Precalculus
Lecture 21

Todor Milev
https://github.com/tmilev/freecalc

2020

Outline

Outline

Outline

License to use and redistribute

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$:

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$:

$$\begin{aligned}(f+g)(x) &= \\ (f-g)(x) &= \\ (f \cdot g)(x) &= \\ \left(\frac{f}{g}\right)(x) &= \end{aligned}$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$:

$$
\begin{aligned}
(f+g)(x) &= f(x)+g(x)\\
(f-g)(x) &=\\
(f \cdot g)(x) &=\\
\left(\frac{f}{g}\right)(x) &=
\end{aligned}
$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$
\begin{array}{rcl}
(f+g)(x) & = & f(x)+g(x) \\
(f-g)(x) & = & ? \\
(f\cdot g)(x) & = & \\
\left(\frac{f}{g}\right)(x) & = &
\end{array}
$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcl}(f+g)(x) & = & f(x)+g(x)\\ (f-g)(x) & = & f(x)-g(x)\\ (f\cdot g)(x) & = & \\ \left(\frac{f}{g}\right)(x) & = & \end{array}$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcl}(f+g)(x) & = & f(x)+g(x)\\ (f-g)(x) & = & f(x)-g(x)\\ (f\cdot g)(x) & = & \textbf{?}\\ \left(\frac{f}{g}\right)(x) & = & \end{array}$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$:

$$\begin{aligned}(f+g)(x) &= f(x)+g(x)\\ (f-g)(x) &= f(x)-g(x)\\ (f\cdot g)(x) &= f(x)\cdot g(x)\\ \left(\frac{f}{g}\right)(x) &= \end{aligned}$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$\begin{aligned}(f+g)(x) &= f(x)+g(x)\\(f-g)(x) &= f(x)-g(x)\\(f\cdot g)(x) &= f(x)\cdot g(x)\\\left(\frac{f}{g}\right)(x) &= \textbf{?}\end{aligned}$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcl|l} (f+g)(x) & = & f(x)+g(x) & \\ (f-g)(x) & = & f(x)-g(x) & \\ (f \cdot g)(x) & = & f(x) \cdot g(x) & \\ \left(\frac{f}{g}\right)(x) & = & \frac{f(x)}{g(x)} & \text{for } g(x) \neq 0 \end{array} .$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$
\begin{array}{rcll}
(f+g)(x) & = & f(x)+g(x) & \\
(f-g)(x) & = & f(x)-g(x) & \\
(f\cdot g)(x) & = & f(x)\cdot g(x) & \\
\left(\frac{f}{g}\right)(x) & = & \frac{f(x)}{g(x)} & \Big| \text{ for } g(x)\neq 0 \quad .
\end{array}
$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcl|l} (f+g)(x) &=& f(x)+g(x) & \\ (f-g)(x) &=& f(x)-g(x) & \\ (f\cdot g)(x) &=& f(x)\cdot g(x) & \\ \left(\frac{f}{g}\right)(x) &=& \frac{f(x)}{g(x)} & \text{for } g(x)\neq 0 \end{array}.$$

Let $\mathrm{Dom}(f)$ denote the domain of f.

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcl} (f+g)(x) & = & f(x)+g(x) \\ (f-g)(x) & = & f(x)-g(x) \\ (f \cdot g)(x) & = & f(x) \cdot g(x) \\ \left(\frac{f}{g}\right)(x) & = & \frac{f(x)}{g(x)} \end{array} \quad \Bigg| \text{ for } g(x) \neq 0 \quad .$$

Let $\mathrm{Dom}(f)$ denote the domain of f.

$$\begin{array}{rcl} \mathrm{Dom}(f+g) & = & \textbf{?} \\ \mathrm{Dom}(f-g) & = & \textbf{?} \\ \mathrm{Dom}(f \cdot g) & = & \textbf{?} \\ \mathrm{Dom}\left(\frac{f}{g}\right) & = & \textbf{?} \end{array}$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcl|l}
(f+g)(x) &=& f(x)+g(x) & \\
(f-g)(x) &=& f(x)-g(x) & \\
(f\cdot g)(x) &=& f(x)\cdot g(x) & \\
\left(\frac{f}{g}\right)(x) &=& \frac{f(x)}{g(x)} & \text{for } g(x)\neq 0 \quad .
\end{array}$$

Let $\mathrm{Dom}(f)$ denote the domain of f. The function $f+g$ is defined only if both f and g are defined, and similarly for the others.

$$\begin{array}{rcl}
\mathrm{Dom}(f+g) &=& \textbf{?} \\
\mathrm{Dom}(f-g) &=& \textbf{?} \\
\mathrm{Dom}(f\cdot g) &=& \textbf{?} \\
\mathrm{Dom}\left(\frac{f}{g}\right) &=& \textbf{?}
\end{array}$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcl}(f+g)(x) &=& f(x)+g(x)\\ (f-g)(x) &=& f(x)-g(x)\\ (f\cdot g)(x) &=& f(x)\cdot g(x)\\ \left(\frac{f}{g}\right)(x) &=& \frac{f(x)}{g(x)} \qquad \Big| \text{ for } g(x)\neq 0 \quad .\end{array}$$

Let $\mathrm{Dom}(f)$ denote the domain of f. The function $f+g$ is defined only if both f and g are defined, and similarly for the others. Therefore

$$\begin{array}{rcl}\mathrm{Dom}(f+g) &=& \textbf{?}\\ \mathrm{Dom}(f-g) &=& \textbf{?}\\ \mathrm{Dom}(f\cdot g) &=& \textbf{?}\\ \mathrm{Dom}\left(\frac{f}{g}\right) &=& \textbf{?}\end{array}$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$
\begin{array}{rcll}
(f+g)(x) &=& f(x)+g(x) & \\
(f-g)(x) &=& f(x)-g(x) & \\
(f\cdot g)(x) &=& f(x)\cdot g(x) & \\
\left(\frac{f}{g}\right)(x) &=& \frac{f(x)}{g(x)} & \Big| \text{ for } g(x)\neq 0 \quad .
\end{array}
$$

Let $\mathrm{Dom}(f)$ denote the domain of f. The function $f+g$ is defined only if both f and g are defined, and similarly for the others. Therefore

$$
\begin{array}{rcl}
\mathrm{Dom}(f+g) &=& \mathrm{Dom}(f)\cap \mathrm{Dom}(g) \\
\mathrm{Dom}(f-g) &=& \textbf{?} \\
\mathrm{Dom}(f\cdot g) &=& \textbf{?} \\
\mathrm{Dom}\left(\frac{f}{g}\right) &=& \textbf{?}
\end{array}
$$

$\cap$ stands for set intersection

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcl} (f+g)(x) & = & f(x)+g(x) \\ (f-g)(x) & = & f(x)-g(x) \\ (f\cdot g)(x) & = & f(x)\cdot g(x) \\ \left(\frac{f}{g}\right)(x) & = & \frac{f(x)}{g(x)} \end{array} \qquad \Bigg| \text{ for } g(x)\neq 0 \quad .$$

Let $\mathrm{Dom}(f)$ denote the domain of f. The function $f+g$ is defined only if both f and g are defined, and similarly for the others. Therefore

$$\begin{array}{rcl} \mathrm{Dom}(f+g) & = & \mathrm{Dom}(f)\cap\mathrm{Dom}(g) \\ \mathrm{Dom}(f-g) & = & \textbf{?} \\ \mathrm{Dom}(f\cdot g) & = & \textbf{?} \\ \mathrm{Dom}\left(\frac{f}{g}\right) & = & \textbf{?} \end{array}$$

$\cap$ stands for set intersection

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcl|l}
(f+g)(x) &=& f(x)+g(x) & \\
(f-g)(x) &=& f(x)-g(x) & \\
(f\cdot g)(x) &=& f(x)\cdot g(x) & \\
\left(\frac{f}{g}\right)(x) &=& \frac{f(x)}{g(x)} & \text{for } g(x)\neq 0
\end{array}.$$

Let Dom(f) denote the domain of f. The function $f+g$ is defined only if both f and g are defined, and similarly for the others. Therefore

$$\begin{array}{rcl}
\mathrm{Dom}(f+g) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g)\\
\mathrm{Dom}(f-g) &=& \textbf{?}\\
\mathrm{Dom}(f\cdot g) &=& \textbf{?}\\
\mathrm{Dom}\left(\frac{f}{g}\right) &=& \textbf{?}
\end{array}$$

$\cap$ stands for set intersection

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcll} (f+g)(x) & = & f(x)+g(x) & \\ (f-g)(x) & = & f(x)-g(x) & \\ (f\cdot g)(x) & = & f(x)\cdot g(x) & \\ \left(\frac{f}{g}\right)(x) & = & \frac{f(x)}{g(x)} & \Big| \text{ for } g(x)\neq 0 \quad . \end{array}$$

Let $\mathrm{Dom}(f)$ denote the domain of f. The function $f+g$ is defined only if both f and g are defined, and similarly for the others. Therefore

$$\begin{array}{rcl} \mathrm{Dom}(f+g) & = & \mathrm{Dom}(f)\cap\mathrm{Dom}(g) \\ \mathrm{Dom}(f-g) & = & \mathrm{Dom}(f)\cap\mathrm{Dom}(g) \\ \mathrm{Dom}(f\cdot g) & = & \mathrm{Dom}(f)\cap\mathrm{Dom}(g) \end{array}$$

$\cap$ stands for set intersection

$$\mathrm{Dom}\left(\frac{f}{g}\right) = \textbf{?}$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcl|l} (f+g)(x) & = & f(x)+g(x) & \\ (f-g)(x) & = & f(x)-g(x) & \\ (f\cdot g)(x) & = & f(x)\cdot g(x) & \\ \left(\frac{f}{g}\right)(x) & = & \frac{f(x)}{g(x)} & \text{for } g(x)\neq 0 \end{array} .$$

Let $\mathrm{Dom}(f)$ denote the domain of f. The function $f+g$ is defined only if both f and g are defined, and similarly for the others. Therefore

$$\begin{array}{rcl} \mathrm{Dom}(f+g) & = & \mathrm{Dom}(f)\cap\mathrm{Dom}(g) \\ \mathrm{Dom}(f-g) & = & \mathrm{Dom}(f)\cap\mathrm{Dom}(g) \\ \mathrm{Dom}(f\cdot g) & = & \mathrm{Dom}(f)\cap\mathrm{Dom}(g) \end{array}$$

$\cap$ stands for set intersection

$$\mathrm{Dom}\left(\frac{f}{g}\right) = \textbf{?}$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$
\begin{array}{rcl}
(f+g)(x) & = & f(x)+g(x) \\
(f-g)(x) & = & f(x)-g(x) \\
(f\cdot g)(x) & = & f(x)\cdot g(x) \\
\left(\frac{f}{g}\right)(x) & = & \frac{f(x)}{g(x)} \qquad \Big| \text{ for } g(x)\neq 0 \quad .
\end{array}
$$

