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The Dice is Right: Engineering Winning Strategies in Yahtzee

https://yahtzee-sim.streamlit.app/

Abstract

Yahtzee is a popular dice game introduced in the mid-20th century where players aim to score points by rolling combinations of five dice over thirteen rounds. While Yahtzee may seem like a game of chance, there are many different strategies involved after the rolling of the dice. The goal of our project was to simulate some of these popular strategies, and examine the results.

The main challenge in Yahtzee is to optimize the scoring strategy given the stochastic nature of dice rolls. Every game unfolds in a different way, influencing the decisions players must make regarding which combinations to pursue and when. Due to the random nature of each game, we plan to use simulation to determine which strategies consistently yield the highest scores. We will evaluate traditional strategies – such as prioritizing straights or going for high scoring categories – as well as more dynamic strategies that adapt based on the state of the game. Through this simulation, we hope to gain insights into decision-making under uncertainty and quantify the effectiveness of various strategic approaches in Yahtzee.

How to Play

At the beginning of each turn, the player rolls all 5 dice. After that, the player has the option to reroll up to two more times to achieve the desired score. The scorecard has 16 slots including for each number 1-6, a few classic poker hands (small and large straights, three and four of a kind, full house), chance, Yahtzee, and then three Yahtzee “bonuses”.

The upper section includes categories representing the numbers one through six, where the player's score is the sum of all dice showing that number. For example, if a player rolls 3-3-3-4-6, they could choose to score in the "Threes" category and receive 9 points (3 + 3 + 3). If the player gets 63 points in the upper section combined, they also get a 35 point bonus.

The lower section gets a little more interesting with the scoring. The score for the three and four of a kind and chance boxes is the sum of all dice. For example, 3-3-3-4-6, they could choose to score in the "Three of a kind" or “chance” categories and receive 19 points (3 + 3 + 3 + 4 + 6). This scoring discrepancy between the lower section and upper section makes scoring decisions much more interesting.

After sixteen rounds, the player with the highest total score wins the game. The strategic aspect of Yahtzee lies in deciding which combination to aim for in each round, considering both the current roll of the dice and the player's overall scorecard.

Code Explanation

The code is split into a few main functions that are combined to simulate games quickly and efficiently. The foundational function is called “roll_dice” and simulates exactly that. It simply generates 5 random integers between 1 and 6 and returns them. The next is a helper function called “decide_dice”. It accepts the current dice, the current scorecard (a dictionary), and the strategy name. Until a score box is filled in, it is denoted as a NoneType so it is easy to distinguish which ones have been used already. This function is the main one that changes with the different strategies, so this will be explained further in future sections. This function returns a vector of length 5 with the dice that were chosen to be kept according to the current strategy.

“Get_scores” is another helper function utilized in the program. It simply accepts a vector of length 5 and returns a dictionary with how many points would be attained if each of the possible score boxes was chosen. “Decide_score” uses the previous two functions in tandem to decide which box is most ideal to choose. To someone who hasn’t played Yahtzee before, it may seem like the best strategy would be to just choose whichever one would yield the highest score, but this is very inefficient. High-level Yahtzeers have pioneered many more beneficial strategies including taking a 0 score in the Yahtzee or “ones” box, saving the chance box, and prioritizing the upper section, among others. Some more of these strategies will be discussed later.

Following the helper functions, the “turn” and “game” functions are self-explanatory and just handle taking care of making the game run properly. One thing we had to consider were cases of “Yahtzee bonuses”. If a player gets a Yahtzee, they get 50 points, but they also gain the opportunity to get further “Yahtzee bonuses” which are 100 points. If the player takes a 0 in the Yahtzee box, they are not able to get these bonuses, so some games have different numbers of turns than others.

Strategies

Strategy 1: Completely Random

We began our simulations with a baseline strategy, where the player keeps the dice from the first roll of every turn. We included this strategy as a control group, measuring the average score when using essentially no strategy and keeping a random dice roll. The selected score box was chosen by checking which of the remaining possible options had the highest score, a very basic strategy.

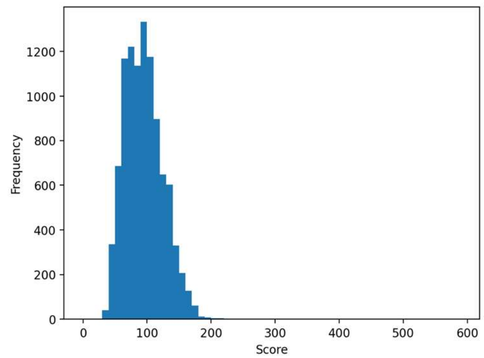


Figure 1: Distribution of points for simulation of 1000 games using Strategy 1

As shown in Figure 1, Strategy 1 achieved an average score of 94.6965 during a simulation of 1000 games. While it did achieve some relatively high scores by chance, our assumption is that this is going to be the worst strategy. We can only improve from here.

