Calculus I
Lecture 2
Trigonometry Review

Todor Milev
https://github.com/tmilev/freecalc

2020

Outline

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Degrees and radians

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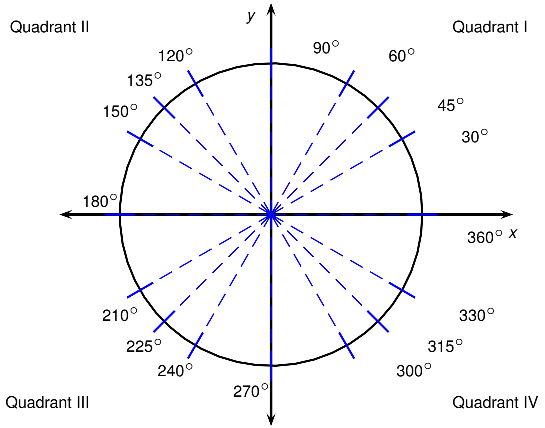
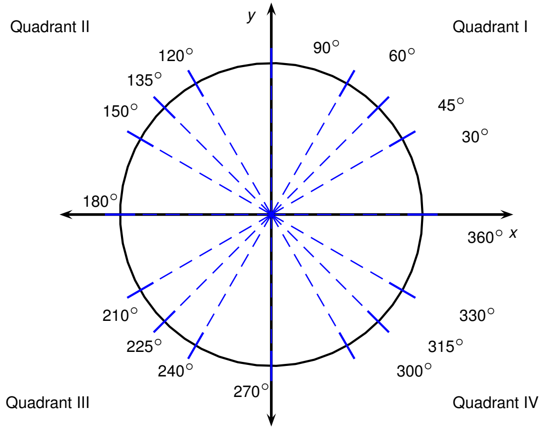
- In other words, a half-turn is measured by πrad or 180°.
- Degrees are useful because the most frequently encountered fractions of a half turn are measured by a whole number of degrees.
- If a measurement unit is not specified, it is implied to be radians. For example, in $\sin 5$, the number 5 stands for 5 radians.



The most frequently encountered angles are given in the table below.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	?										



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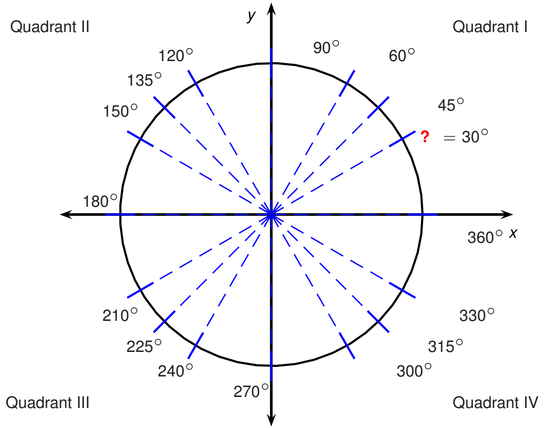

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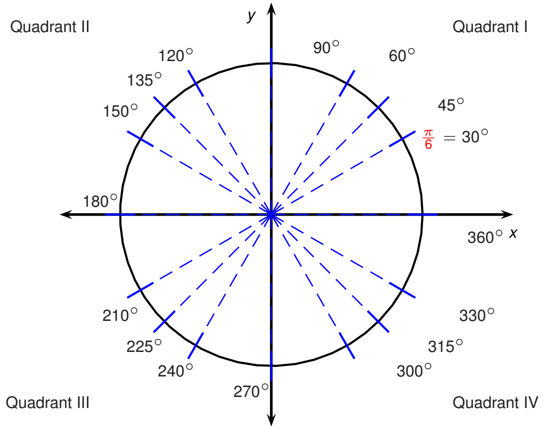

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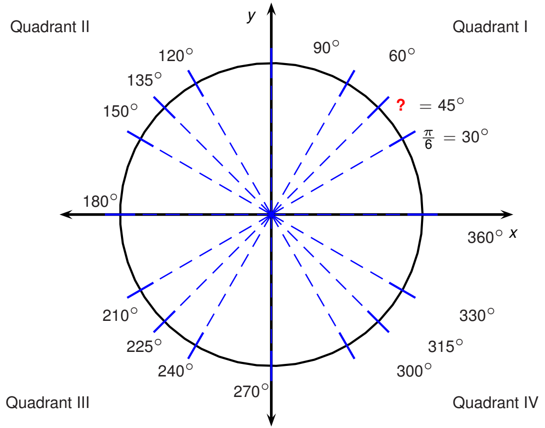

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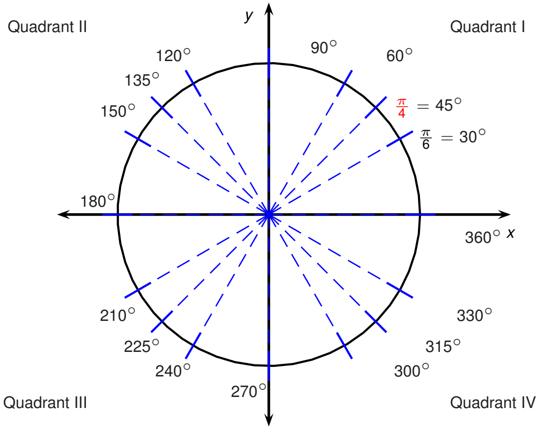

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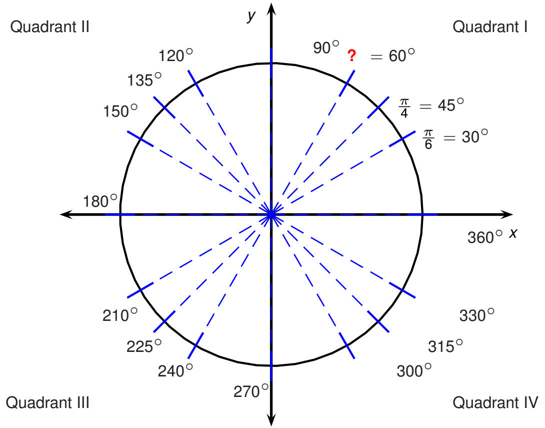

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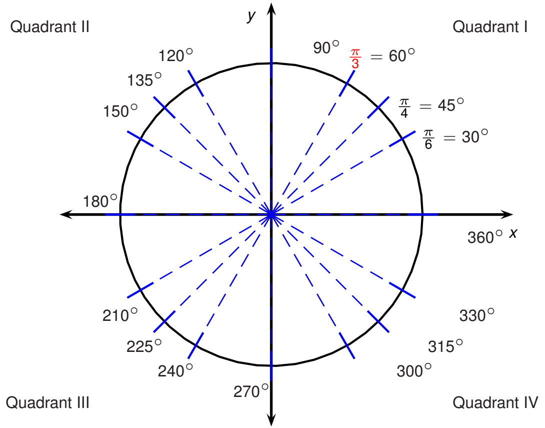

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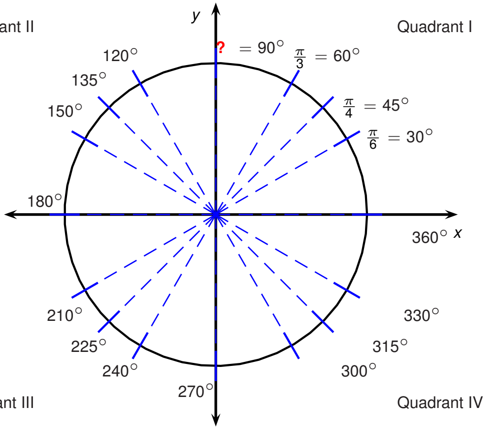

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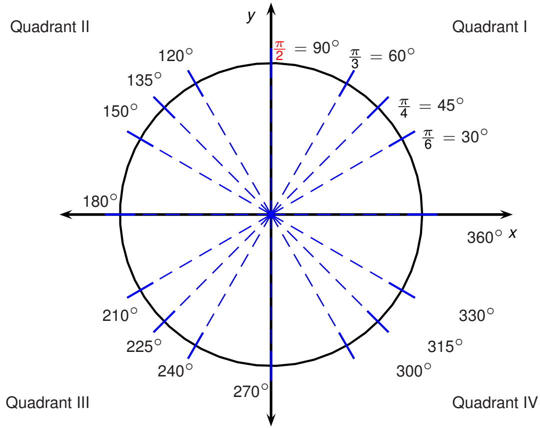

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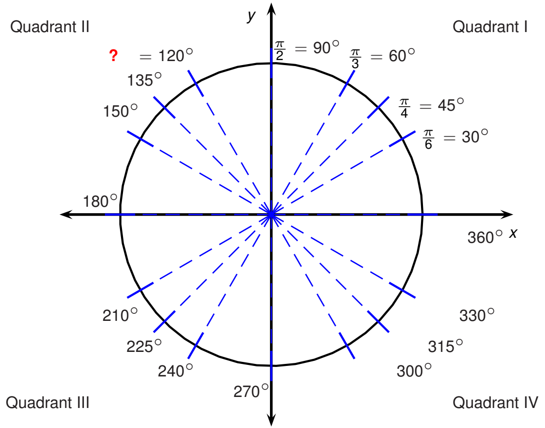

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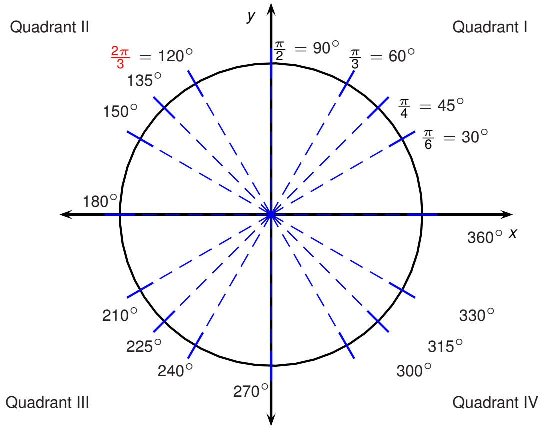

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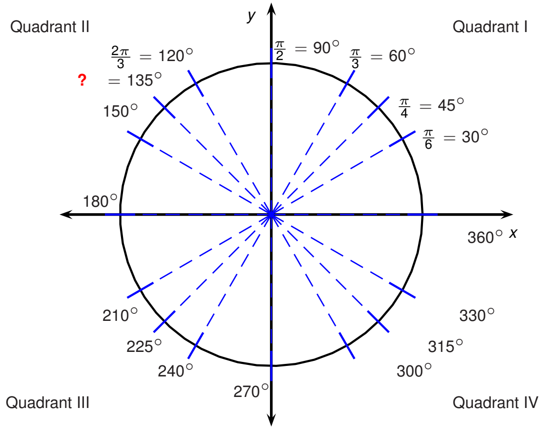

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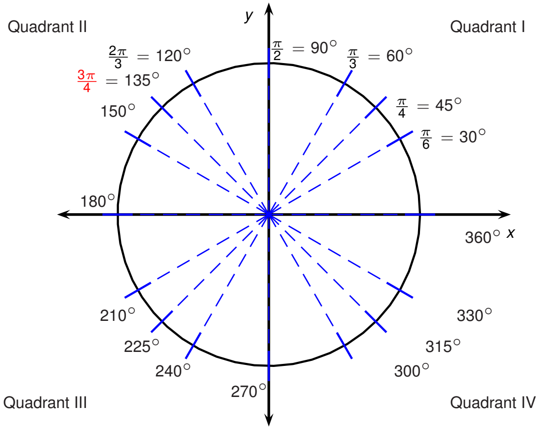

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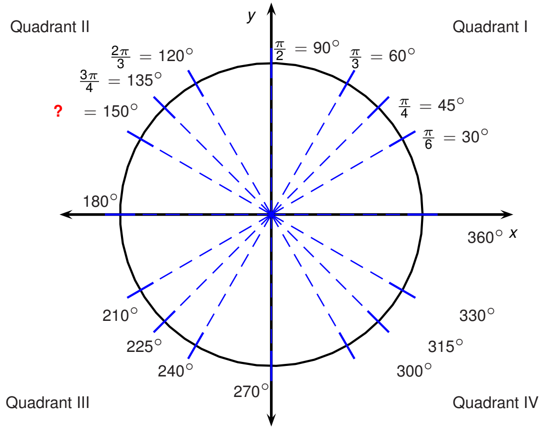

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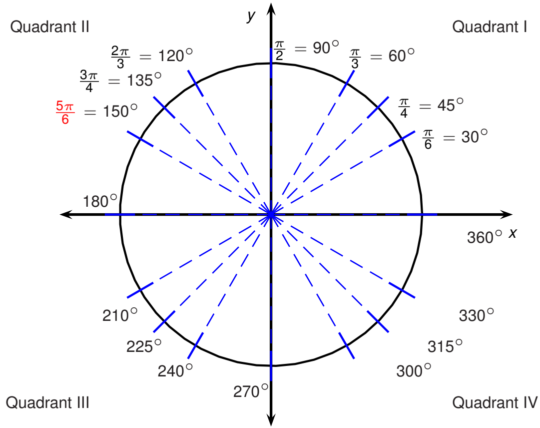

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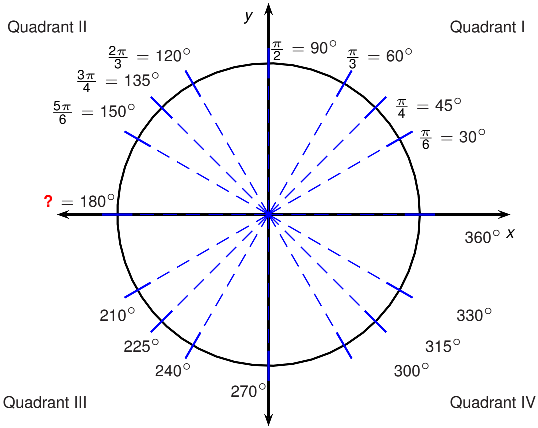

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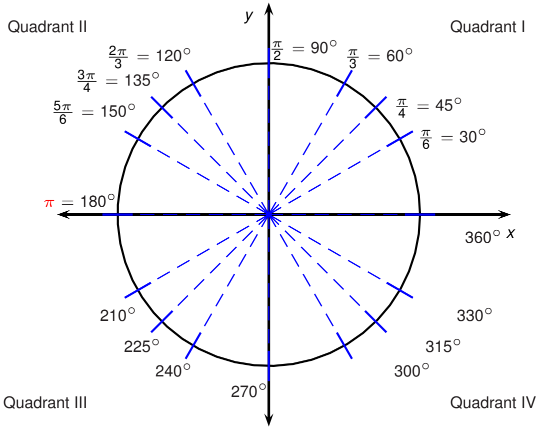

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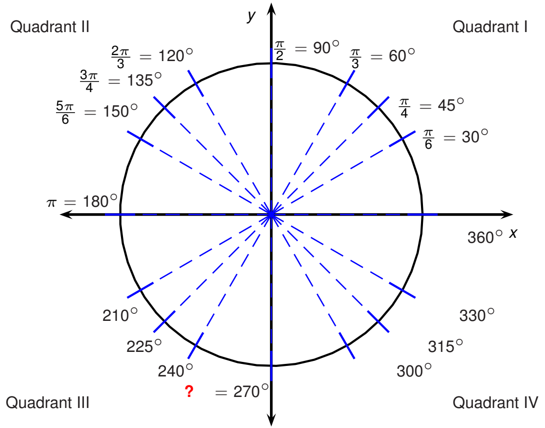

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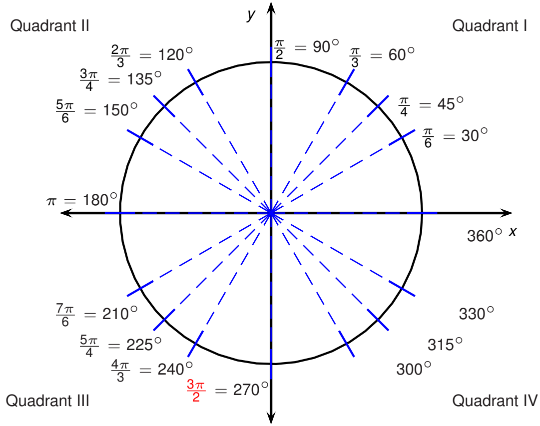

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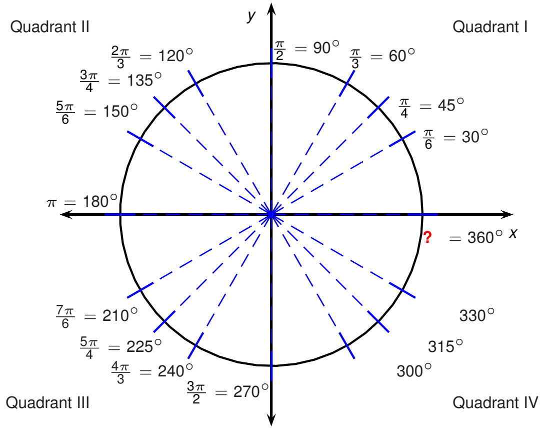

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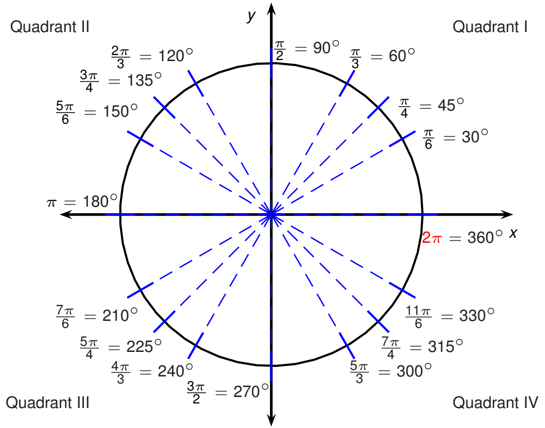

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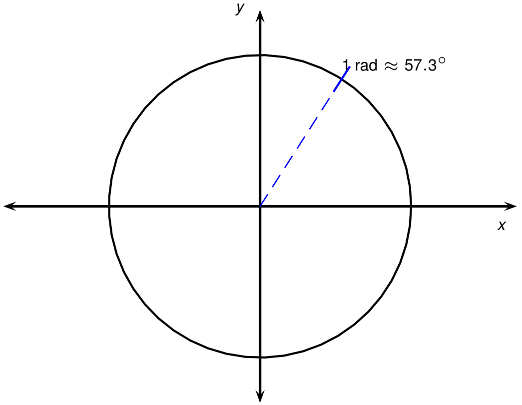
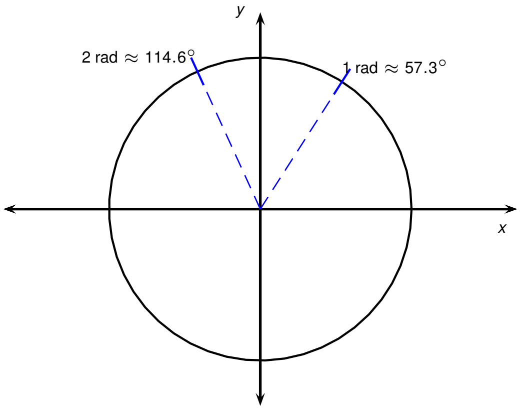


- Integer quantities of radians are not rational multiples of (the measure of) a half-turn and are not easy to compute with.



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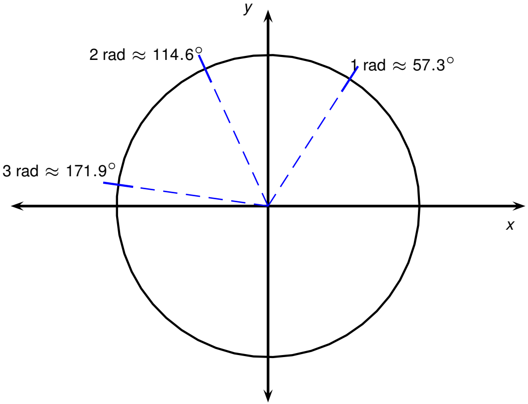

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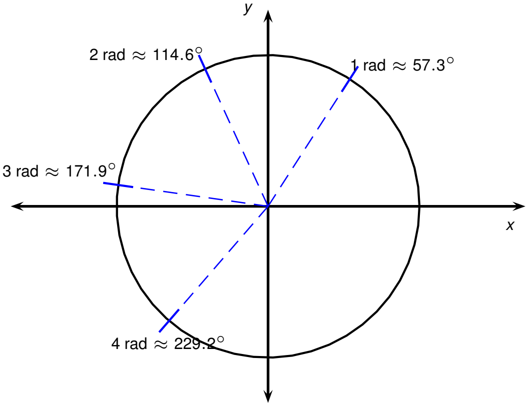

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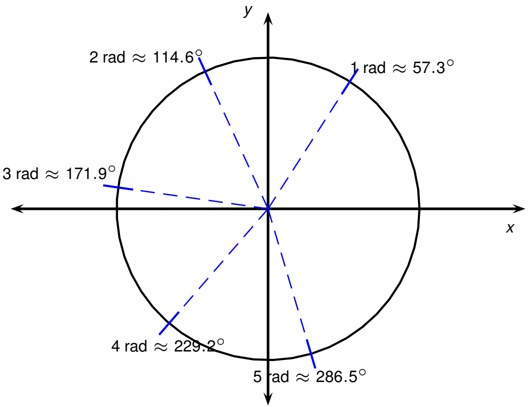

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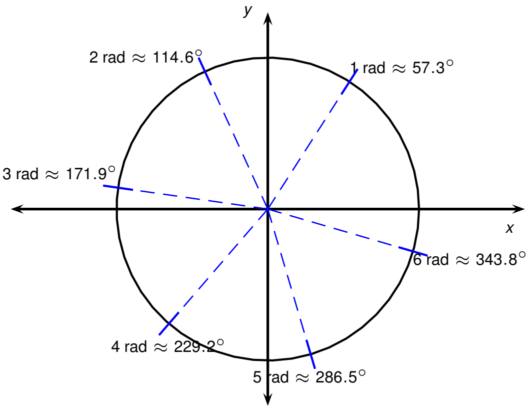

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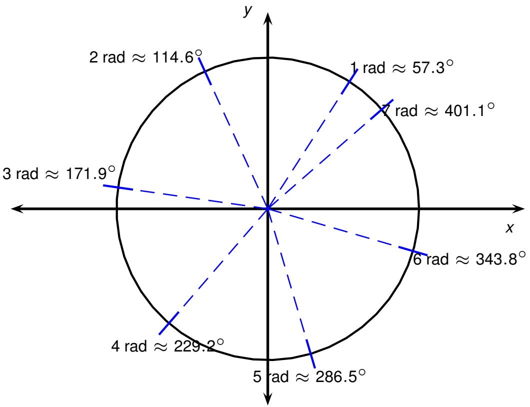

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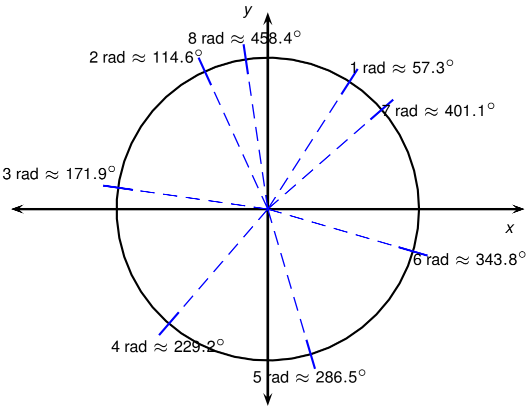

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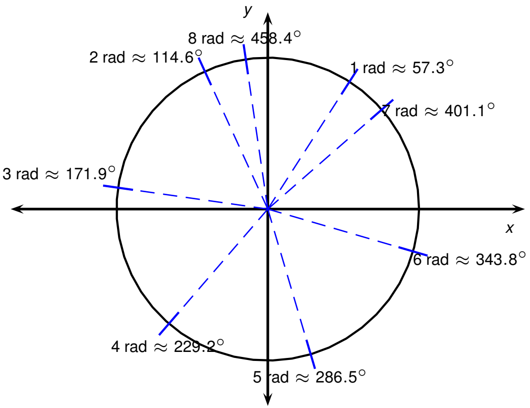
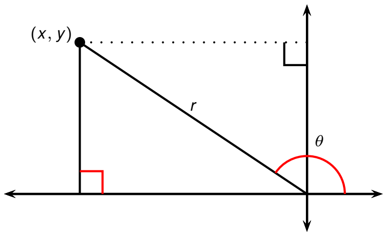
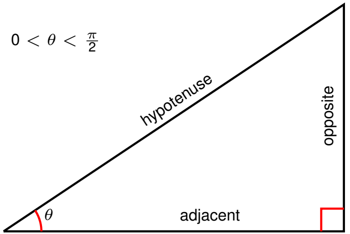


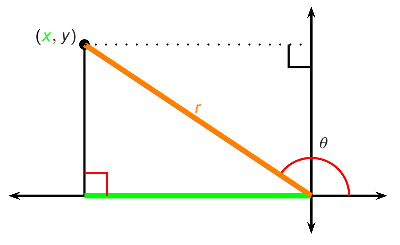
- Integer quantities of radians are not rational multiples of (the measure of) a half-turn and are not easy to compute with.
- For example to determine in which quadrant is an angle of k radians located one needs to know the numerical value of $\frac{k}{\pi}$, which requires knowledge of π with great numerical accuracy.

