Calculus I
Homework Antiderivatives and Integrals
Lecture 21

1. Find all antiderivatives of the functions.

   (a) $f(x) = \sqrt{3} + \pi^2$.

   (b) $f(x) = x - 5$.

   (c) $f(x) = x^2 - 2x + 6$.

   (d) $f(x) = \dfrac{x(x+1)}{2}$.

   (e) $f(x) = x(x+1)(2x+1)$.

   (f) $f(x) = 7x^{\frac{2}{7}} + x^{-\frac{4}{7}}$.

   (g) $f(x) = x^{2.4} - 2x^{\sqrt{3}-1}$.

   (h) $f(x) = \dfrac{8}{x^7}$.

   (i) $f(x) = \dfrac{x+1}{x^3}$.

   (j) $f(x) = \dfrac{1}{x}$.

   (k) $f(x) = \dfrac{x^2+1}{x}$.

   (l) $f(x) = \dfrac{5 - 4x^3 + 2x^6}{x^4}$.

   (m) $g(x) = \dfrac{1 + \sqrt{x} + x}{\sqrt{x^3}}$.

   (n) $f(t) = 3\sin t - 4\cos t$.

   (o) $f(\theta) = \sec^2\theta$.

   (p) $f(\theta) = \csc^2\theta$.

   (q) $f(t) = \sec t \tan t + \csc t \cot t$.

   (r) $f(x) = \dfrac{2 + x\cos x}{x}$.

2. Verify by differentiation that the formula is correct.

   (a) $\displaystyle\int \sqrt{1+x^2}\,\mathrm{d}x = \frac{1}{2}\left(x\sqrt{1+x^2} + \ln\left(x + \sqrt{1+x^2}\right)\right) + C$.

   (b) $\displaystyle\int \sin^2 x\,\mathrm{d}x = -\frac{1}{4}\sin(2x) + \frac{1}{2}x + C$.

   (c) $\displaystyle\int \sin^3 x\,\mathrm{d}x = \frac{1}{3}\cos^3 x - \cos x + C$.

   (d) $\displaystyle\int \frac{x}{\sqrt{1+x}}\,\mathrm{d}x = \frac{2}{3}(x-2)\sqrt{1+x} + C$

3. Evaluate the integral (definite or indefinite).

   (a) $\displaystyle\int_{-2}^{3} \left(x^2 - 1\right)\,\mathrm{d}x$.

   (b) $\displaystyle\int_{1}^{2} \left(4x^3 + 3x^2 + 2x + 1\right)\,\mathrm{d}x$.

   (c) $\displaystyle\int_{0}^{2} (x-1)(x^2+1)\,\mathrm{d}x$.

   (d) $\displaystyle\int_{-1}^{1} \left(\frac{x(x+1)}{2}\right)^2\,\mathrm{d}x$.

   (e) $\displaystyle\int_{0}^{1} (1+x^2)^3\,dx$.

   (f) $\displaystyle\int_{1}^{2} \left(\frac{1}{x} - \frac{4}{x^2}\right)\,\mathrm{d}x$.

   (g) $\displaystyle\int_{1}^{4} \sqrt{x}(1+x)\,\mathrm{d}x$.

   (h) $\displaystyle\int_{1}^{4} \sqrt{\frac{6}{x}}\,\mathrm{d}x$.

   (i) $\displaystyle\int_{1}^{4} \frac{\frac{1}{\sqrt{x}} + 1 + x}{\sqrt{x}}\,\mathrm{d}x$.

   (j) $\displaystyle\int_{1}^{8} \frac{1+x}{\sqrt[3]{x}}\,\mathrm{d}x$.

   (k) $\displaystyle\int_{1}^{64} \frac{\frac{1}{\sqrt[3]{x}} + \sqrt[3]{x}}{\sqrt{x}}\,\mathrm{d}x$.

   (l) $\displaystyle\int_{0}^{1} \left(\sqrt[5]{x^6} + \sqrt[6]{x^5}\right)\,\mathrm{d}x$.

   (m) $\displaystyle\int_{1}^{2} \left(x + \frac{1}{x}\right)^2\,\mathrm{d}x$.

   (n) $\displaystyle\int_{1}^{2} \left(x + \frac{1}{x}\right)^3\,\mathrm{d}x$.

   (o) $\displaystyle\int_{1}^{2} \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2\,\mathrm{d}x$.

   (p) $\displaystyle\int_{1}^{2} \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^3\,\mathrm{d}x$.

   (q) $\displaystyle\int_{0}^{2} |x - 1|\,\mathrm{d}x$.

   (r) $\displaystyle\int_{0}^{1} \left|x - \frac{1}{2}\right|\,\mathrm{d}x$.

   (s) $\displaystyle\int_{-1}^{1} (x - 3|x|)\,\mathrm{d}x$.

   (t) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc^2\theta\,\mathrm{d}\theta$.

   (u) $\displaystyle\int_{0}^{\frac{\pi}{4}} \frac{1 - \cos^2\theta}{\cos^2\theta}\,\mathrm{d}\theta$.

(v) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sin^2 \theta}{\cos^2 \theta}\mathrm{d}\theta.$

(x) $\displaystyle\int_0^{\frac{\pi}{3}} \frac{\sin \theta + \sin \theta \tan^2 \theta}{\sec^2 \theta}\mathrm{d}\theta.$

(z) $\displaystyle\int_0^{\pi} |\sin x|\mathrm{d}x.$

(w) $\displaystyle\int_0^{\frac{\pi}{4}} \tan^2 \theta\mathrm{d}\theta.$

(y) $\displaystyle\int_0^{\pi} (\sin \theta - \cos \theta)\mathrm{d}\theta.$

4. Integrate (definite or indefinite).

(a) $\displaystyle\int_1^8 \frac{t - t^{\frac{1}{3}} + 2}{t^{\frac{4}{3}}}\mathrm{d}t$.

(b) $\displaystyle\int_1^4 \left(x + \sqrt{x}\right)^2 dx$.

(c) $\displaystyle\int \frac{\sqrt[3]{x} - x^{\frac{1}{2}} + 1}{x}\mathrm{d}x.$

(d) $\displaystyle\int \frac{\sqrt[3]{x} - 1}{x}\mathrm{d}x.$