Precalculus Lecture 7 Trigonometric Graphs

Todor Miley

https://github.com/tmilev/freecalc

2020

Outline

- Graphs of the Trigonometric Functions
 - Graphs of sin and cos
 - Graph of $a \sin(bx c)$
 - Graphs of tan, cot, sec, csc

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Outline

- Graphs of the Trigonometric Functions
 - Graphs of sin and cos
 - Graph of $a \sin(bx c)$
 - Graphs of tan, cot, sec, csc
- Inverse Trigonometric Functions
 - Trigonometric Functions with Inverse Trig Arguments

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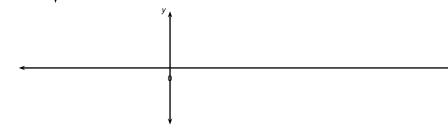
- Latest version of the .tex sources of the slides: https://github.com/tmilev/freecalc
- Should the link be outdated/moved, search for "freecalc project".
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X	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
sin <i>X</i>	?								



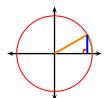
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sin <i>X</i>	0								



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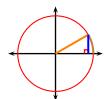
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Trigonometric Graphs

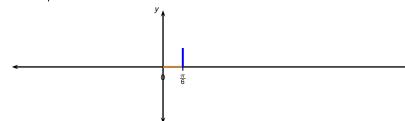


X	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
sin <i>X</i>	0	?							



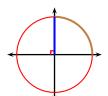


Х	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
sin <i>X</i>	0	$\frac{1}{2}$							

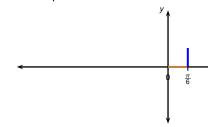


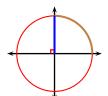
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Graph of sin x

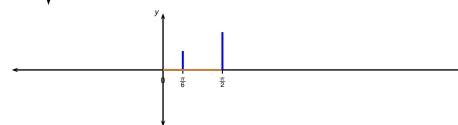


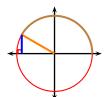
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sin X	0	$\frac{1}{2}$?						



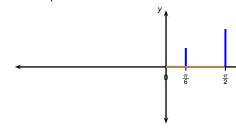


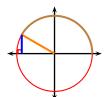
Х	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
sin X	0	$\frac{1}{2}$	1						



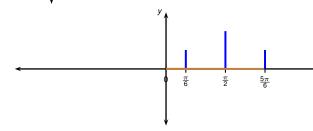


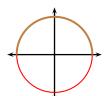
Х	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
sin X	0	$\frac{1}{2}$	1	?					



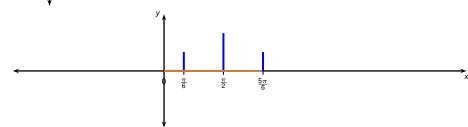


X	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
sin <i>X</i>	0	$\frac{1}{2}$	1	$\frac{1}{2}$					



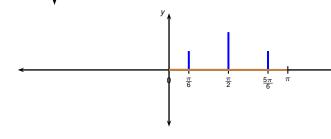


	Х	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
s	in <i>X</i>	0	$\frac{1}{2}$	1	$\frac{1}{2}$?				





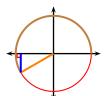
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si	n <i>X</i>	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0				



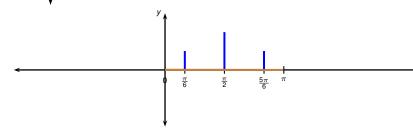
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Trigonometric Graphs

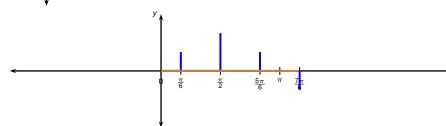


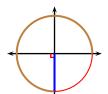
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sin <i>X</i>	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	?			



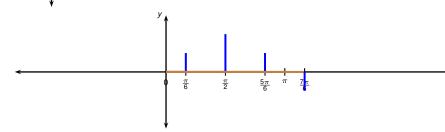


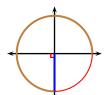
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sin X	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$			



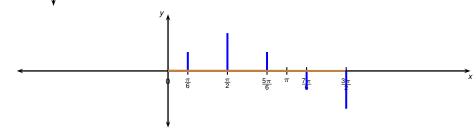


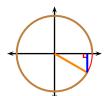
Х	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
sin X	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$?		



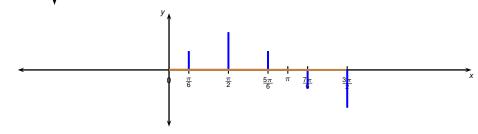


Х	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
sin X	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1		



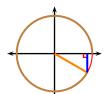


X	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
sin X	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	?	

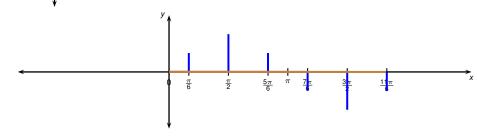


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Graph of sin x

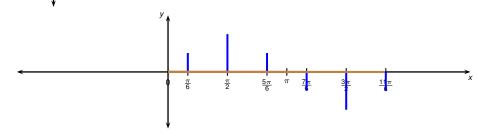


Х	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
sin <i>X</i>	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	





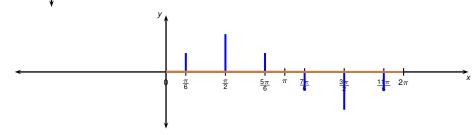
Х	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
sin <i>X</i>	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$?



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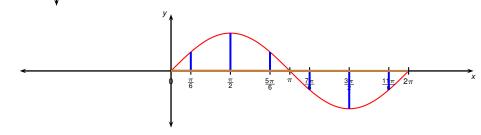


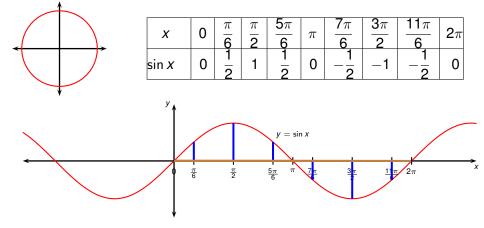
X	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2 π
sin X	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0



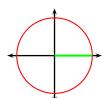


Х	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
sin X	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0

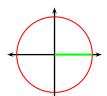




The graph of $\sin x$ is 2π -periodic so the rest of the graph can be inferred from the interval $[0, 2\pi]$.

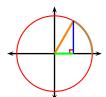


Χ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
cos X	?								

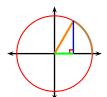


X	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2 π
cos X	1								

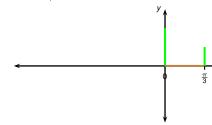
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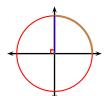


Χ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2 π
cos X	1	?							

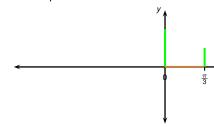


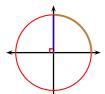
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cos X	1	$\frac{1}{2}$							



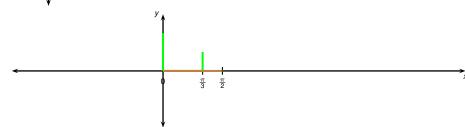


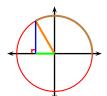
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cos X	•	1	$\frac{1}{2}$?						



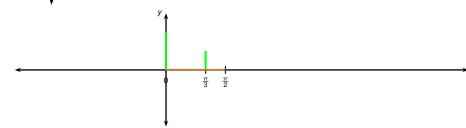


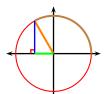
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cos X	1	$\frac{1}{2}$	0						



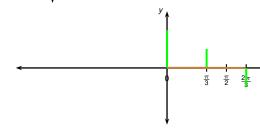


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cos X	1	$\frac{1}{2}$	0	?					



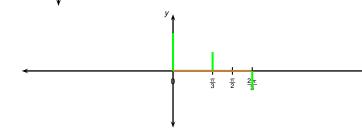


Χ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2 π
cos X	1	$\frac{1}{2}$	0	$-\frac{1}{2}$					



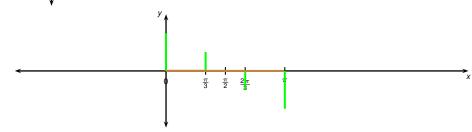


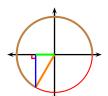
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cos X	1	$\frac{1}{2}$	0	$-\frac{1}{2}$?				



