Calculus I
Lecture 5
Limits Involving Infinity

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https://github.com/tmilev/freecalc

2020

Outline

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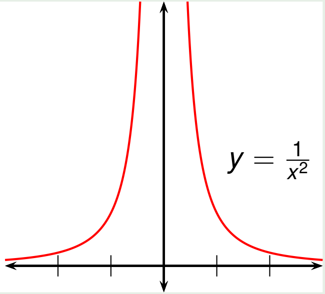
Infinite Limits

Example

Find $\lim_{x\to 0} \frac{1}{x^2}$ if it exists.



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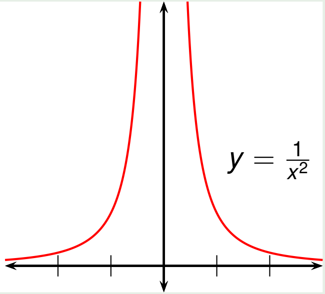

x	$\frac{1}{x^2}$
± 1	1
± 0.5	4
± 0.2	25
± 0.1	100
± 0.05	400
± 0.01	$10,000$
± 0.001	$1,000,000$

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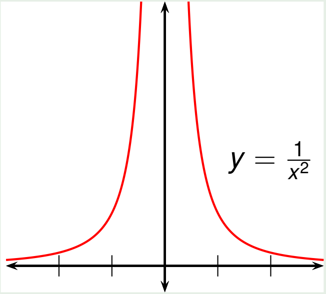

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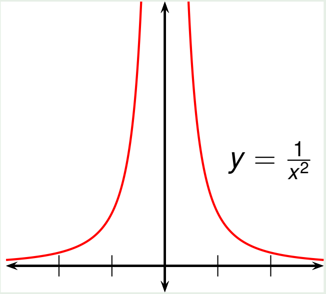

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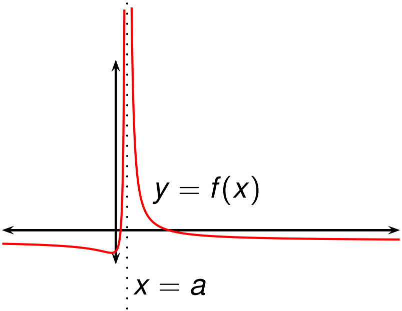
- As x gets close to 0, so does x^2, so $\frac{1}{x^2}$ gets large.
- $\frac{1}{x^2}$ can be made arbitrarily large by taking x close enough to 0.
- $f(x)$ doesn't approach a number, so $\lim_{x\to 0} \frac{1}{x^2}$ doesn't exist.

Definition (Infinite Limit)

Let f be a function defined on both sides of a, except perhaps at a. Then

$$\lim_{x \to a} f(x) = \infty$$

means the values of $f(x)$ can be made arbitrarily large by taking x sufficiently close to a, but not equal to a.



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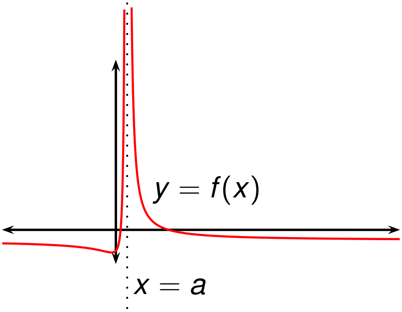

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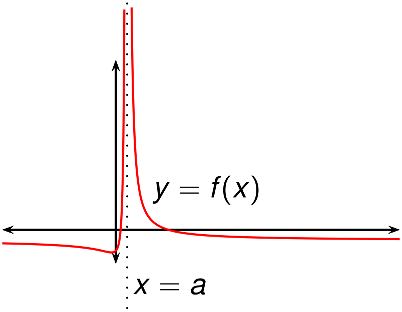

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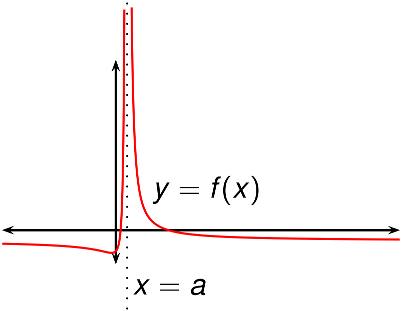

- Other notation: $f(x) \to \infty$ as $x \to a$.
- In such cases, the limit does not exist.
- ∞ is not a number. The notation $\lim_{x \to a} f(x) = \infty$ expresses the particular way in which the limit doesn't exist.

Definition (Infinite Limit)

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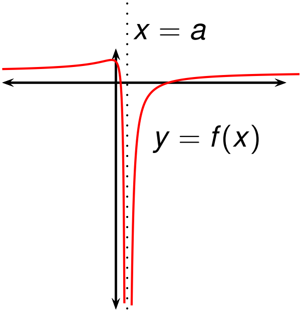

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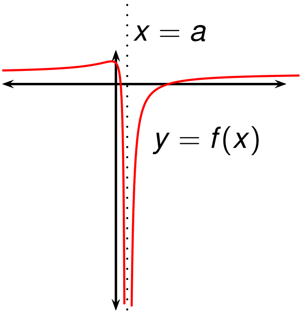

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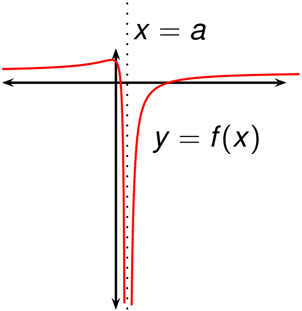

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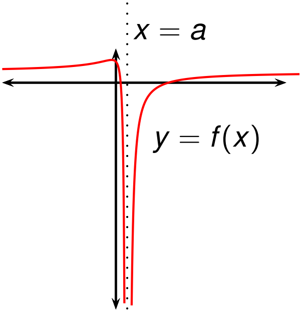
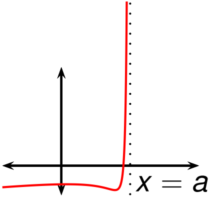
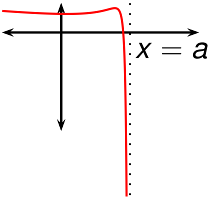


- Here, by "arbitrarily negative" we mean the number is negative with large absolute value.
- In such cases, the limit does not exist.
- $-\infty$ is not a number. The notation $\lim_{x \to a} f(x) = -\infty$ expresses the particular way in which the limit doesn't exist.

There are similar definitions for one-sided limits:

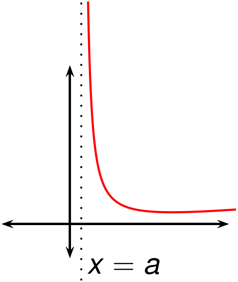


$$\lim_{x \to a^-} f(x) = \infty$$

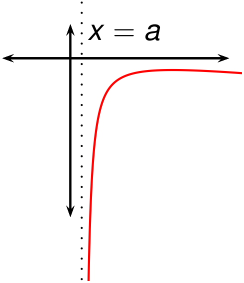


$$\lim_{x \to a^-} f(x) = -\infty$$

$x \to a^-$ means we only consider $x < a$.



$$\lim_{x \to a^+} f(x) = \infty$$



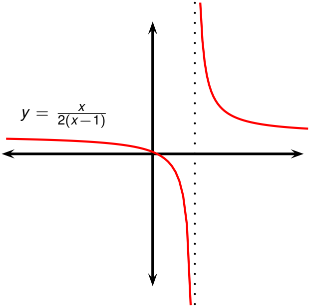
$$\lim_{x \to a^+} f(x) = -\infty$$

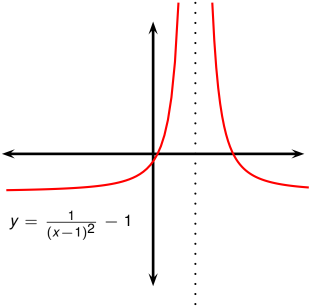
$x \to a^+$ means we only consider $x > a$.

Definition (Vertical Asymptote)

The line $x = a$ is called a vertical asymptote of the curve $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x\to a} f(x) = \infty \qquad \lim_{x\to a^-} f(x) = \infty \qquad \lim_{x\to a^+} f(x) = \infty$$
$$\lim_{x\to a} f(x) = -\infty \qquad \lim_{x\to a^-} f(x) = -\infty \qquad \lim_{x\to a^+} f(x) = -\infty$$





Example

Find $\lim\limits_{x\to 3^+} \frac{2x}{x-3}$ and $\lim\limits_{x\to 3^-} \frac{2x}{x-3}$.

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- If x is near 3 but larger than 3, the denominator $x - 3$ is a small positive number and $2x$ is close to 6.

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- If x is near 3 but smaller than 3, the denominator $x-3$ is a negative number with small absolute value and $2x$ is close to 6.

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- If x is near 3 but smaller than 3, the denominator $x-3$ is a negative number with small absolute value and $2x$ is close to 6.
- So $\frac{2x}{x-3}$ is a negative number with large absolute value.

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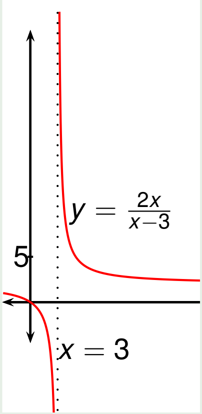

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- So $\frac{2x}{x-3}$ is a negative number with large absolute value.
- $x = 3$ is a vertical asymptote for $f(x) = \frac{2x}{x-3}$.

$$\lim_{x \to a} f(x)$$

If we plug in a and get

$$f(a) = \frac{\text{something different from } 0}{0},$$

then the limit will be DNE, ∞, or $-\infty$.
To determine what the answer is, this is what we do:

1. Factor.
2. Determine if each factor is positive or negative.
3. An odd number of negative factors means the limit is $-\infty$.
4. An even number of negative factors means the limit is ∞.
5. For a two-sided limit, the answer is DNE unless the left limit and the right limit are either both ∞ or both $-\infty$.

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Find $$\lim_{x\to 1^+} \frac{x^2-3x}{x^2-3x+2}$$

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Plug in 1: $\displaystyle \frac{(1)^2-3(1)}{(1)^2-3(1)+2} = \frac{\mathbf{?}}{\mathbf{?}}$

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$$\text{Plug in 1:} \quad \frac{(1)^2-3(1)}{(1)^2-3(1)+2} = \frac{-2}{0}$$

The numerator is non-zero and the denominator is zero. Therefore the answer is DNE, ∞, or $-\infty$.

$$\text{Factor:} \quad \lim_{x\to 1^+} \frac{x^2-3x}{x^2-3x+2} = \lim_{x\to 1^+} \frac{x(x-3)}{(x-2)(x-1)}$$

$$\to \frac{(+)(-)}{(-)(+)}$$

$$= \textbf{?}$$

Example (Infinite Limit)

$$\text{Find} \quad \lim_{x\to 1^+} \frac{x^2-3x}{x^2-3x+2}$$

$$\text{Plug in 1:} \quad \frac{(1)^2-3(1)}{(1)^2-3(1)+2} = \frac{-2}{0}$$

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Example (Infinite Limit)

Find $$\lim_{x\to 1^+} \frac{x^2-3x}{x^2-3x+2}$$

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Factor: $$\begin{aligned}\lim_{x\to 1^+} \frac{x^2-3x}{x^2-3x+2} &= \lim_{x\to 1^+} \frac{x(x-3)}{(x-2)(x-1)} \\ &\to \frac{(+)(-)}{(-)(+)} \\ &= (+)\end{aligned}$$

Therefore $$\lim_{x\to 1^+} \frac{x^2-3x}{x^2-3x+2} = +\infty.$$

Example (Infinite Limit)

Find $\displaystyle \lim_{x \to -1} \frac{x^2 + 5x + 6}{x^3 + 2x^2 + x}$

Example (Infinite Limit)

$$\text{Find} \quad \lim_{x\to -1} \frac{x^2+5x+6}{x^3+2x^2+x}$$

Plug in -1: $$\frac{(-1)^2+5(-1)+6}{(-1)^3+2(-1)^2+(-1)} = \frac{\textbf{?}}{\textbf{?}}$$

