Precalculus
Lecture 16
Factoring Polynomials

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https://github.com/tmilev/freecalc

2020

Outline

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Recall that $i^2 = -1$, $\sqrt{-1} = i$.

Example (Polynomial factorizations)

$$2x^2 + 3x - 5 =$$

$$x^2 + 1 =$$

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Every polynomial can be factored into product of linear terms

$$p(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n = a_0 (x - x_1) \dots (x - x_n),$$

where $x_1, \dots, x_n$ *are the (not necessarily different) roots of* $p(x)$.

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Corollary

Every real polynomial can be factored into a product of real linear terms and real quadratic terms with no real roots, i.e., factors of form

- $(x - r)$, *where r is real and*
- $ax^2 + bx + c$ *with* $b^2 - 4ac < 0$ *where* a, b, c *are real.*

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Example

$$2x^2 + 3x - 5 = (2x+5)(x-1) = 2\left(x - \left(-\tfrac{5}{2}\right)\right)(x-1)$$

$$x^2 + 1 = x^2 - (-1) = x^2 - i^2 = (x - (-i))(x - i)$$

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$$= \left(x - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right)\left(x - \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)\right)$$
$$\left(x - \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right)\left(x - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)\right)$$ complex roots

$$a_0 x^n + a_1 x^{n-1} + \cdots + a_n = a_0(x - x_1)\ldots(x - x_n)$$
$$= \text{prod. real quadratics no roots \& lin. terms.}$$

Example

$$2x^2 + 3x - 5 = (2x+5)(x-1) = 2\left(x - \left(-\tfrac{5}{2}\right)\right)(x-1) \quad \text{real roots}$$

$$x^2 + 1 = x^2 - (-1) = x^2 - i^2 = (x - (-i))(x - i) \quad \text{complex roots}$$

$$x^4 - 1 = (x^2 - 1)(x^2 + 1) = (x-1)(x+1)(x^2+1)$$

$$= (x-1)(x-(-1))(x-i)(x-(-i)) \quad \text{mixed roots}$$

$$x^4 + 1 = (x^2 - \sqrt{2}x + 1)(x^2 + \sqrt{2}x + 1)$$

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$$\left(x - \left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)\right)\left(x - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)\right) \quad \text{complex roots}$$

Factoring polynomials in practice

- In theory every polynomial can be factored.

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- Yes, with extra operations. Difficult: google Galois Theory to get started.

What does factorization mean?

- Based on context, "to factor a polynomial" means one of:

These poly's are equal	Type of factorization
x^4+1	
$(x^2-\sqrt{2}x+1)(x^2+\sqrt{2}x+1)$	
$\left(x-\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)\right)\left(x-\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)\right)$ $\left(x-\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)\right)\left(x-\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)\right)$	

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- If we assume rational roots there are practical algorithms by hand.
- We study those for cubics with the aid of scientific calculator.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

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Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r|l}
 & \;\;\textbf{?} \\ \hline
x - 1 & x^3 + 2x^2 \qquad + 1
\end{array}
$$

Divide x^3 by x.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r|l}
 & x^2 \\
\cline{2-2}
x - 1 & x^3 + 2x^2 \qquad + 1
\end{array}
$$

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Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r|l}
 & x^2 \\
\cline{2-2}
x-1 & x^3 + 2x^2 \qquad + 1 \\
 & \underline{\textbf{?} \quad \textbf{?} \qquad\qquad\qquad}
\end{array}
$$

Multiply x^2 by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r|l}
 & x^2 \\
\cline{2-2}
x - 1 & x^3 + 2x^2 \qquad + 1 \\
 & \underline{x^3 - x^2 \qquad\qquad\quad}
\end{array}
$$

Multiply x^2 by divisor.

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Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r|l}
 & \quad x^2 \\
\hline
x - 1 & x^3 + 2x^2 \qquad + 1 \\
- & x^3 - x^2 \\
\hline
 & \qquad \mathbf{?} \qquad \qquad \mathbf{?}
\end{array}
$$

Subtract last two polynomials.

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Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r|l}
 & \quad x^2 \\
\cline{2-2}
x \underset{-}{-} 1 & x^3 + 2x^2 \qquad + 1 \\
 & x^3 - x^2 \\
\cline{2-2}
 & \qquad 3x^2 \qquad + 1
\end{array}
$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r|l}
 & \quad x^2 \quad \color{red}{\mathbf{?}} \\
\cline{2-2}
\color{red}{x} - 1 & x^3 + 2x^2 \qquad + 1 \\
- & x^3 - x^2 \\
\cline{2-2}
 & \qquad \color{red}{3x^2} \qquad + 1
\end{array}
$$

Divide $\color{red}{3x^2}$ by $\color{red}{x}$.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
 & \quad x^2 + 3x \\
x - 1 & \overline{\big) x^3 + 2x^2 \qquad + 1} \\
- & \quad x^3 - x^2 \\
\hline
 & \qquad 3x^2 \qquad + 1
\end{array}
$$

Divide $3x^2$ by x.

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Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r|l}
 & x^2 + 3x \\
\hline
x - 1 & x^3 + 2x^2 \qquad + 1 \\
- & x^3 - x^2 \\
\hline
 & \quad 3x^2 \qquad + 1 \\
 & \quad ? \quad ? \\
\hline
\end{array}
$$

Multiply $3x$ by divisor.

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Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r|l}
 & x^2 + 3x \\
\hline
x - 1 & x^3 + 2x^2 \qquad + 1 \\
- & x^3 - x^2 \\
\hline
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 & \quad 3x^2 - 3x \\
\hline
\end{array}
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Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

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- & x^3 - x^2 \\
\hline
 & 3x^2 + 1 \\
- & 3x^2 - 3x \\
\hline
 & \mathbf{?} \mathbf{?}
\end{array}
$$

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\begin{array}{r|l}
 & x^2 + 3x \\
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- & x^3 - x^2 \\
\hline
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- & 3x^2 - 3x \\
\hline
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Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
 & \quad x^2 + 3x \quad \textbf{?} \\
x - 1 & \overline{\big)\, x^3 + 2x^2 \qquad + 1} \\
- & \quad x^3 - x^2 \\
\hline
 & \qquad\quad 3x^2 \qquad + 1 \\
- & \qquad\quad 3x^2 - 3x \\
\hline
 & \qquad\qquad\qquad 3x + 1
\end{array}
$$

Divide $3x$ by x.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rr|llll}
 & & x^2 & +\,3x & \color{red}{+\,3} & \\
\cline{3-6}
\color{red}{x} - 1 & & x^3 & +2x^2 & & +1 \\
- & & x^3 & -\ x^2 & & \\
\cline{3-6}
 & & & 3x^2 & & +1 \\
- & & & 3x^2 & -3x & \\
\cline{4-6}
 & & & & \color{red}{3x} & +1
\end{array}
$$

Divide $\color{red}{3x}$ by $\color{red}{x}$.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r|l}
 & \phantom{x^3+{}}x^2 + 3x + 3 \\
\cline{2-2}
x - 1 & x^3 + 2x^2 \phantom{{}-3x} + 1 \\
- & x^3 - x^2 \\
\cline{2-2}
 & \phantom{x^3+{}} 3x^2 \phantom{{}-3x} + 1 \\
- & \phantom{x^3+{}} 3x^2 - 3x \\
\cline{2-2}
 & \phantom{x^3+2x^2-{}} 3x + 1 \\
 & \phantom{x^3+2x^2-{}} \;?\quad ? \\
\cline{2-2}
\end{array}
$$

Multiply 3 by divisor.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rrrrr}
 & & x^2 & +3x & +3 \\
\cline{2-5}
x-1 & \multicolumn{1}{|r}{x^3} & +2x^2 & & +1 \\
- & x^3 & -x^2 & & \\
\cline{2-5}
 & & 3x^2 & & +1 \\
- & & 3x^2 & -3x & \\
\cline{3-5}
 & & & 3x & +1 \\
 & & & 3x & -3 \\
\cline{4-5}
\end{array}
$$

Multiply 3 by divisor.

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Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rrl}
 & & x^2 + 3x + 3 \\
x - 1 & & \overline{)\, x^3 + 2x^2 \qquad + 1} \\
- & & x^3 - x^2 \\
\hline
 & & \quad 3x^2 \qquad + 1 \\
- & & \quad 3x^2 - 3x \\
\hline
 & & \qquad\quad 3x + 1 \\
- & & \qquad\quad 3x - 3 \\
\hline
 & & \qquad\qquad ?
\end{array}
$$

Subtract last two polynomials.

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rl}
 & x^2 + 3x + 3 \\
x - 1 & \overline{)\,x^3 + 2x^2 + 1} \\
- & x^3 - x^2 \\
 & \overline{3x^2 + 1} \\
- & 3x^2 - 3x \\
 & \overline{3x + 1} \\
- & 3x - 3 \\
 & \overline{4}
\end{array}
$$

Subtract last two polynomials.

