

Calculus II

Lecture 11

Todor Milev

<https://github.com/tmilev/freecalc>

2020

Outline

1

Curves

- The Cycloid
- Polar Curves

License to use and redistribute

These lecture slides and their \LaTeX source code are licensed to you under the Creative Commons license CC BY 3.0. You are free

- to Share - to copy, distribute and transmit the work,
- to Remix - to adapt, change, etc., the work,
- to make commercial use of the work,

as long as you reasonably acknowledge the original project.

- Latest version of the .tex sources of the slides:

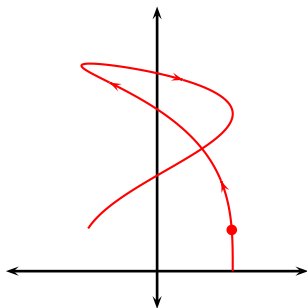
<https://github.com/tmilev/freecalc>

- Should the link be outdated/moved, search for “freecalc project”.
- Creative Commons license CC BY 3.0:

<https://creativecommons.org/licenses/by/3.0/us/>
and the links therein.

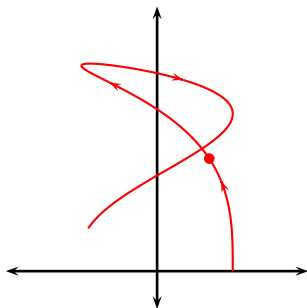
Curves Defined by Parametric Equations

- Suppose a particle moves along the curve in the picture.



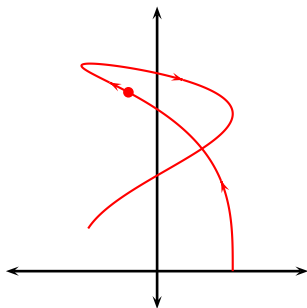
Curves Defined by Parametric Equations

- Suppose a particle moves along the curve in the picture.



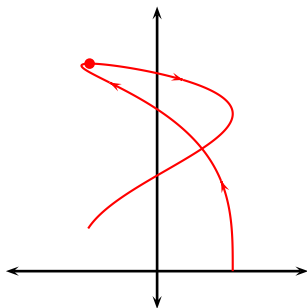
Curves Defined by Parametric Equations

- Suppose a particle moves along the curve in the picture.



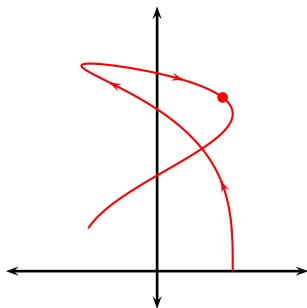
Curves Defined by Parametric Equations

- Suppose a particle moves along the curve in the picture.

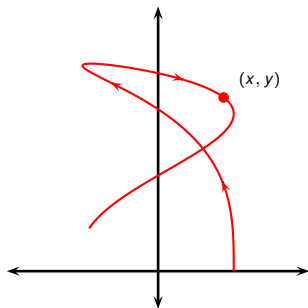


Curves Defined by Parametric Equations

- Suppose a particle moves along the curve in the picture.

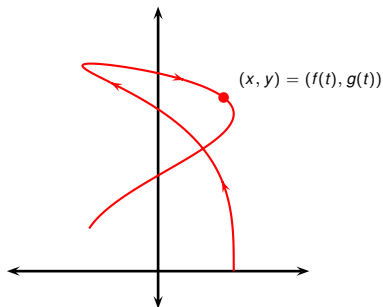


Curves Defined by Parametric Equations



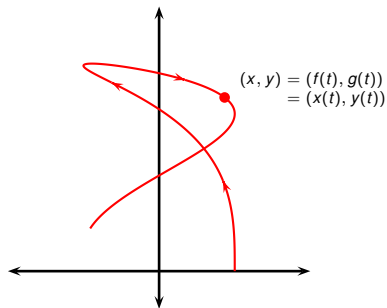
- Suppose a particle moves along the curve in the picture.
- The x -coordinate and y -coordinate of the particle are some functions of the time t .

Curves Defined by Parametric Equations



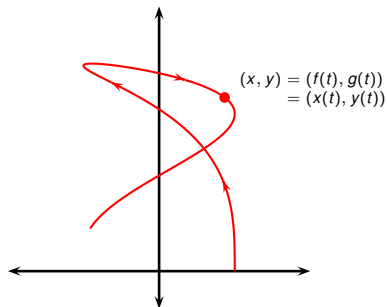
- Suppose a particle moves along the curve in the picture.
- The x -coordinate and y -coordinate of the particle are some functions of the time t .
- We can write $x = f(t)$ and $y = g(t)$.

Curves Defined by Parametric Equations



- Suppose a particle moves along the curve in the picture.
- The x -coordinate and y -coordinate of the particle are some functions of the time t .
- We can write $x = f(t)$ and $y = g(t)$.
- Less formally, we may directly write $(x, y) = (x(t), y(t))$.

Curves Defined by Parametric Equations



- Suppose a particle moves along the curve in the picture.
- The x -coordinate and y -coordinate of the particle are some functions of the time t .
- We can write $x = f(t)$ and $y = g(t)$.
- Less formally, we may directly write $(x, y) = (x(t), y(t))$.
- We say that the equations
$$\begin{cases} x = f(t) \\ y = g(t) \end{cases}$$
are parametric equations of a parametric curve.
- Note that the curve can't be written as $y = f(x)$: it fails the vertical line test.

Definition (Curve in n -dimensional space)

We define an arbitrary n -tuple of functions f_1, \dots, f_n on $[a, b]$ to be a *parametric curve* (or simply *curve*). If C is a curve, we write C as:

$$C : \begin{cases} x_1 = f_1(t) \\ x_2 = f_2(t) \\ \vdots \\ x_n = f_n(t) \end{cases}, t \in [a, b]$$

where x_1, \dots, x_n are the labels of the n -dimensional coordinate system.

Curves in 2- and 3-dimensional space will be of special interest:

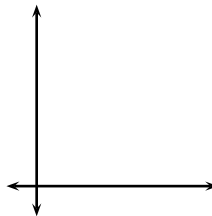
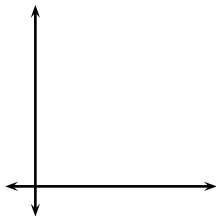
A curve in dimension 2 is given by: A curve in dimension 3 is given by:

$$C : \begin{cases} x = f(t) \\ y = g(t) \end{cases}, t \in [a, b] \quad . \quad C : \begin{cases} x = f(t) \\ y = g(t) \\ z = h(t) \end{cases}, t \in [a, b] \quad .$$

Consider the two parametric curves:

$$\gamma_1 : \begin{cases} x = t^2 \\ y = t^2 \end{cases}, t \in [0, 1]$$

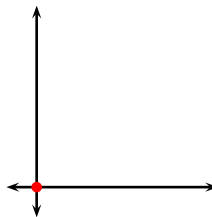
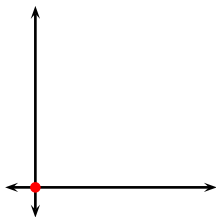
$$\gamma_2 : \begin{cases} x = t \\ y = t \end{cases}, t \in [0, 1]$$



Consider the two parametric curves:

$$\gamma_1 : \begin{cases} x = t^2 \\ y = t^2 \end{cases}, t \in [0, 1]$$

$$\gamma_2 : \begin{cases} x = t \\ y = t \end{cases}, t \in [0, 1]$$

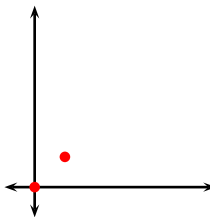
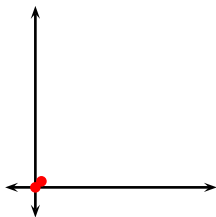


Plug in $t = 0$

Consider the two parametric curves:

$$\gamma_1 : \begin{cases} x = t^2 \\ y = t^2 \end{cases}, t \in [0, 1]$$

$$\gamma_2 : \begin{cases} x = t \\ y = t \end{cases}, t \in [0, 1]$$

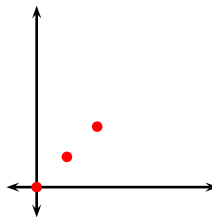
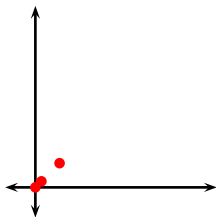


Plug in $t = 0$, $t = 0.2$

Consider the two parametric curves:

$$\gamma_1 : \begin{cases} x = t^2 \\ y = t^2 \end{cases}, t \in [0, 1]$$

$$\gamma_2 : \begin{cases} x = t \\ y = t \end{cases}, t \in [0, 1]$$

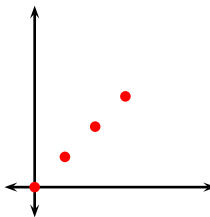
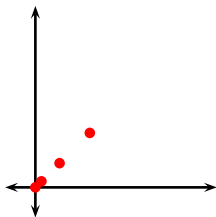


Plug in $t = 0$, $t = 0.2$, $t = 0.4$

Consider the two parametric curves:

$$\gamma_1 : \begin{cases} x = t^2 \\ y = t^2 \end{cases}, t \in [0, 1]$$

$$\gamma_2 : \begin{cases} x = t \\ y = t \end{cases}, t \in [0, 1]$$

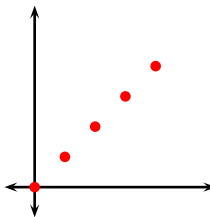
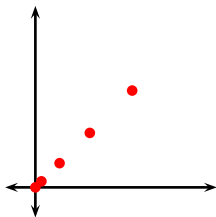


Plug in $t = 0$, $t = 0.2$, $t = 0.4$, $t = 0.6$

Consider the two parametric curves:

$$\gamma_1 : \begin{cases} x = t^2 \\ y = t^2 \end{cases}, t \in [0, 1]$$

$$\gamma_2 : \begin{cases} x = t \\ y = t \end{cases}, t \in [0, 1]$$

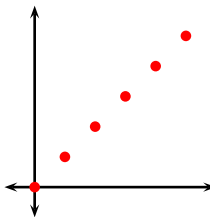
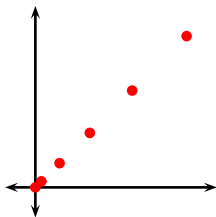


Plug in $t = 0$, $t = 0.2$, $t = 0.4$, $t = 0.6$, $t = 0.8$

Consider the two parametric curves:

$$\gamma_1 : \begin{cases} x = t^2 \\ y = t^2 \end{cases}, t \in [0, 1]$$

$$\gamma_2 : \begin{cases} x = t \\ y = t \end{cases}, t \in [0, 1]$$

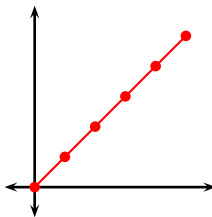
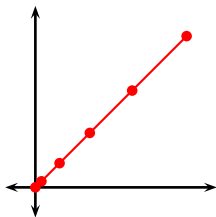


Plug in $t = 0$, $t = 0.2$, $t = 0.4$, $t = 0.6$, $t = 0.8$, $t = 1$.

Consider the two parametric curves:

$$\gamma_1 : \begin{cases} x = t^2 \\ y = t^2 \end{cases}, t \in [0, 1]$$

$$\gamma_2 : \begin{cases} x = t \\ y = t \end{cases}, t \in [0, 1]$$

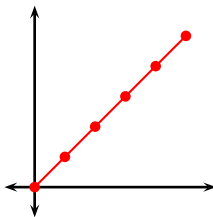
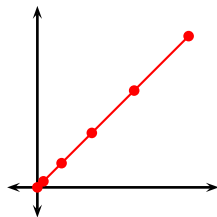


Plug in $t = 0$, $t = 0.2$, $t = 0.4$, $t = 0.6$, $t = 0.8$, $t = 1$.

Consider the two parametric curves:

$$\gamma_1 : \begin{cases} x = t^2 \\ y = t^2 \end{cases}, t \in [0, 1]$$

$$\gamma_2 : \begin{cases} x = t \\ y = t \end{cases}, t \in [0, 1]$$



Plug in $t = 0, t = 0.2, t = 0.4, t = 0.6, t = 0.8, t = 1$.

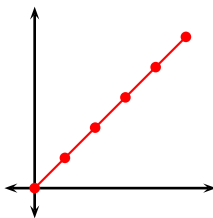
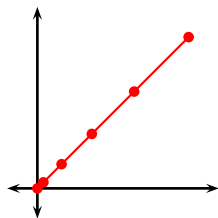
Question

Are the above curves different?

Consider the two parametric curves:

$$\gamma_1 : \begin{cases} x = t^2 \\ y = t^2 \end{cases}, t \in [0, 1]$$

$$\gamma_2 : \begin{cases} x = t \\ y = t \end{cases}, t \in [0, 1]$$



Plug in $t = 0, t = 0.2, t = 0.4, t = 0.6, t = 0.8, t = 1$.

Question

Are the above curves different?

To answer this question we need a definition.

Recall a parametric curve C was defined as the data

$$C : \left\{ \begin{array}{lcl} x_1 & = & f_1(t) \\ x_2 & = & f_2(t) \\ & \vdots & \\ x_n & = & f_n(t) \end{array} \right. , t \in [a, b]$$

Definition

A *curve image* (or simply a curve) is any set of points that arises by traversing some continuous curve. In other words, a curve image is any set that can be written in the form

$$\{(f_1(t), \dots, f_n(t)) \mid t \in [a, b]\} \quad ,$$

for some continuous functions f_1, \dots, f_n .

Recall a parametric curve C was defined as the data

$$C : \begin{cases} x_1 = f_1(t) \\ x_2 = f_2(t) \\ \vdots \\ x_n = f_n(t) \end{cases}, t \in [a, b]$$

Definition

A *curve image* (or simply a curve) is any set of points that arises by traversing some **continuous** curve. In other words, a curve image is any set that can be written in the form

$$\{(f_1(t), \dots, f_n(t)) \mid t \in [a, b]\} \quad ,$$

for some **continuous** functions f_1, \dots, f_n .

If we don't require that the functions be **continuous**, every set of points will be a curve and the definition would be pointless.