Example (Infinite Limit)

$$\text{Find} \quad \lim_{x\to -1} \frac{x^2+5x+6}{x^3+2x^2+x}$$

$$\text{Plug in } -1\text{:} \quad \frac{(-1)^2+5(-1)+6}{(-1)^3+2(-1)^2+(-1)} = \frac{2}{\textbf{?}}$$

Example (Infinite Limit)

Find $$\lim_{x \to -1} \frac{x^2 + 5x + 6}{x^3 + 2x^2 + x}$$

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Example (Infinite Limit)

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The numerator is non-zero and the denominator is zero. Therefore the answer is DNE, ∞, or $-\infty$.

$$\text{Factor:} \quad \lim_{x \to -1} \frac{x^2 + 5x + 6}{x^3 + 2x^2 + x} = \lim_{x \to -1} \frac{(x+2)(x+3)}{\mathbf{?}}$$

Example (Infinite Limit)

$$\text{Find} \quad \lim_{x\to -1} \frac{x^2+5x+6}{x^3+2x^2+x}$$

$$\text{Plug in } -1\text{:} \quad \frac{(-1)^2+5(-1)+6}{(-1)^3+2(-1)^2+(-1)} = \frac{2}{0}$$

The numerator is non-zero and the denominator is zero. Therefore the answer is DNE, ∞, or $-\infty$.

$$\text{Factor:} \quad \lim_{x\to -1} \frac{x^2+5x+6}{\color{red}{x^3+2x^2+x}} = \lim_{x\to -1} \frac{(x+2)(x+3)}{\color{red}{?}}$$

Example (Infinite Limit)

$$\text{Find} \quad \lim_{x\to -1} \frac{x^2+5x+6}{x^3+2x^2+x}$$

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$$\text{Factor:} \quad \lim_{x\to -1} \frac{x^2+5x+6}{x^3+2x^2+x} = \lim_{x\to -1} \frac{(x+2)(x+3)}{x(x+1)^2}$$

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$$\to \frac{(\mathbf{?}\quad)(\mathbf{?}\quad)}{(\mathbf{?}\quad)(\mathbf{?}\quad)}$$

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$$\to \frac{(\ +\ ){\color{red}(\mathbf{?}\ \ )}}{(\mathbf{?}\ \ )(\mathbf{?}\ \ )}$$

Example (Infinite Limit)

$$\text{Find} \quad \lim_{x\to -1} \frac{x^2+5x+6}{x^3+2x^2+x}$$

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Plug in -1: $\displaystyle\frac{(-1)^2+5(-1)+6}{(-1)^3+2(-1)^2+(-1)} = \frac{2}{0}$

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$$\text{Find} \quad \lim_{x\to -1} \frac{x^2+5x+6}{x^3+2x^2+x}$$

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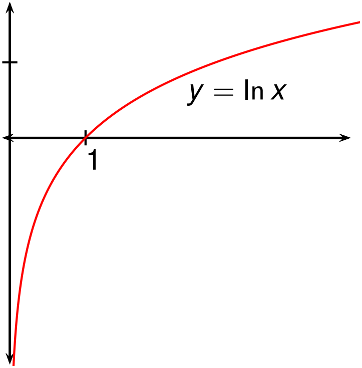
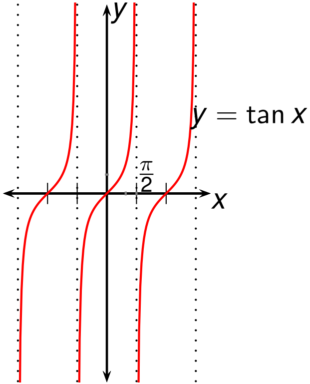
$$\begin{aligned}\text{Factor:} \quad \lim_{x\to -1} \frac{x^2+5x+6}{x^3+2x^2+x} &= \lim_{x\to -1} \frac{(x+2)(x+3)}{x(x+1)^2} \\ &\to \frac{(\ +\ )(\ +\ )}{(\ -\ )(\ +\ )} \\ &= (-)\end{aligned}$$

$$\text{Therefore} \quad \lim_{x\to -1} \frac{x^2+5x+6}{x^3+2x^2+x} = -\infty.$$





$$\lim_{x\to 0^+} \ln x =$$

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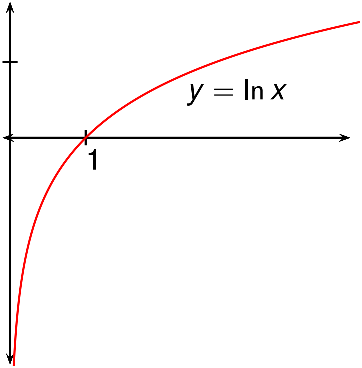

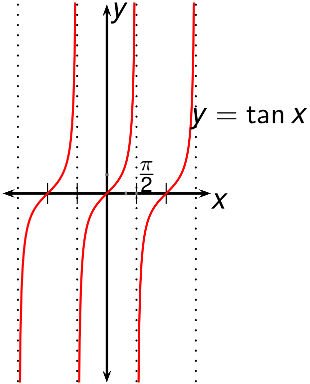

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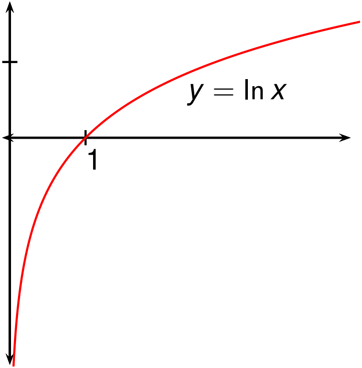

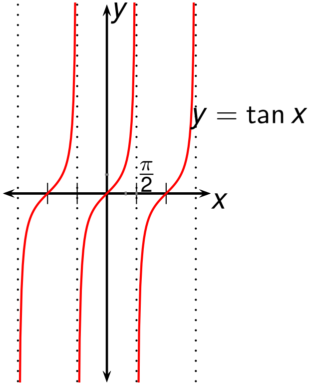

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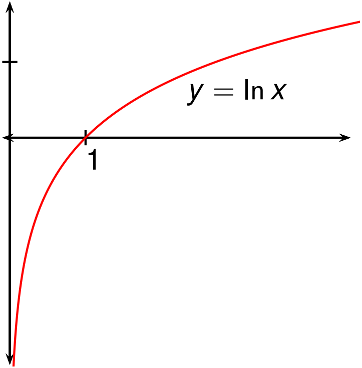

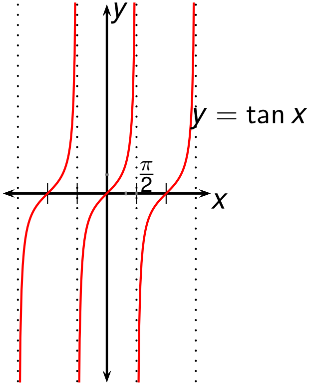

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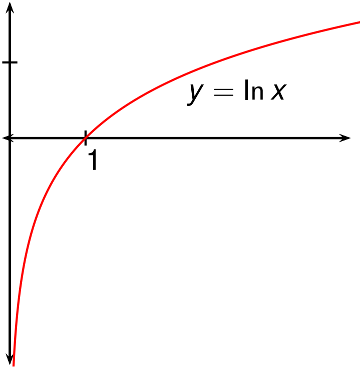

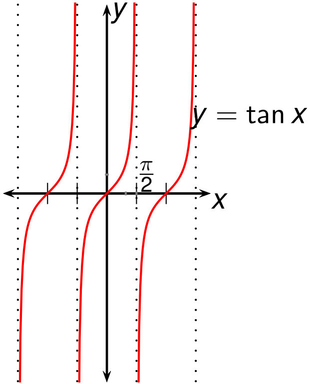

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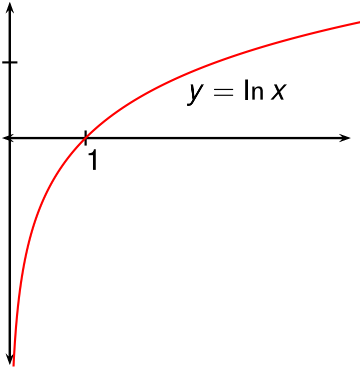

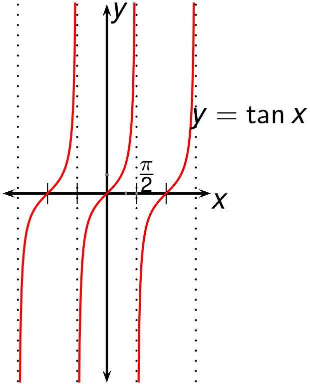

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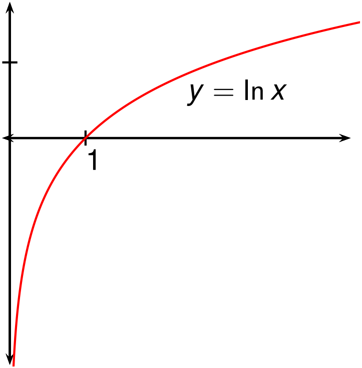

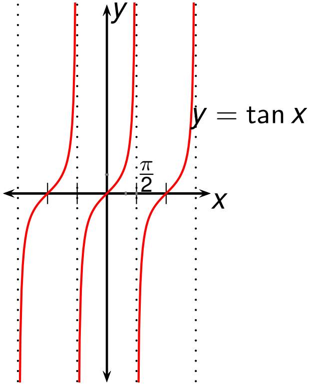

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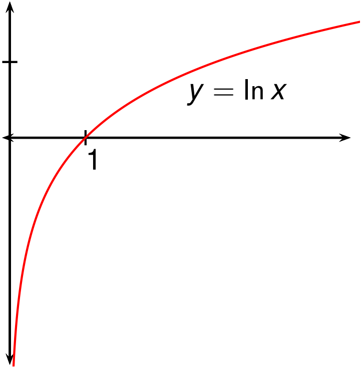

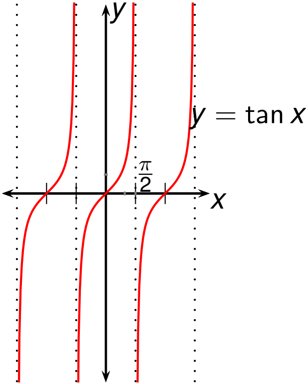

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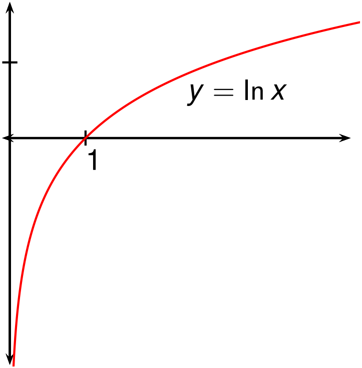

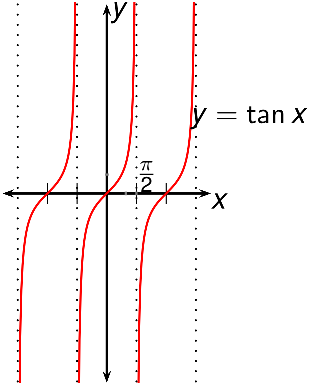

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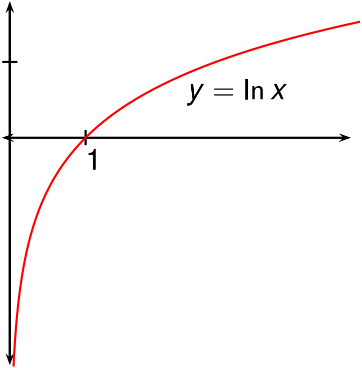

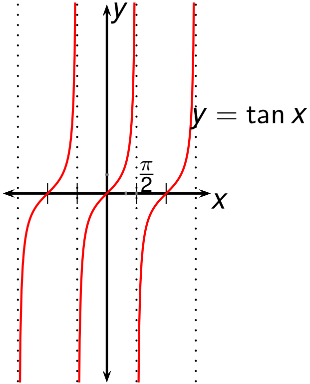

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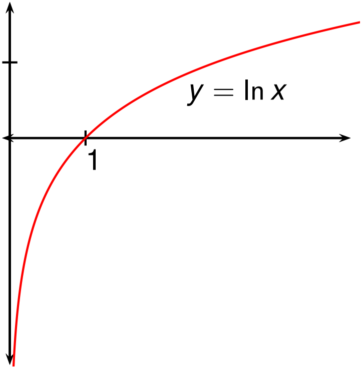