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Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{r|rrrr}
 & x^2 & +\,3x & +\,3 & \\
\cline{2-5}
x-1 & x^3 & +2x^2 & & +1 \\
- & x^3 & -\ x^2 & & \\
\cline{2-5}
 & & 3x^2 & & +1 \\
- & & 3x^2 & -3x & \\
\cline{3-5}
 & & & 3x & +1 \\
- & & & 3x & -3 \\
\cline{4-5}
 & & & & 4
\end{array}
$$

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rr|rrrr}
\textbf{Quotient:} & & x^2 & +\,3x & +\,3 & \\
\hline
x-1 & & x^3 & +2x^2 & & +1 \\
- & & x^3 & -\;x^2 & & \\
\hline
 & & & 3x^2 & & +1 \\
- & & & 3x^2 & -3x & \\
\hline
 & & & & 3x & +1 \\
- & & & & 3x & -3 \\
\hline
 & & & & & 4
\end{array}
$$

$$
\begin{array}{rcl}
\textbf{(Dividend)} & = & \textbf{(Quotient)} \cdot \textbf{(Divisor)} + \textbf{(Remainder)} \\
(x^3 + 2x^2 + 1) & = & (x^2 + 3x + 3) \cdot (x - 1) + 4
\end{array}
$$

Example (Polynomial long division)

Divide with quotient and remainder $x^3 + 2x^2 + 1$ by $x - 1$.

$$
\begin{array}{rrr}
\textbf{Quotient:} & & x^2 + 3x + 3 \\
x - 1 & & \overline{)\,x^3 + 2x^2 \qquad + 1} \\
- & & x^3 - x^2 \\ \hline
 & & 3x^2 \qquad + 1 \\
- & & 3x^2 - 3x \\ \hline
 & & 3x + 1 \\
- & & 3x - 3 \\ \hline
\textbf{Remainder:} & & 4
\end{array}
$$

$$
\begin{array}{rcl}
\textbf{(Dividend)} & = & \textbf{(Quotient)} \cdot \textbf{(Divisor)} + \textbf{(Remainder)} \\
(x^3 + 2x^2 + 1) & = & (x^2 + 3x + 3) \cdot (x - 1) + 4
\end{array}
$$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$2x - 3 \;\big)\overline{\,6x^3 - 19x^2 + 17x - 3\,}$$

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$$2x - 3 \quad \overline{\left) 6x^3 - 19x^2 + 17x - 3 \right.}$$

Example

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$$\begin{array}{r|l} & \quad ? \\ \cline{2-2} 2x - 3 & 6x^3 - 19x^2 + 17x - 3 \end{array}$$

Divide $6x^3$ by $2x$.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r|l}
 & 3x^2 \\
\cline{2-2}
2x - 3 & 6x^3 - 19x^2 + 17x - 3
\end{array}
$$

Divide $6x^3$ by $2x$.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{rl}
 & 3x^2 \\
2x - 3 & \overline{\big)\, 6x^3 - 19x^2 + 17x - 3} \\
 & \underline{\mathbf{?} \qquad \mathbf{?} \qquad\qquad\qquad}
\end{array}
$$

Multiply $3x^2$ by divisor.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r|l}
 & 3x^2 \\
\cline{2-2}
2x-3 & 6x^3-19x^2+17x-3 \\
 & 6x^3-\ 9x^2 \\
\cline{2-2}
\end{array}
$$

Multiply $3x^2$ by divisor.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r|l}
 & 3x^2 \\
\cline{2-2}
2x - 3 & 6x^3 - 19x^2 + 17x - 3 \\
- & 6x^3 - 9x^2 \\
\cline{2-2}
 & \quad ? \quad ? \quad ?
\end{array}
$$

Subtract last two polynomials.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r|l}
 & 3x^2 \\
\cline{2-2}
2x - 3 & 6x^3 - 19x^2 + 17x - 3 \\
- & 6x^3 - 9x^2 \\
\cline{2-2}
 & \quad -10x^2 + 17x - 3
\end{array}
$$

Subtract last two polynomials.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r|l}
 & \quad 3x^2 \quad ? \\
\cline{2-2}
2x - 3 & 6x^3 - 19x^2 + 17x - 3 \\
- & 6x^3 - 9x^2 \\
\cline{2-2}
 & \quad -10x^2 + 17x - 3
\end{array}
$$

Divide $-10x^2$ by $2x$.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r|l}
 & \;\;\;\;\;\;\;\; 3x^2 - 5x \\
\cline{2-2}
2x - 3 & 6x^3 - 19x^2 + 17x - 3 \\
- & 6x^3 - \;\, 9x^2 \\
\cline{2-2}
 & \;\;\;\;\;\;\; -10x^2 + 17x - 3
\end{array}
$$

Divide $-10x^2$ by $2x$.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r|l}
 & \quad 3x^2 - 5x \\
\cline{2-2}
2x - 3 & 6x^3 - 19x^2 + 17x - 3 \\
- & 6x^3 - 9x^2 \\
\cline{2-2}
 & \quad -10x^2 + 17x - 3 \\
 & \quad\quad ? \quad\quad ? \\
\cline{2-2}
\end{array}
$$

Multiply $-5x$ by divisor.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r|l}
 & \quad\; 3x^2 - 5x \\
\hline
2x - 3 & 6x^3 - 19x^2 + 17x - 3 \\
- & 6x^3 - \;\, 9x^2 \\
\hline
 & \qquad -10x^2 + 17x - 3 \\
 & \qquad -10x^2 + 15x \\
\hline
\end{array}
$$

Multiply $-5x$ by divisor.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{rl}
 & 3x^2 - 5x \\
2x - 3 & \overline{\big)\,6x^3 - 19x^2 + 17x - 3} \\
- & 6x^3 - 9x^2 \\
\hline
 & \color{red}{-10x^2 + 17x - 3} \\
\color{red}{-} & \color{red}{-10x^2 + 15x} \\
\hline
 & \color{red}{\mathbf{?}} \quad \color{red}{\mathbf{?}}
\end{array}
$$

Subtract last two polynomials.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{rl}
 & \quad 3x^2 - 5x \\
2x - 3 & \overline{\big)\, 6x^3 - 19x^2 + 17x - 3} \\
- & \quad 6x^3 - 9x^2 \\
\hline
 & \qquad -10x^2 + 17x - 3 \\
- & \qquad -10x^2 + 15x \\
\hline
 & \qquad\qquad 2x - 3
\end{array}
$$

Subtract last two polynomials.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r|l}
 & \quad 3x^2 - 5x \quad \color{red}{\textbf{?}} \\
\cline{2-2}
\color{red}{2x} - 3 & 6x^3 - 19x^2 + 17x - 3 \\
- & 6x^3 - 9x^2 \\
\cline{2-2}
 & \quad -10x^2 + 17x - 3 \\
- & \quad -10x^2 + 15x \\
\cline{2-2}
 & \qquad\qquad \color{red}{2x} - 3
\end{array}
$$

Divide $\color{red}{2x}$ by $\color{red}{2x}$.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{r|l}
 & 3x^2 - 5x + 1 \\
\hline
2x - 3 & 6x^3 - 19x^2 + 17x - 3 \\
- & 6x^3 - 9x^2 \\
\hline
 & -10x^2 + 17x - 3 \\
- & -10x^2 + 15x \\
\hline
 & 2x - 3
\end{array}
$$

Divide $2x$ by $2x$.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{rrrrrrr}
 & & 3x^2 & - & 5x & + & 1 \\
\cline{2-7}
2x - 3 & \big) & 6x^3 & -19x^2 & +17x & -3 & \\
- & & 6x^3 & -9x^2 & & & \\
\cline{2-7}
 & & & -10x^2 & +17x & -3 & \\
- & & & -10x^2 & +15x & & \\
\cline{4-7}
 & & & & 2x & -3 & \\
 & & & & ? & ? & \\
\cline{5-7}
\end{array}
$$

Multiply 1 by divisor.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{rl}
 & 3x^2 - 5x + 1 \\
2x - 3 & \overline{\big)\, 6x^3 - 19x^2 + 17x - 3} \\
- & 6x^3 - 9x^2 \\
 & \overline{ - 10x^2 + 17x - 3} \\
- & - 10x^2 + 15x \\
 & \overline{ 2x - 3} \\
 & 2x - 3 \\
 & \overline{}
\end{array}
$$

Multiply 1 by divisor.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{rl}
 & 3x^2 - 5x + 1 \\
2x - 3 & \overline{\big)\, 6x^3 - 19x^2 + 17x - 3} \\
- & 6x^3 - 9x^2 \\
\hline
 & -10x^2 + 17x - 3 \\
- & -10x^2 + 15x \\
\hline
 & {\color{red}2x - 3} \\
{\color{red}-} & {\color{red}2x - 3} \\
\hline
 & {\color{red}\mathbf{?}}
\end{array}
$$

Subtract last two polynomials.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{rl}
 & 3x^2 - 5x + 1 \\
2x - 3 & \overline{)\,6x^3 - 19x^2 + 17x - 3} \\
- & 6x^3 - 9x^2 \\
\hline
 & -10x^2 + 17x - 3 \\
- & -10x^2 + 15x \\
\hline
 & \color{red}{2x - 3} \\
\color{red}{-} & \color{red}{2x - 3} \\
\hline
 & \color{red}{0}
\end{array}
$$

Subtract last two polynomials.

