Calculus II
Lecture 1

Todor Milev
https://github.com/tmilev/freecalc

2020

Outline

1. Review of trigonometry
   - The Trigonometric Functions
   - Trigonometric Identities
   - Trigonometric Identities and Complex Numbers
   - Graphs of the Trigonometric Functions

Outline

1. Review of trigonometry
   - The Trigonometric Functions
   - Trigonometric Identities
   - Trigonometric Identities and Complex Numbers
   - Graphs of the Trigonometric Functions
2. Inverse Trigonometric Functions

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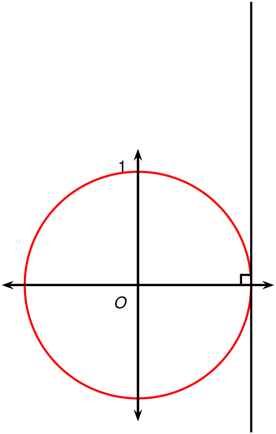
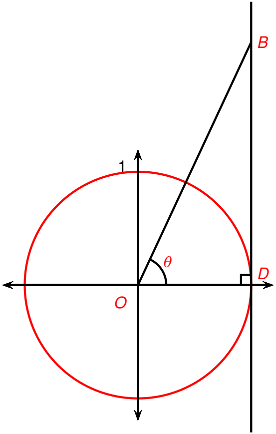
Geometric interpretation of all trigonometric functions



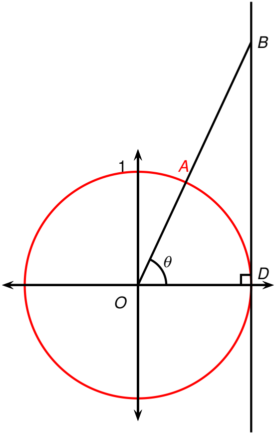
Fix unit circle, center O, coordinates $(0, 0)$.

Geometric interpretation of all trigonometric functions



Fix unit circle, center O, coordinates $(0, 0)$.
Let $\angle DOB = \theta$.

Geometric interpretation of all trigonometric functions



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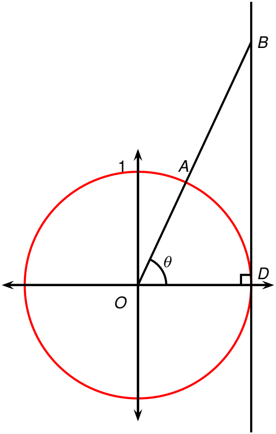

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$\sin\theta$

$\cos\theta$

$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

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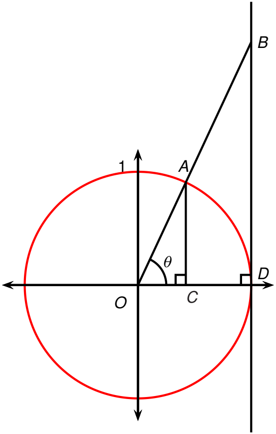

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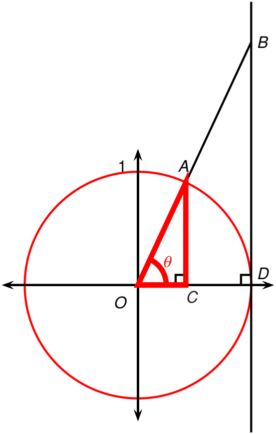

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$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$\cos\theta$

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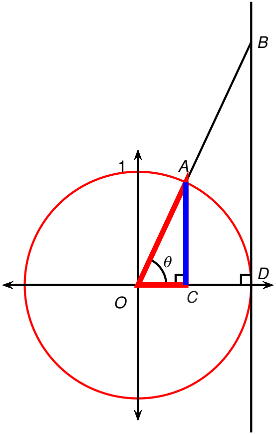
$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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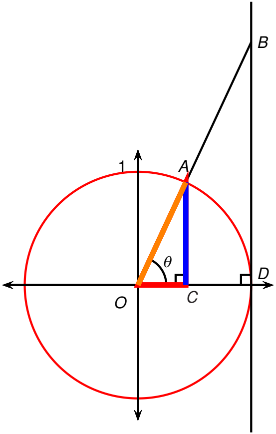

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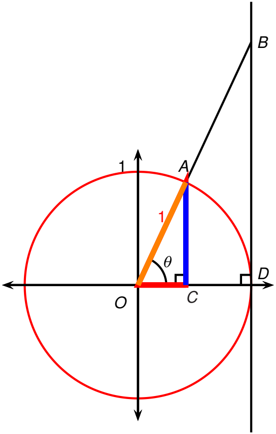

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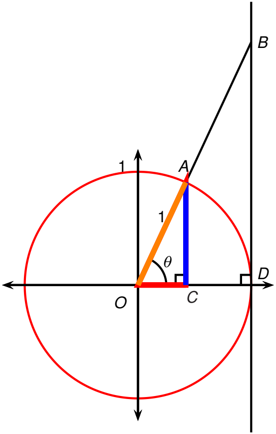

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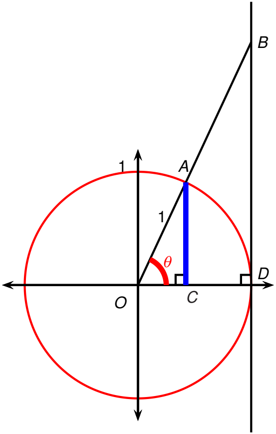

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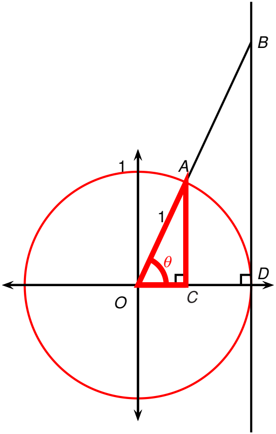

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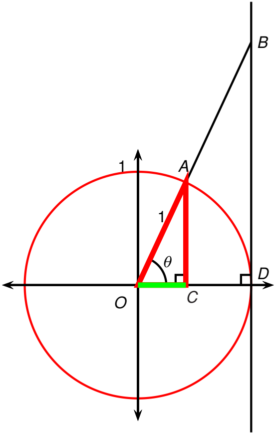

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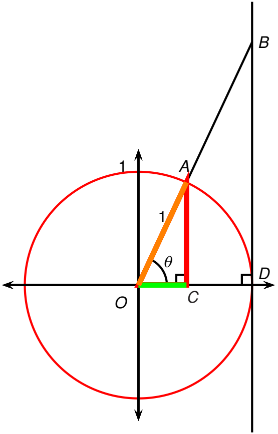

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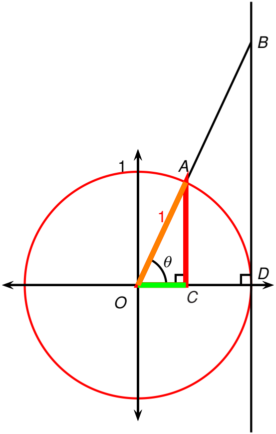

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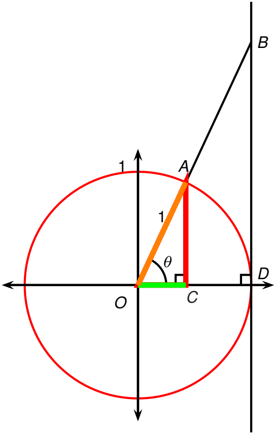
$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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\end{aligned}$$

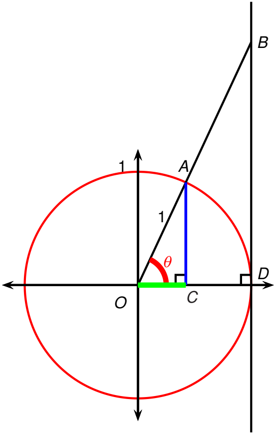
$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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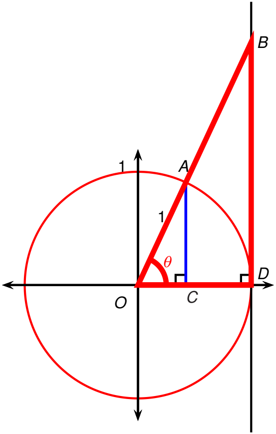
Geometric interpretation of all trigonometric functions



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$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$\cot\theta$

$\sec\theta$

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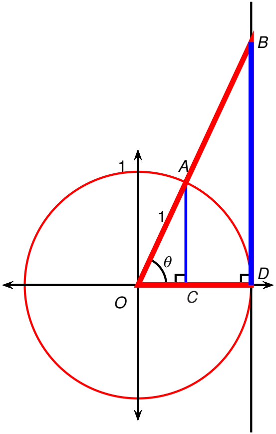
Geometric interpretation of all trigonometric functions



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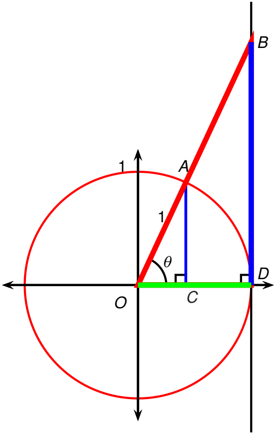
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{|BD|}{|OD|}$$

$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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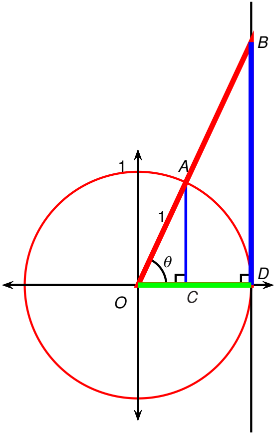
\tan\theta &= \frac{\text{opp}}{\text{adj}} = \frac{|BD|}{|OD|} \\
\cot\theta & \\
\sec\theta & \\
\csc\theta &
\end{aligned}$$

Geometric interpretation of all trigonometric functions



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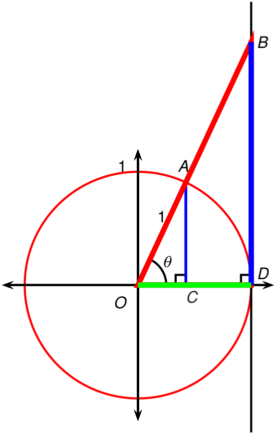
$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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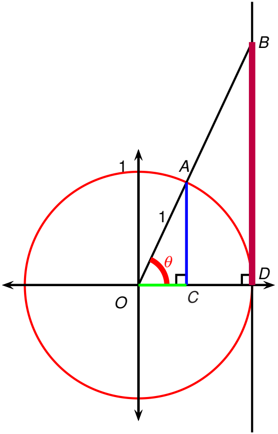

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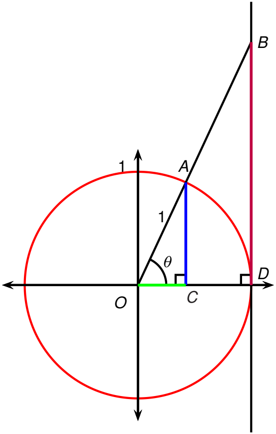
$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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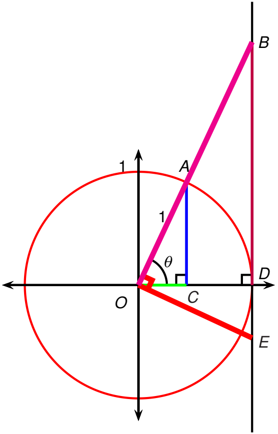
\tan\theta &= \frac{\text{opp}}{\text{adj}} = \frac{|BD|}{|OD|} = \frac{|BD|}{1} = |BD| \\
\cot\theta &= \frac{\text{adj}}{\text{opp}} \\
\sec\theta & \\
\csc\theta &
\end{aligned}$$

Geometric interpretation of all trigonometric functions



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$$\sec\theta$$

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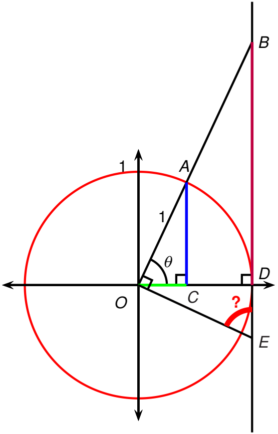

$\angle OED = \mathbf{?}$

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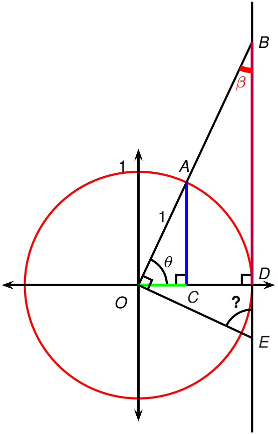

$\beta = \textbf{?}$

$\angle OED = \textbf{?}$

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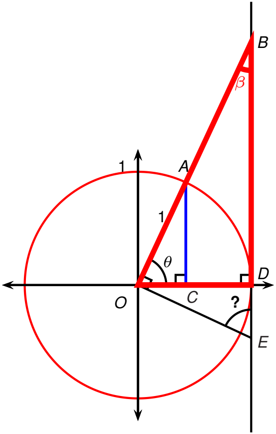

$\beta = 180° - 90° - \theta$

$\angle OED =$ **?**

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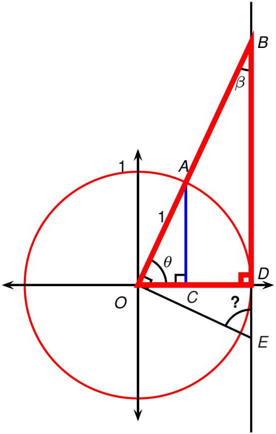

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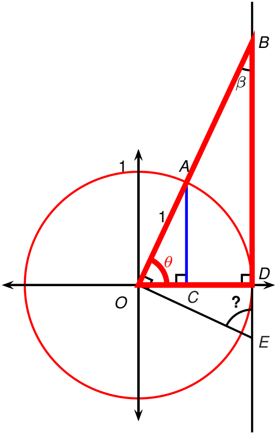

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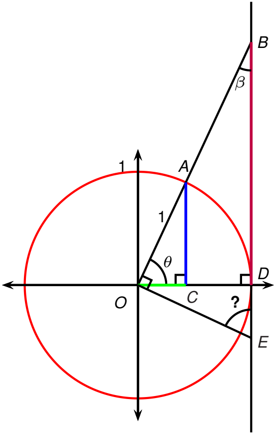
Geometric interpretation of all trigonometric functions



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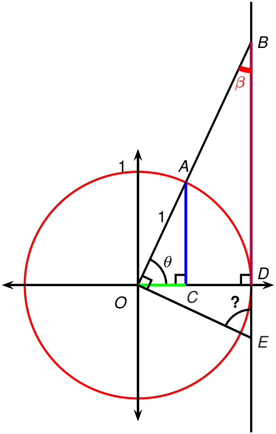

