Calculus III
Lecture 14

Todor Milev
https://github.com/tmilev/freecalc

2020

Outline

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Density and Mass

Question

Let $\mathcal{R}$ be a region in space. Suppose we know the density of $\mathcal{R}$ at every point. Can we find the mass of $\mathcal{R}$?

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- Choose a sample point P_k inside each D_k. Then $\text{mass}(D_k) \approx \rho(P_k)\text{vol}(D_k)$.
- Sum the above approximations to get an approximation for mass$\mathcal{R}$: $\text{mass}(\mathcal{R}) \approx \sum_{k=1}^{N} \rho(P_k)\text{vol}(D_k)$.
- Take the limit as the diameter of the partitions tends to zero:

$$\text{mass}(\mathcal{R}) = \lim_{\max_k \text{diam}(D_k) \to 0} \sum_{k=1}^{N} \rho(P_k)\text{vol}(D_k) .$$

Triple Integrals

Let f be a scalar or vector-valued function on region $\mathcal{R}$.

Definition

If the limit

$$\lim_{\max_k \operatorname{diam}(D_k)\to 0} \sum_{k=1}^{N} f(P_k)\operatorname{vol}(D_k)$$

exists and is finite, its value is called *the integral of f on $\mathcal{R}$ with respect to volume* and is denoted by

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- If f is a scalar function, then the value of the integral is a scalar.
- If f is a vector-valued function, then the integral is a vector.
- If f is continuous, the limit is guaranteed to exist. If f is not continuous, the limit may fail to exist.

Theoretical Examples

- The volume of a region is defined via a triple integral.

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- The average value of a function f with respect to mass distribution:

$$\text{av. value of } f = \frac{1}{\mathrm{m}(\mathcal{R})} \iiint_{\mathcal{R}} f(P)\,\mathrm{d}m = \frac{1}{\mathrm{m}(\mathcal{R})} \iiint_{\mathcal{R}} f(P)\rho(P)\,\mathrm{d}V \ .$$

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- Problem: compute the moment of inertia I
  - of a rectangular box with sides $2a$, $2b$, and $2c$
  - rotating about axis L through center that is perpendicular to a face.
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- Decompose into slices as follows.
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  - For a fixed z, the slice S_z is: $-a \leq x \leq a$, $-b \leq y \leq b$.

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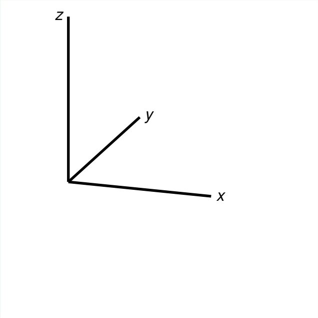
Example (Decomposition into slices)



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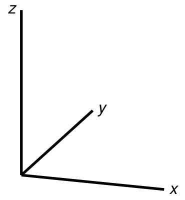

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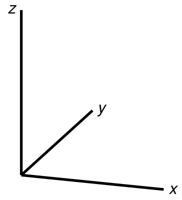

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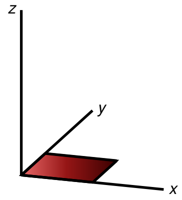

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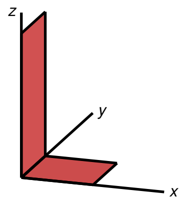

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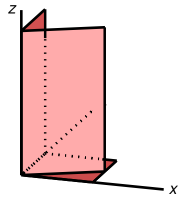

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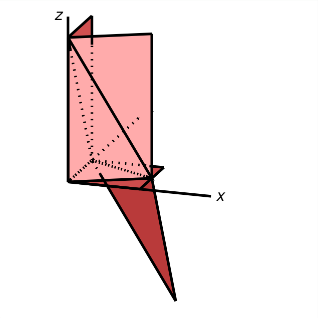

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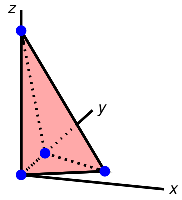

