Calculus I
Lecture 21
The Fundamental Theorem of Calculus
Part I

Todor Milev
https://github.com/tmilev/freecalc

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Outline

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Antiderivatives

Definition (Antiderivative)

A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

Example

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- No. If $G(x) = \frac{1}{3}x^3 + 1$, then $G'(x) = x^2 = f(x)$.

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- $\frac{1}{3}x^3 + 2$ will also work.
- Any function of the form $H(x) = \frac{1}{3}x^3 + C$, where C is a constant, is an antiderivative of f.

Theorem

If F is an antiderivative of f on an interval I, then an arbitrary antiderivative of f on I is of the form

$$F(x) + C$$

where C is an arbitrary constant.

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Find all antiderivatives of each of the following functions.

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$$G(x) = \begin{cases} \ln|x| + C_1 & \text{if} \quad x > 0 \\ \ln|x| + C_2 & \text{if} \quad x < 0 \end{cases}$$

Every differentiation formula gives rise to an antidifferentiation formula. Suppose $F' = f$ and $G' = g$.

Function	Particular Antiderivative
$cf(x)$	
$f(x) + g(x)$	
$x^n (n \neq -1)$	
$\frac{1}{x}$	
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$$= -4\cos x + \frac{2}{5}x^5 - 2\sqrt{x} + C$$

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$$\begin{aligned} f'(x) &= \frac{1}{x\sqrt{x}} = x^{-3/2} \\ f(x) &= \frac{x^{-1/2}}{-\frac{1}{2}} + C \\ &= -\frac{2}{\sqrt{x}} + C \end{aligned}$$

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$$-\frac{2}{\sqrt{1}} + C = 1$$

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To find C, use the fact that $f(1) = 1$.

$$\begin{aligned} f(1) &= 1 \\ -\frac{2}{\sqrt{1}} + C &= 1 \\ C &= 3 \end{aligned}$$

Example

Find f if $f'(x) = \frac{1}{x\sqrt{x}}$ for $x > 0$, and $f(1) = 1$.

$$f'(x) = \frac{1}{x\sqrt{x}} = x^{-3/2}$$

$$f(x) = \frac{x^{-1/2}}{-\frac{1}{2}} + C$$

$$= -\frac{2}{\sqrt{x}} + C$$

To find C, use the fact that $f(1) = 1$.

$$f(1) = 1$$

$$-\frac{2}{\sqrt{1}} + C = 1$$

$$C = 3$$

Therefore

$$f(x) = -\frac{2}{\sqrt{x}} + 3.$$

Theorem (The Evaluation Theorem (FTC part 2))

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a),$$

where F is any antiderivative of f.

$$\int_a^b f(x)dx \text{ exists for any continuous (over } [a,b]) $$ function f.

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Theorem

Let f be a continuous function on $[a, b]$. Then f is integrable over $[a, b]$.

In other words, $\int_a^b f(x)dx$ exists for any continuous (over $[a, b]$) function f.

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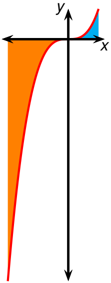
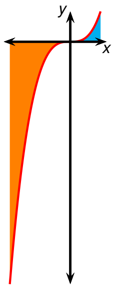


Evaluate the integral $\int_{-2}^{1} x^3 \, \mathrm{d}x$.

Example



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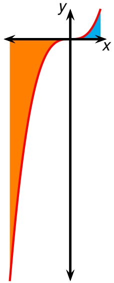

Evaluate the integral $\int_{-2}^{1} x^3 \, dx$.

