IBM Research

Lattice-based cryptography (I)

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Lattices

What is a lattice?

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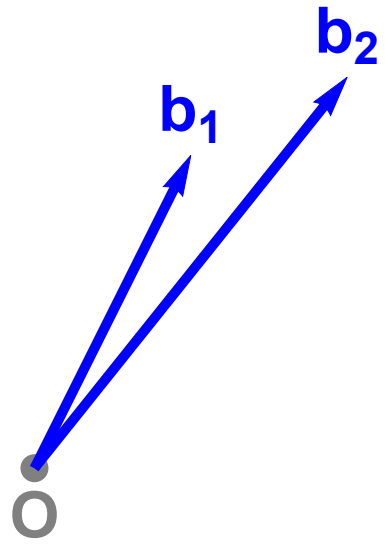

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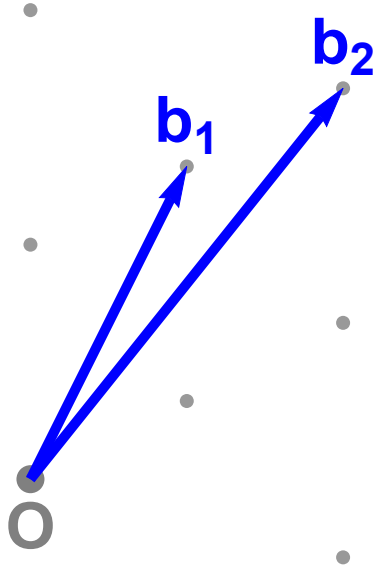
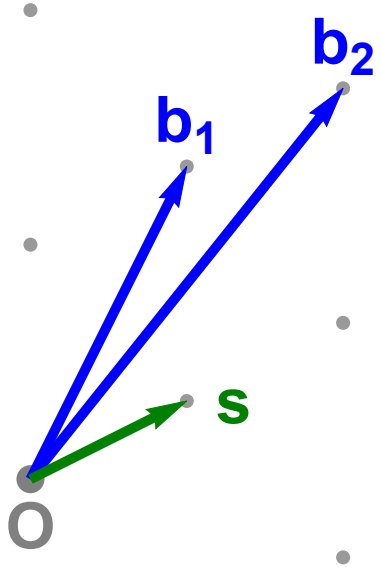


Lattices

Shortest Vector Problem (SVP)



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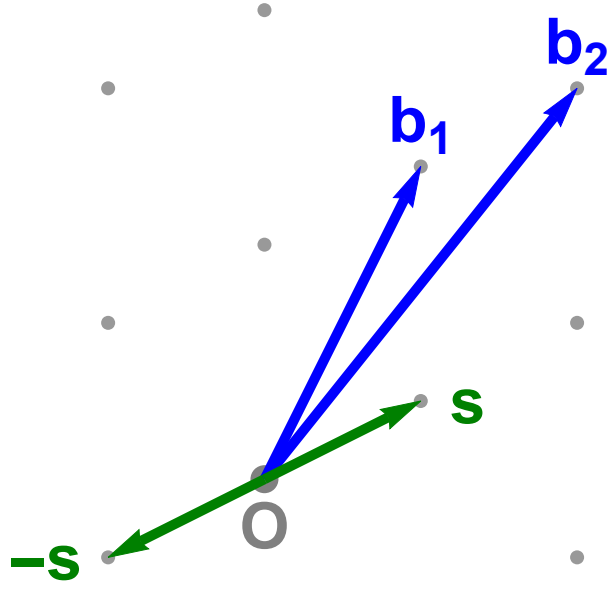

Lattices

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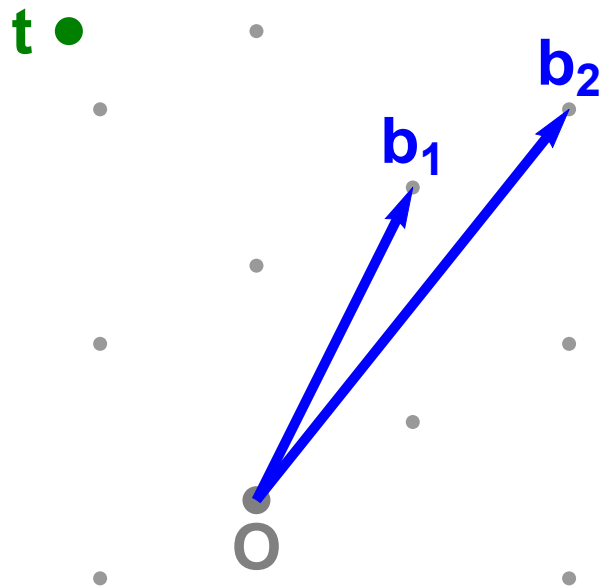

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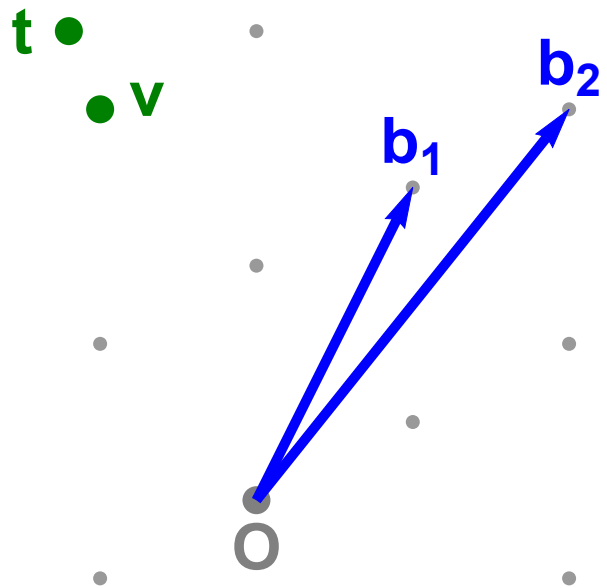
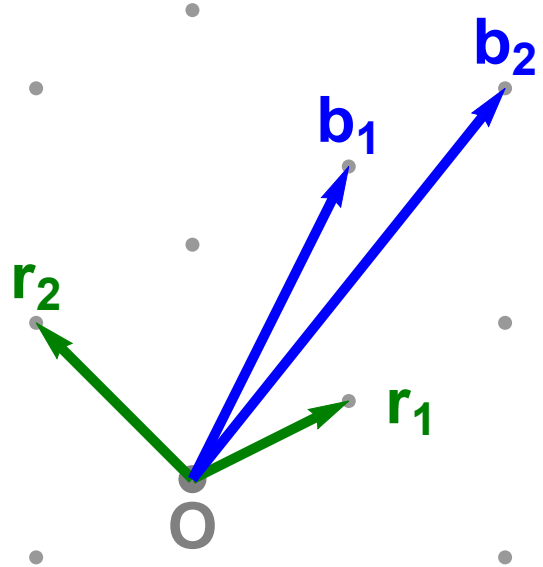


Lattices

Lattice basis reduction



Outline

Motivation: GGH encryption

Lattice basis reduction

- Gauss reduction
- LLL reduction
- BKZ reduction

Exact SVP algorithms

- Enumeration algorithms
- Sieving algorithms
- Practical comparison

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GGH cryptosystem

Overview

Private key: $R = \begin{pmatrix} \boldsymbol{r}_1 \\ \boldsymbol{r}_2 \end{pmatrix}$

Public key: $B = \begin{pmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \end{pmatrix}$

Encrypt $\boldsymbol{m}$:

$$\boldsymbol{v} = \boldsymbol{m}B$$
$$\boldsymbol{c} = \boldsymbol{v} + \boldsymbol{e}$$

Decrypt $\boldsymbol{c}$:

$$\boldsymbol{v}' = \lfloor \boldsymbol{c}R^{-1} \rceil R$$
$$\boldsymbol{m}' = \boldsymbol{v}'B^{-1}$$

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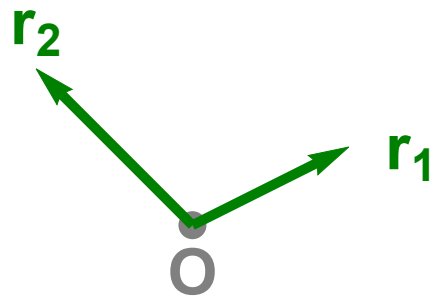

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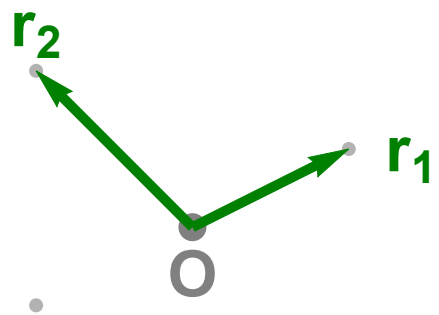

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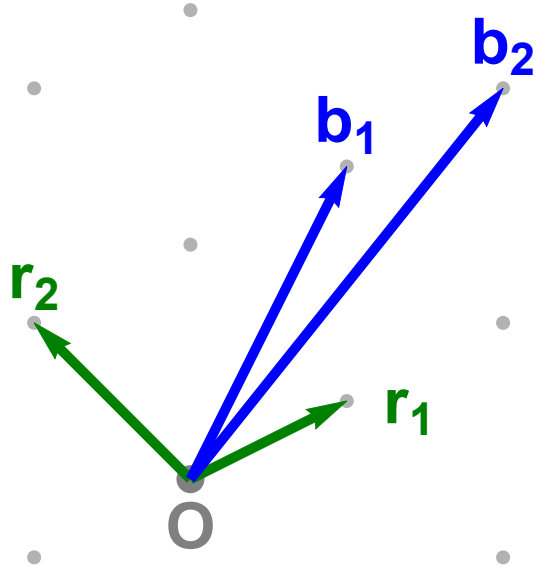

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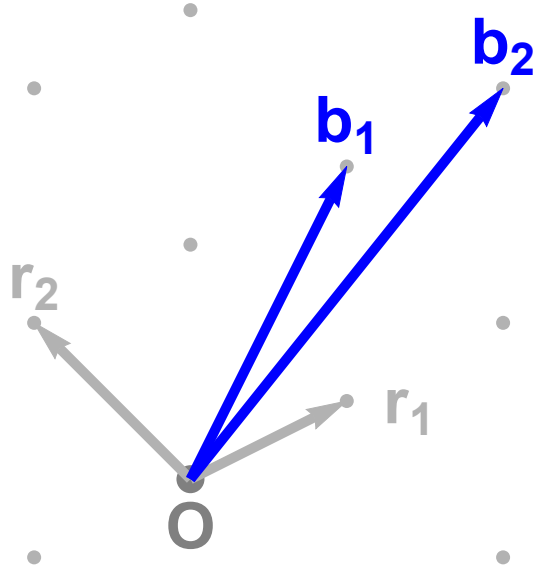

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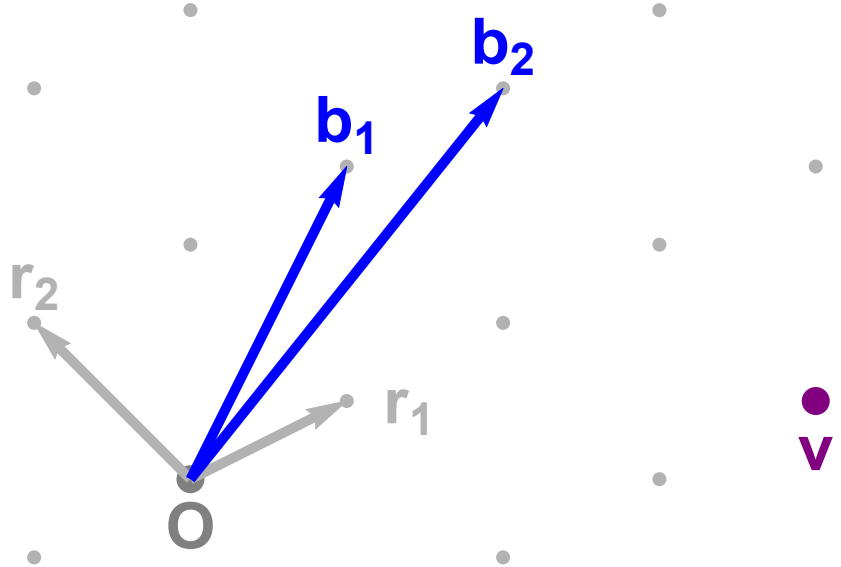

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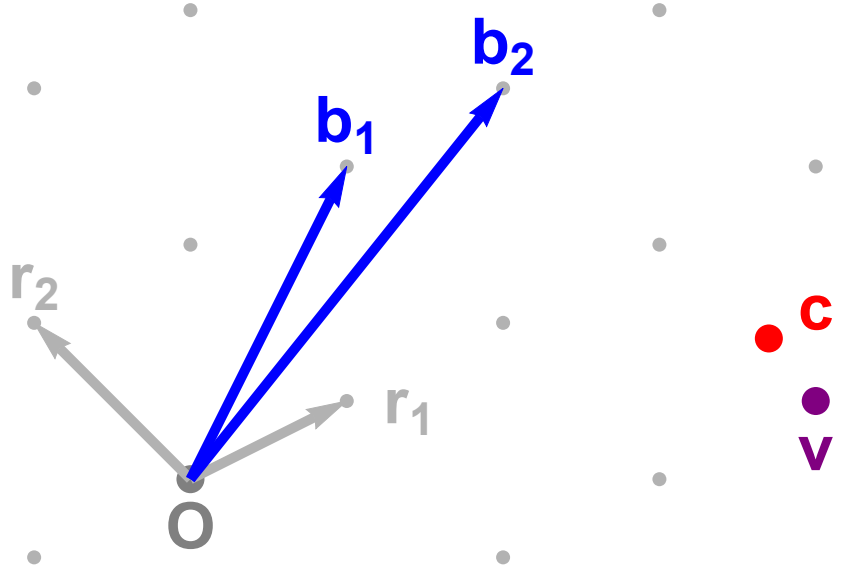

