Precalculus
Lecture 14
Graphing Equations

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https://github.com/tmilev/freecalc

2020

Outline

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- If we set $H(x, y) = F(x, y) - G(x, y)$, we transform an arbitrary equation to an equivalent equation of the form:

$$H(x, y) = 0.$$

Example

Determine which of the following points

- $(-3, -5)$
- $(3, 5)$

is a solution to the equation

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  so $(3, 5)$ is **not** a solution.

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- In particular, while computer algorithms plot graphs of well-behaved equations relatively well, it is not clear why those algorithms work.
- When, using algebra, we can express one variable in terms of the other, it is easy to produce the graph of the equation.

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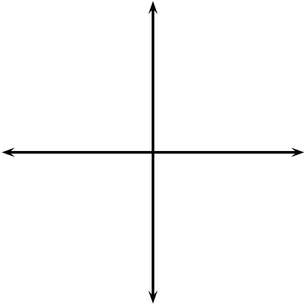
Can we plot the graph of an equation of the form $y = f(x)$ or $x = h(y)$ (for continuous h, f)? Yes.



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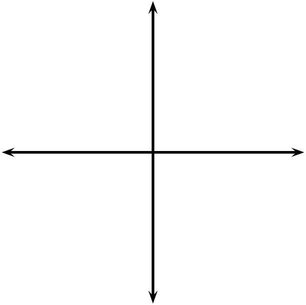

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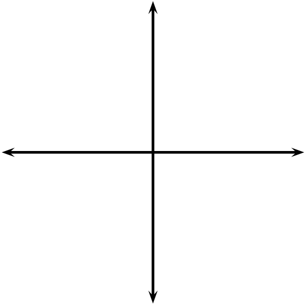

Demonstration of the algorithm for $y = -\frac{1}{3}x + 1$

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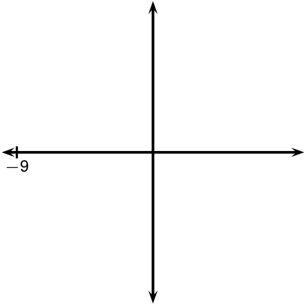

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- Suppose $y = f(x)$.
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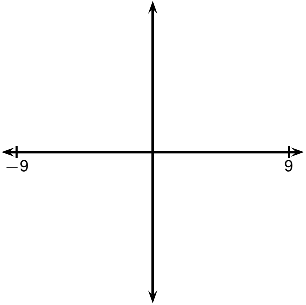

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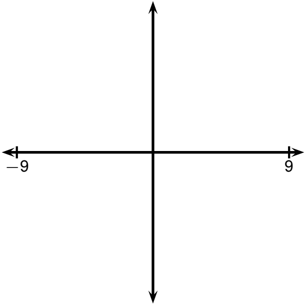

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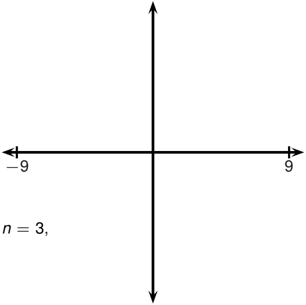

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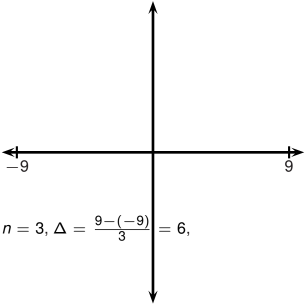

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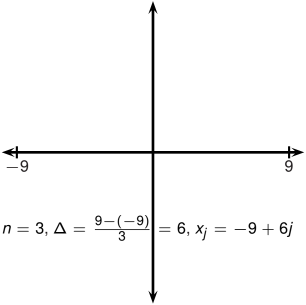

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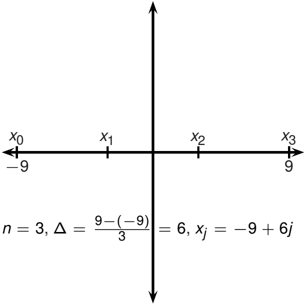

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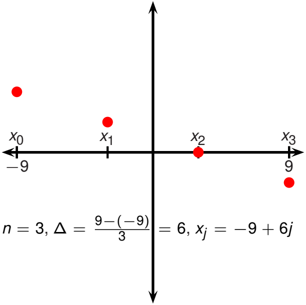

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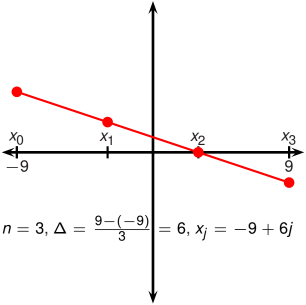
Question

Can we plot the graph of an equation of the form $y = f(x)$ or $x = h(y)$ (for continuous h, f)? Yes.



