Precalculus
Lecture 18

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https://github.com/tmilev/freecalc

2020

Outline

1 Lines
- Slope-intercept Form
- Line intersection

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$\mathbb{R}^2, \mathbb{R}^3, \mathbb{R}^n$

Definition ($\mathbb{R}^2$)

The set of ordered pairs of real numbers is denoted by $\mathbb{R}^2$.

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Proposition

Let (x_1, y_1) *and* (x_2, y_2) *be points and L be the line between them. Then L has equation*

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point-slope form

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Definition (non-vertical line, slope-intercept form)

A line that is the graph of an equation of the form

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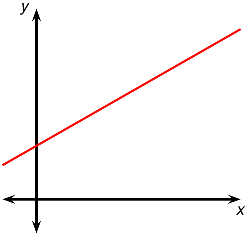
is called a *non-vertical* line.



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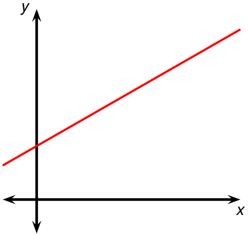

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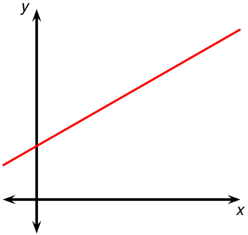

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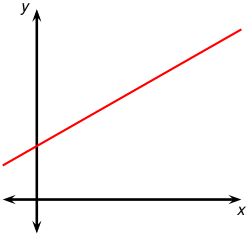
- The number b is the y intercept of the line.



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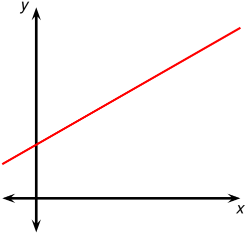

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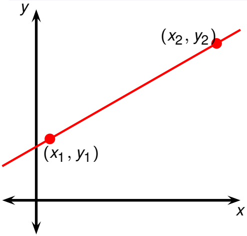

- Fix pts. (x_1, y_1), (x_2, y_2) on the line with $x_2 > x_1$.

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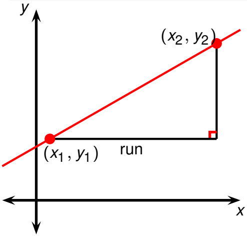

- Fix pts. (x_1, y_1), (x_2, y_2) on the line with $x_2 > x_1$.
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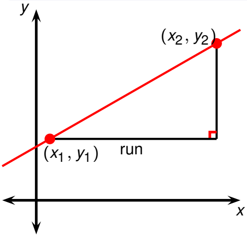

- Fix pts. (x_1, y_1), (x_2, y_2) on the line with $x_2 > x_1$.
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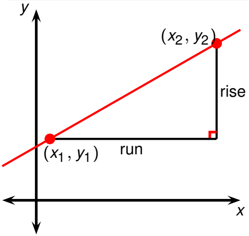

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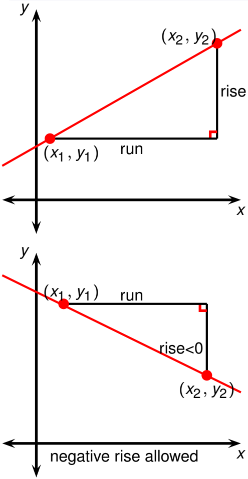

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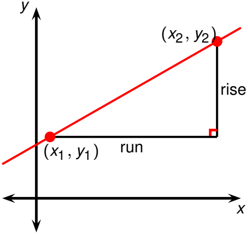

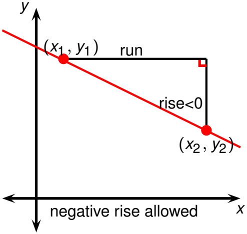

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\hline
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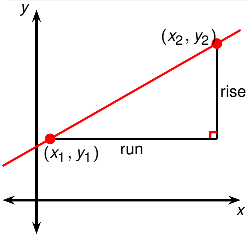

