Calculus III
Lecture 5

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https://github.com/tmilev/freecalc

2020

Outline

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Polar Coordinates

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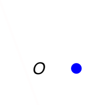
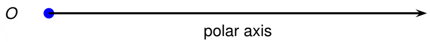
- Choose a point in the plane called O (the origin).



Polar Coordinates

- The polar coordinate system is an alternative to the Cartesian coordinate system.
- Choose a point in the plane called O (the origin).
- Draw a ray starting at O. The ray is called the polar axis. This ray is usually drawn horizontally to the right.



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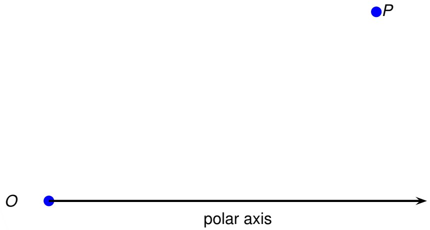

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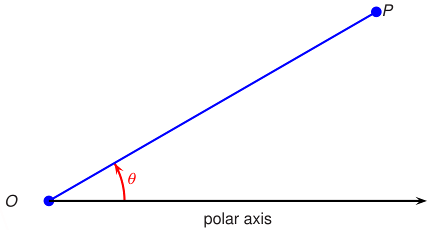

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- Let θ denote the angle between the polar axis and the line OP.

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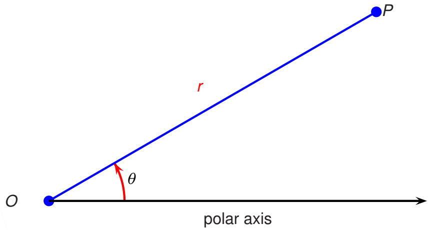

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- Let r denote the length of the segment OP.

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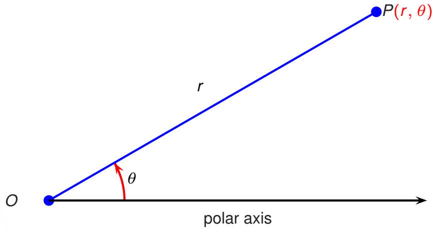

- Let P be a point in the plane.
- Let θ denote the angle between the polar axis and the line OP.
- Let r denote the length of the segment OP.
- Then P is represented by the ordered pair (r, θ).

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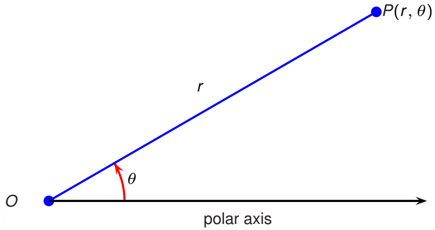

- The letters (x, y) imply Cartesian coordinates and the letters (r, θ)- polar.

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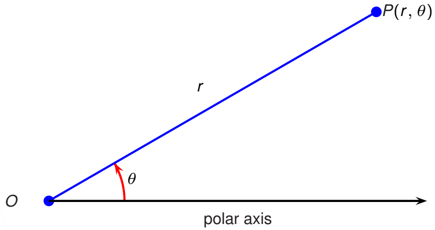

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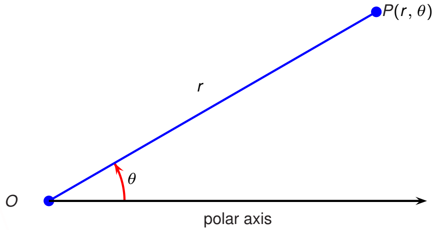

- The letters (x, y) imply Cartesian coordinates and the letters (r, θ)- polar. When we use other letters, it should be clear from context whether we mean Cartesian or polar coordinates. If not, one must request clarification.

1. What if θ is negative?
2. What if r is negative?
3. What if r is 0?

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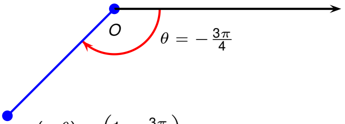

$(r, \theta) = \left(1, -\frac{3\pi}{4}\right)$
$(x, y) = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

1. Positive angles θ are measured in the counterclockwise direction from O. Negative angles are measured in the clockwise direction.

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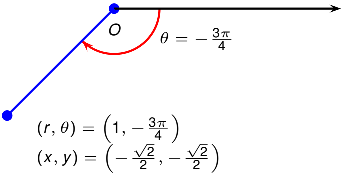

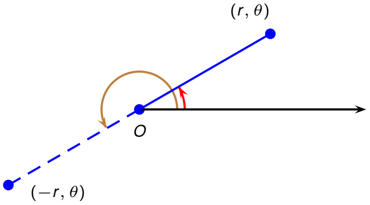

1. Positive angles θ are measured in the counterclockwise direction from O. Negative angles are measured in the clockwise direction.
2. Points with polar coordinates $(-r, \theta)$ and (r, θ) lie on the same line through O and at the same distance from O, but on opposite sides.

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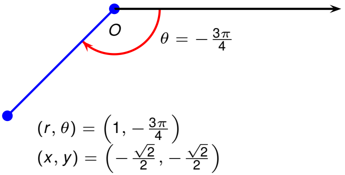
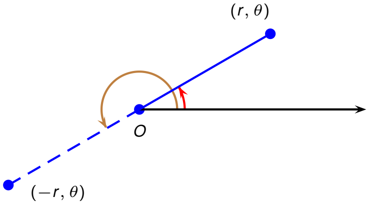
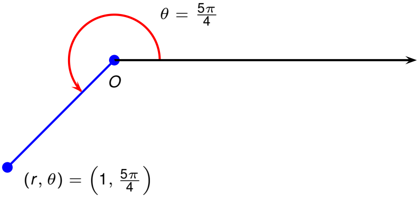
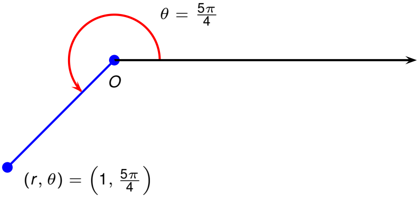


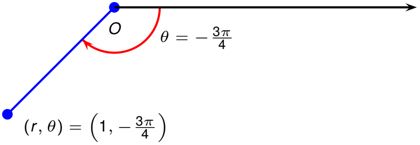


1. Positive angles θ are measured in the counterclockwise direction from O. Negative angles are measured in the clockwise direction.
2. Points with polar coordinates $(-r, \theta)$ and (r, θ) lie on the same line through O and at the same distance from O, but on opposite sides.
3. If $r = 0$, then $(0, \theta)$ represents O for all values of θ.

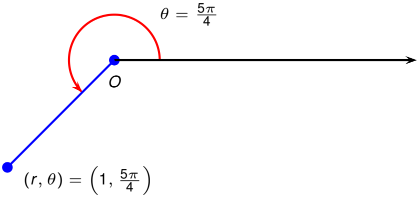


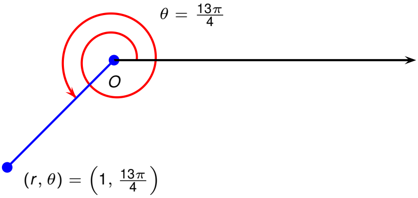
- There are many ways to represent the same point.

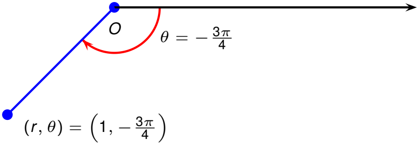




- There are many ways to represent the same point.
- We could use a negative θ.







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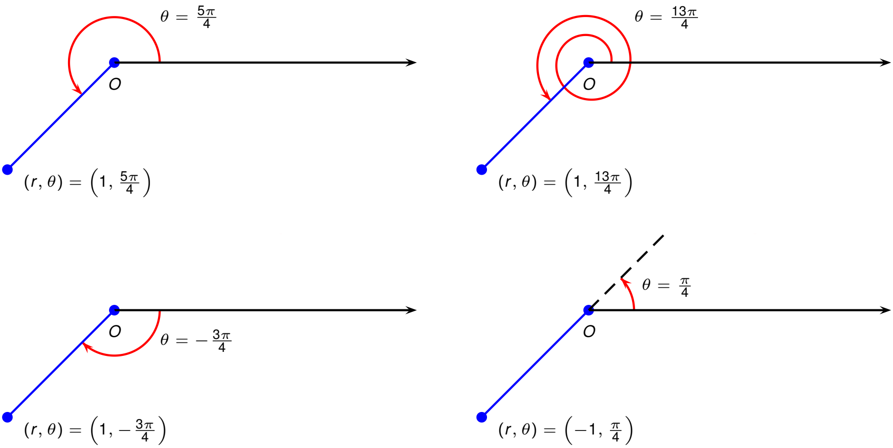
- We could use a negative θ.
- We could go around more than once.



- There are many ways to represent the same point.
- We could use a negative θ.
- We could go around more than once.
- We could use a negative r.

- Let P_1 be point with polar coordinates (r_1, θ_1).
- Let P_2 be point with polar coordinates (r_2, θ_2).

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Observation

P_1 coincides with P_2 if one of the three mutually exclusive possibilities holds:

- $r_1 = r_2 \neq 0$ *and* $\theta_2 = \theta_1 + 2k\pi, k \in \mathbb{Z}$,
- $r_1 = -r_2 \neq 0$ *and* $\theta_2 = \theta_1 + (2k+1)\pi, k \in \mathbb{Z}$,
- $r_1 = r_2 = 0$ *and* θ *is arbitrary.*

- Let P_1 be point with polar coordinates (r_1, θ_1).
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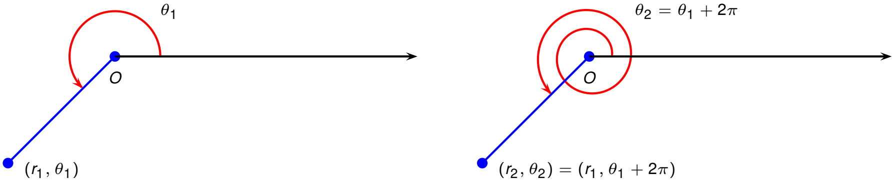

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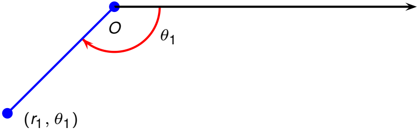
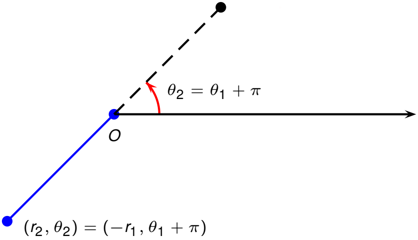
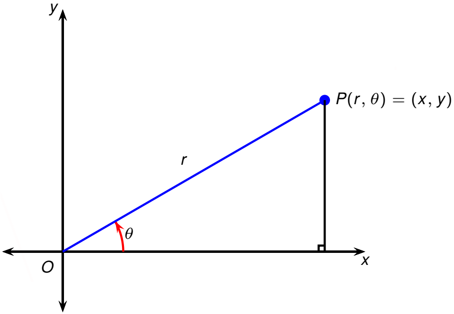




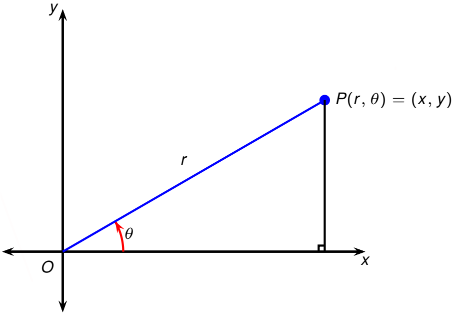
- How do we go from polar coordinates to Cartesian coordinates?



$$x =$$
$$y =$$

$$r =$$
$$\theta =$$

- How do we go from polar coordinates to Cartesian coordinates?
- Suppose a point has polar coordinates (r, θ) and Cartesian coordinates (x, y).