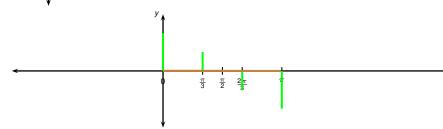


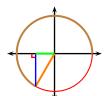
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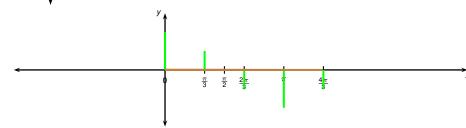


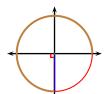
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cos X	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	?			



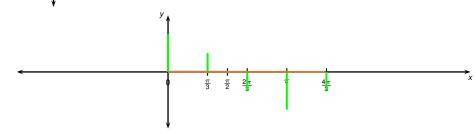


Χ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
cos X	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$			





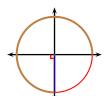
Χ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2 π
cos X	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$?		



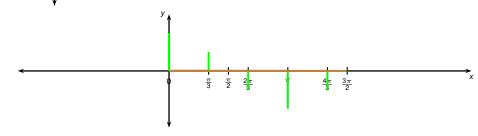
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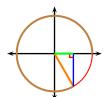


Χ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2 π
cos X	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0		

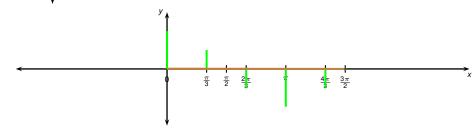


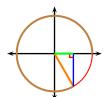
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Graph of cos x

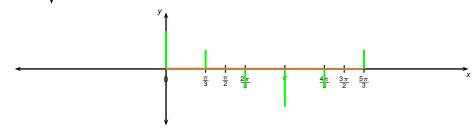


Χ	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
cos X	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	?	





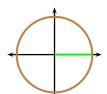
X	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
cos X	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	



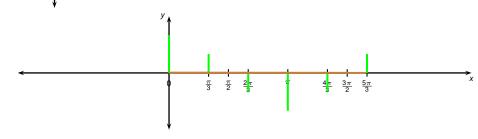
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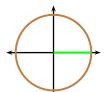
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rigonometric Graphs

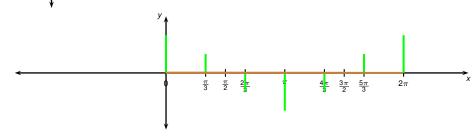


\ \	^	π	π	2π	_	4π	3π	5π	2-
\ \ \ \ \	U	3	2	3	η	3	2	3	271
cos X	1	1 2	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	1 -	?





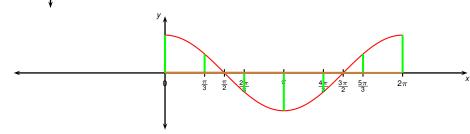
X	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
cos X	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0

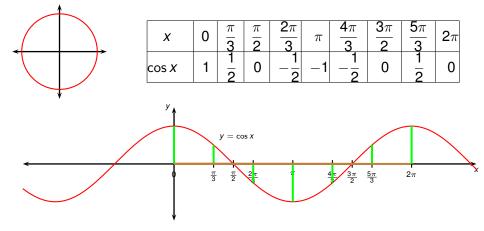


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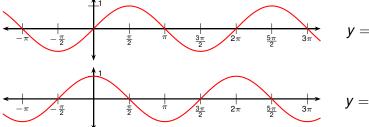


	Х	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
C	cos X	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	0



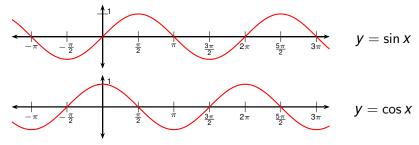


The graph of $\cos x$ is 2π -periodic so the rest of the graph can be inferred from the interval $[0, 2\pi]$.

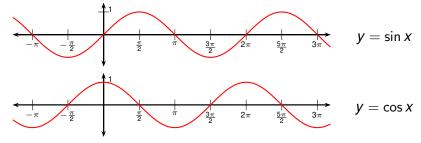


$$y = \sin x$$

 $y = \cos x$



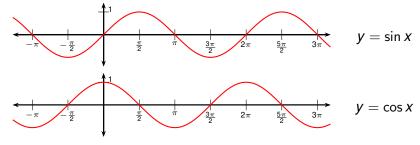
• $\sin x$ has zeroes at $n\pi$ for all integers n.



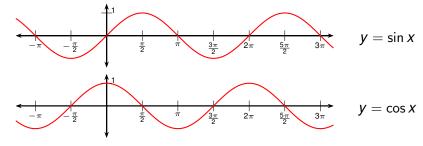
- $\sin x$ has zeroes at $n\pi$ for all integers n.
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.

2020

Graphs of the Trigonometric Functions

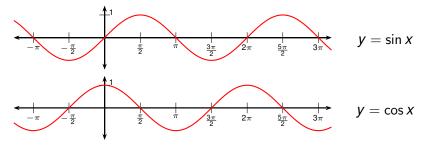


- $\sin x$ has zeroes at $n\pi$ for all integers n.
- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.
- $-1 \le \sin x \le 1.$

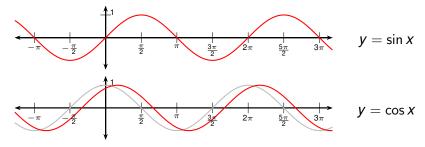


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- $\cos x$ has zeroes at $\frac{\pi}{2} + n\pi$ for all integers n.
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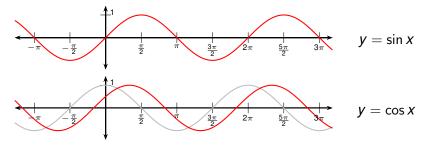
Todor Milev



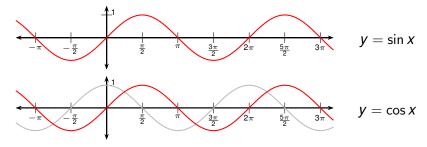
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- -1 ≤ cos x ≤ 1.
- If we translate the graph of $\cos x$ by $\frac{\pi}{2}$ units to the right



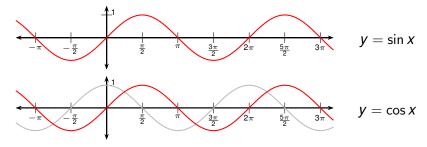
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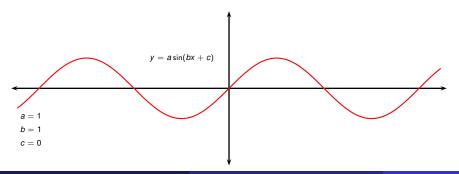


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- $\sin x$ has zeroes at $n\pi$ for all integers n.
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- -1 ≤ $\sin x$ ≤ 1.
- $ext{ } ext{ } e$
- If we translate the graph of $\cos x$ by $\frac{\pi}{2}$ units to the right we get the graph of $\sin x$. This is a consequence of $\cos \left(x \frac{\pi}{2}\right) = \sin x$.

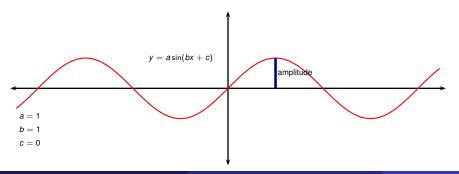
Definition (Phase, period, frequency, amplitude of a wave)



Todor Miley

Definition (Phase, period, frequency, amplitude of a wave)

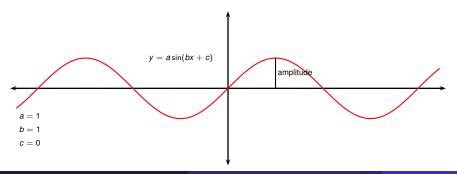
In the function $a\sin(bx+c)$, the number |a| is called the *amplitude* of the wave,



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Definition (Phase, period, frequency, amplitude of a wave)

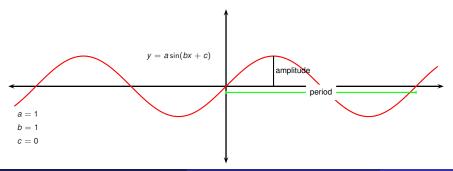
In the function $a\sin(bx+c)$, the number |a| is called the *amplitude* of the wave, the number $\frac{b}{2\pi}$ is called the *frequency* of the wave,



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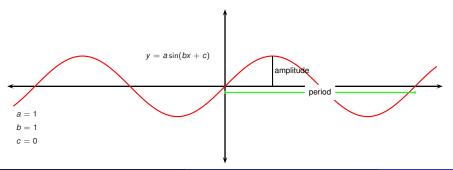
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Definition (Phase, period, frequency, amplitude of a wave)

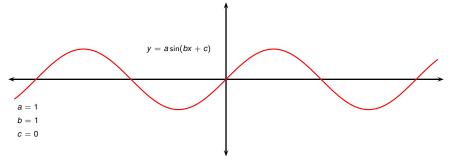
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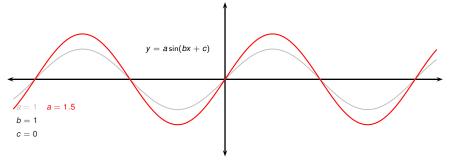
 What happens when we change the amplitude? The frequency/period? The phase?