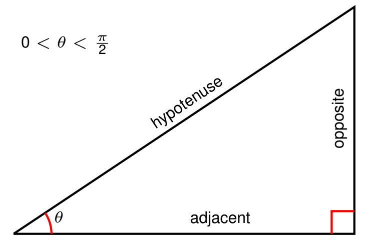
Trigonometric Functions and Right Angle Triangles

	
$\cos\theta$ $\quad$ $\sec\theta$ $\sin\theta$ $\quad$ $\csc\theta$ $\tan\theta$ $\quad$ $\cot\theta$	$\cos\theta$ $\quad$ $\sec\theta$ $\sin\theta$ $\quad$ $\csc\theta$ $\tan\theta$ $\quad$ $\cot\theta$
All angles	Acute angles

- The trigonometric functions can be defined without requesting that the pt. (x, y) on the terminal arm of the angle lie on the unit circle.

Trigonometric Functions and Right Angle Triangles





$\cos\theta = \frac{x}{r}$	$\sec\theta$	$\cos\theta$	$\sec\theta$
$\sin\theta$	$\csc\theta$	$\sin\theta$	$\csc\theta$
$\tan\theta$	$\cot\theta$	$\tan\theta$	$\cot\theta$
All angles		Acute angles	

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- To do so we rescale by the distance r from the origin.

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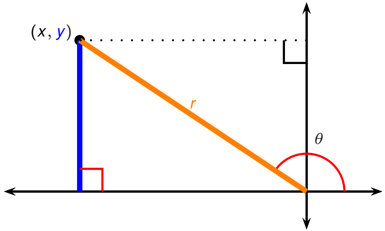
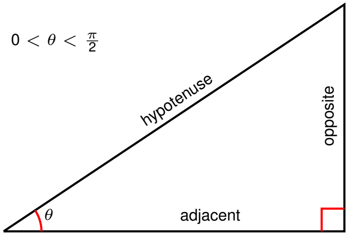


$\cos\theta = \frac{x}{r}$ $\qquad$ $\sec\theta$

$\sin\theta = \frac{y}{r}$ $\qquad$ $\csc\theta$

$\tan\theta$ $\qquad$ $\cot\theta$

All angles



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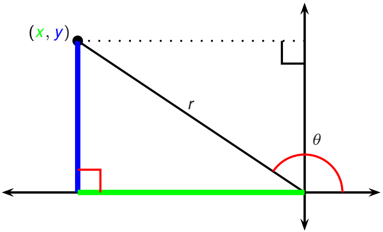

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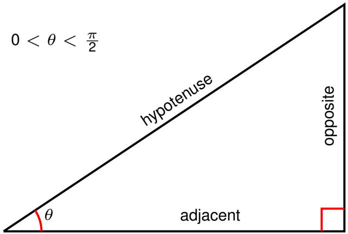
All angles



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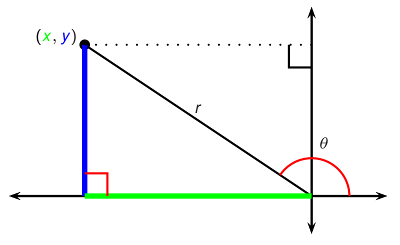
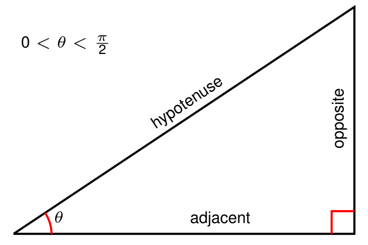
Trigonometric Functions and Right Angle Triangles





$\cos\theta = \frac{x}{r}$	$\sec\theta$	$\cos\theta$	$\sec\theta$
$\sin\theta = \frac{y}{r}$	$\csc\theta$	$\sin\theta$	$\csc\theta$
$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$	$\tan\theta$	$\cot\theta$
All angles		Acute angles	

- The trigonometric functions can be defined without requesting that the pt. (x, y) on the terminal arm of the angle lie on the unit circle.
- To do so we rescale by the distance r from the origin.

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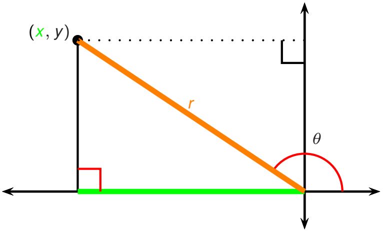

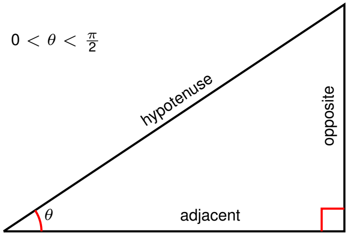

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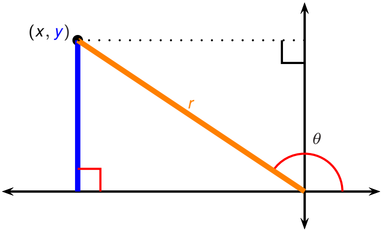

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$
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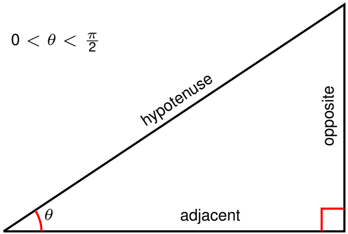
All angles



$$\cos\theta \qquad \sec\theta$$
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$$\tan\theta \qquad \cot\theta$$

Acute angles

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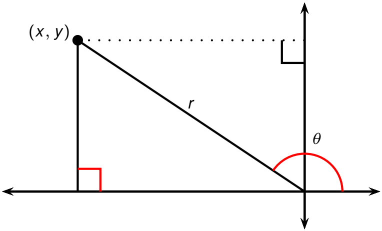

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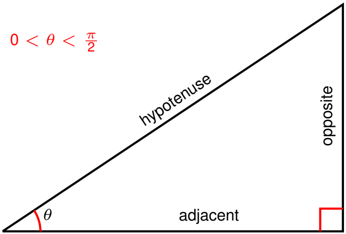
All angles



$\cos\theta$ $\quad$ $\sec\theta$

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$\tan\theta$ $\quad$ $\cot\theta$

Acute angles

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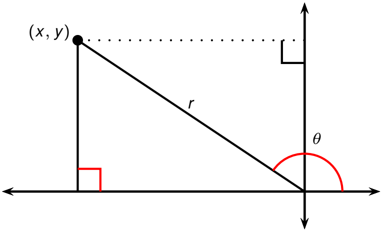
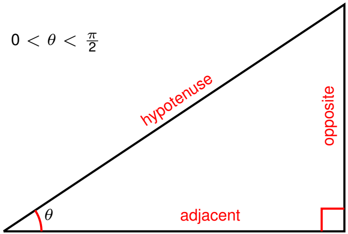
Trigonometric Functions and Right Angle Triangles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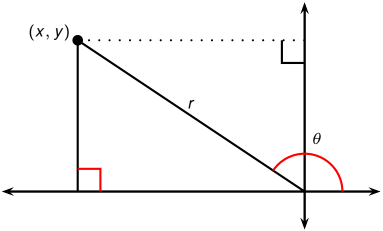

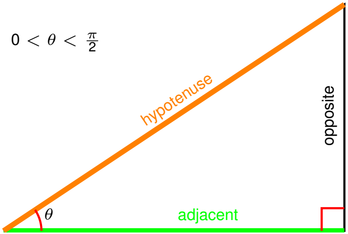

$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\sec\theta$
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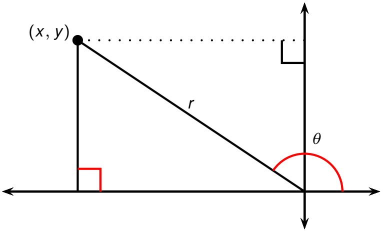

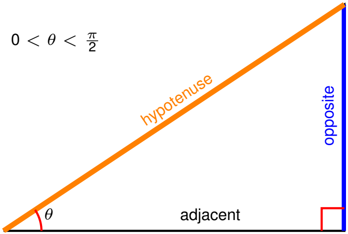

$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\sec\theta$
$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$	$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	$\csc\theta$
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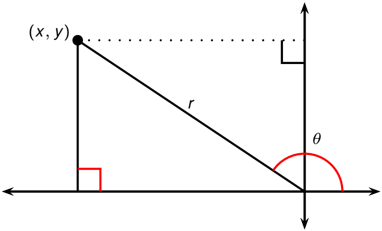

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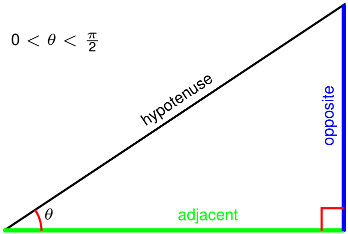
All angles



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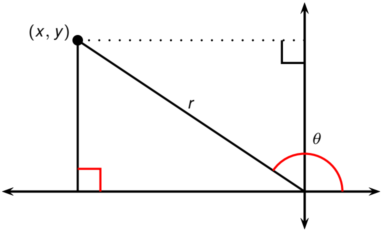

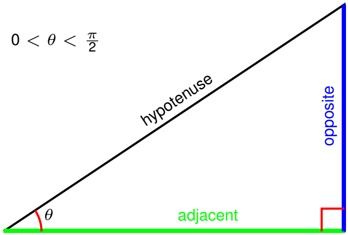

All angles		Acute angles	
$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\sec\theta$
$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$	$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	$\csc\theta$
$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$	$\tan\theta = \frac{\text{opp}}{\text{adj}}$	$\cot\theta = \frac{\text{adj}}{\text{opp}}$

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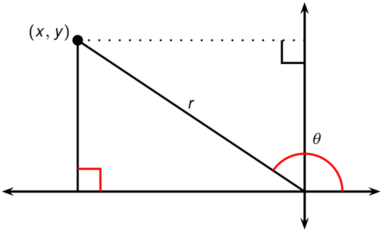

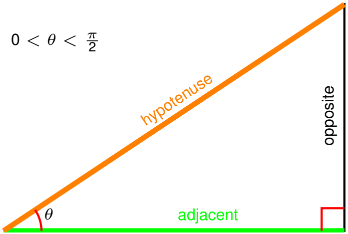

All angles		Acute angles	
$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$	$\cos\theta = \frac{\text{adj}}{\text{hyp}}$	$\sec\theta = \frac{\text{hyp}}{\text{adj}}$
$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$	$\sin\theta = \frac{\text{opp}}{\text{hyp}}$	$\csc\theta$
$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$	$\tan\theta = \frac{\text{opp}}{\text{adj}}$	$\cot\theta = \frac{\text{adj}}{\text{opp}}$

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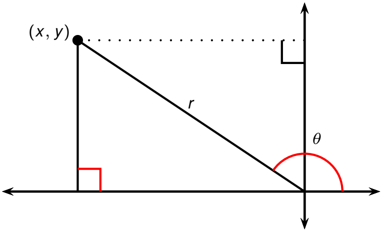

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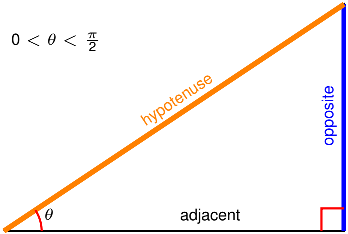
All angles



$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

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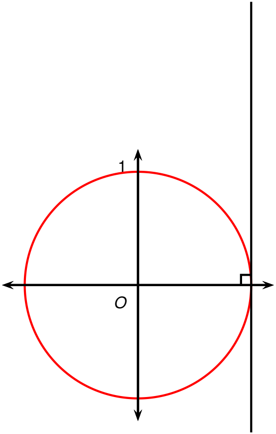
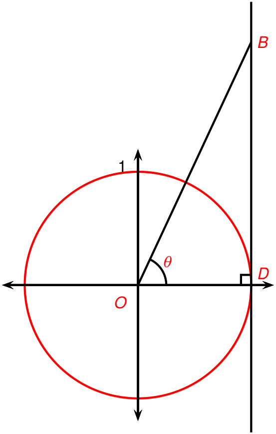
Geometric interpretation of all trigonometric functions



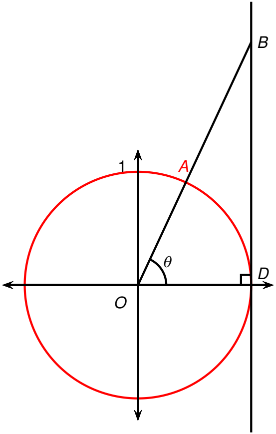
Fix unit circle, center O, coordinates $(0,0)$.

Geometric interpretation of all trigonometric functions



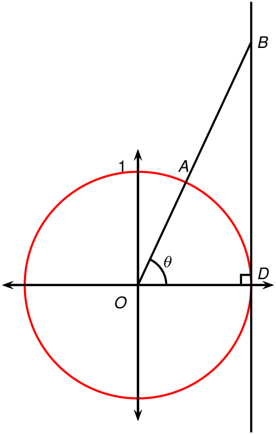
Fix unit circle, center O, coordinates $(0, 0)$.
Let $\angle DOB = \theta$.

Geometric interpretation of all trigonometric functions



Fix unit circle, center O, coordinates $(0, 0)$. Let $\angle DOB = \theta$. Let OB intersect the circle at point A.

Geometric interpretation of all trigonometric functions



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$\sin\theta$

$\cos\theta$

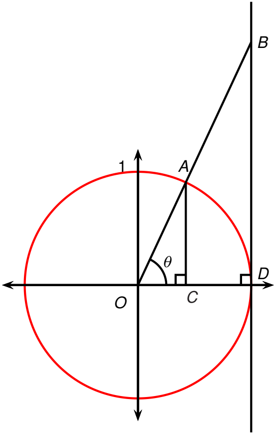
$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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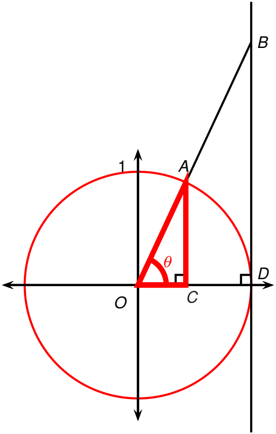


Fix unit circle, center O, coordinates $(0,0)$. Let $\angle DOB = \theta$. Let OB intersect the circle at point A. Coordinates of A are $(\cos\theta, \sin\theta)$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$\cos\theta$

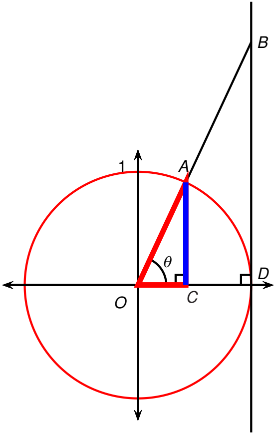
$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|}$$

$\cos\theta$

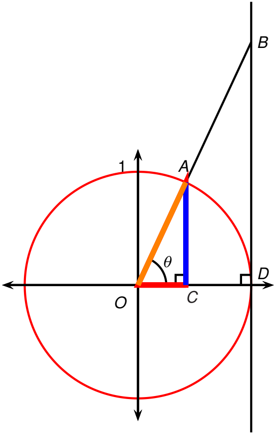
$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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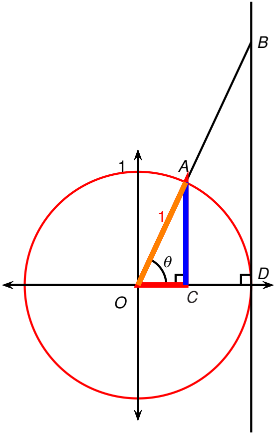

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$\cos\theta$

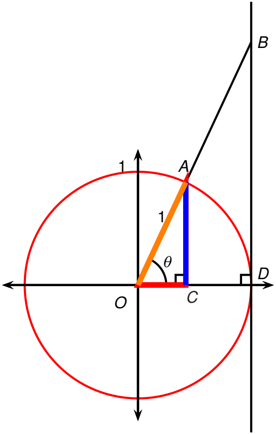
$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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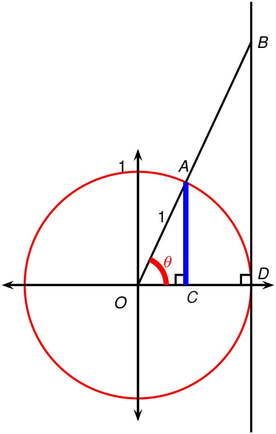

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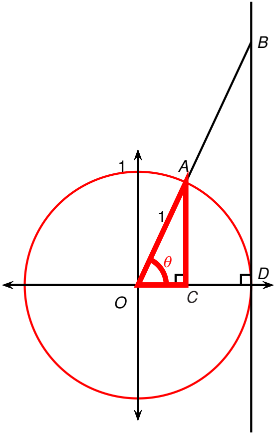

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$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

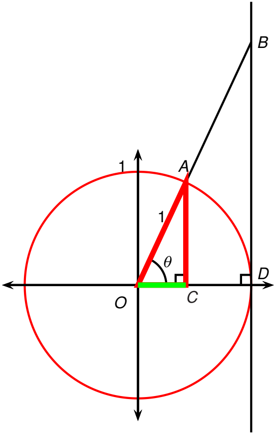
$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|} = \frac{|AC|}{1} = |AC|$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{|OC|}{|OA|}$$

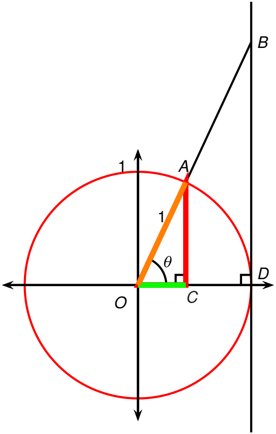
$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{|OC|}{|OA|}$$

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$\cot\theta$

$\sec\theta$

$\csc\theta$

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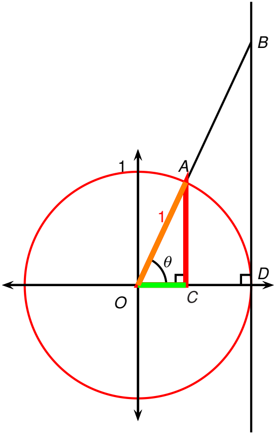

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$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{|OC|}{|OA|} = \frac{|OC|}{1}$$

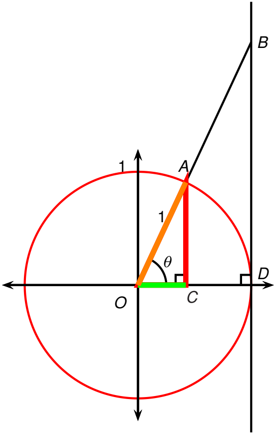
$\tan\theta$

$\cot\theta$

$\sec\theta$

$\csc\theta$

Geometric interpretation of all trigonometric functions



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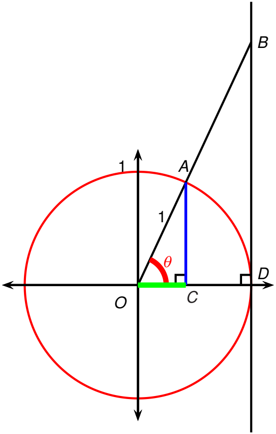
Geometric interpretation of all trigonometric functions



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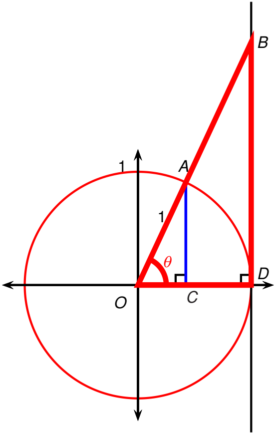

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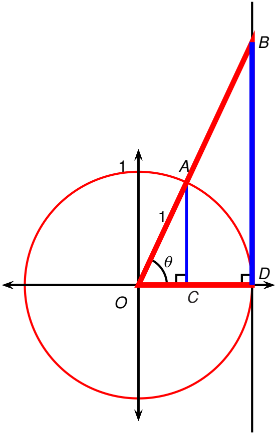
Geometric interpretation of all trigonometric functions



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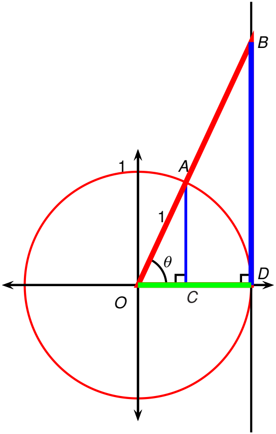
Geometric interpretation of all trigonometric functions



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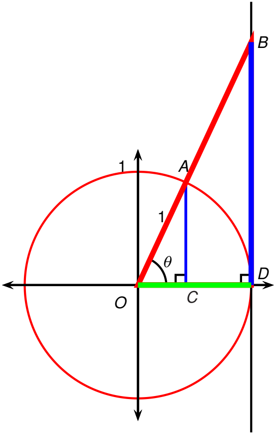
Geometric interpretation of all trigonometric functions



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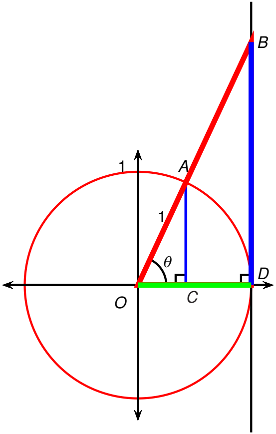
Geometric interpretation of all trigonometric functions



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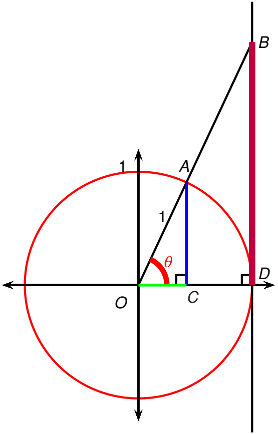
Geometric interpretation of all trigonometric functions



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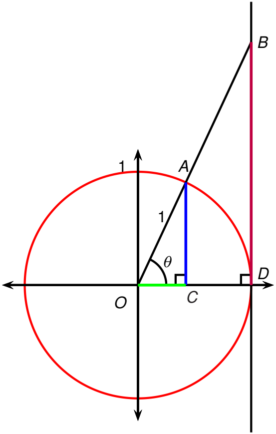
Geometric interpretation of all trigonometric functions



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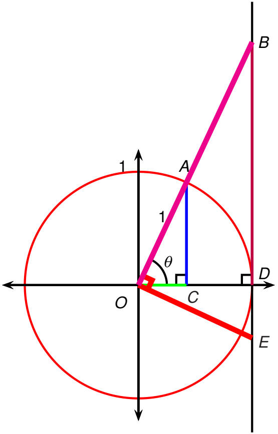
\cot\theta &= \frac{\text{adj}}{\text{opp}} \\
\sec\theta & \\
\csc\theta &
\end{aligned}
$$

