Precalculus
Lecture 20

Todor Milev
https://github.com/tmilev/freecalc

2020

Outline

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Linear Functions

Definition (Linear Function)

A linear function is a function the graph of which is a line. We can write any linear function in slope-intercept form:

$$f(x) = mx + b.$$

m is called the slope, and b is called the y-intercept.

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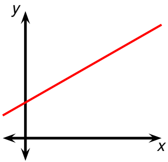

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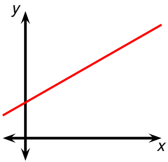

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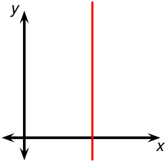
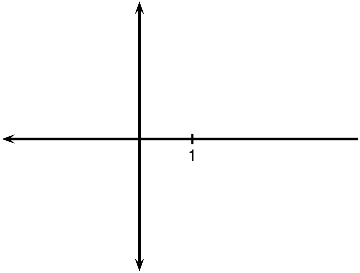
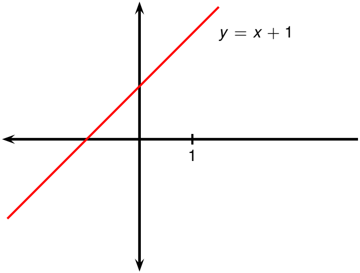


- Vertical lines fail the vertical line test and therefore are not graphs of a function of x.



$f(x)$	Direction	y-intercept
$x+1$		
$-0.5x$		
-1		



$f(x)$	Direction	y-intercept
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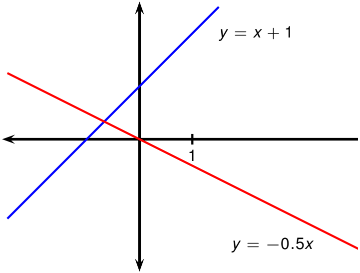
- $m > 0$ means the graph of f points up ($\nearrow$).



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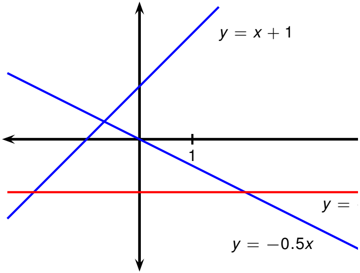

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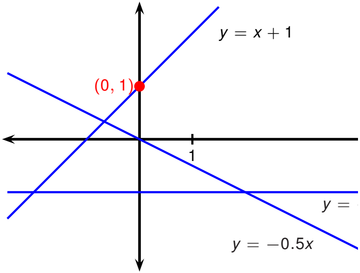

$f(x)$	Direction	y-intercept
$x + 1$	$\nearrow$	1
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-1	$\rightarrow$	

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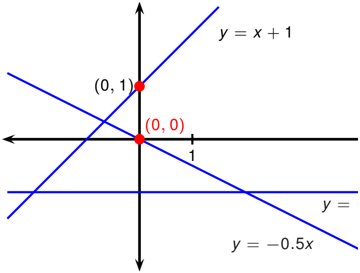
- b tells us the height of the point where the graph hits the y-axis.



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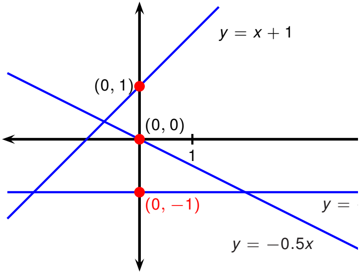

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Definition (Polynomial Function)

A polynomial function is a function f of the form

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1} + a_n x^n,$$

where n is a non-negative integer and $a_0, \ldots, a_n$ are real numbers, called the coefficients. If $a_n \neq 0$ the integer n is called the degree of f.

If we interpret x as an indeterminate formal expression, rather than a number, we say that $f(x)$ is a polynomial (rather than a polynomial function).

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6					
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$f(x)$	Polynomial?	Degree	a_0	a_1	a_2
$x^4 - x + 1$	Yes	4	1	-1	0
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$3x^2 - \frac{1}{2}x + \sqrt{x}$	No				
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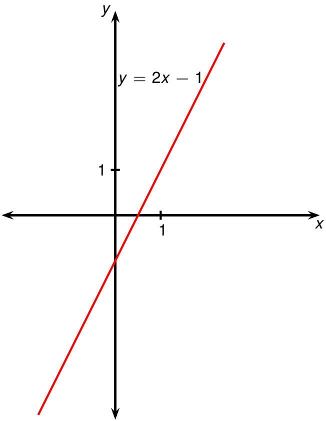
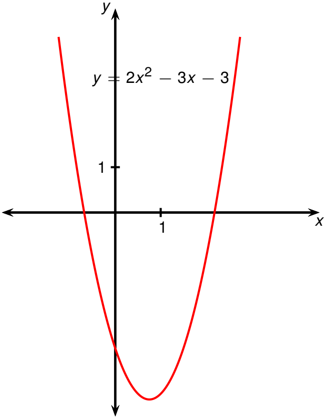
- Linear functions are polynomial (functions).



