Calculus III
Lecture 11

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https://github.com/tmilev/freecalc

2020

Outline

1. Surfaces
   - Quadric Surfaces

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   - Quadric Surfaces
2. Tangent Planes

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- A graph surface $z=f(x,y)$ can be represented as a level surface:
$$z=f(x,y) \Longleftrightarrow F(x,y,z)=0 \quad \text{for} \quad F(x,y,z)=z-f(x,y)\ .$$

Quadratic surfaces

- The level sets for second degree polynomial functions

  $$f(x, y, z) = Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J.$$

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- The coefficients are allowed to be zero.

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Through rigid motions (translations and rotations) a quadratic surface can be reduced to one of the two canonical forms.

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- $$\begin{aligned} Ax^2 + By^2 + Cz^2 + D &= 0, \\ A(-x)^2 + B(-y)^2 + C(-z)^2 + D &= 0, \end{aligned}$$

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The canonical forms above are in addition split into sub-forms depending on the sign of A, B, C, D, I.

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0, B > 0, C > 0, D > 0$

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- Example:

$$x^2 + 2y^2 + 3z^2 + 4 = 0.$$

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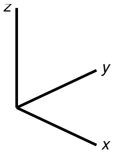
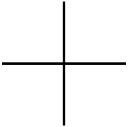
- Let $A > 0, B > 0, C > 0, D < 0$. Rescale so $D = -1$.





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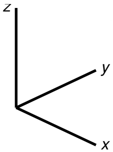

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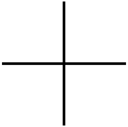

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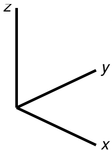

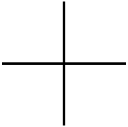

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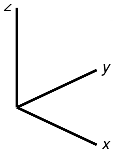

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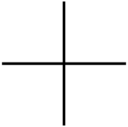

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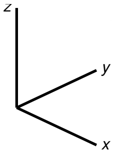

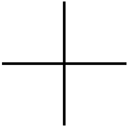

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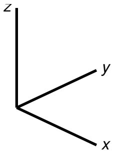

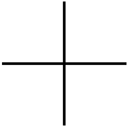

$$\begin{aligned} z &= -3 \\ \frac{1}{2}x^2 + \frac{1}{3}y^2 &= 1 - \frac{z^2}{4} \\ &= \end{aligned}$$

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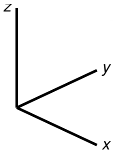

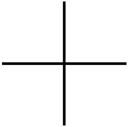

$$\begin{aligned} z &= -3 \\ \frac{1}{2}x^2 + \frac{1}{3}y^2 &= 1 - \frac{z^2}{4} \\ &= -\frac{5}{4} \end{aligned}$$

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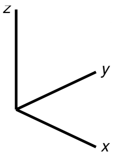

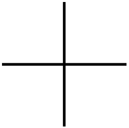

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\begin{aligned}
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&=
\end{aligned}
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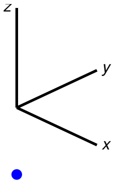

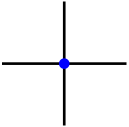

$$\begin{aligned} z &= {\color{red}-2} \\ \frac{1}{2}x^2 + \frac{1}{3}y^2 &= 1 - \frac{z^2}{4} \\ &{\color{red}= 0} \end{aligned}$$

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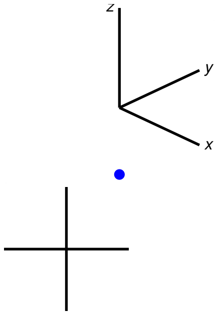

$$\begin{aligned} z &= -1 \\ \frac{1}{2}x^2 + \frac{1}{3}y^2 &= 1 - \frac{z^2}{4} \\ &= \end{aligned}$$

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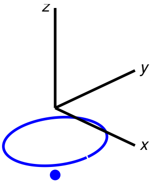
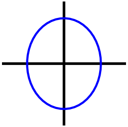


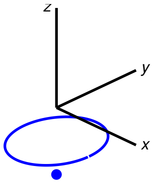


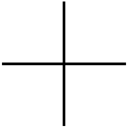
$$\begin{aligned} z &= {\color{red}-1} \\ \frac{1}{2}x^2 + \frac{1}{3}y^2 &= 1 - \frac{z^2}{4} \\ &{\color{red}= \frac{3}{4}} \end{aligned}$$

- Let $A > 0, B > 0, C > 0, D < 0$. Rescale so $D = -1$.
- Set $A = \frac{1}{a^2}$, $B = \frac{1}{b^2}$, $C = \frac{1}{c^2}$.
- Surface becomes: $\left\{(x, y, z) | \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1\right\}$.
- We illustrate the theory on the example: $\frac{1}{2}x^2 + \frac{1}{3}y^2 + \frac{z^2}{4} = 1$.
- Rewrite: $\frac{1}{2}x^2 + \frac{1}{3}y^2 = 1 - \frac{z^2}{4}$.
- The level curves are:
  - None for $z < 2$ and $z > 2$.
  - Two points for $z = \pm 2$.
  - Ellipses for $z \in (-2, 2)$.

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C > 0$, $D > 0$

Consider the surface $\mathcal{C} = \{(x, y, z) | Ax^2 + By^2 + Cz^2 + D = 0\}$.





$$\begin{aligned} z &= 0 \\ \frac{1}{2}x^2 + \frac{1}{3}y^2 &= 1 - \frac{z^2}{4} \\ &= \end{aligned}$$

- Let $A > 0, B > 0, C > 0, D < 0$. Rescale so $D = -1$.
- Set $A = \frac{1}{a^2}$, $B = \frac{1}{b^2}$, $C = \frac{1}{c^2}$.
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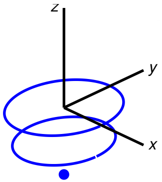
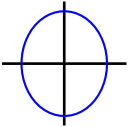
$Ax^2 + By^2 + Cz^2 + D = 0, A > 0, B > 0, C > 0, D > 0$

Consider the surface $\mathcal{C} = \{(x, y, z) | Ax^2 + By^2 + Cz^2 + D = 0\}$.





$$\begin{aligned} z &= 0 \\ \frac{1}{2}x^2 + \frac{1}{3}y^2 &= 1 - \frac{z^2}{4} \\ &= 1 \end{aligned}$$

- Let $A > 0, B > 0, C > 0, D < 0$. Rescale so $D = -1$.
- Set $A = \frac{1}{a^2}, B = \frac{1}{b^2}, C = \frac{1}{c^2}$.
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$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C > 0$, $D > 0$

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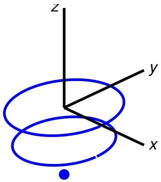

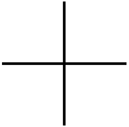

$$\begin{aligned} z &= 1 \\ \frac{1}{2}x^2 + \frac{1}{3}y^2 &= 1 - \frac{z^2}{4} \\ &= \end{aligned}$$

- Let $A > 0, B > 0, C > 0, D < 0$. Rescale so $D = -1$.
- Set $A = \frac{1}{a^2}$, $B = \frac{1}{b^2}$, $C = \frac{1}{c^2}$.
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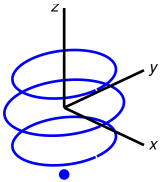

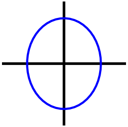

$$\begin{aligned} z &= {\color{red}1} \\ \frac{1}{2}x^2 + \frac{1}{3}y^2 &= 1 - \frac{z^2}{4} \\ &{\color{red}= \frac{3}{4}} \end{aligned}$$

- Let $A > 0, B > 0, C > 0, D < 0$. Rescale so $D = -1$.
- Set $A = \frac{1}{a^2}$, $B = \frac{1}{b^2}$, $C = \frac{1}{c^2}$.
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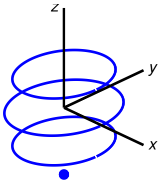
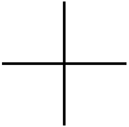
$Ax^2 + By^2 + Cz^2 + D = 0,\ A > 0, B > 0, C > 0, D > 0$

Consider the surface $\mathcal{C} = \{(x, y, z) | Ax^2 + By^2 + Cz^2 + D = 0\}$.





$$\begin{aligned} z &= 2 \\ \tfrac{1}{2}x^2 + \tfrac{1}{3}y^2 &= 1 - \tfrac{z^2}{4} \\ &= \end{aligned}$$

- Let $A > 0, B > 0, C > 0,\ D < 0$. Rescale so $D = -1$.
- Set $A = \frac{1}{a^2}$, $B = \frac{1}{b^2}$, $C = \frac{1}{c^2}$.
- Surface becomes: $\left\{(x, y, z) | \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1\right\}$.
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  - Two points for $z = \pm 2$.
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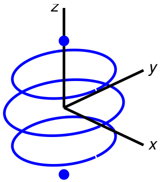
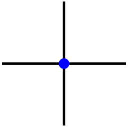
$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0, B > 0, C > 0, D > 0$

Consider the surface $\mathcal{C} = \{(x, y, z) | Ax^2 + By^2 + Cz^2 + D = 0\}$.





$$\begin{aligned} z &= 2 \\ \frac{1}{2}x^2 + \frac{1}{3}y^2 &= 1 - \frac{z^2}{4} \\ &= 0 \end{aligned}$$

- Let $A > 0, B > 0, C > 0, D < 0$. Rescale so $D = -1$.
- Set $A = \frac{1}{a^2}$, $B = \frac{1}{b^2}$, $C = \frac{1}{c^2}$.
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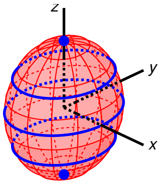
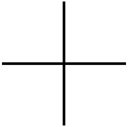
$Ax^2 + By^2 + Cz^2 + D = 0,\ A > 0, B > 0, C > 0, D > 0$

Consider the surface $\mathcal{C} = \{(x, y, z) | Ax^2 + By^2 + Cz^2 + D = 0\}$.

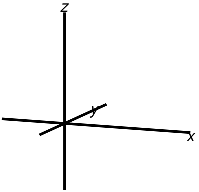


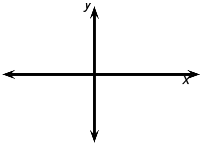


$$\begin{aligned} z &= \\ \frac{1}{2}x^2 + \frac{1}{3}y^2 &= 1 - \frac{z^2}{4} \end{aligned}$$

- Let $A > 0, B > 0, C > 0, D < 0$. Rescale so $D = -1$.
- Set $A = \frac{1}{a^2}$, $B = \frac{1}{b^2}$, $C = \frac{1}{c^2}$.
- Surface becomes: $\left\{(x, y, z) | \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1\right\}$.
- We illustrate the theory on the example: $\frac{1}{2}x^2 + \frac{1}{3}y^2 + \frac{z^2}{4} = 1$.
- Rewrite: $\frac{1}{2}x^2 + \frac{1}{3}y^2 = 1 - \frac{z^2}{4}$.
- The level curves are:
  - None for $z < 2$ and $z > 2$.
  - Two points for $z = \pm 2$.
  - Ellipses for $z \in (-2, 2)$.
- Figure is called ellipsoid.

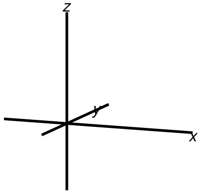
$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D = 0$

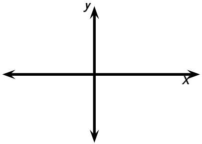




- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2\right\}$

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D = 0$

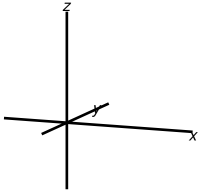




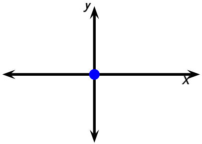
$$\begin{aligned} z &= \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= \end{aligned}$$

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2\right\}$
- The level curves $z = const$ are:

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D = 0$

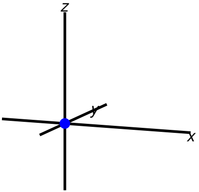


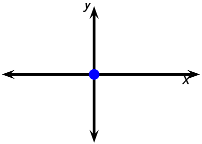
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 \right\}$
- The level curves $z = const$ are:



$$\begin{aligned} z &= 0 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= 0 \end{aligned}$$

$Ax^2 + By^2 + Cz^2 + D = 0, A > 0, B > 0, C < 0, D = 0$

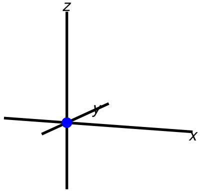




$$\begin{aligned} z &= 0 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= 0 \\ \Rightarrow (x, y, z) &= (0, 0, 0) \end{aligned}$$

- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 \right\}$
- The level curves $z = const$ are:
  - A point for $z = 0$.

$Ax^2 + By^2 + Cz^2 + D = 0,\ A > 0,\ B > 0,\ C < 0,\ D = 0$



- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 \right\}$
- The level curves $z = const$ are:
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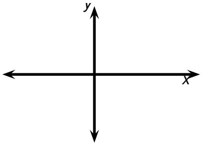
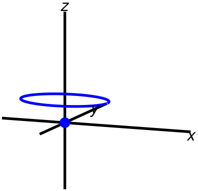


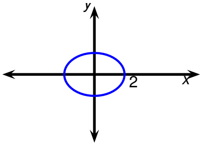
$$z = 1$$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 1$$

$$\Rightarrow (x, y, z) \in$$

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0, B > 0, C < 0, D = 0$

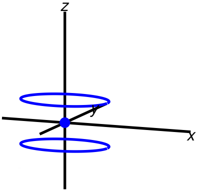


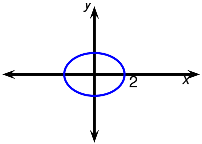


$$\begin{aligned} z &= 1 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= 1 \\ \Rightarrow (x, y, z) &\in \text{ellipse} \end{aligned}$$

- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 \right\}$
- The level curves $z = const$ are:
  - A point for $z = 0$.
  - Ellipses for $z \neq 0$.

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D = 0$

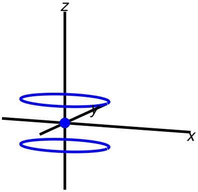




$$\begin{aligned} z &= \pm 1 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= 1 = (\pm 1)^2 \\ \Rightarrow (x, y, z) &\in \text{ellipse} \end{aligned}$$

- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 \right\}$
- The level curves $z = const$ are:
  - A point for $z = 0$.
  - Ellipses for $z \neq 0$.

