Calculus III
Homework on Lecture 9

1. Compute the indicated partial derivatives. Answer key has not been proofread, use with caution.

   (a) $\frac{\partial r}{\partial x}$, $\frac{\partial r}{\partial y}$, $r = \sqrt{x^2+y^2}$.

   (b) $\frac{\partial^2 r}{\partial x^2}$, $\frac{\partial^2 r}{\partial y^2}$, $\frac{\partial^2 r}{\partial y \partial x}$, $r = \sqrt{x^2+y^2}$.

   (c) $\frac{\partial \theta}{\partial x}$, $\frac{\partial \theta}{\partial y}$, $\theta = \arctan\left(\frac{y}{x}\right)$.

   (d) $\frac{\partial^2 \theta}{\partial x^2}$, $\frac{\partial \theta^2}{\partial y \partial x}$, $\frac{\partial^2 \theta}{\partial y^2}$, $\theta = \arctan\left(\frac{y}{x}\right)$.

answer: $\frac{\partial r}{\partial x} = \frac{x}{\sqrt{x^2+y^2}} = \frac{x}{r} \cdot \frac{\partial r}{\partial y} = \frac{y}{\sqrt{x^2+y^2}} = \frac{y}{r}$

answer: $\frac{\partial^2 r}{\partial x^2} = \frac{y^2}{r^3} \cdot \frac{\partial^2 r}{\partial y^2} = \frac{x^2}{r^3} \cdot \frac{\partial^2 r}{\partial y \partial x} = -\frac{xy}{r^3}$

answer: $\frac{\partial \theta}{\partial x} = \frac{-y}{x^2+y^2} \cdot \frac{\partial \theta}{\partial y} = \frac{x}{x^2+y^2}$

answer: Let $r = \sqrt{x^2+y^2}$. $\frac{\partial^2 \theta}{\partial x^2} = \frac{2xy}{(x^2+y^2)^2} \cdot \frac{\partial^2 \theta}{\partial y^2} = \frac{-2xy}{(x^2+y^2)^2} \cdot \frac{\partial \theta^2}{\partial y \partial x} = \frac{y^2-x^2}{(x^2+y^2)^2}$

Solution. 1.c

$$\begin{aligned} \frac{\partial}{\partial x}\left(\arctan\left(\frac{y}{x}\right)\right) &= \frac{\frac{\partial}{\partial x}\left(\frac{y}{x}\right)}{1+\left(\frac{y}{x}\right)^2} = \frac{-\frac{y}{x^2}}{1+\frac{y^2}{x^2}} = \frac{-y}{x^2+y^2} \\ \frac{\partial}{\partial y}\left(\arctan\left(\frac{y}{x}\right)\right) &= \frac{\frac{\partial}{\partial y}\left(\frac{y}{x}\right)}{1+\left(\frac{y}{x}\right)^2} = \frac{\frac{1}{x}}{1+\frac{y^2}{x^2}} = \frac{x}{x^2+y^2} \end{aligned}$$