

Precalculus

Lecture 2

Trigonometry Definitions

Todor Milev

<https://github.com/tmilev/freecalculator>

2020

Outline

1 Trigonometry

- Definition of the Trigonometric Functions
- Basic Computations with Trigonometric Functions
- Reference Angles
- Geometric Interpretation of the Trigonometric Functions
- Periodicity and Symmetries of the Trig Functions

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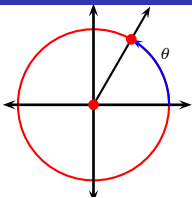
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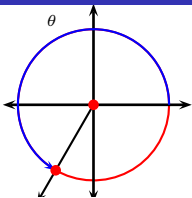
- Latest version of the .tex sources of the slides:
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- Should the link be outdated/moved, search for “freecalc project”.
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and the links therein.

Definition of the trigonometric functions



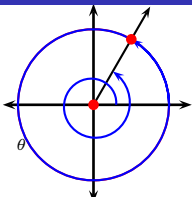
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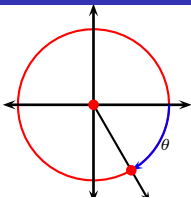
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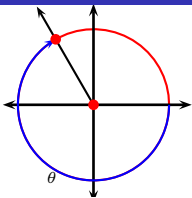
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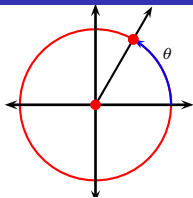
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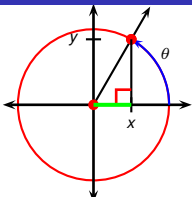
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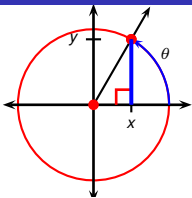
Definition (sin and cos)

The sine and cosine functions of the angle θ , denoted by $\sin \theta$ and $\cos \theta$, are defined by

$$\cos \theta = x$$

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Definition of the trigonometric functions



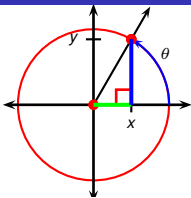
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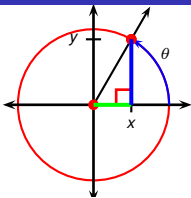
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Definition (additional trigonometric functions)

The functions **tangent**, cotangent, secant and cosecant of the angle θ , denoted by $\tan \theta$, $\cot \theta$, $\sec \theta$, $\csc \theta$, are defined by

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \qquad \cot \theta = \frac{\cos \theta}{\sin \theta} \qquad \sec \theta = \frac{1}{\cos \theta} \qquad \csc \theta = \frac{1}{\sin \theta}.$$

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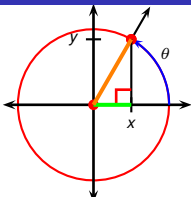
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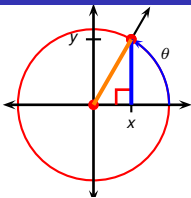
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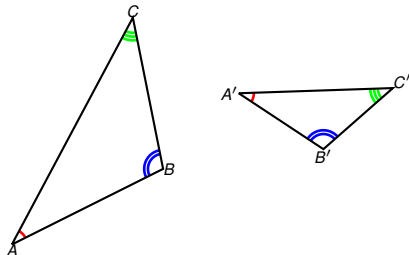
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Definition (Similar triangles)

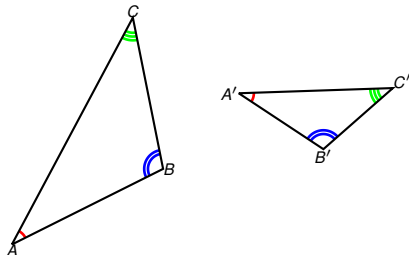
We say that a triangle $\triangle ABC$ is similar to a triangle $\triangle A'B'C'$ if the two triangles have equal angles.



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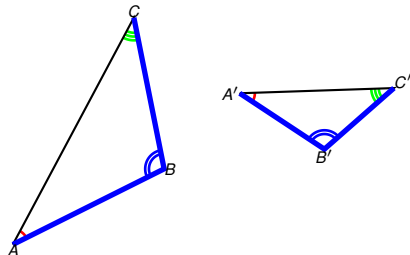
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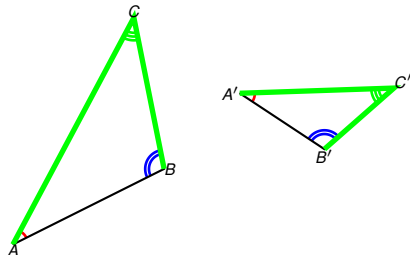
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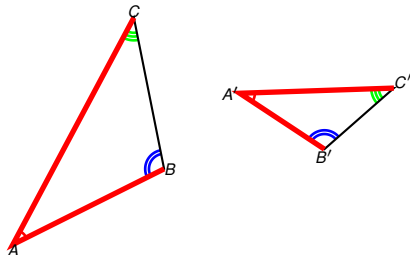
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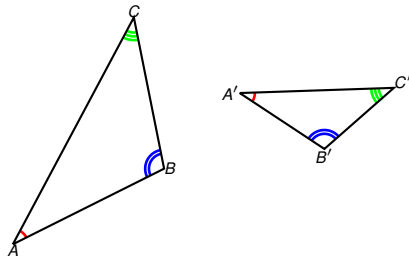


The following statement is proved in the subject of Euclidean (planar) geometry.

Theorem (Similar triangles have equal side ratios)

Let $\triangle ABC$ and $\triangle A'B'C'$ be two similar triangles. Then the ratios of the lengths of the sides of the two triangles are equal, that is

$$\frac{|AB|}{|BC|} = \frac{|A'B'|}{|B'C'|} \quad \frac{|BC|}{|CA|} = \frac{|B'C'|}{|C'A'|} \quad \frac{|CA|}{|AB|} = \frac{|C'A'|}{|A'B'|}$$

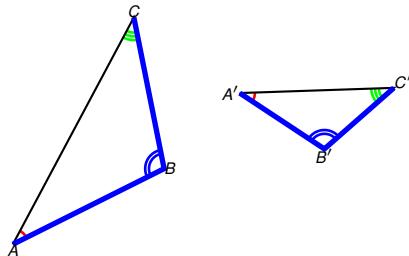


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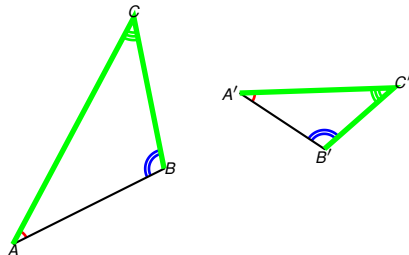


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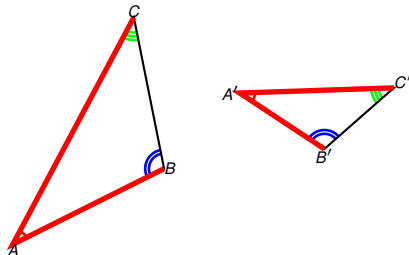


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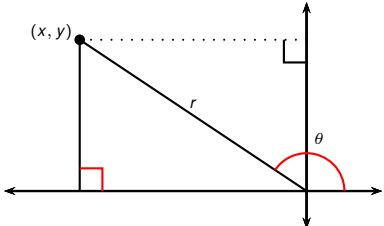
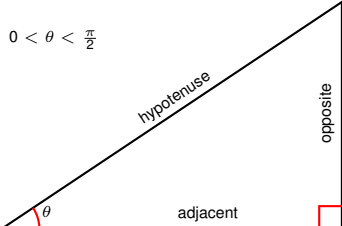
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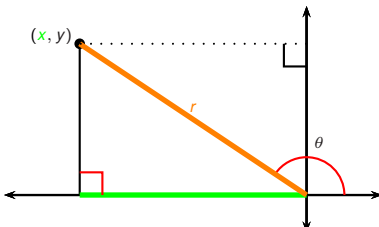
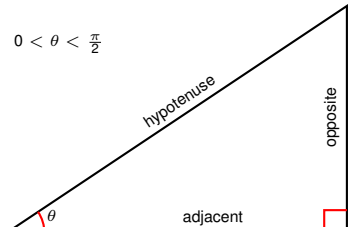


Trigonometric Functions and Right Angle Triangles

	
$\cos \theta$ $\sin \theta$ $\tan \theta$	$\sec \theta$ $\csc \theta$ $\cot \theta$
All angles	Acute angles

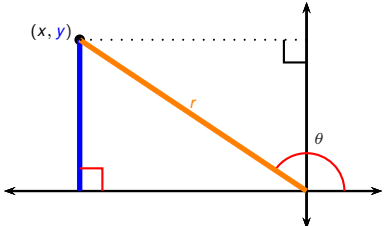
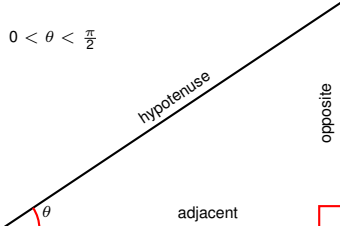
- The trigonometric functions can be defined without requesting that the pt. (x, y) on the terminal arm of the angle lie on the unit circle.

Trigonometric Functions and Right Angle Triangles

 <p>Diagram illustrating the definition of trigonometric functions for an angle θ in standard position. The terminal arm passes through the point (x, y) in the second quadrant. The distance from the origin to the point is r. The horizontal distance from the y-axis to the point is x, and the vertical distance is y. A right angle is shown at the point (x, y) between the vertical and horizontal segments.</p>	 <p>Diagram illustrating the definition of trigonometric functions for an acute angle θ. The angle θ is at the bottom-left vertex. The side adjacent to θ is the horizontal base, the side opposite is the vertical height, and the hypotenuse is the slanted side. A right angle is shown at the bottom-right vertex.</p>
$\cos \theta = \frac{x}{r}$ $\sin \theta$ $\tan \theta$	$\sec \theta$ $\csc \theta$ $\cot \theta$
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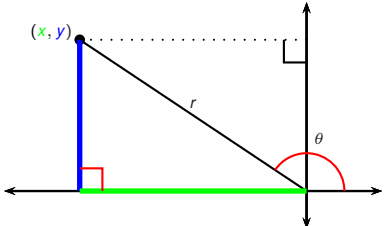
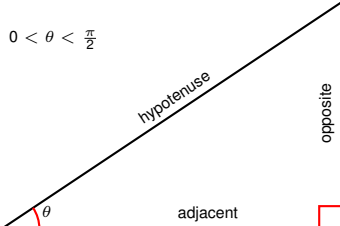
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- To do so we rescale by the distance r from the origin.

Trigonometric Functions and Right Angle Triangles

	
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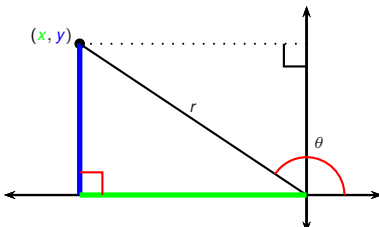
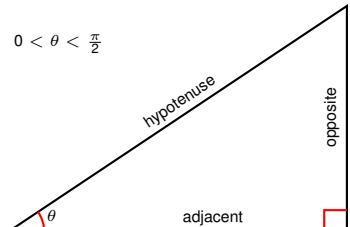
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 <p>Diagram showing an angle θ in standard position. The terminal arm passes through the point (x, y) and has length r. The x-axis is green, and the y-axis is blue. A right angle is shown at the origin.</p>	 <p>Diagram showing a right triangle with angle θ. The hypotenuse is the longest side, the adjacent is the side next to θ, and the opposite is the side opposite θ. A right angle is shown at the bottom right.</p>
$\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$	$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$
All angles	Acute angles

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Trigonometric Functions and Right Angle Triangles

 <p>Diagram showing an angle θ in standard position. The terminal arm passes through the point (x, y) in the second quadrant. The distance from the origin to the point is r. The x-axis is highlighted in green, and the y-axis is highlighted in blue. A right angle is shown at the point $(x, 0)$ on the x-axis.</p>	 <p>Diagram showing a right-angled triangle with angle θ at the bottom-left vertex. The hypotenuse is the longest side, the adjacent side is the base, and the opposite side is the height. A right angle is shown at the bottom-right vertex.</p>
$\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$	$0 < \theta < \frac{\pi}{2}$ $\cos \theta$ $\sin \theta$ $\tan \theta$
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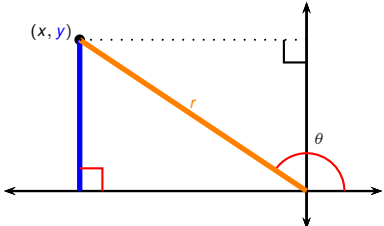
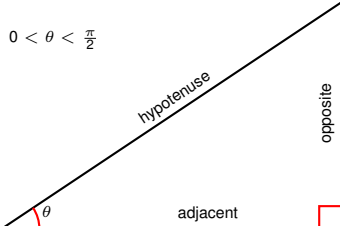
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<p>Diagram showing an angle θ in standard position. The terminal arm passes through point (x, y). The distance from the origin to (x, y) is r. The horizontal distance is x (green) and the vertical distance is y (black). A right angle is shown at $(x, 0)$.</p>	<p>Diagram showing a right triangle with angle θ at the bottom-left vertex. The hypotenuse is the longest side, the adjacent side is horizontal, and the opposite side is vertical. A right angle is shown at the bottom-right vertex.</p>
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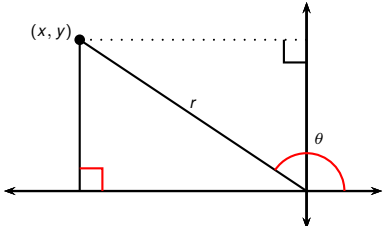
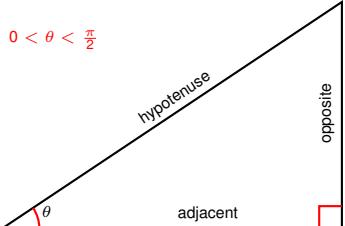
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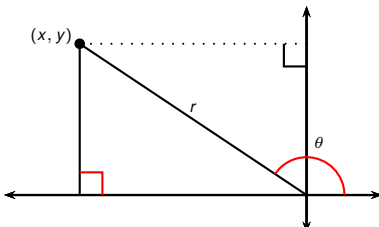
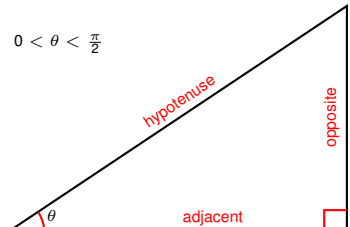
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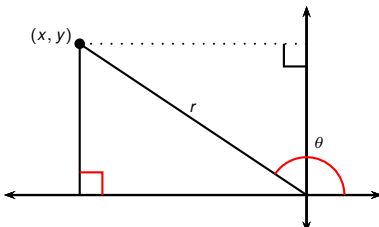
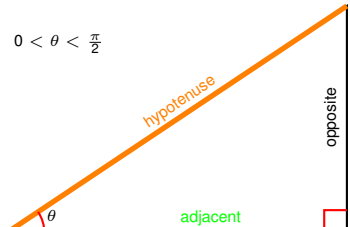
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- To do so we rescale by the distance r from the origin.
- The trig functions of **acute θ (between 0 and $\frac{\pi}{2}$)** can be interpreted as ratios of sides of right angle triangle with angle θ .

Trigonometric Functions and Right Angle Triangles

	
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$\sec \theta = \frac{r}{x}$ $\csc \theta = \frac{r}{y}$ $\cot \theta = \frac{x}{y}$	$\sec \theta$ $\csc \theta$ $\cot \theta$
All angles	Acute angles

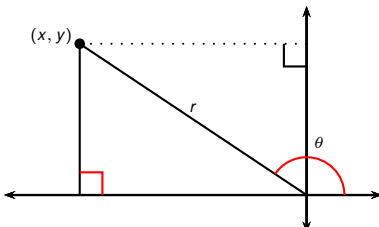
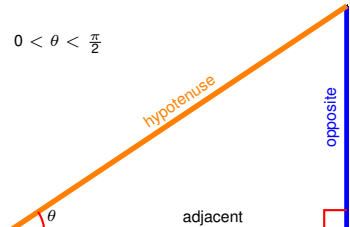
- The trigonometric functions can be defined without requesting that the pt. (x, y) on the terminal arm of the angle lie on the unit circle.
- To do so we rescale by the distance r from the origin.
- The trig functions of acute θ (between 0 and $\frac{\pi}{2}$) can be interpreted as ratios of **sides of right angle triangle** with angle θ .