Linear

- Linear functions are polynomial (functions).
- So are quadratic functions. Their graphs are parabolas.



Quadratic

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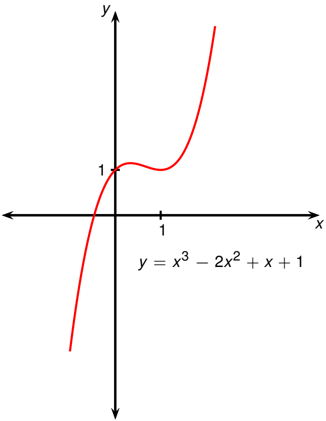
- And there are many more.



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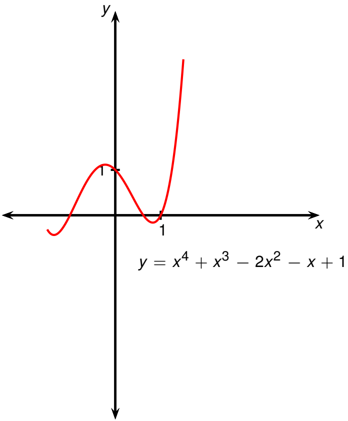

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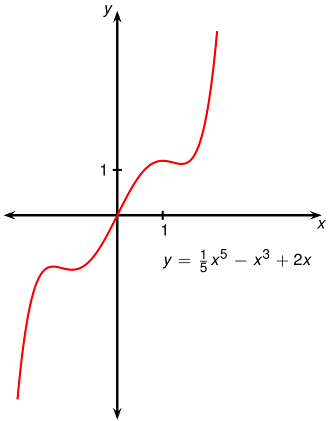

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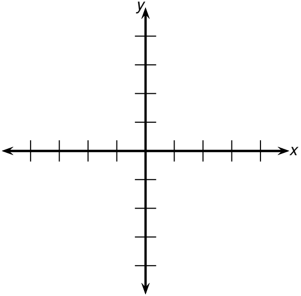

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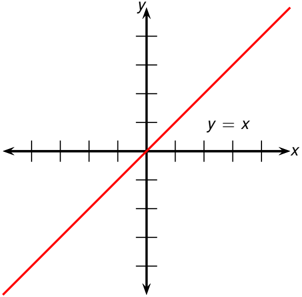

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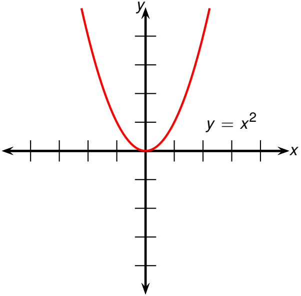

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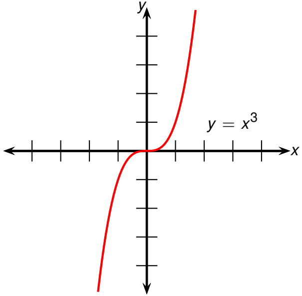

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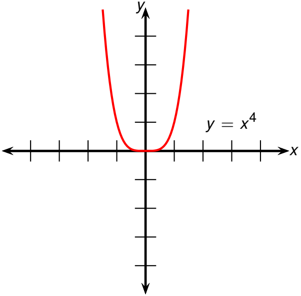

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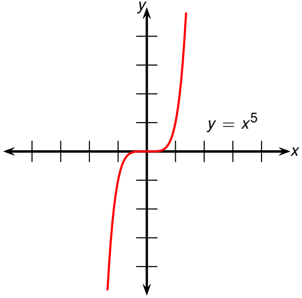

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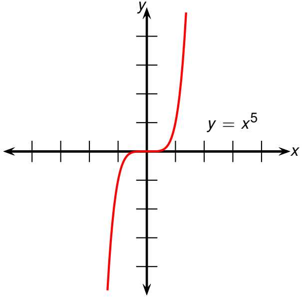

