Calculus I
Lecture 22
The Substitution Rule

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Outline

1. The Substitution Rule
   - Substitution rule and definite Integrals

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The Substitution Rule

- How do we integrate $\int 2x\sqrt{1+x^2}\,\mathrm{d}x$?

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- Substitute into the integral:

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$$\int 2x\sqrt{1+x^2}\,\mathrm{d}x = \int \sqrt{u}\,\mathrm{d}u = \int u^{\frac{1}{2}}\mathrm{d}u = \frac{2}{3}u^{\frac{3}{2}} + C$$

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- Is this procedure justified?

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- Is this procedure justified?
- Take the derivative

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{2}{3}\left(1+x^2\right)^{\frac{3}{2}} + C\right)$$

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Theorem (The Substitution Rule)

Let $u = g(x)$ be a differentiable function whose range is an interval I and let f be a function continuous on I. Then

$$\int f(g(x))g'(x)\,\mathrm{d}x = \int f(u)\,\mathrm{d}u$$

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Let $u = g(x)$ be a differentiable function whose range is an interval I and let f be a function continuous on I. Then

$$\int f(g(x))g'(x)\,\mathrm{d}x = \int f(u)\,\mathrm{d}u$$

This is the integration counterpart of the Chain Rule.

Example (Substitution Rule)

Find $$\int x^3 \cos(x^4 + 3) \mathrm{d}x.$$

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Let $u = \mathbf{?}$

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Example (Substitution Rule)

Find $\displaystyle\int x^3 \cos(x^4+3)\mathrm{d}x.$

Let $u = x^4 + 3.$

Then $\mathrm{d}u = 4x^3\mathrm{d}x$

Example (Substitution Rule)

Find $\displaystyle\int x^3 \cos(x^4 + 3)\mathrm{d}x$.

$$\text{Let} \quad u = x^4 + 3.$$

$$\text{Then} \quad \mathrm{d}u = 4x^3\mathrm{d}x$$

$$x^3\mathrm{d}x = \textbf{?}$$

Example (Substitution Rule)

Find $\displaystyle\int x^3 \cos(x^4+3)\mathrm{d}x.$

$$\begin{aligned} \text{Let} \quad u &= x^4+3. \\ \text{Then} \quad \mathrm{d}u &= 4x^3\mathrm{d}x \\ x^3\mathrm{d}x &= \frac{1}{4}\mathrm{d}u. \end{aligned}$$

Example (Substitution Rule)

Find $\displaystyle\int x^3\cos(x^4+3)\mathrm{d}x.$

$$\text{Let}\quad u = x^4 + 3.$$

$$\text{Then}\quad \mathrm{d}u = 4x^3\mathrm{d}x$$

$$x^3\mathrm{d}x = \frac{1}{4}\mathrm{d}u.$$

Substitute: $\displaystyle\int x^3\cos(x^4+3)\mathrm{d}x = \int \quad \cos u\,\mathbf{?}$

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Find $\displaystyle\int x^3 \cos(x^4+3)\mathrm{d}x.$

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\text{Let}\quad u &= x^4+3.\\
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x^3\mathrm{d}x &= \frac{1}{4}\mathrm{d}u.
\end{aligned}$$

$$\text{Substitute:}\quad \int x^3\cos(x^4+3)\mathrm{d}x = \int \frac{1}{4}\cos u\,\mathrm{d}u$$

$$= \textbf{?}$$

Example (Substitution Rule)

Find $\displaystyle\int x^3\cos(x^4+3)\mathrm{d}x.$

$$
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\text{Then}\quad \mathrm{d}u &= 4x^3\mathrm{d}x\\
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\end{aligned}
$$

$$
\text{Substitute:}\quad \int x^3\cos(x^4+3)\mathrm{d}x = \int \frac{1}{4}\cos u\,\mathrm{d}u = \frac{1}{4}\sin u
$$

Example (Substitution Rule)

