Let's talk about rabbits $\cdots$

| Time | Number of Rabbits |
|---|---|
| 0 | 2 |
| 1 | 6 |
| 2 | 18 |
| 3 | 54 |
| 4 | 162 |
| 5 | 486 |

Nathan has \$100 to open a savings account. He found an account that offers 2.5% interest compounded annually. Write a function to represent the balance in the account as a function of time in years, assuming the initial deposit and all subsequent interest is kept in the account. Graph this function in an appropriate window.

## Things to know about Exponential functions

What is the general form of on exponential function?
What differentiates an exponential function from the other types of functions we have studied?
What is the domain of an exponential function? The range? The intercept(s)? The asymptote(s)?
What determines whether an exponential function is increasing or decreasing?
What is the formula for compound interest if it is compounded annually? $n$ times per year? continuously?

## Examples

Determine the for $y = C \cdot b^x$ that passes through the points $(1, 12)$ and $(3, 192)$.

A person plans to invest \$5,000 into a money market account. Find the interest rate required for the money to grow to \$45,000 in 30 years if the interest is compounded quarterly.

Bacteria from raw eggs has come into contact with the onions and celery you are going to put into your potato salad. Initially there were 500 bacteria present; one hour later there were 4,000 bacteria present in the salad. The population of the bacteria in the salad can be modeled by a exponential function $y = C \cdot b^t$, where $t$ is measured in hours. Create a function that represents this situation, and use the graph to determine the number of hours it takes for the bacteria's population to double in size.

\$100 is invested into an account bearing 12% interest. What will the balance in the account be in 10 years if ...
interest is compounded annually?
interest is compounded quarterly?
interest is compounded monthly?
interest is compounded daily?

A radioactive substance has a half-life of 20 days. If the initial amount is 25 grams, write a function to represent the amount of substance remaining after $t$ days.

**Problem 1.** A population of 800 beetles is growing each month at a rate of 8%. Write a function that represents the number of beetles at time $x$. About how many beetles will there be in 2 years?

**Problem 2.** The half-life of ibuprofen is given by the formula

$$R = M(0.5)^t$$

where $t$ is the time in hours and the dosages are measured in milligrams. If a person took a 200 milligram dose 2 hours ago, how much is currently left in their blood stream? A person has 30 milligrams in their body 1 hour after taking the medication, how much was their original dosage?

**Problem 3.** You recently bought a new computer for \$1,500 and were astounded when you did research that people with the same model computer who bought it 2 years before can only sell their machines for \$800. Write an equation to represent depreciation of your computer. When will the computer be valued at \$200?

**Problem 4.** The half-life of caffeine is 5 hours. A grande French roast has 330 milligrams of caffeine. Using this information write an equation to represent the amount of caffeine in your bloodstream after $t$ hours in the form $y = Pe^{rt}$. At what time will the caffeine in your body no longer be effective (for most adults this occurs when there are about 20 milligrams or less in your bloodstream)?