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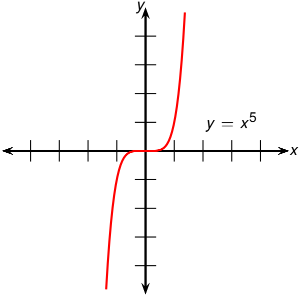

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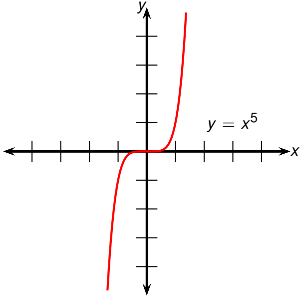

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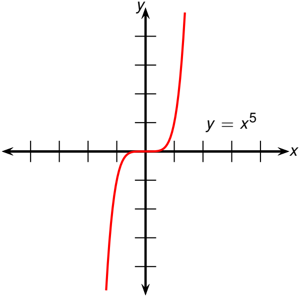

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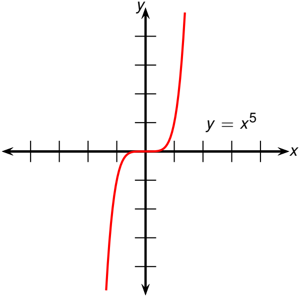

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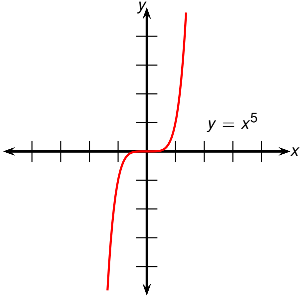

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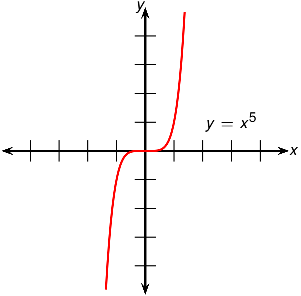

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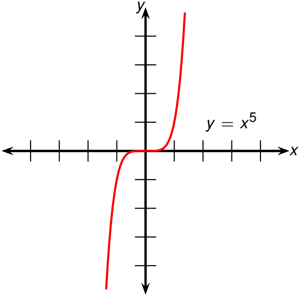

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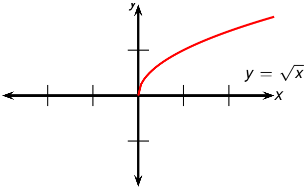

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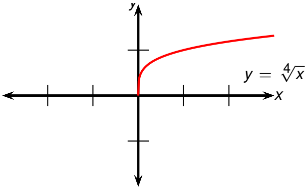

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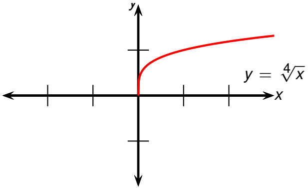

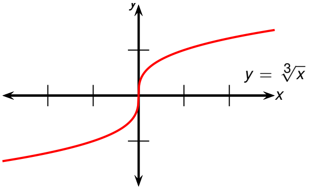

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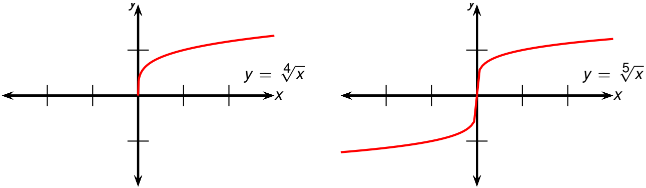
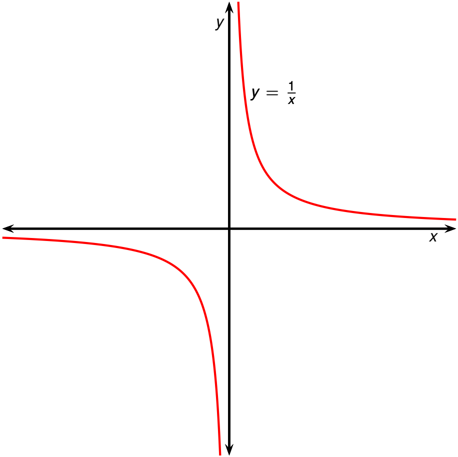


$f(x) = x^{-1} = \frac{1}{x}$ is called the reciprocal function. Its graph has equation $y = \frac{1}{x}$, or $xy = 1$, and is an hyperbola with the coordinate axes as its



asymptotes.