Trigonometric Functions and Right Angle Triangles

	
$\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\sin \theta$ $\tan \theta$
$\sec \theta = \frac{r}{x}$ $\csc \theta = \frac{r}{y}$ $\cot \theta = \frac{x}{y}$	$\sec \theta$ $\csc \theta$ $\cot \theta$
All angles	Acute angles

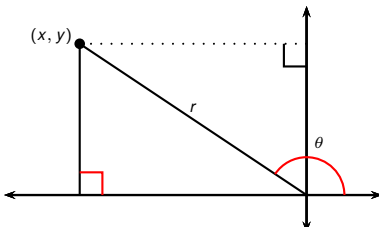
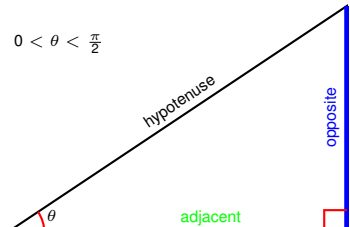
- The trigonometric functions can be defined without requesting that the pt. (x, y) on the terminal arm of the angle lie on the unit circle.
- To do so we rescale by the distance r from the origin.
- The trig functions of acute θ (between 0 and $\frac{\pi}{2}$) can be interpreted as ratios of sides of right angle triangle with angle θ .

Trigonometric Functions and Right Angle Triangles

 <p>Diagram showing an angle θ in standard position. The terminal arm passes through the point (x, y). The distance from the origin to the point is r. The angle θ is measured counter-clockwise from the positive x-axis.</p>	 <p>Diagram showing an acute angle θ in a right triangle. The hypotenuse is labeled <i>hypotenuse</i>, the adjacent side is labeled <i>adjacent</i>, and the opposite side is labeled <i>opposite</i>.</p>
$\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$ $\sec \theta = \frac{r}{x}$ $\csc \theta = \frac{r}{y}$ $\cot \theta = \frac{x}{y}$	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ $\tan \theta = \frac{\text{opp}}{\text{adj}}$ $\sec \theta = \frac{\text{hyp}}{\text{adj}}$ $\csc \theta = \frac{\text{hyp}}{\text{opp}}$ $\cot \theta = \frac{\text{adj}}{\text{opp}}$
All angles	Acute angles

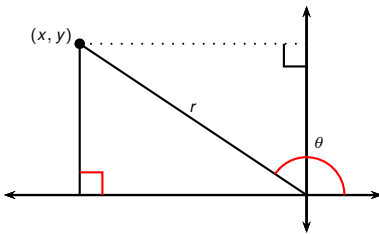
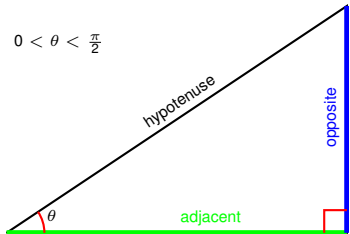
- The trigonometric functions can be defined without requesting that the pt. (x, y) on the terminal arm of the angle lie on the unit circle.
- To do so we rescale by the distance r from the origin.
- The trig functions of acute θ (between 0 and $\frac{\pi}{2}$) can be interpreted as ratios of sides of right angle triangle with angle θ .

Trigonometric Functions and Right Angle Triangles

	
$\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ $\tan \theta = \frac{\text{opp}}{\text{adj}}$
All angles	Acute angles

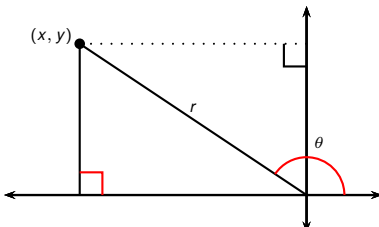
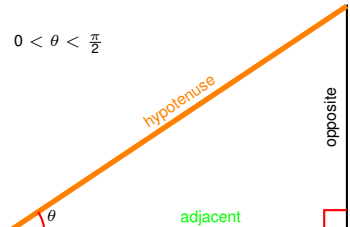
- The trigonometric functions can be defined without requesting that the pt. (x, y) on the terminal arm of the angle lie on the unit circle.
- To do so we rescale by the distance r from the origin.
- The trig functions of acute θ (between 0 and $\frac{\pi}{2}$) can be interpreted as ratios of sides of right angle triangle with angle θ .

Trigonometric Functions and Right Angle Triangles

 $\cos \theta = \frac{x}{r} \quad \sec \theta = \frac{r}{x}$ $\sin \theta = \frac{y}{r} \quad \csc \theta = \frac{r}{y}$ $\tan \theta = \frac{y}{x} \quad \cot \theta = \frac{x}{y}$ <p style="text-align: center;">All angles</p>	 $0 < \theta < \frac{\pi}{2}$ $\cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \sec \theta$ $\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \csc \theta$ $\tan \theta = \frac{\text{opp}}{\text{adj}} \quad \cot \theta = \frac{\text{adj}}{\text{opp}}$ <p style="text-align: center;">Acute angles</p>
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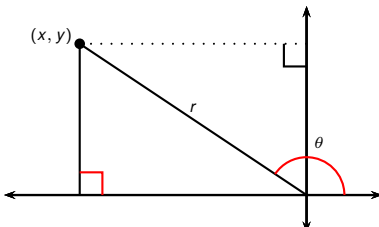
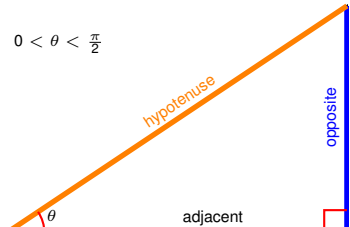
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- To do so we rescale by the distance r from the origin.
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Trigonometric Functions and Right Angle Triangles

	
$\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ $\tan \theta = \frac{\text{opp}}{\text{adj}}$
$\sec \theta = \frac{r}{x}$ $\csc \theta = \frac{r}{y}$ $\cot \theta = \frac{x}{y}$	$\sec \theta = \frac{\text{hyp}}{\text{adj}}$ $\csc \theta = \frac{\text{hyp}}{\text{opp}}$ $\cot \theta = \frac{\text{adj}}{\text{opp}}$
All angles	Acute angles

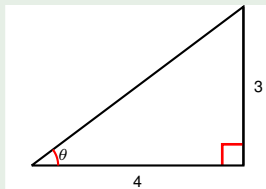
- The trigonometric functions can be defined without requesting that the pt. (x, y) on the terminal arm of the angle lie on the unit circle.
- To do so we rescale by the distance r from the origin.
- The trig functions of acute θ (between 0 and $\frac{\pi}{2}$) can be interpreted as ratios of sides of right angle triangle with angle θ .

Trigonometric Functions and Right Angle Triangles

	
$\cos \theta = \frac{x}{r}$ $\sin \theta = \frac{y}{r}$ $\tan \theta = \frac{y}{x}$	$\cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ $\tan \theta = \frac{\text{opp}}{\text{adj}}$
$\sec \theta = \frac{r}{x}$ $\csc \theta = \frac{r}{y}$ $\cot \theta = \frac{x}{y}$	$\sec \theta = \frac{\text{hyp}}{\text{adj}}$ $\csc \theta = \frac{\text{hyp}}{\text{opp}}$ $\cot \theta = \frac{\text{adj}}{\text{opp}}$
All angles	Acute angles

- The trigonometric functions can be defined without requesting that the pt. (x, y) on the terminal arm of the angle lie on the unit circle.
- To do so we rescale by the distance r from the origin.
- The trig functions of acute θ (between 0 and $\frac{\pi}{2}$) can be interpreted as ratios of sides of right angle triangle with angle θ .

Example

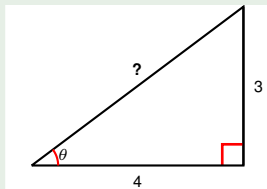


³ Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

$$\sin \theta = \quad \cos \theta = \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

Example



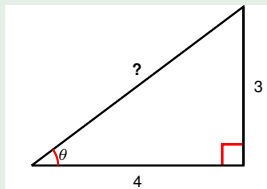
³ Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

To find the trigonometric functions, we need to know the length of the hypotenuse.

$$\sin \theta = \quad \cos \theta = \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

Example



³ Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

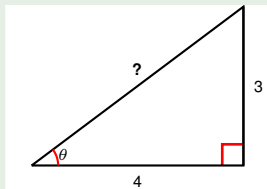
To find the trigonometric functions, we need to know the length of the hypotenuse.

hypotenuse = ?

$$\sin \theta = \quad \cos \theta = \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

Example



³ Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

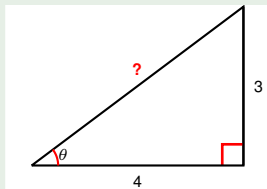
To find the trigonometric functions, we need to know the length of the hypotenuse.

$$\text{hypotenuse} = \sqrt{4^2 + 3^2}$$

$$\sin \theta = \quad \cos \theta = \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

Example



³ Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

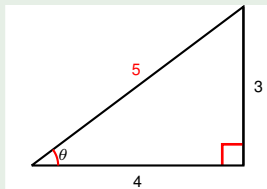
To find the trigonometric functions, we need to know the length of the hypotenuse.

$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25}$$

$$\sin \theta = \quad \cos \theta = \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

Example



³ Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

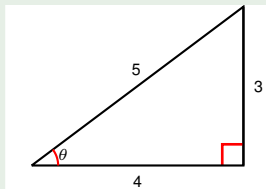
To find the trigonometric functions, we need to know the length of the hypotenuse.

$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

$$\sin \theta = \quad \cos \theta = \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

Example



Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

To find the trigonometric functions, we need to know the length of the hypotenuse.

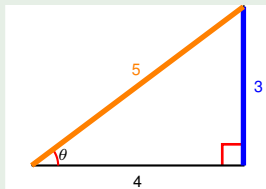
$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

Using the right angle triangle ratio interpretations of the trig functions, we can compute:

$$\sin \theta = ? \quad \cos \theta = \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

Example



Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

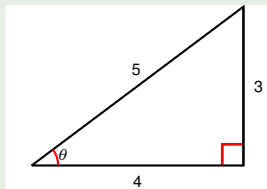
To find the trigonometric functions, we need to know the length of the hypotenuse.

$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

Using the right angle triangle ratio interpretations of the trig functions, we can compute:

$$\begin{array}{lll} \sin \theta = \frac{3}{5} & \cos \theta = & \tan \theta = \\ \csc \theta = & \sec \theta = & \cot \theta = \end{array}$$

Example



Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

To find the trigonometric functions, we need to know the length of the hypotenuse.

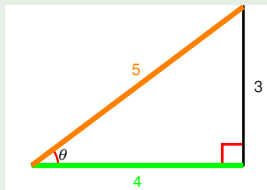
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Using the right angle triangle ratio interpretations of the trig functions, we can compute:

$$\sin \theta = \frac{3}{5} \quad \cos \theta = ? \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

Example



³ Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

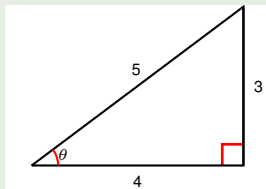
To find the trigonometric functions, we need to know the length of the hypotenuse.

$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

Using the right angle triangle ratio interpretations of the trig functions, we can compute:

$$\begin{array}{lll} \sin \theta = \frac{3}{5} & \cos \theta = \frac{4}{5} & \tan \theta = \\ \csc \theta = & \sec \theta = & \cot \theta = \end{array}$$

Example



Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

To find the trigonometric functions, we need to know the length of the hypotenuse.

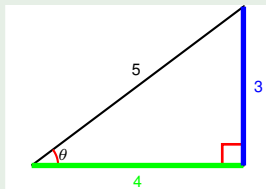
$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

Using the right angle triangle ratio interpretations of the trig functions, we can compute:

$$\sin \theta = \frac{3}{5} \quad \cos \theta = \frac{4}{5} \quad \tan \theta = ?$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

Example



Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

To find the trigonometric functions, we need to know the length of the hypotenuse.

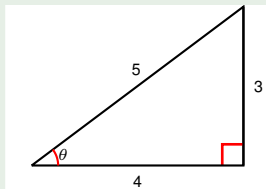
$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

Using the right angle triangle ratio interpretations of the trig functions, we can compute:

$$\sin \theta = \frac{3}{5} \quad \cos \theta = \frac{4}{5} \quad \tan \theta = \frac{3}{4}$$

$$\csc \theta = \quad \sec \theta = \quad \cot \theta =$$

Example



Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

To find the trigonometric functions, we need to know the length of the hypotenuse.

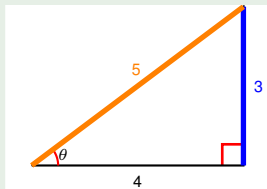
$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

Using the right angle triangle ratio interpretations of the trig functions, we can compute:

$$\sin \theta = \frac{3}{5} \quad \cos \theta = \frac{4}{5} \quad \tan \theta = \frac{3}{4}$$

$$\text{csc } \theta = ? \quad \sec \theta = \quad \cot \theta =$$

Example



Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

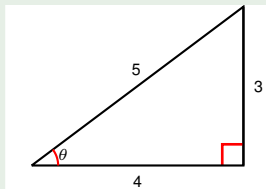
To find the trigonometric functions, we need to know the length of the hypotenuse.

$$\text{hypotenuse} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5.$$

Using the right angle triangle ratio interpretations of the trig functions, we can compute:

$$\begin{array}{lll} \sin \theta = \frac{3}{5} & \cos \theta = \frac{4}{5} & \tan \theta = \frac{3}{4} \\ \text{csc } \theta = \frac{5}{3} & \sec \theta = & \cot \theta = \end{array}$$

Example



Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

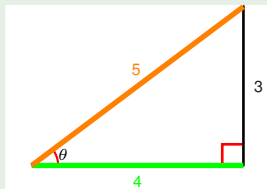
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$$\begin{array}{lll} \sin \theta = \frac{3}{5} & \cos \theta = \frac{4}{5} & \tan \theta = \frac{3}{4} \\ \csc \theta = \frac{5}{3} & \sec \theta = ? & \cot \theta = \end{array}$$

Example



³ Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

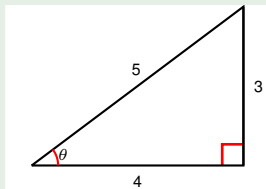
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$$\begin{array}{lll} \sin \theta = \frac{3}{5} & \cos \theta = \frac{4}{5} & \tan \theta = \frac{3}{4} \\ \csc \theta = \frac{5}{3} & \sec \theta = \frac{5}{4} & \cot \theta = \end{array}$$

Example



Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

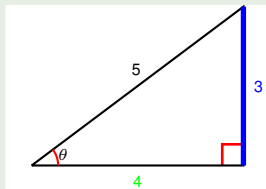
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Using the right angle triangle ratio interpretations of the trig functions, we can compute:

$$\begin{array}{lll} \sin \theta = \frac{3}{5} & \cos \theta = \frac{4}{5} & \tan \theta = \frac{3}{4} \\ \csc \theta = \frac{5}{3} & \sec \theta = \frac{5}{4} & \cot \theta = ? \end{array}$$

Example



Let the angle θ be as indicated in the figure. Find the values of the six trigonometric functions of θ .

To find the trigonometric functions, we need to know the length of the hypotenuse.

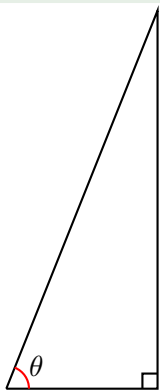
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Using the right angle triangle ratio interpretations of the trig functions, we can compute:

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Example

If $\cos \theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ .



$$\sin \theta =$$

$$\tan \theta =$$

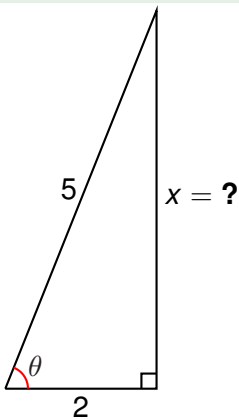
$$\csc \theta =$$

$$\sec \theta =$$

$$\cot \theta =$$

Example

If $\cos \theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ .



- Label the hypotenuse with length 5 and the adjacent side with length 2.

$$\sin \theta =$$

$$\tan \theta =$$

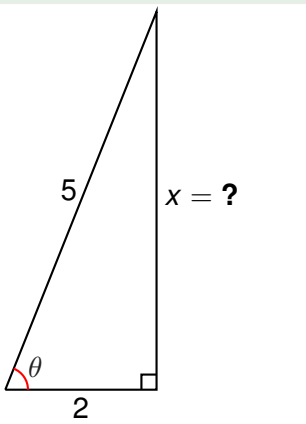
$$\csc \theta =$$

$$\sec \theta =$$

$$\cot \theta =$$

Example

If $\cos \theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ .



- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem: $x^2 + 2^2 = 5^2$.

$$\sin \theta =$$

$$\tan \theta =$$

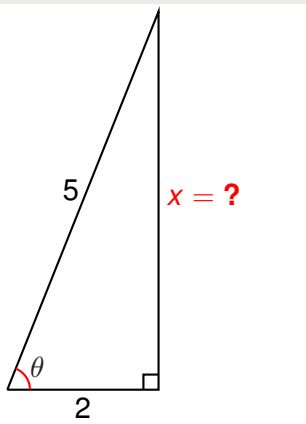
$$\csc \theta =$$

$$\sec \theta =$$

$$\cot \theta =$$

Example

If $\cos \theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ .



- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem: $x^2 + 2^2 = 5^2$.
- Therefore $x^2 = ?$, so $x = ?$.

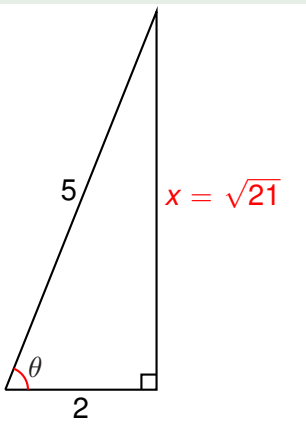
$$\sin \theta = \quad \quad \tan \theta =$$

$$\csc \theta = \quad \quad \sec \theta =$$

$$\cot \theta =$$

Example

If $\cos \theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ .



- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem: $x^2 + 2^2 = 5^2$.
- Therefore $x^2 = 21$, so $x = \sqrt{21}$.

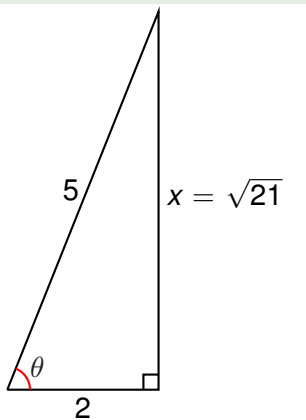
$$\sin \theta = \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta =$$

$$\cot \theta =$$

Example

If $\cos \theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ .



- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem: $x^2 + 2^2 = 5^2$.
- Therefore $x^2 = 21$, so $x = \sqrt{21}$.

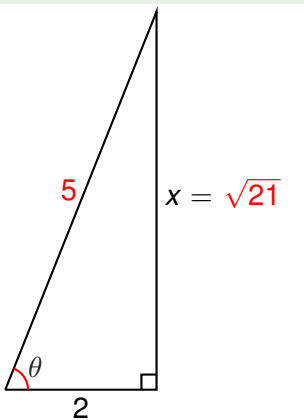
$$\sin \theta = ? \quad \tan \theta =$$

$$\csc \theta = \quad \sec \theta =$$

$$\cot \theta =$$

Example

If $\cos \theta = \frac{2}{5}$ and $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ .



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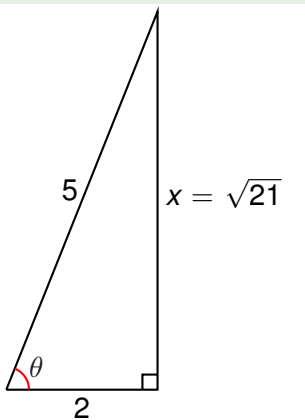
$$\sin \theta = \frac{\sqrt{21}}{5} \quad \tan \theta =$$

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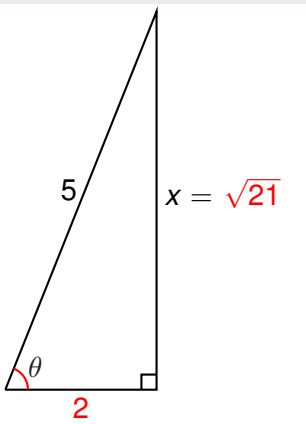
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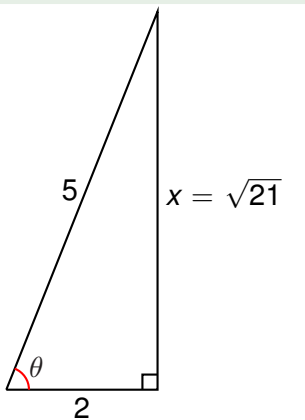
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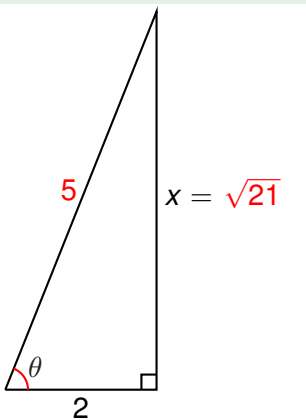
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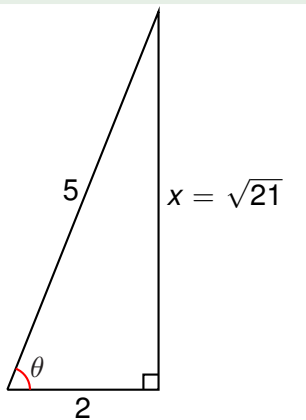
$$\sin \theta = \frac{\sqrt{21}}{5} \quad \tan \theta = \frac{\sqrt{21}}{2}$$

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$$\cot \theta =$$

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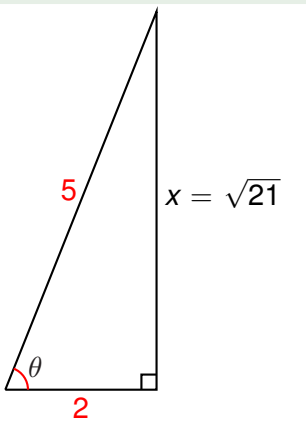
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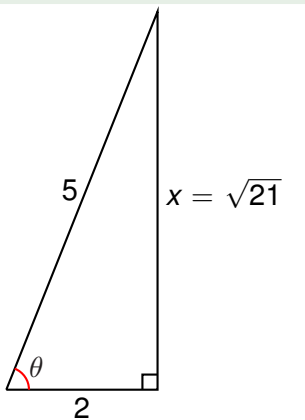
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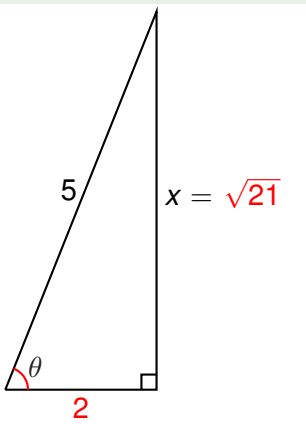
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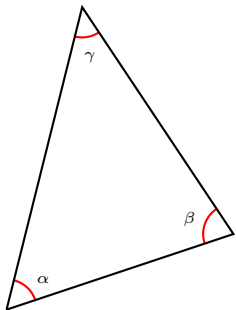


- Label the hypotenuse with length 5 and the adjacent side with length 2.
- Pythagorean theorem: $x^2 + 2^2 = 5^2$.
- Therefore $x^2 = 21$, so $x = \sqrt{21}$.

$$\sin \theta = \frac{\sqrt{21}}{5} \quad \tan \theta = \frac{\sqrt{21}}{2}$$

$$\csc \theta = \frac{5}{\sqrt{21}} \quad \sec \theta = \frac{5}{2}$$

$$\cot \theta = \frac{2}{\sqrt{21}}$$



Proposition

The angles of every triangle sum up to $\pi = 180^\circ$.

In other words, if α, β, γ are the angles indicated in the figure, then we have:

$$\alpha + \beta + \gamma = 180^\circ.$$

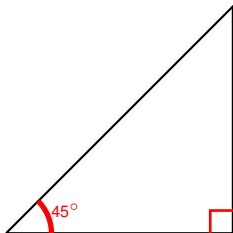
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Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.

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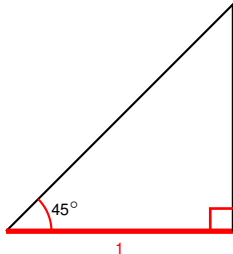
- Draw the 45° angle in right angle triangle,



Example

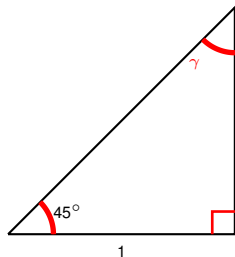
Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.

- Draw the 45° angle in right angle triangle, adjacent side of length **1**.



Example

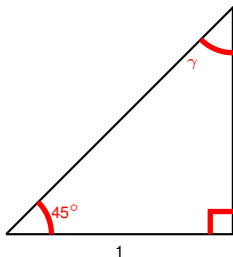
Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.



- Draw the 45° angle in right angle triangle, adjacent side of length 1.
- Let γ be the angle indicated on the plot.

Example

Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.

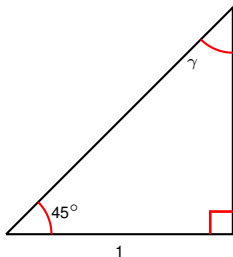


- Draw the 45° angle in right angle triangle, adjacent side of length 1.
- Let γ be the angle indicated on the plot.
- Angles in triangle sum to 180° :

$$45^\circ + 90^\circ + \gamma = 180^\circ$$

Example

Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.

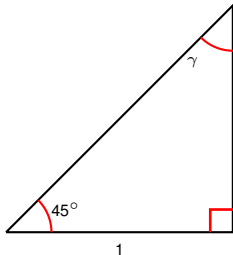


- Draw the 45° angle in right angle triangle, adjacent side of length 1.
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Example

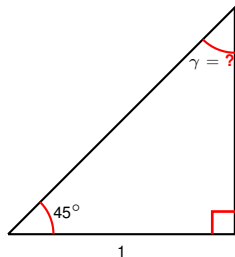
Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.



- Draw the 45° angle in right angle triangle, adjacent side of length 1.
- Let γ be the angle indicated on the plot.
- Angles in triangle sum to 180° :

$$\begin{aligned}45^\circ + 90^\circ + \gamma &= 180^\circ \\ \gamma &= 180^\circ - 90^\circ - 45^\circ\end{aligned}$$

Example



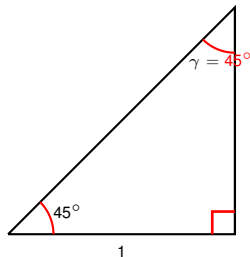
Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.

- Draw the 45° angle in right angle triangle, adjacent side of length 1.
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$$\begin{aligned} 45^\circ + 90^\circ + \gamma &= 180^\circ \\ \gamma &= 180^\circ - 90^\circ - 45^\circ = ? \end{aligned}$$

Example

Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.



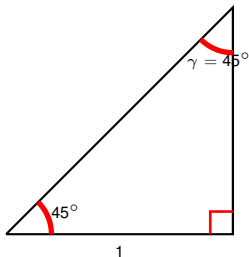
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Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.



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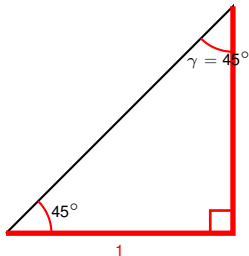
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- Triangle has two equal angles

Example

Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.



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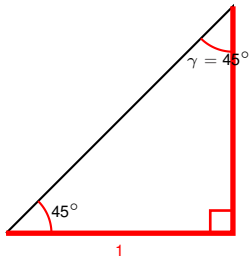
$$45^\circ + 90^\circ + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 90^\circ - 45^\circ = 45^\circ.$$

- Triangle has two equal angles \Rightarrow is **isosceles**

Example

Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.



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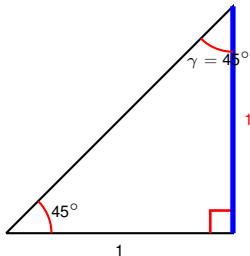
$$45^\circ + 90^\circ + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 90^\circ - 45^\circ = 45^\circ.$$

- Triangle has two equal angles \Rightarrow is **isosceles (has two equal sides)**.

Example

Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.



- Draw the 45° angle in right angle triangle, adjacent side of length 1.
- Let γ be the angle indicated on the plot.
- Angles in triangle sum to 180° :

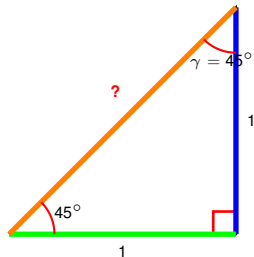
$$45^\circ + 90^\circ + \gamma = 180^\circ$$

$$\gamma = 180^\circ - 90^\circ - 45^\circ = 45^\circ.$$

- Triangle has two equal angles \Rightarrow is isosceles (has two equal sides).
- \Rightarrow **Opposite leg: length 1**

Example

Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.



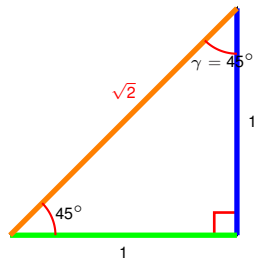
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- Let γ be the angle indicated on the plot.
- Angles in triangle sum to 180° :

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- Triangle has two equal angles \Rightarrow is isosceles (has two equal sides).
- \Rightarrow Opposite leg: length 1 \Rightarrow length(hyp) = ? .

Example



Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.

- Draw the 45° angle in right angle triangle, adjacent side of length 1.
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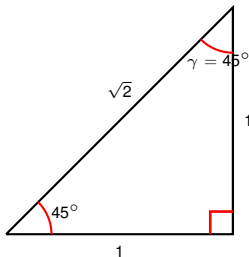
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- Triangle has two equal angles \Rightarrow is isosceles (has two equal sides).
- \Rightarrow Opposite leg: length 1 \Rightarrow length(hyp) = $\sqrt{1^2 + 1^2} = \sqrt{2}$.

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Find the values of $\sin 45^\circ$, $\cos 45^\circ$, $\tan 45^\circ$.



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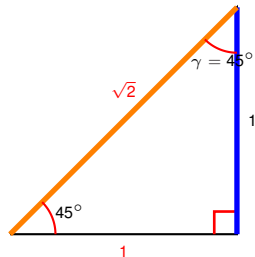
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• $\sin 45^\circ = ?$

$\cos 45^\circ = ?$

$\tan 45^\circ = ?$

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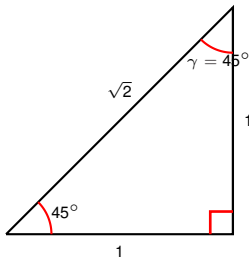
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$$\sin 45^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{2}}{2} \quad \cos 45^\circ = ?$$

$$\tan 45^\circ = ?$$

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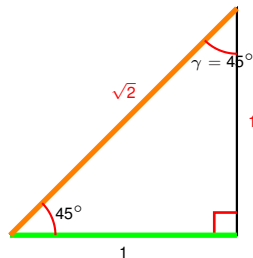
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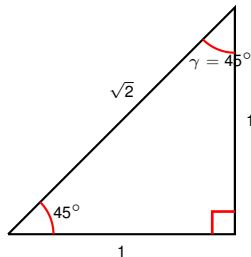
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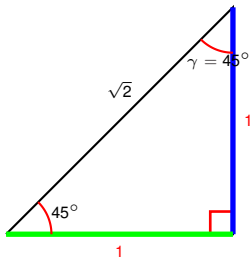
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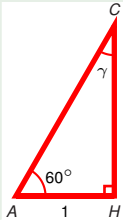
$$\sin 45^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{2}}{2} \qquad \cos 45^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{\text{opp}}{\text{adj}} = \frac{1}{1} = 1.$$

Example

Find the values of $\sin 60^\circ$, $\cos 60^\circ$, $\tan 60^\circ$,
 $\sin 30^\circ$, $\cos 30^\circ$, $\tan 30^\circ$.

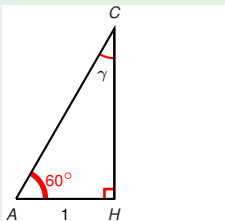
Example



Find the values of $\sin 60^\circ$, $\cos 60^\circ$, $\tan 60^\circ$,
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Construct a right angled $\triangle AHC$ as indicated:

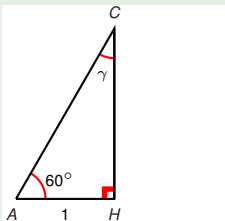
Example



Find the values of $\sin 60^\circ$, $\cos 60^\circ$, $\tan 60^\circ$,
 $\sin 30^\circ$, $\cos 30^\circ$, $\tan 30^\circ$.

Construct a right angled $\triangle AHC$ as indicated: angles
 60° , 90° , γ .

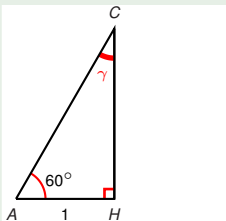
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Find the values of $\sin 60^\circ$, $\cos 60^\circ$, $\tan 60^\circ$,
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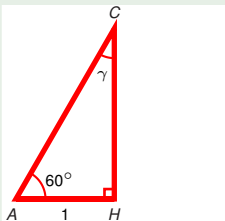
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Example

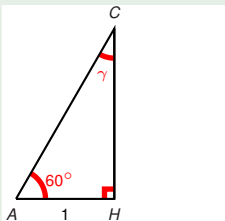


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Construct a right angled $\triangle AHC$ as indicated: angles
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$$60^\circ + 90^\circ + \gamma = 180^\circ$$

Example

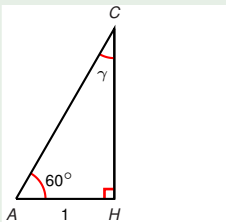


Find the values of $\sin 60^\circ$, $\cos 60^\circ$, $\tan 60^\circ$, $\sin 30^\circ$, $\cos 30^\circ$, $\tan 30^\circ$.

Construct a right angled $\triangle AHC$ as indicated: angles 60° , 90° , γ . Angles in \triangle **sum to 180°** :

$$60^\circ + 90^\circ + \gamma = 180^\circ$$

Example



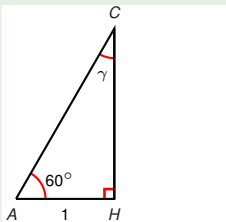
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Example



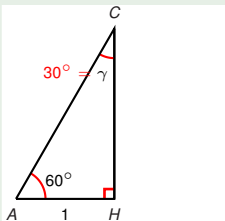
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Example



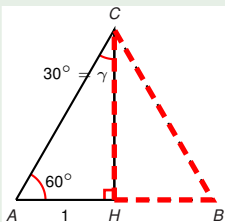
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Example



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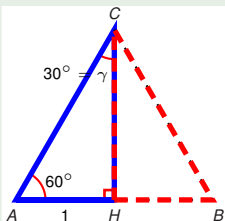
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Construct $\triangle HBC$ as indicated so that $\triangle HBC \cong \triangle HAC$.

