Calculus I
Homework Linear approximations
Lecture 19

1.

(a) Find the linearization of $f(x) = \sqrt{x}$ at $a = 100$ and use it to approximate $\sqrt{99.8}$.

answer: $L(x) = 10 + 0.05(x - 100)$. Therefore $\sqrt{99.8} \approx L(99.8) = 9.99$.

(b) Find the linearization of $f(x) = \sqrt{8 + x}$ at $a = 1$ and use it to approximate $\sqrt{9.02}$.

answer: $f(x) \approx 3 + \frac{1}{6}(x - 1) = \frac{1}{6}x + \frac{17}{6}$. Therefore $\sqrt{9.02} \approx \frac{901}{300} \approx 3.003333$

(c) Find the linearization of $f(x) = \sqrt[3]{8 + x}$ at $a = 0$ and use it to approximate $\sqrt[3]{7.97}$.

answer: $\sqrt[3]{8 + x} \approx \frac{1}{12}x + 2$. Therefore $\sqrt[3]{7.97} \simeq \frac{799}{400} = 1.9975$

(d) Find the linearization of $f(x) = \ln x$ at $a = 1$ and use it to approximate $\ln 1.01$.

answer: $f(x) \approx f(1) + f'(1)(x - 1) = x - 1, \ln 1.01 \approx 0.01.$

(e) Use a linear approximation to estimate $(1.001)^9$.

answer: $(1.001)^9 \approx 1.009.$

(f) Use a linear approximation to estimate $(0.9999)^{2014}$.

answer: $(0.9999)^{2014} \approx 0.7986.$

Solution. 1.f Let $f(x) = x^{2014}$. We are looking to approximate $(0.9999)^{2014} = f(0.9999)$. As $f(1) = 1^{2014} = 1$ is easy to compute, is makes sense to use linear approximation at $a = 1$ to approximate $(0.9999)^{2014}$. We have that

$$f'(x) = 2014x^{2013} \quad .$$

Therefore the linear approximation of $f(x) = x^{2014}$ at $a = 1$ is:

$$f(x) \approx f(1) + f'(1)(x - 1) = 1^{2014} + 2014 \cdot 1^{2013}(x - 1) = 1 + 2014(x - 1) = 2014x - 2013 \quad .$$

Therefore

$$f(0.9999) \approx 2014 \cdot 0.9999 - 2013 = 1 \cdot 0.9999 + 2013(0.9999 - 1) = 0.9999 - 2013 \cdot 0.0001 = 0.9999 - 0.2013 = 0.7986$$

A computation with computer shows that $0.999^{2014} = 0.817577\dots$. While our approximation of 0.7986 is less than perfect, it is within the same order of magnitude. We study techniques for estimating errors in linear approximations later.