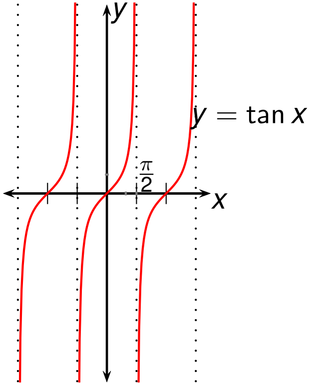

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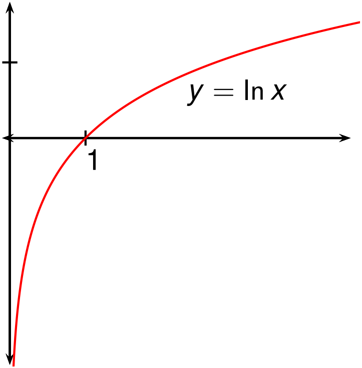

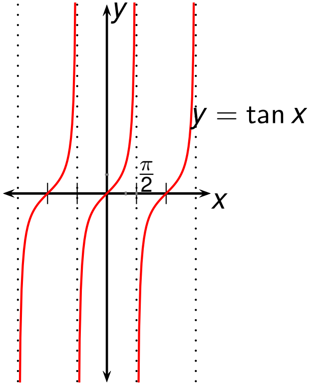

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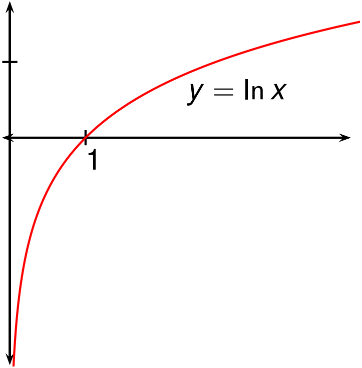
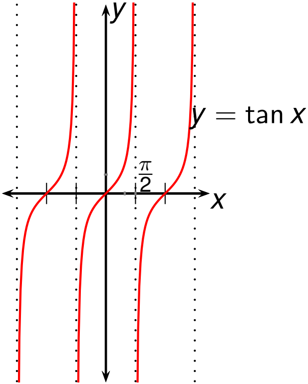
$$\lim_{x \to \frac{\pi}{2}} \tan x = \textbf{?}$$





$$\lim_{x \to 0^+} \ln x = -\infty$$

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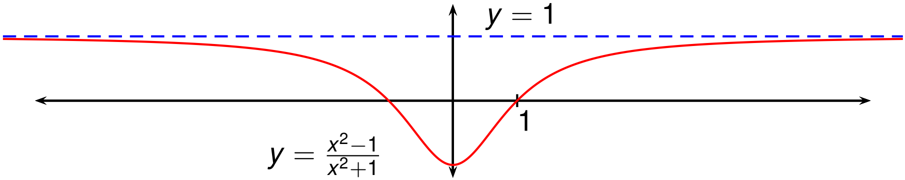
$$\lim_{x \to \frac{\pi}{2}} \tan x = \text{DNE}$$

Limits at Infinity; Horizontal Asymptotes



- Consider $f(x) = \frac{x^2-1}{x^2+1}$ as x becomes large.

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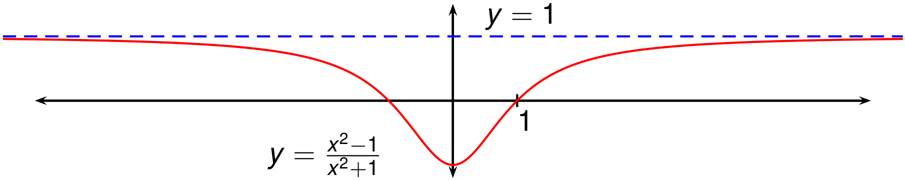

x	$f(x)$
0	-1
± 1	0
± 2	0.600000
± 3	0.800000
± 4	0.882353
± 5	0.923077
± 10	0.980198

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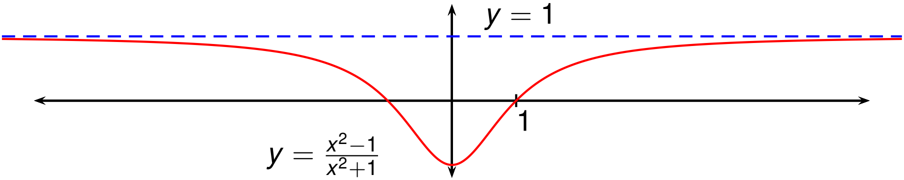
- The values of $f(x)$ get closer and closer to 1.

Limits at Infinity; Horizontal Asymptotes



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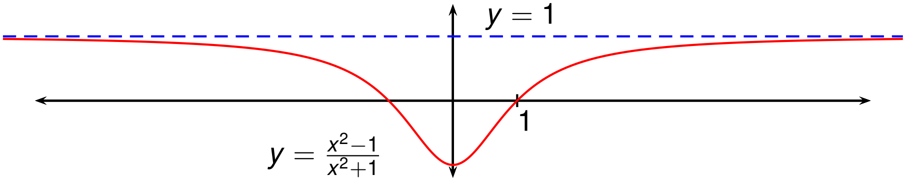

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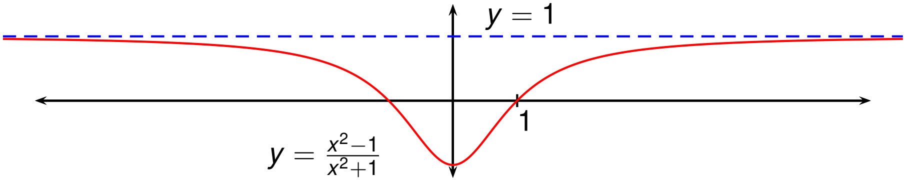

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Definition (Limit at Infinity)

Let f be a function defined on some interval (a, ∞). Then

$$\lim_{x \to \infty} f(x) = L$$

means that the values of f can be made arbitrarily close to L by taking x sufficiently large.

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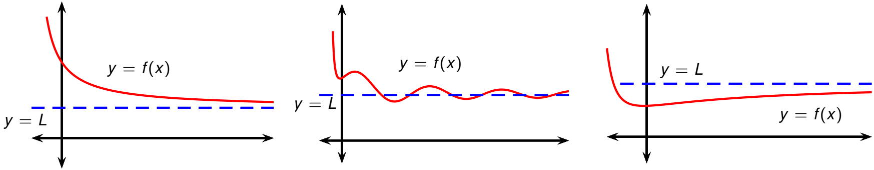

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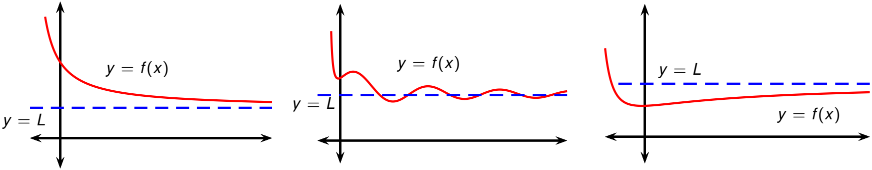

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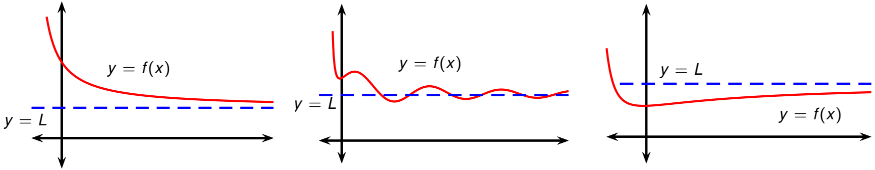

- There are many ways that this can happen.
- Other notation: $f(x) \to L$ as $x \to \infty$.
- ∞ is not a number.

Definition (Limit at Minus Infinity)

Let f be a function defined on some interval $(-\infty, b)$. Then

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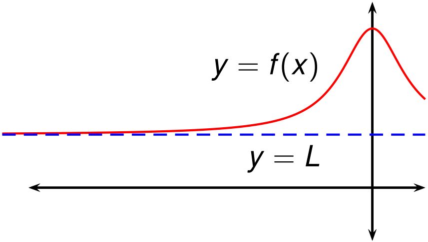
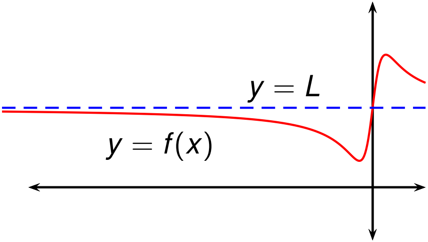
means that the values of f can be made arbitrarily close to L by taking x sufficiently negative.





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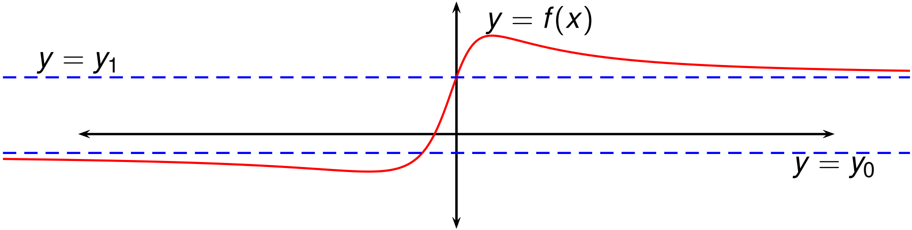
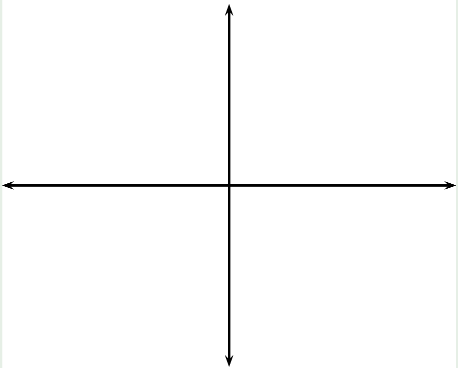
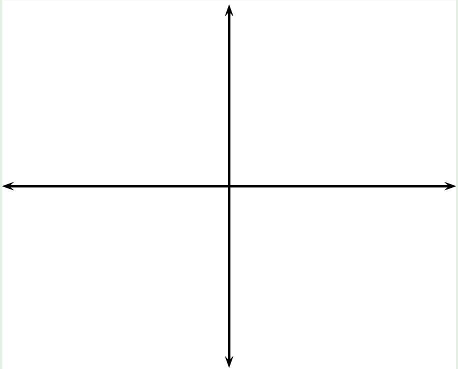


Example



Find $\lim\limits_{x\to\infty} \frac{1}{x}$ and $\lim\limits_{x\to-\infty} \frac{1}{x}$.

Example



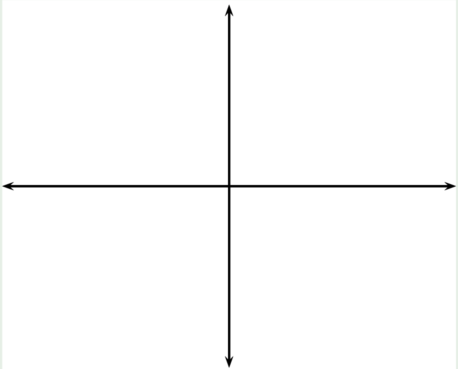
Find $\lim_{x\to\infty} \frac{1}{x}$ and $\lim_{x\to-\infty} \frac{1}{x}$.