Strategy 2: Keeping the most common number

Our next strategy is a very straightforward approach: keeping the dice showing the most frequently occurring number in the roll. For example, if the roll displays 1, 3, 3, 5, 6, our focus would be on preserving the dice showing the threes. If there are multiple numbers with the same number of occurrences, we keep the higher number.

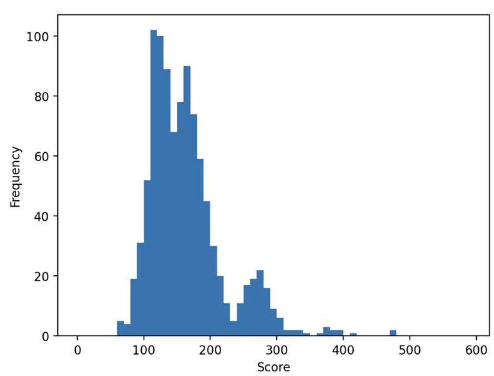


Figure 2: Distribution of points for simulation of 1000 games using Strategy 2

As shown in Figure 2, Strategy 2 achieved an average score of 161.4 in a round of 1000 simulations. We begin to see the appearance of some games where Yahtzee's are scored, which is shown by the small peaks in the distribution around the scores of 280 and 380.

Strategy 3: Going for straights

Our third strategy is similar to strategy 2, with an added aspect of going for straights in some scenarios. If there is a run of at least three consecutive numbers, this strategy prioritizes the pursuit of large or small straights over aiming for pairs. If it's the first turn, the strategy will keep any runs of 3 or more, but after that it will only keep runs of 4+.

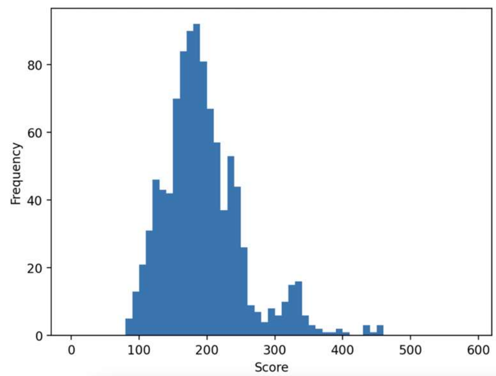


Figure 3: Distribution of points for simulation of 1000 games using Strategy 3

We raised the score again by implementing Strategy 3, achieving an average of 189.8 in a round of 1000 simulated games. This bump in score from Strategy 2 is largely due to the appearance of small and large straights on the scorecard.

Strategy 4: Avoiding Ones

This strategy builds upon the previous approach by incorporating additional constraints to optimize scoring. If the total score for ones is below 10 points, it deliberately chooses not to score in the ones category, opting for a score of zero instead. Because the max number of points that you can realistically get in the ones category is 4 (a 5 would be a Yahtzee), the player loses almost no points by taking a zero and can preserve another category for a future play.

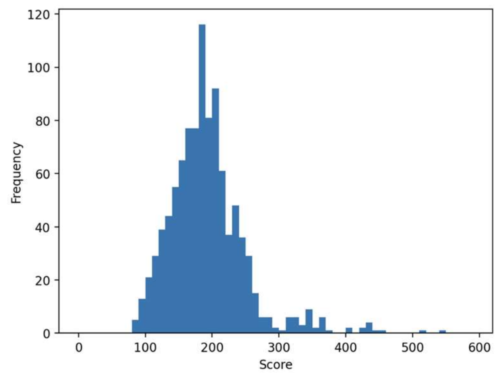


Figure 4: Distribution of points for simulation of 1000 games using Strategy 4

We saw a very small bump in score after the implementation of Strategy 4, reaching 193.1 as depicted in Figure 4.

Strategy 5: Prioritizing the Upper Bonus

In this strategy, the emphasis shifts towards maximizing the potential for earning the upper bonus. The player evaluates the dice roll with the primary goal of securing a substantial score in the upper section of the scorecard. It places significant importance on achieving a high score in the upper section, particularly aiming to surpass the 63 point threshold for the upper bonus. The upper bonus is still difficult to attain and this strategy only actually achieves it ~10% of the time, but the previous strategy only got the bonus ~1% of the time, for reference.

