Calculus I
Lecture 12
More on Derivative Formulas

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https://github.com/tmilev/freecalc

2020

Outline

1. Understanding computations with derivatives

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Rules of differentiation.

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We derived the first set of rules by directly computing limits. The second set of rules can be derived from the first set algebraically.

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Let c be a constant. Derive the constant multiple rule

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Let n be a positive integer. Derive the negative integer power rule

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x^n (x^{-n})' & = & -\frac{n}{x} & \quad \Big| \ \frac{1}{x^n} \\
(x^{-n})' & = & -\frac{n}{x^{n+1}} &
\end{array}$$

Example

For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$

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$$\left(x^{\frac{1}{q}}\right)^q = x$$

Example

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$$
\begin{aligned}
\left(x^{\frac{1}{q}}\right)^q &= x & \Big| \ \frac{\mathrm{d}}{\mathrm{d}x} \\
\left(\left(x^{\frac{1}{q}}\right)^q\right)' &= (x)'
\end{aligned}
$$

Example

For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$ using the rule $(x)' = 1$,

$$\begin{array}{rcl} \left(x^{\frac{1}{q}}\right)^q &=& x \\ \left(\left(x^{\frac{1}{q}}\right)^q\right)' &=& 1 \end{array} \quad \Bigg| \ \frac{\mathrm{d}}{\mathrm{d}x}$$

Example

For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$ using the rule $(x)' = 1$,

$$
\begin{array}{rcl|l}
\left(x^{\frac{1}{q}}\right)^q & = & x & \frac{\mathrm{d}}{\mathrm{d}x} \\
\left(\left(x^{\frac{1}{q}}\right)^q\right)' & = & 1 & \text{Set } u = x^{\frac{1}{q}} \\
(u^q)' & = & 1 &
\end{array}
$$

Example

For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$ using the rule $(x)' = 1$, the chain rule

$$
\begin{array}{rcl}
\left(x^{\frac{1}{q}}\right)^q &=& x \\
\left(\left(x^{\frac{1}{q}}\right)^q\right)' &=& 1 \\
\left(u^q\right)' &=& 1 \\
\frac{\mathrm{d}}{\mathrm{d}u}\left(u^q\right)u' &=& 1
\end{array}
\qquad
\left|
\begin{array}{l}
\frac{\mathrm{d}}{\mathrm{d}x} \\
\text{Set } u = x^{\frac{1}{q}}
\end{array}
\right.
$$

Example

For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$ using the rule $(x)' = 1$, the chain rule and the integer power rule $\frac{\mathrm{d}}{\mathrm{d}u}(u^q) = qu^{q-1}$.

$$\begin{array}{rcl|l}
\left(x^{\frac{1}{q}}\right)^q &=& x & \frac{\mathrm{d}}{\mathrm{d}x} \\
\left(\left(x^{\frac{1}{q}}\right)^q\right)' &=& 1 & \text{Set } u = x^{\frac{1}{q}} \\
(u^q)' &=& 1 & \\
\frac{\mathrm{d}}{\mathrm{d}u}\left(u^q\right) u' &=& 1 & \\
q(u)^{q-1}(u)' &=& 1 &
\end{array}$$

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For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$ using the rule $(x)' = 1$, the chain rule and the integer power rule $\frac{\mathrm{d}}{\mathrm{d}u}(u^q) = qu^{q-1}$.

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(u^q)' & = & 1 & \\
\frac{\mathrm{d}}{\mathrm{d}u}(u^q)\, u' & = & 1 & \\
q(u)^{q-1}(u)' & = & 1 & \\
q(x^{\frac{1}{q}})^{q-1}\left(x^{\frac{1}{q}}\right)' & = & 1 &
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(u^q)' &=& 1 \\
\frac{\mathrm{d}}{\mathrm{d}u}(u^q)\, u' &=& 1 \\
q(u)^{q-1}(u)' &=& 1 \\
q(x^{\frac{1}{q}})^{q-1}\left(x^{\frac{1}{q}}\right)' &=& 1 \\
qx^{\frac{q-1}{q}}\left(x^{\frac{1}{q}}\right)' &=& 1
\end{array}
$$

$\frac{\mathrm{d}}{\mathrm{d}x}$

Set $u = x^{\frac{1}{q}}$

Example

For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$ using the rule $(x)' = 1$, the chain rule and the integer power rule $\frac{\mathrm{d}}{\mathrm{d}u}(u^q) = qu^{q-1}$.