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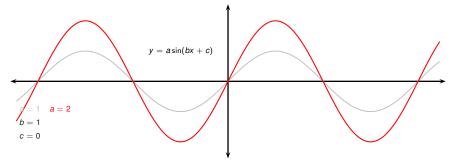
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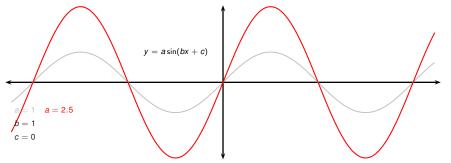
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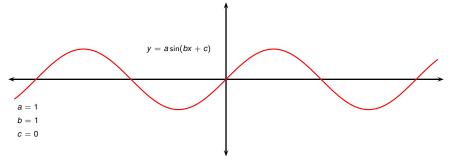
2020

• The graph of $a\sin(bx + c)$ is referred to as a "wave".

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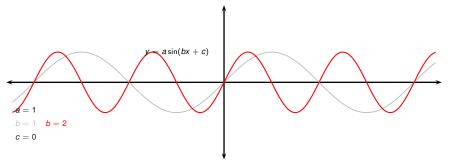
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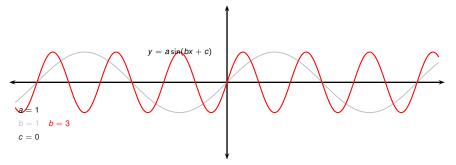
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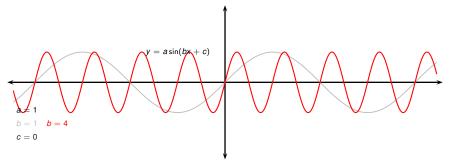
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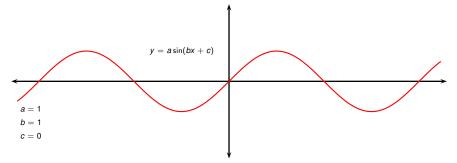
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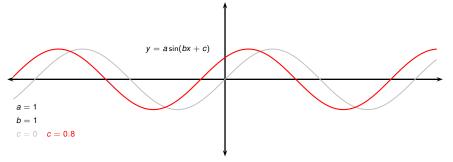
7/26

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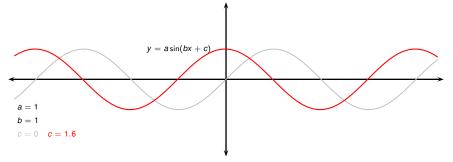
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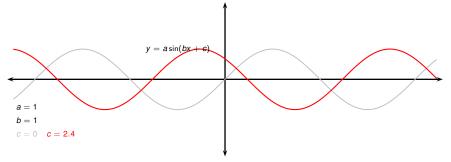
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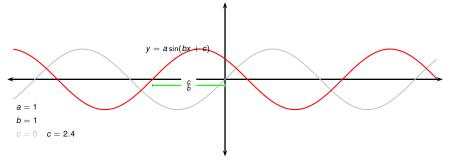
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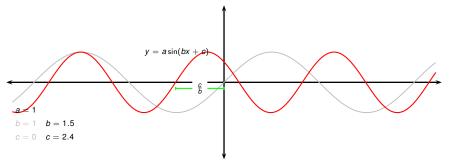


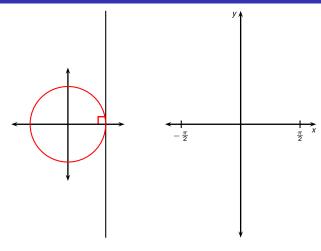
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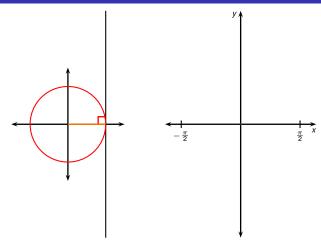
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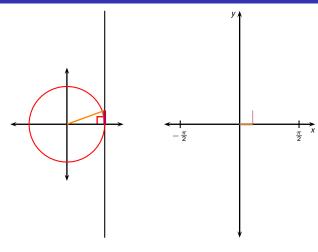




2020

Graph of tan x

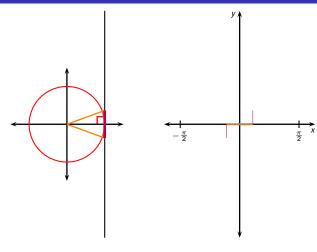




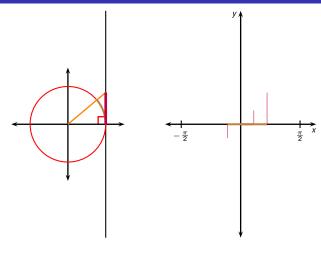
Todor Milev

Lecture 7

Trigonometric Graphs



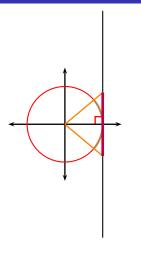
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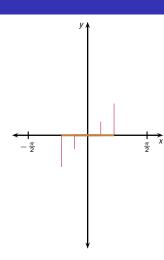


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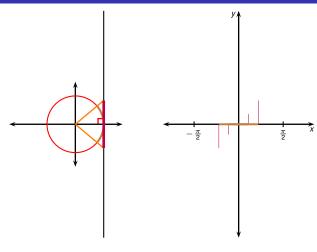
Lecture 7

Trigonometric Graphs

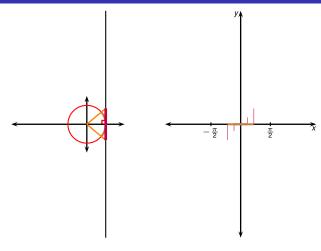


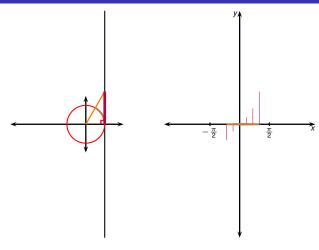


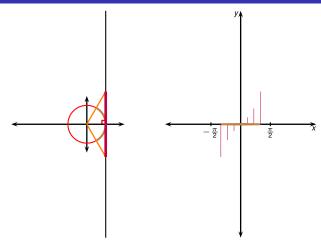
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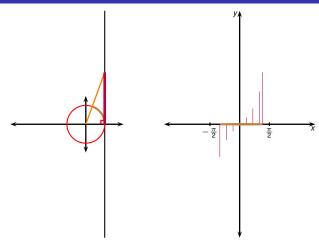


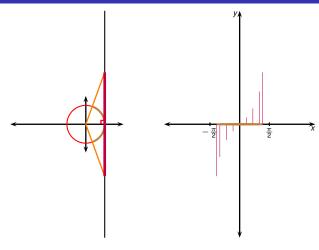
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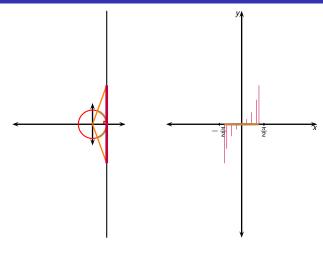