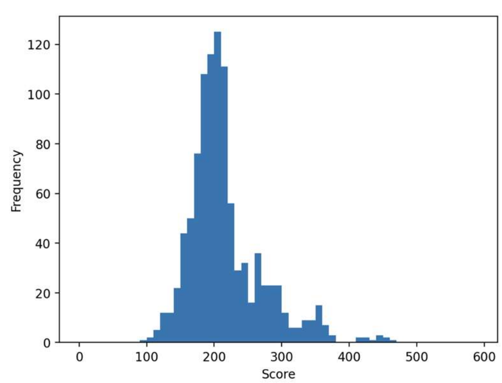


Figure 5: Distribution of points for simulation of 1000 games using Strategy 5

Our greatest success was achieved with the implementation of Strategy 5. In a simulation of 1000 games, an average score of 215.3 was achieved. This was a large jump from Strategy 4, increasing by over 20 points. We saw some games that scored the upper bonus that adds 35 to the card, but there was also an increase in scoring in all categories of the upper scorecard.

App Demo

Our interactive web app provides a user-friendly interface to visualize each of our provided strategies. Users can observe the outcomes of each strategy and gain insights into the effectiveness of different approaches in Yahtzee gameplay.

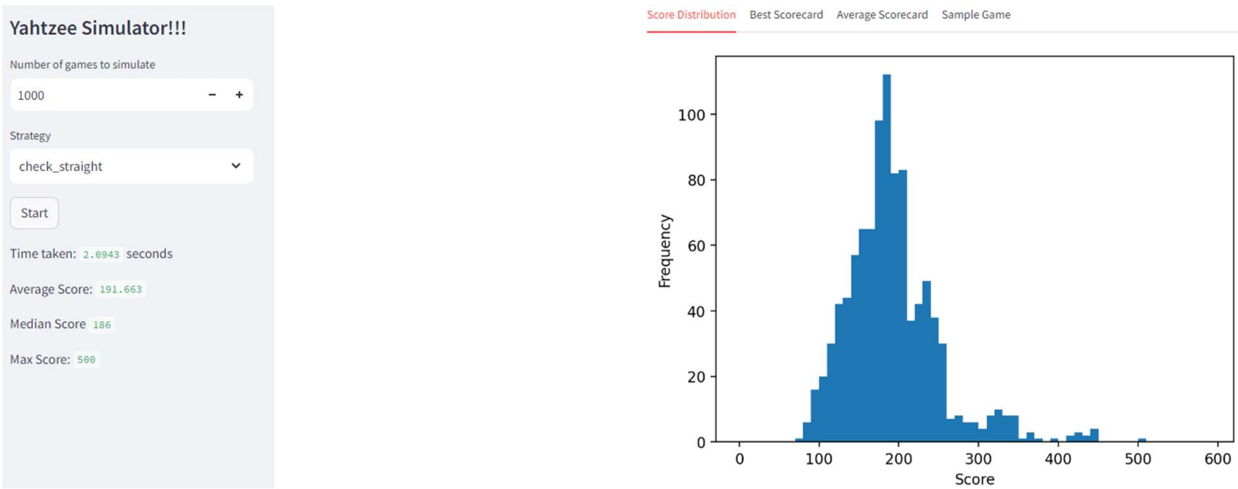
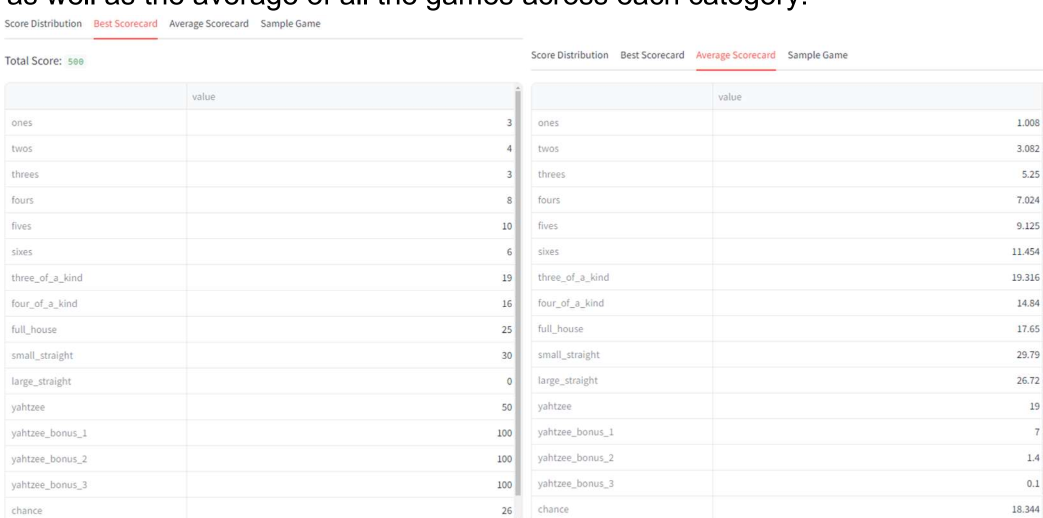