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Decryption with good basis

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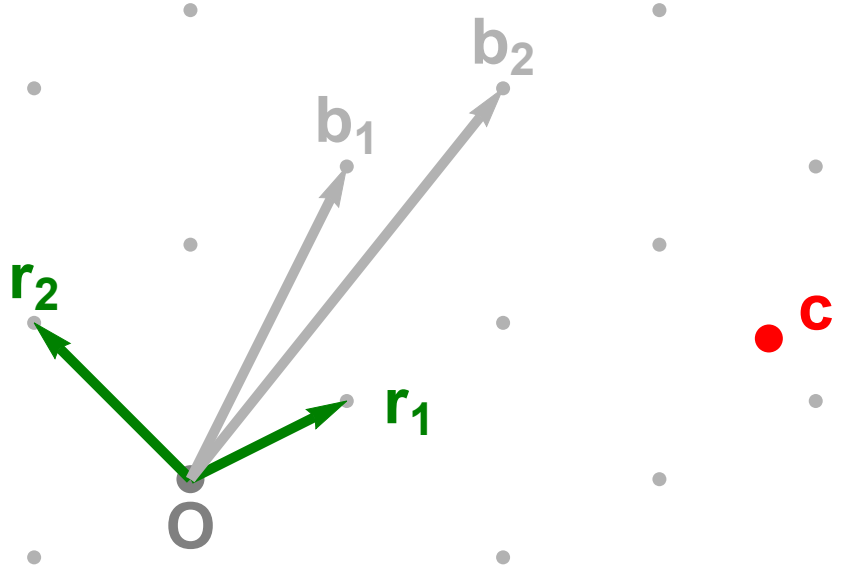

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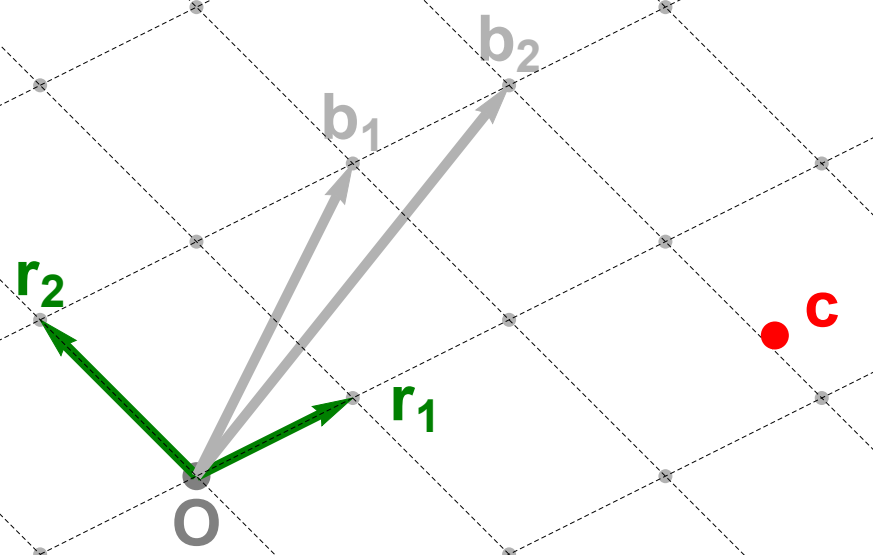

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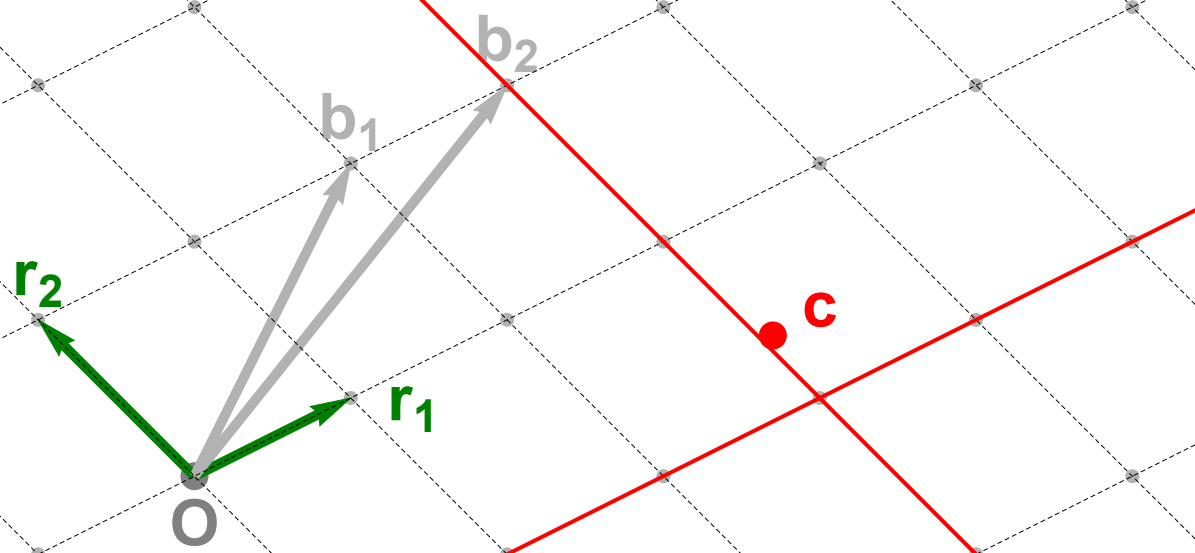

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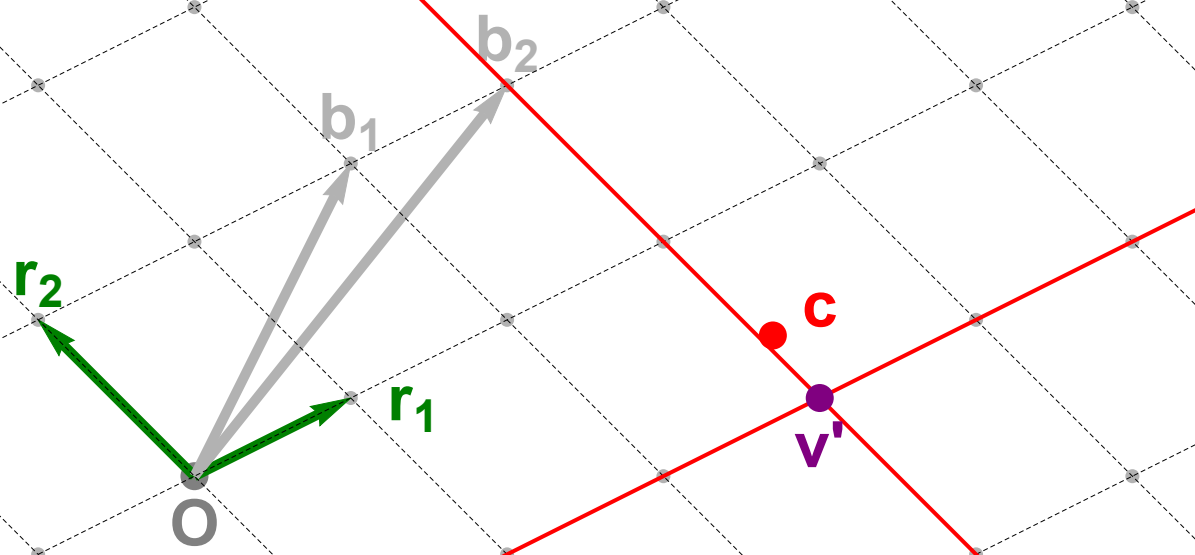

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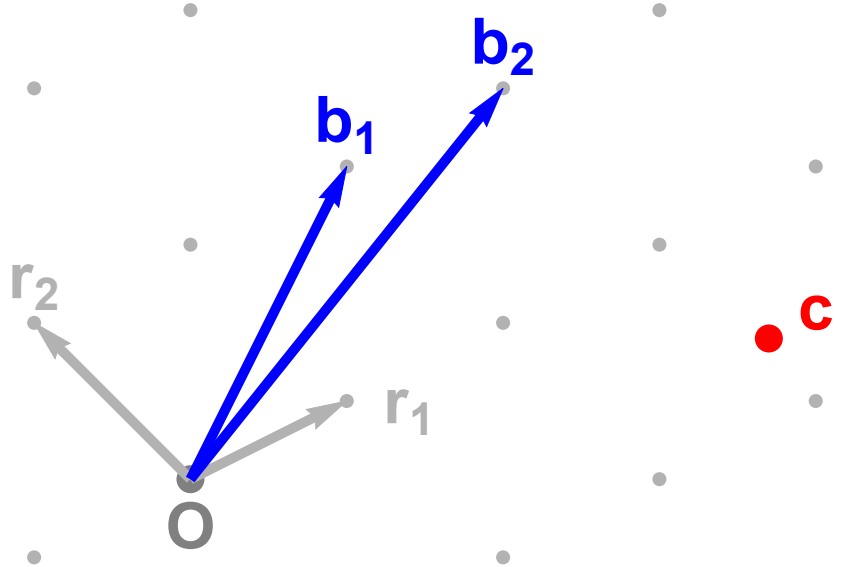

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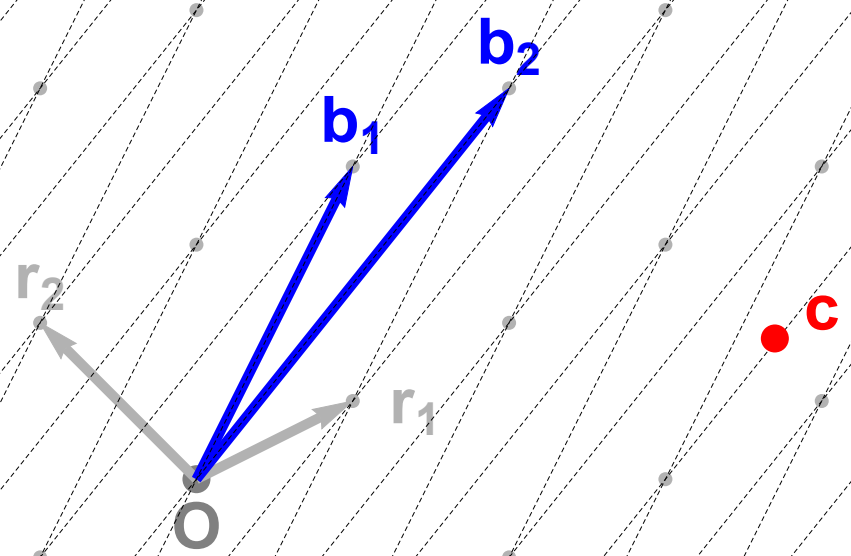

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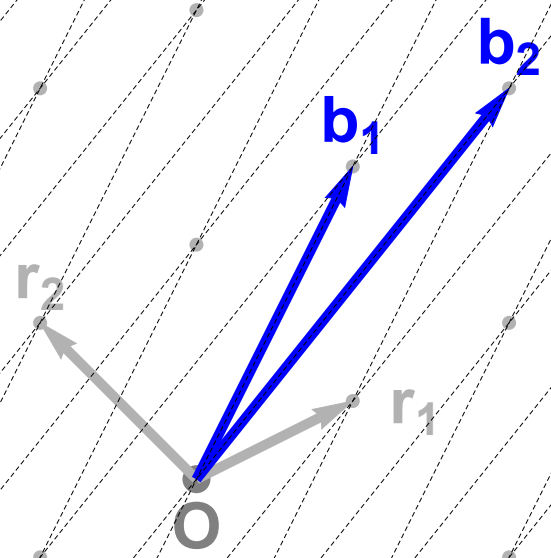

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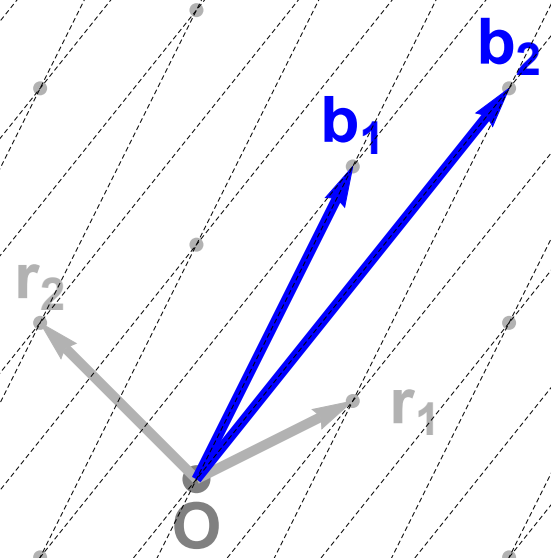

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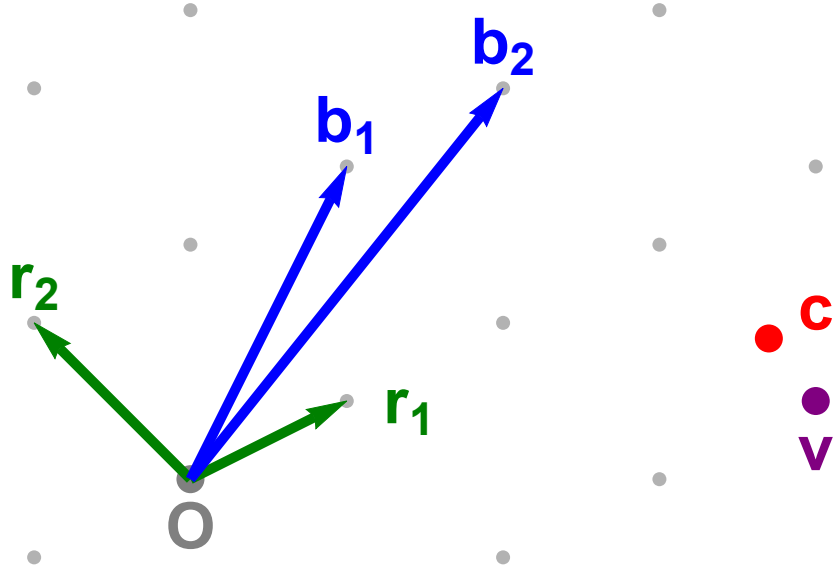

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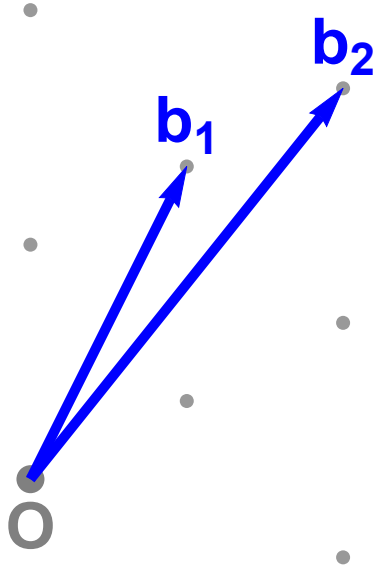
Gauss reduction