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D = 0$



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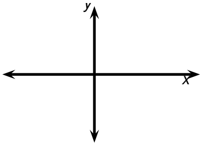
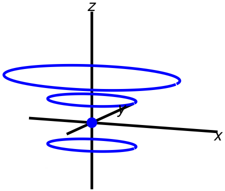
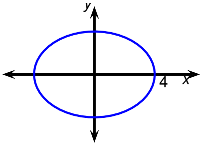


$$\begin{aligned} z &= 2 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= (\quad 2)^2 = 4 \\ \Rightarrow (x, y, z) &\in \end{aligned}$$

$Ax^2 + By^2 + Cz^2 + D = 0, A > 0, B > 0, C < 0, D = 0$





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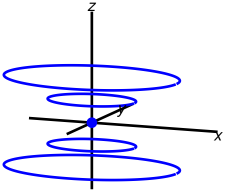
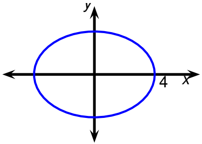
$Ax^2 + By^2 + Cz^2 + D = 0,\ A > 0, B > 0, C < 0, D = 0$

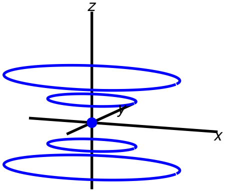




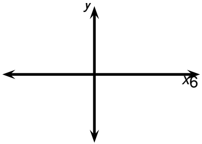
$$\begin{aligned} z &= 2 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= (\pm 2)^2 = 4 \\ \Rightarrow (x, y, z) &\in \text{ellipse} \end{aligned}$$

- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 \right\}$
- The level curves $z = const$ are:
  - A point for $z = 0$.
  - Ellipses for $z \neq 0$.

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D = 0$

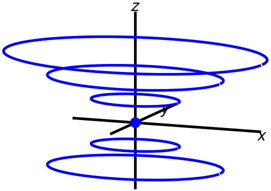


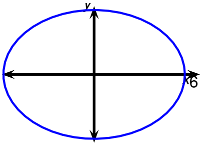
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 \right\}$
- The level curves $z = const$ are:
  - A point for $z = 0$.
  - Ellipses for $z \neq 0$.



$$\begin{aligned} z &= 3 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= (\quad 3)^2 = 9 \\ \Rightarrow (x, y, z) &\in \end{aligned}$$

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D = 0$

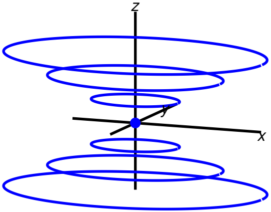


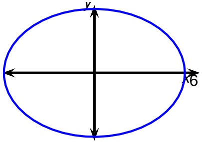


- Consider the surface $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2\right\}$
- The level curves $z = const$ are:
  - A point for $z = 0$.
  - Ellipses for $z \neq 0$.

$$\begin{aligned} z &= 3 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= (3)^2 = 9 \\ \Rightarrow (x, y, z) &\in \text{ellipse} \end{aligned}$$

$Ax^2 + By^2 + Cz^2 + D = 0,\ A > 0, B > 0, C < 0, D = 0$

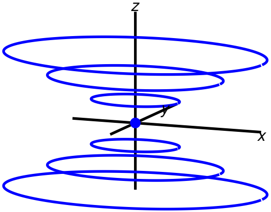


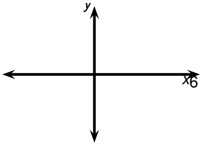


$$
\begin{aligned}
z &= \pm 3\\
\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= (\pm 3)^2 = 9\\
\Rightarrow (x, y, z) &\in \text{ellipse}
\end{aligned}
$$

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2\right\}$
- The level curves $z = const$ are:
  - A point for $z = 0$.
  - Ellipses for $z \neq 0$.

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D = 0$



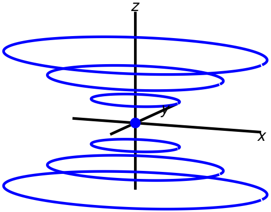


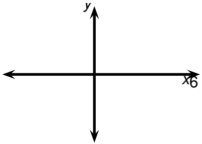
- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2\right\}$
- The level curves $z = const$ are:
  - A point for $z = 0$.
  - Ellipses for $z \neq 0$.

$$\begin{aligned} z &= \pm 3 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= (\pm 3)^2 = 9 \\ \Rightarrow (x, y, z) &\in \text{ellipse} \end{aligned}$$

- $\Rightarrow$ the ellipses are stacked along **?** .

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D = 0$





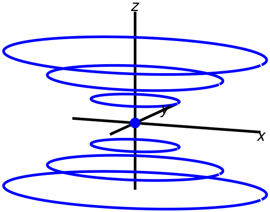
- Consider the surface $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2\right\}$
- The level curves $z = const$ are:
  - A point for $z = 0$.
  - Ellipses for $z \neq 0$.
- For $y = 0$:

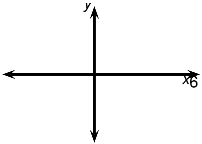
$$\left(\frac{x}{2}\right)^2 = z^2$$

$$
\begin{aligned}
z &= \pm 3\\
\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= (\pm 3)^2 = 9\\
\Rightarrow (x, y, z) &\in \text{ellipse}
\end{aligned}
$$

- $\Rightarrow$ the ellipses are stacked along **?** .

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D = 0$





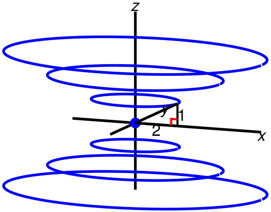
$$\begin{aligned} z &= \pm 3 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= (\pm 3)^2 = 9 \\ \Rightarrow (x, y, z) &\in \text{ellipse} \end{aligned}$$

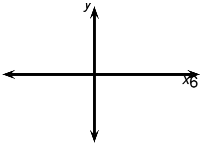
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 \right\}$
- The level curves $z = const$ are:
  - A point for $z = 0$.
  - Ellipses for $z \neq 0$.
- For $y = 0$:

$$\begin{aligned} \left(\frac{x}{2}\right)^2 &= z^2 \\ \frac{x}{2} &= \pm z \end{aligned}$$

- $\Rightarrow$ the ellipses are stacked along **?** .

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0, B > 0, C < 0, D = 0$





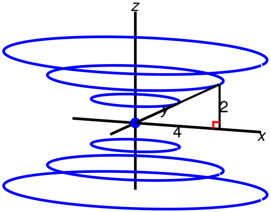
$$z = \pm 3$$
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = (\pm 3)^2 = 9$$
$$\Rightarrow (x, y, z) \in \text{ellipse}$$

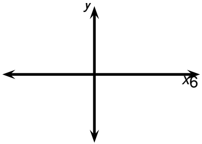
- Consider the surface $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2\right\}$
- The level curves $z = const$ are:
  - A point for $z = 0$.
  - Ellipses for $z \neq 0$.
- For $y = 0$:

$$\begin{aligned} \left(\frac{x}{2}\right)^2 &= z^2 \\ \frac{x}{2} &= \pm z \\ x &= \pm 2z \end{aligned}$$

- $\Rightarrow$ the ellipses are stacked along **?** .

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D = 0$





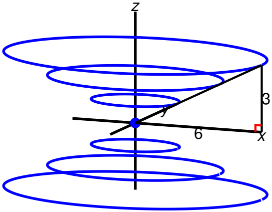
$$\begin{aligned} z &= \pm 3 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= (\pm 3)^2 = 9 \\ \Rightarrow (x, y, z) &\in \text{ellipse} \end{aligned}$$

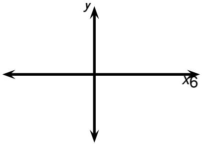
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 \right\}$
- The level curves $z = const$ are:
  - A point for $z = 0$.
  - Ellipses for $z \neq 0$.
- For $y = 0$:

$$\begin{aligned} \left(\frac{x}{2}\right)^2 &= z^2 \\ \frac{x}{2} &= \pm z \\ x &= \pm 2z \end{aligned}$$

- $\Rightarrow$ the ellipses are stacked along **?** .

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D = 0$





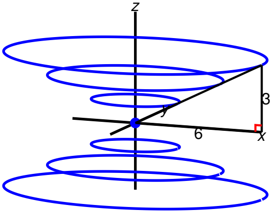
$$
\begin{aligned}
z &= \pm 3\\
\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= (\pm 3)^2 = 9\\
\Rightarrow (x, y, z) &\in \text{ ellipse}
\end{aligned}
$$

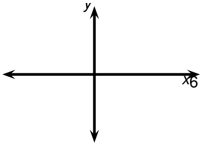
- Consider the surface $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2\right\}$
- The level curves $z = \mathit{const}$ are:
  - A point for $z = 0$.
  - Ellipses for $z \neq 0$.
- For $y = 0$:

$$
\begin{aligned}
\left(\frac{x}{2}\right)^2 &= z^2\\
\frac{x}{2} &= \pm z\\
{\color{red}x} &{\color{red}=} \pm 2z
\end{aligned}
$$

- $\Rightarrow$ the ellipses are stacked along **?** .

$Ax^2 + By^2 + Cz^2 + D = 0,\ A > 0,\ B > 0,\ C < 0,\ D = 0$





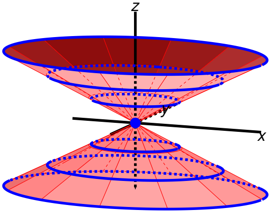
$$\begin{aligned} z &= \pm 3 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= (\pm 3)^2 = 9 \\ \Rightarrow (x, y, z) &\in \text{ellipse} \end{aligned}$$

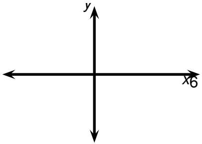
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 \right\}$
- The level curves $z = const$ are:
  - A point for $z = 0$.
  - Ellipses for $z \neq 0$.
- For $y = 0$:

$$\begin{aligned} \left(\frac{x}{2}\right)^2 &= z^2 \\ \frac{x}{2} &= \pm z \\ x &= \pm 2z \end{aligned}$$

- $\Rightarrow$ the ellipses are stacked along lines .

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D = 0$





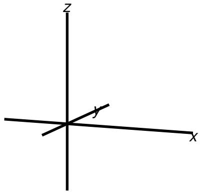
$$\begin{aligned} z &= \pm 3 \\ \left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 &= (\pm 3)^2 = 9 \\ \Rightarrow (x, y, z) &\in \text{ellipse} \end{aligned}$$

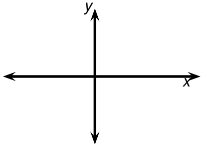
- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2\right\}$
- The level curves $z = const$ are:
  - A point for $z = 0$.
  - Ellipses for $z \neq 0$.
- For $y = 0$:

$$\begin{aligned} \left(\frac{x}{2}\right)^2 &= z^2 \\ \frac{x}{2} &= \pm z \\ x &= \pm 2z \end{aligned}$$

- $\Rightarrow$ the ellipses are stacked along lines .
- $\Rightarrow$ The figure is a ("two-piece") cone.

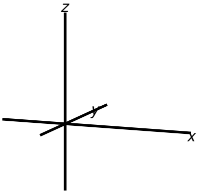
$Ax^2 + By^2 + Cz^2 + D = 0, A > 0, B > 0, C < 0, D < 0$

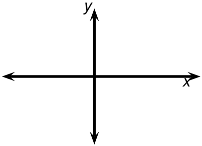




- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 1 \right\}$

$Ax^2 + By^2 + Cz^2 + D = 0, A > 0, B > 0, C < 0, D < 0$



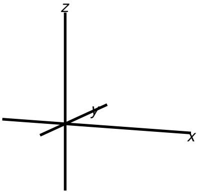


$z =$

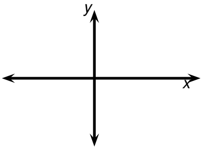
$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 =$

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 1\right\}$
- The level curves $z = const$ are:

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D < 0$



- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 1 \right\}$
- The level curves $z = const$ are:

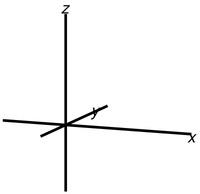


$z=0$

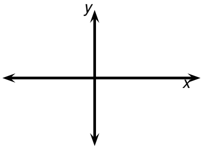
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 1 = 0^2 + 1$$

$$\Rightarrow (x, y, z) \in$$

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D < 0$



- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 1 \right\}$
- The level curves $z = const$ are:

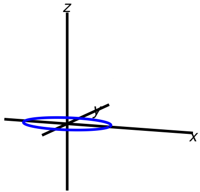


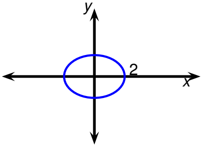
$z{=}0$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 {=} 1 = 0^2 + 1$$

$$\Rightarrow (x, y, z) \in$$

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D < 0$





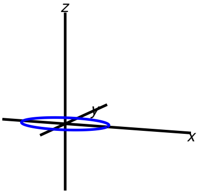
$z=0$

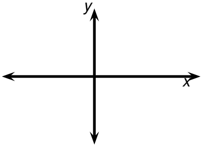
$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 1 = 0^2 + 1$

$\Rightarrow (x, y, z) \in$ ellipse

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 1\right\}$
- The level curves $z = const$ are: Ellipses

$Ax^2 + By^2 + Cz^2 + D = 0, A > 0, B > 0, C < 0, D < 0$





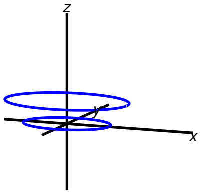
$$z = \quad 1$$

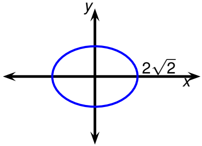
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 2 = 1 + (\quad 1)^2$$

$$\Rightarrow {\color{red}(x, y, z) \in}$$

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 1\right\}$
- The level curves $z = const$ are: Ellipses

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D < 0$





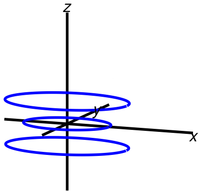
$z = \quad 1$

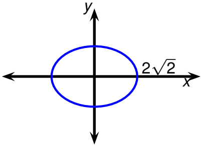
$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 2 = 1 + (\quad 1)^2$

$\Rightarrow (x, y, z) \in$ ellipse

- Consider the surface $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 1\right\}$
- The level curves $z = const$ are: Ellipses

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D < 0$





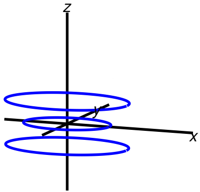
$z = \pm 1$

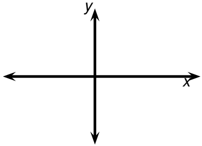
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 2 = 1 + (\pm 1)^2$$

$\Rightarrow (x, y, z) \in$ ellipse

- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 1 \right\}$
- The level curves $z = const$ are: Ellipses

