

# Formelsammlung HSR

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# Vorwort

Die vorliegende Formelsammlung wurde während dem Studium für Elektrotechnik an der Fachhochschule in Rapperswil geschrieben. Ziel war es, den Inhalt an den Prüfungsstoff anzupassen, aber auch ein Werk zu schreiben, das wir später im Berufsleben verwenden können. Obwohl wir den Inhalt sorgfältig zusammengestellt haben, sind Fehler nicht ausschliessbar. Für jegliche Korrektur- oder Verbesserungsvorschläge haben wir immer ein offenes Ohr!

In der Formelsammlung sind die folgenden Fächer enthalten:

- Physik
- Elektrizitätslehre
- Energie und Antriebstechnik
- Elektronik
- Digitale Signalverarbeitung
- Mathematik

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# **Teil I.**

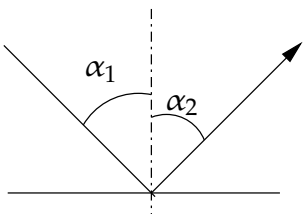
# **Physik**

# 1. Geometrische Optik

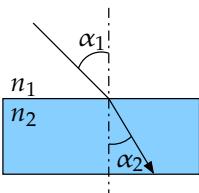
## 1.1. Sichtbares Licht

| Wellenbereich $\lambda/nm$ | Farbe    |
|----------------------------|----------|
| 380 - 435                  | violett  |
| 435 - 465                  | blau     |
| 465 - 485                  | blaugrün |
| 485 - 565                  | grün     |
| 565 - 590                  | gelb     |
| 590 - 630                  | orange   |
| 630 - 780                  | rot      |

## 1.2. Reflexionsgesetz

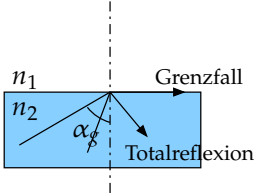
|   |                       |   |
|---|-----------------------|---|
|  | $\alpha_1 = \alpha_2$ | <div> <math>\alpha_1</math>    Einfallswinkel    [rad]         </div> <div> <math>\alpha_2</math>    Ausfallswinkel    [rad]         </div> |
|---|-----------------------|---|

## 1.3. Brechung

|   |   |   |
|---|---|---|
|  | <p>falls <math>n_2 &gt; n_1</math>:</p> $n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2)$ $n = \frac{c}{u}$ $n_2 > n_1 \Rightarrow \alpha_1 > \alpha_2$ | <div> <math>\alpha_1</math>    Einfallswinkel    [rad]         </div> <div> <math>\alpha_2</math>    Brechungswinkel    [rad]         </div> <div> <math>n_{1,2}</math>    Brechungsindex    [1]         </div> |
|---|---|---|



## 1.4. Totalreflexion

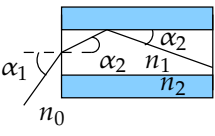
|   |  |  |            |             |       |           |                |     |
|---|--|--|------------|-------------|-------|-----------|----------------|-----|
|  | <p>falls <math>n_2 &gt; n_1</math>:</p> $\alpha_g = \arcsin\left(\frac{n_1}{n_2}\right)$ | <table> <tr> <td><math>\alpha_g</math></td> <td>Grenzwinkel</td> <td>[rad]</td> </tr> <tr> <td><math>n_{1,2}</math></td> <td>Brechungsindex</td> <td>[1]</td> </tr> </table> | $\alpha_g$ | Grenzwinkel | [rad] | $n_{1,2}$ | Brechungsindex | [1] |
| $\alpha_g$  | Grenzwinkel  | [rad]  |            |             |       |           |                |     |
| $n_{1,2}$   | Brechungsindex   | [1]  |            |             |       |           |                |     |

### 1.4.1. Prisma

Diagram illustrating the geometry of light dispersion through a triangular prism. The incident ray makes an angle  $\alpha$  with the normal. The refracted ray inside the prism makes an angle  $\delta$  with the normal. The emergent ray makes an angle  $\beta$  with the normal. The angle of deviation is  $\gamma$ . The minimum deviation angle is  $\gamma_{min}$ . The refractive indices are  $n_1$  for the prism and  $n_2$  for the medium. The diagram also shows the dispersion of white light into red and violet components.

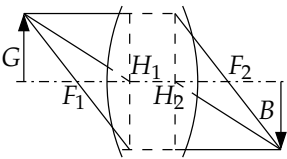
## 1. GEOMETRISCHE OPTIK

### 1.4.2. Lichtwellenleiter

|   |  |  |            |                |       |                 |                         |       |       |          |     |       |        |     |       |          |     |
|---|--|--|------------|----------------|-------|-----------------|-------------------------|-------|-------|----------|-----|-------|--------|-----|-------|----------|-----|
|  | <p>falls <math>n_1 &gt; n_2</math>:</p> $\alpha_{1max} = \arcsin \frac{n_1 \cos \left[ \arcsin \left( \frac{n_2}{n_1} \right) \right]}{n_0}$ $n_0 \sin \alpha_1 = n_1 \sqrt{1 - \cos^2 \alpha_2}$ $n_0 \sin \alpha_1 = n_1 \sqrt{1 - \left( \frac{n_2}{n_1} \right)^2}$ $n_0 \sin \alpha_1 = \sqrt{n_1^2 - n_2^2}$ | <table> <tr> <td><math>\alpha_1</math></td><td>Einfallswinkel</td><td>[rad]</td></tr> <tr> <td><math>\alpha_{1max}</math></td><td>max Einfalls-<br/>winkel</td><td>[rad]</td></tr> <tr> <td><math>n_0</math></td><td>n-Medium</td><td>[1]</td></tr> <tr> <td><math>n_1</math></td><td>n-Kern</td><td>[1]</td></tr> <tr> <td><math>n_2</math></td><td>n-Mantel</td><td>[1]</td></tr> </table> | $\alpha_1$ | Einfallswinkel | [rad] | $\alpha_{1max}$ | max Einfalls-<br>winkel | [rad] | $n_0$ | n-Medium | [1] | $n_1$ | n-Kern | [1] | $n_2$ | n-Mantel | [1] |
| $\alpha_1$  | Einfallswinkel   | [rad]  |            |                |       |                 |                         |       |       |          |     |       |        |     |       |          |     |
| $\alpha_{1max}$   | max Einfalls-<br>winkel  | [rad]  |            |                |       |                 |                         |       |       |          |     |       |        |     |       |          |     |
| $n_0$   | n-Medium   | [1]  |            |                |       |                 |                         |       |       |          |     |       |        |     |       |          |     |
| $n_1$   | n-Kern   | [1]  |            |                |       |                 |                         |       |       |          |     |       |        |     |       |          |     |
| $n_2$   | n-Mantel   | [1]  |            |                |       |                 |                         |       |       |          |     |       |        |     |       |          |     |

## 1.5. Abbildungen

### 1.5.1. Allgemein

|   |   |  |     |                       |     |     |           |     |     |            |     |       |                        |  |       |                        |  |       |                        |  |       |                        |  |     |                        |     |     |            |     |         |                           |     |
|---|---|--|-----|-----------------------|-----|-----|-----------|-----|-----|------------|-----|-------|------------------------|--|-------|------------------------|--|-------|------------------------|--|-------|------------------------|--|-----|------------------------|-----|-----|------------|-----|---------|---------------------------|-----|
|  <p>Vorzeichen:</p> <ul style="list-style-type: none"> <li>Für sammelnde optische Bauelemente ist <math>f &gt; 0</math>.</li> <li>Für zerstreuende optische Bauelemente <math>f &lt; 0</math>.</li> <li>Für virtuelle Bilder ist <math>b &lt; 0</math> und <math>B &lt; 0</math>.</li> <li>Für vortuelle Gegenstände ist <math>g &lt; 0</math> und <math>G &lt; 0</math>.</li> </ul> | $g = \overline{H_1 G}$ $b = \overline{H_2 B}$ $f = \overline{H_1 F_1}$ $f = \overline{H_2 F_2}$ $\frac{1}{f} = \frac{1}{g} + \frac{1}{b}$ $\frac{B}{G} = \frac{b}{g}$ $\beta = \frac{B}{G}$ | <table> <tr> <td><math>g</math></td><td>Gegenstands-<br/>weite</td><td>[m]</td></tr> <tr> <td><math>b</math></td><td>Bildweite</td><td>[m]</td></tr> <tr> <td><math>f</math></td><td>Brennweite</td><td>[m]</td></tr> <tr> <td><math>H_1</math></td><td>vorderer<br/>Hauptpunkt</td><td></td></tr> <tr> <td><math>H_2</math></td><td>hinterer<br/>Hauptpunkt</td><td></td></tr> <tr> <td><math>F_1</math></td><td>vorderer<br/>Brennpunkt</td><td></td></tr> <tr> <td><math>F_2</math></td><td>hinterer<br/>Brennpunkt</td><td></td></tr> <tr> <td><math>G</math></td><td>Gegenstands-<br/>grösse</td><td>[m]</td></tr> <tr> <td><math>B</math></td><td>Bildgrösse</td><td>[m]</td></tr> <tr> <td><math>\beta</math></td><td>Abbildungs-<br/>verhältnis</td><td>[1]</td></tr> </table> | $g$ | Gegenstands-<br>weite | [m] | $b$ | Bildweite | [m] | $f$ | Brennweite | [m] | $H_1$ | vorderer<br>Hauptpunkt |  | $H_2$ | hinterer<br>Hauptpunkt |  | $F_1$ | vorderer<br>Brennpunkt |  | $F_2$ | hinterer<br>Brennpunkt |  | $G$ | Gegenstands-<br>grösse | [m] | $B$ | Bildgrösse | [m] | $\beta$ | Abbildungs-<br>verhältnis | [1] |
| $g$   | Gegenstands-<br>weite   | [m]  |     |                       |     |     |           |     |     |            |     |       |                        |  |       |                        |  |       |                        |  |       |                        |  |     |                        |     |     |            |     |         |                           |     |
| $b$   | Bildweite   | [m]  |     |                       |     |     |           |     |     |            |     |       |                        |  |       |                        |  |       |                        |  |       |                        |  |     |                        |     |     |            |     |         |                           |     |
| $f$   | Brennweite  | [m]  |     |                       |     |     |           |     |     |            |     |       |                        |  |       |                        |  |       |                        |  |       |                        |  |     |                        |     |     |            |     |         |                           |     |
| $H_1$   | vorderer<br>Hauptpunkt  |  |     |                       |     |     |           |     |     |            |     |       |                        |  |       |                        |  |       |                        |  |       |                        |  |     |                        |     |     |            |     |         |                           |     |
| $H_2$   | hinterer<br>Hauptpunkt  |  |     |                       |     |     |           |     |     |            |     |       |                        |  |       |                        |  |       |                        |  |       |                        |  |     |                        |     |     |            |     |         |                           |     |
| $F_1$   | vorderer<br>Brennpunkt  |  |     |                       |     |     |           |     |     |            |     |       |                        |  |       |                        |  |       |                        |  |       |                        |  |     |                        |     |     |            |     |         |                           |     |
| $F_2$   | hinterer<br>Brennpunkt  |  |     |                       |     |     |           |     |     |            |     |       |                        |  |       |                        |  |       |                        |  |       |                        |  |     |                        |     |     |            |     |         |                           |     |
| $G$   | Gegenstands-<br>grösse  | [m]  |     |                       |     |     |           |     |     |            |     |       |                        |  |       |                        |  |       |                        |  |       |                        |  |     |                        |     |     |            |     |         |                           |     |
| $B$   | Bildgrösse  | [m]  |     |                       |     |     |           |     |     |            |     |       |                        |  |       |                        |  |       |                        |  |       |                        |  |     |                        |     |     |            |     |         |                           |     |
| $\beta$   | Abbildungs-<br>verhältnis   | [1]  |     |                       |     |     |           |     |     |            |     |       |                        |  |       |                        |  |       |                        |  |       |                        |  |     |                        |     |     |            |     |         |                           |     |

### 1.5.2. Spiegel

#### Parabolspiegel

Bei Parabolspiegeln treffen sich alle parallel einfallenden Strahlen in einem Punkt (Brennpunkt) auf der optische Achse.

#### Elliptische Spiegel

Alle Strahlen die vom einen Brennpunkt ausgesendet werden, treffen auf den zweiten Brennpunkt. (Ellipse ist der geometrische Ort aller Punkte einer Ebene, für die die Summe ihrer Abstände von zwei festen Punkten  $F_1$  und  $F_2$  konstant ist.)

#### Hyperbolische Spiegel

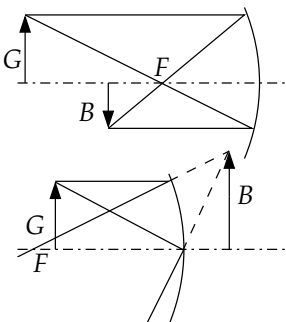
Alle Strahlen, die von einem Brennpunkt ausgesendet werden, verlaufen nach der Reflexion so, als wären sie vom anderen der beiden Brennpunkte ausgesendet worden. (Hyperbel ist der geometrische Ort aller Punkte einer Ebene, für die die Differenz ihrer Abstände von zwei festen Punkten  $F_1$  und  $F_2$  konstant ist.)

#### Sphärische Spiegel

Die spiegelnde Fläche ist ein Teil einer Kugel. Wenn nur ein kleiner Ausschnitt der Kugel- fläche verwendet wird, gehen parallel einfallende Strahlen näherungsweise durch einen Brennpunkt:  $f = r/2$ .

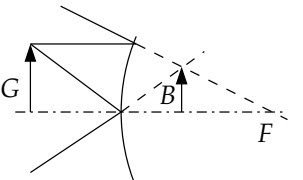
### 1.5.3. Abbildungen durch Spiegel

#### Konkavspiegel

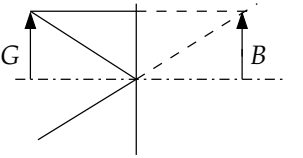
|   |  |  |     |            |       |     |      |       |     |            |  |
|---|--|--|-----|------------|-------|-----|------|-------|-----|------------|--|
|  | <p>Befindet sich der Gegenstand ausserhalb der Brennweite, so entsteht ein reelles Bild, anderseits ist das Bild virtuell.</p> | <table> <tr> <td><math>G</math></td><td>Gegenstand</td><td><math>[m]</math></td></tr> <tr> <td><math>B</math></td><td>Bild</td><td><math>[m]</math></td></tr> <tr> <td><math>F</math></td><td>Brennpunkt</td><td></td></tr> </table> | $G$ | Gegenstand | $[m]$ | $B$ | Bild | $[m]$ | $F$ | Brennpunkt |  |
| $G$   | Gegenstand   | $[m]$  |     |            |       |     |      |       |     |            |  |
| $B$   | Bild   | $[m]$  |     |            |       |     |      |       |     |            |  |
| $F$   | Brennpunkt   |  |     |            |       |     |      |       |     |            |  |

## 1. GEOMETRISCHE OPTIK

### Konvexspiegel

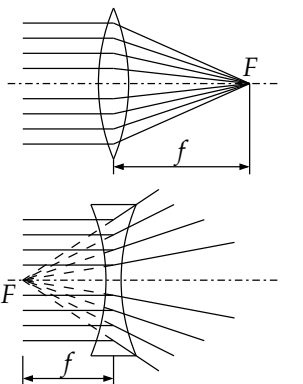
|   |   |  |     |            |       |     |      |       |     |            |  |
|---|---|--|-----|------------|-------|-----|------|-------|-----|------------|--|
|  | <p>Konvexspiegel haben stets virtuelle Bilder bei reellen Gegenständen.</p> | <table> <tr> <td><math>G</math></td><td>Gegenstand</td><td><math>[m]</math></td></tr> <tr> <td><math>B</math></td><td>Bild</td><td><math>[m]</math></td></tr> <tr> <td><math>F</math></td><td>Brennpunkt</td><td></td></tr> </table> | $G$ | Gegenstand | $[m]$ | $B$ | Bild | $[m]$ | $F$ | Brennpunkt |  |
| $G$   | Gegenstand  | $[m]$  |     |            |       |     |      |       |     |            |  |
| $B$   | Bild  | $[m]$  |     |            |       |     |      |       |     |            |  |
| $F$   | Brennpunkt  |  |     |            |       |     |      |       |     |            |  |

### Planspiegel

|   |  |  |     |            |       |     |      |       |     |            |  |
|---|--|--|-----|------------|-------|-----|------|-------|-----|------------|--|
|  | <p>Das Virtuelle Bild ist gleich gross wie der Gegenstand. Der Brennpunkt <math>F</math> liegt im Unendlichen. Der Planspiegel ist ein Spezialfall des Konvexspiegels.</p> | <table> <tr> <td><math>G</math></td><td>Gegenstand</td><td><math>[m]</math></td></tr> <tr> <td><math>B</math></td><td>Bild</td><td><math>[m]</math></td></tr> <tr> <td><math>F</math></td><td>Brennpunkt</td><td></td></tr> </table> | $G$ | Gegenstand | $[m]$ | $B$ | Bild | $[m]$ | $F$ | Brennpunkt |  |
| $G$   | Gegenstand   | $[m]$  |     |            |       |     |      |       |     |            |  |
| $B$   | Bild   | $[m]$  |     |            |       |     |      |       |     |            |  |
| $F$   | Brennpunkt   |  |     |            |       |     |      |       |     |            |  |

## 1.5.4. Linsen

### Linsentypen

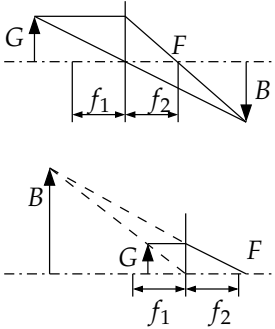
|   |   |   |     |                    |       |     |                    |       |     |            |       |     |            |         |       |            |       |       |         |       |           |              |       |
|---|---|---|-----|--------------------|-------|-----|--------------------|-------|-----|------------|-------|-----|------------|---------|-------|------------|-------|-------|---------|-------|-----------|--------------|-------|
|  | $q = \frac{d}{f}$ $D = \frac{1}{f}$ <p>Linsenschleifergleichung:</p> $D = \left( \frac{n_2}{n_1} - 1 \right) \left( \frac{1}{r_1} + \frac{1}{r_2} \right)$ <p>Falls das Linsenmaterial optisch dichter ist als das umgebende Medium, zeigt die obere Abbildung eine Sammel- und die untere eine Streulinse.</p> | <table> <tr> <td><math>q</math></td><td>Öffnungsverhältnis</td><td><math>[m]</math></td></tr> <tr> <td><math>d</math></td><td>effektiver Durchm.</td><td><math>[m]</math></td></tr> <tr> <td><math>f</math></td><td>Brennweite</td><td><math>[m]</math></td></tr> <tr> <td><math>D</math></td><td>Brechkraft</td><td><math>[dpt]</math></td></tr> <tr> <td><math>n_1</math></td><td>n-Umgebung</td><td><math>[1]</math></td></tr> <tr> <td><math>n_2</math></td><td>n-Linse</td><td><math>[1]</math></td></tr> <tr> <td><math>r_{1,2}</math></td><td>Linsenradien</td><td><math>[m]</math></td></tr> </table> | $q$ | Öffnungsverhältnis | $[m]$ | $d$ | effektiver Durchm. | $[m]$ | $f$ | Brennweite | $[m]$ | $D$ | Brechkraft | $[dpt]$ | $n_1$ | n-Umgebung | $[1]$ | $n_2$ | n-Linse | $[1]$ | $r_{1,2}$ | Linsenradien | $[m]$ |
| $q$   | Öffnungsverhältnis  | $[m]$   |     |                    |       |     |                    |       |     |            |       |     |            |         |       |            |       |       |         |       |           |              |       |
| $d$   | effektiver Durchm.  | $[m]$   |     |                    |       |     |                    |       |     |            |       |     |            |         |       |            |       |       |         |       |           |              |       |
| $f$   | Brennweite  | $[m]$   |     |                    |       |     |                    |       |     |            |       |     |            |         |       |            |       |       |         |       |           |              |       |
| $D$   | Brechkraft  | $[dpt]$   |     |                    |       |     |                    |       |     |            |       |     |            |         |       |            |       |       |         |       |           |              |       |
| $n_1$   | n-Umgebung  | $[1]$   |     |                    |       |     |                    |       |     |            |       |     |            |         |       |            |       |       |         |       |           |              |       |
| $n_2$   | n-Linse   | $[1]$   |     |                    |       |     |                    |       |     |            |       |     |            |         |       |            |       |       |         |       |           |              |       |
| $r_{1,2}$   | Linsenradien  | $[m]$   |     |                    |       |     |                    |       |     |            |       |     |            |         |       |            |       |       |         |       |           |              |       |

## Linsensysteme

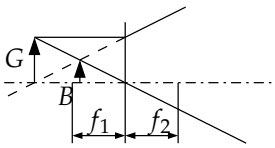
|  |   |  |
|--|---|--|
| Zwei Linsen mit Brennweiten $f_1$ , $f_2$ auf einer Achse ergeben eine Linse mit Brennweite $f$ , falls ihr Abstand $d$ kleiner $f_1$ ist. | $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ $D = D_1 + D_2 - d D_1 D_2$ | $f_{1,2}$ Brennweiten [m]<br>$D$ Brechkraft [dpt]<br>$d$ Linsenabstand [m] |
|--|---|--|

### 1.5.5. Abbildungen durch Linsen

#### Sammellinse

|  |   |   |
|--|---|---|
|  | <p>Der Gegenstand ist innerhalb der Brennweite <math>\Rightarrow</math> reelles Bild.</p> <p>Der Gegenstand ist ausserhalb der Brennweite <math>\Rightarrow</math> virtuelles Bild.</p> | $G$ Gegenstand [m]<br>$B$ Bild [m]<br>$F$ Brennpunkt<br>$f_{1,2}$ Brennweiten [m] |
|--|---|---|

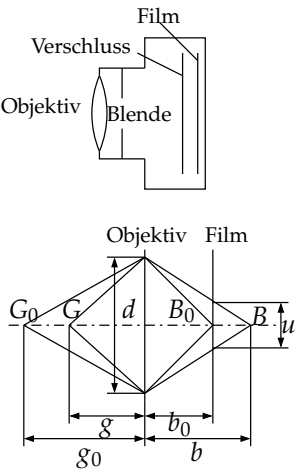
#### Zerstreuungslinse

|   |  |   |
|---|--|---|
|  | Bei Zerstreuungslinsen haben reelle Gegenstände stets virtuelle Bilder, unabhängig von ihrer Position. | $G$ Gegenstand [m]<br>$B$ Bild [m]<br>$F$ Brennpunkt<br>$f_{1,2}$ Brennweiten [m] |
|---|--|---|

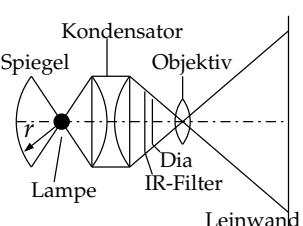
## 1. GEOMETRISCHE OPTIK

### 1.5.6. Optische Geräte

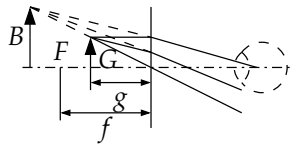
#### Fotoapparat

|   |  |  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |
|---|--|--|-----|------------|-------|-----|------------------|-------|-------|------------------------|-------|-----|-----------|-------|-----|------|-------|-----|------------|-------|-----|------------|-------|-----|--------------------------|-------|-----|------------|-------------------|-----|--------------------|-------|-----|---------------------|-------|-----|------------|-------|-----|-----------------|-------|-----|-------------------------|-------|
|  | $B = \frac{f}{g-f} G$ $I \approx d^2$ $H \approx \frac{1}{B^2} \approx \frac{d^2}{f^2}$ $E = Ht$ $q = \frac{d}{f} \quad Z = \frac{1}{q}$ $E \approx q^2 t$ $\frac{u}{d} = \frac{b - b_0}{b}$ $\frac{1}{g} = \frac{1}{g_0} \pm \frac{u}{q f^2}$ $g > g_0 \Rightarrow - \quad g < g_0 \Rightarrow +$ | <table> <tr> <td><math>G</math></td><td>Gegenstand</td><td><math>[m]</math></td></tr> <tr> <td><math>g</math></td><td>Gegenstandsweite</td><td><math>[m]</math></td></tr> <tr> <td><math>g_0</math></td><td>Schärfentieffenbereich</td><td><math>[m]</math></td></tr> <tr> <td><math>B</math></td><td>Bildweite</td><td><math>[m]</math></td></tr> <tr> <td><math>b</math></td><td>Bild</td><td><math>[m]</math></td></tr> <tr> <td><math>f</math></td><td>Brennweite</td><td><math>[m]</math></td></tr> <tr> <td><math>I</math></td><td>Lichtstrom</td><td><math>[W]</math></td></tr> <tr> <td><math>d</math></td><td>Durchm. Eintrittspupille</td><td><math>[m]</math></td></tr> <tr> <td><math>H</math></td><td>Helligkeit</td><td><math>[\frac{W}{m^2}]</math></td></tr> <tr> <td><math>q</math></td><td>Öffnungsverhältnis</td><td><math>[1]</math></td></tr> <tr> <td><math>Z</math></td><td>Brenndeneinstellung</td><td><math>[1]</math></td></tr> <tr> <td><math>E</math></td><td>Belichtung</td><td><math>[1]</math></td></tr> <tr> <td><math>t</math></td><td>Belichtungszeit</td><td><math>[s]</math></td></tr> <tr> <td><math>u</math></td><td>Durchm. Unschärfenkreis</td><td><math>[m]</math></td></tr> </table> | $G$ | Gegenstand | $[m]$ | $g$ | Gegenstandsweite | $[m]$ | $g_0$ | Schärfentieffenbereich | $[m]$ | $B$ | Bildweite | $[m]$ | $b$ | Bild | $[m]$ | $f$ | Brennweite | $[m]$ | $I$ | Lichtstrom | $[W]$ | $d$ | Durchm. Eintrittspupille | $[m]$ | $H$ | Helligkeit | $[\frac{W}{m^2}]$ | $q$ | Öffnungsverhältnis | $[1]$ | $Z$ | Brenndeneinstellung | $[1]$ | $E$ | Belichtung | $[1]$ | $t$ | Belichtungszeit | $[s]$ | $u$ | Durchm. Unschärfenkreis | $[m]$ |
| $G$   | Gegenstand   | $[m]$  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |
| $g$   | Gegenstandsweite   | $[m]$  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |
| $g_0$   | Schärfentieffenbereich   | $[m]$  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |
| $B$   | Bildweite  | $[m]$  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |
| $b$   | Bild   | $[m]$  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |
| $f$   | Brennweite   | $[m]$  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |
| $I$   | Lichtstrom   | $[W]$  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |
| $d$   | Durchm. Eintrittspupille   | $[m]$  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |
| $H$   | Helligkeit   | $[\frac{W}{m^2}]$  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |
| $q$   | Öffnungsverhältnis   | $[1]$  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |
| $Z$   | Brenndeneinstellung  | $[1]$  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |
| $E$   | Belichtung   | $[1]$  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |
| $t$   | Belichtungszeit  | $[s]$  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |
| $u$   | Durchm. Unschärfenkreis  | $[m]$  |     |            |       |     |                  |       |       |                        |       |     |           |       |     |      |       |     |            |       |     |            |       |     |                          |       |     |            |                   |     |                    |       |     |                     |       |     |            |       |     |                 |       |     |                         |       |

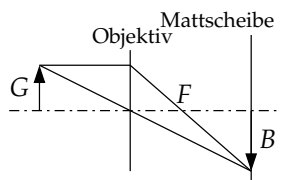
#### Projektor

|   |  |   |       |            |       |       |               |       |
|---|--|---|-------|------------|-------|-------|---------------|-------|
|  | <p>Das Dia wird im Objektiv abgebildet <math>\Rightarrow g_2 = b_1</math><br/>         Das Bild der Lampe muss im Objektiv sein.</p> | <table> <tr> <td><math>g_2</math></td><td>g-Objektiv</td><td><math>[m]</math></td></tr> <tr> <td><math>b_1</math></td><td>b-Kondensator</td><td><math>[m]</math></td></tr> </table> | $g_2$ | g-Objektiv | $[m]$ | $b_1$ | b-Kondensator | $[m]$ |
| $g_2$   | g-Objektiv   | $[m]$   |       |            |       |       |               |       |
| $b_1$   | b-Kondensator  | $[m]$   |       |            |       |       |               |       |

## Lupe

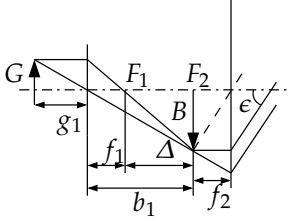
|   |   |   |     |            |       |     |                       |       |     |           |       |     |      |       |     |            |       |            |           |         |  |            |  |              |           |         |  |           |  |     |                         |       |     |              |       |  |               |  |
|---|---|---|-----|------------|-------|-----|-----------------------|-------|-----|-----------|-------|-----|------|-------|-----|------------|-------|------------|-----------|---------|--|------------|--|--------------|-----------|---------|--|-----------|--|-----|-------------------------|-------|-----|--------------|-------|--|---------------|--|
|  <p>Sammellinse zur Vergrößerung des Seh winkels (Bild im Unendlichen)</p> | <p>Gegenstand in Brennweite <math>\Rightarrow</math> Sehwin-<br/>kel <math>\epsilon</math> ist unabhängig von der Augen-<br/>position</p> $V = \frac{\tan \epsilon}{\tan \epsilon_0}$ $V = \frac{s}{g} > V_{normal}$ $\tan \epsilon' = \frac{G}{f}$ $\tan \epsilon_0 = \frac{G}{s}$ $V = \frac{s}{f}$ | <table> <tr> <td><math>G</math></td><td>Gegenstand</td><td><math>[m]</math></td></tr> <tr> <td><math>g</math></td><td>Gegenstands-<br/>weite</td><td><math>[m]</math></td></tr> <tr> <td><math>B</math></td><td>Bildweite</td><td><math>[m]</math></td></tr> <tr> <td><math>b</math></td><td>Bild</td><td><math>[m]</math></td></tr> <tr> <td><math>f</math></td><td>Brennweite</td><td><math>[m]</math></td></tr> <tr> <td><math>\epsilon</math></td><td>Sehwinkel</td><td><math>[rad]</math></td></tr> <tr> <td></td><td>durch Lupe</td><td></td></tr> <tr> <td><math>\epsilon_0</math></td><td>Sehwinkel</td><td><math>[rad]</math></td></tr> <tr> <td></td><td>ohne Lupe</td><td></td></tr> <tr> <td><math>s</math></td><td>deutliche Seh-<br/>weite</td><td><math>[m]</math></td></tr> <tr> <td><math>V</math></td><td>Vergrößerung</td><td><math>[1]</math></td></tr> <tr> <td></td><td>(max. ca. 25)</td><td></td></tr> </table> | $G$ | Gegenstand | $[m]$ | $g$ | Gegenstands-<br>weite | $[m]$ | $B$ | Bildweite | $[m]$ | $b$ | Bild | $[m]$ | $f$ | Brennweite | $[m]$ | $\epsilon$ | Sehwinkel | $[rad]$ |  | durch Lupe |  | $\epsilon_0$ | Sehwinkel | $[rad]$ |  | ohne Lupe |  | $s$ | deutliche Seh-<br>weite | $[m]$ | $V$ | Vergrößerung | $[1]$ |  | (max. ca. 25) |  |
| $G$   | Gegenstand  | $[m]$   |     |            |       |     |                       |       |     |           |       |     |      |       |     |            |       |            |           |         |  |            |  |              |           |         |  |           |  |     |                         |       |     |              |       |  |               |  |
| $g$   | Gegenstands-<br>weite   | $[m]$   |     |            |       |     |                       |       |     |           |       |     |      |       |     |            |       |            |           |         |  |            |  |              |           |         |  |           |  |     |                         |       |     |              |       |  |               |  |
| $B$   | Bildweite   | $[m]$   |     |            |       |     |                       |       |     |           |       |     |      |       |     |            |       |            |           |         |  |            |  |              |           |         |  |           |  |     |                         |       |     |              |       |  |               |  |
| $b$   | Bild  | $[m]$   |     |            |       |     |                       |       |     |           |       |     |      |       |     |            |       |            |           |         |  |            |  |              |           |         |  |           |  |     |                         |       |     |              |       |  |               |  |
| $f$   | Brennweite  | $[m]$   |     |            |       |     |                       |       |     |           |       |     |      |       |     |            |       |            |           |         |  |            |  |              |           |         |  |           |  |     |                         |       |     |              |       |  |               |  |
| $\epsilon$  | Sehwinkel   | $[rad]$   |     |            |       |     |                       |       |     |           |       |     |      |       |     |            |       |            |           |         |  |            |  |              |           |         |  |           |  |     |                         |       |     |              |       |  |               |  |
|   | durch Lupe  |   |     |            |       |     |                       |       |     |           |       |     |      |       |     |            |       |            |           |         |  |            |  |              |           |         |  |           |  |     |                         |       |     |              |       |  |               |  |
| $\epsilon_0$  | Sehwinkel   | $[rad]$   |     |            |       |     |                       |       |     |           |       |     |      |       |     |            |       |            |           |         |  |            |  |              |           |         |  |           |  |     |                         |       |     |              |       |  |               |  |
|   | ohne Lupe   |   |     |            |       |     |                       |       |     |           |       |     |      |       |     |            |       |            |           |         |  |            |  |              |           |         |  |           |  |     |                         |       |     |              |       |  |               |  |
| $s$   | deutliche Seh-<br>weite   | $[m]$   |     |            |       |     |                       |       |     |           |       |     |      |       |     |            |       |            |           |         |  |            |  |              |           |         |  |           |  |     |                         |       |     |              |       |  |               |  |
| $V$   | Vergrößerung  | $[1]$   |     |            |       |     |                       |       |     |           |       |     |      |       |     |            |       |            |           |         |  |            |  |              |           |         |  |           |  |     |                         |       |     |              |       |  |               |  |
|   | (max. ca. 25)   |   |     |            |       |     |                       |       |     |           |       |     |      |       |     |            |       |            |           |         |  |            |  |              |           |         |  |           |  |     |                         |       |     |              |       |  |               |  |

## Mikroprojektor

|  |  |   |     |            |       |     |                       |       |     |           |       |     |      |       |     |              |       |
|--|--|---|-----|------------|-------|-----|-----------------------|-------|-----|-----------|-------|-----|------|-------|-----|--------------|-------|
|  <p>Das reelle Bild einer Sam-<br/>mellinse wird verwendet<br/>und auf einer Mattscheibe<br/>abgebildet</p> | <p>Bild aus deutlicher Sehweite betrach-<br/>tet:</p> $V = \frac{B}{G} = \frac{b}{g}$ <p>Stahlengang siehe Projektor</p> | <table> <tr> <td><math>G</math></td><td>Gegenstand</td><td><math>[m]</math></td></tr> <tr> <td><math>g</math></td><td>Gegenstands-<br/>weite</td><td><math>[m]</math></td></tr> <tr> <td><math>B</math></td><td>Bildweite</td><td><math>[m]</math></td></tr> <tr> <td><math>b</math></td><td>Bild</td><td><math>[m]</math></td></tr> <tr> <td><math>V</math></td><td>Vergrößerung</td><td><math>[1]</math></td></tr> </table> | $G$ | Gegenstand | $[m]$ | $g$ | Gegenstands-<br>weite | $[m]$ | $B$ | Bildweite | $[m]$ | $b$ | Bild | $[m]$ | $V$ | Vergrößerung | $[1]$ |
| $G$  | Gegenstand   | $[m]$   |     |            |       |     |                       |       |     |           |       |     |      |       |     |              |       |
| $g$  | Gegenstands-<br>weite  | $[m]$   |     |            |       |     |                       |       |     |           |       |     |      |       |     |              |       |
| $B$  | Bildweite  | $[m]$   |     |            |       |     |                       |       |     |           |       |     |      |       |     |              |       |
| $b$  | Bild   | $[m]$   |     |            |       |     |                       |       |     |           |       |     |      |       |     |              |       |
| $V$  | Vergrößerung   | $[1]$   |     |            |       |     |                       |       |     |           |       |     |      |       |     |              |       |

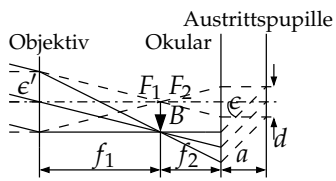
## 1. GEOMETRISCHE OPTIK

### Mikroskop

|   |  |  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |
|---|--|--|-----|------------|-------|-------|------------------|-------|-----|------|-------|-------|-----------|-------|-------|---------------------|--|-------|-------------------|--|-------|---------------------|-------|-------|-------------------|-------|----------|------------|-------|------------|-----------|---------|-----|--------------------|-------|-----|--------------------|-------|-------|------------|-------|-------|----------|-------|
|  <p>Das Objektiv verhält sich wie ein Mikroprojektor. Sein Bild wird durch das Okular, welches sich wie eine Lupe verhält, betrachtet.</p> | $V = V_1 V_2$ $V = \frac{\tan \epsilon}{\tan \epsilon_0}$ $V = \frac{B}{G} \frac{f}{f_2}$ $V = \frac{\Delta}{f_1} \frac{s}{f_2}$ $V_1 = \frac{\Delta}{f_1}$ $V_2 = \frac{s}{f_2}$ $\Delta = b_1 - f_1$ | <table> <tr> <td><math>G</math></td><td>Gegenstand</td><td><math>[m]</math></td></tr> <tr> <td><math>g_1</math></td><td>Gegenstandsweite</td><td><math>[m]</math></td></tr> <tr> <td><math>B</math></td><td>Bild</td><td><math>[m]</math></td></tr> <tr> <td><math>b_1</math></td><td>Bildweite</td><td><math>[m]</math></td></tr> <tr> <td><math>F_1</math></td><td>Brennpunkt Objektiv</td><td></td></tr> <tr> <td><math>F_2</math></td><td>Brennpunkt Okular</td><td></td></tr> <tr> <td><math>f_1</math></td><td>Brennweite Objektiv</td><td><math>[m]</math></td></tr> <tr> <td><math>f_2</math></td><td>Brennweite Okular</td><td><math>[m]</math></td></tr> <tr> <td><math>\Delta</math></td><td>Tubuslänge</td><td><math>[m]</math></td></tr> <tr> <td><math>\epsilon</math></td><td>Sehwinkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>s</math></td><td>deutliche Sehweite</td><td><math>[m]</math></td></tr> <tr> <td><math>V</math></td><td>Vergrößerung total</td><td><math>[1]</math></td></tr> <tr> <td><math>V_1</math></td><td>V-Objektiv</td><td><math>[1]</math></td></tr> <tr> <td><math>V_2</math></td><td>V-Okular</td><td><math>[1]</math></td></tr> </table> | $G$ | Gegenstand | $[m]$ | $g_1$ | Gegenstandsweite | $[m]$ | $B$ | Bild | $[m]$ | $b_1$ | Bildweite | $[m]$ | $F_1$ | Brennpunkt Objektiv |  | $F_2$ | Brennpunkt Okular |  | $f_1$ | Brennweite Objektiv | $[m]$ | $f_2$ | Brennweite Okular | $[m]$ | $\Delta$ | Tubuslänge | $[m]$ | $\epsilon$ | Sehwinkel | $[rad]$ | $s$ | deutliche Sehweite | $[m]$ | $V$ | Vergrößerung total | $[1]$ | $V_1$ | V-Objektiv | $[1]$ | $V_2$ | V-Okular | $[1]$ |
| $G$   | Gegenstand   | $[m]$  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |
| $g_1$   | Gegenstandsweite   | $[m]$  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |
| $B$   | Bild   | $[m]$  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |
| $b_1$   | Bildweite  | $[m]$  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |
| $F_1$   | Brennpunkt Objektiv  |  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |
| $F_2$   | Brennpunkt Okular  |  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |
| $f_1$   | Brennweite Objektiv  | $[m]$  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |
| $f_2$   | Brennweite Okular  | $[m]$  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |
| $\Delta$  | Tubuslänge   | $[m]$  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |
| $\epsilon$  | Sehwinkel  | $[rad]$  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |
| $s$   | deutliche Sehweite   | $[m]$  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |
| $V$   | Vergrößerung total   | $[1]$  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |
| $V_1$   | V-Objektiv   | $[1]$  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |
| $V_2$   | V-Okular   | $[1]$  |     |            |       |       |                  |       |     |      |       |       |           |       |       |                     |  |       |                   |  |       |                     |       |       |                   |       |          |            |       |            |           |         |     |                    |       |     |                    |       |       |            |       |       |          |       |



## Fernrohre



Ein Fernglas mit den Daten  $10 \times 50$  hat eine Vergrößerung von 10 und einen Objektivdurchmesser von 50 mm.

$$V = \frac{\tan \epsilon}{\tan \epsilon'}$$

$$\epsilon = V \epsilon'$$

$$V = \frac{f_1}{f_2}$$

$$\frac{1}{f_1 + f_2} + \frac{1}{a} = \frac{1}{f_2}$$

$$\frac{D}{d} = \frac{f_1 + f_2}{a} = V$$

$$a = \frac{l}{V} \quad d = \frac{D}{V}$$

$$L = d^2$$

$$L = \left( \frac{D}{V} \right)^2$$

$$l = f_1 + f_2$$

|             |                                 |       |
|-------------|---------------------------------|-------|
| $B$         | Bildweite                       | [m]   |
| $f_1$       | Brennweite Objektiv             | [m]   |
| $f_2$       | Brennweite Okular               | [m]   |
| $l$         | Fernrohlänge                    | [m]   |
| $\epsilon$  | Ausfallswinkel                  | [rad] |
| $\epsilon'$ | Einfallswinkel                  | [rad] |
| $s$         | deutliche Sehweite              | [m]   |
| $V$         | Vergrößerung total              | [1]   |
| $L$         | Lichtstärke                     | [1]   |
| $D$         | Durchm. Objektiv                | [m]   |
| $d$         | Durchm. Austrittspupille        | [mm]  |
| $a$         | Abstand Okular Austrittspupille | [m]   |

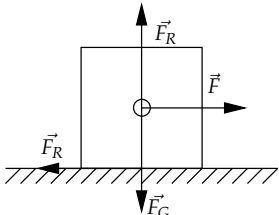
## 2. Statik

### 2.1. Starre Körper im Gleichgewicht

#### 2.1.1. Gleichgewichtsbedingung starrer Körper

|  |  |   |
|--|--|---|
| Ein Körper ist dann im Gleichgewicht, wenn keine resultierende Kraft auf ihn wirkt, d.h. die Summe der ihn angreifenden Kräfte ist null. | <p>Allgemein:</p> $\sum_{i=1}^n \vec{F}_i = 0 \quad \sum_{i=1}^n \vec{M}_i = 0$ <p>In Komponenten:</p> $\sum_{i=1}^n F_{ix} = 0 \quad \sum_{i=1}^n M_{ix} = 0$ $\sum_{i=1}^n F_{iy} = 0 \quad \sum_{i=1}^n M_{iy} = 0$ $\sum_{i=1}^n F_{iz} = 0 \quad \sum_{i=1}^n M_{iz} = 0$ | <p><math>F</math> Kraft [N]<br/> <math>M</math> Drehmoment [Nm]</p> |
|--|--|---|

#### 2.1.2. Haftreibung

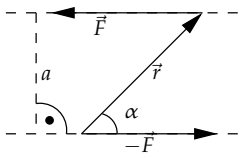
|   |  |   |
|---|--|---|
|  | $\vec{F}_N = \vec{F}_G \quad \vec{F}_R = \vec{F}$ $\vec{F}_R \leq \vec{F}_{Rmax} \leq \mu_H F_N$ | <p><math>F</math> Kraft [N]<br/> <math>F_G</math> Gewichtskraft [N]<br/> <math>F_N</math> Normalkraft [N]<br/> <math>F_R</math> Reibkraft [N]<br/> <math>\mu_H</math> Haftreibungskoeffizient [1]</p> |
|---|--|---|

#### 2.1.3. Reaktionsprinzip

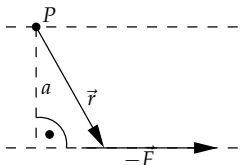
|  |                                |   |
|--|--------------------------------|---|
| Das Reaktionsprinzip gilt, wenn zwei Körper Kräfte auf einander ausüben. | $\vec{F}_{BA} = -\vec{F}_{AB}$ | <p><math>F_{AB}</math> Kraft von Körper A [N]<br/> <math>F_{BA}</math> Kraft von Körper B [N]</p> |
|--|--------------------------------|---|

### 2.1.4. Drehmoment

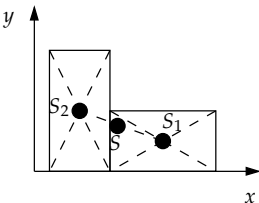
#### Drehmoment eines Kräftepaars

|   |   |   |     |            |        |     |         |       |     |       |       |     |        |       |
|---|---|---|-----|------------|--------|-----|---------|-------|-----|-------|-------|-----|--------|-------|
|  | $M = aF$ $\vec{M} \perp \text{ auf Ebene } \vec{r}, \vec{F} :$ $M = Fr \sin(\alpha)$ <p>Drehmomente nicht in einer Ebene:</p> $\vec{M} = \vec{r} \times \vec{F}$ <p>Drehsinn im Gegenuhrzeigersinn: +</p> | <table> <tr> <td><math>M</math></td><td>Drehmoment</td><td><math>[Nm]</math></td></tr> <tr> <td><math>a</math></td><td>Abstand</td><td><math>[m]</math></td></tr> <tr> <td><math>F</math></td><td>Kraft</td><td><math>[N]</math></td></tr> <tr> <td><math>r</math></td><td>Radius</td><td><math>[m]</math></td></tr> </table> | $M$ | Drehmoment | $[Nm]$ | $a$ | Abstand | $[m]$ | $F$ | Kraft | $[N]$ | $r$ | Radius | $[m]$ |
| $M$   | Drehmoment  | $[Nm]$  |     |            |        |     |         |       |     |       |       |     |        |       |
| $a$   | Abstand   | $[m]$   |     |            |        |     |         |       |     |       |       |     |        |       |
| $F$   | Kraft   | $[N]$   |     |            |        |     |         |       |     |       |       |     |        |       |
| $r$   | Radius  | $[m]$   |     |            |        |     |         |       |     |       |       |     |        |       |

#### Drehmoment einer Einzelkraft

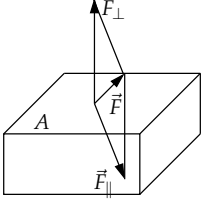
|  |   |   |     |            |        |     |         |       |     |       |       |     |        |       |
|--|---|---|-----|------------|--------|-----|---------|-------|-----|-------|-------|-----|--------|-------|
|  | $M = aF$ $\vec{M} = \vec{r} \times \vec{F}$ | <table> <tr> <td><math>M</math></td><td>Drehmoment</td><td><math>[Nm]</math></td></tr> <tr> <td><math>a</math></td><td>Abstand</td><td><math>[m]</math></td></tr> <tr> <td><math>F</math></td><td>Kraft</td><td><math>[N]</math></td></tr> <tr> <td><math>r</math></td><td>Radius</td><td><math>[m]</math></td></tr> </table> | $M$ | Drehmoment | $[Nm]$ | $a$ | Abstand | $[m]$ | $F$ | Kraft | $[N]$ | $r$ | Radius | $[m]$ |
| $M$  | Drehmoment                                  | $[Nm]$  |     |            |        |     |         |       |     |       |       |     |        |       |
| $a$  | Abstand                                     | $[m]$   |     |            |        |     |         |       |     |       |       |     |        |       |
| $F$  | Kraft                                       | $[N]$   |     |            |        |     |         |       |     |       |       |     |        |       |
| $r$  | Radius                                      | $[m]$   |     |            |        |     |         |       |     |       |       |     |        |       |

## 2.2. Schwerpunkt

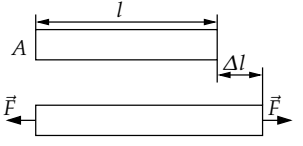
|   |   |   |                 |                                     |       |                 |                                      |       |     |        |       |
|---|---|---|-----------------|-------------------------------------|-------|-----------------|--------------------------------------|-------|-----|--------|-------|
|  | $x_s = \frac{\sum_i x_i m_i}{\sum_i m_i}$ $y_s = \frac{\sum_i y_i m_i}{\sum_i m_i}$ $z_s = \frac{\sum_i z_i m_i}{\sum_i m_i}$ <p>Schwerpunkt eines Halbkreises:</p> $x = 0 \quad y = \frac{4r}{3\pi}$ | <table> <tr> <td><math>x_s, y_s, z_s</math></td><td>Koordinaten des Gesamtschwerpunktes</td><td><math>[m]</math></td></tr> <tr> <td><math>x_i, y_i, z_i</math></td><td>Schwerpunktskoordinaten Teilkörper i</td><td><math>[m]</math></td></tr> <tr> <td><math>r</math></td><td>Radius</td><td><math>[m]</math></td></tr> </table> | $x_s, y_s, z_s$ | Koordinaten des Gesamtschwerpunktes | $[m]$ | $x_i, y_i, z_i$ | Schwerpunktskoordinaten Teilkörper i | $[m]$ | $r$ | Radius | $[m]$ |
| $x_s, y_s, z_s$   | Koordinaten des Gesamtschwerpunktes   | $[m]$   |                 |                                     |       |                 |                                      |       |     |        |       |
| $x_i, y_i, z_i$   | Schwerpunktskoordinaten Teilkörper i  | $[m]$   |                 |                                     |       |                 |                                      |       |     |        |       |
| $r$   | Radius  | $[m]$   |                 |                                     |       |                 |                                      |       |     |        |       |

## 2.3. Deformierung

### 2.3.1. Spannung

|   |   |   |          |             |                              |        |               |                              |     |       |                              |     |        |         |     |       |       |
|---|---|---|----------|-------------|------------------------------|--------|---------------|------------------------------|-----|-------|------------------------------|-----|--------|---------|-----|-------|-------|
|  | $\sigma = \frac{F_{\perp}}{A}$ $\tau = \frac{F_{\parallel}}{A}$ $p = -\sigma$ | <table> <tr> <td><math>\sigma</math></td><td>Zugspannung</td><td><math>\left[\frac{N}{m^2}\right]</math></td></tr> <tr> <td><math>\tau</math></td><td>Schubspannung</td><td><math>\left[\frac{N}{m^2}\right]</math></td></tr> <tr> <td><math>p</math></td><td>Druck</td><td><math>\left[\frac{N}{m^2}\right]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr> <td><math>F</math></td><td>Kraft</td><td><math>[N]</math></td></tr> </table> | $\sigma$ | Zugspannung | $\left[\frac{N}{m^2}\right]$ | $\tau$ | Schubspannung | $\left[\frac{N}{m^2}\right]$ | $p$ | Druck | $\left[\frac{N}{m^2}\right]$ | $A$ | Fläche | $[m^2]$ | $F$ | Kraft | $[N]$ |
| $\sigma$  | Zugspannung   | $\left[\frac{N}{m^2}\right]$  |          |             |                              |        |               |                              |     |       |                              |     |        |         |     |       |       |
| $\tau$  | Schubspannung   | $\left[\frac{N}{m^2}\right]$  |          |             |                              |        |               |                              |     |       |                              |     |        |         |     |       |       |
| $p$   | Druck   | $\left[\frac{N}{m^2}\right]$  |          |             |                              |        |               |                              |     |       |                              |     |        |         |     |       |       |
| $A$   | Fläche  | $[m^2]$   |          |             |                              |        |               |                              |     |       |                              |     |        |         |     |       |       |
| $F$   | Kraft   | $[N]$   |          |             |                              |        |               |                              |     |       |                              |     |        |         |     |       |       |

### 2.3.2. Dehnung

|   |  |   |            |         |       |     |                    |         |     |             |       |     |       |       |     |                   |                              |          |             |                              |
|---|--|---|------------|---------|-------|-----|--------------------|---------|-----|-------------|-------|-----|-------|-------|-----|-------------------|------------------------------|----------|-------------|------------------------------|
|  | $\epsilon = \frac{\Delta l}{l}$ $\Delta l \sim \frac{lF}{A}$ $\epsilon = \frac{1}{E}\sigma = \frac{1}{E}\frac{F}{A}$ | <table> <tr> <td><math>\epsilon</math></td><td>Dehnung</td><td><math>[1]</math></td></tr> <tr> <td><math>A</math></td><td>Querschnittsfläche</td><td><math>[m^2]</math></td></tr> <tr> <td><math>l</math></td><td>Balkenlänge</td><td><math>[m]</math></td></tr> <tr> <td><math>F</math></td><td>Kraft</td><td><math>[N]</math></td></tr> <tr> <td><math>E</math></td><td>Elastizitätsmodul</td><td><math>\left[\frac{N}{m^2}\right]</math></td></tr> <tr> <td><math>\sigma</math></td><td>Zugspannung</td><td><math>\left[\frac{N}{m^2}\right]</math></td></tr> </table> | $\epsilon$ | Dehnung | $[1]$ | $A$ | Querschnittsfläche | $[m^2]$ | $l$ | Balkenlänge | $[m]$ | $F$ | Kraft | $[N]$ | $E$ | Elastizitätsmodul | $\left[\frac{N}{m^2}\right]$ | $\sigma$ | Zugspannung | $\left[\frac{N}{m^2}\right]$ |
| $\epsilon$  | Dehnung  | $[1]$   |            |         |       |     |                    |         |     |             |       |     |       |       |     |                   |                              |          |             |                              |
| $A$   | Querschnittsfläche   | $[m^2]$   |            |         |       |     |                    |         |     |             |       |     |       |       |     |                   |                              |          |             |                              |
| $l$   | Balkenlänge  | $[m]$   |            |         |       |     |                    |         |     |             |       |     |       |       |     |                   |                              |          |             |                              |
| $F$   | Kraft  | $[N]$   |            |         |       |     |                    |         |     |             |       |     |       |       |     |                   |                              |          |             |                              |
| $E$   | Elastizitätsmodul  | $\left[\frac{N}{m^2}\right]$  |            |         |       |     |                    |         |     |             |       |     |       |       |     |                   |                              |          |             |                              |
| $\sigma$  | Zugspannung  | $\left[\frac{N}{m^2}\right]$  |            |         |       |     |                    |         |     |             |       |     |       |       |     |                   |                              |          |             |                              |

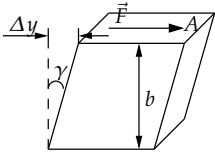
### 2.3.3. Querkontraktion

|  |  |  |              |                 |       |       |             |       |     |                |       |     |       |       |
|--|--|--|--------------|-----------------|-------|-------|-------------|-------|-----|----------------|-------|-----|-------|-------|
| <p>Die Querkontraktion entspricht dem Dünnerwerden eines Materials bei Dehnung</p> | $\epsilon_q = \frac{\Delta d}{d} = \frac{-\mu \Delta l}{l}$ $\epsilon_q = -\mu \epsilon$ | <table> <tr> <td><math>\epsilon_q</math></td><td>Querkontraktion</td><td><math>[1]</math></td></tr> <tr> <td><math>\mu</math></td><td>Poissonzahl</td><td><math>[1]</math></td></tr> <tr> <td><math>d</math></td><td>Dicke Material</td><td><math>[m]</math></td></tr> <tr> <td><math>l</math></td><td>Länge</td><td><math>[m]</math></td></tr> </table> | $\epsilon_q$ | Querkontraktion | $[1]$ | $\mu$ | Poissonzahl | $[1]$ | $d$ | Dicke Material | $[m]$ | $l$ | Länge | $[m]$ |
| $\epsilon_q$   | Querkontraktion  | $[1]$  |              |                 |       |       |             |       |     |                |       |     |       |       |
| $\mu$  | Poissonzahl  | $[1]$  |              |                 |       |       |             |       |     |                |       |     |       |       |
| $d$  | Dicke Material   | $[m]$  |              |                 |       |       |             |       |     |                |       |     |       |       |
| $l$  | Länge  | $[m]$  |              |                 |       |       |             |       |     |                |       |     |       |       |

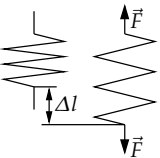
## 2.3.4. Kompression

|   |  |   |
|---|--|---|
| Wird ein Körper einem Druck ausgesetzt, spricht man von Kompression | $\frac{\Delta V}{V} = -\kappa \Delta p$ $\kappa = \frac{3(1-2\mu)}{E}$ | $V$ Volumen $[m^3]$<br>$p$ Druck $[\frac{N}{m^2}]$<br>$\kappa$ Kompressibilität $[\frac{m^2}{N}]$<br>$\mu$ Poissonzahl $[1]$<br>$E$ E-Modul $[\frac{N}{m^2}]$ |
|---|--|---|

## 2.3.5. Schubbeanspruchung

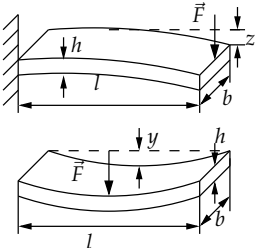
|   |  |  |
|---|--|--|
|  | $\frac{\Delta y}{b} = \frac{1}{G} \frac{F}{A} = \frac{1}{G} \tau$ $G = \frac{E}{2(1+\mu)}$ | $F$ Kraft $[N]$<br>$A$ Fläche $[m^2]$<br>$y$ Spaltbreite $[m]$<br>$\gamma$ Winkel $[rad]$<br>$b$ Körperbreite $[m]$<br>$G$ Schubmodul $[\frac{N}{m^2}]$<br>$\tau$ Schubspan. $[\frac{N}{m^2}]$<br>$\mu$ Poissonzahl $[1]$<br>$E$ E-Modul $[\frac{N}{m^2}]$ |
|---|--|--|

## 2.3.6. Schraubenfeder

|   |   |  |
|---|---|--|
|  | $F = c \Delta l$ $c = \frac{Gr^4}{4nR^3}$ <p>parallel: <math>c = c_1 + c_2 + \dots + c_n</math><br/> seriel: <math>\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2} + \dots + \frac{1}{c_n}</math></p> | $F$ Kraft $[N]$<br>$c$ Federkonst. $[\frac{N}{m}]$<br>$l$ Länge $[N]$<br>$G$ Schubmodul $[\frac{N}{m^2}]$<br>$r$ Radius Draht $[m]$<br>$R$ Radius Feder $[m]$<br>$n$ Windungen $[1]$ |
|---|---|--|

## 2. STATIK

### 2.3.7. Biegung eines Balkens

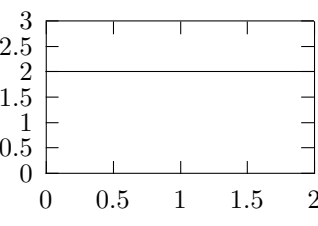
|   |   |   |     |              |       |     |              |       |     |       |       |     |              |       |     |            |       |     |       |       |        |        |                    |     |                   |                   |  |          |  |     |                   |                   |
|---|---|---|-----|--------------|-------|-----|--------------|-------|-----|-------|-------|-----|--------------|-------|-----|------------|-------|-----|-------|-------|--------|--------|--------------------|-----|-------------------|-------------------|--|----------|--|-----|-------------------|-------------------|
|  | $z = \frac{4l^3}{Eb h^3} F$ $y = \frac{5\rho g l^4}{32E h^2}$ | <table> <tr> <td><math>z</math></td><td>Durchbiegung</td><td><math>[m]</math></td></tr> <tr> <td><math>y</math></td><td>Durchbiegung</td><td><math>[m]</math></td></tr> <tr> <td><math>l</math></td><td>Länge</td><td><math>[m]</math></td></tr> <tr> <td><math>b</math></td><td>Balkenbreite</td><td><math>[m]</math></td></tr> <tr> <td><math>h</math></td><td>Balkenhöhe</td><td><math>[m]</math></td></tr> <tr> <td><math>F</math></td><td>Kraft</td><td><math>[N]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>[\frac{kg}{m^3}]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschleunigung</td><td><math>[\frac{m}{s^2}]</math></td></tr> <tr> <td></td><td><math>= 9,81</math></td><td></td></tr> <tr> <td><math>E</math></td><td>Elastizitätsmodul</td><td><math>[\frac{N}{m^2}]</math></td></tr> </table> | $z$ | Durchbiegung | $[m]$ | $y$ | Durchbiegung | $[m]$ | $l$ | Länge | $[m]$ | $b$ | Balkenbreite | $[m]$ | $h$ | Balkenhöhe | $[m]$ | $F$ | Kraft | $[N]$ | $\rho$ | Dichte | $[\frac{kg}{m^3}]$ | $g$ | Erdbeschleunigung | $[\frac{m}{s^2}]$ |  | $= 9,81$ |  | $E$ | Elastizitätsmodul | $[\frac{N}{m^2}]$ |
| $z$   | Durchbiegung  | $[m]$   |     |              |       |     |              |       |     |       |       |     |              |       |     |            |       |     |       |       |        |        |                    |     |                   |                   |  |          |  |     |                   |                   |
| $y$   | Durchbiegung  | $[m]$   |     |              |       |     |              |       |     |       |       |     |              |       |     |            |       |     |       |       |        |        |                    |     |                   |                   |  |          |  |     |                   |                   |
| $l$   | Länge   | $[m]$   |     |              |       |     |              |       |     |       |       |     |              |       |     |            |       |     |       |       |        |        |                    |     |                   |                   |  |          |  |     |                   |                   |
| $b$   | Balkenbreite  | $[m]$   |     |              |       |     |              |       |     |       |       |     |              |       |     |            |       |     |       |       |        |        |                    |     |                   |                   |  |          |  |     |                   |                   |
| $h$   | Balkenhöhe  | $[m]$   |     |              |       |     |              |       |     |       |       |     |              |       |     |            |       |     |       |       |        |        |                    |     |                   |                   |  |          |  |     |                   |                   |
| $F$   | Kraft   | $[N]$   |     |              |       |     |              |       |     |       |       |     |              |       |     |            |       |     |       |       |        |        |                    |     |                   |                   |  |          |  |     |                   |                   |
| $\rho$  | Dichte  | $[\frac{kg}{m^3}]$  |     |              |       |     |              |       |     |       |       |     |              |       |     |            |       |     |       |       |        |        |                    |     |                   |                   |  |          |  |     |                   |                   |
| $g$   | Erdbeschleunigung   | $[\frac{m}{s^2}]$   |     |              |       |     |              |       |     |       |       |     |              |       |     |            |       |     |       |       |        |        |                    |     |                   |                   |  |          |  |     |                   |                   |
|   | $= 9,81$  |   |     |              |       |     |              |       |     |       |       |     |              |       |     |            |       |     |       |       |        |        |                    |     |                   |                   |  |          |  |     |                   |                   |
| $E$   | Elastizitätsmodul   | $[\frac{N}{m^2}]$   |     |              |       |     |              |       |     |       |       |     |              |       |     |            |       |     |       |       |        |        |                    |     |                   |                   |  |          |  |     |                   |                   |

## 2.4. Vorgehen beim Lösen von Statikaufgaben

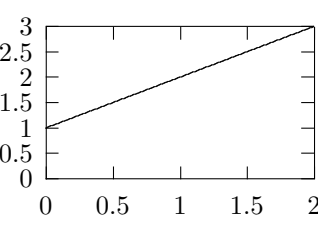
1. Skizze mit allen Kräften aufzeichnen
2. Koordinatensystem einführen
3. Falls notwendig einen Drehpunkt einführen
4. Gleichgewichtsbedingungs - Gleichungssystem aufstellen
5. Gleichungssystem auflösen

## 3. Kinematik

### 3.1. Gleichförmige Bewegung

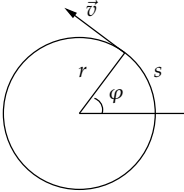
|   |  |   |     |                |                              |     |         |       |     |         |                            |     |      |       |       |                 |       |
|---|--|---|-----|----------------|------------------------------|-----|---------|-------|-----|---------|----------------------------|-----|------|-------|-------|-----------------|-------|
|  | $a = 0$<br>$v = \text{konstant}$<br>$s = vt + s_0$ | <table> <tr> <td><math>a</math></td><td>Beschleunigung</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> <tr> <td><math>s</math></td><td>Strecke</td><td><math>[m]</math></td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>s_0</math></td><td>Anfangs Strecke</td><td><math>[m]</math></td></tr> </table> | $a$ | Beschleunigung | $\left[\frac{m}{s^2}\right]$ | $s$ | Strecke | $[m]$ | $v$ | Geschw. | $\left[\frac{m}{s}\right]$ | $t$ | Zeit | $[s]$ | $s_0$ | Anfangs Strecke | $[m]$ |
| $a$   | Beschleunigung                                     | $\left[\frac{m}{s^2}\right]$  |     |                |                              |     |         |       |     |         |                            |     |      |       |       |                 |       |
| $s$   | Strecke  | $[m]$   |     |                |                              |     |         |       |     |         |                            |     |      |       |       |                 |       |
| $v$   | Geschw.  | $\left[\frac{m}{s}\right]$  |     |                |                              |     |         |       |     |         |                            |     |      |       |       |                 |       |
| $t$   | Zeit   | $[s]$   |     |                |                              |     |         |       |     |         |                            |     |      |       |       |                 |       |
| $s_0$   | Anfangs Strecke                                    | $[m]$   |     |                |                              |     |         |       |     |         |                            |     |      |       |       |                 |       |

### 3.2. Gleichförmig beschleunigte Bewegung

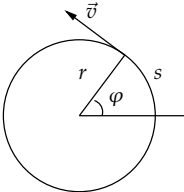
|   |   |   |     |                |                              |     |         |       |     |         |                            |       |               |                            |       |             |                            |     |      |       |       |                 |       |
|---|---|---|-----|----------------|------------------------------|-----|---------|-------|-----|---------|----------------------------|-------|---------------|----------------------------|-------|-------------|----------------------------|-----|------|-------|-------|-----------------|-------|
|  | $a = \text{konstant}$<br>$v = at + v_0$<br>$v = \dot{s}$<br>$s = \frac{a}{2}t^2 + v_0t + s_0$<br>$s = \frac{v_1^2 - v_0^2}{2a}$<br>$a = \frac{v_1^2 - v_0^2}{2s} = \frac{\Delta v}{\Delta t}$<br>$a = \dot{v} = \ddot{s}$<br>$v_2 = \sqrt{v_0^2 + 2as}$ | <table> <tr> <td><math>a</math></td><td>Beschleunigung</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> <tr> <td><math>s</math></td><td>Strecke</td><td><math>[m]</math></td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>v_0</math></td><td>Start-Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>v_1</math></td><td>End-Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>s_0</math></td><td>Anfangs Strecke</td><td><math>[m]</math></td></tr> </table> | $a$ | Beschleunigung | $\left[\frac{m}{s^2}\right]$ | $s$ | Strecke | $[m]$ | $v$ | Geschw. | $\left[\frac{m}{s}\right]$ | $v_0$ | Start-Geschw. | $\left[\frac{m}{s}\right]$ | $v_1$ | End-Geschw. | $\left[\frac{m}{s}\right]$ | $t$ | Zeit | $[s]$ | $s_0$ | Anfangs Strecke | $[m]$ |
| $a$   | Beschleunigung  | $\left[\frac{m}{s^2}\right]$  |     |                |                              |     |         |       |     |         |                            |       |               |                            |       |             |                            |     |      |       |       |                 |       |
| $s$   | Strecke   | $[m]$   |     |                |                              |     |         |       |     |         |                            |       |               |                            |       |             |                            |     |      |       |       |                 |       |
| $v$   | Geschw.   | $\left[\frac{m}{s}\right]$  |     |                |                              |     |         |       |     |         |                            |       |               |                            |       |             |                            |     |      |       |       |                 |       |
| $v_0$   | Start-Geschw.   | $\left[\frac{m}{s}\right]$  |     |                |                              |     |         |       |     |         |                            |       |               |                            |       |             |                            |     |      |       |       |                 |       |
| $v_1$   | End-Geschw.   | $\left[\frac{m}{s}\right]$  |     |                |                              |     |         |       |     |         |                            |       |               |                            |       |             |                            |     |      |       |       |                 |       |
| $t$   | Zeit  | $[s]$   |     |                |                              |     |         |       |     |         |                            |       |               |                            |       |             |                            |     |      |       |       |                 |       |
| $s_0$   | Anfangs Strecke   | $[m]$   |     |                |                              |     |         |       |     |         |                            |       |               |                            |       |             |                            |     |      |       |       |                 |       |

### 3.3. Drehbewegung

#### 3.3.1. Gleichförmige Kreisbewegung

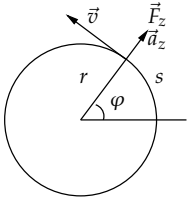
|  |  |  |          |                       |                     |          |                |                   |           |        |         |     |        |       |     |         |       |     |          |                 |     |         |       |     |         |                 |     |      |       |       |                 |       |     |               |                 |
|--|--|--|----------|-----------------------|---------------------|----------|----------------|-------------------|-----------|--------|---------|-----|--------|-------|-----|---------|-------|-----|----------|-----------------|-----|---------|-------|-----|---------|-----------------|-----|------|-------|-------|-----------------|-------|-----|---------------|-----------------|
|  <p>Analogie:</p> <p><math>s \rightarrow \varphi</math></p> <p><math>v \rightarrow \omega</math></p> <p><math>a \rightarrow \alpha</math></p> | $\alpha = 0$<br>$s = r\varphi$<br>$v = r\omega$<br>$\omega = \dot{\varphi} = \frac{v}{r}$<br>$\varphi = \omega t$<br>$f = \frac{1}{T}$<br>$\omega = 2\pi f$<br>$\vec{v} = \vec{\omega} \times \vec{r}$ | <table> <tr> <td><math>\alpha</math></td><td>Winkel-Beschleunigung</td><td><math>[\frac{rad}{s^2}]</math></td></tr> <tr> <td><math>\omega</math></td><td>Winkel-Geschw.</td><td><math>[\frac{rad}{s}]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Winkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr> <td><math>T</math></td><td>Periode</td><td><math>[s]</math></td></tr> <tr> <td><math>f</math></td><td>Frequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>s</math></td><td>Strecke</td><td><math>[m]</math></td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>[\frac{m}{s}]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>s_0</math></td><td>Anfangs Strecke</td><td><math>[m]</math></td></tr> <tr> <td><math>v</math></td><td>Geschw. tang.</td><td><math>[\frac{m}{s}]</math></td></tr> </table> | $\alpha$ | Winkel-Beschleunigung | $[\frac{rad}{s^2}]$ | $\omega$ | Winkel-Geschw. | $[\frac{rad}{s}]$ | $\varphi$ | Winkel | $[rad]$ | $r$ | Radius | $[m]$ | $T$ | Periode | $[s]$ | $f$ | Frequenz | $[\frac{1}{s}]$ | $s$ | Strecke | $[m]$ | $v$ | Geschw. | $[\frac{m}{s}]$ | $t$ | Zeit | $[s]$ | $s_0$ | Anfangs Strecke | $[m]$ | $v$ | Geschw. tang. | $[\frac{m}{s}]$ |
| $\alpha$   | Winkel-Beschleunigung  | $[\frac{rad}{s^2}]$  |          |                       |                     |          |                |                   |           |        |         |     |        |       |     |         |       |     |          |                 |     |         |       |     |         |                 |     |      |       |       |                 |       |     |               |                 |
| $\omega$   | Winkel-Geschw.   | $[\frac{rad}{s}]$  |          |                       |                     |          |                |                   |           |        |         |     |        |       |     |         |       |     |          |                 |     |         |       |     |         |                 |     |      |       |       |                 |       |     |               |                 |
| $\varphi$  | Winkel   | $[rad]$  |          |                       |                     |          |                |                   |           |        |         |     |        |       |     |         |       |     |          |                 |     |         |       |     |         |                 |     |      |       |       |                 |       |     |               |                 |
| $r$  | Radius   | $[m]$  |          |                       |                     |          |                |                   |           |        |         |     |        |       |     |         |       |     |          |                 |     |         |       |     |         |                 |     |      |       |       |                 |       |     |               |                 |
| $T$  | Periode  | $[s]$  |          |                       |                     |          |                |                   |           |        |         |     |        |       |     |         |       |     |          |                 |     |         |       |     |         |                 |     |      |       |       |                 |       |     |               |                 |
| $f$  | Frequenz   | $[\frac{1}{s}]$  |          |                       |                     |          |                |                   |           |        |         |     |        |       |     |         |       |     |          |                 |     |         |       |     |         |                 |     |      |       |       |                 |       |     |               |                 |
| $s$  | Strecke  | $[m]$  |          |                       |                     |          |                |                   |           |        |         |     |        |       |     |         |       |     |          |                 |     |         |       |     |         |                 |     |      |       |       |                 |       |     |               |                 |
| $v$  | Geschw.  | $[\frac{m}{s}]$  |          |                       |                     |          |                |                   |           |        |         |     |        |       |     |         |       |     |          |                 |     |         |       |     |         |                 |     |      |       |       |                 |       |     |               |                 |
| $t$  | Zeit   | $[s]$  |          |                       |                     |          |                |                   |           |        |         |     |        |       |     |         |       |     |          |                 |     |         |       |     |         |                 |     |      |       |       |                 |       |     |               |                 |
| $s_0$  | Anfangs Strecke  | $[m]$  |          |                       |                     |          |                |                   |           |        |         |     |        |       |     |         |       |     |          |                 |     |         |       |     |         |                 |     |      |       |       |                 |       |     |               |                 |
| $v$  | Geschw. tang.  | $[\frac{m}{s}]$  |          |                       |                     |          |                |                   |           |        |         |     |        |       |     |         |       |     |          |                 |     |         |       |     |         |                 |     |      |       |       |                 |       |     |               |                 |

#### 3.3.2. Gleichförmig beschleunigte Kreisbewegung

|   |   |   |          |                       |                     |          |                |                   |            |                      |                   |            |                    |                   |           |        |         |             |              |         |     |        |       |     |      |       |     |                |                   |
|---|---|---|----------|-----------------------|---------------------|----------|----------------|-------------------|------------|----------------------|-------------------|------------|--------------------|-------------------|-----------|--------|---------|-------------|--------------|---------|-----|--------|-------|-----|------|-------|-----|----------------|-------------------|
|  | $\alpha = \text{konstant}$<br>$\alpha = \dot{\omega}$<br>$\alpha = \ddot{\varphi}$<br>$a = r\alpha$<br>$\omega = \alpha t + \omega_0$<br>$\omega = \sqrt{\omega_0^2 + 2\alpha(\varphi - \varphi_0)}$<br>$\varphi = \frac{\alpha}{2}t^2 + \omega_0 t + \varphi_0$<br>$\varphi = \frac{\omega_1^2 - \omega_0^2}{2\alpha}$ | <table> <tr> <td><math>\alpha</math></td><td>Winkel-Beschleunigung</td><td><math>[\frac{rad}{s^2}]</math></td></tr> <tr> <td><math>\omega</math></td><td>Winkel-Geschw.</td><td><math>[\frac{rad}{s}]</math></td></tr> <tr> <td><math>\omega_0</math></td><td>Start-Winkel-Geschw.</td><td><math>[\frac{rad}{s}]</math></td></tr> <tr> <td><math>\omega_1</math></td><td>End-Winkel-Geschw.</td><td><math>[\frac{rad}{s}]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Winkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>\varphi_0</math></td><td>Start-Winkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>a</math></td><td>Beschleunigung</td><td><math>[\frac{m}{s^2}]</math></td></tr> </table> | $\alpha$ | Winkel-Beschleunigung | $[\frac{rad}{s^2}]$ | $\omega$ | Winkel-Geschw. | $[\frac{rad}{s}]$ | $\omega_0$ | Start-Winkel-Geschw. | $[\frac{rad}{s}]$ | $\omega_1$ | End-Winkel-Geschw. | $[\frac{rad}{s}]$ | $\varphi$ | Winkel | $[rad]$ | $\varphi_0$ | Start-Winkel | $[rad]$ | $r$ | Radius | $[m]$ | $t$ | Zeit | $[s]$ | $a$ | Beschleunigung | $[\frac{m}{s^2}]$ |
| $\alpha$  | Winkel-Beschleunigung   | $[\frac{rad}{s^2}]$   |          |                       |                     |          |                |                   |            |                      |                   |            |                    |                   |           |        |         |             |              |         |     |        |       |     |      |       |     |                |                   |
| $\omega$  | Winkel-Geschw.  | $[\frac{rad}{s}]$   |          |                       |                     |          |                |                   |            |                      |                   |            |                    |                   |           |        |         |             |              |         |     |        |       |     |      |       |     |                |                   |
| $\omega_0$  | Start-Winkel-Geschw.  | $[\frac{rad}{s}]$   |          |                       |                     |          |                |                   |            |                      |                   |            |                    |                   |           |        |         |             |              |         |     |        |       |     |      |       |     |                |                   |
| $\omega_1$  | End-Winkel-Geschw.  | $[\frac{rad}{s}]$   |          |                       |                     |          |                |                   |            |                      |                   |            |                    |                   |           |        |         |             |              |         |     |        |       |     |      |       |     |                |                   |
| $\varphi$   | Winkel  | $[rad]$   |          |                       |                     |          |                |                   |            |                      |                   |            |                    |                   |           |        |         |             |              |         |     |        |       |     |      |       |     |                |                   |
| $\varphi_0$   | Start-Winkel  | $[rad]$   |          |                       |                     |          |                |                   |            |                      |                   |            |                    |                   |           |        |         |             |              |         |     |        |       |     |      |       |     |                |                   |
| $r$   | Radius  | $[m]$   |          |                       |                     |          |                |                   |            |                      |                   |            |                    |                   |           |        |         |             |              |         |     |        |       |     |      |       |     |                |                   |
| $t$   | Zeit  | $[s]$   |          |                       |                     |          |                |                   |            |                      |                   |            |                    |                   |           |        |         |             |              |         |     |        |       |     |      |       |     |                |                   |
| $a$   | Beschleunigung  | $[\frac{m}{s^2}]$   |          |                       |                     |          |                |                   |            |                      |                   |            |                    |                   |           |        |         |             |              |         |     |        |       |     |      |       |     |                |                   |

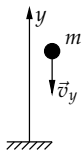


## 3.3.3. Zentripetalbeschleunigung

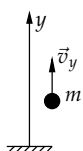
|   |  |  |            |                       |                                |          |                |                              |           |        |         |     |        |       |     |         |       |     |         |                            |     |       |        |
|---|--|--|------------|-----------------------|--------------------------------|----------|----------------|------------------------------|-----------|--------|---------|-----|--------|-------|-----|---------|-------|-----|---------|----------------------------|-----|-------|--------|
|  | $a_z = \frac{v^2}{r}$ $a_z = r\omega^2$ $v = \frac{2\pi r}{T}$ $F_z = ma_z = \frac{mv^2}{r}$ $F_z = m\omega^2 r$ | <table> <tr> <td><math>\alpha_z</math></td><td>Zentr.-Beschleunigung</td><td><math>\left[\frac{rad}{s^2}\right]</math></td></tr> <tr> <td><math>\omega</math></td><td>Winkel-Geschw.</td><td><math>\left[\frac{rad}{s}\right]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Winkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr> <td><math>T</math></td><td>Periode</td><td><math>[s]</math></td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>m</math></td><td>Masse</td><td><math>[kg]</math></td></tr> </table> | $\alpha_z$ | Zentr.-Beschleunigung | $\left[\frac{rad}{s^2}\right]$ | $\omega$ | Winkel-Geschw. | $\left[\frac{rad}{s}\right]$ | $\varphi$ | Winkel | $[rad]$ | $r$ | Radius | $[m]$ | $T$ | Periode | $[s]$ | $v$ | Geschw. | $\left[\frac{m}{s}\right]$ | $m$ | Masse | $[kg]$ |
| $\alpha_z$  | Zentr.-Beschleunigung  | $\left[\frac{rad}{s^2}\right]$   |            |                       |                                |          |                |                              |           |        |         |     |        |       |     |         |       |     |         |                            |     |       |        |
| $\omega$  | Winkel-Geschw.   | $\left[\frac{rad}{s}\right]$   |            |                       |                                |          |                |                              |           |        |         |     |        |       |     |         |       |     |         |                            |     |       |        |
| $\varphi$   | Winkel   | $[rad]$  |            |                       |                                |          |                |                              |           |        |         |     |        |       |     |         |       |     |         |                            |     |       |        |
| $r$   | Radius   | $[m]$  |            |                       |                                |          |                |                              |           |        |         |     |        |       |     |         |       |     |         |                            |     |       |        |
| $T$   | Periode  | $[s]$  |            |                       |                                |          |                |                              |           |        |         |     |        |       |     |         |       |     |         |                            |     |       |        |
| $v$   | Geschw.  | $\left[\frac{m}{s}\right]$   |            |                       |                                |          |                |                              |           |        |         |     |        |       |     |         |       |     |         |                            |     |       |        |
| $m$   | Masse  | $[kg]$   |            |                       |                                |          |                |                              |           |        |         |     |        |       |     |         |       |     |         |                            |     |       |        |

## 3.4. Wurfbahnen

## 3.4.1. Freier Fall

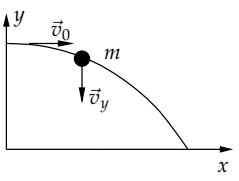
|   |   |  |       |                  |                            |     |      |       |       |           |       |       |          |       |     |      |       |     |                   |                              |
|---|---|--|-------|------------------|----------------------------|-----|------|-------|-------|-----------|-------|-------|----------|-------|-----|------|-------|-----|-------------------|------------------------------|
|  | $a_y = -g$ $v_y = -gt$ $v_e = \sqrt{2gh}$ $y = -\frac{g}{2}t^2$ $t = \sqrt{\frac{2h}{g}}$ | <table> <tr> <td><math>v_0</math></td><td>Abschuss-Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>t_s</math></td><td>Steigzeit</td><td><math>[s]</math></td></tr> <tr> <td><math>t_f</math></td><td>Flugzeit</td><td><math>[s]</math></td></tr> <tr> <td><math>h</math></td><td>Höhe</td><td><math>[m]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl. = 9.81</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> </table> | $v_0$ | Abschuss-Geschw. | $\left[\frac{m}{s}\right]$ | $t$ | Zeit | $[s]$ | $t_s$ | Steigzeit | $[s]$ | $t_f$ | Flugzeit | $[s]$ | $h$ | Höhe | $[m]$ | $g$ | Erdbeschl. = 9.81 | $\left[\frac{m}{s^2}\right]$ |
| $v_0$   | Abschuss-Geschw.  | $\left[\frac{m}{s}\right]$   |       |                  |                            |     |      |       |       |           |       |       |          |       |     |      |       |     |                   |                              |
| $t$   | Zeit  | $[s]$  |       |                  |                            |     |      |       |       |           |       |       |          |       |     |      |       |     |                   |                              |
| $t_s$   | Steigzeit   | $[s]$  |       |                  |                            |     |      |       |       |           |       |       |          |       |     |      |       |     |                   |                              |
| $t_f$   | Flugzeit  | $[s]$  |       |                  |                            |     |      |       |       |           |       |       |          |       |     |      |       |     |                   |                              |
| $h$   | Höhe  | $[m]$  |       |                  |                            |     |      |       |       |           |       |       |          |       |     |      |       |     |                   |                              |
| $g$   | Erdbeschl. = 9.81   | $\left[\frac{m}{s^2}\right]$   |       |                  |                            |     |      |       |       |           |       |       |          |       |     |      |       |     |                   |                              |

## 3.4.2. Senkrechter Wurf

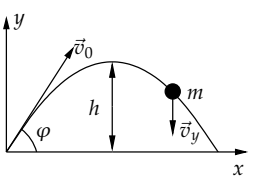
|   |   |  |       |                  |                            |       |           |       |       |          |       |     |      |       |     |                   |                              |
|---|---|--|-------|------------------|----------------------------|-------|-----------|-------|-------|----------|-------|-----|------|-------|-----|-------------------|------------------------------|
|  | $t_s = \left  \frac{v_0}{g} \right $ $h = \frac{v_0 t}{2} = \left  \frac{v_0^2}{2g} \right $ $t_f = 2t_s$ | <table> <tr> <td><math>v_0</math></td><td>Abschuss-Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>t_s</math></td><td>Steigzeit</td><td><math>[s]</math></td></tr> <tr> <td><math>t_f</math></td><td>Flugzeit</td><td><math>[s]</math></td></tr> <tr> <td><math>h</math></td><td>Höhe</td><td><math>[m]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl. = 9.81</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> </table> | $v_0$ | Abschuss-Geschw. | $\left[\frac{m}{s}\right]$ | $t_s$ | Steigzeit | $[s]$ | $t_f$ | Flugzeit | $[s]$ | $h$ | Höhe | $[m]$ | $g$ | Erdbeschl. = 9.81 | $\left[\frac{m}{s^2}\right]$ |
| $v_0$   | Abschuss-Geschw.  | $\left[\frac{m}{s}\right]$   |       |                  |                            |       |           |       |       |          |       |     |      |       |     |                   |                              |
| $t_s$   | Steigzeit   | $[s]$  |       |                  |                            |       |           |       |       |          |       |     |      |       |     |                   |                              |
| $t_f$   | Flugzeit  | $[s]$  |       |                  |                            |       |           |       |       |          |       |     |      |       |     |                   |                              |
| $h$   | Höhe  | $[m]$  |       |                  |                            |       |           |       |       |          |       |     |      |       |     |                   |                              |
| $g$   | Erdbeschl. = 9.81   | $\left[\frac{m}{s^2}\right]$   |       |                  |                            |       |           |       |       |          |       |     |      |       |     |                   |                              |

### 3. KINEMATIK

#### 3.4.3. Horizontaler Wurf

|   |   |   |     |         |                              |     |         |       |     |         |                            |       |               |                            |     |      |       |     |                   |                              |
|---|---|---|-----|---------|------------------------------|-----|---------|-------|-----|---------|----------------------------|-------|---------------|----------------------------|-----|------|-------|-----|-------------------|------------------------------|
|  | $a_x = 0 \rightarrow v_x = v_0$ $s_x = v_0 t$ $s_x = \sqrt{\frac{2v_0^2 y}{g}}$ $a_y = -g \rightarrow v_y = -gt$ $s_y = -\frac{g}{2} t^2$ $s_y = -\frac{g}{2v_0^2} s_x^2$ | <table> <tr> <td><math>a</math></td><td>Beschl.</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> <tr> <td><math>s</math></td><td>Strecke</td><td><math>[m]</math></td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>v_0</math></td><td>Start-Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl. = 9.81</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> </table> | $a$ | Beschl. | $\left[\frac{m}{s^2}\right]$ | $s$ | Strecke | $[m]$ | $v$ | Geschw. | $\left[\frac{m}{s}\right]$ | $v_0$ | Start-Geschw. | $\left[\frac{m}{s}\right]$ | $t$ | Zeit | $[s]$ | $g$ | Erdbeschl. = 9.81 | $\left[\frac{m}{s^2}\right]$ |
| $a$   | Beschl.   | $\left[\frac{m}{s^2}\right]$  |     |         |                              |     |         |       |     |         |                            |       |               |                            |     |      |       |     |                   |                              |
| $s$   | Strecke   | $[m]$   |     |         |                              |     |         |       |     |         |                            |       |               |                            |     |      |       |     |                   |                              |
| $v$   | Geschw.   | $\left[\frac{m}{s}\right]$  |     |         |                              |     |         |       |     |         |                            |       |               |                            |     |      |       |     |                   |                              |
| $v_0$   | Start-Geschw.   | $\left[\frac{m}{s}\right]$  |     |         |                              |     |         |       |     |         |                            |       |               |                            |     |      |       |     |                   |                              |
| $t$   | Zeit  | $[s]$   |     |         |                              |     |         |       |     |         |                            |       |               |                            |     |      |       |     |                   |                              |
| $g$   | Erdbeschl. = 9.81   | $\left[\frac{m}{s^2}\right]$  |     |         |                              |     |         |       |     |         |                            |       |               |                            |     |      |       |     |                   |                              |

#### 3.4.4. Schiefer Wurf

|   |   |  |     |         |                              |     |             |       |     |         |       |     |         |                            |       |               |                            |     |      |       |           |                |         |     |                   |                              |
|---|---|--|-----|---------|------------------------------|-----|-------------|-------|-----|---------|-------|-----|---------|----------------------------|-------|---------------|----------------------------|-----|------|-------|-----------|----------------|---------|-----|-------------------|------------------------------|
|  | $a_y = -g \quad a_x = 0$ $d = \frac{v_0^2}{g} \sin(2\varphi)$ $h = \frac{v_0^2}{2g} \sin^2(\varphi)$ $t = \frac{2v_0 \sin(\varphi)}{g}$ $\Delta y = v_0 \sin(\varphi) t - \frac{gt^2}{2}$ $\Delta x = v_0 \cos(\varphi) t$ <p>Parabelgleichung:</p> $y = \tan(\varphi) s_x - \frac{gs_x^2}{2v_0^2 \cos^2(\varphi)}$ | <table> <tr> <td><math>a</math></td><td>Beschl.</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> <tr> <td><math>d</math></td><td>Wurfdistanz</td><td><math>[m]</math></td></tr> <tr> <td><math>s</math></td><td>Strecke</td><td><math>[m]</math></td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>v_0</math></td><td>Start-Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Abschusswinkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl. = 9.81</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> </table> | $a$ | Beschl. | $\left[\frac{m}{s^2}\right]$ | $d$ | Wurfdistanz | $[m]$ | $s$ | Strecke | $[m]$ | $v$ | Geschw. | $\left[\frac{m}{s}\right]$ | $v_0$ | Start-Geschw. | $\left[\frac{m}{s}\right]$ | $t$ | Zeit | $[s]$ | $\varphi$ | Abschusswinkel | $[rad]$ | $g$ | Erdbeschl. = 9.81 | $\left[\frac{m}{s^2}\right]$ |
| $a$   | Beschl.   | $\left[\frac{m}{s^2}\right]$   |     |         |                              |     |             |       |     |         |       |     |         |                            |       |               |                            |     |      |       |           |                |         |     |                   |                              |
| $d$   | Wurfdistanz   | $[m]$  |     |         |                              |     |             |       |     |         |       |     |         |                            |       |               |                            |     |      |       |           |                |         |     |                   |                              |
| $s$   | Strecke   | $[m]$  |     |         |                              |     |             |       |     |         |       |     |         |                            |       |               |                            |     |      |       |           |                |         |     |                   |                              |
| $v$   | Geschw.   | $\left[\frac{m}{s}\right]$   |     |         |                              |     |             |       |     |         |       |     |         |                            |       |               |                            |     |      |       |           |                |         |     |                   |                              |
| $v_0$   | Start-Geschw.   | $\left[\frac{m}{s}\right]$   |     |         |                              |     |             |       |     |         |       |     |         |                            |       |               |                            |     |      |       |           |                |         |     |                   |                              |
| $t$   | Zeit  | $[s]$  |     |         |                              |     |             |       |     |         |       |     |         |                            |       |               |                            |     |      |       |           |                |         |     |                   |                              |
| $\varphi$   | Abschusswinkel  | $[rad]$  |     |         |                              |     |             |       |     |         |       |     |         |                            |       |               |                            |     |      |       |           |                |         |     |                   |                              |
| $g$   | Erdbeschl. = 9.81   | $\left[\frac{m}{s^2}\right]$   |     |         |                              |     |             |       |     |         |       |     |         |                            |       |               |                            |     |      |       |           |                |         |     |                   |                              |

# 4. Dynamik

## 4.1. Newtonsche Gesetze

### 4.1.1. Erstes Newtonsches Gesetz (Trägheitsgesetz)

Ein Körper verharrt im Zustand der Ruhe oder der gleichförmigen Bewegung, wenn er nicht durch einwirkende Kräfte gezwungen wird, seinen Zustand zu ändern. Die Gesamtsumme der Kräfte in einem abgeschlossenen System ist unveränderlich:

$$\vec{F} = \sum_i \vec{F}_i = 0$$

### 4.1.2. Zweites Newtonsches Gesetz (Aktionsgesetz)

Die Beschleunigung eines Körpers ist umgekehrt proportional zu seiner Masse und direkt proportional zur Kraft, die auf ihn wirkt.

$$\vec{F} = m\vec{a}$$

### 4.1.3. Drittes Newtonsches Gesetz (Actio = Reactio)

Wirkt ein Körper A auf einen Körper B mit der Kraft  $\vec{F}_{AB}$ , so wirkt der Körper B mit der entgegengesetzt gerichteten, gleich grossen Kraft  $\vec{F}_{BA}$ .

$$\sum_{i=1}^n F_{ix} = ma_x \vec{a} \quad \sum_{i=1}^n F_{iy} = ma_y \vec{a} \quad \sum_{i=1}^n F_{iz} = ma_z \vec{a}$$

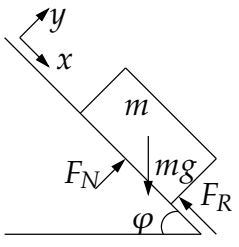
### 4.1.4. Allgemeines Vorgehen beim lösen von Bewegungsproblemen

1. Zeichnung anfertigen
2. Für jeden Körper, der untersucht werden soll, wird ein Kräftediagramm eingezeichnet
3. Ein geeignetes Koordinatensystem einführen
4. Das entstandene Gleichungssystem auflösen
5. Ergebnisse mit gesundem Menschenverstand auflösen

## 4. DYNAMIK

### 4.2. Masse und Gewicht

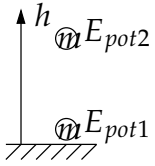
#### 4.2.1. Spezielle Kräfte, Masse, Dichte und Reibung

|   |  |   |       |               |     |       |               |     |       |             |     |        |        |                    |       |         |     |         |              |     |         |             |     |                  |                   |       |            |                        |     |     |                |                   |     |                    |                   |     |       |      |     |         |         |
|---|--|---|-------|---------------|-----|-------|---------------|-----|-------|-------------|-----|--------|--------|--------------------|-------|---------|-----|---------|--------------|-----|---------|-------------|-----|------------------|-------------------|-------|------------|------------------------|-----|-----|----------------|-------------------|-----|--------------------|-------------------|-----|-------|------|-----|---------|---------|
|  | $F_G = mg$ $F_R = \mu_G F_N$ $\rho = \frac{m}{V}$ $\mu = -\frac{a}{g}$ $\mu_H = \tan(\varphi_{krit})$ $F_{Hmax} = \mu_H F_N$ | <table> <tr><td><math>F_G</math></td><td>Gewichtskraft</td><td>[N]</td></tr> <tr><td><math>F_R</math></td><td>Reibungskraft</td><td>[N]</td></tr> <tr><td><math>F_N</math></td><td>Normalkraft</td><td>[N]</td></tr> <tr><td><math>\rho</math></td><td>Dichte</td><td><math>[\frac{kg}{m^3}]</math></td></tr> <tr><td><math>\mu</math></td><td>Reibung</td><td>[1]</td></tr> <tr><td><math>\mu_G</math></td><td>Gleitreibung</td><td>[1]</td></tr> <tr><td><math>\mu_H</math></td><td>Haftreibung</td><td>[1]</td></tr> <tr><td><math>\varphi_{krit}</math></td><td>kritischer Winkel</td><td>[rad]</td></tr> <tr><td><math>F_{Hmax}</math></td><td>Max. Haftreibungskraft</td><td>[N]</td></tr> <tr><td><math>a</math></td><td>Beschleunigung</td><td><math>[\frac{m}{s^2}]</math></td></tr> <tr><td><math>g</math></td><td>Fallbeschleunigung</td><td><math>[\frac{m}{s^2}]</math></td></tr> <tr><td><math>m</math></td><td>Masse</td><td>[kg]</td></tr> <tr><td><math>V</math></td><td>Volumen</td><td><math>[m^3]</math></td></tr> </table> | $F_G$ | Gewichtskraft | [N] | $F_R$ | Reibungskraft | [N] | $F_N$ | Normalkraft | [N] | $\rho$ | Dichte | $[\frac{kg}{m^3}]$ | $\mu$ | Reibung | [1] | $\mu_G$ | Gleitreibung | [1] | $\mu_H$ | Haftreibung | [1] | $\varphi_{krit}$ | kritischer Winkel | [rad] | $F_{Hmax}$ | Max. Haftreibungskraft | [N] | $a$ | Beschleunigung | $[\frac{m}{s^2}]$ | $g$ | Fallbeschleunigung | $[\frac{m}{s^2}]$ | $m$ | Masse | [kg] | $V$ | Volumen | $[m^3]$ |
| $F_G$   | Gewichtskraft  | [N]   |       |               |     |       |               |     |       |             |     |        |        |                    |       |         |     |         |              |     |         |             |     |                  |                   |       |            |                        |     |     |                |                   |     |                    |                   |     |       |      |     |         |         |
| $F_R$   | Reibungskraft  | [N]   |       |               |     |       |               |     |       |             |     |        |        |                    |       |         |     |         |              |     |         |             |     |                  |                   |       |            |                        |     |     |                |                   |     |                    |                   |     |       |      |     |         |         |
| $F_N$   | Normalkraft  | [N]   |       |               |     |       |               |     |       |             |     |        |        |                    |       |         |     |         |              |     |         |             |     |                  |                   |       |            |                        |     |     |                |                   |     |                    |                   |     |       |      |     |         |         |
| $\rho$  | Dichte   | $[\frac{kg}{m^3}]$  |       |               |     |       |               |     |       |             |     |        |        |                    |       |         |     |         |              |     |         |             |     |                  |                   |       |            |                        |     |     |                |                   |     |                    |                   |     |       |      |     |         |         |
| $\mu$   | Reibung  | [1]   |       |               |     |       |               |     |       |             |     |        |        |                    |       |         |     |         |              |     |         |             |     |                  |                   |       |            |                        |     |     |                |                   |     |                    |                   |     |       |      |     |         |         |
| $\mu_G$   | Gleitreibung   | [1]   |       |               |     |       |               |     |       |             |     |        |        |                    |       |         |     |         |              |     |         |             |     |                  |                   |       |            |                        |     |     |                |                   |     |                    |                   |     |       |      |     |         |         |
| $\mu_H$   | Haftreibung  | [1]   |       |               |     |       |               |     |       |             |     |        |        |                    |       |         |     |         |              |     |         |             |     |                  |                   |       |            |                        |     |     |                |                   |     |                    |                   |     |       |      |     |         |         |
| $\varphi_{krit}$  | kritischer Winkel  | [rad]   |       |               |     |       |               |     |       |             |     |        |        |                    |       |         |     |         |              |     |         |             |     |                  |                   |       |            |                        |     |     |                |                   |     |                    |                   |     |       |      |     |         |         |
| $F_{Hmax}$  | Max. Haftreibungskraft   | [N]   |       |               |     |       |               |     |       |             |     |        |        |                    |       |         |     |         |              |     |         |             |     |                  |                   |       |            |                        |     |     |                |                   |     |                    |                   |     |       |      |     |         |         |
| $a$   | Beschleunigung   | $[\frac{m}{s^2}]$   |       |               |     |       |               |     |       |             |     |        |        |                    |       |         |     |         |              |     |         |             |     |                  |                   |       |            |                        |     |     |                |                   |     |                    |                   |     |       |      |     |         |         |
| $g$   | Fallbeschleunigung   | $[\frac{m}{s^2}]$   |       |               |     |       |               |     |       |             |     |        |        |                    |       |         |     |         |              |     |         |             |     |                  |                   |       |            |                        |     |     |                |                   |     |                    |                   |     |       |      |     |         |         |
| $m$   | Masse  | [kg]  |       |               |     |       |               |     |       |             |     |        |        |                    |       |         |     |         |              |     |         |             |     |                  |                   |       |            |                        |     |     |                |                   |     |                    |                   |     |       |      |     |         |         |
| $V$   | Volumen  | $[m^3]$   |       |               |     |       |               |     |       |             |     |        |        |                    |       |         |     |         |              |     |         |             |     |                  |                   |       |            |                        |     |     |                |                   |     |                    |                   |     |       |      |     |         |         |

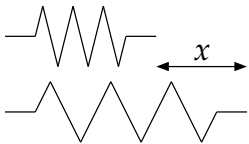
### 4.3. Arbeit und Energie, Energieerhaltung

|  |   |   |     |        |     |     |         |     |     |       |     |     |     |     |     |      |     |
|--|---|---|-----|--------|-----|-----|---------|-----|-----|-------|-----|-----|-----|-----|-----|------|-----|
| <p>Energie ist die Fähigkeit Arbeit zu leisten.<br/>Arbeit = überwinden eines Widerstandes</p> | $dW = \vec{F} \cdot d\vec{s}$ $W = Pt$ <p>Energieerhaltung im abgeschlossenen System:</p> $\sum_i E_i = const.$ | <table> <tr><td><math>W</math></td><td>Arbeit</td><td>[J]</td></tr> <tr><td><math>E</math></td><td>Energie</td><td>[J]</td></tr> <tr><td><math>F</math></td><td>Kraft</td><td>[N]</td></tr> <tr><td><math>s</math></td><td>Weg</td><td>[m]</td></tr> <tr><td><math>t</math></td><td>Zeit</td><td>[s]</td></tr> </table> | $W$ | Arbeit | [J] | $E$ | Energie | [J] | $F$ | Kraft | [N] | $s$ | Weg | [m] | $t$ | Zeit | [s] |
| $W$  | Arbeit  | [J]   |     |        |     |     |         |     |     |       |     |     |     |     |     |      |     |
| $E$  | Energie   | [J]   |     |        |     |     |         |     |     |       |     |     |     |     |     |      |     |
| $F$  | Kraft   | [N]   |     |        |     |     |         |     |     |       |     |     |     |     |     |      |     |
| $s$  | Weg   | [m]   |     |        |     |     |         |     |     |       |     |     |     |     |     |      |     |
| $t$  | Zeit  | [s]   |     |        |     |     |         |     |     |       |     |     |     |     |     |      |     |

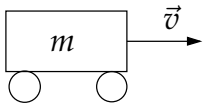
### 4.3.1. Hubarbeit, Potentielle Energie

|   |   |   |           |                     |     |       |           |     |     |       |      |     |                    |                              |     |      |     |
|---|---|---|-----------|---------------------|-----|-------|-----------|-----|-----|-------|------|-----|--------------------|------------------------------|-----|------|-----|
|  | $E_{pot} = mgh$ $W_H \vec{F} \cdot \vec{h}$ $E_{pot} = W_H$ | <table> <tr> <td><math>E_{pot}</math></td><td>potentielle Energie</td><td>[J]</td></tr> <tr> <td><math>W_H</math></td><td>Hubarbeit</td><td>[J]</td></tr> <tr> <td><math>m</math></td><td>Masse</td><td>[kg]</td></tr> <tr> <td><math>g</math></td><td>Fallbeschleunigung</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> <tr> <td><math>h</math></td><td>Höhe</td><td>[m]</td></tr> </table> | $E_{pot}$ | potentielle Energie | [J] | $W_H$ | Hubarbeit | [J] | $m$ | Masse | [kg] | $g$ | Fallbeschleunigung | $\left[\frac{m}{s^2}\right]$ | $h$ | Höhe | [m] |
| $E_{pot}$   | potentielle Energie   | [J]   |           |                     |     |       |           |     |     |       |      |     |                    |                              |     |      |     |
| $W_H$   | Hubarbeit   | [J]   |           |                     |     |       |           |     |     |       |      |     |                    |                              |     |      |     |
| $m$   | Masse   | [kg]  |           |                     |     |       |           |     |     |       |      |     |                    |                              |     |      |     |
| $g$   | Fallbeschleunigung  | $\left[\frac{m}{s^2}\right]$  |           |                     |     |       |           |     |     |       |      |     |                    |                              |     |      |     |
| $h$   | Höhe  | [m]   |           |                     |     |       |           |     |     |       |      |     |                    |                              |     |      |     |


### 4.3.2. Spannarbeit, Spannenergie

|   |  |  |       |              |     |       |             |     |     |             |                            |     |           |     |     |            |     |
|---|--|--|-------|--------------|-----|-------|-------------|-----|-----|-------------|----------------------------|-----|-----------|-----|-----|------------|-----|
|  | $E_s = \frac{cx^2}{2}$ $W_s = \vec{F} \cdot \vec{x}$ $E_s = W_s$ | <table> <tr> <td><math>E_s</math></td><td>Spannenergie</td><td>[J]</td></tr> <tr> <td><math>W_s</math></td><td>Spannarbeit</td><td>[J]</td></tr> <tr> <td><math>c</math></td><td>Federkonst.</td><td><math>\left[\frac{N}{m}\right]</math></td></tr> <tr> <td><math>x</math></td><td>Spann-Weg</td><td>[m]</td></tr> <tr> <td><math>F</math></td><td>Spannkraft</td><td>[N]</td></tr> </table> | $E_s$ | Spannenergie | [J] | $W_s$ | Spannarbeit | [J] | $c$ | Federkonst. | $\left[\frac{N}{m}\right]$ | $x$ | Spann-Weg | [m] | $F$ | Spannkraft | [N] |
| $E_s$   | Spannenergie   | [J]  |       |              |     |       |             |     |     |             |                            |     |           |     |     |            |     |
| $W_s$   | Spannarbeit  | [J]  |       |              |     |       |             |     |     |             |                            |     |           |     |     |            |     |
| $c$   | Federkonst.  | $\left[\frac{N}{m}\right]$   |       |              |     |       |             |     |     |             |                            |     |           |     |     |            |     |
| $x$   | Spann-Weg  | [m]  |       |              |     |       |             |     |     |             |                            |     |           |     |     |            |     |
| $F$   | Spannkraft   | [N]  |       |              |     |       |             |     |     |             |                            |     |           |     |     |            |     |

### 4.3.3. Beschleunigungsarbeit, Kinetische Energie

|   |  |   |             |                     |     |       |                       |     |     |       |      |     |         |                            |
|---|--|---|-------------|---------------------|-----|-------|-----------------------|-----|-----|-------|------|-----|---------|----------------------------|
|  | $E_{kin} = \frac{mv^2}{2}$ $W_B = \frac{m(\Delta v)^2}{2}$ | <table> <tr> <td><math>E_{trans}</math></td><td>Translationsenergie</td><td>[J]</td></tr> <tr> <td><math>W_B</math></td><td>Beschleunigungsarbeit</td><td>[J]</td></tr> <tr> <td><math>m</math></td><td>Masse</td><td>[kg]</td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> </table> | $E_{trans}$ | Translationsenergie | [J] | $W_B$ | Beschleunigungsarbeit | [J] | $m$ | Masse | [kg] | $v$ | Geschw. | $\left[\frac{m}{s}\right]$ |
| $E_{trans}$   | Translationsenergie  | [J]   |             |                     |     |       |                       |     |     |       |      |     |         |                            |
| $W_B$   | Beschleunigungsarbeit                                      | [J]   |             |                     |     |       |                       |     |     |       |      |     |         |                            |
| $m$   | Masse  | [kg]  |             |                     |     |       |                       |     |     |       |      |     |         |                            |
| $v$   | Geschw.  | $\left[\frac{m}{s}\right]$  |             |                     |     |       |                       |     |     |       |      |     |         |                            |

### 4.3.4. Rotationsenergie

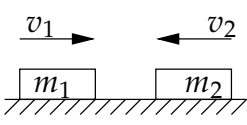
|   |                                  |  |           |                  |     |     |                |           |          |               |                              |
|---|----------------------------------|--|-----------|------------------|-----|-----|----------------|-----------|----------|---------------|------------------------------|
|  | $E_{rot} = \frac{1}{2}J\omega^2$ | <table> <tr> <td><math>E_{rot}</math></td><td>Rotationsenergie</td><td>[J]</td></tr> <tr> <td><math>J</math></td><td>Massenträgheit</td><td><math>[kgm^2]</math></td></tr> <tr> <td><math>\omega</math></td><td>Winkelgeschw.</td><td><math>\left[\frac{rad}{s}\right]</math></td></tr> </table> | $E_{rot}$ | Rotationsenergie | [J] | $J$ | Massenträgheit | $[kgm^2]$ | $\omega$ | Winkelgeschw. | $\left[\frac{rad}{s}\right]$ |
| $E_{rot}$   | Rotationsenergie                 | [J]  |           |                  |     |     |                |           |          |               |                              |
| $J$   | Massenträgheit                   | $[kgm^2]$  |           |                  |     |     |                |           |          |               |                              |
| $\omega$  | Winkelgeschw.                    | $\left[\frac{rad}{s}\right]$   |           |                  |     |     |                |           |          |               |                              |

## 4. DYNAMIK

### 4.3.5. Reibungsarbeit

|   |               |  |
|---|---------------|--|
|  | $W_R = F_R s$ | $W_R$ Reibarbeit [J]<br>$F_R$ Reibkraft [N]<br>$s$ Strecke [m] |
|---|---------------|--|

### 4.3.6. Verformungsarbeit

|   |   |   |
|---|---|---|
|  | <p>Inelastisch:</p> $W_D = \frac{m_1 m_2 (v_1 - v_2)^2}{2(m_1 + m_2)}$ <p>Elastisch:</p> $W_D = \frac{F_1 + F_2}{2} \Delta s$ | $W_D$ Deformationsarbeit [J]<br>$m_1, m_2$ Massen [kg]<br>$v_1, v_2$ Geschw. [ $\frac{m}{s}$ ]<br>$F_1, F_2$ Kräfte [N] |
|---|---|---|

### 4.3.7. Einstein, Kernbindungsenergie

|  |            |  |
|--|------------|--|
|  | $E = mc^2$ | $E$ Energie [J]<br>$m$ Masse [kg]<br>$c$ $v_{\text{Licht}}$ [ $\frac{m}{s}$ ]<br>$= 299'792'458$<br>(Vakuum) |
|--|------------|--|