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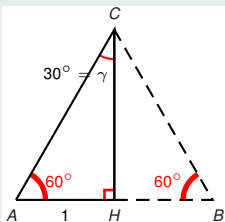
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Example



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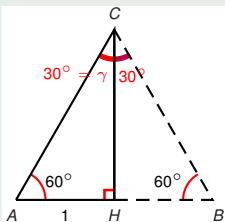
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Example



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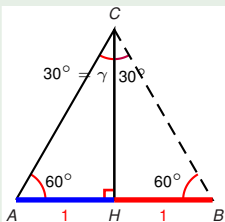
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Example



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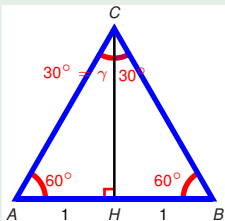
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Example



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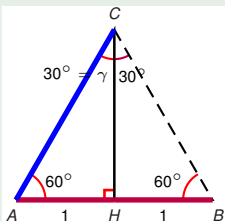
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Construct $\triangle HBC$ as indicated so that $\triangle HBC \cong \triangle HAC$. $\triangle ABC$ has
 three equal angles ($= 60^\circ$)

Example



Find the values of $\sin 60^\circ$, $\cos 60^\circ$, $\tan 60^\circ$,
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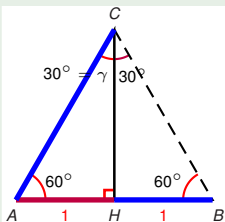
$$60^\circ + 90^\circ + \gamma = 180^\circ$$

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Construct $\triangle HBC$ as indicated so that $\triangle HBC \cong \triangle HAC$. $\triangle ABC$ has
 three equal angles ($= 60^\circ$) \Rightarrow its sides are of equal length. Therefore

$$|AC| = |AB|$$

Example



Find the values of $\sin 60^\circ$, $\cos 60^\circ$, $\tan 60^\circ$,
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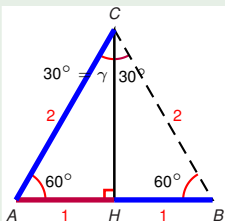
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$$|AC| = |AB| = 1 + 1$$

Example



Find the values of $\sin 60^\circ$, $\cos 60^\circ$, $\tan 60^\circ$,
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Construct a right angled $\triangle AHC$ as indicated: angles 60° , 90° , γ . Angles in \triangle sum to 180° :

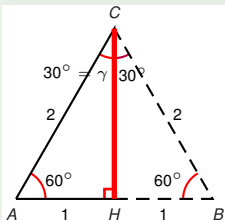
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Construct $\triangle HBC$ as indicated so that $\triangle HBC \cong \triangle HAC$. $\triangle ABC$ has three equal angles ($= 60^\circ$) \Rightarrow its sides are of equal length. Therefore

$$|AC| = |AB| = 1 + 1 = 2$$

Example



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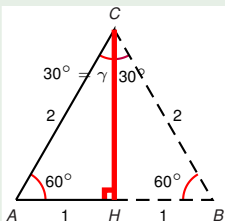
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$$|CH| = ?$$

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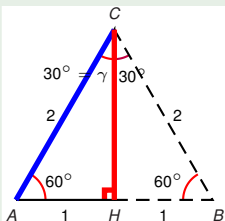
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$$|CH| = \sqrt{|AC|^2 - |AH|^2} \quad \bigg| \quad \text{Pythagorean theorem}$$

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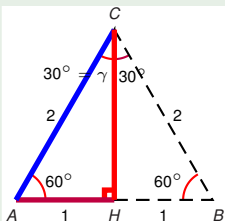
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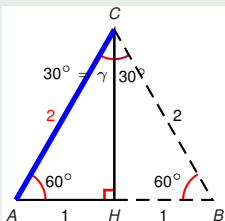
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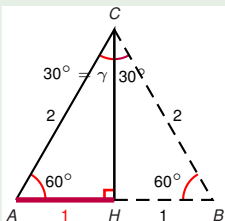
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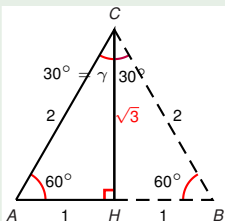
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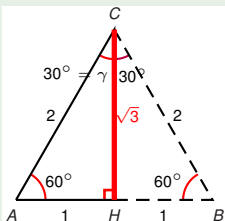
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$$= \sqrt{2^2 - 1^2} = \sqrt{3}$$

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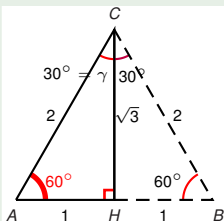
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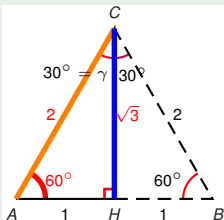
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$$\sin 60^\circ = ?$$

$$\cos 60^\circ = ?$$

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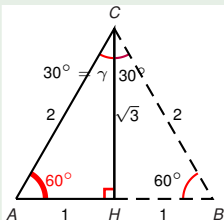
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$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = ?$$

$$\tan 60^\circ = ?$$

Example



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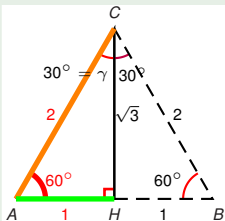
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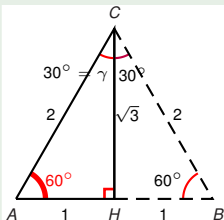
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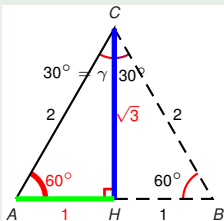
$$= \sqrt{2^2 - 1^2} = \sqrt{3}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = ?$$

Example



Find the values of $\sin 60^\circ$, $\cos 60^\circ$, $\tan 60^\circ$,
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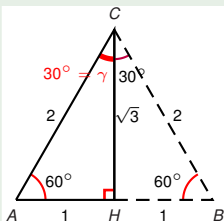
$$|AC| = |AB| = 1 + 1 = 2$$

$$\begin{aligned} |CH| &= \sqrt{|AC|^2 - |AH|^2} && \text{Pythagorean theorem} \\ &= \sqrt{2^2 - 1^2} = \sqrt{3} \end{aligned}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Example



Find the values of $\sin 60^\circ$, $\cos 60^\circ$, $\tan 60^\circ$,
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$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

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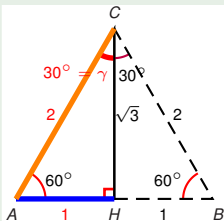
$$\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

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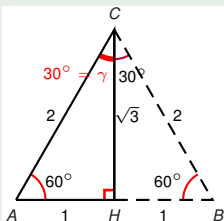
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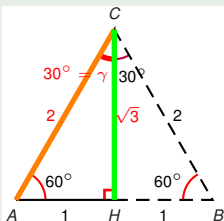
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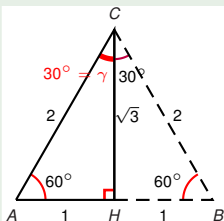
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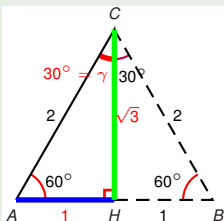
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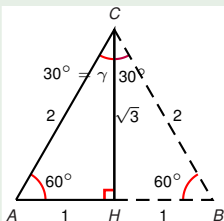
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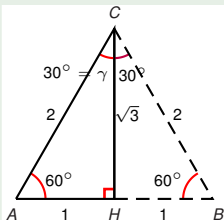
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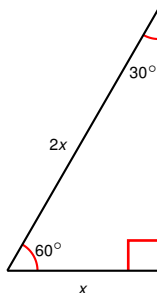
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Observation

- *If the hypotenuse of a right angle triangle is twice larger than one of the sides, then the angle opposite to that side is 30° .*
- *Conversely, in a right angle triangle with angle 30° , the hypotenuse is twice longer than the shorter of the two legs.*

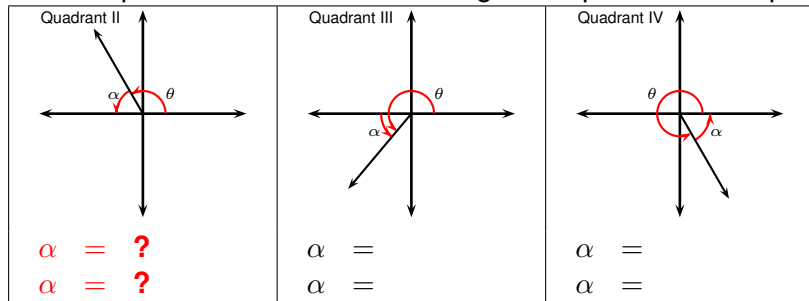


To compute trigonometric functions from obtuse ($> 90^\circ$) or negative angles, we can use the following visual aid.

Definition (Reference Angle)

Let θ be an angle in standard position. Its reference angle is the acute positive angle formed by the terminal arm and the x axis.

The computation of the reference angle α depends on the quadrant.

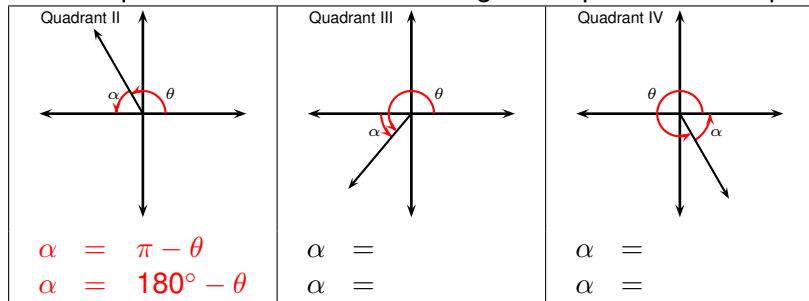


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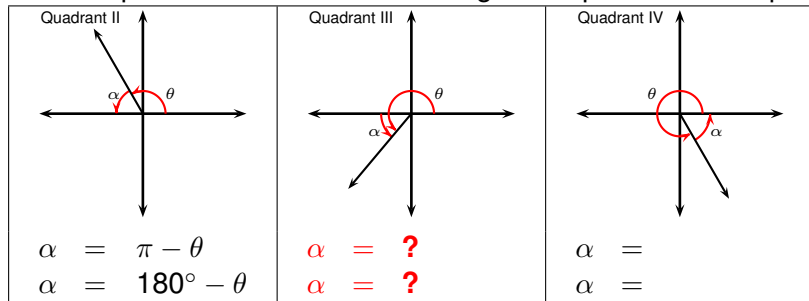


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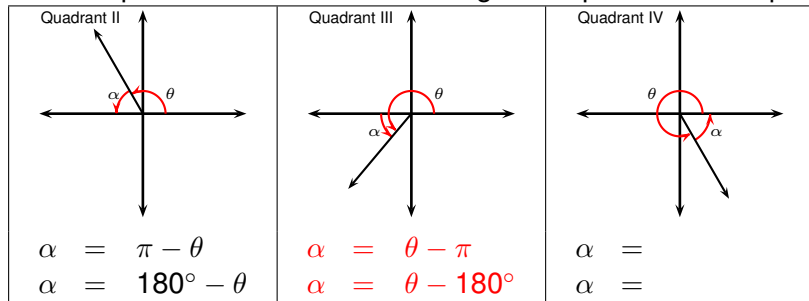


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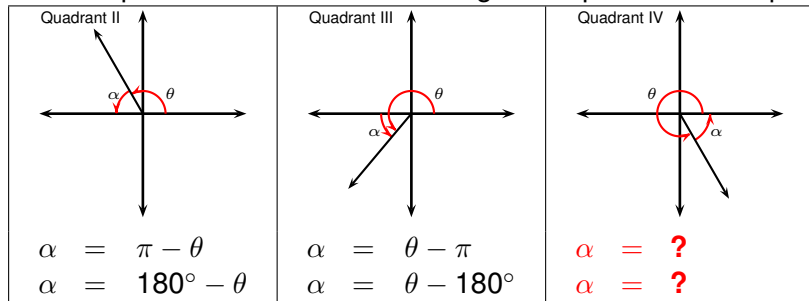


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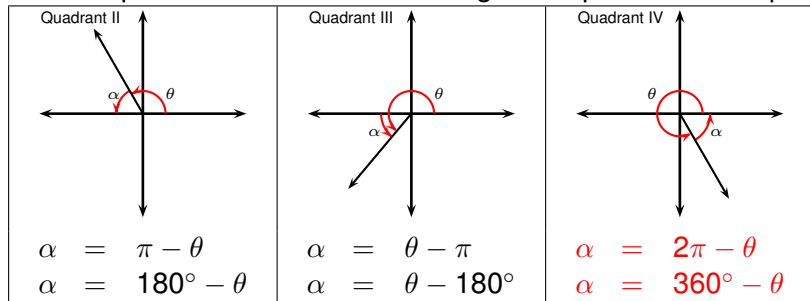


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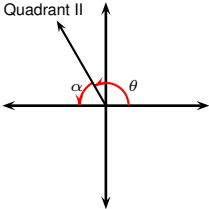
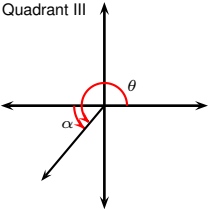
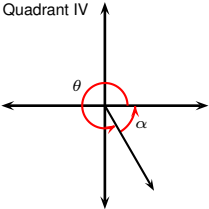


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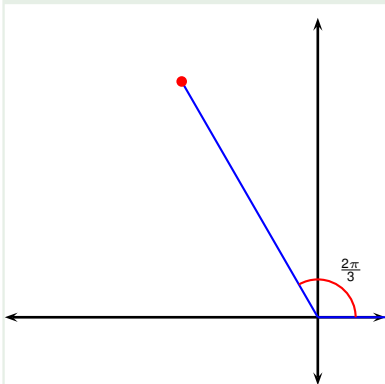
<p>Quadrant II</p>  <p>$\alpha = \pi - \theta$ $\alpha = 180^\circ - \theta$</p>	<p>Quadrant III</p>  <p>$\alpha = \theta - \pi$ $\alpha = \theta - 180^\circ$</p>	<p>Quadrant IV</p>  <p>$\alpha = 2\pi - \theta$ $\alpha = 360^\circ - \theta$</p>
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Observation

One can find the value of a trigonometric function of θ as follows.

- *Find the reference angle α associated to θ .*
- *Find the trig function of α .*
- *Use the quadrant in which θ lies to affix an appropriate sign to the function value.*

