Calculus I
Homework Review: Function Composition
Lecture 1

1. Find the implied domain of the function.

   (a) $f(x) = \dfrac{x+4}{x^2-4}$.

   answer: $x \neq \pm 2$, alternatively: $x \in (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

   (b) $f(x) = \dfrac{2x^3-5}{x^2+5x+6}$.

   answer: $x \neq -2, -3$, alternatively: $x \in (-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$

   (c) $f(t) = \sqrt[3]{3t-1}$.

   answer: $x \in \mathbb{R}$ (the domain is all real numbers)

   (d) $g(t) = \sqrt{5-t} - \sqrt{1+t}$.

   answer: $x \in [-1, 5]$.

   (e) $h(x) = \dfrac{1}{\sqrt[6]{x^2-7x}}$.

   answer: $x \in (-\infty, 0) \cup (7, \infty)$.

   (f) $f(u) = \dfrac{u+1}{1+\frac{1}{u+1}}$.

   answer: $u \neq -1, -2$ or $u \in (-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$..

   (g) $F(x) = \sqrt{10-\sqrt{x}}$.

   answer: $x \in [0, 100]$

2. Compute the composite functions $(f \circ g)(x)$, $(g \circ f)(x)$. Simplify your answer to a single fraction. Find the domain of the composite function.

   (a) $f(x) = \dfrac{x+2}{x-2}, g(x) = \dfrac{x-1}{x+2}$.

   answer: $(f \circ g)(x) = \frac{3+3x}{-5-x}$ $x \neq -2, -5$; $(g \circ f)(x) = \frac{4}{-2+3x}$ $x \neq 2, \frac{2}{3}$

   (b) $f(x) = \dfrac{x+1}{3x-2}, g(x) = \dfrac{x-2}{x-1}$.

   answer: $(f \circ g)(x) = \frac{-3+2x}{-4+x}$ $x \neq 4, 1$; $(g \circ f)(x) = \frac{5-5x}{3-2x}$ $x \neq \frac{2}{3}, \frac{3}{2}$

   (c) $f(x) = \dfrac{2x+1}{3x-1}, g(x) = \dfrac{x-2}{2x-1}$.

   answer: $(f \circ g)(x) = \frac{-5+4x}{-5+x}$ $x \neq 5, \frac{1}{2}$; $(g \circ f)(x) = \frac{3-4x}{3+x}$ $x \neq -3, \frac{1}{3}$

   (d) $f(x) = \dfrac{x+1}{x-2}, g(x) = \dfrac{x+2}{2x-1}$.

   answer: $(f \circ g)(x) = \frac{1+3x}{4-3x}$ $x \neq \frac{4}{3}, \frac{1}{2}$; $(g \circ f)(x) = \frac{-3+3x}{4+x}$ $x \neq -4, 2$

   (e) $f(x) = \dfrac{5x+1}{4x-1}, g(x) = \dfrac{4x-1}{3x+1}$.

   answer: $(f \circ g)(x) = \frac{-4+23x}{-5+13x}$ $x \neq \frac{5}{13}, -\frac{1}{3}$; $(g \circ f)(x) = \frac{5+16x}{2+19x}$ $x \neq -\frac{2}{19}, \frac{1}{4}$

   (f) $f(x) = \dfrac{3x-5}{x-2}, g(x) = \dfrac{x-2}{x-4}$.

   answer: $(f \circ g)(x) = \frac{-2x+14}{-x+6}$ $x \neq 6, 4$; $(g \circ f)(x) = \frac{x-1}{-x+3}$ $x \neq 3, 2$

   (g) $f(x) = \dfrac{x-3}{x+2}, g(y) = \dfrac{y+3}{y-4}$.

   answer: $(f \circ g)(x) = \frac{-2x+15}{3x-5}$ $x \neq \frac{5}{3}, 4$; $(g \circ f)(x) = \frac{4x+3}{-3x-11}$ $x \neq -\frac{11}{3}, -2$

3. Find the functions $f \circ g$, $g \circ f$, $f \circ f$ and $g \circ g$ and their implied domains. The answer key has not been proofread, use with caution.

(a) $f(x) = x^2 + 1$, $g(x) = x + 1$.

answer: Domain, all 4 cases: $x \in \mathbb{R}$ (all reals)
in some order: $(1+x)^2 + 1$, $(x)^2 + 2$, $((x)^2 + 1)^2 + 1$, $2 + x$

(b) $f(x) = \sqrt{x+1}$, $g(x) = x + 1$.

answer: Domain of $f \circ g$ is $x \geq -2$. Domain of $g \circ f$ is $x \geq -1$
Domain of $f \circ f$ is $x \geq -1$. Domain of $g \circ g$ is all reals ($x \in \mathbb{R}$).
in some order: $\sqrt{2+x}$, $1 + \sqrt{1+x}$, $\sqrt{1 + \sqrt{1+x}}$, $2 + x$

(c) $f(x) = 2x$, $g(x) = \tan x$.

In this subproblem, you are not required to find the domain.

answer: Domain $f \circ f$: all reals ($x \in \mathbb{R}$). Domain $g \circ f$: $x \neq (2k+1)\frac{\pi}{2}$ for all $k \in \mathbb{Z}$
Domain $f \circ g$: $x \neq (4k+1)\frac{\pi}{4}$, $x \neq (4k+3)\frac{\pi}{4}$ for all $k \in \mathbb{Z}$
Domain $g \circ g$: $x \neq (2k+1)\frac{\pi}{2}$ and $x \neq k\pi + \arctan\left(\frac{\pi}{2}\right)$ for all $k \in \mathbb{Z}$
in some order:$2\tan x$, $\tan(2x)$, $4x$, $\tan(\tan x)$

(d) $f(x) = \dfrac{x+1}{x-1}$, $g(x) = \dfrac{x-1}{x+1}$.

answer: Domain $f \circ f$: $x \neq 1$. Domain $g \circ g$: $x \neq 0, x \neq -1$
Domain $f \circ g$: $x \neq -1$. Domain $g \circ f$: $x \neq 0, x \neq 1$
in some order: $-x$, $\frac{1}{x}$, x, $-\frac{1}{x}$