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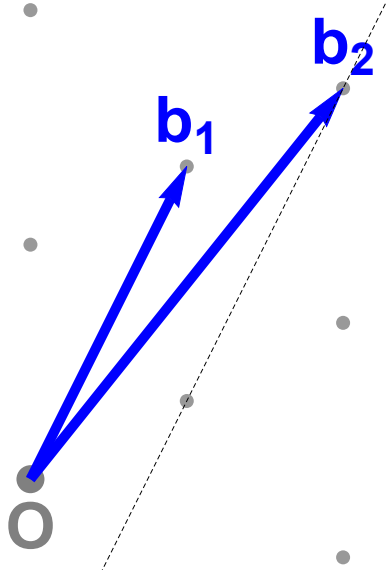

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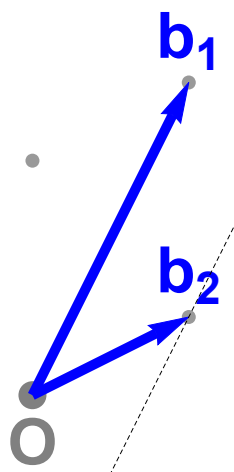

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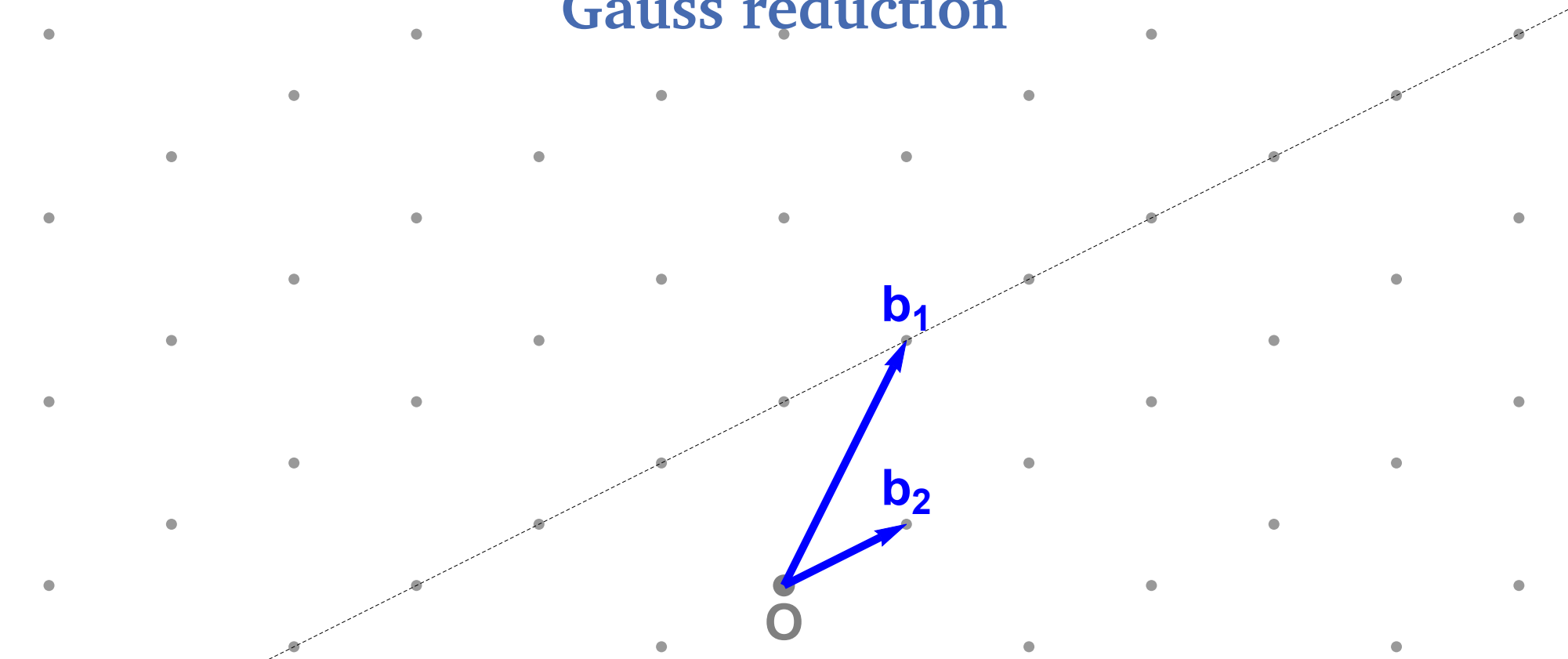

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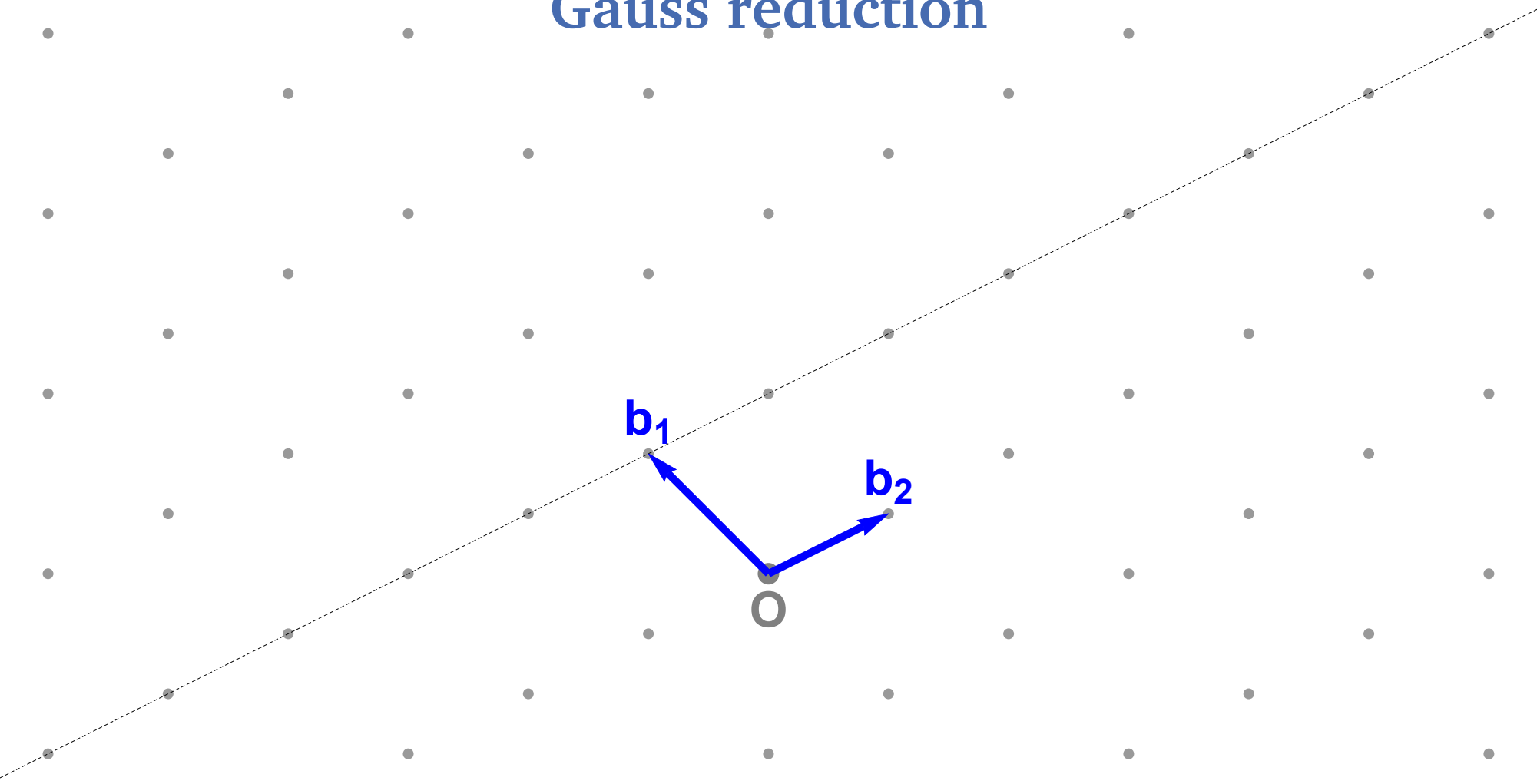

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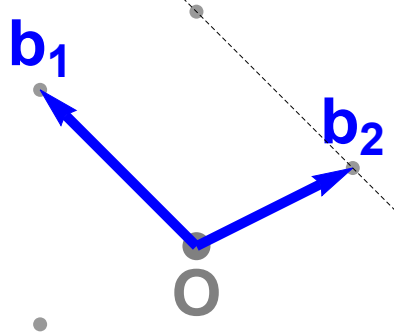

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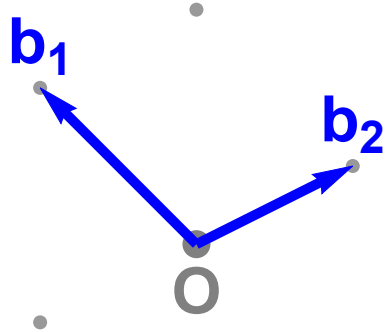

Gauss reduction

Given $\mathbf{B} = \{\boldsymbol{b}_1, \boldsymbol{b}_2\}$, repeat two steps:

- **Swap**: If $\|\boldsymbol{b}_1\| > \|\boldsymbol{b}_2\|$, then swap $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$.
- **Reduce**: While $\|\boldsymbol{b}_2 \pm \boldsymbol{b}_1\| < \|\boldsymbol{b}_2\|$, replace $\boldsymbol{b}_2 \leftarrow \boldsymbol{b}_2 \pm \boldsymbol{b}_1$.

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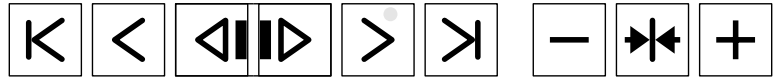
At the end, $\boldsymbol{b}_1$ is a shortest (non-zero) lattice vector and $\boldsymbol{b}_2$ a "second shortest" (non-zero) lattice vector.

Gauss reduction

3141592653	0
5897932384	1



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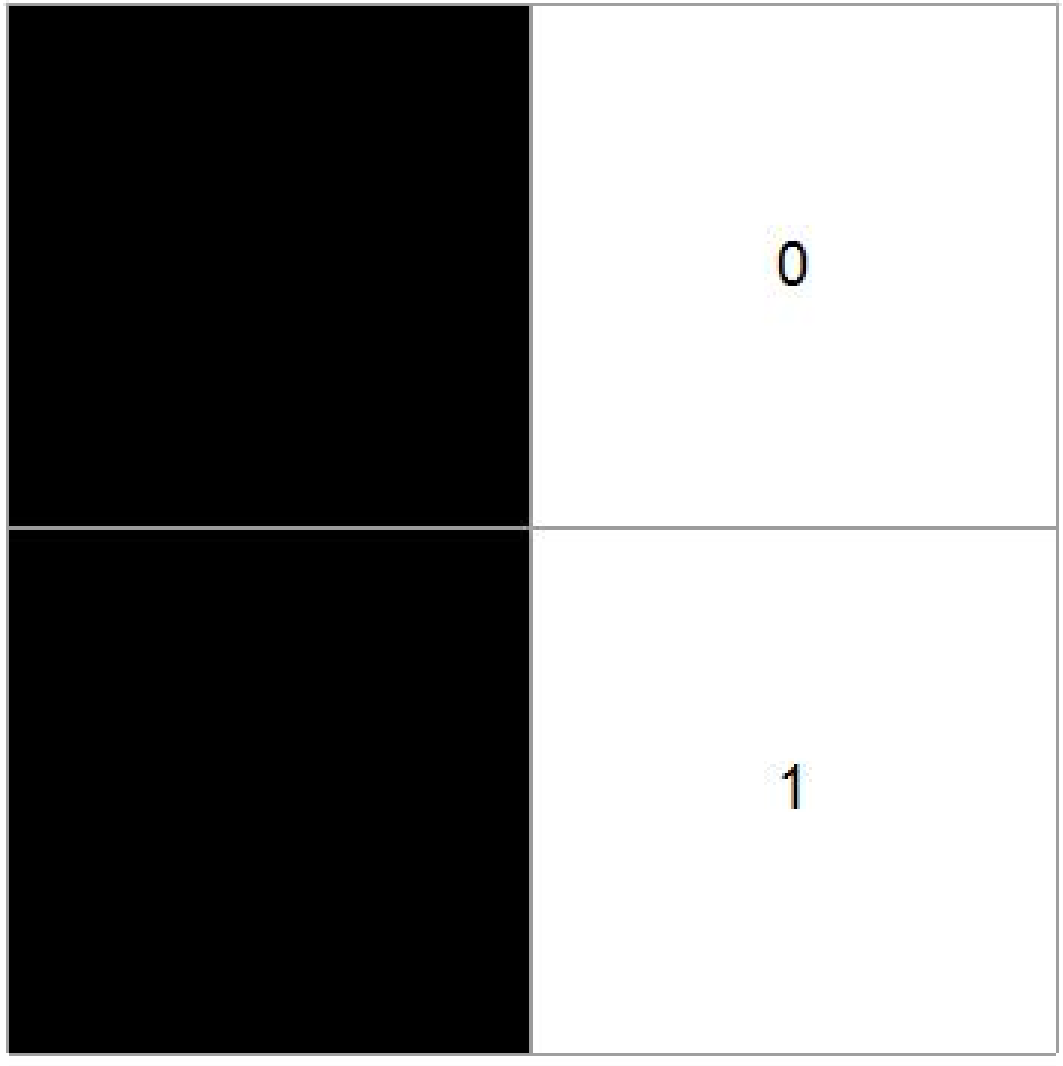


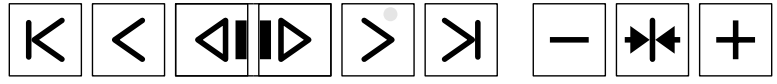
LLL algorithm

Lenstra-Lenstra-Lovasz (LLL) algorithm [LLL82]

- Blockwise generalization of Gauss reduction
- Do reductions/swaps on $(\boldsymbol{b}_i, \boldsymbol{b}_{i+1})$ for $i = 1, \ldots, n-1$

LLL algorithm

	0	0	0	0	0	0	0
	1	0	0	0	0	0	0
	0	1	0	0	0	0	0
	0	0	1	0	0	0	0
	0	0	0	1	0	0	0
	0	0	0	0	1	0	0
	0	0	0	0	0	1	0
	0	0	0	0	0	0	1



