Calculus I
Lecture 1
Construcing Functions

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https://github.com/tmilev/freecalc

2020

Outline

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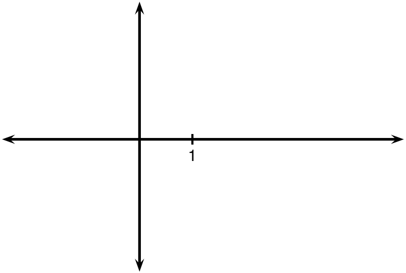
Linear Functions

Definition (Linear Function)

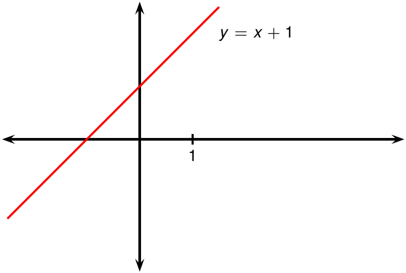
A linear function is a function the graph of which is a line. We can write any linear function in slope-intercept form:

$$f(x) = mx + b.$$

m is called the slope, and b is called the y-intercept.

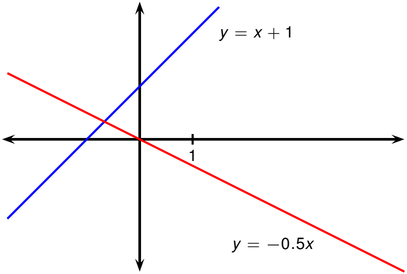


$f(x)$	Direction	y-intercept
$x+1$		
$-0.5x$		
-1		



$f(x)$	Direction	y-intercept
$x + 1$	$\nearrow$	
$-0.5x$		
-1		

- $m > 0$ means the graph of f points up ($\nearrow$).



$f(x)$	Direction	y-intercept
$x + 1$	$\nearrow$	
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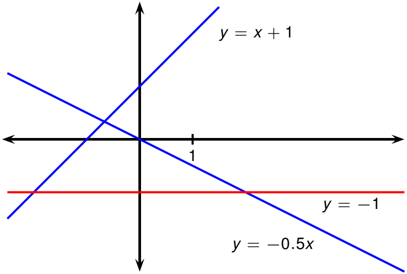
- $m > 0$ means the graph of f points up ($\nearrow$).
- $m < 0$ means the graph of f points down ($\searrow$).



$f(x)$	Direction	y-intercept
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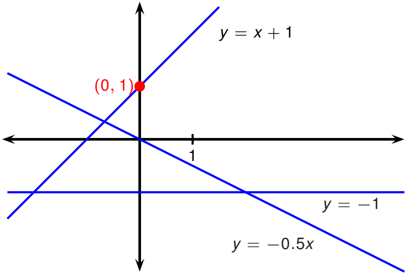
- $m = 0$ means the graph of f is horizontal ($\rightarrow$).



$f(x)$	Direction	y-intercept
$x + 1$	$\nearrow$	1
$-0.5x$	$\searrow$	
-1	$\rightarrow$	

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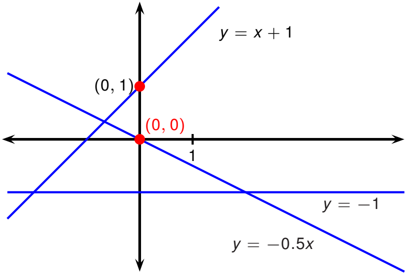
- b tells us the height of the point where the graph hits the y-axis.



$f(x)$	Direction	y-intercept
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$-0.5x + 0$	$\searrow$	0
-1	$\rightarrow$	

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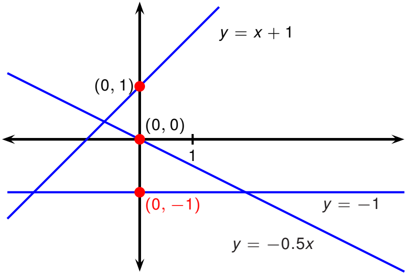

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Polynomials

Definition (Polynomial Function)

A polynomial function is a function f of the form

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1} + a_n x^n,$$

where n is a non-negative integer and $a_0, \ldots, a_n$ are real numbers, called the coefficients. If $a_n \neq 0$ the integer n is called the degree of f.

If we interpret x as an indeterminate formal expression, rather than a number, we say that $f(x)$ is a polynomial (rather than a polynomial function).

