Precalculus
Lecture 19

Todor Milev
https://github.com/tmilev/freecalc

2020

Outline

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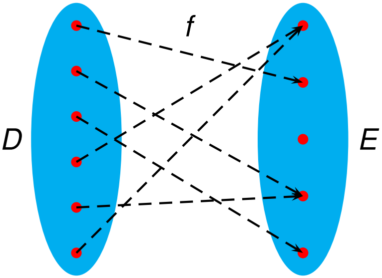

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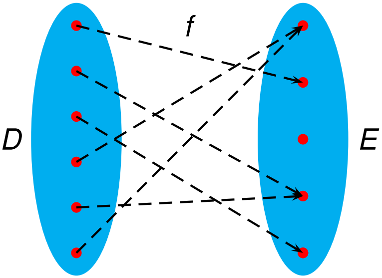

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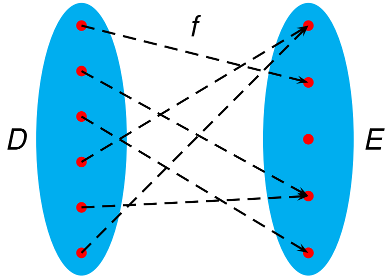

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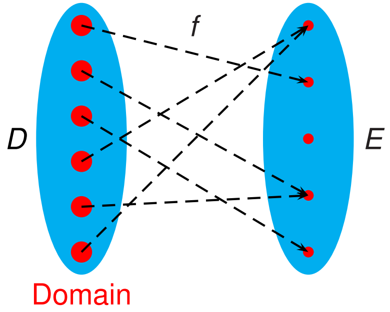

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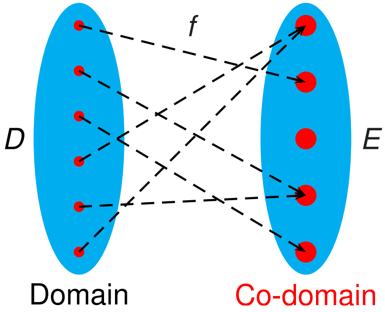

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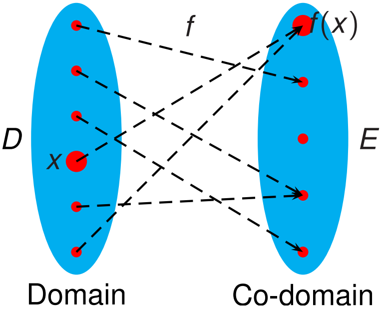

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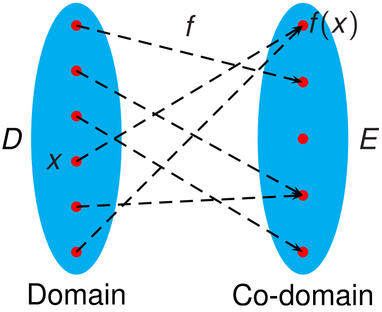

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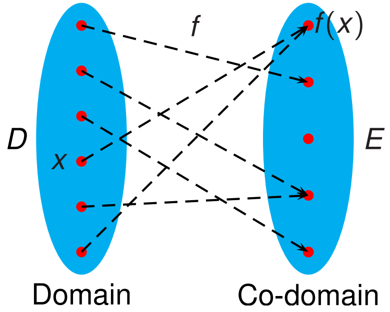

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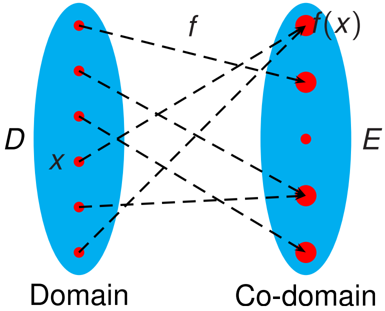

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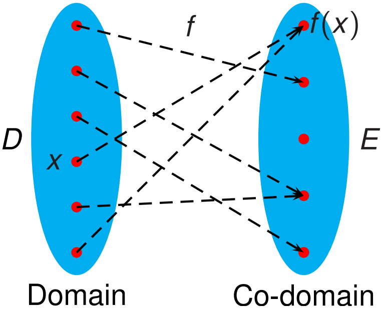

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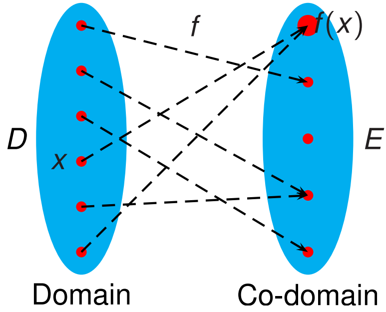

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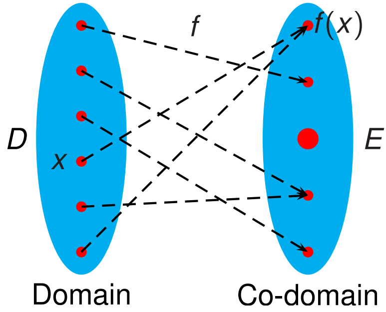

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- It is allowed to have an element in the co-domain without arrows pointing to it.