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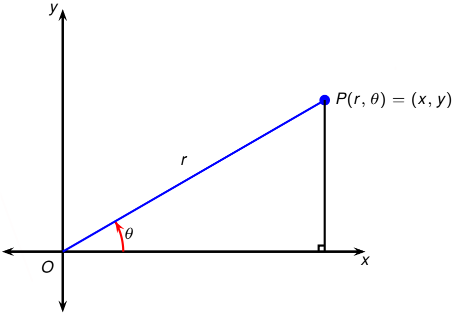

$$x =$$
$$y =$$

$$\cos\theta =$$
$$\sin\theta =$$

$$r =$$
$$\theta =$$

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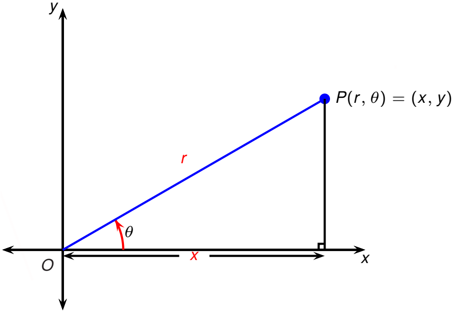

$$
\begin{aligned}
x &= \\
y &= \\
\hline
\cos\theta &= \frac{x}{r} \\
\sin\theta &= \\
\hline
r &= \\
\theta &=
\end{aligned}
$$

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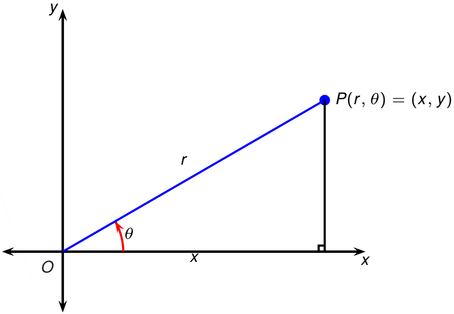

$$
\begin{aligned}
x &= \\
y &= \\
\hline
\cos\theta &= \frac{x}{r} \\
\color{red}{\sin\theta} &\color{red}{=} \\
\hline
r &= \\
\theta &=
\end{aligned}
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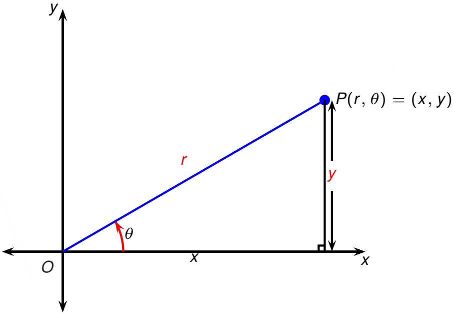

$$x =$$
$$y =$$

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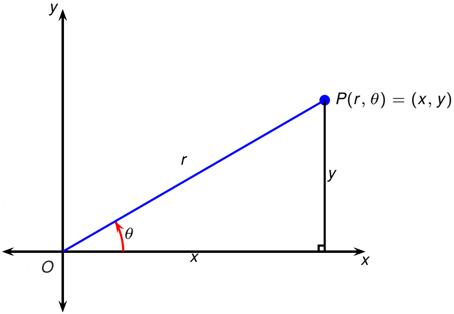

$$
\begin{aligned}
x &= r\cos\theta \\
y &= r\sin\theta \\
\hline
\cos\theta &= \frac{x}{r} \\
\sin\theta &= \frac{y}{r} \\
\hline
r &= \\
\theta &=
\end{aligned}
$$

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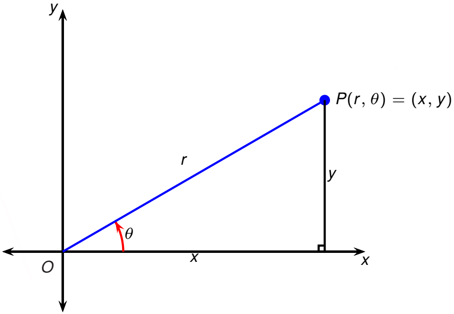
- Suppose a point has polar coordinates (r, θ) and Cartesian coordinates (x, y).
- How do we go from Cartesian coordinates to polar coordinates?



$$
\begin{aligned}
x &= r\cos\theta \\
y &= r\sin\theta \\
\hline
\cos\theta &= \frac{x}{r} \\
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r &= \\
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\end{aligned}
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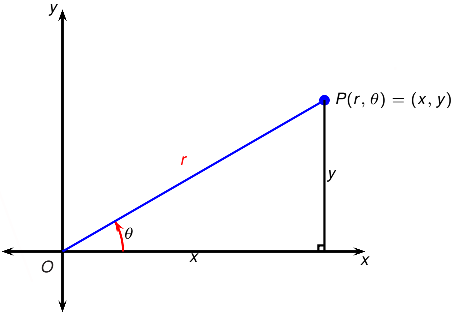

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ \hline \cos\theta &= \frac{x}{r} \\ \sin\theta &= \frac{y}{r} \\ r^2 &= \\ \hline r &= \\ \theta &= \end{aligned}$$

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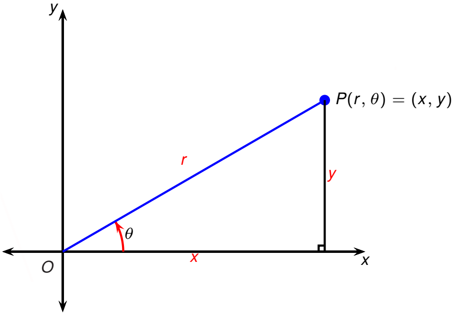

$$
\begin{aligned}
x &= r\cos\theta \\
y &= r\sin\theta \\
\hline
\cos\theta &= \frac{x}{r} \\
\sin\theta &= \frac{y}{r} \\
r^2 &= x^2 + y^2 \\
\hline
r &= \\
\theta &=
\end{aligned}
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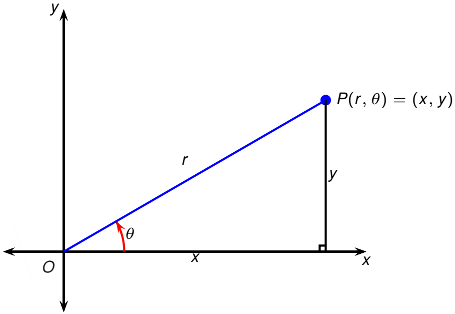

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x &= r\cos\theta \\
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r^2 &= x^2 + y^2 \\
\hline
r &= \sqrt{x^2 + y^2} \\
\theta &=
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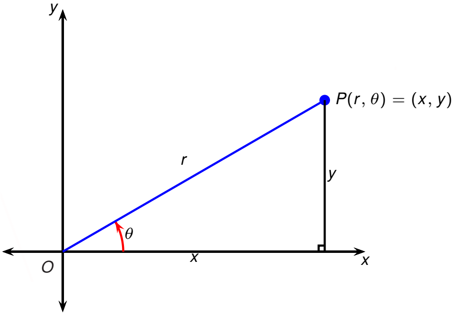

$$
\begin{array}{rcl}
x & = & r\cos\theta \\
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\hline
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\theta & = &
\end{array}
$$

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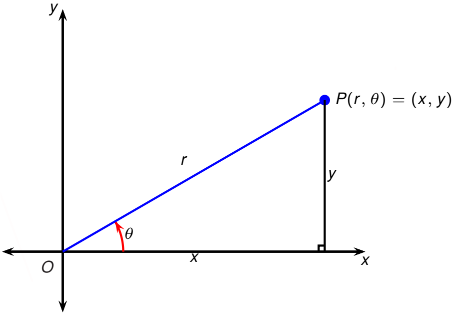

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\hline
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r^2 &= x^2 + y^2 \\
\hline
r &= \sqrt{x^2 + y^2} \\
\theta &= \arcsin\left(\frac{y}{r}\right) && \text{if } x > 0 \\
&= \arccos\left(\frac{x}{r}\right) && \text{if } y > 0 \\
&= \arctan\left(\frac{y}{x}\right) && \text{if } x > 0
\end{aligned}
$$

Example

Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

Example

Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$x = r\cos\theta =$$

$$y = r\sin\theta =$$

Example

Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$x = r\cos\theta = \quad \cos$$

$$y = r\sin\theta =$$

Example

Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$x = r\cos\theta = 2\cos$$

$$y = r\sin\theta =$$

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Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$x = r\cos\theta = 2\cos$$

$$y = r\sin\theta =$$

Example

Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$x = r\cos\theta = 2\cos\frac{\pi}{3}$$

$$y = r\sin\theta =$$

Example

Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2\left(\quad \right)$$

$$y = r\sin\theta =$$

Example

Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2\left(\frac{1}{2}\right)$$

$$y = r\sin\theta =$$

Example

Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2\left(\frac{1}{2}\right) = 1$$

$$y = r\sin\theta =$$

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Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

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Example

Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2\left(\frac{1}{2}\right) = 1$$

$$y = r\sin\theta = 2\sin\frac{\pi}{3} = 2\left(\quad\right)$$

Example

Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2\left(\frac{1}{2}\right) = 1$$

$$y = r\sin\theta = 2\sin\frac{\pi}{3} = 2\left(\frac{\sqrt{3}}{2}\right)$$

Example

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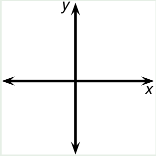
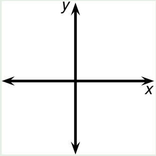
Therefore the point with polar coordinates $(2, \frac{\pi}{3})$ has Cartesian coordinates $(1, \sqrt{3})$.