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$f(x)$	Polynomial?	Degree	a_0	a_1	a_2
$x^4 - x + 1$					
6					
$3x^2 - \frac{1}{2}x + \sqrt{x}$					
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$f(x)$	Polynomial?	Degree	a_0	a_1	a_2
$x^4 - x + 1$	Yes	4			
6					
$3x^2 - \frac{1}{2}x + \sqrt{x}$					
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where n is a non-negative integer and $a_0, \ldots, a_n$ are real numbers, called the coefficients. If $a_n \neq 0$ the integer n is called the degree of f.

$f(x)$	Polynomial?	Degree	a_0	a_1	a_2
$x^4 - x + 1$	Yes	4	1	-1	0
6	Yes	0	6	0	0
$3x^2 - \frac{1}{2}x + \sqrt{x}$	No				
$3x^2 - \frac{1}{2}x + \sqrt{2}$	Yes	2			
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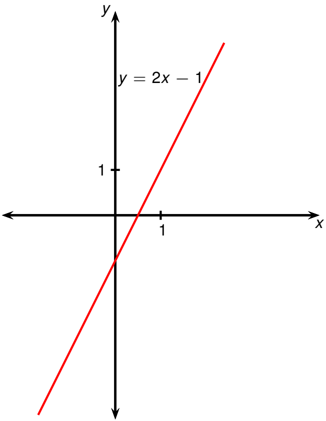
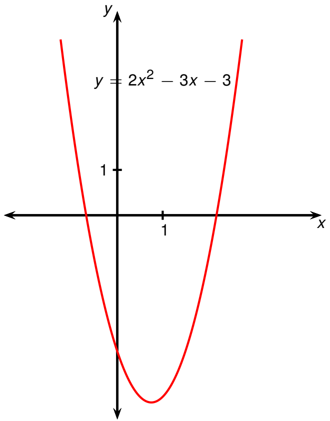
- Linear functions are polynomial (functions).



Linear

- Linear functions are polynomial (functions).
- So are quadratic functions. Their graphs are parabolas.



Quadratic

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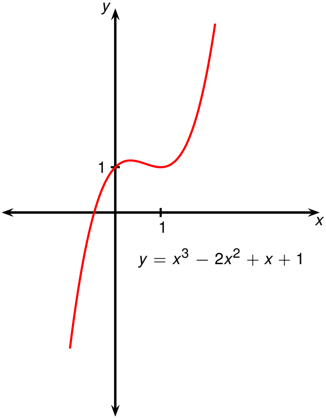
- And there are many more.



Cubic

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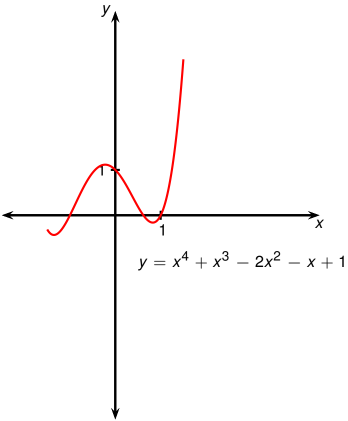

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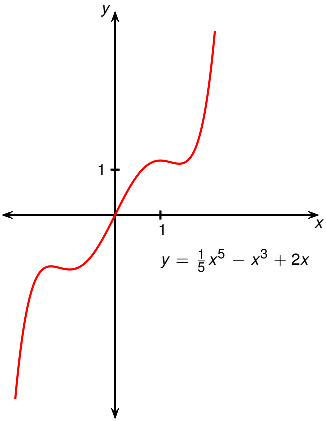

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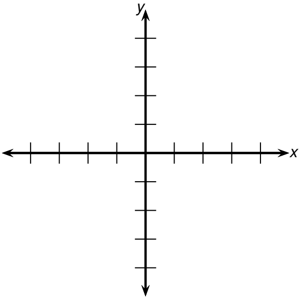

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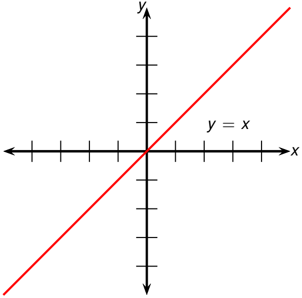