Find $\displaystyle\int x^3\cos(x^4+3)\mathrm{d}x.$

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$$\text{Substitute:}\quad \int x^3\cos(x^4+3)\mathrm{d}x = \int\frac{1}{4}\cos u\,\mathrm{d}u = \frac{1}{4}\sin u + C$$

Example (Substitution Rule)

Find $\displaystyle\int x^3\cos(x^4+3)\mathrm{d}x.$

$$\begin{aligned}
\text{Let}\quad u &= x^4+3.\\
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$$\text{Substitute:}\quad \int x^3\cos(x^4+3)\mathrm{d}x = \int\frac{1}{4}\cos u\,\mathrm{d}u$$

$$= \frac{1}{4}\sin u + C$$

$$= \frac{1}{4}\sin(x^4+3) + C.$$

Example (Substitution Rule)

Find $\displaystyle\int \sqrt{2x+1}\mathrm{d}x.$

Example (Substitution Rule)

$$\text{Find} \quad \int \sqrt{2x+1}\,dx.$$

$$\text{Let} \quad u = \textbf{?}$$

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Find $$\int \sqrt{2x+1}\mathrm{d}x.$$

Let $u = 2x + 1.$

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Find $\displaystyle\int \sqrt{2x+1}\,\mathrm{d}x.$

Let $u = 2x + 1.$

Then $\mathrm{d}u = \mathbf{?}$

Example (Substitution Rule)

$$\text{Find} \quad \int \sqrt{2x+1}\,\mathrm{d}x.$$

$$\text{Let} \quad u = 2x + 1.$$

$$\text{Then} \quad \mathrm{d}u = 2\mathrm{d}x$$

Example (Substitution Rule)

Find $\displaystyle\int \sqrt{2x+1}\,\mathrm{d}x.$

$$\text{Let}\quad u = 2x+1.$$
$$\text{Then}\quad \mathrm{d}u = 2\mathrm{d}x$$
$$\mathrm{d}x = \textbf{?}$$

Example (Substitution Rule)

Find $\displaystyle\int \sqrt{2x+1}\,\mathrm{d}x.$

$$\text{Let}\quad u = 2x + 1.$$

$$\text{Then}\quad \mathrm{d}u = 2\mathrm{d}x$$

$$\mathrm{d}x = \frac{1}{2}\mathrm{d}u.$$

Example (Substitution Rule)

Find $\displaystyle\int \sqrt{2x+1}\,\mathrm{d}x.$

Let $u = 2x + 1.$

Then $\mathrm{d}u = 2\mathrm{d}x$

$\mathrm{d}x = \frac{1}{2}\mathrm{d}u.$

Substitute: $\displaystyle\int \sqrt{2x+1}\,\mathrm{d}x = \int \sqrt{u}$ **?**

Example (Substitution Rule)

Find $\displaystyle\int \sqrt{2x+1}\,\mathrm{d}x.$

$$\begin{aligned} \text{Let} \quad u &= 2x+1. \\ \text{Then} \quad \mathrm{d}u &= 2\mathrm{d}x \\ \mathrm{d}x &= \frac{1}{2}\mathrm{d}u. \end{aligned}$$

Substitute: $\displaystyle\int \sqrt{2x+1}\,\mathrm{d}x = \int \sqrt{u}\,\mathbf{?}$

Example (Substitution Rule)

$$\text{Find} \quad \int \sqrt{2x+1}\,\mathrm{d}x.$$

$$\text{Let} \quad u = 2x+1.$$

$$\text{Then} \quad \mathrm{d}u = 2\mathrm{d}x$$

$$\mathrm{d}x = \frac{1}{2}\mathrm{d}u.$$

$$\text{Substitute:} \quad \int \sqrt{2x+1}\,\mathrm{d}x = \int \frac{1}{2}\sqrt{u}\,\mathrm{d}u$$

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Find $\displaystyle\int \sqrt{2x+1}\,\mathrm{d}x.$

$$\begin{aligned} \text{Let} \quad u &= 2x+1. \\ \text{Then} \quad \mathrm{d}u &= 2\mathrm{d}x \\ \mathrm{d}x &= \frac{1}{2}\mathrm{d}u. \end{aligned}$$

