Calculus III
Homework on Lecture 5

1. Find polar equations of the line given below.

   (a) The line $x + y = 1$.

   answer: $r(\cos\theta + \sin\theta) = 1$ or $r = \frac{\sqrt{2}}{2}\sec\left(\theta - \frac{\pi}{4}\right)$

   (b) The line $x + \sqrt{3}y = 2$.

   answer: $r\left(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right) = 1$ or $r = \sec\left(\theta - \frac{\pi}{3}\right)$

   (c) The line passing through $(3, 5)$ and $(5, 7)$.

   answer: $x - y = -2$, $r(\cos\theta - \sin\theta) = -2$ or $r = -\sqrt{2}\sec\left(\theta + \frac{\pi}{4}\right)$

   (d) The line passing through $(2, 3)$ and $(-3, -2)$.

   answer: $x - y = -1$, $r(\cos\theta - \sin\theta) = -1$ or $r = -\frac{\sqrt{2}}{2}\sec\left(\theta + \frac{\pi}{4}\right)$

2. **Solution.** 1.b

   Polar coordinates are given by

$$\left| \begin{array}{rcl} x &=& r\cos\theta \\ y &=& r\sin\theta \end{array} \right. .$$

   All we need to do to obtain polar equations for our line is substitute the above expressions in the equation for the line.

$$r\cos\theta + \sqrt{3}r\sin\theta = 2.$$

   This is a perfectly good answer, but we can transform the equation to make it look more compact:

$$\begin{array}{rcl} r\cos\theta + \sqrt{3}r\sin\theta &=& 2 \\ r\underbrace{\frac{1}{2}}_{=\cos\left(\frac{\pi}{3}\right)}\cos\theta + r\underbrace{\frac{\sqrt{3}}{2}}_{=\sin\left(\frac{\pi}{3}\right)}\sin\theta &=& 1 \\ r\cos\left(-\frac{\pi}{3}\right)\cos\theta - \sin\left(-\frac{\pi}{3}\right)\sin\theta &=& 1 \qquad \left| \text{ use } \cos(a+b) = \cos a\cos b - \sin a\sin b \right. \\ r\cos(\theta - \frac{\pi}{3}) &=& 1 \\ r &=& \frac{1}{\cos\left(\theta - \frac{\pi}{3}\right)} \\ &=& \sec\left(\theta - \frac{\pi}{3}\right) \quad . \end{array}$$

3. Find polar equations of the circle given below.

   (a) The circle given by $(x-1)^2 + y^2 = 1$.

   answer: $r = 2\cos\theta$

   (b) The circle given by $x^2 + x + y^2 = 1$.

   answer: $r^2 + r\cos\alpha - 1 = 0$

   (c) The circle with center $(1, 2)$ and radius 3.

   answer: $r^2 - 4r\sin\theta - 2r\cos\theta - 4 = 0$

   (d) The circle with center $(2, 3)$ and radius 4.

   answer: $r^2 - 6r\sin\theta - 4r\cos\theta - 3 = 0$

4. Find an equation of the plane in cylindrical coordinates.

   (a) The plane given by $x + y + z = 1$.

   (b) The plane given by $2x + 3y - 5z = 0$.

   (c) The plane passing through $(-1, 1, 1)$, $(1, 1, -1)$ and $(1, -1, 1)$.

   (d) The plane passing through $(2, 3, 5)$, $(3, 5, 2)$ and $(5, 2, 3)$.

5. Find an equation of the sphere in cylindrical coordinates.

   (a) The unit sphere.
   (b) The sphere with equation $x^2 + x + y^2 + 2y + z^2 + 3z = 0$.
   (c) The sphere with center $(1, 2, 3)$ and radius 5.

6. Find an equation of the plane in spherical coordinates.

   (a) The plane given by $x + y + z = 1$.
   (b) The plane given by $2x + 3y - 5z = 0$.
   (c) The plane passing through $(-1, 1, 1)$, $(1, 1, -1)$ and $(1, -1, 1)$.
   (d) The plane passing through $(2, 3, 5)$, $(3, 5, 2)$ and $(5, 2, 3)$.

7. Find an equation of the sphere in spherical coordinates.

   (a) The unit sphere.
   (b) The sphere with equation $x^2 + x + y^2 + 2y + z^2 + 3z = 0$.
   (c) The sphere with center $(1, 2, 3)$ and radius 5.