$Ax^2 + By^2 + Cz^2 + D = 0,\ A > 0,\ B > 0,\ C < 0,\ D < 0$





$z = \quad 2$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 5 = 1 + (\quad 2)^2$

$\Rightarrow (x, y, z) \in$

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 1\right\}$
- The level curves $z = const$ are: Ellipses

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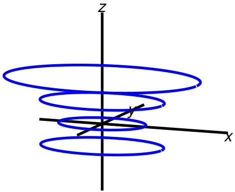

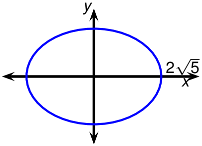

$z = \quad 2$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 5 = 1 + (\quad 2)^2$

$\Rightarrow (x, y, z) \in$ ellipse

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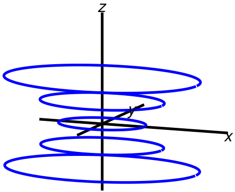

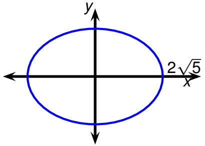

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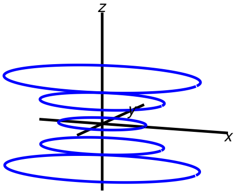

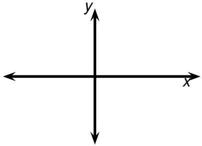

$$z = \quad 3$$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 10 = 1 + (\quad 3)^2$$

$$\Rightarrow (x, y, z) \in$$

- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 1 \right\}$
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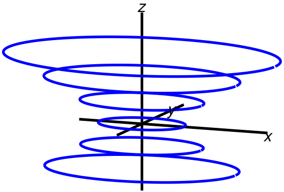

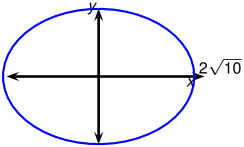

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$\Rightarrow (x, y, z) \in$ ellipse

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 1\right\}$
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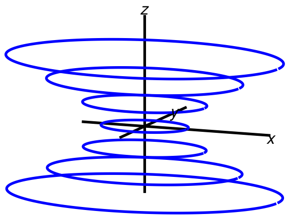

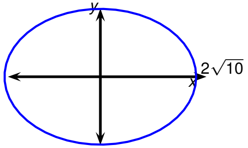

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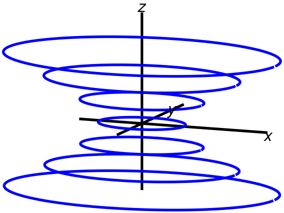
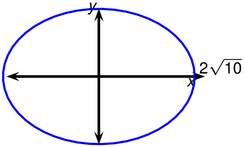
$\Rightarrow (x, y, z) \in$ ellipse

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 1\right\}$
- The level curves $z = const$ are: Ellipses for all z.

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D < 0$





$$z = \pm 3$$
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 10 = 1 + (\pm 3)^2$$
$$\Rightarrow (x, y, z) \in \text{ellipse}$$

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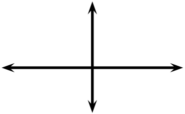
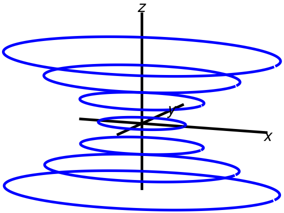
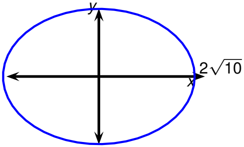


- $\Rightarrow$ ellipses: stacked along **?**

$Ax^2 + By^2 + Cz^2 + D = 0, A > 0, B > 0, C < 0, D < 0$





$$z = \pm 3$$

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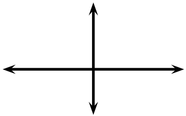

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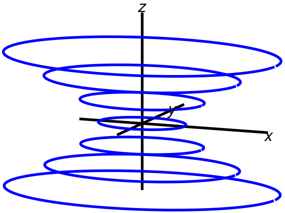

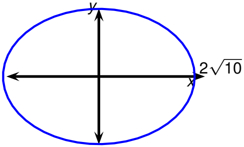

$z = \pm 3$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 10 = 1 + (\pm 3)^2$

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- Consider the surface $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 1\right\}$
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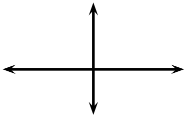

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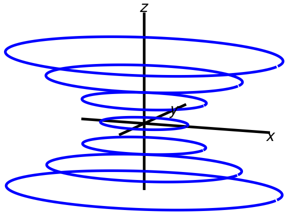

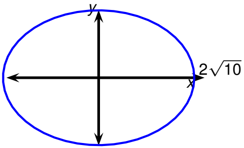

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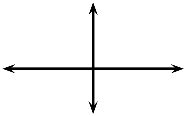

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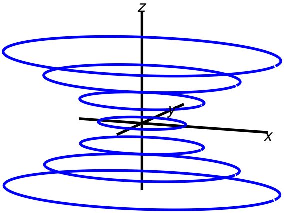

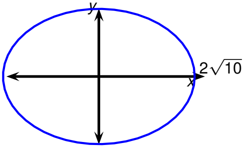

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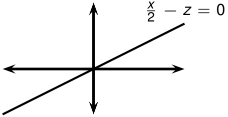

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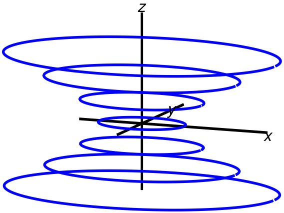

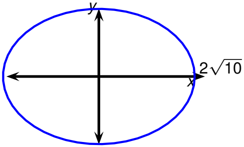

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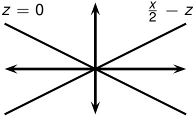

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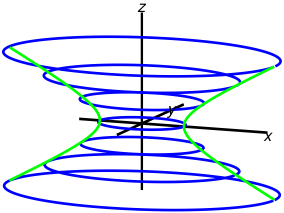

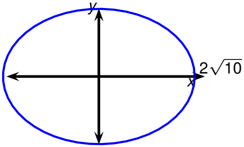

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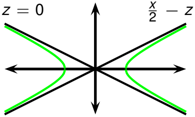

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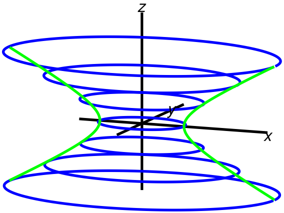

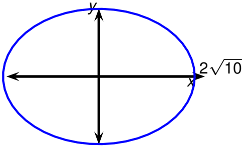

$z = \pm 3$

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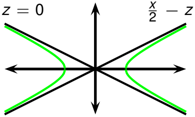

- $\Rightarrow$ ellipses: stacked along hyperbolas.

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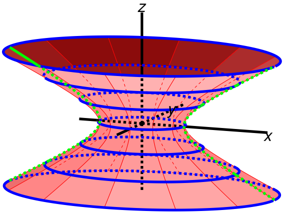

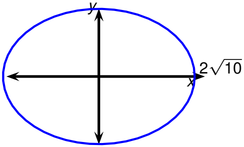

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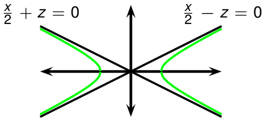

- $\Rightarrow$ ellipses: stacked along hyperbolas.
- Figure called: one-sheet hyperboloid.

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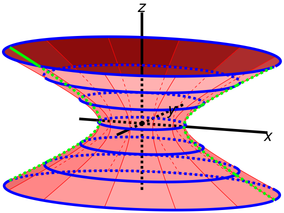
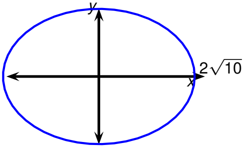




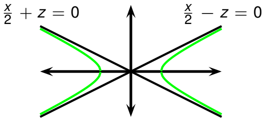
$z = \pm 3$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 10 = 1 + (\pm 3)^2$

$\Rightarrow (x, y, z) \in$ ellipse

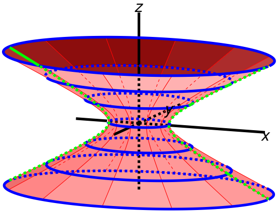
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 1 \right\}$
- The level curves $z = const$ are: Ellipses for all z. For $y = 0$:

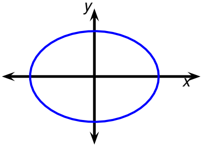
$$\begin{aligned} \left(\frac{x}{2}\right)^2 &= z^2 + 1 \\ \left(\frac{x}{2}\right)^2 - z^2 &= 1 \\ \left(\frac{x}{2} - z\right)\left(\frac{x}{2} + z\right) &= 1 \\ \left(\frac{x}{2} - z\right) &= \frac{1}{\left(\frac{x}{2} + z\right)} \end{aligned}$$



- $\Rightarrow$ ellipses: stacked along hyperbolas.
- Figure called: one-sheet hyperboloid.

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D < 0$





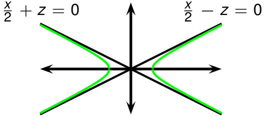
$$z = \pm 3$$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = \frac{1}{2} + (\pm 3)^2$$

$\Rightarrow (x, y, z) \in$ ellipse

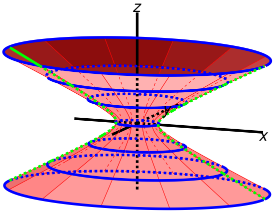
- Consider the surface $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 + \frac{1}{2}\right\}$
- The level curves $z = const$ are: Ellipses for all z. For $y = 0$:

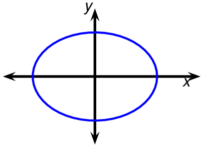
$$\begin{aligned}
\left(\frac{x}{2}\right)^2 &= z^2 + \frac{1}{2}\\
\left(\frac{x}{2}\right)^2 - z^2 &= \frac{1}{2}\\
\left(\frac{x}{2} - z\right)\left(\frac{x}{2} + z\right) &= \frac{1}{2}\\
\left(\frac{x}{2} - z\right) &= \frac{\frac{1}{2}}{\left(\frac{x}{2} + z\right)}
\end{aligned}$$



- $\Rightarrow$ ellipses: stacked along hyperbolas.
- Figure called: one-sheet hyperboloid.

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D < 0$





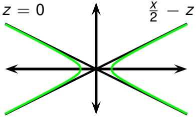
$$z = \pm 3$$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = \frac{1}{4} + (\pm 3)^2$$

$$\Rightarrow (x, y, z) \in \text{ellipse}$$

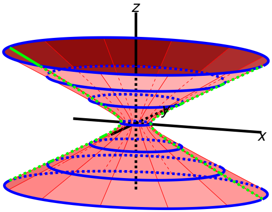
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 + \frac{1}{4} \right\}$
- The level curves $z = const$ are: Ellipses for all z. For $y = 0$:

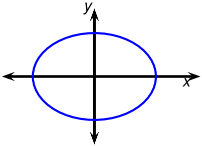
$$\begin{aligned} \left(\frac{x}{2}\right)^2 &= z^2 + \frac{1}{4} \\ \left(\frac{x}{2}\right)^2 - z^2 &= \frac{1}{4} \\ \left(\frac{x}{2} - z\right)\left(\frac{x}{2} + z\right) &= \frac{1}{4} \\ \left(\frac{x}{2} - z\right) &= \frac{\frac{1}{4}}{\left(\frac{x}{2} + z\right)} \end{aligned}$$



- $\Rightarrow$ ellipses: stacked along hyperbolas.
- Figure called: one-sheet hyperboloid.

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D < 0$





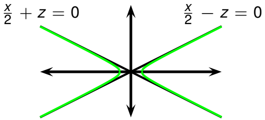
$z = \pm 3$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = \frac{1}{8} + (\pm 3)^2$$

$\Rightarrow (x, y, z) \in$ ellipse

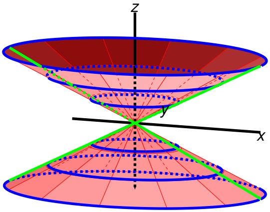
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 + \frac{1}{8} \right\}$
- The level curves $z = const$ are: Ellipses for all z. For $y = 0$:

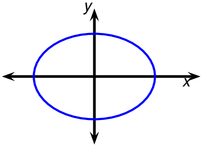
$$\begin{aligned} \left(\frac{x}{2}\right)^2 &= z^2 + \frac{1}{8} \\ \left(\frac{x}{2}\right)^2 - z^2 &= \frac{1}{8} \\ \left(\frac{x}{2} - z\right)\left(\frac{x}{2} + z\right) &= \frac{1}{8} \\ \left(\frac{x}{2} - z\right) &= \frac{\frac{1}{8}}{\left(\frac{x}{2} + z\right)} \end{aligned}$$



- $\Rightarrow$ ellipses: stacked along hyperbolas.
- Figure called: one-sheet hyperboloid.

$Ax^2 + By^2 + Cz^2 + D = 0, A > 0, B > 0, C < 0, D = 0$





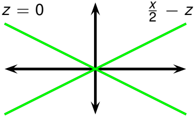
$z = \pm 3$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 0 + (\pm 3)^2$

$\Rightarrow (x, y, z) \in$ ellipse

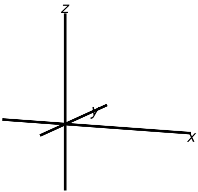
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 + 0 \right\}$
- The level curves $z = const$ are: Ellipses for all z. For $y = 0$:

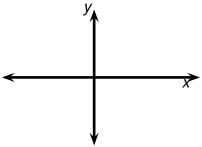
$$\begin{aligned} \left(\frac{x}{2}\right)^2 &= z^2 + 0 \\ \left(\frac{x}{2}\right)^2 - z^2 &= 0 \\ \left(\frac{x}{2} - z\right)\left(\frac{x}{2} + z\right) &= 0 \\ \left(\frac{x}{2} - z\right) &= \frac{0}{\left(\frac{x}{2} + z\right)} \end{aligned}$$



- $\Rightarrow$ ellipses: stacked along hyperbolas.
- Figure called: cone.