Recall a parametric curve C was defined as the data

$$C : \begin{cases} x_1 = f_1(t) \\ x_2 = f_2(t) \\ \vdots \\ x_n = f_n(t) \end{cases}, t \in [a, b]$$

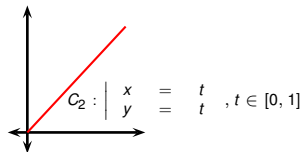
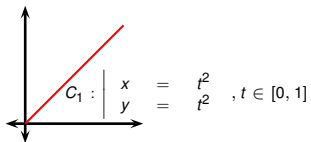
Definition

A *curve image* (or simply a curve) is any set of points that arises by traversing some continuous curve. In other words, a curve image is any set that can be written in the form

$$\{(f_1(t), \dots, f_n(t)) \mid t \in [a, b]\} \quad ,$$

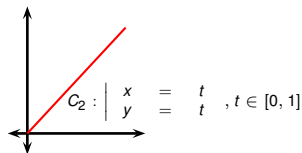
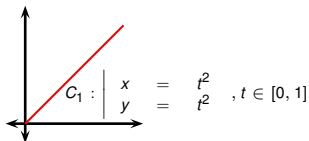
for some continuous functions f_1, \dots, f_n .

Informally, a curve image “remembers” only the points lying on the curve but forgets the “speed” with which each point was visited and “how many times” each point was visited.



Question

Are the above curves different?

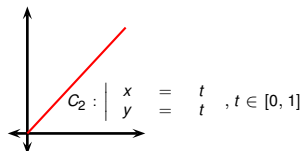
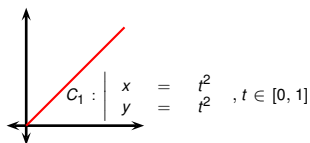


Question

Are the above curves different?

Are the above parametric curves different? Yes.

- As parametric curves, C_1 and C_2 are different: C_1, C_2 are given by different functions.



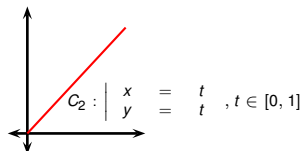
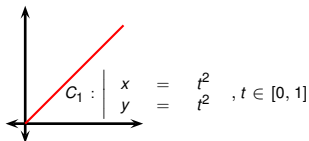
Question

Are the above curves different?

Are the above parametric curves different? Yes.

Are the above curve images different? No.

- As parametric curves, C_1 and C_2 are different: C_1, C_2 are given by different functions.
- As curve images, C_1, C_2 coincide.



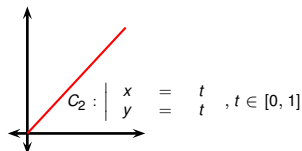
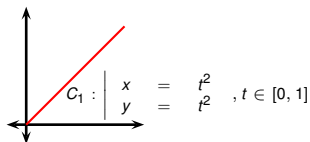
Question

~~Are the above curves different?~~

Are the above parametric curves different? Yes.

Are the above curve images different? No.

- As parametric curves, C_1 and C_2 are different: C_1, C_2 are given by different functions.
- As curve images, C_1, C_2 coincide.
- The original question is incorrectly posed: the word “curve” does not have a mathematical definition without the words “parametric” or “image” attached to it.



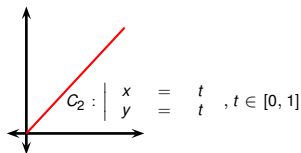
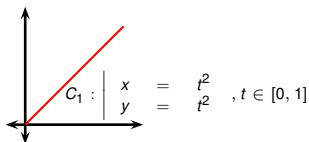
Question

~~Are the above curves different?~~

Are the above parametric curves different? Yes.

Are the above curve images different? No.

- Nonetheless we sometimes use the word “curve” **informally**, without specifying “parametric curve” or “curve image”.



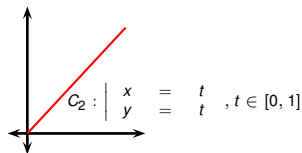
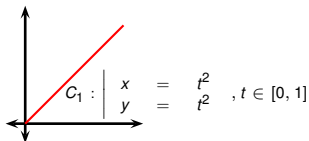
Question

~~Are the above curves different?~~

Are the above parametric curves different? Yes.

Are the above curve images different? No.

- Nonetheless we sometimes use the word “curve” informally, without specifying “parametric curve” or “curve image”.
- In this case, whether we mean “parametric curve” or “curve image” should be clear from the context.



Question

~~Are the above curves different?~~

Are the above parametric curves different? Yes.

Are the above curve images different? No.

- Nonetheless we sometimes use the word “curve” informally, without specifying “parametric curve” or “curve image”.
- In this case, whether we mean “parametric curve” or “curve image” should be clear from the context. **If not, we are using mathematical language incorrectly.**

Graphs of functions as curve images

- Consider a graph of a function given by

$$y = f(x)$$

Graphs of functions as curve images

- Consider a graph of a function given by

$$y = f(x)$$

- Write $x = t$. Then $y = f(x)$

Graphs of functions as curve images

- Consider a graph of a function given by

$$y = f(x)$$

- Write $x = t$. Then $y = f(x) = f(t)$

Graphs of functions as curve images

- Consider a graph of a function given by

$$y = f(x)$$

- Write $x = t$. Then $y = f(x) = f(t)$, so we get the system

$$C : \begin{cases} x = t \\ y = f(t) \end{cases}, t \in [a, b]$$

Graphs of functions as curve images

- Consider a graph of a function given by

$$y = f(x)$$

- Write $x = t$. Then $y = f(x) = f(t)$, so we get the system

$$C : \begin{cases} x = t \\ y = f(t) \end{cases}, t \in [a, b]$$

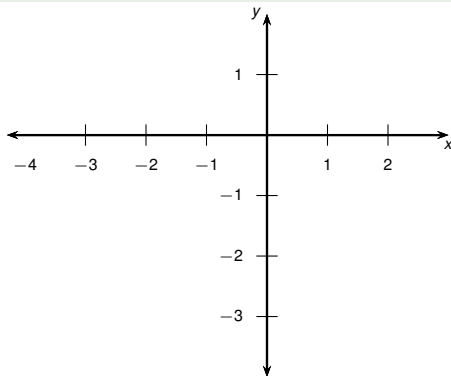
Observation

The graph of an arbitrary function can be written as the image of a curve C using the above transformation.

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

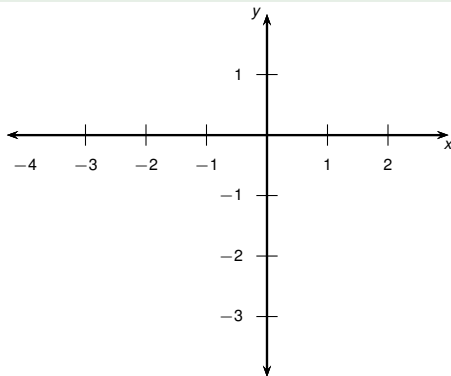


t	x	y
-2		
-1		
0		
1		
2		

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

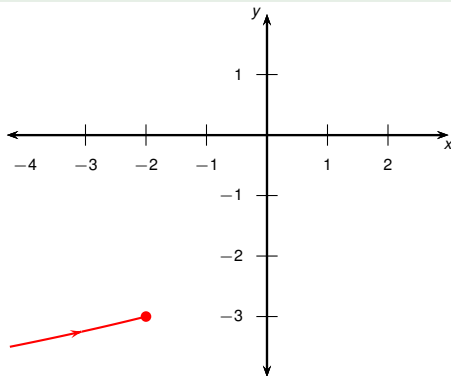


t	x	y
-2		
-1		
0		
1		
2		

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

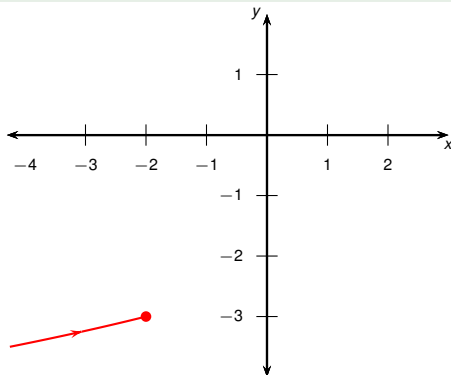


t	x	y
-2	-2	-3
-1		
0		
1		
2		

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

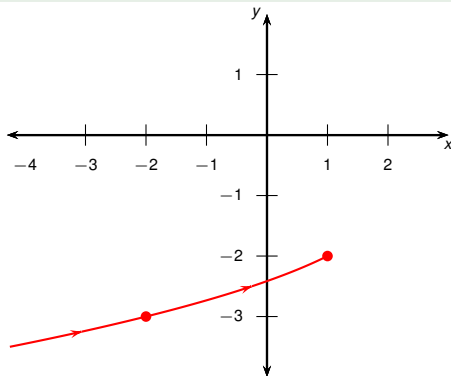


t	x	y
-2	-2	-3
-1		
0		
1		
2		

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

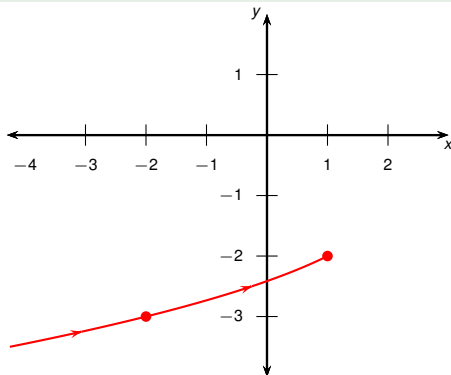


t	x	y
-2	-2	-3
-1	1	-2
0		
1		
2		

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

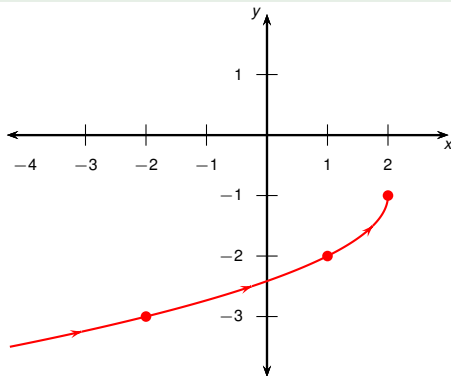


t	x	y
-2	-2	-3
-1	1	-2
0		
1		
2		

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

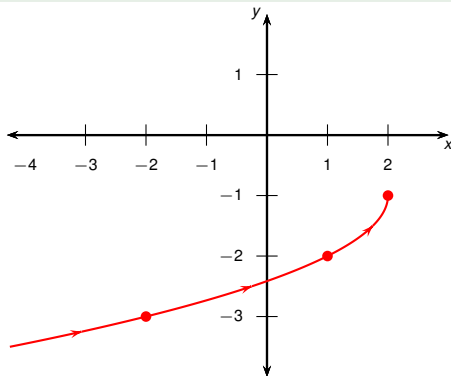


t	x	y
-2	-2	-3
-1	1	-2
0	2	-1
1		
2		

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

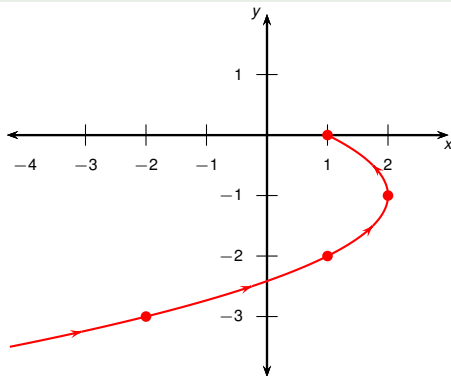


t	x	y
-2	-2	-3
-1	1	-2
0	2	-1
1		
2		

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

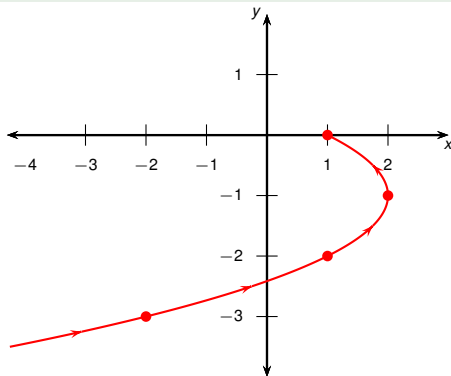


t	x	y
-2	-2	-3
-1	1	-2
0	2	-1
1	1	0
2		

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

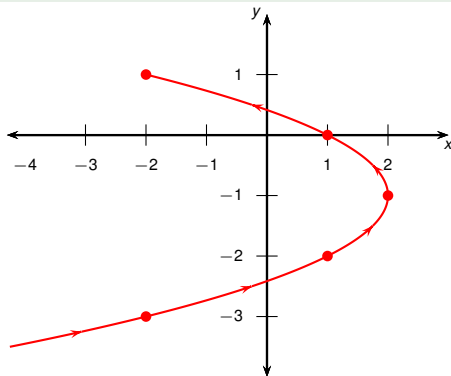


t	x	y
-2	-2	-3
-1	1	-2
0	2	-1
1	1	0
2	-2	1

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

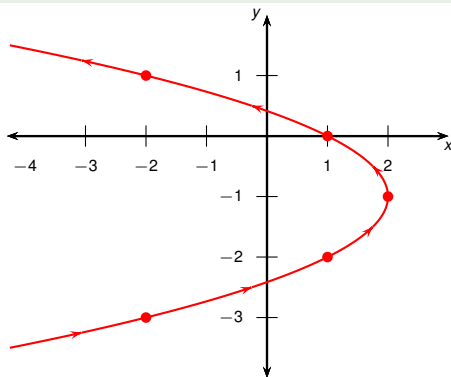


t	x	y
-2	-2	-3
-1	1	-2
0	2	-1
1	1	0
2	-2	1

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

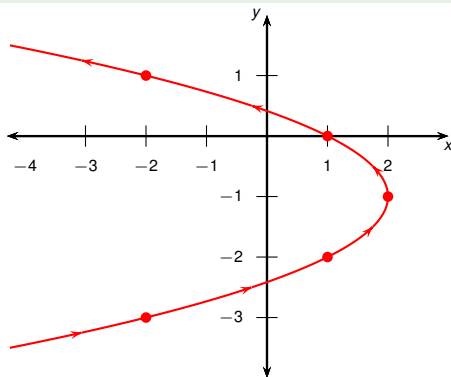


t	x	y
-2	-2	-3
-1	-1	-2
0	2	-1
1	1	0
2	-2	1

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

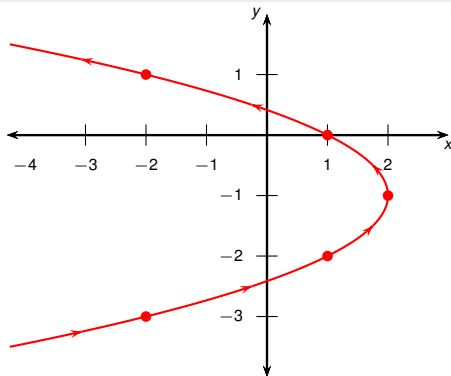


t	x	y
-2	-2	-3
-1	-1	-2
0	2	-1
1	1	0
2	-2	1

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$



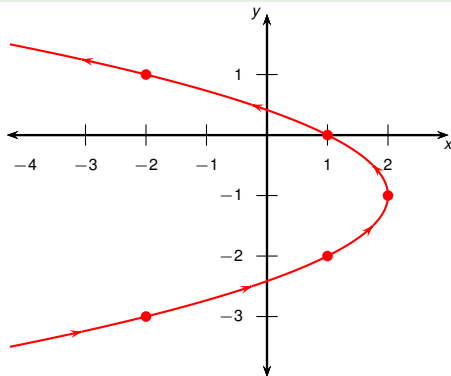
t	x	y
-2	-2	-3
-1	1	-2
0	2	-1
1	1	0
2	-2	1

