Precalculus
Lecture 8
Trigonometric Equations

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https://github.com/tmilev/freecalc

2020

Outline

1. Trigonometric equations and inequalities
   - The Equations $\sin x = A$, $\cos x = B$
   - Equations that reduce to $\sin x = A$, $\cos x = B$

Outline

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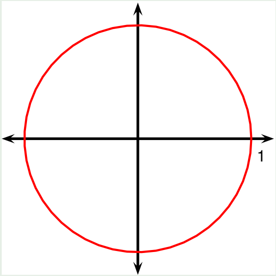
Trigonometric equations

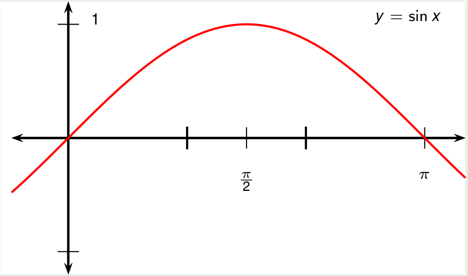
- Some problems will not ask you to prove a trigonometric identity, but rather to solve a trigonometric equation.
- Consider the problem of finding all values of x for which $\sin x = \sin(2x) = 2\sin x \cos x$.
- This is not a trigonometric identity - the two sides are different.
- However, there are values for x which the above equality holds.

Example

Find all solutions and then find those that lie between $-360°$ and $360°$.

$$\sin\theta = \frac{\sqrt{3}}{2}$$





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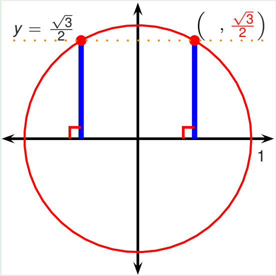

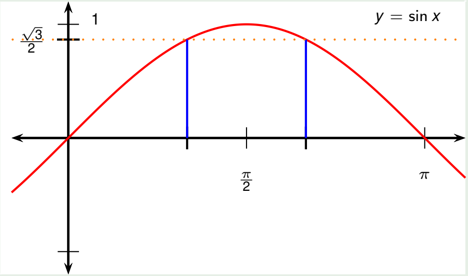

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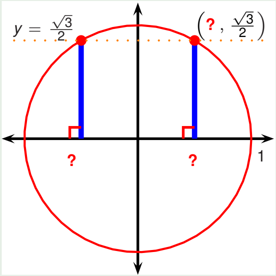

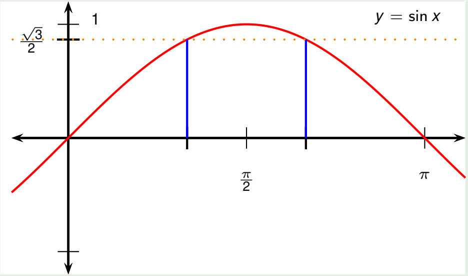

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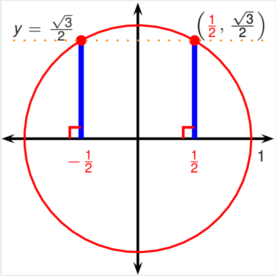
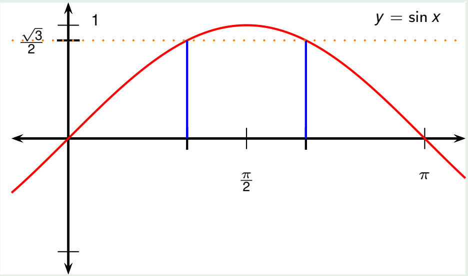
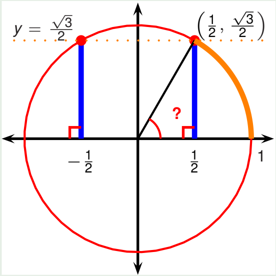


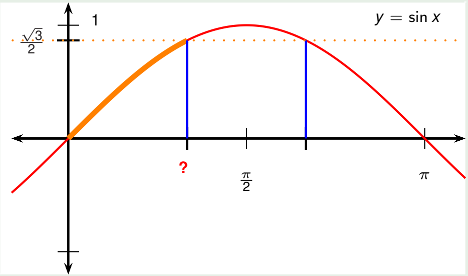


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned} \sin\theta &= \frac{\sqrt{3}}{2} \\ \theta &= \textbf{?} \end{aligned}$$

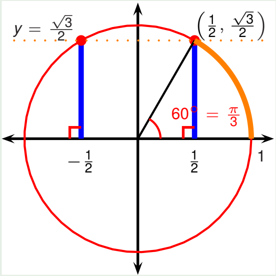


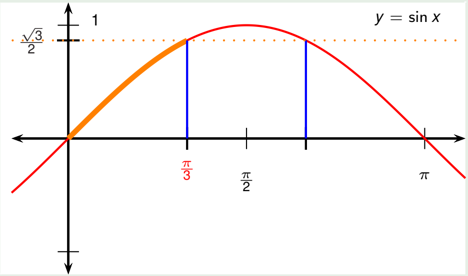


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned}\sin\theta &= \frac{\sqrt{3}}{2}\\ \theta &= 60^\circ\end{aligned}$$





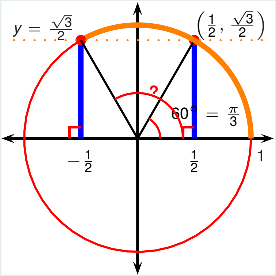
Example

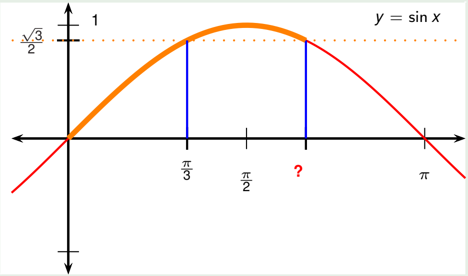
Find all solutions and then find those that lie between $-360°$ and $360°$.

$$\sin\theta = \frac{\sqrt{3}}{2}$$
$$\theta = 60°$$

or

?

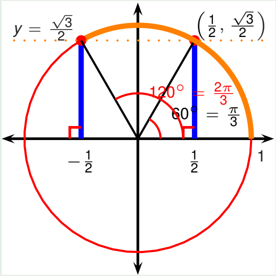


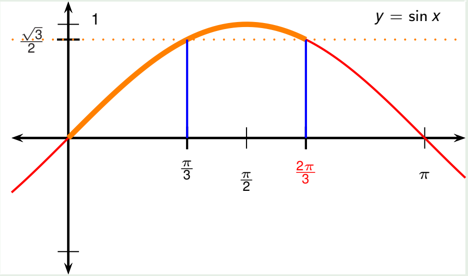


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned} \sin\theta &= \frac{\sqrt{3}}{2} \\ \theta &= 60^\circ \\ &\quad \textbf{or} \\ &\quad 120^\circ \end{aligned}$$

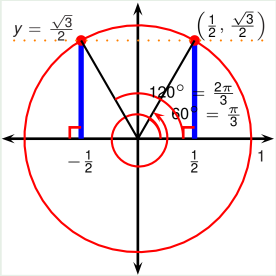


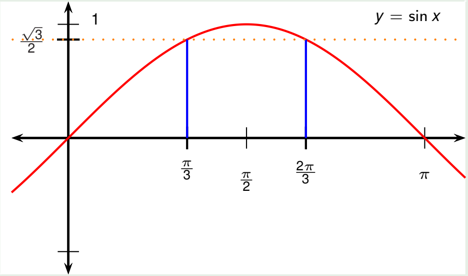


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned} \sin\theta &= \frac{\sqrt{3}}{2} \\ \theta &= 60^\circ + k\cdot 360^\circ \\ &\quad \textbf{or} \\ &\quad 120^\circ \end{aligned}$$

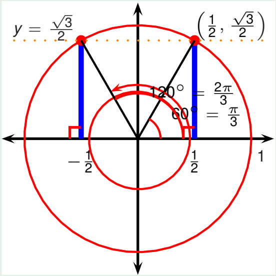


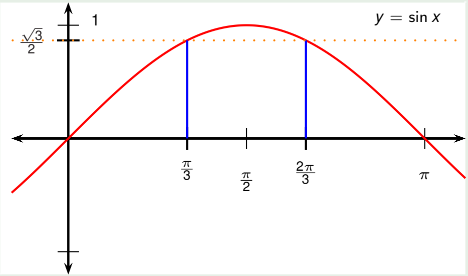


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned} \sin\theta &= \frac{\sqrt{3}}{2} \\ \theta &= 60^\circ + k\cdot 360^\circ \\ &\quad \textbf{or} \\ &\quad 120^\circ + k\cdot 360^\circ \end{aligned}$$

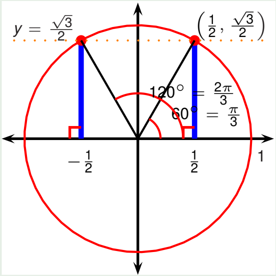


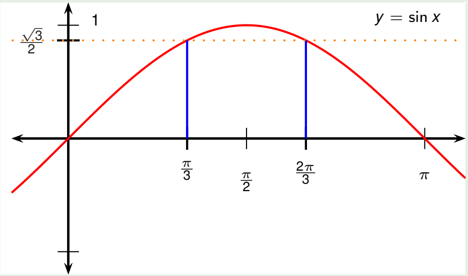


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned} \sin\theta &= \frac{\sqrt{3}}{2} \\ \theta &= 60^\circ + k\cdot 360^\circ = \quad \dots \quad -660^\circ, \\ &\quad \textbf{or} \qquad\qquad\qquad\quad \dots \quad k=-2 \\ &\quad 120^\circ + k\cdot 360^\circ = \dots \quad -600^\circ, \end{aligned}$$

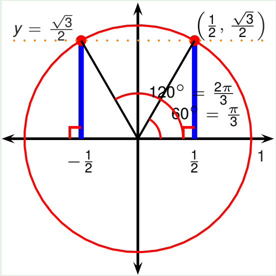


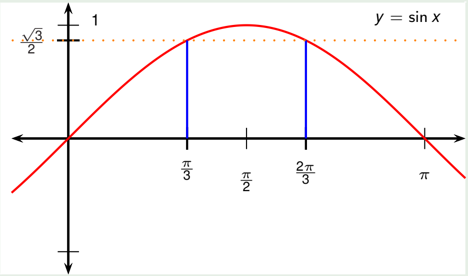


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned}\sin\theta &= \frac{\sqrt{3}}{2}\\ \theta &= 60^\circ + k\cdot 360^\circ = \dots\ \underset{k=-2}{-660^\circ},\ \underset{k=-1}{-300^\circ},\\ &\textbf{or}\\ &\quad 120^\circ + k\cdot 360^\circ = \dots\ -600^\circ,\ -240^\circ,\end{aligned}$$





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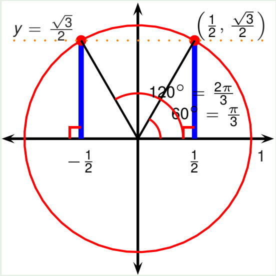
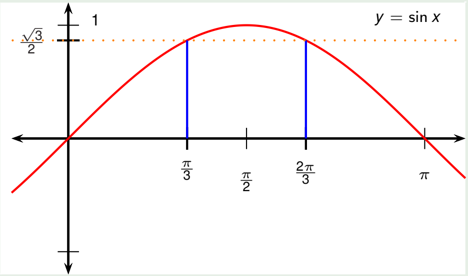
&\quad 120^\circ + k\cdot 360^\circ = \dots\ -600^\circ,\ -240^\circ,\ 120^\circ,
\end{aligned}$$

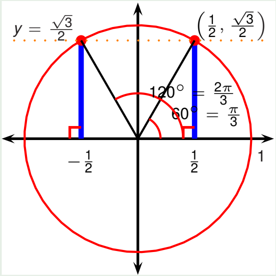


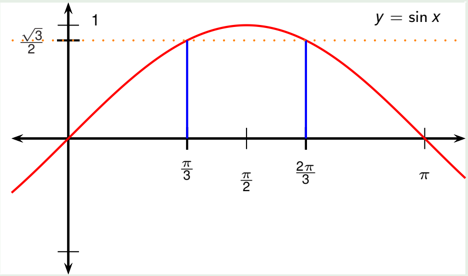


Example

Find all solutions and then find those that lie between -360° and 360°.