Demonstration of the algorithm for $y = -\frac{1}{3}x + 1$ from $x = -9$ to $x = 9$.

- Suppose $y = f(x)$.
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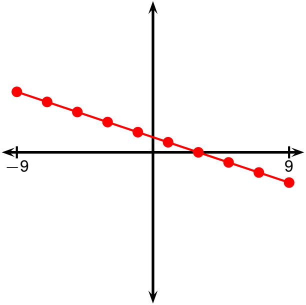

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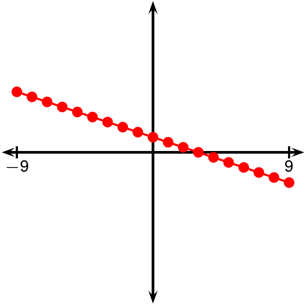

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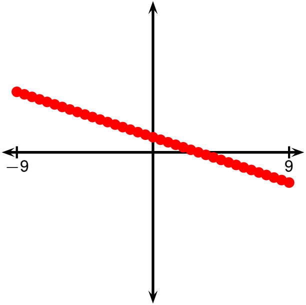

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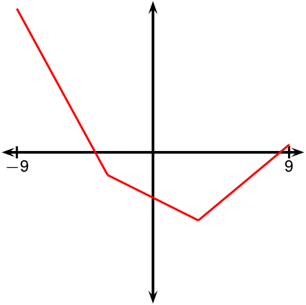

Demonstration of the algorithm for $y = \frac{1}{9}x^2 - \frac{1}{2}x + 4$ from $x = -9$ to $x = 9$.

- Suppose $y = f(x)$.
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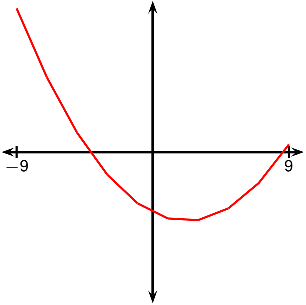

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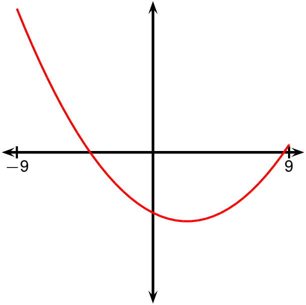

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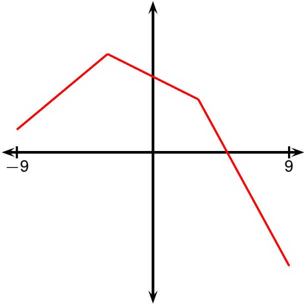

Demonstration of the algorithm for $y = -\frac{1}{9}x^2 - \frac{1}{2}x + 6$ from $x = -9$ to $x = 9$.

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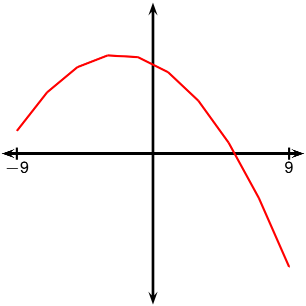

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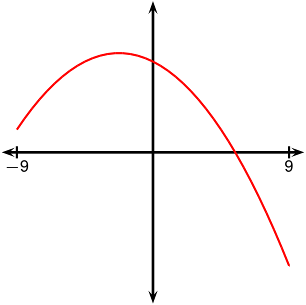

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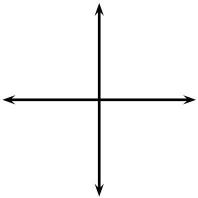
You will not be tested on the material in the following slide.

(Elementary Computer algorithm for sketching graphs)

Let H-continuous; is there simple algorithm to sketch $H(x, y) = 0$?



