Calculus I Lecture 13 Implicit Differentiation

Todor Miley

https://github.com/tmilev/freecalc

2020

Outline

Outline

Implicit Differentiation

2 Related Rates

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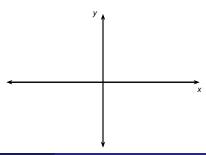
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Implicit Differentiation

 So far, we have seen functions with formulas that express one varable explicitly in terms of the other.

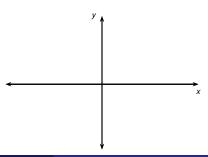


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Implicit Differentiation

 So far, we have seen functions with formulas that express one varable explicitly in terms of the other.

• $y = \sqrt{x^3 + 1}$, $y = x \sin x$, etc.



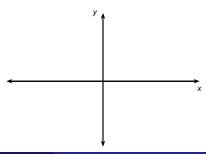
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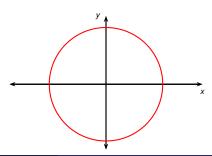


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- $y = \sqrt{x^3 + 1}$, $y = x \sin x$, etc.
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- $x^2 + y^2 = 1$ isn't the equation of any one function.

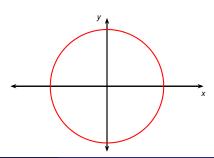


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- Implicitly it gives two functions:



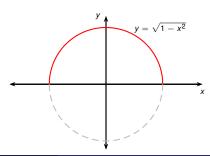
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Lecture 13

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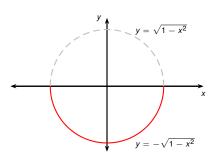
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Lecture 13

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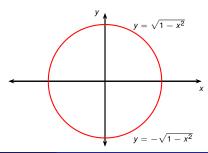
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Lecture 13

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- How do we differentiate these functions?

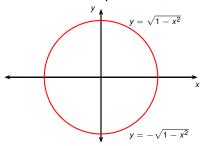


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Implicit Differentiation

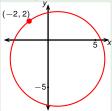
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- How do we differentiate these functions?
- Differentiate both sides with respect to x, and then solve for y'.

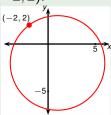


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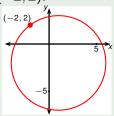
Find an equation of the tangent line to $(x-1)^2 + (y+2)^2 = 25$ at (-2,2).



Lecture 13

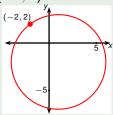


Find
$$\frac{dy}{dx}$$
, given $(x-1)^2 + (y+2)^2 = 25$:



Find
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 $\frac{d}{dx} \left((x-1)^2 \right) + \frac{d}{dx} \left((y+2)^2 \right) = \frac{d}{dx} (25)$
+?

Find an equation of the tangent line to $(x-1)^2 + (y+2)^2 = 25$ at (-2,2).



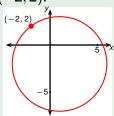
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$$2(x-1)\frac{d}{dx} (x-1) + ? = ?$$

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Find an equation of the tangent line to $(x-1)^2 + (y+2)^2 = 25$ at (-2,2).

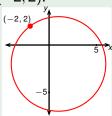


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2020

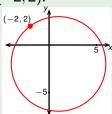


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$$2(x-1)\frac{d}{dx}(x-1) + 2(y+2)\frac{d}{dx}(y+2) = ?$$

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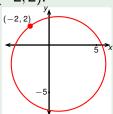
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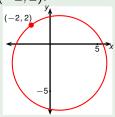


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2020

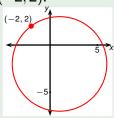


Find
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$$\frac{d}{dx} \left((x-1)^2 \right) + \frac{d}{dx} \left((y+2)^2 \right) = \frac{d}{dx} (25)$$

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$$2(x-1)(?) + 2(y+2) \left(? \right) = 0$$

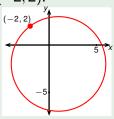


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$$2(x-1)\frac{d}{dx}(x-1) + 2(y+2) (?) = 0$$

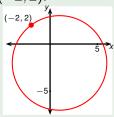


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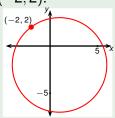


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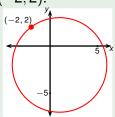
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$$2(y+2)\left(\frac{dy}{dx}\right) = 2(1-x)$$



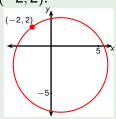
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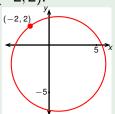
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$$\frac{dy}{dx} = \frac{1-x}{y+2}$$

Find an equation of the tangent line to $(x-1)^2 + (y+2)^2 = 25$ at (-2,2).



