Precalculus
Lecture 12
Equations with Logarithms and Exponents

Todor Milev
https://github.com/tmilev/freecalc

2020

Outline

Outline

Outline

Outline

License to use and redistribute

These lecture slides and their LaTeX source code are licensed to you under the Creative Commons license CC BY 3.0. You are free

- to Share - to copy, distribute and transmit the work,
- to Remix - to adapt, change, etc., the work,
- to make commercial use of the work,

as long as you reasonably acknowledge the original project.

- Latest version of the .tex sources of the slides: `https://github.com/tmilev/freecalc`
- Should the link be outdated/moved, search for "freecalc project".
- Creative Commons license CC BY 3.0: `https://creativecommons.org/licenses/by/3.0/us/` and the links therein.

Example

Solve the equation.

$$\log_3(2x^2+1) = 2$$

Example

Solve the equation.

$$\begin{array}{rcl} \log_3(2x^2+1) &=& 2 \\ 3^{\log_3(2x^2+1)} &=& 3^2 \end{array}$$

| Exponentiate base 3

Example

Solve the equation.

$$
\begin{aligned}
\log_3(2x^2+1) &= 2 \\
{\color{red}3}^{\log_3(2x^2+1)} &= 3^2 \\
2x^2+1 &= 9
\end{aligned}
$$

| Exponentiate base 3

Example

Solve the equation.

$$
\begin{aligned}
\log_3(2x^2+1) &= 2 \\
3^{\log_3(2x^2+1)} &= 3^2 \\
2x^2+1 &= 9 \\
2x^2 &= 8
\end{aligned}
$$

| Exponentiate base 3

Example

Solve the equation.

$$
\begin{aligned}
\log_3(2x^2+1) &= 2 \\
3^{\log_3(2x^2+1)} &= 3^2 \\
2x^2+1 &= 9 \\
2x^2 &= 8 \\
x^2 &= \frac{8}{2} = 4
\end{aligned}
$$

| Exponentiate base 3

Example

Solve the equation.

$$
\begin{array}{rcll}
\log_3(2x^2+1) & = & 2 & \Big|\ \text{Exponentiate base 3} \\
3^{\log_3(2x^2+1)} & = & 3^2 & \\
2x^2+1 & = & 9 & \\
2x^2 & = & 8 & \\
x^2 & = & \frac{8}{2} = 4 & \\
x & = & \pm\sqrt{4} = \pm 2 &
\end{array}
$$

Example

Solve the equation.

$$
\begin{array}{rcll}
\log_3(2x^2+1) & = & 2 & \Big| \text{ Exponentiate base 3} \\
3^{\log_3(2x^2+1)} & = & 3^2 & \\
2x^2+1 & = & 9 & \\
2x^2 & = & 8 & \\
x^2 & = & \frac{8}{2} = 4 & \\
x & = & \pm\sqrt{4} = \pm 2 & \\
x = 2 & \text{or} & x = -2 & \Big| \text{ final answer}
\end{array}
$$

The logarithmic property $\log_a(xy) = \log_a x + \log_a y$ holds only for positive x, y. Failure to check the positivity of x, y can result in extraneous (fake) solutions to logarithmic equations.

Example

Solve the equation.

$$\log_2(x+2) + \log_2(x-1) \quad = \quad 2$$

The logarithmic property $\log_a(xy) = \log_a x + \log_a y$ holds only for positive x, y. Failure to check the positivity of x, y can result in extraneous (fake) solutions to logarithmic equations.

Example

Solve the equation.

$$\begin{array}{rcl|l} \log_2(x+2) + \log_2(x-1) & = & 2 & \text{Domain: } x > 1 \\ \log_2\left((x+2)(x-1)\right) & = & 2 & \text{Exponentiate base 2} \\ (x+2)(x-1) & = & 2^2 & \\ x^2 + x - 2 & = & 4 & \\ x^2 + x - 6 & = & 0 & \\ (x-2)(x+3) & = & 0 & \\ x = 2 & \text{or} & \cancel{x = -3} & \end{array}$$

$x = -3$ not a solution (outside of domain)

Example (Solve exponential equation without logarithms)

Solve for t.

$$16^{4t} = 8^{t-2}$$

Example (Solve exponential equation without logarithms)

Solve for t.

$$
\begin{array}{rrcl}
 & 16^{4t} & = & 8^{t-2} \\
\text{Find a common base:} & (\mathbf{?}\ )^{4t} & = & (\mathbf{?}\ )^{t-2}
\end{array}
$$

Example (Solve exponential equation without logarithms)

Solve for t.

$$\begin{array}{rrcl} & 16^{4t} & = & 8^{t-2} \\ \text{Find a common base:} & \left(2^4\right)^{4t} & = & \left(2^3\right)^{t-2} \end{array}$$

Example (Solve exponential equation without logarithms)

Solve for t.

$$\begin{array}{rrcl} & 16^{4t} & = & 8^{t-2} \\ \text{Find a common base:} & \left(2^4\right)^{4t} & = & \left(2^3\right)^{t-2} \\ & 2^{16t} & = & 2^{3t-6} \end{array}$$

Example (Solve exponential equation without logarithms)

Solve for t.

Find a common base:
$$
\begin{array}{rcl}
16^{4t} & = & 8^{t-2} \\
\left(2^4\right)^{4t} & = & \left(2^3\right)^{t-2} \\
2^{16t} & = & 2^{3t-6} \\
16t & = & 3t-6
\end{array}
$$

Example (Solve exponential equation without logarithms)

Solve for t.

Find a common base:

$$\begin{array}{rcl} 16^{4t} &=& 8^{t-2} \\ \left(2^4\right)^{4t} &=& \left(2^3\right)^{t-2} \\ 2^{16t} &=& 2^{3t-6} \\ 16t &=& 3t-6 \\ 13t &=& -6 \end{array}$$

Example (Solve exponential equation without logarithms)

Solve for t.

Find a common base:

$$\begin{array}{rcl} 16^{4t} &=& 8^{t-2} \\ \left(2^4\right)^{4t} &=& \left(2^3\right)^{t-2} \\ 2^{16t} &=& 2^{3t-6} \\ 16t &=& 3t-6 \\ 13t &=& -6 \\ t &=& -\dfrac{6}{13}. \end{array}$$

Example

Solve the equation.

$$2^{1-5x} = 12$$

Example

Solve the equation.

$$\begin{aligned} 2^{1-5x} &= 12 && \Big\vert \text{ apply } \log_2 \\ \log_2(2^{1-5x}) &= \log_2 12 \end{aligned}$$

Example

Solve the equation.

$$\begin{aligned} 2^{1-5x} &= 12 && \Big| \text{ apply } \log_2 \\ \color{red}{\log_2}(\color{red}{2}^{1-5x}) &= \log_2 12 \\ 1-5x &= \log_2 12 \end{aligned}$$

Example

Solve the equation.

$$\begin{array}{rcl} 2^{1-5x} &=& 12 \\ \log_2(2^{1-5x}) &=& \log_2 12 \\ 1-5x &=& \log_2 12 = \textbf{?} \end{array} \quad \Big| \text{ apply } \log_2$$

Example

Solve the equation.

$$\begin{array}{rcll} 2^{1-5x} &=& 12 & \Big|\ \text{apply } \log_2 \\ \log_2(2^{1-5x}) &=& \log_2 12 & \\ 1-5x &=& \log_2 12 = \log_2(4\cdot 3) & \end{array}$$

Example

Solve the equation.

$$\begin{array}{rcll} 2^{1-5x} &=& 12 & \Big| \text{ apply } \log_2 \\ \log_2(2^{1-5x}) &=& \log_2 12 & \\ 1-5x &=& \log_2 12 = \log_2(4\cdot 3) & \\ 1-5x &=& \log_2 4 + \log_2 3 & \end{array}$$

Example

Solve the equation.

$$
\begin{array}{rcll}
2^{1-5x} & = & 12 & \Big| \text{ apply } \log_2 \\
\log_2(2^{1-5x}) & = & \log_2 12 & \\
1-5x & = & \log_2 12 = \log_2(4\cdot 3) & \\
1-5x & = & \log_2 4 + \log_2 3 & \\
1-5x & = & \mathbf{?} + \log_2 3 &
\end{array}
$$

Example

Solve the equation.

$$
\begin{array}{rcl}
2^{1-5x} & = & 12 \qquad\qquad\qquad\qquad \Big|\ \text{apply } \log_2 \\
\log_2(2^{1-5x}) & = & \log_2 12 \\
1-5x & = & \log_2 12 = \log_2(4\cdot 3) \\
1-5x & = & \log_2 4 + \log_2 3 \\
1-5x & = & 2 + \log_2 3
\end{array}
$$

Example

Solve the equation.

