Calculus II
Lecture 14

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https://github.com/tmilev/freecalc

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Outline

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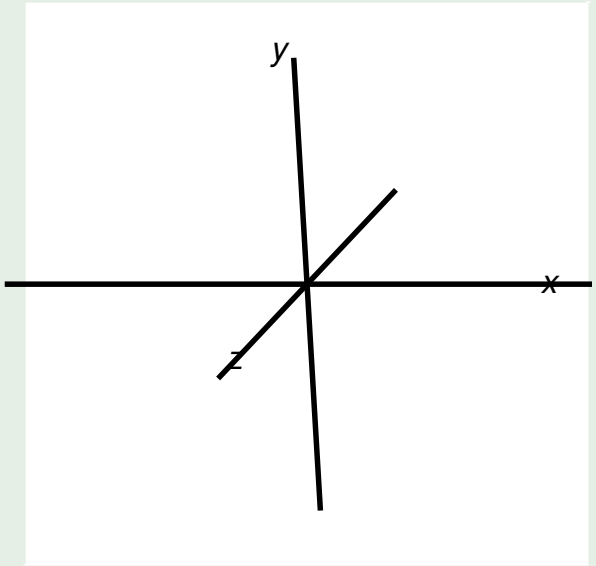
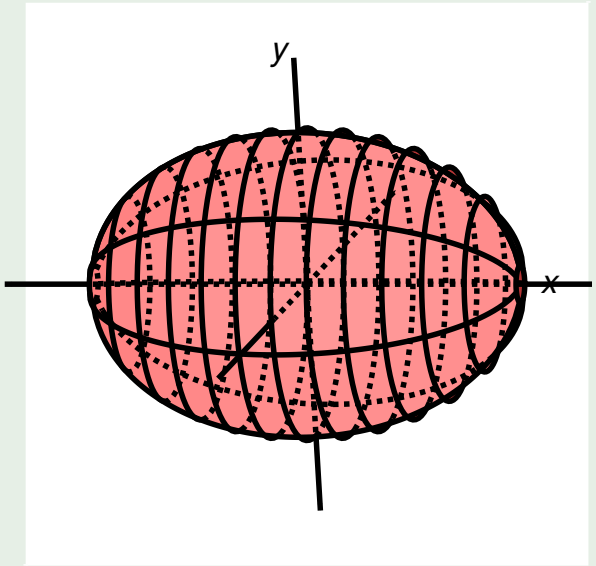
Example



Find the surface area of the ellipsoid obtained by rotating $y = \sqrt{1 - \frac{x^2}{2}}$ about the x axis.

Example



Find the surface area of the ellipsoid obtained by rotating $y = \sqrt{1 - \frac{x^2}{2}}$ about the x axis.

$$\begin{aligned} y' &= \frac{1}{2}\frac{(-x)}{\sqrt{1-\frac{x^2}{2}}} = -\frac{x}{2y} \\ (y')^2 &= \frac{x^2}{4y^2} \end{aligned}$$

$$\begin{aligned} \textit{Area} &= \int_{-2}^{2} 2\pi y\sqrt{1+\frac{x^2}{4y^2}}\mathrm{d}x \\ &= \int_{-2}^{2} 2\pi y\sqrt{\frac{4y^2+x^2}{4y^2}}\mathrm{d}x \\ &= \int_{-2}^{2} 2\pi y\sqrt{\frac{4y^2+x^2}{4y^2}}\mathrm{d}x \\ &= \int_{-2}^{2} \pi\sqrt{4\left(1-\frac{x^2}{2}\right)+x^2}\mathrm{d}x \\ &= \int_{-2}^{2} \pi\sqrt{4-x^2}\mathrm{d}x \end{aligned}$$