Calculus I
Lecture 23
The Fundamental Theorem Of Calculus Part II

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https://github.com/tmilev/freecalc

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Outline

1. The Fundamental Theorem of Calculus
   - Proof of FTC, part 1

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- Part 2 of the FTC was already studied as the Evaluation Theorem. It allows us to compute integrals by finding antiderivatives, without writing limits of Riemann sums.
- Part 1 of the FTC roughly says “differentiation undoes integration.”
- Part 1 of the FTC deals with functions of the form

$$g(x) = \int_a^x f(t)\mathrm{d}t$$

where f is a continuous function on $[a, b]$ and x varies between a and b.

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- If we let x vary, then $\int_a^x f(t)\mathrm{d}t$ varies.
- If f is positive, then g can be interpreted as the area under f from a to x.

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Theorem (The Fundamental Theorem of Calculus, Part 1)

If f is continuous on $[a,b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\mathrm{d}t$$

is continuous on $[a,b]$ and differentiable on (a,b), and $g'(x) = f(x)$.

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Find the derivative of $g(x) = \int_0^x \sqrt{1+t^2} \mathrm{d}t$.

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$$g'(x) = \sqrt{1+x^2}$$

Example (FTC, Part 1)

For each formula $g(x)$, find the derivative $g'(x)$.

$g(x)$	$g'(x)$
$\displaystyle\int_0^x \sin(t^2+1)\cos(t^3+2)\mathrm{d}t$	
$\displaystyle\int_{35}^x \frac{1+r^2+4r^3}{1-r^4}\mathrm{d}r$	
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Theorem (The Fundamental Theorem of Calculus)

Suppose f is continuous on $[a, b]$. Then

1. *If* $G(x) = \int_a^x f(t)\mathrm{d}t$, *then* $G'(x) = f(x)$.
2. $\int_a^b f(x)\mathrm{d}x = F(b) - F(a)$, *where F is any antiderivative of f.*

We already studied part 2 of the FTC as the Evaluation Theorem.

Theorem

Let A, B-numbers, $a(x), b(x)$ -differentiable functions with $A < a(x) < B$, $A < b(x) < B$. Let f - continuous on $[A, B]$ and

$$G(x) = \int_{a(x)}^{b(x)} f(t)dt.$$

Then $G'(x) = f(b(x))b'(x) - f(a(x))a'(x)$.

Proof.

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Let $c \in (A, B)$.

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Proof.

Let $c \in (A, B)$. Set $h(u) = \int_{c}^{u} f(t)dt$.

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$$G(x) = \int_{a(x)}^{b(x)} f(t)dt = \int_{c}^{b(x)} f(t)dt + \int_{a(x)}^{c} f(t)dt$$

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Then

$G'(x) = (h(b(x)) - h(a(x)))'$

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Then using the chain rule we get
$G'(x) = (h(b(x)) - h(a(x)))' = h'(b(x))b'(x) - h'(a(x))a'(x)$

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Let $c \in (A, B)$. Set $h(u) = \int_{c}^{u} f(t)dt$. FTC part 1 states that $h'(u) = f(u)$.

$$\begin{aligned} G(x) &= \int_{a(x)}^{b(x)} f(t)dt = \int_{c}^{b(x)} f(t)dt + \int_{a(x)}^{c} f(t)dt \\ &= \int_{c}^{b(x)} f(t)dt - \int_{c}^{a(x)} f(t)dt = h(b(x)) - h(a(x)) \quad . \end{aligned}$$

Then using the chain rule we get

$G'(x) = (h(b(x)) - h(a(x)))' = h'(b(x))b'(x) - h'(a(x))a'(x) = f(b(x))b'(x) - f(a(x))a'(x)$, as desired. $\square$

Problems similar to the following often appear on Calculus I exams.

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Theorem (The Fundamental Theorem of Calculus part 1)

Let f be a function continuous on $[a, b]$ and let $G(x) = \int_a^x f(t)\mathrm{d}t$ for all $x \in [a, b]$. Then G is differentiable and $G'(x) = f(x)$.