Example



Find the exact values of the trigonometric functions of

$$\theta = \frac{2\pi}{3} = 120^\circ.$$

$$\sin\left(\frac{2\pi}{3}\right) =$$

$$\cos\left(\frac{2\pi}{3}\right) =$$

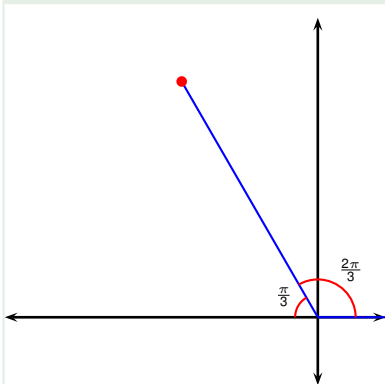
$$\tan\left(\frac{2\pi}{3}\right) =$$

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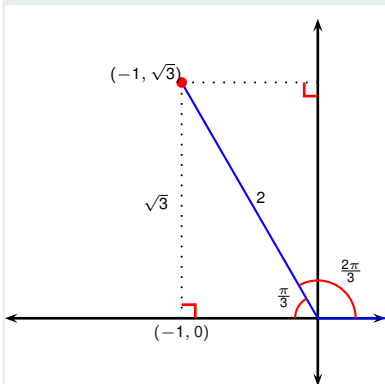
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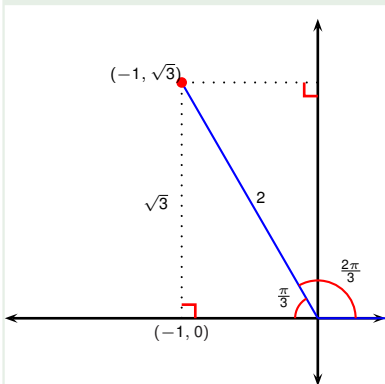
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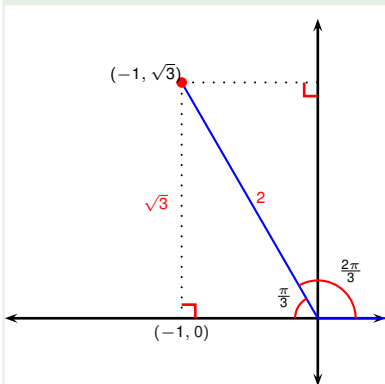
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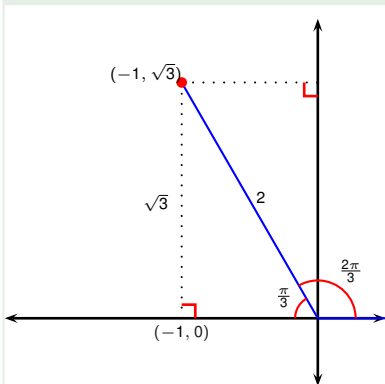
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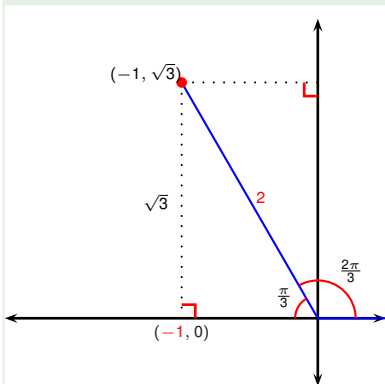
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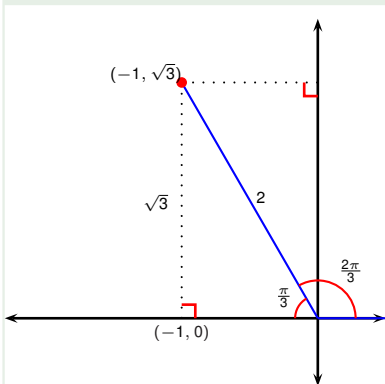
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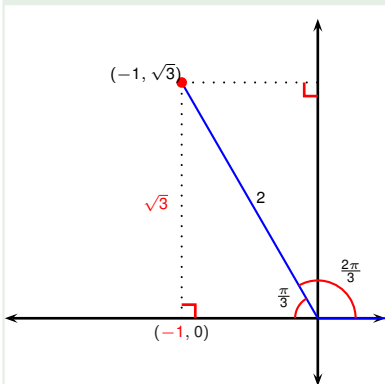
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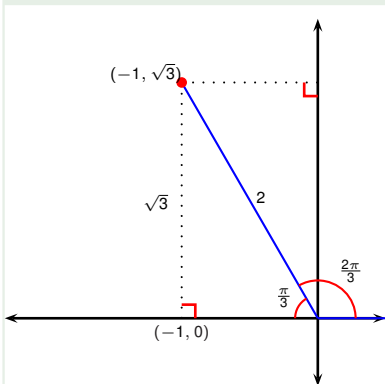
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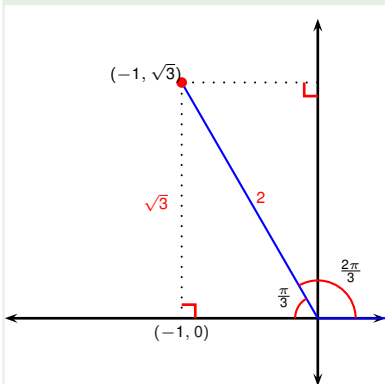
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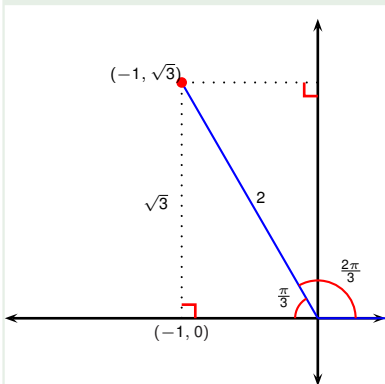
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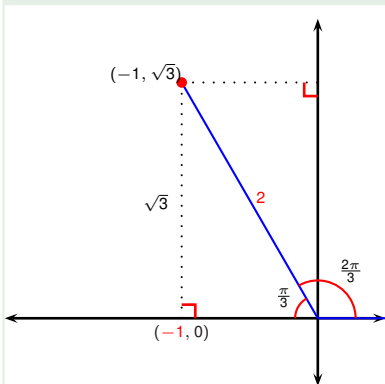
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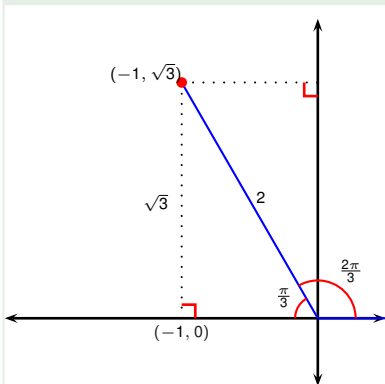


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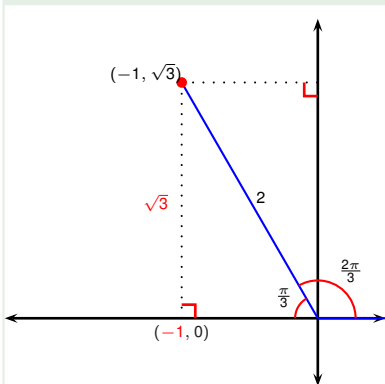


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$$\theta = \frac{2\pi}{3} = 120^\circ.$$

$$\begin{aligned} \sin\left(\frac{2\pi}{3}\right) &= \frac{\sqrt{3}}{2} & \cos\left(\frac{2\pi}{3}\right) &= -\frac{1}{2} & \tan\left(\frac{2\pi}{3}\right) &= \frac{\sqrt{3}}{-1} = -\sqrt{3} \\ \csc\left(\frac{2\pi}{3}\right) &= \frac{2}{\sqrt{3}} & \sec\left(\frac{2\pi}{3}\right) &= -\frac{2}{1} = -2 & \cot\left(\frac{2\pi}{3}\right) &= ? \end{aligned}$$

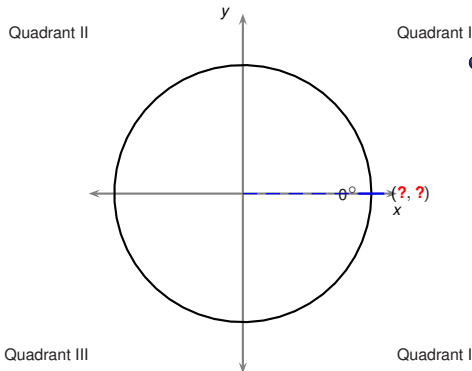
Example



Find the exact values of the trigonometric functions of

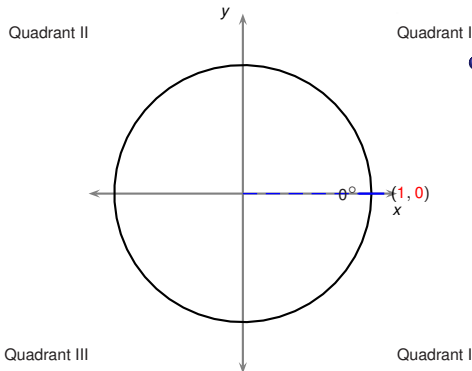
$$\theta = \frac{2\pi}{3} = 120^\circ.$$

$$\begin{aligned} \sin\left(\frac{2\pi}{3}\right) &= \frac{\sqrt{3}}{2} & \cos\left(\frac{2\pi}{3}\right) &= -\frac{1}{2} & \tan\left(\frac{2\pi}{3}\right) &= \frac{\sqrt{3}}{-1} = -\sqrt{3} \\ \csc\left(\frac{2\pi}{3}\right) &= \frac{2}{\sqrt{3}} & \sec\left(\frac{2\pi}{3}\right) &= -\frac{2}{1} = -2 & \cot\left(\frac{2\pi}{3}\right) &= -\frac{1}{\sqrt{3}} \end{aligned}$$



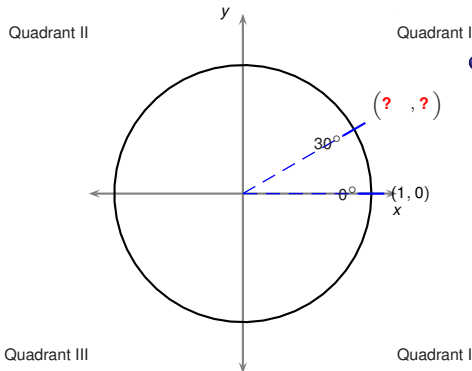
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	?										
cos	?										



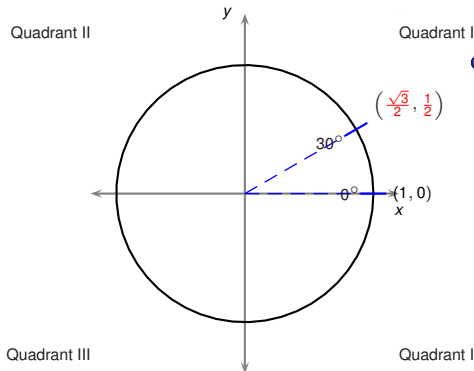
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0										
cos	1										



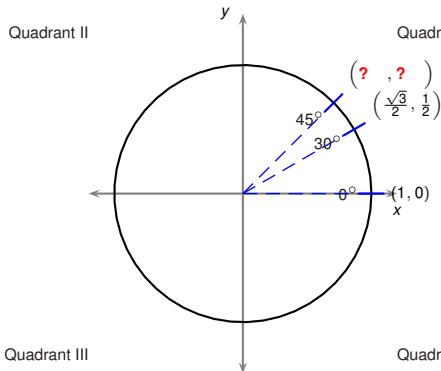
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	?									
cos	1	?									



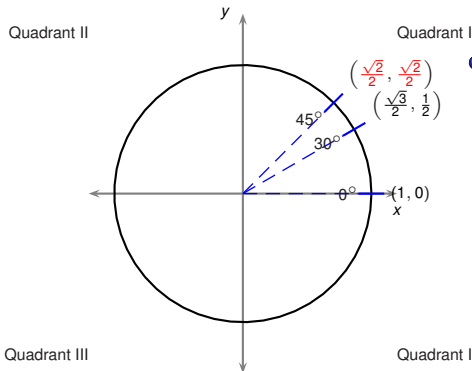
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$									
cos	1	$\frac{\sqrt{3}}{2}$									



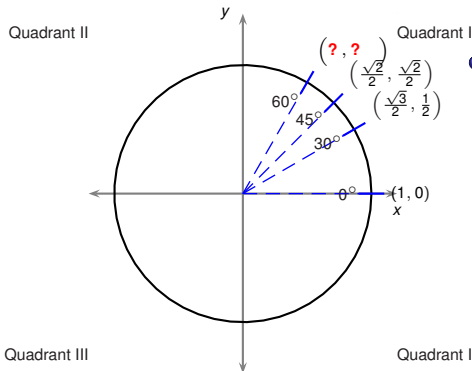
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$?								
cos	1	$\frac{\sqrt{3}}{2}$?								



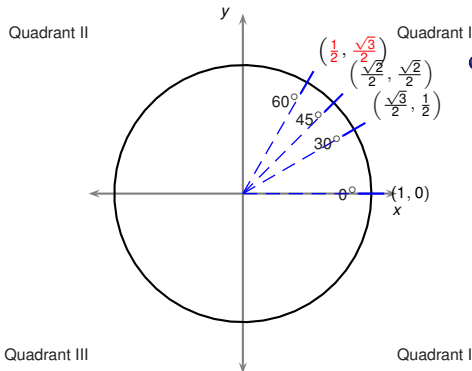
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$								
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$								



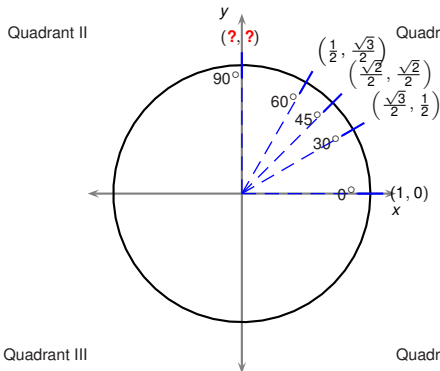
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$?							
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$?							



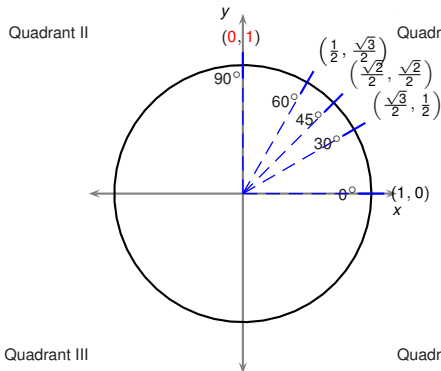
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$							
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$							



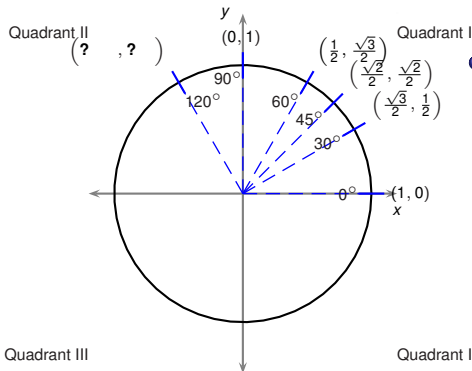
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$?						
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$?						



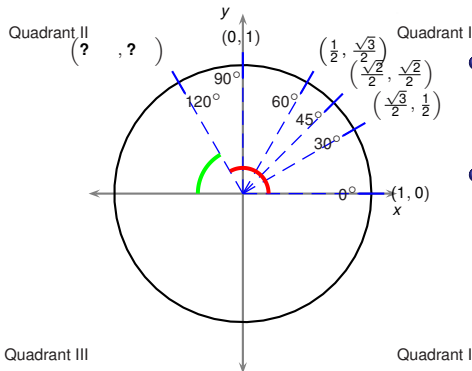
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1						
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0						



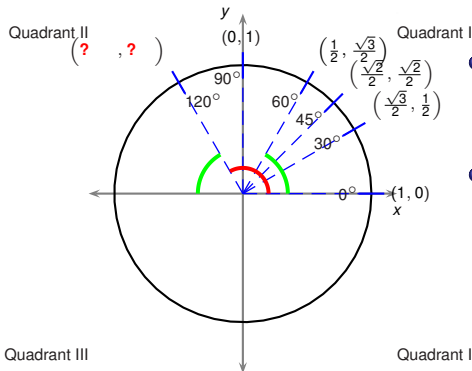
- One only needs to memorize sines and cosines in Quadrant I and on the axes.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	?					
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	?					



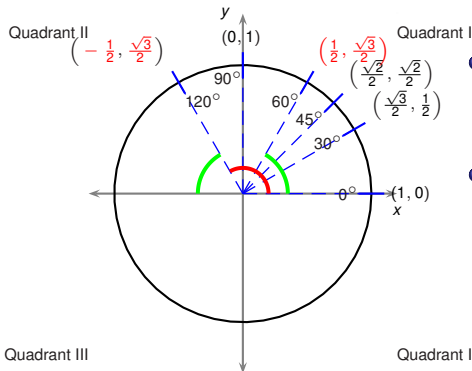
- One only needs to memorize sines and cosines in Quadrant I and on the axes.
- The remaining sines and cosines are extracted by
 - taking the sine/cosine of **the reference angle**

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	?					
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	?					



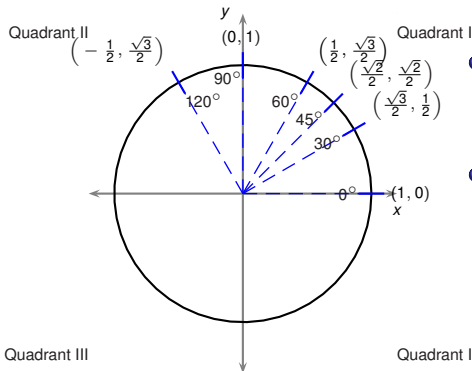
- One only needs to memorize sines and cosines in Quadrant I and on the axes.
- The remaining sines and cosines are extracted by
 - taking the sine/cosine of **the reference angle**

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	?					
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	?					



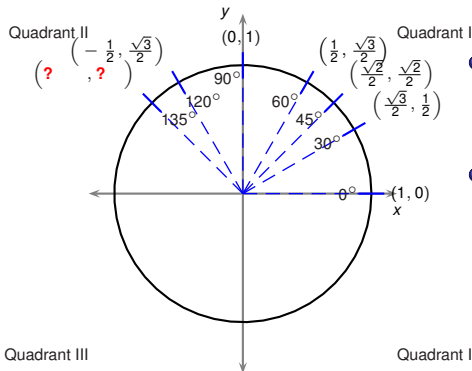
- One only needs to memorize sines and cosines in Quadrant I and on the axes.
- The remaining sines and cosines are extracted by
 - taking the **sine/cosine of the reference angle**

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$					
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$					



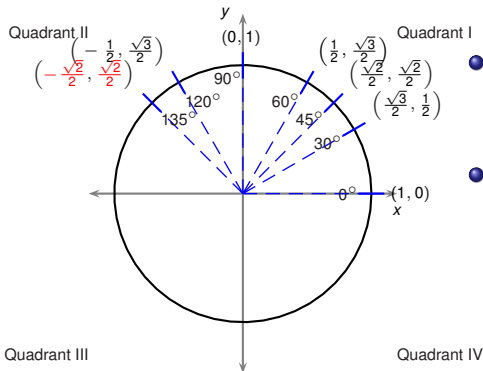
- One only needs to memorize sines and cosines in Quadrant I and on the axes.
- The remaining sines and cosines are extracted by
 - taking the sine/cosine of the reference angle
 - and **adjusting the sign according to the quadrant.**

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$					
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$					



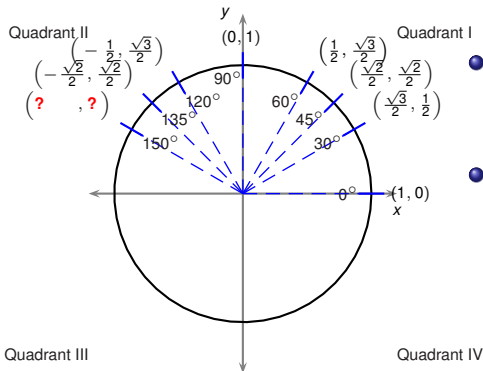
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 - taking the sine/cosine of the reference angle
 - and adjusting the sign according to the quadrant.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$?				
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$?				



