Precalculus
Lecture 9
Laws of Sines and Cosines

Todor Milev
https://github.com/tmilev/freecalc

2020

Outline

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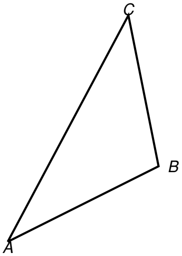
Triangle area = $\frac{1}{2}$base · height

Proposition (Triangle area)

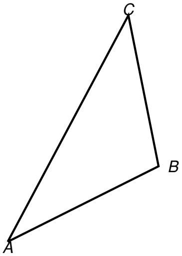
$$Area(\triangle ABC) = \textbf{?} \quad .$$



Triangle area = $\frac{1}{2}$base · height

Proposition (Triangle area)

$$Area(\triangle ABC) = \frac{1}{2} height \cdot base \quad .$$



Triangle area = $\frac{1}{2}$base · height

Let $\triangle ABC$ have side length a and height length h_a, as indicated - side a is opposite to vertex A and h_a starts at A

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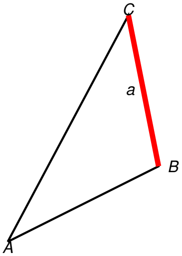
$$Area(\triangle ABC) = \frac{1}{2} height \cdot base = \frac{1}{2} h_a a \quad .$$



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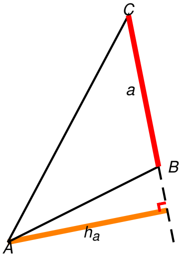

Triangle area = $\frac{1}{2}$base · height

Let $\triangle ABC$ have side lengths a, b and height lengths h_a, h_b, as indicated - side a is opposite to vertex A and h_a starts at A, and so on.

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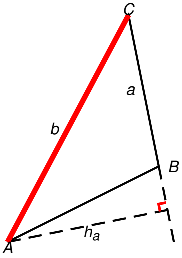
$$Area(\triangle ABC) = \frac{1}{2} height \cdot base = \frac{1}{2} h_a a = \frac{1}{2} h_b b .$$



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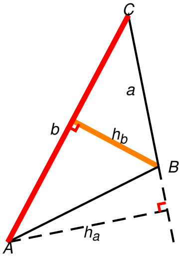

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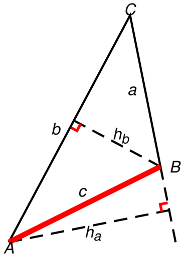

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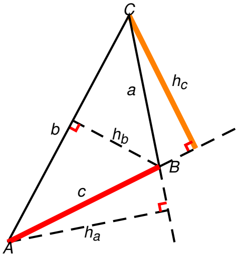


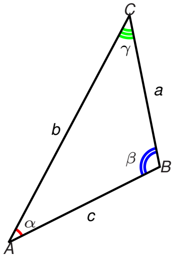
Triangle area from two sides and angle between them

Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ, as indicated: α is opposite to a, β is opposite to b, γ is opposite to c.

Proposition ($\triangle$ area from two sides and angle between them)

The area of a triangle is half the product of the lengths of two of its sides times the sine of the angle between them. In other words,

$$Area(\triangle ABC) = \frac{ab\sin\gamma}{2} = \frac{bc\sin\alpha}{2} = \frac{ca\sin\beta}{2}$$



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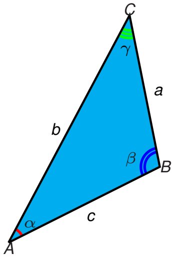

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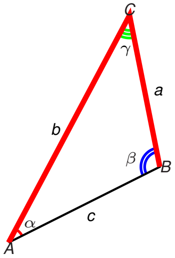

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Let $\triangle ABC$ have sides lengths *a*, *b*, *c* angles α, β, γ, as indicated: α is opposite to *a*, β is opposite to *b*, γ is opposite to *c*.