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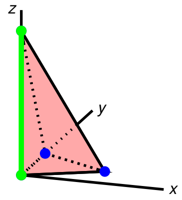

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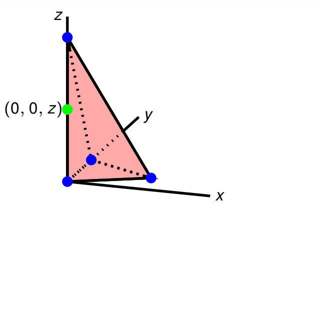

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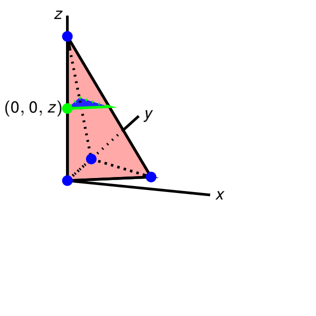

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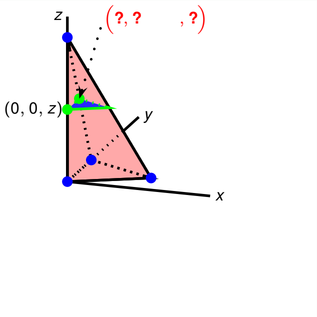
Example (Decomposition into slices)



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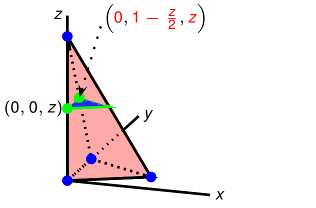

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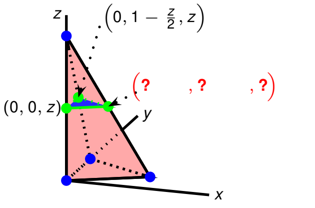

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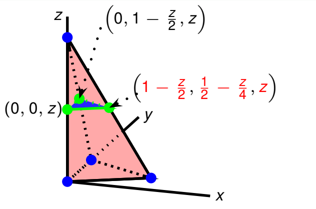

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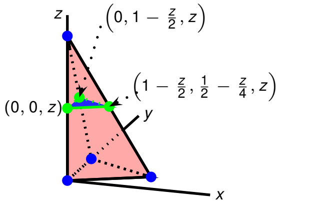

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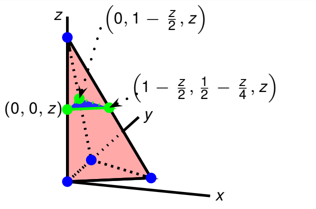

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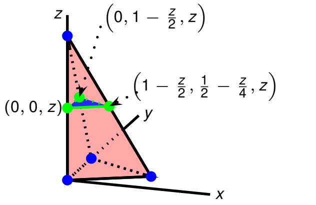

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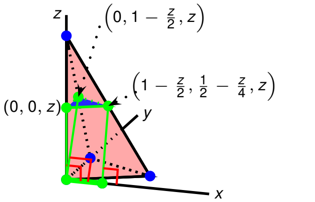

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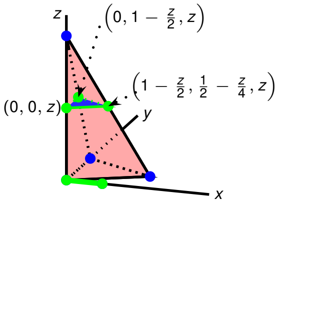

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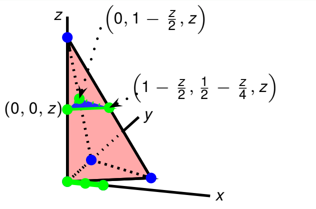

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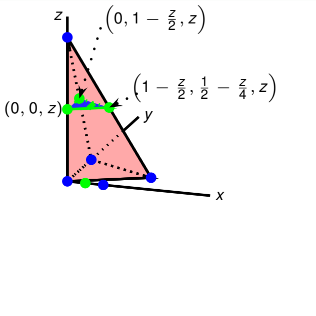