## 4.4. Leistung

|  |  |  |
|--|--|--|
|  | $P = \frac{dW}{dt}$<br>$P = \frac{F ds}{dt} = \vec{F} \vec{s}$<br>$P = M \omega$ | $P$ Leistung [W]<br>$W$ Energie [J]<br>$t$ Zeit [s]<br>$F$ Kraft [N]<br>$s$ Strecke [m]<br>$M$ Drehmoment [Nm]<br>$\omega$ Winkelgeschwindigkeit [ $\frac{rad}{s}$ ] |
|--|--|--|

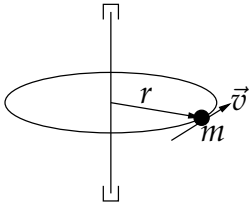
## 4.5. Wirkungsgrad

|  |   |   |
|--|---|---|
|  | $\eta = \frac{W_{ab}}{W_{zu}} = \frac{p_{ab}}{p_{zu}}$ $\eta_{tot} = \eta_1 \cdot \eta_2 \cdot \dots$ | $\eta$ Wirkungsgrad [1]<br>$P_{ab}$ P-Abgeg. [W]<br>$P_{zu}$ P-Aufge. [W]<br>$W_{ab}$ W-Abgeg. [J]<br>$W_{zu}$ W-Aufge. [J] |
|--|---|---|

## 4.6. Impuls, Impulserhaltung

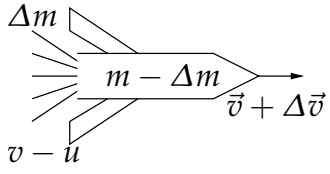
|   |  |   |
|---|--|---|
| Impulserhaltungssatz:<br>Im abgeschlossenen<br>System bleibt der<br>Impuls konstant | $\vec{p} = m\vec{v}$ $\vec{F} = \dot{\vec{p}}$ $p_{ges} = \sum_{i=1}^n m_i v_i$ Kraftstoss:<br>$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$ | $p$ Impuls [ $\frac{kgm}{s}$ ]<br>$m$ Masse [kg]<br>$v$ Geschw. [ $\frac{m}{s}$ ]<br>$F$ Kraft [N]<br>$\Delta t$ Wirkungs-<br>dauer [s] |
|---|--|---|

### 4.6.1. Drehimpuls

|   |  |  |
|---|--|--|
|  | $L = mvr \sin(\varphi)$ $\vec{L} = \vec{r} \times \vec{p}$ $L = J\omega$ $\vec{M} = \frac{d\vec{p}}{dt}$ | $L$ Drehimpuls [ $\frac{kgm^2}{s}$ ]<br>$m$ Masse [kg]<br>$r$ Radius [m]<br>$v$ Geschw. [ $\frac{m}{s}$ ]<br>$\omega$ Winkelgeschw. [ $\frac{rad}{s}$ ]<br>$M$ Drehmoment [Nm] |
|---|--|--|

## 4. DYNAMIK

### 4.6.2. Raketenantrieb

|   |   |   |     |         |                            |     |                            |                            |       |            |        |     |       |        |       |            |       |     |      |       |     |            |                              |
|---|---|---|-----|---------|----------------------------|-----|----------------------------|----------------------------|-------|------------|--------|-----|-------|--------|-------|------------|-------|-----|------|-------|-----|------------|------------------------------|
|  | $v = u \ln \frac{m_0}{m} + v_0$ $F_s = \frac{dm}{dt} u$ <p>Spezifischer Impuls:</p> $T = \frac{u}{g}$ | <table> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>u</math></td><td>v-Strahl relativ zu Rakete</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>m_0</math></td><td>Startmasse</td><td><math>[kg]</math></td></tr> <tr> <td><math>m</math></td><td>Masse</td><td><math>[kg]</math></td></tr> <tr> <td><math>F_s</math></td><td>Schubkraft</td><td><math>[N]</math></td></tr> <tr> <td><math>T</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl.</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> </table> | $v$ | Geschw. | $\left[\frac{m}{s}\right]$ | $u$ | v-Strahl relativ zu Rakete | $\left[\frac{m}{s}\right]$ | $m_0$ | Startmasse | $[kg]$ | $m$ | Masse | $[kg]$ | $F_s$ | Schubkraft | $[N]$ | $T$ | Zeit | $[s]$ | $g$ | Erdbeschl. | $\left[\frac{m}{s^2}\right]$ |
| $v$   | Geschw.   | $\left[\frac{m}{s}\right]$  |     |         |                            |     |                            |                            |       |            |        |     |       |        |       |            |       |     |      |       |     |            |                              |
| $u$   | v-Strahl relativ zu Rakete  | $\left[\frac{m}{s}\right]$  |     |         |                            |     |                            |                            |       |            |        |     |       |        |       |            |       |     |      |       |     |            |                              |
| $m_0$   | Startmasse  | $[kg]$  |     |         |                            |     |                            |                            |       |            |        |     |       |        |       |            |       |     |      |       |     |            |                              |
| $m$   | Masse   | $[kg]$  |     |         |                            |     |                            |                            |       |            |        |     |       |        |       |            |       |     |      |       |     |            |                              |
| $F_s$   | Schubkraft  | $[N]$   |     |         |                            |     |                            |                            |       |            |        |     |       |        |       |            |       |     |      |       |     |            |                              |
| $T$   | Zeit  | $[s]$   |     |         |                            |     |                            |                            |       |            |        |     |       |        |       |            |       |     |      |       |     |            |                              |
| $g$   | Erdbeschl.  | $\left[\frac{m}{s^2}\right]$  |     |         |                            |     |                            |                            |       |            |        |     |       |        |       |            |       |     |      |       |     |            |                              |

### 4.6.3. Inelastischer Stoss

|           |  |   |      |                    |                            |           |             |                            |           |        |        |
|-----------|--|---|------|--------------------|----------------------------|-----------|-------------|----------------------------|-----------|--------|--------|
|           | $v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$ | <table> <tr> <td><math>v'</math></td><td>Geschw. nach Stoss</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>v_{1,2}</math></td><td>v vor Stoss</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>m_{1,2}</math></td><td>Massen</td><td><math>[kg]</math></td></tr> </table> | $v'$ | Geschw. nach Stoss | $\left[\frac{m}{s}\right]$ | $v_{1,2}$ | v vor Stoss | $\left[\frac{m}{s}\right]$ | $m_{1,2}$ | Massen | $[kg]$ |
| $v'$      | Geschw. nach Stoss                         | $\left[\frac{m}{s}\right]$  |      |                    |                            |           |             |                            |           |        |        |
| $v_{1,2}$ | v vor Stoss                                | $\left[\frac{m}{s}\right]$  |      |                    |                            |           |             |                            |           |        |        |
| $m_{1,2}$ | Massen                                     | $[kg]$  |      |                    |                            |           |             |                            |           |        |        |

### 4.6.4. Elastischer Stoss

|            |  |   |            |                    |                            |           |             |                            |           |        |        |
|------------|--|---|------------|--------------------|----------------------------|-----------|-------------|----------------------------|-----------|--------|--------|
|            | $v_1 - v_2 = -(v'_1 - v'_2)$ $v'_1 = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2}$ $v'_2 = \frac{(m_2 - m_1)v_2 + 2m_1 v_1}{m_1 + m_2}$ $m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$ | <table> <tr> <td><math>v'_{1,2}</math></td><td>Geschw. nach Stoss</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>v_{1,2}</math></td><td>v vor Stoss</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>m_{1,2}</math></td><td>Massen</td><td><math>[kg]</math></td></tr> </table> | $v'_{1,2}$ | Geschw. nach Stoss | $\left[\frac{m}{s}\right]$ | $v_{1,2}$ | v vor Stoss | $\left[\frac{m}{s}\right]$ | $m_{1,2}$ | Massen | $[kg]$ |
| $v'_{1,2}$ | Geschw. nach Stoss   | $\left[\frac{m}{s}\right]$  |            |                    |                            |           |             |                            |           |        |        |
| $v_{1,2}$  | v vor Stoss  | $\left[\frac{m}{s}\right]$  |            |                    |                            |           |             |                            |           |        |        |
| $m_{1,2}$  | Massen   | $[kg]$  |            |                    |                            |           |             |                            |           |        |        |

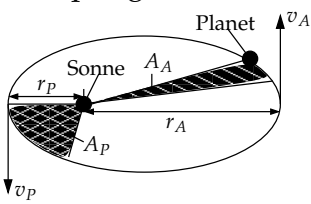


## 4.7. Analogie Translation und Rotation

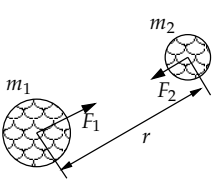
| Translation |                     |                              | Rotation  |                       |                                |
|-------------|---------------------|------------------------------|-----------|-----------------------|--------------------------------|
| Symb        | Grösse              | Beziehung                    | Symb      | Grösse                | Beziehung                      |
| $s$         | Weg                 |                              | $\varphi$ | Winkel                |                                |
| $v$         | Geschwindigkeit     | $v = \frac{ds}{dt}$          | $\omega$  | Winkelgeschwindigkeit | $\omega = \frac{d\varphi}{dt}$ |
| $a$         | Beschleunigung      | $a = \frac{dv}{dt}$          | $\alpha$  | Winkelbeschleunigung  | $\alpha = \frac{d\omega}{dt}$  |
| $m$         | Masse               |                              | $J$       | Trägheitsmoment       | $J = \int r^2 dm$              |
| $p$         | Impuls              | $p = mv$                     | $L$       | Drehimpuls            | $L = J\omega$                  |
| $F$         | Kraft               | $F = \frac{dp}{dt}$          | $M$       | Drehmoment            | $M = \frac{dL}{dt}$            |
| $dW$        | Arbeit              | $dW = \vec{F}d\vec{s}$       | $dW$      | Arbeit                | $dW = Md\varphi$               |
| $P$         | Leistung            | $P = \vec{F}\vec{v}$         | $P$       | Leistung              | $P = M\omega$                  |
| $E_{trans}$ | Translationsenergie | $E_{trans} = \frac{mv^2}{2}$ | $E_{rot}$ | Rotationsenergie      | $E_{rot} = J\omega^2/2$        |

## 4.8. Gravitation und Masse

### 4.8.1. Keplersche Gesetze (→ Bewegung der Planeten)

|  |  |   |
|--|--|---|
| 1. Keplergesetz  | Die Planeten bewegen sich auf Elypsen, in deren einem Brennpunkt die Sonne steht. (Bahn ist eben)  | $v_{P,A}$ Bahngeschw. $\left[\frac{m}{s}\right]$<br>$r_{P,A}$ Elypsen-Radien $\left[m\right]$<br>$T$ Umlaufdauer Planet $[s]$<br>$C$ Konstante $[1]$<br>$r$ mittlerer Abstand $\left[m\right]$<br>$v_K$ Kreisbahngeschwindigkeit $\left[\frac{m}{s}\right]$<br>$G$ Gravitationskonstante $= 6.673 \cdot 10^{-11} \left[\frac{m^3}{kg s^2}\right]$ |
| 2. Keplergesetz<br> | Der Fahrstrahl des Planeten überstreicht in gleichen Zeiten gleiche Flächen.<br><br>$v_P r_P = v_A r_A$<br>$A_P = A_A$   |   |
| 3. Keplergesetz  | Das Quadrat der Umlaufdauer eines Planeten ist proportional zur dritten Potenz seiner mittleren Entfernung zur Sonne.<br><br>$T^2 = C r^3$<br>$t = \frac{4\pi}{GM_{Sonne}} r^3$<br>Planetengeschwindigkeit:<br>$v_K = \frac{2\pi r}{T}$<br>$v_K = \sqrt{\frac{GM_{Sonne}}{r}}$ |   |

### 4.8.2. Newtonsches Gravitationsgesetz

|   |   |   |
|---|---|---|
|  | $F = G \frac{m_1 m_2}{r^2}$ $g = G \frac{m_E}{r^2} = \frac{F}{m}$ | $F$ Anziehungskraft $[N]$<br>$m_{1,2}$ Körpermasse $[kg]$<br>$r$ Abstand $[m]$<br>$G$ Gravitationskonstante $= 6.673 \cdot 10^{-11} \left[\frac{m^3}{kg s^2}\right]$<br>$g$ Erdbeschl. $\left[\frac{m}{s^2}\right]$ |
|---|---|---|

## 4.8.3. Potentielle Energie im Gravitationsfeld einer Zentralmasse

|  |   |  |
|--|---|--|
|  | $E_{pot} = -G \frac{m_Z m}{r}$ $\varphi = -\frac{G m_Z}{r}$ | $m_Z$ m-Zentralmasse [kg]<br>$m$ Körpermasse [kg]<br>$\varphi$ Gravitationspotential [ $\frac{m^2}{s^2}$ ]<br>$G$ Gravitationskonstante = [ $\frac{m^3}{kg s^2}$ ]<br>$6.673 \cdot 10^{-11}$ |
|--|---|--|

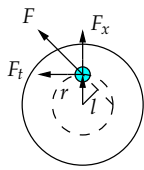
## 4.8.4. Fluchtgeschwindigkeit

|                           |   |   |
|---------------------------|---|---|
| Die Bahn ist eine Parabel | $v_F = \sqrt{2 \frac{G m_Z}{r_0}}$ $v_F = \sqrt{2} v_K$ | $v_F$ Fluchtgeschw. [ $\frac{m}{s}$ ]<br>$v_K$ Kreisbahngeschwindigkeit [ $\frac{m}{s}$ ]<br>$m_Z$ m-Zentralmasse [kg]<br>$r_0$ Abstand [m]<br>$G$ Gravitationskonstante = [ $\frac{m^3}{kg s^2}$ ]<br>$6.673 \cdot 10^{-11}$ |
|---------------------------|---|---|


## 4.8.5. Geostationäre Bahn

|   |  |   |
|---|--|---|
| Ein geostationärer Satellit scheint von der Erde aus gesehen still zu stehen. | $r = \sqrt[3]{\frac{G m_p t^2}{4\pi^2}}$ | $r$ Bahnradius [m]<br>$m_p$ m-Planet [kg]<br>$t$ Umlaufzeit [s]<br>$G$ Gravitationskonstante = [ $\frac{m^3}{kg s^2}$ ]<br>$6.673 \cdot 10^{-11}$ |
|---|--|---|

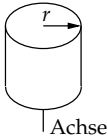
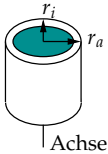

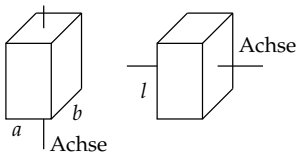
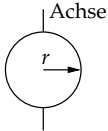
## 4.9. Rotation und Massenträgheitsmoment

|   |   |   |     |            |        |     |       |       |       |              |       |     |          |       |     |                       |           |       |                |           |       |              |           |          |               |                   |     |      |       |     |              |       |
|---|---|---|-----|------------|--------|-----|-------|-------|-------|--------------|-------|-----|----------|-------|-----|-----------------------|-----------|-------|----------------|-----------|-------|--------------|-----------|----------|---------------|-------------------|-----|------|-------|-----|--------------|-------|
|  | $M = F_t l = F r \sin(\varphi)$ $M = J \frac{d\omega}{dt} = J \alpha$ $J = \sum_i m_i r_i^2$ <p>Satz von Steiner:</p> $J_A = J_S + m d^2$ | <table> <tr> <td><math>M</math></td><td>Drehmoment</td><td><math>[Nm]</math></td></tr> <tr> <td><math>F</math></td><td>Kraft</td><td><math>[N]</math></td></tr> <tr> <td><math>F_t</math></td><td>F-tangential</td><td><math>[N]</math></td></tr> <tr> <td><math>l</math></td><td>Hebelarm</td><td><math>[m]</math></td></tr> <tr> <td><math>J</math></td><td>Massenträgheitsmoment</td><td><math>[kgm^2]</math></td></tr> <tr> <td><math>J_A</math></td><td>J bez. Achse A</td><td><math>[kgm^2]</math></td></tr> <tr> <td><math>J_S</math></td><td>J bez. Schwp</td><td><math>[kgm^2]</math></td></tr> <tr> <td><math>\omega</math></td><td>Winkelgeschw.</td><td><math>[\frac{rad}{s}]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>d</math></td><td>Abst. Drehp.</td><td><math>[m]</math></td></tr> </table> | $M$ | Drehmoment | $[Nm]$ | $F$ | Kraft | $[N]$ | $F_t$ | F-tangential | $[N]$ | $l$ | Hebelarm | $[m]$ | $J$ | Massenträgheitsmoment | $[kgm^2]$ | $J_A$ | J bez. Achse A | $[kgm^2]$ | $J_S$ | J bez. Schwp | $[kgm^2]$ | $\omega$ | Winkelgeschw. | $[\frac{rad}{s}]$ | $t$ | Zeit | $[s]$ | $d$ | Abst. Drehp. | $[m]$ |
| $M$   | Drehmoment  | $[Nm]$  |     |            |        |     |       |       |       |              |       |     |          |       |     |                       |           |       |                |           |       |              |           |          |               |                   |     |      |       |     |              |       |
| $F$   | Kraft   | $[N]$   |     |            |        |     |       |       |       |              |       |     |          |       |     |                       |           |       |                |           |       |              |           |          |               |                   |     |      |       |     |              |       |
| $F_t$   | F-tangential  | $[N]$   |     |            |        |     |       |       |       |              |       |     |          |       |     |                       |           |       |                |           |       |              |           |          |               |                   |     |      |       |     |              |       |
| $l$   | Hebelarm  | $[m]$   |     |            |        |     |       |       |       |              |       |     |          |       |     |                       |           |       |                |           |       |              |           |          |               |                   |     |      |       |     |              |       |
| $J$   | Massenträgheitsmoment   | $[kgm^2]$   |     |            |        |     |       |       |       |              |       |     |          |       |     |                       |           |       |                |           |       |              |           |          |               |                   |     |      |       |     |              |       |
| $J_A$   | J bez. Achse A  | $[kgm^2]$   |     |            |        |     |       |       |       |              |       |     |          |       |     |                       |           |       |                |           |       |              |           |          |               |                   |     |      |       |     |              |       |
| $J_S$   | J bez. Schwp  | $[kgm^2]$   |     |            |        |     |       |       |       |              |       |     |          |       |     |                       |           |       |                |           |       |              |           |          |               |                   |     |      |       |     |              |       |
| $\omega$  | Winkelgeschw.   | $[\frac{rad}{s}]$   |     |            |        |     |       |       |       |              |       |     |          |       |     |                       |           |       |                |           |       |              |           |          |               |                   |     |      |       |     |              |       |
| $t$   | Zeit  | $[s]$   |     |            |        |     |       |       |       |              |       |     |          |       |     |                       |           |       |                |           |       |              |           |          |               |                   |     |      |       |     |              |       |
| $d$   | Abst. Drehp.  | $[m]$   |     |            |        |     |       |       |       |              |       |     |          |       |     |                       |           |       |                |           |       |              |           |          |               |                   |     |      |       |     |              |       |

### 4.9.1. Massenträgheitsmoment bei Getriebe

|  |  |   |     |                |           |          |               |                   |     |          |       |     |             |       |
|--|--|---|-----|----------------|-----------|----------|---------------|-------------------|-----|----------|-------|-----|-------------|-------|
|  | $J_1 = \frac{J_2}{\eta_G} \left( \frac{\omega_2}{\omega_1} \right)^2 = \frac{J_2}{\eta_G} \left( \frac{n_2}{n_1} \right)^2$ $J_1 = \frac{J_2}{\eta_G i^2}$ | <table> <tr> <td><math>J</math></td><td>Massenträgheit</td><td><math>[kgm^2]</math></td></tr> <tr> <td><math>\omega</math></td><td>Winkelgeschw.</td><td><math>[\frac{rad}{s}]</math></td></tr> <tr> <td><math>n</math></td><td>Drehzahl</td><td><math>[1]</math></td></tr> <tr> <td><math>i</math></td><td>Übersetzung</td><td><math>[1]</math></td></tr> </table> | $J$ | Massenträgheit | $[kgm^2]$ | $\omega$ | Winkelgeschw. | $[\frac{rad}{s}]$ | $n$ | Drehzahl | $[1]$ | $i$ | Übersetzung | $[1]$ |
| $J$  | Massenträgheit   | $[kgm^2]$   |     |                |           |          |               |                   |     |          |       |     |             |       |
| $\omega$   | Winkelgeschw.  | $[\frac{rad}{s}]$   |     |                |           |          |               |                   |     |          |       |     |             |       |
| $n$  | Drehzahl   | $[1]$   |     |                |           |          |               |                   |     |          |       |     |             |       |
| $i$  | Übersetzung  | $[1]$   |     |                |           |          |               |                   |     |          |       |     |             |       |

### 4.9.2. Massenträgheitsmomente oft verwendeter Körper

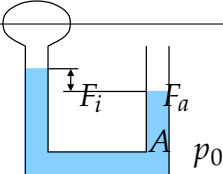
|   |   |   |
|---|---|---|
|   | <p>Allgemein:</p> $J = \int r^2 dm$   | <p><math>J</math> Massenträgheitsmoment <math>[kgm^2]</math></p> <p><math>m</math> Masse <math>[kg]</math></p> <p><math>r</math> Radius <math>[m]</math></p> <p><math>a, b</math> Seite <math>[m]</math></p> <p><math>l</math> Länge <math>[m]</math></p> |
|    | <p>Vollzylinder:</p> $J = \frac{mr^2}{2}$   |   |
|    | <p>Hohlzylinder:</p> $J = \frac{m(r_a^2 + r_i^2)}{2}$   |   |
|    | <p>Kugel:</p> $J = \frac{2}{5}mr^2$   |   |
|  | <p>Quader: <math>J = \frac{m(a^2 + b^2)}{12}</math></p> <p>Stange: <math>J = \frac{ml^2}{12}</math></p> |   |
|  | <p>Kreisscheibe:</p> $J = \frac{mr^2}{4} = \frac{md^2}{16}$   |   |

## 5. Mechanik deformierbarer Körper

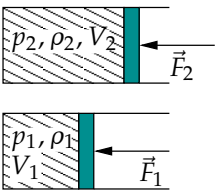
### 5.1. Druck

|   |  |  |     |       |        |     |       |       |     |        |         |        |               |        |
|---|--|--|-----|-------|--------|-----|-------|-------|-----|--------|---------|--------|---------------|--------|
|  | $p = \frac{F}{A}$ $\tau = \frac{F}{A}$ | <table> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>F</math></td><td>Kraft</td><td><math>[N]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr> <td><math>\tau</math></td><td>Schubspannung</td><td><math>[Pa]</math></td></tr> </table> | $p$ | Druck | $[Pa]$ | $F$ | Kraft | $[N]$ | $A$ | Fläche | $[m^2]$ | $\tau$ | Schubspannung | $[Pa]$ |
| $p$   | Druck                                  | $[Pa]$   |     |       |        |     |       |       |     |        |         |        |               |        |
| $F$   | Kraft                                  | $[N]$  |     |       |        |     |       |       |     |        |         |        |               |        |
| $A$   | Fläche                                 | $[m^2]$  |     |       |        |     |       |       |     |        |         |        |               |        |
| $\tau$  | Schubspannung                          | $[Pa]$   |     |       |        |     |       |       |     |        |         |        |               |        |

#### 5.1.1. Absoluter Druck Überdruck

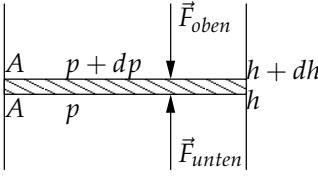
|  |  |   |     |       |        |       |             |        |  |                      |  |            |                     |       |     |        |         |
|--|--|---|-----|-------|--------|-------|-------------|--------|--|----------------------|--|------------|---------------------|-------|-----|--------|---------|
|  | $\Delta p = p - p_0$ $F = F_i - F_a$ $F = pA - p_0A$ | <table> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>p_0</math></td><td>Aussendruck</td><td><math>[Pa]</math></td></tr> <tr> <td></td><td><math>= 1,013 \cdot 10^5</math></td><td></td></tr> <tr> <td><math>F_i, F_a</math></td><td>Kraft innen, aussen</td><td><math>[N]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> </table> | $p$ | Druck | $[Pa]$ | $p_0$ | Aussendruck | $[Pa]$ |  | $= 1,013 \cdot 10^5$ |  | $F_i, F_a$ | Kraft innen, aussen | $[N]$ | $A$ | Fläche | $[m^2]$ |
| $p$  | Druck  | $[Pa]$  |     |       |        |       |             |        |  |                      |  |            |                     |       |     |        |         |
| $p_0$  | Aussendruck  | $[Pa]$  |     |       |        |       |             |        |  |                      |  |            |                     |       |     |        |         |
|  | $= 1,013 \cdot 10^5$                                 |   |     |       |        |       |             |        |  |                      |  |            |                     |       |     |        |         |
| $F_i, F_a$   | Kraft innen, aussen                                  | $[N]$   |     |       |        |       |             |        |  |                      |  |            |                     |       |     |        |         |
| $A$  | Fläche   | $[m^2]$   |     |       |        |       |             |        |  |                      |  |            |                     |       |     |        |         |

### 5.2. Kompression

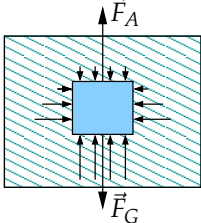
|  |  |   |          |                  |                  |     |                   |        |     |       |        |     |         |         |        |        |                    |
|--|--|---|----------|------------------|------------------|-----|-------------------|--------|-----|-------|--------|-----|---------|---------|--------|--------|--------------------|
|  <p>Ideale Fluide sind inkompressibel und reibungsfrei, Gase können zusammengeedrückt werden.</p> | $\kappa = \frac{-1}{V} \frac{\Delta V}{\Delta p}$ $K = \frac{1}{\kappa}$ <p>ideales Gas:</p> $pV = konst.$ $p_1 V_1 = p_2 V_2$ | <table> <tr> <td><math>\kappa</math></td><td>Kompressibilität</td><td><math>[\frac{1}{Pa}]</math></td></tr> <tr> <td><math>K</math></td><td>Kompressionsmodul</td><td><math>[Pa]</math></td></tr> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>V</math></td><td>Volumen</td><td><math>[m^3]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>[\frac{kg}{m^3}]</math></td></tr> </table> | $\kappa$ | Kompressibilität | $[\frac{1}{Pa}]$ | $K$ | Kompressionsmodul | $[Pa]$ | $p$ | Druck | $[Pa]$ | $V$ | Volumen | $[m^3]$ | $\rho$ | Dichte | $[\frac{kg}{m^3}]$ |
| $\kappa$   | Kompressibilität   | $[\frac{1}{Pa}]$  |          |                  |                  |     |                   |        |     |       |        |     |         |         |        |        |                    |
| $K$  | Kompressionsmodul  | $[Pa]$  |          |                  |                  |     |                   |        |     |       |        |     |         |         |        |        |                    |
| $p$  | Druck  | $[Pa]$  |          |                  |                  |     |                   |        |     |       |        |     |         |         |        |        |                    |
| $V$  | Volumen  | $[m^3]$   |          |                  |                  |     |                   |        |     |       |        |     |         |         |        |        |                    |
| $\rho$   | Dichte   | $[\frac{kg}{m^3}]$  |          |                  |                  |     |                   |        |     |       |        |     |         |         |        |        |                    |

## 5.3. Hydrostatik

### 5.3.1. Schweredruck

|   |   |   |     |       |        |       |                   |        |        |        |                               |          |                    |                               |     |      |       |     |                   |                              |  |  |          |
|---|---|---|-----|-------|--------|-------|-------------------|--------|--------|--------|-------------------------------|----------|--------------------|-------------------------------|-----|------|-------|-----|-------------------|------------------------------|--|--|----------|
|  | $dp = -\rho \cdot g \cdot dh \quad (h \text{ positiv nach oben})$ <p>Bei Flüssigkeiten:</p> $p = \rho gh + p_0 \quad (h \text{ positiv nach unten})$ $\frac{p_0}{\rho_0} = \frac{p(h)}{\rho(h)}$ <p>Bei Gasen:</p> $p = p_0 e^{-\frac{\rho_0}{p_0} gh}$ | <table> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>p_0</math></td><td>Druck bei <math>h = 0</math></td><td><math>[Pa]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> <tr> <td><math>\rho_0</math></td><td>Dichte bei <math>h = 0</math></td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> <tr> <td><math>h</math></td><td>Höhe</td><td><math>[m]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschleunigung</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> <tr> <td></td><td></td><td><math>= 9.81</math></td></tr> </table> | $p$ | Druck | $[Pa]$ | $p_0$ | Druck bei $h = 0$ | $[Pa]$ | $\rho$ | Dichte | $\left[\frac{kg}{m^3}\right]$ | $\rho_0$ | Dichte bei $h = 0$ | $\left[\frac{kg}{m^3}\right]$ | $h$ | Höhe | $[m]$ | $g$ | Erdbeschleunigung | $\left[\frac{m}{s^2}\right]$ |  |  | $= 9.81$ |
| $p$   | Druck   | $[Pa]$  |     |       |        |       |                   |        |        |        |                               |          |                    |                               |     |      |       |     |                   |                              |  |  |          |
| $p_0$   | Druck bei $h = 0$   | $[Pa]$  |     |       |        |       |                   |        |        |        |                               |          |                    |                               |     |      |       |     |                   |                              |  |  |          |
| $\rho$  | Dichte  | $\left[\frac{kg}{m^3}\right]$   |     |       |        |       |                   |        |        |        |                               |          |                    |                               |     |      |       |     |                   |                              |  |  |          |
| $\rho_0$  | Dichte bei $h = 0$  | $\left[\frac{kg}{m^3}\right]$   |     |       |        |       |                   |        |        |        |                               |          |                    |                               |     |      |       |     |                   |                              |  |  |          |
| $h$   | Höhe  | $[m]$   |     |       |        |       |                   |        |        |        |                               |          |                    |                               |     |      |       |     |                   |                              |  |  |          |
| $g$   | Erdbeschleunigung   | $\left[\frac{m}{s^2}\right]$  |     |       |        |       |                   |        |        |        |                               |          |                    |                               |     |      |       |     |                   |                              |  |  |          |
|   |   | $= 9.81$  |     |       |        |       |                   |        |        |        |                               |          |                    |                               |     |      |       |     |                   |                              |  |  |          |

### 5.3.2. Statischer Auftrieb

|   |  |  |       |                |       |             |              |                               |          |               |                               |          |             |        |       |              |        |     |               |         |     |                   |                              |  |  |          |
|---|--|--|-------|----------------|-------|-------------|--------------|-------------------------------|----------|---------------|-------------------------------|----------|-------------|--------|-------|--------------|--------|-----|---------------|---------|-----|-------------------|------------------------------|--|--|----------|
|  | $F_A = \rho_{Fl} V_K g - \rho_K V_K g$ $F_A = m_{Fl} g - m_K g$ $F_A = A \rho_{Fl} g \Delta h$ | <table> <tr> <td><math>F_A</math></td><td>Auftriebskraft</td><td><math>[N]</math></td></tr> <tr> <td><math>\rho_{Fl}</math></td><td>Dichte Fluid</td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> <tr> <td><math>\rho_K</math></td><td>Dichte Körper</td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> <tr> <td><math>m_{Fl}</math></td><td>Masse Fluid</td><td><math>[kg]</math></td></tr> <tr> <td><math>m_K</math></td><td>Masse Körper</td><td><math>[kg]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche Körper</td><td><math>[m^2]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschleunigung</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> <tr> <td></td><td></td><td><math>= 9.81</math></td></tr> </table> | $F_A$ | Auftriebskraft | $[N]$ | $\rho_{Fl}$ | Dichte Fluid | $\left[\frac{kg}{m^3}\right]$ | $\rho_K$ | Dichte Körper | $\left[\frac{kg}{m^3}\right]$ | $m_{Fl}$ | Masse Fluid | $[kg]$ | $m_K$ | Masse Körper | $[kg]$ | $A$ | Fläche Körper | $[m^2]$ | $g$ | Erdbeschleunigung | $\left[\frac{m}{s^2}\right]$ |  |  | $= 9.81$ |
| $F_A$   | Auftriebskraft   | $[N]$  |       |                |       |             |              |                               |          |               |                               |          |             |        |       |              |        |     |               |         |     |                   |                              |  |  |          |
| $\rho_{Fl}$   | Dichte Fluid   | $\left[\frac{kg}{m^3}\right]$  |       |                |       |             |              |                               |          |               |                               |          |             |        |       |              |        |     |               |         |     |                   |                              |  |  |          |
| $\rho_K$  | Dichte Körper  | $\left[\frac{kg}{m^3}\right]$  |       |                |       |             |              |                               |          |               |                               |          |             |        |       |              |        |     |               |         |     |                   |                              |  |  |          |
| $m_{Fl}$  | Masse Fluid  | $[kg]$   |       |                |       |             |              |                               |          |               |                               |          |             |        |       |              |        |     |               |         |     |                   |                              |  |  |          |
| $m_K$   | Masse Körper   | $[kg]$   |       |                |       |             |              |                               |          |               |                               |          |             |        |       |              |        |     |               |         |     |                   |                              |  |  |          |
| $A$   | Fläche Körper  | $[m^2]$  |       |                |       |             |              |                               |          |               |                               |          |             |        |       |              |        |     |               |         |     |                   |                              |  |  |          |
| $g$   | Erdbeschleunigung  | $\left[\frac{m}{s^2}\right]$   |       |                |       |             |              |                               |          |               |                               |          |             |        |       |              |        |     |               |         |     |                   |                              |  |  |          |
|   |  | $= 9.81$   |       |                |       |             |              |                               |          |               |                               |          |             |        |       |              |        |     |               |         |     |                   |                              |  |  |          |

### 5.3.3. Druckwandler

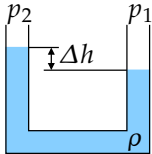
|   |                                     |  |     |       |        |     |        |         |
|---|-------------------------------------|--|-----|-------|--------|-----|--------|---------|
|  | $\frac{p_1}{p_2} = \frac{A_2}{A_1}$ | <table> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> </table> | $p$ | Druck | $[Pa]$ | $A$ | Fläche | $[m^2]$ |
| $p$   | Druck                               | $[Pa]$   |     |       |        |     |        |         |
| $A$   | Fläche                              | $[m^2]$  |     |       |        |     |        |         |

### 5.3.4. Kraftwandler

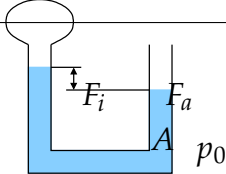
|   |                                     |   |     |       |       |     |        |         |
|---|-------------------------------------|---|-----|-------|-------|-----|--------|---------|
|  | $\frac{F_1}{F_2} = \frac{A_1}{A_2}$ | <table> <tr> <td><math>F</math></td><td>Kraft</td><td><math>[N]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> </table> | $F$ | Kraft | $[N]$ | $A$ | Fläche | $[m^2]$ |
| $F$   | Kraft                               | $[N]$   |     |       |       |     |        |         |
| $A$   | Fläche                              | $[m^2]$   |     |       |       |     |        |         |

### 5.3.5. Druckmessung

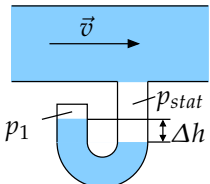
#### Manometer

|   |   |   |     |       |        |     |      |       |        |        |                               |
|---|---|---|-----|-------|--------|-----|------|-------|--------|--------|-------------------------------|
|  | $\Delta p = p_1 - p_2$ $\Delta p = \rho g \Delta h$ | <table> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>h</math></td><td>Höhe</td><td><math>[m]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> </table> | $p$ | Druck | $[Pa]$ | $h$ | Höhe | $[m]$ | $\rho$ | Dichte | $\left[\frac{kg}{m^3}\right]$ |
| $p$   | Druck   | $[Pa]$  |     |       |        |     |      |       |        |        |                               |
| $h$   | Höhe  | $[m]$   |     |       |        |     |      |       |        |        |                               |
| $\rho$  | Dichte  | $\left[\frac{kg}{m^3}\right]$   |     |       |        |     |      |       |        |        |                               |

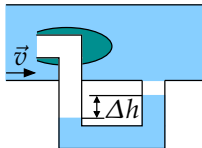
#### Absoluter Druck

|   |                         |  |     |       |        |     |      |       |        |        |                               |     |              |                              |  |  |      |
|---|-------------------------|--|-----|-------|--------|-----|------|-------|--------|--------|-------------------------------|-----|--------------|------------------------------|--|--|------|
|  | $p_a = \rho g \Delta h$ | <table> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>h</math></td><td>Höhe</td><td><math>[m]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl. =</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> <tr> <td></td><td></td><td>9.81</td></tr> </table> | $p$ | Druck | $[Pa]$ | $h$ | Höhe | $[m]$ | $\rho$ | Dichte | $\left[\frac{kg}{m^3}\right]$ | $g$ | Erdbeschl. = | $\left[\frac{m}{s^2}\right]$ |  |  | 9.81 |
| $p$   | Druck                   | $[Pa]$   |     |       |        |     |      |       |        |        |                               |     |              |                              |  |  |      |
| $h$   | Höhe                    | $[m]$  |     |       |        |     |      |       |        |        |                               |     |              |                              |  |  |      |
| $\rho$  | Dichte                  | $\left[\frac{kg}{m^3}\right]$  |     |       |        |     |      |       |        |        |                               |     |              |                              |  |  |      |
| $g$   | Erdbeschl. =            | $\left[\frac{m}{s^2}\right]$   |     |       |        |     |      |       |        |        |                               |     |              |                              |  |  |      |
|   |                         | 9.81   |     |       |        |     |      |       |        |        |                               |     |              |                              |  |  |      |

#### Statischer Druck (Druck auf Rohrwand)

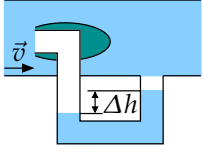
|   |   |   |     |       |        |       |             |        |  |  |                      |     |      |       |        |        |                               |     |              |                              |  |  |      |
|---|---|---|-----|-------|--------|-------|-------------|--------|--|--|----------------------|-----|------|-------|--------|--------|-------------------------------|-----|--------------|------------------------------|--|--|------|
|  | $p_{stat} = p_1 + \rho g \Delta h$ <p>Falls oben offen:</p> $p_1 = p_0$ | <table> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>p_0</math></td><td>Aussendruck</td><td><math>[Pa]</math></td></tr> <tr> <td></td><td></td><td><math>= 1.013 \cdot 10^5</math></td></tr> <tr> <td><math>h</math></td><td>Höhe</td><td><math>[m]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl. =</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> <tr> <td></td><td></td><td>9.81</td></tr> </table> | $p$ | Druck | $[Pa]$ | $p_0$ | Aussendruck | $[Pa]$ |  |  | $= 1.013 \cdot 10^5$ | $h$ | Höhe | $[m]$ | $\rho$ | Dichte | $\left[\frac{kg}{m^3}\right]$ | $g$ | Erdbeschl. = | $\left[\frac{m}{s^2}\right]$ |  |  | 9.81 |
| $p$   | Druck   | $[Pa]$  |     |       |        |       |             |        |  |  |                      |     |      |       |        |        |                               |     |              |                              |  |  |      |
| $p_0$   | Aussendruck   | $[Pa]$  |     |       |        |       |             |        |  |  |                      |     |      |       |        |        |                               |     |              |                              |  |  |      |
|   |   | $= 1.013 \cdot 10^5$  |     |       |        |       |             |        |  |  |                      |     |      |       |        |        |                               |     |              |                              |  |  |      |
| $h$   | Höhe  | $[m]$   |     |       |        |       |             |        |  |  |                      |     |      |       |        |        |                               |     |              |                              |  |  |      |
| $\rho$  | Dichte  | $\left[\frac{kg}{m^3}\right]$   |     |       |        |       |             |        |  |  |                      |     |      |       |        |        |                               |     |              |                              |  |  |      |
| $g$   | Erdbeschl. =  | $\left[\frac{m}{s^2}\right]$  |     |       |        |       |             |        |  |  |                      |     |      |       |        |        |                               |     |              |                              |  |  |      |
|   |   | 9.81  |     |       |        |       |             |        |  |  |                      |     |      |       |        |        |                               |     |              |                              |  |  |      |

#### Dynamischer Druck

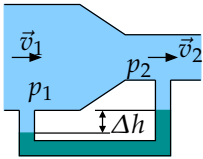
|   |                             |  |     |       |        |     |      |       |        |        |                               |     |              |                              |  |  |      |
|---|-----------------------------|--|-----|-------|--------|-----|------|-------|--------|--------|-------------------------------|-----|--------------|------------------------------|--|--|------|
|  | $p_{ges} = \rho g \Delta h$ | <table> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>h</math></td><td>Höhe</td><td><math>[m]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl. =</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> <tr> <td></td><td></td><td>9.81</td></tr> </table> | $p$ | Druck | $[Pa]$ | $h$ | Höhe | $[m]$ | $\rho$ | Dichte | $\left[\frac{kg}{m^3}\right]$ | $g$ | Erdbeschl. = | $\left[\frac{m}{s^2}\right]$ |  |  | 9.81 |
| $p$   | Druck                       | $[Pa]$   |     |       |        |     |      |       |        |        |                               |     |              |                              |  |  |      |
| $h$   | Höhe                        | $[m]$  |     |       |        |     |      |       |        |        |                               |     |              |                              |  |  |      |
| $\rho$  | Dichte                      | $\left[\frac{kg}{m^3}\right]$  |     |       |        |     |      |       |        |        |                               |     |              |                              |  |  |      |
| $g$   | Erdbeschl. =                | $\left[\frac{m}{s^2}\right]$   |     |       |        |     |      |       |        |        |                               |     |              |                              |  |  |      |
|   |                             | 9.81   |     |       |        |     |      |       |        |        |                               |     |              |                              |  |  |      |



## Gesamtdruck

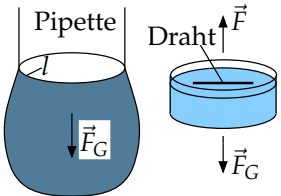
|   |   |  |     |       |        |     |      |       |        |        |                               |     |            |                                |  |      |  |     |         |                            |
|---|---|--|-----|-------|--------|-----|------|-------|--------|--------|-------------------------------|-----|------------|--------------------------------|--|------|--|-----|---------|----------------------------|
|  | $p_{dyn} = \rho g \Delta h$ <p>Strömungsgeschwindigkeit:</p> $v = \sqrt{\frac{2p_{dyn}}{\rho}}$ | <table> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>h</math></td><td>Höhe</td><td><math>[m]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl.</td><td><math>= \left[\frac{m}{s^2}\right]</math></td></tr> <tr> <td></td><td>9.81</td><td></td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> </table> | $p$ | Druck | $[Pa]$ | $h$ | Höhe | $[m]$ | $\rho$ | Dichte | $\left[\frac{kg}{m^3}\right]$ | $g$ | Erdbeschl. | $= \left[\frac{m}{s^2}\right]$ |  | 9.81 |  | $v$ | Geschw. | $\left[\frac{m}{s}\right]$ |
| $p$   | Druck   | $[Pa]$   |     |       |        |     |      |       |        |        |                               |     |            |                                |  |      |  |     |         |                            |
| $h$   | Höhe  | $[m]$  |     |       |        |     |      |       |        |        |                               |     |            |                                |  |      |  |     |         |                            |
| $\rho$  | Dichte  | $\left[\frac{kg}{m^3}\right]$  |     |       |        |     |      |       |        |        |                               |     |            |                                |  |      |  |     |         |                            |
| $g$   | Erdbeschl.  | $= \left[\frac{m}{s^2}\right]$   |     |       |        |     |      |       |        |        |                               |     |            |                                |  |      |  |     |         |                            |
|   | 9.81  |  |     |       |        |     |      |       |        |        |                               |     |            |                                |  |      |  |     |         |                            |
| $v$   | Geschw.   | $\left[\frac{m}{s}\right]$   |     |       |        |     |      |       |        |        |                               |     |            |                                |  |      |  |     |         |                            |

## Druckdifferenzen

|  |   |  |     |       |        |     |      |       |     |        |         |        |        |                               |     |            |                                |  |      |  |     |         |                            |
|--|---|--|-----|-------|--------|-----|------|-------|-----|--------|---------|--------|--------|-------------------------------|-----|------------|--------------------------------|--|------|--|-----|---------|----------------------------|
|  <p>Venturirohr</p> | $\Delta p = p_1 - p_2$ $\Delta p = \rho g \Delta h$ <p>Strömungsgeschwindigkeit:</p> $v_1 = \sqrt{\frac{2\Delta p}{\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]\rho}}$ | <table> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>h</math></td><td>Höhe</td><td><math>[m]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl.</td><td><math>= \left[\frac{m}{s^2}\right]</math></td></tr> <tr> <td></td><td>9.81</td><td></td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> </table> | $p$ | Druck | $[Pa]$ | $h$ | Höhe | $[m]$ | $A$ | Fläche | $[m^2]$ | $\rho$ | Dichte | $\left[\frac{kg}{m^3}\right]$ | $g$ | Erdbeschl. | $= \left[\frac{m}{s^2}\right]$ |  | 9.81 |  | $v$ | Geschw. | $\left[\frac{m}{s}\right]$ |
| $p$  | Druck   | $[Pa]$   |     |       |        |     |      |       |     |        |         |        |        |                               |     |            |                                |  |      |  |     |         |                            |
| $h$  | Höhe  | $[m]$  |     |       |        |     |      |       |     |        |         |        |        |                               |     |            |                                |  |      |  |     |         |                            |
| $A$  | Fläche  | $[m^2]$  |     |       |        |     |      |       |     |        |         |        |        |                               |     |            |                                |  |      |  |     |         |                            |
| $\rho$   | Dichte  | $\left[\frac{kg}{m^3}\right]$  |     |       |        |     |      |       |     |        |         |        |        |                               |     |            |                                |  |      |  |     |         |                            |
| $g$  | Erdbeschl.  | $= \left[\frac{m}{s^2}\right]$   |     |       |        |     |      |       |     |        |         |        |        |                               |     |            |                                |  |      |  |     |         |                            |
|  | 9.81  |  |     |       |        |     |      |       |     |        |         |        |        |                               |     |            |                                |  |      |  |     |         |                            |
| $v$  | Geschw.   | $\left[\frac{m}{s}\right]$   |     |       |        |     |      |       |     |        |         |        |        |                               |     |            |                                |  |      |  |     |         |                            |

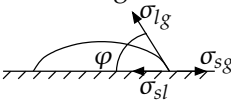
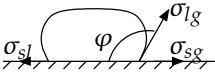
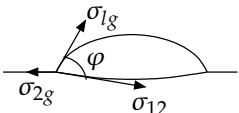
## 5.3.6. Grenzflächeneffekte

## Oberflächenspannung

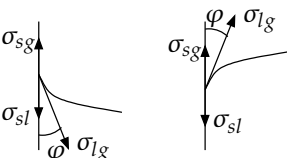
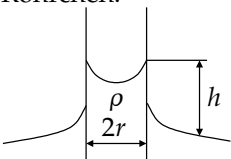
|   |  |   |          |                          |                            |     |       |       |     |       |       |     |               |         |     |        |       |     |            |                                |  |      |  |
|---|--|---|----------|--------------------------|----------------------------|-----|-------|-------|-----|-------|-------|-----|---------------|---------|-----|--------|-------|-----|------------|--------------------------------|--|------|--|
|  | $\sigma = \frac{F}{l}$ $\sigma = \frac{\Delta W}{\Delta A}$ <p>Kraft um Draht zu heben:</p> $F = 2\sigma l + m_{Draht}g$ | <table> <tr> <td><math>\sigma</math></td><td>Oberflächen-<br/>spannung</td><td><math>\left[\frac{N}{m}\right]</math></td></tr> <tr> <td><math>F</math></td><td>Kraft</td><td><math>[N]</math></td></tr> <tr> <td><math>l</math></td><td>Länge</td><td><math>[m]</math></td></tr> <tr> <td><math>A</math></td><td>Kontaktfläche</td><td><math>[m^2]</math></td></tr> <tr> <td><math>W</math></td><td>Arbeit</td><td><math>[J]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl.</td><td><math>= \left[\frac{m}{s^2}\right]</math></td></tr> <tr> <td></td><td>9.81</td><td></td></tr> </table> | $\sigma$ | Oberflächen-<br>spannung | $\left[\frac{N}{m}\right]$ | $F$ | Kraft | $[N]$ | $l$ | Länge | $[m]$ | $A$ | Kontaktfläche | $[m^2]$ | $W$ | Arbeit | $[J]$ | $g$ | Erdbeschl. | $= \left[\frac{m}{s^2}\right]$ |  | 9.81 |  |
| $\sigma$  | Oberflächen-<br>spannung   | $\left[\frac{N}{m}\right]$  |          |                          |                            |     |       |       |     |       |       |     |               |         |     |        |       |     |            |                                |  |      |  |
| $F$   | Kraft  | $[N]$   |          |                          |                            |     |       |       |     |       |       |     |               |         |     |        |       |     |            |                                |  |      |  |
| $l$   | Länge  | $[m]$   |          |                          |                            |     |       |       |     |       |       |     |               |         |     |        |       |     |            |                                |  |      |  |
| $A$   | Kontaktfläche  | $[m^2]$   |          |                          |                            |     |       |       |     |       |       |     |               |         |     |        |       |     |            |                                |  |      |  |
| $W$   | Arbeit   | $[J]$   |          |                          |                            |     |       |       |     |       |       |     |               |         |     |        |       |     |            |                                |  |      |  |
| $g$   | Erdbeschl.   | $= \left[\frac{m}{s^2}\right]$  |          |                          |                            |     |       |       |     |       |       |     |               |         |     |        |       |     |            |                                |  |      |  |
|   | 9.81   |   |          |                          |                            |     |       |       |     |       |       |     |               |         |     |        |       |     |            |                                |  |      |  |

## 5. MECHANIK DEFORMIERBARER KÖRPER

### Grenzflächenspannung

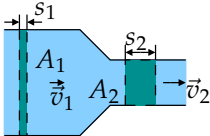
|  |  |   |
|--|--|---|
| <p>Flüssigkeit auf Festkörper</p> <p>Benetzung:</p>  <p>nicht Benetzung:</p>  <p>Flüssigkeit auf Flüssigkeit</p>  | <p>Benetzung: <math>\varphi &lt; 90^\circ</math><br/> Nicht Benetzung: <math>\varphi &gt; 90^\circ</math><br/> Flüssigkeit auf Festkörper</p> $\cos(\varphi) = \frac{\sigma_{sg} - \sigma_{sl}}{\sigma_{lg}}$ <p>Flüssigkeit auf Flüssigkeit</p> $\cos(\varphi) = \frac{\sigma_{2g}^2 - \sigma_{lg}^2 - \sigma_{-2}}{2\sigma_{lg}\sigma_{l2}}$ | <p><math>\varphi</math>    Kontaktwinkel <math>[rad]</math><br/> <math>\sigma</math>    Zugspannung <math>\left[\frac{N}{m}\right]</math><br/> <math>\sigma_{sl}</math>    <math>\sigma</math> fest, flüssig <math>\left[\frac{N}{m}\right]</math><br/> <math>\sigma</math>    <math>\sigma</math> fest, Gas <math>\left[\frac{N}{m}\right]</math><br/> <math>\sigma</math>    <math>\sigma</math> flüssig, Gas <math>\left[\frac{N}{m}\right]</math></p> |
|--|--|---|

### Kapillarität

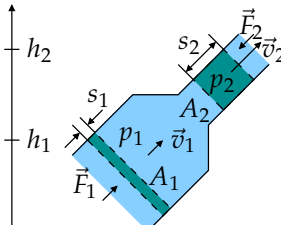
|  |                                |   |
|--|--------------------------------|---|
| <p>Benetzung:    Nicht Benetzung:</p>  <p>Röhrchen:</p>  | $h = \frac{2\sigma}{\rho g r}$ | <p><math>\sigma</math>    Zugspannung <math>\left[\frac{N}{m}\right]</math><br/> <math>h</math>    Höhe <math>[m]</math><br/> <math>r</math>    Radius <math>[m]</math><br/> <math>\rho</math>    Dichte <math>\left[\frac{kg}{m^3}\right]</math><br/> <math>g</math>    Erdbeschl. = <math>\left[\frac{m}{s^2}\right]</math><br/> 9.81</p> |
|--|--------------------------------|---|

## 5.4. Hydrodynamik

### 5.4.1. Kontinuitätsgleichung

|   |  |   |     |        |         |     |         |                 |     |         |       |     |         |         |     |      |       |
|---|--|---|-----|--------|---------|-----|---------|-----------------|-----|---------|-------|-----|---------|---------|-----|------|-------|
|  | $A_1 v_1 = A_2 v_2$ $\Delta V = A_1 v_1 \Delta t$ $\dot{V} = v A = \text{konst}$ $s_1 = v_1 \Delta t$ $s_2 = v_2 \Delta t$ | <table> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>[\frac{m}{s}]</math></td></tr> <tr> <td><math>s</math></td><td>Strecke</td><td><math>[m]</math></td></tr> <tr> <td><math>V</math></td><td>Volumen</td><td><math>[m^3]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> </table> | $A$ | Fläche | $[m^2]$ | $v$ | Geschw. | $[\frac{m}{s}]$ | $s$ | Strecke | $[m]$ | $V$ | Volumen | $[m^3]$ | $t$ | Zeit | $[s]$ |
| $A$   | Fläche   | $[m^2]$   |     |        |         |     |         |                 |     |         |       |     |         |         |     |      |       |
| $v$   | Geschw.  | $[\frac{m}{s}]$   |     |        |         |     |         |                 |     |         |       |     |         |         |     |      |       |
| $s$   | Strecke  | $[m]$   |     |        |         |     |         |                 |     |         |       |     |         |         |     |      |       |
| $V$   | Volumen  | $[m^3]$   |     |        |         |     |         |                 |     |         |       |     |         |         |     |      |       |
| $t$   | Zeit   | $[s]$   |     |        |         |     |         |                 |     |         |       |     |         |         |     |      |       |

### 5.4.2. Bernoulli Gleichung (Energieerhaltung)

|  |  |  |     |       |        |     |         |                 |     |      |       |        |        |                    |     |       |       |          |                 |       |
|--|--|--|-----|-------|--------|-----|---------|-----------------|-----|------|-------|--------|--------|--------------------|-----|-------|-------|----------|-----------------|-------|
|  | $\underbrace{p_1}_{p_{\text{statisch}}} + \underbrace{\frac{1}{2} \rho v_1^2}_{p_{\text{dynamisch}}} + \underbrace{gh_1 \rho}_{p_{\text{schwer}}} = \underbrace{p_2}_{p_{\text{statisch}}} + \underbrace{\frac{1}{2} \rho v_2^2}_{p_{\text{dynamisch}}} + \underbrace{gh_2 \rho}_{p_{\text{schwer}}}$ $p_1 + \frac{1}{2} \rho v^2 + gh\rho = \text{konst.}$ <p>Mit Berücksichtigung der Reibung:</p> $\underbrace{p_1}_{p_{\text{statisch}}} + \underbrace{\alpha_1 \frac{\rho v_1^2}{2}}_{p_{\text{dynamisch}}} + \underbrace{gh_1 \rho}_{p_{\text{schwer}}} = \underbrace{p_2}_{p_{\text{statisch}}} + \underbrace{\alpha_2 \frac{\rho v_2^2}{2}}_{p_{\text{dynamisch}}} + \underbrace{gh_2 \rho}_{p_{\text{schwer}}}$ <p>Laminar: <math>\alpha = 2</math><br/> Turbulent: <math>\alpha \approx 1</math></p> | <table> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>[\frac{m}{s}]</math></td></tr> <tr> <td><math>h</math></td><td>Höhe</td><td><math>[m]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>[\frac{kg}{m^3}]</math></td></tr> <tr> <td><math>F</math></td><td>Kraft</td><td><math>[N]</math></td></tr> <tr> <td><math>\alpha</math></td><td>Korrekturfaktor</td><td><math>[1]</math></td></tr> </table> | $p$ | Druck | $[Pa]$ | $v$ | Geschw. | $[\frac{m}{s}]$ | $h$ | Höhe | $[m]$ | $\rho$ | Dichte | $[\frac{kg}{m^3}]$ | $F$ | Kraft | $[N]$ | $\alpha$ | Korrekturfaktor | $[1]$ |
| $p$  | Druck  | $[Pa]$   |     |       |        |     |         |                 |     |      |       |        |        |                    |     |       |       |          |                 |       |
| $v$  | Geschw.  | $[\frac{m}{s}]$  |     |       |        |     |         |                 |     |      |       |        |        |                    |     |       |       |          |                 |       |
| $h$  | Höhe   | $[m]$  |     |       |        |     |         |                 |     |      |       |        |        |                    |     |       |       |          |                 |       |
| $\rho$   | Dichte   | $[\frac{kg}{m^3}]$   |     |       |        |     |         |                 |     |      |       |        |        |                    |     |       |       |          |                 |       |
| $F$  | Kraft  | $[N]$  |     |       |        |     |         |                 |     |      |       |        |        |                    |     |       |       |          |                 |       |
| $\alpha$   | Korrekturfaktor  | $[1]$  |     |       |        |     |         |                 |     |      |       |        |        |                    |     |       |       |          |                 |       |

## 5.5. Reale Strömung

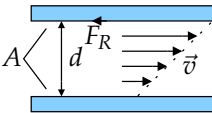
### 5.5.1. Zirkulation

|  |                                   |   |
|--|-----------------------------------|---|
|  | $\Gamma = \oint \vec{v} d\vec{s}$ | $\Gamma$ Zirkulation $\left[\frac{m^2}{s}\right]$ |
|  |                                   | $v$ Geschw. $\left[\frac{m}{s}\right]$            |
|  |                                   | $s$ Strecke $[m]$                                 |

### 5.5.2. Vortizität

|  |                                     |  |
|--|-------------------------------------|--|
|  | $\vec{\omega} = \text{rot} \vec{v}$ | $\omega$ Vortizität $\left[\frac{1}{s}\right]$ |
|  | Rotation der Geschwindigkeit        | $v$ Geschw. $\left[\frac{m}{s}\right]$         |

### 5.5.3. Newtonsches Reibungsgesetz

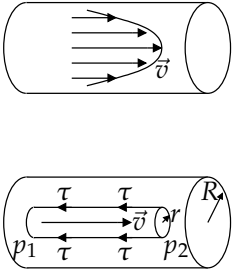
|  |                             |  |
|--|-----------------------------|--|
|  | $\tau = \eta \frac{dv}{dz}$ | $\tau$ Schubspan. $\left[\frac{N}{m^2}\right]$           |
|  | $\nu = \frac{\eta}{\rho}$   | $v$ Geschw. $\left[\frac{m}{s}\right]$                   |
|  | $\varphi = \frac{1}{\eta}$  | $\eta$ Dynamische Viskosität $[Pas]$                     |
|  |                             | $\nu$ Kinetische Viskosität $\left[\frac{m^2}{s}\right]$ |
|  |                             | $\rho$ Dichte $\left[\frac{kg}{m^3}\right]$              |
|  |                             | $z$ Abst. Platten $[m]$                                  |
|  |                             | $\varphi$ Fluidität $\left[\frac{1}{Pas}\right]$         |

## 5.6. Strömungsformen

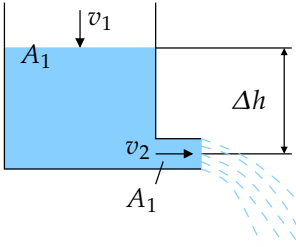
### 5.6.1. Reynolds-Zahl

|  |  |   |
|--|--|---|
|  | $Re = \frac{\rho v L}{\eta} = \frac{v L}{\nu}$ | $Re$ Reynolds-Zahl $[1]$                      |
|  | Im Rohr:                                       | $\rho$ Dichte $\left[\frac{kg}{m^3}\right]$   |
|  | $Re = \frac{\rho v d}{\eta} = \frac{v d}{\nu}$ | $v$ Geschw. $\left[\frac{m}{s}\right]$        |
|  |  | $L$ Linearabm. $[m]$                          |
|  |  | $d$ Rohr-Ø $[m]$                              |
|  |  | $\eta$ Dyn. Visk. $[Pas]$                     |
|  | $Re_{kritisch} = 2320$                         | $\nu$ Kin. Visk. $\left[\frac{m^2}{s}\right]$ |

5.6.2. Laminare Strömung ( $Re < 2320$ )

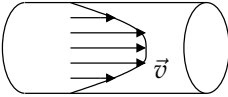
|  |   |  |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
|--|---|--|-------|---------------|-------|-----|---------|-----------------|-----|-----------------|-------|-----|--------------------|-------|--------|-----------------------|---------|--------|--------|--------------------|------|---------------|-------|--------|--------|--------------------|-----|-----------|-------|-----|---------|---------|-----|--------|-------|-----------|-----------------|-------|-----|-------|--------|-----|------------|-------------------|--|------|--|
| <p>Die Strömung ist laminar, wenn die Reynolds-Zahl <math>Re &lt; 2320</math> ist.</p>  | <p>Umströmte Kugel:</p> $F_R = 6\pi\eta Rv$ <p>Kugelgeschwindigkeit:</p> $v_{Kugel} = \frac{2R^2g(\rho_K - \rho_{Fl})}{9\eta}$ <p>Fluidzylinder in Fluid:</p> $v_{Zylinder} = \frac{p_1 - p_2}{4l\eta}(R^2 - r^2)$ <p>Durchflussmenge:</p> $\dot{V} = \frac{\pi\Delta p R^4}{8\eta l}$ $\Delta p = p_1 - p_2$ <p>Volumenfluss:</p> $V = \frac{\pi\Delta p R^4}{8\eta l}t$ <p>Druckabfall im glatten Rohr:</p> $\Delta p = \lambda_l \frac{l}{d} \frac{\rho v^2}{2}, \quad \lambda_l = \frac{64}{Re}$ <p>Reibungskraft auf Rohr:</p> $F_R = \Delta p R^2 \pi = 8\pi\eta l v$ | <table> <tr> <td><math>F_R</math></td><td>Reibungskraft</td><td><math>[N]</math></td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>[\frac{m}{s}]</math></td></tr> <tr> <td><math>r</math></td><td>Radius Zylinder</td><td><math>[m]</math></td></tr> <tr> <td><math>R</math></td><td>Radius Kugel, Rohr</td><td><math>[m]</math></td></tr> <tr> <td><math>\eta</math></td><td>Dynamische Viskosität</td><td><math>[Pas]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>[\frac{kg}{m^3}]</math></td></tr> <tr> <td><math>Re</math></td><td>Raynolds-Zahl</td><td><math>[1]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>[\frac{kg}{m^3}]</math></td></tr> <tr> <td><math>l</math></td><td>Rohrlänge</td><td><math>[m]</math></td></tr> <tr> <td><math>V</math></td><td>Volumen</td><td><math>[m^3]</math></td></tr> <tr> <td><math>d</math></td><td>Rohr-Ø</td><td><math>[m]</math></td></tr> <tr> <td><math>\lambda</math></td><td>Widerstandszahl</td><td><math>[1]</math></td></tr> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl.</td><td><math>[\frac{m}{s^2}]</math></td></tr> <tr> <td></td><td>9.81</td><td></td></tr> </table> | $F_R$ | Reibungskraft | $[N]$ | $v$ | Geschw. | $[\frac{m}{s}]$ | $r$ | Radius Zylinder | $[m]$ | $R$ | Radius Kugel, Rohr | $[m]$ | $\eta$ | Dynamische Viskosität | $[Pas]$ | $\rho$ | Dichte | $[\frac{kg}{m^3}]$ | $Re$ | Raynolds-Zahl | $[1]$ | $\rho$ | Dichte | $[\frac{kg}{m^3}]$ | $l$ | Rohrlänge | $[m]$ | $V$ | Volumen | $[m^3]$ | $d$ | Rohr-Ø | $[m]$ | $\lambda$ | Widerstandszahl | $[1]$ | $p$ | Druck | $[Pa]$ | $g$ | Erdbeschl. | $[\frac{m}{s^2}]$ |  | 9.81 |  |
| $F_R$  | Reibungskraft   | $[N]$  |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
| $v$  | Geschw.   | $[\frac{m}{s}]$  |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
| $r$  | Radius Zylinder   | $[m]$  |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
| $R$  | Radius Kugel, Rohr  | $[m]$  |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
| $\eta$   | Dynamische Viskosität   | $[Pas]$  |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
| $\rho$   | Dichte  | $[\frac{kg}{m^3}]$   |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
| $Re$   | Raynolds-Zahl   | $[1]$  |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
| $\rho$   | Dichte  | $[\frac{kg}{m^3}]$   |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
| $l$  | Rohrlänge   | $[m]$  |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
| $V$  | Volumen   | $[m^3]$  |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
| $d$  | Rohr-Ø  | $[m]$  |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
| $\lambda$  | Widerstandszahl   | $[1]$  |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
| $p$  | Druck   | $[Pa]$   |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
| $g$  | Erdbeschl.  | $[\frac{m}{s^2}]$  |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |
|  | 9.81  |  |       |               |       |     |         |                 |     |                 |       |     |                    |       |        |                       |         |        |        |                    |      |               |       |        |        |                    |     |           |       |     |         |         |     |        |       |           |                 |       |     |       |        |     |            |                   |  |      |  |

## 5.6.3. Volumenstrom

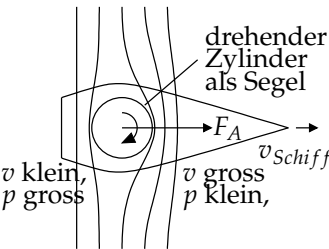
|   |   |  |     |      |       |     |         |                 |     |         |         |     |        |         |     |      |       |     |            |                   |  |      |  |
|---|---|--|-----|------|-------|-----|---------|-----------------|-----|---------|---------|-----|--------|---------|-----|------|-------|-----|------------|-------------------|--|------|--|
|  | <p><math>A_1 \gg A_2</math></p> <p><math>v_1 \approx 0</math></p> <p><math>v_2 = \sqrt{2gh}</math></p> <p>Volumenstrom:</p> $\dot{V} = a_2 v_2$ | <table> <tr> <td><math>h</math></td><td>Höhe</td><td><math>[m]</math></td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>[\frac{m}{s}]</math></td></tr> <tr> <td><math>V</math></td><td>Volumen</td><td><math>[m^3]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl.</td><td><math>[\frac{m}{s^2}]</math></td></tr> <tr> <td></td><td>9.81</td><td></td></tr> </table> | $h$ | Höhe | $[m]$ | $v$ | Geschw. | $[\frac{m}{s}]$ | $V$ | Volumen | $[m^3]$ | $A$ | Fläche | $[m^2]$ | $t$ | Zeit | $[s]$ | $g$ | Erdbeschl. | $[\frac{m}{s^2}]$ |  | 9.81 |  |
| $h$   | Höhe  | $[m]$  |     |      |       |     |         |                 |     |         |         |     |        |         |     |      |       |     |            |                   |  |      |  |
| $v$   | Geschw.   | $[\frac{m}{s}]$  |     |      |       |     |         |                 |     |         |         |     |        |         |     |      |       |     |            |                   |  |      |  |
| $V$   | Volumen   | $[m^3]$  |     |      |       |     |         |                 |     |         |         |     |        |         |     |      |       |     |            |                   |  |      |  |
| $A$   | Fläche  | $[m^2]$  |     |      |       |     |         |                 |     |         |         |     |        |         |     |      |       |     |            |                   |  |      |  |
| $t$   | Zeit  | $[s]$  |     |      |       |     |         |                 |     |         |         |     |        |         |     |      |       |     |            |                   |  |      |  |
| $g$   | Erdbeschl.  | $[\frac{m}{s^2}]$  |     |      |       |     |         |                 |     |         |         |     |        |         |     |      |       |     |            |                   |  |      |  |
|   | 9.81  |  |     |      |       |     |         |                 |     |         |         |     |        |         |     |      |       |     |            |                   |  |      |  |

## 5. MECHANIK DEFORMIERBARER KÖRPER

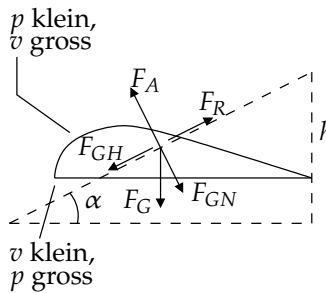
### 5.6.4. Turbulente Strömung ( $Re > 2320$ )

|  |  |  |       |               |       |     |         |                            |     |                |         |       |                         |       |        |        |                               |      |               |       |        |        |                               |     |           |       |     |        |       |           |                  |       |     |       |        |     |              |                              |  |      |  |
|--|--|--|-------|---------------|-------|-----|---------|----------------------------|-----|----------------|---------|-------|-------------------------|-------|--------|--------|-------------------------------|------|---------------|-------|--------|--------|-------------------------------|-----|-----------|-------|-----|--------|-------|-----------|------------------|-------|-----|-------|--------|-----|--------------|------------------------------|--|------|--|
| <p>Die Strömung ist turbulent, wenn die Reynolds-Zahl <math>Re &gt; 2320</math> ist.</p>  | <p>Druckwiderstand:</p> $F_D = \frac{c_W \rho v^2}{2} A$ <p>Druckabfall im glatten Rohr:</p> $\Delta p = \lambda_t \frac{l}{d} \frac{\rho v^2}{2}, \quad \lambda_t = \frac{0.316}{\sqrt[4]{Re}}$ | <table> <tr> <td><math>F_D</math></td><td>Druckwiderst.</td><td><math>[N]</math></td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>A</math></td><td>Angriffsfläche</td><td><math>[m^2]</math></td></tr> <tr> <td><math>c_W</math></td><td>Widerstands-koeffizient</td><td><math>[1]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> <tr> <td><math>Re</math></td><td>Raynolds-Zahl</td><td><math>[1]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> <tr> <td><math>l</math></td><td>Rohrlänge</td><td><math>[m]</math></td></tr> <tr> <td><math>d</math></td><td>Rohr-Ø</td><td><math>[m]</math></td></tr> <tr> <td><math>\lambda</math></td><td>Widerstands-zahl</td><td><math>[1]</math></td></tr> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl. =</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr> <tr> <td></td><td>9.81</td><td></td></tr> </table> | $F_D$ | Druckwiderst. | $[N]$ | $v$ | Geschw. | $\left[\frac{m}{s}\right]$ | $A$ | Angriffsfläche | $[m^2]$ | $c_W$ | Widerstands-koeffizient | $[1]$ | $\rho$ | Dichte | $\left[\frac{kg}{m^3}\right]$ | $Re$ | Raynolds-Zahl | $[1]$ | $\rho$ | Dichte | $\left[\frac{kg}{m^3}\right]$ | $l$ | Rohrlänge | $[m]$ | $d$ | Rohr-Ø | $[m]$ | $\lambda$ | Widerstands-zahl | $[1]$ | $p$ | Druck | $[Pa]$ | $g$ | Erdbeschl. = | $\left[\frac{m}{s^2}\right]$ |  | 9.81 |  |
| $F_D$  | Druckwiderst.  | $[N]$  |       |               |       |     |         |                            |     |                |         |       |                         |       |        |        |                               |      |               |       |        |        |                               |     |           |       |     |        |       |           |                  |       |     |       |        |     |              |                              |  |      |  |
| $v$  | Geschw.  | $\left[\frac{m}{s}\right]$   |       |               |       |     |         |                            |     |                |         |       |                         |       |        |        |                               |      |               |       |        |        |                               |     |           |       |     |        |       |           |                  |       |     |       |        |     |              |                              |  |      |  |
| $A$  | Angriffsfläche   | $[m^2]$  |       |               |       |     |         |                            |     |                |         |       |                         |       |        |        |                               |      |               |       |        |        |                               |     |           |       |     |        |       |           |                  |       |     |       |        |     |              |                              |  |      |  |
| $c_W$  | Widerstands-koeffizient  | $[1]$  |       |               |       |     |         |                            |     |                |         |       |                         |       |        |        |                               |      |               |       |        |        |                               |     |           |       |     |        |       |           |                  |       |     |       |        |     |              |                              |  |      |  |
| $\rho$   | Dichte   | $\left[\frac{kg}{m^3}\right]$  |       |               |       |     |         |                            |     |                |         |       |                         |       |        |        |                               |      |               |       |        |        |                               |     |           |       |     |        |       |           |                  |       |     |       |        |     |              |                              |  |      |  |
| $Re$   | Raynolds-Zahl  | $[1]$  |       |               |       |     |         |                            |     |                |         |       |                         |       |        |        |                               |      |               |       |        |        |                               |     |           |       |     |        |       |           |                  |       |     |       |        |     |              |                              |  |      |  |
| $\rho$   | Dichte   | $\left[\frac{kg}{m^3}\right]$  |       |               |       |     |         |                            |     |                |         |       |                         |       |        |        |                               |      |               |       |        |        |                               |     |           |       |     |        |       |           |                  |       |     |       |        |     |              |                              |  |      |  |
| $l$  | Rohrlänge  | $[m]$  |       |               |       |     |         |                            |     |                |         |       |                         |       |        |        |                               |      |               |       |        |        |                               |     |           |       |     |        |       |           |                  |       |     |       |        |     |              |                              |  |      |  |
| $d$  | Rohr-Ø   | $[m]$  |       |               |       |     |         |                            |     |                |         |       |                         |       |        |        |                               |      |               |       |        |        |                               |     |           |       |     |        |       |           |                  |       |     |       |        |     |              |                              |  |      |  |
| $\lambda$  | Widerstands-zahl   | $[1]$  |       |               |       |     |         |                            |     |                |         |       |                         |       |        |        |                               |      |               |       |        |        |                               |     |           |       |     |        |       |           |                  |       |     |       |        |     |              |                              |  |      |  |
| $p$  | Druck  | $[Pa]$   |       |               |       |     |         |                            |     |                |         |       |                         |       |        |        |                               |      |               |       |        |        |                               |     |           |       |     |        |       |           |                  |       |     |       |        |     |              |                              |  |      |  |
| $g$  | Erdbeschl. =   | $\left[\frac{m}{s^2}\right]$   |       |               |       |     |         |                            |     |                |         |       |                         |       |        |        |                               |      |               |       |        |        |                               |     |           |       |     |        |       |           |                  |       |     |       |        |     |              |                              |  |      |  |
|  | 9.81   |  |       |               |       |     |         |                            |     |                |         |       |                         |       |        |        |                               |      |               |       |        |        |                               |     |           |       |     |        |       |           |                  |       |     |       |        |     |              |                              |  |      |  |

### 5.7. Dynamischer Auftrieb

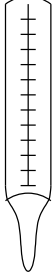
|   |  |   |       |                |       |     |               |       |     |        |       |     |              |                            |     |                       |                            |        |        |                               |          |            |                              |
|---|--|---|-------|----------------|-------|-----|---------------|-------|-----|--------|-------|-----|--------------|----------------------------|-----|-----------------------|----------------------------|--------|--------|-------------------------------|----------|------------|------------------------------|
| <p>Beispiel: Schiff mit Zylinder-Segel:</p>  | <p>Bei Zylinderform:</p> $F_A = \rho l v \Gamma$ $\Gamma = 4\pi^2 r^2 f$ | <table> <tr> <td><math>F_A</math></td><td>Auftriebskraft</td><td><math>[N]</math></td></tr> <tr> <td><math>l</math></td><td>Zylinderlänge</td><td><math>[m]</math></td></tr> <tr> <td><math>r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr> <td><math>v</math></td><td>Fluidgeschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>f</math></td><td>Drehfrequenz Zylinder</td><td><math>\left[\frac{1}{s}\right]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> <tr> <td><math>\Gamma</math></td><td>Zirkulaion</td><td><math>\left[\frac{m^2}{s}\right]</math></td></tr> </table> | $F_A$ | Auftriebskraft | $[N]$ | $l$ | Zylinderlänge | $[m]$ | $r$ | Radius | $[m]$ | $v$ | Fluidgeschw. | $\left[\frac{m}{s}\right]$ | $f$ | Drehfrequenz Zylinder | $\left[\frac{1}{s}\right]$ | $\rho$ | Dichte | $\left[\frac{kg}{m^3}\right]$ | $\Gamma$ | Zirkulaion | $\left[\frac{m^2}{s}\right]$ |
| $F_A$   | Auftriebskraft   | $[N]$   |       |                |       |     |               |       |     |        |       |     |              |                            |     |                       |                            |        |        |                               |          |            |                              |
| $l$   | Zylinderlänge  | $[m]$   |       |                |       |     |               |       |     |        |       |     |              |                            |     |                       |                            |        |        |                               |          |            |                              |
| $r$   | Radius   | $[m]$   |       |                |       |     |               |       |     |        |       |     |              |                            |     |                       |                            |        |        |                               |          |            |                              |
| $v$   | Fluidgeschw.   | $\left[\frac{m}{s}\right]$  |       |                |       |     |               |       |     |        |       |     |              |                            |     |                       |                            |        |        |                               |          |            |                              |
| $f$   | Drehfrequenz Zylinder  | $\left[\frac{1}{s}\right]$  |       |                |       |     |               |       |     |        |       |     |              |                            |     |                       |                            |        |        |                               |          |            |                              |
| $\rho$  | Dichte   | $\left[\frac{kg}{m^3}\right]$   |       |                |       |     |               |       |     |        |       |     |              |                            |     |                       |                            |        |        |                               |          |            |                              |
| $\Gamma$  | Zirkulaion   | $\left[\frac{m^2}{s}\right]$  |       |                |       |     |               |       |     |        |       |     |              |                            |     |                       |                            |        |        |                               |          |            |                              |

## 5.7.1. Tragflügel

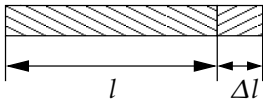
|  |  |   |
|--|--|---|
| <p>Tragflügel:</p>  <p><math>p</math> klein,<br/><math>v</math> gross</p> <p><math>v</math> klein,<br/><math>p</math> gross</p> | <p>Auftrieb:</p> $F_A = c_A \frac{\rho v^2}{2} A_T$ <p>Induzierter Widerstand:</p> $F_W = c_W \frac{\rho v^2}{2} A_T$ $F_R = F_{GH} = F_G \sin(\alpha)$ $F_A = F_{GN} = F_G \cos(\alpha)$ $\frac{c_W}{c_A} = \frac{\text{Höhenverlust}}{\text{MeterFlug}}$ | <p><math>F_A</math> Auftriebskraft [N]<br/> <math>F_W</math> Widerstandskraft [N]<br/> <math>c_A</math> Auftriebskoeffizient [1]<br/> <math>c_W</math> Widerstandskoeffizient [1]<br/> <math>v</math> Geschw. [<math>\frac{m}{s}</math>]<br/> <math>A_T</math> Fläche aus Anströmrichtung gesehen [<math>m^2</math>]<br/> <math>\rho</math> Dichte Fluid [<math>\frac{kg}{m^3}</math>]<br/> <math>\alpha</math> Gleitwinkel [rad]</p> |
|--|--|---|

## 6. Wärmelehre

### 6.1. Temperatur

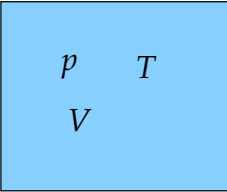
|   |  |  |     |            |       |     |                  |                 |  |  |                |     |       |        |           |                                 |       |     |                 |                 |  |        |       |  |         |               |  |            |       |
|---|--|--|-----|------------|-------|-----|------------------|-----------------|--|--|----------------|-----|-------|--------|-----------|---------------------------------|-------|-----|-----------------|-----------------|--|--------|-------|--|---------|---------------|--|------------|-------|
|  | <p>Absolute Temperatur:</p> $T = \frac{2}{3k} \bar{E}_{kin}$ $\bar{E}_{kin} = \frac{1}{2} m \bar{v}^2$ <p>Umrechnungen:</p> $T(K) = T(C) + 273.15$ $T(F) = \frac{9}{5} T(C) + 32$ $T(C) = \frac{5}{9} (T(F) - 32)$ | <table> <tr> <td><math>T</math></td><td>Temperatur</td><td><math>[K]</math></td></tr> <tr> <td><math>k</math></td><td>Boltzmann-konst.</td><td><math>[\frac{J}{K}]</math></td></tr> <tr> <td></td><td></td><td><math>1.381E^{-23}</math></td></tr> <tr> <td><math>m</math></td><td>Masse</td><td><math>[kg]</math></td></tr> <tr> <td><math>E_{kin}</math></td><td>kinetische Energie der Gasatome</td><td><math>[J]</math></td></tr> <tr> <td><math>v</math></td><td>Geschwindigkeit</td><td><math>[\frac{m}{s}]</math></td></tr> <tr> <td></td><td>Kelvin</td><td><math>[K]</math></td></tr> <tr> <td></td><td>Celcius</td><td><math>[^{\circ}C]</math></td></tr> <tr> <td></td><td>Fahrenheit</td><td><math>[F]</math></td></tr> </table> | $T$ | Temperatur | $[K]$ | $k$ | Boltzmann-konst. | $[\frac{J}{K}]$ |  |  | $1.381E^{-23}$ | $m$ | Masse | $[kg]$ | $E_{kin}$ | kinetische Energie der Gasatome | $[J]$ | $v$ | Geschwindigkeit | $[\frac{m}{s}]$ |  | Kelvin | $[K]$ |  | Celcius | $[^{\circ}C]$ |  | Fahrenheit | $[F]$ |
| $T$   | Temperatur   | $[K]$  |     |            |       |     |                  |                 |  |  |                |     |       |        |           |                                 |       |     |                 |                 |  |        |       |  |         |               |  |            |       |
| $k$   | Boltzmann-konst.   | $[\frac{J}{K}]$  |     |            |       |     |                  |                 |  |  |                |     |       |        |           |                                 |       |     |                 |                 |  |        |       |  |         |               |  |            |       |
|   |  | $1.381E^{-23}$   |     |            |       |     |                  |                 |  |  |                |     |       |        |           |                                 |       |     |                 |                 |  |        |       |  |         |               |  |            |       |
| $m$   | Masse  | $[kg]$   |     |            |       |     |                  |                 |  |  |                |     |       |        |           |                                 |       |     |                 |                 |  |        |       |  |         |               |  |            |       |
| $E_{kin}$   | kinetische Energie der Gasatome  | $[J]$  |     |            |       |     |                  |                 |  |  |                |     |       |        |           |                                 |       |     |                 |                 |  |        |       |  |         |               |  |            |       |
| $v$   | Geschwindigkeit  | $[\frac{m}{s}]$  |     |            |       |     |                  |                 |  |  |                |     |       |        |           |                                 |       |     |                 |                 |  |        |       |  |         |               |  |            |       |
|   | Kelvin   | $[K]$  |     |            |       |     |                  |                 |  |  |                |     |       |        |           |                                 |       |     |                 |                 |  |        |       |  |         |               |  |            |       |
|   | Celcius  | $[^{\circ}C]$  |     |            |       |     |                  |                 |  |  |                |     |       |        |           |                                 |       |     |                 |                 |  |        |       |  |         |               |  |            |       |
|   | Fahrenheit   | $[F]$  |     |            |       |     |                  |                 |  |  |                |     |       |        |           |                                 |       |     |                 |                 |  |        |       |  |         |               |  |            |       |

### 6.2. Ausdehnung von Materialien

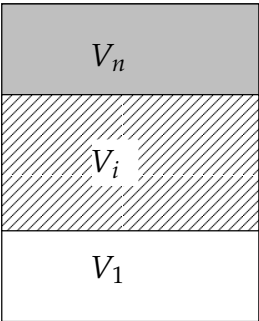
|   |  |   |     |       |       |     |         |         |          |                   |                 |          |                    |         |            |            |       |          |                     |                   |     |                   |                   |
|---|--|---|-----|-------|-------|-----|---------|---------|----------|-------------------|-----------------|----------|--------------------|---------|------------|------------|-------|----------|---------------------|-------------------|-----|-------------------|-------------------|
|  | <p>Längenausdehnung:</p> $\Delta l = \alpha l \Delta T$ <p>Volumenausdehnung:</p> $\Delta V = \gamma V \Delta T$ $\gamma = 3\alpha = \frac{\Delta V}{V \Delta T}$ $\alpha = \frac{\Delta \gamma}{\gamma \Delta T}$ $\sigma = E \alpha \Delta T = E \frac{\Delta l}{l}$ | <table> <tr> <td><math>l</math></td><td>Länge</td><td><math>[m]</math></td></tr> <tr> <td><math>V</math></td><td>Volumen</td><td><math>[m^3]</math></td></tr> <tr> <td><math>\alpha</math></td><td>Längenausd. koef.</td><td><math>[\frac{1}{K}]</math></td></tr> <tr> <td><math>\gamma</math></td><td>Volumenausd. koef.</td><td><math>[m^3]</math></td></tr> <tr> <td><math>\Delta T</math></td><td>Temperatur</td><td><math>[K]</math></td></tr> <tr> <td><math>\sigma</math></td><td>Spannung bzw. Druck</td><td><math>[\frac{N}{m^2}]</math></td></tr> <tr> <td><math>E</math></td><td>Elastizitätsmodul</td><td><math>[\frac{N}{m^2}]</math></td></tr> </table> | $l$ | Länge | $[m]$ | $V$ | Volumen | $[m^3]$ | $\alpha$ | Längenausd. koef. | $[\frac{1}{K}]$ | $\gamma$ | Volumenausd. koef. | $[m^3]$ | $\Delta T$ | Temperatur | $[K]$ | $\sigma$ | Spannung bzw. Druck | $[\frac{N}{m^2}]$ | $E$ | Elastizitätsmodul | $[\frac{N}{m^2}]$ |
| $l$   | Länge  | $[m]$   |     |       |       |     |         |         |          |                   |                 |          |                    |         |            |            |       |          |                     |                   |     |                   |                   |
| $V$   | Volumen  | $[m^3]$   |     |       |       |     |         |         |          |                   |                 |          |                    |         |            |            |       |          |                     |                   |     |                   |                   |
| $\alpha$  | Längenausd. koef.  | $[\frac{1}{K}]$   |     |       |       |     |         |         |          |                   |                 |          |                    |         |            |            |       |          |                     |                   |     |                   |                   |
| $\gamma$  | Volumenausd. koef.   | $[m^3]$   |     |       |       |     |         |         |          |                   |                 |          |                    |         |            |            |       |          |                     |                   |     |                   |                   |
| $\Delta T$  | Temperatur   | $[K]$   |     |       |       |     |         |         |          |                   |                 |          |                    |         |            |            |       |          |                     |                   |     |                   |                   |
| $\sigma$  | Spannung bzw. Druck  | $[\frac{N}{m^2}]$   |     |       |       |     |         |         |          |                   |                 |          |                    |         |            |            |       |          |                     |                   |     |                   |                   |
| $E$   | Elastizitätsmodul  | $[\frac{N}{m^2}]$   |     |       |       |     |         |         |          |                   |                 |          |                    |         |            |            |       |          |                     |                   |     |                   |                   |



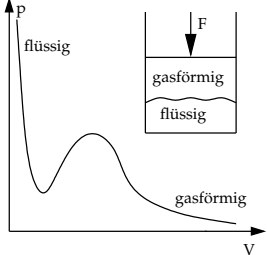
## 6.3. Ideale Gase

|   |  |   |
|---|--|---|
|  | $pV = \text{konst.}$<br>$\frac{V}{T} = \text{konst.}$<br>$\frac{pV}{T} = \text{konst.}$<br>$pV = NkT$<br>$N = nN_A$<br>$R = N_A k$<br>$pV = nRT$<br>$m = nM$<br>$\rho = \frac{m}{V} = \frac{pM}{RT}$<br><br>Volumen eines idealen Gases: $22.4 \cdot 10^{-3} \frac{\text{m}^3}{\text{mol}}$ bei<br>$p = 10133 \text{ Pa}$ und $T = 273.15 \text{ K}$ | <p> <math>p</math> Druck <math>[\frac{\text{N}}{\text{m}^2}]</math><br/> <math>V</math> Volumen <math>[\text{m}^3]</math><br/> <math>T</math> Temperatur <math>[\text{K}]</math><br/> <math>N</math> Anz. Moleküle<br/> <math>n</math> Anz. Mol<br/> <math>m</math> Gasmasse <math>[\text{kg}]</math><br/> <math>M</math> Molmasse <math>[\text{kg}]</math><br/> <math>\rho</math> Dichte <math>[\frac{\text{kg}}{\text{m}^3}]</math><br/> <math>N_A</math> Anz. Atome pro 12g C<br/> <math>= 6,022 \cdot 10^{23}</math><br/> <math>k</math> Boltzmann-konst. <math>[\frac{\text{J}}{\text{K}}]</math><br/> <math>= 1,381 \cdot 10^{-23}</math><br/> <math>R</math> Univers.-Gaskonst. <math>[\frac{\text{J}}{\text{molK}}]</math><br/> <math>= 8,314</math> </p> |
|---|--|---|

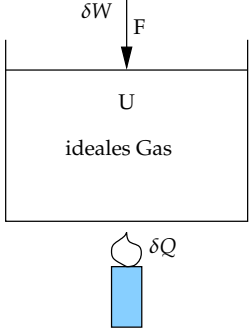
## 6.4. Gemische idealer Gase

|   |  |  |
|---|--|--|
|  | $\sum_{i=1}^z p_i = p$<br>$p_i = q_i p$<br>$M = \sum_{i=1}^z q_i M_i$<br>$\mu_i = \frac{M_i}{M} q_i$ | <p> <math>p</math> (Gesamt-) Druck <math>[\frac{\text{N}}{\text{m}^2}]</math><br/> <math>p_i</math> Partialdruck <math>[\frac{\text{N}}{\text{m}^2}]</math><br/> <math>q</math> Konzentration <math>[\frac{1}{\text{K}}]</math><br/> <math>M</math> Molmasse <math>[\text{kg}]</math><br/> <math>\mu_i</math> Massenkonzentration </p> |
|---|--|--|

## 6.5. Reale Gase

|   |   |   |     |       |                              |       |            |         |     |                    |                               |     |            |       |     |          |  |     |                     |        |     |                     |         |       |                      |       |       |                  |                              |          |                    |         |
|---|---|---|-----|-------|------------------------------|-------|------------|---------|-----|--------------------|-------------------------------|-----|------------|-------|-----|----------|--|-----|---------------------|--------|-----|---------------------|---------|-------|----------------------|-------|-------|------------------|------------------------------|----------|--------------------|---------|
|  | $\left(p + \frac{a}{V_m^2}\right)(V_m - b) = RT$ $p = n \frac{RT}{V - nb} - n^2 \frac{a}{V^2}$ $a = 3p_k V_{mk}^2 = \frac{9}{8} RT_k V_{mk}$ $b = \frac{V_{mk}}{3} = \frac{RT_k}{8p_k}$ $T_k = \frac{8a}{27Rb}$ $p_k = \frac{a}{27b^2}$ $V_{mk} = 3b$ | <table> <tr> <td><math>p</math></td><td>Druck</td><td><math>\left[\frac{N}{m^2}\right]</math></td></tr> <tr> <td><math>V_m</math></td><td>Molvolumen</td><td><math>[m^3]</math></td></tr> <tr> <td><math>R</math></td><td>Univers.-Gaskonst.</td><td><math>\left[\frac{J}{molK}\right]</math></td></tr> <tr> <td><math>T</math></td><td>Temperatur</td><td><math>[K]</math></td></tr> <tr> <td><math>n</math></td><td>Anz. Mol</td><td></td></tr> <tr> <td><math>a</math></td><td>Van der Waal-konst.</td><td><math>[Nm]</math></td></tr> <tr> <td><math>b</math></td><td>Van der Waal-konst.</td><td><math>[m^3]</math></td></tr> <tr> <td><math>T_k</math></td><td>Kritische Temperatur</td><td><math>[K]</math></td></tr> <tr> <td><math>p_k</math></td><td>Kritischer Druck</td><td><math>\left[\frac{N}{m^2}\right]</math></td></tr> <tr> <td><math>V_{mk}</math></td><td>Kritisches Volumen</td><td><math>[m^3]</math></td></tr> </table> | $p$ | Druck | $\left[\frac{N}{m^2}\right]$ | $V_m$ | Molvolumen | $[m^3]$ | $R$ | Univers.-Gaskonst. | $\left[\frac{J}{molK}\right]$ | $T$ | Temperatur | $[K]$ | $n$ | Anz. Mol |  | $a$ | Van der Waal-konst. | $[Nm]$ | $b$ | Van der Waal-konst. | $[m^3]$ | $T_k$ | Kritische Temperatur | $[K]$ | $p_k$ | Kritischer Druck | $\left[\frac{N}{m^2}\right]$ | $V_{mk}$ | Kritisches Volumen | $[m^3]$ |
| $p$   | Druck   | $\left[\frac{N}{m^2}\right]$  |     |       |                              |       |            |         |     |                    |                               |     |            |       |     |          |  |     |                     |        |     |                     |         |       |                      |       |       |                  |                              |          |                    |         |
| $V_m$   | Molvolumen  | $[m^3]$   |     |       |                              |       |            |         |     |                    |                               |     |            |       |     |          |  |     |                     |        |     |                     |         |       |                      |       |       |                  |                              |          |                    |         |
| $R$   | Univers.-Gaskonst.  | $\left[\frac{J}{molK}\right]$   |     |       |                              |       |            |         |     |                    |                               |     |            |       |     |          |  |     |                     |        |     |                     |         |       |                      |       |       |                  |                              |          |                    |         |
| $T$   | Temperatur  | $[K]$   |     |       |                              |       |            |         |     |                    |                               |     |            |       |     |          |  |     |                     |        |     |                     |         |       |                      |       |       |                  |                              |          |                    |         |
| $n$   | Anz. Mol  |   |     |       |                              |       |            |         |     |                    |                               |     |            |       |     |          |  |     |                     |        |     |                     |         |       |                      |       |       |                  |                              |          |                    |         |
| $a$   | Van der Waal-konst.   | $[Nm]$  |     |       |                              |       |            |         |     |                    |                               |     |            |       |     |          |  |     |                     |        |     |                     |         |       |                      |       |       |                  |                              |          |                    |         |
| $b$   | Van der Waal-konst.   | $[m^3]$   |     |       |                              |       |            |         |     |                    |                               |     |            |       |     |          |  |     |                     |        |     |                     |         |       |                      |       |       |                  |                              |          |                    |         |
| $T_k$   | Kritische Temperatur  | $[K]$   |     |       |                              |       |            |         |     |                    |                               |     |            |       |     |          |  |     |                     |        |     |                     |         |       |                      |       |       |                  |                              |          |                    |         |
| $p_k$   | Kritischer Druck  | $\left[\frac{N}{m^2}\right]$  |     |       |                              |       |            |         |     |                    |                               |     |            |       |     |          |  |     |                     |        |     |                     |         |       |                      |       |       |                  |                              |          |                    |         |
| $V_{mk}$  | Kritisches Volumen  | $[m^3]$   |     |       |                              |       |            |         |     |                    |                               |     |            |       |     |          |  |     |                     |        |     |                     |         |       |                      |       |       |                  |                              |          |                    |         |

## 6.6. Wärme

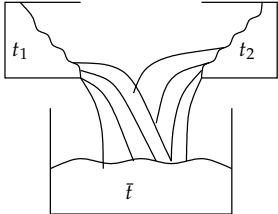
|   |  |   |     |                |       |     |        |       |     |       |       |     |                |                            |       |                       |                               |          |                              |                               |          |                             |                               |     |                      |                              |     |       |        |     |          |                               |     |          |  |     |                      |                               |
|---|--|---|-----|----------------|-------|-----|--------|-------|-----|-------|-------|-----|----------------|----------------------------|-------|-----------------------|-------------------------------|----------|------------------------------|-------------------------------|----------|-----------------------------|-------------------------------|-----|----------------------|------------------------------|-----|-------|--------|-----|----------|-------------------------------|-----|----------|--|-----|----------------------|-------------------------------|
|  | $dU = \delta W + \delta Q$ $\delta Q = CdT$ $\delta Q = cmdT$ $\delta Q = C_m n dT$ $C = mc$ $C_m = Mc$ $C_{mp} - C_{mv} = R$ $m = nM$ | <table> <tr> <td><math>U</math></td><td>Innere Energie</td><td><math>[J]</math></td></tr> <tr> <td><math>W</math></td><td>Arbeit</td><td><math>[J]</math></td></tr> <tr> <td><math>Q</math></td><td>Wärme</td><td><math>[J]</math></td></tr> <tr> <td><math>C</math></td><td>Wärmekapazität</td><td><math>\left[\frac{J}{K}\right]</math></td></tr> <tr> <td><math>C_m</math></td><td>Molare Wärmekapazität</td><td><math>\left[\frac{J}{molK}\right]</math></td></tr> <tr> <td><math>C_{mv}</math></td><td><math>C_m</math>, isochor<sup>1</sup></td><td><math>\left[\frac{J}{molK}\right]</math></td></tr> <tr> <td><math>C_{mp}</math></td><td><math>C_m</math>, isobar<sup>2</sup></td><td><math>\left[\frac{J}{molK}\right]</math></td></tr> <tr> <td><math>c</math></td><td>spez. Wärmekapazität</td><td><math>\left[\frac{J}{kgK}\right]</math></td></tr> <tr> <td><math>m</math></td><td>Masse</td><td><math>[kg]</math></td></tr> <tr> <td><math>M</math></td><td>Molmasse</td><td><math>\left[\frac{kg}{mol}\right]</math></td></tr> <tr> <td><math>n</math></td><td>Anz. Mol</td><td></td></tr> <tr> <td><math>R</math></td><td>Univers.-Gk. = 8.314</td><td><math>\left[\frac{J}{molK}\right]</math></td></tr> </table> | $U$ | Innere Energie | $[J]$ | $W$ | Arbeit | $[J]$ | $Q$ | Wärme | $[J]$ | $C$ | Wärmekapazität | $\left[\frac{J}{K}\right]$ | $C_m$ | Molare Wärmekapazität | $\left[\frac{J}{molK}\right]$ | $C_{mv}$ | $C_m$ , isochor <sup>1</sup> | $\left[\frac{J}{molK}\right]$ | $C_{mp}$ | $C_m$ , isobar <sup>2</sup> | $\left[\frac{J}{molK}\right]$ | $c$ | spez. Wärmekapazität | $\left[\frac{J}{kgK}\right]$ | $m$ | Masse | $[kg]$ | $M$ | Molmasse | $\left[\frac{kg}{mol}\right]$ | $n$ | Anz. Mol |  | $R$ | Univers.-Gk. = 8.314 | $\left[\frac{J}{molK}\right]$ |
| $U$   | Innere Energie   | $[J]$   |     |                |       |     |        |       |     |       |       |     |                |                            |       |                       |                               |          |                              |                               |          |                             |                               |     |                      |                              |     |       |        |     |          |                               |     |          |  |     |                      |                               |
| $W$   | Arbeit   | $[J]$   |     |                |       |     |        |       |     |       |       |     |                |                            |       |                       |                               |          |                              |                               |          |                             |                               |     |                      |                              |     |       |        |     |          |                               |     |          |  |     |                      |                               |
| $Q$   | Wärme  | $[J]$   |     |                |       |     |        |       |     |       |       |     |                |                            |       |                       |                               |          |                              |                               |          |                             |                               |     |                      |                              |     |       |        |     |          |                               |     |          |  |     |                      |                               |
| $C$   | Wärmekapazität   | $\left[\frac{J}{K}\right]$  |     |                |       |     |        |       |     |       |       |     |                |                            |       |                       |                               |          |                              |                               |          |                             |                               |     |                      |                              |     |       |        |     |          |                               |     |          |  |     |                      |                               |
| $C_m$   | Molare Wärmekapazität  | $\left[\frac{J}{molK}\right]$   |     |                |       |     |        |       |     |       |       |     |                |                            |       |                       |                               |          |                              |                               |          |                             |                               |     |                      |                              |     |       |        |     |          |                               |     |          |  |     |                      |                               |
| $C_{mv}$  | $C_m$ , isochor <sup>1</sup>   | $\left[\frac{J}{molK}\right]$   |     |                |       |     |        |       |     |       |       |     |                |                            |       |                       |                               |          |                              |                               |          |                             |                               |     |                      |                              |     |       |        |     |          |                               |     |          |  |     |                      |                               |
| $C_{mp}$  | $C_m$ , isobar <sup>2</sup>  | $\left[\frac{J}{molK}\right]$   |     |                |       |     |        |       |     |       |       |     |                |                            |       |                       |                               |          |                              |                               |          |                             |                               |     |                      |                              |     |       |        |     |          |                               |     |          |  |     |                      |                               |
| $c$   | spez. Wärmekapazität   | $\left[\frac{J}{kgK}\right]$  |     |                |       |     |        |       |     |       |       |     |                |                            |       |                       |                               |          |                              |                               |          |                             |                               |     |                      |                              |     |       |        |     |          |                               |     |          |  |     |                      |                               |
| $m$   | Masse  | $[kg]$  |     |                |       |     |        |       |     |       |       |     |                |                            |       |                       |                               |          |                              |                               |          |                             |                               |     |                      |                              |     |       |        |     |          |                               |     |          |  |     |                      |                               |
| $M$   | Molmasse   | $\left[\frac{kg}{mol}\right]$   |     |                |       |     |        |       |     |       |       |     |                |                            |       |                       |                               |          |                              |                               |          |                             |                               |     |                      |                              |     |       |        |     |          |                               |     |          |  |     |                      |                               |
| $n$   | Anz. Mol   |   |     |                |       |     |        |       |     |       |       |     |                |                            |       |                       |                               |          |                              |                               |          |                             |                               |     |                      |                              |     |       |        |     |          |                               |     |          |  |     |                      |                               |
| $R$   | Univers.-Gk. = 8.314   | $\left[\frac{J}{molK}\right]$   |     |                |       |     |        |       |     |       |       |     |                |                            |       |                       |                               |          |                              |                               |          |                             |                               |     |                      |                              |     |       |        |     |          |                               |     |          |  |     |                      |                               |

<sup>1</sup>Isochore Prozesse sind Zustandsänderungen bei konstantem Volumen<sup>2</sup>Isobare Prozesse sind Zustandsänderungen bei konstantem Druck

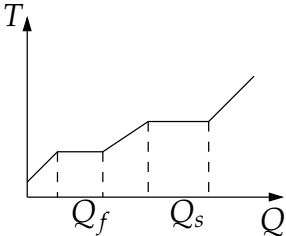
### 6.6.1. Molare Wärme kristalliner Festkörper

|  |   |  |
|--|---|--|
|  | falls $T > \Theta_D$ :<br>$C_{mv} = 3R$<br>falls $T \ll \Theta_D$ :<br>$C_{mv} = \frac{12\pi^4}{5} R \left( \frac{T}{\Theta_D} \right)^3$ | $C_{mv}$ $C_m$ , isochor $\left[ \frac{J}{molK} \right]$<br>$T$ Temperatur $[K]$<br>$\Theta_D$ Debye-Temp. $[K]$<br>$R$ Univers.-Gaskonst. $\left[ \frac{J}{molK} \right]$ |
|--|---|--|

### 6.6.2. Austausch von Wärmemengen

|  |   |   |
|--|---|---|
|  | $\bar{t} = \frac{m_1 c_1 t_1 + m_2 c_2 t_2 + \dots}{m_1 c_1 + m_2 c_2 + \dots}$ | $\bar{t}$ Mischtemp. $[K]$<br>$t$ Temperatur $[K]$<br>$m$ Masse $[kg]$<br>$c$ spez. Wärmekapazität $\left[ \frac{J}{kgK} \right]$ |
|--|---|---|

## 6.7. Phasen und Phasenübergänge

|   |  |   |
|---|--|---|
|  | Schmelzen:<br>$Q = mQ_s \quad q_s = \frac{Q_s}{m}$<br>Verdampfen:<br>$Q = mQ_v \quad q_v = \frac{Q_v}{m}$<br>$\frac{dp_s}{dT} = \frac{q_s}{T \left( \frac{1}{\rho_g} - \frac{1}{\rho_f} \right)}$ $\frac{dp_f}{dT} = \frac{q_f}{T \left( \frac{1}{\rho_f} - \frac{1}{\rho_s} \right)}$ | $Q$ Wärme $[J]$<br>$m$ Masse $[kg]$<br>$Q_s$ Schmelzenergie $[J]$<br>$Q_v$ Verdampfenergie $[J]$<br>$q_s$ spez. Schmelz $Q- \left[ \frac{J}{kg} \right]$<br>$q_v$ spez. Verdampf $Q- \left[ \frac{J}{kg} \right]$<br>$p_s$ Dampfdruck $[Pa]$<br>$p_f$ Schmelzdruck $[Pa]$<br>$T$ Temperatur $[K]$<br>$\rho_g$ Dichte Phase gasförmig $\left[ \frac{kg}{m^3} \right]$<br>$\rho_f$ Dichte Phase flüssig $\left[ \frac{kg}{m^3} \right]$<br>$\rho_s$ Dichte Phase fest $\left[ \frac{kg}{m^3} \right]$ |
|---|--|---|

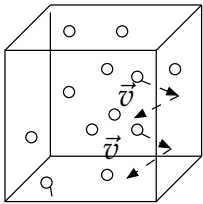
## 6.7.1. Schmelz- und Verdampfungsenergien

| Substanz     | $T_{\text{schmelz}} [K]$ | $Q_s [\frac{kJ}{kg}]$ | $T_{\text{verdampf}} [K]$ | $Q_v [\frac{kJ}{kg}]$ |
|--------------|--------------------------|-----------------------|---------------------------|-----------------------|
| Blei         | 600                      | 24,7                  | 2023                      | 858                   |
| Brom         | 266                      | 67,4                  | 332                       | 369                   |
| Ethanol      | 159                      | 109                   | 351                       | 879                   |
| Gold         | 1336                     | 62,8                  | 3081                      | 1701                  |
| Helium       | -                        | -                     | 4,2                       | 21                    |
| Kohlendioxid | -                        | -                     | 194,6                     | 573                   |
| Kupfer       | 1356                     | 205                   | 2839                      | 4726                  |
| Quecksilber  | 234                      | 11,3                  | 630                       | 296                   |
| Sauerstoff   | 54,4                     | 13,8                  | 90,2                      | 213                   |
| Schwefel     | 388                      | 38,5                  | 717,75                    | 287                   |
| Silber       | 1234                     | 105                   | 2436                      | 2323                  |
| Stickstoff   | 63                       | 25,7                  | 77,35                     | 199                   |
| Wasser       | 273,15                   | 333,5                 | 373,15                    | 2257                  |
| Zink         | 692                      | 102                   | 1184                      | 1768                  |

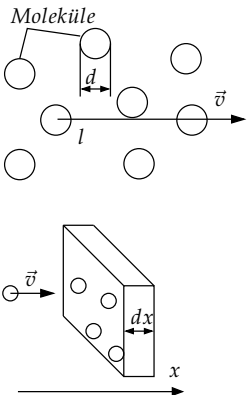
## 6.8. Luftfeuchtigkeit

|  |  |  |
|--|--|--|
|  | $f = \frac{m_W}{V}$ $f_r = \frac{m_W}{m_s} = \frac{p_D}{p_s} (\cdot 100\%)$ $p_s = p_{s0} 10^{\frac{7,5\vartheta}{\vartheta+237}} \quad , \vartheta \geq 0^\circ\text{C}$ $p_s = p_{s0} 10^{\frac{9,5\vartheta}{\vartheta+265,5}} \quad , \vartheta \leq 0^\circ\text{C}$ $p_D = p_s(\vartheta_d)$ $\vartheta = \frac{237 \log \frac{p_s}{6,107}}{7,5 - \log \frac{p_s}{6,107}} , p_s \geq 610,7 \text{ Pa}$ $\vartheta = \frac{265,5 \log \frac{p_s}{6,107}}{9,5 - \log \frac{p_s}{6,107}} , p_s \leq 610,7 \text{ Pa}$ | $f$ Luftfeuchtigkeit absolut $[\frac{kg}{m^3}]$<br>$f_r$ Luftfeuchtigkeit relativ $[1]$<br>$m_W$ Wasserdampfmasse $[kg]$<br>$m_s$ Dampfmasse im Sättigungszustand $[\frac{kg}{m^3}]$<br>$V$ Volumen $[m^3]$<br>$p_D$ Partialdruck Wasserdampf $[Pa]$<br>$p_s$ Sättigungsdruck Wasserdampf $[Pa]$<br>$\vartheta$ Temperatur $[^\circ\text{C}]$<br>$p_{s0}$ 61070 $[Pa]$ |
|--|--|--|

