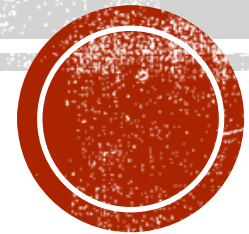


CSPs TUTORIAL VOL. 2

CSPs Algorithms



EXAMPE MODEL & CONSTRAINTS

- Variables: $V1, V2, \dots, V10$
- Domains: $D1 - D10 = \{1, 2, 3\}$
- Constraints:
 - $V1 = V4$
 - $V4 > V7$
 - $V7 = V10 + 1$
- A possible solution is $[3, 1, 1, 3, 1, 1, 2, 1, 1, 1]$



SEARCH ALGORITHMS FOR CSPs

- BT (Back Tracking)
- FC (Forward Checking), FC+MRV (FC + Minimum-Remaining Values)



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 =$$

$$V3 =$$

$$V4 =$$

$$V5 =$$

$$V6 =$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 =$$

$$V4 =$$

$$V5 =$$

$$V6 =$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 =$$

$$V5 =$$

$$V6 =$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 =$$

$$V6 =$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 =$$

$$V6 =$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 1$$

$$V6 =$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 1$$

$$V6 = 1$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 1$$

$$V6 = 1$$

$$V7 = 1$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 1$$

$$V6 = 1$$

$$V7 = 2$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 1$$

$$V6 = 1$$

$$V7 = 3$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 1$$

$$V6 = 2$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 1$$

$$V6 = 2$$

$$V7 = 1$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 1$$

$$V6 = 2$$

$$V7 = 2$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 1$$

$$V6 = 2$$

$$V7 = 3$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 1$$

$$V6 = 3$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 1$$

$$V6 = 3$$

$$V7 = 1$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 1$$

$$V6 = 3$$

$$V7 = 2$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 1$$

$$V6 = 3$$

$$V7 = 3$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 1$$

$$V6 = 3$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 2$$

$$V6 =$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 2$$

$$V6 = 1$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 = 2$$

$$V6 = 1$$

$$V7 = 1$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 1$$

$$V5 =$$

$$V6 =$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$

After some steps, we are back in V4...



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 2$$

$$V5 =$$

$$V6 =$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



BT ALGORITHM

VARIABLES

$$V1 = 1$$

$$V2 = 1$$

$$V3 = 1$$

$$V4 = 3$$

$$V5 =$$

$$V6 =$$

$$V7 =$$

$$V8 =$$

$$V9 =$$

$$V10 =$$

CONSTRAINTS

$$V1 = V4$$

$$V4 > V7$$

$$V7 = V10 + 1$$



FC ALGORITHM

VARIABLES
V1 =
V2 =
V3 =
V4 =
V5 =
V6 =
V7 =
V8 =
V9 =
V10 =

DOMAINS
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {1,2,3}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {1,2,3}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {1,2,3}

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1



FC ALGORITHM

VARIABLES
V1 = 1
V2 = 1
V3 =
V4 =
V5 =
V6 =
V7 =
V8 =
V9 =
V10 =

DOMAINS
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {1}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {1,2,3}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {1,2,3}

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1



FC ALGORITHM

VARIABLES
$V1 = 1$
$V2 = 1$
$V3 = 1$
$V4 =$
$V5 =$
$V6 =$
$V7 =$
$V8 =$
$V9 =$
$V10 =$

DOMAINS
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{1\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{1,2,3\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{1,2,3\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$



FC ALGORITHM

VARIABLES
$V1 = 1$
$V2 = 1$
$V3 = 1$
$V4 = 1$
$V5 =$
$V6 =$
$V7 =$
$V8 =$
$V9 =$
$V10 =$

DOMAINS
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{1\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{1,2,3\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$

! D7 is empty and backtrack to V3



FC ALGORITHM

VARIABLES
$V1 = 1$
$V2 = 1$
$V3 = 2$
$V4 =$
$V5 =$
$V6 =$
$V7 =$
$V8 =$
$V9 =$
$V10 =$

DOMAINS
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{1\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{1,2,3\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{1,2,3\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$



FC ALGORITHM

VARIABLES
V1 = 1
V2 = 1
V3 = 2
V4 = 1
V5 =
V6 =
V7 =
V8 =
V9 =
V10 =

DOMAINS
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {1}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {1,2,3}