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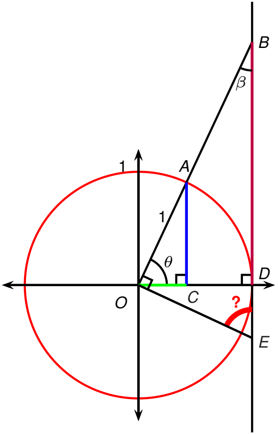

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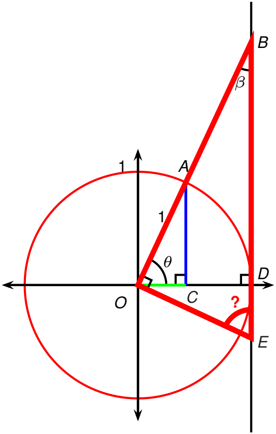
Geometric interpretation of all trigonometric functions



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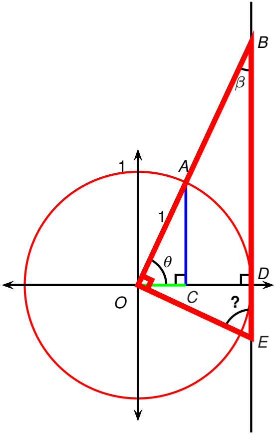

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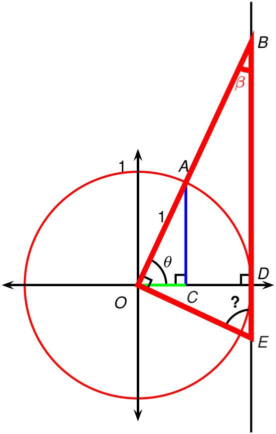

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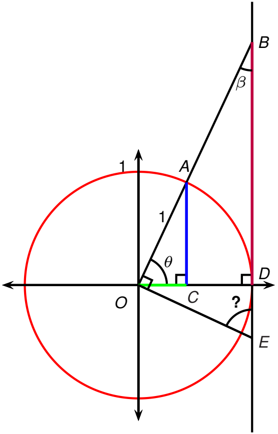

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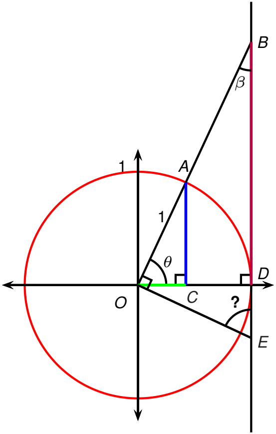

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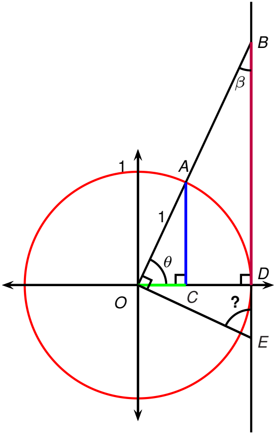

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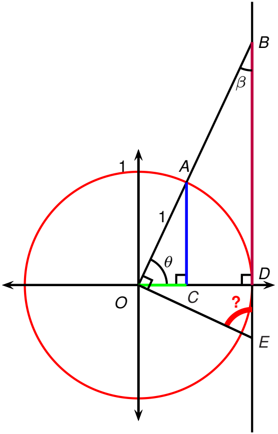

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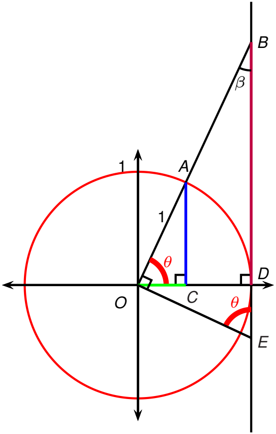

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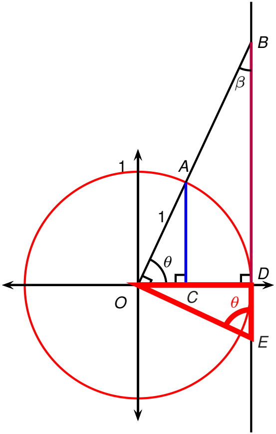

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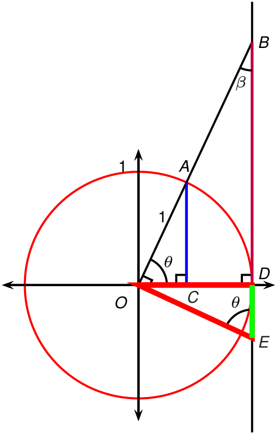

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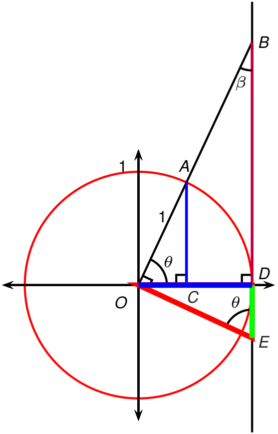

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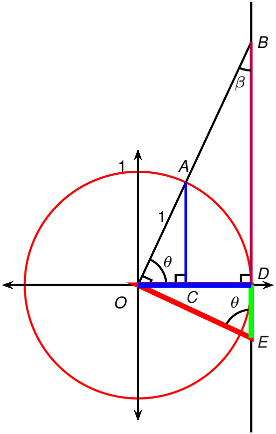

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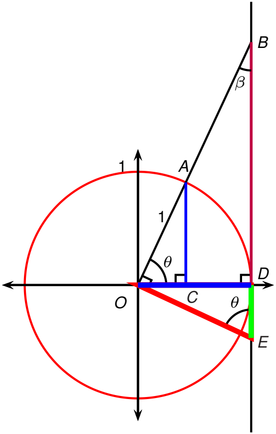

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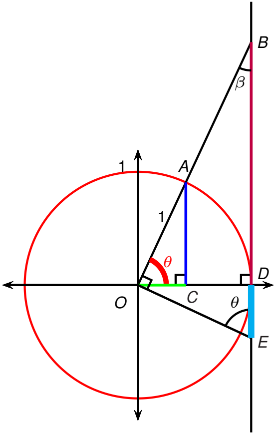

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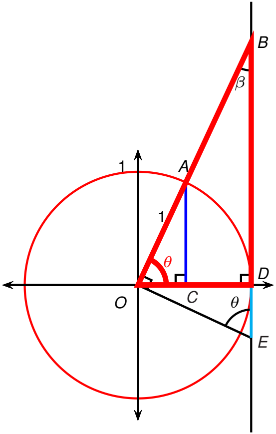

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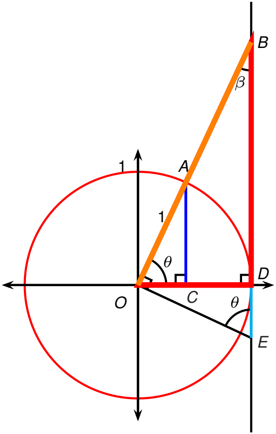
Geometric interpretation of all trigonometric functions



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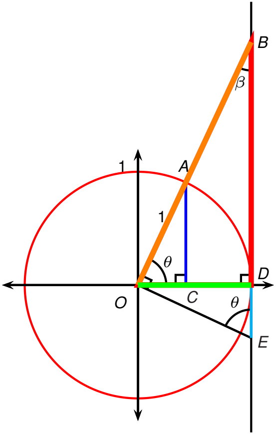

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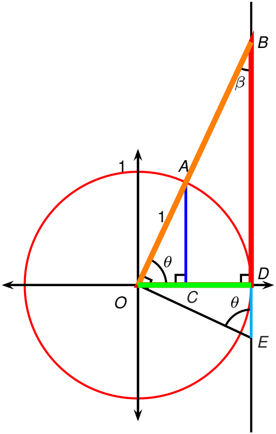

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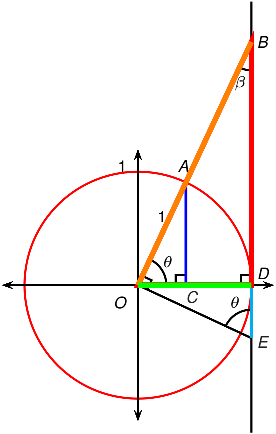
$$\csc\theta$$

Geometric interpretation of all trigonometric functions



$$\begin{aligned}\beta &= 180^\circ - 90^\circ - \theta\\ &= 90^\circ - \theta\\ \angle OED &= 180^\circ - 90^\circ - \beta\\ &= 90^\circ - (90^\circ - \theta)\\ &= \theta\end{aligned}$$

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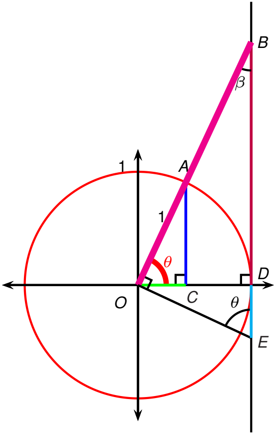

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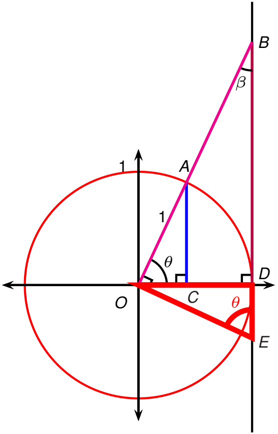

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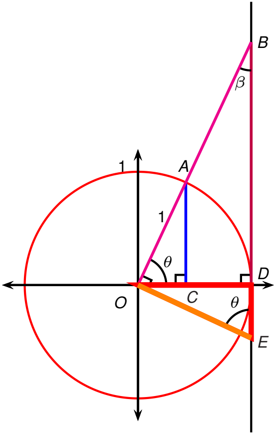

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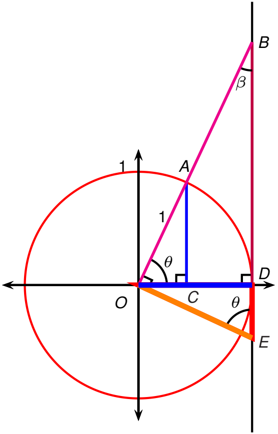

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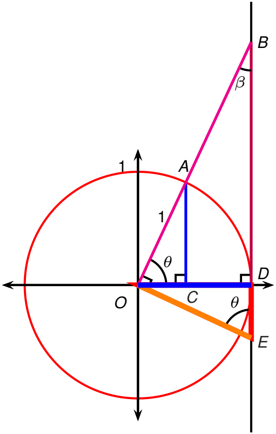

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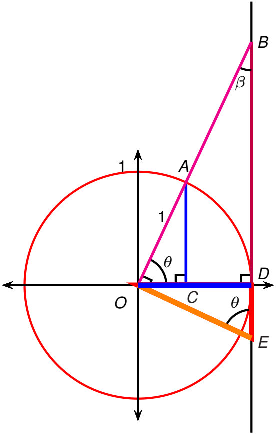

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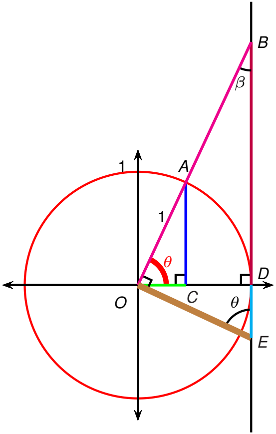

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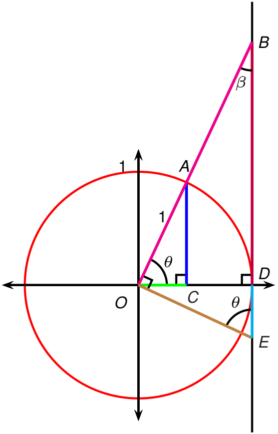

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Trigonometric Identities

Definition (Trigonometric Identity)

A trigonometric identity is an equality between the trigonometric functions in one or more variables that holds for all values of the involved variables in the domains of all of the expressions.

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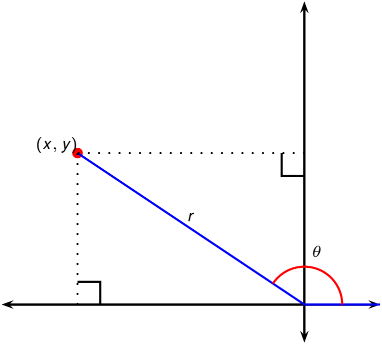
- By convention, when dealing with trigonometric identities we do not account for the domains of the involved expressions.
- For example, $\frac{\sin\theta}{\sin\theta} = 1$ is considered a valid trigonometric identity, although, when considered as a function, the left hand side is not defined for $\theta \neq 0$.



$$\sin\theta = \frac{y}{r} \quad \csc\theta = \frac{r}{y}$$
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- $\csc\theta = \frac{1}{\sin\theta}$
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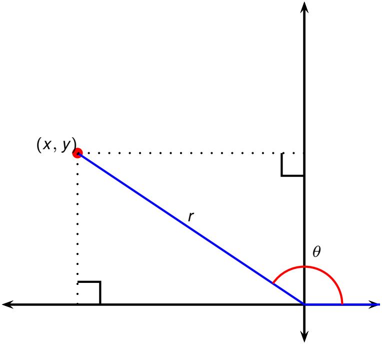

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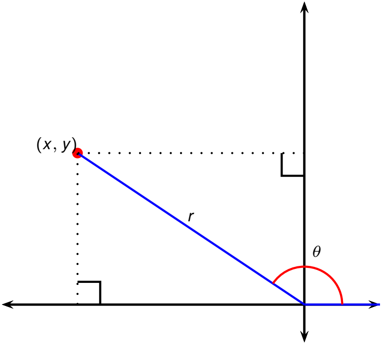

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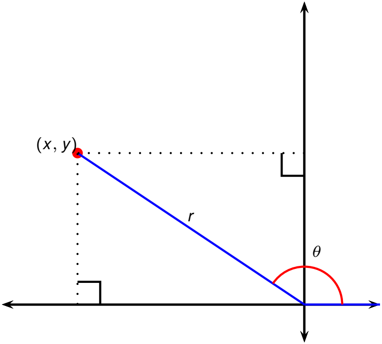
$$\sin^2\theta + \cos^2\theta$$



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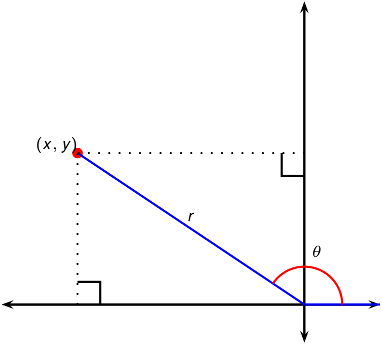
$$\begin{aligned} & \sin^2\theta + \cos^2\theta \\ = \quad & \frac{y^2}{r^2} + \frac{x^2}{r^2} \end{aligned}$$



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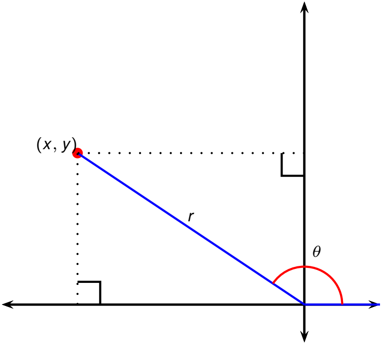

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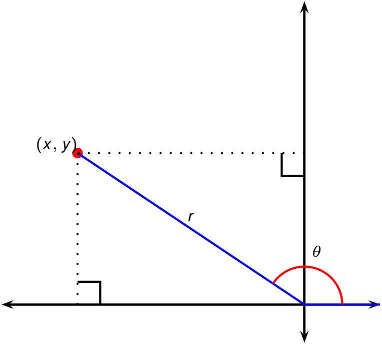

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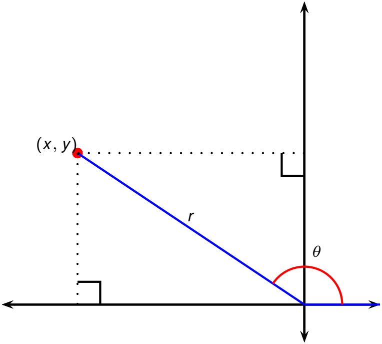

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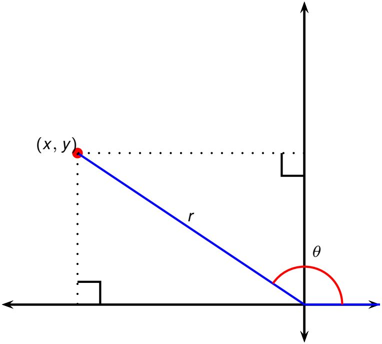
Therefore $\sin^2\theta + \cos^2\theta = 1$.