Plug in (-2,2):

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1 - (-2)}{2 + 2}$$

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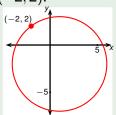
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Lecture 13



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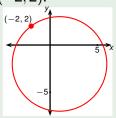
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$$2(x-1)(1) + 2(y+2)\left(\frac{dy}{dx}\right) = 0$$

$$= \frac{3}{4}$$

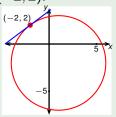
$$2(y+2)\left(\frac{dy}{dx}\right) = 2(1-x)$$

Plug in
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$$\frac{dy}{dx} = \frac{1 - (-2)}{2 + 2} = \frac{3}{4}$$

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$$\frac{dy}{dx} = \frac{1 - (-2)}{2 + 2} = \frac{3}{4}$$

Point-slope form:

$$y-\frac{2}{2}=\frac{3}{4}(x+2)$$

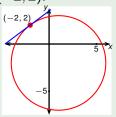
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Plug in
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:

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Point-slope form:

$$y-2=\frac{3}{4}(x+2)$$

$$\frac{d}{dx}\left((x-1)^2\right) + \frac{d}{dx}\left((y+2)^2\right) = \frac{d}{dx}(25)$$

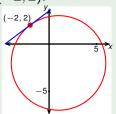
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$$2(x-1)(1) + 2(y+2)\left(\frac{dy}{dx}\right) = 0$$

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Point-slope form:

$$y-2=\frac{3}{4}(x+2)$$

$$\frac{dx}{dx}(x-1) + dx(y+2) = 0$$

$$\frac{d}{dx}(x-1) + 2(y+2)\frac{d}{dx}(y+2) = 0$$

$$2(x-1)(1) + 2(y+2)\left(\frac{dy}{dx}\right) = 0$$

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$$\frac{dy}{dx} = \frac{1-x}{y+2}$$



Find y' as an expression of x and y.

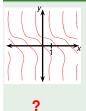
$$\sin(2(x+y))=y^2\cos(2x).$$



Find y' as an expression of x and y.

$$\sin(2(x+y)) = y^{2}\cos(2x).$$

$$\frac{d}{dx}(\sin(2(x+y))) = \frac{d}{dx}(y^{2}\cos(2x))$$



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-2



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$$\cos(2(x+y))\frac{d}{dx}(2(x+y)) = ?$$



$$\sin(2(x+y)) = y^{2} \cos(2x).$$

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$$\sin(2(x+y)) = y^{2} \cos(2x).$$

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$$\cos(2(x+y)) \frac{d}{dx}(2(x+y)) = \frac{d}{dx}(y^{2}) \cos(2x) + (y^{2}) \frac{d}{dx}(\cos(2x))$$

$$\cos(2(x+y)) ? \qquad = ? \cos(2x) + y^{2}?$$



$$\sin(2(x+y)) = y^{2} \cos(2x).$$

$$\frac{d}{dx}(\sin(2(x+y))) = \frac{d}{dx}(y^{2} \cos(2x))$$

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$$\cos(2(x+y)) (2+2y') = ? \cos(2x) + y^{2}?$$



$$\sin(2(x+y)) = y^{2} \cos(2x).$$

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2020



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Implicit Differentiation 6/10

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Implicit Differentiation 6/10

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Implicit Differentiation 6/10

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$$y' = \frac{-\cos(2(x+y)) - y^{2}\sin(2x)}{\cos(2(x+y)) - y\cos(2x)}.$$

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Implicit Differentiation 7/10

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Lecture 13

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Implicit Differentiation 7/1

Example

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Todor Miley

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Related Rates

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- The balloon's volume is increasing.
- The balloon's radius is increasing.
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- In a related rates problem, we compute the rate of change of one quantity in terms of the rate of change of another (which may be more easily measured).
- Procedure:
 - Find an equation relating the two quantities.
 - 2 Use the Chain Rule to differentiate both sides with respect to time.

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Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm³/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

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Lecture 13

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- Let V denote the balloon's volume.
- $V=\frac{4}{3}\pi r^3$

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$$\frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi r^2 \frac{\mathrm{d}r}{\mathrm{d}t}$$

Example

- Let V denote the balloon's volume.
- Let *r* denote its radius.
- Given: $\frac{dV}{dt} = 100 \text{ cm}^3/\text{s}$.
- Unknown: $\frac{dr}{dt}$ when r = 25 cm.
- Find an equation relating the two quantities.
- Use the Chain Rule to differentiate both sides.

$$V = \frac{1}{3}\pi r^{3}$$

$$\frac{dV}{dt} = \frac{d}{dt} \left(\frac{4}{3}\pi r^{3} \right)$$

$$\frac{dV}{dt} = \frac{d}{dr} \left(\frac{4}{3}\pi r^{3} \right) \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^{2} \frac{dr}{dt}$$