Geometric interpretation of all trigonometric functions



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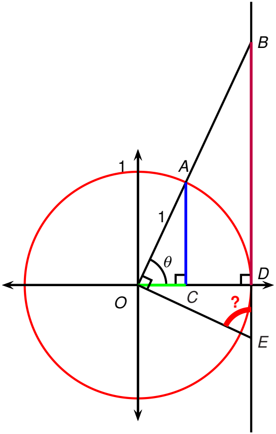
Geometric interpretation of all trigonometric functions



$\angle OED = \mathbf{?}$

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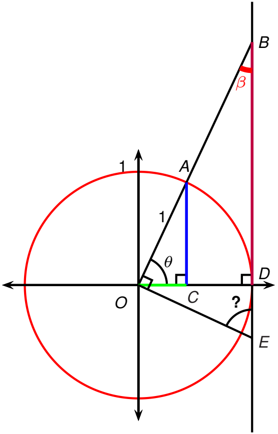
Geometric interpretation of all trigonometric functions



$\beta = ?$

$\angle OED = ?$

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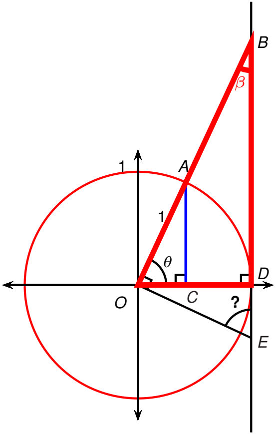
Geometric interpretation of all trigonometric functions



$\beta = 180^\circ - 90^\circ - \theta$

$\angle OED =$ **?**

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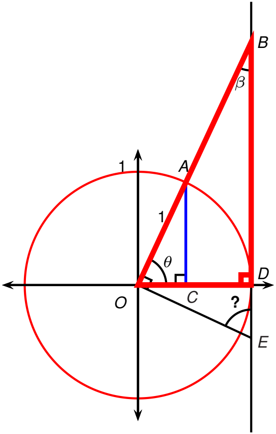
Geometric interpretation of all trigonometric functions



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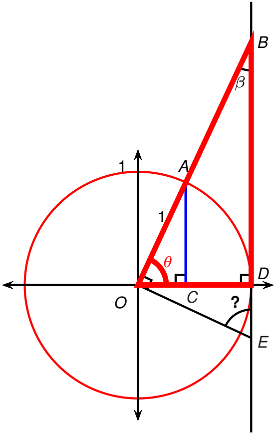
Geometric interpretation of all trigonometric functions



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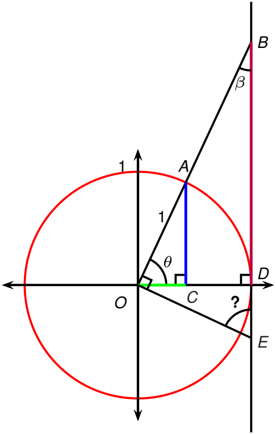
Geometric interpretation of all trigonometric functions



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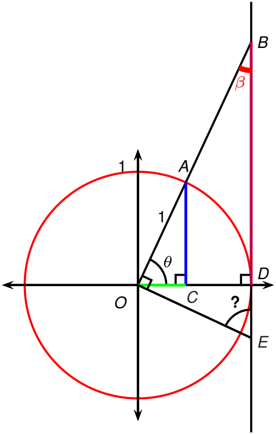
Geometric interpretation of all trigonometric functions



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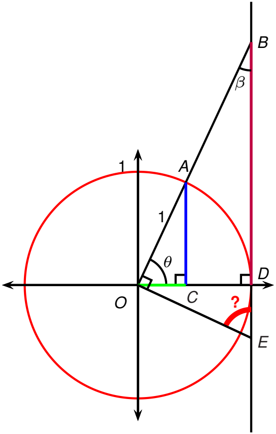
Geometric interpretation of all trigonometric functions



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\csc\theta &
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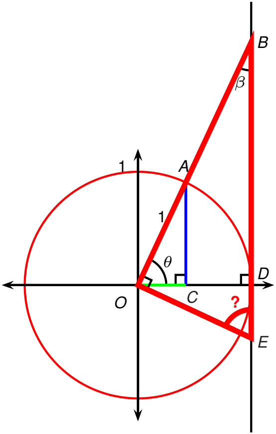
Geometric interpretation of all trigonometric functions



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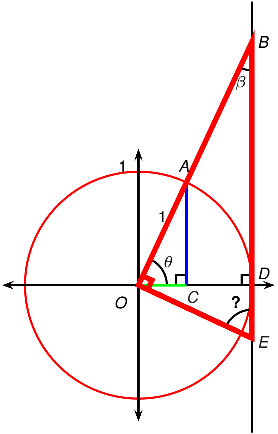
Geometric interpretation of all trigonometric functions



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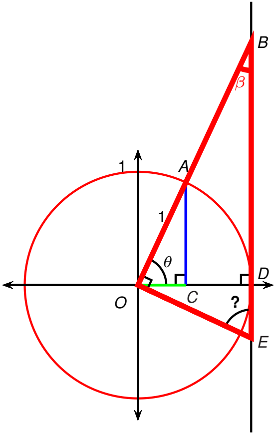
Geometric interpretation of all trigonometric functions



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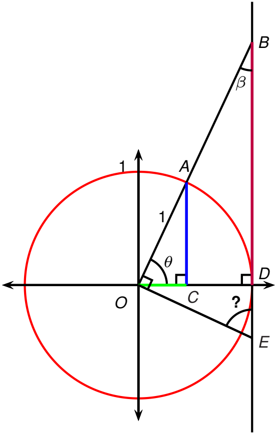
Geometric interpretation of all trigonometric functions



$$\beta = 180^\circ - 90^\circ - \theta = 90^\circ - \theta$$

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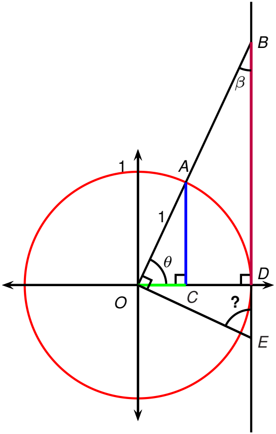
Geometric interpretation of all trigonometric functions



$$\beta = 180^\circ - 90^\circ - \theta = 90^\circ - \theta$$

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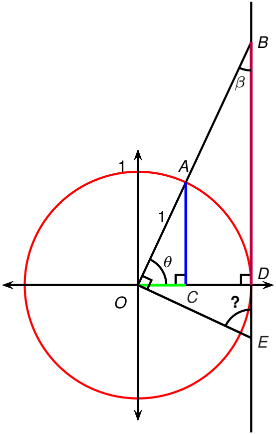
Geometric interpretation of all trigonometric functions



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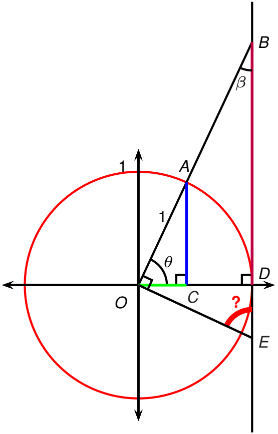
$\csc\theta$

Geometric interpretation of all trigonometric functions



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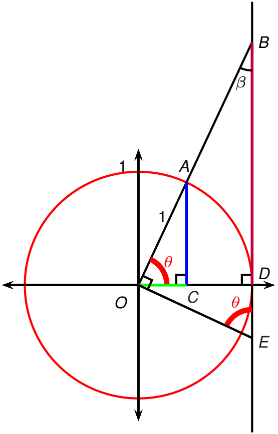
Geometric interpretation of all trigonometric functions



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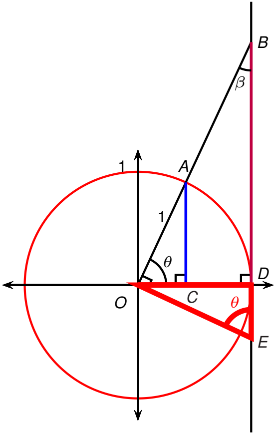

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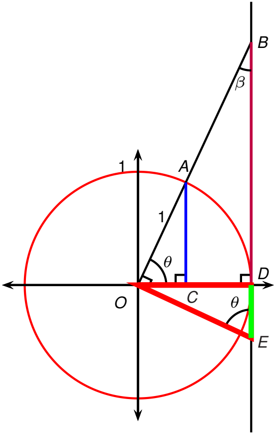

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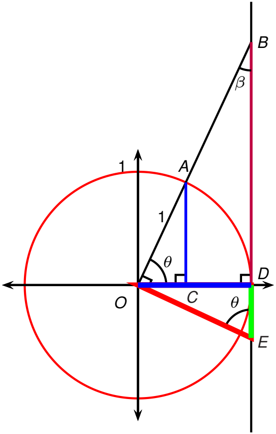

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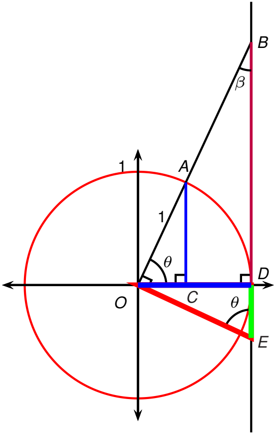

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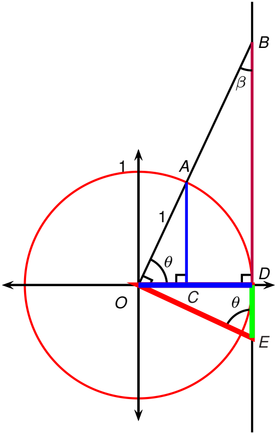

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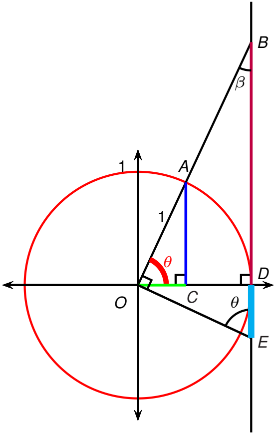
Geometric interpretation of all trigonometric functions



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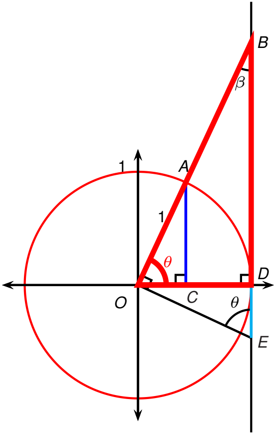
Geometric interpretation of all trigonometric functions



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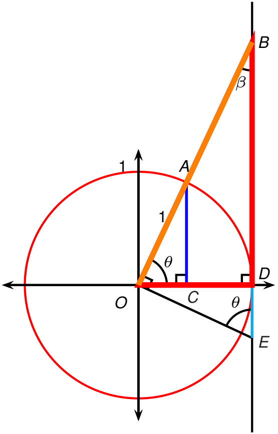
Geometric interpretation of all trigonometric functions



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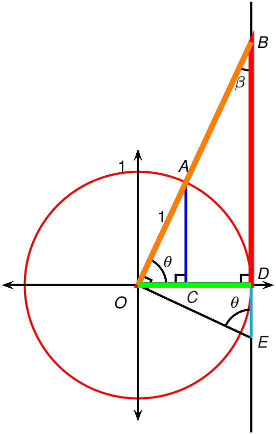
Geometric interpretation of all trigonometric functions



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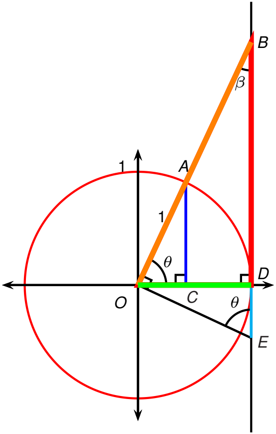
Geometric interpretation of all trigonometric functions



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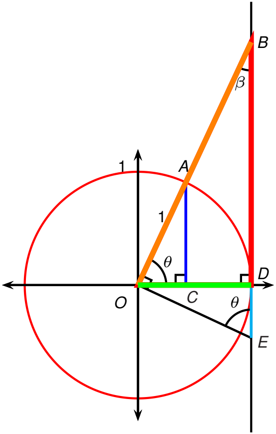

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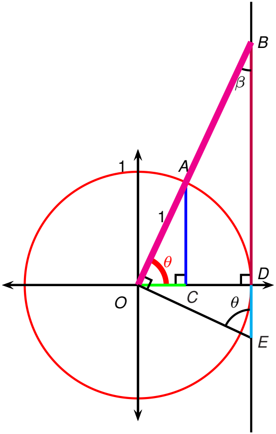

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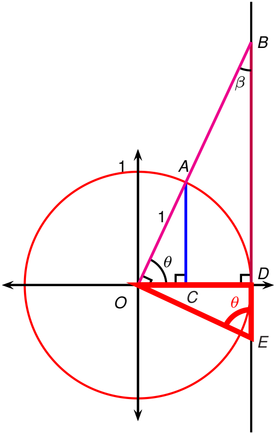

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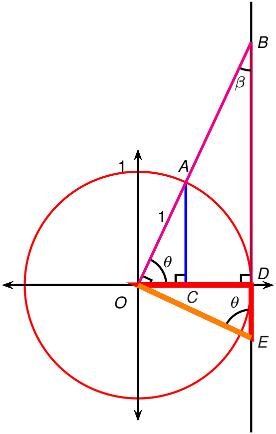
Geometric interpretation of all trigonometric functions



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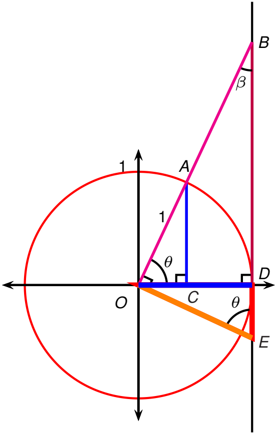
Geometric interpretation of all trigonometric functions



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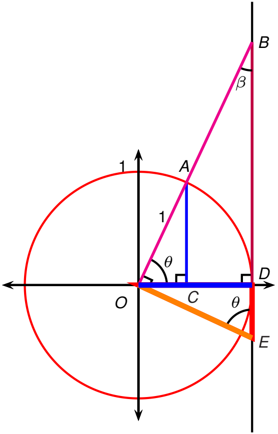
Geometric interpretation of all trigonometric functions



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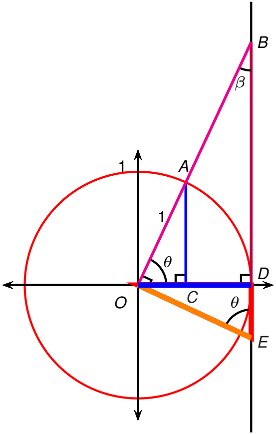
Geometric interpretation of all trigonometric functions



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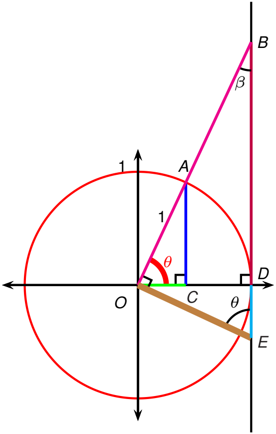
Geometric interpretation of all trigonometric functions



$$\begin{aligned}\beta &= 180^\circ - 90^\circ - \theta\\ &= 90^\circ - \theta\\ \angle OED &= 180^\circ - 90^\circ - \beta\\ &= 90^\circ - (90^\circ - \theta)\\ &= \theta\end{aligned}$$

Fix unit circle, center O, coordinates $(0, 0)$. Let $\angle DOB = \theta$. Let OB intersect the circle at point A. Coordinates of A are $(\cos\theta, \sin\theta)$.

$$\begin{aligned}
\sin\theta &= \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|} = \frac{|AC|}{1} = |AC|\\
\cos\theta &= \frac{\text{adj}}{\text{hyp}} = \frac{|OC|}{|OA|} = \frac{|OC|}{1} = |OC|\\
\tan\theta &= \frac{\text{opp}}{\text{adj}} = \frac{|BD|}{|OD|} = \frac{|BD|}{1} = |BD|\\
\cot\theta &= \frac{\text{adj}}{\text{opp}} = \frac{|DE|}{|OD|} = \frac{|DE|}{1} = |DE|\\
\sec\theta &= \frac{\text{hyp}}{\text{adj}} = \frac{|OB|}{|OD|} = \frac{|OB|}{1} = |OB|\\
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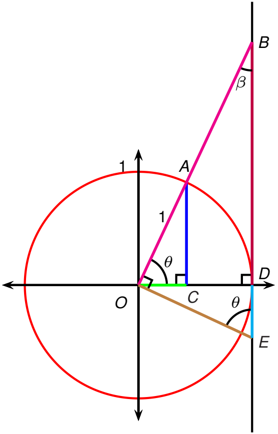
Geometric interpretation of all trigonometric functions



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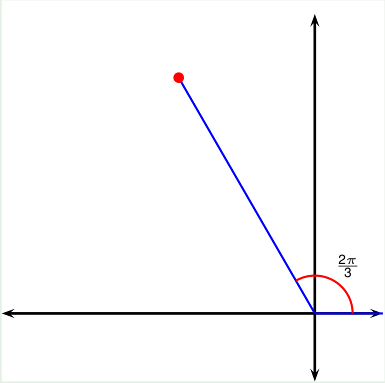
Example



Find the exact values of the trigonometric functions of $\theta = \dfrac{2\pi}{3} = 120^\circ$.

$$\sin\left(\frac{2\pi}{3}\right) = \qquad \cos\left(\frac{2\pi}{3}\right) = \qquad \tan\left(\frac{2\pi}{3}\right) =$$

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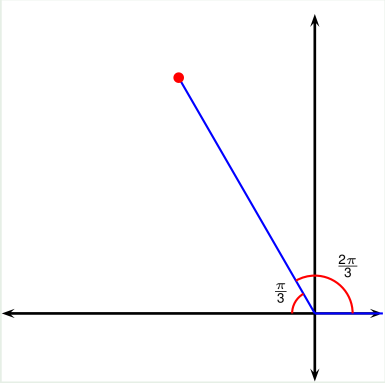
Example



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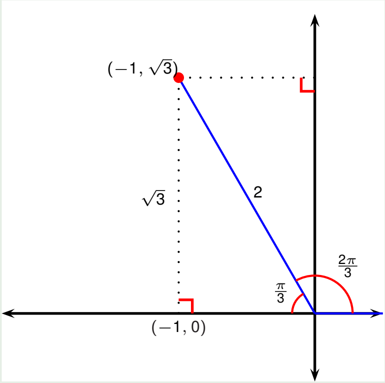

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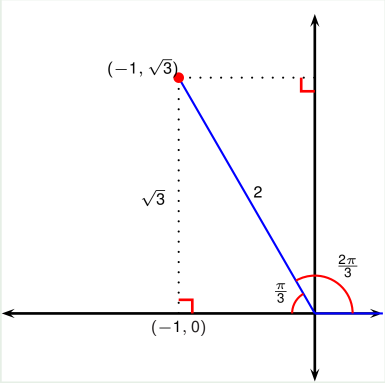

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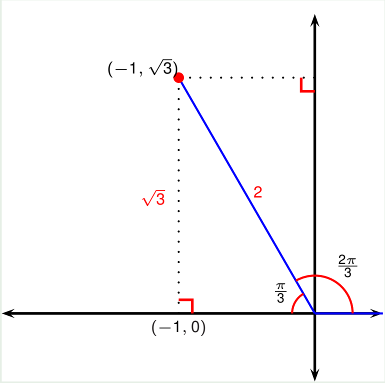

Find the exact values of the trigonometric functions of $\theta = \dfrac{2\pi}{3} = 120^\circ$.

$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \cos\left(\frac{2\pi}{3}\right) = \quad \tan\left(\frac{2\pi}{3}\right) =$$

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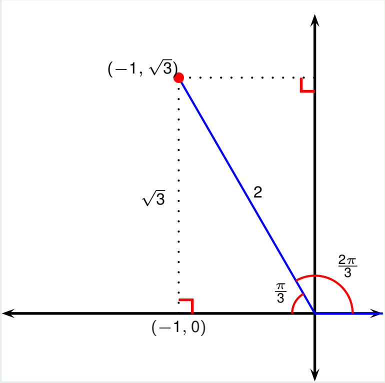

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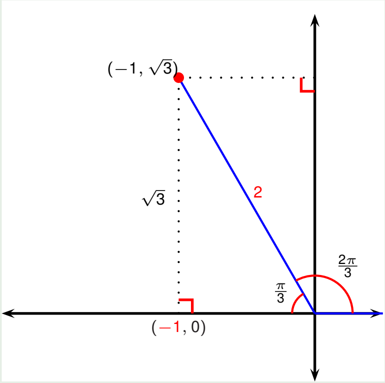

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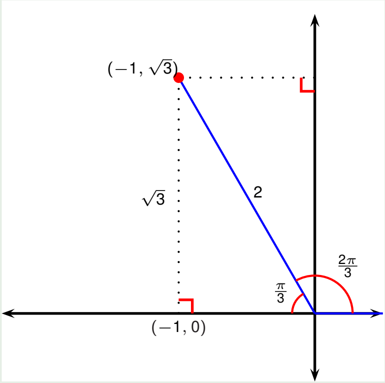

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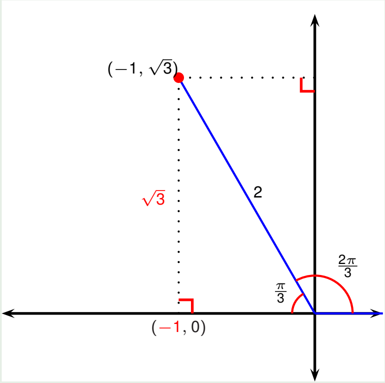

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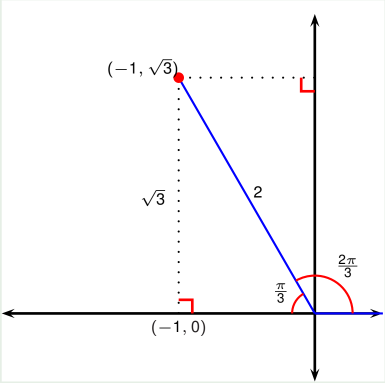

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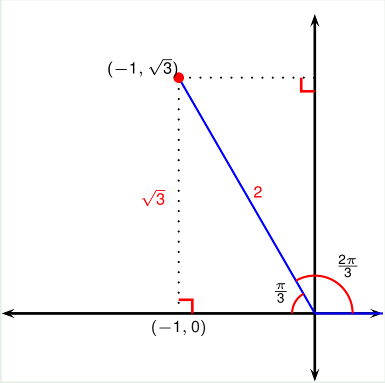

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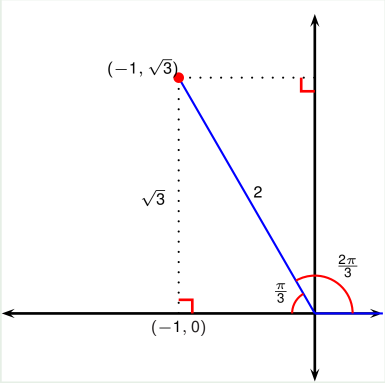

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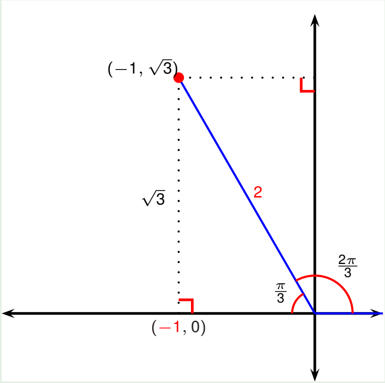

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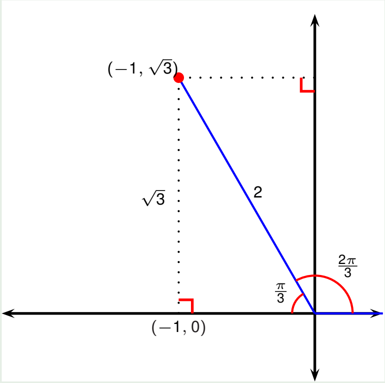
Example



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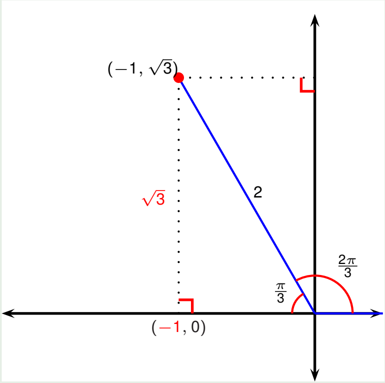

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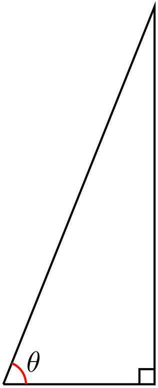
Example

If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



$\sin\theta =$ $\qquad$ $\tan\theta =$

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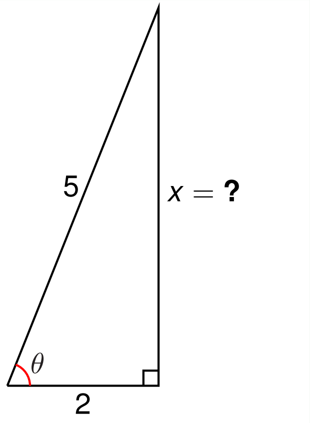

- Label the hypotenuse with length 5 and the adjacent side with length 2.

