Warm-Up

Complete the following problems:

1. Calculate the difference quotient $\frac{f(x+h)-f(x)}{h}$ for $f(x) = 3x^2 + 5$.

   Solution. First we solve $f(x+h)$, by substituting $x+h$ into $f(x)$ wherever we see an x:

   $$\begin{aligned} f(x+h) &= 3(x+h)^2 + 5 \\ &= 3(x^2 + 2xh + h^2) + 5 \\ &= 3x^2 + 6xh + 3h^2 + 5 \end{aligned}$$

   Next we solve $f(x+h) - f(x)$:

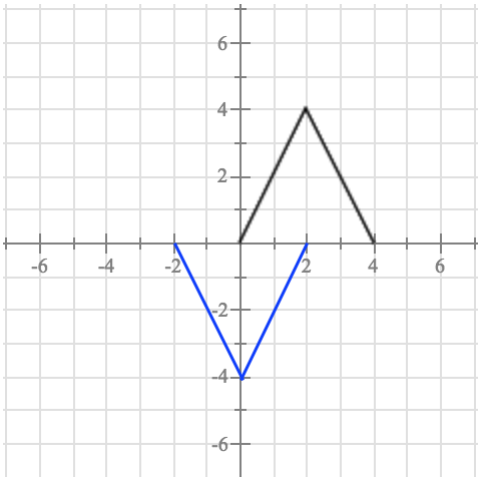
   $$\begin{aligned} f(x+h) - f(x) &= 3x^2 + 6xh + 3h^2 + 5 - (3x^2 + 5) \\ &= 3x^2 + 6xh + 3h^2 - 3x^2 - 5 \\ &= 3h^2 + 6xh \end{aligned}$$

   Finally we take the above solution to $f(x+h) - f(x)$ and divide by h:

   $$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{3h^2 + 6xh}{h} \\ &= 3h + 6x \end{aligned}$$

   This gives us the final answer of $\frac{f(x+h)-f(x)}{h} = 3h + 6x$. □

2. Transform the following graph via $y = -f(x+2)$

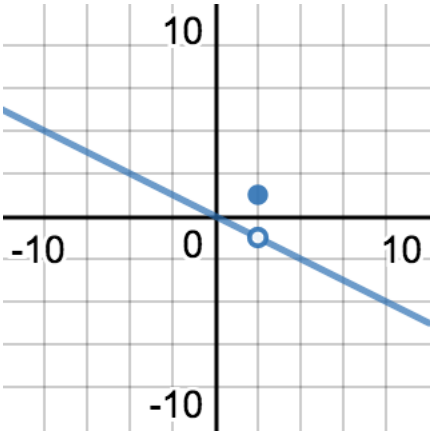


   Solution. The original graph is the graph drawn in the black line and the solution is the graph drawn in the blue line. The transformations that were applied was a reflection over the x-axis and a horizontal shift left by 2. □

3. Graph the following function:

   $$g(x) = \begin{cases} -\frac{x}{2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

Solution.



□

4. This is similar to Friday's warm-up For the following function state the base function and list the transformations in the order in which they were performed.

$$f(x) = 3\sqrt{-x} + 5.$$

Solution. Base Function: $f(x) = \sqrt{x}$
Transformations: Reflection over the y-axis, vertical stretch by 3, vertical shift up 5. □

Example 1

The local jazz society puts on a series of weekly concerts during the spring. When concert tickets are priced at \$15, the average attendance is 400 people. This past fall the society tried out different ticket prices and found that for every \$2 increase in ticket price, approximately 25 fewer people come to the show.

- Write an equation to represent the attendance, $A(x)$, as a function of ticket price x.
- Write a function that represents revenue generated by selling tickets as a function of the selling price x.
- When (at what selling price) is revenue maximized?

Things to know about combining functions

- $(f+g)(x) =$
- $(f-g)(x) =$
- $(f \cdot g)(x) =$
- $\frac{f(x)}{g(x)} =$
- $(f \circ g)(x) =$
- $(g \circ f)(x) =$

Example 2

The sales tax at a certain store is 9%.

- Write a function for the total price paid, $P(x)$, for an item priced at x dollars after factoring in sales tax.
- The store is having a sale and offering \$5 off every purchase. Write an equation, $S(x)$ for an item originally priced at x dollars.
- Find the composition $P(S(x))$.

Example 3

1. Suppose $h(x) = \frac{3}{\sqrt{x-7}}$. What are two functions $f(x)$ and $g(x)$ such that $(f \circ g)(x) = h(x)$?
2. The following tables show the unemployment rate and crime rates in a certain city. Use these tables to answer some questions.

t, time in years	U, unemployment rate
0	0.02
1	0.023
1	0.03
3	0.032

U, unemployment rate	Crime rate
0.01	0.015
0.02	0.021
0.03	0.028
0.04	0.031
0.05	0.037