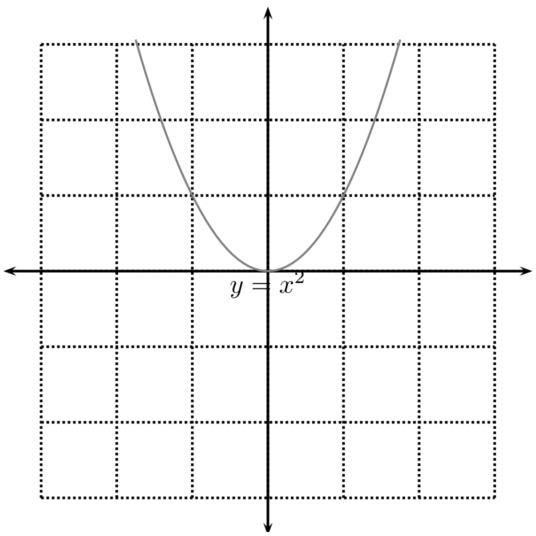
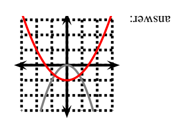
Precalculus
Homework Lecture 12

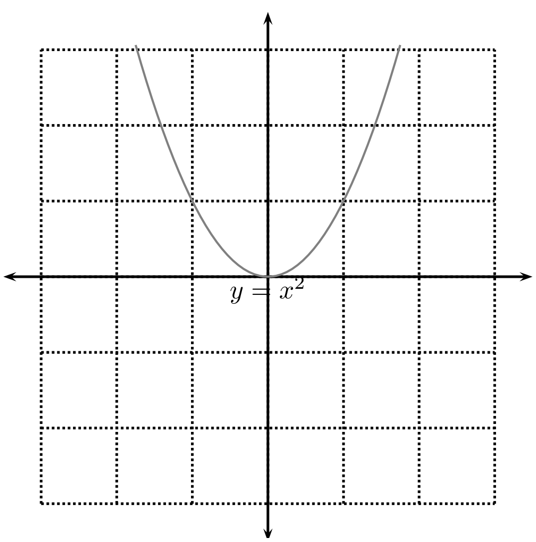
1. Sketch by hand approximately the given function. The function is obtained by transforming linearly the graph of a known function. The known function has been sketched for you by computer.

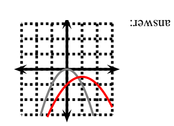
   (a) $f(x) = -\dfrac{1}{2}x^2 + 1.$



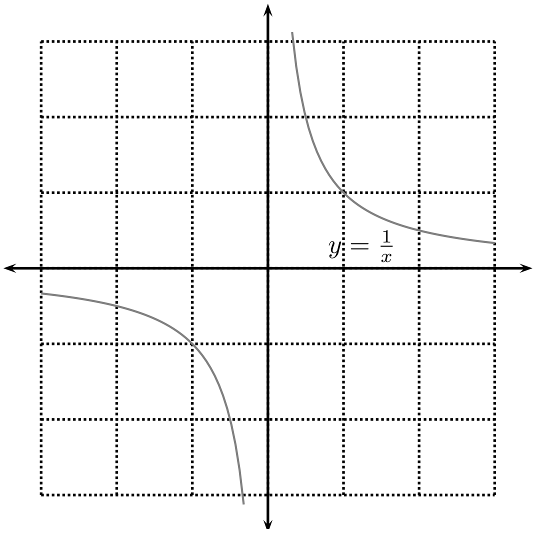


   (b) $f(x) = \dfrac{1}{2}x^2 + x - 1.$

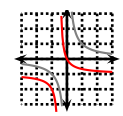




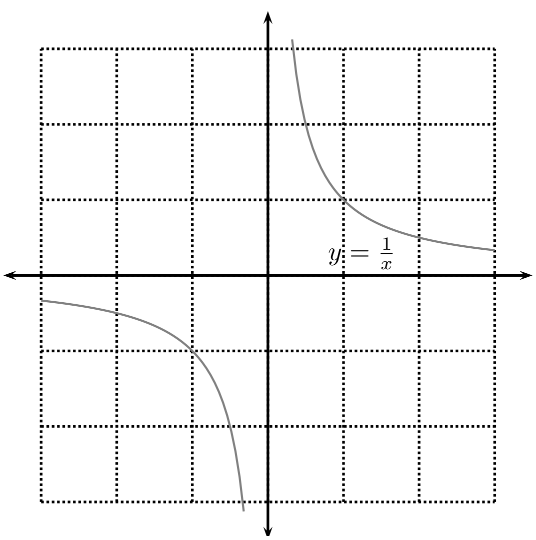
(c) $f(x) = \dfrac{1}{2x-1} + 1.$



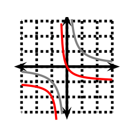
answer:



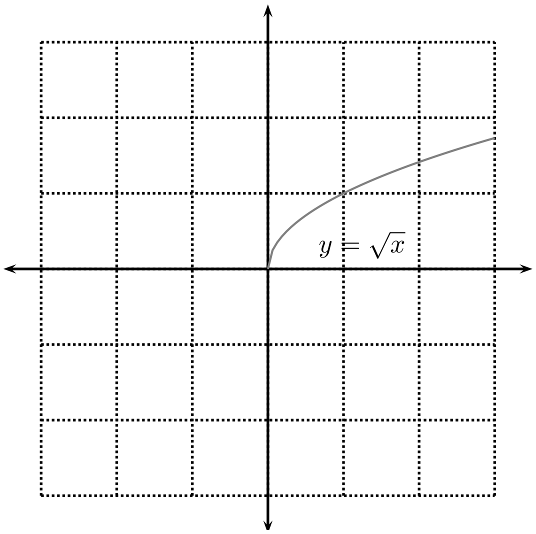
(d) $f(x) = \dfrac{\frac{1}{2}x + \frac{1}{4}}{x - \frac{1}{2}} + \dfrac{1}{2}.$

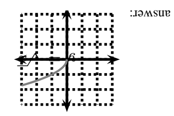


answer:

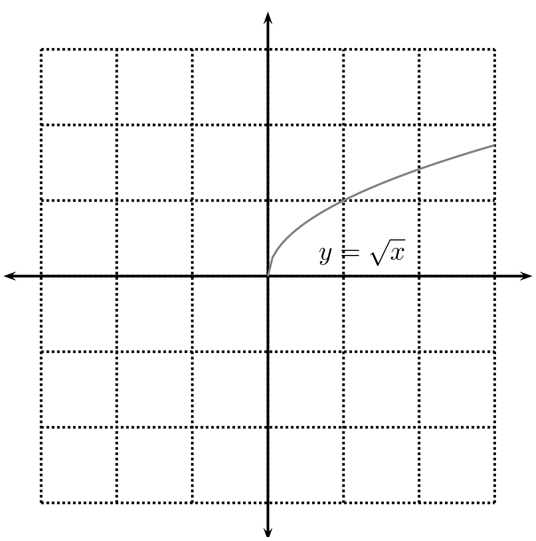


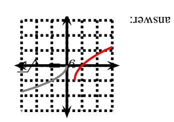
(e) $f(x) = -\sqrt{2x-1} - 1$





(f) $f(x) = -\sqrt{-2x-1} + 1$





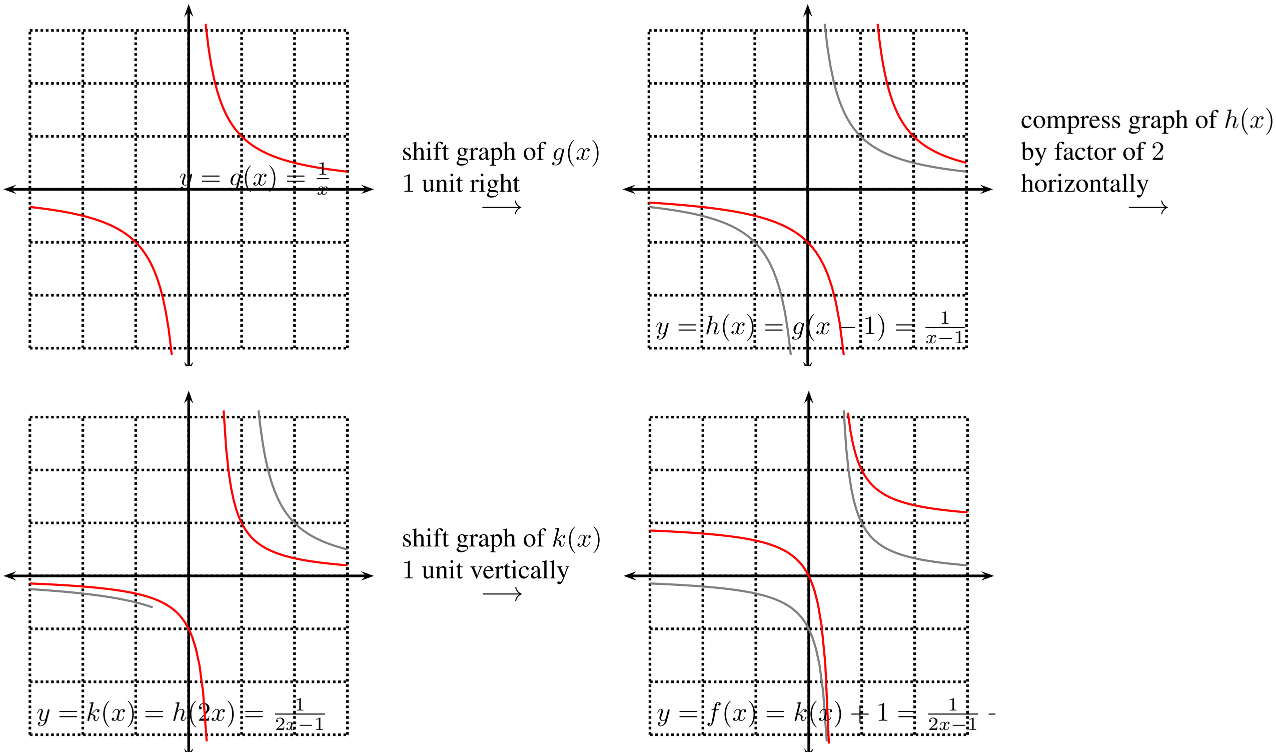
Solution. 1.c.

