Calculus I
Homework Continuity
Lecture 4

1. Find the (implied) domain of $f(x)$. Extend the definition of f at $x = 3$ to make f continuous at 3.

   (a) $f(x) = \dfrac{x^2 - x - 6}{x - 3}$.

   (b) $f(x) = \dfrac{x^3 - 27}{x^2 - 9}$.

2. Use the Intermediate Value Theorem to show that there is a real number solution of the given equation in the specified interval.

   (a) $x^5 + x - 3 = 0$ where $x \in (1, 2)$.

   (b) $\sqrt[4]{x} = 1 - x$ where $x \in \mathbb{R}$ (i.e., x is an arbitrary real number).

   (c) $\cos x = 2x$, where $x \in (0, 1)$.

   (d) $\sin x = x^2 - x - 1$, where $x \in \mathbb{R}$ (i.e., x is an arbitrary real number).

   (e) $\cos x = x^4$, where $x \in \mathbb{R}$ (i.e., x is an arbitrary real number).

   (f) $x^5 - x^2 + x + 3 = 0$, where $x \in \mathbb{R}$.

3.

   (a) i. Solve the equation $x^2 + 13x + 41 = 1$.

   ii. Use the intermediate value theorem to prove that the equation $x^2 + 13x + 41 = \sin x$ has at least two solutions, lying between the two solutions to 3.a.i.

   (b) i. Solve the equation $x^2 - 15x + 55 = 1$.

   ii. Use the intermediate value theorem to prove that the equation $x^2 - 15x + 55 = \cos x$ has at least two solutions, lying between the two solutions to the equation in the preceding item.

4. **This problem will not appear on the quiz.** For which values of x is f continuous?

   - $f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$
   - $f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ x & \text{if } x \text{ is irrational} \end{cases}$

5. **This problem is too difficult for a test or a quiz.** Show that $f(x)$ is continuous at all irrational points and discontinuous at all rational ones.

$$f(x) = \begin{cases} \frac{1}{q^2} & \text{if } x \text{ is rational and } x = \frac{p}{q} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

   where in the first item p, q are relatively prime integers (i.e., integers without a common divisor).