Calculus II
Lecture 3

Todor Milev

https://github.com/tmilev/freecalc

2020

Outline

License to use and redistribute

These lecture slides and their LaTeX source code are licensed to you under the Creative Commons license CC BY 3.0. You are free

- to Share - to copy, distribute and transmit the work,
- to Remix - to adapt, change, etc., the work,
- to make commercial use of the work,

as long as you reasonably acknowledge the original project.

- Latest version of the .tex sources of the slides:
  `https://github.com/tmilev/freecalc`
- Should the link be outdated/moved, search for “freecalc project”.
- Creative Commons license CC BY 3.0:
  `https://creativecommons.org/licenses/by/3.0/us/`
  and the links therein.

Integration by Parts

Every differentiation rule

Integration by Parts

Every differentiation rule

$$(uv)' = u'v + uv' \quad \Big| \text{ Product Rule}$$

Integration by Parts

Every differentiation rule corresponds to a differential form rule

$$\begin{array}{rcl|l} (uv)' & = & u'v + uv' & \text{Product Rule} \\ \mathrm{d}(uv) & = & v\mathrm{d}u + u\mathrm{d}v & \text{Differential Prod. Rule} \end{array}$$

Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

$$\begin{array}{rcl|l} (uv)' & = & u'v + uv' & \text{Product Rule} \\ \mathrm{d}(uv) & = & v\mathrm{d}u + u\mathrm{d}v & \text{Differential Prod. Rule} \\ \int \mathrm{d}(uv) & = & \int v\mathrm{d}u + \int u\mathrm{d}v & \text{integration of the above} \end{array}$$

Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

$$
\begin{array}{rcl|l}
(uv)' & = & u'v + uv' & \text{Product Rule} \\
\mathrm{d}(uv) & = & v\mathrm{d}u + u\mathrm{d}v & \text{Differential Prod. Rule} \\
\int \mathrm{d}(uv) & = & \int v\mathrm{d}u + \int u\mathrm{d}v & \text{integration of the above} \\
uv & = & \int v\mathrm{d}u + \int u\mathrm{d}v &
\end{array}
$$

Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

$$
\begin{array}{rcl|l}
(uv)' & = & u'v + uv' & \text{Product Rule} \\
\mathrm{d}(uv) & = & v\mathrm{d}u + u\mathrm{d}v & \text{Differential Prod. Rule} \\
\int \mathrm{d}(uv) & = & \int v\mathrm{d}u + \int u\mathrm{d}v & \text{integration of the above} \\
uv & = & \int v\mathrm{d}u + \int u\mathrm{d}v & \text{rearrange} \\
\int u\mathrm{d}v & = & uv - \int v\mathrm{d}u &
\end{array}
$$

Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

$$
\begin{array}{rcl|l}
(uv)' & = & u'v + uv' & \text{Product Rule} \\
\mathrm{d}(uv) & = & v\mathrm{d}u + u\mathrm{d}v & \text{Differential Prod. Rule} \\
\int \mathrm{d}(uv) & = & \int v\mathrm{d}u + \int u\mathrm{d}v & \text{integration of the above} \\
uv & = & \int v\mathrm{d}u + \int u\mathrm{d}v & \text{rearrange} \\
\int u\mathrm{d}v & = & uv - \int v\mathrm{d}u &
\end{array}
$$

Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

$$
\begin{array}{rcl|l}
(uv)' & = & u'v + uv' & \text{Product Rule} \\
\mathrm{d}(uv) & = & v\mathrm{d}u + u\mathrm{d}v & \text{Differential Prod. Rule} \\
\int \mathrm{d}(uv) & = & \int v\mathrm{d}u + \int u\mathrm{d}v & \text{integration of the above} \\
uv & = & \int v\mathrm{d}u + \int u\mathrm{d}v & \text{rearrange} \\
\int u\mathrm{d}v & = & uv - \int v\mathrm{d}u &
\end{array}
$$

Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

$$
\begin{array}{rcl|l}
(uv)' & = & u'v + uv' & \text{Product Rule} \\
\mathrm{d}(uv) & = & v\mathrm{d}u + u\mathrm{d}v & \text{Differential Prod. Rule} \\
\int \mathrm{d}(uv) & = & \int v\mathrm{d}u + \int u\mathrm{d}v & \text{integration of the above} \\
uv & = & \int v\mathrm{d}u + \int u\mathrm{d}v & \text{rearrange} \\
\int u\mathrm{d}v & = & uv - \int v\mathrm{d}u &
\end{array}
$$

We just proved the following.

