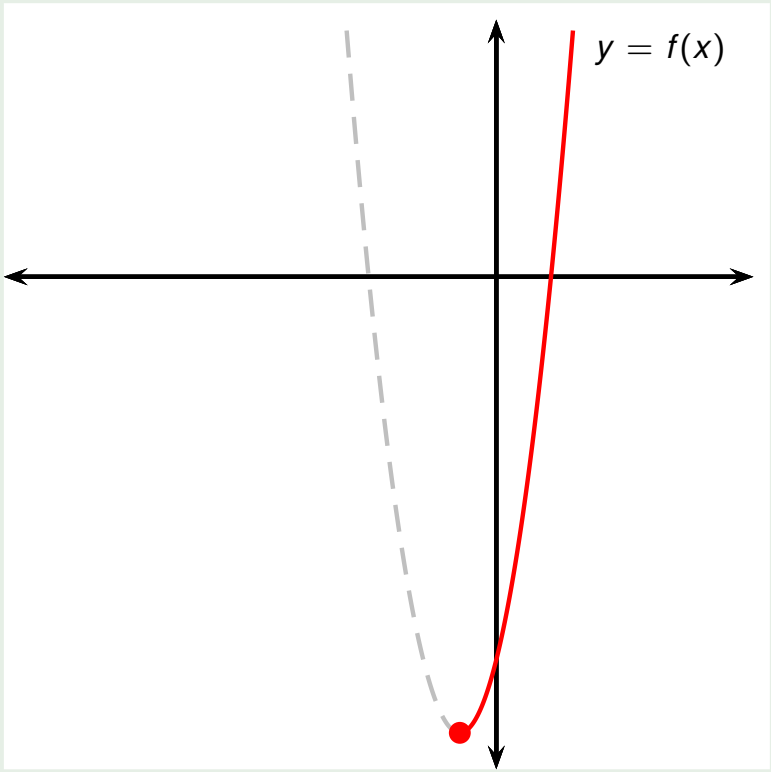
Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



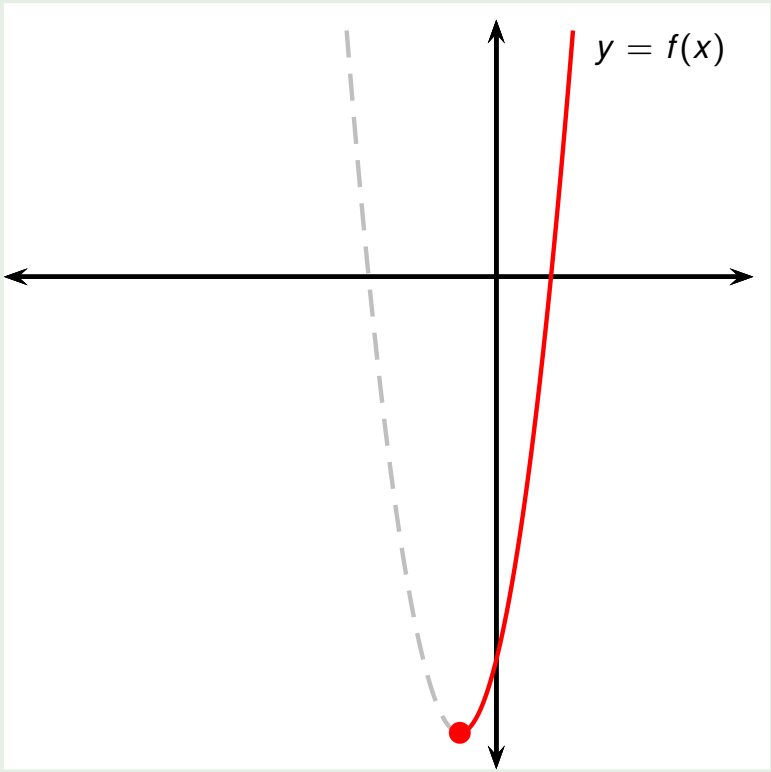
Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



Example ( )

