Properties of Logarithms

Introductory Example

For the introduction to this section the students will work on a worksheet to come up with the common rules for logs "multiplication turns into addition" etc. The students will compute this without using their calculators as it will be good practice for them to turn the logs into exponentials and reason their way through the problems since we technically don't know change of base.

Things to know about the Properties of Logarithms

What is the property for a logarithm of an input that is multiplied? $\log_b(x \cdot y) = \log_b(x) + \log_b(y)$
What is the property for a logarithm of an input that is divided? $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
What is the property for a logarithm of a power? $\log_b(x^p) = p \cdot \log_b(x)$
What is the inverse property of logs/exponential functions? $f(x) = b^x$ and $g(x) = \log_b(x)$ are inverses of each other so ... $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$ go through the steps to get these results

Practice Practice Practice

The remainder of the class will be spent working on using these properties productively and correctly. I most likely will have the students work in small groups on these problems and will have the students present their solutions to each other and have them tell me what to do next as we go through them

Use the properties of logs to rewrite the following as a single logarithmic expression:

$\ln(6x) + \frac{1}{2}\ln(x) - \ln(2x)$

$\log(5z) - \log(x) - 3\log(3y) + \log(t)$

$2log_2(x^2) + \log_2(y) - 4\log_2(P) - \frac{1}{3}\log_2(Q) + \log_2(z)$

Use the properties of logarithms to expand each expression as much as possible:

$\ln(10xe^3x)$

$\log\left(\frac{2x^4}{y\sqrt{z}}\right)$

$\log_5(\sqrt{5z})$

A couple more problems to try:

Use the natural logarithm and a property of logarithms to solve: $4(3)^x = 20$.

Use property of logarithms to solve:

$\log(-x-2) + \log(1-x) = 1$

$\log_3(3x+17) - \log_3(x+1) = 2$