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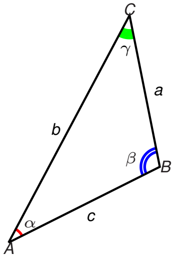

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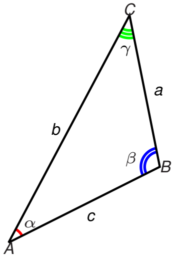

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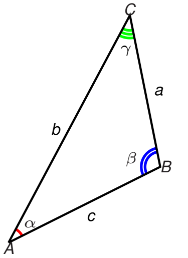

Proof.

$$Area(\triangle ABC) = \frac{\text{base} \cdot \text{height}}{2}$$

$\square$

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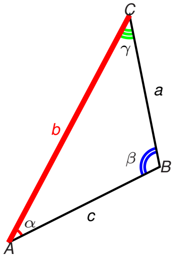

Proof.

$$\mathrm{Area}(\triangle ABC) = \frac{\text{base}\cdot\text{height}}{2} = \frac{bh_b}{2}$$

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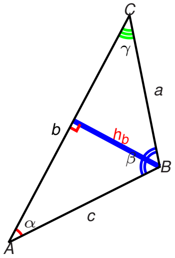

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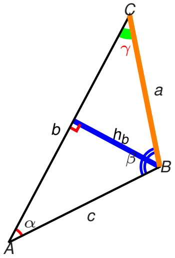

Proof.

$$\begin{aligned} \text{Area}(\triangle ABC) &= \frac{\text{base}\cdot\text{height}}{2} = \frac{bh_b}{2} \\ &= \frac{ba\sin\gamma}{2}. \end{aligned}$$

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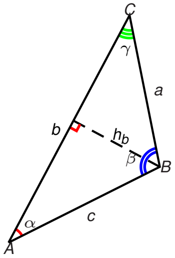

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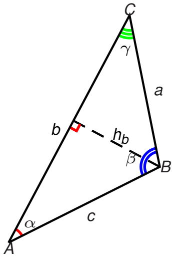

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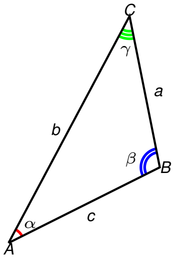
The proof of the other two cases is similar. $\square$

Law of sines

Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ, as indicated: α is opposite to a, β is opposite to b, γ is opposite to c.

Proposition (Law of Sines)

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}.$$



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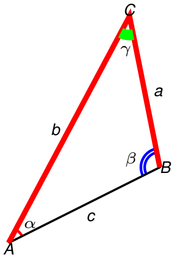

Proof.

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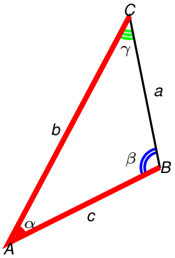

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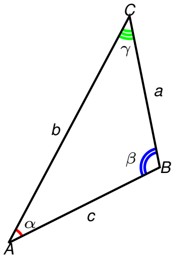

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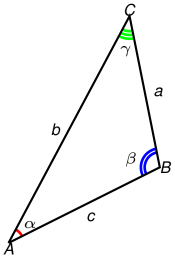

Proof.

$$\begin{aligned} \text{Area}(\triangle ABC) = \frac{ab\sin\gamma}{2} &= \frac{bc\sin\alpha}{2} && \Big| \text{ Div. by } \frac{b}{2} \\ a\sin\gamma &= c\sin\alpha \end{aligned}$$

$\square$

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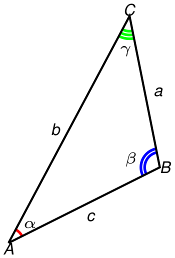

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a\sin\gamma & = & c\sin\alpha & \\
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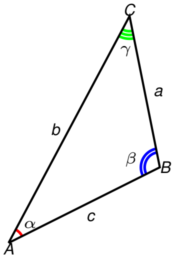

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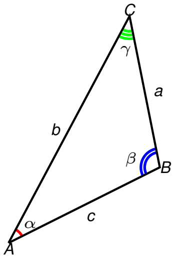

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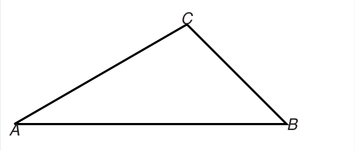
The remaining cases are similar. $\square$

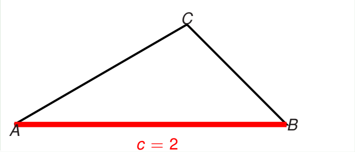
Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
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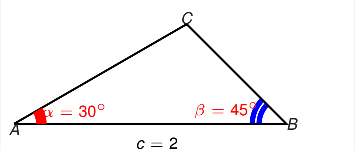
- Let the known side be $c = 2cm$.

