Calculus III
Homework on Lecture 12

1. Using the second derivative test, find the local minima and maxima as well as the saddle points of the function.

   (a) $f(x, y) = 1 + x^3 + y^3 - 3xy$.

   (b) $f(x, y) = x^3 y + x^2 - 27y$.

   (c) $f(x, y) = e^{2y - x^2 - y^2}$.

   (d) $f(x, y) = e^x \sin y$.

   (e) $f(x, y) = x^2 + y^2 + \frac{1}{x^2 y^2}$.

   (f) $f(x, y) = x^2 + x^2 y + y^3 - 4y$.

2. Find the maximum of the function subject to the given restriction, or show the maximum does not exist.

   The problems don't have an answer key yet. If you think that a problem is incorrectly posed, make a clean argument why that is the case.

   (a) $f(x, y) = x^2 + 2y^2$, $xy = 1$.

   (b) $f(x, y) = 4x + 5y$, $x^2 + y^2 = 13$.

   (c) $f(x, y) = x^2 y$, $x^2 + 2y^2 = 1$.

   (d) $f(x, y) = e^{xy}$, $x^3 + y^3 = 2$.

   (e) $f(x, y) = x + 3y + 5z$, $x^2 + y^2 + z^2 = 35$.

   (f) $f(x, y) = x - z$, $x^2 + 3y^2 + z^2 = 1$.

   (g) $f(x, y) = xyz$, $x^2 + 3y^2 + 5z^2 = 8$.

   (h) $f(x, y) = x^2 y^2 z^2$, $x^2 + y^2 + z^2 = 1$.

   (i) $f(x, y) = x^2 + y^2 + z^2$, $x^4 + y^4 + z^4 = 1$.

   (j) $f(x, y) = x^4 + y^4 + z^4$, $x^2 + y^2 + z^2 = 1$.

   (k) $f(x_1, \dots, x_n) = x_1 + \dots + x_n$, $x_1^2 + \dots + x_n^2 = 1$.

   (l) Find the local extrema of $f(x, y) = y + x$ when x, y satisfy the restriction $y^2 + y + x^2 + x = 1$.