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{rrl}
 & & 3x^2 - 5x + 1 \\
2x - 3 & \Big| & 6x^3 - 19x^2 + 17x - 3 \\
- & & 6x^3 - 9x^2 \\
\hline
 & & -10x^2 + 17x - 3 \\
- & & -10x^2 + 15x \\
\hline
 & & 2x - 3 \\
- & & 2x - 3 \\
\hline
 & & 0
\end{array}
$$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{rl}
\textbf{Quotient:} & \quad 3x^2 - 5x + 1 \\
2x - 3 & \overline{\big)\, 6x^3 - 19x^2 + 17x - 3} \\
- & \;\; 6x^3 - 9x^2 \\
\hline
 & \qquad\; -10x^2 + 17x - 3 \\
- & \qquad\; -10x^2 + 15x \\
\hline
 & \qquad\qquad\qquad\; 2x - 3 \\
- & \qquad\qquad\qquad\; 2x - 3 \\
\hline
 & \qquad\qquad\qquad\qquad\; 0
\end{array}
$$

(Dividend)$=$**(Quotient)** $\cdot$ **(Divisor)** $+$ **(Remainder)**

$(6x^3 - 19x^2 + 17x - 3) = (3x^2 - 5x + 1) \cdot (2x - 3)$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{array}{rr}
\textbf{Quotient:} & 3x^2 - 5x + 1 \\
2x - 3 & \overline{\big) 6x^3 - 19x^2 + 17x - 3} \\
- & 6x^3 - 9x^2 \\
\hline
 & -10x^2 + 17x - 3 \\
- & -10x^2 + 15x \\
\hline
 & 2x - 3 \\
- & 2x - 3 \\
\hline
\textbf{Remainder:} & 0
\end{array}
$$

(Dividend)$=$(Quotient) $\cdot$ (Divisor) $+$ (Remainder)

$(6x^3 - 19x^2 + 17x - 3) = (3x^2 - 5x + 1) \cdot (2x - 3)$

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$$
\begin{array}{rrl}
\textbf{Quotient:} & & 3x^2 - 5x + 1 \\
2x-3 & & \overline{\big)\, 6x^3 - 19x^2 + 17x - 3} \\
- & & 6x^3 - 9x^2 \\
\hline
 & & -10x^2 + 17x - 3 \\
- & & -10x^2 + 15x \\
\hline
 & & 2x - 3 \\
- & & 2x - 3 \\
\hline
\textbf{Remainder:} & & 0
\end{array}
$$

$$\textbf{(Dividend)} = \textbf{(Quotient)} \cdot \textbf{(Divisor)} + \textbf{(Remainder)}$$

$$(6x^3 - 19x^2 + 17x - 3) = (3x^2 - 5x + 1) \cdot (2x - 3)$$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

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$$\begin{aligned}(6x^3 - 19x^2 + 17x - 3) &= (3x^2 - 5x + 1) \cdot (2x - 3)\\ &= 3\left(x - \textbf{?} \qquad\qquad\right)\left(x - \textbf{?} \qquad\qquad\right)(2x - 3)\end{aligned}$$

No easy factorization of quadratic, so use formula:

$x_1, x_2 = \textbf{?}$

Example

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$$
\begin{aligned}
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&= 3\left(x - \textbf{?} \qquad\qquad \right)\left(x - \textbf{?} \qquad\qquad \right)(2x - 3)
\end{aligned}
$$

No easy factorization of quadratic, so use formula:

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
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\end{aligned}
$$

No easy factorization of quadratic, so use formula:

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 3 \cdot 1}}{2 \cdot 3}$$

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$$\begin{aligned}(6x^3 - 19x^2 + 17x - 3) &= (3x^2 - 5x + 1) \cdot (2x - 3)\\ &= 3\left(x - \left(\frac{5+\sqrt{13}}{6}\right)\right)\left(x - \left(\frac{5-\sqrt{13}}{6}\right)\right)(2x - 3)\end{aligned}$$

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Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{aligned}
(6x^3 - 19x^2 + 17x - 3) &= (3x^2 - 5x + 1) \cdot (2x - 3) \\
&= 3\left(x - \left(\frac{5+\sqrt{13}}{6}\right)\right)\left(x - \left(\frac{5-\sqrt{13}}{6}\right)\right)(2x - 3)
\end{aligned}
$$

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$$6x^3 - 19x^2 + 17x - 3 = 0$$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

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$$\begin{aligned}6x^3 - 19x^2 + 17x - 3 &= 0\\ 3\left(x - \left(\frac{5+\sqrt{13}}{6}\right)\right)\left(x - \left(\frac{5-\sqrt{13}}{6}\right)\right)(2x-3) &= 0\end{aligned}$$

$$2x - 3 = 0 \quad \textbf{or} \quad x = \left(\frac{5+\sqrt{13}}{6}\right) \quad \textbf{or} \quad x = \left(\frac{5-\sqrt{13}}{6}\right)$$

Example

Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

$$
\begin{aligned}
(6x^3 - 19x^2 + 17x - 3) &= (3x^2 - 5x + 1) \cdot (2x - 3)\\
&= 3\left(x - \left(\frac{5+\sqrt{13}}{6}\right)\right)\left(x - \left(\frac{5-\sqrt{13}}{6}\right)\right)(2x - 3)
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$$
\begin{aligned}
6x^3 - 19x^2 + 17x - 3 &= 0\\
3\left(x - \left(\frac{5+\sqrt{13}}{6}\right)\right)\left(x - \left(\frac{5-\sqrt{13}}{6}\right)\right)(2x - 3) &= 0
\end{aligned}
$$

$2x - 3 = 0$ **or** $x = \left(\frac{5+\sqrt{13}}{6}\right)$ **or** $x = \left(\frac{5-\sqrt{13}}{6}\right)$

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Demonstrate that $6x^3 - 19x^2 + 17x - 3$ is divisible by $2x - 3$ using polynomial long division. Use your work to factor the cubic. Solve the equation $6x^3 - 19x^2 + 17x - 3 = 0$.

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(6x^3 - 19x^2 + 17x - 3) &= (3x^2 - 5x + 1) \cdot (2x - 3) \\
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No easy factorization of quadratic, so use formula:

$x_1, x_2 = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-5)\pm\sqrt{(-5)^2-4\cdot3\cdot1}}{2\cdot3} = \frac{5\pm\sqrt{13}}{6}$ We are ready to solve the equation.

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Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$2x^3 + x^2 - 7x - 6 = 0$$

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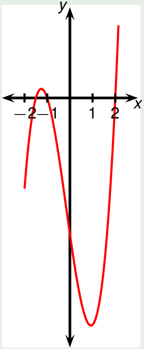

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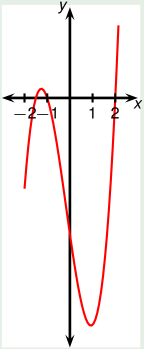

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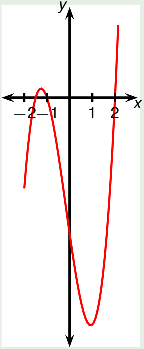

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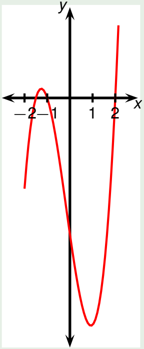

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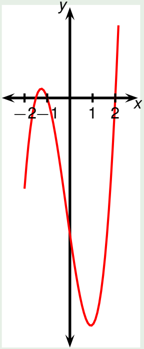

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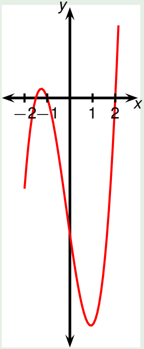

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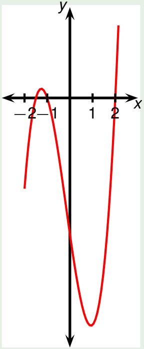

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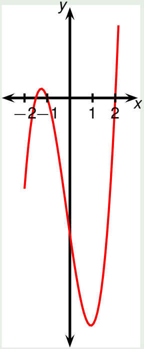

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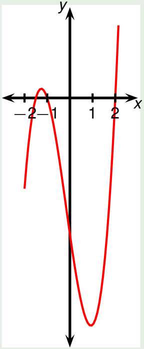

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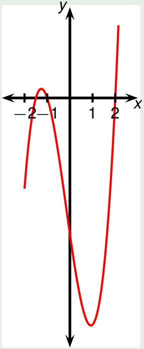

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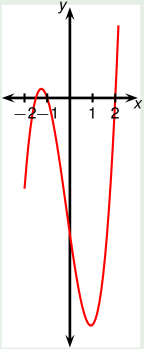

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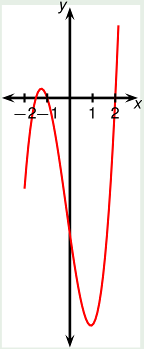

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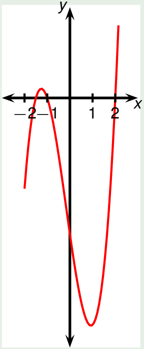