## 6.9. Kinetische Gastheorie

|  |   |   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
|--|---|---|-----|-------|--------|-----|---------|---------|-----|---------------|-------|-----|----------|-------|-----|-------------------------|-----------------|-----------|---------------------------------|-------|-----------|---------------------|-------|-----|----------------|-------|-----------|--------------------|-------|-----|------------|-------|-----|---------------|-------|-----|-------|--------|----------|-----------------------|--------------------|-------|----------------------|-----------------|--|--|-------------------------|-----|------------------|-----------------|--|--|--------------------------|-----|--------------|--------------------|--|--|-----------|
|  <p>Molekül</p> <p>Einatomige Moleküle haben keine Rotationsenergie, deshalb ist in diesem Fall: <math>E_{kin} = E_{trans}</math></p> | <p>Translationsenergie:</p> $\bar{E}_{kin} = N \frac{m \bar{v}^2}{2} = \frac{3}{2} N k T = \frac{3}{2} n R T$ $U = N_A \bar{E} = N_A \frac{f}{2} k T = \frac{f}{2} R T$ $U = E_{kin} + E_{pot}$ $pV = \frac{2}{3} N_A \frac{m \bar{v}^2}{2}$ <p>Mittlere Energie pro Molekül:</p> $\bar{E} = \frac{f}{2} k T$ $\bar{E}_{kin} \approx T_{abs}$ $C_{mv} = \frac{f}{2} R$ <p>f=3 bei einatomigen Molekülen<br/>f=5 bei zweiatomigen Molekülen<br/>f=6 bei mehratomigen Molekülen</p> <p>Volumen eines idealen Gases: <math>22.4 \cdot 10^{-3} \frac{m^3}{mol}</math> bei <math>p = 10133 Pa</math> und <math>T = 273.15 K</math></p> | <table> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>V</math></td><td>Volumen</td><td><math>[m^3]</math></td></tr> <tr> <td><math>N</math></td><td>Anz. Moleküle</td><td><math>[1]</math></td></tr> <tr> <td><math>n</math></td><td>Anz. Mol</td><td><math>[1]</math></td></tr> <tr> <td><math>v</math></td><td>Molekül Geschwindigkeit</td><td><math>[\frac{m}{s}]</math></td></tr> <tr> <td><math>E_{kin}</math></td><td>Kinetische Energie der Moleküle</td><td><math>[J]</math></td></tr> <tr> <td><math>E_{pot}</math></td><td>Potentielle Energie</td><td><math>[J]</math></td></tr> <tr> <td><math>U</math></td><td>Innere Energie</td><td><math>[J]</math></td></tr> <tr> <td><math>T_{abs}</math></td><td>Temperatur absolut</td><td><math>[K]</math></td></tr> <tr> <td><math>T</math></td><td>Temperatur</td><td><math>[K]</math></td></tr> <tr> <td><math>f</math></td><td>Freiheitsgrad</td><td><math>[1]</math></td></tr> <tr> <td><math>m</math></td><td>Masse</td><td><math>[kg]</math></td></tr> <tr> <td><math>C_{mv}</math></td><td>Molare Wärmekapazität</td><td><math>[\frac{J}{molK}]</math></td></tr> <tr> <td><math>N_A</math></td><td>Anz. Atome pro 12g C</td><td><math>\frac{1}{mol}</math></td></tr> <tr> <td></td><td></td><td><math>= 6,022 \cdot 10^{23}</math></td></tr> <tr> <td><math>k</math></td><td>Boltzmann-konst.</td><td><math>[\frac{J}{K}]</math></td></tr> <tr> <td></td><td></td><td><math>= 1,381 \cdot 10^{-23}</math></td></tr> <tr> <td><math>R</math></td><td>Univers.-Gk.</td><td><math>[\frac{J}{molK}]</math></td></tr> <tr> <td></td><td></td><td><math>= 8.314</math></td></tr> </table> | $p$ | Druck | $[Pa]$ | $V$ | Volumen | $[m^3]$ | $N$ | Anz. Moleküle | $[1]$ | $n$ | Anz. Mol | $[1]$ | $v$ | Molekül Geschwindigkeit | $[\frac{m}{s}]$ | $E_{kin}$ | Kinetische Energie der Moleküle | $[J]$ | $E_{pot}$ | Potentielle Energie | $[J]$ | $U$ | Innere Energie | $[J]$ | $T_{abs}$ | Temperatur absolut | $[K]$ | $T$ | Temperatur | $[K]$ | $f$ | Freiheitsgrad | $[1]$ | $m$ | Masse | $[kg]$ | $C_{mv}$ | Molare Wärmekapazität | $[\frac{J}{molK}]$ | $N_A$ | Anz. Atome pro 12g C | $\frac{1}{mol}$ |  |  | $= 6,022 \cdot 10^{23}$ | $k$ | Boltzmann-konst. | $[\frac{J}{K}]$ |  |  | $= 1,381 \cdot 10^{-23}$ | $R$ | Univers.-Gk. | $[\frac{J}{molK}]$ |  |  | $= 8.314$ |
| $p$  | Druck   | $[Pa]$  |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $V$  | Volumen   | $[m^3]$   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $N$  | Anz. Moleküle   | $[1]$   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $n$  | Anz. Mol  | $[1]$   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $v$  | Molekül Geschwindigkeit   | $[\frac{m}{s}]$   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $E_{kin}$  | Kinetische Energie der Moleküle   | $[J]$   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $E_{pot}$  | Potentielle Energie   | $[J]$   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $U$  | Innere Energie  | $[J]$   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $T_{abs}$  | Temperatur absolut  | $[K]$   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $T$  | Temperatur  | $[K]$   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $f$  | Freiheitsgrad   | $[1]$   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $m$  | Masse   | $[kg]$  |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $C_{mv}$   | Molare Wärmekapazität   | $[\frac{J}{molK}]$  |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $N_A$  | Anz. Atome pro 12g C  | $\frac{1}{mol}$   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
|  |   | $= 6,022 \cdot 10^{23}$   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $k$  | Boltzmann-konst.  | $[\frac{J}{K}]$   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
|  |   | $= 1,381 \cdot 10^{-23}$  |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
| $R$  | Univers.-Gk.  | $[\frac{J}{molK}]$  |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |
|  |   | $= 8.314$   |     |       |        |     |         |         |     |               |       |     |          |       |     |                         |                 |           |                                 |       |           |                     |       |     |                |       |           |                    |       |     |            |       |     |               |       |     |       |        |          |                       |                    |       |                      |                 |  |  |                         |     |                  |                 |  |  |                          |     |              |                    |  |  |           |

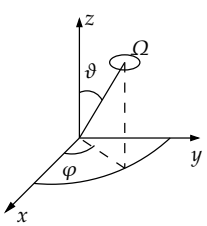
## 6.9.1. Mittlere freie Weglänge, Wärmeleitung, Diffusion und Viskosität

|   |  |  |                 |   |       |     |                             |       |     |           |       |     |                                  |       |          |             |         |                   |                     |                    |     |              |         |     |               |       |     |   |                 |     |                  |                   |        |            |                   |        |        |                    |
|---|--|--|-----------------|---|-------|-----|-----------------------------|-------|-----|-----------|-------|-----|----------------------------------|-------|----------|-------------|---------|-------------------|---------------------|--------------------|-----|--------------|---------|-----|---------------|-------|-----|---|-----------------|-----|------------------|-------------------|--------|------------|-------------------|--------|--------|--------------------|
|  | $\bar{\lambda} = \frac{1}{\sqrt{2}n\pi d^2}$ $N = N_0 e^{-x/\bar{\lambda}}$ <p>Wahrscheinlichkeit <math>f(x)dx</math>, dass ein Molekül einen freien Weg auf der Strecke <math>dx</math> hat:</p> $f(x)dx = n\sigma e^{-x/\bar{\lambda}}dx$ $\sigma = \pi d^2$ $\lambda_Q = \frac{1}{6}n\bar{v}\bar{\lambda}fk$ $D = \frac{1}{3}\bar{v}\bar{\lambda}$ $\eta = \frac{1}{3}\bar{v}\bar{\lambda}\rho$ | <table> <tr> <td><math>\bar{\lambda}</math></td><td>Mittlere freie Weglänge zwischen Molekülzusammenstoss</td><td><math>[m]</math></td></tr> <tr> <td><math>n</math></td><td>Anz. Moleküle (≠ Anz. Mole)</td><td><math>[1]</math></td></tr> <tr> <td><math>d</math></td><td>Ø-Molekül</td><td><math>[m]</math></td></tr> <tr> <td><math>N</math></td><td>Anz. Moleküle durch Schicht <math>dx</math></td><td><math>[1]</math></td></tr> <tr> <td><math>\sigma</math></td><td>Querschnitt</td><td><math>[m^2]</math></td></tr> <tr> <td><math>\bar{\lambda}_Q</math></td><td>Wärmeleitungskoeff.</td><td><math>[\frac{W}{m^2K}]</math></td></tr> <tr> <td><math>v</math></td><td>v - Moleküle</td><td><math>[m/s]</math></td></tr> <tr> <td><math>f</math></td><td>Freiheitsgrad</td><td><math>[1]</math></td></tr> <tr> <td><math>k</math></td><td>Boltzmann-konst. = <math>1,381 \cdot 10^{-23}</math></td><td><math>[\frac{J}{K}]</math></td></tr> <tr> <td><math>D</math></td><td>Diffusionskonst.</td><td><math>[\frac{m^2}{s}]</math></td></tr> <tr> <td><math>\eta</math></td><td>Viskosität</td><td><math>[\frac{kg}{ms}]</math></td></tr> <tr> <td><math>\rho</math></td><td>Dichte</td><td><math>[\frac{kg}{m^3}]</math></td></tr> </table> | $\bar{\lambda}$ | Mittlere freie Weglänge zwischen Molekülzusammenstoss | $[m]$ | $n$ | Anz. Moleküle (≠ Anz. Mole) | $[1]$ | $d$ | Ø-Molekül | $[m]$ | $N$ | Anz. Moleküle durch Schicht $dx$ | $[1]$ | $\sigma$ | Querschnitt | $[m^2]$ | $\bar{\lambda}_Q$ | Wärmeleitungskoeff. | $[\frac{W}{m^2K}]$ | $v$ | v - Moleküle | $[m/s]$ | $f$ | Freiheitsgrad | $[1]$ | $k$ | Boltzmann-konst. = $1,381 \cdot 10^{-23}$ | $[\frac{J}{K}]$ | $D$ | Diffusionskonst. | $[\frac{m^2}{s}]$ | $\eta$ | Viskosität | $[\frac{kg}{ms}]$ | $\rho$ | Dichte | $[\frac{kg}{m^3}]$ |
| $\bar{\lambda}$   | Mittlere freie Weglänge zwischen Molekülzusammenstoss  | $[m]$  |                 |   |       |     |                             |       |     |           |       |     |                                  |       |          |             |         |                   |                     |                    |     |              |         |     |               |       |     |   |                 |     |                  |                   |        |            |                   |        |        |                    |
| $n$   | Anz. Moleküle (≠ Anz. Mole)  | $[1]$  |                 |   |       |     |                             |       |     |           |       |     |                                  |       |          |             |         |                   |                     |                    |     |              |         |     |               |       |     |   |                 |     |                  |                   |        |            |                   |        |        |                    |
| $d$   | Ø-Molekül  | $[m]$  |                 |   |       |     |                             |       |     |           |       |     |                                  |       |          |             |         |                   |                     |                    |     |              |         |     |               |       |     |   |                 |     |                  |                   |        |            |                   |        |        |                    |
| $N$   | Anz. Moleküle durch Schicht $dx$   | $[1]$  |                 |   |       |     |                             |       |     |           |       |     |                                  |       |          |             |         |                   |                     |                    |     |              |         |     |               |       |     |   |                 |     |                  |                   |        |            |                   |        |        |                    |
| $\sigma$  | Querschnitt  | $[m^2]$  |                 |   |       |     |                             |       |     |           |       |     |                                  |       |          |             |         |                   |                     |                    |     |              |         |     |               |       |     |   |                 |     |                  |                   |        |            |                   |        |        |                    |
| $\bar{\lambda}_Q$   | Wärmeleitungskoeff.  | $[\frac{W}{m^2K}]$   |                 |   |       |     |                             |       |     |           |       |     |                                  |       |          |             |         |                   |                     |                    |     |              |         |     |               |       |     |   |                 |     |                  |                   |        |            |                   |        |        |                    |
| $v$   | v - Moleküle   | $[m/s]$  |                 |   |       |     |                             |       |     |           |       |     |                                  |       |          |             |         |                   |                     |                    |     |              |         |     |               |       |     |   |                 |     |                  |                   |        |            |                   |        |        |                    |
| $f$   | Freiheitsgrad  | $[1]$  |                 |   |       |     |                             |       |     |           |       |     |                                  |       |          |             |         |                   |                     |                    |     |              |         |     |               |       |     |   |                 |     |                  |                   |        |            |                   |        |        |                    |
| $k$   | Boltzmann-konst. = $1,381 \cdot 10^{-23}$  | $[\frac{J}{K}]$  |                 |   |       |     |                             |       |     |           |       |     |                                  |       |          |             |         |                   |                     |                    |     |              |         |     |               |       |     |   |                 |     |                  |                   |        |            |                   |        |        |                    |
| $D$   | Diffusionskonst.   | $[\frac{m^2}{s}]$  |                 |   |       |     |                             |       |     |           |       |     |                                  |       |          |             |         |                   |                     |                    |     |              |         |     |               |       |     |   |                 |     |                  |                   |        |            |                   |        |        |                    |
| $\eta$  | Viskosität   | $[\frac{kg}{ms}]$  |                 |   |       |     |                             |       |     |           |       |     |                                  |       |          |             |         |                   |                     |                    |     |              |         |     |               |       |     |   |                 |     |                  |                   |        |            |                   |        |        |                    |
| $\rho$  | Dichte   | $[\frac{kg}{m^3}]$   |                 |   |       |     |                             |       |     |           |       |     |                                  |       |          |             |         |                   |                     |                    |     |              |         |     |               |       |     |   |                 |     |                  |                   |        |            |                   |        |        |                    |

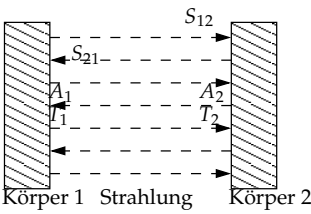
## 6.9.2. Maxwellsche Geschwindigkeitsverteilung

|        |  |  |        |                |  |     |              |        |     |              |         |     |                |       |     |   |                 |       |                    |                 |     |                 |                 |
|--------|--|--|--------|----------------|--|-----|--------------|--------|-----|--------------|---------|-----|----------------|-------|-----|---|-----------------|-------|--------------------|-----------------|-----|-----------------|-----------------|
|        | <p>Wahrscheinlichkeit, dass ein Molekül eine Geschwindigkeit zwischen <math>v</math> und <math>v + dv</math> aufweist:</p> $f(v)dv = \sqrt{\frac{2m^3}{\pi k^3 T^3}} v^2 e^{-\frac{mv^2}{2kT}} dv$ $v_0 = \sqrt{\frac{2kT}{m}}$ $u = \sqrt{\frac{3kT}{m}} = \sqrt{\bar{v}^2}$ $\bar{v} = \sqrt{\frac{8kT}{\pi m}} = 2\sqrt{\frac{2RT}{\pi M}}$ | <table> <tr> <td><math>f(v)</math></td><td>Dichtefunktion</td><td></td></tr> <tr> <td><math>m</math></td><td>Molekülmasse</td><td><math>[kg]</math></td></tr> <tr> <td><math>v</math></td><td>v - Moleküle</td><td><math>[m/s]</math></td></tr> <tr> <td><math>T</math></td><td>freie Weglänge</td><td><math>[m]</math></td></tr> <tr> <td><math>k</math></td><td>Boltzmann-konst. = <math>1,381 \cdot 10^{-23}</math></td><td><math>[\frac{J}{K}]</math></td></tr> <tr> <td><math>v_0</math></td><td>v-wahrscheinlichst</td><td><math>[\frac{m}{s}]</math></td></tr> <tr> <td><math>u</math></td><td>spez. <math>\bar{v}</math></td><td><math>[\frac{m}{s}]</math></td></tr> </table> | $f(v)$ | Dichtefunktion |  | $m$ | Molekülmasse | $[kg]$ | $v$ | v - Moleküle | $[m/s]$ | $T$ | freie Weglänge | $[m]$ | $k$ | Boltzmann-konst. = $1,381 \cdot 10^{-23}$ | $[\frac{J}{K}]$ | $v_0$ | v-wahrscheinlichst | $[\frac{m}{s}]$ | $u$ | spez. $\bar{v}$ | $[\frac{m}{s}]$ |
| $f(v)$ | Dichtefunktion   |  |        |                |  |     |              |        |     |              |         |     |                |       |     |   |                 |       |                    |                 |     |                 |                 |
| $m$    | Molekülmasse   | $[kg]$   |        |                |  |     |              |        |     |              |         |     |                |       |     |   |                 |       |                    |                 |     |                 |                 |
| $v$    | v - Moleküle   | $[m/s]$  |        |                |  |     |              |        |     |              |         |     |                |       |     |   |                 |       |                    |                 |     |                 |                 |
| $T$    | freie Weglänge   | $[m]$  |        |                |  |     |              |        |     |              |         |     |                |       |     |   |                 |       |                    |                 |     |                 |                 |
| $k$    | Boltzmann-konst. = $1,381 \cdot 10^{-23}$  | $[\frac{J}{K}]$  |        |                |  |     |              |        |     |              |         |     |                |       |     |   |                 |       |                    |                 |     |                 |                 |
| $v_0$  | v-wahrscheinlichst   | $[\frac{m}{s}]$  |        |                |  |     |              |        |     |              |         |     |                |       |     |   |                 |       |                    |                 |     |                 |                 |
| $u$    | spez. $\bar{v}$  | $[\frac{m}{s}]$  |        |                |  |     |              |        |     |              |         |     |                |       |     |   |                 |       |                    |                 |     |                 |                 |

## 6.10. Temperaturstrahlung, Strahlungsgesetze

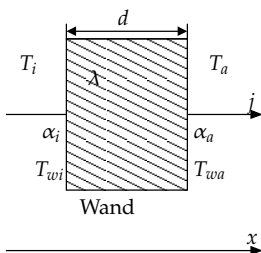
|   |   |   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
|---|---|---|---|------------|------|---|---------------------------|-------------------|---|-------------|-----|---|--------------|-----|---|-----------------|-----|---|--------------------|---------------------|---|-------------------|---------------------|---|--------------|-----------------------|----------------|---|-----|----------------|--------------------|-----|---|-------------|-----|---|----------|------|---|--------------|-----|----------------|-----------------|-----|----------------|--------------------|-----|------------------|-------------------|-----|---|------------------|-------|--|--|-----------------------------|---|------------------|----------------------|--|--|----------------------------|---|-------------------|------|--|--|-----------------------------|---|--------|------|--|--|----------------------------|----------------|-----------------------|-------|--|--|---------------|
|  <p>0 ≤ A<sub>λ</sub> ≤ 1</p> | $\Omega = \frac{A}{R^2} \quad I = \frac{\Phi}{\Omega}$ $E = \frac{\Phi}{A}$ $A_\lambda = \frac{\Phi_{\lambda a}}{\Phi_{\lambda e}}$ $K = \int_{HR} L(\vartheta, \varphi) \cos(\vartheta) d\Omega$ <p>Diffuse Strahlung:</p> $K = L \int_{HR} L \cos(\vartheta) d\Omega = L\pi$ <p>HR = Halbraum : z &gt; 0</p> $\frac{K_\lambda(\lambda, T)}{A_\lambda(\lambda, T)} = f(\lambda, T)$ $K_\lambda = \epsilon_\lambda(\lambda, T) K_{\lambda s}(\lambda, T)$ $\epsilon_\lambda(\lambda, T) \equiv A_\lambda(\lambda, T)$ <p>Körper schwarz: K<sub>s</sub> = σT<sup>4</sup></p> <p>Körper grau: K = εσT<sup>4</sup>, A = ε</p> $P_e = \epsilon_\lambda \sigma A T^4$ $P_{e\text{Netto}} = \epsilon_\lambda \sigma A (T^4 - T_0^4)$ $K_{\nu s}(\nu, T) d\nu = \frac{2\pi h \nu^3}{c^2 (e^{\frac{h\nu}{kT}} - 1)} d\nu$ $K_{\lambda s}(\lambda, T) d\lambda = \frac{2\pi h c^2}{\lambda^5 (e^{\frac{hc}{\lambda kT}} - 1)} d\lambda$ $\lambda_{\max} T = b$ $\nu = c/\lambda$ $E_{Str} = \frac{1.05 \cdot 10^{-34} c_0}{\lambda}$ | <table> <tr> <td>Ω</td><td>Raumwinkel</td><td>[sr]</td></tr> <tr> <td>A</td><td>Fläche, Flächenausschnitt</td><td>[m<sup>2</sup>]</td></tr> <tr> <td>R</td><td>Kugelradius</td><td>[m]</td></tr> <tr> <td>I</td><td>Strahlstärke</td><td>[W]</td></tr> <tr> <td>Φ</td><td>Strahlungsstrom</td><td>[W]</td></tr> <tr> <td>E</td><td>Bestrahlungsstärke</td><td>[W/m<sup>2</sup>]</td></tr> <tr> <td>K</td><td>Emmisionsvermögen</td><td>[W/m<sup>2</sup>]</td></tr> <tr> <td>L</td><td>Strahldichte</td><td>[W/m<sup>2</sup>sr]</td></tr> <tr> <td>A<sub>λ</sub></td><td>Absorptionszahl (Schwarzer Körper ⇒ A<sub>λ</sub> = 1)</td><td>[1]</td></tr> <tr> <td>ε<sub>λ</sub></td><td>Emissionsverhältn.</td><td>[1]</td></tr> <tr> <td>λ</td><td>Wellenlänge</td><td>[m]</td></tr> <tr> <td>ν</td><td>Frequenz</td><td>[Hz]</td></tr> <tr> <td>T</td><td>Temp. Körper</td><td>[K]</td></tr> <tr> <td>T<sub>0</sub></td><td>Umgebungs-temp.</td><td>[K]</td></tr> <tr> <td>P<sub>e</sub></td><td>Strahlungsleistung</td><td>[W]</td></tr> <tr> <td>E<sub>Str</sub></td><td>Strahlungsenergie</td><td>[J]</td></tr> <tr> <td>k</td><td>Boltzmann-konst.</td><td>[J/K]</td></tr> <tr> <td></td><td></td><td>= 1,381 · 10<sup>-23</sup></td></tr> <tr> <td>σ</td><td>Boltzmann-konst.</td><td>[W/m<sup>2</sup>K]</td></tr> <tr> <td></td><td></td><td>= 5,671 · 10<sup>-8</sup></td></tr> <tr> <td>h</td><td>Plancksche Konst.</td><td>[Js]</td></tr> <tr> <td></td><td></td><td>= 6,626 · 10<sup>-34</sup></td></tr> <tr> <td>b</td><td>Konst.</td><td>[mK]</td></tr> <tr> <td></td><td></td><td>= 2,898 · 10<sup>-3</sup></td></tr> <tr> <td>c<sub>0</sub></td><td>Lichtgeschw. (Vakuum)</td><td>[m/s]</td></tr> <tr> <td></td><td></td><td>= 299'792'458</td></tr> </table> | Ω | Raumwinkel | [sr] | A | Fläche, Flächenausschnitt | [m <sup>2</sup> ] | R | Kugelradius | [m] | I | Strahlstärke | [W] | Φ | Strahlungsstrom | [W] | E | Bestrahlungsstärke | [W/m <sup>2</sup> ] | K | Emmisionsvermögen | [W/m <sup>2</sup> ] | L | Strahldichte | [W/m <sup>2</sup> sr] | A <sub>λ</sub> | Absorptionszahl (Schwarzer Körper ⇒ A <sub>λ</sub> = 1) | [1] | ε <sub>λ</sub> | Emissionsverhältn. | [1] | λ | Wellenlänge | [m] | ν | Frequenz | [Hz] | T | Temp. Körper | [K] | T <sub>0</sub> | Umgebungs-temp. | [K] | P <sub>e</sub> | Strahlungsleistung | [W] | E <sub>Str</sub> | Strahlungsenergie | [J] | k | Boltzmann-konst. | [J/K] |  |  | = 1,381 · 10 <sup>-23</sup> | σ | Boltzmann-konst. | [W/m <sup>2</sup> K] |  |  | = 5,671 · 10 <sup>-8</sup> | h | Plancksche Konst. | [Js] |  |  | = 6,626 · 10 <sup>-34</sup> | b | Konst. | [mK] |  |  | = 2,898 · 10 <sup>-3</sup> | c <sub>0</sub> | Lichtgeschw. (Vakuum) | [m/s] |  |  | = 299'792'458 |
| Ω   | Raumwinkel  | [sr]  |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| A   | Fläche, Flächenausschnitt   | [m <sup>2</sup> ]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| R   | Kugelradius   | [m]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| I   | Strahlstärke  | [W]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| Φ   | Strahlungsstrom   | [W]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| E   | Bestrahlungsstärke  | [W/m <sup>2</sup> ]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| K   | Emmisionsvermögen   | [W/m <sup>2</sup> ]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| L   | Strahldichte  | [W/m <sup>2</sup> sr]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| A <sub>λ</sub>  | Absorptionszahl (Schwarzer Körper ⇒ A <sub>λ</sub> = 1)   | [1]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| ε <sub>λ</sub>  | Emissionsverhältn.  | [1]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| λ   | Wellenlänge   | [m]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| ν   | Frequenz  | [Hz]  |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| T   | Temp. Körper  | [K]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| T <sub>0</sub>  | Umgebungs-temp.   | [K]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| P <sub>e</sub>  | Strahlungsleistung  | [W]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| E <sub>Str</sub>  | Strahlungsenergie   | [J]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| k   | Boltzmann-konst.  | [J/K]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
|   |   | = 1,381 · 10 <sup>-23</sup>   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| σ   | Boltzmann-konst.  | [W/m <sup>2</sup> K]  |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
|   |   | = 5,671 · 10 <sup>-8</sup>  |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| h   | Plancksche Konst.   | [Js]  |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
|   |   | = 6,626 · 10 <sup>-34</sup>   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| b   | Konst.  | [mK]  |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
|   |   | = 2,898 · 10 <sup>-3</sup>  |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
| c <sub>0</sub>  | Lichtgeschw. (Vakuum)   | [m/s]   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |
|   |   | = 299'792'458   |   |            |      |   |                           |                   |   |             |     |   |              |     |   |                 |     |   |                    |                     |   |                   |                     |   |              |                       |                |   |     |                |                    |     |   |             |     |   |          |      |   |              |     |                |                 |     |                |                    |     |                  |                   |     |   |                  |       |  |  |                             |   |                  |                      |  |  |                            |   |                   |      |  |  |                             |   |        |      |  |  |                            |                |                       |       |  |  |               |

6.10.1. Strahlungsaustausch

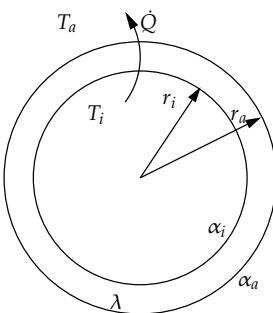
|   |  |  |          |                         |                                  |     |            |       |     |          |                              |     |                |       |            |                     |       |     |                  |                              |          |                 |                             |  |  |                         |
|---|--|--|----------|-------------------------|----------------------------------|-----|------------|-------|-----|----------|------------------------------|-----|----------------|-------|------------|---------------------|-------|-----|------------------|------------------------------|----------|-----------------|-----------------------------|--|--|-------------------------|
|  | $j = C_{12}(T_1^4 - T_2^4) = S_{12} - S_{21}$ $j = \frac{A_1 A_2}{A_1 + A_2 - A_1 A_2} \sigma$ $C_{12} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} \sigma$ | <table><tr><td><math>C_{12}</math></td><td>Strahlungsaustauschzahl</td><td><math>\left[\frac{W}{m^2 K^4}\right]</math></td></tr><tr><td><math>T</math></td><td>Temperatur</td><td><math>[K]</math></td></tr><tr><td><math>S</math></td><td>Entropie</td><td><math>\left[\frac{J}{m^2}\right]</math></td></tr><tr><td><math>A</math></td><td>Absortionszahl</td><td><math>[1]</math></td></tr><tr><td><math>\epsilon</math></td><td>Emissionsverhältnis</td><td><math>[K]</math></td></tr><tr><td><math>j</math></td><td>Wärmestromdichte</td><td><math>\left[\frac{W}{m^2}\right]</math></td></tr><tr><td><math>\sigma</math></td><td>Boltzmannkonst.</td><td><math>\left[\frac{W}{mK}\right]</math></td></tr><tr><td></td><td></td><td><math>= 5,671 \cdot 10^{-8}</math></td></tr></table> | $C_{12}$ | Strahlungsaustauschzahl | $\left[\frac{W}{m^2 K^4}\right]$ | $T$ | Temperatur | $[K]$ | $S$ | Entropie | $\left[\frac{J}{m^2}\right]$ | $A$ | Absortionszahl | $[1]$ | $\epsilon$ | Emissionsverhältnis | $[K]$ | $j$ | Wärmestromdichte | $\left[\frac{W}{m^2}\right]$ | $\sigma$ | Boltzmannkonst. | $\left[\frac{W}{mK}\right]$ |  |  | $= 5,671 \cdot 10^{-8}$ |
| $C_{12}$  | Strahlungsaustauschzahl  | $\left[\frac{W}{m^2 K^4}\right]$   |          |                         |                                  |     |            |       |     |          |                              |     |                |       |            |                     |       |     |                  |                              |          |                 |                             |  |  |                         |
| $T$   | Temperatur   | $[K]$  |          |                         |                                  |     |            |       |     |          |                              |     |                |       |            |                     |       |     |                  |                              |          |                 |                             |  |  |                         |
| $S$   | Entropie   | $\left[\frac{J}{m^2}\right]$   |          |                         |                                  |     |            |       |     |          |                              |     |                |       |            |                     |       |     |                  |                              |          |                 |                             |  |  |                         |
| $A$   | Absortionszahl   | $[1]$  |          |                         |                                  |     |            |       |     |          |                              |     |                |       |            |                     |       |     |                  |                              |          |                 |                             |  |  |                         |
| $\epsilon$  | Emissionsverhältnis  | $[K]$  |          |                         |                                  |     |            |       |     |          |                              |     |                |       |            |                     |       |     |                  |                              |          |                 |                             |  |  |                         |
| $j$   | Wärmestromdichte   | $\left[\frac{W}{m^2}\right]$   |          |                         |                                  |     |            |       |     |          |                              |     |                |       |            |                     |       |     |                  |                              |          |                 |                             |  |  |                         |
| $\sigma$  | Boltzmannkonst.  | $\left[\frac{W}{mK}\right]$  |          |                         |                                  |     |            |       |     |          |                              |     |                |       |            |                     |       |     |                  |                              |          |                 |                             |  |  |                         |
|   |  | $= 5,671 \cdot 10^{-8}$  |          |                         |                                  |     |            |       |     |          |                              |     |                |       |            |                     |       |     |                  |                              |          |                 |                             |  |  |                         |



## 6.11. Wärmetransport



| Wärmeübergang           | $\alpha$ |
|-------------------------|----------|
| Wandflächen innen       | 8        |
| Wandflächen aussen      | 20       |
| Boden, Decke nach oben  | 8        |
| Boden, Decke nach unten | 6        |



$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c} \frac{\partial^2 T}{\partial x^2} = \frac{\lambda}{\rho c} \Delta T$$

$$I = \dot{Q} = \lambda A \frac{dT}{dx} = jA$$

$$\Delta T = IR$$

$$R = \frac{\Delta x}{\lambda A}$$

$$R = R_1 + R_2 + \dots + R_n$$

Wandschicht:

$$j = -\lambda \frac{dT}{dx} = \lambda \frac{T_{wi} - T_{wa}}{d}$$

Übergangsschicht innen:

$$j = \alpha_i (T_i - T_{wi})$$

Übergangsschicht aussen:

$$j = \alpha_a (T_{wa} - T_a)$$

$$j = k (T_i - T_a)$$

$$\dot{Q}_w = Aj = Ak\Delta T$$

$$k = \frac{1}{\frac{1}{\alpha_i} + \sum_s \frac{d_s}{\lambda_s} + \frac{1}{\alpha_a}} = \frac{j}{\Delta T}$$

Für zylinderförmige Wand:

$$\dot{Q} = 2\pi r l j = 2\pi r_a l k_a \Delta T$$

$$k_a = \frac{1}{r_a \left( \frac{1}{r_i \alpha_i} + \sum_s \frac{1}{\lambda_s} \ln \frac{r_{sa}}{r_{si}} + \frac{1}{r_a \alpha_a} \right)}$$

Wärmebedarf eines Gebäudes:

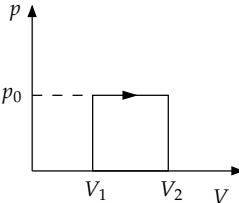
$$Q = \left( \sum_w A_w k_w + \rho c_p \dot{V} \right) G$$

$$G = \int_{\text{Heizsaison}} \Delta T dt$$

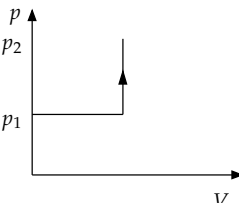
|           |                             |                               |
|-----------|-----------------------------|-------------------------------|
| $I$       | Wärmestrom                  | $[W]$                         |
| $R$       | Wärmewiderstand             | $\left[\frac{K}{W}\right]$    |
| $j$       | Wärmestromdichte            | $\left[\frac{W}{m^2}\right]$  |
| $\lambda$ | Wärmeleitungskoeff.         | $\left[\frac{W}{mK}\right]$   |
| $T_L$     | Lufttemperatur              | $[K]$                         |
| $T_W$     | Wandtemperatur              | $[K]$                         |
| $T_{i,a}$ | Innen- / Aussen- Temperatur | $[K]$                         |
| $t$       | Zeit                        | $[s]$                         |
| $\rho$    | Dichte                      | $\left[\frac{kg}{m^3}\right]$ |
| $d$       | Wanddichm.                  | $[m]$                         |
| $\alpha$  | Wärmeübergang               | $\left[\frac{W}{m^2K}\right]$ |
| $k$       | k-Wert, Wärmedurchgangszahl | $\left[\frac{W}{m^2K}\right]$ |
| $Q$       | Wärmebedarf                 | $[J]$                         |
| $A$       | Wandfläche                  | $[m^2]$                       |
| $\dot{V}$ | Luftaustausch               | $\left[\frac{m^3}{s}\right]$  |
| $G$       | Heiztage                    | $[Kd]$                        |
| $r$       | Zylinderradius              | $[m]$                         |
| $l$       | Zylinderlänge               | $[m]$                         |

## 6.12. Zustandsänderungen

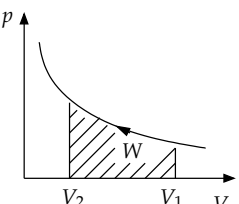
### 6.12.1. Isobare Zustandsänderung

|   |   |  |     |       |     |     |       |      |     |         |                   |          |                       |                      |     |            |     |     |              |     |     |        |     |     |                    |                      |
|---|---|--|-----|-------|-----|-----|-------|------|-----|---------|-------------------|----------|-----------------------|----------------------|-----|------------|-----|-----|--------------|-----|-----|--------|-----|-----|--------------------|----------------------|
|  <p>Isobar: <math>p = \text{konstant}</math></p> | $Q = nC_{mp}(T_2 - T_1)$ $W = p(V_2 - V_1) = nR(T_2 - T_1)$ $W = p_0 \int_{V_1}^{V_2} dV$ | <table> <tr><td><math>Q</math></td><td>Wärme</td><td>[J]</td></tr> <tr><td><math>p</math></td><td>Druck</td><td>[Pa]</td></tr> <tr><td><math>V</math></td><td>Volumen</td><td>[m<sup>3</sup>]</td></tr> <tr><td><math>C_{mp}</math></td><td>Molare Wärmekapazität</td><td>[<math>\frac{J}{molK}</math>]</td></tr> <tr><td><math>T</math></td><td>Temperatur</td><td>[K]</td></tr> <tr><td><math>n</math></td><td>Anz. Mol Gas</td><td>[1]</td></tr> <tr><td><math>W</math></td><td>Arbeit</td><td>[J]</td></tr> <tr><td><math>R</math></td><td>Univers.-Gaskonst.</td><td>[<math>\frac{J}{molK}</math>]</td></tr> </table> | $Q$ | Wärme | [J] | $p$ | Druck | [Pa] | $V$ | Volumen | [m <sup>3</sup> ] | $C_{mp}$ | Molare Wärmekapazität | [ $\frac{J}{molK}$ ] | $T$ | Temperatur | [K] | $n$ | Anz. Mol Gas | [1] | $W$ | Arbeit | [J] | $R$ | Univers.-Gaskonst. | [ $\frac{J}{molK}$ ] |
| $Q$   | Wärme   | [J]  |     |       |     |     |       |      |     |         |                   |          |                       |                      |     |            |     |     |              |     |     |        |     |     |                    |                      |
| $p$   | Druck   | [Pa]   |     |       |     |     |       |      |     |         |                   |          |                       |                      |     |            |     |     |              |     |     |        |     |     |                    |                      |
| $V$   | Volumen   | [m <sup>3</sup> ]  |     |       |     |     |       |      |     |         |                   |          |                       |                      |     |            |     |     |              |     |     |        |     |     |                    |                      |
| $C_{mp}$  | Molare Wärmekapazität   | [ $\frac{J}{molK}$ ]   |     |       |     |     |       |      |     |         |                   |          |                       |                      |     |            |     |     |              |     |     |        |     |     |                    |                      |
| $T$   | Temperatur  | [K]  |     |       |     |     |       |      |     |         |                   |          |                       |                      |     |            |     |     |              |     |     |        |     |     |                    |                      |
| $n$   | Anz. Mol Gas  | [1]  |     |       |     |     |       |      |     |         |                   |          |                       |                      |     |            |     |     |              |     |     |        |     |     |                    |                      |
| $W$   | Arbeit  | [J]  |     |       |     |     |       |      |     |         |                   |          |                       |                      |     |            |     |     |              |     |     |        |     |     |                    |                      |
| $R$   | Univers.-Gaskonst.  | [ $\frac{J}{molK}$ ]   |     |       |     |     |       |      |     |         |                   |          |                       |                      |     |            |     |     |              |     |     |        |     |     |                    |                      |

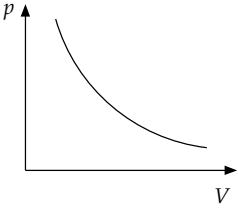
### 6.12.2. Isochore Zustandsänderungen

|   |                                  |   |     |       |     |          |                       |                      |     |            |     |     |              |     |     |        |     |
|---|----------------------------------|---|-----|-------|-----|----------|-----------------------|----------------------|-----|------------|-----|-----|--------------|-----|-----|--------|-----|
|  <p>Isochor: <math>V = \text{konstant}</math></p> | $Q = nC_{mv}(T_2 - T_1)$ $W = 0$ | <table> <tr><td><math>Q</math></td><td>Wärme</td><td>[J]</td></tr> <tr><td><math>C_{mv}</math></td><td>Molare Wärmekapazität</td><td>[<math>\frac{J}{molK}</math>]</td></tr> <tr><td><math>T</math></td><td>Temperatur</td><td>[K]</td></tr> <tr><td><math>n</math></td><td>Anz. Mol Gas</td><td>[1]</td></tr> <tr><td><math>W</math></td><td>Arbeit</td><td>[J]</td></tr> </table> | $Q$ | Wärme | [J] | $C_{mv}$ | Molare Wärmekapazität | [ $\frac{J}{molK}$ ] | $T$ | Temperatur | [K] | $n$ | Anz. Mol Gas | [1] | $W$ | Arbeit | [J] |
| $Q$   | Wärme                            | [J]   |     |       |     |          |                       |                      |     |            |     |     |              |     |     |        |     |
| $C_{mv}$  | Molare Wärmekapazität            | [ $\frac{J}{molK}$ ]  |     |       |     |          |                       |                      |     |            |     |     |              |     |     |        |     |
| $T$   | Temperatur                       | [K]   |     |       |     |          |                       |                      |     |            |     |     |              |     |     |        |     |
| $n$   | Anz. Mol Gas                     | [1]   |     |       |     |          |                       |                      |     |            |     |     |              |     |     |        |     |
| $W$   | Arbeit                           | [J]   |     |       |     |          |                       |                      |     |            |     |     |              |     |     |        |     |

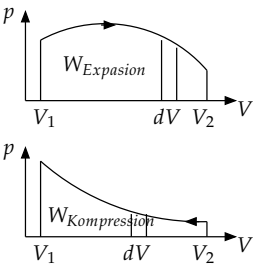
### 6.12.3. Isotherme Zustandsänderungen

|   |   |  |     |       |     |     |         |                   |     |            |     |     |              |     |     |        |     |     |       |      |     |                    |                      |
|---|---|--|-----|-------|-----|-----|---------|-------------------|-----|------------|-----|-----|--------------|-----|-----|--------|-----|-----|-------|------|-----|--------------------|----------------------|
|  <p>isotherm: <math>T = \text{konst.}</math></p> | <p>Expansion:</p> $Q_{ab} = nRT \ln \left( \frac{V_2}{V_1} \right) = W_{ab}$ <p>Kompression:</p> $Q_{zu} = nRT \ln \left( \frac{V_1}{V_2} \right) = W_{zu}$ $\frac{V_1}{V_2} = \frac{p_2}{p_1}$ $ W_{zu}  =  Q_{ab} $ | <table> <tr><td><math>Q</math></td><td>Wärme</td><td>[J]</td></tr> <tr><td><math>V</math></td><td>Volumen</td><td>[m<sup>3</sup>]</td></tr> <tr><td><math>T</math></td><td>Temperatur</td><td>[K]</td></tr> <tr><td><math>n</math></td><td>Anz. Mol Gas</td><td>[1]</td></tr> <tr><td><math>W</math></td><td>Arbeit</td><td>[J]</td></tr> <tr><td><math>p</math></td><td>Druck</td><td>[Pa]</td></tr> <tr><td><math>R</math></td><td>Univers.-Gaskonst.</td><td>[<math>\frac{J}{molK}</math>]</td></tr> </table> | $Q$ | Wärme | [J] | $V$ | Volumen | [m <sup>3</sup> ] | $T$ | Temperatur | [K] | $n$ | Anz. Mol Gas | [1] | $W$ | Arbeit | [J] | $p$ | Druck | [Pa] | $R$ | Univers.-Gaskonst. | [ $\frac{J}{molK}$ ] |
| $Q$   | Wärme   | [J]  |     |       |     |     |         |                   |     |            |     |     |              |     |     |        |     |     |       |      |     |                    |                      |
| $V$   | Volumen   | [m <sup>3</sup> ]  |     |       |     |     |         |                   |     |            |     |     |              |     |     |        |     |     |       |      |     |                    |                      |
| $T$   | Temperatur  | [K]  |     |       |     |     |         |                   |     |            |     |     |              |     |     |        |     |     |       |      |     |                    |                      |
| $n$   | Anz. Mol Gas  | [1]  |     |       |     |     |         |                   |     |            |     |     |              |     |     |        |     |     |       |      |     |                    |                      |
| $W$   | Arbeit  | [J]  |     |       |     |     |         |                   |     |            |     |     |              |     |     |        |     |     |       |      |     |                    |                      |
| $p$   | Druck   | [Pa]   |     |       |     |     |         |                   |     |            |     |     |              |     |     |        |     |     |       |      |     |                    |                      |
| $R$   | Univers.-Gaskonst.  | [ $\frac{J}{molK}$ ]   |     |       |     |     |         |                   |     |            |     |     |              |     |     |        |     |     |       |      |     |                    |                      |

## 6.12.4. Adiabatische Zustandsänderungen

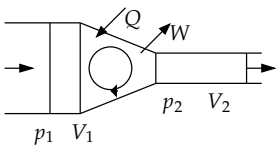
|  |   |   |     |                |       |     |       |        |     |         |         |          |                     |       |          |                               |                    |          |                              |                    |     |            |       |     |               |       |     |          |       |     |        |       |
|--|---|---|-----|----------------|-------|-----|-------|--------|-----|---------|---------|----------|---------------------|-------|----------|-------------------------------|--------------------|----------|------------------------------|--------------------|-----|------------|-------|-----|---------------|-------|-----|----------|-------|-----|--------|-------|
|  <p>Adiabatisch:<br/> <math>Q = konst.</math> (kein Wärmeaustausch)</p> | $dU = \delta Q - \delta W$ $pV^\kappa = konst.$ $\rightarrow p_2 = p_1 \left( \frac{V_1}{V_2} \right)^\kappa$ $TV^{\kappa-1} = konst.$ $\rightarrow T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{\kappa-1}$ $T^\kappa p^{1-\kappa} \text{ und } Tp^{\frac{1}{\kappa}-1} = konst.$ $\rightarrow T_2 = T_1 \left( \frac{p_1}{p_2} \right)^{\frac{1-\kappa}{\kappa}}$ $\kappa = \frac{C_{mp}}{C_{mv}}$ $\kappa = \frac{f+2}{f} \quad C_{mv} = \frac{f}{2}R$ $W = nC_{mv}(T_1 - T_2)$ $\Delta W = \frac{p_2V_2 - p_1V_1}{\kappa - 1}$ $\Delta W = C_{mv}(p_1V_1 - p_2V_2)$ | <table> <tr> <td><math>U</math></td><td>Innere Energie</td><td><math>[J]</math></td></tr> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>V</math></td><td>Volumen</td><td><math>[m^3]</math></td></tr> <tr> <td><math>\kappa</math></td><td>Adiabatene exponent</td><td><math>[1]</math></td></tr> <tr> <td><math>C_{mv}</math></td><td>Molare Wärmekapazität isochor</td><td><math>[\frac{J}{molK}]</math></td></tr> <tr> <td><math>C_{mp}</math></td><td>Molare Wärmekapazität isobar</td><td><math>[\frac{J}{molK}]</math></td></tr> <tr> <td><math>T</math></td><td>Temperatur</td><td><math>[K]</math></td></tr> <tr> <td><math>f</math></td><td>Freiheitsgrad</td><td><math>[1]</math></td></tr> <tr> <td><math>n</math></td><td>Anz. Mol</td><td><math>[1]</math></td></tr> <tr> <td><math>W</math></td><td>Arbeit</td><td><math>[J]</math></td></tr> </table> | $U$ | Innere Energie | $[J]$ | $p$ | Druck | $[Pa]$ | $V$ | Volumen | $[m^3]$ | $\kappa$ | Adiabatene exponent | $[1]$ | $C_{mv}$ | Molare Wärmekapazität isochor | $[\frac{J}{molK}]$ | $C_{mp}$ | Molare Wärmekapazität isobar | $[\frac{J}{molK}]$ | $T$ | Temperatur | $[K]$ | $f$ | Freiheitsgrad | $[1]$ | $n$ | Anz. Mol | $[1]$ | $W$ | Arbeit | $[J]$ |
| $U$  | Innere Energie  | $[J]$   |     |                |       |     |       |        |     |         |         |          |                     |       |          |                               |                    |          |                              |                    |     |            |       |     |               |       |     |          |       |     |        |       |
| $p$  | Druck   | $[Pa]$  |     |                |       |     |       |        |     |         |         |          |                     |       |          |                               |                    |          |                              |                    |     |            |       |     |               |       |     |          |       |     |        |       |
| $V$  | Volumen   | $[m^3]$   |     |                |       |     |       |        |     |         |         |          |                     |       |          |                               |                    |          |                              |                    |     |            |       |     |               |       |     |          |       |     |        |       |
| $\kappa$   | Adiabatene exponent   | $[1]$   |     |                |       |     |       |        |     |         |         |          |                     |       |          |                               |                    |          |                              |                    |     |            |       |     |               |       |     |          |       |     |        |       |
| $C_{mv}$   | Molare Wärmekapazität isochor   | $[\frac{J}{molK}]$  |     |                |       |     |       |        |     |         |         |          |                     |       |          |                               |                    |          |                              |                    |     |            |       |     |               |       |     |          |       |     |        |       |
| $C_{mp}$   | Molare Wärmekapazität isobar  | $[\frac{J}{molK}]$  |     |                |       |     |       |        |     |         |         |          |                     |       |          |                               |                    |          |                              |                    |     |            |       |     |               |       |     |          |       |     |        |       |
| $T$  | Temperatur  | $[K]$   |     |                |       |     |       |        |     |         |         |          |                     |       |          |                               |                    |          |                              |                    |     |            |       |     |               |       |     |          |       |     |        |       |
| $f$  | Freiheitsgrad   | $[1]$   |     |                |       |     |       |        |     |         |         |          |                     |       |          |                               |                    |          |                              |                    |     |            |       |     |               |       |     |          |       |     |        |       |
| $n$  | Anz. Mol  | $[1]$   |     |                |       |     |       |        |     |         |         |          |                     |       |          |                               |                    |          |                              |                    |     |            |       |     |               |       |     |          |       |     |        |       |
| $W$  | Arbeit  | $[J]$   |     |                |       |     |       |        |     |         |         |          |                     |       |          |                               |                    |          |                              |                    |     |            |       |     |               |       |     |          |       |     |        |       |

## 6.12.5. Expansion und Kompression

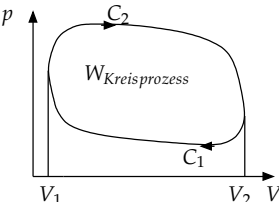
|   |  |   |     |        |       |     |       |        |     |         |         |
|---|--|---|-----|--------|-------|-----|-------|--------|-----|---------|---------|
|  | <p>Expansion:</p> $W = \int_{V_1}^{V_2} p dV = A_1$ <p>Kompression:</p> $W = \int_{V_2}^{V_1} p dV = -A_2$ | <table> <tr> <td><math>W</math></td><td>Arbeit</td><td><math>[J]</math></td></tr> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>V</math></td><td>Volumen</td><td><math>[m^3]</math></td></tr> </table> | $W$ | Arbeit | $[J]$ | $p$ | Druck | $[Pa]$ | $V$ | Volumen | $[m^3]$ |
| $W$   | Arbeit   | $[J]$   |     |        |       |     |       |        |     |         |         |
| $p$   | Druck  | $[Pa]$  |     |        |       |     |       |        |     |         |         |
| $V$   | Volumen  | $[m^3]$   |     |        |       |     |       |        |     |         |         |

## 6. WÄRMELEHRE

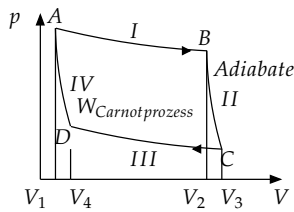
### 6.12.6. Enthalpie

|   |   |   |     |           |       |     |        |       |       |                             |       |     |       |       |     |       |        |     |         |         |          |                |       |
|---|---|---|-----|-----------|-------|-----|--------|-------|-------|-----------------------------|-------|-----|-------|-------|-----|-------|--------|-----|---------|---------|----------|----------------|-------|
|  <p>Maschine</p> | $H = U + pV$ $W = Q - \Delta H$ $W_{tot} = W_g + p_2 V_2 - p_1 V_1$ <p>Isotherm: <math>W = W_g</math>    Adiabatisch: <math>W = \kappa W_g</math></p> | <table> <tr> <td><math>H</math></td><td>Enthalpie</td><td><math>[J]</math></td></tr> <tr> <td><math>W</math></td><td>Arbeit</td><td><math>[J]</math></td></tr> <tr> <td><math>W_g</math></td><td>W im abgeschlossenen System</td><td><math>[J]</math></td></tr> <tr> <td><math>Q</math></td><td>Wärme</td><td><math>[J]</math></td></tr> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>V</math></td><td>Volumen</td><td><math>[m^3]</math></td></tr> <tr> <td><math>\kappa</math></td><td>Adiabaten exp.</td><td><math>[1]</math></td></tr> </table> | $H$ | Enthalpie | $[J]$ | $W$ | Arbeit | $[J]$ | $W_g$ | W im abgeschlossenen System | $[J]$ | $Q$ | Wärme | $[J]$ | $p$ | Druck | $[Pa]$ | $V$ | Volumen | $[m^3]$ | $\kappa$ | Adiabaten exp. | $[1]$ |
| $H$   | Enthalpie   | $[J]$   |     |           |       |     |        |       |       |                             |       |     |       |       |     |       |        |     |         |         |          |                |       |
| $W$   | Arbeit  | $[J]$   |     |           |       |     |        |       |       |                             |       |     |       |       |     |       |        |     |         |         |          |                |       |
| $W_g$   | W im abgeschlossenen System   | $[J]$   |     |           |       |     |        |       |       |                             |       |     |       |       |     |       |        |     |         |         |          |                |       |
| $Q$   | Wärme   | $[J]$   |     |           |       |     |        |       |       |                             |       |     |       |       |     |       |        |     |         |         |          |                |       |
| $p$   | Druck   | $[Pa]$  |     |           |       |     |        |       |       |                             |       |     |       |       |     |       |        |     |         |         |          |                |       |
| $V$   | Volumen   | $[m^3]$   |     |           |       |     |        |       |       |                             |       |     |       |       |     |       |        |     |         |         |          |                |       |
| $\kappa$  | Adiabaten exp.  | $[1]$   |     |           |       |     |        |       |       |                             |       |     |       |       |     |       |        |     |         |         |          |                |       |

### 6.13. Kreisprozesse

|  |   |  |     |        |       |     |       |       |     |       |        |     |         |         |
|--|---|--|-----|--------|-------|-----|-------|-------|-----|-------|--------|-----|---------|---------|
|  <p>Rechtslaufend (Uhrzeigersinn):<br/>Wärmekraftmaschine<br/>Linkslaufend: Wärmepumpe</p> | $W = \underbrace{\int_{V_1}^{V_2} p dV}_{\text{Kurve } C_1} + \underbrace{\int_{V_2}^{V_1} p dV}_{\text{Kurve } C_2}$ $W = \oint \delta W = \oint \delta Q$ $Q = W$ | <table> <tr> <td><math>W</math></td><td>Arbeit</td><td><math>[J]</math></td></tr> <tr> <td><math>Q</math></td><td>Wärme</td><td><math>[J]</math></td></tr> <tr> <td><math>p</math></td><td>Druck</td><td><math>[Pa]</math></td></tr> <tr> <td><math>V</math></td><td>Volumen</td><td><math>[m^3]</math></td></tr> </table> | $W$ | Arbeit | $[J]$ | $Q$ | Wärme | $[J]$ | $p$ | Druck | $[Pa]$ | $V$ | Volumen | $[m^3]$ |
| $W$  | Arbeit  | $[J]$  |     |        |       |     |       |       |     |       |        |     |         |         |
| $Q$  | Wärme   | $[J]$  |     |        |       |     |       |       |     |       |        |     |         |         |
| $p$  | Druck   | $[Pa]$   |     |        |       |     |       |       |     |       |        |     |         |         |
| $V$  | Volumen   | $[m^3]$  |     |        |       |     |       |       |     |       |        |     |         |         |

## 6.13.1. Carnotprozess



Beispiel Motor:

$$T_1 = T_{\text{Zylinder}}$$

$$T_2 = T_{\text{Abgas}}$$

I: Isotherme Expansion:

$$W_{ab} = RT_1 \ln \frac{V_2}{V_1} = Q_{zu}$$

$$U = \text{konst.}$$

II: Adiabatische Expansion:

$$W_{ab} = C_{mv}(T_1 - T_2)$$

$$Q_{zu} = 0$$

III: Isotherme Kompression:

$$W_{zu} = RT_2 \ln \frac{V_3}{V_4} = Q_{ab}$$

$$U = \text{konst.}$$

IV: Adiabatische Kompression:

$$W_{zu} = C_{mv}(T_1 - T_2)$$

$$Q_{zu} = 0$$

$$\eta_C = \frac{T_1 - T_2}{T_1}$$

Carnot-Wärmepumpe:

$$\epsilon_C = \frac{T_1}{T_1 - T_2}$$

Carnot-Kältemaschine:

$$\epsilon_C = \frac{T_2}{T_1 - T_2}$$

|              |  |                    |
|--------------|--|--------------------|
| $W$          | Arbeit                                       | $[J]$              |
| $Q$          | Wärme  | $[J]$              |
| $U$          | Innere Energie                               | $[J]$              |
| $V$          | Volumen                                      | $[m^3]$            |
| $C_{mv}$     | Molare Wärmekapazität isochor                | $[\frac{J}{molK}]$ |
| $R$          | Univers.-Gaskonst.                           | $[\frac{J}{molK}]$ |
| $\eta_C$     | Carnot-Wirkungsgrad (bei Wärmekraftmaschine) | $[1]$              |
| $\epsilon_C$ | Carnot-Leistungszahl (bei Wärmepumpe)        | $[1]$              |

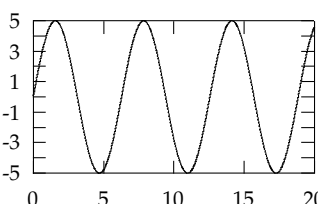
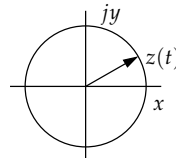
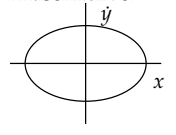
## 6.14. Entropie

|   |   |   |
|---|---|---|
| <p>Im abgeschlossenen System gilt:</p> <ul style="list-style-type: none"> <li>• S kann niemals abnehmen.</li> <li>• Bei allen Vorgängen nimmt S zu oder bleibt gleich.</li> <li>• Der Zustand wo S maximal ist, ist der stabile Zustand.</li> </ul> | $S = S_0 + \int_0^P \frac{\delta Q_r}{T}$ $dS = \frac{\delta Q_r}{T}$ $\Delta S = S_2 - S_1 = \int_1^2 \frac{\delta Q_r}{T}$ $S = k \cdot \ln(W)$ | <p>S Entropie <math>[\frac{J}{K}]</math><br/> P Punkt P <math>[1]</math><br/> <math>Q_r</math> Wärme <math>[J]</math><br/> T Temperatur <math>[K]</math><br/> W Wahrscheinlichkeit <math>[1]</math></p> |
|---|---|---|

# 7. Schwingungen

## 7.1. Freie Schwingungen

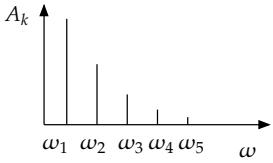
### 7.1.1. Ungedämpfte, harmonische Schwingung

|  |  |   |     |                    |         |     |           |       |          |               |                 |           |                  |         |     |      |       |     |         |       |     |          |                 |     |                |                   |     |         |                 |     |         |       |     |           |                 |
|--|--|---|-----|--------------------|---------|-----|-----------|-------|----------|---------------|-----------------|-----------|------------------|---------|-----|------|-------|-----|---------|-------|-----|----------|-----------------|-----|----------------|-------------------|-----|---------|-----------------|-----|---------|-------|-----|-----------|-----------------|
| <p>Zeitfunktion:</p>  <p>Zeigerbild:</p>  <p>Phasenkurve:</p>  | <p>Funktion:</p> $y = A \sin(\omega t + \varphi)$ $T = \frac{2\pi}{\omega}$ $f = \frac{1}{T}$ $\omega = 2\pi f$ $\ddot{y} + \omega^2 y = 0$ <p>Bei einer harmonischen Schwingung ist die Beschleunigung proportional zur Auslenkung:</p> $a(t) = \ddot{y} = -A\omega_0^2 \sin(\omega_0 t)$ $v(t) = \dot{y} = A\omega_0 \cos(\omega_0 t)$ <p>Energie bleibt konstant:</p> $E_{ges} = \frac{1}{2}cA^2 = E_{pot} + E_{kin}$ $E_{pot} = \frac{1}{2}cA^2 \cos^2(\omega t + \varphi)$ $E_{kin} = \frac{1}{2}cA^2 \sin^2(\omega t + \varphi)$ | <table> <tr> <td><math>y</math></td><td>schwingende Grösse</td><td><math>[m]^1</math></td></tr> <tr> <td><math>A</math></td><td>Amplitude</td><td><math>[1]</math></td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Nullphasenwinkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>T</math></td><td>Periode</td><td><math>[s]</math></td></tr> <tr> <td><math>f</math></td><td>Frequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>a</math></td><td>Beschleunigung</td><td><math>[\frac{m}{s^2}]</math></td></tr> <tr> <td><math>v</math></td><td>Geschw.</td><td><math>[\frac{m}{s}]</math></td></tr> <tr> <td><math>E</math></td><td>Energie</td><td><math>[J]</math></td></tr> <tr> <td><math>c</math></td><td>Konstante</td><td><math>[\frac{N}{m}]</math></td></tr> </table> | $y$ | schwingende Grösse | $[m]^1$ | $A$ | Amplitude | $[1]$ | $\omega$ | Kreisfrequenz | $[\frac{1}{s}]$ | $\varphi$ | Nullphasenwinkel | $[rad]$ | $t$ | Zeit | $[s]$ | $T$ | Periode | $[s]$ | $f$ | Frequenz | $[\frac{1}{s}]$ | $a$ | Beschleunigung | $[\frac{m}{s^2}]$ | $v$ | Geschw. | $[\frac{m}{s}]$ | $E$ | Energie | $[J]$ | $c$ | Konstante | $[\frac{N}{m}]$ |
| $y$  | schwingende Grösse   | $[m]^1$   |     |                    |         |     |           |       |          |               |                 |           |                  |         |     |      |       |     |         |       |     |          |                 |     |                |                   |     |         |                 |     |         |       |     |           |                 |
| $A$  | Amplitude  | $[1]$   |     |                    |         |     |           |       |          |               |                 |           |                  |         |     |      |       |     |         |       |     |          |                 |     |                |                   |     |         |                 |     |         |       |     |           |                 |
| $\omega$   | Kreisfrequenz  | $[\frac{1}{s}]$   |     |                    |         |     |           |       |          |               |                 |           |                  |         |     |      |       |     |         |       |     |          |                 |     |                |                   |     |         |                 |     |         |       |     |           |                 |
| $\varphi$  | Nullphasenwinkel   | $[rad]$   |     |                    |         |     |           |       |          |               |                 |           |                  |         |     |      |       |     |         |       |     |          |                 |     |                |                   |     |         |                 |     |         |       |     |           |                 |
| $t$  | Zeit   | $[s]$   |     |                    |         |     |           |       |          |               |                 |           |                  |         |     |      |       |     |         |       |     |          |                 |     |                |                   |     |         |                 |     |         |       |     |           |                 |
| $T$  | Periode  | $[s]$   |     |                    |         |     |           |       |          |               |                 |           |                  |         |     |      |       |     |         |       |     |          |                 |     |                |                   |     |         |                 |     |         |       |     |           |                 |
| $f$  | Frequenz   | $[\frac{1}{s}]$   |     |                    |         |     |           |       |          |               |                 |           |                  |         |     |      |       |     |         |       |     |          |                 |     |                |                   |     |         |                 |     |         |       |     |           |                 |
| $a$  | Beschleunigung   | $[\frac{m}{s^2}]$   |     |                    |         |     |           |       |          |               |                 |           |                  |         |     |      |       |     |         |       |     |          |                 |     |                |                   |     |         |                 |     |         |       |     |           |                 |
| $v$  | Geschw.  | $[\frac{m}{s}]$   |     |                    |         |     |           |       |          |               |                 |           |                  |         |     |      |       |     |         |       |     |          |                 |     |                |                   |     |         |                 |     |         |       |     |           |                 |
| $E$  | Energie  | $[J]$   |     |                    |         |     |           |       |          |               |                 |           |                  |         |     |      |       |     |         |       |     |          |                 |     |                |                   |     |         |                 |     |         |       |     |           |                 |
| $c$  | Konstante  | $[\frac{N}{m}]$   |     |                    |         |     |           |       |          |               |                 |           |                  |         |     |      |       |     |         |       |     |          |                 |     |                |                   |     |         |                 |     |         |       |     |           |                 |

<sup>1</sup>m gilt nur bei mechanischen Schwingungen

## 7. SCHWINGUNGEN

### 7.1.2. Ungedämpfte, periodische Schwingung

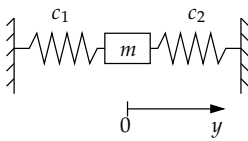
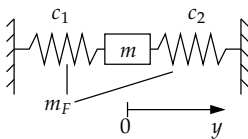
|   |   |   |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |                             |       |
|---|---|---|-----|--------------------|-------|-----|-----------|-------|----------|---------------|----------------------------|-----------|------------------|---------|-----|-----------------------------|-------|
|  | <p>Fourierreihe:</p> $y = A_0 + \sum_{k=1}^{\infty} A_k \sin(\omega_k t + \varphi_k)$ $\omega_k = k\omega_1$ $\omega_1 = \frac{2\pi}{T}$ $y = A_0 + \sum_{k=1}^{\infty} [a_k \sin(\omega_k t) + b_k \cos(\omega_k t)]$ $A_k = \sqrt{a_k^2 + b_k^2}$ $\varphi_k = \arctan\left(\frac{b_k}{a_k}\right)$ | <table> <tr> <td><math>y</math></td><td>schwingende Grösse</td><td><math>[m]</math></td></tr> <tr> <td><math>A</math></td><td>Amplitude</td><td><math>[1]</math></td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>\left[\frac{1}{s}\right]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Nullphasenwinkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>T</math></td><td>Periode der Grundschwingung</td><td><math>[s]</math></td></tr> </table> | $y$ | schwingende Grösse | $[m]$ | $A$ | Amplitude | $[1]$ | $\omega$ | Kreisfrequenz | $\left[\frac{1}{s}\right]$ | $\varphi$ | Nullphasenwinkel | $[rad]$ | $T$ | Periode der Grundschwingung | $[s]$ |
| $y$   | schwingende Grösse  | $[m]$   |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |                             |       |
| $A$   | Amplitude   | $[1]$   |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |                             |       |
| $\omega$  | Kreisfrequenz   | $\left[\frac{1}{s}\right]$  |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |                             |       |
| $\varphi$   | Nullphasenwinkel  | $[rad]$   |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |                             |       |
| $T$   | Periode der Grundschwingung   | $[s]$   |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |                             |       |

### 7.1.3. Ungedämpfte, nicht periodische Schwingung

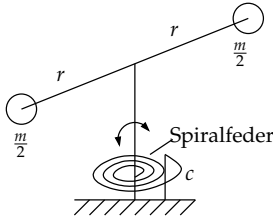
|          |  |  |     |                    |       |     |           |       |          |               |                            |     |      |       |
|----------|--|--|-----|--------------------|-------|-----|-----------|-------|----------|---------------|----------------------------|-----|------|-------|
|          | <p>Fourierreihe:</p> $y = \int_{-\infty}^{\infty} A(\omega) e^{j\omega t} d\omega$ | <table> <tr> <td><math>y</math></td><td>schwingende Grösse</td><td><math>[m]</math></td></tr> <tr> <td><math>A</math></td><td>Amplitude</td><td><math>[1]</math></td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>\left[\frac{1}{s}\right]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> </table> | $y$ | schwingende Grösse | $[m]$ | $A$ | Amplitude | $[1]$ | $\omega$ | Kreisfrequenz | $\left[\frac{1}{s}\right]$ | $t$ | Zeit | $[s]$ |
| $y$      | schwingende Grösse   | $[m]$  |     |                    |       |     |           |       |          |               |                            |     |      |       |
| $A$      | Amplitude  | $[1]$  |     |                    |       |     |           |       |          |               |                            |     |      |       |
| $\omega$ | Kreisfrequenz  | $\left[\frac{1}{s}\right]$   |     |                    |       |     |           |       |          |               |                            |     |      |       |
| $t$      | Zeit   | $[s]$  |     |                    |       |     |           |       |          |               |                            |     |      |       |



## 7.1.4. Federpendel

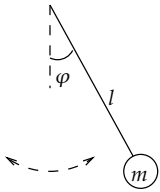
|   |   |   |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |      |       |     |         |       |     |          |                            |     |               |        |       |                 |        |     |                              |                            |     |                |                              |
|---|---|---|-----|--------------------|-------|-----|-----------|-------|----------|---------------|----------------------------|-----------|------------------|---------|-----|------|-------|-----|---------|-------|-----|----------|----------------------------|-----|---------------|--------|-------|-----------------|--------|-----|------------------------------|----------------------------|-----|----------------|------------------------------|
|    | <p>Federmasse vernachlässigt:</p> $m\ddot{y} + cy = 0$ $y = A \sin(\omega_0 t + \varphi)$ $\omega_0 = \sqrt{\frac{c}{m}}$ $T = 2\pi \sqrt{\frac{m}{c}}$ $a(t) = -\left(\frac{c}{m}\right) y(t)$ <p>Energiesatz:</p> $\frac{1}{2}cA^2 = \frac{1}{2}cy^2(t) + \frac{1}{2}mv^2(t)$ | <table><tr><td><math>y</math></td><td>schwingende Grösse</td><td><math>[m]</math></td></tr><tr><td><math>A</math></td><td>Amplitude</td><td><math>[1]</math></td></tr><tr><td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>\left[\frac{1}{s}\right]</math></td></tr><tr><td><math>\varphi</math></td><td>Nullphasenwinkel</td><td><math>[rad]</math></td></tr><tr><td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr><tr><td><math>T</math></td><td>Periode</td><td><math>[s]</math></td></tr><tr><td><math>f</math></td><td>Frequenz</td><td><math>\left[\frac{1}{s}\right]</math></td></tr><tr><td><math>m</math></td><td>Bewegte Masse</td><td><math>[kg]</math></td></tr><tr><td><math>m_F</math></td><td>Masse der Feder</td><td><math>[kg]</math></td></tr><tr><td><math>c</math></td><td>Federkonstante (siehe S. 15)</td><td><math>\left[\frac{N}{m}\right]</math></td></tr><tr><td><math>a</math></td><td>Beschleunigung</td><td><math>\left[\frac{m}{s^2}\right]</math></td></tr></table> | $y$ | schwingende Grösse | $[m]$ | $A$ | Amplitude | $[1]$ | $\omega$ | Kreisfrequenz | $\left[\frac{1}{s}\right]$ | $\varphi$ | Nullphasenwinkel | $[rad]$ | $t$ | Zeit | $[s]$ | $T$ | Periode | $[s]$ | $f$ | Frequenz | $\left[\frac{1}{s}\right]$ | $m$ | Bewegte Masse | $[kg]$ | $m_F$ | Masse der Feder | $[kg]$ | $c$ | Federkonstante (siehe S. 15) | $\left[\frac{N}{m}\right]$ | $a$ | Beschleunigung | $\left[\frac{m}{s^2}\right]$ |
| $y$   | schwingende Grösse  | $[m]$   |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |      |       |     |         |       |     |          |                            |     |               |        |       |                 |        |     |                              |                            |     |                |                              |
| $A$   | Amplitude   | $[1]$   |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |      |       |     |         |       |     |          |                            |     |               |        |       |                 |        |     |                              |                            |     |                |                              |
| $\omega$  | Kreisfrequenz   | $\left[\frac{1}{s}\right]$  |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |      |       |     |         |       |     |          |                            |     |               |        |       |                 |        |     |                              |                            |     |                |                              |
| $\varphi$   | Nullphasenwinkel  | $[rad]$   |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |      |       |     |         |       |     |          |                            |     |               |        |       |                 |        |     |                              |                            |     |                |                              |
| $t$   | Zeit  | $[s]$   |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |      |       |     |         |       |     |          |                            |     |               |        |       |                 |        |     |                              |                            |     |                |                              |
| $T$   | Periode   | $[s]$   |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |      |       |     |         |       |     |          |                            |     |               |        |       |                 |        |     |                              |                            |     |                |                              |
| $f$   | Frequenz  | $\left[\frac{1}{s}\right]$  |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |      |       |     |         |       |     |          |                            |     |               |        |       |                 |        |     |                              |                            |     |                |                              |
| $m$   | Bewegte Masse   | $[kg]$  |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |      |       |     |         |       |     |          |                            |     |               |        |       |                 |        |     |                              |                            |     |                |                              |
| $m_F$   | Masse der Feder   | $[kg]$  |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |      |       |     |         |       |     |          |                            |     |               |        |       |                 |        |     |                              |                            |     |                |                              |
| $c$   | Federkonstante (siehe S. 15)  | $\left[\frac{N}{m}\right]$  |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |      |       |     |         |       |     |          |                            |     |               |        |       |                 |        |     |                              |                            |     |                |                              |
| $a$   | Beschleunigung  | $\left[\frac{m}{s^2}\right]$  |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |      |       |     |         |       |     |          |                            |     |               |        |       |                 |        |     |                              |                            |     |                |                              |
|  | <p>Mit Federmasse:</p> $T = 2\pi \sqrt{\frac{m + \frac{m_F}{3}}{c}}$  |   |     |                    |       |     |           |       |          |               |                            |           |                  |         |     |      |       |     |         |       |     |          |                            |     |               |        |       |                 |        |     |                              |                            |     |                |                              |

## 7.1.5. Drehpendel

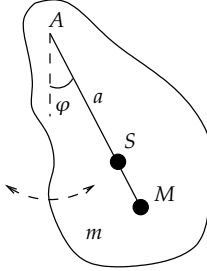
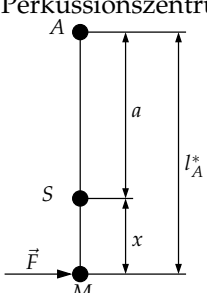
|   |   |  |     |                    |       |          |               |                            |           |                  |         |     |         |       |     |                |                            |     |                       |                               |
|---|---|--|-----|--------------------|-------|----------|---------------|----------------------------|-----------|------------------|---------|-----|---------|-------|-----|----------------|----------------------------|-----|-----------------------|-------------------------------|
|  | $J\ddot{\varphi} + c\varphi = 0$ $\omega_0 = \sqrt{\frac{c}{J}}$ $T = 2\pi \sqrt{\frac{J}{c}}$ <p>bestimmung von Massenträgheitsmomenten:</p> $J_{unbek} = J_{bek} \frac{T_{unbek}^2}{T_{bek}^2}$ | <table> <tr> <td><math>y</math></td><td>schwingende Grösse</td><td><math>[m]</math></td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>\left[\frac{1}{s}\right]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Nullphasenwinkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>T</math></td><td>Periode</td><td><math>[s]</math></td></tr> <tr> <td><math>c</math></td><td>Federkonstante</td><td><math>\left[\frac{N}{m}\right]</math></td></tr> <tr> <td><math>J</math></td><td>Massenträgheitsmoment</td><td><math>\left[\frac{kg}{m^2}\right]</math></td></tr> </table> | $y$ | schwingende Grösse | $[m]$ | $\omega$ | Kreisfrequenz | $\left[\frac{1}{s}\right]$ | $\varphi$ | Nullphasenwinkel | $[rad]$ | $T$ | Periode | $[s]$ | $c$ | Federkonstante | $\left[\frac{N}{m}\right]$ | $J$ | Massenträgheitsmoment | $\left[\frac{kg}{m^2}\right]$ |
| $y$   | schwingende Grösse  | $[m]$  |     |                    |       |          |               |                            |           |                  |         |     |         |       |     |                |                            |     |                       |                               |
| $\omega$  | Kreisfrequenz   | $\left[\frac{1}{s}\right]$   |     |                    |       |          |               |                            |           |                  |         |     |         |       |     |                |                            |     |                       |                               |
| $\varphi$   | Nullphasenwinkel  | $[rad]$  |     |                    |       |          |               |                            |           |                  |         |     |         |       |     |                |                            |     |                       |                               |
| $T$   | Periode   | $[s]$  |     |                    |       |          |               |                            |           |                  |         |     |         |       |     |                |                            |     |                       |                               |
| $c$   | Federkonstante  | $\left[\frac{N}{m}\right]$   |     |                    |       |          |               |                            |           |                  |         |     |         |       |     |                |                            |     |                       |                               |
| $J$   | Massenträgheitsmoment   | $\left[\frac{kg}{m^2}\right]$  |     |                    |       |          |               |                            |           |                  |         |     |         |       |     |                |                            |     |                       |                               |

## 7. SCHWINGUNGEN

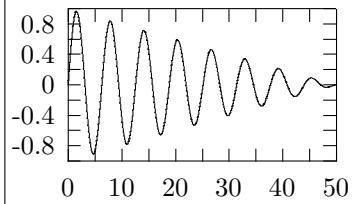
### 7.1.6. Mathematisches Pendel

|   |   |  |     |                    |       |          |               |                 |           |        |         |     |         |       |     |             |       |     |              |                   |  |      |  |
|---|---|--|-----|--------------------|-------|----------|---------------|-----------------|-----------|--------|---------|-----|---------|-------|-----|-------------|-------|-----|--------------|-------------------|--|------|--|
|  | $l\ddot{\varphi} + g \sin(\varphi) = 0$ $l\ddot{\varphi} + g\varphi = 0$ $\omega_0 = \sqrt{\frac{g}{l}}$ $T = 2\pi\sqrt{\frac{l}{g}}$ | <table> <tr> <td><math>y</math></td><td>schwingende Grösse</td><td><math>[m]</math></td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Winkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>T</math></td><td>Periode</td><td><math>[s]</math></td></tr> <tr> <td><math>l</math></td><td>Pendellänge</td><td><math>[m]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl. =</td><td><math>[\frac{m}{s^2}]</math></td></tr> <tr> <td></td><td>9.81</td><td></td></tr> </table> | $y$ | schwingende Grösse | $[m]$ | $\omega$ | Kreisfrequenz | $[\frac{1}{s}]$ | $\varphi$ | Winkel | $[rad]$ | $T$ | Periode | $[s]$ | $l$ | Pendellänge | $[m]$ | $g$ | Erdbeschl. = | $[\frac{m}{s^2}]$ |  | 9.81 |  |
| $y$   | schwingende Grösse  | $[m]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |     |             |       |     |              |                   |  |      |  |
| $\omega$  | Kreisfrequenz   | $[\frac{1}{s}]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |     |             |       |     |              |                   |  |      |  |
| $\varphi$   | Winkel  | $[rad]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |     |             |       |     |              |                   |  |      |  |
| $T$   | Periode   | $[s]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |     |             |       |     |              |                   |  |      |  |
| $l$   | Pendellänge   | $[m]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |     |             |       |     |              |                   |  |      |  |
| $g$   | Erdbeschl. =  | $[\frac{m}{s^2}]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |     |             |       |     |              |                   |  |      |  |
|   | 9.81  |  |     |                    |       |          |               |                 |           |        |         |     |         |       |     |             |       |     |              |                   |  |      |  |

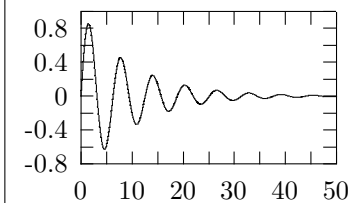
### 7.1.7. Physikalisches Pendel

|  |   |   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
|--|---|---|-------|----------------------------|-----------|-------|---|-----------|-----|-------------|-------|-----|--------------|-------|-----|---------|-------|-------|--------------------------------------|-------|-----|--------------------|--|-----|-----------|-------|----------|---------------|-----------------|-----------|--------|---------|-----|------|-------|-----|----------|-----------------|-----|---------------|--------|-----|---------|-------|-----|---|--|-----|-------------|-------|-----|--------------|-------------------|--|------|--|
|  <p>Perkussionszentrum</p>  | $J_A \ddot{\varphi} + mga \sin(\varphi) = 0$ $l^* = \frac{J_A}{ma}$ $J_A \ddot{\varphi} + mga\varphi = 0$ $\omega_0 = \sqrt{\frac{mga}{J_A}}$ $T = 2\pi\sqrt{\frac{J_A}{mga}} = 2\pi\sqrt{\frac{J_S + ma^2}{mga}}$ $T = 2\pi\sqrt{\frac{J_{A1} + J_{A2}}{g(m_1 a_1 + m_2 a_2)}}$ $J_A = J_S + ma^2$ $\omega_{max} = \sqrt{\frac{g}{2}} \sqrt{\frac{m}{J_S}}$ $\omega(A) = \omega(M)$ $x = \frac{J_S}{ma}$ $E = \frac{m\omega_0^2 A^2}{2}$ | <table> <tr> <td><math>J_A</math></td><td>Masseträgheit bez. A-Achse</td><td><math>[kgm^2]</math></td></tr> <tr> <td><math>J_S</math></td><td>Masseträgheit bez. Achse    a (siehe S. 30)</td><td><math>[kgm^2]</math></td></tr> <tr> <td><math>S</math></td><td>Schwerpunkt</td><td><math>[1]</math></td></tr> <tr> <td><math>a</math></td><td>Anstand zu S</td><td><math>[m]</math></td></tr> <tr> <td><math>T</math></td><td>Periode</td><td><math>[s]</math></td></tr> <tr> <td><math>l^*</math></td><td>Pendellänge des entspr. math. Pendel</td><td><math>[m]</math></td></tr> <tr> <td><math>y</math></td><td>schwingende Grösse</td><td></td></tr> <tr> <td><math>A</math></td><td>Amplitude</td><td><math>[1]</math></td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Winkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>f</math></td><td>Frequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>m</math></td><td>Bewegte Masse</td><td><math>[kg]</math></td></tr> <tr> <td><math>E</math></td><td>Energie</td><td><math>[J]</math></td></tr> <tr> <td><math>M</math></td><td>Schwing-Mittelpunkt, Perkussionszentrum</td><td></td></tr> <tr> <td><math>x</math></td><td>Abstand M,S</td><td><math>[m]</math></td></tr> <tr> <td><math>g</math></td><td>Erdbeschl. =</td><td><math>[\frac{m}{s^2}]</math></td></tr> <tr> <td></td><td>9.81</td><td></td></tr> </table> | $J_A$ | Masseträgheit bez. A-Achse | $[kgm^2]$ | $J_S$ | Masseträgheit bez. Achse    a (siehe S. 30) | $[kgm^2]$ | $S$ | Schwerpunkt | $[1]$ | $a$ | Anstand zu S | $[m]$ | $T$ | Periode | $[s]$ | $l^*$ | Pendellänge des entspr. math. Pendel | $[m]$ | $y$ | schwingende Grösse |  | $A$ | Amplitude | $[1]$ | $\omega$ | Kreisfrequenz | $[\frac{1}{s}]$ | $\varphi$ | Winkel | $[rad]$ | $t$ | Zeit | $[s]$ | $f$ | Frequenz | $[\frac{1}{s}]$ | $m$ | Bewegte Masse | $[kg]$ | $E$ | Energie | $[J]$ | $M$ | Schwing-Mittelpunkt, Perkussionszentrum |  | $x$ | Abstand M,S | $[m]$ | $g$ | Erdbeschl. = | $[\frac{m}{s^2}]$ |  | 9.81 |  |
| $J_A$  | Masseträgheit bez. A-Achse  | $[kgm^2]$   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $J_S$  | Masseträgheit bez. Achse    a (siehe S. 30)   | $[kgm^2]$   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $S$  | Schwerpunkt   | $[1]$   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $a$  | Anstand zu S  | $[m]$   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $T$  | Periode   | $[s]$   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $l^*$  | Pendellänge des entspr. math. Pendel  | $[m]$   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $y$  | schwingende Grösse  |   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $A$  | Amplitude   | $[1]$   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $\omega$   | Kreisfrequenz   | $[\frac{1}{s}]$   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $\varphi$  | Winkel  | $[rad]$   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $t$  | Zeit  | $[s]$   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $f$  | Frequenz  | $[\frac{1}{s}]$   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $m$  | Bewegte Masse   | $[kg]$  |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $E$  | Energie   | $[J]$   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $M$  | Schwing-Mittelpunkt, Perkussionszentrum   |   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $x$  | Abstand M,S   | $[m]$   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
| $g$  | Erdbeschl. =  | $[\frac{m}{s^2}]$   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |
|  | 9.81  |   |       |                            |           |       |   |           |     |             |       |     |              |       |     |         |       |       |                                      |       |     |                    |  |     |           |       |          |               |                 |           |        |         |     |      |       |     |          |                 |     |               |        |     |         |       |     |   |  |     |             |       |     |              |                   |  |      |  |

## 7.1.8. Gedämpfte Schwingung mit konstanter Reibung

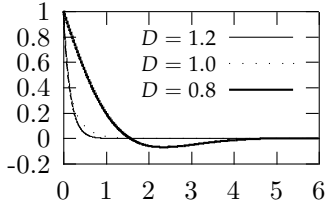
|   |   |  |            |                    |       |  |             |  |       |           |                 |     |                |                 |
|---|---|--|------------|--------------------|-------|--|-------------|--|-------|-----------|-----------------|-----|----------------|-----------------|
|  | $\Delta A = 4 \frac{F_R}{c}$ $m\ddot{y} + cy + F_R = 0$ | <table> <tr> <td><math>\Delta A</math></td><td><math>\Delta</math> Amplitude</td><td><math>[m]</math></td></tr> <tr> <td></td><td>pro Periode</td><td></td></tr> <tr> <td><math>F_R</math></td><td>Reibkraft</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>c</math></td><td>Federkonstante</td><td><math>[\frac{N}{m}]</math></td></tr> </table> | $\Delta A$ | $\Delta$ Amplitude | $[m]$ |  | pro Periode |  | $F_R$ | Reibkraft | $[\frac{1}{s}]$ | $c$ | Federkonstante | $[\frac{N}{m}]$ |
| $\Delta A$  | $\Delta$ Amplitude                                      | $[m]$  |            |                    |       |  |             |  |       |           |                 |     |                |                 |
|   | pro Periode   |  |            |                    |       |  |             |  |       |           |                 |     |                |                 |
| $F_R$   | Reibkraft   | $[\frac{1}{s}]$  |            |                    |       |  |             |  |       |           |                 |     |                |                 |
| $c$   | Federkonstante  | $[\frac{N}{m}]$  |            |                    |       |  |             |  |       |           |                 |     |                |                 |

7.1.9. Schwingung mit geschwindigkeitsproportionaler Dämpfung  
( $D < 1$ )

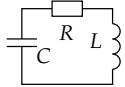
|  |  |  |     |                    |       |          |               |                 |           |        |         |     |         |       |          |               |       |     |                  |                  |     |       |        |     |             |                 |     |                |       |           |                 |       |     |           |       |     |         |       |
|--|--|--|-----|--------------------|-------|----------|---------------|-----------------|-----------|--------|---------|-----|---------|-------|----------|---------------|-------|-----|------------------|------------------|-----|-------|--------|-----|-------------|-----------------|-----|----------------|-------|-----------|-----------------|-------|-----|-----------|-------|-----|---------|-------|
|  | $m\ddot{y} + b\dot{y} + cy = 0$ $y = Ae^{-\delta t} \sin(\omega_d t + \phi_0)$ $\delta = \frac{b}{2m} \quad F_R = -b\dot{y}$ $\omega_d = \sqrt{\omega_0^2 - \delta^2}$ $\omega_0 = \sqrt{\frac{c}{m}}$ $D = \frac{\delta}{\omega_0}$ $D = \frac{\frac{\Lambda}{2\pi}}{\sqrt{1 + (\frac{\Lambda}{2\pi})^2}}$ $\omega_d = \omega_0 \sqrt{1 - D^2}$ $\Lambda = \frac{2\pi D}{\sqrt{1 - D^2}}$ $\Lambda = \delta T$ $\Lambda = \ln \frac{A_n}{A_{n+1}} \quad \frac{A_n}{A_{n+1}} = e^{\delta T}$ $\frac{E_1}{E_2} = \frac{A_1^2}{A_2^2}$ | <table> <tr> <td><math>y</math></td><td>schwingende Grösse</td><td><math>[m]</math></td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Winkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>T</math></td><td>Periode</td><td><math>[s]</math></td></tr> <tr> <td><math>\delta</math></td><td>Abklingkonst.</td><td><math>[1]</math></td></tr> <tr> <td><math>b</math></td><td>Dämpfungs-konst.</td><td><math>[\frac{kg}{s}]</math></td></tr> <tr> <td><math>m</math></td><td>Masse</td><td><math>[kg]</math></td></tr> <tr> <td><math>c</math></td><td>Federkonst.</td><td><math>[\frac{N}{m}]</math></td></tr> <tr> <td><math>D</math></td><td>Dämpfungs-grad</td><td><math>[1]</math></td></tr> <tr> <td><math>\Lambda</math></td><td>log. Dekre-ment</td><td><math>[1]</math></td></tr> <tr> <td><math>A</math></td><td>Amplitude</td><td><math>[1]</math></td></tr> <tr> <td><math>E</math></td><td>Energie</td><td><math>[J]</math></td></tr> </table> | $y$ | schwingende Grösse | $[m]$ | $\omega$ | Kreisfrequenz | $[\frac{1}{s}]$ | $\varphi$ | Winkel | $[rad]$ | $T$ | Periode | $[s]$ | $\delta$ | Abklingkonst. | $[1]$ | $b$ | Dämpfungs-konst. | $[\frac{kg}{s}]$ | $m$ | Masse | $[kg]$ | $c$ | Federkonst. | $[\frac{N}{m}]$ | $D$ | Dämpfungs-grad | $[1]$ | $\Lambda$ | log. Dekre-ment | $[1]$ | $A$ | Amplitude | $[1]$ | $E$ | Energie | $[J]$ |
| $y$  | schwingende Grösse   | $[m]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |          |               |       |     |                  |                  |     |       |        |     |             |                 |     |                |       |           |                 |       |     |           |       |     |         |       |
| $\omega$   | Kreisfrequenz  | $[\frac{1}{s}]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |          |               |       |     |                  |                  |     |       |        |     |             |                 |     |                |       |           |                 |       |     |           |       |     |         |       |
| $\varphi$  | Winkel   | $[rad]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |          |               |       |     |                  |                  |     |       |        |     |             |                 |     |                |       |           |                 |       |     |           |       |     |         |       |
| $T$  | Periode  | $[s]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |          |               |       |     |                  |                  |     |       |        |     |             |                 |     |                |       |           |                 |       |     |           |       |     |         |       |
| $\delta$   | Abklingkonst.  | $[1]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |          |               |       |     |                  |                  |     |       |        |     |             |                 |     |                |       |           |                 |       |     |           |       |     |         |       |
| $b$  | Dämpfungs-konst.   | $[\frac{kg}{s}]$   |     |                    |       |          |               |                 |           |        |         |     |         |       |          |               |       |     |                  |                  |     |       |        |     |             |                 |     |                |       |           |                 |       |     |           |       |     |         |       |
| $m$  | Masse  | $[kg]$   |     |                    |       |          |               |                 |           |        |         |     |         |       |          |               |       |     |                  |                  |     |       |        |     |             |                 |     |                |       |           |                 |       |     |           |       |     |         |       |
| $c$  | Federkonst.  | $[\frac{N}{m}]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |          |               |       |     |                  |                  |     |       |        |     |             |                 |     |                |       |           |                 |       |     |           |       |     |         |       |
| $D$  | Dämpfungs-grad   | $[1]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |          |               |       |     |                  |                  |     |       |        |     |             |                 |     |                |       |           |                 |       |     |           |       |     |         |       |
| $\Lambda$  | log. Dekre-ment  | $[1]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |          |               |       |     |                  |                  |     |       |        |     |             |                 |     |                |       |           |                 |       |     |           |       |     |         |       |
| $A$  | Amplitude  | $[1]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |          |               |       |     |                  |                  |     |       |        |     |             |                 |     |                |       |           |                 |       |     |           |       |     |         |       |
| $E$  | Energie  | $[J]$  |     |                    |       |          |               |                 |           |        |         |     |         |       |          |               |       |     |                  |                  |     |       |        |     |             |                 |     |                |       |           |                 |       |     |           |       |     |         |       |

## 7. SCHWINGUNGEN

### 7.1.10. Aperiodeische Lösung ( $D > 1$ )

|   |  |   |     |                    |       |          |               |                 |          |               |       |     |                  |                  |     |                |       |     |       |        |     |                |                 |
|---|--|---|-----|--------------------|-------|----------|---------------|-----------------|----------|---------------|-------|-----|------------------|------------------|-----|----------------|-------|-----|-------|--------|-----|----------------|-----------------|
|  | $y = b_1 e^{\lambda_1 t} + b_2 e^{\lambda_2 t}$ $\lambda_1 = -\omega_0 (D + \sqrt{D^2 - 1})$ $\lambda_2 = -\omega_0 (D - \sqrt{D^2 - 1})$ <p>Grenzfall <math>D = 1</math></p> $\frac{c}{m} = \frac{b^2}{4m^2}$ $y = (b_1 + b_2 t) e^{-\delta t}$ | <table> <tr> <td><math>y</math></td><td>schwingende Grösse</td><td><math>[m]</math></td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>\delta</math></td><td>Abklingkonst.</td><td><math>[1]</math></td></tr> <tr> <td><math>b</math></td><td>Dämpfungs-konst.</td><td><math>[\frac{kg}{s}]</math></td></tr> <tr> <td><math>D</math></td><td>Dämpfungs-grad</td><td><math>[1]</math></td></tr> <tr> <td><math>m</math></td><td>Masse</td><td><math>[kg]</math></td></tr> <tr> <td><math>c</math></td><td>Federkonstante</td><td><math>[\frac{N}{m}]</math></td></tr> </table> | $y$ | schwingende Grösse | $[m]$ | $\omega$ | Kreisfrequenz | $[\frac{1}{s}]$ | $\delta$ | Abklingkonst. | $[1]$ | $b$ | Dämpfungs-konst. | $[\frac{kg}{s}]$ | $D$ | Dämpfungs-grad | $[1]$ | $m$ | Masse | $[kg]$ | $c$ | Federkonstante | $[\frac{N}{m}]$ |
| $y$   | schwingende Grösse   | $[m]$   |     |                    |       |          |               |                 |          |               |       |     |                  |                  |     |                |       |     |       |        |     |                |                 |
| $\omega$  | Kreisfrequenz  | $[\frac{1}{s}]$   |     |                    |       |          |               |                 |          |               |       |     |                  |                  |     |                |       |     |       |        |     |                |                 |
| $\delta$  | Abklingkonst.  | $[1]$   |     |                    |       |          |               |                 |          |               |       |     |                  |                  |     |                |       |     |       |        |     |                |                 |
| $b$   | Dämpfungs-konst.   | $[\frac{kg}{s}]$  |     |                    |       |          |               |                 |          |               |       |     |                  |                  |     |                |       |     |       |        |     |                |                 |
| $D$   | Dämpfungs-grad   | $[1]$   |     |                    |       |          |               |                 |          |               |       |     |                  |                  |     |                |       |     |       |        |     |                |                 |
| $m$   | Masse  | $[kg]$  |     |                    |       |          |               |                 |          |               |       |     |                  |                  |     |                |       |     |       |        |     |                |                 |
| $c$   | Federkonstante   | $[\frac{N}{m}]$   |     |                    |       |          |               |                 |          |               |       |     |                  |                  |     |                |       |     |       |        |     |                |                 |

### 7.1.11. Elektrischer Schwingkreis

|  |   |  |     |       |       |     |            |            |     |              |       |          |               |                 |          |               |       |     |      |       |     |                |       |
|--|---|--|-----|-------|-------|-----|------------|------------|-----|--------------|-------|----------|---------------|-----------------|----------|---------------|-------|-----|------|-------|-----|----------------|-------|
|  | $I = I_0 e^{-\delta t} \sin(\omega_d t + \phi_0)$ $\delta = \frac{R}{2L}$ $\omega_d = \omega_0 \sqrt{1 - D^2}$ $\omega_0 = \frac{1}{\sqrt{LC}}$ $D = \frac{R}{2} \sqrt{\frac{C}{L}}$ $\omega_d = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{R^2 C}{4L}}$ | <table> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>L</math></td><td>Induktivität</td><td><math>[H]</math></td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>\delta</math></td><td>Abklingkonst.</td><td><math>[1]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>D</math></td><td>Dämpfungs-grad</td><td><math>[1]</math></td></tr> </table> | $I$ | Strom | $[A]$ | $R$ | Widerstand | $[\Omega]$ | $L$ | Induktivität | $[H]$ | $\omega$ | Kreisfrequenz | $[\frac{1}{s}]$ | $\delta$ | Abklingkonst. | $[1]$ | $t$ | Zeit | $[s]$ | $D$ | Dämpfungs-grad | $[1]$ |
| $I$  | Strom   | $[A]$  |     |       |       |     |            |            |     |              |       |          |               |                 |          |               |       |     |      |       |     |                |       |
| $R$  | Widerstand  | $[\Omega]$   |     |       |       |     |            |            |     |              |       |          |               |                 |          |               |       |     |      |       |     |                |       |
| $L$  | Induktivität  | $[H]$  |     |       |       |     |            |            |     |              |       |          |               |                 |          |               |       |     |      |       |     |                |       |
| $\omega$   | Kreisfrequenz   | $[\frac{1}{s}]$  |     |       |       |     |            |            |     |              |       |          |               |                 |          |               |       |     |      |       |     |                |       |
| $\delta$   | Abklingkonst.   | $[1]$  |     |       |       |     |            |            |     |              |       |          |               |                 |          |               |       |     |      |       |     |                |       |
| $t$  | Zeit  | $[s]$  |     |       |       |     |            |            |     |              |       |          |               |                 |          |               |       |     |      |       |     |                |       |
| $D$  | Dämpfungs-grad  | $[1]$  |     |       |       |     |            |            |     |              |       |          |               |                 |          |               |       |     |      |       |     |                |       |

# 8. Wellenlehre

## 8.1. Wellengeschwindigkeiten

|   |   |   |
|---|---|---|
| Elastische Longitudinalwellen                     | $u_L = \sqrt{\frac{E}{\rho}}$                 | $u$ Wellengeschw. $[\frac{m}{s}]$                     |
| Elastische Transversalwellen                      | $u_T = \sqrt{\frac{G}{\rho}}$                 | $A$ Fläche $[m^2]$                                    |
| Transversalwellen auf einem Seil oder einer Saite | $u_T = \sqrt{\frac{F}{\rho A}}$               | $E$ Elastizitätsmodul $[\frac{N}{m^2}]$               |
| Schwerewellen in tiefem Wasser                    | $u_S = \sqrt{\frac{g\lambda}{2\pi}}$          | $F$ Spannkraft $[N]$                                  |
| Schwerewellen in flachem Wasser                   | $u_S = \sqrt{gh}$                             | $G$ Schubmodul $[\frac{N}{m^2}]$                      |
| Kapillarwellen                                    | $u_K = \sqrt{\frac{2\pi\sigma}{\rho\lambda}}$ | $h$ Wassertiefe $[m]$                                 |
| Schallwellen in Fluiden                           | $u = \sqrt{\frac{1}{\rho\kappa}}$             | $M$ Molmasse $[\frac{kg}{mol}]$                       |
| Schallwellen in Gasen                             | $u_G = \sqrt{\frac{\kappa p}{\rho}}$          | $p$ Druck $[Pa]$                                      |
|   | $u_G = \sqrt{\frac{\kappa RT}{M}}$            | $T$ abs. Temp. $[K]$                                  |
|   |   | $\kappa$ Kompressibilität $[\frac{m^2}{N}]$           |
|   |   | $\kappa$ Adiabatenexponent $[1]$                      |
|   |   | $\lambda$ Wellenlänge $[m]$                           |
|   |   | $\rho$ Dichte $[\frac{kg}{m^3}]$                      |
|   |   | $\sigma$ Oberflächenspannung $[\frac{N}{m}]$          |
|   |   | $g$ Erdbeschl. = $9.81$ $[\frac{m}{s^2}]$             |
|   |   | $R$ univers. Gas-Konst. = $8.3145$ $[\frac{J}{Kmol}]$ |

### 8.1.1. Zusammenhänge der verschiedenen Wellen

|                         |                                       |  |
|-------------------------|---------------------------------------|--|
| Gilt nur bei einem Stab | $u_T = \sqrt{\frac{1}{2(1+\mu)}} u_L$ | $u$ Wellengeschw. $\left[\frac{m}{s}\right]$<br>$u_T$ u longitudinal $\left[\frac{m}{s}\right]$<br>$u_L$ u transversal $\left[\frac{m}{s}\right]$<br>$\mu$ Poissonzahl $[1]$ |
|-------------------------|---------------------------------------|--|

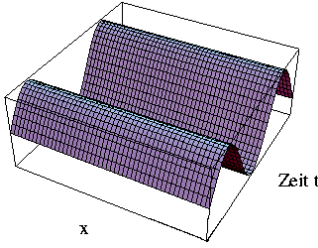
## 8.2. Wellengleichung

|  |  |   |
|--|--|---|
| Bei Wellengleichungen (lineare Dgl) gilt das Superpositionsprinzip, d.h. die Summe zweier Lösungen ist wieder eine Lösung. | <p>Eindimensional:</p> $\frac{\partial^2 \xi}{\partial x^2} = \frac{1}{u^2} \frac{\partial^2 \xi}{\partial t^2}$ <p>Zweidimensional:</p> $\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} = \frac{1}{u^2} \frac{\partial^2 \xi}{\partial t^2}$ <p>Dreidimensional:</p> $\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} + \frac{\partial^2 \xi}{\partial z^2} = \frac{1}{u^2} \frac{\partial^2 \xi}{\partial t^2}$ <p>oder: <math>\Delta \xi = \frac{1}{u^2} \frac{\partial^2 \xi}{\partial t^2}</math></p> <p>wobei: <math>\Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}</math></p> | $u$ Wellengeschw. $\left[\frac{m}{s}\right]$<br>$\xi$ Störung $[...]$<br>$t$ Zeit $[s]$<br>$\Delta$ Laplace-OP $[\ ]$ |
|--|--|---|

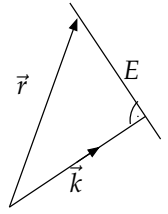
## 8.3. Intensität

|  |   |  |
|--|---|--|
|  | $I = \frac{1}{2} \rho u \omega^2 \xi_0^2$ | $I$ Intensität $\left[\frac{W}{m^2}\right]$<br>$u$ Wellengeschw. $\left[\frac{m}{s}\right]$<br>$\xi$ Störung $[...]$<br>$\rho$ Dichte $\left[\frac{kg}{m^3}\right]$<br>$\omega$ Winkelgeschw. $\left[\frac{1}{s}\right]$ |
|--|---|--|

## 8.4. Harmonische Wellen

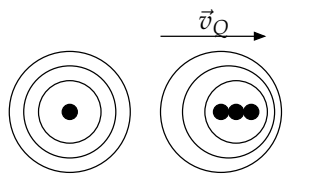
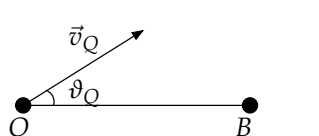
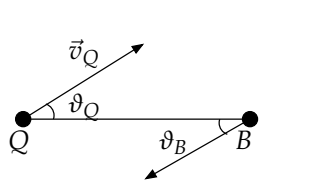
|   |   |  |     |               |                 |       |         |         |     |      |       |          |               |                 |     |         |       |           |             |       |           |       |         |     |            |                 |     |          |        |
|---|---|--|-----|---------------|-----------------|-------|---------|---------|-----|------|-------|----------|---------------|-----------------|-----|---------|-------|-----------|-------------|-------|-----------|-------|---------|-----|------------|-----------------|-----|----------|--------|
|  <p> <math>\xi = f(x - ut) \rightarrow</math><br/> <i>Ausbreitung pos. x - Koord.</i><br/> <math>\xi = f(x + ut) \rightarrow</math><br/> <i>Ausbreitung neg. x - Koord.</i><br/>           Bei EM - Wellen:<br/> <math>u = c = 299'792'458 \frac{m}{s}</math> </p> | $\omega = 2\pi f = \frac{2\pi}{T} = ku$ $k = \frac{\omega}{u} = \frac{2\pi}{\lambda}$ $\xi(x, t) = \xi_0 \sin(kx - \omega t + \varphi)$ $\xi(x, t) = \xi_0 \sin(\omega t - kx)$ $T = \frac{2\pi}{\omega} = \frac{1}{f}$ $f = \frac{\omega}{2\pi} = \frac{1}{T} = \frac{u}{\lambda}$ $\lambda = \frac{2\pi}{k} = \frac{u}{f}$ $u = \frac{\omega}{k} = \lambda f$ | <table> <tr> <td><math>u</math></td><td>Wellengeschw.</td><td><math>[\frac{m}{s}]</math></td></tr> <tr> <td><math>\xi</math></td><td>Störung</td><td><math>[...]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>T</math></td><td>Periode</td><td><math>[s]</math></td></tr> <tr> <td><math>\lambda</math></td><td>Wellenlänge</td><td><math>[m]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Phase</td><td><math>[rad]</math></td></tr> <tr> <td><math>k</math></td><td>Wellenzahl</td><td><math>[\frac{1}{m}]</math></td></tr> <tr> <td><math>f</math></td><td>Frequenz</td><td><math>[Hz]</math></td></tr> </table> | $u$ | Wellengeschw. | $[\frac{m}{s}]$ | $\xi$ | Störung | $[...]$ | $t$ | Zeit | $[s]$ | $\omega$ | Kreisfrequenz | $[\frac{1}{s}]$ | $T$ | Periode | $[s]$ | $\lambda$ | Wellenlänge | $[m]$ | $\varphi$ | Phase | $[rad]$ | $k$ | Wellenzahl | $[\frac{1}{m}]$ | $f$ | Frequenz | $[Hz]$ |
| $u$   | Wellengeschw.   | $[\frac{m}{s}]$  |     |               |                 |       |         |         |     |      |       |          |               |                 |     |         |       |           |             |       |           |       |         |     |            |                 |     |          |        |
| $\xi$   | Störung   | $[...]$  |     |               |                 |       |         |         |     |      |       |          |               |                 |     |         |       |           |             |       |           |       |         |     |            |                 |     |          |        |
| $t$   | Zeit  | $[s]$  |     |               |                 |       |         |         |     |      |       |          |               |                 |     |         |       |           |             |       |           |       |         |     |            |                 |     |          |        |
| $\omega$  | Kreisfrequenz   | $[\frac{1}{s}]$  |     |               |                 |       |         |         |     |      |       |          |               |                 |     |         |       |           |             |       |           |       |         |     |            |                 |     |          |        |
| $T$   | Periode   | $[s]$  |     |               |                 |       |         |         |     |      |       |          |               |                 |     |         |       |           |             |       |           |       |         |     |            |                 |     |          |        |
| $\lambda$   | Wellenlänge   | $[m]$  |     |               |                 |       |         |         |     |      |       |          |               |                 |     |         |       |           |             |       |           |       |         |     |            |                 |     |          |        |
| $\varphi$   | Phase   | $[rad]$  |     |               |                 |       |         |         |     |      |       |          |               |                 |     |         |       |           |             |       |           |       |         |     |            |                 |     |          |        |
| $k$   | Wellenzahl  | $[\frac{1}{m}]$  |     |               |                 |       |         |         |     |      |       |          |               |                 |     |         |       |           |             |       |           |       |         |     |            |                 |     |          |        |
| $f$   | Frequenz  | $[Hz]$   |     |               |                 |       |         |         |     |      |       |          |               |                 |     |         |       |           |             |       |           |       |         |     |            |                 |     |          |        |

## 8.5. Räumliche Ausbreitung von Wellen

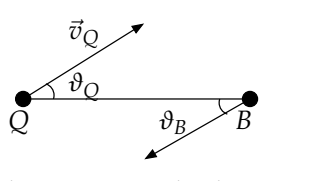
|   |   |  |       |         |         |     |      |       |          |               |                 |     |            |                 |     |        |       |     |           |         |
|---|---|--|-------|---------|---------|-----|------|-------|----------|---------------|-----------------|-----|------------|-----------------|-----|--------|-------|-----|-----------|---------|
|  | <p>Ebene Welle:</p> $\xi(x, y, z, t) = \xi_0 e^{i(\omega t - \vec{k} \cdot \vec{r})}$ $\vec{k} \cdot \vec{r} = konst.$ <p>Kugel Welle:</p> $\xi(r, t) = \frac{A}{r} e^{i(\omega t - kr)}$ | <table> <tr> <td><math>\xi</math></td><td>Störung</td><td><math>[...]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>k</math></td><td>Wellenzahl</td><td><math>[\frac{1}{m}]</math></td></tr> <tr> <td><math>r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr> <td><math>A</math></td><td>Amplitude</td><td><math>[...]</math></td></tr> </table> | $\xi$ | Störung | $[...]$ | $t$ | Zeit | $[s]$ | $\omega$ | Kreisfrequenz | $[\frac{1}{s}]$ | $k$ | Wellenzahl | $[\frac{1}{m}]$ | $r$ | Radius | $[m]$ | $A$ | Amplitude | $[...]$ |
| $\xi$   | Störung   | $[...]$  |       |         |         |     |      |       |          |               |                 |     |            |                 |     |        |       |     |           |         |
| $t$   | Zeit  | $[s]$  |       |         |         |     |      |       |          |               |                 |     |            |                 |     |        |       |     |           |         |
| $\omega$  | Kreisfrequenz   | $[\frac{1}{s}]$  |       |         |         |     |      |       |          |               |                 |     |            |                 |     |        |       |     |           |         |
| $k$   | Wellenzahl  | $[\frac{1}{m}]$  |       |         |         |     |      |       |          |               |                 |     |            |                 |     |        |       |     |           |         |
| $r$   | Radius  | $[m]$  |       |         |         |     |      |       |          |               |                 |     |            |                 |     |        |       |     |           |         |
| $A$   | Amplitude   | $[...]$  |       |         |         |     |      |       |          |               |                 |     |            |                 |     |        |       |     |           |         |

## 8.6. Doppler-Effekt

### 8.6.1. Akustischer Doppler-Effekt

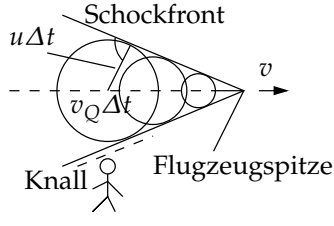
|  |  |   |
|--|--|---|
|  <p>ruhende und bewegte Punktquelle</p><br> <p>bewegte Punktquelle</p><br> <p>bewegter Beobachter und bewegte Punktquelle</p> | <p>bewegte Quelle, ruhender Beobachter:</p> $f' = \frac{1}{1 \mp \frac{v_Q}{u}} f \quad \text{– auf Hörer zu}$ $f' = \frac{1}{1 - \frac{v_Q}{u} \cos(\vartheta_Q)} f$ <p>ruhende Quelle, bewegter Beobachter:</p> $f' = (1 \pm \frac{v_B}{u}) f \quad \text{+ auf Quelle zu}$ $f' = (1 + \frac{v_B}{u} \cos(\vartheta_B)) f$ <p>Allgemein:</p> $f_B = \frac{u + v_B \cos(\vartheta_B)}{u - v_Q \cos(\vartheta_Q)} f_Q$ | <p><math>u</math> Wellengeschw. <math>[\frac{m}{s}]</math></p> <p><math>f</math> Frequenz <math>[Hz]</math></p> <p><math>f'</math> gehörte Frequenz <math>[Hz]</math></p> <p><math>v_Q</math> Geschw. Quelle <math>[\frac{m}{s}]</math></p> <p><math>v_B</math> Geschw. Beobachter <math>[\frac{m}{s}]</math></p> <p><math>\vartheta</math> Winkel <math>[rad]</math></p> |
|--|--|---|

