Precalculus
Homework Lecture 4

1. Prove the trigonometry identities.

   (a) $\sin\theta\cot\theta = \cos\theta$.

   (b) $(\sin\theta + \cos\theta)^2 = 1 + \sin(2\theta)$.

   (c) $\sec\theta - \cos\theta = \tan\theta\sin\theta$.

   (d) $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$.

   (e) $\cot^2\theta + \sec^2\theta = \tan^2\theta + \csc^2\theta$.

   (f) $2\csc(2\theta) = \sec\theta\csc\theta$.

   (g) $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$.

   (h) $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} = 2\sec^2\theta$.

   (i) $\tan\alpha + \tan\beta = \dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$.

   (j) $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$.

   (k) $\sin(3\theta) + \sin\theta = 2\sin(2\theta)\cos\theta$.

   (l) $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$.

   (m) $1 + \tan^2\theta = \sec^2\theta$.

   (n) $1 + \csc^2\theta = \cot^2\theta$.

   (o) $2\cos^2(2x) = 2\sin^4\theta + 2\cos^4\theta - \sin^2(2\theta)$.

   (p) $\dfrac{1+\tan\left(\frac{\theta}{2}\right)}{1-\tan\left(\frac{\theta}{2}\right)} = \tan\theta + \sec\theta$.