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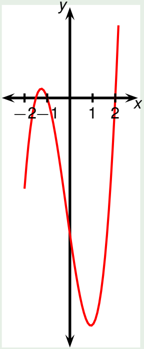

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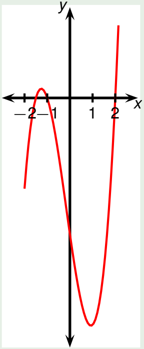

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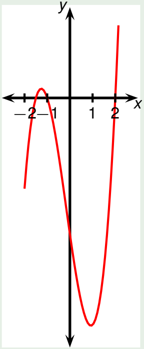

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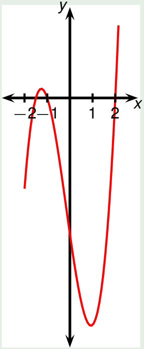

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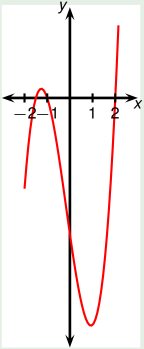

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Example

Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$x^3 - x^2 - 8x + 6 = 0$$

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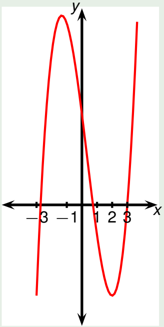

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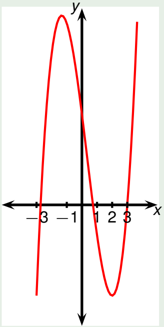

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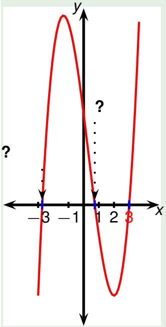

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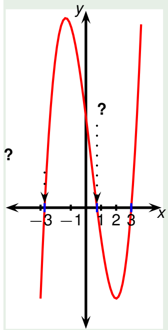

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$$x - 3 \quad \overline{\left| x^3 - x^2 - 8x + 6 \right.}$$

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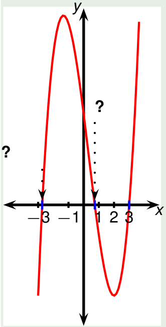

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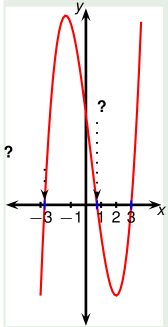

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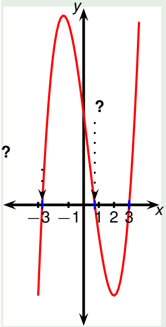

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Divide x^3 by x.

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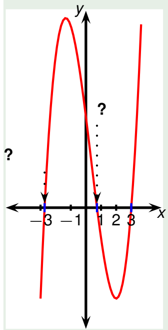

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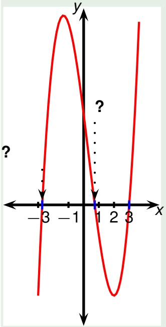
Example



Plot the left hand side of the equation with a graphing calculator. Solve the equation.

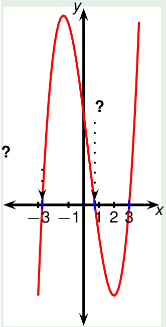
$$x^3 - x^2 - 8x + 6 \quad = \quad 0$$

The graph appears to intersect the x axis at:
? , **?** , 3. What are the two roots besides 3?

$$\begin{array}{r|l} & \color{red}{x^2} \\ \color{red}{x-3} & x^3 - x^2 - 8x + 6 \\ & \color{red}{?} \quad \color{red}{?} \\ \hline \end{array}$$

Multiply $\color{red}{x^2}$ by divisor.

Example



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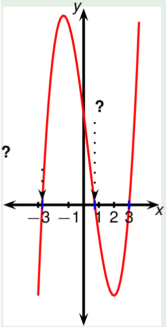
Multiply x^2 by divisor.

Example



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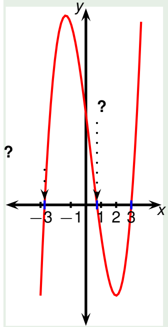
Subtract last two polynomials.

Example



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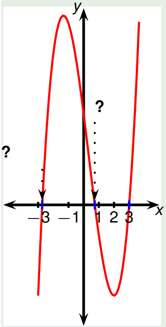

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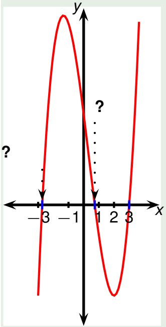
Divide $2x^2$ by x.

Example



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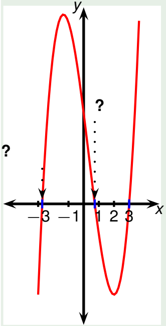

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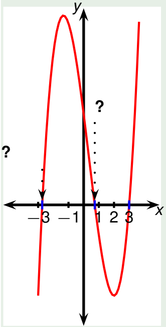
Multiply $2x$ by divisor.

Example



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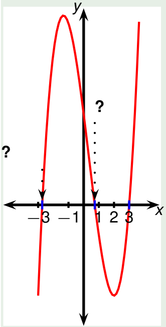

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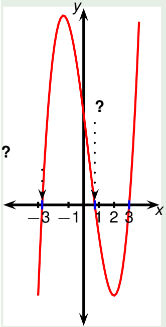
$$\begin{array}{r|l} & \phantom{x^3-{}} x^2 + 2x \\ \hline x - 3 & x^3 - x^2 - 8x + 6 \\ - & x^3 - 3x^2 \\ \hline & \phantom{x^3-{}} 2x^2 - 8x + 6 \\ - & \phantom{x^3-{}} 2x^2 - 6x \\ \hline & \phantom{x^3-2x^2-{}} ? \quad ? \end{array}$$

Subtract last two polynomials.

Example



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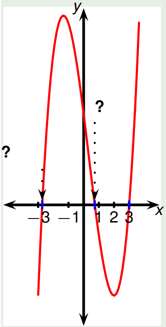
Subtract last two polynomials.

Example



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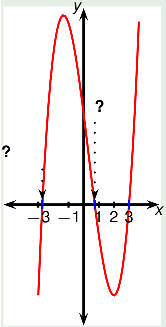
The graph appears to intersect the x axis at:

? , **?** , 3. What are the two roots besides 3?

$$
\begin{array}{r|l}
 & x^2 + 2x \quad \color{red}{?} \\
\hline
\color{red}{x} - 3 & x^3 - x^2 - 8x + 6 \\
- & x^3 - 3x^2 \\
\hline
 & 2x^2 - 8x + 6 \\
- & 2x^2 - 6x \\
\hline
 & \color{red}{-2x} + 6
\end{array}
$$

Divide $\color{red}{-2x}$ by $\color{red}{x}$.

Example



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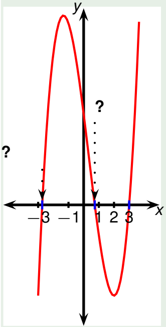
$$\begin{array}{rl}
 & x^2 + 2x - 2 \\
x - 3 & \overline{\big) x^3 - x^2 - 8x + 6} \\
- & x^3 - 3x^2 \\
\hline
 & \quad 2x^2 - 8x + 6 \\
- & \quad 2x^2 - 6x \\
\hline
 & \qquad -2x + 6
\end{array}$$

Divide $-2x$ by x.

Example



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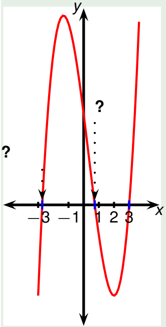
$$
\begin{array}{rl}
 & x^2 + 2x - 2 \\
x - 3 & \overline{)\, x^3 - x^2 - 8x + 6} \\
- & \underline{x^3 - 3x^2} \\
 & \phantom{x^3 -{}} 2x^2 - 8x + 6 \\
- & \phantom{x^3 -{}} \underline{2x^2 - 6x} \\
 & \phantom{x^3 - x^2 -{}} -2x + 6 \\
 & \phantom{x^3 - x^2 -{}} \underline{\ ?\quad ?\ }
\end{array}
$$

Multiply -2 by divisor.

Example



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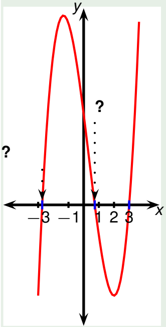
Multiply -2 by divisor.

Example



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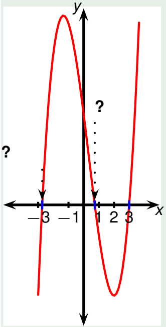
Subtract last two polynomials.

