Calculus I
Lecture 4
Continuity

Todor Milev
https://github.com/tmilev/freecalc

2020

Outline

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- Should the link be outdated/moved, search for "freecalc project".
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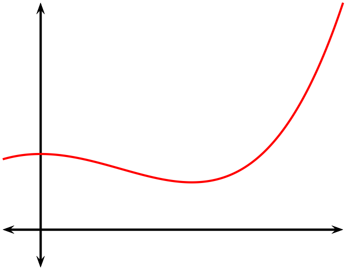
Continuity

- Let f be a function and a be a point in its domain.



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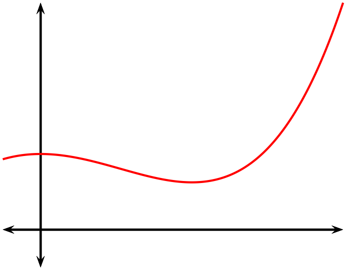

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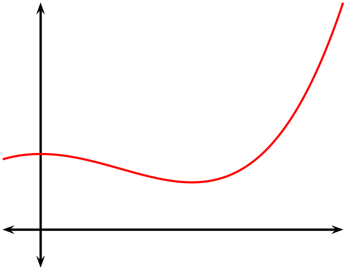
Definition (Continuous at a Number)

We say that f is continuous at a if

$$\lim_{x\to a} f(x) = f(a).$$



Definition (Discontinuous at a Number)

Suppose f is defined at a. We say f is discontinuous at a if it is not continuous at a.

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- Motion of a vehicle with respect to time without sudden brakes.
- Orbits of planets and celestial bodies with respect to time.
- A person's height with respect to time.
- And many more.

Discontinuous phenomena examples:

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- Motion of a vehicle with respect to time without sudden brakes.
- Orbits of planets and celestial bodies with respect to time.
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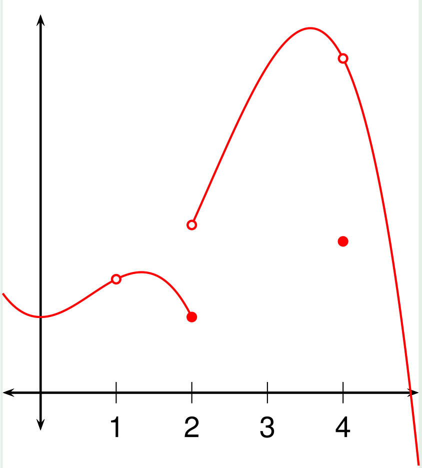
- And many more.

Discontinuous phenomena examples:

- Particle velocities during collisions and explosions.
- Electric current phenomena, gating events in porins (the event of a molecule passing in and out of a cell).
- Particle physics phenomena.
- And many more.

Example

The picture below shows a graph of a function f. At which numbers is f either discontinuous or not defined? Why?



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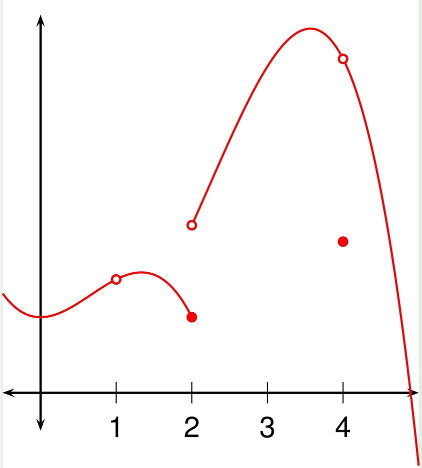

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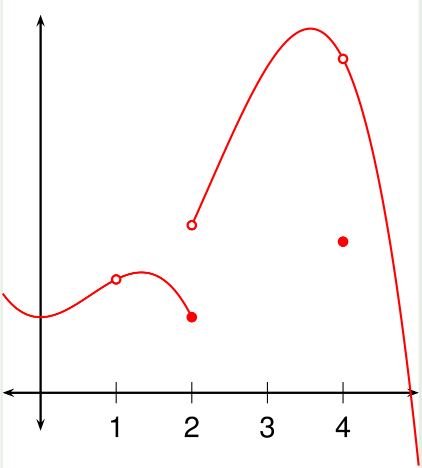

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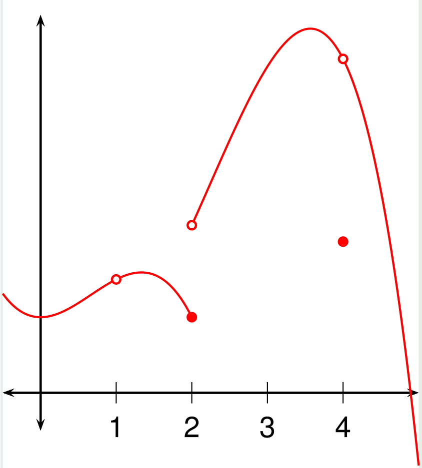

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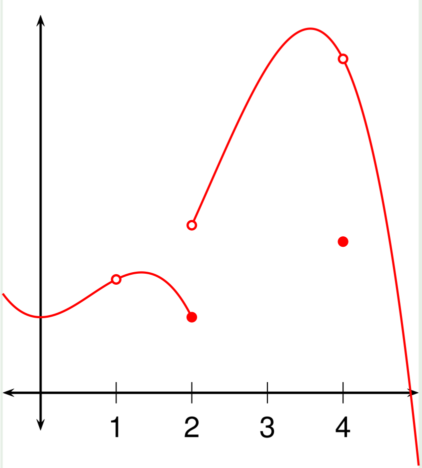

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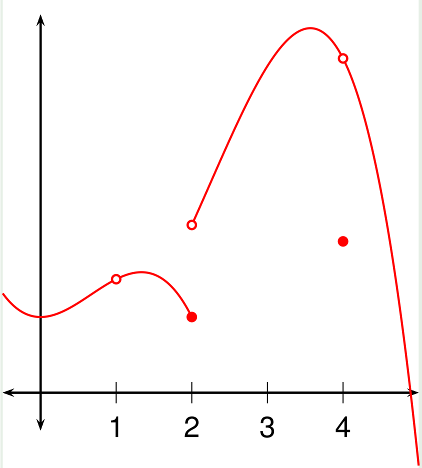

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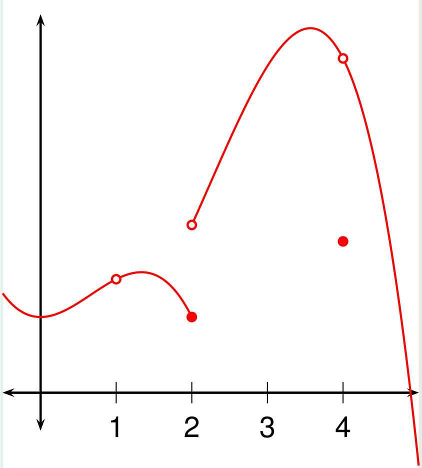

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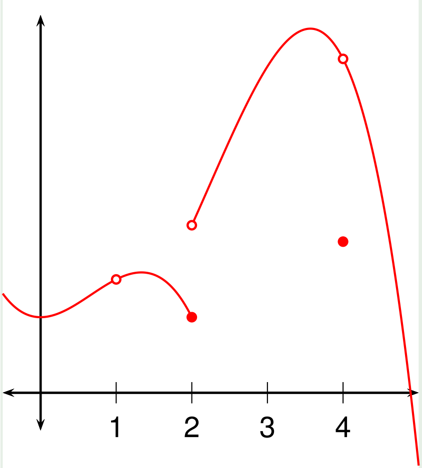