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(u^q)' &=& 1 & \\
\frac{\mathrm{d}}{\mathrm{d}u}(u^q)\, u' &=& 1 & \\
q(u)^{q-1}(u)' &=& 1 & \\
q(x^{\frac{1}{q}})^{q-1}\left(x^{\frac{1}{q}}\right)' &=& 1 & \\
{\color{red}qx^{\frac{q-1}{q}}}\left(x^{\frac{1}{q}}\right)' &=& 1 & {\color{red}\text{divide by } qx^{\frac{q-1}{q}}} \\
\left(x^{\frac{1}{q}}\right)' &=& \frac{1}{{\color{red}qx^{\frac{q-1}{q}}}} = &
\end{array}
$$

Example

For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$ using the rule $(x)' = 1$, the chain rule and the integer power rule $\frac{\mathrm{d}}{\mathrm{d}u}(u^q) = qu^{q-1}$.

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(u^q)' &=& 1 & \\
\frac{\mathrm{d}}{\mathrm{d}u}\left(u^q\right)u' &=& 1 & \\
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q(x^{\frac{1}{q}})^{q-1}\left(x^{\frac{1}{q}}\right)' &=& 1 & \\
qx^{\frac{q-1}{q}}\left(x^{\frac{1}{q}}\right)' &=& 1 & \Big|\ \text{divide by } qx^{\frac{q-1}{q}} \\
\left(x^{\frac{1}{q}}\right)' &=& \frac{1}{qx^{\frac{q-1}{q}}} = \frac{x^{-\frac{q-1}{q}}}{q} = &
\end{array}
$$

Example

For positive integer q, derive the power rule $\left(x^{\frac{1}{q}}\right)' = \frac{1}{q}x^{\frac{1}{q}-1}$ using the rule $(x)' = 1$, the chain rule and the integer power rule $\frac{\mathrm{d}}{\mathrm{d}u}(u^q) = qu^{q-1}$.

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qx^{\frac{q-1}{q}}\left(x^{\frac{1}{q}}\right)' &=& 1 & \Big| \ \text{divide by } qx^{\frac{q-1}{q}} \\
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\end{array}
$$

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qx^{\frac{q-1}{q}}\left(x^{\frac{1}{q}}\right)' &=& 1 & \text{divide by } qx^{\frac{q-1}{q}} \\
\left(x^{\frac{1}{q}}\right)' &=& \frac{1}{qx^{\frac{q-1}{q}}} = \frac{x^{-\frac{q-1}{q}}}{q} = \frac{1}{q}x^{\frac{1}{q}-1} & \text{as desired}
\end{array}$$

Example

Derive the quotient rules

$$\begin{aligned}\left(\frac{1}{g}\right)' &= -\frac{g'}{g^2}\\ \left(\frac{f}{g}\right)' &= \frac{f'g-fg'}{g^2}\end{aligned}$$

Example

Derive the quotient rules

$$\begin{aligned} \left(\frac{1}{g}\right)' &= -\frac{g'}{g^2} \\ \left(\frac{f}{g}\right)' &= \frac{f'g - fg'}{g^2} \end{aligned}$$

$$\left(\frac{1}{g}\right)' =$$

Example

Derive the quotient rules

$$\begin{aligned}\left(\frac{1}{g}\right)' &= -\frac{g'}{g^2}\\ \left(\frac{f}{g}\right)' &= \frac{f'g-fg'}{g^2}\end{aligned}$$

using the chain rule,

$$\left(\frac{1}{g}\right)' = \frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{1}{g}\right)g' =$$

Example

Derive the quotient rules

$$\begin{aligned}\left(\frac{1}{g}\right)' &= -\frac{g'}{g^2}\\ \left(\frac{f}{g}\right)' &= \frac{f'g - fg'}{g^2}\end{aligned}$$

using the chain rule, the negative power rule

$$\left(\frac{1}{g}\right)' = \frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{1}{g}\right)g' = -\frac{1}{g^2}g'$$

