Calculus II
Homework Prerequisite Exponential Function Topics

1. Differentiate.

   (a) 10^{x^3}.

   (b) $2^{\tan x}$.

   (c) x^x.

   (d) x^{x^x}.

   (e) $(\sin x)^{\cos x}$.

   (f) $(\ln x)^{\ln x}$.

2. Compute the limit.

   (a) $\displaystyle\lim_{x\to\infty} \left(\frac{x-2}{x}\right)^x$.

   (b) $\displaystyle\lim_{x\to\infty} \left(\frac{x-2}{x}\right)^{2x}$

   (c) $\displaystyle\lim_{x\to\infty} \left(\frac{x}{x+3}\right)^{2x}$

3. (a) A sum is held under a yearly compound interest of 1%. Make an approximation by hand (no calculators allowed) by what factor will have the money increased after 200 years. Can you do the computation in your head?

   (b) Decide, without using a calculator, which is more profitable: earning a yearly compound interest of 2% for 150 years or earning yearly simple interest of 11% for 150 years?

4. $1,000,000$ servers are handling Internet users. Suppose we distribute the computation load as follows. The computation load distributing program directs every new user to a server chosen at random (one server is allowed to process more than one user at a time). Suppose one server has defective hardware and crashes. We are testing the system by simulating X Internet users.

   - What is the chance we catch the defective server using 1 simulated user?

   - Without using a calculator, estimate the chance we fail to catch the defective server using $1,000,000$ simulated users.

   - Using a calculator, estimate the chance we fail to catch the defective server using $100,000$ simulated users. Write an expression using e which approximates this chance. Evaluate the latter with a calculator. Are the two numbers close?

   Remark. While such a simple system architecture would not be practical, it is not to be immediately dismissed as terrible. For example, if we need to handle 2 million users per second, our load distributing mechanism might not be fast enough to keep track of each server's load. On the other hand, an inexpensive modern pc will easily generate 2 million random numbers per second.