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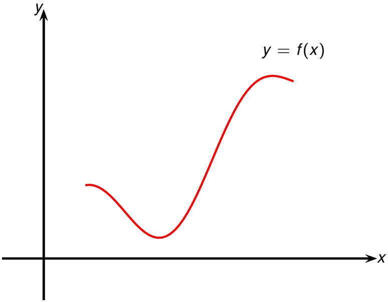

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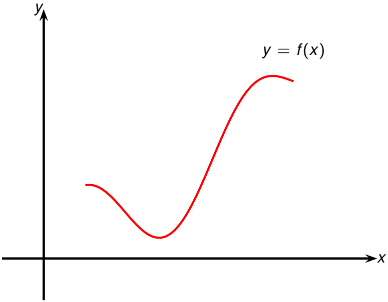

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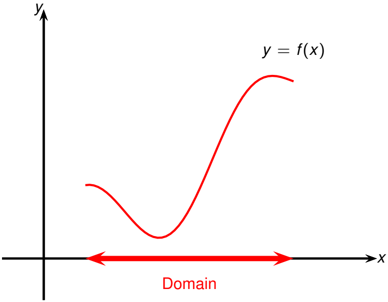

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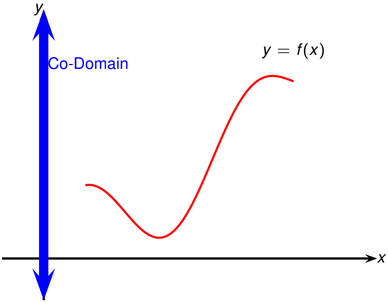

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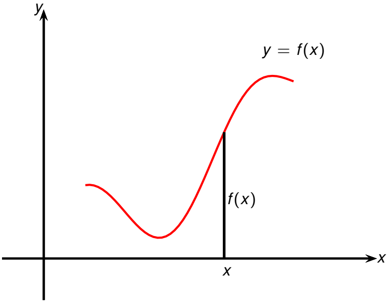

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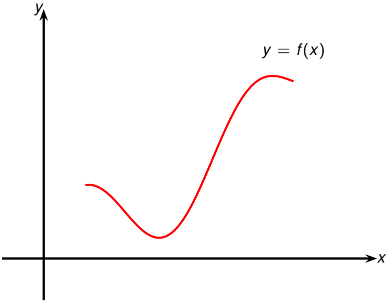

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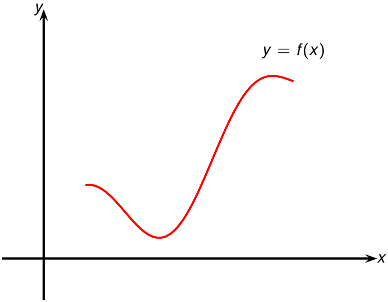

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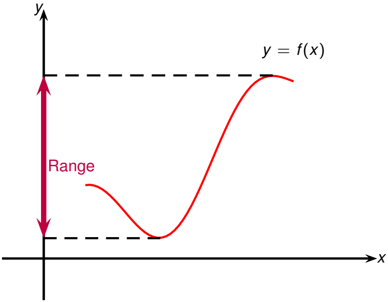

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- However if no such clarification is present (as often is the case in mathematical exercises/tests), the matter is up to the reader's intelligent interpretation.

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- In computer programming, the issues described here are addressed via "variable scope rules".

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&= \frac{2^2 + 2 \cdot 2h + h^2 - 2 - h - 1 - 2^2 + 2 + 1}{h}
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- Taking $\log x$ if $x \leq 0$ is not allowed in this course; taking $\log 0$ is not allowed in any course.

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Find the implied domains of the given functions.

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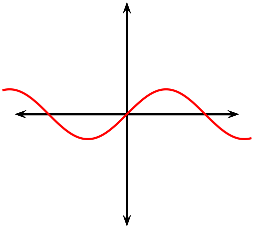
Domain is all real numbers except 3 and -2; that is, $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$.

The Vertical Line Test

Question

Given a curve in the plane, is it the graph of a function or not?



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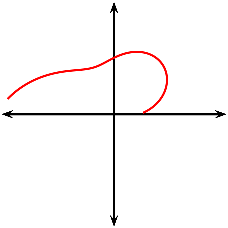

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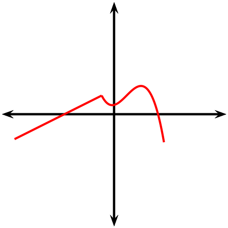

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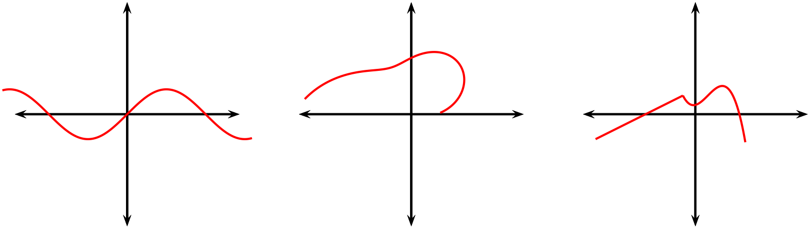
Proposition (The Vertical Line Test)

A curve in the plane is the graph of a function if and only if no vertical line intersects it more than once.



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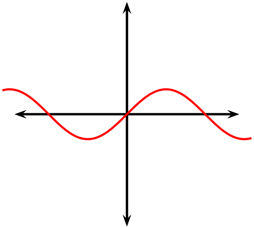

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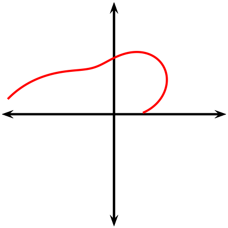

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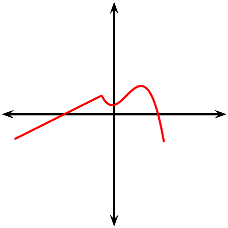

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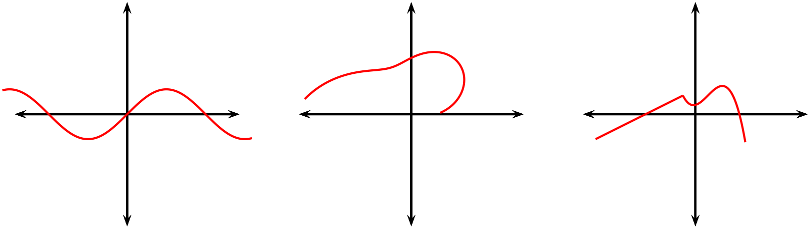

