

Calculus I

Homework Trig Derivatives

Lecture 10

1. Compute the derivative.

(a) $f(x) = 2x^3 - 3 \cos x$.

ANSWER: $6x^2 + 3 \sin x$

(b) $f(x) = \sqrt{x} \cos x$.

ANSWER: $\frac{\sqrt{x}}{2} - \frac{1}{2}x + \frac{\sqrt{x}}{2} \cos x$

(c) $f(x) = \sin x + \frac{1}{3} \cot x$.

ANSWER: $-\frac{1}{3} - \frac{\cos x}{x^2} + \sin x$

(d) $y = 2 \sec x - \csc x$.

ANSWER: $\frac{\cos^3 x + 2 \sin^3 x}{x^3}$

(e) $y = \frac{1 + \sin^2 \theta}{\cos^3 \theta}$.

ANSWER: $\frac{\theta \sec^2 \theta}{2 \sin \theta \cos^3 \theta + 3 \sin^2 \theta + 3 \sin \theta}$

(f) $g(t) = 4 \sec t + \tan t - \csc t + 3 \cot t$.

ANSWER: $4 \sec^2 t \tan t + \sec^2 t + \csc^2 t \cot t - 3 \csc^2 t$

(g) $y = c \cos t + t^2 \sin t$.

ANSWER: $2t \sin t + c \sin t - \cos t$

(h) $y = u(a \cos u + b \cot u)$.

ANSWER: $-\frac{a \sin^3 u + b \cos u \sin^2 u}{\cos^2 u} + a \cos u + b \cot u$

(i) $y = \frac{x}{2 - \tan x}$.

ANSWER: $\frac{x^2 \cos x \sin x - \cos^2 x}{x^2}$

(j) $y = \sin \theta \cos \theta$.

ANSWER: $\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$

(k) $f(\theta) = \frac{\sec \theta}{1 + \sec \theta}$.

ANSWER: $\frac{\sin(\theta + \cos \theta + 1)}{\theta}$

(l) $y = \frac{\cos x}{1 - \sin x}$.

ANSWER: $\frac{x \sin x - 1}{1}$

(m) $y = \frac{t \sin t}{1 + t}$.

ANSWER: $\frac{t^{(t+1)} \sin t + t \cos t}{t^2}$

(n) $y = \frac{1 - \sec x}{\tan x}$.

ANSWER: $\frac{x \sin x}{\cos x - 1}$

(o) $h(\theta) = \theta \csc \theta - \cot \theta$.

ANSWER: $\frac{\theta \sin^2 \theta}{1 + \sin \theta - \theta \cos \theta}$

(p) $y = x^2 \sin x \tan x$.

ANSWER: $\frac{2x^2 \cos x \sin^2 x + 2x^2 \sin x \cos^2 x + x^3 \sin^3 x}{x^3}$

2. Differentiate.

(a) $\tan x$.

ANSWER: $\sec^2 x$

(b) $\cot x$.

ANSWER: $-\csc^2 x$

(c) $\sec x$.

ANSWER: $\sec x \tan x = \frac{\sin x}{\cos^2 x}$

(d) $\csc x$.

ANSWER: $-\csc x \cot x = -\frac{x \sin x}{\cos^2 x}$

(e) $\sec x \tan x$.

ANSWER: $\sec^2 x \tan x + \sec^3 x$

(f) $\sec x + \tan x$.

ANSWER: $\sec x (\tan x + \sec x)$

(g) $\sec^2 x$.

ANSWER: $2 \tan x \sec^2 x$

(h) $\csc^2 x$.

ANSWER: $-\frac{2 \cot x \csc^2 x}{x}$

(i) $f(x) = (\sec x)e^x$.

ANSWER: $\sec x \tan x e^x + \sec^2 x e^x = e^x (\tan x + \sec^2 x)$

(j) $f(x) = (\tan x)e^x$.

ANSWER: $\sec^2 x e^x + \tan x e^x$

(k) $\frac{\sin x}{x}$.

ANSWER: $\frac{x^2 \cos x - \sin x}{x^2}$

(l) $\frac{\sin x}{e^x}$.

ANSWER: $\frac{x^2 \cos x - \sin x}{e^x}$

(m) $x(\cos x)e^x$.

ANSWER: $e^x (x \cos x - x \sin x + \cos x)$

(n) $\frac{e^x}{\tan x}$.

ANSWER: $e^x (\cot x - \csc^2 x)$

(o) $\frac{e^x}{\sec x} + \sec x$.

ANSWER: $e^x (\cos x - \sin x) + \sec x \tan x$

Solution. 2i

$$\begin{aligned}\frac{d}{dx} ((\sec x)e^x) &= \left(\frac{d}{dx} (\sec x) \right) e^x + (\sec x) \frac{d}{dx} (e^x) && \left| \text{product rule} \right. \\ &= \sec x \tan x e^x + \sec x e^x \\ &= (\tan x + 1) \sec x e^x\end{aligned}$$

Solution. 2j

$$\begin{aligned}\frac{d}{dx} ((\tan x)e^x) &= \left(\frac{d}{dx} (\tan x) \right) e^x + (\tan x) \frac{d}{dx} (e^x) && \left| \text{product rule} \right. \\ &= (\sec^2 x) e^x + (\tan x) e^x \\ &= (\sec^2 x + \tan x) e^x.\end{aligned}$$