Precalculus
Lecture 7
Trigonometric Graphs

Todor Milev
https://github.com/tmilev/freecalc

2020

Outline

1. Graphs of the Trigonometric Functions
   - Graphs of $\sin$ and $\cos$
   - Graph of $a\sin(bx - c)$
   - Graphs of $\tan, \cot, \sec, \csc$

Outline

1. Graphs of the Trigonometric Functions
   - Graphs of $\sin$ and $\cos$
   - Graph of $a\sin(bx - c)$
   - Graphs of $\tan, \cot, \sec, \csc$
2. Inverse Trigonometric Functions
   - Trigonometric Functions with Inverse Trig Arguments

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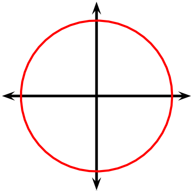
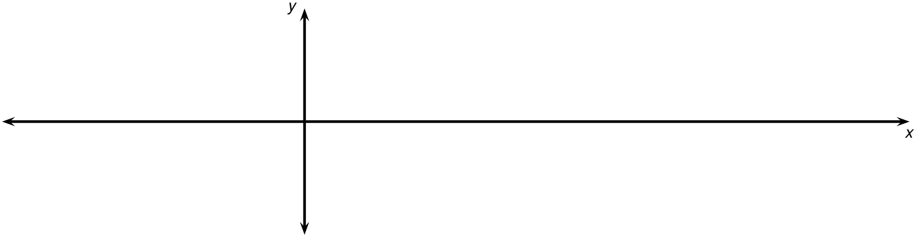
Graph of $\sin x$



x	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
$\sin x$	**?**								



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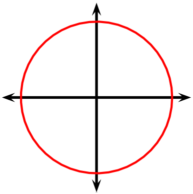

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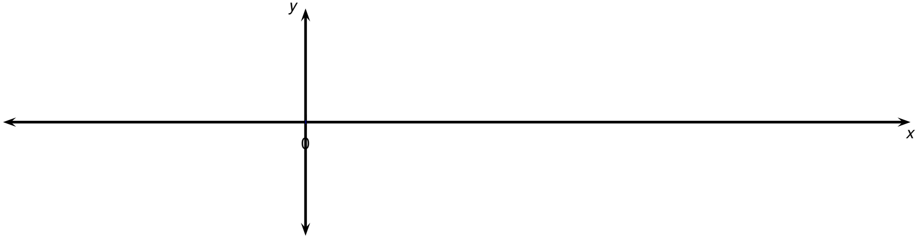
$\sin x$	0								



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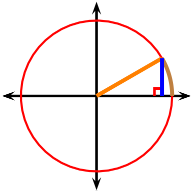

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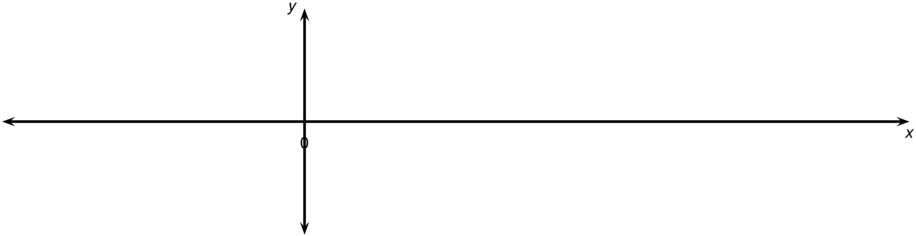

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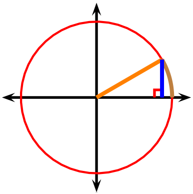

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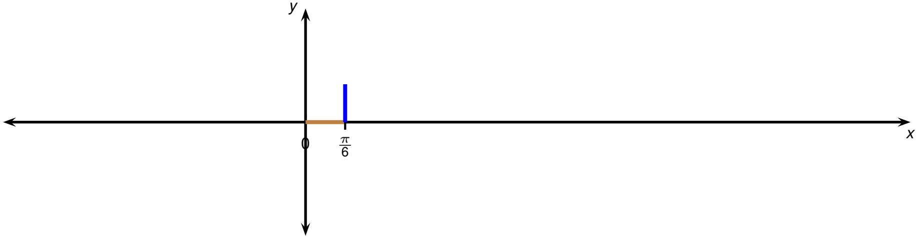
$\sin x$	0	$\frac{1}{2}$							



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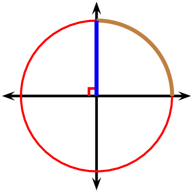

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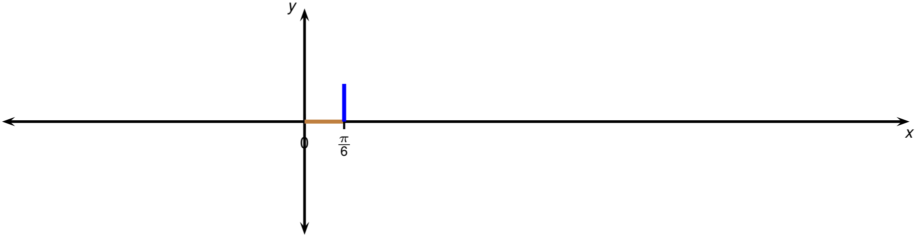

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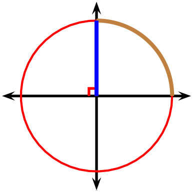

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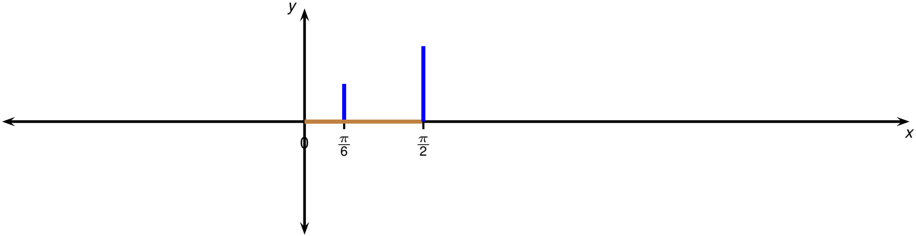

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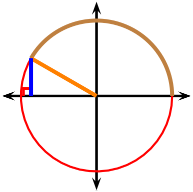

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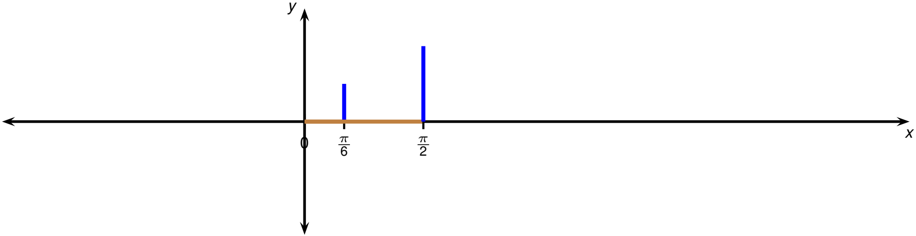

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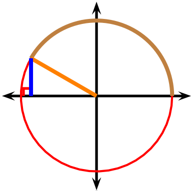

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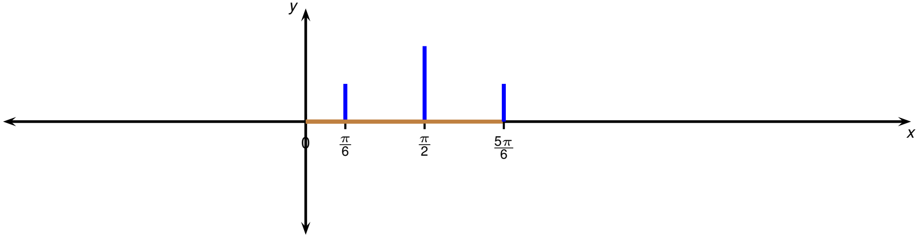

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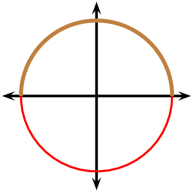

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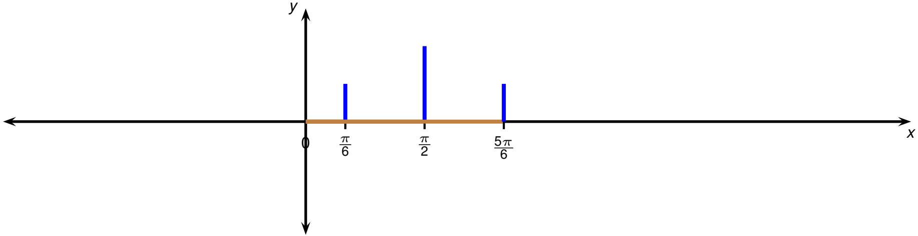

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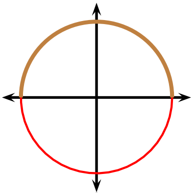

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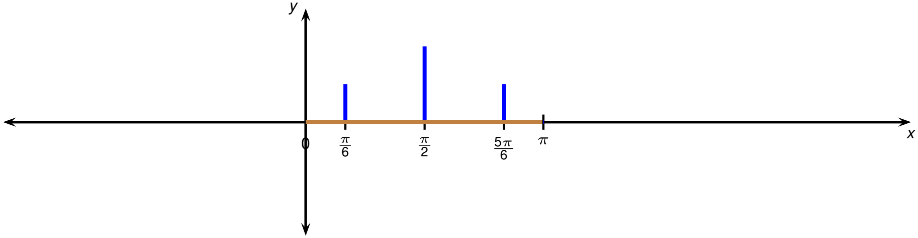

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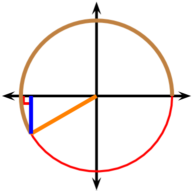

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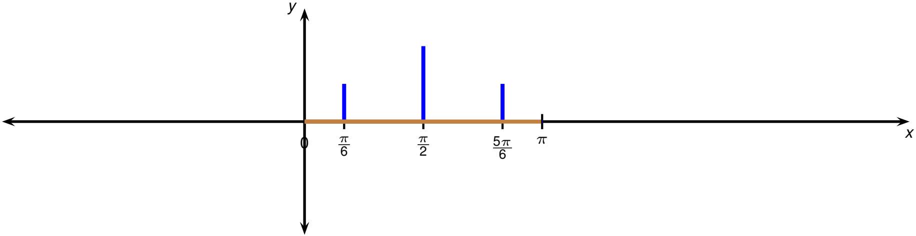

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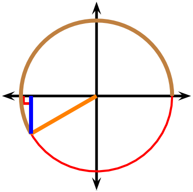

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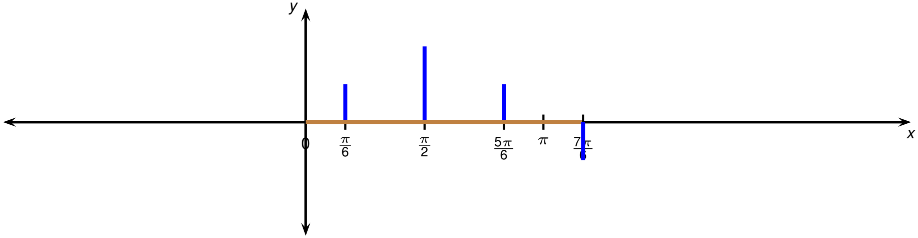

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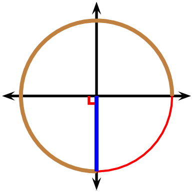

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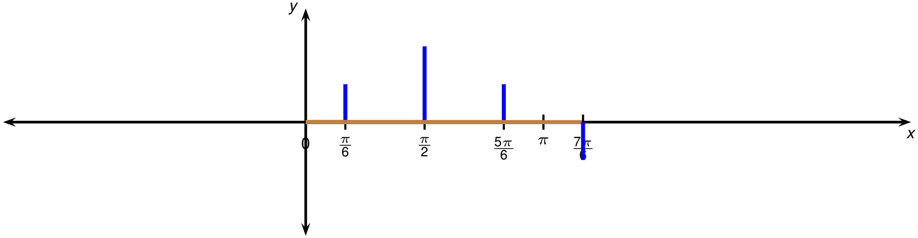

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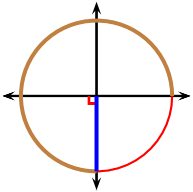

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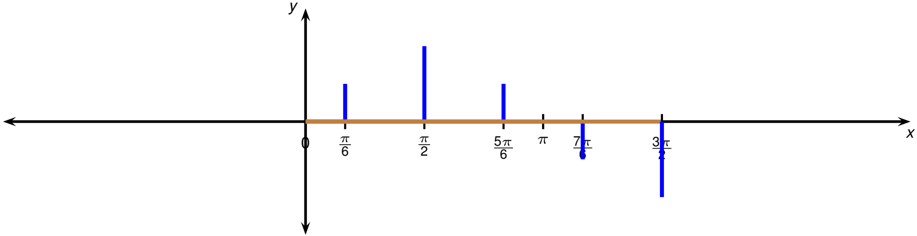

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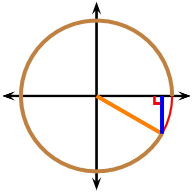

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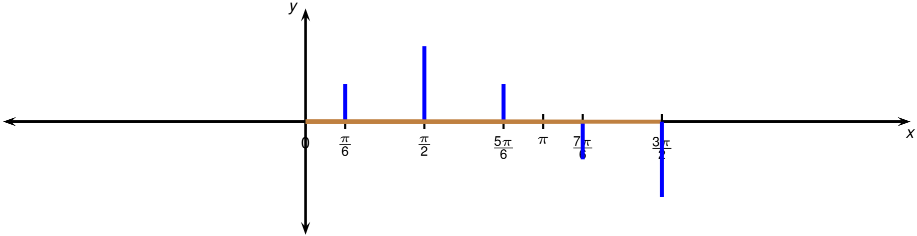

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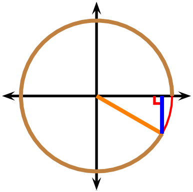

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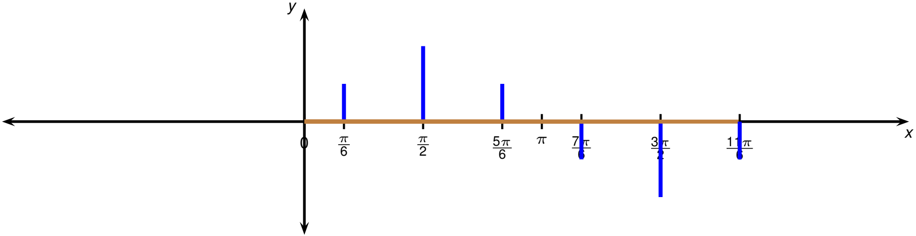

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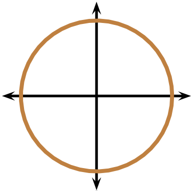

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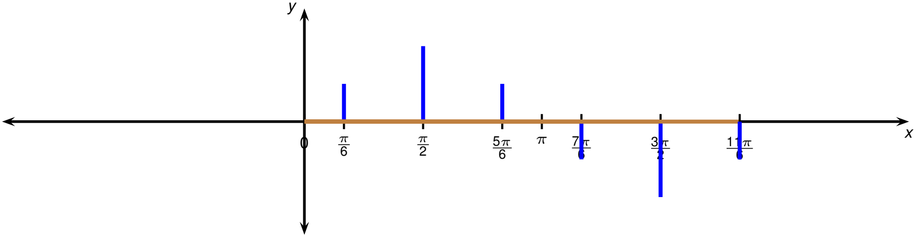

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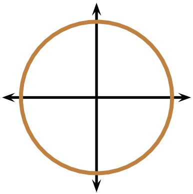

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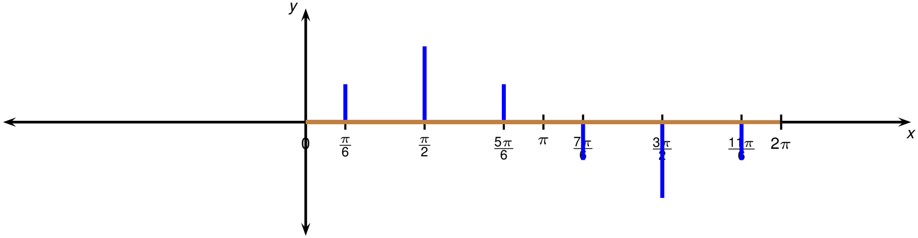

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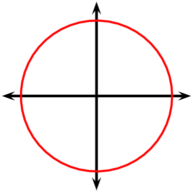

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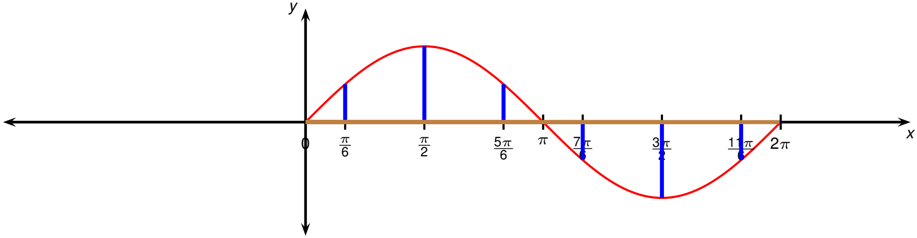

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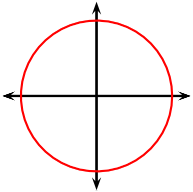

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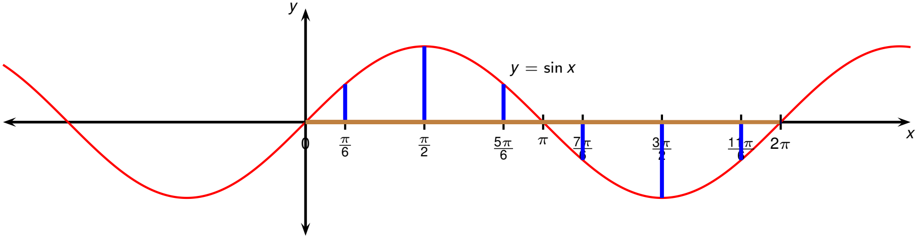
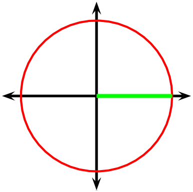
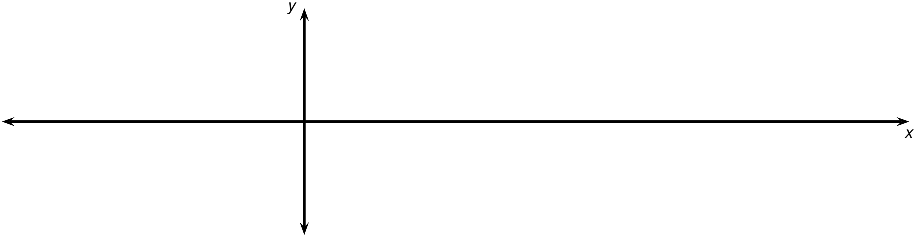


The graph of $\sin x$ is 2π-periodic so the rest of the graph can be inferred from the interval $[0, 2\pi]$.

Graph of $\cos x$



x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	?								