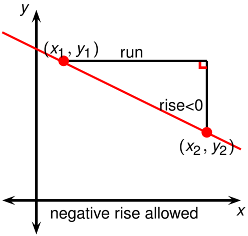

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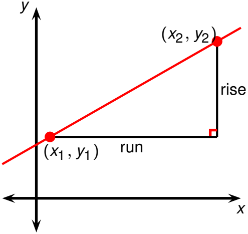

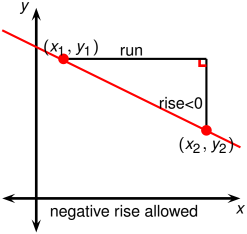

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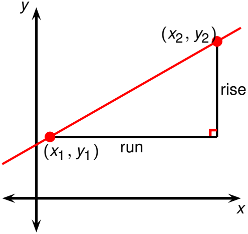

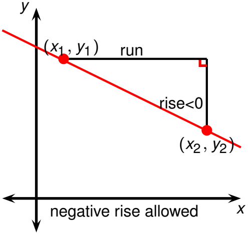

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\begin{array}{rrcl}
 & y_2 & = & mx_2 + b \\
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\hline
 & y_2 - y_1 & = & {\color{red}m}x_2 + \cancel{b} - {\color{red}m}x_1 - \cancel{b} \\
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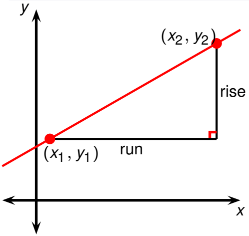

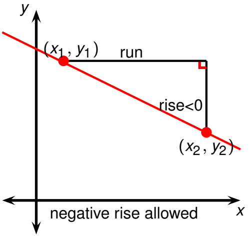

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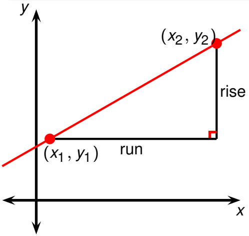

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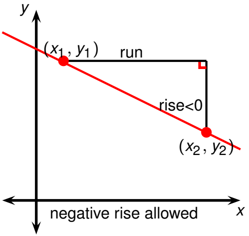

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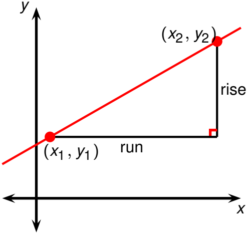

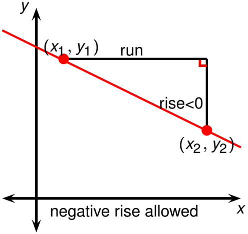

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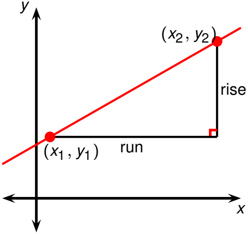

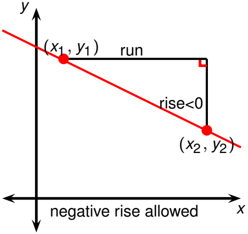

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m & = & \dfrac{\text{rise}}{\text{run}}
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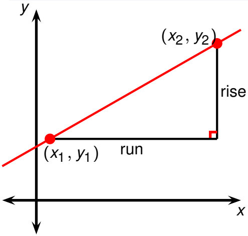

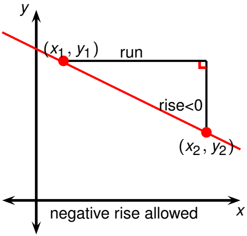

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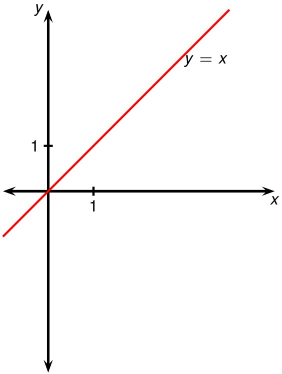
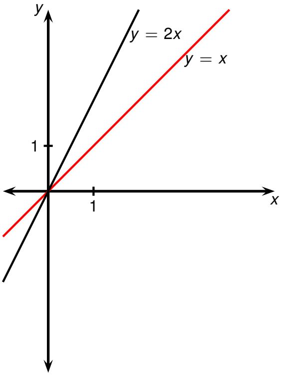
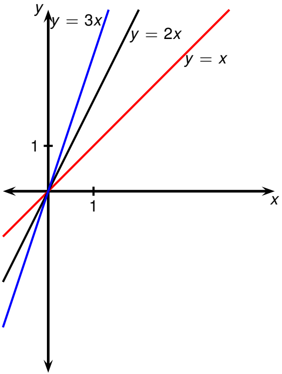


- If two linear functions have positive slopes, the one with the bigger slope increases faster.



