Calculus III
Homework on Lecture 2

1. Carry out the indicated operations between the indicated vectors.

$$\begin{aligned} \mathbf{u} &= (-1, 2, 3) \\ \mathbf{v} &= (2, -3, -5) \\ \mathbf{w} &= (3, 5, -7). \end{aligned}$$

   (a) $-\mathbf{u}$

   answer: $(1, -2, -3)$

   (b) $\mathbf{u} + \mathbf{v}$

   answer: $(1, -1, -2)$

   (c) $\mathbf{u} - 2\mathbf{w}$

   answer: $(-7, -8, 17)$

   (d) $-3\mathbf{w} + \frac{\mathbf{v}}{2}$

   answer: $\left(-8, -\frac{33}{2}, \frac{37}{2}\right)$

   (e) $\frac{\mathbf{w}+2\mathbf{u}+3\mathbf{v}}{6}$

   answer: $(\frac{7}{6}, 0, -\frac{8}{3})$

   (f) $\mathbf{u} + \mathbf{w} - (2\mathbf{v} + 3\mathbf{u})$

   answer: $(1, 7, -3)$

2. Compute the dot product.

   (a) $\mathbf{u} = (2, -3, 5)$, $\mathbf{v} = (-3, 5, 7)$.

   answer: $-6 - 15 + 35 = 14$

   (b) $\mathbf{u} = (\frac{1}{2}, \frac{1}{3}, \frac{1}{4})$, $\mathbf{v} = (\frac{1}{3}, \frac{1}{4}, \frac{1}{5})$.

   answer: $\frac{3}{10}$

3. Determine if the vectors are orthogonal.

   (a) $\mathbf{u} = (1, 2, 3)$, $\mathbf{v} = (-1, 2, -1)$.

   answer: $\mathbf{u} \perp \mathbf{v}$

   (b) $\mathbf{u} = (1, 0, 1)$, $\mathbf{v} = (-1, 1, 1)$.

   answer: $\mathbf{u} \perp \mathbf{v}$

   (c) $\mathbf{u} = (-1, 0, 1)$, $\mathbf{v} = (-1, 1, 1)$.

   answer: $\mathbf{u} \not\perp \mathbf{v}$

4. Find the angles between the vectors. You may use a calculator to get a numerical approximation.

   (a) $\mathbf{u} = (1, 2, 3)$, $\mathbf{v} = (3, 1, 2)$.

   answer: $\arccos\left(\frac{11}{14}\right) \approx 0.666946$

   (b) $\mathbf{u} = (-1, -1, -1)$, $\mathbf{v} = (0, 0, 1)$

   answer: $\arccos\left(-\frac{\sqrt{3}}{3}\right) \approx 2.186276$

5. A tetrahedron is a pyramid whose base is a triangle. The 8 points $(1, 1, 1)$, $(-1, 1, 1)$, $(1, -1, 1)$, $(-1, -1, 1)$, $(1, 1, -1)$, $(-1, 1, -1)$, $(1, -1, -1)$, $(-1, -1, -1)$ (all possible sign combinations) give the vertices of a cube with edge 2 units.

   (a) Find 4 vertices of the cube so they form a regular tetrahedron, i.e., 4 points that are not in the same plane and such that the distance between any two is equal.

   (b) Form two vectors, $\mathbf{u}$ and $\mathbf{v}$, by connecting the origin with any two of the 4 points you found.

   (c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

(d) What is the angle between the two bonds of hydrogen atoms in the methane molecule CH_4?

answer: $\arccos\left(-\frac{1}{3}\right) \approx 109.471207^\circ$

6. Find the

   - Scalar projection $\text{comp}_{\mathbf{v}}\mathbf{u}$ of $\mathbf{u}$ onto $\mathbf{v}$.
   - The vector projection $\mathbf{proj}_{\mathbf{v}}\mathbf{u}$ of $\mathbf{u}$ along $\mathbf{v}$.
   - The component $\mathbf{orth}_{\mathbf{v}}\mathbf{u}$ of $\mathbf{u}$ orthogonal of $\mathbf{v}$.

   The answer key has not been proofread, use with caution.

   (a) $\mathbf{v} = (2, 3, 5)$, $\mathbf{u} = (3, 5, 7)$.

   answer: $\text{comp}_{\mathbf{v}}\mathbf{u} = \frac{28}{19}\sqrt{38}$, $\mathbf{proj}_{\mathbf{v}}\mathbf{u} = \left(\frac{56}{19}, \frac{84}{19}, \frac{140}{19}\right)$, $\mathbf{orth}_{\mathbf{v}}\mathbf{u} = \left(\frac{1}{19}, \frac{11}{19}, -\frac{7}{19}\right)$

   (b) $\mathbf{v} = (5, 1, -3)$, $\mathbf{u} = (2, 3, 5)$.

   answer: $\text{comp}_{\mathbf{v}}\mathbf{u} = -\frac{2}{35}\sqrt{35}$, $\mathbf{proj}_{\mathbf{v}}\mathbf{u} = \left(-\frac{2}{7}, -\frac{2}{35}, \frac{6}{35}\right)$, $\mathbf{orth}_{\mathbf{v}}\mathbf{u} = \left(\frac{16}{7}, \frac{107}{35}, \frac{169}{35}\right)$