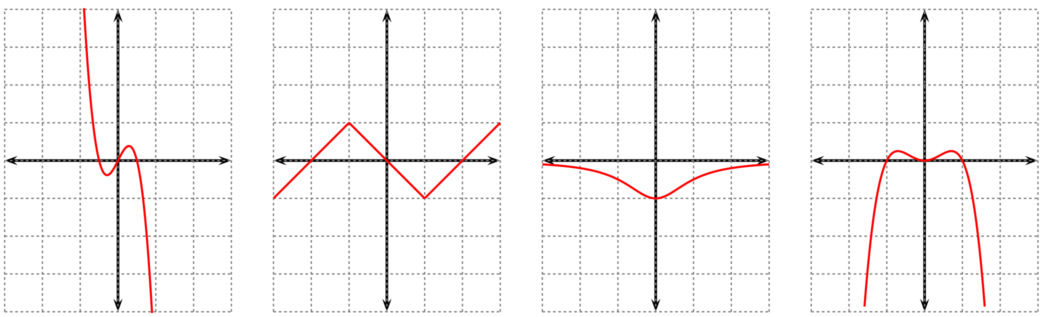
Calculus I
Homework Curve Sketching
Lecture 17

1.

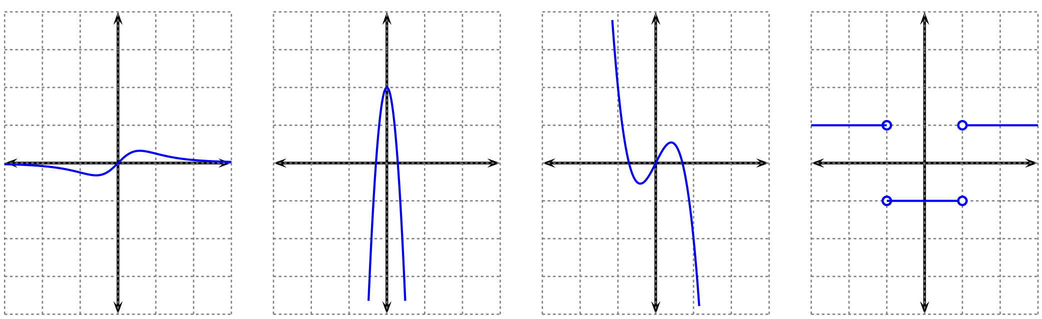
Match each of the following function plots:

1. 2. 3. 4.



to their derivative plots:

(a) (b) (c) (d)

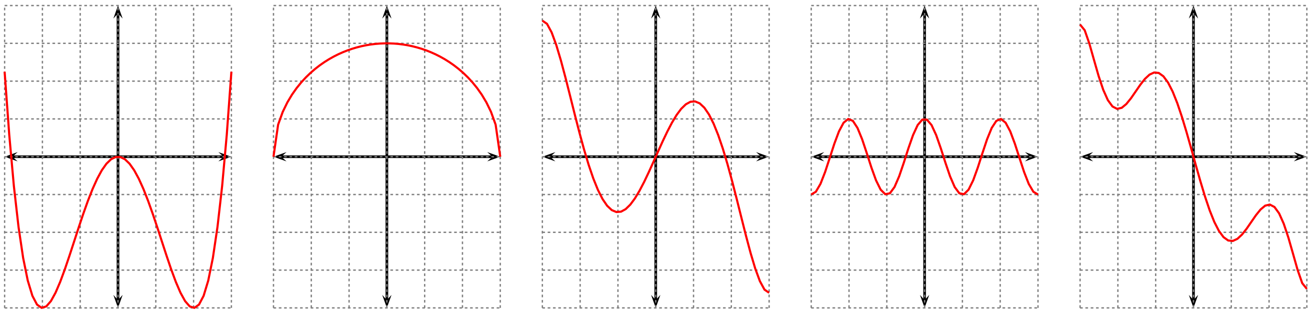


Give reasons for your choices. Can you guess formulas that would give a similar (or precisely the same) graph, and confirm visually your guess using a graphing device?

2.