- If two linear functions have positive slopes, the one with the bigger slope increases faster.
- $y = 2x$ increases twice as fast as $y = x$.



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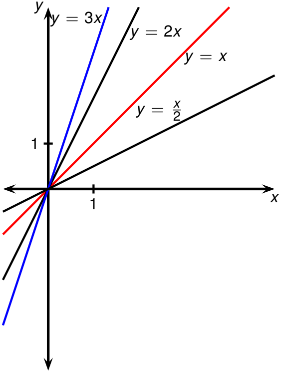
- $y = 3x$ increases three times as fast as $y = x$.



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- $y = 2x$ increases twice as fast as $y = x$.
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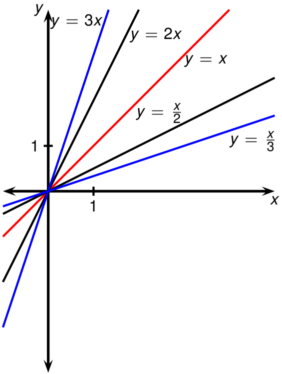
- $y = \frac{x}{2}$ increases half as fast as $y = x$.



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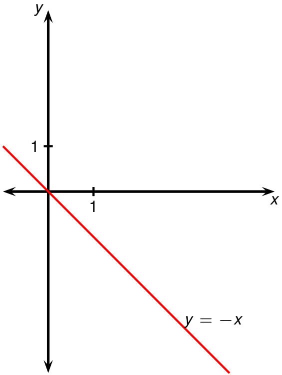
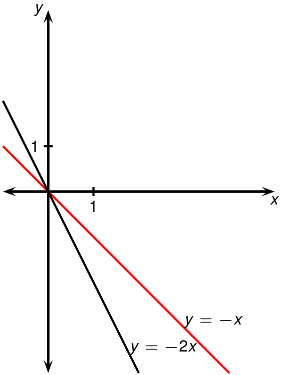
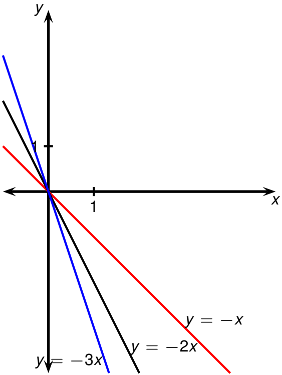
- $y = \frac{x}{2}$ increases half as fast as $y = x$.
- $y = \frac{x}{3}$ increases one third as fast as $y = x$.



- If two linear functions have negative slopes, the one with the lower slope decreases faster.



- If two linear functions have negative slopes, the one with the lower slope decreases faster.
- $y = -2x$ decreases twice as fast as $y = -x$.



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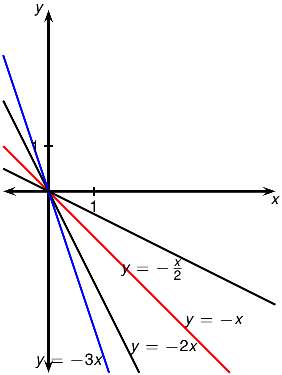

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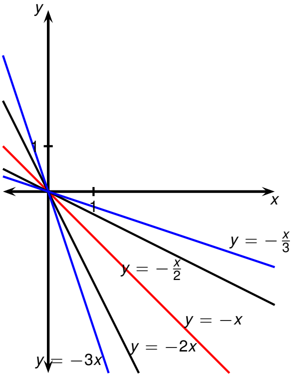
- $y = -\frac{1}{2}x$ decreases half as fast as $y = -x$.