Let $\mathrm{Dom}(f)$ denote the domain of f. The function $f+g$ is defined only if both f and g are defined, and similarly for the others. Therefore

$$
\begin{array}{rcl}
\mathrm{Dom}(f+g) & = & \mathrm{Dom}(f)\cap\mathrm{Dom}(g) \\
\mathrm{Dom}(f-g) & = & \mathrm{Dom}(f)\cap\mathrm{Dom}(g) \\
\mathrm{Dom}(f\cdot g) & = & \mathrm{Dom}(f)\cap\mathrm{Dom}(g)
\end{array}
$$

$\cap$ stands for set intersection

$$
\mathrm{Dom}\left(\frac{f}{g}\right) = \mathrm{Dom}(f)\cap\mathrm{Dom}(g)\cap\{x|g(x)\neq 0\}
$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcl}(f+g)(x) &=& f(x)+g(x)\\ (f-g)(x) &=& f(x)-g(x)\\ (f\cdot g)(x) &=& f(x)\cdot g(x)\\ \left(\frac{f}{g}\right)(x) &=& \frac{f(x)}{g(x)} \qquad \Big| \text{ for } g(x)\neq 0 \quad .\end{array}$$

Let $\mathrm{Dom}(f)$ denote the domain of f. The function $f+g$ is defined only if both f and g are defined, and similarly for the others. Therefore

$$\begin{array}{rcl}\mathrm{Dom}(f+g) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g)\\ \mathrm{Dom}(f-g) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g)\\ \mathrm{Dom}(f\cdot g) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g)\\ \mathrm{Dom}\left(\frac{f}{g}\right) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g)\cap\{x|g(x)\neq 0\}\end{array}$$

$\cap$ stands for set intersection

right expr. stands for set where $g(x)\neq 0$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcl|l} (f+g)(x) &=& f(x)+g(x) & \\ (f-g)(x) &=& f(x)-g(x) & \\ (f\cdot g)(x) &=& f(x)\cdot g(x) & \\ \left(\frac{f}{g}\right)(x) &=& \frac{f(x)}{g(x)} & \text{for } g(x)\neq 0 \end{array}.$$

Let $\mathrm{Dom}(f)$ denote the domain of f. The function $f+g$ is defined only if both f and g are defined, and similarly for the others. Therefore

$$\begin{array}{rcll} \mathrm{Dom}(f+g) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g) & \Big| \cap \text{ stands for set intersection} \\ \mathrm{Dom}(f-g) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g) & \\ \mathrm{Dom}(f\cdot g) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g) & \\ \mathrm{Dom}\left(\frac{f}{g}\right) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g)\cap\{x|g(x)\neq 0\} & \Big| \text{right expr. stands for set where } g(x)\neq 0 \end{array}$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f\cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcll} (f+g)(x) &=& f(x)+g(x) & \\ (f-g)(x) &=& f(x)-g(x) & \\ (f\cdot g)(x) &=& f(x)\cdot g(x) & \\ \left(\frac{f}{g}\right)(x) &=& \frac{f(x)}{g(x)} & \Big| \text{ for } g(x)\neq 0 \end{array} .$$

Let $\mathrm{Dom}(f)$ denote the domain of f. The function $f+g$ is defined only if both f and g are defined, and similarly for the others. Therefore

$$\begin{array}{rcl|l} \mathrm{Dom}(f+g) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g) & \cap \text{ stands for} \\ \mathrm{Dom}(f-g) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g) & \text{set intersection} \\ \mathrm{Dom}(f\cdot g) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g) & \\ \mathrm{Dom}\left(\frac{f}{g}\right) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g)\cap\{x|g(x)\neq 0\} & \text{right expr. stands for set where } g(x)\neq 0 \end{array}$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$:

$$\begin{array}{rcl|l} (f+g)(x) & = & f(x)+g(x) & \\ (f-g)(x) & = & f(x)-g(x) & \\ (f \cdot g)(x) & = & f(x) \cdot g(x) & \\ \left(\frac{f}{g}\right)(x) & = & \frac{f(x)}{g(x)} & \text{for } g(x) \neq 0 \end{array} .$$

Let $\mathrm{Dom}(f)$ denote the domain of f. The function $f+g$ is defined only if both f and g are defined, and similarly for the others. Therefore

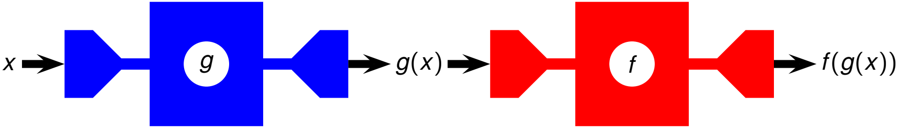
$$\begin{array}{rcl|l} \mathrm{Dom}(f+g) & = & \mathrm{Dom}(f) \cap \mathrm{Dom}(g) & \cap \text{ stands for} \\ \mathrm{Dom}(f-g) & = & \mathrm{Dom}(f) \cap \mathrm{Dom}(g) & \text{set intersection} \\ \mathrm{Dom}(f \cdot g) & = & \mathrm{Dom}(f) \cap \mathrm{Dom}(g) & \\ & & & \text{right expr.} \\ \mathrm{Dom}\left(\frac{f}{g}\right) & = & \mathrm{Dom}(f) \cap \mathrm{Dom}(g) \cap \{x | g(x) \neq 0\} & \text{stands for set} \\ & & & \text{where } g(x) \neq 0 \end{array}$$

Definition (Composition of f and g)

If f and g are two functions, then the composition of f and g is written $f \circ g$ and is defined by the formula

$$(f \circ g)(x) = f(g(x)).$$

Imagine f and g as machines taking some input and producing some output. Then $f \circ g$ corresponds to attaching both machines end-to-end so that the output of g becomes the input of f.

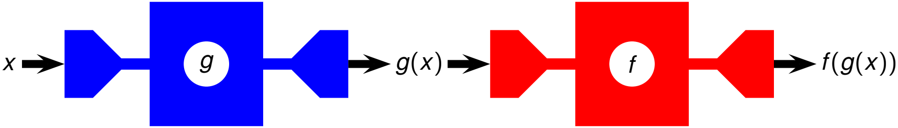


Definition (Composition of f and g)

If f and g are two functions, then the composition of f and g is written $f \circ g$ and is defined by the formula

$$(f \circ g)(x) = f(g(x)).$$

Imagine f and g as machines taking some input and producing some output. Then $f \circ g$ corresponds to attaching both machines end-to-end so that the output of g becomes the input of f.



The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f. If the domain of f is A and the domain of g is B, we write this as

$$\{x \in B | \; g(x) \in A\}.$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$(f \circ g)(x)$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) \quad = \quad f(g(x))$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(f \circ g)(x) \quad = \quad f(g(x)) = f\left(\sqrt{3 - x}\right)$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) \quad = \quad f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(f \circ g)(x) \quad = \quad f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) \quad = \quad f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$3 - x \geq 0$$

Example

Find $f \circ g$, $g \circ f$, $g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) \quad = \quad f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) \quad = \quad f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \\ \color{red}{x} &\color{red}{\in} \color{red}{\textbf{?}} \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \\ x &\in (-\infty, 3]. \end{aligned}$$

Example

Find $f \circ g$, $g \circ f$, $g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(f \circ g)(x) \quad = \quad f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{array}{rcl} 3 - x & \geq & 0 \\ -x & \geq & -3 \\ x & \leq & 3 \\ x & \in & (-\infty, 3]. \end{array}$$

$(g \circ f)(x)$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \\ x &\in (-\infty, 3]. \end{aligned}$$

$$(g \circ f)(x) = g(f(x))$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{aligned} 3-x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \\ x &\in (-\infty, 3]. \end{aligned}$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x})$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$
\begin{aligned}
(f \circ g)(x) &= f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}
\end{aligned}
$$

Domain:

$$
\begin{aligned}
3 - x &\geq 0 \\
-x &\geq -3 \\
x &\leq 3 \\
x &\in (-\infty, 3].
\end{aligned}
$$

$$
(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{3 - \sqrt{x}}
$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \\ x &\in (-\infty, 3]. \end{aligned}$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{3 - \sqrt{x}}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) \quad = \quad f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \\ x &\in (-\infty, 3]. \end{aligned}$$

$$(g \circ f)(x) \quad = \quad g(f(x)) = g(\sqrt{x}) = \sqrt{3 - \sqrt{x}}$$

Domain :

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \\ x &\in (-\infty, 3]. \end{aligned}$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{3 - \sqrt{x}}$$

Domain :

$$x \geq 0$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$
\begin{array}{rcl}
(f \circ g)(x) & = & f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}
\end{array}
$$

Domain:

$$
\begin{array}{rcl}
3 - x & \geq & 0 \\
-x & \geq & -3 \\
x & \leq & 3 \\
x & \in & (-\infty, 3].
\end{array}
$$

$$
\begin{array}{rcl}
(g \circ f)(x) & = & g(f(x)) = g(\sqrt{x}) = \sqrt{3 - \sqrt{x}}
\end{array}
$$

Domain :

$$
\begin{array}{rcl}
x & \geq & 0 \\
3 - \sqrt{x} & \geq & 0
\end{array}
$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \\ x &\in (-\infty, 3]. \end{aligned}$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{3-\sqrt{x}}$$

Domain :

$$\begin{aligned} x &\geq 0 \\ {\color{red}3} - \sqrt{x} &\geq 0 \\ -\sqrt{x} &\geq {\color{red}-3} \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) \quad = \quad f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \\ x &\in (-\infty, 3]. \end{aligned}$$

$$(g \circ f)(x) \quad = \quad g(f(x)) = g(\sqrt{x}) = \sqrt{3-\sqrt{x}}$$

Domain :

$$\begin{aligned} x &\geq 0 \\ 3 - \sqrt{x} &\geq 0 \\ -\sqrt{x} &\geq -3 \\ \sqrt{x} &\leq 3 \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{array}{rcl} 3-x & \geq & 0 \\ -x & \geq & -3 \\ x & \leq & 3 \\ x & \in & (-\infty, 3]. \end{array}$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{3-\sqrt{x}}$$

Domain :

$$\begin{array}{rcl} x & \geq & 0 \\ 3-\sqrt{x} & \geq & 0 \\ -\sqrt{x} & \geq & -3 \\ \sqrt{x} & \leq & 3 \\ x & \leq & 9 \end{array}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$
\begin{array}{rcl}
(f \circ g)(x) & = & f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}\\
\text{Domain:} & & \\
3-x & \geq & 0\\
-x & \geq & -3\\
x & \leq & 3\\
x & \in & (-\infty, 3].\\
(g \circ f)(x) & = & g(f(x)) = g(\sqrt{x}) = \sqrt{3-\sqrt{x}}\\
\text{Domain :} & & \\
x & \geq & 0\\
3-\sqrt{x} & \geq & 0\\
-\sqrt{x} & \geq & -3\\
\sqrt{x} & \leq & 3\\
x & \leq & 9\\
x & \in & \textbf{?}
\end{array}
$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \\ x &\in (-\infty, 3]. \end{aligned}$$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{3 - \sqrt{x}}$$

