

# Calculus III

## Homework on Lecture 3

- Find the area of the triangle  $\triangle ABC$ . The answer key has not been proofread, use with caution.
  - $A(1, 0, 0), B(0, 1, 0), C(0, 0, 1)$ .
  - $A(1, -1, 0), B(0, 1, -1), C(-1, 0, 1)$ .
  - $A(1, 2, 3), B(5, 7, 11), C(13, 17, 19)$ .
- Find a vector orthogonal to the two given vectors. The answer key has not been proofread, use with caution.
  - $\mathbf{u} = (2, 3, 5), \mathbf{v} = (3, 5, 7)$ .
  - $\mathbf{u} = (2, -5, -3), \mathbf{v} = (3, 5, 7)$ .
- Let the 4 vertices of a tetrahedron be  $O, A, B, C$ . Let  $\mathbf{v}_1 = \mathbf{OA}, \mathbf{v}_2 = \mathbf{OB}, \mathbf{v}_3 = \mathbf{OC}$  (the vectors given by the edges of the tetrahedron that pass through  $O$ ). It can be shown that the volume of the tetrahedron equals  $\frac{1}{3!} = \frac{1}{6}$  of the volume of the slanted box spanned by  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ . Using that information find the volumes of the following tetrahedra.
  - The volume of the tetrahedron with vertices  $(1, 1, 1), (1, -1, -1), (-1, 1, -1), (-1, -1, 1)$ .
  - The volume of the tetrahedron with vertices  $(1, 2, 3), (2, 3, 5), (3, 5, 7), (5, 7, 13)$ .
  - The volume of the tetrahedron with vertices  $A(1, 2, 2), B(1, 3, 3), C(1, 0, 2), D(-2, 3, 2)$ .
- Do the points  $(1, 2, 3), (2, 3, 5), (3, 5, 7), (5, 7, 11)$  lie in one plane?
- Let  $\mathbf{u}, \mathbf{v}, \mathbf{w}$  be arbitrary vectors. Show that the Jacobi identity for the cross product holds, i.e., show that

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) + \mathbf{v} \times (\mathbf{w} \times \mathbf{u}) + \mathbf{w} \times (\mathbf{u} \times \mathbf{v}) = \mathbf{0} \quad .$$

This problem will not appear on the quiz.