Calculus III Homework on Lecture 13

1. Evaluate the double integral.

(a)
$$\iint_D x^3 y^2 dx dy$$
, $D = \{(x, y) | 0 \le x \le 2, -x \le y \le x\}$.

(b)
$$\iint_{D} \frac{4y}{x^3 + 2} dx dy, D = \{(x, y) | 1 \le x \le 2, 0 \le y \le 2x \}.$$

(c)
$$\iint_D \frac{2y}{x^2 + 1} dx dy$$
, $D = \{(x, y) | 0 \le x \le 1, 0 \le y \le \sqrt{x} \}$.

(d)
$$\iint_D e^{y^2} dxdy$$
, $D = \{(x, y) | 0 \le y \le 1, 0 \le x \le y\}$.

(e)
$$\iint_D x \cos y dx dy$$
, D bounded by $y = 0$, $y = x^2$, $x = 1$.

(f)
$$\iint_D (x+y) dxdy$$
, D bounded by $y = \sqrt{x}$ and $y = x^2$.

(g)
$$\iint_D y^3 dx dy$$
, D - triangle with vertices $(0,2),(1,1),(3,2)$.

(h)
$$\iint_D xy^2 dxdy$$
, D enclosed by $x = 0$ and $x^2 + y^2 = 1$.

(i)
$$\iint_D (2x-y) dxdy$$
, D bounded by circle with radius 2 centered at the origin.

(j)
$$\iint\limits_{D}2xy\mathrm{d}x\mathrm{d}y,$$
 D - triangular region with vertices $(0,0),(1,2),(0,3).$

2. Evaluate the double integral. The answer key has not been proofread, use with caution.

(a)
$$\iint_{\mathcal{R}} xy dx dy$$
 where \mathcal{R} is bounded by the curves $x=3, x+1=y^2, x=y^2+2y+3$.

(b)
$$\iint_{\mathcal{R}} xy \mathrm{d}x \mathrm{d}y$$
 . where \mathcal{R} is the region enclosed by $y=x^2+1$ and $y=2x^2-x-1$.

3. Integrate.

(a)
$$\int_{y=0}^{y=\sqrt{\pi}} \int_{x=y}^{x=\sqrt{\pi}} \cos\left(x^2\right) dx dy.$$

(b)
$$\int_{y=0}^{y=1} \int_{x=\sqrt{y}}^{x=\sqrt[5]{y}} e^{-x^3} \mathrm{d}x \mathrm{d}y.$$