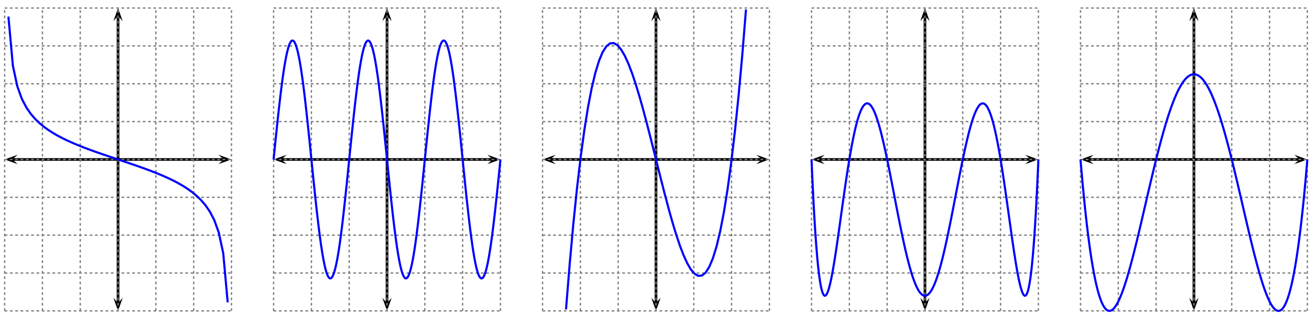
Match each of the following function plots:

1. 2. 3. 4. 5.



to their derivative plots:

(a) (b) (c) (d) (e)

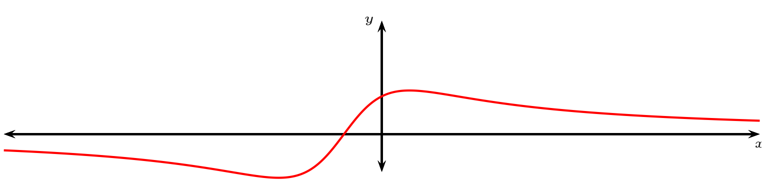


3. Find the

- the implied domain of f,
- x and y intercepts of f,
- horizontal and vertical asymptotes,
- intervals of increase and decrease,
- local and global minima, maxima,
- intervals of concavity,
- points of inflection.

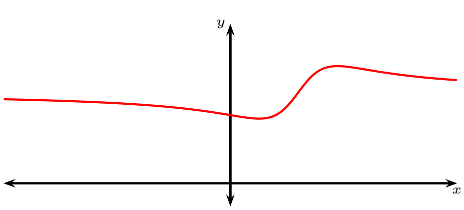
Label all relevant points on the graph. Show all of your computations.

(a) $f(x) = \dfrac{x + \frac{1}{2}}{x^2 + x + 1}$

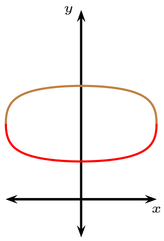


(b) $f(x) = \dfrac{2x^2 - 5x + \frac{9}{2}}{x^2 - 3x + 3}$. **For this problem, indicate only the x-coordinates of the local maxima/minima and inflection points; you do not need to compute the y-coordinates of those points.**

Computation shows that $f'(x) = \dfrac{-x^2 + 3x - \frac{3}{2}}{(x^2 - 3x + 3)^2}$ and that $f''(x) = \dfrac{(2x-3)x(x-3)}{(x^2 - 3x + 3)^3}$; you may use those computations without further justification.

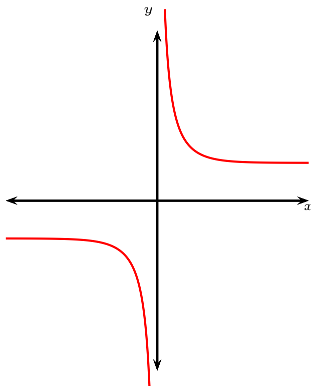


(c) $f(x) = \dfrac{2\sqrt{-x^2+1}+1}{\sqrt{-x^2+1}+1}$, $f(x) = \dfrac{1}{\sqrt{-x^2+1}+1}$

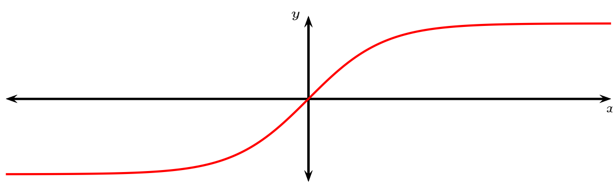


The two functions are plotted simultaneously in the x, y-plane. Indicate which part of the graph is the graph of which function.