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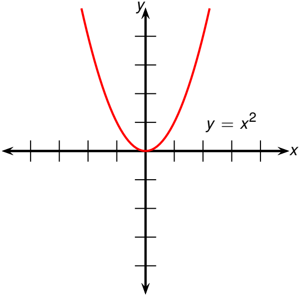

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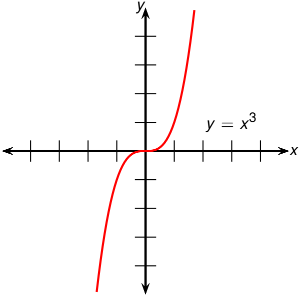

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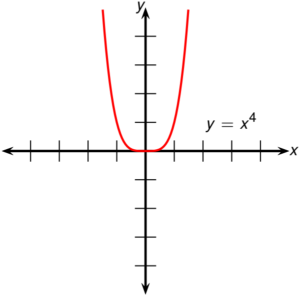

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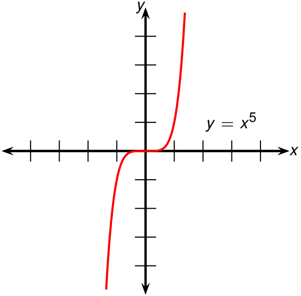

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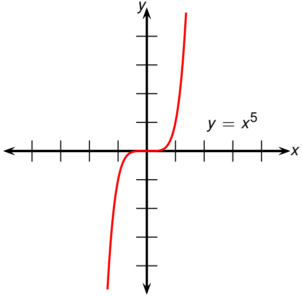

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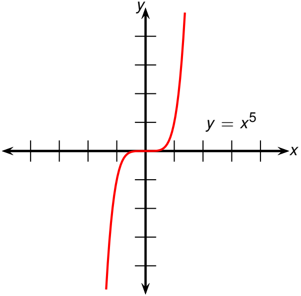

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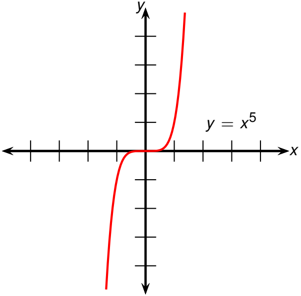

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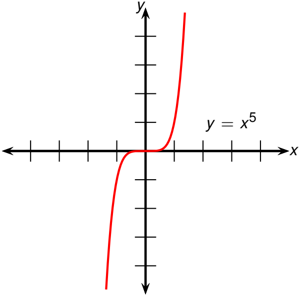

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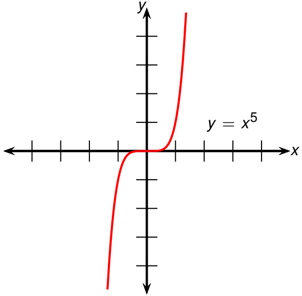

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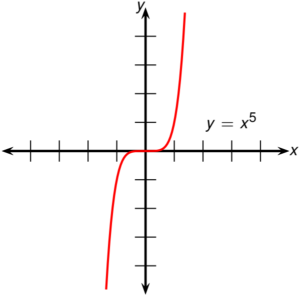

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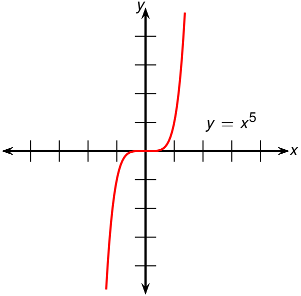

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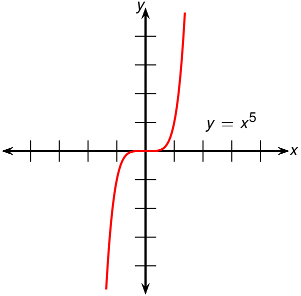

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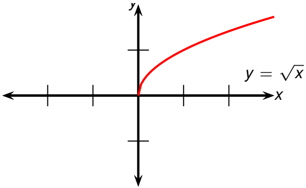

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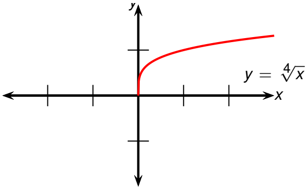

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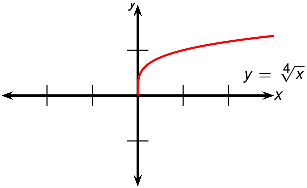

