Calculus II
Homework on Lecture 20

1. (a) A tank contains 30 kg of salt dissolved in 10000 liters of water and salt solution. Brine that contains 0.05 kg of salt per liter enters the tank at a rate of 10 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate (10 liters per minute). Determine how much salt remains in the tank after 45 minutes.

   (b) A tank contains 1000 kg of salt dissolved in 10000 liters of water. Brine that contains 0.05 kg of salt per liter of water enters the tank at a rate of 30 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate (30 liters per minute).

   i. Determine how much salt remains in the tank after an hour. The answer key has not been proofread, use with caution.

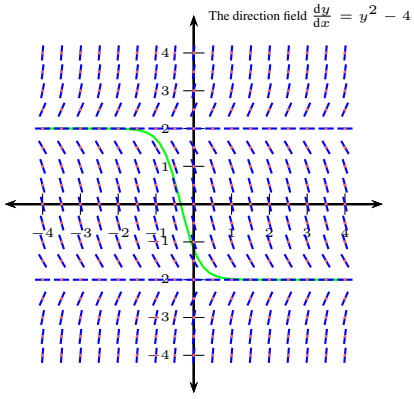
   ii. Determine how much time will be needed in order to have the concentration of salt in the tank reach 0.0501kg/liter. The answer key has not been proofread, use with caution.

2. (a)
$$\frac{\mathrm{d}y}{\mathrm{d}x} = y^2 - 1 \quad . \tag{1}$$

   i. Find all solutions of the differential equation above.

   ii. Find a solution for which $y(0) = -\frac{3}{5}$.

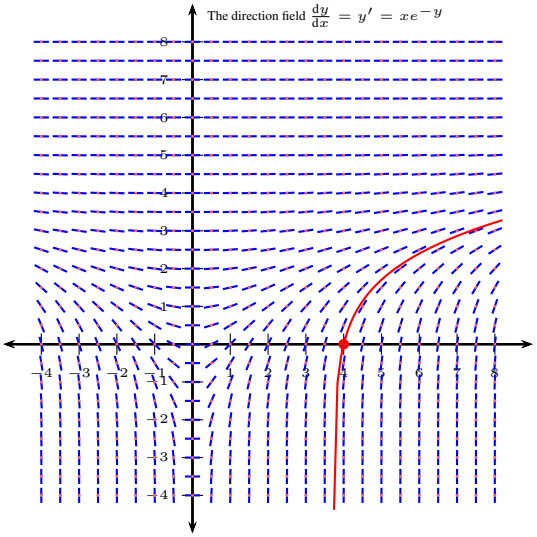
   (b) i. Find the general solution to the differential equation
$$\frac{\mathrm{d}y}{\mathrm{d}x} = y^2 - 4 \quad .$$
   Below is a computer-generated plot of the direction field $\frac{\mathrm{d}y}{\mathrm{d}x} = y^2 - 4$, you may use it to get a feeling for what your answer should look like.



   ii. Find a solution of the above equation for which $y(0) = -\frac{6}{5}$.

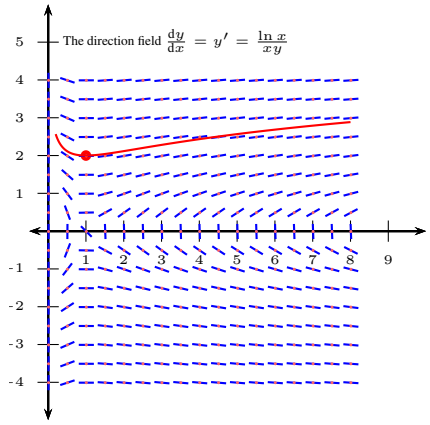
   (c) Solve the initial-value differential equation $y' = y^2(1 + x)$, $y(0) = 3$.

   (d) Solve the initial-value differential equation problem
$$y' = xe^{-y} \quad , \qquad y(4) = 0.$$
   Below is a computer-generated plot of the corresponding direction field, you may use it to get a feeling for what your answer should look like.



   (e) Solve the initial-value differential equation problem
$$y' = \frac{\ln x}{xy} \quad , \qquad y(1) = 2.$$
   Below is a computer-generated plot of the corresponding direction field, you may use it to get a feeling for what

your answer should look like.



(f) i. Solve the initial-value differential equation problem

$$y' = x \tan y \quad , \qquad y(0) = \arcsin\left(\frac{1}{e}\right) \approx 0.376728.$$

ii. Solve the same differential equation with initial condition $y(0) = \pi + \arcsin\left(-\frac{1}{e}\right) \approx 2.764865$.

Below is a computer-generated plot of corresponding direction field, you may use it to get a feeling for what your answer should look like.