Precalculus
Find all angles coterminal to a given one

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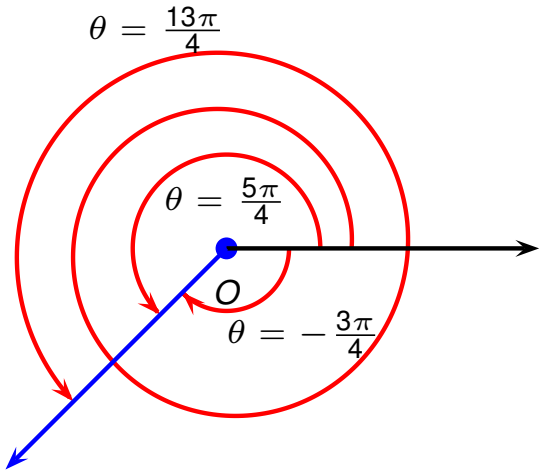
Definition (Coterminal Angles)

Two angles (angle measures) are called coterminal if the corresponding geometric angles have the same initial and terminal sides.



Observation

The set of angles coterminal with α consists of the angles $\alpha + 2k\pi$, where k runs over the set of integers. In other words, the angles coterminal with α are the angles:

$$\ldots, \alpha - 6\pi, \alpha - 4\pi, \alpha - 2\pi, \alpha, \alpha + 2\pi, \alpha + 4\pi, \alpha + 6\pi, \ldots.$$

Example

- Find all angles that are coterminal to $\frac{\pi}{4}$.
- Find all angles in the interval $[-2\pi, \pi]$ that are coterminal to $\frac{\pi}{4}$.

By theory, the angles coterminal with $\frac{\pi}{4}$ are all angles of the form

$$\frac{\pi}{4} + 2k\pi.$$

To find which among the angles $\frac{\pi}{4} + 2k\pi$ lie in the interval $[-2\pi, \pi]$, we write them as an infinite list (we indicate the unboundedness of the list by ellipsis dots) and cross out the angles that lie outside of the desired interval.

$$\cancel{\ldots}, \cancel{\frac{\pi}{4} - 4\pi}, \frac{\pi}{4} - 2\pi, \frac{\pi}{4}, \cancel{\frac{\pi}{4} + 2\pi}, \cancel{\frac{\pi}{4} + 4\pi}, \cancel{\ldots}$$

Our final answer is $-\frac{7\pi}{4}, \frac{\pi}{4}$