

Calculus I

Lecture 18

Todor Milev

`https://github.com/tmilev/freecalc`

2020

Outline

1 Newton's Method

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$$x^3 - 5x^2 - 6x = 0$$

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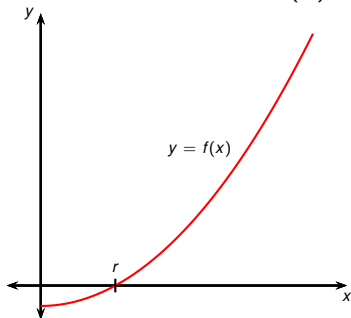
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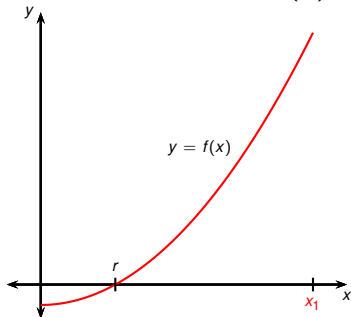
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- Roots: $x = 0, -1$, or 6 .
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- How does the computer find the root?
- Probably using Newton's Method.

Goal: find a root r of $f(x)$.

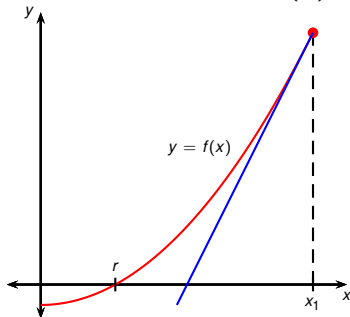


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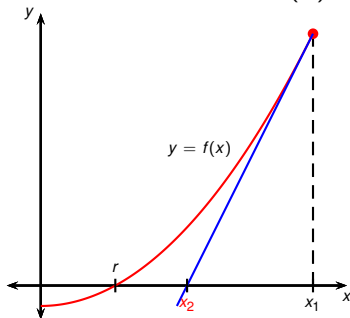
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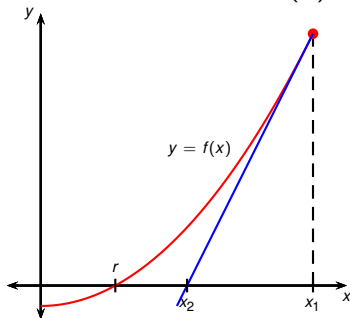
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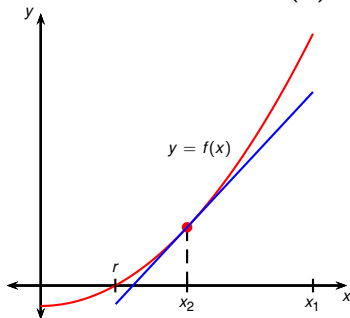
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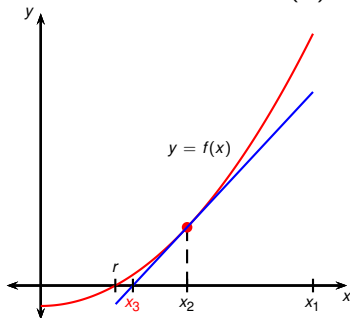
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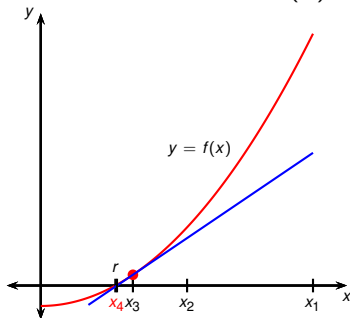
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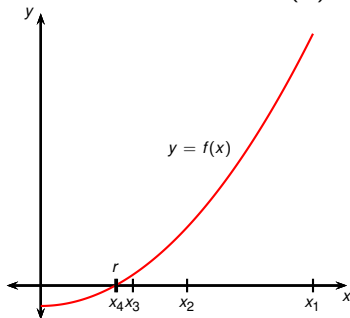
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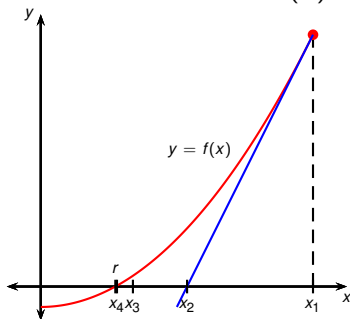
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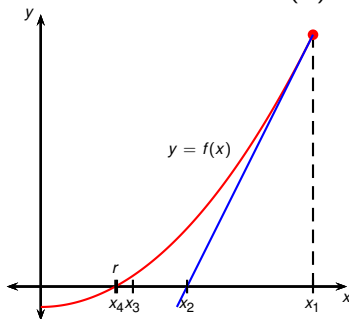
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Equation: $y - ? = ? (x - ?)$

