Calculus III
Homework on Lecture 16

1. Compute the double integral. The integrals are set up to be easy in polar coordinates.

   (a) $\iint_{\mathcal{S}}(x+y)\mathrm{d}x\mathrm{d}y$, where $\mathcal{S}$ is the region left of the $y-$axis and between the circles $x^2+y^2=1$ and $x^2+y^2=4$.

   (b) $\iint_{\mathcal{S}}(x+y)\mathrm{d}x\mathrm{d}y$, where $\mathcal{S}$ is the sector region in the first quadrant locked between the circles $x^2+y^2=1$ and $x^2+y^2=4$, and the lines $-\sqrt{3}x+y=0$, $x-\sqrt{3}y=0$.

2. Compute the triple integral. The integrals are set up to be easy in cylindrical coordinates.

   (a) $\iiint_{\mathcal{S}}\sqrt{x^2+y^2}\mathrm{d}x\mathrm{d}y\mathrm{d}z$, where $\mathcal{S}$ is the solid conical body with vertical axis along the z axis, pointing upwards, of height 1 and with circular base of radius 1 lying on the xy-plane.

3. (a) Find the centroid of a semi-ball of radius R whose base is a circle in the x,y-plane.

   (b) Integrate $\iiint_{\mathcal{S}} z\mathrm{d}x\mathrm{d}y\mathrm{d}z$, where $\mathcal{S}$ is the semi-ball from the previous point.