Example



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- Find the other two sides of the triangle.
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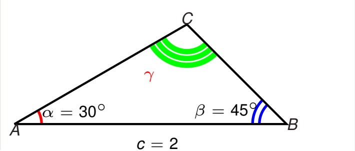
- Let the known side be $c = 2cm$.
- Let the known angles 30°, 45° be arranged as in the figure

Example

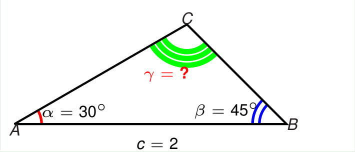


A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

- Let the known side be $c = 2cm$.
- Let the known angles 30°, 45° be arranged as in the figure, and let the third angle be γ .

Example



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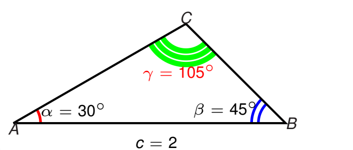
- Find the other two sides of the triangle.
- Find the area of the triangle.

- Let the known side be $c = 2cm$.
- Let the known angles 30°, 45° be arranged as in the figure, and let the third angle be $\gamma = ?$.

Example

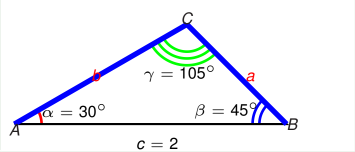


A triangle has a side of length $2cm$; the two angles adjacent to it are $30°$ and $45°$.

- Find the other two sides of the triangle.
- Find the area of the triangle.

- Let the known side be $c = 2cm$.
- Let the known angles $30°$, $45°$ be arranged as in the figure, and let the third angle be $\gamma = 180° - 30° - 45° = 180° - 75° = 105°$.

Example



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- Find the other two sides of the triangle.
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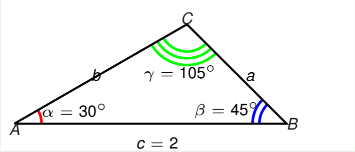
- Let the known side be $c = 2cm$.
- Let the known angles $30^\circ, 45^\circ$ be arranged as in the figure, and let the third angle be $\gamma = 180^\circ - 30^\circ - 45^\circ = 180^\circ - 75^\circ = 105^\circ$.
- Label the unknown sides a, b as indicated.

Example



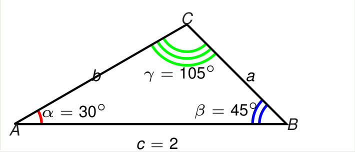
A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

|Law of sines

Example



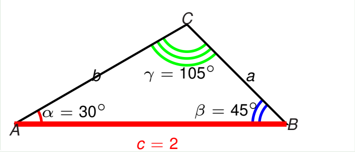
A triangle has a side of length $2cm$; the two angles adjacent to it are $30°$ and $45°$.

- Find the other two sides of the triangle.
- Find the area of the triangle.

$$\begin{aligned} \frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} \\ a &= \frac{c\sin\alpha}{\sin\gamma} \end{aligned}$$

|Law of sines

Example



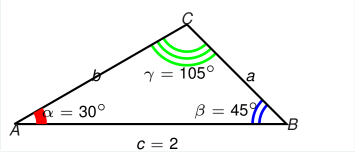
A triangle has a side of length $2cm$; the two angles adjacent to it are $30°$ and $45°$.

- Find the other two sides of the triangle.
- Find the area of the triangle.

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \qquad |\text{Law of sines}$$

$$a = \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30°}{\sin 105°}$$

Example



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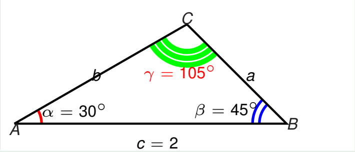
Example



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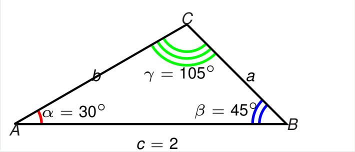

A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

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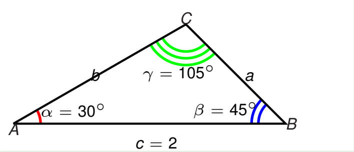
- Find the area of the triangle.