$\sin\theta =$ $\tan\theta =$

$\csc\theta =$ $\sec\theta =$

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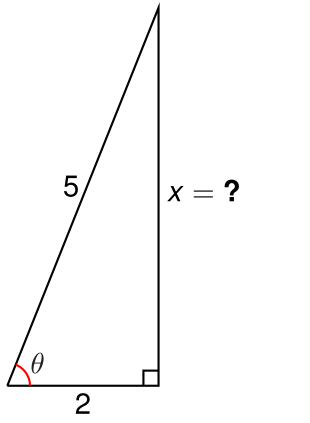
If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem: $x^2 + 2^2 = 5^2$.

$$\sin\theta = \qquad \tan\theta =$$

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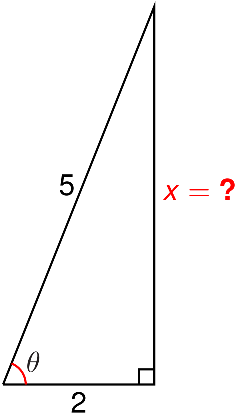

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- Therefore $x^2 =$ **?** , so $x =$ **?** .

$\sin\theta =$ $\tan\theta =$

$\csc\theta =$ $\sec\theta =$

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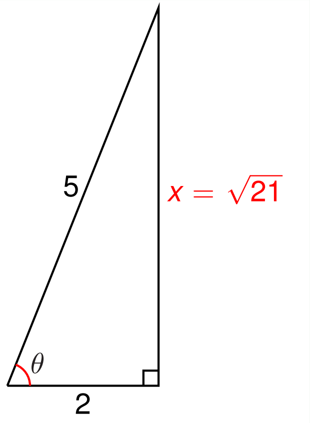

- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem: $x^2 + 2^2 = 5^2$.
- Therefore $x^2 = 21$, so $x = \sqrt{21}$.

$\sin\theta =$ $\qquad$ $\tan\theta =$

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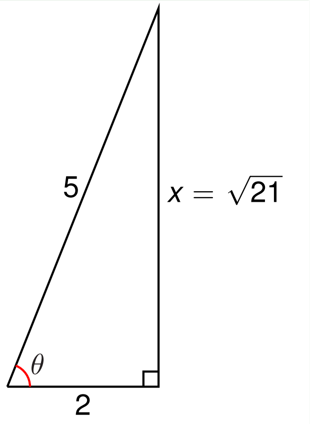
If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



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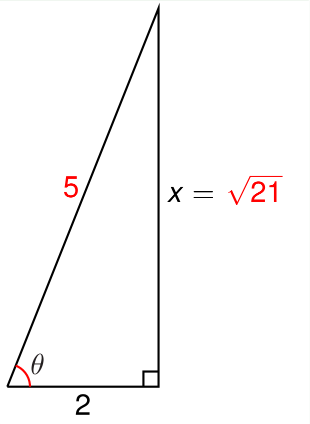

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$\sin\theta = \frac{\sqrt{21}}{5}$ $\quad$ $\tan\theta =$

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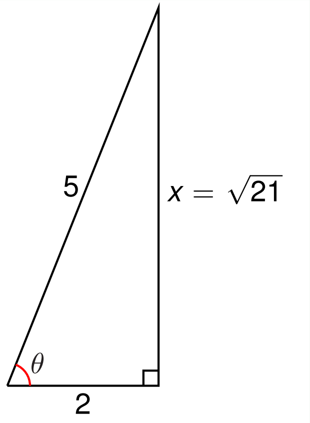

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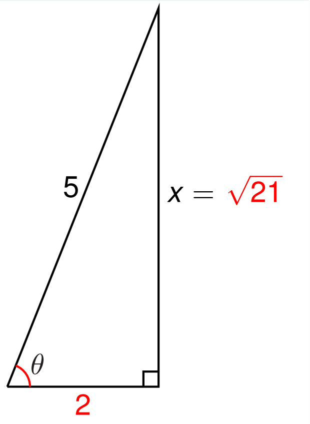
If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



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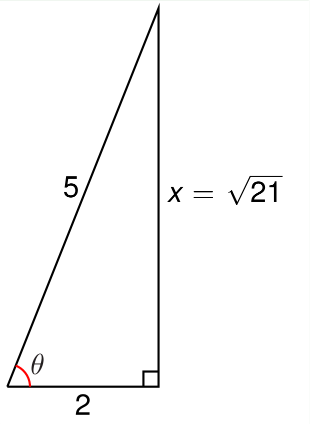

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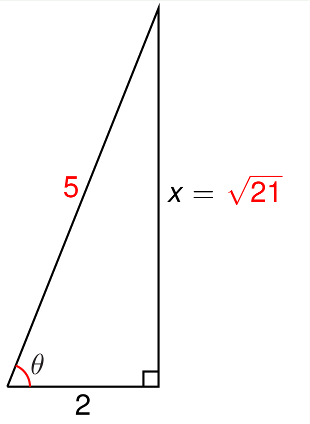

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$$\sin\theta = \frac{\sqrt{21}}{5} \quad \tan\theta = \frac{\sqrt{21}}{2}$$

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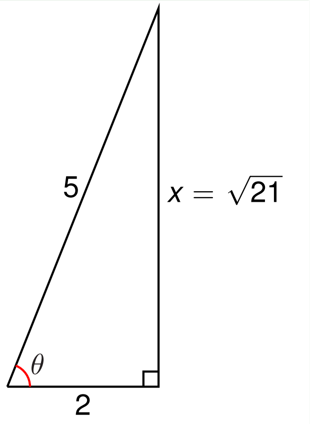
If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



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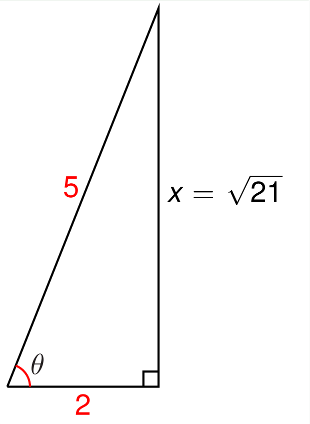
Example

If $\cos\theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ.



- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem: $x^2 + 2^2 = 5^2$.
- Therefore $x^2 = 21$, so $x = \sqrt{21}$.

$$\sin\theta = \frac{\sqrt{21}}{5} \qquad \tan\theta = \frac{\sqrt{21}}{2}$$

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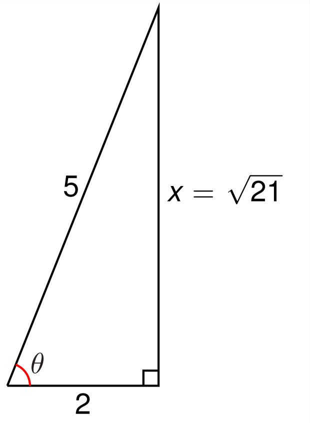

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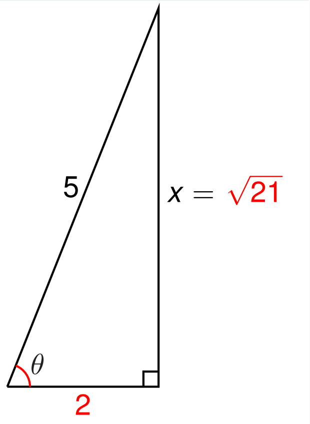

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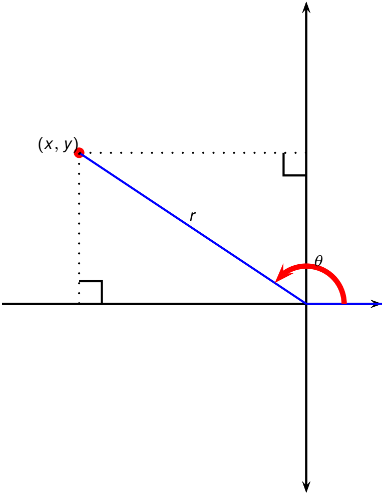

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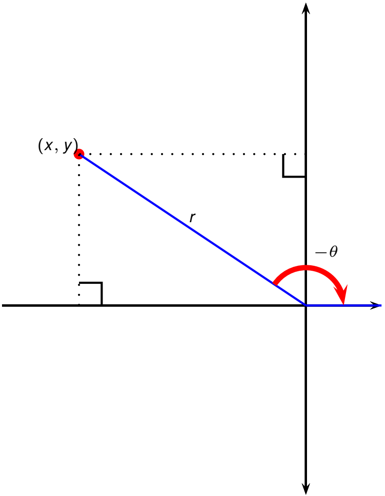

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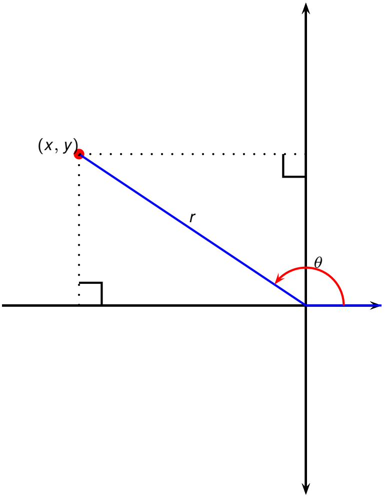

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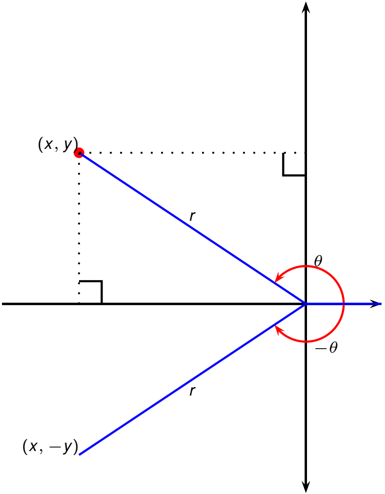

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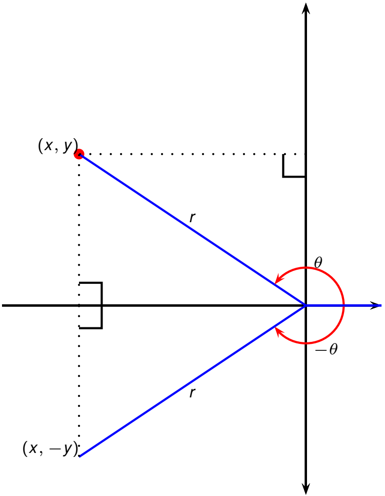

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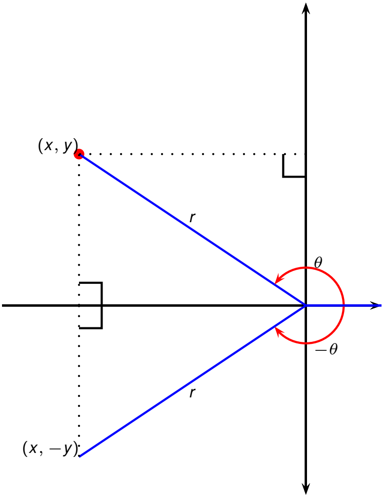

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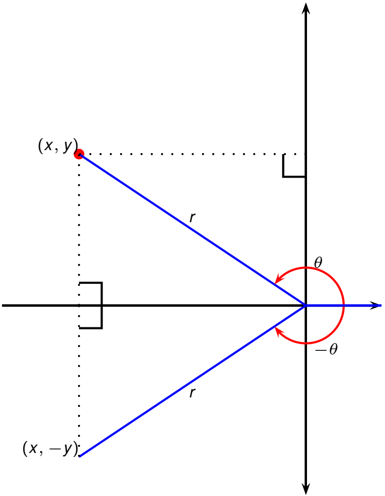

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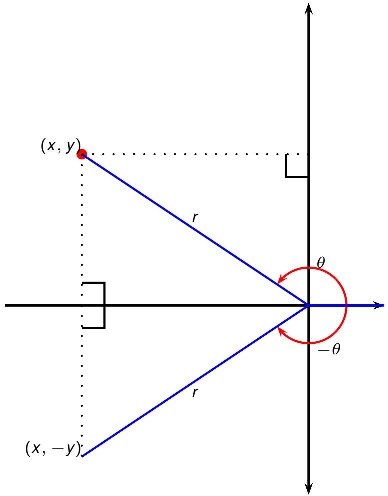
- sin is an odd function.



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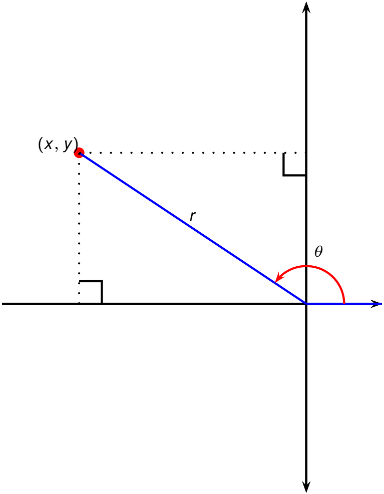
- sin is an odd function.
- cos is an even function.



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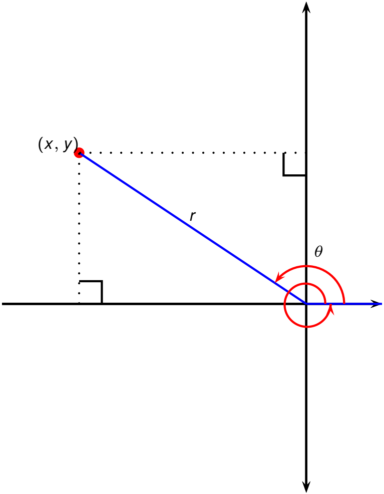

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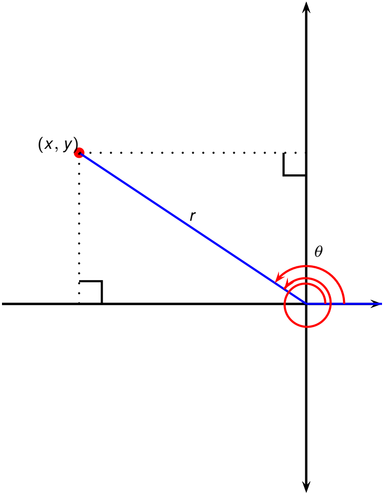

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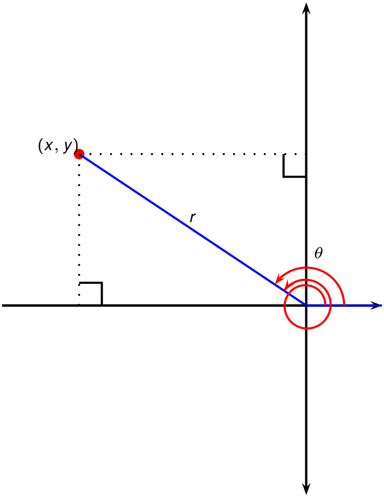

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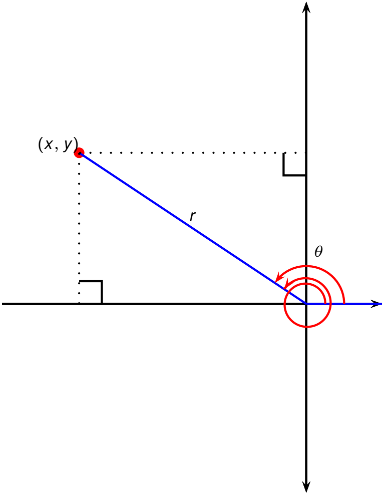

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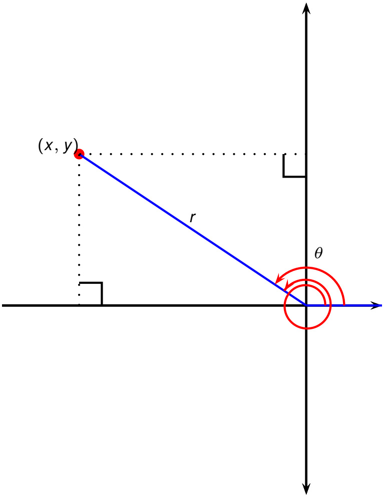

$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$
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- We say $\sin$ and $\cos$ are 2π-periodic.

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Definition (Trigonometric Identity)

A trigonometric identity is an equality between the trigonometric functions in one or more variables that holds for all values of the involved variables in the domains of all of the expressions.

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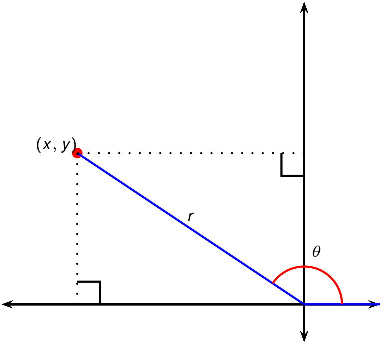
- By convention, when dealing with trigonometric identities we do not account for the domains of the involved expressions.
- For example, $\frac{\sin\theta}{\sin\theta} = 1$ is considered a valid trigonometric identity, although, when considered as a function, the left hand side is not defined for $\theta \neq 0$.



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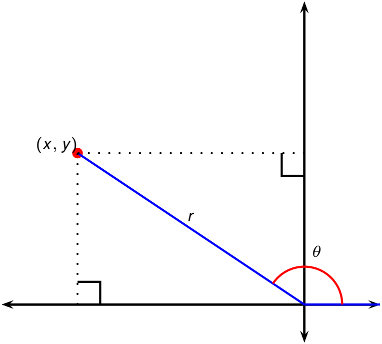

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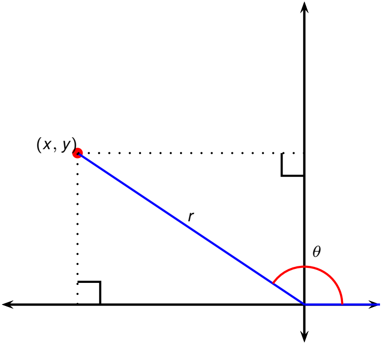

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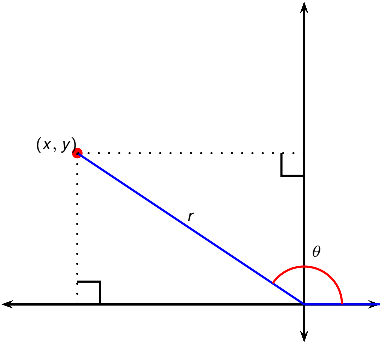
$$\sin^2\theta + \cos^2\theta$$



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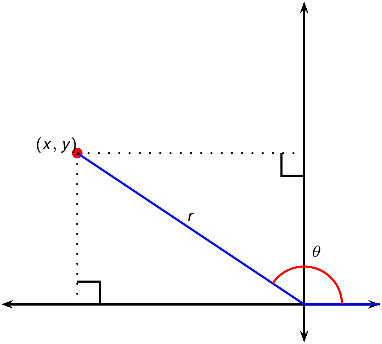
$$\begin{aligned} & \sin^2\theta + \cos^2\theta \\ = \quad & \frac{y^2}{r^2} + \frac{x^2}{r^2} \end{aligned}$$



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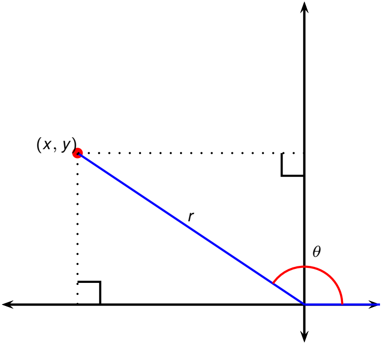

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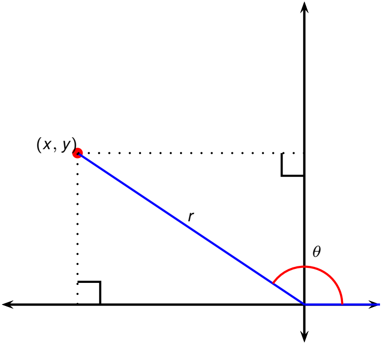

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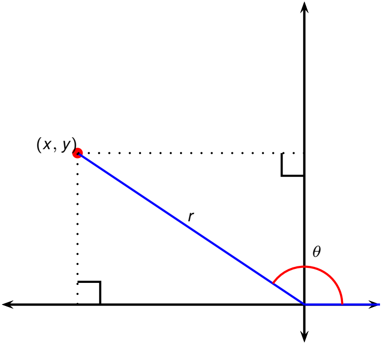

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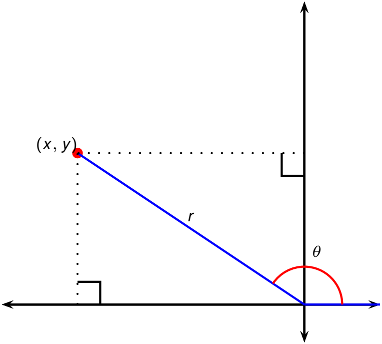
Therefore $\sin^2\theta + \cos^2\theta = 1$.



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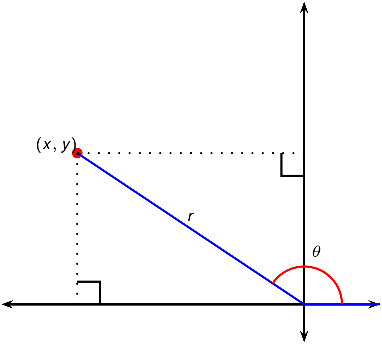
Example ($\tan^2\theta + 1 = \sec^2\theta$)

Prove the identity
$\tan^2\theta + 1 = \sec^2\theta$.



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Prove the identity
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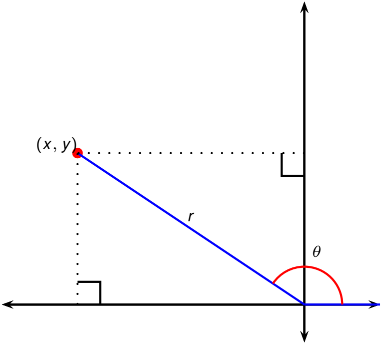
$$\sin^2\theta + \cos^2\theta = 1$$



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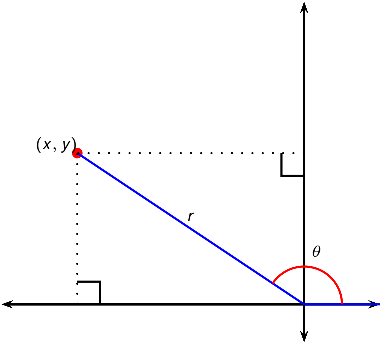
Example ($\tan^2\theta + 1 = \sec^2\theta$)

Prove the identity $\tan^2\theta + 1 = \sec^2\theta$.

$$\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} &= \frac{1}{\cos^2\theta} \end{aligned}$$



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Example ($\tan^2\theta + 1 = \sec^2\theta$)

Prove the identity
$\tan^2\theta + 1 = \sec^2\theta$.