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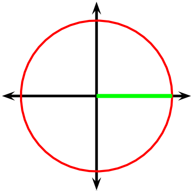
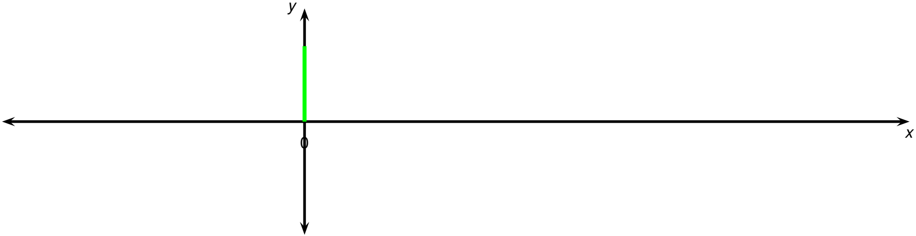


x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1								



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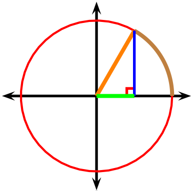

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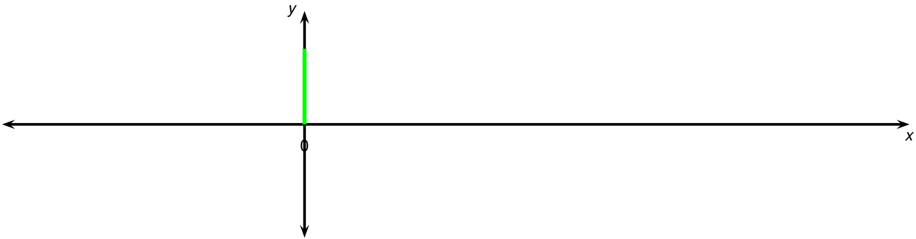

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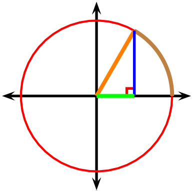

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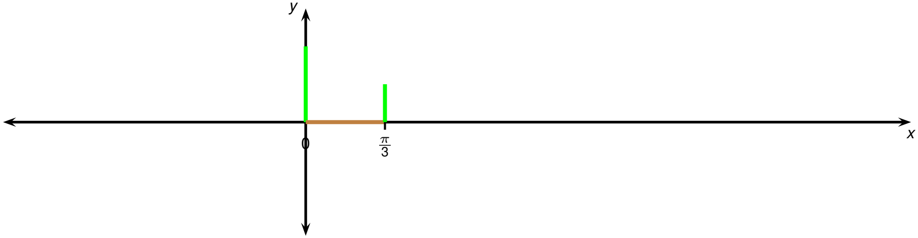
$\cos x$	1	$\frac{1}{2}$							



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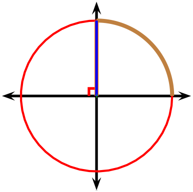

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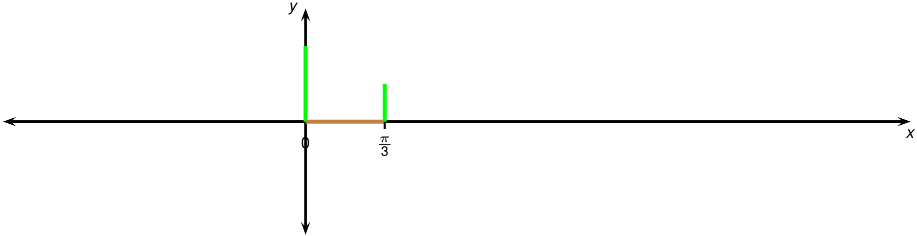

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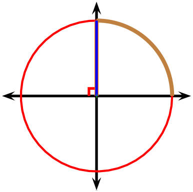
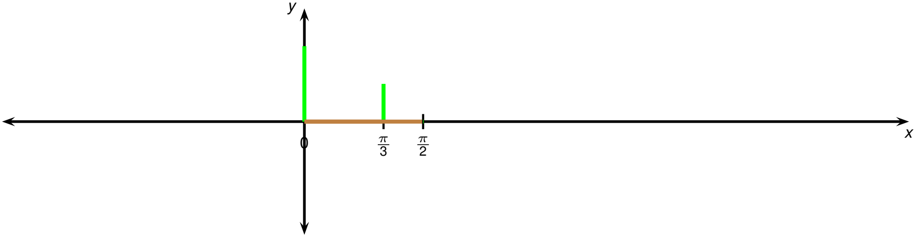
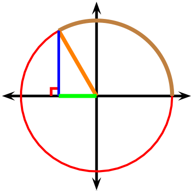


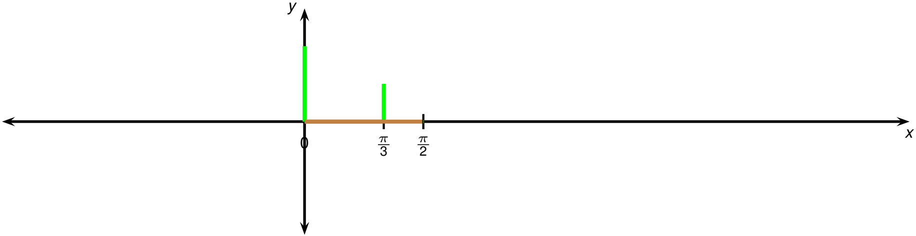
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0						



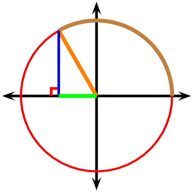
Graph of $\cos x$



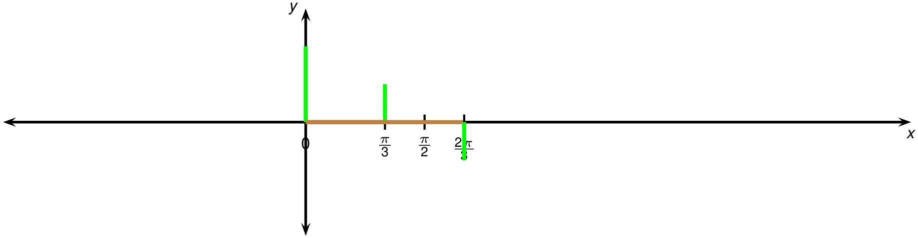
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	**?**					



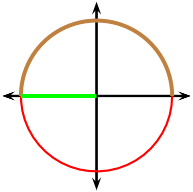
Graph of $\cos x$



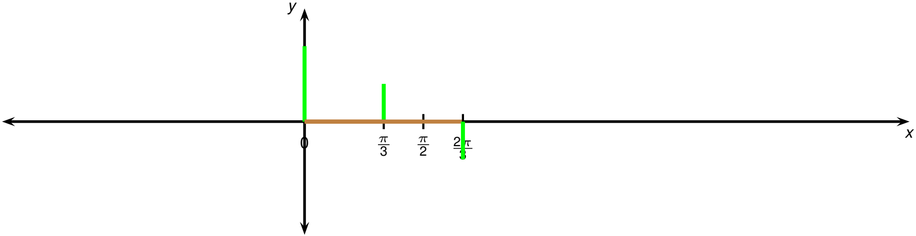
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$					



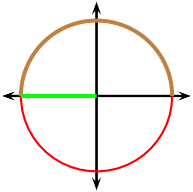
Graph of $\cos x$



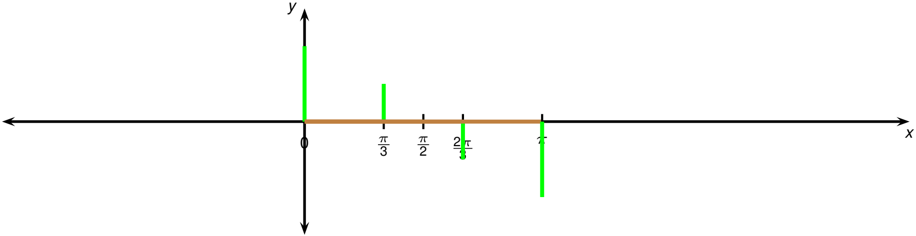
x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	?				



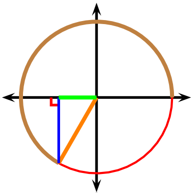
Graph of $\cos x$



x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1				



Graph of $\cos x$



x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
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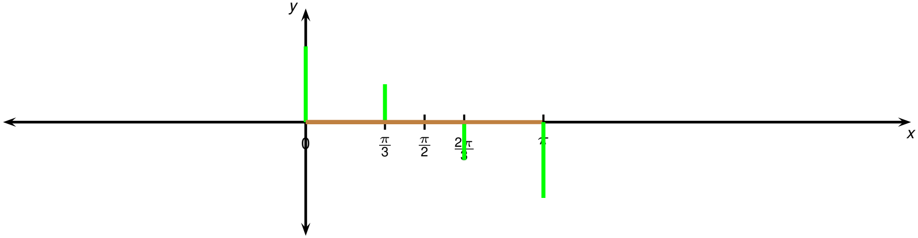
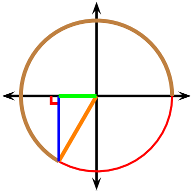


Graph of $\cos x$



x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
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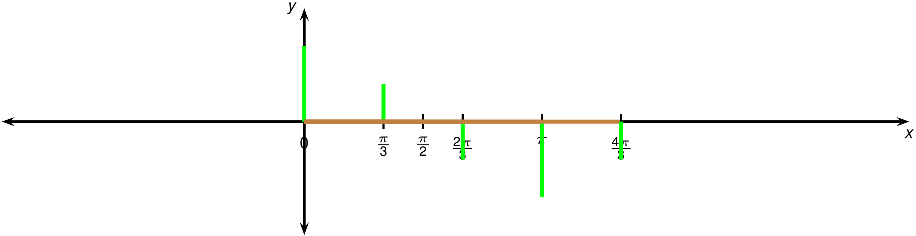
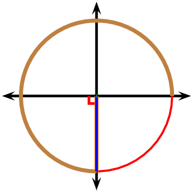


Graph of $\cos x$



x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
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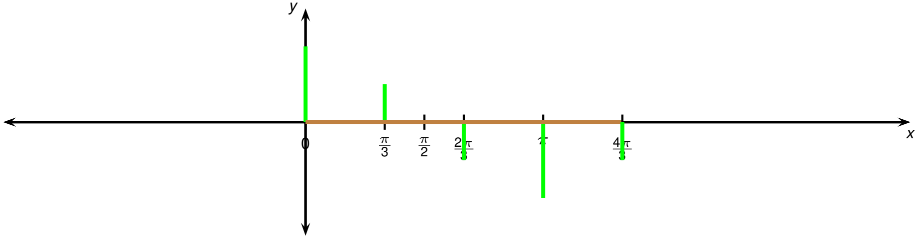
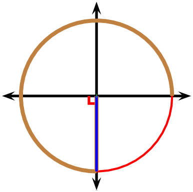


Graph of $\cos x$



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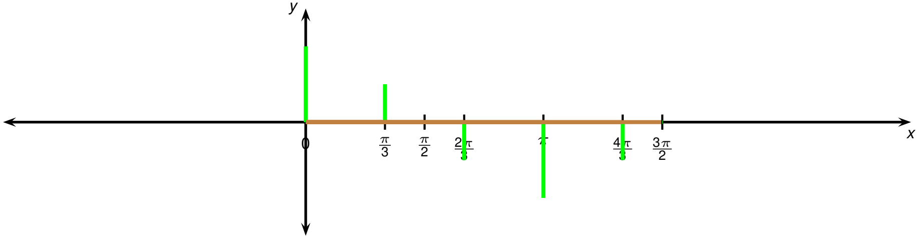
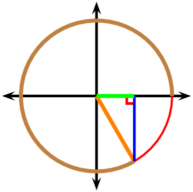
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0		



Graph of $\cos x$



x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
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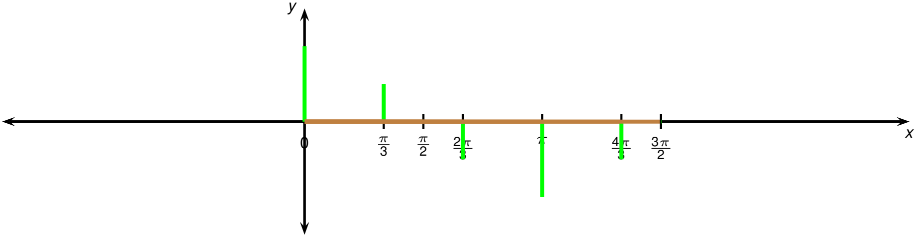
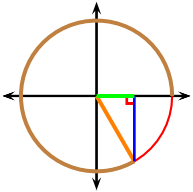
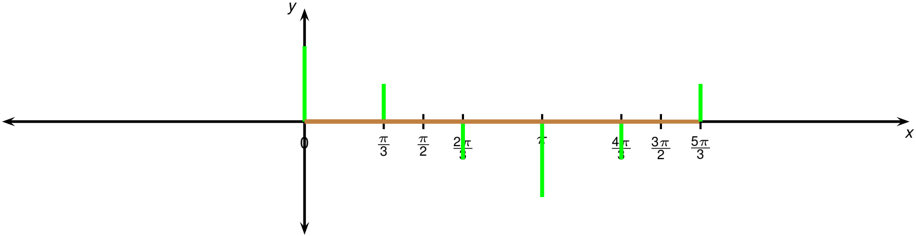


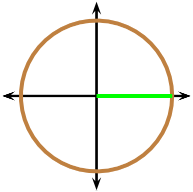
Graph of $\cos x$



x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	



Graph of $\cos x$



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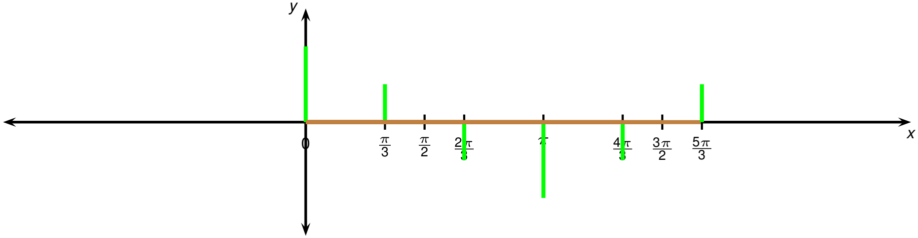

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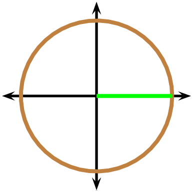

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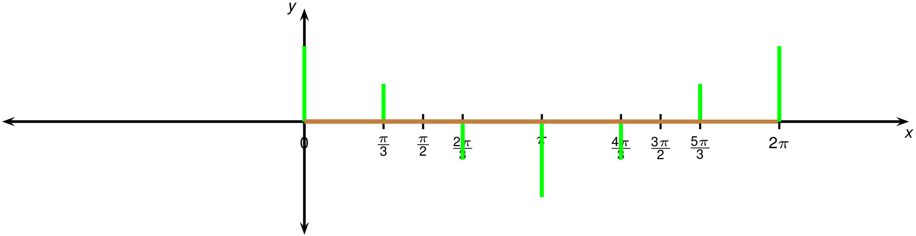

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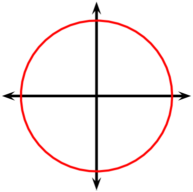
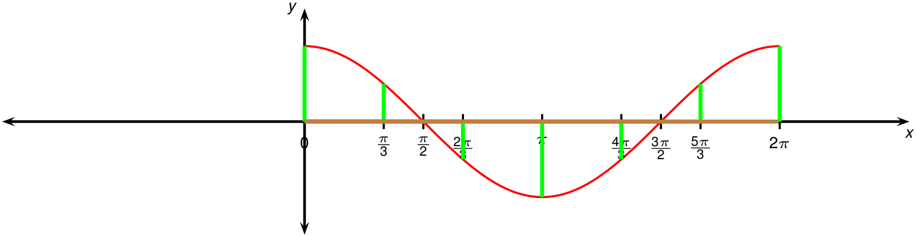


x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0



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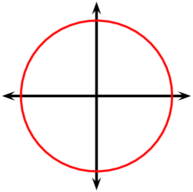
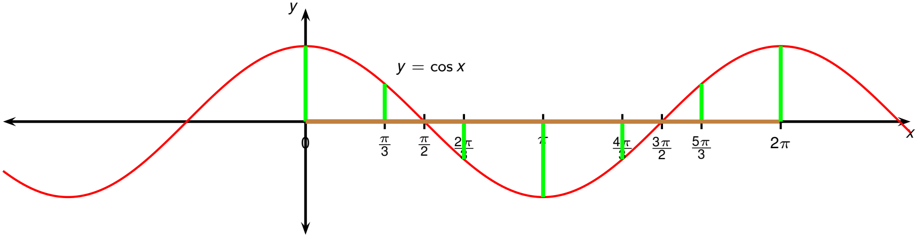
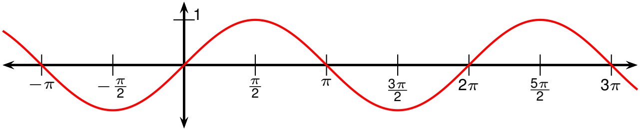


x	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\cos x$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0

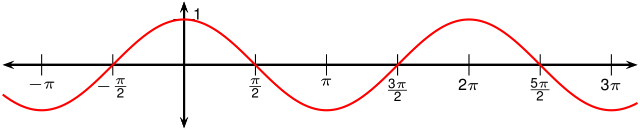


The graph of $\cos x$ is 2π-periodic so the rest of the graph can be inferred from the interval $[0, 2\pi]$.

Graphs of the Trigonometric Functions



$y = \sin x$



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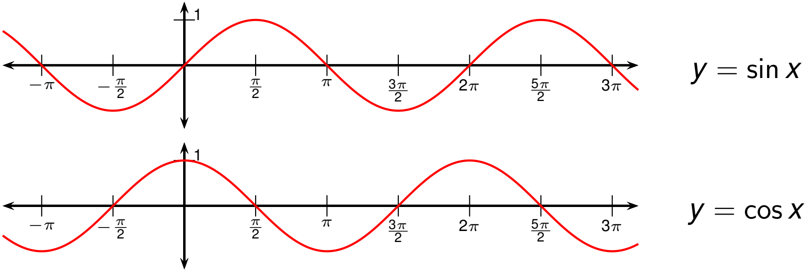
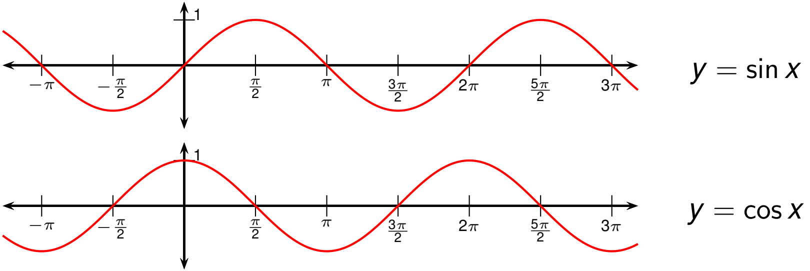
Graphs of the Trigonometric Functions



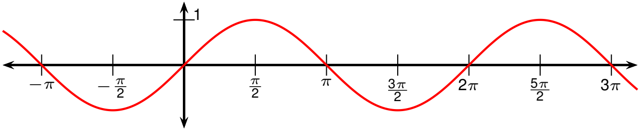
- $\sin x$ has zeroes at $n\pi$ for all integers n.

Graphs of the Trigonometric Functions



- $\sin x$ has zeroes at $n\pi$ for all integers n.
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.

Graphs of the Trigonometric Functions



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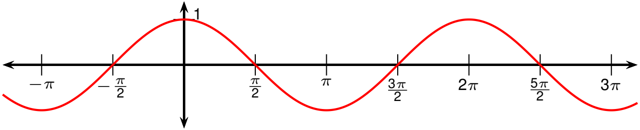
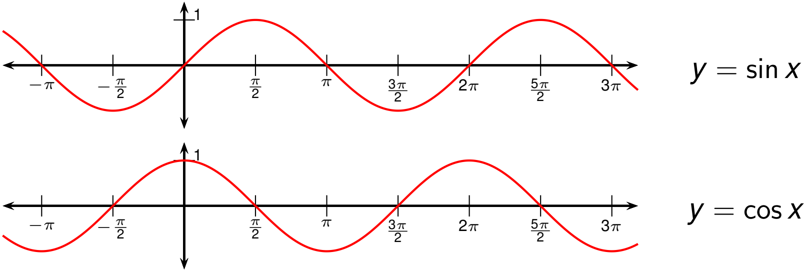


$y = \cos x$

- $\sin x$ has zeroes at $n\pi$ for all integers n.
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.
- $-1 \leq \sin x \leq 1$.