LLL algorithm

	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
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	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
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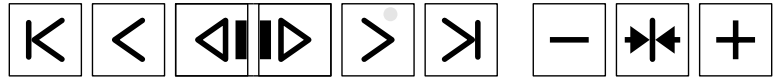
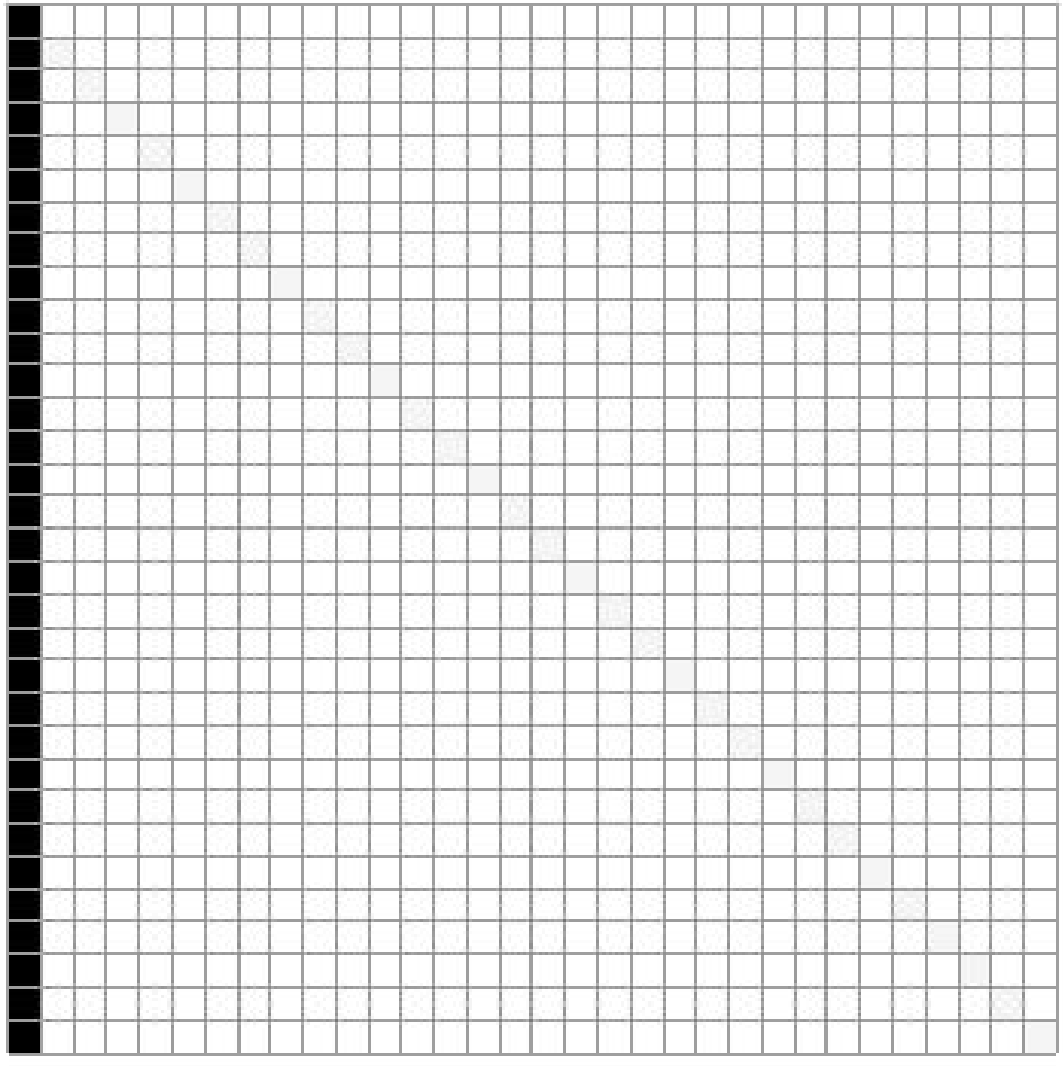
	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1



LLL algorithm



BKZ algorithm

Lenstra-Lenstra-Lovasz (LLL) algorithm [LLL82]

- Blockwise generalization of Gauss reduction
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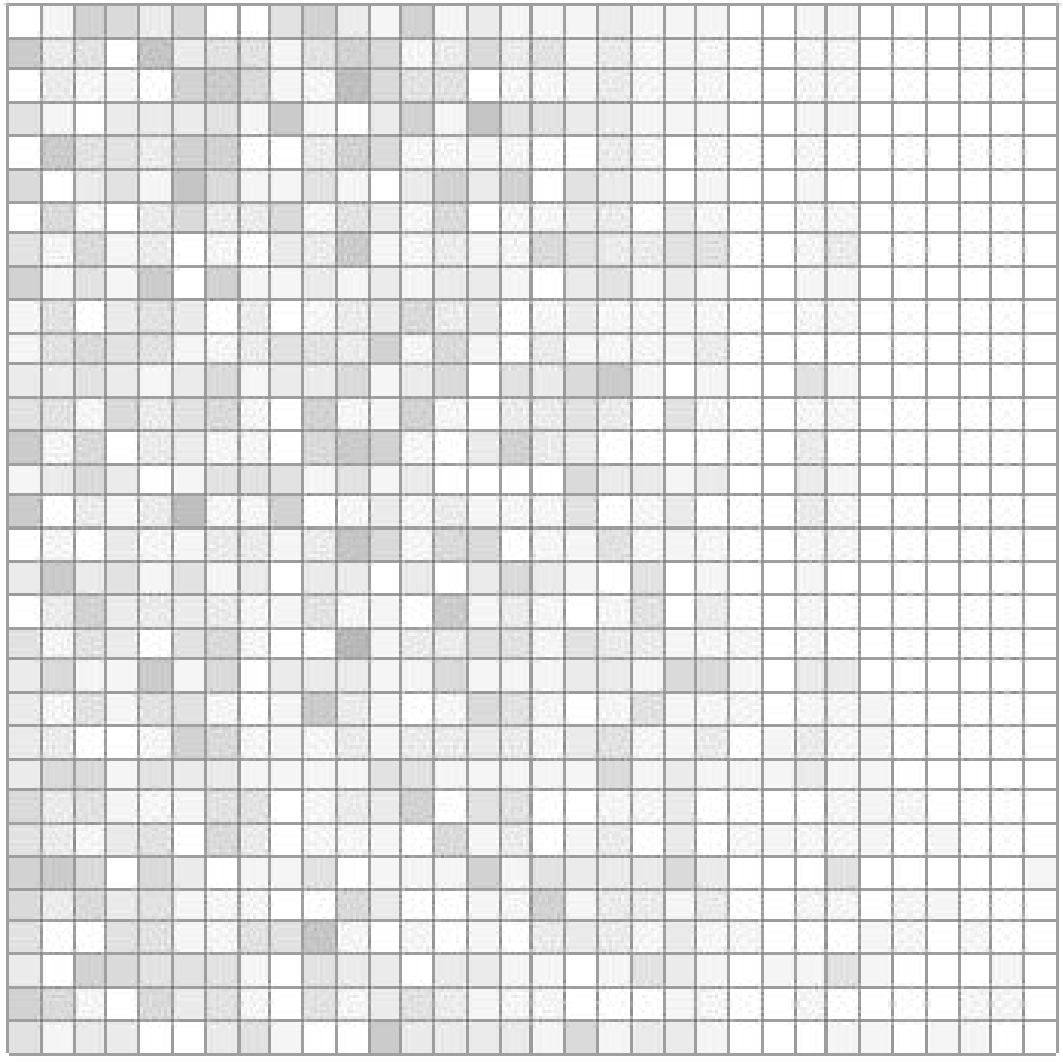

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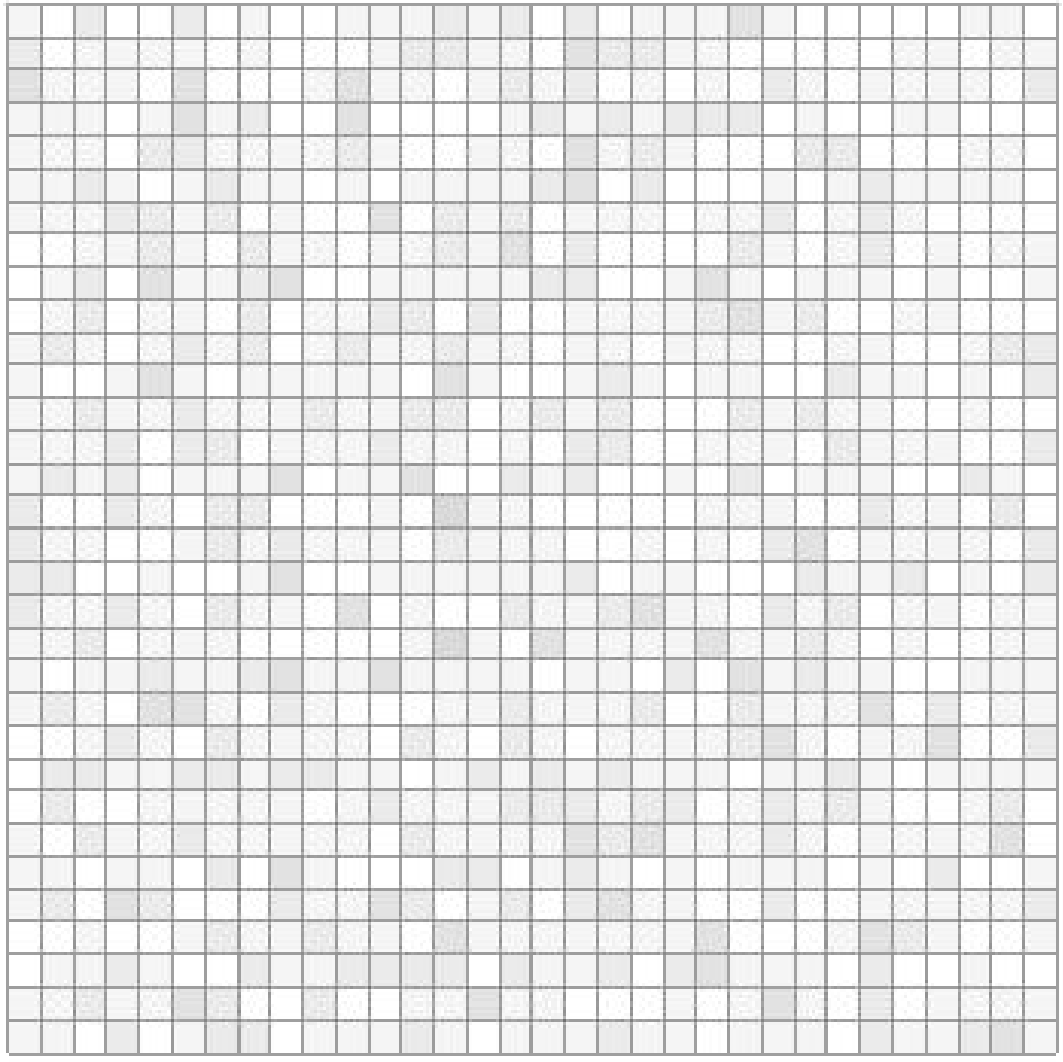

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Question: How to solve exact SVP in high dimensions?

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Outline

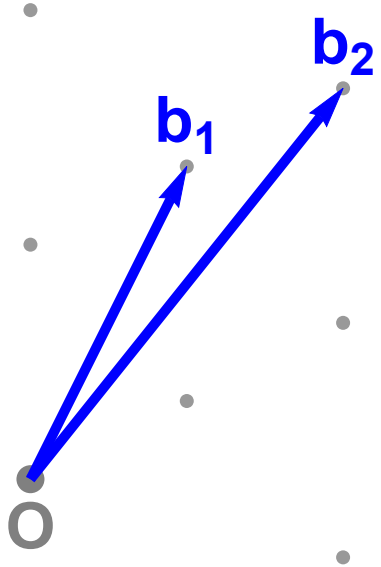
Motivation: GGH encryption

Lattice basis reduction
- Gauss reduction
- LLL reduction
- BKZ reduction

Exact SVP algorithms
- Enumeration algorithms
- Sieving algorithms
- Practical comparison

Fincke-Pohst enumeration

1. Determine possible coefficients of b_2



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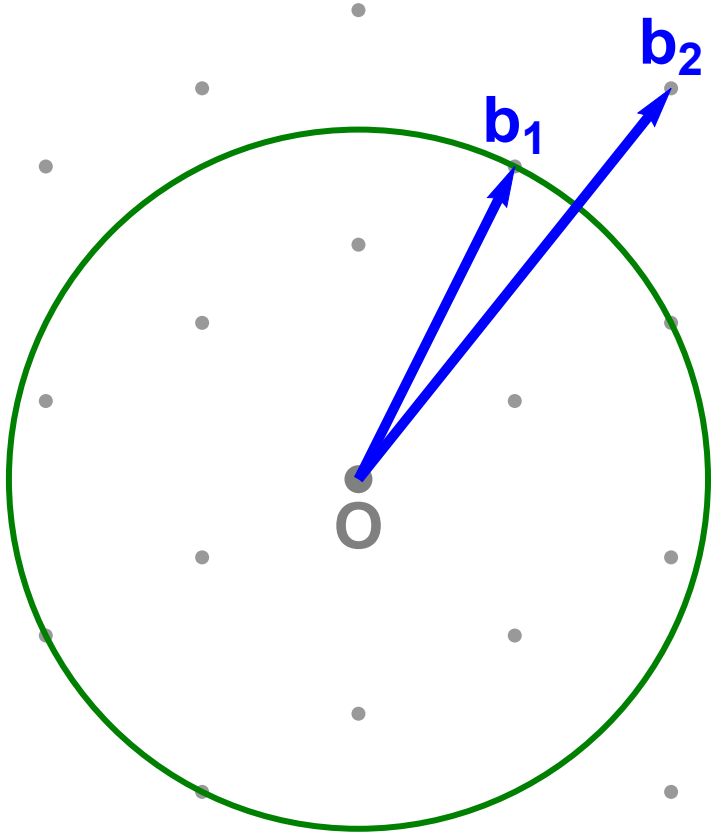

