Calculus I
Homework Riemann Sums
Lecture 20

1. Estimate the integral using a Riemann sum using the indicated sample points and interval length.

   (a) $\displaystyle\int_0^4 \left(\sqrt{8x+1}\right) \mathrm{d}x$. Use four intervals of equal width, choose the sample point to be the left endpoint of each interval.

   answer: $\Delta x = 1$ and $f(x) = \sqrt{8x+1}$. Thus $\int_0^4 f(x)\mathrm{d}x \approx 9 + \sqrt{17}$.

   (b) $\displaystyle\int_0^6 \frac{1}{x^2+1}\mathrm{d}x$. Use three intervals of equal width, choose the sample point to be the left endpoint.

   answer: $\Delta x = 2$ and $f(x) = \frac{1}{x^2+1}$. Thus $\int_0^6 f(x)\mathrm{d}x \approx \frac{214}{85}$.

   (c) $\displaystyle\int_{-3.5}^{-0.5} \frac{\mathrm{d}x}{x^2+1}$. Use three intervals of equal width, choose the sample point to be the midpoint of each interval.

   answer: $\Delta x = 1$ and $f(x) = \frac{1}{x^2+1}$. Thus $\displaystyle\int_{-3.5}^{-0.5} f(x)\mathrm{d}x \approx \Delta x \left(f(-3) + f(-2) + f(-1)\right) = \frac{4}{5} = 0.8$.

   (d) $\displaystyle\int_0^2 \frac{\mathrm{d}x}{1+x+x^3}$. Use $\Delta x = \frac{1}{2}$ and right endpoint sampling points.

   answer: $\frac{1}{2}\left(\frac{8}{13} + \frac{1}{3} + \frac{8}{47} + \frac{1}{11}\right) = \frac{12197}{20163} \approx 0.604920$

   (e) $\displaystyle\int_{-2}^0 \frac{\mathrm{d}x}{1+x+x^2}$. Use $\Delta x = \frac{2}{3}$ and left endpoint sampling points.

   answer: $\frac{2}{3}\left(\frac{1}{3} + \frac{9}{13} + \frac{9}{7}\right) = \frac{1262}{819} \approx 1.540904$

   (f) $\displaystyle\int_0^2 \frac{\mathrm{d}x}{1+x^3}$. Use four intervals of equal width, choose the sample point to be the left endpoint of each interval.

   answer: $\Delta x = 0.5$ and $f(x) = \frac{1}{1+x^3}$. Thus $\displaystyle\int_0^2 f(x)\mathrm{d}x \approx \Delta x \left(f(0) + f(1) + f\left(\frac{1}{2}\right) + f\left(\frac{3}{2}\right)\right) = \frac{1649}{1260} \approx 1.30873$.

   (g) $\displaystyle\int_{-2}^0 \frac{\mathrm{d}x}{x^4+1}$. Use four intervals of equal width, choose the sample point to be the right endpoint.

   answer: $\Delta x = 0.5$ and $f(x) = \frac{1}{1+x^4}$. Thus $\displaystyle\int_{-2}^0 f(x)\mathrm{d}x \approx \Delta x \left(f\left(-\frac{3}{2}\right) + f(-1) + f\left(-\frac{1}{2}\right) + f(0)\right) = \frac{8595}{6596} \approx 1.303062$.

   (h) $\displaystyle\int_{-1}^0 \frac{1}{3x^2+1}\mathrm{d}x$. Use 3 **intervals** of equal width, choose the sampling points to be the **left endpoints** of each interval. Simplify your answer to a rational number (single fraction of two integers).

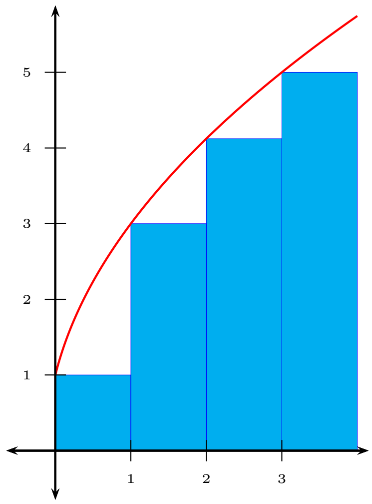
   answer: $\Delta x = \frac{1}{3}$ and $f(x) = \frac{1}{3x^2+1}$. Thus $\displaystyle\int_{-1}^0 f(x)\mathrm{d}x$ is approximated by $\Delta x \left(f(-1) + f\left(-\frac{2}{3}\right) + f\left(-\frac{1}{3}\right)\right) = \frac{10}{21}$.

Solution. 1.a. The interval $[0, 4]$ is subdivided into $n = 4$ intervals, therefore the length of each is $\Delta x = 1$. The intervals are therefore

$$[0, 1], [1, 2], [2, 3], [3, 4] \quad .$$

The problem asks us to use the left endpoints of each interval as sampling points. Therefore our sampling points are $0, 1, 2, 3$. Therefore the Riemann sum we are looking for is

$$\Delta x \left(f(0) + f(1) + f(2) + f(3)\right) = 1 \cdot \left(\sqrt{8 \cdot 0 + 1} + \sqrt{8 \cdot 1 + 1} + \sqrt{8 \cdot 2 + 1} + \sqrt{8 \cdot 3 + 1}\right) = 9 + \sqrt{17} \approx 13.1231$$

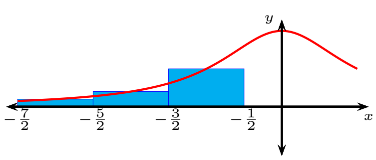


Solution. 1.c. The interval $[-3.5, -0.5]$ is subdivided into $n = 3$ intervals, therefore the length of each is $\Delta x = \frac{-0.5-(-3.5)}{3} = \frac{3}{3} = 1$. The intervals are therefore

$$[-3.5, -2.5], [-2.5, -1.5], [-1.5, -0.5] \quad .$$

The problem asks us to use the midpoint of each interval as a sampling point. Therefore our sampling points are $-3, -2, -1$. Therefore the Riemann sum we are looking for is

$$\Delta x \left(f(-3) + f(-2) + f(-1) \right) = 1 \cdot \left(\frac{1}{10} + \frac{1}{5} + \frac{1}{2} \right) = 0.8 \quad .$$



Solution. 1.h

$\Delta x = \frac{1}{3}$ and $f(x) = \frac{1}{3x^2+1}$. Thus $\int\limits_{-1}^{0} f(x) \mathrm{d}x$ is approximated by $\Delta x \left(f\left(-1\right) + f\left(-\frac{2}{3}\right) + f\left(-\frac{1}{3}\right) \right) = \frac{10}{21}$.