Calculus I
Homework Derivatives of Involving Logarithms and Arbitrary Exponents
Lecture 14

1. Compute the derivative.

(a) $\ln(4x)$

answer: $\frac{1}{x}$

(b) $\ln(-13x)$

answer: $\frac{1}{x}$

(c) $\log_2(5x)$

answer: $\frac{1}{\ln(2)x}$

(d) $\log_{10}(-3x)$

answer: $\frac{1}{\ln(10)x}$

(e) $x^6\ln(2x)$

answer: $6x^5 \ln\ (2x) + x^5$

(f) $x^4\ln(2x)$

answer: $4x^3 \ln\ (2x) + x^3$

(g) $\ln\left(x^4\right)$

answer: $\frac{4}{x}$

(h) $(\ln\ (x))^4$

answer: $\frac{4(\ln x)^3}{x}$

(i) $\ln\ (7x+1)$

answer: $\frac{7}{7x+1}$

(j) $\ln\ (-6x+2)$

answer: $\frac{3}{3x-1}$

(k) $\ln\left(\dfrac{3x-2}{-2x+3}\right)$

answer: $\frac{-5}{6x^2-13x+6} = -\frac{3}{(-3x+2)} - \frac{2}{(2x-3)}$

(l) $\ln\left(\dfrac{5x-4}{-x-5}\right)$

answer: $\frac{29}{5x^2+21x-20} = \frac{5}{5x-4} - \frac{1}{x+5}$

(m) $\ln\left(\dfrac{3x+1}{4x-5}\right)$.

answer: $\frac{3}{3x+1} - \frac{4}{4x-5} = \frac{-19}{12x^2-11x-5}$

(n) $\ln\ (\cot x)$

answer: $-\csc x \sec x$

(o) $\ln\ (\sec(2x))$

answer: $2\tan(2x)$

(p) $f(x) = \ln(\sec x) + \ln(\cot x)$.

answer: $-\cot x$

Solution. 1.k

$$\begin{array}{rcll}
\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(\dfrac{3x-2}{-2x+3}\right)\right) & = & \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\ln(3x-2)-\ln(-2x+3)\right) & \text{logarithm properties} \\
& = & \dfrac{(3x-2)'}{(3x-2)} - \dfrac{(-2x+3)'}{-2x+3} & \text{chain rule} \\
& = & \dfrac{3}{3x-2} - \dfrac{-2}{-2x+3} & \\
& = & \dfrac{3}{3x-2} - \dfrac{2}{2x-3} & \\
& = & \dfrac{-5}{6x^2-13x+6} & \text{combine fractions (optional).}
\end{array}$$

Solution. 1.m

$$\begin{array}{rcll}
\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(\dfrac{3x+1}{4x-5}\right)\right) & = & \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\ln(3x+1)-\ln(4x-5)\right) & \\
& = & \dfrac{(3x+1)'}{3x+1} - \dfrac{(4x-5)'}{4x-5} & \\
& = & \dfrac{3}{3x+1} - \dfrac{4}{4x-5} & \\
& = & \dfrac{-19}{12x^2-11x-5} & \text{step optional.}
\end{array}$$

Solution. 1.p

$$
\begin{array}{rcll}
\frac{\mathrm{d}}{\mathrm{d}x}(\ln(\sec x)+\ln(\cot x)) &=& \frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(\frac{1}{\cos x}\right)+\ln\left(\frac{\cos x}{\sin x}\right)\right) & \\
&=& \frac{\mathrm{d}}{\mathrm{d}x}\left(\cancel{-\ln(\cos x)}+\cancel{\ln(\cos x)}-\ln(\sin x)\right) & \\
&=& -\frac{\mathrm{d}}{\mathrm{d}x}(\ln(\sin x)) & \Big| \text{ chain rule} \\
&=& -\frac{1}{\sin x}\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) & \\
&=& -\frac{\cos x}{\sin x} & \\
&=& -\cot x &
\end{array}
$$

2. Differentiate.

(a) 10^{x^3}. answer: $3(\ln 10)x^2(10)^{x^3}$

(b) $2^{\tan x}$. answer: $(\ln 2)2^{\tan x}\sec^2 x$

(c) x^x. answer: $x^x(\log(x)+1)$

(d) x^{x^x}. answer: $(\ln(x))^2 x^{x^x+x} + x^{x^x+x-1} + (\ln x)x^{x^x+x}$

(e) $(\sin x)^{\cos x}$. answer: $\frac{-\ln(\sin x)(\sin x)^{\cos x+2}+(\sin x)^{\cos x}\cos^2 x}{\sin x}$

(f) $(\ln x)^{\ln x}$. answer: $\ln(\ln(x))x^{-1}(\ln(x))^{\ln(x)} + x^{-1}(\ln(x))^{\ln(x)}$

3. Find the limit.

(a) $\lim\limits_{x\to\infty}\left(1-\frac{2}{x}\right)^x$. answer: e^{-2}

(b) $\lim\limits_{x\to 0}(1-x)^{\frac{1}{x}}$. answer: e^{-1}

(c) $\lim\limits_{x\to\infty}\left(\frac{x}{x-5}\right)^x$. answer: e^5

(d) $\lim\limits_{x\to\infty}\left(\frac{x}{x-2}\right)^{3x+2}$. answer: e^6