$$
\begin{array}{rcl}
2^{1-5x} &=& 12 \\
\log_2(2^{1-5x}) &=& \log_2 12 \\
1-5x &=& \log_2 12 = \log_2(4\cdot 3) \\
1-5x &=& \log_2 4 + \log_2 3 \\
1-5x &=& 2+\log_2 3 \\
5x &=& 1-(2+\log_2 3)
\end{array}
\quad \Big| \text{ apply } \log_2
$$

Example

Solve the equation.

$$\begin{array}{rcll} 2^{1-5x} &=& 12 & \Big|\ \text{apply } \log_2 \\ \log_2(2^{1-5x}) &=& \log_2 12 & \\ 1-5x &=& \log_2 12 = \log_2(4\cdot 3) & \\ 1-5x &=& \log_2 4 + \log_2 3 & \\ 1-5x &=& 2+\log_2 3 & \\ 5x &=& 1-(2+\log_2 3) & \\ x &=& \dfrac{-1}{} & \end{array}$$

Example

Solve the equation.

$$
\begin{aligned}
2^{1-5x} &= 12 && \Big| \text{ apply } \log_2 \\
\log_2(2^{1-5x}) &= \log_2 12 \\
1-5x &= \log_2 12 = \log_2(4\cdot 3) \\
1-5x &= \log_2 4 + \log_2 3 \\
1-5x &= 2 + \log_2 3 \\
5x &= 1 - (2 + \log_2 3) \\
x &= \frac{-1 - \log_2 3}{}
\end{aligned}
$$

Example

Solve the equation.

$$
\begin{array}{rcll}
2^{1-5x} & = & 12 & \Big|\ \text{apply } \log_2 \\
\log_2(2^{1-5x}) & = & \log_2 12 & \\
1-5x & = & \log_2 12 = \log_2(4\cdot 3) & \\
1-5x & = & \log_2 4 + \log_2 3 & \\
1-5x & = & 2+\log_2 3 & \\
{\color{red}5}x & = & 1-(2+\log_2 3) & \\
x & = & \dfrac{-1-\log_2 3}{{\color{red}5}} &
\end{array}
$$

Example

Solve the equation.

$$
\begin{array}{rcll}
2^{1-5x} &=& 12 & \Big| \text{ apply } \log_2 \\
\log_2(2^{1-5x}) &=& \log_2 12 & \\
1-5x &=& \log_2 12 = \log_2(4\cdot 3) & \\
1-5x &=& \log_2 4 + \log_2 3 & \\
1-5x &=& 2+\log_2 3 & \\
5x &=& 1-(2+\log_2 3) & \\
x &=& \dfrac{-1-\log_2 3}{5} & \\
\text{Calculator:}\quad x &\approx& -0.516993. &
\end{array}
$$

Example

Solve the equation.

$$e^{x-3} = 2e^{2x-1}$$

Example

Solve the equation.

$$\begin{aligned} e^{x-3} &= 2e^{2x-1} && \Big| \text{ Divide by } e^{2x-1} \\ \frac{e^{x-3}}{e^{2x-1}} &= 2 \end{aligned}$$

Example

Solve the equation.

$$\begin{aligned} e^{x-3} &= 2e^{2x-1} \\ \frac{e^{x-3}}{e^{2x-1}} &= 2 \\ e^{x-3-(2x-1)} &= 2 \end{aligned}$$

Divide by e^{2x-1}

Example

Solve the equation.

$$\begin{aligned} e^{x-3} &= 2e^{2x-1} \\ \frac{e^{x-3}}{e^{2x-1}} &= 2 \\ e^{x-3-(2x-1)} &= 2 \end{aligned}$$

Divide by e^{2x-1}

Example

Solve the equation.

$$
\begin{aligned}
e^{x-3} &= 2e^{2x-1} \\
\frac{e^{x-3}}{e^{2x-1}} &= 2 \\
e^{x-3-(2x-1)} &= 2 \\
e^{-x-2} &= 2
\end{aligned}
$$

| Divide by e^{2x-1}

Example

Solve the equation.

$$
\begin{aligned}
e^{x-3} &= 2e^{2x-1} && \Big|\ \text{Divide by } e^{2x-1}\\
\frac{e^{x-3}}{e^{2x-1}} &= 2\\
e^{x-3-(2x-1)} &= 2\\
e^{-x-2} &= 2
\end{aligned}
$$

Example

Solve the equation.

$$
\begin{aligned}
e^{x-3} &= 2e^{2x-1} && \Big| \text{ Divide by } e^{2x-1} \\
\frac{e^{x-3}}{e^{2x-1}} &= 2 \\
e^{x-3-(2x-1)} &= 2 \\
{\color{red}e}^{-x-2} &= 2 && \Big| \text{ Apply } {\color{red}\ln} \\
-x-2 &= \ln 2
\end{aligned}
$$

Example

Solve the equation.

$$\begin{aligned} e^{x-3} &= 2e^{2x-1} && \Big| \text{ Divide by } e^{2x-1} \\ \frac{e^{x-3}}{e^{2x-1}} &= 2 \\ e^{x-3-(2x-1)} &= 2 \\ e^{-x-2} &= 2 && \Big| \text{ Apply } \ln \\ -x-2 &= \ln 2 \end{aligned}$$

Example

Solve the equation.

$$
\begin{aligned}
e^{x-3} &= 2e^{2x-1} && \Big| \text{ Divide by } e^{2x-1} \\
\frac{e^{x-3}}{e^{2x-1}} &= 2 \\
e^{x-3-(2x-1)} &= 2 \\
e^{-x-2} &= 2 && \Big| \text{ Apply } \ln \\
-x-2 &= \ln 2 \\
-x &= \ln 2 + 2
\end{aligned}
$$

Example

Solve the equation.

$$\begin{aligned}
e^{x-3} &= 2e^{2x-1} && \Big| \text{ Divide by } e^{2x-1}\\
\frac{e^{x-3}}{e^{2x-1}} &= 2\\
e^{x-3-(2x-1)} &= 2\\
e^{-x-2} &= 2 && \Big| \text{ Apply } \ln\\
-x-2 &= \ln 2\\
-x &= \ln 2 + 2\\
x &= -(\ln 2 + 2)
\end{aligned}$$

Example

Solve the equation.

$$\begin{aligned}
e^{x-3} &= 2e^{2x-1} && \Big| \text{ Divide by } e^{2x-1}\\
\frac{e^{x-3}}{e^{2x-1}} &= 2\\
e^{x-3-(2x-1)} &= 2\\
e^{-x-2} &= 2 && \Big| \text{ Apply } \ln\\
-x-2 &= \ln 2\\
-x &= \ln 2 + 2\\
x &= -(\ln 2 + 2)\\
x &= -\ln 2 - 2 && \Big| \text{ Final answer}
\end{aligned}$$

Example

Solve the equation.

$$
\begin{array}{rcl|l}
e^{x-3} & = & 2e^{2x-1} & \text{Divide by } e^{2x-1} \\
\frac{e^{x-3}}{e^{2x-1}} & = & 2 & \\
e^{x-3-(2x-1)} & = & 2 & \\
e^{-x-2} & = & 2 & \text{Apply } \ln \\
-x-2 & = & \ln 2 & \\
-x & = & \ln 2 + 2 & \\
x & = & -(\ln 2 + 2) & \\
x & = & -\ln 2 - 2 & \text{Final answer} \\
x & \approx & -2.693 & \text{Calculator}
\end{array}
$$

Example

Solve.

$$3^{2x+5} = 5 \cdot 2^{-x+1}$$

Example

Solve.

$$3^{2x+5} = 5 \cdot 2^{-x+1}$$

Common base

Example

Solve.

$$
\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1}\\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1}
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Example

Solve.

$$\begin{aligned} 3^{2x+5} &= 5\cdot 2^{-x+1} \\ \left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\ 2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \end{aligned}$$

Common base

$a = b^{\log_b a}$

Example

Solve.

$$\begin{aligned} 3^{2x+5} &= 5\cdot 2^{-x+1} \\ \left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\ 2^{(\log_2 3)(2x+5)} &= 5\cdot {\color{red}2^{-x+1}} \\ \frac{2^{(\log_2 3)(2x+5)}}{{\color{red}2^{-x+1}}} &= 5 \end{aligned}$$

Common base

$$a = b^{\log_b a}$$

Example

Solve.

$$\begin{aligned} 3^{2x+5} &= 5\cdot 2^{-x+1} \\ \left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\ 2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\ \frac{{\color{red}2}^{(\log_2 3)(2x+5)}}{{\color{red}2}^{-x+1}} &= 5 \\ {\color{red}2}^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \end{aligned}$$

Common base

$a = b^{\log_b a}$

Example

Solve.

$$\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1}\\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1}\\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1}\\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5\\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5
\end{aligned}$$

Common base

$a = b^{\log_b a}$

Example

Solve.