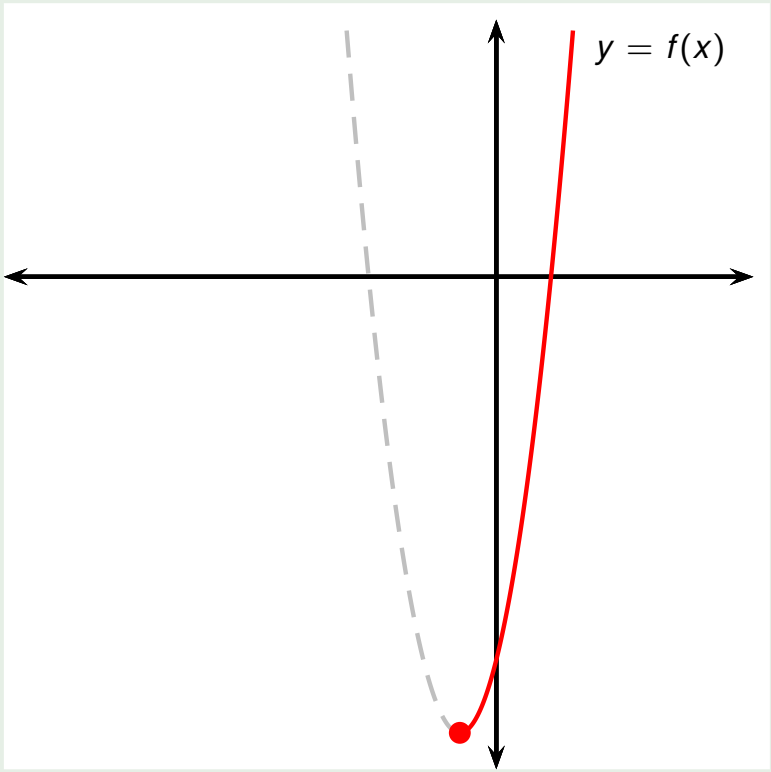
Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



$$\begin{aligned} 3x^2 + 4x - 7 &= y \\ 3x^2 + 4x + (-7 - y) &= 0 \end{aligned}$$

Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



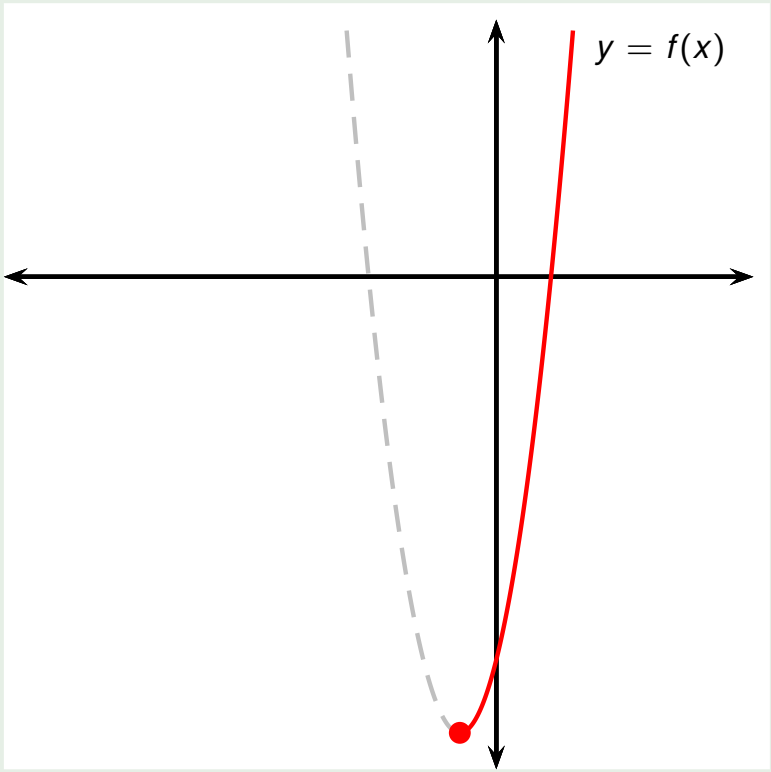
$$\begin{aligned} 3x^2 + 4x - 7 &= y \\ 3x^2 + 4x + (-7 - y) &= 0 \end{aligned}$$

That's a quadratic equation in x. Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