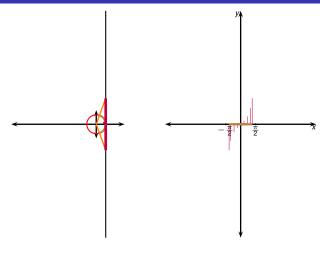


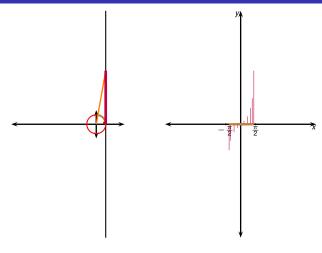


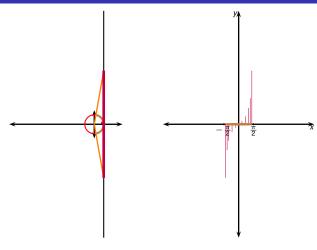


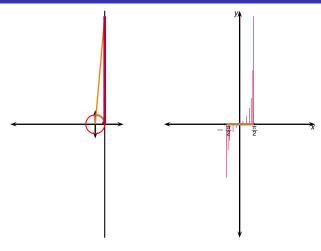


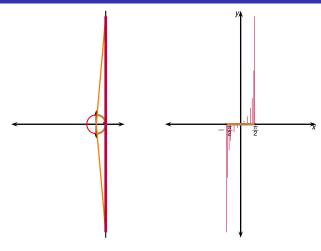


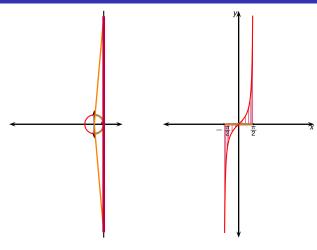


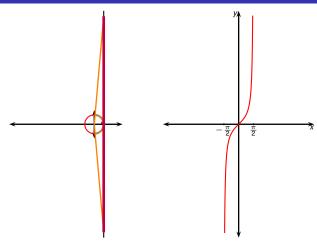


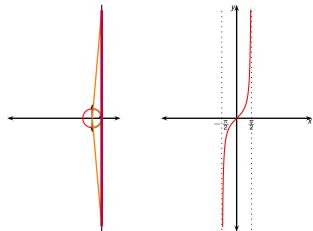




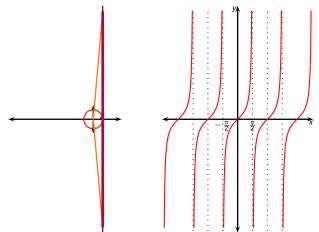




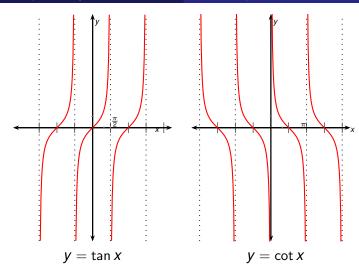




Near $\pm \frac{\pi}{2}$ the graph of $\tan x$ approaches $\pm \infty$.



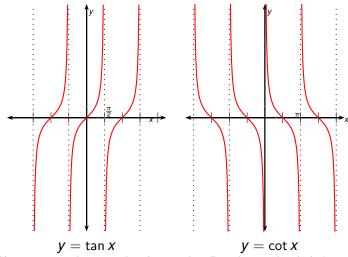
Near $\pm \frac{\pi}{2}$ the graph of $\tan x$ approaches $\pm \infty$. The graph of $\tan x$ is π -periodic so the rest of the graph can be inferred from the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.



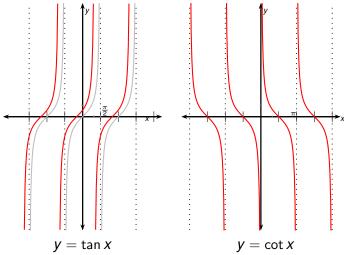
Todor Milev

Lecture 7

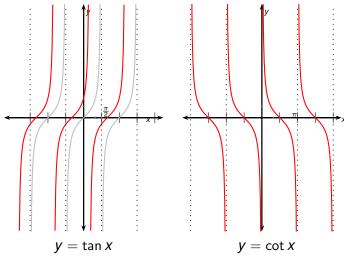
Trigonometric Graphs



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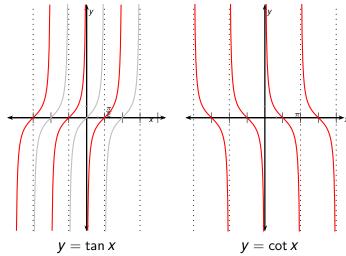
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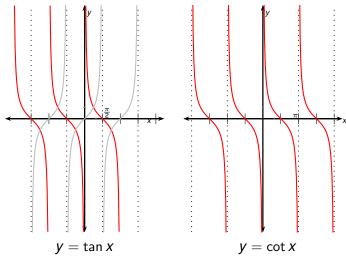
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Lecture 7

Trigonometric Graphs

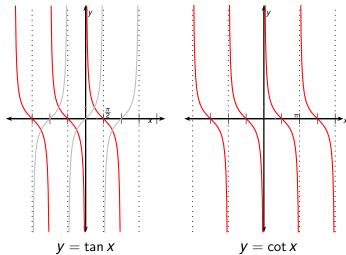


Todor Miley



If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis, we get the graph of $\cot x$.

Todor Miley

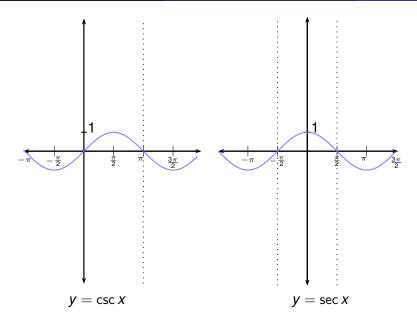


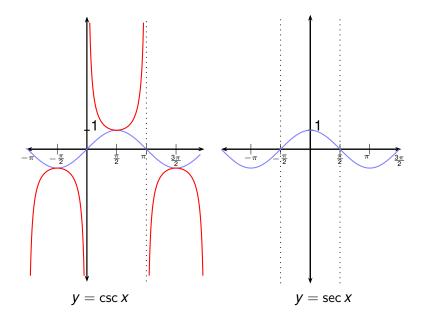
If we move the graph of $\tan x$ by $\frac{\pi}{2}$ units to the left (or right) and reflect across the x axis, we get the graph of $\cot x$. This follows from $\tan \left(x \pm \frac{\pi}{2}\right) = -\cot x$.

Todor Miley

Lecture 7

Trigonometric Graphs

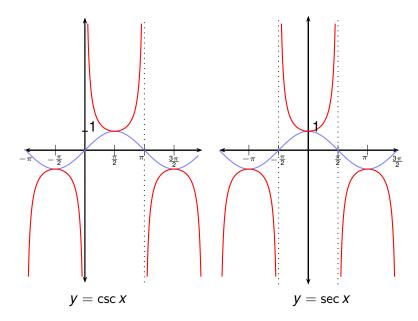




Todor Milev

Lectu

Trigonometric Graph

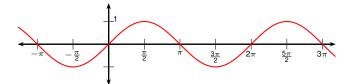


Todor Milev

Lectu

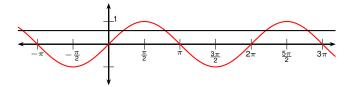
Trigonometric Graph

Inverse Trigonometric Functions



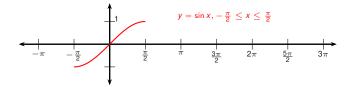
2020

Inverse Trigonometric Functions



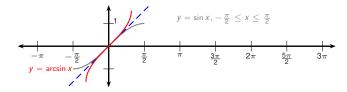
• sin x isn't one-to-one.

Inverse Trigonometric Functions



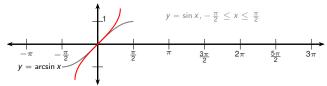
- sin x isn't one-to-one.
- It is if we restrict the domain to $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

Inverse Trigonometric Functions

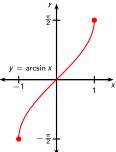


- sin x isn't one-to-one.
- It is if we restrict the domain to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
- Then it has an inverse function.
- We call it arcsin or sin⁻¹.

Inverse Trigonometric Functions

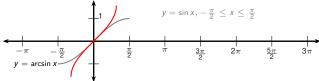


- sin x isn't one-to-one.
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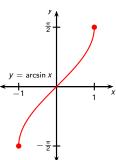


Todor Milev

Inverse Trigonometric Functions



- sin x isn't one-to-one.
- It is if we restrict the domain to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
- Then it has an inverse function.
- We call it arcsin or sin⁻¹.
- $\arcsin x = y \Leftrightarrow \sin y = x$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.



Find
$$\arcsin\left(\frac{1}{2}\right)$$
.

• arcsin y = the appropriate angle whose sine equals y.

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$$\bullet \sin\left(\frac{?}{?}\right) = \frac{1}{2}.$$

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Find
$$\arcsin\left(\frac{1}{2}\right)$$
.

•
$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$
.

- arcsin y = the appropriate angle whose sine equals y.
- Important: the output angle must lie in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Find
$$\arcsin\left(\frac{1}{2}\right)$$
.

- $\bullet -\frac{\pi}{2} \leq \frac{\pi}{6} \leq \frac{\pi}{2}.$

- arcsin y = the appropriate angle whose sine equals y.
- Important: the output angle must lie in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Find
$$\arcsin\left(\frac{1}{2}\right)$$
.