Proposition ((Rule of) Integration by Parts)

$$\int u\mathrm{d}v = uv - \int v\mathrm{d}u$$

Integration by parts: strategy for applying

Integration by parts:

$$\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$$

Generally: Choose u in this order: **LIPET**

Integration by parts: strategy for applying

Integration by parts:

$$\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$$

Generally: Choose u in this order: **LIPET**

Logs, Inverse trig, Polynomial, Exponential, Trig

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x \sin x \mathrm{d}x =$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$\int x \sin x \mathrm{d}x =$ | $\sin x \mathrm{d}x = \mathrm{d}(\textbf{?} \qquad)$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x \sin x \mathrm{d}x = \quad \Big| \quad \sin x \mathrm{d}x = \mathrm{d}(-\cos x)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x \sin x \mathrm{d}x = \int x \mathrm{d}(-\cos x) \quad \Big| \quad \sin x \mathrm{d}x = \mathrm{d}(-\cos x)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x\sin x\mathrm{d}x = \int x\mathrm{d}(-\cos x) \quad \Big| \quad \sin x\mathrm{d}x = \mathrm{d}(-\cos x)$$
$$= x(-\cos x) - \int (-\cos x)\mathrm{d}x$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x\sin x\mathrm{d}x = \int x\mathrm{d}(-\cos x) \quad \Big| \quad \sin x\mathrm{d}x = \mathrm{d}(-\cos x)$$
$$= x(-\cos x) - \int (-\cos x)\mathrm{d}x$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x\sin x\mathrm{d}x = \int x\mathrm{d}(-\cos x) \quad \Big| \quad \sin x\mathrm{d}x = \mathrm{d}(-\cos x)$$

$$= x(-\cos x) - \int (-\cos x)\mathrm{d}x$$

$$= -x\cos x + \int \cos x\mathrm{d}x$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x\sin x\mathrm{d}x = \int x\mathrm{d}(-\cos x) \quad \Big| \quad \sin x\mathrm{d}x = \mathrm{d}(-\cos x)$$

$$= x(-\cos x) - \int (-\cos x)\mathrm{d}x$$

$$= -x\cos x + \int \cos x\mathrm{d}x$$

$$= -x\cos x + \textbf{?}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int x\sin x\mathrm{d}x &= \int x\mathrm{d}(-\cos x) \qquad \Big| \ \sin x\mathrm{d}x = \mathrm{d}(-\cos x)\\ &= x(-\cos x) - \int(-\cos x)\mathrm{d}x\\ &= -x\cos x + \int \cos x\mathrm{d}x\\ &= -x\cos x + \sin x + C\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int \ln x\mathrm{d}x \quad =$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int \ln x \mathrm{d}x = (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int \ln x \mathrm{d}x = (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned} \int \ln x \mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts} \\ &= x\ln x - \int x(\ln x)'\mathrm{d}x \end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int \ln x \mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts} \\
&= x\ln x - \int x(\ln x)'\mathrm{d}x \\
&= x\ln x - \int x\textbf{?}\ \mathrm{d}x
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int \ln x \mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts} \\
&= x\ln x - \int x(\ln x)'\mathrm{d}x \\
&= x\ln x - \int x\frac{1}{x}\mathrm{d}x
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int \ln x \mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts} \\
&= x\ln x - \int x(\ln x)'\mathrm{d}x \\
&= x\ln x - \int x\frac{1}{x}\mathrm{d}x \\
&= x\ln x - \int \mathrm{d}x
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int \ln x \mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts} \\
&= x\ln x - \int x(\ln x)'\mathrm{d}x \\
&= x\ln x - \int x\frac{1}{x}\mathrm{d}x \\
&= x\ln x - \int \mathrm{d}x \\
&= x\ln x - \textbf{?}
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int \ln x \mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts} \\
&= x\ln x - \int x(\ln x)'\mathrm{d}x \\
&= x\ln x - \int x\frac{1}{x}\mathrm{d}x \\
&= x\ln x - \int \mathrm{d}x \\
&= x\ln x - x + C \quad .
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int t^2 e^t \mathrm{d}t$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int t^2 e^t \mathrm{d}t \quad = \quad \int t^2 \mathrm{d}\left(\textbf{?}\right)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int t^2 e^t \mathrm{d}t = \int t^2 \mathrm{d}\left(e^t\right)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int t^2 e^t \mathrm{d}t = \int t^2 \mathrm{d}\left(e^t\right) \quad \Big| \quad \text{integrate by parts}$$