Example

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100 \text{ cm}^3/\text{s}$.
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$$\frac{dV}{dt} = \frac{d}{dr} \left(\frac{4}{3}\pi r^{3}\right) \frac{dr}{dt}$$

$$\frac{dV}{dt} = \frac{4\pi r^{2}}{dt} \frac{dr}{dt}$$

$$\frac{dV}{dt} = \frac{1}{dt} \frac{dV}{dt}$$

Example

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100 \text{ cm}^3/\text{s}$.
- Unknown: $\frac{dr}{dt}$ when r = 25cm.
- Find an equation relating the two quantities.
- Use the Chain Rule to differentiate both sides.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{4}{3} \pi r^3 \right)$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}r} \left(\frac{4}{3} \pi r^3 \right) \frac{\mathrm{d}r}{\mathrm{d}t}$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi r^2 \frac{\mathrm{d}r}{\mathrm{d}t}$$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{4\pi r^2} \frac{\mathrm{d}V}{\mathrm{d}t}$$

$$\frac{dr}{dt} = \frac{1}{4\pi (25\text{cm})^2} 100 \frac{\text{cm}^3}{\text{s}}$$

Example

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100 \text{ cm}^3/\text{s}$.
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$$V = \frac{1}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{d}{dt} \left(\frac{4}{3}\pi r^3 \right)$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}r} \left(\frac{4}{3} \pi r^3 \right) \frac{\mathrm{d}r}{\mathrm{d}t}$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi r^2 \frac{\mathrm{d}r}{\mathrm{d}t}$$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{4\pi r^2} \frac{\mathrm{d}V}{\mathrm{d}t}$$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{4\pi (25\mathrm{cm})^2} 100 \frac{\mathrm{cm}^3}{\mathrm{s}}$$

Example

- Let *V* denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100 \text{ cm}^3/\text{s}$.
- Unknown: $\frac{dr}{dt}$ when r = 25 cm.
- Find an equation relating the two quantities.
- Use the Chain Rule to differentiate both sides.

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi (4)$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} \left(\frac{4}{3} \pi r^3 \right)$$

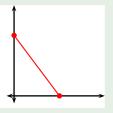
$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}r} \left(\frac{4}{3} \pi r^3 \right) \frac{\mathrm{d}r}{\mathrm{d}t}$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi r^2 \frac{\mathrm{d}r}{\mathrm{d}t}$$

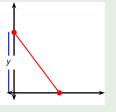
$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{4\pi r^2} \frac{\mathrm{d}V}{\mathrm{d}t}$$

$$\frac{dr}{dt} = \frac{1}{4\pi (25\text{cm})^2} 100 \frac{\text{cm}^3}{\text{s}} = \frac{1}{25\pi} \text{cm/s}$$

Example

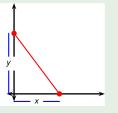


Example



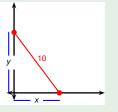
- Let y = dist. from top to ground.
- Let x = dist. from bottom to wall.

Example



- Let y = dist. from top to ground.
- Let x= dist, from bottom to wall.

Example

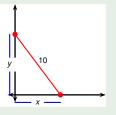


10 ft ladder rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 ft/s. How fast is the ladder top sliding down when the bottom is 6 ft from the wall?

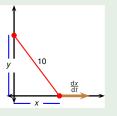
2020

- Let y = dist. from top to ground.
- Let x= dist, from bottom to wall.

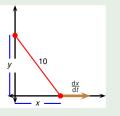
Example



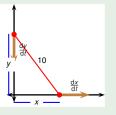
- Let y = dist. from top to ground.
- Let x = dist. from bottom to wall.
- Given: ?
- Unknown: ?



- Let y = dist. from top to ground.
- Let x = dist. from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: ?

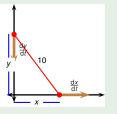


- Let y = dist. from top to ground.
- Let x = dist. from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: ?



- Let y = dist. from top to ground.
- Let x = dist. from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when x = 6 ft.

Example



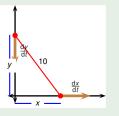
10 ft ladder rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 ft/s. How fast is the ladder top sliding down when the bottom is 6 ft from the wall?

- Let y = dist. from top to ground.
- Let x = dist. from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when x = 6 ft.

- Relationship b/n quantities.
- Differentiate (use Chain Rule).

Todor Milev Lecture 13 Implicit Differentiation 2020

Example

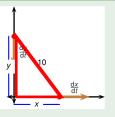


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Todor Miley Lecture 13 Implicit Differentiation 2020



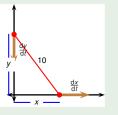
10 ft ladder rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 ft/s. How fast is the ladder top sliding down when the bottom is 6 ft from the wall?

$$x^2 + y^2 = 10^2 = 100$$

- Let y = dist. from top to ground.
- Let x = dist. from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when x = 6 ft.