Graphs of the Trigonometric Functions



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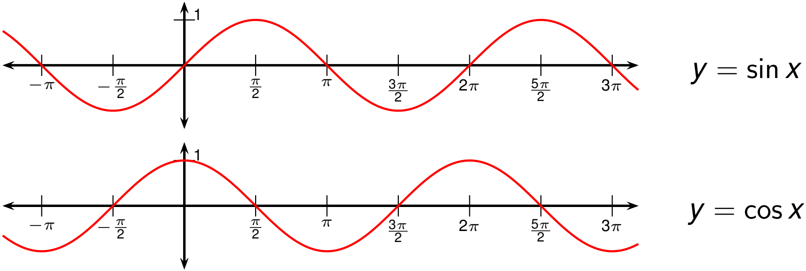

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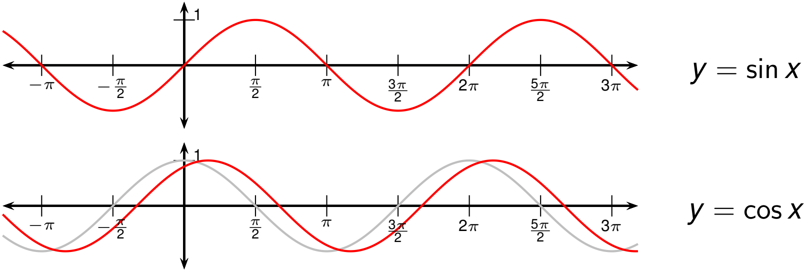
- If we translate the graph of $\cos x$ by $\frac{\pi}{2}$ units to the right

Graphs of the Trigonometric Functions



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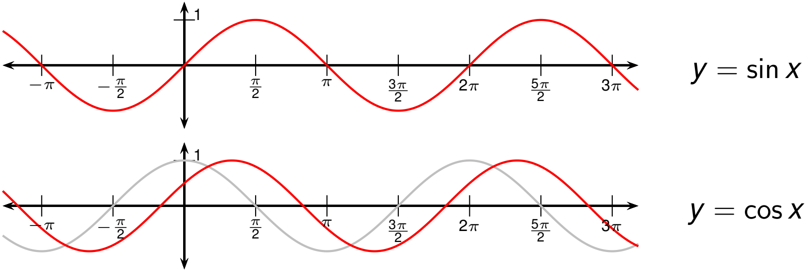
Graphs of the Trigonometric Functions



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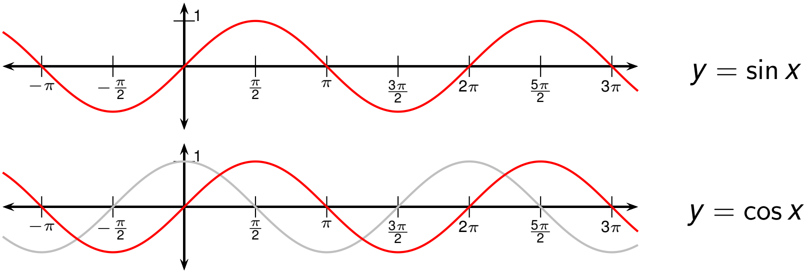

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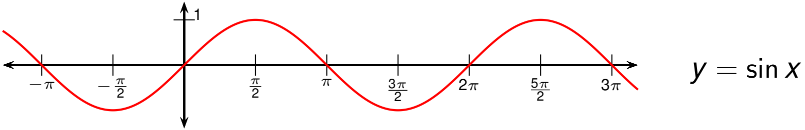
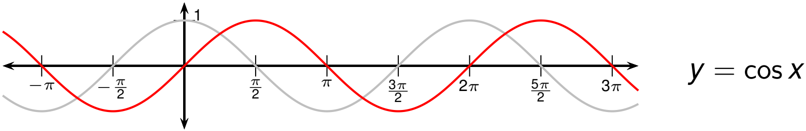
Graphs of the Trigonometric Functions





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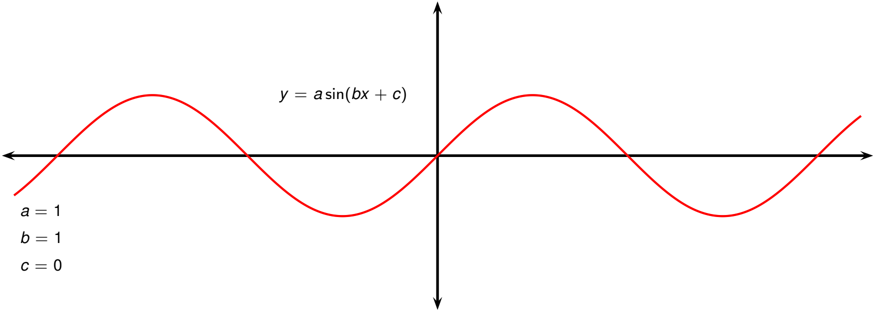
- $-1 \leq \sin x \leq 1$.
- $-1 \leq \cos x \leq 1$.
- If we translate the graph of $\cos x$ by $\frac{\pi}{2}$ units to the right we get the graph of $\sin x$. This is a consequence of $\cos\left(x - \frac{\pi}{2}\right) = \sin x$.

- The graph of $a\sin(bx+c)$ is referred to as a "wave".

Definition (Phase, period, frequency, amplitude of a wave)



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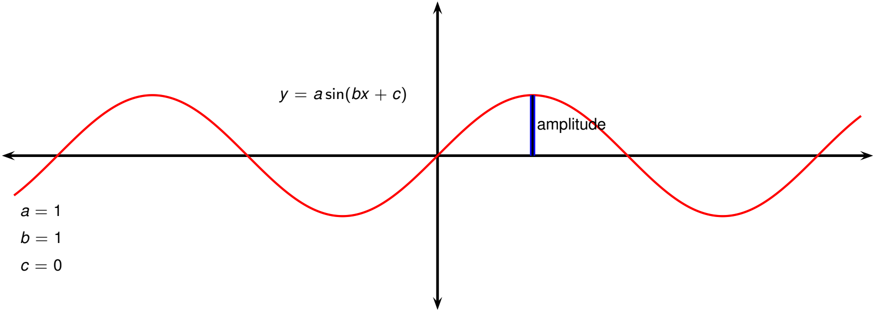
In the function $a\sin(bx+c)$, the number $|a|$ is called the *amplitude* of the wave,



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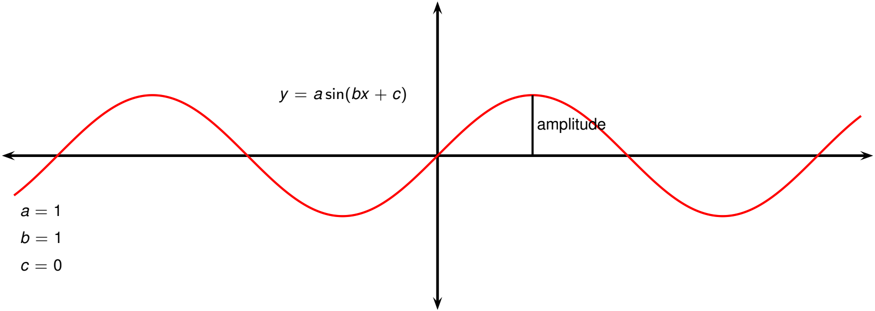

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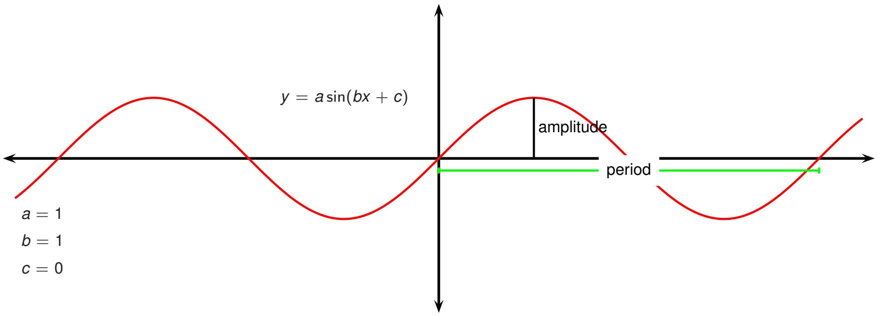

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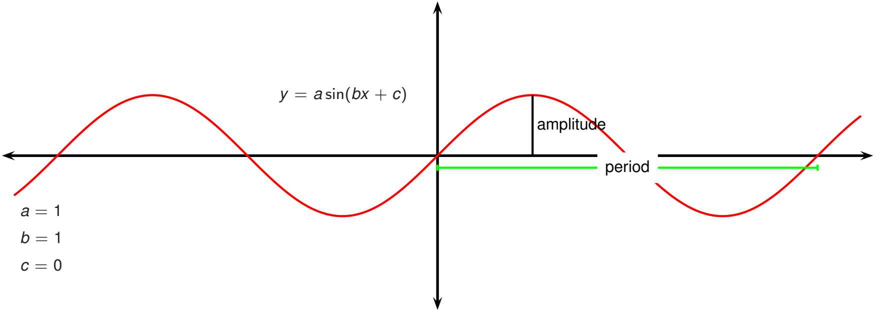

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Definition (Phase, period, frequency, amplitude of a wave)

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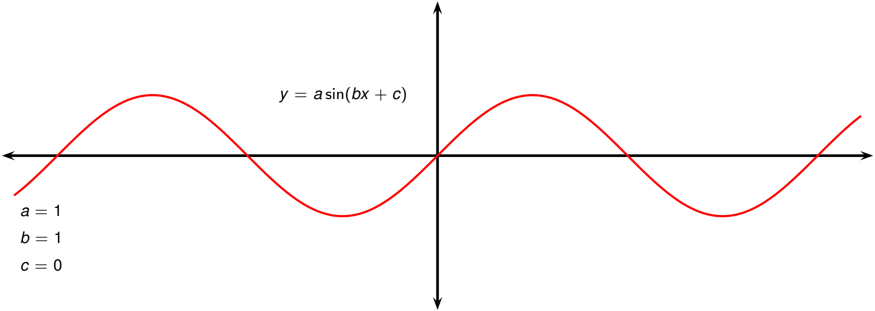
- What happens when we change the amplitude? The frequency/period? The phase?



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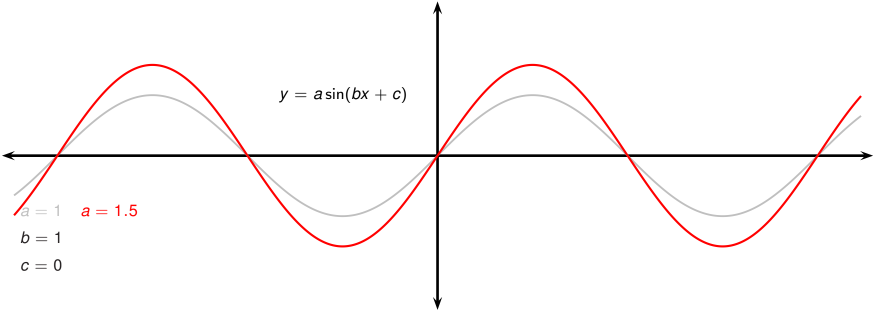

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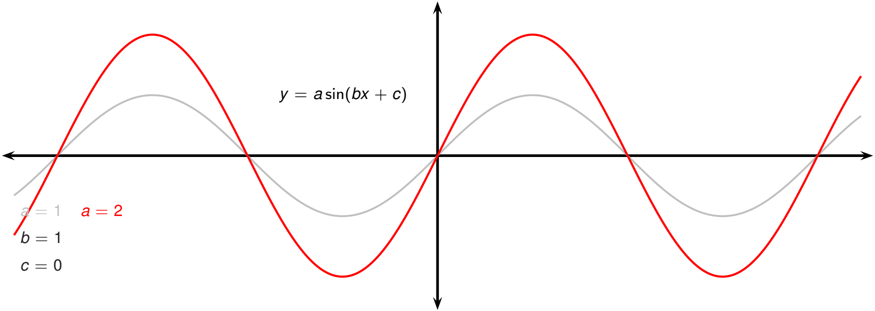

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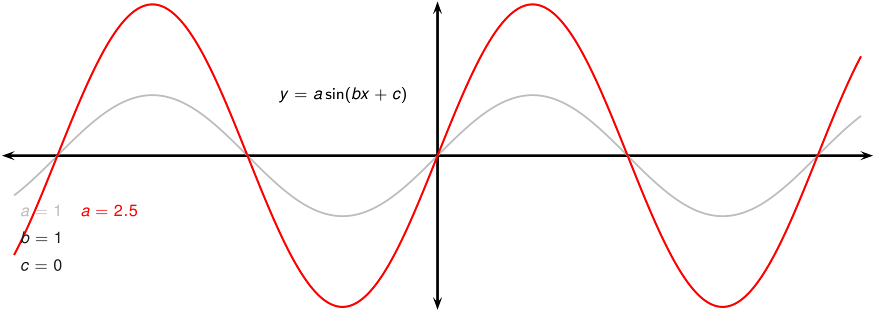

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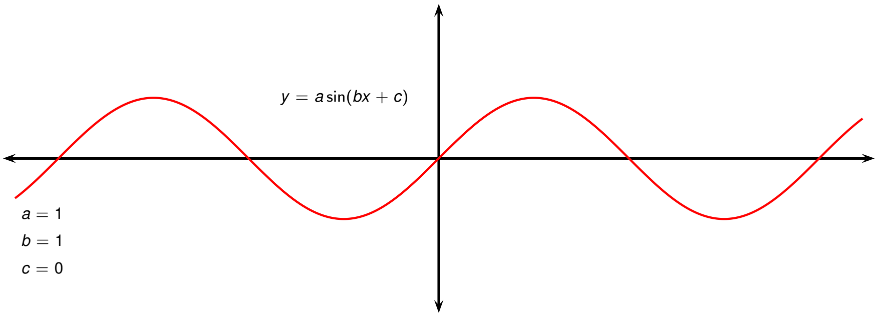

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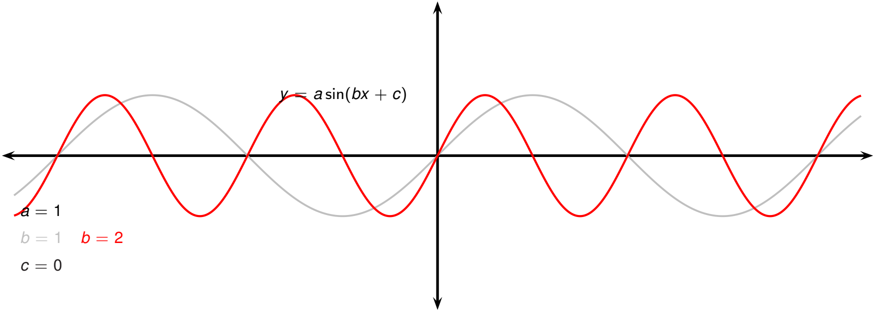

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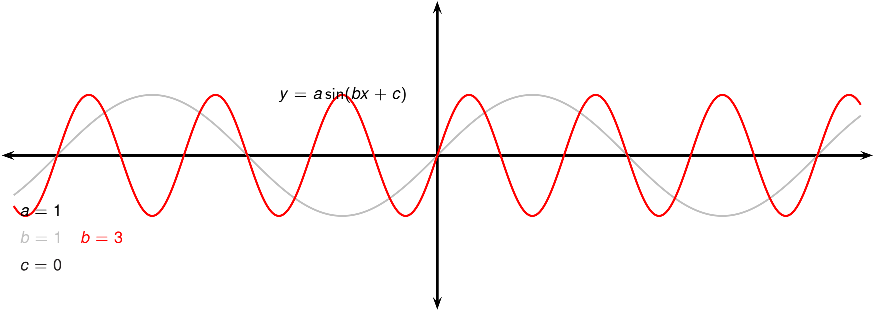

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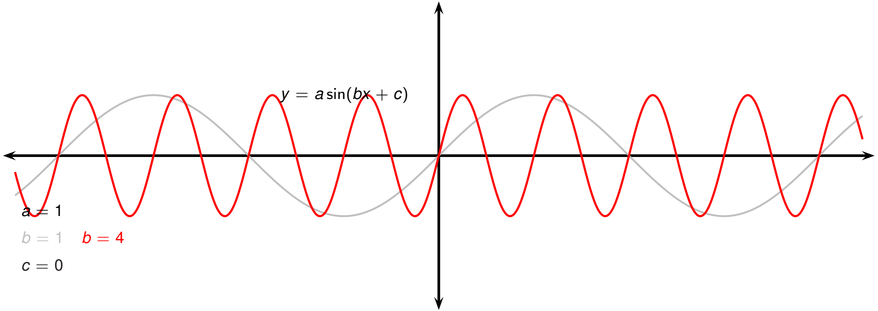

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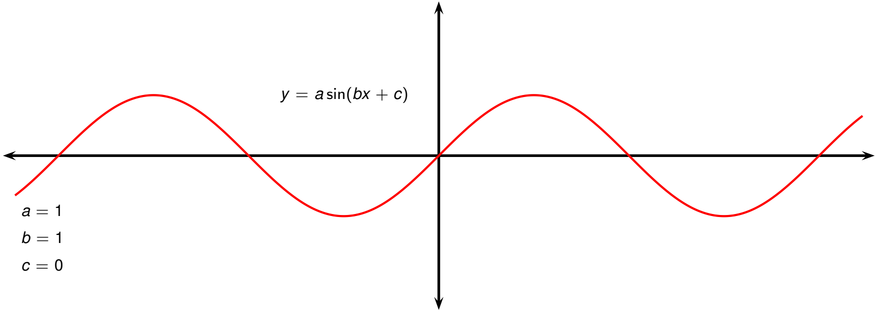

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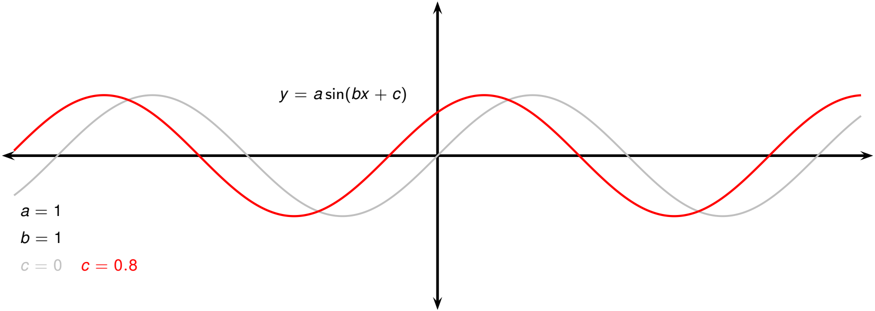

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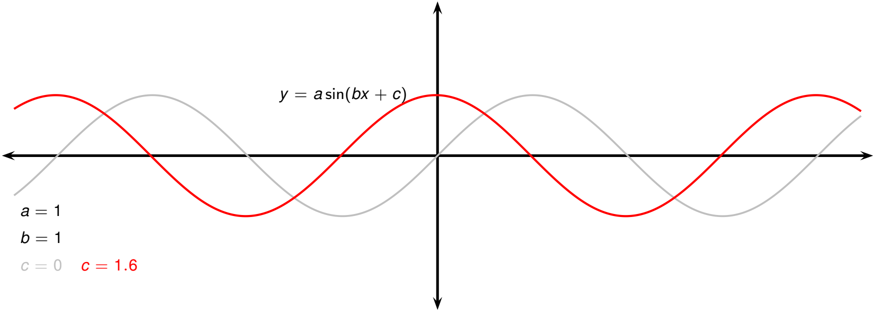

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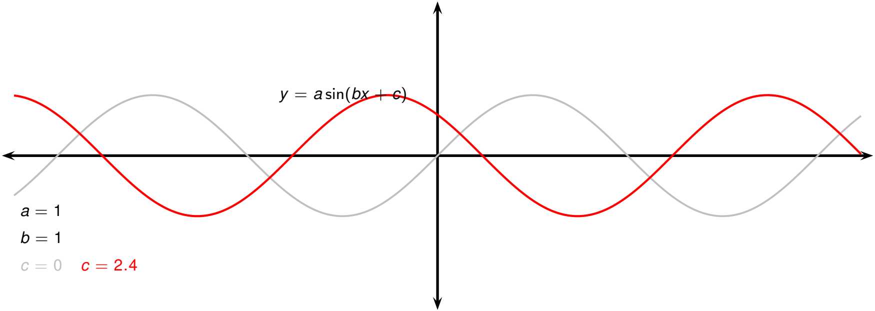

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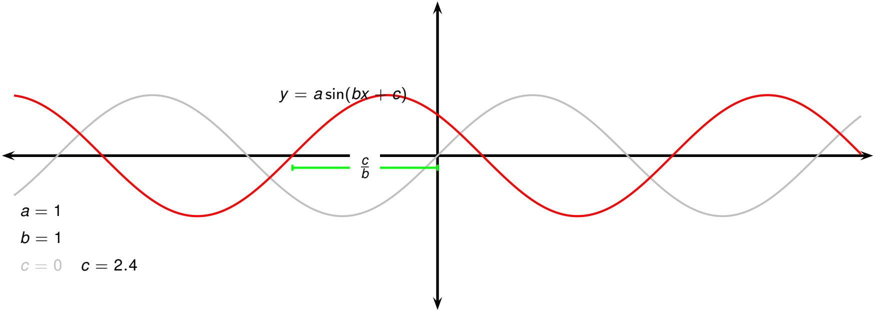

