# Calculus III
# Homework on Lecture 10

1. Recall that the directional derivative $D_{\mathbf{u}}$ in the direction $\mathbf{u}$ is defined as the covariant derivative $D_{\mathbf{u}} f = \nabla_{\frac{\mathbf{u}}{|\mathbf{u}|}} f$. Find the covariant derivative $\nabla_{\mathbf{u}} f$ and the directional derivative $D_{\mathbf{u}} f$ at the indicated point.

   (a) $f(x, y) = x^2 + y^2$, $\mathbf{u} = (1, 2)$, $(x, y) = P = (2, 1)$.

   (b) $f(x, y) = e^{x+y}$, $\mathbf{u} = (1, 1)$, $(x, y) = P = (0, 0)$.

   (c) $f(x, y, z) = \ln \sqrt{x^2 + y^2 + z^2}$, $\mathbf{u} = (1, -1, 1)$, $(x, y, z) = P = (1, 1, 1)$.

   (d) $f(x, y, z) = \ln \sqrt{x^2 - 2y^2 + z^2}$, $\mathbf{u} = (1, -1, 2)$, $(x, y, z) = (1, 1, 2)$

   (e) $f(x, y, z) = xyz$, $\mathbf{u} = (-1, -2, 3)$, $(x, y, z) = (1, 1, 1)$.

2. (a) Let the variables $b, c, x_1, x_2$ be related via $b = -x_1 - x_2$ and $c = x_1 x_2$.

   i. Express the differential operators $\frac{\partial}{\partial c}$ and $\frac{\partial}{\partial b}$ via $\frac{\partial}{\partial x_1}$ and $\frac{\partial}{\partial x_2}$.

   ii. Express the differential operators $\frac{\partial}{\partial x_1}$ and $\frac{\partial}{\partial x_2}$ via $\frac{\partial}{\partial c}$ and $\frac{\partial}{\partial b}$.

   (b) Let $x, y, z$ and $\rho, \phi, \theta$ be related via the usual spherical coordinates equations i.e., $x =$

   i. Express the differential operators $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ via $\frac{\partial}{\partial \rho}, \frac{\partial}{\partial \phi}, \frac{\partial}{\partial \theta}$.

   ii. Express the differential operators $\frac{\partial}{\partial \rho}, \frac{\partial}{\partial \phi}, \frac{\partial}{\partial \theta}$ via $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$.

   iii. Express the Laplace differential operator $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ via $\frac{\partial}{\partial \rho}, \frac{\partial}{\partial \phi}, \frac{\partial}{\partial \theta}$ (in other words, write the 3 dimensional Laplace operator in spherical coordinates).