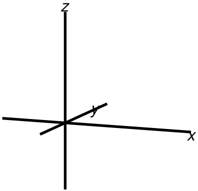
$Ax^2 + By^2 + Cz^2 + D = 0,\ A > 0,\ B > 0,\ C < 0,\ D > 0$

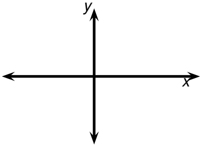




- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 1 \right\}$

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D > 0$



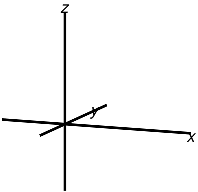


$z =$

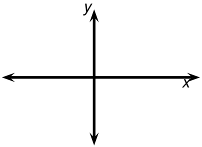
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 =$$

- Consider the surface $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 1\right\}$
- When $z = const$:

$Ax^2 + By^2 + Cz^2 + D = 0, A > 0, B > 0, C < 0, D > 0$



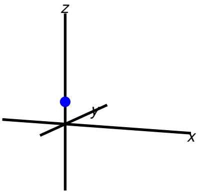
- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 1\right\}$
- When $z = const$:

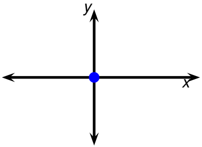


$z = \quad 1$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 0 = 1^2 - 1$$

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D > 0$





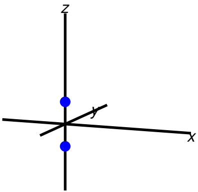
$$z = \quad 1$$

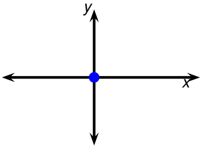
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 0 = 1^2 - 1$$

$$\Rightarrow (x, y, z) = (0, 0, \quad 1)$$

- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 1 \right\}$
- When $z = const$:

$Ax^2 + By^2 + Cz^2 + D = 0, A > 0, B > 0, C < 0, D > 0$





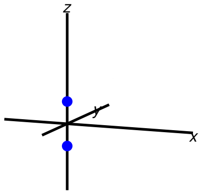
$$z = \pm 1$$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 0 = 1^2 - 1$$

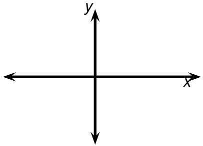
$$\Rightarrow (x, y, z) = (0, 0, \pm 1)$$

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 1\right\}$
- When $z = const$: Two pts. for $z = \pm 1$. Ellipses

$Ax^2 + By^2 + Cz^2 + D = 0,\ A > 0,\ B > 0,\ C < 0,\ D > 0$



- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 1 \right\}$
- When $z = const$: Two pts. for $z = \pm 1$. Ellipses

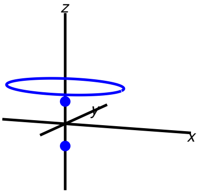


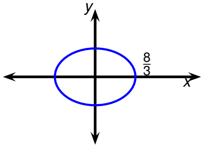
$$z = \frac{5}{3}$$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = \frac{16}{9} = \left(\frac{5}{3}\right)^2 - 1$$

$\Rightarrow (x, y, z) \in$

$Ax^2 + By^2 + Cz^2 + D = 0,\ A > 0, B > 0, C < 0, D > 0$





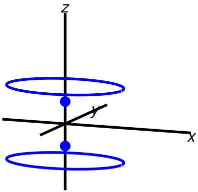
$$z = \frac{5}{3}$$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = \frac{16}{9} = \left(\frac{5}{3}\right)^2 - 1$$

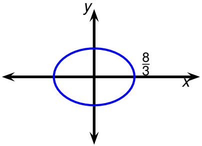
$\Rightarrow (x, y, z) \in$ ellipse

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 1\right\}$
- When $z = const$: Two pts. for $z = \pm 1$. Ellipses

$Ax^2 + By^2 + Cz^2 + D = 0,\ A > 0,\ B > 0,\ C < 0,\ D > 0$



- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 1 \right\}$
- When $z = const$: Two pts. for $z = \pm 1$. Ellipses

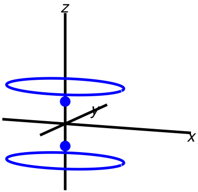


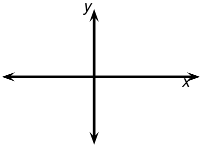
$$z = \pm \frac{5}{3}$$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = \frac{16}{9} = \left(\pm \frac{5}{3}\right)^2 - 1$$

$$\Rightarrow (x, y, z) \in \text{ellipse}$$

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D > 0$





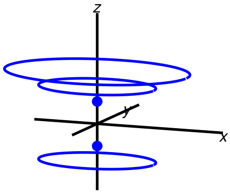
$$z = \quad \frac{7}{3}$$

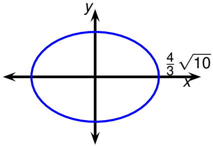
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = \frac{40}{9} = \left(\quad \frac{7}{3}\right)^2 - 1$$

$$\Rightarrow (x, y, z) \in$$

- Consider the surface $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 1\right\}$
- When $z = const$: Two pts. for $z = \pm 1$. Ellipses

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D > 0$





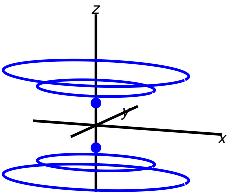
$$z = \quad \frac{7}{3}$$

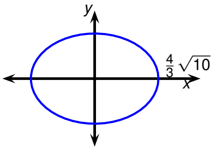
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = \frac{40}{9} = \left(\quad \frac{7}{3}\right)^2 - 1$$

$\Rightarrow (x, y, z) \in$ ellipse

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 1\right\}$
- When $z = const$: Two pts. for $z = \pm 1$. Ellipses

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D > 0$





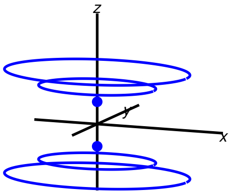
$$z = \pm\frac{7}{3}$$

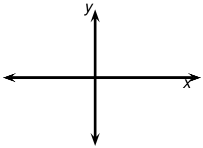
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = \frac{40}{9} = \left(\pm\frac{7}{3}\right)^2 - 1$$

$\Rightarrow (x, y, z) \in$ ellipse

- Consider the surface $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 1\right\}$
- When $z = const$: Two pts. for $z = \pm 1$. Ellipses

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D > 0$





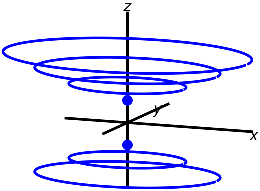
$z = \quad 3$

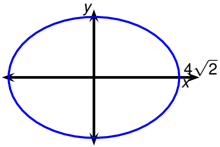
$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 8 = (\quad 3)^2 - 1$

$\Rightarrow (x, y, z) \in$

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 1\right\}$
- When $z = const$: Two pts. for $z = \pm 1$. Ellipses

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D > 0$





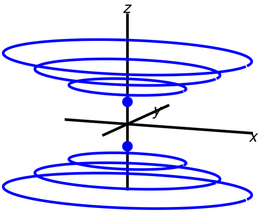
$$z = \quad 3$$

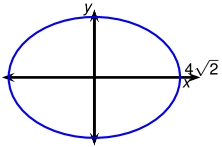
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 8 = (\quad 3)^2 - 1$$

$\Rightarrow (x, y, z) \in$ ellipse

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 1\right\}$
- When $z = const$: Two pts. for $z = \pm 1$. Ellipses

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D > 0$





$$z = \pm 3$$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 8 = (\pm 3)^2 - 1$$

$\Rightarrow (x, y, z) \in$ ellipse

- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 1 \right\}$
- When $z = const$: Two pts. for $z = \pm 1$. Ellipses for $|z| > 1$.

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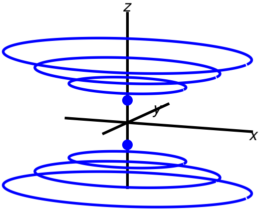

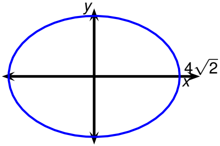

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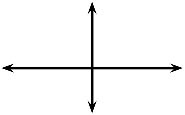

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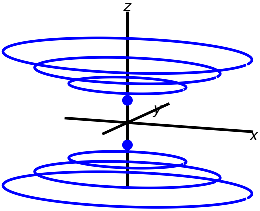

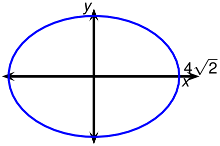

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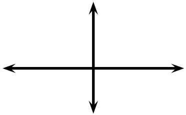

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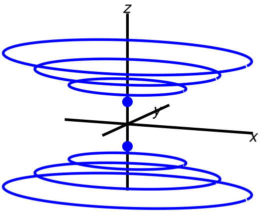

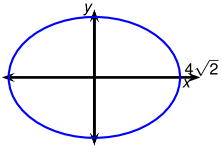

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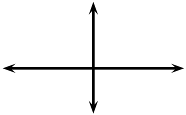

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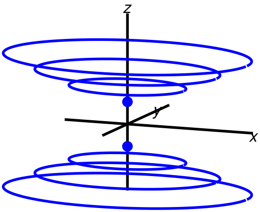

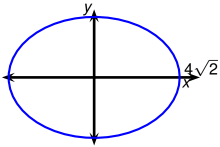

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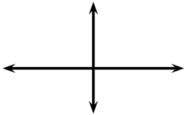

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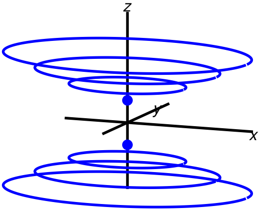

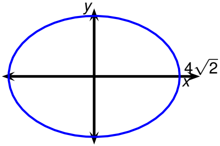

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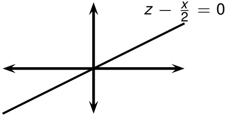

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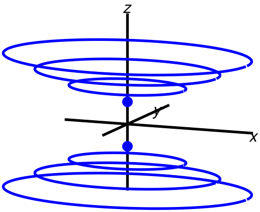

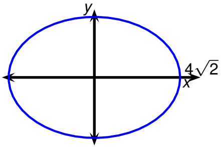

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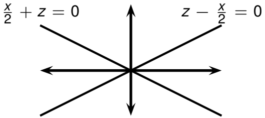

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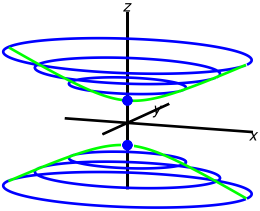

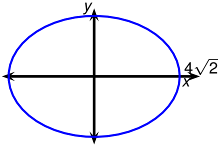

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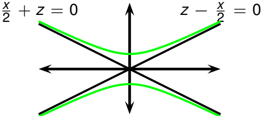

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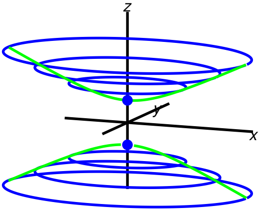

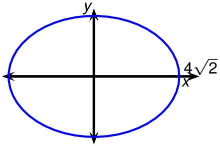

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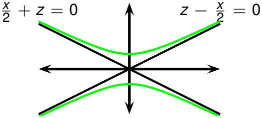

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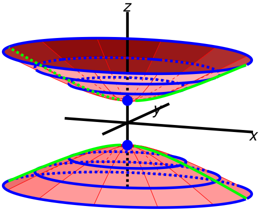

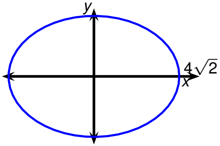

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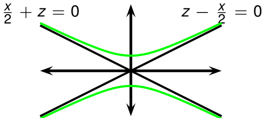

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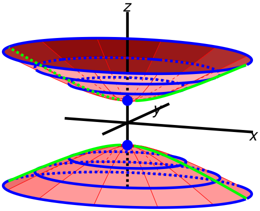

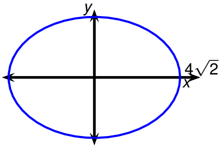

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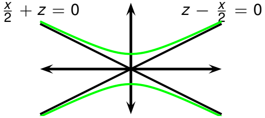

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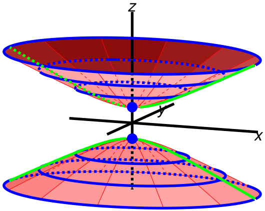

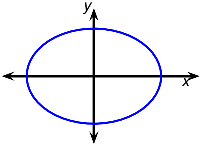

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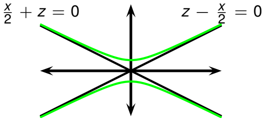

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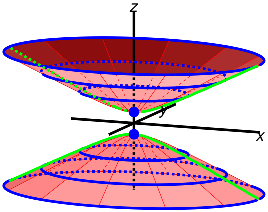

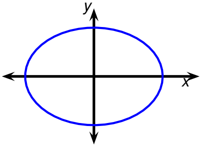

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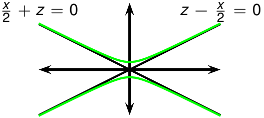

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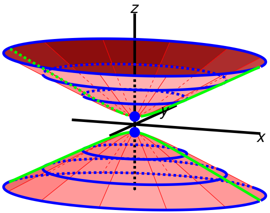
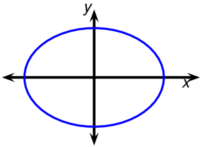




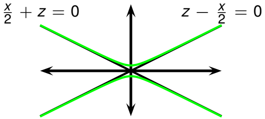
$$z = \pm 3$$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = (\pm 3)^2 - \frac{1}{8}$$

$\Rightarrow (x, y, z) \in$ ellipse

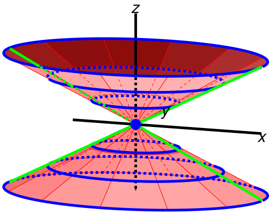
- Consider the surface $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z^2 - \frac{1}{8}\right\}$
- When $z = const$: Two pts. for $z = \pm 1$. Ellipses for $|z| > 1$. When

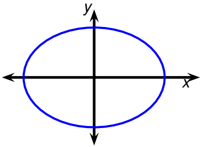
$$y = 0: \quad \begin{aligned} \left(\frac{x}{2}\right)^2 &= z^2 - \frac{1}{8} \\ z^2 - \left(\frac{x}{2}\right)^2 &= \frac{1}{8} \\ \left(z - \frac{x}{2}\right)\left(\frac{x}{2} + z\right) &= \frac{1}{8} \\ \left(z - \frac{x}{2}\right) &= \frac{\frac{1}{8}}{\left(\frac{x}{2} + z\right)} \end{aligned}$$



- $\Rightarrow$ ellipses: stacked along hyperbolas.
- Figure called: two-sheet hyperboloid.