- When x is large, $\frac{1}{x}$ is small.

$$\frac{1}{100} = 0.01, \qquad \frac{1}{10,000} = 0.0001$$

$$\frac{1}{1,000,000} = 0.000001$$

Example



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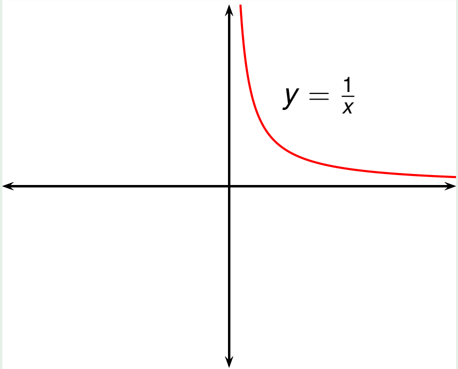

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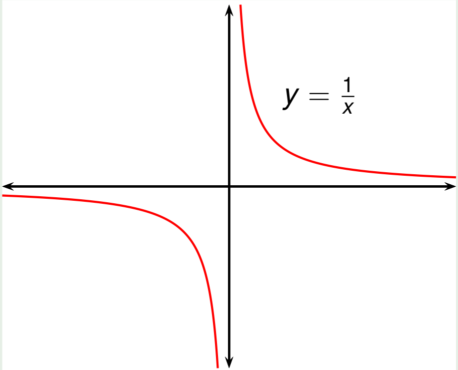

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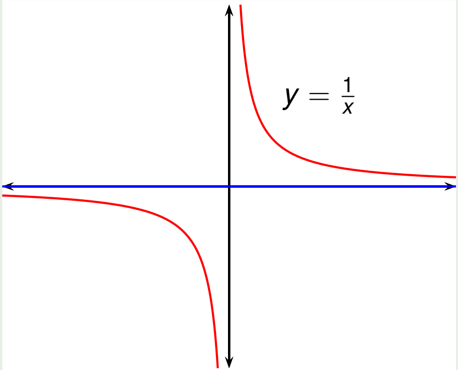

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- Therefore $\lim_{x\to\infty} \frac{1}{x} = 0$.
- Similarly, $\lim_{x\to-\infty} \frac{1}{x} = 0$.
- $y = 0$ (the x-axis) is a horizontal asymptote for the curve $y = \frac{1}{x}$.

We can generalize the previous example to other powers of x:

Theorem (Infinite Limits of $\frac{1}{x^r}$)

If $r > 0$ is a rational number, then

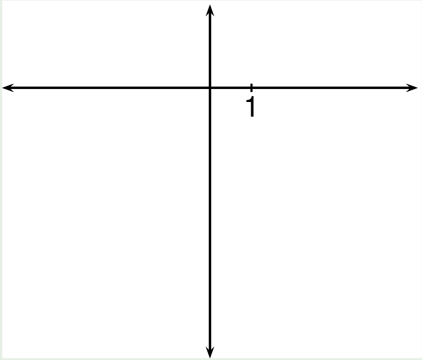
$$\lim_{x\to\infty} \frac{1}{x^r} = 0.$$

If $r > 0$ is a rational number such that x^r is defined for all x, then

$$\lim_{x\to-\infty} \frac{1}{x^r} = 0.$$

Example

Evaluate $\lim\limits_{x\to\infty} \frac{3x^2-x-2}{5x^2+4x+1}$.



$$\lim_{x\to\infty} \frac{(3x^2 - x - 2)}{(5x^2 + 4x + 1)}$$

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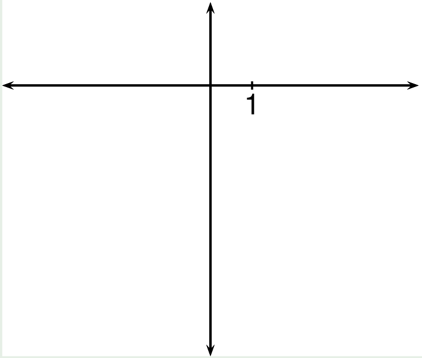

Standard approach: divide top and bottom by the highest power of x in the denominator.

$$\lim_{x\to\infty} \frac{\left(3x^2 - x - 2\right)}{\left(5x^2 + 4x + 1\right)}$$

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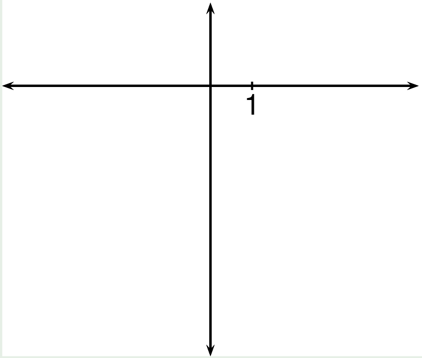

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$$\lim_{x\to\infty} \frac{(3x^2-x-2)}{(5x^2+4x+1)} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

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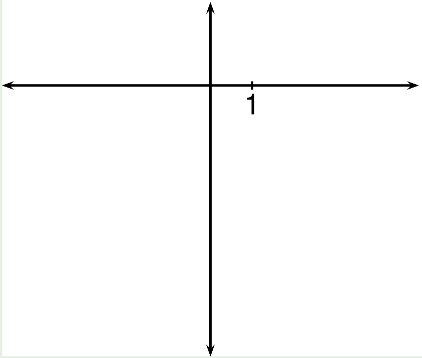

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$$\lim_{x\to\infty} \frac{(3x^2-x-2)}{(5x^2+4x+1)} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

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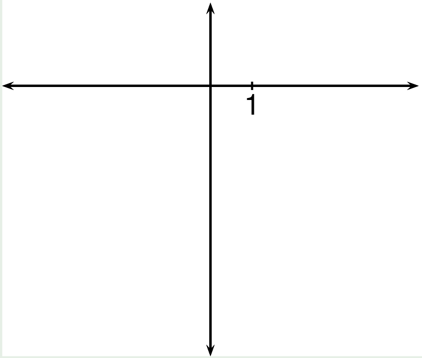

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$$= \lim_{x\to\infty} \frac{3 - \frac{1}{x} - \frac{2}{x^2}}{}$$

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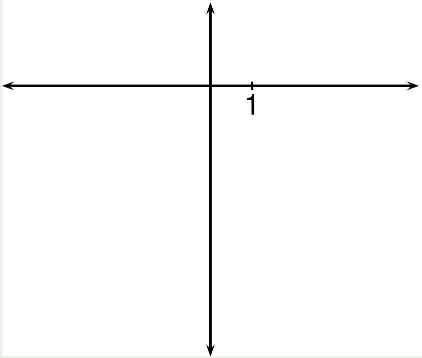

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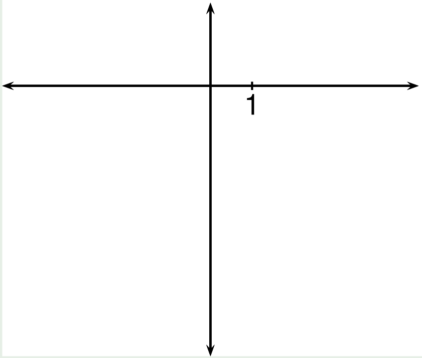

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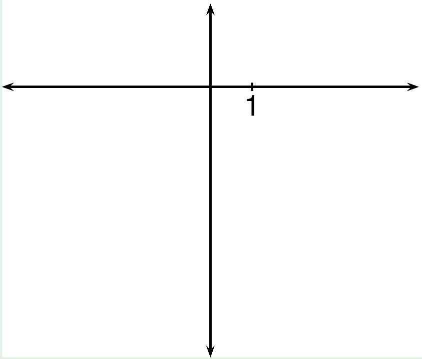

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$$
\begin{aligned}
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= \quad & \frac{\lim\limits_{x\to\infty} 3 - \lim\limits_{x\to\infty} \dfrac{1}{x} - 2\lim\limits_{x\to\infty} \dfrac{1}{x^2}}{\lim\limits_{x\to\infty} 5 + 4\lim\limits_{x\to\infty} \dfrac{1}{x} + \lim\limits_{x\to\infty} \dfrac{1}{x^2}}
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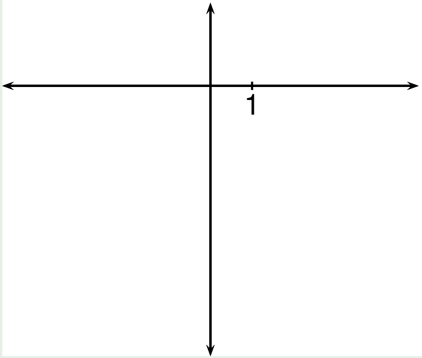

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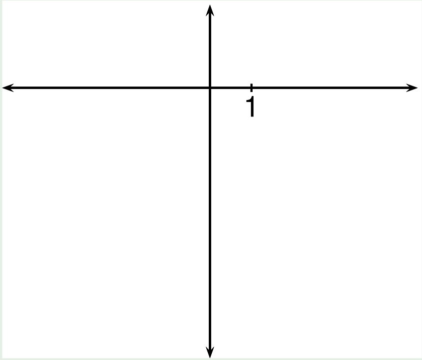

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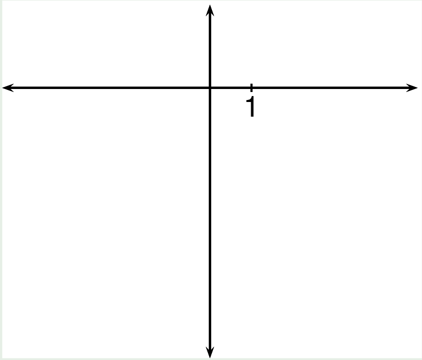

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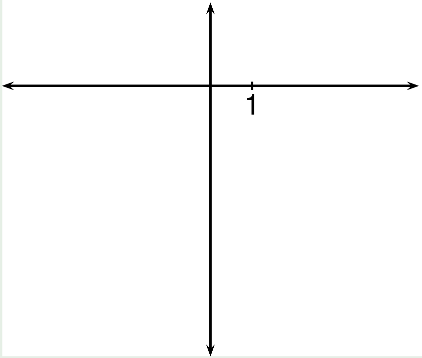

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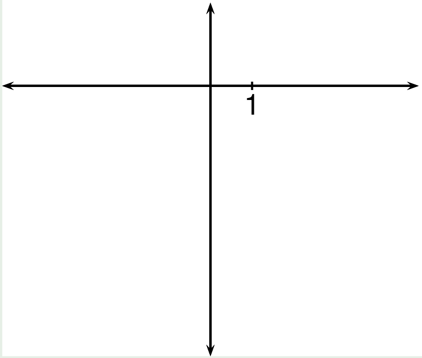
$$= \frac{\lim\limits_{x\to\infty} 3 - \lim\limits_{x\to\infty} \frac{1}{x} - 2 \lim\limits_{x\to\infty} \frac{1}{x^2}}{\lim\limits_{x\to\infty} 5 + 4 \lim\limits_{x\to\infty} \frac{1}{x} + \lim\limits_{x\to\infty} \frac{1}{x^2}}$$

$$= \frac{3 - 0 - \mathbf{?}}{\mathbf{?} + \mathbf{?} + \mathbf{?}}$$

Example

Evaluate $\lim\limits_{x\to\infty} \frac{3x^2-x-2}{5x^2+4x+1}$.