Example



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$$
\begin{array}{rr}
 & x^2 + 2x - 2 \\
\cline{2-2}
x - 3 & \big)\, x^3 - x^2 - 8x + 6 \\
- & x^3 - 3x^2 \phantom{{}- 8x + 6} \\
\cline{2-2}
 & 2x^2 - 8x + 6 \\
- & 2x^2 - 6x \phantom{{}+ 6} \\
\cline{2-2}
 & {\color{red}-2x + 6} \\
{\color{red}-} & {\color{red}-2x + 6} \\
\cline{2-2}
 & {\color{red}0}
\end{array}
$$

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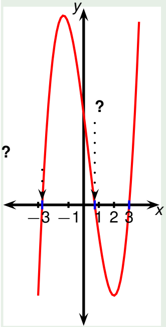
Example



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- & \qquad\qquad -2x + 6 \\
\hline
 & \qquad\qquad\qquad 0
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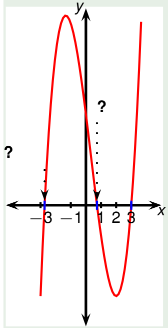
$$

Example



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$$\begin{aligned} x^3 - x^2 - 8x + 6 &= 0 \\ (x-3)({\color{red}x^2 + 2x - 2}) + 0 &= 0 \end{aligned}$$

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$$\begin{array}{rl}
\textbf{Quotient:} & {\color{red}x^2 + 2x - 2} \\
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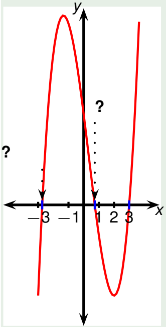
- & \underline{x^3 - 3x^2} \\
 & \quad 2x^2 - 8x + 6 \\
- & \quad \underline{2x^2 - 6x} \\
 & \qquad -2x + 6 \\
- & \qquad \underline{-2x + 6} \\
 & \qquad\qquad {\color{orange}0}
\end{array}$$

Example



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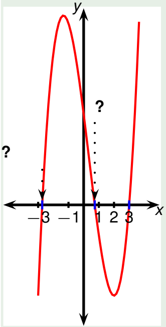

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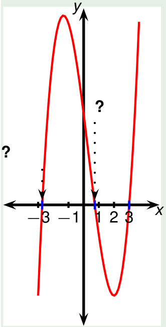

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$x - 3 = 0$ **or** $x =$

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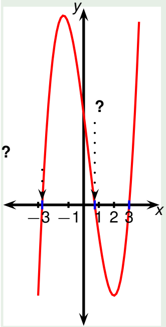

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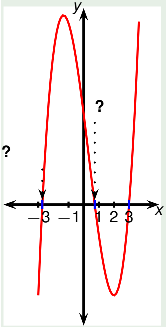

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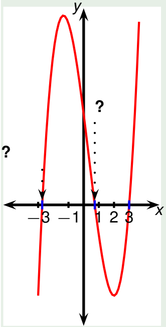

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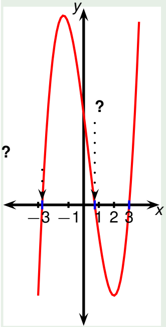

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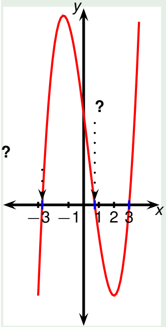

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$$x = 3 \qquad\qquad x = \frac{-2 \pm \sqrt{12}}{2}$$

The graph appears to intersect the x axis at:

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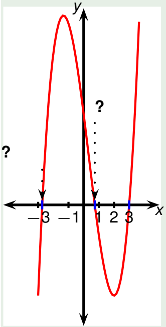
Example



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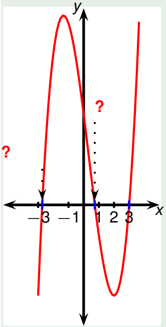
Example



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$$x^3 - x^2 - 8x + 6 = 0$$
$$(x-3)(x^2+2x-2) = 0$$

$$x - 3 = 0 \quad \textbf{or} \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4\cdot 1\cdot(-2)}}{2\cdot 1}$$
$$x = 3 \qquad x = \frac{-2 \pm \sqrt{12}}{2}$$
$$x = \frac{-2 \pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}.$$

The graph appears to intersect the x axis at:

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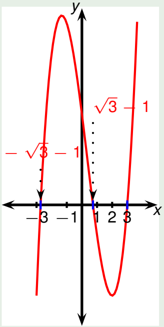
Example



Plot the left hand side of the equation with a graphing calculator. Solve the equation.

$$\begin{aligned} x^3 - x^2 - 8x + 6 &= 0 \\ (x-3)(x^2+2x-2) &= 0 \end{aligned}$$

$x - 3 = 0$ **or** $x = \dfrac{-2 \pm \sqrt{(2)^2 - 4\cdot 1\cdot(-2)}}{2\cdot 1}$

$x = 3$ $\quad x = \dfrac{-2\pm\sqrt{12}}{2}$

$x = \dfrac{-2\pm 2\sqrt{3}}{2} = -1 \pm \sqrt{3}.$

The graph appears to intersect the x axis at: $-\sqrt{3}-1, \sqrt{3}-1, 3$. What are the two roots besides 3?

Final answer:

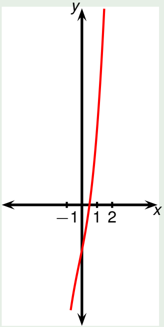
$x = 3$ **or** $x = -1-\sqrt{3}$ **or** $x = -1+\sqrt{3}$.

Example

Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 \quad = \quad 0$$

Example



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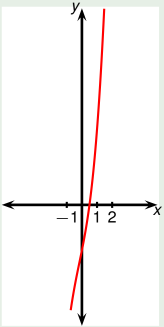
Example

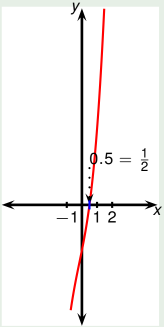


Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = \textbf{?}$.

Example

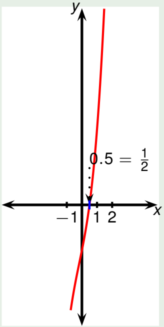


Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$.

Example



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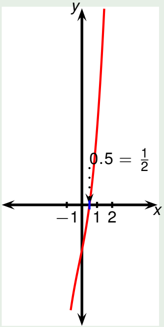
$$2x^3 + x^2 + 5x - 3 \quad = \quad 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)?

Example



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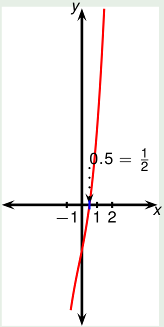

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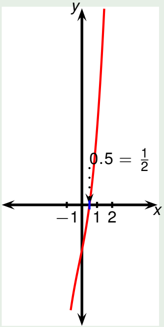
$$2x^3 + x^2 + 5x - 3 = 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$x - \frac{1}{2} \quad \overline{\left)\, 2x^3 + x^2 + 5x - 3 \right.}$$

Example



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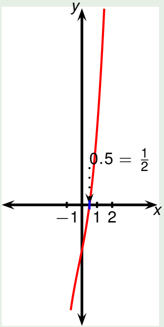

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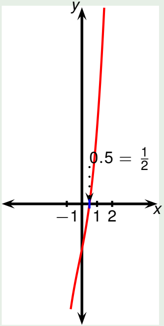

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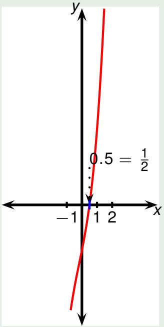
$$\begin{array}{r|l} & \quad ? \\ x - \frac{1}{2} & 2x^3 + x^2 + 5x - 3 \end{array}$$

Divide $2x^3$ by x.

Example



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$$\begin{array}{r|l} & 2x^2 \\ \hline x - \frac{1}{2} & 2x^3 + x^2 + 5x - 3 \end{array}$$

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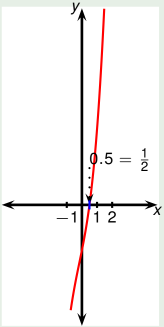
Example



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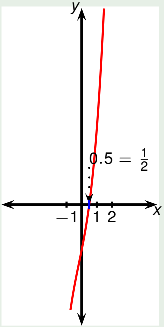
$$\begin{array}{r|l} & 2x^2 \\ \hline x - \frac{1}{2} & 2x^3 + x^2 + 5x - 3 \\ & \textbf{?} \qquad \textbf{?} \\ \hline \end{array}$$

Multiply $2x^2$ by divisor.

Example



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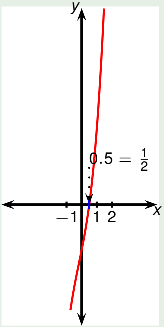
$$\begin{array}{r|l} & 2x^2 \\ \hline x - \frac{1}{2} & 2x^3 + x^2 + 5x - 3 \\ & \underline{2x^3 - x^2} \end{array}$$

Multiply $2x^2$ by divisor.

Example



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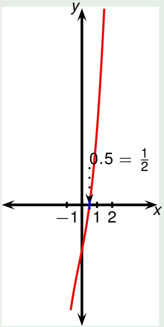
Subtract last two polynomials.

Example



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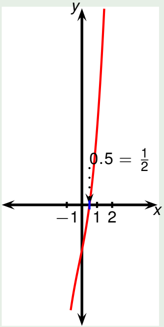

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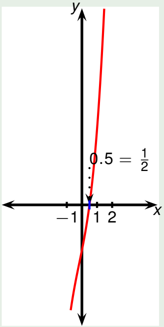
$$\begin{array}{r|l} & 2x^2 \quad \textbf{?} \\ \hline x - \frac{1}{2} & 2x^3 + x^2 + 5x - 3 \\ & 2x^3 - x^2 \\ \hline & \quad\quad 2x^2 + 5x - 3 \end{array}$$

Divide $2x^2$ by x.

