Calculus III
Homework on Lecture 17 and 18

1. Show the field $\mathbf{F}$ has scalar potential f; find f. Compute the indicated line integral.
   We recall that f is scalar potential for $\mathbf{F}$ if $\mathbf{F} = \nabla f = (f_x, f_y)$.
   (a) $\mathbf{F} = (3 + 2xy)\mathbf{i} + (x^2 - 3y^2)\mathbf{j}$. Compute $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$, where C is any curve from $(1, 0)$ to $(0, 1)$.
   (b) $\mathbf{F} = (e^{xy}y + 2x)\,\mathbf{i} + (e^{xy}x + 1)\,\mathbf{j}$. Compute $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$, where C is any curve from $(1, 0)$ to $(0, 1)$.

2. Compute the line integral.
   (a) $\displaystyle\int_C \left(-y^3 + e^{x^3+\arctan x}\right) \mathrm{d}x + \left(x^3 + \arctan\left(e^{y^2} + y\right)\right) \mathrm{d}y$, where C is the positively oriented boundary of the unit disk.
   (b) $\displaystyle\int_C (\sin(\cos x) + y)\mathrm{d}x + (\cos(\sin y) + 2x)\mathrm{d}y$, where C is the oriented curve consisting of the four segments connecting $(0, -1)$, $(1, 0)$, $(0, 1)$, $(-1, 0)$, $(0, -1)$ in this order.