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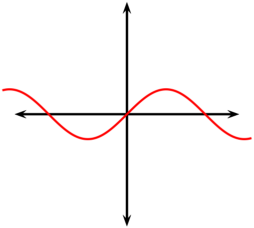

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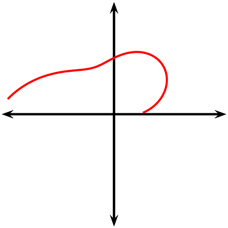

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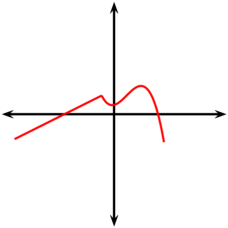

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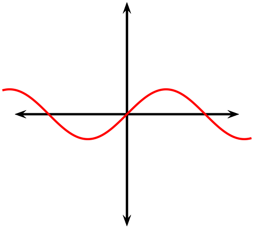

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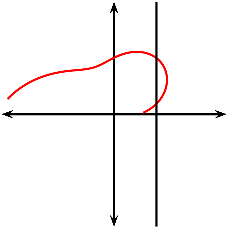

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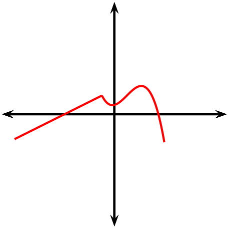

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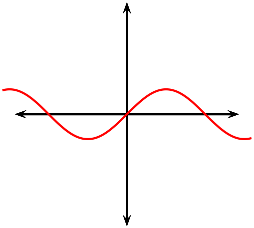

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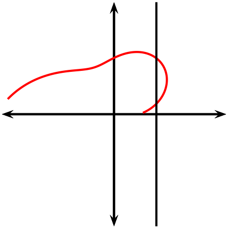

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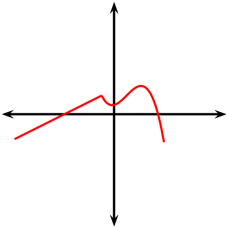

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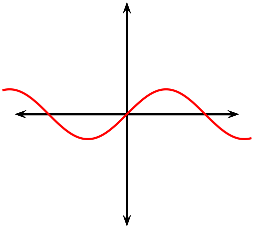

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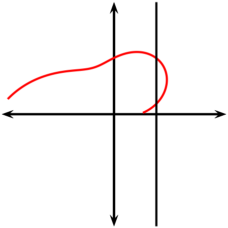

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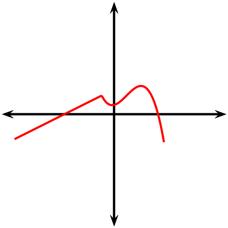

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Piecewise Defined Functions

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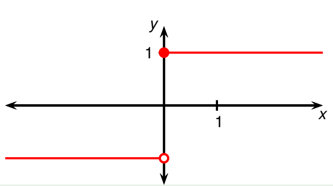


$$f(x) = \begin{cases} 1 & \text{if} \quad x \geq 0 \\ -1 & \text{if} \quad x < 0 \end{cases}$$

The filled red circle means $(0, 1)$ is on the curve.

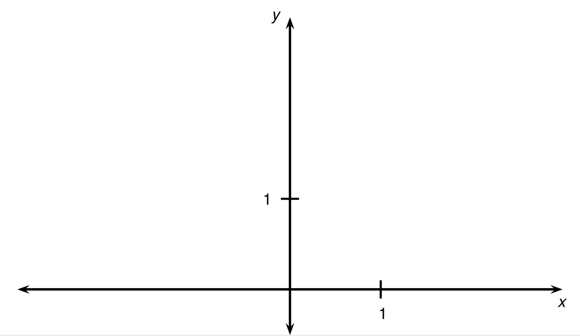
The open circle means $(0, -1)$ is not on the curve.

Example

The absolute value $|x|$ of a number a is defined to be

$$|x| = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0. \end{cases}$$

Sketch a graph of the function $f(x) = |x|$.



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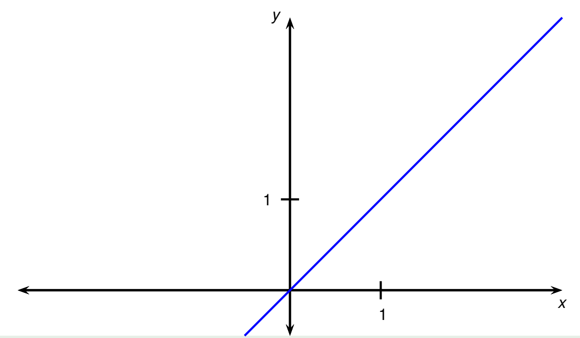

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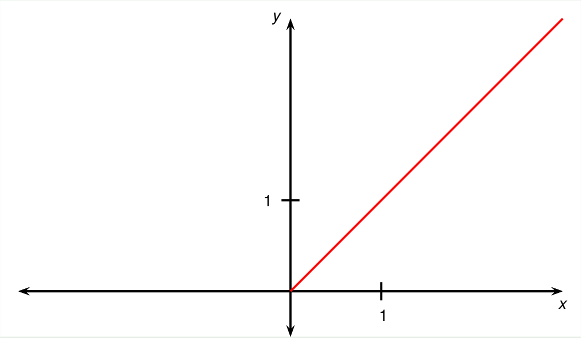