Substitute: $\displaystyle\int \sqrt{2x+1}\,\mathrm{d}x = \int \frac{1}{2}\sqrt{u}\,\mathrm{d}u$

$$= \textbf{?}$$

Example (Substitution Rule)

Find $\displaystyle\int \sqrt{2x+1}\mathrm{d}x.$

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$$\text{Substitute:} \quad \int \sqrt{2x+1}\mathrm{d}x = \int \frac{1}{2}\sqrt{u}\mathrm{d}u = \frac{1}{2} \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}}$$

Example (Substitution Rule)

Find $\displaystyle\int \sqrt{2x+1}\mathrm{d}x.$

$$\begin{aligned} \text{Let} \quad u &= 2x+1. \\ \text{Then} \quad \mathrm{d}u &= 2\mathrm{d}x \\ \mathrm{d}x &= \frac{1}{2}\mathrm{d}u. \end{aligned}$$

$$\text{Substitute:} \quad \int \sqrt{2x+1}\mathrm{d}x = \int \frac{1}{2}\sqrt{u}\mathrm{d}u$$

$$= \frac{1}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

Example (Substitution Rule)

Find $\displaystyle\int \sqrt{2x+1}\,\mathrm{d}x.$

$$\text{Let}\quad u = 2x+1.$$

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\text{Substitute:}\quad \int \sqrt{2x+1}\,\mathrm{d}x &= \int \frac{1}{2}\sqrt{u}\,\mathrm{d}u\\
&= \frac{1}{\cancel{2}}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{\cancel{2}}}+C\\
&= \frac{1}{3}(2x+1)^{\frac{3}{2}}+C.
\end{aligned}$$

Example (Substitution Rule)

Find $\displaystyle\int \frac{x}{\sqrt{3-4x^2}} \mathrm{d}x.$

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Example (Substitution Rule)

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Let $u = 3 - 4x^2$.

Example (Substitution Rule)

$$\text{Find} \quad \int \frac{x}{\sqrt{3-4x^2}} \mathrm{d}x.$$

$$\text{Let} \quad u = 3 - 4x^2.$$

$$\text{Then} \quad \mathrm{d}u = \mathbf{?}$$

Example (Substitution Rule)

Find $\displaystyle\int \frac{x}{\sqrt{3-4x^2}}\mathrm{d}x.$

Let $u = 3 - 4x^2.$

Then $\mathrm{d}u = -8x\mathrm{d}x$

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$$\text{Substitute:} \quad \int \frac{x}{\sqrt{3-4x^2}} \mathrm{d}x = \int \left(-\frac{1}{8}\right) \frac{1}{\sqrt{u}} \mathrm{d}u$$

$$= \mathbf{?}$$

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\end{aligned}
$$

$$
\text{Substitute:}\quad \int \frac{x}{\sqrt{3-4x^2}}\mathrm{d}x = \int \left(-\frac{1}{8}\right)\frac{1}{\sqrt{u}}\mathrm{d}u = -\frac{1}{8}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}}
$$

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\text{Substitute:} \quad \int \frac{x}{\sqrt{3-4x^2}}\mathrm{d}x &= \int \left(-\frac{1}{8}\right)\frac{1}{\sqrt{u}}\mathrm{d}u\\
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&= -\frac{1}{4}\sqrt{3-4x^2} + C.
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Example (Substitution Rule)

Find $\displaystyle\int e^{3x} \mathrm{d}x.$

Example (Substitution Rule)

$$\text{Find} \quad \int e^{3x} \mathrm{d}x.$$

$$\text{Let} \quad u = \textbf{?}$$

Example (Substitution Rule)

$$\text{Find} \quad \int e^{3x} dx.$$

$$\text{Let} \quad u = 3x.$$

Example (Substitution Rule)

$$\text{Find} \quad \int e^{3x} \mathrm{d}x.$$

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Example (Substitution Rule)