$Ax^2 + By^2 + Cz^2 + D = 0$, $A > 0$, $B > 0$, $C < 0$, $D = 0$

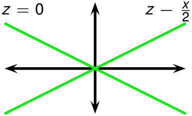




$$z = \pm 3$$
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = (\pm 3)^2 - 0$$
$\Rightarrow (x, y, z) \in$ ellipse

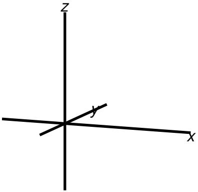
- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z^2 - 0\right\}$
- When $z = const$: Two pts. for $z = \pm 1$. Ellipses for $|z| > 1$. When

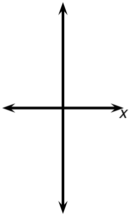
$$y = 0: \quad \begin{aligned} \left(\frac{x}{2}\right)^2 &= z^2 - 0 \\ z^2 - \left(\frac{x}{2}\right)^2 &= 0 \\ \left(z - \frac{x}{2}\right)\left(\frac{x}{2} + z\right) &= 0 \\ \left(z - \frac{x}{2}\right) &= \frac{0}{\left(\frac{x}{2} + z\right)} \end{aligned}$$



- $\Rightarrow$ ellipses: stacked along hyperbolas.
- Figure called: cone.

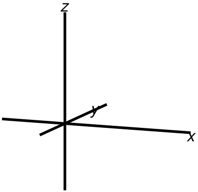
$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$



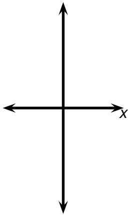


- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z \right\}$

$Ax^2 + By^2 + Iz = 0, A > 0, B > 0, I \neq 0$



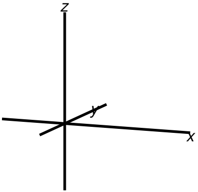
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z \right\}$
- The level curves $z = const$ are:

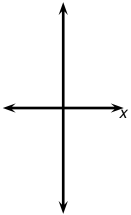


$z =$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 =$

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$



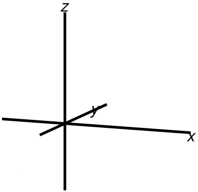


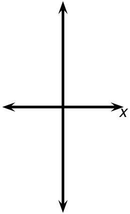
$z=0$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 0$$

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z\right\}$
- The level curves $z = const$ are:

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$



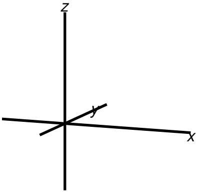


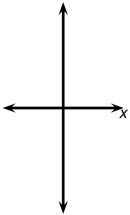
$z = 0$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 0$

- Consider the surface $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z\right\}$
- The level curves $z = const$ are:

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$





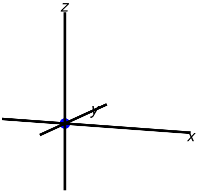
$z=0$

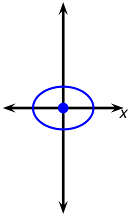
$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2=0$

$(x, y, z)=(0, 0, 0)$

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z\right\}$
- The level curves $z = const$ are: a point for $z = 0$;

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$





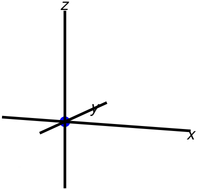
$z = 1$

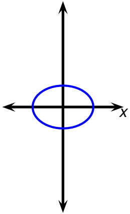
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 1$$

$\Rightarrow (x, y, z) \in$

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z\right\}$
- The level curves $z = const$ are: a point for $z = 0$;

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$





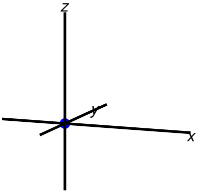
$z = 1$

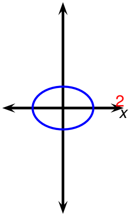
$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 1$

$\Rightarrow (x, y, z) \in$

- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z \right\}$
- The level curves $z = const$ are: a point for $z = 0$;

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$





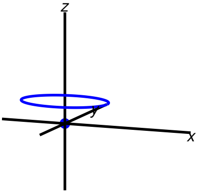
$z=1$

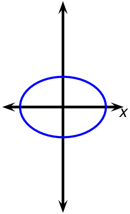
$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 1$

$\Rightarrow (x, y, z) \in$ ellipse

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z\right\}$
- The level curves $z = const$ are: a point for $z = 0$;

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$





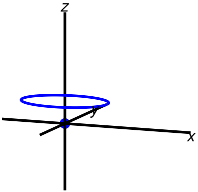
$$z = 2$$

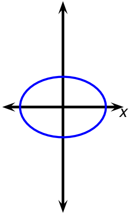
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 2$$

$$\Rightarrow (x, y, z) \in$$

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z\right\}$
- The level curves $z = const$ are: a point for $z = 0$;

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$





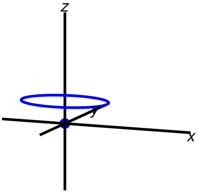
$z = 2$

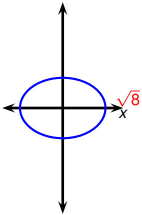
$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 2$

$\Rightarrow (x, y, z) \in$

- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z \right\}$
- The level curves $z = const$ are: a point for $z = 0$;

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$





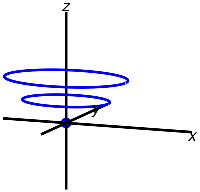
$z = 2$

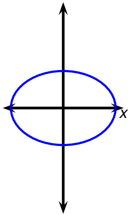
$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 2$

$\Rightarrow (x, y, z) \in$ ellipse

- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z \right\}$
- The level curves $z = const$ are: a point for $z = 0$;

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$





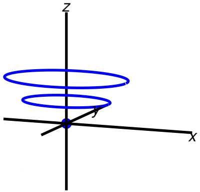
$z = 3$

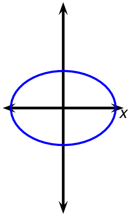
$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 3$

$\Rightarrow (x, y, z) \in$

- Consider the surface
  $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z\right\}$
- The level curves $z = const$ are: a point for $z = 0$;

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$





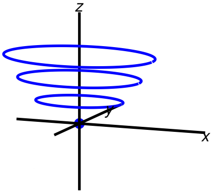
$z = 3$

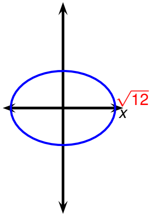
$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 3$

$\Rightarrow (x, y, z) \in$

- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z \right\}$
- The level curves $z = const$ are: a point for $z = 0$;

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$





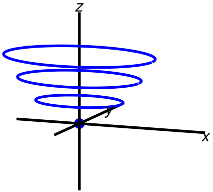
$z = 3$

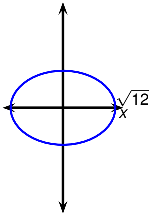
$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 3$

$\Rightarrow (x, y, z) \in$ ellipse

- Consider the surface $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z\right\}$
- The level curves $z = const$ are: a point for $z = 0$; ellipses for $z > 0$.

$Ax^2 + By^2 + Iz = 0, A > 0, B > 0, I \neq 0$



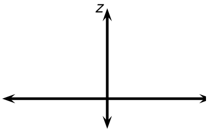


$$z = 3$$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 3$$

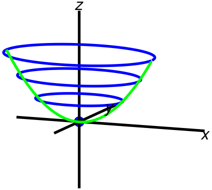
$$\Rightarrow (x, y, z) \in \text{ellipse}$$

- Consider the surface $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{4} + \frac{y^2}{2} = z \right\}$
- The level curves $z = const$ are: a point for $z = 0$; ellipses for $z > 0$. For $y = 0$: $\left(\frac{x}{2}\right)^2 = z$

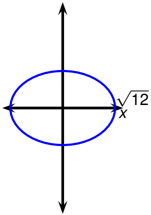


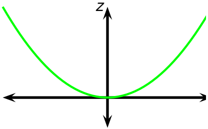
- $\Rightarrow$ ellipses: stacked along **?**

$Ax^2 + By^2 + Iz = 0,\ A > 0,\ B > 0,\ I \neq 0$



- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z \right\}$
- The level curves $z = const$ are: a point for $z = 0$; ellipses for $z > 0$. For $y = 0$: $\left(\frac{x}{2}\right)^2 = z$

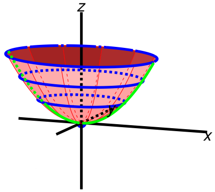


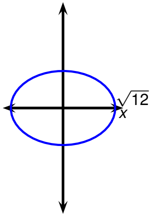


$$z = 3$$
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 3$$
$$\Rightarrow (x, y, z) \in \text{ellipse}$$

- $\Rightarrow$ ellipses: stacked along parabola.

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$



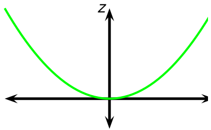


$$z = 3$$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 3$$

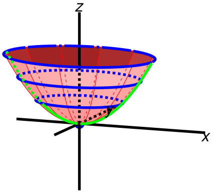
$$\Rightarrow (x, y, z) \in \text{ellipse}$$

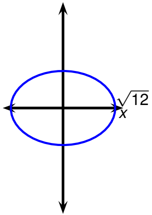
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z \right\}$
- The level curves $z = const$ are: a point for $z = 0$; ellipses for $z > 0$. For $y = 0$: $\left(\frac{x}{2}\right)^2 = z$



- $\Rightarrow$ ellipses: stacked along parabola.
- Surface name: paraboloid. If $A \neq B$: elliptic paraboloid.

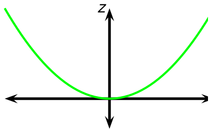
$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$





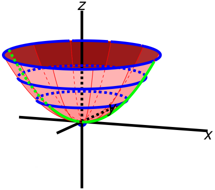
$$z = 3$$
$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{2}}\right)^2 = 3$$
$$\Rightarrow (x, y, z) \in \text{ellipse}$$

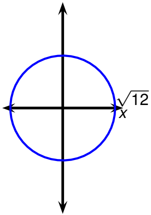
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{2} = z \right\}$
- The level curves $z = const$ are: a point for $z = 0$; ellipses for $z > 0$. For $y = 0$: $\left(\frac{x}{2}\right)^2 = z$



- $\Rightarrow$ ellipses: stacked along parabola.
- Surface name: paraboloid. If $A \neq B$: elliptic paraboloid.
- What happens if we decrease B?

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$



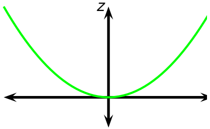


$$z = 3$$

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 = 3$$

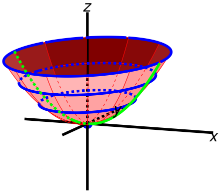
$$\Rightarrow (x, y, z) \in \text{ellipse}$$

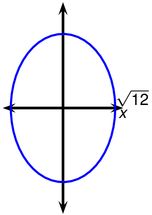
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{4} = z \right\}$
- The level curves $z = const$ are: a point for $z = 0$; ellipses for $z > 0$. For $y = 0$: $\left(\frac{x}{2}\right)^2 = z$



- $\Rightarrow$ ellipses: stacked along parabola.
- Surface name: paraboloid. If $A \neq B$: elliptic paraboloid.
- What happens if we decrease B?

$Ax^2 + By^2 + Iz = 0,\ A > 0,\ B > 0,\ I \neq 0$



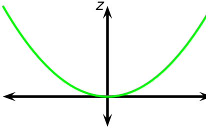


$z=3$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\sqrt{8}}\right)^2 = 3$

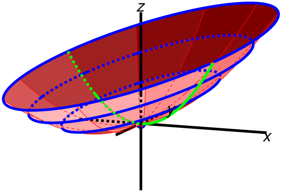
$\Rightarrow (x, y, z) \in$ ellipse

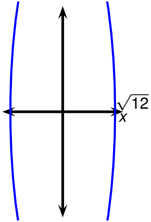
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{8} = z \right\}$
- The level curves $z = const$ are: a point for $z = 0$; ellipses for $z > 0$. For $y = 0$: $\left(\frac{x}{2}\right)^2 = z$



- $\Rightarrow$ ellipses: stacked along parabola.
- Surface name: paraboloid. If $A \neq B$: elliptic paraboloid.
- What happens if we decrease B?

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$



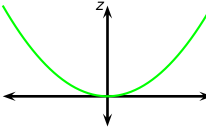


$z=3$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{8}\right)^2 = 3$

$\Rightarrow (x, y, z) \in$ ellipse

- Consider the surface $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{64} = z\right\}$
- The level curves $z = const$ are: a point for $z = 0$; ellipses for $z > 0$. For $y = 0$: $\left(\frac{x}{2}\right)^2 = z$



- $\Rightarrow$ ellipses: stacked along parabola.
- Surface name: paraboloid. If $A \neq B$: elliptic paraboloid.
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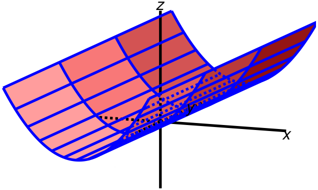
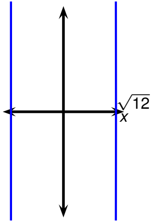
$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B > 0$, $I \neq 0$



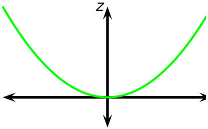


$z=3$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{\text{"}\infty\text{"}}\right)^2 = 3$

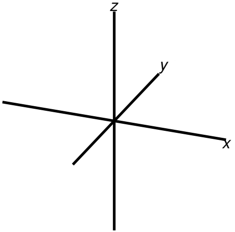
$\Rightarrow (x, y, z) \in \text{ellipse}$

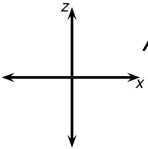
- Consider the surface $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{4} + \frac{y^2}{\text{"}\infty\text{"}} = z \right\}$
- The level curves $z = const$ are: a point for $z = 0$; ellipses for $z > 0$. For $y = 0$: $\left(\frac{x}{2}\right)^2 = z$



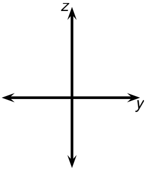
- $\Rightarrow$ ellipses: stacked along parabola.
- Surface name: paraboloid. If $A \neq B$: elliptic paraboloid.
- What happens if we decrease B?

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$

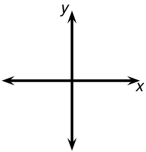




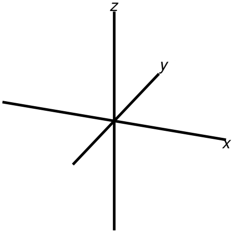
$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.



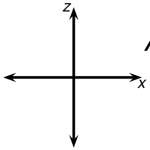
Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.

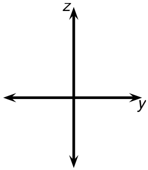


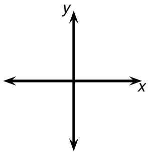
$Ax^2 + By^2 + Iz = 0,\ A > 0,\ B < 0,\ C = 0,\ I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.