Eliminate t : from second equation
we have $t = y + 1$

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$



t	x	y
-2	-2	-3
-1	-1	-2
0	2	-1
1	1	0
2	-2	1

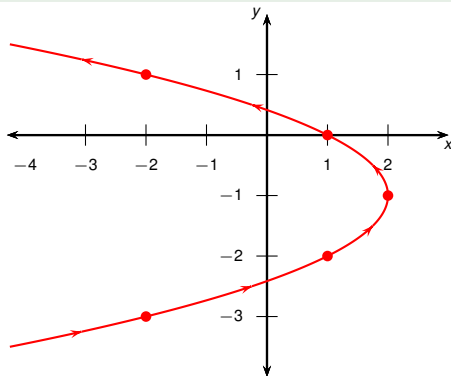
Eliminate t : from second equation we have $t = y + 1$ and therefore:

$$x = -t^2 + 2$$

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$



t	x	y
-2	-2	-3
-1	-1	-2
0	2	-1
1	1	0
2	-2	1

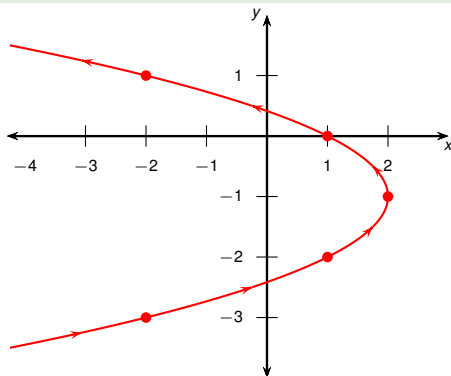
Eliminate t : from second equation we have $t = y + 1$ and therefore:

$$\begin{aligned} x &= -t^2 + 2 \\ &= -(y + 1)^2 + 2 \end{aligned}$$

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$



t	x	y
-2	-2	-3
-1	-1	-2
0	2	-1
1	1	0
2	-2	1

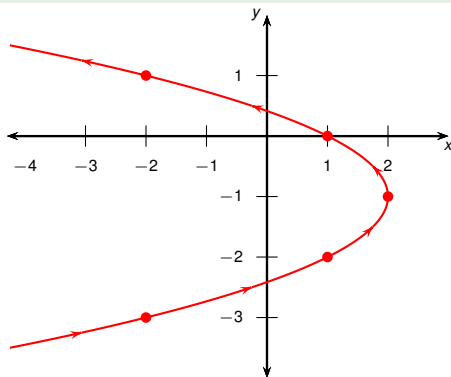
Eliminate t : from second equation we have $t = y + 1$ and therefore:

$$\begin{aligned} x &= -t^2 + 2 \\ &= -(y + 1)^2 + 2 \\ &= -y^2 - 2y + 1 \end{aligned}$$

Example

Sketch and identify the curve image defined by the equations

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$



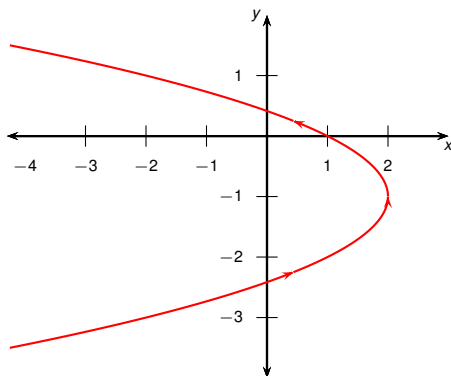
t	x	y
-2	-2	-3
-1	1	-2
0	2	-1
1	1	0
2	-2	1

Eliminate t : from second equation we have $t = y + 1$ and therefore:

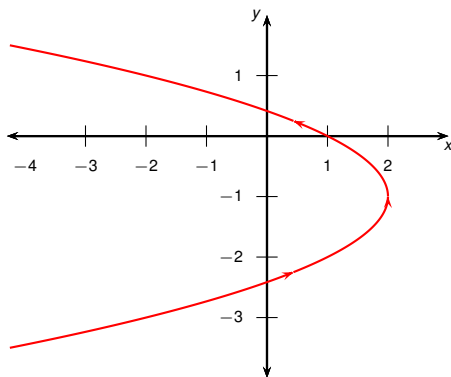
$$\begin{aligned} x &= -t^2 + 2 \\ &= -(y + 1)^2 + 2 \\ &= -y^2 - 2y + 1 \end{aligned}$$

Thus our curve image is a parabola, as expected.

- There was no restriction placed on t in the last example.

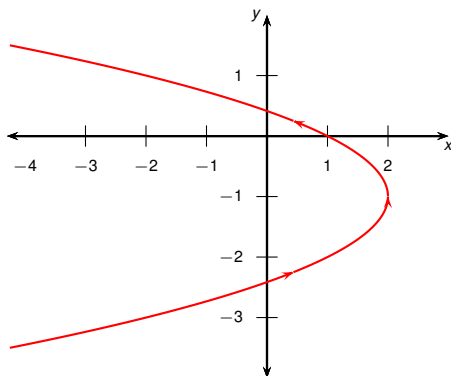


$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$



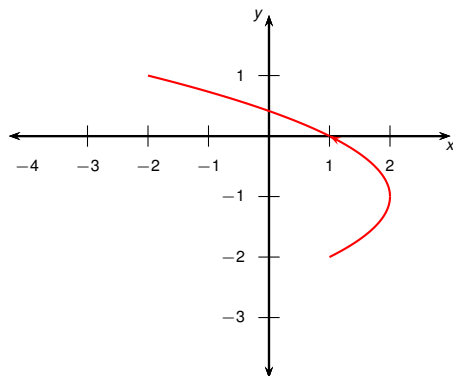
$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

- There was no restriction placed on t in the last example.
- In such a case we assume $t \in (-\infty, \infty)$, i.e., t runs over all real numbers.



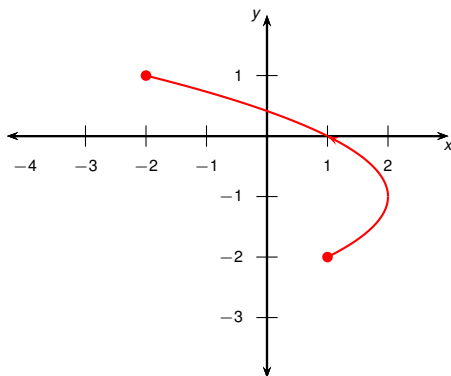
$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}$$

- There was no restriction placed on t in the last example.
- In such a case we assume $t \in (-\infty, \infty)$, i.e., t runs over all real numbers.
- In general we are expected to specify the interval in which t lies.



$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}, -1 \leq t \leq 2$$

- There was no restriction placed on t in the last example.
- In such a case we assume $t \in (-\infty, \infty)$, i.e., t runs over all real numbers.
- In general we are expected to specify the interval in which t lies.
- For example, if we restrict the previous example to $t \in [-1, 2]$, we get the part of the parabola that begins at $(1, -2)$ and ends at $(-2, 1)$.



$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}, -1 \leq t \leq 2$$

- There was no restriction placed on t in the last example.
- In such a case we assume $t \in (-\infty, \infty)$, i.e., t runs over all real numbers.
- In general we are expected to specify the interval in which t lies.
- For example, if we restrict the previous example to $t \in [-1, 2]$, we get the part of the parabola that begins at $(1, -2)$ and ends at $(-2, 1)$.
- We say that $(1, -2)$ is the initial point and $(-2, 1)$ is the terminal point of the curve.

Implicit vs Explicit (Parametric) Curve Equations

- Consider the parametric curve

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}.$$

Implicit vs Explicit (Parametric) Curve Equations

- Consider the parametric curve

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}.$$

- As we saw in preceding slides/lectures, all points (x, y) on the image of this curve satisfy the equation

$$x + (y + 1)^2 - 2 = 0$$

Implicit vs Explicit (Parametric) Curve Equations

- Consider the parametric curve

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}.$$

- As we saw in preceding slides/lectures, all points (x, y) on the image of this curve satisfy the equation

$$x + (y + 1)^2 - 2 = 0$$

- Equations of the first form are called explicit (parametric) curve equations.

Implicit vs Explicit (Parametric) Curve Equations

- Consider the parametric curve

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}.$$

- As we saw in preceding slides/lectures, all points (x, y) on the image of this curve satisfy the equation

$$x + (y + 1)^2 - 2 = 0$$

- Equations of the first form are called explicit (parametric) curve equations.
- Equations of the second form are called implicit equations of the curve image.

Implicit vs Explicit (Parametric) Curve Equations

- Consider the parametric curve

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}.$$

- As we saw in preceding slides/lectures, all points (x, y) on the image of this curve satisfy the equation

$$x + (y + 1)^2 - 2 = 0$$

- Equations of the first form are called explicit (parametric) curve equations.
- Equations of the second form are called implicit equations of the curve image.
- Explicit (parametric) curve equations have the advantage that it is easy to generate points on the curve.

Implicit vs Explicit (Parametric) Curve Equations

- Consider the parametric curve

$$\begin{cases} x = -t^2 + 2 \\ y = t - 1 \end{cases}.$$

- As we saw in preceding slides/lectures, all points (x, y) on the image of this curve satisfy the equation

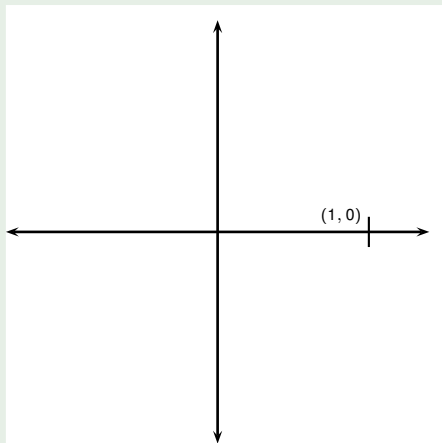
$$x + (y + 1)^2 - 2 = 0$$

- Equations of the first form are called explicit (parametric) curve equations.
- Equations of the second form are called implicit equations of the curve image.
- Explicit (parametric) curve equations have the advantage that it is easy to generate points on the curve.
- Implicit curve equations have the advantage that it is easy to check whether a point is on the curve.

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

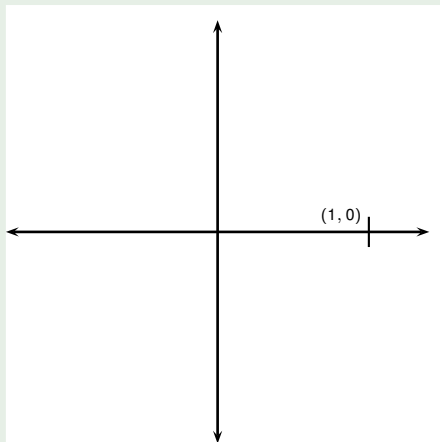


t	x	y
0		
$\frac{\pi}{6}$		
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		
π		
$\frac{3\pi}{2}$		
2π		

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

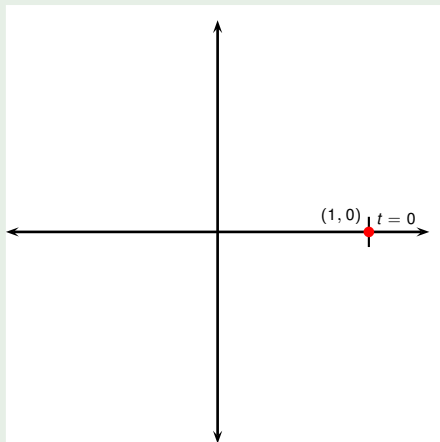


t	x	y
0		
$\frac{\pi}{6}$		
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		
π		
$\frac{3\pi}{2}$		
2π		

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

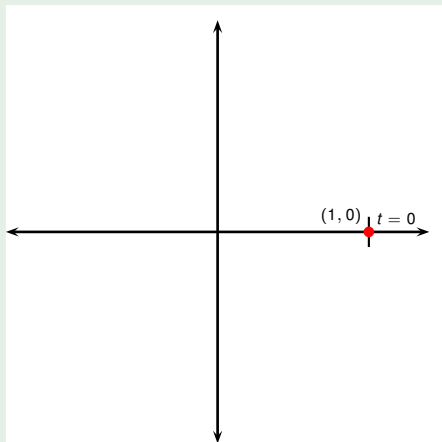


t	x	y
0	1	0
$\frac{\pi}{6}$		
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		
π		
$\frac{3\pi}{2}$		
2π		

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

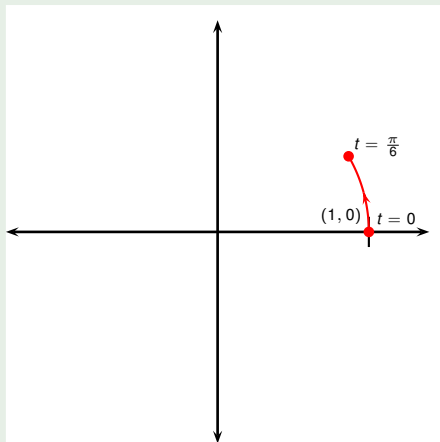


t	x	y
0	1	0
$\frac{\pi}{6}$		
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		
π		
$\frac{3\pi}{2}$		
2π		

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

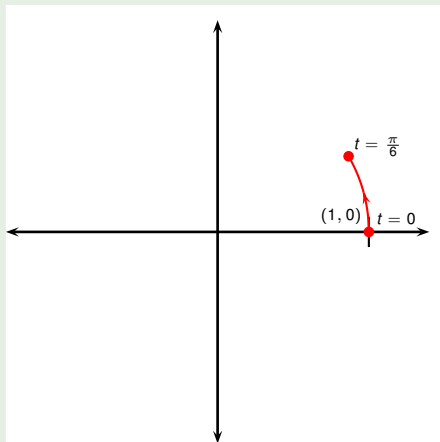


t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		
π		
$\frac{3\pi}{2}$		
2π		

