Calculus II
Lecture 8

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https://github.com/tmilev/freecalc

2020

Outline

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Example

Find $\lim_{x\to 1} \frac{\ln x}{x-1}$.

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- We don't get any cancellation between top and bottom.
- We need new techniques.

Theorem (L'Hospital's Rule)

Suppose that f and g are differentiable and $g'(x) \neq 0$ on an open interval that contains a (except possibly at a). Suppose that

$$\lim_{x\to a} f(x) = 0 \qquad \textit{and} \qquad \lim_{x\to a} g(x) = 0$$

$$\textit{or that} \qquad \lim_{x\to a} f(x) = \pm\infty \qquad \textit{and} \qquad \lim_{x\to a} g(x) = \pm\infty$$

(In other words, we have an indeterminate form of type $0/0$ or ∞/∞.) Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

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$$fg = \frac{f}{1/g} \qquad \text{or} \qquad fg = \frac{g}{1/f}.$$

This converts the given limit into an indeterminate form of type $0/0$ or ∞/∞.

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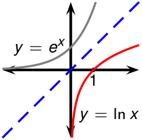

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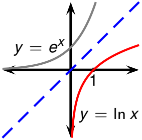

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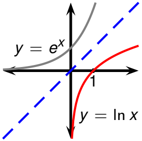

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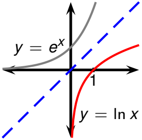

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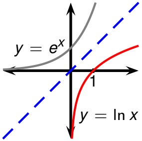

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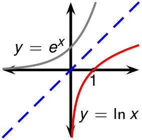

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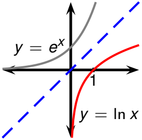

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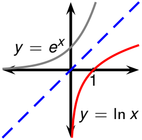

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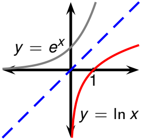

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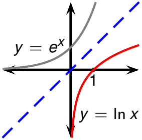

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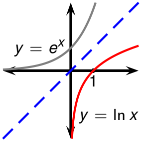

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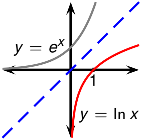

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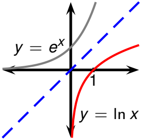

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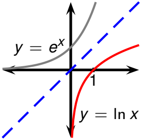

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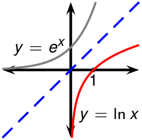

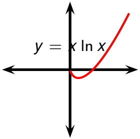

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Indeterminate Differences

If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$, then the limit

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To compute such a limit, try to convert it into a quotient (by using a common denominator, or by rationalizing, or by factoring out a common factor).

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Evaluate $\lim_{x\to(\pi/2)^-}(\sec x - \tan x)$.

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Indeterminate Powers

Several indeterminate forms arise from the limit $\lim_{x\to a} f(x)^{g(x)}$.

$$\lim_{x\to a} f(x) = 0 \quad \text{and} \quad \lim_{x\to a} g(x) = 0 \qquad \text{type } 0^0$$

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These can all be solved either by taking the natural logarithm:

$$\text{let } \; y = [f(x)]^{g(x)}, \quad \text{then } \; \ln y = g(x) \ln f(x)$$

or by writing the function as an exponential:

$$[f(x)]^{g(x)} = e^{g(x) \ln f(x)}.$$

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Example

$$\lim_{x \to \infty} \left(1 + \frac{k}{x}\right)^x$$

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\lim_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x &= \lim_{x\to\infty} {\color{red}x}\ln\left(1+\frac{k}{x}\right)\\
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&= \lim_{x\to\infty}\frac{\left(\ln\left(1+\frac{k}{x}\right)\right)}{\left(\frac{1}{x}\right)} && \Big|\ \text{form “}\frac{0}{0}\text{”,}
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form "$\frac{0}{0}$", use L'Hospital

$$= \lim_{x\to\infty}\frac{\textbf{?}}{\textbf{?}}$$

Example

$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} e^{\ln\left(1+\frac{k}{x}\right)^x}$$ | exponent= continuous f-n

$$= e^{\lim\limits_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x}$$

$$\lim_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} x\ln\left(1+\frac{k}{x}\right)$$

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$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} e^{\ln\left(1+\frac{k}{x}\right)^x} \quad \Big| \text{ exponent= continuous f-n}$$

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$$= \lim_{x\to\infty}\frac{\frac{1}{1+\frac{k}{x}}\left(1+\frac{k}{x}\right)'}{-\frac{1}{x^2}}$$

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Example

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Example

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$$= e^{\lim\limits_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x} = e^k$$ limit computed below

$$\lim_{x\to\infty}\ln\left(1+\frac{k}{x}\right)^x = \lim_{x\to\infty} x\ln\left(1+\frac{k}{x}\right)$$

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