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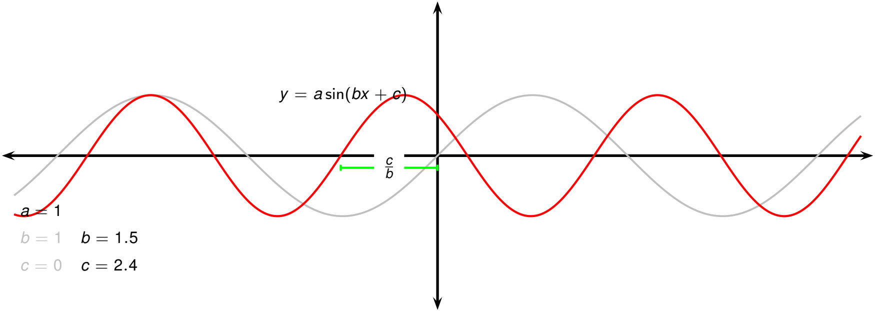
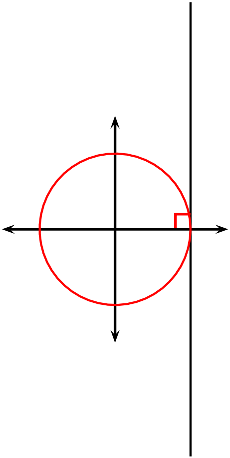
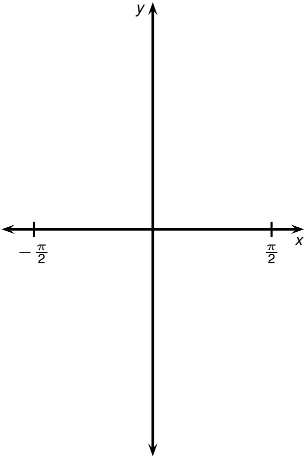


Graph of $\tan x$





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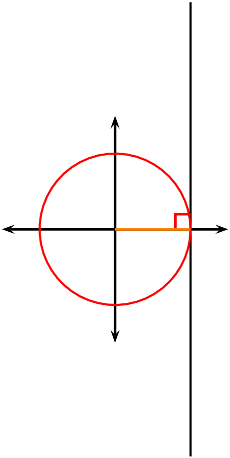

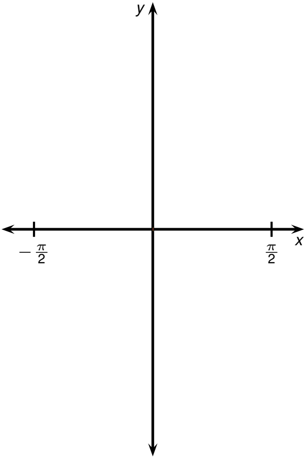

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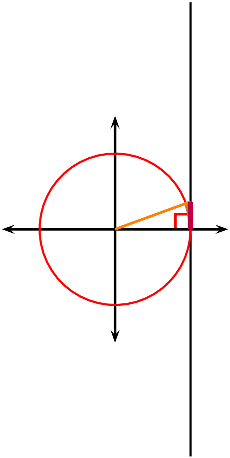

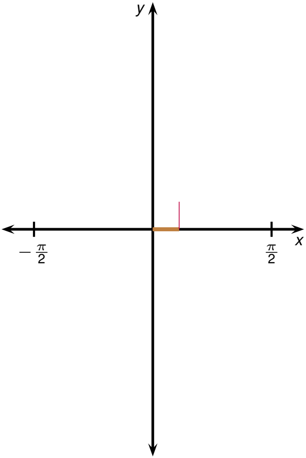

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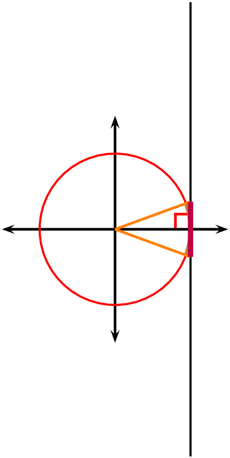

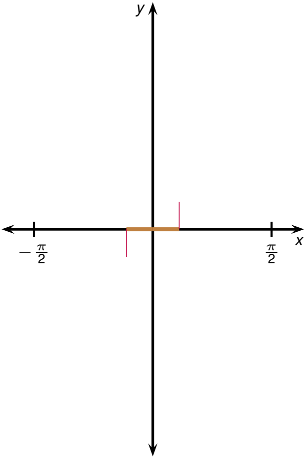

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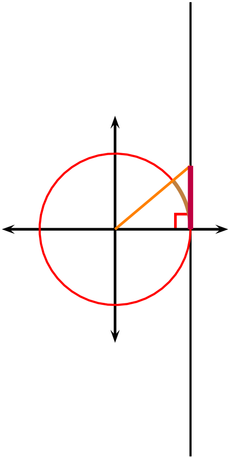

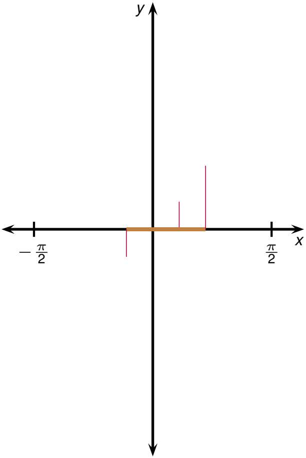

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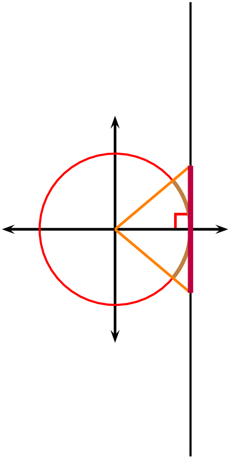

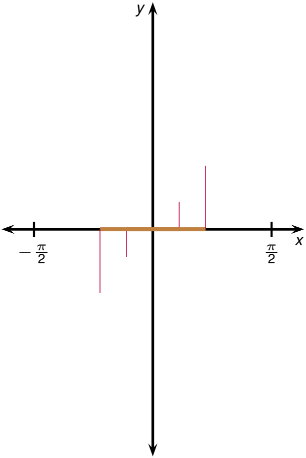

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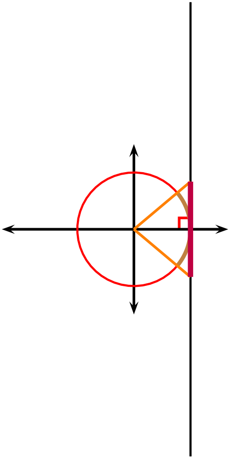

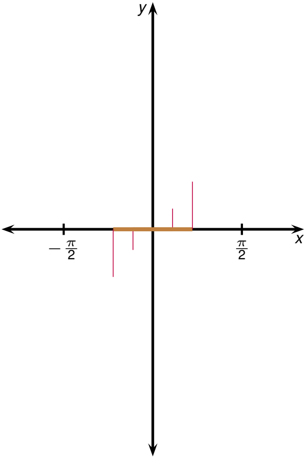

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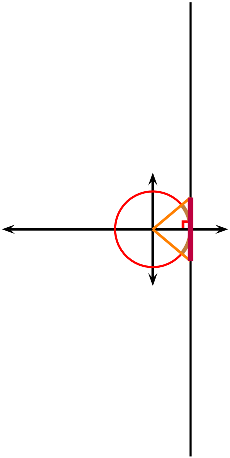

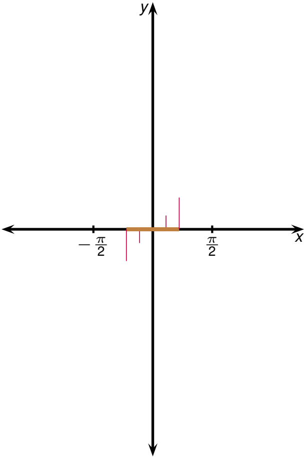

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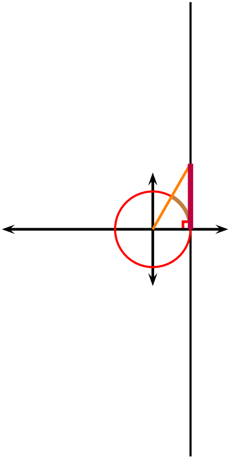

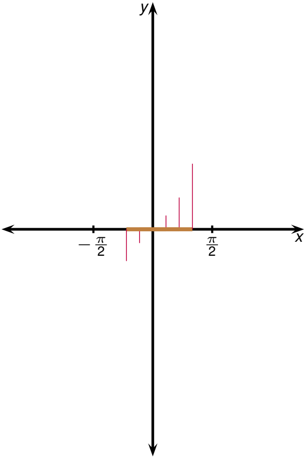

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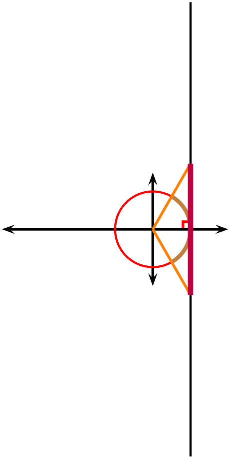

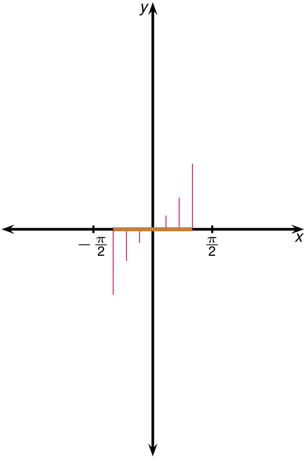

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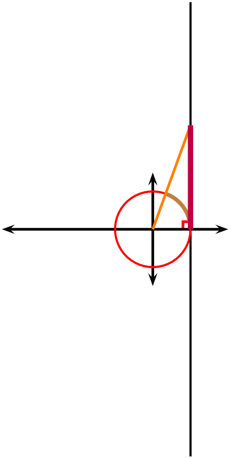

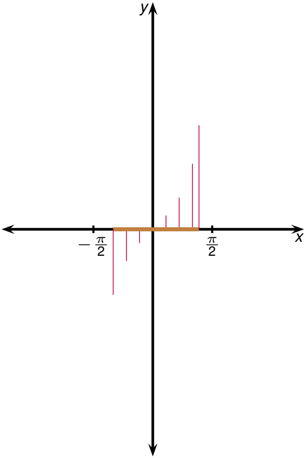

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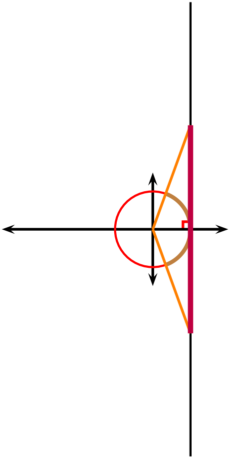

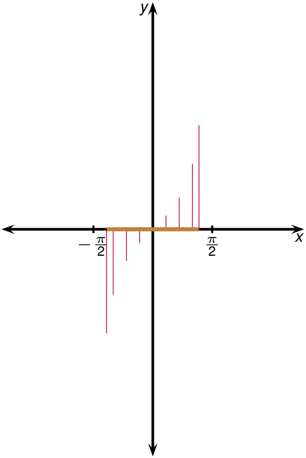

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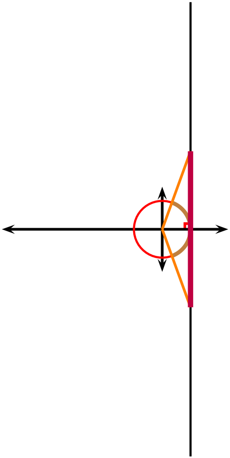

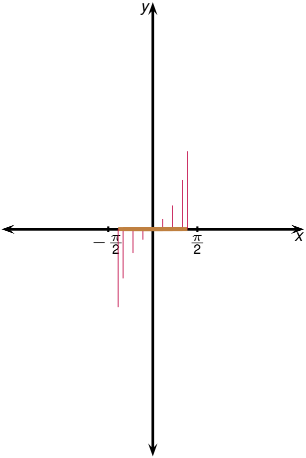

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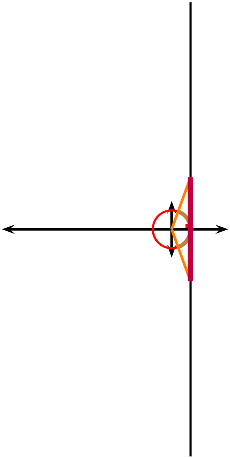

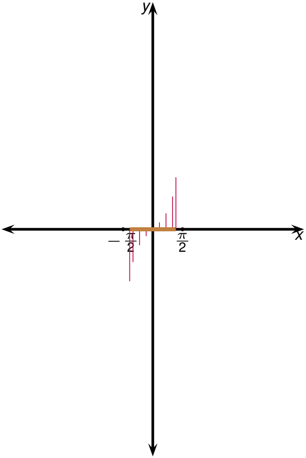

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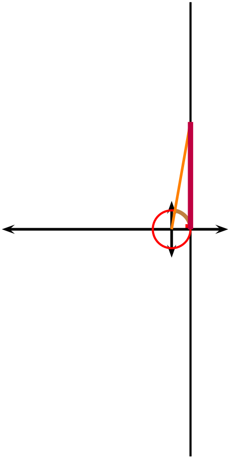

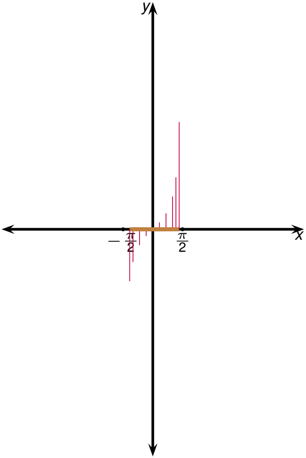

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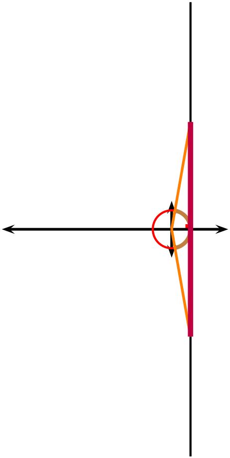

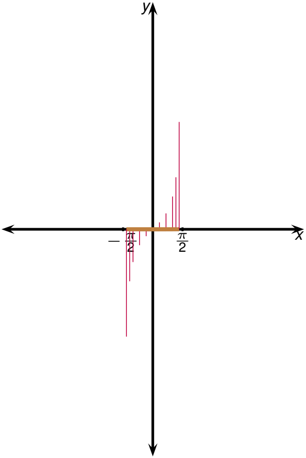

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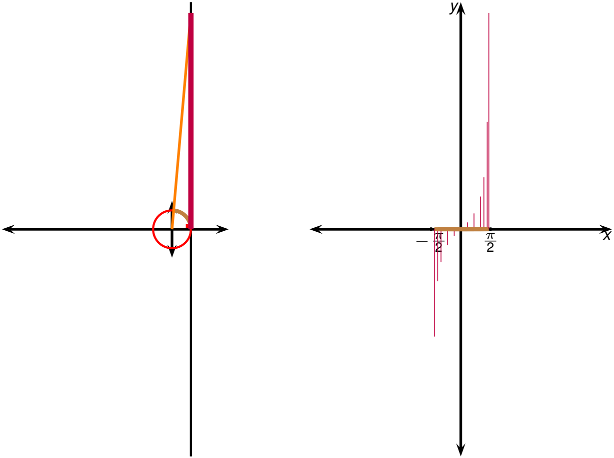

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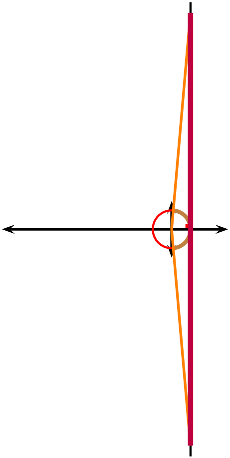

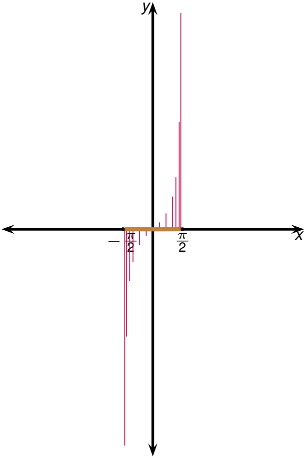

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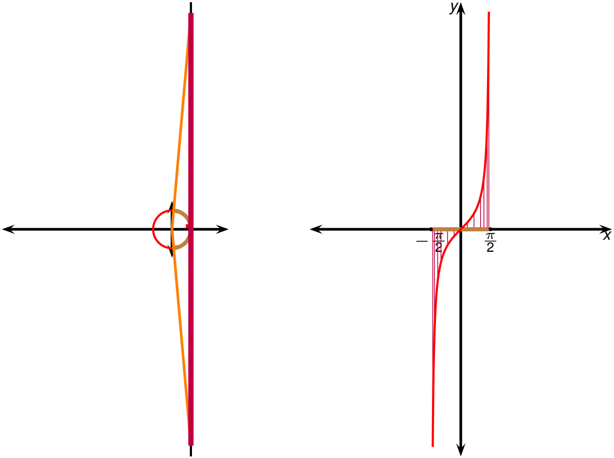

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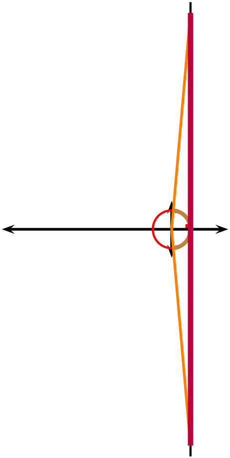

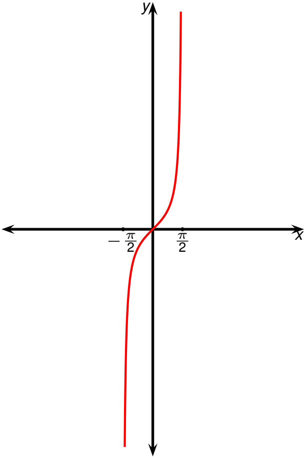

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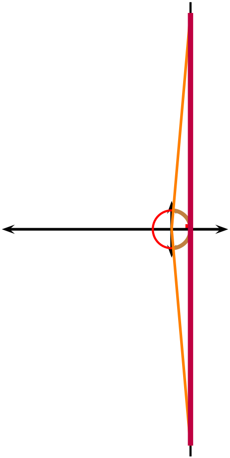
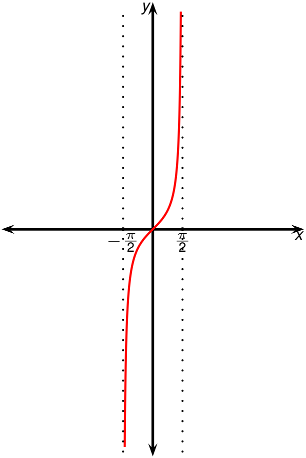
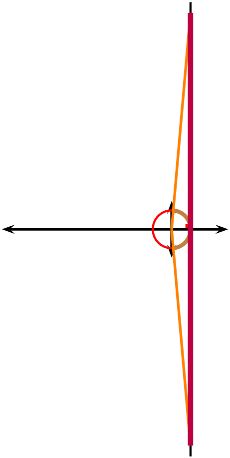
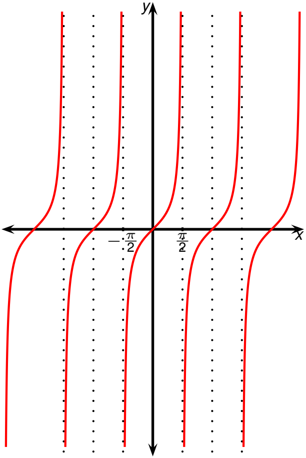




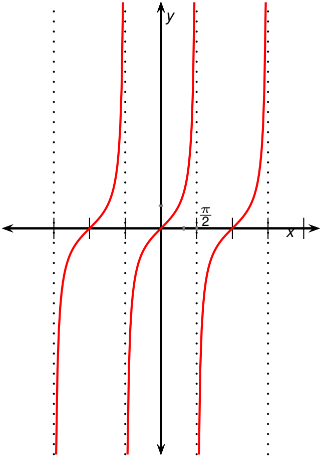
Near $\pm\frac{\pi}{2}$ the graph of $\tan x$ approaches $\pm\infty$.

Graph of $\tan x$

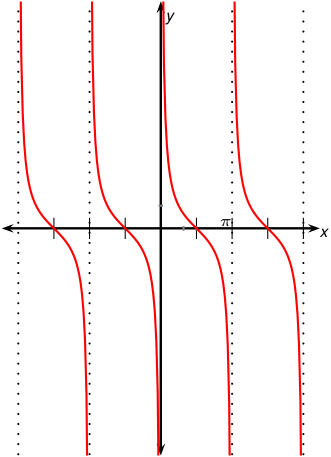




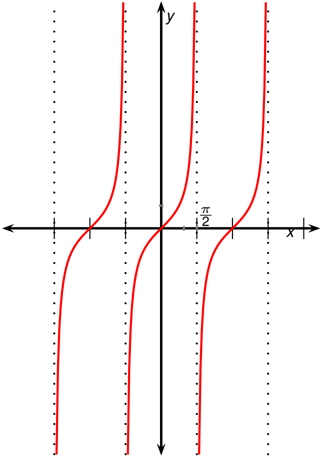
Near $\pm\frac{\pi}{2}$ the graph of $\tan x$ approaches $\pm\infty$. The graph of $\tan x$ is π-periodic so the rest of the graph can be inferred from the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.