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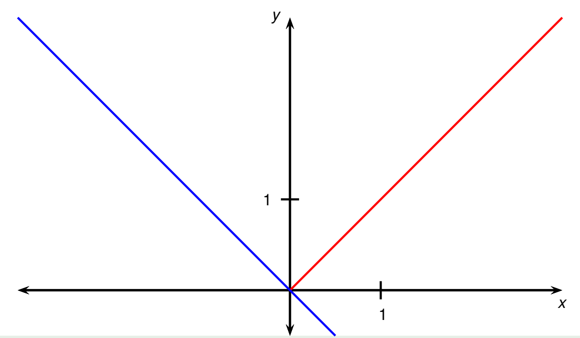

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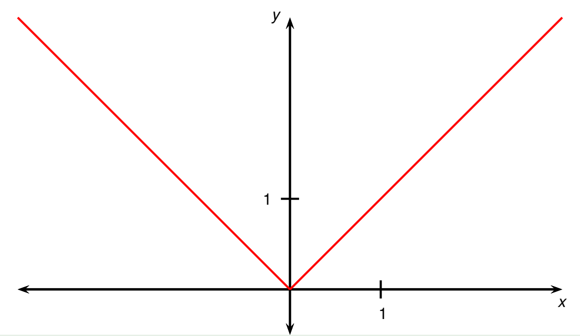

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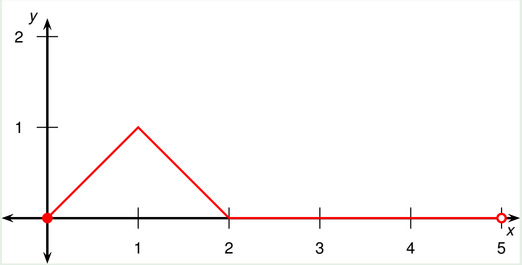

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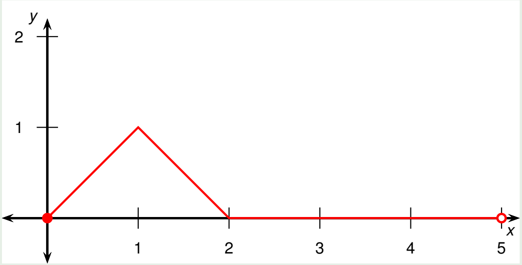

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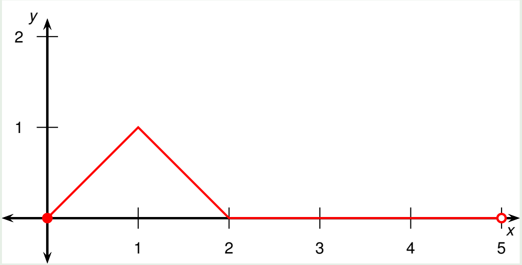

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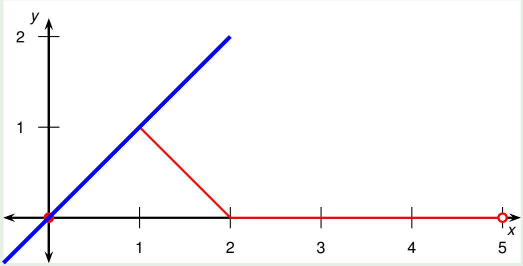
$$f(x) = \begin{cases} & \text{if } 0 \leq x < 1 \\ & \text{if } 1 \leq x < 2 \\ & \text{if } 2 \leq x < 5 \end{cases}$$

Example

Find a formula for the function f whose graph is given below.



Different formulas on $[0, 1)$, $[1, 2)$, and $[2, 5)$.

$$f(x) = \begin{cases} ? & \text{if } 0 \leq x < 1 \\ & \text{if } 1 \leq x < 2 \\ & \text{if } 2 \leq x < 5 \end{cases}$$

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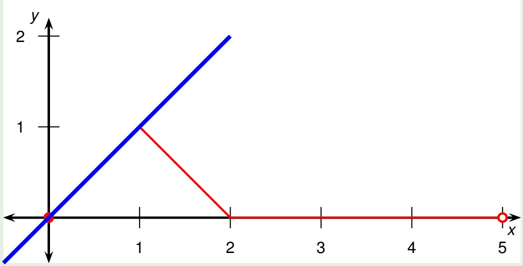

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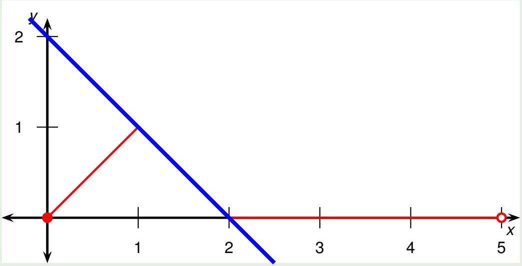

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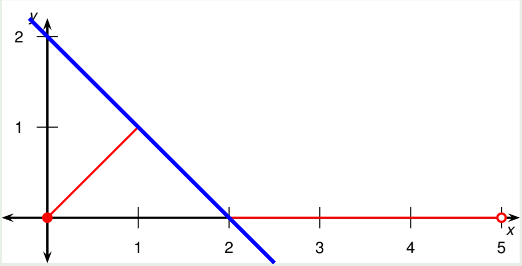

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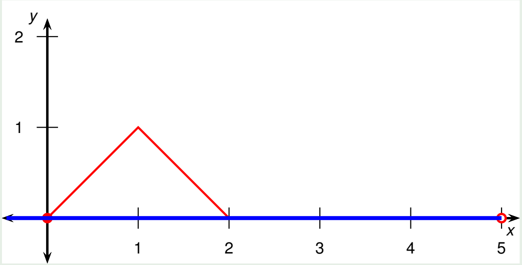