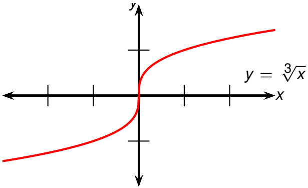

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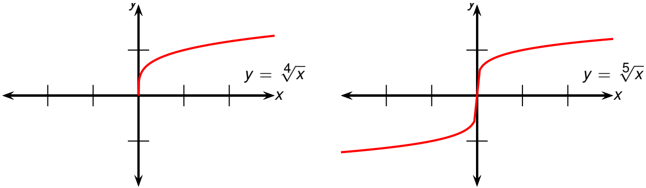
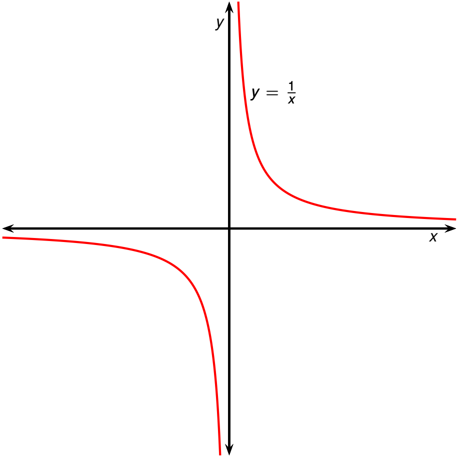


$f(x) = x^{-1} = \frac{1}{x}$ is called the reciprocal function. Its graph has equation $y = \frac{1}{x}$, or $xy = 1$, and is an hyperbola with the coordinate axes as its



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Rational Functions

Definition (Rational Function)

A rational function is a quotient of two polynomials; that is, a function of the form

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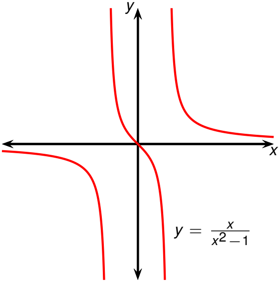

Example ($x/(x^2-1)$)

The function

$$f(x) = \frac{x}{x^2-1}$$

is a rational function.

Algebraic Functions

(Algebraic Function)

A function in x that can be constructed using x, constants, and finitely many of the operations $+, -, *, /$, and $\sqrt[n]{\ }$ is an algebraic function.

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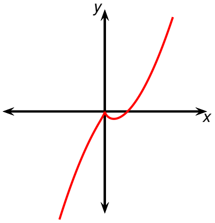
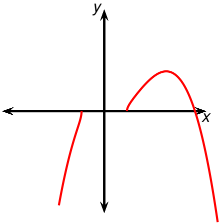
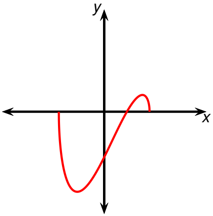
Examples.



$y = (x-1)\sqrt[3]{x^2}$



$y = \frac{1}{5}(4x - x^2)\sqrt{x^2 - 1}$



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$$\begin{array}{rcll}\mathrm{Dom}(f+g) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g) & \Big| \ \cap \text{ stands for} \\ \mathrm{Dom}(f-g) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g) & \Big| \ \text{set intersectio} \\ \mathrm{Dom}(f\cdot g) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g) & \\ & & & \Big| \ \text{right expr.} \\ \mathrm{Dom}\left(\frac{f}{g}\right) &=& \mathrm{Dom}(f)\cap\mathrm{Dom}(g)\cap\{x — g(x)\neq 0\} & \Big| \ \text{stands for set} \\ & & & \Big| \ \text{where } g(x)\neq 0\end{array}$$

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$:

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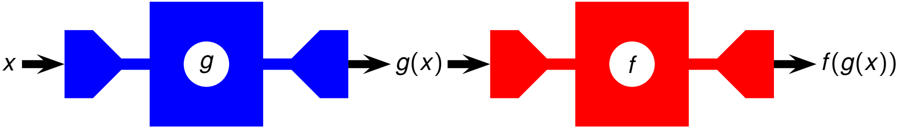
Definition (Composition of f and g)

If f and g are two functions, then the composition of f and g is written $f \circ g$ and is defined by the formula

$$(f \circ g)(x) = f(g(x)).$$

Imagine f and g as machines taking some input and producing some output. Then $f \circ g$ corresponds to attaching both machines end-to-end so that the output of g becomes the input of f.