$$\begin{aligned}
\sin\theta &= \frac{\sqrt{3}}{2}\\
\theta &= 60^\circ + k\cdot 360^\circ = \quad \dots\ -660^\circ,\ -300^\circ,\ 60^\circ,\ 420^\circ,\ \dots\\
&\textbf{or} \qquad\qquad\qquad\quad \cdots\ k=-2\quad k=-1\quad k=0\quad k=1\ \cdots\\
& 120^\circ + k\cdot 360^\circ = \dots\ -600^\circ,\ -240^\circ,\ 120^\circ,\ 480^\circ,\ \dots
\end{aligned}$$





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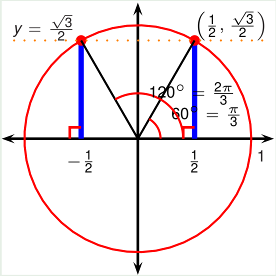

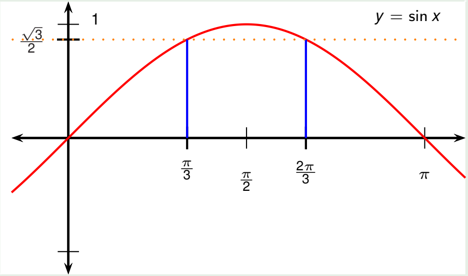

Example

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$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = 60^\circ + k\cdot 360^\circ = \underset{\cdots}{\dots} \underset{k=-2}{-660^\circ}, \underset{k=-1}{-300^\circ}, \underset{k=0}{60^\circ}, \underset{k=1}{420^\circ}, \underset{\cdots}{\dots}$$

or

$$120^\circ + k\cdot 360^\circ = \dots -600^\circ, -240^\circ, 120^\circ, 480^\circ, \dots$$

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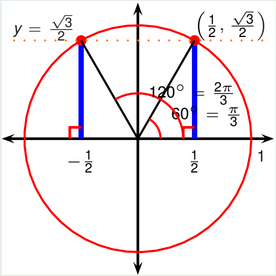

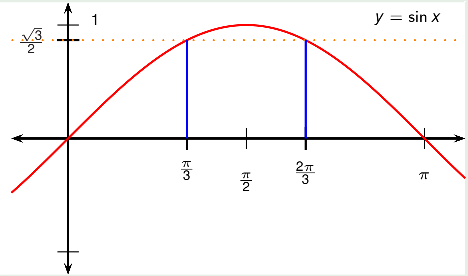

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$$
\begin{array}{rcl}
\sin\theta &=& \dfrac{\sqrt{3}}{2}\\
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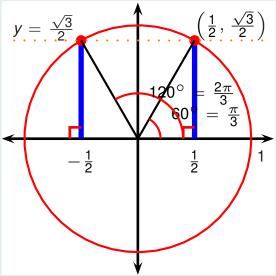
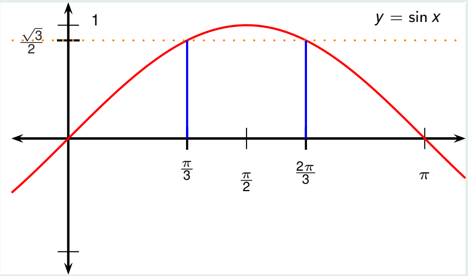
&& \textbf{or}\\
&& 120^\circ + k\cdot 360^\circ = \dots -600^\circ, -240^\circ, 120^\circ, 480^\circ, \dots\\
\hline
\theta &=& \cancel{\dots} \cancel{-660^\circ}, -300^\circ, 60^\circ, \cancel{420^\circ}, \cancel{\dots}\\
&& \cancel{\dots} \cancel{-600^\circ}, -240^\circ, 120^\circ, \cancel{480^\circ}, \cancel{\dots}
\end{array}
$$

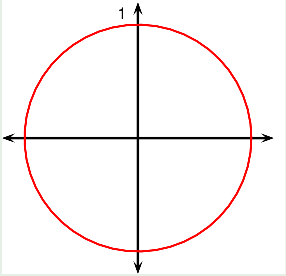


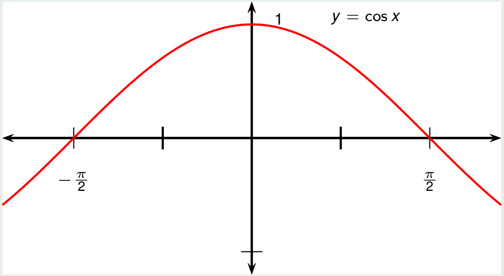


Example

Find all solutions and then find those that lie between -180° and 180°.

$$\cos\theta = \frac{\sqrt{2}}{2}$$





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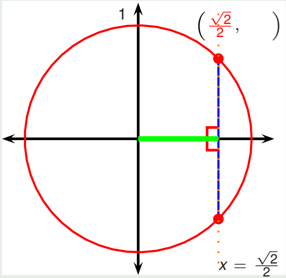

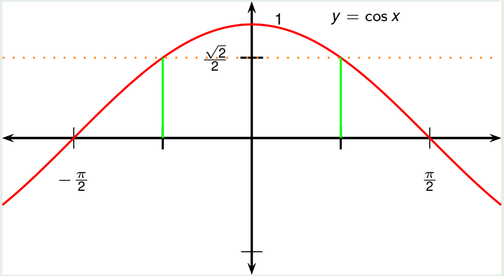

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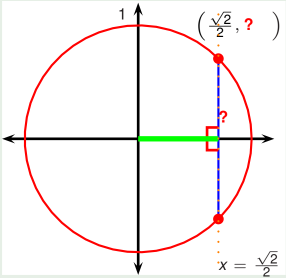

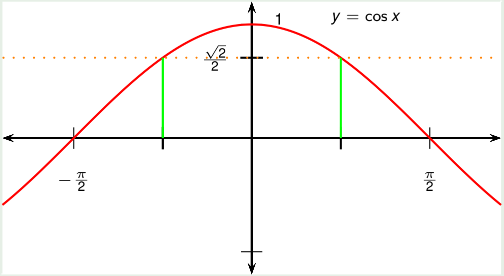

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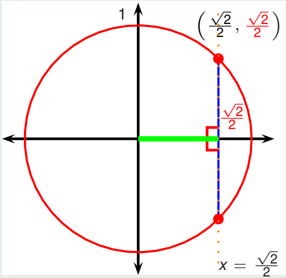
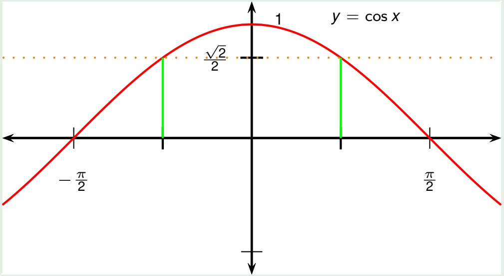
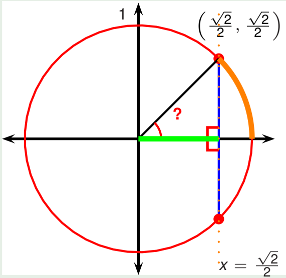


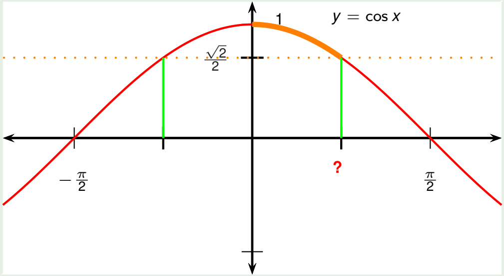


Example

Find all solutions and then find those that lie between -180° and 180°.

$$\cos\theta = \frac{\sqrt{2}}{2}$$
$$\theta = \textbf{?}$$

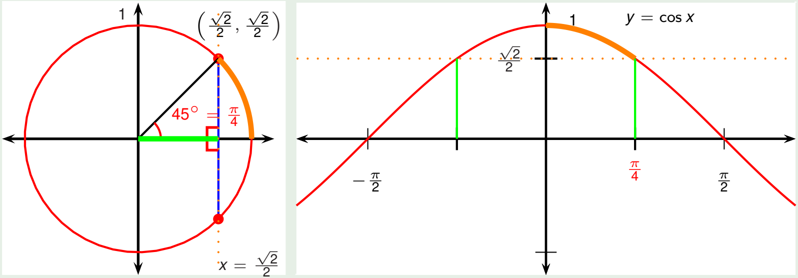




Example

Find all solutions and then find those that lie between -180° and 180°.

$$\cos\theta = \frac{\sqrt{2}}{2}$$
$$\theta = 45^\circ$$



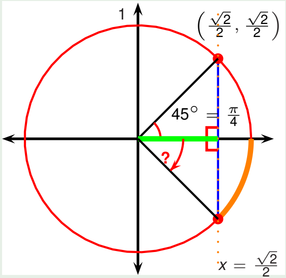
Example

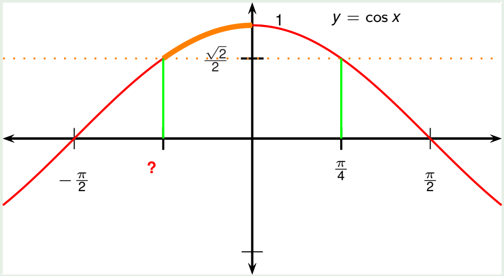
Find all solutions and then find those that lie between -180° and 180°.

$$\cos\theta = \frac{\sqrt{2}}{2}$$
$$\theta = 45^\circ$$

or

?





Example

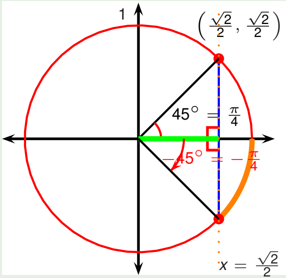
Find all solutions and then find those that lie between -180° and 180°.

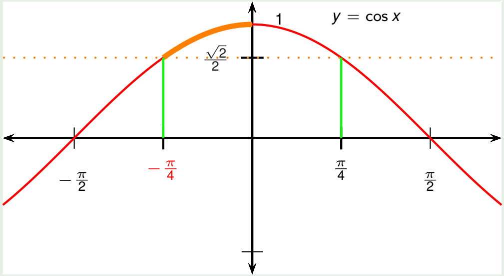
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ$$

or

$$-45^\circ$$





Example

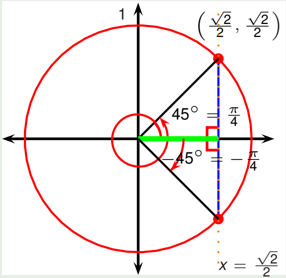
Find all solutions and then find those that lie between -180° and 180°.

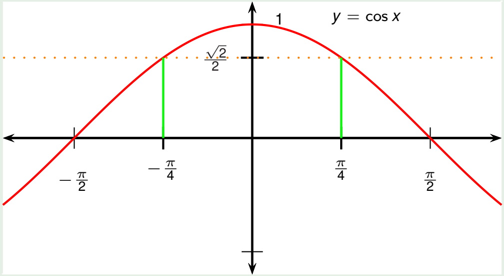
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ + k \cdot 360^\circ$$

or

$$-45^\circ$$





Example

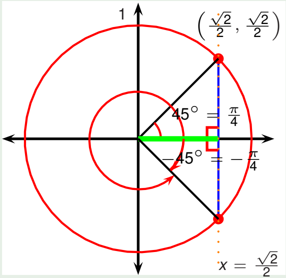
Find all solutions and then find those that lie between -180° and 180°.

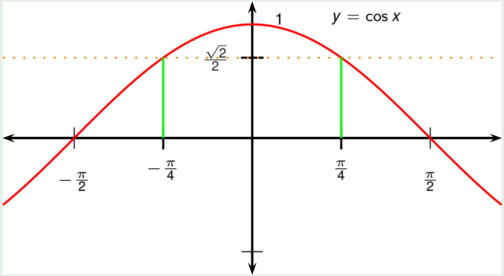
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ + k\cdot 360^\circ$$

or

$$-45^\circ + k\cdot 360^\circ$$





Example

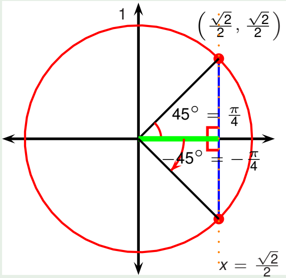
Find all solutions and then find those that lie between -180° and 180°.