$$= \textbf{?}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned} \int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) \\ &= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \end{aligned}$$

integrate by parts

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned} \int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) \\ &= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \end{aligned} \quad \Big| \quad \text{integrate by parts}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned} \int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) \\ &= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \end{aligned}$$

integrate by parts

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) \\ &= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\ &= t^2 e^t - \int e^t \textbf{?}\ \mathrm{d}t\end{aligned}$$

integrate by parts

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) \\ &= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\ &= t^2 e^t - \int e^t 2t\mathrm{d}t\end{aligned}$$

integrate by parts

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
&= t^2 e^t - \int e^t 2t\mathrm{d}t \\
&= t^2 e^t - 2\int t\mathrm{d}(\textbf{?}\ )
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
&= t^2 e^t - \int e^t 2t\mathrm{d}t \\
&= t^2 e^t - 2\int t\mathrm{d}(e^t)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
&= t^2 e^t - \int e^t 2t\mathrm{d}t \\
&= t^2 e^t - 2\int t\mathrm{d}(e^t) && \text{integrate by parts} \\
&= t^2 e^t - 2\textbf{?}
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
&= t^2 e^t - \int e^t 2t\mathrm{d}t \\
&= t^2 e^t - 2\int t\mathrm{d}(e^t) && \text{integrate by parts} \\
&= t^2 e^t - 2\left(te^t - \int e^t \mathrm{d}t\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
&= t^2 e^t - \int e^t 2t \mathrm{d}t \\
&= t^2 e^t - 2\int t\mathrm{d}(e^t) && \text{integrate by parts} \\
&= t^2 e^t - 2\left(te^t - \int e^t \mathrm{d}t\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
&= t^2 e^t - \int e^t 2t \mathrm{d}t \\
&= t^2 e^t - 2\int t\mathrm{d}(e^t) && \text{integrate by parts} \\
&= t^2 e^t - 2\left(te^t - \int e^t \mathrm{d}t\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$
\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
&= t^2 e^t - \int e^t 2t \mathrm{d}t \\
&= t^2 e^t - 2\int t\mathrm{d}(e^t) && \text{integrate by parts} \\
&= t^2 e^t - 2\left(te^t - \int e^t \mathrm{d}t\right) \\
&= t^2 e^t - 2te^t + 2e^t + C
\end{aligned}
$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$
\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
&= t^2 e^t - \int e^t 2t \mathrm{d}t \\
&= t^2 e^t - 2\int t\mathrm{d}(e^t) && \text{integrate by parts} \\
&= t^2 e^t - 2\left(te^t - \int e^t \mathrm{d}t\right) \\
&= t^2 e^t - 2te^t + 2e^t + C
\end{aligned}
$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int e^x \sin x \mathrm{d}x \quad =$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int e^x \sin x \mathrm{d}x \quad = \quad \int \sin x \, \mathrm{d}(\textbf{?}\ )$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int e^x \sin x \mathrm{d}x = \int \sin x \, \mathrm{d}\left(e^x\right)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int e^x \sin x \mathrm{d}x = \int \sin x \, \mathrm{d}\left(e^x\right)$$