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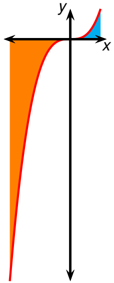
- An antiderivative is $F(x) =$ **?**

Example



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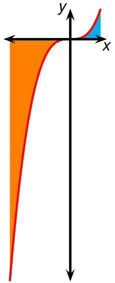

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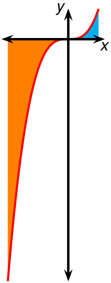

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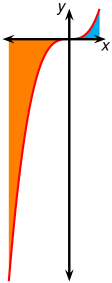

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$$\int_{-2}^{1} x^3 \, \mathrm{d}x = F(1) - F(-2) = \frac{1}{4}(1)^4 - \frac{1}{4}(-2)^4 = \frac{1}{4} - \frac{16}{4} = -\frac{15}{4}$$

We often use the notation

$$F(x)]_a^b = F(b) - F(a)$$

or

$$[F(x)]_a^b = F(b) - F(a)$$

Therefore we can write

$$\int_a^b f(x)\mathrm{d}x = F(x)]_a^b$$

or

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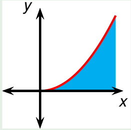
Example

Find the area under the parabola $y = x^2$ from 0 to 1.



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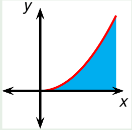

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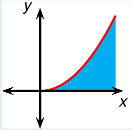

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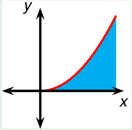

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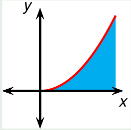

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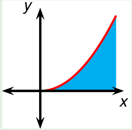

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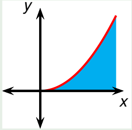

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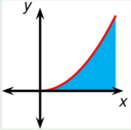

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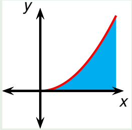

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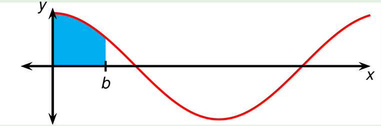

Find the area under the cosine curve from 0 to b, where $0 \leq b \leq \frac{\pi}{2}$.

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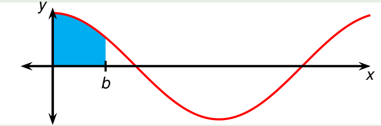

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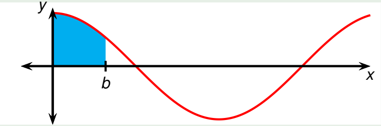

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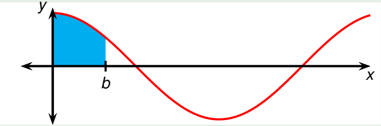

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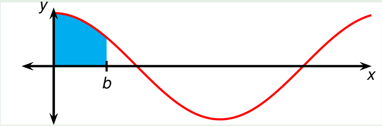

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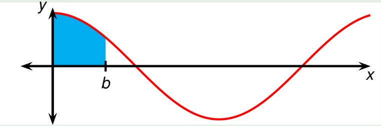

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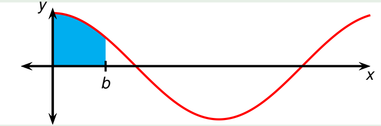

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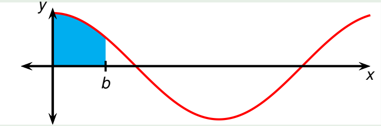

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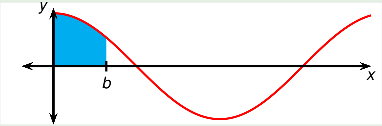

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Indefinite Integrals

- The Evaluation Theorem establishes a connection between antiderivatives and definite integrals.
- It says that $\int_a^b f(x)\mathrm{d}x$ equals $F(b) - F(a)$, where F is an antiderivative of f.
- We need convenient notation for writing antiderivatives.
- This is what the indefinite integral is.

Definition (Indefinite Integral)

The indefinite integral of f is another way of saying the antiderivative of f, and is written $\int f(x)\mathrm{d}x$. In other words,

$$\int f(x)\mathrm{d}x = F(x) \qquad \text{means} \qquad F'(x) = f(x).$$

Example

$$\int x^4 \mathrm{d}x = \textbf{?}$$

Example

$$\int x^4 \mathrm{d}x = \frac{x^5}{5}$$

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- The indefinite integral represents a whole family of functions.
- Example: the general antiderivative of $\frac{1}{x}$ is
$$F(x) = \begin{cases} \ln|x| + C_1 & \text{if} \quad x > 0 \\ \ln|x| + C_2 & \text{if} \quad x < 0 \end{cases}$$

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- $\int \frac{1}{x}\mathrm{d}x = \ln|x| + C$, and this is valid either on $(-\infty, 0)$ or $(0, \infty)$.