- Relationship b/n quantities.
- Differentiate (use Chain Rule).

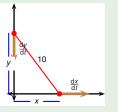
Todor Milev Lecture 13 Implicit Differentiation



- Let y = dist. from top to ground.
- Let x= dist, from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when x = 6 ft.

- Relationship b/n quantities.
- Differentiate (use Chain Rule).

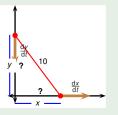
$$x^2 + y^2 = 10^2 = 100$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$



- Let y = dist. from top to ground.
- Let x= dist, from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when x = 6 ft.

$$x^{2} + y^{2} = 10^{2} = 100$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
$$\frac{dy}{dt} = -\frac{x}{v}\frac{dx}{dt}$$

- Relationship b/n quantities.
- Differentiate (use Chain Rule).



- Let y= dist. from top to ground.
- Let x = dist. from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
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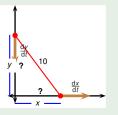
$$x^{2} + y^{2} = 10^{2} = 100$$

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$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{?}{2}$$

- Relationship b/n quantities.
- Differentiate (use Chain Rule).



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- Let x = dist. from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
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$$x^{2} + y^{2} = 10^{2} = 100$$

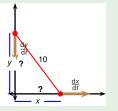
$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{?}{2}$$

- Relationship b/n quantities.
- Differentiate (use Chain Rule).

Example



10 ft ladder rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 ft/s. How fast is the ladder top sliding down when the bottom is 6 ft from the wall?

- Let y = dist. from top to ground.
- Let x = dist. from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when x = 6 ft.

$$x^{2} + y^{2} = 10^{2} = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

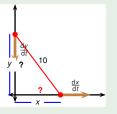
$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{?}{2} \cdot 1 \text{ ft/s}$$

- Relationship b/n quantities.
- Differentiate (use Chain Rule).

Todor Milev

Example



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- Let x = dist. from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when x = 6 ft.

$$x^{2} + y^{2} = 10^{2} = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

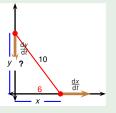
$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{?}{2} \cdot 1 \text{ ft/s}$$

- Relationship b/n quantities.
- Differentiate (use Chain Rule).

Lecture 13

Example



- Let y = dist. from top to ground.
- Let x= dist, from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when x = 6 ft.

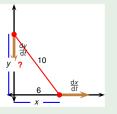
$$x^{2} + y^{2} = 10^{2} = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{6}{2}t \cdot 1 \text{ ft/s}$$

- Relationship b/n quantities.
- Differentiate (use Chain Rule).



- Let y = dist. from top to ground.
- Let x = dist. from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when x = 6 ft.

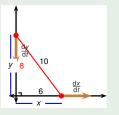
$$x^{2} + y^{2} = 10^{2} = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{6}{2} \cdot 1 \text{ ft/s}$$

- Relationship b/n quantities.
- Differentiate (use Chain Rule).



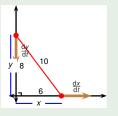
- Let y = dist. from top to ground.
- Let x = dist. from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when x = 6 ft.
- Pythagorean Therem: $y = \sqrt{10^2 6^2} = 8$.
- Relationship b/n quantities.
- Differentiate (use Chain Rule).

$$x^{2} + y^{2} = 10^{2} = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{6}{8}\frac{ft}{ft} \cdot 1 \text{ ft/s}$$



- Let y= dist. from top to ground.
- Let x = dist. from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
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- Pythagorean Therem: $y = \sqrt{10^2 6^2} = 8$.
- Relationship b/n quantities.
- Differentiate (use Chain Rule).

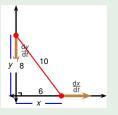
$$x^{2} + y^{2} = 10^{2} = 100$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{6}{8} \frac{ft}{ft} \cdot 1 \text{ ft/s}$$

$$= -3/4 \text{ ft/s}.$$



10 ft ladder rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 ft/s. How fast is the ladder top sliding down when the bottom is 6 ft from the wall?

- Let y = dist. from top to ground.
- Let x= dist, from bottom to wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when x = 6 ft.
- Pythagorean Therem: $v = \sqrt{10^2 - 6^2} = 8$.
- Relationship b/n quantities.
- Differentiate (use Chain Rule).

$$x^{2} + y^{2} = 10^{2} = 100$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y} \frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{6}{8} \frac{ft}{ft} \cdot 1 \text{ ft/s}$$

$$= -3/4 \text{ ft/s}.$$

Therefore the top of the ladder is falling at a rate of 3/4 ft/s.