Standard approach: divide top and bottom by the highest power of x in the denominator.

$$\begin{aligned}
& \lim_{x\to\infty} \frac{(3x^2-x-2)}{(5x^2+4x+1)} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \\
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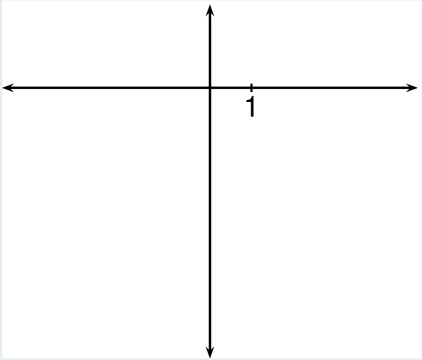

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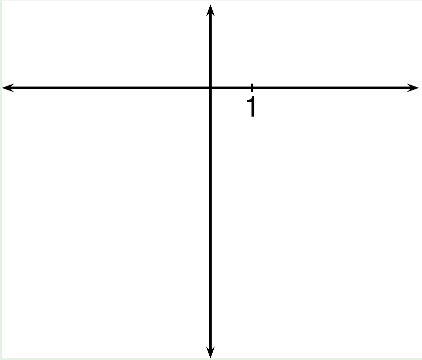

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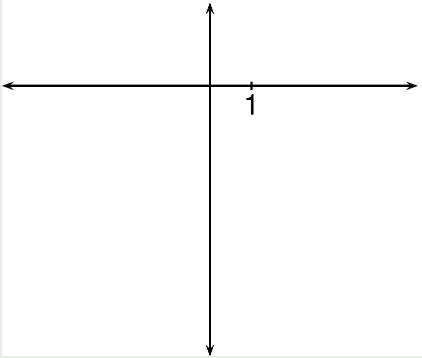

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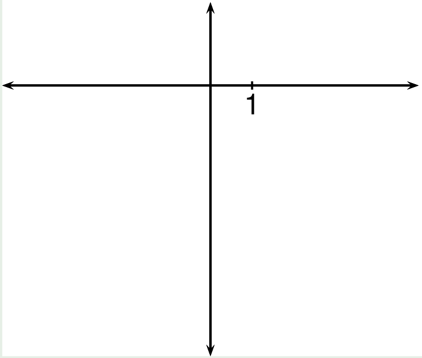

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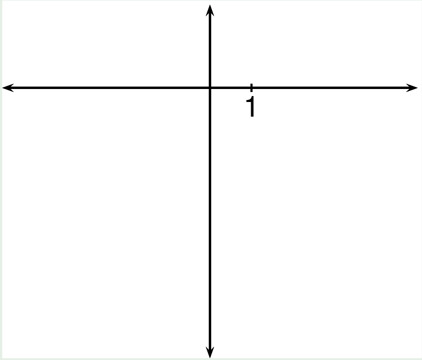

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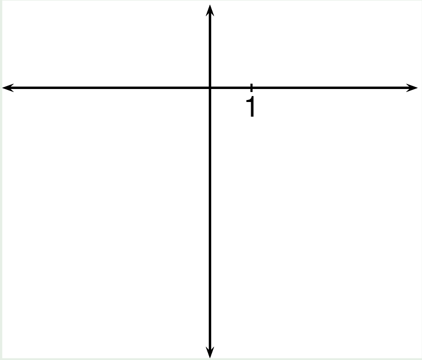

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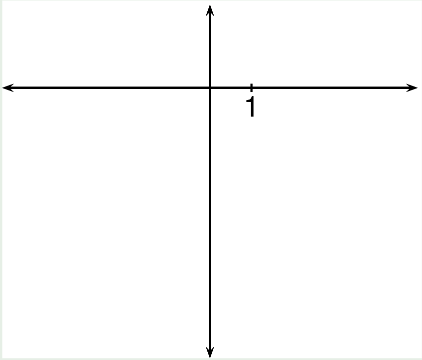

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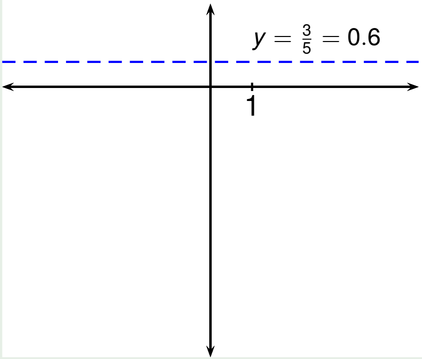

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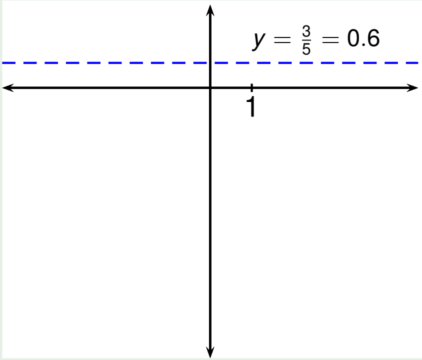

A similar calculation shows that the limit as $x \to -\infty$ is also $\frac{3}{5}$.

Standard approach: divide top and bottom by the highest power of x in the denominator.

$$
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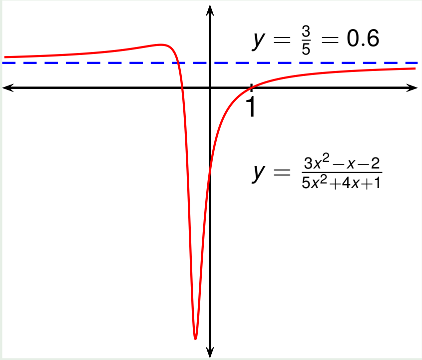
Example

Evaluate $\lim\limits_{x\to\infty} \frac{3x^2-x-2}{5x^2+4x+1}$.



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= & \frac{\lim\limits_{x\to\infty} 3 - \lim\limits_{x\to\infty} \frac{1}{x} - 2\lim\limits_{x\to\infty} \frac{1}{x^2}}{\lim\limits_{x\to\infty} 5 + 4\lim\limits_{x\to\infty} \frac{1}{x} + \lim\limits_{x\to\infty} \frac{1}{x^2}} \\
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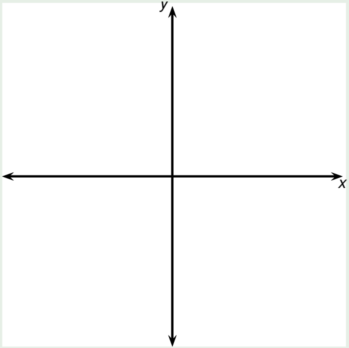
\end{aligned}$$

Example

Find the horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{3x^2+1}}{2x-3}$.



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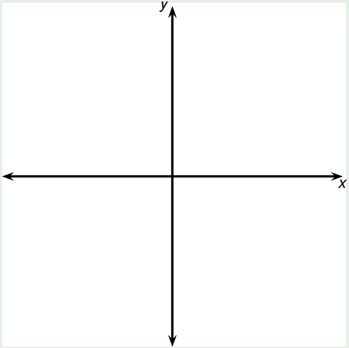
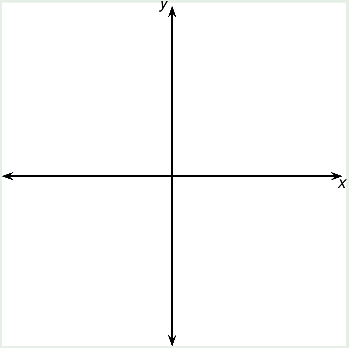


$$\lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-3}$$

$$\lim_{x\to-\infty} \frac{\sqrt{3x^2+1}}{2x-3}$$

Example

Find the horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{3x^2+1}}{2x-3}$.



$$\lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$\lim_{x\to-\infty} \frac{\sqrt{3x^2+1}}{2x-3}$$

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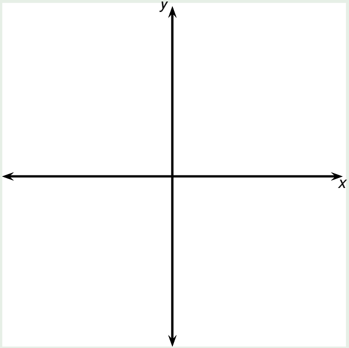

$$\lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{x}}$$

If $x > 0$ then $x = \sqrt{x^2}$.

$$\lim_{x\to-\infty} \frac{\sqrt{3x^2+1}}{2x-3}$$

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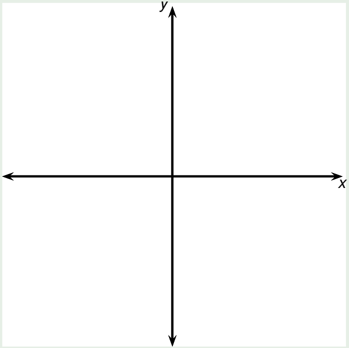

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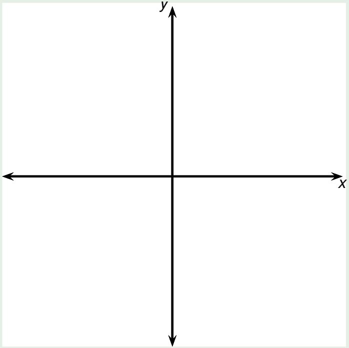
$$= \lim_{x\to\infty} \frac{\sqrt{\textbf{?}}}{\textbf{?}}$$

If $x > 0$ then $x = \sqrt{x^2}$.

$$\lim_{x\to-\infty} \frac{\sqrt{3x^2+1}}{2x-3}$$

Example

Find the horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{3x^2+1}}{2x-3}$.



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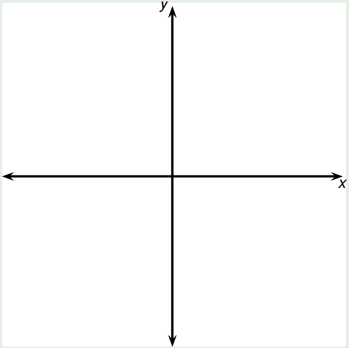
$$= \lim_{x\to\infty} \frac{\sqrt{3+\frac{1}{x^2}}}{\textbf{?}}$$

If $x > 0$ then $x = \sqrt{x^2}$.

$$\lim_{x\to-\infty} \frac{\sqrt{3x^2+1}}{2x-3}$$

Example

Find the horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{3x^2+1}}{2x-3}$.



$$\lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{3x^2+1}}{2x-3} \cdot \frac{\frac{1}{\sqrt{x^2}}}{\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{3+\frac{1}{x^2}}}{?}$$

If $x > 0$ then $x = \sqrt{x^2}$.

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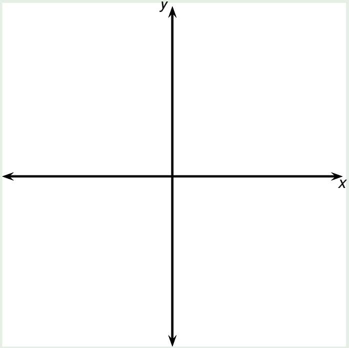

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$$= \lim_{x\to\infty} \frac{\sqrt{3+\frac{1}{x^2}}}{2-\frac{3}{x}}$$

If $x > 0$ then $x = \sqrt{x^2}$.

$$\lim_{x\to-\infty} \frac{\sqrt{3x^2+1}}{2x-3}$$

Example

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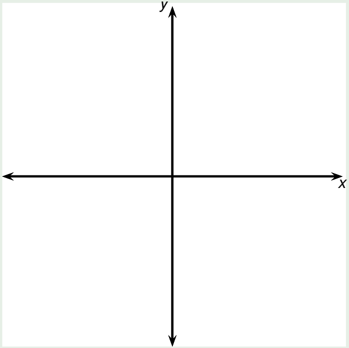

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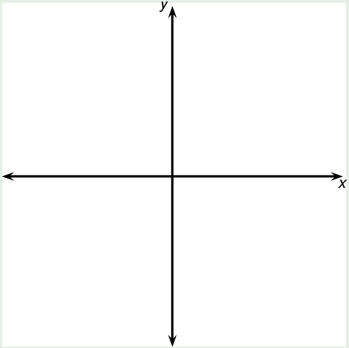
If $x > 0$ then $x = \sqrt{x^2}$.

$$\lim_{x\to-\infty} \frac{\sqrt{3x^2+1}}{2x-3}$$

Example

Find the horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{3x^2+1}}{2x-3}$.



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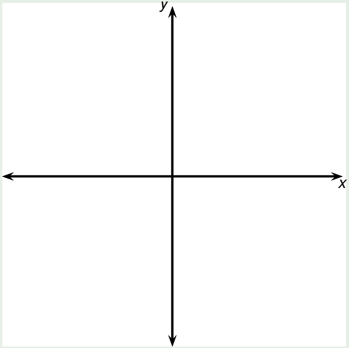
$$= \frac{\sqrt{\mathbf{?} + \mathbf{?}}}{\mathbf{?} - \mathbf{?}}$$

$$\lim_{x\to-\infty} \frac{\sqrt{3x^2+1}}{2x-3}$$

Example

Find the horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{3x^2+1}}{2x-3}$.



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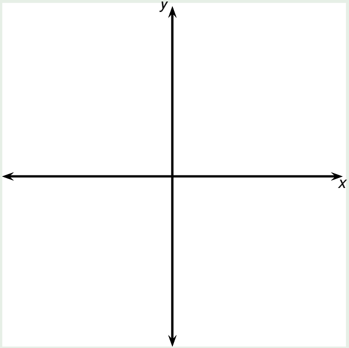
If $x > 0$ then $x = \sqrt{x^2}$.

$$\lim_{x\to-\infty} \frac{\sqrt{3x^2+1}}{2x-3}$$

Example

Find the horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{3x^2+1}}{2x-3}$.



