Calculus I
Homework Review: Function Basics
Lecture 0

1. Evaluate the difference quotient and simplify your answer.

   (a) $\dfrac{f(2+h)-f(2)}{h}$, where $f(x)=x^2-x-1$.

   answer: $h+3$

   (b) $\dfrac{f(a+h)-f(a)}{h}$, where $f(x)=x^2$.

   answer: $h+2a$

   (c) $\dfrac{f(a+h)-f(a)}{h}$, where $f(x)=x^3$.

   answer: h^2+3a^2+3ah

   (d) $\dfrac{f(a+h)-f(a)}{h}$, where $f(x)=x^4$.

   answer: $6a^2h+4ah^2+h^3+4a^3$

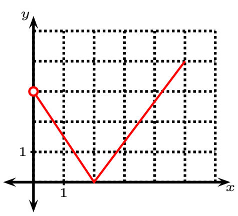
   (e) $\dfrac{f(x)-f(a)}{x-a}$, where $f(x)=\frac{1}{x}$.

   answer: $-\frac{1}{ax}$.

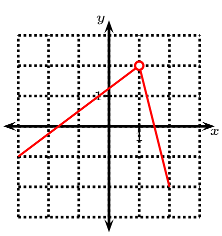
   (f) $\dfrac{f(x)-f(1)}{x-1}$, where $f(x)=\frac{x-1}{x+1}$.

   answer: $\frac{1}{x+1}$.

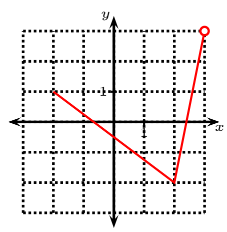
2. Write down a formula for a function whose graphs is given below. The graphs are up to scale. Please note that there is more than one way to write down a correct answer.

   (a) 

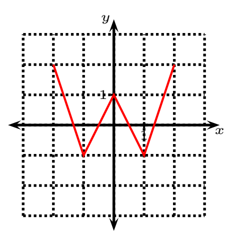
   answer: $y=\begin{cases} -\frac{3}{2}x+3 & \text{if } 0<x\leq 2 \\ \frac{4}{3}x-\frac{8}{3} & \text{if } 2<x\leq 6 \end{cases}$

   (b) 

   answer: $y=\begin{cases} \frac{3}{4}x+\frac{5}{4} & \text{if } -3\leq x<1 \\ -4x+6 & \text{if } 1<x\leq 2 \end{cases}$

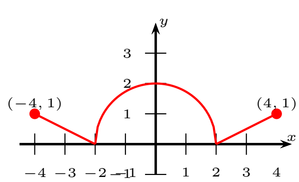
   (c) 

   answer: $y=\begin{cases} -\frac{3}{4}x-\frac{1}{2} & \text{if } -2\leq x\leq 2 \\ 5x-12 & \text{if } 2<x<3 \end{cases}$

   (d) 

   answer: $y=\begin{cases} -3x-4 & \text{if } -2\leq x<-1 \\ 2x+1 & \text{if } -1\leq x<0 \\ -2x+1 & \text{if } 0\leq x<1 \\ 3x-4 & \text{if } 1\leq x\leq 2 \end{cases}$

3. Write down formulas for function whose graphs are as follows. The graphs are up to scale. All arcs are parts of circles.

   (a) 

4. Evaluate the difference quotient and simplify your answer.

(a) $\dfrac{f(2+h)-f(2)}{h}$, where $f(x)=x^2-x-1$.

answer: $h+3$

(b) $\dfrac{f(a+h)-f(a)}{h}$, where $f(x)=x^2$.

answer: $h+2a$

(c) $\dfrac{f(a+h)-f(a)}{h}$, where $f(x)=x^3$.

answer: h^2+3a^2+3ah

(d) $\dfrac{f(a+h)-f(a)}{h}$, where $f(x)=x^4$.

answer: $6a^2h+4ah^2+h^3+4a^3$

(e) $\dfrac{f(x)-f(a)}{x-a}$, where $f(x)=\frac{1}{x}$.

answer: $-\frac{1}{ax}$.

(f) $\dfrac{f(x)-f(1)}{x-1}$, where $f(x)=\frac{x-1}{x+1}$.

answer: $\frac{1}{x+1}$.

5. Find the implied domain of the function.

(a) $f(x)=\dfrac{x+4}{x^2-4}$.

answer: $x\neq\pm 2$, alternatively: $x\in(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

(b) $f(x)=\dfrac{2x^3-5}{x^2+5x+6}$.

answer: $x\neq -2,-3$, alternatively: $x\in(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$

(c) $f(t)=\sqrt[3]{3t-1}$.

answer: $x\in\mathbb{R}$ (the domain is all real numbers)

(d) $g(t)=\sqrt{5-t}-\sqrt{1+t}$.

answer: $x\in[-1,5]$.

(e) $h(x)=\dfrac{1}{\sqrt[6]{x^2-7x}}$.

answer: $x\in(-\infty,0)\cup(7,\infty)$.

(f) $f(u)=\dfrac{u+1}{1+\frac{1}{u+1}}$.

answer: $u\neq -1,-2$ or $u\in(-\infty,-2)\cup(-2,-1)\cup(-1,\infty)$.

(g) $F(x)=\sqrt{10-\sqrt{x}}$.

answer: $x\in[0,100]$