Example

Derive the quotient rules

$$\left(\frac{1}{g}\right)' = -\frac{g'}{g^2}$$
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$$\left(\frac{1}{g}\right)' = \frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{1}{g}\right)g' = -\frac{1}{g^2}g' \quad \Big| \text{ as desired}$$

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$$\left(\frac{f}{g}\right)'$$

Example

Derive the quotient rules

$$\begin{aligned}\left(\frac{1}{g}\right)' &= -\frac{g'}{g^2}\\ \left(\frac{f}{g}\right)' &= \frac{f'g-fg'}{g^2}\end{aligned}$$

using the chain rule, the negative power rule

$$\left(\frac{1}{g}\right)' = \frac{d}{dg}\left(\frac{1}{g}\right)g' = -\frac{1}{g^2}g' \qquad \Big| \text{ as desired}$$

$$\left(\frac{f}{g}\right)' = \left(f\frac{1}{g}\right)' =$$

Example

Derive the quotient rules

$$\left(\frac{1}{g}\right)' = -\frac{g'}{g^2}$$
$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

using the chain rule, the negative power rule and the product rule.

$$\left(\frac{1}{g}\right)' = \frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{1}{g}\right)g' = -\frac{1}{g^2}g'$$ as desired

$$\left(\frac{f}{g}\right)' = \left(f\frac{1}{g}\right)' = f'\frac{1}{g} + f\left(\frac{1}{g}\right)' =$$

Example

Derive the quotient rules

$$\left(\frac{1}{g}\right)' = -\frac{g'}{g^2}$$
$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

using the chain rule, the negative power rule and the product rule.

$$\left(\frac{1}{g}\right)' = \frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{1}{g}\right)g' = -\frac{1}{g^2}g' \quad \Big| \text{ as desired}$$

$$\left(\frac{f}{g}\right)' = \left(f\frac{1}{g}\right)' = f'\frac{1}{g} + f\left(\frac{1}{g}\right)' = \frac{f'}{g} + f\left(-\frac{g'}{g^2}\right)$$

Example

Derive the quotient rules

$$\begin{aligned}\left(\frac{1}{g}\right)' &= -\frac{g'}{g^2}\\ \left(\frac{f}{g}\right)' &= \frac{f'g-fg'}{g^2}\end{aligned}$$

using the chain rule, the negative power rule and the product rule.

$$\left(\frac{1}{g}\right)' = \frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{1}{g}\right)g' = -\frac{1}{g^2}g'$$

as desired

$$\begin{aligned}\left(\frac{f}{g}\right)' &= \left(f\frac{1}{g}\right)' = f'\frac{1}{g}+f\left(\frac{1}{g}\right)' = \frac{f'}{g}+f\left(-\frac{g'}{g^2}\right)\\ &= \frac{f'g-fg'}{g^2}\end{aligned}$$

Example

Derive the quotient rules

$$\begin{aligned}\left(\frac{1}{g}\right)' &= -\frac{g'}{g^2}\\ \left(\frac{f}{g}\right)' &= \frac{f'g - fg'}{g^2}\end{aligned}$$

using the chain rule, the negative power rule and the product rule.

$$\left(\frac{1}{g}\right)' = \frac{\mathrm{d}}{\mathrm{d}g}\left(\frac{1}{g}\right)g' = -\frac{1}{g^2}g' \quad \Big| \text{ as desired}$$

$$\begin{aligned}\left(\frac{f}{g}\right)' &= \left(f\frac{1}{g}\right)' = f'\frac{1}{g} + f\left(\frac{1}{g}\right)' = \frac{f'}{g} + f\left(-\frac{g'}{g^2}\right)\\ &= \frac{f'g - fg'}{g^2} \quad \Big| \text{ as desired}\end{aligned}$$

You will not be tested on the material in the following slide.