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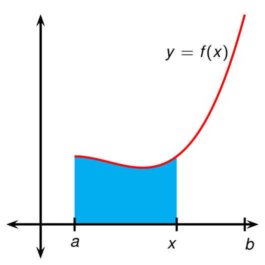

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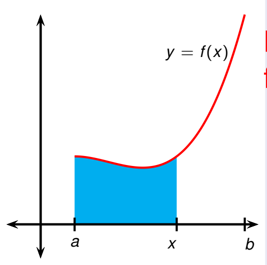

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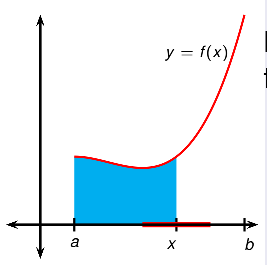

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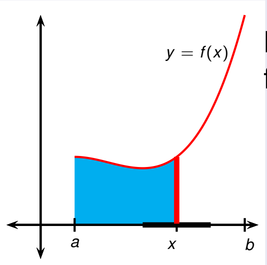

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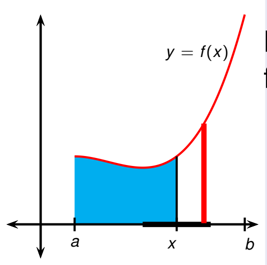

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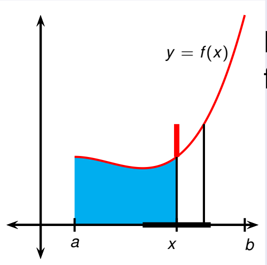

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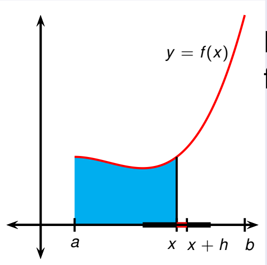

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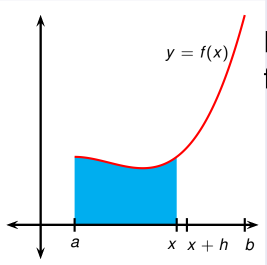

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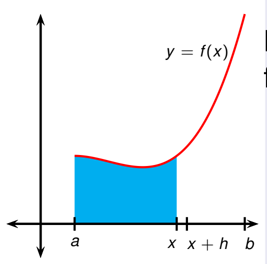

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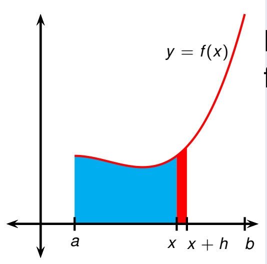

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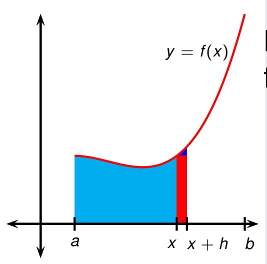

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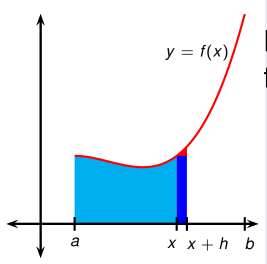

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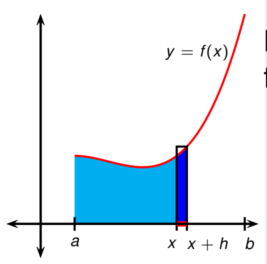

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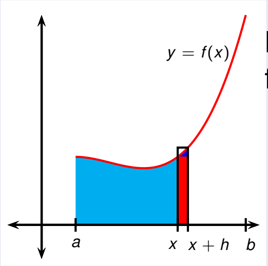

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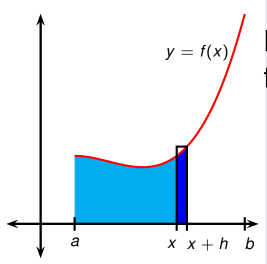