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

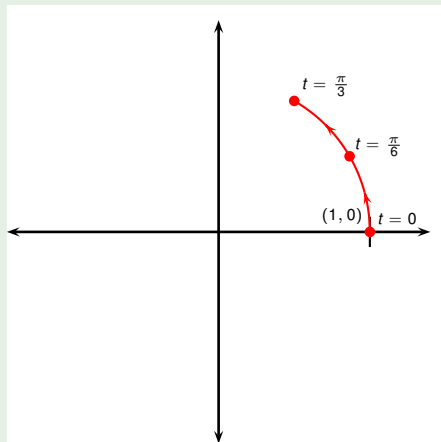


t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		
π		
$\frac{3\pi}{2}$		
2π		

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

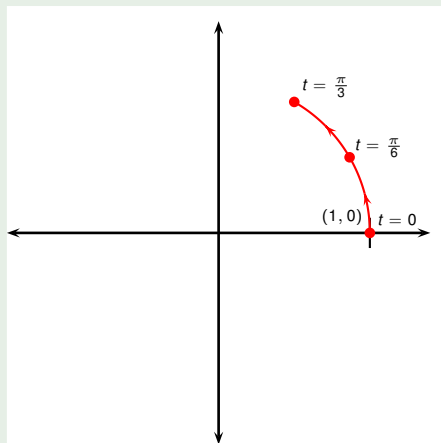


t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$		
π		
$\frac{3\pi}{2}$		
2π		

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

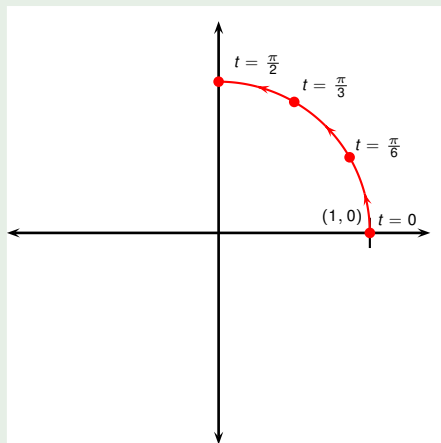


t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$		
π		
$\frac{3\pi}{2}$		
2π		

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

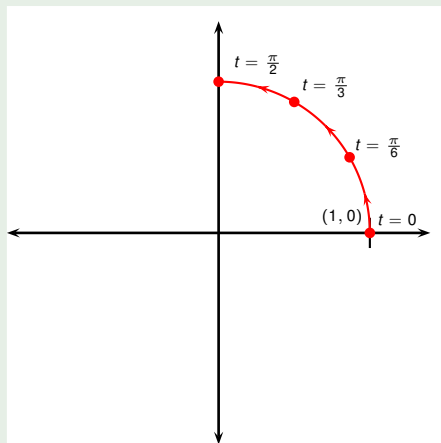


t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
π		
$\frac{3\pi}{2}$		
2π		

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

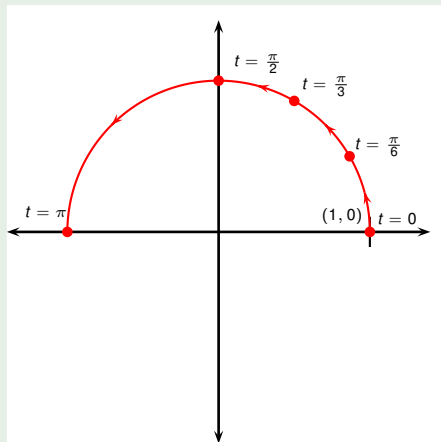


t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
π		
$\frac{3\pi}{2}$		
2π		

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

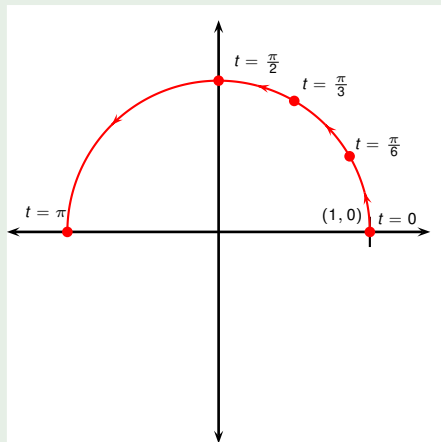


t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$		
2π		

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

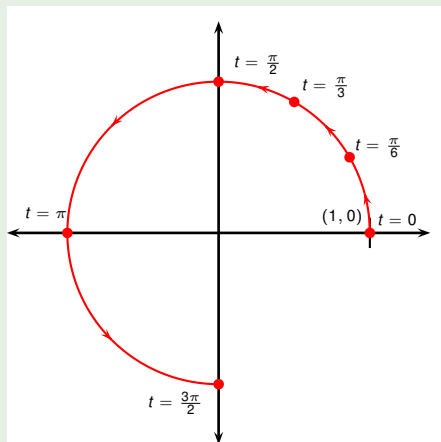


t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$		
2π		

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

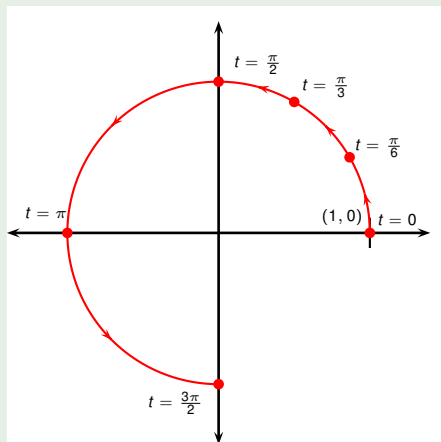


t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	-1
2π		

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

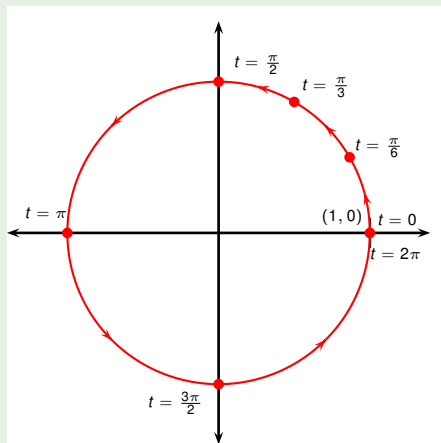


t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	-1
2π		

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$

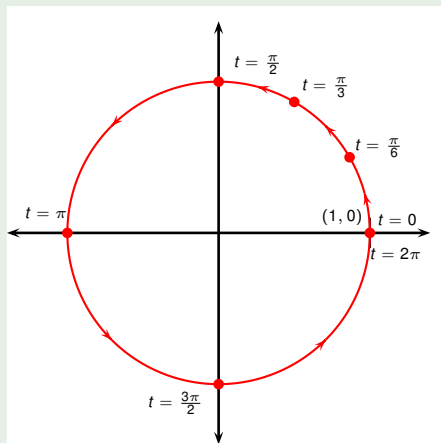


t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	-1
2π	1	0

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$



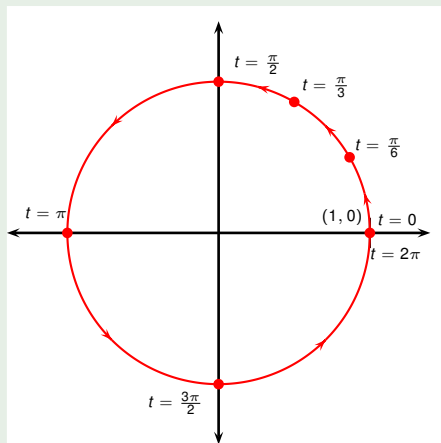
t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	-1
2π	1	0

$$x^2 + y^2 =$$

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$



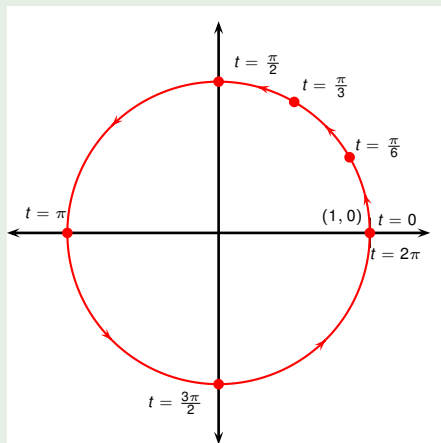
t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	-1
2π	1	0

$$x^2 + y^2 = \cos^2 t + \sin^2 t =$$

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$



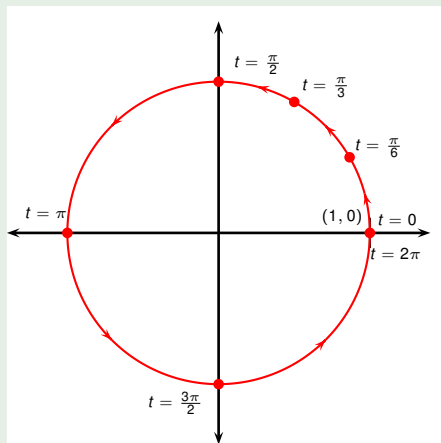
t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	-1
2π	1	0

$$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$$

Example

Sketch and identify the curve defined by the parametric equations

$$x = \cos t, \quad y = \sin t.$$



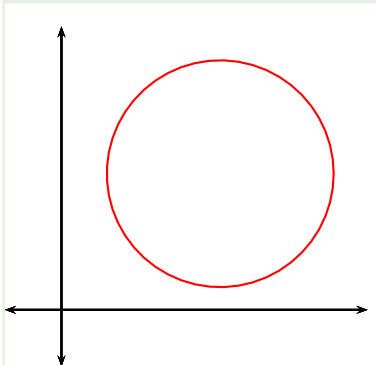
t	x	y
0	1	0
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	-1
2π	1	0

$$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$$

Therefore (x, y) travels on the unit circle $x^2 + y^2 = 1$.

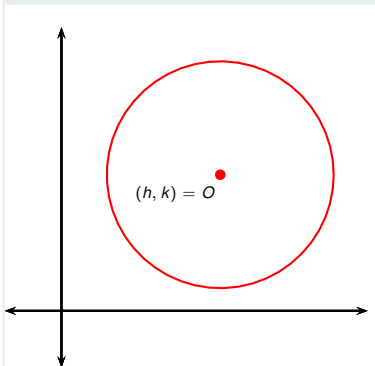
Example

Find parametric equations for the circle with center (h, k) and radius r .



Example

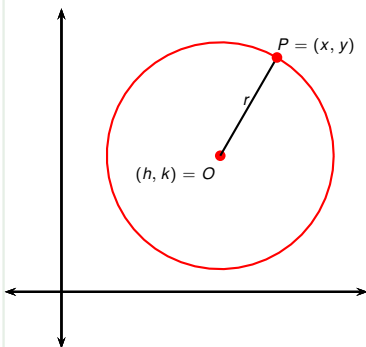
Find parametric equations for the circle with center (h, k) and radius r .



- Let O be the center of the circle with coordinates (h, k) .

Example

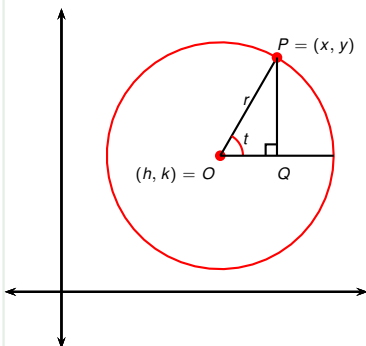
Find parametric equations for the circle with center (h, k) and radius r .



- Let O be the center of the circle with coordinates (h, k) .
- Let P be a point on the circle with coordinates (x, y) .

Example

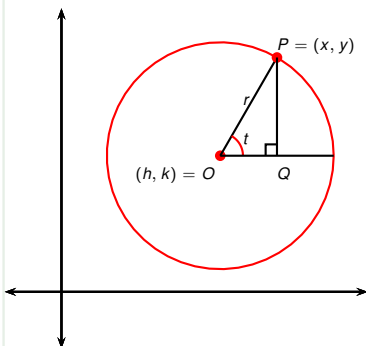
Find parametric equations for the circle with center (h, k) and radius r .



- Let O be the center of the circle with coordinates (h, k) .
- Let P be a point on the circle with coordinates (x, y) .
- Let t, Q be as indicated on the figure.

Example

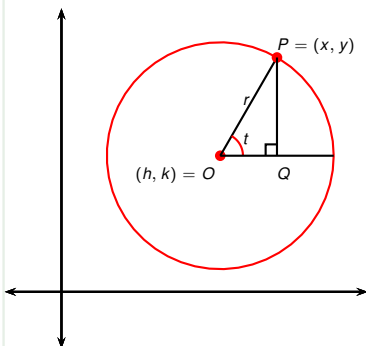
Find parametric equations for the circle with center (h, k) and radius r .



- Let O be the center of the circle with coordinates (h, k) .
- Let P be a point on the circle with coordinates (x, y) .
- Let t , Q be as indicated on the figure.
- Then $|OQ| = ?$.

Example

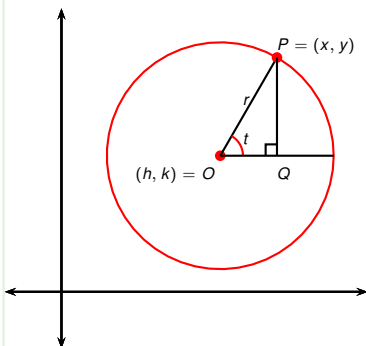
Find parametric equations for the circle with center (h, k) and radius r .



- Let O be the center of the circle with coordinates (h, k) .
- Let P be a point on the circle with coordinates (x, y) .
- Let t , Q be as indicated on the figure.
- Then $|OQ| = r \cos t$.

Example

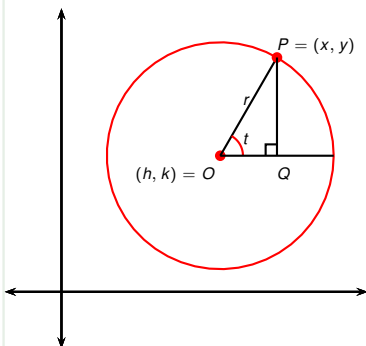
Find parametric equations for the circle with center (h, k) and radius r .



- Let O be the center of the circle with coordinates (h, k) .
- Let P be a point on the circle with coordinates (x, y) .
- Let t , Q be as indicated on the figure.
- Then $|OQ| = r \cos t$.
- $|PQ| = ?$.

Example

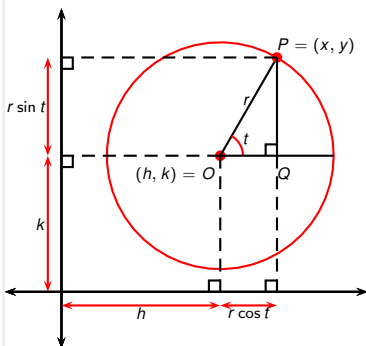
Find parametric equations for the circle with center (h, k) and radius r .