$$\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5
\end{aligned}$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$
\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1}\\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1}\\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1}\\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5\\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5\\
(\log_2 3)(2x+5)+x-1 &= \log_2 5
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5 \\
x\,(\qquad + \quad)+ \qquad &= \log_2 5
\end{aligned}$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1}\\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1}\\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1}\\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5\\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5\\
(\log_2 3)(2x+5)+x-1 &= \log_2 5\\
x\left(2\log_2 3 + \quad\right) + &= \log_2 5
\end{aligned}$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5 \\
x\left(2\log_2 3+1\right)+ &= \log_2 5
\end{aligned}$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$
\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5 \\
x\left(2\log_2 3+1\right)+5\log_2 3 &= \log_2 5
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$
\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5 \\
x\left(2\log_2 3+1\right)+5\log_2 3 &= \log_2 5+1
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$
\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5 \\
x\left(2\log_2 3+1\right)+5\log_2 3 &= \log_2 5+1 \\
x &= \frac{\log_2 5+1}{}
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$
\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5 \\
x\left(2\log_2 3+1\right)+{\color{red}5\log_2 3} &= \log_2 5+1 \\
x &= \frac{\log_2 5+1{\color{red}-5\log_2 3}}{}
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$
\begin{aligned}
3^{2x+5} &= 5 \cdot 2^{-x+1} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5 \cdot 2^{-x+1} \\
2^{(\log_2 3)(2x+5)} &= 5 \cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 \\
(\log_2 3)(2x+5) + x - 1 &= \log_2 5 \\
x\left(2\log_2 3 + 1\right) + 5\log_2 3 &= \log_2 5 + 1 \\
x &= \frac{\log_2 5 + 1 - 5\log_2 3}{2\log_2 3 + 1}
\end{aligned}
$$

Common base

$a = b^{\log_b a}$

Apply $\log_2$

Example

Solve.

$$\begin{aligned}
3^{2x+5} &= 5\cdot 2^{-x+1} && \text{Common base} \\
\left(2^{\log_2 3}\right)^{2x+5} &= 5\cdot 2^{-x+1} && a = b^{\log_b a} \\
2^{(\log_2 3)(2x+5)} &= 5\cdot 2^{-x+1} \\
\frac{2^{(\log_2 3)(2x+5)}}{2^{-x+1}} &= 5 \\
2^{(\log_2 3)(2x+5)-(-x+1)} &= 5 && \text{Apply } \log_2 \\
(\log_2 3)(2x+5)+x-1 &= \log_2 5 \\
x\left(2\log_2 3+1\right)+5\log_2 3 &= \log_2 5+1 \\
x &= \frac{\log_2 5+1-5\log_2 3}{2\log_2 3+1} \\
x &\approx -1.1038 && \text{Calculator}
\end{aligned}$$

Example

Solve the equation.

$$e^{5-3x} = 10$$

Example

Solve the equation.

$$\begin{array}{rcll} e^{5-3x} & = & 10 & \text{apply } \ln \\ \ln(e^{5-3x}) & = & \ln 10 & \end{array}$$

Example

Solve the equation.

$$\begin{array}{rcll} e^{5-3x} & = & 10 & \text{apply } \ln \\ \ln(e^{5-3x}) & = & \ln 10 & \\ 5-3x & = & \ln 10 & \end{array}$$

Example

Solve the equation.

$$\begin{aligned} e^{5-3x} &= 10 && \text{apply } \ln \\ \ln(e^{5-3x}) &= \ln 10 \\ 5-3x &= \ln 10 \\ 3x &= 5-\ln 10 \end{aligned}$$

Example

Solve the equation.

$$\begin{array}{rcll} e^{5-3x} & = & 10 & \text{apply } \ln \\ \ln(e^{5-3x}) & = & \ln 10 & \\ 5-3x & = & \ln 10 & \\ 3x & = & 5-\ln 10 & \\ x & = & \dfrac{5-\ln 10}{3} & \end{array}$$

Example

Solve the equation.

$$\begin{array}{rcll} e^{5-3x} & = & 10 & \text{apply } \ln \\ \ln(e^{5-3x}) & = & \ln 10 & \\ 5-3x & = & \ln 10 & \\ 3x & = & 5-\ln 10 & \\ x & = & \dfrac{5-\ln 10}{3} & \\ \text{Calculator:} \quad x & \approx & 0.8991. & \end{array}$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

Solve for t:

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

$$\text{Solve for } t: \quad c(t)$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

Solve for t: $c(t) =$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

Solve for t: $\quad c(t) = r(t)$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

$$\text{Solve for } t: \quad c(t) = r(t)$$
$$=$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

$$\text{Solve for } t: \quad c(t) = r(t)$$

$$48 \cdot 2^t =$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

$$\text{Solve for } t: \quad c(t) = r(t)$$

$$48 \cdot 2^t =$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

$$\text{Solve for } t: \quad c(t) = r(t)$$
$$48 \cdot 2^t = 6 \cdot 4^t$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

$$\text{Solve for } t: \quad c(t) = r(t)$$
$$48 \cdot 2^t = 6 \cdot 4^t$$
$$8 \cdot 2^t = 4^t$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

$$\begin{aligned} \text{Solve for } t: \quad c(t) &= r(t) \\ 48 \cdot 2^t &= 6 \cdot 4^t \\ {\color{red}8} \cdot 2^t &= 4^t \\ \text{Find a common base:} \quad {\color{red}2^{\mathbf{?}}} \cdot 2^t &= 2^{\mathbf{?}} \end{aligned}$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

$$\begin{aligned} \text{Solve for } t: \quad c(t) &= r(t) \\ 48 \cdot 2^t &= 6 \cdot 4^t \\ 8 \cdot 2^t &= 4^t \\ \text{Find a common base:} \quad 2^3 \cdot 2^t &= 2^{\mathbf{?}} \end{aligned}$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

$$
\begin{aligned}
\text{Solve for } t: \quad c(t) &= r(t)\\
48 \cdot 2^t &= 6 \cdot 4^t\\
8 \cdot 2^t &= 4^t\\
\text{Find a common base:} \quad 2^3 \cdot 2^t &= 2^{\mathbf{?}}
\end{aligned}
$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

$$\begin{aligned} \text{Solve for } t: \quad c(t) &= r(t) \\ 48 \cdot 2^t &= 6 \cdot 4^t \\ 8 \cdot 2^t &= 4^t \\ \text{Find a common base:} \quad 2^3 \cdot 2^t &= 2^{2t} \end{aligned}$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

$$
\begin{aligned}
\text{Solve for } t: \quad c(t) &= r(t)\\
48 \cdot 2^t &= 6 \cdot 4^t\\
8 \cdot 2^t &= 4^t\\
\text{Find a common base:} \quad 2^3 \cdot 2^t &= 2^{2t}\\
2^{t+3} &= 2^{2t}
\end{aligned}
$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

$$
\begin{aligned}
\text{Solve for } t: \quad c(t) &= r(t)\\
48 \cdot 2^t &= 6 \cdot 4^t\\
8 \cdot 2^t &= 4^t\\
\text{Find a common base:} \quad 2^3 \cdot 2^t &= 2^{2t}\\
2^{t+3} &= 2^{2t}\\
t + 3 &= 2t
\end{aligned}
$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

$$\begin{aligned}
\text{Solve for } t: \quad c(t) &= r(t)\\
48 \cdot 2^t &= 6 \cdot 4^t\\
8 \cdot 2^t &= 4^t\\
\text{Find a common base:} \quad 2^3 \cdot 2^t &= 2^{2t}\\
2^{t+3} &= 2^{2t}\\
t + 3 &= 2t\\
t &= 3.
\end{aligned}$$

Example (Solving an exponential word problem)

A farmer buys 48 chickens and 6 rabbits. The chicken population doubles each year, and the rabbit population doubles every six months. When does the farmer have the same number of chickens as rabbits?

Let $c(t)$ denote the number of chickens after t years, and let $r(t)$ denote the number of rabbits after t years.

$$
\begin{aligned}
\text{Solve for } t: \quad c(t) &= r(t)\\
48\cdot 2^t &= 6\cdot 4^t\\
8\cdot 2^t &= 4^t\\
\text{Find a common base:} \quad 2^3\cdot 2^t &= 2^{2t}\\
2^{t+3} &= 2^{2t}\\
t+3 &= 2t\\
t &= 3.
\end{aligned}
$$

Therefore the chicken and rabbit populations are equal after 3 years.