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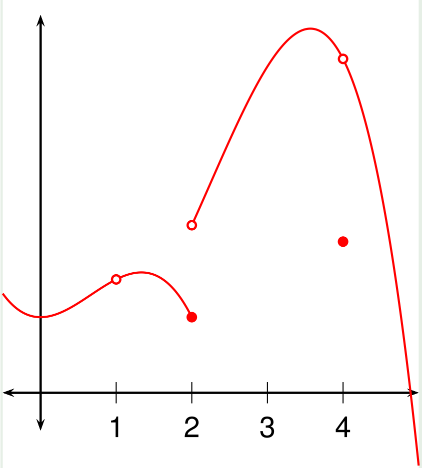

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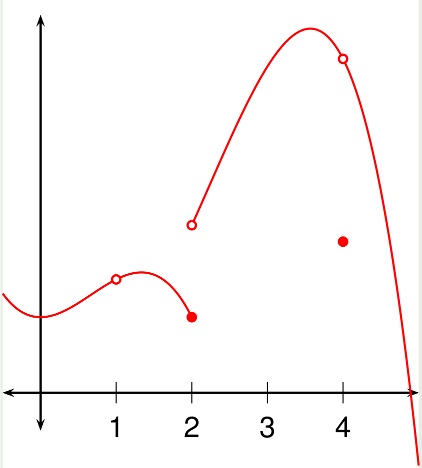

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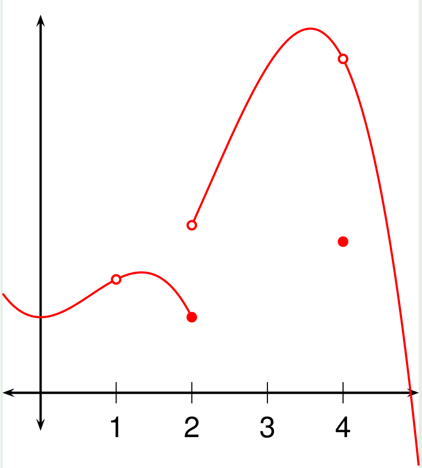

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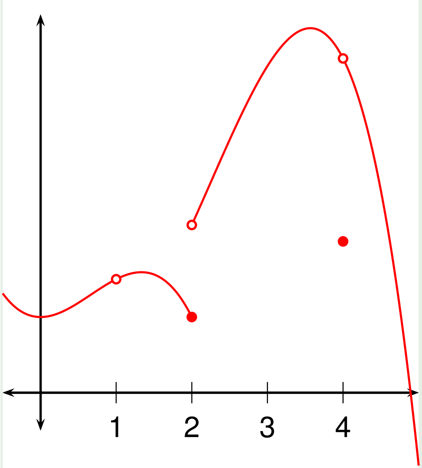

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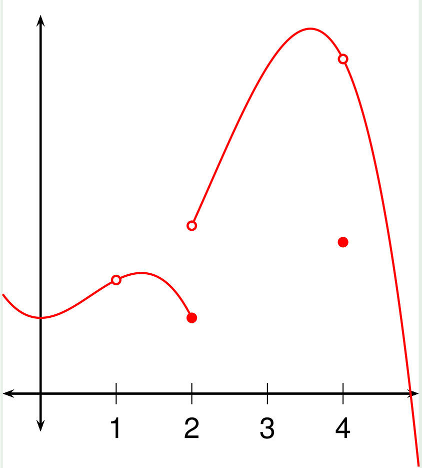

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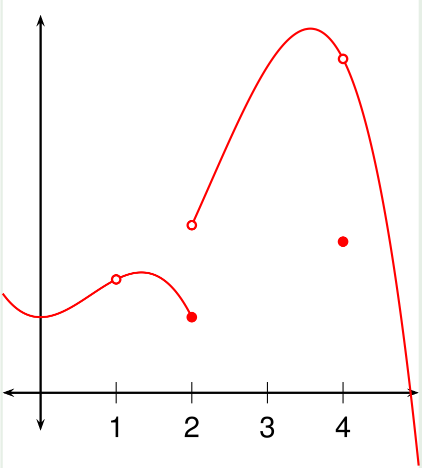

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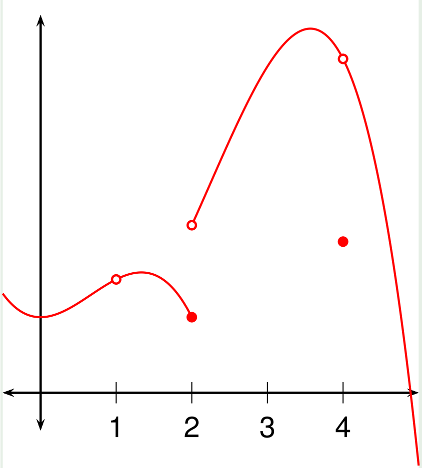

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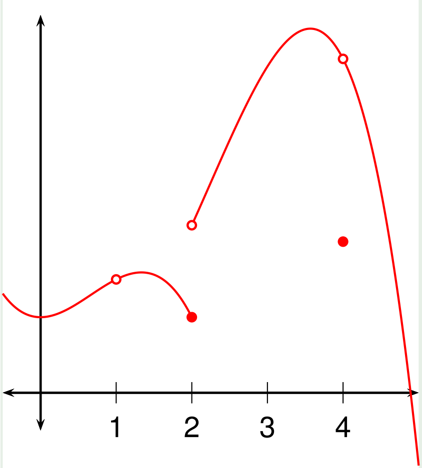

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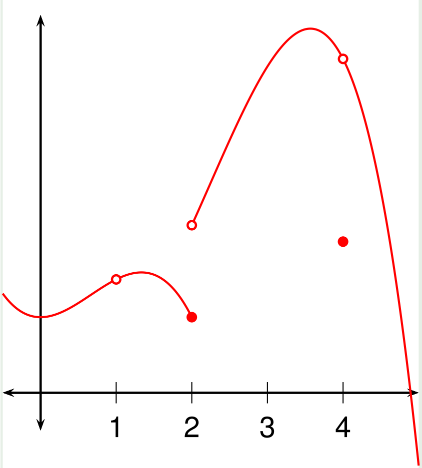

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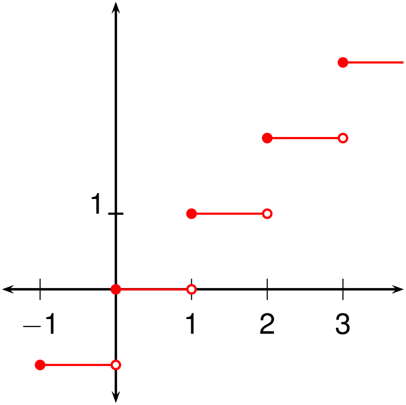
The *greatest integer function* $\lfloor x \rfloor$ is defined as the largest integer that is less than or equal to x.

In computer science this function is called the *floor* function.