Example



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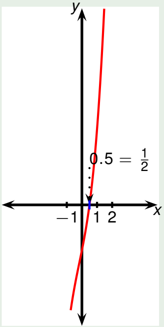
$$2x^3 + x^2 + 5x - 3 \quad = \quad 0$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$\begin{array}{r|l} & 2x^2 + 2x \\ \hline x - \frac{1}{2} & 2x^3 + x^2 + 5x - 3 \\ - & 2x^3 - x^2 \\ \hline & \qquad 2x^2 + 5x - 3 \end{array}$$

Divide $2x^2$ by x.

Example



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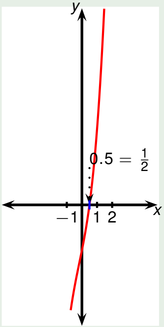
Multiply $2x$ by divisor.

Example



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\hline
 & \quad 2x^2 + 5x - 3 \\
 & \quad 2x^2 - x \\
\hline
\end{array}$$

Multiply $2x$ by divisor.

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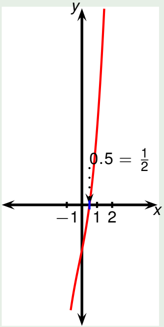

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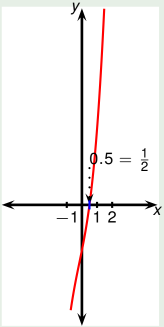
Subtract last two polynomials.

Example



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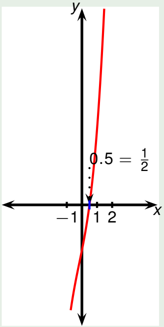
x - \frac{1}{2} & \overline{\left| 2x^3 + x^2 + 5x - 3 \right.} \\
- & 2x^3 - x^2 \\
\hline
 & 2x^2 + 5x - 3 \\
- & 2x^2 - x \\
\hline
 & 6x - 3
\end{array}$$

Subtract last two polynomials.

Example



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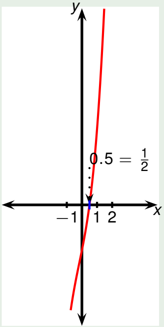
$$
\begin{array}{rl}
 & 2x^2 + 2x \quad \textcolor{red}{\textbf{?}} \\
\textcolor{red}{x} - \frac{1}{2} & \overline{\big) 2x^3 + x^2 + 5x - 3} \\
- & 2x^3 - x^2 \\
\hline
 & \qquad 2x^2 + 5x - 3 \\
- & \qquad 2x^2 - x \\
\hline
 & \qquad\qquad \textcolor{red}{6x} - 3
\end{array}
$$

Divide $\textcolor{red}{6x}$ by $\textcolor{red}{x}$.

Example



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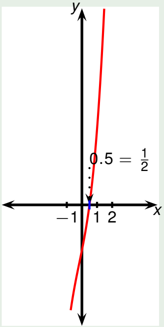
$$\begin{array}{r@{\;}l}
 & 2x^2 + 2x + 6 \\
x - \frac{1}{2} & \overline{\big)\,2x^3 + x^2 + 5x - 3} \\
- & 2x^3 - x^2 \\ \hline
 & \quad 2x^2 + 5x - 3 \\
- & \quad 2x^2 - x \\ \hline
 & \qquad\quad 6x - 3
\end{array}$$

Divide $6x$ by x.

Example



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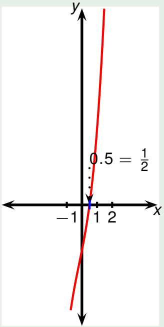
$$
\begin{array}{rr}
 & 2x^2 + 2x + 6 \\
\cline{2-2}
x - \frac{1}{2} & \big) \; 2x^3 + x^2 + 5x - 3 \\
- & 2x^3 - x^2 \qquad\qquad \\
\cline{2-2}
 & 2x^2 + 5x - 3 \\
- & 2x^2 - x \qquad \\
\cline{2-2}
 & 6x - 3 \\
 & ? \quad ? \\
\cline{2-2}
\end{array}
$$

Multiply 6 by divisor.

Example



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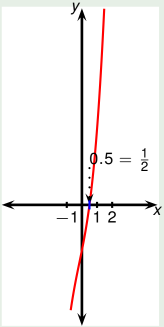
 & 2x^2 + 2x + 6 \\ \cline{2-2}
x - \frac{1}{2} & \big) \; 2x^3 + x^2 + 5x - 3 \\
- & 2x^3 - x^2 \\ \cline{2-2}
 & \qquad 2x^2 + 5x - 3 \\
- & \qquad 2x^2 - x \\ \cline{2-2}
 & \qquad\qquad 6x - 3 \\
 & \qquad\qquad 6x - 3 \\ \cline{2-2}
\end{array}$$

Multiply 6 by divisor.

Example



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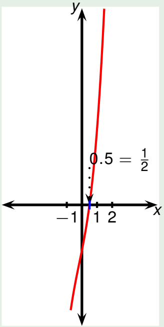
$$\begin{array}{r|l} & \quad 2x^2 + 2x + 6 \\ x - \frac{1}{2} & \overline{2x^3 + x^2 + 5x - 3} \\ - & \underline{2x^3 - x^2} \\ & \qquad 2x^2 + 5x - 3 \\ - & \qquad \underline{2x^2 - x} \\ & \qquad\qquad 6x - 3 \\ - & \qquad\qquad \underline{6x - 3} \\ & \qquad\qquad\quad ? \end{array}$$

Subtract last two polynomials.

Example



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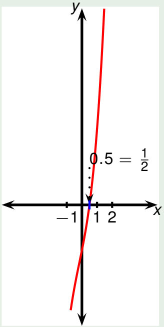
$$\begin{array}{rr}
 & 2x^2 + 2x + 6 \\
x - \frac{1}{2} & \overline{\big)\, 2x^3 + x^2 + 5x - 3} \\
- & 2x^3 - x^2 \\
\hline
 & 2x^2 + 5x - 3 \\
- & 2x^2 - x \\
\hline
 & {\color{red}6x - 3} \\
{\color{red}-} & {\color{red}6x - 3} \\
\hline
 & {\color{red}0}
\end{array}$$

Subtract last two polynomials.

Example



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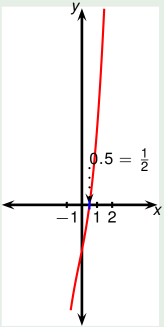
- & 2x^3 - x^2 \\
\hline
 & \qquad 2x^2 + 5x - 3 \\
- & \qquad 2x^2 - x \\
\hline
 & \qquad\qquad 6x - 3 \\
- & \qquad\qquad 6x - 3 \\
\hline
 & \qquad\qquad\quad 0
\end{array}$$

Example



Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

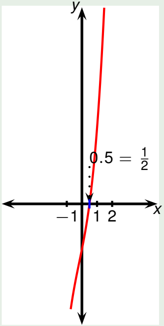
$$\begin{aligned} 2x^3 + x^2 + 5x - 3 &= 0 \\ \left(x - \tfrac{1}{2}\right)\left(2x^2 + 2x + 6\right) + 0 &= 0 \end{aligned}$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

Quotient: $2x^2 + 2x + 6$

$$\begin{array}{r|l} x - \frac{1}{2} & 2x^3 + x^2 + 5x - 3 \\ - & 2x^3 - x^2 \\ \hline & \quad 2x^2 + 5x - 3 \\ - & \quad 2x^2 - x \\ \hline & \qquad 6x - 3 \\ - & \qquad 6x - 3 \\ \hline & \qquad\quad 0 \end{array}$$

Example



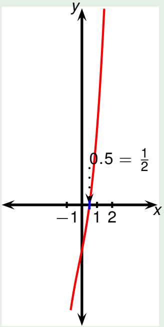
Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$\begin{aligned} 2x^3 + x^2 + 5x - 3 &= 0 \\ \left(x - \tfrac{1}{2}\right)\left(2x^2 + 2x + 6\right) + 0 &= 0 \end{aligned}$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)? Factor:

$$\begin{array}{rr} \textbf{Quotient:} & 2x^2 + 2x + 6 \\ x - \frac{1}{2} & \overline{)\,2x^3 + x^2 + 5x - 3} \\ - & 2x^3 - x^2 \qquad\qquad \\ \hline & 2x^2 + 5x - 3 \\ - & 2x^2 - x \qquad \\ \hline & 6x - 3 \\ - & 6x - 3 \\ \hline \textbf{Remainder:} & 0 \end{array}$$

Example



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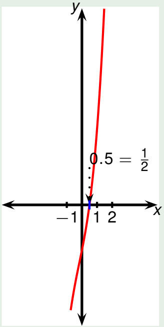
\textbf{Quotient:} & & 2x^2 + 2x + 6 \\
x - \frac{1}{2} & & 2x^3 + x^2 + 5x - 3 \\
- & & 2x^3 - x^2 \\
\hline
 & & 2x^2 + 5x - 3 \\
- & & 2x^2 - x \\
\hline
 & & 6x - 3 \\
- & & 6x - 3 \\
\hline
\textbf{Remainder:} & & 0
\end{array}$$

Example



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$x - \frac{1}{2} = 0$ **or** $x =$

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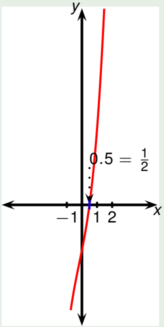
Example



Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

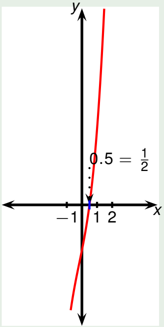
$$\begin{aligned} 2x^3 + x^2 + 5x - 3 &= 0 \\ \left(x - \tfrac{1}{2}\right)\left(2x^2 + 2x + 6\right) &= 0 \end{aligned}$$

$$x - \frac{1}{2} = 0 \quad \textbf{or} \quad x =$$

$$x = \frac{1}{2}$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)?