Example (Solving a quadratic exponential equation)

Solve for x.

$$9^x = 2 \cdot 3^x + 63$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$\begin{aligned} 9^x &= 2\cdot 3^x + 63 \\ 9^x - 2\cdot 3^x - 63 &= 0 \end{aligned}$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$\begin{aligned} 9^x &= 2\cdot 3^x + 63 \\ 9^x - 2\cdot 3^x - 63 &= 0 && \Big|\ \text{Substitute } u = 3^x \\ \textbf{?}\ - 2u - 63 &= 0 \end{aligned}$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$\begin{aligned} 9^x &= 2\cdot 3^x + 63 \\ 9^x - 2\cdot 3^x - 63 &= 0 && \Big|\ \text{Substitute } u = 3^x \\ \mathbf{?}\ -2u - 63 &= 0 \end{aligned}$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$\begin{array}{rcll} 9^x &=& 2\cdot 3^x + 63 & \\ 9^x - 2\cdot 3^x - 63 &=& 0 & \Big| \text{ Substitute } u = 3^x \\ u^2 - 2u - 63 &=& 0 & \end{array}$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$\begin{aligned} 9^x &= 2\cdot 3^x + 63 \\ 9^x - 2\cdot 3^x - 63 &= 0 & \Big|\ \text{Substitute } u = 3^x \\ u^2 - 2u - 63 &= 0 \\ (\mathbf{?}\qquad)(\mathbf{?}\qquad) &= 0 \end{aligned}$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$\begin{aligned} 9^x &= 2\cdot 3^x + 63 \\ 9^x - 2\cdot 3^x - 63 &= 0 && \Big| \text{ Substitute } u = 3^x \\ u^2 - 2u - 63 &= 0 \\ (u-9)(u+7) &= 0 \end{aligned}$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$
\begin{aligned}
9^x &= 2\cdot 3^x + 63 \\
9^x - 2\cdot 3^x - 63 &= 0 && \Big|\ \text{Substitute } u = 3^x \\
u^2 - 2u - 63 &= 0 \\
(u-9)(u+7) &= 0
\end{aligned}
$$

$$u = 9 \quad \text{or} \quad u = -7$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$\begin{aligned} 9^x &= 2\cdot 3^x + 63 \\ 9^x - 2\cdot 3^x - 63 &= 0 && \Big| \text{ Substitute } u = 3^x \\ u^2 - 2u - 63 &= 0 \\ (u-9)(u+7) &= 0 \end{aligned}$$

$$\begin{aligned} u &= 9 \quad \text{or} \quad u = -7 \\ 3^x &= 9 \quad \text{or} \quad 3^x = -7 \end{aligned}$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$\begin{aligned} 9^x &= 2\cdot 3^x + 63 \\ 9^x - 2\cdot 3^x - 63 &= 0 && \Big| \text{ Substitute } u = 3^x \\ u^2 - 2u - 63 &= 0 \\ (u-9)(u+7) &= 0 \end{aligned}$$

$$\begin{aligned} u &= 9 \quad \text{or} \quad u = -7 \\ 3^x &= 9 \quad \text{or} \quad 3^x = -7 \\ {\color{red} x} &{\color{red} = \mathbf{?}} \end{aligned}$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$\begin{aligned} 9^x &= 2\cdot 3^x + 63 \\ 9^x - 2\cdot 3^x - 63 &= 0 && \Big| \text{ Substitute } u = 3^x \\ u^2 - 2u - 63 &= 0 \\ (u-9)(u+7) &= 0 \end{aligned}$$

$$\begin{array}{ccccccc} u & = & 9 & \text{or} & u & = & -7 \\ 3^x & = & 9 & \text{or} & 3^x & = & -7 \\ x & = & 2 \end{array}$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$\begin{aligned} 9^x &= 2\cdot 3^x + 63 \\ 9^x - 2\cdot 3^x - 63 &= 0 && \Big| \text{ Substitute } u = 3^x \\ u^2 - 2u - 63 &= 0 \\ (u-9)(u+7) &= 0 \end{aligned}$$

$$\begin{aligned} u &= 9 \quad \text{or} \quad u = -7 \\ 3^x &= 9 \quad \text{or} \quad 3^x = -7 \\ x &= 2 \qquad\qquad \textbf{?} \end{aligned}$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$\begin{aligned} 9^x &= 2\cdot 3^x + 63 \\ 9^x - 2\cdot 3^x - 63 &= 0 \qquad \Big|\ \text{Substitute } u = 3^x \\ u^2 - 2u - 63 &= 0 \\ (u-9)(u+7) &= 0 \end{aligned}$$

$$\begin{array}{rclcrcl} u &=& 9 & \text{or} & u &=& -7 \\ 3^x &=& 9 & \text{or} & 3^x &=& -7 \\ x &=& 2 & & & & \text{no real solution} \end{array}$$

Example (Solving a quadratic exponential equation)

Solve for x.

$$\begin{aligned} 9^x &= 2\cdot 3^x + 63 \\ 9^x - 2\cdot 3^x - 63 &= 0 && \Big| \text{ Substitute } u = 3^x \\ u^2 - 2u - 63 &= 0 \\ (u-9)(u+7) &= 0 \end{aligned}$$

$$\begin{array}{ccccccc} u & = & 9 & \text{or} & u & = & -7 \\ 3^x & = & 9 & \text{or} & 3^x & = & -7 \\ x & = 2 & & & \text{no real solution} & & \end{array}$$

Therefore $x = 2$ is the solution.

Example

Solve the equation

$$e^{2x} - 3e^{x} - 4 = 0$$

Example

Solve the equation

$$e^{2x} - 3e^{x} - 4 = 0$$

Set $e^{x} = u$.

Example

Solve the equation

$$e^{2x} - 3e^x - 4 = 0$$

Set $e^x = u$. Then $e^{2x} = \textbf{?}$.

Example

Solve the equation

$$e^{2x} - 3e^{x} - 4 = 0$$

Set $e^{x} = u$. Then $e^{2x} = u^{2}$.

Example

Solve the equation

$$e^{2x} - 3e^{x} - 4 = 0$$

Set $e^x = u$. Then $e^{2x} = u^2$.

$$u^2 - 3u - 4 = 0$$

Example

Solve the equation

$$e^{2x} - 3e^{x} - 4 = 0$$

Set $e^x = u$. Then $e^{2x} = u^2$.

$$u^2 - 3u - 4 = 0$$

$$(\textbf{?} \quad\quad)(\textbf{?} \quad\quad) = 0$$

Example

Solve the equation

$$e^{2x} - 3e^{x} - 4 = 0$$

Set $e^x = u$. Then $e^{2x} = u^2$.

$$u^2 - 3u - 4 = 0$$

$$(u - 4)(u + 1) = 0$$

Example

Solve the equation

$$e^{2x} - 3e^{x} - 4 = 0$$

Set $e^x = u$. Then $e^{2x} = u^2$.

$$u^2 - 3u - 4 = 0$$
$$(u - 4)(u + 1) = 0$$

$u = 4$ or $u = -1$

Example

Solve the equation

$$e^{2x} - 3e^x - 4 = 0$$

Set $e^x = u$. Then $e^{2x} = u^2$.

$$u^2 - 3u - 4 = 0$$
$$(u-4)(u+1) = 0$$

$$u = 4 \quad \text{or} \quad u = -1$$
$$e^x = 4 \quad \text{or} \quad e^x = -1$$

Example

Solve the equation

$$e^{2x} - 3e^{x} - 4 = 0$$

Set $e^x = u$. Then $e^{2x} = u^2$.

$$\begin{aligned} u^2 - 3u - 4 &= 0 \\ (u-4)(u+1) &= 0 \end{aligned}$$

$$\begin{array}{rcr} u = 4 & \text{or} & u = -1 \\ e^x = 4 & \text{or} & e^x = -1 \\ x = \ln 4 & \text{or} & \text{no real solution} \end{array}$$

Example

Solve the equation

$$e^{2x} - 3e^{x} - 4 = 0$$

Set $e^x = u$. Then $e^{2x} = u^2$.

$$\begin{aligned} u^2 - 3u - 4 &= 0 \\ (u-4)(u+1) &= 0 \end{aligned}$$

$$\begin{array}{rcr} u = 4 & \text{or} & u = -1 \\ e^x = 4 & \text{or} & e^x = -1 \\ x = \ln 4 & \text{or} & \text{no real solution} \\ x \approx 1.3863 & & \end{array}$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 \quad = \quad 0$$

Set $u = \textbf{?}$.

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$.

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 \quad = \quad 0$$

Set $u = 2^x$.

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = \mathbf{?}$,

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 \quad = \quad 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$,

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = \textbf{?}$.