- Let O be the center of the circle with coordinates (h, k) .
- Let P be a point on the circle with coordinates (x, y) .
- Let t , Q be as indicated on the figure.
- Then $|OQ| = r \cos t$.
- $|PQ| = r \sin t$.

Example

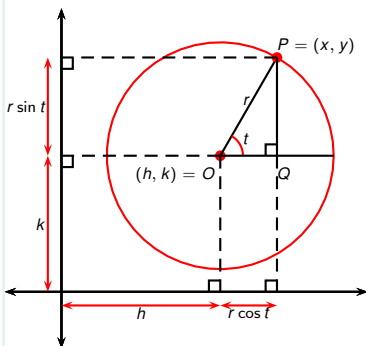
Find parametric equations for the circle with center (h, k) and radius r .



- Let O be the center of the circle with coordinates (h, k) .
- Let P be a point on the circle with coordinates (x, y) .
- Let t , Q be as indicated on the figure.
- Then $|OQ| = r \cos t$.
- $|PQ| = r \sin t$.
- Then the coordinates of P are $(h + r \cos t, k + r \sin t)$.

Example

Find parametric equations for the circle with center (h, k) and radius r .

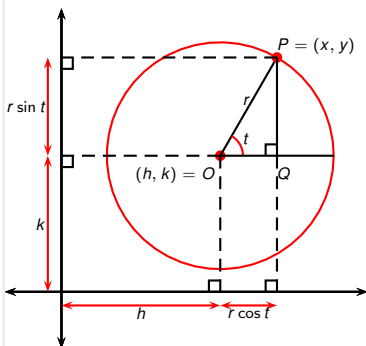


- Let O be the center of the circle with coordinates (h, k) .
- Let P be a point on the circle with coordinates (x, y) .
- Let t , Q be as indicated on the figure.
- Then $|OQ| = r \cos t$.
- $|PQ| = r \sin t$.
- Then the coordinates of P are $(h + r \cos t, k + r \sin t)$.
- In this way we get the parametric equations

$$\begin{cases} x = h + r \cos t \\ y = k + r \sin t \end{cases}, t \in [0, 2\pi]$$

Example

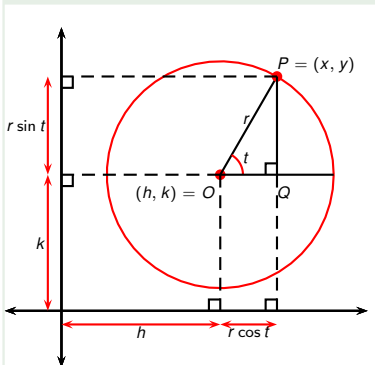
Find parametric equations for the circle with center (h, k) and radius r .



- Alternative solution: $x = \cos t$, $y = \sin t$ are parametric equations of the unit circle.

Example

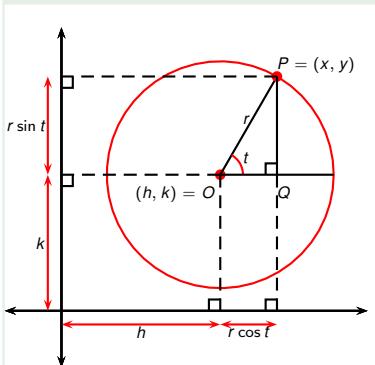
Find parametric equations for the circle with center (h, k) and radius r .



- Alternative solution: $x = \cos t$, $y = \sin t$ are parametric equations of the unit circle.
- Multiply by r to scale the circle to have radius r : $x = r \cos t$, $y = r \sin t$.

Example

Find parametric equations for the circle with center (h, k) and radius r .



- Alternative solution: $x = \cos t$, $y = \sin t$ are parametric equations of the unit circle.
- Multiply by r to scale the circle to have radius r : $x = r \cos t$, $y = r \sin t$.
- Add h to x and k to y to translate the circle h units to the left and k units up: $x = h + r \cos t$, $y = k + r \sin t$

The Cycloid



Definition (Cycloid)

The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a cycloid.

The Cycloid



Definition (Cycloid)

The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a cycloid.

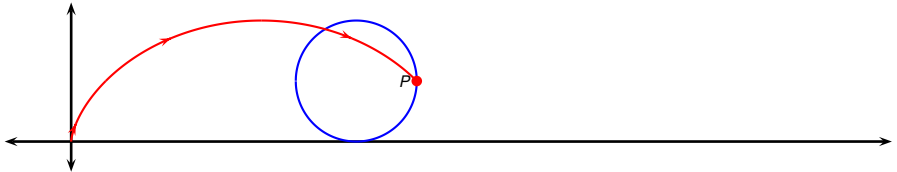
The Cycloid



Definition (Cycloid)

The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a cycloid.

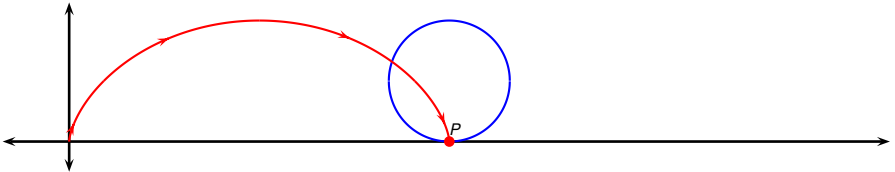
The Cycloid



Definition (Cycloid)

The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a cycloid.

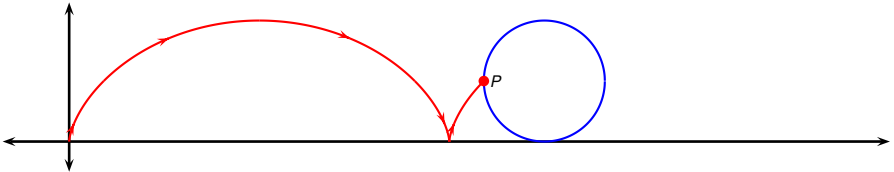
The Cycloid



Definition (Cycloid)

The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a cycloid.

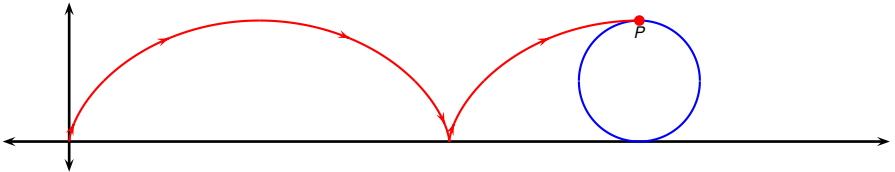
The Cycloid



Definition (Cycloid)

The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a cycloid.

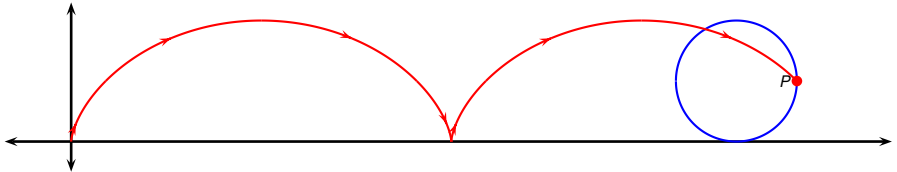
The Cycloid



Definition (Cycloid)

The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a cycloid.

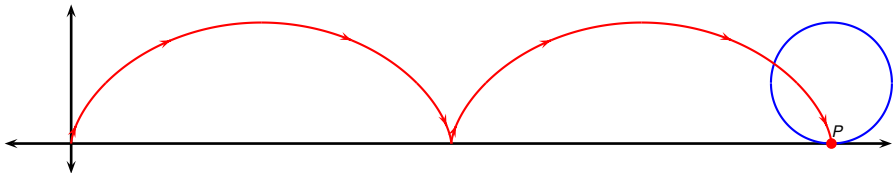
The Cycloid



Definition (Cycloid)

The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a cycloid.

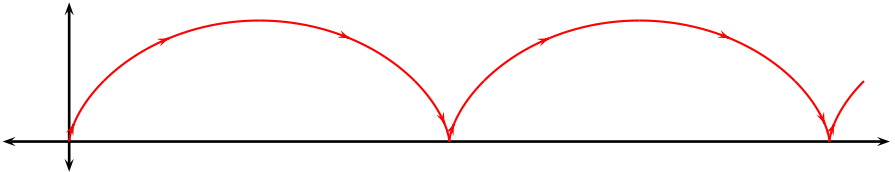
The Cycloid



Definition (Cycloid)

The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a cycloid.

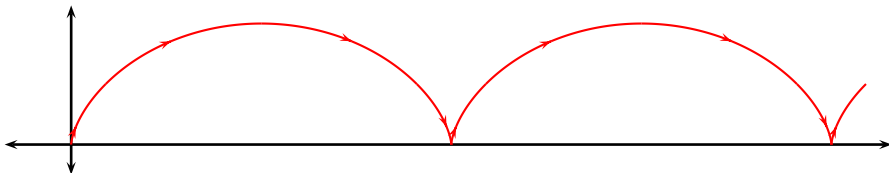
The Cycloid



Definition (Cycloid)

The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a cycloid.

The Cycloid

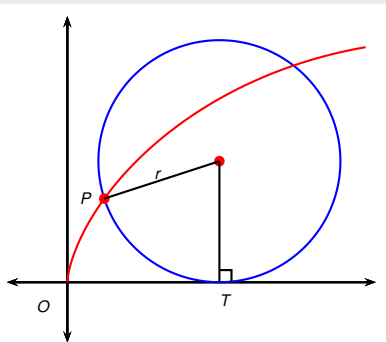


Definition (Cycloid)

The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a cycloid.

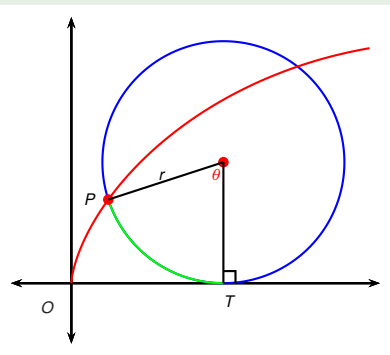
Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



Example

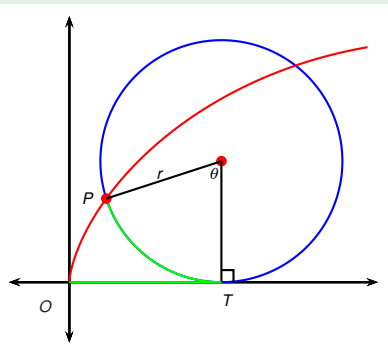
Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



- We choose our parameter to be θ , the angle of rotation of the circle.

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.

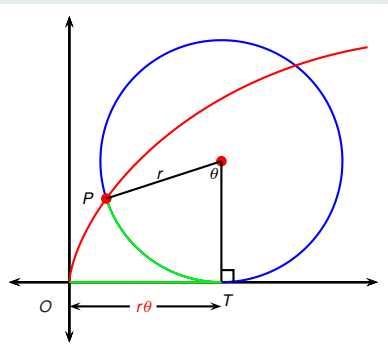


- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT$$

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.

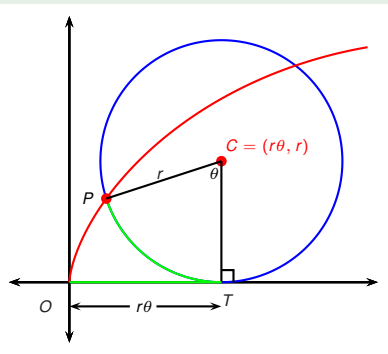


- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT = r\theta$$

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



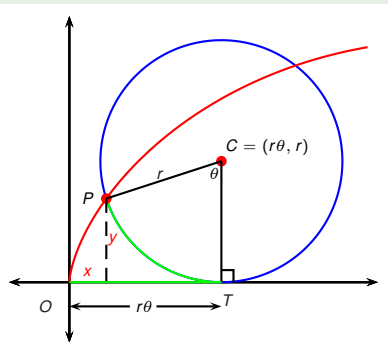
- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT = r\theta$$

- Then the center is $C = (r\theta, r)$.

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT = r\theta$$

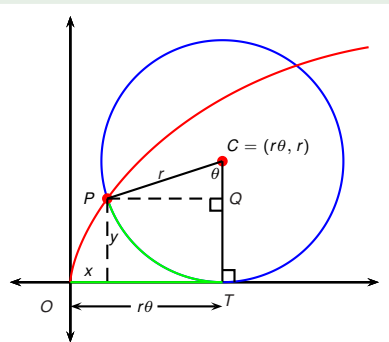
- Then the center is $C = (r\theta, r)$.
- Let the coordinates of P be (x, y) .

$$x =$$

$$y =$$

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT = r\theta$$

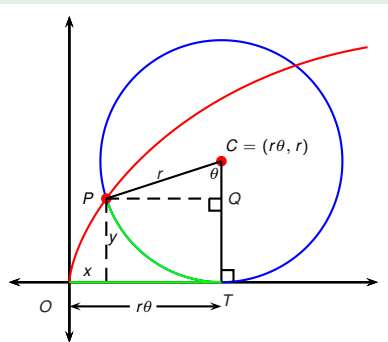
- Then the center is $C = (r\theta, r)$.
- Let the coordinates of P be (x, y) .

$$x =$$

$$y =$$

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT = r\theta$$

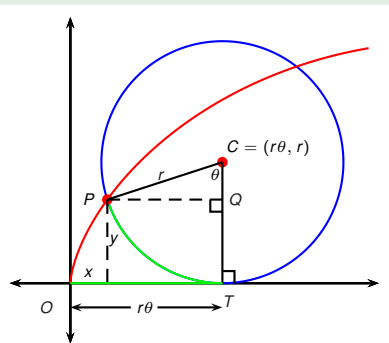
- Then the center is $C = (r\theta, r)$.
- Let the coordinates of P be (x, y) .

$$x = |OT| - |PQ|$$

$$y =$$

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT = r\theta$$

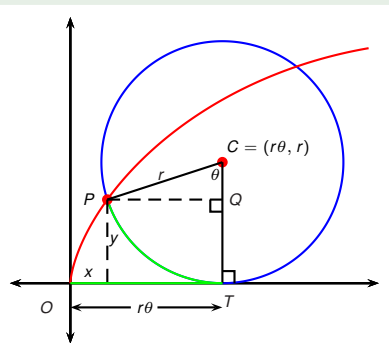
- Then the center is $C = (r\theta, r)$.
- Let the coordinates of P be (x, y) .

