Calculus III
Homework on Lecture 3

1. Find the area of the triangle $\triangle ABC$. The answer key has not been proofread, use with caution.

   (a) $A(1,0,0)$, $B(0,1,0)$, $C(0,0,1)$.

   answer: $\frac{\sqrt{3}}{2}$

   (b) $A(1,-1,0)$, $B(0,1,-1)$, $C(-1,0,1)$.

   answer: $\frac{3}{2}\sqrt{3}$

   (c) $A(1,2,3)$, $B(5,7,11)$, $C(13,17,19)$.

   answer: $4\sqrt{41}$

2. Find a vector orthogonal to the two given vectors. The answer key has not been proofread, use with caution.

   (a) $\mathbf{u} = (2,3,5)$, $\mathbf{v} = (3,5,7)$.

   answer: $\mathbf{u} \times \mathbf{v} = (-4, 1, 1)$

   (b) $\mathbf{u} = (2,-5,-3)$, $\mathbf{v} = (3,5,7)$.

   answer: $\mathbf{u} \times \mathbf{v} = (-20, -23, 25)$

3. Let the 4 vertices of a tetrahedron be O, A, B, C. Let $\mathbf{v}_1 = \mathbf{OA}$, $\mathbf{v}_2 = \mathbf{OB}$, $\mathbf{v}_3 = \mathbf{OC}$ (the vectors given by the edges of the tetrahedron that pass through O). It can be shown that the volume of the tetrahedron equals $\frac{1}{3!} = \frac{1}{6}$ of the volume of the slanted box spanned by $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$. Using that information find the volumes of the following tetrahedra.

   (a) The volume of the tetrahedron with vertices $(1,1,1)$, $(1,-1,-1)$, $(-1,1,-1)$, $(-1,-1,1)$.

   answer: $\frac{8}{3}$

   (b) The volume of the tetrahedron with vertices $(1,2,3)$, $(2,3,5)$, $(3,5,7)$, $(5,7,13)$.

   answer: $\frac{1}{3}$

   (c) The volume of the tetrahedron with vertices $A(1,2,2)$, $B(1,3,3)$, $C(1,0,2)$, $D(-2,3,2)$.

4. Do the points $(1,2,3)$, $(2,3,5)$ $(3,5,7)$ $(5,7,11)$ lie in one plane?

   answer: yes.

5. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be arbitrary vectors. Show that the Jacobi identity for the cross product holds, i.e., show that

   $$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) + \mathbf{v} \times (\mathbf{w} \times \mathbf{u}) + \mathbf{w} \times (\mathbf{u} \times \mathbf{v}) = \mathbf{0} \quad .$$

   This problem will not appear on the quiz.