Example

Derive the exponent rule $(e^x)' = e^x$

Example

Derive the exponent rule $(e^x)' = e^x$ using the Calc II formula below,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots,$$

where $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$.

$$(e^x)' = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\right)'$$

Example

Derive the exponent rule $(e^x)' = e^x$ using the Calc II formula below, the infinite (both sides uniformly convergent) sum rule
$(f_1 + f_2 + f_3 + \dots)' = f_1' + f_2' + f_3' + \dots$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots,$$

where $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$.

$$\begin{aligned}(e^x)' &= \left(1 + x + \tfrac{x^2}{2!} + \tfrac{x^3}{3!} + \dots\right)' \\ &= (1)' + (x)' + \frac{\left(x^2\right)'}{2!} + \frac{\left(x^3\right)'}{3!} + \dots + \frac{(x^n)'}{n!} + \dots\end{aligned}$$

Example

Derive the exponent rule $(e^x)' = e^x$ using the Calc II formula below, the infinite (both sides uniformly convergent) sum rule $(f_1 + f_2 + f_3 + \dots)' = f_1' + f_2' + f_3' + \dots$ and the power rule $(x^n)' = nx^{n-1}$.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots,$$

where $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$.

$$\begin{aligned}
(e^x)' &= \left(1 + x + \tfrac{x^2}{2!} + \tfrac{x^3}{3!} + \dots\right)' \\
&= (1)' + (x)' + \frac{(x^2)'}{2!} + \frac{(x^3)'}{3!} + \dots + \frac{(x^n)'}{n!} + \dots \\
&= 0 + 1 + \tfrac{2x}{2!} + \tfrac{3x^2}{3!} + \dots + \frac{nx^{n-1}}{n!} + \dots
\end{aligned}$$

Example

Derive the exponent rule $(e^x)' = e^x$ using the Calc II formula below, the infinite (both sides uniformly convergent) sum rule $(f_1 + f_2 + f_3 + \dots)' = f_1' + f_2' + f_3' + \dots$ and the power rule $(x^n)' = nx^{n-1}$.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots,$$

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$\frac{n}{n!} =$

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(e^x)' &= \left(1 + x + \tfrac{x^2}{2!} + \tfrac{x^3}{3!} + \dots\right)' \\
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Derive the exponent rule $(e^x)' = e^x$ using the Calc II formula below, the infinite (both sides uniformly convergent) sum rule $(f_1 + f_2 + f_3 + \dots)' = f_1' + f_2' + f_3' + \dots$ and the power rule $(x^n)' = nx^{n-1}$.

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&= 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \dots + \frac{nx^{n-1}}{n!} + \dots \\
&= 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^{n-1}}{(n-1)!} + \dots = e^x
\end{aligned}$$

Example

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(e^x)' &= \left(1 + x + \tfrac{x^2}{2!} + \tfrac{x^3}{3!} + \dots\right)' \\
&= (1)' + (x)' + \frac{\left(x^2\right)'}{2!} + \frac{\left(x^3\right)'}{3!} + \dots + \frac{(x^n)'}{n!} + \dots \\
&= 0 + 1 + \tfrac{2x}{2!} + \tfrac{3x^2}{3!} + \dots + \tfrac{nx^{n-1}}{n!} + \dots \\
&= \quad 1 + \tfrac{x}{1!} + \tfrac{x^2}{2!} + \dots + \tfrac{x^{n-1}}{(n-1)!} + \dots = e^x
\end{aligned}$$

as desired.

Example

Derive the logarithm derivative rules

$$\begin{aligned}(\ln x)' &= \frac{1}{x}\\ (\log_a x)' &= \frac{1}{x \ln a}\end{aligned}$$

Example

Derive the logarithm derivative rules

$$\begin{aligned}(\ln x)' &= \frac{1}{x}\\ (\log_a x)' &= \frac{1}{x\ln a}\end{aligned}$$

$$e^{\ln x} = x$$

Example

Derive the logarithm derivative rules

$$\begin{aligned}(\ln x)' &= \frac{1}{x}\\ (\log_a x)' &= \frac{1}{x\ln a}\end{aligned}$$

$$\begin{aligned} e^{\ln x} &= x && \Big|\ \text{set}\quad u=\ln x\\ e^{u} &= x \end{aligned}$$

Example

Derive the logarithm derivative rules

$$\begin{aligned}(\ln x)' &= \frac{1}{x}\\ (\log_a x)' &= \frac{1}{x\ln a}\end{aligned}$$

$$\begin{aligned}e^{\ln x} &= x \\ e^{u} &= x\end{aligned} \qquad \left|\ \begin{array}{l}\text{set}\quad u = \ln x\\ \frac{d}{dx}\end{array}\right.$$

