Quadratic Functions

Introductory Example

A business has created a mathematical model based on market data for its profit, P (in dollars), as function of the number of items sold, x. The model is given by the function

$$P(x) = -0.1x^2 + 150x - 1400.$$

How many units must be sold to maximize profit? What is the maximum profit?

Students will work on this question with their partners. It is hoped that the students will graph it on their graphing calculator. To help them with the graphing calculator they should use the table feature to identify the zeros. Most students will probably use the maximize feature on their calculator. To hold them all accountable, I am going to ask each pair of students to write the window that they used on a whiteboard and hold it up for me to see the window. This way I know which students I should target to come to my office for extra help.

Some questions to ask students:

1. What is the shape of this function? How would you describe it to someone?
2. Does the parabola have a maximum or minimum? What is it?
3. What does the first coordinate of the vertex represent? The second coordinate?
4. How can we identify this point using just the equation, without the graph? *This will be useful as some of the students are struggling with finding appropriate windows*
5. What is the relationship between the zeros and the vertex?
6. What is the formula for finding a vertex?

Things you need to know about Quadratic Functions

In general quadratic functions are always parabolas and are symmetric about the vertex.

General form: $f(x) = ax^2 + bx + c$
a is the leading coefficient, $a > 0$ opens up, $a < 0$ opens down, y-intercept $(0, c)$, vertex at $x = -\frac{b}{2a}$ to find y-coordinate plug in x and solve.

Vertex form: $f(x) = a(x-h)^2 + k$
Think of this as a transformation of $y = x^2$, a does the same thing as above, vertex (h, k)

Factored form: $f(x) = a(x-r_1)(x-r_2)$
a does the same thing as above, zeros: r_1, r_2, x-coordinate of the vertex $\frac{r_1+r_2}{2}$

Practice Problems

$f(x) = 2x^2 + 5x + 3$

Describe the following features of the quadratic function.

- The shape
- y-intercept

- Write it in vertex form
- Write it in factored form

Find a formula for the parabola that goes through the points $(-5, 0)$, $(3, 0)$ and $(4, 12)$
If students are struggling, ask them to sketch a graph of it. Then ask them what form fits best based on what we were given. Have them find the vertex (x-coordinate).

Example 1

A concert venue holds a maximum of 1,000 people with ticket prices at \$30, the average attendance is 650 people. It is predicted that for every dollar the ticket price is lowered approximately 25 more people will attend. Create a function to represent the revenue generated from ticket sales and use this to find the maximum possible revenue.

I want students to work and struggle with this problem on their own for a bit. This is similar to questions that they will be asked to do on the test so using their brains and thinking about it for a bit will be useful to them. I anticipate that the students will have a hard time with this problem, so some questions to have prepped to ask them follow here.

Some questions to ask stuck students:

- How do you find revenue in this problem?
- Make a table to find an equation for the price and average attendance.
- Can you find a function that represents attendance based on price?
- Now use your equation for attendance and what you told me before to find your equation for Revenue.
- How do you find the maximum revenue? What ticket price should be used to maximize revenue?

Example 2

Suppose a sunglass manufacturer determines the demand function for a certain line of sunglasses is given by $p = 50 - \frac{1}{4000}x$, where p is the price per pair, and x is the number of pairs sold. The fixed cost of producing this line of sunglasses is \$25,000 and each pair of sunglasses costs \$3 to produce. How many pairs of sunglasses should be produced and sold in order to maximize profits?

Again I want students to work through this on their own, thinking about Revenue, Profit, Cost will be very helpful for the students during the exam.

A couple of questions to ask the students:

- How do we find profit?
- Can you write an equation for the cost?
- Can you write an equation for the revenue?