

Precalculus

Lecture 4

Complex Numbers

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<https://github.com/tmilev/freecalc>

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Outline

1 Complex Numbers

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Definition (Complex numbers)

The set of complex numbers \mathbb{C} is defined as the set

$$\{a + bi \mid a, b - \text{real numbers}\},$$

where the number i is a number for which

$$i^2 = -1 \quad .$$

The number i is called the imaginary unit. By definition, $\sqrt{-1} = i$.

- Complex addition/subtraction

$$(a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i \quad .$$

- Complex multiplication

$$\begin{aligned}(a + bi)(c + di) &= ac + adi + bci + bdi^2 = ac + adi + bci - bd \\ &= (ac - bd) + i(ad + bc)\end{aligned}$$

Let $u = 2 + 3i$, $v = 5 - 7i$.

Example (Addition)

$$u + v = (2 + 3i) + (5 - 7i) = (2 + 5) + (3 - 7)i = 7 - 4i.$$

Example (Subtraction)

$$u - v = (2 + 3i) - (5 - 7i) = (2 - 5) + (3 - (-7))i = -3 + 10i.$$

Example (Multiplication)

$$\begin{aligned}u \cdot v &= (2 + 3i) \cdot (5 - 7i) \\&= 2 \cdot 5 + 2 \cdot (-7)i + 3i \cdot 5 + 3i(-7i) \\&= 10 - 14i + 15i - 21i^2 \\&= 10 + i - (-21) \\&= 31 + i\end{aligned}$$

Example (Complex multiplication)

Multiply $u = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ by $v = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$.

$$\begin{aligned}u \cdot v &= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i \right) \cdot \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \right) \\&= \frac{\sqrt{2}^2}{2^2} - \frac{\sqrt{2}^2}{2^2}i + \frac{\sqrt{2}^2}{2^2}i \cancel{\frac{\sqrt{2}^2}{2^2}i} + \cancel{\frac{\sqrt{2}^2}{2^2}i} - \frac{\sqrt{2}^2}{2^2}i^2 \\&= \frac{2}{4} - \frac{2}{4}(-1) \\&= \frac{1}{2} + \frac{1}{2} \\&= 1\end{aligned}$$

Review of the basic types of numbers

- An integer, or whole number, is one of the numbers:

$$\dots, -2, -1, 0, 1, 2, \dots$$

- A rational number is the quotient of two integers, for example:

$$\frac{1}{2}, \quad \frac{2}{-3} = -\frac{2}{3}, \quad \frac{8}{12} = \frac{4}{6} = \frac{2}{3}.$$

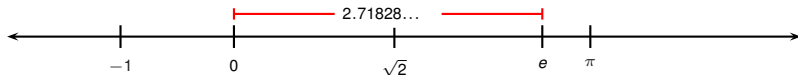
- A real number measures the location of a point on the real line:

$$\sqrt{2} = 1.414213562373095048801688724209698 \dots$$

$$\pi = 3.141592653589793238462643383279502 \dots$$

$$e = 2.718281828459045235360287471352662 \dots$$

$$-1$$



- A number is complex if it equals $a + bi$ with a, b - real, $\sqrt{-1} = i$:

$$2 + 3i, \quad -i, \quad 1 + \sqrt{2}i$$

- Geometric interpretation of complex numbers: beyond our scope.