Precalculus
Homework Lecture 15

1. Use the definition of a logarithm to evaluate each of the following without using a calculator. The answer key has not been proofread, use with caution.

   (a) $\log_2 16$.

   (b) $\log_3\left(\dfrac{1}{9}\right)$.

   (c) $\log_{10} 1000$.

   (d) $\log_6 36^{-\frac{2}{3}}$.

   (e) $\log_2(8\sqrt{2})$.

   (f) $\log_{\frac{1}{2}}(4)$.

   (g) $\log_{\frac{1}{9}}(\sqrt{3})$.

2. Find the exact value of each expression.

   (a) $\log_5 125$.

   (b) $\log_3 \dfrac{1}{27}$.

   (c) $\ln\left(\dfrac{1}{e}\right)$.

   (d) $\log_{10}\sqrt{10}$.

   (e) $e^{\ln 4.5}$.

   (f) $\log_{10} 0.0001$.

   (g) $\log_{1.5} 2.25$.

   (h) $\log_5 4 - \log_5 500$.

   (i) $\log_2 6 - \log_2 15 + \log_2 20$.

   (j) $\log_3 100 - \log_3 18 - \log_3 50$.

   (k) $e^{-2\ln 5}$.

   (l) $\ln\left(\ln e^{e^{10}}\right)$.

   (m) $\log_7\left(\dfrac{49^x}{343^y}\right)$

3. Using only the $\ln$ operation of your calculator compute the indicated logarithm. Confirm your computation numerically by exponentiation.

   (a) $\log_5(13)$.

   (b) $\log_{12}(9)$.

   (c) $\log_{13}(101)$.

   (d) $\log_{10}(2015)$.

4. Express each of the following as a single logarithm. If possible, compute the logarithm without using a calculator. The answer key has not been proofread, use with caution.

   (a) $\ln 4 + \ln 6 - \ln 5$.

   (b) $2\ln 2 - 3\ln 3 + 4\ln 4$.

   (c) $\ln 36 - 2\ln 3 - 3\ln 2$.

   (d) $\log_2(24) - \log_4 9$.

   (e) $\log_7(24) + \log_{\frac{1}{7}} 3 - \log_{49}(64)$.

   (f) $\log_3(24) + \log_3\left(\frac{3}{8}\right)$.

5. Demonstrate the identity(s).

   (a) $-\ln\left(\sqrt{1+x^2} - x\right) = \ln\left(x + \sqrt{1+x^2}\right)$

6. Solve each equation for x. If available, use a calculator to give an ($\approx$) answer in decimal notation. If available, use a calculator to verify your approximate solutions.

(a) $e^{7-4x} = 7$.

(b) $\ln(2x - 9) = 2$.

(c) $\ln(x^2 - 2) = 3$.

(d) $2^{x-3} = 5$.

(e) $\ln x + \ln(x - 1) = 1$.

(f) $e^{2x+1} = t$.

(g) $\log_2(mx) = c$.

(h) $e - e^{-2x} = 1$.

(i) $8(1 + e^{-x})^{-1} = 3$.

(j) $\ln(\ln x) = 1$.

(k) $e^{e^x} = 10$.

(l) $\ln(2x + 1) = 3 - \ln x$.

(m) $e^{2x} - 4e^x + 3 = 0$.

(n) $e^{4x} + 3e^{2x} - 4 = 0$.

(o) $e^{2x} - e^x - 6 = 0$.

(p) $4^{3x} - 2^{3x+2} - 5 = 0$.

(q) $3 \cdot 2^x + 2 \left(\frac{1}{2}\right)^{x-1} - 7 = 0$.