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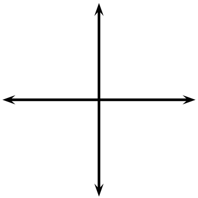

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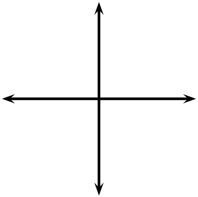

We illustrate the algorithm for:
$$x^2 + 2y^2 = 1$$

(Elementary Computer algorithm for sketching graphs)

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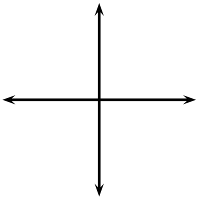

We illustrate the algorithm for:

$$x^2 + 2y^2 = 1$$

$$x^2 + 2y^2 - 1 = 0$$

(Elementary Computer algorithm for sketching graphs)

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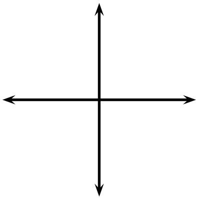


We illustrate the algorithm for:

$$x^2 + 2y^2 = 1$$

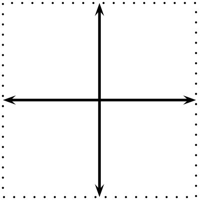
$$x^2 + 2y^2 - 1 = 0$$

Set $H(x, y) = x^2 + 2y^2 - 1$

(Elementary Computer algorithm for sketching graphs)

Let H-continuous; is there simple algorithm to sketch $H(x, y) = 0$? Yes.

- Elementary algorithm: fix large rectangle.



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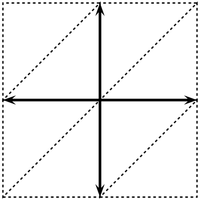

- Elementary algorithm: fix large rectangle.
- Split the grid in triangular mesh. One strategy to do that is shown.

We illustrate the algorithm for:

$$x^2 + 2y^2 = 1$$
$$x^2 + 2y^2 - 1 = 0$$

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(Elementary Computer algorithm for sketching graphs)

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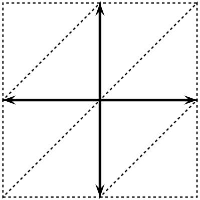

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- Elementary algorithm: fix large rectangle.
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- For each triangle:

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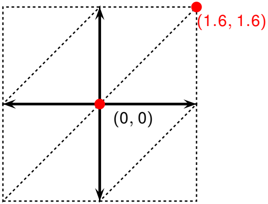

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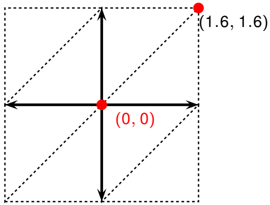

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$$x^2 + 2y^2 = 1$$
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(Elementary Computer algorithm for sketching graphs)

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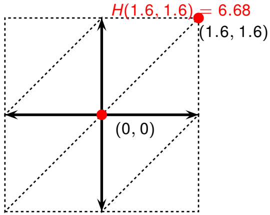

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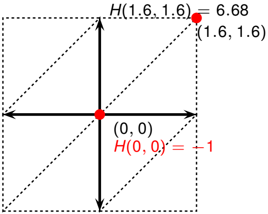
  - If $H(x_P, y_P)$ and $H(x_Q, y_Q)$ have different sign then H must become zero somewhere on the segment between P and Q.

(Elementary Computer algorithm for sketching graphs)

Let H-continuous; is there simple algorithm to sketch $H(x, y) = 0$? Yes.



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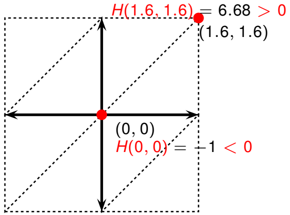

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$$x^2 + 2y^2 = 1$$
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- Elementary algorithm: fix large rectangle.
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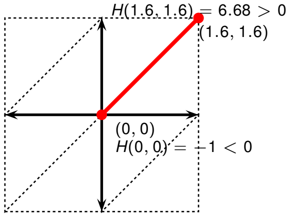
(Elementary Computer algorithm for sketching graphs)

Let H-continuous; is there simple algorithm to sketch $H(x,y)=0$? Yes.



