Warm-Up

Use these functions to answer the following questions:

$$f(x) = \frac{x}{x-1} \qquad g(x) = \frac{4}{x+7}$$

(a) Find $\frac{f}{g}$

Solution.

$$\begin{aligned}\frac{f}{g} &= \frac{\frac{x}{x-1}}{\frac{4}{x+7}} \\ &= \frac{x}{x-1} \cdot \frac{x+7}{4} \\ &= \frac{x(x+7)}{4(x-1)} \\ &= \frac{x^2+7x}{4x-4}\end{aligned}$$

□

(b) What is the domain of $\frac{f}{g}$?

Solution. The domain of f is $(-\infty,1)\cup(1,\infty)$, the domain for g is $(-\infty,-7)\cup(-7,\infty)$. The domain of $\frac{f}{g}$ is the intersection of the domain of the two and the "bad" points for the new denominator. In this case, $(-\infty,-7)\cup(-7,1)\cup(1,\infty)$. □

Introductory Example

Suppose that the demand for a certain pair of sunglasses can be expresses as a function of the purchase price using the equation $q = f(p) = -4000p + 200,000$. Use this equation to answer these questions:

(a) What is the demand when the price is \$40?

(b) What is $f(0)$ and what does this mean in practical terms?

(c) Graph this function in a reasonable window.

(d) What are the domain and range of this function?

(e) What price corresponds to a demand of 80,000 pairs of sunglasses?

Things you need to know about inverse functions

When does a function have an inverse function?

How do you find an inverse function, for a function that has one?

Notation:

Domain/Range:

Practice Example

Find the inverse function and evaluate $f^{-1}(57)$ for the following function:

$$y = f(x) = 7x^3 + 1$$

Does an inverse exist?

Below is a verbal description of several functions. Determine whether the function described has an inverse function, and if it does, explain what the inverse function tells you.

- $T = f(p)$ represents the city sales tax paid on an item that sells for p dollars
- $C = f(t)$ represents the average cost of a gallon of unleaded gasoline in Tucson t days after January 1
- The profit function $P(x)$, found from the revenue function $R(x) = x\left(\frac{100{,}000-x}{2000}\right)$ and the cost function $C(x) = 1200 + 10x$, where x is the number of units sold

Example

Suppose a cost-benefit model is given by $C = f(x) = \frac{6.6}{100-x}$ where C is the cost, in thousands of dollars, of removing x percent of a given pollutant. Find the inverse function, and explain what it represents.

Concept Check

The functions u and w are defined as follows:

$$u(x) = -x - 2$$

$$v(x) = 2x^2 + 1$$

1. Find $u(v(-2))$.

   Solution. $v(2) = 2(-2)^2 + 1 = 9$
   $u(v(-2)) = u(9) = -9 - 2 = -11$ □

2. Find $v(u(x))$.

   Solution.

   $$\begin{aligned} v(u(x)) &= 2(-x-2)^2 + 1 \\ &= 2(x^2 + 4x + 4) + 1 \\ &= 2x^2 + 8x + 8 + 1 \\ &= 2x^2 + 8x + 9 \end{aligned}$$

   □

3. Find $(u + v)(1)$.

   Solution. $u(1) = -1 - 2 = -3$,
   $v(1) = 2(1)^2 + 1 = 2 + 1 = 3$,
   so $(u + v)(1) = u(1) + v(1) = -3 + 3 = 0$ □

Warm-Up

1. Suppose the functions g and h are defined as follows:

$$g(x) = \frac{8}{x}, x \neq 0$$

$$h(x) = x^2 - 7$$

Find the compositions $g \circ g$ and $h \circ h$.

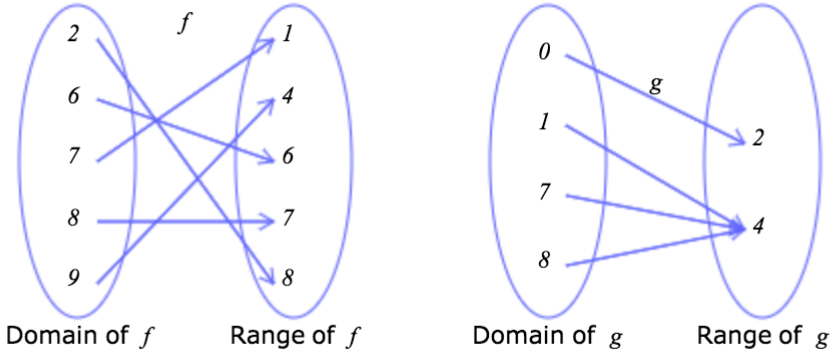
Solution.

$$\begin{aligned} g \circ g &= \frac{\frac{8}{x}}{\frac{8}{x}} \\ &= \frac{8}{x} \cdot \frac{x}{8} \\ &= \frac{8x}{8x} \\ &= 1 \end{aligned}$$

$$\begin{aligned} h \circ h &= (x^2 - 7)^2 - 7 \\ &= x^4 - 14x^2 + 49 - 7 \\ &= x^4 - 14x + 42 \end{aligned}$$

□

2. Two functions f and g are defined in the figure below.



Find the domain and range of the composition $g \circ f$. Write your answers in set notation.

Solution. Domain: $\{2,7,8\}$ this is found by founding the domain of f and then seeing which of these values are inputs for g.
Range: $\{4\}$ this is found by evaluating $g \circ f$ with the domain we found □

Warm-Up

The volume $V(r)$ in cubic meters of a spherical balloon with a radius r is given by the formula

$$V(r) = \frac{4}{3}\pi r^3$$

The radius $W(t)$ in meters after t seconds is given by

$$W(t) = 5t + 3$$

Write a formula, $M(t)$ for the volume of the balloon after t seconds.

Solution. The formula asks for the volume of the balloon after t seconds. We know that the volume of the balloon $V(r) = \frac{4}{3}\pi r^2$, and we know that the radius in terms of time is given by $W(t) = 5t + 3$. This means that $M(t) = (V \circ W)(t) = V(W(t)) = \frac{4}{3}\pi(5t - 3)^3$. □

Practice Problems

Complete the following problems:

1. For the following find the inverse of the function and evaluate $f^{-1}(2)$

n	3	-4	1	2
$f(n)$	2	6	-1	0

2. Find the inverse of $f(x) = \frac{6x+2}{4x-5}$

3. Let $f(x)$ be a one-to-one function. What point must be on the graph if $f(9) = 11$?

Concept Check

The average cost function of producing purple striped berets is given by the function $A = f(x) = 3.5 + \frac{2500}{x}$ where x is the number of hats and A is the average cost per hat when producing x hats.

Give the input, output, domain and range for this function.

Find the inverse of this function.

Evaluate $f^{-1}(16)$, and describe what this tells you about this problem.