Calculus I
Homework Review Trigonometry
Lecture 2

1. **The problem is too easy to appear on a quiz or test.** Convert from degrees to radians.

   (a) 15°. answer: $\frac{\pi}{12} \approx 0.261799388$

   (b) 30°. answer: $\frac{\pi}{6} \approx 0.523598776$

   (c) 36°. answer: $\frac{\pi}{5} \approx 0.628318531$

   (d) 45°. answer: $\frac{\pi}{4} \approx 0.785398163$

   (e) 60°. answer: $\frac{\pi}{3} \approx 1.047197551$

   (f) 75°. answer: $\frac{5\pi}{12} \approx 1.308997$

   (g) 90°. answer: $\frac{\pi}{2}$

   (h) 120°. answer: $\frac{2\pi}{3}$

   (i) 135°. answer: $\frac{3\pi}{4}$

   (j) 150°. answer: $\frac{5\pi}{6}$

   (k) 180°. answer: π

   (l) 225°. answer: $\frac{5\pi}{4}$

   (m) 270°. answer: $\frac{3\pi}{2}$

   (n) 305°. answer: $\frac{61\pi}{36} \approx 5.323254$

   (o) 360°. answer: 2π

   (p) 405°. answer: $\frac{9\pi}{4}$

   (q) 1200°. answer: $\frac{20\pi}{3}$

   (r) -900°. answer: -5π

   (s) -2014°. answer: $-\frac{1007}{90}\pi \approx -35.150931$

2. **The problem is too easy to appear on a quiz or test.** Convert from radians to degrees. The answer key has not been proofread, use with caution.

   (a) 4π. answer: 720°

   (b) $-\frac{7}{6}\pi$. answer: -210°

   (c) $\frac{7}{12}\pi$. answer: 105°

   (d) $\frac{4}{3}\pi$. answer: 240°

   (e) $-\frac{3}{8}\pi$. answer: -67.5°

   (f) 2014π. answer: 362520°

   (g) 5. answer: $\left(\frac{900}{\pi}\right)^\circ \approx 286^\circ$

   (h) -2014. answer: -362520°

3. Prove the trigonometry identities.

   (a) $\sin\theta\cot\theta = \cos\theta$.

   (b) $(\sin\theta + \cos\theta)^2 = 1 + \sin(2\theta)$.

   (c) $\sec\theta - \cos\theta = \tan\theta\sin\theta$.

   (d) $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$.

   (e) $\cot^2\theta + \sec^2\theta = \tan^2\theta + \csc^2\theta$.

   (f) $2\csc(2\theta) = \sec\theta\csc\theta$.

   (g) $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$.

   (h) $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} = 2\sec^2\theta$.

   (i) $\tan\alpha + \tan\beta = \dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$.

   (j) $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$.

   (k) $\sin(3\theta) + \sin\theta = 2\sin(2\theta)\cos\theta$.

   (l) $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$.

   (m) $1 + \tan^2\theta = \sec^2\theta$.

   (n) $1 + \csc^2\theta = \cot^2\theta$.

   (o) $2\cos^2(2x) = 2\sin^4\theta + 2\cos^4\theta - \sin^2(2\theta)$.

   (p) $\dfrac{1+\tan\left(\frac{\theta}{2}\right)}{1-\tan\left(\frac{\theta}{2}\right)} = \tan\theta + \sec\theta$.

4. Find all values of x in the interval $[0, 2\pi]$ that satisfy the equation.

(a) $2\cos x - 1 = 0$.

answer: $x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$

(b) $\sin(2x) = \cos x$.

answer: $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$, $x = \frac{\pi}{6}$, or $x = \frac{5\pi}{6}$

(c) $\sqrt{3}\sin x = \sin(2x)$.

answer: $x = \frac{\pi}{6}$, $\frac{11\pi}{6}$, 0, π, 2π

(d) $2\sin^2 x = 1$.

answer: $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$, $x = \frac{5\pi}{4}$, or $x = \frac{7\pi}{4}$

(e) $2 + \cos(2x) = 3\cos x$.

answer: $x = 0$, $x = 2\pi$, $x = \frac{\pi}{3}$, or $x = \frac{5\pi}{3}$

(f) $2\cos x + \sin(2x) = 0$.

answer: $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$

(g) $2\cos^2 x - (1+\sqrt{2})\cos x + \dfrac{\sqrt{2}}{2} = 0$.

answer: $x = \frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{5\pi}{3}$, $\frac{7\pi}{4}$

(h) $|\tan x| = 1$.

answer: $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$, $x = \frac{5\pi}{4}$, or $x = \frac{7\pi}{4}$

(i) $3\cot^2 x = 1$.

answer: $x = \frac{\pi}{3}$, $x = \frac{2\pi}{3}$, $x = \frac{4\pi}{3}$, or $x = \frac{5\pi}{3}$

(j) $\sin x = \tan x$.

answer: $x = 0$, $x = \pi$, or $x = 2\pi$

Solution. 4.g Set $\cos x = u$. Then

$$2\cos^2 x - (1+\sqrt{2})\cos x + \frac{\sqrt{2}}{2} = 0$$

becomes

$$2u^2 - (1+\sqrt{2})u + \frac{\sqrt{2}}{2} = 0.$$

This is a quadratic equation in u and therefore has solutions

$$\begin{aligned} u_1, u_2 &= \frac{1+\sqrt{2} \pm \sqrt{(1+\sqrt{2})^2 - 4\sqrt{2}}}{4} \\ &= \frac{1+\sqrt{2} \pm \sqrt{1 - 2\sqrt{2} + 2}}{4} \\ &= \frac{1+\sqrt{2} \pm \sqrt{(1-\sqrt{2})^2}}{4} \\ &= \frac{1+\sqrt{2} \pm (1-\sqrt{2})}{4} = \left\{ \begin{array}{l} \frac{1}{2} \quad \text{or} \\ \frac{\sqrt{2}}{2} \end{array} \right. \end{aligned}$$

Therefore $u = \cos x = \frac{1}{2}$ or $u = \cos x = \frac{\sqrt{2}}{2}$, and, as x is in the interval $[0, 2\pi]$, we get $x = \frac{\pi}{3}$, $\frac{5\pi}{3}$ (for $\cos x = \frac{1}{2}$) or $x = \frac{\pi}{4}$, $\frac{7\pi}{4}$ (for $\cos x = \frac{\sqrt{2}}{2}$).