Find $\displaystyle\int e^{3x} \mathrm{d}x.$

Let $u = 3x.$

Then $\mathrm{d}u = 3\mathrm{d}x$

Example (Substitution Rule)

Find $\displaystyle\int e^{3x} \mathrm{d}x.$

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$$\text{Substitute:} \quad \int e^{3x} \mathrm{d}x = \int \quad e^{u} \textbf{?}$$

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Example (Substitution Rule, more factors)

Evaluate $\displaystyle\int 3x^5\sqrt{1+x^3}\mathrm{d}x$.

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Evaluate $\int 3x^5\sqrt{1+x^3}\mathrm{d}x = \int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x.$

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\begin{aligned}
\text{Let } u &= 1+x^3.\\
\text{Then } \mathrm{d}u &= 3x^2\mathrm{d}x.\\
x^3 &= u-1.\\
\int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x &= \int (u-1)\sqrt{u}\,\mathrm{d}u\\
&= \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u
\end{aligned}
$$

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$$
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\text{Evaluate } \int 3x^5\sqrt{1+x^3}\mathrm{d}x &= \int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x.\\
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&= \left(\mathbf{?} \quad - \mathbf{?} \quad\right)
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&= \left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right)
\end{aligned}$$

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&= \left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right) + C
\end{aligned}$$

Example (Substitution Rule, more factors)

$$\text{Evaluate } \int 3x^5\sqrt{1+x^3}\mathrm{d}x = \int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x.$$

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&= \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right)\mathrm{d}u \\
&= \left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right) + C \\
&= \frac{2}{5}\left(1+x^3\right)^{\frac{5}{2}} - \frac{2}{3}\left(1+x^3\right)^{\frac{3}{2}} + C.
\end{aligned}$$

Definite Integrals

There are two ways to find a definite integral with the Substitution Rule:

1. First evaluate the indefinite integral, then use the FTC.

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&= \frac{1}{3}(9)^{3/2} - \frac{1}{3}(1)^{3/2} = \frac{1}{3}(27-1) = \frac{26}{3}
\end{aligned}
$$

2. Change the limits of integration when the variable is changed.

Definite Integrals

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$$\int_0^4 \sqrt{2x+1}\,\mathrm{d}x = \left[\int \sqrt{2x+1}\,\mathrm{d}x\right]_0^4 = \left[\frac{1}{3}(2x+1)^{3/2}\right]_0^4$$
$$= \frac{1}{3}(2\cdot 4+1)^{3/2} - \frac{1}{3}(2\cdot 0+1)^{3/2}$$
$$= \frac{1}{3}(9)^{3/2} - \frac{1}{3}(1)^{3/2} = \frac{1}{3}(27-1) = \frac{26}{3}$$

2. Change the limits of integration when the variable is changed.

Theorem (The Substitution Rule for Definite Integrals)

If g' is continuous on $[a, b]$ and f is continuous on the range of g, then letting $u = g(x)$ we get

$$\int_a^b f(g(x))g'(x)\,\mathrm{d}x = \int_{g(a)}^{g(b)} f(u)\,\mathrm{d}u$$

Example

Find $\displaystyle\int_0^4 \sqrt{2x+1}\,\mathrm{d}x$.

Example

Find $\int_0^4 \sqrt{2x+1}\, dx$.

- Let $u =$ **?** .
- Then $du =$ **?** .

Example

Find $\int_0^4 \sqrt{2x+1}\, \mathrm{d}x$.

- Let $u = \textbf{?}$.
- Then $\mathrm{d}u = \textbf{?}$.

Example

Find $\int_0^4 \sqrt{2x+1}\,\mathrm{d}x$.

- Let $u = 2x + 1$.
- Then $\mathrm{d}u =$ **?** .

Example

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- Let $u = 2x + 1$.
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Example

Find $\int_0^4 \sqrt{2x+1}\,\mathrm{d}x$.