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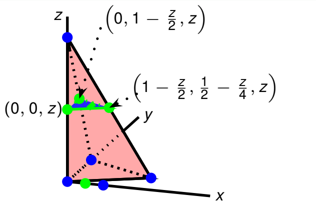

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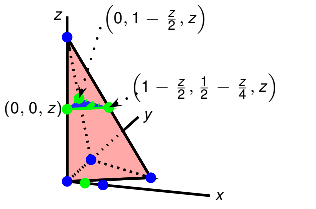

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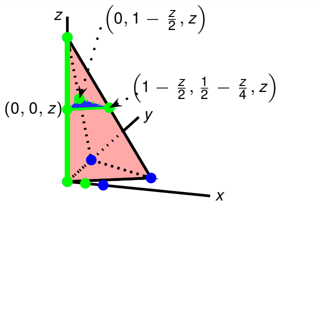

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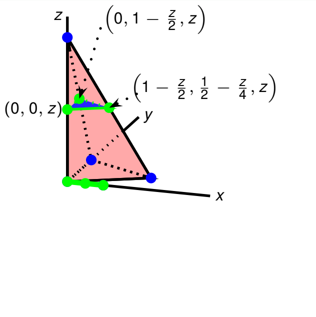

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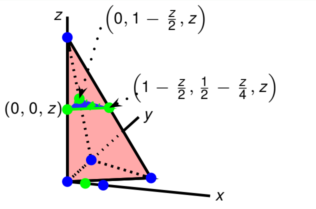

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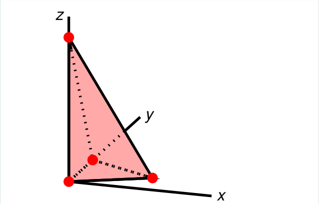
Example (Decomposition into rods)



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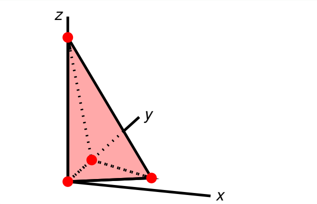

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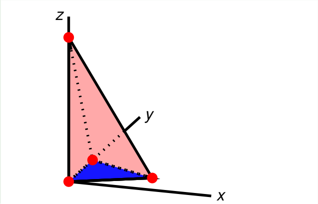

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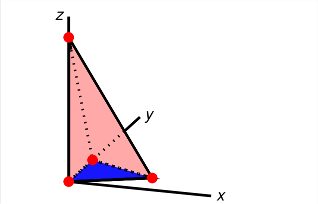
Project the region onto the $xy-$plane to get triangle D with vertices **?** , **?** and **?** .

Example (Decomposition into rods)



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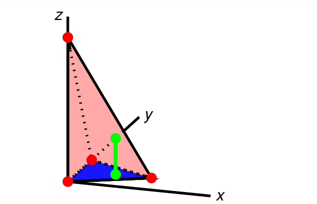

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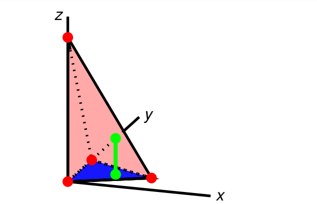
Project the region onto the $xy-$plane to get triangle D with vertices $(0,0,0)$, $(0,1,0)$ and $\left(1, \frac{1}{2}, 0\right)$. Fix $(x, y) \in D$; the vertical rod is segment with endpoints $z = ?$ and $z = ?$.

Example (Decomposition into rods)



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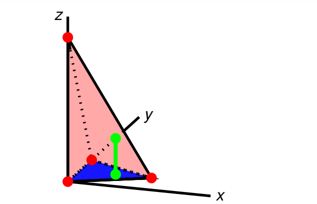
Project the region onto the $xy-$plane to get triangle D with vertices $(0,0,0)$, $(0,1,0)$ and $\left(1, \frac{1}{2}, 0\right)$. Fix $(x, y) \in D$; the vertical rod is segment with endpoints $z = 0$ and $z = 2 - x - 2y$.

