Calculus II
Homework on Lecture 21

1. Plot the number z on the complex plane (you may use one drawing only for all the numbers). Find all real numbers φ and ρ for which $z = e^{\rho + i\varphi}$. Your answer may contain expressions of the form $\arcsin x$, $\arccos x$, $\arctan x$, $\ln x$, only if x is a real number.

   (a) $z = 1 + i\sqrt{3}$.

   (b) $z = -2 - 3i$.

   (c) $z = 1 - i\sqrt{3}$.

   (d) $z = 1 + i$.

   (e) $z = -1 - i$.

   (f) $z = \frac{\sqrt{3}+i}{4}$.

   (g) $z = -i$.

   (h) $z = 3 + 4i$.

2. Carry out the operations. For some of the problems you may want to review the Newton Binomial formula.

   (a) $(5 + 3i)^2$.

   (b) $\dfrac{5 + 3i}{2 - 3i}$.

   (c) $(5 + 3i)^{-2}$.

   (d) $(1 + i)^3$.

   (e) $(1 + i)^4$.

   (f) $(1 + i)^5$.

   (g) $(1 + i)^{-5}$.

3. Find all complex solutions of the equation. The answer key has not been proofread. Use with caution.

   (a) $z^3 = i$.

   (b) $z^3 = -\dfrac{i}{8}$.

   (c) $z^4 = -16$.

   (d) $z^3 = -27$.

   (e) $z^8 = 1$.

4. Express the number in polar form and compute the indicated power. The answer key has not been proofread, use with caution.

   (a) $z = \sqrt{3} + i$, find z^3.

   (b) $z = \sqrt{3}i - 1$, find z^{10}.

   (c) $z = -1 - i$, find z^{21}.

5. The de Moivre follows directly from Euler's formula and states that $(\cos(n\alpha) + i\sin(n\alpha)) = (\cos\alpha + i\sin\alpha)^n$. Expand the indicated expression and use it to express $\cos(n\alpha)$ and $\sin(n\alpha)$ via $\cos\alpha$ and $\sin\alpha$.

   You may want to use the Newton binomial formulas (derived, say, via Pascal's triangle). The formulas you may want to use are:

$$
\begin{aligned}
(a+b)^2 &= a^2 + 2ab + b^2 \\
(a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
(a+b)^4 &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 \quad .
\end{aligned}
$$

   (a) Expand $(\cos\alpha + i\sin\alpha)^2$. Express $\cos(2\alpha)$ and $\sin(2\alpha)$ via $\cos\alpha$ and $\sin\alpha$.

   (b) Expand $(\cos\alpha + i\sin\alpha)^3$. Express $\cos(3\alpha)$ and $\sin(3\alpha)$ via $\cos\alpha$ and $\sin\alpha$.

   (c) Expand $(\cos\alpha + i\sin\alpha)^4$. Express $\cos(4\alpha)$ and $\sin(4\alpha)$ via $\cos\alpha$ and $\sin\alpha$.