Precalculus
Lecture 4
Complex Numbers

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https://github.com/tmilev/freecalc

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Outline

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Definition (Complex numbers)

The set of complex numbers $\mathbb{C}$ is defined as the set

$$\{a + bi | a, b - \text{real numbers}\},$$

where the number i is a number for which

$$i^2 = -1 \quad .$$

The number i is called the imaginary unit. .

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- Complex addition/subtraction

$$(a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i \quad .$$

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Let $u = 2 + 3i$, $v = 5 - 7i$.

Example (Addition)

$u + v =$.

Example (Subtraction)

$u - v =$.

Example (Multiplication)

$u \cdot v \quad =$

Let $u = 2 + 3i$, $v = 5 - 7i$.

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$u + v = (2 + 3i) + (5 - 7i) = \mathbf{?}$.

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$u + v = (2 + 3i) + (5 - 7i) = (2 + 5) + (3 - 7)i = 7 - 4i.$

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$u - v = (2 + 3i) - (5 - 7i) = (2 - 5) + (3 - (-7))i = -3 + 10i.$

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$$u \cdot v = (2 + 3i) \cdot (5 - 7i)$$

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$$\begin{aligned} u \cdot v &= (2 + 3i) \cdot (5 - 7i) \\ &= 2 \cdot 5 + 2 \cdot (-7)i + 3i \cdot 5 + 3i(-7i) \end{aligned}$$

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Example (Multiplication)

$$\begin{aligned} u \cdot v &= (2 + {\color{red}3i}) {\color{red}\cdot} ({\color{red}5} - 7i) \\ &= 2 \cdot 5 + 2 \cdot (-7)i + {\color{red}3i \cdot 5} + 3i(-7i) \end{aligned}$$

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$$\begin{aligned} u \cdot v &= (2 + 3i) \cdot (5 - 7i) \\ &= {\color{red}2 \cdot 5} + 2 \cdot (-7)i + 3i \cdot 5 + 3i(-7i) \\ &= {\color{red}10} - 14i + 15i - 21i^2 \end{aligned}$$

Let $u = 2 + 3i$, $v = 5 - 7i$.

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$$\begin{aligned} u \cdot v &= (2 + 3i) \cdot (5 - 7i) \\ &= 2 \cdot 5 + 2 \cdot (-7)i + 3i \cdot 5 + 3i(-7i) \\ &= 10 - 14i + 15i - 21i^2 \end{aligned}$$

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&= 10 - 14i + 15i - 21i^2 \\
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Example (Complex multiplication)

Multiply $u = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ by $v = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$.

$$u \cdot v$$

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Multiply $u = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$ by $v = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$.

$$u \cdot v = \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \right) \cdot \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i \right)$$

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&= \frac{\sqrt{2}^2}{2^2} - \frac{\sqrt{2}^2}{2^2} i + \frac{\sqrt{2}^2}{2^2} i - \frac{\sqrt{2}^2}{2^2} i^2
\end{aligned}
$$

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Multiply $u = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$ by $v = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i$.

$$\begin{array}{rcl} u \cdot v & = & \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i \right) \cdot \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i \right) \\ & = & \frac{\sqrt{2}^2}{2^2} - \frac{\sqrt{2}^2}{2^2} i + \frac{\sqrt{2}^2}{2^2} i - \frac{\sqrt{2}^2}{2^2} i^2 \end{array}$$

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&= \frac{2}{4} - \frac{2}{4}(-1)
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&= {\color{red}\frac{2}{4}} - \frac{2}{4}(-1) \\
&= {\color{red}\frac{1}{2}} + \frac{1}{2}
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&= \frac{2}{4} - \frac{2}{4}(-1) \\
&= \frac{1}{2} + \frac{1}{2} \\
&= 1
\end{aligned}$$

Review of the basic types of numbers

- An integer, or whole number, is one of the numbers:
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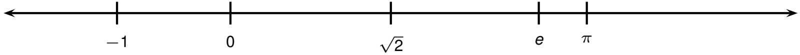
$\sqrt{2}$

π

e

-1



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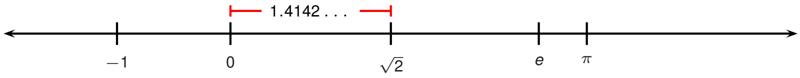

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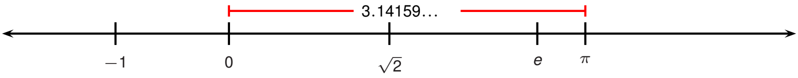

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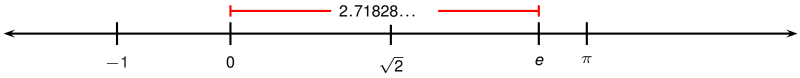

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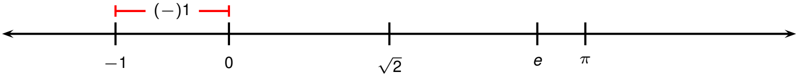

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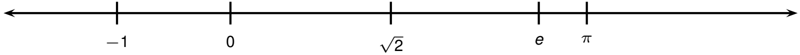

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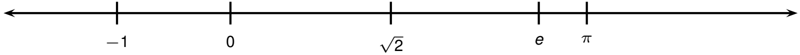

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- Geometric interpretation of complex numbers: beyond our scope.