Calculus III
Homework on Lecture 6

1. Compute the tangent vector, the normal vector and the curvature at each point on the curve.

   (a) The equator of the unit sphere $\mathbf{r}(t) = (\cos t, \sin t, 0)$.

   (b) The equator of the unit sphere $\mathbf{r}(t) = (\cos t, \sin t, 0)$.

   (c) The loxodromic curve $\mathbf{r}(t) = (\cos(10t)\sin(t), \sin(10t)\sin(t), \cos(t))$. A loxodromic curve is a curve obtained by taking a straight line in the $\{(\rho, \phi, \theta) | \rho = \text{const}\}$-plane in spherical coordinates and mapping it into the x, y, z-coordinates. Loxodromic curves were used in navigation: maintaining a course on a loxodromic curve requires only keeping a constant angle with the north direction (which one approximately obtained via compass).

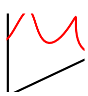
   

   (d) The ellipse $\mathbf{r}(t) = (a\cos t, b\sin t)$. Where is the curvature largest? Where smallest? Can you answer without computation, and does your answer match your computation?

   (e) The loxodromic meridian $\mathbf{r}(t) = (\sin(at)\cos(bt), \sin(at)\sin(bt), \rho\cos(at))$.

   (f) The trefoil (torus) knot $\mathbf{r} = ((R + r\sin(3t))\cos(2t), (R + r\sin(3t)\sin(2t), r\cos(3t))$.

   (g) The torus curve $\mathbf{r} = ((R + r\sin(20t))\cos(t), (R + r\sin(20t)\sin(2t), r\cos(20t))$.

   (h) The helix $\mathbf{r} = (\cos t, \sin t, t)$.

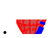
   (i) The cone curve $\mathbf{r} = (t\cos t, t\sin t, -t)$.

2. Find the length of the curve.

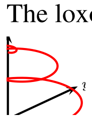
   (a) The helix $\mathbf{r} = (\cos t, \sin t, t)$, $t \in [0, 2\pi]$.

   (b) The cone curve $\mathbf{r} = (t\cos t, t\sin t, t)$, $t \in [0, 2\pi]$. 

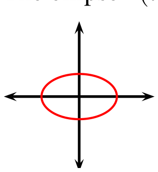
   (c) The paraboloid curve $\mathbf{r} = (t\cos t, t\sin t, t^2)$, $t \in [0, 2\pi]$. 

3. Write down the integral expressing the length of the curve. Please do not try to solve the integrals by hand. Optionally, for this exercise only, you may type the integrals in an on-line computer algebra system and see what you get.

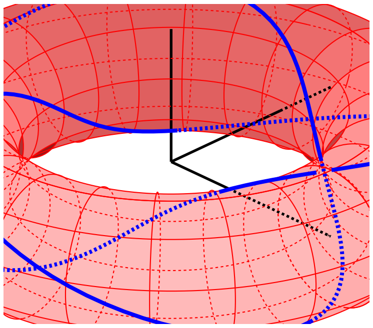
   (a) The loxodromic curve $\mathbf{r}(t) = (\cos(10t)\sin(t), \sin(10t)\sin(t), \cos(t))$ from $t = 0$ to $t = t_0$.

   (b) The ellipse $\mathbf{r}(t) = (a\cos t, b\sin t)$ from $t = 0$ to $t = t_0$.

(c) The trefoil (torus) knot $\mathbf{r} = ((3 + \sin(3t))\cos(2t), (3 + \sin(3t)\sin(2t), \cos(3t))$ from $t = 0$ to $t = 2\pi$.



(d) The torus curve $\mathbf{r} = ((R + r\sin(20t))\cos(t), (R + r\sin(20t)\sin(2t), r\cos(20t))$ from $t = 0$ to $t = 2\pi$.