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Example ($\tan^2\theta + 1 = \sec^2\theta$)

Prove the identity
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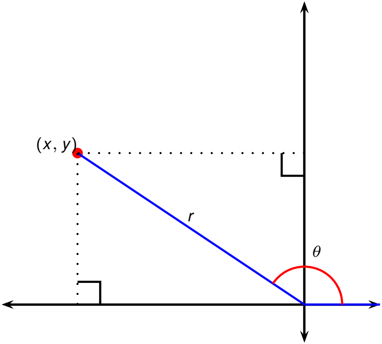

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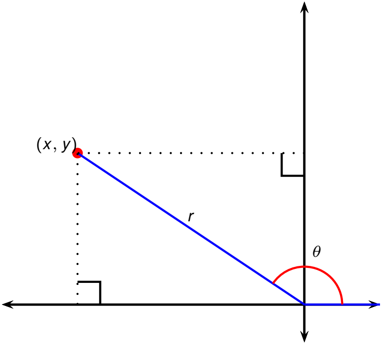
Example ($\tan^2\theta + 1 = \sec^2\theta$)

Prove the identity $\tan^2\theta + 1 = \sec^2\theta$.

$$\sin^2\theta + \cos^2\theta = 1$$



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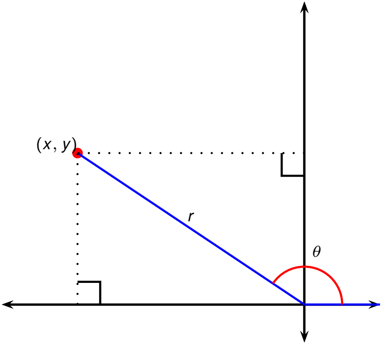
Example ($\tan^2\theta + 1 = \sec^2\theta$)

Prove the identity $\tan^2\theta + 1 = \sec^2\theta$.

$$\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} &= \frac{1}{\cos^2\theta} \end{aligned}$$



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Prove the identity $\tan^2\theta + 1 = \sec^2\theta$.

$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1\\
\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} &= \frac{1}{\cos^2\theta}\\
\tan^2\theta + 1 &= \sec^2\theta
\end{aligned}$$

The remaining identities are consequences of the addition formulas:

$$\begin{array}{rcrcl}\sin(x+y) &=& \sin x\cos y &+& \cos x\sin y\\ \cos(x+y) &=& \cos x\cos y &-& \sin x\sin y\end{array}$$

The remaining identities are consequences of the addition formulas:

$$\begin{aligned} \sin(x+y) &= \sin x \cos y + \cos x \sin y \\ \cos(x+y) &= \cos x \cos y - \sin x \sin y \end{aligned}$$

Substitute $-y$ for y, and use the fact that $\sin(-y) = -\sin y$ and $\cos(-y) = \cos y$:

$$\begin{aligned} \sin(x-y) &= \sin x \cos y - \cos x \sin y \\ \cos(x-y) &= \cos x \cos y + \sin x \sin y \end{aligned}$$

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$$\begin{aligned}\sin(x+y) &= \sin x\cos y + \cos x\sin y\\ \cos(x+y) &= \cos x\cos y - \sin x\sin y\end{aligned}$$

To get the double angle formulas, substitute x for y:

$$\begin{aligned}\sin(2x) &= 2\sin x\cos x\\ \cos(2x) &= \cos^2 x - \sin^2 x\end{aligned}$$

The remaining identities are consequences of the addition formulas:

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Rewrite the second double angle formula in two ways, using $\cos^2 x = 1-\sin^2 x$ and $\sin^2 x = 1-\cos^2 x$:

$$\begin{aligned}
\cos(2x) &= 2\cos^2 x - 1\\
\cos(2x) &= 1 - 2\sin^2 x
\end{aligned}$$

The remaining identities are consequences of the addition formulas:

$$\begin{array}{rcrcl}\sin(x+y) & = & \sin x\cos y & + & \cos x\sin y\\ \cos(x+y) & = & \cos x\cos y & - & \sin x\sin y\end{array}$$

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To get the half-angle formulas, solve these equations for $\cos^2 x$ and $\sin^2 x$ respectively.

$$\cos^2 x=\frac{1+\cos(2x)}{2},\qquad \sin^2 x=\frac{1-\cos(2x)}{2}$$

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Divide the first equation by the second, and then cancel $\cos x \cos y$ from the top and bottom:

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

The remaining identities are consequences of the addition formulas:

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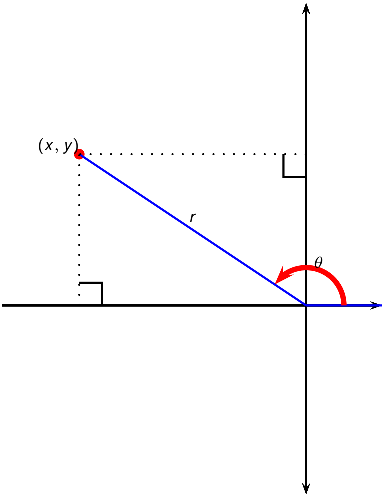
Divide the first equation by the second, and then cancel $\cos x\cos y$ from the top and bottom:

$$\tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y}$$

Do the same for the subtraction formulas:

$$\tan(x-y) = \frac{\tan x-\tan y}{1+\tan x\tan y}$$



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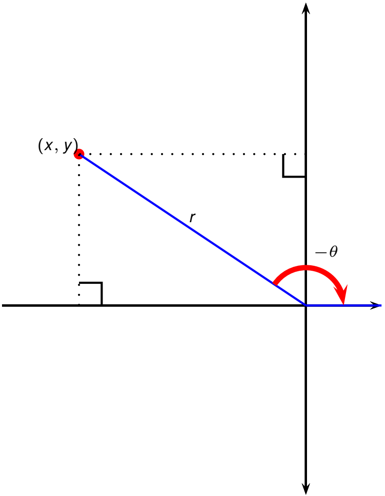
- Positive angles are obtained by rotating counterclockwise.



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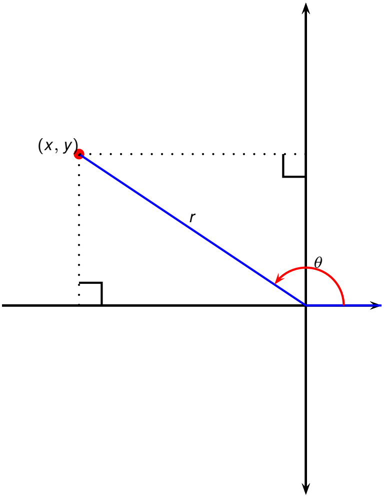

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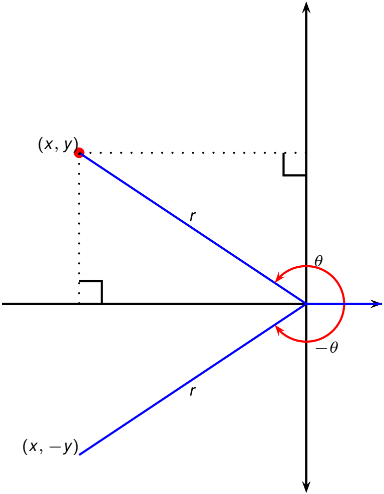
- If (x, y) is on the terminal arm of the angle θ, then $(x, -y)$ is on the terminal arm of $-\theta$.



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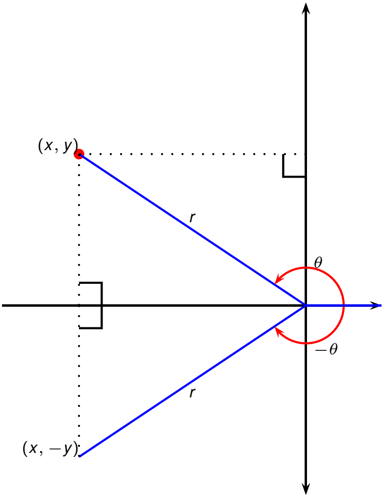

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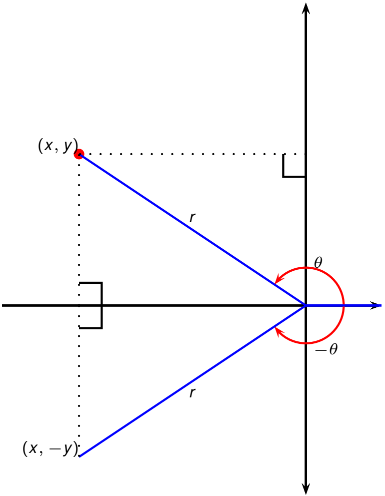
- $\sin(-\theta) = \frac{-y}{r} = -\frac{y}{r} = -\sin\theta$.



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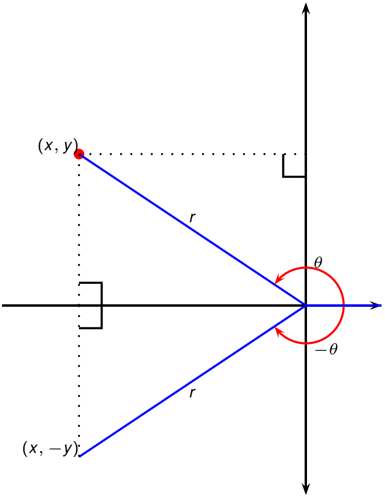

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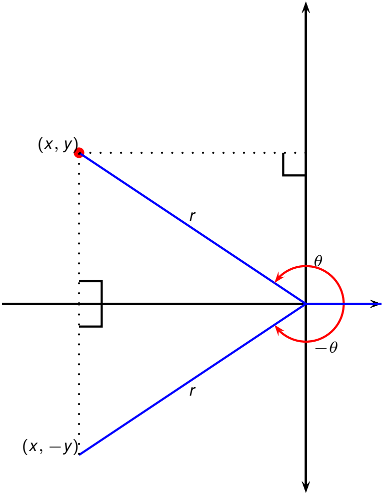
- sin is an odd function.



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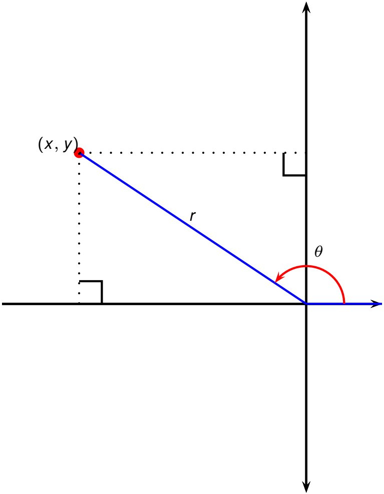
- sin is an odd function.
- cos is an even function.



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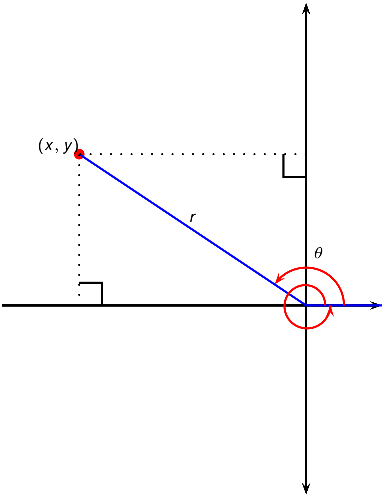

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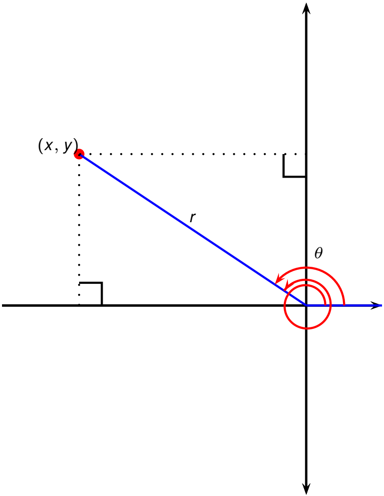
- 2π represents a full rotation.



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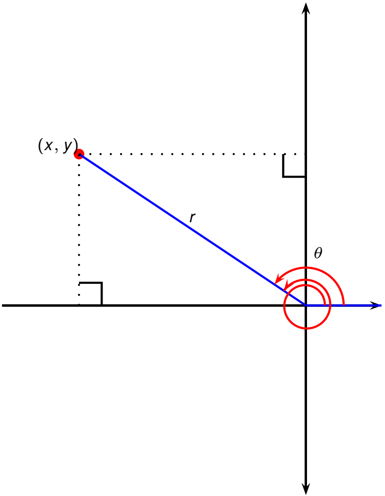
- 2π represents a full rotation.
- $\theta + 2\pi$ has the same terminal arm as θ.