$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} &= \frac{1}{\cos^2\theta} \\
\tan^2\theta + 1 &= \sec^2\theta
\end{aligned}$$

The remaining identities are consequences of the addition formulas:

$$\begin{array}{rcccc}\sin(x+y) &=& \sin x\cos y &+& \cos x\sin y\\ \cos(x+y) &=& \cos x\cos y &-& \sin x\sin y\end{array}$$

The remaining identities are consequences of the addition formulas:

$$\begin{aligned}\sin(x+y) &= \sin x\cos y + \cos x\sin y\\ \cos(x+y) &= \cos x\cos y - \sin x\sin y\end{aligned}$$

Substitute $-y$ for y, and use the fact that $\sin(-y) = -\sin y$ and $\cos(-y) = \cos y$:

$$\begin{aligned}\sin(x-y) &= \sin x\cos y - \cos x\sin y\\ \cos(x-y) &= \cos x\cos y + \sin x\sin y\end{aligned}$$

The remaining identities are consequences of the addition formulas:

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To get the double angle formulas, substitute x for y:

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$$

To get the half-angle formulas, solve these equations for $\cos^2 x$ and $\sin^2 x$ respectively.

$$
\cos^2 x = \frac{1+\cos(2x)}{2}, \qquad \sin^2 x = \frac{1-\cos(2x)}{2}
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Do the same for the subtraction formulas:

$$\tan(x-y) = \frac{\tan x-\tan y}{1+\tan x\tan y}$$

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Prove the trigonometric identity.

$$(\sin\theta + \cos\theta)^2 = 1 + \sin(2\theta)$$

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Strategy for proving trigonometric identities

An expression is rational trigonometric if it is written using $\sin\theta, \cos\theta$ and the four arithmetic operations.

Question

Is there a general method for proving all rational trigonometric identities in one variable?

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    - You may need to use trig functions of angles smaller than θ, for example $\sin\left(\frac{\theta}{2}\right), \cos\left(\frac{\theta}{2}\right)$.
    - A fraction of θ such that all appearing angles are integer multiples of it will always work.

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Prove the identity $\tan\theta + \sec\theta = \dfrac{1+\tan\left(\frac{\theta}{2}\right)}{1-\tan\left(\frac{\theta}{2}\right)}$

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as desired.

Trigonometric equations

- Some problems will not ask you to prove a trigonometric identity, but rather to solve a trigonometric equation.
- Consider the problem of finding all values of x for which $\sin x = \sin(2x) = 2\sin x \cos x$.
- This is not a trigonometric identity - the two sides are different.
- However, there are values for x which the above equality holds.

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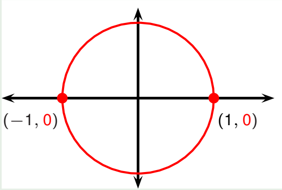

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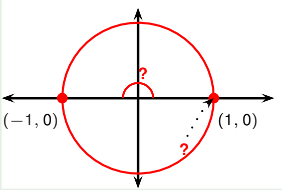

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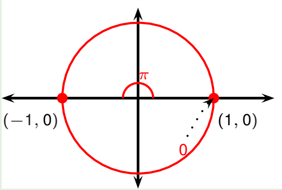

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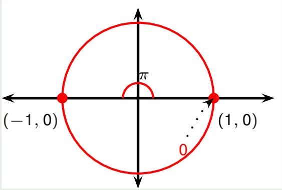

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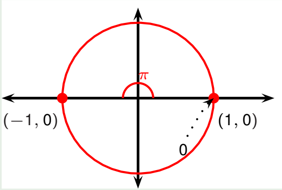

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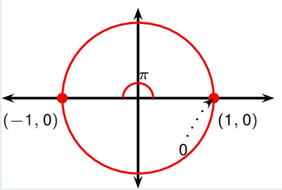

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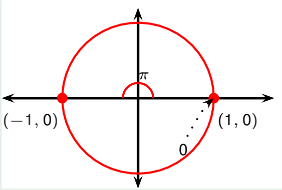

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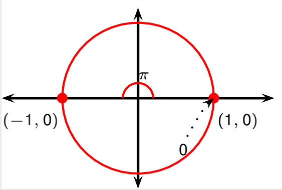

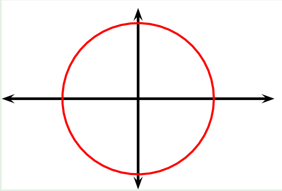

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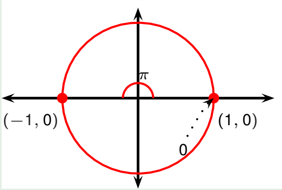

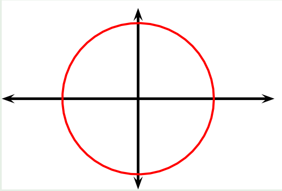

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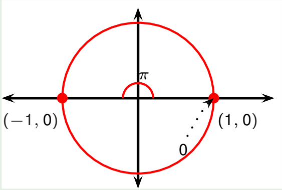

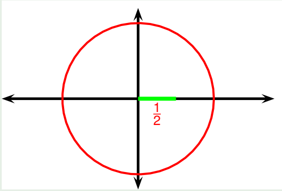

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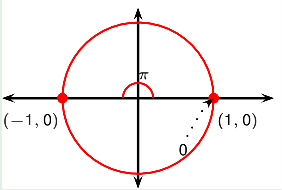

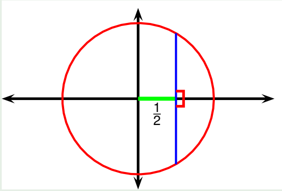

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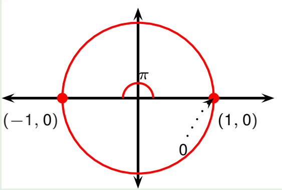

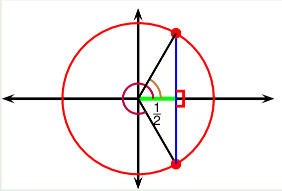

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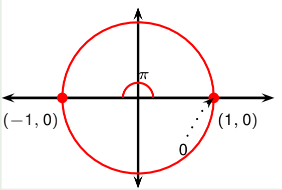

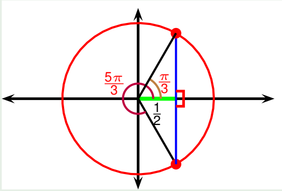

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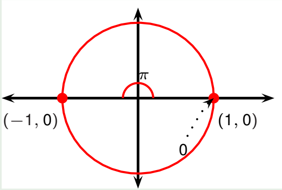

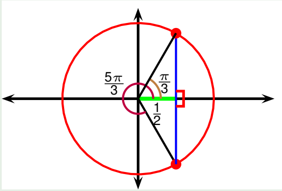

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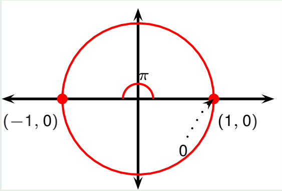
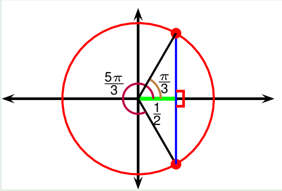
$$\begin{aligned}2\cos\theta - 1 &= 0\\ \cos\theta &= \frac{1}{2}\\ \theta &= \frac{\pi}{3} + 2k\pi \text{ or } \frac{5\pi}{3} + 2k\pi\\ \theta &= \frac{\pi}{3} \text{ or } \frac{5\pi}{3}\end{aligned}$$





Strategy for solving trigonometric equations

- Suppose we want to solve an algebraic trigonometric equation.
- More precisely, the equation should be an algebraic expressions of the trigonometric functions of a single variable.
- Here is a general strategy for solving such a problem:
  - Using trig identities, rewrite in terms of $\sin x$ and $\cos x$ only.
  - Suppose $x \in [2n\pi, (2n+1)\pi]$.
    - Set $\sin x = \sqrt{1-\cos^2 x}$ (allowed due to restrictions on x).
    - Set $\cos x = u$. Solve the resulting algebraic equation for u.
    - For the found solutions for u, solve $\cos x = u$.
    - Check whether your solutions satisfy $x \in [2n\pi, (2n+1)\pi]$.
  - Suppose $x \in [(2n-1)\pi, 2n\pi]$.
    - Set $\sin x = -\sqrt{1-\cos^2 x}$ (allowed due to restrictions on x).
    - Set $\cos x = u$. Solve the resulting algebraic equation for u.
    - For the found solutions for u, solve $\cos x = u$.
    - Check whether your solutions satisfy $x \in [(2n-1)\pi, 2n\pi]$.
- A similar strategy exists for $u = \sin x$ instead of $u = \cos x$.
- Problems requiring full algorithm may be too hard for Calc exams.

Example

Find all values of θ in the interval $\theta \in [0, 2\pi]$ for which

$$\cos(2\theta) = \cos\theta$$

Example

Find all values of θ in the interval $\theta \in [0, 2\pi]$ for which

$$
\begin{array}{lrcl}
 & \cos(2\theta) & = & \cos\theta \\
\textbf{?} & -\cos\theta & = & 0
\end{array}
$$

Example

Find all values of θ in the interval $\theta \in [0, 2\pi]$ for which

$$\begin{aligned} \cos(2\theta) &= \cos\theta \\ ? \quad -\cos\theta &= 0 \end{aligned}$$

Example

Find all values of θ in the interval $\theta \in [0, 2\pi]$ for which

$$\begin{aligned}\cos(2\theta) &= \cos\theta \\ \cos^2\theta - \sin^2\theta - \cos\theta &= 0\end{aligned}$$

Example

Find all values of θ in the interval $\theta \in [0, 2\pi]$ for which

$$\begin{aligned} \cos(2\theta) &= \cos\theta \\ \cos^2\theta - \sin^2\theta - \cos\theta &= 0 \quad \Big| \text{ Express via } \cos\theta \\ \cos^2\theta - (\textbf{?} \qquad\qquad) - \cos\theta &= 0 \end{aligned}$$

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Find all values of θ in the interval $\theta \in [0, 2\pi]$ for which

$$\begin{array}{rcll} \cos(2\theta) & = & \cos\theta & \\ \cos^2\theta - \sin^2\theta - \cos\theta & = & 0 & \Big| \text{ Express via } \cos\theta \\ \cos^2\theta - (1 - \cos^2\theta) - \cos\theta & = & 0 & \end{array}$$

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$$\begin{array}{rcll}
\cos(2\theta) &=& \cos\theta & \\
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\cos^2\theta - (1 - \cos^2\theta) - \cos\theta &=& 0 & \\
2\cos^2\theta - \cos\theta - 1 &=& 0 & \Big|\ \text{Set } \cos\theta = u \\
2u^2 - u - 1 &=& 0 &
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(u - 1)(2u + 1) &= 0
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or

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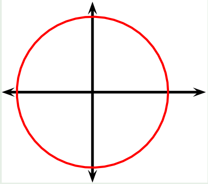

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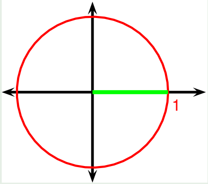

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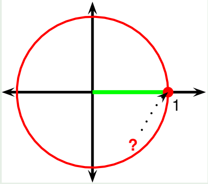

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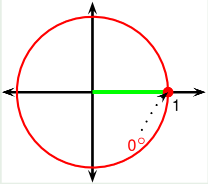

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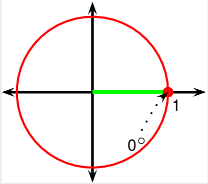

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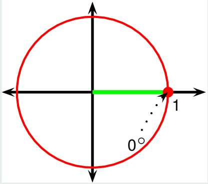

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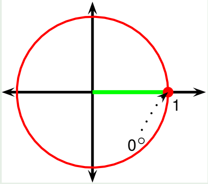

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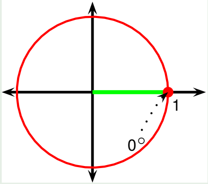

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2\cos^2\theta - \cos\theta - 1 &= 0 && \Big|\ \text{Set } \color{red}{\cos\theta = u} \\
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\end{aligned}$$

$$\begin{aligned}
u - 1 &= 0 \\
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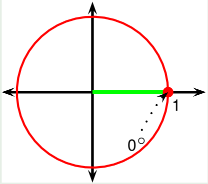
\theta &= 0 \text{ or } 2\pi
\end{aligned}
\quad \text{or} \quad
\begin{aligned}
2\color{red}{u} + 1 &= 0 \\
\color{red}{\cos\theta} &= -\frac{1}{2}
\end{aligned}$$



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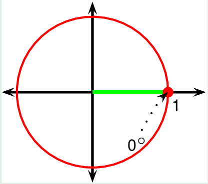

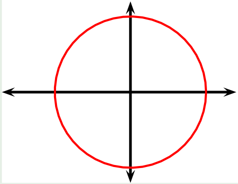

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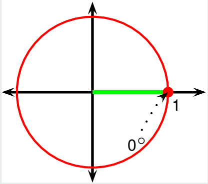

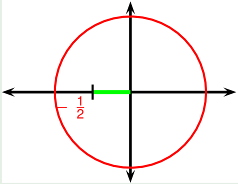

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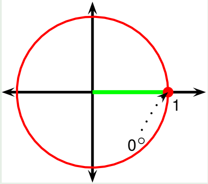

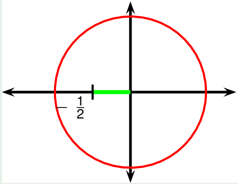

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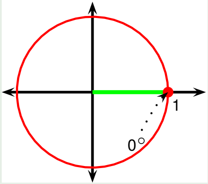

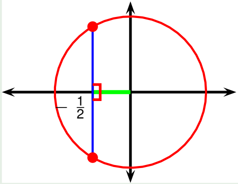

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$$\begin{array}{rcl}
u - 1 &=& 0 \\
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\theta &=& 0 \text{ or } 2\pi
\end{array}
\quad \text{or} \quad
\begin{array}{rcl}
2u + 1 &=& 0 \\
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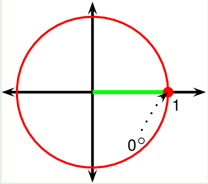
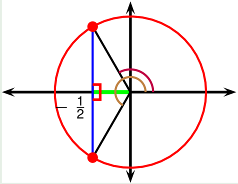
\theta &=& ?
\end{array}$$





Example

Find all values of θ in the interval $\theta \in [0, 2\pi]$ for which

$$
\begin{aligned}
\cos(2\theta) &= \cos\theta \\
\cos^2\theta - \sin^2\theta - \cos\theta &= 0 && \Big| \text{ Express via } \cos\theta \\
\cos^2\theta - (1 - \cos^2\theta) - \cos\theta &= 0 \\
2\cos^2\theta - \cos\theta - 1 &= 0 && \Big| \text{ Set } \cos\theta = u \\
2u^2 - u - 1 &= 0 \\
(u-1)(2u+1) &= 0
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\begin{aligned}
2u + 1 &= 0 \\
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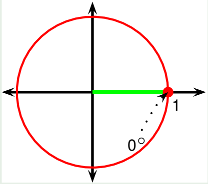
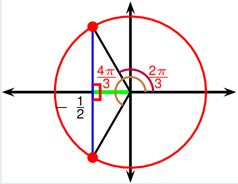
\theta &= \frac{2\pi}{3} + 2k\pi \text{ or } \frac{4\pi}{3} + 2k\pi
\end{aligned}
$$





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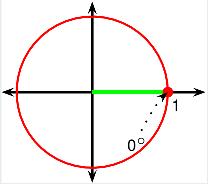

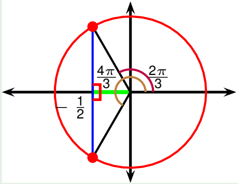

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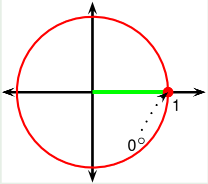

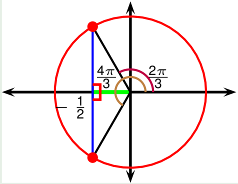

Definition (Complex numbers)

The set of complex numbers $\mathbb{C}$ is defined as the set

$$\{a + bi | a, b - \text{real numbers}\},$$

where the number i is a number for which

$$i^2 = -1 \quad .$$

The number i is called the imaginary unit.

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You will not be tested on the material in the following slide.

Euler's Formula

Theorem (Euler's Formula)

$$e^{ix} = \cos x + i \sin x,$$

where $e \approx 2.71828$ *is Euler's/Napier's constant .*

Proof.

Recall $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n$. Borrow from Calc II the f-las:

$\square$

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$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + \frac{(-1)^n x^{2n}}{(2n)!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

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\sin x & = & & x & & -\frac{x^3}{3!} & & +\frac{x^5}{5!} & -\dots \\
\cos x & = & 1 & & -\frac{x^2}{2!} & & +\frac{x^4}{4!} & & +\dots \\
\hline
e^z & = & 1 & +z & +\frac{z^2}{2!} & +\frac{z^3}{3!} & +\frac{z^4}{4!} & +\frac{z^5}{5!} & +\dots
\end{array}$$

Rearrange.

$\square$

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Rearrange. Plug-in $z = ix$.

□

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\hline
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Rearrange. Plug-in $z = ix$. Use $i^2 = -1$.

$\square$

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Rearrange. Plug-in $z = ix$. Use $i^2 = -1$. Multiply $\sin x$ by i.

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\end{array}$$

Rearrange. Plug-in $z = ix$. Use $i^2 = -1$. Multiply $\sin x$ by i. Add to get $e^{ix} = \cos x + i \sin x$. □

You will not be tested on the material in the following slide.

Trigonometric Identities Revisited

- $e^{ix} = \cos x + i \sin x$ (Euler's Formula).
- $e^{ix} e^{iy} = e^{ix+iy} = e^{i(x+y)}$ (exponentiation rule: valid for $\mathbb{C}$).
- $e^0 = 1$ (exponentiation rule).

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All trigonometric formulas can be easily derived using the above formulas.

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Example

$$\begin{array}{rcl} \sin(x+y) & = & \sin x \cos y + \sin y \cos x \\ \cos(x+y) & = & \cos x \cos y - \sin x \sin y \end{array} .$$

Proof.

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$$e^{i(x+y)} =$$

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$$\begin{array}{rcl} e^{i(x+y)} & = & \cos(x+y) + i\sin(x+y) \\ e^{ix}e^{iy} & = & \cos(x+y) + i\sin(x+y) \\ (\cos x + i\sin x)(\cos y + i \sin y) & = & \cos(x+y) + i\sin(x+y) \\ \cos x\cos y - \sin x \sin y + i(\sin x \cos y + \sin y \cos x) & = & \cos(x+y) + i\sin(x+y) \end{array}$$

Compare coefficient in front of i and

Trigonometric Identities Revisited

- $e^{ix} = \cos x + i \sin x$ (Euler's Formula).
- $e^{ix} e^{iy} = e^{ix+iy} = e^{i(x+y)}$ (exponentiation rule: valid for $\mathbb{C}$).
- $e^0 = 1$ (exponentiation rule).
- $\sin(-x) = -\sin x$, $\cos(-x) = \cos x$ (easy to remember).

Example

$$\begin{array}{rcl} \sin(x+y) & = & \sin x \cos y + \sin y \cos x \\ \cos(x+y) & = & \cos x \cos y - \sin x \sin y \end{array} .$$

Proof.

$$\begin{array}{rcl} e^{i(x+y)} & = & \cos(x+y) + i \sin(x+y) \\ e^{ix} e^{iy} & = & \cos(x+y) + i \sin(x+y) \\ (\cos x + i \sin x)(\cos y + i \sin y) & = & \cos(x+y) + i \sin(x+y) \\ \cos x \cos y - \sin x \sin y + i(\sin x \cos y + \sin y \cos x) & = & \cos(x+y) + i \sin(x+y) \end{array}$$

Compare coefficient in front of i and remaining terms to get the desired equalities. □

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$$\sin^2 x + \cos^2 x = 1$$

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$\square$

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$$\begin{array}{rcl} \sin(2x) & = & 2 \sin x \cos x \\ \cos(2x) & = & \cos^2 x - \sin^2 x \end{array} .$$

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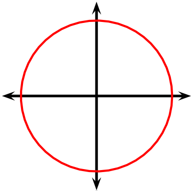
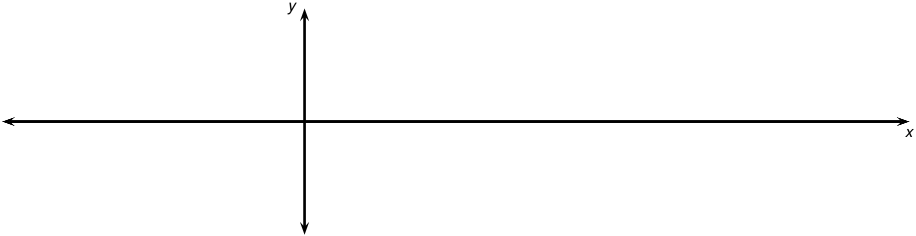
□

Graph of $\sin x$



x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	**?**								



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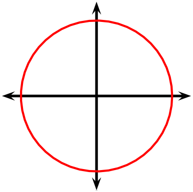

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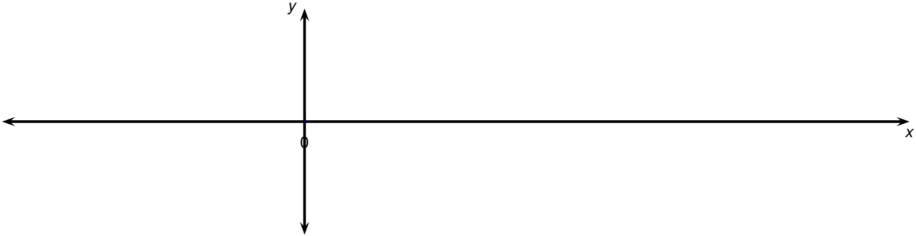

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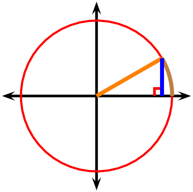

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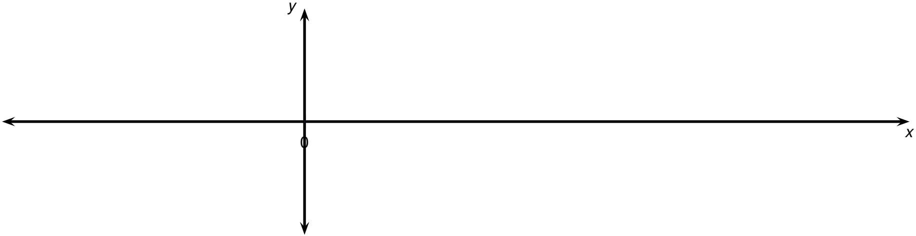

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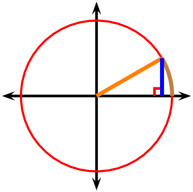

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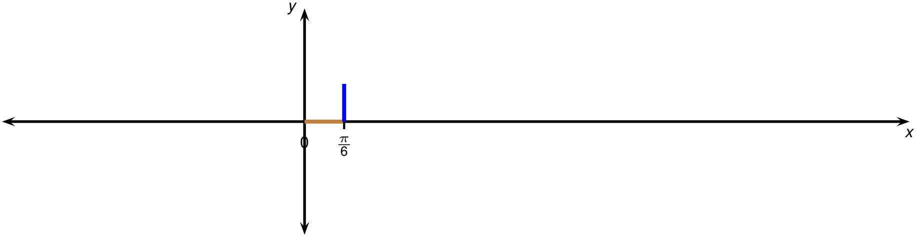

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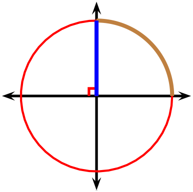