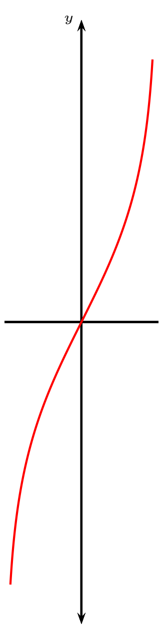
(d) $f(x) = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$



(e) $f(x) = \dfrac{-e^{-x} + e^x}{e^{-x} + e^x}$

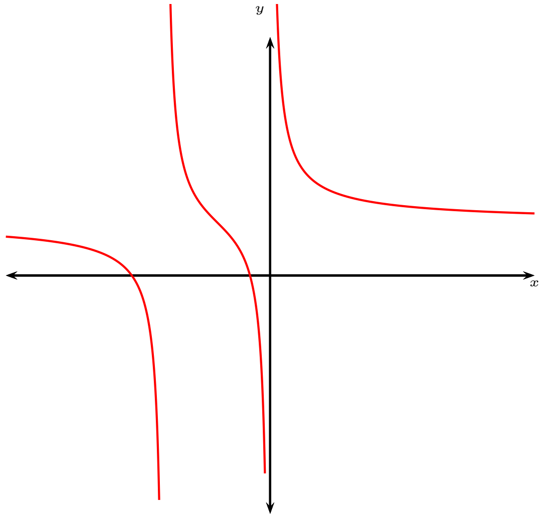


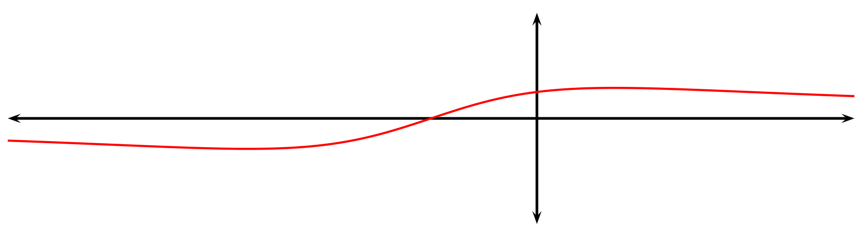
(f) $f(x) = \ln\left(\dfrac{x+1}{-x+1}\right)$



(g) $f(x) = \dfrac{x^2+3x+1}{x^2+2x}$. **For this problem, indicate only the x-coordinates of the local maxima/minima and inflection points; you do not need to compute the y-coordinates of those points.**

Computation shows that $f'(x) = \dfrac{-x^2-2x-2}{\left(x^2+2x\right)^2}$ and that $f''(x) = \dfrac{2x^3+6x^2+12x+8}{\left(x^2+2x\right)^3} = \dfrac{(x+1)\left(2x^2+4x+8\right)}{\left(x^2+2x\right)^3}$; you may use those computations without further justification.

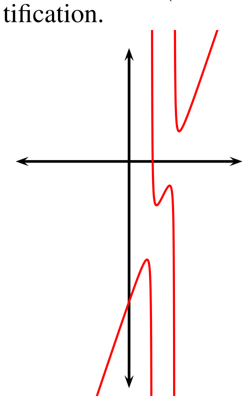




(h) $f(x) = \dfrac{x+1}{x^2+2x+4}$

(i) $f(x) = \dfrac{3x^3-30x^2+97x-99}{x^2-6x+8}$. **For this problem, do not find the x-intercepts of the function. Indicate only the x-coordinates of the local maxima/minima and inflection points; you do not need to compute the y-coordinates of those points.**

Computation shows that $f'(x) = \dfrac{3x^4-36x^3+155x^2-282x+182}{\left(x^2-6x+8\right)^2} = \dfrac{\left(x^2-6x+7\right)\left(3x^2-18x+26\right)}{\left(x^2-6x+8\right)^2}$ and that $f''(x) = \dfrac{2x^3-18x^2+60x-72}{\left(x^2-6x+8\right)^3} = \dfrac{(x-3)\left(2x^2-12x+24\right)}{\left(x^2-6x+8\right)^3}$; you may use those computations without further justification.