### 8.6.2. Optischer Doppler-Effekt

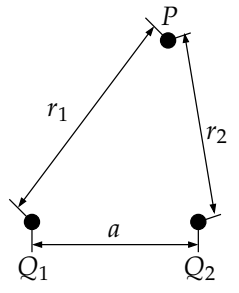
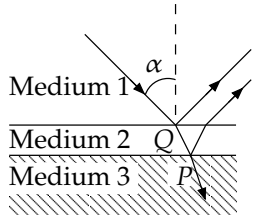
|  |   |  |
|--|---|--|
|  <p>bewegter Beobachter und bewegte Punktquelle</p> | $f' = \frac{\sqrt{1 - \beta^2}}{1 - \beta \cos(\vartheta)} f$ $\beta = \frac{v}{c}$ <p>falls <math>f \gg c</math>:</p> $\frac{f - f'}{f} = \frac{\Delta f}{f} = \frac{v}{c}$ <p>falls <math>\vartheta = 0^\circ</math> oder <math>\vartheta = 180^\circ</math>:</p> $\frac{\Delta \lambda}{\lambda} = -\frac{v}{c}$ | <p><math>f</math> Frequenz <math>[Hz]</math></p> <p><math>f'</math> gesehene Frequenz <math>[Hz]</math></p> <p><math>v</math> Geschw. relativ Beobachter Quelle <math>[\frac{m}{s}]</math></p> <p><math>\vartheta</math> Winkel <math>[rad]</math></p> <p><math>c</math> Lichtgeschwindigkeit <math>[\frac{m}{s}]</math></p> <p><math>= 299'792'458</math></p> |
|--|---|--|



### 8.6.3. Machscher Kegel

|   |   |   |
|---|---|---|
|  | <p>Falls <math>v &gt; u</math> entsteht ein Machscher Kegel</p> $\sin(\vartheta) = \frac{u}{v}$ $M = \frac{v}{u}$ | <p><math>u</math> Wellengeschw. <math>\left[\frac{m}{s}\right]</math><br/> <math>v</math> Geschw. Flugzeug <math>\left[\frac{m}{s}\right]</math><br/> <math>\vartheta</math> Winkel des Kegels <math>[rad]</math><br/> <math>M</math> Machzahl <math>[1]</math></p> |
|---|---|---|

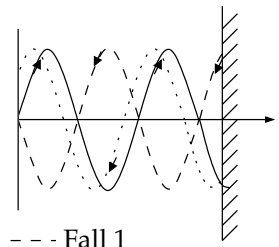
## 8.7. Überlagerung von Wellen gleicher Frequenz

|   |  |   |
|---|--|---|
|   | $l = nr$ <p>In 1s geht Energie S durch <math>1m^2</math>:</p> $S = \frac{\delta \xi^2 \omega^2}{2} u$ <p>Prinzip von Huygens:<br/>         Jedes Flächenelement auf einer Welle kann als Zentrum einer Kugelwelle betrachtet werden. Die Wellenfläche zu einem späteren Zeitpunkt ist die Einhüllende all dieser Elementarwellen.</p>                                  | <p><math>u</math> Wellengeschw. <math>\left[\frac{m}{s}\right]</math><br/> <math>l</math> Optische Weglänge <math>[m]</math><br/> <math>n</math> Brechungsindex <math>[1]</math><br/> <math>S</math> Energie <math>\left[\frac{J}{m^2s}\right]</math><br/> <math>\xi</math> Störung <math>[...]</math><br/> <math>\omega</math> Kreisfrequenz <math>\left[\frac{1}{s}\right]</math></p> |
|  | <p>Bei der Reflexion an einem optisch dichteren Medium findet ein Phasensprung von <math>\pi</math> statt.<br/>         Beispiel: Falls Medium 1 dichter Medium 2 dichter Medium 3: Phasensprung in P und Q.</p> $\Rightarrow +\frac{\lambda}{2}$ <p><math>\rightarrow</math> Auslöschung bei <math>m\frac{\lambda}{2}</math>, <math>m = \{1, 3, 5, \dots\}</math></p> |   |

## 8.8. Optische Länge


|  |   |   |
|--|---|---|
| Durchqueren Wellen Medien muss mit optischen Längen gerechnet werden | $s \rightarrow ns$<br>$\lambda \rightarrow \frac{\lambda}{n}$ | $n$ Brech-Index $[1]$<br>$s$ Strecke $[m]$<br>$\lambda$ Wellenlänge $[m]$ |
|--|---|---|

## 8.9. Stehende Wellen

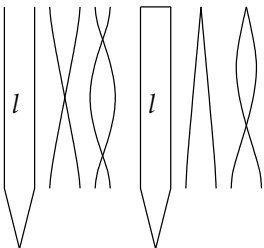
|  |  |   |
|--|--|---|
|  <p>       - - - Fall 1<br/>       ···· Fall 2<br/>       Einfallende Welle wird von Grenzfläche reflektiert     </p> | <p>1. Fall: Phasensprung <math>\pi</math> bei Reflexion</p> $\xi_0 \sin(k_x - \omega t) + \xi_0 \sin(k_x + \omega t) =$ $2\xi \sin(k_x) \cos(\omega t)$ <p>       Knoten bei <math>k_x = 0, \pi, 2\pi \dots</math><br/>       Bäuche bei: <math>k_x = \frac{1}{2}\pi, \frac{3}{2}\pi \dots</math> </p> <p>2. Fall: Kein Phasensprung</p> $\xi_0 \sin(k_x - \omega t) - \xi_0 \sin(k_x + \omega t) =$ $-2\xi \cos(k_x) \sin(\omega t)$ <p>       Knoten bei <math>k_x = \frac{1}{2}\pi, \frac{3}{2}\pi \dots</math><br/>       Bäuche bei: <math>k_x = 0, \pi, 2\pi \dots</math> </p> | $\xi$ Störung $[...]$<br>$\omega$ Kreisfrequenz $[\frac{1}{s}]$<br>$t$ Zeit $[s]$<br>$k$ Wellenzahl $[\frac{1}{m}]$ |
|--|--|---|

## 8.10. Eigenschwingungen

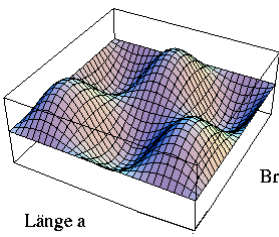
### 8.10.1. Saite

|   |  |  |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |
|---|--|--|-----|---------------|----------------------------|-----|--------|---------|-----|------------|-------|-----------|-------------|-------|--------|--------------|-------------------------------|-------|---------|---------|----------|---------------|----------------------------|-----|----------|--------|-------|---------------|--------|-----|-------------|-------|-----|------------------|-------|----------|-------------------|----------------------------|--------|--------|-------------------------------|-----|-------------------|------------------------------|
|  <p>— <math>\lambda = 2l</math><br/>- - - <math>\lambda = l</math></p> <p>Die Saite ist zweiseitig fixiert</p> | $\lambda_n = \frac{2l}{n}$ $f_n = \frac{u}{\lambda_n} = \frac{u}{2l}n = nf_1$ $f_1 = \frac{1}{2l} \sqrt{\frac{F}{\rho A}}$ $u = \sqrt{\frac{F}{\rho A}}$ $F = \frac{4l^2}{n^2} \rho A f^2$ <p>Bei Temperatur Änderung:</p> $\Delta f = \left( \frac{E_{Sa}(\alpha_{Trag} - \alpha_{Sa})}{8\rho_{Sa}l^2f^2} - \alpha_{Trag} \right) \Delta T f$ | <table> <tr><td><math>u</math></td><td>Wellengeschw.</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr><td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr><td><math>F</math></td><td>Spannkraft</td><td><math>[N]</math></td></tr> <tr><td><math>\lambda</math></td><td>Wellenlänge</td><td><math>[m]</math></td></tr> <tr><td><math>\rho</math></td><td>Dichte Saite</td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> <tr><td><math>\xi</math></td><td>Störung</td><td><math>[...]</math></td></tr> <tr><td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>\left[\frac{1}{s}\right]</math></td></tr> <tr><td><math>f</math></td><td>Frequenz</td><td><math>[Hz]</math></td></tr> <tr><td><math>f_1</math></td><td>Grundfrequenz</td><td><math>[Hz]</math></td></tr> <tr><td><math>l</math></td><td>Saitenlänge</td><td><math>[m]</math></td></tr> <tr><td><math>n</math></td><td>n-te Harmonische</td><td><math>[1]</math></td></tr> <tr><td><math>\alpha</math></td><td>Längenausd. koef.</td><td><math>\left[\frac{1}{K}\right]</math></td></tr> <tr><td><math>\rho</math></td><td>Dichte</td><td><math>\left[\frac{kg}{m^3}\right]</math></td></tr> <tr><td><math>E</math></td><td>Elastizitätsmodul</td><td><math>\left[\frac{N}{m^2}\right]</math></td></tr> </table> | $u$ | Wellengeschw. | $\left[\frac{m}{s}\right]$ | $A$ | Fläche | $[m^2]$ | $F$ | Spannkraft | $[N]$ | $\lambda$ | Wellenlänge | $[m]$ | $\rho$ | Dichte Saite | $\left[\frac{kg}{m^3}\right]$ | $\xi$ | Störung | $[...]$ | $\omega$ | Kreisfrequenz | $\left[\frac{1}{s}\right]$ | $f$ | Frequenz | $[Hz]$ | $f_1$ | Grundfrequenz | $[Hz]$ | $l$ | Saitenlänge | $[m]$ | $n$ | n-te Harmonische | $[1]$ | $\alpha$ | Längenausd. koef. | $\left[\frac{1}{K}\right]$ | $\rho$ | Dichte | $\left[\frac{kg}{m^3}\right]$ | $E$ | Elastizitätsmodul | $\left[\frac{N}{m^2}\right]$ |
| $u$   | Wellengeschw.  | $\left[\frac{m}{s}\right]$   |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |
| $A$   | Fläche   | $[m^2]$  |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |
| $F$   | Spannkraft   | $[N]$  |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |
| $\lambda$   | Wellenlänge  | $[m]$  |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |
| $\rho$  | Dichte Saite   | $\left[\frac{kg}{m^3}\right]$  |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |
| $\xi$   | Störung  | $[...]$  |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |
| $\omega$  | Kreisfrequenz  | $\left[\frac{1}{s}\right]$   |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |
| $f$   | Frequenz   | $[Hz]$   |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |
| $f_1$   | Grundfrequenz  | $[Hz]$   |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |
| $l$   | Saitenlänge  | $[m]$  |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |
| $n$   | n-te Harmonische   | $[1]$  |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |
| $\alpha$  | Längenausd. koef.  | $\left[\frac{1}{K}\right]$   |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |
| $\rho$  | Dichte   | $\left[\frac{kg}{m^3}\right]$  |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |
| $E$   | Elastizitätsmodul  | $\left[\frac{N}{m^2}\right]$   |     |               |                            |     |        |         |     |            |       |           |             |       |        |              |                               |       |         |         |          |               |                            |     |          |        |       |               |        |     |             |       |     |                  |       |          |                   |                            |        |        |                               |     |                   |                              |

### 8.10.2. Pfeife

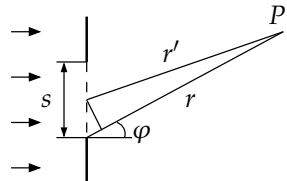
|   |   |  |     |          |        |       |               |        |     |          |                               |     |            |       |          |                     |       |     |                     |                               |  |  |            |     |             |       |           |             |       |
|---|---|--|-----|----------|--------|-------|---------------|--------|-----|----------|-------------------------------|-----|------------|-------|----------|---------------------|-------|-----|---------------------|-------------------------------|--|--|------------|-----|-------------|-------|-----------|-------------|-------|
|  <p>offen      gedackt</p> | <p>offene Pfeife:</p> $f_1 = \frac{1}{2l} \sqrt{\frac{\kappa RT}{M}} = \frac{u}{2l}$ $f_n = nf_1$ $\lambda_n = \frac{4l}{n} \quad n = 1, 3, 5, \dots$ <p>gedackte Pfeife:</p> $f_1 = \frac{1}{4l} \sqrt{\frac{\kappa RT}{M}} = \frac{u}{4l}$ $f_n = nf_1$ $\lambda_n = \frac{4l}{n} \quad n = 2, 4, 6, \dots$ | <table> <tr><td><math>f</math></td><td>Frequenz</td><td><math>[Hz]</math></td></tr> <tr><td><math>f_1</math></td><td>Grundfrequenz</td><td><math>[Hz]</math></td></tr> <tr><td><math>M</math></td><td>Molmasse</td><td><math>\left[\frac{kg}{mol}\right]</math></td></tr> <tr><td><math>T</math></td><td>abs. Temp.</td><td><math>[K]</math></td></tr> <tr><td><math>\kappa</math></td><td>Adiabatene exponent</td><td><math>[1]</math></td></tr> <tr><td><math>R</math></td><td>univers. Gas-Konst.</td><td><math>\left[\frac{J}{Kmol}\right]</math></td></tr> <tr><td></td><td></td><td><math>= 8.3145</math></td></tr> <tr><td><math>l</math></td><td>Saitenlänge</td><td><math>[m]</math></td></tr> <tr><td><math>\lambda</math></td><td>Wellenlänge</td><td><math>[m]</math></td></tr> </table> | $f$ | Frequenz | $[Hz]$ | $f_1$ | Grundfrequenz | $[Hz]$ | $M$ | Molmasse | $\left[\frac{kg}{mol}\right]$ | $T$ | abs. Temp. | $[K]$ | $\kappa$ | Adiabatene exponent | $[1]$ | $R$ | univers. Gas-Konst. | $\left[\frac{J}{Kmol}\right]$ |  |  | $= 8.3145$ | $l$ | Saitenlänge | $[m]$ | $\lambda$ | Wellenlänge | $[m]$ |
| $f$   | Frequenz  | $[Hz]$   |     |          |        |       |               |        |     |          |                               |     |            |       |          |                     |       |     |                     |                               |  |  |            |     |             |       |           |             |       |
| $f_1$   | Grundfrequenz   | $[Hz]$   |     |          |        |       |               |        |     |          |                               |     |            |       |          |                     |       |     |                     |                               |  |  |            |     |             |       |           |             |       |
| $M$   | Molmasse  | $\left[\frac{kg}{mol}\right]$  |     |          |        |       |               |        |     |          |                               |     |            |       |          |                     |       |     |                     |                               |  |  |            |     |             |       |           |             |       |
| $T$   | abs. Temp.  | $[K]$  |     |          |        |       |               |        |     |          |                               |     |            |       |          |                     |       |     |                     |                               |  |  |            |     |             |       |           |             |       |
| $\kappa$  | Adiabatene exponent   | $[1]$  |     |          |        |       |               |        |     |          |                               |     |            |       |          |                     |       |     |                     |                               |  |  |            |     |             |       |           |             |       |
| $R$   | univers. Gas-Konst.   | $\left[\frac{J}{Kmol}\right]$  |     |          |        |       |               |        |     |          |                               |     |            |       |          |                     |       |     |                     |                               |  |  |            |     |             |       |           |             |       |
|   |   | $= 8.3145$   |     |          |        |       |               |        |     |          |                               |     |            |       |          |                     |       |     |                     |                               |  |  |            |     |             |       |           |             |       |
| $l$   | Saitenlänge   | $[m]$  |     |          |        |       |               |        |     |          |                               |     |            |       |          |                     |       |     |                     |                               |  |  |            |     |             |       |           |             |       |
| $\lambda$   | Wellenlänge   | $[m]$  |     |          |        |       |               |        |     |          |                               |     |            |       |          |                     |       |     |                     |                               |  |  |            |     |             |       |           |             |       |

### 8.10.3. Rechteckige Membrane

|   |  |   |          |               |      |     |            |     |       |                  |                    |       |         |       |        |              |       |     |       |     |     |        |     |        |                  |     |
|---|--|---|----------|---------------|------|-----|------------|-----|-------|------------------|--------------------|-------|---------|-------|--------|--------------|-------|-----|-------|-----|-----|--------|-----|--------|------------------|-----|
|  <p>Länge a<br/>m=3, n=2</p> | $f(x, y) = \xi_0 \sin(k_x x + \varphi_x) \sin(k_y y + \varphi_y)$ $k_x a = m\pi \quad k_y b = n\pi$ $f_{mn} = \frac{1}{2} \sqrt{\frac{F}{\mu}} \sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$ | <table> <tr> <td><math>f_{mn}</math></td><td>Eigenfrequenz</td><td>[Hz]</td></tr> <tr> <td><math>F</math></td><td>Spannkraft</td><td>[N]</td></tr> <tr> <td><math>\mu</math></td><td>Masse pro Fläche</td><td><math>[\frac{kg}{m^2}]</math></td></tr> <tr> <td><math>\xi</math></td><td>Störung</td><td>[...]</td></tr> <tr> <td><math>x, y</math></td><td>Richtung x,y</td><td>[...]</td></tr> <tr> <td><math>a</math></td><td>Länge</td><td>[m]</td></tr> <tr> <td><math>b</math></td><td>Breite</td><td>[m]</td></tr> <tr> <td><math>m, n</math></td><td>Anz. Oberflächen</td><td>[1]</td></tr> </table> | $f_{mn}$ | Eigenfrequenz | [Hz] | $F$ | Spannkraft | [N] | $\mu$ | Masse pro Fläche | $[\frac{kg}{m^2}]$ | $\xi$ | Störung | [...] | $x, y$ | Richtung x,y | [...] | $a$ | Länge | [m] | $b$ | Breite | [m] | $m, n$ | Anz. Oberflächen | [1] |
| $f_{mn}$  | Eigenfrequenz  | [Hz]  |          |               |      |     |            |     |       |                  |                    |       |         |       |        |              |       |     |       |     |     |        |     |        |                  |     |
| $F$   | Spannkraft   | [N]   |          |               |      |     |            |     |       |                  |                    |       |         |       |        |              |       |     |       |     |     |        |     |        |                  |     |
| $\mu$   | Masse pro Fläche   | $[\frac{kg}{m^2}]$  |          |               |      |     |            |     |       |                  |                    |       |         |       |        |              |       |     |       |     |     |        |     |        |                  |     |
| $\xi$   | Störung  | [...]   |          |               |      |     |            |     |       |                  |                    |       |         |       |        |              |       |     |       |     |     |        |     |        |                  |     |
| $x, y$  | Richtung x,y   | [...]   |          |               |      |     |            |     |       |                  |                    |       |         |       |        |              |       |     |       |     |     |        |     |        |                  |     |
| $a$   | Länge  | [m]   |          |               |      |     |            |     |       |                  |                    |       |         |       |        |              |       |     |       |     |     |        |     |        |                  |     |
| $b$   | Breite   | [m]   |          |               |      |     |            |     |       |                  |                    |       |         |       |        |              |       |     |       |     |     |        |     |        |                  |     |
| $m, n$  | Anz. Oberflächen   | [1]   |          |               |      |     |            |     |       |                  |                    |       |         |       |        |              |       |     |       |     |     |        |     |        |                  |     |

## 8.11. Beugung

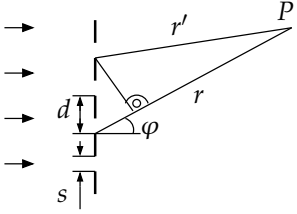
### 8.11.1. Beugung am Spalt

|  |  |   |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |            |                 |           |                    |       |       |            |                   |           |             |     |     |         |     |
|--|--|---|-----|-----------|-------|-------|------------|-----|-----|-------------|-----|-----|---------|-----|-----|------|-----|-------|---------|-------|-----|------------|-----------------|-----------|--------------------|-------|-------|------------|-------------------|-----------|-------------|-----|-----|---------|-----|
|  | $\xi = \frac{A}{r} A_s \cos(\omega t - kr_s)$ $A_s = \frac{\sin\left(\frac{ks \sin(\varphi)}{2}\right)}{\frac{ks \sin(\varphi)}{2}}$ $I_s \sim \xi^2$ <p>Nullstelle n-ter Ordnung:</p> $\sin(\varphi_n) = n \frac{\lambda}{s}$ | <table> <tr> <td><math>A</math></td><td>Amplitude</td><td>[...]</td></tr> <tr> <td><math>A_s</math></td><td>Formfaktor</td><td>[1]</td></tr> <tr> <td><math>s</math></td><td>Spaltbreite</td><td>[m]</td></tr> <tr> <td><math>r</math></td><td>Abstand</td><td>[m]</td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td>[s]</td></tr> <tr> <td><math>\xi</math></td><td>Störung</td><td>[...]</td></tr> <tr> <td><math>k</math></td><td>Wellenzahl</td><td><math>[\frac{1}{m}]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Betrachtungswinkel</td><td>[rad]</td></tr> <tr> <td><math>I_s</math></td><td>Intensität</td><td><math>[\frac{W}{m^2}]</math></td></tr> <tr> <td><math>\lambda</math></td><td>Wellenlänge</td><td>[m]</td></tr> <tr> <td><math>n</math></td><td>Ordnung</td><td>[1]</td></tr> </table> | $A$ | Amplitude | [...] | $A_s$ | Formfaktor | [1] | $s$ | Spaltbreite | [m] | $r$ | Abstand | [m] | $t$ | Zeit | [s] | $\xi$ | Störung | [...] | $k$ | Wellenzahl | $[\frac{1}{m}]$ | $\varphi$ | Betrachtungswinkel | [rad] | $I_s$ | Intensität | $[\frac{W}{m^2}]$ | $\lambda$ | Wellenlänge | [m] | $n$ | Ordnung | [1] |
| $A$  | Amplitude  | [...]   |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |            |                 |           |                    |       |       |            |                   |           |             |     |     |         |     |
| $A_s$  | Formfaktor   | [1]   |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |            |                 |           |                    |       |       |            |                   |           |             |     |     |         |     |
| $s$  | Spaltbreite  | [m]   |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |            |                 |           |                    |       |       |            |                   |           |             |     |     |         |     |
| $r$  | Abstand  | [m]   |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |            |                 |           |                    |       |       |            |                   |           |             |     |     |         |     |
| $t$  | Zeit   | [s]   |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |            |                 |           |                    |       |       |            |                   |           |             |     |     |         |     |
| $\xi$  | Störung  | [...]   |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |            |                 |           |                    |       |       |            |                   |           |             |     |     |         |     |
| $k$  | Wellenzahl   | $[\frac{1}{m}]$   |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |            |                 |           |                    |       |       |            |                   |           |             |     |     |         |     |
| $\varphi$  | Betrachtungswinkel   | [rad]   |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |            |                 |           |                    |       |       |            |                   |           |             |     |     |         |     |
| $I_s$  | Intensität   | $[\frac{W}{m^2}]$   |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |            |                 |           |                    |       |       |            |                   |           |             |     |     |         |     |
| $\lambda$  | Wellenlänge  | [m]   |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |            |                 |           |                    |       |       |            |                   |           |             |     |     |         |     |
| $n$  | Ordnung  | [1]   |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |            |                 |           |                    |       |       |            |                   |           |             |     |     |         |     |

### 8.11.2. Beugung an Kreisförmiger Öffnung

|           |   |  |     |                     |     |           |                    |       |           |             |     |     |                            |     |
|-----------|---|--|-----|---------------------|-----|-----------|--------------------|-------|-----------|-------------|-----|-----|----------------------------|-----|
|           | $\sin(\varphi_1) = 1.22 \frac{\lambda}{D}$ $a = 1.22 f \frac{\lambda}{D}$ | <table> <tr> <td><math>D</math></td><td>Öffnungsdurchmesser</td><td>[m]</td></tr> <tr> <td><math>\varphi</math></td><td>Betrachtungswinkel</td><td>[rad]</td></tr> <tr> <td><math>\lambda</math></td><td>Wellenlänge</td><td>[m]</td></tr> <tr> <td><math>a</math></td><td>Radius erster dunkler Ring</td><td>[m]</td></tr> </table> | $D$ | Öffnungsdurchmesser | [m] | $\varphi$ | Betrachtungswinkel | [rad] | $\lambda$ | Wellenlänge | [m] | $a$ | Radius erster dunkler Ring | [m] |
| $D$       | Öffnungsdurchmesser   | [m]  |     |                     |     |           |                    |       |           |             |     |     |                            |     |
| $\varphi$ | Betrachtungswinkel  | [rad]  |     |                     |     |           |                    |       |           |             |     |     |                            |     |
| $\lambda$ | Wellenlänge   | [m]  |     |                     |     |           |                    |       |           |             |     |     |                            |     |
| $a$       | Radius erster dunkler Ring  | [m]  |     |                     |     |           |                    |       |           |             |     |     |                            |     |

## 8.11.3. Beugung am Gitter

|   |   |  |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |              |     |           |                    |       |       |            |                              |           |             |     |     |                      |     |
|---|---|--|-----|-----------|-------|-------|------------|-----|-----|-------------|-----|-----|---------|-----|-----|------|-----|-------|---------|-------|-----|--------------|-----|-----------|--------------------|-------|-------|------------|------------------------------|-----------|-------------|-----|-----|----------------------|-----|
|  | $I_g \sim \frac{A^2}{r^2} A_s^2 B^2$ $B = \frac{\sin\left(\frac{kd \sin(\varphi)}{2} Z\right)}{\sin\left(\frac{kd \sin(\varphi)}{2}\right)}$ <p>Hauptmaximum n-ter Ordnung:</p> $\sin(\varphi_n) = n \frac{\lambda}{d}$ $\frac{\lambda}{\Delta \lambda} = nZ$ | <table> <tr> <td><math>A</math></td><td>Amplitude</td><td>[...]</td></tr> <tr> <td><math>A_s</math></td><td>Formfaktor</td><td>[1]</td></tr> <tr> <td><math>s</math></td><td>Spaltbreite</td><td>[m]</td></tr> <tr> <td><math>r</math></td><td>Abstand</td><td>[m]</td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td>[s]</td></tr> <tr> <td><math>\xi</math></td><td>Störung</td><td>[...]</td></tr> <tr> <td><math>d</math></td><td>Gitterkonst.</td><td>[m]</td></tr> <tr> <td><math>\varphi</math></td><td>Betrachtungswinkel</td><td>[rad]</td></tr> <tr> <td><math>I_s</math></td><td>Intensität</td><td><math>\left[\frac{W}{m^2}\right]</math></td></tr> <tr> <td><math>\lambda</math></td><td>Wellenlänge</td><td>[m]</td></tr> <tr> <td><math>Z</math></td><td>Zahl Gitteröffnungen</td><td>[1]</td></tr> </table> | $A$ | Amplitude | [...] | $A_s$ | Formfaktor | [1] | $s$ | Spaltbreite | [m] | $r$ | Abstand | [m] | $t$ | Zeit | [s] | $\xi$ | Störung | [...] | $d$ | Gitterkonst. | [m] | $\varphi$ | Betrachtungswinkel | [rad] | $I_s$ | Intensität | $\left[\frac{W}{m^2}\right]$ | $\lambda$ | Wellenlänge | [m] | $Z$ | Zahl Gitteröffnungen | [1] |
| $A$   | Amplitude   | [...]  |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |              |     |           |                    |       |       |            |                              |           |             |     |     |                      |     |
| $A_s$   | Formfaktor  | [1]  |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |              |     |           |                    |       |       |            |                              |           |             |     |     |                      |     |
| $s$   | Spaltbreite   | [m]  |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |              |     |           |                    |       |       |            |                              |           |             |     |     |                      |     |
| $r$   | Abstand   | [m]  |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |              |     |           |                    |       |       |            |                              |           |             |     |     |                      |     |
| $t$   | Zeit  | [s]  |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |              |     |           |                    |       |       |            |                              |           |             |     |     |                      |     |
| $\xi$   | Störung   | [...]  |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |              |     |           |                    |       |       |            |                              |           |             |     |     |                      |     |
| $d$   | Gitterkonst.  | [m]  |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |              |     |           |                    |       |       |            |                              |           |             |     |     |                      |     |
| $\varphi$   | Betrachtungswinkel  | [rad]  |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |              |     |           |                    |       |       |            |                              |           |             |     |     |                      |     |
| $I_s$   | Intensität  | $\left[\frac{W}{m^2}\right]$   |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |              |     |           |                    |       |       |            |                              |           |             |     |     |                      |     |
| $\lambda$   | Wellenlänge   | [m]  |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |              |     |           |                    |       |       |            |                              |           |             |     |     |                      |     |
| $Z$   | Zahl Gitteröffnungen  | [1]  |     |           |       |       |            |     |     |             |     |     |         |     |     |      |     |       |         |       |     |              |     |           |                    |       |       |            |                              |           |             |     |     |                      |     |

# **Teil II.**

## **Elektrizitätslehre**

# 9. Grundlagen

## 9.1. Grundgrößen

|                          |   |   |
|--------------------------|---|---|
| Ladung $Q$               | $\Delta Q = I(t) \cdot \Delta t = \int I(t)$ $\Delta Q = \frac{\Delta W(t)}{\Delta U(t)}$           | $Q$ Ladung $[C], [As]$<br>$I$ Strom $[A]$<br>$J$ Stromdichte $[\frac{A}{m^2}]$<br>$E$ el. Feldstärke $[\frac{V}{m}]$<br>$v$ Driftgeschwindigkeit $[\frac{m}{s}]$                        |
| Strom $I$                | $I = \frac{\Delta Q}{\Delta t}$ $I = \frac{U}{R} = \frac{P}{U}$                                     | $U$ Spannung $[V]$<br>$W$ Arbeit $[Ws], [J]$<br>$P$ Leistung $[W]$<br>$R$ Widerstand $[\Omega]$<br>$\rho$ Spez. Widerstand $[\frac{\Omega mm^2}{m}]$                                    |
| Driftgeschwindigkeit $v$ | $v = \frac{I}{neA}$   | $G$ Leitwert $[S]$<br>$\kappa$ spez. Leitwert $[\frac{S}{m}]$<br>$t$ Zeit $[t]$<br>$A$ Fläche $[m^2]$<br>$F$ Kraft $[N]$<br>$m$ Masse $[kg]$<br>$g$ Erdbeschleunigung $[\frac{m}{s^2}]$ |
| Spannung $U$             | $U = RI$ $U(t) = \frac{\Delta W(t)}{\Delta Q}$ $U = \frac{P}{I} = \sqrt{PR}$ $\Delta U = E\Delta x$ | $l$ Länge $[m]$<br>$\alpha$ Temp. Koeff. $[\frac{1}{^\circ C}]$<br>$\vartheta$ Temperatur $[^\circ C]$<br>$n$ Elektronendichte $[\frac{1}{m^3}]$  |
| Energie $W$              | $W = Fh = mgh$ $\Delta W(t) = U(t)I(t)\Delta t$   | $e$ Elementarladung $[C]$<br>$1.602 \cdot 10^{19} C$  |

## 9. GRUNDLAGEN

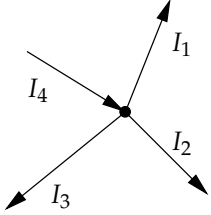
|                         |   |  |
|-------------------------|---|--|
| Leistung $P$            | $P(t) = \frac{\Delta W(t)}{\Delta t}$ $P(t) = U(t)I(t)$ $P(t) = I^2(t)R = \frac{U^2(t)}{R}$   | $Q$ Ladung $[C], [As]$<br>$I$ Strom $[A]$<br>$J$ Stromdichte $[\frac{A}{m^2}]$<br>$E$ el. Feldstärke $[\frac{V}{m}]$<br>$v$ DriftGeschwindigkeit $[\frac{m}{s}]$   |
| Widerstand $R$          | $R = \frac{U}{I} = \frac{U^2}{P} = \frac{P}{I^2}$ $R = \frac{\rho l}{A} = \frac{l}{\kappa A}$ | $U$ Spannung $[V]$<br>$W$ Arbeit $[Ws], [J]$<br>$P$ Leistung $[W]$<br>$R$ Widerstand $[\Omega]$<br>$\rho$ Spez. Widerstand $[\frac{\Omega mm^2}{m}]$<br>$G$ Leitwert $[S]$<br>$\kappa$ spez. Leitwert $[\frac{S}{m}]$<br>$t$ Zeit $[t]$<br>$A$ Fläche $[m^2]$<br>$F$ Kraft $[N]$<br>$m$ Masse $[kg]$<br>$g$ Erdberschleunigung $[\frac{m}{s^2}]$ |
| Spez. Widerstand $\rho$ | $\rho = \frac{1}{\kappa A}$ $\rho = \rho_{20}(1 + \alpha_{20})\Delta\vartheta$                | $l$ Länge $[m]$<br>$\alpha$ Temp. Koeff. $[\frac{1}{^\circ C}]$<br>$\vartheta$ Temperatur $[^\circ C]$<br>$n$ Elektronendichte $[\frac{1}{m^3}]$<br>$e$ Elementarladung $[C]$<br>$1.602 \cdot 10^{19} C$   |
| Leitwert $G$            | $G = \frac{\kappa A}{l} = \frac{1}{R}$  |  |
| Spez. Leitwert $\kappa$ | $\kappa = \frac{1}{\rho}$   |  |
| Stromdichte $J$         | $J(t) = \frac{I(t)}{A} = \frac{\Delta I(x, y)}{\Delta A}$ $\vec{J} = \kappa \vec{E}$          |  |
| Feldstärke $E$          | $E(x) = \frac{\Delta U}{\Delta x}$ $E = \frac{F}{Q}$ $\vec{E} = \rho \vec{J}$                 |  |



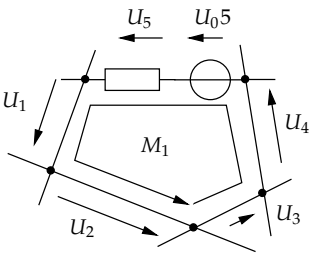
## 9.2. Netzwerke bei Gleichstrom

### 9.2.1. Kirchoffsche Gesetze

#### Stromgesetz

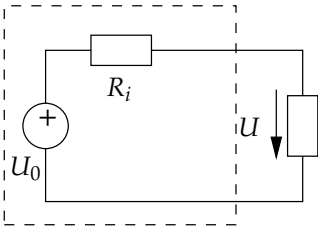
|   |  |                                       |
|---|--|---------------------------------------|
|  | <p>Die Summe aller zufließenden Ströme ist gleich der Summe aller Abfließenden Ströme</p> $\sum_{k=1}^N I_k = 0$ | <p><math>I</math>    Strom    [A]</p> |
|---|--|---------------------------------------|

#### Spannungsgesetz

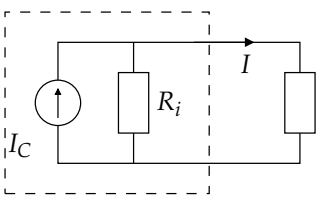
|  |   |  |
|--|---|--|
|  | <p>Die Summe aller Spannungen im Kreis ist null.</p> $\sum_{k=1}^N U_k = 0$ | <p><math>U</math>    Spannung    [V]</p> |
|--|---|--|

## 9.3. Reale Quellen

### 9.3.1. Reale Spannungsquelle

|   |   |   |
|---|---|---|
|  | $U = U_0 - R_i I$ $R_i = \frac{U_0}{I_C}$ | <p><math>U</math>    Spannung    [V]<br/> <math>I</math>    Strom    [A]<br/> <math>R</math>    Widerstand    [<math>\Omega</math>]</p> |
|---|---|---|

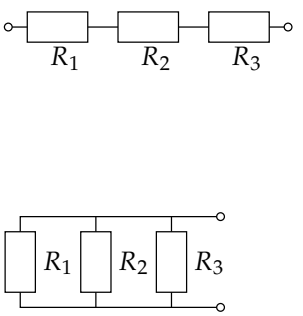
### 9.3.2. Reale Stromquelle

|   |   |  |     |          |       |     |       |       |     |            |            |
|---|---|--|-----|----------|-------|-----|-------|-------|-----|------------|------------|
|  | $I = I_C - G_i U$ $G_i = \frac{1}{R_i} = \frac{I_C}{U_0}$ | <table> <tr> <td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[I]</math></td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td><math>[\Omega]</math></td></tr> </table> | $U$ | Spannung | $[V]$ | $I$ | Strom | $[I]$ | $R$ | Widerstand | $[\Omega]$ |
| $U$   | Spannung  | $[V]$  |     |          |       |     |       |       |     |            |            |
| $I$   | Strom   | $[I]$  |     |          |       |     |       |       |     |            |            |
| $R$   | Widerstand  | $[\Omega]$   |     |          |       |     |       |       |     |            |            |

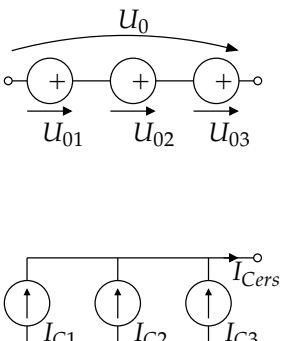
## 9.4. Netzwerkanalyse

### 9.4.1. Netzwerkumwandlung

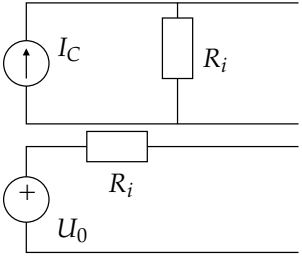
#### Widerstandsschaltungen

|  |  |  |     |          |       |     |       |       |     |            |            |
|--|--|--|-----|----------|-------|-----|-------|-------|-----|------------|------------|
|  | <p>Serieschaltung:</p> $R_{\text{ers}} = \sum_{k=1}^n R_k$ <p>Parallelschaltung:</p> $R_{\text{ers}} = \frac{1}{\sum_{k=1}^n \frac{1}{R_k}}$ | <table> <tr> <td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[I]</math></td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td><math>[\Omega]</math></td></tr> </table> | $U$ | Spannung | $[V]$ | $I$ | Strom | $[I]$ | $R$ | Widerstand | $[\Omega]$ |
| $U$  | Spannung   | $[V]$  |     |          |       |     |       |       |     |            |            |
| $I$  | Strom  | $[I]$  |     |          |       |     |       |       |     |            |            |
| $R$  | Widerstand   | $[\Omega]$   |     |          |       |     |       |       |     |            |            |

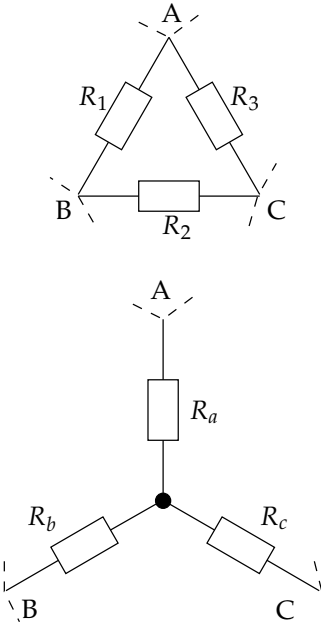
#### Mehrere Quellen

|   |  |   |     |          |       |     |       |       |
|---|--|---|-----|----------|-------|-----|-------|-------|
|  | <p>Serieschaltung von Spannungsquellen:</p> $U_{0\text{ers}} = \sum_{k=1}^n U_{0k}$ <p>Parallelschaltung von Stromquellen:</p> $I_{C\text{ers}} = \sum_{k=1}^n I_{Ck}$ | <table> <tr> <td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[I]</math></td></tr> </table> | $U$ | Spannung | $[V]$ | $I$ | Strom | $[I]$ |
| $U$   | Spannung   | $[V]$   |     |          |       |     |       |       |
| $I$   | Strom  | $[I]$   |     |          |       |     |       |       |

## Quellenumwandlung

|   |   |  |     |          |       |     |       |       |     |            |            |
|---|---|--|-----|----------|-------|-----|-------|-------|-----|------------|------------|
|  | <p>U-Quelle <math>\rightarrow</math> I-Quelle:</p> $R_i = R_i \quad I_C = \frac{U_0}{R_i}$ <p>I-Quelle <math>\rightarrow</math> U-Quelle:</p> $R_i = R_i \quad U_0 = I_C R_i$ | <table> <tr> <td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[I]</math></td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td><math>[\Omega]</math></td></tr> </table> | $U$ | Spannung | $[V]$ | $I$ | Strom | $[I]$ | $R$ | Widerstand | $[\Omega]$ |
| $U$   | Spannung  | $[V]$  |     |          |       |     |       |       |     |            |            |
| $I$   | Strom   | $[I]$  |     |          |       |     |       |       |     |            |            |
| $R$   | Widerstand  | $[\Omega]$   |     |          |       |     |       |       |     |            |            |

## Stern – Dreieck Umwandlung

|  |   |  |     |          |       |     |       |       |     |            |            |     |          |       |
|--|---|--|-----|----------|-------|-----|-------|-------|-----|------------|------------|-----|----------|-------|
|  | <p>Dreieck <math>\rightarrow</math> Stern:</p> $R_a = \frac{R_1 R_2}{R_0}$ $R_b = \frac{R_2 R_3}{R_0}$ $R_c = \frac{R_1 R_3}{R_0}$ $R_0 = R_1 + R_2 + R_3$ <p>Stern <math>\rightarrow</math> Dreieck:</p> $R_1 = R_a R_b B_0 \quad G_1 = \frac{G_a G_b}{G_0}$ $R_2 = R_b R_c B_0 \quad G_1 = \frac{G_b G_c}{G_0}$ $R_3 = R_a R_c B_0 \quad G_1 = \frac{G_a G_c}{G_0}$ $G_0 = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ | <table> <tr> <td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[I]</math></td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>G</math></td><td>Leitwert</td><td><math>[S]</math></td></tr> </table> | $U$ | Spannung | $[V]$ | $I$ | Strom | $[I]$ | $R$ | Widerstand | $[\Omega]$ | $G$ | Leitwert | $[S]$ |
| $U$  | Spannung  | $[V]$  |     |          |       |     |       |       |     |            |            |     |          |       |
| $I$  | Strom   | $[I]$  |     |          |       |     |       |       |     |            |            |     |          |       |
| $R$  | Widerstand  | $[\Omega]$   |     |          |       |     |       |       |     |            |            |     |          |       |
| $G$  | Leitwert  | $[S]$  |     |          |       |     |       |       |     |            |            |     |          |       |

## Überlagerungsprinzip (Superposition)

Die Wirkungen der entsprechenden Ursachen werden einzeln betrachtet. In einer Schaltung mit mehreren Quellen wird jede Quelle einzeln betrachtet. Die übrigen Spannungsquellen werden durch einen Kurzschluss und die restlichen Stromquellen durch einen Unterbruch ersetzt. Die Summen der einzelnen Teilwirkungen ergibt die gesamte Wirkung.

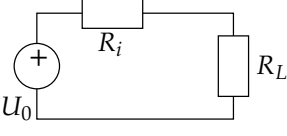
## 9. GRUNDLAGEN

(Voraussetzung: lineares System)

### Nichtlinearer Verbraucher an linearer Schaltung (Thévenin)

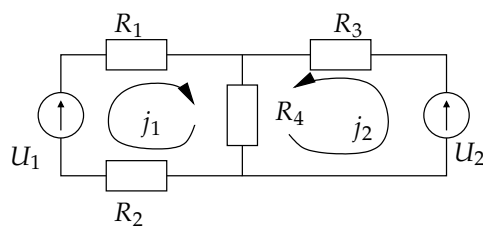
Die gesamte Schaltung muss in eine Ersatzquelle umgeformt werden. Das Ersatzschema gilt für  $U$  und  $I$ . (Achtung: z.B.  $P_{\text{Quellen}} \neq U_0 I$ )

### 9.4.2. Wirkungsgrad und Leistungsanpassung

|   |   |   |     |          |       |     |          |       |     |            |            |
|---|---|---|-----|----------|-------|-----|----------|-------|-----|------------|------------|
|  | $\eta = \frac{P_L}{P_0} = \frac{R_L}{\underbrace{R_L + R_i}_{(*)}}$ <p>(*) gilt nicht für Ersatzschaltungen</p> | <table> <tr> <td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr> <td><math>P</math></td><td>Leistung</td><td><math>[W]</math></td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td><math>[\Omega]</math></td></tr> </table> | $U$ | Spannung | $[V]$ | $P$ | Leistung | $[W]$ | $R$ | Widerstand | $[\Omega]$ |
| $U$   | Spannung  | $[V]$   |     |          |       |     |          |       |     |            |            |
| $P$   | Leistung  | $[W]$   |     |          |       |     |          |       |     |            |            |
| $R$   | Widerstand  | $[\Omega]$  |     |          |       |     |          |       |     |            |            |

### 9.4.3. Systematische Analyse linearer Netzwerke

#### Kreisströme als Variablen (Kreistrom-Methode)



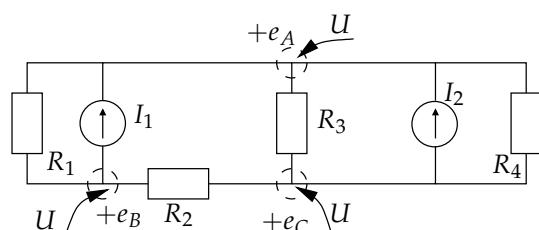
$\alpha$  = Anzahl Knoten  
 $\beta$  = Anzahl Zweige

$\beta - \alpha + 1$  unabhängige Gleichungen

$$\begin{aligned} j_1(R_1 + R_2 + R_4) + j_2 R_4 &= U_1 \\ j_1 R_4 + j_2(R_3 + R_4) &= U_2 \end{aligned}$$

$$\begin{bmatrix} R_1 + R_2 + R_4 & R_4 \\ R_4 & R_3 + R_4 \end{bmatrix} \begin{bmatrix} j_1 \\ j_2 \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

#### Trennspannungen als Variable (Knotenspannungsmethode)



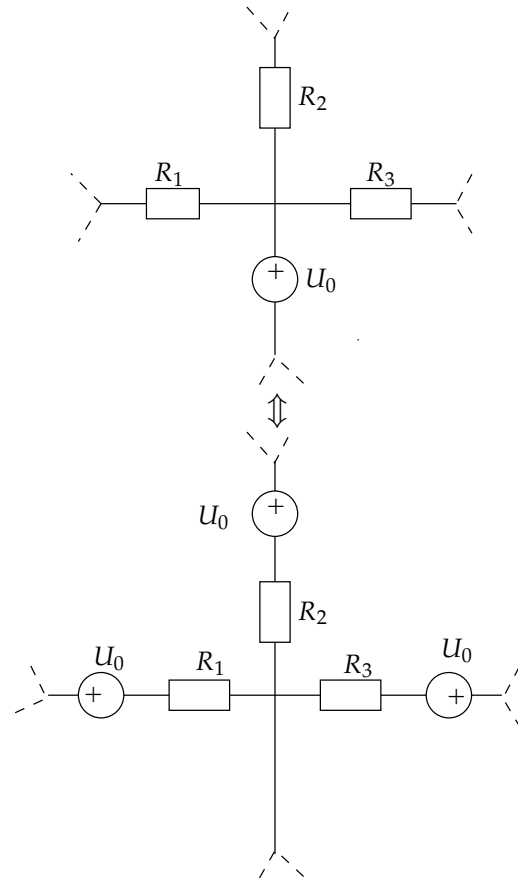
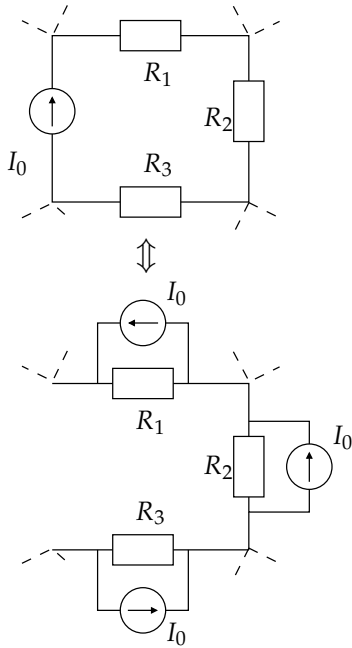
$\alpha$  = Anzahl Knoten  
 $\alpha - 1$  unabhängige Gleichungen

$$\begin{aligned} e_A(G_1 + G_3 + G_4) - e_B G_1 &= -I_1 - I_2 \\ e_A G_1 + e_B(G_1 + G_2) &= I_1 \end{aligned}$$

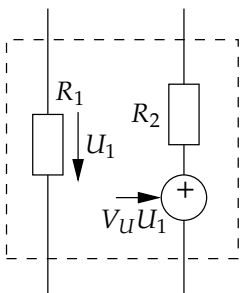
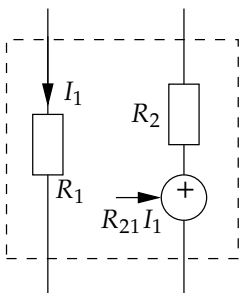
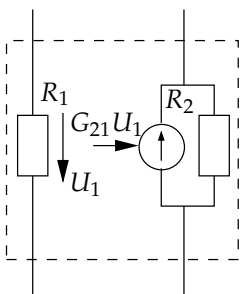
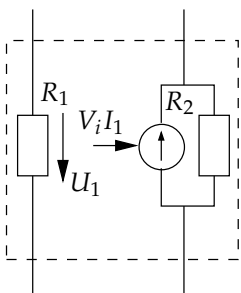
$$\begin{bmatrix} G_1 + G_3 + G_4 & -G_1 \\ G_1 & G_1 + G_2 \end{bmatrix} \begin{bmatrix} e_A \\ e_B \end{bmatrix} = \begin{bmatrix} -I_1 - I_2 \\ I_1 \end{bmatrix}$$

### 9.4.4. Quellenverschiebung

Es werden gleiche Quellen so in die Schaltung eingefügt, dass die Wirkung der ursprünglichen Quelle aufgehoben wird.

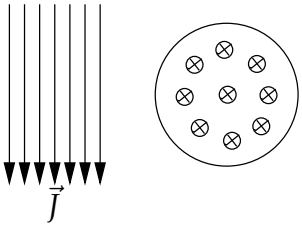


### 9.4.5. Netzwerke mit gesteuerten Quellen

|   |   |  |     |       |       |     |          |       |     |            |            |     |          |       |     |             |       |
|---|---|--|-----|-------|-------|-----|----------|-------|-----|------------|------------|-----|----------|-------|-----|-------------|-------|
|    | <p>Spannungsgesteuerte Spannungsquelle</p> $U_{02} = V_U U_1$ | <table><tr><td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr><tr><td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr><tr><td><math>R</math></td><td>Widerstand</td><td><math>[\Omega]</math></td></tr><tr><td><math>G</math></td><td>Leitwert</td><td><math>[S]</math></td></tr><tr><td><math>V</math></td><td>Verstärkung</td><td><math>[1]</math></td></tr></table> | $I$ | Strom | $[A]$ | $U$ | Spannung | $[V]$ | $R$ | Widerstand | $[\Omega]$ | $G$ | Leitwert | $[S]$ | $V$ | Verstärkung | $[1]$ |
| $I$   | Strom   | $[A]$  |     |       |       |     |          |       |     |            |            |     |          |       |     |             |       |
| $U$   | Spannung  | $[V]$  |     |       |       |     |          |       |     |            |            |     |          |       |     |             |       |
| $R$   | Widerstand  | $[\Omega]$   |     |       |       |     |          |       |     |            |            |     |          |       |     |             |       |
| $G$   | Leitwert  | $[S]$  |     |       |       |     |          |       |     |            |            |     |          |       |     |             |       |
| $V$   | Verstärkung   | $[1]$  |     |       |       |     |          |       |     |            |            |     |          |       |     |             |       |
|    | <p>Stromgesteuerte Spannungsquelle</p> $U_{02} = R_{12} U_1$  |  |     |       |       |     |          |       |     |            |            |     |          |       |     |             |       |
|  | <p>Spannungsgesteuerte Stromquelle</p> $I_{C2} = G_{12} U_1$  |  |     |       |       |     |          |       |     |            |            |     |          |       |     |             |       |
|  | <p>Stromgesteuerte Stromquelle</p> $I_{C2} = V_i U_1$         |  |     |       |       |     |          |       |     |            |            |     |          |       |     |             |       |

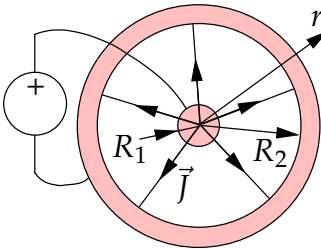
# 10. Das elektrische Strömungsfeld

## 10.1. Allgemein

|   |   |  |     |       |       |     |             |                   |     |        |         |
|---|---|--|-----|-------|-------|-----|-------------|-------------------|-----|--------|---------|
|  | $J = \frac{\Delta I}{\Delta A}$ $I = \int \vec{J} d\vec{A} = \vec{J} \vec{A}$ | <table> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>J</math></td><td>Stromdichte</td><td><math>[\frac{A}{m^2}]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> </table> | $I$ | Strom | $[A]$ | $J$ | Stromdichte | $[\frac{A}{m^2}]$ | $A$ | Fläche | $[m^2]$ |
| $I$   | Strom   | $[A]$  |     |       |       |     |             |                   |     |        |         |
| $J$   | Stromdichte   | $[\frac{A}{m^2}]$  |     |       |       |     |             |                   |     |        |         |
| $A$   | Fläche  | $[m^2]$  |     |       |       |     |             |                   |     |        |         |

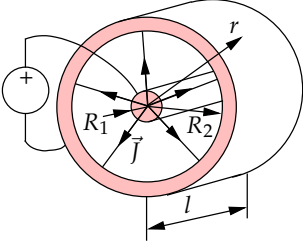
## 10.2. Spezielle Felder

### 10.2.1. Räumliches Zentralfeld (Kugelanordnung)

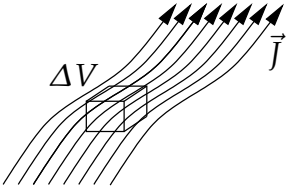
|   |   |  |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                                 |          |                |                 |     |          |       |        |        |       |     |           |       |     |        |         |
|---|---|--|-----|-------|-------|-----|-------------|-------------------|-----|----------------|-----------------|-----|----------|-------|--------|------------------|---------------------------------|----------|----------------|-----------------|-----|----------|-------|--------|--------|-------|-----|-----------|-------|-----|--------|---------|
|  | $J(r) = \frac{I}{A_{\text{Kugel}}} = \frac{I}{4\pi r^2}$ $I = J(r)4\pi r^2 - I = 0$ $J(r) = \kappa E(r) \quad E(r) = \rho E(r)$ $E(r) = \frac{I}{4\pi \kappa r^2}$ $U_{12} = \frac{I}{4\pi \kappa} \left( \frac{1}{r_1} - \frac{1}{r_2} \right)$ $V(r) = \int_r^{R_2} \vec{E}(x) d\vec{x} = \frac{I}{4\pi \kappa} \left( \frac{1}{r} - \frac{1}{R_2} \right)$ $G = 4\pi \kappa \frac{R_1 R_2}{R_2 - R_1}$ | <table> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>J</math></td><td>Stromdichte</td><td><math>[\frac{A}{m^2}]</math></td></tr> <tr> <td><math>E</math></td><td>el. Feldstärke</td><td><math>[\frac{V}{m}]</math></td></tr> <tr> <td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr> <td><math>\rho</math></td><td>Spez. Widerstand</td><td><math>[\frac{\Omega \cdot mm^2}{m}]</math></td></tr> <tr> <td><math>\kappa</math></td><td>spez. Leitwert</td><td><math>[\frac{S}{m}]</math></td></tr> <tr> <td><math>G</math></td><td>Leitwert</td><td><math>[S]</math></td></tr> <tr> <td><math>R, r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr> <td><math>V</math></td><td>Potential</td><td><math>[V]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> </table> | $I$ | Strom | $[A]$ | $J$ | Stromdichte | $[\frac{A}{m^2}]$ | $E$ | el. Feldstärke | $[\frac{V}{m}]$ | $U$ | Spannung | $[V]$ | $\rho$ | Spez. Widerstand | $[\frac{\Omega \cdot mm^2}{m}]$ | $\kappa$ | spez. Leitwert | $[\frac{S}{m}]$ | $G$ | Leitwert | $[S]$ | $R, r$ | Radius | $[m]$ | $V$ | Potential | $[V]$ | $A$ | Fläche | $[m^2]$ |
| $I$   | Strom   | $[A]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                                 |          |                |                 |     |          |       |        |        |       |     |           |       |     |        |         |
| $J$   | Stromdichte   | $[\frac{A}{m^2}]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                                 |          |                |                 |     |          |       |        |        |       |     |           |       |     |        |         |
| $E$   | el. Feldstärke  | $[\frac{V}{m}]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                                 |          |                |                 |     |          |       |        |        |       |     |           |       |     |        |         |
| $U$   | Spannung  | $[V]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                                 |          |                |                 |     |          |       |        |        |       |     |           |       |     |        |         |
| $\rho$  | Spez. Widerstand  | $[\frac{\Omega \cdot mm^2}{m}]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                                 |          |                |                 |     |          |       |        |        |       |     |           |       |     |        |         |
| $\kappa$  | spez. Leitwert  | $[\frac{S}{m}]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                                 |          |                |                 |     |          |       |        |        |       |     |           |       |     |        |         |
| $G$   | Leitwert  | $[S]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                                 |          |                |                 |     |          |       |        |        |       |     |           |       |     |        |         |
| $R, r$  | Radius  | $[m]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                                 |          |                |                 |     |          |       |        |        |       |     |           |       |     |        |         |
| $V$   | Potential   | $[V]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                                 |          |                |                 |     |          |       |        |        |       |     |           |       |     |        |         |
| $A$   | Fläche  | $[m^2]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                                 |          |                |                 |     |          |       |        |        |       |     |           |       |     |        |         |

## 10. DAS ELEKTRISCHE STRÖMUNGSFELD

### 10.2.2. Zylindrisches Zentralfeld

|   |   |  |     |       |       |     |             |                   |     |                |                 |     |          |       |     |           |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
|---|---|--|-----|-------|-------|-----|-------------|-------------------|-----|----------------|-----------------|-----|----------|-------|-----|-----------|-------|--------|------------------|---------------------------|----------|----------------|-----------------|-----|----------|-------|--------|--------|-------|-----|--------|---------|
|  | $J(r) = \frac{I}{A_{\text{Kugel}}} = \frac{I}{4\pi r^2}$ $J(r) = \frac{I}{2\pi r l}$ $J(r) = \kappa E(r) \quad E(r) = \rho E(r)$ $E(r) = \frac{I}{2\pi \kappa r l}$ $U_{12} = \frac{I}{2\pi \kappa l} \ln \frac{r_2}{r_1}$ $U = \frac{I}{2\pi \kappa l} \ln \frac{R_2}{R_1}$ $V(r) = \frac{I}{2\pi \kappa l} \ln \frac{R_2}{r}$ $G = \frac{2\pi \kappa l}{\ln \frac{R_2}{R_1}}$ | <table> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>J</math></td><td>Stromdichte</td><td><math>[\frac{A}{m^2}]</math></td></tr> <tr> <td><math>E</math></td><td>el. Feldstärke</td><td><math>[\frac{V}{m}]</math></td></tr> <tr> <td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr> <td><math>V</math></td><td>Potential</td><td><math>[V]</math></td></tr> <tr> <td><math>\rho</math></td><td>Spez. Widerstand</td><td><math>[\frac{\Omega mm^2}{m}]</math></td></tr> <tr> <td><math>\kappa</math></td><td>spez. Leitwert</td><td><math>[\frac{S}{m}]</math></td></tr> <tr> <td><math>G</math></td><td>Leitwert</td><td><math>[S]</math></td></tr> <tr> <td><math>R, r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> </table> | $I$ | Strom | $[A]$ | $J$ | Stromdichte | $[\frac{A}{m^2}]$ | $E$ | el. Feldstärke | $[\frac{V}{m}]$ | $U$ | Spannung | $[V]$ | $V$ | Potential | $[V]$ | $\rho$ | Spez. Widerstand | $[\frac{\Omega mm^2}{m}]$ | $\kappa$ | spez. Leitwert | $[\frac{S}{m}]$ | $G$ | Leitwert | $[S]$ | $R, r$ | Radius | $[m]$ | $A$ | Fläche | $[m^2]$ |
| $I$   | Strom   | $[A]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |     |           |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $J$   | Stromdichte   | $[\frac{A}{m^2}]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |     |           |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $E$   | el. Feldstärke  | $[\frac{V}{m}]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |     |           |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $U$   | Spannung  | $[V]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |     |           |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $V$   | Potential   | $[V]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |     |           |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $\rho$  | Spez. Widerstand  | $[\frac{\Omega mm^2}{m}]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |     |           |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $\kappa$  | spez. Leitwert  | $[\frac{S}{m}]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |     |           |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $G$   | Leitwert  | $[S]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |     |           |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $R, r$  | Radius  | $[m]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |     |           |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $A$   | Fläche  | $[m^2]$  |     |       |       |     |             |                   |     |                |                 |     |          |       |     |           |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |

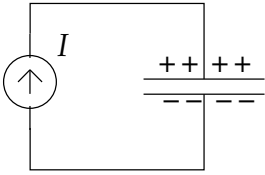
### 10.2.3. Leistung und räumliche Leistungsdichte

|   |  |   |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
|---|--|---|-----|-------|-------|-----|-------------|-------------------|-----|----------------|-----------------|-----|----------|-------|--------|------------------|---------------------------|----------|----------------|-----------------|-----|----------|-------|--------|--------|-------|-----|--------|---------|
|  | $p(x, y, z) = \frac{\Delta P}{\Delta V}$ $p(x, y, z) = E(x, y, z) J(x, y, z)$ $= \kappa(x, y, z) E^2(x, y, z)$ $= \rho(x, y, z) J^2(x, y, z)$ <p>Gesamtleistung <math>P</math> aus <math>p</math>:</p> $P = \sum \Delta p = \sum_n P_n \Delta V$ | <table> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>J</math></td><td>Stromdichte</td><td><math>[\frac{A}{m^2}]</math></td></tr> <tr> <td><math>E</math></td><td>el. Feldstärke</td><td><math>[\frac{V}{m}]</math></td></tr> <tr> <td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr> <td><math>\rho</math></td><td>Spez. Widerstand</td><td><math>[\frac{\Omega mm^2}{m}]</math></td></tr> <tr> <td><math>\kappa</math></td><td>spez. Leitwert</td><td><math>[\frac{S}{m}]</math></td></tr> <tr> <td><math>G</math></td><td>Leitwert</td><td><math>[S]</math></td></tr> <tr> <td><math>R, r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> </table> | $I$ | Strom | $[A]$ | $J$ | Stromdichte | $[\frac{A}{m^2}]$ | $E$ | el. Feldstärke | $[\frac{V}{m}]$ | $U$ | Spannung | $[V]$ | $\rho$ | Spez. Widerstand | $[\frac{\Omega mm^2}{m}]$ | $\kappa$ | spez. Leitwert | $[\frac{S}{m}]$ | $G$ | Leitwert | $[S]$ | $R, r$ | Radius | $[m]$ | $A$ | Fläche | $[m^2]$ |
| $I$   | Strom  | $[A]$   |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $J$   | Stromdichte  | $[\frac{A}{m^2}]$   |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $E$   | el. Feldstärke   | $[\frac{V}{m}]$   |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $U$   | Spannung   | $[V]$   |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $\rho$  | Spez. Widerstand   | $[\frac{\Omega mm^2}{m}]$   |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $\kappa$  | spez. Leitwert   | $[\frac{S}{m}]$   |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $G$   | Leitwert   | $[S]$   |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $R, r$  | Radius   | $[m]$   |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |
| $A$   | Fläche   | $[m^2]$   |     |       |       |     |             |                   |     |                |                 |     |          |       |        |                  |                           |          |                |                 |     |          |       |        |        |       |     |        |         |

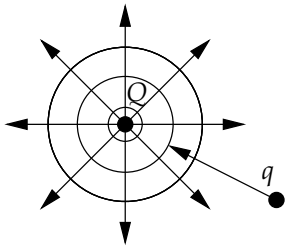


# 11. Elektrostatik

## 11.1. Das Coulombsche Gesetz

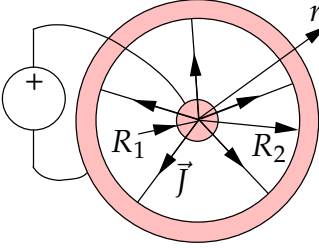
|   |  |   |     |       |       |     |        |       |     |        |       |            |                |                  |              |                     |  |              |                          |                  |
|---|--|---|-----|-------|-------|-----|--------|-------|-----|--------|-------|------------|----------------|------------------|--------------|---------------------|--|--------------|--------------------------|------------------|
|  | $F = \frac{Q_1 Q_2}{4\pi\epsilon r^2}$ $F > 0 \rightarrow \text{Abstossung}$ $\epsilon = \epsilon_0 \epsilon_r$ $\epsilon_0 = 8.854 \cdot 10^{-12} \frac{C}{Nm}$ | <table> <tr> <td><math>F</math></td><td>Kraft</td><td><math>[N]</math></td></tr> <tr> <td><math>Q</math></td><td>Ladung</td><td><math>[C]</math></td></tr> <tr> <td><math>r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr> <td><math>\epsilon</math></td><td>Dielektrizität</td><td><math>[\frac{C}{Nm}]</math></td></tr> <tr> <td><math>\epsilon_r</math></td><td>rel. Dielektrizität</td><td></td></tr> <tr> <td><math>\epsilon_0</math></td><td>Dielektrizitätskonstante</td><td><math>[\frac{C}{Nm}]</math></td></tr> </table> | $F$ | Kraft | $[N]$ | $Q$ | Ladung | $[C]$ | $r$ | Radius | $[m]$ | $\epsilon$ | Dielektrizität | $[\frac{C}{Nm}]$ | $\epsilon_r$ | rel. Dielektrizität |  | $\epsilon_0$ | Dielektrizitätskonstante | $[\frac{C}{Nm}]$ |
| $F$   | Kraft  | $[N]$   |     |       |       |     |        |       |     |        |       |            |                |                  |              |                     |  |              |                          |                  |
| $Q$   | Ladung   | $[C]$   |     |       |       |     |        |       |     |        |       |            |                |                  |              |                     |  |              |                          |                  |
| $r$   | Radius   | $[m]$   |     |       |       |     |        |       |     |        |       |            |                |                  |              |                     |  |              |                          |                  |
| $\epsilon$  | Dielektrizität   | $[\frac{C}{Nm}]$  |     |       |       |     |        |       |     |        |       |            |                |                  |              |                     |  |              |                          |                  |
| $\epsilon_r$  | rel. Dielektrizität  |   |     |       |       |     |        |       |     |        |       |            |                |                  |              |                     |  |              |                          |                  |
| $\epsilon_0$  | Dielektrizitätskonstante   | $[\frac{C}{Nm}]$  |     |       |       |     |        |       |     |        |       |            |                |                  |              |                     |  |              |                          |                  |

## 11.2. Das elektrostatische Feld (Allgemein)

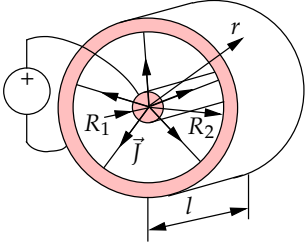
|   |  |   |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
|---|--|---|-----|-------|-------|-----|--------|-------|-----|-------------|-------|-----|--------|-------|-----|-----------|-------|-----|----------------|-----------------|-----|----------|-------|-----|-----------|-------|----------|-----------------------|-------------------|-----|--------|---------|-------------|-----------|-------|------------|----------------|------------------|--------------|---------------------|--|--------------|--------------------------|------------------|
|  <p>Bei mehreren Ladungen gilt das Superpositionsprinzip</p> | $\vec{E} = \frac{\vec{F}}{q} = \frac{Q}{4\pi\epsilon r^2} \vec{e}_r$ $V(r) = \frac{Q}{4\pi\epsilon r}$ $U_{AB} = V_A - V_B = \int_A^B \vec{E} d\vec{s}$ $\vec{D} = \epsilon \vec{E}$ $\psi_{el} = \int_A \vec{D} d\vec{A}$ $\sigma = \frac{\Delta Q}{\Delta A}$ $Q = CU$ | <table> <tr> <td><math>F</math></td><td>Kraft</td><td><math>[N]</math></td></tr> <tr> <td><math>Q</math></td><td>Ladung</td><td><math>[C]</math></td></tr> <tr> <td><math>q</math></td><td>Probeladung</td><td><math>[C]</math></td></tr> <tr> <td><math>r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr> <td><math>C</math></td><td>Kapazität</td><td><math>[F]</math></td></tr> <tr> <td><math>E</math></td><td>el. Feldstärke</td><td><math>[\frac{V}{m}]</math></td></tr> <tr> <td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr> <td><math>V</math></td><td>Potential</td><td><math>[V]</math></td></tr> <tr> <td><math>\sigma</math></td><td>Oberfl. Ladungsdichte</td><td><math>[\frac{C}{m^2}]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr> <td><math>\psi_{el}</math></td><td>el. Fluss</td><td><math>[C]</math></td></tr> <tr> <td><math>\epsilon</math></td><td>Dielektrizität</td><td><math>[\frac{C}{Nm}]</math></td></tr> <tr> <td><math>\epsilon_r</math></td><td>rel. Dielektrizität</td><td></td></tr> <tr> <td><math>\epsilon_0</math></td><td>Dielektrizitätskonstante</td><td><math>[\frac{C}{Nm}]</math></td></tr> </table> | $F$ | Kraft | $[N]$ | $Q$ | Ladung | $[C]$ | $q$ | Probeladung | $[C]$ | $r$ | Radius | $[m]$ | $C$ | Kapazität | $[F]$ | $E$ | el. Feldstärke | $[\frac{V}{m}]$ | $U$ | Spannung | $[V]$ | $V$ | Potential | $[V]$ | $\sigma$ | Oberfl. Ladungsdichte | $[\frac{C}{m^2}]$ | $A$ | Fläche | $[m^2]$ | $\psi_{el}$ | el. Fluss | $[C]$ | $\epsilon$ | Dielektrizität | $[\frac{C}{Nm}]$ | $\epsilon_r$ | rel. Dielektrizität |  | $\epsilon_0$ | Dielektrizitätskonstante | $[\frac{C}{Nm}]$ |
| $F$   | Kraft  | $[N]$   |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $Q$   | Ladung   | $[C]$   |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $q$   | Probeladung  | $[C]$   |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $r$   | Radius   | $[m]$   |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $C$   | Kapazität  | $[F]$   |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $E$   | el. Feldstärke   | $[\frac{V}{m}]$   |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $U$   | Spannung   | $[V]$   |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $V$   | Potential  | $[V]$   |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\sigma$  | Oberfl. Ladungsdichte  | $[\frac{C}{m^2}]$   |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $A$   | Fläche   | $[m^2]$   |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\psi_{el}$   | el. Fluss  | $[C]$   |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\epsilon$  | Dielektrizität   | $[\frac{C}{Nm}]$  |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\epsilon_r$  | rel. Dielektrizität  |   |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\epsilon_0$  | Dielektrizitätskonstante   | $[\frac{C}{Nm}]$  |     |       |       |     |        |       |     |             |       |     |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |

## 11.3. Spezielle Felder

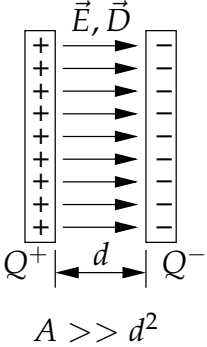
### 11.3.1. Räumliches Zentralfeld (Kugelanordnung)

|   |   |  |     |        |       |        |        |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
|---|---|--|-----|--------|-------|--------|--------|-------|-----|-----------|-------|-----|----------------|----------------------------|-----|----------|-------|-----|-----------|-------|----------|-----------------------|--------------------------------|-----|--------|---------|-------------|-----------|-------|------------|----------------|-----------------------------|--------------|---------------------|--|--------------|--------------------------|-----------------------------|
|  | $\sigma = \frac{Q}{4\pi R^2} = \text{const}$ <p>für <math>R_1 &lt; r &lt; R_2</math> gilt:</p> $E(r) = \frac{Q}{4\pi\epsilon r^2} = \frac{1}{\epsilon}D(r)$ $D(r) = \frac{Q}{4\pi r^2} = \frac{QR^2}{4\pi r^2} = \frac{\sigma R^2}{r^2}$ $V(r) = \frac{Q}{4\pi\epsilon} \left( \frac{1}{r} - \frac{1}{R_2} \right)$ <p>Hülle bei <math>r</math>:</p> $\psi_{el} = D(r)4\pi r^2 = Q$ <p>Kugelkondensator:</p> $U = \frac{Q}{4\pi\epsilon} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$ $C = \frac{Q}{U} = \frac{\psi_{el}}{U} = 4\pi\epsilon \frac{R_1 R_2}{R_2 - R_1}$ | <table> <tr> <td><math>Q</math></td><td>Ladung</td><td><math>[C]</math></td></tr> <tr> <td><math>R, r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr> <td><math>C</math></td><td>Kapazität</td><td><math>[F]</math></td></tr> <tr> <td><math>E</math></td><td>el. Feldstärke</td><td><math>\left[\frac{V}{m}\right]</math></td></tr> <tr> <td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr> <td><math>V</math></td><td>Potential</td><td><math>[V]</math></td></tr> <tr> <td><math>\sigma</math></td><td>Oberfl. Ladungsdichte</td><td><math>\left[\frac{Qmm^2}{m}\right]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr> <td><math>\psi_{el}</math></td><td>el. Fluss</td><td><math>[C]</math></td></tr> <tr> <td><math>\epsilon</math></td><td>Dielektrizität</td><td><math>\left[\frac{C}{Nm}\right]</math></td></tr> <tr> <td><math>\epsilon_r</math></td><td>rel. Dielektrizität</td><td></td></tr> <tr> <td><math>\epsilon_0</math></td><td>Dielektrizitätskonstante</td><td><math>\left[\frac{C}{Nm}\right]</math></td></tr> </table> | $Q$ | Ladung | $[C]$ | $R, r$ | Radius | $[m]$ | $C$ | Kapazität | $[F]$ | $E$ | el. Feldstärke | $\left[\frac{V}{m}\right]$ | $U$ | Spannung | $[V]$ | $V$ | Potential | $[V]$ | $\sigma$ | Oberfl. Ladungsdichte | $\left[\frac{Qmm^2}{m}\right]$ | $A$ | Fläche | $[m^2]$ | $\psi_{el}$ | el. Fluss | $[C]$ | $\epsilon$ | Dielektrizität | $\left[\frac{C}{Nm}\right]$ | $\epsilon_r$ | rel. Dielektrizität |  | $\epsilon_0$ | Dielektrizitätskonstante | $\left[\frac{C}{Nm}\right]$ |
| $Q$   | Ladung  | $[C]$  |     |        |       |        |        |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $R, r$  | Radius  | $[m]$  |     |        |       |        |        |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $C$   | Kapazität   | $[F]$  |     |        |       |        |        |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $E$   | el. Feldstärke  | $\left[\frac{V}{m}\right]$   |     |        |       |        |        |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $U$   | Spannung  | $[V]$  |     |        |       |        |        |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $V$   | Potential   | $[V]$  |     |        |       |        |        |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $\sigma$  | Oberfl. Ladungsdichte   | $\left[\frac{Qmm^2}{m}\right]$   |     |        |       |        |        |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $A$   | Fläche  | $[m^2]$  |     |        |       |        |        |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $\psi_{el}$   | el. Fluss   | $[C]$  |     |        |       |        |        |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $\epsilon$  | Dielektrizität  | $\left[\frac{C}{Nm}\right]$  |     |        |       |        |        |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $\epsilon_r$  | rel. Dielektrizität   |  |     |        |       |        |        |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $\epsilon_0$  | Dielektrizitätskonstante  | $\left[\frac{C}{Nm}\right]$  |     |        |       |        |        |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |

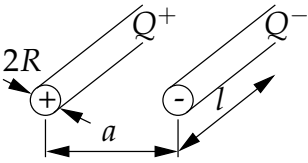
## 11.3.2. Zylindrisches Zentralfeld

|   |  |  |     |        |       |        |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
|---|--|--|-----|--------|-------|--------|--------|-------|-----|-----------|-------|-----|----------------|-----------------|-----|----------|-------|-----|-----------|-------|----------|-----------------------|-------------------|-----|--------|---------|-------------|-----------|-------|------------|----------------|------------------|--------------|---------------------|--|--------------|--------------------------|------------------|
|  | $\sigma = \frac{Q}{2\pi R_1 l} = \text{const}$ <p>für <math>R_1 &lt; r &lt; R_2</math> gilt:</p> $E(r) = \frac{Q}{2\pi\epsilon r l} = \frac{1}{\epsilon} D(r)$ $D(r) = \frac{Q}{2\pi r l} = \frac{\sigma R}{r}$ $V(r) = \frac{Q}{2\pi\epsilon l} \ln \frac{R_2}{r}$ <p>Hülle bei <math>r</math>:</p> $\psi_{el} = D(r) 2\pi r l = Q$ <p>Kondensator:</p> $U = \frac{Q}{2\pi\epsilon l} \ln \frac{R_1}{R_2}$ $C = \frac{Q}{U} = \frac{\psi_{el}}{U} = \frac{2\pi\epsilon l}{\ln \frac{R_1}{R_2}}$ | <table> <tr><td><math>Q</math></td><td>Ladung</td><td><math>[C]</math></td></tr> <tr><td><math>R, r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr><td><math>C</math></td><td>Kapazität</td><td><math>[F]</math></td></tr> <tr><td><math>E</math></td><td>el. Feldstärke</td><td><math>[\frac{V}{m}]</math></td></tr> <tr><td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr><td><math>V</math></td><td>Potential</td><td><math>[V]</math></td></tr> <tr><td><math>\sigma</math></td><td>Oberfl. Ladungsdichte</td><td><math>[\frac{C}{m^2}]</math></td></tr> <tr><td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr><td><math>\psi_{el}</math></td><td>el. Fluss</td><td><math>[C]</math></td></tr> <tr><td><math>\epsilon</math></td><td>Dielektrizität</td><td><math>[\frac{C}{Nm}]</math></td></tr> <tr><td><math>\epsilon_r</math></td><td>rel. Dielektrizität</td><td></td></tr> <tr><td><math>\epsilon_0</math></td><td>Dielektrizitätskonstante</td><td><math>[\frac{C}{Nm}]</math></td></tr> </table> | $Q$ | Ladung | $[C]$ | $R, r$ | Radius | $[m]$ | $C$ | Kapazität | $[F]$ | $E$ | el. Feldstärke | $[\frac{V}{m}]$ | $U$ | Spannung | $[V]$ | $V$ | Potential | $[V]$ | $\sigma$ | Oberfl. Ladungsdichte | $[\frac{C}{m^2}]$ | $A$ | Fläche | $[m^2]$ | $\psi_{el}$ | el. Fluss | $[C]$ | $\epsilon$ | Dielektrizität | $[\frac{C}{Nm}]$ | $\epsilon_r$ | rel. Dielektrizität |  | $\epsilon_0$ | Dielektrizitätskonstante | $[\frac{C}{Nm}]$ |
| $Q$   | Ladung   | $[C]$  |     |        |       |        |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $R, r$  | Radius   | $[m]$  |     |        |       |        |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $C$   | Kapazität  | $[F]$  |     |        |       |        |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $E$   | el. Feldstärke   | $[\frac{V}{m}]$  |     |        |       |        |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $U$   | Spannung   | $[V]$  |     |        |       |        |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $V$   | Potential  | $[V]$  |     |        |       |        |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\sigma$  | Oberfl. Ladungsdichte  | $[\frac{C}{m^2}]$  |     |        |       |        |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $A$   | Fläche   | $[m^2]$  |     |        |       |        |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\psi_{el}$   | el. Fluss  | $[C]$  |     |        |       |        |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\epsilon$  | Dielektrizität   | $[\frac{C}{Nm}]$   |     |        |       |        |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\epsilon_r$  | rel. Dielektrizität  |  |     |        |       |        |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\epsilon_0$  | Dielektrizitätskonstante   | $[\frac{C}{Nm}]$   |     |        |       |        |        |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |

## 11.3.3. Homogenes Feld (Plattenkondensator)

|   |   |  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
|---|---|--|-----|--------|-------|-----|---------|-------|-----|-----------|-------|-----|----------------|-----------------|-----|----------|-------|-----|-----------|-------|----------|-----------------------|-------------------|-----|--------|---------|-------------|-----------|-------|------------|----------------|------------------|--------------|---------------------|--|--------------|--------------------------|------------------|
|  <p><math>A \gg d^2</math></p> | $\sigma = D = \frac{Q}{A}$ $E = \frac{\sigma}{\epsilon} = \frac{Q}{A\epsilon}$ <p>Kondensator:</p> $U = \frac{\sigma}{\epsilon} d = \frac{Q}{A\epsilon} d$ $C = \frac{Q}{U} = \frac{\epsilon A}{d}$ | <table> <tr><td><math>Q</math></td><td>Ladung</td><td><math>[C]</math></td></tr> <tr><td><math>d</math></td><td>Abstand</td><td><math>[m]</math></td></tr> <tr><td><math>C</math></td><td>Kapazität</td><td><math>[F]</math></td></tr> <tr><td><math>E</math></td><td>el. Feldstärke</td><td><math>[\frac{V}{m}]</math></td></tr> <tr><td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr><td><math>V</math></td><td>Potential</td><td><math>[V]</math></td></tr> <tr><td><math>\sigma</math></td><td>Oberfl. Ladungsdichte</td><td><math>[\frac{C}{m^2}]</math></td></tr> <tr><td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr><td><math>\psi_{el}</math></td><td>el. Fluss</td><td><math>[C]</math></td></tr> <tr><td><math>\epsilon</math></td><td>Dielektrizität</td><td><math>[\frac{C}{Nm}]</math></td></tr> <tr><td><math>\epsilon_r</math></td><td>rel. Dielektrizität</td><td></td></tr> <tr><td><math>\epsilon_0</math></td><td>Dielektrizitätskonstante</td><td><math>[\frac{C}{Nm}]</math></td></tr> </table> | $Q$ | Ladung | $[C]$ | $d$ | Abstand | $[m]$ | $C$ | Kapazität | $[F]$ | $E$ | el. Feldstärke | $[\frac{V}{m}]$ | $U$ | Spannung | $[V]$ | $V$ | Potential | $[V]$ | $\sigma$ | Oberfl. Ladungsdichte | $[\frac{C}{m^2}]$ | $A$ | Fläche | $[m^2]$ | $\psi_{el}$ | el. Fluss | $[C]$ | $\epsilon$ | Dielektrizität | $[\frac{C}{Nm}]$ | $\epsilon_r$ | rel. Dielektrizität |  | $\epsilon_0$ | Dielektrizitätskonstante | $[\frac{C}{Nm}]$ |
| $Q$   | Ladung  | $[C]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $d$   | Abstand   | $[m]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $C$   | Kapazität   | $[F]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $E$   | el. Feldstärke  | $[\frac{V}{m}]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $U$   | Spannung  | $[V]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $V$   | Potential   | $[V]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\sigma$  | Oberfl. Ladungsdichte   | $[\frac{C}{m^2}]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $A$   | Fläche  | $[m^2]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\psi_{el}$   | el. Fluss   | $[C]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\epsilon$  | Dielektrizität  | $[\frac{C}{Nm}]$   |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\epsilon_r$  | rel. Dielektrizität   |  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |
| $\epsilon_0$  | Dielektrizitätskonstante  | $[\frac{C}{Nm}]$   |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                       |                   |     |        |         |             |           |       |            |                |                  |              |                     |  |              |                          |                  |

### 11.3.4. Paralleldrahtleitung

|   |  |  |     |        |       |     |         |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
|---|--|--|-----|--------|-------|-----|---------|-------|-----|-----------|-------|-----|----------------|----------------------------|-----|----------|-------|-----|-----------|-------|----------|-----------------------|--------------------------------|-----|--------|---------|-------------|-----------|-------|------------|----------------|-----------------------------|--------------|---------------------|--|--------------|--------------------------|-----------------------------|
|  | <p>Es gilt das Superpositionsprinzip:</p> $E_{tot} = E_{Leiter_1} + E_{Leiter_2}$ <p>Kondensator:</p> $C = \frac{Q}{U} = \frac{\pi \epsilon l}{\ln \frac{a-R}{R}}$ $C' = \frac{Q}{U} = \frac{\pi \epsilon}{\ln \frac{a-R}{R}}$ | <table> <tr> <td><math>Q</math></td><td>Ladung</td><td><math>[C]</math></td></tr> <tr> <td><math>d</math></td><td>Abstand</td><td><math>[m]</math></td></tr> <tr> <td><math>C</math></td><td>Kapazität</td><td><math>[F]</math></td></tr> <tr> <td><math>E</math></td><td>el. Feldstärke</td><td><math>\left[\frac{V}{m}\right]</math></td></tr> <tr> <td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr> <td><math>V</math></td><td>Potential</td><td><math>[V]</math></td></tr> <tr> <td><math>\sigma</math></td><td>Oberfl. Ladungsdichte</td><td><math>\left[\frac{Cmm^2}{m}\right]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr> <td><math>\psi_{el}</math></td><td>el. Fluss</td><td><math>[C]</math></td></tr> <tr> <td><math>\epsilon</math></td><td>Dielektrizität</td><td><math>\left[\frac{C}{Nm}\right]</math></td></tr> <tr> <td><math>\epsilon_r</math></td><td>rel. Dielektrizität</td><td></td></tr> <tr> <td><math>\epsilon_0</math></td><td>Dielektrizitätskonstante</td><td><math>\left[\frac{C}{Nm}\right]</math></td></tr> </table> | $Q$ | Ladung | $[C]$ | $d$ | Abstand | $[m]$ | $C$ | Kapazität | $[F]$ | $E$ | el. Feldstärke | $\left[\frac{V}{m}\right]$ | $U$ | Spannung | $[V]$ | $V$ | Potential | $[V]$ | $\sigma$ | Oberfl. Ladungsdichte | $\left[\frac{Cmm^2}{m}\right]$ | $A$ | Fläche | $[m^2]$ | $\psi_{el}$ | el. Fluss | $[C]$ | $\epsilon$ | Dielektrizität | $\left[\frac{C}{Nm}\right]$ | $\epsilon_r$ | rel. Dielektrizität |  | $\epsilon_0$ | Dielektrizitätskonstante | $\left[\frac{C}{Nm}\right]$ |
| $Q$   | Ladung   | $[C]$  |     |        |       |     |         |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $d$   | Abstand  | $[m]$  |     |        |       |     |         |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $C$   | Kapazität  | $[F]$  |     |        |       |     |         |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $E$   | el. Feldstärke   | $\left[\frac{V}{m}\right]$   |     |        |       |     |         |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $U$   | Spannung   | $[V]$  |     |        |       |     |         |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $V$   | Potential  | $[V]$  |     |        |       |     |         |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $\sigma$  | Oberfl. Ladungsdichte  | $\left[\frac{Cmm^2}{m}\right]$   |     |        |       |     |         |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $A$   | Fläche   | $[m^2]$  |     |        |       |     |         |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $\psi_{el}$   | el. Fluss  | $[C]$  |     |        |       |     |         |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $\epsilon$  | Dielektrizität   | $\left[\frac{C}{Nm}\right]$  |     |        |       |     |         |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $\epsilon_r$  | rel. Dielektrizität  |  |     |        |       |     |         |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |
| $\epsilon_0$  | Dielektrizitätskonstante   | $\left[\frac{C}{Nm}\right]$  |     |        |       |     |         |       |     |           |       |     |                |                            |     |          |       |     |           |       |          |                       |                                |     |        |         |             |           |       |            |                |                             |              |                     |  |              |                          |                             |

## 11.4. Energie im elektrischen Feld

|     |  |  |     |         |       |     |               |           |     |          |       |     |           |       |     |         |         |
|-----|--|--|-----|---------|-------|-----|---------------|-----------|-----|----------|-------|-----|-----------|-------|-----|---------|---------|
|     | $W = \frac{CU^2}{2}$ $w = \frac{W}{V}$ <p>Allgemein:</p> $w(x, y, z) = \frac{dW(x, y, z)}{dV}$ | <table> <tr> <td><math>W</math></td><td>Energie</td><td><math>[J]</math></td></tr> <tr> <td><math>w</math></td><td>Energiedichte</td><td><math>[J/m^3]</math></td></tr> <tr> <td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr> <td><math>C</math></td><td>Kapazität</td><td><math>[F]</math></td></tr> <tr> <td><math>V</math></td><td>Volumen</td><td><math>[m^3]</math></td></tr> </table> | $W$ | Energie | $[J]$ | $w$ | Energiedichte | $[J/m^3]$ | $U$ | Spannung | $[V]$ | $C$ | Kapazität | $[F]$ | $V$ | Volumen | $[m^3]$ |
| $W$ | Energie  | $[J]$  |     |         |       |     |               |           |     |          |       |     |           |       |     |         |         |
| $w$ | Energiedichte  | $[J/m^3]$  |     |         |       |     |               |           |     |          |       |     |           |       |     |         |         |
| $U$ | Spannung   | $[V]$  |     |         |       |     |               |           |     |          |       |     |           |       |     |         |         |
| $C$ | Kapazität  | $[F]$  |     |         |       |     |               |           |     |          |       |     |           |       |     |         |         |
| $V$ | Volumen  | $[m^3]$  |     |         |       |     |               |           |     |          |       |     |           |       |     |         |         |

## 11.5. Kräfte im elektrischen Feld

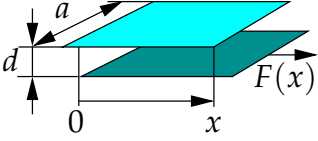
### 11.5.1. Allgemein

$$\Delta W = F \Delta x \Leftrightarrow F = \frac{\Delta W}{\Delta x} \Leftrightarrow F(x) = \frac{dW(x)}{dx}$$

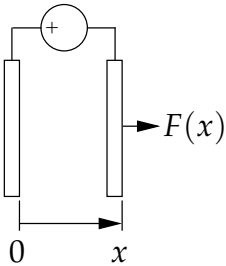
#### Prinzip der virtuellen Verschiebung

Man denkt sich den Leiter, auf den die Kraft berechnet werden soll, um  $\Delta x$  in diejenige Richtung verschoben, in welche die Kraft berechnet werden soll:  $\rightarrow$  Energiedifferenz  $\Delta W$

### 11.5.2. Verschiebung

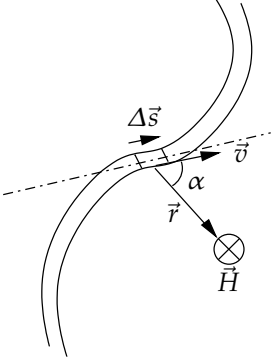
|   |  |  |     |        |       |     |         |       |     |             |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
|---|--|--|-----|--------|-------|-----|---------|-------|-----|-------------|-------|-----|-----------|-------|-----|----------------|-----------------|-----|----------|-------|-----|-----------|-------|----------|----------------------------|-------------------|-----|--------|---------|-------------|-----------|-------|------------|----------------|------------------|--------------|--------------------------|--|--------------|-------------------------------|------------------|
|  | <p>Mit eingeschalteter Quelle:</p> $W(x) = \frac{CU^2}{2} = \frac{\epsilon AU^2}{2d} = \frac{\epsilon axU^2}{2d}$ $F(x) = \frac{dW(x)}{dx} = \frac{\epsilon aU^2}{2d}$ <p>Mit ausgeschalteter Quelle:</p> $W(x) = \frac{\epsilon AU^2}{2d} = \frac{\epsilon ax_0^2 U^2}{2xd}$ $F(x) = \frac{dW(x)}{dx} = \frac{\epsilon ax_0^2 U^2}{2x^2 d}$ | <table> <tr><td><math>Q</math></td><td>Ladung</td><td><math>[C]</math></td></tr> <tr><td><math>d</math></td><td>Abstand</td><td><math>[m]</math></td></tr> <tr><td><math>x</math></td><td>Überlappung</td><td><math>[m]</math></td></tr> <tr><td><math>C</math></td><td>Kapazität</td><td><math>[F]</math></td></tr> <tr><td><math>E</math></td><td>el. Feldstärke</td><td><math>[\frac{V}{m}]</math></td></tr> <tr><td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr><td><math>V</math></td><td>Potential</td><td><math>[V]</math></td></tr> <tr><td><math>\sigma</math></td><td>Oberfl. La-<br/>dungsdichte</td><td><math>[\frac{C}{m^2}]</math></td></tr> <tr><td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr><td><math>\psi_{el}</math></td><td>el. Fluss</td><td><math>[C]</math></td></tr> <tr><td><math>\epsilon</math></td><td>Dielektrizität</td><td><math>[\frac{C}{Nm}]</math></td></tr> <tr><td><math>\epsilon_r</math></td><td>rel. Dielektri-<br/>zität</td><td></td></tr> <tr><td><math>\epsilon_0</math></td><td>Dielektrizitäts-<br/>konstante</td><td><math>[\frac{C}{Nm}]</math></td></tr> </table> | $Q$ | Ladung | $[C]$ | $d$ | Abstand | $[m]$ | $x$ | Überlappung | $[m]$ | $C$ | Kapazität | $[F]$ | $E$ | el. Feldstärke | $[\frac{V}{m}]$ | $U$ | Spannung | $[V]$ | $V$ | Potential | $[V]$ | $\sigma$ | Oberfl. La-<br>dungsdichte | $[\frac{C}{m^2}]$ | $A$ | Fläche | $[m^2]$ | $\psi_{el}$ | el. Fluss | $[C]$ | $\epsilon$ | Dielektrizität | $[\frac{C}{Nm}]$ | $\epsilon_r$ | rel. Dielektri-<br>zität |  | $\epsilon_0$ | Dielektrizitäts-<br>konstante | $[\frac{C}{Nm}]$ |
| $Q$   | Ladung   | $[C]$  |     |        |       |     |         |       |     |             |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $d$   | Abstand  | $[m]$  |     |        |       |     |         |       |     |             |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $x$   | Überlappung  | $[m]$  |     |        |       |     |         |       |     |             |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $C$   | Kapazität  | $[F]$  |     |        |       |     |         |       |     |             |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $E$   | el. Feldstärke   | $[\frac{V}{m}]$  |     |        |       |     |         |       |     |             |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $U$   | Spannung   | $[V]$  |     |        |       |     |         |       |     |             |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $V$   | Potential  | $[V]$  |     |        |       |     |         |       |     |             |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $\sigma$  | Oberfl. La-<br>dungsdichte   | $[\frac{C}{m^2}]$  |     |        |       |     |         |       |     |             |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $A$   | Fläche   | $[m^2]$  |     |        |       |     |         |       |     |             |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $\psi_{el}$   | el. Fluss  | $[C]$  |     |        |       |     |         |       |     |             |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $\epsilon$  | Dielektrizität   | $[\frac{C}{Nm}]$   |     |        |       |     |         |       |     |             |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $\epsilon_r$  | rel. Dielektri-<br>zität   |  |     |        |       |     |         |       |     |             |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $\epsilon_0$  | Dielektrizitäts-<br>konstante  | $[\frac{C}{Nm}]$   |     |        |       |     |         |       |     |             |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |

### 11.5.3. Anziehung

|   |   |  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
|---|---|--|-----|--------|-------|-----|---------|-------|-----|-----------|-------|-----|----------------|-----------------|-----|----------|-------|-----|-----------|-------|----------|----------------------------|-------------------|-----|--------|---------|-------------|-----------|-------|------------|----------------|------------------|--------------|--------------------------|--|--------------|-------------------------------|------------------|
|  | $W(x) = \frac{CU^2}{2} = \frac{\epsilon AU^2}{2x}$ $F(x) = \frac{dW(x)}{dx} = \frac{\epsilon AU^2}{2x^2}$ | <table> <tr><td><math>Q</math></td><td>Ladung</td><td><math>[C]</math></td></tr> <tr><td><math>x</math></td><td>Abstand</td><td><math>[m]</math></td></tr> <tr><td><math>C</math></td><td>Kapazität</td><td><math>[F]</math></td></tr> <tr><td><math>E</math></td><td>el. Feldstärke</td><td><math>[\frac{V}{m}]</math></td></tr> <tr><td><math>U</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr><td><math>V</math></td><td>Potential</td><td><math>[V]</math></td></tr> <tr><td><math>\sigma</math></td><td>Oberfl. La-<br/>dungsdichte</td><td><math>[\frac{C}{m^2}]</math></td></tr> <tr><td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr><td><math>\psi_{el}</math></td><td>el. Fluss</td><td><math>[C]</math></td></tr> <tr><td><math>\epsilon</math></td><td>Dielektrizität</td><td><math>[\frac{C}{Nm}]</math></td></tr> <tr><td><math>\epsilon_r</math></td><td>rel. Dielektri-<br/>zität</td><td></td></tr> <tr><td><math>\epsilon_0</math></td><td>Dielektrizitäts-<br/>konstante</td><td><math>[\frac{C}{Nm}]</math></td></tr> </table> | $Q$ | Ladung | $[C]$ | $x$ | Abstand | $[m]$ | $C$ | Kapazität | $[F]$ | $E$ | el. Feldstärke | $[\frac{V}{m}]$ | $U$ | Spannung | $[V]$ | $V$ | Potential | $[V]$ | $\sigma$ | Oberfl. La-<br>dungsdichte | $[\frac{C}{m^2}]$ | $A$ | Fläche | $[m^2]$ | $\psi_{el}$ | el. Fluss | $[C]$ | $\epsilon$ | Dielektrizität | $[\frac{C}{Nm}]$ | $\epsilon_r$ | rel. Dielektri-<br>zität |  | $\epsilon_0$ | Dielektrizitäts-<br>konstante | $[\frac{C}{Nm}]$ |
| $Q$   | Ladung  | $[C]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $x$   | Abstand   | $[m]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $C$   | Kapazität   | $[F]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $E$   | el. Feldstärke  | $[\frac{V}{m}]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $U$   | Spannung  | $[V]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $V$   | Potential   | $[V]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $\sigma$  | Oberfl. La-<br>dungsdichte  | $[\frac{C}{m^2}]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $A$   | Fläche  | $[m^2]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $\psi_{el}$   | el. Fluss   | $[C]$  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $\epsilon$  | Dielektrizität  | $[\frac{C}{Nm}]$   |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $\epsilon_r$  | rel. Dielektri-<br>zität  |  |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |
| $\epsilon_0$  | Dielektrizitäts-<br>konstante   | $[\frac{C}{Nm}]$   |     |        |       |     |         |       |     |           |       |     |                |                 |     |          |       |     |           |       |          |                            |                   |     |        |         |             |           |       |            |                |                  |              |                          |  |              |                               |                  |

# 12. Magnetismus

## 12.1. Feldstärke

|  |   |   |     |            |                            |     |       |       |      |                             |       |     |        |       |     |                 |                            |          |        |         |
|--|---|---|-----|------------|----------------------------|-----|-------|-------|------|-----------------------------|-------|-----|--------|-------|-----|-----------------|----------------------------|----------|--------|---------|
|  <p>siehe spezielle Anordnungen ab S. 98.</p> | <p>Biot-Savart</p> $\vec{H} = \frac{Q}{4\pi r^3} (\vec{v} \times \vec{r})$ $H = \frac{Qv}{4\pi r^2} \sin \alpha$ <p>Leiterbezogen</p> $d\vec{H} = \frac{I}{4\pi r^3} (d\vec{s} \times \vec{r})$ $dH = \frac{I ds}{4\pi r^2} \sin \alpha$ $H = \int d\vec{H} = \int \frac{I}{4\pi r^2} \sin \alpha ds$ | <table> <tr> <td><math>H</math></td><td>Feldstärke</td><td><math>\left[\frac{A}{m}\right]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>ds</math></td><td>infinitesimales Leiterstück</td><td><math>[m]</math></td></tr> <tr> <td><math>r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr> <td><math>v</math></td><td>Geschwindigkeit</td><td><math>\left[\frac{m}{s}\right]</math></td></tr> <tr> <td><math>\alpha</math></td><td>Winkel</td><td><math>[rad]</math></td></tr> </table> | $H$ | Feldstärke | $\left[\frac{A}{m}\right]$ | $I$ | Strom | $[A]$ | $ds$ | infinitesimales Leiterstück | $[m]$ | $r$ | Radius | $[m]$ | $v$ | Geschwindigkeit | $\left[\frac{m}{s}\right]$ | $\alpha$ | Winkel | $[rad]$ |
| $H$  | Feldstärke  | $\left[\frac{A}{m}\right]$  |     |            |                            |     |       |       |      |                             |       |     |        |       |     |                 |                            |          |        |         |
| $I$  | Strom   | $[A]$   |     |            |                            |     |       |       |      |                             |       |     |        |       |     |                 |                            |          |        |         |
| $ds$   | infinitesimales Leiterstück   | $[m]$   |     |            |                            |     |       |       |      |                             |       |     |        |       |     |                 |                            |          |        |         |
| $r$  | Radius  | $[m]$   |     |            |                            |     |       |       |      |                             |       |     |        |       |     |                 |                            |          |        |         |
| $v$  | Geschwindigkeit   | $\left[\frac{m}{s}\right]$  |     |            |                            |     |       |       |      |                             |       |     |        |       |     |                 |                            |          |        |         |
| $\alpha$   | Winkel  | $[rad]$   |     |            |                            |     |       |       |      |                             |       |     |        |       |     |                 |                            |          |        |         |

## 12.2. Permeabilität

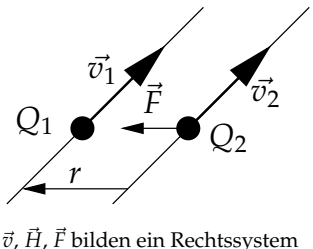
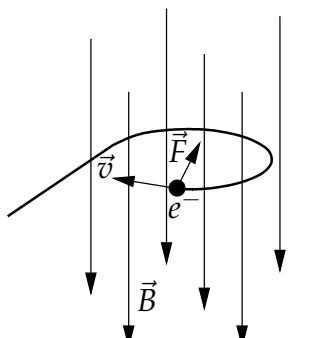
|         |   |  |       |               |                            |         |                    |       |         |                           |                            |
|---------|---|--|-------|---------------|----------------------------|---------|--------------------|-------|---------|---------------------------|----------------------------|
|         | $\mu = \mu_0 \mu_r$ $\mu_0 = \frac{4\pi}{10} \cdot 10^{-6} \frac{Vs}{Am} = 1.257 \cdot 10^{-6} \frac{H}{m}$ | <table> <tr> <td><math>\mu</math></td><td>Permeabilität</td><td><math>\left[\frac{H}{m}\right]</math></td></tr> <tr> <td><math>\mu_r</math></td><td>Permeabilitätszahl</td><td><math>[1]</math></td></tr> <tr> <td><math>\mu_0</math></td><td>Permeabilität des Vakuums</td><td><math>\left[\frac{H}{m}\right]</math></td></tr> </table> | $\mu$ | Permeabilität | $\left[\frac{H}{m}\right]$ | $\mu_r$ | Permeabilitätszahl | $[1]$ | $\mu_0$ | Permeabilität des Vakuums | $\left[\frac{H}{m}\right]$ |
| $\mu$   | Permeabilität   | $\left[\frac{H}{m}\right]$   |       |               |                            |         |                    |       |         |                           |                            |
| $\mu_r$ | Permeabilitätszahl  | $[1]$  |       |               |                            |         |                    |       |         |                           |                            |
| $\mu_0$ | Permeabilität des Vakuums   | $\left[\frac{H}{m}\right]$   |       |               |                            |         |                    |       |         |                           |                            |

## 12.3. Magnetische Flussdichte

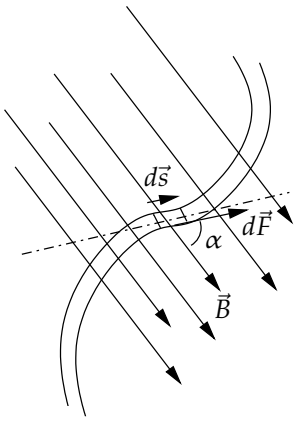
|                                       |                         |   |
|---------------------------------------|-------------------------|---|
| siehe spezielle Anordnungen ab S. 98. | $\vec{B} = \mu \vec{H}$ | $H$ Feldstärke $\left[\frac{A}{m}\right]$<br>$\mu$ Permeabilität $\left[\frac{H}{m}\right]$<br>$B$ Flussdichte, Induktion $\left[\frac{T}{m}\right], \left[\frac{Vs}{m^2}\right]$ |
|---------------------------------------|-------------------------|---|

## 12.4. Kräfte im Magnetischen Feld

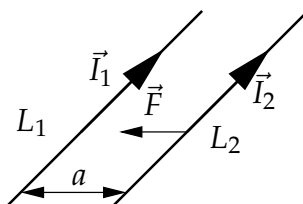
### 12.4.1. Kräfte auf Ladungen

|  |   |   |
|--|---|---|
|  <p><math>\vec{v}, \vec{H}, \vec{F}</math> bilden ein Rechtssystem</p> | <p>Für parallele Bahnen</p> $F_A = \frac{\mu_0 Q_1 Q_2 v_1 v_2}{4\pi r^2}$ <p>Allgemein</p> $F = Q_2 \vec{v}_2 \times \left( \frac{\mu}{4\pi} \frac{Q_1 \vec{v}_1 \times \frac{\vec{r}}{r}}{r^2} \right)$ | $H$ Feldstärke $\left[\frac{A}{m}\right]$<br>$F$ Kraft auf Ladung $Q_1$ $[N]$<br>$Q_{1,2}$ Ladung $[C]$<br>$v_{1,2}$ Geschwindigkeit $\left[\frac{m}{s}\right]$<br>$r$ Radius $[m]$<br>$\mu$ Permeabilität $\left[\frac{H}{m}\right]$<br>$B$ Flussdichte, Induktion $\left[\frac{T}{m}\right], \left[\frac{Vs}{m^2}\right]$ |
|   | $F = Q(\vec{v} \times \vec{B})$ $F = Q(\vec{v} \times \mu \vec{H})$ $F = Qv\mu H \sin \alpha$   |   |

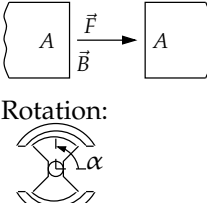
## 12.4.2. Kraft auf Leiter im B-Feld

|   |   |   |     |                        |                         |     |                  |       |     |        |       |     |       |       |          |        |         |      |                             |       |     |       |       |
|---|---|---|-----|------------------------|-------------------------|-----|------------------|-------|-----|--------|-------|-----|-------|-------|----------|--------|---------|------|-----------------------------|-------|-----|-------|-------|
|  | $d\vec{F} = \frac{dQ}{dt}(\vec{ds} \times \vec{B}) = I(d\vec{s} \times \vec{B})$ $dF = IBds \sin \alpha$ <p>für geraden Leiter:</p> $F = IBl \sin \alpha$ | <table> <tr> <td><math>B</math></td><td>Flussdichte, Induktion</td><td><math>[T], [\frac{Vs}{m^2}]</math></td></tr> <tr> <td><math>F</math></td><td>Kraft auf Leiter</td><td><math>[N]</math></td></tr> <tr> <td><math>Q</math></td><td>Ladung</td><td><math>[C]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>\alpha</math></td><td>Winkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>ds</math></td><td>infinitesimales Leiterstück</td><td><math>[m]</math></td></tr> <tr> <td><math>l</math></td><td>Länge</td><td><math>[m]</math></td></tr> </table> | $B$ | Flussdichte, Induktion | $[T], [\frac{Vs}{m^2}]$ | $F$ | Kraft auf Leiter | $[N]$ | $Q$ | Ladung | $[C]$ | $I$ | Strom | $[A]$ | $\alpha$ | Winkel | $[rad]$ | $ds$ | infinitesimales Leiterstück | $[m]$ | $l$ | Länge | $[m]$ |
| $B$   | Flussdichte, Induktion  | $[T], [\frac{Vs}{m^2}]$   |     |                        |                         |     |                  |       |     |        |       |     |       |       |          |        |         |      |                             |       |     |       |       |
| $F$   | Kraft auf Leiter  | $[N]$   |     |                        |                         |     |                  |       |     |        |       |     |       |       |          |        |         |      |                             |       |     |       |       |
| $Q$   | Ladung  | $[C]$   |     |                        |                         |     |                  |       |     |        |       |     |       |       |          |        |         |      |                             |       |     |       |       |
| $I$   | Strom   | $[A]$   |     |                        |                         |     |                  |       |     |        |       |     |       |       |          |        |         |      |                             |       |     |       |       |
| $\alpha$  | Winkel  | $[rad]$   |     |                        |                         |     |                  |       |     |        |       |     |       |       |          |        |         |      |                             |       |     |       |       |
| $ds$  | infinitesimales Leiterstück   | $[m]$   |     |                        |                         |     |                  |       |     |        |       |     |       |       |          |        |         |      |                             |       |     |       |       |
| $l$   | Länge   | $[m]$   |     |                        |                         |     |                  |       |     |        |       |     |       |       |          |        |         |      |                             |       |     |       |       |

## 12.4.3. Kräfte auf parallele Leiter

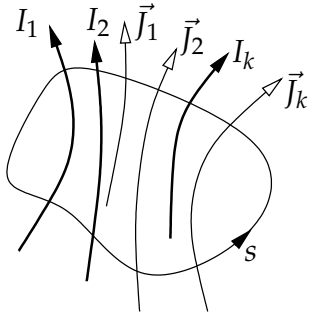
|  |   |   |       |                            |       |           |        |       |           |       |       |     |         |       |       |               |                 |
|--|---|---|-------|----------------------------|-------|-----------|--------|-------|-----------|-------|-------|-----|---------|-------|-------|---------------|-----------------|
|  | $F = \frac{\mu I_1 I_2}{2\pi a}$ $\vec{I}_1 \uparrow \downarrow \vec{I}_2 \Rightarrow \text{Abstossung}$ $\vec{I}_1 \uparrow \uparrow \vec{I}_2 \Rightarrow \text{Anziehung}$ | <table> <tr> <td><math>F_A</math></td><td>Kraft zwischen den Leitern</td><td><math>[N]</math></td></tr> <tr> <td><math>L_{1,2}</math></td><td>Leiter</td><td><math>[C]</math></td></tr> <tr> <td><math>I_{1,2}</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>a</math></td><td>Abstand</td><td><math>[m]</math></td></tr> <tr> <td><math>\mu</math></td><td>Permeabilität</td><td><math>[\frac{H}{m}]</math></td></tr> </table> | $F_A$ | Kraft zwischen den Leitern | $[N]$ | $L_{1,2}$ | Leiter | $[C]$ | $I_{1,2}$ | Strom | $[A]$ | $a$ | Abstand | $[m]$ | $\mu$ | Permeabilität | $[\frac{H}{m}]$ |
| $F_A$  | Kraft zwischen den Leitern  | $[N]$   |       |                            |       |           |        |       |           |       |       |     |         |       |       |               |                 |
| $L_{1,2}$  | Leiter  | $[C]$   |       |                            |       |           |        |       |           |       |       |     |         |       |       |               |                 |
| $I_{1,2}$  | Strom   | $[A]$   |       |                            |       |           |        |       |           |       |       |     |         |       |       |               |                 |
| $a$  | Abstand   | $[m]$   |       |                            |       |           |        |       |           |       |       |     |         |       |       |               |                 |
| $\mu$  | Permeabilität   | $[\frac{H}{m}]$   |       |                            |       |           |        |       |           |       |       |     |         |       |       |               |                 |