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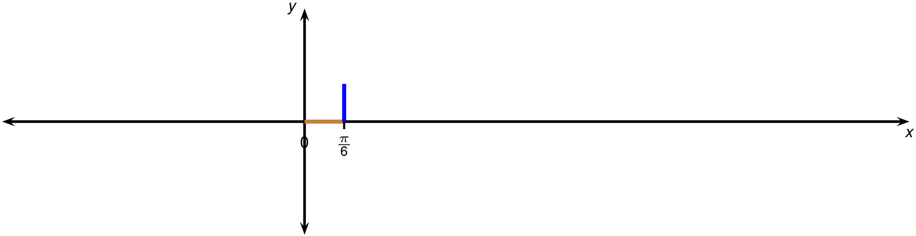

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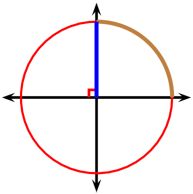

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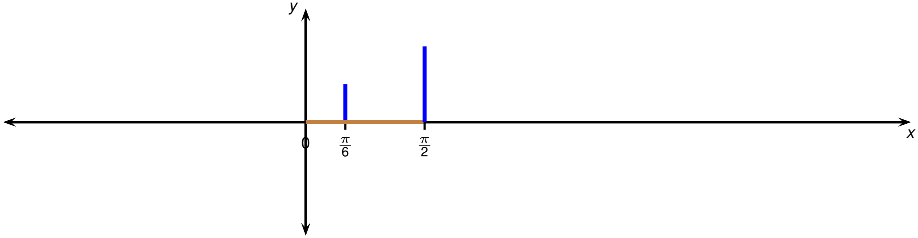

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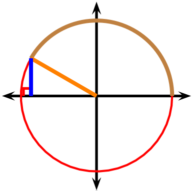

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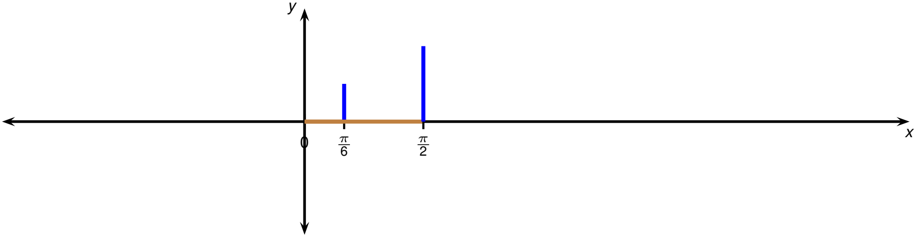

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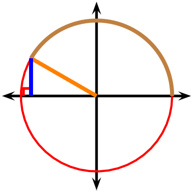

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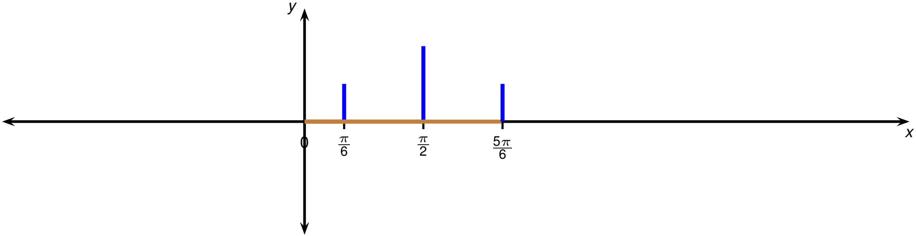

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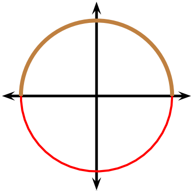

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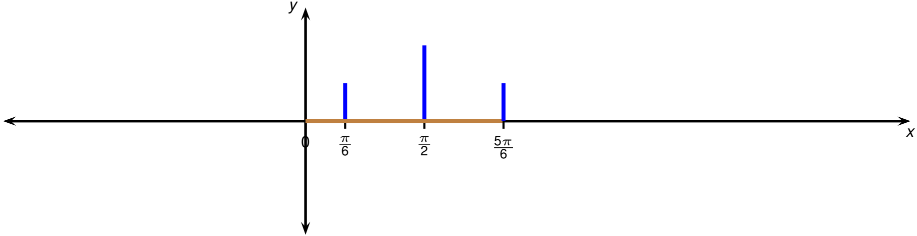

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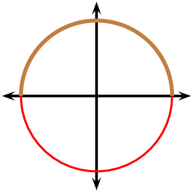

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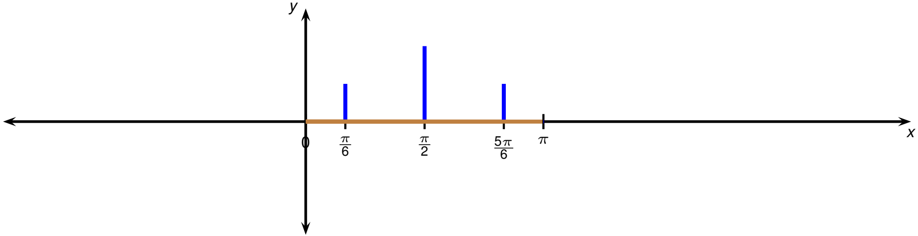

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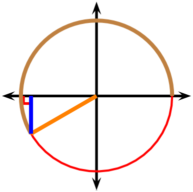

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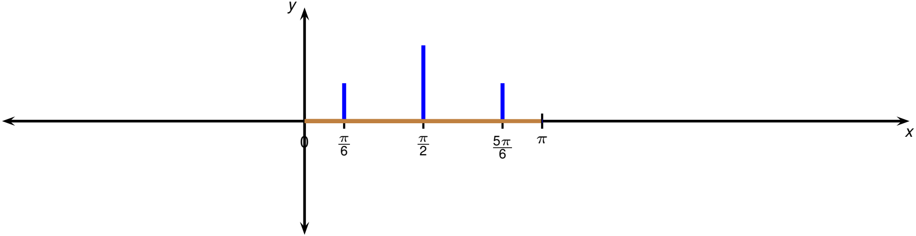

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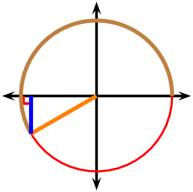

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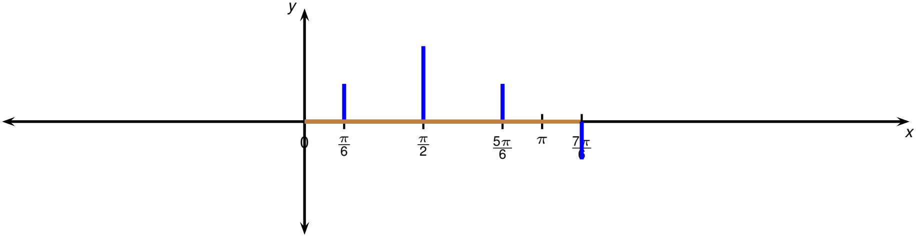

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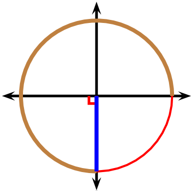

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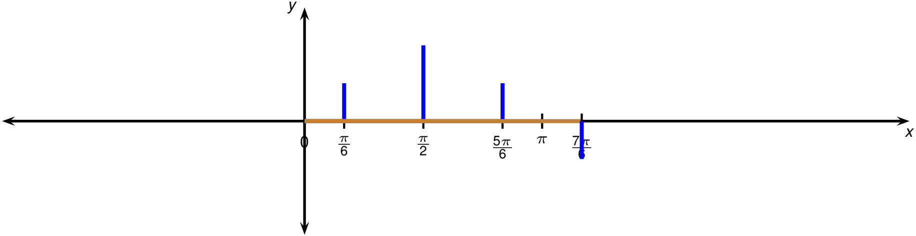

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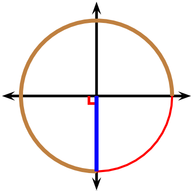
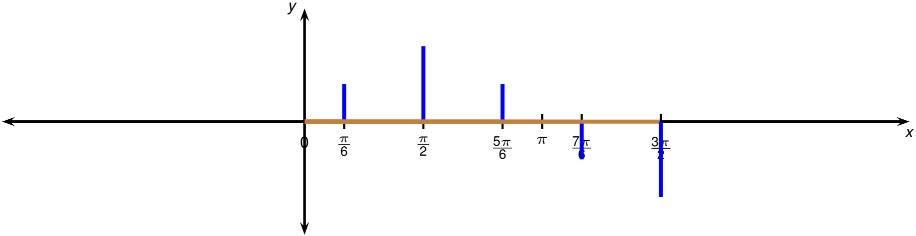
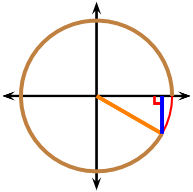


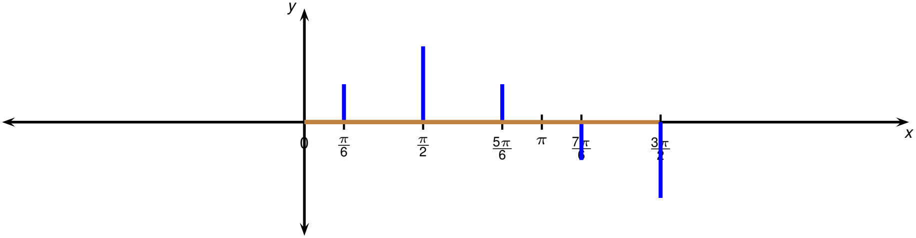
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1		



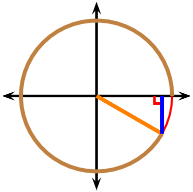
Graph of $\sin x$



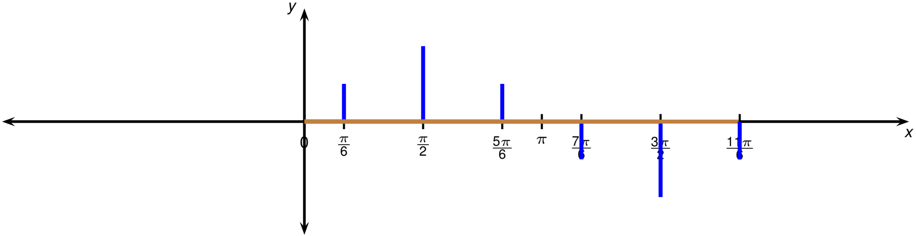
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	**?**	



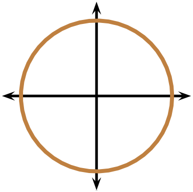
Graph of $\sin x$



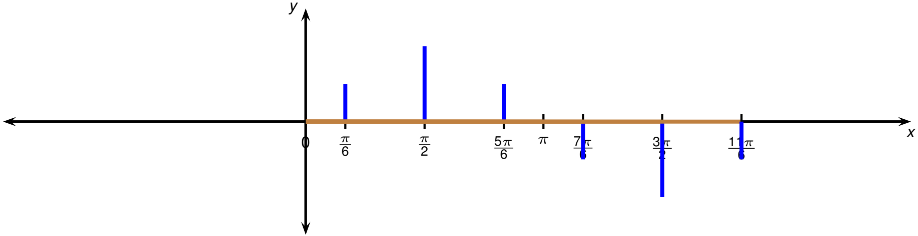
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	



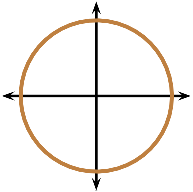
Graph of $\sin x$



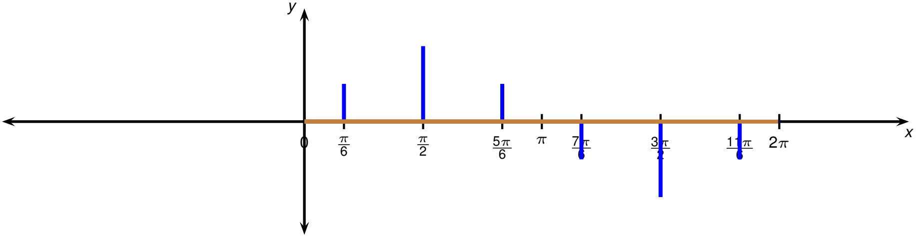
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	**?**



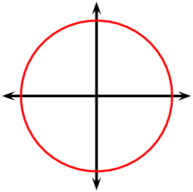
Graph of $\sin x$



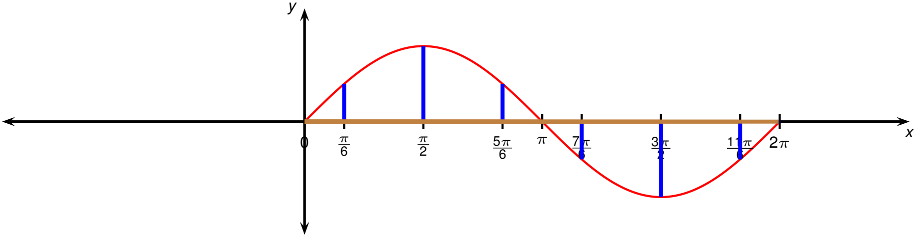
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0



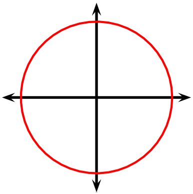
Graph of $\sin x$



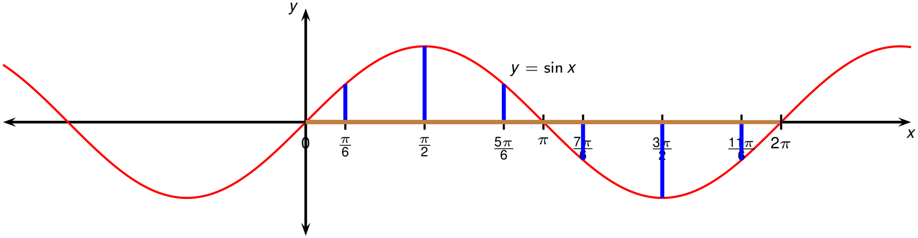
x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0



Graph of $\sin x$

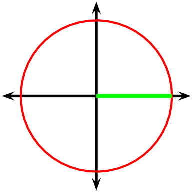


x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0

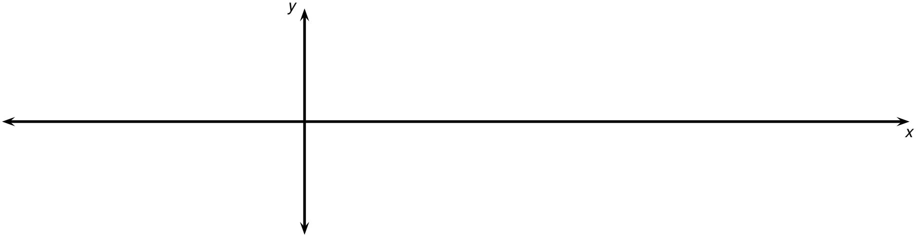


The graph of $\sin x$ is 2π-periodic so the rest of the graph can be inferred from the interval $[0, 2\pi]$.

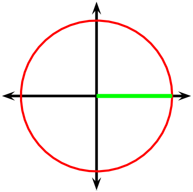
Graph of $\cos x$



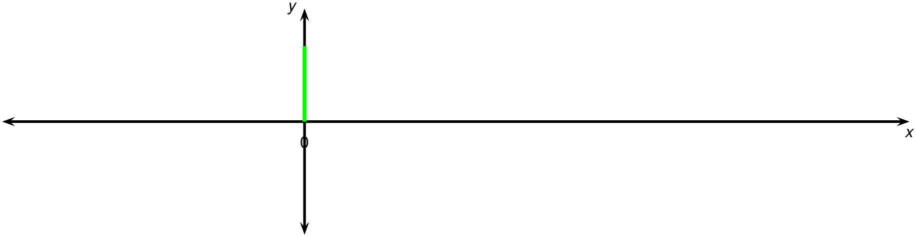
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	**?**								



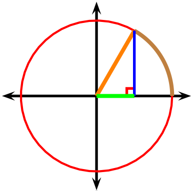
Graph of $\cos x$



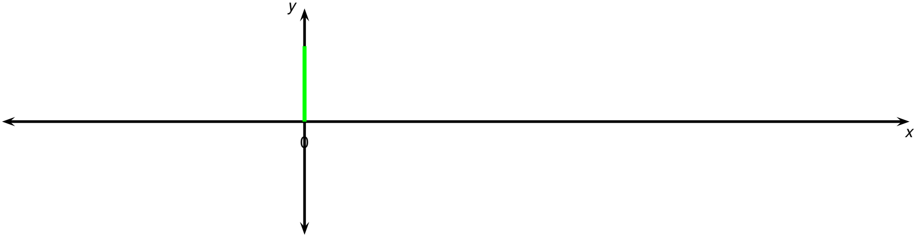
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1								



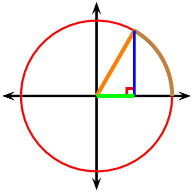
Graph of $\cos x$



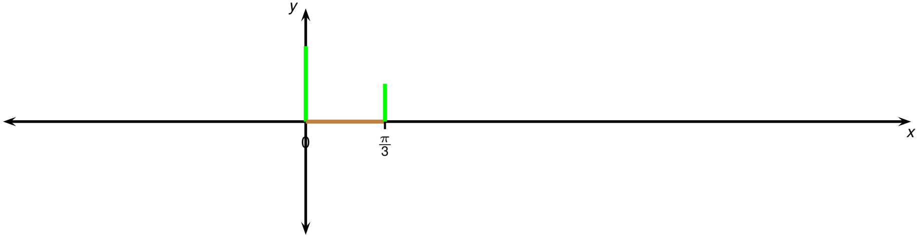
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	**?**							



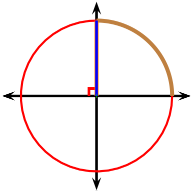
Graph of $\cos x$



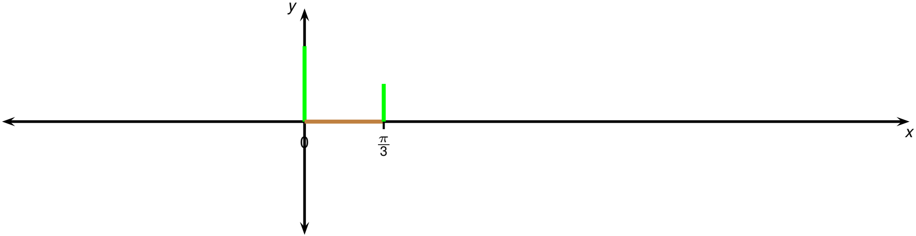
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$							



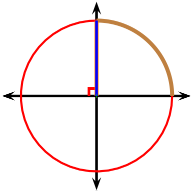
Graph of $\cos x$



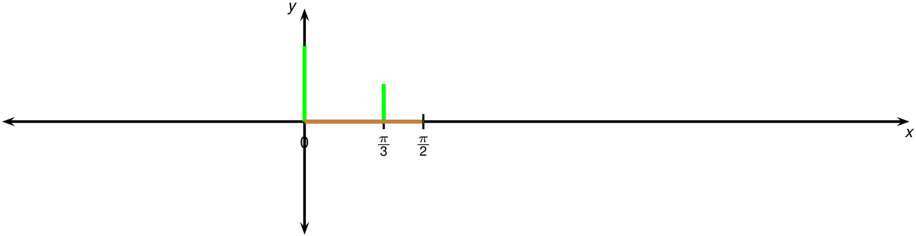
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	**?**						



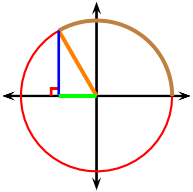
Graph of $\cos x$



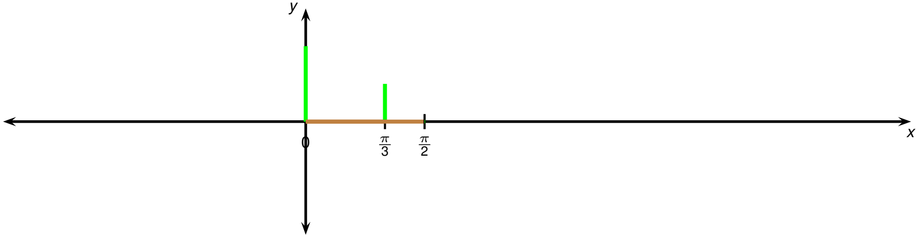
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0						



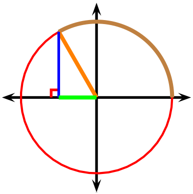
Graph of $\cos x$



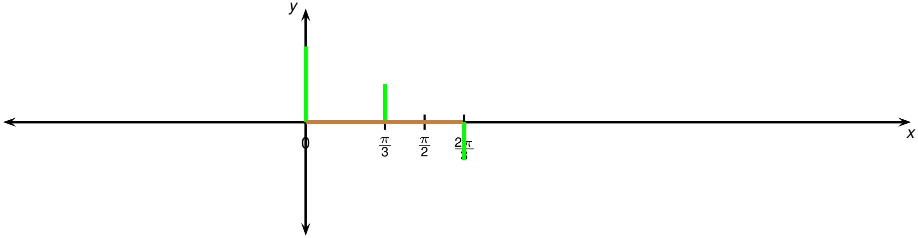
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	?					



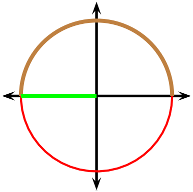
Graph of $\cos x$



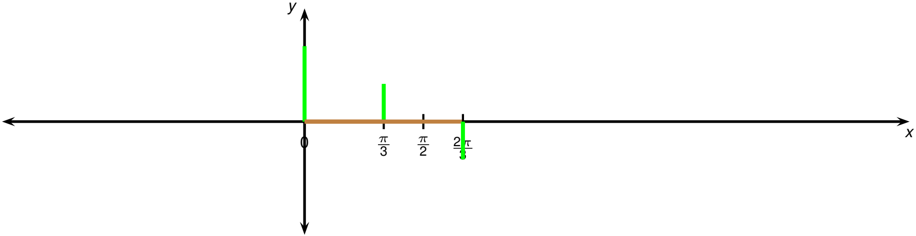
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$					



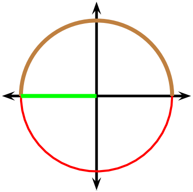
Graph of $\cos x$



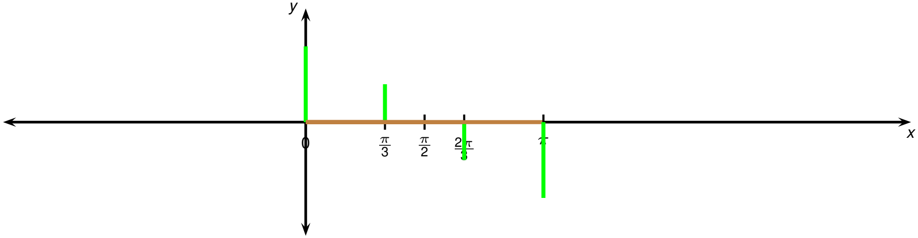
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	**?**				



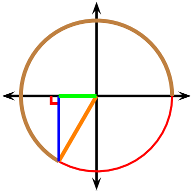
Graph of $\cos x$



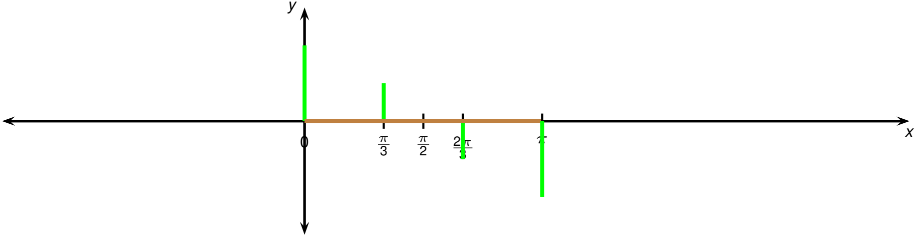
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1				



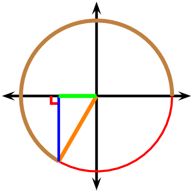
Graph of $\cos x$



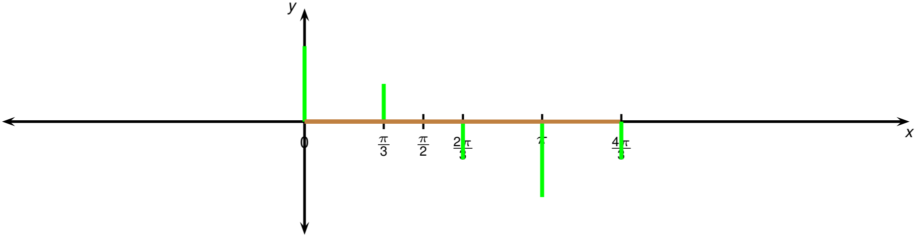
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	?			