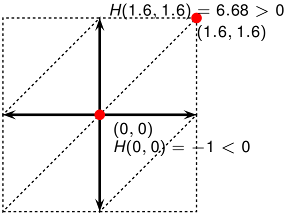
- Elementary algorithm: fix large rectangle.
- Split the grid in triangular mesh. One strategy to do that is shown.
- For each triangle:
  - Fix two corners $P(x_P, y_P)$ and $Q(x_Q, y_Q)$.
  - If $H(x_P, y_P)$ and $H(x_Q, y_Q)$ have different sign then H must become zero somewhere on the segment between P and Q.

We illustrate the algorithm for:

$$x^2+2y^2=1$$
$$x^2+2y^2-1=0$$
Set $H(x,y)=x^2+2y^2-1$

(Elementary Computer algorithm for sketching graphs)

Let H-continuous; is there simple algorithm to sketch $H(x, y) = 0$? Yes.



We illustrate the algorithm for:

$$x^2 + 2y^2 = 1$$
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Set $H(x, y) = x^2 + 2y^2 - 1$

- Elementary algorithm: fix large rectangle.
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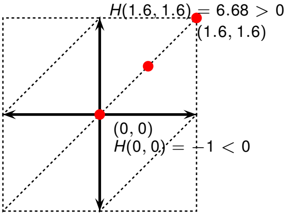
  - If $H(x_P, y_P)$ and $H(x_Q, y_Q)$ have different sign then H must become zero somewhere on the segment between P and Q.
  - Select a point between P and Q and "guess" that H is zero there.

(Elementary Computer algorithm for sketching graphs)

Let H-continuous; is there simple algorithm to sketch $H(x, y) = 0$? Yes.



We illustrate the algorithm for:

$$x^2 + 2y^2 = 1$$
$$x^2 + 2y^2 - 1 = 0$$
$$\text{Set } H(x, y) = x^2 + 2y^2 - 1$$

- Elementary algorithm: fix large rectangle.
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- For each triangle:
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  - If $H(x_P, y_P)$ and $H(x_Q, y_Q)$ have different sign then H must become zero somewhere on the segment between P and Q.
  - Select a point between P and Q and "guess" that H is zero there.
    - In our implementation, we select the midpoint (i.e., $\frac{1}{2}P + \frac{1}{2}Q$).

(Elementary Computer algorithm for sketching graphs)

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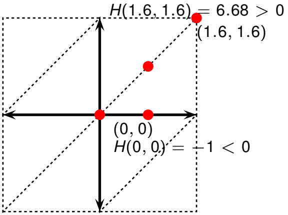

We illustrate the algorithm for:

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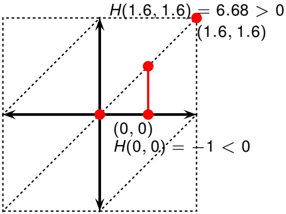

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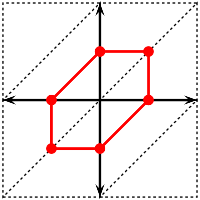

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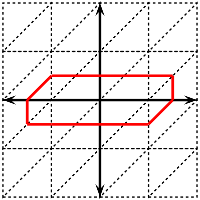

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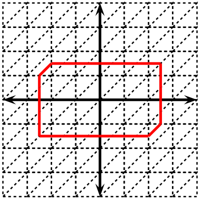

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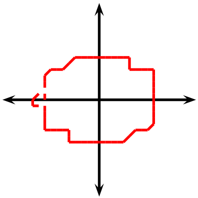

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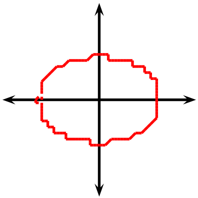

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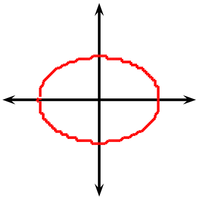

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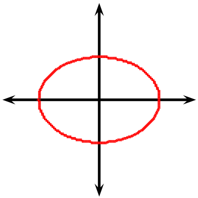

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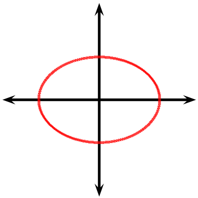