## 12.4.4. Kräfte auf Randflächen eines Feldes

|  |   |   |     |       |       |     |     |       |     |         |       |     |                        |                         |     |            |                 |     |        |         |           |            |        |          |        |         |
|--|---|---|-----|-------|-------|-----|-----|-------|-----|---------|-------|-----|------------------------|-------------------------|-----|------------|-----------------|-----|--------|---------|-----------|------------|--------|----------|--------|---------|
| <p>Energie <math>W</math> siehe S. 94</p>  <p>Rotation:</p> | $F = \frac{dW(s)}{ds}$ <p>Prinzip der virtuellen Verschiebung: Fläche um <math>ds</math> verschoben (s-Richtung = Kraftrichtung)</p> $F = \frac{1}{2}BHA$ <p>Bei Drehbewegung:</p> $M_{rot} = \frac{dW(\alpha)}{d\alpha}$ | <table> <tr> <td><math>F</math></td><td>Kraft</td><td><math>[N]</math></td></tr> <tr> <td><math>s</math></td><td>Weg</td><td><math>[m]</math></td></tr> <tr> <td><math>W</math></td><td>Energie</td><td><math>[J]</math></td></tr> <tr> <td><math>B</math></td><td>Flussdichte, Induktion</td><td><math>[T], [\frac{Vs}{m^2}]</math></td></tr> <tr> <td><math>H</math></td><td>Feldstärke</td><td><math>[\frac{A}{m}]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr> <td><math>M_{rot}</math></td><td>Drehmoment</td><td><math>[Nm]</math></td></tr> <tr> <td><math>\alpha</math></td><td>Winkel</td><td><math>[rad]</math></td></tr> </table> | $F$ | Kraft | $[N]$ | $s$ | Weg | $[m]$ | $W$ | Energie | $[J]$ | $B$ | Flussdichte, Induktion | $[T], [\frac{Vs}{m^2}]$ | $H$ | Feldstärke | $[\frac{A}{m}]$ | $A$ | Fläche | $[m^2]$ | $M_{rot}$ | Drehmoment | $[Nm]$ | $\alpha$ | Winkel | $[rad]$ |
| $F$  | Kraft   | $[N]$   |     |       |       |     |     |       |     |         |       |     |                        |                         |     |            |                 |     |        |         |           |            |        |          |        |         |
| $s$  | Weg   | $[m]$   |     |       |       |     |     |       |     |         |       |     |                        |                         |     |            |                 |     |        |         |           |            |        |          |        |         |
| $W$  | Energie   | $[J]$   |     |       |       |     |     |       |     |         |       |     |                        |                         |     |            |                 |     |        |         |           |            |        |          |        |         |
| $B$  | Flussdichte, Induktion  | $[T], [\frac{Vs}{m^2}]$   |     |       |       |     |     |       |     |         |       |     |                        |                         |     |            |                 |     |        |         |           |            |        |          |        |         |
| $H$  | Feldstärke  | $[\frac{A}{m}]$   |     |       |       |     |     |       |     |         |       |     |                        |                         |     |            |                 |     |        |         |           |            |        |          |        |         |
| $A$  | Fläche  | $[m^2]$   |     |       |       |     |     |       |     |         |       |     |                        |                         |     |            |                 |     |        |         |           |            |        |          |        |         |
| $M_{rot}$  | Drehmoment  | $[Nm]$  |     |       |       |     |     |       |     |         |       |     |                        |                         |     |            |                 |     |        |         |           |            |        |          |        |         |
| $\alpha$   | Winkel  | $[rad]$   |     |       |       |     |     |       |     |         |       |     |                        |                         |     |            |                 |     |        |         |           |            |        |          |        |         |



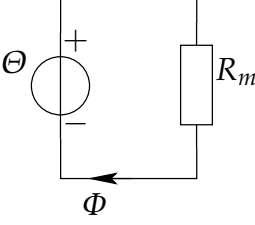
## 12.5. Durchflutung

|   |  |  |          |              |       |     |             |                   |     |       |       |     |                    |       |     |            |                 |          |                      |       |     |       |       |
|---|--|--|----------|--------------|-------|-----|-------------|-------------------|-----|-------|-------|-----|--------------------|-------|-----|------------|-----------------|----------|----------------------|-------|-----|-------|-------|
|  | $\Theta = \oint_s \vec{H} ds = I$ $\Theta = \sum_{k=1}^n I_k + \iint_{A_s} \vec{J} d\vec{A}$ | <table><tr><td><math>\Theta</math></td><td>Durchflutung</td><td><math>[A]</math></td></tr><tr><td><math>J</math></td><td>Stromdichte</td><td><math>[\frac{A}{m^2}]</math></td></tr><tr><td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr><tr><td><math>s</math></td><td>Geschlossene Kurve</td><td><math>[m]</math></td></tr><tr><td><math>H</math></td><td>Feldstärke</td><td><math>[\frac{A}{m}]</math></td></tr><tr><td><math>U_{mg}</math></td><td>Magnetische Spannung</td><td><math>[A]</math></td></tr><tr><td><math>l</math></td><td>Länge</td><td><math>[m]</math></td></tr></table> | $\Theta$ | Durchflutung | $[A]$ | $J$ | Stromdichte | $[\frac{A}{m^2}]$ | $I$ | Strom | $[A]$ | $s$ | Geschlossene Kurve | $[m]$ | $H$ | Feldstärke | $[\frac{A}{m}]$ | $U_{mg}$ | Magnetische Spannung | $[A]$ | $l$ | Länge | $[m]$ |
| $\Theta$  | Durchflutung   | $[A]$  |          |              |       |     |             |                   |     |       |       |     |                    |       |     |            |                 |          |                      |       |     |       |       |
| $J$   | Stromdichte  | $[\frac{A}{m^2}]$  |          |              |       |     |             |                   |     |       |       |     |                    |       |     |            |                 |          |                      |       |     |       |       |
| $I$   | Strom  | $[A]$  |          |              |       |     |             |                   |     |       |       |     |                    |       |     |            |                 |          |                      |       |     |       |       |
| $s$   | Geschlossene Kurve   | $[m]$  |          |              |       |     |             |                   |     |       |       |     |                    |       |     |            |                 |          |                      |       |     |       |       |
| $H$   | Feldstärke   | $[\frac{A}{m}]$  |          |              |       |     |             |                   |     |       |       |     |                    |       |     |            |                 |          |                      |       |     |       |       |
| $U_{mg}$  | Magnetische Spannung   | $[A]$  |          |              |       |     |             |                   |     |       |       |     |                    |       |     |            |                 |          |                      |       |     |       |       |
| $l$   | Länge  | $[m]$  |          |              |       |     |             |                   |     |       |       |     |                    |       |     |            |                 |          |                      |       |     |       |       |
| <b>Nicht geschlossener Weg <math>A \rightarrow B</math></b>                       | $U_{mgAB} = \int_A^B \vec{H} d\vec{s}$ <p>z.B Luftspalt: <math>U_{mgAB} = Hl</math></p>      |  |          |              |       |     |             |                   |     |       |       |     |                    |       |     |            |                 |          |                      |       |     |       |       |
| <b>Feld um Leiter</b>   | $\Theta = \oint_s \vec{H} ds = I$  |  |          |              |       |     |             |                   |     |       |       |     |                    |       |     |            |                 |          |                      |       |     |       |       |
| <b>Spule</b>  | $\Theta = NI$  |  |          |              |       |     |             |                   |     |       |       |     |                    |       |     |            |                 |          |                      |       |     |       |       |

## 12.6. Magnetischer Fluss

|                                       |   |   |        |                    |              |     |        |         |     |                        |                         |          |              |       |           |                       |                              |
|---------------------------------------|---|---|--------|--------------------|--------------|-----|--------|---------|-----|------------------------|-------------------------|----------|--------------|-------|-----------|-----------------------|------------------------------|
| siehe spezielle Anordnungen ab S. 98. | $\Phi = \iint_A \vec{B} dA$ $\Phi = \Lambda \Theta = \frac{\Theta}{R_m}$ <p>Homogenes Feld:</p> $\Phi = BA$ | <table> <tr> <td><math>\Phi</math></td><td>Magnetischer Fluss</td><td><math>[Vs], [Wb]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> <tr> <td><math>B</math></td><td>Flussdichte, Induktion</td><td><math>[T], [\frac{Vs}{m^2}]</math></td></tr> <tr> <td><math>\Theta</math></td><td>Durchflutung</td><td><math>[A]</math></td></tr> <tr> <td><math>\Lambda</math></td><td>Magnetischer Leitwert</td><td><math>[\frac{Vs}{A}], [\Omega s]</math></td></tr> </table> | $\Phi$ | Magnetischer Fluss | $[Vs], [Wb]$ | $A$ | Fläche | $[m^2]$ | $B$ | Flussdichte, Induktion | $[T], [\frac{Vs}{m^2}]$ | $\Theta$ | Durchflutung | $[A]$ | $\Lambda$ | Magnetischer Leitwert | $[\frac{Vs}{A}], [\Omega s]$ |
| $\Phi$                                | Magnetischer Fluss  | $[Vs], [Wb]$  |        |                    |              |     |        |         |     |                        |                         |          |              |       |           |                       |                              |
| $A$                                   | Fläche  | $[m^2]$   |        |                    |              |     |        |         |     |                        |                         |          |              |       |           |                       |                              |
| $B$                                   | Flussdichte, Induktion  | $[T], [\frac{Vs}{m^2}]$   |        |                    |              |     |        |         |     |                        |                         |          |              |       |           |                       |                              |
| $\Theta$                              | Durchflutung  | $[A]$   |        |                    |              |     |        |         |     |                        |                         |          |              |       |           |                       |                              |
| $\Lambda$                             | Magnetischer Leitwert   | $[\frac{Vs}{A}], [\Omega s]$  |        |                    |              |     |        |         |     |                        |                         |          |              |       |           |                       |                              |

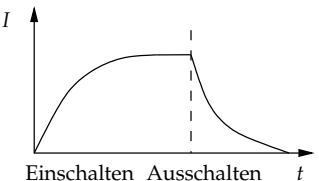
## 12.7. Ohmsches Gesetz des magnetischen Kreises

|   |  |  |       |                         |                  |        |                    |              |          |              |       |           |                       |                              |     |       |       |     |             |         |       |               |                 |
|---|--|--|-------|-------------------------|------------------|--------|--------------------|--------------|----------|--------------|-------|-----------|-----------------------|------------------------------|-----|-------|-------|-----|-------------|---------|-------|---------------|-----------------|
|  | $R_m = \frac{\Theta}{\Phi}$ $\Lambda = \frac{1}{R_m}$ <p>für homogenes Feld:</p> $R_{m_n} = \frac{l_n}{\mu_n A_n}$ | <table> <tr> <td><math>R_m</math></td><td>Magnetischer Widerstand</td><td><math>[\frac{A}{Vs}]</math></td></tr> <tr> <td><math>\Phi</math></td><td>Magnetischer Fluss</td><td><math>[Vs], [Wb]</math></td></tr> <tr> <td><math>\Theta</math></td><td>Durchflutung</td><td><math>[A]</math></td></tr> <tr> <td><math>\Lambda</math></td><td>Magnetischer Leitwert</td><td><math>[\frac{Vs}{A}], [\Omega s]</math></td></tr> <tr> <td><math>l</math></td><td>Länge</td><td><math>[m]</math></td></tr> <tr> <td><math>A</math></td><td>Querschnitt</td><td><math>[m^2]</math></td></tr> <tr> <td><math>\mu</math></td><td>Permeabilität</td><td><math>[\frac{H}{m}]</math></td></tr> </table> | $R_m$ | Magnetischer Widerstand | $[\frac{A}{Vs}]$ | $\Phi$ | Magnetischer Fluss | $[Vs], [Wb]$ | $\Theta$ | Durchflutung | $[A]$ | $\Lambda$ | Magnetischer Leitwert | $[\frac{Vs}{A}], [\Omega s]$ | $l$ | Länge | $[m]$ | $A$ | Querschnitt | $[m^2]$ | $\mu$ | Permeabilität | $[\frac{H}{m}]$ |
| $R_m$   | Magnetischer Widerstand  | $[\frac{A}{Vs}]$   |       |                         |                  |        |                    |              |          |              |       |           |                       |                              |     |       |       |     |             |         |       |               |                 |
| $\Phi$  | Magnetischer Fluss   | $[Vs], [Wb]$   |       |                         |                  |        |                    |              |          |              |       |           |                       |                              |     |       |       |     |             |         |       |               |                 |
| $\Theta$  | Durchflutung   | $[A]$  |       |                         |                  |        |                    |              |          |              |       |           |                       |                              |     |       |       |     |             |         |       |               |                 |
| $\Lambda$   | Magnetischer Leitwert  | $[\frac{Vs}{A}], [\Omega s]$   |       |                         |                  |        |                    |              |          |              |       |           |                       |                              |     |       |       |     |             |         |       |               |                 |
| $l$   | Länge  | $[m]$  |       |                         |                  |        |                    |              |          |              |       |           |                       |                              |     |       |       |     |             |         |       |               |                 |
| $A$   | Querschnitt  | $[m^2]$  |       |                         |                  |        |                    |              |          |              |       |           |                       |                              |     |       |       |     |             |         |       |               |                 |
| $\mu$   | Permeabilität  | $[\frac{H}{m}]$  |       |                         |                  |        |                    |              |          |              |       |           |                       |                              |     |       |       |     |             |         |       |               |                 |

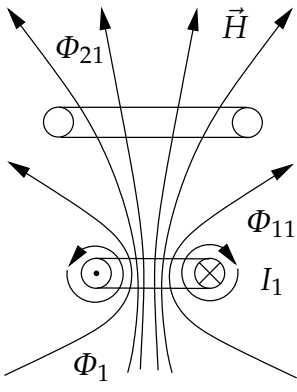
## 12.8. Spulenfluss

|  |  |  |        |             |        |        |                             |              |     |              |                  |     |              |       |     |       |       |           |                       |                              |
|--|--|--|--------|-------------|--------|--------|-----------------------------|--------------|-----|--------------|------------------|-----|--------------|-------|-----|-------|-------|-----------|-----------------------|------------------------------|
| <p>Flüsse durch alle Einzelwindungsflächen aufsummiert<br/>(verketteter Fluss)</p> | $\Psi = N\Phi = \Lambda N^2 I$ $\Psi = LI$ | <table> <tr> <td><math>\Psi</math></td><td>Spulenfluss</td><td><math>[Vs]</math></td></tr> <tr> <td><math>\Phi</math></td><td>Magnetischer Fluss Einzelw.</td><td><math>[Vs], [Wb]</math></td></tr> <tr> <td><math>L</math></td><td>Induktivität</td><td><math>[\frac{Vs}{A}]</math></td></tr> <tr> <td><math>N</math></td><td>Windungszahl</td><td><math>[1]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>\Lambda</math></td><td>Magnetischer Leitwert</td><td><math>[\frac{Vs}{A}], [\Omega s]</math></td></tr> </table> | $\Psi$ | Spulenfluss | $[Vs]$ | $\Phi$ | Magnetischer Fluss Einzelw. | $[Vs], [Wb]$ | $L$ | Induktivität | $[\frac{Vs}{A}]$ | $N$ | Windungszahl | $[1]$ | $I$ | Strom | $[A]$ | $\Lambda$ | Magnetischer Leitwert | $[\frac{Vs}{A}], [\Omega s]$ |
| $\Psi$   | Spulenfluss                                | $[Vs]$   |        |             |        |        |                             |              |     |              |                  |     |              |       |     |       |       |           |                       |                              |
| $\Phi$   | Magnetischer Fluss Einzelw.                | $[Vs], [Wb]$   |        |             |        |        |                             |              |     |              |                  |     |              |       |     |       |       |           |                       |                              |
| $L$  | Induktivität                               | $[\frac{Vs}{A}]$   |        |             |        |        |                             |              |     |              |                  |     |              |       |     |       |       |           |                       |                              |
| $N$  | Windungszahl                               | $[1]$  |        |             |        |        |                             |              |     |              |                  |     |              |       |     |       |       |           |                       |                              |
| $I$  | Strom                                      | $[A]$  |        |             |        |        |                             |              |     |              |                  |     |              |       |     |       |       |           |                       |                              |
| $\Lambda$  | Magnetischer Leitwert                      | $[\frac{Vs}{A}], [\Omega s]$   |        |             |        |        |                             |              |     |              |                  |     |              |       |     |       |       |           |                       |                              |

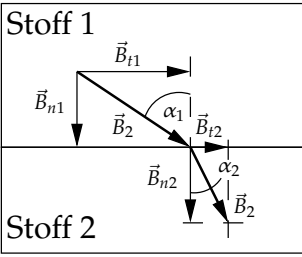
## 12.9. Induktivität

|   |   |   |     |                    |             |     |              |       |        |                             |              |     |              |       |     |       |       |           |                       |                              |       |                         |                  |        |             |        |
|---|---|---|-----|--------------------|-------------|-----|--------------|-------|--------|-----------------------------|--------------|-----|--------------|-------|-----|-------|-------|-----------|-----------------------|------------------------------|-------|-------------------------|------------------|--------|-------------|--------|
|  | $L = \frac{N\Phi}{I} = \frac{\Psi}{I}$ $L = N^2 \Lambda = \frac{N^2}{R_m}$ $L = \frac{2W}{I^2}$ | <table> <tr> <td><math>W</math></td><td>Energie des Feldes</td><td><math>[Ws], [J]</math></td></tr> <tr> <td><math>L</math></td><td>Induktivität</td><td><math>[H]</math></td></tr> <tr> <td><math>\Phi</math></td><td>Magnetischer Fluss Einzelw.</td><td><math>[Vs], [Wb]</math></td></tr> <tr> <td><math>N</math></td><td>Windungszahl</td><td><math>[1]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>\Lambda</math></td><td>Magnetischer Leitwert</td><td><math>[\frac{Vs}{A}], [\Omega s]</math></td></tr> <tr> <td><math>R_m</math></td><td>Magnetischer Widerstand</td><td><math>[\frac{A}{Vs}]</math></td></tr> <tr> <td><math>\Psi</math></td><td>Spulenfluss</td><td><math>[Wb]</math></td></tr> </table> | $W$ | Energie des Feldes | $[Ws], [J]$ | $L$ | Induktivität | $[H]$ | $\Phi$ | Magnetischer Fluss Einzelw. | $[Vs], [Wb]$ | $N$ | Windungszahl | $[1]$ | $I$ | Strom | $[A]$ | $\Lambda$ | Magnetischer Leitwert | $[\frac{Vs}{A}], [\Omega s]$ | $R_m$ | Magnetischer Widerstand | $[\frac{A}{Vs}]$ | $\Psi$ | Spulenfluss | $[Wb]$ |
| $W$   | Energie des Feldes  | $[Ws], [J]$   |     |                    |             |     |              |       |        |                             |              |     |              |       |     |       |       |           |                       |                              |       |                         |                  |        |             |        |
| $L$   | Induktivität  | $[H]$   |     |                    |             |     |              |       |        |                             |              |     |              |       |     |       |       |           |                       |                              |       |                         |                  |        |             |        |
| $\Phi$  | Magnetischer Fluss Einzelw.   | $[Vs], [Wb]$  |     |                    |             |     |              |       |        |                             |              |     |              |       |     |       |       |           |                       |                              |       |                         |                  |        |             |        |
| $N$   | Windungszahl  | $[1]$   |     |                    |             |     |              |       |        |                             |              |     |              |       |     |       |       |           |                       |                              |       |                         |                  |        |             |        |
| $I$   | Strom   | $[A]$   |     |                    |             |     |              |       |        |                             |              |     |              |       |     |       |       |           |                       |                              |       |                         |                  |        |             |        |
| $\Lambda$   | Magnetischer Leitwert   | $[\frac{Vs}{A}], [\Omega s]$  |     |                    |             |     |              |       |        |                             |              |     |              |       |     |       |       |           |                       |                              |       |                         |                  |        |             |        |
| $R_m$   | Magnetischer Widerstand   | $[\frac{A}{Vs}]$  |     |                    |             |     |              |       |        |                             |              |     |              |       |     |       |       |           |                       |                              |       |                         |                  |        |             |        |
| $\Psi$  | Spulenfluss   | $[Wb]$  |     |                    |             |     |              |       |        |                             |              |     |              |       |     |       |       |           |                       |                              |       |                         |                  |        |             |        |

## 12.10. Gegeninduktivität und induktive Kopplung

|  |   |  |        |             |        |        |                                  |              |        |                        |              |     |              |                  |     |              |       |     |       |       |     |               |       |          |             |       |     |                   |                  |
|--|---|--|--------|-------------|--------|--------|----------------------------------|--------------|--------|------------------------|--------------|-----|--------------|------------------|-----|--------------|-------|-----|-------|-------|-----|---------------|-------|----------|-------------|-------|-----|-------------------|------------------|
|  <p>1. Index: Wirkungsort<br/>2. Index: Ursachort</p> | $M_{21} = \frac{\Psi_{21}}{I_1} = \frac{N_2 \Phi_{21}}{I_1}$ $M_{12} = \frac{\Psi_{12}}{I_2} = \frac{N_1 \Phi_{12}}{I_2}$ $M = \sqrt{L_1 L_2} \quad \text{ohne Streufluss}$ $M = k \sqrt{L_1 L_2} \quad \text{mit Streufluss}$ $k_{12} = \frac{\Phi_{12}}{\Phi_{22}} \quad k_{21} = \frac{\Phi_{21}}{\Phi_{11}}$ $\sigma = 1 - \frac{M^2}{L_1 L_2} = 1 - k^2$ | <table> <tr> <td><math>\Psi</math></td><td>Spulenfluss</td><td><math>[Vs]</math></td></tr> <tr> <td><math>\Phi</math></td><td>Magnetischer Fluss durch Windung</td><td><math>[Vs], [Wb]</math></td></tr> <tr> <td><math>\Phi</math></td><td>Magnetischer Streuluss</td><td><math>[Vs], [Wb]</math></td></tr> <tr> <td><math>L</math></td><td>Induktivität</td><td><math>[\frac{Vs}{A}]</math></td></tr> <tr> <td><math>N</math></td><td>Windungszahl</td><td><math>[1]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>k</math></td><td>Kopplungsfak.</td><td><math>[1]</math></td></tr> <tr> <td><math>\sigma</math></td><td>Streukoeff.</td><td><math>[1]</math></td></tr> <tr> <td><math>M</math></td><td>Gegeninduktivität</td><td><math>[\frac{Vs}{A}]</math></td></tr> </table> | $\Psi$ | Spulenfluss | $[Vs]$ | $\Phi$ | Magnetischer Fluss durch Windung | $[Vs], [Wb]$ | $\Phi$ | Magnetischer Streuluss | $[Vs], [Wb]$ | $L$ | Induktivität | $[\frac{Vs}{A}]$ | $N$ | Windungszahl | $[1]$ | $I$ | Strom | $[A]$ | $k$ | Kopplungsfak. | $[1]$ | $\sigma$ | Streukoeff. | $[1]$ | $M$ | Gegeninduktivität | $[\frac{Vs}{A}]$ |
| $\Psi$   | Spulenfluss   | $[Vs]$   |        |             |        |        |                                  |              |        |                        |              |     |              |                  |     |              |       |     |       |       |     |               |       |          |             |       |     |                   |                  |
| $\Phi$   | Magnetischer Fluss durch Windung  | $[Vs], [Wb]$   |        |             |        |        |                                  |              |        |                        |              |     |              |                  |     |              |       |     |       |       |     |               |       |          |             |       |     |                   |                  |
| $\Phi$   | Magnetischer Streuluss  | $[Vs], [Wb]$   |        |             |        |        |                                  |              |        |                        |              |     |              |                  |     |              |       |     |       |       |     |               |       |          |             |       |     |                   |                  |
| $L$  | Induktivität  | $[\frac{Vs}{A}]$   |        |             |        |        |                                  |              |        |                        |              |     |              |                  |     |              |       |     |       |       |     |               |       |          |             |       |     |                   |                  |
| $N$  | Windungszahl  | $[1]$  |        |             |        |        |                                  |              |        |                        |              |     |              |                  |     |              |       |     |       |       |     |               |       |          |             |       |     |                   |                  |
| $I$  | Strom   | $[A]$  |        |             |        |        |                                  |              |        |                        |              |     |              |                  |     |              |       |     |       |       |     |               |       |          |             |       |     |                   |                  |
| $k$  | Kopplungsfak.   | $[1]$  |        |             |        |        |                                  |              |        |                        |              |     |              |                  |     |              |       |     |       |       |     |               |       |          |             |       |     |                   |                  |
| $\sigma$   | Streukoeff.   | $[1]$  |        |             |        |        |                                  |              |        |                        |              |     |              |                  |     |              |       |     |       |       |     |               |       |          |             |       |     |                   |                  |
| $M$  | Gegeninduktivität   | $[\frac{Vs}{A}]$   |        |             |        |        |                                  |              |        |                        |              |     |              |                  |     |              |       |     |       |       |     |               |       |          |             |       |     |                   |                  |

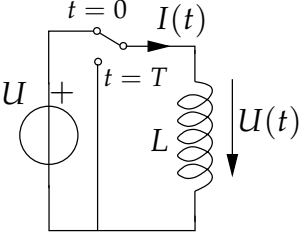
## 12.11. Brechung magnetischer Feldlinien

|   |   |  |     |            |                 |     |                        |                         |          |        |         |       |               |                 |
|---|---|--|-----|------------|-----------------|-----|------------------------|-------------------------|----------|--------|---------|-------|---------------|-----------------|
|  | $B_{n1} = B_{n2}$ $\frac{H_{n1}}{H_{n2}} = \frac{\mu_{r2}}{\mu_{r1}}$ $H_{t1} = H_{t2}$ $\frac{B_{t1}}{B_{t2}} = \frac{\mu_{r1}}{\mu_{r2}}$ $\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{\mu_{r1}}{\mu_{r2}}$ | <table> <tr> <td><math>H</math></td><td>Feldstärke</td><td><math>[\frac{A}{m}]</math></td></tr> <tr> <td><math>B</math></td><td>Flussdichte, Induktion</td><td><math>[T], [\frac{Vs}{m^2}]</math></td></tr> <tr> <td><math>\alpha</math></td><td>Winkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>\mu</math></td><td>Permeabilität</td><td><math>[\frac{H}{m}]</math></td></tr> </table> | $H$ | Feldstärke | $[\frac{A}{m}]$ | $B$ | Flussdichte, Induktion | $[T], [\frac{Vs}{m^2}]$ | $\alpha$ | Winkel | $[rad]$ | $\mu$ | Permeabilität | $[\frac{H}{m}]$ |
| $H$   | Feldstärke  | $[\frac{A}{m}]$  |     |            |                 |     |                        |                         |          |        |         |       |               |                 |
| $B$   | Flussdichte, Induktion  | $[T], [\frac{Vs}{m^2}]$  |     |            |                 |     |                        |                         |          |        |         |       |               |                 |
| $\alpha$  | Winkel  | $[rad]$  |     |            |                 |     |                        |                         |          |        |         |       |               |                 |
| $\mu$   | Permeabilität   | $[\frac{H}{m}]$  |     |            |                 |     |                        |                         |          |        |         |       |               |                 |

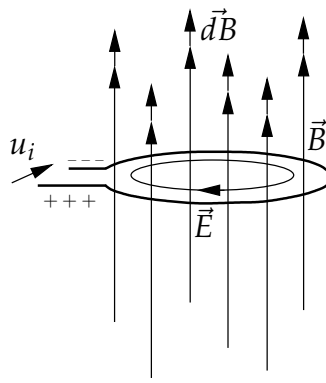
## 12.12. Räumliche Energiedichte

|  |   |       |                        |   |
|--|---|-------|------------------------|---|
|  | Inhomogenes Feld:                                     | $W$   | Energiedichte          | $\left[\frac{Ws}{m^3}\right]$   |
|  | $W_{mg(x,y,z)} = \frac{1}{2} B_{(x,y,z)} H_{(x,y,z)}$ | $H$   | Feldstärke             | $\left[\frac{J}{m^3}\right]$  |
|  | Homogenes Feld:                                       | $B$   | Flussdichte, Induktion | $\left[\frac{A}{m}\right], \left[\frac{T}{m}\right], \left[\frac{Vs}{m^2}\right]$ |
|  | $W_{mg(x,y,z)} = \frac{\mu}{2} H_{(x,y,z)}^2$         | $\mu$ | Permeabilität          | $\left[\frac{H}{m}\right]$  |

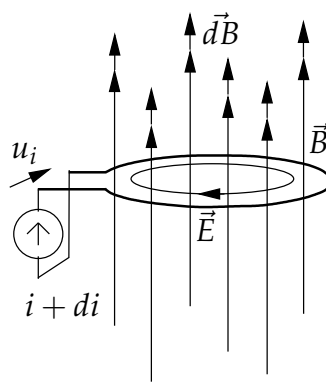
## 12.13. Energie im magnetischen Feld

|  |                                  |           |                         |   |
|--|----------------------------------|-----------|-------------------------|---|
|  | für $t \geq T$ :                 | $W$       | Energie                 | $[Ws]$                                  |
|  | $W = \frac{U^2 T^2}{2L}$         | $\mu$     | Permeabilität           | $\left[\frac{H}{m}\right]$              |
|  | $W = \frac{\psi^2}{2L}$          | $\Psi$    | Spulenfluss             | $[Vs]$                                  |
|  | $W = \frac{LI^2}{2}$             | $\Phi$    | Magnetischer Fluss      | $[Vs], [Wb]$                            |
|  | $W = \frac{I\Psi}{2}$            | $L$       | Induktivität            | $\left[\frac{Vs}{A}\right]$             |
|  | $W = \frac{\Phi^2 R_m}{2}$       | $I$       | Strom                   | $[A]$                                   |
|  | $W = \frac{\Theta^2 \Lambda}{2}$ | $R_m$     | Magnetischer Widerstand | $\left[\frac{A}{Vs}\right]$             |
|  |                                  | $U$       | Spannung                | $[V]$                                   |
|  |                                  | $t$       | Zeit                    | $[s]$                                   |
|  |                                  | $T$       | Zeitpunkt               | $[s]$                                   |
|  |                                  | $\Theta$  | Durchflutung            | $[A]$                                   |
|  |                                  | $\Lambda$ | Magnetischer Leitwert   | $\left[\frac{Vs}{A}\right], [\Omega s]$ |

## 12.14. Induktionsgesetz

|   |   |   |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |      |                             |       |     |                        |                         |
|---|---|---|--------|-------------|--------|-----|------|-------|-----|------------------------|-----------------|-------|--------------------|-------|--------|--------------------|--------------|------|-----------------------------|-------|-----|------------------------|-------------------------|
|  <p>The diagram shows a horizontal circular loop. Vertical arrows pointing upwards represent the magnetic field <math>\vec{B}</math>. A dashed line with '+' signs on the left and '-' signs on the right represents the induced electric field <math>\vec{E}</math> around the loop. A curved arrow on the loop indicates the direction of induced current. The text below states that <math>\vec{E}</math> and <math>d\vec{B}</math> form a left-hand screw.</p> | $u_i = \frac{d\Psi}{dt}$ $\oint \vec{E} d\vec{s} = -\frac{d\Phi}{dt} = -u_i$ <p><math>\vec{E}</math> bildet mit <math>d\vec{B}</math> eine Links-schraube</p> | <table> <tr> <td><math>\Psi</math></td><td>Spulenfluss</td><td><math>[Vs]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>E</math></td><td>Elektrostatisches Feld</td><td><math>[\frac{V}{m}]</math></td></tr> <tr> <td><math>u_i</math></td><td>Induktionsspannung</td><td><math>[V]</math></td></tr> <tr> <td><math>\Phi</math></td><td>Magnetischer Fluss</td><td><math>[Vs], [Wb]</math></td></tr> <tr> <td><math>ds</math></td><td>infinitesimales Leiterstück</td><td><math>[m]</math></td></tr> <tr> <td><math>B</math></td><td>Flussdichte, Induktion</td><td><math>[T], [\frac{Vs}{m^2}]</math></td></tr> </table> | $\Psi$ | Spulenfluss | $[Vs]$ | $t$ | Zeit | $[s]$ | $E$ | Elektrostatisches Feld | $[\frac{V}{m}]$ | $u_i$ | Induktionsspannung | $[V]$ | $\Phi$ | Magnetischer Fluss | $[Vs], [Wb]$ | $ds$ | infinitesimales Leiterstück | $[m]$ | $B$ | Flussdichte, Induktion | $[T], [\frac{Vs}{m^2}]$ |
| $\Psi$  | Spulenfluss   | $[Vs]$  |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |      |                             |       |     |                        |                         |
| $t$   | Zeit  | $[s]$   |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |      |                             |       |     |                        |                         |
| $E$   | Elektrostatisches Feld  | $[\frac{V}{m}]$   |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |      |                             |       |     |                        |                         |
| $u_i$   | Induktionsspannung  | $[V]$   |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |      |                             |       |     |                        |                         |
| $\Phi$  | Magnetischer Fluss  | $[Vs], [Wb]$  |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |      |                             |       |     |                        |                         |
| $ds$  | infinitesimales Leiterstück   | $[m]$   |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |      |                             |       |     |                        |                         |
| $B$   | Flussdichte, Induktion  | $[T], [\frac{Vs}{m^2}]$   |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |      |                             |       |     |                        |                         |

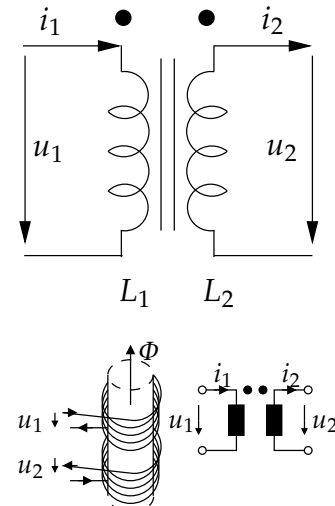
## 12.15. Selbstinduktion

|  |  |   |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |     |              |                  |     |       |       |     |                        |                         |
|--|--|---|--------|-------------|--------|-----|------|-------|-----|------------------------|-----------------|-------|--------------------|-------|--------|--------------------|--------------|-----|--------------|------------------|-----|-------|-------|-----|------------------------|-------------------------|
|  <p>The diagram shows a horizontal circular loop connected to a current source (a circle with an upward arrow). The current is labeled <math>i + di</math>. Vertical arrows pointing upwards represent the magnetic field <math>\vec{B}</math>. A curved arrow on the loop indicates the direction of induced current. The text below states <math>di</math> während <math>dt</math>.</p> | <p>Für Schleife:</p> $u_i = \frac{d\Phi}{dt} = L \frac{di}{dt}$ <p>Für Spule:</p> $u_i = \frac{d\Psi}{dt} = L \frac{di}{dt}$ | <table> <tr> <td><math>\Psi</math></td><td>Spulenfluss</td><td><math>[Vs]</math></td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td><math>[s]</math></td></tr> <tr> <td><math>E</math></td><td>Elektrostatisches Feld</td><td><math>[\frac{V}{m}]</math></td></tr> <tr> <td><math>u_i</math></td><td>Induktionsspannung</td><td><math>[V]</math></td></tr> <tr> <td><math>\Phi</math></td><td>Magnetischer Fluss</td><td><math>[Vs], [Wb]</math></td></tr> <tr> <td><math>L</math></td><td>Induktivität</td><td><math>[\frac{Vs}{A}]</math></td></tr> <tr> <td><math>i</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>B</math></td><td>Flussdichte, Induktion</td><td><math>[T], [\frac{Vs}{m^2}]</math></td></tr> </table> | $\Psi$ | Spulenfluss | $[Vs]$ | $t$ | Zeit | $[s]$ | $E$ | Elektrostatisches Feld | $[\frac{V}{m}]$ | $u_i$ | Induktionsspannung | $[V]$ | $\Phi$ | Magnetischer Fluss | $[Vs], [Wb]$ | $L$ | Induktivität | $[\frac{Vs}{A}]$ | $i$ | Strom | $[A]$ | $B$ | Flussdichte, Induktion | $[T], [\frac{Vs}{m^2}]$ |
| $\Psi$   | Spulenfluss  | $[Vs]$  |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |     |              |                  |     |       |       |     |                        |                         |
| $t$  | Zeit   | $[s]$   |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |     |              |                  |     |       |       |     |                        |                         |
| $E$  | Elektrostatisches Feld   | $[\frac{V}{m}]$   |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |     |              |                  |     |       |       |     |                        |                         |
| $u_i$  | Induktionsspannung   | $[V]$   |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |     |              |                  |     |       |       |     |                        |                         |
| $\Phi$   | Magnetischer Fluss   | $[Vs], [Wb]$  |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |     |              |                  |     |       |       |     |                        |                         |
| $L$  | Induktivität   | $[\frac{Vs}{A}]$  |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |     |              |                  |     |       |       |     |                        |                         |
| $i$  | Strom  | $[A]$   |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |     |              |                  |     |       |       |     |                        |                         |
| $B$  | Flussdichte, Induktion   | $[T], [\frac{Vs}{m^2}]$   |        |             |        |     |      |       |     |                        |                 |       |                    |       |        |                    |              |     |              |                  |     |       |       |     |                        |                         |

## 12.16. Serie- und Parallelschaltung von Induktivitäten

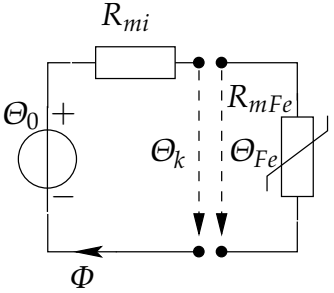
|  |  |   |
|--|--|---|
|  | <p>Serieschaltung</p> $L_{Ers.} = L_1 + L_2 + \dots + L_n$ <p>Parallelschaltung</p> $L_{Ers.} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_n}}$ | <p><math>L</math> Induktivität <math>\left[\frac{Vs}{A}\right]</math></p> |
|--|--|---|

## 12.17. Trafogleichungen

|  |   |   |
|--|---|---|
|  | $u_1 = u_{11} + u_{12} = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$ $u_2 = u_{21} + u_{22} = M \frac{di_1}{dt} - L_2 \frac{di_2}{dt}$ $u_1 = j\omega L_1 i_1 - j\omega M i_2$ $u_2 = j\omega M i_1 - j\omega L_2 i_2$ | <p><math>u</math> Induktionsspannung <math>[V]</math></p> <p><math>i</math> Strom <math>[A]</math></p> <p><math>L</math> Induktivität <math>\left[\frac{Vs}{A}\right]</math></p> <p><math>t</math> Zeit <math>[s]</math></p> <p><math>M</math> Gegeninduktivität <math>\left[\frac{Vs}{A}\right]</math></p> <p><math>\omega</math> Kreisfrequenz <math>\left[\frac{1}{s}\right]</math></p> <p><math>R</math> (Kupfer-) Widerstand <math>[\Omega]</math></p> |
|--|---|---|

## 12.18. Nichtlinearität

### 12.18.1. $B(H)$ -Kurve in $\Phi(\Theta)$ -Kurve umrechnen

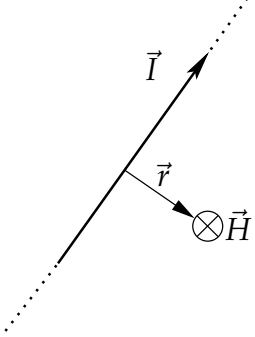
|   |  |  |     |                        |                 |     |            |         |       |               |         |          |              |       |     |       |       |     |             |         |        |                    |              |
|---|--|--|-----|------------------------|-----------------|-----|------------|---------|-------|---------------|---------|----------|--------------|-------|-----|-------|-------|-----|-------------|---------|--------|--------------------|--------------|
|  | <p>Leerlauf</p> $B_k(0) = B_c = \frac{\mu_0 A_L \Theta_0}{l_L A_{Fe}}$ <p>Kurzschluss</p> $H_0 = \frac{\Theta_0}{l_{Fe}}$ <p>Umrechnung:</p> $\Phi_{Fe} = A_{Fe} B_{Fe}$ $\Theta_{Fe} = l_{Fe} H_{Fe}$ $B_L = \frac{A_{Fe}}{A_L} B_{Fe}$ | <table> <tr> <td><math>B</math></td><td>Flussdichte, Induktion</td><td><math>[T], [Vs/m^2]</math></td></tr> <tr> <td><math>H</math></td><td>Feldstärke</td><td><math>[A/m]</math></td></tr> <tr> <td><math>\mu</math></td><td>Permeabilität</td><td><math>[H/m]</math></td></tr> <tr> <td><math>\Theta</math></td><td>Durchflutung</td><td><math>[A]</math></td></tr> <tr> <td><math>l</math></td><td>Länge</td><td><math>[m]</math></td></tr> <tr> <td><math>A</math></td><td>Querschnitt</td><td><math>[m^2]</math></td></tr> <tr> <td><math>\Phi</math></td><td>Magnetischer Fluss</td><td><math>[Vs], [Wb]</math></td></tr> </table> | $B$ | Flussdichte, Induktion | $[T], [Vs/m^2]$ | $H$ | Feldstärke | $[A/m]$ | $\mu$ | Permeabilität | $[H/m]$ | $\Theta$ | Durchflutung | $[A]$ | $l$ | Länge | $[m]$ | $A$ | Querschnitt | $[m^2]$ | $\Phi$ | Magnetischer Fluss | $[Vs], [Wb]$ |
| $B$   | Flussdichte, Induktion   | $[T], [Vs/m^2]$  |     |                        |                 |     |            |         |       |               |         |          |              |       |     |       |       |     |             |         |        |                    |              |
| $H$   | Feldstärke   | $[A/m]$  |     |                        |                 |     |            |         |       |               |         |          |              |       |     |       |       |     |             |         |        |                    |              |
| $\mu$   | Permeabilität  | $[H/m]$  |     |                        |                 |     |            |         |       |               |         |          |              |       |     |       |       |     |             |         |        |                    |              |
| $\Theta$  | Durchflutung   | $[A]$  |     |                        |                 |     |            |         |       |               |         |          |              |       |     |       |       |     |             |         |        |                    |              |
| $l$   | Länge  | $[m]$  |     |                        |                 |     |            |         |       |               |         |          |              |       |     |       |       |     |             |         |        |                    |              |
| $A$   | Querschnitt  | $[m^2]$  |     |                        |                 |     |            |         |       |               |         |          |              |       |     |       |       |     |             |         |        |                    |              |
| $\Phi$  | Magnetischer Fluss   | $[Vs], [Wb]$   |     |                        |                 |     |            |         |       |               |         |          |              |       |     |       |       |     |             |         |        |                    |              |

### 12.18.2. Luftspaltkennwert $\alpha$

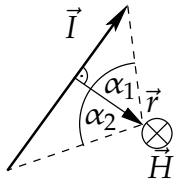
|          |   |   |          |                     |       |       |               |         |     |       |       |     |             |         |
|----------|---|---|----------|---------------------|-------|-------|---------------|---------|-----|-------|-------|-----|-------------|---------|
|          | $\alpha = \frac{A_L l_{Fe}}{A_{Fe} l_L}$ $\frac{1}{\mu_{reff}} = \frac{1}{\mu_{rFe}} + \frac{1}{\alpha}$ $\mu_{reff} = \frac{\mu_{rFe} \alpha}{\mu_{rFe} + \alpha}$ | <table> <tr> <td><math>\alpha</math></td><td>Luftspaltkenngrösse</td><td><math>[1]</math></td></tr> <tr> <td><math>\mu</math></td><td>Permeabilität</td><td><math>[H/m]</math></td></tr> <tr> <td><math>l</math></td><td>Länge</td><td><math>[m]</math></td></tr> <tr> <td><math>A</math></td><td>Querschnitt</td><td><math>[m^2]</math></td></tr> </table> | $\alpha$ | Luftspaltkenngrösse | $[1]$ | $\mu$ | Permeabilität | $[H/m]$ | $l$ | Länge | $[m]$ | $A$ | Querschnitt | $[m^2]$ |
| $\alpha$ | Luftspaltkenngrösse   | $[1]$   |          |                     |       |       |               |         |     |       |       |     |             |         |
| $\mu$    | Permeabilität   | $[H/m]$   |          |                     |       |       |               |         |     |       |       |     |             |         |
| $l$      | Länge   | $[m]$   |          |                     |       |       |               |         |     |       |       |     |             |         |
| $A$      | Querschnitt   | $[m^2]$   |          |                     |       |       |               |         |     |       |       |     |             |         |

## 12.19. Spezielle Anordnungen

### 12.19.1. Langer gerader Leiter $l \gg d$

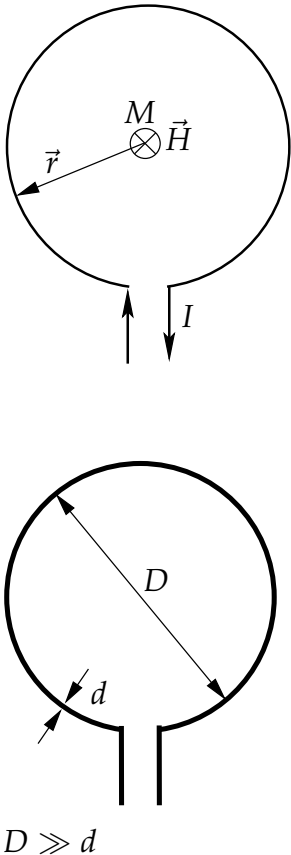
|   |  |  |     |            |                            |     |                    |       |     |       |       |
|---|--|--|-----|------------|----------------------------|-----|--------------------|-------|-----|-------|-------|
|  | <p>Bezugspunkt ausserhalb des Leiters im Abstand <math>r</math></p> $H = \frac{I}{2\pi r}$ $\vec{H} = \frac{I}{2\pi \vec{r} ^2}(\vec{e}_1 \times \vec{r})$ | <table> <tr> <td><math>H</math></td><td>Feldstärke</td><td><math>\left[\frac{A}{m}\right]</math></td></tr> <tr> <td><math>r</math></td><td>Abstand vom Leiter</td><td><math>[m]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> </table> | $H$ | Feldstärke | $\left[\frac{A}{m}\right]$ | $r$ | Abstand vom Leiter | $[m]$ | $I$ | Strom | $[A]$ |
| $H$   | Feldstärke   | $\left[\frac{A}{m}\right]$   |     |            |                            |     |                    |       |     |       |       |
| $r$   | Abstand vom Leiter   | $[m]$  |     |            |                            |     |                    |       |     |       |       |
| $I$   | Strom  | $[A]$  |     |            |                            |     |                    |       |     |       |       |

### 12.19.2. Kurzer, gerader Leiter

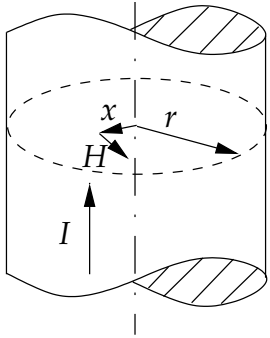
|  |  |   |     |            |                            |     |                    |       |     |       |       |          |        |         |
|--|--|---|-----|------------|----------------------------|-----|--------------------|-------|-----|-------|-------|----------|--------|---------|
|  | $H = \frac{I}{4\pi r} \int_{\alpha_2}^{\alpha_1} \cos \varphi d\varphi$ $H = \frac{I}{4\pi r} (\sin \alpha_1 - \sin \alpha_2)$ | <table> <tr> <td><math>H</math></td><td>Feldstärke</td><td><math>\left[\frac{A}{m}\right]</math></td></tr> <tr> <td><math>r</math></td><td>Abstand vom Leiter</td><td><math>[m]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>\alpha</math></td><td>Winkel</td><td><math>[rad]</math></td></tr> </table> | $H$ | Feldstärke | $\left[\frac{A}{m}\right]$ | $r$ | Abstand vom Leiter | $[m]$ | $I$ | Strom | $[A]$ | $\alpha$ | Winkel | $[rad]$ |
| $H$  | Feldstärke   | $\left[\frac{A}{m}\right]$  |     |            |                            |     |                    |       |     |       |       |          |        |         |
| $r$  | Abstand vom Leiter   | $[m]$   |     |            |                            |     |                    |       |     |       |       |          |        |         |
| $I$  | Strom  | $[A]$   |     |            |                            |     |                    |       |     |       |       |          |        |         |
| $\alpha$   | Winkel   | $[rad]$   |     |            |                            |     |                    |       |     |       |       |          |        |         |



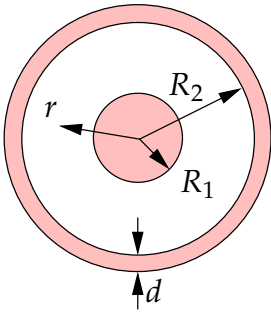
## 12.19.3. Kreisförmige Drahtschleife

|  |  |   |     |            |                 |     |             |  |     |       |       |     |        |       |       |               |                 |     |                   |       |     |                       |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |                          |       |
|--|--|---|-----|------------|-----------------|-----|-------------|--|-----|-------|-------|-----|--------|-------|-------|---------------|-----------------|-----|-------------------|-------|-----|-----------------------|-------|-----------|-----------------------|------------------------------|-------|-------------------------|------------------|-----|--------------|------------------|--------|--------------------|--------------|----------|--------------|-------|-----|--------------------------|-------|
|  <p><math>D \gg d</math></p> | <p>Bezugspunkt: M Teilkreis:</p> $H = \frac{I}{4\pi r} \int_0^\alpha d\varphi$ <p>Vollkreis:</p> $H = \frac{I}{2r}$ <p>Feld auf der Achse:</p> $H = \frac{ I r^2}{2(x^2 + r^2)^{\frac{3}{2}}}$ $\Phi = \frac{\mu D}{2} \ln \frac{D}{d} \Theta$ $\Lambda = \mu \frac{D}{2} \ln \frac{D}{d}$ $R_m = \frac{2}{\mu D \ln \frac{D}{d}}$ $L = \mu \frac{D}{2} \ln \frac{D}{d}$ | <table> <tr> <td><math>H</math></td><td>Feldstärke</td><td><math>[\frac{A}{m}]</math></td></tr> <tr> <td><math>M</math></td><td>Mittelpunkt</td><td></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr> <td><math>\mu</math></td><td>Permeabilität</td><td><math>[\frac{H}{m}]</math></td></tr> <tr> <td><math>d</math></td><td>Draht Durchmesser</td><td><math>[m]</math></td></tr> <tr> <td><math>D</math></td><td>Schleifen Durchmesser</td><td><math>[m]</math></td></tr> <tr> <td><math>\Lambda</math></td><td>Magnetischer Leitwert</td><td><math>[\frac{Vs}{A}], [\Omega s]</math></td></tr> <tr> <td><math>R_m</math></td><td>Magnetischer Widerstand</td><td><math>[\frac{A}{Vs}]</math></td></tr> <tr> <td><math>L</math></td><td>Induktivität</td><td><math>[\frac{Vs}{A}]</math></td></tr> <tr> <td><math>\Phi</math></td><td>Magnetischer Fluss</td><td><math>[Vs], [Wb]</math></td></tr> <tr> <td><math>\Theta</math></td><td>Durchflutung</td><td><math>[A]</math></td></tr> <tr> <td><math>x</math></td><td>senkrechter Abstand zu M</td><td><math>[m]</math></td></tr> </table> | $H$ | Feldstärke | $[\frac{A}{m}]$ | $M$ | Mittelpunkt |  | $I$ | Strom | $[A]$ | $r$ | Radius | $[m]$ | $\mu$ | Permeabilität | $[\frac{H}{m}]$ | $d$ | Draht Durchmesser | $[m]$ | $D$ | Schleifen Durchmesser | $[m]$ | $\Lambda$ | Magnetischer Leitwert | $[\frac{Vs}{A}], [\Omega s]$ | $R_m$ | Magnetischer Widerstand | $[\frac{A}{Vs}]$ | $L$ | Induktivität | $[\frac{Vs}{A}]$ | $\Phi$ | Magnetischer Fluss | $[Vs], [Wb]$ | $\Theta$ | Durchflutung | $[A]$ | $x$ | senkrechter Abstand zu M | $[m]$ |
| $H$  | Feldstärke   | $[\frac{A}{m}]$   |     |            |                 |     |             |  |     |       |       |     |        |       |       |               |                 |     |                   |       |     |                       |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |                          |       |
| $M$  | Mittelpunkt  |   |     |            |                 |     |             |  |     |       |       |     |        |       |       |               |                 |     |                   |       |     |                       |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |                          |       |
| $I$  | Strom  | $[A]$   |     |            |                 |     |             |  |     |       |       |     |        |       |       |               |                 |     |                   |       |     |                       |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |                          |       |
| $r$  | Radius   | $[m]$   |     |            |                 |     |             |  |     |       |       |     |        |       |       |               |                 |     |                   |       |     |                       |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |                          |       |
| $\mu$  | Permeabilität  | $[\frac{H}{m}]$   |     |            |                 |     |             |  |     |       |       |     |        |       |       |               |                 |     |                   |       |     |                       |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |                          |       |
| $d$  | Draht Durchmesser  | $[m]$   |     |            |                 |     |             |  |     |       |       |     |        |       |       |               |                 |     |                   |       |     |                       |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |                          |       |
| $D$  | Schleifen Durchmesser  | $[m]$   |     |            |                 |     |             |  |     |       |       |     |        |       |       |               |                 |     |                   |       |     |                       |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |                          |       |
| $\Lambda$  | Magnetischer Leitwert  | $[\frac{Vs}{A}], [\Omega s]$  |     |            |                 |     |             |  |     |       |       |     |        |       |       |               |                 |     |                   |       |     |                       |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |                          |       |
| $R_m$  | Magnetischer Widerstand  | $[\frac{A}{Vs}]$  |     |            |                 |     |             |  |     |       |       |     |        |       |       |               |                 |     |                   |       |     |                       |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |                          |       |
| $L$  | Induktivität   | $[\frac{Vs}{A}]$  |     |            |                 |     |             |  |     |       |       |     |        |       |       |               |                 |     |                   |       |     |                       |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |                          |       |
| $\Phi$   | Magnetischer Fluss   | $[Vs], [Wb]$  |     |            |                 |     |             |  |     |       |       |     |        |       |       |               |                 |     |                   |       |     |                       |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |                          |       |
| $\Theta$   | Durchflutung   | $[A]$   |     |            |                 |     |             |  |     |       |       |     |        |       |       |               |                 |     |                   |       |     |                       |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |                          |       |
| $x$  | senkrechter Abstand zu M   | $[m]$   |     |            |                 |     |             |  |     |       |       |     |        |       |       |               |                 |     |                   |       |     |                       |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |                          |       |

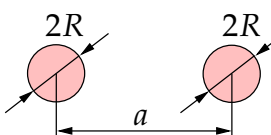
## 12.19.4. Voller Leiter

|   |   |   |     |            |                 |     |                             |       |     |       |       |     |                       |       |
|---|---|---|-----|------------|-----------------|-----|-----------------------------|-------|-----|-------|-------|-----|-----------------------|-------|
|  | $H = \frac{Ix}{2\pi r^2}$ <p>gilt nur für <math>x \leq r</math></p> | <table> <tr> <td><math>H</math></td><td>Feldstärke</td><td><math>[\frac{A}{m}]</math></td></tr> <tr> <td><math>r</math></td><td>Abstand von der Leiterachse</td><td><math>[m]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>x</math></td><td>Abstand von der Achse</td><td><math>[m]</math></td></tr> </table> | $H$ | Feldstärke | $[\frac{A}{m}]$ | $r$ | Abstand von der Leiterachse | $[m]$ | $I$ | Strom | $[A]$ | $x$ | Abstand von der Achse | $[m]$ |
| $H$   | Feldstärke  | $[\frac{A}{m}]$   |     |            |                 |     |                             |       |     |       |       |     |                       |       |
| $r$   | Abstand von der Leiterachse   | $[m]$   |     |            |                 |     |                             |       |     |       |       |     |                       |       |
| $I$   | Strom   | $[A]$   |     |            |                 |     |                             |       |     |       |       |     |                       |       |
| $x$   | Abstand von der Achse   | $[m]$   |     |            |                 |     |                             |       |     |       |       |     |                       |       |

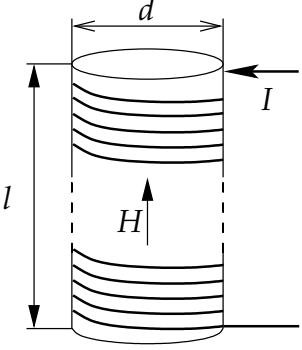
## 12.19.5. Koaxialkabel

|  |   |   |     |            |                              |     |                             |                    |     |       |                    |     |        |                    |     |             |                    |       |               |                              |     |       |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |        |                    |  |          |              |                    |
|--|---|---|-----|------------|------------------------------|-----|-----------------------------|--------------------|-----|-------|--------------------|-----|--------|--------------------|-----|-------------|--------------------|-------|---------------|------------------------------|-----|-------|--------------------|-----------|-----------------------|--|-------|-------------------------|-------------------------------|------|--------------------|--------------------------------|-----|--------------|-------------------------------|--------|--------------------|--|----------|--------------|--------------------|
|  <p><math>l \gg R_1, R_2</math></p> | $H = \frac{I}{2\pi R_1^2} r \quad \text{für } 0 \leq r < R_1$ $H = \frac{I}{2\pi r} \quad \text{für } R_1 \leq r \leq R_2$ $H = \frac{I}{2\pi r} \left( 1 - \frac{r - R_2}{d} \right) \quad \text{für } R_2 < r \leq R_2 + d$ $\Phi = \frac{\mu l}{2\pi} \ln \frac{R_2}{R_1} \Theta$ $\Lambda = \mu \frac{l}{2\pi} \ln \frac{R_2}{R_1}$ $R_m = \frac{2\pi}{\mu l \ln \frac{R_2}{R_1}}$ $L = \mu \frac{l}{2\pi} \ln \frac{R_2}{R_1}$ $L' = \frac{\mu}{2\pi} \ln \frac{R_2}{R_1}$ | <table> <tr> <td><math>H</math></td><td>Feldstärke</td><td><math>\left[ \frac{A}{m} \right]</math></td></tr> <tr> <td><math>r</math></td><td>Abstand von der Leiterachse</td><td><math>\left[ m \right]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>\left[ A \right]</math></td></tr> <tr> <td><math>R</math></td><td>Radius</td><td><math>\left[ m \right]</math></td></tr> <tr> <td><math>d</math></td><td>Manteldicke</td><td><math>\left[ m \right]</math></td></tr> <tr> <td><math>\mu</math></td><td>Permeabilität</td><td><math>\left[ \frac{H}{m} \right]</math></td></tr> <tr> <td><math>l</math></td><td>Länge</td><td><math>\left[ m \right]</math></td></tr> <tr> <td><math>\Lambda</math></td><td>Magnetischer Leitwert</td><td><math>\left[ \frac{Vs}{A} \right], \left[ \Omega s \right]</math></td></tr> <tr> <td><math>R_m</math></td><td>Magnetischer Widerstand</td><td><math>\left[ \frac{A}{Vs} \right]</math></td></tr> <tr> <td><math>L'</math></td><td>Induktivitätsbelag</td><td><math>\left[ \frac{Vs}{Am} \right]</math></td></tr> <tr> <td><math>L</math></td><td>Induktivität</td><td><math>\left[ \frac{Vs}{A} \right]</math></td></tr> <tr> <td><math>\Phi</math></td><td>Magnetischer Fluss</td><td><math>\left[ Vs \right], \left[ Wb \right]</math></td></tr> <tr> <td><math>\Theta</math></td><td>Durchflutung</td><td><math>\left[ A \right]</math></td></tr> </table> | $H$ | Feldstärke | $\left[ \frac{A}{m} \right]$ | $r$ | Abstand von der Leiterachse | $\left[ m \right]$ | $I$ | Strom | $\left[ A \right]$ | $R$ | Radius | $\left[ m \right]$ | $d$ | Manteldicke | $\left[ m \right]$ | $\mu$ | Permeabilität | $\left[ \frac{H}{m} \right]$ | $l$ | Länge | $\left[ m \right]$ | $\Lambda$ | Magnetischer Leitwert | $\left[ \frac{Vs}{A} \right], \left[ \Omega s \right]$ | $R_m$ | Magnetischer Widerstand | $\left[ \frac{A}{Vs} \right]$ | $L'$ | Induktivitätsbelag | $\left[ \frac{Vs}{Am} \right]$ | $L$ | Induktivität | $\left[ \frac{Vs}{A} \right]$ | $\Phi$ | Magnetischer Fluss | $\left[ Vs \right], \left[ Wb \right]$ | $\Theta$ | Durchflutung | $\left[ A \right]$ |
| $H$  | Feldstärke  | $\left[ \frac{A}{m} \right]$  |     |            |                              |     |                             |                    |     |       |                    |     |        |                    |     |             |                    |       |               |                              |     |       |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |        |                    |  |          |              |                    |
| $r$  | Abstand von der Leiterachse   | $\left[ m \right]$  |     |            |                              |     |                             |                    |     |       |                    |     |        |                    |     |             |                    |       |               |                              |     |       |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |        |                    |  |          |              |                    |
| $I$  | Strom   | $\left[ A \right]$  |     |            |                              |     |                             |                    |     |       |                    |     |        |                    |     |             |                    |       |               |                              |     |       |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |        |                    |  |          |              |                    |
| $R$  | Radius  | $\left[ m \right]$  |     |            |                              |     |                             |                    |     |       |                    |     |        |                    |     |             |                    |       |               |                              |     |       |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |        |                    |  |          |              |                    |
| $d$  | Manteldicke   | $\left[ m \right]$  |     |            |                              |     |                             |                    |     |       |                    |     |        |                    |     |             |                    |       |               |                              |     |       |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |        |                    |  |          |              |                    |
| $\mu$  | Permeabilität   | $\left[ \frac{H}{m} \right]$  |     |            |                              |     |                             |                    |     |       |                    |     |        |                    |     |             |                    |       |               |                              |     |       |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |        |                    |  |          |              |                    |
| $l$  | Länge   | $\left[ m \right]$  |     |            |                              |     |                             |                    |     |       |                    |     |        |                    |     |             |                    |       |               |                              |     |       |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |        |                    |  |          |              |                    |
| $\Lambda$  | Magnetischer Leitwert   | $\left[ \frac{Vs}{A} \right], \left[ \Omega s \right]$  |     |            |                              |     |                             |                    |     |       |                    |     |        |                    |     |             |                    |       |               |                              |     |       |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |        |                    |  |          |              |                    |
| $R_m$  | Magnetischer Widerstand   | $\left[ \frac{A}{Vs} \right]$   |     |            |                              |     |                             |                    |     |       |                    |     |        |                    |     |             |                    |       |               |                              |     |       |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |        |                    |  |          |              |                    |
| $L'$   | Induktivitätsbelag  | $\left[ \frac{Vs}{Am} \right]$  |     |            |                              |     |                             |                    |     |       |                    |     |        |                    |     |             |                    |       |               |                              |     |       |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |        |                    |  |          |              |                    |
| $L$  | Induktivität  | $\left[ \frac{Vs}{A} \right]$   |     |            |                              |     |                             |                    |     |       |                    |     |        |                    |     |             |                    |       |               |                              |     |       |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |        |                    |  |          |              |                    |
| $\Phi$   | Magnetischer Fluss  | $\left[ Vs \right], \left[ Wb \right]$  |     |            |                              |     |                             |                    |     |       |                    |     |        |                    |     |             |                    |       |               |                              |     |       |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |        |                    |  |          |              |                    |
| $\Theta$   | Durchflutung  | $\left[ A \right]$  |     |            |                              |     |                             |                    |     |       |                    |     |        |                    |     |             |                    |       |               |                              |     |       |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |        |                    |  |          |              |                    |

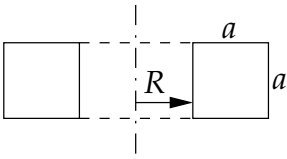
## 12.19.6. Paralleldrahtleitung

|  |   |  |       |               |                              |     |       |                    |     |         |                    |     |        |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |
|--|---|--|-------|---------------|------------------------------|-----|-------|--------------------|-----|---------|--------------------|-----|--------|--------------------|-----------|-----------------------|--|-------|-------------------------|-------------------------------|------|--------------------|--------------------------------|-----|--------------|-------------------------------|
|  <p><math>l \gg a</math><br/><math>a \gg R</math></p> | $\Lambda = \mu \frac{l}{\pi} \ln \frac{a - R}{R}$ $R_m = \frac{\pi}{\mu l \ln \frac{a - R}{R}}$ $L = \mu \frac{l}{\pi} \ln \frac{a - R}{R}$ $L' = \frac{\mu}{2\pi} \ln \frac{a - R}{R}$ | <table> <tr> <td><math>\mu</math></td><td>Permeabilität</td><td><math>\left[ \frac{H}{m} \right]</math></td></tr> <tr> <td><math>l</math></td><td>Länge</td><td><math>\left[ m \right]</math></td></tr> <tr> <td><math>a</math></td><td>Abstand</td><td><math>\left[ m \right]</math></td></tr> <tr> <td><math>R</math></td><td>Radius</td><td><math>\left[ m \right]</math></td></tr> <tr> <td><math>\Lambda</math></td><td>Magnetischer Leitwert</td><td><math>\left[ \frac{Vs}{A} \right], \left[ \Omega s \right]</math></td></tr> <tr> <td><math>R_m</math></td><td>Magnetischer Widerstand</td><td><math>\left[ \frac{A}{Vs} \right]</math></td></tr> <tr> <td><math>L'</math></td><td>Induktivitätsbelag</td><td><math>\left[ \frac{Vs}{Am} \right]</math></td></tr> <tr> <td><math>L</math></td><td>Induktivität</td><td><math>\left[ \frac{Vs}{A} \right]</math></td></tr> </table> | $\mu$ | Permeabilität | $\left[ \frac{H}{m} \right]$ | $l$ | Länge | $\left[ m \right]$ | $a$ | Abstand | $\left[ m \right]$ | $R$ | Radius | $\left[ m \right]$ | $\Lambda$ | Magnetischer Leitwert | $\left[ \frac{Vs}{A} \right], \left[ \Omega s \right]$ | $R_m$ | Magnetischer Widerstand | $\left[ \frac{A}{Vs} \right]$ | $L'$ | Induktivitätsbelag | $\left[ \frac{Vs}{Am} \right]$ | $L$ | Induktivität | $\left[ \frac{Vs}{A} \right]$ |
| $\mu$  | Permeabilität   | $\left[ \frac{H}{m} \right]$   |       |               |                              |     |       |                    |     |         |                    |     |        |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |
| $l$  | Länge   | $\left[ m \right]$   |       |               |                              |     |       |                    |     |         |                    |     |        |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |
| $a$  | Abstand   | $\left[ m \right]$   |       |               |                              |     |       |                    |     |         |                    |     |        |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |
| $R$  | Radius  | $\left[ m \right]$   |       |               |                              |     |       |                    |     |         |                    |     |        |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |
| $\Lambda$  | Magnetischer Leitwert   | $\left[ \frac{Vs}{A} \right], \left[ \Omega s \right]$   |       |               |                              |     |       |                    |     |         |                    |     |        |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |
| $R_m$  | Magnetischer Widerstand   | $\left[ \frac{A}{Vs} \right]$  |       |               |                              |     |       |                    |     |         |                    |     |        |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |
| $L'$   | Induktivitätsbelag  | $\left[ \frac{Vs}{Am} \right]$   |       |               |                              |     |       |                    |     |         |                    |     |        |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |
| $L$  | Induktivität  | $\left[ \frac{Vs}{A} \right]$  |       |               |                              |     |       |                    |     |         |                    |     |        |                    |           |                       |  |       |                         |                               |      |                    |                                |     |              |                               |

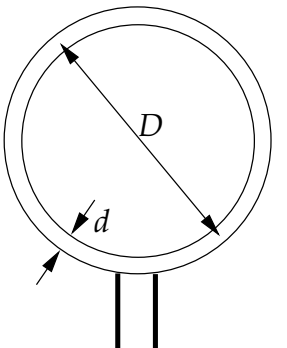
## 12.19.7. Zylinderspule

|  |  |   |
|--|--|---|
|   | <p>Bezugspunkt: Mittelpunkt der Achse im Innern</p> $H = \frac{IN}{\sqrt{l^2 + d^2}}$ <p>Bezugspunkt: Mittelpunkt der Stirnflächen</p> $H = \frac{IN}{2\sqrt{l^2 + d^2}}$  | <p><math>H</math> Feldstärke <math>[\frac{A}{m}]</math><br/> <math>l</math> Länge bzw. mittl. Umfang der Spule <math>[m]</math><br/> <math>d</math> Durchmesser <math>[m]</math><br/> <math>I</math> Strom <math>[A]</math><br/> <math>A</math> Stirnfläche <math>[m^2]</math><br/> <math>\mu</math> Permeabilität <math>[\frac{H}{m}]</math><br/> <math>N</math> Windungszahl <math>[1]</math><br/> <math>\Lambda</math> Magnetischer Leitwert <math>[\frac{Vs}{A}], [\Omega s]</math></p> |
| <p><b>Sehr lange Zylinder-<br/>spule</b> (<math>l \gg d</math>) und<br/><b>Ringspule</b> (mittlerer<br/>Umfang <math>l</math>)</p> | <p>Bezugspunkt für <math>H</math>-Feld:<br/>im Inneren der Spule</p> $H = \frac{IN}{l}$ $\Phi = \frac{\mu A}{l} \Theta = \mu \frac{\pi d^2}{4l} \Theta$ $\Lambda = \mu \frac{A}{l} = \mu \frac{\pi d^2}{4l}$ $R_m = \frac{l}{\mu A} = \frac{4l}{\mu \pi d^2}$ $L = \mu N^2 \frac{A}{l} = \mu N^2 \frac{\pi d^2}{4l}$ | <p><math>R_m</math> Magnetischer Widerstand <math>[\frac{A}{Vs}]</math><br/> <math>L</math> Induktivität <math>[\frac{Vs}{A}]</math><br/> <math>\Phi</math> Magnetischer Fluss <math>[Vs], [Wb]</math><br/> <math>\Theta</math> Durchflutung <math>[A]</math></p>   |

## 12.19.8. Ringspule (Toroid)

|   |  |   |     |                     |       |     |             |       |       |               |                 |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |        |         |
|---|--|---|-----|---------------------|-------|-----|-------------|-------|-------|---------------|-----------------|-----|--------------|-------|-----------|-----------------------|------------------------------|-------|-------------------------|------------------|-----|--------------|------------------|--------|--------------------|--------------|----------|--------------|-------|-----|--------|---------|
|  | $\Lambda = \mu \frac{a}{2\pi} \ln \frac{R+a}{R}$ $R_m = \frac{2\pi}{\mu a \ln \frac{R+a}{R}}$ $L = \mu N^2 \frac{a}{2\pi} \ln \frac{R+a}{R}$ $\Phi = \frac{\mu a}{2\pi} \ln \frac{R_1+a}{R_1} \Theta$ $H = \frac{NI}{R + \frac{a}{2}n}$ <p>für <math>a \ll R</math>:</p> $L = \frac{\mu N^2 A}{2R\pi}$ $\Phi \approx \frac{\mu A}{l_{\text{mittl.}}} \Theta$ | <table> <tr> <td><math>a</math></td><td>Spulen Breite, Höhe</td><td><math>[m]</math></td></tr> <tr> <td><math>R</math></td><td>Innenradius</td><td><math>[m]</math></td></tr> <tr> <td><math>\mu</math></td><td>Permeabilität</td><td><math>[\frac{H}{m}]</math></td></tr> <tr> <td><math>N</math></td><td>Windungszahl</td><td><math>[1]</math></td></tr> <tr> <td><math>\Lambda</math></td><td>Magnetischer Leitwert</td><td><math>[\frac{Vs}{A}], [\Omega s]</math></td></tr> <tr> <td><math>R_m</math></td><td>Magnetischer Widerstand</td><td><math>[\frac{A}{Vs}]</math></td></tr> <tr> <td><math>L</math></td><td>Induktivität</td><td><math>[\frac{Vs}{A}]</math></td></tr> <tr> <td><math>\Phi</math></td><td>Magnetischer Fluss</td><td><math>[Vs], [Wb]</math></td></tr> <tr> <td><math>\Theta</math></td><td>Durchflutung</td><td><math>[A]</math></td></tr> <tr> <td><math>A</math></td><td>Fläche</td><td><math>[m^2]</math></td></tr> </table> | $a$ | Spulen Breite, Höhe | $[m]$ | $R$ | Innenradius | $[m]$ | $\mu$ | Permeabilität | $[\frac{H}{m}]$ | $N$ | Windungszahl | $[1]$ | $\Lambda$ | Magnetischer Leitwert | $[\frac{Vs}{A}], [\Omega s]$ | $R_m$ | Magnetischer Widerstand | $[\frac{A}{Vs}]$ | $L$ | Induktivität | $[\frac{Vs}{A}]$ | $\Phi$ | Magnetischer Fluss | $[Vs], [Wb]$ | $\Theta$ | Durchflutung | $[A]$ | $A$ | Fläche | $[m^2]$ |
| $a$   | Spulen Breite, Höhe  | $[m]$   |     |                     |       |     |             |       |       |               |                 |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |        |         |
| $R$   | Innenradius  | $[m]$   |     |                     |       |     |             |       |       |               |                 |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |        |         |
| $\mu$   | Permeabilität  | $[\frac{H}{m}]$   |     |                     |       |     |             |       |       |               |                 |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |        |         |
| $N$   | Windungszahl   | $[1]$   |     |                     |       |     |             |       |       |               |                 |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |        |         |
| $\Lambda$   | Magnetischer Leitwert  | $[\frac{Vs}{A}], [\Omega s]$  |     |                     |       |     |             |       |       |               |                 |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |        |         |
| $R_m$   | Magnetischer Widerstand  | $[\frac{A}{Vs}]$  |     |                     |       |     |             |       |       |               |                 |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |        |         |
| $L$   | Induktivität   | $[\frac{Vs}{A}]$  |     |                     |       |     |             |       |       |               |                 |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |        |         |
| $\Phi$  | Magnetischer Fluss   | $[Vs], [Wb]$  |     |                     |       |     |             |       |       |               |                 |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |        |         |
| $\Theta$  | Durchflutung   | $[A]$   |     |                     |       |     |             |       |       |               |                 |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |        |         |
| $A$   | Fläche   | $[m^2]$   |     |                     |       |     |             |       |       |               |                 |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |        |                    |              |          |              |       |     |        |         |

## 12.19.9. Kreisrahmenspule

|   |   |   |     |                     |       |       |               |                 |     |                   |       |     |                       |       |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |
|---|---|---|-----|---------------------|-------|-------|---------------|-----------------|-----|-------------------|-------|-----|-----------------------|-------|-----|--------------|-------|-----------|-----------------------|------------------------------|-------|-------------------------|------------------|-----|--------------|------------------|
|  <p><math>D \gg d</math></p> | $\Lambda = \mu \frac{D}{2} \ln \frac{D}{d}$ $R_m = \frac{2}{\mu D \ln \frac{D}{d}}$ $L = \mu N^2 \frac{D}{2} \ln \frac{D}{d}$ | <table> <tr> <td><math>a</math></td><td>Spulen Breite, Höhe</td><td><math>[m]</math></td></tr> <tr> <td><math>\mu</math></td><td>Permeabilität</td><td><math>[\frac{H}{m}]</math></td></tr> <tr> <td><math>d</math></td><td>Draht Durchmesser</td><td><math>[m]</math></td></tr> <tr> <td><math>D</math></td><td>Schleifen Durchmesser</td><td><math>[m]</math></td></tr> <tr> <td><math>N</math></td><td>Windungszahl</td><td><math>[1]</math></td></tr> <tr> <td><math>\Lambda</math></td><td>Magnetischer Leitwert</td><td><math>[\frac{Vs}{A}], [\Omega s]</math></td></tr> <tr> <td><math>R_m</math></td><td>Magnetischer Widerstand</td><td><math>[\frac{A}{Vs}]</math></td></tr> <tr> <td><math>L</math></td><td>Induktivität</td><td><math>[\frac{Vs}{A}]</math></td></tr> </table> | $a$ | Spulen Breite, Höhe | $[m]$ | $\mu$ | Permeabilität | $[\frac{H}{m}]$ | $d$ | Draht Durchmesser | $[m]$ | $D$ | Schleifen Durchmesser | $[m]$ | $N$ | Windungszahl | $[1]$ | $\Lambda$ | Magnetischer Leitwert | $[\frac{Vs}{A}], [\Omega s]$ | $R_m$ | Magnetischer Widerstand | $[\frac{A}{Vs}]$ | $L$ | Induktivität | $[\frac{Vs}{A}]$ |
| $a$   | Spulen Breite, Höhe   | $[m]$   |     |                     |       |       |               |                 |     |                   |       |     |                       |       |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |
| $\mu$   | Permeabilität   | $[\frac{H}{m}]$   |     |                     |       |       |               |                 |     |                   |       |     |                       |       |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |
| $d$   | Draht Durchmesser   | $[m]$   |     |                     |       |       |               |                 |     |                   |       |     |                       |       |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |
| $D$   | Schleifen Durchmesser   | $[m]$   |     |                     |       |       |               |                 |     |                   |       |     |                       |       |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |
| $N$   | Windungszahl  | $[1]$   |     |                     |       |       |               |                 |     |                   |       |     |                       |       |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |
| $\Lambda$   | Magnetischer Leitwert   | $[\frac{Vs}{A}], [\Omega s]$  |     |                     |       |       |               |                 |     |                   |       |     |                       |       |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |
| $R_m$   | Magnetischer Widerstand   | $[\frac{A}{Vs}]$  |     |                     |       |       |               |                 |     |                   |       |     |                       |       |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |
| $L$   | Induktivität  | $[\frac{Vs}{A}]$  |     |                     |       |       |               |                 |     |                   |       |     |                       |       |     |              |       |           |                       |                              |       |                         |                  |     |              |                  |

# 13. Wechselstromlehre

## 13.1. Mittel- und Kennwerte

### 13.1.1. Linearer Mittelwert

|  |  |   |
|--|--|---|
|  | $A_m = \frac{1}{T} \int_{t_1}^{t_1+T} a(t) dt$ | $A$ Amplitude [...]<br>$a(t)$ Signalfunktion [...]<br>$T$ Periodendauer [s]<br>$t$ Zeit [s] |
|--|--|---|

### 13.1.2. Betragsmittelwert

|  |  |   |
|--|--|---|
|  | $A_{ m } = \frac{1}{T} \int_{t_1}^{t_1+T}  a(t)  dt$ | $A$ Amplitude [...]<br>$a(t)$ Signalfunktion [...]<br>$T$ Periodendauer [s]<br>$t$ Zeit [s] |
|--|--|---|

### 13.1.3. Halbwellenmittelwert

|  |   |  |
|--|---|--|
|  | $A_{2m} = \frac{2}{T} \int_{t_1}^{t_1+T} a(t) dt$<br>für $a(t) > 0$ | $A_{2m}$ Halbwellen- [...]<br>mittelwert<br>$a(t)$ Signalfunktion [...]<br>$T$ Periodendauer [s]<br>$t$ Zeit [s] |
|--|---|--|

### 13.1.4. Quadratischer Mittelwert (Effektivwert, RMS)

|  |  |   |
|--|--|---|
|  | $A_{\text{eff}} = \sqrt{\frac{1}{T} \int_{t_1}^{t_1+T} a^2(t) dt}$ <p>für sinunsförmige Signale:</p> $A_{\text{eff}} = \frac{A}{\sqrt{2}}$ | $A$ Amplitude [...]<br>$a(t)$ Signalfunktion [...]<br>$T$ Periodendauer [s]<br>$t$ Zeit [s] |
|--|--|---|

### 13.1.5. Scheitelfaktor (Crestfaktor)

|  |   |  |
|--|---|--|
|  | $k_s = \frac{a_{\text{max}}}{A_{\text{eff}}}$ | $A_{\text{eff}}$ Effektivwert [...]<br>$a_{\text{max}}$ Spitzenwert [...]<br>$k_s$ Crestfaktor [1] |
|--|---|--|

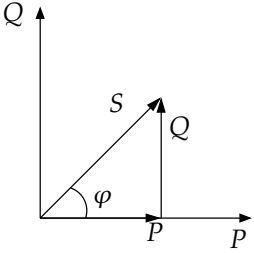
### 13.1.6. Formfaktor

|  |   |   |
|--|---|---|
|  | $k_f = \frac{A_{\text{eff}}}{A_{ \text{m} }}$ | $A_{\text{eff}}$ Effektivwert [...]<br>$A_{ \text{m} }$ Betragsmittelwert [...]<br>$k_f$ Formfaktor [1] |
|--|---|---|

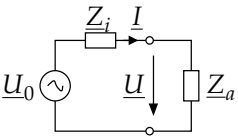
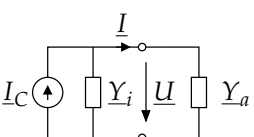
### 13.1.7. Effektivwert eines zusammengesetzten, mehrfrequenten Signals

|  |   |                                     |
|--|---|-------------------------------------|
|  | $A_{\text{eff}} = \sqrt{\sum_{n=0}^N A_{\text{eff}_n}^2}$ | $A_{\text{eff}}$ Effektivwert [...] |
|--|---|-------------------------------------|

## 13.2. Leistung

|   |  |  |     |                |      |     |              |     |     |               |       |     |       |     |     |          |     |           |       |       |     |          |     |
|---|--|--|-----|----------------|------|-----|--------------|-----|-----|---------------|-------|-----|-------|-----|-----|----------|-----|-----------|-------|-------|-----|----------|-----|
|  <p>Beispiel mit Induktiver Last</p> | $\underline{S} = \underline{U} \underline{I}^* = \frac{U^2}{\underline{Z}^*}$ $\underline{S} = P + jQ$ $P = UI \cos(\varphi) = \operatorname{Re}(S)$ $Q = UI \sin(\varphi) = \operatorname{Im}(S)$ $\cos(\varphi) = \frac{P}{S} \quad \sin(\varphi) = \frac{Q}{S}$ | <table> <tr><td><math>S</math></td><td>Scheinleistung</td><td>[VA]</td></tr> <tr><td><math>P</math></td><td>Wirkleistung</td><td>[W]</td></tr> <tr><td><math>Q</math></td><td>Blindleistung</td><td>[Var]</td></tr> <tr><td><math>I</math></td><td>Strom</td><td>[A]</td></tr> <tr><td><math>U</math></td><td>Spannung</td><td>[V]</td></tr> <tr><td><math>\varphi</math></td><td>Phase</td><td>[rad]</td></tr> <tr><td><math>Z</math></td><td>Impedanz</td><td>[Ω]</td></tr> </table> | $S$ | Scheinleistung | [VA] | $P$ | Wirkleistung | [W] | $Q$ | Blindleistung | [Var] | $I$ | Strom | [A] | $U$ | Spannung | [V] | $\varphi$ | Phase | [rad] | $Z$ | Impedanz | [Ω] |
| $S$   | Scheinleistung   | [VA]   |     |                |      |     |              |     |     |               |       |     |       |     |     |          |     |           |       |       |     |          |     |
| $P$   | Wirkleistung   | [W]  |     |                |      |     |              |     |     |               |       |     |       |     |     |          |     |           |       |       |     |          |     |
| $Q$   | Blindleistung  | [Var]  |     |                |      |     |              |     |     |               |       |     |       |     |     |          |     |           |       |       |     |          |     |
| $I$   | Strom  | [A]  |     |                |      |     |              |     |     |               |       |     |       |     |     |          |     |           |       |       |     |          |     |
| $U$   | Spannung   | [V]  |     |                |      |     |              |     |     |               |       |     |       |     |     |          |     |           |       |       |     |          |     |
| $\varphi$   | Phase  | [rad]  |     |                |      |     |              |     |     |               |       |     |       |     |     |          |     |           |       |       |     |          |     |
| $Z$   | Impedanz   | [Ω]  |     |                |      |     |              |     |     |               |       |     |       |     |     |          |     |           |       |       |     |          |     |

### 13.2.1. Leistung und Leistungsanpassung bei Quellen

|   |   |   |     |                |      |     |              |     |     |       |     |     |          |     |     |          |     |     |           |     |     |          |     |     |            |     |     |          |     |
|---|---|---|-----|----------------|------|-----|--------------|-----|-----|-------|-----|-----|----------|-----|-----|----------|-----|-----|-----------|-----|-----|----------|-----|-----|------------|-----|-----|----------|-----|
|   | $\underline{S} = U_0^2 \frac{Z_a}{ Z_i + Z_a ^2}$ $P = U_0^2 \frac{R_a}{(R_a + R_i)^2 + (X_a + X_i)^2}$ <p>Bei Leistungsanpassung:</p> $X_a = -X_i \quad \text{bzw.} \quad R_a = R_i$ $\underline{Z}_a = \underline{Z}_i^*$ $P_{\max} = \frac{U_0^2}{4R_i}$ | <table> <tr><td><math>S</math></td><td>Scheinleistung</td><td>[VA]</td></tr> <tr><td><math>P</math></td><td>Wirkleistung</td><td>[W]</td></tr> <tr><td><math>I</math></td><td>Strom</td><td>[A]</td></tr> <tr><td><math>U</math></td><td>Spannung</td><td>[V]</td></tr> <tr><td><math>Z</math></td><td>Impedanz</td><td>[Ω]</td></tr> <tr><td><math>Y</math></td><td>Admittanz</td><td>[S]</td></tr> <tr><td><math>X</math></td><td>Reaktanz</td><td>[Ω]</td></tr> <tr><td><math>R</math></td><td>Widerstand</td><td>[Ω]</td></tr> <tr><td><math>G</math></td><td>Leitwert</td><td>[S]</td></tr> </table> | $S$ | Scheinleistung | [VA] | $P$ | Wirkleistung | [W] | $I$ | Strom | [A] | $U$ | Spannung | [V] | $Z$ | Impedanz | [Ω] | $Y$ | Admittanz | [S] | $X$ | Reaktanz | [Ω] | $R$ | Widerstand | [Ω] | $G$ | Leitwert | [S] |
| $S$   | Scheinleistung  | [VA]  |     |                |      |     |              |     |     |       |     |     |          |     |     |          |     |     |           |     |     |          |     |     |            |     |     |          |     |
| $P$   | Wirkleistung  | [W]   |     |                |      |     |              |     |     |       |     |     |          |     |     |          |     |     |           |     |     |          |     |     |            |     |     |          |     |
| $I$   | Strom   | [A]   |     |                |      |     |              |     |     |       |     |     |          |     |     |          |     |     |           |     |     |          |     |     |            |     |     |          |     |
| $U$   | Spannung  | [V]   |     |                |      |     |              |     |     |       |     |     |          |     |     |          |     |     |           |     |     |          |     |     |            |     |     |          |     |
| $Z$   | Impedanz  | [Ω]   |     |                |      |     |              |     |     |       |     |     |          |     |     |          |     |     |           |     |     |          |     |     |            |     |     |          |     |
| $Y$   | Admittanz   | [S]   |     |                |      |     |              |     |     |       |     |     |          |     |     |          |     |     |           |     |     |          |     |     |            |     |     |          |     |
| $X$   | Reaktanz  | [Ω]   |     |                |      |     |              |     |     |       |     |     |          |     |     |          |     |     |           |     |     |          |     |     |            |     |     |          |     |
| $R$   | Widerstand  | [Ω]   |     |                |      |     |              |     |     |       |     |     |          |     |     |          |     |     |           |     |     |          |     |     |            |     |     |          |     |
| $G$   | Leitwert  | [S]   |     |                |      |     |              |     |     |       |     |     |          |     |     |          |     |     |           |     |     |          |     |     |            |     |     |          |     |
|  | $\underline{Y}_a = \underline{Y}_i^*$ $P_{\max} = \frac{I_C^2}{4G_i}$   |   |     |                |      |     |              |     |     |       |     |     |          |     |     |          |     |     |           |     |     |          |     |     |            |     |     |          |     |

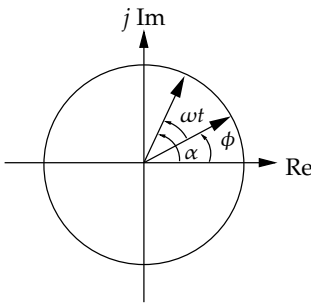
### 13.2.2. Effektivwert und Leistung

|     |   |  |     |          |     |     |            |     |     |          |     |
|-----|---|--|-----|----------|-----|-----|------------|-----|-----|----------|-----|
|     | $P = \frac{U_{\text{eff}}^2}{R} = I_{\text{eff}}^2 R$ | <table> <tr><td><math>P</math></td><td>Leistung</td><td>[W]</td></tr> <tr><td><math>R</math></td><td>Widerstand</td><td>[Ω]</td></tr> <tr><td><math>U</math></td><td>Spannung</td><td>[V]</td></tr> </table> | $P$ | Leistung | [W] | $R$ | Widerstand | [Ω] | $U$ | Spannung | [V] |
| $P$ | Leistung  | [W]  |     |          |     |     |            |     |     |          |     |
| $R$ | Widerstand  | [Ω]  |     |          |     |     |            |     |     |          |     |
| $U$ | Spannung  | [V]  |     |          |     |     |            |     |     |          |     |

### 13.3. Energie

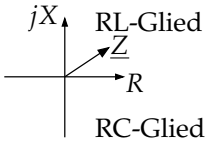
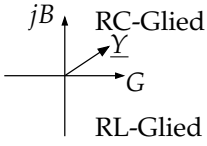
|  |                                 |   |
|--|---------------------------------|---|
|  | $W(t) = \int_0^t P(\tau) d\tau$ | $W$ Energie [J]<br>$P$ Leistung [W]<br>$t$ Zeit [s] |
|--|---------------------------------|---|

### 13.4. Komplexe Darstellung sinusförmiger Vorgänge

|   |  |  |
|---|--|--|
|  | <p>Hintransformation:</p> $a(t) = A \cos(\omega t + \phi)$ $\underline{a}(t) = A \cos(\omega t + \phi) + jA \sin(\omega t + \phi)$ $\underline{a}(t) = A e^{j(\omega t + \phi)} = A e^{j\phi} e^{j\omega t} = \underline{A} e^{j\omega t}$ $\underline{A} = A e^{j\phi}$ <p>Rücktransformation:</p> $\underline{B} = B e^{j\beta}$ $\underline{b}(t) = \underline{B} e^{j\omega t} = B e^{j\beta} e^{j\omega t} = \underline{A} e^{j\omega t}$ $\underline{b}(t) = B \cos(\omega t + \beta) + jB \sin(\omega t + \beta)$ $b(t) = \operatorname{Re}\{\underline{b}(t)\} = B \cos(\omega t + \beta)$ | $A, B$ Amplitude [V]<br>$a, b$ Signal [V]<br>$\phi, \beta$ Phase [rad]<br>$\omega$ Winkelgeschwindigkeit [ $\frac{1}{s}$ ]<br>$t$ Zeit [s] |
|---|--|--|



## 13.5. Komplexe Darstellung von Impedanz und Admittanz

|  |  |   |
|--|--|---|
| <p>Impedanz-Ebene:</p>  <p>Admittanz-Ebene:</p>  | $\underline{Z} = \frac{u(t)}{\underline{i}(t)} = \frac{\underline{U}}{\underline{I}} = \frac{\underline{U}_{\text{eff}}}{\underline{I}_{\text{eff}}} = R + jX$ $\underline{Y} = \frac{1}{\underline{Z}} = G + jB$ $\underline{U} = \underline{Z} \underline{I} \quad \text{bzw.} \quad \underline{I} = \underline{Y} \underline{U}$ $\sum_{\text{Kreis}} \underline{U}_i = 0 \quad \sum_{\text{Trennbündel}} \underline{I}_i = 0$ <p>Serieschaltung: <math>\underline{Z}_s = \sum_{i=1}^N \underline{Z}_i</math></p> <p>Parallelschaltung: <math>\underline{Y}_p = \sum_i^N \underline{Y}_i</math></p> | <p><math>U, u</math> Spannung [V]<br/> <math>I, i</math> Strom [A]<br/> <math>R</math> Widerstand [<math>\Omega</math>]<br/> <math>G</math> Leitwert [S]<br/> <math>Z</math> Impedanz [<math>\Omega</math>]<br/> <math>Y</math> Admittanz [S]<br/> <math>X</math> Reaktanz [<math>\Omega</math>]<br/> <math>B</math> Suszeptanz [S]</p> |
|--|--|---|

## 13.6. Klemmgrößen von Schaltelementen

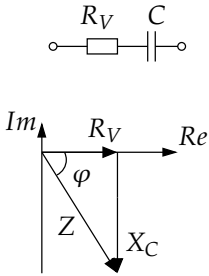
### 13.6.1. Allgemein

|  |  |
|--|--|
| $q(t) = \int_{t_a}^t i(\tau) d\tau$ $i(t) = \frac{dq(t)}{dt}$ $p(t) = \frac{dW}{dt}$ $P_{at} = \frac{1}{t - t_a} \int_{t_a}^t p(\tau) d\tau$ $w_{at} = \int_{t_a}^t p(\tau) d\tau$ | <p><math>i</math> Strom [A]<br/> <math>u</math> Spannung [V]<br/> <math>q</math> Ladung [C]<br/> <math>p</math> Leistung [W]<br/> <math>w</math> Gespeicherte Energie [J]<br/> <math>t, \tau</math> Zeit [s]</p> |
|--|--|

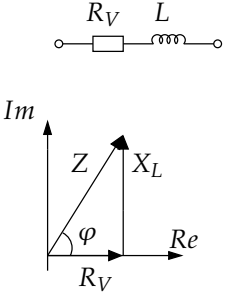
### 13.6.2. Ohm'sche Widerstände

|  |   |   |
|--|---|---|
|  | $u(t) = Ri(t)$<br>$i(t) = Gu(t)$<br>$p(t) = u(t)i(t)$<br>$\underline{Z}_R = R$<br>$\underline{Y}_R = \frac{1}{R} = G$ | $i$ Strom [A]<br>$u$ Spannung [V]<br>$R$ Widerstand [ $\Omega$ ]<br>$G$ Leitwert [S]<br>$p$ Leistung [W]<br>$Z$ Impedanz [ $\Omega$ ]<br>$t, \tau$ Zeit [s] |
|--|---|---|

### 13.6.3. Kapazitäten

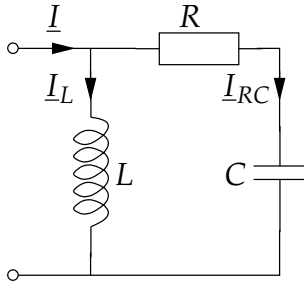
|  |   |  |
|--|---|--|
|  | <p>Zeitbereich:</p> $q(t) = Cu(t)$<br>$i(t) = \frac{dq(t)}{dt} = C \frac{du(t)}{dt}$<br>$u(t) = \frac{1}{C} \int_{t_a}^t i(\tau) d\tau + u(t_a)$<br><p>Frequenzbereich:</p> $\underline{Z}_C = \frac{1}{j\omega C} = \frac{1}{sC} = -j \frac{1}{\omega C}$<br>$\underline{Y}_C = j\omega C = sC$<br><p>Leistung, Energie:</p> $p(t) = u(t)i(t)$<br>$w(t) = \frac{1}{2} Cu^2(t)$ | $i$ Strom [A]<br>$u$ Spannung [V]<br>$q$ Ladung [C]<br>$C$ Kapazität [F]<br>$p$ Leistung [W]<br>$Z$ Impedanz [ $\Omega$ ]<br>$Y$ Admitanz [S]<br>$\omega$ Kreisfrequenz [ $\frac{1}{s}$ ]<br>$s$ Laplaceoperator [1]<br>$w$ Gespeicherte Energie [J]<br>$t, \tau$ Zeit [s] |
|--|---|--|

## 13.6.4. Induktivitäten

|   |   |   |     |       |       |     |          |       |     |        |       |     |          |            |     |          |       |          |               |                 |     |                 |       |     |                   |                  |     |              |                  |        |             |        |     |          |       |     |                      |       |           |      |       |
|---|---|---|-----|-------|-------|-----|----------|-------|-----|--------|-------|-----|----------|------------|-----|----------|-------|----------|---------------|-----------------|-----|-----------------|-------|-----|-------------------|------------------|-----|--------------|------------------|--------|-------------|--------|-----|----------|-------|-----|----------------------|-------|-----------|------|-------|
|  | <p>Zeitbereich:</p> $\Psi(t) = Li(t)$ $u(t) = L \frac{di(t)}{dt}$ $i(t) = \frac{1}{L} \int_{t_a}^t u(\tau) d\tau + i(t_a)$ $p(t) = u(t)i(t)$ $w(t) = \frac{1}{2} Li^2(t)$ <p>Frequenzbereich:</p> $\underline{Z}_L = j\omega L = sL$ $\underline{Y}_L = \frac{1}{j\omega L} = \frac{1}{sL} = -j \frac{1}{\omega L}$ <p>Gegeninduktion:</p> $u_{12}(t) = \pm M \frac{di_2(t)}{dt}$ $\underline{Z}_M = j\omega M$ $\underline{Y}_M = \frac{1}{j\omega M} = -j \frac{1}{\omega M}$ <p>induktive Kopplung:</p> $u_1(t) = L_1 \frac{di_1(t)}{dt} \pm M \frac{di_2(t)}{dt}$ | <table> <tr> <td><math>i</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>u</math></td><td>Spannung</td><td><math>[V]</math></td></tr> <tr> <td><math>q</math></td><td>Ladung</td><td><math>[C]</math></td></tr> <tr> <td><math>Z</math></td><td>Impedanz</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>Y</math></td><td>Admitanz</td><td><math>[S]</math></td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>s</math></td><td>Laplaceoperator</td><td><math>[1]</math></td></tr> <tr> <td><math>M</math></td><td>Gegeninduktivität</td><td><math>[\frac{Vs}{A}]</math></td></tr> <tr> <td><math>L</math></td><td>Induktivität</td><td><math>[\frac{Vs}{A}]</math></td></tr> <tr> <td><math>\Psi</math></td><td>Spulenfluss</td><td><math>[Vs]</math></td></tr> <tr> <td><math>p</math></td><td>Leistung</td><td><math>[W]</math></td></tr> <tr> <td><math>w</math></td><td>Gespeicherte Energie</td><td><math>[J]</math></td></tr> <tr> <td><math>t, \tau</math></td><td>Zeit</td><td><math>[s]</math></td></tr> </table> | $i$ | Strom | $[A]$ | $u$ | Spannung | $[V]$ | $q$ | Ladung | $[C]$ | $Z$ | Impedanz | $[\Omega]$ | $Y$ | Admitanz | $[S]$ | $\omega$ | Kreisfrequenz | $[\frac{1}{s}]$ | $s$ | Laplaceoperator | $[1]$ | $M$ | Gegeninduktivität | $[\frac{Vs}{A}]$ | $L$ | Induktivität | $[\frac{Vs}{A}]$ | $\Psi$ | Spulenfluss | $[Vs]$ | $p$ | Leistung | $[W]$ | $w$ | Gespeicherte Energie | $[J]$ | $t, \tau$ | Zeit | $[s]$ |
| $i$   | Strom   | $[A]$   |     |       |       |     |          |       |     |        |       |     |          |            |     |          |       |          |               |                 |     |                 |       |     |                   |                  |     |              |                  |        |             |        |     |          |       |     |                      |       |           |      |       |
| $u$   | Spannung  | $[V]$   |     |       |       |     |          |       |     |        |       |     |          |            |     |          |       |          |               |                 |     |                 |       |     |                   |                  |     |              |                  |        |             |        |     |          |       |     |                      |       |           |      |       |
| $q$   | Ladung  | $[C]$   |     |       |       |     |          |       |     |        |       |     |          |            |     |          |       |          |               |                 |     |                 |       |     |                   |                  |     |              |                  |        |             |        |     |          |       |     |                      |       |           |      |       |
| $Z$   | Impedanz  | $[\Omega]$  |     |       |       |     |          |       |     |        |       |     |          |            |     |          |       |          |               |                 |     |                 |       |     |                   |                  |     |              |                  |        |             |        |     |          |       |     |                      |       |           |      |       |
| $Y$   | Admitanz  | $[S]$   |     |       |       |     |          |       |     |        |       |     |          |            |     |          |       |          |               |                 |     |                 |       |     |                   |                  |     |              |                  |        |             |        |     |          |       |     |                      |       |           |      |       |
| $\omega$  | Kreisfrequenz   | $[\frac{1}{s}]$   |     |       |       |     |          |       |     |        |       |     |          |            |     |          |       |          |               |                 |     |                 |       |     |                   |                  |     |              |                  |        |             |        |     |          |       |     |                      |       |           |      |       |
| $s$   | Laplaceoperator   | $[1]$   |     |       |       |     |          |       |     |        |       |     |          |            |     |          |       |          |               |                 |     |                 |       |     |                   |                  |     |              |                  |        |             |        |     |          |       |     |                      |       |           |      |       |
| $M$   | Gegeninduktivität   | $[\frac{Vs}{A}]$  |     |       |       |     |          |       |     |        |       |     |          |            |     |          |       |          |               |                 |     |                 |       |     |                   |                  |     |              |                  |        |             |        |     |          |       |     |                      |       |           |      |       |
| $L$   | Induktivität  | $[\frac{Vs}{A}]$  |     |       |       |     |          |       |     |        |       |     |          |            |     |          |       |          |               |                 |     |                 |       |     |                   |                  |     |              |                  |        |             |        |     |          |       |     |                      |       |           |      |       |
| $\Psi$  | Spulenfluss   | $[Vs]$  |     |       |       |     |          |       |     |        |       |     |          |            |     |          |       |          |               |                 |     |                 |       |     |                   |                  |     |              |                  |        |             |        |     |          |       |     |                      |       |           |      |       |
| $p$   | Leistung  | $[W]$   |     |       |       |     |          |       |     |        |       |     |          |            |     |          |       |          |               |                 |     |                 |       |     |                   |                  |     |              |                  |        |             |        |     |          |       |     |                      |       |           |      |       |
| $w$   | Gespeicherte Energie  | $[J]$   |     |       |       |     |          |       |     |        |       |     |          |            |     |          |       |          |               |                 |     |                 |       |     |                   |                  |     |              |                  |        |             |        |     |          |       |     |                      |       |           |      |       |
| $t, \tau$   | Zeit  | $[s]$   |     |       |       |     |          |       |     |        |       |     |          |            |     |          |       |          |               |                 |     |                 |       |     |                   |                  |     |              |                  |        |             |        |     |          |       |     |                      |       |           |      |       |

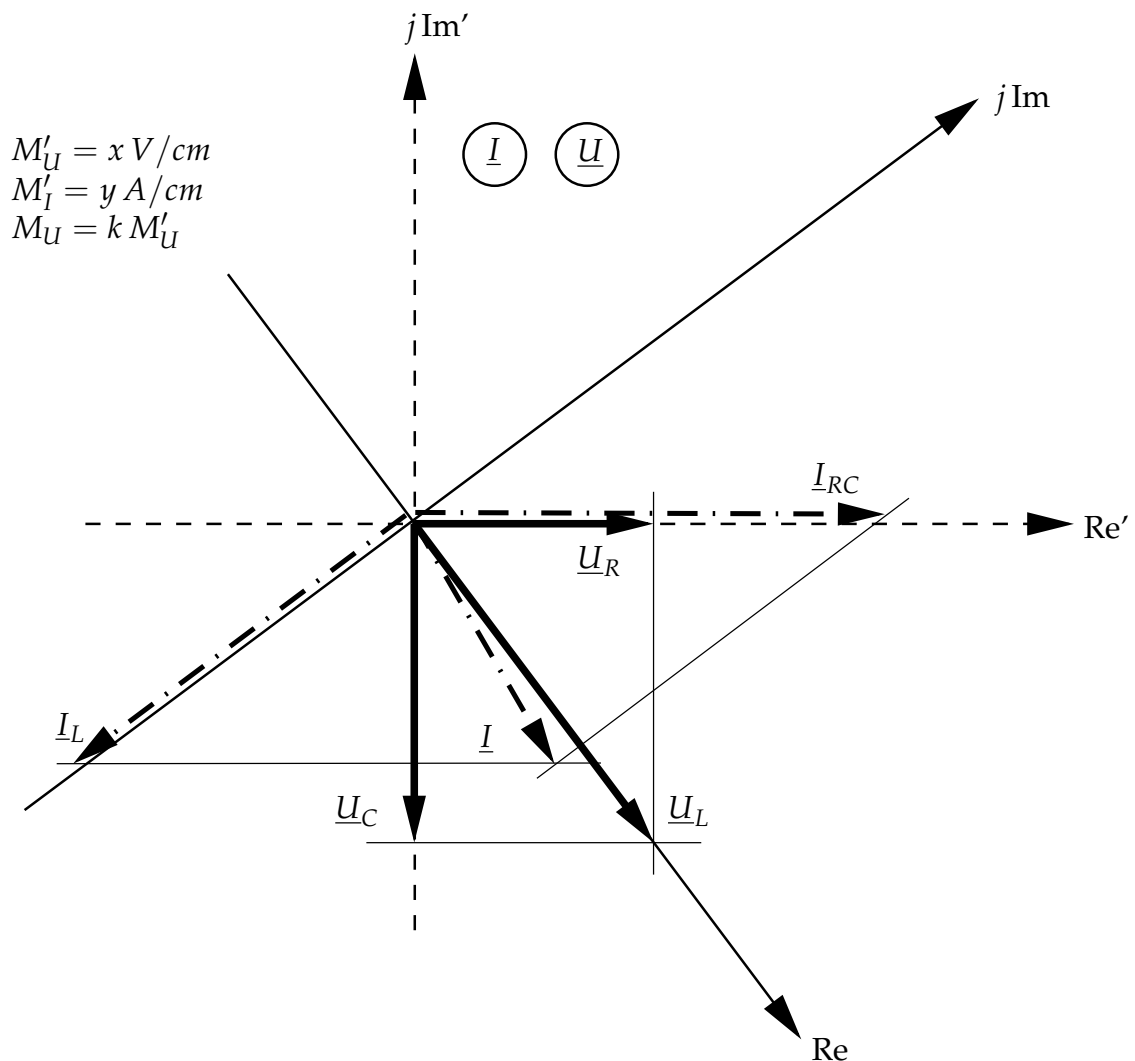
## 13.7. Zeigerdarstellung Komplexer Klemmgrößen

Alle Spannungen und Ströme am folgenden Netzwerk sind graphisch mittels Zeigerdiagramm darzustellen.

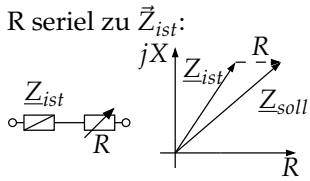
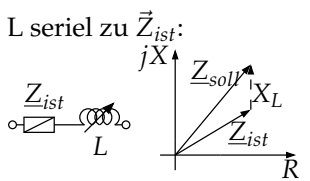
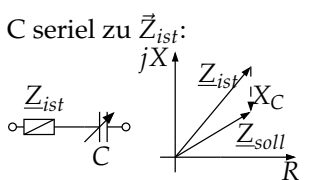
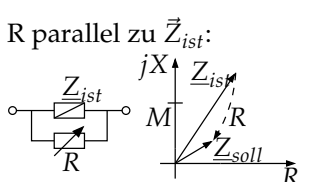
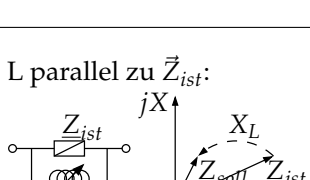
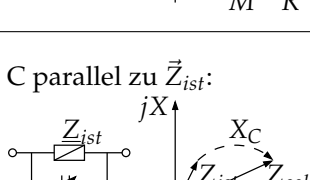


1. Impedanzen  $\underline{Z}$  aller Elemente berechnen.
2. Strom  $\underline{I}_{RC}$  auf reeller Achse  $\text{Re}'$  wählen.
3. Spannungen an  $\underline{R}$  und  $\underline{C}$  aus  $\underline{I}_{RC}$  und  $\underline{Z}$  berechnen und einzeichnen.
4. Spannung  $\underline{U}_L$  entspricht der Summe von  $\underline{U}_R$  und  $\underline{U}_C$ .
5. Strom  $\underline{I}_L$  aus  $\underline{U}_L$  und  $\underline{Z}_L$  berechnen und einzeichnen.
6. Strom  $\underline{I}$  entspricht der Summe von  $\underline{I}_{RC}$  und  $\underline{I}_L$ .
7. Achsen neu bestimmen:  $\text{Re}$  in Richtung  $\underline{U}_L$ .