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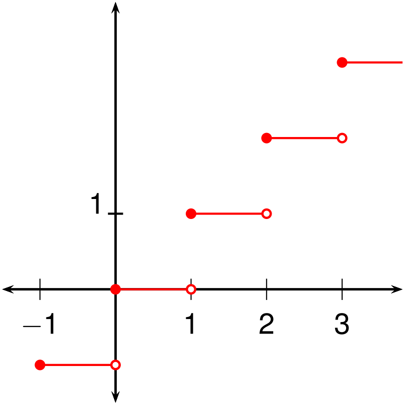

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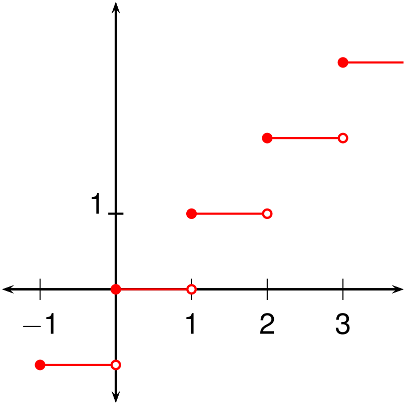

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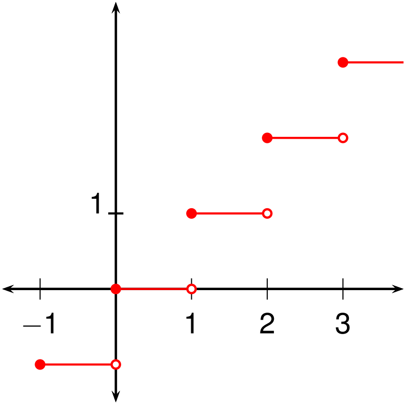

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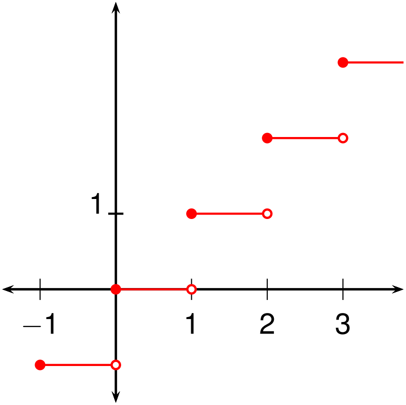

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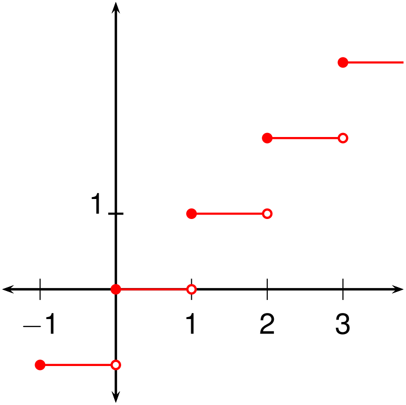

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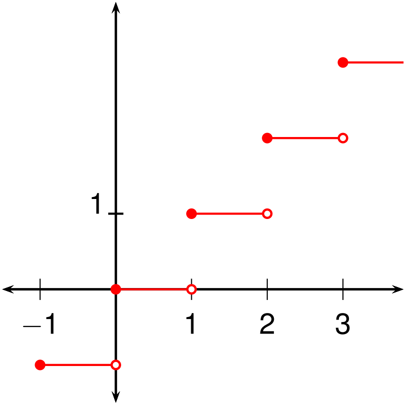

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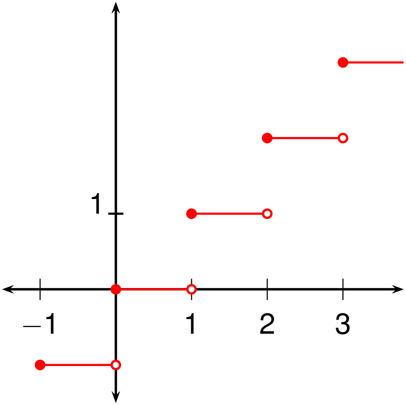

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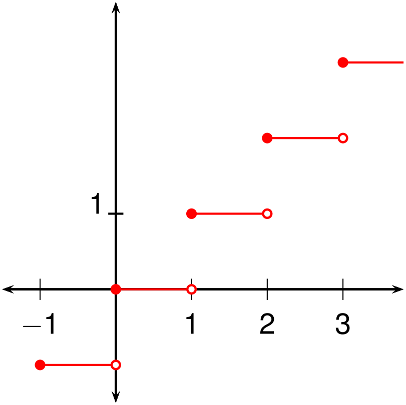

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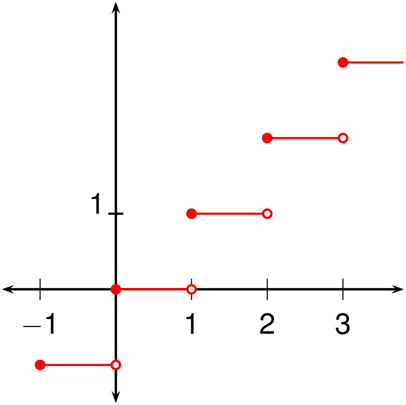

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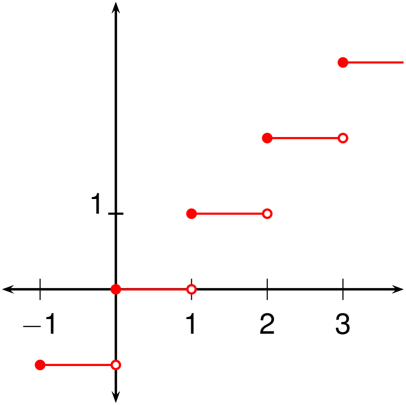

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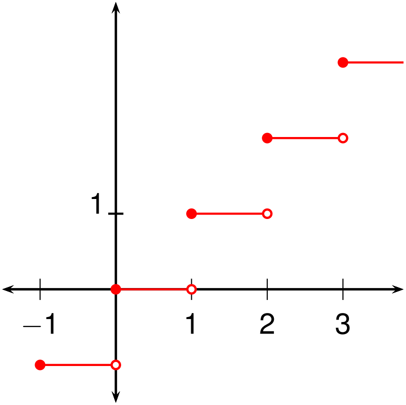

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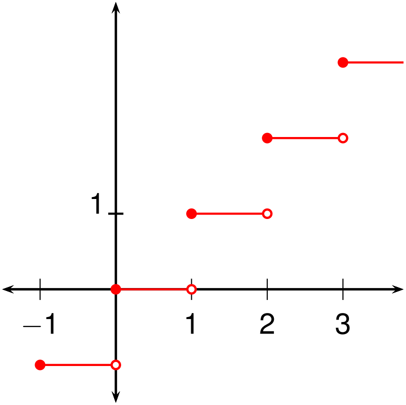

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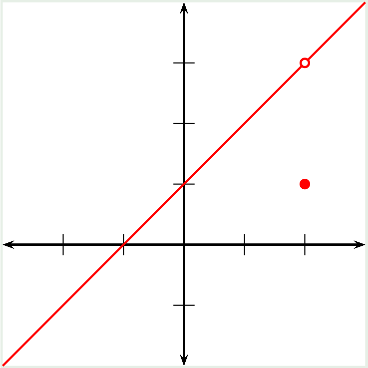
Where is this function discontinuous?

$$f(x) = \begin{cases} \frac{x^2-x-2}{x-2} & \text{if} \quad x \neq 2 \\ 1 & \text{if} \quad x = 2 \end{cases}$$



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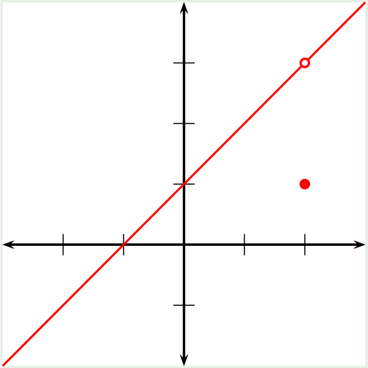