$y = \tan x$



$y = \cot x$



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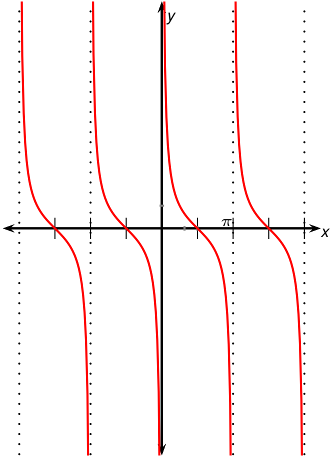
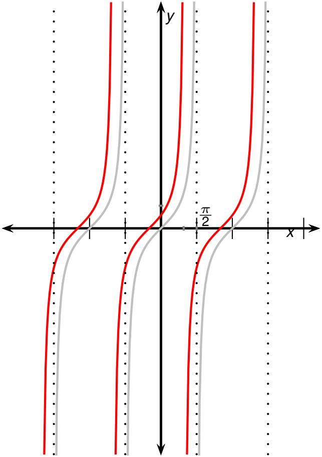
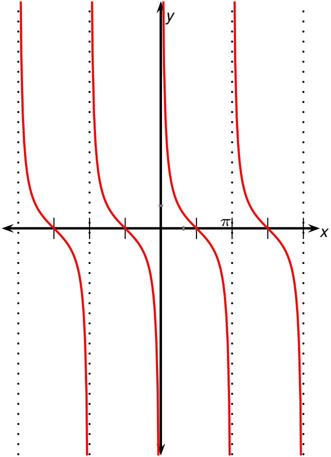


$y = \cot x$

If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis





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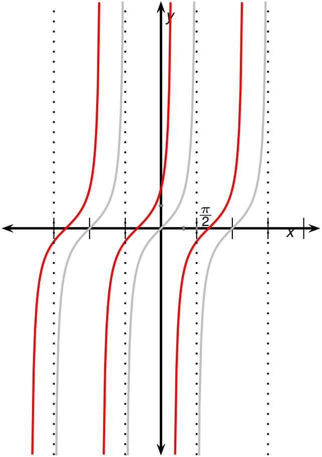

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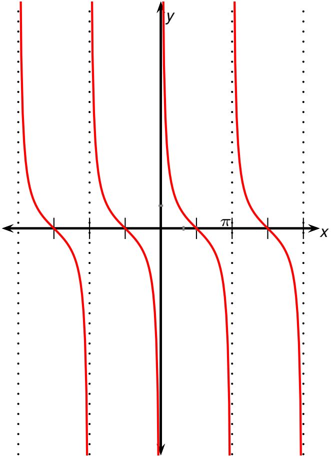

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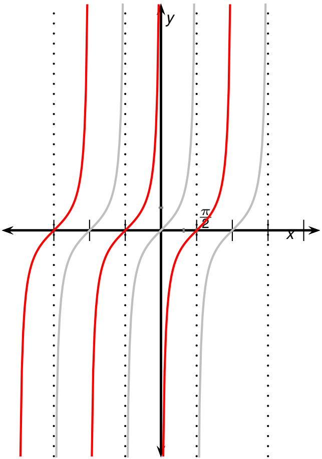

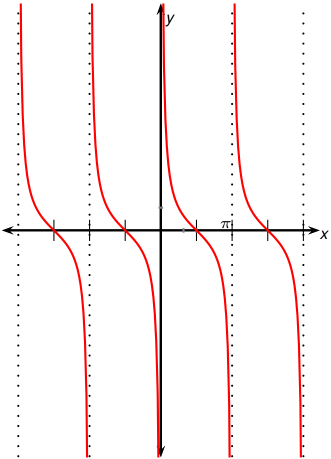

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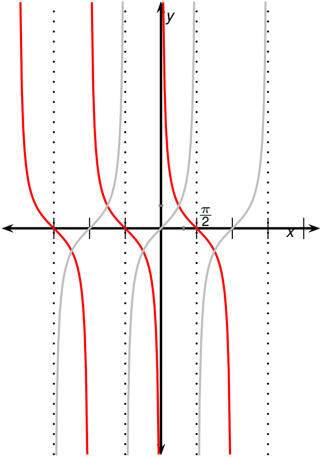

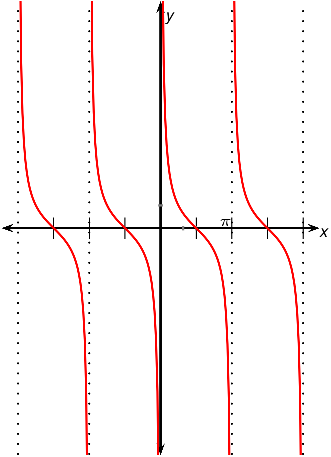

$y = \tan x$ $\qquad$ $y = \cot x$

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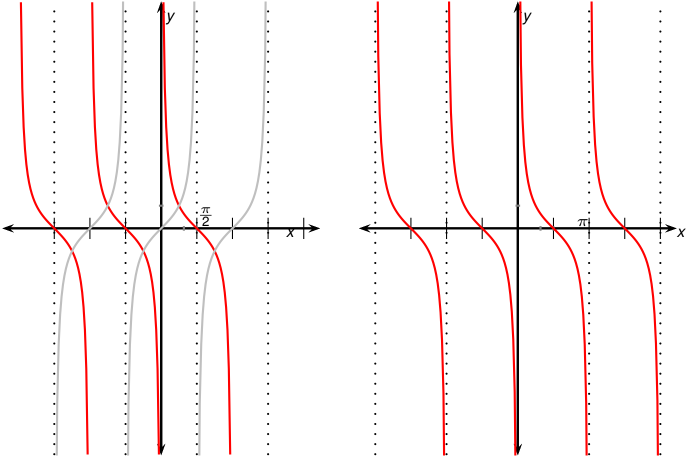
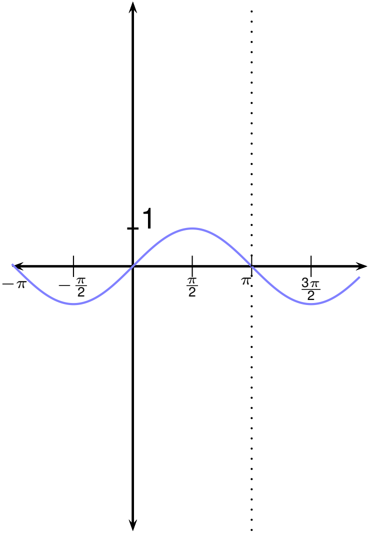
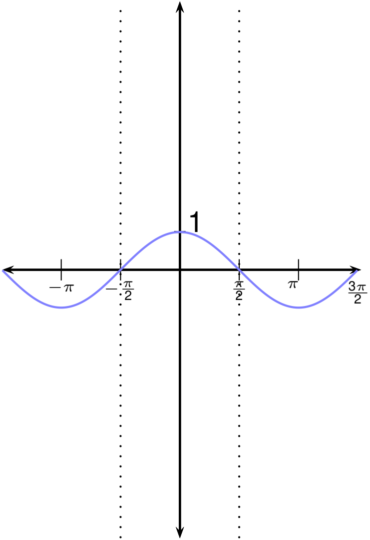


$y = \tan x$ $\qquad$ $y = \cot x$

If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis, we get the graph of $\cot x$. This follows from $\tan\left(x \pm \frac{\pi}{2}\right) = -\cot x$.


1
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$

$y = \csc x$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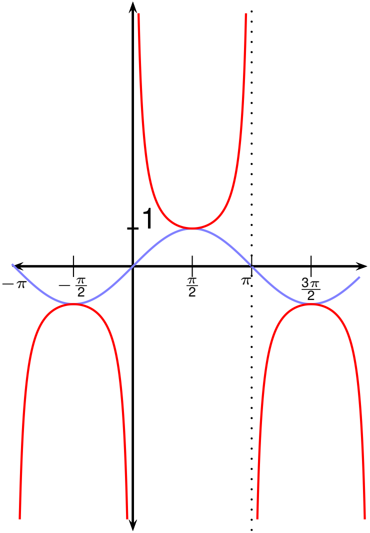
$y = \sec x$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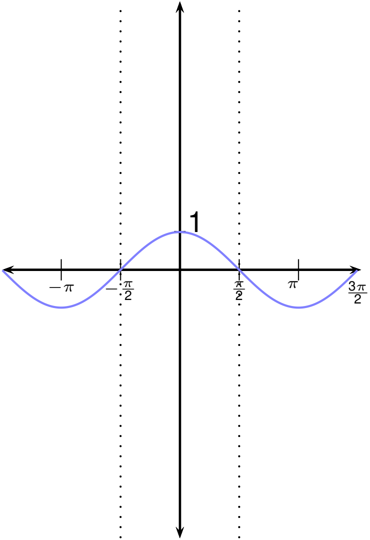
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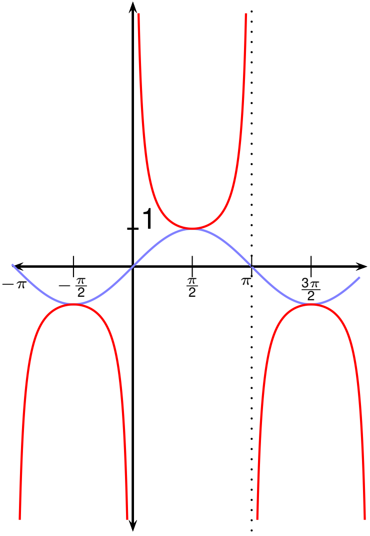
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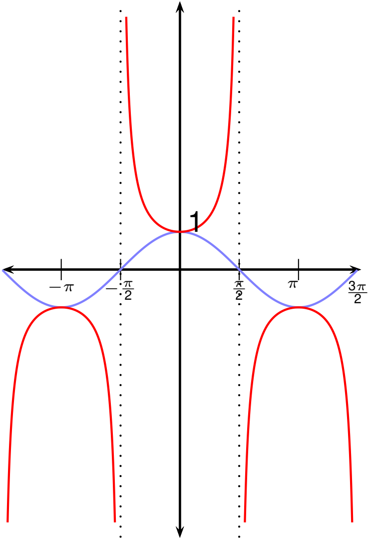
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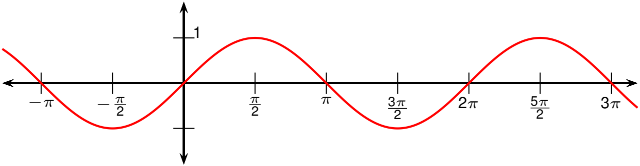
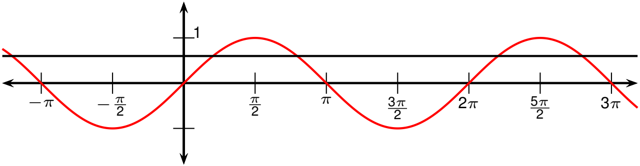
Inverse Trigonometric Functions

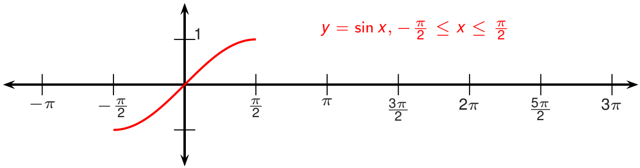


Inverse Trigonometric Functions



- $\sin x$ isn't one-to-one.

Inverse Trigonometric Functions



- $\sin x$ isn't one-to-one.
- It is if we restrict the domain to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

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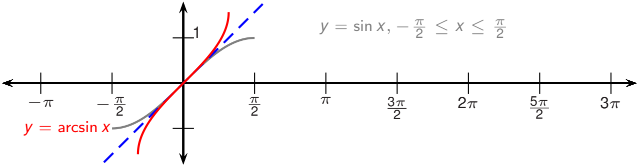

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- Then it has an inverse function.
- We call it $\arcsin$ or $\sin^{-1}$.

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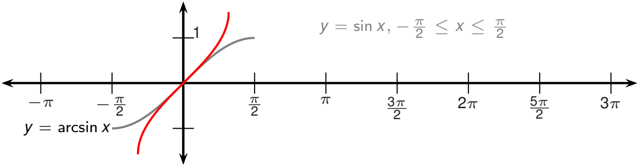

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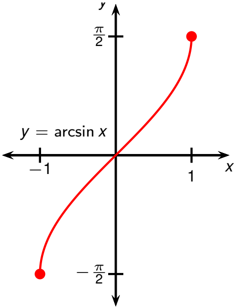

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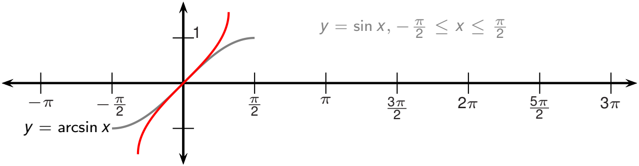

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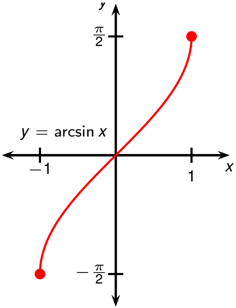
- $\arcsin x = y \Leftrightarrow \sin y = x$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.



Example

Find $\arcsin\left(\frac{1}{2}\right)$.

Observation

- $\arcsin y =$ *the appropriate angle whose sine equals y.*

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$$\text{Find } \arcsin\left(\frac{1}{2}\right).$$

- $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.

Observation

- $\arcsin y =$ *the appropriate angle whose sine equals* y.
- *Important: the output angle must lie in the interval* $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Example

Find $\arcsin\left(\frac{1}{2}\right)$.

- $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.
- $-\frac{\pi}{2} \leq \frac{\pi}{6} \leq \frac{\pi}{2}$.

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Example

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- $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.
- $-\frac{\pi}{2} \leq \frac{\pi}{6} \leq \frac{\pi}{2}$.
- Therefore $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

Example

Find $\tan\left(\arcsin\left(\frac{1}{3}\right)\right)$.

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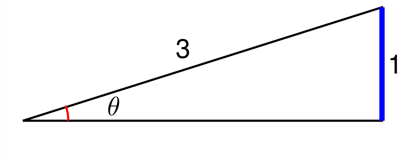

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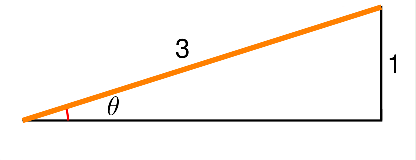

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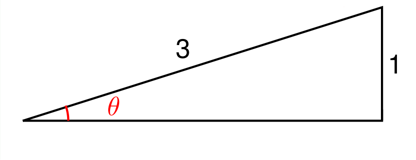

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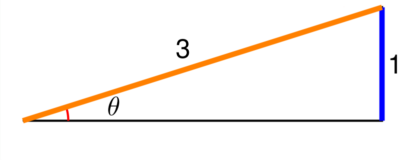

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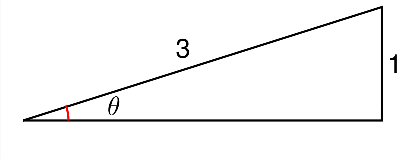

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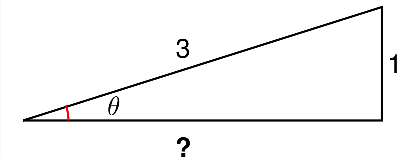
- Length of adjacent side $=$ **?**



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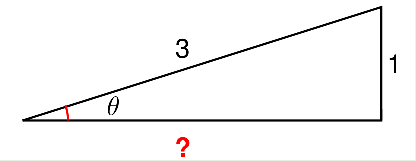
- Length of adjacent side $= \sqrt{3^2 - 1^2}$



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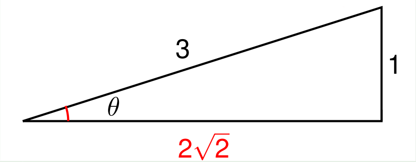
- Let the angle θ be as labeled. Then $\sin\theta = \frac{1}{3}$ and so $\theta = \arcsin\left(\frac{1}{3}\right)$.
- Length of adjacent side $= \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$.



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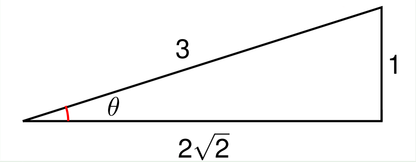

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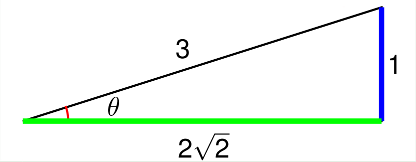
- Length of adjacent side $= \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$.
- Then $\tan\left(\arcsin\left(\frac{1}{3}\right)\right) = \frac{1}{2\sqrt{2}}$.



Example

Find $\arcsin(\sin(1.5))$.

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- $\frac{\pi}{2} \approx$ **?**

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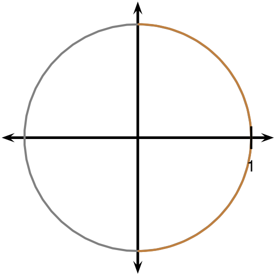
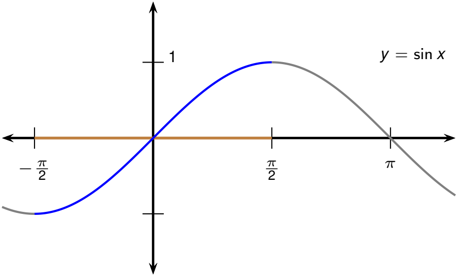
- $\frac{\pi}{2} \approx 1.57.$
- Therefore $-\frac{\pi}{2} \leq 1.5 \leq \frac{\pi}{2}$.
- Therefore $\arcsin(\sin 1.5) = 1.5$.

Example

Find $\arcsin(\sin 2)$.





Example

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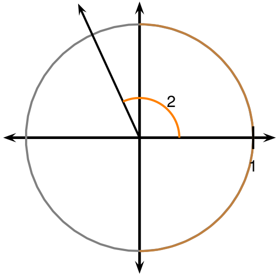
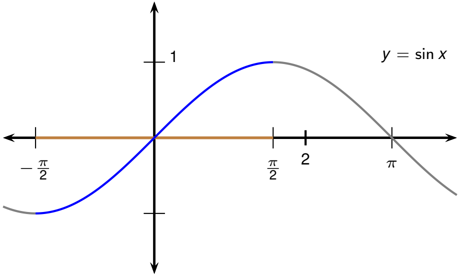
- 2 is not between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

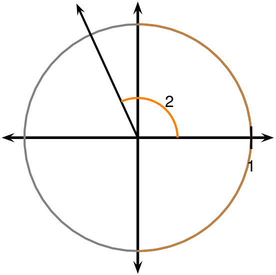


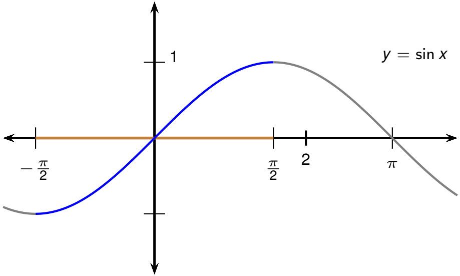


Example

Find $\arcsin(\sin 2)$.

- 2 is not between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.
- We need the angle a between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ for which $\sin 2 = \sin a$.