Example

Derive the logarithm derivative rules

$$\begin{aligned}(\ln x)' &= \frac{1}{x}\\ (\log_a x)' &= \frac{1}{x \ln a}\end{aligned}$$

using the chain rule,

$$\begin{aligned} e^{\ln x} &= x \\ e^{u} &= x \\ \frac{d}{du}(e^u)u' &= (x)' \end{aligned} \qquad \left| \begin{array}{l} \text{set} \quad u = \ln x \\ \frac{d}{dx} \end{array}\right.$$

Example

Derive the logarithm derivative rules

$$\begin{aligned}(\ln x)' &= \frac{1}{x}\\ (\log_a x)' &= \frac{1}{x\ln a}\end{aligned}$$

using the chain rule, the exponent derivative rule $(e^x)' = e^x$,

$$\begin{aligned} e^{\ln x} &= x \\ e^u &= x \\ \frac{\mathrm{d}}{\mathrm{d}u}(e^u)u' &= (x)' \\ e^u u' &= 1 \end{aligned} \qquad \left| \begin{array}{l} \text{set} \quad u = \ln x \\ \frac{\mathrm{d}}{\mathrm{d}x} \\ \\ \\ \end{array}\right.$$

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$$\begin{aligned}(\ln x)' &= \frac{1}{x}\\ (\log_a x)' &= \frac{1}{x\ln a}\end{aligned}$$

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| set $u = \ln x$
| $\frac{d}{dx}$

Example

Derive the logarithm derivative rules

$$\begin{aligned}(\ln x)' &= \frac{1}{x}\\ (\log_a x)' &= \frac{1}{x\ln a}\end{aligned}$$

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$$\begin{aligned}
e^{\ln x} &= x \\
e^{u} &= x \\
\frac{\mathrm{d}}{\mathrm{d}u}(e^u)u' &= (x)' \\
e^u u' &= 1 \\
\color{red}{e^{\ln x}}(\ln x)' &= 1 \\
\color{red}{x}(\ln x)' &= 1
\end{aligned}
\qquad
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e^u u' &= 1\\
e^{\ln x}(\ln x)' &= 1\\
x(\ln x)' &= 1 && \Big|\ \cdot\frac{1}{x}\\
(\ln x)' &= \frac{1}{x}
\end{aligned}$$

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Derive the logarithm derivative rules

$$\begin{aligned} (\ln x)' &= \frac{1}{x} \\ (\log_a x)' &= \frac{1}{x \ln a} \end{aligned}$$

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e^{\ln x} &= x & &\Big|\ \text{set} \quad u = \ln x \\
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e^u u' &= 1 \\
e^{\ln x}(\ln x)' &= 1 \\
x(\ln x)' &= 1 & &\Big|\ \cdot \frac{1}{x} \\
(\ln x)' &= \frac{1}{x} & &\Big|\ \text{as desired}
\end{aligned}$$

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$$\begin{aligned}(\ln x)' &= \frac{1}{x}\\ (\log_a x)' &= \frac{1}{x\ln a}\end{aligned}$$

using the chain rule, the exponent derivative rule $(e^x)' = e^x$, the rule $(x)' = 1$

$$\begin{array}{rcl|l} e^{\ln x} &=& x & \text{set} \quad u = \ln x \\ e^u &=& x & \frac{\mathrm{d}}{\mathrm{d}x} \\ \frac{\mathrm{d}}{\mathrm{d}u}(e^u)u' &=& (x)' & \\ e^u u' &=& 1 & \\ e^{\ln x}(\ln x)' &=& 1 & \\ x(\ln x)' &=& 1 & \cdot\frac{1}{x} \\ (\ln x)' &=& \frac{1}{x} & \text{as desired} \\ \hline (\log_a x)' &=& & \end{array}$$

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\hline
(\log_a x)' &=& \left(\frac{\ln x}{\ln a}\right)' = &
\end{array}$$

Example

Derive the logarithm derivative rules

$$\begin{aligned}(\ln x)' &= \frac{1}{x}\\ (\log_a x)' &= \frac{1}{x\ln a}\end{aligned}$$

using the chain rule, the exponent derivative rule $(e^x)' = e^x$, the rule $(x)' = 1$ and the constant multiple rule $(cf)' = cf'$.