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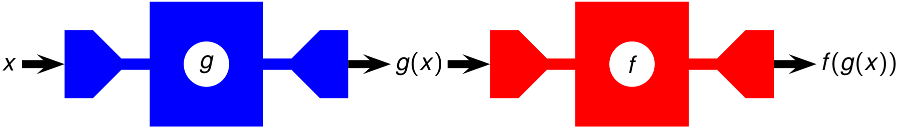

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f. If the domain of f is A and the domain of g is B, we write this as

$$\{x \in B |\ g(x) \in A\}.$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

Example

Find $f \circ g$, $g \circ f$, $g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$(f \circ g)(x)$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(f \circ g)(x) = f(g(x))$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) \quad = \quad f(g(x)) = f\left(\sqrt{3-x}\right)$$

Example

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$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Example

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$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

Example

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$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$3 - x \geq 0$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

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Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

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Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \\ x &\in \textbf{?} \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}$$

Domain:

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \\ x &\in (-\infty, 3]. \end{aligned}$$

Example

Find $f \circ g$, $g \circ f$, $g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

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Domain:

$$\begin{array}{rcl} 3-x & \geq & 0 \\ -x & \geq & -3 \\ x & \leq & 3 \\ x & \in & (-\infty, 3]. \end{array}$$

$(g \circ f)(x)$

Example

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$$
\begin{aligned}
(f \circ g)(x) &= f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}
\end{aligned}
$$

Domain:

$$
\begin{aligned}
3 - x &\geq 0\\
-x &\geq -3\\
x &\leq 3\\
x &\in (-\infty, 3].
\end{aligned}
$$

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(g \circ f)(x) = g(f(x))
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$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{3 - \sqrt{x}}$$

Example

Find $f\circ g, g\circ f, g\circ g$ and their domains, where $f(x)=\sqrt{x}$ and $g(x)=\sqrt{3-x}$.

$$
\begin{array}{rcl}
(f\circ g)(x) &=& f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{\sqrt{3-x}} = \sqrt[4]{3-x}
\end{array}
$$

Domain:

$$
\begin{array}{rcl}
3-x &\geq& 0\\
-x &\geq& -3\\
x &\leq& 3\\
x &\in& (-\infty,3].
\end{array}
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$$
\begin{aligned}
3 - x &\geq 0\\
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$$

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(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{3 - \sqrt{x}}
$$

Domain :

$$
x \geq 0
$$

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$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{3 - \sqrt{x}}$$

Domain :

$$\begin{aligned} x &\geq 0 \\ 3 - \sqrt{x} &\geq 0 \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

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Domain :

$$\begin{aligned} x &\geq 0 \\ 3 - \sqrt{x} &\geq 0 \\ -\sqrt{x} &\geq -3 \end{aligned}$$

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Domain:

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$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{3 - \sqrt{x}}$$

Domain :

$$\begin{aligned} x &\geq 0 \\ 3 - \sqrt{x} &\geq 0 \\ -\sqrt{x} &\geq -3 \\ \sqrt{x} &\leq 3 \\ x &\leq 9 \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

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Domain:

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$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{3 - \sqrt{x}}$$

Domain :

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Domain:

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$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{3-\sqrt{x}}$$

Domain :

$$\begin{aligned} x &\geq 0 \\ 3 - \sqrt{x} &\geq 0 \\ -\sqrt{x} &\geq -3 \\ \sqrt{x} &\leq 3 \\ x &\leq 9 \\ x &\in [0, 9] \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$(g \circ g)(x)$

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$$(g \circ g)(x) \quad = \quad g(g(x))$$

Example

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$$(g \circ g)(x) = g(g(x)) = g\left(\sqrt{3-x}\right)$$

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$$(g \circ g)(x) = g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3 - \sqrt{3 - x}}$$

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Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

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$$(g \circ g)(x) = g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3-\sqrt{3-x}}$$

Domain :

Example

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$$(g \circ g)(x) = g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3 - \sqrt{3-x}}$$

Domain :

$$3 - x \geq 0$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

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Domain :

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(g \circ g)(x) = g(g(x)) = g\left(\sqrt{3 - x}\right) = \sqrt{3 - \sqrt{3 - x}}$$

Domain :

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(g \circ g)(x) = g(g(x)) = g\left(\sqrt{3 - x}\right) = \sqrt{3 - \sqrt{3 - x}}$$