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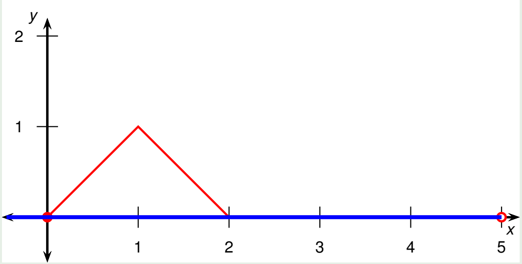

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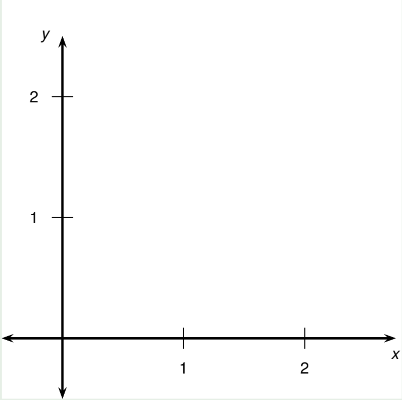
Example

Sketch the function $f(x) = |2x - 3|$.



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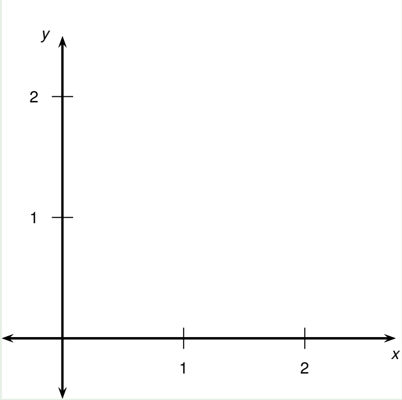

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$$

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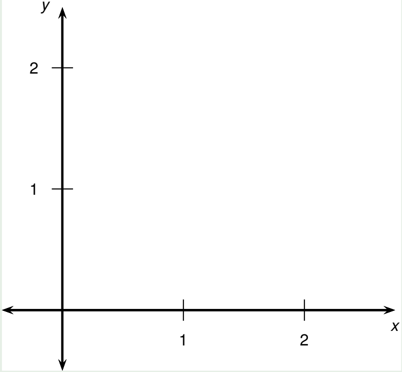

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$$

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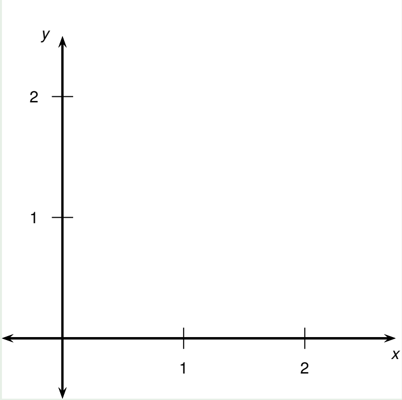

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$$= \begin{cases} 2x - 3 & \text{if } 2x \geq 3 \\ -2x + 3 & \text{if } 2x < 3 \end{cases}$$

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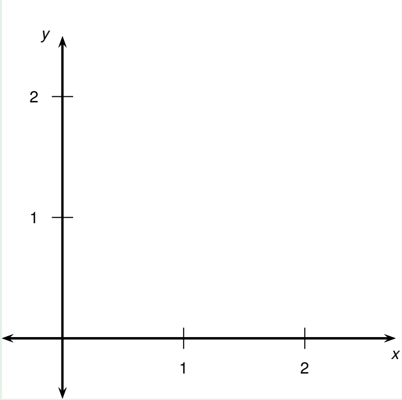

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$$\begin{aligned} |2x-3| &= \begin{cases} 2x-3 & \text{if } 2x-3 \geq 0 \\ -(2x-3) & \text{if } 2x-3 < 0 \end{cases} \\ &= \begin{cases} 2x-3 & \text{if } 2x \geq 3 \\ -2x+3 & \text{if } 2x < 3 \end{cases} \\ &= \begin{cases} 2x-3 & \text{if } x \geq 3/2 \\ -2x+3 & \text{if } x < 3/2. \end{cases} \end{aligned}$$

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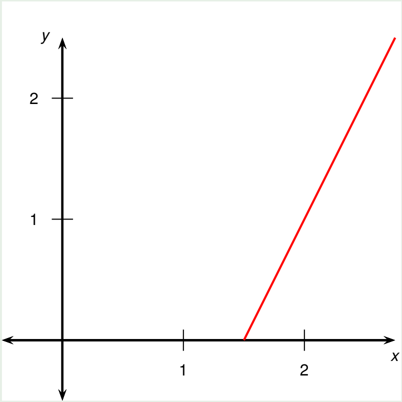

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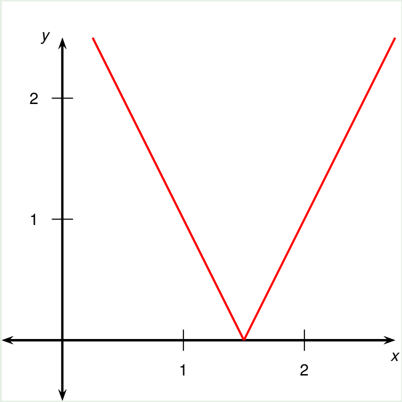

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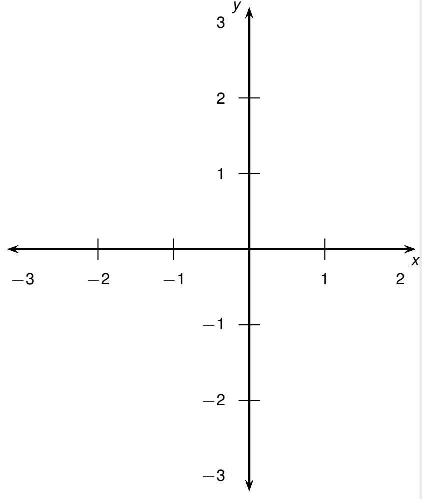