4. (a) Sketch the graph of $y = x^4 - 8x^2 + 8$ by determining the intervals of increase and decrease, finding the local mins and maxes, determining where the graph is concave up and concave down, and plotting a few key points.

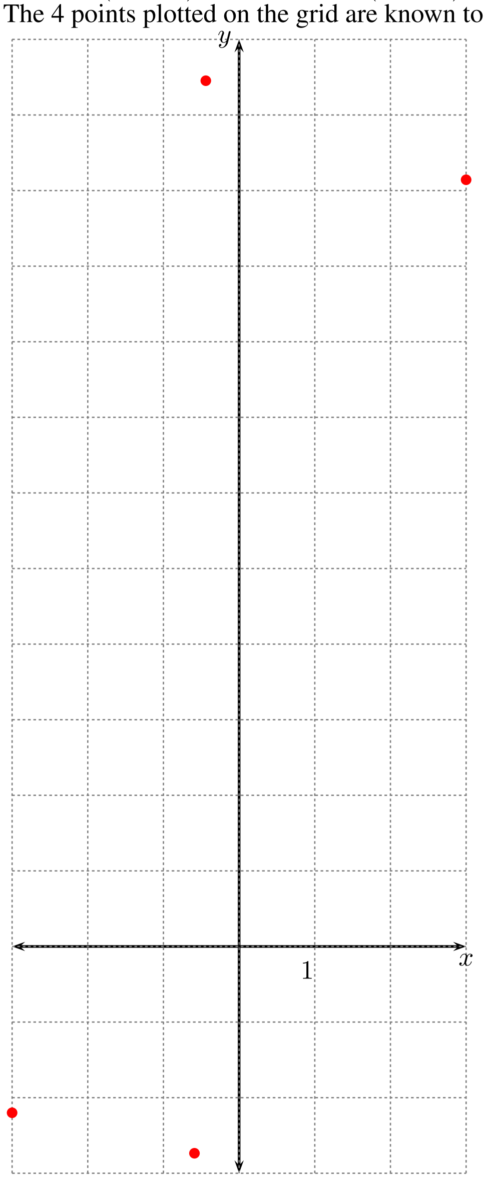
(b) Sketch the graph of $y = \frac{x-1}{x^2-9}$ by graphing any vertical and horizontal asymptotes, finding the x- and y-intercepts, and then sketching a graph that fits this information.

(c) Consider the function $f(x) = \dfrac{4x^2+10x+5}{2x+1}$. Computation shows that $f'(x) = \dfrac{8x^2+8x}{(2x+1)^2}$ and $f''(x) = \dfrac{8}{(2x+1)^3}$.

- Find the intervals of increase and intervals of decrease of f.
- Find the local maxima and minima of f.
- Find where the function is concave up and where it is concave down.
- Sketch the function $f(x)$ roughly by hand. Make sure that your plot matches your computations from the preceding parts of the problem.

  You may use the provided grid and coordinate system. From the previous page, we recall that $f(x) = \dfrac{4x^2+10x+5}{2x+1}$, $f'(x) = \dfrac{8x^2+8x}{(2x+1)^2}$ and $f''(x) = \dfrac{8}{(2x+1)^3}$.

  The 4 points plotted on the grid are known to lie on the curve.



(d) Consider the function $f(x) = \dfrac{2x^2-4x+2}{x^2-2x}$.

- Find the vertical asymptotes of f. **For this particular sub-question, and for this sub-question alone, no justification is required (just write the answer).**
- Computation shows that $f'(x) = \dfrac{-4x+4}{(x^2-2x)^2}$. Find the intervals of increase and decrease of f.
- Find the local maxima and minima of f.
- Computation shows that $f''(x) = \dfrac{12x^2-24x+16}{(x^2-2x)^3}$. Find where the function is concave up and where it is concave down.
- Sketch the function $f(x)$ roughly by hand. Make sure that your plot matches your computations from the preceding parts of the problem.

You may use the provided grid and coordinate system. We recall that

$$f(x) = \frac{2x^2 - 4x + 2}{x^2 - 2x},$$

$$f'(x) = \frac{-4x + 4}{\left(x^2 - 2x\right)^2},$$

$$f''(x) = \frac{12x^2 - 24x + 16}{\left(x^2 - 2x\right)^3}.$$

The points plotted below are known to lie on the curve.