$$\begin{array}{rcll}
e^{\ln x} &=& x & \Big| \text{ set } \quad u = \ln x \\
e^{u} &=& x & \Big| \ \frac{\mathrm{d}}{\mathrm{d}x} \\
\frac{\mathrm{d}}{\mathrm{d}u}(e^u)u' &=& (x)' & \\
e^u u' &=& 1 & \\
e^{\ln x}(\ln x)' &=& 1 & \\
x(\ln x)' &=& 1 & \Big| \cdot\frac{1}{x} \\
(\ln x)' &=& \frac{1}{x} & \Big| \text{ as desired} \\
\hline
(\log_a x)' &=& \left(\frac{\ln x}{\ln a}\right)' = \frac{(\ln x)'}{\ln a} = \frac{1}{x\ln a} &
\end{array}$$

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$$\begin{aligned} (\ln x)' &= \frac{1}{x} \\ (\log_a x)' &= \frac{1}{x \ln a} \end{aligned}$$

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e^{\ln x} &=& x & \text{set} \quad u = \ln x\\
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e^u u' &=& 1 & \\
e^{\ln x}(\ln x)' &=& 1 & \\
x(\ln x)' &=& 1 & \cdot\frac{1}{x}\\
(\ln x)' &=& \frac{1}{x} & \text{as desired}\\
\hline
(\log_a x)' &=& \left(\frac{\ln x}{\ln a}\right)' = \frac{(\ln x)'}{\ln a} = \frac{1}{x\ln a} & \text{as desired}
\end{array}$$

Example

Derive the power rule

$$\left(x^{r}\right)' = rx^{r-1}, \qquad x > 0$$

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Derive the power rule

$$\left(x^r\right)' = rx^{r-1}, \qquad x > 0$$

$$\left(x^r\right)' = \left(\left(e^{\ln x}\right)^r\right)' = \left(e^{r \ln x}\right)'$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

$$\begin{aligned}(x^r)' &= \left((e^{\ln x})^r\right)' = \left(e^{r\ln x}\right)' \\ &= (e^u)' = \end{aligned}$$

Set $u = r\ln x$

Example

Derive the power rule

$$\left(x^r\right)' = rx^{r-1}, \qquad x > 0$$

using the chain rule,

$$\begin{aligned} \left(x^r\right)' &= \left(\left(e^{\ln x}\right)^r\right)' = \left(e^{r \ln x}\right)' && \Big| \text{ Set } u = r \ln x \\ &= \left(e^u\right)' = \frac{\mathrm{d}}{\mathrm{d}u}\left(e^u\right) u' = \end{aligned}$$

Example

Derive the power rule

$$(x^r)' = rx^{r-1}, \qquad x > 0$$

using the chain rule, the the rule $(e^x)' = e^x$,

$$\begin{aligned} (x^r)' &= \left((e^{\ln x})^r\right)' = \left(e^{r \ln x}\right)' && \Big| \text{ Set } u = r\ln x \\ &= (e^u)' = \frac{\mathrm{d}}{\mathrm{d}u}(e^u)\, u' = e^u u' = \end{aligned}$$

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&= e^{r\ln x}(r\ln x)' =
\end{aligned}$$

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&= e^{r\ln x}(r\ln x)' = \left(e^{\ln x}\right)^r r(\ln x)'
\end{aligned}
$$

Example

Derive the power rule

$$\left(x^r\right)' = rx^{r-1}, \qquad x > 0$$

using the chain rule, the the rule $(e^x)' = e^x$, the constant multiple derivative rule

$$\begin{aligned}
\left(x^r\right)' &= \left(\left(e^{\ln x}\right)^r\right)' = \left(e^{r \ln x}\right)' && \Big|\ \text{Set } u = r \ln x \\
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&= e^{r \ln x}(r \ln x)' = \left(e^{\ln x}\right)^r r(\ln x)'
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&= e^{r\ln x}(r\ln x)' = \left(e^{\ln x}\right)^r r(\ln x)' \\
&= x^r r\tfrac{1}{x} =
\end{aligned}$$

Example

Derive the power rule

$$\left(x^r\right)' = rx^{r-1}, \qquad x > 0$$

using the chain rule, the the rule $(e^x)' = e^x$, the constant multiple derivative rule and the logarithm derivative rule $(\ln x)' = \frac{1}{x}$.

