Dear Editors of **Physical Review X,**

We are pleased to submit our manuscript entitled **“Thermal Hall Transport in Kitaev spin liquids”** for consideration in Physical Review X.

Quantum spin liquids are exotic states of matter characterized by long-range entanglement and fractionalized excitations. The Kitaev model offers a paradigmatic platform for realizing such states, hosting itinerant Majorana fermions and localized fluxes. A key experimental signature of these excitations is the thermal Hall conductivity, which is predicted to exhibit half-integer quantization at zero temperature under a magnetic field. Remarkably, this quantization has been reported in the Kitaev candidate material α-RuCl₃, suggesting the realization of a topological Majorana Chern insulator. However, the experimental situation remains highly controversial and is currently the subject of intense debate. While some studies report quantized thermal Hall conductivity, others observe deviations, with possible contributions from phonons, magnons, or visons being proposed. These conflicting results raise fundamental questions about the nature of the heat-carrying quasiparticles and the validity of the topological interpretation, particularly at finite temperatures and in the presence of additional interactions beyond the ideal Kitaev model, where no unbiased method has been available to calculate the thermal Hall effect without approximations.

In this work, we investigate the thermal Hall conductivity in the Kitaev model and its extensions including symmetric off-diagonal interactions, Γ and Γ′ terms, under magnetic fields. To this end, we have developed a new numerical approach based on unbiased finite-temperature tensor network simulations, benchmarked by thermal pure quantum state methods. This framework enables the computation of the thermal Hall conductivity in quantum spin systems without assuming specific types of heat-carrying quasiparticles. We find that the thermal Hall conductivity divided by temperature, κxy/T, exhibits a pronounced overshooting behavior beyond the half-integer quantized value, closely resembling experimental observations in α-RuCl₃. Furthermore, we show that the sign of κxy/T changes systematically with the direction of the applied magnetic field, in agreement with topological predictions from perturbation theory as well as experimental results. Importantly, these features persist even in the polarized regime beyond the critical field, indicating that the topological Majorana fermion picture remains relevant across a wide range of magnetic fields and interaction strengths.

Our findings reveal that the thermal Hall effect in the extended Kitaev models is a rich finite-temperature phenomenon, with topological Majorana fermion excitations remaining robust even in regimes traditionally considered dominated by conventional magnons. By reproducing key experimental features, including the overshooting behavior and field-direction-dependent sign change, and offering a consistent topological interpretation, our study provides a pathway toward resolving conflicting experimental observations. We expect these results, together with the developed methodology, to have a strong impact not only on the ongoing debate over the Kitaev materials but also on the broader study of quantum spin systems, offering a solid framework for interpreting thermal transport and guiding the identification of topological phases in strongly correlated quantum materials.

We confirm that this manuscript has not been published elsewhere and is not under consideration by another journal. All authors have approved the manuscript and agree with its submission to Physical Review X.

Thank you for considering our work. We look forward to your response.

Sincerely,  
Tsuyoshi Okubo, Joji Nasu, Takahiro Misawa, and Yukitoshi Motome

100-word justification:（100 words）

We present a comprehensive study of thermal Hall transport in extended Kitaev models, using unbiased finite-temperature tensor network simulations. Our results successfully reproduce key experimental features observed in the prime candidate α-RuCl3, including the overshoot beyond the half-integer quantization and field-direction dependence. Notably, we uncover robust topological Majorana signatures across a wide parameter range, persisting even in the high-field polarized regime beyond the quantum critical point. Our work provides timely insight into an active experimental and theoretical debate on the origin of thermal Hall transport in Kitaev magnets and advances our understanding of topological excitations relevant to future quantum technologies.

**[Referee]**

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Prof. Achim Rosch (University of Cologne, Germany) and his collaborators in Refs. [31,32]

Prof. Leon Balents (UC Santa Barbara, USA) and his collaborators in Ref. [30]

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Research Areas  
   • Quantum spin liquid  
   • Thermal Hall effect  
Techniques>Theoretical & Computational Techniques  
   • Kitaev model  
   • Monte Carlo methods  
   • Tensor network methods  
Physical Systems  
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