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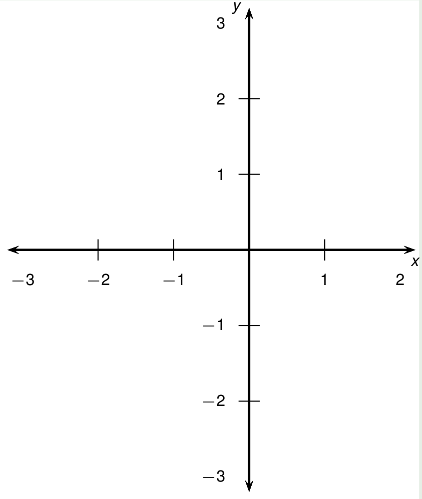

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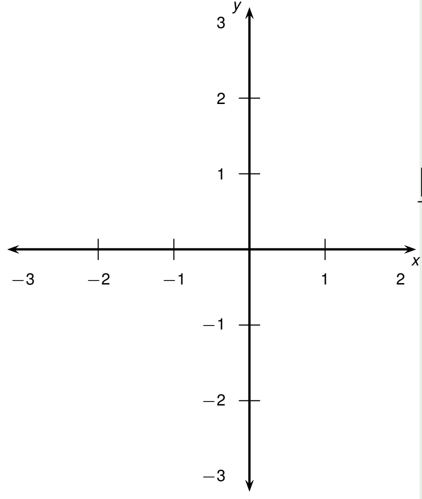

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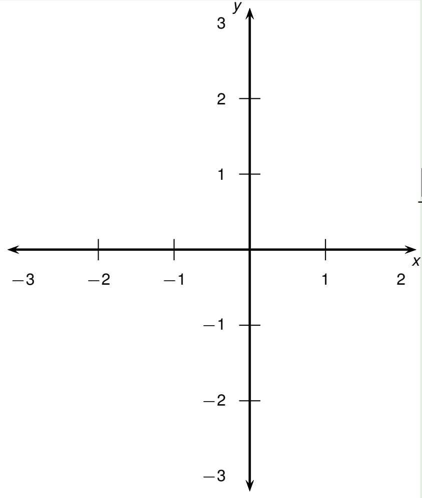

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$$= \begin{cases} \frac{?}{2x+1} & \text{if } 4x > -2 \\ \frac{?}{2x+1} & \text{if } 4x < -2 \end{cases}$$

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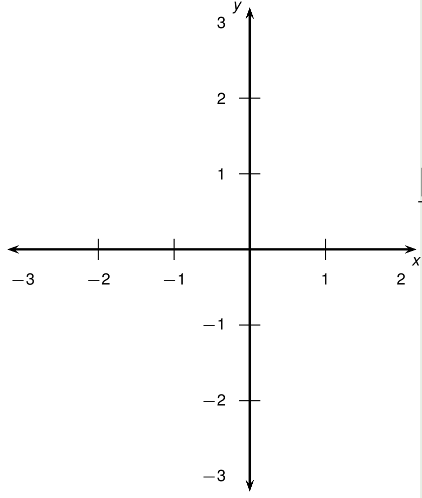

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$$= \begin{cases} \frac{2(2x+1)}{2x+1} & \text{if } 4x > -2 \\ \frac{\textbf{?}}{2x+1} & \text{if } 4x < -2 \end{cases}$$

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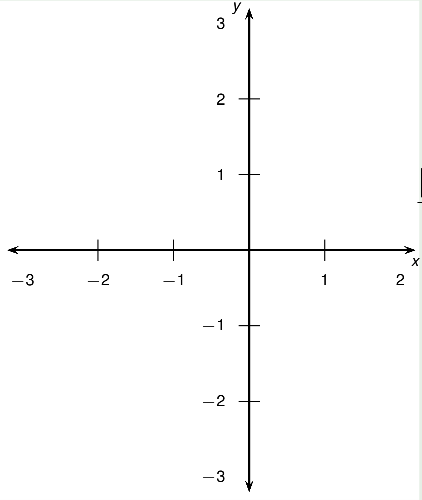

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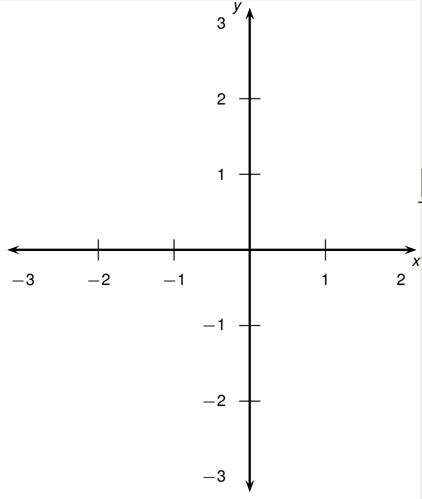

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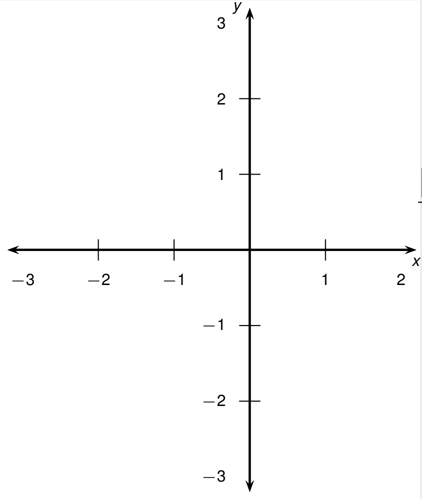