Example (Decomposition into rods)



Compute the volume of the region $\mathcal{R}$ bounded by $x + 2y + z = 2$, $x = 2y$, $x = 0$, $z = 0$.

$$\mathrm{vol}(\mathcal{R}) = \iiint_{\mathcal{R}} 1 \cdot \mathrm{d}V$$

$\mathcal{R}$ is a tetrahedron with vertices at $(0,0,0)$, $(0,1,0)$, $(0,0,2)$, and $\left(1, \frac{1}{2}, 0\right)$.

$$\begin{aligned}\mathrm{vol}(\mathcal{R}) &= \iiint_{\mathcal{R}} 1 \cdot \mathrm{d}V = \iint_D \left(\int_{z=0}^{z=2-x-2y} 1 \cdot \mathrm{d}z \right) \mathrm{d}x\mathrm{d}y \\ &= \iint_D (2 - x - 2y)\mathrm{d}x\mathrm{d}y\end{aligned}$$

Project the region onto the $xy-$plane to get triangle D with vertices $(0,0,0)$, $(0,1,0)$ and $\left(1, \frac{1}{2}, 0\right)$. Fix $(x, y) \in D$; the vertical rod is segment with endpoints $z = 0$ and $z = 2 - x - 2y$.

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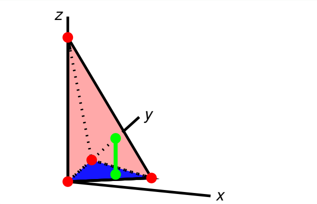

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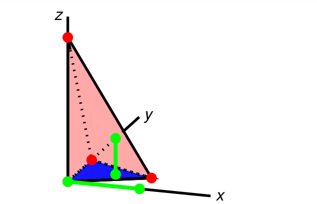

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Project D on the $x-$axis to get segment from $x = 0$ to $x = 1$.

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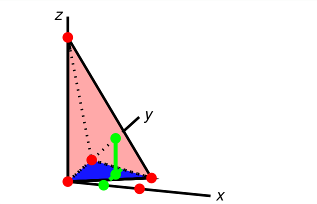

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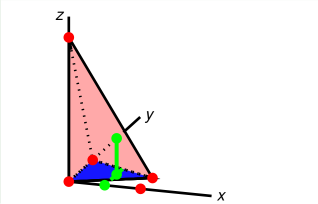

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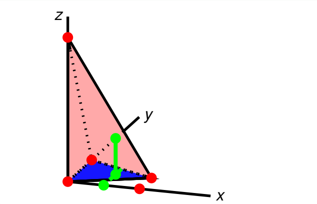

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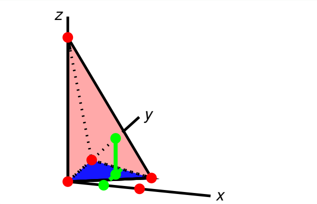

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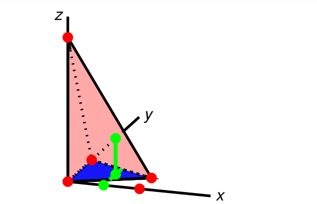

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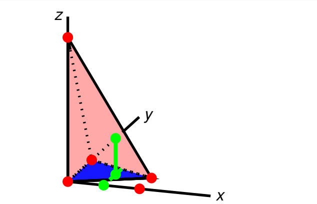

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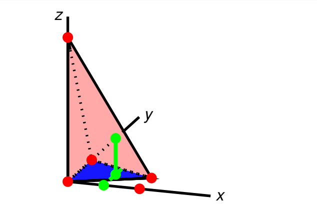

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&= \int_0^1 \left(\left[(2-x)y - y^2 \right]_{y=\frac{x}{2}}^{y=1-\frac{x}{2}} \right) \mathrm{d}x = \int_0^1 (x^2 - 2x + 1)\mathrm{d}x = \frac{1}{3}.
\end{aligned}$$

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