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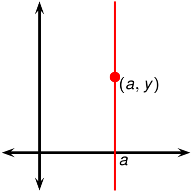
- $y = -\frac{1}{3}x$ decreases one third as fast as $y = -x$.

Definition (Vertical line)

A line of the form $x = a$ is called a vertical line.



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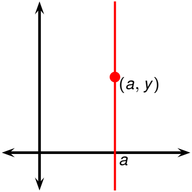

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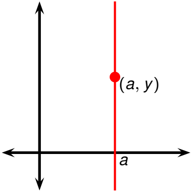

- y does not participate directly in the equation $x = a$.
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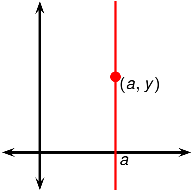

- y does not participate directly in the equation $x = a$.
- Therefore the equation cannot be rewritten in slope-intercept form ($y = \mathbf{?}x + \mathbf{?}$).
- Consequently the notion of a slope is not undefined for vertical lines.

Plotting Lines from line equation

To plot a line from its equation $ax + by = c$ do the following.

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To plot a line from its equation $ax + by = c$ do the following.

- If $b = 0$, the line is vertical through $x = \frac{c}{a}$. Suppose $b \neq 0$.
    - Plug in arbitrary number for x and find y from $y = \frac{c - ax}{b}$.

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To plot a line from its equation $ax + by = c$ do the following.

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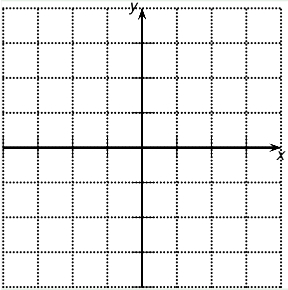
  - Use same procedure to find a second point on the line.
  - If $a \neq 0$: can also plug in values for y to find x.
  - Draw a line between the two dots.

Example



Plot the line with the given equation.

equation	pt.	another pt.
$x + y = 1$		
$2x - y = 3$		
$y = 2$		
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Other points can be used as well.

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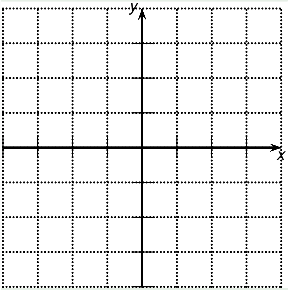

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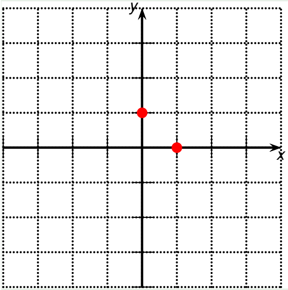

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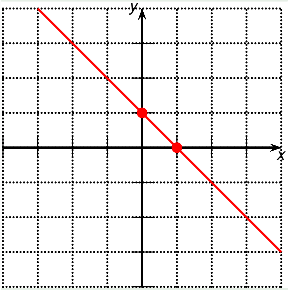

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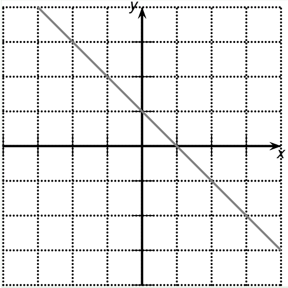

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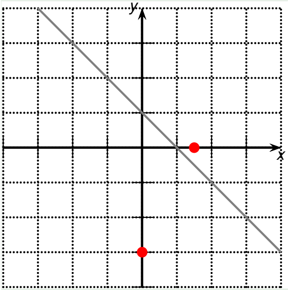

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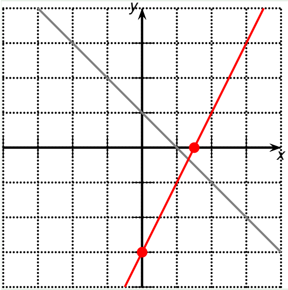

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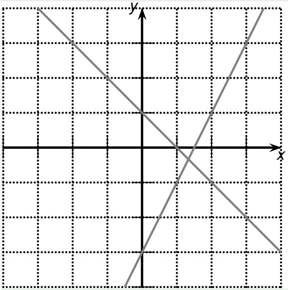