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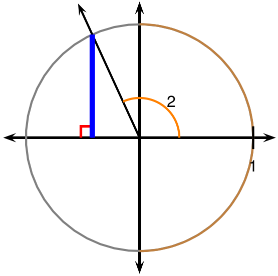

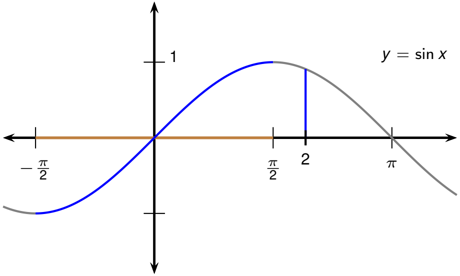

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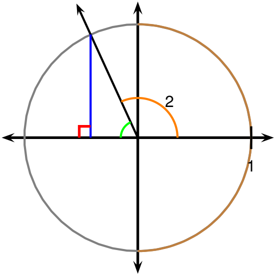

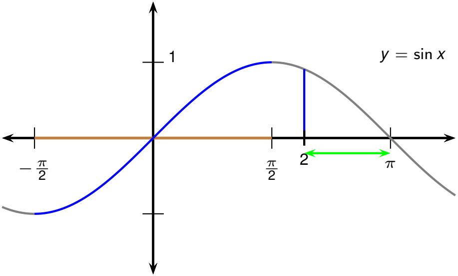

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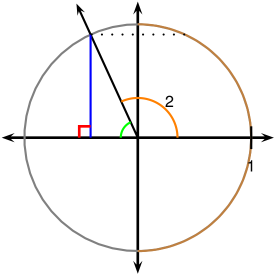

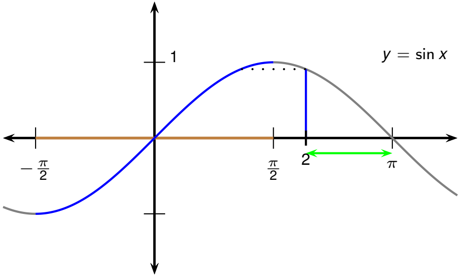

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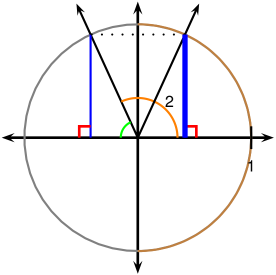

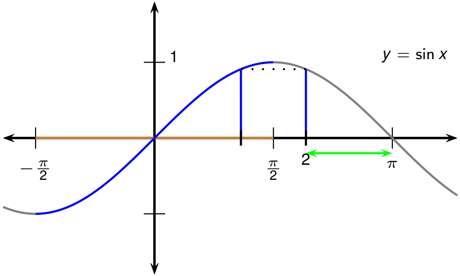

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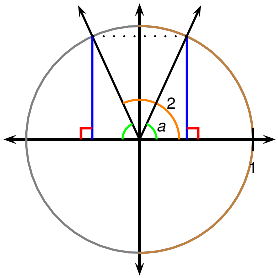

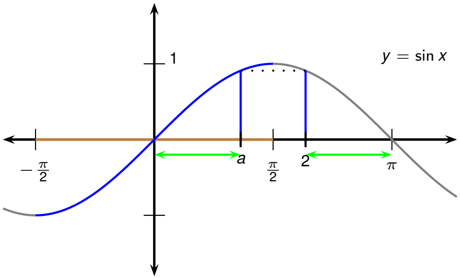

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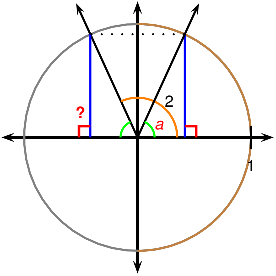
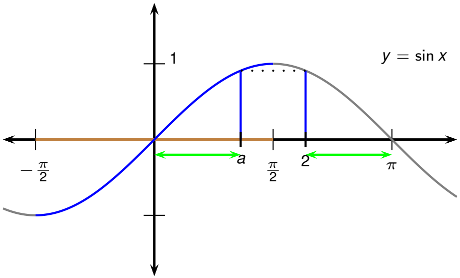
$$a = \textbf{?}$$





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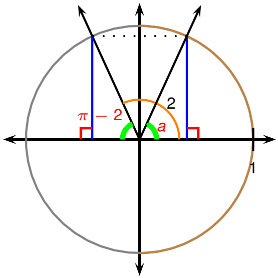
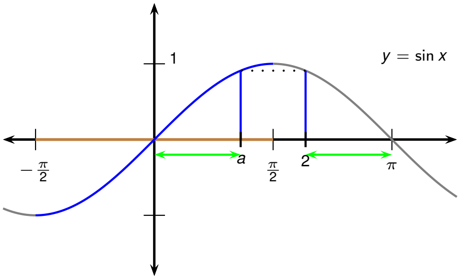
$$a = \pi - 2.$$





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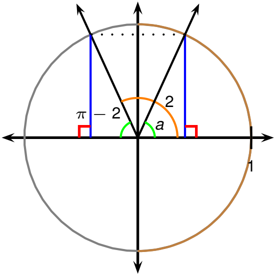
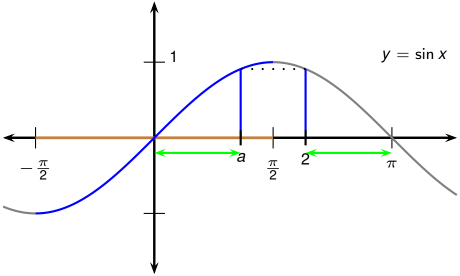
$$\begin{aligned} a &= \pi - 2. \\ \text{Therefore} \quad \arcsin(\sin 2) &= \arcsin(\sin a) \end{aligned}$$





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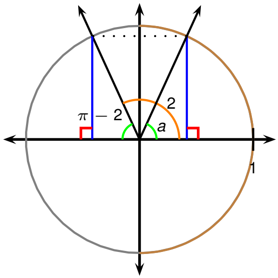

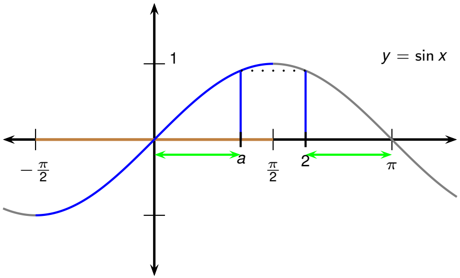

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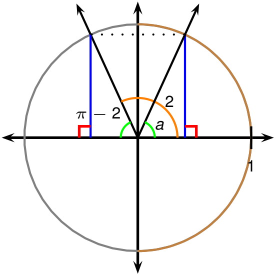
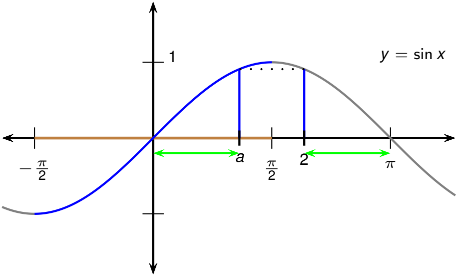
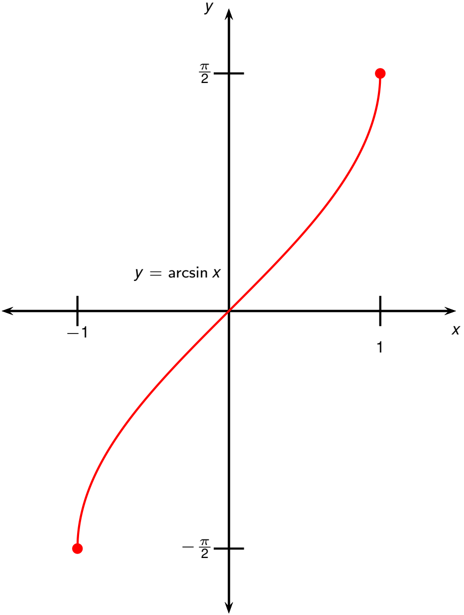




Important facts about arcsin:



1. Domain: **?**
2. Range: **?**
3. $\arcsin x = y \Leftrightarrow \sin y = x$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.
4. $\arcsin(\sin x) = x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.
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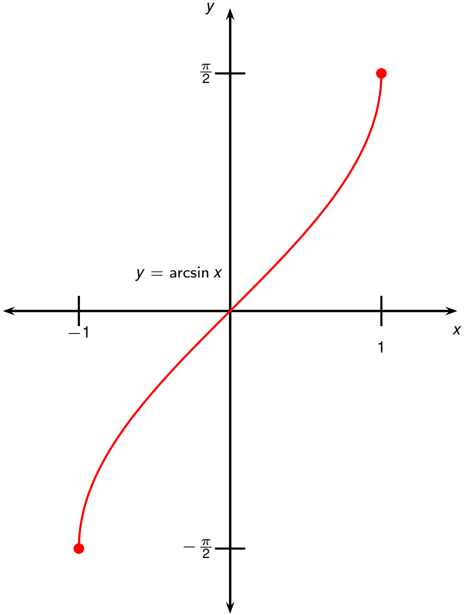

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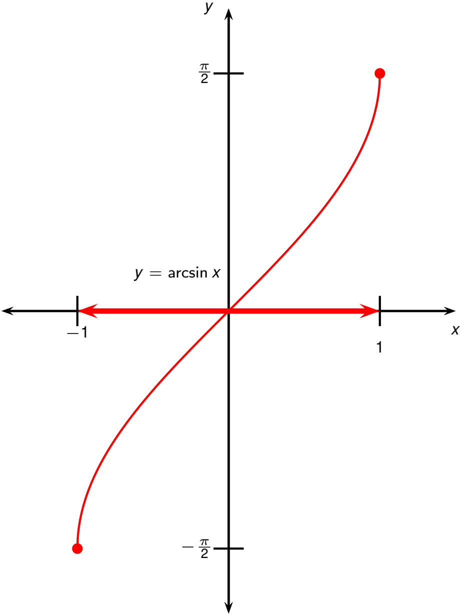

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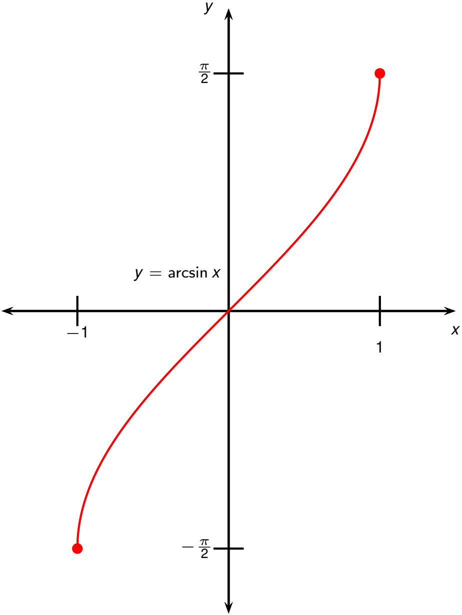

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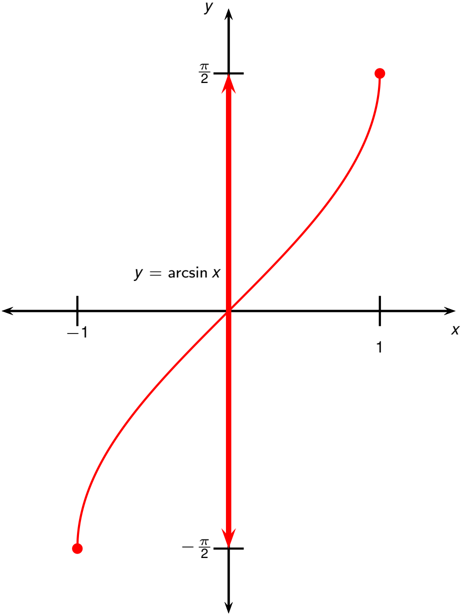

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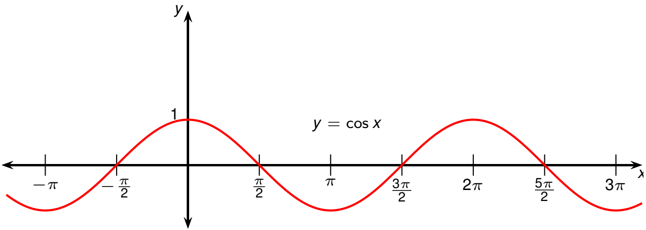
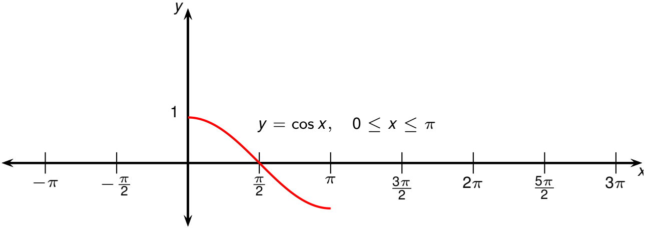
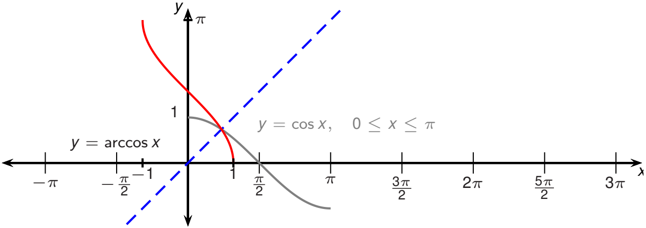


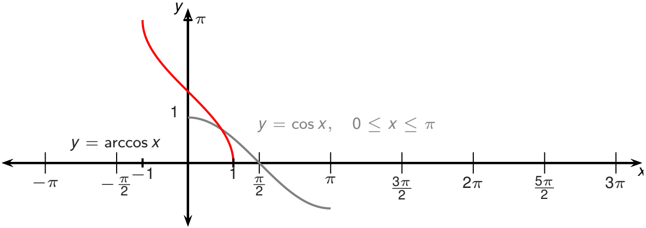
- Same for $\cos x$.



- Same for $\cos x$.
- Restrict the domain to $[0, \pi]$.



- Same for $\cos x$.
- Restrict the domain to $[0, \pi]$.
- The inverse is called $\arccos$ or $\cos^{-1}$.



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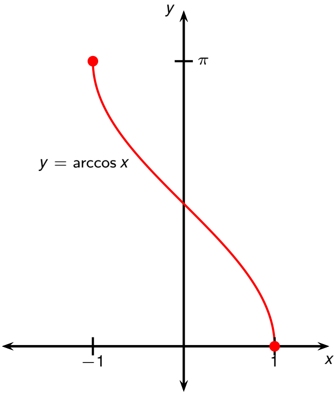
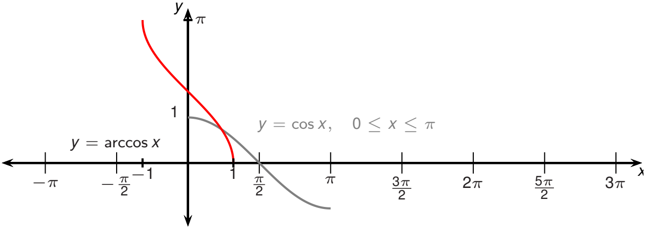
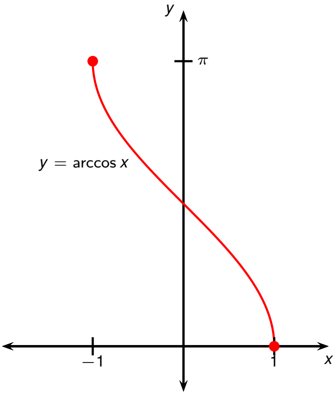




- Same for $\cos x$.
- Restrict the domain to $[0, \pi]$.
- The inverse is called arccos or $\cos^{-1}$.
- $\arccos(x) = y \Leftrightarrow \cos y = x$ and $0 \leq y \leq \pi$.



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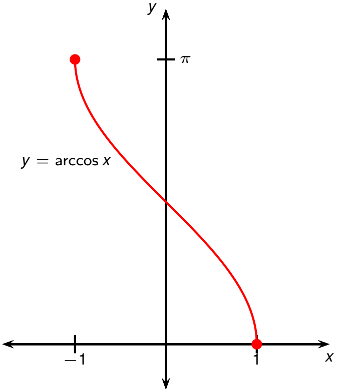

1. Domain:
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4. $\arccos(\cos x) = x$ for $0 \leq x \leq \pi$.
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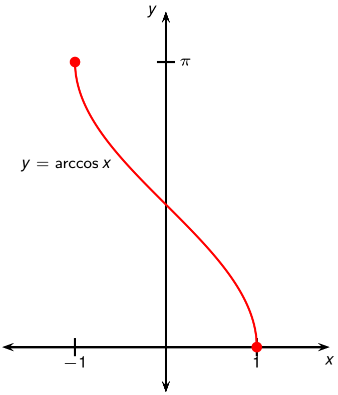

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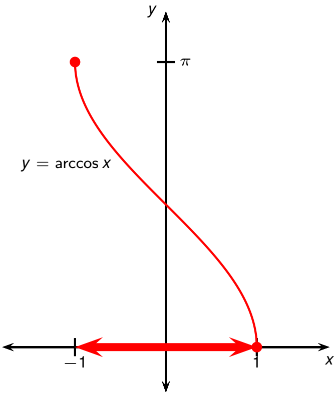

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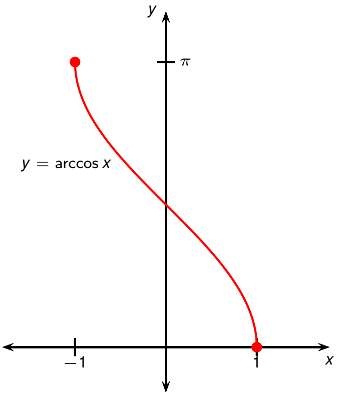

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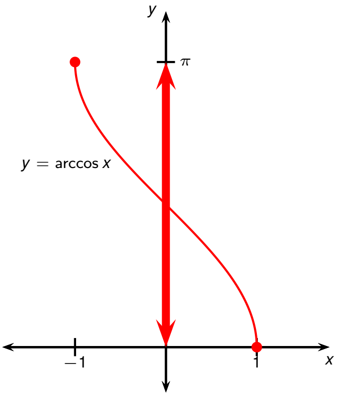

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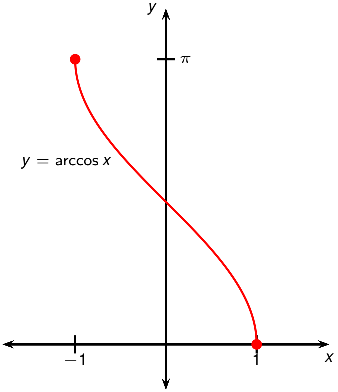

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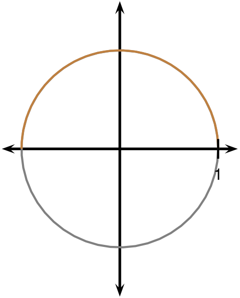
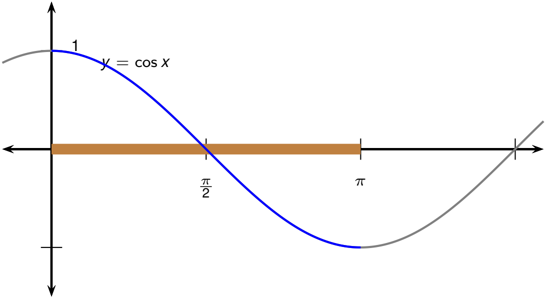
(The proof is similar to the proof of the formula for the derivative of $\arcsin x$.)

Example

Find $\arccos(\cos 4)$.





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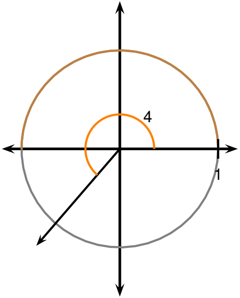
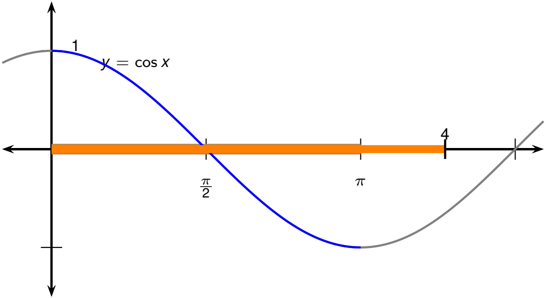
- 4 is not between 0 and π.





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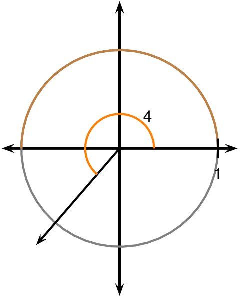
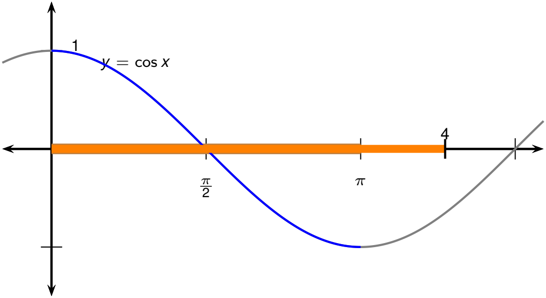
- 4 is not between 0 and π.
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.