$\sin 105^\circ$

$$
\begin{aligned}
\frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} && |\text{Law of sines}\\
a &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ}
\end{aligned}
$$

Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

$$\sin 105^\circ = \sin(60^\circ + 45^\circ)$$

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \qquad |\text{Law of sines}$$

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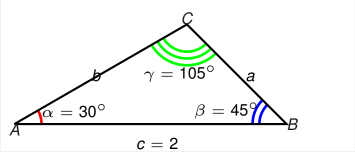
Example



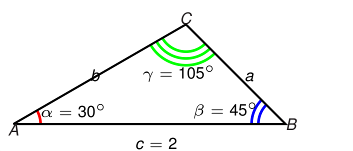
A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

$$\sin 105^\circ = \sin(60^\circ + 45^\circ) = \textbf{?}$$

$$\begin{aligned} \frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} && |\text{Law of sines} \\ a &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ} \end{aligned}$$

Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
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$$\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \qquad |\text{Law of sines}$$

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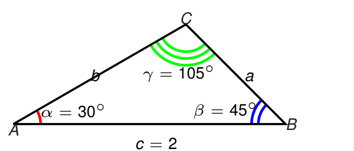
Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
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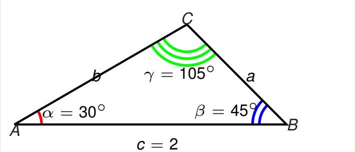
$$
\begin{aligned}
\sin 105° &= \sin(60° + 45°) = \sin 60° \cos 45° + \cos 60° \sin 45° \\
&= ?\quad ?\quad + ?\ ? \\
\frac{a}{\sin \alpha} &= \frac{c}{\sin \gamma} \qquad\qquad |\text{Law of sines} \\
a &= \frac{c \sin \alpha}{\sin \gamma} = \frac{2 \sin 30°}{\sin 105°}
\end{aligned}
$$

Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

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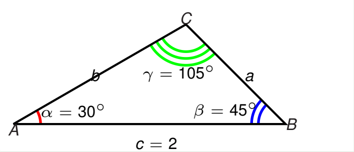
$$\begin{aligned}
\sin 105^\circ &= \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ \\
&= \frac{\sqrt{3}}{2} \textbf{?} \quad + \textbf{? ?} \\
\frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} && |\text{Law of sines} \\
a &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ}
\end{aligned}$$

Example



A triangle has a side of length $2cm$; the two angles adjacent to it are $30°$ and $45°$.

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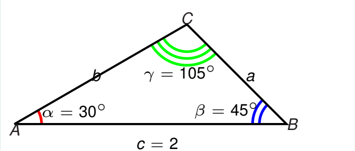
\end{aligned}$$

Example

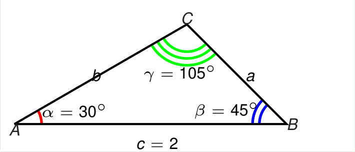


A triangle has a side of length 2*cm*; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

$$\begin{aligned}
\sin 105° &= \sin(60° + 45°) = \sin 60° \cos 45° + \cos 60° \sin 45° \\
&= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + ??\\
\frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} && |\text{Law of sines}\\
a &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30°}{\sin 105°}
\end{aligned}$$

Example



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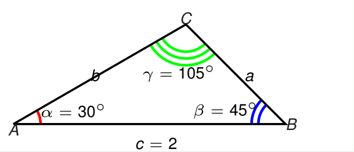
\end{aligned}$$

Example



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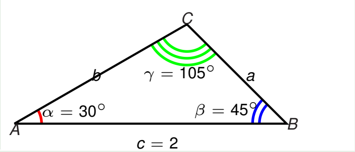
Example



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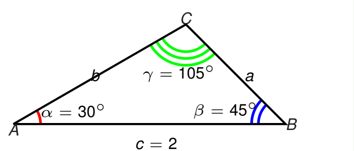
Example



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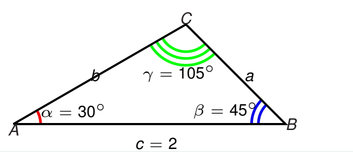
Example



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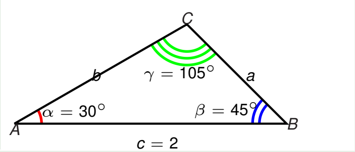
Example



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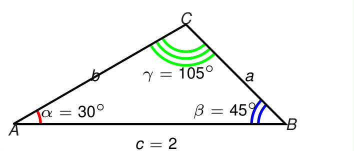
Example