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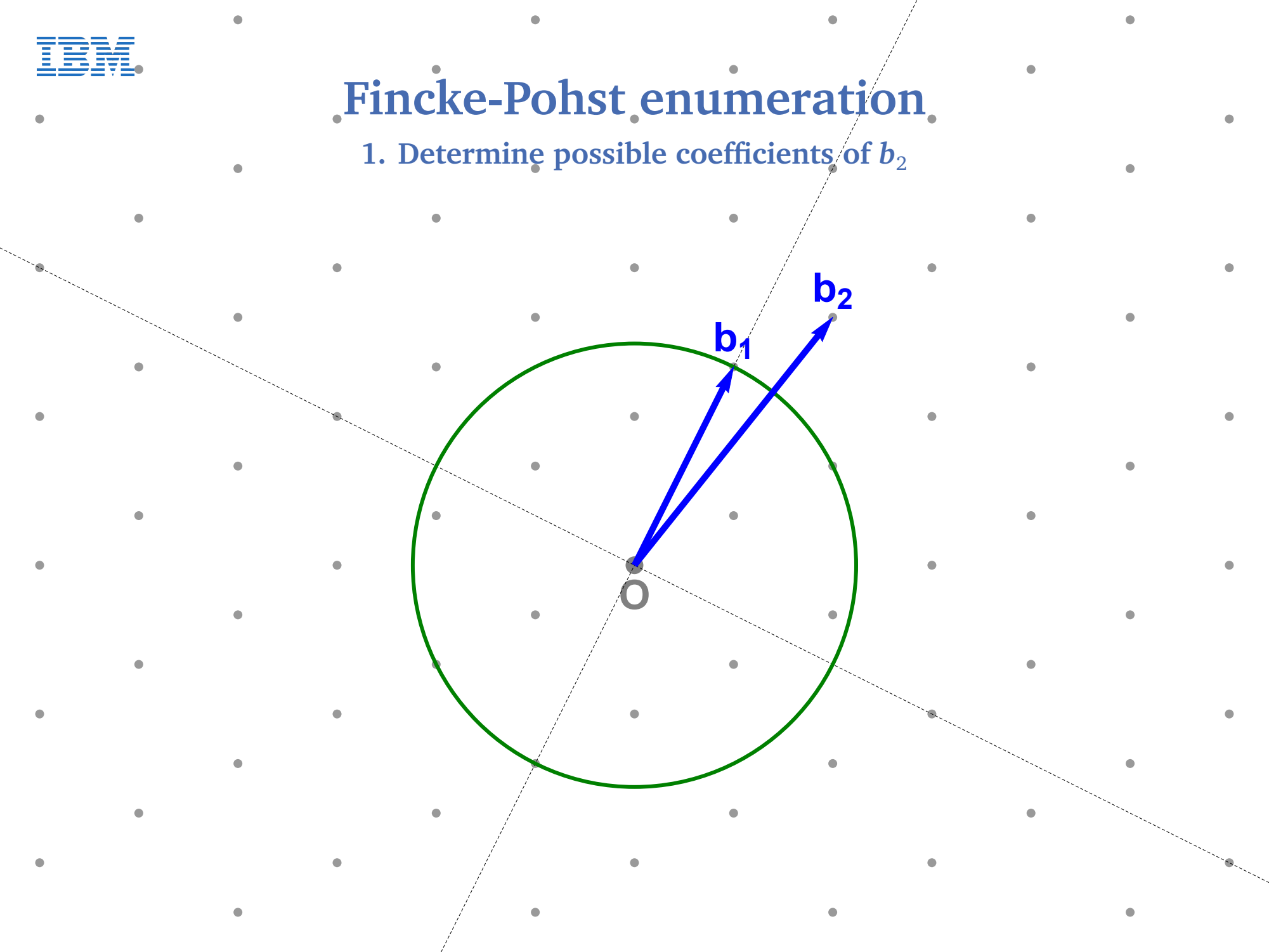

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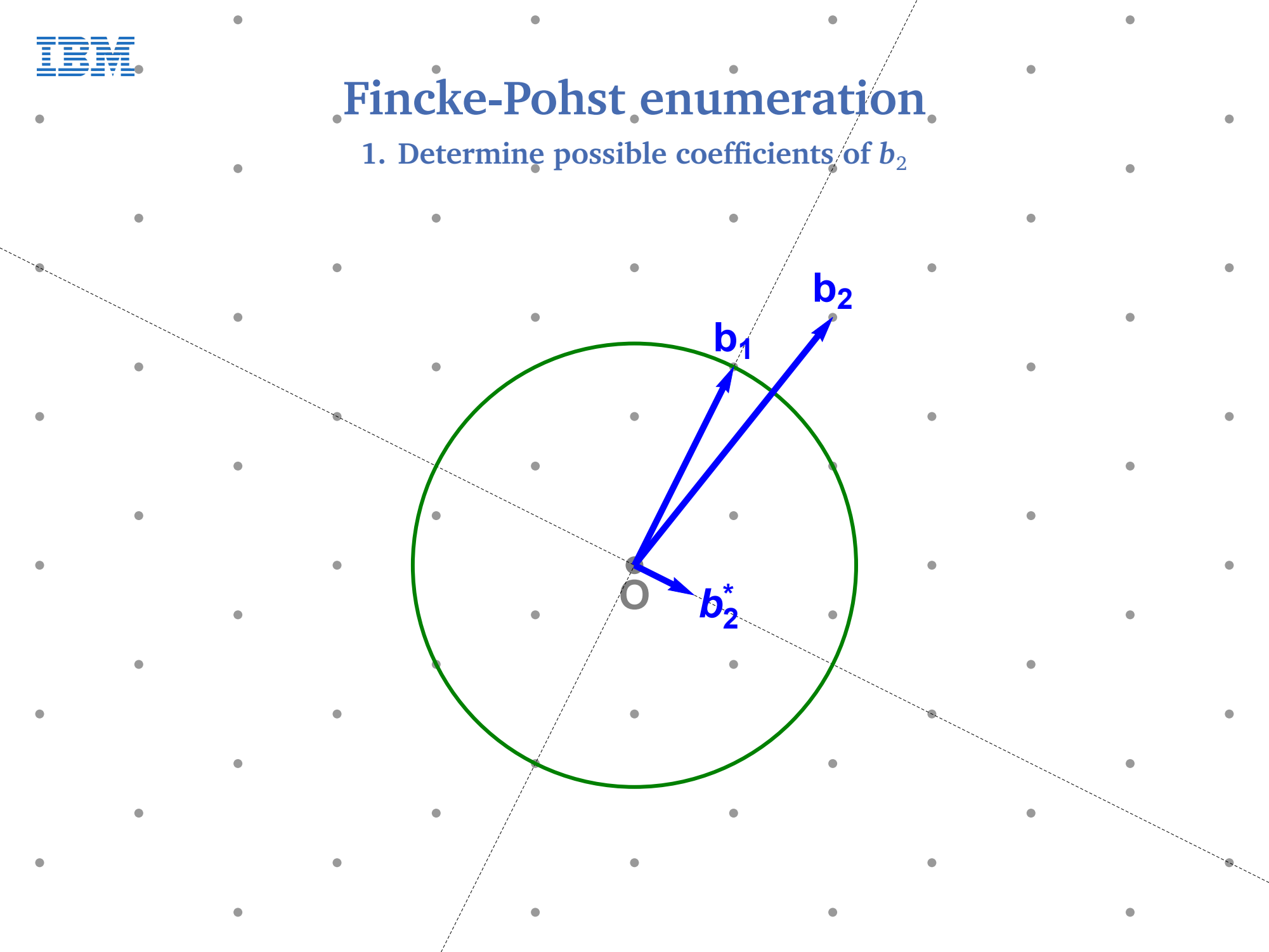

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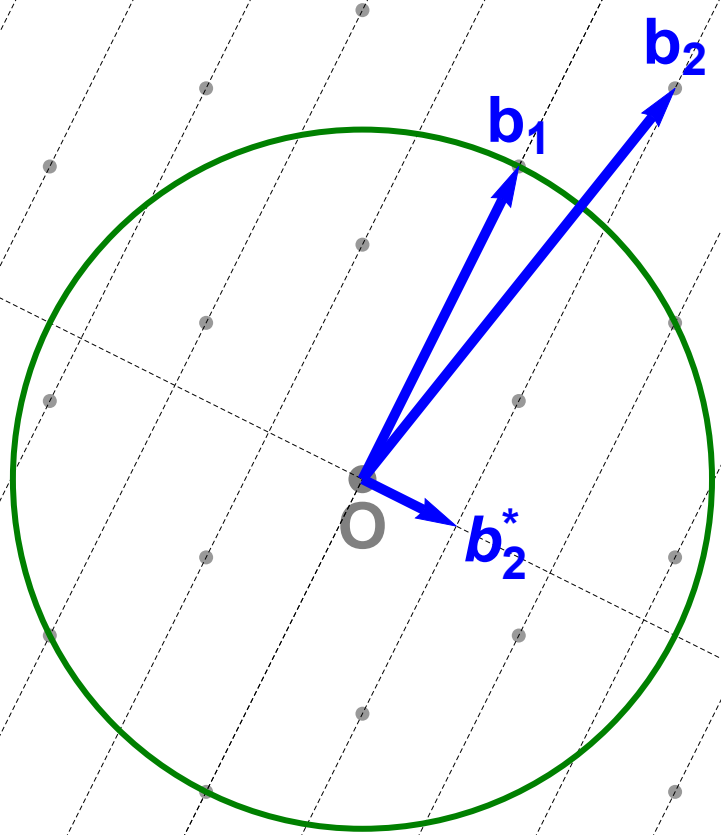
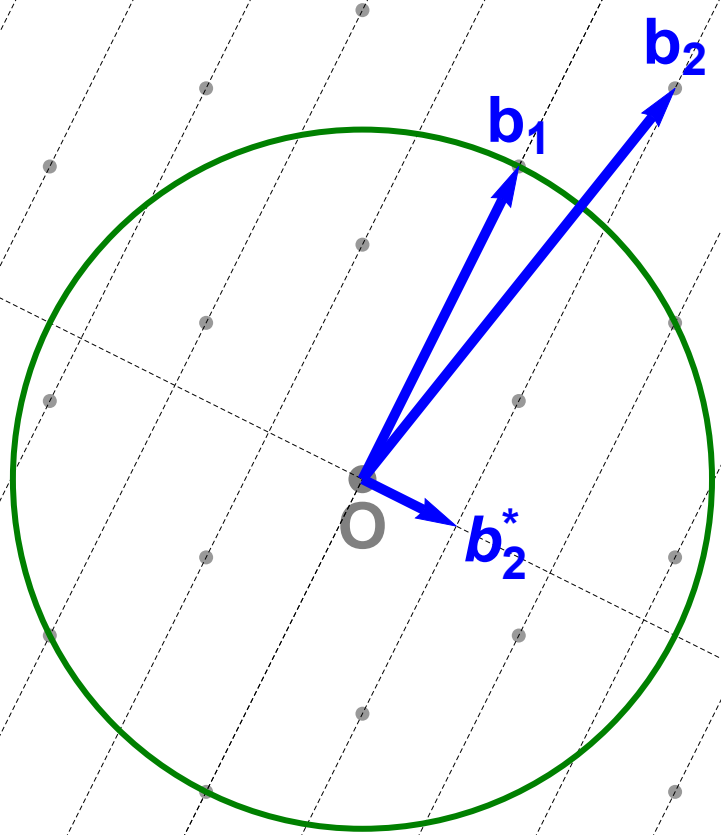


Fincke-Pohst enumeration

2. Find short vectors for each coefficient of b_2



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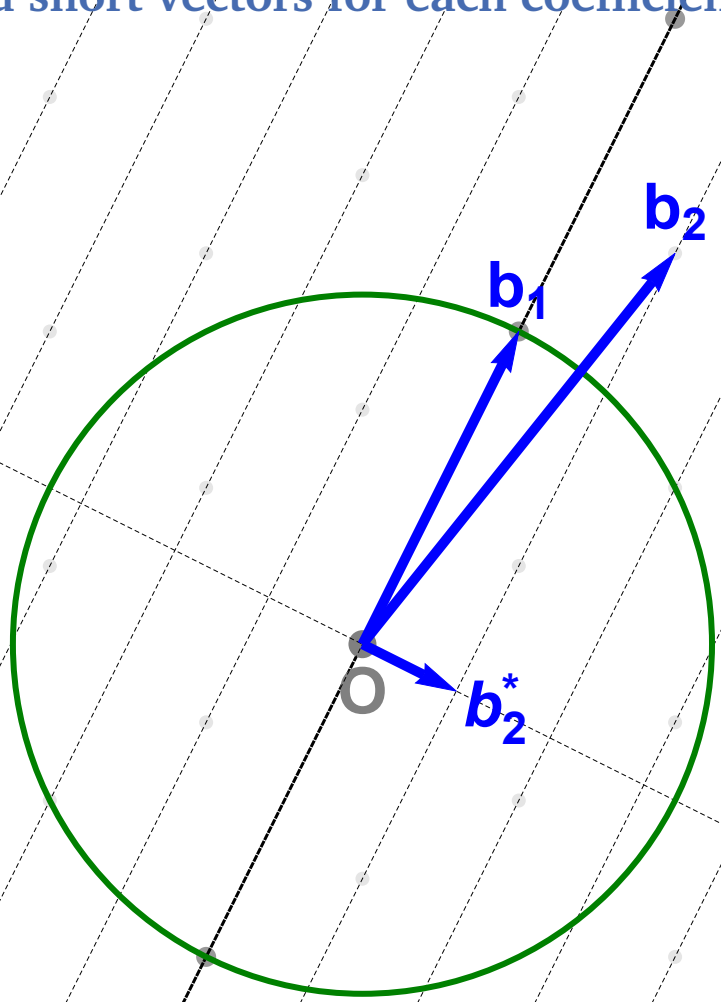