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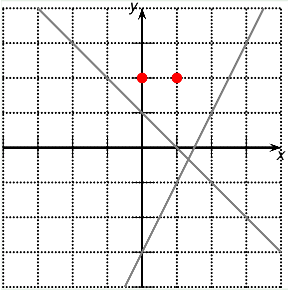

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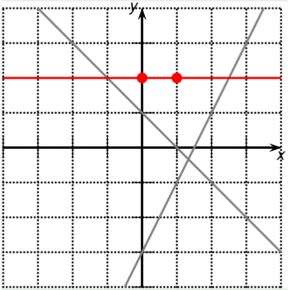

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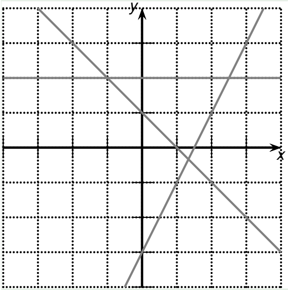

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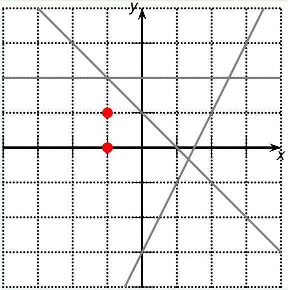

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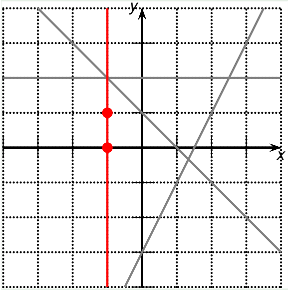

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$x = -1$	$(-1, 0)$	$(-1, 1)$

Other points can be used as well.

Example

Find an equation of a line passing though the indicated pairs of points.

- $(1, 2)$ and $(2, -1)$.
- $(1, 1)$ and $(2, -2)$.
- $(0, 1)$ and $(1, 0)$.
- $(3, 5)$ and $(7, -11)$.

Example

Find an equation of the line passing through $(1, 2)$ with slope $-\frac{1}{2}$.

To find the intersection of two lines (if they do intersect) with equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ we need to solve the system of equations

$$\left| \begin{array}{lcl} a_1x + b_1y + c_1 & = & 0 \\ a_2x + b_2y + c_2 & = & 0 \end{array} \right.$$

Example

Find the intersection of the following lines.

1. $x - y = 3$ and $x + 2y = 10$.
2. $3x - y = 3$ and $x = 1 - 3y$.
3. Line $x = 3$ and $x = 1 - 2y$
4. Line through $(2, 0)$ and $(1, 2)$ and line through $(3, 7)$ and $(2, 5)$.
5. Line through $(3, -1)$ and $(-1, 3)$ and line through $(1, 1)$ and $(2, 3)$.

Definition

Two lines are parallel if they have no common point.

Proposition

Two non-vertical lines are parallel if and only if they have equal slopes and different y intercepts.

Proof $\Leftarrow$.

- Suppose the two lines have different y intercepts and have the same slope m.
- Then the lines have equations as shown below.

$$\left|\begin{array}{rcl} y & = & mx+b_1 \\ y & = & mx+b_2 \end{array}\right.$$

- System has no solutions as $b_1 \neq b_2 \Rightarrow$ the lines don't intersect. $\square$

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$$
\begin{array}{rll}
- \quad y = & m_1 x + b_1 & \\
y = & m_2 x + b_2 & \\
\hline
0 = & (m_1 - m_2)x + b_1 - b_2 & \\
(m_1 - m_2)x = & b_2 - b_1 & \Big| \text{ Div. by } m_1 - m_2 \neq 0 \\
x = & \dfrac{b_2 - b_1}{m_1 - m_2} &
\end{array}
$$

- The system has solution $x = \frac{b_2 - b_1}{m_1 - m_2}$, $y = m_1 \frac{b_2 - b_1}{m_1 - m_2} + b_1 \Rightarrow$ the lines intersect. $\square$