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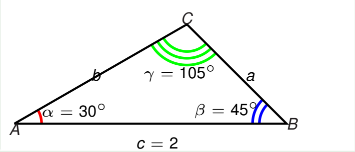
Example



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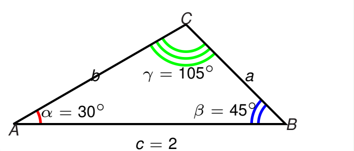
Example



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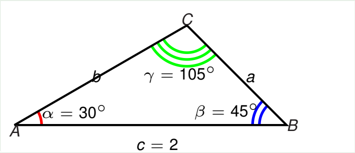
Example



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\end{aligned}
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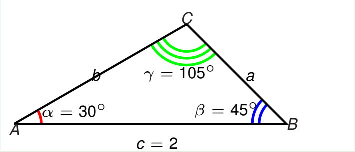
Example



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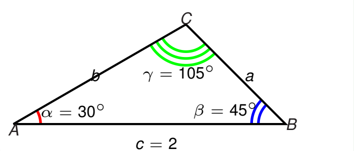
\frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} \qquad |\text{Law of sines} \\
a &= \frac{c \sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ} = \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{4}{(\sqrt{6}+\sqrt{2})}
\end{aligned}$$

Example



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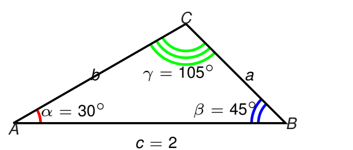
\frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} \qquad |\text{Law of sines} \\
a &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30°}{\sin 105°} = \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{4(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})}
\end{aligned}$$

Example



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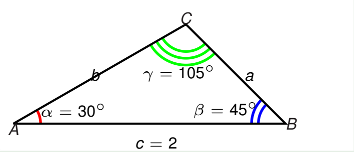
a &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ} = \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{4(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})} \\
&= \frac{4(\sqrt{6}-\sqrt{2})}{6-2}
\end{aligned}$$

Example



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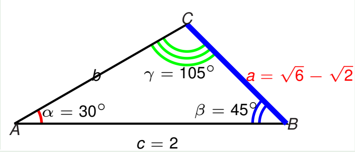
a &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ} = \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{4(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})} \\
&= \frac{4(\sqrt{6}-\sqrt{2})}{6-2} = \sqrt{6}-\sqrt{2}
\end{aligned}$$

Example



A triangle has a side of length $2cm$; the two angles adjacent to it are 30° and 45°.

- Find the other two sides of the triangle.
- Find the area of the triangle.

$$\begin{aligned}
\sin 105^\circ &= \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ \\
&= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4} \\
\frac{a}{\sin\alpha} &= \frac{c}{\sin\gamma} \qquad | \text{Law of sines} \\
{\color{red}a} &= \frac{c\sin\alpha}{\sin\gamma} = \frac{2\sin 30^\circ}{\sin 105^\circ} = \frac{\cancel{2}\cdot\frac{1}{\cancel{2}}}{\frac{\sqrt{6}+\sqrt{2}}{4}} = \frac{4(\sqrt{6}-\sqrt{2})}{(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})} \\
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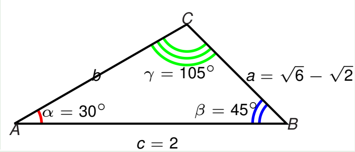

$$\frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

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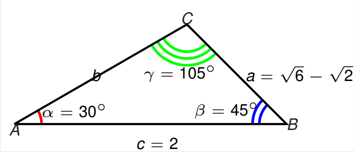
|Law of sines

Example



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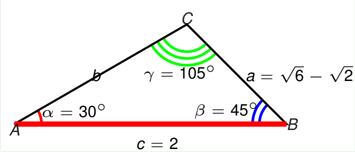
Example



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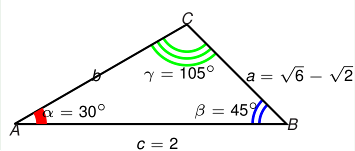
Example



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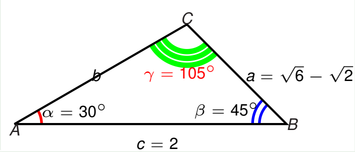