We are asked to plot $f(x) = \dfrac{1}{2x-1} + 1$ by linearly transforming the graph of $g(x) = \frac{1}{x}$ to the graph of $f(x)$. To do that we have to compose g with a sequence of linear transformations to obtain $f(x)$. There are two natural ways to do that; we show both by presenting two different solutions.

Solution I. We show how to get from $g(x) = \frac{1}{x}$ to $f(x)$ by composing g with a sequence of linear transformations.

$$\begin{array}{r|rcl} g(x) & & = & \frac{1}{x} \\ \text{Define } h(x) \text{ via:} & h(x) & = & g(x+1) = \dfrac{1}{x-1} \\ \text{Define } k(x) \text{ via:} & k(x) & = & h(2x) = \dfrac{1}{2x-1} \\ \text{Therefore} & f(x) & = & k(x) + 1 = \dfrac{1}{2x-1} + 1 \end{array}$$

We plot consecutively the functions $g(x)$, $h(x)$, $k(x)$ and $f(x)$. We start from the given graph of $g(x)$.



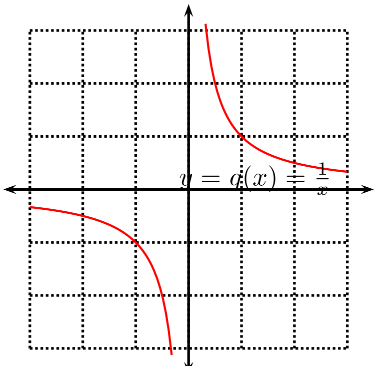
Solution II. In the previous solution we used horizontal stretch to transform the graph of $h(x)$ to the graph of $k(x) = h(2x)$. Algebra suggests a second way to transform the graph of $g(x)$ to the graph of $f(x)$, this time using a vertical stretch. Indeed, we have the equality

$$f(x) = \frac{1}{2x-1} + 1 = \frac{1}{2} \cdot \frac{1}{x - \frac{1}{2}} + 1.$$

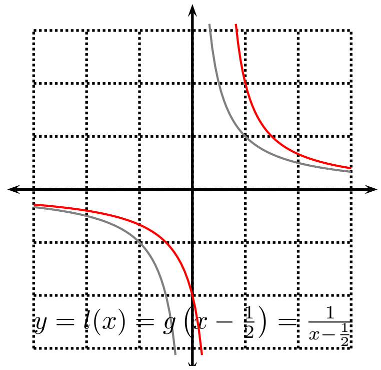
Therefore we can carry out the sequence of transformations shown below.

$$\begin{array}{r|rcl}
g(x) & & = & \frac{1}{x} \\
\text{Define } l(x) \text{ via:} & l(x) & = & g\left(x - \dfrac{1}{2}\right) = \dfrac{1}{x - \frac{1}{2}} \\
\text{Define } k(x) \text{ via:} & k(x) & = & \dfrac{1}{2}h(x) = \dfrac{1}{2} \cdot \dfrac{1}{\left(x - \frac{1}{2}\right)} = \dfrac{1}{(2x-1)} \\
\text{Therefore} & f(x) & = & k(x) + 1 = \dfrac{1}{2x-1} + 1
\end{array}$$

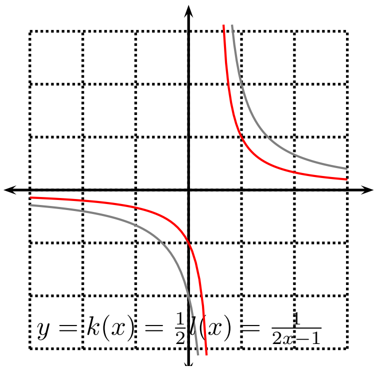
We plot consecutively the functions $g(x)$, $l(x)$, $k(x)$ and $f(x)$. We start from the given graph of $g(x)$.



shift graph of $g(x)$
$\frac{1}{2}$ units right
$\longrightarrow$



compress graph of $h(x)$
by factor of 2
vertically
$\longrightarrow$



shift graph of $k(x)$
1 unit vertically
$\longrightarrow$