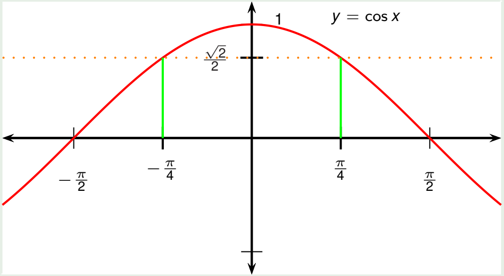
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ + k\cdot 360^\circ = \quad \dots \quad -675^\circ,$$

or $\quad \dots \quad k=-2$

$$-45^\circ + k\cdot 360^\circ = \dots \quad -765^\circ,$$





Example

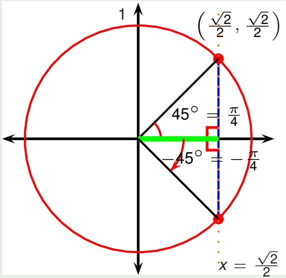
Find all solutions and then find those that lie between -180° and 180°.

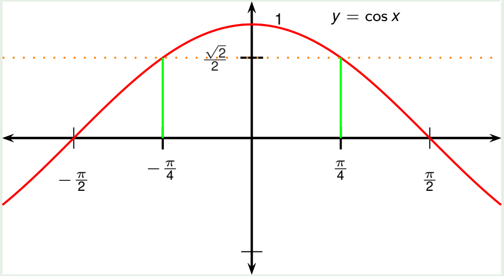
$$\cos\theta = \frac{\sqrt{2}}{2}$$

$$\theta = 45^\circ + k\cdot 360^\circ = \quad \dots \quad -675^\circ, \quad -315^\circ,$$

or $\qquad \cdots \qquad k=-2 \qquad k=-1$

$$-45^\circ + k\cdot 360^\circ = \dots -765^\circ, \quad -405^\circ,$$





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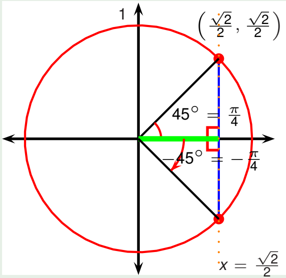

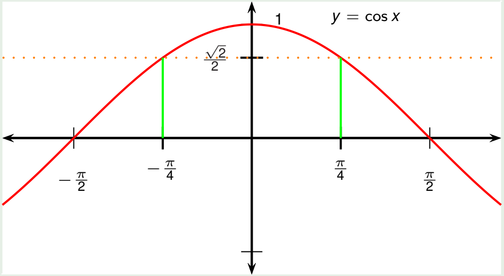

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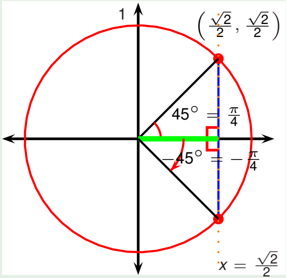

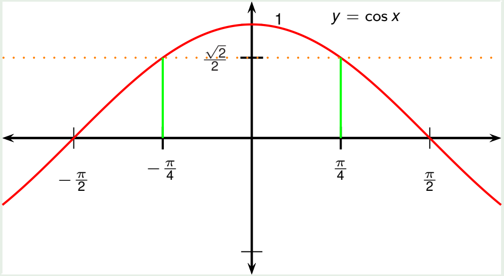

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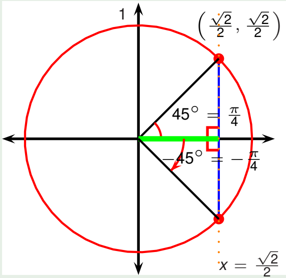

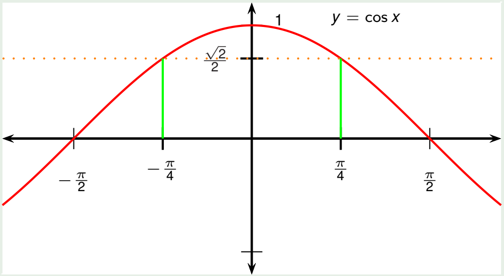

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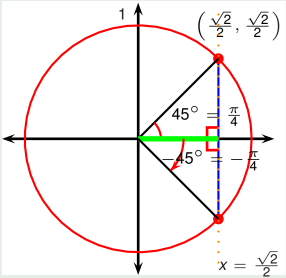

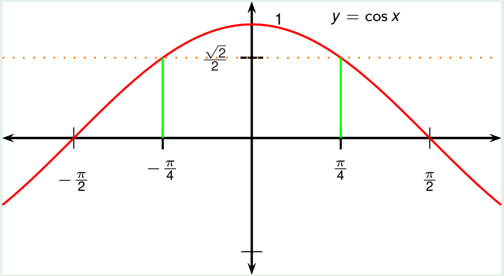

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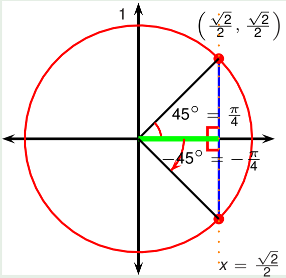
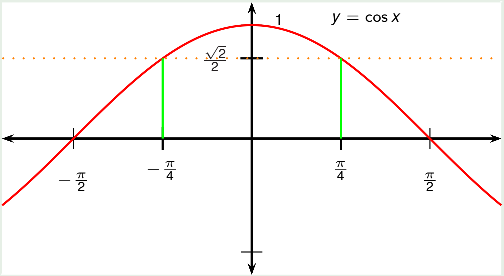
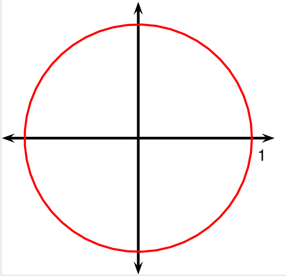


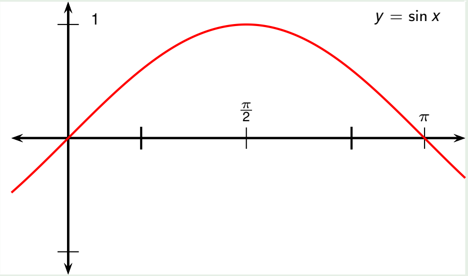


Example

Find all solutions of the equation.

$$\sin\theta = \frac{3}{5}$$





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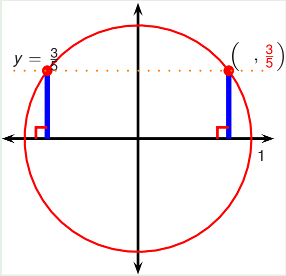

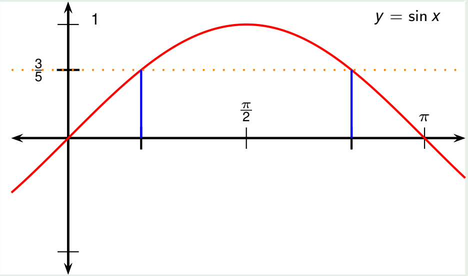

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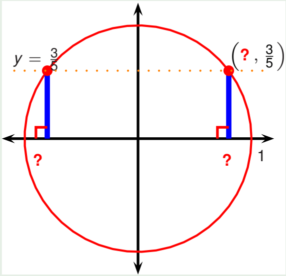

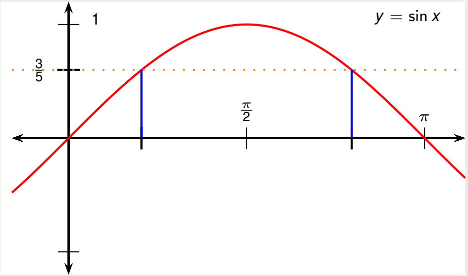

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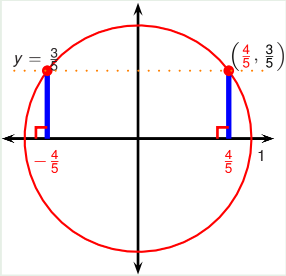
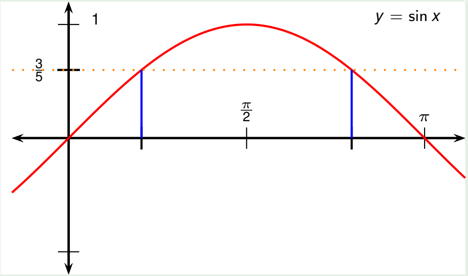


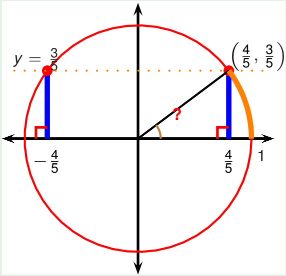


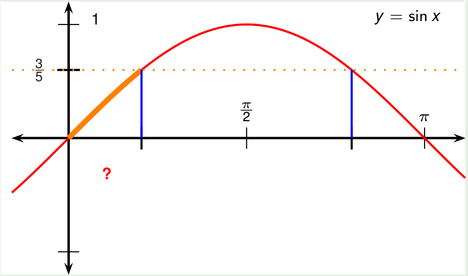
Example

Find all solutions of the equation.

$$\sin\theta = \frac{3}{5}$$

$$\theta = \textbf{?}$$



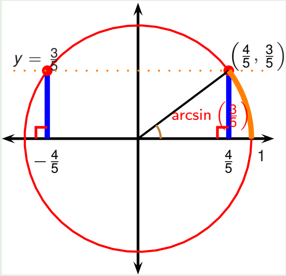


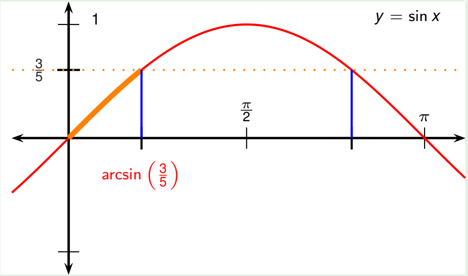
Example

Find all solutions of the equation.

$$\begin{aligned}\sin\theta &= \frac{3}{5}\\ \theta &= \arcsin\left(\frac{3}{5}\right)\end{aligned}$$

arcsin implies radians





Example

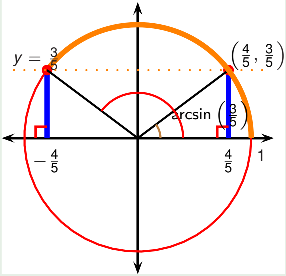
Find all solutions of the equation.

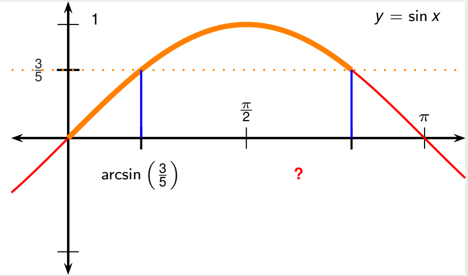
$$\sin\theta = \frac{3}{5}$$

$$\theta = \arcsin\left(\frac{3}{5}\right) \qquad \Big|\ \arcsin \text{ implies radians}$$

or

?





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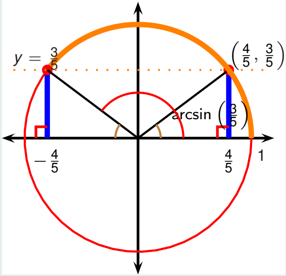

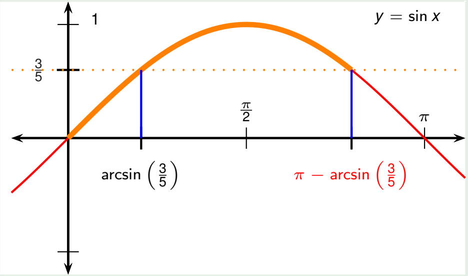

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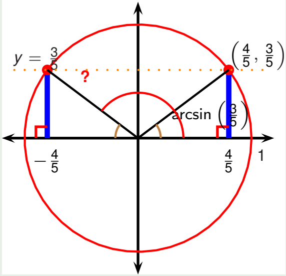

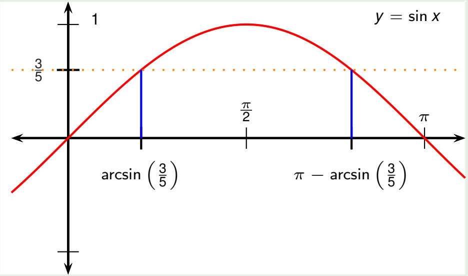

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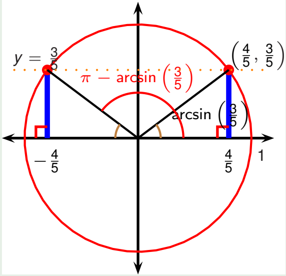
Find all solutions of the equation.