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$$= \frac{\sqrt{3+\mathbf{?}}}{\mathbf{?}-\mathbf{?}}$$

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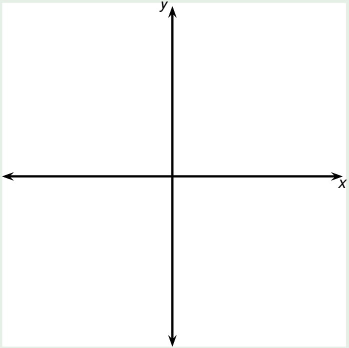

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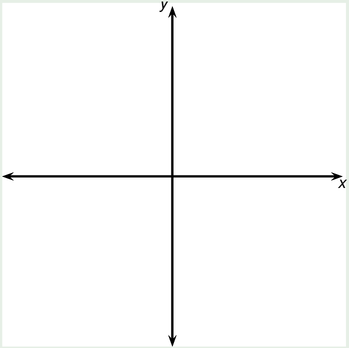

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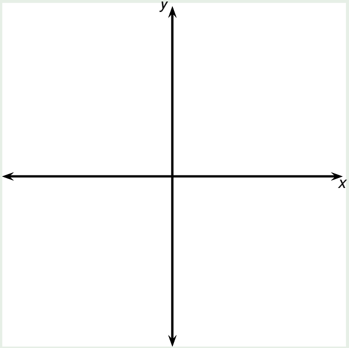

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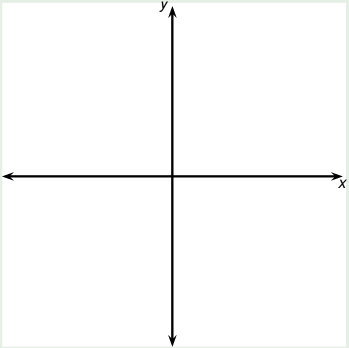

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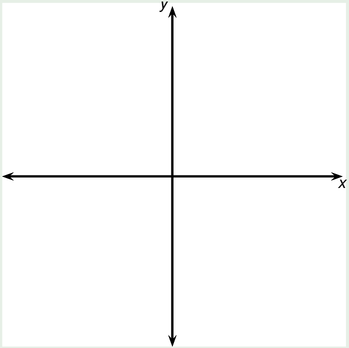

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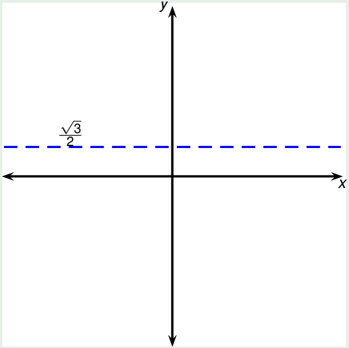

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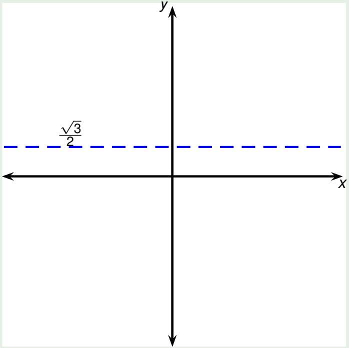

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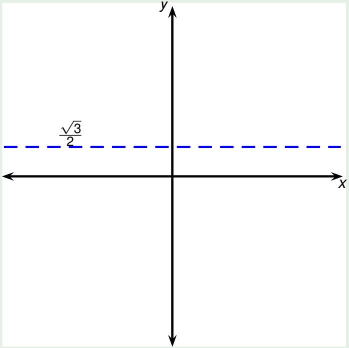

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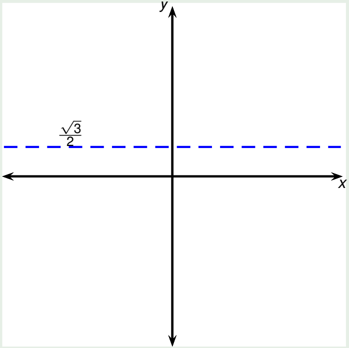

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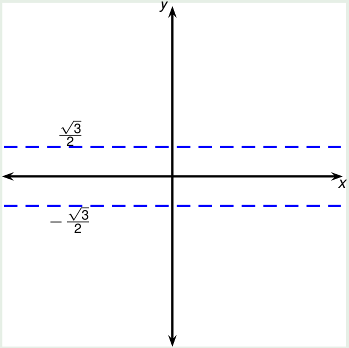

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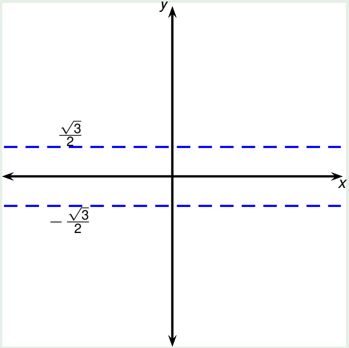

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Vertical Asymptote:

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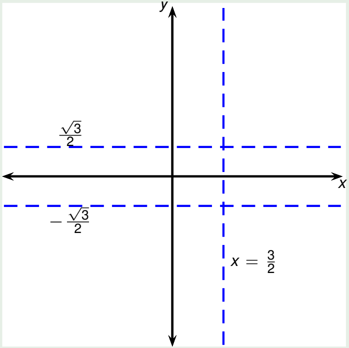

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Vertical Asymptote:
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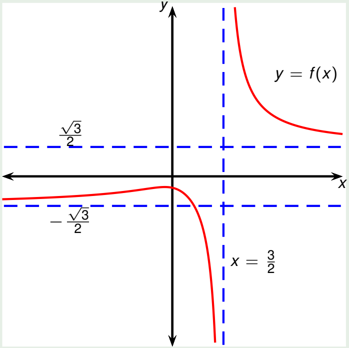

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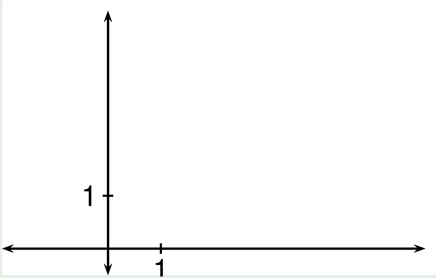
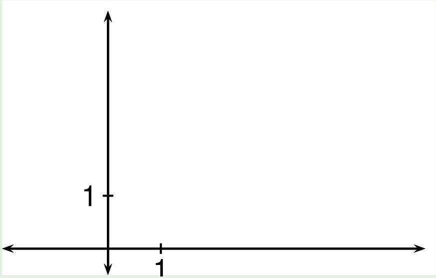
Example

Evaluate $\lim_{x\to\infty} \sqrt{x^2+1} - x$.



Example

Evaluate $\lim_{x\to\infty} \sqrt{x^2+1} - x$.

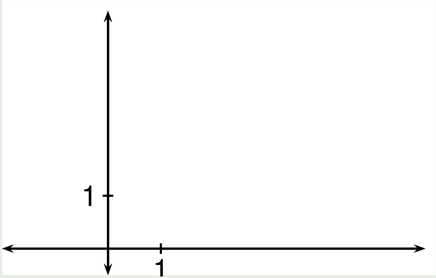


$$\lim_{x\to\infty} \left(\sqrt{x^2+1} - x\right)$$

- $\sqrt{x^2+1} \to \infty$ and $x \to \infty$ as $x \to \infty$.
- It isn't clear what happens to the difference.

Example

Evaluate $\lim_{x\to\infty} \sqrt{x^2+1} - x$.



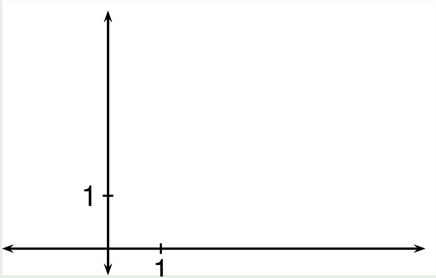
- $\sqrt{x^2+1} \to \infty$ and $x \to \infty$ as $x \to \infty$.
- It isn't clear what happens to the difference.

- Standard approach: multiply top and bottom by $\pm$conjugate radical.

$$\lim_{x\to\infty} \left(\sqrt{x^2+1} - x\right)$$

Example

Evaluate $\lim_{x\to\infty} \sqrt{x^2+1} - x$.



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$$\lim_{x\to\infty} \left(\sqrt{x^2+1} - x\right) \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}$$

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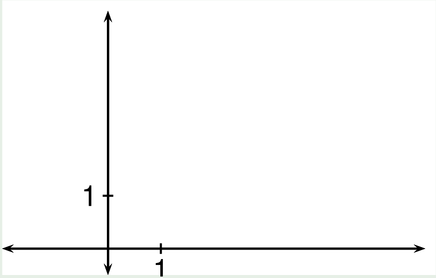

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$$\lim_{x\to\infty} \left(\sqrt{x^2+1} - x\right) \frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1} + x}$$
$$= \lim_{x\to\infty} \frac{(x^2+1) - x^2}{\sqrt{x^2+1} + x}$$

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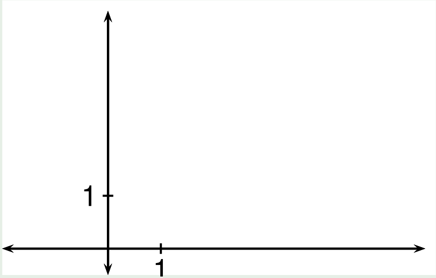
Evaluate $\lim_{x\to\infty} \sqrt{x^2+1} - x$.



- Standard approach: multiply top and bottom by $\pm$conjugate radical.

$$\lim_{x\to\infty} \left(\sqrt{x^2+1} - x\right) \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}$$
$$= \lim_{x\to\infty} \frac{(\cancel{x^2}+1) - \cancel{x^2}}{\sqrt{x^2+1}+x}$$
$$= \lim_{x\to\infty} \frac{1}{\left(\sqrt{x^2+1}+x\right)}$$

- $\sqrt{x^2+1} \to \infty$ and $x \to \infty$ as $x \to \infty$.
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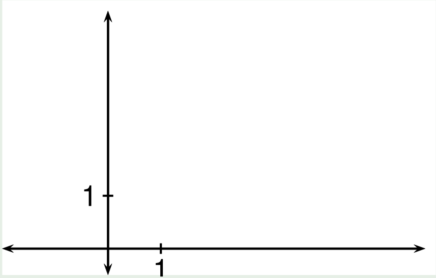
Example

Evaluate $\lim\limits_{x\to\infty} \sqrt{x^2+1} - x$.



- $\sqrt{x^2+1} \to \infty$ and $x \to \infty$ as $x \to \infty$.
- It isn't clear what happens to the difference.
- Divide top & bottom by x.

- Standard approach: multiply top and bottom by $\pm$conjugate radical.

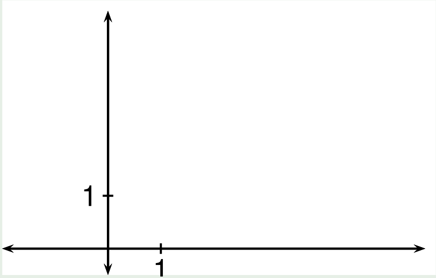
$$\lim_{x\to\infty} \left(\sqrt{x^2+1} - x\right) \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}$$

$$= \lim_{x\to\infty} \frac{(\cancel{x^2}+1) - \cancel{x^2}}{\sqrt{x^2+1}+x}$$

$$= \lim_{x\to\infty} \frac{1}{\left(\sqrt{x^2+1}+x\right)} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

Example

Evaluate $\lim\limits_{x\to\infty} \sqrt{x^2+1} - x$.



- $\sqrt{x^2+1} \to \infty$ and $x \to \infty$ as $x \to \infty$.
- It isn't clear what happens to the difference.
- Divide top & bottom by x.