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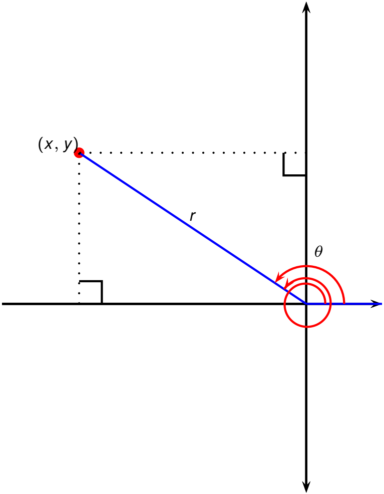
- $\theta + 2\pi$ uses the same point (x, y) and the same length r.



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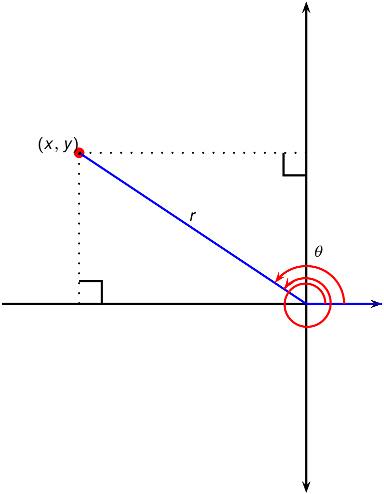

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- We say $\sin$ and $\cos$ are 2π-periodic.

You will not be tested on the material in the following slide.

Definition (Complex numbers)

The set of complex numbers $\mathbb{C}$ is defined as the set

$$\{a + bi | a, b - \text{real numbers}\},$$

where the number i is a number for which

$$i^2 = -1 \quad .$$

The number i is called the imaginary unit.

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$$(a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i \quad .$$

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You will not be tested on the material in the following slide.

Euler's Formula

Theorem (Euler's Formula)

$$e^{ix} = \cos x + i \sin x,$$

where $e \approx 2.71828$ *is Euler's/Napier's constant .*

Proof.

Recall $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n$. Borrow from Calc II the f-las:

□

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$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + \frac{(-1)^n x^{2n}}{(2n)!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

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\begin{array}{rclllllll}
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\cos x & = & 1 & & -\frac{x^2}{2!} & & +\frac{x^4}{4!} & & + \dots \\
\hline
e^z & = & 1 & +z & +\frac{z^2}{2!} & +\frac{z^3}{3!} & +\frac{z^4}{4!} & +\frac{z^5}{5!} & + \dots
\end{array}
$$

Rearrange.

$\square$

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\hline
e^{ix} & = & 1 & +ix & +\frac{(ix)^2}{2!} & +\frac{(ix)^3}{3!} & +\frac{(ix)^4}{4!} & +\frac{(ix)^5}{5!} & +\dots
\end{array}$$

Rearrange. Plug-in $z = ix$.

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\sin x &=& & x & & -\frac{x^3}{3!} & & +\frac{x^5}{5!} & -\dots \\
\cos x &=& 1 & & -\frac{x^2}{2!} & & +\frac{x^4}{4!} & & +\dots \\
\hline
e^{ix} &=& 1 & +ix & \color{red}{-}\frac{\color{red}{x^2}}{2!} & \color{red}{-i}\frac{x^3}{3!} & +\frac{\color{red}{x^4}}{4!} & +\color{red}{i}\frac{x^5}{5!} & \color{red}{-}\dots
\end{array}$$

Rearrange. Plug-in $z = ix$. Use $i^2 = -1$.

$\square$

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where $e \approx 2.71828$ is Euler's/Napier's constant .

Proof.

Recall $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n$. Borrow from Calc II the f-las:

$$\begin{array}{rcllllllll}
i \sin x &=& & ix & & -i\frac{x^3}{3!} & & +i\frac{x^5}{5!} & - \dots \\
\cos x &=& 1 & & -\frac{x^2}{2!} & & +\frac{x^4}{4!} & & + \dots \\
\hline
e^{ix} &=& 1 & +ix & -\frac{x^2}{2!} & -i\frac{x^3}{3!} & +\frac{x^4}{4!} & +i\frac{x^5}{5!} & - \dots
\end{array}$$

Rearrange. Plug-in $z = ix$. Use $i^2 = -1$. Multiply $\sin x$ by i.

$\square$

Euler's Formula

Theorem (Euler's Formula)

$$e^{ix} = \cos x + i \sin x,$$

where $e \approx 2.71828$ is Euler's/Napier's constant .

Proof.

Recall $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n$. Borrow from Calc II the f-las:

$$\begin{array}{rcccccccc}
i \sin x & = & & ix & & -i\frac{x^3}{3!} & & +i\frac{x^5}{5!} & - \dots \\
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e^{ix} & = & 1 & +ix & -\frac{x^2}{2!} & -i\frac{x^3}{3!} & +\frac{x^4}{4!} & +i\frac{x^5}{5!} & - \dots
\end{array}$$

Rearrange. Plug-in $z = ix$. Use $i^2 = -1$. Multiply $\sin x$ by i. Add to get $e^{ix} = \cos x + i \sin x$. □

You will not be tested on the material in the following slide.

Trigonometric Identities Revisited

- $e^{ix} = \cos x + i \sin x$ (Euler's Formula).
- $e^{ix} e^{iy} = e^{ix+iy} = e^{i(x+y)}$ (exponentiation rule: valid for $\mathbb{C}$).
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All trigonometric formulas can be easily derived using the above formulas.

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Example

$$\begin{aligned} \sin(x+y) &= \sin x \cos y + \sin y \cos x \\ \cos(x+y) &= \cos x \cos y - \sin x \sin y \quad . \end{aligned}$$

Proof.

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$$e^{i(x+y)} =$$

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$$e^{i(x+y)} = \cos(x+y) + i\sin(x+y)$$

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Compare coefficient in front of i and

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Compare coefficient in front of i and remaining terms to get the desired equalities. □

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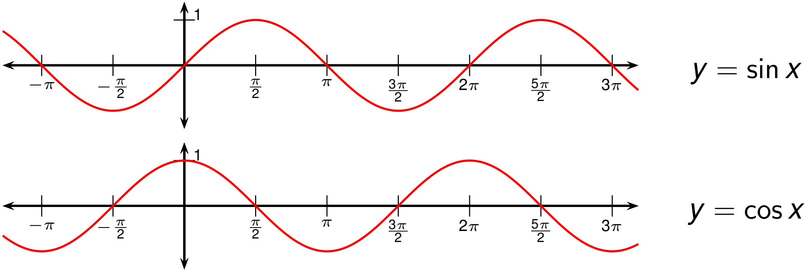
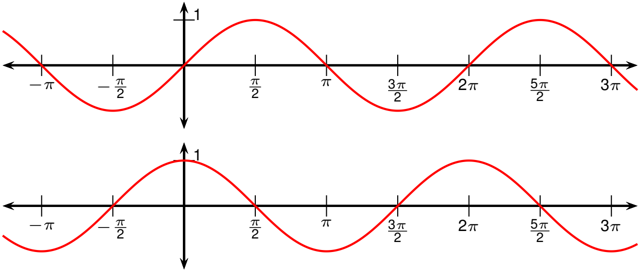
Graphs of the Trigonometric Functions



Graphs of the Trigonometric Functions



$y = \sin x$

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- $\sin x$ has zeroes at $n\pi$ for all integers n.

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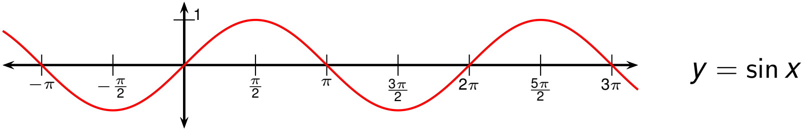

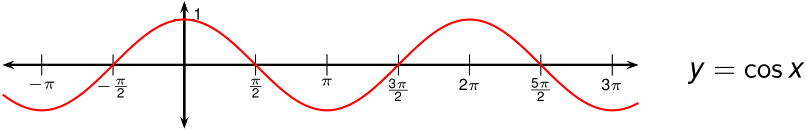

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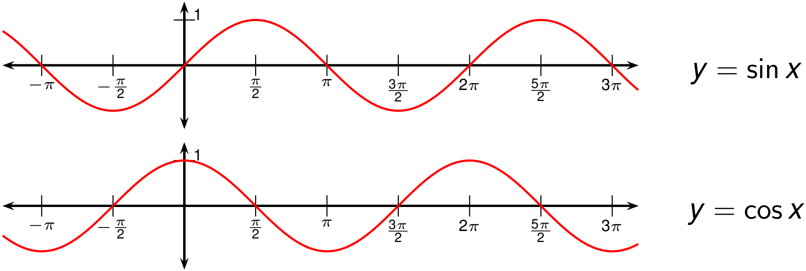

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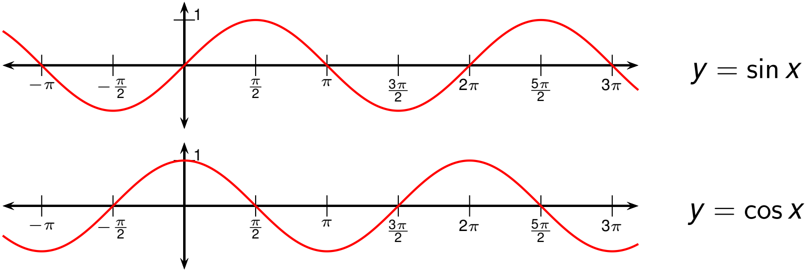

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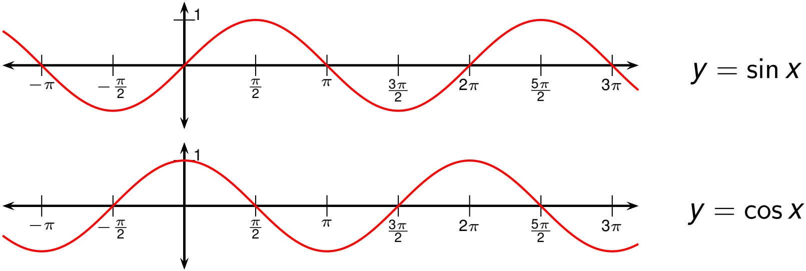

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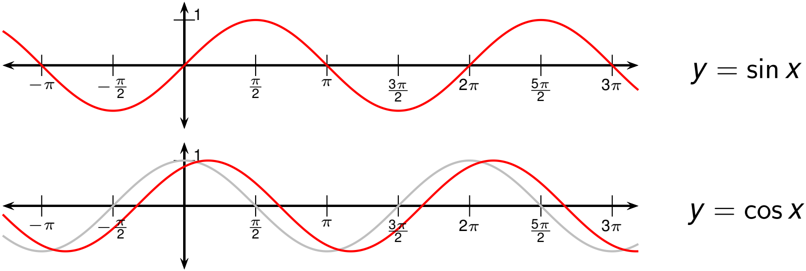

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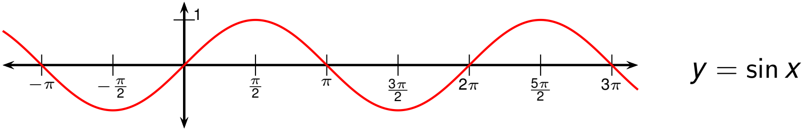

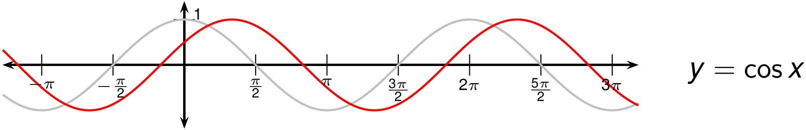

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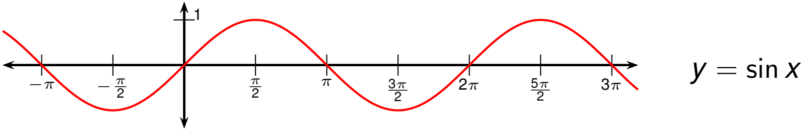
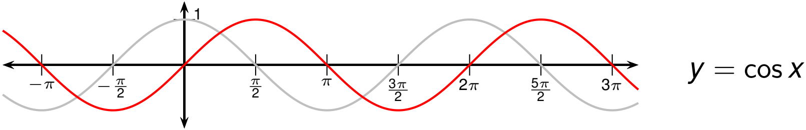
Graphs of the Trigonometric Functions





- $\sin x$ has zeroes at $n\pi$ for all integers n.
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.
- $-1 \leq \sin x \leq 1$.
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- If we translate the graph of $\cos x$ by $\frac{\pi}{2}$ units to the right we get the graph of $\sin x$.

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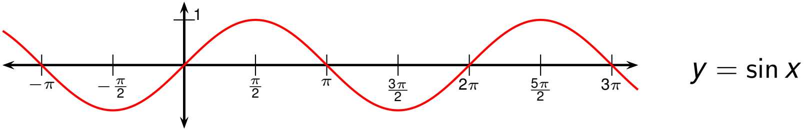

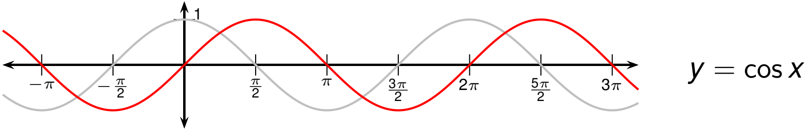

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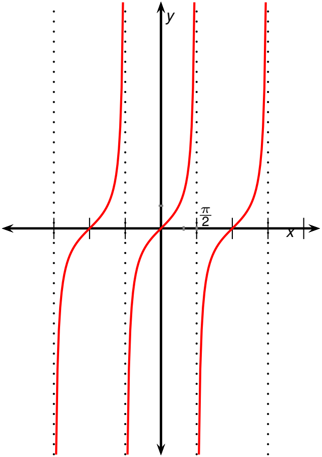
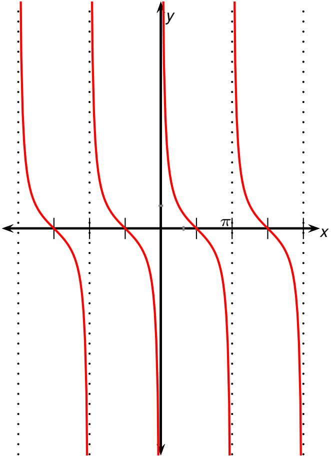
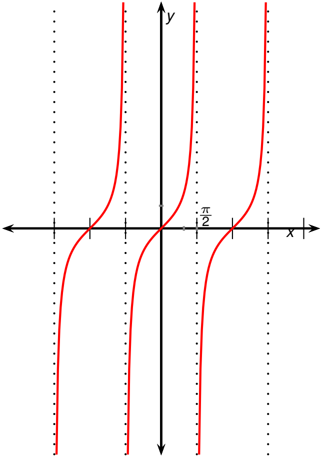


