Polynomial Functions

Things to know about Polynomial Functions

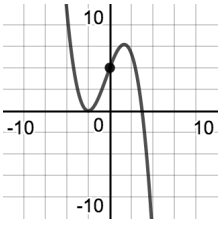
- How does the degree of a polynomial function affect the ends of the graph?
- How does the leading coefficient affect the ends of the graph?
- What generalization can you make about the ends of the graph based on the degree of the polynomial and the leading coefficient?
- How does the degree of the polynomial relate to the number of zeros it has?
- How does the degree of the polynomial relate to the number of turns a graph has?

Examples

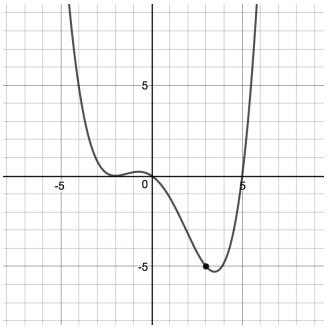
1. Determine the degree, leading coefficient, and zeros of the following polynomial equations. Sketch a rough graph by hand and check your answers on your graphing calculator.
   a) $y = x^3 - x^2 - 6x$
   b) $y = -\frac{1}{2}(x+3)(x-2)(x+1)(x-5)$
   c) $y = -x^4 + 13x^2 - 36$

2. Determine a possible equation for the following graphs. Verify your answers on your graphing calculator.

a)



b)



3. Based on actuarial life tables, the average number of years of life remaining for a female of age x (up to 100) can be approximated by the function

$$N(x) = 0.0000004x^4 - 0.00004x^3 + 0.002x^2 - 1.0191x + 81.955.$$

Use this to answer the following questions:

a) Graph this function in an appropriate window.
b) What does this graph tell us in practical terms?
c) What is the y-intercept? What does it mean in practical terms?
d) What is the degree of this function?

e) What is the leading coefficient of this function?

f) Is this function increasing or decreasing on the implied domain? What does this tell us in practical terms?

g) Why does it make sense that life expectancy increases as the age of the female increases?

4. A 12×8 rectangular piece of cardboard is folded into an open top box. This is possible by cutting squares from each of the corners.

   a) Determine a formula for the volume of the box.

   b) Find the maximum possible volume.

5. For a particular pair of sunglasses, it is determined that the cost function is given by

$$C(x) = 0.00003x^3 + 7x + 1,500$$

   and the revenue function is given by

$$R(x) = -0.01x^2 + 40x$$

   where x is the number of sunglasses produced and sold. Find a function to represent the profit as a function of x. Determine the number of units that should be sold in order to maximize profit. What is the maximum profit?