- $f(2)$
- $\lim_{x \to 2} f(x)$

Example

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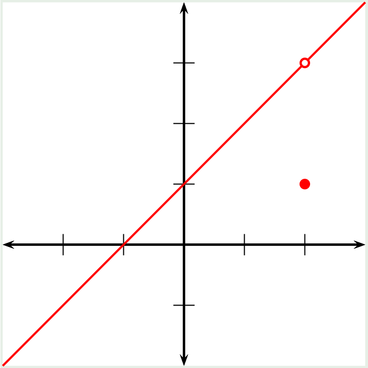

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Example

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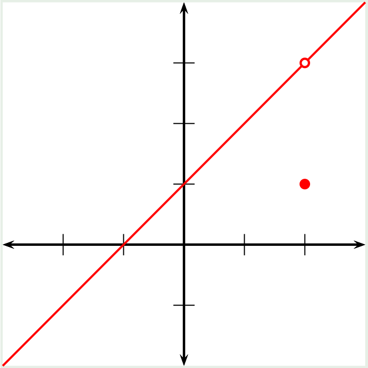

- $f(2)$ is defined ($f(2) = 1$).
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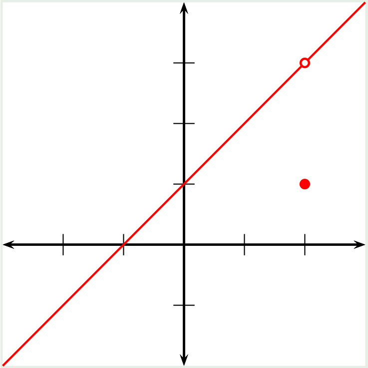

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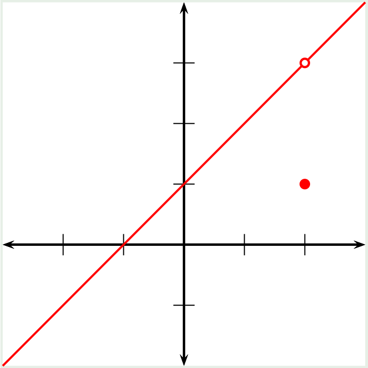

- $f(2)$ is defined ($f(2) = 1$).
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Example

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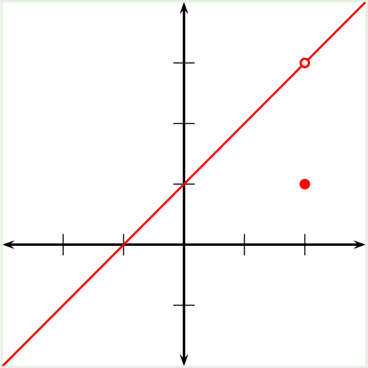

- $f(2)$ is defined ($f(2) = 1$).
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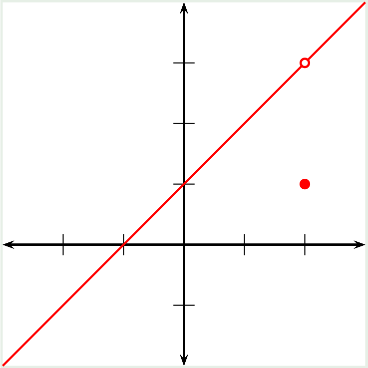

- $f(2)$ is defined ($f(2) = 1$).
- $\lim\limits_{x\to 2} f(x)$ exists ($= 3$).
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- Discontinuous at 2.

Example

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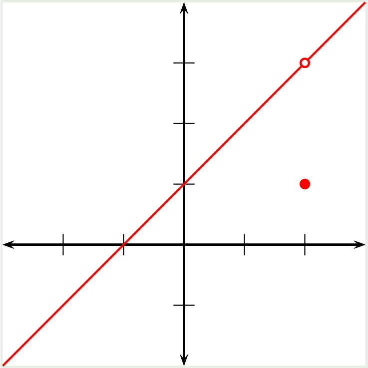
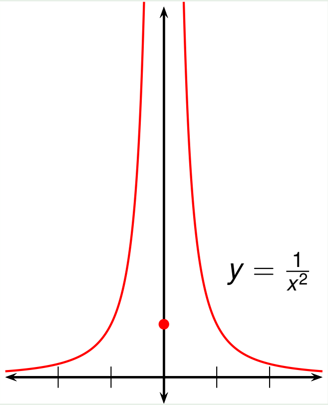


- $f(2)$ is defined ($f(2) = 1$).
- $\lim_{x\to 2} f(x)$ exists ($= 3$).
- $\lim_{x\to 2} f(x) \neq f(2)$.
- Discontinuous at 2.
- This is called a removable discontinuity because we can redefine f at one point to make f continuous.

Example

Where is this function discontinuous?

$$f(x) = \begin{cases} \frac{1}{x^2} & \text{if} \quad x \neq 0 \\ 1 & \text{if} \quad x = 0 \end{cases}$$



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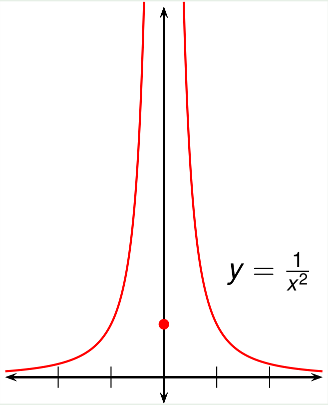

- $f(0)$
- $\lim\limits_{x \to 0} f(x)$

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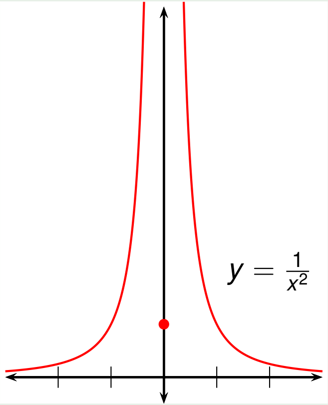

- $f(0)$ **?**
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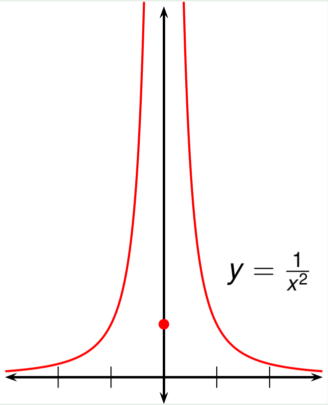

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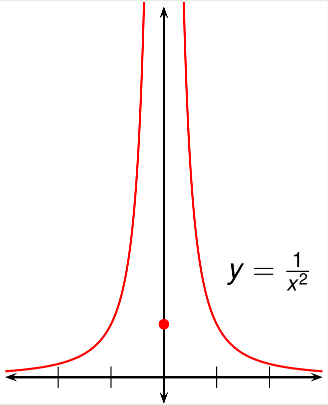

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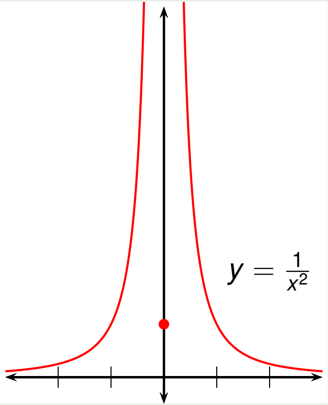

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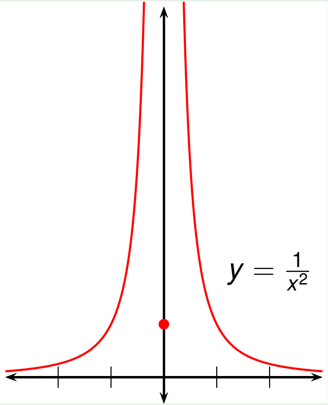

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- Discontinuous at 0.

