Applications of Exponential and Logarithmic Functions

Problem 1. A cup of coffee that is initially 125°F is placed in a room kept at a constant 72°F. The temperature of the coffee, T as a function of time is given by $T(x) = 72 + 53(0.8)^x$, where x is measured in minutes. How long will it take the coffee to cool to 100°F?

Problem 2. A radioactive isotope of bismuth has a half-life of 5 days. After how many days will 20% of the isotope remain?

Problem 3. The world population has been growing roughly exponentially for the past 30 years. In 1987, the world population was approximately 5 billion. In 1998, the world population was approximately 6 billion. Find an exponential equation of the form $y = Ce^{kt}$ which models the population with t representing the number of years 1987. Use at least 6 decimal places for the value of k. What does the model predict the population was in 2000?

Problem 4. Suppose Matt wants to have \$10,000 saved in 9 years. How much should he invest today at 3.4% compounded continuously in order to reach his goal?

Problem 5. A radioactive isotope of bismuth has a half-life of 5 days. After how many days will a 200-gram sample decay to 20 grams of radioactive material?

Problem 6. The doubling time for an investment is given by the equation $T = \frac{\ln(2)}{\ln(1+r)}$ where r is the interest rate in decimal form. At what interest rate would you need to invest in order to double your investment in 10 years?

Problem 7. Suppose after 2500 years an initial amount of 1000 grams of radioactive substance has decayed to 75 grams. What is the half-life of the substance?

Problem 8. The concentration of a pollutant in the atmosphere increases according to the exponential growth model $A = Pe^{rt}$, where time is measured in years and $r = 0.0035$. In 1990, the concentration was 56 parts per million. Clean-up procedures will be initiated when the concentration reaches 70 parts per million. According to the model, in what year will that occur?

Problem 9. Air pressure, P, decreases exponentially with the height above the surface of the earth. At the top of Mount Everest, height 8848 meters, the air pressure is about 34.6% of the air pressure at sea level. Approximate the air pressure as a percentage of the sea level value at the top of Mount Kilimanjaro, height 5895 meters.

Problem 10. What is the doubling time for a population of rabbits that grows from 120 to 500 in 12 months?

Problem 11. A pork roast is removed from the freezer (25°F) and placed in a room (75°F). The number of minutes it takes the temperature of the roast to reach x degrees F is given by the formula $T = f(x) = \frac{100}{3}\ln\left(\frac{50}{75-x}\right)$. Find and interpret $f^{-1}(70)$.

Problem 12. New employees are given an initial exam and then retested monthly with an equivalent exam. The average scores for the employees can be expressed as $S(t) = 76 - 6 \cdot \log_3(t+1)$ where t is measured in months since the initial exam. After approximately how many months would the average score be 54?

Problem 13. The temperature of a cup of hot tea, in °F, is represented by the function $H(t) = 72 + 102e^{-0.023t}$, where t represents time in minutes since the cup of tea was placed on the kitchen counter. Use the function to determine how long it will take for the cup of tea to cool to 120°F.

Problem 14. A population of ladybugs grows according to the limited growth model $A = 400 - 400e^{-0.04t}$ where t is measured in weeks, $t \geq 1$. After how many weeks will the population be approximately 300 ladybugs?

Problem 15. A colony of bacteria grows from 300 to 1300 in 7 hours. Assume the population can be modeled by $A = pe^{rt}$, where t is measured in hours. Determine the tripling time of the bacteria?

Problem 16. A population of lemmings declined rom 100,000 to 75,000 in 3 months. Assuming an exponential pattern of decline, approximately how many will be remaining after another 3 months pass?