Domain :

$$\begin{aligned} x &\geq 0 \\ 3 - \sqrt{x} &\geq 0 \\ -\sqrt{x} &\geq -3 \\ \sqrt{x} &\leq 3 \\ x &\leq 9 \\ x &\in [0, 9] \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$(g \circ g)(x)$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(g \circ g)(x) \quad = \quad g(g(x))$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(g \circ g)(x) = g(g(x)) = g\left(\sqrt{3 - x}\right)$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(g \circ g)(x) = g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3-\sqrt{3-x}}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(g \circ g)(x) \quad = \quad g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3 - \sqrt{3 - x}}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(g \circ g)(x) \quad = \quad g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3 - \sqrt{3 - x}}$$

Domain :

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(g \circ g)(x) = g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3 - \sqrt{3 - x}}$$

Domain :

$$3 - x \geq 0$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(g \circ g)(x) = g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3 - \sqrt{3-x}}$$

Domain :

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(g \circ g)(x) = g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3 - \sqrt{3 - x}}$$

Domain :

$$\begin{array}{rcl} 3 - x & \geq & 0 \\ -x & \geq & -3 \\ x & \leq & 3 \end{array}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$
\begin{array}{rcl}
(g \circ g)(x) & = & g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3-\sqrt{3-x}} \\
\text{Domain :} & & \\
3 - x & \geq & 0 \\
-x & \geq & -3 \\
x & \leq & 3 \\
3 - \sqrt{3-x} & \geq & 0
\end{array}
$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$
\begin{array}{rcl}
(g \circ g)(x) & = & g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3-\sqrt{3-x}} \\
\text{Domain}: & & \\
3-x & \geq & 0 \\
-x & \geq & -3 \\
x & \leq & 3 \\
3-\sqrt{3-x} & \geq & 0 \\
-\sqrt{3-x} & \geq & -3
\end{array}
$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$
\begin{aligned}
(g \circ g)(x) &= g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3-\sqrt{3-x}} \\
\text{Domain :} & \\
3 - x &\geq 0 \\
-x &\geq -3 \\
x &\leq 3 \\
3 - \sqrt{3-x} &\geq 0 \\
-\sqrt{3-x} &\geq -3 \\
\sqrt{3-x} &\leq 3
\end{aligned}
$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$
\begin{array}{rcl}
(g \circ g)(x) & = & g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3-\sqrt{3-x}} \\
\text{Domain :} & & \\
3-x & \geq & 0 \\
-x & \geq & -3 \\
x & \leq & 3 \\
3-\sqrt{3-x} & \geq & 0 \\
-\sqrt{3-x} & \geq & -3 \\
\sqrt{3-x} & \leq & 3 \\
3-x & \leq & 9
\end{array}
$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$
\begin{array}{rcl}
(g \circ g)(x) & = & g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3-\sqrt{3-x}} \\
\text{Domain :} & & \\
3-x & \geq & 0 \\
-x & \geq & -3 \\
x & \leq & 3 \\
3-\sqrt{3-x} & \geq & 0 \\
-\sqrt{3-x} & \geq & -3 \\
\sqrt{3-x} & \leq & 3 \\
3-x & \leq & 9 \\
-x & \leq & 6
\end{array}
$$

Example

Find $f \circ g$, $g \circ f$, $g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$
\begin{aligned}
(g \circ g)(x) &= g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3-\sqrt{3-x}}\\
\text{Domain :}&\\
3-x &\geq 0\\
-x &\geq -3\\
x &\leq 3\\
3-\sqrt{3-x} &\geq 0\\
-\sqrt{3-x} &\geq -3\\
\sqrt{3-x} &\leq 3\\
3-x &\leq 9\\
-x &\leq 6\\
x &\geq -6
\end{aligned}
$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$\begin{array}{rcl} (g \circ g)(x) & = & g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3-\sqrt{3-x}} \\ \text{Domain :} & & \\ 3 - x & \geq & 0 \\ -x & \geq & -3 \\ x & \leq & 3 \\ 3-\sqrt{3-x} & \geq & 0 \\ -\sqrt{3-x} & \geq & -3 \\ \sqrt{3-x} & \leq & 3 \\ 3-x & \leq & 9 \\ -x & \leq & 6 \\ x & \geq & -6 \\ x & \in & \textbf{?} \end{array}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(g \circ g)(x) = g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3-\sqrt{3-x}}$$

Domain :

$$\begin{aligned}
3-x &\geq 0\\
-x &\geq -3\\
x &\leq 3\\
3-\sqrt{3-x} &\geq 0\\
-\sqrt{3-x} &\geq -3\\
\sqrt{3-x} &\leq 3\\
3-x &\leq 9\\
-x &\leq 6\\
x &\geq -6\\
x &\in [-6,3].
\end{aligned}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2}$$

$$g(x) = \frac{2x+3}{5x-7}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big|\ x \neq \textbf{?}$$

$$g(x) = \frac{2x+3}{5x-7}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x - 1}{x + 2}$$

$$g(x) = \frac{2x + 3}{5x - 7}$$

$\Big| \; x \neq -2$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad x \neq \textbf{?}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad x \neq \frac{7}{5}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big| \; x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big| \; x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x))$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big| \; x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big| \; x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right)$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big|\; x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big|\; x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x - 1}{x + 2} \qquad x \neq -2$$

$$g(x) = \frac{2x + 3}{5x - 7} \qquad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big| \; x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big| \; x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x - 1}{x + 2} \qquad \Big| \; x \neq -2$$

$$g(x) = \frac{2x + 3}{5x - 7} \qquad \Big| \; x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x + 3}{5x - 7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{5x-7}}{\frac{2x+3+(10x-14)}{5x-7}}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{5x-7}}{\frac{2x+3+(10x-14)}{5x-7}}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right)-1}{\frac{2x+3}{5x-7}+2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{5x-7}}{\frac{2x+3+(10x-14)}{5x-7}}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big| \quad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big| \quad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right)-1}{\frac{2x+3}{5x-7}+2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11} \qquad x \neq \textbf{?}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big| \; x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big| \; x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right)-1}{\frac{2x+3}{5x-7}+2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11} \qquad \Big| \; x \neq \frac{11}{12}, \frac{7}{5}$$

Example

Give simplified f-las for $f\circ g$, $f\circ f$, $g\circ f$, $g\circ g$. Find the implied domains.

$$f(x)=\frac{2x-1}{x+2} \qquad \Big| \; x\neq -2$$

$$g(x)=\frac{2x+3}{5x-7} \qquad \Big| \; x\neq \frac{7}{5}$$

$$(f\circ g)(x)=f(g(x))=f\left(\frac{2x+3}{5x-7}\right)=\frac{2\left(\frac{2x+3}{5x-7}\right)-1}{\frac{2x+3}{5x-7}+2}$$

$$=\frac{\frac{2(2x+3)}{5x-7}-\frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7}+\frac{2(5x-7)}{5x-7}}=\frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}}=\frac{-x+13}{12x-11} \qquad \Big| \; x\neq \frac{11}{12}, \frac{7}{5}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big| \quad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big| \quad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11} \qquad \Big| \quad x \neq \frac{11}{12}, \frac{7}{5}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big| \; x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big| \; x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11} \qquad \Big| \; x \neq \frac{11}{12}, \frac{7}{5}$$

$$(f \circ f)(x) = f(f(x))$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11} \qquad x \neq \frac{11}{12}, \frac{7}{5}$$

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{2x-1}{x+2}\right)$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11} \qquad x \neq \frac{11}{12}, \frac{7}{5}$$

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{2x-1}{x+2}\right) = \frac{2\left(\frac{2x-1}{x+2}\right) - 1}{\frac{2x-1}{x+2} + 2}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big| \; x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big| \; x \neq \frac{7}{5}$$

$$(f\circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right)-1}{\frac{2x+3}{5x-7}+2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11} \qquad \Big| \; x \neq \frac{11}{12}, \frac{7}{5}$$

$$(f\circ f)(x) = f(f(x)) = f\left(\frac{2x-1}{x+2}\right) = \frac{2\left(\frac{2x-1}{x+2}\right)-1}{\frac{2x-1}{x+2}+2}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big|\; x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big|\; x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11} \qquad \Big|\; x \neq \frac{11}{12}, \frac{7}{5}$$

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{2x-1}{x+2}\right) = \frac{2\left(\frac{2x-1}{x+2}\right) - 1}{\frac{2x-1}{x+2} + 2}$$

$$= \mathbf{?}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x)=\frac{2x-1}{x+2} \qquad \Big|\; x \neq -2$$

$$g(x)=\frac{2x+3}{5x-7} \qquad \Big|\; x \neq \frac{7}{5}$$

$$(f\circ g)(x)=f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right)-1}{\frac{2x+3}{5x-7}+2}$$

$$=\frac{\frac{2(2x+3)}{5x-7}-\frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7}+\frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11} \qquad \Big|\; x \neq \frac{11}{12}, \frac{7}{5}$$

$$(f\circ f)(x)=f(f(x)) = f\left(\frac{2x-1}{x+2}\right) = \frac{2\left(\frac{2x-1}{x+2}\right)-1}{\frac{2x-1}{x+2}+2}$$

$$=\frac{3x-4}{4x+3}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big|\; x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big|\; x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right)-1}{\frac{2x+3}{5x-7}+2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11} \qquad \Big|\; x \neq \frac{11}{12}, \frac{7}{5}$$

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{2x-1}{x+2}\right) = \frac{2\left(\frac{2x-1}{x+2}\right)-1}{\frac{2x-1}{x+2}+2}$$

$$= \frac{3x-4}{4x+3} \qquad \Big|\; x \neq \textbf{?}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big| \; x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big| \; x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11} \qquad \Big| \; x \neq \frac{11}{12}, \frac{7}{5}$$

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{2x-1}{x+2}\right) = \frac{2\left(\frac{2x-1}{x+2}\right) - 1}{\frac{2x-1}{x+2} + 2}$$

$$= \frac{3x-4}{4x+3} \qquad \Big| \; x \neq -2, -\frac{3}{4}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big| \; x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big| \; x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11} \qquad \Big| \; x \neq \frac{11}{12}, \frac{7}{5}$$

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{2x-1}{x+2}\right) = \frac{2\left(\frac{2x-1}{x+2}\right) - 1}{\frac{2x-1}{x+2} + 2}$$

$$= \frac{3x-4}{4x+3} \qquad \Big| \; x \neq -2, -\frac{3}{4}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big| \; x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big| \; x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11} \qquad \Big| \; x \neq \frac{11}{12}, \frac{7}{5}$$