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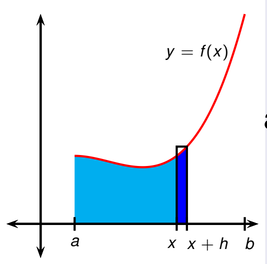

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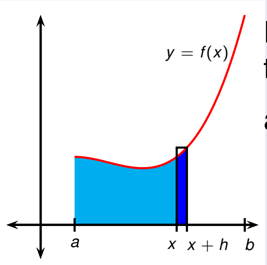

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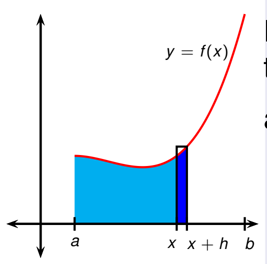

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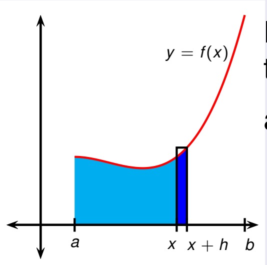

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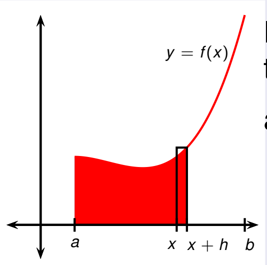

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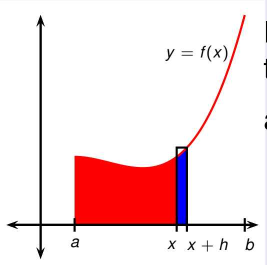

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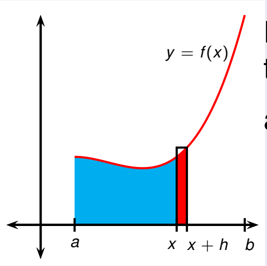

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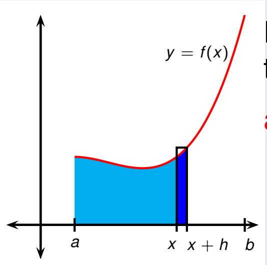

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- $F(b) - F(a)$ is the net change in y as x changes from a to b.

Theorem (The Net Change Theorem)

The integral of the rate of change is the net change:
$$\int_a^b F'(x)\mathrm{d}x = F(b) - F(a).$$

- If an object moves along a straight line with position function $s(t)$, then its velocity is $v(t) = s'(t)$.
- In this case, the Net Change Theorem says

$$\int_{t_1}^{t_2} v(t)\mathrm{d}t = s(t_2) - s(t_1).$$

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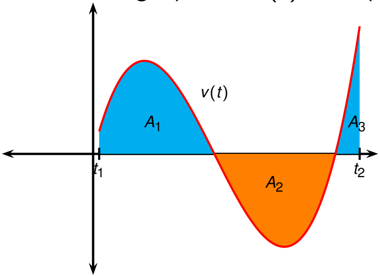

$$\begin{aligned} \text{displacement} &= \int_{t_1}^{t_2} v(t)\mathrm{d}t \\ &= A_1 - A_2 + A_3 \\ \text{distance} &= \int_{t_1}^{t_2} |v(t)|\mathrm{d}t \\ &= A_1 + A_2 + A_3 \end{aligned}$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

1. Find the displacement of the particle during the time period $1 \leq t \leq 4$.
2. Find the distance traveled during this time period.

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Therefore the particle moves 4.5m to the left.

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&= \left[-\frac{t^3}{3}+\frac{t^2}{2}+6t\right]_1^3 + \left[\frac{t^3}{3}-\frac{t^2}{2}-6t\right]_3^4\\
&= \frac{61}{6} \approx 10.17\mathrm{m}
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- Velocity is the antiderivative of acceleration.
- If we know the acceleration and the initial values $s(0)$ and $v(0)$ for position and velocity, then we can find $s(t)$ by antidifferentiating twice.

An object near the Earth is subject to a gravitational force that produces a downward acceleration of 32 ft/s^2 (or 9.8 m/s^2).

Example

A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 ft above the ground. Find its height above the ground t seconds later.

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