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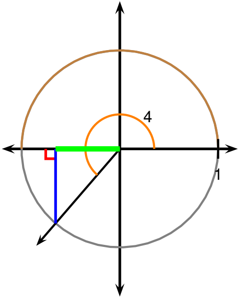

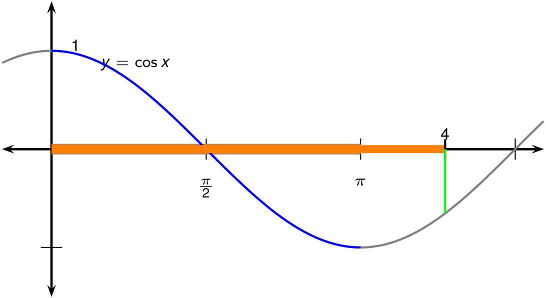

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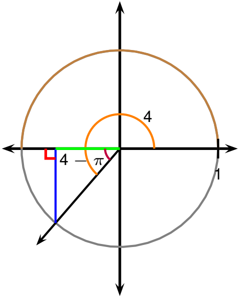

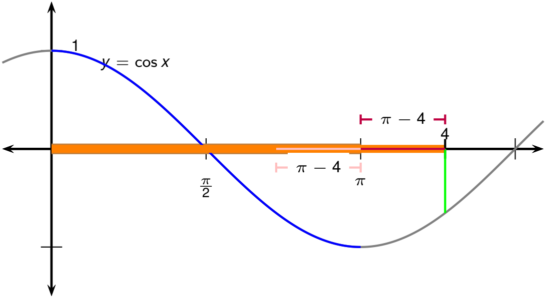

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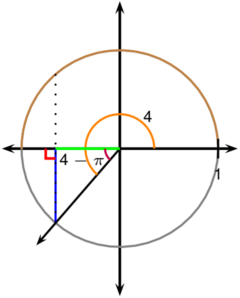

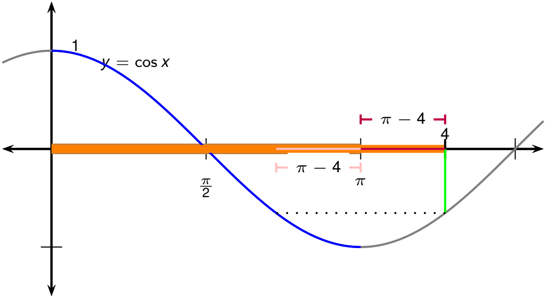

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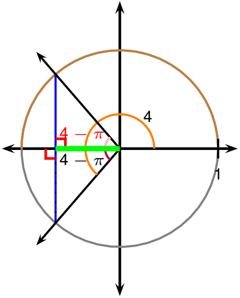

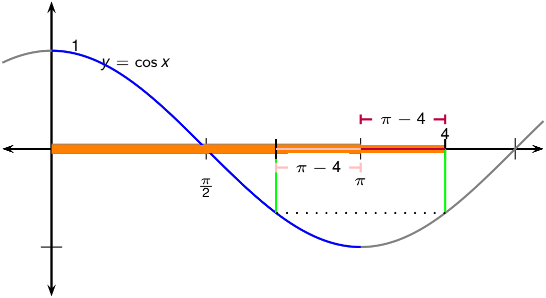

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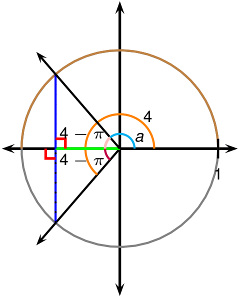
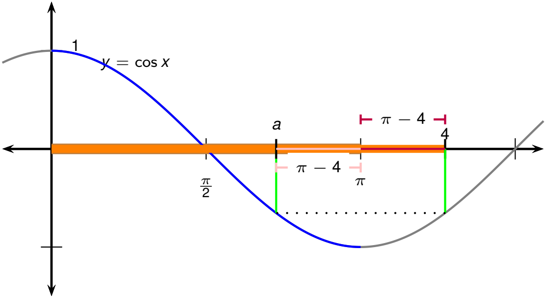


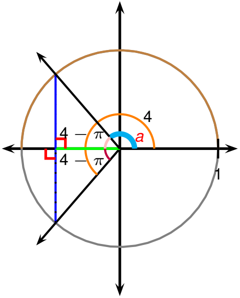


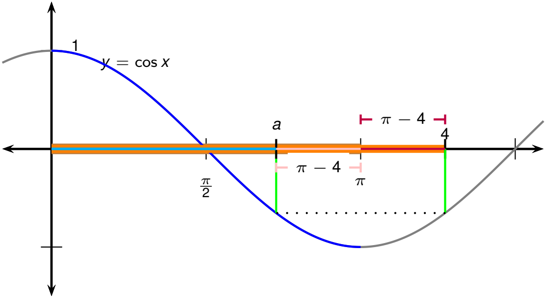
Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$a = ?$$



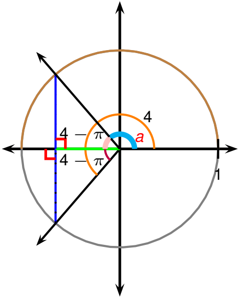


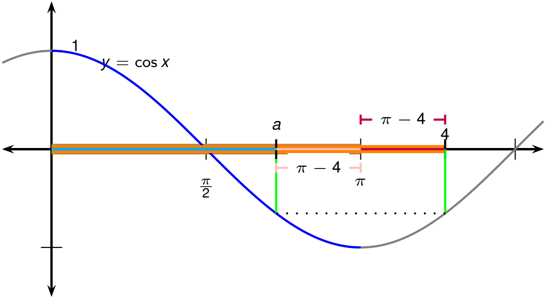
Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$a = \pi - (4 - \pi)$$



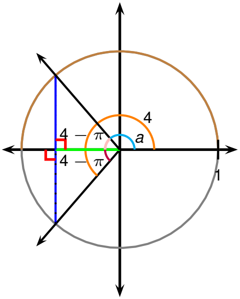


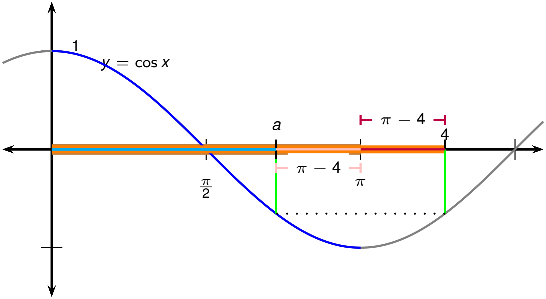
Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$a = \pi - (4 - \pi) = 2\pi - 4$$



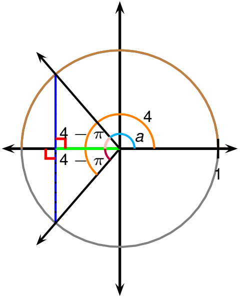


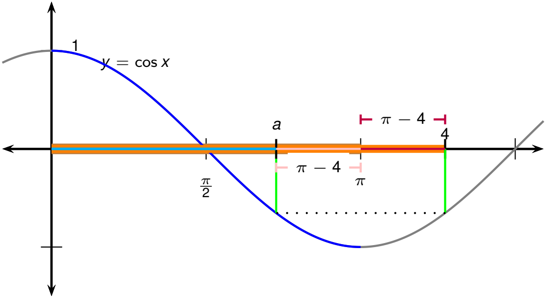
Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$
\begin{aligned}
a &= \pi - (4 - \pi) = 2\pi - 4 \\
\text{Therefore} \quad \arccos(\cos 4) &= \arccos(\cos a)
\end{aligned}
$$





Example

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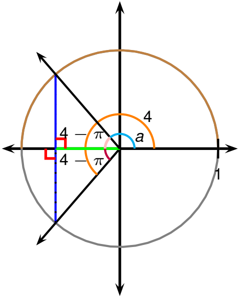
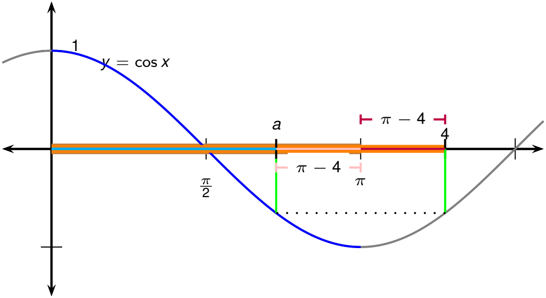
$$\begin{aligned} & & a &= \pi - (4-\pi) = 2\pi - 4 \\ \text{Therefore} & & \arccos(\cos 4) &= \arccos(\cos a) \\ & & &= a \end{aligned}$$



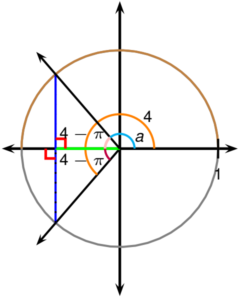


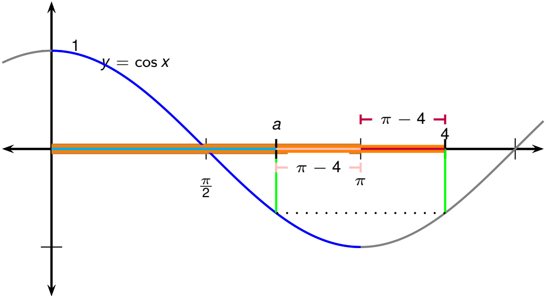
Example

Find $\arccos(\cos 4)$.

- 4 is not between 0 and π.
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$\begin{aligned} a &= \pi - (4-\pi) = 2\pi - 4 \\ \text{Therefore} \quad \arccos(\cos 4) &= \arccos(\cos a) \\ &= a = 2\pi - 4. \end{aligned}$$

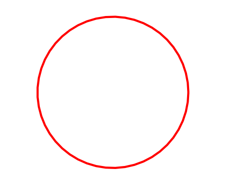




Example

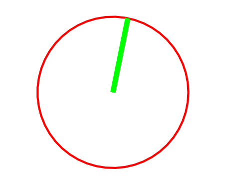
The horizon at sea is viewed at from a ship height of 10m. How far does the ship need to travel to reach the horizon just observed?

Example



The horizon at sea is viewed at from a ship height of 10m. How far does the ship need to travel to reach the horizon just observed? Assume earth is round with radius 6371 km and that the ship sails along the shortest curved path.

Example



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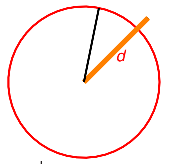
Example

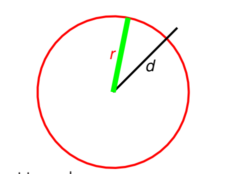


not to scale

The horizon at sea is viewed at from a ship height of 10m. How far does the ship need to travel to reach the horizon just observed? Assume earth is round with radius 6371 km and that the ship sails along the shortest curved path.

- Let d be the distance from eyes of seaman to the center of earth.

Example



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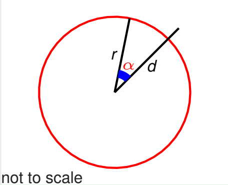
- Let d be the distance from eyes of seaman to the center of earth.
- Let r be the radius of earth.

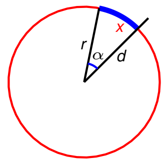
Example



The horizon at sea is viewed at from a ship height of 10m. How far does the ship need to travel to reach the horizon just observed? Assume earth is round with radius 6371 km and that the ship sails along the shortest curved path.

- Let *d* be the distance from eyes of seaman to the center of earth.
- Let *r* be the radius of earth. Let α be the indicated angle.

Example



not to scale

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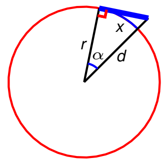
- Let d be the distance from eyes of seaman to the center of earth.
- Let r be the radius of earth. Let α be the indicated angle.
- Let the distance to the horizon be x.

Example



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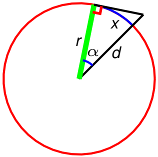

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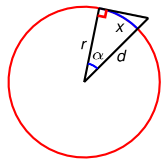
- Let the distance to the horizon be x.

$r = 6371\text{km}$

Example



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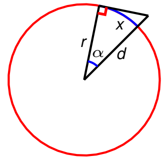
r=6371km
d=**?**

Example



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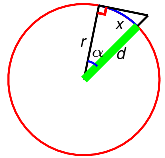
$r{=}6371\text{km}$
$d{=}6371\text{km} + 0.01\text{km}$

Example



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$$r = 6371\text{km}$$
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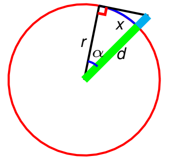
Example



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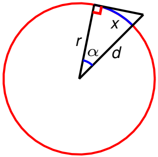
$$r = 6371\text{km}$$
$$d = 6371\text{km} + 0.01\text{km}$$

Example



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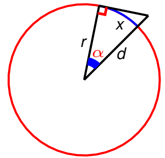
$$r = 6371\text{km}$$
$$d = 6371\text{km} + 0.01\text{km} = 6371.01\text{km}$$

Example



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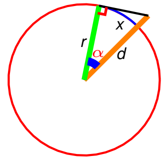
$$r = 6371\text{km}$$
$$d = 6371\text{km} + 0.01\text{km} = 6371.01\text{km}$$

$\cos\alpha =$ **?**

Example



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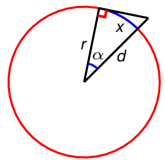
$$r = 6371\text{km}$$
$$d = 6371\text{km} + 0.01\text{km} = 6371.01\text{km}$$
$$\cos\alpha = \frac{r}{d}$$

Example



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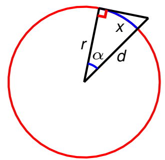
$$\begin{aligned} r &= 6371\text{km} \\ d &= 6371\text{km} + 0.01\text{km} = 6371.01\text{km} \\ \cos\alpha &= \frac{r}{d} \\ \alpha &= \arccos\left(\frac{r}{d}\right) \end{aligned}$$

Example



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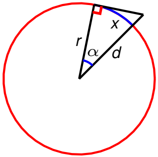
\cos\alpha &= \frac{r}{d} \\
\alpha &= \arccos\left(\frac{r}{d}\right) \\
x &= \textbf{?}
\end{aligned}$$

Example



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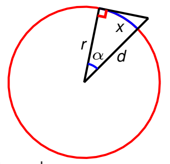
\cos\alpha &= \frac{r}{d}\\
\alpha &= \arccos\left(\frac{r}{d}\right)\\
x &= r\alpha
\end{aligned}
$$

Example



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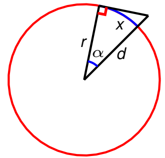
Example



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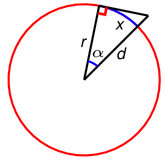
\cos\alpha &= \frac{r}{d}\\
\alpha &= \arccos\left(\frac{r}{d}\right)\\
x &= r\alpha = r\arccos\left(\frac{r}{d}\right) = 6371\text{km}\arccos\left(\frac{6371\text{km}}{6371.01\text{km}}\right)
\end{aligned}
$$

Example



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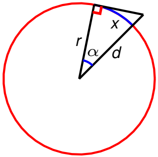
$$\alpha = \arccos\left(\frac{r}{d}\right)$$
$$x = r\alpha = r\arccos\left(\frac{r}{d}\right) = 6371\text{km}\arccos\left(\frac{6371\text{km}}{6371.01\text{km}}\right)$$

Example



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$$\begin{aligned} r &= 6371\text{km} \\ d &= 6371\text{km} + 0.01\text{km} = 6371.01\text{km} \\ \cos\alpha &= \frac{r}{d} \\ \alpha &= \arccos\left(\frac{r}{d}\right) \\ x &= r\alpha = r\arccos\left(\frac{r}{d}\right) = 6371\text{km}\arccos\left(\frac{6371\text{km}}{6371.01\text{km}}\right) \approx 11.29\text{km} \end{aligned}$$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

$\sin(2\arccos(x))$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.
To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$.

$$\sin(2\arccos(x))$$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.
To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\sin(2\arccos(x))$$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\sin(2\arccos(x)) = \sin(2y) \qquad \Big| \text{ Set } y = \arccos x$$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\begin{aligned}\sin(2\arccos(x)) &= \sin(2y) \\ &= \textbf{?}\end{aligned}$$

Set $y = \arccos x$
Express via $\sin y, \cos y$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\begin{aligned}\sin(2\arccos(x)) &= \sin(2y) \\ &= 2\cos y \sin y\end{aligned}$$

Set $y = \arccos x$

Express via $\sin y, \cos y$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$
\begin{aligned}
\sin(2\arccos(x)) &= \sin(2y) && \text{Set } y = \arccos x \\
&= 2\cos y \sin y && \text{Express via } \sin y, \cos y \\
&= 2\cos y \left(\pm\sqrt{1-\cos^2 y}\right) && \text{Express } \sin y \text{ via } \cos y
\end{aligned}
$$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

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&= 2\cos y\left(\pm\sqrt{1-\cos^2 y}\right) && \text{Express } \sin y \text{ via } \cos y \\
&= 2\cos y\sqrt{1-\cos^2 y} && \sin y > 0 \text{ because } 0 \le y \le \pi
\end{aligned}
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Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\begin{aligned}\sin(2\arccos(x)) &= \sin(2y) && \text{Set } y = \arccos x\\ &= 2\cos y \sin y && \text{Express via } \sin y, \cos y\\ &= 2\cos y\left(\pm\sqrt{1-\cos^2 y}\right) && \text{Express } \sin y \text{ via } \cos y\\ &= 2\cos y\sqrt{1-\cos^2 y} && \sin y > 0 \text{ because } 0 \leq y \leq \pi\end{aligned}$$

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Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

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\sin(2\arccos(x)) &= \sin(2y) && \text{Set } y = \arccos x \\
&= 2\cos y \sin y && \text{Express via } \sin y, \cos y \\
&= 2\cos y\left(\pm\sqrt{1-\cos^2 y}\right) && \text{Express } \sin y \text{ via } \cos y \\
&= 2\cos y\sqrt{1-\cos^2 y} && \sin y > 0 \text{ because } 0 \le y \le \pi \\
&= 2x\sqrt{1-x^2} && \text{use } x = \cos y
\end{aligned}
$$

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.
To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$\sin(2\arccos(x)) = \sin(2y)$	Set $y = \arccos x$
$= 2\cos y \sin y$	Express via $\sin y, \cos y$
$= 2\cos y\left(\pm\sqrt{1-\cos^2 y}\right)$	Express $\sin y$ via $\cos y$
$= 2\cos y\sqrt{1-\cos^2 y}$	$\sin y > 0$ because $0 \leq y \leq \pi$
$= 2x\sqrt{1-x^2}$	use $x = \cos y$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

$$\cos(3\arccos(x))$$

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Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

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Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\cos(3\arccos(x)) = \cos(3y) \qquad \Big| \; y = \arccos x$$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\cos(3\arccos(x)) = \cos(3y) = \cos(2y + y) \qquad \Big|\ y = \arccos x$$

Example

Rewrite $\cos(3 \arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\begin{aligned} \cos(3\arccos(x)) &= \cos(3y) = \cos(2y+y) \\ &= \mathbf{?} \end{aligned} \qquad \left| \begin{array}{l} y = \arccos x \\ \text{Angle sum f-la} \end{array} \right.$$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.
To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$$\begin{aligned} \cos(3\arccos(x)) &= \cos(3y) = \cos(2y+y) && \Big|\ y = \arccos x \\ &= \cos(2y)\cos y - \sin(2y)\sin y && \Big|\ \text{Angle sum f-la} \end{aligned}$$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$\cos(3\arccos(x)) = \cos(3y) = \cos(2y+y)$	$y = \arccos x$
$= \cos(2y)\cos y - \sin(2y)\sin y$	Angle sum f-la
$= (\textbf{?} \qquad)\cos y$	Express via
$- \textbf{?} \qquad \sin y$	$\sin y, \cos y$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

$\cos(3\arccos(x)) = \cos(3y) = \cos(2y+y)$	$y = \arccos x$
$= \cos(2y)\cos y - \sin(2y)\sin y$	Angle sum f-la
$= (\cos^2 y - \sin^2 y)\cos y$	Express via
$-\,\mathbf{?} \qquad \sin y$	$\sin y, \cos y$

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Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

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&= \cos(2y)\cos y - \sin(2y)\sin y && \Big|\ \text{Angle sum f-la} \\
&= (\cos^2 y - \sin^2 y)\cos y && \Big|\ \text{Express via} \\
&\quad -\textbf{?} \qquad\qquad \sin y && \Big|\ \sin y, \cos y
\end{aligned}$$

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Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.
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&\quad - 2\sin y \cos y \sin y && \Big| \; \sin y, \cos y
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$$\begin{array}{rl|l}
\cos(3\arccos(x)) &= \cos(3y) = \cos(2y+y) & y = \arccos x \\
&= \cos(2y)\cos y - \sin(2y)\sin y & \text{Angle sum f-la} \\
&= (\cos^2 y - \sin^2 y)\cos y & \text{Express via} \\
&\quad -2\sin y\cos y\sin y & \sin y, \cos y \\
&= \cos^3 y - \sin^2 y\cos y - 2\sin^2 y\cos y &
\end{array}$$

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&\quad - 2 \sin y \cos y \sin y && \Big|\; \sin y, \cos y \\
&= \cos^3 y - \sin^2 y \cos y - 2\sin^2 y \cos y
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&= (\cos^2 y - \sin^2 y)\cos y && \Big|\ \text{Express via} \\
&\quad - 2\sin y \cos y \sin y && \Big|\ \sin y, \cos y \\
&= \cos^3 y - \sin^2 y \cos y - 2\sin^2 y \cos y \\
&= \cos^3 y - 3\sin^2 y \cos y
\end{aligned}
$$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.
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&= (\cos^2 y - \sin^2 y)\cos y && \Big| \; \text{Express via} \\
&\quad - 2\sin y \cos y \sin y && \Big| \; \sin y, \cos y \\
&= \cos^3 y - \sin^2 y \cos y - 2\sin^2 y \cos y \\
&= \cos^3 y - 3\sin^2 y \cos y && \Big| \; \text{Express } \sin y \text{ via } \cos y \\
&= \cos^3 y - 3(\textbf{?} \qquad\qquad)\cos y
\end{aligned}
$$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

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&= (\cos^2 y - \sin^2 y)\cos y && \big|\ \text{Express via} \\
&\quad - 2\sin y\cos y\sin y && \big|\ \sin y, \cos y \\
&= \cos^3 y - \sin^2 y\cos y - 2\sin^2 y\cos y \\
&= \cos^3 y - 3\sin^2 y\cos y && \big|\ \text{Express } \sin y \text{ via } \cos y \\
&= \cos^3 y - 3(1-\cos^2 y)\cos y
\end{aligned}
$$

Example

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the $\cos$ function.

