Calculus III
Homework on Lecture 11

1. Determine the type of the quadratic surface given by the equation. The answer key has not been proofread, use with extreme caution.

   (a) $x^2 + y^2 + z^2 + x + 2y + 3z = 0$.

   (b) $x^2 + 2y^2 + z^2 + x + 2y + 3z = 0$.

   (c) $x^2 + 2y^2 + 3z^2 + x + 2y + 3z = 0$.

   (d) $z^2 + 2y^2 - 3x^2 + x + y + 1 = 0$.

   (e) $z^2 - y^2 + \frac{1}{4}x^2 + x - y + 1 = 0$.

   (f) $x^2 + y^2 - \frac{1}{4}z^2 + x - y + 5 = 0$.

   (g) $\frac{1}{4}x^2 - y^2 + z^2 - x + 1 = 0$

   (h) $-\frac{1}{4}x^2 + y^2 + z^2 - x - 1 = 0$

   (i) $xy + z^2 + 1 = 0$. Hint: write $x = \frac{1}{\sqrt{2}}(u + v)$, $y = \frac{1}{\sqrt{2}}(u - v)$ for some new variables u, v. Solve the problem in the z, u, v -coordinates. Argue that the (axes of the) u, v, z-coordinate system can be obtained from the x, y, z-coordinate system via rotation.

   (j) $x^2 + 2y^2 + z = 0$.

   (k) $x^2 + y^2 + 2xy + z = 0$.

   (l) $x^2 - y^2 + 2x + z = 0$.

2. Find an equation of the tangent plane to the surface at the given point. The surface is given via an implicit equation.

   (a) The sphere $x^2 + y^2 + z^2 = 1$ at $(x, y, z) = (\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$.

   (b) The two-sheet hyperboloid $x^2 + y^2 - z^2 = -3$ at $(x, y, z) = (2, 3, 4)$.

   (c) The ellipsoid $x^2 + 2y^2 + 3z^2 = 20$ at $(x, y, z) = (3, 2, 1)$.

   Find the equation of the tangent plane to the graph of the function at the indicated point.

3. (a) $z = x^2 - y^2$, at the point $(1, 1, 0)$.

   (b) $z = e^{-x^2-y^2}$, at the point $(0, 0, 1)$

   (c) $z = e^{x^2-y^2}$, at the point $(1, -1, 1)$.

   (d) $z = \sqrt{3 - x^2 - y^2}$, at the point $(1, 1, 1)$.