Calculus III
Homework on Lecture 8

1. Find the limit or show that it does not exist. The answer key has not been proofread, use with caution.

   (a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{y^4}{x^4+2y^2}$.

   answer: 0

   (b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+(\ln(1+y))^2}{x^2+y^2}$.

   answer: the limit does not exist.

   (c) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2\ln(1+y)}{x^2+y^2}$.

   answer: 0

   (d) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^4}{x^4+y^8}$.

   answer: the limit does not exist.

   Solution. 1.a Limits respect inequalities, therefore

$$0 \le \lim_{(x,y)\to 0} \frac{y^4}{x^4+2y^2} \le \lim_{(x,y)\to 0} \frac{y^4}{2y^2} = \lim_{(x,y)\to 0} \frac{1}{2}y^2 = 0.$$

   Therefore $\lim\limits_{(x,y)\to 0} \frac{y^4}{x^4+2y^2} = 0$.