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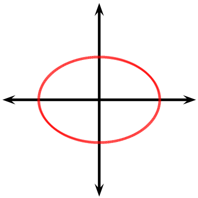

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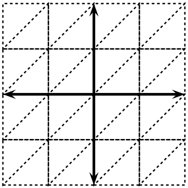

Illustrate the algorithm for:

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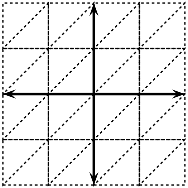

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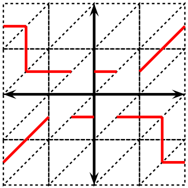

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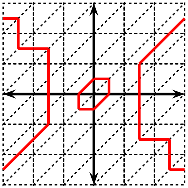

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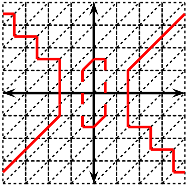

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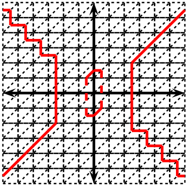

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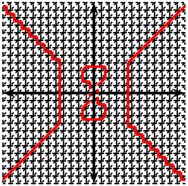

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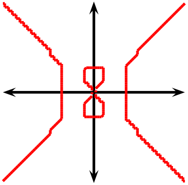

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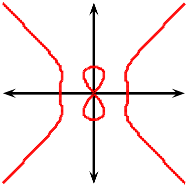

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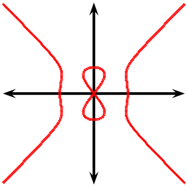

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  - If $H(x_P, y_P)$ and $H(x_Q, y_Q)$ have different sign then H must become zero somewhere on the segment between P and Q.
  - Select a point between P and Q and "guess" that H is zero there.
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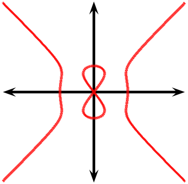
(Elementary Computer algorithm for sketching graphs)

Let H-continuous; is there simple algorithm to sketch $H(x, y) = 0$? Yes.



Illustrate the algorithm for:

$$y^2(y^2-3)=x^2(x^2-5)$$
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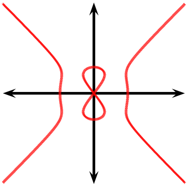

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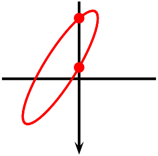
Definition

Points on the graph of $F(x, y) = G(x, y)$ for which

- $x = 0$ are called y-intercepts (those lie on the y axis)



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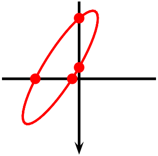

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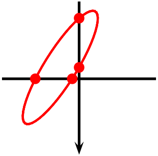

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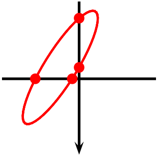

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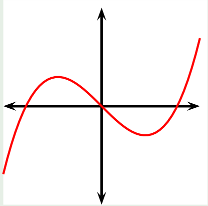
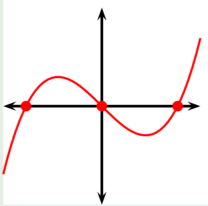
Example



Find the x and y intercepts of the graph of the equation $y = x^3 - x$.

Example

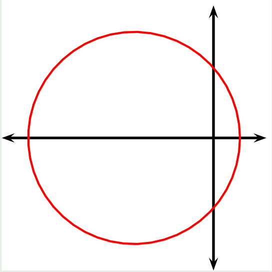


Find the x and y intercepts of the graph of the equation $y = x^3 - x$.
To find the y intercept, set $x = 0$ to get $y = 0$. To find the x intercepts, set $y = 0$ and solve

$$\begin{aligned} x^3 - x &= 0 \\ x(x^2-1) &= 0 \\ x(x-1)(x+1) &= 0 \\ x = 0 \text{ or } x = 1 &\quad \text{ or } x = -1. \end{aligned}$$

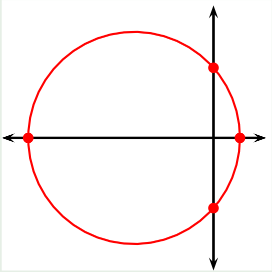
The x-intercepts are: $(-1, 0), (0, 0), (1, 0)$, the y-intercept is $(0, 0)$.