Example



Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$
\begin{aligned}
2x^3 + x^2 + 5x - 3 &= 0 \\
\left(x - \frac{1}{2}\right)\left(2x^2 + 2x + 6\right) &= 0
\end{aligned}
$$

$$x - \frac{1}{2} = 0 \quad \textbf{or} \quad x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2}$$

$$x = \frac{1}{2}$$

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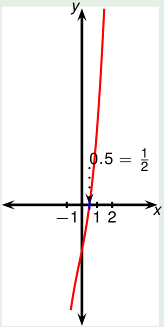
Example



Plot the left hand side of the equation with a graphing calculator. Find all real solutions of the equation.

$$\begin{aligned} 2x^3 + x^2 + 5x - 3 &= 0 \\ \left(x - \tfrac{1}{2}\right)\left(2x^2 + {\color{red}2}x + 6\right) &= 0 \end{aligned}$$

$$x - \frac{1}{2} = 0 \quad \textbf{or} \quad x = \frac{-{\color{red}2} \pm \sqrt{{\color{red}2}^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2}$$

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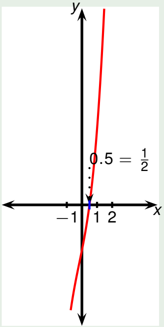

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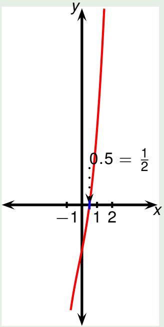

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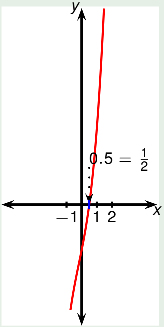
$$x = \frac{1}{2} \qquad\qquad x = \frac{-2 \pm \sqrt{-44}}{2 \cdot 2}$$

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)?

Example



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$$x = \frac{1}{2} \qquad\qquad x = \frac{-2 \pm \sqrt{-44}}{2 \cdot 2}$$

no real solution

We see only one root, $x = 0.5 = \frac{1}{2}$. Is our guess correct? Is there another root (far away from 0)?

Example

Solve the inequality.

$$2x^2 + 3x - 5 \quad \geq \quad 0$$

Example

Solve the inequality.

$$
\begin{array}{rcl}
2x^2+3x-5 & \geq & 0\\
(\mathbf{?} \qquad)(\mathbf{?} \qquad) & \geq & 0
\end{array}
$$

Example

Solve the inequality.

$$\begin{aligned} 2x^2+3x-5 &\geq 0 \\ (2x+5)(x-1) &\geq 0 \end{aligned}$$

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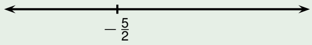
Left hand side vanishes when $x = -\frac{5}{2}$ and when $x = 1$.



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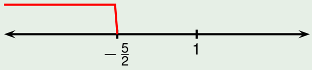

Interval	Factor signs	Final sign	
$\left(-\infty, -\frac{5}{2}\right)$			

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Solve the inequality.

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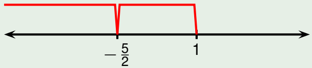

Interval	Factor signs	Final sign	
$\left(-\infty, -\frac{5}{2}\right)$			
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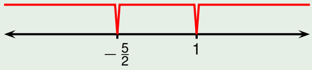

Interval	Factor signs	Final sign	
$\left(-\infty, -\frac{5}{2}\right)$			
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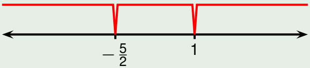

Interval	Factor signs	Final sign	
$\left(-\infty, -\frac{5}{2}\right)$			
$\left(-\frac{5}{2}, 1\right)$			
$(1, \infty)$	$(\mathbf{?}\ )(\mathbf{?}\ \ )$		

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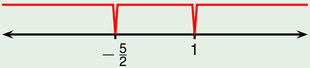

Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$			
$\left(-\frac{5}{2},1\right)$			
$(1,\infty)$	$(+)($**?** $)$		

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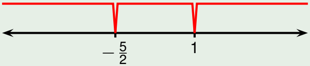

Interval	Factor signs	Final sign	
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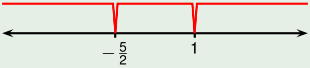

Interval	Factor signs	Final sign	
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$\left(-\frac{5}{2}, 1\right)$			
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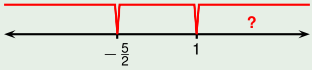

Interval	Factor signs	Final sign	
$\left(-\infty, -\frac{5}{2}\right)$			
$\left(-\frac{5}{2}, 1\right)$			
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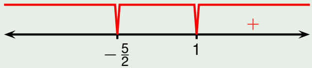

Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$			
$\left(-\frac{5}{2},1\right)$			
$(1,\infty)$	$(+)(+)$	$+$	

Example

Solve the inequality.

$$\begin{array}{rcl} 2x^2+3x-5 & \geq & 0 \\ ({\color{red}2x+5})(x-1) & \geq & 0 \end{array}$$

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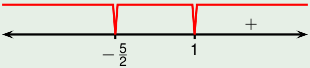

Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$			
$\color{red}\left(-\frac{5}{2},1\right)$	$(\color{red}\mathbf{?}\color{black})(\mathbf{?}\ \ )$		
$(1,\infty)$	$(+)(+)$	$+$	

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Solve the inequality.

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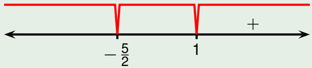

Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$			
$\left(-\frac{5}{2},1\right)$	$(+)(\textbf{?}\ )$		
$(1,\infty)$	$(+)(+)$	$+$	

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Solve the inequality.

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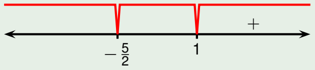

Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$			
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$(1,\infty)$	$(+)(+)$	$+$	

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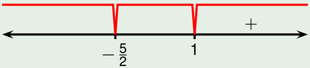

Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$			
$\left(-\frac{5}{2},1\right)$	$(+)(-)$		
$(1,\infty)$	$(+)(+)$	$+$	

Example

Solve the inequality.

$$\begin{array}{rcl} 2x^2 + 3x - 5 & \geq & 0 \\ (2x+5)(x-1) & \geq & 0 \end{array}$$

Left hand side vanishes when $x = -\frac{5}{2}$ and when $x = 1$.
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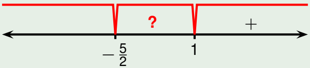

Interval	Factor signs	Final sign	
$\left(-\infty, -\frac{5}{2}\right)$			
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$(1, \infty)$	$(+)(+)$	$+$	

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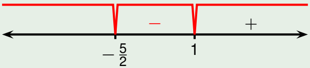

Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$			
$\left(-\frac{5}{2},1\right)$	$(+)(-)$	$-$	
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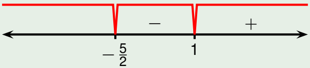

Interval	Factor signs	Final sign	
${\color{red}\left(-\infty,-\frac{5}{2}\right)}$	(**?**)(**?**)		
$\left(-\frac{5}{2},1\right)$	$(+)(-)$	$-$	
$(1,\infty)$	$(+)(+)$	$+$	

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Solve the inequality.

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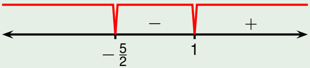

Interval	Factor signs	Final sign	
$\left(-\infty, -\frac{5}{2}\right)$	$(-)($**?**$\ )$		
$\left(-\frac{5}{2}, 1\right)$	$(+)(-)$	$-$	
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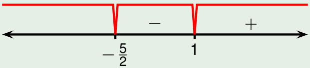

Interval	Factor signs	Final sign	
$\left(-\infty,-\frac{5}{2}\right)$	$(-)(\textbf{?}\ )$		
$\left(-\frac{5}{2},1\right)$	$(+)(-)$	$-$	
$(1,\infty)$	$(+)(+)$	$+$	

Example

Solve the inequality.