$$x = |OT| - |PQ| = \quad -$$

$$y =$$

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT = r\theta$$

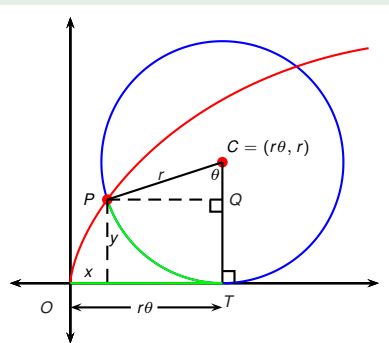
- Then the center is $C = (r\theta, r)$.
- Let the coordinates of P be (x, y) .

$$x = |OT| - |PQ| = r\theta -$$

$$y =$$

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT = r\theta$$

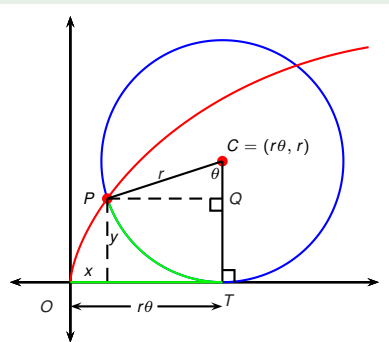
- Then the center is $C = (r\theta, r)$.
- Let the coordinates of P be (x, y) .

$$x = |OT| - |PQ| = r\theta -$$

$$y =$$

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT = r\theta$$

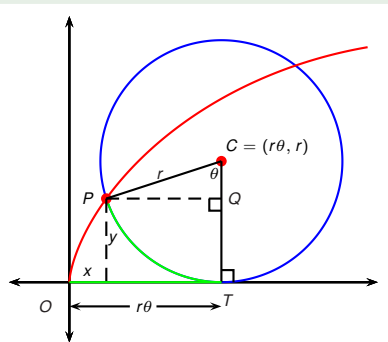
- Then the center is $C = (r\theta, r)$.
- Let the coordinates of P be (x, y) .

$$x = |OT| - |PQ| = r\theta - r \sin \theta$$

$$y =$$

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT = r\theta$$

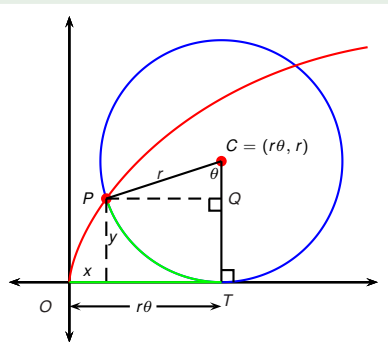
- Then the center is $C = (r\theta, r)$.
- Let the coordinates of P be (x, y) .

$$x = |OT| - |PQ| = r\theta - r \sin \theta$$

$$y = |CT| - |CQ|$$

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT = r\theta$$

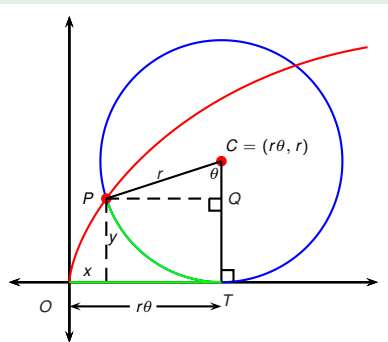
- Then the center is $C = (r\theta, r)$.
- Let the coordinates of P be (x, y) .

$$x = |OT| - |PQ| = r\theta - r \sin \theta$$

$$y = \textcolor{red}{|CT|} - |CQ| = r - r \cos \theta$$

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT = r\theta$$

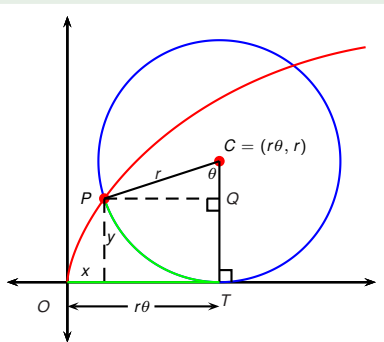
- Then the center is $C = (r\theta, r)$.
- Let the coordinates of P be (x, y) .

$$x = |OT| - |PQ| = r\theta - r \sin \theta$$

$$y = |CT| - |CQ| = r - r \cos \theta$$

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT = r\theta$$

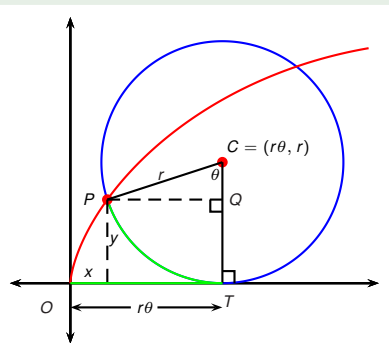
- Then the center is $C = (r\theta, r)$.
- Let the coordinates of P be (x, y) .

$$x = |OT| - |PQ| = r\theta - r \sin \theta$$

$$y = |CT| - |CQ| = r - r \cos \theta$$

Example

Find parametric equations of a cycloid made using a circle with radius r that rolls along the x -axis such that P hits the origin.



- We choose our parameter to be θ , the angle of rotation of the circle.
- How far has the circle moved if it has rolled through θ radians?

$$|OT| = \text{arc } PT = r\theta$$

- Then the center is $C = (r\theta, r)$.
- Let the coordinates of P be (x, y) .

$$x = |OT| - |PQ| = r\theta - r \sin \theta$$

$$y = |CT| - |CQ| = r - r \cos \theta$$

Therefore the equations are

$$x = r(\theta - \sin \theta), \quad y = r(1 - \cos \theta), \quad \theta \in \mathbb{R}$$

- Recall polar coordinates:

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

- Recall polar coordinates:

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

- A curve in polar coordinates is given by specifying explicit or implicit equations in polar coordinates.

Example

What curve is represented by the polar equation $r = 2$?



Example

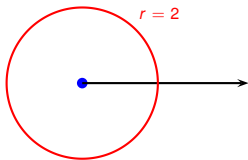
What curve is represented by the polar equation $r = 2$?

- The equation describes all points that are 2 units away from O .



Example

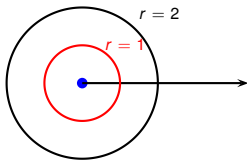
What curve is represented by the polar equation $r = 2$?



- The equation describes all points that are 2 units away from O .
- This is the circle with center O and radius 2.

Example

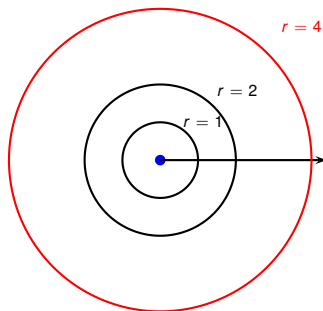
What curve is represented by the polar equation $r = 2$?



- The equation describes all points that are 2 units away from O .
- This is the circle with center O and radius 2.
- The equation $r = 1$ describes the unit circle.

Example

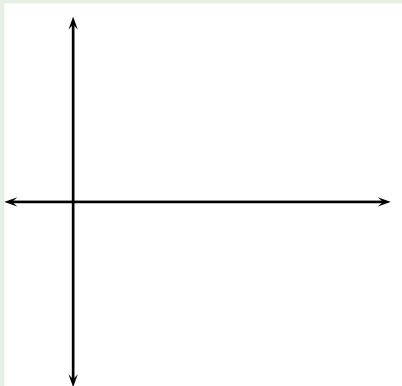
What curve is represented by the polar equation $r = 2$?



- The equation describes all points that are 2 units away from O .
- This is the circle with center O and radius 2.
- The equation $r = 1$ describes the unit circle.
- The equation $r = 4$ describes the circle with center O and radius 4.

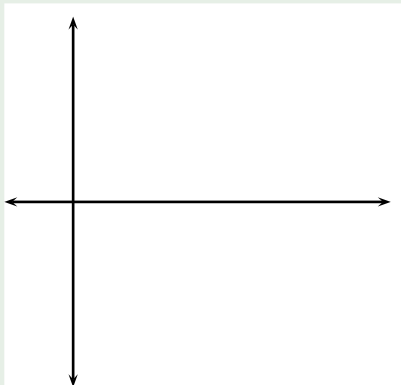
Example

- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



Example

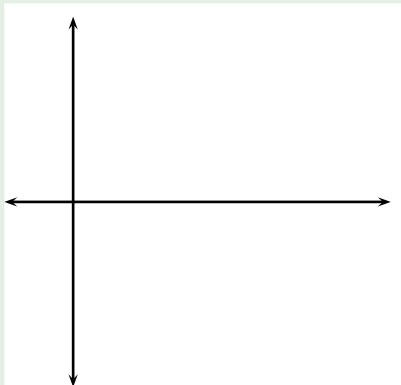
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	
$\pi/6$	
$\pi/4$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

Example

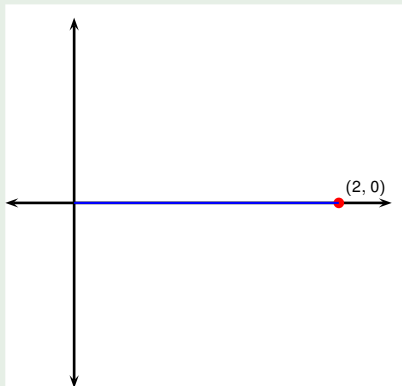
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	
$\pi/6$	
$\pi/4$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

Example

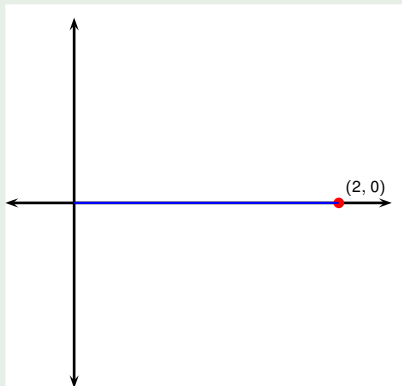
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	
$\pi/4$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

Example

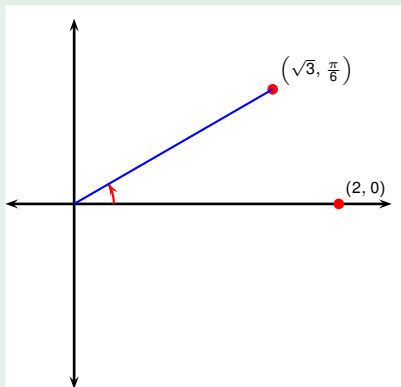
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	
$\pi/4$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

Example

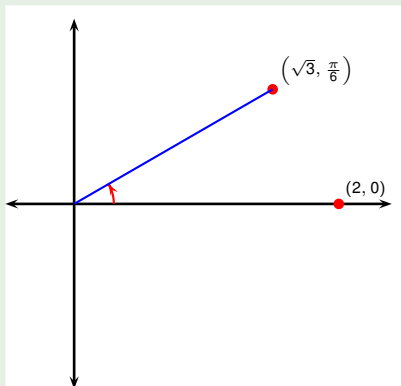
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

Example

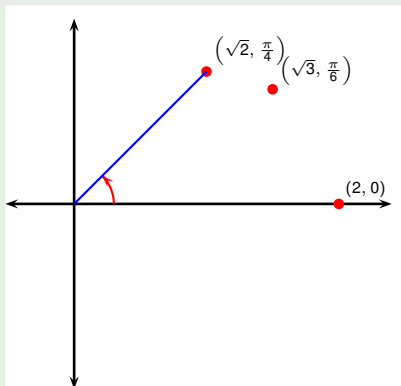
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

Example

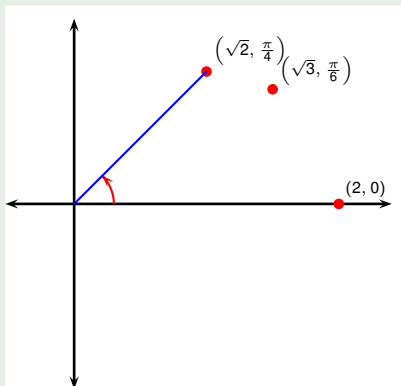
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

Example

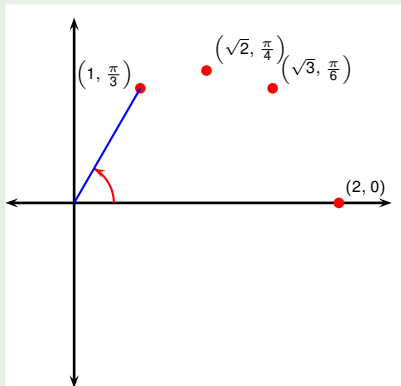
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

Example

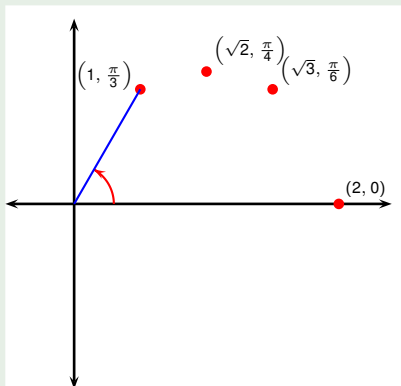
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

Example

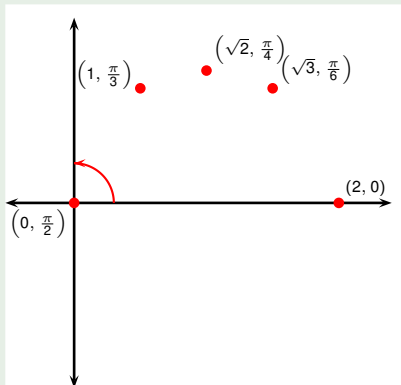
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

Example

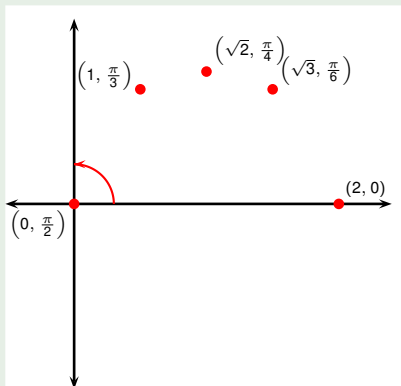
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

Example

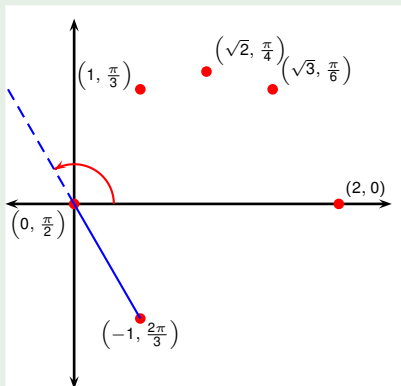
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