- Let $u = 2x + 1$.
- Then $\mathrm{d}u = 2\mathrm{d}x$.

Example

Find $\int_0^4 \sqrt{2x+1}\, \mathrm{d}x$.

- Let $u = 2x + 1$.
- Then $\mathrm{d}u = 2\mathrm{d}x$.
- Therefore $\mathrm{d}x = \mathbf{?}$.

Example

Find $\int_0^4 \sqrt{2x+1}\,\mathrm{d}x$.

- Let $u = 2x + 1$.
- Then $\mathrm{d}u = 2\mathrm{d}x$.
- Therefore $\mathrm{d}x = \frac{1}{2}\mathrm{d}u$.

Example

Find $\displaystyle\int_0^4 \sqrt{2x+1}\, \mathrm{d}x$.

- Let $u = 2x + 1$.
- Then $\mathrm{d}u = 2\mathrm{d}x$.
- Therefore $\mathrm{d}x = \frac{1}{2}\mathrm{d}u$.
- When $x = 0$, $u = \mathbf{?}$.
- When $x = 4$, $u = \mathbf{?}$.

Example

Find $\int_0^4 \sqrt{2x+1}\, \mathrm{d}x$.

- Let $u = 2x + 1$.
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$$\int\limits_{x=0}^{x=4} \sqrt{2x+1}\,\mathrm{d}x = \int \quad \sqrt{}$$

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$$\int_{x=0}^{x=4} \sqrt{2x+1}\,\mathrm{d}x = \int \quad \sqrt{u}$$

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$$\int_{x=0}^{x=4} \sqrt{2x+1}\,\mathrm{d}x = \int_{u=1}^{u=9} \frac{1}{2}\sqrt{u}\,\mathrm{d}u$$

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$$\int\limits_{x=0}^{x=4} \sqrt{2x+1}\,\mathrm{d}x = \int\limits_{u=1}^{u=9} \frac{1}{2}\sqrt{u}\,\mathrm{d}u = \int_1^9 \frac{1}{2}u^{\frac{1}{2}}\mathrm{d}u$$

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Find $\int_0^4 \sqrt{2x+1}\,\mathrm{d}x$.

- Let $u = 2x+1$.
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$$= \left[\frac{1}{2} \cdot \textbf{?} \qquad \right]_1^9$$

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$$= \left[\frac{1}{2}\cdot\frac{2}{3}(u)^{\frac{3}{2}}\right]_1^9$$

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$$= \frac{1}{3}(9)^{\frac{3}{2}} - \frac{1}{3}(1)^{\frac{3}{2}}$$

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Example

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$$= \frac{1}{3}(9)^{\frac{3}{2}} - \frac{1}{3}(1)^{\frac{3}{2}} = \frac{1}{3}(27-1) = \frac{26}{3}$$

Example

Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

Example

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- Let $u =$ **?** .
- Then $\mathrm{d}u =$ **?** .

Example

Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

- Let $u = \mathbf{?}$.
- Then $\mathrm{d}u = \mathbf{?}$.

Example

Find $\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = \textbf{?}$.

Example

Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $\mathrm{d}u = \textbf{?}$.

Example

Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $\mathrm{d}u = -3\,\mathrm{d}x$.

Example

Find $\displaystyle\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = \ -3\, dx$.
- Therefore $dx = $ **?** .

Example

Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $\mathrm{d}u = \ -3\ \mathrm{d}x$.
- Therefore $\mathrm{d}x = -\frac{1}{3}\mathrm{d}u$.

Example

Find $\int_{1}^{2} \frac{\mathrm{d}x}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $\mathrm{d}u = \ -3\ \mathrm{d}x$.
- Therefore $\mathrm{d}x = -\frac{1}{3}\mathrm{d}u$.
- When $x = 1$, $u = \textbf{?}$.
- When $x = 2$, $u = \textbf{?}$.