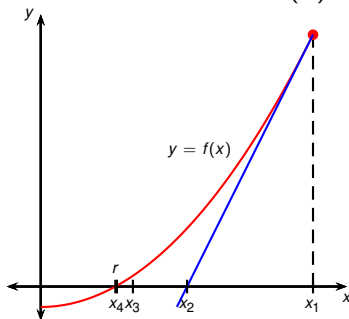
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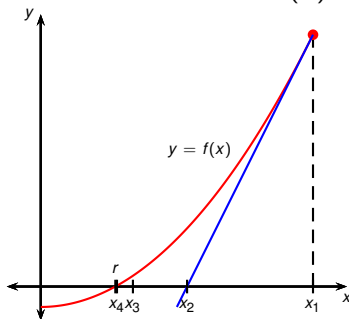
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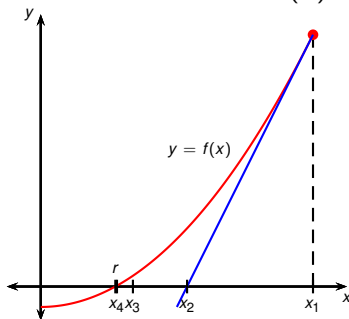
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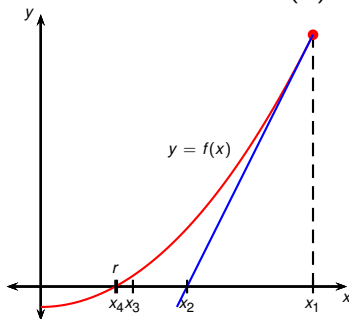
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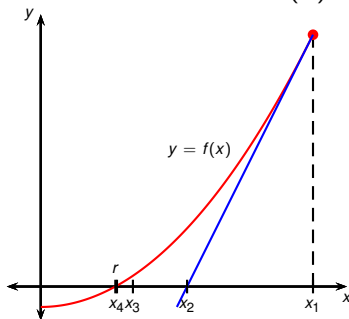
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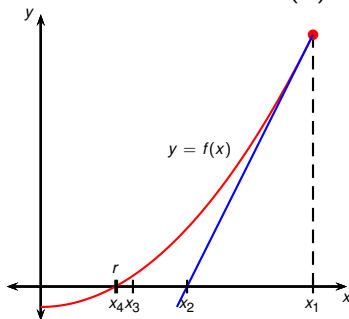
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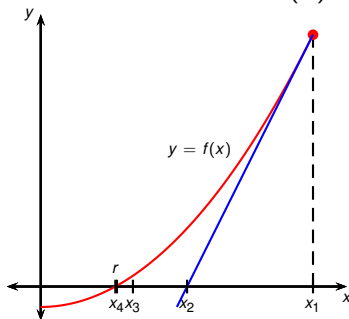
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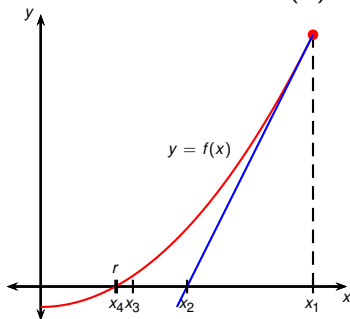
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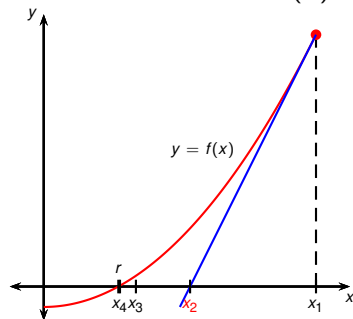
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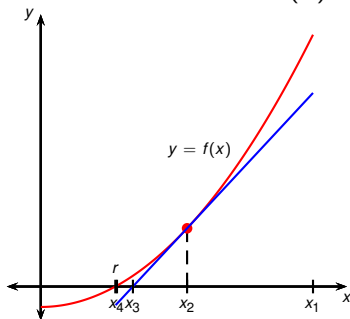