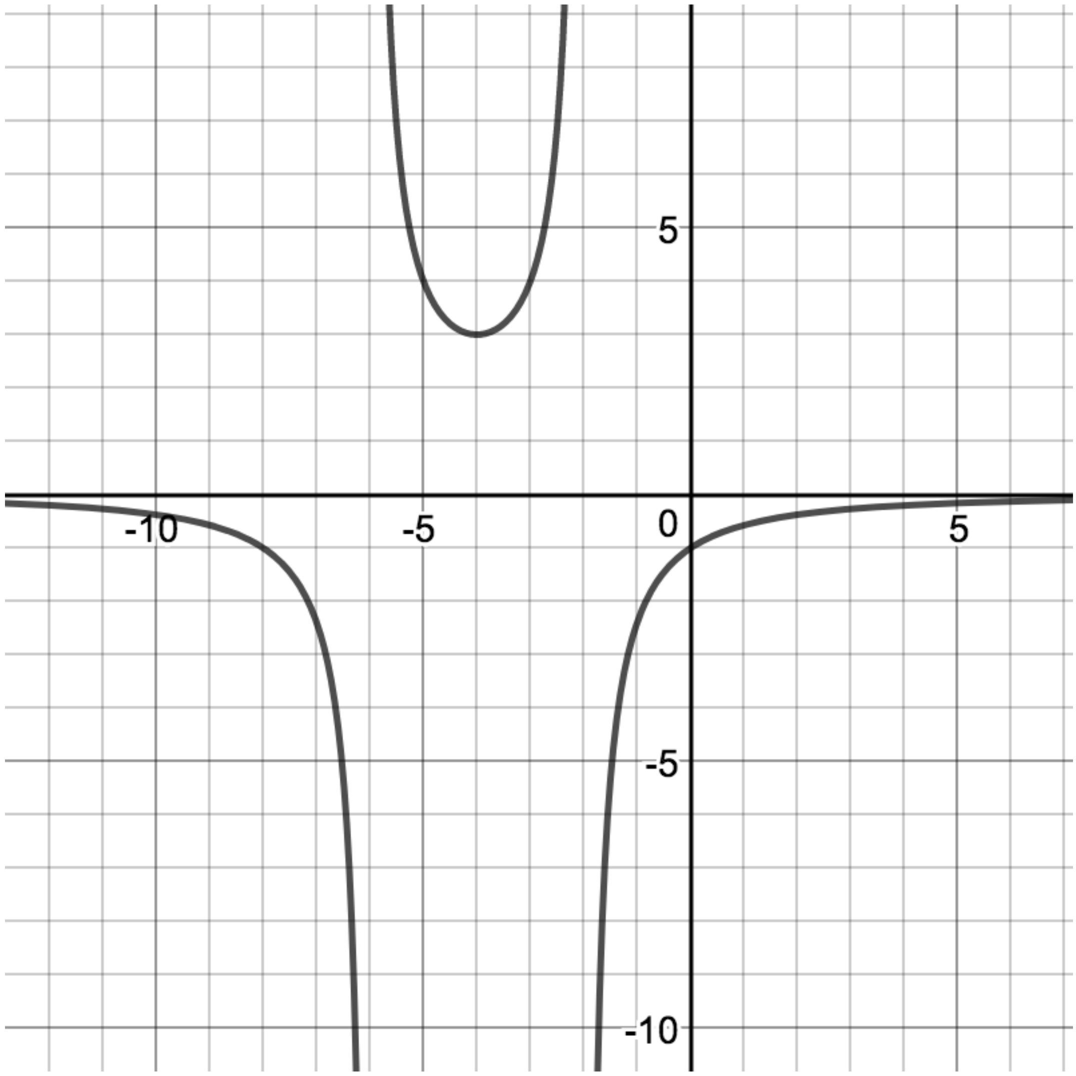
**Problem 5.** A certain termite colony triples in size every 2 weeks. The pest control came to your house a month ago and counted that you had 500 termites in your house. Write a function that represents the number of termites in your house in terms of $t$ where $t$ represents the number of weeks since the initial inspection. Since that inspection it has been 6 weeks and they termite people have come back to spray termite spray. How many termites are they exterminators going to exterminate?

## Warm-up Questions

**Problem 6.** Graph the following rational function:

$$f(x) = \frac{-12}{x^2 + 8x + 12}$$

*Solution.* Here is the graph that you should have seen. This was created using Desmos.




□

Identify the:

- Horizontal Asymptote:

  *Solution.* Since the degree in the denominator is bigger than the degree in the numerator, the horizontal asymptote is $y = 0$.This can also be seen in the graph above. □

- Vertical Asymptote(s):

  *Solution.* To find the vertical asymptotes, we set the denominator equal to zero and solve for $x$. That is

$$\begin{aligned} 0 &= x^2 + 8x + 12 \\ &= (x+2)(x+6) \end{aligned}$$

  This implies that there are vertical asymptotes at $x = -6$ and $x = -2$. This can also be seen in the graph of the function above. □

- Zeros of the function

  *Solution.* To find the zeros we set the numerator equal to 0 and solve. That is we have $0 = -12$. This does not make sense which implies this function has no zeros. □

- $y$-intercept

  *Solution.* The $y$-intercept occurs when we plug in 0 for $x$ and simplify. We see that

$$\begin{aligned} f(0) &= \frac{-12}{0^2+8(0)+12} \\ &= \frac{-12}{0+0+12} \\ &= \frac{-12}{12} \\ &= -1 \end{aligned}$$

Thus the $y$-intercept is $(0,-1)$. □