

Calculus II

Homework on Lecture 16

1. Express the infinite decimal number as a rational number.

(a) $0.\overline{9} = 0.99999 \dots$

(b) $1.\overline{6} = 1.6666 \dots$

(c) $1.\overline{3} = 1.3333 \dots$

(d) $1.\overline{19} = 1.191919 \dots$

(e) $0.\overline{09} = 0.09090909 \dots$

(f) $2.\overline{16} = 2.16161616 \dots$

(g) $2014.\overline{2014} = 2014.2014201420142014 \dots$

2. Express the sum of the series as a rational number.

(a) $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{5^n}$

(b) $\sum_{n=0}^{\infty} \frac{2^n + 5^n}{10^n}$

(c) $\sum_{n=1}^{\infty} \frac{5^n - 3^n}{7^n}$

(d) $\sum_{n=1}^{\infty} \frac{3^{n+1} + 7^{n-1}}{21^n}$

(e) $\sum_{n=0}^{\infty} \frac{2^{n+1} + (-3)^{n-1}}{5^n}$

3. Sum the telescoping series (a sum is “telescoping” if it can be broken into summands so that consecutive terms cancel).

(a) $\sum_{n=0}^{\infty} \frac{-6}{9n^2 + 3n - 2} \quad .$

(b) $\sum_{n=3}^{\infty} \frac{3}{n^2 - 3n + 2} \quad .$

(c) $\sum_{n=2}^{\infty} \ln \left(1 - \frac{1}{n^2} \right) .$ (Hint: Use the properties of the logarithm to aim for a telescoping series).

4. **Problems b, c, d will not appear on the quiz.** Use partial fractions to sum the telescoping series (a sum is “telescoping” if it can be broken into summands so that consecutive terms cancel).

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$

(b) $\sum_{n=2}^{\infty} \frac{2n + 1}{n^4 + 2n^3 - n^2 - 2n}$

(c) $\sum_{n=1}^{\infty} \frac{2n}{n^4 - 3n^2 + 1}$

(d) $\sum_{n=3}^{\infty} \frac{n^2 + n + 2}{n^4 - 5n^2 + 4}$