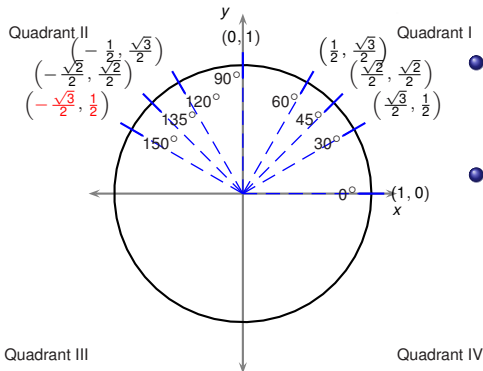
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 - and adjusting the sign according to the quadrant.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$				
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$				



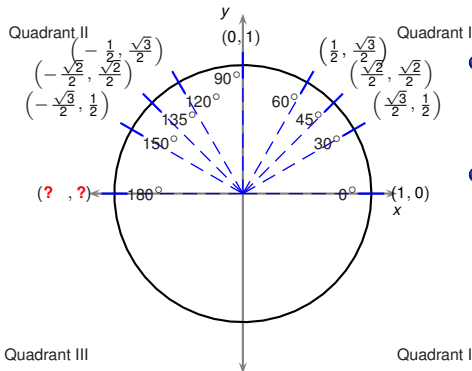
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Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$?			
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$?			



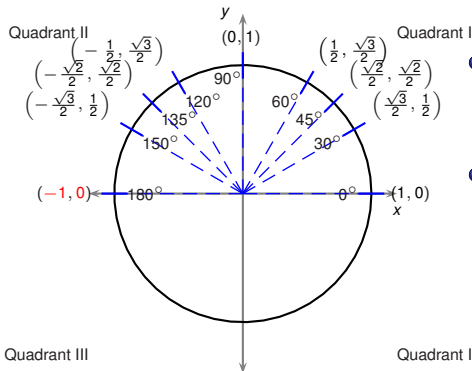
- One only needs to memorize sines and cosines in Quadrant I and on the axes.
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Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$			
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$			



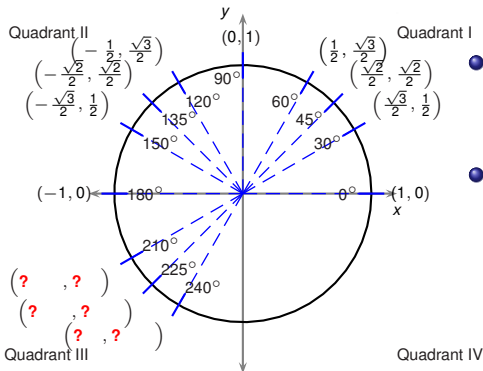
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Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$?		
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$?		



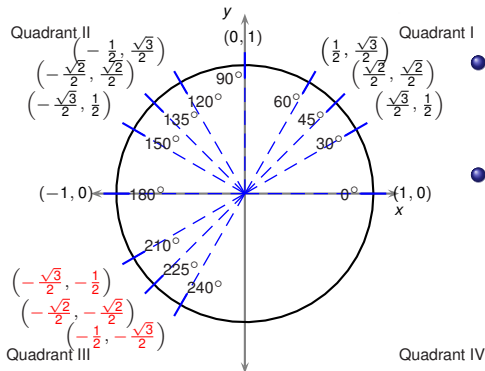
- One only needs to memorize sines and cosines in Quadrant I and on the axes.
- The remaining sines and cosines are extracted by
 - taking the sine/cosine of the reference angle
 - and adjusting the sign according to the quadrant.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0		
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1		



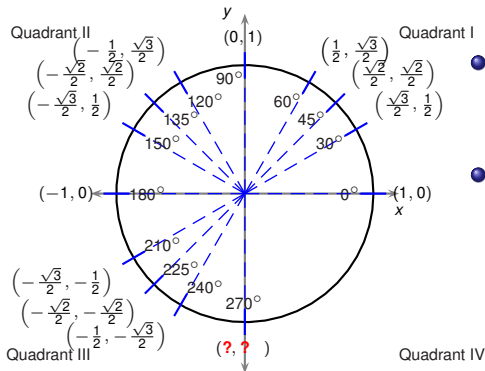
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Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0		
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1		



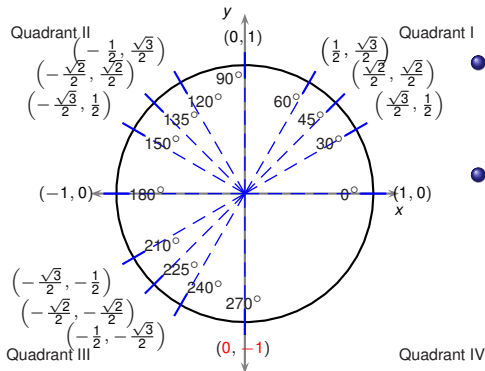
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Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0		
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1		



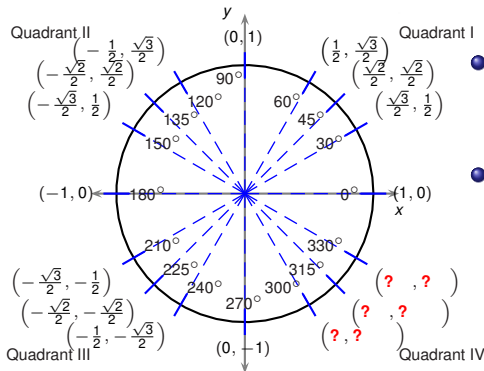
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 - taking the sine/cosine of the reference angle
 - and adjusting the sign according to the quadrant.

Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	?	
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	?	



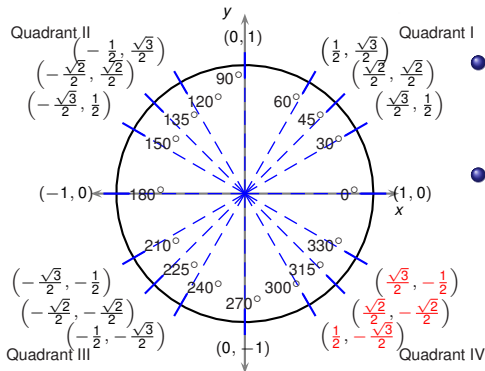
- One only needs to memorize sines and cosines in Quadrant I and on the axes.
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sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	



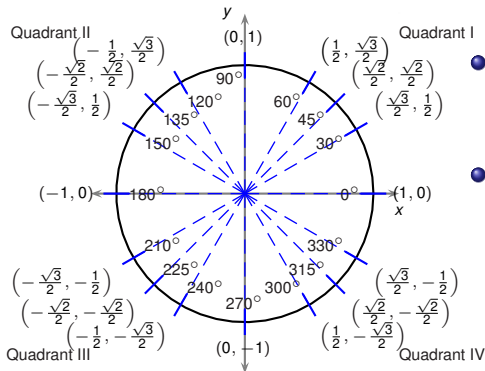
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sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	



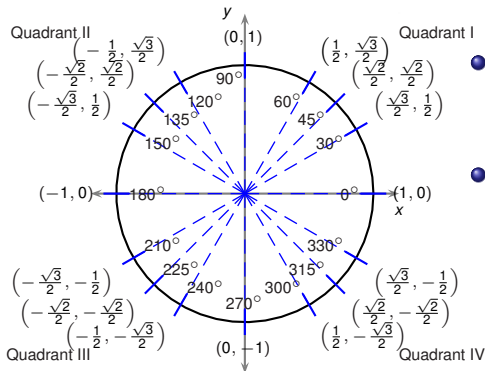
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sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	



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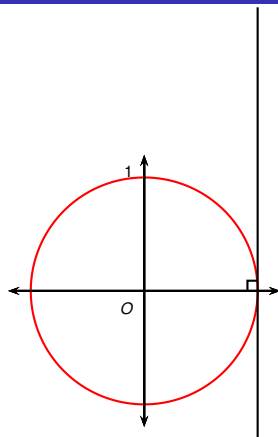
Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	?
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	?



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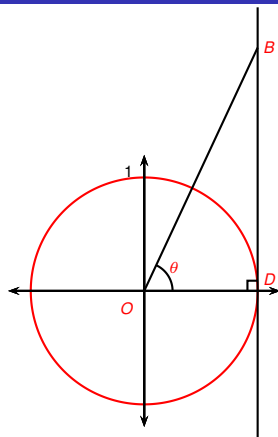
Deg.	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Rad.	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	0	1

Geometric interpretation of all trigonometric functions



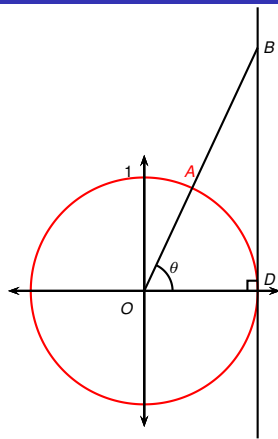
Fix unit circle, center O , coordinates $(0, 0)$.

Geometric interpretation of all trigonometric functions



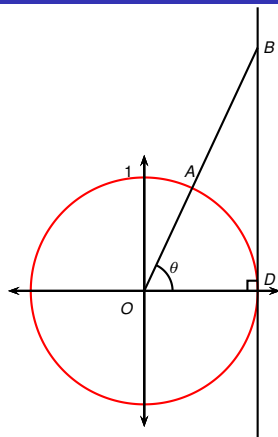
Fix unit circle, center O , coordinates $(0, 0)$.
Let $\angle DOB = \theta$.

Geometric interpretation of all trigonometric functions



Fix unit circle, center O , coordinates $(0, 0)$.
Let $\angle DOB = \theta$. Let OB intersect the circle at point A .

Geometric interpretation of all trigonometric functions



Fix unit circle, center O , coordinates $(0, 0)$.
Let $\angle DOB = \theta$. Let OB intersect the circle at point A . Coordinates of A are $(\cos \theta, \sin \theta)$.

$\sin \theta$

$\cos \theta$

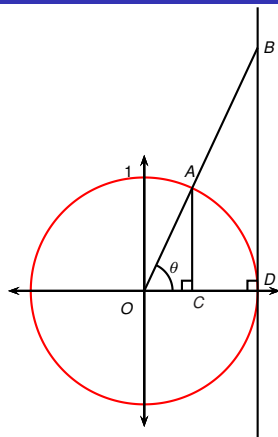
$\tan \theta$

$\cot \theta$

$\sec \theta$

$\csc \theta$

Geometric interpretation of all trigonometric functions



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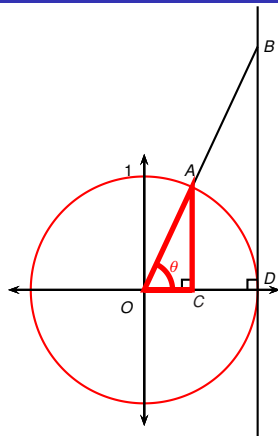
$\tan \theta$

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Geometric interpretation of all trigonometric functions



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$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta$$

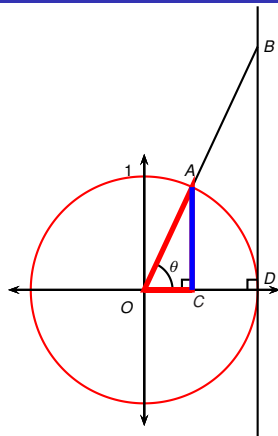
$$\tan \theta$$

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Geometric interpretation of all trigonometric functions



Fix unit circle, center O , coordinates $(0, 0)$.
 Let $\angle DOB = \theta$. Let OB intersect the circle at point A . Coordinates of A are $(\cos \theta, \sin \theta)$.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{|AC|}{|OA|}$$

$$\cos \theta$$

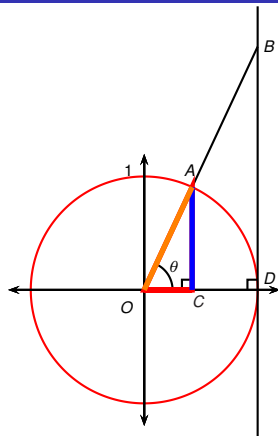
$$\tan \theta$$

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Geometric interpretation of all trigonometric functions



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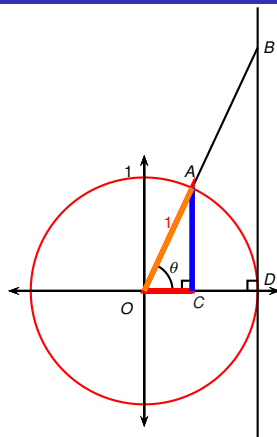
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Geometric interpretation of all trigonometric functions



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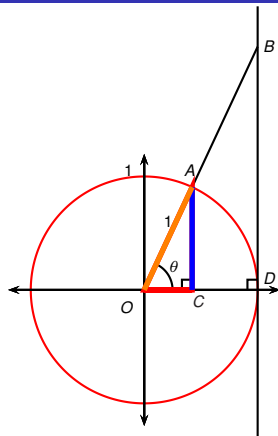
$$\tan \theta$$

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Geometric interpretation of all trigonometric functions



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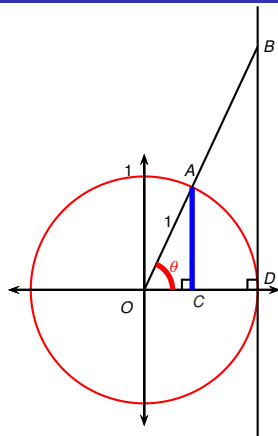
$$\tan \theta$$

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Geometric interpretation of all trigonometric functions



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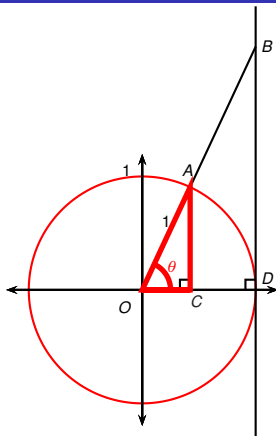
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Geometric interpretation of all trigonometric functions



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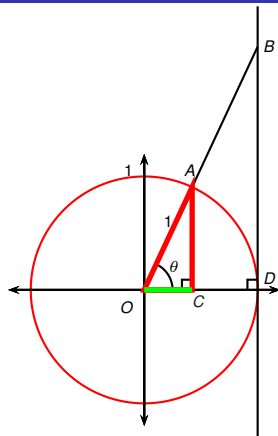
$$\tan \theta$$

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Geometric interpretation of all trigonometric functions



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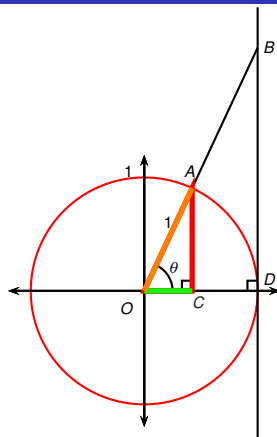
$$\tan \theta$$

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Geometric interpretation of all trigonometric functions



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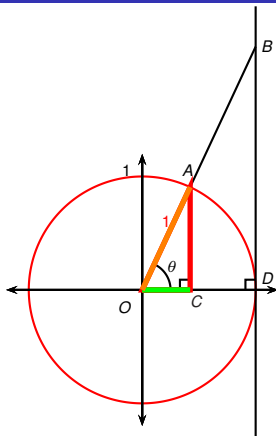
$$\tan \theta$$

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Geometric interpretation of all trigonometric functions



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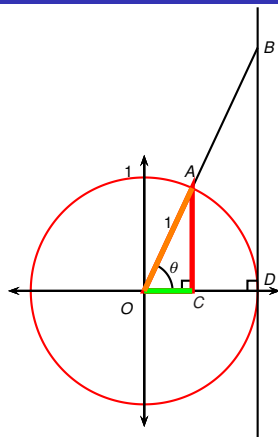
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Geometric interpretation of all trigonometric functions



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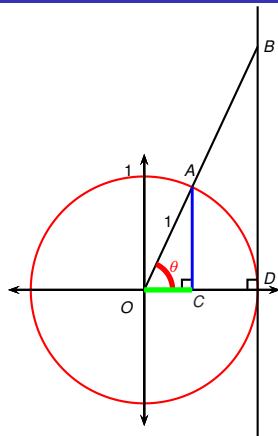
$$\tan \theta$$

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Geometric interpretation of all trigonometric functions



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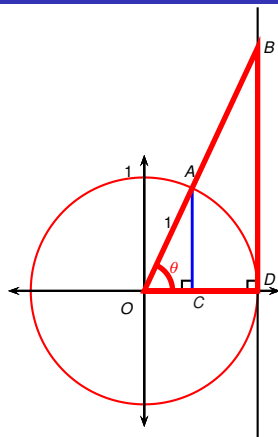
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Geometric interpretation of all trigonometric functions