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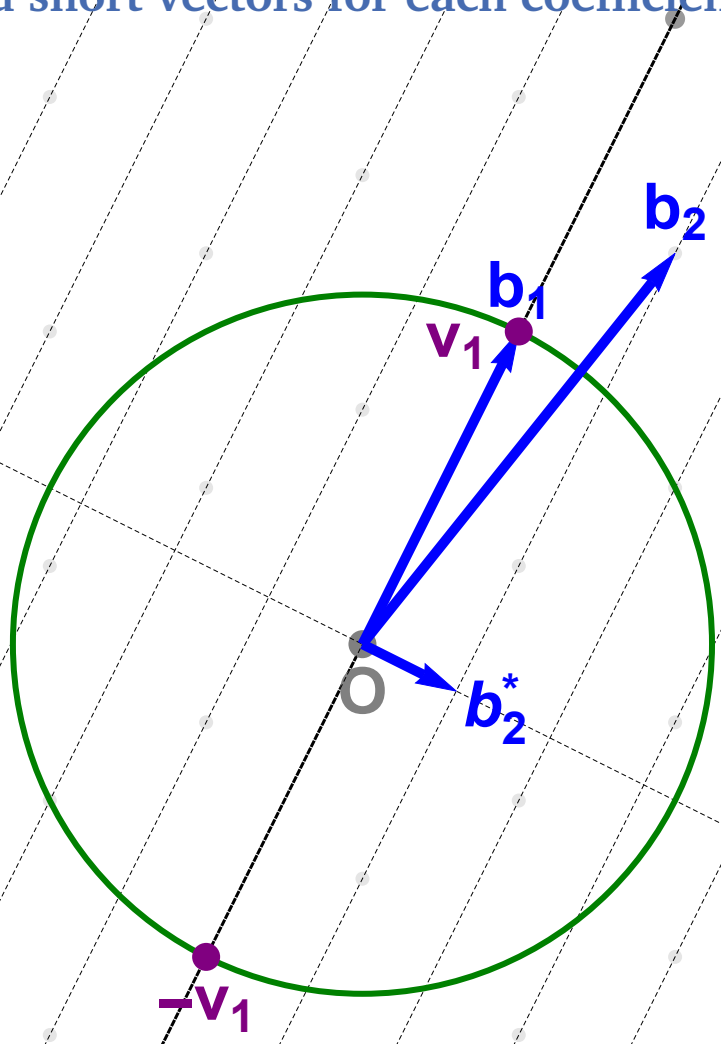

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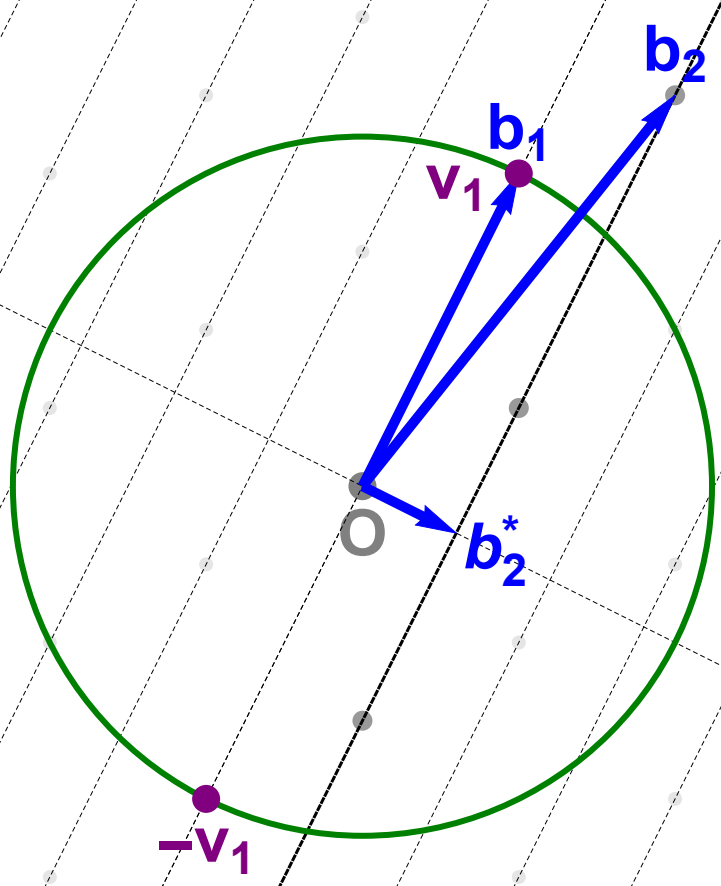

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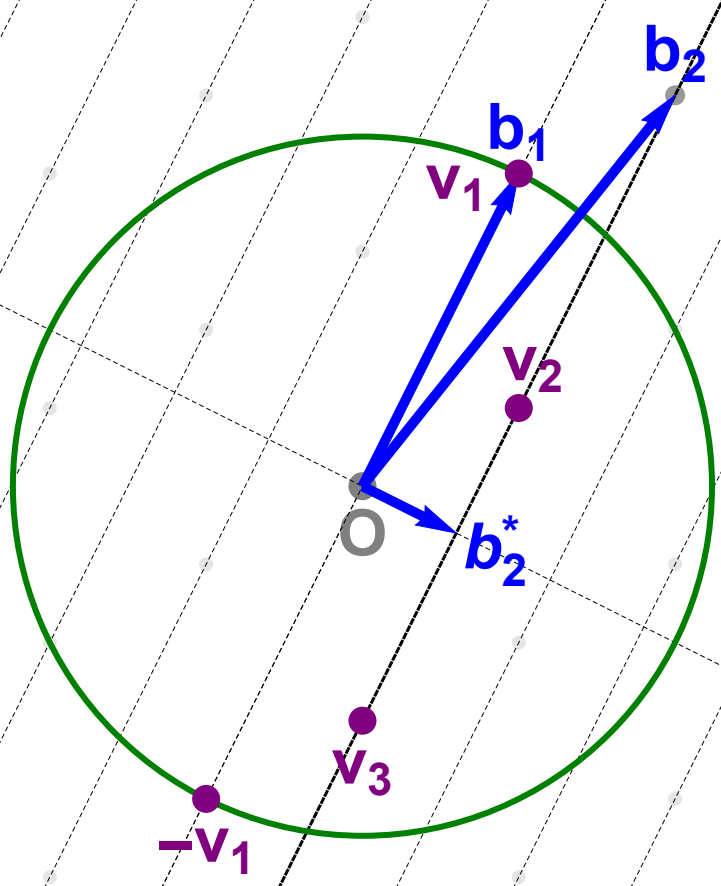

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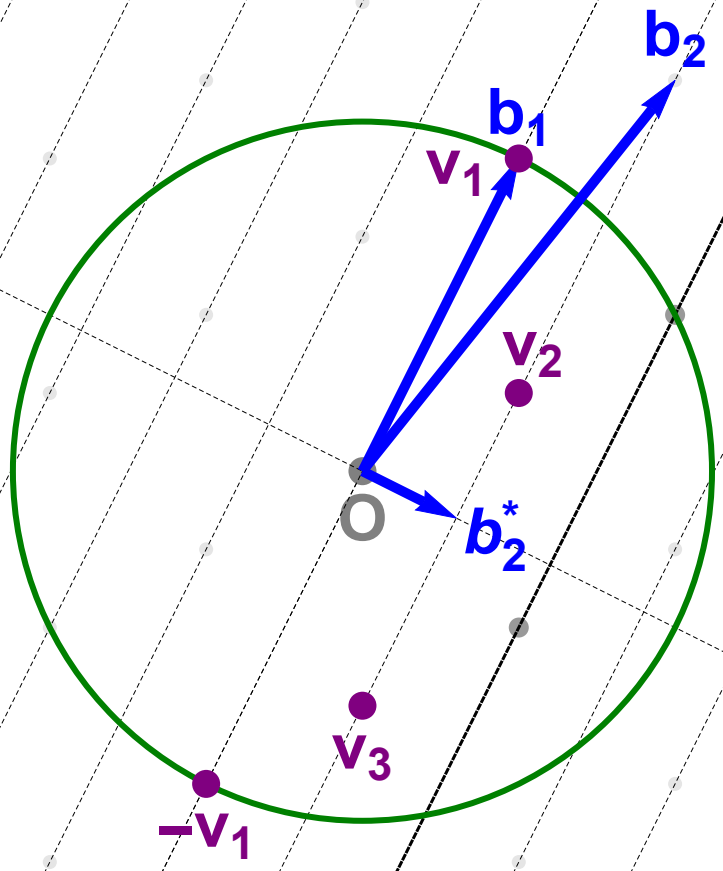

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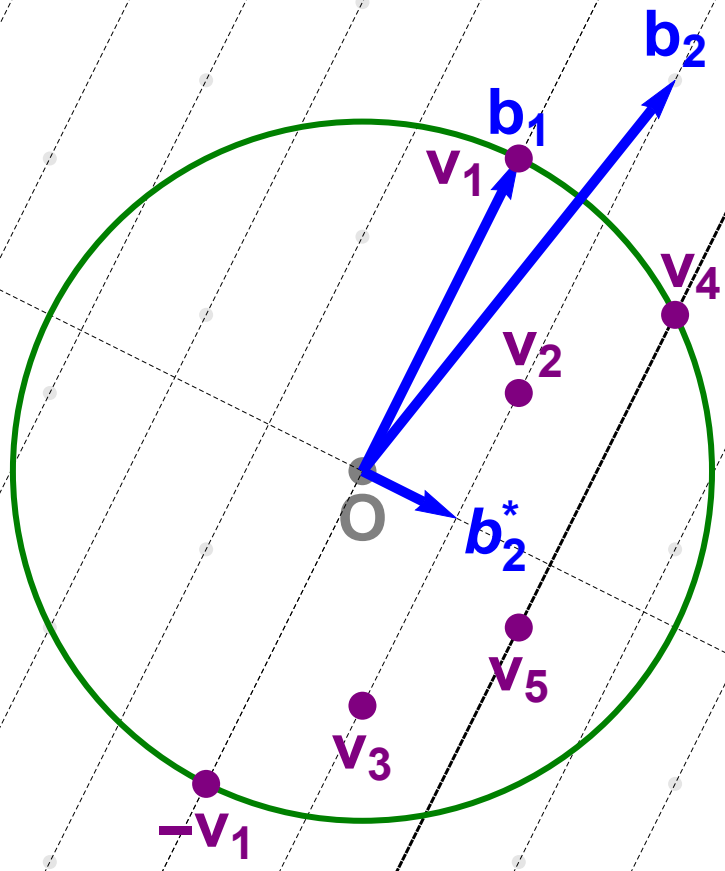

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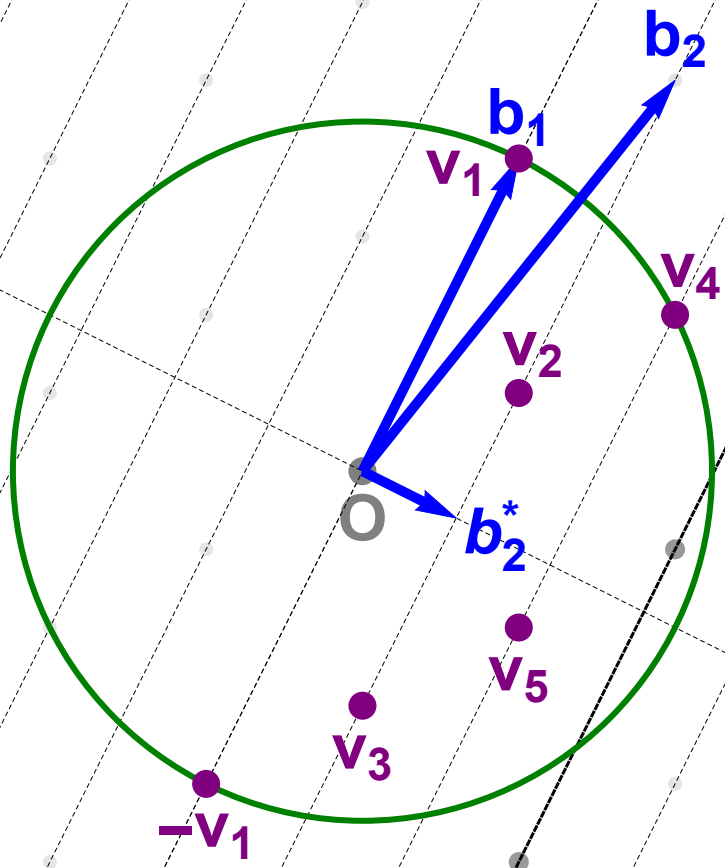

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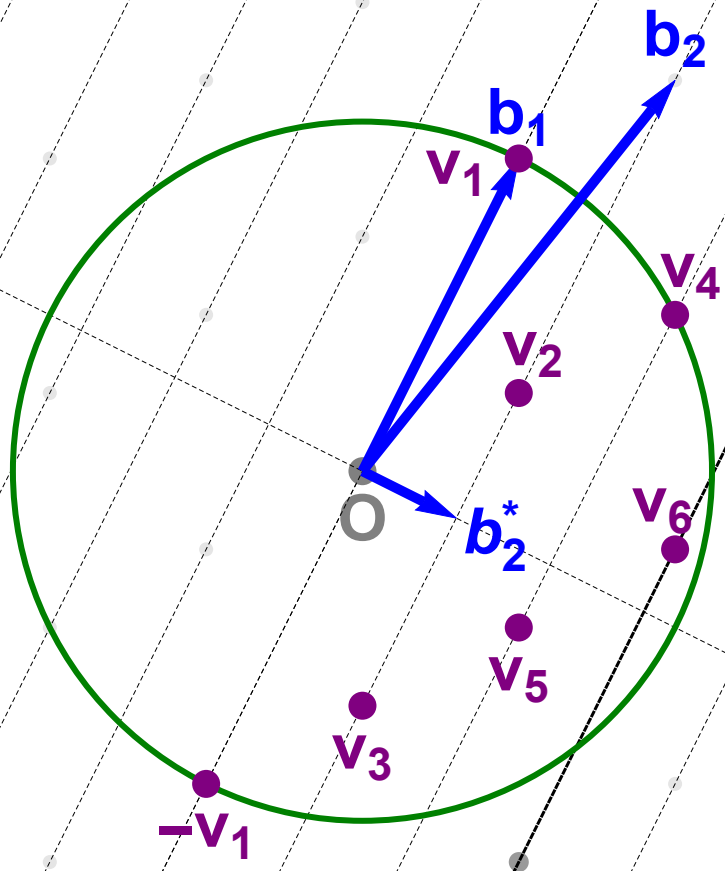