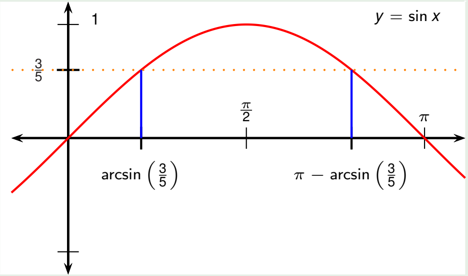
$$\sin\theta = \frac{3}{5}$$

$$\theta = \arcsin\left(\frac{3}{5}\right) \quad \Big| \ \arcsin \text{ implies radians}$$

or

$$\pi - \arcsin\left(\frac{3}{5}\right)$$





Example

Find all solutions of the equation.

$$\sin\theta = \frac{3}{5}$$

$$\theta = \arcsin\left(\frac{3}{5}\right) + k\cdot(2\pi) \qquad \Big| \ \arcsin \text{ implies radians}$$

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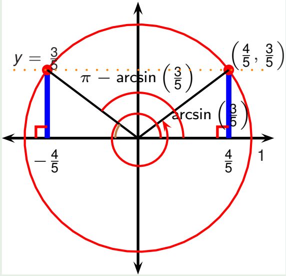

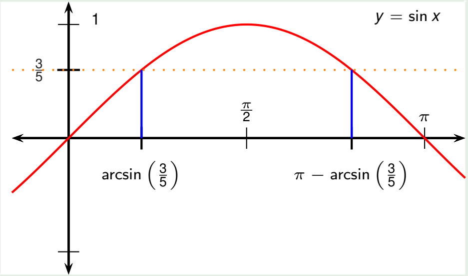

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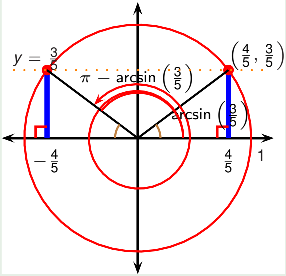

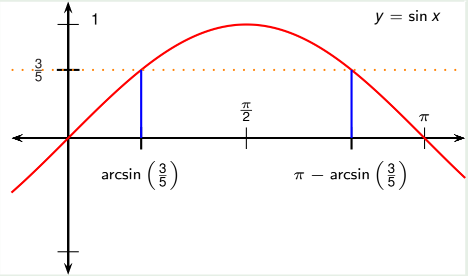

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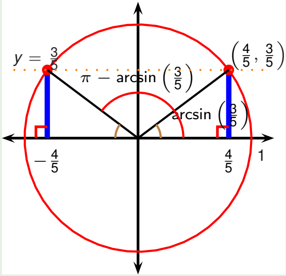

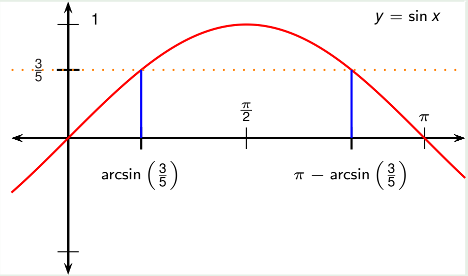

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Find all values of θ in the interval $[0, 2\pi]$ such that $\sin\theta = \sin(2\theta)$.

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$$\begin{aligned} \sin\theta &= \sin(2\theta) \\ \sin\theta &= \textbf{?} \end{aligned}$$

Example

Find all values of θ in the interval $[0, 2\pi]$ such that $\sin\theta = \sin(2\theta)$.

$$\begin{aligned} \sin\theta &= \sin(2\theta) \\ \sin\theta &= 2\sin\theta\cos\theta \end{aligned}$$

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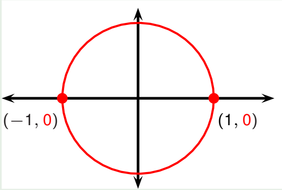

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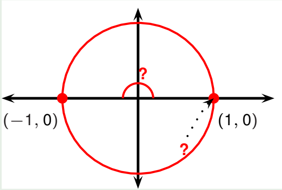

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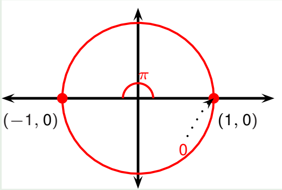

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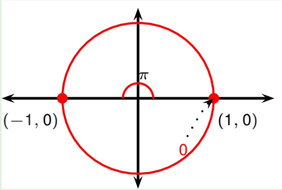

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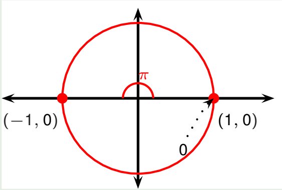

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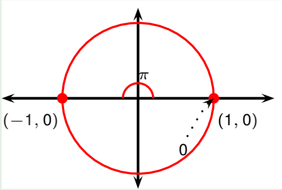

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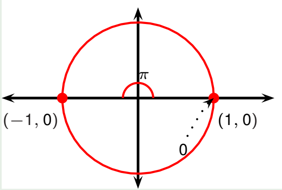

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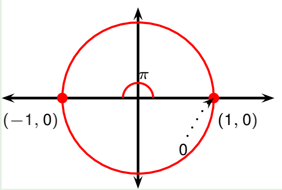

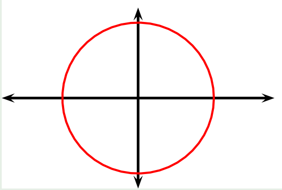

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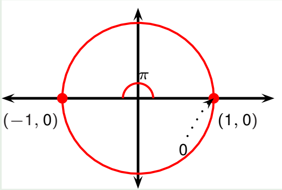

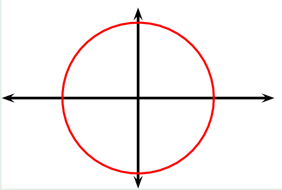

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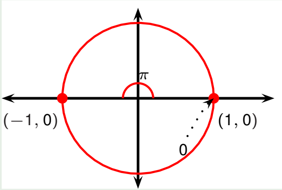

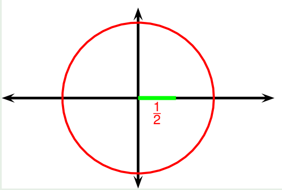

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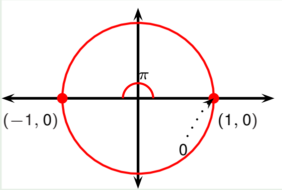

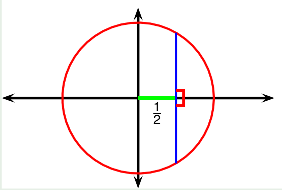

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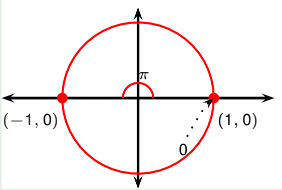

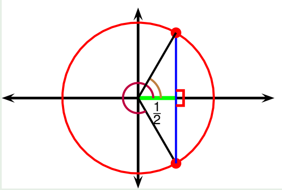

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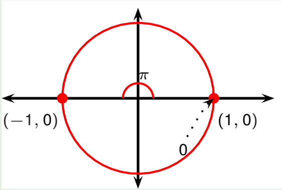

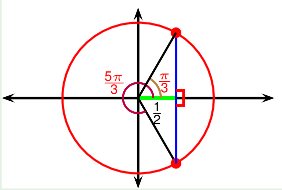

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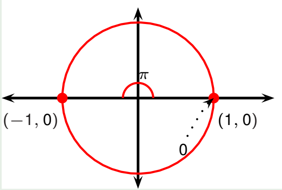

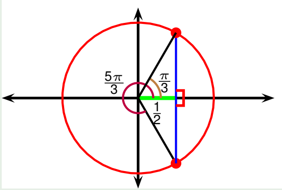

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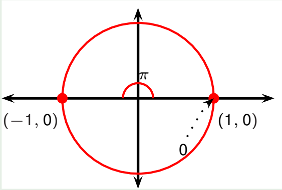

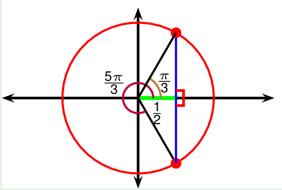

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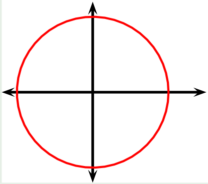

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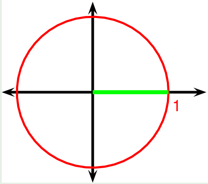

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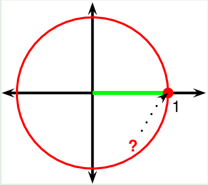

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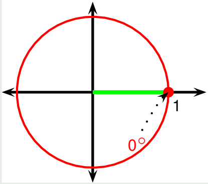

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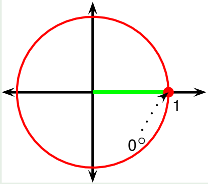

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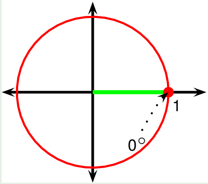

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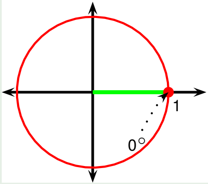

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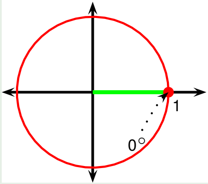

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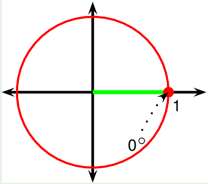

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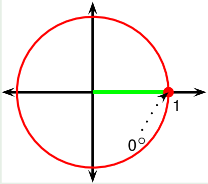

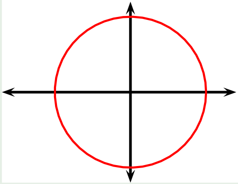

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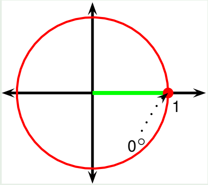

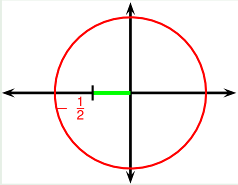

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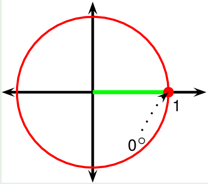

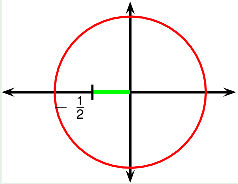

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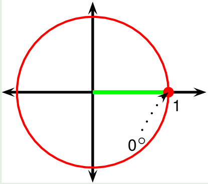

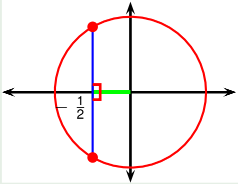

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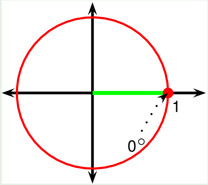

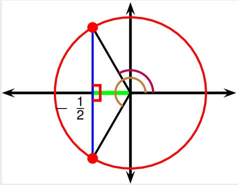

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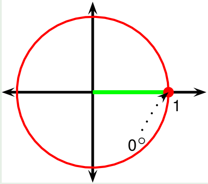

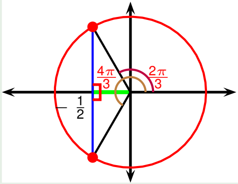

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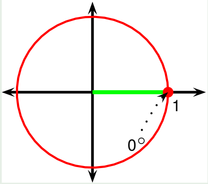

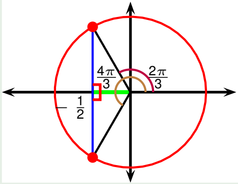

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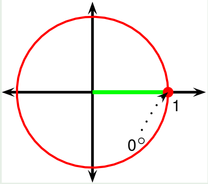

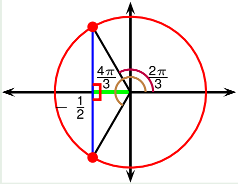

Strategy for solving trigonometric equations

- Suppose we want to solve an algebraic trigonometric equation.
- More precisely, the equation should be an algebraic expressions of the trigonometric functions of a single variable.
- Here is a general strategy for solving such a problem:
  - Using trig identities, rewrite in terms of $\sin x$ and $\cos x$ only.
  - Suppose $x \in [2n\pi, (2n+1)\pi]$.
    - Set $\sin x = \sqrt{1-\cos^2 x}$ (allowed due to restrictions on x).
    - Set $\cos x = u$. Solve the resulting algebraic equation for u.
    - For the found solutions for u, solve $\cos x = u$.
    - Check whether your solutions satisfy $x \in [2n\pi, (2n+1)\pi]$.
  - Suppose $x \in [(2n-1)\pi, 2n\pi]$.
    - Set $\sin x = -\sqrt{1-\cos^2 x}$ (allowed due to restrictions on x).
    - Set $\cos x = u$. Solve the resulting algebraic equation for u.
    - For the found solutions for u, solve $\cos x = u$.
    - Check whether your solutions satisfy $x \in [(2n-1)\pi, 2n\pi]$.
- A similar strategy exists for $u = \sin x$ instead of $u = \cos x$.
- Problems requiring full algorithm may be too hard for Calc exams.