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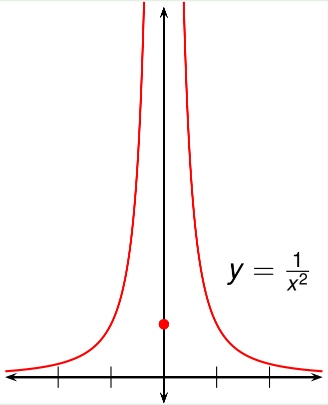
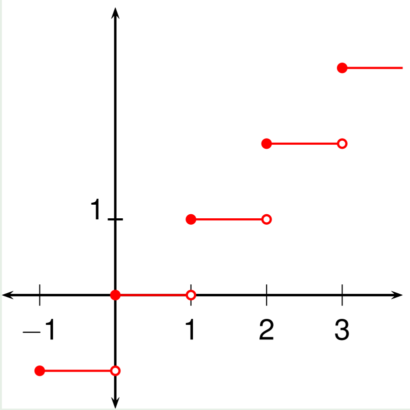


- $f(0)$ is defined ($f(0) = 1$).
- $\lim_{x \to 0} f(x)$ doesn't exist (∞).
- Discontinuous at 0.
- This is called an infinite discontinuity.

Example

Where is this function discontinuous?

$$f(x) = \lfloor x \rfloor$$



Example

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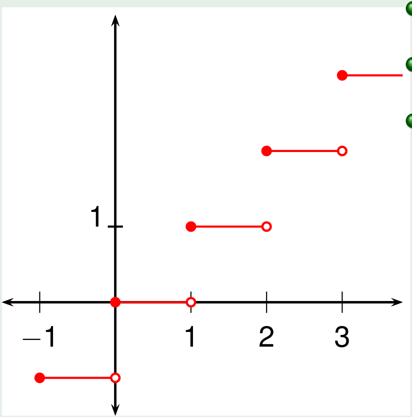

- $f(1)$ **?**
- $\lim\limits_{x\to 1^+} f(x) =$ **?**.
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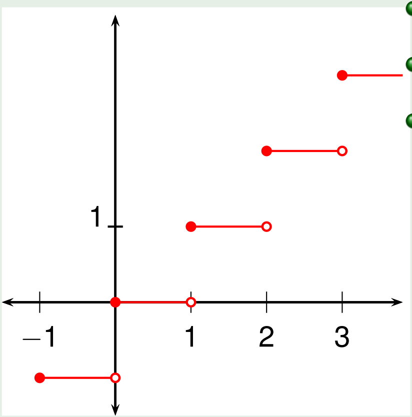

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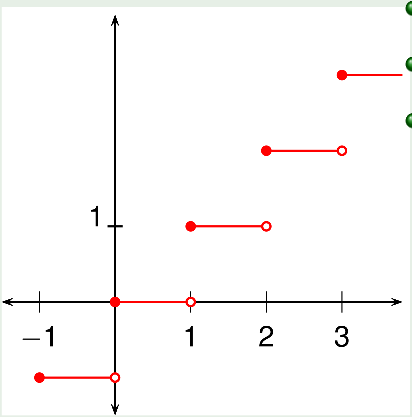

- $f(1)$ exists ($f(1) = 1$).
- $\lim\limits_{x \to 1^+} f(x) =$ **?**.
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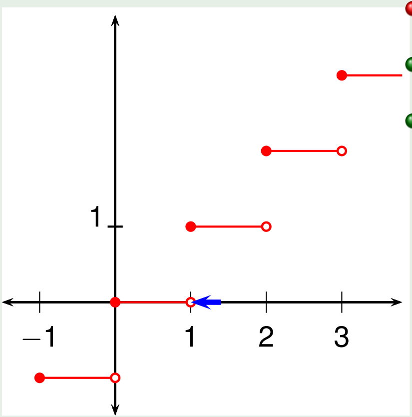

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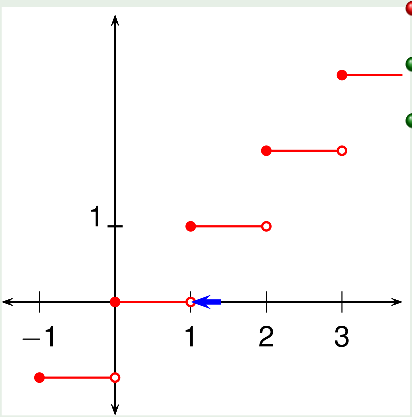

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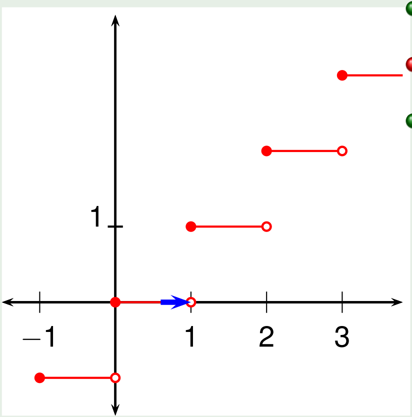

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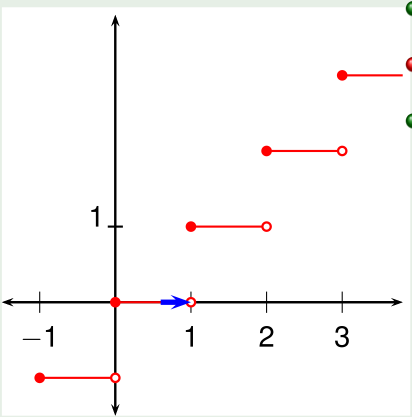

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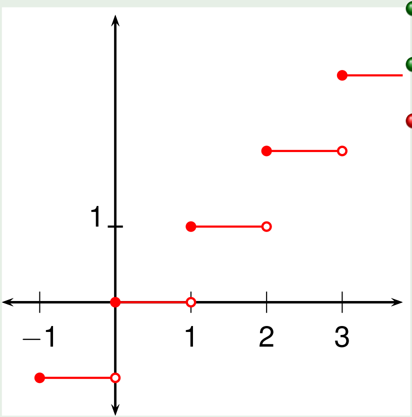

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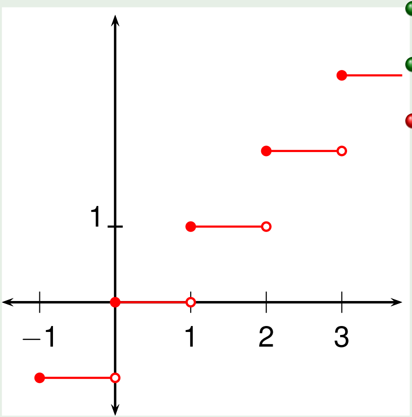

- $f(1)$ exists ($f(1) = 1$).
- $\lim_{x \to 1^+} f(x) = 1.$
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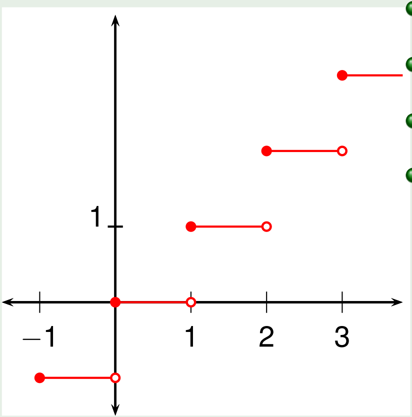

- $f(1)$ exists ($f(1) = 1$).
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- Discontinuous at 1.