Figure 6: App Demo of 1000 game simulation on the Strategy 3

Our app interface is shown in Figure 6. On the left side, the user is given an option of what strategy to select and how many games to simulate, with a maximum of 100000. After running the simulation, the time taken and average, median, and max score are shown, as well as the score distribution for all of the games. The example in this demo shows a simulation of 1000 games using Strategy 3.

Our app also provides the feature of viewing the scorecard of the game with the highest points scored, as well as the average of all the games across each category.



Score Distribution | Best Scorecard | Average Scorecard | Sample Game

Total Score: 500

	value
ones	3
twos	4
threes	3
fours	8
fives	10
sixes	6
three_of_a_kind	19
four_of_a_kind	16
full_house	25
small_straight	30
large_straight	0
yahtzee	50
yahtzee_bonus_1	100
yahtzee_bonus_2	100
yahtzee_bonus_3	100
chance	26

Score Distribution | Best Scorecard | Average Scorecard | Sample Game

	value
ones	1.008
twos	3.082
threes	5.25
fours	7.024
fives	9.125
sixes	11.454
three_of_a_kind	19.316
four_of_a_kind	14.84
full_house	17.65
small_straight	29.79
large_straight	26.72
yahtzee	19
yahtzee_bonus_1	7
yahtzee_bonus_2	1.4
yahtzee_bonus_3	0.1
chance	18.344

Figure 7: Best Scorecard and Average Scorecard for simulation of 1000 games using Strategy 3

As shown on the left panel in Figure 7, the best game from the simulation achieved a score of 500 points. This was largely due to successfully hitting all three of the Yahtzee bonus. On the right panel, the average score across all of the games is shown. This can be useful in examining where a certain strategy excels versus where it may be missing out on points.

Lastly, our app provides an option to see the decisions being made at each roll for a sample game from the simulation.

Figure 8: Sample game from simulation of 1000 games using Strategy 3

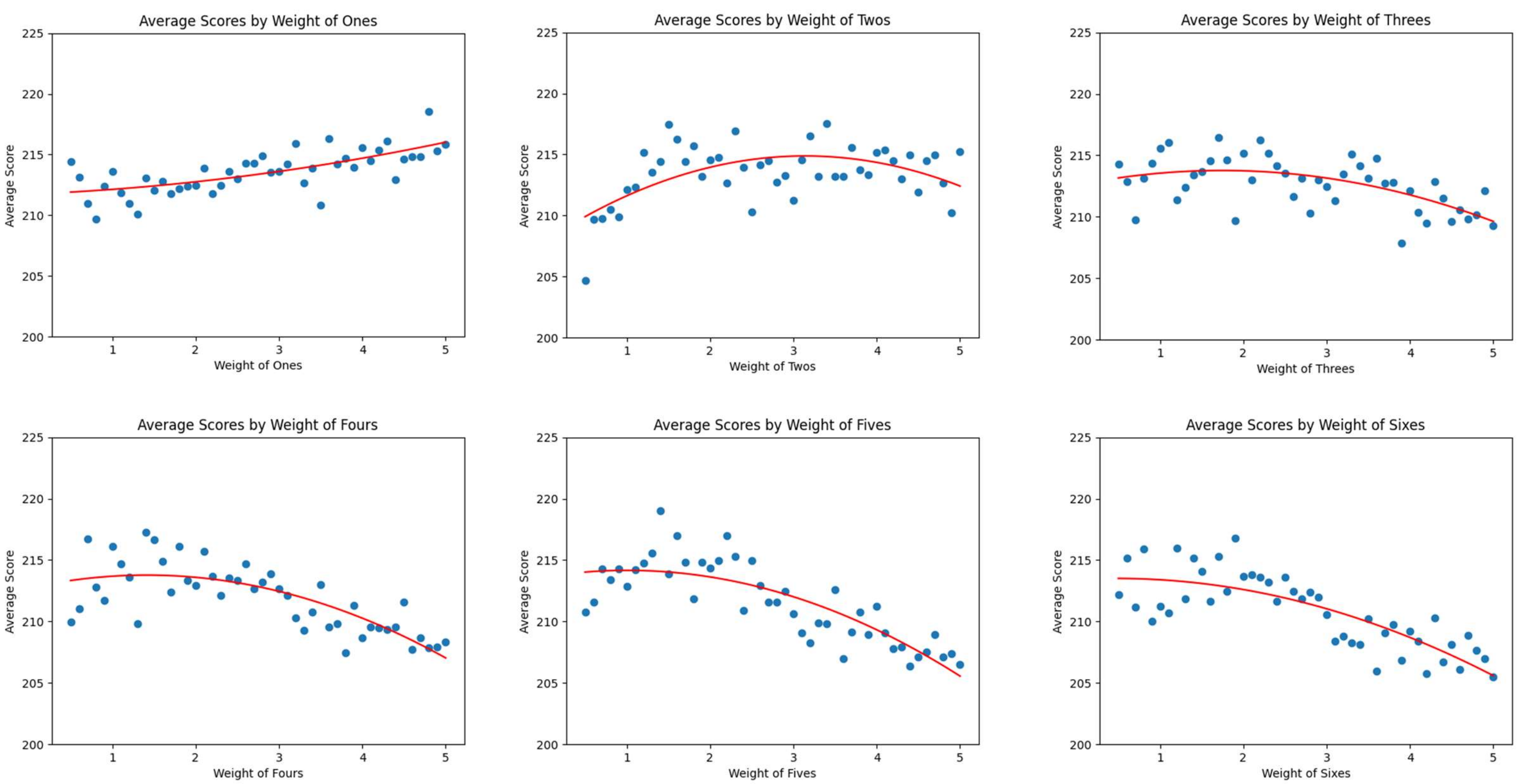
In Figure 8, the first turn from a sample game of the simulation of Strategy 3 is displayed. This makes it easy to see exactly what is going on behind the scenes of each strategy. It also makes it apparent where a certain strategy may be behaving sub-optimally.