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1

! D7 is empty and backtrack to V3



FC ALGORITHM

VARIABLES
$V1 = 1$
$V2 = 1$
$V3 = 3$
$V4 =$
$V5 =$
$V6 =$
$V7 =$
$V8 =$
$V9 =$
$V10 =$

DOMAINS
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{1\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{1,2,3\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{1,2,3\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$



FC ALGORITHM

VARIABLES
V1 = 1
V2 = 1
V3 = 3
V4 = 1
V5 =
V6 =
V7 =
V8 =
V9 =
V10 =

DOMAINS
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {1}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {1,2,3}

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1

! D7 is empty and backtrack to V2



FC ALGORITHM

VARIABLES
$V1 = 1$
$V2 = 2$
$V3 =$
$V4 =$
$V5 =$
$V6 =$
$V7 =$
$V8 =$
$V9 =$
$V10 =$

DOMAINS
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{1\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{1,2,3\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{1,2,3\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$



FC ALGORITHM

VARIABLES
V1 = 1
V2 = 2
V3 = 1
V4 = 1
V5 =
V6 =
V7 =
V8 =
V9 =
V10 =

DOMAINS
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {1}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {1,2,3}

! D7 is empty and backtrack to V2

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1



FC ALGORITHM

VARIABLES
$V1 = 1$
$V2 = 3$
$V3 = 3$
$V4 = 1$
$V5 =$
$V6 =$
$V7 =$
$V8 =$
$V9 =$
$V10 =$

DOMAINS
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{1\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{1,2,3\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$

! D7 is empty and backtrack to V1



FC ALGORITHM

VARIABLES
V1 = 2
V2 =
V3 =
V4 =
V5 =
V6 =
V7 =
V8 =
V9 =
V10 =

DOMAINS
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {2}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {1,2,3}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {1,2,3}

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1



FC ALGORITHM

VARIABLES
$V1 = 2$
$V2 = 1$
$V3 =$
$V4 =$
$V5 =$
$V6 =$
$V7 =$
$V8 =$
$V9 =$
$V10 =$

DOMAINS
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{2\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{1,2,3\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{1,2,3\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$



FC ALGORITHM

VARIABLES
$V1 = 2$
$V2 = 1$
$V3 = 1$
$V4 =$
$V5 =$
$V6 =$
$V7 =$
$V8 =$
$V9 =$
$V10 =$

DOMAINS
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{2\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{1,2,3\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{1,2,3\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$



FC ALGORITHM

VARIABLES
$V1 = 2$
$V2 = 1$
$V3 = 1$
$V4 = 2$
$V5 =$
$V6 =$
$V7 =$
$V8 =$
$V9 =$
$V10 =$

DOMAINS
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{2\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{1\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{1,2,3\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$



FC ALGORITHM

VARIABLES
$V1 = 2$
$V2 = 1$
$V3 = 1$
$V4 = 2$
$V5 = 1$
$V6 =$
$V7 =$
$V8 =$
$V9 =$
$V10 =$

DOMAINS
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{2\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{1\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{1,2,3\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$



FC ALGORITHM

VARIABLES
$V1 = 2$
$V2 = 1$
$V3 = 1$
$V4 = 2$
$V5 = 1$
$V6 = 1$
$V7 =$
$V8 =$
$V9 =$
$V10 =$

DOMAINS
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{2\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{1\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{1,2,3\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$



FC ALGORITHM

VARIABLES
$V1 = 2$
$V2 = 1$
$V3 = 1$
$V4 = 2$
$V5 = 1$
$V6 = 1$
$V7 = 1$
$V8 =$
$V9 =$
$V10 =$

DOMAINS
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{2\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{1\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{\}$

! D10 is empty and backtrack to V6

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$



FC ALGORITHM

VARIABLES
$V1 = 2$
$V2 = 1$
$V3 = 1$
$V4 = 2$
$V5 = 1$
$V6 = 2$
$V7 =$
$V8 =$
$V9 =$
$V10 =$

DOMAINS
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{2\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{1\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{1,2,3\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$



FC ALGORITHM

VARIABLES
$V1 = 2$
$V2 = 1$
$V3 = 1$
$V4 = 2$
$V5 = 1$
$V6 = 2$
$V7 = 1$
$V8 =$
$V9 =$
$V10 =$

DOMAINS
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{2\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{1\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$