Example

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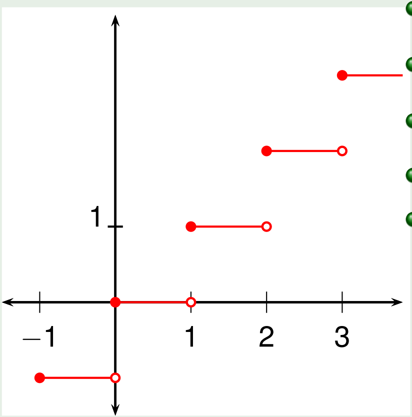

- $f(1)$ exists ($f(1) = 1$).
- $\lim\limits_{x\to 1^+} f(x) = 1$.
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- Discontinuous at 1.
- Discontinuous at every integer n.

Example

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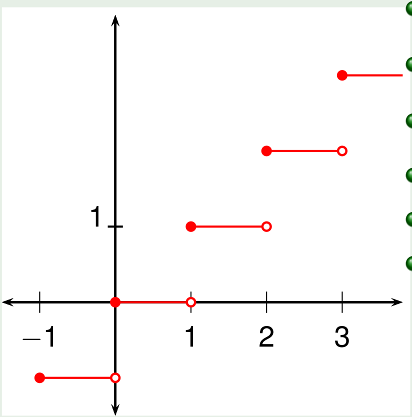

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- Discontinuous at every integer n.
- The left and right limits both exist but are not equal.

Example

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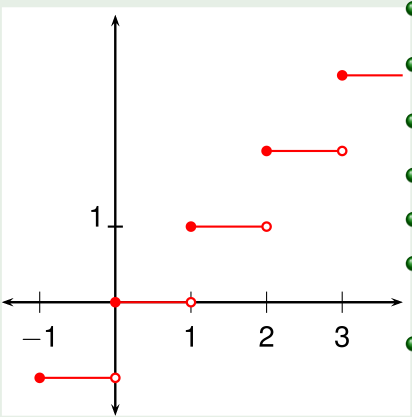

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- $\lim\limits_{x\to 1} f(x)$ doesn't exist.
- Discontinuous at 1.
- Discontinuous at every integer n.
- The left and right limits both exist but are not equal.
- Such discontinuities are called jump discontinuities (the function appears to "jump").

Definition (Continuous from the Right or Left)

A function f is continuous from the right at a number a if

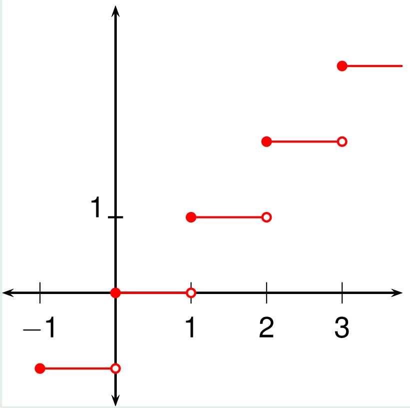
$$\lim_{x \to a^+} f(x) = f(a)$$

and f is continuous from the left at a if

$$\lim_{x \to a^-} f(x) = f(a).$$

Example

Consider $f(x) = \lfloor x \rfloor$, and pick any integer n.



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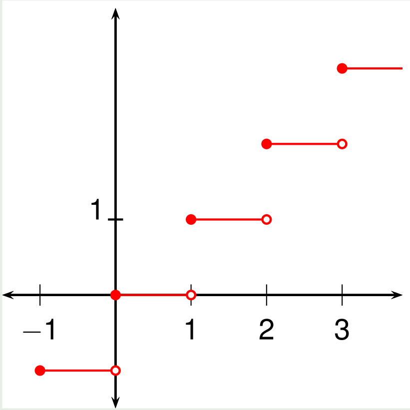

- $f(n) = \quad .$
- $\lim\limits_{x\to n^+} f(x) = \mathbf{?}.$
- $\lim\limits_{x\to n^-} f(x) = \mathbf{?} \qquad .$

Example

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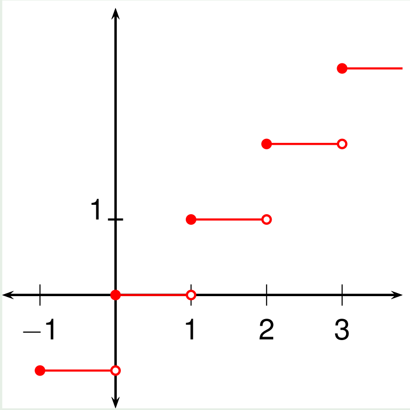

- $f(n) = \mathbf{?}$.
- $\displaystyle\lim_{x \to n^+} f(x) = \mathbf{?}$.
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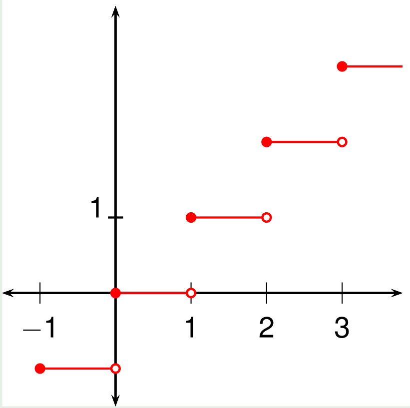

- $f(n) = n$.
- $\lim\limits_{x \to n^+} f(x) = \textbf{?}$.
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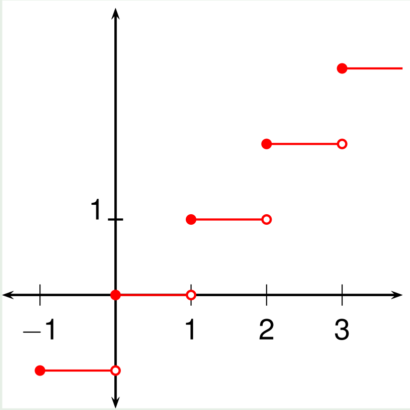

- $f(n) = n$.
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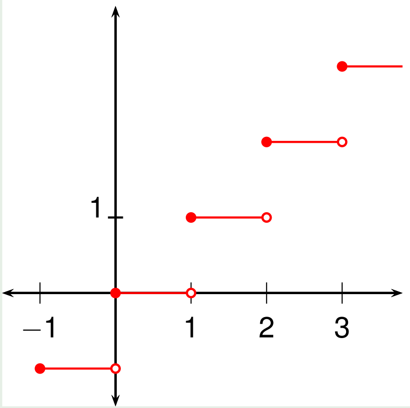

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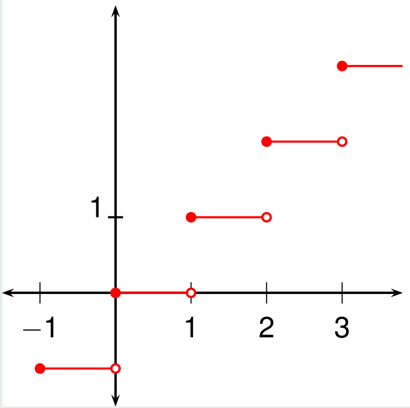
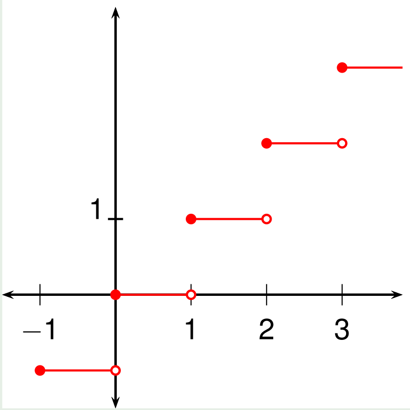


- $f(n) = n$.
- $\lim_{x \to n^+} f(x) = n$.
- Continuous from the right at n.
- $\lim_{x \to n^-} f(x) =$ **?** .