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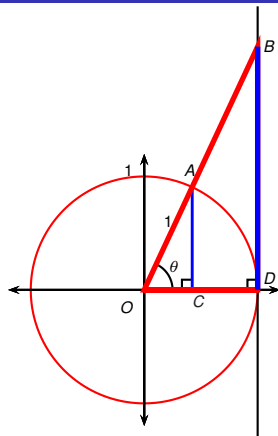
$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

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$$\sec \theta$$

$$\csc \theta$$

Geometric interpretation of all trigonometric functions



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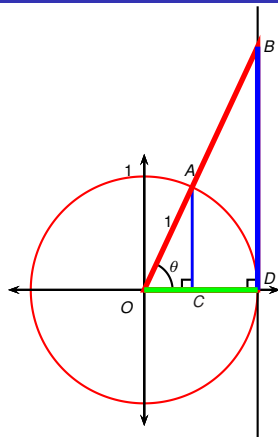
$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{|BD|}{|OD|}$$

$$\cot \theta$$

$$\sec \theta$$

$$\csc \theta$$

Geometric interpretation of all trigonometric functions



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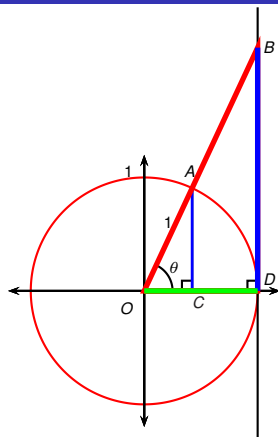
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Geometric interpretation of all trigonometric functions



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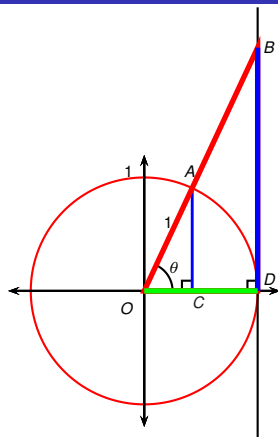
$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{|BD|}{|OD|} = \frac{|BD|}{1}$$

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Geometric interpretation of all trigonometric functions



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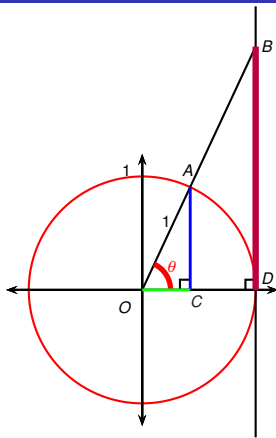
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Geometric interpretation of all trigonometric functions



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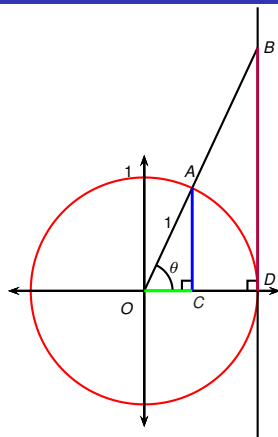
$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{|BD|}{|OD|} = \frac{|BD|}{1} = |BD|$$

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Geometric interpretation of all trigonometric functions



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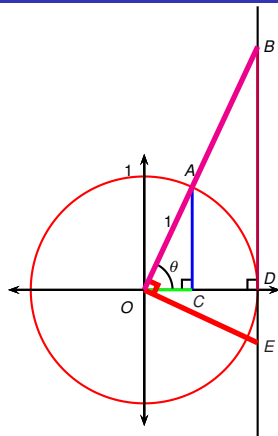
$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{|BD|}{|OD|} = \frac{|BD|}{1} = |BD|$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$

$$\sec \theta$$

$$\csc \theta$$

Geometric interpretation of all trigonometric functions



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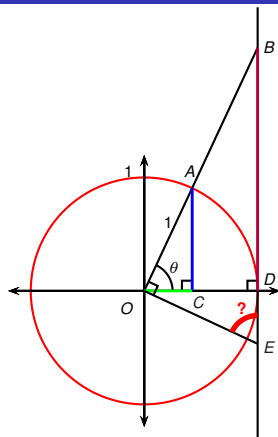
$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{|BD|}{|OD|} = \frac{|BD|}{1} = |BD|$$

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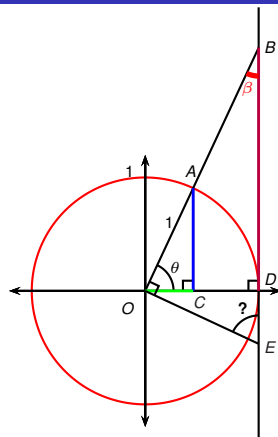
$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$

$$\sec \theta$$

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$$\angle OED = ?$$

Geometric interpretation of all trigonometric functions



$\beta = ?$

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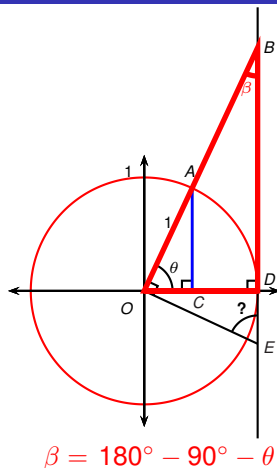
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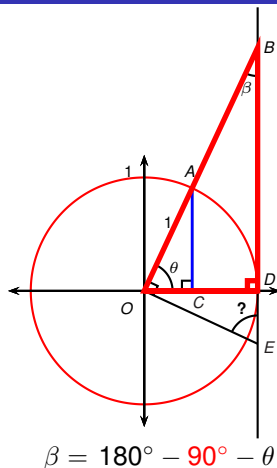
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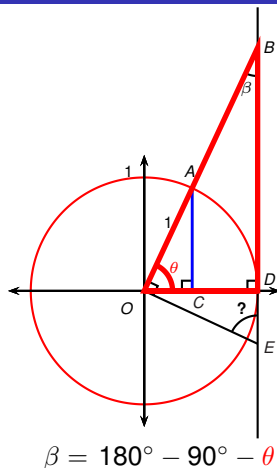
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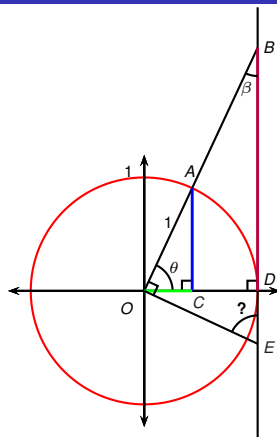
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Geometric interpretation of all trigonometric functions



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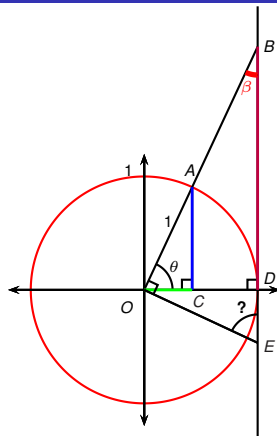
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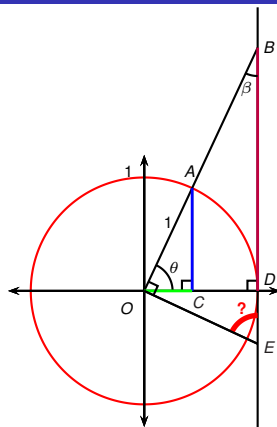
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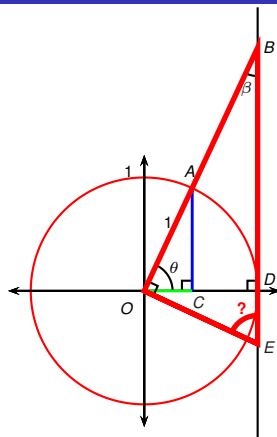
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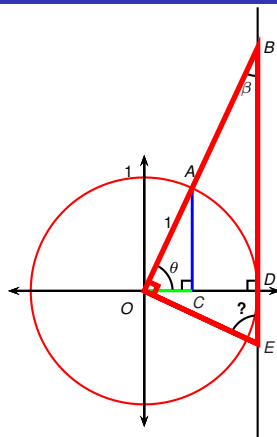
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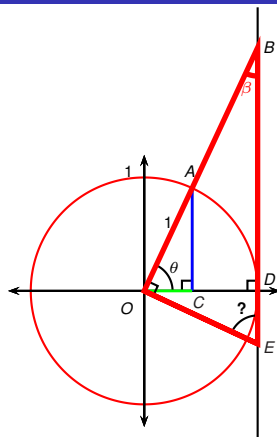
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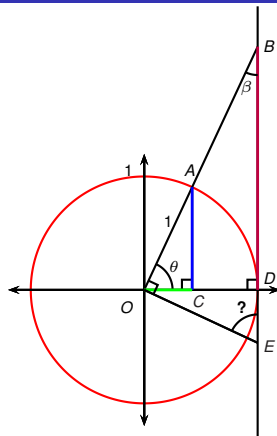
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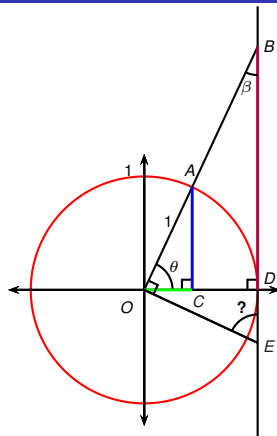
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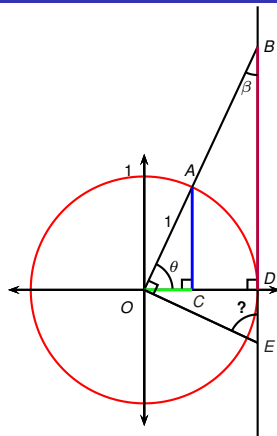
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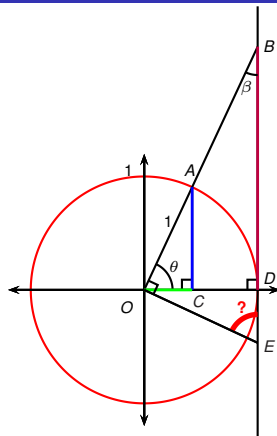
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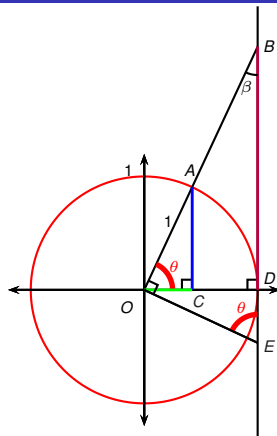
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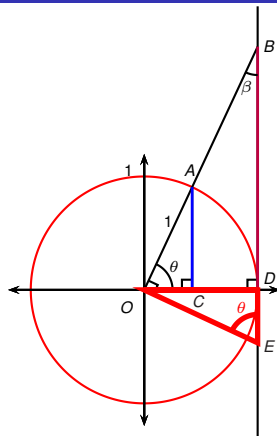
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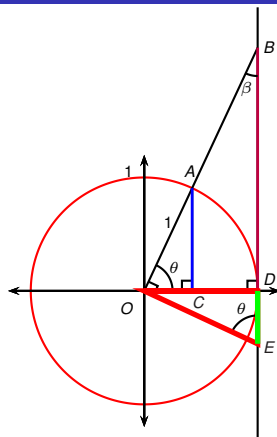
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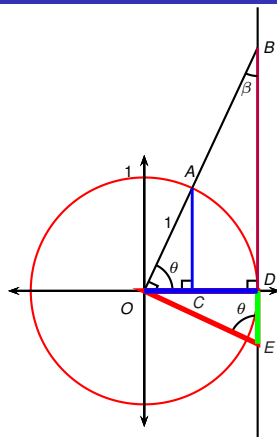
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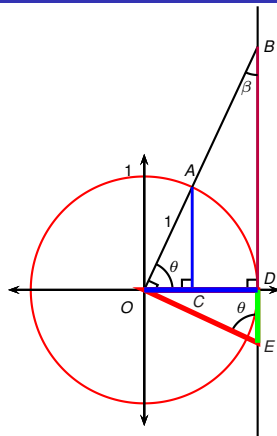
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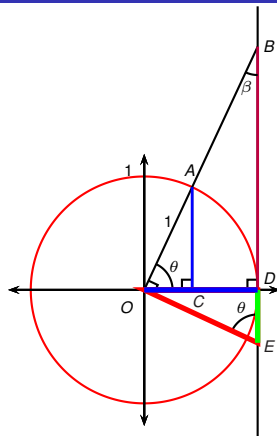
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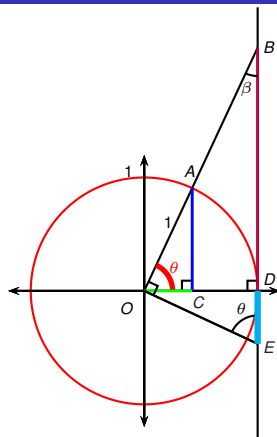
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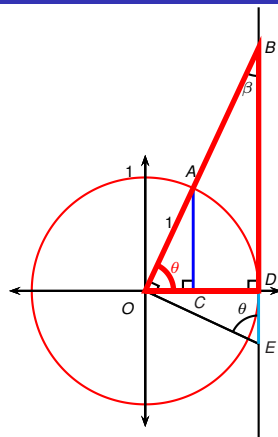
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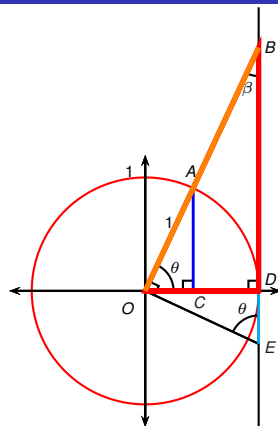
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Geometric interpretation of all trigonometric functions



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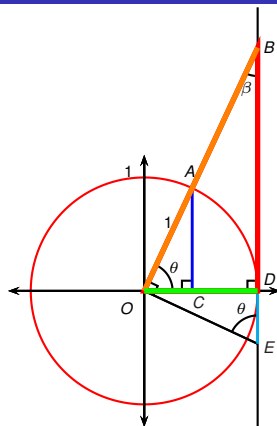
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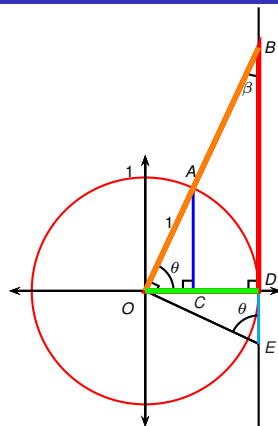
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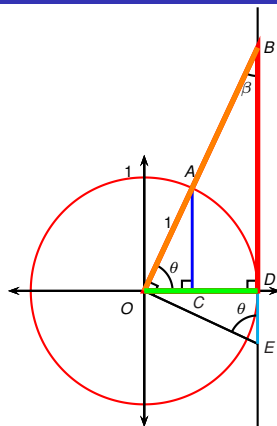
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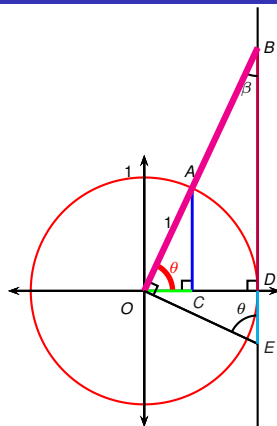
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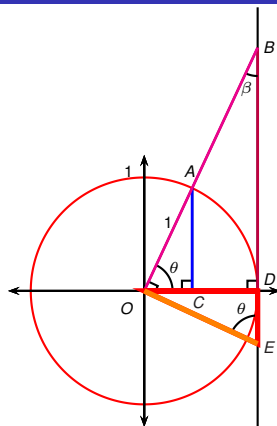
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$\sin \theta$	$= \frac{\text{opp}}{\text{hyp}} = \frac{ AC }{ OA } = \frac{ AC }{1} = AC $
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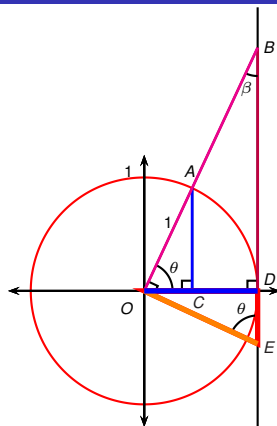
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Geometric interpretation of all trigonometric functions