$y = \tan x$



$y = \cot x$



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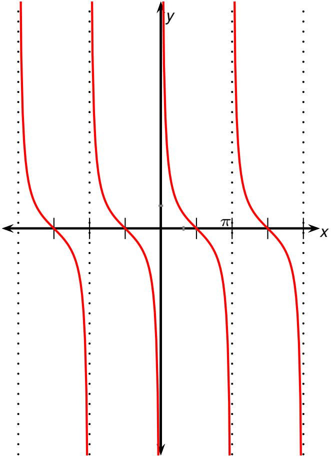

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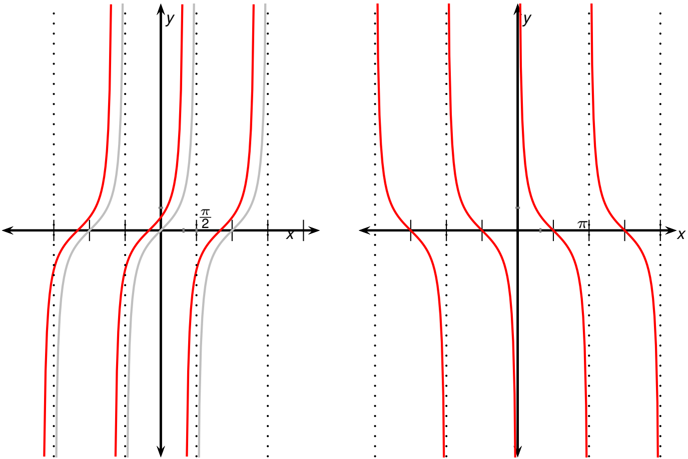
If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis



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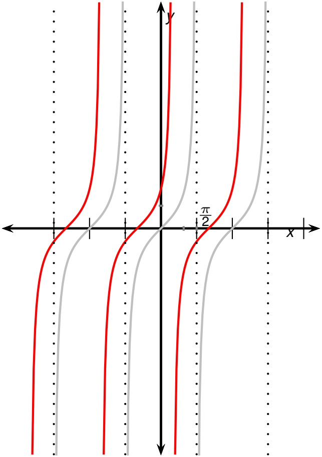

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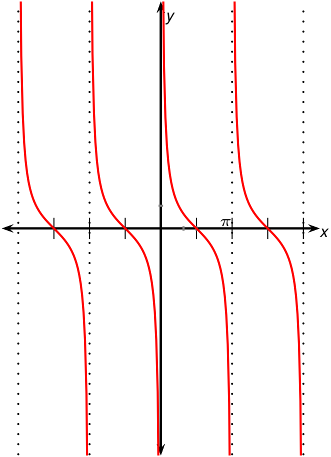

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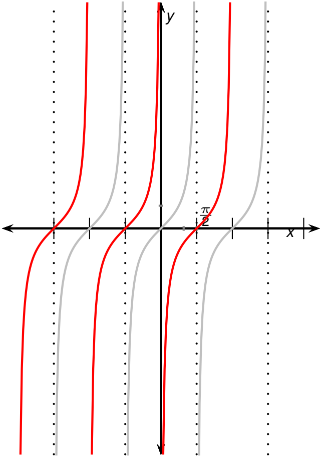

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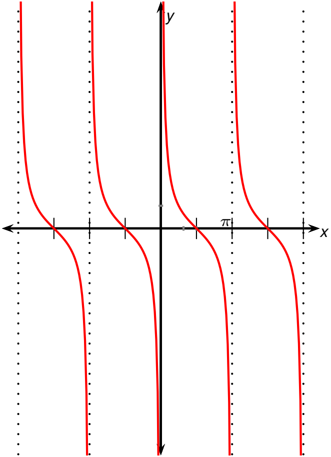

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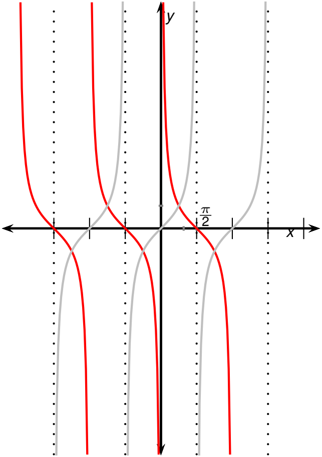

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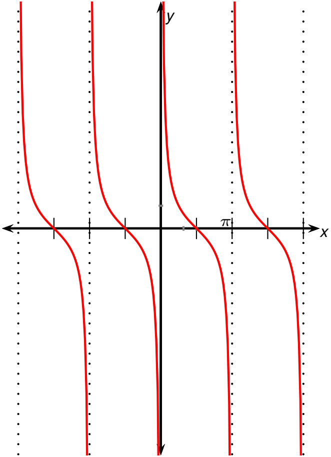
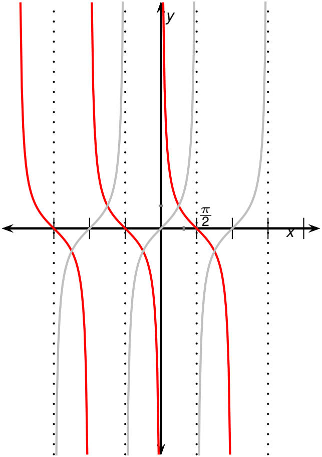


$y = \cot x$

If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis, we get the graph of $\cot x$.



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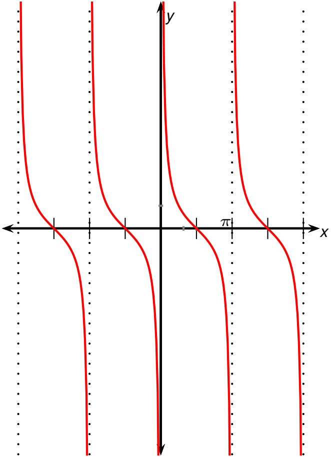
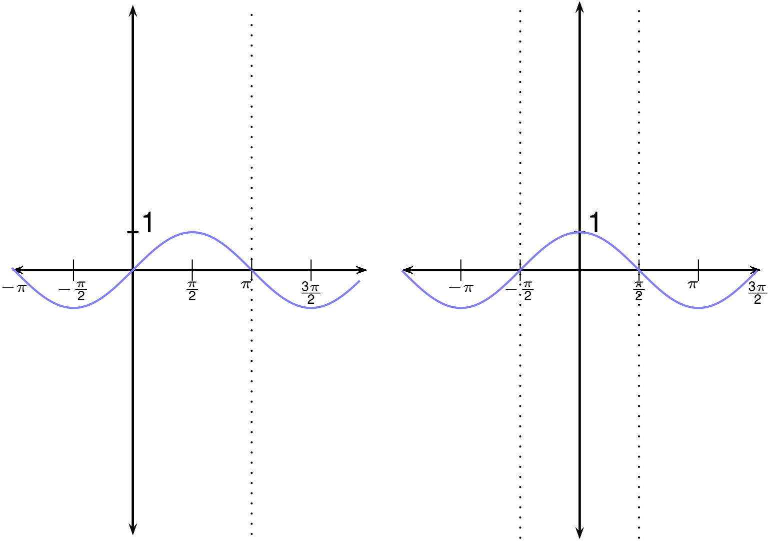
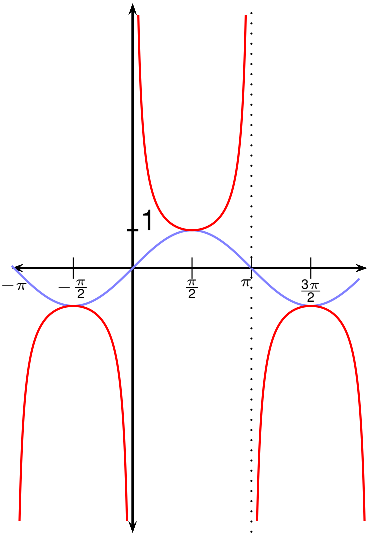


$y = \cot x$

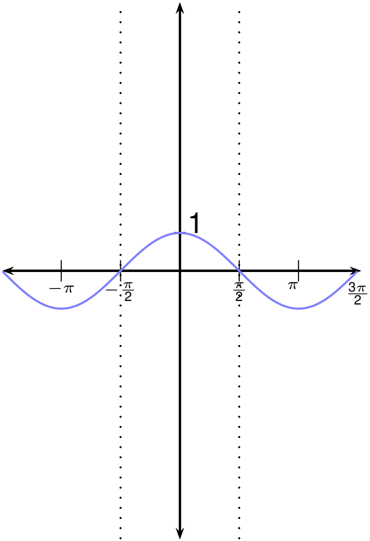
If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis, we get the graph of $\cot x$. This follows from $\tan\left(x \pm \frac{\pi}{2}\right) = -\cot x$.



$y = \csc x$ $\qquad$ $y = \sec x$


1
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$

$y = \csc x$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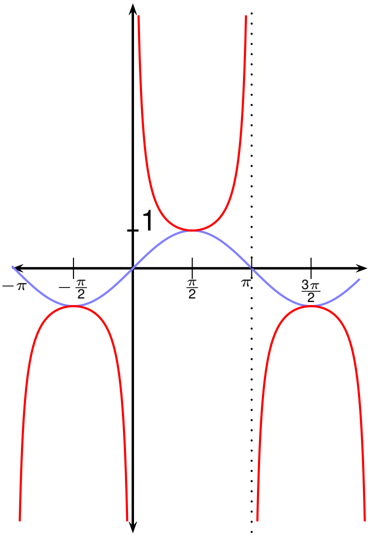
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$-\pi$
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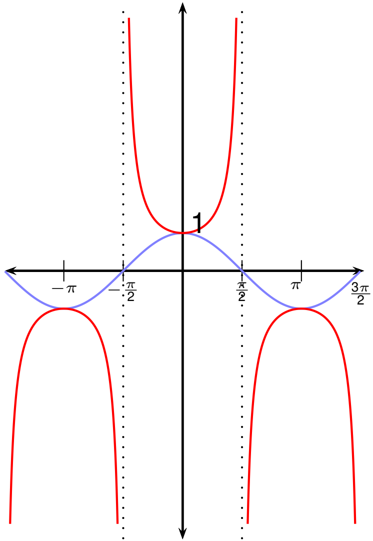
1
$-\pi$
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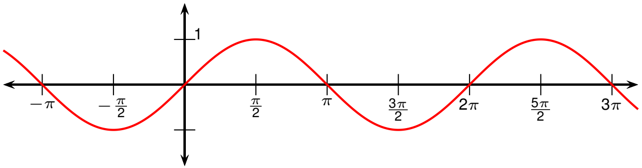
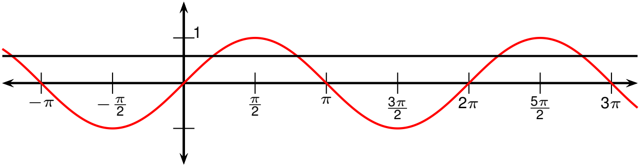
Inverse Trigonometric Functions

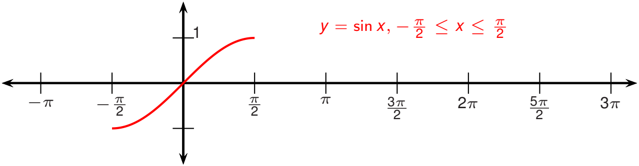


Inverse Trigonometric Functions



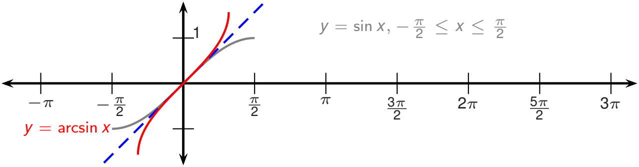
- $\sin x$ isn't one-to-one.

Inverse Trigonometric Functions



- $\sin x$ isn't one-to-one.
- It is if we restrict the domain to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Inverse Trigonometric Functions



- $\sin x$ isn't one-to-one.
- It is if we restrict the domain to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
- Then it has an inverse function.
- We call it $\arcsin$ or $\sin^{-1}$.

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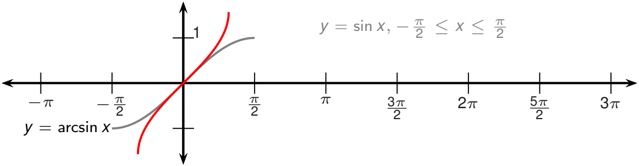

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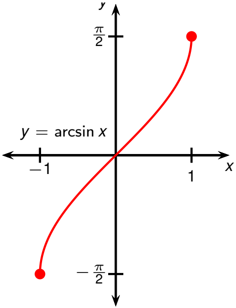

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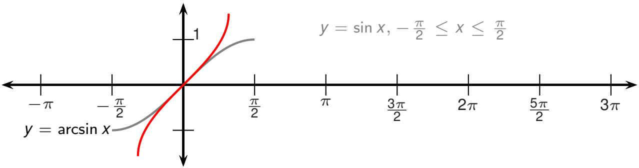

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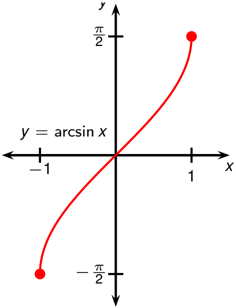
- Then it has an inverse function.
- We call it arcsin or $\sin^{-1}$.
- $\arcsin x = y \Leftrightarrow \sin y = x$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.



Example

Find $\arcsin\left(\frac{1}{2}\right)$.

Find $\tan\left(\arcsin\left(\frac{1}{3}\right)\right)$.

Example

Find $\arcsin\left(\frac{1}{2}\right)$.

- $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.

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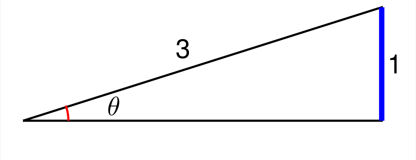

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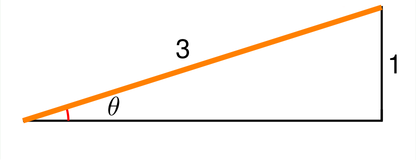

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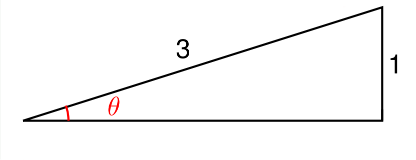

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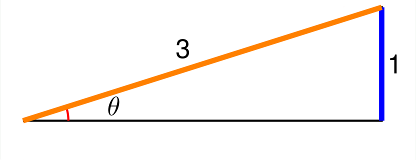
- Let the angle θ be as labeled. Then $\sin\theta = \frac{1}{3}$.



