**PROBLEM SET #5** **NAME:**______________________________

**Math 196L (Spring 2018)**

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**Instructions:** Read each problem. Write a sentence or two about the approach you might take to solve each problem. Draw a picture to illustrate the scenario. Write a formula that might be needed to help set up or solve the problem. **DO NOT SOLVE THE PROBLEMS**.

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## Problem 1

A suspension bridge is a type of bridge in which the deck (the roadway) is hung below suspension cables on vertical suspenders. Consider a bridge with 2 suspension cables, each of which form a curve that can be approximated by a parabola, its lowest point 10 feet above the horizontal roadway. The suspension cables are attached to twin towers on the roadway that rise 98.2 feet from the roadway and 420 feet apart. Between the towers, equally spaced vertical suspenders lines are used to hang the deck from the suspension cables. On each suspension cable, you can use six stronger vertical suspenders lines which cost $120 per foot or nine cheaper (but weaker) vertical suspenders lines costing $72 per foot. Which should you use, and what is the cost of the suspender lines?

## Problem 2

A firework can be created in the following way. Load a spherical shell with burst stars (a chemical mixture that when ignited produces the colorful bursts of light) and run a wick into a mortar, an object similar to a small cannon. Black powder is loaded below the shell and once ignited, will propel the shell with some initial velocity. Once the shell is airborne, the wick burns and is timed so the explosion of the shell and contents is delayed.

Suppose we plan on shooting such a firework from a rudimentary mortar which points straight up from the roof of a 30 foot building. The shell is fired with an initial velocity of 112 feet per second.

It will be helpful to use the following equation for the height in feet, $p$, as a function of time, t, in seconds:

$$p(t) = -16t^2 + v_0 t + s_0$$

where $v_0$represents initial velocity, and $s_0$ represents initial position.

**A)** Determine a function that models the height of the firework.

B) Write the function from part A in vertex/standard form.
What do the constants in your equation represent?

C) Write the average velocity (average rate of change) formula.

D) Find the average velocity of the shell between zero and 1,
between 0.5 and 1,
between 1 and 1.5 seconds
and between 1 and 2.

E) Write the average rate of change of position between $t = 1 \textit{ and } t = 1 + h$, for both equations (general and standard form), and simplify. Verify your answers in part D will work for this simplified version.

F) Find the average rate of change of position between $t = 1 \textit{ and } t = 1 + h$,
When $h = .5,\ 0.1,\ \ 0.001,\ \ \ 0.0001,\ \ 0.000001$
Explain what you observe.

G) If the pyro technician wants to time the explosion to occur when the shell is the farthest away, how long after the shell is airborne should he time the shell to explode? What would the velocity of the shell be near that instant (just prior to exploding)?

## Problem 3

Suppose the rate at which a rumor is spread in a town with a population of 1000 people is (jointly) proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. Assume that the constant of proportionality is positive. Use $k$ for the proportional constant.

(A) Write an equation for the rate as a function of the number of people who have heard the rumor.

(B) What is the rate of spread when the entire town has heard the rumor?

(C) How many people have heard the rumor at the instant when the rate of spread is at its maximum?

## Problem 4

Suppose a closed box with a square base has a surface area of ***A***.

(A) Determine the volume of the box in terms of the side of the square base. *This fix variable **A** will be part of your equation.*

(B) If the surface area is 1000, find the maximum volume of the box.

## Problem 5

A quadratic function $Q(x)$ passes through the points $(1,2)$ and $(-3,6)$.

(A) Write an expression for $Q(x)$ if $(1,2)$ is the vertex.

(B) Write an expression for $Q(x)$ if $(-3,6)$ is the vertex.

(C) Write an expression for $Q(x)$ if the graph is symmetric with respect to the $y$-axis.

(D) Write an expression for $Q(x)$ if $Q(x)$ has a zero at $x = 2$

## Problem 6

The following statements about $f(x)$ are true:

$f(x)$ is a polynomial function.
$f(x) = 0$ at exactly four different values of $x$.
$f(x) \to -\infty$ as $x \to \pm\infty$.

For each of the following statements decide if the statement is **A**lways true, **N**ever true or **S**ometimes true. For **A**lways true and **N**ever true, justify your answer. If **S**ometimes true give an example of when it could be true and when it might not be true.

(A) $f(x)$ is an odd function.

(B) $f(x)$ is an even function.

(C) $f(x)$ is a fourth degree polynomial.

(D) $f(x)$ is a fifth degree polynomial.

(E) $f(-x) \to -\infty$ as $x \to \pm\infty$.

(F) $f(x)$ is a 1-1 function.

(G) The lead coefficient is $-0.001$.

## Problem 7

$g(t) = -2(t-a)^2(t-b)(t-c)$ where $a < b < 0 < c$. What is the vertical intercept? What are the horizontal intercepts? On what interval(s) is/are $g(t) > 0$?

## Problem 8

Consider the function in Figure 1.89b: Find the coordinates of $C$ in terms of $b$.