Calculus III
Homework on Lecture 2

1. Carry out the indicated operations between the indicated vectors.

$$\begin{aligned} \mathbf{u} &= (-1, 2, 3) \\ \mathbf{v} &= (2, -3, -5) \\ \mathbf{w} &= (3, 5, -7). \end{aligned}$$

   (a) $-\mathbf{u}$
   (b) $\mathbf{u} + \mathbf{v}$
   (c) $\mathbf{u} - 2\mathbf{w}$
   (d) $-3\mathbf{w} + \frac{\mathbf{v}}{2}$
   (e) $\frac{\mathbf{w}+2\mathbf{u}+3\mathbf{v}}{6}$
   (f) $\mathbf{u} + \mathbf{w} - (2\mathbf{v} + 3\mathbf{u})$

2. Compute the dot product.

   (a) $\mathbf{u} = (2, -3, 5)$, $\mathbf{v} = (-3, 5, 7)$.
   (b) $\mathbf{u} = (\frac{1}{2}, \frac{1}{3}, \frac{1}{4})$, $\mathbf{v} = (\frac{1}{3}, \frac{1}{4}, \frac{1}{5})$.

3. Determine if the vectors are orthogonal.

   (a) $\mathbf{u} = (1, 2, 3)$, $\mathbf{v} = (-1, 2, -1)$.
   (b) $\mathbf{u} = (1, 0, 1)$, $\mathbf{v} = (-1, 1, 1)$.
   (c) $\mathbf{u} = (-1, 0, 1)$, $\mathbf{v} = (-1, 1, 1)$.

4. Find the angles between the vectors. You may use a calculator to get a numerical approximation.

   (a) $\mathbf{u} = (1, 2, 3)$, $\mathbf{v} = (3, 1, 2)$.
   (b) $\mathbf{u} = (-1, -1, -1)$, $\mathbf{v} = (0, 0, 1)$

5. A tetrahedron is a pyramid whose base is a triangle. The 8 points $(1, 1, 1)$, $(-1, 1, 1)$, $(1, -1, 1)$, $(-1, -1, 1)$, $(1, 1, -1)$, $(-1, 1, -1)$, $(1, -1, -1)$, $(-1, -1, -1)$ (all possible sign combinations) give the vertices of a cube with edge 2 units.

   (a) Find 4 vertices of the cube so they form a regular tetrahedron, i.e., 4 points that are not in the same plane and such that the distance between any two is equal.
   (b) Form two vectors, $\mathbf{u}$ and $\mathbf{v}$, by connecting the origin with any two of the 4 points you found.
   (c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$.
   (d) What is the angle between the two bonds of hydrogen atoms in the methane molecule CH_4?

6. Find the

   - Scalar projection $\text{comp}_{\mathbf{v}}\mathbf{u}$ of $\mathbf{u}$ onto $\mathbf{v}$.
   - The vector projection $\mathbf{proj}_{\mathbf{v}}\mathbf{u}$ of $\mathbf{u}$ along $\mathbf{v}$.
   - The component $\mathbf{orth}_{\mathbf{v}}\mathbf{u}$ of $\mathbf{u}$ orthogonal of $\mathbf{v}$.

   The answer key has not been proofread, use with caution.

   (a) $\mathbf{v} = (2, 3, 5)$, $\mathbf{u} = (3, 5, 7)$.
   (b) $\mathbf{v} = (5, 1, -3)$, $\mathbf{u} = (2, 3, 5)$.