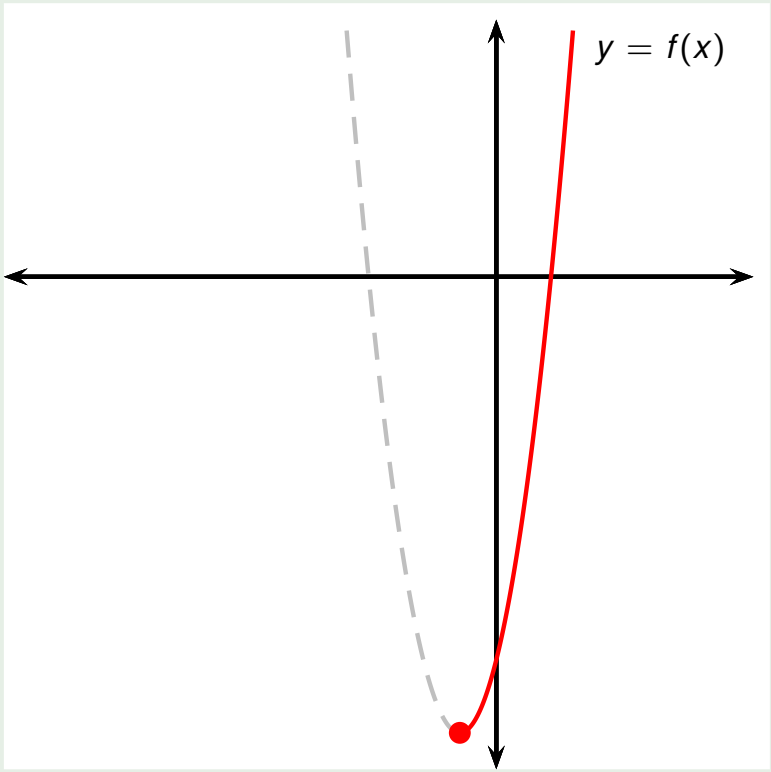
$$\begin{aligned} 3x^2 + 4x - 7 &= y \\ 3x^2 + 4x + (-7 - y) &= 0 \end{aligned}$$

That's a quadratic equation in x. Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



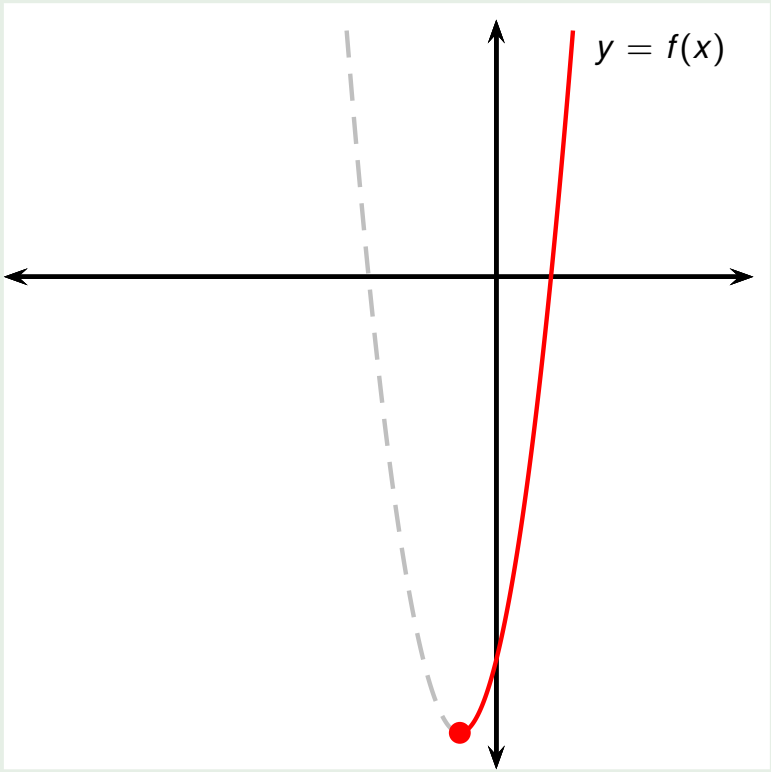
$$\begin{aligned} 3x^2 + 4x - 7 &= y \\ 3x^2 + 4x + (-7 - y) &= 0 \end{aligned}$$

That's a quadratic equation in x. Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