Example



Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

Example

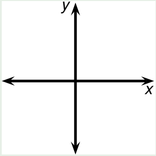


Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

$$r = \pm\sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

Example



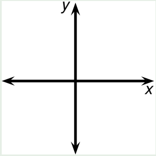
Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.

$$r = \pm\sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

Example



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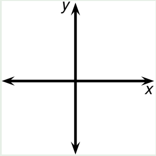


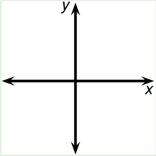
Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{2} \end{aligned}$$

$$\tan\theta = \frac{y}{x}$$

Example



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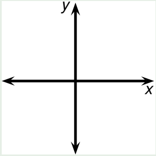
Example



Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.
- $\tan\theta = -1$ for $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, and many other angles.

$$
\begin{aligned}
r &= \sqrt{x^2+y^2} \\
&= \sqrt{1^2+(-1)^2} \\
&= \sqrt{2}
\end{aligned}
$$

$$
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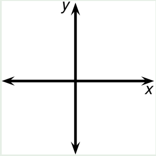
Example



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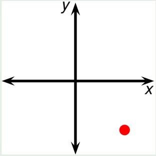
- Suppose r is positive.
- $\tan\theta = -1$ for $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, and many other angles.
- $(1, -1)$ is in the quadrant.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{y}{x} \\ &= -1 \end{aligned}$$

Example



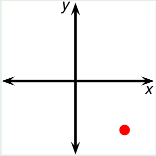
Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.
- $\tan\theta = -1$ for $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, and many other angles.
- $(1, -1)$ is in the fourth quadrant.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{y}{x} \\ &= -1 \end{aligned}$$

Example



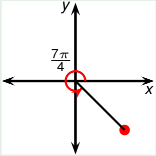
Represent the point with Cartesian coordinates $(1,-1)$ in terms of polar coordinates.

- Suppose r is positive.
- $\tan\theta = -1$ for $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, and many other angles.
- $(1,-1)$ is in the fourth quadrant.
- Of the two values above, only $\theta =$ gives a point in the fourth quadrant.

$$\begin{aligned} r &= \sqrt{x^2+y^2} \\ &= \sqrt{1^2+(-1)^2} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{y}{x} \\ &= -1 \end{aligned}$$

Example

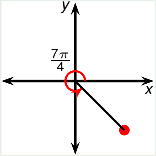


Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.
- $\tan\theta = -1$ for $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, and many other angles.
- $(1, -1)$ is in the fourth quadrant.
- Of the two values above, only $\theta = \frac{7\pi}{4}$ gives a point in the fourth quadrant.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{2} \\ \tan\theta &= \frac{y}{x} \\ &= -1 \end{aligned}$$

Example



Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.
- $\tan\theta = -1$ for $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, and many other angles.
- $(1, -1)$ is in the fourth quadrant.
- Of the two values above, only $\theta = \frac{7\pi}{4}$ gives a point in the fourth quadrant.
- $\Rightarrow$ one representation of $(1, -1)$ in polar coordinates is $\left(\sqrt{2}, \frac{7\pi}{4}\right)$.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{2} \end{aligned}$$

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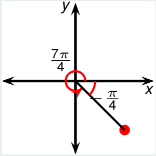
Example



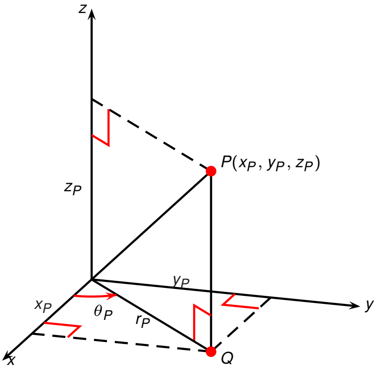
Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.
- $\tan\theta = -1$ for $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, and many other angles.
- $(1, -1)$ is in the fourth quadrant.
- Of the two values above, only $\theta = \frac{7\pi}{4}$ gives a point in the fourth quadrant.
- $\Rightarrow$ one representation of $(1, -1)$ in polar coordinates is $\left(\sqrt{2}, \frac{7\pi}{4}\right)$.
- $\left(\sqrt{2}, -\frac{\pi}{4}\right)$ is another.

$$\begin{aligned} r &= \sqrt{x^2+y^2} \\ &= \sqrt{1^2+(-1)^2} \\ &= \sqrt{2} \end{aligned}$$

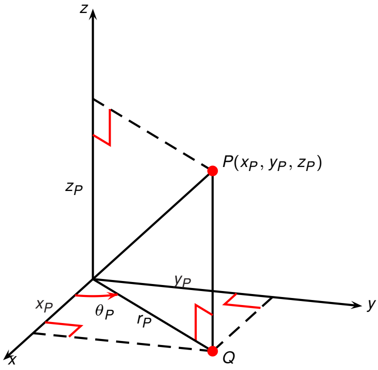
$$\begin{aligned} \tan\theta &= \frac{y}{x} \\ &= -1 \end{aligned}$$

Cylindrical coordinates



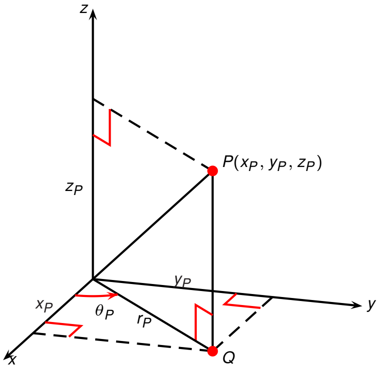
- In Cartesian coordinates, a point P is given by triple (x_P, y_P, z_P).

Cylindrical coordinates



- In Cartesian coordinates, a point P is given by triple (x_P, y_P, z_P).
- We introduce alternative cylindrical coordinates (r_P, θ_P, z_P).

Cylindrical coordinates



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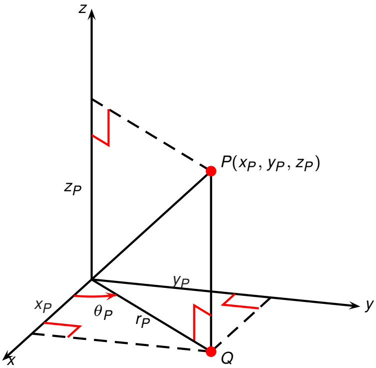
Cylindrical coordinates



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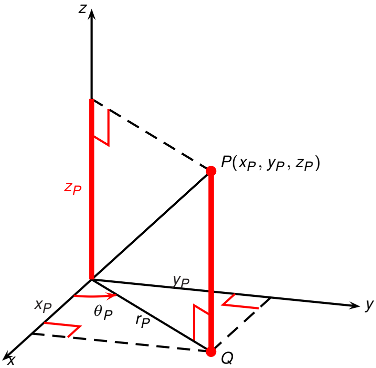
- More precisely, to P we assign triple (r_P, θ_P, z_P), where:

Cylindrical coordinates



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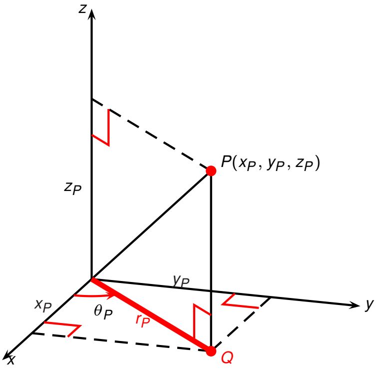
  - z_P equals the z-coordinate of P,

Cylindrical coordinates



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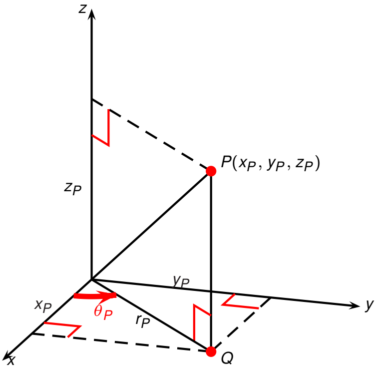
  - z_P equals the z-coordinate of P,
  - r_P is the distance $|OQ|$, where Q is the projection of P in the xy-plane and O-origin,

Cylindrical coordinates



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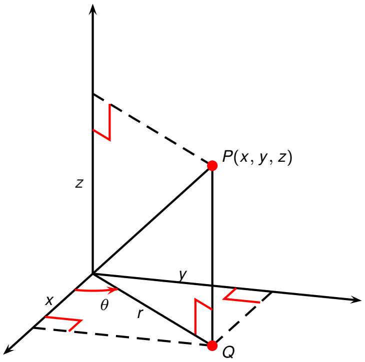
- More precisely, to P we assign triple (r_P, θ_P, z_P), where:
    - z_P equals the z-coordinate of P,
    - r_P is the distance $|OQ|$, where Q is the projection of P in the xy-plane and O-origin,
    - θ_P is an angle between the x-axis and **OQ**.

Cylindrical to and from cartesian coordinates

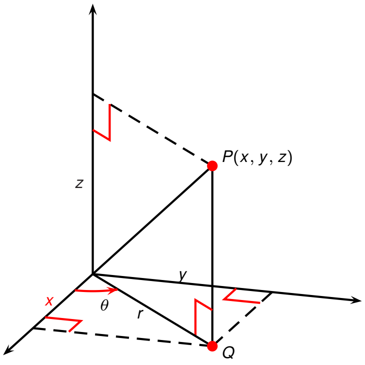


To transform cylindrical to rectangular coordinates:

To transform rectangular to cylindrical:

$$
\begin{aligned}
r &= \\
\cos\theta &= \\
\sin\theta &= \\
\theta &= \\
z_{cylindirical} &=
\end{aligned}
$$

Cylindrical to and from cartesian coordinates



To transform cylindrical to rectangular coordinates:

$$\begin{aligned} {\color{red}x} &{\color{red}=} \\ y &= \\ z_{rectangular} &= \end{aligned}$$

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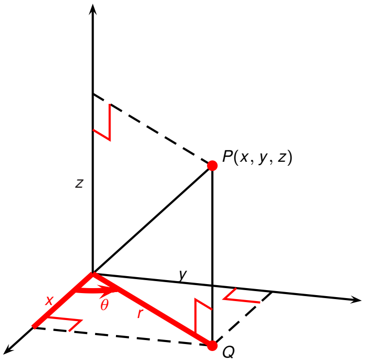
Cylindrical to and from cartesian coordinates