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{2x-1}{x+2}\right) = \frac{2\left(\frac{2x-1}{x+2}\right) - 1}{\frac{2x-1}{x+2} + 2}$$

$$= \frac{3x-4}{4x+3} \qquad \Big| \; x \neq -2, -\frac{3}{4}$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2} \qquad \Big| \; x \neq -2$$

$$g(x) = \frac{2x+3}{5x-7} \qquad \Big| \; x \neq \frac{7}{5}$$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+3}{5x-7}\right) = \frac{2\left(\frac{2x+3}{5x-7}\right) - 1}{\frac{2x+3}{5x-7} + 2}$$

$$= \frac{\frac{2(2x+3)}{5x-7} - \frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7} + \frac{2(5x-7)}{5x-7}} = \frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}} = \frac{-x+13}{12x-11} \qquad \Big| \; x \neq \frac{11}{12}, \frac{7}{5}$$

$$(f \circ f)(x) = f(f(x)) = f\left(\frac{2x-1}{x+2}\right) = \frac{2\left(\frac{2x-1}{x+2}\right) - 1}{\frac{2x-1}{x+2} + 2}$$

$$= \frac{3x-4}{4x+3} \qquad \Big| \; x \neq -2, -\frac{3}{4}$$

$(g \circ f)(x) = $ **?** $\qquad \Big| \; x \neq$ **?**

$(g \circ g)(x) = $ **?** $\qquad \Big| \; x \neq$ **?**

Example

Give simplified f-las for $f\circ g$, $f\circ f$, $g\circ f$, $g\circ g$. Find the implied domains.

$$f(x)=\frac{2x-1}{x+2} \qquad \Big| \quad x\neq -2$$

$$g(x)=\frac{2x+3}{5x-7} \qquad \Big| \quad x\neq \frac{7}{5}$$

$$(f\circ g)(x)=f(g(x))=f\left(\frac{2x+3}{5x-7}\right)=\frac{2\left(\frac{2x+3}{5x-7}\right)-1}{\frac{2x+3}{5x-7}+2}$$

$$=\frac{\frac{2(2x+3)}{5x-7}-\frac{5x-7}{5x-7}}{\frac{2x+3}{5x-7}+\frac{2(5x-7)}{5x-7}}=\frac{\frac{4x+6-(5x-7)}{\cancel{5x-7}}}{\frac{2x+3+(10x-14)}{\cancel{5x-7}}}=\frac{-x+13}{12x-11} \qquad \Big| \quad x\neq \frac{11}{12},\frac{7}{5}$$

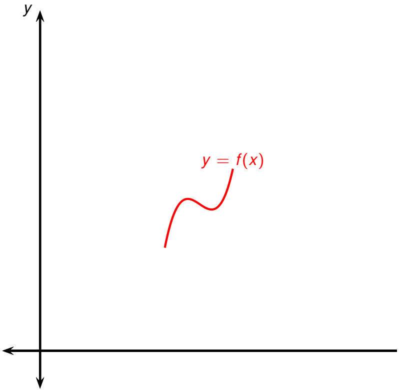
$$(f\circ f)(x)=f(f(x))=f\left(\frac{2x-1}{x+2}\right)=\frac{2\left(\frac{2x-1}{x+2}\right)-1}{\frac{2x-1}{x+2}+2}$$

$$=\frac{3x-4}{4x+3} \qquad \Big| \quad x\neq -2,-\frac{3}{4}$$

$$(g\circ f)(x)=\frac{7x+4}{3x-19} \qquad \Big| \quad x\neq -2,\frac{19}{3}$$

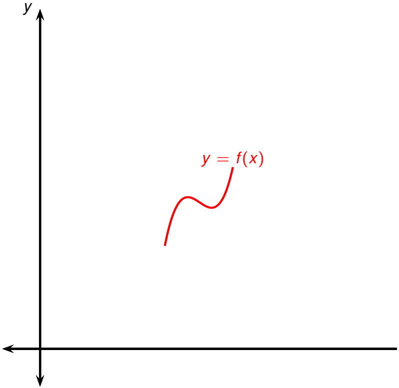
$$(g\circ g)(x)=\frac{19x-15}{-25x+64} \qquad \Big| \quad x\neq \frac{7}{5},\frac{64}{25}$$

Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

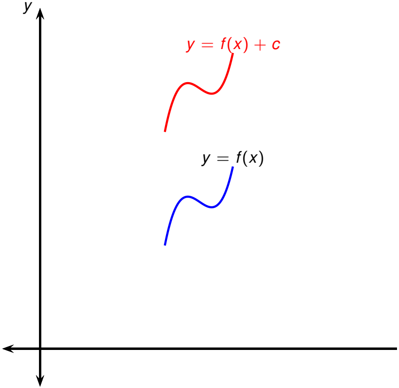
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	
$f(x) - c$	
$f(x - c)$	
$f(x + c)$	

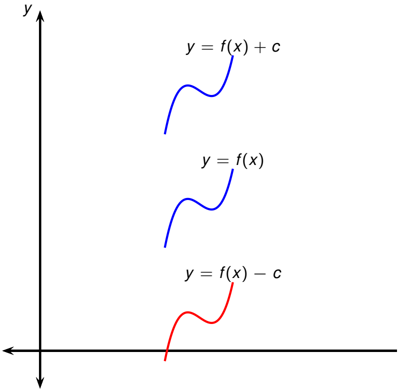
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	
$f(x - c)$	
$f(x + c)$	

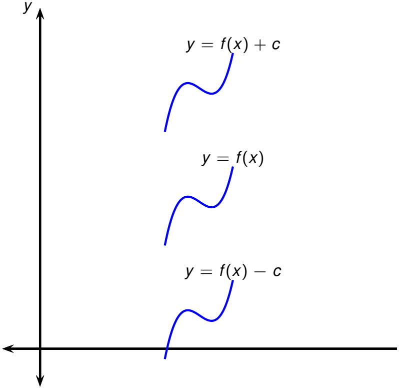
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	
$f(x + c)$	

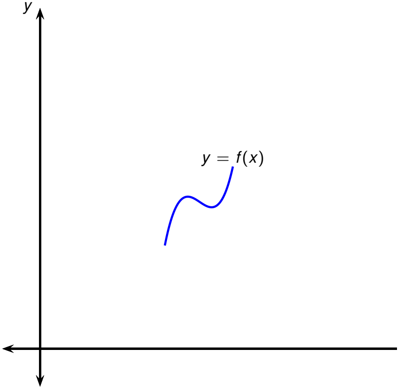
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	
$f(x + c)$	

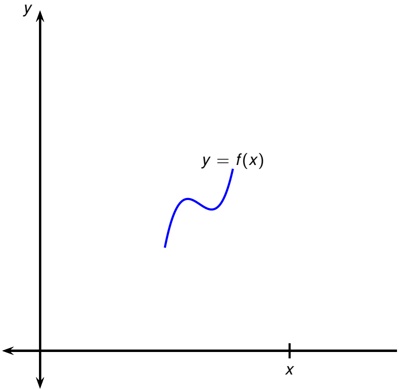
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	Shift the graph of $f(x)$ **?** .
$f(x + c)$	

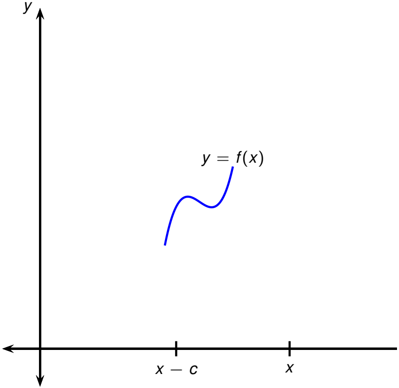
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	Shift the graph of $f(x)$ **?** .
$f(x + c)$	

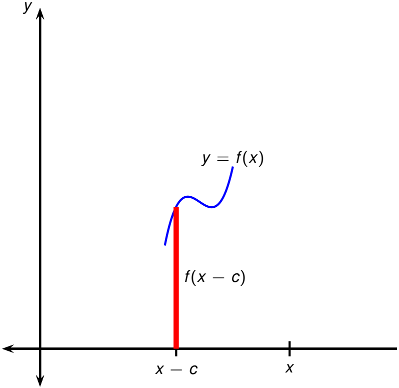
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	Shift the graph of $f(x)$ **?** .
$f(x + c)$	

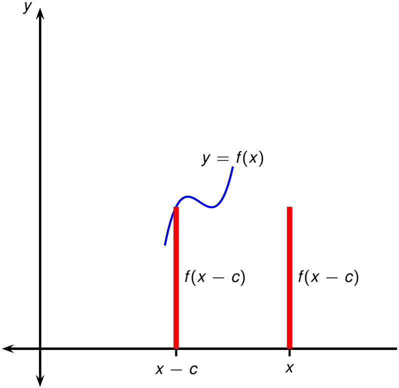
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x)+c$	Shift the graph of $f(x)$ c units up.
$f(x)-c$	Shift the graph of $f(x)$ c units down.
$f(x-c)$	Shift the graph of $f(x)$ **?** .
$f(x+c)$	

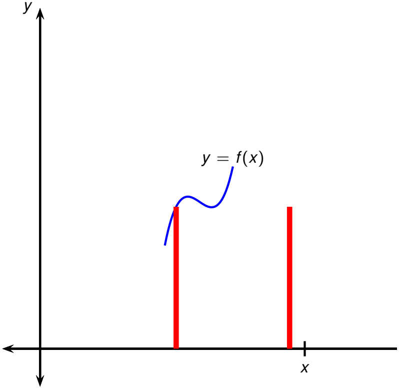
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x)+c$	Shift the graph of $f(x)$ c units up.
$f(x)-c$	Shift the graph of $f(x)$ c units down.
$f(x-c)$	Shift the graph of $f(x)$ **?** .
$f(x+c)$	

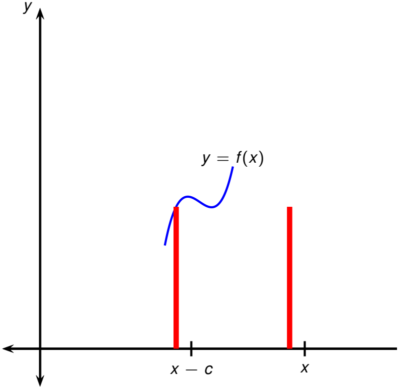
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	Shift the graph of $f(x)$ **?** .
$f(x + c)$	

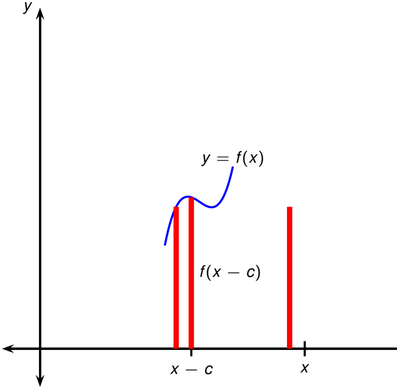
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	Shift the graph of $f(x)$ **?** .
$f(x + c)$	

