Calculus I
Lecture 13
Implicit Differentiation

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https://github.com/tmilev/freecalc

2020

Outline

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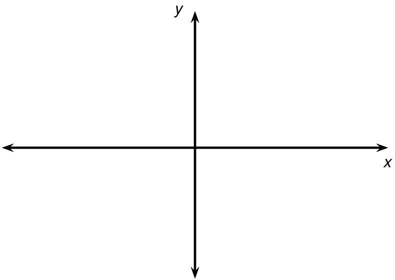
Implicit Differentiation

- So far, we have seen functions with formulas that express one varable explicitly in terms of the other.



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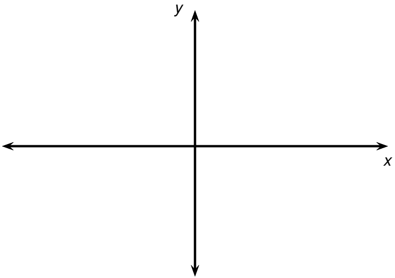

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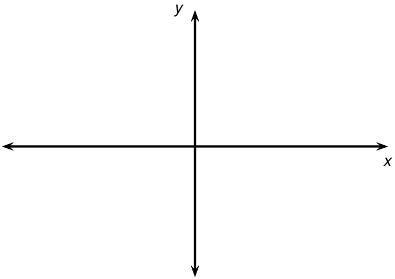

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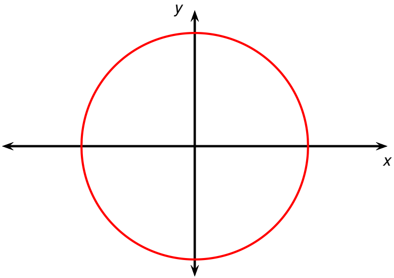

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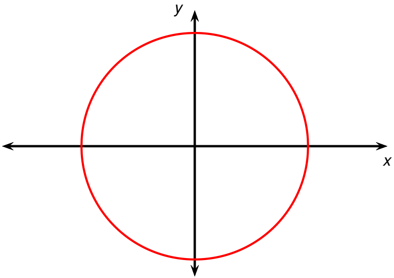

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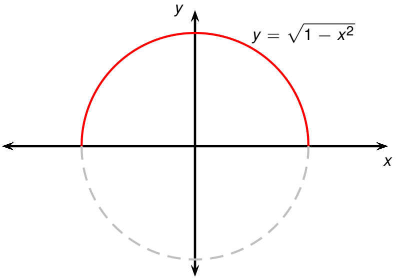

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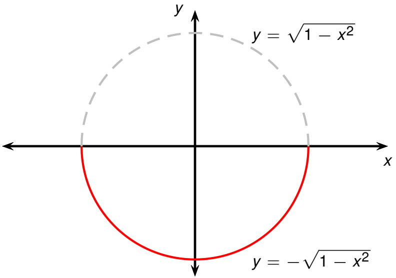

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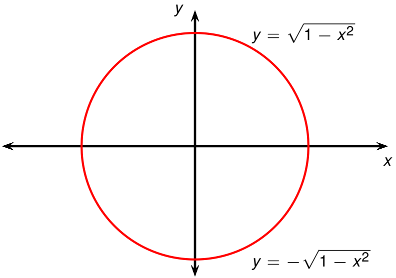

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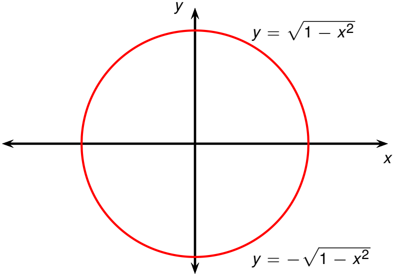
- How do we differentiate these functions?
- Differentiate both sides with respect to x, and then solve for y'.



Example

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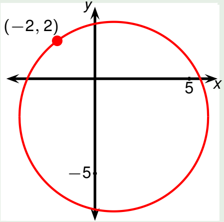

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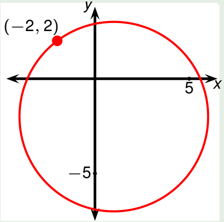

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, given $(x-1)^2\ +(y+2)^2=25$:

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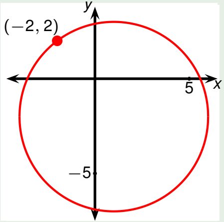

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$$? \qquad + ? \qquad = ?$$

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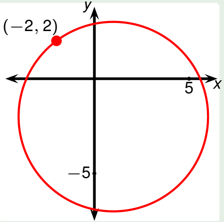

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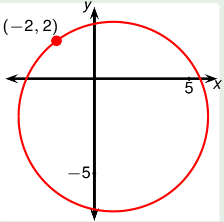

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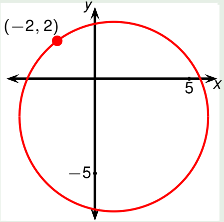