Example

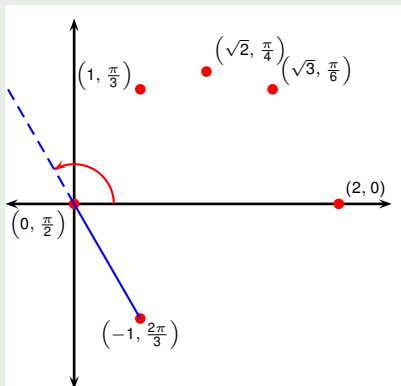
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	
$5\pi/6$	
π	

Example

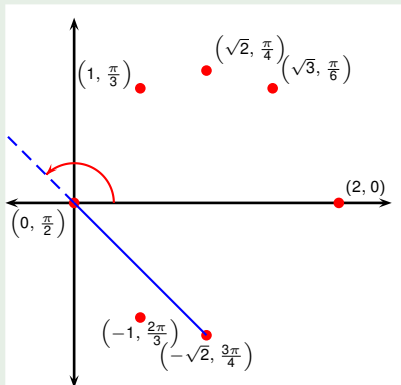
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	
$5\pi/6$	
π	

Example

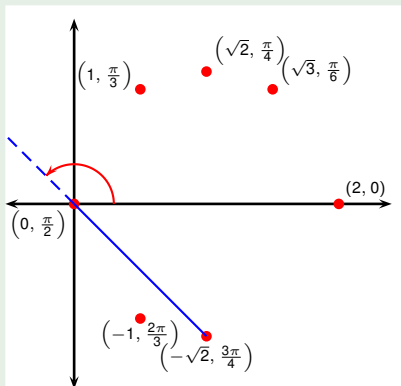
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	
π	

Example

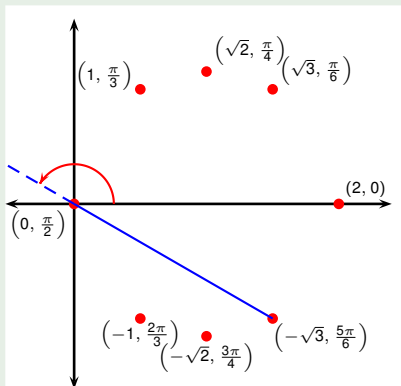
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	
π	

Example

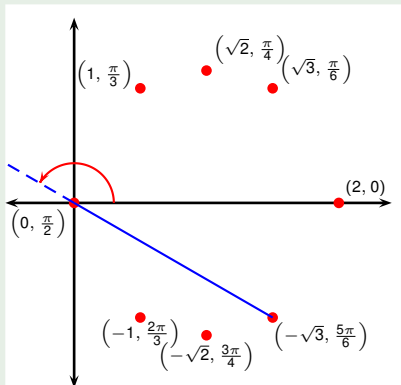
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	

Example

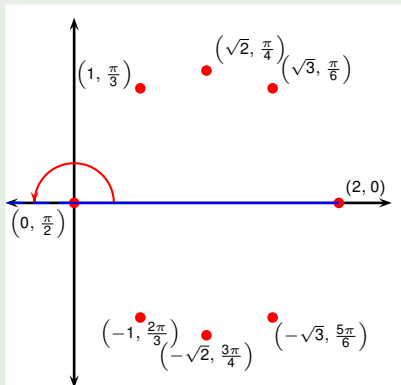
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	

Example

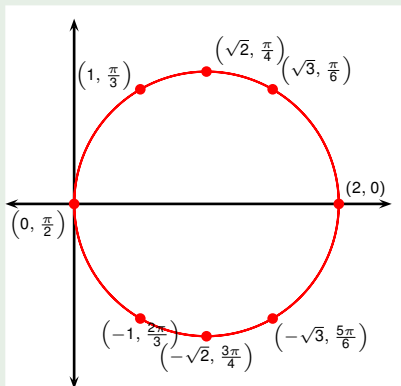
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	-2

Example

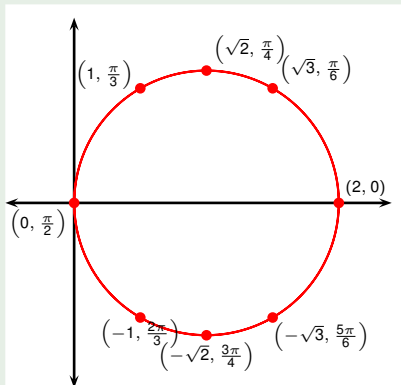
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	-2

Example

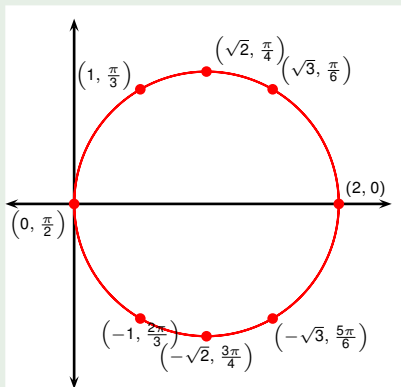
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



● $X =$

Example

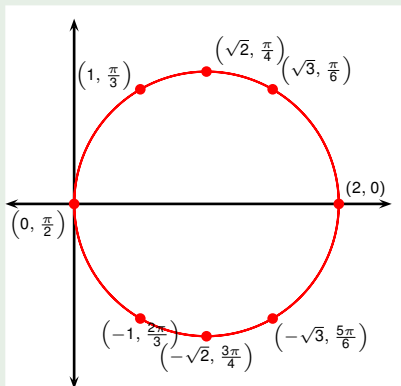
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



• $x = r \cos \theta$.

Example

- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.

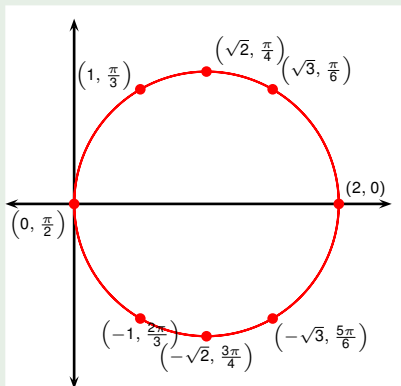


● $x = r \cos \theta.$

● $\cos \theta =$

Example

- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.

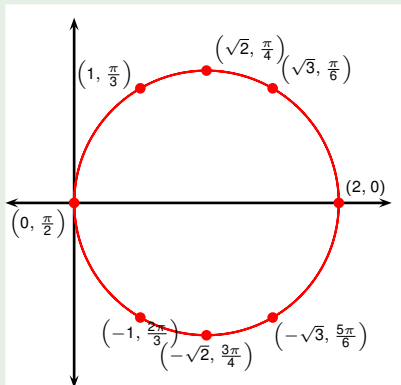


● $x = r \cos \theta.$

● $\cos \theta = x/r.$

Example

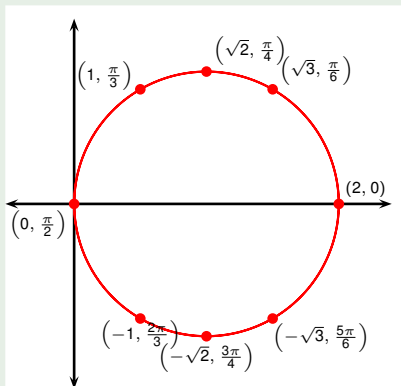
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



- $x = r \cos \theta$.
- $\cos \theta = x/r$.
- $r = 2 \cos \theta =$

Example

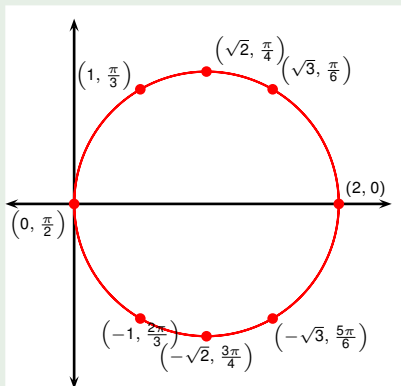
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



- $x = r \cos \theta$.
- $\cos \theta = x/r$.
- $r = 2 \cos \theta = 2x/r$.

Example

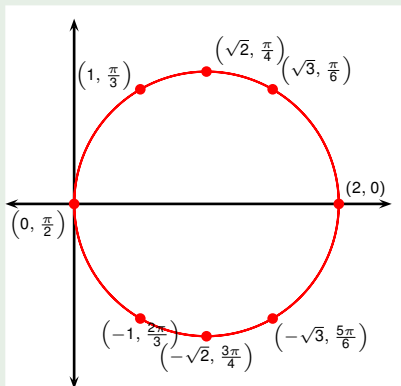
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



- $x = r \cos \theta$.
- $\cos \theta = x/r$.
- $r = 2 \cos \theta = 2x/r$.
- $2x =$

Example

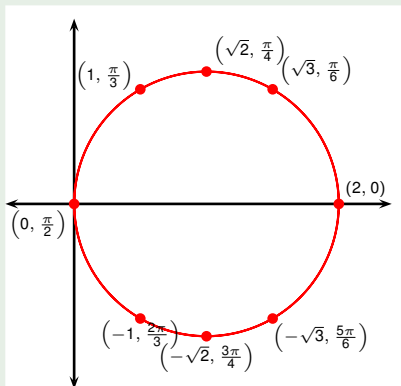
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



- $x = r \cos \theta$.
- $\cos \theta = x/r$.
- $r = 2 \cos \theta = 2x/r$.
- $2x = r^2 =$

Example

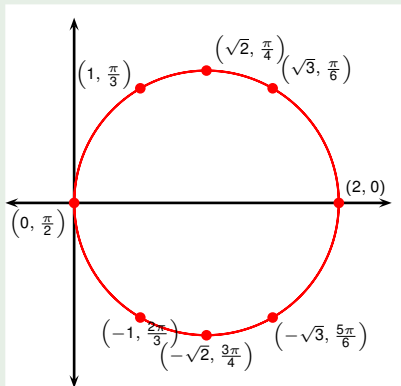
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



- $x = r \cos \theta$.
- $\cos \theta = x/r$.
- $r = 2 \cos \theta = 2x/r$.
- $2x = r^2 = x^2 + y^2$.

Example

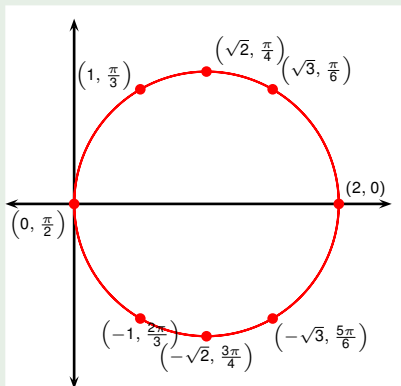
- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.



- $x = r \cos \theta$.
- $\cos \theta = x/r$.
- $r = 2 \cos \theta = 2x/r$.
- $2x = r^2 = x^2 + y^2$.
- $x^2 + y^2 - 2x = 0$.

Example

- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.

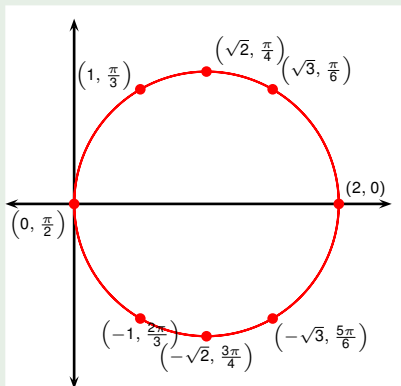


- $x = r \cos \theta$.
- $\cos \theta = x/r$.
- $r = 2 \cos \theta = 2x/r$.
- $2x = r^2 = x^2 + y^2$.
- $x^2 + y^2 - 2x = 0$.
- Complete the square:

$$(x^2 - 2x \quad) + y^2 = 0$$

Example

- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.

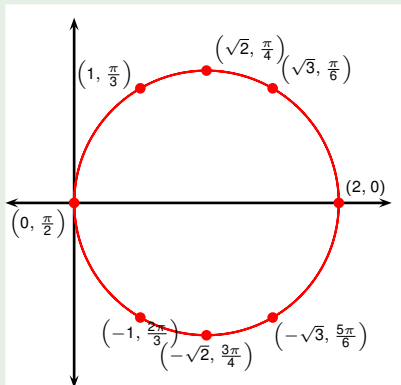


- $x = r \cos \theta$.
- $\cos \theta = x/r$.
- $r = 2 \cos \theta = 2x/r$.
- $2x = r^2 = x^2 + y^2$.
- $x^2 + y^2 - 2x = 0$.
- Complete the square:

$$(x^2 - 2x + 1) + y^2 = 0 + 1$$

Example

- 1 Sketch the curve with polar equation $r = 2 \cos \theta$.
- 2 Find a Cartesian equation for this curve.

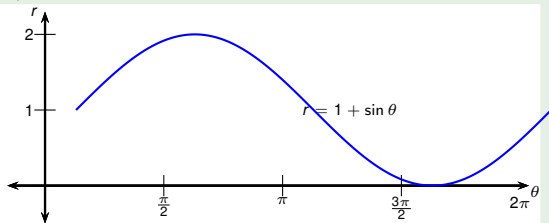
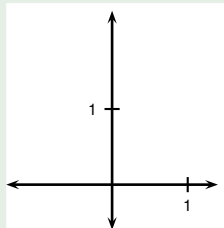


- $x = r \cos \theta$.
- $\cos \theta = x/r$.
- $r = 2 \cos \theta = 2x/r$.
- $2x = r^2 = x^2 + y^2$.
- $x^2 + y^2 - 2x = 0$.
- Complete the square:

$$\begin{aligned}(x^2 - 2x + 1) + y^2 &= 0 + 1 \\(x - 1)^2 + y^2 &= 1\end{aligned}$$

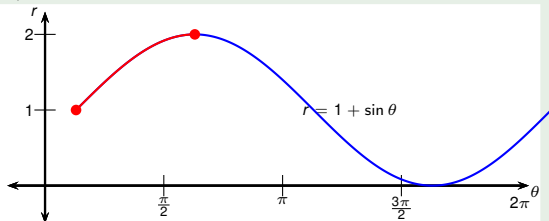
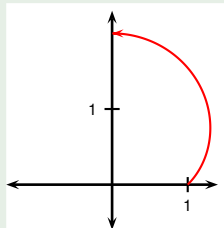
Example ()

Sketch the curve $r = 1 + \sin \theta$.