Proposition (Product to sum formulas)

$$\begin{aligned}
\sin\alpha\sin\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)\\
\cos\alpha\cos\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)+\cos(\alpha+\beta)\right)\\
\sin\alpha\cos\beta &= \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)
\end{aligned}$$

Proof.

$\square$

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? $= \cos(\alpha+\beta)$

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$$\cos\alpha\cos\beta-\sin\alpha\sin\beta = \cos(\alpha+\beta)$$

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$$
\begin{aligned}
\sin\alpha\sin\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)\\
\cos\alpha\cos\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)+\cos(\alpha+\beta)\right)\\
\sin\alpha\cos\beta &= \frac{1}{2}\left(\sin(\alpha+\beta)+\sin(\alpha-\beta)\right)
\end{aligned}
$$

Proof.

$$
\begin{array}{rrcl}
+ & \cos\alpha\cos\beta + \cancel{\sin\alpha\sin\beta} &=& \cos(\alpha-\beta)\\
 & \cos\alpha\cos\beta - \cancel{\sin\alpha\sin\beta} &=& \cos(\alpha+\beta)\\
 & 2\cos\alpha\cos\beta &=& \cos(\alpha-\beta)+\cos(\alpha+\beta)\\
\hline
- & \cos\alpha\cos\beta + \sin\alpha\sin\beta &=& \cos(\alpha-\beta)\\
 & \cos\alpha\cos\beta - \sin\alpha\sin\beta &=& \cos(\alpha+\beta)\\
 & 2\sin\alpha\sin\beta &=& \cos(\alpha-\beta)-\cos(\alpha+\beta)
\end{array}
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\sin\alpha\sin\beta &= \frac{1}{2}\left(\cos(\alpha-\beta)-\cos(\alpha+\beta)\right)\\
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Proof.

$$\begin{array}{crcl}
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\end{array}$$

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\end{aligned}$$

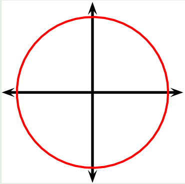
- Product to sum formulas are used when integrating (a topic to be studied later/in another course).

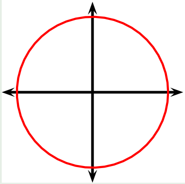
Proposition (Sum to product formulas)

$$
\begin{aligned}
\sin\alpha + \sin\beta &= 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\\
\sin\alpha - \sin\beta &= 2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)\\
\cos\alpha + \cos\beta &= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\\
\cos\alpha - \cos\beta &= 2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)
\end{aligned}
$$

Example

Find all solutions in the interval $[0, 2\pi)$.



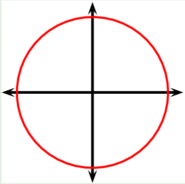


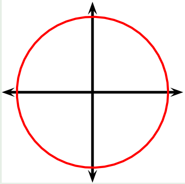
$$\sin(2x) + \sin(5x) = 0$$

Recall the formula $\sin\alpha + \sin\beta = \mathbf{?}$

Example

Find all solutions in the interval $[0, 2\pi)$.





$\sin(2x) + \sin(5x) = 0 \;\Big|\;$ use f-la

Recall the formula $\sin\alpha+\sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

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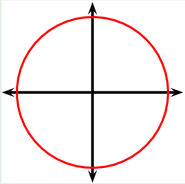

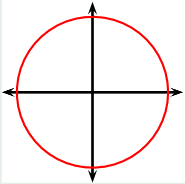

$\sin(2x)+\sin(5x)=0$ | use f-la

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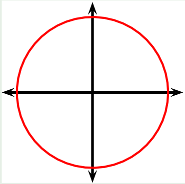

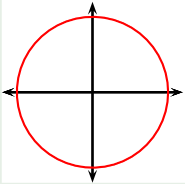

$$\sin(2x) + \sin(5x) = 0 \quad \Big| \text{ use f-la}$$

$$2\sin\left(\frac{2x+5x}{2}\right)\cos\left(\frac{2x-5x}{2}\right) = 0$$

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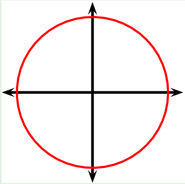

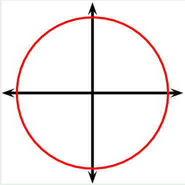

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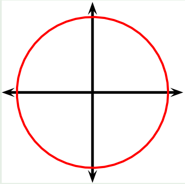

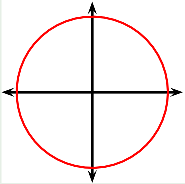

$$
\begin{aligned}
\sin(2x) + \sin(5x) &= 0 \quad \Big| \text{ use f-la} \\
2\sin\left(\frac{2x+5x}{2}\right)\cos\left(\frac{2x-5x}{2}\right) &= 0 \\
2\sin\left(\frac{7}{2}x\right)\cos\left(-\frac{3}{2}x\right) &= 0
\end{aligned}
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Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

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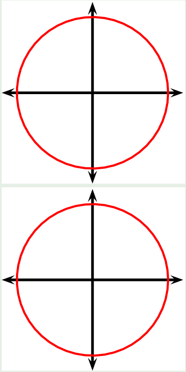

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$$2\sin\left(\frac{7}{2}x\right)\cos\left(\frac{3}{2}x\right) = 0$$

Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

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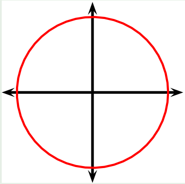

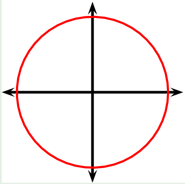

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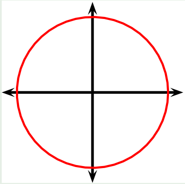

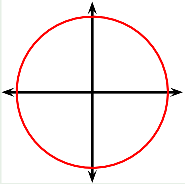

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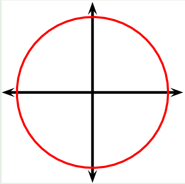
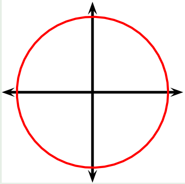


$\sin\left(\frac{7}{2}x\right) = 0$

or



$\cos\left(\frac{3}{2}x\right) = 0$

Recall the formula $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

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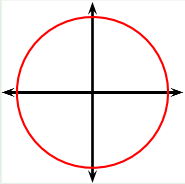

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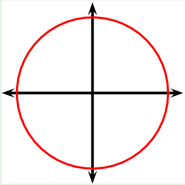

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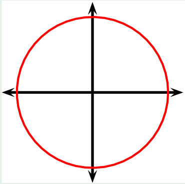

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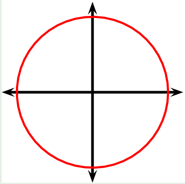

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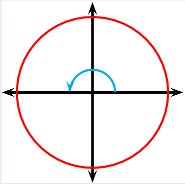

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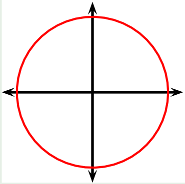

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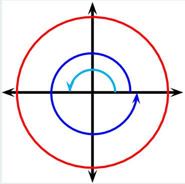

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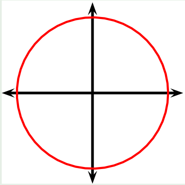

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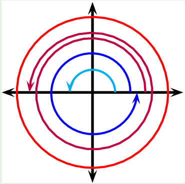

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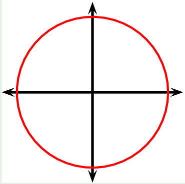

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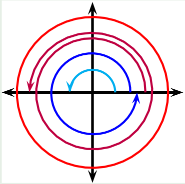

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$$\frac{7}{2}x = k\pi \qquad k - \text{integer}$$

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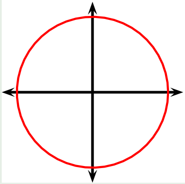

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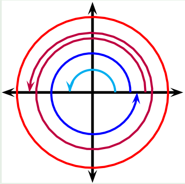
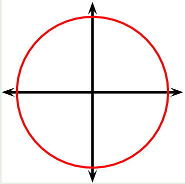


$$\sin\left(\tfrac{7}{2}x\right) = 0$$
$$\tfrac{7}{2}x = k\pi \qquad k - \text{integer}$$
$$x = \frac{2k\pi}{7}$$

or



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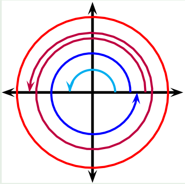

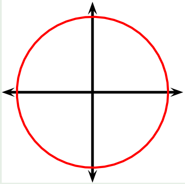

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$$x = \ldots, \frac{-2\pi}{7}, 0, \frac{2\pi}{7}, \frac{4\pi}{7}, \frac{6\pi}{7}, \frac{8\pi}{7}, \frac{10\pi}{7}, \frac{12\pi}{7}, \frac{14\pi}{7}, \ldots$$

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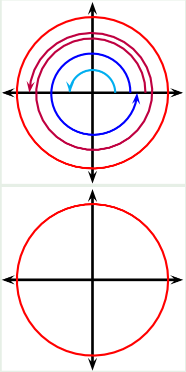

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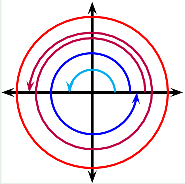

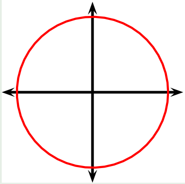

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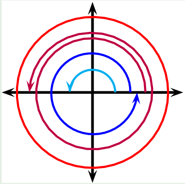

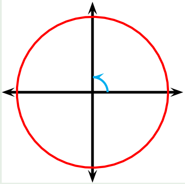

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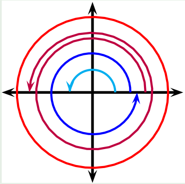

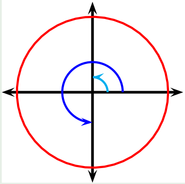

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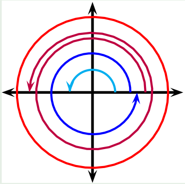

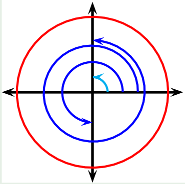

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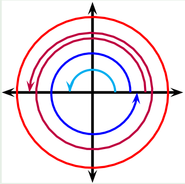

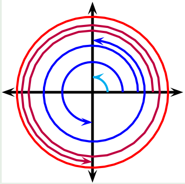

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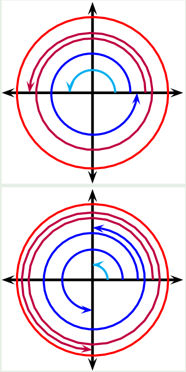

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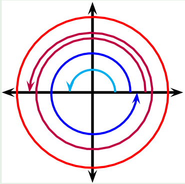

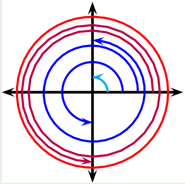

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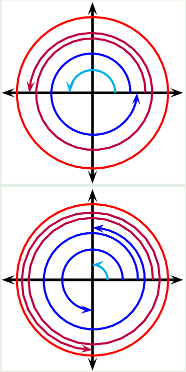

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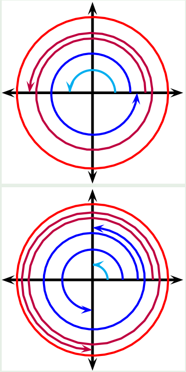

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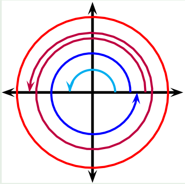

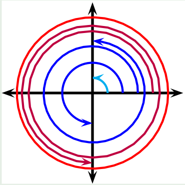

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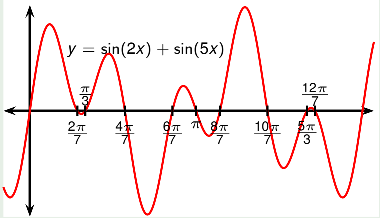
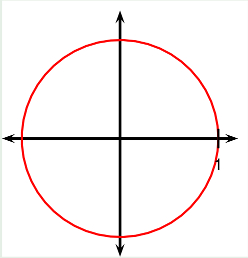
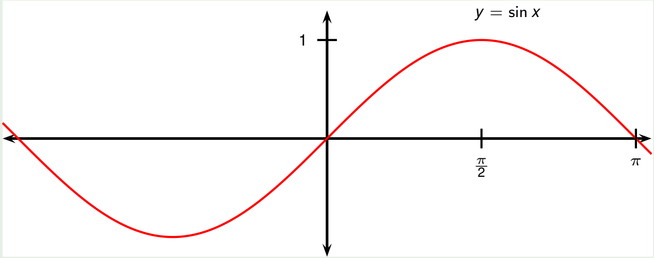


Example

Solve. Among your solutions, find those between -360° and 450°.

