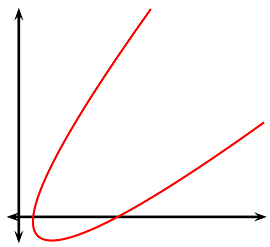
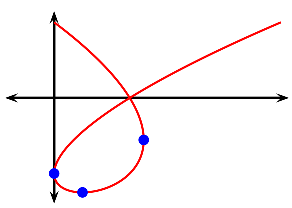
Calculus II
Homework on Lecture 12

1. Find the values of the parameter t for which the curve has horizontal and vertical tangents.

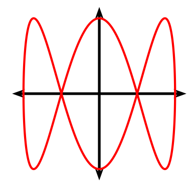
   (a) $x = t^2 - t + 1, y = t^2 + t - 1$

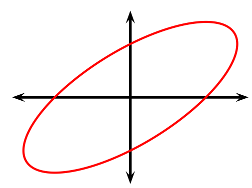
   (b) $x = t^3 - t^2 - t + 1, y = t^2 - t - 1$.

   (c) $x = \cos(t), y = \sin(3t)$

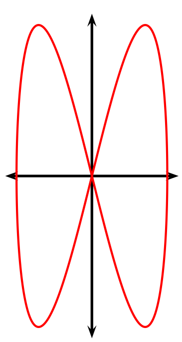
   

   (d) $x = \cos(t) + \sin(t)$, $y = \sin(t)$.

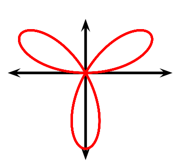
   

2. Show that the parametric curve has multiple tangents at the point and find their slopes.

   (a) $x = \cos t, y = 2\sin(2t)$, two tangents at $(x, y) = (0, 0)$.

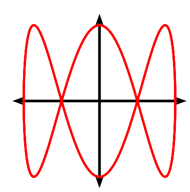
   

   (b) $x = \cos t \sin(3t)$, $y = \sin(t)\sin(3t)$, six tangents at

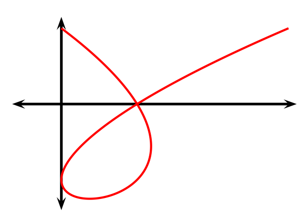
   $(x, y) = (0, 0)$.

   (c) $x = \cos t, y = \sin(3t)$, find the two points at which the curve has double tangent and find the slopes of both pairs

   of tangents.

   (d) $x = t^3 - t^2 - t + 1, y = t^2 - t - 1$, find a point where the curve has double tangent and find the slopes of the tangents.

3. Find the length of the curve.

   (a) $y = x^2, x \in [1, 2]$.

   (b) $y = \sqrt{x}, x \in [1, 2]$.

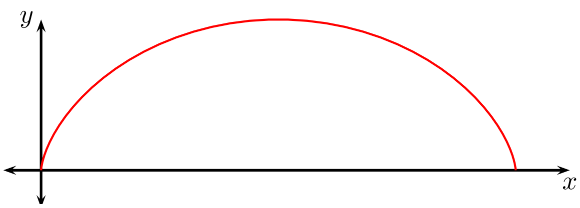
(c) $x = \sqrt{t} - 2t$ and $y = \dfrac{8}{3}t^{\frac{3}{4}}$ from $t = 1$ to $t = 4$.

(d) $\gamma : \left| \begin{array}{lcl} x(t) & = & \frac{1}{t} + \frac{t^3}{3} \\ y(t) & = & 2t \end{array} \right. , t \in [1, 2] \quad .$

(e) $\gamma : \left| \begin{array}{lcl} x(t) & = & \frac{1}{t} + t \\ y(t) & = & 2 \ln t \end{array} \right. , t \in [1, 2] \quad .$

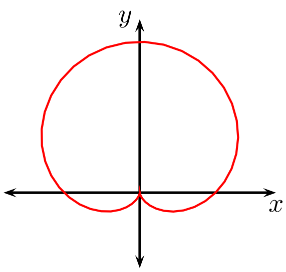
(f) One arch of the cycloid

$$\gamma : \left| \begin{array}{lcl} x(t) & = & t - \sin t \\ y(t) & = & 1 - \cos t \end{array} \right. , t \in [0, 2\pi]$$



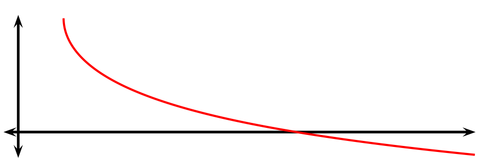
(g) The cardioid

$$\gamma : \left| \begin{array}{lcl} x(t) & = & (1 + \sin t)\cos t \\ y(t) & = & (1 + \sin t)\sin t \end{array} \right. , t \in [0, 2\pi]$$



4. Set up an integral that expresses the length of the curve and find the length of the curve.

(a) $\left| \begin{array}{lcl} x(t) & = & e^t + e^{-t} \\ y(t) & = & 5 - 2t \end{array} \right. , t \in [0, 3]$



(b) $\left| \begin{array}{lcl} x(t) & = & \sin t + \cos t \\ y(t) & = & \sin t - \cos t \end{array} \right. , t \in [0, \pi]$