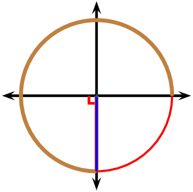
Graph of $\cos x$



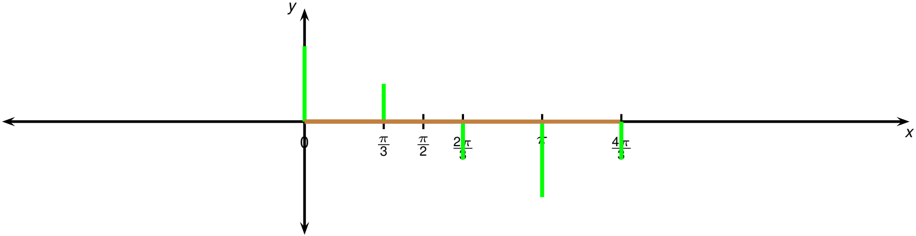
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$			



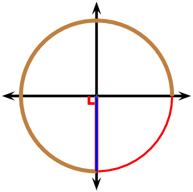
Graph of $\cos x$



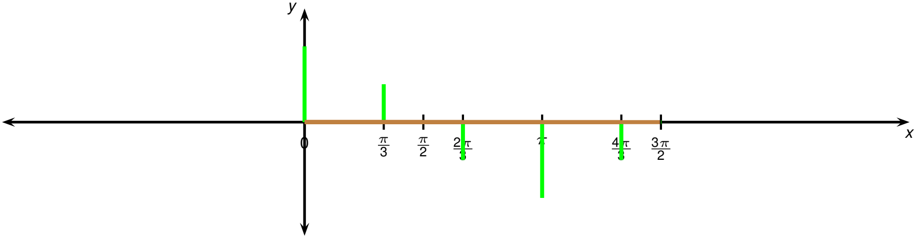
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	**?**		



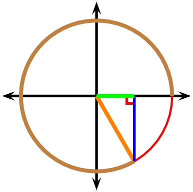
Graph of $\cos x$



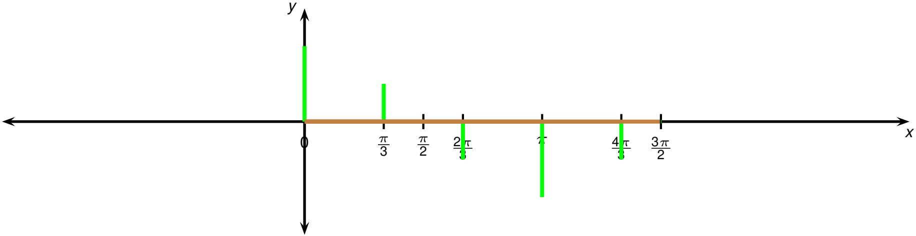
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0		



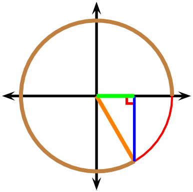
Graph of $\cos x$



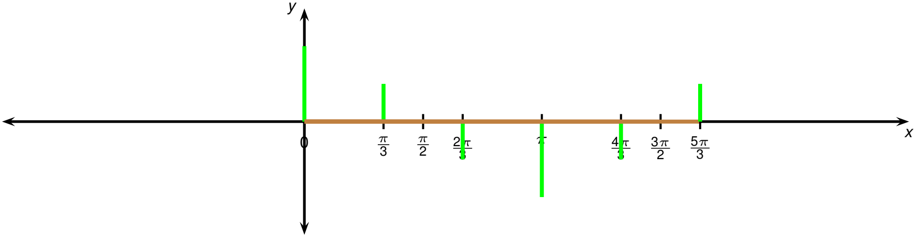
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	**?**	



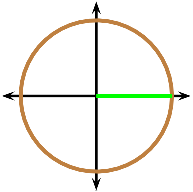
Graph of $\cos x$



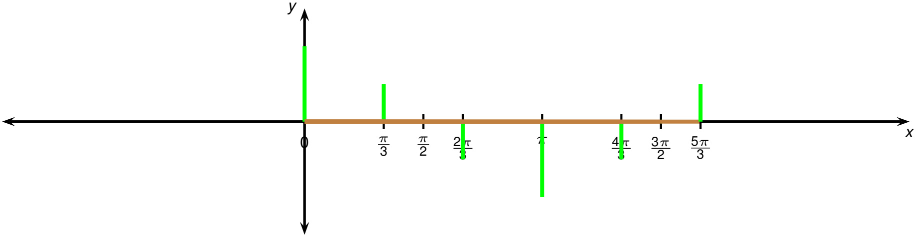
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	



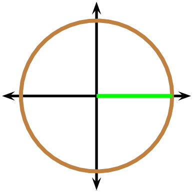
Graph of $\cos x$



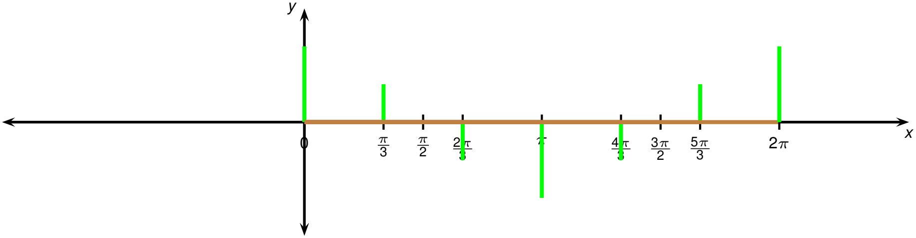
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	**?**



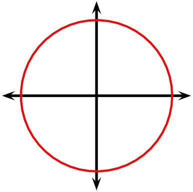
Graph of $\cos x$



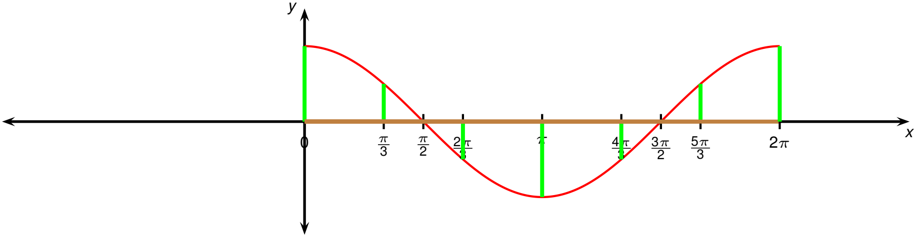
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0



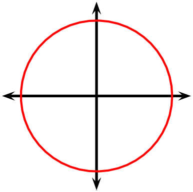
Graph of $\cos x$



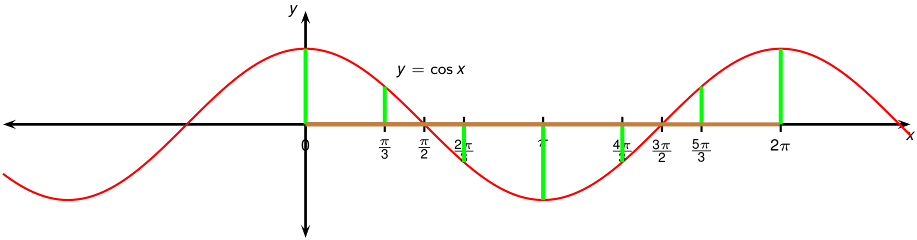
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0



Graph of $\cos x$

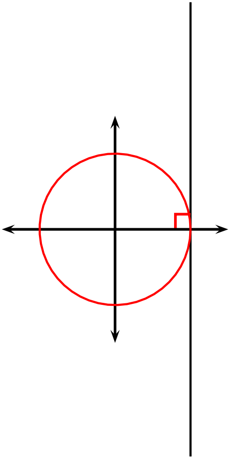


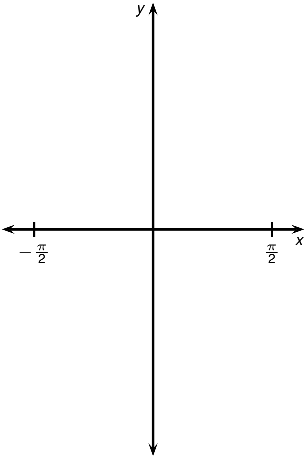
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0



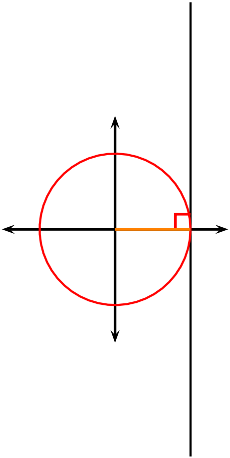
The graph of $\cos x$ is 2π-periodic so the rest of the graph can be inferred from the interval $[0, 2\pi]$.

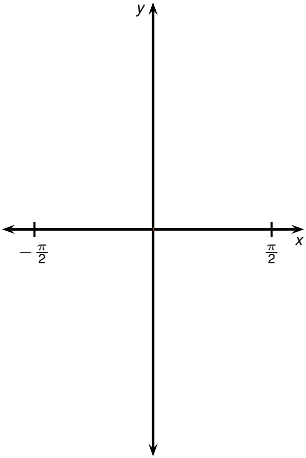
Graph of $\tan x$



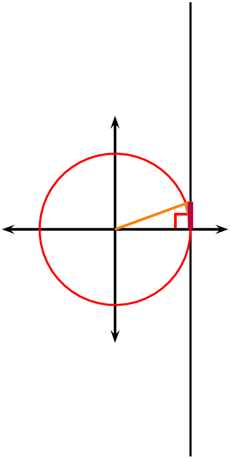


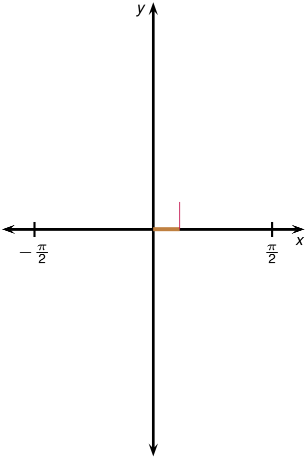
Graph of $\tan x$



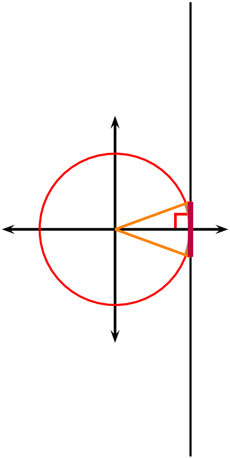


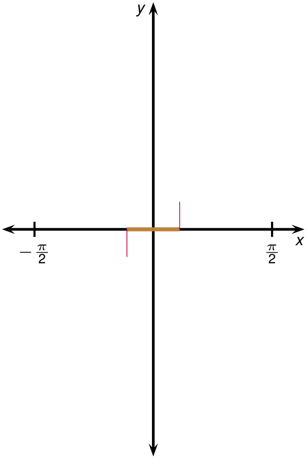
Graph of $\tan x$



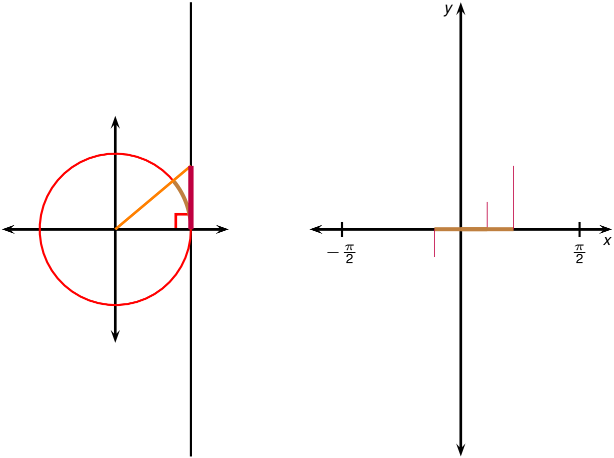


Graph of $\tan x$

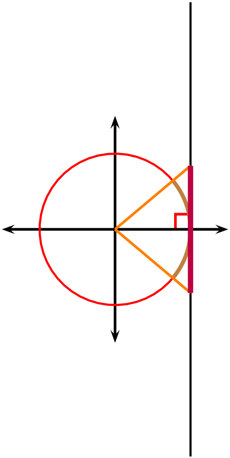


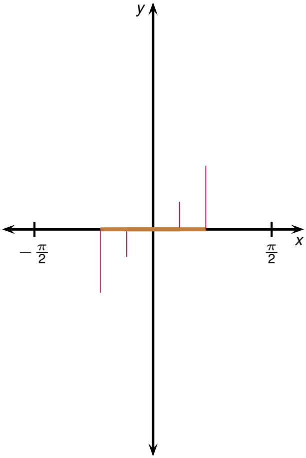


Graph of tan x

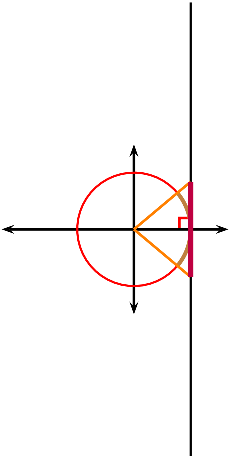


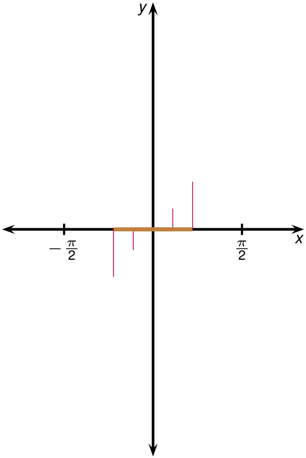
Graph of $\tan x$



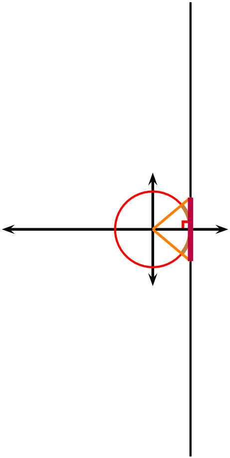


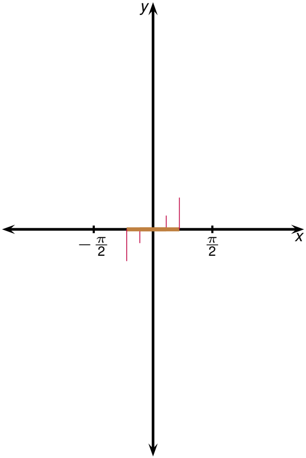
Graph of $\tan x$



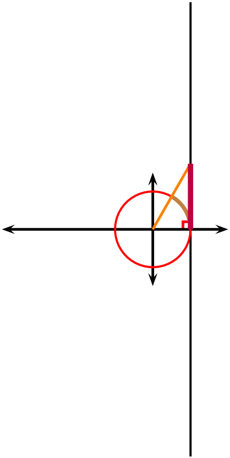


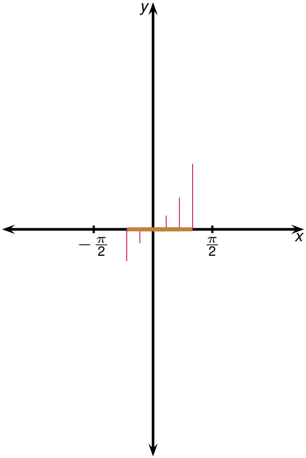
Graph of $\tan x$



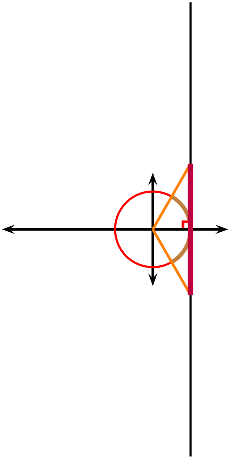


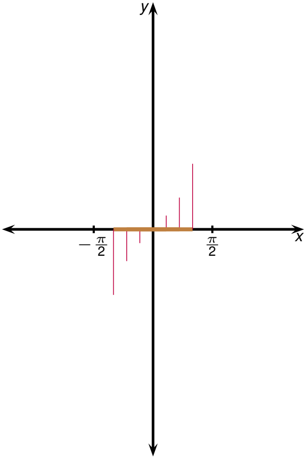
Graph of $\tan x$



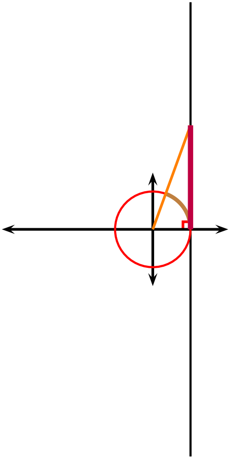


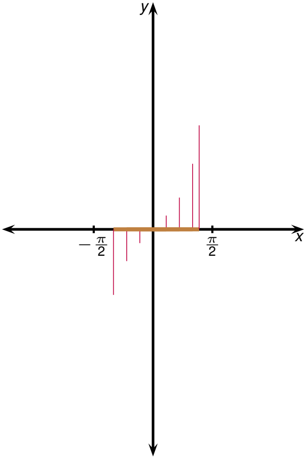
Graph of $\tan x$



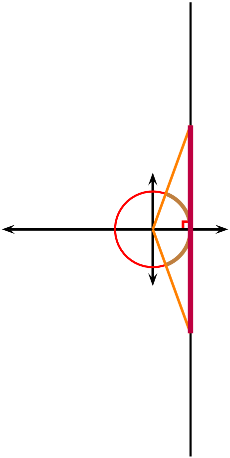


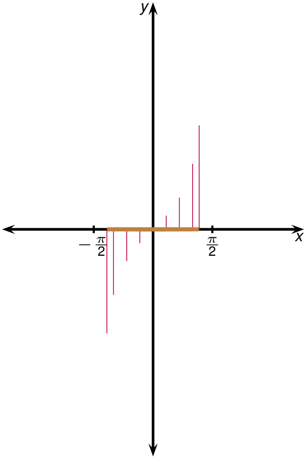
Graph of $\tan x$



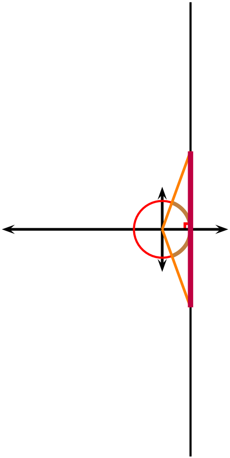


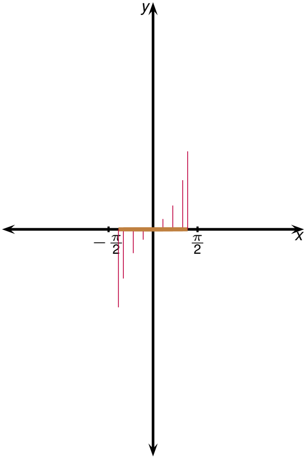
Graph of $\tan x$



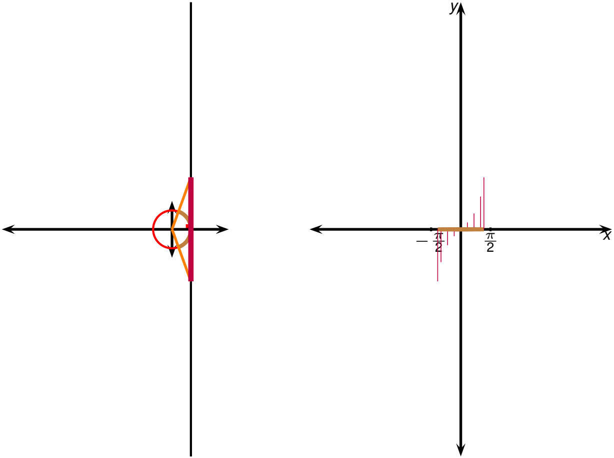


Graph of $\tan x$





Graph of $\tan x$



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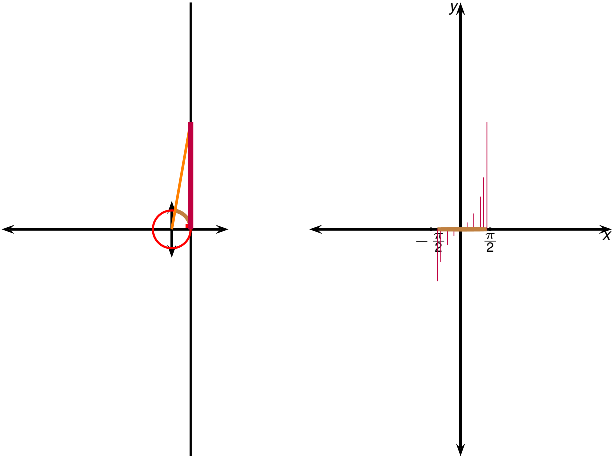

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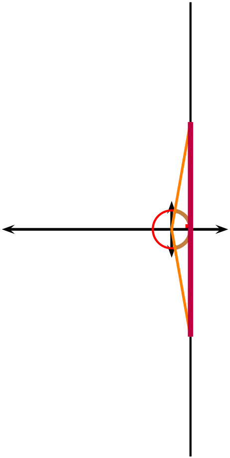
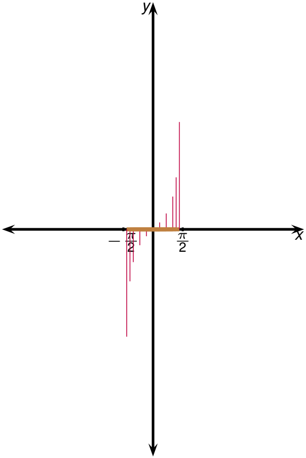
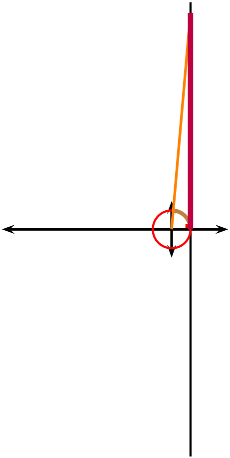
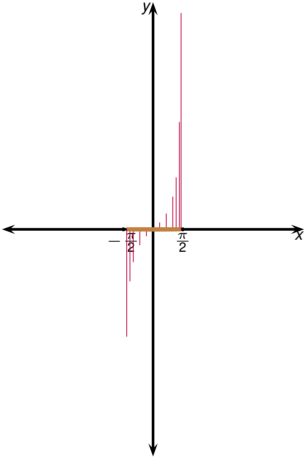




Graph of $\tan x$





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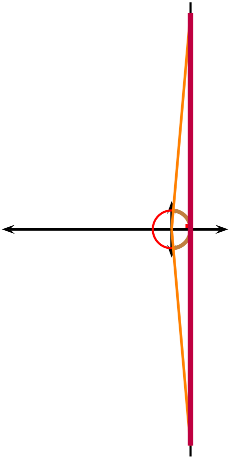

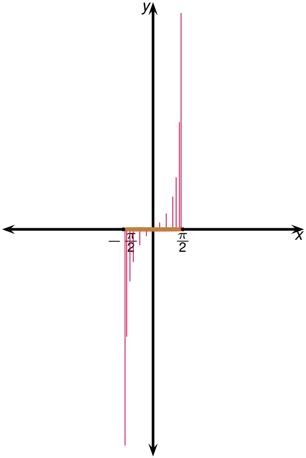

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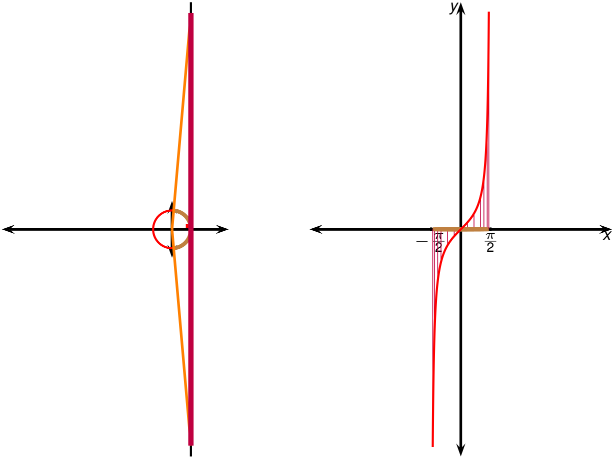

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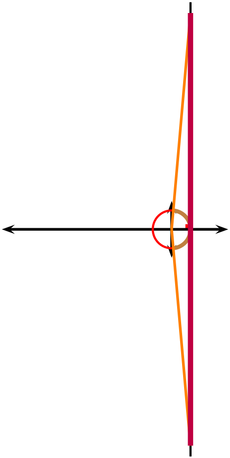
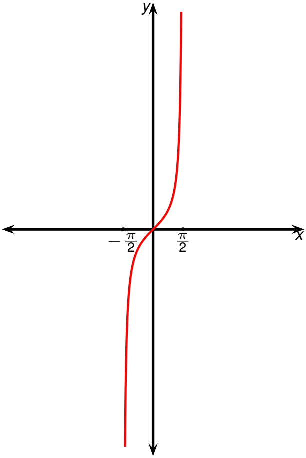
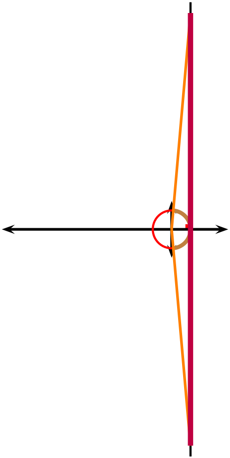
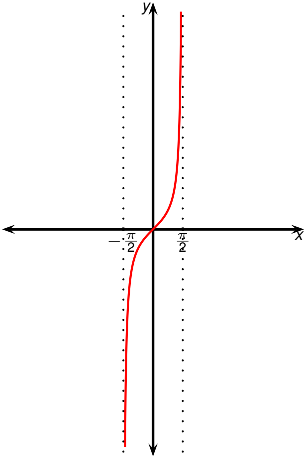




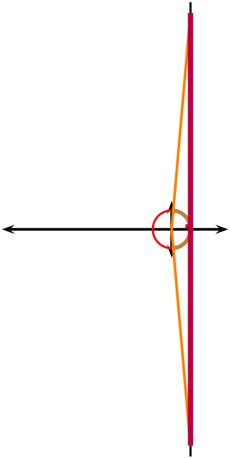
Graph of $\tan x$

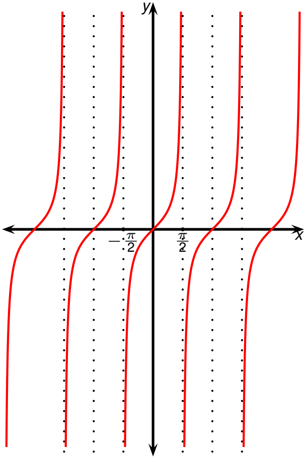




Near $\pm\frac{\pi}{2}$ the graph of $\tan x$ approaches $\pm\infty$.