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 \quad = \quad 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$4u^2 - 4u - 3 = 0$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$4u^2 - 4u - 3 = 0$$

$$(\textbf{?} \qquad)(\textbf{?} \qquad) = 0$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$4u^2 - 4u - 3 = 0$$

$$(2u - 3)(2u + 1) = 0$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$4u^2 - 4u - 3 = 0$$

$$(2u - 3)(2u + 1) = 0$$

$$2u - 3 = 0 \quad \text{or} \quad 2u + 1 = 0$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$4u^2 - 4u - 3 = 0$$

$$(2u - 3)(2u + 1) = 0$$

$$2u - 3 = 0 \quad \text{or} \quad 2u + 1 = 0$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$4u^2 - 4u - 3 = 0$$

$$(2u - 3)(2u + 1) = 0$$

$$2u - 3 = 0 \quad \text{or} \quad 2u + 1 = 0$$

$$u = \frac{3}{} \quad \text{or} \quad u =$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$\begin{aligned} 4u^2 - 4u - 3 &= 0 \\ (2u-3)(2u+1) &= 0 \end{aligned}$$

$$2u - 3 = 0 \quad \text{or} \quad 2u + 1 = 0$$

$$u = \frac{3}{2} \quad \text{or} \quad u =$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$\begin{aligned} 4u^2 - 4u - 3 &= 0 \\ (2u-3)(2u+1) &= 0 \end{aligned}$$

$$2u - 3 = 0 \quad \text{or} \quad 2u + 1 = 0$$

$$u = \frac{3}{2} \quad \text{or} \quad u = -\frac{1}{}$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$\begin{aligned} 4u^2 - 4u - 3 &= 0 \\ (2u - 3)(2u + 1) &= 0 \end{aligned}$$

$$2u - 3 = 0 \quad \text{or} \quad 2u + 1 = 0$$

$$u = \frac{3}{2} \quad \text{or} \quad u = -\frac{1}{2}$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2, 2^{x+2} = 4u$.

$$\begin{aligned} 4u^2 - 4u - 3 &= 0 \\ (2u-3)(2u+1) &= 0 \\ 2u - 3 = 0 \quad &\text{or} \quad 2u + 1 = 0 \\ u = \frac{3}{2} \quad &\text{or} \quad u = -\frac{1}{2} \\ 2^x = \frac{3}{2} \quad &\text{or} \quad 2^x = -\frac{1}{2} \end{aligned}$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$\begin{aligned} 4u^2 - 4u - 3 &= 0 \\ (2u-3)(2u+1) &= 0 \end{aligned}$$

$$2u - 3 = 0 \quad \text{or} \quad 2u + 1 = 0$$

$$u = \frac{3}{2} \quad \text{or} \quad u = -\frac{1}{2}$$

$$2^x = \frac{3}{2} \quad \text{or} \quad 2^x = -\frac{1}{2}$$

$$x = \log_2\left(\frac{3}{2}\right)$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$\begin{aligned} 4u^2 - 4u - 3 &= 0 \\ (2u-3)(2u+1) &= 0 \end{aligned}$$

$$\begin{aligned} 2u - 3 = 0 \quad &\text{or} \quad 2u + 1 = 0 \\ u = \frac{3}{2} \quad &\text{or} \quad u = -\frac{1}{2} \\ 2^x = \frac{3}{2} \quad &\text{or} \quad 2^x = -\frac{1}{2} \\ x = \log_2\left(\frac{3}{2}\right) \quad &\text{or} \quad \text{no real solution} \end{aligned}$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$4u^2 - 4u - 3 = 0$$

$$(2u - 3)(2u + 1) = 0$$

$$2u - 3 = 0 \quad \text{or} \quad 2u + 1 = 0$$

$$u = \frac{3}{2} \quad \text{or} \quad u = -\frac{1}{2}$$

$$2^x = \frac{3}{2} \quad \text{or} \quad 2^x = -\frac{1}{2}$$

$$x = \log_2\left(\frac{3}{2}\right) = \frac{\ln(\mathbf{?})}{\ln \mathbf{?}} \quad \text{or} \quad \text{no real solution}$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$4u^2 - 4u - 3 = 0$$

$$(2u - 3)(2u + 1) = 0$$

$$2u - 3 = 0 \quad \text{or} \quad 2u + 1 = 0$$

$$u = \frac{3}{2} \quad \text{or} \quad u = -\frac{1}{2}$$

$$2^x = \frac{3}{2} \quad \text{or} \quad 2^x = -\frac{1}{2}$$

$$x = \log_2\left(\frac{3}{2}\right) = \frac{\ln\left(\frac{3}{2}\right)}{\ln 2} \quad \text{or} \quad \text{no real solution}$$

Example

Solve the equation

$$4^{x+1} - 2^{x+2} - 3 = 0$$

Set $u = 2^x$. Then $4^{x+1} = 4u^2$, $2^{x+2} = 4u$.

$$\begin{aligned} 4u^2 - 4u - 3 &= 0 \\ (2u-3)(2u+1) &= 0 \\ 2u - 3 = 0 \quad &\text{or} \quad 2u + 1 = 0 \\ u = \frac{3}{2} \quad &\text{or} \quad u = -\frac{1}{2} \\ 2^x = \frac{3}{2} \quad &\text{or} \quad 2^x = -\frac{1}{2} \\ x = \log_2\left(\frac{3}{2}\right) = \frac{\ln\left(\frac{3}{2}\right)}{\ln 2} \approx 0.58496 \quad &\text{or} \quad \text{no real solution} \end{aligned}$$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$3^{2x} = 2 + 63 \cdot 3^{-2x}$$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$3^{2x} = 2 + 63 \cdot 3^{-2x}$$

> 2 terms $\Rightarrow$ transfer one side

$$3^{2x} - 2 - 63 \cdot 3^{-2x} = 0$$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$\begin{aligned} 3^{2x} &= 2 + 63 \cdot 3^{-2x} \\ 3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \end{aligned}$$

>2 terms $\Rightarrow$ transfer one side

$3^{2x} = u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$\begin{aligned} 3^{2x} &= 2 + 63 \cdot 3^{-2x} \\ 3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \end{aligned}$$

>2 terms $\Rightarrow$
transfer one side
$3^{2x} = u$
$3^{-2x} = $ **?**

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$\begin{aligned} 3^{2x} &= 2 + 63 \cdot 3^{-2x} \\ 3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \end{aligned}$$

>2 terms $\Rightarrow$
transfer one side
$3^{2x} = u$
$3^{-2x} = \left(3^{2x}\right)^{-1}$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$\begin{aligned} 3^{2x} &= 2 + 63 \cdot 3^{-2x} \\ 3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \end{aligned}$$

>2 terms $\Rightarrow$
transfer one side

$3^{2x} = u$

$3^{-2x} = \left(3^{2x}\right)^{-1}$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{aligned}
3^{2x} &= 2 + 63 \cdot 3^{-2x} \\
3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0
\end{aligned}
$$

$>$ 2 terms $\Rightarrow$
transfer one side

$3^{2x} = u$

$3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1}$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{aligned}
3^{2x} &= 2 + 63 \cdot 3^{-2x} \\
3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \\
u - 2 - 63u^{-1} &= 0
\end{aligned}
$$

>2 terms $\Rightarrow$ transfer one side

$3^{2x} = u$

$3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1}$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{aligned}
3^{2x} &= 2 + 63 \cdot 3^{-2x} \\
3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \\
u - 2 - 63u^{-1} &= 0
\end{aligned}
$$

>2 terms $\Rightarrow$ transfer one side

$3^{2x} = u$

$3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1}$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$\begin{aligned} 3^{2x} &= 2 + 63 \cdot 3^{-2x} \\ 3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \\ u - 2 - 63u^{-1} &= 0 \\ u^2 - 2u - 63 &= 0 \end{aligned}$$

$>$ 2 terms $\Rightarrow$ transfer one side

$3^{2x} = u$

$3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1}$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{aligned}
3^{2x} &= 2 + 63 \cdot 3^{-2x} \\
3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \\
u - {\color{red}2} - 63u^{-1} &= 0 \\
u^2 - {\color{red}2u} - 63 &= 0
\end{aligned}
$$

>2 terms $\Rightarrow$ transfer one side

$3^{2x} = u$

$3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1}$

Multiply ${\color{red}\cdot\, u}$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{aligned}
3^{2x} &= 2 + 63 \cdot 3^{-2x} \\
3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \\
u - 2 - 63u^{-1} &= 0 \\
u^2 - 2u - 63 &= 0
\end{aligned}
$$

>2 terms $\Rightarrow$ transfer one side

$3^{2x} = u$

$3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1}$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{aligned}
3^{2x} &= 2 + 63 \cdot 3^{-2x} \\
3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \\
u - 2 - 63u^{-1} &= 0 \\
u^2 - 2u - 63 &= 0 \\
(\mathbf{?} \quad)(\mathbf{?} \quad) &= 0
\end{aligned}
$$

>2 terms $\Rightarrow$ transfer one side

$3^{2x} = u$

$3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1}$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{aligned}
3^{2x} &= 2+63\cdot 3^{-2x} \\
3^{2x}-2-63\cdot 3^{-2x} &= 0 \\
u-2-63u^{-1} &= 0 \\
u^2-2u-63 &= 0 \\
(u-9)(u+7) &= 0
\end{aligned}
$$

>2 terms $\Rightarrow$
transfer one side
$3^{2x}=u$
$3^{-2x}=\left(3^{2x}\right)^{-1}=u^{-1}$
Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$\begin{aligned} 3^{2x} &= 2 + 63 \cdot 3^{-2x} \\ 3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \\ u - 2 - 63u^{-1} &= 0 \\ u^2 - 2u - 63 &= 0 \\ (u - 9)(u + 7) &= 0 \\ u - 9 = 0 \quad &\text{or} \quad u + 7 = 0 \end{aligned}$$