- Standard approach: multiply top and bottom by $\pm$conjugate radical.

$$
\begin{aligned}
&\lim_{x\to\infty} \left(\sqrt{x^2+1} - x\right) \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}\\
=& \lim_{x\to\infty} \frac{(\cancel{x^2}+1) - \cancel{x^2}}{\sqrt{x^2+1}+x}\\
=& \lim_{x\to\infty} \frac{1}{\left(\sqrt{x^2+1}+x\right)} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\\
=& \lim_{x\to\infty} \frac{\frac{1}{x}}{\sqrt{1+\frac{1}{x^2}}+1}
\end{aligned}
$$

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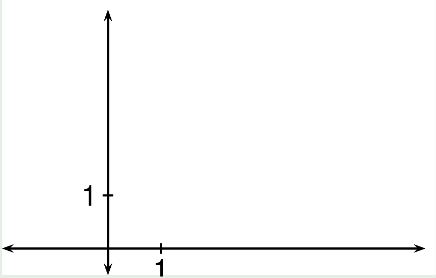

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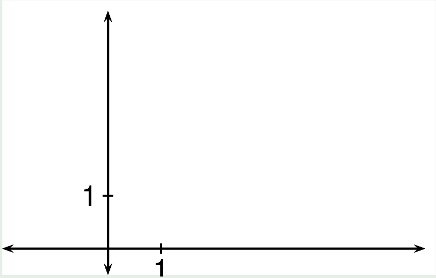

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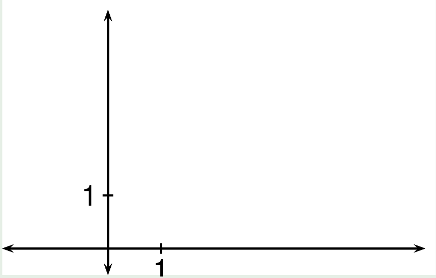
$$\lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right)\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}$$
$$=\lim_{x\to\infty}\frac{(\cancel{x^2}+1)-\cancel{x^2}}{\sqrt{x^2+1}+x}$$
$$=\lim_{x\to\infty}\frac{1}{\left(\sqrt{x^2+1}+x\right)}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}$$
$$=\lim_{x\to\infty}\frac{\frac{1}{x}}{\sqrt{1+\frac{1}{x^2}}+1}$$

Example

Evaluate $\lim\limits_{x\to\infty} \sqrt{x^2+1} - x$.



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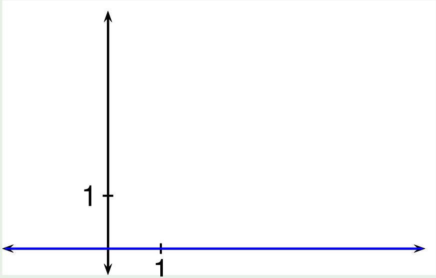
Example

Evaluate $\lim\limits_{x\to\infty} \sqrt{x^2+1} - x$.



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$$\begin{aligned}
&\lim_{x\to\infty} \left(\sqrt{x^2+1} - x\right) \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x} \\
=& \lim_{x\to\infty} \frac{(\cancel{x^2}+1) - \cancel{x^2}}{\sqrt{x^2+1}+x} \\
=& \lim_{x\to\infty} \frac{1}{\left(\sqrt{x^2+1}+x\right)} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \\
=& \lim_{x\to\infty} \frac{\frac{1}{x}}{\sqrt{1+\frac{1}{x^2}}+1} \\
=& \frac{0}{\sqrt{1+0}+1}
\end{aligned}$$

Example

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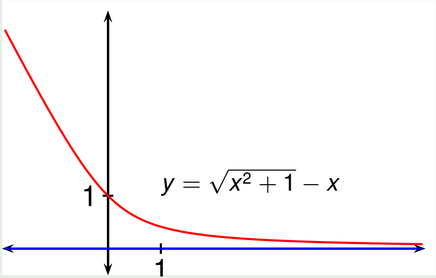

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$$\begin{aligned}
&\lim_{x\to\infty} \left(\sqrt{x^2+1} - x\right) \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}\\
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&= \lim_{x\to\infty} \frac{1}{\left(\sqrt{x^2+1}+x\right)} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}\\
&= \lim_{x\to\infty} \frac{\frac{1}{x}}{\sqrt{1+\frac{1}{x^2}}+1}\\
&= \frac{0}{\sqrt{1+0}+1} = 0
\end{aligned}$$

Infinite Limits at Infinity

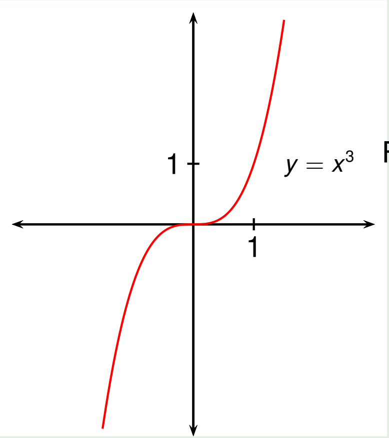
We write

$$\lim_{x\to\infty} f(x) = \infty$$

to mean that $f(x)$ becomes large as x becomes large. We attach similar meaning to

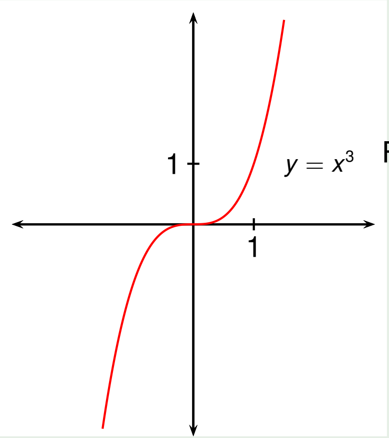
$$\lim_{x\to\infty} f(x) = -\infty, \qquad \lim_{x\to-\infty} f(x) = \infty, \qquad \lim_{x\to-\infty} = -\infty$$

Example



Find $\lim_{x\to\infty} x^3$ and $\lim_{x\to-\infty} x^3$.

Example

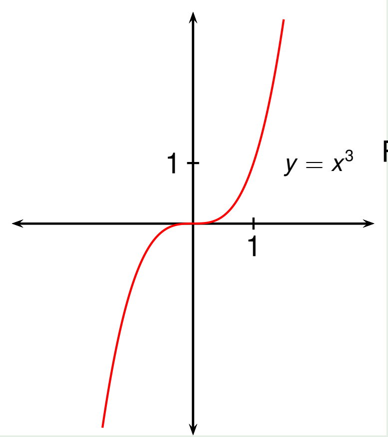


Find $\lim_{x\to\infty} x^3$ and $\lim_{x\to-\infty} x^3$.

- When x is large, so is x^3.

$10^3 = 1000, \qquad 100^3 = 1,000,000,$
$1000^3 = 1,000,000,000$

Example

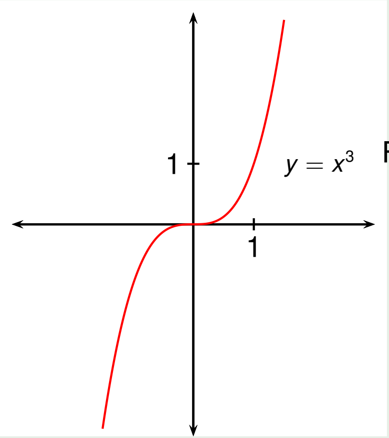


Find $\lim_{x\to\infty} x^3$ and $\lim_{x\to-\infty} x^3$.

- When x is large, so is x^3.
- By taking x large enough, we can make x^3 arbitrarily large.

$10^3 = 1000, \qquad 100^3 = 1,000,000,$
$1000^3 = 1,000,000,000$

Example



Find $\lim_{x\to\infty} x^3$ and $\lim_{x\to-\infty} x^3$.

- When x is large, so is x^3.
- By taking x large enough, we can make x^3 arbitrarily large.
- Therefore $\lim_{x\to\infty} x^3 = \infty$.

$10^3 = 1000, \qquad 100^3 = 1,000,000,$
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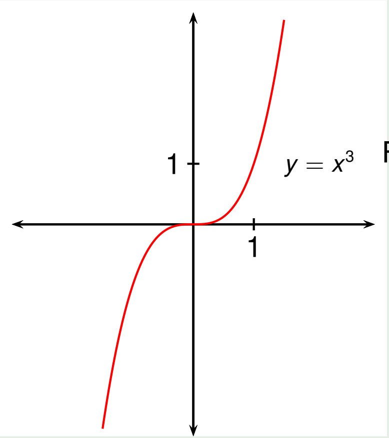
Example



Find $\lim_{x\to\infty} x^3$ and $\lim_{x\to-\infty} x^3$.

- When x is large, so is x^3.
- By taking x large enough, we can make x^3 arbitrarily large.
- Therefore $\lim_{x\to\infty} x^3 = \infty$.
- Similarly, $\lim_{x\to-\infty} x^3 = -\infty$.

$10^3 = 1000, \qquad 100^3 = 1,000,000,$
$1000^3 = 1,000,000,000$

Example

Find $\lim_{x \to \infty} (x^2 - x)$.

Example

Find $\lim\limits_{x\to\infty}(x^2 - x)$.

- WRONG: $\lim\limits_{x\to\infty}(x^2 - x) = \lim\limits_{x\to\infty} x^2 - \lim\limits_{x\to\infty} x = \infty - \infty = 0$.

Example

Find $\lim_{x\to\infty}(x^2 - x)$.

- WRONG: $\lim_{x\to\infty}(x^2 - x) = \lim_{x\to\infty} x^2 - \lim_{x\to\infty} x = \infty - \infty = 0$.
- The limit laws don't apply here as the limits on the right don't exist (recall: limits equal to ∞ don't exist).

Example

Find $\lim_{x \to \infty} (x^2 - x)$.

- WRONG: $\lim_{x \to \infty} (x^2 - x) = \lim_{x \to \infty} x^2 - \lim_{x \to \infty} x = \infty - \infty = 0$.
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- Furthermore arithmetics with ∞ is not allowed: ∞ isn't a number.

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- Furthermore arithmetics with ∞ is not allowed: ∞ isn't a number.
- Instead: $\lim_{x\to\infty}(x^2 - x) = \lim_{x\to\infty} x(x-1) = \infty$.

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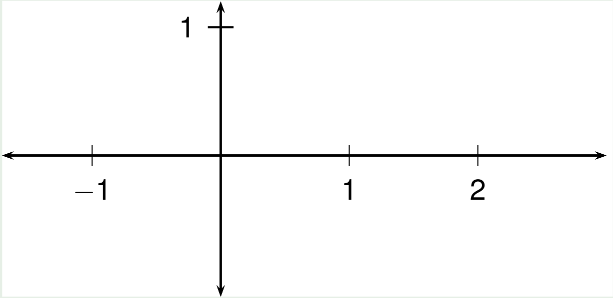
- WRONG: $\lim_{x\to\infty}(x^2 - x) = \lim_{x\to\infty} x^2 - \lim_{x\to\infty} x = \infty - \infty = 0$.
- The limit laws don't apply here as the limits on the right don't exist (recall: limits equal to ∞ don't exist).
- Furthermore arithmetics with ∞ is not allowed: ∞ isn't a number.
- Instead: $\lim_{x\to\infty}(x^2 - x) = \lim_{x\to\infty} x(x-1) = \infty$.
- This is because x and $x - 1$ both become arbitrarily large as $x \to \infty$.