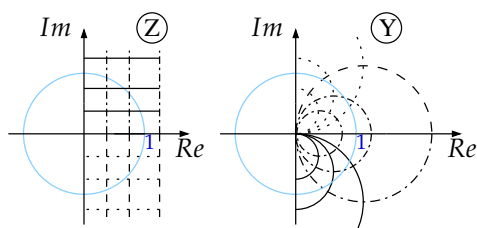
$$\text{Korrekturfaktor: } k = \frac{\underline{U}_{\text{Nenn}}}{\underline{U}_{\text{gemessen}}}$$



### 13.7.1. Impedanztransformation

|  |   |  |
|--|---|--|
| <p>R seriel zu <math>\vec{Z}_{ist}</math>:</p>      | <p><math>\vec{Z}_{soll}</math> bewegt sich auf einer Geraden parallel zur R-Achse nach rechts.</p>  | <p><math>\vec{Z}_{ist}</math> Impedanz, <math>[\Omega]</math> die transformiert werden soll</p> <p><math>\vec{Z}_{soll}</math> Impedanz, <math>[\Omega]</math> nach Transformation</p> <p><math>X</math> Blindwiderstand <math>[\Omega]</math></p> <p><math>R</math> Widerstand <math>[\Omega]</math></p> <p><math>L</math> Induktivität <math>[H]</math></p> <p><math>C</math> Kapazität <math>[F]</math></p> |
| <p>L seriel zu <math>\vec{Z}_{ist}</math>:</p>      | <p><math>\vec{Z}_{soll}</math> bewegt sich auf einer Geraden parallel zur X-Achse nach oben.</p>  |  |
| <p>C seriel zu <math>\vec{Z}_{ist}</math>:</p>      | <p><math>\vec{Z}_{soll}</math> bewegt sich auf einer Geraden parallel zur X-Achse nach unten.</p>   |  |
| <p>R parallel zu <math>\vec{Z}_{ist}</math>:</p>   | <p><math>\vec{Z}_{soll}</math> bewegt sich auf einem Halbkreis, welcher auf der X-Achse beginnt, durch den Endpunkt des <math>\vec{Z}_{ist}</math>-Vektors geht um im Nullpunkt endet. Falls <math>R = 0 \rightarrow \vec{Z}_{soll} = \vec{0}</math>. Falls <math>R = \infty \rightarrow \vec{Z}_{soll} = \vec{Z}_{ist}</math>.</p> |  |
| <p>L parallel zu <math>\vec{Z}_{ist}</math>:</p>  | <p><math>\vec{Z}_{soll}</math> bewegt sich auf einem Kreis mit Mittelpunkt M, welcher durch den Nullpunkt sowie durch den Endpunkt des <math>\vec{Z}_{ist}</math>-Vektors geht. Für <math>L \rightarrow 0 \rightarrow \vec{Z}_{soll} \rightarrow 0</math></p>   |  |
| <p>C parallel zu <math>\vec{Z}_{ist}</math>:</p>  | <p><math>\vec{Z}_{soll}</math> bewegt sich auf einem Kreis mit Mittelpunkt M, welcher durch den Nullpunkt und den Endpunkt des <math>\vec{Z}_{ist}</math>-Vektors geht. Für <math>C \rightarrow \infty \rightarrow \vec{Z}_{soll} \rightarrow 0</math></p>  |  |

### 13.7.2. Transformation von Z-Ebene zu Y-Ebene

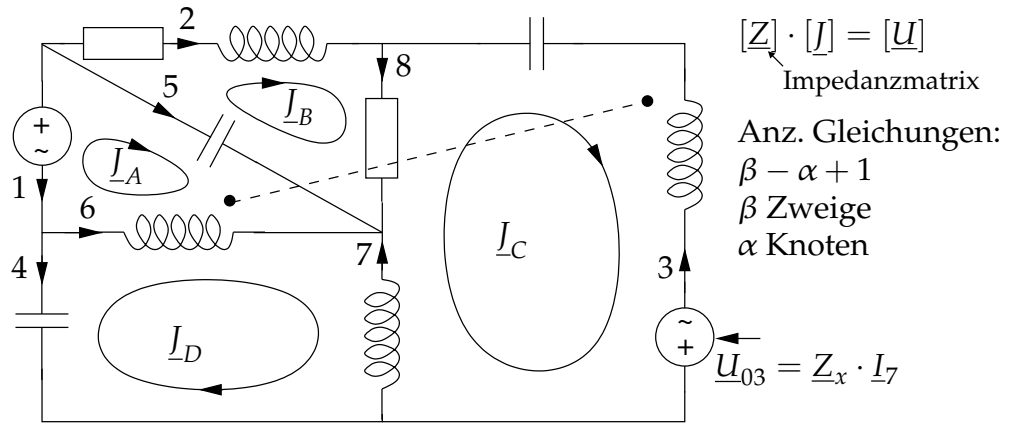


Im Bild ist zu sehen wie gewisse Punktmengen von der Z-Ebene auf die Y-Ebene abgebildet werden.

## 13.8. Netzwerkanalyse

### 13.8.1. Maschenmethode / Kreisstrommethode

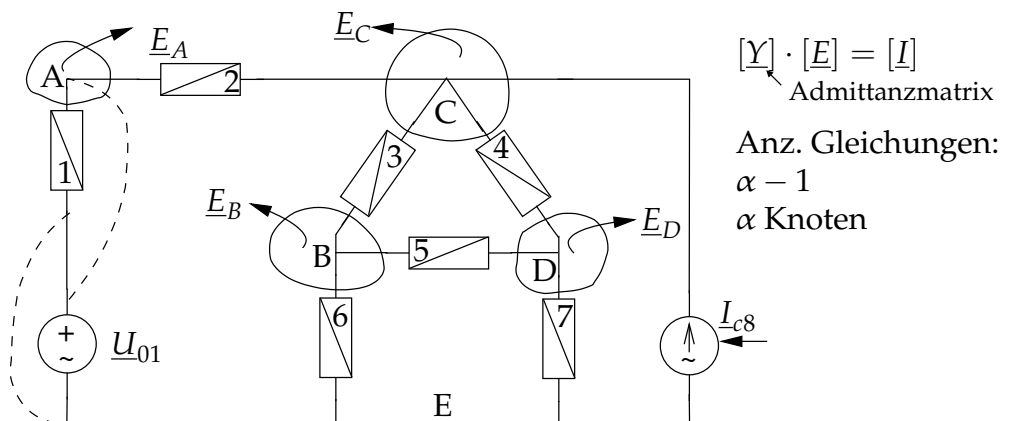
Es dürfen nur Spannungsquellen vorkommen, vorhandene Stromquellen sind zuerst umzuwandeln.



|  |  |  |  |                        |
|--|--|--|--|------------------------|
| $\underline{I}_A$  | $\underline{I}_B$  | $\underline{I}_C$  | $\underline{I}_D$  |                        |
| $+ \underline{I}_A(j\omega L_6 - j\frac{1}{\omega C_5})$ | $- \underline{I}_B(-j\frac{1}{\omega C_5})$                          | $+ \underline{I}_C j\omega M$  | $- \underline{I}_D j\omega L_6$  | $= \underline{U}_{01}$ |
| $- \underline{I}_A(-j\frac{1}{\omega C_5})$              | $+ \underline{I}_B(R_8 + R_2 + j\omega L_2 - j\frac{1}{\omega C_5})$ | $- \underline{I}_C R_8$  | $- 0$  | $= 0$                  |
| $+ \underline{I}_A j\omega M$                            | $- \underline{I}_B R_8$  | $+ \underline{I}_C(R_8 + j\omega L_7 + j\omega L_3 - j\frac{1}{\omega C_3} + \underline{Z}_x)$ | $- \underline{I}_D(\underline{Z}_x + j\omega M + j\omega L_7)$         | $= 0$                  |
| $- \underline{I}_A j\omega L_6$                          | $- 0$  | $- \underline{I}_C(j\omega L_7 + j\omega M)$   | $+ \underline{I}_D(j\omega L_6 + j\omega L_7 - j\frac{1}{\omega C_4})$ | $= 0$                  |

### 13.8.2. Trennbündelmethode / Knotenspannungsmethode

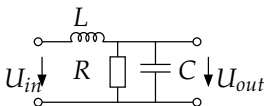
Es dürfen nur Stromquellen vorkommen, vorhandene Spannungsquellen sind zuerst umzuwandeln.



|  |  |  |  |  |
|--|--|--|--|--|
| $\underline{E}_A$                                      | $\underline{E}_B$  | $\underline{E}_C$  | $\underline{E}_D$  |  |
| $+ \underline{E}_A(\underline{Y}_1 + \underline{Y}_2)$ | $- 0$  | $- \underline{E}_C \underline{Y}_2$                                      | $- 0$  | $= \underline{U}_{01} \underline{Y}_1$ |
| $- 0$  | $+ \underline{E}_B(\underline{Y}_3 + \underline{Y}_5 + \underline{Y}_6)$ | $- \underline{E}_C \underline{Y}_3$                                      | $- \underline{E}_D \underline{Y}_5$                                      | $= 0$                                  |
| $- \underline{E}_A \underline{Y}_2$                    | $- \underline{E}_B(\underline{Y}_3 + k \underline{Y}_6)$                 | $+ \underline{E}_C(\underline{Y}_2 + \underline{Y}_3 + \underline{Y}_4)$ | $- \underline{E}_D \underline{Y}_4$                                      | $= 0$                                  |
| $- 0$  | $- \underline{E}_B \underline{Y}_5$                                      | $- \underline{E}_C \underline{Y}_4$                                      | $+ \underline{E}_D(\underline{Y}_4 + \underline{Y}_5 + \underline{Y}_7)$ | $= 0$                                  |

## 13.9. Darstellungsformen

### 13.9.1. Beispiel: Nyquistdiagramm, Ortskurve

|  |   |   |     |                   |  |             |          |                               |     |                |     |                    |     |                 |     |                       |
|--|---|---|-----|-------------------|--|-------------|----------|-------------------------------|-----|----------------|-----|--------------------|-----|-----------------|-----|-----------------------|
|  <p><math>R = 2k\Omega, L = 10mH, C = 10nF</math></p> | $\underline{G} = \frac{U_{out}}{U_{in}}$ $\underline{G} = \frac{1}{1 - \omega^2 LC + j\omega \frac{L}{R}}$ <p>tiefe f (<math>\omega \ll \frac{1}{\sqrt{LC}}</math>): <math>\underline{G} \approx 1 \angle 0^\circ</math><br/>hohe f (<math>\omega \gg \frac{1}{\sqrt{LC}}</math>): <math>\underline{G} \approx \frac{1}{\omega^2 LC} \angle -180^\circ</math><br/>Achsenschnittpunkte:<br/><math>\omega = \frac{1}{\sqrt{LC}}</math>: <math>Re(\underline{G}) = 0 \rightarrow \omega_0 \rightarrow \underline{G} = -2j</math><br/><math>Im(\underline{G}) = 0 \rightarrow \omega \rightarrow 1</math></p> | <table><tr><td><math>G</math></td><td>Frequenzgang, [1]</td></tr><tr><td></td><td>Verstärkung</td></tr><tr><td><math>\omega</math></td><td>Kreisfrequenz <math>[\frac{1}{s}]</math></td></tr><tr><td><math>U</math></td><td>Spannung <math>[V]</math></td></tr><tr><td><math>L</math></td><td>Induktivität <math>[H]</math></td></tr><tr><td><math>C</math></td><td>Kapazität <math>[C]</math></td></tr><tr><td><math>R</math></td><td>Widerstand <math>[\Omega]</math></td></tr></table> | $G$ | Frequenzgang, [1] |  | Verstärkung | $\omega$ | Kreisfrequenz $[\frac{1}{s}]$ | $U$ | Spannung $[V]$ | $L$ | Induktivität $[H]$ | $C$ | Kapazität $[C]$ | $R$ | Widerstand $[\Omega]$ |
| $G$  | Frequenzgang, [1]   |   |     |                   |  |             |          |                               |     |                |     |                    |     |                 |     |                       |
|  | Verstärkung   |   |     |                   |  |             |          |                               |     |                |     |                    |     |                 |     |                       |
| $\omega$   | Kreisfrequenz $[\frac{1}{s}]$   |   |     |                   |  |             |          |                               |     |                |     |                    |     |                 |     |                       |
| $U$  | Spannung $[V]$  |   |     |                   |  |             |          |                               |     |                |     |                    |     |                 |     |                       |
| $L$  | Induktivität $[H]$  |   |     |                   |  |             |          |                               |     |                |     |                    |     |                 |     |                       |
| $C$  | Kapazität $[C]$   |   |     |                   |  |             |          |                               |     |                |     |                    |     |                 |     |                       |
| $R$  | Widerstand $[\Omega]$   |   |     |                   |  |             |          |                               |     |                |     |                    |     |                 |     |                       |

### 13.9.2. Bodediagramm

Vorgehen beim Erstellen eines Bodediagramms:

Netzwerkfunktion aufstellen 
$$F(\omega) = \frac{a_0 + a_1 j\omega + a_2 (j\omega)^2 + \dots + a_n (j\omega)^n}{b_0 + b_1 j\omega + b_2 (j\omega)^2 + \dots + b_n (j\omega)^n}$$

In Produktform 
$$F(\omega) = K_1 \prod_{i=1}^r \text{Standardterme}, \quad K_1 = \frac{a_0}{b_0}$$

Standardterme 
$$\begin{cases} (j\omega T)^n \\ (1 + j\omega T)^n \\ [1 + 2\xi j\omega T + (j\omega)^2 T^2]^n \end{cases} \quad n \pm 1, \pm 2 \dots$$

$$1 + 2\xi j\omega T + (j\omega)^2 T^2 = \begin{cases} (1 + j\omega T_1)(1 + j\omega T_2) & \text{für } \xi > 1 \\ (1 + j\omega T)^2 & \text{für } \xi = 1 \\ \text{nicht aufspaltbar} & \text{für } \xi < 1 \end{cases}$$

Normierung

- Frequenz: Bezugsfrequenz  $\omega_0 = \frac{1}{T_0} \Rightarrow$  normierte Frequenz  $\Omega = \frac{\omega}{\omega_0} = \omega T_0$   
Beispiele:  $\omega_0 = \frac{1}{T} \Rightarrow \Omega = \omega T$  oder  $\omega_0 = 1 \frac{1}{s} \Rightarrow \Omega = \omega 1s$
- Wert: Bezugswert  $K_0 \Rightarrow$  normierte Konstante  $K = \frac{K_1}{K_0}$   
Beispiel:  $K_0 = K_1 \Rightarrow K = 1$

### 13. WECHSELSTROMLEHRE

Normierte Netzwerkfunktion  $F(\omega) \Rightarrow F_n(\Omega) = F_n(\omega T_0) = F_n(\frac{\omega}{\omega_0})$

Normierte Standardterme

$$\begin{aligned} (j\omega T)^n &\Rightarrow (j\Omega \frac{T}{T_0})^n \\ (1 + j\omega T)^n &\Rightarrow (1 + j\Omega \frac{T}{T_0})^n \\ [1 + 2\xi j\omega T + (j\omega)^2 T^2]^n &\Rightarrow \left[ 1 + 2\xi j\Omega \frac{T}{T_0} + (j\Omega)^2 \left(\frac{T}{T_0}\right)^2 \right]^n \end{aligned}$$

Bodediagramm

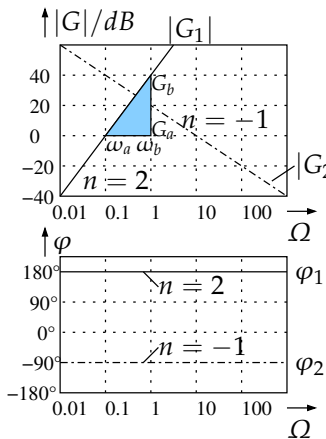
- Betrag, Amplitudengang:  $|G|/dB = \sum_{i=1}^r 20 \log_{10}\{|norm.Standardterme|\} + 20 \log_{10} K$
- Argument, Phasengang:  $\varphi = \sum_{i=1}^r \arg\{norm.Standardterme\}$

**P-Glied: Standardterm  $K$**

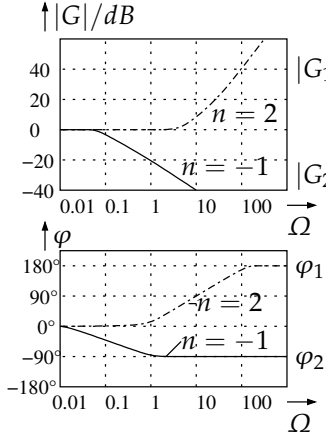
|           |  |   |     |           |       |     |             |        |           |       |            |
|-----------|--|---|-----|-----------|-------|-----|-------------|--------|-----------|-------|------------|
|           | <p>Form:</p> <p><math>K</math></p> <p>Steigung: <math>0 \frac{dB}{DK}</math></p> <p><math> G  = 20 \log_{10}(K)</math></p> <p><math>\varphi = const_0</math></p> | <table> <tr> <td><math>K</math></td><td>Konstante</td><td><math>[1]</math></td></tr> <tr> <td><math>G</math></td><td>Verstärkung</td><td><math>[dB]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Phase</td><td><math>[^\circ]</math></td></tr> </table> | $K$ | Konstante | $[1]$ | $G$ | Verstärkung | $[dB]$ | $\varphi$ | Phase | $[^\circ]$ |
| $K$       | Konstante  | $[1]$   |     |           |       |     |             |        |           |       |            |
| $G$       | Verstärkung  | $[dB]$  |     |           |       |     |             |        |           |       |            |
| $\varphi$ | Phase  | $[^\circ]$  |     |           |       |     |             |        |           |       |            |



**I-Glied: Standardterm**  $(j\omega T)^n$ 

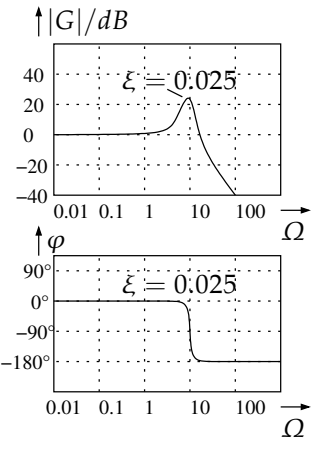
|   |   |  |     |           |     |     |             |      |           |       |     |     |          |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
|---|---|--|-----|-----------|-----|-----|-------------|------|-----------|-------|-----|-----|----------|-----|-----|---------|-----|-------|---------|-----|----------|---------------|-------------------|----------|--------------------|-----|
|  <p>Siehe auch S.165</p> | <p>Form:</p> $(j\omega T)^n$ <p>Normalisiert:</p> $(j\omega \frac{T}{T_0})^n$ <p>Amplitude: <math> G  = \omega^n T^n</math><br/> <math> G  \Rightarrow</math> Gerade<br/> Steigung: <math>n \cdot 20 \frac{dB}{DK}</math><br/> Falls <math>\Omega = \frac{T_0}{T} \Rightarrow  G  = 0</math></p> <p>Phase <math>\varphi</math>:<br/> <math>\varphi = n \cdot 90^\circ</math></p> $\frac{G_a}{G_b} = \frac{\omega_a^n}{\omega_b^n}, \quad n = \pm 1$ | <table> <tr> <td><math>K</math></td><td>Konstante</td><td>[1]</td></tr> <tr> <td><math>G</math></td><td>Verstärkung</td><td>[dB]</td></tr> <tr> <td><math>\varphi</math></td><td>Phase</td><td>[°]</td></tr> <tr> <td><math>n</math></td><td>Exponent</td><td>[1]</td></tr> <tr> <td><math>T</math></td><td>Periode</td><td>[s]</td></tr> <tr> <td><math>T_0</math></td><td>Periode</td><td>[s]</td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td>[<math>\frac{1}{s}</math>]</td></tr> <tr> <td><math>\Omega</math></td><td>Normierte Frequenz</td><td>[1]</td></tr> </table> | $K$ | Konstante | [1] | $G$ | Verstärkung | [dB] | $\varphi$ | Phase | [°] | $n$ | Exponent | [1] | $T$ | Periode | [s] | $T_0$ | Periode | [s] | $\omega$ | Kreisfrequenz | [ $\frac{1}{s}$ ] | $\Omega$ | Normierte Frequenz | [1] |
| $K$   | Konstante   | [1]  |     |           |     |     |             |      |           |       |     |     |          |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
| $G$   | Verstärkung   | [dB]   |     |           |     |     |             |      |           |       |     |     |          |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
| $\varphi$   | Phase   | [°]  |     |           |     |     |             |      |           |       |     |     |          |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
| $n$   | Exponent  | [1]  |     |           |     |     |             |      |           |       |     |     |          |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
| $T$   | Periode   | [s]  |     |           |     |     |             |      |           |       |     |     |          |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
| $T_0$   | Periode   | [s]  |     |           |     |     |             |      |           |       |     |     |          |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
| $\omega$  | Kreisfrequenz   | [ $\frac{1}{s}$ ]  |     |           |     |     |             |      |           |       |     |     |          |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
| $\Omega$  | Normierte Frequenz  | [1]  |     |           |     |     |             |      |           |       |     |     |          |     |     |         |     |       |         |     |          |               |                   |          |                    |     |

**PT<sub>1</sub>-Glied: Standardterm**  $(1 + j\omega T)^n$ 

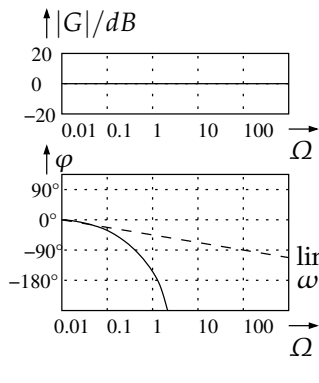
|   |   |   |     |           |     |     |             |      |           |       |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
|---|---|---|-----|-----------|-----|-----|-------------|------|-----------|-------|-----|-----|---------|-----|-------|---------|-----|----------|---------------|-------------------|----------|--------------------|-----|
|  | <p>Form:</p> $(1 + j\omega T)^n$ <p>Normalisiert:</p> $\left(1 + j\Omega \frac{T}{T_0}\right)^n$ <p>Amplitude: <math> G  = \sqrt{1 + \omega^2 T^2}</math><br/> Für <math>\Omega \ll \frac{T_0}{T} :  G  \approx 0 \frac{dB}{DK}</math><br/> Für <math>\Omega \gg \frac{T_0}{T} :  G  \approx n \cdot 20 \frac{dB}{DK}</math><br/> Knick:<br/> Bei <math>\Omega = \frac{T_0}{T} :  G  = n \cdot 3dB</math></p> <p>Phase: <math>\varphi = \arctan(\omega T)</math><br/> Für <math>\Omega \ll \frac{T_0}{T} : \varphi \approx 0^\circ</math><br/> Für <math>\Omega \gg \frac{T_0}{T} : \varphi \approx n \cdot 90^\circ</math><br/> Für <math>\Omega = \frac{T_0}{T} : \varphi = n \cdot 45^\circ</math></p> | <table> <tr> <td><math>K</math></td><td>Konstante</td><td>[1]</td></tr> <tr> <td><math>G</math></td><td>Verstärkung</td><td>[dB]</td></tr> <tr> <td><math>\varphi</math></td><td>Phase</td><td>[°]</td></tr> <tr> <td><math>T</math></td><td>Periode</td><td>[s]</td></tr> <tr> <td><math>T_0</math></td><td>Periode</td><td>[s]</td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td>[<math>\frac{1}{s}</math>]</td></tr> <tr> <td><math>\Omega</math></td><td>Normierte Frequenz</td><td>[1]</td></tr> </table> | $K$ | Konstante | [1] | $G$ | Verstärkung | [dB] | $\varphi$ | Phase | [°] | $T$ | Periode | [s] | $T_0$ | Periode | [s] | $\omega$ | Kreisfrequenz | [ $\frac{1}{s}$ ] | $\Omega$ | Normierte Frequenz | [1] |
| $K$   | Konstante   | [1]   |     |           |     |     |             |      |           |       |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
| $G$   | Verstärkung   | [dB]  |     |           |     |     |             |      |           |       |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
| $\varphi$   | Phase   | [°]   |     |           |     |     |             |      |           |       |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
| $T$   | Periode   | [s]   |     |           |     |     |             |      |           |       |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
| $T_0$   | Periode   | [s]   |     |           |     |     |             |      |           |       |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
| $\omega$  | Kreisfrequenz   | [ $\frac{1}{s}$ ]   |     |           |     |     |             |      |           |       |     |     |         |     |       |         |     |          |               |                   |          |                    |     |
| $\Omega$  | Normierte Frequenz  | [1]   |     |           |     |     |             |      |           |       |     |     |         |     |       |         |     |          |               |                   |          |                    |     |

### 13. WECHSELSTROMLEHRE

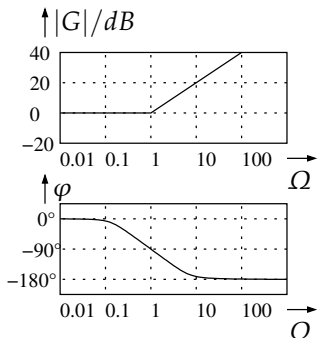
**PT<sub>2</sub>-Glied: Standardterm**  $\frac{1}{1+2\xi j\omega T+(j\omega T)^2}$

|   |   |  |     |           |     |     |             |      |           |       |     |     |         |     |               |                  |       |       |         |     |       |          |     |          |               |       |          |                    |     |
|---|---|--|-----|-----------|-----|-----|-------------|------|-----------|-------|-----|-----|---------|-----|---------------|------------------|-------|-------|---------|-----|-------|----------|-----|----------|---------------|-------|----------|--------------------|-----|
|  <p>Je kleiner <math>\xi</math> ist, desto schneller springt die Phase</p> | <p>Form:</p> $\frac{1}{1 + 2\xi j\omega T + (j\omega)^2 T^2}$ <p>Normalisiert:</p> $\frac{1}{1 + 2\xi j\Omega \frac{T}{T_0} + (j\Omega)^2 \left(\frac{T}{T_0}\right)^2}$ <p>Amplitude:</p> $ G  = \sqrt{(1 - \omega^2 T^2)^2 + (\omega 2\xi T)^2}$ <p>Für <math>\Omega \ll \frac{T_0}{T} :  G  \approx 0 \frac{dB}{DK}</math><br/>         Für <math>\Omega \gg \frac{T_0}{T} :  G  \approx -40 \frac{dB}{DK}</math><br/>         Überspringen, Knick:<br/> <math>\Omega = \frac{T_0}{T} :  G  = -20 \log_{10}(2\xi)</math></p> <p>Phase: <math>\varphi = \arctan\left(\frac{\omega 2\xi T}{1 - \omega^2 T^2}\right)</math><br/>         Für <math>\Omega \ll \frac{T_0}{T} : \varphi \approx 0^\circ</math><br/>         Für <math>\Omega \gg \frac{T_0}{T} : \varphi \approx -180^\circ</math><br/>         Für <math>\Omega = \frac{T_0}{T} : \varphi = -90^\circ</math></p> | <table> <tr> <td><math>K</math></td><td>Konstante</td><td>[1]</td></tr> <tr> <td><math>G</math></td><td>Verstärkung</td><td>[dB]</td></tr> <tr> <td><math>\varphi</math></td><td>Phase</td><td>[°]</td></tr> <tr> <td><math>T</math></td><td>Periode</td><td>[s]</td></tr> <tr> <td><math>\frac{1}{T}</math></td><td>Resonanzfrequenz</td><td>[1/s]</td></tr> <tr> <td><math>T_0</math></td><td>Periode</td><td>[s]</td></tr> <tr> <td><math>\xi</math></td><td>Dämpfung</td><td>[1]</td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td>[1/s]</td></tr> <tr> <td><math>\Omega</math></td><td>Normierte Frequenz</td><td>[1]</td></tr> </table> | $K$ | Konstante | [1] | $G$ | Verstärkung | [dB] | $\varphi$ | Phase | [°] | $T$ | Periode | [s] | $\frac{1}{T}$ | Resonanzfrequenz | [1/s] | $T_0$ | Periode | [s] | $\xi$ | Dämpfung | [1] | $\omega$ | Kreisfrequenz | [1/s] | $\Omega$ | Normierte Frequenz | [1] |
| $K$   | Konstante   | [1]  |     |           |     |     |             |      |           |       |     |     |         |     |               |                  |       |       |         |     |       |          |     |          |               |       |          |                    |     |
| $G$   | Verstärkung   | [dB]   |     |           |     |     |             |      |           |       |     |     |         |     |               |                  |       |       |         |     |       |          |     |          |               |       |          |                    |     |
| $\varphi$   | Phase   | [°]  |     |           |     |     |             |      |           |       |     |     |         |     |               |                  |       |       |         |     |       |          |     |          |               |       |          |                    |     |
| $T$   | Periode   | [s]  |     |           |     |     |             |      |           |       |     |     |         |     |               |                  |       |       |         |     |       |          |     |          |               |       |          |                    |     |
| $\frac{1}{T}$   | Resonanzfrequenz  | [1/s]  |     |           |     |     |             |      |           |       |     |     |         |     |               |                  |       |       |         |     |       |          |     |          |               |       |          |                    |     |
| $T_0$   | Periode   | [s]  |     |           |     |     |             |      |           |       |     |     |         |     |               |                  |       |       |         |     |       |          |     |          |               |       |          |                    |     |
| $\xi$   | Dämpfung  | [1]  |     |           |     |     |             |      |           |       |     |     |         |     |               |                  |       |       |         |     |       |          |     |          |               |       |          |                    |     |
| $\omega$  | Kreisfrequenz   | [1/s]  |     |           |     |     |             |      |           |       |     |     |         |     |               |                  |       |       |         |     |       |          |     |          |               |       |          |                    |     |
| $\Omega$  | Normierte Frequenz  | [1]  |     |           |     |     |             |      |           |       |     |     |         |     |               |                  |       |       |         |     |       |          |     |          |               |       |          |                    |     |

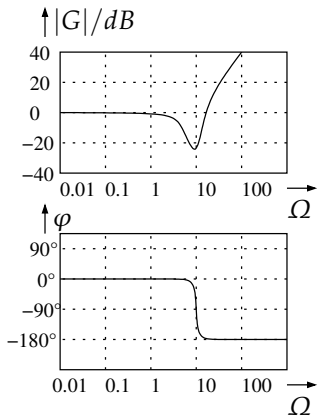
**Totzeitglied: Standardterm**  $e^{-j\omega T_t}$

|   |   |   |       |         |     |     |             |      |           |       |     |
|---|---|---|-------|---------|-----|-----|-------------|------|-----------|-------|-----|
|  | <p>Form:</p> $e^{-j\omega T_t}$ <p>Steigung: <math>0 \frac{dB}{DK}</math></p> <p><math> G  = 1</math></p> <p><math>\varphi = -\omega T_t</math></p> | <table> <tr> <td><math>T_t</math></td><td>Totzeit</td><td>[s]</td></tr> <tr> <td><math>G</math></td><td>Verstärkung</td><td>[dB]</td></tr> <tr> <td><math>\varphi</math></td><td>Phase</td><td>[°]</td></tr> </table> | $T_t$ | Totzeit | [s] | $G$ | Verstärkung | [dB] | $\varphi$ | Phase | [°] |
| $T_t$   | Totzeit   | [s]   |       |         |     |     |             |      |           |       |     |
| $G$   | Verstärkung   | [dB]  |       |         |     |     |             |      |           |       |     |
| $\varphi$   | Phase   | [°]   |       |         |     |     |             |      |           |       |     |

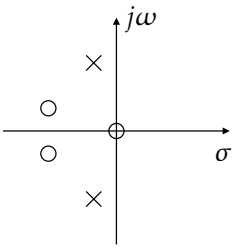
**Irregulärer Aufwärtsknick 1. Ordnung: Standardterm  $(1 - j\omega T)^n$** 

|   |  |  |     |           |       |     |             |        |           |       |            |     |         |       |       |         |       |          |               |                 |          |                    |       |
|---|--|--|-----|-----------|-------|-----|-------------|--------|-----------|-------|------------|-----|---------|-------|-------|---------|-------|----------|---------------|-----------------|----------|--------------------|-------|
|  | <p>Form:</p> $(1 - j\omega T)^n$ <p>Normalisiert:</p> $\left(1 - j\Omega \frac{T}{T_0}\right)^n$ <p>Amplitude: <math> G  = \sqrt{1 + \omega^2 T^2}</math><br/> Für <math>\Omega \ll \frac{T_0}{T}</math>: <math> G  \approx 0 \frac{dB}{DK}</math><br/> Für <math>\Omega \gg \frac{T_0}{T}</math>: <math> G  \approx n \cdot 20 \frac{dB}{DK}</math><br/> Knick:<br/> Bei <math>\Omega = \frac{T_0}{T}</math>: <math> G  = n \cdot 3dB</math></p> <p>Phase: <math>\varphi = -\arctan(\omega T)</math><br/> Für <math>\Omega \ll \frac{T_0}{T}</math>: <math>\varphi \approx 0^\circ</math><br/> Für <math>\Omega \gg \frac{T_0}{T}</math>: <math>\varphi \approx n \cdot 90^\circ</math><br/> Für <math>\Omega = \frac{T_0}{T}</math>: <math>\varphi = n \cdot 45^\circ</math></p> | <table> <tr> <td><math>K</math></td><td>Konstante</td><td><math>[1]</math></td></tr> <tr> <td><math>G</math></td><td>Verstärkung</td><td><math>[dB]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Phase</td><td><math>[^\circ]</math></td></tr> <tr> <td><math>T</math></td><td>Periode</td><td><math>[s]</math></td></tr> <tr> <td><math>T_0</math></td><td>Periode</td><td><math>[s]</math></td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>\Omega</math></td><td>Normierte Frequenz</td><td><math>[1]</math></td></tr> </table> | $K$ | Konstante | $[1]$ | $G$ | Verstärkung | $[dB]$ | $\varphi$ | Phase | $[^\circ]$ | $T$ | Periode | $[s]$ | $T_0$ | Periode | $[s]$ | $\omega$ | Kreisfrequenz | $[\frac{1}{s}]$ | $\Omega$ | Normierte Frequenz | $[1]$ |
| $K$   | Konstante  | $[1]$  |     |           |       |     |             |        |           |       |            |     |         |       |       |         |       |          |               |                 |          |                    |       |
| $G$   | Verstärkung  | $[dB]$   |     |           |       |     |             |        |           |       |            |     |         |       |       |         |       |          |               |                 |          |                    |       |
| $\varphi$   | Phase  | $[^\circ]$   |     |           |       |     |             |        |           |       |            |     |         |       |       |         |       |          |               |                 |          |                    |       |
| $T$   | Periode  | $[s]$  |     |           |       |     |             |        |           |       |            |     |         |       |       |         |       |          |               |                 |          |                    |       |
| $T_0$   | Periode  | $[s]$  |     |           |       |     |             |        |           |       |            |     |         |       |       |         |       |          |               |                 |          |                    |       |
| $\omega$  | Kreisfrequenz  | $[\frac{1}{s}]$  |     |           |       |     |             |        |           |       |            |     |         |       |       |         |       |          |               |                 |          |                    |       |
| $\Omega$  | Normierte Frequenz   | $[1]$  |     |           |       |     |             |        |           |       |            |     |         |       |       |         |       |          |               |                 |          |                    |       |

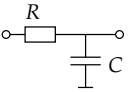
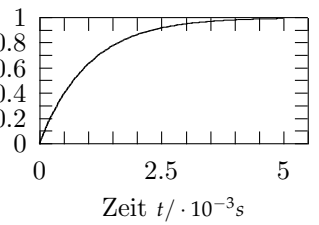
**Irregulärer Aufwärtsknick 2. Ordnung: Standardterm  $1 - 2\xi j\omega T + (j\omega T)^2$** 

|   |  |  |     |           |       |     |             |        |           |       |            |     |         |       |               |                  |                 |       |         |       |       |          |       |          |               |                 |          |                    |       |
|---|--|--|-----|-----------|-------|-----|-------------|--------|-----------|-------|------------|-----|---------|-------|---------------|------------------|-----------------|-------|---------|-------|-------|----------|-------|----------|---------------|-----------------|----------|--------------------|-------|
|  <p>Je kleiner <math>\xi</math> ist, desto schneller springt die Phase</p> | <p>Form:</p> $1 - 2\xi j\omega T + (j\omega T)^2$ <p>Normalisiert:</p> $1 - 2\xi j\Omega \frac{T}{T_0} + (j\Omega)^2 \left(\frac{T}{T_0}\right)^2$ <p>Amplitude<br/> <math> G  = \sqrt{(1 - \omega^2 T^2)^2 + (\omega 2\xi T)^2}</math><br/> Für <math>\Omega \ll \frac{T_0}{T}</math>: <math> G  \approx 0 \frac{dB}{DK}</math><br/> Für <math>\Omega \gg \frac{T_0}{T}</math>: <math> G  \approx +40 \frac{dB}{DK}</math><br/> Überschwingen, Knick:<br/> <math>\Omega = \frac{T_0}{T}</math>: <math> G  = -20 \log_{10}(2\xi)</math></p> <p>Phase: <math>\varphi = -\arctan\left(\frac{\omega 2\xi T}{1 - \omega^2 T^2}\right)</math><br/> Für <math>\Omega \ll \frac{T_0}{T}</math>: <math>\varphi \approx 0^\circ</math><br/> Für <math>\Omega \gg \frac{T_0}{T}</math>: <math>\varphi \approx -180^\circ</math><br/> Für <math>\Omega = \frac{T_0}{T}</math>: <math>\varphi = -90^\circ</math></p> | <table> <tr> <td><math>K</math></td><td>Konstante</td><td><math>[1]</math></td></tr> <tr> <td><math>G</math></td><td>Verstärkung</td><td><math>[dB]</math></td></tr> <tr> <td><math>\varphi</math></td><td>Phase</td><td><math>[^\circ]</math></td></tr> <tr> <td><math>T</math></td><td>Periode</td><td><math>[s]</math></td></tr> <tr> <td><math>\frac{1}{T}</math></td><td>Resonanzfrequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>T_0</math></td><td>Periode</td><td><math>[s]</math></td></tr> <tr> <td><math>\xi</math></td><td>Dämpfung</td><td><math>[1]</math></td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>\Omega</math></td><td>Normierte Frequenz</td><td><math>[1]</math></td></tr> </table> | $K$ | Konstante | $[1]$ | $G$ | Verstärkung | $[dB]$ | $\varphi$ | Phase | $[^\circ]$ | $T$ | Periode | $[s]$ | $\frac{1}{T}$ | Resonanzfrequenz | $[\frac{1}{s}]$ | $T_0$ | Periode | $[s]$ | $\xi$ | Dämpfung | $[1]$ | $\omega$ | Kreisfrequenz | $[\frac{1}{s}]$ | $\Omega$ | Normierte Frequenz | $[1]$ |
| $K$   | Konstante  | $[1]$  |     |           |       |     |             |        |           |       |            |     |         |       |               |                  |                 |       |         |       |       |          |       |          |               |                 |          |                    |       |
| $G$   | Verstärkung  | $[dB]$   |     |           |       |     |             |        |           |       |            |     |         |       |               |                  |                 |       |         |       |       |          |       |          |               |                 |          |                    |       |
| $\varphi$   | Phase  | $[^\circ]$   |     |           |       |     |             |        |           |       |            |     |         |       |               |                  |                 |       |         |       |       |          |       |          |               |                 |          |                    |       |
| $T$   | Periode  | $[s]$  |     |           |       |     |             |        |           |       |            |     |         |       |               |                  |                 |       |         |       |       |          |       |          |               |                 |          |                    |       |
| $\frac{1}{T}$   | Resonanzfrequenz   | $[\frac{1}{s}]$  |     |           |       |     |             |        |           |       |            |     |         |       |               |                  |                 |       |         |       |       |          |       |          |               |                 |          |                    |       |
| $T_0$   | Periode  | $[s]$  |     |           |       |     |             |        |           |       |            |     |         |       |               |                  |                 |       |         |       |       |          |       |          |               |                 |          |                    |       |
| $\xi$   | Dämpfung   | $[1]$  |     |           |       |     |             |        |           |       |            |     |         |       |               |                  |                 |       |         |       |       |          |       |          |               |                 |          |                    |       |
| $\omega$  | Kreisfrequenz  | $[\frac{1}{s}]$  |     |           |       |     |             |        |           |       |            |     |         |       |               |                  |                 |       |         |       |       |          |       |          |               |                 |          |                    |       |
| $\Omega$  | Normierte Frequenz   | $[1]$  |     |           |       |     |             |        |           |       |            |     |         |       |               |                  |                 |       |         |       |       |          |       |          |               |                 |          |                    |       |

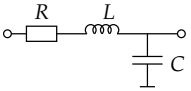
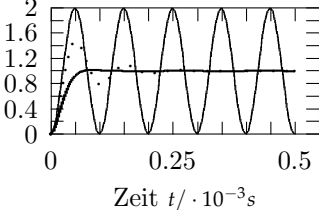
## 13.9.3. Pol- Nullstellendiagramm

|   |   |  |     |           |     |     |                             |     |          |         |     |          |               |                 |     |             |     |     |             |     |
|---|---|--|-----|-----------|-----|-----|-----------------------------|-----|----------|---------|-----|----------|---------------|-----------------|-----|-------------|-----|-----|-------------|-----|
|  <p>Ausser <math>K</math> ist die gesamte Netzwerkfunktion aus dem Pol- Nullstellendiagramm ersichtlich.</p> | $s = \sigma + j\omega$ (Frequenzgang: $\sigma=0$ )<br>Netzwerkfunktion:<br>$F(s) = K \frac{(s - p_1)(s - p_2) \dots (s - p_n)}{(s - q_1)(s - q_2) \dots (s - q_n)}$ Nullstellen $\Rightarrow \times$ in Diagramm<br>Polstellen $\Rightarrow \circ$ in Diagramm<br><br>Pol nahe an $j\omega$ -Achse $\Rightarrow$ Überhöhung im Amplitudengang | <table> <tr> <td><math>K</math></td><td>Konstante</td><td>[1]</td></tr> <tr> <td><math>s</math></td><td>komplexe Frequenz (Laplace)</td><td>[1]</td></tr> <tr> <td><math>\sigma</math></td><td><math>Re(s)</math></td><td>[1]</td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>p</math></td><td>von Polynom</td><td>[1]</td></tr> <tr> <td><math>q</math></td><td>von Polynom</td><td>[1]</td></tr> </table> | $K$ | Konstante | [1] | $s$ | komplexe Frequenz (Laplace) | [1] | $\sigma$ | $Re(s)$ | [1] | $\omega$ | Kreisfrequenz | $[\frac{1}{s}]$ | $p$ | von Polynom | [1] | $q$ | von Polynom | [1] |
| $K$   | Konstante   | [1]  |     |           |     |     |                             |     |          |         |     |          |               |                 |     |             |     |     |             |     |
| $s$   | komplexe Frequenz (Laplace)   | [1]  |     |           |     |     |                             |     |          |         |     |          |               |                 |     |             |     |     |             |     |
| $\sigma$  | $Re(s)$   | [1]  |     |           |     |     |                             |     |          |         |     |          |               |                 |     |             |     |     |             |     |
| $\omega$  | Kreisfrequenz   | $[\frac{1}{s}]$  |     |           |     |     |                             |     |          |         |     |          |               |                 |     |             |     |     |             |     |
| $p$   | von Polynom   | [1]  |     |           |     |     |                             |     |          |         |     |          |               |                 |     |             |     |     |             |     |
| $q$   | von Polynom   | [1]  |     |           |     |     |                             |     |          |         |     |          |               |                 |     |             |     |     |             |     |

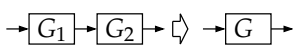
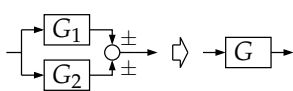
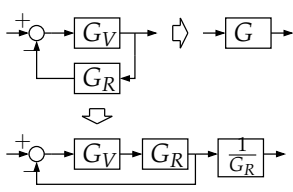
13.10. Eigenschaften des PT<sub>1</sub>-Glied

|  |  |   |     |             |      |     |         |     |          |               |                 |     |            |            |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
|--|--|---|-----|-------------|------|-----|---------|-----|----------|---------------|-----------------|-----|------------|------------|-----|-----------|-----|-------|-----------|-----|----------|-----------|-----|-----|------|-----|-----|--------|-----|
| <p>Beispielschaltung</p>  <p>Sprungantwort:</p>  | $G = \frac{1}{1 + j\omega T}$ <p>Beispiel:</p> $T = RC$ <p>Sprungantwort:</p> $u_o = k \left[ 1 - e^{-\frac{t}{T}} \right]$ $T \dot{u}_o + u_o = k u_{in}$ | <table> <tr> <td><math>G</math></td><td>Verstärkung</td><td>[dB]</td></tr> <tr> <td><math>T</math></td><td>Periode</td><td>[s]</td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td><math>[\frac{1}{s}]</math></td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>C</math></td><td>Kapazität</td><td>[F]</td></tr> <tr> <td><math>u_o</math></td><td>u-Ausgang</td><td>[V]</td></tr> <tr> <td><math>u_{in}</math></td><td>u-Eingang</td><td>[V]</td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td>[s]</td></tr> <tr> <td><math>k</math></td><td>Faktor</td><td>[1]</td></tr> </table> | $G$ | Verstärkung | [dB] | $T$ | Periode | [s] | $\omega$ | Kreisfrequenz | $[\frac{1}{s}]$ | $R$ | Widerstand | $[\Omega]$ | $C$ | Kapazität | [F] | $u_o$ | u-Ausgang | [V] | $u_{in}$ | u-Eingang | [V] | $t$ | Zeit | [s] | $k$ | Faktor | [1] |
| $G$  | Verstärkung  | [dB]  |     |             |      |     |         |     |          |               |                 |     |            |            |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $T$  | Periode  | [s]   |     |             |      |     |         |     |          |               |                 |     |            |            |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $\omega$   | Kreisfrequenz  | $[\frac{1}{s}]$   |     |             |      |     |         |     |          |               |                 |     |            |            |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $R$  | Widerstand   | $[\Omega]$  |     |             |      |     |         |     |          |               |                 |     |            |            |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $C$  | Kapazität  | [F]   |     |             |      |     |         |     |          |               |                 |     |            |            |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $u_o$  | u-Ausgang  | [V]   |     |             |      |     |         |     |          |               |                 |     |            |            |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $u_{in}$   | u-Eingang  | [V]   |     |             |      |     |         |     |          |               |                 |     |            |            |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $t$  | Zeit   | [s]   |     |             |      |     |         |     |          |               |                 |     |            |            |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $k$  | Faktor   | [1]   |     |             |      |     |         |     |          |               |                 |     |            |            |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |

## 13.11. Eigenschaften des PT<sub>2</sub>-Glied

|   |   |  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
|---|---|--|-----|-------------|------|-----------|-------|-------|-----|---------|-----|-------|----------|-----|----------|---------------|-------|------------|---------------|-------|------------|---------------|-------|------------|------------------|-------|-----|------------|-----|-----|--------------|-----|-----|-----------|-----|-------|-----------|-----|----------|-----------|-----|-----|------|-----|-----|--------|-----|
| <p>Beispielschaltung</p>  <p>Sprungantwort:</p> <p><math>R = 100\Omega</math> ———</p> <p><math>R = 30\Omega</math> ·····</p> <p><math>R = 0\Omega</math> ———</p>  | $G = \frac{1}{1 + 2\xi j\omega T + (j\omega)^2 T^2}$ <p>Beispiel:</p> $\xi = \frac{R}{2} \sqrt{\frac{C}{L}} \quad T = \sqrt{LC}$ <p>Je kleiner <math>\xi</math> desto mehr schwingt die Schaltung. Bei aktiven Schaltungen kann <math>\xi &lt; 0</math> werden.</p> $\omega_e = \omega_0 \sqrt{1 - D^2}, \quad 0 < D < 1$ $\omega_0 = \frac{1}{T}$ $\omega_r = \omega_0 \sqrt{1 - 2D^2}, \quad D < 0.707$ <p>Sprungantwort:</p> $u_o = k \left[ 1 - \frac{1}{\sqrt{1 - \xi^2}} e^{-\xi \omega_0 t} x \right]$ $x = \sin \left\{ \sqrt{1 - \xi^2} \omega_0 t + \arccos(\xi) \right\}$ $T^2 \ddot{u}_o + 2\xi T \dot{u}_o + u_o = k u_{in}$ | <table> <tr> <td><math>G</math></td><td>Verstärkung</td><td>[dB]</td></tr> <tr> <td><math>\varphi</math></td><td>Phase</td><td>[rad]</td></tr> <tr> <td><math>T</math></td><td>Periode</td><td>[s]</td></tr> <tr> <td><math>\xi</math></td><td>Dämpfung</td><td>[1]</td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td>[1/s]</td></tr> <tr> <td><math>\omega_e</math></td><td>Eigenfrequenz</td><td>[1/s]</td></tr> <tr> <td><math>\omega_0</math></td><td>Knickfrequenz</td><td>[1/s]</td></tr> <tr> <td><math>\omega_r</math></td><td>Resonanzfrequenz</td><td>[1/s]</td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td>[Ω]</td></tr> <tr> <td><math>L</math></td><td>Induktivität</td><td>[H]</td></tr> <tr> <td><math>C</math></td><td>Kapazität</td><td>[F]</td></tr> <tr> <td><math>u_o</math></td><td>u-Ausgang</td><td>[V]</td></tr> <tr> <td><math>u_{in}</math></td><td>u-Eingang</td><td>[V]</td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td>[s]</td></tr> <tr> <td><math>k</math></td><td>Faktor</td><td>[1]</td></tr> </table> | $G$ | Verstärkung | [dB] | $\varphi$ | Phase | [rad] | $T$ | Periode | [s] | $\xi$ | Dämpfung | [1] | $\omega$ | Kreisfrequenz | [1/s] | $\omega_e$ | Eigenfrequenz | [1/s] | $\omega_0$ | Knickfrequenz | [1/s] | $\omega_r$ | Resonanzfrequenz | [1/s] | $R$ | Widerstand | [Ω] | $L$ | Induktivität | [H] | $C$ | Kapazität | [F] | $u_o$ | u-Ausgang | [V] | $u_{in}$ | u-Eingang | [V] | $t$ | Zeit | [s] | $k$ | Faktor | [1] |
| $G$   | Verstärkung   | [dB]   |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $\varphi$   | Phase   | [rad]  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $T$   | Periode   | [s]  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $\xi$   | Dämpfung  | [1]  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $\omega$  | Kreisfrequenz   | [1/s]  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $\omega_e$  | Eigenfrequenz   | [1/s]  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $\omega_0$  | Knickfrequenz   | [1/s]  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $\omega_r$  | Resonanzfrequenz  | [1/s]  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $R$   | Widerstand  | [Ω]  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $L$   | Induktivität  | [H]  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $C$   | Kapazität   | [F]  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $u_o$   | u-Ausgang   | [V]  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $u_{in}$  | u-Eingang   | [V]  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $t$   | Zeit  | [s]  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |
| $k$   | Faktor  | [1]  |     |             |      |           |       |       |     |         |     |       |          |     |          |               |       |            |               |       |            |               |       |            |                  |       |     |            |     |     |              |     |     |           |     |       |           |     |          |           |     |     |      |     |     |        |     |

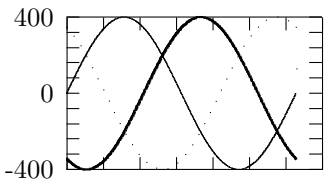
## 13.12. Verknüpfung von Blockdiagrammen

|   |  |                                   |
|---|--|-----------------------------------|
|  | $G = G_1 \cdot G_2$  | $G$ Übertragungsfunktion      [1] |
|  | $G = G_1 \pm G_2$  |                                   |
|  | $G = \frac{G_V}{1 + G_V G_R}$<br>$G = \frac{1}{G_R} \frac{G_V G_R}{1 + G_V G_R}$ |                                   |

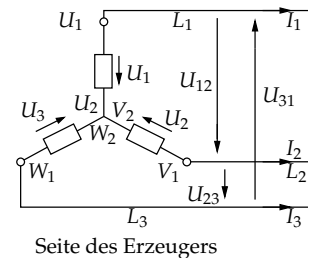
**Teil III.**

**Energie und Antriebstechnik**

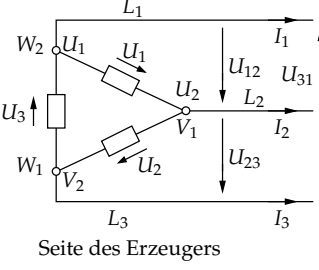
# 14. Dreiphasensysteme

|   |   |  |
|---|---|--|
|  | <p>Maschensatz:</p> $\underline{U}_1 + \underline{U}_2 + \underline{U}_3 = 0$ $\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$ | <p><math>\underline{U}</math> Spannung [V]<br/>(komplex)</p> <p><math>\underline{I}</math> Strom (komplex) [A]</p> |
|---|---|--|

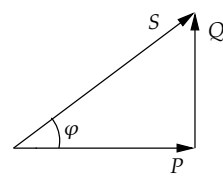
## 14.1. Sternschaltung

|  |   |   |
|--|---|---|
|  <p>Seite des Erzeugers</p> | <p>Strang-Sternspannungen:</p> $\underline{U}_{Str1} = \underline{U}_1 = \underline{U}_1 - \underline{U}_2$ $\underline{U}_{Str2} = \underline{U}_2 = \underline{U}_2 - \underline{U}_3$ $\underline{U}_{Str3} = \underline{U}_3 = \underline{U}_3 - \underline{U}_1$ <p>Aussenleiterspannungen:</p> $\underline{U}_{12} = \underline{U}_1 - \underline{U}_2 \quad \angle(\underline{U}_1, \underline{U}_2) = 120^\circ$ $\underline{U}_{23} = \underline{U}_2 - \underline{U}_3$ $\underline{U}_{31} = \underline{U}_3 - \underline{U}_1$ $U = U_{Str} \sqrt{3}$ $I = I_{Str}$ | <p><math>\underline{U}</math> Spannung [V]</p> <p><math>\underline{U}_{Str}</math> Strangspannung [V]</p> <p><math>\underline{I}</math> Strom [A]</p> |
|--|---|---|

## 14.2. Dreieckschaltung

|  |  |  |                 |          |     |           |                |     |                 |       |     |
|--|--|--|-----------------|----------|-----|-----------|----------------|-----|-----------------|-------|-----|
|  <p>Seite des Erzeugers</p> | <p>Strang-Sternspannungen:</p> $U_{Str1} = \underline{U}_1 = \underline{U}_1 - \underline{U}_2$ $U_{Str2} = \underline{U}_2 = \underline{V}_1 - \underline{V}_2$ $U_{Str3} = \underline{U}_3 = \underline{W}_1 - \underline{W}_2$ <p>Aussenleiterspannungen:</p> $\underline{U}_{12} = \underline{U}_1 \quad \angle(\underline{U}_1, \underline{U}_2) = 120^\circ$ $\underline{U}_{23} = \underline{U}_2$ $\underline{U}_{31} = \underline{U}_3$ $U = U_{Str}$ $I = 2I_{Str} \cos(30^\circ) = I_{Str}\sqrt{3}$ | <table> <tr> <td><math>\underline{U}</math></td><td>Spannung</td><td>[V]</td></tr> <tr> <td><math>U_{Str}</math></td><td>Strangspannung</td><td>[V]</td></tr> <tr> <td><math>\underline{I}</math></td><td>Strom</td><td>[A]</td></tr> </table> | $\underline{U}$ | Spannung | [V] | $U_{Str}$ | Strangspannung | [V] | $\underline{I}$ | Strom | [A] |
| $\underline{U}$  | Spannung   | [V]  |                 |          |     |           |                |     |                 |       |     |
| $U_{Str}$  | Strangspannung   | [V]  |                 |          |     |           |                |     |                 |       |     |
| $\underline{I}$  | Strom  | [A]  |                 |          |     |           |                |     |                 |       |     |

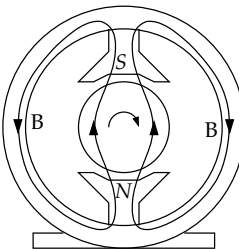
### 14.2.1. Leistungen bei Stern- und Dreieckschaltung

|  |   |  |     |          |     |     |       |     |     |                |      |     |              |     |     |               |       |     |            |      |       |             |        |     |      |     |
|--|---|--|-----|----------|-----|-----|-------|-----|-----|----------------|------|-----|--------------|-----|-----|---------------|-------|-----|------------|------|-------|-------------|--------|-----|------|-----|
|  | $S_{Str} = U_{Str} I_{Str}$ $S = 3S_{Str} = \sqrt{3}UI$ $P = S \cos(\varphi) = \sqrt{3}UI \cos(\varphi)$ $Q = S \sin(\varphi) = \sqrt{3}UI \sin(\varphi)$ $W = Pt = \sqrt{3}UI \cos(\varphi)t$ $W_b = Qt = \sqrt{3}UI \sin(\varphi)t$ | <table> <tr> <td><math>U</math></td><td>Spannung</td><td>[V]</td></tr> <tr> <td><math>I</math></td><td>Strom</td><td>[A]</td></tr> <tr> <td><math>S</math></td><td>Scheinleistung</td><td>[VA]</td></tr> <tr> <td><math>P</math></td><td>Wirkleistung</td><td>[W]</td></tr> <tr> <td><math>Q</math></td><td>Blindleistung</td><td>[Var]</td></tr> <tr> <td><math>W</math></td><td>Wirkarbeit</td><td>[Ws]</td></tr> <tr> <td><math>W_b</math></td><td>Blindarbeit</td><td>[Vars]</td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td>[s]</td></tr> </table> | $U$ | Spannung | [V] | $I$ | Strom | [A] | $S$ | Scheinleistung | [VA] | $P$ | Wirkleistung | [W] | $Q$ | Blindleistung | [Var] | $W$ | Wirkarbeit | [Ws] | $W_b$ | Blindarbeit | [Vars] | $t$ | Zeit | [s] |
| $U$  | Spannung  | [V]  |     |          |     |     |       |     |     |                |      |     |              |     |     |               |       |     |            |      |       |             |        |     |      |     |
| $I$  | Strom   | [A]  |     |          |     |     |       |     |     |                |      |     |              |     |     |               |       |     |            |      |       |             |        |     |      |     |
| $S$  | Scheinleistung  | [VA]   |     |          |     |     |       |     |     |                |      |     |              |     |     |               |       |     |            |      |       |             |        |     |      |     |
| $P$  | Wirkleistung  | [W]  |     |          |     |     |       |     |     |                |      |     |              |     |     |               |       |     |            |      |       |             |        |     |      |     |
| $Q$  | Blindleistung   | [Var]  |     |          |     |     |       |     |     |                |      |     |              |     |     |               |       |     |            |      |       |             |        |     |      |     |
| $W$  | Wirkarbeit  | [Ws]   |     |          |     |     |       |     |     |                |      |     |              |     |     |               |       |     |            |      |       |             |        |     |      |     |
| $W_b$  | Blindarbeit   | [Vars]   |     |          |     |     |       |     |     |                |      |     |              |     |     |               |       |     |            |      |       |             |        |     |      |     |
| $t$  | Zeit  | [s]  |     |          |     |     |       |     |     |                |      |     |              |     |     |               |       |     |            |      |       |             |        |     |      |     |

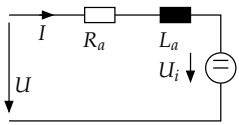


# 15. Elektromotoren und Generatoren

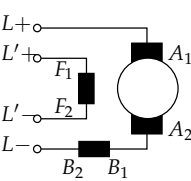
## 15.1. Allgemein

|   |   |   |     |        |       |     |       |       |     |           |                  |     |       |       |     |             |       |     |             |       |          |                     |         |       |                 |       |        |             |        |          |               |                   |       |                     |       |     |          |       |
|---|---|---|-----|--------|-------|-----|-------|-------|-----|-----------|------------------|-----|-------|-------|-----|-------------|-------|-----|-------------|-------|----------|---------------------|---------|-------|-----------------|-------|--------|-------------|--------|----------|---------------|-------------------|-------|---------------------|-------|-----|----------|-------|
|  | $M = Fr$ $F = BIlz\alpha$ $M = BIlz\alpha r = BIC_m$ $C_m = lz\alpha r$ $U_i = Bl\omega z\alpha$ $\omega = 2\pi n$ $U_i \approx \phi n C_n$ $C_n = 2\pi r z \alpha$ | <table> <tr> <td><math>r</math></td><td>Radius</td><td><math>[m]</math></td></tr> <tr> <td><math>F</math></td><td>Kraft</td><td><math>[N]</math></td></tr> <tr> <td><math>B</math></td><td>Induktion</td><td><math>[\frac{Vs}{m}]</math></td></tr> <tr> <td><math>I</math></td><td>Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>l</math></td><td>Leiterlänge</td><td><math>[m]</math></td></tr> <tr> <td><math>z</math></td><td>Anz. Leiter</td><td><math>[1]</math></td></tr> <tr> <td><math>\alpha</math></td><td>Polbedeckungswinkel</td><td><math>[rad]</math></td></tr> <tr> <td><math>C_m</math></td><td>Maschinenkonst.</td><td><math>[1]</math></td></tr> <tr> <td><math>\Phi</math></td><td>magn. Fluss</td><td><math>[Wb]</math></td></tr> <tr> <td><math>\omega</math></td><td>Winkelgeschw.</td><td><math>[\frac{rad}{s}]</math></td></tr> <tr> <td><math>U_i</math></td><td>U-Generator / Motor</td><td><math>[V]</math></td></tr> <tr> <td><math>n</math></td><td>Drehzahl</td><td><math>[1]</math></td></tr> </table> | $r$ | Radius | $[m]$ | $F$ | Kraft | $[N]$ | $B$ | Induktion | $[\frac{Vs}{m}]$ | $I$ | Strom | $[A]$ | $l$ | Leiterlänge | $[m]$ | $z$ | Anz. Leiter | $[1]$ | $\alpha$ | Polbedeckungswinkel | $[rad]$ | $C_m$ | Maschinenkonst. | $[1]$ | $\Phi$ | magn. Fluss | $[Wb]$ | $\omega$ | Winkelgeschw. | $[\frac{rad}{s}]$ | $U_i$ | U-Generator / Motor | $[V]$ | $n$ | Drehzahl | $[1]$ |
| $r$   | Radius  | $[m]$   |     |        |       |     |       |       |     |           |                  |     |       |       |     |             |       |     |             |       |          |                     |         |       |                 |       |        |             |        |          |               |                   |       |                     |       |     |          |       |
| $F$   | Kraft   | $[N]$   |     |        |       |     |       |       |     |           |                  |     |       |       |     |             |       |     |             |       |          |                     |         |       |                 |       |        |             |        |          |               |                   |       |                     |       |     |          |       |
| $B$   | Induktion   | $[\frac{Vs}{m}]$  |     |        |       |     |       |       |     |           |                  |     |       |       |     |             |       |     |             |       |          |                     |         |       |                 |       |        |             |        |          |               |                   |       |                     |       |     |          |       |
| $I$   | Strom   | $[A]$   |     |        |       |     |       |       |     |           |                  |     |       |       |     |             |       |     |             |       |          |                     |         |       |                 |       |        |             |        |          |               |                   |       |                     |       |     |          |       |
| $l$   | Leiterlänge   | $[m]$   |     |        |       |     |       |       |     |           |                  |     |       |       |     |             |       |     |             |       |          |                     |         |       |                 |       |        |             |        |          |               |                   |       |                     |       |     |          |       |
| $z$   | Anz. Leiter   | $[1]$   |     |        |       |     |       |       |     |           |                  |     |       |       |     |             |       |     |             |       |          |                     |         |       |                 |       |        |             |        |          |               |                   |       |                     |       |     |          |       |
| $\alpha$  | Polbedeckungswinkel   | $[rad]$   |     |        |       |     |       |       |     |           |                  |     |       |       |     |             |       |     |             |       |          |                     |         |       |                 |       |        |             |        |          |               |                   |       |                     |       |     |          |       |
| $C_m$   | Maschinenkonst.   | $[1]$   |     |        |       |     |       |       |     |           |                  |     |       |       |     |             |       |     |             |       |          |                     |         |       |                 |       |        |             |        |          |               |                   |       |                     |       |     |          |       |
| $\Phi$  | magn. Fluss   | $[Wb]$  |     |        |       |     |       |       |     |           |                  |     |       |       |     |             |       |     |             |       |          |                     |         |       |                 |       |        |             |        |          |               |                   |       |                     |       |     |          |       |
| $\omega$  | Winkelgeschw.   | $[\frac{rad}{s}]$   |     |        |       |     |       |       |     |           |                  |     |       |       |     |             |       |     |             |       |          |                     |         |       |                 |       |        |             |        |          |               |                   |       |                     |       |     |          |       |
| $U_i$   | U-Generator / Motor   | $[V]$   |     |        |       |     |       |       |     |           |                  |     |       |       |     |             |       |     |             |       |          |                     |         |       |                 |       |        |             |        |          |               |                   |       |                     |       |     |          |       |
| $n$   | Drehzahl  | $[1]$   |     |        |       |     |       |       |     |           |                  |     |       |       |     |             |       |     |             |       |          |                     |         |       |                 |       |        |             |        |          |               |                   |       |                     |       |     |          |       |

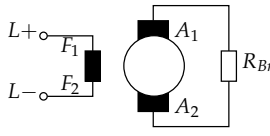
## 15.2. Gleichstrommaschine

|  |   |  |
|--|---|--|
|  <p>Ersatzschaltbild Ankerkreis</p> | <p>Falls <math>U &gt; U_i \rightarrow</math> Motorbetrieb, sonst Genratorbetrieb</p> $U_i = k_1 \Phi n$ $U = U_i + R_A I L_a \frac{dI}{dt}$ $I = \frac{U - U_i}{R_A} (\text{stationär})$ $n_0 = \frac{U}{k_1 \Phi}$ $P_{el} = U_i I \pm \overset{Mot}{Gen} (I^2 R_A)$ $M = \frac{k_1}{2\pi} \Phi I = \frac{P_{mech}}{2\pi n}$ $M = k_2 \Phi I$ $n = \underbrace{\frac{U}{k_1 \Phi}}_{\text{Leerlaufterm}} - \underbrace{\frac{R_A M}{k_1 k_2 \Phi^2}}_{\text{Lastterm}}$ $M_A = \frac{k_2 \Phi U}{R_A}$ | <p><math>U_i</math> Ankerspannung induziert [V]</p> <p><math>U</math> Ankerspannung [V]</p> <p><math>I</math> Strom [A]</p> <p><math>n</math> Drehzahl [1]</p> <p><math>n_0</math> n-Leerlauf [1]</p> <p><math>P</math> Leistung [W]</p> <p><math>R_A</math> R-Anker [<math>\Omega</math>]</p> <p><math>L_a</math> L-Anker [<math>\Omega</math>]</p> <p><math>\Phi</math> magn. Fluss [Wb]</p> <p><math>M</math> Drehmoment [Nm]</p> <p><math>M_A</math> M-Anlauf [Nm]</p> <p><math>k_1</math> Maschinenkonst. [1]</p> <p><math>k_2</math> Maschinenkonst. [1]</p> |
|--|---|--|

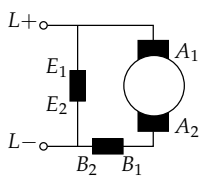
### 15.2.1. Fremderregte Gleichstrommaschine (GNSM)

|   |   |   |
|---|---|---|
|  | $M = \frac{k_2 \Phi U}{R_A} - \frac{k_1 k_2 \Phi^2 n}{R_A}$ <p>Drehzahlsteuerung:</p> <ol style="list-style-type: none"> <li>1. Änderung des Erregerfeldes</li> <li>2. Änderung der Ankerspannung</li> <li>3. Vergrößerung des Ankerwiderstandes</li> </ol> | <p><math>U</math> Ankerspannung [V]</p> <p><math>R_A</math> R-Anker [<math>\Omega</math>]</p> <p><math>M</math> Drehmoment [Nm]</p> <p><math>\Phi</math> magn. Fluss [Wb]</p> <p><math>k_1</math> Maschinenkonst. [1]</p> <p><math>k_2</math> Maschinenkonst. [1]</p> |
|---|---|---|

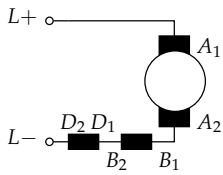
### 15.2.2. Nutzbremung mit fremderregter Gleichstrommaschine

|   |   |  |     |          |     |       |         |              |          |         |              |     |            |      |        |             |      |       |                      |     |       |                      |     |
|---|---|--|-----|----------|-----|-------|---------|--------------|----------|---------|--------------|-----|------------|------|--------|-------------|------|-------|----------------------|-----|-------|----------------------|-----|
|  | $n = \frac{(R_A + R_{Br})M_{Br}}{k_1 k_2 \Phi^2}$ $M = \frac{k_1 k_2 \Phi^2 n}{R_A + R_{Br}}$ | <table> <tr> <td><math>n</math></td><td>Drehzahl</td><td>[1]</td></tr> <tr> <td><math>R_A</math></td><td>R-Anker</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>R_{Br}</math></td><td>R-Brems</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>M</math></td><td>Drehmoment</td><td>[Nm]</td></tr> <tr> <td><math>\Phi</math></td><td>magn. Fluss</td><td>[Wb]</td></tr> <tr> <td><math>k_1</math></td><td>Maschinen-<br/>konst.</td><td>[1]</td></tr> <tr> <td><math>k_2</math></td><td>Maschinen-<br/>konst.</td><td>[1]</td></tr> </table> | $n$ | Drehzahl | [1] | $R_A$ | R-Anker | [ $\Omega$ ] | $R_{Br}$ | R-Brems | [ $\Omega$ ] | $M$ | Drehmoment | [Nm] | $\Phi$ | magn. Fluss | [Wb] | $k_1$ | Maschinen-<br>konst. | [1] | $k_2$ | Maschinen-<br>konst. | [1] |
| $n$   | Drehzahl  | [1]  |     |          |     |       |         |              |          |         |              |     |            |      |        |             |      |       |                      |     |       |                      |     |
| $R_A$   | R-Anker   | [ $\Omega$ ]   |     |          |     |       |         |              |          |         |              |     |            |      |        |             |      |       |                      |     |       |                      |     |
| $R_{Br}$  | R-Brems   | [ $\Omega$ ]   |     |          |     |       |         |              |          |         |              |     |            |      |        |             |      |       |                      |     |       |                      |     |
| $M$   | Drehmoment  | [Nm]   |     |          |     |       |         |              |          |         |              |     |            |      |        |             |      |       |                      |     |       |                      |     |
| $\Phi$  | magn. Fluss   | [Wb]   |     |          |     |       |         |              |          |         |              |     |            |      |        |             |      |       |                      |     |       |                      |     |
| $k_1$   | Maschinen-<br>konst.  | [1]  |     |          |     |       |         |              |          |         |              |     |            |      |        |             |      |       |                      |     |       |                      |     |
| $k_2$   | Maschinen-<br>konst.  | [1]  |     |          |     |       |         |              |          |         |              |     |            |      |        |             |      |       |                      |     |       |                      |     |

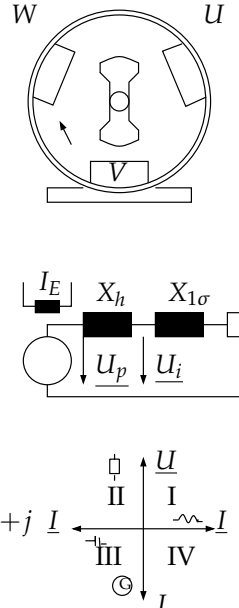
### 15.3. Gleichstrom Nebenschlussmaschine (GNSM)

|  |  |  |
|--|--|--|
|  | <p>Es gelten die selben Regeln wie bei der fremderregten Gleichstrommaschine, jedoch kann die Drehzahl nicht durch erniedrigen der Spannung gesenkt werden.</p> <p>Bei Widerstandsbremung ist nur der Anker an <math>R_{Br}</math> anzuschliessen.</p> |  |
|--|--|--|

## 15.4. Gleichstrom Reihenschlussmaschine (GRSM)

|   |  |   |     |          |     |       |         |              |       |                         |              |       |                                   |              |       |                                   |     |     |          |     |     |            |      |       |          |      |     |                      |     |
|---|--|---|-----|----------|-----|-------|---------|--------------|-------|-------------------------|--------------|-------|-----------------------------------|--------------|-------|-----------------------------------|-----|-----|----------|-----|-----|------------|------|-------|----------|------|-----|----------------------|-----|
|  | $\sum R_A = R_A + R_B + R_D$ $U_i = k_1 c * I n = k_3 I n$ $M = I^2 \frac{k_3}{2\pi} = I^2 k_4$ $n = \frac{U}{\sqrt{2\pi k_3 M}} - \frac{\sum R_A}{k_3}$ $M = \frac{k_3}{2\pi} \left( \frac{U}{k_3 n + \sum R_A} \right)^2$ $M_A = \frac{k_3}{2\pi} \left( \frac{U}{\sum R_A} \right)^2$ <p>Die Änderung der Drehzahl ist wie bei GNSM</p> | <table> <tr> <td><math>n</math></td><td>Drehzahl</td><td>[1]</td></tr> <tr> <td><math>R_A</math></td><td>R-Anker</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>R_B</math></td><td>R-Wendepol-<br/>wicklung</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>R_D</math></td><td>R-Reihen-<br/>schlusswick-<br/>lung</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>U_i</math></td><td>Ankerspan-<br/>nung indu-<br/>ziert</td><td>[V]</td></tr> <tr> <td><math>U</math></td><td>Spannung</td><td>[V]</td></tr> <tr> <td><math>M</math></td><td>Drehmoment</td><td>[Nm]</td></tr> <tr> <td><math>M_A</math></td><td>M-Anlauf</td><td>[Nm]</td></tr> <tr> <td><math>k</math></td><td>Maschinen-<br/>konst.</td><td>[1]</td></tr> </table> | $n$ | Drehzahl | [1] | $R_A$ | R-Anker | [ $\Omega$ ] | $R_B$ | R-Wendepol-<br>wicklung | [ $\Omega$ ] | $R_D$ | R-Reihen-<br>schlusswick-<br>lung | [ $\Omega$ ] | $U_i$ | Ankerspan-<br>nung indu-<br>ziert | [V] | $U$ | Spannung | [V] | $M$ | Drehmoment | [Nm] | $M_A$ | M-Anlauf | [Nm] | $k$ | Maschinen-<br>konst. | [1] |
| $n$   | Drehzahl   | [1]   |     |          |     |       |         |              |       |                         |              |       |                                   |              |       |                                   |     |     |          |     |     |            |      |       |          |      |     |                      |     |
| $R_A$   | R-Anker  | [ $\Omega$ ]  |     |          |     |       |         |              |       |                         |              |       |                                   |              |       |                                   |     |     |          |     |     |            |      |       |          |      |     |                      |     |
| $R_B$   | R-Wendepol-<br>wicklung  | [ $\Omega$ ]  |     |          |     |       |         |              |       |                         |              |       |                                   |              |       |                                   |     |     |          |     |     |            |      |       |          |      |     |                      |     |
| $R_D$   | R-Reihen-<br>schlusswick-<br>lung  | [ $\Omega$ ]  |     |          |     |       |         |              |       |                         |              |       |                                   |              |       |                                   |     |     |          |     |     |            |      |       |          |      |     |                      |     |
| $U_i$   | Ankerspan-<br>nung indu-<br>ziert  | [V]   |     |          |     |       |         |              |       |                         |              |       |                                   |              |       |                                   |     |     |          |     |     |            |      |       |          |      |     |                      |     |
| $U$   | Spannung   | [V]   |     |          |     |       |         |              |       |                         |              |       |                                   |              |       |                                   |     |     |          |     |     |            |      |       |          |      |     |                      |     |
| $M$   | Drehmoment   | [Nm]  |     |          |     |       |         |              |       |                         |              |       |                                   |              |       |                                   |     |     |          |     |     |            |      |       |          |      |     |                      |     |
| $M_A$   | M-Anlauf   | [Nm]  |     |          |     |       |         |              |       |                         |              |       |                                   |              |       |                                   |     |     |          |     |     |            |      |       |          |      |     |                      |     |
| $k$   | Maschinen-<br>konst.   | [1]   |     |          |     |       |         |              |       |                         |              |       |                                   |              |       |                                   |     |     |          |     |     |            |      |       |          |      |     |                      |     |

## 15.5. Drehstrom Synchrongenerator (DSG)

|  |   |  |
|--|---|--|
|  <p>I, IV: Motorbetrieb<br/>II, III: Generatorbetrieb<br/>I, II: Abgabe induktive Blindleistung = Übererregt<br/>III, IV: Aufnahme kapazitive Blindleistung = Untererregt</p> | $n_{syn} = \frac{60f}{p}$ $U_i = z \frac{d\Phi}{dt}$ $ U_i  = Blv_R z$ $I_w = I \cos(\varphi) \quad I_b = I \sin(\varphi)$ $I^2 = I_w^2 + I_b^2$ $\underline{I} = I_w + jI_b = \frac{j}{X_d}(\underline{U}_p - \underline{U}_{Kl})$ $\underline{U}_{Kl} = \underline{U}_p + jX_d \underline{I}$ $X_d = X_H + X_\sigma$ <p>Leerlauf:</p> $\frac{I_E}{I_{E0N}} = \frac{U_p \sqrt{3}}{U_N}$ <p>Kurzschluss:</p> $X_d = \frac{U_p}{I_{K0}}$ $x_d = X_d \frac{I_N \sqrt{3}}{U_N} = X_d \frac{I_N}{U_{Kl}} = \frac{1}{k_0}$ | <p><math>n_{syn}</math> Drehzahl <math>\left[\frac{1}{min}\right]</math><br/> <math>f</math> (Netz-) Frequenz <math>[Hz]</math><br/> <math>p</math> Polpaarzahl <math>[1]</math><br/> <math>U_i</math> Span. induziert <math>[V]</math><br/> <math>U_{Kl}</math> Span. Klemmen <math>[V]</math><br/> <math>l</math> Leiterlänge <math>[m]</math><br/> <math>z</math> Anz. Windungen <math>[1]</math><br/> <math>v_R</math> Luftspaltgeschwindigkeit <math>\left[\frac{m}{s}\right]</math><br/> <math>B</math> Induktion <math>\left[\frac{Vs}{m^2}\right]</math><br/> <math>\Phi</math> magn. Fluss <math>[Wb]</math><br/> <math>I_w</math> Wirkstrom <math>[A]</math><br/> <math>I_b</math> Blindstrom <math>[A]</math><br/> <math>X_d</math> synch.Reakt <math>[\Omega]</math><br/> <math>x_d</math> relative synch.Reakt <math>[\Omega]</math><br/> <math>I_E</math> Erregerstrom <math>[A]</math><br/> <math>U_N</math> U-Nenn verkettet <math>[V]</math><br/> <math>k_0</math> Leerlauf-Kurzschluss Verhältnis <math>[1]</math></p> |
|--|---|--|

## 15.6. DSG im Inselbetrieb

|  |   |   |
|--|---|---|
|  | $\underline{U}_p = \underline{U}_{Kl} - jX_d \underline{I}$ | <p><math>U_p</math> Span. Polrad <math>[V]</math><br/> <math>U_{Kl}</math> Span. Klemmen <math>[V]</math><br/> <math>X_d</math> synch.Reakt <math>[\Omega]</math></p> |
|--|---|---|

## 15.7. Belastung des DSG am starren Netz

|   |  |
|---|--|
| $U_p = \sqrt{\frac{U_{Netz}^2}{3} + X_d^2 I^2 + 2 \frac{U_{Netz}}{\sqrt{3}} X_d I \sin(\varphi)}$ <p>falls <math>U_N = U_{Netz}</math> :</p> $\frac{U_p \sqrt{3}}{U_N} = \sqrt{1 + x_d^2 \left(\frac{I}{I_N}\right)^2 + 2 x_d \frac{I}{I_N} \sin(\varphi)}$ | $U_{Netz}$ Netzspannung [V]<br>$U_{Kl}$ Span. Klemmen [V]<br>$U_p$ Span. Polrad [V]<br>$U_N$ U-Nenn verkettet [V]<br>$X_d$ synch.Reakt [ $\Omega$ ]<br>$x_d$ rel. synch.Reakt [ $\Omega$ ]<br>$I$ Laststrom [A]<br>$\varphi$ Phase [rad] |
|---|--|

## 15.8. Drehmoment und Stabilität des DSG am starren Netz

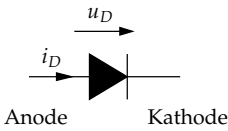
|   |   |
|---|---|
| $P_{el} = \sqrt{3} U_{Netz} I \cos(\varphi)$ $P_{el} = 3 U_{Str} I_{Str} \cos(\varphi)$ $U_p \sin(\vartheta) = I X_d \cos(\varphi)$ $P_{el} = \sqrt{3} U_{Netz} \frac{U_p}{X_d} \sin(\vartheta)$ $M_{mech} = \frac{P_{mech}}{2\pi n}$ $M_{el} = \frac{P_{el}}{2\pi n}$ $M_{el} = \frac{\sqrt{3}}{2\pi n} U_{Netz} \frac{U_p}{X_d} \sin(\vartheta)$ $M = c_i x l$ $c_i = \frac{3}{X_d 2\pi n}$ $x = I X_d$ $I = \frac{U_{Netz}}{\sqrt{3}} \cos(\varphi)$ | $U_{Netz}$ Netzspannung [V]<br>$U_{Str}$ Strangspan. [V]<br>$U_p$ Span. Polrad [V]<br>$P_{el}$ El-Wirkleistung [W]<br>$M_{el}$ Generatormoment [Nm]<br>$n$ Drehzahl [ $\frac{1}{min}$ ]<br>$X_d$ synch.Reakt [ $\Omega$ ]<br>$I$ Laststrom [A]<br>$\varphi$ Phase [rad]<br>$\vartheta$ Lastwinkel [rad] |
|---|---|

# **Teil IV.**

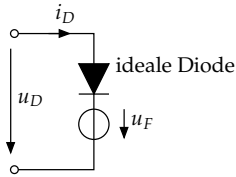
## **Elektronik**

# 16. Diode

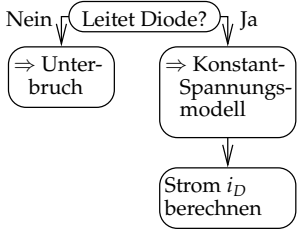
## 16.1. Ideale Diode

|   |   |  |
|---|---|--|
|  | <p>Sperrbereich (SB) :</p> $i_D = 0, u_D < 0$ <p>Durchlassbereich (DB):</p> $i_D \geq 0, u_D = 0$ | <p><math>u_D</math>    Spannungs    [V]<br/>         über Diode</p> <p><math>i_D</math>    Strom    durch [A]<br/>         Diode</p> |
|---|---|--|

## 16.2. Konstantspannungsmodell

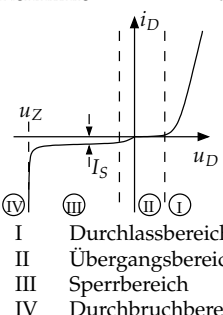
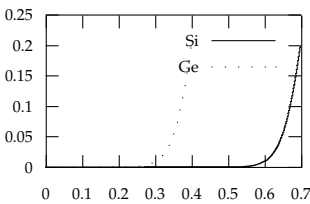
|   |  |  |
|---|--|--|
|  | $u_{FSiDiode} = 0.6$ $u_{FGermaniumDiode} = 0.3$ $u_{FShottkyDiode} = 0.1$ $u_{FLEDrot} = 1.6$ | <p><math>u_F</math>    Forwärts-    [V]<br/>         Spannungs</p> |
|---|--|--|

## 16.3. Arbeitspunktberechnung

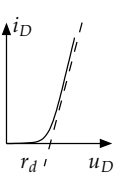
|   |   |  |
|---|---|--|
|  | <p>Teilweise ist es nur durch ausprobieren möglich zu sagen, ob die Diode leitet oder sperrt.</p> | <p><math>i_D</math>    Strom    durch [A]<br/>         Diode</p> |
|---|---|--|



## 16.4. Kennlinie

|  |  |  |
|--|--|--|
| <p>Kennlinie allgemein:</p>  <p>I Durchlassbereich<br/>II Übergangsbereich<br/>III Sperrbereich<br/>IV Durchbruchbereich</p> <p>Kennlinie Ge- und Si-Diode:</p>  | $i_D = I_S(e^{\frac{u_D}{U_T}} - 1)$ $i_D = I_S(e^{\frac{u_D - r_b i_D}{m U_T}} - 1)$ $U_T = \frac{kT}{e} = 8.6 \cdot 10^{-5} \cdot T$ <p> <math>U_T(300K) = 26mV</math> , <math>U_T(348K) = 30mV</math> ,<br/> <math>U_T(393K) = 34mV</math><br/>         Für normale Si-Diode gilt:<br/> <math>I_S = 10^{-12} A</math> , <math>r_b = 0.1\Omega</math> , <math>m = 1</math> </p> <p>Die vier Bereiche der Kennlinie:</p> <p>I: <math>-0.1V &lt; u_D &lt; 0.1V</math><br/> <math>\Rightarrow</math> Diodengleichung exakt verwenden</p> <p>II: <math>u_D &gt; 0.1V</math><br/> <math>\Rightarrow</math> Diodengleichung wird: <math>i_D =  I_S e^{\frac{u_D}{U_T}}</math><br/>         Verhältnis zweier Spannungen:<br/> <math>\frac{I_{D2}}{I_{D1}} = e^{\frac{u_{D2} - u_{D1}}{U_T}} \rightarrow u_{D2} - u_{D1} = U_T \ln \frac{I_{D2}}{I_{D1}}</math> </p> <p>III: <math>u_D &lt; -0.1V</math><br/> <math>i_D = -I_S</math> oder <math>i_D =  I_S </math></p> <p>IV: Siehe Zehner-Diode</p> | <p><math>i_D</math> Strom durch Diode [A]</p> <p><math>u_D</math> Spannung über Diode [V]</p> <p><math>U_T</math> Temperaturspannung [V]</p> <p><math>T</math> Temperatur [K]</p> <p><math>I_S</math> Sättigungsstrom [A]</p> <p><math>I_R</math> Sperrstrom [A]</p> <p><math>r_b</math> Bahnwiderstand [<math>\Omega</math>]</p> <p><math>m</math> Korrekturfakt. [1]</p> <p><math>k</math> Boltzmannkonst. [<math>\frac{Ws}{K}</math>]</p> <p><math>e</math> Elementarladung [As]</p> <p><math>= 1.38 \cdot 10^{-23}</math></p> <p><math>= 1.602 \cdot 10^{-19}</math></p> |
|--|--|--|

## 16.4.1. Differentieller Widerstand

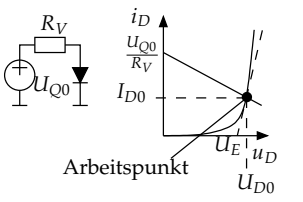
|   |   |   |
|---|---|---|
|  <p>Für kleine Signale wird die Kennlinie der Diode durch eine Tangente (= <math>r_d</math>) approximiert.</p> | $r_d = \frac{du_D}{di_D} = \frac{1}{g_d} \approx \frac{U_T}{I_{D0}}$ <p>Falls <math>m = 1</math> und <math>r_b = 0</math> gilt:</p> $g_d = \frac{di_D}{du_D} = I_S e^{\frac{u_D}{U_T}} \frac{1}{U_T} = \frac{i_D}{U_T}$ | <p><math>r_d</math> Differentieller Widerstand [<math>\Omega</math>]</p> <p><math>d_d</math> Differentieller Leitwert [S]</p> <p><math>i_D</math> Strom durch Diode [A]</p> <p><math>u_D</math> Spannung über Diode [V]</p> <p><math>U_T</math> Temperaturspannung [V]</p> <p><math>m</math> Korrekturfakt. [1]</p> <p><math>I_{D0}</math> DC-Strom im Arbeitspunkt [A]</p> |
|---|---|---|

## 16.5. DC- und AC-Analyse von Diodenschaltungen

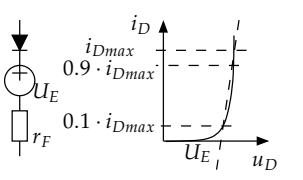
### 16.5.1. Vorgehen

1. Schaltung aufteilen in AC- und DC-Ersatzschltbild
2. In DC-Ersatzschaltung den Arbeitspunkt bestimmen (Konstantspannungsmodell)
3. Berechnen der dynamischen Widerstände im Arbeitspunkt (approximieren der Diodenkennlinie)
4. Kleinsignalanalyse (Lineare Netzwerktheorie)
5. Gesamtlösung setzt sich aus Arbeitspunkt und Wechselstromlösung zusammen

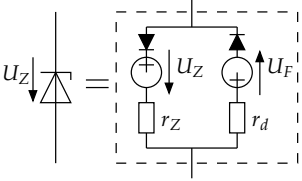
### 16.5.2. Kleinsignalanalyse

|  |  |   |       |            |     |       |               |              |          |            |     |          |              |     |          |             |     |       |                    |     |       |                     |     |       |                   |     |       |                 |     |       |             |     |
|--|--|---|-------|------------|-----|-------|---------------|--------------|----------|------------|-----|----------|--------------|-----|----------|-------------|-----|-------|--------------------|-----|-------|---------------------|-----|-------|-------------------|-----|-------|-----------------|-----|-------|-------------|-----|
|  <p>Arbeitspunktbestimmung</p> | <p>DC-Ersatzschaltung: Konstantspannungsmodell (siehe S. 130)</p> <p>AC-Ersatzschaltung: Differentieller Widerstand (siehe S. 131)</p> <p>Resultierendes Gleichungssystem:</p> $\begin{cases} i_D = \frac{U_{Q0} - u_D}{R_V} \\ i_D = I_S e^{\frac{u_D}{U_T}} \end{cases}$ $i_D = 0 \quad u_D \leq U_E$ $i_D = \frac{1}{r_D(u_D - U_E)} \quad u_D > U_E$ $U_E = U_{D0} - I_{D0} r_D$ | <table> <tr><td><math>U_F</math></td><td>Flussspan.</td><td>[V]</td></tr> <tr><td><math>R_V</math></td><td>Vorwiderstand</td><td>[<math>\Omega</math>]</td></tr> <tr><td><math>U_{Q0}</math></td><td>Quellspan.</td><td>[V]</td></tr> <tr><td><math>I_{D0}</math></td><td>Arbeitsstrom</td><td>[A]</td></tr> <tr><td><math>U_{D0}</math></td><td>Arbeitspan.</td><td>[V]</td></tr> <tr><td><math>U_T</math></td><td>Temperaturspannung</td><td>[V]</td></tr> <tr><td><math>u_D</math></td><td>Spannung über Diode</td><td>[V]</td></tr> <tr><td><math>i_D</math></td><td>Strom durch Diode</td><td>[A]</td></tr> <tr><td><math>I_S</math></td><td>Sättigungsstrom</td><td>[A]</td></tr> <tr><td><math>U_E</math></td><td>Gleichspan.</td><td>[V]</td></tr> </table> | $U_F$ | Flussspan. | [V] | $R_V$ | Vorwiderstand | [ $\Omega$ ] | $U_{Q0}$ | Quellspan. | [V] | $I_{D0}$ | Arbeitsstrom | [A] | $U_{D0}$ | Arbeitspan. | [V] | $U_T$ | Temperaturspannung | [V] | $u_D$ | Spannung über Diode | [V] | $i_D$ | Strom durch Diode | [A] | $I_S$ | Sättigungsstrom | [A] | $U_E$ | Gleichspan. | [V] |
| $U_F$  | Flussspan.   | [V]   |       |            |     |       |               |              |          |            |     |          |              |     |          |             |     |       |                    |     |       |                     |     |       |                   |     |       |                 |     |       |             |     |
| $R_V$  | Vorwiderstand  | [ $\Omega$ ]  |       |            |     |       |               |              |          |            |     |          |              |     |          |             |     |       |                    |     |       |                     |     |       |                   |     |       |                 |     |       |             |     |
| $U_{Q0}$   | Quellspan.   | [V]   |       |            |     |       |               |              |          |            |     |          |              |     |          |             |     |       |                    |     |       |                     |     |       |                   |     |       |                 |     |       |             |     |
| $I_{D0}$   | Arbeitsstrom   | [A]   |       |            |     |       |               |              |          |            |     |          |              |     |          |             |     |       |                    |     |       |                     |     |       |                   |     |       |                 |     |       |             |     |
| $U_{D0}$   | Arbeitspan.  | [V]   |       |            |     |       |               |              |          |            |     |          |              |     |          |             |     |       |                    |     |       |                     |     |       |                   |     |       |                 |     |       |             |     |
| $U_T$  | Temperaturspannung   | [V]   |       |            |     |       |               |              |          |            |     |          |              |     |          |             |     |       |                    |     |       |                     |     |       |                   |     |       |                 |     |       |             |     |
| $u_D$  | Spannung über Diode  | [V]   |       |            |     |       |               |              |          |            |     |          |              |     |          |             |     |       |                    |     |       |                     |     |       |                   |     |       |                 |     |       |             |     |
| $i_D$  | Strom durch Diode  | [A]   |       |            |     |       |               |              |          |            |     |          |              |     |          |             |     |       |                    |     |       |                     |     |       |                   |     |       |                 |     |       |             |     |
| $I_S$  | Sättigungsstrom  | [A]   |       |            |     |       |               |              |          |            |     |          |              |     |          |             |     |       |                    |     |       |                     |     |       |                   |     |       |                 |     |       |             |     |
| $U_E$  | Gleichspan.  | [V]   |       |            |     |       |               |              |          |            |     |          |              |     |          |             |     |       |                    |     |       |                     |     |       |                   |     |       |                 |     |       |             |     |

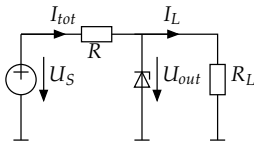
### 16.5.3. Grosssignalanalyse

|   |  |  |       |                     |     |       |                   |     |       |                  |              |
|---|--|--|-------|---------------------|-----|-------|-------------------|-----|-------|------------------|--------------|
|  <p>Für Grosssignalanalyse wird die Kennlinie durch eine Gerade durch die Punkte <math>0.1 i_{Dmax}</math> und <math>0.9 i_{Dmax}</math> approximiert.</p> | $U_E = u_D(0.1 I_{Dmax}) - 0.1 I_{Dmax} r_F$ $r_F = \frac{\Delta u_D}{0.8 I_{Dmax}}$ | <table> <tr><td><math>u_D</math></td><td>Spannung über Diode</td><td>[V]</td></tr> <tr><td><math>i_D</math></td><td>Strom durch Diode</td><td>[A]</td></tr> <tr><td><math>r_F</math></td><td>Diodenwiderstand</td><td>[<math>\Omega</math>]</td></tr> </table> | $u_D$ | Spannung über Diode | [V] | $i_D$ | Strom durch Diode | [A] | $r_F$ | Diodenwiderstand | [ $\Omega$ ] |
| $u_D$   | Spannung über Diode  | [V]  |       |                     |     |       |                   |     |       |                  |              |
| $i_D$   | Strom durch Diode  | [A]  |       |                     |     |       |                   |     |       |                  |              |
| $r_F$   | Diodenwiderstand   | [ $\Omega$ ]   |       |                     |     |       |                   |     |       |                  |              |

## 16.6. Z-Dioden

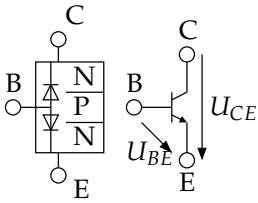
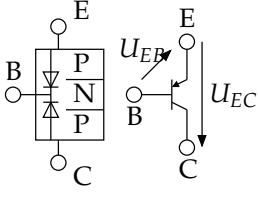
|   |   |   |       |              |            |       |                            |            |       |           |       |       |          |       |     |            |       |
|---|---|---|-------|--------------|------------|-------|----------------------------|------------|-------|-----------|-------|-------|----------|-------|-----|------------|-------|
|  | $r_Z = \frac{du_Z}{di_Z}$ <p>Temperaturkoeffizient:</p> $\alpha = \frac{\frac{dU_Z}{dT}}{U_Z}$ <p> <math>\alpha &lt; 0</math> bei <math>U_Z &lt; 5.6V</math><br/> <math>\alpha \approx 0</math> bei <math>U_Z \approx 5.6V</math><br/> <math>\alpha &gt; 0</math> bei <math>U_Z &gt; 5.6V</math> </p> <p>Temperaturkompensation durch Serieschaltung: <math>\alpha_1 U_{Z1} = -\alpha_2 U_{Z2}</math></p> | <table> <tr> <td><math>r_Z</math></td><td>Z-Widerstand</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>r_d</math></td><td>Differentieller Widerstand</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>U_Z</math></td><td>Zehnersp.</td><td><math>[V]</math></td></tr> <tr> <td><math>U_F</math></td><td>Flusssp.</td><td><math>[V]</math></td></tr> <tr> <td><math>T</math></td><td>Temperatur</td><td><math>[K]</math></td></tr> </table> | $r_Z$ | Z-Widerstand | $[\Omega]$ | $r_d$ | Differentieller Widerstand | $[\Omega]$ | $U_Z$ | Zehnersp. | $[V]$ | $U_F$ | Flusssp. | $[V]$ | $T$ | Temperatur | $[K]$ |
| $r_Z$   | Z-Widerstand  | $[\Omega]$  |       |              |            |       |                            |            |       |           |       |       |          |       |     |            |       |
| $r_d$   | Differentieller Widerstand  | $[\Omega]$  |       |              |            |       |                            |            |       |           |       |       |          |       |     |            |       |
| $U_Z$   | Zehnersp.   | $[V]$   |       |              |            |       |                            |            |       |           |       |       |          |       |     |            |       |
| $U_F$   | Flusssp.  | $[V]$   |       |              |            |       |                            |            |       |           |       |       |          |       |     |            |       |
| $T$   | Temperatur  | $[K]$   |       |              |            |       |                            |            |       |           |       |       |          |       |     |            |       |

### 16.6.1. Z-Dioden zur Spannungsstabilisierung

|  |  |  |           |           |       |       |           |       |       |           |       |           |             |       |     |               |            |       |                |            |       |            |       |       |                            |            |       |                |          |           |                |          |
|--|--|--|-----------|-----------|-------|-------|-----------|-------|-------|-----------|-------|-----------|-------------|-------|-----|---------------|------------|-------|----------------|------------|-------|------------|-------|-------|----------------------------|------------|-------|----------------|----------|-----------|----------------|----------|
|  | $I_{totmin} = \frac{U_{Smin} - U_{outmin}}{R}$ $I_{totmax} = \frac{U_{Smax} - U_{outmin}}{R}$ $I_{outmin} = I_{totmin} - I_{Lmax}$ $I_{outmax} = I_{totmax} - I_{Lmin}$ $P_{Zmax} = U_{outnom} I_{outmax}$ <p>Rippelunterdrückung:</p> $u_{out} = u_S \frac{r_Z \parallel R_L}{R + (r_Z \parallel R_L)}$ | <table> <tr> <td><math>I_{tot}</math></td><td>I-Eingang</td><td><math>[A]</math></td></tr> <tr> <td><math>U_S</math></td><td>Speisesp.</td><td><math>[V]</math></td></tr> <tr> <td><math>I_L</math></td><td>Laststrom</td><td><math>[A]</math></td></tr> <tr> <td><math>U_{out}</math></td><td>Ausgangssp.</td><td><math>[V]</math></td></tr> <tr> <td><math>R</math></td><td>Vorwiderstand</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>R_L</math></td><td>Lastwiderstand</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>P_Z</math></td><td>P-Verslust</td><td><math>[W]</math></td></tr> <tr> <td><math>r_Z</math></td><td>Differentieller Widerstand</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>u_S</math></td><td>Rippel Eingang</td><td>am <math>[V]</math></td></tr> <tr> <td><math>u_{out}</math></td><td>Rippel Ausgang</td><td>am <math>[V]</math></td></tr> </table> | $I_{tot}$ | I-Eingang | $[A]$ | $U_S$ | Speisesp. | $[V]$ | $I_L$ | Laststrom | $[A]$ | $U_{out}$ | Ausgangssp. | $[V]$ | $R$ | Vorwiderstand | $[\Omega]$ | $R_L$ | Lastwiderstand | $[\Omega]$ | $P_Z$ | P-Verslust | $[W]$ | $r_Z$ | Differentieller Widerstand | $[\Omega]$ | $u_S$ | Rippel Eingang | am $[V]$ | $u_{out}$ | Rippel Ausgang | am $[V]$ |
| $I_{tot}$  | I-Eingang  | $[A]$  |           |           |       |       |           |       |       |           |       |           |             |       |     |               |            |       |                |            |       |            |       |       |                            |            |       |                |          |           |                |          |
| $U_S$  | Speisesp.  | $[V]$  |           |           |       |       |           |       |       |           |       |           |             |       |     |               |            |       |                |            |       |            |       |       |                            |            |       |                |          |           |                |          |
| $I_L$  | Laststrom  | $[A]$  |           |           |       |       |           |       |       |           |       |           |             |       |     |               |            |       |                |            |       |            |       |       |                            |            |       |                |          |           |                |          |
| $U_{out}$  | Ausgangssp.  | $[V]$  |           |           |       |       |           |       |       |           |       |           |             |       |     |               |            |       |                |            |       |            |       |       |                            |            |       |                |          |           |                |          |
| $R$  | Vorwiderstand  | $[\Omega]$   |           |           |       |       |           |       |       |           |       |           |             |       |     |               |            |       |                |            |       |            |       |       |                            |            |       |                |          |           |                |          |
| $R_L$  | Lastwiderstand   | $[\Omega]$   |           |           |       |       |           |       |       |           |       |           |             |       |     |               |            |       |                |            |       |            |       |       |                            |            |       |                |          |           |                |          |
| $P_Z$  | P-Verslust   | $[W]$  |           |           |       |       |           |       |       |           |       |           |             |       |     |               |            |       |                |            |       |            |       |       |                            |            |       |                |          |           |                |          |
| $r_Z$  | Differentieller Widerstand   | $[\Omega]$   |           |           |       |       |           |       |       |           |       |           |             |       |     |               |            |       |                |            |       |            |       |       |                            |            |       |                |          |           |                |          |
| $u_S$  | Rippel Eingang   | am $[V]$   |           |           |       |       |           |       |       |           |       |           |             |       |     |               |            |       |                |            |       |            |       |       |                            |            |       |                |          |           |                |          |
| $u_{out}$  | Rippel Ausgang   | am $[V]$   |           |           |       |       |           |       |       |           |       |           |             |       |     |               |            |       |                |            |       |            |       |       |                            |            |       |                |          |           |                |          |

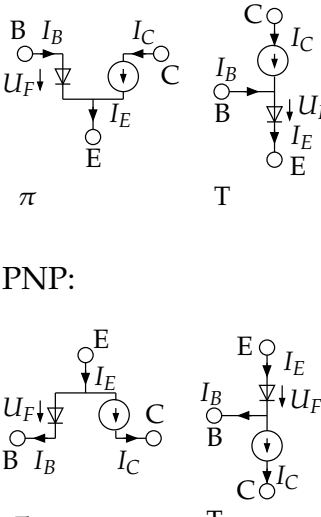
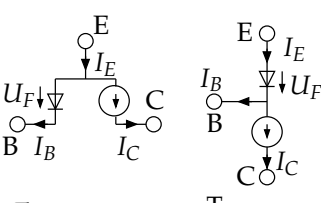
# 17. Bipolar Transistor

## 17.1. NPN- und PNP-Transistor

|  |  |  |
|--|--|--|
| <p>NPN:</p>  <p>PNP:</p>  | $i_E = i_C + i_B$ $i_C = A i_E$ $B = \frac{A}{1 - A} = \frac{i_C}{i_B}$ <p>DC-Ersatzschaltung:</p> $i_B = I_{SB} e^{\frac{u_{BE}}{U_T}}$ $i_C = B I_{SB} e^{\frac{u_{BE}}{U_T}}$ | <p><math>A</math> Stromverstärkung in B-Schaltung <math>[1]</math><br/> <math>= 0.9 \dots 0.998</math></p> <p><math>B</math> Stromverstärkung <math>[1]</math></p> <p><math>i_B</math> Basisstrom <math>[A]</math></p> <p><math>i_C</math> Kollektorstrom <math>[A]</math></p> <p><math>i_E</math> Emitterstrom <math>[A]</math></p> <p><math>u_{BE}</math> Span. B <math>\rightarrow</math> E <math>[V]</math></p> <p><math>u_T</math> Temp.-Span. Diode B <math>\rightarrow</math> E <math>[V]</math><br/> <math>\approx 0.026</math></p> <p><math>i_{SB}</math> Stromquelle zw. C <math>\rightarrow</math> B <math>[A]</math></p> |
|--|--|--|

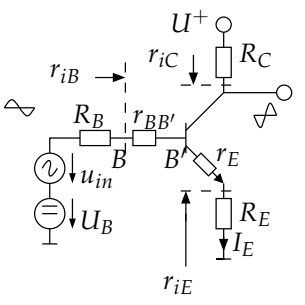
## 17.2. Der ideale Transistor bei Gleichspannung

### 17.2.1. DC-Ersatzschaltung

|   |   |   |
|---|---|---|
| <p>NPN:</p>  <p>PNP:</p>  | $B = \frac{I_C}{I_B}$ $I_C = B I_B$ $I_E = I_B + I_C = I_B + B I_B$ $I_E = (1 + B) I_B$ <p>falls <math>B \gg 1</math> gilt:</p> $I_C \approx I_E \approx B I_B$ $A = \frac{I_C}{I_E} = \frac{B I_B}{(1 + B) I_B} = \frac{B}{1 + B}$ | <p><math>A</math> Stromverstärkung in B-Schaltung [1]<br/> <math>\approx 1</math> falls <math>B \gg 1</math></p> <p><math>B</math> DC-Stromverstärkung [1]</p> <p><math>I_B</math> Basisstrom [A]<br/> <math>I_C</math> Kollektorstrom [A]<br/> <math>I_E</math> Emitterstrom [A]<br/> <math>U_F</math> Span. Diode [V]<br/> <math>\approx 0.6</math></p> |
|---|---|---|

## 17.3. Verstärkerschaltungen

### 17.3.1. Dynamische Innenwiderstände des Transistors

|   |   |   |
|---|---|---|
|  | $r_{iB} = r_{BB'} + (\beta + 1)(R_E + r_E)$ $r_{iE} = r'_E = r_E + \frac{r_{BB'} + R_B}{\beta + 1}$ $r_{iC} = r_{CE} \left( 1 + \frac{\beta R_E}{R_E + R_B + r_{BB'} + r_{B'E}} \right)$ $r_{iC} \approx R_C$ $r_E = \frac{U_{temp}}{I_E} = \frac{r_{B'E}}{\beta + 1} = \frac{\frac{U_{temp}}{I_B}}{\beta + 1}$ $r_{CE} = \frac{U_{Early} + U_{CE}}{I_C} \approx \frac{U_{Early}}{I_C}$ | <p><math>r_{iB}</math> r-Basis [<math>\Omega</math>]<br/> <math>r_{iE}</math> r-Emitter [<math>\Omega</math>]<br/> <math>r_{iC}</math> r-Kollektor [<math>\Omega</math>]<br/> <math>r_{BB'}</math> Basisbahnwiderstand [<math>\Omega</math>]<br/> <math>r_E</math> innerer r-Emitter [<math>\Omega</math>]<br/> <math>R_E</math> R-Emitter [<math>\Omega</math>]<br/> <math>\beta</math> AC-Stromverstärkung [1]<br/> <math>R_B</math> R-Basis [<math>\Omega</math>]<br/> <math>I_C</math> Kollektorstrom [A]<br/> <math>I_E</math> Emitterstrom [A]<br/> <math>U_{Early}</math> Early-Span. = [V]<br/> 20...400, typ.100<br/> <math>U_{temp}</math> Temp-Span. [V]<br/> <math>\approx 0.026</math></p> |
|---|---|---|

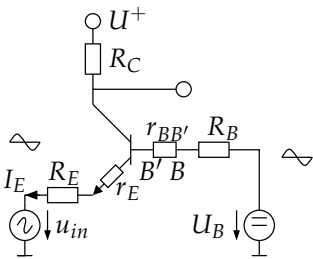
### 17.3.2. Emitterschaltug

|                      |  |   |
|----------------------|--|---|
| <p>invertierend:</p> | $A = \frac{u_{out}}{u_{in}} = -\frac{R_C}{R_E + r'_E + \frac{R_C}{\mu}}$ $A \approx -\frac{R_C}{R_E + r'_E}$ <p>Falls <math>R_E = 0</math>: <math>A = \frac{R_C \parallel r_{CE}}{r'_E}</math></p> $\mu = \frac{r_{CE}}{r'_E} \approx \frac{U_{Early}}{U_{temp}}$ $r_E = \frac{U_{temp}}{I_E} = \frac{r_{B'E}}{\beta + 1}$ $r'_E = r_E + \frac{r_{BB'} + R_B}{\beta + 1}$ $r_{CE} = \frac{U_{Early} + U_{CE}}{I_C} \approx \frac{U_{Early}}{I_C}$ <p><math>r_{0C} \approx R_C \quad r_{0E} = r_{iE} \parallel R_E \quad r_{0B} = r_{iB} \parallel R_1 \parallel R_2</math></p> | <p><math>A</math> Verstärkung [1]</p> <p><math>\mu</math> max. theoretisch A [1]</p> <p><math>\beta</math> AC-Stromverstärkung [1]</p> <p><math>R_C</math> R-Kollektor [<math>\Omega</math>]</p> <p><math>R_E</math> R-Emitter [<math>\Omega</math>]</p> <p><math>R_B</math> R-Basis [<math>\Omega</math>]</p> <p><math>r_{BB'}</math> Basisbahnwiderstand [<math>\Omega</math>]</p> <p><math>r_E</math> innerer r-Emitter [<math>\Omega</math>]</p> <p><math>I_C</math> I-Kollektor [A]</p> <p><math>I_E</math> Emitterstrom [A]</p> <p><math>U_{Early}</math> Early-Span. = 20...400, typ. 100 [V]</p> <p><math>U_{temp}</math> Temp-Span. <math>\approx 0.026</math> [V]</p> |
|----------------------|--|---|

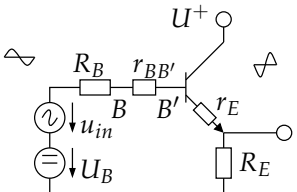
### Arbeitspunktberechnung

|  |  |  |
|--|--|--|
|  | $U_{0_{Ersatz-Quelle}} = U_0 = \frac{U^+}{R_1 + R_2} R_2$ $R_{i_{Ersatz-Quelle}} = R_1 \parallel R_2$ $U_{R_E} = \frac{(U_0 - U_{BE})(\beta + 1)R_E}{(R_1 \parallel R_2) + (\beta + 1)R_E}$ <p>Falls <math>I_B = 0</math>: <math>U_{RE} = U_0 - U_{BE}</math></p> $I_B = \frac{(U_0 - U_{BE} - U_{R_E})}{R_1 \parallel R_2}$ $I_C = I_E - I_B$ $U_{R_E} = I_C R_C \rightarrow U_C$ | <p><math>\beta</math> AC-Stromverstärkung [1]</p> <p><math>R_C</math> R-Kollektor [<math>\Omega</math>]</p> <p><math>R_E</math> R-Emitter [<math>\Omega</math>]</p> <p><math>I_C</math> I-Kollektor [A]</p> <p><math>I_E</math> Emitterstrom [A]</p> <p><math>U^+</math> Speise-Span. [V]</p> <p><math>U_{BE}</math> B-E-Span. <math>\approx 0.6</math> [V]</p> <p><math>U_0</math> Span. der gedachten Quelle des Basisspannungsteilers [V]</p> <p><math>R_i</math> R-Innen [<math>\Omega</math>]</p> |
|--|--|--|

## 17.3.3. Basisschaltung

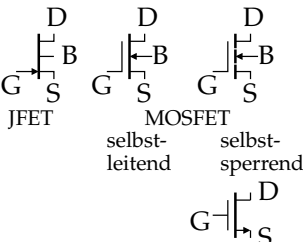
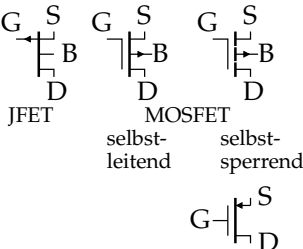
|  |  |  |
|--|--|--|
| <p>nicht invertierend:</p>  | $A = \frac{u_{out}}{u_{in}} = \frac{R_C}{R_E + r'_E + \frac{R_C}{\mu}}$ $A \approx \frac{R_C}{R_E + r'_E}$ <p>Falls <math>R_E = 0</math> :</p> $A = \frac{R_C \parallel r_{CE}}{r'_E}$ $\mu = \frac{r_{CE}}{r'_E} \approx \frac{U_{Early}}{U_{temp}}$ $r_E = \frac{U_{temp}}{I_E} = \frac{r_{B'E}}{\beta + 1}$ $r'_E = r_E + \frac{r_{BB'} + R_B}{\beta + 1}$ $r_{CE} = \frac{U_{Early} + U_{CE}}{I_C} \approx \frac{U_{Early}}{I_C}$ <p><math>r_{0C} \approx R_C \quad r_{0E} = r_{iE} \parallel R_E \quad r_{0B} = r_{iB} \parallel R_1 \parallel R_2</math></p> | <p><math>A</math> Verstärkung [1]</p> <p><math>\mu</math> max. theoretisch A [1]</p> <p><math>\beta</math> AC-Stromverstärkung [1]</p> <p><math>R_C</math> R-Kollektor [<math>\Omega</math>]</p> <p><math>R_E</math> R-Emitter [<math>\Omega</math>]</p> <p><math>R_B</math> R-Basis [<math>\Omega</math>]</p> <p><math>r_{BB'}</math> Basisbahnwiderstand [<math>\Omega</math>]</p> <p><math>r_E</math> innerer Emitter r- [<math>\Omega</math>]</p> <p><math>I_C</math> I-Kollektor [A]</p> <p><math>I_E</math> Emitterstrom [A]</p> <p><math>U_{Early}</math> Early-Span. = [V]<br/>20...400, typ. 100</p> <p><math>U_{temp}</math> Temp-Span. [V]<br/><math>\approx 0.026</math></p> |
|--|--|--|

## 17.3.4. Kollektorschaltung (Emitterfolger)

|  |  |  |
|--|--|--|
| <p>nicht invertierend:</p>  | $A = \frac{u_{out}}{u_{in}} = \frac{R_E}{R_E + r'_E}$ <p>Falls <math>R_E \gg r'_E</math> gilt :</p> $A \approx 1$ $r'_E = r_E + \frac{r_{BB'} + R_B}{\beta + 1}$ | <p><math>A</math> Verstärkung [1]</p> <p><math>R_E</math> R-Emitter [<math>\Omega</math>]</p> <p><math>R_B</math> R-Basis [<math>\Omega</math>]</p> <p><math>r_E</math> innerer Emitter r- [<math>\Omega</math>]</p> <p><math>r_{BB'}</math> Basisbahnwiderstand [<math>\Omega</math>]</p> <p><math>\beta</math> AC-Stromverstärkung [1]</p> |
|--|--|--|

# 18. Feldeffekt Transistor

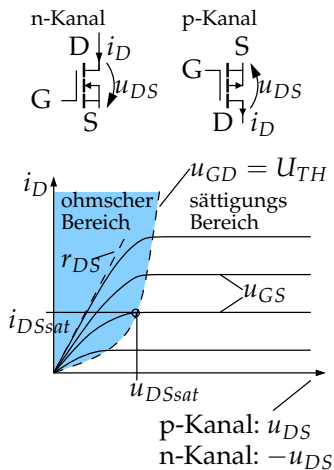
## 18.1. Verschiedene Typen

|  |  |   |
|--|--|---|
| <p>n-Kanal:</p>  <p>p-Kanal:</p>  | <p>JFET: Die Isolation zwischen Kanal und Gate besteht aus einer pn-Sperrschicht (Diode).</p> <p>MOSFET: Die Isolation zwischen Kanal und Gate besteht aus einer <math>SiO_2</math>-Schicht.</p> <p>Der selbstsperrende MOSFET sowie der JFET werden mit der Gatespannung gesperrt.</p> <p>Bulk ist meistens mit Source verbunden.</p> | <p>G Gate<br/>D Drain<br/>B Bulk oder Substrat<br/>S Source</p> |
|--|--|---|



## 18.2. Der ideale MOSFET (Handrechnung)

Im gesättigten Bereich verhält sich ein FET annähernd wie eine Stromquelle, im ungesättigten Bereich stellt er einen Widerstand dar. Die Steuergrösse ist  $u_{GS}$ .