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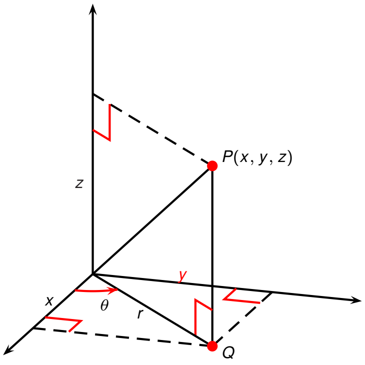

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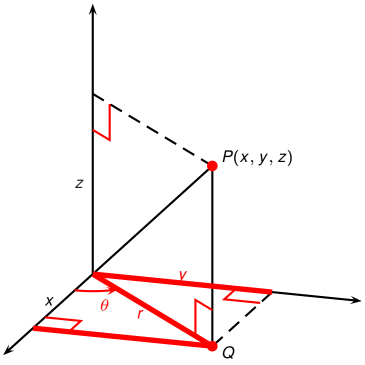

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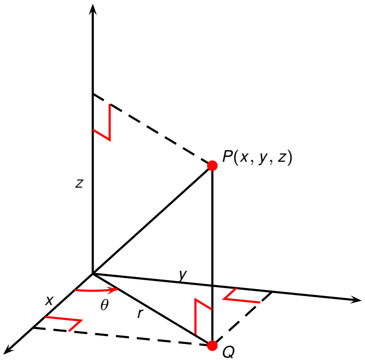
Cylindrical to and from cartesian coordinates



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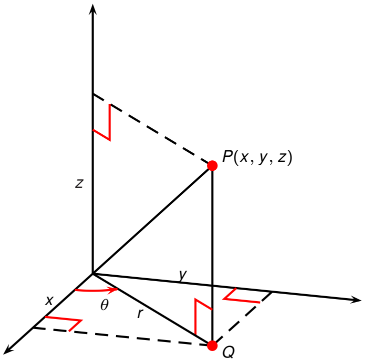
Cylindrical to and from cartesian coordinates



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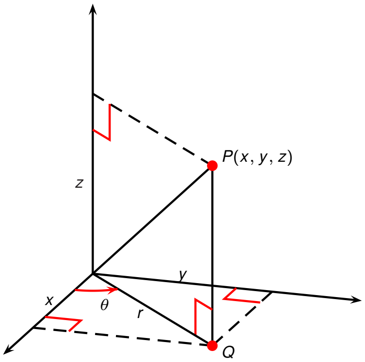
Cylindrical to and from cartesian coordinates



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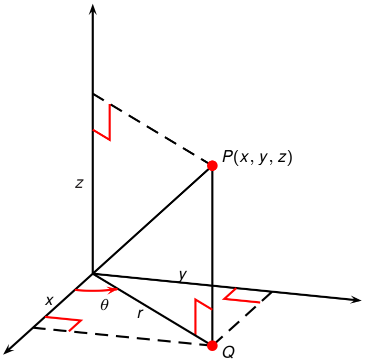

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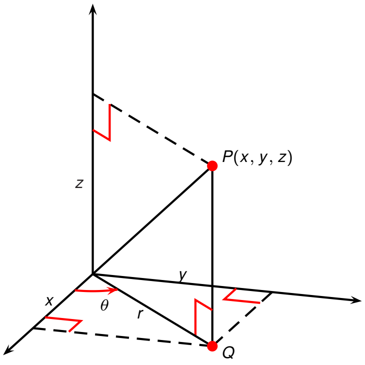
Cylindrical to and from cartesian coordinates



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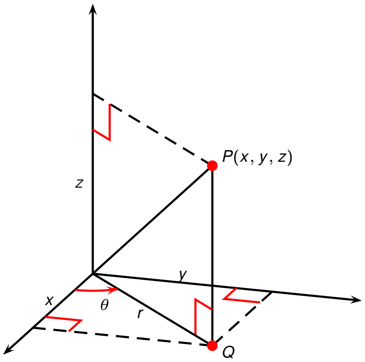
Cylindrical to and from cartesian coordinates



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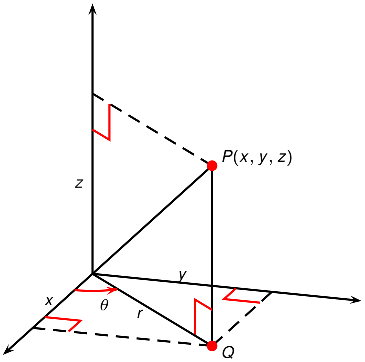
$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z_{rectangular} &= z_{cylindrical} \end{aligned}$$

To transform rectangular to cylindrical:

$$\begin{aligned} r &= \sqrt{x^2+y^2} \\ \cos\theta &= \frac{x}{r} \\ \sin\theta &= \frac{y}{r} \\ \theta &= \\ z_{cylindirical} &= \end{aligned}$$

Cylindrical to and from cartesian coordinates



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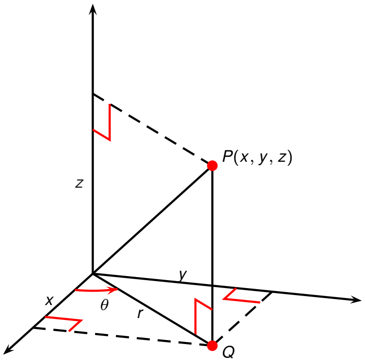
$$z_{cylindirical} =$$

Cylindrical to and from cartesian coordinates



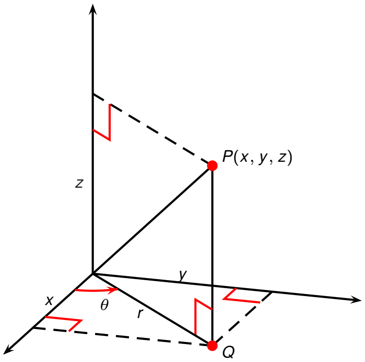
To transform cylindrical to rectangular coordinates:

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z_{rectangular} &= z_{cylindrical} \end{aligned}$$

To transform rectangular to cylindrical:

$$\begin{aligned} r &= \sqrt{x^2+y^2} \\ \cos\theta &= \frac{x}{r} \\ \sin\theta &= \frac{y}{r} \\ \theta &= \arcsin\left(\frac{y}{r}\right) && \text{if } x>0 \\ &= \arccos\left(\frac{x}{r}\right) && \text{if } y>0 \\ &= \arctan\left(\frac{y}{x}\right) && \text{if } x>0 \\ z_{cylindirical} &= \end{aligned}$$

Cylindrical to and from cartesian coordinates



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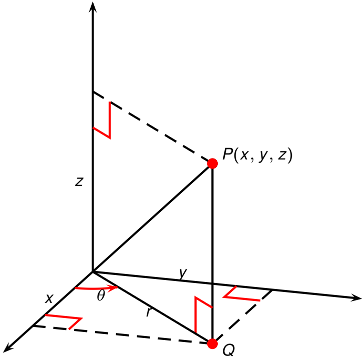
$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ z_{rectangular} &= z_{cylindrical} \end{aligned}$$

To transform rectangular to cylindrical:

$$\begin{aligned} r &= \sqrt{x^2+y^2} \\ \cos\theta &= \frac{x}{r} \\ \sin\theta &= \frac{y}{r} \\ \theta &= \arcsin\left(\frac{y}{r}\right) && \text{if } x>0 \\ &= \arccos\left(\frac{x}{r}\right) && \text{if } y>0 \\ &= \arctan\left(\frac{y}{x}\right) && \text{if } x>0 \\ z_{cylindirical} &= z_{rectangular} \end{aligned}$$

Constant Coordinate Sets



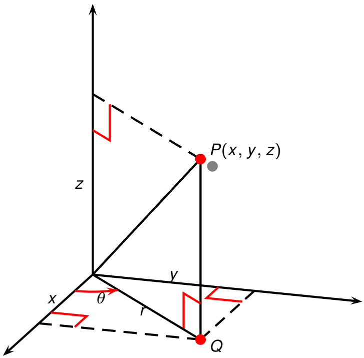
What curve is traced when:

- keep θ, z constant, let r vary:
- keep r, z constant, let θ vary:
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



What curve is traced when:

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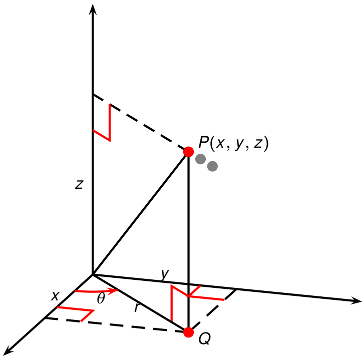
- keep r, z constant, let θ vary:
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



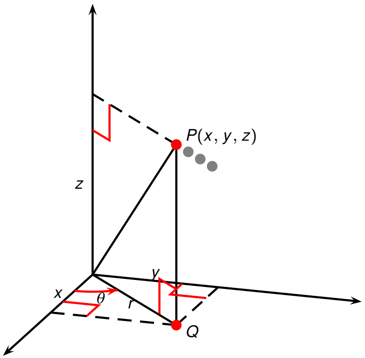
What curve is traced when:

- keep θ, z constant, let r vary:
- keep r, z constant, let θ vary:
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



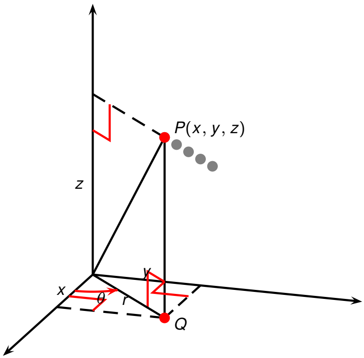
What curve is traced when:

- keep θ, z constant, let r vary:
- keep r, z constant, let θ vary:
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



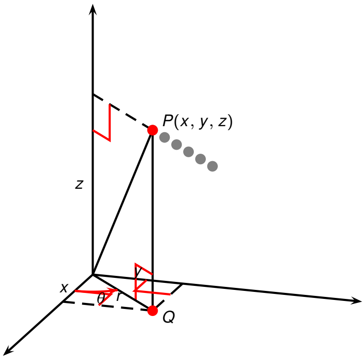
What curve is traced when:

- keep θ, z constant, let r vary:
- keep r, z constant, let θ vary:
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



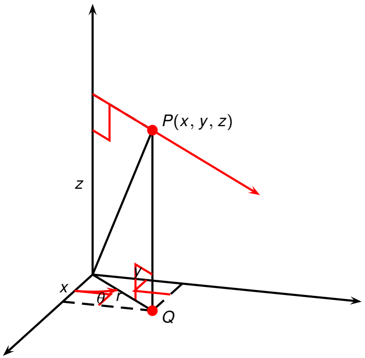
What curve is traced when:

- keep θ, z constant, let r vary:
- keep r, z constant, let θ vary:
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