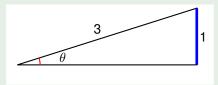
- $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.
- $-\frac{\pi}{2} \le \frac{\pi}{6} \le \frac{\pi}{2}$.
- Therefore $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$.

Find
$$\tan \left(\arcsin \left(\frac{1}{3}\right)\right)$$
.

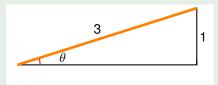
Find $\tan \left(\arcsin \left(\frac{1}{3}\right)\right)$.

• Let $\theta = \arcsin\left(\frac{1}{3}\right)$, so $\sin \theta = \frac{1}{3}$.

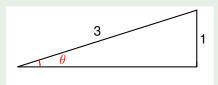
- Let $\theta = \arcsin\left(\frac{1}{3}\right)$, so $\sin \theta = \frac{1}{3}$.
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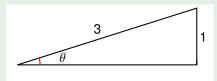
- Let $\theta = \arcsin\left(\frac{1}{3}\right)$, so $\sin \theta = \frac{1}{3}$.
- Draw a right triangle with opposite side 1 and hypotenuse 3.
- Let the angle θ be as labeled.



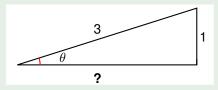
- Let $\theta = \arcsin\left(\frac{1}{3}\right)$, so $\sin \theta = \frac{1}{3}$.
- Draw a right triangle with opposite side 1 and hypotenuse 3.
- Let the angle θ be as labeled. Then $\sin \theta = \frac{1}{3}$



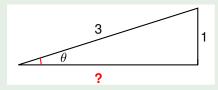
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- Draw a right triangle with opposite side 1 and hypotenuse 3.
- Let the angle θ be as labeled. Then $\sin \theta = \frac{1}{3}$ and so $\theta = \arcsin \left(\frac{1}{3}\right)$.



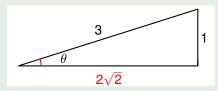
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- Length of adjacent side = ?



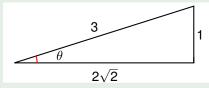
- Let $\theta = \arcsin\left(\frac{1}{3}\right)$, so $\sin \theta = \frac{1}{3}$.
- Draw a right triangle with opposite side 1 and hypotenuse 3.
- Let the angle θ be as labeled. Then $\sin \theta = \frac{1}{3}$ and so $\theta = \arcsin \left(\frac{1}{3}\right)$.
- Length of adjacent side = $\sqrt{3^2 1^2}$



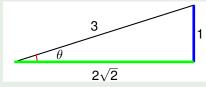
- Let $\theta = \arcsin\left(\frac{1}{3}\right)$, so $\sin \theta = \frac{1}{3}$.
- Draw a right triangle with opposite side 1 and hypotenuse 3.
- Let the angle θ be as labeled. Then $\sin \theta = \frac{1}{3}$ and so $\theta = \arcsin \left(\frac{1}{3}\right)$.
- Length of adjacent side = $\sqrt{3^2 1^2} = \sqrt{8} = 2\sqrt{2}$.



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- Then tan $\left(\arcsin\left(\frac{1}{3}\right)\right) = ?$



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- Length of adjacent side = $\sqrt{3^2 1^2} = \sqrt{8} = 2\sqrt{2}$.
- Then $\tan \left(\arcsin \left(\frac{1}{3}\right)\right) = \frac{1}{2\sqrt{2}}$.



Find arcsin(sin(1.5)).

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• $\frac{\pi}{2} \approx$?

Find arcsin(sin(1.5)).

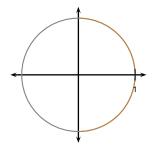
• $\frac{\pi}{2} \approx 1.57$.

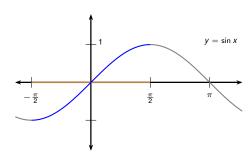
Find $\arcsin(\sin(1.5))$.

- $\frac{\pi}{2} \approx 1.57$.
- Therefore $-\frac{\pi}{2} \le 1.5 \le \frac{\pi}{2}$.

Find $\arcsin(\sin(1.5))$.

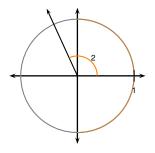
- $\frac{\pi}{2} \approx 1.57$.
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- Therefore $\arcsin(\sin 1.5) = 1.5$.

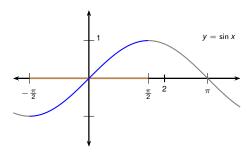




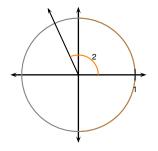
Find arcsin(sin 2).

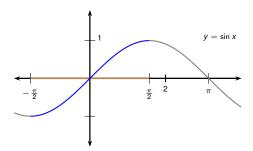
• 2 is not between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.



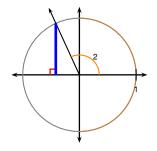


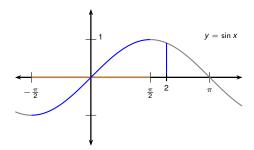
- 2 is not between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.
- We need the angle a between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ for which $\sin 2 = \sin a$.



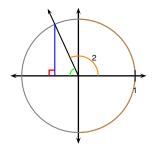


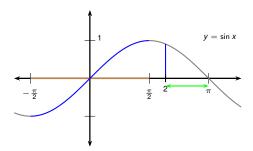
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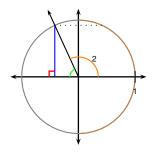


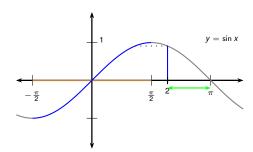
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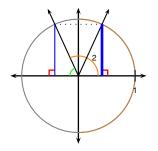


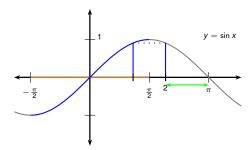
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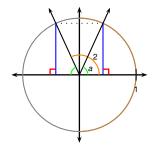
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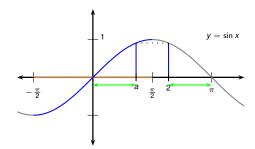




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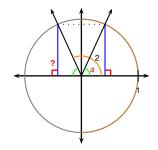


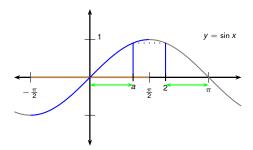


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$$a = ?$$



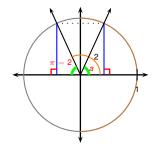


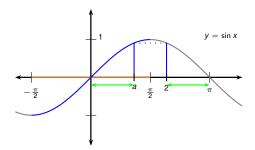
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Lecture 7

$$a = \pi - 2$$
.



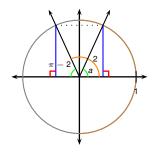


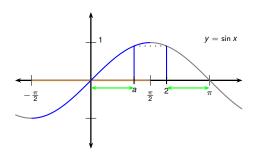
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Therefore $\arcsin(\sin 2) = \arcsin(\sin a)$

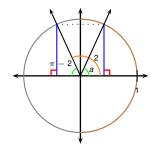


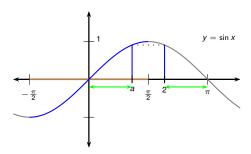


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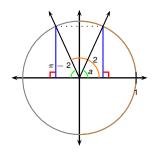


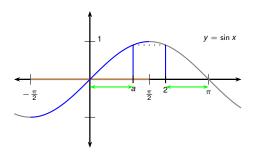
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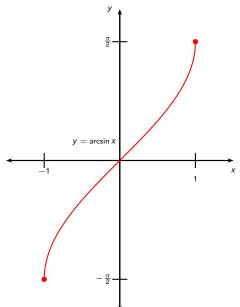
- 2 is not between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.
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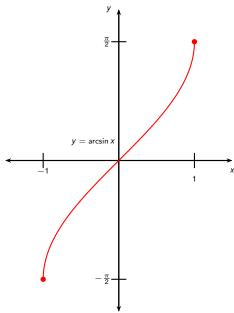
Therefore $\arcsin(\sin 2) = \arcsin(\sin a)$ = $a = \pi - 2$.