Example



Find the x and y intercepts of the graph of the equation $x^2 + 3x + y^2 = \frac{7}{4}$.

Example



Answer: the y-intercepts are: $\left(0, \sqrt{\frac{7}{4}}\right)$, $\left(0, -\sqrt{\frac{7}{4}}\right)$; the x intercepts are: $(\frac{1}{2}, 0))$ and $(-\frac{7}{2}, 0)$

Find the x and y intercepts of the graph of the equation $x^2 + 3x + y^2 = \frac{7}{4}$.

To find the y intercept, set $x = 0$ and solve:

$$y^2 = \frac{7}{4} \Rightarrow y = \pm\sqrt{\frac{7}{4}}$$

To find the x intercepts, set $y = 0$ and solve:

$$\begin{aligned} x^2 + 3x &= \frac{7}{4} \\ x^2 + 3x - \frac{7}{4} &= 0 \\ 4x^2 + 12x - 7 &= 0 \\ (2x-1)(2x+7) &= 0 \\ 2x - 1 = 0 \quad &\text{or} \quad 2x + 7 = 0 \\ x = \frac{1}{2} \quad &\text{or} \quad x = -\frac{7}{2} \end{aligned}$$

- A graph is symmetric with respect to the x axis for each (x, y) lying on the graph $(x, -y)$ also lies on the graph.
- A graph is symmetric with respect to the y axis for each (x, y) lying on the graph $(-x, y)$ also lies on the graph.
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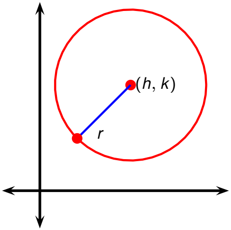

Observation

The graph of the equation

$$(x-h)^2+(y-k)^2=r^2$$

is a circle with radius r and center (h, k).

Definition (Completing the square)

Let $a \neq 0$. To *complete the square* means to carry out the following algebraic manipulation.

$$ax^2 + bx + c$$

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Add & subtract $\left(\quad\right)^2$

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Add & subtract $\left(\frac{b}{2a}\right)^2$

use $(A+B)^2 = A^2+2AB+B^2$

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&= {\color{red}a}\left(\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right) + c \\
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&= a\left(\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}\right)+c\\
&= a\left(x+\frac{b}{2a}\right)^2-\cancel{a}\cdot\frac{b^2}{4a^{\cancel{2}}}+c\\
&= a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}.
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Example (Completing the square)

Complete the square.

$$3x^2 - 5x + 1$$

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$$3x^2 - 5x + 1 \quad = \quad 3\left(x^2 - \mathbf{?}\, x\right) + 1$$

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$$3x^2 - 5x + 1 = 3\left(x^2 - \frac{5}{3}x\right) + 1$$

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\begin{aligned}
3x^2 - 5x + 1 &= 3\left(x^2 - \frac{5}{3}x\right) + 1 \\
&= 3\left(x^2 - 2 \cdot \frac{5}{2 \cdot 3}x\right) + 1
\end{aligned}
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Complete the square.

$$\begin{aligned} 3x^2 - 5x + 1 &= 3\left(x^2 - \frac{5}{3}x\right) + 1 \\ &= 3\left(x^2 - 2\cdot\frac{5}{\color{red}{2\cdot 3}}x\right) + 1 \\ &= 3\left(x^2 - 2\cdot\frac{5}{\color{red}{6}}x + \mathbf{?} \qquad - \mathbf{?} \qquad\right) + 1 \end{aligned}$$

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&= 3\left(\mathbf{?} \qquad - \mathbf{?}\right) + 1
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&= 3\left(x-\frac{5}{6}\right)^2-\frac{25}{12}+1\\
&= 3\left(x-\frac{5}{6}\right)^2-\frac{13}{12}.
\end{aligned}$$

Example

Show that the graph of the given equation is a circle. Find the center and radius of the circle.

- $x^2 + 2x + y^2 = 1$.
- $x^2 + x + 2y^2 + y = 1 + y^2$.
- $x^2 = 3x - y^2 - 2y$.
- $3x^2 + y = -3y^2$.
- $2x^2 + y = \frac{1}{2}x - 2y^2$.

Example

Find an equation of a circle with center $(2, 3)$ and passing through the point $(-1, 1)$.