$>$ 2 terms $\Rightarrow$
transfer one side
$3^{2x} = u$
$3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1}$
Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{array}{rcl}
3^{2x} & = & 2+63\cdot 3^{-2x} \\
3^{2x}-2-63\cdot 3^{-2x} & = & 0 \\
u-2-63u^{-1} & = & 0 \\
u^2-2u-63 & = & 0 \\
(u-9)(u+7) & = & 0 \\
u-9=0 & \text{or} & u+7=0
\end{array}
$$

>2 terms $\Rightarrow$
transfer one side
$3^{2x}=u$
$3^{-2x}=\left(3^{2x}\right)^{-1}=u^{-1}$
Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{array}{rcl}
3^{2x} & = & 2+63\cdot 3^{-2x} \\
3^{2x}-2-63\cdot 3^{-2x} & = & 0 \\
u-2-63u^{-1} & = & 0 \\
u^2-2u-63 & = & 0 \\
(u-9)(u+7) & = & 0 \\
u-9=0 & \text{or} & u+{\color{red}7}=0 \\
 & \text{or} & u={\color{red}-7}
\end{array}
$$

>2 terms $\Rightarrow$
transfer one side

$3^{2x}=u$

$3^{-2x}=\left(3^{2x}\right)^{-1}=u^{-1}$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{array}{rcl}
3^{2x} & = & 2 + 63 \cdot 3^{-2x} \\
3^{2x} - 2 - 63 \cdot 3^{-2x} & = & 0 \\
u - 2 - 63u^{-1} & = & 0 \\
u^2 - 2u - 63 & = & 0 \\
(u-9)(u+7) & = & 0 \\
u - 9 = 0 & \text{or} & u + 7 = 0 \\
 & \text{or} & u = -7 \\
 & \text{or} & \text{no real solution}
\end{array}
$$

>2 terms $\Rightarrow$ transfer one side

$3^{2x} = u$

$3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1}$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{array}{rcl}
3^{2x} &=& 2+63\cdot 3^{-2x} \\
3^{2x}-2-63\cdot 3^{-2x} &=& 0 \\
u-2-63u^{-1} &=& 0 \\
u^2-2u-63 &=& 0 \\
(u-9)(u+7) &=& 0 \\
u-9=0 &\text{or}& u+7=0 \\
u=9 &\text{or}& u=-7 \\
 &\text{or}& \text{no real solution}
\end{array}
$$

>2 terms $\Rightarrow$ transfer one side

$3^{2x}=u$

$3^{-2x}=\left(3^{2x}\right)^{-1}=u^{-1}$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$\begin{aligned}
3^{2x} &= 2 + 63 \cdot 3^{-2x} \\
3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \\
u - 2 - 63u^{-1} &= 0 \\
u^2 - 2u - 63 &= 0 \\
(u - 9)(u + 7) &= 0 \\
u - 9 = 0 \quad &\text{or} \quad u + 7 = 0 \\
u = 9 \quad &\text{or} \quad u = -7 \\
3^{2x} = 9 \quad &\text{or} \quad \text{no real solution}
\end{aligned}$$

>2 terms $\Rightarrow$ transfer one side

$3^{2x} = u$

$3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1}$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{array}{rcl}
3^{2x} & = & 2 + 63 \cdot 3^{-2x} \\
3^{2x} - 2 - 63 \cdot 3^{-2x} & = & 0 \\
u - 2 - 63u^{-1} & = & 0 \\
u^2 - 2u - 63 & = & 0 \\
(u-9)(u+7) & = & 0 \\
u - 9 = 0 & \text{or} & u + 7 = 0 \\
u = 9 & \text{or} & u = -7 \\
3^{2x} = 9 & \text{or} & \text{no real solution} \\
2x = \log_3 9 & &
\end{array}
$$

>2 terms $\Rightarrow$ transfer one side

$3^{2x} = u$

$3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1}$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{aligned}
3^{2x} &= 2 + 63 \cdot 3^{-2x} \\
3^{2x} - 2 - 63 \cdot 3^{-2x} &= 0 \\
u - 2 - 63u^{-1} &= 0 \\
u^2 - 2u - 63 &= 0 \\
(u-9)(u+7) &= 0 \\
u - 9 = 0 \quad &\text{or} \quad u + 7 = 0 \\
u = 9 \quad &\text{or} \quad u = -7 \\
3^{2x} = 9 \quad &\text{or} \quad \text{no real solution} \\
2x = \log_3 9 & \\
2x = \mathbf{?} &
\end{aligned}
$$

> 2 terms $\Rightarrow$ transfer one side

$3^{2x} = u$

$3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1}$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{array}{rcl}
3^{2x} & = & 2+63\cdot 3^{-2x} \\
3^{2x}-2-63\cdot 3^{-2x} & = & 0 \\
u-2-63u^{-1} & = & 0 \\
u^2-2u-63 & = & 0 \\
(u-9)(u+7) & = & 0 \\
u-9=0 & \text{or} & u+7=0 \\
u=9 & \text{or} & u=-7 \\
3^{2x}=9 & \text{or} & \text{no real solution} \\
2x=\log_3 9 & & \\
2x=2 & &
\end{array}
$$

>2 terms $\Rightarrow$ transfer one side

$3^{2x}=u$

$3^{-2x}=\left(3^{2x}\right)^{-1}=u^{-1}$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{array}{rcl}
3^{2x} & = & 2+63\cdot 3^{-2x} \\
3^{2x}-2-63\cdot 3^{-2x} & = & 0 \\
u-2-63u^{-1} & = & 0 \\
u^2-2u-63 & = & 0 \\
(u-9)(u+7) & = & 0 \\
u-9=0 & \text{or} & u+7=0 \\
u=9 & \text{or} & u=-7 \\
3^{2x}=9 & \text{or} & \text{no real solution} \\
2x=\log_3 9 & & \\
{\color{red}2}x={\color{red}2} & & \\
x={\color{red}1} & &
\end{array}
$$

>2 terms $\Rightarrow$ transfer one side

$3^{2x}=u$

$3^{-2x}=\left(3^{2x}\right)^{-1}=u^{-1}$

Multiply $\cdot u$

Example (Exponential equation that reduces to quadratic)

Solve the equation.

$$
\begin{array}{rcl}
3^{2x} & = & 2 + 63 \cdot 3^{-2x} \\
3^{2x} - 2 - 63 \cdot 3^{-2x} & = & 0 \\
u - 2 - 63u^{-1} & = & 0 \\
u^2 - 2u - 63 & = & 0 \\
(u-9)(u+7) & = & 0 \\
u - 9 = 0 & \text{or} & u + 7 = 0 \\
u = 9 & \text{or} & u = -7 \\
3^{2x} = 9 & \text{or} & \text{no real solution} \\
2x = \log_3 9 & & \\
2x = 2 & & \\
x = 1 & &
\end{array}
$$

> 2 terms $\Rightarrow$ transfer one side

$3^{2x} = u$

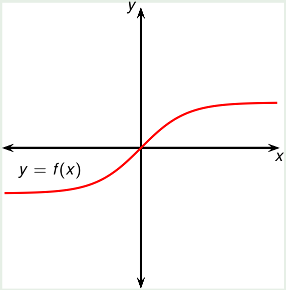
$3^{-2x} = \left(3^{2x}\right)^{-1} = u^{-1}$

Multiply $\cdot u$

Example

Find $f^{-1}(x)$ for

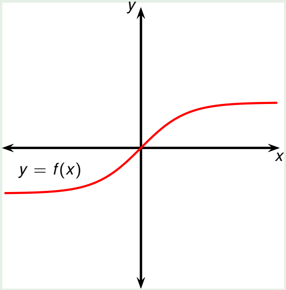
$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



Example

Find $f^{-1}(x)$ for
$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$



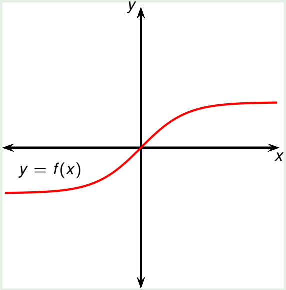
Example

Find $f^{-1}(x)$ for
$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{\left(u - \mathbf{?}\right)}{\left(u + \mathbf{?}\right)} = y$$

Set $u = e^x$



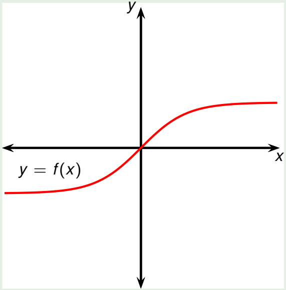
Example

Find $f^{-1}(x)$ for
$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{(u - \mathbf{?})}{(u + \mathbf{?})} = y$$