$\frac{1}{2} \leq \sin\theta < \frac{\sqrt{3}}{2}$





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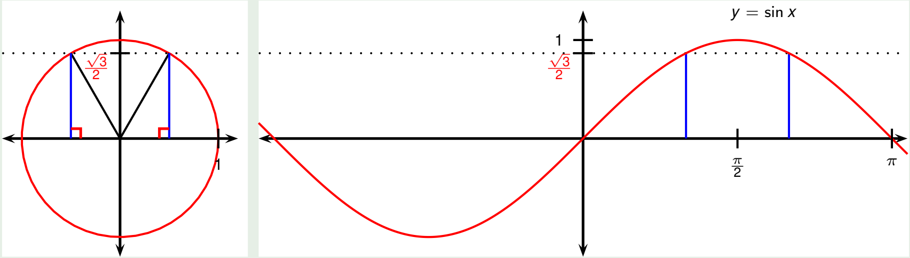

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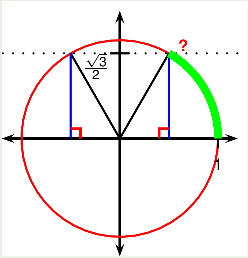

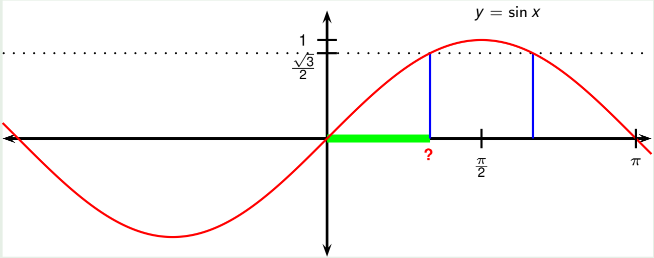

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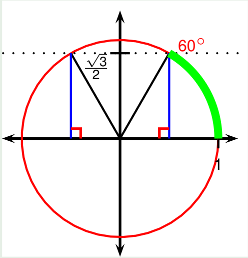

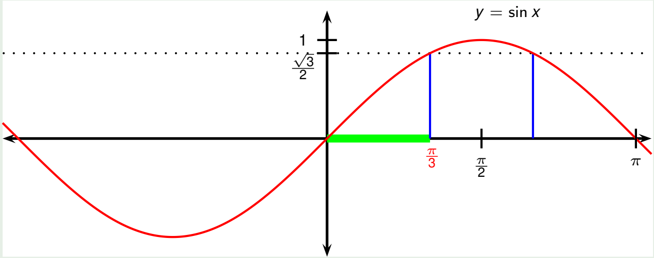

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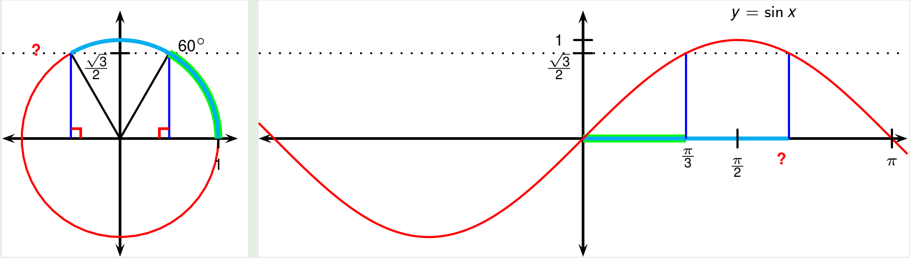

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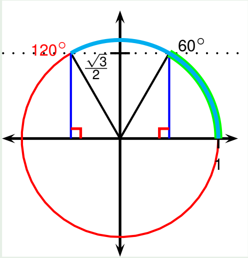

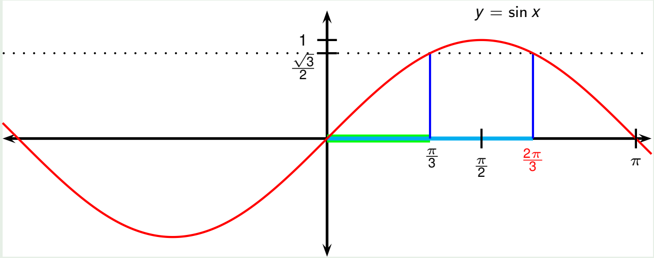

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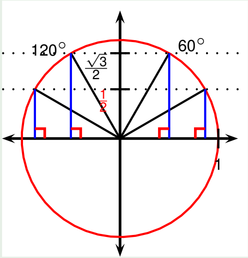

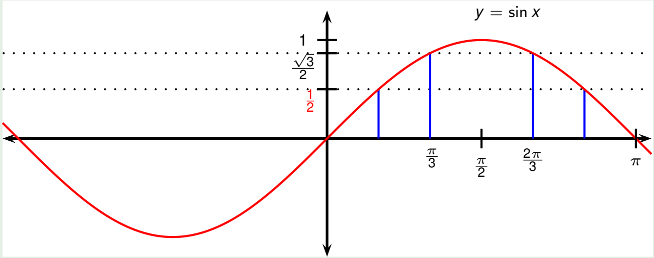

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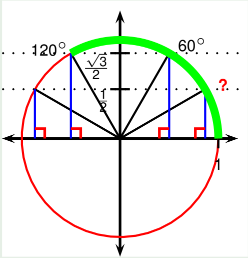

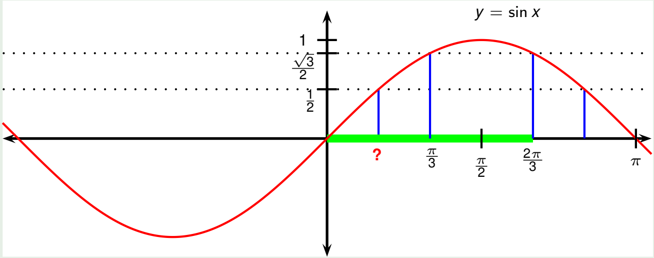

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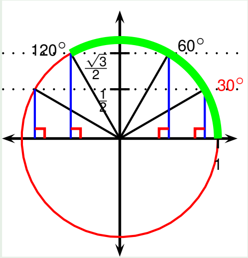

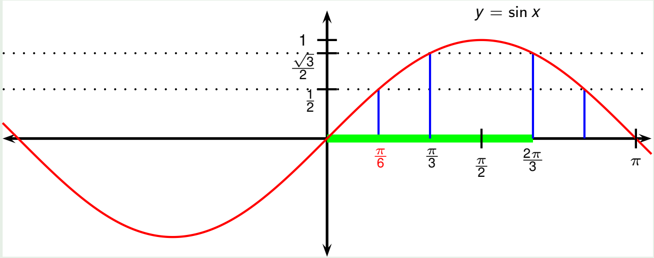

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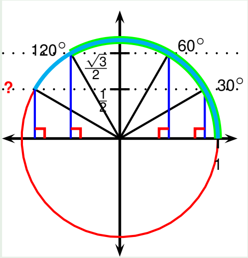

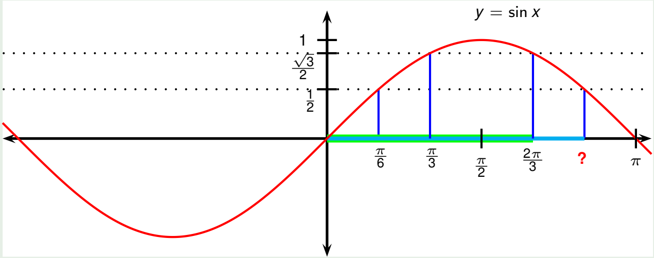

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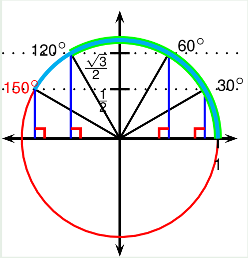

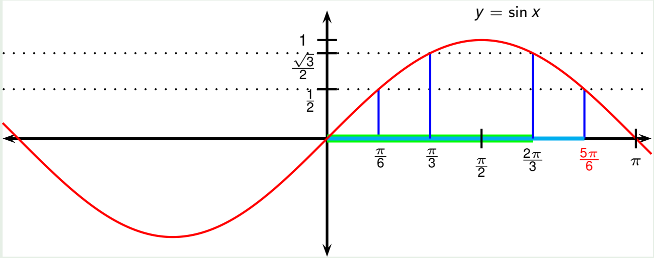

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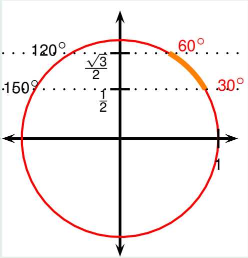
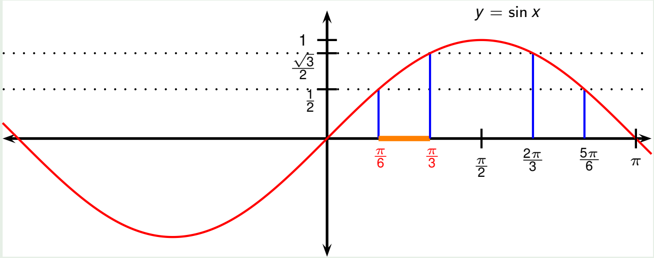


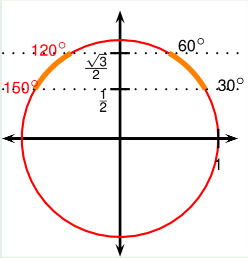


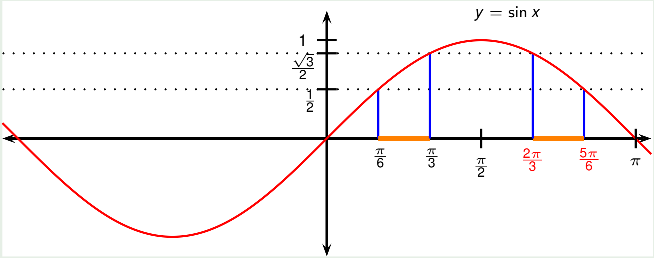
Example

Solve. Among your solutions, find those between -360° and 450°.