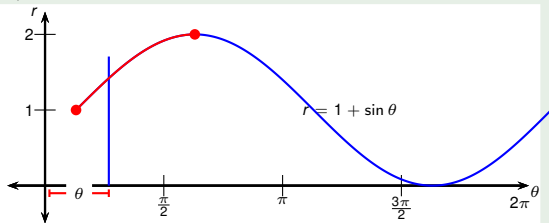
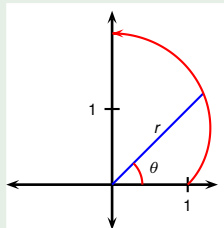
Example ()

Sketch the curve $r = 1 + \sin \theta$.



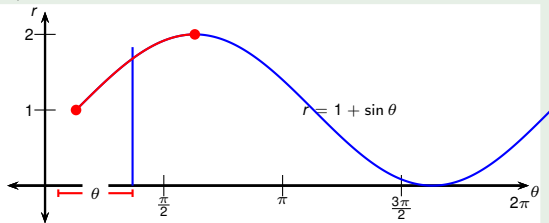
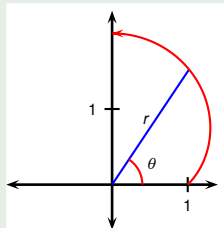
Example ()

Sketch the curve $r = 1 + \sin \theta$.



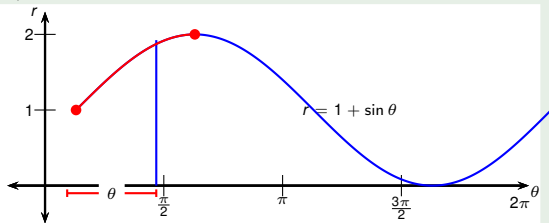
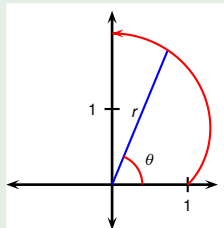
Example ()

Sketch the curve $r = 1 + \sin \theta$.



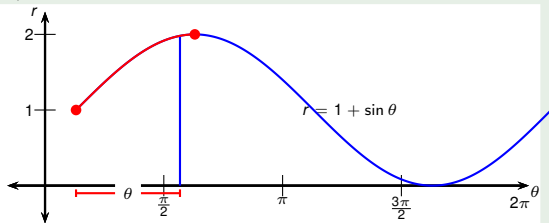
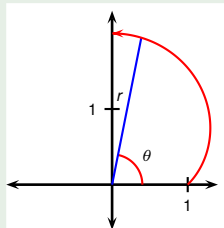
Example ()

Sketch the curve $r = 1 + \sin \theta$.



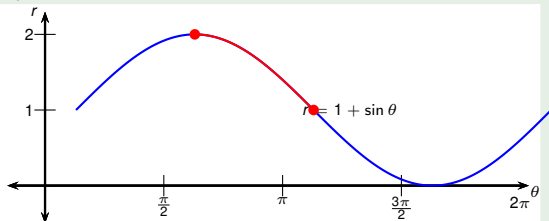
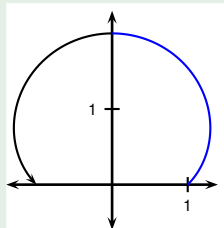
Example ()

Sketch the curve $r = 1 + \sin \theta$.



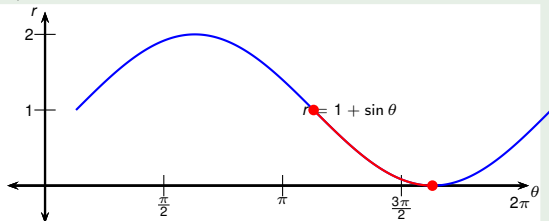
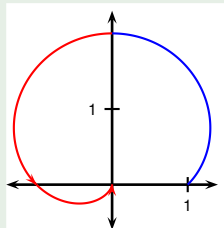
Example ()

Sketch the curve $r = 1 + \sin \theta$.



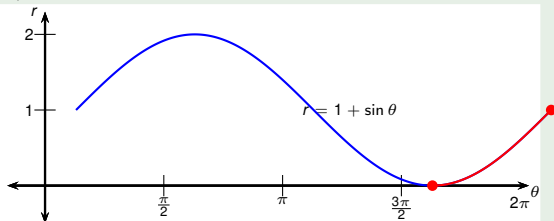
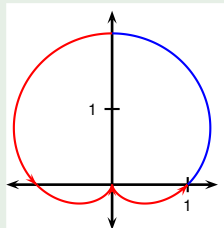
Example ()

Sketch the curve $r = 1 + \sin \theta$.



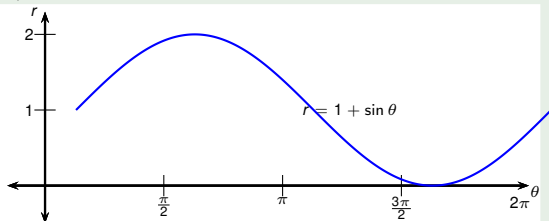
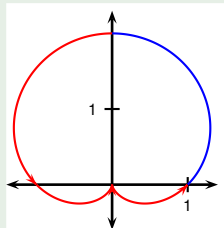
Example ()

Sketch the curve $r = 1 + \sin \theta$.



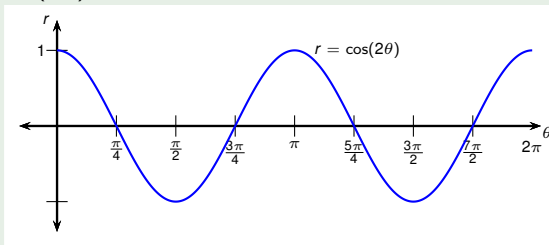
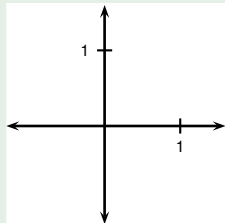
Example (Cardioid)

Sketch the curve $r = 1 + \sin \theta$.



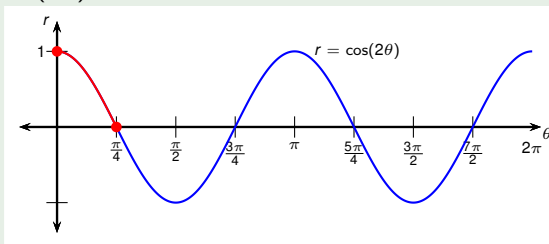
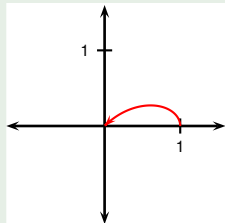
Example

Sketch the curve $r = \cos(2\theta)$.



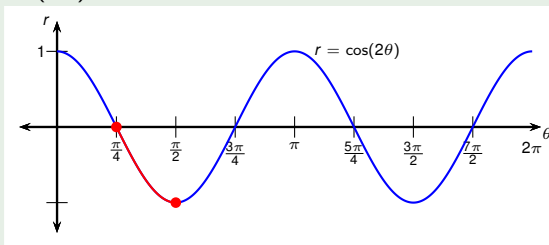
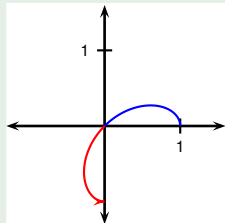
Example

Sketch the curve $r = \cos(2\theta)$.



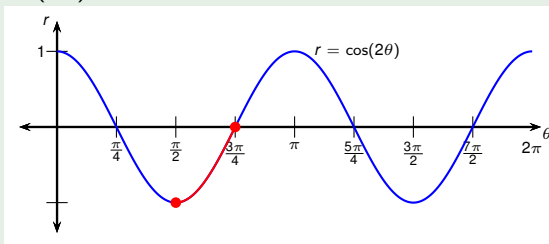
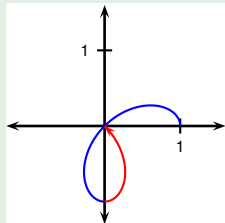
Example

Sketch the curve $r = \cos(2\theta)$.



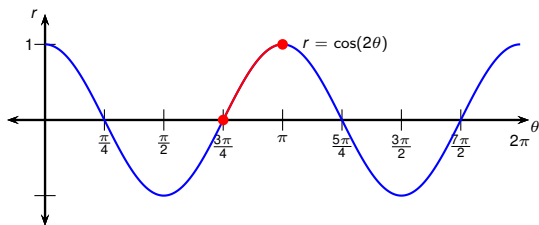
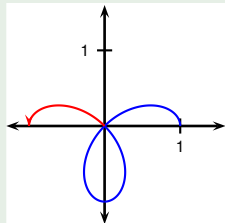
Example

Sketch the curve $r = \cos(2\theta)$.



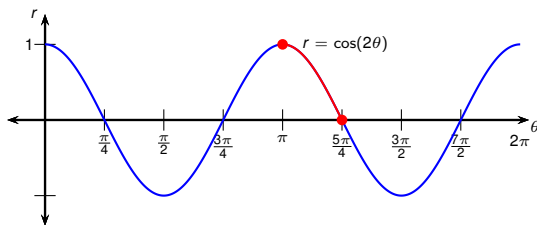
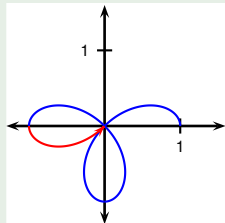
Example

Sketch the curve $r = \cos(2\theta)$.



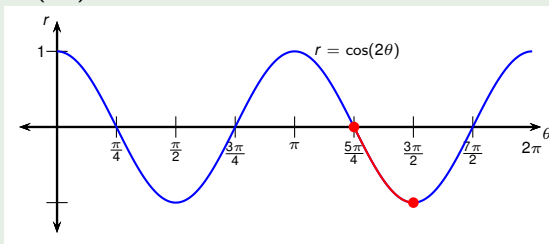
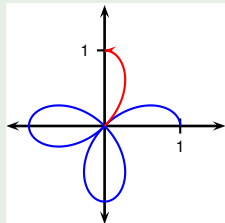
Example

Sketch the curve $r = \cos(2\theta)$.



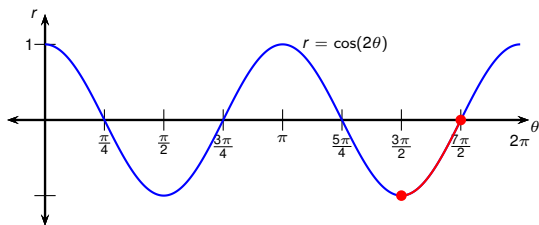
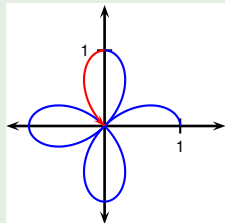
Example

Sketch the curve $r = \cos(2\theta)$.



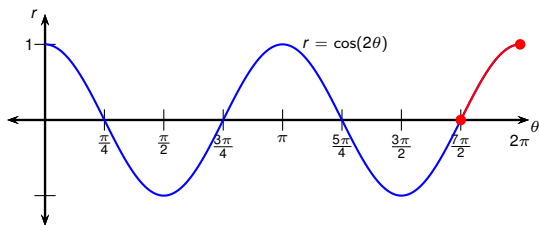
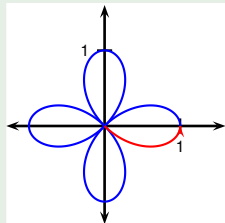
Example

Sketch the curve $r = \cos(2\theta)$.



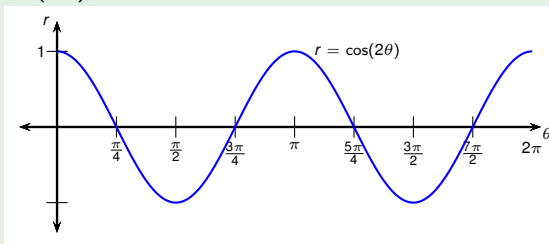
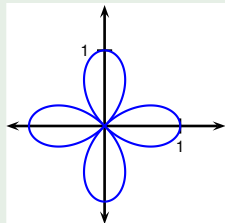
Example

Sketch the curve $r = \cos(2\theta)$.



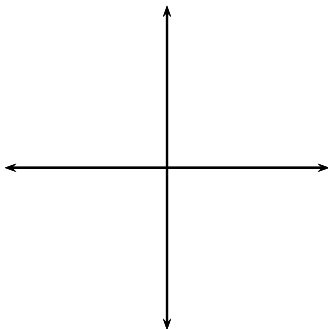
Example

Sketch the curve $r = \cos(2\theta)$.



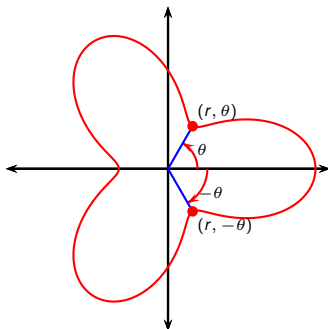
Symmetry

- If the polar equation is unchanged when θ is replaced by $-\theta$, the curve is symmetric about the polar axis.
- If the equation is unchanged when θ is replaced by $\pi + \theta$, the curve is symmetric under rotation about the pole.
- If the equation is unchanged when θ is replaced by $\pi - \theta$, the curve is symmetric about the vertical line $\theta = \frac{\pi}{2}$.



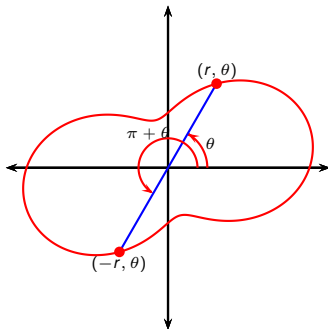
Symmetry

- If the polar equation is unchanged when θ is replaced by $-\theta$, the curve is symmetric about the polar axis.
- If the equation is unchanged when θ is replaced by $\pi + \theta$, the curve is symmetric under rotation about the pole.
- If the equation is unchanged when θ is replaced by $\pi - \theta$, the curve is symmetric about the vertical line $\theta = \frac{\pi}{2}$.



Symmetry

- If the polar equation is unchanged when θ is replaced by $-\theta$, the curve is symmetric about the polar axis.
- If the equation is unchanged when θ is replaced by $\pi + \theta$, the curve is symmetric under rotation about the pole.
- If the equation is unchanged when θ is replaced by $\pi - \theta$, the curve is symmetric about the vertical line $\theta = \frac{\pi}{2}$.



Symmetry

- If the polar equation is unchanged when θ is replaced by $-\theta$, the curve is symmetric about the polar axis.
- If the equation is unchanged when θ is replaced by $\pi + \theta$, the curve is symmetric under rotation about the pole.
- If the equation is unchanged when θ is replaced by $\pi - \theta$, the curve is symmetric about the vertical line $\theta = \frac{\pi}{2}$.