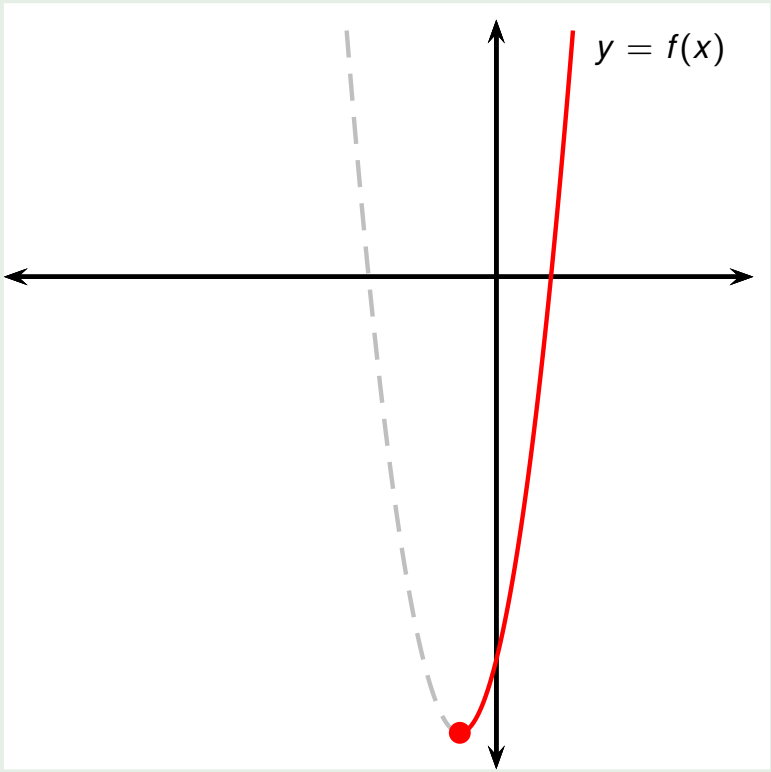
$$\begin{aligned} 3x^2 + 4x - 7 &= y \\ 3x^2 + 4x + (-7 - y) &= 0 \end{aligned}$$

That's a quadratic equation in x. Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



$$\begin{aligned} 3x^2 + 4x - 7 &= y \\ 3x^2 + 4x + (-7 - y) &= 0 \end{aligned}$$

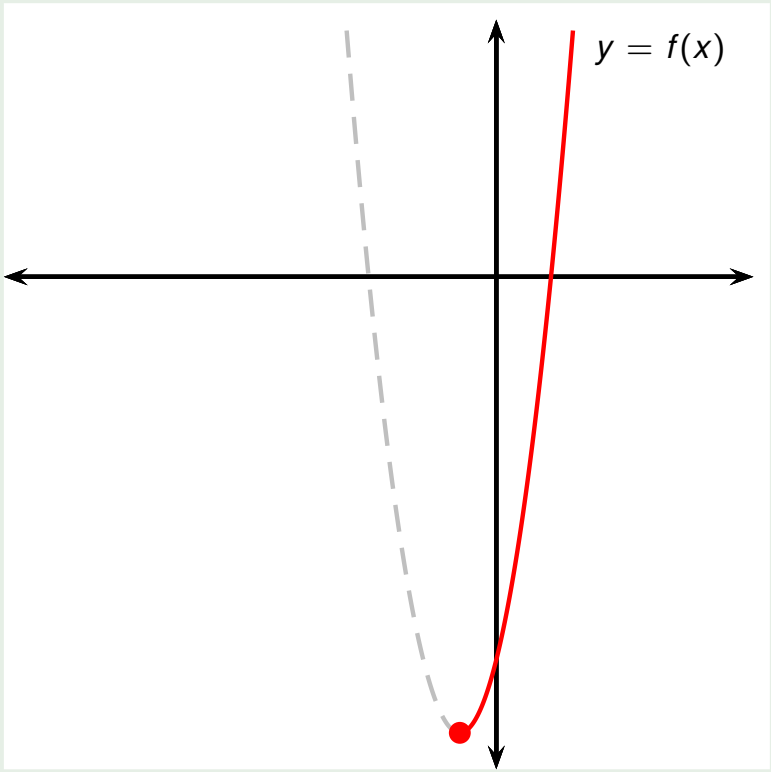
That's a quadratic equation in x. Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

$$= -\frac{2 \pm \sqrt{25 + 3y}}{3} =$$

Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



$$\begin{aligned} 3x^2 + 4x - 7 &= y \\ 3x^2 + 4x + (-7 - y) &= 0 \end{aligned}$$