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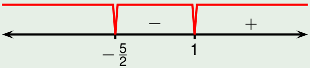

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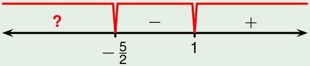

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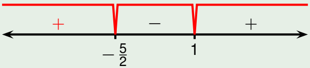

Interval	Factor signs	Final sign	
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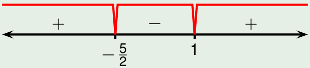

Interval	Factor signs	Final sign	Sample pt	Value at sample pt
$\left(-\infty,-\frac{5}{2}\right)$	$(-)(-)$	$+$	-100	$f(-100)>0$
$\left(-\frac{5}{2},1\right)$	$(+)(-)$	$-$	0	$f(0)=-5<0$
$(1,\infty)$	$(+)(+)$	$+$	100	$f(100)>0$

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Solve the inequality.

$$\begin{aligned} 2x^2+3x-5 &\geq 0 \\ (2x+5)(x-1) &\geq 0 \end{aligned}$$

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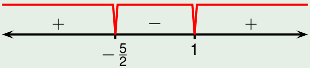

Interval	Factor signs	Final sign	Sample pt	Value at sample pt
$\left(-\infty,-\frac{5}{2}\right)$	$(-)(-)$	$+$	-100	$f(-100)>0$
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Example

Solve the inequality.

$$\begin{aligned} 2x^2 + 3x - 5 &\geq 0 \\ (2x+5)(x-1) &\geq 0 \\ x \in \left(-\infty, -\tfrac{5}{2}\right] \cup [1, \infty) \end{aligned}$$

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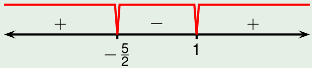

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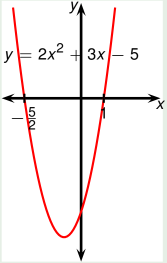

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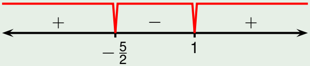

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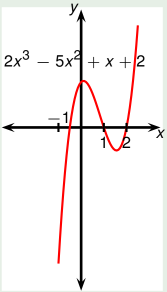
Example

Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality.

$$2x^3 - 5x^2 + x + 2 \quad > \quad 0$$

Example



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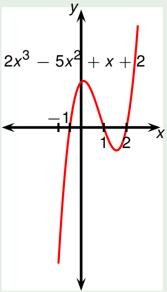
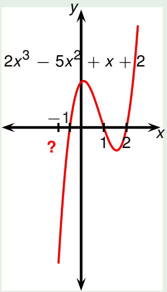


Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality.

$$\begin{array}{rcl} \color{red}{2x^3 - 5x^2 + x + 2} & > & 0 \\ \textbf{?}\ (x - \qquad\quad)\,(x - \quad)(x - \quad) & > & 0 \end{array}$$

Example



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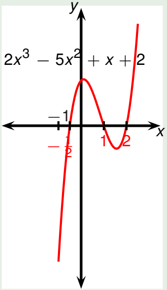
$$\begin{array}{rcl} 2x^3 - 5x^2 + x + 2 & > & 0 \\ \mathbf{?}\,(x - \mathbf{?} \quad)(x - \mathbf{?})(x - \mathbf{?}) & > & 0 \end{array}$$

Example

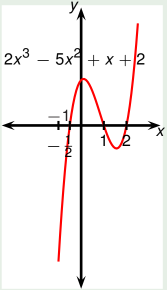


Plot the function $2x^3 - 5x^2 + x + 2$. Solve the inequality.

$$2x^3 - 5x^2 + x + 2 \quad > \quad 0$$

$$\textbf{?}\ \left(x - \left(-\frac{1}{2}\right)\right)(x - 1)(x - 2) \quad > \quad 0$$

Example



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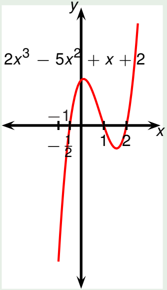

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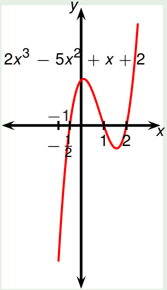

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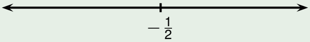

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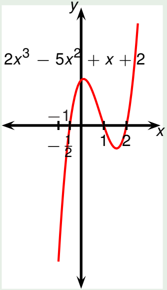

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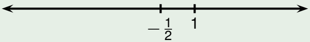
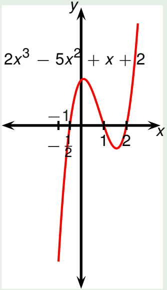


Example



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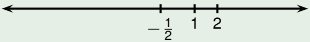
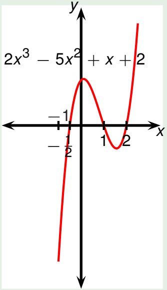


Example



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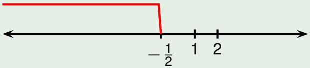
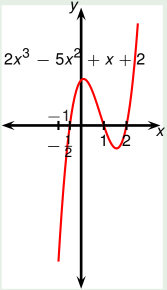


Interval	Factor signs	Final sign from plot
$\left(-\infty, -\frac{1}{2}\right)$		

Example



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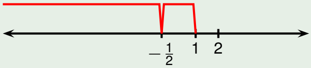

Interval	Factor signs	Final sign from plot
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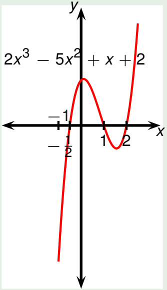

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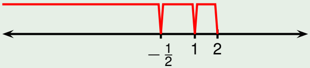

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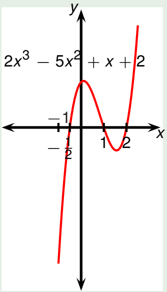
$(1, 2)$		

Example



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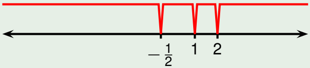

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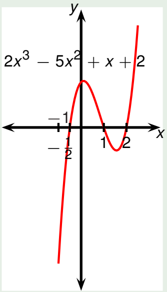

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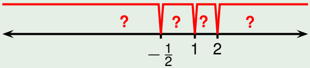

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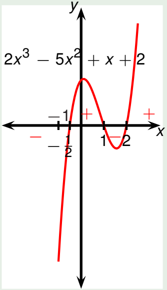
Example



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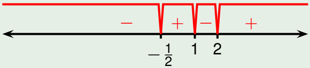

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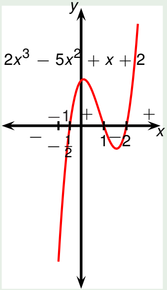

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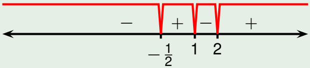

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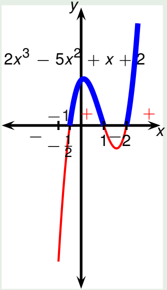
Example



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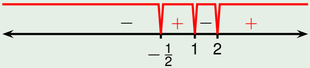
$$\begin{aligned} 2x^3 - 5x^2 + x + 2 &> 0 \\ 2\left(x - \left(-\tfrac{1}{2}\right)\right)(x-1)(x-2) &> 0 \end{aligned}$$

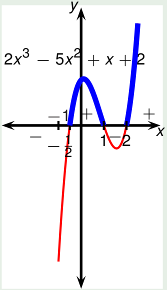
$$x \in \left(-\tfrac{1}{2}, 1\right) \cup (2, \infty)$$

Left hand side vanishes when $x = -\frac{1}{2}$, when $x = 1$ and when $x = 2$. The two roots split the real line into four intervals: $\left(-\infty, -\frac{1}{2}\right), \left(-\frac{1}{2}, 1\right), (1, 2), (2, \infty)$.



Interval	Factor signs	Final sign from plot
$\left(-\infty, -\frac{1}{2}\right)$	$(-)(-)(-)$	$-$
$\left(-\frac{1}{2}, 1\right)$	$(+)(-)(-)$	$+$
$(1, 2)$	$(+)(+)(-)$	$-$
$(2, \infty)$	$(+)(+)(+)$	$+$

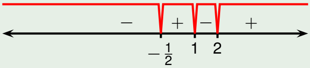
Example



Plot the function $2x^3-5x^2+x+2$. Solve the inequality.

$$\begin{array}{rcl} 2x^3-5x^2+x+2 & > & 0 \\ 2\left(x-\left(-\frac{1}{2}\right)\right)(x-1)(x-2) & > & 0 \\ x\in\left(-\frac{1}{2},1\right)\cup(2,\infty) & & \end{array}$$

Left hand side vanishes when $x=-\frac{1}{2}$, when $x=1$ and when $x=2$. The two roots split the real line into four intervals: $\left(-\infty,-\frac{1}{2}\right),\left(-\frac{1}{2},1\right),(1,2),(2,\infty)$.



Interval	Factor signs	Final sign from plot
$\left(-\infty,-\frac{1}{2}\right)$	$(-)(-)(-)$	$-$
$\left(-\frac{1}{2},1\right)$	$(+)(-)(-)$	$+$
$(1,2)$	$(+)(+)(-)$	$-$
$(2,\infty)$	$(+)(+)(+)$	$+$