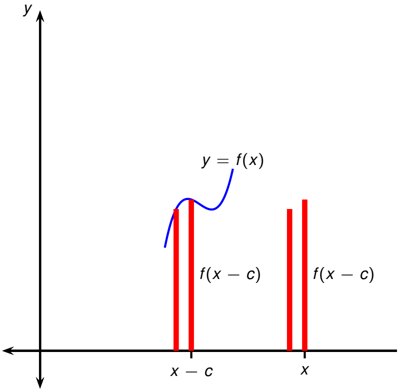
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	Shift the graph of $f(x)$ **?** .
$f(x + c)$	

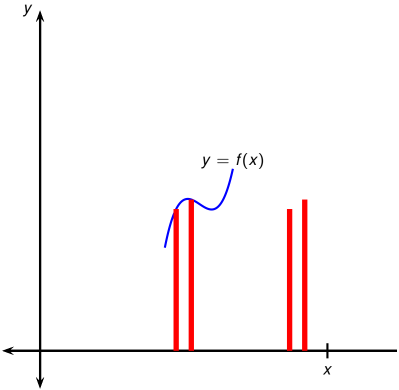
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x)+c$	Shift the graph of $f(x)$ c units up.
$f(x)-c$	Shift the graph of $f(x)$ c units down.
$f(x-c)$	Shift the graph of $f(x)$ **?** .
$f(x+c)$	

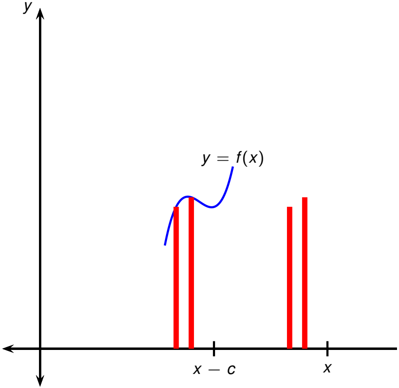
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	Shift the graph of $f(x)$ **?** .
$f(x + c)$	

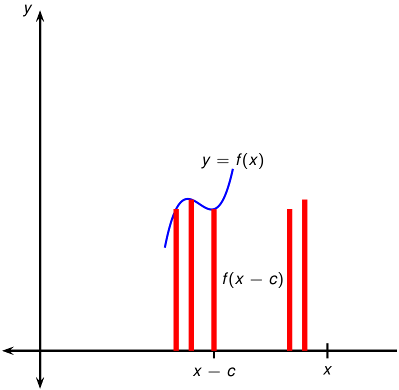
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	Shift the graph of $f(x)$ **?** .
$f(x + c)$	

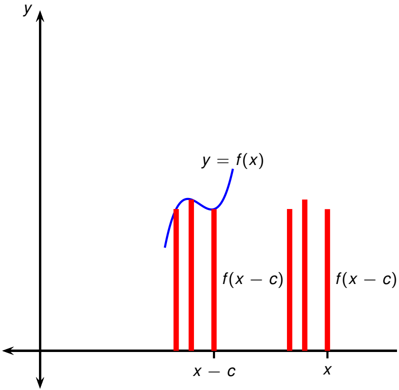
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	Shift the graph of $f(x)$ **?** .
$f(x + c)$	

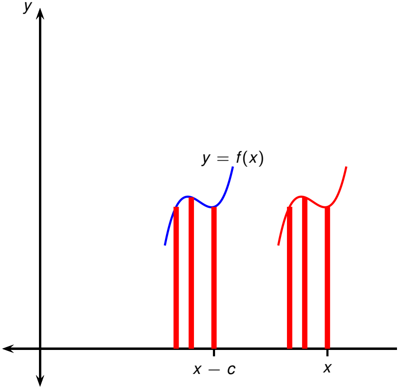
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	Shift the graph of $f(x)$ **?** .
$f(x + c)$	

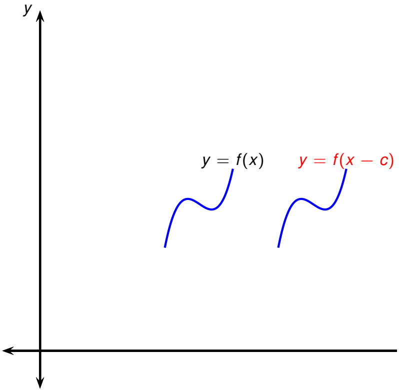
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	Shift the graph of $f(x)$ **?** .
$f(x + c)$	

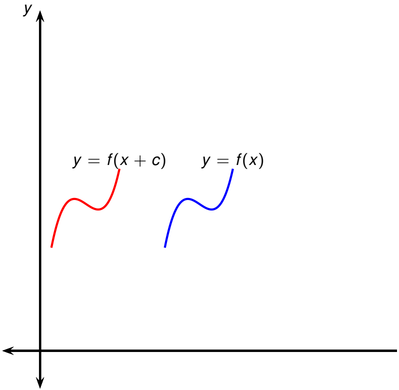
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x)+c$	Shift the graph of $f(x)$ c units up.
$f(x)-c$	Shift the graph of $f(x)$ c units down.
$f(x-c)$	Shift the graph of $f(x)$ c units right .
$f(x+c)$	

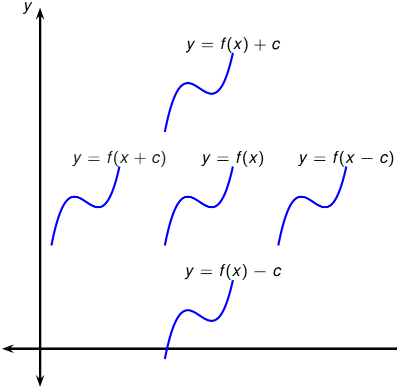
Transformations of Functions



- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	Shift the graph of $f(x)$ c units right .
$f(x + c)$	Shift the graph of $f(x)$ c units left.

Transformations of Functions

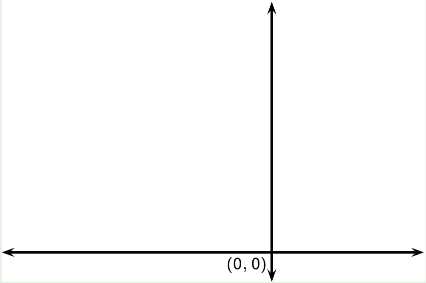


- What happens to the graph if we add/subtract a positive constant c in the equation of a function f?
- What happens if we add or subtract c from x before applying the function f?

$f(x) + c$	Shift the graph of $f(x)$ c units up.
$f(x) - c$	Shift the graph of $f(x)$ c units down.
$f(x - c)$	Shift the graph of $f(x)$ c units right .
$f(x + c)$	Shift the graph of $f(x)$ c units left.

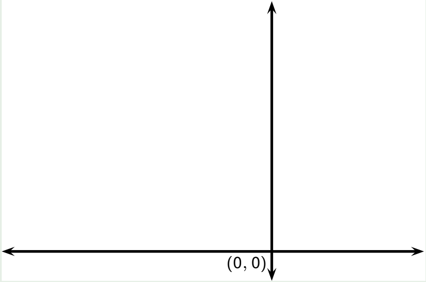
Example

Relative to the graph of $f(x) = x^2$, draw a graph of $f(x) = x^2 + 6x + 10$. Assume the graph of $f(x) = x^2$ given.



Example

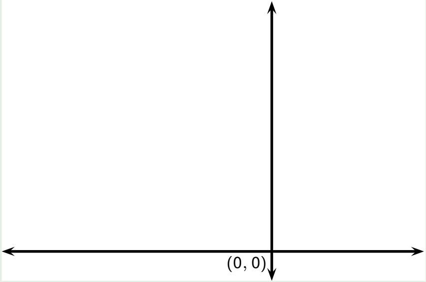
Relative to the graph of $f(x) = x^2$, draw a graph of $f(x) = x^2 + 6x + 10$. Assume the graph of $f(x) = x^2$ given.



Complete the square:

Example

Relative to the graph of $f(x) = x^2$, draw a graph of $f(x) = x^2 + 6x + 10$. Assume the graph of $f(x) = x^2$ given.

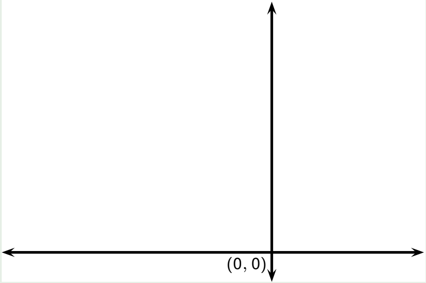


Complete the square:

$$f(x) = x^2 + 6x + 10$$

Example

Relative to the graph of $f(x) = x^2$, draw a graph of $f(x) = x^2 + 6x + 10$. Assume the graph of $f(x) = x^2$ given.

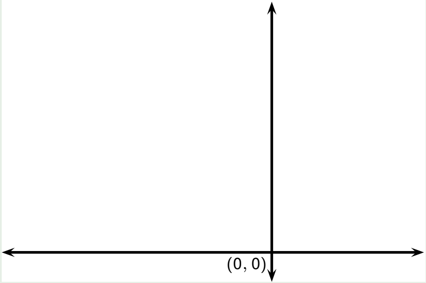


Complete the square:

$$\begin{aligned} f(x) &= x^2 + 6x + 10 \\ &= \left(x^2 + 6x + \mathbf{?}\right) + 10 - \mathbf{?} \end{aligned}$$

Example

Relative to the graph of $f(x) = x^2$, draw a graph of $f(x) = x^2 + 6x + 10$. Assume the graph of $f(x) = x^2$ given.

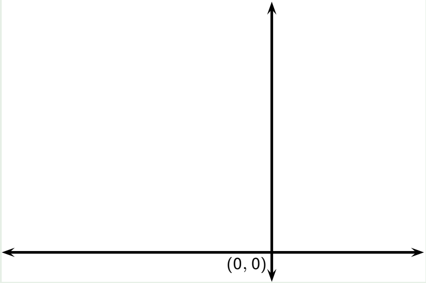


Complete the square:

$$\begin{aligned} f(x) &= x^2 + 6x + 10 \\ &= \left(x^2 + 6x + 9\right) + 10 - 9 \end{aligned}$$

Example

Relative to the graph of $f(x) = x^2$, draw a graph of $f(x) = x^2 + 6x + 10$. Assume the graph of $f(x) = x^2$ given.

