Calculus II
Homework on Lecture 1

1. Let $x \in (0,1)$. Express the following using x and $\sqrt{1-x^2}$.

   (a) $\sin(\arcsin(x))$.

   (b) $\sin(2\arcsin(x))$.

   (c) $\sin(3\arcsin(x))$.

   (d) $\sin(\arccos(x))$.

   (e) $\sin(2\arccos(x))$.

   (f) $\sin(3\arccos(x))$.

   (g) $\cos(2\arcsin(x))$.

   (h) $\cos(3\arccos(x))$.

2. Express as the following as an algebraic expression of x. In other words, "get rid" of the trigonometric and inverse trigonometric expressions.

   (a) $\cos^2(\arctan x)$.

   (b) $-\sin^2(\operatorname{arccot} x)$.

   (c) $\dfrac{1}{\cos(\arcsin x)}$.

   (d) $-\dfrac{1}{\sin(\arccos x)}$.

3. Rewrite as a rational function of t. This problem will be later used to derive the Euler substitutions (an important technique for integrating).

   (a) $\cos(2\arctan t)$.

   (b) $\sin(2\arctan t)$.

   (c) $\tan(2\arctan t)$.

   (d) $\cot(2\arctan t)$.

   (e) $\csc(2\arctan t)$.

   (f) $\sec(2\arctan t)$.

   (g) $\cos(2\operatorname{arccot} t)$.

   (h) $\sin(2\operatorname{arccot} t)$.

   (i) $\tan(2\operatorname{arccot} t)$.

   (j) $\cot(2\operatorname{arccot} t)$.

   (k) $\csc(2\operatorname{arccot} t)$.

   (l) $\sec(2\operatorname{arccot} t)$.

4. Compute the derivative (derive the formula).

   (a) $(\arctan x)'$.

   (b) $(\operatorname{arccot} x)'$.

   (c) $(\arcsin x)'$.

   (d) $(\arccos x)'$.

   (e) Let arcsec denote the inverse of the secant function. Compute $(\operatorname{arcsec} x)'$.

5. (a) Let $a+b \neq k\pi$, $a \neq k\pi + \frac{\pi}{2}$ and $b \neq k\pi + \frac{\pi}{2}$ for any $k \in \mathbb{Z}$ (integers). Prove that

   $$\frac{\tan a + \tan b}{1 - \tan a \tan b} = \tan(a+b) \quad .$$

   (b) Let x and y be real. Prove that, for $xy \neq 1$, we have

   $$\arctan x + \arctan y = \arctan\left(\frac{x+y}{1-xy}\right)$$

   if the left hand side lies between $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.