Rational Functions

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A rational function is a quotient of two polynomials; that is, a function of the form

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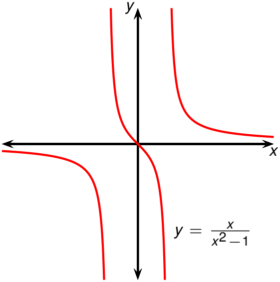

Example ($x/(x^2-1)$)

The function

$$f(x) = \frac{x}{x^2-1}$$

is a rational function.

Algebraic Functions

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A function in x that can be constructed using x, constants, and finitely many of the operations $+, -, *, /$, and $\sqrt[n]{}$ is an algebraic function.

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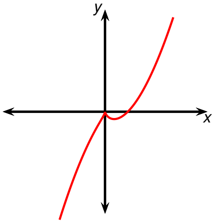
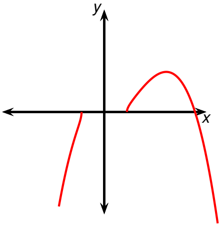
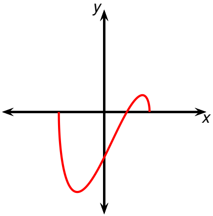
Examples.



$y = (x-1)\sqrt[3]{x^2}$



$y = \frac{1}{5}(4x - x^2)\sqrt{x^2 - 1}$



$y = (x-1)\sqrt{4 - x^2}$

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- Outside of the present course: by definition, a function is transcendental if it is not algebraic, i.e., if it satisfies no polynomial equation with polynomial coefficients.

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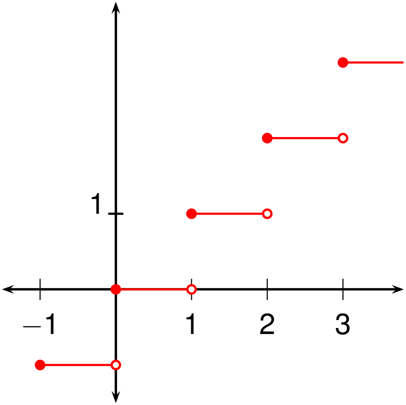
The *greatest integer function* $\lfloor x \rfloor$ is defined as the largest integer that is less than or equal to x.

In computer science this function is called the *floor* function.



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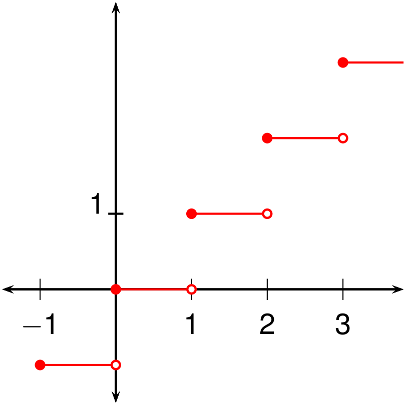

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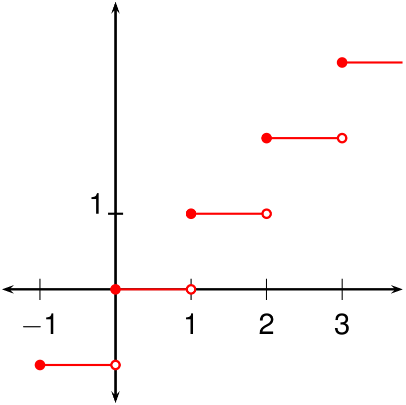

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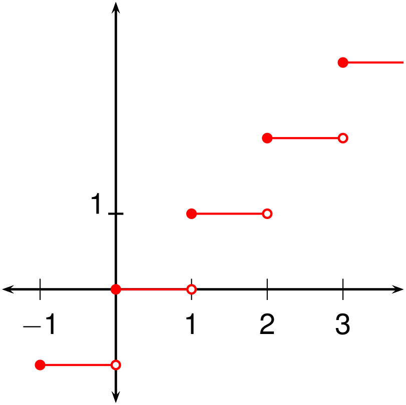

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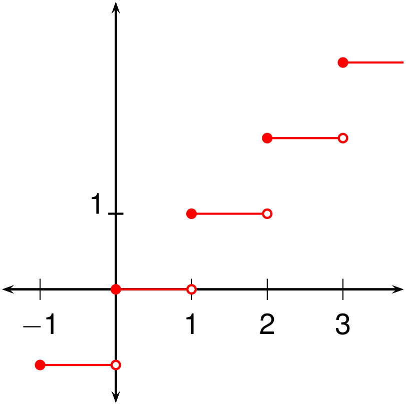

