Precalculus
Homework Lecture 11

1. Compute the composite functions $(f \circ g)(x)$, $(g \circ f)(x)$. Simplify your answer to a single fraction. Find the domain of the composite function.

   (a) $f(x) = \dfrac{x+2}{x-2}, g(x) = \dfrac{x-1}{x+2}$.

   answer: $(f \circ g)(x) = \frac{3+3x}{-5-x}$ $\quad x \neq -2, -5$
   $(g \circ f)(x) = \frac{4}{-2+3x}$ $\quad x \neq 2, \frac{2}{3}$

   (b) $f(x) = \dfrac{x+1}{3x-2}, g(x) = \dfrac{x-2}{x-1}$.

   answer: $(f \circ g)(x) = \frac{-3+2x}{-4+x}$ $\quad x \neq 4, 1$
   $(g \circ f)(x) = \frac{5-5x}{3-2x}$ $\quad x \neq \frac{2}{3}, \frac{3}{2}$

   (c) $f(x) = \dfrac{2x+1}{3x-1}, g(x) = \dfrac{x-2}{2x-1}$.

   answer: $(f \circ g)(x) = \frac{-5+4x}{-5+x}$ $\quad x \neq 5, \frac{1}{2}$
   $(g \circ f)(x) = \frac{3-4x}{3+x}$ $\quad x \neq -3, \frac{1}{3}$

   (d) $f(x) = \dfrac{x+1}{x-2}, g(x) = \dfrac{x+2}{2x-1}$.

   answer: $(f \circ g)(x) = \frac{1+3x}{4-3x}$ $\quad x \neq \frac{4}{3}, \frac{1}{2}$
   $(g \circ f)(x) = \frac{-3+3x}{4+x}$ $\quad x \neq -4, 2$

   (e) $f(x) = \dfrac{5x+1}{4x-1}, g(x) = \dfrac{4x-1}{3x+1}$.

   answer: $(f \circ g)(x) = \frac{-4+23x}{-5+13x}$ $\quad x \neq \frac{5}{13}, -\frac{1}{3}$
   $(g \circ f)(x) = \frac{5+16x}{2+19x}$ $\quad x \neq -\frac{2}{19}, \frac{1}{4}$

   (f) $f(x) = \dfrac{3x-5}{x-2}, g(x) = \dfrac{x-2}{x-4}$.

   answer: $(f \circ g)(x) = \frac{-2x+14}{-x+6}$ $\quad x \neq 6, 4$
   $(g \circ f)(x) = \frac{x-1}{-x+3}$ $\quad x \neq 3, 2$

   (g) $f(x) = \dfrac{x-3}{x+2}, g(y) = \dfrac{y+3}{y-4}$.

   answer: $(f \circ g)(x) = \frac{-2x+15}{3x-5}$ $\quad x \neq \frac{5}{3}, 4$
   $(g \circ f)(x) = \frac{4x+3}{-3x-11}$ $\quad x \neq -\frac{11}{3}, -2$

2. Find the functions $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$ and their implied domains. The answer key has not been proofread, use with caution.

   (a) $f(x) = x^2 + 1, g(x) = x + 1$.

   answer: Domain, all 4 cases: $x \in \mathbb{R}$ (all reals)
   in some order: $(1+x)^2 + 1$, $(x)^2 + 2$, $((x)^2 + 1)^2 + 1$, $2 + x$

   (b) $f(x) = \sqrt{x+1}, g(x) = x + 1$.

   answer: Domain of $f \circ g$ is $x \geq -2$. Domain of $g \circ f$ is $x \geq -1$
   Domain of $f \circ f$ is $x \geq -1$. Domain of $g \circ g$ is all reals ($x \in \mathbb{R}$).
   in some order: $\sqrt{2+x}$, $1 + \sqrt{1+x}$, $\sqrt{1+\sqrt{1+x}}$, $2 + x$

   (c) $f(x) = 2x, g(x) = \tan x$.

   In this subproblem, you are not required to find the domain.

   answer: Domain $f \circ f$: all reals ($x \in \mathbb{R}$). Domain $g \circ f$: $x \neq (2k+1)\frac{\pi}{2}$ for all $k \in \mathbb{Z}$
   Domain $f \circ g$: $x \neq (4k+1)\frac{\pi}{4}$, $x \neq (4k+3)\frac{\pi}{4}$ for all $k \in \mathbb{Z}$
   Domain $g \circ g$: $x \neq (2k+1)\frac{\pi}{2}$ and $x \neq k\pi + \arctan\left(\frac{\pi}{2}\right)$ for all $k \in \mathbb{Z}$
   in some order: $2\tan x$, $\tan(2x)$, $4x$, $\tan(\tan x)$

   (d) $f(x) = \dfrac{x+1}{x-1}, g(x) = \dfrac{x-1}{x+1}$.

   answer: Domain $f \circ f$: $x \neq 1$. Domain $g \circ g$: $x \neq 0, x \neq -1$
   Domain $f \circ g$: $x \neq -1$. Domain $g \circ f$: $x \neq 0, x \neq 1$
   in some order: $-x$, $\frac{1}{x}$, x, $-\frac{1}{x}$