Example

Find the limits as $x \to \infty$ and $x \to -\infty$ of

$$y = \frac{1}{24}(x-2)^4(x+1)^3(x-1).$$



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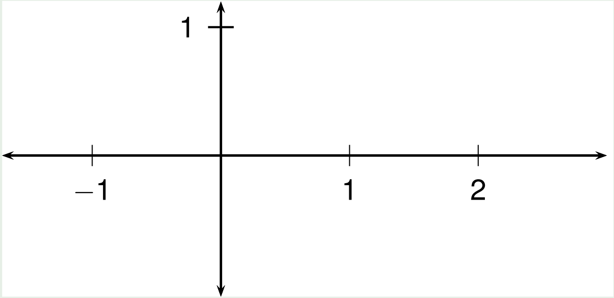

$$\lim_{x\to\infty} \frac{1}{24}(x-2)^4(x+1)^3(x-1) =$$

$$(\quad) \quad (\quad) \quad (\quad)$$

$$\lim_{x\to-\infty} \frac{1}{24}(x-2)^4(x+1)^3(x-1) =$$

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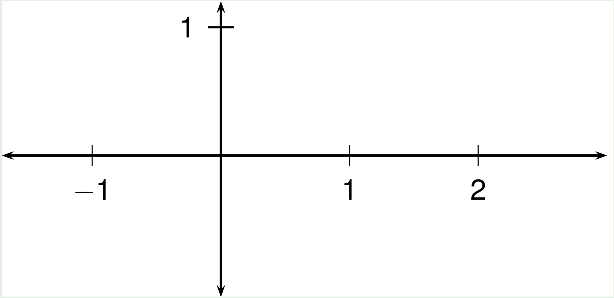

$$\lim_{x\to\infty} \frac{1}{24}(x-2)^4(x+1)^3(x-1) =$$

$$(\mathbf{?}\ )\qquad (\mathbf{?}\ )\quad (\mathbf{?}\ )$$

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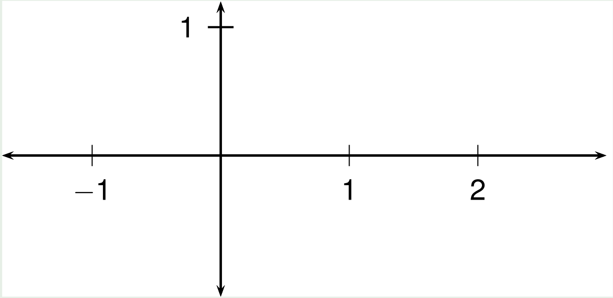

$$\lim_{x\to\infty} \frac{1}{24}(x-2)^4(x+1)^3(x-1) \quad =$$

$$(+) \qquad (\textbf{?}\ ) \qquad (\textbf{?}\ )$$

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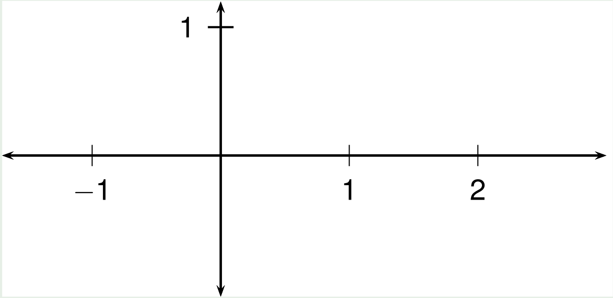

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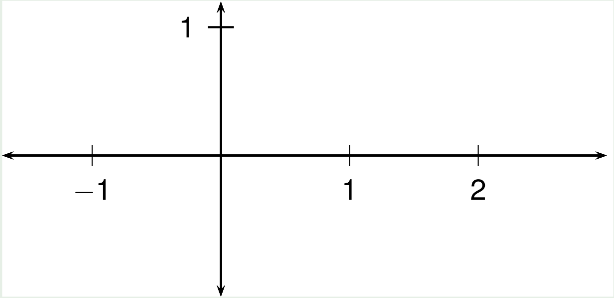

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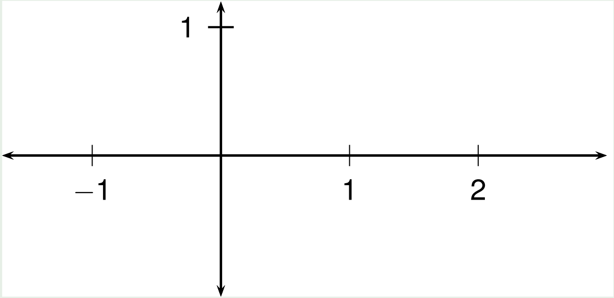

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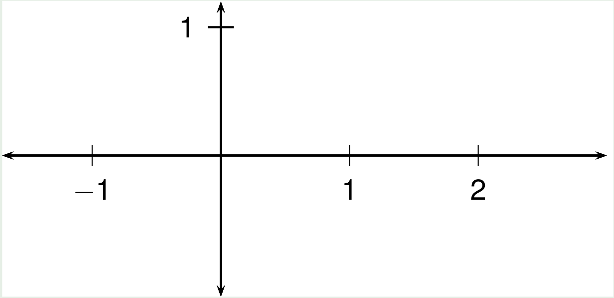

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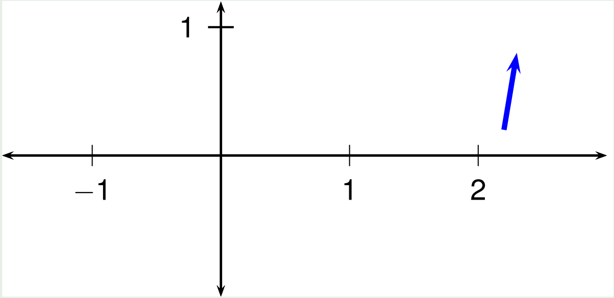

$$\lim_{x\to\infty} \frac{1}{24}(x-2)^4(x+1)^3(x-1) = \infty$$

$$(+) \quad (+) \quad (+)$$

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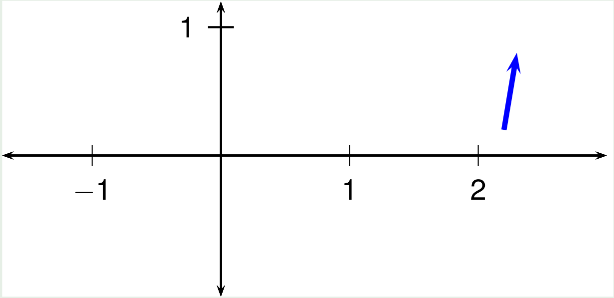

$$\lim_{x\to\infty} \frac{1}{24}(x-2)^4(x+1)^3(x-1) \quad = \quad \infty$$

$$(+) \qquad (+) \qquad (+)$$

$$\lim_{x\to-\infty} \frac{1}{24}{\color{red}(x-2)^4}(x+1)^3(x-1) \quad =$$

$$({\color{red}?}\ ) \qquad (?\ ) \qquad (?\ )$$

Example

Find the limits as $x \to \infty$ and $x \to -\infty$ of

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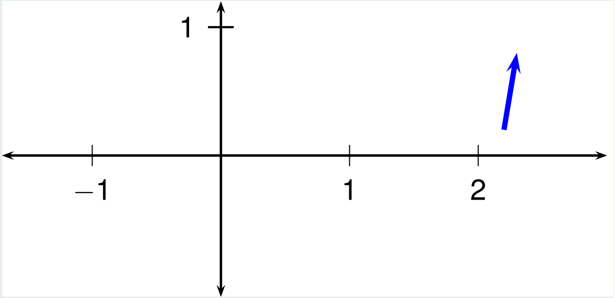

$$\lim_{x\to\infty} \frac{1}{24}\underset{(+)}{(x-2)^4}\underset{(+)}{(x+1)^3}\underset{(+)}{(x-1)} = \infty$$

$$\lim_{x\to-\infty} \frac{1}{24}\underset{(+)}{(x-2)^4}\underset{(?)}{(x+1)^3}\underset{(?)}{(x-1)} =$$

Example

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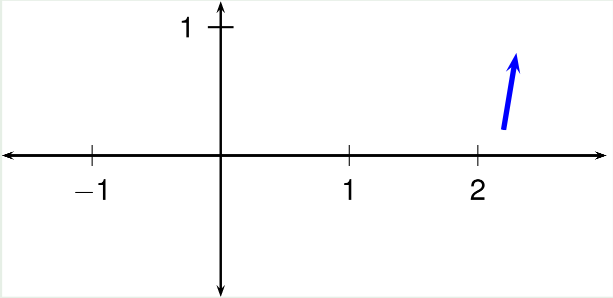

$$\lim_{x\to\infty} \frac{1}{24}\underbrace{(x-2)^4}_{(+)}\underbrace{(x+1)^3}_{(+)}\underbrace{(x-1)}_{(+)} = \infty$$

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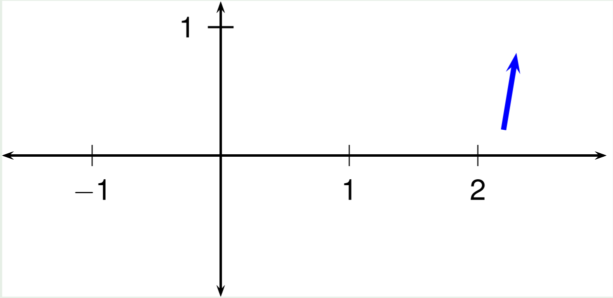

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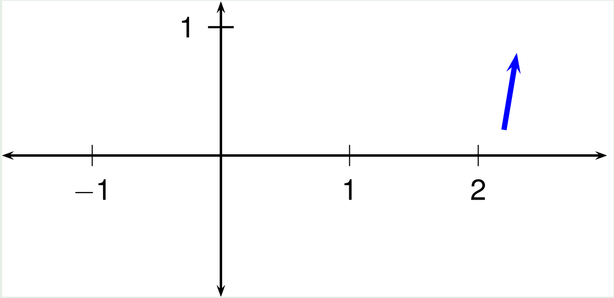

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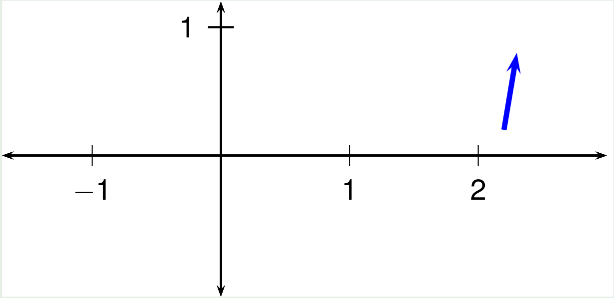


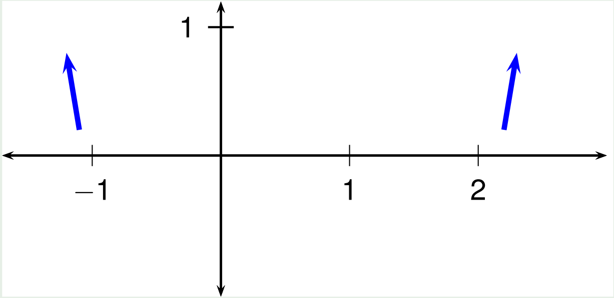
$$\lim_{x\to\infty} \frac{1}{24}\underset{(+)}{(x-2)^4}\underset{(+)}{(x+1)^3}\underset{(+)}{(x-1)} = \infty$$

$$\lim_{x\to-\infty} \frac{1}{24}\underset{(+)}{(x-2)^4}\underset{(-)}{(x+1)^3}\underset{(-)}{(x-1)} =$$

Example

Find the limits as $x \to \infty$ and $x \to -\infty$ of

$$y = \frac{1}{24}(x-2)^4(x+1)^3(x-1).$$



$$\lim_{x\to\infty} \frac{1}{24}\underset{(+)}{(x-2)^4}\underset{(+)}{(x+1)^3}\underset{(+)}{(x-1)} = \infty$$

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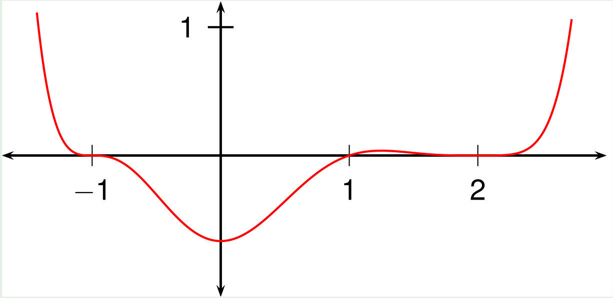

$$\lim_{x\to\infty} \underset{(+)}{\frac{1}{24}(x-2)^4}\underset{(+)}{(x+1)^3}\underset{(+)}{(x-1)} = \infty$$

$$\lim_{x\to-\infty} \underset{(+)}{\frac{1}{24}(x-2)^4}\underset{(-)}{(x+1)^3}\underset{(-)}{(x-1)} = \infty$$