- Drainstrom fließt nur falls  $|u_{GS}| > |U_T|$ .
- Gatestrom ist 0.

$$U_{DSsat} = U_{GS} - U_T$$

$$I_{Dsat} = K \frac{U_{DS}^2}{2}$$

$$I_D = I'_D \frac{W}{L}$$

$$K = \frac{2I_{DSS}^*}{U_T^2}$$

$$K = K' \frac{W}{L}$$

$$K' = \mu C_{ox}$$

$$k' \approx K'$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

$$U_A \approx aL$$

Im ohmschen Bereich gilt:

$$U_{DS} < U_{DSsat}$$

$$I_D = K \left[ (U_{GS} - U_T) U_{DS} - \frac{U_{DS}^2}{2} \right]$$

$$r_{DS} = \frac{dV_{DS}}{dI_D} = \frac{|U_A| + U_{DS}}{I_D}$$

Im gesättigten Bereich gilt:

$$U_{DS} > U_{DSsat}$$

$$I_D = \frac{k}{2} (U_{GS} - U_T)^2$$

Nur bei n-Kanal:

$$U_T > 0$$

$$U_{GS} > U_T \Rightarrow I_D > 0$$

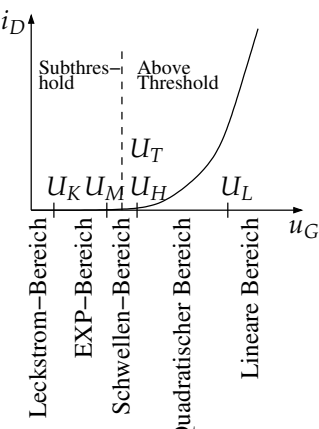
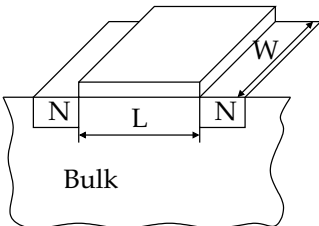
Nur bei p-Kanal:

$$U_T < 0$$

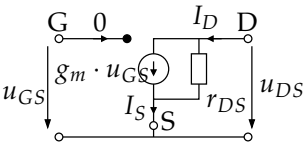
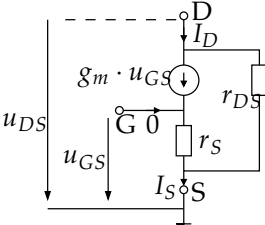
$$U_{GS} < U_T \Rightarrow I_D > 0$$

|                 |   |                     |
|-----------------|---|---------------------|
| $I_D$           | Drainstrom  | [A]                 |
| $U_T$           | Schwellspannung (0.6...8)   | [V]                 |
| $K$             | Transkonduktanzparameter  | $[\frac{A}{V^2}]$   |
| $k'$            | spez. $k$   | $[\frac{A}{V^2}]$   |
|                 | $(k'_N \approx 44 \cdot 10^{-6}, k'_p \approx 17 \cdot 10^{-6})$              |                     |
| $k$             | wie $K$ jedoch gesättigt  | $[\frac{A}{V^2}]$   |
| $U_{DS}$        | DS-Spannung   | [V]                 |
| $U_{GS}$        | GS-Spannung   | [V]                 |
| $I_{DSS}^*$     | ev. anstelle von $K$ gegeben  | [A]                 |
| $W$             | Kanalbreite   | [m]                 |
| $L$             | Kanallänge  | [m]                 |
| $C_{ox}$        | spez. Kapazität Kanal-Gate  | $[\frac{F}{m^2}]$   |
| $\epsilon_{ox}$ | Dielektrizitätskonst. ( $SiO_2 = 3.9 \cdot 8.86 \cdot 10^{-3}$ )              | $[\frac{F}{m}]$     |
| $t_{ox}$        | Dicke Isolation Kanal-Gate  | [m]                 |
| $\mu$           | Beweglichkeit Ladungsträger im Kanal (Für Si: $\mu_p = 580$ , $\mu_n = 230$ ) | $[\frac{cm^2}{sV}]$ |
| $r_{DS}$        | dyn. Drain-Source Widerstand  | $[\Omega]$          |
| $U_A$           | Earlyspannung   | [V]                 |
| $L$             | Gatelänge   | [m]                 |
| $a$             | Early Faktor $\approx 6$  | $[\frac{V}{\mu m}]$ |

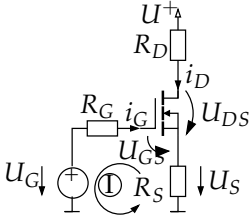
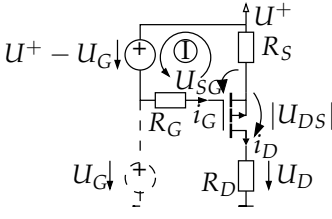
## 18.3. Der reale MOSFET

|  |  |   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
|--|--|---|-------|------------|-----|-------|------------------|-----|----------|---------------|-----|----------|--------------|-----|-------|------------|-----|----------|---|-----|----------|---------------|-----|------------|---------------------------------------|-----|-------|-------------|-----|----------|--------------------------------|-----|-----|------------|-------------------|-----|---------------|-------------------|----------|--|-------------------|----------|--------------------------------------|-----|-----|--|-----|----------|----------------------|--------------|-----------|----------------------------|-----------------|-----|-------------|-----|-----|--------------|-----|-----|------------|-----|-------|--|---------------------|
| <p>Im Exp-Bereich, bei schwacher Inversion:<br/> <math>0 &lt; U_{GS} &lt; (U_T - 60mV)</math><br/>         Im Quad-Bereich, bei starker Inversion:<br/> <math>U_{GS} &gt; (U_T - 60mV)</math><br/>         Dazwischen: Moderate Inversion</p>  | <p>Im EXP-Bereich gilt:</p> $U_{DSsat} \approx 5U_{temp} \approx 130mV$ <p>ungesättigt: <math>U_{DS} \leq U_{DSsat}</math></p> $I_D = I_M e^{\frac{U_{GS}-U_M}{nU_{temp}}} \left(1 - e^{\frac{-U_{DS}}{U_{temp}}}\right) (1 + \lambda U_{DS})$ <p>gesättigt: <math>U_{DS} \geq U_{DSsat}</math></p> $I_D = I_M e^{\frac{U_{GS}-U_M}{nU_{temp}}} (1 + \lambda U_{DS})$ <p>Im Quadratischen Bereich gilt:</p> $U_{DSsat} = U_{GS} - U_T = \sqrt{2 \frac{I_D}{k}}$ <p>ungesättigt: <math>U_{DS} \leq U_{DSsat}</math></p> $I_D = K \left[ (U_{GS} - U_T) U_{DS} - \frac{U_{DS}^2}{2} \right] (1 + \lambda U_{DS})$ <p>gesättigt: <math>U_{DS} \geq U_{DSsat}</math></p> $I_D = \frac{k}{2} (U_{GS} - U_T)^2 (1 + \lambda U_{DS})$ $\lambda = \frac{1}{U_A}$ $U_T = T_{T0} \pm \Delta U_T \quad N \rightarrow +$ $\Delta U_T = \gamma (\sqrt{U_{SB} \pm \Phi_0} - \sqrt{\Phi_0})$ $U_{temp} = \frac{kT}{e} = 86 \frac{\mu V}{K} T$ $I_M = I'_M \frac{W}{L}$ $n = 1 + \frac{\gamma}{2\sqrt{U_{SB} + \Phi_0}}$ $K = K' \frac{W}{L} = \mu C_{oc} \frac{W}{L}$ $k = k' \frac{W}{L} = \mu C_{ox} \frac{W}{L}, \quad \alpha = \frac{K'}{k'}$ | <table> <tr> <td><math>I_D</math></td><td>Drainstrom</td><td>[A]</td></tr> <tr> <td><math>I_M</math></td><td>Drainstromgrenze</td><td>[A]</td></tr> <tr> <td><math>U_{GS}</math></td><td>U Gate Source</td><td>[V]</td></tr> <tr> <td><math>U_{DS}</math></td><td>Drain-Source</td><td>[V]</td></tr> <tr> <td><math>U_T</math></td><td>Schwellsp.</td><td>[V]</td></tr> <tr> <td><math>U_{T0}</math></td><td><math>U_{T0N} \approx 0.6</math><br/><math>U_{T0P} \approx 0.65</math></td><td>[V]</td></tr> <tr> <td><math>U_{SB}</math></td><td>U Source Bulk</td><td>[V]</td></tr> <tr> <td><math>U_{temp}</math></td><td>U-Temp.<br/><math>\approx 26 \cdot 10^{-3}</math></td><td>[V]</td></tr> <tr> <td><math>U_A</math></td><td>Early-Span.</td><td>[V]</td></tr> <tr> <td><math>\Phi_0</math></td><td>Fermi-Pot.=<br/><math>2\Phi_F = 0.6</math></td><td>[V]</td></tr> <tr> <td><math>k</math></td><td>Transkond.</td><td><math>[\frac{A}{V^2}]</math></td></tr> <tr> <td><math>K</math></td><td>k ungesättigt</td><td><math>[\frac{A}{V^2}]</math></td></tr> <tr> <td><math>K', k'</math></td><td>bei quadratischem Kanal<br/>typisch: <math>k_N = 44 \cdot 10^{-6}</math>,<br/><math>k_P = 17 \cdot 10^{-6}</math></td><td><math>[\frac{A}{V^2}]</math></td></tr> <tr> <td><math>\alpha</math></td><td>Transkond. Verhältnis<br/><math>\approx 1</math></td><td>[1]</td></tr> <tr> <td><math>n</math></td><td>Subthreshold Slope Faktor<br/><math>\approx 1.5</math></td><td>[1]</td></tr> <tr> <td><math>\gamma</math></td><td><math>\gamma \approx 0.6</math></td><td><math>[\sqrt{V}]</math></td></tr> <tr> <td><math>\lambda</math></td><td>Mod-fakt.<br/>(0.01...0.05)</td><td><math>[\frac{1}{V}]</math></td></tr> <tr> <td><math>W</math></td><td>Kanal-Länge</td><td>[m]</td></tr> <tr> <td><math>L</math></td><td>Kanal-Breite</td><td>[m]</td></tr> <tr> <td><math>T</math></td><td>Temperatur</td><td>[K]</td></tr> <tr> <td><math>\mu</math></td><td>Beweglichkeit<br/>Ladungsträger im Kanal (Für Si:<br/><math>\mu_p = 580</math>,<br/><math>\mu_n = 230</math>)</td><td><math>[\frac{cm^2}{sV}]</math></td></tr> </table> | $I_D$ | Drainstrom | [A] | $I_M$ | Drainstromgrenze | [A] | $U_{GS}$ | U Gate Source | [V] | $U_{DS}$ | Drain-Source | [V] | $U_T$ | Schwellsp. | [V] | $U_{T0}$ | $U_{T0N} \approx 0.6$<br>$U_{T0P} \approx 0.65$ | [V] | $U_{SB}$ | U Source Bulk | [V] | $U_{temp}$ | U-Temp.<br>$\approx 26 \cdot 10^{-3}$ | [V] | $U_A$ | Early-Span. | [V] | $\Phi_0$ | Fermi-Pot.=<br>$2\Phi_F = 0.6$ | [V] | $k$ | Transkond. | $[\frac{A}{V^2}]$ | $K$ | k ungesättigt | $[\frac{A}{V^2}]$ | $K', k'$ | bei quadratischem Kanal<br>typisch: $k_N = 44 \cdot 10^{-6}$ ,<br>$k_P = 17 \cdot 10^{-6}$ | $[\frac{A}{V^2}]$ | $\alpha$ | Transkond. Verhältnis<br>$\approx 1$ | [1] | $n$ | Subthreshold Slope Faktor<br>$\approx 1.5$ | [1] | $\gamma$ | $\gamma \approx 0.6$ | $[\sqrt{V}]$ | $\lambda$ | Mod-fakt.<br>(0.01...0.05) | $[\frac{1}{V}]$ | $W$ | Kanal-Länge | [m] | $L$ | Kanal-Breite | [m] | $T$ | Temperatur | [K] | $\mu$ | Beweglichkeit<br>Ladungsträger im Kanal (Für Si:<br>$\mu_p = 580$ ,<br>$\mu_n = 230$ ) | $[\frac{cm^2}{sV}]$ |
| $I_D$  | Drainstrom   | [A]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $I_M$  | Drainstromgrenze   | [A]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $U_{GS}$   | U Gate Source  | [V]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $U_{DS}$   | Drain-Source   | [V]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $U_T$  | Schwellsp.   | [V]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $U_{T0}$   | $U_{T0N} \approx 0.6$<br>$U_{T0P} \approx 0.65$  | [V]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $U_{SB}$   | U Source Bulk  | [V]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $U_{temp}$   | U-Temp.<br>$\approx 26 \cdot 10^{-3}$  | [V]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $U_A$  | Early-Span.  | [V]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $\Phi_0$   | Fermi-Pot.=<br>$2\Phi_F = 0.6$   | [V]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $k$  | Transkond.   | $[\frac{A}{V^2}]$   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $K$  | k ungesättigt  | $[\frac{A}{V^2}]$   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $K', k'$   | bei quadratischem Kanal<br>typisch: $k_N = 44 \cdot 10^{-6}$ ,<br>$k_P = 17 \cdot 10^{-6}$   | $[\frac{A}{V^2}]$   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $\alpha$   | Transkond. Verhältnis<br>$\approx 1$   | [1]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $n$  | Subthreshold Slope Faktor<br>$\approx 1.5$   | [1]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $\gamma$   | $\gamma \approx 0.6$   | $[\sqrt{V}]$  |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $\lambda$  | Mod-fakt.<br>(0.01...0.05)   | $[\frac{1}{V}]$   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $W$  | Kanal-Länge  | [m]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $L$  | Kanal-Breite   | [m]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $T$  | Temperatur   | [K]   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| $\mu$  | Beweglichkeit<br>Ladungsträger im Kanal (Für Si:<br>$\mu_p = 580$ ,<br>$\mu_n = 230$ )   | $[\frac{cm^2}{sV}]$   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |
| <p>Im Leckstrombereich, im Schwellenbereich und im linearen Bereich existieren keine handlichen Formeln.</p>    |  |   |       |            |     |       |                  |     |          |               |     |          |              |     |       |            |     |          |   |     |          |               |     |            |                                       |     |       |             |     |          |                                |     |     |            |                   |     |               |                   |          |  |                   |          |                                      |     |     |  |     |          |                      |              |           |                            |                 |     |             |     |     |              |     |     |            |     |       |  |                     |

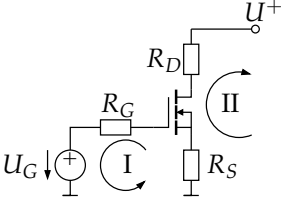
## 18.4. Kleinsignal Ersatzschaltbild für tiefe Frequenzen

|  |   |  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
|--|---|--|-------|-----------|-----|--|-----------------------|--|----------|------------------|-----|-------|------------------------|--------------|-------|------------|-----|-------|---------------------------|-----|-------|-------------|-----|------------|---------|-----|--|----------------------------|--|----------|-----------------------|-----|----------|----------------------|-----|----------|----------------------|-----|----------|----------------------|-----|-----|--------------------------|---------------------|-----|---------------------------|-----|--|---------------|--|-----------|-----------|-------------------|--|---------------|--|----------|-------------|-----|--|-----------------|--|----------|----------------------|----------------|
| <p><math>\pi</math>-Ersatzschaltbild:</p>  <p>T-Ersatzschaltbild:</p>  | <p>Steilheit im Stromquellenbetrieb bei starker Inversion:</p> $g_m = \frac{dI_D}{dU_{GS}} \approx K(U_{GS} - U_T)$ $g_m = K(U_{GS} - U_T)(1 + \lambda U_{DS})$ $g_m = \sqrt{2kI_D(1 + \lambda U_{DS})}$ $g_m \approx \sqrt{2kI_D}$ $\frac{g_{m1}}{g_{m2}} = \sqrt{\frac{I_{D1}}{I_{D2}}}$ $r_S = \frac{1}{g_m}$ <p>Ausgangswiderstand:</p> $r_{DS} = \frac{U_A +  U_{DS} }{ I_D } \approx \frac{ U_A }{ I_D }$ <p>Steilheit im Stromquellenbetrieb bei schwacher Inversion:</p> $g_m = \frac{dI_D}{dU_{GS}} = \frac{I_D}{nU_{temp}}$ $r_S = \frac{1}{g_m} = \frac{nU_{temp}}{I_D}$ <p>Body Steilheit im Stromquellenbetrieb bei starker Inversion:</p> $g_{mB} = \frac{dI_D}{dU_{SB}}$ $g_{mB} = -g_m \frac{\gamma}{2\sqrt{U_{SB} + \Phi_0}}$ $g_{mB} = -g_{mB}(n - 1)$ <p><math>\Rightarrow</math> Back-Gate hat die halbe Wirkung des Gate bei <math>U_{SB} = 0</math></p> | <table> <tr> <td><math>g_m</math></td><td>Steilheit</td><td>[1]</td></tr> <tr> <td></td><td>Übertragungskennlinie</td><td></td></tr> <tr> <td><math>g_{mB}</math></td><td>Body - Steilheit</td><td>[1]</td></tr> <tr> <td><math>r_S</math></td><td>int. Source Widerstand</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>I_D</math></td><td>Drainstrom</td><td>[A]</td></tr> <tr> <td><math>U_T</math></td><td>Schwellspannung (0.6...8)</td><td>[V]</td></tr> <tr> <td><math>U_A</math></td><td>Early-Span.</td><td>[V]</td></tr> <tr> <td><math>U_{temp}</math></td><td>U-Temp.</td><td>[V]</td></tr> <tr> <td></td><td><math>\approx 26 \cdot 10^{-3}</math></td><td></td></tr> <tr> <td><math>U_{DS}</math></td><td>Drain-Source-Spannung</td><td>[V]</td></tr> <tr> <td><math>U_{GS}</math></td><td>Gate-Source-Spannung</td><td>[V]</td></tr> <tr> <td><math>U_{SG}</math></td><td>Source-Gate-Spannung</td><td>[V]</td></tr> <tr> <td><math>U_{SB}</math></td><td>Source-Bulk-Spannung</td><td>[V]</td></tr> <tr> <td><math>K</math></td><td>Transkonduktanzparameter</td><td>[<math>\frac{A}{V^2}</math>]</td></tr> <tr> <td><math>n</math></td><td>Subthreshold Slope Faktor</td><td>[1]</td></tr> <tr> <td></td><td><math>\approx 1.5</math></td><td></td></tr> <tr> <td><math>\lambda</math></td><td>Mod-fakt.</td><td>[<math>\frac{1}{V}</math>]</td></tr> <tr> <td></td><td>(0.01...0.05)</td><td></td></tr> <tr> <td><math>\Phi_0</math></td><td>Fermi-Pot.=</td><td>[V]</td></tr> <tr> <td></td><td><math>2\Phi_F = 0.6</math></td><td></td></tr> <tr> <td><math>\gamma</math></td><td><math>\gamma \approx 0.6</math></td><td>[<math>\sqrt{V}</math>]</td></tr> </table> | $g_m$ | Steilheit | [1] |  | Übertragungskennlinie |  | $g_{mB}$ | Body - Steilheit | [1] | $r_S$ | int. Source Widerstand | [ $\Omega$ ] | $I_D$ | Drainstrom | [A] | $U_T$ | Schwellspannung (0.6...8) | [V] | $U_A$ | Early-Span. | [V] | $U_{temp}$ | U-Temp. | [V] |  | $\approx 26 \cdot 10^{-3}$ |  | $U_{DS}$ | Drain-Source-Spannung | [V] | $U_{GS}$ | Gate-Source-Spannung | [V] | $U_{SG}$ | Source-Gate-Spannung | [V] | $U_{SB}$ | Source-Bulk-Spannung | [V] | $K$ | Transkonduktanzparameter | [ $\frac{A}{V^2}$ ] | $n$ | Subthreshold Slope Faktor | [1] |  | $\approx 1.5$ |  | $\lambda$ | Mod-fakt. | [ $\frac{1}{V}$ ] |  | (0.01...0.05) |  | $\Phi_0$ | Fermi-Pot.= | [V] |  | $2\Phi_F = 0.6$ |  | $\gamma$ | $\gamma \approx 0.6$ | [ $\sqrt{V}$ ] |
| $g_m$  | Steilheit   | [1]  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
|  | Übertragungskennlinie   |  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $g_{mB}$   | Body - Steilheit  | [1]  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $r_S$  | int. Source Widerstand  | [ $\Omega$ ]   |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $I_D$  | Drainstrom  | [A]  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $U_T$  | Schwellspannung (0.6...8)   | [V]  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $U_A$  | Early-Span.   | [V]  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $U_{temp}$   | U-Temp.   | [V]  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
|  | $\approx 26 \cdot 10^{-3}$  |  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $U_{DS}$   | Drain-Source-Spannung   | [V]  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $U_{GS}$   | Gate-Source-Spannung  | [V]  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $U_{SG}$   | Source-Gate-Spannung  | [V]  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $U_{SB}$   | Source-Bulk-Spannung  | [V]  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $K$  | Transkonduktanzparameter  | [ $\frac{A}{V^2}$ ]  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $n$  | Subthreshold Slope Faktor   | [1]  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
|  | $\approx 1.5$   |  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $\lambda$  | Mod-fakt.   | [ $\frac{1}{V}$ ]  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
|  | (0.01...0.05)   |  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $\Phi_0$   | Fermi-Pot.=   | [V]  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
|  | $2\Phi_F = 0.6$   |  |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |
| $\gamma$   | $\gamma \approx 0.6$  | [ $\sqrt{V}$ ]   |       |           |     |  |                       |  |          |                  |     |       |                        |              |       |            |     |       |                           |     |       |             |     |            |         |     |  |                            |  |          |                       |     |          |                      |     |          |                      |     |          |                      |     |     |                          |                     |     |                           |     |  |               |  |           |           |                   |  |               |  |          |             |     |  |                 |  |          |                      |                |

## 18.5. DC-Berechnung mit idealen MOSFET Gleichungen

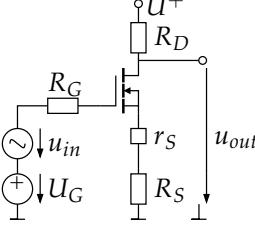
|  |   |  |
|--|---|--|
|  | Bei Verstärkern muss der Arbeitspunkt im Sättigungsbereich liegen!<br>⇒ prüfen ob $u_{DS} > u_{GS} - U_T$   | $i_D$ Drainstrom [A]<br>$U^+$ Speisespannung [V]<br>$U_T$ Schwellspannung (0.6...8) [V]<br>$U_G$ Gate-Spannung [V]<br>$u_{DS}$ Drain-Source-Spannung [V]<br>$u_{GS}$ Gate-Source-Spannung [V]<br>$u_{SG}$ Source-Gate-Spannung [V]<br>$R_G$ R-Gate [ $\Omega$ ]<br>$R_D$ R-Drain [ $\Omega$ ]<br>$R_S$ R-Source [ $\Omega$ ]<br>$k$ Transkonduktanzparameter [ $\frac{A}{V^2}$ ] |
| n-Kanal:<br>  | $i_D = \frac{k}{2}(u_{GS} - U_T)^2$<br>$I: U_G - i_D R_S - u_{GS} = 0$<br>$u_{GS} = \left( U_T - \frac{1}{k R_S} \right) + \sqrt{\frac{2}{k R_S} (U_G - U_T) + \frac{1}{(k R_S)^2}}$                                |  |
| p-Kanal:<br> | $i_D = \frac{k}{2}(u_{GS} - U_T)^2$<br>$I: U_G - i_D R_S - u_{GS} = 0$<br>$u_{SG} = \left(  U_T  - \frac{1}{k R_S} \right) + \sqrt{\frac{2}{k R_S} (U_{GP} -  U_T ) + \frac{1}{(k R_S)^2}}$<br>$U_{GP} = U^+ - U_G$ |  |
| Arbeitspunkt:  | $u_{DS} = U^+ - i_D (R_S + R_D)$  |  |

## 18.6. Der FET als Schalter

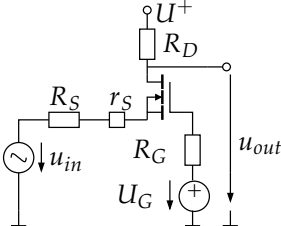
|   |   |   |       |                |     |       |             |     |       |            |     |          |                      |     |          |                       |     |       |            |     |          |                       |     |           |                     |     |  |              |  |       |            |     |       |                           |     |     |            |                     |
|---|---|---|-------|----------------|-----|-------|-------------|-----|-------|------------|-----|----------|----------------------|-----|----------|-----------------------|-----|-------|------------|-----|----------|-----------------------|-----|-----------|---------------------|-----|--|--------------|--|-------|------------|-----|-------|---------------------------|-----|-----|------------|---------------------|
|  <p>meistens: <math>R_S = 0</math></p> <p>Der Fet muss im ohmschen Bereich betrieben werden</p> <p>Schalter offen wenn <math> u_{GS}  &lt;  U_T </math></p> <p>Schalter geschlossen wenn <math> u_{GS}  \gg  U_T </math></p> | <p>Aus I und II:</p> $\begin{vmatrix} u_G - i_D R_S - u_{GS} & = & 0 \\ U^+ - i_D (R_S + R_D) - u_{DS} & = & 0 \end{vmatrix}$ $\frac{di_D}{du_{DS}} = \frac{1}{r_{DS}}$ $\frac{di_D}{du_{DS}} = K(u_{GS} - U_T) - Ku_{DS}$ $r_{DS0} = \frac{1}{K(u_{GS} - U_T)}$ <p>eingeschaltet und <math>R_S = 0</math>:</p> $i_D = \frac{U^+}{R_D + r_{DS0}}$ <p>eingeschaltet und <math>R_S \neq 0</math>:</p> $\begin{vmatrix} u_G - \frac{R_S U^+}{R_S + R_D + r_{DS0}} - u_{GS} & = & 0 \\ r_{DS0} = \frac{1}{K(u_{GS} - U_T)} & = & 0 \end{vmatrix}$ | <table> <tr> <td><math>U^+</math></td><td>Speisespannung</td><td>[V]</td></tr> <tr> <td><math>R_S</math></td><td>R an Source</td><td>[Ω]</td></tr> <tr> <td><math>R_D</math></td><td>R an Drain</td><td>[Ω]</td></tr> <tr> <td><math>u_{GS}</math></td><td>Gate-Source-Spannung</td><td>[V]</td></tr> <tr> <td><math>u_{DS}</math></td><td>Drain-Source-Spannung</td><td>[V]</td></tr> <tr> <td><math>u_G</math></td><td>Gate-Span.</td><td>[V]</td></tr> <tr> <td><math>r_{DS}</math></td><td>dyn.Source Widerstand</td><td>[Ω]</td></tr> <tr> <td><math>r_{DS0}</math></td><td>Einschaltwiderstand</td><td>[Ω]</td></tr> <tr> <td></td><td><math>u_{DS} = 0</math></td><td></td></tr> <tr> <td><math>i_D</math></td><td>Drainstrom</td><td>[A]</td></tr> <tr> <td><math>U_T</math></td><td>Schwellspannung (0.6...8)</td><td>[V]</td></tr> <tr> <td><math>K</math></td><td>Transkond.</td><td>[<math>\frac{A}{V^2}</math>]</td></tr> </table> | $U^+$ | Speisespannung | [V] | $R_S$ | R an Source | [Ω] | $R_D$ | R an Drain | [Ω] | $u_{GS}$ | Gate-Source-Spannung | [V] | $u_{DS}$ | Drain-Source-Spannung | [V] | $u_G$ | Gate-Span. | [V] | $r_{DS}$ | dyn.Source Widerstand | [Ω] | $r_{DS0}$ | Einschaltwiderstand | [Ω] |  | $u_{DS} = 0$ |  | $i_D$ | Drainstrom | [A] | $U_T$ | Schwellspannung (0.6...8) | [V] | $K$ | Transkond. | [ $\frac{A}{V^2}$ ] |
| $U^+$   | Speisespannung  | [V]   |       |                |     |       |             |     |       |            |     |          |                      |     |          |                       |     |       |            |     |          |                       |     |           |                     |     |  |              |  |       |            |     |       |                           |     |     |            |                     |
| $R_S$   | R an Source   | [Ω]   |       |                |     |       |             |     |       |            |     |          |                      |     |          |                       |     |       |            |     |          |                       |     |           |                     |     |  |              |  |       |            |     |       |                           |     |     |            |                     |
| $R_D$   | R an Drain  | [Ω]   |       |                |     |       |             |     |       |            |     |          |                      |     |          |                       |     |       |            |     |          |                       |     |           |                     |     |  |              |  |       |            |     |       |                           |     |     |            |                     |
| $u_{GS}$  | Gate-Source-Spannung  | [V]   |       |                |     |       |             |     |       |            |     |          |                      |     |          |                       |     |       |            |     |          |                       |     |           |                     |     |  |              |  |       |            |     |       |                           |     |     |            |                     |
| $u_{DS}$  | Drain-Source-Spannung   | [V]   |       |                |     |       |             |     |       |            |     |          |                      |     |          |                       |     |       |            |     |          |                       |     |           |                     |     |  |              |  |       |            |     |       |                           |     |     |            |                     |
| $u_G$   | Gate-Span.  | [V]   |       |                |     |       |             |     |       |            |     |          |                      |     |          |                       |     |       |            |     |          |                       |     |           |                     |     |  |              |  |       |            |     |       |                           |     |     |            |                     |
| $r_{DS}$  | dyn.Source Widerstand   | [Ω]   |       |                |     |       |             |     |       |            |     |          |                      |     |          |                       |     |       |            |     |          |                       |     |           |                     |     |  |              |  |       |            |     |       |                           |     |     |            |                     |
| $r_{DS0}$   | Einschaltwiderstand   | [Ω]   |       |                |     |       |             |     |       |            |     |          |                      |     |          |                       |     |       |            |     |          |                       |     |           |                     |     |  |              |  |       |            |     |       |                           |     |     |            |                     |
|   | $u_{DS} = 0$  |   |       |                |     |       |             |     |       |            |     |          |                      |     |          |                       |     |       |            |     |          |                       |     |           |                     |     |  |              |  |       |            |     |       |                           |     |     |            |                     |
| $i_D$   | Drainstrom  | [A]   |       |                |     |       |             |     |       |            |     |          |                      |     |          |                       |     |       |            |     |          |                       |     |           |                     |     |  |              |  |       |            |     |       |                           |     |     |            |                     |
| $U_T$   | Schwellspannung (0.6...8)   | [V]   |       |                |     |       |             |     |       |            |     |          |                      |     |          |                       |     |       |            |     |          |                       |     |           |                     |     |  |              |  |       |            |     |       |                           |     |     |            |                     |
| $K$   | Transkond.  | [ $\frac{A}{V^2}$ ]   |       |                |     |       |             |     |       |            |     |          |                      |     |          |                       |     |       |            |     |          |                       |     |           |                     |     |  |              |  |       |            |     |       |                           |     |     |            |                     |

## 18.7. Des FET als AC-Verstärker

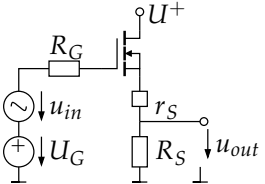
### 18.7.1. Sourceschaltung

|  |  |  |       |                |     |          |             |     |           |             |     |     |             |     |       |        |              |       |         |              |       |          |              |       |                       |              |       |                           |     |       |                     |     |             |               |     |          |                       |     |       |            |     |
|--|--|--|-------|----------------|-----|----------|-------------|-----|-----------|-------------|-----|-----|-------------|-----|-------|--------|--------------|-------|---------|--------------|-------|----------|--------------|-------|-----------------------|--------------|-------|---------------------------|-----|-------|---------------------|-----|-------------|---------------|-----|----------|-----------------------|-----|-------|------------|-----|
|  <ul style="list-style-type: none"> <li>• invertierend</li> <li>• Für tiefe bis mittlere Frequenzen</li> <li>• <math>r_{in}</math> gross</li> <li>• <math>r_{out}</math> gross</li> </ul> | $A = \frac{u_{out}}{u_{in}}$ $A = -\frac{R_D}{R_S + r_S + \frac{R_S + R_D}{\mu}}$ $\mu = \frac{r_{DS}}{r_S} = A_{max}$ <p>Für grosses <math>\mu</math>:</p> $A \approx -\frac{R_D}{R_S + r_S}$ <p>Bei <math>R_S = 0</math> gilt:</p> $A = -\frac{R_D \parallel r_{DS}}{r_S}$ <p>Bei <math>R_S = 0</math> und <math>R_D = \infty</math> gilt:</p> $ A  = \left  \frac{r_{DS}}{r_S} \right  = \mu$ $r_S = \frac{1}{g_m}$ $r_{DS} = \frac{U_{Early} + U_{DS}}{I_D} \approx \frac{U_{Early}}{I_D}$ | <table> <tr> <td><math>U^+</math></td><td>Speisespannung</td><td>[V]</td></tr> <tr> <td><math>u_{in}</math></td><td>Eingangssp.</td><td>[V]</td></tr> <tr> <td><math>u_{out}</math></td><td>Ausgangssp.</td><td>[V]</td></tr> <tr> <td><math>A</math></td><td>Verstärkung</td><td>[1]</td></tr> <tr> <td><math>R_G</math></td><td>R-Gate</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>R_D</math></td><td>R-Drain</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>R_S</math></td><td>R-Source</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>r_S</math></td><td>dyn.Source Widerstand</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>\mu</math></td><td>Max A bei Sourceschaltung</td><td>[1]</td></tr> <tr> <td><math>g_m</math></td><td>Steilheit Kennlinie</td><td>[1]</td></tr> <tr> <td><math>U_{Early}</math></td><td>Early 5...100</td><td>[V]</td></tr> <tr> <td><math>U_{DS}</math></td><td>Drain-Source-Spannung</td><td>[V]</td></tr> <tr> <td><math>I_D</math></td><td>Drainstrom</td><td>[A]</td></tr> </table> | $U^+$ | Speisespannung | [V] | $u_{in}$ | Eingangssp. | [V] | $u_{out}$ | Ausgangssp. | [V] | $A$ | Verstärkung | [1] | $R_G$ | R-Gate | [ $\Omega$ ] | $R_D$ | R-Drain | [ $\Omega$ ] | $R_S$ | R-Source | [ $\Omega$ ] | $r_S$ | dyn.Source Widerstand | [ $\Omega$ ] | $\mu$ | Max A bei Sourceschaltung | [1] | $g_m$ | Steilheit Kennlinie | [1] | $U_{Early}$ | Early 5...100 | [V] | $U_{DS}$ | Drain-Source-Spannung | [V] | $I_D$ | Drainstrom | [A] |
| $U^+$  | Speisespannung   | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |        |              |       |         |              |       |          |              |       |                       |              |       |                           |     |       |                     |     |             |               |     |          |                       |     |       |            |     |
| $u_{in}$   | Eingangssp.  | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |        |              |       |         |              |       |          |              |       |                       |              |       |                           |     |       |                     |     |             |               |     |          |                       |     |       |            |     |
| $u_{out}$  | Ausgangssp.  | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |        |              |       |         |              |       |          |              |       |                       |              |       |                           |     |       |                     |     |             |               |     |          |                       |     |       |            |     |
| $A$  | Verstärkung  | [1]  |       |                |     |          |             |     |           |             |     |     |             |     |       |        |              |       |         |              |       |          |              |       |                       |              |       |                           |     |       |                     |     |             |               |     |          |                       |     |       |            |     |
| $R_G$  | R-Gate   | [ $\Omega$ ]   |       |                |     |          |             |     |           |             |     |     |             |     |       |        |              |       |         |              |       |          |              |       |                       |              |       |                           |     |       |                     |     |             |               |     |          |                       |     |       |            |     |
| $R_D$  | R-Drain  | [ $\Omega$ ]   |       |                |     |          |             |     |           |             |     |     |             |     |       |        |              |       |         |              |       |          |              |       |                       |              |       |                           |     |       |                     |     |             |               |     |          |                       |     |       |            |     |
| $R_S$  | R-Source   | [ $\Omega$ ]   |       |                |     |          |             |     |           |             |     |     |             |     |       |        |              |       |         |              |       |          |              |       |                       |              |       |                           |     |       |                     |     |             |               |     |          |                       |     |       |            |     |
| $r_S$  | dyn.Source Widerstand  | [ $\Omega$ ]   |       |                |     |          |             |     |           |             |     |     |             |     |       |        |              |       |         |              |       |          |              |       |                       |              |       |                           |     |       |                     |     |             |               |     |          |                       |     |       |            |     |
| $\mu$  | Max A bei Sourceschaltung  | [1]  |       |                |     |          |             |     |           |             |     |     |             |     |       |        |              |       |         |              |       |          |              |       |                       |              |       |                           |     |       |                     |     |             |               |     |          |                       |     |       |            |     |
| $g_m$  | Steilheit Kennlinie  | [1]  |       |                |     |          |             |     |           |             |     |     |             |     |       |        |              |       |         |              |       |          |              |       |                       |              |       |                           |     |       |                     |     |             |               |     |          |                       |     |       |            |     |
| $U_{Early}$  | Early 5...100  | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |        |              |       |         |              |       |          |              |       |                       |              |       |                           |     |       |                     |     |             |               |     |          |                       |     |       |            |     |
| $U_{DS}$   | Drain-Source-Spannung  | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |        |              |       |         |              |       |          |              |       |                       |              |       |                           |     |       |                     |     |             |               |     |          |                       |     |       |            |     |
| $I_D$  | Drainstrom   | [A]  |       |                |     |          |             |     |           |             |     |     |             |     |       |        |              |       |         |              |       |          |              |       |                       |              |       |                           |     |       |                     |     |             |               |     |          |                       |     |       |            |     |

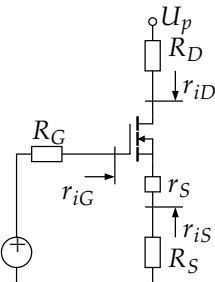
## 18.7.2. Gateschaltung

|  |   |  |       |                |     |          |             |     |           |             |     |     |             |     |       |         |              |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
|--|---|--|-------|----------------|-----|----------|-------------|-----|-----------|-------------|-----|-----|-------------|-----|-------|---------|--------------|-------|----------|--------------|-------|-----------------------|--------------|-------|---------------------|-----|-------------|---------------|-----|----------|--------------|-----|-------|------------|-----|-------|---------------------------|-----|
|  <ul style="list-style-type: none"> <li>• nicht invertierend</li> <li>• Für hohe Frequenzen</li> <li>• <math>r_{in}</math> klein</li> <li>• <math>r_{out}</math> gross</li> </ul> | $A = \frac{u_{out}}{u_{in}}$ $A = \frac{R_D \left(1 + \frac{1}{\mu}\right)}{r_s + R_S + \frac{R_D + R_S}{\mu}} = \frac{R_D}{R_S + r_s}$ $r_s = \frac{1}{g_m}$ $r_{DS} = \frac{U_{Early} + U_{DS}}{I_D} \approx \frac{U_{Early}}{I_D}$ | <table> <tr> <td><math>U^+</math></td><td>Speisespannung</td><td>[V]</td></tr> <tr> <td><math>u_{in}</math></td><td>Eingangssp.</td><td>[V]</td></tr> <tr> <td><math>u_{out}</math></td><td>Ausgangssp.</td><td>[V]</td></tr> <tr> <td><math>A</math></td><td>Verstärkung</td><td>[1]</td></tr> <tr> <td><math>R_D</math></td><td>R-Drain</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>R_S</math></td><td>R-Source</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>r_s</math></td><td>dyn.Source Widerstand</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>g_m</math></td><td>Steilheit Kennlinie</td><td>[1]</td></tr> <tr> <td><math>U_{Early}</math></td><td>Early 5...100</td><td>[V]</td></tr> <tr> <td><math>U_{DS}</math></td><td>Drain-Source</td><td>[V]</td></tr> <tr> <td><math>I_D</math></td><td>Drainstrom</td><td>[A]</td></tr> <tr> <td><math>\mu</math></td><td>Max A bei Sourceschaltung</td><td>[1]</td></tr> </table> | $U^+$ | Speisespannung | [V] | $u_{in}$ | Eingangssp. | [V] | $u_{out}$ | Ausgangssp. | [V] | $A$ | Verstärkung | [1] | $R_D$ | R-Drain | [ $\Omega$ ] | $R_S$ | R-Source | [ $\Omega$ ] | $r_s$ | dyn.Source Widerstand | [ $\Omega$ ] | $g_m$ | Steilheit Kennlinie | [1] | $U_{Early}$ | Early 5...100 | [V] | $U_{DS}$ | Drain-Source | [V] | $I_D$ | Drainstrom | [A] | $\mu$ | Max A bei Sourceschaltung | [1] |
| $U^+$  | Speisespannung  | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |         |              |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $u_{in}$   | Eingangssp.   | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |         |              |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $u_{out}$  | Ausgangssp.   | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |         |              |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $A$  | Verstärkung   | [1]  |       |                |     |          |             |     |           |             |     |     |             |     |       |         |              |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $R_D$  | R-Drain   | [ $\Omega$ ]   |       |                |     |          |             |     |           |             |     |     |             |     |       |         |              |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $R_S$  | R-Source  | [ $\Omega$ ]   |       |                |     |          |             |     |           |             |     |     |             |     |       |         |              |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $r_s$  | dyn.Source Widerstand   | [ $\Omega$ ]   |       |                |     |          |             |     |           |             |     |     |             |     |       |         |              |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $g_m$  | Steilheit Kennlinie   | [1]  |       |                |     |          |             |     |           |             |     |     |             |     |       |         |              |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $U_{Early}$  | Early 5...100   | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |         |              |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $U_{DS}$   | Drain-Source  | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |         |              |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $I_D$  | Drainstrom  | [A]  |       |                |     |          |             |     |           |             |     |     |             |     |       |         |              |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $\mu$  | Max A bei Sourceschaltung   | [1]  |       |                |     |          |             |     |           |             |     |     |             |     |       |         |              |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |

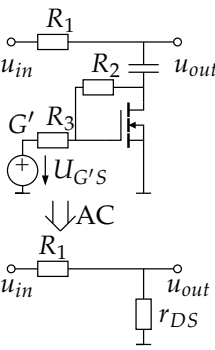
## 18.7.3. Drainschaltung

|  |   |  |       |                |     |          |             |     |           |             |     |     |             |     |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
|--|---|--|-------|----------------|-----|----------|-------------|-----|-----------|-------------|-----|-----|-------------|-----|-------|----------|--------------|-------|-----------------------|--------------|-------|---------------------|-----|-------------|---------------|-----|----------|--------------|-----|-------|------------|-----|-------|---------------------------|-----|
|  <ul style="list-style-type: none"> <li>• nicht invertierend</li> <li>• Spannungsfolger (<math>A = 1</math>), Impedanzwandler, Leistungstreiber</li> <li>• <math>r_{in}</math> gross</li> <li>• <math>r_{out}</math> klein</li> </ul> | $A = \frac{u_{out}}{u_{in}}$ $A = -\frac{R_S}{R_S + r_s \left(1 + \frac{R_S}{r_{ds}}\right)}$ $r_s = \frac{1}{g_m}$ $r_{DS} = \frac{U_{Early} + U_{DS}}{I_D} \approx \frac{U_{Early}}{I_D}$ | <table> <tr> <td><math>U^+</math></td><td>Speisespannung</td><td>[V]</td></tr> <tr> <td><math>u_{in}</math></td><td>Eingangssp.</td><td>[V]</td></tr> <tr> <td><math>u_{out}</math></td><td>Ausgangssp.</td><td>[V]</td></tr> <tr> <td><math>A</math></td><td>Verstärkung</td><td>[1]</td></tr> <tr> <td><math>R_S</math></td><td>R-Source</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>r_s</math></td><td>dyn.Source Widerstand</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>g_m</math></td><td>Steilheit Kennlinie</td><td>[1]</td></tr> <tr> <td><math>U_{Early}</math></td><td>Early 5...100</td><td>[V]</td></tr> <tr> <td><math>U_{DS}</math></td><td>Drain-Source</td><td>[V]</td></tr> <tr> <td><math>I_D</math></td><td>Drainstrom</td><td>[A]</td></tr> <tr> <td><math>\mu</math></td><td>Max A bei Sourceschaltung</td><td>[1]</td></tr> </table> | $U^+$ | Speisespannung | [V] | $u_{in}$ | Eingangssp. | [V] | $u_{out}$ | Ausgangssp. | [V] | $A$ | Verstärkung | [1] | $R_S$ | R-Source | [ $\Omega$ ] | $r_s$ | dyn.Source Widerstand | [ $\Omega$ ] | $g_m$ | Steilheit Kennlinie | [1] | $U_{Early}$ | Early 5...100 | [V] | $U_{DS}$ | Drain-Source | [V] | $I_D$ | Drainstrom | [A] | $\mu$ | Max A bei Sourceschaltung | [1] |
| $U^+$  | Speisespannung  | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $u_{in}$   | Eingangssp.   | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $u_{out}$  | Ausgangssp.   | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $A$  | Verstärkung   | [1]  |       |                |     |          |             |     |           |             |     |     |             |     |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $R_S$  | R-Source  | [ $\Omega$ ]   |       |                |     |          |             |     |           |             |     |     |             |     |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $r_s$  | dyn.Source Widerstand   | [ $\Omega$ ]   |       |                |     |          |             |     |           |             |     |     |             |     |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $g_m$  | Steilheit Kennlinie   | [1]  |       |                |     |          |             |     |           |             |     |     |             |     |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $U_{Early}$  | Early 5...100   | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $U_{DS}$   | Drain-Source  | [V]  |       |                |     |          |             |     |           |             |     |     |             |     |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $I_D$  | Drainstrom  | [A]  |       |                |     |          |             |     |           |             |     |     |             |     |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |
| $\mu$  | Max A bei Sourceschaltung   | [1]  |       |                |     |          |             |     |           |             |     |     |             |     |       |          |              |       |                       |              |       |                     |     |             |               |     |          |              |     |       |            |     |       |                           |     |

## 18.8. Dynamische Innenwiderstände des MOS-Transistors


|   |  |   |          |             |            |          |              |            |          |               |            |       |                       |            |          |                |            |             |       |             |          |              |     |       |            |     |     |            |            |
|---|--|---|----------|-------------|------------|----------|--------------|------------|----------|---------------|------------|-------|-----------------------|------------|----------|----------------|------------|-------------|-------|-------------|----------|--------------|-----|-------|------------|-----|-----|------------|------------|
|  | $r_{iG} \rightarrow \infty$<br>$r_{iS} = r_S \parallel r_{DS} \approx r_S \quad (R_D = 0)$<br>$r_{iD} = r_{DS} \left( 1 + \frac{R_S}{r_S} + \frac{R_S}{r_{DS}} \right)$<br>Falls $r_{DS} \gg R_S$ :<br>$r_{iD} \approx r_{DS} \left( 1 + \frac{R_S}{r_S} \right)$<br>$r_{DS} = \frac{U_{DS} + U_{Early}}{I_D} \approx \frac{U_{Early}}{I_D}$ | <table> <tr><td><math>r_{iG}</math></td><td>dyn. r-Gate</td><td><math>[\Omega]</math></td></tr> <tr><td><math>r_{iD}</math></td><td>dyn. r-Drain</td><td><math>[\Omega]</math></td></tr> <tr><td><math>r_{iS}</math></td><td>dyn. r-Source</td><td><math>[\Omega]</math></td></tr> <tr><td><math>r_S</math></td><td>siehe 18.4 auf S. 141</td><td><math>[\Omega]</math></td></tr> <tr><td><math>r_{DS}</math></td><td>r-Drain-Source</td><td><math>[\Omega]</math></td></tr> <tr><td><math>U_{Early}</math></td><td>Early</td><td>5...100 [V]</td></tr> <tr><td><math>U_{DS}</math></td><td>Drain-Source</td><td>[V]</td></tr> <tr><td><math>I_D</math></td><td>Drainstrom</td><td>[A]</td></tr> <tr><td><math>R</math></td><td>Widerstand</td><td><math>[\Omega]</math></td></tr> </table> | $r_{iG}$ | dyn. r-Gate | $[\Omega]$ | $r_{iD}$ | dyn. r-Drain | $[\Omega]$ | $r_{iS}$ | dyn. r-Source | $[\Omega]$ | $r_S$ | siehe 18.4 auf S. 141 | $[\Omega]$ | $r_{DS}$ | r-Drain-Source | $[\Omega]$ | $U_{Early}$ | Early | 5...100 [V] | $U_{DS}$ | Drain-Source | [V] | $I_D$ | Drainstrom | [A] | $R$ | Widerstand | $[\Omega]$ |
| $r_{iG}$  | dyn. r-Gate  | $[\Omega]$  |          |             |            |          |              |            |          |               |            |       |                       |            |          |                |            |             |       |             |          |              |     |       |            |     |     |            |            |
| $r_{iD}$  | dyn. r-Drain   | $[\Omega]$  |          |             |            |          |              |            |          |               |            |       |                       |            |          |                |            |             |       |             |          |              |     |       |            |     |     |            |            |
| $r_{iS}$  | dyn. r-Source  | $[\Omega]$  |          |             |            |          |              |            |          |               |            |       |                       |            |          |                |            |             |       |             |          |              |     |       |            |     |     |            |            |
| $r_S$   | siehe 18.4 auf S. 141  | $[\Omega]$  |          |             |            |          |              |            |          |               |            |       |                       |            |          |                |            |             |       |             |          |              |     |       |            |     |     |            |            |
| $r_{DS}$  | r-Drain-Source   | $[\Omega]$  |          |             |            |          |              |            |          |               |            |       |                       |            |          |                |            |             |       |             |          |              |     |       |            |     |     |            |            |
| $U_{Early}$   | Early  | 5...100 [V]   |          |             |            |          |              |            |          |               |            |       |                       |            |          |                |            |             |       |             |          |              |     |       |            |     |     |            |            |
| $U_{DS}$  | Drain-Source   | [V]   |          |             |            |          |              |            |          |               |            |       |                       |            |          |                |            |             |       |             |          |              |     |       |            |     |     |            |            |
| $I_D$   | Drainstrom   | [A]   |          |             |            |          |              |            |          |               |            |       |                       |            |          |                |            |             |       |             |          |              |     |       |            |     |     |            |            |
| $R$   | Widerstand   | $[\Omega]$  |          |             |            |          |              |            |          |               |            |       |                       |            |          |                |            |             |       |             |          |              |     |       |            |     |     |            |            |

## 18.9. Der FET als Spannungsgesteuerter Widerstand

|   |  |   |       |            |            |           |                |            |          |                       |            |          |              |     |           |              |     |           |             |     |          |                         |            |           |                     |            |              |  |  |       |            |     |       |                           |     |     |            |                   |
|---|--|---|-------|------------|------------|-----------|----------------|------------|----------|-----------------------|------------|----------|--------------|-----|-----------|--------------|-----|-----------|-------------|-----|----------|-------------------------|------------|-----------|---------------------|------------|--------------|--|--|-------|------------|-----|-------|---------------------------|-----|-----|------------|-------------------|
|  | <p>ohne Linearisierung <math>R_2 = \infty</math> und <math>R_3 = 0</math>:<br/>         für kleine AC-Spannung:<br/> <math>r_{DS}(t) = r_{DS0}</math><br/> <math>u_{DS}(t) = \frac{u_{in}(t)r_{DS}(t)}{R_1 + r_{DS}(t)}</math></p> <p>für grössere AC-Spannung:<br/> <math>r_{DS}(t) = \frac{1}{K(U_{GS} - U_T) - u_{DS}(t)}</math><br/>         mit Linearisierung <math>R_2 = R_3 \gg r_{DS}</math>:<br/> <math>\frac{u_{out}}{u_{in}} = \frac{r_{DS}}{R_1 + r_{DS}}</math><br/> <math>r_{DS} = \frac{du_{DS}}{di_{DS}} = \frac{2}{K(U_{G'S} - 2U_T)}</math></p> | <table> <tr><td><math>R_1</math></td><td>Widerstand</td><td><math>[\Omega]</math></td></tr> <tr><td><math>R_{2,3}</math></td><td>Linearisierung</td><td><math>[\Omega]</math></td></tr> <tr><td><math>r_{DS}</math></td><td>dyn.Source Widerstand</td><td><math>[\Omega]</math></td></tr> <tr><td><math>u_{in}</math></td><td>Eingangs-Sp.</td><td>[V]</td></tr> <tr><td><math>u_{out}</math></td><td>Ausgangs-Sp.</td><td>[V]</td></tr> <tr><td><math>u_{G'S}</math></td><td>Gate'-Span.</td><td>[V]</td></tr> <tr><td><math>r_{DS}</math></td><td>Drain-Source-Widerstand</td><td><math>[\Omega]</math></td></tr> <tr><td><math>r_{DS0}</math></td><td>Einschaltwiderstand</td><td><math>[\Omega]</math></td></tr> <tr><td><math>u_{DS} = 0</math></td><td></td><td></td></tr> <tr><td><math>i_D</math></td><td>Drainstrom</td><td>[A]</td></tr> <tr><td><math>U_T</math></td><td>Schwellspannung (0.6...8)</td><td>[V]</td></tr> <tr><td><math>K</math></td><td>Transkond.</td><td><math>[\frac{A}{V^2}]</math></td></tr> </table> | $R_1$ | Widerstand | $[\Omega]$ | $R_{2,3}$ | Linearisierung | $[\Omega]$ | $r_{DS}$ | dyn.Source Widerstand | $[\Omega]$ | $u_{in}$ | Eingangs-Sp. | [V] | $u_{out}$ | Ausgangs-Sp. | [V] | $u_{G'S}$ | Gate'-Span. | [V] | $r_{DS}$ | Drain-Source-Widerstand | $[\Omega]$ | $r_{DS0}$ | Einschaltwiderstand | $[\Omega]$ | $u_{DS} = 0$ |  |  | $i_D$ | Drainstrom | [A] | $U_T$ | Schwellspannung (0.6...8) | [V] | $K$ | Transkond. | $[\frac{A}{V^2}]$ |
| $R_1$   | Widerstand   | $[\Omega]$  |       |            |            |           |                |            |          |                       |            |          |              |     |           |              |     |           |             |     |          |                         |            |           |                     |            |              |  |  |       |            |     |       |                           |     |     |            |                   |
| $R_{2,3}$   | Linearisierung   | $[\Omega]$  |       |            |            |           |                |            |          |                       |            |          |              |     |           |              |     |           |             |     |          |                         |            |           |                     |            |              |  |  |       |            |     |       |                           |     |     |            |                   |
| $r_{DS}$  | dyn.Source Widerstand  | $[\Omega]$  |       |            |            |           |                |            |          |                       |            |          |              |     |           |              |     |           |             |     |          |                         |            |           |                     |            |              |  |  |       |            |     |       |                           |     |     |            |                   |
| $u_{in}$  | Eingangs-Sp.   | [V]   |       |            |            |           |                |            |          |                       |            |          |              |     |           |              |     |           |             |     |          |                         |            |           |                     |            |              |  |  |       |            |     |       |                           |     |     |            |                   |
| $u_{out}$   | Ausgangs-Sp.   | [V]   |       |            |            |           |                |            |          |                       |            |          |              |     |           |              |     |           |             |     |          |                         |            |           |                     |            |              |  |  |       |            |     |       |                           |     |     |            |                   |
| $u_{G'S}$   | Gate'-Span.  | [V]   |       |            |            |           |                |            |          |                       |            |          |              |     |           |              |     |           |             |     |          |                         |            |           |                     |            |              |  |  |       |            |     |       |                           |     |     |            |                   |
| $r_{DS}$  | Drain-Source-Widerstand  | $[\Omega]$  |       |            |            |           |                |            |          |                       |            |          |              |     |           |              |     |           |             |     |          |                         |            |           |                     |            |              |  |  |       |            |     |       |                           |     |     |            |                   |
| $r_{DS0}$   | Einschaltwiderstand  | $[\Omega]$  |       |            |            |           |                |            |          |                       |            |          |              |     |           |              |     |           |             |     |          |                         |            |           |                     |            |              |  |  |       |            |     |       |                           |     |     |            |                   |
| $u_{DS} = 0$  |  |   |       |            |            |           |                |            |          |                       |            |          |              |     |           |              |     |           |             |     |          |                         |            |           |                     |            |              |  |  |       |            |     |       |                           |     |     |            |                   |
| $i_D$   | Drainstrom   | [A]   |       |            |            |           |                |            |          |                       |            |          |              |     |           |              |     |           |             |     |          |                         |            |           |                     |            |              |  |  |       |            |     |       |                           |     |     |            |                   |
| $U_T$   | Schwellspannung (0.6...8)  | [V]   |       |            |            |           |                |            |          |                       |            |          |              |     |           |              |     |           |             |     |          |                         |            |           |                     |            |              |  |  |       |            |     |       |                           |     |     |            |                   |
| $K$   | Transkond.   | $[\frac{A}{V^2}]$   |       |            |            |           |                |            |          |                       |            |          |              |     |           |              |     |           |             |     |          |                         |            |           |                     |            |              |  |  |       |            |     |       |                           |     |     |            |                   |

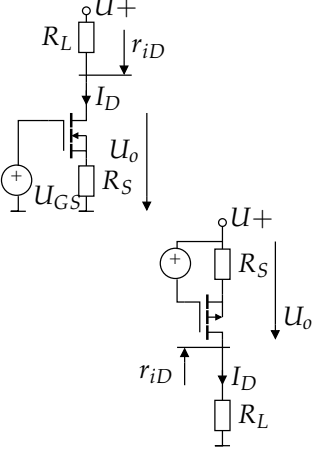


## 18.10. MOS-Diode

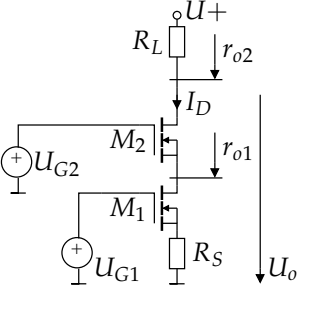
|   |   |  |       |            |     |       |                           |     |     |                          |                   |          |                      |     |          |                       |     |          |                        |     |       |                                 |     |       |                        |     |       |             |     |     |             |     |     |            |     |
|---|---|--|-------|------------|-----|-------|---------------------------|-----|-----|--------------------------|-------------------|----------|----------------------|-----|----------|-----------------------|-----|----------|------------------------|-----|-------|---------------------------------|-----|-------|------------------------|-----|-------|-------------|-----|-----|-------------|-----|-----|------------|-----|
|  <p>Wichtig, alle Substrate auf gleichem Potential!</p> | $I_D = \frac{k}{2}(U_{GS} - U_T)^2$ $r_{MD} = r_S \parallel r_{DS}$ $r_{MD} = \frac{u_{DS}}{i_D}$ $r_{DS} = \frac{U_A + U_{DS}}{I_D}$ $r_S = \frac{1}{g_m} = \frac{1}{\sqrt{2I_D k}}$ $U_{GS} = U_T + \sqrt{\frac{2I_D}{k(1 + \lambda U_{DS})}}$ $U_{GS} \approx U_T + \sqrt{\frac{2I_D}{k}}$ <p>Spannungsteiler</p> $\frac{U_{GS1} - U_{T1}}{U_{GS2} - U_{T2}} = \sqrt{\frac{\frac{W_2}{L_2}}{\frac{W_1}{L_1}}}$ | <table> <tr> <td><math>I_D</math></td><td>Drainstrom</td><td>[A]</td></tr> <tr> <td><math>U_T</math></td><td>Schwellspannung (0.6...8)</td><td>[V]</td></tr> <tr> <td><math>k</math></td><td>Transkonduktanzparameter</td><td><math>[\frac{A}{V^2}]</math></td></tr> <tr> <td><math>U_{GS}</math></td><td>Gate-Source-Spannung</td><td>[V]</td></tr> <tr> <td><math>U_{DS}</math></td><td>Drain-Source-Spannung</td><td>[V]</td></tr> <tr> <td><math>r_{MD}</math></td><td>Dynamischer Widerstand</td><td>[Ω]</td></tr> <tr> <td><math>g_m</math></td><td>Steilheit Übertragungskennlinie</td><td>[1]</td></tr> <tr> <td><math>r_S</math></td><td>int. Source Widerstand</td><td>[Ω]</td></tr> <tr> <td><math>U_A</math></td><td>Early-Span.</td><td>[V]</td></tr> <tr> <td><math>W</math></td><td>Gate-Breite</td><td>[m]</td></tr> <tr> <td><math>L</math></td><td>Gate-Länge</td><td>[m]</td></tr> </table> | $I_D$ | Drainstrom | [A] | $U_T$ | Schwellspannung (0.6...8) | [V] | $k$ | Transkonduktanzparameter | $[\frac{A}{V^2}]$ | $U_{GS}$ | Gate-Source-Spannung | [V] | $U_{DS}$ | Drain-Source-Spannung | [V] | $r_{MD}$ | Dynamischer Widerstand | [Ω] | $g_m$ | Steilheit Übertragungskennlinie | [1] | $r_S$ | int. Source Widerstand | [Ω] | $U_A$ | Early-Span. | [V] | $W$ | Gate-Breite | [m] | $L$ | Gate-Länge | [m] |
| $I_D$   | Drainstrom  | [A]  |       |            |     |       |                           |     |     |                          |                   |          |                      |     |          |                       |     |          |                        |     |       |                                 |     |       |                        |     |       |             |     |     |             |     |     |            |     |
| $U_T$   | Schwellspannung (0.6...8)   | [V]  |       |            |     |       |                           |     |     |                          |                   |          |                      |     |          |                       |     |          |                        |     |       |                                 |     |       |                        |     |       |             |     |     |             |     |     |            |     |
| $k$   | Transkonduktanzparameter  | $[\frac{A}{V^2}]$  |       |            |     |       |                           |     |     |                          |                   |          |                      |     |          |                       |     |          |                        |     |       |                                 |     |       |                        |     |       |             |     |     |             |     |     |            |     |
| $U_{GS}$  | Gate-Source-Spannung  | [V]  |       |            |     |       |                           |     |     |                          |                   |          |                      |     |          |                       |     |          |                        |     |       |                                 |     |       |                        |     |       |             |     |     |             |     |     |            |     |
| $U_{DS}$  | Drain-Source-Spannung   | [V]  |       |            |     |       |                           |     |     |                          |                   |          |                      |     |          |                       |     |          |                        |     |       |                                 |     |       |                        |     |       |             |     |     |             |     |     |            |     |
| $r_{MD}$  | Dynamischer Widerstand  | [Ω]  |       |            |     |       |                           |     |     |                          |                   |          |                      |     |          |                       |     |          |                        |     |       |                                 |     |       |                        |     |       |             |     |     |             |     |     |            |     |
| $g_m$   | Steilheit Übertragungskennlinie   | [1]  |       |            |     |       |                           |     |     |                          |                   |          |                      |     |          |                       |     |          |                        |     |       |                                 |     |       |                        |     |       |             |     |     |             |     |     |            |     |
| $r_S$   | int. Source Widerstand  | [Ω]  |       |            |     |       |                           |     |     |                          |                   |          |                      |     |          |                       |     |          |                        |     |       |                                 |     |       |                        |     |       |             |     |     |             |     |     |            |     |
| $U_A$   | Early-Span.   | [V]  |       |            |     |       |                           |     |     |                          |                   |          |                      |     |          |                       |     |          |                        |     |       |                                 |     |       |                        |     |       |             |     |     |             |     |     |            |     |
| $W$   | Gate-Breite   | [m]  |       |            |     |       |                           |     |     |                          |                   |          |                      |     |          |                       |     |          |                        |     |       |                                 |     |       |                        |     |       |             |     |     |             |     |     |            |     |
| $L$   | Gate-Länge  | [m]  |       |            |     |       |                           |     |     |                          |                   |          |                      |     |          |                       |     |          |                        |     |       |                                 |     |       |                        |     |       |             |     |     |             |     |     |            |     |

## 18.11. Stromquellen

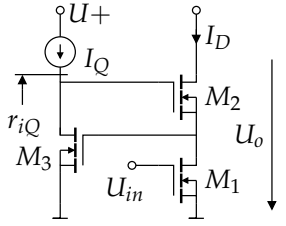
### 18.11.1. Einfache Stromquelle

|   |  |  |       |            |     |          |                 |     |       |                  |     |       |             |     |          |                |     |          |                         |     |       |                           |     |     |            |     |     |            |                              |           |           |                            |
|---|--|--|-------|------------|-----|----------|-----------------|-----|-------|------------------|-----|-------|-------------|-----|----------|----------------|-----|----------|-------------------------|-----|-------|---------------------------|-----|-----|------------|-----|-----|------------|------------------------------|-----------|-----------|----------------------------|
|  <p>Die Schaltung ist für extrem kleine Betriebsspannungen ungeeignet, da über <math>R_S</math> eine Spannung abfallen muss. Für diesen Einsatzbereich eignet sich die Kaskodeschaltung, bei der <math>R_S</math> durch einen Transistor ersetzt wird.</p> | <p>Der Fet muss im gesättigten Bereich (siehe Kapitel 18.3) betrieben werden.</p> <p>Für <math>R_S = 0</math>:</p> $I_D = \frac{k}{2}(U_{GS} - U_T)^2(1 + \lambda U_{DS})$ $r_{iD} = r_{DS} = \frac{U_A + U_{DS}}{I_D} \approx \frac{U_A}{I_D}$ $U_o \geq U_{DSsat}$ <p>Für <math>R_S \neq 0</math>:</p> $I_D = \frac{U_G - U_{GS} - U_{SS}}{R_S}$ $r_{iD} = r_{DS} \left( 1 + \frac{R_S}{r_s} + \frac{R_S}{r_{DS}} \right)$ $r_s = \frac{1}{g_m} = \frac{1}{\sqrt{2I_D k}}, \quad r_{DS} \approx \frac{U_A}{I_D}$ $U_o > R_S I_D + U_{DSsat}$ <p><math>R_S</math> dient als Gegenkopplung</p> | <table> <tr> <td><math>I_D</math></td><td>Drainstrom</td><td>[A]</td></tr> <tr> <td><math>r_{iD}</math></td><td>Innenwiderstand</td><td>[Ω]</td></tr> <tr> <td><math>U_o</math></td><td>Ausgangsspannung</td><td>[V]</td></tr> <tr> <td><math>U_A</math></td><td>Early-Span.</td><td>[V]</td></tr> <tr> <td><math>U_{DS}</math></td><td>U-Drain-Source</td><td>[V]</td></tr> <tr> <td><math>U_{SS}</math></td><td>Negative Speisespannung</td><td>[V]</td></tr> <tr> <td><math>U_T</math></td><td>Schwellspannung (0.6...8)</td><td>[V]</td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td>[Ω]</td></tr> <tr> <td><math>k</math></td><td>Transkond.</td><td><math>\left[\frac{A}{V^2}\right]</math></td></tr> <tr> <td><math>\lambda</math></td><td>Mod-fakt.</td><td><math>\left[\frac{1}{V}\right]</math></td></tr> </table> | $I_D$ | Drainstrom | [A] | $r_{iD}$ | Innenwiderstand | [Ω] | $U_o$ | Ausgangsspannung | [V] | $U_A$ | Early-Span. | [V] | $U_{DS}$ | U-Drain-Source | [V] | $U_{SS}$ | Negative Speisespannung | [V] | $U_T$ | Schwellspannung (0.6...8) | [V] | $R$ | Widerstand | [Ω] | $k$ | Transkond. | $\left[\frac{A}{V^2}\right]$ | $\lambda$ | Mod-fakt. | $\left[\frac{1}{V}\right]$ |
| $I_D$   | Drainstrom   | [A]  |       |            |     |          |                 |     |       |                  |     |       |             |     |          |                |     |          |                         |     |       |                           |     |     |            |     |     |            |                              |           |           |                            |
| $r_{iD}$  | Innenwiderstand  | [Ω]  |       |            |     |          |                 |     |       |                  |     |       |             |     |          |                |     |          |                         |     |       |                           |     |     |            |     |     |            |                              |           |           |                            |
| $U_o$   | Ausgangsspannung   | [V]  |       |            |     |          |                 |     |       |                  |     |       |             |     |          |                |     |          |                         |     |       |                           |     |     |            |     |     |            |                              |           |           |                            |
| $U_A$   | Early-Span.  | [V]  |       |            |     |          |                 |     |       |                  |     |       |             |     |          |                |     |          |                         |     |       |                           |     |     |            |     |     |            |                              |           |           |                            |
| $U_{DS}$  | U-Drain-Source   | [V]  |       |            |     |          |                 |     |       |                  |     |       |             |     |          |                |     |          |                         |     |       |                           |     |     |            |     |     |            |                              |           |           |                            |
| $U_{SS}$  | Negative Speisespannung  | [V]  |       |            |     |          |                 |     |       |                  |     |       |             |     |          |                |     |          |                         |     |       |                           |     |     |            |     |     |            |                              |           |           |                            |
| $U_T$   | Schwellspannung (0.6...8)  | [V]  |       |            |     |          |                 |     |       |                  |     |       |             |     |          |                |     |          |                         |     |       |                           |     |     |            |     |     |            |                              |           |           |                            |
| $R$   | Widerstand   | [Ω]  |       |            |     |          |                 |     |       |                  |     |       |             |     |          |                |     |          |                         |     |       |                           |     |     |            |     |     |            |                              |           |           |                            |
| $k$   | Transkond.   | $\left[\frac{A}{V^2}\right]$   |       |            |     |          |                 |     |       |                  |     |       |             |     |          |                |     |          |                         |     |       |                           |     |     |            |     |     |            |                              |           |           |                            |
| $\lambda$   | Mod-fakt.  | $\left[\frac{1}{V}\right]$   |       |            |     |          |                 |     |       |                  |     |       |             |     |          |                |     |          |                         |     |       |                           |     |     |            |     |     |            |                              |           |           |                            |

### 18.11.2. Stromquelle mit Kaskode-Schaltung

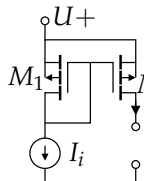
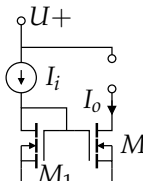
|   |  |  |          |                 |     |       |                  |     |     |            |     |       |                                  |     |
|---|--|--|----------|-----------------|-----|-------|------------------|-----|-----|------------|-----|-------|----------------------------------|-----|
|  | $r_{o2} = r_{DS2} \left( 1 + \frac{r_{DS1}}{r_{S2}} + \frac{r_{DS1}}{r_{DS2}} \right)$ <p>Falls <math>\text{Typ}(M_1) = \text{Typ}(M_2)</math>:</p> $r_{o2} \approx \frac{r_{DS}^2}{r_{D2}} = \mu r_{DS}$ $U_o \geq U_{G2} - U_{GS2} + U_{DSsat2}$ | <table> <tr> <td><math>r_{iD}</math></td><td>Innenwiderstand</td><td>[Ω]</td></tr> <tr> <td><math>U_o</math></td><td>Ausgangsspannung</td><td>[V]</td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td>[Ω]</td></tr> <tr> <td><math>\mu</math></td><td>Max Verstärkung Source Schaltung</td><td>[1]</td></tr> </table> | $r_{iD}$ | Innenwiderstand | [Ω] | $U_o$ | Ausgangsspannung | [V] | $R$ | Widerstand | [Ω] | $\mu$ | Max Verstärkung Source Schaltung | [1] |
| $r_{iD}$  | Innenwiderstand  | [Ω]  |          |                 |     |       |                  |     |     |            |     |       |                                  |     |
| $U_o$   | Ausgangsspannung   | [V]  |          |                 |     |       |                  |     |     |            |     |       |                                  |     |
| $R$   | Widerstand   | [Ω]  |          |                 |     |       |                  |     |     |            |     |       |                                  |     |
| $\mu$   | Max Verstärkung Source Schaltung   | [1]  |          |                 |     |       |                  |     |     |            |     |       |                                  |     |

### 18.11.3. Stromquelle mit geregelter Kaskode-Schaltung

|  |  |  |       |                 |            |       |                  |       |          |                |       |     |            |            |     |                |       |
|--|--|--|-------|-----------------|------------|-------|------------------|-------|----------|----------------|-------|-----|------------|------------|-----|----------------|-------|
|  <p>Die Schaltung kann auch als Supertransistor interpretiert werden.</p> | $r_o = r_{DS1} r_{DS2} g_{m1} g_{m3} (r_{DS3} \parallel r_{iQ})$ $U_o \geq 2U_{DSsat}$ <p>Strom kann wie in vorhergehender Schaltung berechnet werden.</p> | <table> <tr> <td><math>r_o</math></td><td>Innenwiderstand</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>U_o</math></td><td>Ausgangsspannung</td><td><math>[V]</math></td></tr> <tr> <td><math>U_{DS}</math></td><td>U-Drain-Source</td><td><math>[V]</math></td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>g</math></td><td>Gate-Steilheit</td><td><math>[1]</math></td></tr> </table> | $r_o$ | Innenwiderstand | $[\Omega]$ | $U_o$ | Ausgangsspannung | $[V]$ | $U_{DS}$ | U-Drain-Source | $[V]$ | $R$ | Widerstand | $[\Omega]$ | $g$ | Gate-Steilheit | $[1]$ |
| $r_o$  | Innenwiderstand  | $[\Omega]$   |       |                 |            |       |                  |       |          |                |       |     |            |            |     |                |       |
| $U_o$  | Ausgangsspannung   | $[V]$  |       |                 |            |       |                  |       |          |                |       |     |            |            |     |                |       |
| $U_{DS}$   | U-Drain-Source   | $[V]$  |       |                 |            |       |                  |       |          |                |       |     |            |            |     |                |       |
| $R$  | Widerstand   | $[\Omega]$   |       |                 |            |       |                  |       |          |                |       |     |            |            |     |                |       |
| $g$  | Gate-Steilheit   | $[1]$  |       |                 |            |       |                  |       |          |                |       |     |            |            |     |                |       |

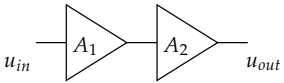
## 18.12. Stromspiegel

### 18.12.1. Widlar Stromspiegel

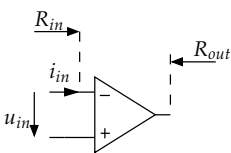
|   |  |   |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
|---|--|---|-------|-----------------|------------|-------|--------------------|------------|----------|----------|------------|-------|----------------|------------|-------|------------------|-------|----------|----------------|-------|----------|---------------|-------|-------|---------------------------|-------|-------|-------------|-------|-------|-------------|-------|-----|------------|------------|-----|----------------|-------|-----|------------|-------------------|-----------|-----------|-----------------|-----|-------------|-------|-----|------------|-------|
| <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>P-Kanal</p>  </div> <div style="text-align: center;"> <p>N-Kanal</p>  </div> </div> <p>Ein Stromspiegel kann auch mehrere Ausgänge haben.<br/>Der Eingangstransistor ist als MOS-Diode geschaltet.</p> <p>Die Genauigkeit der Schaltung hängt sehr von den Exemplaren der Transistoren ab.</p> | $U_{omin} = U_{DSsat} = U_{GS2} - U_{T2}$ $U_{omin} = \sqrt{\frac{2I_{D2}}{k}}$ $n = \frac{I_o}{I_i} \approx i_o i_i$ $n = \frac{\frac{W_2}{L_2}}{\frac{W_1}{L_1}} \cdot \frac{1 + \lambda_2 U_{DS2}}{1 + \lambda U_{DS1}}$ $r_o = r_{DS2} = \frac{U_{A2} + U_{DS2}}{I_{D2}}$ $r_o \approx \frac{U_{A1}}{I_D} = \frac{1}{\lambda I_D}$ $r_i = r_{S1} \parallel r_{DS1} \approx r_{S1} = \frac{1}{g_{m1}}$ $r_i = \frac{1}{\sqrt{2I_D k}}$ $U_i = U_{GS1} = \sqrt{\frac{2I_D}{k}} + U_{T1}$ | <table> <tr> <td><math>r_o</math></td><td>Innenwiderstand</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>r_i</math></td><td>Ausgangswiderstand</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>r_{DS}</math></td><td>r-Source</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>r_S</math></td><td>r-Drain-Source</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>U_o</math></td><td>Ausgangsspannung</td><td><math>[V]</math></td></tr> <tr> <td><math>U_{DS}</math></td><td>U-Drain-Source</td><td><math>[V]</math></td></tr> <tr> <td><math>U_{GS}</math></td><td>U-Gate-Source</td><td><math>[V]</math></td></tr> <tr> <td><math>U_T</math></td><td>Schwellspannung (0.6...8)</td><td><math>[V]</math></td></tr> <tr> <td><math>U_A</math></td><td>Early-Span.</td><td><math>[V]</math></td></tr> <tr> <td><math>I_D</math></td><td>Drain-Strom</td><td><math>[A]</math></td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td><math>[\Omega]</math></td></tr> <tr> <td><math>g</math></td><td>Gate-Steilheit</td><td><math>[1]</math></td></tr> <tr> <td><math>k</math></td><td>Transkond.</td><td><math>[\frac{A}{V^2}]</math></td></tr> <tr> <td><math>\lambda</math></td><td>Mod-fakt.</td><td><math>[\frac{1}{V}]</math></td></tr> <tr> <td><math>W</math></td><td>Gate-Breite</td><td><math>[m]</math></td></tr> <tr> <td><math>L</math></td><td>Gate-Länge</td><td><math>[m]</math></td></tr> </table> | $r_o$ | Innenwiderstand | $[\Omega]$ | $r_i$ | Ausgangswiderstand | $[\Omega]$ | $r_{DS}$ | r-Source | $[\Omega]$ | $r_S$ | r-Drain-Source | $[\Omega]$ | $U_o$ | Ausgangsspannung | $[V]$ | $U_{DS}$ | U-Drain-Source | $[V]$ | $U_{GS}$ | U-Gate-Source | $[V]$ | $U_T$ | Schwellspannung (0.6...8) | $[V]$ | $U_A$ | Early-Span. | $[V]$ | $I_D$ | Drain-Strom | $[A]$ | $R$ | Widerstand | $[\Omega]$ | $g$ | Gate-Steilheit | $[1]$ | $k$ | Transkond. | $[\frac{A}{V^2}]$ | $\lambda$ | Mod-fakt. | $[\frac{1}{V}]$ | $W$ | Gate-Breite | $[m]$ | $L$ | Gate-Länge | $[m]$ |
| $r_o$   | Innenwiderstand  | $[\Omega]$  |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $r_i$   | Ausgangswiderstand   | $[\Omega]$  |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $r_{DS}$  | r-Source   | $[\Omega]$  |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $r_S$   | r-Drain-Source   | $[\Omega]$  |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $U_o$   | Ausgangsspannung   | $[V]$   |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $U_{DS}$  | U-Drain-Source   | $[V]$   |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $U_{GS}$  | U-Gate-Source  | $[V]$   |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $U_T$   | Schwellspannung (0.6...8)  | $[V]$   |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $U_A$   | Early-Span.  | $[V]$   |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $I_D$   | Drain-Strom  | $[A]$   |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $R$   | Widerstand   | $[\Omega]$  |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $g$   | Gate-Steilheit   | $[1]$   |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $k$   | Transkond.   | $[\frac{A}{V^2}]$   |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $\lambda$   | Mod-fakt.  | $[\frac{1}{V}]$   |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $W$   | Gate-Breite  | $[m]$   |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |
| $L$   | Gate-Länge   | $[m]$   |       |                 |            |       |                    |            |          |          |            |       |                |            |       |                  |       |          |                |       |          |               |       |       |                           |       |       |             |       |       |             |       |     |            |            |     |                |       |     |            |                   |           |           |                 |     |             |       |     |            |       |

# 19. Operationsverstärker

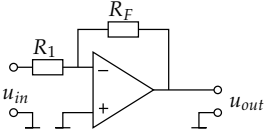
## 19.1. Verstärkung

|   |   |  |
|---|---|--|
|  | <p>Spannungsverstärkung:</p> $A_U[dB] = 20 \log \frac{u_{out}}{u_{in}}$ <p>Leistungsverstärkung:</p> $A_P[dB] = 10 \log \frac{P_{out}}{P_{in}}$ $A_{tot}[dB] = \sum_i A_i[dB]$ $A_{tot}[1] = A_1[1]A_2[1] \dots A_n[1]$ | <p><math>A_U</math> Spannungsverstärkung [1]</p> <p><math>A_P</math> Leistungsverstärkung [1]</p> <p><math>A_{tot}</math> A total [1]</p> <p><math>u_{in}</math> Eingangsspannung [V]</p> <p><math>u_{out}</math> Ausgangsspannung [V]</p> |
|---|---|--|

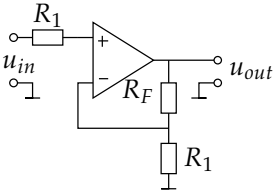
## 19.2. Idealer OP

|   |   |   |
|---|---|---|
|  | <p><math>R_{out} = 0</math></p> <p><math>R_{in} = \infty</math></p> <p><math>i_{in} = 0</math></p> <p><math>u_{in} = 0</math></p> | <p><math>R_{out}</math> Ausgangswiderstand [<math>\Omega</math>]</p> <p><math>R_{in}</math> Eingangswiderstand [<math>\Omega</math>]</p> <p><math>i_{in}</math> Eingangsstrom [A]</p> <p><math>u_{in}</math> Eingangsspannung [V]</p> |
|---|---|---|

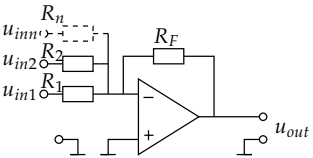
### 19.2.1. Invertierender Verstärker

|   |  |  |           |                       |     |          |                       |     |       |            |              |       |                                   |              |          |                     |     |
|---|--|--|-----------|-----------------------|-----|----------|-----------------------|-----|-------|------------|--------------|-------|-----------------------------------|--------------|----------|---------------------|-----|
|  | $A_{CL} = \frac{u_{out}}{u_{in}} = -\frac{R_F}{R_1}$ | <table> <tr> <td><math>u_{out}</math></td><td>Ausgangs-<br/>spannung</td><td>[V]</td></tr> <tr> <td><math>u_{in}</math></td><td>Eingangs-<br/>spannung</td><td>[V]</td></tr> <tr> <td><math>R_1</math></td><td>Widerstand</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>R_F</math></td><td>Rückkopp-<br/>lungswider-<br/>stand</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>A_{CL}</math></td><td>Closed Loop<br/>Gain</td><td>[1]</td></tr> </table> | $u_{out}$ | Ausgangs-<br>spannung | [V] | $u_{in}$ | Eingangs-<br>spannung | [V] | $R_1$ | Widerstand | [ $\Omega$ ] | $R_F$ | Rückkopp-<br>lungswider-<br>stand | [ $\Omega$ ] | $A_{CL}$ | Closed Loop<br>Gain | [1] |
| $u_{out}$   | Ausgangs-<br>spannung                                | [V]  |           |                       |     |          |                       |     |       |            |              |       |                                   |              |          |                     |     |
| $u_{in}$  | Eingangs-<br>spannung                                | [V]  |           |                       |     |          |                       |     |       |            |              |       |                                   |              |          |                     |     |
| $R_1$   | Widerstand   | [ $\Omega$ ]   |           |                       |     |          |                       |     |       |            |              |       |                                   |              |          |                     |     |
| $R_F$   | Rückkopp-<br>lungswider-<br>stand                    | [ $\Omega$ ]   |           |                       |     |          |                       |     |       |            |              |       |                                   |              |          |                     |     |
| $A_{CL}$  | Closed Loop<br>Gain                                  | [1]  |           |                       |     |          |                       |     |       |            |              |       |                                   |              |          |                     |     |

### 19.2.2. Nichtinvertierender Verstärker

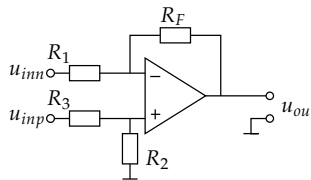
|  |   |  |           |                       |     |          |                       |     |       |            |              |       |                                   |              |
|--|---|--|-----------|-----------------------|-----|----------|-----------------------|-----|-------|------------|--------------|-------|-----------------------------------|--------------|
|  | $A_{CL} = \frac{u_{out}}{u_{in}} = \frac{R_F}{R_1} + 1$ <p>Impedanzwandler:</p> $R_F = 0, R_1 = \infty \Rightarrow A = 1$ | <table> <tr> <td><math>u_{out}</math></td><td>Ausgangs-<br/>spannung</td><td>[V]</td></tr> <tr> <td><math>u_{in}</math></td><td>Eingangs-<br/>spannung</td><td>[V]</td></tr> <tr> <td><math>R_1</math></td><td>Widerstand</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>R_F</math></td><td>Rückkopp-<br/>lungswider-<br/>stand</td><td>[<math>\Omega</math>]</td></tr> </table> | $u_{out}$ | Ausgangs-<br>spannung | [V] | $u_{in}$ | Eingangs-<br>spannung | [V] | $R_1$ | Widerstand | [ $\Omega$ ] | $R_F$ | Rückkopp-<br>lungswider-<br>stand | [ $\Omega$ ] |
| $u_{out}$  | Ausgangs-<br>spannung   | [V]  |           |                       |     |          |                       |     |       |            |              |       |                                   |              |
| $u_{in}$   | Eingangs-<br>spannung   | [V]  |           |                       |     |          |                       |     |       |            |              |       |                                   |              |
| $R_1$  | Widerstand  | [ $\Omega$ ]   |           |                       |     |          |                       |     |       |            |              |       |                                   |              |
| $R_F$  | Rückkopp-<br>lungswider-<br>stand   | [ $\Omega$ ]   |           |                       |     |          |                       |     |       |            |              |       |                                   |              |

### 19.2.3. Addierer

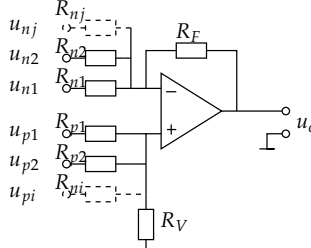
|   |  |  |           |                       |     |          |                       |     |     |             |     |       |                                   |              |       |            |              |
|---|--|--|-----------|-----------------------|-----|----------|-----------------------|-----|-----|-------------|-----|-------|-----------------------------------|--------------|-------|------------|--------------|
|  | $u_{out} = A_1 u_{in1} + A_2 u_{in2} + \dots + A_n u_{inn}$ $A_n = -\frac{R_F}{R_n}$ | <table> <tr> <td><math>u_{out}</math></td><td>Ausgangs-<br/>spannung</td><td>[V]</td></tr> <tr> <td><math>u_{in}</math></td><td>Eingangs-<br/>spannung</td><td>[V]</td></tr> <tr> <td><math>A</math></td><td>Verstärkung</td><td>[1]</td></tr> <tr> <td><math>R_F</math></td><td>Rückkopp-<br/>lungswider-<br/>stand</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>R_n</math></td><td>Widerstand</td><td>[<math>\Omega</math>]</td></tr> </table> | $u_{out}$ | Ausgangs-<br>spannung | [V] | $u_{in}$ | Eingangs-<br>spannung | [V] | $A$ | Verstärkung | [1] | $R_F$ | Rückkopp-<br>lungswider-<br>stand | [ $\Omega$ ] | $R_n$ | Widerstand | [ $\Omega$ ] |
| $u_{out}$   | Ausgangs-<br>spannung  | [V]  |           |                       |     |          |                       |     |     |             |     |       |                                   |              |       |            |              |
| $u_{in}$  | Eingangs-<br>spannung  | [V]  |           |                       |     |          |                       |     |     |             |     |       |                                   |              |       |            |              |
| $A$   | Verstärkung  | [1]  |           |                       |     |          |                       |     |     |             |     |       |                                   |              |       |            |              |
| $R_F$   | Rückkopp-<br>lungswider-<br>stand  | [ $\Omega$ ]   |           |                       |     |          |                       |     |     |             |     |       |                                   |              |       |            |              |
| $R_n$   | Widerstand   | [ $\Omega$ ]   |           |                       |     |          |                       |     |     |             |     |       |                                   |              |       |            |              |

## 19. OPERATIONSVERSTÄRKER

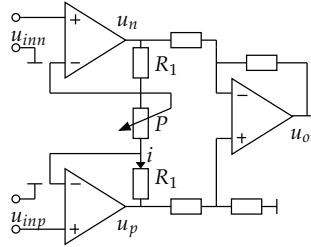
### 19.2.4. Subtrahierer

|   |   |   |           |                       |     |          |                       |     |            |                        |     |     |             |     |       |                                   |     |       |            |     |
|---|---|---|-----------|-----------------------|-----|----------|-----------------------|-----|------------|------------------------|-----|-----|-------------|-----|-------|-----------------------------------|-----|-------|------------|-----|
|  | $u_{out} = A_1 u_{inn} + A_2 u_{inp}$ $A_n = -\frac{R_F}{R_1}$ $A_p = \frac{R_2}{R_2 + R_3} \left( \frac{R_F}{R_1} + 1 \right)$ <p>Differenzverst.: <math> A_1  =  A_2 </math></p> $\frac{R_2}{R_3} = \frac{R_F}{R_1} = u_{Diff}$ $u_{out} = u_{Diff}(u_{inp} - u_{inn})$ <p>meist: <math>R_1 = R_3, R_F = R_2</math></p> | <table> <tr> <td><math>u_{out}</math></td><td>Ausgangs-<br/>spannung</td><td>[V]</td></tr> <tr> <td><math>u_{in}</math></td><td>Eingangs-<br/>spannung</td><td>[V]</td></tr> <tr> <td><math>u_{Diff}</math></td><td>Differenz-<br/>spannung</td><td>[V]</td></tr> <tr> <td><math>A</math></td><td>Verstärkung</td><td>[1]</td></tr> <tr> <td><math>R_F</math></td><td>Rückkopp-<br/>lungswider-<br/>stand</td><td>[Ω]</td></tr> <tr> <td><math>R_n</math></td><td>Widerstand</td><td>[Ω]</td></tr> </table> | $u_{out}$ | Ausgangs-<br>spannung | [V] | $u_{in}$ | Eingangs-<br>spannung | [V] | $u_{Diff}$ | Differenz-<br>spannung | [V] | $A$ | Verstärkung | [1] | $R_F$ | Rückkopp-<br>lungswider-<br>stand | [Ω] | $R_n$ | Widerstand | [Ω] |
| $u_{out}$   | Ausgangs-<br>spannung   | [V]   |           |                       |     |          |                       |     |            |                        |     |     |             |     |       |                                   |     |       |            |     |
| $u_{in}$  | Eingangs-<br>spannung   | [V]   |           |                       |     |          |                       |     |            |                        |     |     |             |     |       |                                   |     |       |            |     |
| $u_{Diff}$  | Differenz-<br>spannung  | [V]   |           |                       |     |          |                       |     |            |                        |     |     |             |     |       |                                   |     |       |            |     |
| $A$   | Verstärkung   | [1]   |           |                       |     |          |                       |     |            |                        |     |     |             |     |       |                                   |     |       |            |     |
| $R_F$   | Rückkopp-<br>lungswider-<br>stand   | [Ω]   |           |                       |     |          |                       |     |            |                        |     |     |             |     |       |                                   |     |       |            |     |
| $R_n$   | Widerstand  | [Ω]   |           |                       |     |          |                       |     |            |                        |     |     |             |     |       |                                   |     |       |            |     |

### 19.2.5. Mehrfach Addierer und Subtrahierer

|  |  |   |           |                       |     |           |                       |     |     |             |     |       |                                   |     |       |            |     |
|--|--|---|-----------|-----------------------|-----|-----------|-----------------------|-----|-----|-------------|-----|-------|-----------------------------------|-----|-------|------------|-----|
|  <p>Dummy input</p> | <p>falls: <math>A_{n1} + A_{n2} + \dots = A_{p1} + A_{p2} + \dots</math></p> $u_{out} = \sum_i A_{pi} u_{pi} - \sum_j A_{nj} u_{nj}$ <p>Beispiele :</p> $R_{n1} = \frac{R_F}{ A_{n1} }, R_{p1} = \frac{R_F}{ A_{p1} }$ $A_{n1} = \frac{R_F}{R_{n1}}$ $A_{p1} = \left( \frac{R_F}{R_{n1} \parallel \dots \parallel R_{nj}} + 1 \right) \left( \frac{(R_{p2} \parallel \dots \parallel R_{pi}) \parallel R_V}{R_{p1} + (R_{p2} \parallel \dots \parallel R_{pi}) \parallel R_V} \right)$ | <table> <tr> <td><math>u_{out}</math></td><td>Ausgangs-<br/>spannung</td><td>[V]</td></tr> <tr> <td><math>u_{n,p}</math></td><td>Eingangs-<br/>spannung</td><td>[V]</td></tr> <tr> <td><math>A</math></td><td>Verstärkung</td><td>[1]</td></tr> <tr> <td><math>R_F</math></td><td>Rückkopp-<br/>lungswider-<br/>stand</td><td>[Ω]</td></tr> <tr> <td><math>R_n</math></td><td>Widerstand</td><td>[Ω]</td></tr> </table> | $u_{out}$ | Ausgangs-<br>spannung | [V] | $u_{n,p}$ | Eingangs-<br>spannung | [V] | $A$ | Verstärkung | [1] | $R_F$ | Rückkopp-<br>lungswider-<br>stand | [Ω] | $R_n$ | Widerstand | [Ω] |
| $u_{out}$  | Ausgangs-<br>spannung  | [V]   |           |                       |     |           |                       |     |     |             |     |       |                                   |     |       |            |     |
| $u_{n,p}$  | Eingangs-<br>spannung  | [V]   |           |                       |     |           |                       |     |     |             |     |       |                                   |     |       |            |     |
| $A$  | Verstärkung  | [1]   |           |                       |     |           |                       |     |     |             |     |       |                                   |     |       |            |     |
| $R_F$  | Rückkopp-<br>lungswider-<br>stand  | [Ω]   |           |                       |     |           |                       |     |     |             |     |       |                                   |     |       |            |     |
| $R_n$  | Widerstand   | [Ω]   |           |                       |     |           |                       |     |     |             |     |       |                                   |     |       |            |     |

### 19.2.6. Instrumentationsverstärker

|   |  |  |           |                       |     |           |                       |     |           |                       |     |            |             |     |
|---|--|--|-----------|-----------------------|-----|-----------|-----------------------|-----|-----------|-----------------------|-----|------------|-------------|-----|
|  | $u_{out} = (u_{inp} - u_{inn}) \frac{2R_1 + P}{P}$ $A_{Diff} = 1 + \frac{2R_1}{P}$ $u_p - u_n = i(2R_1 + P)$ | <table> <tr> <td><math>u_{out}</math></td><td>Ausgangs-<br/>spannung</td><td>[V]</td></tr> <tr> <td><math>u_{inn}</math></td><td>Eingangs-<br/>spannung</td><td>[V]</td></tr> <tr> <td><math>u_{inp}</math></td><td>Eingangs-<br/>spannung</td><td>[V]</td></tr> <tr> <td><math>A_{Diff}</math></td><td>Diff-Verst.</td><td>[1]</td></tr> </table> | $u_{out}$ | Ausgangs-<br>spannung | [V] | $u_{inn}$ | Eingangs-<br>spannung | [V] | $u_{inp}$ | Eingangs-<br>spannung | [V] | $A_{Diff}$ | Diff-Verst. | [1] |
| $u_{out}$   | Ausgangs-<br>spannung  | [V]  |           |                       |     |           |                       |     |           |                       |     |            |             |     |
| $u_{inn}$   | Eingangs-<br>spannung  | [V]  |           |                       |     |           |                       |     |           |                       |     |            |             |     |
| $u_{inp}$   | Eingangs-<br>spannung  | [V]  |           |                       |     |           |                       |     |           |                       |     |            |             |     |
| $A_{Diff}$  | Diff-Verst.  | [1]  |           |                       |     |           |                       |     |           |                       |     |            |             |     |

## 19.2.7. Stromquelle

|  |  |   |
|--|--|---|
|  | <p>Variante 1 und 2:</p> $i_L = \frac{u_{ref}}{R_1}$   | <p><math>u_{ref}</math> Referenzspannung [V]<br/> <math>i_L</math> Strom durch <math>R_L</math> [A]<br/> <math>R_L</math> Lastwiderstand [<math>\Omega</math>]<br/> <math>R</math> Widerstand [<math>\Omega</math>]</p> |
|  | <p>Variante 3:</p> $i_L = -\frac{u_{ref}}{R_1} \cdot \frac{R_2 + R_3}{R_3}$                              |   |
|  | <p>Variante 4:<br/>falls <math>R_{2a} = R_{2b} = R_{2c} = R_{2d}</math>:</p> $i_L = \frac{u_{ref}}{R_1}$ |   |

## 19.2.8. Stromspiegel

|  |  |  |
|--|--|--|
|  | <p><math>n = \frac{i_{out}}{i_{in}} = \frac{R_1}{R_2}</math></p> <p><math>u_{R_1} = u_{R_2}, R_1 i_{in} = R_2 i_{out}</math></p> | <p><math>i_{in}</math> Referenzstrom [A]<br/> <math>i_{out}</math> Strom durch <math>R_L</math> [A]<br/> <math>R_L</math> Lastwiderstand [<math>\Omega</math>]<br/> <math>R</math> Widerstand [<math>\Omega</math>]<br/> <math>n</math> Teilverhältnis [1]</p> |
|--|--|--|

### 19.2.9. Differentieller UI-Wandler

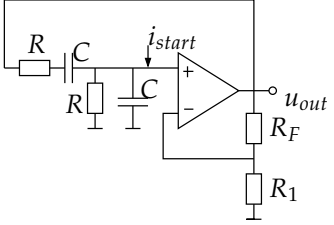
|       |   |  |       |               |     |       |               |     |       |                |              |     |            |              |
|-------|---|--|-------|---------------|-----|-------|---------------|-----|-------|----------------|--------------|-----|------------|--------------|
|       | $i_L = \frac{u_1 - u_2}{R}$ $R = R_a = R_b = R_c = R_d$ | <table> <tr> <td><math>u_1</math></td><td>Eingangssp. 1</td><td>[V]</td></tr> <tr> <td><math>u_2</math></td><td>Eingangssp. 2</td><td>[V]</td></tr> <tr> <td><math>R_L</math></td><td>Lastwiderstand</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td>[<math>\Omega</math>]</td></tr> </table> | $u_1$ | Eingangssp. 1 | [V] | $u_2$ | Eingangssp. 2 | [V] | $R_L$ | Lastwiderstand | [ $\Omega$ ] | $R$ | Widerstand | [ $\Omega$ ] |
| $u_1$ | Eingangssp. 1   | [V]  |       |               |     |       |               |     |       |                |              |     |            |              |
| $u_2$ | Eingangssp. 2   | [V]  |       |               |     |       |               |     |       |                |              |     |            |              |
| $R_L$ | Lastwiderstand  | [ $\Omega$ ]   |       |               |     |       |               |     |       |                |              |     |            |              |
| $R$   | Widerstand  | [ $\Omega$ ]   |       |               |     |       |               |     |       |                |              |     |            |              |

### 19.2.10. Schmitt-Trigger

|   |  |   |         |                  |     |         |                  |     |       |                    |     |           |                   |     |              |             |     |              |             |     |       |                            |              |       |            |              |
|---|--|---|---------|------------------|-----|---------|------------------|-----|-------|--------------------|-----|-----------|-------------------|-----|--------------|-------------|-----|--------------|-------------|-----|-------|----------------------------|--------------|-------|------------|--------------|
| <p>Nicht invertierend:</p> <p>Invertierend:</p> | <p>Nicht invertierend:</p> $u_T^+ = u_{ref} + \frac{R_1}{R_F}(u_{ref} - u_{outmin})$ $u_T^- = u_{ref} - \frac{R_1}{R_F}(u_{outmax} - u_{ref})$ $u_H = u_T^+ - u_T^-$ $u_H = (u_{outmax} - u_{outmin}) \frac{R_1}{R_F}$ <p>Invertierend:</p> $u_T^+ = u_{ref} + \frac{R_1(u_{outmax} - u_{ref})}{R_1 + R_F}$ $u_T^- = u_{ref} - \frac{R_1(u_{ref} - u_{outmin})}{R_1 + R_F}$ $u_H = \frac{R_1(u_{outmax} - u_{outmin})}{R_1 + R_F}$ | <table> <tr> <td><math>u_T^+</math></td><td>Sprungspannung ↗</td><td>[V]</td></tr> <tr> <td><math>u_T^-</math></td><td>Sprungspannung ↘</td><td>[V]</td></tr> <tr> <td><math>u_H</math></td><td>Hysterese-spannung</td><td>[V]</td></tr> <tr> <td><math>u_{ref}</math></td><td>Referenz-spannung</td><td>[V]</td></tr> <tr> <td><math>u_{outmax}</math></td><td>→ +Speisung</td><td>[V]</td></tr> <tr> <td><math>u_{outmin}</math></td><td>→ -Speisung</td><td>[V]</td></tr> <tr> <td><math>R_F</math></td><td>Rückkopp-lungs-Wider-stand</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>R_1</math></td><td>Widerstand</td><td>[<math>\Omega</math>]</td></tr> </table> | $u_T^+$ | Sprungspannung ↗ | [V] | $u_T^-$ | Sprungspannung ↘ | [V] | $u_H$ | Hysterese-spannung | [V] | $u_{ref}$ | Referenz-spannung | [V] | $u_{outmax}$ | → +Speisung | [V] | $u_{outmin}$ | → -Speisung | [V] | $R_F$ | Rückkopp-lungs-Wider-stand | [ $\Omega$ ] | $R_1$ | Widerstand | [ $\Omega$ ] |
| $u_T^+$   | Sprungspannung ↗   | [V]   |         |                  |     |         |                  |     |       |                    |     |           |                   |     |              |             |     |              |             |     |       |                            |              |       |            |              |
| $u_T^-$   | Sprungspannung ↘   | [V]   |         |                  |     |         |                  |     |       |                    |     |           |                   |     |              |             |     |              |             |     |       |                            |              |       |            |              |
| $u_H$   | Hysterese-spannung   | [V]   |         |                  |     |         |                  |     |       |                    |     |           |                   |     |              |             |     |              |             |     |       |                            |              |       |            |              |
| $u_{ref}$                                       | Referenz-spannung  | [V]   |         |                  |     |         |                  |     |       |                    |     |           |                   |     |              |             |     |              |             |     |       |                            |              |       |            |              |
| $u_{outmax}$                                    | → +Speisung  | [V]   |         |                  |     |         |                  |     |       |                    |     |           |                   |     |              |             |     |              |             |     |       |                            |              |       |            |              |
| $u_{outmin}$                                    | → -Speisung  | [V]   |         |                  |     |         |                  |     |       |                    |     |           |                   |     |              |             |     |              |             |     |       |                            |              |       |            |              |
| $R_F$   | Rückkopp-lungs-Wider-stand   | [ $\Omega$ ]  |         |                  |     |         |                  |     |       |                    |     |           |                   |     |              |             |     |              |             |     |       |                            |              |       |            |              |
| $R_1$   | Widerstand   | [ $\Omega$ ]  |         |                  |     |         |                  |     |       |                    |     |           |                   |     |              |             |     |              |             |     |       |                            |              |       |            |              |