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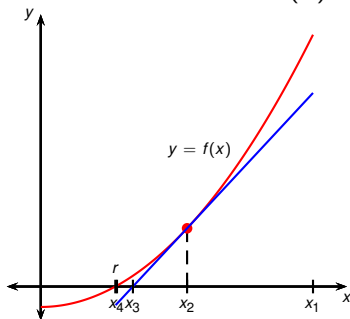
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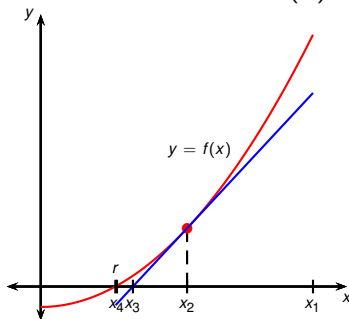


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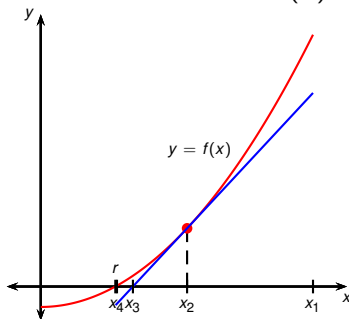


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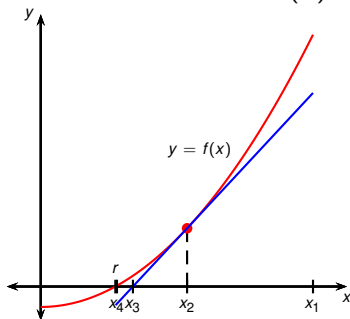


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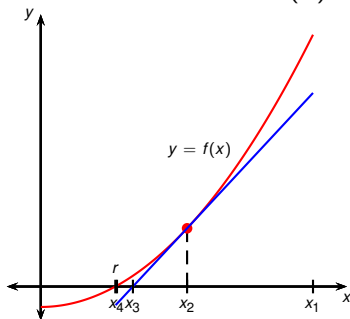


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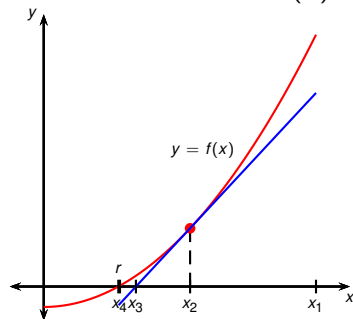


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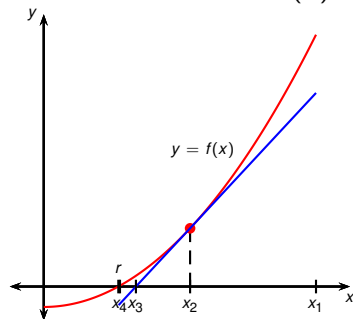


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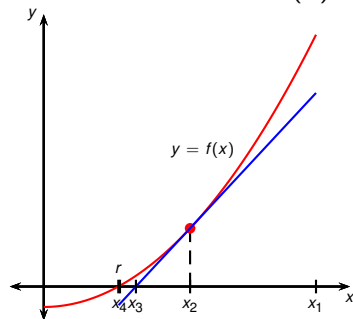


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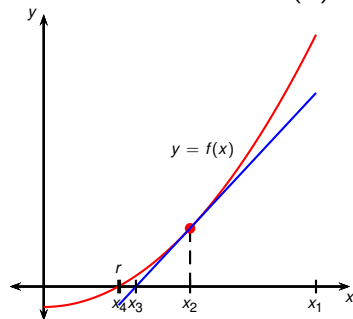
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$$x_3 = \frac{f'(x_2)x_2 - f(x_2)}{f'(x_2)}$$

Goal: find a root r of $f(x)$.