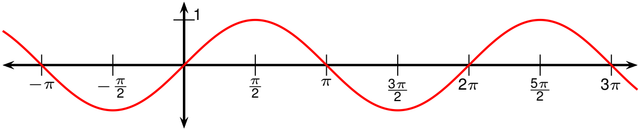
Graph of $\tan x$



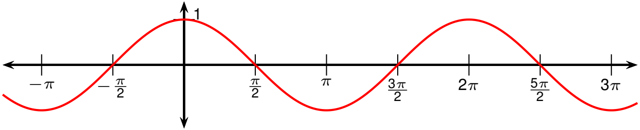


Near $\pm\frac{\pi}{2}$ the graph of $\tan x$ approaches $\pm\infty$. The graph of $\tan x$ is π-periodic so the rest of the graph can be inferred from the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Graphs of the Trigonometric Functions

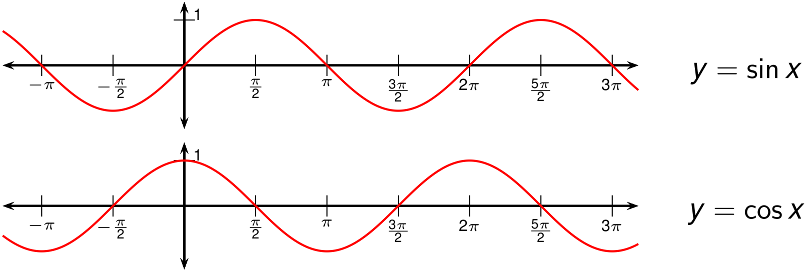


$y = \sin x$



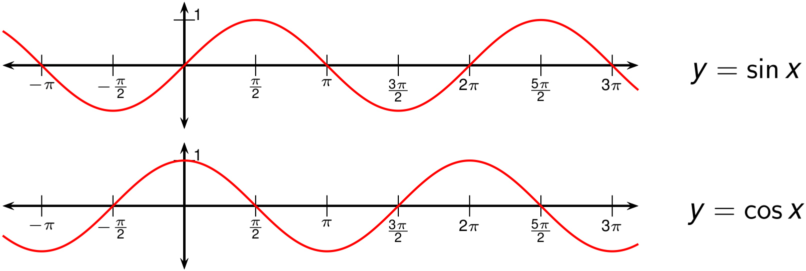
$y = \cos x$

Graphs of the Trigonometric Functions



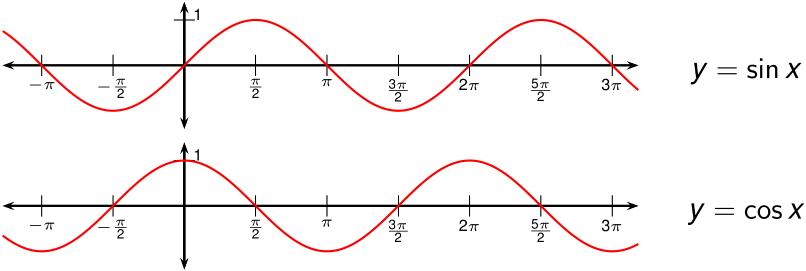
- $\sin x$ has zeroes at $n\pi$ for all integers n.

Graphs of the Trigonometric Functions



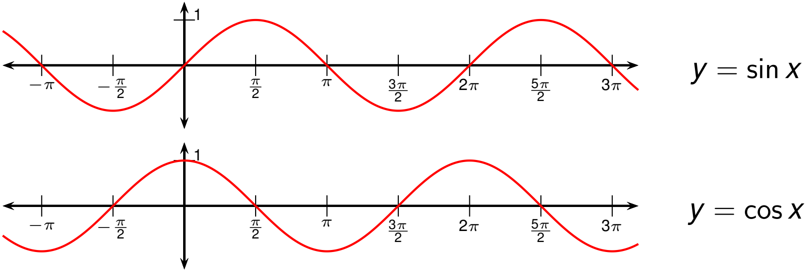
- $\sin x$ has zeroes at $n\pi$ for all integers n.
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.

Graphs of the Trigonometric Functions



- $\sin x$ has zeroes at $n\pi$ for all integers n.
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.
- $-1 \leq \sin x \leq 1$.

Graphs of the Trigonometric Functions



- $\sin x$ has zeroes at $n\pi$ for all integers n.
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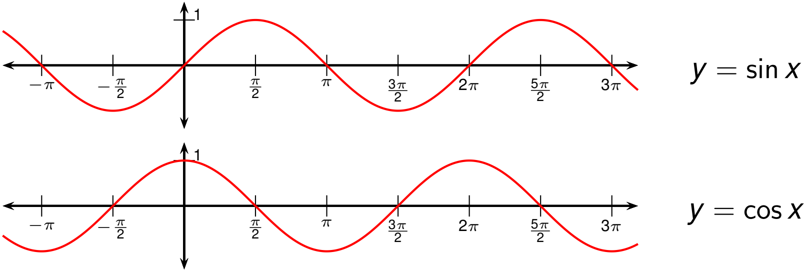
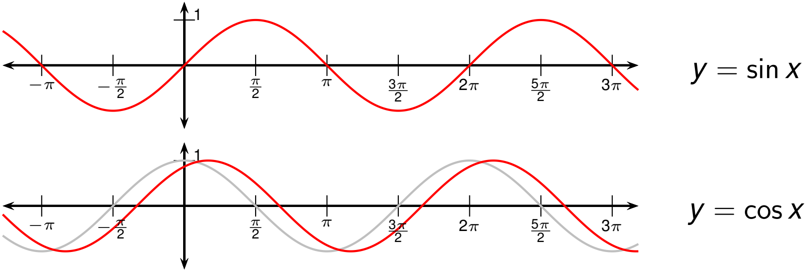
- $-1 \leq \sin x \leq 1$.
- $-1 \leq \cos x \leq 1$.

Graphs of the Trigonometric Functions



- $\sin x$ has zeroes at $n\pi$ for all integers n.
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.
- $-1 \leq \sin x \leq 1$.
- $-1 \leq \cos x \leq 1$.
- If we translate the graph of $\cos x$ by $\frac{\pi}{2}$ units to the right

Graphs of the Trigonometric Functions



- $\sin x$ has zeroes at $n\pi$ for all integers n.
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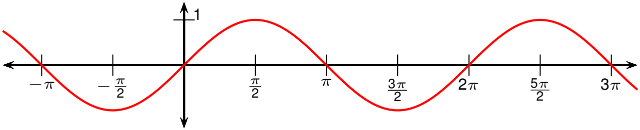
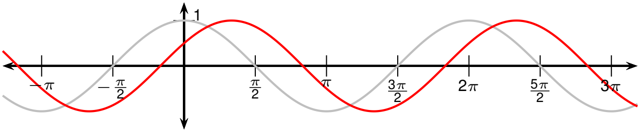
Graphs of the Trigonometric Functions



$y = \sin x$



$y = \cos x$

- $\sin x$ has zeroes at $n\pi$ for all integers n.
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.
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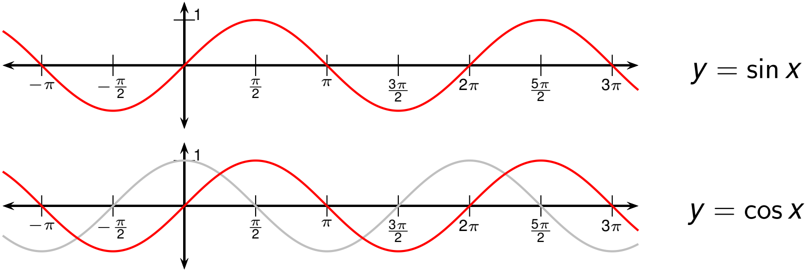
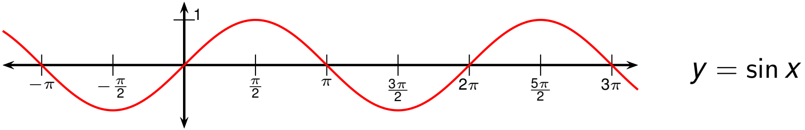
- If we translate the graph of $\cos x$ by $\frac{\pi}{2}$ units to the right

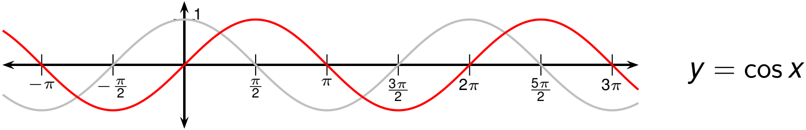
Graphs of the Trigonometric Functions



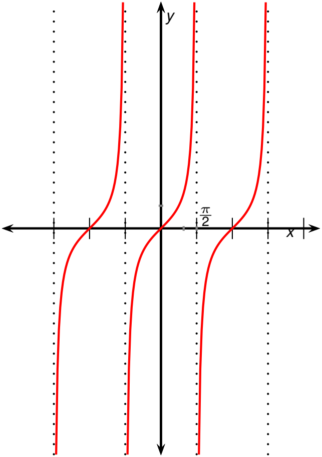
- $\sin x$ has zeroes at $n\pi$ for all integers n.
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.
- $-1 \leq \sin x \leq 1$.
- $-1 \leq \cos x \leq 1$.
- If we translate the graph of $\cos x$ by $\frac{\pi}{2}$ units to the right we get the graph of $\sin x$.

Graphs of the Trigonometric Functions

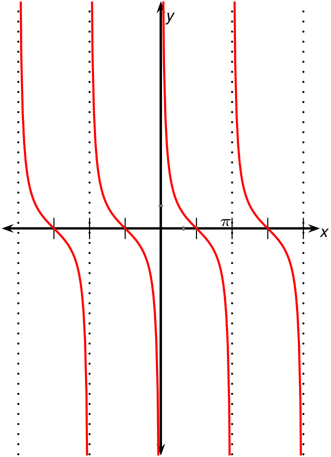




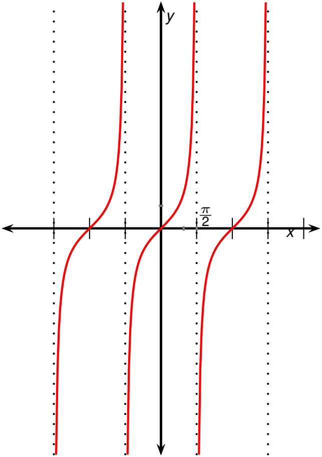
- $\sin x$ has zeroes at $n\pi$ for all integers n.
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.
- $-1 \leq \sin x \leq 1$.
- $-1 \leq \cos x \leq 1$.
- If we translate the graph of $\cos x$ by $\frac{\pi}{2}$ units to the right we get the graph of $\sin x$. This is a consequence of $\cos\left(x - \frac{\pi}{2}\right) = \sin x$.



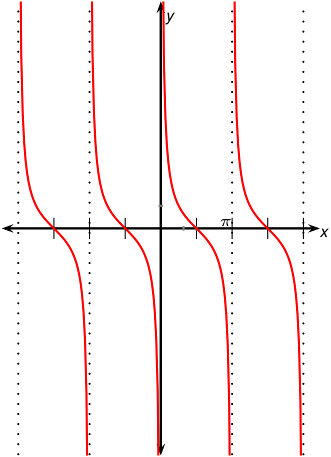
$y = \tan x$



$y = \cot x$

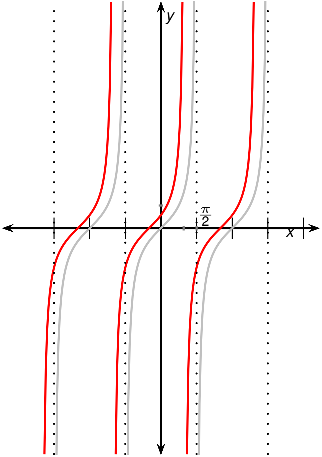


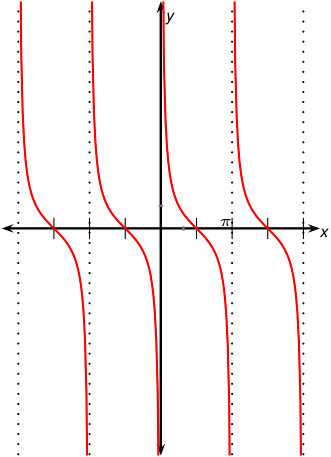
$y = \tan x$



$y = \cot x$

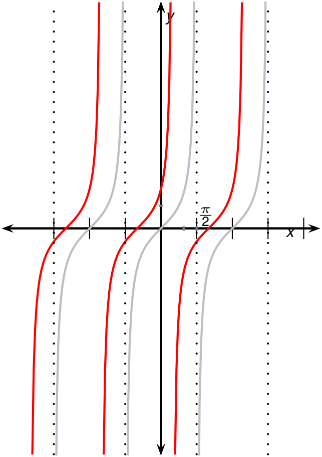
If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis

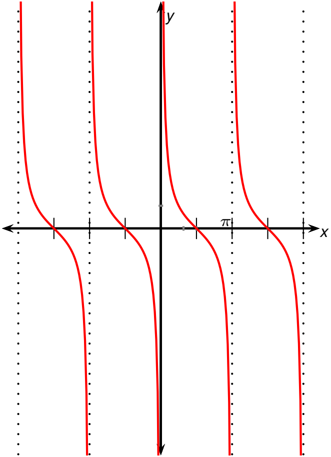




$y = \tan x$ $\quad$ $y = \cot x$

If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis





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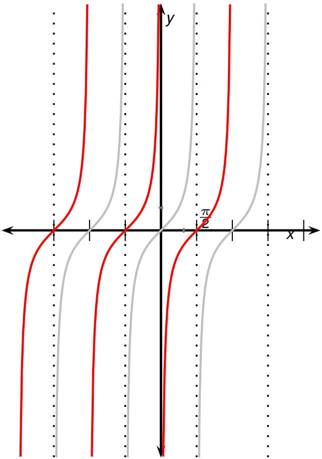
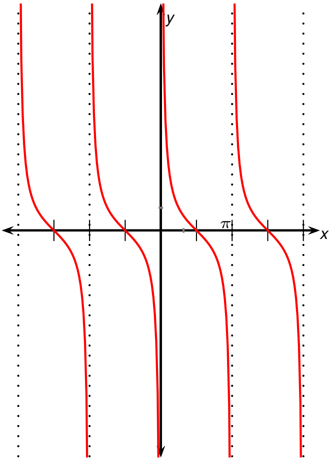
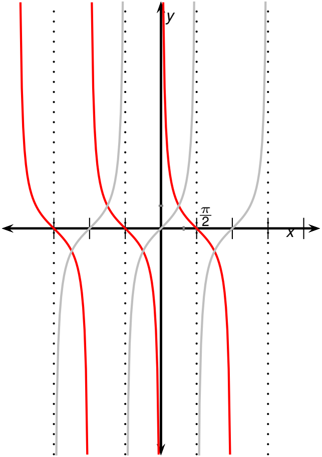


$y = \tan x$

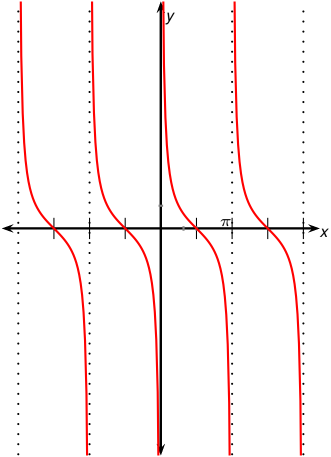


$y = \cot x$

If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis

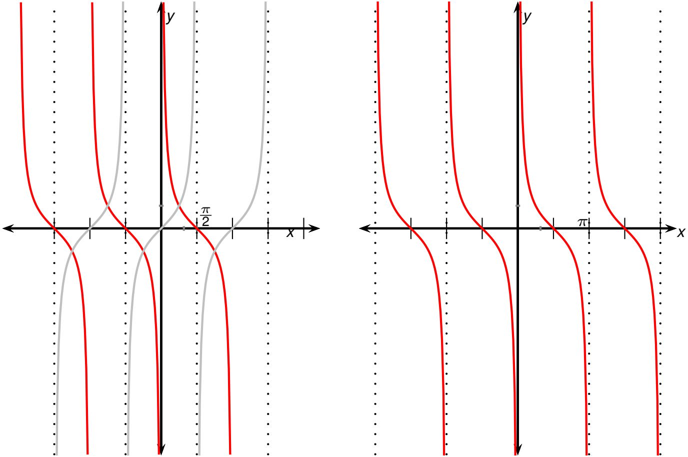


$y = \tan x$



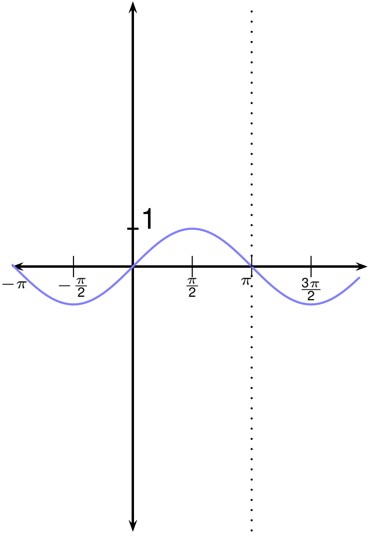
$y = \cot x$

If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis, we get the graph of $\cot x$.

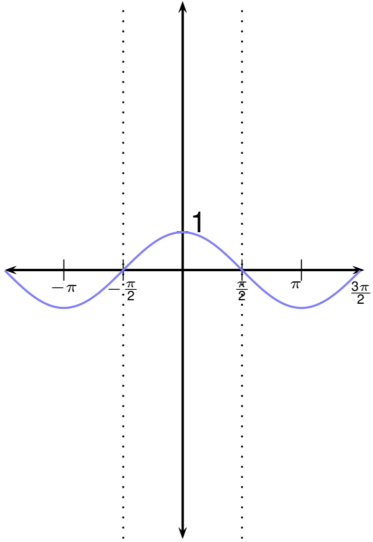


$y = \tan x$ $\qquad\qquad$ $y = \cot x$

If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis, we get the graph of $\cot x$. This follows from $\tan\left(x \pm \frac{\pi}{2}\right) = -\cot x$.


1
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$

$y = \csc x$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$-\frac{\pi}{2}$
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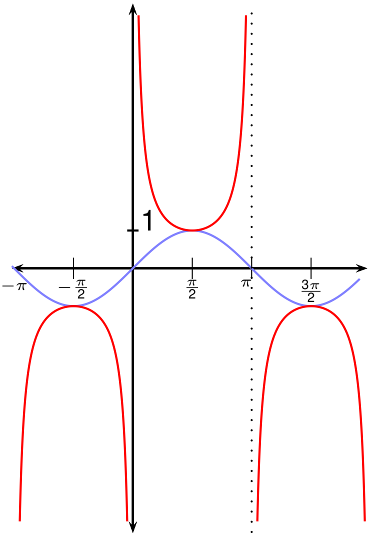
π
$\frac{3\pi}{2}$

$y = \sec x$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π
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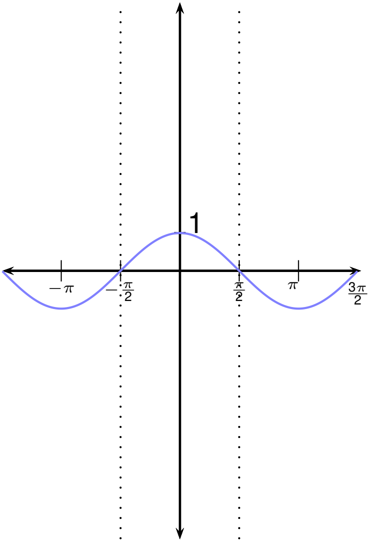
$y = \csc x$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π
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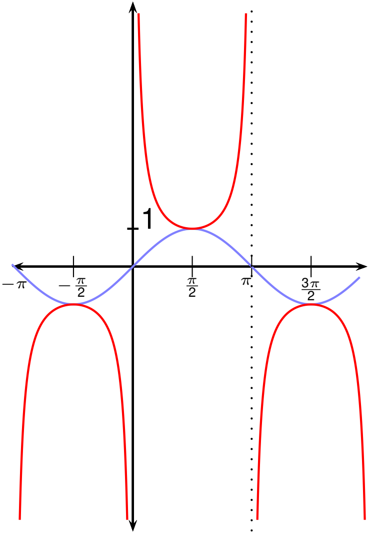
$y = \sec x$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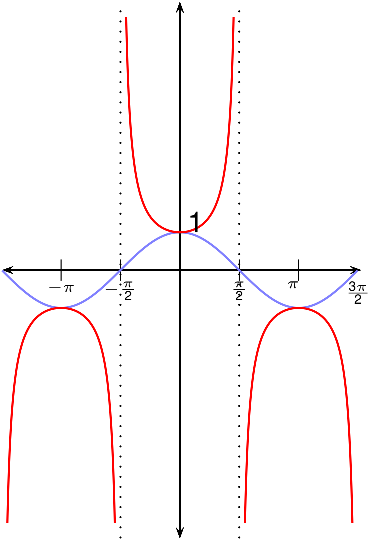
$y = \csc x$


1
$-\pi$
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π
$\frac{3\pi}{2}$

$y = \sec x$