$$= \textbf{?}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\ &= (\sin x)e^x - \int e^x \mathrm{d}(\sin x)\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\ &= (\sin x)e^x - \int e^x \mathrm{d}(\sin x)\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\ &= (\sin x)e^x - \int e^x \mathrm{d}(\sin x)\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\ &= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\ &= e^x \sin x - \int e^x \textbf{?}\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\ &= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\ &= e^x \sin x - \int e^x \cos x \mathrm{d}x\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(\textbf{?}\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(e^x\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(e^x\right) \\
&= e^x \sin x - \left(\textbf{?} \right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(e^x\right) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x)\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(e^x\right) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x)\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(e^x\right) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x)\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(e^x\right) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x)\right) \\
&= e^x \sin x - \cos x e^x + \int e^x \textbf{?}
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(e^x\right) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x)\right) \\
&= e^x \sin x - \cos x e^x + \int e^x \textbf{?}
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(e^x\right) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x)\right) \\
&= e^x \sin x - \cos x e^x + \int e^x \left(-\sin x\right)\mathrm{d}x
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(e^x\right) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x)\right) \\
&= e^x \sin x - \cos x e^x + \int e^x (-\sin x)\mathrm{d}x \\
&= e^x \sin x - \cos x e^x - \int e^x \sin x \mathrm{d}x
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}\left(e^x\right) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}\left(e^x\right) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x)\right) \\
&= e^x \sin x - \cos x e^x + \int e^x (-\sin x)\mathrm{d}x \\
&= e^x \sin x - \cos x e^x - \int e^x \sin x \mathrm{d}x
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int e^x \sin x\mathrm{d}x = e^x \sin x - \cos x e^x - \int e^x \sin x\mathrm{d}x$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned} \int e^x \sin x \mathrm{d}x &= e^x \sin x - \cos x e^x - \int e^x \sin x \mathrm{d}x \\ 2\int e^x \sin x \mathrm{d}x &= e^x \sin x - \cos x e^x \end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= e^x \sin x - \cos x e^x - \int e^x \sin x \mathrm{d}x \\
2\int e^x \sin x \mathrm{d}x &= e^x \sin x - \cos x e^x \\
\int e^x \sin x \mathrm{d}x &= \frac{1}{2}\left(e^x \sin x - \cos x e^x\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= e^x \sin x - \cos x e^x - \int e^x \sin x \mathrm{d}x \\
2\int e^x \sin x \mathrm{d}x &= e^x \sin x - \cos x e^x \\
\int e^x \sin x \mathrm{d}x &= \frac{1}{2}\left(e^x \sin x - \cos x e^x\right) + C
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int_0^1 \arctan x\mathrm{d}x \quad =$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int_0^1 \arctan x \mathrm{d}x \quad = \quad \textbf{?}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int_0^1 \arctan x\mathrm{d}x = [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int_0^1 \arctan x\mathrm{d}x = [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int_0^1 \arctan x\mathrm{d}x = [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right)\\ &= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\textbf{?}\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right)\\ &= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\textbf{?}\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\ &= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\textbf{?}\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\ &= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\,\frac{1}{1+x^2}\mathrm{d}x\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x \frac{1}{1+x^2}\mathrm{d}x \\
&= \textbf{?} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\textbf{?}\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x\,\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\textbf{?}\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\,\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\ ?\ \right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\,\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1}\frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1}\frac{1}{1+x^2}\mathrm{d}(\quad x^2)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1}\frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1}\frac{1}{1+x^2}\mathrm{d}(1+x^2)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

Set $w = 1 + x^2$.

$$\begin{aligned}
\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

Set $w = 1 + x^2$.

$$\begin{aligned}
\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{w}\mathrm{d}w
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

Set $w = 1 + x^2$.

$$\begin{aligned}
\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{w}\mathrm{d}w = \frac{\pi}{4} - \frac{1}{2}\left[\ln|w|\right]_{x=0}^{x=1}
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

Set $w = 1 + x^2$.

$$\begin{aligned}
\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{w}\mathrm{d}w = \frac{\pi}{4} - \frac{1}{2}\left[\ln|w|\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}\left[\ln\left(1+x^2\right)\right]_{x=0}^{x=1}
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

Set $w = 1 + x^2$.

$$\begin{aligned}
\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{w}\mathrm{d}w = \frac{\pi}{4} - \frac{1}{2}\left[\ln|w|\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}\left[\ln\left(1+x^2\right)\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}\left(\ln 2 - \ln 1\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

Set $w = 1 + x^2$.

$$\begin{aligned}
\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{w}\mathrm{d}w = \frac{\pi}{4} - \frac{1}{2}\left[\ln|w|\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}\left[\ln\left(1+x^2\right)\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}\left(\ln 2 - \ln 1\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

Set $w = 1 + x^2$.

$$\begin{aligned}
\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\left(\arctan x\right) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x\frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{w}\mathrm{d}w = \frac{\pi}{4} - \frac{1}{2}\left[\ln|w|\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}\left[\ln\left(1+x^2\right)\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}\left(\ln 2 - \ln 1\right) = \frac{\pi}{4} - \frac{1}{2}\ln 2 \quad .
\end{aligned}$$