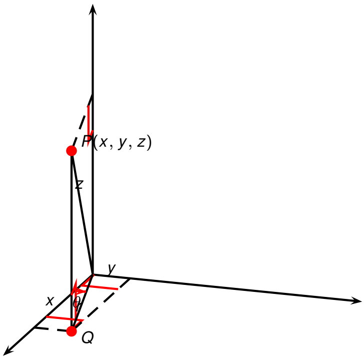
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary:
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



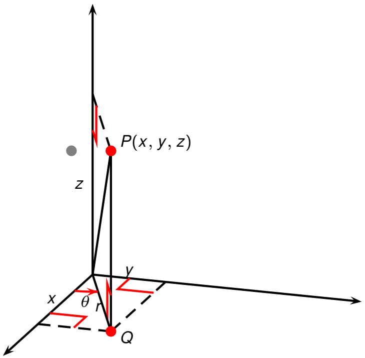
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary:
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



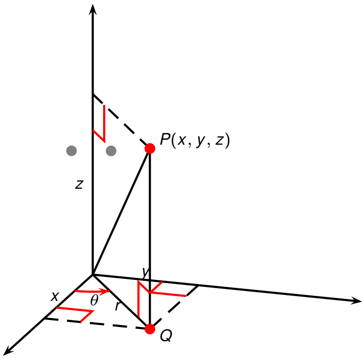
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary:
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



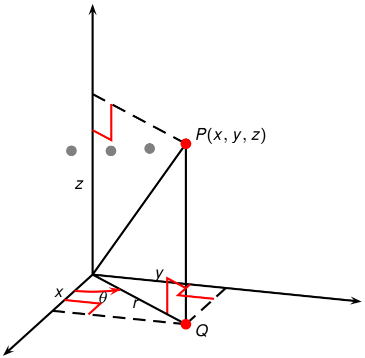
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary:
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



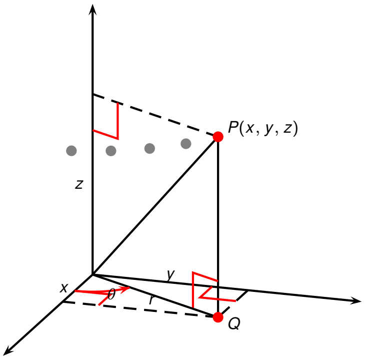
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary:
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



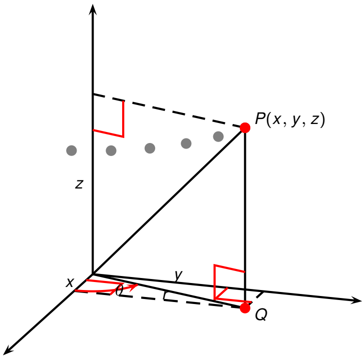
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary:
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



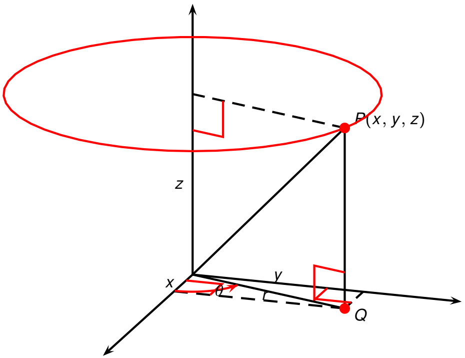
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary:
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



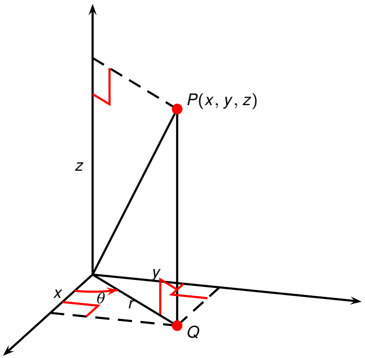
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary: horizontal circle;
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



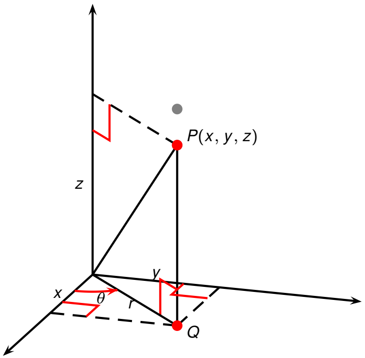
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary: horizontal circle;
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



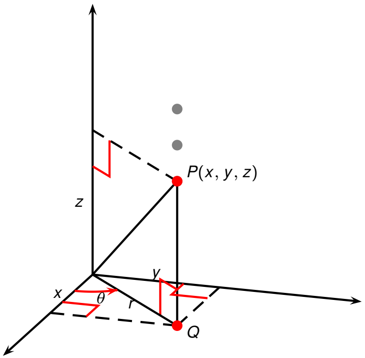
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary: horizontal circle;
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



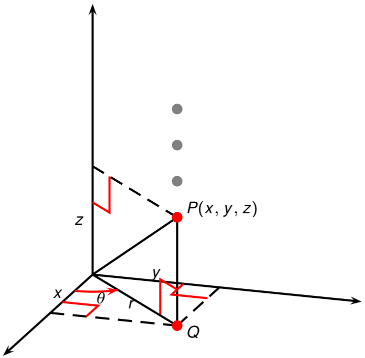
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary: horizontal circle;
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



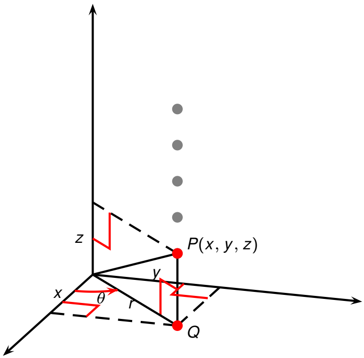
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary: horizontal circle;
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



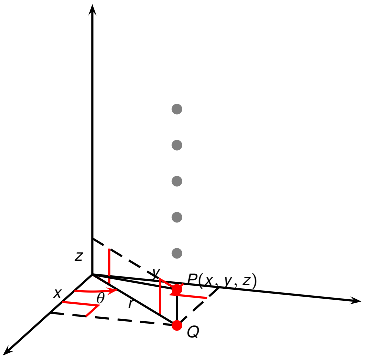
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary: horizontal circle;
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



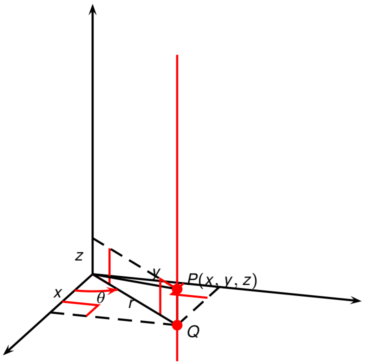
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary: horizontal circle;
- keep r, θ constant, let z vary:

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



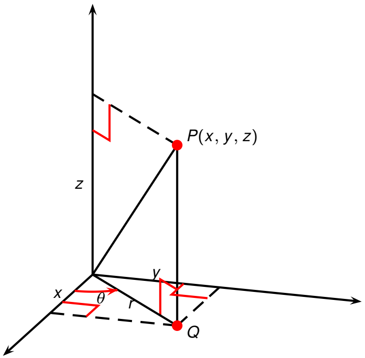
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary: horizontal circle;
- keep r, θ constant, let z vary: vertical line.

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



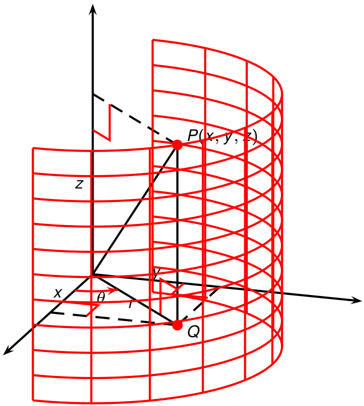
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary: horizontal circle;
- keep r, θ constant, let z vary: vertical line.

What surface is traced when:

- keep r constant, let θ, z vary:
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



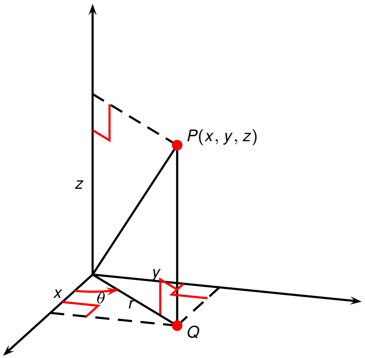
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary: horizontal circle;
- keep r, θ constant, let z vary: vertical line.

What surface is traced when:

- keep r constant, let θ, z vary: vertical cylinder;
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



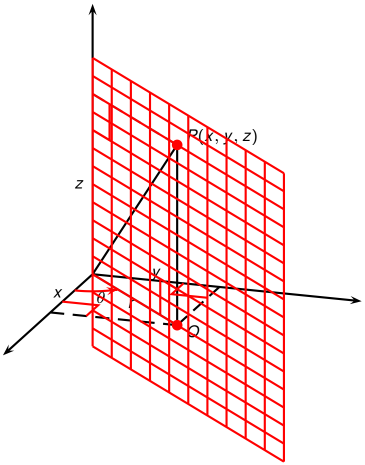
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary: horizontal circle;
- keep r, θ constant, let z vary: vertical line.

What surface is traced when:

- keep r constant, let θ, z vary: vertical cylinder;
- keep θ constant, let r, z vary:
- keep z constant, let r, θ vary:

Constant Coordinate Sets



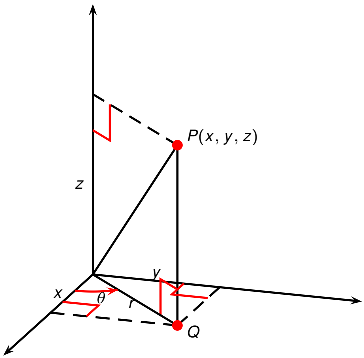
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary: horizontal circle;
- keep r, θ constant, let z vary: vertical line.

What surface is traced when:

- keep r constant, let θ, z vary: vertical cylinder;
- keep θ constant, let r, z vary: vertical half plane;
- keep z constant, let r, θ vary:

Constant Coordinate Sets



What curve is traced when:

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- keep r, θ constant, let z vary: vertical line.