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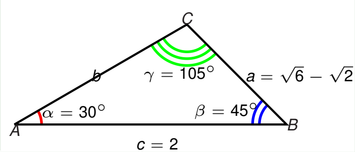

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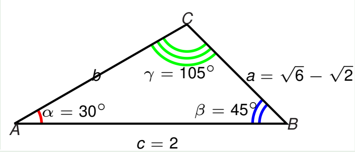
Example



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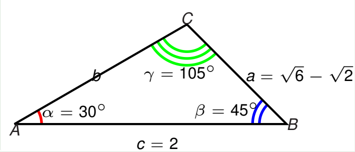

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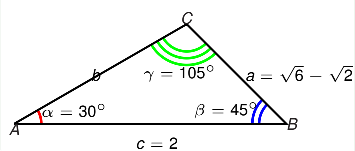
Example



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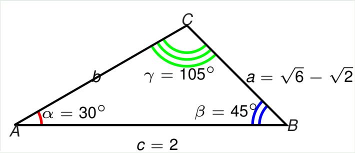
Example



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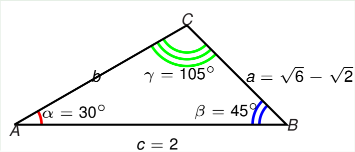

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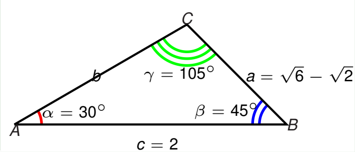
Example



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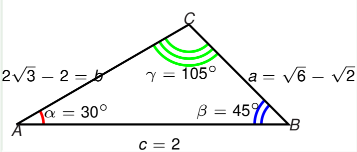

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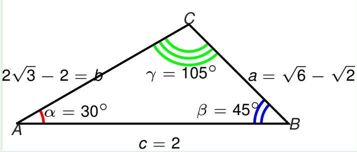
Example



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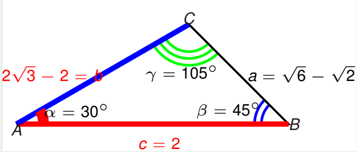

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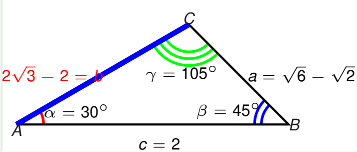
$$\text{Area} = \frac{bc\sin\alpha}{2}$$

Example



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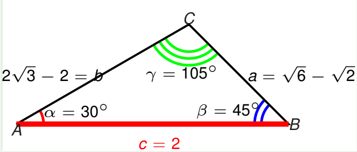
Example



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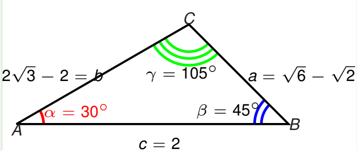

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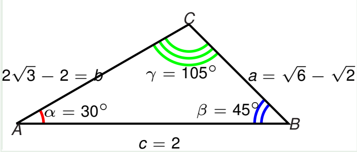

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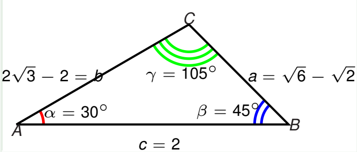

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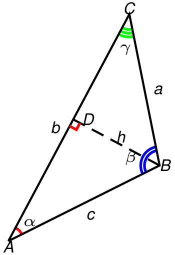
Let $\triangle ABC$ have sides lengths a, b, c angles α, β, γ, as indicated.

Proposition (Law of Cosines)

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos\gamma \\ a^2 &= b^2 + c^2 - 2bc\cos\alpha \\ b^2 &= c^2 + a^2 - 2ca\cos\beta \end{aligned}$$



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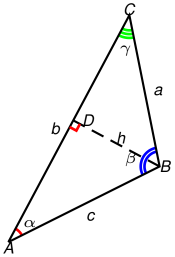

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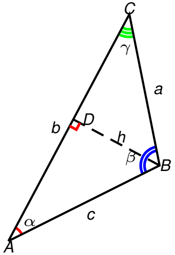

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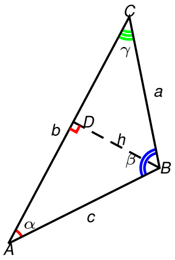

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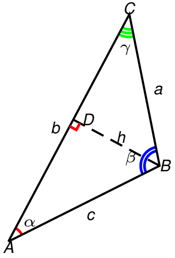


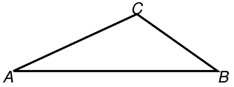
Proof if $\gamma < 90^\circ$.