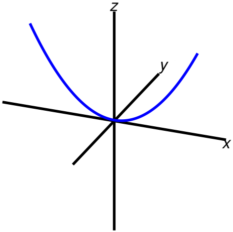


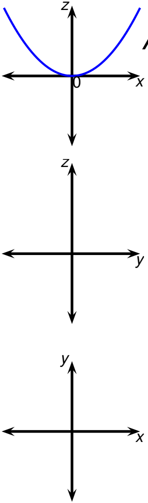
$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = 0$

$\frac{x^2}{3} - 0 = z$ | parab.

$Ax^2 + By^2 + Iz = 0, A > 0, B < 0, C = 0, I \neq 0$





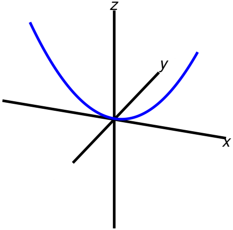
$A = \frac{1}{3}, |B| = \frac{1}{4}$, $I = -1$.

Set: $y=0$

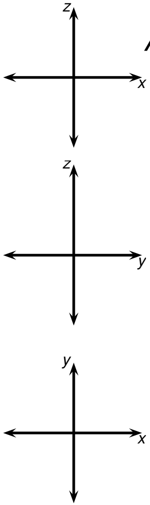
$\frac{x^2}{3} - 0 = z$ | parab.

Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.

$Ax^2 + By^2 + Iz = 0, A > 0, B < 0, C = 0, I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.

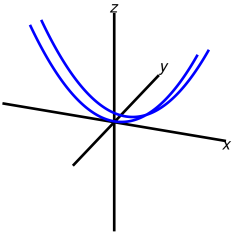


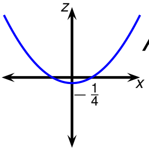
$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y =$ 1

$\frac{x^2}{3} - \frac{(\ \ 1)^2}{4} = z$ | parab.

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



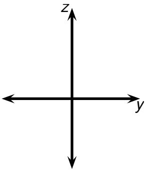


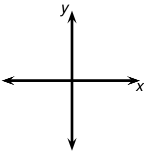
$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y =\quad 1$

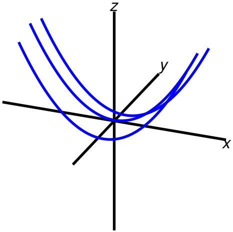
$\frac{x^2}{3} - \frac{(\quad 1)^2}{4} = z$ | parab.

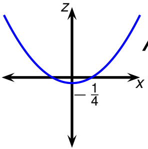
Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.





$Ax^2 + By^2 + Iz = 0, A > 0, B < 0, C = 0, I \neq 0$



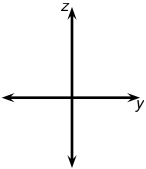


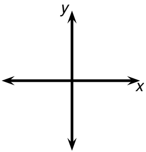
$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 1$

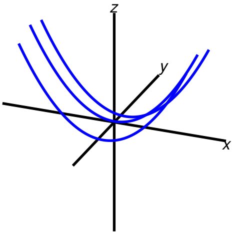
$\frac{x^2}{3} - \frac{(\pm 1)^2}{4} = z$ | parab.

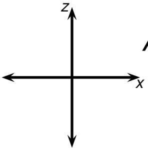
Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.





$Ax^2 + By^2 + Iz = 0, A > 0, B < 0, C = 0, I \neq 0$

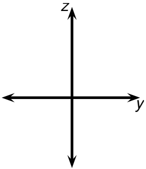




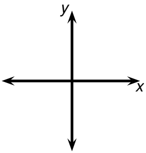
$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \quad 2$

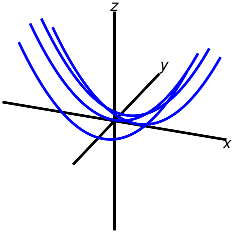
$\frac{x^2}{3} - \frac{(\ \ 2)^2}{4} \quad = z \qquad \Big|$ parab.

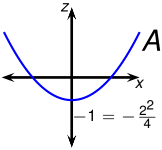


Surface: $\mathcal{C} = \left\{ (x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z \right\}$.



$Ax^2 + By^2 + Iz = 0,\ A > 0,\ B < 0,\ C = 0,\ I \neq 0$



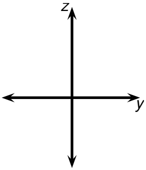


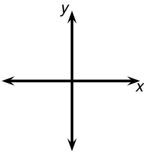
$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \quad 2$

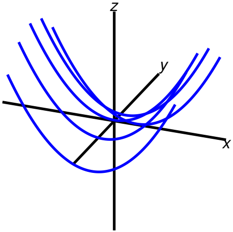
$\frac{x^2}{3} - \frac{(\quad 2)^2}{4} = z \qquad \Big|$ parab.

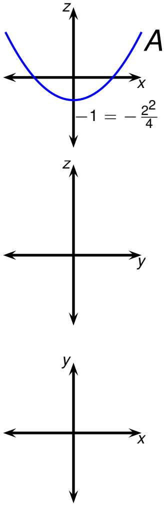
Surface: $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{3} - \frac{y^2}{4} = z \right\}$.





$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$





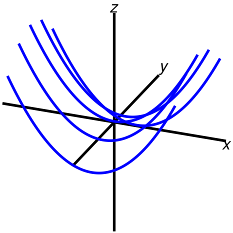
$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 2$

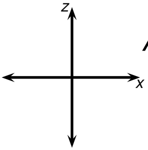
$\frac{x^2}{3} - \frac{(\pm 2)^2}{4} = z$ | parab.

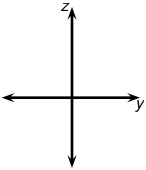
Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.

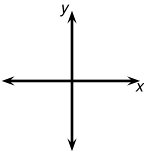
$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.





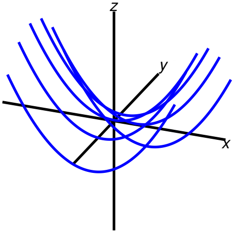


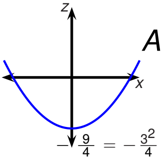
$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y =$ 3

$\frac{x^2}{3} - \frac{(\ 3)^2}{4} = z$ | parab.

$Ax^2 + By^2 + Iz = 0,\ A > 0,\ B < 0,\ C = 0,\ I \neq 0$

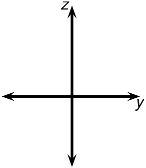




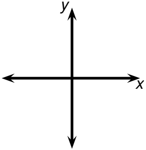
$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y =$ 3

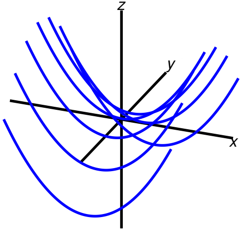
$\frac{x^2}{3} - \frac{(\ 3)^2}{4} = z$ | parab.

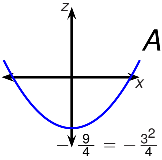


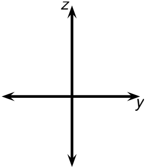
Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.

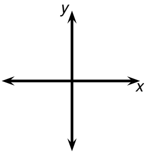


$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$









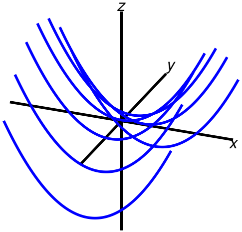
$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



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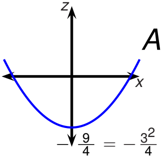
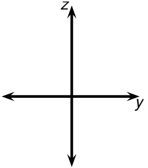
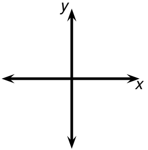






$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

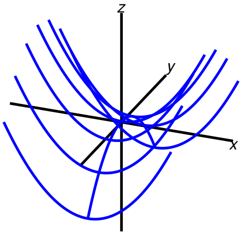
Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

Set: $x = 0$

$0 - \frac{y^2}{4} = z$ | parab.

$Ax^2 + By^2 + Iz = 0,\ A > 0,\ B < 0,\ C = 0,\ I \neq 0$



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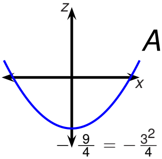

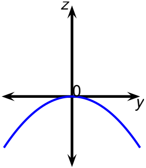

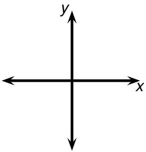

$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

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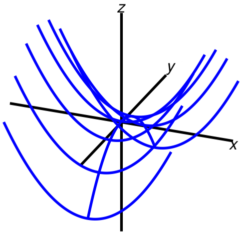

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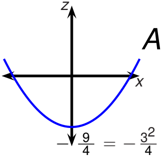
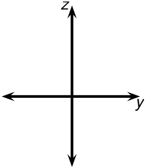
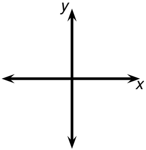






$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

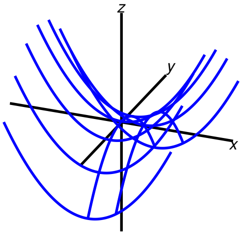
Set: $y = \pm 3$

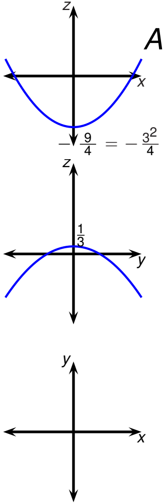
$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

Set: $x = 1$

$\frac{(1)^2}{3} - \frac{y^2}{4} = z$ | parab.

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$





$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

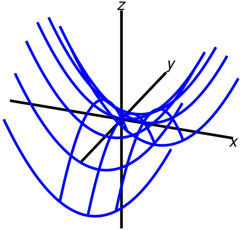
$$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z \qquad \Big| \text{ parab.}$$

Set: $x = \quad 1$

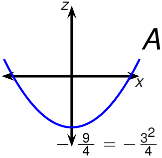
$$\frac{(\quad 1)^2}{3} - \frac{y^2}{4} = z \qquad \Big| \text{ parab.}$$

Surface: $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



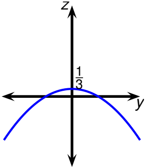
Surface: $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.



$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

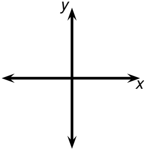
Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

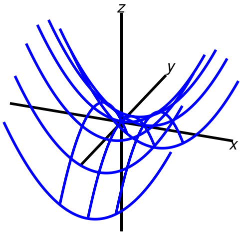


Set: $x = \pm 1$

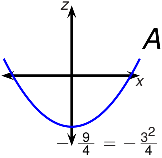
$\frac{(\pm 1)^2}{3} - \frac{y^2}{4} = z$ | parab.

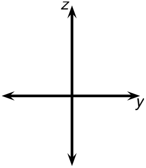


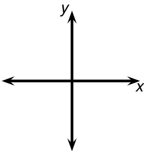
$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.







$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

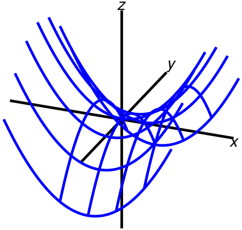
Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

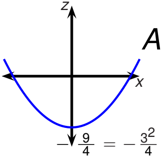
Set: $x = 2$

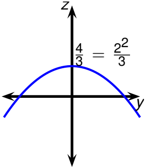
$\frac{(2)^2}{3} - \frac{y^2}{4} = z$ | parab.

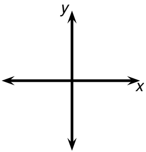
$Ax^2 + By^2 + Iz = 0,\ A > 0,\ B < 0,\ C = 0,\ I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.







$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

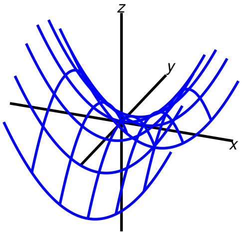
Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

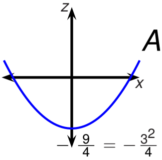
Set: $x = 2$

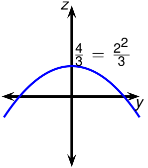
$\frac{(2)^2}{3} - \frac{y^2}{4} = z$ | parab.

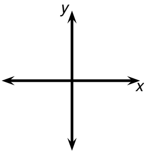
$Ax^2 + By^2 + Iz = 0,\ A > 0,\ B < 0,\ C = 0,\ I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.







$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

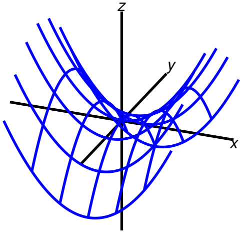
Set: $y = \pm 3$

$$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z \qquad \Big| \text{ parab.}$$

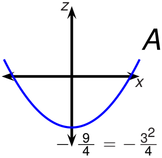
Set: $x = \pm 2$

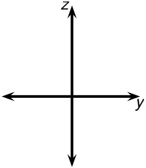
$$\frac{(\pm 2)^2}{3} - \frac{y^2}{4} = z \qquad \Big| \text{ parab.}$$

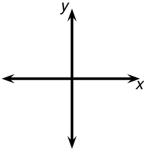
$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.







$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

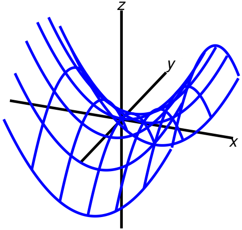
Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

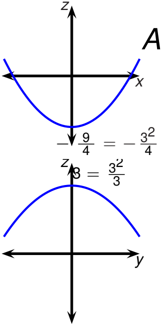
Set: $x = \ \ 3$

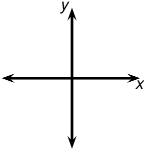
$\frac{(\ \ 3)^2}{3} - \frac{y^2}{4} = z$ | parab.

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.





$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

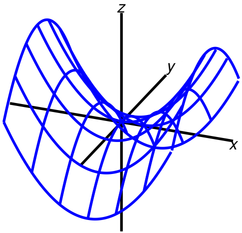
Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

Set: $x = \quad 3$

$\frac{(\quad 3)^2}{3} - \frac{y^2}{4} = z$ | parab.

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



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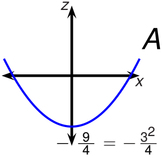
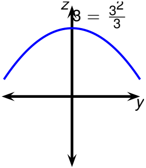
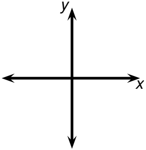






$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

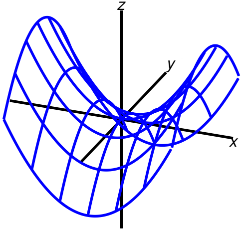
$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

Set: $x = \pm 3$

$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z$ | parab.