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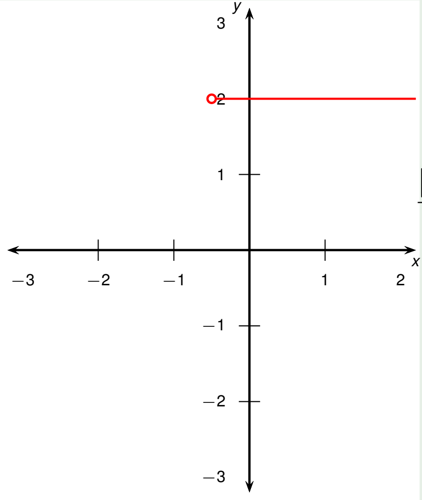

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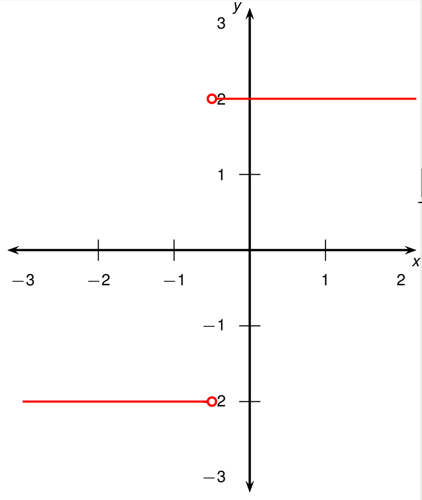

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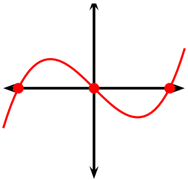
Definition

The zeros of a function f are the values of the argument x for which $f(x) = 0$.

Observation

The zeros of a function are the x-coordinates of the x intercepts of the graph of the function.



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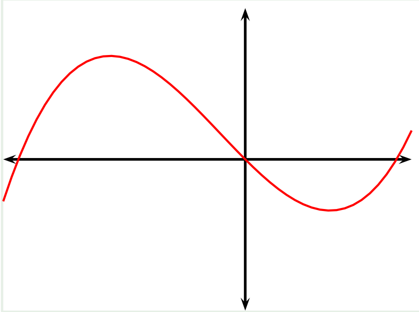
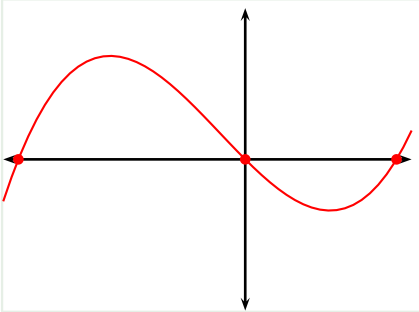


Find the zeroes of
$f(x) = \frac{1}{6}x^3 + \frac{1}{6}x^2 - x$.

Example



Find the zeroes of
$$f(x) = \frac{1}{6}x^3 + \frac{1}{6}x^2 - x.$$

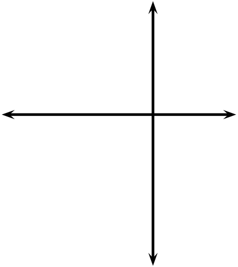
Example

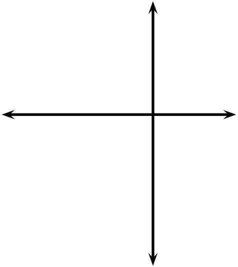
- Find when $f(x) = g(x)$, where

$$f(x) = \frac{1}{6}x^3 + \frac{1}{6}x^2 \qquad\qquad g(x) = x$$

- Find the intersections of the graphs of f and g.

Let g of x and f of x be functions.

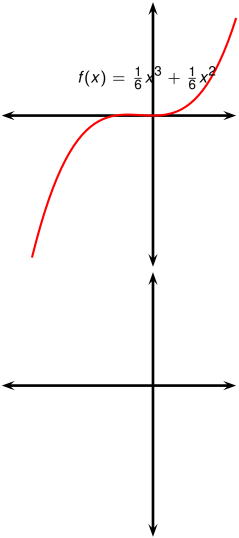




Observation

- *To solve $f(x) = g(x)$ means to find the x coordinates of the intersections of the graphs of f and g.*

Let g of x and f of x be functions.



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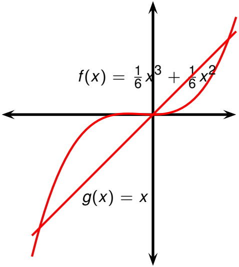

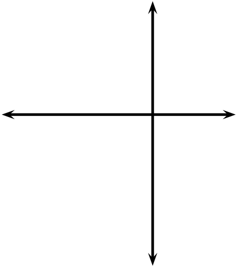

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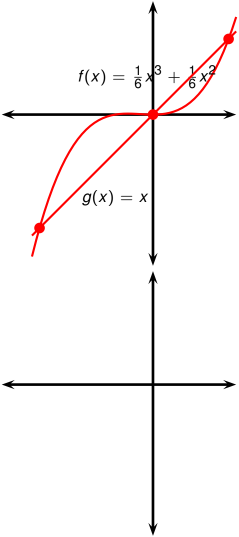

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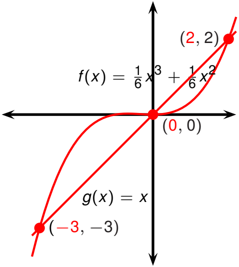

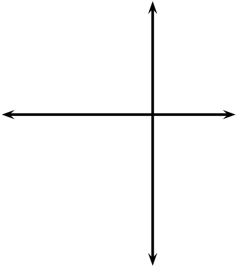

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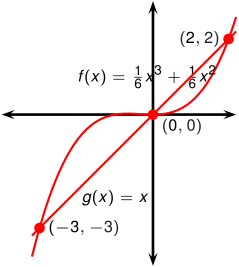

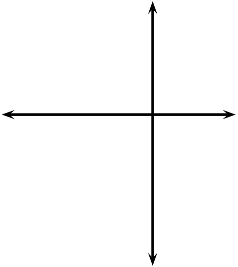

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- *To solve $f(x) = g(x)$ means to find the x coordinates of the intersections of the graphs of f and g.*
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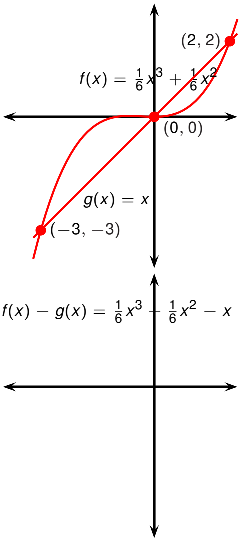
Let g of x and f of x be functions.