Domain :

$$\begin{aligned} 3 - x &\geq 0 \\ -x &\geq -3 \\ x &\leq 3 \\ 3 - \sqrt{3 - x} &\geq 0 \end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$\begin{aligned}
(g \circ g)(x) &= g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3 - \sqrt{3 - x}}
\end{aligned}$$

Domain :

$$\begin{aligned}
3 - x &\geq 0 \\
-x &\geq -3 \\
x &\leq 3 \\
{\color{red}3} - \sqrt{3 - x} &\geq 0 \\
-\sqrt{3 - x} &\geq {\color{red}-3}
\end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$(g \circ g)(x) = g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3 - \sqrt{3 - x}}$$

Domain :

$$\begin{aligned}
3 - x &\geq 0 \\
-x &\geq -3 \\
x &\leq 3 \\
3 - \sqrt{3-x} &\geq 0 \\
-\sqrt{3-x} &\geq -3 \\
\sqrt{3-x} &\leq 3
\end{aligned}$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$
\begin{array}{rcl}
(g \circ g)(x) & = & g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3-\sqrt{3-x}}
\end{array}
$$

Domain :

$$
\begin{array}{rcl}
3-x & \geq & 0 \\
-x & \geq & -3 \\
x & \leq & 3 \\
3-\sqrt{3-x} & \geq & 0 \\
-\sqrt{3-x} & \geq & -3 \\
\sqrt{3-x} & \leq & 3 \\
3-x & \leq & 9
\end{array}
$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$
\begin{array}{rcl}
(g \circ g)(x) & = & g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3-\sqrt{3-x}} \\
\text{Domain}: & & \\
3-x & \geq & 0 \\
-x & \geq & -3 \\
x & \leq & 3 \\
3-\sqrt{3-x} & \geq & 0 \\
-\sqrt{3-x} & \geq & -3 \\
\sqrt{3-x} & \leq & 3 \\
3-x & \leq & 9 \\
-x & \leq & 6
\end{array}
$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$
\begin{aligned}
(g \circ g)(x) &= g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3-\sqrt{3-x}} \\
\text{Domain :} & \\
3 - x &\geq 0 \\
-x &\geq -3 \\
x &\leq 3 \\
3 - \sqrt{3-x} &\geq 0 \\
-\sqrt{3-x} &\geq -3 \\
\sqrt{3-x} &\leq 3 \\
3 - x &\leq 9 \\
-x &\leq 6 \\
x &\geq -6
\end{aligned}
$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3 - x}$.

$$
\begin{aligned}
(g \circ g)(x) &= g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3 - \sqrt{3-x}} \\
\text{Domain :} & \\
3 - x &\geq 0 \\
-x &\geq -3 \\
x &\leq 3 \\
3 - \sqrt{3-x} &\geq 0 \\
-\sqrt{3-x} &\geq -3 \\
\sqrt{3-x} &\leq 3 \\
3 - x &\leq 9 \\
-x &\leq 6 \\
x &\geq -6 \\
x &\in \textbf{?}
\end{aligned}
$$

Example

Find $f \circ g, g \circ f, g \circ g$ and their domains, where $f(x) = \sqrt{x}$ and $g(x) = \sqrt{3-x}$.

$$
\begin{aligned}
(g \circ g)(x) &= g(g(x)) = g\left(\sqrt{3-x}\right) = \sqrt{3-\sqrt{3-x}} \\
\text{Domain}: & \\
3 - x &\geq 0 \\
-x &\geq -3 \\
x &\leq 3 \\
3 - \sqrt{3-x} &\geq 0 \\
-\sqrt{3-x} &\geq -3 \\
\sqrt{3-x} &\leq 3 \\
3 - x &\leq 9 \\
-x &\leq 6 \\
x &\geq -6 \\
x &\in [-6, 3].
\end{aligned}
$$

Example

Give simplified f-las for $f \circ g$, $f \circ f$, $g \circ f$, $g \circ g$. Find the implied domains.

$$f(x) = \frac{2x-1}{x+2}$$
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$$= \frac{3x-4}{4x+3} \qquad \Big| \; x \neq ?$$

Example

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$$(g\circ f)(x)=\frac{7x+4}{3x-19} \qquad \Big| \; x\neq -2,\frac{19}{3}$$

$$(g\circ g)(x)=\frac{19x-15}{-25x+64} \qquad \Big| \; x\neq \frac{7}{5},\frac{64}{25}$$