Example

Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $\mathrm{d}u = \ -3\ \mathrm{d}x$.
- Therefore $\mathrm{d}x = -\frac{1}{3}\mathrm{d}u$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u =$ **?** .

Example

Find $\displaystyle\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = -3\, dx$.
- Therefore $dx = -\frac{1}{3}du$.
- When $x = 1$, $u = -1$.
- When $x = 2$, $u = \textbf{?}$.

Example

Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

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- Then $\mathrm{d}u = -3\,\mathrm{d}x$.
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- When $x = 2$, $u = -4$.

Example

Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $\mathrm{d}u = \; -3\,\mathrm{d}x$.
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$$\int_{x=1}^{x=2} \frac{\mathrm{d}x}{(2-3x)^2} = \qquad \int \qquad \text{—}$$

Example

Find $\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

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- When $x = 2$, $u = \; -4$.

$$\int_{x=1}^{x=2} \frac{\mathrm{d}x}{(2-3x)^2} = -\frac{1}{3}\int \qquad \frac{\mathrm{d}u}{u^2}$$

Example

Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

- Let $u = 2-3x$.
- Then $\mathrm{d}u = \ -3\,\mathrm{d}x$.
- Therefore $\mathrm{d}x = -\frac{1}{3}\mathrm{d}u$.
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$$\int_{x=1}^{x=2} \frac{\mathrm{d}x}{(2-3x)^2} = -\frac{1}{3}\int_{u=-1} \frac{\mathrm{d}u}{u^2}$$

Example

Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

- Let $u = 2-3x$.
- Then $\mathrm{d}u = -3\,\mathrm{d}x$.
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$$\int_{x=1}^{x=2} \frac{\mathrm{d}x}{(2-3x)^2} = -\frac{1}{3}\int_{u=-1}^{u=-4} \frac{\mathrm{d}u}{u^2}$$

Example

Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $\mathrm{d}u = -3\,\mathrm{d}x$.
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$$\int_{x=1}^{x=2} \frac{\mathrm{d}x}{(2-3x)^2} = -\frac{1}{3}\int_{u=-1}^{u=-4} \frac{\mathrm{d}u}{u^2} = -\frac{1}{3}\int_{-1}^{-4} u^{-2}\mathrm{d}u$$

Example

Find $\displaystyle\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = \ -3\,dx$.
- Therefore $dx = -\frac{1}{3}du$.
- When $x = 1$, $u = -1$.
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$$\int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} = -\frac{1}{3}\int_{u=-1}^{u=-4} \frac{du}{u^2} = -\frac{1}{3}\int_{-1}^{-4} u^{-2}du$$

$$= -\frac{1}{3}\cdot\Big[\ ?\ \Big]_{-1}^{-4}$$

Example

Find $\displaystyle\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
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Example

Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

- Let $u = 2-3x$.
- Then $\mathrm{d}u = -3\,\mathrm{d}x$.
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$$= -\frac{1}{3}\cdot\left[-\frac{1}{u}\right]_{-1}^{-4} = \frac{1}{3}\left[\frac{1}{u}\right]_{-1}^{-4}$$
$$= \frac{1}{3}\left(\frac{1}{-4}-\frac{1}{-1}\right)$$

Example

Find $\displaystyle\int_1^2 \frac{dx}{(2-3x)^2}$.

- Let $u = 2 - 3x$.
- Then $du = \;-3\,dx$.
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- When $x = 1$, $u = -1$.
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$$\int_{x=1}^{x=2} \frac{dx}{(2-3x)^2} = -\frac{1}{3}\int_{u=-1}^{u=-4} \frac{du}{u^2} = -\frac{1}{3}\int_{-1}^{-4} u^{-2}du$$

$$= -\frac{1}{3}\cdot\left[-\frac{1}{u}\right]_{-1}^{-4} = \frac{1}{3}\left[\frac{1}{u}\right]_{-1}^{-4}$$

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Example

Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

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Example

Find $\displaystyle\int_1^2 \frac{\mathrm{d}x}{(2-3x)^2}$.

- Let $u = 2-3x$.
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