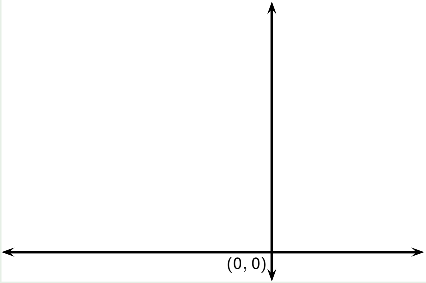


Complete the square:

$$\begin{aligned} f(x) &= x^2 + 6x + 10 \\ &= \left(x^2 + 6x + 9\right) + 10 - 9 \\ &= (x+3)^2 + 1 \end{aligned}$$

Example

Relative to the graph of $f(x) = x^2$, draw a graph of $f(x) = x^2 + 6x + 10$. Assume the graph of $f(x) = x^2$ given.

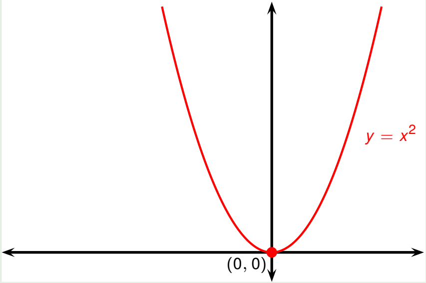


Complete the square:

$$
\begin{aligned}
f(x) &= x^2 + 6x + 10 \\
&= \left(x^2 + 6x + 9\right) + 10 - 9 \\
&= (x + 3)^2 + 1 \\
&= (x - (-3))^2 + 1
\end{aligned}
$$

Example

Relative to the graph of $f(x) = x^2$, draw a graph of $f(x) = x^2 + 6x + 10$. Assume the graph of $f(x) = x^2$ given.

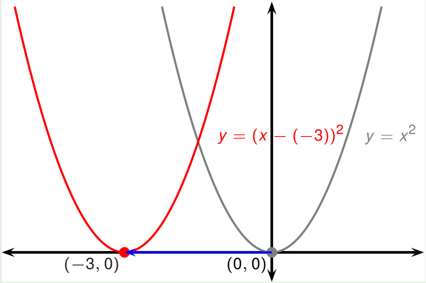


Complete the square:

$$
\begin{aligned}
f(x) &= x^2 + 6x + 10 \\
&= \left(x^2 + 6x + 9\right) + 10 - 9 \\
&= (x + 3)^2 + 1 \\
&= (x - (-3))^2 + 1
\end{aligned}
$$

Example

Relative to the graph of $f(x) = x^2$, draw a graph of $f(x) = x^2 + 6x + 10$. Assume the graph of $f(x) = x^2$ given.

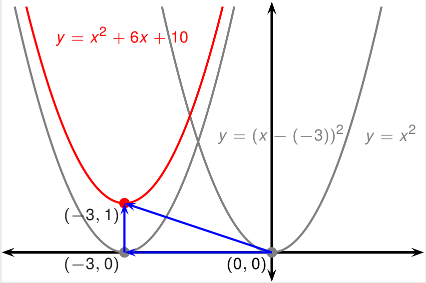


Complete the square:

$$\begin{aligned} f(x) &= x^2 + 6x + 10 \\ &= \left(x^2 + 6x + 9\right) + 10 - 9 \\ &= (x+3)^2 + 1 \\ &= (x - (-3))^2 + 1 \end{aligned}$$

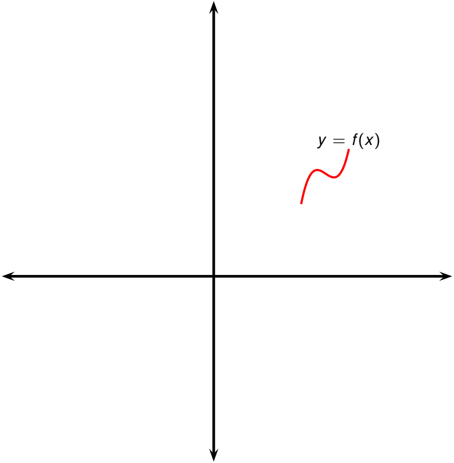
Example

Relative to the graph of $f(x) = x^2$, draw a graph of $f(x) = x^2 + 6x + 10$. Assume the graph of $f(x) = x^2$ given.



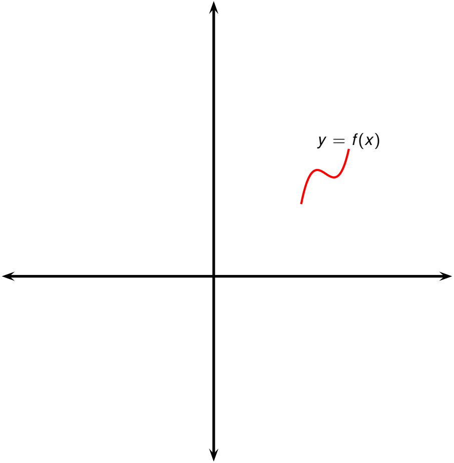
Complete the square:

$$
\begin{aligned}
f(x) &= x^2 + 6x + 10 \\
&= \left(x^2 + 6x + 9\right) + 10 - 9 \\
&= (x+3)^2 + 1 \\
&= {\color{red}(x - (-3))^2 + 1}
\end{aligned}
$$



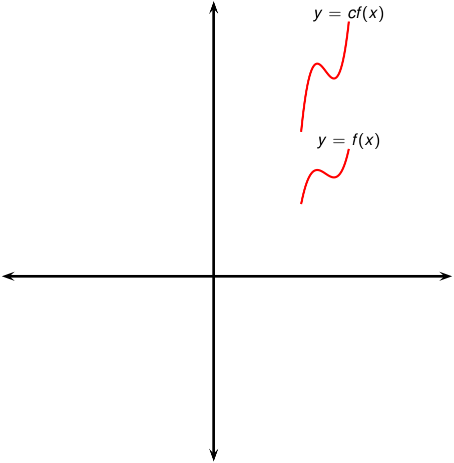
- What happens if we multiply or divide by a constant $c > 1$ in the equation of a function f?
- What happens if we multiply f by -1?
- What happens if we multiply x by -1 before applying f?

$cf(x)$ $\frac{1}{c}f(x)$ $-f(x)$ $f(-x)$	



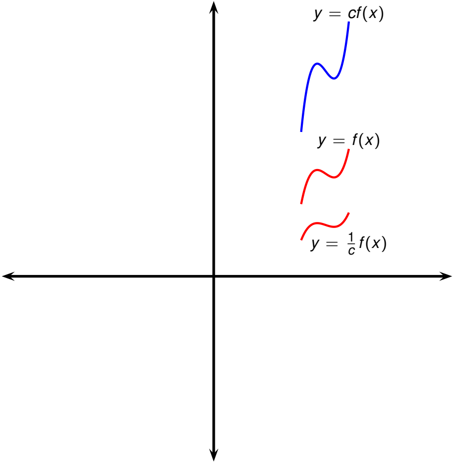
- What happens if we multiply or divide by a constant $c > 1$ in the equation of a function f?
- What happens if we multiply f by -1?
- What happens if we multiply x by -1 before applying f?

$cf(x)$ $\frac{1}{c}f(x)$ $-f(x)$ $f(-x)$	



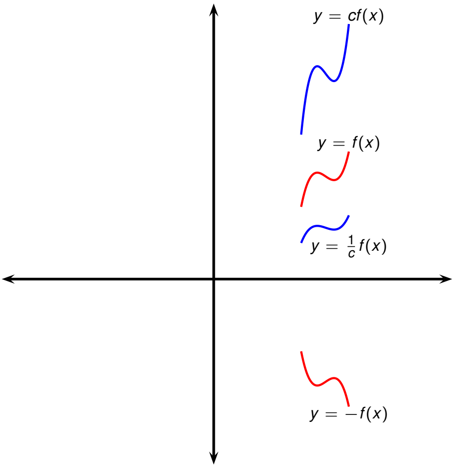
- What happens if we multiply or divide by a constant $c > 1$ in the equation of a function f?
- What happens if we multiply f by -1?
- What happens if we multiply x by -1 before applying f?

$cf(x)$	Stretch the graph of $f(x)$ vertically by a factor of c.
$\frac{1}{c}f(x)$	
$-f(x)$	
$f(-x)$	



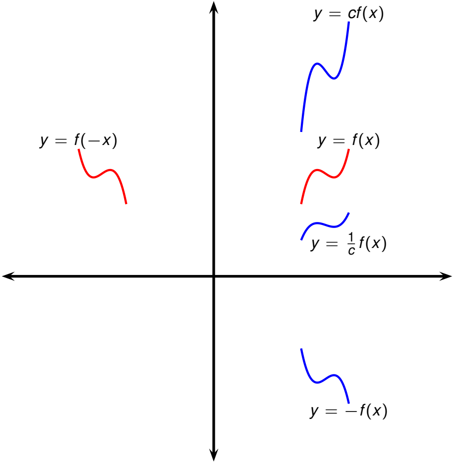
- What happens if we multiply or divide by a constant $c > 1$ in the equation of a function f?
- What happens if we multiply f by -1?
- What happens if we multiply x by -1 before applying f?

$cf(x)$	Stretch the graph of $f(x)$ vertically by a factor of c.
$\frac{1}{c}f(x)$	Compress the graph of $f(x)$ vertically by a factor of c.
$-f(x)$	
$f(-x)$	



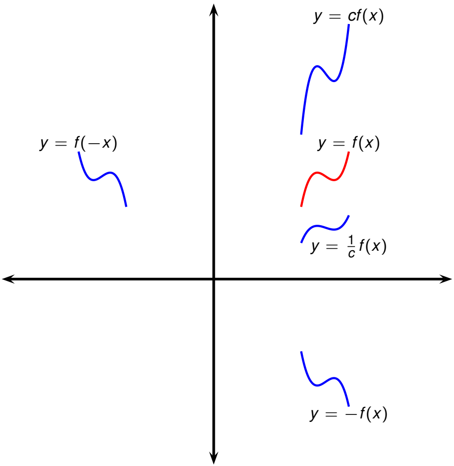
- What happens if we multiply or divide by a constant $c > 1$ in the equation of a function f?
- What happens if we multiply f by -1?
- What happens if we multiply x by -1 before applying f?

$cf(x)$	Stretch the graph of $f(x)$ vertically by a factor of c.
$\frac{1}{c}f(x)$	Compress the graph of $f(x)$ vertically by a factor of c.
$-f(x)$	Reflect the graph of $f(x)$ in the x-axis.
$f(-x)$	



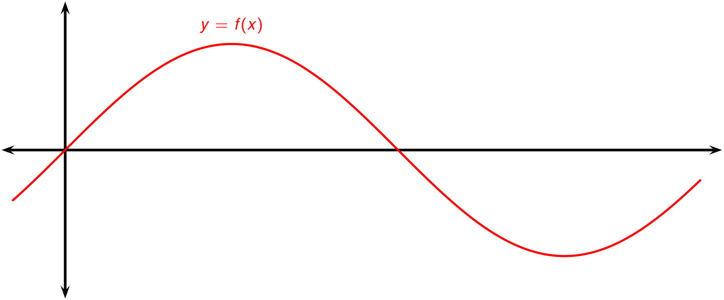
- What happens if we multiply or divide by a constant $c > 1$ in the equation of a function f?
- What happens if we multiply f by -1?
- What happens if we multiply x by -1 before applying f?