! D10 is empty and backtrack to V6



FC+MRV ALGORITHM

VARIABLES
V1 =
V2 =
V3 =
V4 =
V5 =
V6 =
V7 =
V8 =
V9 =
V10 =

CONSTRAINT
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {1,2,3}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {1,2,3}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {1,2,3}

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1



FC+MRV ALGORITHM

VARIABLES
V1 = 1
V2 =
V3 =
V4 =
V5 =
V6 =
V7 =
V8 =
V9 =
V10 =

CONSTRAINT
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {1}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {1,2,3}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {1,2,3}

D4 is the smaller domain

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1



FC+MRV ALGORITHM

VARIABLES
V1 = 1
V2 =
V3 =
V4 = 1
V5 =
V6 =
V7 =
V8 =
V9 =
V10 =

CONSTRAINT
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {1}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {1,2,3}

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1

! D7 is empty and backtrack to V1



FC+MRV ALGORITHM

VARIABLES
V1 = 2
V2 =
V3 =
V4 =
V5 =
V6 =
V7 =
V8 =
V9 =
V10 =

CONSTRAINT
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {2}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {1,2,3}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {1,2,3}

D4 is the smaller domain

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1



FC+MRV ALGORITHM

VARIABLES
V1 = 2
V2 =
V3 =
V4 = 2
V5 =
V6 =
V7 =
V8 =
V9 =
V10 =

CONSTRAINT
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {2}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {1}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {1,2,3}

D7 is the smaller domain

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1



FC+MRV ALGORITHM

VARIABLES
V1 = 2
V2 =
V3 =
V4 = 2
V5 =
V6 =
V7 = 1
V8 =
V9 =
V10 =

CONSTRAINT
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {2}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {1}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {}

! D10 is empty and backtrack to V1

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1



FC+MRV ALGORITHM

VARIABLES
V1 = 3
V2 =
V3 =
V4 =
V5 =
V6 =
V7 =
V8 =
V9 =
V10 =

CONSTRAINT
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {3}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {1,2,3}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {1,2,3}

D4 is the smaller domain

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1



FC+MRV ALGORITHM

VARIABLES
V1 = 3
V2 =
V3 =
V4 = 3
V5 =
V6 =
V7 =
V8 =
V9 =
V10 =

CONSTRAINT
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {3}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {1,2}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {1,2,3}

D7 is the smaller domain

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1



FC+MRV ALGORITHM

VARIABLES
V1 = 3
V2 =
V3 =
V4 = 3
V5 =
V6 =
V7 = 1
V8 =
V9 =
V10 =

CONSTRAINT
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {3}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {1,2}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {}

! D10 is empty and backtrack to V7

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1



FC+MRV ALGORITHM

VARIABLES
V1 = 3
V2 =
V3 =
V4 = 3
V5 =
V6 =
V7 = 2
V8 =
V9 =
V10 =

CONSTRAINT
D1 = {1,2,3}
D2 = {1,2,3}
D3 = {1,2,3}
D4 = {3}
D5 = {1,2,3}
D6 = {1,2,3}
D7 = {1,2}
D8 = {1,2,3}
D9 = {1,2,3}
D10 = {1}

D10 is the smaller domain

CONSTRAINTS
V1 = V4
V4 > V7
V7 = V10 + 1



FC+MRV ALGORITHM

VARIABLES
$V1 = 3$
$V2 =$
$V3 =$
$V4 = 3$
$V5 =$
$V6 =$
$V7 = 2$
$V8 =$
$V9 =$
$V10 = 1$

CONSTRAINT
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{3\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{1,2\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{1\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$



FC+MRV ALGORITHM

VARIABLES
$V1 = 3$
$V2 = 1$
$V3 = 1$
$V4 = 3$
$V5 = 1$
$V6 = 1$
$V7 = 2$
$V8 = 1$
$V9 = 1$
$V10 = 1$

CONSTRAINT
$D1 = \{1,2,3\}$
$D2 = \{1,2,3\}$
$D3 = \{1,2,3\}$
$D4 = \{3\}$
$D5 = \{1,2,3\}$
$D6 = \{1,2,3\}$
$D7 = \{1,2\}$
$D8 = \{1,2,3\}$
$D9 = \{1,2,3\}$
$D10 = \{1\}$

CONSTRAINTS
$V1 = V4$
$V4 > V7$
$V7 = V10 + 1$

After 6 more steps, we have a solution!