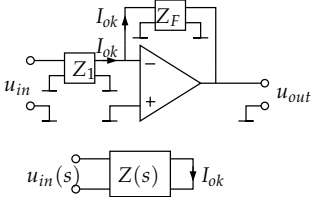


## 19.2.11. Wien-Robinson Oszillator

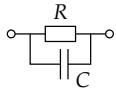
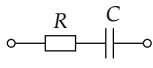
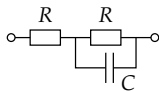
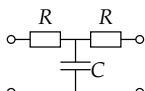
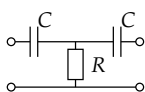
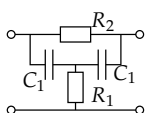
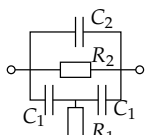
|   |   |  |     |         |     |     |          |     |           |         |     |     |            |     |     |           |     |
|---|---|--|-----|---------|-----|-----|----------|-----|-----------|---------|-----|-----|------------|-----|-----|-----------|-----|
|  | $ G  = \frac{\omega T}{\sqrt{(1 - \omega^2 T^2)^2 + (\omega 2DT)^2}}$ $T = RC, \quad D = \frac{3}{2}$ $\varphi = 90^\circ - \arctan\left(\frac{\omega 2DT}{1 - \omega^2 T^2}\right)$ <p>Amplitudenbedingung:<br/> <math>A_{Loop} \geq 1</math><br/>         Phasenbedingung:<br/> <math>\varphi_{Loop} = n \cdot 360^\circ,</math><br/> <math>n = 0, \pm 1, \pm 2, \dots</math></p> | <table> <tr> <td><math>T</math></td><td>Periode</td><td>[s]</td></tr> <tr> <td><math>D</math></td><td>Dämpfung</td><td>[1]</td></tr> <tr> <td><math>u_{out}</math></td><td>Ausgang</td><td>[V]</td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td>[Ω]</td></tr> <tr> <td><math>C</math></td><td>Kapazität</td><td>[F]</td></tr> </table> | $T$ | Periode | [s] | $D$ | Dämpfung | [1] | $u_{out}$ | Ausgang | [V] | $R$ | Widerstand | [Ω] | $C$ | Kapazität | [F] |
| $T$   | Periode   | [s]  |     |         |     |     |          |     |           |         |     |     |            |     |     |           |     |
| $D$   | Dämpfung  | [1]  |     |         |     |     |          |     |           |         |     |     |            |     |     |           |     |
| $u_{out}$   | Ausgang   | [V]  |     |         |     |     |          |     |           |         |     |     |            |     |     |           |     |
| $R$   | Widerstand  | [Ω]  |     |         |     |     |          |     |           |         |     |     |            |     |     |           |     |
| $C$   | Kapazität   | [F]  |     |         |     |     |          |     |           |         |     |     |            |     |     |           |     |

## 19. OPERATIONSVERSTÄRKER

### 19.2.12. Beschaltung des OPs mit Zweitoren

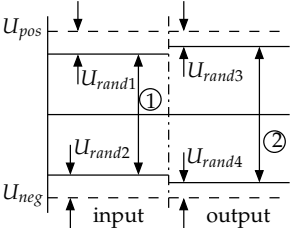
|   |  |   |     |             |     |          |         |     |           |         |     |     |          |              |          |                  |     |     |                  |     |
|---|--|---|-----|-------------|-----|----------|---------|-----|-----------|---------|-----|-----|----------|--------------|----------|------------------|-----|-----|------------------|-----|
|  | <p>Gilt nur für invertierenden Verstärker, da sonst kein virtueller Kurzschluss am Ausgang der Zweitore ist!</p> $A(s) = \frac{u_{in}}{u_{out}} = -\frac{Z_F(s)}{Z_1(s)}$ $Z(s) = \frac{u_{in}(s)}{i_{ok}(s)}$ | <table> <tr> <td><math>A</math></td><td>Verstärkung</td><td>[1]</td></tr> <tr> <td><math>u_{in}</math></td><td>Eingang</td><td>[V]</td></tr> <tr> <td><math>u_{out}</math></td><td>Ausgang</td><td>[V]</td></tr> <tr> <td><math>Z</math></td><td>Impedanz</td><td>[<math>\Omega</math>]</td></tr> <tr> <td><math>i_{ok}</math></td><td>Kurzschlussstrom</td><td>[A]</td></tr> <tr> <td><math>s</math></td><td>Laplace Operator</td><td>[1]</td></tr> </table> | $A$ | Verstärkung | [1] | $u_{in}$ | Eingang | [V] | $u_{out}$ | Ausgang | [V] | $Z$ | Impedanz | [ $\Omega$ ] | $i_{ok}$ | Kurzschlussstrom | [A] | $s$ | Laplace Operator | [1] |
| $A$   | Verstärkung  | [1]   |     |             |     |          |         |     |           |         |     |     |          |              |          |                  |     |     |                  |     |
| $u_{in}$  | Eingang  | [V]   |     |             |     |          |         |     |           |         |     |     |          |              |          |                  |     |     |                  |     |
| $u_{out}$   | Ausgang  | [V]   |     |             |     |          |         |     |           |         |     |     |          |              |          |                  |     |     |                  |     |
| $Z$   | Impedanz   | [ $\Omega$ ]  |     |             |     |          |         |     |           |         |     |     |          |              |          |                  |     |     |                  |     |
| $i_{ok}$  | Kurzschlussstrom   | [A]   |     |             |     |          |         |     |           |         |     |     |          |              |          |                  |     |     |                  |     |
| $s$   | Laplace Operator   | [1]   |     |             |     |          |         |     |           |         |     |     |          |              |          |                  |     |     |                  |     |

#### Häufig verwendete Zweitore

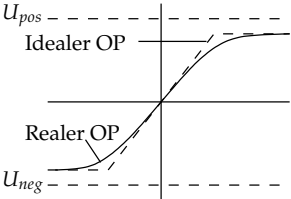
|   |  |  |
|---|--|--|
|    | $\frac{R}{1 + sRC}$  | <div><math>R</math> Kapazität [F]</div> <div><math>C</math> Widerstand [<math>\Omega</math>]</div> <div><math>Z</math> Impedanz [<math>\Omega</math>]</div> <div><math>s</math> Laplace Operator [1]</div> |
|  | $\frac{1}{sC}(1 + sRC)$  |  |
|  | $(R_1 + R_2) \frac{1 + s \frac{R_1 R_2 C}{R_1 + R_2}}{1 + sR_2C}$              |  |
|  | $R(2 + sRC)$   |  |
|  | $\frac{1}{sC} \frac{1 + s2RC}{sRC}$  |  |
|  | $R_2 \frac{1 + s2R_1C_1}{1 + s2R_1C_1 + s^2R_1R_2C_1^2}$                       |  |
|  | $\frac{R_2(1 + s2R_1C_1)}{1 + s(2R_1C_2 + R_2C_2) + s^2R_1R_2C_1(C_1 + 2C_2)}$ |  |

## 19.3. Realer Operationsverstärker

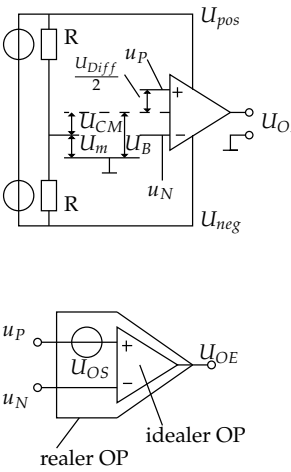
### 19.3.1. Ein- und Ausgangsspannungsbereich

|  |  |   |
|--|--|---|
|  <p>1: Eingangsspannungsbereich<br/>2: Ausgangsspannungsbereich</p> | <p>input rail to rail (IRR):<br/><math>U_{rand1}, U_{rand2} = 0</math></p> <p>output rail to rail (ORR):<br/><math>U_{rand3}, U_{rand4} = 0</math></p> | <p><math>U_{rand}</math> Randspannung [V]<br/><math>U_{pos}</math> Positive Speisespannung [V]<br/><math>U_{neg}</math> Negative Speisespannung [V]</p> |
|--|--|---|

### 19.3.2. Übertragungskennlinie

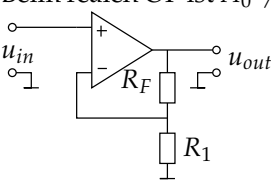
|  |  |   |
|--|--|---|
|  | <p><math>U_O = A_O U_{Diff}</math></p> <p><math>A_O = \frac{dU_O}{dU_D}</math></p> | <p><math>U_O</math> Ausgangsspannung [V]<br/><math>U_{Diff}</math> Differenzspannung [V]<br/><math>A_O</math> Open Loop Verstärkung [1]</p> |
|--|--|---|

### 19.3.3. Gleichtaktfehler (Common Mode Error)

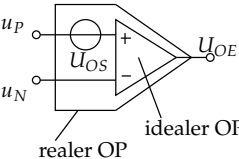
|   |  |   |
|---|--|---|
|  <p>idealer OP<br/>realer OP</p> | <p><math>U_m = \frac{U_{pos} + U_{neg}}{2}</math></p> <p><math>U_B = \frac{U_P + U_N}{2}</math></p> <p><math>U_{CM} = U_B - U_m</math></p> <p><math>U_{OS} = \frac{ U_{CM} }{CMRR}</math></p> <p><math>U_{OE} =  U_{OS}  A_{pos}</math></p> <p><math>U_{OE} = A_O \left( U_{Diff} + \frac{U_{CM}}{CMRR} + \frac{U_m}{A_O} \right)</math></p> <p><math>CMRR_{max} = \frac{U_{Diff}}{U_{CM}} = \frac{U_{Diff} + 1}{4\epsilon}</math></p> | <p><math>U_m</math> Mittenspan. [V]<br/><math>U_B</math> Biasspan. [V]<br/><math>U_{CM}</math> Common Mode Spannung [V]<br/><math>U_{OS}</math> Offsetspannung [V]<br/><math>U_{pos}</math> Speisespannung [V]<br/><math>U_{neg}</math> Speisespannung [V]<br/><math>U_P</math> Spannung am pos. Eingang [V]<br/><math>U_N</math> Spannung am neg. Eingang [V]<br/><math>A_O</math> Open Loop Verstärkung [1]<br/><math>CMRR</math> Gleichtaktunterdrückung [1]<br/><math>\epsilon</math> Widerstandstoleranz [1]</p> |
|---|--|---|

## 19. OPERATIONSVERSTÄRKER

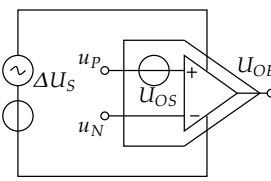
### 19.3.4. Effektive, geschlossene Verstärkung

|  |   |   |
|--|---|---|
| <p>Beim realen OP ist <math>A_0 \neq \infty</math></p>  | <p>Nicht invertierend:</p> $\frac{1}{ A_{Clreal} } = \frac{1}{ A_{Clid} } + \frac{1}{ A_0 }$ <p>Invertierend:</p> $\frac{1}{ A_{Clreal} } = \frac{1}{ A_{Clid} } + \frac{1}{ \eta A_0 }$ $\eta = \frac{R_F}{R_F + R_1}$ | <p><math>A_{Clreal}</math> Effektive Verstärkung real [1]</p> <p><math>A_{Clid}</math> Closed Loop Verstärkung ideal [1]</p> <p><math>A_0</math> Open Loop Verstärkung [1]</p> <p><math>R</math> Widerstand [<math>\Omega</math>]</p> |
|--|---|---|

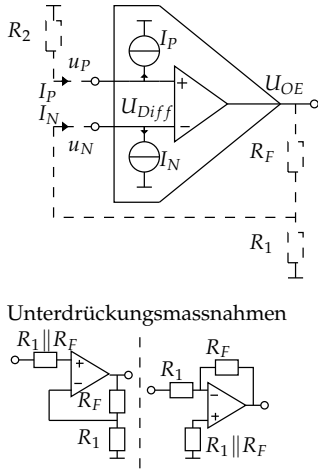
### 19.3.5. Offsetfehler

|  |  |  |
|--|--|--|
|  | $U_{OE} = U_{OS} A_{pos}$ $U_{OE} = U_{OS} \left( 1 + \frac{R_F}{R_1} \right)$ | <p><math>U_{OE}</math> Offset-Fehler-Spannung [V]</p> <p><math>U_{OS}</math> Offsetspannung [V]</p> <p><math>A_{pos}</math> pos. Verstärkung (DC) [1]</p> <p><math>R_F, R_1</math> siehe Seite 151 [<math>\Omega</math>]</p> |
|--|--|--|

### 19.3.6. Versorgungsspannungsfehler (Power supply error)

|   |  |  |
|---|--|--|
|  | $U_{OS} = \frac{\Delta U_S}{PSRR}$ $U_{OE} =  U_{OS}  A_{pos}$ | <p><math>U_S</math> Speisung [V]</p> <p><math>U_{OE}</math> Offset-Fehler-Spannung [V]</p> <p><math>U_{OS}</math> Offsetspannung [V]</p> <p><math>A_{pos}</math> pos. Verstärkung [1]</p> <p><math>PSRR</math> Unterdrückung der Speisungsspannungseinflüsse [1]</p> |
|---|--|--|

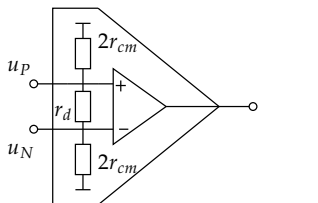
### 19.3.7. Eingangsströme (Bias- und Offsetstrom)

|  |  |   |
|--|--|---|
|  <p>Unterdrückungsmaßnahmen</p> | $I_{OS} =  I_P - I_N $ $I_B = \frac{I_P + I_N}{2}$ $U_{OE} =  -I_P R_2 A_{pos} + I_N R_F $ <p>Bester Fall (Einfluss <math>I_B = 0</math>):</p> $R_2 = R_F \parallel R_1$ $\Downarrow$ $U_{OE} =  -I_{OS} R_F $ | $I_{OS}$ Offsetstrom [A]<br>$I_{P,N}$ Strom am pos, neg Eingang [A]<br>$I_B$ Biasstrom [A]<br>$U_{OE}$ Offset-Fehler-Spannung [V]<br>$A_{pos}$ pos. Verstärkung [1]<br>$R_{1,2}$ Widerstand nach GND [ $\Omega$ ] |
|--|--|---|

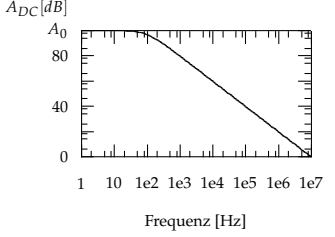
### 19.3.8. Kombination der statischen Fehler

|  |   |   |
|--|---|---|
|  | $U_{OE} = A_{pos} ( \text{Offsetfehler}  +  \text{Versorgungsspannungsfehler}  +  \text{Gleichtaktfehler} ) + \text{Eingangsstromfehler}$ $U_{OE} = A_{pos} \left[  U_{OS}  + \left  \frac{\Delta U_S}{PSRR} \right  + \left  \frac{\Delta U_{CM}}{CMRR} \right  \right] + *$ <p>Worst-Case:</p> $* = \left( I_N R_F - I_P R_2 \frac{R_F + R_1}{R_1} \right)$ <p>Bei unterdrücktem Biasstrom - Fehler:</p> $* =  I_{OS}  R_F$ | $I_{OS}$ Offsetstrom [A]<br>$I_{P,N}$ Strom am pos, neg Eingang [A]<br>$I_B$ Biasstrom [A]<br>$U_{OE}$ Offset-Fehler-Spannung [V]<br>$A_{pos}$ pos. Verstärkung [1] |
|--|---|---|

### 19.3.9. Dynamischer Eingangswiderstand

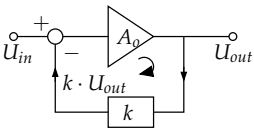
|   |  |   |
|---|--|---|
|  | <p>Messung bei verbundenen Eingängen:</p> $r_{cm} = 2r_{cm} \parallel 2r_{cm}$ | $r_d$ Dynamischer Widerstand [ $\Omega$ ]<br>$r_{cm}$ Common Mode Resistance [ $\Omega$ ] |
|---|--|---|

### 19.3.10. Frequenzgang

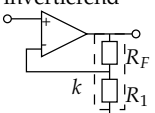
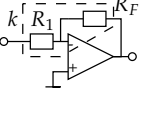
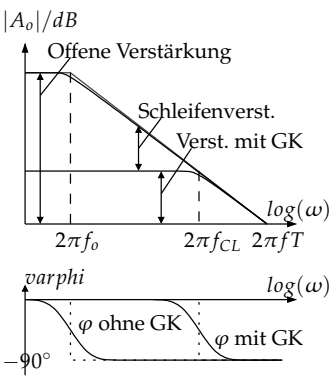
|   |  |  |
|---|--|--|
|  | <p>Knick:</p> $f_0 : A_{OL} = A_{DC} - 3dB$ <p>( ca. 100 Hz in Grafik )</p> $f_T : A_{OL} = 0dB = 1$ <p>( ca. <math>10^7</math> Hz in Grafik )</p> $f_0 = \frac{f_T}{A_0}$ <p>Der Verstärkungsabfall beträgt - 20 <math>\frac{dB}{Dec}</math></p> $A_{CLreal}(s) = \frac{A_{CLDC}}{1 + sT_{neu}}$ $A_{CLDC} = \frac{A_{OLDC}}{1 + k(s)A_{OLDC}}$ $T_{neu} = \frac{T_0}{1 + k(s)A_{OLDC}}$ $\omega_{neu} = \omega_0[1 + k(s)A_{OLDC}]$ <p>Nichtinvertierender Verstärker:</p> $k(s) = \frac{R_1 + R_F}{R_1}$ $f_{neu} = f_0(1 + kA_{OLDC})$ $f_{neu}A_{CL}^+ = GBP(= f_T)$ $f_{neu} = f_0A_{OLDC}$ <p>Invertierenden Verstärker:</p> $f_{neu} = k \cdot BGP = \frac{GBP}{A_{CL}^- + 1}$ $f_{neu}(A_{CL}^- + 1) = GBP(= f_T)$ $f_{neu} = \frac{1}{2}f_0A_{OLDC}$ | <p><math>f_0</math> Kleinsignal [Hz]<br/>Bandbreite</p> <p><math>f_T</math> Transittfrequenz, [Hz]<br/>Verstärkungs-<br/>Bandbreiten-<br/>Produkt</p> <p><math>A_{OL}</math> Open Loop [1]<br/>Gain</p> <p><math>A_{CL}</math> Closed Loop [1]<br/>Gain</p> <p><math>A_{CL}^+</math> <math>A_{CL}</math> nichtin- [1]<br/>vertierender<br/>Verstärker</p> <p><math>s</math> Laplace Ope- [1]<br/>rator</p> <p><math>T_{neu}</math> Closed Loop [s]<br/>Zeitkonst.</p> <p><math>k</math> Faktor des [1]<br/>Spannungs-<br/>teilers</p> <p><math>\omega</math> Knickfrequenz [Hz]</p> <p><math>f_{neu}</math> Knickfrequenz [Hz]</p> <p><math>GBP</math> Verstärkungs [1]<br/>Bandbreiten-<br/>produkt</p> |
|---|--|--|

# 20. Gegengekoppelte Verstärker

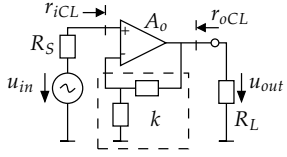
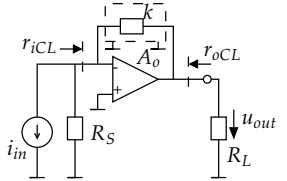
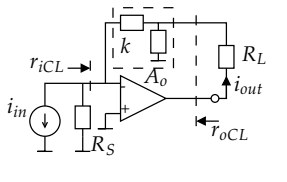
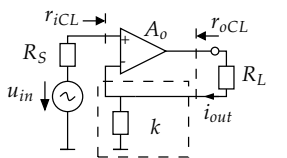
## 20.1. Mit- und Gegenkopplung

|   |   |  |
|---|---|--|
|  | <p>Gegenkopplung:</p> $A_{CL} = \frac{U_{out}}{U_{in}} = \frac{A_o}{1 + kA_o}$ <p>Mitkopplung:</p> $A_{CL} = \frac{U_{out}}{U_{in}} = \frac{A_o}{1 - kA_o}$ | <p><math>A_{CL}</math> Closed Loop [1]<br/>Verstärkung</p> <p><math>A_o</math> Open Loop [1]<br/>Verstärkung</p> <p><math>U</math> Spannung [V]</p> <p><math>k</math> Faktor [1]</p> |
|---|---|--|

### 20.1.1. Gegenkopplung beim OP

|  |   |  |
|--|---|--|
| <p>Nicht-invertierend</p>  <p>Invertierend</p>  <p>Bodeplot:</p>  | <p>Ideal:</p> $A_{CL} = \frac{nA_o}{1 + kA_o}$ <p>Nicht invertierend:</p> $n = 1$ $ A_{CLideal}  = \frac{R_F + R_1}{R_1} = \frac{1}{k}$ <p>Invertierend:</p> $n = \frac{R_F}{R_1 + R_F} \quad k = \frac{R_1}{R_1 + R_F}$ $ A_{CLideal}  = \frac{R_F}{R_1}$ <p>Real:</p> $A_{CLreal} = nA_o \parallel A_{CLideal}$ | <p><math>A_{CL}</math> Closed Loop [1]<br/>Verstärkung</p> <p><math>A_o</math> Open Loop [1]<br/>Verstärkung</p> <p><math>k</math> GK-Faktor [1]</p> <p><math>n</math> Faktor [1]</p> <p><math>R</math> Widerstand [<math>\Omega</math>]</p> |
|--|---|--|

## 20.2. Gegenkopplungsarten

|  |  |  |
|--|--|--|
| <p>Serie-Parallel</p>     | <p>Eingang: Seriell<br/>Ausgang: Parallel</p> $r_{iCL} \rightarrow \infty \quad r_{oCL} \rightarrow 0$ $r_{iCL} = r_i(1 + kA_o)$ $r_{oCL} = \frac{u_{out}}{i_{out}} = \frac{r_o}{1 + kA_o}$    | <p><math>A_o</math> Open Loop [1]<br/>Verstärkung</p> <p><math>k</math> Faktor [1]</p> <p><math>r_i</math> Open Loop r- [Ω]<br/>Eingang</p> <p><math>r_o</math> Open Loop r- [Ω]<br/>Ausgang</p> <p><math>r_{iCL}</math> Closed Loop [Ω]<br/>r-Eingang</p> <p><math>r_{oCL}</math> Closed Loop [Ω]<br/>r-Ausgang</p> <p><math>u_{out}</math> u-Ausgang [V]</p> <p><math>i_{out}</math> i-Ausgang [A]</p> <p><math>R_S</math> Quell- [Ω]<br/>Widerst.</p> <p><math>R_L</math> Last-Widerst. [Ω]</p> |
| <p>Parallel-Parallel</p>  | <p>Eingang: Parallel<br/>Ausgang: Parallel</p> $r_{iCL} \rightarrow 0 \quad r_{oCL} \rightarrow 0$ $r_{iCL} = \frac{r_i}{1 + kA_o}$ $r_{oCL} = \frac{u_{out}}{i_{out}} = \frac{r_o}{1 + kA_o}$ |  |
| <p>Parallel-Serie</p>   | <p>Eingang: Parallel<br/>Ausgang: Seriell</p> $r_{iCL} \rightarrow 0 \quad r_{oCL} \rightarrow \infty$ $r_{iCL} = \frac{r_i}{1 + kA_o}$ $r_{oCL} = \frac{u_{out}}{i_{out}} = r_o(1 + kA_o)$    |  |
| <p>Serie-Serie</p>      | <p>Eingang: Seriell<br/>Ausgang: Seriell</p> $r_{iCL} \rightarrow \infty \quad r_{oCL} \rightarrow \infty$ $r_{iCL} = r_i(1 + kA_o)$ $r_{oCL} = \frac{u_{out}}{i_{out}} = r_o(1 + kA_o)$       |  |

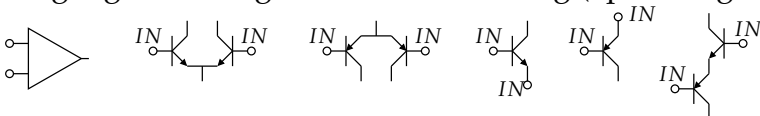


### 20.2.1. Bestimmung der Gegenkopplungsart

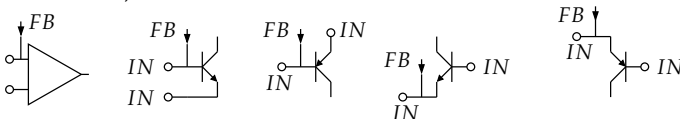
1. Forwärtspfad, Rückwärtspfad und Gegenkopplungsschleife einzeichnen.
2. Anzahl Inversionen im Vorwärtspfad ( $\Rightarrow$  Invertierend oder nicht invertierend) bzw. in der Schleife bestimmen ( $\Rightarrow$  Gegenkopplung bei ungerade Anzahl bzw. Mittkopplung bei gerader Anzahl).
3. Knoten (out, in+ und in- ) der Äquivalenten OP-Schaltung bestimmen.
4. Äquivalenten OP-Schaltung zeichnen.

### 20.2.2. Eingangsschaltungen

Eingangsschaltungen bei Serieschaltung (Spannungsaddition)

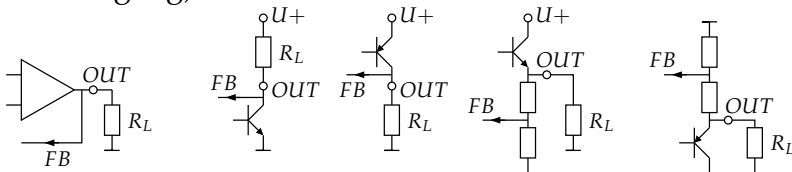


Eingangsschaltungen bei Parallelschaltung von Verstärkereingang und Ausgang (Stromaddition)

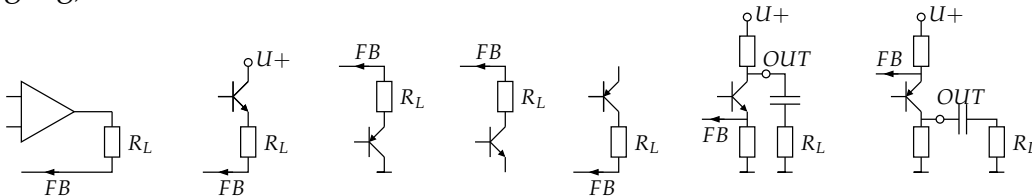


### 20.2.3. Ausgangsschaltungen

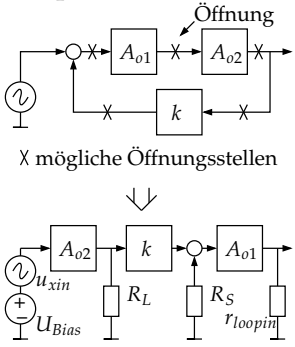
Ausgangsschaltungen bei Parallelschaltung von Last und Eingang (Spannungsabnahme am Ausgang)



Ausgangsschaltungen bei Serieschaltung von Last und Eingang (Stromabnahme am Ausgang)



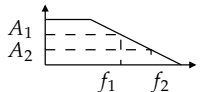
## 20.3. Schleifenverstärkung

|   |   |   |       |                       |     |       |                       |     |     |          |     |     |        |     |     |            |              |
|---|---|---|-------|-----------------------|-----|-------|-----------------------|-----|-----|----------|-----|-----|--------|-----|-----|------------|--------------|
| <p>Beispiel:</p>  <p>Öffnung</p> <p>X mögliche Öffnungsstellen</p> | $A_L = kA_o = \frac{u_{xout}}{u_{xin}}$ <p>Gegenkopplungsgrad:</p> $1 + A_L = 1 + kA_L$ <p><math>U_{Bias}</math> legt den Arbeitspunkt fest.<br/>Es soll eine Trennstelle gewählt werden bei der <math>r_{loopout} \gg r_{loopin}</math> gilt <math>\Rightarrow</math> Belastung des Schleifenausganges kann vernachlässigt werden.</p> | <table> <tr> <td><math>A_L</math></td><td>Schleifen-Verstärkung</td><td>[1]</td></tr> <tr> <td><math>A_o</math></td><td>Open Loop Verstärkung</td><td>[1]</td></tr> <tr> <td><math>U</math></td><td>Spannung</td><td>[V]</td></tr> <tr> <td><math>k</math></td><td>Faktor</td><td>[1]</td></tr> <tr> <td><math>R</math></td><td>Widerstand</td><td>[<math>\Omega</math>]</td></tr> </table> | $A_L$ | Schleifen-Verstärkung | [1] | $A_o$ | Open Loop Verstärkung | [1] | $U$ | Spannung | [V] | $k$ | Faktor | [1] | $R$ | Widerstand | [ $\Omega$ ] |
| $A_L$   | Schleifen-Verstärkung   | [1]   |       |                       |     |       |                       |     |     |          |     |     |        |     |     |            |              |
| $A_o$   | Open Loop Verstärkung   | [1]   |       |                       |     |       |                       |     |     |          |     |     |        |     |     |            |              |
| $U$   | Spannung  | [V]   |       |                       |     |       |                       |     |     |          |     |     |        |     |     |            |              |
| $k$   | Faktor  | [1]   |       |                       |     |       |                       |     |     |          |     |     |        |     |     |            |              |
| $R$   | Widerstand  | [ $\Omega$ ]  |       |                       |     |       |                       |     |     |          |     |     |        |     |     |            |              |

## 20.4. Wirkung der GK auf die Sensivität der Verstärkung

|  |  |  |     |              |     |       |                       |     |       |                       |     |     |        |     |     |                       |       |     |                     |       |
|--|--|--|-----|--------------|-----|-------|-----------------------|-----|-------|-----------------------|-----|-----|--------|-----|-----|-----------------------|-------|-----|---------------------|-------|
| <p>Die Sensitivität <math>S_x^N</math> ist ein Mass für die Empfindlichkeit einer Schaltungseigenschaft N gegenüber Schwankungen eines Parameters x.</p> | $S_x^N = \frac{\frac{dN}{N}}{\frac{dx}{x}}$ $S_{A_o}^{A_{CL}} = \frac{\frac{dA_{CL}}{A_{CL}}}{\frac{dA_o}{A_o}} = \frac{A_o}{A_{CL}} \frac{dA_{CL}}{dA_o}$ $S_{A_o}^{A_{CL}} = \frac{1}{1 + kA_o}$ | <table> <tr> <td><math>S</math></td><td>Sensitivität</td><td>[1]</td></tr> <tr> <td><math>A_L</math></td><td>Schleifen-Verstärkung</td><td>[1]</td></tr> <tr> <td><math>A_o</math></td><td>Open Loop Verstärkung</td><td>[1]</td></tr> <tr> <td><math>k</math></td><td>Faktor</td><td>[1]</td></tr> <tr> <td><math>x</math></td><td>veränderter Parameter</td><td>[...]</td></tr> <tr> <td><math>N</math></td><td>Beeinflusste Grösse</td><td>[...]</td></tr> </table> | $S$ | Sensitivität | [1] | $A_L$ | Schleifen-Verstärkung | [1] | $A_o$ | Open Loop Verstärkung | [1] | $k$ | Faktor | [1] | $x$ | veränderter Parameter | [...] | $N$ | Beeinflusste Grösse | [...] |
| $S$  | Sensitivität   | [1]  |     |              |     |       |                       |     |       |                       |     |     |        |     |     |                       |       |     |                     |       |
| $A_L$  | Schleifen-Verstärkung  | [1]  |     |              |     |       |                       |     |       |                       |     |     |        |     |     |                       |       |     |                     |       |
| $A_o$  | Open Loop Verstärkung  | [1]  |     |              |     |       |                       |     |       |                       |     |     |        |     |     |                       |       |     |                     |       |
| $k$  | Faktor   | [1]  |     |              |     |       |                       |     |       |                       |     |     |        |     |     |                       |       |     |                     |       |
| $x$  | veränderter Parameter  | [...]  |     |              |     |       |                       |     |       |                       |     |     |        |     |     |                       |       |     |                     |       |
| $N$  | Beeinflusste Grösse  | [...]  |     |              |     |       |                       |     |       |                       |     |     |        |     |     |                       |       |     |                     |       |

## 20.5. Das Verstärkungs-Bandbreiten-Produkt

|   |  |   |
|---|--|---|
| <p>Für alle Punkte die auf einer Amplitudengeraden mit einer Neigung von <math>\pm 20 \frac{dB}{Dek}</math> liegen gilt das Gesetz vom konstanten Verstärkungs-Bandbreiten-Produkt. Siehe auch S. 115</p>  | $Af = f_T = GBP$<br>$A_1 f_1 = A_2 f_2$<br>$A_{oDC} = f_o = GBP$ | $f_T$ Transitfrequenz $\left[\frac{1}{s}\right]$<br>= Amplitude<br>$\cap$ 0dB-Achse<br>$f$ Frequenz $\left[\frac{1}{s}\right]$<br>$A$ Verstärkung $[1]$<br>$A_{oDC}$ Open-Loop $[1]$<br>DC-Gain |
|---|--|---|

**Teil V.**

**Digitale Signalverarbeitung**

# 21. Stochastische Signale

## 21.1. Allgemein

|       |          |     |        |       |
|-------|----------|-----|--------|-------|
| hallo | $M = Fr$ | $r$ | Radius | $[m]$ |
|-------|----------|-----|--------|-------|

## 22. Abtastung

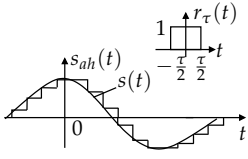
### 22.1. Ideale Abtastung

|   |   |  |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |     |              |     |          |               |                   |            |                |                   |
|---|---|--|-----|--------|-------|-------|--------------|-------|-----|----------------|-------|-----|------|-----|-----|---------|-----|-----|--------------|-----|----------|---------------|-------------------|------------|----------------|-------------------|
| <p><b>Zeitbereich:</b></p> <p><b>Frequenzbereich:</b></p> | $s_a(t) = s(t)T\delta_p(t)$ $s_a(t) = Ts(t) \sum_{m=-\infty}^{\infty} \delta(t - mT)$ $s_a(t) = T \sum_{m=-\infty}^{\infty} s(mT)\delta(t - mT)$ $S_a(\omega) = S(\omega) * \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_c)$ $S_a(\omega) = \sum_{k=-\infty}^{\infty} S(\omega - k\omega_c)$ $\omega_c = \frac{2\pi}{T}$ | <table> <tr> <td><math>s</math></td><td>Signal</td><td>[...]</td></tr> <tr> <td><math>s_a</math></td><td>s abgetastet</td><td>[...]</td></tr> <tr> <td><math>S</math></td><td>Spektrum von s</td><td>[...]</td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td>[s]</td></tr> <tr> <td><math>T</math></td><td>Periode</td><td>[s]</td></tr> <tr> <td><math>m</math></td><td>m-te Periode</td><td>[1]</td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td>[<math>\frac{1}{s}</math>]</td></tr> <tr> <td><math>\omega_c</math></td><td>Abtastfrequenz</td><td>[<math>\frac{1}{s}</math>]</td></tr> </table> | $s$ | Signal | [...] | $s_a$ | s abgetastet | [...] | $S$ | Spektrum von s | [...] | $t$ | Zeit | [s] | $T$ | Periode | [s] | $m$ | m-te Periode | [1] | $\omega$ | Kreisfrequenz | [ $\frac{1}{s}$ ] | $\omega_c$ | Abtastfrequenz | [ $\frac{1}{s}$ ] |
| $s$   | Signal  | [...]  |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |     |              |     |          |               |                   |            |                |                   |
| $s_a$   | s abgetastet  | [...]  |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |     |              |     |          |               |                   |            |                |                   |
| $S$   | Spektrum von s  | [...]  |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |     |              |     |          |               |                   |            |                |                   |
| $t$   | Zeit  | [s]  |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |     |              |     |          |               |                   |            |                |                   |
| $T$   | Periode   | [s]  |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |     |              |     |          |               |                   |            |                |                   |
| $m$   | m-te Periode  | [1]  |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |     |              |     |          |               |                   |            |                |                   |
| $\omega$  | Kreisfrequenz   | [ $\frac{1}{s}$ ]  |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |     |              |     |          |               |                   |            |                |                   |
| $\omega_c$  | Abtastfrequenz  | [ $\frac{1}{s}$ ]  |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |     |              |     |          |               |                   |            |                |                   |

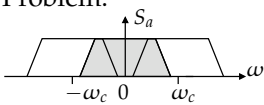
### 22.2. Flat Top Sampling

|   |   |   |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |        |                |     |     |              |     |          |               |                   |
|---|---|---|-----|--------|-------|-------|--------------|-------|-----|----------------|-------|-----|------|-----|-----|---------|-----|--------|----------------|-----|-----|--------------|-----|----------|---------------|-------------------|
| <p>Signal wird verzerrt durch <math>G_\tau(\omega)</math></p> | $s_a(t) = \sum_{m=-\infty}^{\infty} s(mT)r_\tau(t - mT)$ $S_a(\omega) = G_\tau(\omega) \sum_{k=-\infty}^{\infty} S(\omega - k\omega_c)$ $G_\tau(\omega) = \frac{1}{T}R_\tau(\omega) = \frac{\tau}{T} \frac{\sin(\frac{\tau}{2}\omega)}{\frac{\tau}{2}\omega}$ <p>Je kürzer die Abtast-Pulse desto breiter die <math>\frac{\sin(x)}{x}</math> Kurve.</p> | <table> <tr> <td><math>s</math></td><td>Signal</td><td>[...]</td></tr> <tr> <td><math>s_a</math></td><td>s abgetastet</td><td>[...]</td></tr> <tr> <td><math>S</math></td><td>Spektrum von s</td><td>[...]</td></tr> <tr> <td><math>t</math></td><td>Zeit</td><td>[s]</td></tr> <tr> <td><math>T</math></td><td>Periode</td><td>[s]</td></tr> <tr> <td><math>\tau</math></td><td>Rechteckbreite</td><td>[s]</td></tr> <tr> <td><math>m</math></td><td>m-te Periode</td><td>[1]</td></tr> <tr> <td><math>\omega</math></td><td>Kreisfrequenz</td><td>[<math>\frac{1}{s}</math>]</td></tr> </table> | $s$ | Signal | [...] | $s_a$ | s abgetastet | [...] | $S$ | Spektrum von s | [...] | $t$ | Zeit | [s] | $T$ | Periode | [s] | $\tau$ | Rechteckbreite | [s] | $m$ | m-te Periode | [1] | $\omega$ | Kreisfrequenz | [ $\frac{1}{s}$ ] |
| $s$   | Signal  | [...]   |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |        |                |     |     |              |     |          |               |                   |
| $s_a$   | s abgetastet  | [...]   |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |        |                |     |     |              |     |          |               |                   |
| $S$   | Spektrum von s  | [...]   |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |        |                |     |     |              |     |          |               |                   |
| $t$   | Zeit  | [s]   |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |        |                |     |     |              |     |          |               |                   |
| $T$   | Periode   | [s]   |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |        |                |     |     |              |     |          |               |                   |
| $\tau$  | Rechteckbreite  | [s]   |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |        |                |     |     |              |     |          |               |                   |
| $m$   | m-te Periode  | [1]   |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |        |                |     |     |              |     |          |               |                   |
| $\omega$  | Kreisfrequenz   | [ $\frac{1}{s}$ ]   |     |        |       |       |              |       |     |                |       |     |      |     |     |         |     |        |                |     |     |              |     |          |               |                   |

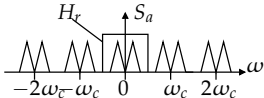
## 22.3. Sample and Hold

|   |  |   |
|---|--|---|
|  | <p>Entspricht Flat Top Sampling (S. 168) bei <math>\tau = T</math></p> <p>Die <math>\frac{\sin(x)}{x}</math> Kurve hat die Nullgänge bei <math>k\frac{2\pi}{T}, k = \{1, 2, \dots\}</math></p> | <p><math>T</math> Periode [s]</p> <p><math>\tau</math> Rechteckbreite [s]</p> |
|---|--|---|

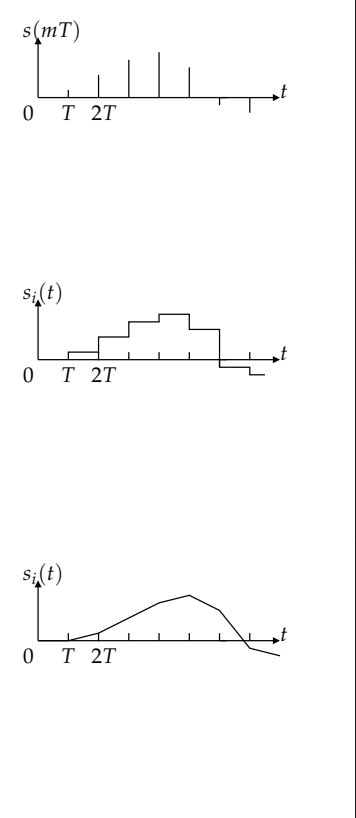
## 22.4. Abtasttheorem

|   |  |  |
|---|--|--|
| <p>Problem:</p>  <p><math>\Rightarrow</math> Rekonstruktion ist nicht möglich.</p> | <p><math>\omega_c &gt; 2\omega_{max}</math></p> <p><math>\Rightarrow</math> Praktisch muss immer ein analoger Tiefpass vorgeschaltet werden.</p> | <p><math>\omega_c</math> Abtastfrequenz <math>\left[\frac{1}{s}\right]</math></p> <p><math>\omega_{max}</math> max Frequenz <math>\left[\frac{1}{s}\right]</math> in <math>s(t)</math></p> |
|---|--|--|

## 22.5. Rekonstruktion

|   |   |  |
|---|---|--|
| <p>Ist das Abtasttheorem erfüllt, so ist das ursprüngliche Signal exakt reproduzierbar.</p>  | $s_r(t) = T \sum_{m=-\infty}^{\infty} s(mT)h_r(t - mT)$ $s_r(t) = T \sum_{m=-\infty}^{\infty} s(mT)\delta(t - mT) * h_r(t)$ $h_r(t) = \frac{\omega_c}{2\pi} \frac{\sin\left(\frac{\omega_c}{2}t\right)}{\frac{\omega_c}{2}t}$ | <p><math>s_r</math> Signal re- konst. [...]</p> <p><math>h_r</math> Stossantw. [...]</p> <p>Rekonstruktions-Tiefpass</p> <p><math>t</math> Zeit [s]</p> <p><math>T</math> Periode [s]</p> <p><math>m</math> m-te Periode [1]</p> <p><math>\omega_c</math> Abtastfrequenz <math>\left[\frac{1}{s}\right]</math></p> |
|---|---|--|

## 22.5.1. Interpolation

|  |  |   |
|--|--|---|
|  | $s_i(t) = \sum_{m=-\infty}^{\infty} s(mT)h_i(t - mT)$ $S_i(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} S(\omega - k\omega_c)H_i(\omega)$ <p>Halteglied nullter Ordnung</p> $h_i(t) = \text{Rechteck}, h = 1, \tau = T$ $H_i(\omega) = T \frac{\sin\left(\frac{T}{2}\omega\right)}{\frac{T}{2}\omega} e^{-j\frac{T}{2}\omega}$ <p>Lineare Interpolation</p> $h_i(t) = \text{Dreieck}, h = 1, \tau = 2T$ $H_i(\omega) = T \left( \frac{\sin\left(\frac{T}{2}\omega\right)}{\frac{T}{2}\omega} \right)^2 e^{-jT\omega}$ | <p><math>s_i</math> Signal interpoliert [...]</p> <p><math>h_i</math> Interpolationsfunktion [1]</p> <p><math>t</math> Zeit [s]</p> <p><math>T</math> Periode [s]</p> <p><math>\tau</math> Pulsbreite [s]</p> <p><math>m, k</math> m, k-te Periode [1]</p> <p><math>\omega</math> Kreisfrequenz [<math>\frac{1}{s}</math>]</p> <p><math>\omega_c</math> Abtastfrequenz [<math>\frac{1}{s}</math>]</p> |
|--|--|---|

## 22.6. Energie und Leistung bandbegrenzter Signale

|  |   |  |
|--|---|--|
| <p>Falls das Abtasttheorem, <math>T &lt; \frac{1}{2}f_{max}</math> eingehalten wird, hat das abgetastete Signal die selbe Energie bzw. Leistung wie das Original. Siehe Parsevalsches Theorem S. 188</p> | $W = \int_{-\infty}^{\infty} s^2(t)dt$ $W = T \sum_{m=-\infty}^{\infty} s^2(mT)$ $P = \frac{1}{T_{per}} \int_0^{T_{per}} s^2(t)dt$ $P = \frac{1}{N} \sum_{m=0}^{N-1} s^2(mT)$ $N = \frac{T_{per}}{T}$ | <p><math>W</math> Energie [Ws]</p> <p><math>P</math> Leistung [W]</p> <p><math>s</math> Signal [...]</p> <p><math>t</math> Zeit [s]</p> <p><math>T</math> Periode [s]</p> <p><math>T_{per}</math> Periodenintervall [s]</p> <p><math>m</math> m-te Periode [1]</p> <p><math>\omega_c</math> Abtastfrequenz [<math>\frac{1}{s}</math>]</p> <p><math>N</math> Abtastwertzahl [1]</p> |
|--|---|--|



**Teil VI.**  
**Mathematik**

## 23. Grundlagen

## 23.1. Allgemeines

### 23.1.1. Binome

$$\binom{n}{k} = \frac{n!}{k! (n-k)!}$$

$$(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

$$(a - b)^n = \sum_{k=0}^n (-1)^k \binom{n}{k} a^{n-k} b^k$$

$$(a^2 - b^2) = (a - b)(a + b)$$

$$(a^3 \pm b^3) = (a \pm b) (a^2 \mp ab + b^2)$$

$$\begin{array}{cccccccccccccccc}
 & & & & & & 1 & & & & & & & & & \\
 & & & & & 1 & 1 & & & & & & & & & \\
 & & & 1 & 2 & 1 & & & & & & & & & & \\
 & & 1 & 3 & 3 & 1 & & & & & & & & & & \\
 & 1 & 4 & 6 & 4 & 1 & & & & & & & & & & \\
 & 1 & 5 & 10 & 10 & 5 & 1 & & & & & & & & & \\
 & 1 & 6 & 15 & 20 & 15 & 6 & 1 & & & & & & & & \\
 & 1 & 7 & 21 & 35 & 35 & 21 & 7 & 1 & & & & & & & \\
 & 1 & 8 & 28 & 56 & 70 & 56 & 28 & 8 & 1 & & & & & & \\
 & 1 & 9 & 36 & 84 & 126 & 126 & 84 & 36 & 9 & 1 & & & & & \\
 & 1 & 10 & 45 & 120 & 210 & 252 & 210 & 120 & 45 & 10 & 1 & & & & \\
 & 1 & 11 & 55 & 165 & 330 & 462 & 462 & 330 & 165 & 55 & 11 & 1 & & & \\
 1 & 12 & 66 & 220 & 495 & 792 & 924 & 792 & 495 & 220 & 66 & 12 & 1 & & & 
 \end{array}$$

### 23.1.2. Faktorzerlegungen

$$a^2 - b^2 = (a + b)(a - b)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + \dots + ab^{n-2} + b^{n-1})$$

für n gerade:

$$a^n - b^n = (a + b)(a^{n-1} - a^{n-2}b + \dots + ab^{n-2} - b^{n-1})$$

für n ungerade:

$$a^n + b^n = (a + b)(a^{n-1} - a^{n-2}b + \dots - ab^{n-2} + b^{n-1})$$

$$s^2 + 1 = (s - j)(s + j)$$

**23.1.3. Quadratische Gleichung**

$$ax^2 + bx + c = 0 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**23.1.4. Arithmetische Folge**

$$a_{n+1} - a_n = d, \quad d \text{ const.}$$

$$a_n = a_1 + (n-1)d$$

$$s_n = n \frac{a_1 + a_n}{2} = a_1 n + \frac{n(n-1)}{2} d$$

**23.1.5. Geometrische Folge**

$$a_{n+1}/a_n = q, \quad q \text{ const.}$$

$$a_n = a_1 q^{n-1}$$

$$s_n = a_1 \frac{1 - q^n}{1 - q}$$

$$s = \lim_{n \rightarrow \infty} s_n = \frac{a_1}{1 - q}, \quad \text{falls } |q| < 1$$

**23.1.6. Partialbruchzerlegung**

$$r(z) = \frac{r_1(z)}{(z-a)(z-b)^3((z-c)^2 + d^2)^3}$$

$$r(z) = \frac{\alpha}{z-a} + \frac{\beta_1}{z-b} + \frac{\beta_2}{(z-b)^2} + \frac{\beta_3}{(z-b)^3} +$$

$$\frac{\gamma_1 z + \delta_1}{(z-c)^2 + d^2} + \frac{\gamma_2 z + \delta_2}{((z-c)^2 + d^2)^2} + \frac{\gamma_3 z + \delta_3}{((z-c)^2 + d^2)^3}$$

**23.2. Matrizen und Determinanten****23.2.1.  $2 \times 2$  Matrizen**

$$\det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} a_{11}c_{11} + a_{12}c_{21} & a_{11}c_{12} + a_{12}c_{22} \\ a_{21}c_{11} + a_{22}c_{21} & a_{21}c_{12} + a_{22}c_{22} \end{bmatrix}$$

Achtung:  $AB \neq BA$  !

Inverse: (falls  $a_{11}a_{22} - a_{12}a_{21} \neq 0$ )

$$A^{-1} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

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### 23.2.2. $3 \times 3$ Matrizen

$$\det \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} =$$
$$= a_{11} \det \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12} \det \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13} \det \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

### 23.2.3. Transponierte einer Matrix

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix} \quad A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} & a_{41} \\ a_{12} & a_{22} & a_{32} & a_{42} \end{bmatrix}$$

$$(A \cdot B)^T = B^T \cdot A^T$$

$$(A \cdot B \cdot C)^T = C^T \cdot B^T \cdot A^T$$

$$(A^T)^{-1} = (A^{-1})^T$$

## 23.3. Vektorrechnung

### 23.3.1. Grundlagen

#### Skalarprodukt

$$\vec{x} \cdot \vec{y} = xy \cos \alpha$$

$$\vec{x} \perp \vec{y} \Leftrightarrow \vec{x} \cdot \vec{y} = 0$$

#### Skalare Projektion von $\vec{b}$ auf $\vec{a}$

$$b_a = \vec{b} \vec{e}_a$$

#### Vektorielle Projektion von $\vec{b}$ auf $\vec{a}$

$$\vec{b}_a = b_a \vec{e}_a = (\vec{b} \vec{e}_a) \vec{e}_a$$

#### Vektorprodukt

$$|\vec{a} \times \vec{b}| = a b \sin \alpha$$

$$\vec{a} \times \vec{b} = [a_1, a_2, a_3] \times [b_1, b_2, b_3] = \left[ \det \begin{bmatrix} a_2 & a_3 \\ b_2 & b_3 \end{bmatrix}, -\det \begin{bmatrix} a_3 & a_1 \\ b_3 & b_1 \end{bmatrix}, \det \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \right]$$

$$\vec{a} = \lambda \cdot \vec{b} \Leftrightarrow \vec{a} \times \vec{b} = 0$$

$$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$$

**Steigung eines Vektors**

$$\vec{x} = (x_1, x_2, x_3)$$

$$\tan \alpha = \frac{x_3}{\sqrt{x_1^2 + x_2^2}}$$

**23.3.2. Lineare Abbildungen**

**Drehung der XY-Ebene um den Ursprung mit Drehwinkel  $\varphi$**

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

**Spiegelung der XY-Ebene an der Geraden g durch den Ursprung mit den Steigungswinkel  $\varphi$**

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = \begin{bmatrix} \cos 2\varphi & \sin 2\varphi \\ \sin 2\varphi & -\cos 2\varphi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

**Drehung des Raumes um die X-Achse**

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & -\sin \varphi \\ 0 & \sin \varphi & \cos \varphi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

**Drehung des Raumes um die Y-Achse**

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = \begin{bmatrix} \cos \varphi & 0 & \sin \varphi \\ 0 & 1 & 0 \\ -\sin \varphi & 0 & \cos \varphi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

**Drehung des Raumes um die Z-Achse**

$$\begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = \begin{bmatrix} \cos \varphi & -\sin \varphi & 0 \\ \sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

**23.4. Trigonometrie****23.4.1. Komplementwinkel**

$$\sin \alpha = \cos\left(\frac{\pi}{2} - \alpha\right) \quad \cos \alpha = \sin\left(\frac{\pi}{2} - \alpha\right)$$

$$\tan \alpha = \cot\left(\frac{\pi}{2} - \alpha\right) \quad \cot \alpha = \tan\left(\frac{\pi}{2} - \alpha\right)$$

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### 23.4.2. Sinussatz

$$\sin \alpha = \sin(\pi - \alpha)$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2r$$

wobei  $r$  = Umkreisradius

### 23.4.3. Cosinussatz

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

## 23.5. Goniometrie

### 23.5.1. Serien (Lösungsmengen)

$$\alpha_1 = \arcsin x, \quad \alpha_2 = \pi - \alpha_1$$

$$\alpha_{1n} = \alpha_1 + n2\pi, \quad \alpha_{2n} = \alpha_2 + n2\pi$$

$$\pm \alpha = \arccos x, \quad \alpha_n = \pm \alpha + n2\pi$$

$$\alpha_0 = \arctan x, \quad \alpha_n = \alpha_0 + n\pi, \quad n \in \mathbb{Z}$$

### 23.5.2. Potenzen

$$1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin^3 \alpha = \frac{1}{4} (3 \sin \alpha - \sin 3\alpha)$$

$$\cos^3 \alpha = \frac{1}{4} (3 \cos \alpha + \cos 3\alpha)$$

$$\sin^4 \alpha = \frac{1}{8} (\cos 4\alpha - 4 \cos 2\alpha + 3)$$

$$\cos^4 \alpha = \frac{1}{8} (\cos 4\alpha + 4 \cos 2\alpha + 3)$$

**23.5.3. Additionstheoreme**

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

**23.5.4. Doppelwinkel**

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2\cos^2(\alpha) - 1 = 1 - \sin^2(\alpha)$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

**23.5.5. Dreifachwinkel**

$$\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$$

$$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

$$\tan 3\alpha = \frac{3 \tan \alpha - \tan^3 \alpha}{1 - 3 \tan^2 \alpha}$$

**23.5.6. Halbwinkel**

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2}$$

$$\cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$$

$$\tan^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{1 + \cos \alpha}$$

**23.5.7. Summen und Produkte**

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\sin \alpha \sin \beta = \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\cos \alpha \cos \beta = \frac{1}{2} (\cos(\alpha - \beta) + \cos(\alpha + \beta))$$

$$\sin \alpha \cos \beta = \frac{1}{2} (\sin(\alpha - \beta) + \sin(\alpha + \beta))$$

**23.5.8. Genaue Funktionswerte**

| $\alpha$      | 0 | $\frac{\pi}{6}$      | $\frac{\pi}{4}$      | $\frac{\pi}{3}$      | $\frac{\pi}{2}$ |
|---------------|---|----------------------|----------------------|----------------------|-----------------|
| $\sin \alpha$ | 0 | $\frac{1}{2}$        | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1               |
| $\cos \alpha$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$        | 0               |
| $\tan \alpha$ | 0 | $\frac{\sqrt{3}}{3}$ | 1                    | $\sqrt{3}$           | —               |
| $\cot \alpha$ | — | $\sqrt{3}$           | 1                    | $\frac{\sqrt{3}}{3}$ | 0               |

**23.6. Logarithmen**

$$\log(u \cdot v) = \log u + \log v$$

$$\log\left(\frac{u}{v}\right) = \log u - \log v$$

$$\log(u^k) = k \log u$$

$$\log \sqrt[k]{u} = \frac{1}{k} \log u$$

$$\log_b r = \frac{\log_a r}{\log_a b}$$

**23.7. Komplexe Zahlen****23.7.1. Allgemeines**

$$j^2 = -1, \quad \frac{1}{j} = -j, \quad (-1)^j = \left(e^{j\pi}\right)^j = e^{-\pi}$$

$$\underline{z} \in \mathbb{C}, \quad \bar{\underline{z}} : \text{konjugiertkomplex}$$

$$\text{karthesisch} : \underline{z} = a + jb, \quad \bar{\underline{z}} = a - jb$$

$$\text{polar} : \underline{z} = r \cdot e^{j\varphi}, \quad \bar{\underline{z}} = r \cdot e^{-j\varphi}$$

$$\underline{z} = r(\cos \varphi + j \sin \varphi) = r \cdot e^{j\varphi} = a + jb$$

$$a = r \cos \varphi, \quad b = r \sin \varphi$$

$$r = |\underline{z}| = \sqrt{a^2 + b^2}, \quad \varphi = \begin{cases} \text{I.} & \text{Quadrant} & \arctan \frac{b}{a} \\ \text{II.} & \text{Quadrant} & \arctan \frac{b}{a} + \pi \\ \text{III.} & \text{Quadrant} & \arctan \frac{b}{a} + \pi \\ \text{IV.} & \text{Quadrant} & \arctan \frac{b}{a} + 2\pi \end{cases}$$



### 23.7.2. Rechenregeln

$$(a_1 + jb_1) \pm (a_2 + jb_2) = a_1 \pm a_2 + j(b_1 \pm b_2)$$

$$(a_1 + jb_1)(a_2 + jb_2) = (a_1a_2 - b_1b_2) + j(a_1b_2 + b_1a_2)$$

$$z_1 \cdot z_2 = r_1 r_2 \cdot e^{j(\varphi_1 + \varphi_2)}$$

$$\frac{(a_1 + jb_1)}{(a_2 + jb_2)} = \frac{(a_1 + jb_1)(a_2 - jb_2)}{(a_2^2 + b_2^2)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \cdot e^{j(\varphi_1 - \varphi_2)}$$

$$\sqrt[n]{z} = \sqrt[n]{r} \left( \cos \frac{\varphi}{n} + j \sin \frac{\varphi}{n} \right)$$

$$\sqrt[n]{z} = e^{\frac{1}{n} \ln z} + (n-1) \text{ weitere Lösungen gleichmässig verteilt auf einem Kreis mit Radius } \sqrt[n]{r}$$

### 23.7.3. Euler

$$e^{\pm jkt} = \cos kt \pm j \sin kt$$

$$e^{\pm jk\pi} = (-1)^k, \quad e^{t+j2\pi} = e^t$$

$$\cos kt = \frac{1}{2} (e^{jkt} + e^{-jkt})$$

$$\sin kt = \frac{1}{2j} (e^{jkt} - e^{-jkt})$$

$$\cosh kt = \frac{1}{2} (e^{kt} + e^{-kt})$$

$$\sinh kt = \frac{1}{2} (e^{kt} - e^{-kt})$$

## 23.8. Ableiten

### 23.8.1. Rechenregeln

$$(\lambda f)' = \lambda f'$$

$$(f \pm g)' = f' \pm g'$$

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$\left( \frac{f}{g} \right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$$

$$f'^{-1} = \frac{1}{f' \circ f^{-1}}$$

$$(f \circ g)' = (f' \circ g) \cdot g'$$

**Elementare Funktionen**

$$\text{pot}'_k x = k \text{pot}_{k-1} x$$

$$\sin' kx = k \cos kx$$

$$\cos' kx = -k \sin kx$$

$$\exp' kx = k \exp kx$$

$$\log' x = \frac{1}{x}$$

$$\ln' |f| = \frac{f'}{f}$$

$$(a^{kx})' = (k \ln a) a^{kx}$$

$$\tan' x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$$

$$\cot' x = -\frac{1}{\sin^2 x} = -1 - \cot^2 x$$

$$\sqrt{x}' = \frac{1}{2\sqrt{x}}$$

$$\arcsin' x = \frac{1}{\sqrt{1-x^2}}$$

$$\arccos' x = -\frac{1}{\sqrt{1-x^2}}$$

$$\arctan' x = \frac{1}{1+x^2}$$

$$\text{arccot}' x = -\frac{1}{1+x^2}$$

$$\cosh' x = \sinh x = \frac{e^x - e^{-x}}{2}$$

$$\sinh' x = \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\text{arcosh}' x = \frac{1}{\sqrt{1+x^2}}$$

$$\text{arsinh}' x = \frac{1}{\sqrt{x^2-1}}$$

**Satz von Bernoulli und de l'Hospital**

$$\lim_{t \rightarrow t_0} \frac{f(x)}{g(x)} = \lim_{t \rightarrow t_0} \frac{f'(x)}{g'(x)}$$

**Beispiel:**

$$\lim_{t \rightarrow \infty} \frac{t}{e^t} = \lim_{t \rightarrow \infty} \frac{1}{e^t} = 0$$

**23.9. Integrieren****23.9.1. Rechenregeln**

$$\int \lambda f = \lambda \int f$$

$$\int (f \pm g) = \int f \pm \int g$$

$$\int f \cdot g' = f \cdot g - \int f' \cdot g$$

**23.9.2. Substitution**

$$\int f(x) dx$$

Aufstellen der Substitutionsgleichung:

$$u = g(x), \frac{du}{dx} = g'(x), dx = \frac{du}{g'(x)} \quad \text{bzw.} \quad x = h(u), \frac{dx}{du} = h'(u), dx = \frac{h'(u)}{du}$$

(  $u = g(x)$  bzw.  $x = h(u)$  müssen monotone Funktionen sein)

Substitution:

$$\int f(x) dx = \int \varphi(u) du$$

Integration:

$$\int \varphi(u) du = \Phi(u)$$

Rücksubstitution:

$$\int f(x) dx = \int \varphi(u) du = \Phi(u) = \Phi(g(x)) = F(x)$$

**Beispiel:**

$$\int_0^2 x \sqrt{3x^2 + 4} dx$$

$$\text{Subst: } u = 3x^2 + 4 \Leftrightarrow u' = \frac{du}{dx} = 6x$$

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Die neuen Grenzen erhalten wir durch Einsetzen der ursprünglichen Grenzen in die Substitutionsgleichung, die Rücksubstitution entfällt:

$$\begin{aligned} 2 &\mapsto 16 \\ 0 &\mapsto 4 \end{aligned}$$

$$\Rightarrow \int_4^{16} \sqrt{u} \, du$$

### 23.9.3. Sätze

$$\int_a^b f = - \int_b^a f$$

$$\int_a^b f(t) \, dt = - \int_{-a}^{-b} f(-t) \, dt$$

$$\int_a^b f = \int_a^c f + \int_c^b f$$

$$\int_a^b f(t) = \int_{a+c}^{b+c} f(t-c)$$

$$f \text{ stetig} \Rightarrow \int_a^b f = \int_a^b f(b) - \int_a^b f(a) = F(b) - F(a)$$

$$f \text{ stetig in } [a, b] \Rightarrow \exists \xi \in [a, b] \text{ mit } \int_a^b f = (b-a) f(\xi)$$

### 23.9.4. Integration rationaler Funktionen

Rationale Funktionen können integriert werden, indem man Division der Polynome durchführt

Beispiel:  $\int \frac{x^2}{x^2+1}$

$$x^2 : (x^2 + 1) = 1 + \frac{1}{x^2 + 1}$$

$$\int 1 + \frac{1}{x^2 + 1} \, dx = x + \arctan x$$

### 23.9.5. Rationalisierungsformeln

Für Rationale Funktionen von  $\sin x$  und  $\cos x$

- Beispiel

$$\int \frac{1 + \cos x}{\sin x} dx$$

- Substitution

$$u = \tan x/2$$

$$\Rightarrow dx = \frac{2}{1+u^2} du \quad \sin x = \frac{2u}{1+u^2} \quad \cos x = \frac{1-u^2}{1+u^2}$$

Weitere Rationalisierungsformeln siehe Papula Seite 148

### 23.9.6. Spezielle Integrale

$$\int \text{pot}_k = \frac{1}{k+1} \text{pot}_{k+1}$$

$$\int \exp kx dx = \frac{1}{k} \exp kx$$

$$\int a^{cx} dx = \frac{1}{c \ln a} a^{cx}$$

$$\int \frac{1}{x} dx = \ln x$$

$$\int \ln |x| dx = x (\ln |x| - 1)$$

$$\int \frac{1}{x \ln x} dx = \ln |\ln |x||$$

$$\int \log_a |x| dx = x (\log_a |x| - \log_a e)$$

$$\int x^k \ln x dx = \frac{x^{k+1}}{k+1} \left( \ln x - \frac{1}{k+1} \right), \quad k \neq -1, \quad x > 0$$

$$\int \frac{\ln x}{x} dx = \frac{1}{2} (\ln x)^2$$

$$\int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b)$$

$$\int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b)$$

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$$\int \tan x \, dx = -\ln |\cos x|$$

$$\int \cot x \, dx = \ln |\sin x|$$

$$\int \frac{1}{\sin x} \, dx = \ln \left| \tan \frac{x}{2} \right|$$

$$\int \frac{1}{\cos x} \, dx = \ln \left| \tan \left( \frac{x}{2} + \frac{\pi}{4} \right) \right|$$

$$\int \sin^2 x \, dx = \frac{1}{2} (x - \sin x \cos x)$$

$$\int \cos^2 x \, dx = \frac{1}{2} (x + \sin x \cos x)$$

$$\int \tan^2 x \, dx = \tan x - x$$

$$\int \cot^2 x \, dx = -\cot x - x$$

$$\int \frac{1}{x^2} \sin \frac{1}{x} \, dx = \cos \frac{1}{x}$$

$$\int \arcsin x \, dx = x \arcsin x + \sqrt{1-x^2}$$

$$\int \arccos x \, dx = x \arccos x - \sqrt{1-x^2}$$

$$\int \arctan x \, dx = x \arctan x - \frac{1}{2} \ln (1+x^2)$$

$$\int \operatorname{arccot} x \, dx = x \operatorname{arccot} x + \frac{1}{2} \ln (1+x^2)$$

$$\int (ax+b)^k \, dx = \frac{(ax+b)^{k+1}}{a(k+1)}, \quad k \neq -1$$

$$\int (ax^p+b)^k x^{p-1} \, dx = \frac{(ax^p+b)^{k+1}}{ap(k+1)}, \quad k \neq -1, \quad ap \neq 0$$

$$\int \frac{1}{ax+b} \, dx = \frac{1}{a} \ln |ax+b|$$

$$\int \frac{ax+b}{cx+d} \, dx = \frac{ax+b}{c} - \frac{ad-bc}{c^2} \ln |cx+d|$$

$$\int \frac{x^{p-1}}{ax^p+b} \, dx = \frac{1}{ap} \ln |ax^p+b|, \quad ap \neq 0$$

$$\int \frac{ax+b}{cx+d} dx = \frac{ax+b}{c} - \frac{ad-bc}{c^2} \ln |cx+d|$$

$$\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \arctan \frac{x}{a}$$

$$\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right|$$

$$\int \frac{x^2}{x^2+a^2} dx = x - a \arctan \frac{x}{a}$$

$$\int \frac{2x}{1-x^2} dx = -\ln |1-x^2|$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln \left( x + \sqrt{x^2 \pm a^2} \right)$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln \left( x + \sqrt{x^2 \pm a^2} \right)$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{|a|}$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{|a|}$$

$$\int e^{cx} \sin(ax+b) dx = \frac{e^{cx}}{a^2+c^2} (c \sin(ax+b) - a \cos(ax+b))$$

$$\int e^{cx} \cos(ax+b) dx = \frac{e^{cx}}{a^2+c^2} (c \cos(ax+b) + a \sin(ax+b))$$

$$\int \exp_k \sin_l dx = \frac{\exp_k}{l^2-k^2} (jk \sin_l - l \cos_l)$$

$$\int \exp_k \cos_l dx = \frac{\exp_k}{l^2-k^2} (jk \cos_l - l \sin_l)$$

$$\int x^n \sin kx dx = -\frac{x^n}{k} \cos kx + \frac{n}{k} \int x^{n-1} \cos kx dx \quad n \in \mathbb{N}$$

$$\int x^n \cos kx dx = +\frac{x^n}{k} \sin kx - \frac{n}{k} \int x^{n-1} \sin kx dx \quad n \in \mathbb{N}$$

## 24. Fourierreihen

### 24.1. Bezeichnungen

Vektorraum der trigonometrischen Polynome:  $\mathbb{P} \subset \mathbb{V}$

$$\mathbb{P} = \{a_0 \cos_0 + \sum_{k=1}^n a_k \cos_k + b_k \sin_k \mid n \in \mathbb{N}^*\}$$

Vektorraum der Exponentialpolynome:  $\mathbb{E} \subset \mathbb{V}$

$$\mathbb{E} = \left\{ \sum_{k=-n}^m c_k \exp_k \mid c_k \in \mathbb{C} \right\}$$

$$\sin_k = \sin kt$$

$$\cos_k = \cos kt$$

$$\exp_k = e^{jkt}$$

### 24.2. Skalarprodukt

#### 24.2.1. Eigenschaften

$$[a, b] = [b, a]$$

$$[a + b, c] = [a, c] + [b, c]$$

$$[\lambda a, b] = \lambda [a, b]$$

$$[a, a] \geq 0 \quad [a, a] = 0 \Leftrightarrow a = 0$$

#### 24.2.2. Definitionen in $\mathbb{P}$ und $\mathbb{E}$

$$[f, g] = \frac{1}{\pi} \int_{-\pi}^{\pi} f \cdot g \quad f, g \in \mathbb{P} \text{ und STF}$$

$$[f, g] = \frac{1}{2\pi} \int_0^{2\pi} f \cdot \bar{g} \quad f, g \in \mathbb{E}$$

$$[f, g] = \overline{[g, f]} \quad f, g \in \mathbb{E}$$



**24.2.3. Für orthonormierte Basis**

$$[\cos_k, \sin_l] = 0 \quad k \in \mathbb{N}_0, l \in \mathbb{N}$$

$$[\cos_k, \cos_l] = \begin{cases} 1 & k = l \\ 0 & k \neq l \end{cases} \quad k, l \in \mathbb{N}_0$$

$$[\sin_k, \sin_l] = \begin{cases} 1 & k = l \\ 0 & k \neq l \end{cases} \quad k, l \in \mathbb{N}$$

$$[\exp_k, \exp_l] = \begin{cases} 1 & k = l \\ 0 & k \neq l \end{cases} \quad k, l \in \mathbb{Z}$$

**24.3. Norm in  $\mathbb{P}$  und  $\mathbb{E}$** 

$$\|p\| = \sqrt{[p, p]} \quad p \in \mathbb{P} \text{ und } STF$$

$$\|p\|^2 = [p, p] = a_0^2 + \sum_{k=1}^n a_k^2 + b_k^2$$

$$\|e\| = \sqrt{[e, e]} \quad e \in \mathbb{E}$$

$$\|e\|^2 = [e, e] = \sum_{k=-n}^n |c_k|^2$$

**24.4. Cauchy-Schwarzsche Ungleichung**

$$[f, g]^2 \leq [f, f] \cdot [g, g]$$

**24.5. Abstand**

$$d(f, g) = \|f - g\|$$

**24.6. Fourierreihe reell****24.6.1. Fourierkoeffizienten**

$$a_k = [f, \cos_k] = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos kt \, dt \quad f \in STF$$

$$b_k = [f, \sin_k] = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin kt \, dt \quad b_0 = 0$$

gerade Funktion:

$$g(-t) = g(t) \Rightarrow b_k = 0$$

ungerade Funktion:

$$u(-t) = -u(t) \Rightarrow a_k = 0$$

## 24. FOURIERREIHEN

### 24.6.2. Fourierreihe der Funktion $f \in \mathbb{P}$

$$f = \sum_{k=0}^{\infty} (a_k \cos_k + b_k \sin_k)$$

$$\cos_0 = \frac{1}{\sqrt{2}} \quad a_0 = [f, \cos_0] \quad b_0 = 0$$

## 24.7. Fourierreihe komplex

### 24.7.1. Fourierkoeffizienten

$$c_0 = \frac{a_0}{\sqrt{2}} \quad c_{-k} = \overline{c_k}$$

$$c_k = \frac{1}{2} (a_k - j b_k) \quad a_k = 2 \operatorname{Re}(c_k) = c_k + c_{-k}$$

$$c_{-k} = \frac{1}{2} (a_k + j b_k) \quad b_k = -2 \operatorname{Im}(c_k) = j(c_k - c_{-k})$$

$$c_k = [f, \exp_k] = \frac{1}{2\pi} \int_{-\pi}^{\pi} f \exp_{-k} = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) e^{-jkt} dt$$

### 24.7.2. Fourierreihe der Funktion $f \in \mathbb{E}$

$$f = \sum_{k=-\infty}^{\infty} c_k \exp_k = c_0 + \sum_{k=1}^{\infty} (c_k \exp_k + c_{-k} \exp_{-k})$$

## 24.8. Parsevalsches Theorem

$$\left\| f - \sum_{k=0}^n (a_k \cos_k + b_k \sin_k) \right\|^2 = \|f\|^2 - \sum_{k=0}^n (a_k^2 + b_k^2) = \|f\|^2 - \sum_{k=-n}^n |c_k|^2$$

Leistung periodischer Signale:

$$\frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} |f(t)|^2 dt = \sum_{n=-\infty}^{\infty} |c_n|^2$$

## 24.9. Durchgang durch LTI-System

gegeben:  $f(t) = c_k e^{jkt} + c_{-k} e^{-jkt} = a_k \cos_k + b_k \sin_k; \quad H(\omega)$

gesucht:  $T(f(t)) = \tilde{f}$

$$\tilde{f} = \sum_{-\infty}^{\infty} c_k e^{jkt} H(k)$$

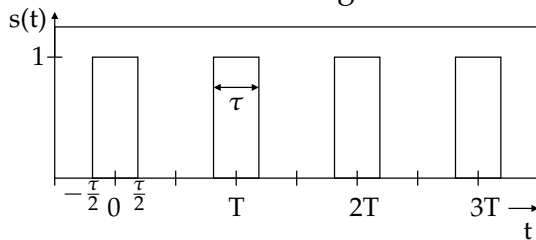
$$\tilde{a}_k = \operatorname{Re}(H(k)(a_k - jb_k))$$

$$\tilde{b}_k = -\operatorname{Im}(H(k)(a_k - jb_k))$$

$$\tilde{f} = \tilde{a}_k \cos_k + \tilde{b}_k \sin_k$$

## 24.10. Fourierkoeffizienten wichtiger periodischer Signale

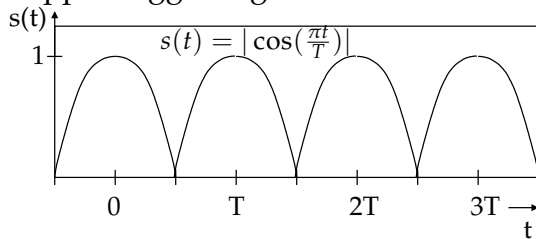
Periodische Rechteckfolge



$$c_n = a_n = \frac{\tau}{T} \frac{\sin\left(\frac{n\pi\tau}{T}\right)}{\frac{n\pi\tau}{T}}$$

$$b_n = 0$$

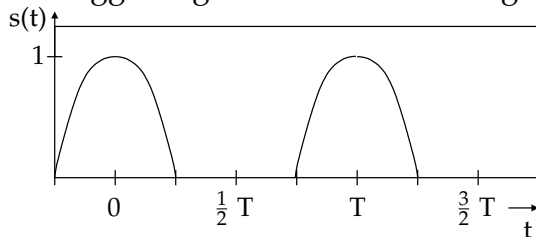
Doppelweggleichgerichtete cos-Schwingung



$$c_n = a_n = \frac{2}{\pi} (-1)^{n+1} \frac{1}{4n^2 - 1}$$

$$b_n = 0$$

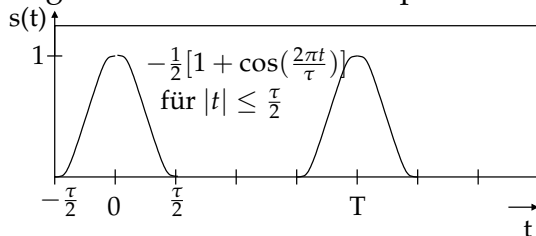
Einweggleichgerichtete cos-Schwingung



$$c_n = a_n = \frac{1}{\pi} \frac{\cos\left(\frac{n\pi}{2}\right)}{1 - n^2}$$

$$b_n = 0$$

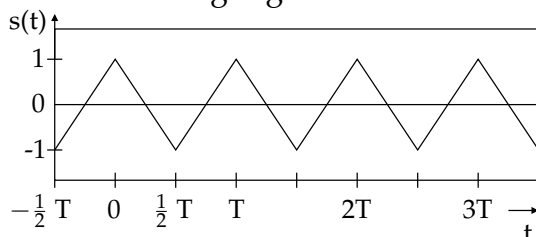
Folge von Raised-Cosine-Impulsen



$$c_n = a_n = \frac{\tau}{2T} \frac{\sin\left(\frac{n\pi\tau}{T}\right)}{\frac{n\pi\tau}{T}} \frac{1}{1 - \left(\frac{n\tau}{T}\right)^2}$$

$$b_n = 0$$

Dreieckschwingung DC-frei

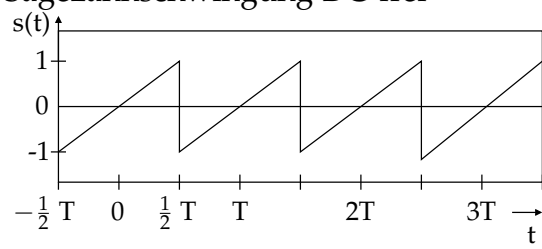


$$c_n = a_n = \frac{2[1 - (-1)^n]}{(n\pi)^2}, \quad c_0 = 0$$

$$b_n = 0$$

## 24. FOURIERREIHEN

Sägezahnschwingung DC-frei



$$c_n = -jb_n, \quad c_0 = 0$$

$$a_n = 0, \quad b_n = \frac{(-1)^{n+1}}{n\pi}$$

## 25. Fouriertransformation

$$\begin{array}{ccc}
 f & \xrightarrow{\quad} & \tilde{f} = f * h \\
 \mathcal{F} \downarrow & & \uparrow \mathcal{F}^{-1} \\
 F & \xrightarrow{\quad} & \tilde{F} = F \cdot H
 \end{array}$$

### 25.1. Fouriertransformation

$$\mathcal{F}(f(t)) = F(\omega), \quad \mathcal{F}^{-1}(F(\omega)) = f(t)$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

Wichtig: Sonderfälle beachten! (Division durch 0 in der Lösung separat behandeln)

### 25.2. Fourier-Cosinustransformation

Für gerade Funktionen

$$\mathcal{F}_c(f(t)) = F_c(\omega), \quad \mathcal{F}_c^{-1}(F_c(\omega)) = f(t)$$

$$F_c(\omega) = \int_0^{\infty} f(t) \cos \omega t dt$$

$$f(t) = \frac{1}{\pi} \int_0^{\infty} F_c(\omega) \cos \omega t d\omega$$

$$F = 2F_c$$

## 25.3. Fourier-Sinustransformation

Für ungerade Funktionen

$$\mathcal{F}_s(f(t)) = F_s(\omega), \quad \mathcal{F}_s^{-1}(F_s(\omega)) = f(t)$$

$$F_s(\omega) = \int_0^{\infty} f(t) \sin \omega t dt$$

$$f(t) = \frac{j}{\pi} \int_0^{\infty} F_s(\omega) \sin \omega t d\omega$$

$$F = -2jF_s$$

## 25.4. Faltung

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau) \cdot g(t - \tau) d\tau$$

$$f * g = g * f \quad (f * g) * k = f * (g * k)$$

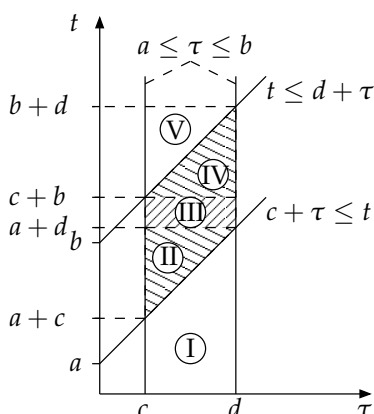
$$\mathcal{F}(f * g) = \mathcal{F}(f) \cdot \mathcal{F}(g) = F \cdot G$$

### 25.4.1. Fallunterscheidung bei Definitionsbereichen

$$p(t) = (f * g)$$

$$D(g(t)) = [a|b]$$

$$D(f(t)) = [c|d]$$



1. Fall:  $c + b < a + d$

|     |                            |   |
|-----|----------------------------|---|
| I   | $t < a + c:$               | $p(t) = 0$  |
| II  | $a + c \leq t \leq b + c:$ | $p(t) = \int_a^{t-c} f(\tau) \cdot g(t - \tau) d\tau$ |
| III | $b + c \leq t \leq a + d:$ | $p(t) = \int_a^b f(\tau) \cdot g(t - \tau) d\tau$     |
| IV  | $a + d \leq t \leq b + d:$ | $p(t) = \int_{t-d}^b f(\tau) \cdot g(t - \tau) d\tau$ |
| V   | $b + d < t:$               | $p(t) = 0$  |

2. Fall:  $c + b > a + d$

|     |                            |   |
|-----|----------------------------|---|
| I   | $t < a + c:$               | $p(t) = 0$  |
| II  | $a + c \leq t \leq a + d:$ | $p(t) = \int_a^{t-c} f(\tau) \cdot g(t - \tau) d\tau$     |
| III | $a + d \leq t \leq b + c:$ | $p(t) = \int_{t-d}^{t-c} f(\tau) \cdot g(t - \tau) d\tau$ |
| IV  | $b + c \leq t \leq b + d:$ | $p(t) = \int_{t-d}^b f(\tau) \cdot g(t - \tau) d\tau$     |
| V   | $b + d < t:$               | $p(t) = 0$  |

3. Fall:  $c + b = a + d$

|     |                      |                   |
|-----|----------------------|-------------------|
| III | $a + d = t = b + c:$ | $p(t) = p(a + d)$ |
|-----|----------------------|-------------------|

## 25.5. Eigenschaften

|  |   |
|--|---|
| $t \mapsto f(t)$                           | $\omega \mapsto \overline{F(-\omega)}$                        |
| $t \mapsto f(-t)$                          | $\omega \mapsto F(-\omega)$                                   |
| $t \mapsto f(at)$                          | $\omega \mapsto \frac{1}{ a } F\left(\frac{\omega}{a}\right)$ |
| $t \mapsto f(t - t_0)$                     | $\omega \mapsto F(\omega) e^{-j\omega t_0}$                   |
| $t \mapsto e^{j\omega_0 t} f(t)$           | $\omega \mapsto F(\omega - \omega_0)$                         |
| $t \mapsto F(t)$                           | $\omega \mapsto 2\pi f(-\omega)$                              |
| $t \mapsto f^{(n)}(t)$                     | $\omega \mapsto (j\omega)^n F(\omega)$                        |
| $t \mapsto (-jt)^n f(t)$                   | $\omega \mapsto F^{(n)}(\omega)$                              |
| $t \mapsto \int_{-\infty}^t f(\tau) d\tau$ | $\omega \mapsto \frac{1}{j\omega} F(\omega)$                  |

## 25.6. Fouriertransformationen mit Diracdelta

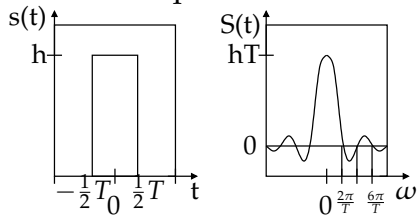
| <i>Funktion</i>              | <i>Fourier – Transformierte</i>  |
|------------------------------|--|
| $t \mapsto \delta(t)$        | $\omega \mapsto 1$   |
| $t \mapsto 1$                | $\omega \mapsto 2\pi\delta(\omega)$  |
| $t \mapsto \delta(t - t_0)$  | $\omega \mapsto e^{-j\omega t_0}$  |
| $t \mapsto e^{j\omega_0 t}$  | $\omega \mapsto 2\pi\delta(\omega - \omega_0)$                               |
| $t \mapsto \sin(\omega_0 t)$ | $\omega \mapsto j\pi(\delta(\omega + \omega_0) - \delta(\omega - \omega_0))$ |
| $t \mapsto \cos(\omega_0 t)$ | $\omega \mapsto \pi(\delta(\omega + \omega_0) + \delta(\omega - \omega_0))$  |
| $t \mapsto \delta^{(n)}(t)$  | $\omega \mapsto (j\omega)^n$   |
| $t \mapsto \text{sign}(t)$   | $\omega \mapsto \frac{2}{j\omega}$   |
| $t \mapsto \frac{1}{\pi t}$  | $\omega \mapsto -j\pi \text{sign}(\omega)$                                   |
| <i>us</i>                    | $\omega \mapsto \frac{1}{j\omega} + \pi\delta(\omega)$                       |

Faltung mit Dirac:

$$(f(t) * \delta(t_0)) = \int_{-\infty}^{\infty} f(t) \delta(t_0 - t) dt = f(t_0)$$

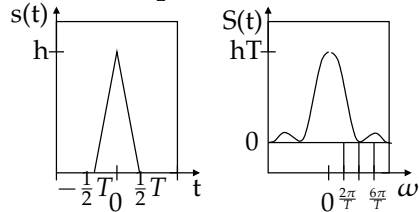
## 25.7. Fouriertransformationen wichtiger Impulse

### Rechteckimpuls



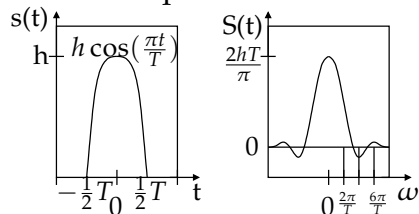
$$S(\omega) = hT \frac{\sin\left(\frac{T\omega}{2}\right)}{\left(\frac{T\omega}{2}\right)}$$

### Dreieckimpuls



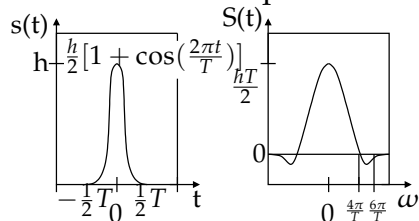
$$S(\omega) = \frac{hT}{2} \left[ \frac{\sin\left(\frac{T\omega}{4}\right)}{\frac{T\omega}{4}} \right]^2$$

### Cosinusimpuls



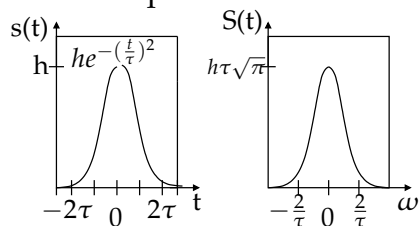
$$S(\omega) = \frac{2hT}{\pi} \frac{\cos\left(\frac{T\omega}{2}\right)}{1 - \left(\frac{T\omega}{\pi}\right)^2}$$

### Raised-Cosine-Impuls



$$S(\omega) = \frac{hT}{2} \frac{\sin\left(\frac{T\omega}{2}\right)}{\frac{T\omega}{2} \left[ 1 - \left(\frac{T\omega}{2\pi}\right)^2 \right]}$$

### Gauss-Impuls



$$S(\omega) = h\tau\sqrt{\pi} e^{-\frac{\omega^2\tau^2}{4}}$$



## 26. Laplace

$$\begin{array}{ccc}
 f & \xrightarrow{\quad} & \tilde{f} = f * g \\
 \mathcal{L} \downarrow & & \uparrow \mathcal{L}^{-1} \\
 F & \xrightarrow{\quad} & \tilde{F} = F \cdot G
 \end{array}
 \quad G = \frac{1}{cp(s)}$$

### 26.1. Laplacetransformation

$$\mathcal{L}(f(t)) = F(s), \quad \mathcal{L}^{-1}(F(s)) = f(t), \quad s \in \mathbb{C}$$

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

$$f(t) = \frac{1}{2\pi j} \int_{x-j\infty}^{x+j\infty} F(s) e^{st} ds, \quad \text{falls } t \geq 0$$

$$f(t) = 0, \quad \text{falls } t < 0$$

## 26.2. Rechenregeln

$$\begin{array}{ll}
 t \mapsto f(at) & s \mapsto \frac{1}{a} F\left(\frac{s}{a}\right) \quad a > 0 \\
 t \mapsto \frac{1}{a} f\left(\frac{t}{a}\right) & (s \mapsto F(as)) \\
 t \mapsto u(t-a) \cdot f(t-a) & s \mapsto e^{-as} F(s) \quad a > 0 \\
 t \mapsto f(t+a) & s \mapsto e^{as} \left( F(s) - \int_0^a f(t) e^{-st} dt \right) \quad a > 0 \\
 t \mapsto e^{-bt} f(t) & s \mapsto F(s+b) \quad c \in \mathbb{C} \\
 t \mapsto f'(t) & s \mapsto s F(s) - f(0) \\
 t \mapsto f^{(2)}(t) & s \mapsto s^2 F(s) - s f(0) - f'(0) \\
 t \mapsto f^{(3)}(t) & s \mapsto s^3 F(s) - s^2 f(0) - s f'(0) - f^{(2)}(0) \\
 t \mapsto f^{(n)}(t) & s \mapsto s^n F(s) - \sum_{k=0}^{n-1} s^{n-1-k} f^{(k)}(0) \\
 t \mapsto -t f(t) & s \mapsto F'(s) \\
 t \mapsto +t^2 f(t) & s \mapsto F^{(2)}(s) \\
 t \mapsto -t^3 f(t) & s \mapsto F^{(3)}(s) \\
 t \mapsto (-1)^n t^n f(t) & s \mapsto F^{(n)}(s) \\
 t \mapsto \int_0^t f(\tau) d\tau & s \mapsto \frac{1}{s} F(s)
 \end{array}$$

## 26.3. Spezielle Laplacetransformationen

$$\mathcal{L}(\delta(t)) = 1$$

$$\mathcal{L}(u(t)) = \frac{1}{s} \quad \operatorname{Re}(s) > 0$$

$$\mathcal{L}(e^{at}) = \frac{1}{s-a} \quad \operatorname{Re}(s) > \operatorname{Re}(a)$$

$$\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}(t^n e^{at}) = \frac{n!}{(s-a)^{n+1}}$$

$$\mathcal{L}(\sin at) = \frac{a}{s^2 + a^2}$$

$$\mathcal{L}(\cos at) = \frac{s}{s^2 + a^2}$$

$$\mathcal{L}\left(\frac{1}{d} e^{ct} \sin ct\right) = \frac{1}{(s-c)^2 + d^2}$$

$$\mathcal{L}\left(e^{ct} \left(\frac{c}{d} \sin dt + \cos dt\right)\right) = \frac{s}{(s-c)^2 + d^2}$$

## 26.4. Faltung

$$(f * g)(t) = \int_0^t f(\tau) \cdot g(t - \tau) d\tau$$

$$f * g = g * f \quad f(t) = g(t) = 0 \quad \text{falls } t < 0$$

$$\mathcal{L}(f * g) = \mathcal{L}(f) \cdot \mathcal{L}(g) = F \cdot G$$

Die Fallunterscheidung bei eingeschränkten Definitionsbereichen der Funktionen ist die selbe wie bei der Fourier-Theorie in Abschnitt 25.4.1 auf S. 192

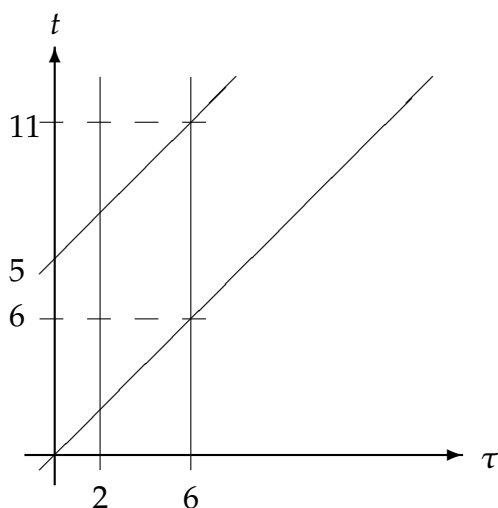
**Beispiel:**

Geg:  $g(t) = u(t) - u(t - 5)$  und  $f(t) = u(t - 2) - u(t - 6)$

Ges:  $\tilde{f} = (f * g)(t)$

$$\tilde{f} = \int_0^t f(\tau) \cdot g(t - \tau) d\tau$$

$$g(t - \tau) = 1 \quad \text{falls} \quad 0 \leq t - \tau \leq 5 \quad \Leftrightarrow \quad \tau \leq t \leq 5 + \tau$$



## 26.5. Periodische Funktionen

$f$  auf einer Periode  $T$  vorgeben.

$$F(s) = \int_0^T f(t) e^{-st} dt$$

Periodische Fortsetzung:

$$F_{per}(s) = F(s) \frac{1}{1 - e^{-sT}}$$

# 27. Differentialgleichungen

## 27.1. 1. Ordnung

### 27.1.1. Homogene

#### Separierbar

Praktisches Vorgehen beim Lösen der separierbaren Differentialgleichungen:

$$y' = \frac{g(x)}{h(y)} \Leftrightarrow \frac{dy}{dx} = \frac{g(x)}{h(y)}$$

$$\Leftrightarrow h(y) dy = g(x) dx \Leftrightarrow \int h(y) dy = \int g(x) dx$$

$$\Leftrightarrow H(y) = G(x) + c$$

Wenn durch ein Ausdruck, der die unbekannte Funktion enthält zu dividieren ist, so ist zu prüfen ob sein Verschwinden eine Lösung der DGL ergibt.

#### Substitution

Gegeben:

$$y'(x) = (x + y(x))^2$$

Substitution:

$$z = x + y(x) \Rightarrow z' = 1 + y'(x) \Leftrightarrow y'(x) = z' - 1$$

Einsetzen:

$$z' - 1 = z^2 \Leftrightarrow \frac{dz}{dx} - 1 = z^2$$

$$\Leftrightarrow \frac{1}{1 - z^2} dz = dx \Rightarrow \text{separierbar}$$

### 27.1.2. Partikuläre

$$\text{DGL: } y' + y = q$$

**Ansatz**

Ansatz für partikuläre Lösung: «Ähnlich» wie die Störfunktion ( $q$ ), jedoch nicht in der homogenen Lösung enthalten.

| Störfunktion     | Ansatz                  |
|------------------|-------------------------|
| $\sin t, \cos t$ | $a \sin t + b \cos t$   |
| $e^{-t}$         | $a e^{-t}$              |
| $t e^{-t}$       | $a e^{-t} + b t e^{-t}$ |
| $t$              | $at + b$                |

Ansatz in DGL einsetzen und Koeffizientenvergleich durchführen.

**Variation der Konstanten**

Homogene Lsg:  $y_h = c p(x)$

Ansatz:  $y_p = g(x) p(x)$  (c wird durch  $g(x)$  ersetzt)

Ansatz in DGL einsetzen und nach  $g(x)$  auflösen

**27.1.3. Lösung**

Gesamtlösungsmenge:  $y = y_h + y_p$

**27.2. Höhere Ordnung****27.2.1. Homogen, linear mit konstanten Koeffizienten**

DGL:  $y^{(4)} + 6y^{(3)} + 22y'' + 30y' + 13y = 0$

$\Rightarrow$  charakteristisches Polynom:  $p(t) = t^4 + 6t^3 + 22t^2 + 30t + 13$

$\Leftrightarrow p(t) = (t+1)^2(t+2-3j)(t+2+3j)$

$\mathbb{N}(p) = \{-1; -1; -2+3j; -2-3j\}$

Aus den Nullstellen des charakteristischen Polynoms ergeben sich die Lösungen.

Ordnung DGL = Anzahl Lösungen

$y_1(t) = e^{-t}$      $y_2(t) = t e^{-t}$      $y_3(t) = e^{t(-2+3j)}$      $y_4 = e^{t(-2-3j)}$

Linearkombinationen aus Lösungen komplexer Nullstellen ergibt reelle Lösungen:

$$\frac{1}{2} (y_3(t) + y_4(t)) = e^{-2t} \cos 3t$$

$$\frac{1}{2j} (y_3(t) - y_4(t)) = e^{-2t} \sin 3t$$

**27.2.2. Partikuläre****Ansatz**

$\Rightarrow$  Siehe 27.1.1 Homogene S. 198

**Variation der Konstanten**Störfunktion:  $q(x)$ Homogene Lsg:  $y_1(t) \quad y_2(t)$ Ansatz:  $y_p = g_1(t) y_1(t) + g_2(t) y_2(t)$  $\Rightarrow$  Gleichungssystem:

$$\begin{aligned} g_1'(t) y_1(t) + g_2'(t) y_2(t) &= 0 \\ g_1'(t) y_1'(t) + g_2'(t) y_2'(t) &= q(x) \end{aligned}$$

Dieses Gleichungssystem liefert  $g_1(t)$  und  $g_2(t)$ **27.3. Laplace****27.3.1. Lineare Übertragung**Übertragungsfunktion:  $G(s) = \frac{1}{cp(s)}$ Stossantwort:  $g(t) = \mathcal{L}^{-1}(G(s)) = \tilde{u}'$ wobei  $cp$  = Charakteristisches Polynom und  $\tilde{u}$  = Sprungantwort

$$y(0) = 0, \quad y'(0) = 0, \quad y^{(2)}(0) = 0, \quad \dots, \quad y^{(n)}(0) = 0$$

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_1 y' + a_0 y = q$$

 $\Downarrow \mathcal{L}$ 

$$Y(s) \cdot cp(s) = F(s) \quad \Leftrightarrow \quad Y(s) = \frac{F(s)}{cp(s)} = F(s) \cdot G(s)$$

 $\Downarrow \mathcal{L}^{-1}$ 

$$y(t) = (f * g)(t)$$

**Beispiel:**

$$y^{(2)} + 5y' + 6y = u, \quad y(0) = 0, \quad y'(0) = 0$$

 $\Downarrow \mathcal{L}$ 

$$Y(s) \cdot (s^2 + 5s + 6) = Y(s) \cdot (s + 2)(s + 3) = \frac{1}{s}$$

$$Y(s) = \frac{1}{s(s+2)(s+3)} = \frac{\alpha}{s} + \frac{\beta}{s+2} + \frac{\gamma}{s+3}$$

$$1 = \alpha(s+2)(s+3) + \beta(s+3)s + \gamma(s+2)s$$

$$\begin{aligned} s = 0 : \quad 1 &= 6\alpha \quad \Rightarrow \quad \alpha = \frac{1}{6} \\ s = -2 : \quad 1 &= -2\beta \quad \Rightarrow \quad \beta = -\frac{1}{2} \\ s = -3 : \quad 1 &= 3\gamma \quad \Rightarrow \quad \gamma = \frac{1}{3} \end{aligned}$$

$$Y(s) = \frac{1}{6} \frac{1}{s} - \frac{1}{2} \frac{1}{s+2} + \frac{1}{3} \frac{1}{s+3}$$

$$\Downarrow \mathcal{L}^{-1}$$

$$y(t) = \frac{1}{6}u(t) - \frac{1}{2}e^{-2t}u(t) + \frac{1}{3}e^{-3t}u(t)$$

$$y(t) = \left( \frac{1}{6} - \frac{1}{2}e^{-2t} + \frac{1}{3}e^{-3t} \right) u(t)$$

### 27.3.2. Nichtlineare Übertragung

**Beispiel:** Geg:  $g(t) = 1 - \cos t$

Ges:  $\tilde{v}$  auf  $v = \sin t$

mit  $\tilde{v}''(0) = 1$ ,  $\tilde{v}'(0) = 0$ ,  $\tilde{v}(0) = 0$

$$g(t) = 1 - \cos t$$

$$\Downarrow \mathcal{L}$$

$$G(s) = \frac{1}{s} - \frac{s}{s^2+1} = \frac{1}{s^3+s} = \frac{1}{cp(s)}$$

$$\Rightarrow \text{DGL: } y^{(3)} + y' = \sin t$$

$$\Downarrow \mathcal{L}$$

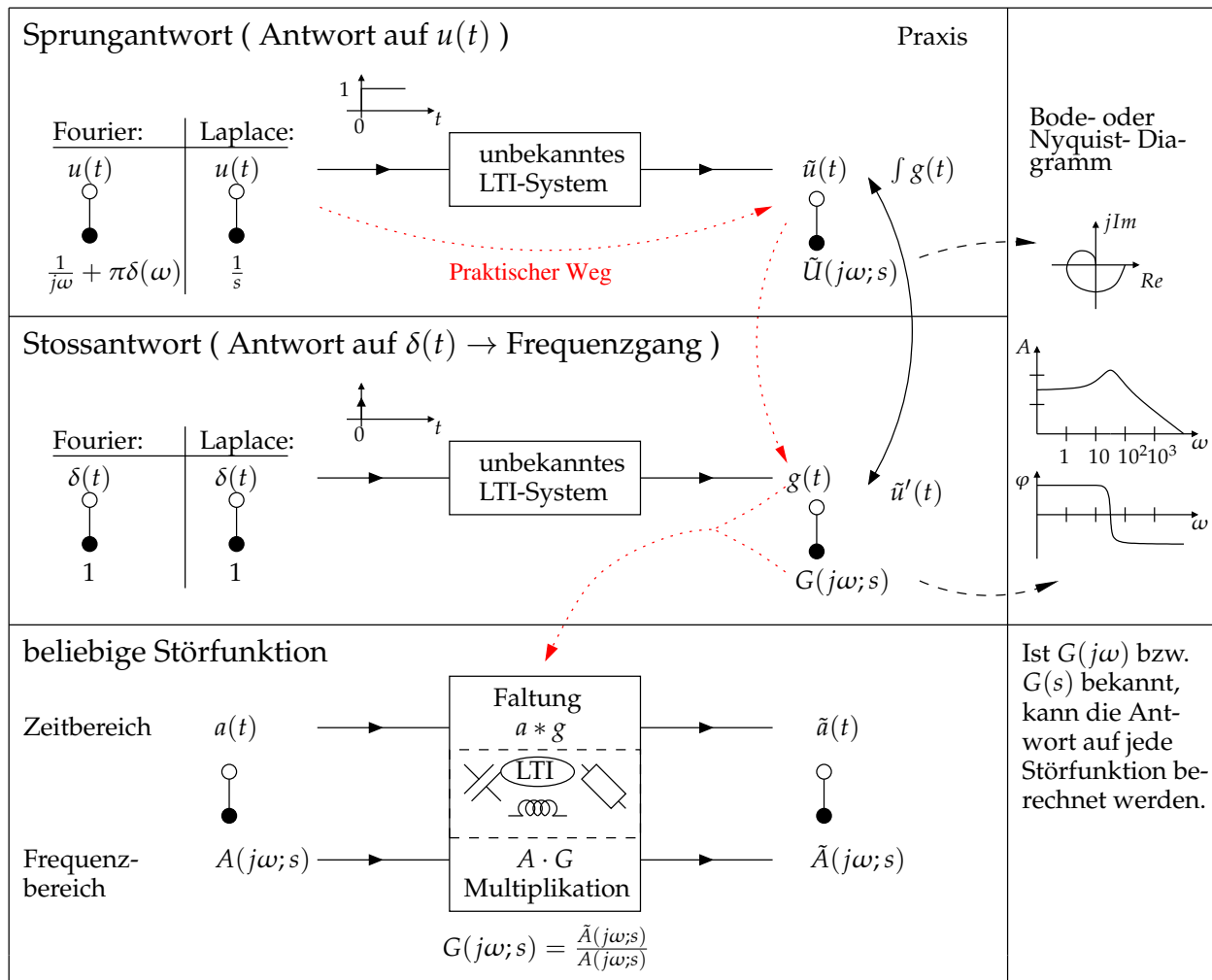
$$s^3 Y(s) - 1 + s Y(s) = \frac{1}{s^2+1} \Leftrightarrow Y(s)(s^3+s) = \frac{1}{s^2+1} + 1$$

$$\Leftrightarrow Y(s) = \frac{1}{s^3+s} \frac{1}{s^2+1} + \frac{1}{s^3+1}$$

$$\Downarrow \mathcal{L}^{-1}$$

$$y(t) = (g * \sin)(t) + g(t)$$

## 27.4. Übersicht Laplace und Fourier





# 28. Funktionsdiskussion

## 28.1. Funktionen mit einer Variablen

### 28.1.1. Zu beantwortende Fragen

1. Definitionsbereich  $D(f)$
2. Bild von  $f$
3. Hat der Graph von  $f$ ,  $G(f)$  Symmetrien?  
Gerade  $f(-x) = f(x)$  oder Ungerade  $f(-x) = -f(x)$
4. Gibt es Polstellen?
5. Gibt es Gebiete der Koordinatenebene wo der Graph keine Punkte hat?  
(Achtung beim kürzen)
6. Gibt es Schranken für die Funktionswerte?
7. Welches sind die Nullstellen von  $f$ ?
8. Welches sind die Nullstellen der Ableitungen von  $f$ ?
9. Wo steigt  $f$ , wo fällt  $f$ ?
10. Gibt es Grenzwerte für Argumente gegen  $\pm\infty$ ?
11. Gibt es Asymptoten?

$$m = \lim_{|x| \rightarrow \infty} \left( \frac{f(x)}{x} \right) \quad q = \lim_{|x| \rightarrow \infty} (f(x) - mx)$$

Asymptote:  $mx + q$

Bei Brüchen mit Polynomen ergibt eine Division mit Rest die Asymptote:  
Beispiel:

$$(x^3 - 4x^2 - 17x + 60) \div (x^2 - 4) = \underbrace{x - 4}_{\text{Asymptote}} + \underbrace{\frac{44 - 13x}{x^2 - 4}}_{\text{Rest}}$$

Die Nullstellen des Zählerpolynoms im Rest ergeben die Schnittpunkte zwischen der Asymptote und der Funktion.

12. Gibt es absolute Maximal- oder Minimalstellen?

### 28.1.2. Gerade (2-Punkte-Form)

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1$$

### 28.1.3. Abstand eines Punktes von einer Geraden

Gegeben: Gerade  $Ax + By + C = 0$ , Punkt  $P = (p_1, p_2)$

$$d = \left| \frac{Ap_1 + Bp_2 + C}{\sqrt{A^2 + B^2}} \right| \quad (A^2 + B^2 \neq 0)$$

## 28.2. Funktionen mit mehreren Variablen

### 28.2.1. Bezeichnungen

$$f_1(x, y) = \frac{\partial f}{\partial x}$$

$$f_2(x, y) = \frac{\partial f}{\partial y}$$

#### Richtungsvektoren an die Parameterlinien

Richtungsvektor an die Abszissenlinie:  $(1, 0, f_1(x, y))$

Richtungsvektor an die Ordinatenlinie:  $(0, 1, f_2(x, y))$

#### Tangentialebene

$$\varepsilon: \quad \vec{p} = (p_1, p_2, p_3) = (x, y, f(x, y)) + \alpha(1, 0, f_1(x, y)) + \beta(0, 1, f_2(x, y))$$

$$\vec{n}_\varepsilon = (f_1(x, y), f_2(x, y), -1)$$

#### Gradient

Wir betrachten die Funktion  $f : (x_1, x_2) \mapsto f(x_1, x_2)$ . Sie ist in einer gewissen Umgebung  $U$  von  $(x_0, y_0)$  definiert.

Die Richtung des stärksten Anstiegs von  $f$  in  $(x_0, y_0)$  ist

$$\text{grad} f(x_0, y_0) = (f_1(x_0, y_0), f_2(x_0, y_0)) = \vec{v}$$

( $\Rightarrow$  Richtung der Fallgeraden in der Grundrissebene)

Richtungsvektor der Fallgerade der Tangentialebene:

$$(f_1(x_0, y_0), f_2(x_0, y_0), f_1(x_0, y_0)^2 + f_2(x_0, y_0)^2)$$

#### Richtungsableitung

$$D_{\vec{v}} f(x_0, y_0) = \text{grad} f(x_0, y_0) \cdot \vec{e}_v$$

wobei  $\vec{e}_v$  der Einheitsvektor in Richtung  $\vec{v}$  ist

**Totales Differential**

$$df = h \cdot f_1(x, y) + k \cdot f_2(x, y)$$

wobei h und k die Inkremente sind

**Kettenregel**

Vollständig differenzierbare Funktionen:

$$f : (x_1, x_2) \mapsto f(x_1, x_2)$$

$$u : (y_1, y_2) \mapsto u(y_1, y_2)$$

$$v : (y_1, y_2) \mapsto v(y_1, y_2)$$

$$\tilde{f} : (y_1, y_2) \mapsto f(u(y_1, y_2), v(y_1, y_2))$$

Dann sind

$$\tilde{f}_1(y_1, y_2) = f_1(u(y_1, y_2), v(y_1, y_2)) \cdot u_1(y_1, y_2) + f_2(u(y_1, y_2), v(y_1, y_2)) \cdot v_1(y_1, y_2)$$

$$\tilde{f}_2(y_1, y_2) = f_1(u(y_1, y_2), v(y_1, y_2)) \cdot u_2(y_1, y_2) + f_2(u(y_1, y_2), v(y_1, y_2)) \cdot v_2(y_1, y_2)$$

**28.3. Kegelschnitte****28.3.1. Kreis**

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

$$M = (x_0, y_0)$$

**28.3.2. Ellipse**

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$$

$$M = (x_0, y_0)$$

**28.3.3. Hyperbel**

$$\frac{(x - x_0)^2}{a^2} - \frac{(y - y_0)^2}{b^2} = 1$$

$$M = (x_0, y_0)$$

**28.3.4. Parabel**

$$(y - y_0)^2 = 2p(x - x_0)$$

$$S = (x_0, y_0)$$

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