Drop a perpendicular h from B to AC.

$$\begin{aligned} |CD| &= a\cos\gamma \\ h &= a\sin\gamma \\ |AD| &= b - |CD| = b - a\cos\gamma \\ c^2 &= |AD|^2 + h^2 && \text{Pyth. thm. } \triangle BDA \\ &= (b - a\cos\gamma)^2 + (a\sin\gamma)^2 \\ &= b^2 - 2ab\cos\gamma + a^2\cos^2\gamma + a^2\sin^2\gamma \\ &= b^2 - 2ab\cos\gamma + a^2. \end{aligned}$$

$\square$

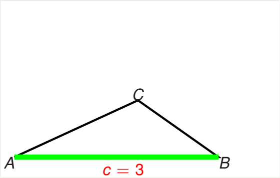
Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

Example



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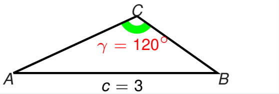
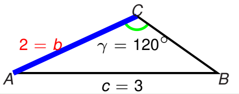
Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

- Find the length of the third side.
- Find the area of the triangle.

Example



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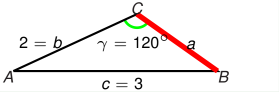

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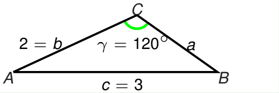

?

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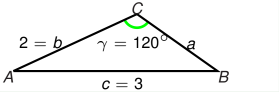
Law of cosines

Example



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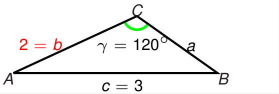
$$a^2 + b^2 - 2ab\cos\gamma = c^2$$ Law of cosines

Example



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$$a^2 + 2^2 - 2a \cdot 2 \cdot \cos 120^\circ = 3^2$$

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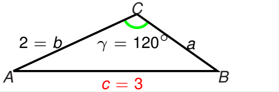

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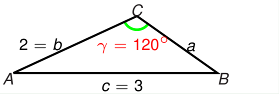

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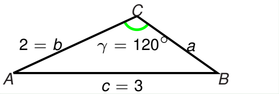

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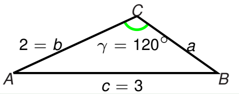

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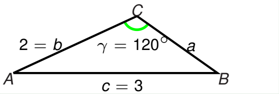

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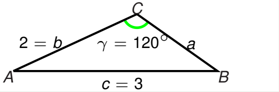

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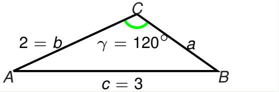

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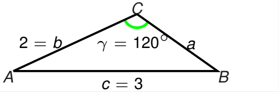
Example



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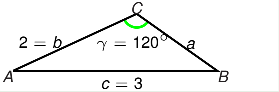

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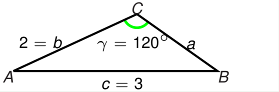

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$$a = \textbf{?}$$

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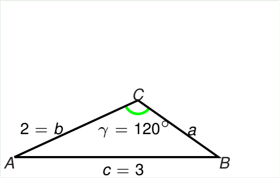

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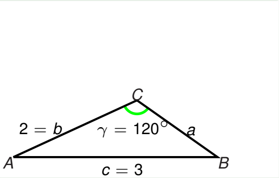
$$a^2 + 2a - 5 = 0$$

$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$

Example



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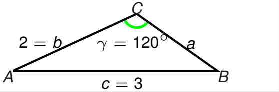
a^2 - 4a\left(-\frac{1}{2}\right) - 5 &= 0 \\
a^2 + {\color{red}2}a - 5 &= 0 \\
a &= \frac{-{\color{red}2} \pm \sqrt{{\color{red}2}^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}
\end{aligned}
$$

Example



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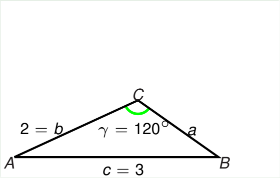
$$

Example



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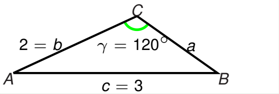