Sensitivity Analysis

One of the parts of our best strategy that is most prone to mistakes is where some of the score boxes are given “weights” to adjust the way that the algorithm sees them. For example, because this strategy is going for the upper bonus, the scores in the top section are given extra weight:

ones: 4
twos: 1.4
threes: 1.4
fours: 1.3
fives: 1.3
sixes: 1.2

Let’s see how much these weights affect the final scores.



Each of the values in
these charts depicts the average of 1000 independently simulated games while keeping all other values the same as they were initially

As you can see from these, the weights do have a significant impact (that becomes more significant as the number increases) but doesn’t change the average score by more than 5 or 10 between the best and worst weights. This is an example of a place where a neural network could be used in the future to use the optimal combination of weights to maximize score.

Comparative Analysis

To test our engine, we both played 5 games of Yahtzee online at solitaire.com/yahtzee to test ourselves against it. Our scores were as follows:

Tyson: 199, 385, 210, 263, 187
Josh: 281, 213, 250, 186, 310

Score Comparison

Strategy	Mean Points	Median Points
Random	110.1	110
Most Common	161.4	146
Straights	189.8	183
No Ones	193.1	183
Upper Bonus	215.3	204
Tyson	248.8	210
Josh	281	250

As you can see from this table, our best strategy for deciding which dice to keep and which score box to use are decent, but not comparable to a competent human player. This is to be expected as there are an infinite number of possible situations that can arise during a game of Yahtzee and not all can be accounted for using a simple system like this.

Conclusions

Our research into the strategy of Yahtzee through simulation has provided several valuable insights into decision-making and strategy optimization for stochastic games like Yahtzee. Through the simulations of each strategy, we have found the importance of adaptive strategy over random or overly simplistic approaches.

Our first finding came with the implementation of Strategy 2. We were able to achieve a significantly higher score than Strategy 1 by focusing on keeping the most frequently occurring numbers. This was a very simple yet effective change. We then implemented more structured strategies in Strategy 3 and 4, such as aiming for straights or deliberately avoiding scoring in low-yield categories like ones. Adding these approaches slowly but surely raised the average scores that we were seeing in our simulations. Finally, we implemented the strategy of prioritizing the upper section. This proved to be very beneficial, and we immediately saw a large jump in average score. Despite the bonus being achieved in only about 10% of the games, it led to the highest average and median scores, demonstrating the power of aiming for cumulative advantages in gameplay.

Looking ahead, our study opens up possibilities for further exploration in the realm of Yahtzee strategy. Future studies could look deeper into the optimization of specific strategies or further investigate how different strategies interact with each other. By continuing to unravel the complexities of Yahtzee strategy, we can improve our understanding of decision-making under uncertainty and pave the way for more informed gameplay of this dice game.