Set: $z = 2$

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



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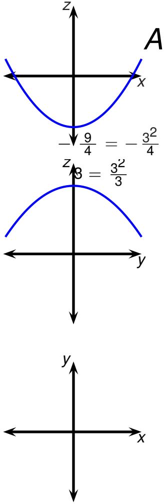


$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

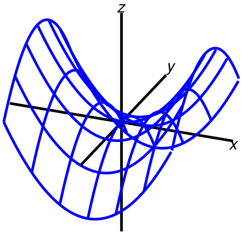
Set: $x = \pm 3$

$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z$ | parab.

Set: $z = 2$

$\frac{x^2}{3} - \frac{y^2}{4} = 2$

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



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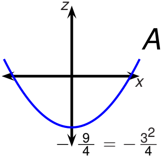
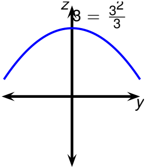
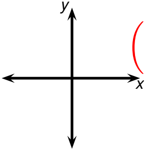






$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

Set: $x = \pm 3$

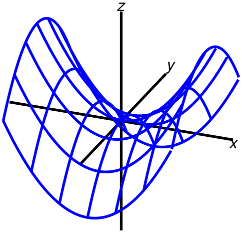
$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z$ | parab.

Set: $z = 2$

$\frac{x^2}{3} - \frac{y^2}{4} = 2$

$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right) = 2$

$Ax^2 + By^2 + Iz = 0,\ A > 0,\ B < 0,\ C = 0,\ I \neq 0$



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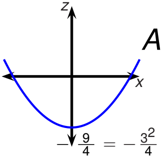
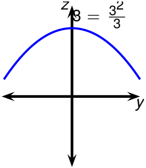
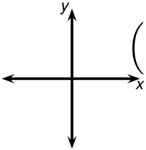






$A = \frac{1}{3}, |B| = \frac{1}{4}, I = -1.$

Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

Set: $x = \pm 3$

$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z$ | parab.

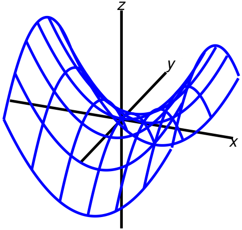
Set: $z = 2$

$$\frac{x^2}{3} - \frac{y^2}{4} = 2$$

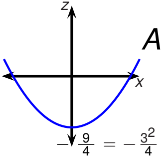
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right) = 2$$

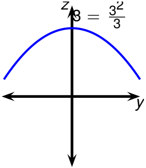
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right) = \frac{2}{\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right)}$$

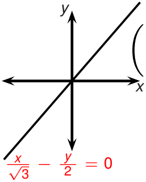
$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.







$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

$$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z \quad \Big| \text{ parab.}$$

Set: $x = \pm 3$

$$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z \quad \Big| \text{ parab.}$$

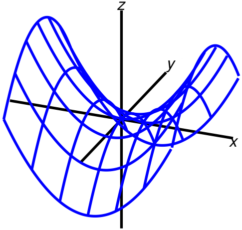
Set: $z = 2$

$$\frac{x^2}{3} - \frac{y^2}{4} = 2$$

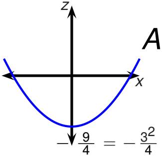
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right) = 2$$

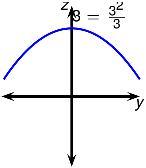
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right) = \frac{2}{\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right)}$$

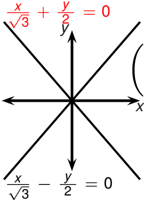
$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.







$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

$$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z \qquad \Big| \text{ parab.}$$

Set: $x = \pm 3$

$$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z \qquad \Big| \text{ parab.}$$

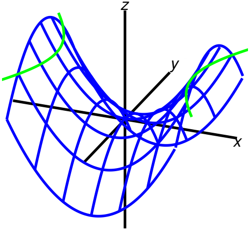
Set: $z = 2$

$$\frac{x^2}{3} - \frac{y^2}{4} = 2$$

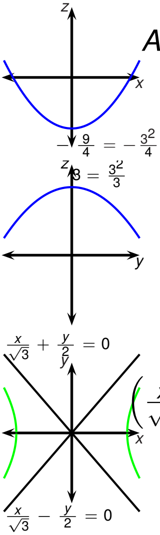
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right) = 2$$

$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right) = \frac{2}{\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right)}$$

$Ax^2 + By^2 + Iz = 0,\ A > 0,\ B < 0,\ C = 0,\ I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.



$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

$$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$$ | parab.

Set: $x = \pm 3$

$$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z$$ | parab.

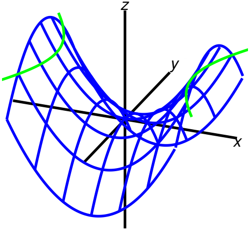
Set: $z = 2$

$$\frac{x^2}{3} - \frac{y^2}{4} = 2$$

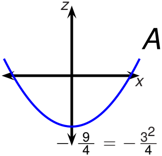
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right) = 2$$

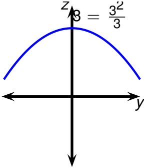
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right) = \frac{2}{\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right)}$$ | hyperb.

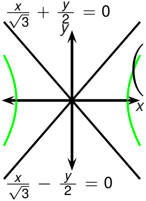
$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.







$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

$$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z \qquad \text{parab.}$$

Set: $x = \pm 3$

$$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z \qquad \text{parab.}$$

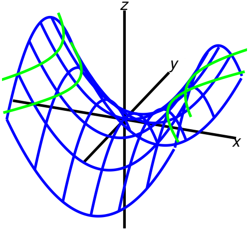
Set: $z = 2$

$$\frac{x^2}{3} - \frac{y^2}{4} = 2$$

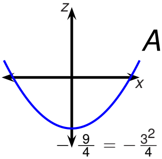
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right) = 2$$

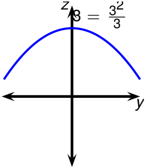
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right) = \frac{2}{\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right)} \qquad \text{hyperb.}$$

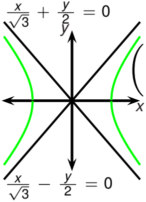
$Ax^2 + By^2 + Iz = 0,\ A > 0,\ B < 0,\ C = 0,\ I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.







$A = \frac{1}{3}, |B| = \frac{1}{4} \quad , I = -1.$

Set: $y = \pm 3$

$$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z \qquad \Big| \text{ parab.}$$

Set: $x = \pm 3$

$$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z \qquad \Big| \text{ parab.}$$

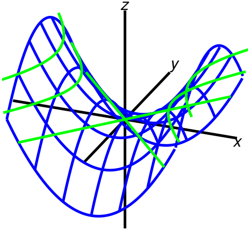
Set: $z = 1$

$$\frac{x^2}{3} - \frac{y^2}{4} = 1$$

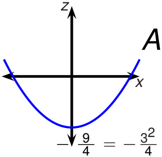
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right) = 1$$

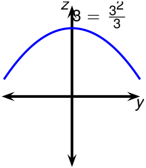
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right) = \frac{1}{\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right)} \Big| \text{hyperb.}$$

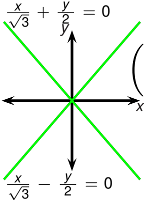
$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.







$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

$$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z \qquad \Big| \text{ parab.}$$

Set: $x = \pm 3$

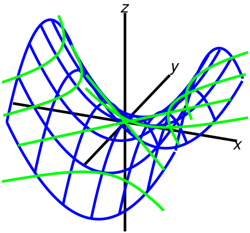
$$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z \qquad \Big| \text{ parab.}$$

Set: $z = 0$

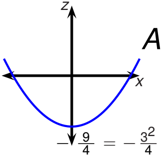
$$\frac{x^2}{3} - \frac{y^2}{4} = 0$$

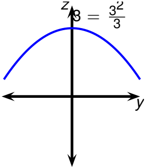
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right) = 0 \qquad \Big| \text{ two lines}$$

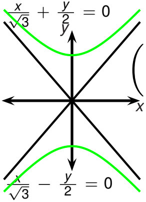
$Ax^2 + By^2 + Iz = 0,\ A > 0,\ B < 0,\ C = 0,\ I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.







$A = \frac{1}{3},\ |B| = \frac{1}{4}\qquad, I = -1.$

Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z \quad \Big|$ parab.

Set: $x = \pm 3$

$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z \quad \Big|$ parab.

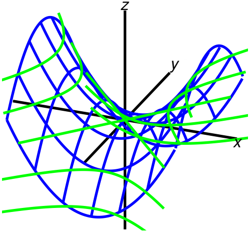
Set: $z = -1$

$\frac{x^2}{3} - \frac{y^2}{4} = -1$

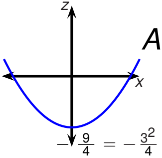
$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right) = -1$

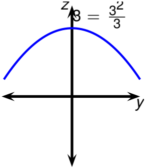
$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right) = \frac{-1}{\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right)}\Bigg|$

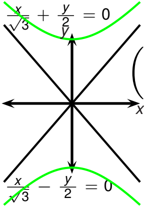
$Ax^2 + By^2 + Iz = 0,\ A > 0,\ B < 0,\ C = 0,\ I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.







$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

Set: $x = \pm 3$

$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z$ | parab.

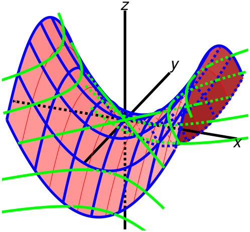
Set: $z = -2$

$$\frac{x^2}{3} - \frac{y^2}{4} = -2$$

$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right) = -2$$

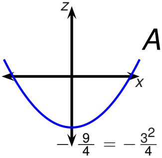
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right) = \frac{-2}{\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right)}$$

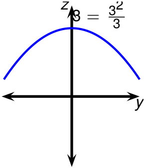
$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$

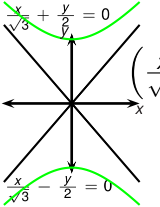


Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.

- Name: hyperbolic paraboloid.







$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

$$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z \qquad \Big| \text{ parab.}$$

Set: $x = \pm 3$

$$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z \qquad \Big| \text{ parab.}$$

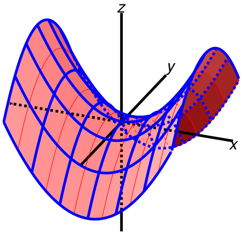
Set: $z = -2$

$$\frac{x^2}{3} - \frac{y^2}{4} = -2$$

$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right) = -2$$

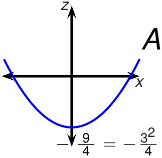
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right) = \frac{-2}{\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right)} \Big|$$

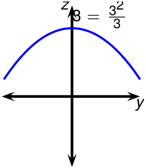
$Ax^2 + By^2 + Iz = 0,\ A > 0,\ B < 0,\ C = 0,\ I \neq 0$

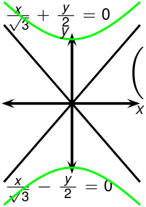


Surface: $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.

- Name: hyperbolic paraboloid.







$A = \frac{1}{3},\ |B| = \frac{1}{4},\ I = -1.$

Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

Set: $x = \pm 3$

$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z$ | parab.

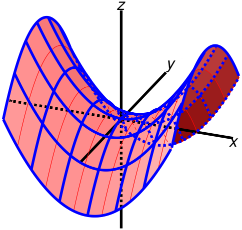
Set: $z = -2$

$$\frac{x^2}{3} - \frac{y^2}{4} = -2$$

$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right) = -2$$

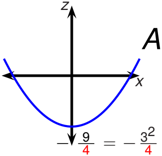
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right) = \frac{-2}{\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right)}$$

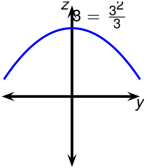
$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$

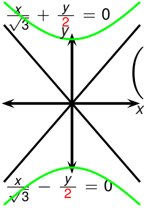


Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.

- Name: hyperbolic paraboloid.
- What happens if $|B|$ decreases?







$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z$ | parab.

Set: $x = \pm 3$

$\frac{(\pm 3)^2}{3} - \frac{y^2}{4} = z$ | parab.

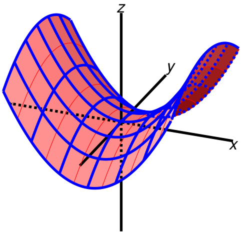
Set: $z = -2$

$$\frac{x^2}{3} - \frac{y^2}{4} = -2$$

$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right) = -2$$

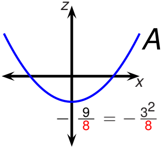
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{2}\right) = \frac{-2}{\left(\frac{x}{\sqrt{3}} + \frac{y}{2}\right)}$$

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$



Surface: $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{3} - \frac{y^2}{8} = z\right\}$.

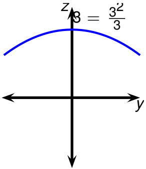
- Name: hyperbolic paraboloid.
- What happens if $|B|$ decreases?



$A = \frac{1}{3}$, $|B| = \frac{1}{8}$, $I = -1$.

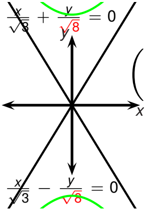
Set: $y = \pm 3$

$$\frac{x^2}{3} - \frac{(\pm 3)^2}{8} = z \qquad \Big| \text{ parab.}$$



Set: $x = \pm 3$

$$\frac{(\pm 3)^2}{3} - \frac{y^2}{8} = z \qquad \Big| \text{ parab.}$$



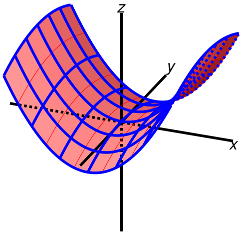
Set: $z = -2$

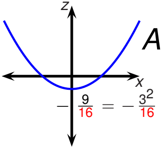
$$\frac{x^2}{3} - \frac{y^2}{8} = -2$$

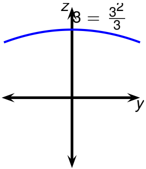
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{\sqrt{8}}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{\sqrt{8}}\right) = -2$$

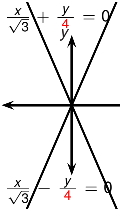
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{\sqrt{8}}\right) = \frac{-2}{\left(\frac{x}{\sqrt{3}} + \frac{y}{\sqrt{8}}\right)} \Bigg|$$

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$









$A = \frac{1}{3}$, $|B| = \frac{1}{16}$, $I = -1$.