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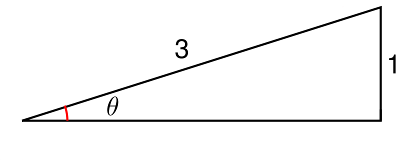

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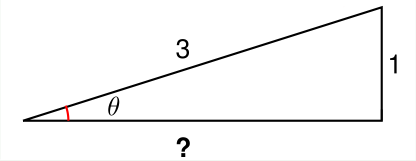
- Length of adjacent side $=$ **?**



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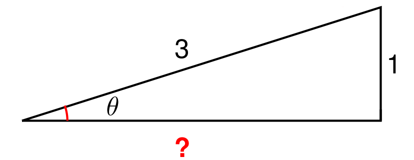
- Length of adjacent side $= \sqrt{3^2 - 1^2}$



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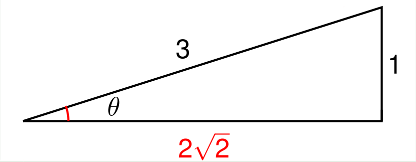
- Let $\theta = \arcsin\left(\frac{1}{3}\right)$, so $\sin\theta = \frac{1}{3}$.
- Draw a right triangle with opposite side 1 and hypotenuse 3.
- Let the angle θ be as labeled. Then $\sin\theta = \frac{1}{3}$ and so $\theta = \arcsin\left(\frac{1}{3}\right)$.
- Length of adjacent side $= \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$.



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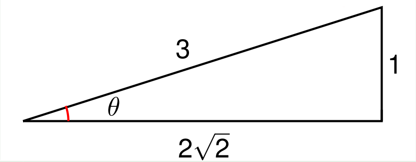
- Then $\tan\left(\arcsin\left(\frac{1}{3}\right)\right) =$ **?**



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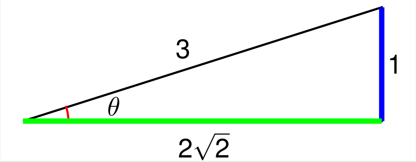
- Length of adjacent side $= \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$.
- Then $\tan\left(\arcsin\left(\frac{1}{3}\right)\right) = \frac{1}{2\sqrt{2}}$.



Example

Find $\arcsin(\sin(1.5))$.

Example

Find $\arcsin(\sin(1.5))$.

- $\frac{\pi}{2} \approx$ **?**

Example

Find $\arcsin(\sin(1.5))$.

- $\frac{\pi}{2} \approx 1.57$.

Example

Find $\arcsin(\sin(1.5))$.

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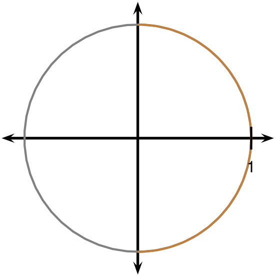
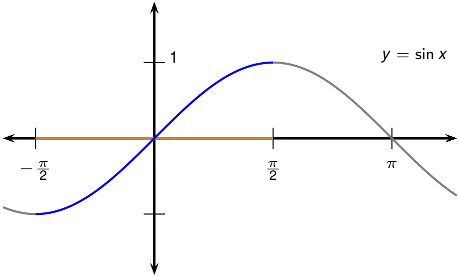
- Therefore $\arcsin(\sin 1.5) = 1.5$.

Example

Find $\arcsin(\sin 2)$.





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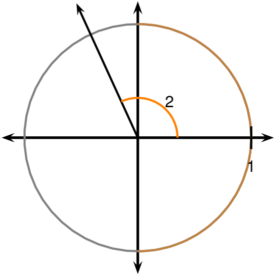

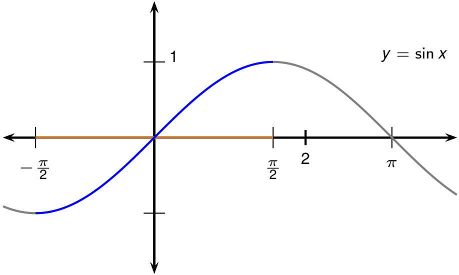

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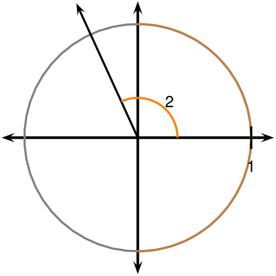

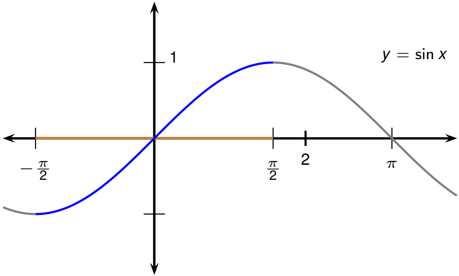

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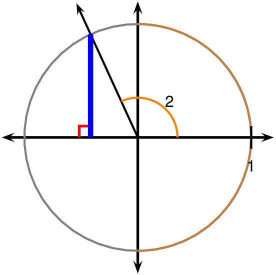

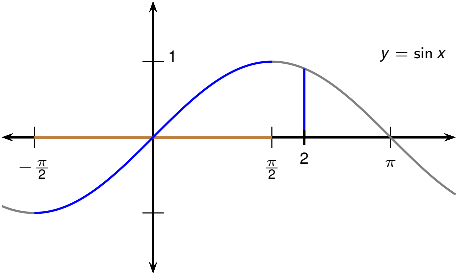

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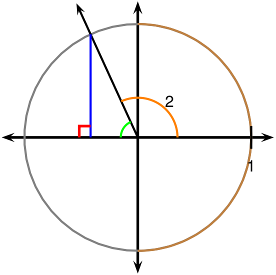

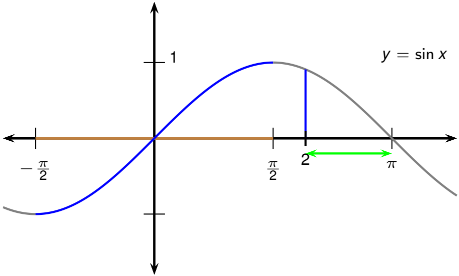

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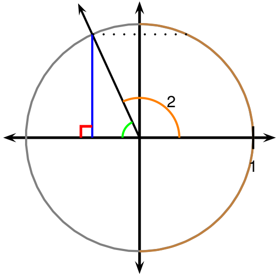

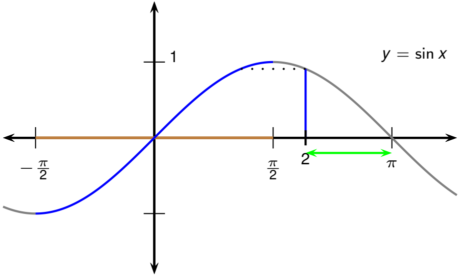

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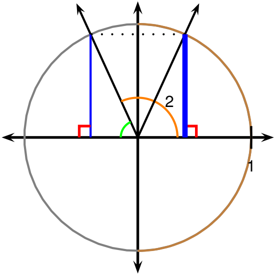

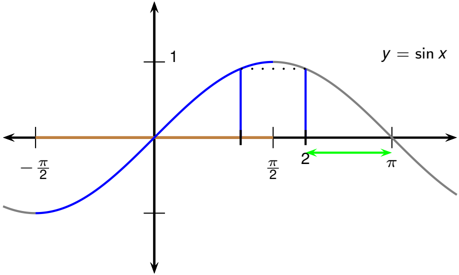

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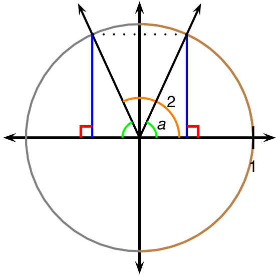

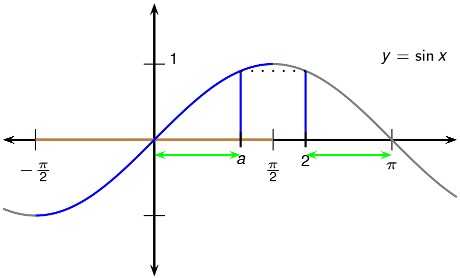

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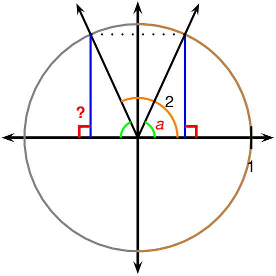

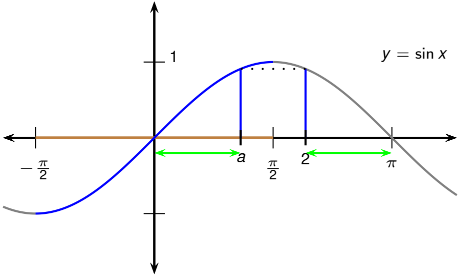

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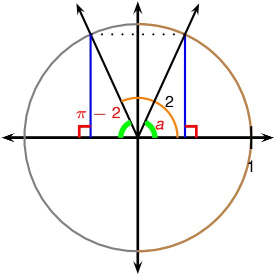

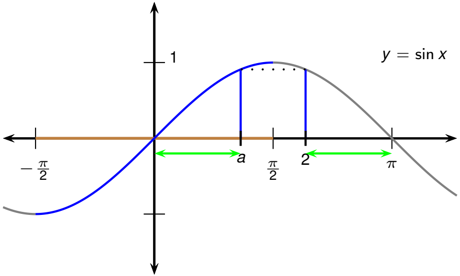

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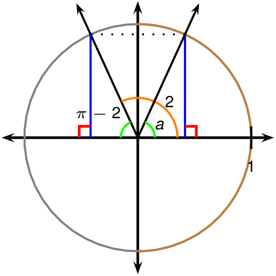

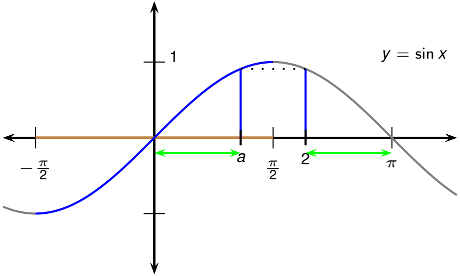

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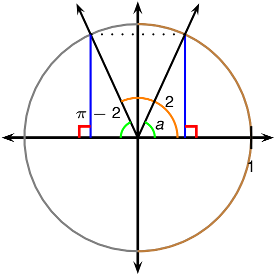

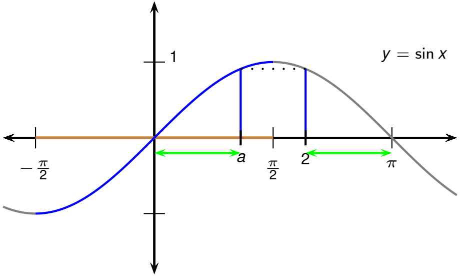

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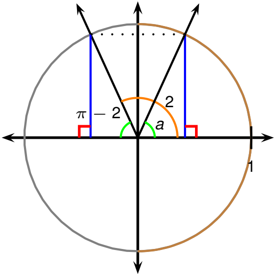

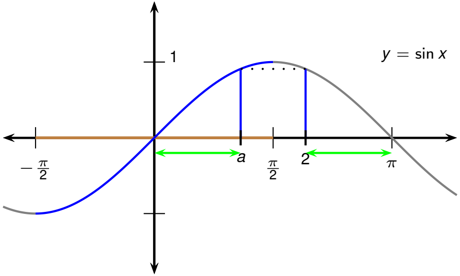

Theorem (The Derivative of $\arcsin x$)

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Proof.

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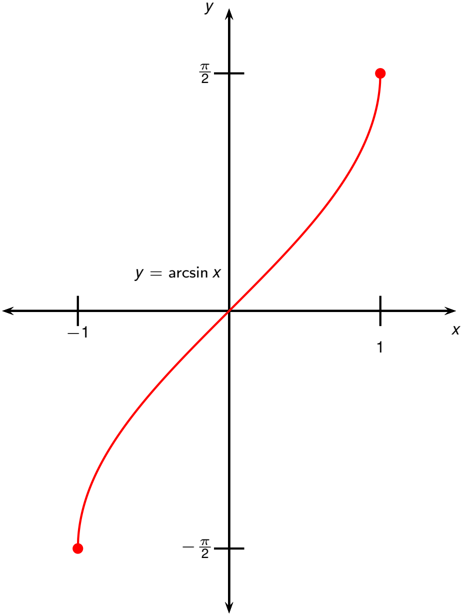
Important facts about arcsin:



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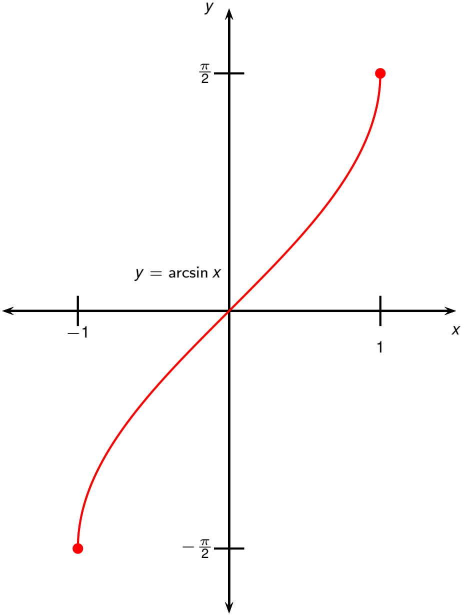

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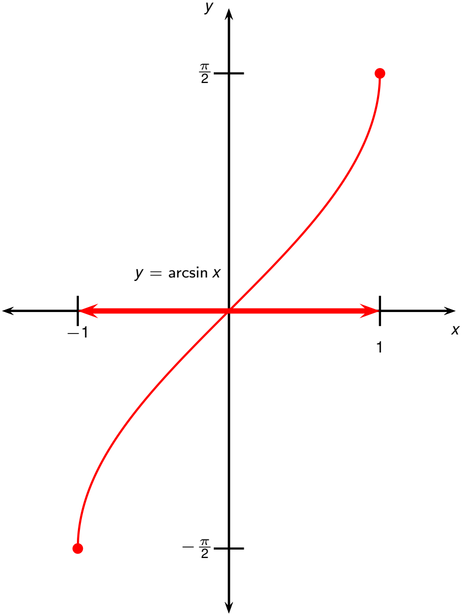

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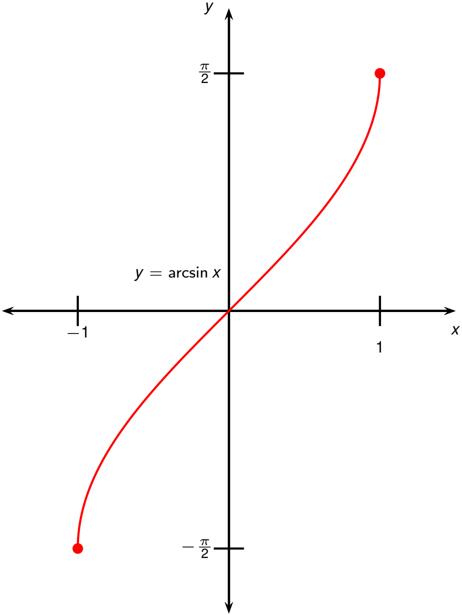