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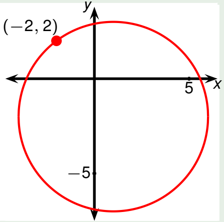

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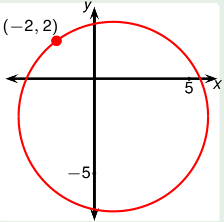

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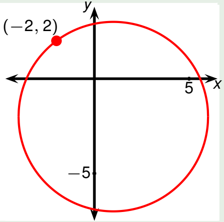

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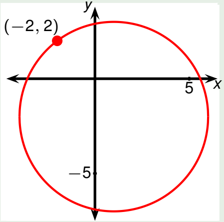

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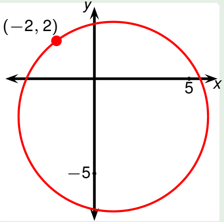

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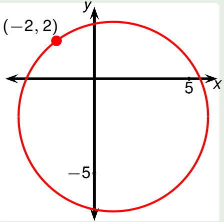

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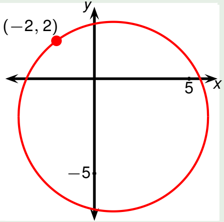

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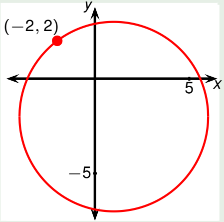

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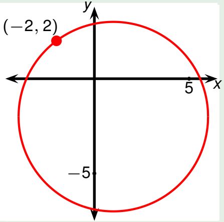

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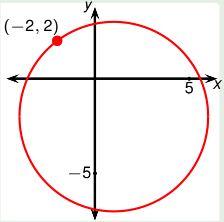

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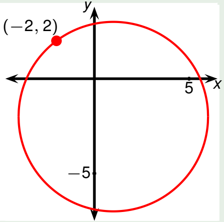

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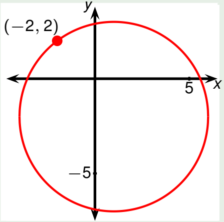

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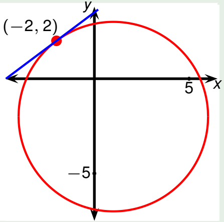

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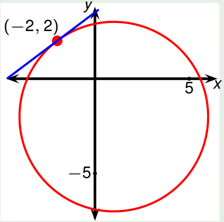

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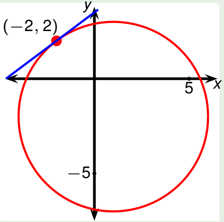

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$$2(y+2)\left(\frac{dy}{dx}\right) = 2(1-x)$$

$$\frac{dy}{dx} = \frac{1-x}{y+2}$$

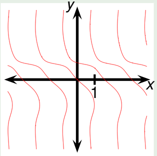
Plug in $(-2,2)$:

$$\frac{dy}{dx} = \frac{1-(-2)}{2+2} = \frac{3}{4}$$

Point-slope form:

$$y-2 = \frac{3}{4}(x+2)$$

Example



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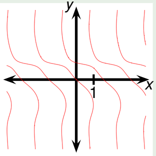

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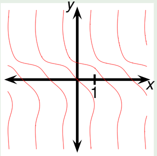

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?

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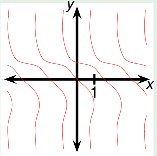

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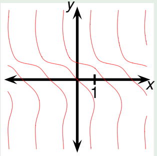

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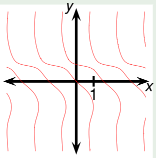

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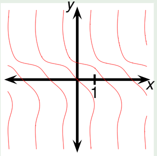

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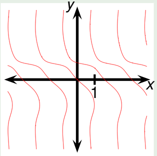

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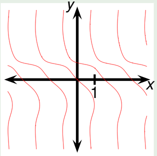

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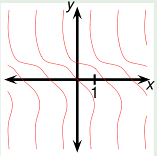

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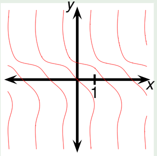

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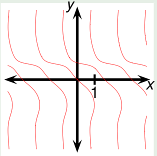

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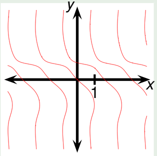

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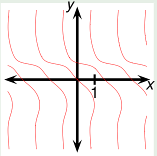

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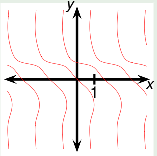

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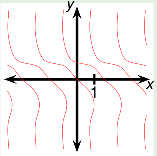

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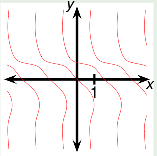

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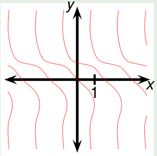

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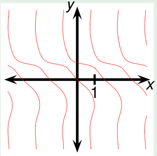