Set: $y = \pm 3$

$$\frac{x^2}{3} - \frac{(\pm 3)^2}{16} = z \quad \Big| \text{ parab.}$$

Set: $x = \pm 3$

$$\frac{(\pm 3)^2}{3} - \frac{y^2}{16} = z \quad \Big| \text{ parab.}$$

Set: $z = -2$

$$\frac{x^2}{3} - \frac{y^2}{16} = -2$$

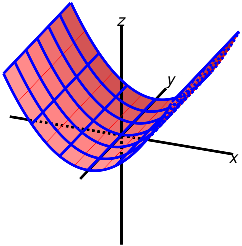
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{4}\right)\left(\frac{x}{\sqrt{3}} + \frac{y}{4}\right) = -2$$

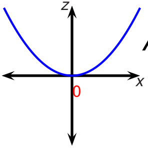
$$\left(\frac{x}{\sqrt{3}} - \frac{y}{4}\right) = \frac{-2}{\left(\frac{x}{\sqrt{3}} + \frac{y}{4}\right)} \Big|$$

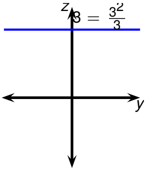
Surface: $\mathcal{C} = \left\{(x, y, z) \middle| \frac{x^2}{3} - \frac{y^2}{16} = z\right\}$.

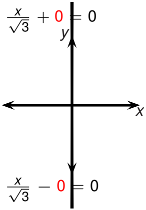
- Name: hyperbolic paraboloid.
- What happens if $|B|$ decreases?

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$









$A = \frac{1}{3}$, $|B| = \frac{1}{\infty} = 0$, $I = -1$.

Set: $y = \pm 3$

$\frac{x^2}{3} - \quad 0 = z$ | parab.

Set: $x = \pm 3$

$\frac{(\pm 3)^2}{3} - 0 = z$ | parab.

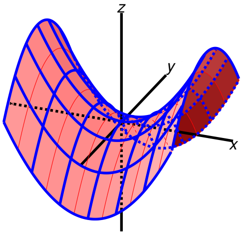
Set: $z = -2$

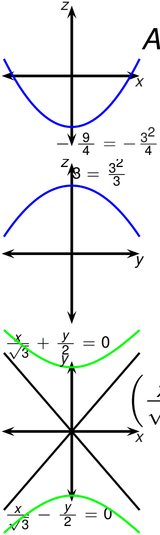
$\frac{x^2}{3} - 0 = -2$

Surface: $\mathcal{C} = \left\{ (x, y, z) \middle| \frac{x^2}{3} - 0 = z \right\}$.

- Name: hyperbolic paraboloid.
- What happens if $|B|$ decreases?

$Ax^2 + By^2 + Iz = 0$, $A > 0$, $B < 0$, $C = 0$, $I \neq 0$





$A = \frac{1}{3}$, $|B| = \frac{1}{4}$, $I = -1$.

Set: $y = \pm 3$

$$\frac{x^2}{3} - \frac{(\pm 3)^2}{4} = z \quad \Big| \text{ parab.}$$

Set: $x = \pm 3$

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Surface: $\mathcal{C} = \left\{(x, y, z) | \frac{x^2}{3} - \frac{y^2}{4} = z\right\}$.

- Name: hyperbolic paraboloid.
- What happens if $|B|$ decreases?

Summary: surfaces of form $Ax^2 + By^2 + Cz^2 + D = 0$

A	B	C	D	$x = x_0$	$y = y_0$	$z = z_0$	Example	Name
> 0	> 0	> 0	> 0	empty	empty	empty	$x^2 + 2y^2 + 3z^2 + 4 = 0$	empty
> 0	> 0	> 0	$= 0$					
> 0	> 0	> 0	< 0	ellipse	ellipse	ellipse	$x^2 + 2y^2 + 3z^2 - 4 = 0$	Ellipsoid
> 0	> 0	$= 0$	> 0					
> 0	> 0	$= 0$	$= 0$					
> 0	> 0	$= 0$	< 0					
> 0	> 0	< 0	> 0					
> 0	> 0	< 0	$= 0$					
> 0	> 0	< 0	< 0					
> 0	$= 0$	$= 0$	> 0					
> 0	$= 0$	$= 0$	$= 0$					
> 0	$= 0$	$= 0$	< 0					

Fill in the rest of the table.

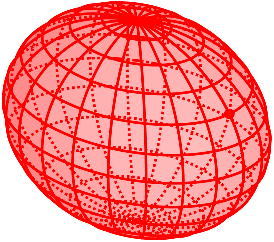
Quadratics $Ax^2 + By^2 + Iz = 0$ (no central symmetry)

$$Ax^2 + By^2 + Iz = 0$$

A	B	$x = x_0$	$y = y_0$	$z = z_0$	Example	Name
> 0	> 0	parabola	parabola	ellipse, point, or empty	$x^2 + 2y^2 + 3z = 0$	Elliptic paraboloid
> 0	$= 0$					
> 0	< 0					

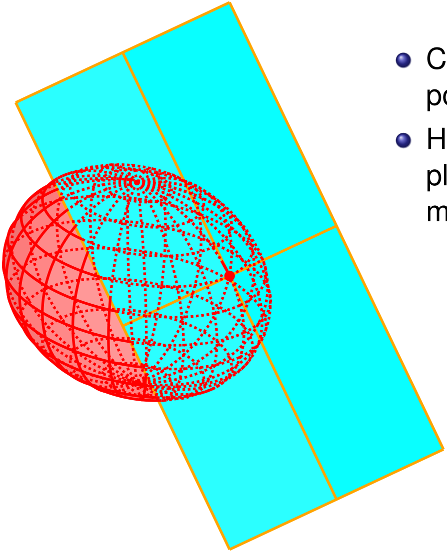
Fill in the rest of the table.

Tangent Plane



- Consider a surface S in space and a point P on the surface.

Tangent Plane



- Consider a surface S in space and a point P on the surface.
- How should we define the notion of "a plane tangent to S at P" so that it matches our geometric intuition?

Tangent Plane

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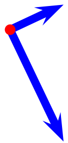

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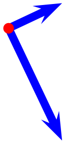
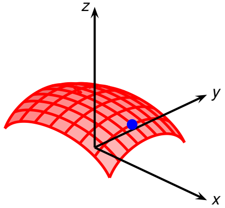
Tangent Plane



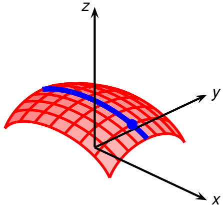
- Consider a surface S in space and a point P on the surface.
- How should we define the notion of “a plane tangent to S at P” so that it matches our geometric intuition?
- Intuitively, it should include all tangents at P to curves passing through P and contained in the surface.
- Therefore it should be the plane
  - passing through P;
  - parallel to the directions of all tangent vectors of curves passing through P and contained in the surface.

Tangent Plane to a Graph Surface



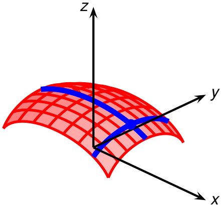
- Graph surface $z = f(x, y)$, point $P(x_0, y_0, z_0)$ on the surface.

Tangent Plane to a Graph Surface



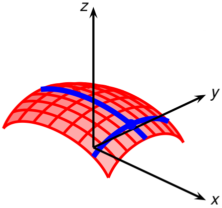
- Graph surface $z = f(x, y)$, point $P(x_0, y_0, z_0)$ on the surface.
- Call $\mathbf{p}(x)$ the curve given by $f(x, y)$ by keeping $y = y_0$ constant;

Tangent Plane to a Graph Surface



- Graph surface $z = f(x, y)$, point $P(x_0, y_0, z_0)$ on the surface.
- Call $\mathbf{p}(x)$ the curve given by $f(x, y)$ by keeping $y = y_0$ constant; call $\mathbf{q}(y)$ the curve given by $f(x, y)$ by keeping $x = x_0$ constant.

Tangent Plane to a Graph Surface



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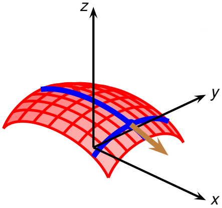
- Call $\mathbf{p}(x)$ the curve given by $f(x, y)$ by keeping $y = y_0$ constant; call $\mathbf{q}(y)$ the curve given by $f(x, y)$ by keeping $x = x_0$ constant.
- $$\begin{array}{rcl} \mathbf{p}(x) & = & (x, y_0, f(x, y_0)) \\ \mathbf{q}(y) & = & (x_0, y, f(x_0, y)) \end{array}$$.

Tangent Plane to a Graph Surface



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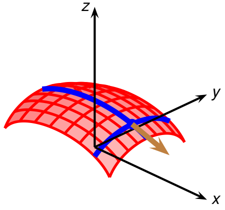
- $$\begin{aligned} \mathbf{p}(x) &= (x, y_0, f(x, y_0)) & \mathbf{p}'(x_0) &= (1, 0, f_x(x_0, y_0)) \\ \mathbf{q}(y) &= (x_0, y, f(x_0, y)) & & . \end{aligned}$$

Tangent Plane to a Graph Surface



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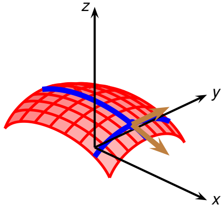
Tangent Plane to a Graph Surface



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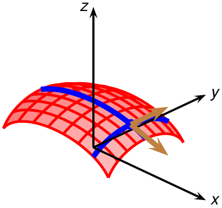
Tangent Plane to a Graph Surface



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- Normal to tangent plane at P: $\mathbf{n} = \mathbf{p}'(x_0) \times \mathbf{q}'(y_0) =$

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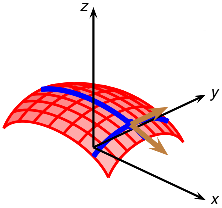
Tangent Plane to a Graph Surface



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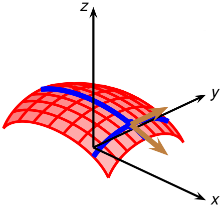
- Normal to tangent plane at P: $\mathbf{n} = \mathbf{p}'(x_0) \times \mathbf{q}'(y_0) = (1, 0, f_x(x_0, y_0)) \times (0, 1, f_y(x_0, y_0))$.

Tangent Plane to a Graph Surface



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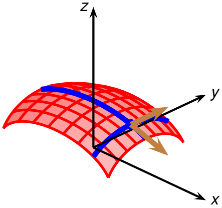
Tangent Plane to a Graph Surface



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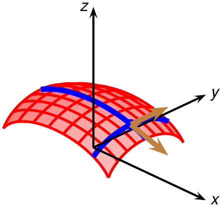
Tangent Plane to a Graph Surface



- Graph surface $z = f(x, y)$, point $P(x_0, y_0, z_0)$ on the surface.
- Call $\mathbf{p}(x)$ the curve given by $f(x, y)$ by keeping $y = y_0$ constant; call $\mathbf{q}(y)$ the curve given by $f(x, y)$ by keeping $x = x_0$ constant.
- $$\begin{array}{rclcrcl} \mathbf{p}(x) & = & (x, y_0, f(x, y_0)) & & \mathbf{p}'(x_0) & = & (1, 0, f_x(x_0, y_0)) \\ \mathbf{q}(y) & = & (x_0, y, f(x_0, y)) & & \mathbf{q}'(y_0) & = & (0, 1, f_y(x_0, y_0)) \ . \end{array}$$
- Normal to tangent plane at P: $\mathbf{n} = \mathbf{p}'(x_0) \times \mathbf{q}'(y_0) =$ $(1, 0, f_x(x_0, y_0)) \times (0, 1, f_y(x_0, y_0)) = (-f_x(x_0, y_0), -f_y(x_0, y_0), 1)$.

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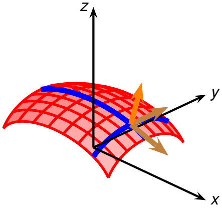

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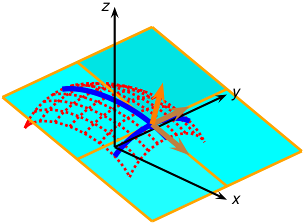

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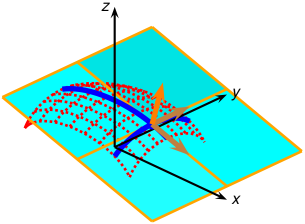

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$-f_x(x_0, y_0)(x - x_0) - f_y(x_0, y_0)(y - y_0) + (z - f(x_0, y_0)) = 0$

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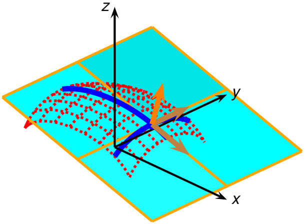

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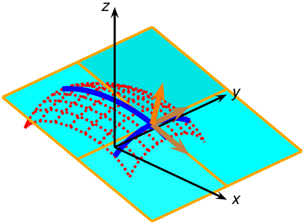

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  $$\boxed{z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)} \; .$$

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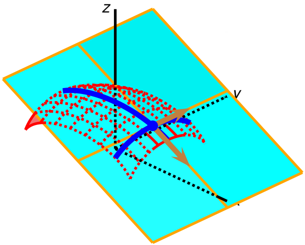

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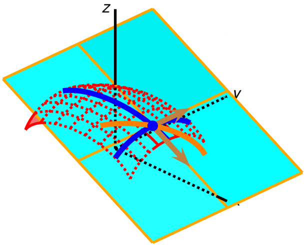

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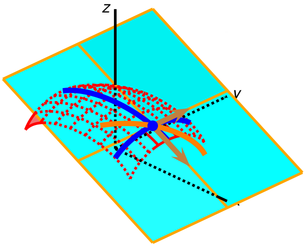

- $\mathbf{q}(t) = (x(t), y(t), z(t))$ - smooth curve in $S = \{(x, y, f(x, y))\}$,
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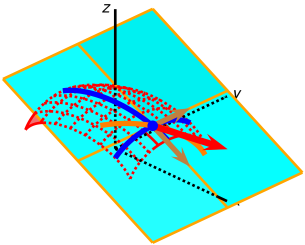
Corollary (Justification of tangent plane definition)

The tangent vector to any curve passing through (x_0, y_0, z_0) is a linear combination of the vectors $\left(1, 0, \frac{\partial f}{\partial x}\right)$ and $\left(0, 1, \frac{\partial f}{\partial y}\right)$.
In particular the tangent space at (x_0, y_0, z_0) is a plane.



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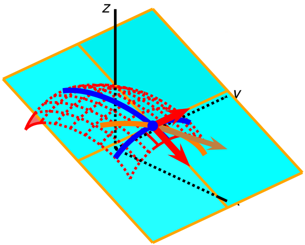

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  - Near $P(0, 0, 1)$, the surface is the graph surface of $z = \sqrt{1 - x^2 - y^2}$.
  - Near $P(1, 0, 0)$, the surface is not a graph surface w.r.t. to z.

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- If the level surface is a graph surface, we say that the equation $F(x, y, z) = k$ implicitly defines $z = f(x, y)$ satisfying the condition $f(x_0, y_0) = z_0$.