Precalculus
Homework Lecture 9

1. Write down an equation for the line passing through the indicated points. The answer key has not been proofread, use with caution.

   (a) L_1 passing through $(1, 2)$ and $(2, -1)$.

   answer: $y = -3x + 5$ or $y + 3x - 5 = 0$

   (b) L_2 passing through $(1, 1)$ and $(2, -2)$.

   answer: $y = -3x + 4$ or $y + 3x - 4 = 0$

   (c) L_3 passing through $(0, 1)$ and $(1, 0)$.

   answer: $y = -x + 1$ or $y + x - 1 = 0$

   (d) L_4 passing through $(3, 5)$ and $(7, -11)$.

   answer: $y = -4x + 17$ or $y + 4x - 17 = 0$

2. A set of lines is given by a pair of points. The answer key has not been proofread, use with caution.

Line name	Point on the line	Second point
L_1	$(1, 2)$	$(2, -1)$
L_2	$(1, 1)$	$(2, -2)$
L_3	$(0, 1)$	$(1, 0)$
L_4	$(3, 5)$	$(7, -11)$

   Find the intersection of the indicated pair of lines, or show that no such intersection exists (i.e., the lines are parallel).

   (a) L_1 and L_2.

   answer: The lines are parallel and do not intersect.

   (b) L_1 and L_3.

   answer: Intersection: $(2, -1)$.

   (c) L_1 and L_4.

   answer: point $(12, -31)$

   (d) L_2 and L_3.

   answer: Intersection: $\left(\frac{3}{2}, -\frac{1}{2}\right)$.

   (e) L_2 and L_4.

   answer: Intersection: $(13, -35)$.

   (f) L_3 and L_4.

   answer: Intersection: $\left(\frac{16}{3}, -\frac{13}{3}\right)$.

Solution. 2.c

An equation of a line passing through the points (x_1, y_1) and (x_2, y_2) is given by:

$$
\begin{aligned}
(x_2 - x_1)(y - y_1) &= (y_2 - y_1)(x - x_1) \\
&\text{or alternatively if } x_1 \neq x_2: \\
y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \\
&\text{or also} \\
y - y_2 &= \frac{y_2 - y_1}{x_2 - x_1}(x - x_2).
\end{aligned}
$$

We recall that if $x_1 \neq x_2$, the number $m = \frac{y_2 - y_1}{x_2 - x_1}$ is called the slope of the line. Therefore an equation of L_1 is given by:

$$\begin{aligned} m_1 &= \frac{-1-2}{2-1} = -3 \\ y - 2 &= m_1(x-1) = -3(x-1) \\ y + 3x - 5 &= 0. \end{aligned}$$

Similarly, an equation of L_4 is given by:

$$\begin{aligned} m_4 &= \frac{-11-5}{7-3} = -4 \\ y - 5 &= m_4(x-3) = -4(x-3) \\ y + 4x - 17 &= 0 \end{aligned}$$

To find the intersection of the two lines, we need to solve the system

$$\left| \begin{aligned} y + 4x - 17 &= 0 \\ y + 3x - 5 &= 0. \end{aligned} \right.$$

A standard method for solving such a system of equation is studied in the subject of Linear algebra. Alternatively, we can solve this system as follows. Observe that the second equation gives $y = -3x + 5$. Substitute that into the first equation to get:

$$\begin{aligned} y + 4x - 17 &= 0 && \Big| \text{ Substitute } y = -3x + 5 \\ -3x + 5 + 4x - 17 &= 0 \\ x &= 12. \end{aligned}$$

Therefore $y = -3x + 5 = -3(12) + 5 = -36 + 5 = 31$. Therefore two lines intersect at the point with coordinates $(12, 31)$.

To check our work, we can substitute $x = 12, y = -31$ in the equation $y + 4x - 17 = 0$ of L_1 to get that $-31 + 4 \cdot 12 - 17 = -31 + 48 - 17 = 0$, as expected. Similarly, we can use the equation $y + 3x - 5 = 0$ of L_4 to check our work: $-31 + 3 \cdot 12 - 5 = -31 + 36 - 5 = 0$, as expected. Finally, a computer-generated plot gives a visual confirmation of our computations.