

Calculus I

Homework Exponents, Logarithms

Lecture 7

1. **The problem is too easy to appear on a quiz or test.** Find the exact value of each expression.

(a) $\log_5 125$.

(g) $\log_{1.5} 2.25$.

(b) $\log_3 \frac{1}{27}$.

(h) $\log_5 4 - \log_5 500$.

(c) $\ln \left(\frac{1}{e} \right)$.

(i) $\log_2 6 - \log_2 15 + \log_2 20$.

(d) $\log_{10} \sqrt{10}$.

(j) $\log_3 100 - \log_3 18 - \log_3 50$.

(e) $e^{\ln 4.5}$.

(k) $e^{-2 \ln 5}$.

(f) $\log_{10} 0.0001$.

(l) $\ln \left(\ln e^{e^{10}} \right)$.

2. **The problem is too easy to appear on a quiz or test.** Use the definition of a logarithm to evaluate each of the following without using a calculator.

(a) $\log_2 16$

(b) $\log_3 \left(\frac{1}{9} \right)$

(c) $\log_{10} 1000$

(d) $\log_6 36^{-\frac{2}{3}}$

(e) $\log_2 (8\sqrt{2})$

(f) $\log_7 \left(\frac{49^x}{343^y} \right)$

Solution.

$$\begin{aligned} \log_7 \left(\frac{49^x}{343^y} \right) &= \log_7 49^x - \log_7 343^y \\ &= x \log_7 49 - y \log_7 343 \end{aligned}$$

$$\text{But } 49 = 7^2 \text{ and } 343 = 7^3, \text{ therefore } \log_7 \left(\frac{49^x}{343^y} \right) = 2x - 3y.$$

3. **The problem is too easy to appear on a quiz or test.** Express each of the following as a single logarithm.

(a) $\ln 4 + \ln 6 - \ln 5$

(b) $2 \ln 2 - 3 \ln 3 + 4 \ln 4$

Solution.

$$\begin{aligned}
 2 \ln 2 - 3 \ln 3 + 4 \ln 4 &= \ln 2^2 - \ln 3^3 + \ln 4^4 \\
 &= \ln 4 - \ln 27 + \ln 256 \\
 &= \ln \left(\frac{4}{27} \right) + \ln 256 \\
 &= \ln \left(\frac{4 \cdot 256}{27} \right) \\
 &= \ln \left(\frac{1024}{27} \right).
 \end{aligned}$$

(c) $\ln 36 - 2 \ln 3 - 3 \ln 2$

4. Solve each equation for x . If available, use a calculator to give an (\approx) answer in decimal notation. If available, use a calculator to verify your approximate solutions.

(a) $e^{7-4x} = 7$.

(j) $\ln(\ln x) = 1$.

(b) $\ln(2x - 9) = 2$.

(k) $e^{e^x} = 10$.

(c) $\ln(x^2 - 2) = 3$.

(l) $\ln(2x + 1) = 3 - \ln x$.

(d) $2^{x-3} = 5$.

(m) $e^{2x} - 4e^x + 3 = 0$.

(e) $\ln x + \ln(x - 1) = 1$.

(n) $e^{4x} + 3e^{2x} - 4 = 0$.

(f) $e^{2x+1} = t$.

(o) $e^{2x} - e^x - 6 = 0$.

(g) $\log_2(mx) = c$.

(p) $4^{3x} - 2^{3x+2} - 5 = 0$.

(h) $e - e^{-2x} = 1$.

(q) $3 \cdot 2^x + 2 \left(\frac{1}{2} \right)^{x-1} - 7 = 0$.

(i) $8(1 + e^{-x})^{-1} = 3$.

Solution. 4.d

$2^{x-3} = 5$	take \log_2 add 3 to both sides answer is complete optional step: convert to ln calculator
$x - 3 = \log_2(5)$	
$x = \log_2(5) + 3$	
$= \frac{\ln 5}{\ln 2} + 3$	
≈ 5.321928095	

Solution. 4.h

$e - e^{-2x} = 1$	apply \ln
$e^{-2x} = e - 1$	
$\ln e^{-2x} = \ln(e - 1)$	
$-2x = \ln(e - 1)$	
$x = -\frac{1}{2} \ln(e - 1)$	calculator
≈ -0.270662427	

Solution. 4.e

$$\begin{aligned}
 \ln x + \ln(x-1) &= 1 \\
 \ln(x^2 - x) &= 1 \\
 e^{\ln(x^2 - x)} &= e^1 \\
 x^2 - x &= e \\
 x^2 - x - e &= 0 \\
 \text{Quadratic formula: } x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-e)}}{2(1)} \\
 &= \frac{1 \pm \sqrt{1+4e}}{2}.
 \end{aligned}$$

However $\frac{1-\sqrt{1+4e}}{2}$ is negative, so $\ln\left(\frac{1-\sqrt{1+4e}}{2}\right)$ is undefined. Hence the only solution is $x = \frac{1+\sqrt{1+4e}}{2} \approx 2.2229$.

Solution. 4.p

$$\begin{aligned}
 4^{3x} - 2^{3x+2} - 5 &= 0 \\
 4^{3x} - 4 \cdot 2^{3x} - 5 &= 0 & \left| \begin{array}{l} \text{Set } 2^{3x} = u \\ 4^{3x} = u^2 \end{array} \right. \\
 u^2 - 4u - 5 &= 0 \\
 (u-5)(u+1) &= 0 \\
 u = 5 &\text{ or } u = -1 \\
 2^{3x} = 5 & \quad 2^{3x} = -1 \\
 3x = \log_2(5) & \quad \text{no real solution} \\
 x = \frac{\log_2 5}{3} \\
 \text{Calculator: } x &\approx 0.773976
 \end{aligned}$$

Solution. 4.q

$$\begin{aligned}
 3 \cdot 2^x + 2 \left(\frac{1}{2}\right)^{x-1} - 7 &= 0 \\
 3 \cdot 2^x + 2 \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{-1} - 7 &= 0 \\
 3 \cdot 2^x + 4 \left(\frac{1}{2}\right)^x - 7 &= 0 & \left| \begin{array}{l} \text{Set } 2^x = u \\ \text{Multiply by } u \end{array} \right. \\
 3u + \frac{4}{u} - 7 &= 0 \\
 3u^2 - 7u + 4 &= 0 \\
 (u-1)(3u-4) &= 0 \\
 u = 1 &\text{ or } 3u - 4 = 0 \\
 2^x = 1 & \quad u = \frac{4}{3} \\
 x = 0 & \quad 2^x = \frac{4}{3} \\
 & \quad x = \log_2 \frac{4}{3} = \log_2 4 - \log_2 3 \\
 & \quad x = 2 - \log_2 3 \\
 \text{Calculator: } & x \approx 0.415037
 \end{aligned}$$