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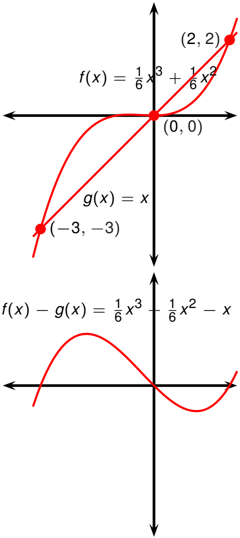
Let g of x and f of x be functions.



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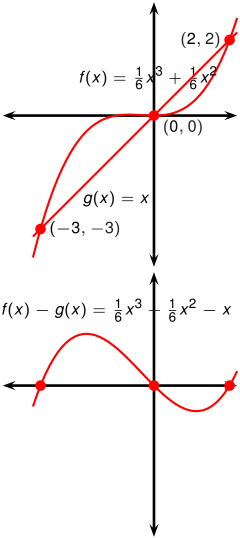

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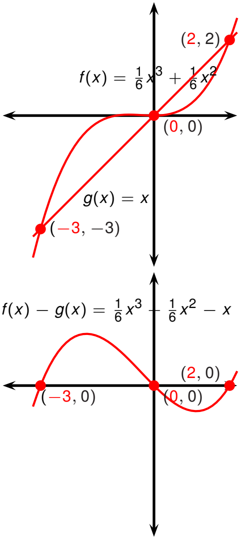

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Definition (Even and Odd Functions)

A function f is called even if $f(-x) = f(x)$ for all x in its domain. A function f is called odd if $f(-x) = -f(x)$ for all x in its domain.

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The function $f(x) = x^2$ is even:

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Therefore f is odd.

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Therefore g is even.

$$h(-x) =$$

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Therefore f is odd.

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Therefore g is even.

$$h(-x) = 2(-x) - 1$$

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Therefore g is even.

Therefore f is odd.

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Therefore f is odd.

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$$\begin{aligned} h(-x) &= 2(-x) - 1 \\ &= -2x - 1 \\ &\neq h(x), -h(x) \end{aligned}$$

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$$g(x) = 1 - x^4$$

$$\begin{aligned} g(-x) &= 1 - (-x)^4 \\ &= 1 - x^4 \\ &= g(x) \end{aligned}$$

Therefore g is even.

$$h(x) = 2x - 1$$

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Therefore h is neither even nor odd.

Increasing and Decreasing Functions

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A function f is called increasing on an interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.
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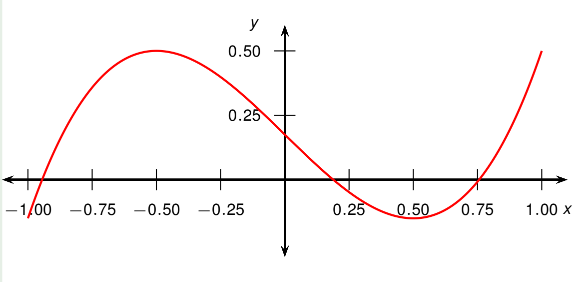

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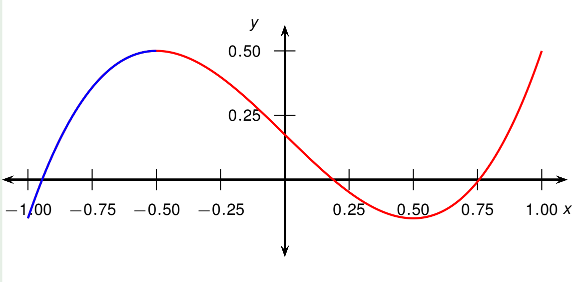

- f is increasing on $[-1, -\frac{1}{2}]$.

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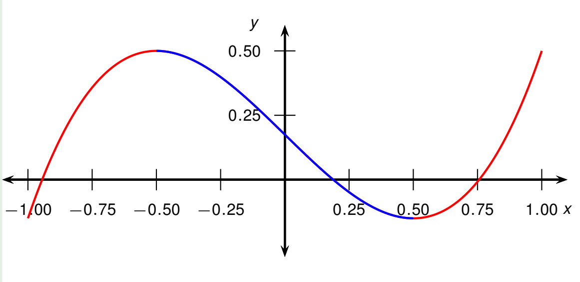

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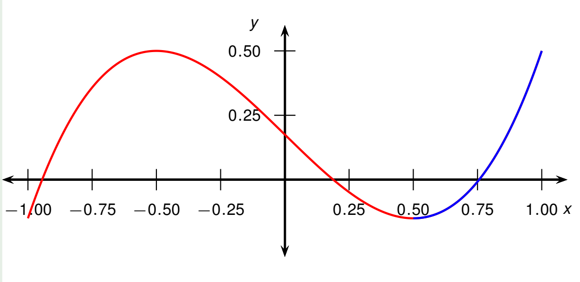

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- f is increasing on $[\frac{1}{2}, 1]$.