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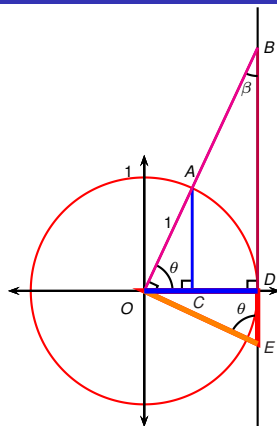
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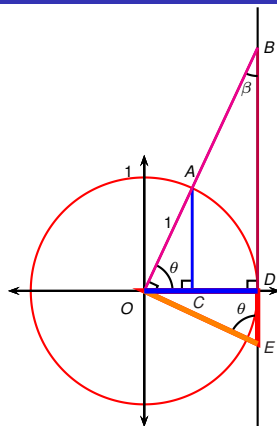
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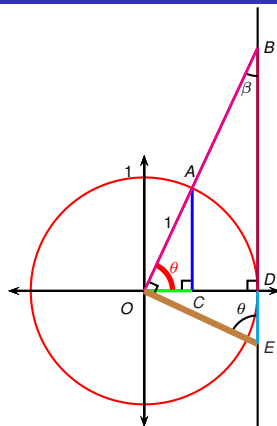
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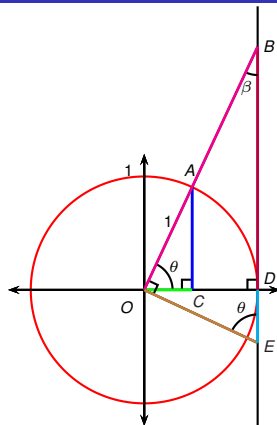
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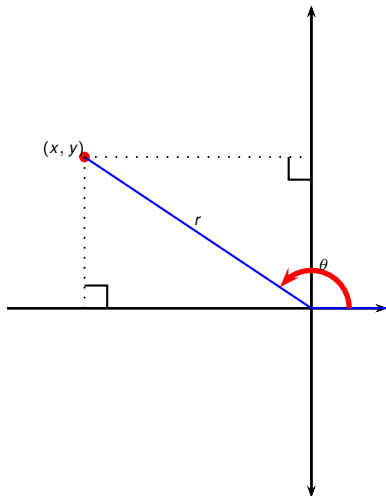
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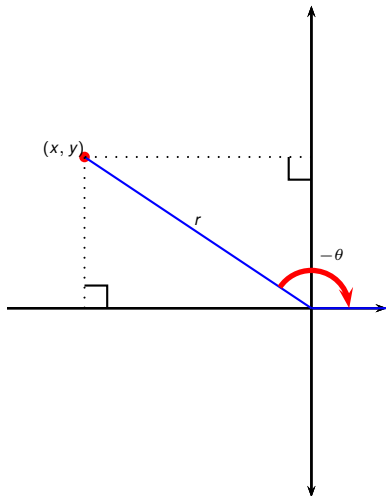
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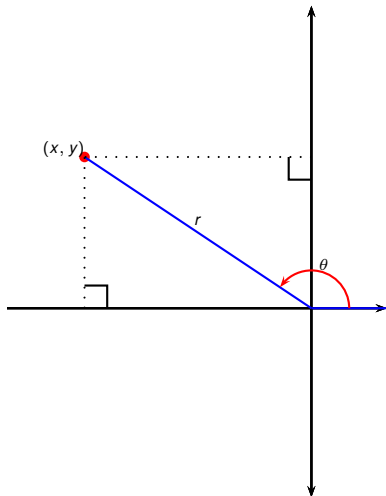
- Positive angles are obtained by rotating counterclockwise.

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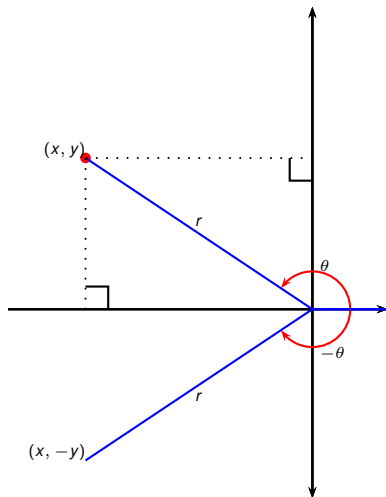
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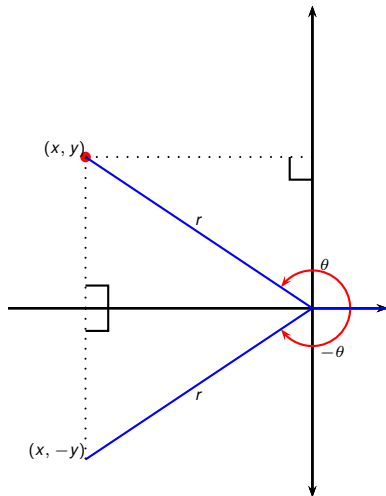
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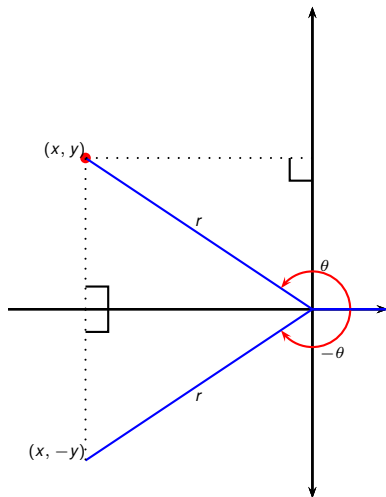
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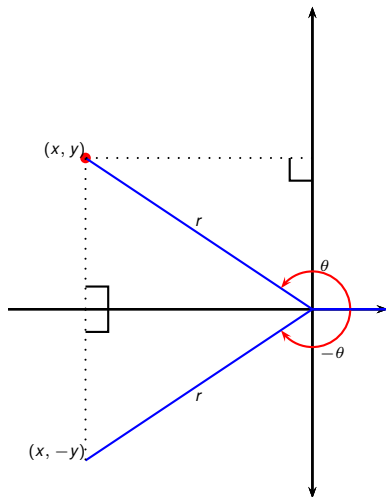
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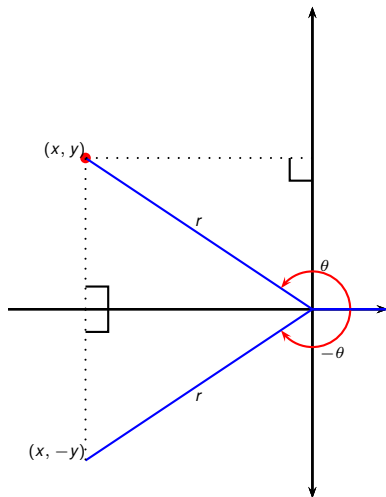
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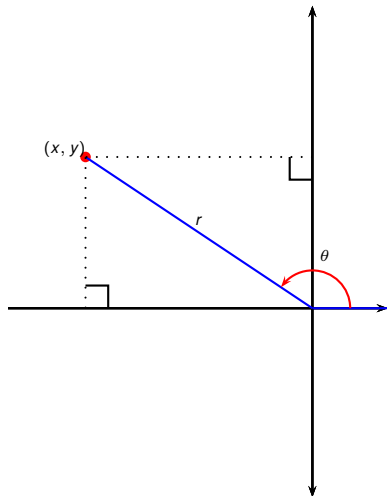
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- \sin is an **odd function**.

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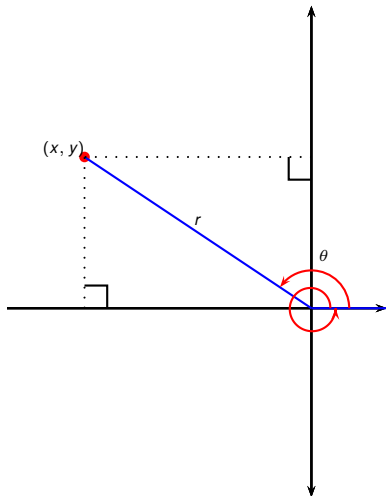


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- \sin is an odd function.
- \cos is an even function.

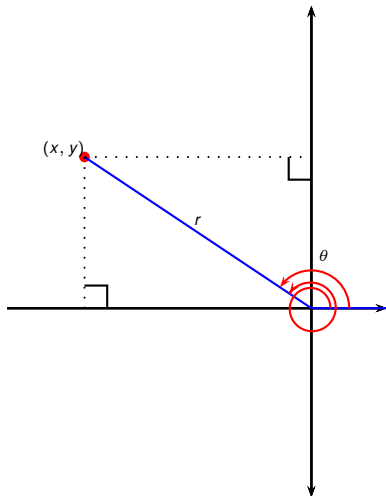


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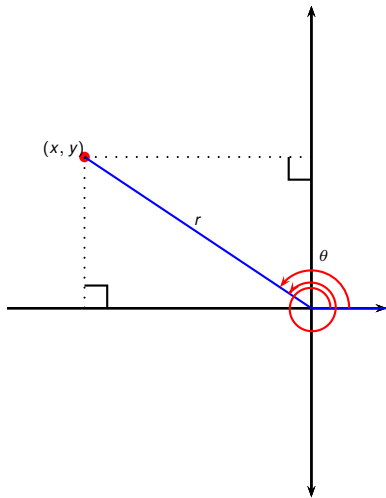
- 2π represents a full rotation.

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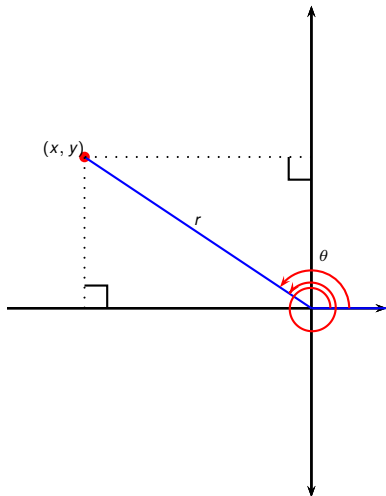
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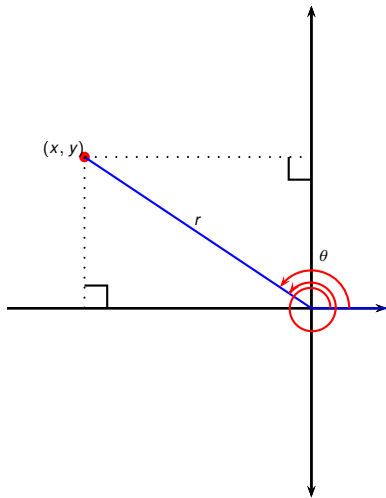
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- 2π represents a full rotation.
- $\theta + 2\pi$ has the same terminal arm as θ .
- $\theta + 2\pi$ uses the same point (x, y) and the same length r .
- $\sin(\theta + 2\pi) = \sin \theta$.
- $\cos(\theta + 2\pi) = \cos \theta$.

$$\begin{array}{ll} \sin \theta = \frac{y}{r} & \csc \theta = \frac{r}{y} \\ \cos \theta = \frac{x}{r} & \sec \theta = \frac{r}{x} \\ \tan \theta = \frac{y}{x} & \cot \theta = \frac{x}{y} \end{array}$$



- 2π represents a full rotation.
- $\theta + 2\pi$ has the same terminal arm as θ .
- $\theta + 2\pi$ uses the same point (x, y) and the same length r .
- $\sin(\theta + 2\pi) = \sin \theta$.
- $\cos(\theta + 2\pi) = \cos \theta$.
- We say \sin and \cos are 2π -periodic.

$$\begin{array}{ll} \sin \theta = \frac{y}{r} & \csc \theta = \frac{r}{y} \\ \cos \theta = \frac{x}{r} & \sec \theta = \frac{r}{x} \\ \tan \theta = \frac{y}{x} & \cot \theta = \frac{x}{y} \end{array}$$

Trigonometric Identities

Definition (Trigonometric Identity)

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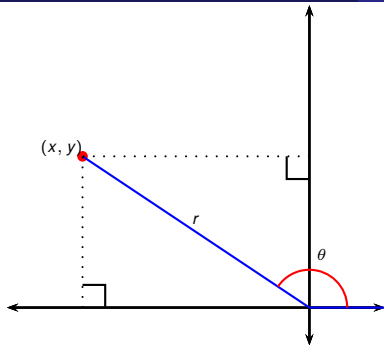
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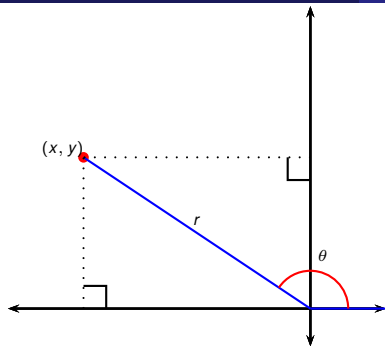
A trigonometric identity is an equality between the trigonometric functions in one or more variables that holds for all values of the involved variables in the domains of all of the expressions.

- By convention, when dealing with trigonometric identities we do not account for the domains of the involved expressions.
- For example, $\frac{\sin \theta}{\sin \theta} = 1$ is considered a valid trigonometric identity, although, when considered as a function, the left hand side is not defined for $\theta \neq 0$.

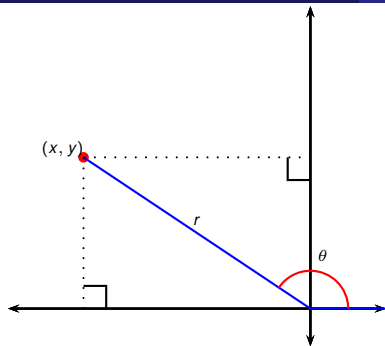


$$\begin{aligned}\sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$

- $\csc \theta = \frac{1}{\sin \theta}$
- $\sec \theta = \frac{1}{\cos \theta}$
- $\cot \theta = \frac{1}{\tan \theta}$
- $\tan \theta = \frac{\sin \theta}{\cos \theta}$
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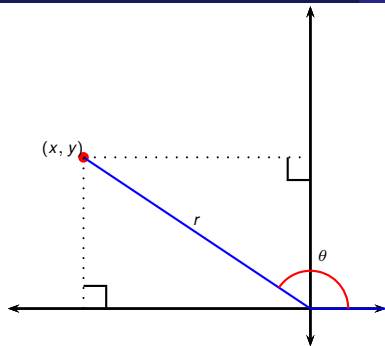


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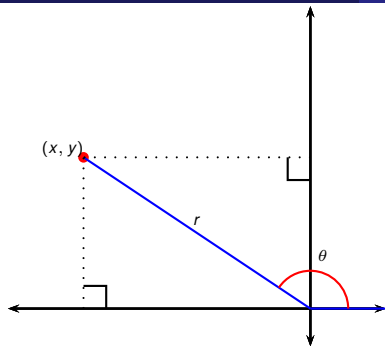
$$\sin^2 \theta + \cos^2 \theta$$

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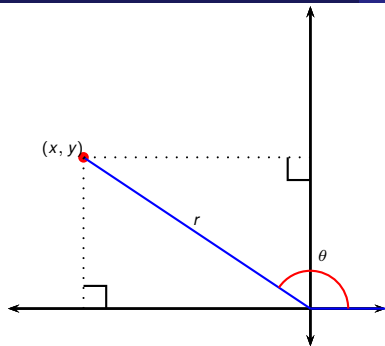
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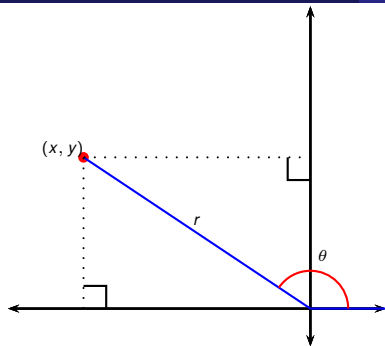
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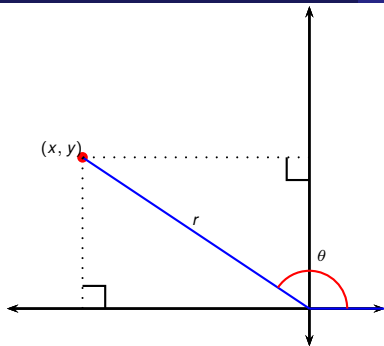
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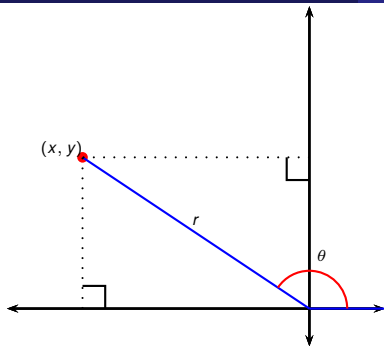
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Therefore $\sin^2 \theta + \cos^2 \theta = 1$.

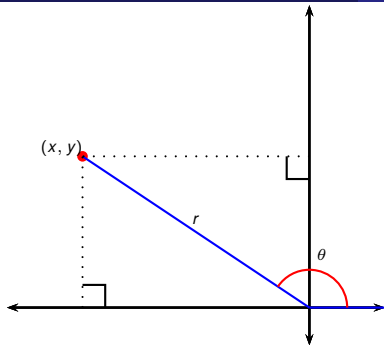


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Example ($\tan^2 \theta + 1 = \sec^2 \theta$)

Prove the identity

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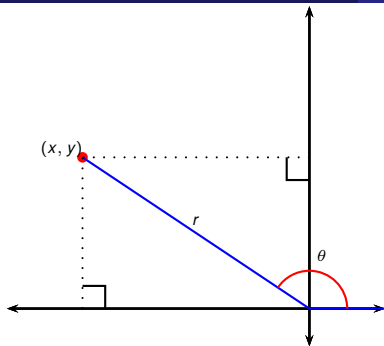
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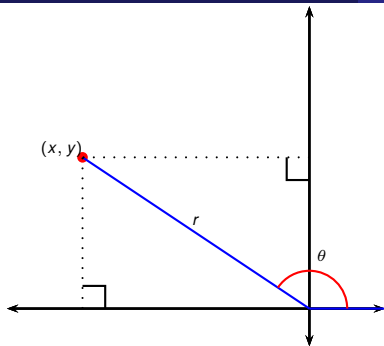
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