$\frac{1}{2} \leq \sin\theta < \frac{\sqrt{3}}{2}$

$x \in [30^\circ \quad , 60^\circ \quad) \cup (120^\circ \quad , 150^\circ \quad]$





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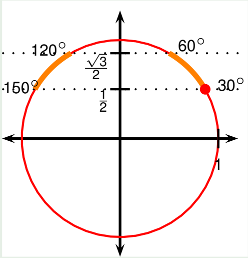

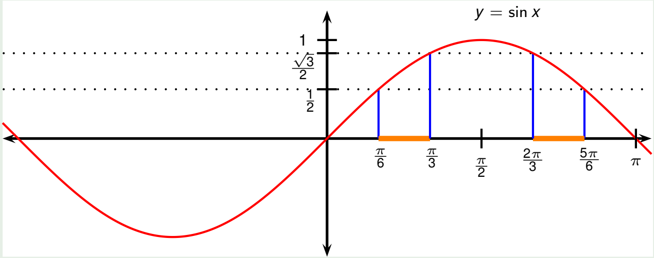

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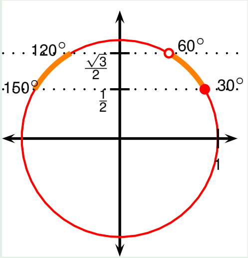

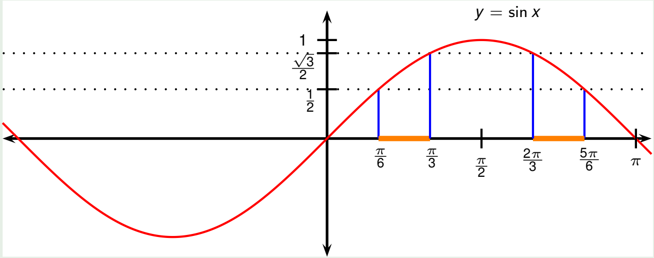

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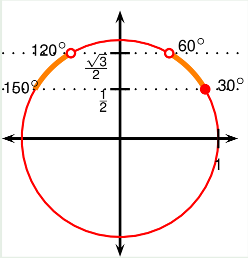

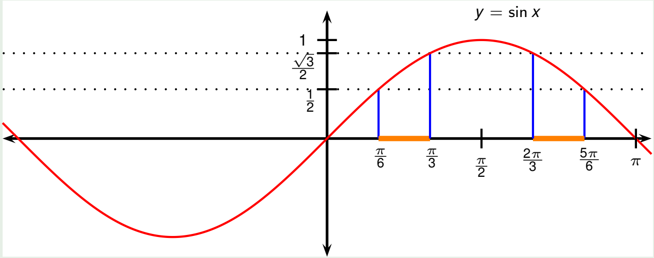

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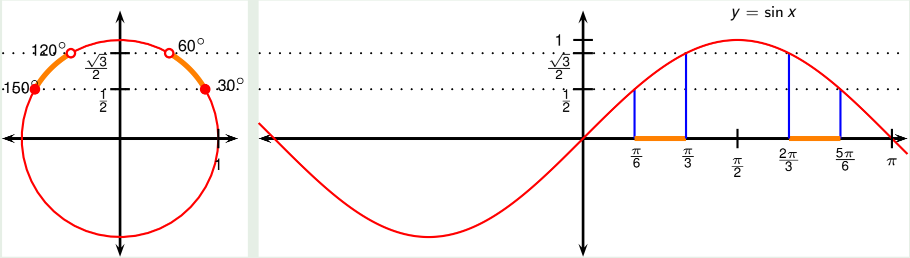

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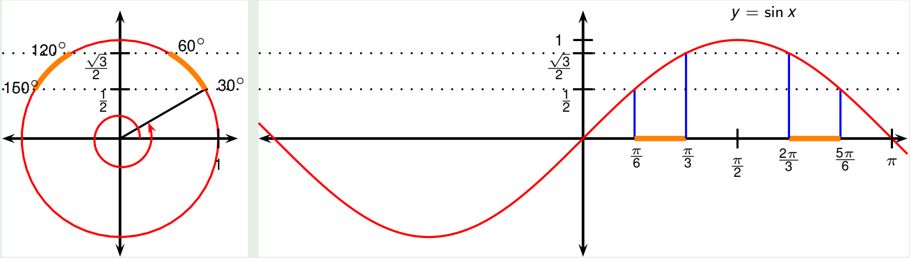

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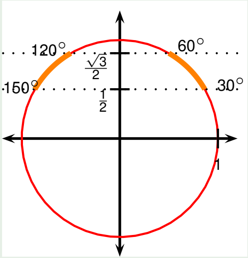

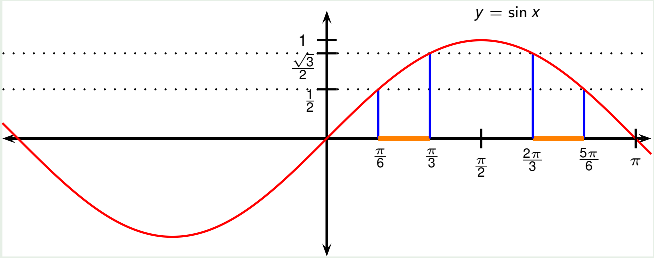

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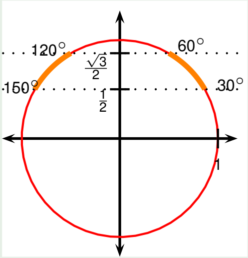

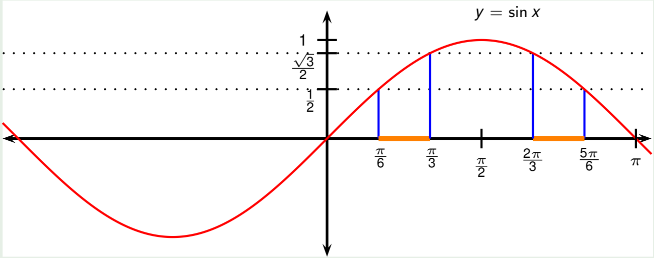

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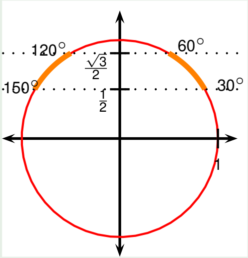

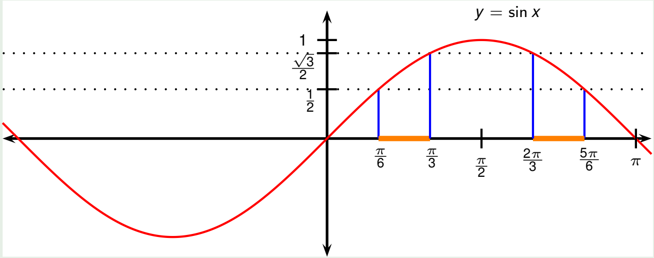

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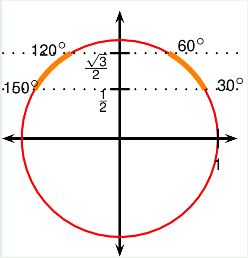
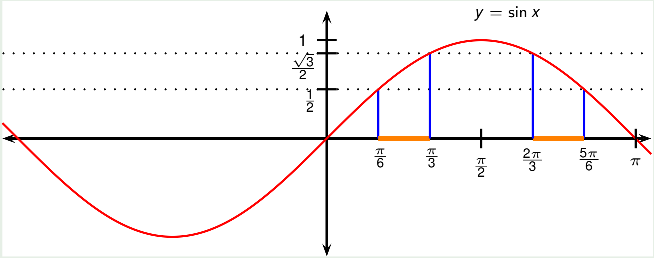
$\ldots$





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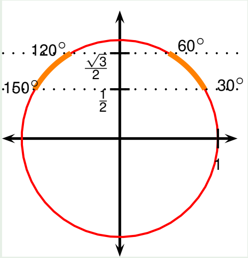

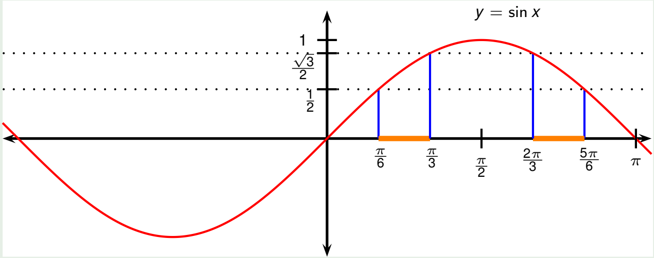

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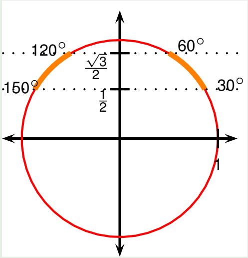

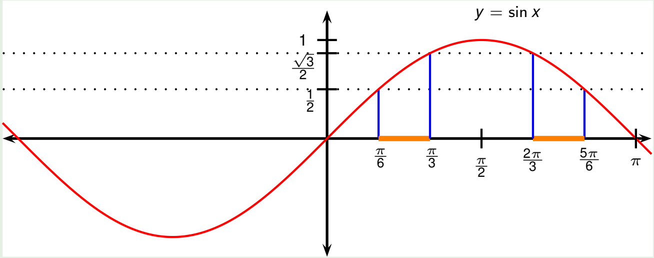

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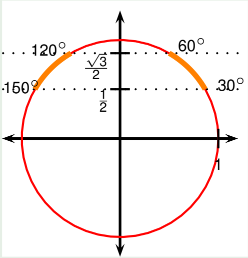

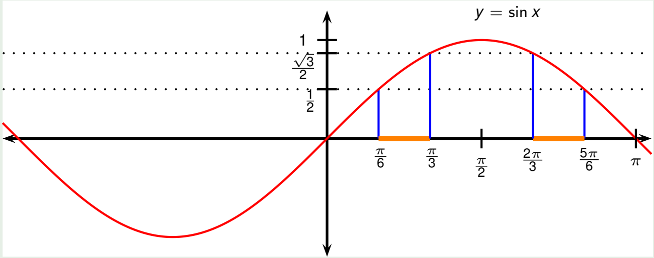

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In radians:

$x \in \left[-\frac{11\pi}{6}, -\frac{5\pi}{3}\right) \cup \left[-\frac{4\pi}{3}, -\frac{7\pi}{6}\right) \cup \left[\frac{\pi}{6}, \frac{\pi}{3}\right) \cup \left[\frac{2\pi}{3}, \frac{5\pi}{6}\right) \cup \left[\frac{13\pi}{6}, \frac{7\pi}{3}\right)$

Example

- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2\cos^2\theta + 2\cos\theta + 1 \leq \cos\theta + 2$ lying in $[-360^\circ, 360^\circ]$.

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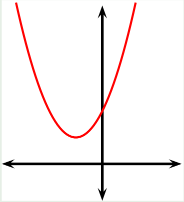

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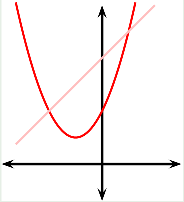

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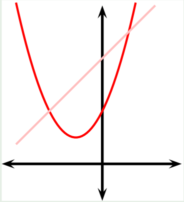

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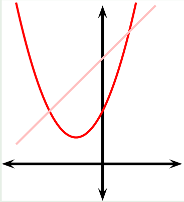

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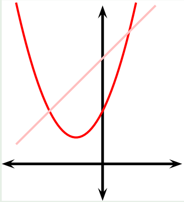

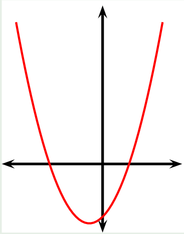

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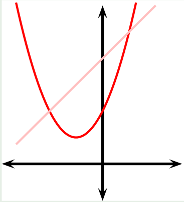

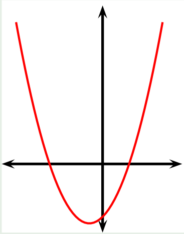

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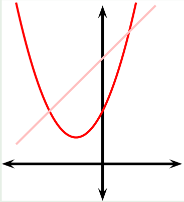
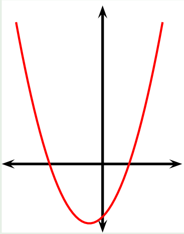
Example





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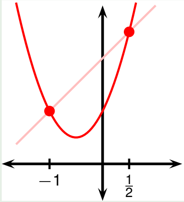
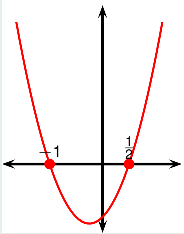
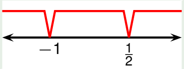
Example







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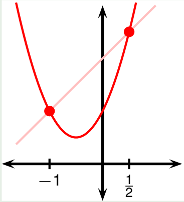

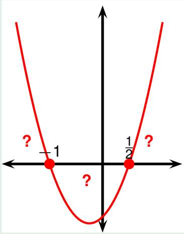

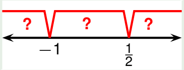

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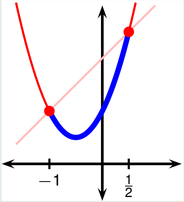
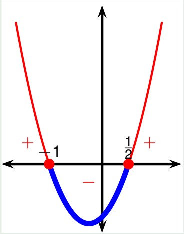
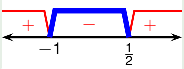
Example







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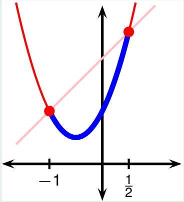
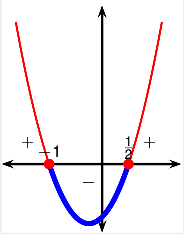
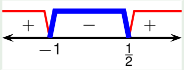
$$\begin{array}{rcl} 2u^2 + 2u + 1 & \leq & u + 2 \\ 2u^2 + u - 1 & \leq & 0 \\ 2\left(u - \frac{1}{2}\right)(u + 1) & \leq & 0 \\ u & \in & \left[-1, \frac{1}{2}\right] \end{array}$$