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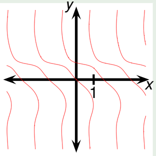

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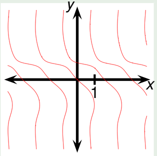

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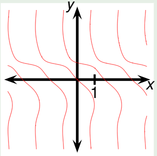

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$$
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2\cos(2(x+y))\left(1+y'\right) &= 2yy'\cos(2x)-y^2\sin(2x)2\\
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y'\cos(2(x+y))-yy'\cos(2x) &= -\cos(2(x+y))-y^2\sin(2x)\\
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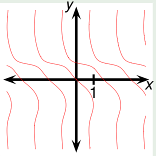

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Let $x^4 + y^4 = 16$. Find y''.

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$4x^3 + 4y^3 y' = 0$

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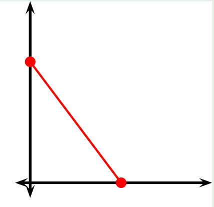

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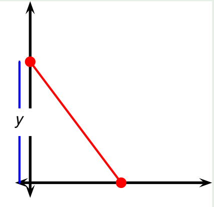

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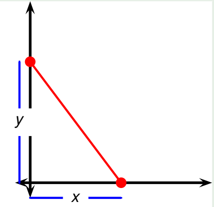

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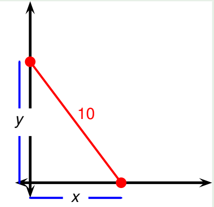

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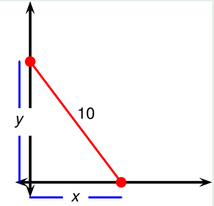

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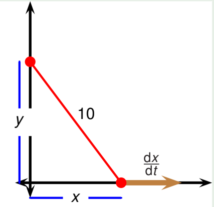

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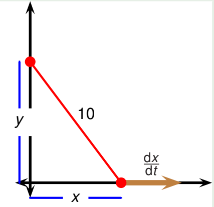

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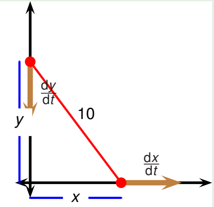

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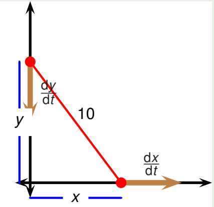

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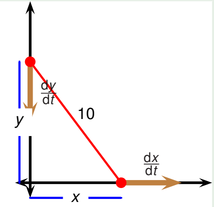

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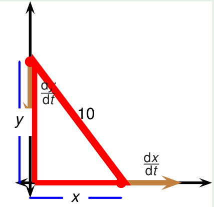

10 ft ladder rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 ft/s. How fast is the ladder top sliding down when the bottom is 6 ft from the wall?

$$x^2 + y^2 = 10^2 = 100$$

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- Relationship b/n quantities.
- Differentiate (use Chain Rule).

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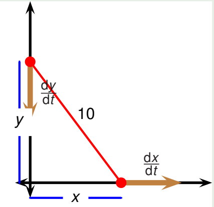

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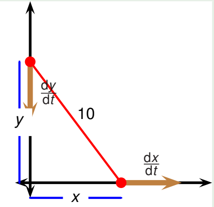

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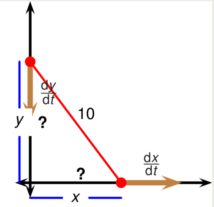

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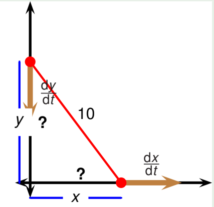

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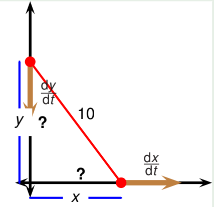

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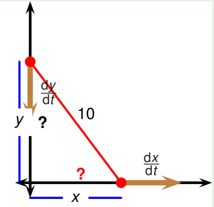

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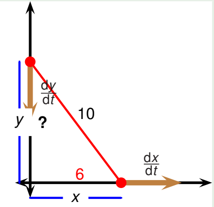

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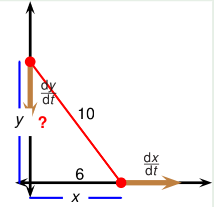

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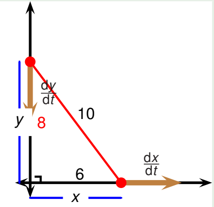

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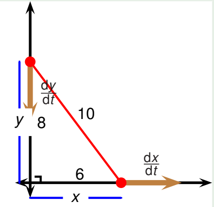

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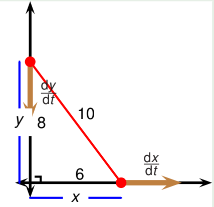

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Therefore the top of the ladder is falling at a rate of $3/4$ ft/s.