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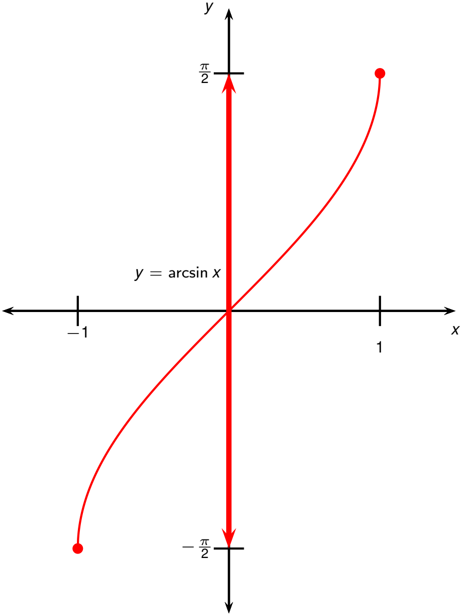

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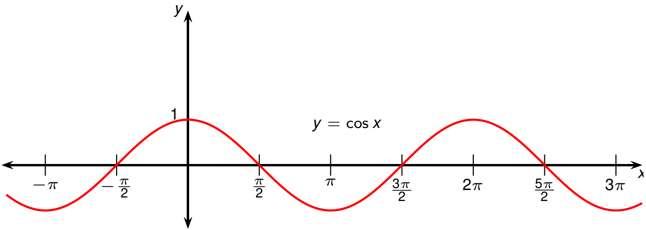

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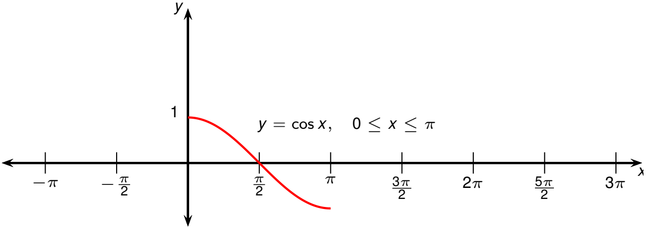

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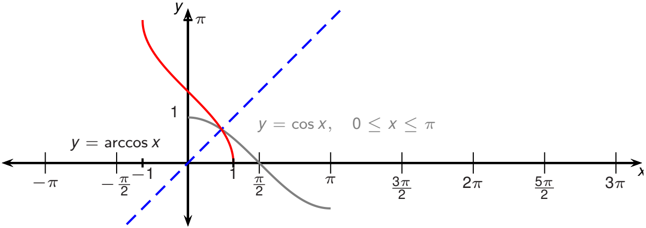

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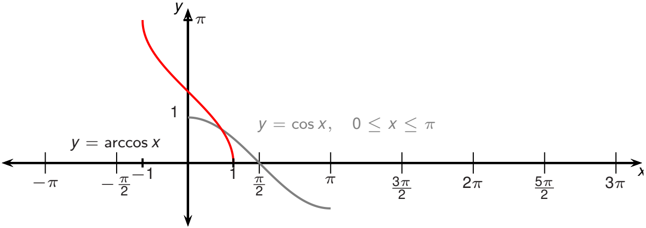

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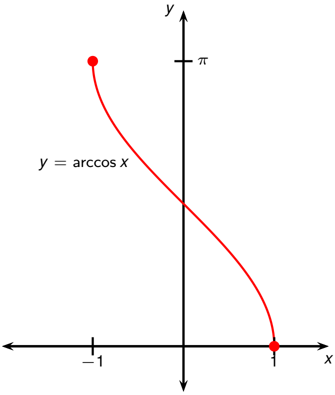

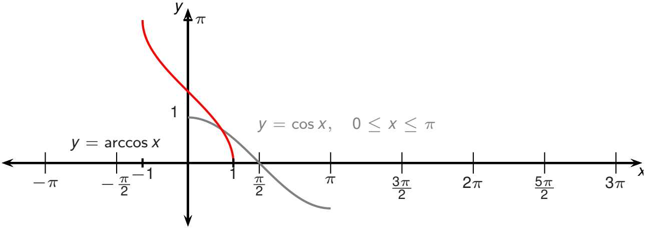

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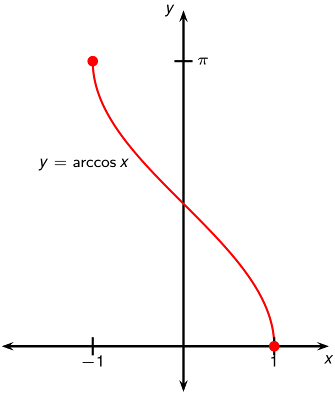

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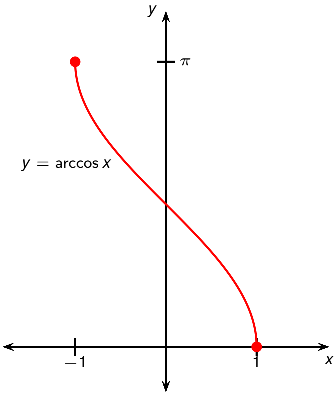

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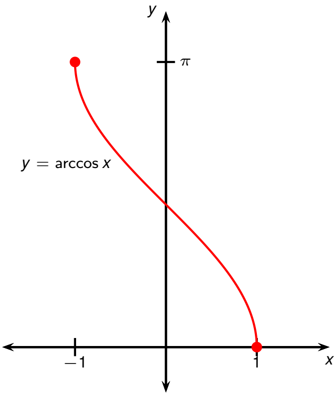

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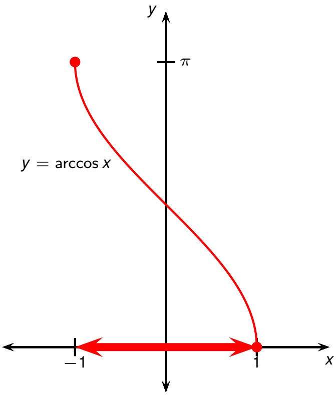

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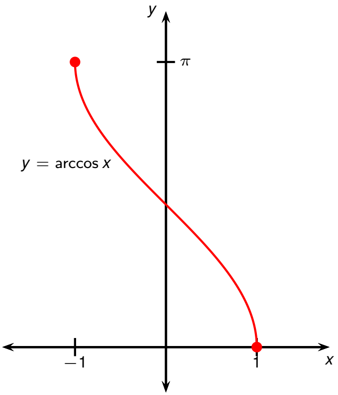
4. $\arccos(\cos x) = x$ for $0 \leq x \leq \pi$.
5. $\cos(\arccos x) = x$ for $-1 \leq x \leq 1$.
6. $\dfrac{d}{dx}(\arccos x) = -\dfrac{1}{\sqrt{1 - x^2}}$.

Important facts about arccos:



1. Domain: $[-1, 1]$.
2. Range: **?**
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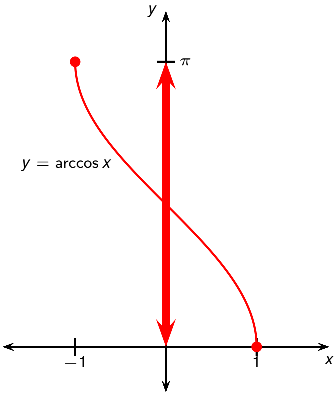

1. Domain: $[-1, 1]$.
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3. $\arccos x = y \Leftrightarrow \cos y = x$ and $0 \leq y \leq \pi$.
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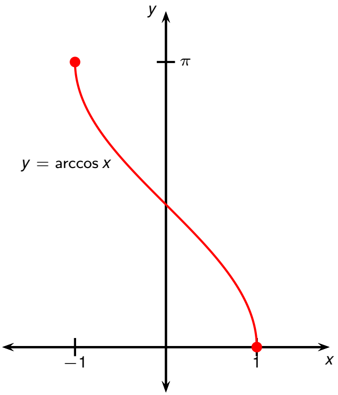

1. Domain: $[-1, 1]$.
2. Range: $[0, \pi]$.
3. $\arccos x = y \Leftrightarrow \cos y = x$ and $0 \leq y \leq \pi$.
4. $\arccos(\cos x) = x$ for $0 \leq x \leq \pi$.
5. $\cos(\arccos x) = x$ for $-1 \leq x \leq 1$.
6. $\dfrac{\mathrm{d}}{\mathrm{d}x}(\arccos x) = -\dfrac{1}{\sqrt{1-x^2}}$.
   (The proof is similar to the proof of the formula for the derivative of $\arcsin x$.)

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

$$\sin(2\arccos(x))$$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.
To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$.

$$\sin(2\arccos(x))$$

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$\sin(2\arccos(x)) = \sin(2y)$ | Set $y = \arccos x$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\begin{aligned}\sin(2\arccos(x)) &= \sin(2y)\\ &= \textbf{?}\end{aligned}$$

Set $y = \arccos x$
Express via $\sin y, \cos y$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\begin{aligned} \sin(2\arccos(x)) &= \sin(2y) \\ &= 2\cos y \sin y \end{aligned}$$

Set $y = \arccos x$
Express via $\sin y, \cos y$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$
\begin{array}{rl|l}
\sin(2\arccos(x)) & = \sin(2y) & \text{Set } y = \arccos x \\
& = 2\cos y \sin y & \text{Express via } \sin y, \cos y \\
& = 2\cos y \left(\pm\sqrt{1-\cos^2 y}\right) & \text{Express } \sin y \text{ via } \cos y
\end{array}
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Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

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&= 2\cos y\sqrt{1-\cos^2 y} && \sin y > 0 \text{ because } 0 \leq y \leq \pi
\end{aligned}$$

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&= 2\cos y\sqrt{1-\cos^2 y} && \sin y > 0 \text{ because } 0 \leq y \leq \pi \\
&= 2x\sqrt{1-x^2} && \text{use } x = \cos y
\end{aligned}
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Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.
To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$\sin(2\arccos(x)) = \sin(2y)$	Set $y = \arccos x$
$= 2\cos y \sin y$	Express via $\sin y, \cos y$
$= 2\cos y\left(\pm\sqrt{1-\cos^2 y}\right)$	Express $\sin y$ via $\cos y$
$= 2\cos y\sqrt{1-\cos^2 y}$	$\sin y > 0$ because $0 \leq y \leq \pi$
$= 2x\sqrt{1-x^2}$	use $x = \cos y$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

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$$\cos(3\arccos(x)) = \cos(3y) \qquad \Big| \; y = \arccos x$$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\cos(3\arccos(x)) = \cos(3y) = \cos(2y + y) \qquad \Big|\ y = \arccos x$$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\begin{aligned}\cos(3\arccos(x)) &= \cos(3y) = \cos(2y+y) \\ &= \textbf{?}\end{aligned}$$

| $y = \arccos x$
| Angle sum f-la

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\begin{aligned} \cos(3\arccos(x)) &= \cos(3y) = \cos(2y+y) && \quad \Big| \; y = \arccos x \\ &= \cos(2y)\cos y - \sin(2y)\sin y && \quad \Big| \; \text{Angle sum f-la} \end{aligned}$$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

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\begin{aligned}
\cos(3\arccos(x)) &= \cos(3y) = \cos(2y+y) && y = \arccos x \\
&= \cos(2y)\cos y - \sin(2y)\sin y && \text{Angle sum f-la} \\
&= (\textbf{?} \qquad\qquad\qquad)\cos y && \text{Express via} \\
&\quad -\textbf{?} \qquad\qquad\qquad \sin y && \sin y, \cos y
\end{aligned}
$$

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Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

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&= \cos^3 y - \sin^2 y \cos y - 2\sin^2 y \cos y \\
&= \cos^3 y - 3\sin^2 y \cos y
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&= (\cos^2 y - \sin^2 y)\cos y && \text{Express via} \\
&\quad - 2\sin y \cos y \sin y && \sin y, \cos y \\
&= \cos^3 y - \sin^2 y \cos y - 2\sin^2 y \cos y && \\
&= \cos^3 y - 3\sin^2 y \cos y && \text{Express } \sin y \text{ via } \cos y \\
&= \cos^3 y - 3(\text{?} \qquad)\cos y &&
\end{aligned}
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Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

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&\quad - 2\sin y \cos y \sin y & \mid\; \sin y, \cos y \\
&= \cos^3 y - \sin^2 y \cos y - 2\sin^2 y \cos y & \\
&= \cos^3 y - 3\sin^2 y \cos y & \mid\; \text{Express } \sin y \text{ via } \cos y \\
&= \cos^3 y - 3(1-\cos^2 y)\cos y &
\end{array}
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&= \cos^3 y - \sin^2 y \cos y - 2\sin^2 y \cos y \\
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&= \cos^3 y - 3(1-\cos^2 y)\cos y \\
&= 4\cos^3 y - 3\cos y
\end{aligned}$$

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&= \cos^3 y - 3(1-\cos^2 y)\cos y \\
&= 4\cos^3 y - 3\cos y \\
&= 4x^3 - 3x && \Big| \; x = \cos y
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& = \cos^3 y - \sin^2 y\cos y - 2\sin^2 y\cos y & & \\
& = \cos^3 y - 3\sin^2 y\cos y & & \text{Express } \sin y \text{ via } \cos y \\
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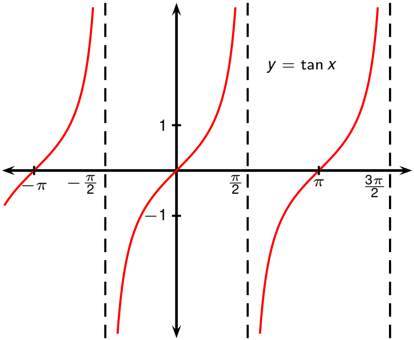
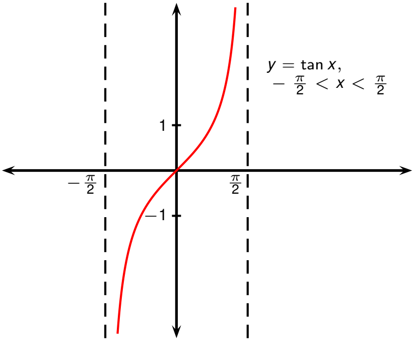
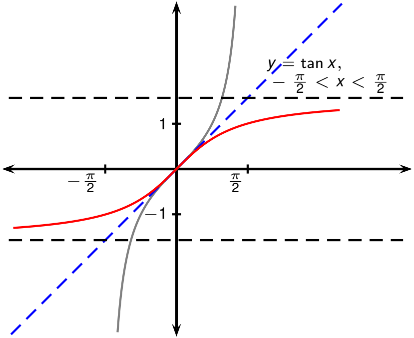


- $\tan x$ isn't one-to-one.