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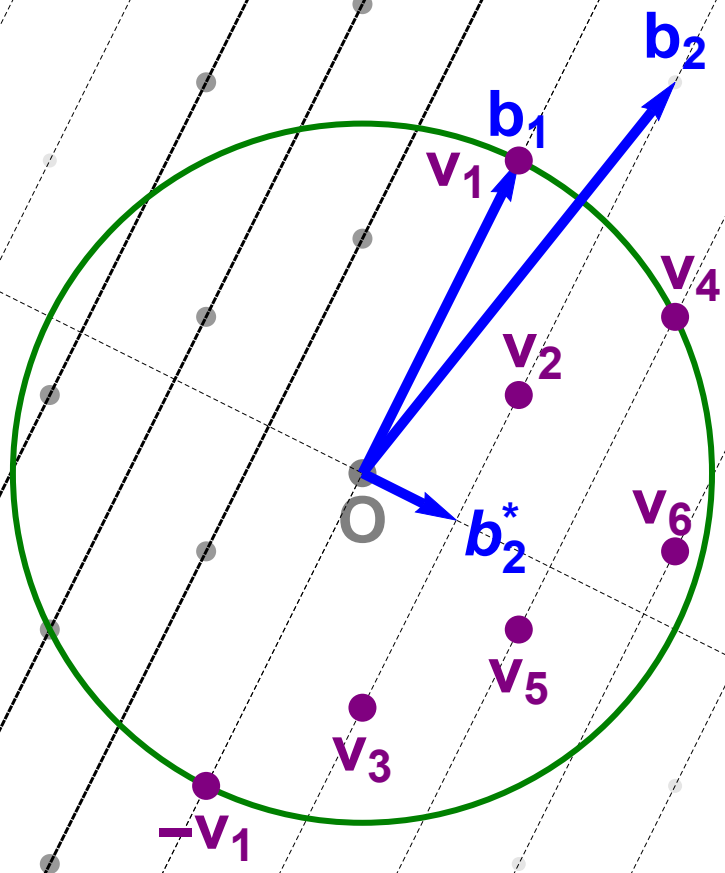

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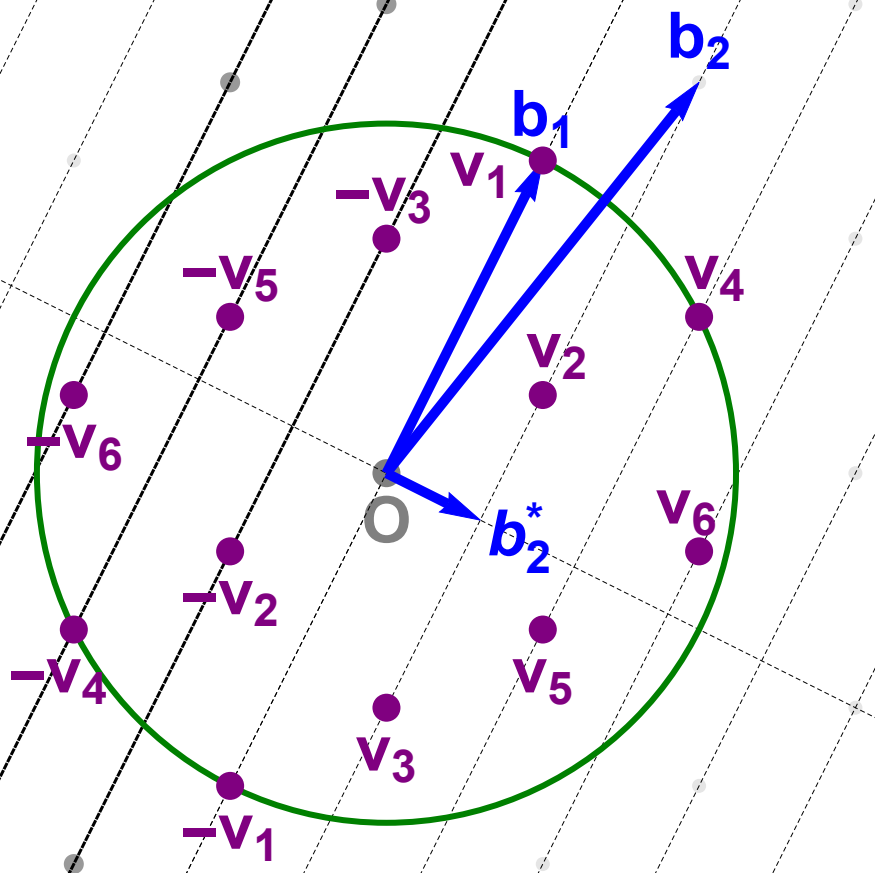

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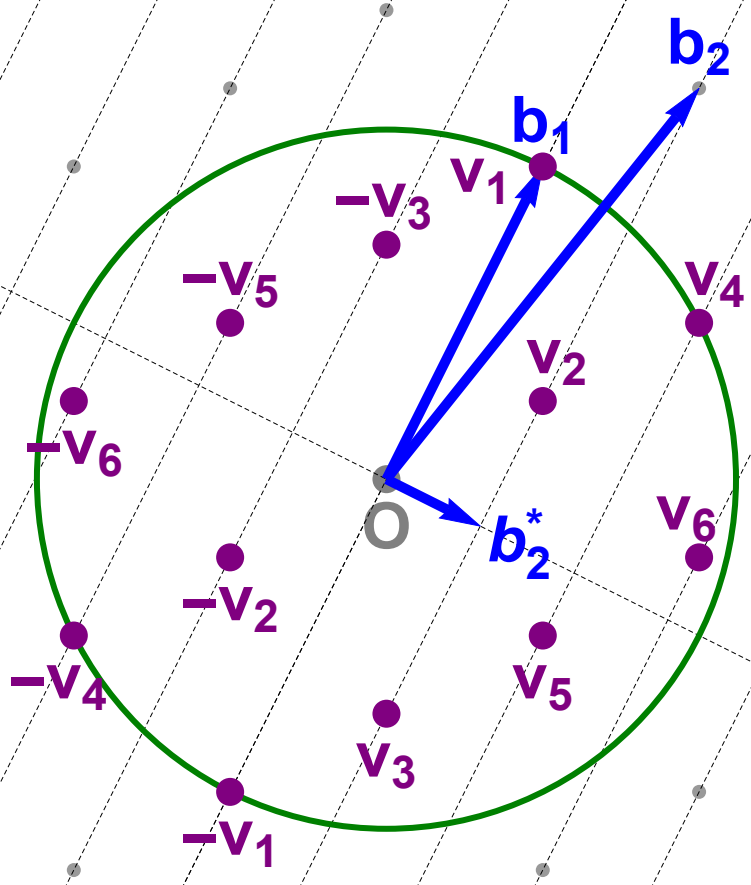

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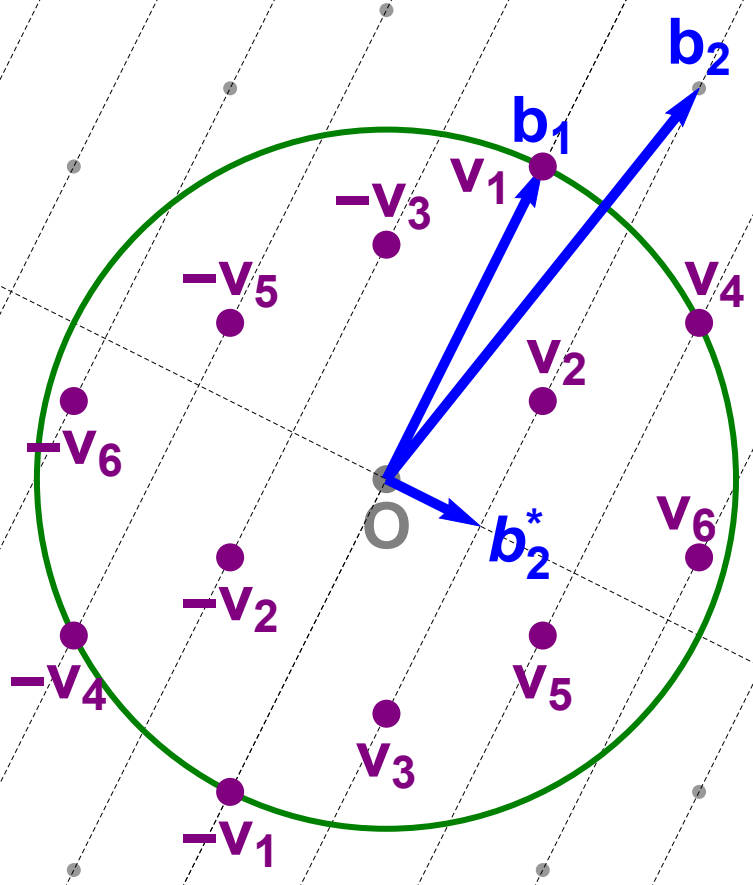

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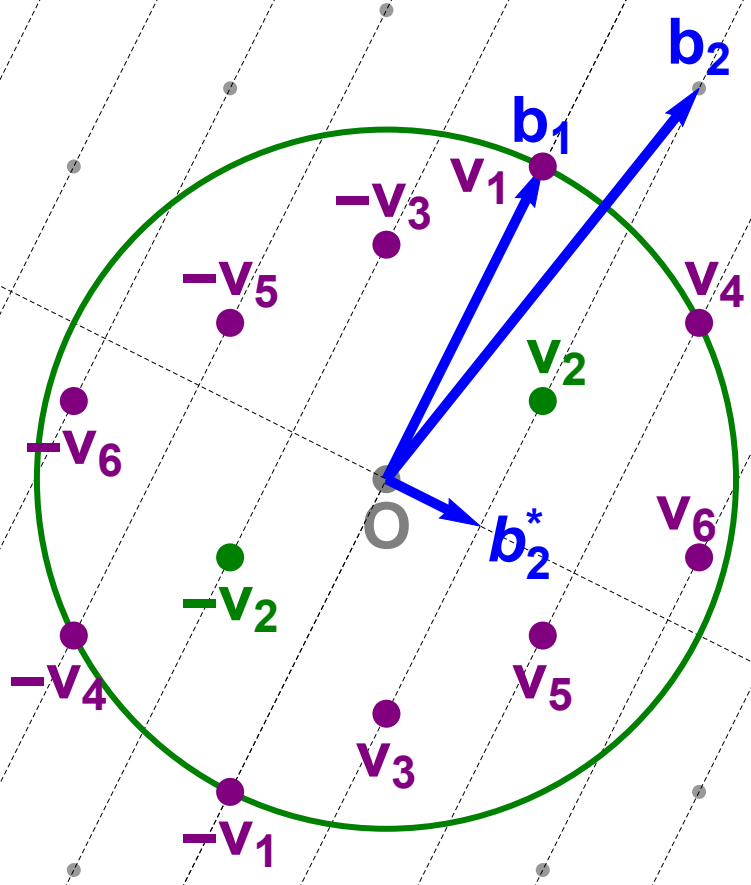
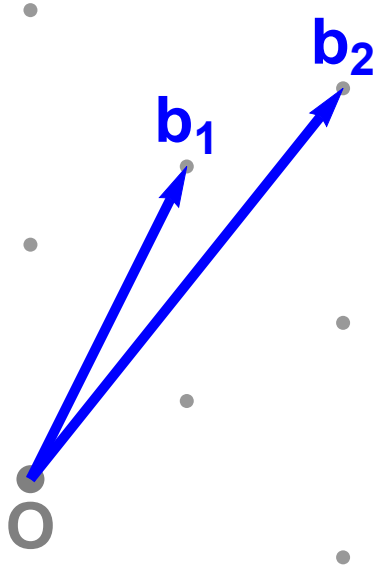


Kannan enumeration

Better bases



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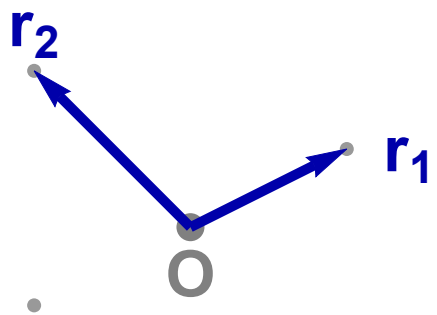

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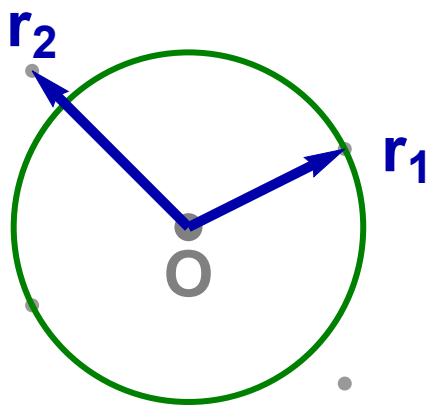

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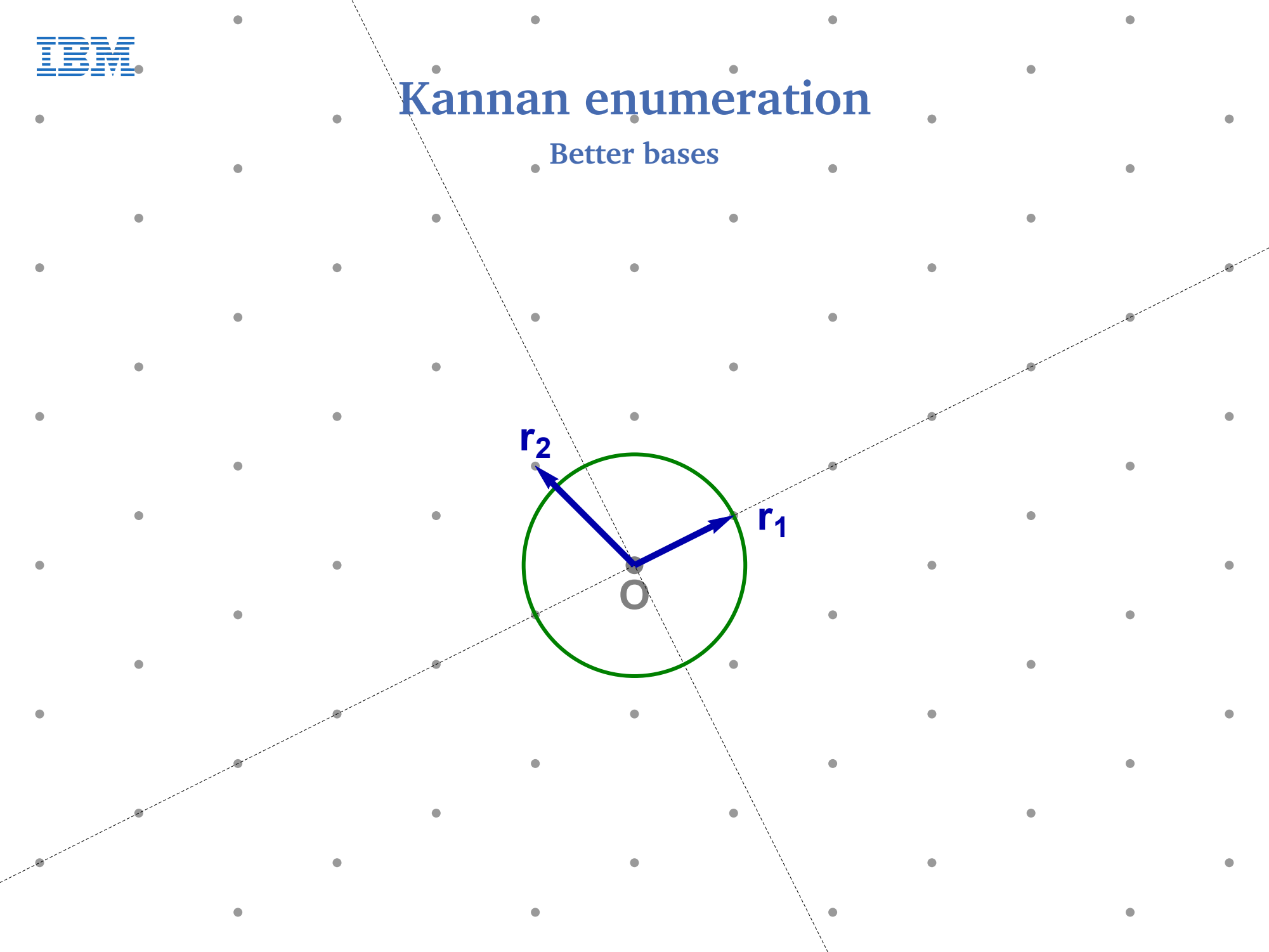