Example

Consider $f(x) = \lfloor x \rfloor$, and pick any integer n.



- $f(n) = n$.
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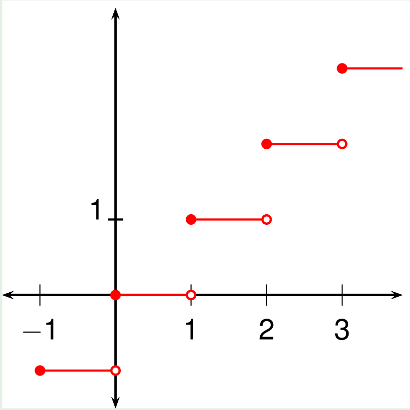
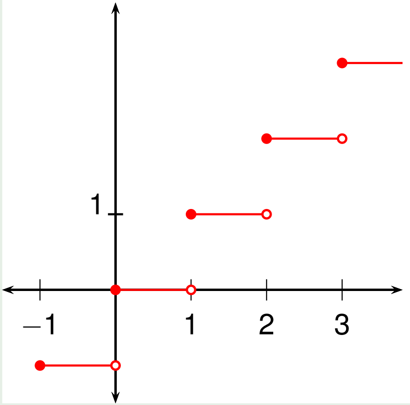
Consider $f(x) = \lfloor x \rfloor$, and pick any integer n.



- $f(n) = n$.
- $\lim\limits_{x \to n^+} f(x) = n$.
- Continuous from the right at n.
- $\lim\limits_{x \to n^-} f(x) = n - 1$.

Example

Consider $f(x) = \lfloor x \rfloor$, and pick any integer n.



- $f(n) = n$.
- $\lim\limits_{x \to n^+} f(x) = n$.
- Continuous from the right at n.
- $\lim\limits_{x \to n^-} f(x) = n - 1$.
- Discontinuous from the left at n.

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- Think of a function that is continuous on an interval as a function that has no breaks in its graph, and so can be drawn "without lifting your pen".

Theorem (Algebra of Continuous Functions)

If f and g are continuous at a and c is a constant, then the following functions are also continuous at a:

1. $f+g$
2. $f-g$
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5. $\frac{f}{g}$ if $g(a)\neq 0$.

Proof.

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Theorem (Classes of Continuous Functions)

The following types of functions are continuous at every number in their domains:

polynomials	*rational functions*
root functions	*trigonometric functions*

Theorem (Compositions of Continuous Functions)

If g is continuous at a and f is continuous at $g(a)$, then the composition function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a.

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- Its domain is given by $x \neq 3$ and $x \neq -3$.
- Therefore F is continuous on $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$.

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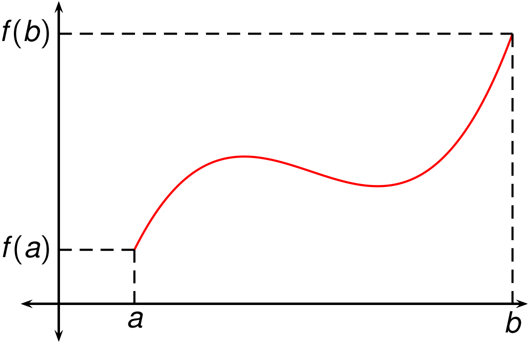
Suppose f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.



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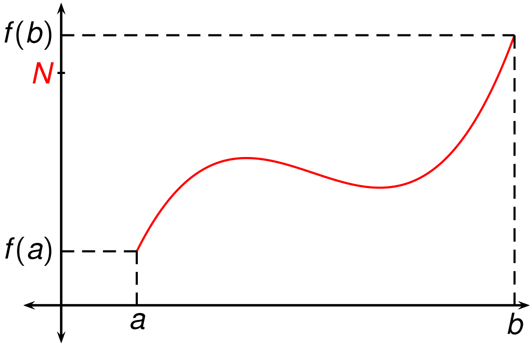

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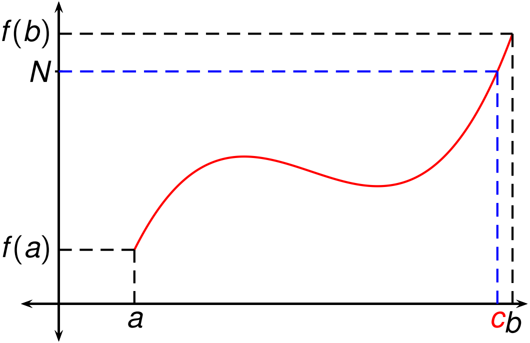

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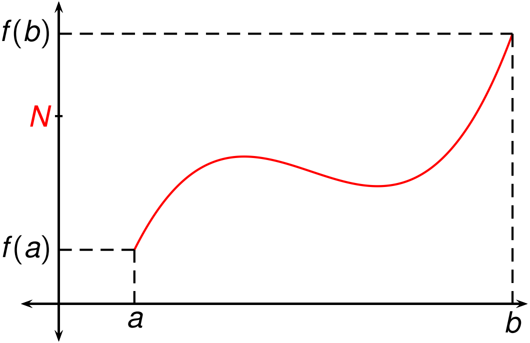

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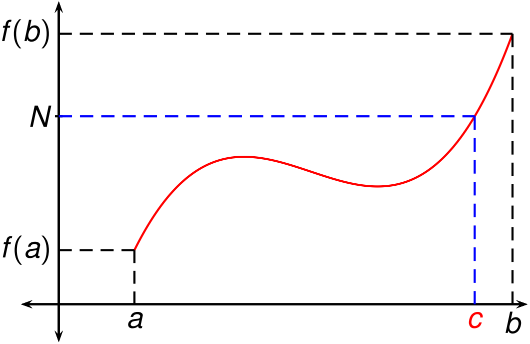

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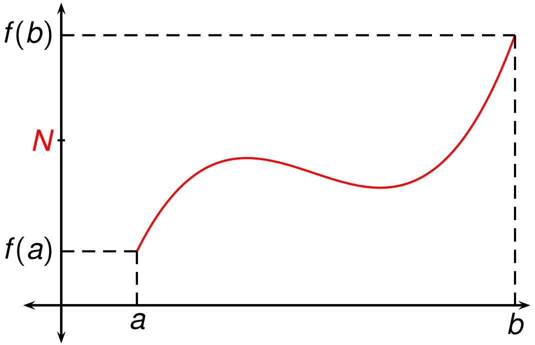

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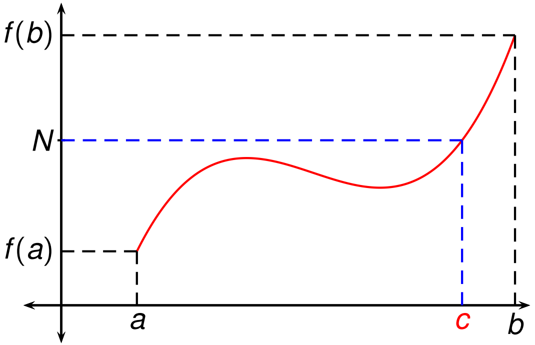