Example







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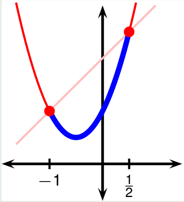
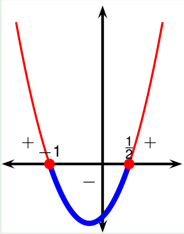
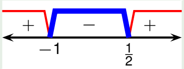
Example







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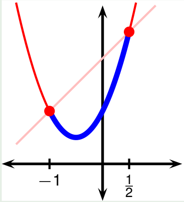
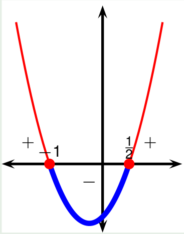
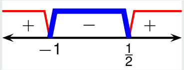
Example







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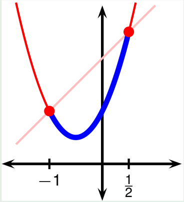
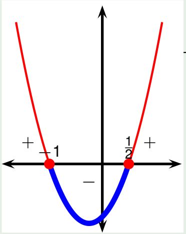
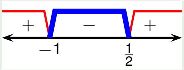
Example







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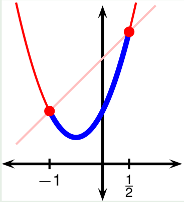
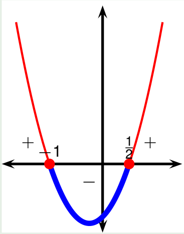
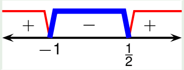
Example







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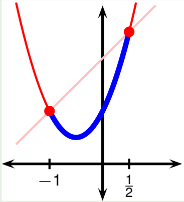
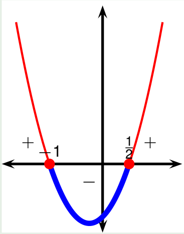
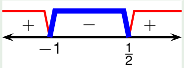
Example







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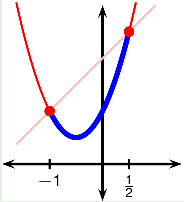
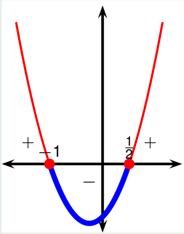
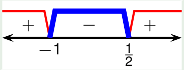
Example







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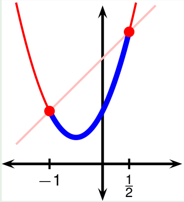
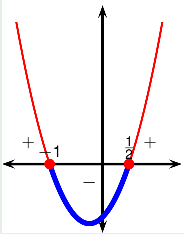
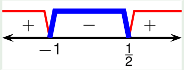
Example







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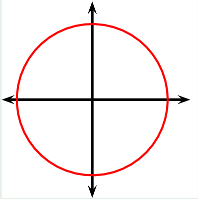
Example



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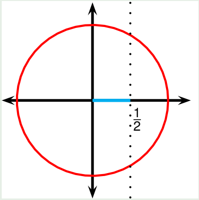
$\theta \in$ [**?** , **?**]

Example



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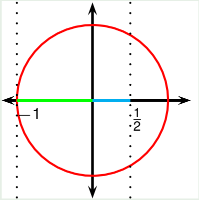
$\theta \in$ $[\mathbf{?}, \mathbf{?}]$

Example



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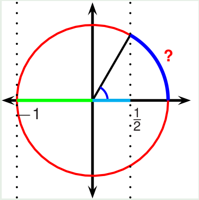
Example



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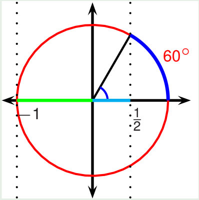
Example



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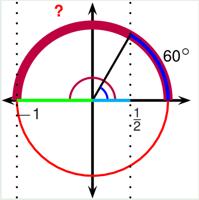
$$\begin{aligned} \cos\theta &\in \left[-1, \tfrac{1}{2}\right] \\ -1 \leq \cos\theta &\leq \tfrac{1}{2} \end{aligned}$$

$\theta \in \quad [60^\circ \quad , \mathbf{?} \quad]$

Example



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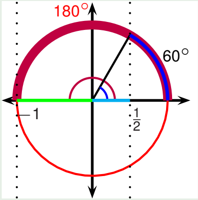
$\theta \in \quad [60° \quad , \textbf{?} \quad]$

Example



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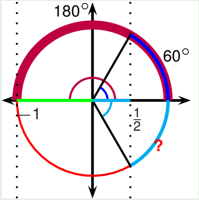
$\theta \in$ $[60°$, $180°$ $]$

Example



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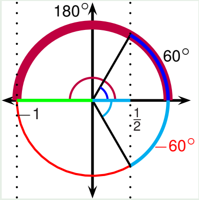
$\theta \in [$**?** , **?** $] \cup [60° , 180°]$

Example



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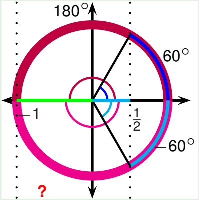
$\theta \in [\textbf{?} \quad , -60° \quad] \cup [60° \quad , 180° \quad]$

Example



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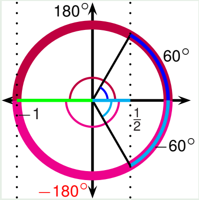
$\theta \in [$**?** $, -60° \quad] \cup [60° \quad , 180° \quad]$

Example



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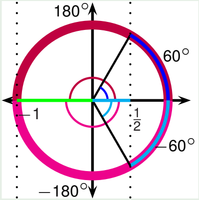
$\theta \in [-180° \quad , -60° \quad] \cup [60° \quad , 180° \quad]$

Example



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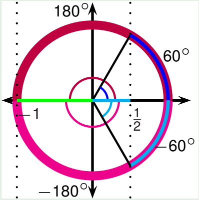
$$\theta \in [-180^\circ + k360^\circ, -60^\circ + k360^\circ] \cup [60^\circ + k360^\circ, 180^\circ + k360^\circ]$$

Example



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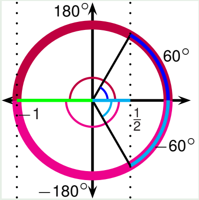
$$\theta \in [-180° + k360°, -60° + k360°] \cup [60° + k360°, 180° + k360°]$$

Example



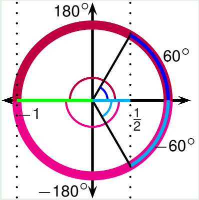
- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2\cos^2\theta + 2\cos\theta + 1 \leq \cos\theta + 2$ lying in $[-360°, 360°]$.

$$\begin{aligned} \cos\theta &\in \left[-1, \tfrac{1}{2}\right] \\ -1 \leq \cos\theta &\leq \frac{1}{2} \end{aligned}$$

$$\theta \in [-180°+k360°, -60°+k360°] \cup [60°+k360°, 180°+k360°]$$

$$\theta \in [-180°, -60°] \cup [60°, 180°] \qquad k = 0$$

Example



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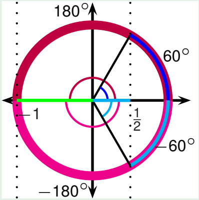
$$\begin{array}{lcr} \theta \in & [-180°, -60°] \cup [60°, 180°] & k = 0 \\ & \cup [180°, 300°] \cup [420°, 540°] & k = 1 \end{array}$$

Example



- Solve the inequality $2u^2 + 2u + 1 \leq u + 2$.
- Find all solutions of $2\cos^2\theta + 2\cos\theta + 1 \leq \cos\theta + 2$ lying in $[-360^\circ, 360^\circ]$.

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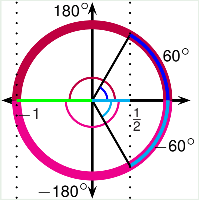
$$\begin{aligned} \theta \in \quad & [-180^\circ, -60^\circ] \cup [60^\circ, 180^\circ] && k = 0 \\ & \cup [180^\circ, 300^\circ] \cup [420^\circ, 540^\circ] && k = 1 \end{aligned}$$

...

Example



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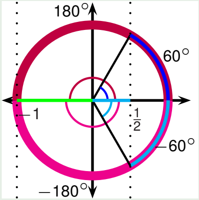
$$\theta \in \begin{array}{rl} [-540°, -420°] \cup [-300°, -180°] & k = -1 \\ \cup [-180°, -60°] \cup [60°, 180°] & k = 0 \\ \cup [180°, 300°] \cup [420°, 540°] & k = 1 \\ \dots & \end{array}$$

Example



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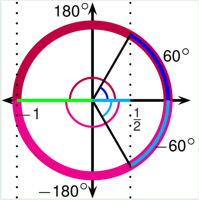
$$\theta\in\begin{array}{rcl} & \ldots & \\ \cup[-540^\circ,-420^\circ]\cup[-300^\circ,-180^\circ] & & k=-1\\ \cup[-180^\circ,-60^\circ]\cup[60^\circ,180^\circ] & & k=0\\ \cup[180^\circ,300^\circ]\cup[420^\circ,540^\circ] & & k=1\\ & \ldots & \end{array}$$

Example



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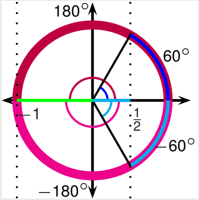
Example



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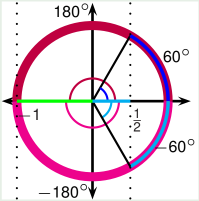
$$\theta \in [-300^\circ, -60^\circ] \cup [60^\circ, 300^\circ]$$

Example



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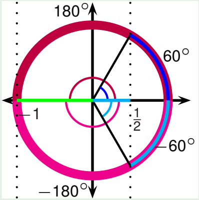
Example



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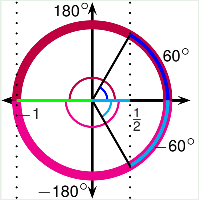
Example



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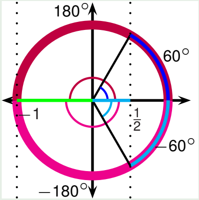
Example



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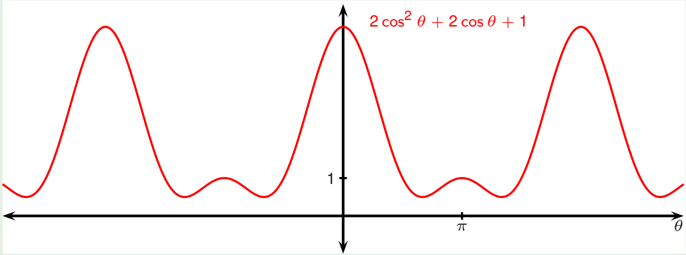
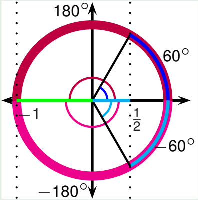


Example



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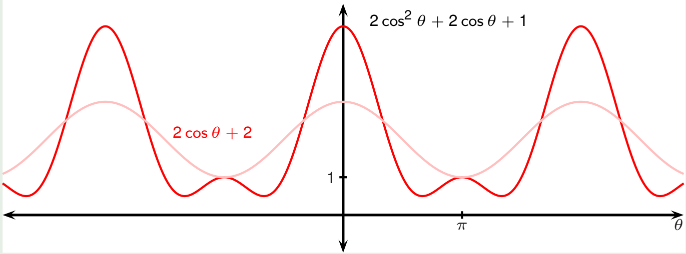
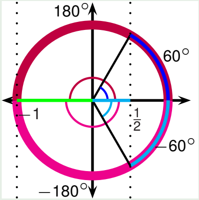


Example



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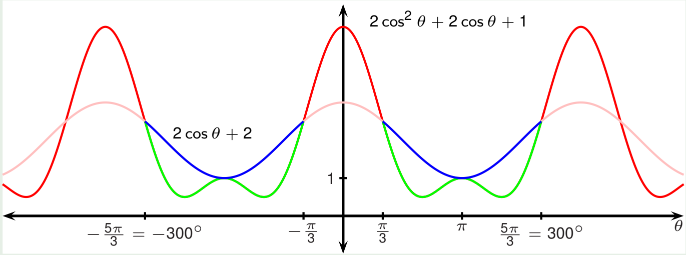
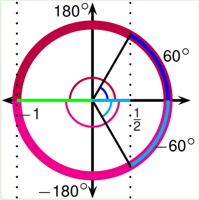


Example



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