Set $u = e^x$
$e^{-x} = \mathbf{?}$



Example

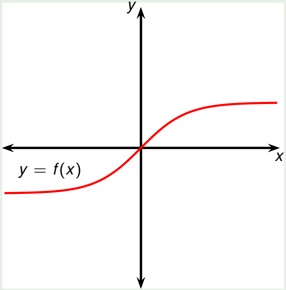
Find $f^{-1}(x)$ for $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{\left(u - \frac{1}{u}\right)}{\left(u + \frac{1}{u}\right)} = y$$

Set $u = e^x$

$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$



Example

Find $f^{-1}(x)$ for

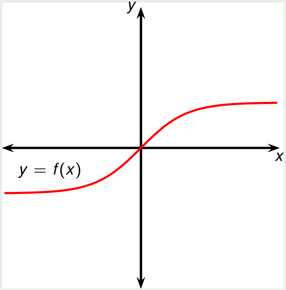
$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{\left(u - \frac{1}{u}\right) u}{\left(u + \frac{1}{u}\right) u} = y$$

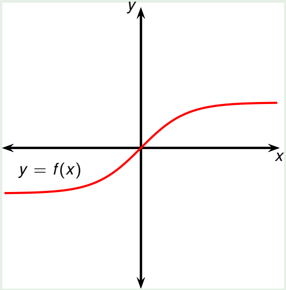
Set $u = e^x$

$$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$$



Example

Find $f^{-1}(x)$ for
$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.



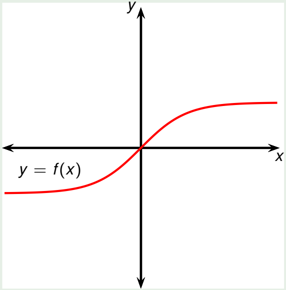
$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$
$$\frac{\left(u - \frac{1}{u}\right)u}{\left(u + \frac{1}{u}\right)u} = y$$
$$\frac{u^2 - 1}{u^2 + 1} = y$$

Set $u = e^x$
$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{\left(u - \frac{1}{u}\right) u}{\left(u + \frac{1}{u}\right) u} = y$$

$$\frac{u^2 - 1}{u^2 + 1} = y$$

$$u^2 - 1 = y(u^2 + 1)$$

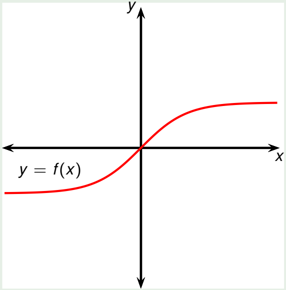
Set $u = e^x$

$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$\frac{\left(u - \frac{1}{u}\right) u}{\left(u + \frac{1}{u}\right) u} = y$$

$$\frac{u^2 - 1}{u^2 + 1} = y$$

$$u^2 - 1 = y(u^2 + 1)$$

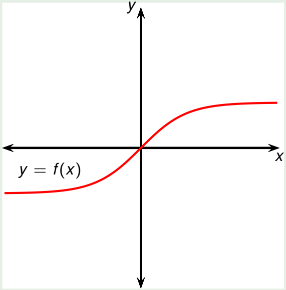
$$u^2(1 - y) = 1 + y$$

Set $u = e^x$

$$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$$

Example

Find $f^{-1}(x)$ for
$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.

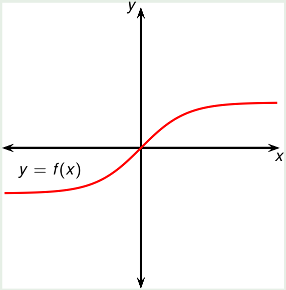


$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$
$$\frac{\left(u - \frac{1}{u}\right) u}{\left(u + \frac{1}{u}\right) u} = y$$
$$\frac{u^2 - 1}{u^2 + 1} = y$$
$$u^2 - 1 = y(u^2 + 1)$$
$$u^2(1 - y) = 1 + y$$

Set $u = e^x$
$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

Example

Find $f^{-1}(x)$ for
$f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

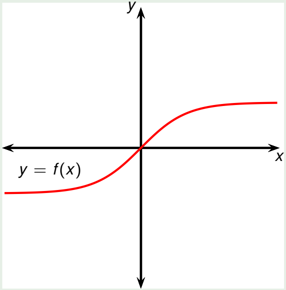


$$\begin{aligned}
\frac{e^x - e^{-x}}{e^x + e^{-x}} &= y \\
\frac{\left(u - \frac{1}{u}\right) u}{\left(u + \frac{1}{u}\right) u} &= y \\
\frac{u^2 - 1}{u^2 + 1} &= y \\
u^2 - 1 &= y(u^2 + 1) \\
u^2(1 - y) &= 1 + y \\
u^2 &= \frac{1 + y}{1 - y}
\end{aligned}$$

Set $u = e^x$
$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

Example

Find $f^{-1}(x)$ for
$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

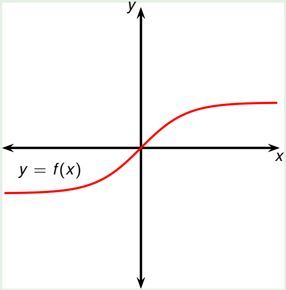


$$\begin{aligned}
\frac{e^x - e^{-x}}{e^x + e^{-x}} &= y \\
\frac{\left(u - \frac{1}{u}\right) u}{\left(u + \frac{1}{u}\right) u} &= y \\
\frac{u^2 - 1}{u^2 + 1} &= y \\
u^2 - 1 &= y(u^2 + 1) \\
u^2(1 - y) &= 1 + y \\
u^2 &= \frac{1 + y}{1 - y} \\
(e^x)^2 &= \frac{1 + y}{1 - y}
\end{aligned}$$

Set $u = e^x$
$$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$$

Example

Find $f^{-1}(x)$ for
$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



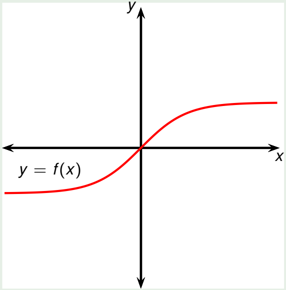
$$\begin{aligned}
\frac{e^x - e^{-x}}{e^x + e^{-x}} &= y \\
\frac{\left(u - \frac{1}{u}\right) u}{\left(u + \frac{1}{u}\right) u} &= y \\
\frac{u^2 - 1}{u^2 + 1} &= y \\
u^2 - 1 &= y(u^2 + 1) \\
u^2(1 - y) &= 1 + y \\
u^2 &= \frac{1 + y}{1 - y} \\
(e^{x})^{2} &= \frac{1 + y}{1 - y} \\
e^{2x} &= \frac{1 + y}{1 - y}
\end{aligned}$$

Set $u = e^x$
$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



$$\begin{aligned}
\frac{e^x - e^{-x}}{e^x + e^{-x}} &= y \\
\frac{\left(u - \frac{1}{u}\right) u}{\left(u + \frac{1}{u}\right) u} &= y \\
\frac{u^2 - 1}{u^2 + 1} &= y \\
u^2 - 1 &= y(u^2 + 1) \\
u^2(1 - y) &= 1 + y \\
u^2 &= \frac{1 + y}{1 - y} \\
(e^x)^2 &= \frac{1 + y}{1 - y} \\
e^{2x} &= \frac{1 + y}{1 - y} \\
2x &= \ln\left(\frac{1 + y}{1 - y}\right)
\end{aligned}$$

Set $u = e^x$

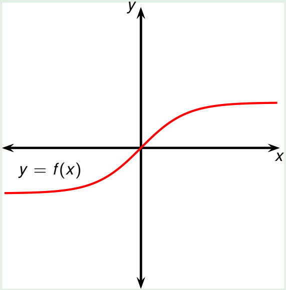
$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

Take ln

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y \qquad \Big| \quad \text{Set } u = e^x,\ e^{-x} = \frac{1}{e^x} = \frac{1}{u}$$

$$\frac{\left(u - \frac{1}{u}\right) u}{\left(u + \frac{1}{u}\right) u} = y$$

$$\frac{u^2 - 1}{u^2 + 1} = y$$

$$u^2 - 1 = y(u^2 + 1)$$

$$u^2(1 - y) = 1 + y$$

$$u^2 = \frac{1+y}{1-y}$$

$$(e^x)^2 = \frac{1+y}{1-y}$$

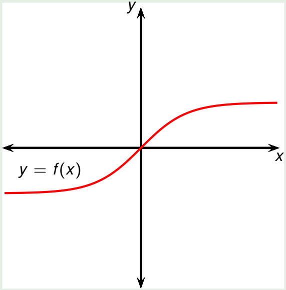
$$e^{2x} = \frac{1+y}{1-y} \qquad \Big| \quad \text{Take } \ln$$

$$\color{red}{2}x = \ln\left(\frac{1+y}{1-y}\right)$$

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



answer

$$f^{-1}(y) = \frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)$$

$$\begin{aligned}
\frac{e^x - e^{-x}}{e^x + e^{-x}} &= y \\
\frac{\left(u - \frac{1}{u}\right)u}{\left(u + \frac{1}{u}\right)u} &= y \\
\frac{u^2 - 1}{u^2 + 1} &= y \\
u^2 - 1 &= y(u^2 + 1) \\
u^2(1 - y) &= 1 + y \\
u^2 &= \frac{1+y}{1-y} \\
(e^x)^2 &= \frac{1+y}{1-y} \\
e^{2x} &= \frac{1+y}{1-y} \\
x &= \frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)
\end{aligned}$$