$$\begin{aligned}
\left(x^r\right)' &= \left(\left(e^{\ln x}\right)^r\right)' = \left(e^{r\ln x}\right)' && \Big| \text{ Set } u = r\ln x \\
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&= e^{r\ln x}(r\ln x)' = \left(e^{\ln x}\right)^r r(\ln x)' \\
&= x^r r\frac{1}{x} = rx^{r-1}
\end{aligned}$$

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(x^r)' & = & \left((e^{\ln x})^r\right)' = \left(e^{r\ln x}\right)' & \Big| \text{ Set } u = r\ln x \\
& = & (e^u)' = \frac{\mathrm{d}}{\mathrm{d}u}(e^u)\, u' = e^u u' = & \\
& = & e^{r\ln x}(r\ln x)' = \left(e^{\ln x}\right)^r r(\ln x)' & \\
& = & x^r r\frac{1}{x} = rx^{r-1} & \Big| \text{ as desired}
\end{array}$$

Example

Derive the sine and cosine rules

$$\begin{aligned}(\sin x)' &= \cos x\\ (\cos x)' &= -\sin x\end{aligned}$$

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using Euler's formula,

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Example

Derive the sine and cosine rules

$$\begin{aligned}(\sin x)' &= \cos x\\ (\cos x)' &= -\sin x\end{aligned}$$

using Euler's formula, the exponent derivative rule, the chain rule,

$$\begin{aligned}e^{ix} &= \cos x + i\sin x && \Big| \frac{\mathrm{d}}{\mathrm{d}x}\\ \frac{\mathrm{d}}{\mathrm{d}x}\left(e^{ix}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\cos x + i\sin x\right)\\ e^{ix}(ix)' &= \end{aligned}$$

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$$\begin{aligned}(\sin x)' &= \cos x\\ (\cos x)' &= -\sin x\end{aligned}$$

using Euler's formula, the exponent derivative rule, the chain rule, Assume all rules are valid over the complex numbers $\mathbb{C}$.

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using Euler's formula, the exponent derivative rule, the chain rule, the sum rule and the constant multiple rule. Assume all rules are valid over the complex numbers $\mathbb{C}$.

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e^{ix} &= \cos x + i\sin x && \Big|\ \frac{\mathrm{d}}{\mathrm{d}x}\\
\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{ix}\right) &= \frac{\mathrm{d}}{\mathrm{d}x}\left(\cos x + i\sin x\right)\\
e^{ix}(ix)' &= (\cos x)' + i(\sin x)'\\
ie^{ix} &= (\cos x)' + i(\sin x)'\\
i(\cos x + i\sin x) &= (\cos x)' + i(\sin x)'
\end{aligned}$$

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Derive the sine and cosine rules

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e^{ix}(ix)' &= (\cos x)' + i(\sin x)'\\
ie^{ix} &= (\cos x)' + i(\sin x)'\\
i^2\sin x + i\cos x = i(\cos x + i\sin x) &= (\cos x)' + i(\sin x)'
\end{aligned}$$

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Derive the sine and cosine rules

$$\begin{aligned}(\sin x)' &= \cos x\\ (\cos x)' &= -\sin x\end{aligned}$$

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e^{ix}(ix)' &= (\cos x)' + i(\sin x)'\\
ie^{ix} &= (\cos x)' + i(\sin x)'\\
{\color{red}i^2}\sin x + i\cos x = i(\cos x + i\sin x) &= (\cos x)' + i(\sin x)'\\
{\color{red}-}\sin x + i\cos x &= (\cos x)' + i(\sin x)'
\end{aligned}$$

Example

Derive the sine and cosine rules

$$\begin{array}{rcl} (\sin x)' &=& \cos x \\ (\cos x)' &=& -\sin x \end{array}$$

using Euler's formula, the exponent derivative rule, the chain rule, the sum rule and the constant multiple rule. Assume all rules are valid over the complex numbers $\mathbb{C}$.

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Compare real part to get the desired equalities.

Example

Derive the sine and cosine rules

$$\begin{aligned} (\sin x)' &= \cos x \\ (\cos x)' &= -\sin x \end{aligned}$$

using Euler's formula, the exponent derivative rule, the chain rule, the sum rule and the constant multiple rule. Assume all rules are valid over the complex numbers $\mathbb{C}$.

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Compare real part and coefficients of i to get the desired equalities.