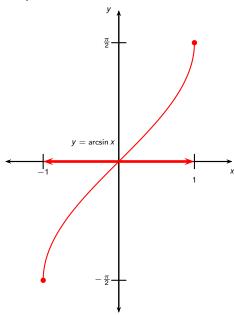




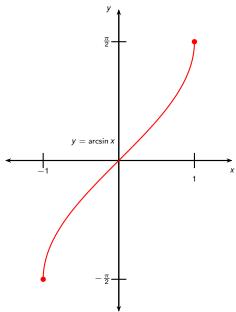
- Domain: ?
- Range: ?
- arcsin $x = y \Leftrightarrow \sin y = x$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.
- arcsin(sin x) = x for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.
- $\sin(\arcsin x) = x$ for $-1 \le x \le 1$.



- Domain: ?
- Range: ?
- arcsin(sin X) = X for $-\frac{\pi}{2} \le X \le \frac{\pi}{2}$.
- $\sin(\arcsin x) = x$ for $-1 \le x \le 1$.

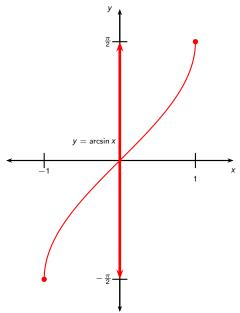


- Domain: [-1,1].
- Range: ?
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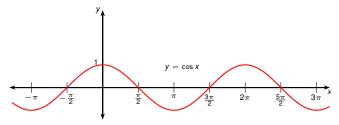


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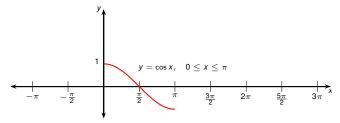
Lecture 7



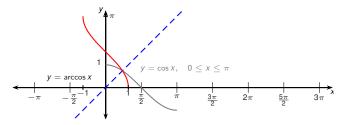
- Domain: [-1,1].
- **2** Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
- arcsin $x = y \Leftrightarrow \sin y = x$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.
- arcsin(sin X) = X for $-\frac{\pi}{2} \le X \le \frac{\pi}{2}$.
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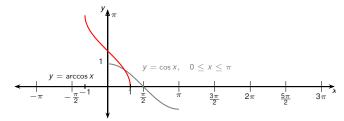
• Same for cos x.

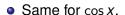


- Same for cos x.
- Restrict the domain to $[0, \pi]$.

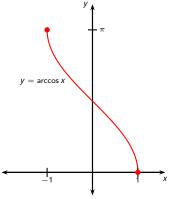


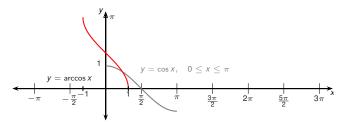
- Same for cos x.
- Restrict the domain to $[0, \pi]$.
- The inverse is called arccos or cos⁻¹.

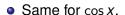




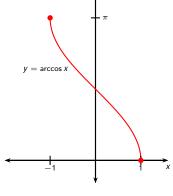
- Restrict the domain to $[0, \pi]$.
- The inverse is called arccos or cos⁻¹.

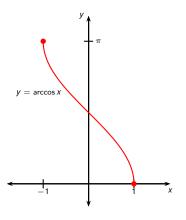




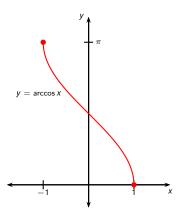


- Restrict the domain to $[0, \pi]$.
- The inverse is called arccos or cos⁻¹.
- $\operatorname{arccos}(x) = y \Leftrightarrow \cos y = x$ and $0 \le y \le \pi$.

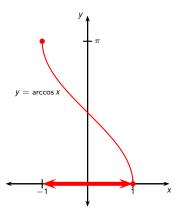




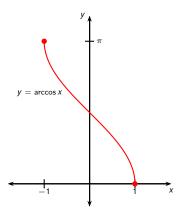
- Domain:
- Range:
- arccos $x = y \Leftrightarrow \cos y = x$ and $0 \le y \le \pi$.
- arccos(cos x) = x for $0 \le x \le \pi$.
- $\begin{array}{l}
 \text{os} (\arccos x) = x \text{ for} \\
 -1 \le x \le 1.
 \end{array}$



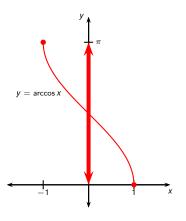
- Domain: ?
- Range:
- arccos $x = y \Leftrightarrow \cos y = x$ and $0 \le y \le \pi$.
- arccos(cos x) = x for $0 \le x \le \pi$.
- $d(\operatorname{arccos} x) = -\frac{1}{\sqrt{1-x^2}}.$



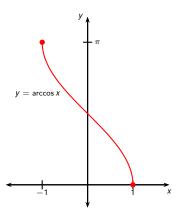
- **●** Domain: [−1,1].
- ② Range:
- arccos $x = y \Leftrightarrow \cos y = x$ and $0 \le y \le \pi$.
- arccos(cos x) = x for $0 \le x \le \pi$.
- $\begin{array}{l}
 \text{os} (\arccos x) = x \text{ for} \\
 -1 \le x \le 1.
 \end{array}$



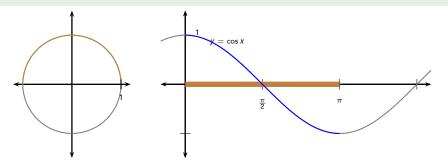
- Domain: [-1,1].
- Range: ?
- arccos $x = y \Leftrightarrow \cos y = x$ and $0 \le y \le \pi$.
- arccos(cos x) = x for $0 \le x \le \pi$.
- $\begin{array}{l}
 \text{5} & \cos(\arccos x) = x \text{ for} \\
 -1 \le x \le 1.
 \end{array}$
- $d(\operatorname{arccos} x) = -\frac{1}{\sqrt{1-x^2}}.$



- Domain: [-1,1].
- **2** Range: $[0, \pi]$.
- arccos $x = y \Leftrightarrow \cos y = x$ and $0 \le y \le \pi$.
- arccos(cos x) = x for $0 \le x \le \pi$.
- $\begin{array}{l} \mathbf{5} & \cos(\arccos x) = x \text{ for} \\ -1 \leq x \leq 1. \end{array}$

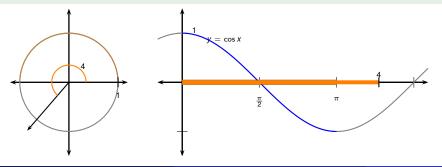


- Domain: [-1,1].
- **2** Range: $[0, \pi]$.
- arccos $x = y \Leftrightarrow \cos y = x$ and $0 \le y \le \pi$.
- arccos(cos x) = x for $0 \le x \le \pi$.
- $\begin{array}{l} \mathbf{5} & \cos(\arccos x) = x \text{ for} \\ -1 \leq x \leq 1. \end{array}$
- of $\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$. (The proof is similar to the proof of the formula for the derivative of $\arcsin x$.)

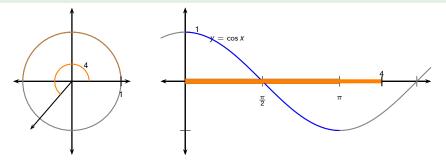


Find arccos(cos 4).

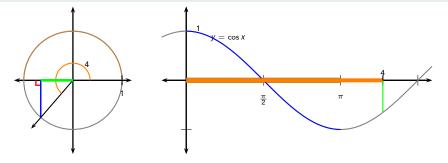
• 4 is not between 0 and π .



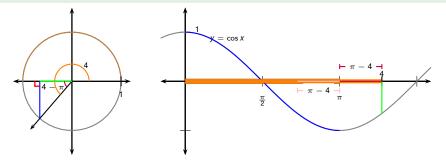
- 4 is not between 0 and π .
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.



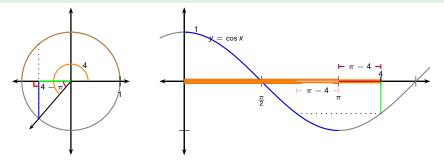
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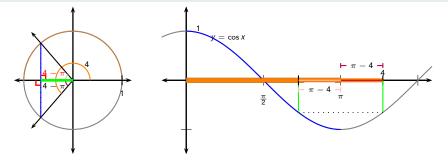
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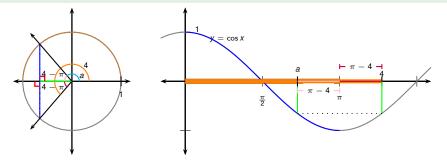
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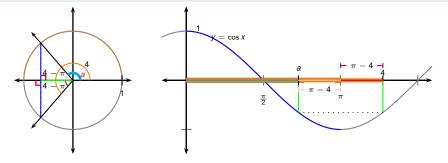


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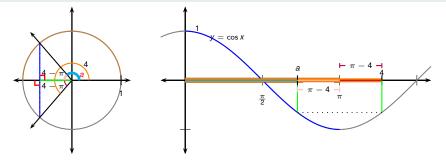
- 4 is not between 0 and π .
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$a = ?$$



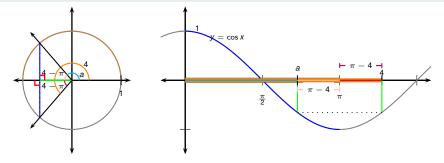
- 4 is not between 0 and π .
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$a = \pi - (4 - \pi)$$



- 4 is not between 0 and π .
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$a = \pi - (4 - \pi) = 2\pi - 4$$

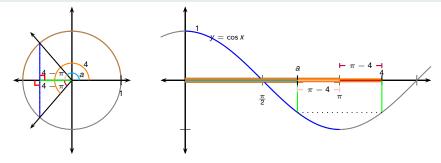


Find arccos(cos 4).