Example

Find the indefinite integral.

$$\int (8x^3 - 3\sec^2 x)\mathrm{d}x$$

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$$\int (8x^3 - 3\sec^2 x)\mathrm{d}x = 8\int x^3\mathrm{d}x - 3\int \sec^2 x\mathrm{d}x$$

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$$\int (8x^3 - 3\sec^2 x)\mathrm{d}x = 8\int x^3 \mathrm{d}x - 3\int \sec^2 x \mathrm{d}x$$
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Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} \mathrm{d}\theta$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} \mathrm{d}\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) \mathrm{d}\theta$$

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Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x$$

Example

$$\int_{0}^{3}(x^3-6x)\mathrm{d}x=\left[\int(x^3-6x)\mathrm{d}x\right]_{0}^{3}$$

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$$\begin{aligned}\int_0^3 (x^3 - 6x)\mathrm{d}x &= \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3\\ &= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3\\ &= \left[\frac{x^4}{4} - 6\textbf{?}\quad\right]_0^3\end{aligned}$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x = \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3$$
$$= \left[\int x^3 \mathrm{d}x - 6 \int x \mathrm{d}x\right]_0^3$$
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Example

$$
\begin{aligned}
\int_0^3 (x^3 - 6x)\mathrm{d}x &= \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3 \\
&= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3 \\
&= \left[\frac{x^4}{4} - 6\,\frac{x^2}{2}\right]_0^{3} \\
&= \left(\frac{1}{4}\cdot 3^4 - 3\cdot 3^2\right) - \left(\frac{1}{4}\cdot 0^4 - 3\cdot 0^2\right)
\end{aligned}
$$

Example

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&= \left(\frac{1}{4}\cdot 3^4 - 3\cdot 3^2\right) - \left(\frac{1}{4}\cdot 0^4 - 3\cdot 0^2\right) \\
&= \frac{81}{4} - 27 - 0 + 0
\end{aligned}
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&= \left[\frac{x^4}{4} - 6\,\frac{x^2}{2}\right]_0^3 \\
&= \left(\frac{1}{4}\cdot 3^4 - 3\cdot 3^2\right) - \left(\frac{1}{4}\cdot 0^4 - 3\cdot 0^2\right) \\
&= \frac{81}{4} - 27 - 0 + 0 = -\frac{27}{4}.
\end{aligned}
$$

Example

Evaluate: $$\int_1^9 \frac{2t^3 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$$

Example

Evaluate: $$\int_1^9 \frac{2t^3 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$$

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$$= \left[\int 2t\mathrm{d}t + \int t^{\frac{1}{2}}\mathrm{d}t - \int t^{-2}\mathrm{d}t\right]_1^9$$

$$= \left[? + ? - ?\right]_1^9$$

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$$= \left[\int 2t\mathrm{d}t + \int t^{\frac{1}{2}}\mathrm{d}t - \int t^{-2}\mathrm{d}t\right]_1^9$$

$$= \left[t^2 + \textbf{?} \quad - \textbf{?} \quad\quad \right]_1^9$$

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$$= \left(9^2 + \frac{2}{3}\cdot 9^{\frac{3}{2}} + \frac{1}{9}\right) - \left(1^2 + \frac{2}{3}\cdot 1^{\frac{3}{2}} + \frac{1}{1}\right)$$

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$$= 81 + 18 + \frac{1}{9} - 1 - \frac{2}{3} - 1$$

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Evaluate: $\displaystyle\int_1^9 \frac{2t^3 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$

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$$= \left(9^2 + \frac{2}{3} \cdot 9^{\frac{3}{2}} + \frac{1}{9}\right) - \left(1^2 + \frac{2}{3} \cdot 1^{\frac{3}{2}} + \frac{1}{1}\right)$$

$$= 81 + 18 + \frac{1}{9} - 1 - \frac{2}{3} - 1 = \frac{868}{9}.$$