Calculus II Lecture 8

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https://github.com/tmilev/freecalc

2020

Outline

- Indeterminate Forms and L'Hospital's Rule
 - Indeterminate Products
 - Indeterminate Differences
 - Indeterminate Powers

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 and the links therein.

Find
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- $\lim_{x\to 1}(x-1)=0$.
- We don't get any cancellation between top and bottom.
- We need new techniques.

Theorem (L'Hospital's Rule)

Suppose that f and g are differentiable and $g'(x) \neq 0$ on an open interval that contains a (except possibly at a). Suppose that

and $\lim_{x\to a} g(x) = 0$

or that
$$\lim_{x\to a} f(x) = \pm \infty$$
 and $\lim_{x\to a} g(x) = \pm \infty$

 $\lim_{x\to a} f(x) = 0$

(In other words, we have an indeterminate form of type 0/0 or ∞/∞ .) Then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}.$$

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Indeterminate Products

If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = \pm \infty$, then it isn't clear what $\lim_{x\to a} (fg)(x)$ will be.

Indeterminate Products

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In such a case, write the product fg as a quotient:

$$fg = \frac{f}{1/g}$$
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In such a case, write the product fg as a quotient:

$$fg = \frac{f}{1/g}$$
 or $fg = \frac{g}{1/f}$.

This converts the given limit into an indeterminate form of type 0/0 or ∞/∞ .

Evaluate $\lim_{x\to 0^+} x \ln x$.

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- $\bullet \lim_{x\to 0^+} x = .$

Evaluate $\lim_{x\to 0^+} x \ln x$.

- $\bullet \lim_{x\to 0^+} \ln x = ? \quad .$
- $\bullet \lim_{x\to 0^+} x = .$



Evaluate $\lim_{x\to 0^+} x \ln x$.

- $\bullet \lim_{x\to 0^+} \ln x = -\infty.$
- $\bullet \lim_{x\to 0^+} x = .$



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Evaluate $\lim x \ln x$. $x\rightarrow 0^+$

- $\lim \ln x = -\infty$. $x\rightarrow 0^+$
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Indeterminate Products

Apply L'Hospital's rule:

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}} =$$

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$$= \lim_{x \to 0^{+}} \frac{\frac{1}{x}}{-\frac{1}{x^{2}}} = \lim_{x \to 0^{+}} (-x)$$

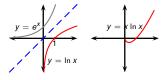


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- $\bullet \lim_{x \to 0^+} x = 0.$
- This is an indeterminate form of type $0(-\infty)$ (or $-\infty/(1/0)$).
- Apply L'Hospital's rule:

$$\lim_{x \to 0^{+}} x \ln x = \lim_{x \to 0^{+}} \frac{\ln x}{\frac{1}{x}} = \lim_{x \to 0^{+}} \frac{\frac{d}{dx} (\ln x)}{\frac{d}{dx} (\frac{1}{x})}$$

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Indeterminate Differences

If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$, then the limit

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To compute such a limit, try to convert it into a quotient (by using a common denominator, or by rationalizing, or by factoring out a common factor).

Evaluate $\lim_{x\to(\pi/2)^-}(\sec x - \tan x)$.

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Evaluate $\lim_{x\to(\pi/2)^-} (\sec x - \tan x)$.

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Indeterminate Powers

Several indeterminate forms arise from the limit $\lim_{x\to a} f(x)^{g(x)}$.

$$\lim_{x\to a} f(x) = 0$$
 and $\lim_{x\to a} g(x) = 0$ type 0^0 $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = 0$ type ∞^0

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These can all be solved either by taking the natural logarithm:

let
$$y = [f(x)]^{g(x)}$$
, then $\ln y = g(x) \ln f(x)$

or by writing the function as an exponential:

$$[f(x)]^{g(x)} = e^{g(x)\ln f(x)}.$$

Find $\lim_{x\to 0^+} x^x$.

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- Therefore

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{x \ln x} = e^0 = 1$$

$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x$$

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$$\lim_{x \to \infty} \left(1 + \frac{k}{x} \right)^x = \lim_{x \to \infty} e^{\ln \left(1 + \frac{k}{x} \right)^x}$$

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 form "\frac{0}{0}",

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 form "\frac{0}{0}", use L'Hospital

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 form "\frac{0}{0}", use L'Hospital
$$= \lim_{x \to \infty} \frac{2}{2}$$

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 exponent= continuous f-n
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$$= \lim_{x \to \infty} \frac{?}{1}$$

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$$= \lim_{x \to \infty} \frac{k}{1 + \frac{k}{x}} = k$$

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$$\lim_{x \to \infty} \left(1 + \frac{k}{x} \right)^x = \lim_{x \to \infty} e^{\ln\left(1 + \frac{k}{x}\right)^x}$$
 exponent= continuous formula to the exponent of the expon

exponent= continuous f-n

$$\lim_{x \to \infty} \left(1 + \frac{k}{x} \right)^x = \lim_{x \to \infty} e^{\ln\left(1 + \frac{k}{x}\right)^x}$$
 exponent= continuous from
$$= e^{\lim_{x \to \infty} \ln\left(1 + \frac{k}{x}\right)^x} = e^k$$

$$\lim_{x \to \infty} \ln\left(1 + \frac{k}{x}\right)^x = \lim_{x \to \infty} x \ln\left(1 + \frac{k}{x}\right)$$

$$= \lim_{x \to \infty} \frac{\frac{d}{dx} \left(\ln\left(1 + \frac{k}{x}\right)\right)}{\frac{d}{dx} \left(\frac{1}{x}\right)}$$
 form "\frac{0}{0}", use L'Hospital
$$= \lim_{x \to \infty} \frac{\frac{1}{1 + \frac{k}{x}} \left(1 + \frac{k}{x}\right)'}{-\frac{1}{x^2}}$$

$$= \lim_{x \to \infty} \frac{\frac{1}{1 + \frac{k}{x}} \left(-\frac{k}{x^2}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x \to \infty} \frac{k}{1 + \frac{k}{x}} = k$$

exponent= continuous f-n

$$\lim_{x \to \infty} \left(1 + \frac{k}{x} \right)^x = \lim_{x \to \infty} e^{\ln(1 + \frac{k}{x})^x}$$
 exponent= continuous formula to the second seco

exponent= continuous f-n

limit computed below