- $\tan x$ isn't one-to-one.
- Restrict the domain to $(-\frac{\pi}{2}, \frac{\pi}{2})$.



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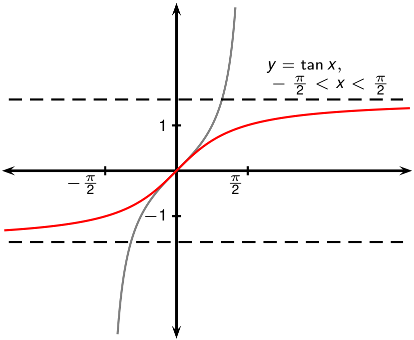

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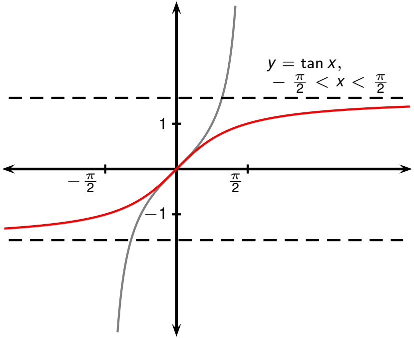

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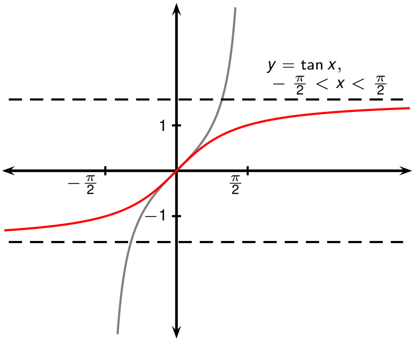
- Domain of $\arctan$: **?**
- Range of $\arctan$:



- $\tan x$ isn't one-to-one.
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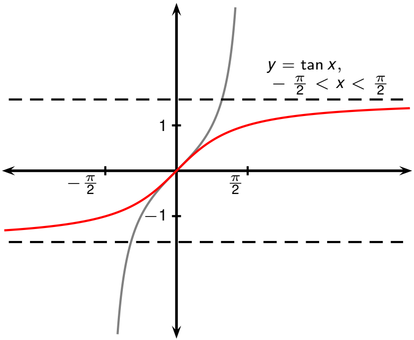
- The inverse is called $\tan^{-1}$ or $\arctan$.
- $\arctan x = y \Leftrightarrow \tan y = x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$.
- Domain of $\arctan$: $(-\infty, \infty)$.
- Range of $\arctan$:



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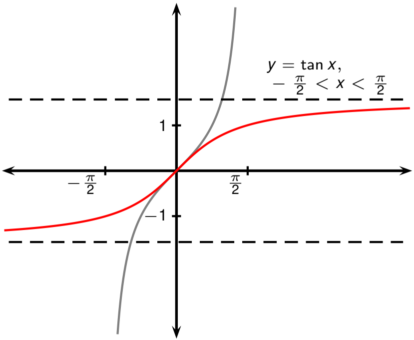

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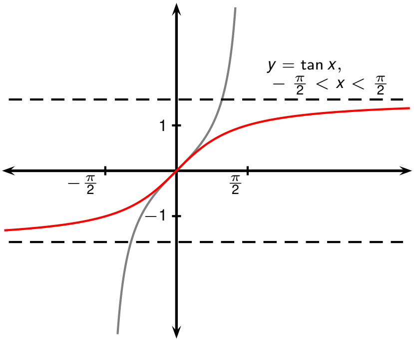

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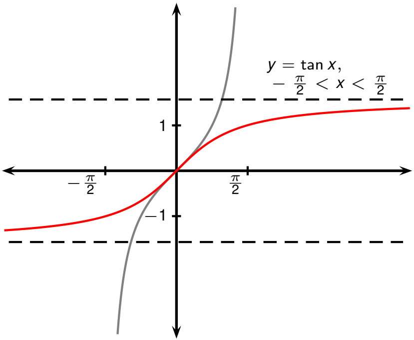

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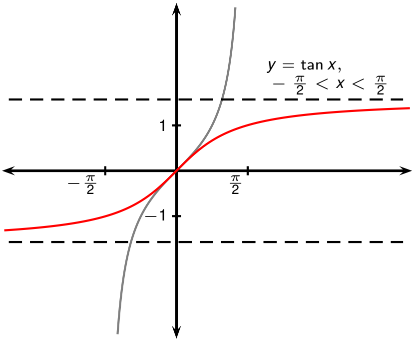

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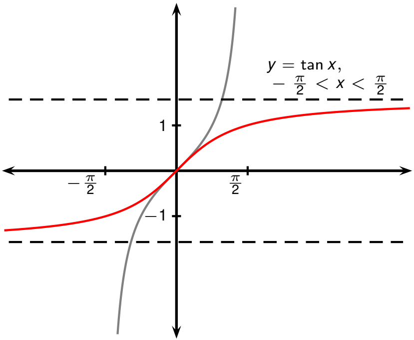

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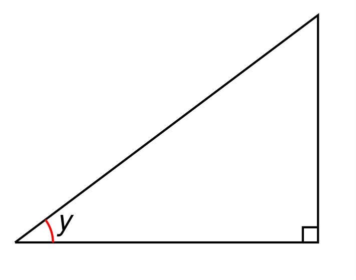
Example

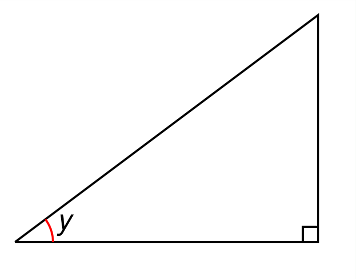
Simplify the expression $\cos(\arctan x)$.



Example

Simplify the expression $\cos(\arctan x)$.

- Let $y = \arctan x$, so $\tan y = x$.



Example

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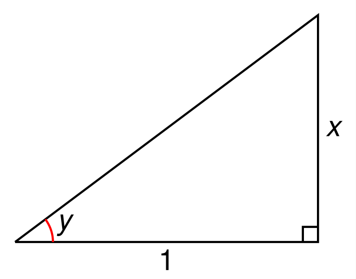
- Let $y = \arctan x$, so $\tan y = x$.
- Draw a right triangle with opposite x and adjacent 1.



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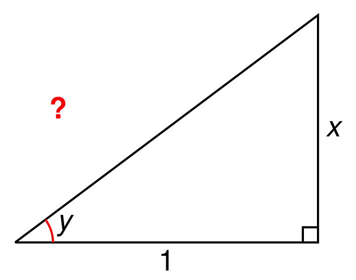
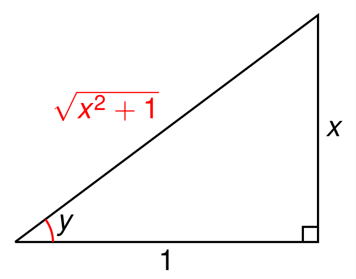
- Let $y = \arctan x$, so $\tan y = x$.
- Draw a right triangle with opposite x and adjacent 1.
- Length of hypotenuse $=$ **?**



Example

Simplify the expression $\cos(\arctan x)$.

- Let $y = \arctan x$, so $\tan y = x$.
- Draw a right triangle with opposite x and adjacent 1.
- Length of hypotenuse $= \sqrt{1^2 + x^2}$.



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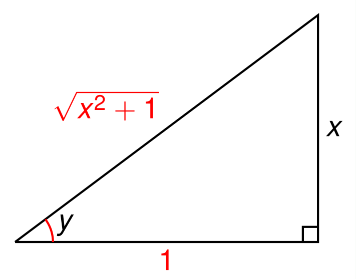

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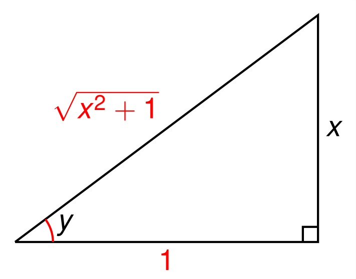

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$$\lim_{x \to 2^+} \arctan\left(\frac{1}{x-2}\right).$$

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Theorem (The Derivative of $\arctan x$)

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}.$$

Proof.

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The remaining inverse trigonometric functions aren't used as often:

$$\begin{array}{llcll} y = \operatorname{arccsc} x & (|x| \geq 1) & \Leftrightarrow & \csc y = x & \text{and} \quad y \in \left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right] \\ y = \operatorname{arcsec} x & (|x| \geq 1) & \Leftrightarrow & \sec y = x & \text{and} \quad y \in \left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right) \\ y = \operatorname{arccot} x & (|x| \in \mathbb{R}) & \Leftrightarrow & \cot y = x & \text{and} \quad y \in (0, \pi) \end{array}$$

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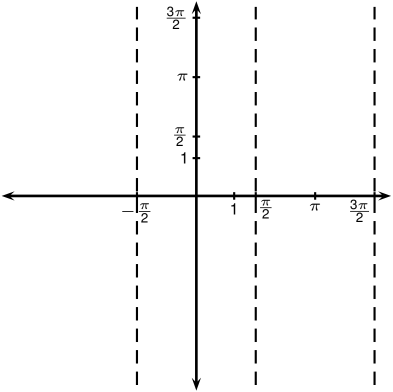

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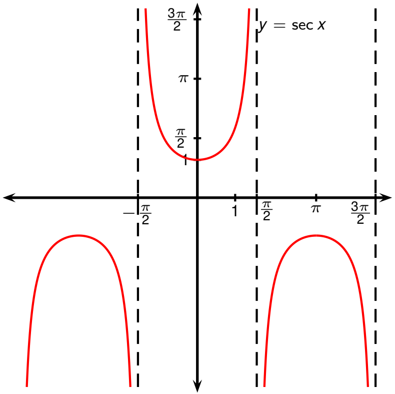

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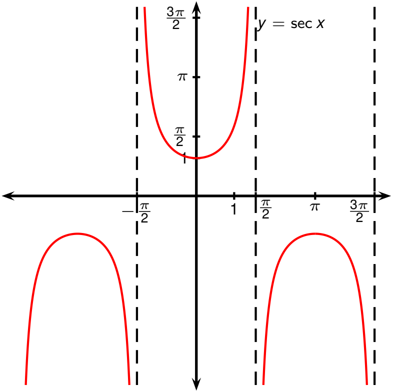

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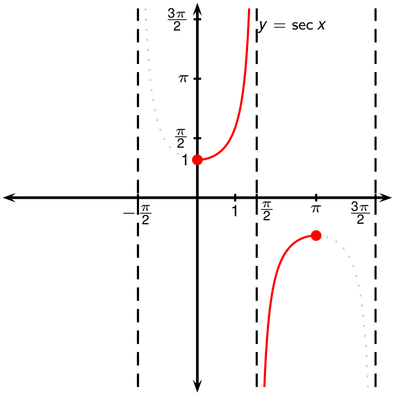

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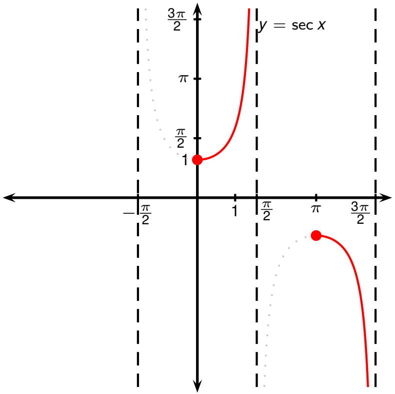

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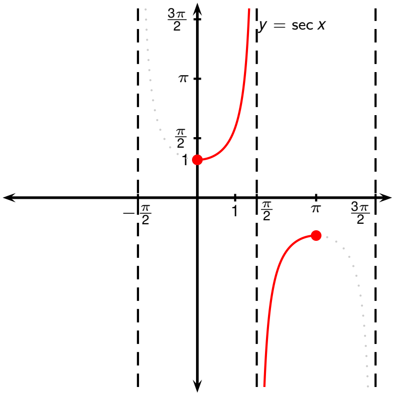

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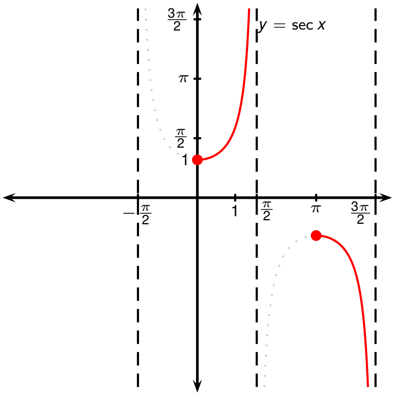

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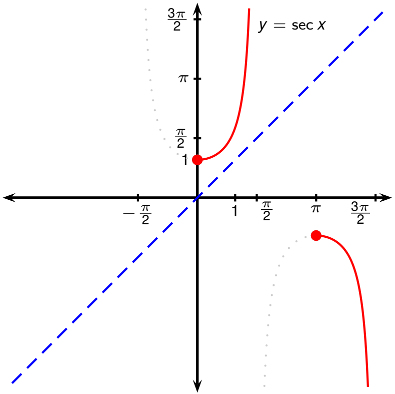

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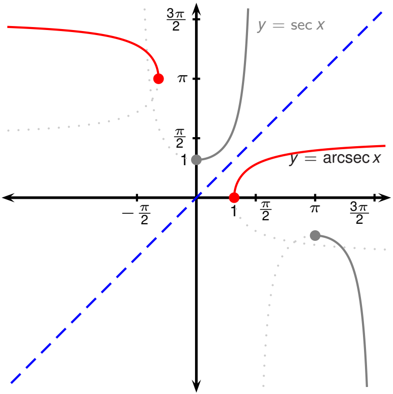

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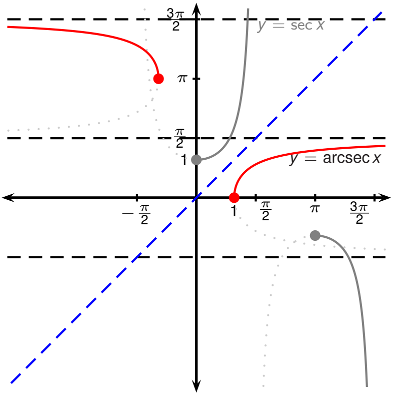

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Table of derivatives of inverse trigonometric functions:

$$\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{arccsc} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\arccos x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{arcsec} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\arctan x) = \frac{1}{1+x^2} \qquad \frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{arccot} x) = -\frac{1}{1+x^2}$$

Example (Chain Rule)

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$$= -\frac{1}{(\arcsin x)^2\sqrt{1-x^2}}.$$

All of the inverse trigonometric derivatives also give rise to integration formulas. These two are the most important:

$$\int \frac{1}{\sqrt{1-x^2}} \mathrm{d}x = \arcsin x + C.$$

$$\int \frac{1}{x^2+1} \mathrm{d}x = \arctan x + C.$$