$cf(x)$	Stretch the graph of $f(x)$ vertically by a factor of c.
$\frac{1}{c}f(x)$	Compress the graph of $f(x)$ vertically by a factor of c.
$-f(x)$	Reflect the graph of $f(x)$ in the x-axis.
$f(-x)$	Reflect the graph of $f(x)$ in the y-axis.

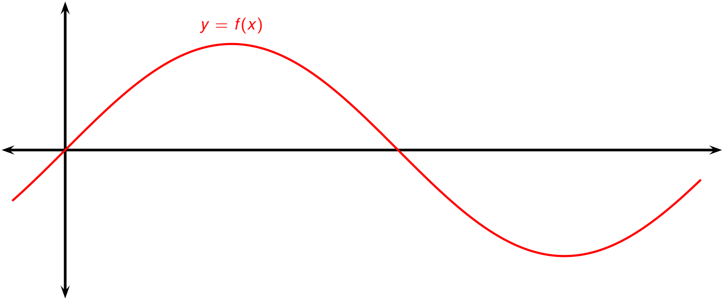


- What happens if we multiply or divide by a constant $c > 1$ in the equation of a function f?
- What happens if we multiply f by -1?
- What happens if we multiply x by -1 before applying f?

$cf(x)$	Stretch the graph of $f(x)$ vertically by a factor of c.
$\frac{1}{c}f(x)$	Compress the graph of $f(x)$ vertically by a factor of c.
$-f(x)$	Reflect the graph of $f(x)$ in the x-axis.
$f(-x)$	Reflect the graph of $f(x)$ in the y-axis.

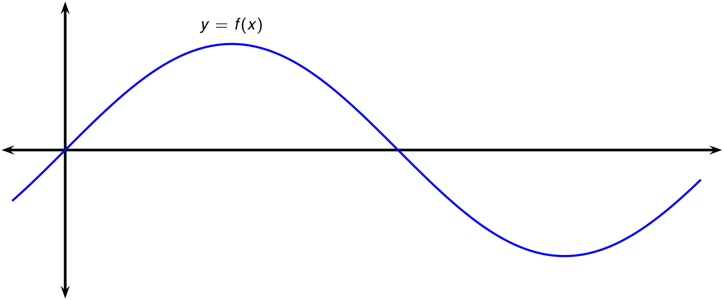


- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?



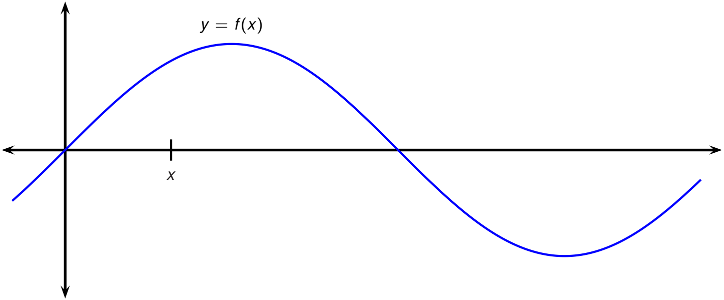
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	



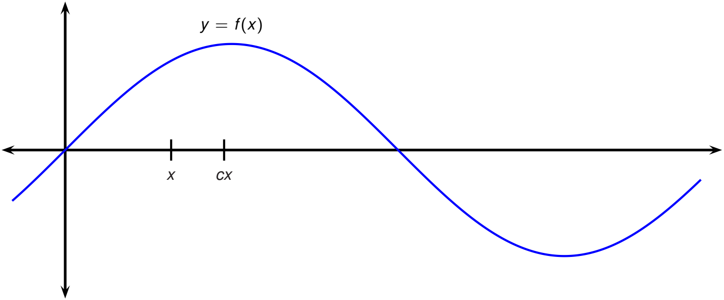
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



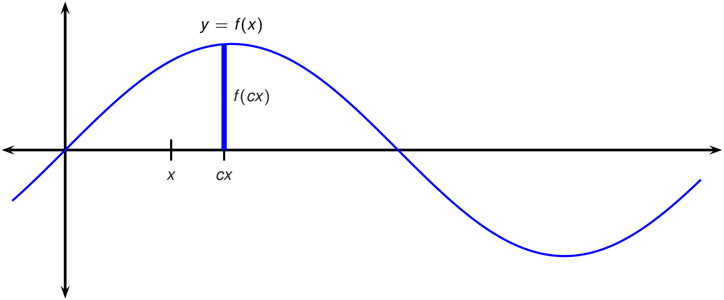
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



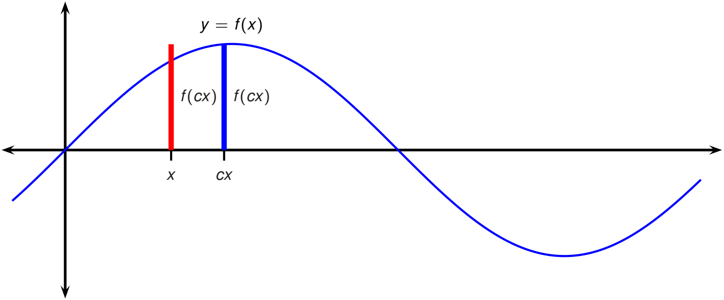
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



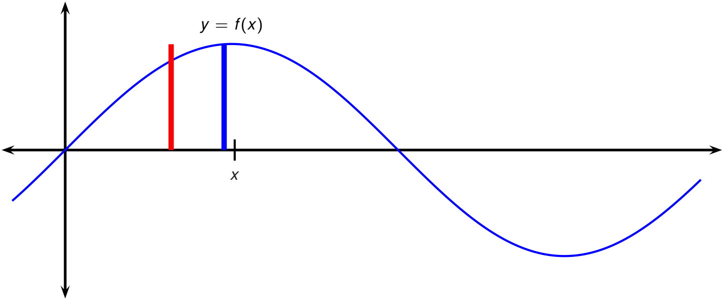
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



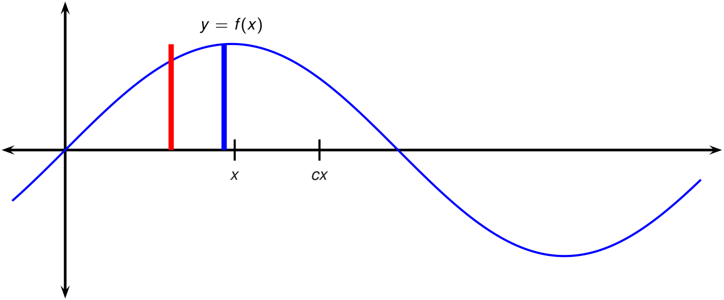
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



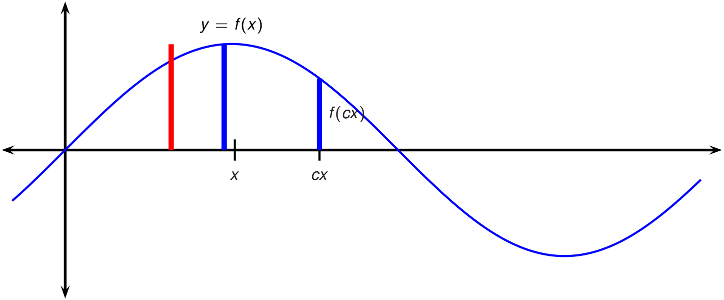
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



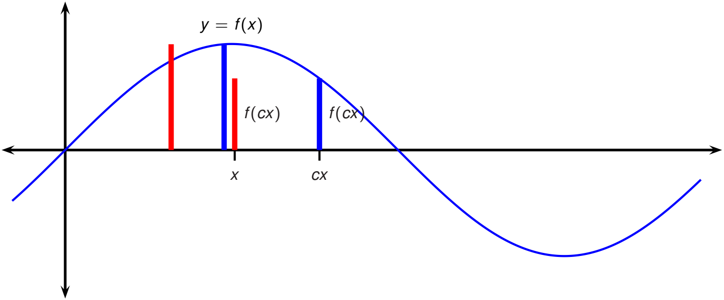
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



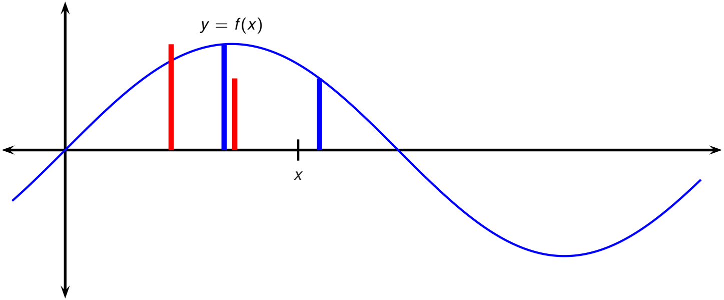
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



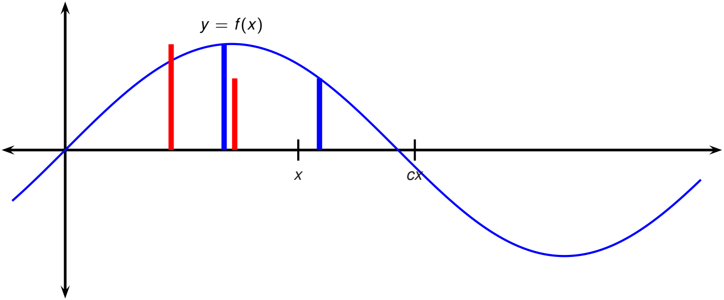
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



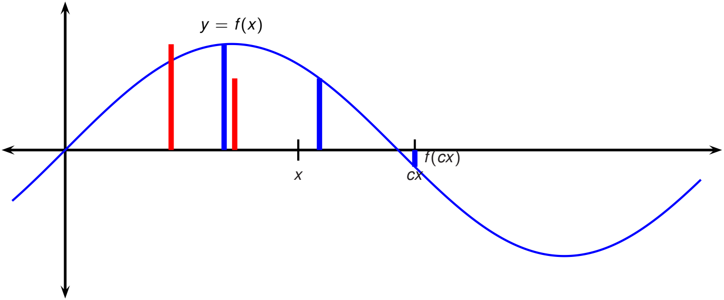
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