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&= (\cos^2 y - \sin^2 y)\cos y && \Big|\; \text{Express via} \\
&\quad - 2\sin y \cos y \sin y && \Big|\; \sin y, \cos y \\
&= \cos^3 y - \sin^2 y \cos y - 2\sin^2 y \cos y && \\
&= \cos^3 y - 3\sin^2 y \cos y && \Big|\; \text{Express } \sin y \text{ via } \cos y \\
&= \cos^3 y - 3(1-\cos^2 y)\cos y && \\
&= 4\cos^3 y - 3\cos y &&
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Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.
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\begin{aligned}
\cos(3\arccos(x)) &= \cos(3y) = \cos(2y+y) && \quad y = \arccos x\\
&= \cos(2y)\cos y - \sin(2y)\sin y && \quad \text{Angle sum f-la}\\
&= (\cos^2 y - \sin^2 y)\cos y && \quad \text{Express via}\\
&\quad - 2\sin y\cos y\sin y && \quad \sin y, \cos y\\
&= \cos^3 y - \sin^2 y\cos y - 2\sin^2 y\cos y &&\\
&= \cos^3 y - 3\sin^2 y\cos y && \quad \text{Express } \sin y \text{ via } \cos y\\
&= \cos^3 y - 3(1-\cos^2 y)\cos y &&\\
&= 4\cos^3 y - 3\cos y &&\\
&= 4x^3 - 3x && \quad x = \cos y
\end{aligned}
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Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

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&= (\cos^2 y - \sin^2 y)\cos y && \Big| \; \text{Express via} \\
&\quad - 2\sin y \cos y \sin y && \Big| \; \sin y, \cos y \\
&= \cos^3 y - \sin^2 y \cos y - 2\sin^2 y \cos y && \\
&= \cos^3 y - 3\sin^2 y \cos y && \Big| \; \text{Express } \sin y \text{ via } \cos y \\
&= \cos^3 y - 3(1-\cos^2 y)\cos y && \\
&= 4\cos^3 y - 3\cos y && \\
&= 4x^3 - 3x && \Big| \; x = \cos y
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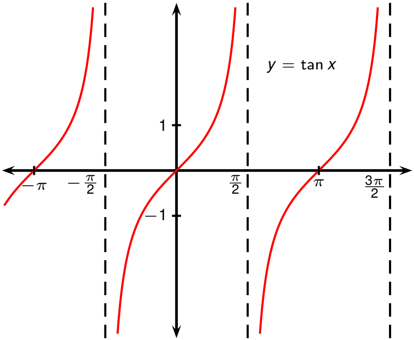
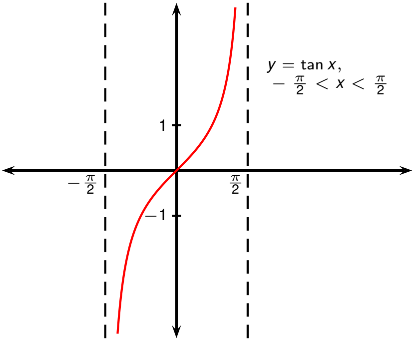
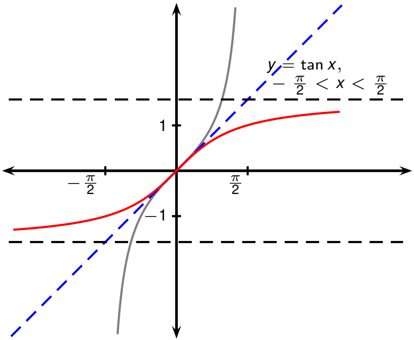


- $\tan x$ isn't one-to-one.



- $\tan x$ isn't one-to-one.
- Restrict the domain to $(-\frac{\pi}{2}, \frac{\pi}{2})$.



- $\tan x$ isn't one-to-one.
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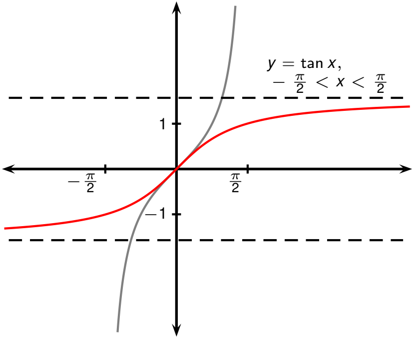

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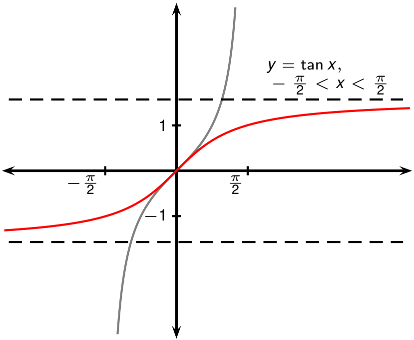

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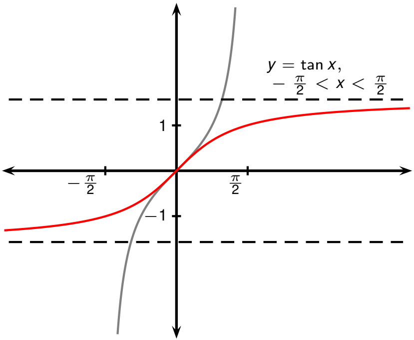
- Domain of arctan: **?**
- Range of arctan:



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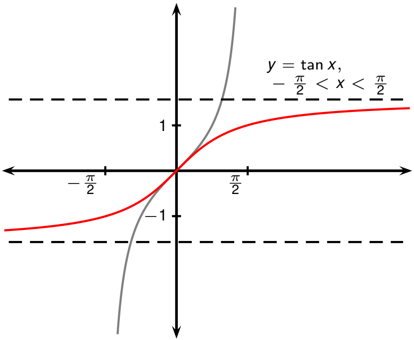
- The inverse is called $\tan^{-1}$ or $\arctan$.
- $\arctan x = y \Leftrightarrow \tan y = x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$.
- Domain of $\arctan$: $(-\infty, \infty)$.
- Range of $\arctan$:



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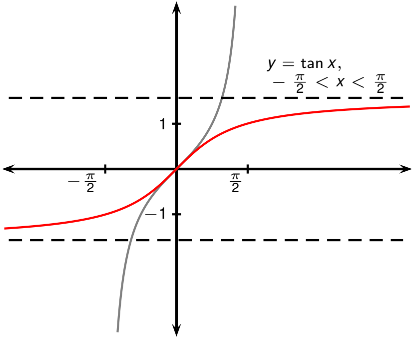

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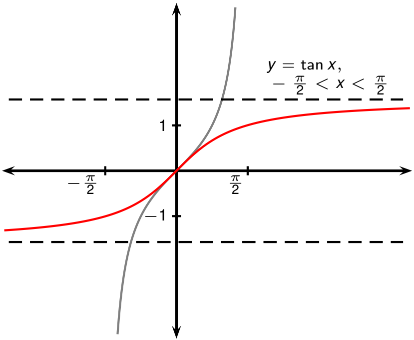

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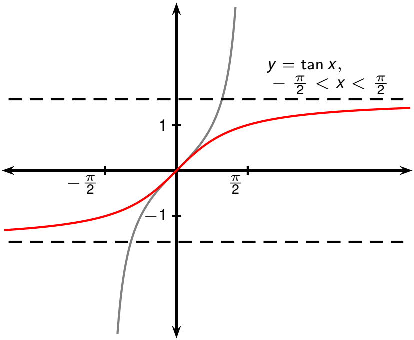
- Domain of arctan: $(-\infty, \infty)$.
- Range of arctan: $(-\frac{\pi}{2}, \frac{\pi}{2})$.
- $\lim_{x\to\infty} \arctan x =$ **?**
- $\lim_{x\to-\infty} \arctan x =$



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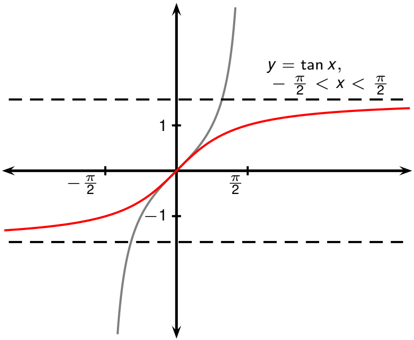
- $\lim_{x \to \infty} \arctan x = \frac{\pi}{2}$.
- $\lim_{x \to -\infty} \arctan x =$



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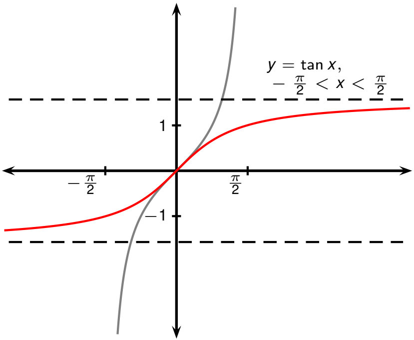
- Domain of arctan: $(-\infty, \infty)$.
- Range of arctan: $(-\frac{\pi}{2}, \frac{\pi}{2})$.
- $\lim\limits_{x\to\infty} \arctan x = \dfrac{\pi}{2}$.
- $\lim\limits_{x\to-\infty} \arctan x =$ **?**



- $\tan x$ isn't one-to-one.
- Restrict the domain to $(-\frac{\pi}{2}, \frac{\pi}{2})$.
- The inverse is called $\tan^{-1}$ or $\arctan$.
- $\arctan x = y \Leftrightarrow \tan y = x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$.
- Domain of $\arctan$: $(-\infty, \infty)$.
- Range of $\arctan$: $(-\frac{\pi}{2}, \frac{\pi}{2})$.
- $\lim\limits_{x\to\infty} \arctan x = \dfrac{\pi}{2}$.
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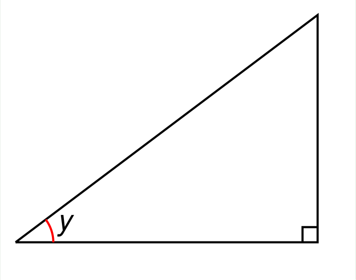
Example

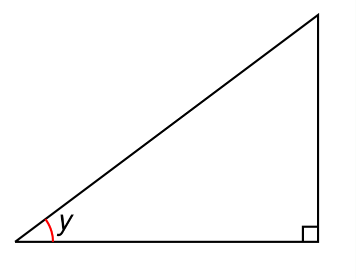
Simplify the expression $\cos(\arctan x)$.



Example

Simplify the expression $\cos(\arctan x)$.

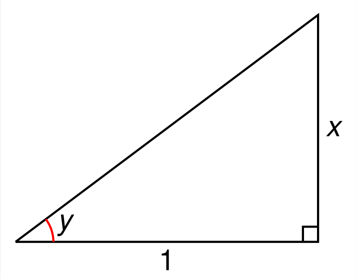
- Let $y = \arctan x$, so $\tan y = x$.



Example

Simplify the expression $\cos(\arctan x)$.

- Let $y = \arctan x$, so $\tan y = x$.
- Draw a right triangle with opposite x and adjacent 1.



Example

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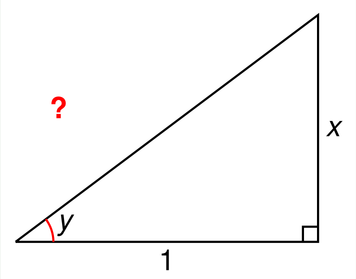
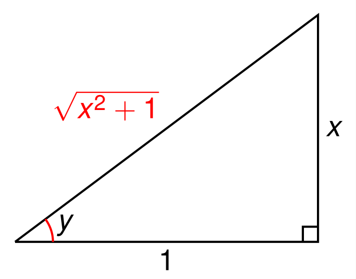
- Let $y = \arctan x$, so $\tan y = x$.
- Draw a right triangle with opposite x and adjacent 1.
- Length of hypotenuse $=$ **?**



Example

Simplify the expression $\cos(\arctan x)$.

- Let $y = \arctan x$, so $\tan y = x$.
- Draw a right triangle with opposite x and adjacent 1.
- Length of hypotenuse $= \sqrt{1^2 + x^2}$.



Example

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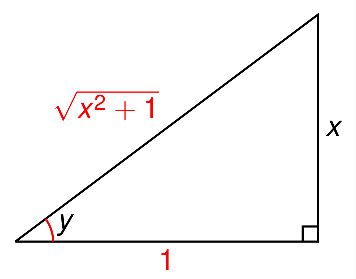

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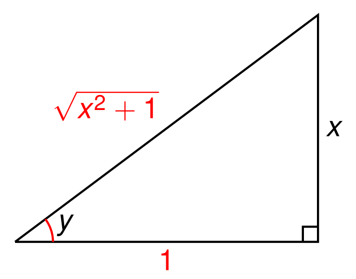

The remaining inverse trigonometric functions aren't used as often:

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y = \operatorname{arccsc} x & (|x| \geq 1) & \Leftrightarrow & \csc y = x & \text{and} & y \in \left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right] \\
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We will however make use of $\operatorname{arcsec} x$: we discuss in detail its domain.

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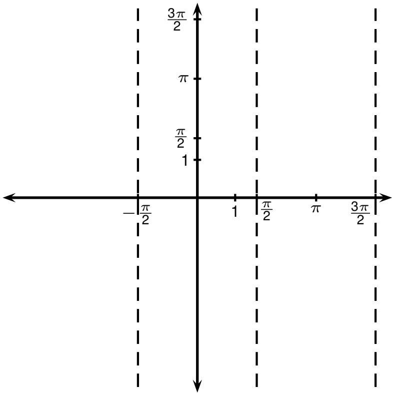

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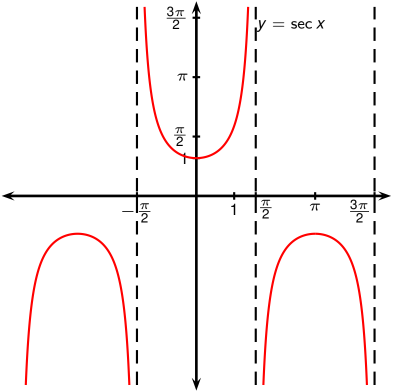

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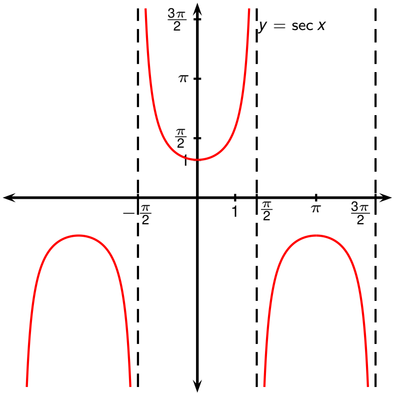

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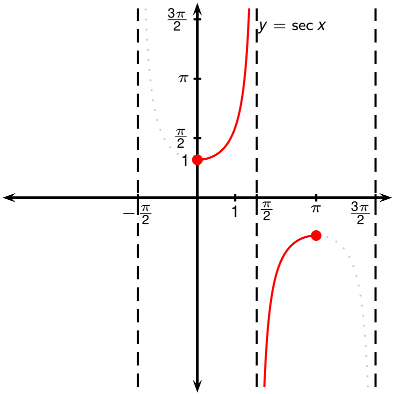

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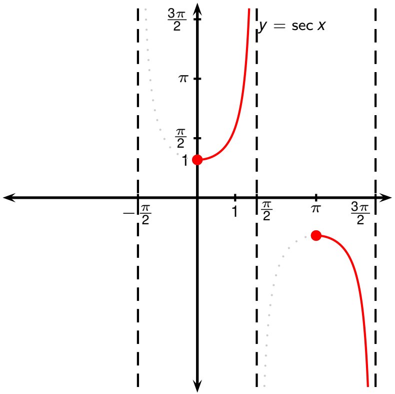

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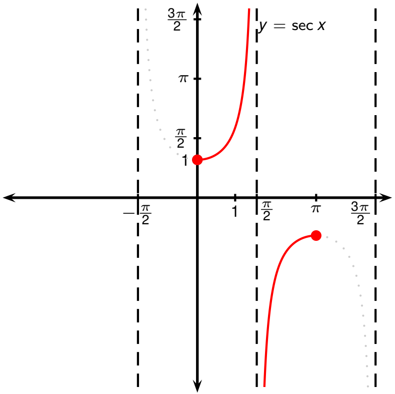

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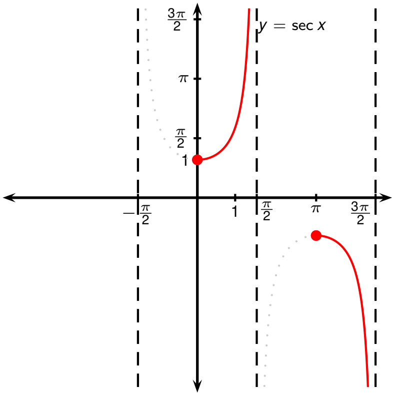

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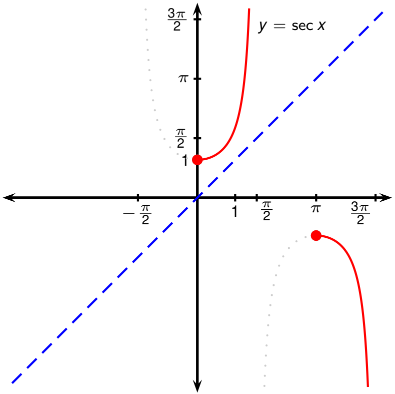

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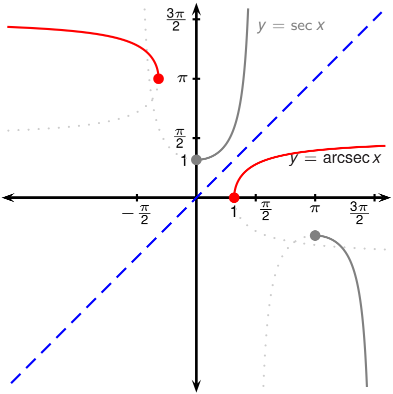

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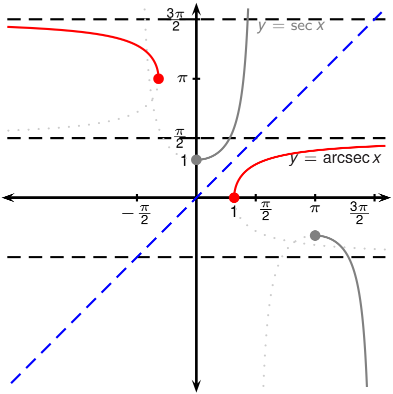

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