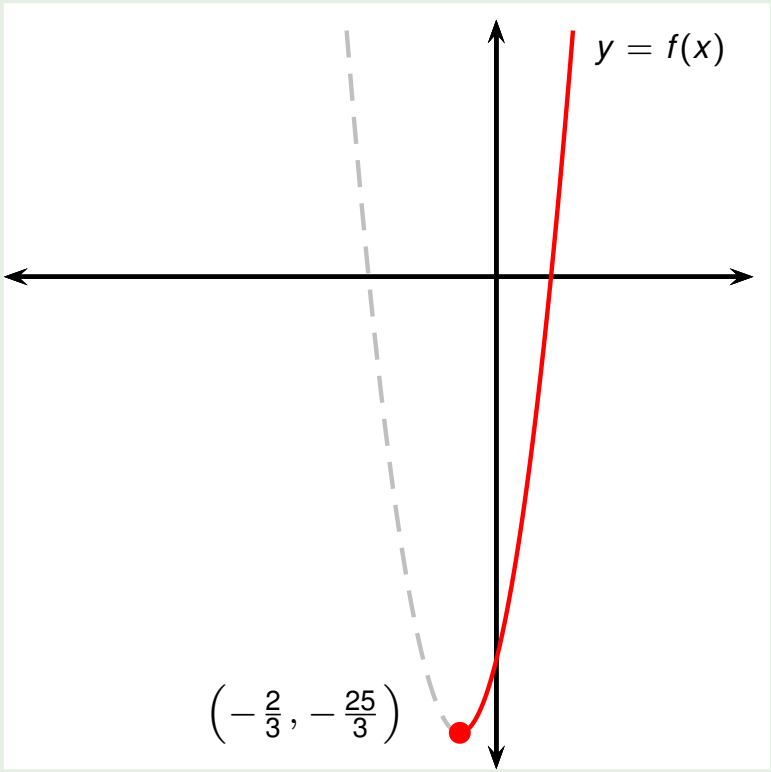
That's a quadratic equation in x. Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

$$= -\frac{2 \pm \sqrt{25 + 3y}}{3} = -\frac{2}{3} \pm \frac{\sqrt{25 + 3y}}{3}$$

Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



$$\begin{aligned} 3x^2 + 4x - 7 &= y \\ 3x^2 + 4x + (-7 - y) &= 0 \end{aligned}$$

That's a quadratic equation in x. Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

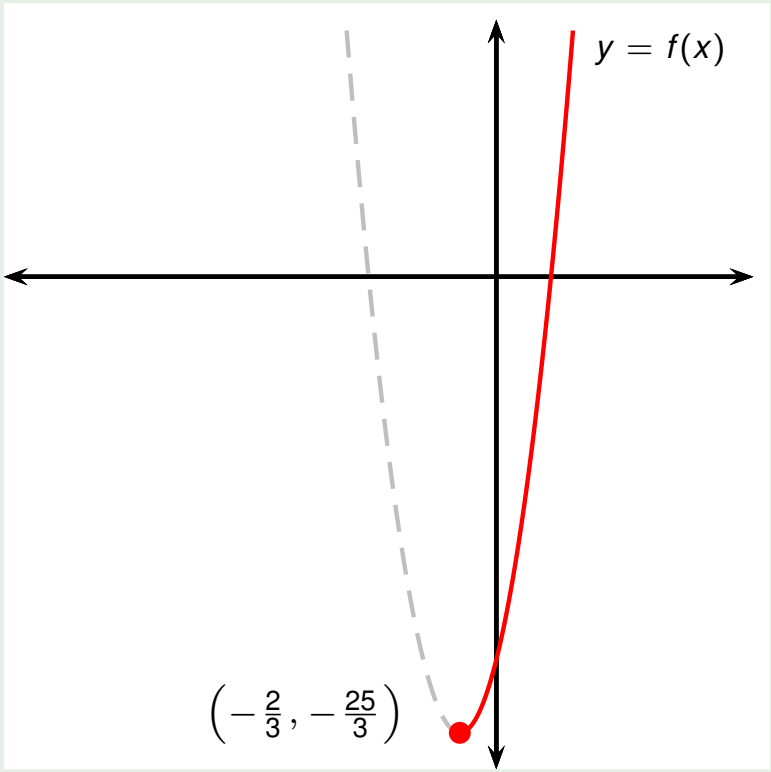
$$= -\frac{2 \pm \sqrt{25 + 3y}}{3} = -\frac{2}{3} \pm \frac{\sqrt{25 + 3y}}{3}$$

We are given $x \geq -\frac{2}{3}$, therefore

$$x = -\frac{2}{3} + \frac{\sqrt{25+3y}}{3} = f^{-1}(y).$$

Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



$$\begin{aligned} 3x^2 + 4x - 7 &= y \\ 3x^2 + 4x + (-7 - y) &= 0 \end{aligned}$$

That's a quadratic equation in x. Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

$$= -\frac{2 \pm \sqrt{25 + 3y}}{3} = -\frac{2}{3} \pm \frac{\sqrt{25 + 3y}}{3}$$