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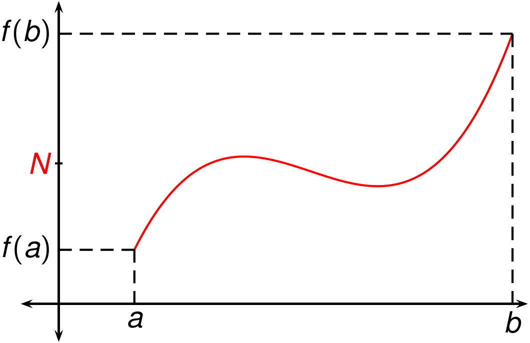

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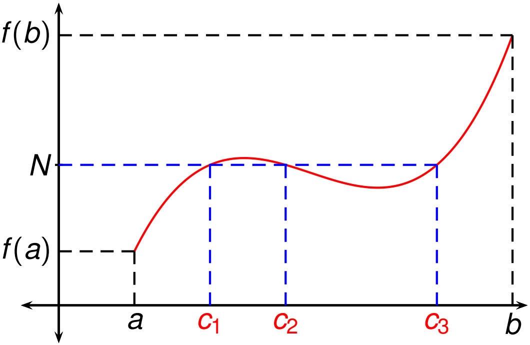

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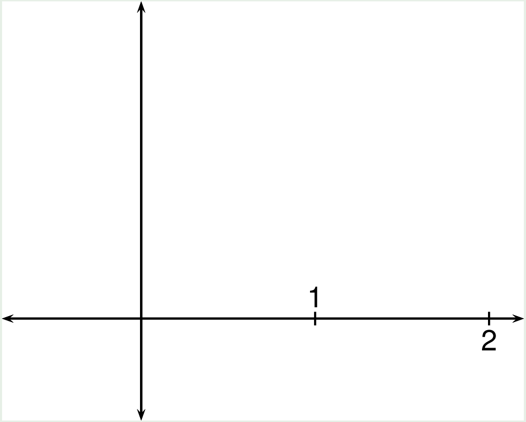
Show that there is a root of the equation

$$4x^3 - 6x^2 + 3x - 2 = 0$$

between 1 and 2.



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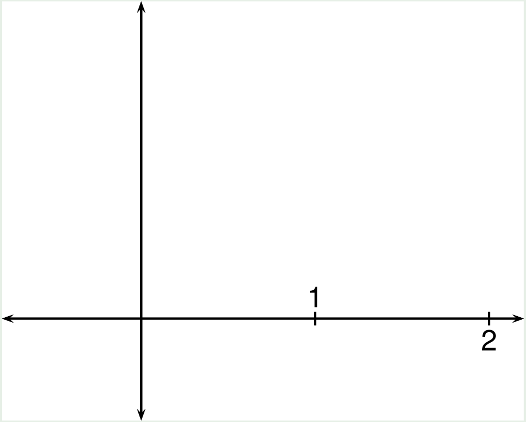

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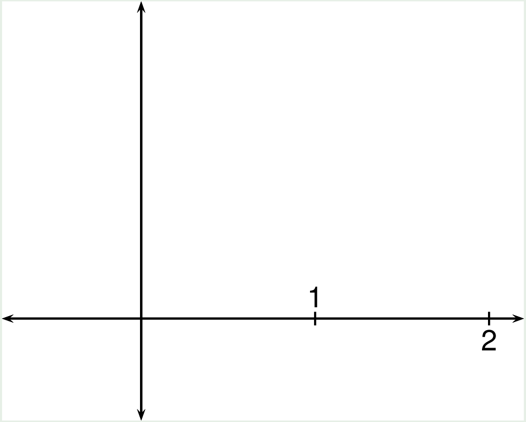

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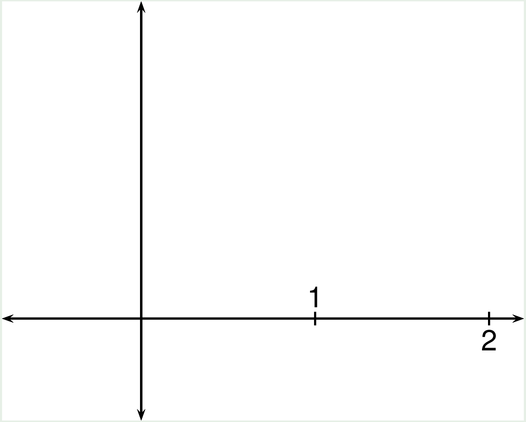

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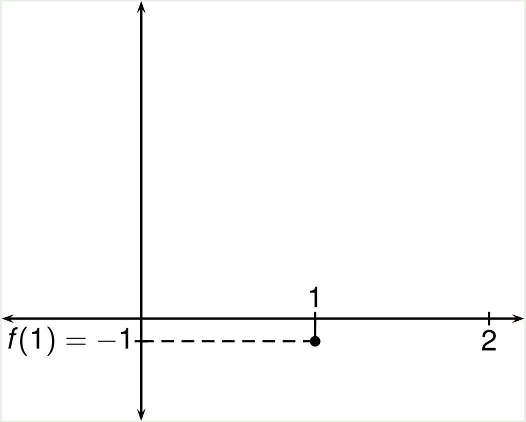
- $f(1) = -1$.



Example

Show that there is a root of the equation

$$4x^3 - 6x^2 + 3x - 2 = 0$$

between 1 and 2.

- Let $f(x) = 4x^3 - 6x^2 + 3x - 2$.
- f is continuous.
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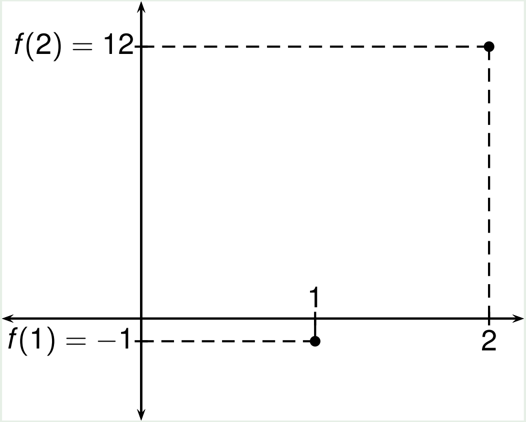
- $f(1) = -1$.
- $f(2) = 12$.



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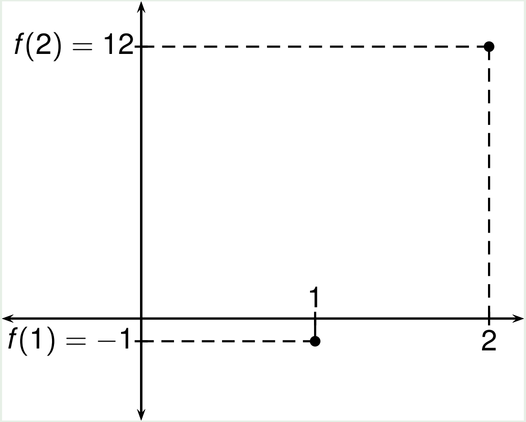

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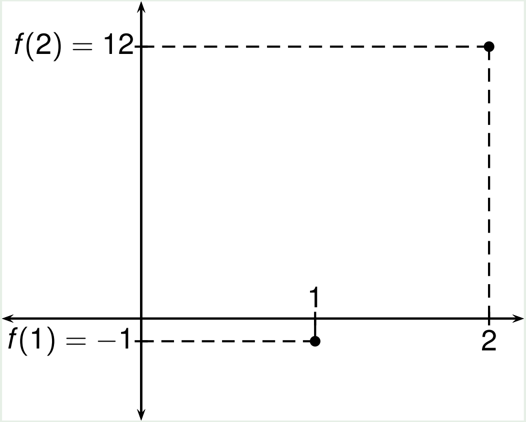

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