What surface is traced when:

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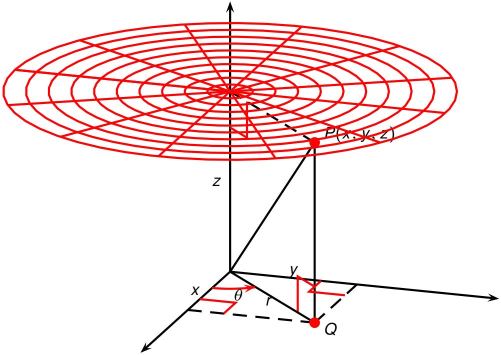
- keep θ constant, let r, z vary: vertical half plane;
- keep z constant, let r, θ vary:

Constant Coordinate Sets



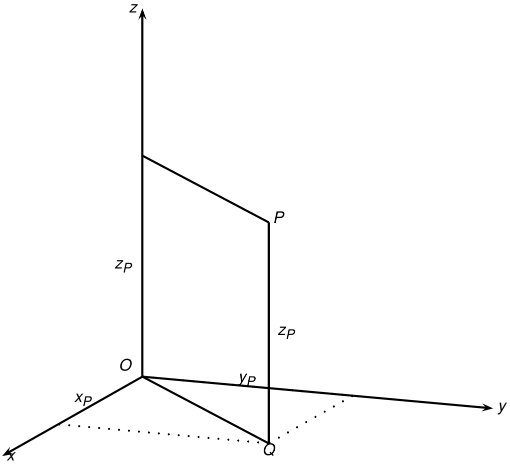
What curve is traced when:

- keep θ, z constant, let r vary: horizontal ray;
- keep r, z constant, let θ vary: horizontal circle;
- keep r, θ constant, let z vary: vertical line.

What surface is traced when:

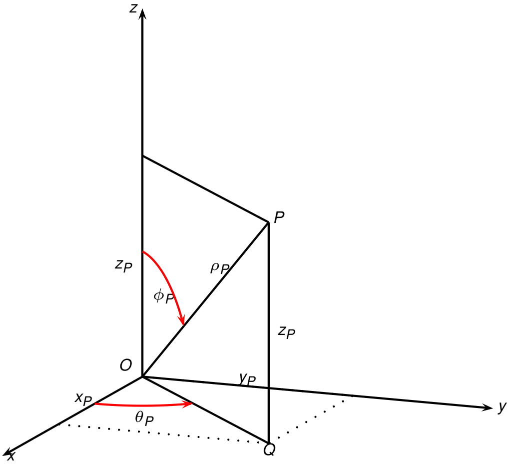
- keep r constant, let θ, z vary: vertical cylinder;
- keep θ constant, let r, z vary: vertical half plane;
- keep z constant, let r, θ vary: horizontal plane.

Spherical Coordinates



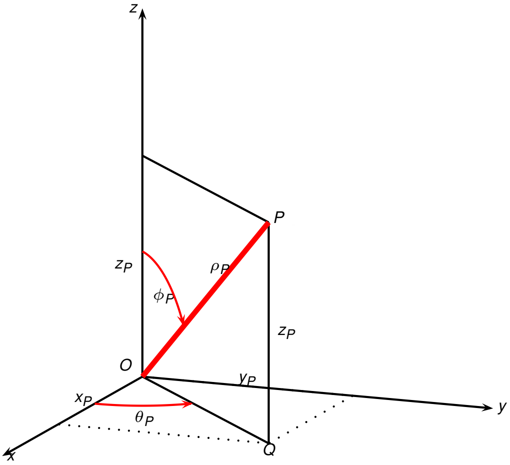
- In Cartesian coordinates, a point P is given by triple (x_P, y_P, z_P).

Spherical Coordinates



- In Cartesian coordinates, a point P is given by triple (x_P, y_P, z_P).
- We introduce alternative spherical coordinates $(\rho_P, \phi_P, \theta_P)$.
  - ρ_P: distance $|OP|$;
  - ϕ_P: angle Oz to OP;
  - θ_P: angle Ox to OP_{xy}.

Spherical Coordinates



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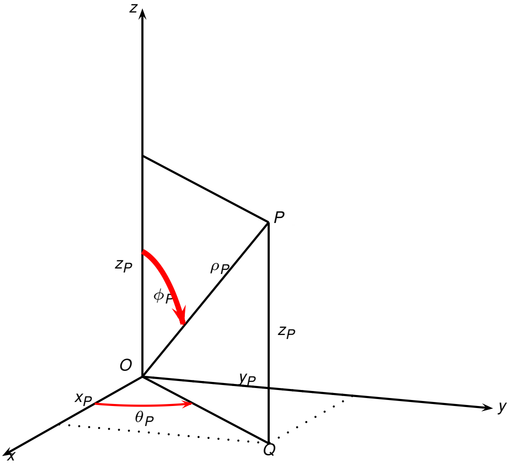
Spherical Coordinates



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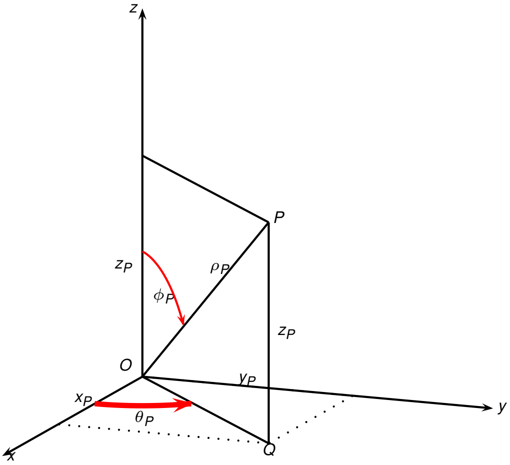
Spherical Coordinates



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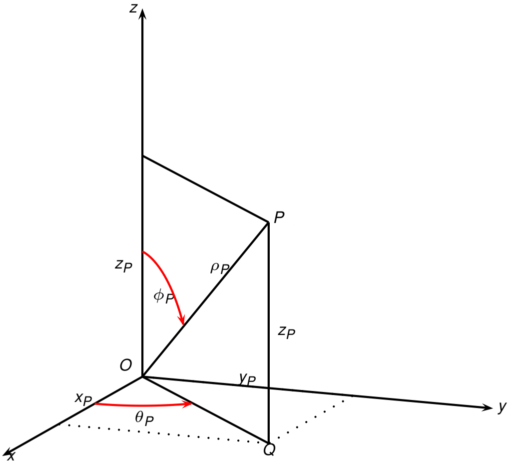
Spherical Coordinates



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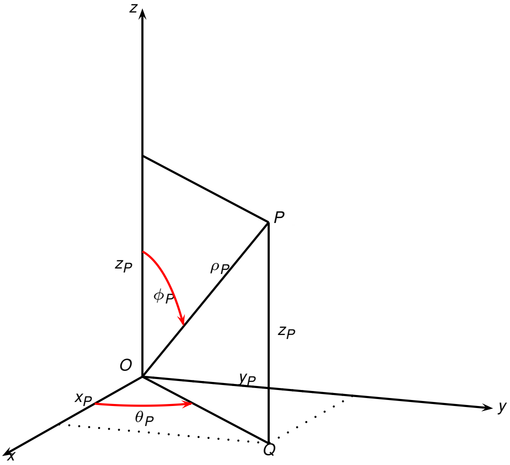
- Coordinates range:
  - ρ:
  - ϕ:
  - θ:

Spherical Coordinates



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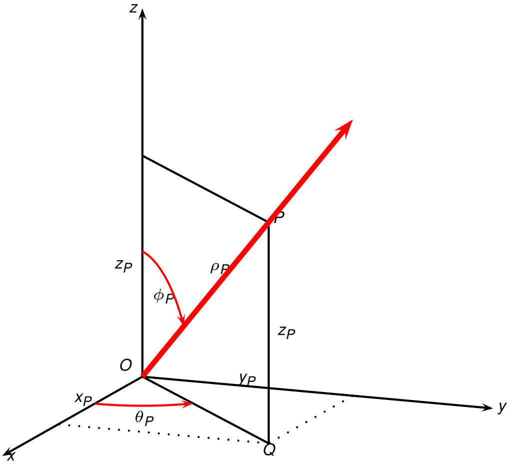
Spherical Coordinates



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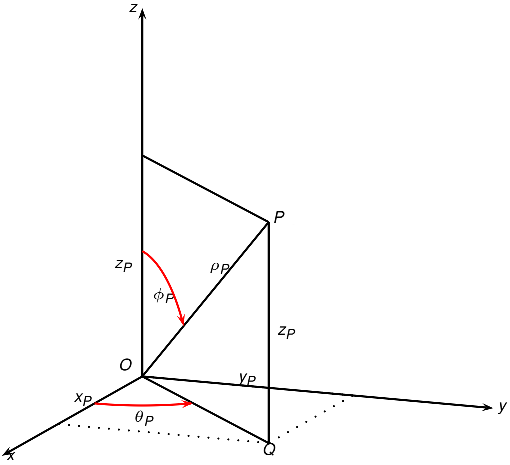
  - ϕ_P: angle Oz to OP;
  - θ_P: angle Ox to OP_{xy}.
- Coordinates range:
  - ρ: $[0, \infty)$;
  - ϕ:
  - θ:

Spherical Coordinates



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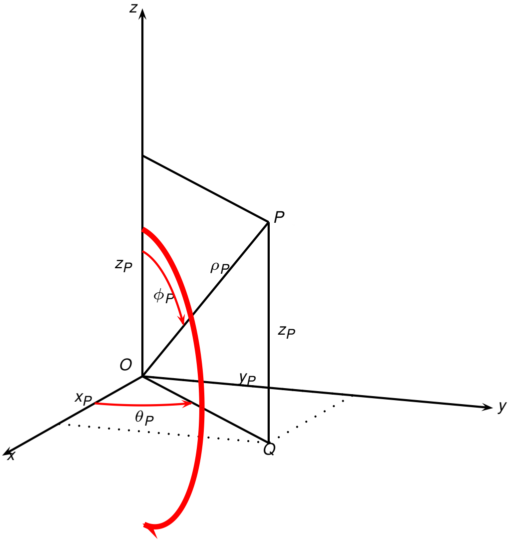
  - ρ: $[0, \infty)$;
  - ϕ:
  - θ:

Spherical Coordinates



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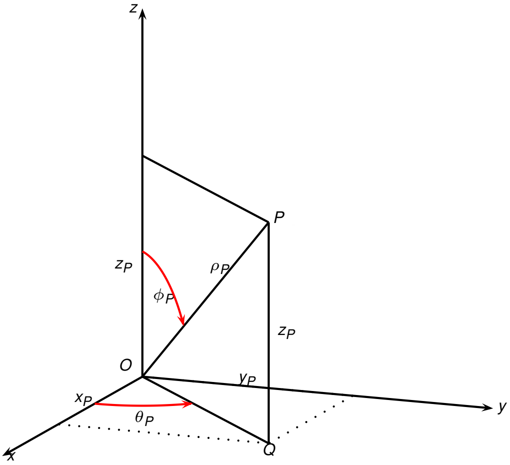
- Coordinates range:
  - ρ: $[0, \infty)$;
  - ϕ: $[0, \pi]$;
  - θ:

Spherical Coordinates



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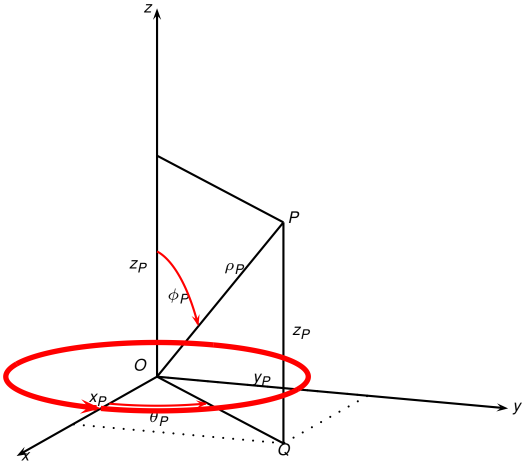
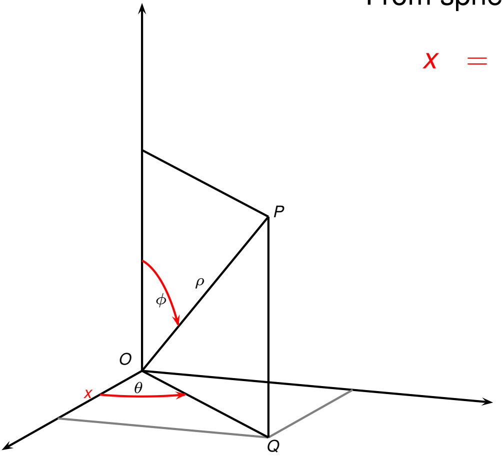
  - ϕ_P: angle Oz to OP;
  - θ_P: angle Ox to OP_{xy}.
- Coordinates range:
  - ρ: $[0, \infty)$;
  - ϕ: $[0, \pi]$;
  - θ:

Spherical Coordinates



- In Cartesian coordinates, a point P is given by triple (x_P, y_P, z_P).
- We introduce alternative spherical coordinates $(\rho_P, \phi_P, \theta_P)$.
  - ρ_P: distance $|OP|$;
  - ϕ_P: angle Oz to OP;
  - θ_P: angle Ox to OP_{xy}.
- Coordinates range:
  - ρ: $[0, \infty)$;
  - ϕ: $[0, \pi]$;
  - θ: $[0, 2\pi)$.

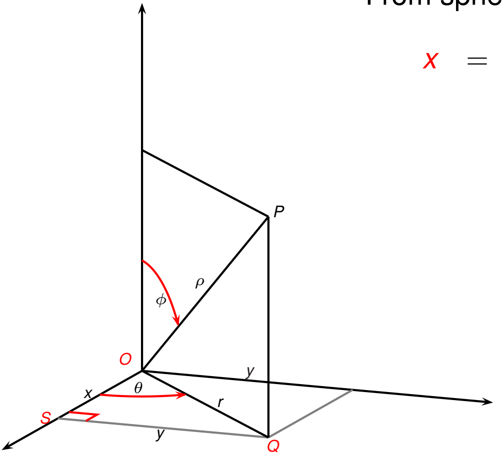
Transition Spherical - Rectangular coordinates



From spherical to rectangular coords:

$$x =$$

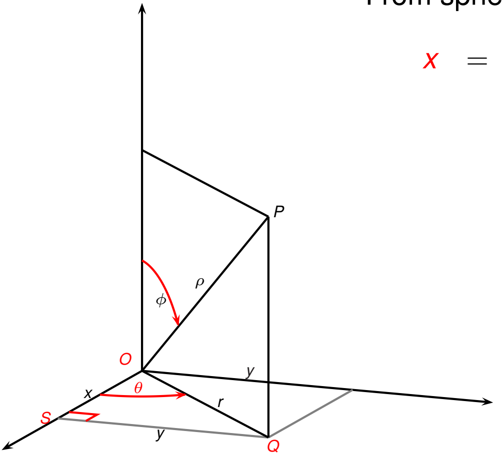
Transition Spherical - Rectangular coordinates



From spherical to rectangular coords:

$$x = \qquad \Big| \text{ use } \triangle SQO$$

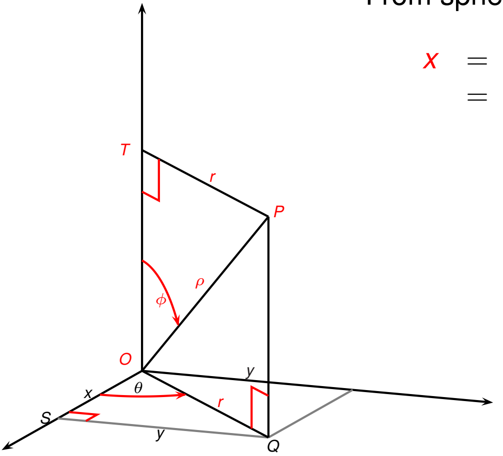
Transition Spherical - Rectangular coordinates



From spherical to rectangular coords:

$$x = r\cos\theta \qquad \Big| \text{ use } \triangle SQO$$

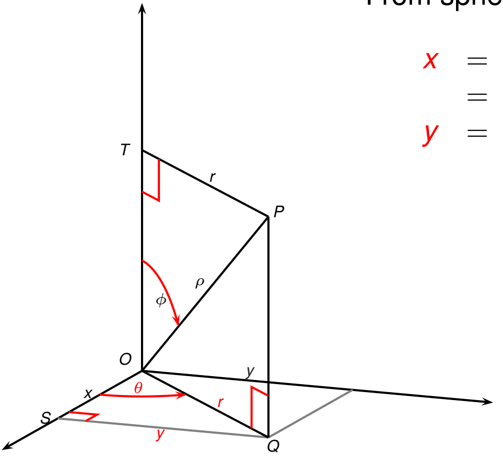
Transition Spherical - Rectangular coordinates



From spherical to rectangular coords:

$$\begin{array}{rcl|l} x & = & r\cos\theta & \text{use } \triangle SQO \\ & = & \rho\sin\phi\cos\theta & \text{use } \triangle OPT \end{array}$$

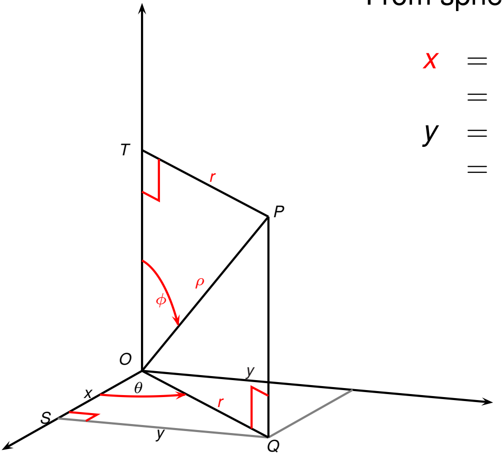
Transition Spherical - Rectangular coordinates



From spherical to rectangular coords:

$$
\begin{array}{rcl|l}
x & = & r\cos\theta & \text{use } \triangle SQO \\
 & = & \rho\sin\phi\cos\theta & \text{use } \triangle OPT \\
y & = & r\sin\theta &
\end{array}
$$

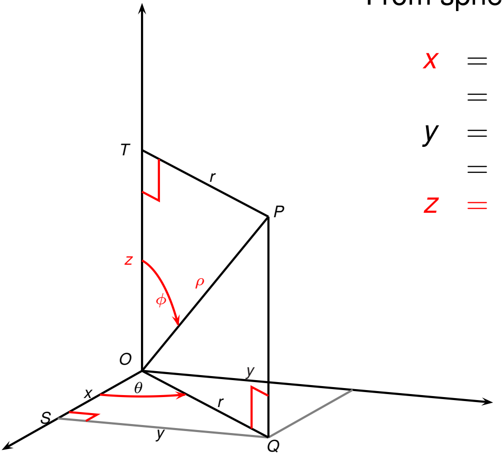
Transition Spherical - Rectangular coordinates



From spherical to rectangular coords:

$$\begin{array}{rcl|l} x & = & r\cos\theta & \text{use } \triangle SQO \\ & = & \rho\sin\phi\cos\theta & \text{use } \triangle OPT \\ y & = & r\sin\theta & \\ & = & \rho\sin\phi\sin\theta & \end{array}$$

Transition Spherical - Rectangular coordinates

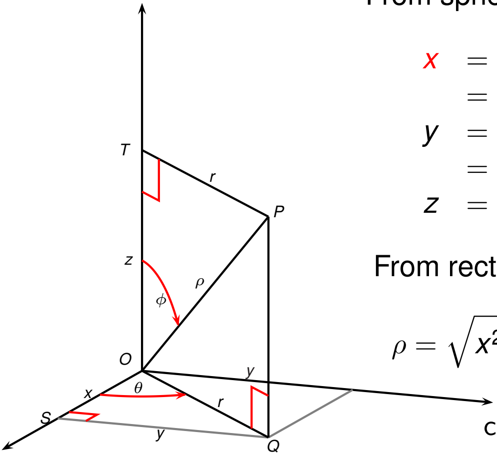


From spherical to rectangular coords:

$$
\begin{aligned}
x &= r\cos\theta \\
&= \rho\sin\phi\cos\theta \\
y &= r\sin\theta \\
&= \rho\sin\phi\sin\theta \\
z &= \rho\cos\phi
\end{aligned}
$$

use $\triangle SQO$
use $\triangle OPT$

Transition Spherical - Rectangular coordinates



From spherical to rectangular coords:

$$
\begin{array}{rcl|l}
x & = & r\cos\theta & \text{use } \triangle SQO \\
 & = & \rho\sin\phi\cos\theta & \text{use } \triangle OPT \\
y & = & r\sin\theta & \\
 & = & \rho\sin\phi\sin\theta & \\
z & = & \rho\cos\phi &
\end{array}
$$

From rectangular to spherical coords:

$$\rho = \sqrt{x^2+y^2+z^2} \quad , \quad r = \sqrt{x^2+y^2}$$

$$\cos\phi = \frac{z}{\rho} \quad , \quad \sin\phi = \frac{r}{\rho}$$

$$\cos\theta = \frac{x}{r} \quad , \quad \sin\theta = \frac{y}{r}$$

Constant Coordinate Sets

What curve is traced when:

- keep θ, ϕ constant, let ρ vary:
- keep ρ, ϕ constant, let θ vary:
- keep ρ, θ constant, let ϕ vary:

Constant Coordinate Sets

What curve is traced when:

- keep θ, ϕ constant, let ρ vary:
- keep ρ, ϕ constant, let θ vary:
- keep ρ, θ constant, let ϕ vary:

Constant Coordinate Sets

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- keep θ, ϕ constant, let ρ vary:
- keep ρ, ϕ constant, let θ vary:
- keep ρ, θ constant, let ϕ vary:

Constant Coordinate Sets

What curve is traced when:

- keep θ, ϕ constant, let ρ vary:
- keep ρ, ϕ constant, let θ vary:
- keep ρ, θ constant, let ϕ vary:

Constant Coordinate Sets

What curve is traced when:

- keep θ, ϕ constant, let ρ vary: ray through the origin;
- keep ρ, ϕ constant, let θ vary:
- keep ρ, θ constant, let ϕ vary:

Constant Coordinate Sets

What curve is traced when:

- keep θ, ϕ constant, let ρ vary: ray through the origin;
- keep ρ, ϕ constant, let θ vary:
- keep ρ, θ constant, let ϕ vary:

Constant Coordinate Sets

What curve is traced when:

- keep θ, ϕ constant, let ρ vary: ray through the origin;
- keep ρ, ϕ constant, let θ vary:
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Constant Coordinate Sets

What curve is traced when:

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Constant Coordinate Sets

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Constant Coordinate Sets

What curve is traced when:

- keep θ, ϕ constant, let ρ vary: ray through the origin;
- keep ρ, ϕ constant, let θ vary:
- keep ρ, θ constant, let ϕ vary:

Constant Coordinate Sets

What curve is traced when:

- keep θ, ϕ constant, let ρ vary: ray through the origin;
- keep ρ, ϕ constant, let θ vary: circle parallel to the *xy*-plane, “parallel”;
- keep ρ, θ constant, let ϕ vary:

Constant Coordinate Sets

What curve is traced when:

- keep θ, ϕ constant, let ρ vary: ray through the origin;
- keep ρ, ϕ constant, let θ vary: circle parallel to the *xy*-plane, "parallel";
- keep ρ, θ constant, let ϕ vary:

Constant Coordinate Sets

What curve is traced when:

- keep θ, ϕ constant, let ρ vary: ray through the origin;
- keep ρ, ϕ constant, let θ vary: circle parallel to the *xy*-plane, “parallel”;
- keep ρ, θ constant, let ϕ vary:

Constant Coordinate Sets

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- keep ρ, ϕ constant, let θ vary: circle parallel to the *xy*-plane, “parallel”;
- keep ρ, θ constant, let ϕ vary:

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What curve is traced when:

- keep θ, ϕ constant, let ρ vary: ray through the origin;
- keep ρ, ϕ constant, let θ vary: circle parallel to the *xy*-plane, “parallel”;
- keep ρ, θ constant, let ϕ vary:

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What curve is traced when:

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- keep ρ, ϕ constant, let θ vary: circle parallel to the *xy*-plane, "parallel";
- keep ρ, θ constant, let ϕ vary:

Constant Coordinate Sets

What curve is traced when:

- keep θ, ϕ constant, let ρ vary: ray through the origin;
- keep ρ, ϕ constant, let θ vary: circle parallel to the *xy*-plane, "parallel";
- keep ρ, θ constant, let ϕ vary:

Constant Coordinate Sets

What curve is traced when:

- keep θ, ϕ constant, let ρ vary: ray through the origin;
- keep ρ, ϕ constant, let θ vary: circle parallel to the *xy*-plane, “parallel”;
- keep ρ, θ constant, let ϕ vary: circle passing through *z* axis, “meridian”.

Constant Coordinate Sets

What surface is traced when:

- keep ϕ constant, let θ, ρ vary:
- keep θ constant, let ρ, ϕ vary:
- keep ρ constant, let ϕ, θ vary:

Constant Coordinate Sets

What surface is traced when:

- keep ϕ constant, let θ, ρ vary: cone;
- keep θ constant, let ρ, ϕ vary:
- keep ρ constant, let ϕ, θ vary:

Constant Coordinate Sets

What surface is traced when:

- keep ϕ constant, let θ, ρ vary: cone;
- keep θ constant, let ρ, ϕ vary:
- keep ρ constant, let ϕ, θ vary:

Constant Coordinate Sets

What surface is traced when:

- keep ϕ constant, let θ, ρ vary: cone;
- keep θ constant, let ρ, ϕ vary: vertical half plane;
- keep ρ constant, let ϕ, θ vary:

Constant Coordinate Sets

What surface is traced when:

- keep ϕ constant, let θ, ρ vary: cone;
- keep θ constant, let ρ, ϕ vary: vertical half plane;
- keep ρ constant, let ϕ, θ vary:

Constant Coordinate Sets

What surface is traced when:

- keep ϕ constant, let θ, ρ vary: cone;
- keep θ constant, let ρ, ϕ vary: vertical half plane;
- keep ρ constant, let ϕ, θ vary: sphere.

Polar curvilinear "boxes"

Polar "wedge":

$$C = \{P(r, \theta) \mid r_0 \leqslant r \leqslant r_0 + \Delta r, \theta_0 \leqslant \theta \leqslant \theta_0 + \Delta\theta\} .$$

Shape?

Polar curvilinear “boxes”

Polar “wedge”:

$$C = \{P(r,\theta) \mid r_0 \leqslant r \leqslant r_0 + \Delta r, \theta_0 \leqslant \theta \leqslant \theta_0 + \Delta\theta\} \ .$$

Shape? Area = ...?

Cylindrical curvilinear "boxes"

Cylindrical "box":

$$X = \{P(r, \theta, z) \mid 0 \leqslant r \leqslant r_0 \, , 0 \leqslant \theta \leqslant \theta_0 \, , 0 \leqslant z \leqslant z_0\}$$

Cylindrical curvilinear "boxes"

Cylindrical "box":

$$X = \{P(r, \theta, z) \mid 0 \leqslant r \leqslant r_0\,, 0 \leqslant \theta \leqslant \theta_0\,, 0 \leqslant z \leqslant z_0\}$$

Shape ?

Cylindrical curvilinear “boxes”

Cylindrical “box”:

$$X = \{P(r, \theta, z) \mid 0 \leqslant r \leqslant r_0 \,, 0 \leqslant \theta \leqslant \theta_0 \,, 0 \leqslant z \leqslant z_0\}$$

Shape ? Volume = ...?

Spherical curvilinear "boxes"

- Cut off a rectangular box B in the ρ, ϕ, θ-coordinates. $B :=$
$$\left\{ (\rho,\phi,\theta) \left| \begin{array}{ccccc} \rho_{min} & \leq & \rho & \leq & \rho_{max} \\ \phi_{min} & \leq & \phi & \leq & \phi_{max} \\ \theta_{min} & \leq & \theta & \leq & \theta_{max} \end{array} \right. \right\}$$

Spherical curvilinear "boxes"

- Cut off a rectangular box B in the ρ, ϕ, θ-coordinates. $B :=$
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- As (ρ, ϕ, θ) traverse B, the point $P(\rho, \phi, \theta)$ traverses curvilinear "box" Y:

$$Y = \left\{ P(\rho, \phi, \theta) | (\rho, \phi, \theta) \in B \right\}.$$

Spherical curvilinear "boxes"

- Cut off a rectangular box B in the ρ, ϕ, θ-coordinates. $B :=$
$$\left\{ (\rho, \phi, \theta) \middle| \begin{array}{ccccc} \rho_{min} & \leq & \rho & \leq & \rho_{max} \\ \phi_{min} & \leq & \phi & \leq & \phi_{max} \\ \theta_{min} & \leq & \theta & \leq & \theta_{max} \end{array} \right\}$$
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$$Y = \left\{ P(\rho, \phi, \theta) | (\rho, \phi, \theta) \in B \right\}.$$
- What is the shape of that curvilinear box?

Spherical curvilinear “boxes”

- Cut off a rectangular box B in the ρ,ϕ,θ-coordinates. $B :=$
$$\left\{(\rho,\phi,\theta) \middle| \begin{array}{ccccc} \rho_{min} & \leq & \rho & \leq & \rho_{max} \\ \phi_{min} & \leq & \phi & \leq & \phi_{max} \\ \theta_{min} & \leq & \theta & \leq & \theta_{max} \end{array} \right\}$$
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- Cut off a rectangular box B in the ρ, ϕ, θ-coordinates. $B :=$
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Spherical curvilinear "boxes"

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- As (ρ, ϕ, θ) traverse B, the point $P(\rho, \phi, \theta)$ traverses curvilinear "box" Y: $$Y = \{P(\rho, \phi, \theta) | (\rho, \phi, \theta) \in B\} .$$
- What is the shape of that curvilinear box?

Spherical curvilinear “boxes”

- Cut off a rectangular box B in the ρ, ϕ, θ-coordinates. $B :=$
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- As (ρ, ϕ, θ) traverse B, the point $P(\rho, \phi, \theta)$ traverses curvilinear “box” Y:
$$Y = \{P(\rho, \phi, \theta) | (\rho, \phi, \theta) \in B\} .$$
- What is the shape of that curvilinear box?

Spherical curvilinear "boxes"

- Cut off a rectangular box B in the ρ, ϕ, θ-coordinates. $B :=$
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- As (ρ, ϕ, θ) traverse B, the point $P(\rho, \phi, \theta)$ traverses curvilinear "box" Y:
$$Y = \{ P(\rho, \phi, \theta) | (\rho, \phi, \theta) \in B \} .$$
- What is the shape of that curvilinear box?

Spherical curvilinear "boxes"

- Cut off a rectangular box B in the ρ, ϕ, θ-coordinates. $B :=$
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- As (ρ, ϕ, θ) traverse B, the point $P(\rho, \phi, \theta)$ traverses curvilinear "box" Y:
$$Y = \{P(\rho, \phi, \theta) | (\rho, \phi, \theta) \in B\} .$$
- What is the shape of that curvilinear box?

Spherical curvilinear "boxes"

- Cut off a rectangular box B in the ρ, ϕ, θ-coordinates. $B :=$
$$\left\{ (\rho,\phi,\theta) \middle| \begin{array}{ccccc} \rho_{min} & \leq & \rho & \leq & \rho_{max} \\ \phi_{min} & \leq & \phi & \leq & \phi_{max} \\ \theta_{min} & \leq & \theta & \leq & \theta_{max} \end{array} \right\}$$
- As (ρ, ϕ, θ) traverse B, the point $P(\rho, \phi, \theta)$ traverses curvilinear "box" Y:
$$Y = \{P(\rho,\phi,\theta) | (\rho,\phi,\theta) \in B\} .$$
- What is the shape of that curvilinear box?
- What is the volume?