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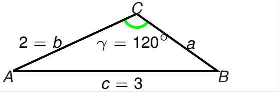
$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1}$$
$$= \frac{-2 \pm \sqrt{24}}{2}$$

Example



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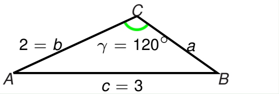
\begin{aligned}
a &= \frac{-2 \pm \sqrt{2^2 - 4\cdot(-5)\cdot 1}}{2\cdot 1}\\
&= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 + 2\sqrt{6}}{2}
\end{aligned}
$$

Example



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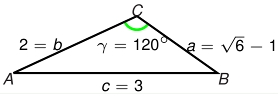
$$= \frac{-2 \pm \sqrt{24}}{2} = \frac{-2 + 2\sqrt{6}}{2} \qquad a > 0$$

Example



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a &= \frac{-2 \pm \sqrt{2^2 - 4\cdot(-5)\cdot 1}}{2\cdot 1}\\
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&= -1 + \sqrt{6}
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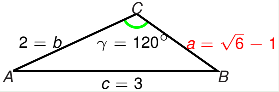
Example



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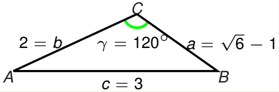

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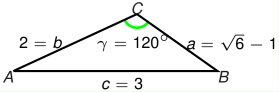
$$a = \frac{-2 \pm \sqrt{2^2 - 4\cdot(-5)\cdot 1}}{2\cdot 1} = -1+\sqrt{6}$$

Example



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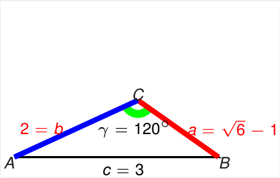
Area = ?

Example



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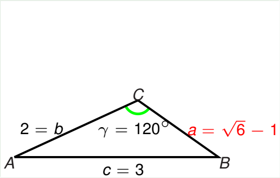
$$= -1 + \sqrt{6}$$

$$\textbf{Area} = \frac{ab\sin\gamma}{2}$$

Example



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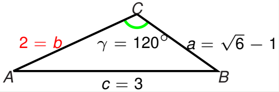
$$\textbf{Area} = \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6} - 1\right)2}{2}\textbf{?}$$

Example



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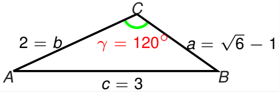
Example



The longest side of a triangle has length 3 and the angle opposite to it is 120°. Another side of that triangle has length 2.

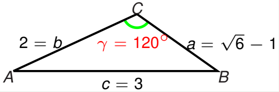
- Find the length of the third side.
- Find the area of the triangle.

$$\begin{array}{rcl|l} a^2 + b^2 - 2ab\cos\gamma &=& c^2 & \text{Law of cosines} \\ a^2 + 2^2 - 2a\cdot 2\cdot \cos 120^\circ &=& 3^2 & \text{Solve for } a: \end{array}$$

$$\begin{aligned} a &= \frac{-2 \pm \sqrt{2^2 - 4\cdot(-5)\cdot 1}}{2\cdot 1} \\ &= -1 + \sqrt{6} \end{aligned}$$

$$\textbf{Area} = \frac{ab\,{\color{red}\sin\gamma}}{2} = \frac{\left(\sqrt{6}-1\right)2}{2}{\color{red}\textbf{?}}$$

Example



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$$a = \frac{-2 \pm \sqrt{2^2 - 4 \cdot (-5) \cdot 1}}{2 \cdot 1} = -1 + \sqrt{6}$$

$$\textbf{Area} = \frac{ab\sin\gamma}{2} = \frac{\left(\sqrt{6} - 1\right) 2}{2} \frac{\sqrt{3}}{2}$$

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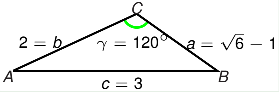

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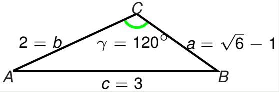

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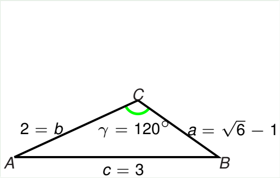

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Law of cosines
Solve for a:

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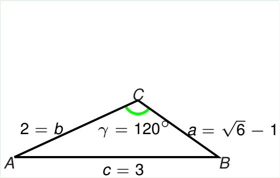

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