Set $u = e^x$

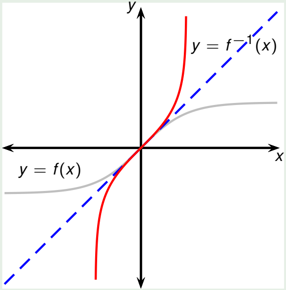
$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

Take ln

Example

Find $f^{-1}(x)$ for

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$



$$\frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

Set $u = e^x$

$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

$$\frac{\left(u - \frac{1}{u}\right)u}{\left(u + \frac{1}{u}\right)u} = y$$

$$\frac{u^2 - 1}{u^2 + 1} = y$$

$$u^2 - 1 = y(u^2 + 1)$$

$$u^2(1 - y) = 1 + y$$

$$u^2 = \frac{1 + y}{1 - y}$$

$$(e^x)^2 = \frac{1 + y}{1 - y}$$

$$e^{2x} = \frac{1 + y}{1 - y}$$

Take ln

$$x = \frac{1}{2}\ln\left(\frac{1 + y}{1 - y}\right)$$

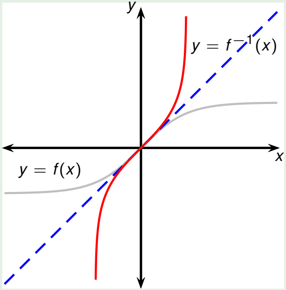
Final answer, relabeled:

$$f^{-1}(x) = \frac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right)$$

Example

Find $f^{-1}(x)$ for $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.

$f = \mathtt{tanh}$ = hyperbolic tangent function.



Final answer, relabeled:

$$f^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$$\begin{aligned}
\frac{e^x - e^{-x}}{e^x + e^{-x}} &= y \\
\frac{\left(u - \frac{1}{u}\right)u}{\left(u + \frac{1}{u}\right)u} &= y \\
\frac{u^2 - 1}{u^2 + 1} &= y \\
u^2 - 1 &= y(u^2 + 1) \\
u^2(1-y) &= 1 + y \\
u^2 &= \frac{1+y}{1-y} \\
(e^x)^2 &= \frac{1+y}{1-y} \\
e^{2x} &= \frac{1+y}{1-y} \\
x &= \frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)
\end{aligned}$$

Set $u = e^x$

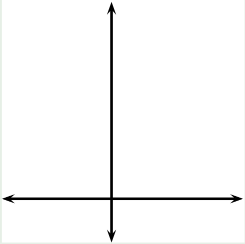
$e^{-x} = \frac{1}{e^x} = \frac{1}{u}$

Take ln

Example

Solve the inequality.

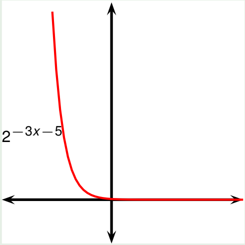
$$2^{-3x-5} < 7$$



Example

Solve the inequality.

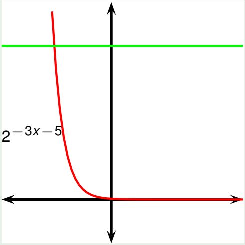
$$2^{-3x-5} < 7$$



Example

Solve the inequality.

$$2^{-3x-5} \quad < \quad 7$$



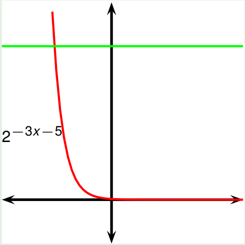
Example

Solve the inequality.

$$2^{-3x-5} < 7$$

$$\log_2 2^{-3x-5} < \log_2 7$$

Logarithms preserve inequalities: apply $\log_2$



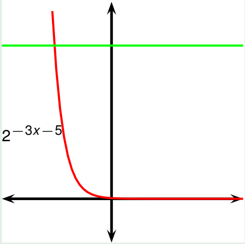
Example

Solve the inequality.

$$2^{-3x-5} < 7$$

$$\log_2 2^{-3x-5} < \log_2 7$$

Logarithms preserve inequalities: apply $\log_2$

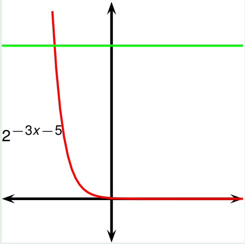


Example

Solve the inequality.

$$
\begin{aligned}
2^{-3x-5} &< 7\\
\log_2 2^{-3x-5} &< \log_2 7\\
-3x-5 &< \log_2 7
\end{aligned}
$$

Logarithms preserve inequalities: apply $\log_2$

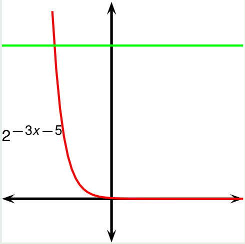


Example

Solve the inequality.

$$\begin{aligned} 2^{-3x-5} &< 7 \\ \log_2 2^{-3x-5} &< \log_2 7 \\ -3x-5 &< \log_2 7 \\ -3x &< \log_2 7 + 5 \end{aligned}$$

Logarithms preserve inequalities: apply $\log_2$



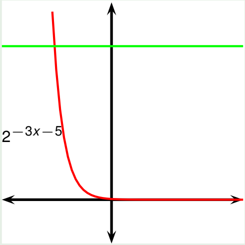
Example

Solve the inequality.

$$
\begin{aligned}
2^{-3x-5} &< 7 \\
\log_2 2^{-3x-5} &< \log_2 7 \\
-3x-5 &< \log_2 7 \\
-3x &< \log_2 7 + 5 \\
x &> -\frac{\log_2 7 + 5}{3}
\end{aligned}
$$

Logarithms preserve inequalities: apply $\log_2$

Division by negative number flips inequalities



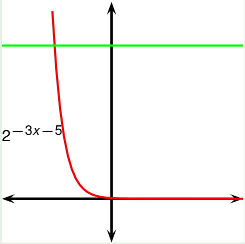
Example

Solve the inequality.

$$\begin{aligned} 2^{-3x-5} &< 7 \\ \log_2 2^{-3x-5} &< \log_2 7 \\ -3x-5 &< \log_2 7 \\ -3x &< \log_2 7 + 5 \\ x &> -\frac{\log_2 7 + 5}{3} \end{aligned}$$

Logarithms preserve inequalities: apply $\log_2$

Division by negative number flips inequalities



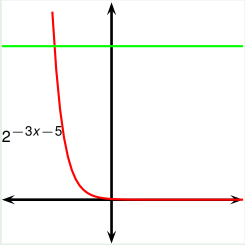
Example

Solve the inequality.

$$
\begin{aligned}
2^{-3x-5} &< 7 \\
\log_2 2^{-3x-5} &< \log_2 7 \\
-3x-5 &< \log_2 7 \\
-3x &< \log_2 7 + 5 \\
x &> -\frac{\log_2 7 + 5}{3} \\
x &\in \left(-\frac{5+\log_2 7}{3}, \infty\right)
\end{aligned}
$$

Logarithms preserve inequalities: apply $\log_2$

Division by negative number flips inequalities



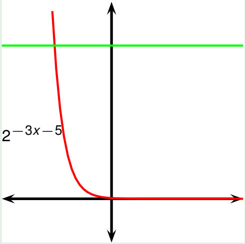
Example

Solve the inequality.

$$\begin{aligned}
2^{-3x-5} &< 7 \\
\log_2 2^{-3x-5} &< \log_2 7 \\
-3x-5 &< \log_2 7 \\
-3x &< \log_2 7 + 5 \\
x &> -\frac{\log_2 7 + 5}{3} \\
x &\in \left(-\frac{5+\log_2 7}{3}, \infty\right)
\end{aligned}$$

Logarithms preserve inequalities: apply $\log_2$

Division by negative number flips inequalities



Example

Solve the inequality.

$$
\begin{aligned}
2^{-3x-5} &< 7 \\
\log_2 2^{-3x-5} &< \log_2 7 \\
-3x-5 &< \log_2 7 \\
-3x &< \log_2 7 + 5 \\
x &> -\frac{\log_2 7 + 5}{3} \\
x &\in \left(-\frac{5+\log_2 7}{3}, \infty\right)
\end{aligned}
$$

Logarithms preserve inequalities: apply $\log_2$

Division by negative number flips inequalities