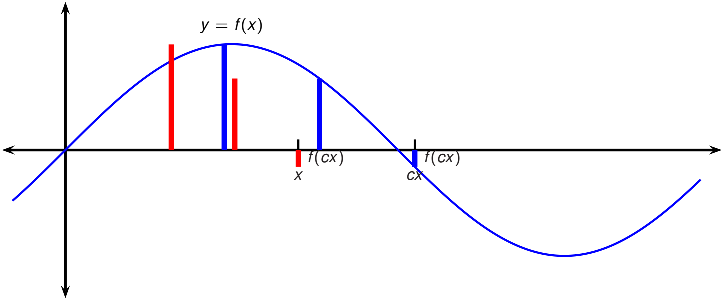
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



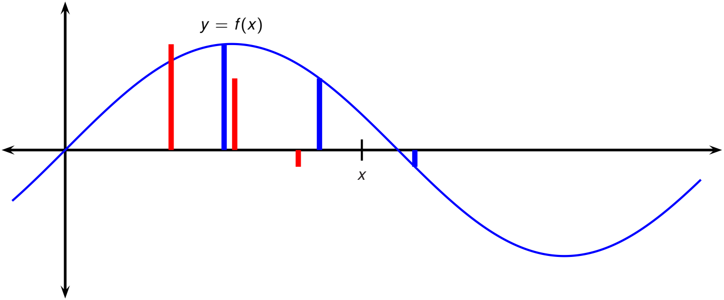
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



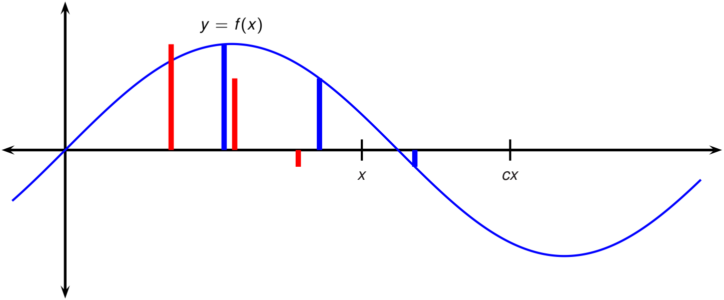
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



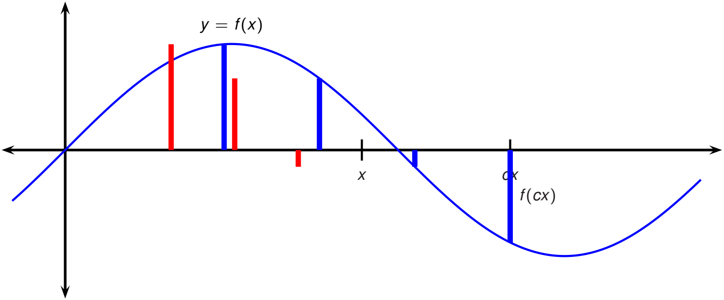
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



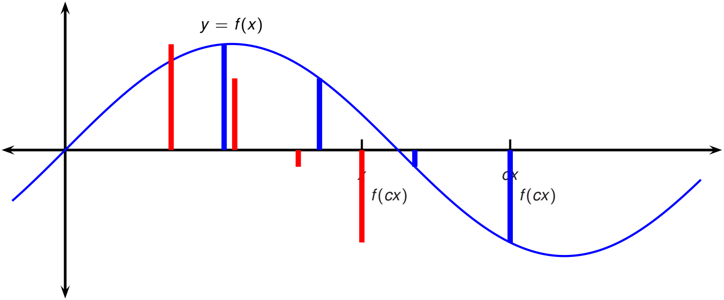
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



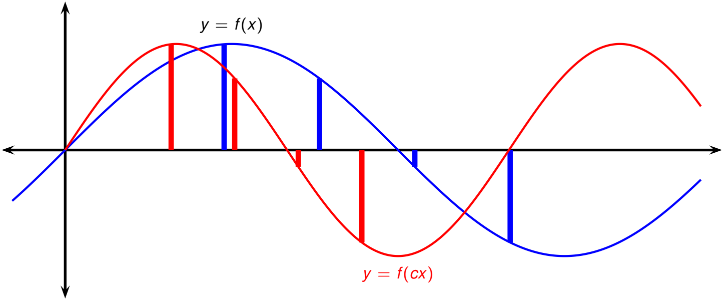
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



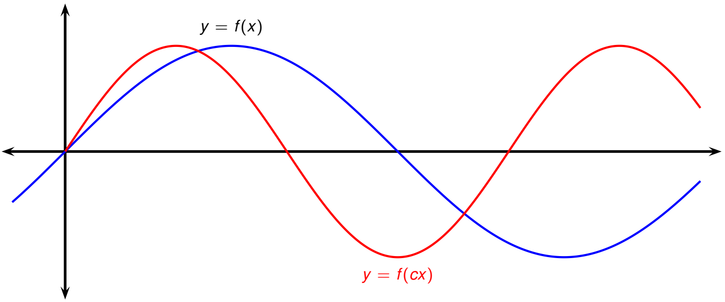
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



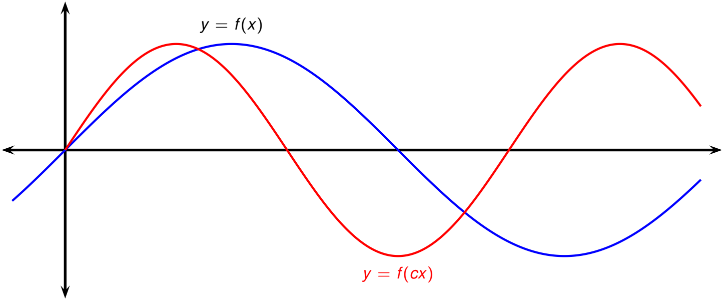
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	**?**



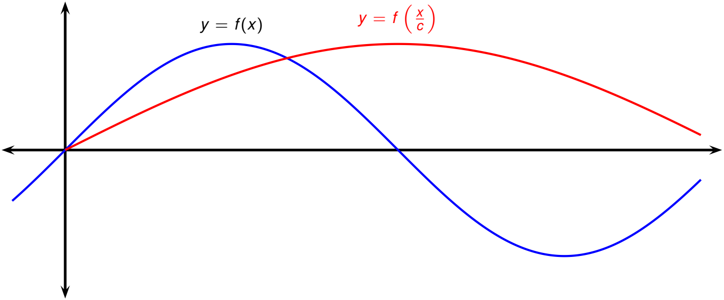
- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$	**?**
$f\left(\frac{1}{c}x\right)$	



- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$ $f\left(\frac{1}{c}x\right)$	Compress the graph of $f(x)$ horizontally by a factor of c.



- What happens if we multiply x by const. $c > 1$ before applying f?
- What happens if we divide x by const. $c > 1$ before applying f?

$f(cx)$	Compress the graph of $f(x)$ horizontally by a factor of c.
$f\left(\frac{1}{c}x\right)$	Stretch the graph of $f(x)$ horizontally by a factor of c.

What happens when we take the absolute value of a function?

What happens when we take the absolute value of a function?

$$|f(x)| = \begin{cases} f(x) & \text{if} \quad f(x) \geq 0 \\ -f(x) & \text{if} \quad f(x) < 0 \end{cases}$$

What happens when we take the absolute value of a function?

$$|f(x)| = \begin{cases} f(x) & \text{if} \quad f(x) \geq 0 \\ -f(x) & \text{if} \quad f(x) < 0 \end{cases}$$

This tells us how to draw the graph of $y = |f(x)|$: the part of the graph above the x-axis remains the same; the part below the x-axis is reflected about the x-axis.

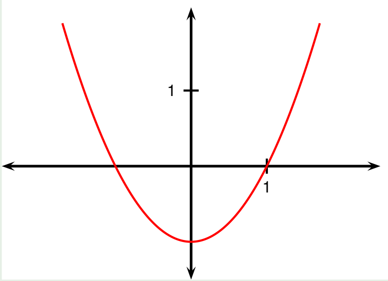
What happens when we take the absolute value of a function?

$$|f(x)| = \begin{cases} f(x) & \text{if} \quad f(x) \geq 0 \\ -f(x) & \text{if} \quad f(x) < 0 \end{cases}$$

This tells us how to draw the graph of $y = |f(x)|$: the part of the graph above the x-axis remains the same; the part below the x-axis is reflected about the x-axis.

Example

Draw the graph of the function $f(x) = |x^2 - 1|$.



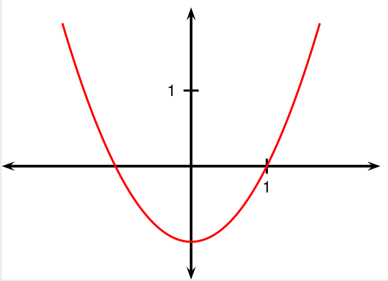
What happens when we take the absolute value of a function?

$$|f(x)| = \begin{cases} f(x) & \text{if} \quad f(x) \geq 0 \\ -f(x) & \text{if} \quad f(x) < 0 \end{cases}$$

This tells us how to draw the graph of $y = |f(x)|$: the part of the graph above the x-axis remains the same; the part below the x-axis is reflected about the x-axis.

Example

Draw the graph of the function $f(x) = |x^2 - 1|$.



- Draw the graph of $f(x) = x^2 - 1$.

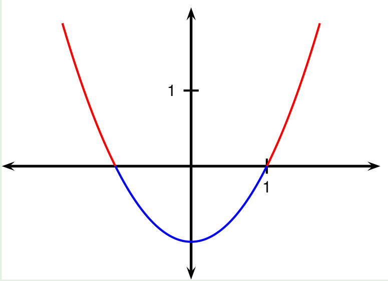
What happens when we take the absolute value of a function?

$$|f(x)| = \begin{cases} f(x) & \text{if} \quad f(x) \geq 0 \\ -f(x) & \text{if} \quad f(x) < 0 \end{cases}$$

This tells us how to draw the graph of $y = |f(x)|$: the part of the graph above the x-axis remains the same; the part below the x-axis is reflected about the x-axis.

Example

Draw the graph of the function $f(x) = |x^2 - 1|$.



- Draw the graph of $f(x) = x^2 - 1$.
- Identify the part(s) below the x-axis.

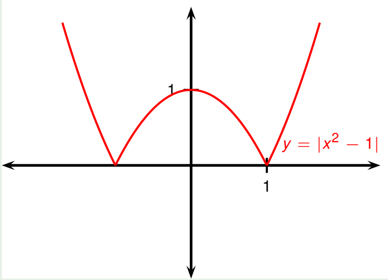
What happens when we take the absolute value of a function?

$$|f(x)| = \begin{cases} f(x) & \text{if} \quad f(x) \geq 0 \\ -f(x) & \text{if} \quad f(x) < 0 \end{cases}$$

This tells us how to draw the graph of $y = |f(x)|$: the part of the graph above the x-axis remains the same; the part below the x-axis is reflected about the x-axis.

Example

Draw the graph of the function $f(x) = |x^2 - 1|$.



- Draw the graph of $f(x) = x^2 - 1$.
- Identify the part(s) below the x-axis.
- Flip those parts over the x-axis.