Precalculus
Homework Lecture 3

1. Use the known values of $\sin 30°, \cos 30°, \sin 45°, \cos 45°, \sin 60°, \cos 60°, \ldots$, the angle sum formulas and the cofunction identities to find an exact value (using radicals) for the trigonometric function.

   (a) The six trigonometric functions of $105° = 45° + 60°$:

   - $\sin(105°)$.

     answer: $\frac{\sqrt{6}+\sqrt{2}}{4}$

   - $\cos(105°)$. Should your answer be a positive or a negative number?

     answer: $\frac{\sqrt{2}-\sqrt{6}}{4}$

   - $\tan(105°)$.

     answer: $-\sqrt{3}-2$

   - $\cot(105°)$.

     answer: $\sqrt{3}-2$

   - $\sec(105°)$.

     answer: $-\sqrt{6}-\sqrt{2}$

   - $\csc(105°)$.

     answer: $\sqrt{6}-\sqrt{2}$

   (b) The six trigonometric functions of $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$:

   - $\sin\left(\frac{\pi}{12}\right)$.

     answer: $\frac{\sqrt{6}-\sqrt{2}}{4}$

   - $\cos\left(\frac{\pi}{12}\right)$. Should $\sin\left(\frac{\pi}{12}\right)$ be larger or smaller than $\cos\left(\frac{\pi}{12}\right)$?

     answer: $\frac{\sqrt{2}+\sqrt{6}}{4}$

   - $\tan\left(\frac{\pi}{12}\right)$.

     answer: $-\sqrt{3}+2$

   - $\cot\left(\frac{\pi}{12}\right)$.

     answer: $\sqrt{3}+2$

   - $\sec\left(\frac{\pi}{12}\right)$.

     answer: $\sqrt{6}-\sqrt{2}$

   - $\csc\left(\frac{\pi}{12}\right)$.

     answer: $\sqrt{6}+\sqrt{2}$

2. Simplify to a trigonometric function of the angle θ. The answer key has not been proofread, use with caution.

   (a) $\sin\left(\frac{\pi}{2} - \theta\right)$.

   answer: $\cos\theta$

   (b) $\cos\left(\frac{13\pi}{2} - \theta\right)$.

   answer: $\sin\theta$

   (c) $\tan(\pi - \theta)$

   answer: $-\tan\theta$

   (d) $\cot\left(\frac{3\pi}{2} - \theta\right)$

   answer: $\tan\theta$

   (e) $\csc\left(\frac{3\pi}{2} + \theta\right)$

   answer: $\sec\theta$

3. **Problems 3.c and 3.d are considered challenge problems and will not be tested/quizzed upon.** Using the power-reducing formulas, rewrite the expression in terms of first powers of the cosines and sines of multiples of the angle θ.

   (a) $\sin^4\theta$.

   answer: $\frac{1}{8}\cos(4\theta) - \frac{1}{2}\cos(2\theta) + \frac{3}{8}$

   (b) $\cos^4\theta$.

   answer: $\frac{1}{8}\cos(4\theta) + \frac{1}{2}\cos(2\theta) + \frac{3}{8}$

   (c) $\sin^6\theta$.

   answer: $\sin^6\theta = -\frac{1}{32}\cos(6\theta) + \frac{3}{16}\cos(4\theta) - \frac{15}{32}\cos(2\theta) + \frac{5}{16}$

(d) $\cos^6\theta$.

answer: $\cos^6\theta = \frac{1}{32}\cos(6\theta) + \frac{3}{16}\cos(4\theta) + \frac{15}{32}\cos(2\theta) + \frac{5}{16}$

4. Use the sum-to-product formulas to find all solutions of the trigonometric equation in the interval $[0, 2\pi)$.

   Please note that typing a query such as "solve(sin(x)+sin (3x)=0)" at `www.wolframalpha.com` will provide you with a correct answer and a function plot.

   (a) $\sin(x) + \sin(3x) = 0$.

   answer: $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

   (b) $\cos(x) + \cos(-3x) = 0$.

   answer: $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

   (c) $\sin(x) - \sin(3x) = 0$.

   answer: $x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}$

   (d) $\cos(2x) - \cos(3x) = 0$.

   answer: $x = 0, \frac{2}{5}\pi, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$