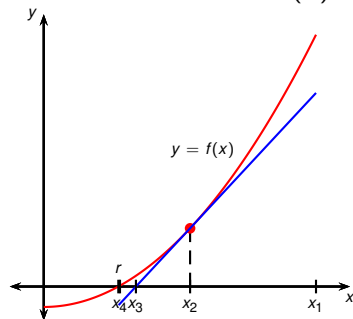


$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

- Pick a number x_1 .
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- Call the x-intercept of this line x_2 .
- Repeat the process using x_2 .
- Find the tangent to f at $(x_2, f(x_2))$.
- **Call the x-intercept of this line x_3 , and so on.**

$$\begin{aligned} \text{Equation: } y - f(x_2) &= f'(x_2)(x - x_2) \\ \text{x-intercept: } 0 - f(x_2) &= f'(x_2)(x_3 - x_2) \\ f'(x_2)x_2 - f(x_2) &= f'(x_2)x_3 \\ x_3 &= \frac{f'(x_2)x_2 - f(x_2)}{f'(x_2)} \end{aligned}$$

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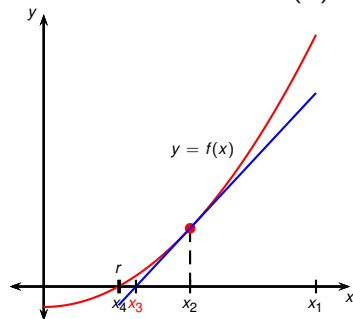


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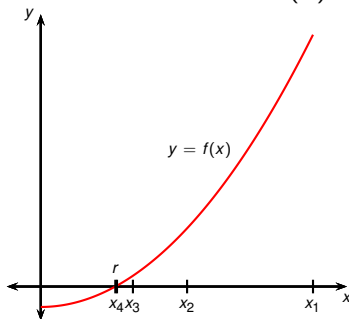
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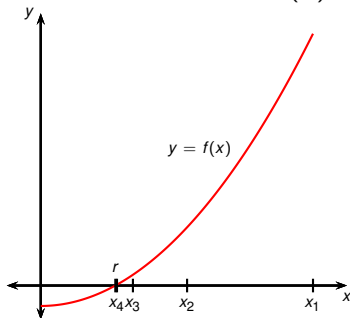
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$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

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 $x_{n+1} = \frac{f'(x_n)x_n - f(x_n)}{f'(x_n)}$
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- Newton's Method gives us a sequence x_1, x_2, x_3, \dots of approximations to a root r of a function $f(x)$.
- If the n th approximation is x_n and $f'(x_n) \neq 0$, then the $(n+1)$ st approximation is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- If the numbers x_n become closer and closer to r as n becomes large, we say that the sequence converges to r .
- The sequence does not always converge.

Example (Newton's Method, Example 1, p. 313)

Starting with $x_1 = 2$, find the third approximation x_3 to the root of the equation $x^3 - 2x - 5 = 0$.

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Example (Newton's Method)

Starting with $x_1 = 5$, use two steps of Newton's Method to approximate $\sqrt{28}$.

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Starting with $x_1 = 5$, use two steps of Newton's Method to approximate $\sqrt{28}$.

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Newton's Method:
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Example (Newton's Method)

Starting with $x_1 = 5$, use two steps of Newton's Method to approximate $\sqrt{28}$.

$$f(x) = x^2 - 28.$$

$$f'(x) = 2x.$$

Newton's Method:
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 28}{2x_n}$$

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