- 4 is not between 0 and π .
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$a = \pi - (4 - \pi) = 2\pi - 4$$

Therefore arccos(cos 4) = arccos(cos a)

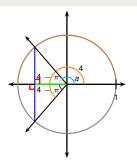


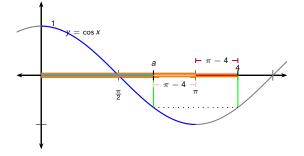
Find arccos(cos 4).

- 4 is not between 0 and π .
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$$a = \pi - (4 - \pi) = 2\pi - 4$$

Therefore $\arccos(\cos 4) = \arccos(\cos a)$





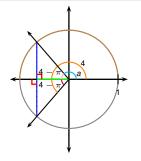
Find arccos(cos 4).

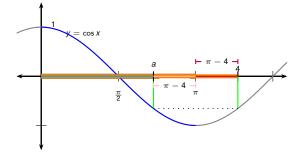
- 4 is not between 0 and π .
- We need the angle a between 0 and π for which $\cos 4 = \cos a$.

$$a = \pi - (4 - \pi) = \frac{2\pi}{4}$$

Therefore $\arccos(\cos 4) = \arccos(\cos a)$

$$= a = 2\pi - 4.$$

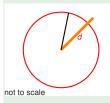




The horizon at sea is viewed at from a ship height of 10m. How far does the ship need to travel to reach the horizon just observed?

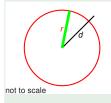




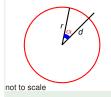


The horizon at sea is viewed at from a ship height of 10m. How far does the ship need to travel to reach the horizon just observed? Assume earth is round with radius 6371 km and that the ship sails along the shortest curved path.

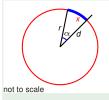
Let d be the distance from eyes of seaman to the center of earth.



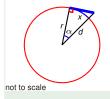
- Let d be the distance from eyes of seaman to the center of earth.
- Let *r* be the radius of earth.



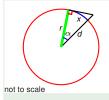
- Let d be the distance from eyes of seaman to the center of earth.
- Let *r* be the radius of earth. Let α be the indicated angle.



- Let d be the distance from eyes of seaman to the center of earth.
- Let r be the radius of earth. Let α be the indicated angle.
- Let the distance to the horizon be x.



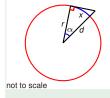
- Let d be the distance from eyes of seaman to the center of earth.
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- Let the distance to the horizon be *x*.



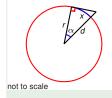
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- Let d be the distance from eyes of seaman to the center of earth.
- Let r be the radius of earth. Let α be the indicated angle.
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r = 6371 km

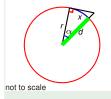


- Let d be the distance from eyes of seaman to the center of earth.
- Let r be the radius of earth. Let α be the indicated angle.
- Let the distance to the horizon be x.



