

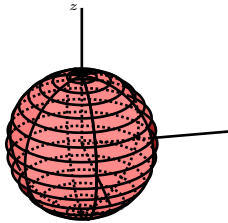
Calculus III

Homework on Lecture 15

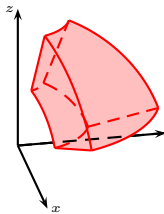
1. Problem 1.e is of higher difficulty.

- Write the Jacobian matrix of the indicated variable change.
- Set up an integral expressing the volume of the region using the indicated variable change and the multivariable integral substitution rule.
- Integrate to find the volume of the region.

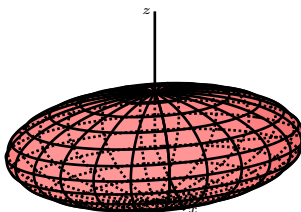
(a) Spherical coordinates; use to find the volume of a ball of radius R .



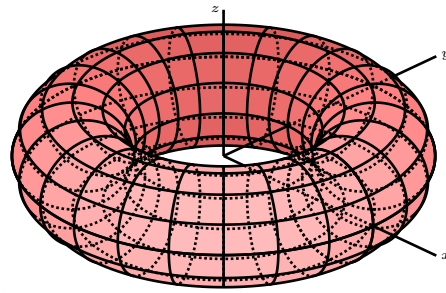
(b) Spherical coordinates; use to find the volume of a curvilinear spherical box, given in spherical coordinates by $\rho_{min} \leq \rho \leq \rho_{max}$, $\phi_{min} \leq \phi \leq \phi_{max}$, $\theta_{min} \leq \theta \leq \theta_{max}$.



(c) Ellipsoidal coordinates: $\mathbf{f} : \begin{cases} x = a\rho \sin \phi \cos \theta \\ y = b\rho \sin \phi \sin \theta \\ z = c\rho \cos \phi \end{cases}$; use to find the volume of an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, $a, b, c > 0$.

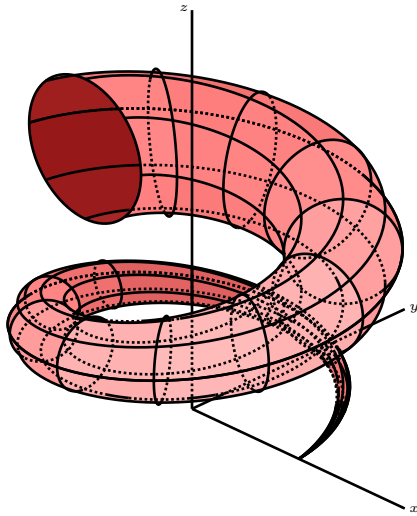


(d) Variable change: $T : \begin{cases} x = (R + \rho \cos \theta) \cos \phi \\ y = (R + \rho \cos \theta) \sin \phi \\ z = \rho \sin \theta \end{cases}$; use to find the volume of a torus with major radius R and minor radius r ,



i.e., the figure given by $\rho \in [0, r], \phi \in [0, 2\pi], \theta \in [0, 2\pi]$.

- (e) Variable change:
$$\begin{cases} x = (2 + \rho \cos \theta) \cos \phi \\ y = (2 + \rho \cos \theta) \sin \phi \\ z = \rho \sin \theta + \frac{\phi}{3} \end{cases}, \text{ use to find the volume of the horn given by } \theta \in [0, 2\pi], \phi \in [0, 3\pi], \rho \in \left[0, \frac{\phi}{9}\right].$$



- (f) Variable change:
$$\begin{cases} x = (2 + \phi/3 + \rho \cos \theta) \cos \phi \\ y = (2 + \phi/3 + \rho \cos \theta) \sin \phi \\ z = \rho \sin \theta + \frac{\phi}{3} \end{cases}, \text{ use to find the volume of the horn given by } \theta \in [0, 2\pi], \phi \in [0, 3\pi], \rho \in \left[0, \frac{\phi}{9}\right].$$