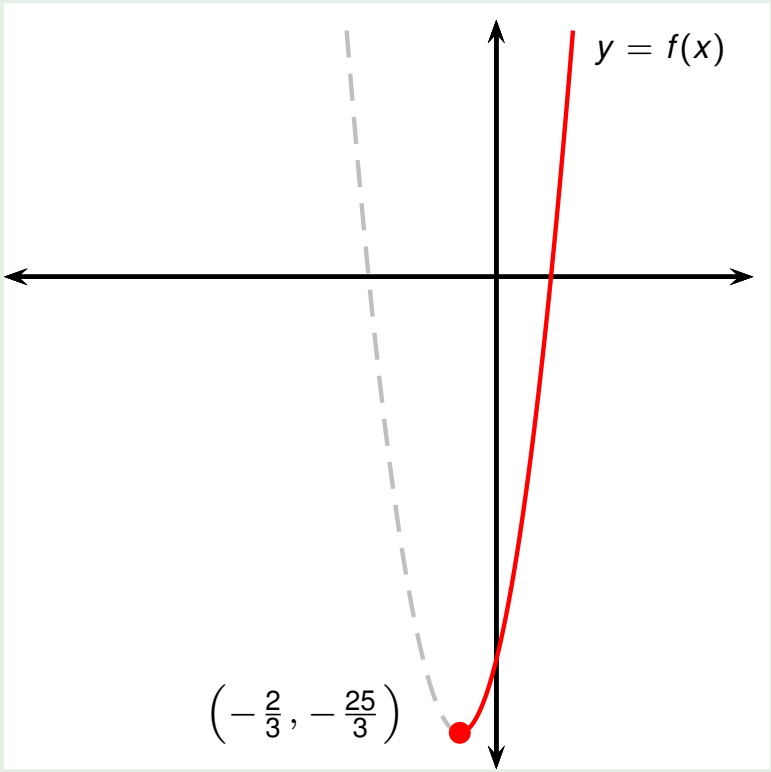
answer

$$f^{-1}(y) = -\frac{2}{3} + \frac{\sqrt{25 + 3y}}{3}$$

We are given $x \geq -\frac{2}{3}$, therefore $x = -\frac{2}{3} + \frac{\sqrt{25+3y}}{3} = f^{-1}(y)$.

Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



$$\begin{aligned} 3x^2 + 4x - 7 &= y \\ 3x^2 + 4x + (-7 - y) &= 0 \end{aligned}$$

That's a quadratic equation in x. Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

$$= -\frac{2 \pm \sqrt{25 + 3y}}{3} = -\frac{2}{3} \pm \frac{\sqrt{25 + 3y}}{3}$$

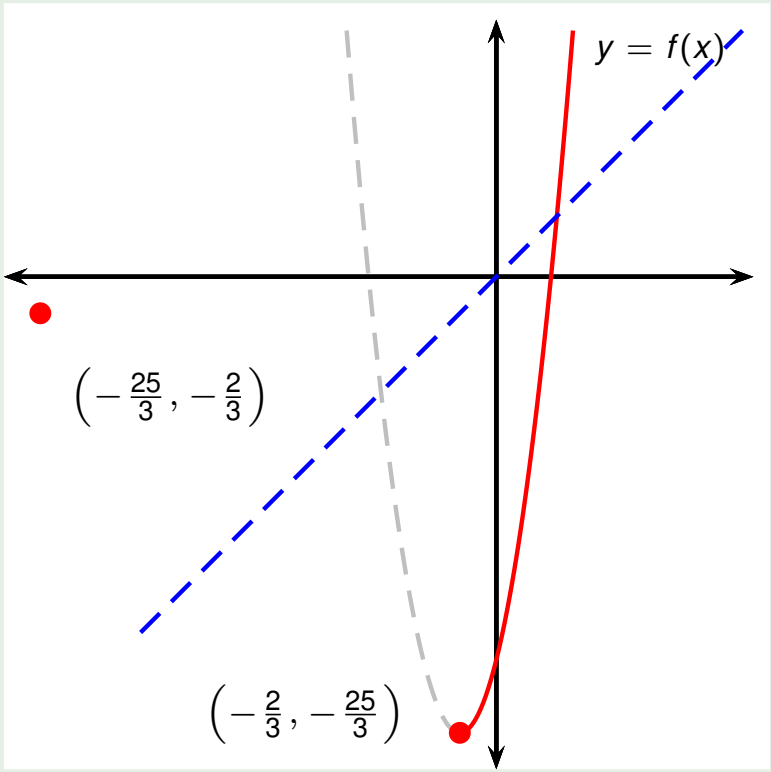
Final answer, relabelled:

$$f^{-1}(x) = -\frac{2}{3} + \frac{\sqrt{25 + 3x}}{3}$$

We are given $x \geq -\frac{2}{3}$, therefore $x = -\frac{2}{3} + \frac{\sqrt{25+3y}}{3} = f^{-1}(y)$.

Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



$$\begin{aligned} 3x^2 + 4x - 7 &= y \\ 3x^2 + 4x + (-7 - y) &= 0 \end{aligned}$$

That's a quadratic equation in x. Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

$$= -\frac{2 \pm \sqrt{25 + 3y}}{3} = -\frac{2}{3} \pm \frac{\sqrt{25 + 3y}}{3}$$

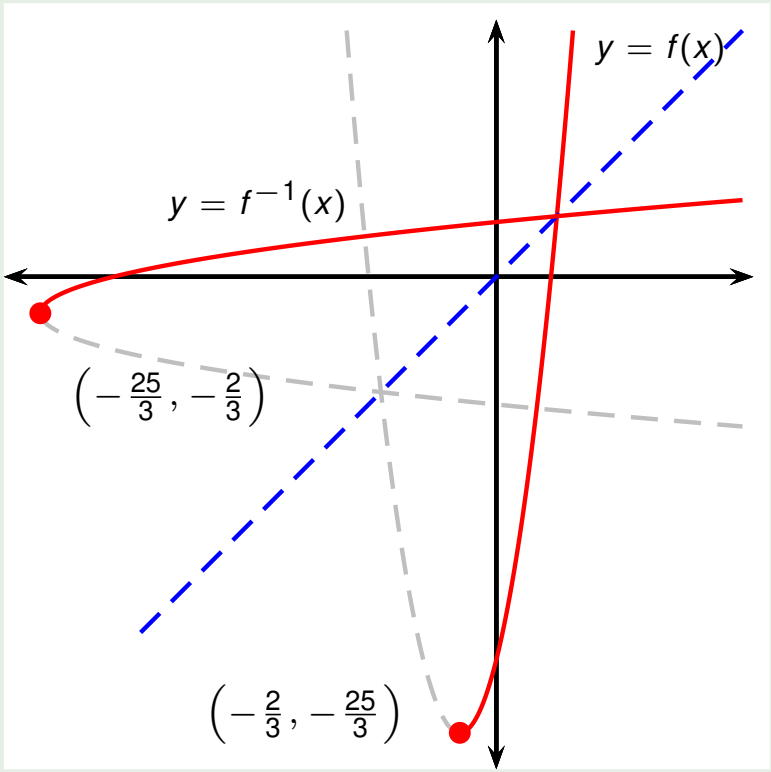
We are given $x \geq -\frac{2}{3}$, therefore $x = -\frac{2}{3} + \frac{\sqrt{25+3y}}{3} = f^{-1}(y)$.

Final answer, relabelled:

$$f^{-1}(x) = -\frac{2}{3} + \frac{\sqrt{25 + 3x}}{3}$$

Example ( )

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



$$\begin{aligned} 3x^2 + 4x - 7 &= y \\ 3x^2 + 4x + (-7 - y) &= 0 \end{aligned}$$

That's a quadratic equation in x. Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

$$= -\frac{2 \pm \sqrt{25 + 3y}}{3} = -\frac{2}{3} \pm \frac{\sqrt{25 + 3y}}{3}$$

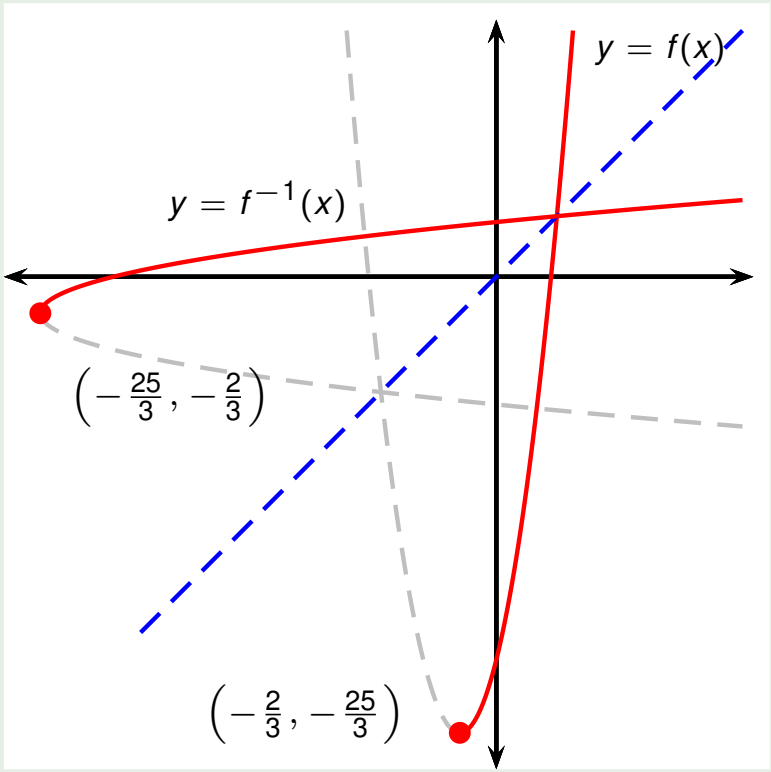
Final answer, relabelled:

$$f^{-1}(x) = -\frac{2}{3} + \frac{\sqrt{25 + 3x}}{3}$$

We are given $x \geq -\frac{2}{3}$, therefore $x = -\frac{2}{3} + \frac{\sqrt{25+3y}}{3} = f^{-1}(y)$.

Example (What if we change the problem to $x \leq -\frac{2}{3}$?)

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \geq -\frac{2}{3}$. Find $f^{-1}(x)$.



$$\begin{aligned} 3x^2 + 4x - 7 &= y \\ 3x^2 + 4x + (-7 - y) &= 0 \end{aligned}$$

That's a quadratic equation in x. Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

$$= -\frac{2 \pm \sqrt{25 + 3y}}{3} = -\frac{2}{3} \pm \frac{\sqrt{25 + 3y}}{3}$$

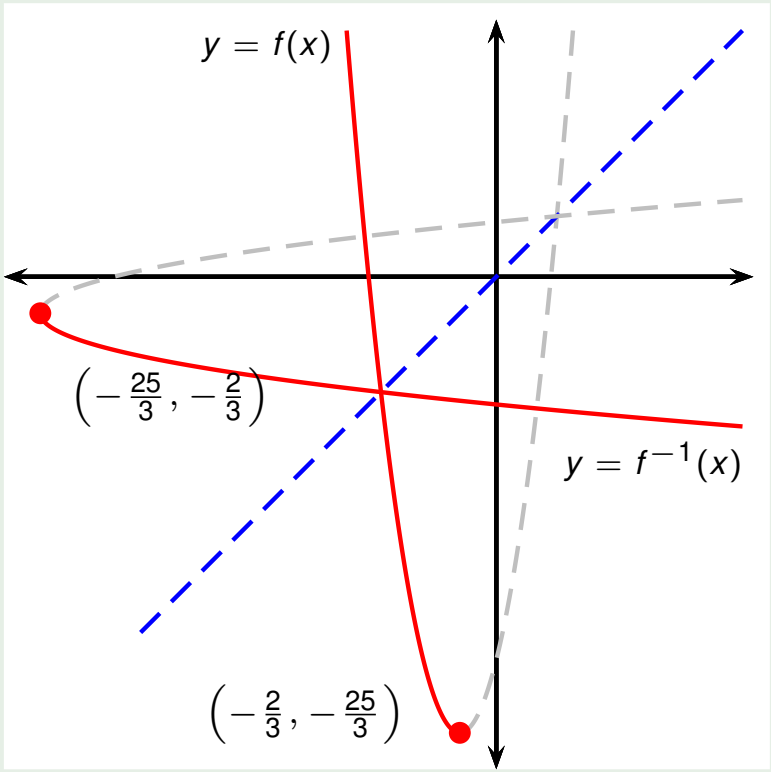
Final answer, relabelled:

$$f^{-1}(x) = -\frac{2}{3} + \frac{\sqrt{25 + 3x}}{3}$$

We are given $x \geq -\frac{2}{3}$, therefore $x = -\frac{2}{3} + \frac{\sqrt{25+3y}}{3} = f^{-1}(y)$.

Example (What if we change the problem to $x \leq -\frac{2}{3}$?)

Given: $f(x) = 3x^2 + 4x - 7$ with domain $x \leq -\frac{2}{3}$. Find $f^{-1}(x)$.



$$\begin{aligned} 3x^2 + 4x - 7 &= y \\ 3x^2 + 4x + (-7 - y) &= 0 \end{aligned}$$

That's a quadratic equation in x. Solve:

$$\frac{-4 \pm \sqrt{4^2 - 4 \cdot 3 \cdot (-y - 7)}}{2 \cdot 3}$$

$$= -\frac{2 \pm \sqrt{25 + 3y}}{3} = -\frac{2}{3} \pm \frac{\sqrt{25 + 3y}}{3}$$

Final answer, relabelled:

$$f^{-1}(x) = -\frac{2}{3} - \frac{\sqrt{25 + 3x}}{3}$$

We are given $x \leq -\frac{2}{3}$, therefore $x = -\frac{2}{3} - \frac{\sqrt{25+3y}}{3} = f^{-1}(y)$.