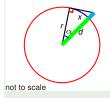
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- Let r be the radius of earth. Let α be the indicated angle.
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```
r=6371km
d=6371km + 0.01km
```



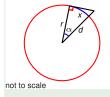
- Let d be the distance from eyes of seaman to the center of earth.
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$$r$$
=6371km
 d =6371km + 0.01km



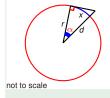
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$$r$$
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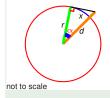
$$r$$
=6371km
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- Let r be the radius of earth. Let α be the indicated angle.
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$$r$$
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 d =6371km + 0.01km = 6371.01km

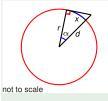
$$\cos \alpha = ?$$



- Let d be the distance from eyes of seaman to the center of earth.
- Let r be the radius of earth. Let α be the indicated angle.
- Let the distance to the horizon be x.

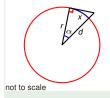
$$r$$
=6371km
 d =6371km + 0.01km = 6371.01km

$$\cos \alpha = \frac{I}{C}$$



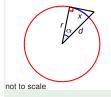
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$$r$$
=6371km
 d =6371km + 0.01km = 6371.01km
 $\cos \alpha = \frac{r}{d}$
 $\alpha = \arccos \left(\frac{r}{d}\right)$



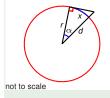
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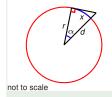
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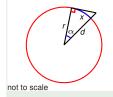
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 $\alpha = \arccos \left(\frac{r}{d}\right)$
 $x = r\alpha = r \arccos \left(\frac{r}{d}\right)$



- Let d be the distance from eyes of seaman to the center of earth.
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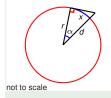
$$r=6371 \text{km}$$
 $d=6371 \text{km} + 0.01 \text{km} = 6371.01 \text{km}$
 $\cos \alpha = \frac{r}{d}$
 $\alpha = \arccos \left(\frac{r}{d}\right)$

$$x = r\alpha = r \arccos\left(\frac{r}{d}\right) = 6371 \text{km} \arccos\left(\frac{6371 \text{km}}{6371.01 \text{km}}\right)$$



- Let d be the distance from eyes of seaman to the center of earth.
- Let r be the radius of earth. Let α be the indicated angle.
- Let the distance to the horizon be x.

$$r=6371 \, \mathrm{km}$$
 $d=6371 \, \mathrm{km} + 0.01 \, \mathrm{km} = 6371.01 \, \mathrm{km}$
 $\cos \alpha = \frac{r}{d}$
 $\alpha = \arccos \left(\frac{r}{d}\right)$
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$$r$$
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 d =6371km + 0.01km = 6371.01km
 $\cos \alpha = \frac{r}{d}$
 $\alpha = \arccos \left(\frac{r}{d}\right)$

$$x = r\alpha = r \arccos\left(\frac{r}{d}\right) = 6371 \text{km} \arccos\left(\frac{6371 \text{km}}{6371.01 \text{km}}\right) \approx 11.29 \text{km}$$

2020

Example

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

sin(2 arccos(x))

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$.

sin(2 arccos(x))

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

sin(2 arccos(x))

2020

Example

$$\sin(2\arccos(x)) = \sin(2y)$$

Set
$$y = \arccos x$$

$$\sin(2\arccos(x)) = \sin(2y)$$
= ?

Set
$$y = \arccos x$$

Express via $\sin y, \cos y$

$$sin(2 arccos(x)) = sin(2y)$$
= 2 cos y sin y

Set
$$y = \arccos x$$

Express via $\sin y, \cos y$

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\sin(2\arccos(x)) = \sin(2y)$$

$$= 2\cos y \sin y$$

$$= 2\cos y \left(\pm\sqrt{1-\cos^2 y}\right)$$
Set $y = \arccos x$
Express via $\sin y$, $\cos y$
Express $\sin y$ via $\cos y$

Todor Milev Lecture 7 Trigonometric Graphs 2020

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\sin(2 \arccos(x)) = \sin(2y)$$

$$= 2 \cos y \sin y$$

$$= 2 \cos y \left(\pm \sqrt{1 - \cos^2 y}\right)$$

$$= 2 \cos y \sqrt{1 - \cos^2 y}$$

Set
$$y = \arccos x$$

Express via $\sin y$, $\cos y$
Express $\sin y$ via $\cos y$
 $\sin y > 0$ because
 $0 < y < \pi$

Todor Milev

$$sin(2 \operatorname{arccos}(x)) = sin(2y)$$

$$= 2 \cos y \sin y$$

$$= 2 \cos y \left(\pm \sqrt{1 - \cos^2 y} \right)$$

$$= 2 \cos y \sqrt{1 - \cos^2 y}$$

Set
$$y = \arccos x$$

Express via $\sin y$, $\cos y$
Express $\sin y$ via $\cos y$
 $\sin y > 0$ because
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Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$sin(2 \arccos(x)) = \sin(2y)
= 2 \cos y \sin y
= 2 \cos y \left(\pm \sqrt{1 - \cos^2 y}\right)
= 2 \cos y \sqrt{1 - \cos^2 y}
= 2x\sqrt{1 - x^2}$$
Set $y = \arccos x$
Express via $\sin y, \cos y$
Express $\sin y$ via $\cos y$

$$0 \le y \le \pi$$

use $x = \cos y$

Todor Milev

Rewrite $\sin(2\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the cos function.

Set $y = \arccos x$ Express via $\sin y$, $\cos y$ Express sin y via cos y $\sin y > 0$ because $0 \le y \le \pi$

Rewrite $cos(3 \operatorname{arccos}(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$.

cos(3 arccos(x))

Rewrite $cos(3 \operatorname{arccos}(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify arccos x we try to use cos(arccos x) = x.

cos(3 arccos(x))

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

cos(3 arccos(x))

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\cos(3\arccos(x)) = \cos(3y)$$

$$y = \arccos x$$

Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the \cos function.

$$\cos(3\arccos(x)) = \cos(3y) = \cos(2y + y)$$
 | $y = \arccos x$

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Rewrite $\cos(3\arccos(x))$ as an algebraic expression of x and $\sqrt{1-x^2}$. To simplify $\arccos x$ we try to use $\cos(\arccos x) = x$. Therefore our aim is to rewrite the expression only using the cos function.

$$cos(3 \operatorname{arccos}(x)) = cos(3y) = cos(2y + y)$$

=? $y = \operatorname{arccos} x$
Angle sum f-la

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Angle sum f-la
$$Express via$$

$$\sin y, \cos y$$

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$$= \cos^3 y - 3(?)$$

$$\cos y$$

$$y = \arccos x$$
Angle sum f-late to the following sin y, cos y
$$\sin y + \cos y$$
Express sin y
$$\sin x + \cos y$$

$$\cos y + \cos y + \cos y + \cos y$$

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Angle sum f-la

Trigonometric Graphs

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Angle sum f-la
Express via
$$\sin y, \cos y$$
Express $\sin y$
via $\cos y$

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$$= 4x^3 - 3x$$

$$x = \cos y$$

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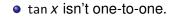
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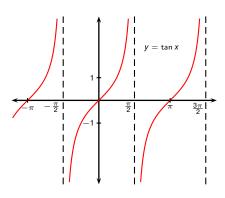
$$y = \arccos x$$
Angle sum f-la
Express via
$$\sin y, \cos y$$
Express $\sin y$
via $\cos y$

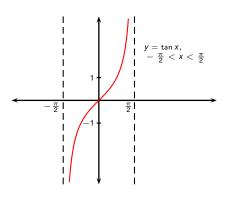
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Lecture 7

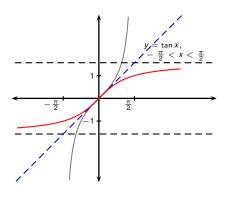
Trigonometric Graphs





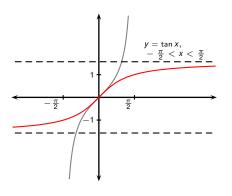


- tan x isn't one-to-one.
- Restrict the domain to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

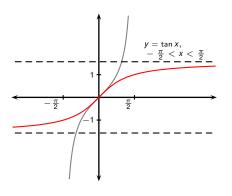


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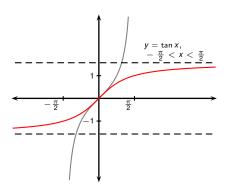
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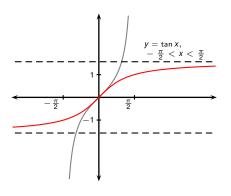
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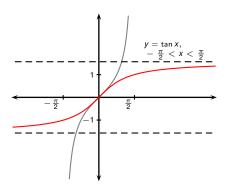
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- Range of arctan:



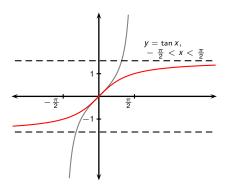
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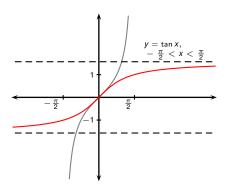
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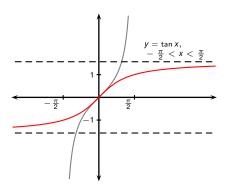


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- $\lim_{x \to -\infty} \arctan x =$

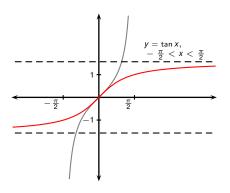


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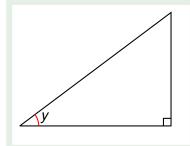
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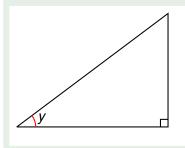


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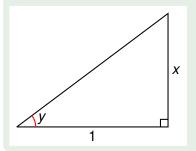


Simplify the expression cos(arctan x).

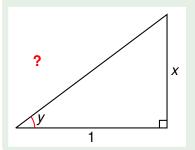
• Let $y = \arctan x$, so $\tan y = x$.



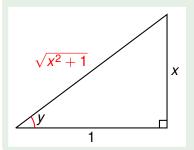
- Let $y = \arctan x$, so $\tan y = x$.
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- Length of hypotenuse = ?



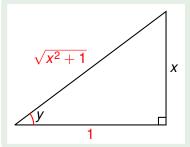
- Let $y = \arctan x$, so $\tan y = x$.
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- Length of hypotenuse = $\sqrt{1^2 + x^2}$.



Example

Simplify the expression cos(arctan x).

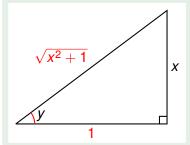
- Let $y = \arctan x$, so $\tan y = x$.
- Draw a right triangle with opposite *x* and adjacent 1.
- Length of hypotenuse = $\sqrt{1^2 + x^2}$.
- Then cos(arctan x) = ?



Example

Simplify the expression cos(arctan x).

- Let $y = \arctan x$, so $\tan y = x$.
- Draw a right triangle with opposite *x* and adjacent 1.
- Length of hypotenuse = $\sqrt{1^2 + x^2}$.
- Then $\cos(\arctan x) = \frac{1}{\sqrt{1+x^2}}$.



The remaining inverse trigonometric functions aren't used as often:

$$y = \operatorname{arccsc} x \quad (|x| \ge 1) \quad \Leftrightarrow \quad \operatorname{csc} y = x \quad \text{ and } \quad y \in \left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right]$$

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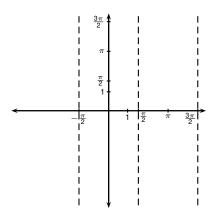
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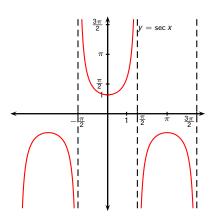
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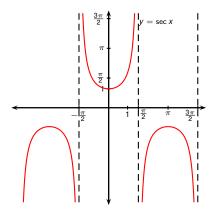
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Plot sec x.



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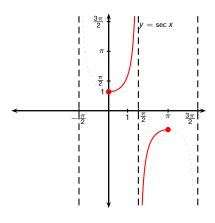
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- Plot sec x.
- Restrict domain to make one-to-one: Two common choices: $x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ and $x \in \left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right)$.

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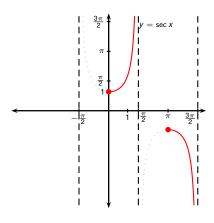


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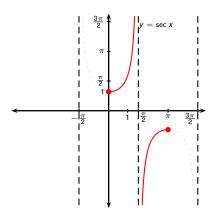
Lecture 7

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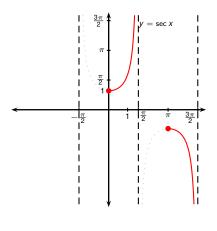
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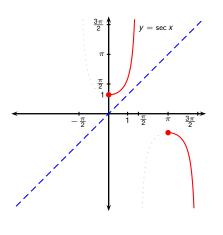
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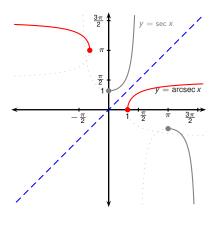
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Lecture 7

Trigonometric Graphs

2020

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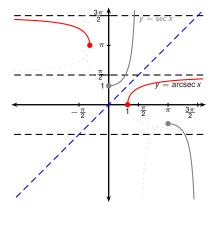


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