Precalculus
Homework Lecture 8

1. Show that the graph of the equation is a circle. Find the center of the circle and its radius. Plot the circle by hand (roughly). The answer key has not been proofread, use with great caution.

   (a) $3x - x^2 = y^2 - 1.$

   answer: radius: $\frac{\sqrt{13}}{2}$, center: $\left(\frac{3}{2}, 0\right)$

   (b) $x^2 + y^2 - x - 2y = 0$

   answer: radius: $\frac{\sqrt{5}}{2}$ center: $\left(\frac{1}{2}, 1\right)$

   (c) $\frac{1}{2}\left((x - y)^2 + (x + y)^2\right) - 1 = 0$

   answer: radius: 1, center: $(0, 0)$

   (d) $2x^2 + y^2 = 2 - y^2$

   answer: radius: 1, center: $(0, 0)$

   (e) $2x^2 + 2y^2 - x + 2y = 3$

   answer: radius: $\frac{\sqrt{29}}{4}$, center: $\left(\frac{1}{4}, -\frac{1}{2}\right)$

Solution. 1e

The standard equation of a circle with center (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2.$$

Our strategy consists in transforming the equation $2x^2 + 2y^2 - x + 2y = 3$ into that form. This is done by completing the square, as we show below. We have included all computations with extreme detail. Such detail is not needed when solving test problems; students are encouraged to carry out some of the steps "in the head".

$$
\begin{aligned}
2x^2+2y^2-x+2y &= 3 \\
x^2+y^2-\frac{1}{2}x+y &= \frac{3}{2} && \text{Divide by 2 to ensure } x^2 \text{ has coeff. } 1 \\
x^2-\frac{1}{2}x+y^2+y &= \frac{3}{2} && \text{group the } x\text{'s and the } y\text{'s} \\
x^2-2\cdot\frac{1}{4}x+y^2+2\cdot\frac{1}{2}y &= \frac{3}{2} && \text{multiply and divide the linear terms by 2} \\
\underbrace{x^2-2\cdot\frac{1}{4}x+\left(\frac{1}{4}\right)^2}_{\text{square}}-\left(\frac{1}{4}\right)^2+\underbrace{y^2+2\cdot\frac{1}{2}y+\left(\frac{1}{2}\right)^2}_{\text{square}}-\left(\frac{1}{2}\right)^2 &= \frac{3}{2} && \text{Add and subtract appropriate constants to facilitate completing the square} \\
\left(x-\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2+\left(y+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2 &= \frac{3}{2} && \text{Complete square using: } (p\pm q)^2=p^2\pm 2pq+q^2 \\
\left(x-\frac{1}{4}\right)^2+\left(y+\frac{1}{2}\right)^2 &= \frac{3}{2}+\left(\frac{1}{4}\right)^2+\left(\frac{1}{2}\right)^2 && \text{transfer constants to the other side} \\
\left(x-\frac{1}{4}\right)^2+\left(y-\left(-\frac{1}{2}\right)\right)^2 &= \frac{3}{2}+\left(\frac{1}{4}\right)^2+\left(\frac{1}{2}\right)^2 && \text{Ensure squares are of form } (x-h)^2+(y-k)^2 \\
\left(x-\frac{1}{4}\right)^2+\left(y-\left(-\frac{1}{2}\right)\right)^2 &= \frac{3}{2}+\frac{1}{4}+\frac{1}{16} \\
&= \frac{24+4+1}{16} \\
&= \frac{29}{16} \\
\left(x-\frac{1}{4}\right)^2+\left(y-\left(-\frac{1}{2}\right)\right)^2 &= \left(\sqrt{\frac{29}{16}}\right)^2 && \text{final step: rewrite constant using } a=\left(\sqrt{a}\right)^2.
\end{aligned}
$$

The equation above is in the standard equation form for a circle $(x-h)^2+(y-k)^2=r^2$. It follows that the equation is of a circle with center $\left(\frac{1}{4},-\frac{1}{2}\right)$ and radius $\frac{\sqrt{29}}{4}$.

2. Write the equation of the circle with the indicated center and passing through the indicated point.
   (a) Center: $(1,2)$, passing through: $(0,0)$.
   (b) Center: $(-1,-2)$, passing through: $(1,1)$.
   (c) Center: $(3,5)$, passing through: $(5,7)$.
3. Find the x and y intercepts (if any) of the indicated circle.
   (a) Circle with center $(1,2)$ and radius 3.
   (b) Circle with center $(-1,2)$ and radius 2.
   (c) Circle with center $(1,-3)$ and passing through $(0,1)$.
   (d) Circle with center $(2,3)$ and passing through $(0,0)$.