Calculus III
Homework on Lecture 4

1. Write vectorial and scalar equations of the line L passing through the given point and with the given direction.
   (a) $P_0 = (1, 2, 3)$, $\mathbf{u} = (-3, -2, -1)$.
   (b) $P_0 = (3, 5, 7)$, $\mathbf{u} = (2, 3, 4)$.

2. Write vectorial and scalar equations of the line passing L through the given points.
   (a) $(2, 3, 5)$ and $(3, 5, 7)$.
   (b) $(-1, -1, 1)$ and $(-1, 1, -1)$.

3. We recall that the 8 points $(1, 1, 1), (-1, 1, 1), (1, -1, 1), (-1, -1, 1), (1, 1, -1), (-1, 1, -1), (1, -1, -1), (-1, -1, -1)$ (all possible sign combinations) give the vertices of a cube with edge 2 units.

   Find equations for all lines connecting two vertices in the cube above that pass through the origin (how many connecting two vertices of a cube are there? How many of them are edges?).

4. Find an equation of the plane $\mathcal{P}$ passing through the given point and with the given normal. Find parametric vectorial equations of the plane.
   (a) $P_0(1, 2, 3)$, $\mathbf{n} = (4, 5, 6)$.
   (b) $P_0(2, 3, 5)$, $\mathbf{n} = (-3, -5, -7)$.
   (c) $P_0(1, 1, 1)$, $\mathbf{n} = (1, 1, 1)$.

5. Find an equation of plane $\mathcal{P}$ passing through the point and parallel to the given directions.
   (a) $P_0(1, 2, 3)$, $\mathbf{u} = (2, 3, 5)$, $\mathbf{v} = (3, 5, 7)$.
   (b) $P_0(1, 1, 1)$, $\mathbf{u} = (1, -1, 0)$, $\mathbf{v} = (0, 1, -1)$.

6. Find an equation of the plane $\mathcal{P}$ passing through the given points.
   (a) $P_0(2, 3, 5)$, $P_1(3, 5, 7)$, $P_2(5, 7, 11)$.
   (b) $P_0(1, 1, 1)$, $P_1(1, -1, -1)$, $P_2(-1, -1, 1)$.

7. Find the distance between the line and the point.
   (a) The line passing through $P_0(1, 1, 1)$ and $P_1(-1, -1, -1)$ and the point $Q(1, 0, 0)$.
   (b) The line passing through $P_0(-2, 3, -5)$ and $P_1(3, 4, 5)$ and the point $Q(2, -2, 2)$.

8. Find the distance between the plane and the point.
   (a) The plane passing through $P_0(1, 2, 3)$, $P_1(2, 3, 5)$ and $P_2(3, 5, 7)$ and the point $Q(2, -2, 2)$.
   (b) The plane passing through $P_0(1, 2, 3)$, $P_1(2, 3, 5)$ and $P_2(3, 5, 7)$ and the point $Q(5, 7, 11)$.
   (c) The plane passing through the points $P_0(1, 1, 1)$, $P_1(1, -1, -1)$, $P_2(-1, -1, 1)$ and the point $Q(-1, 1, -1)$.

9. Recall that a regular tetrahedron can be realized using 4 vertices of a cube.
   (a) In a regular tetrahedron, find the angle between two edges that share a common vertex.
   (b) In a regular tetrahedron, find the angle between two edges that share a common vertex.

10. Recall that a regular tetrahedron can be realized using 4 vertices of a cube.

    Find the distance between two opposite edges of a regular tetrahedron inscribed in a 2x2x2 cm cube.

11. Find the distance between the lines.

    (a) The line passing through $Q_0(1, 2, 3)$ and $Q_1(6, 5, 4)$ and the line passing through $P_0(1, 3, 5)$ and $P_1(2, 4, 6)$.
    (b) The line passing through $Q_0(1, 2, 3)$ and $Q_1(2, 3, 5)$ and the line passing through $P_0(3, 5, 7)$ and $P_1(5, 7, 11)$.
    (c) The line passing through $Q_0(1, 1, 1)$ and $Q_1(-1, -1, -1)$ and the line passing through $P_0(1, -1, -1)$ and $P_1(-1, 1, -1)$.
    (d) The line passing through $(1, 3, 4)$ and $(2, 3, 1)$ and the line passing through $(1, 2, 2)$ and $(0, 2, 5)$.
    (e) The line passing through $(1, 3, 4)$ and $(2, 3, 1)$ and the line passing through $(1, 2, 2)$ and $(0, 2, 4)$.

12. Find the angle between the line and the plane.

    (a) The line passing through $(-1, -1, -1)$ and $(1, 1, 1)$ and the plane with equation $z = -1$.
    (b) The line passing through $(2, 3, 5)$ and $(3, 5, 7)$ and the plane passing through $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.

13. Recall that a regular tetrahedron can be realized using 4 vertices of a cube. Find the angle between an edge of a regular tetrahedron and one of the two sides of the tetrahedron not containing the edge.

14. Recall that a regular tetrahedron can be realized using 4 vertices of a cube.

    Find the angle between two faces of a regular tetrahedron.