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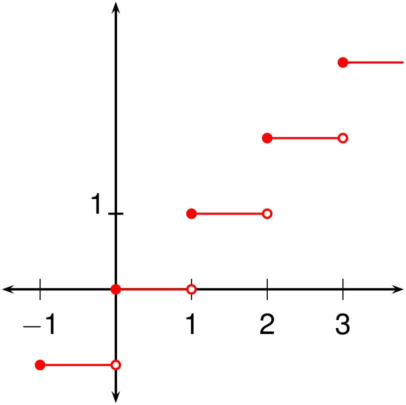

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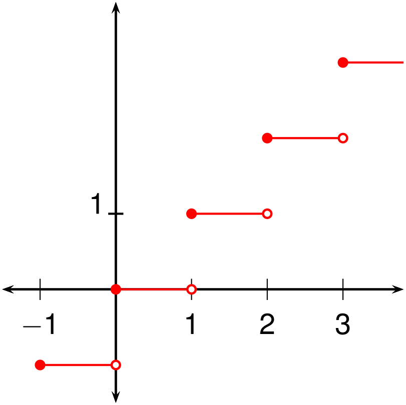

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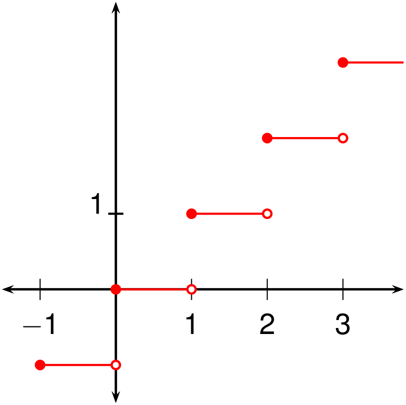

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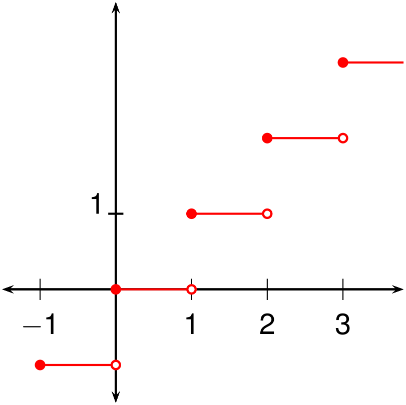

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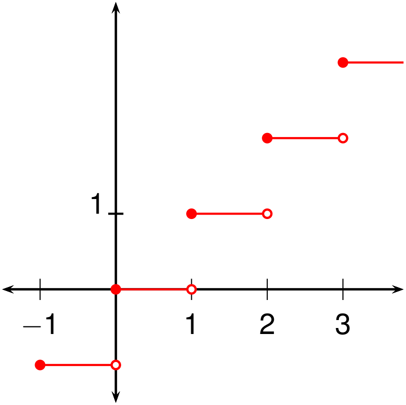

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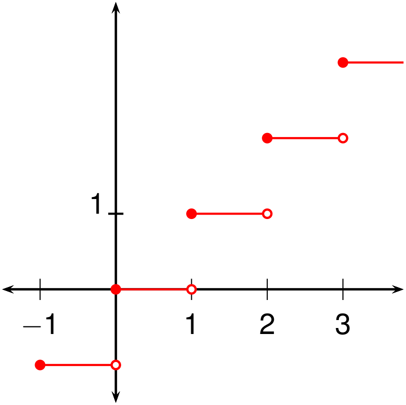

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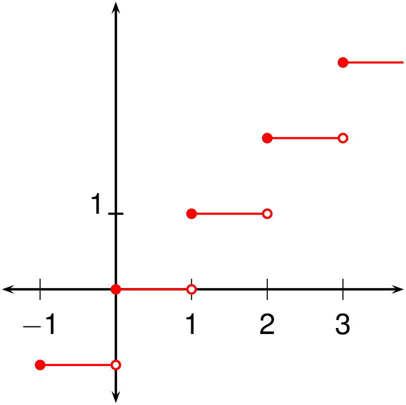

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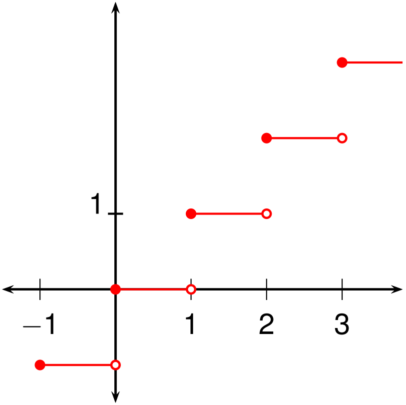

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