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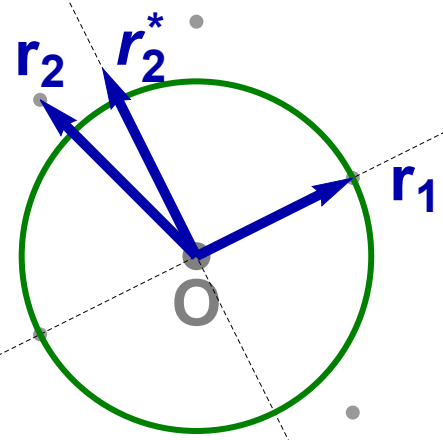

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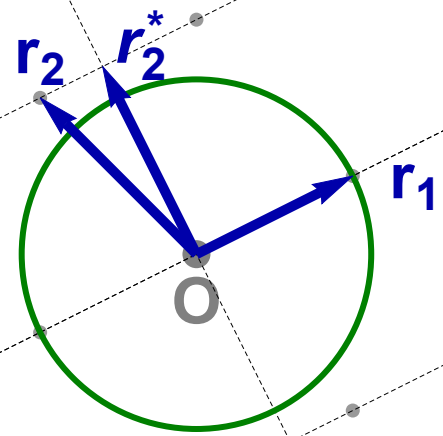

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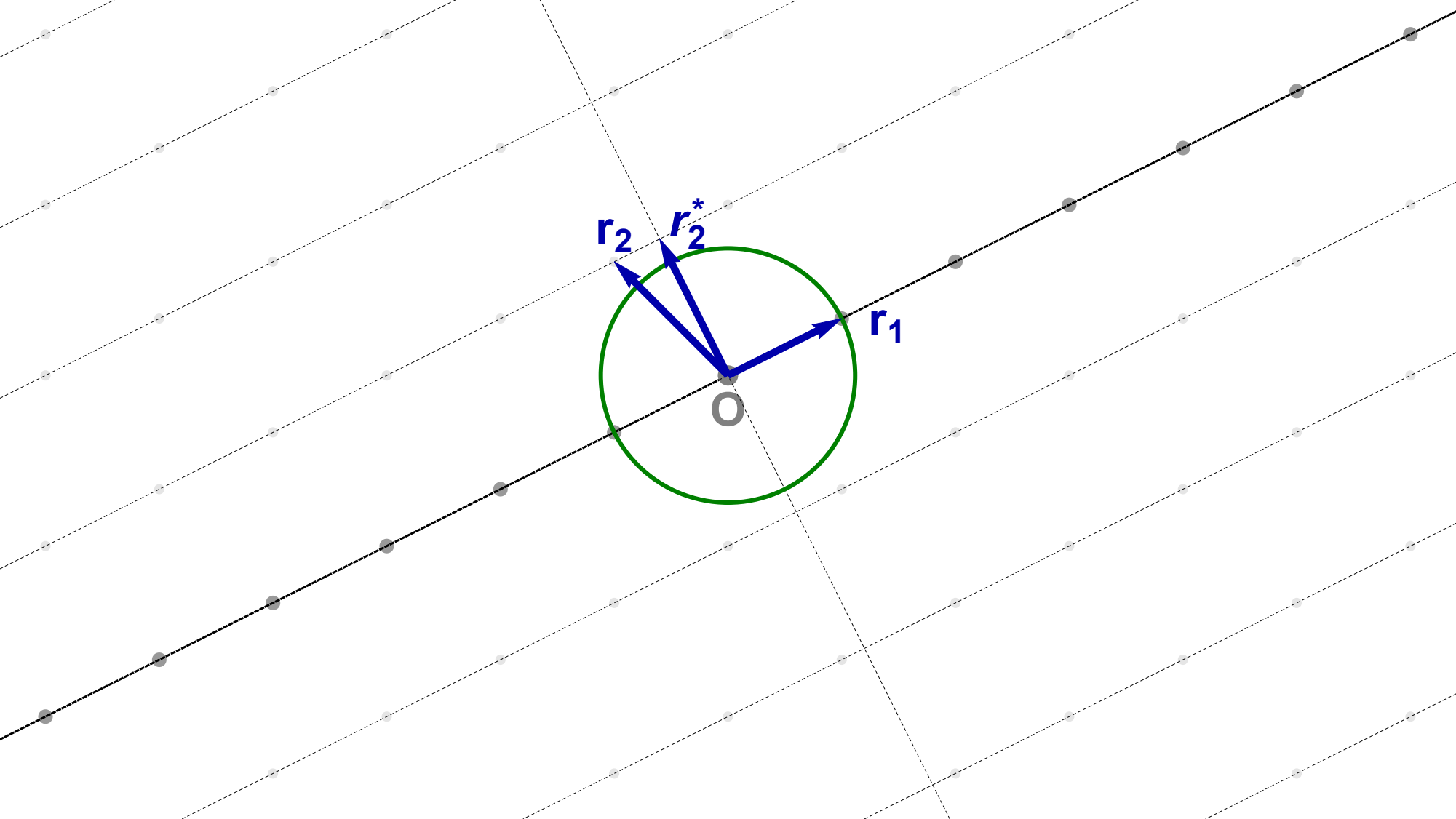

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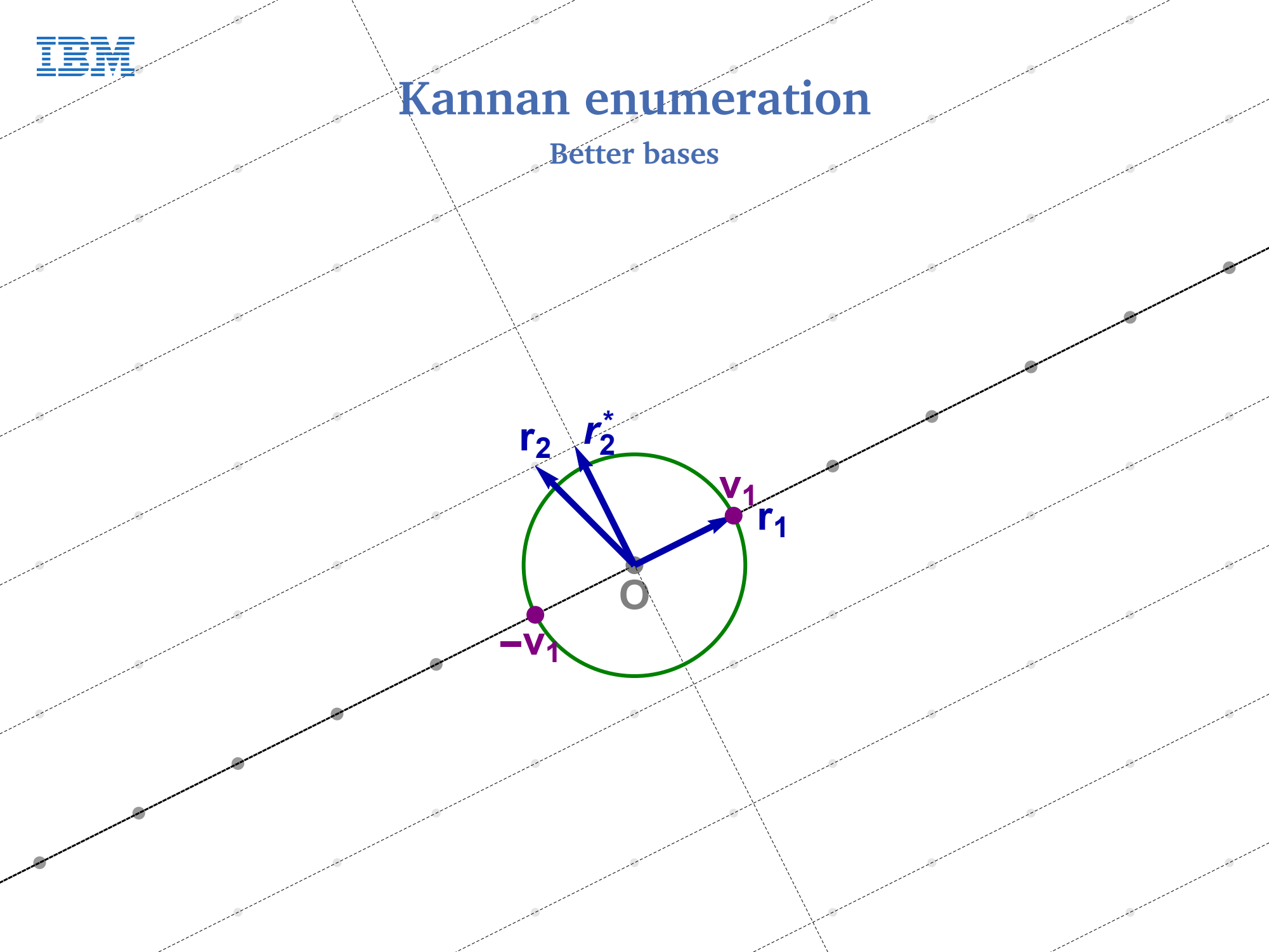

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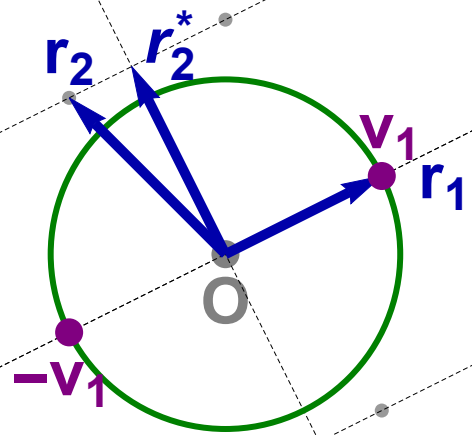

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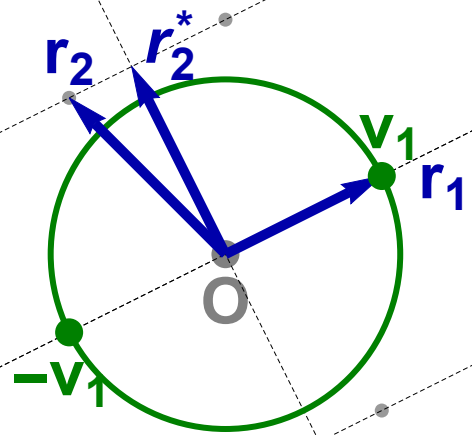
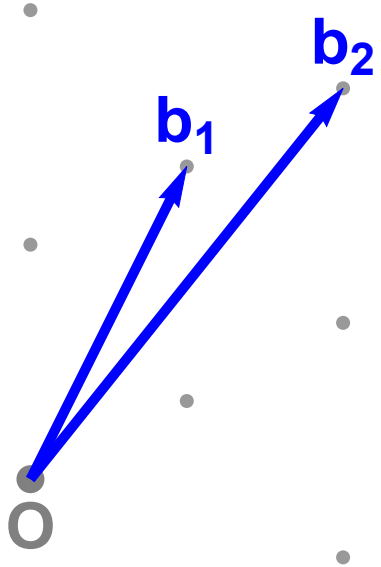


Pruned enumeration

Reducing the search space



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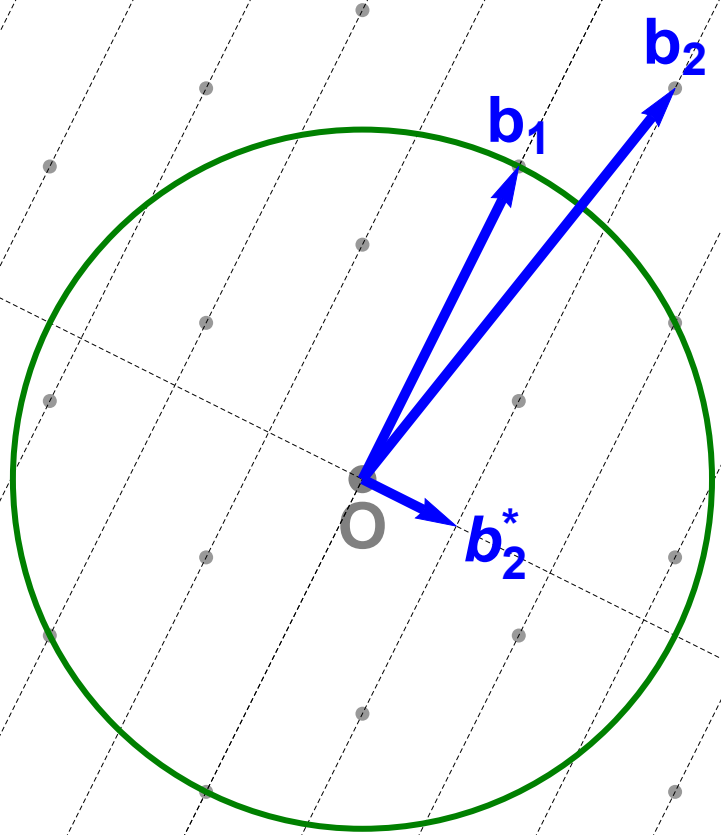

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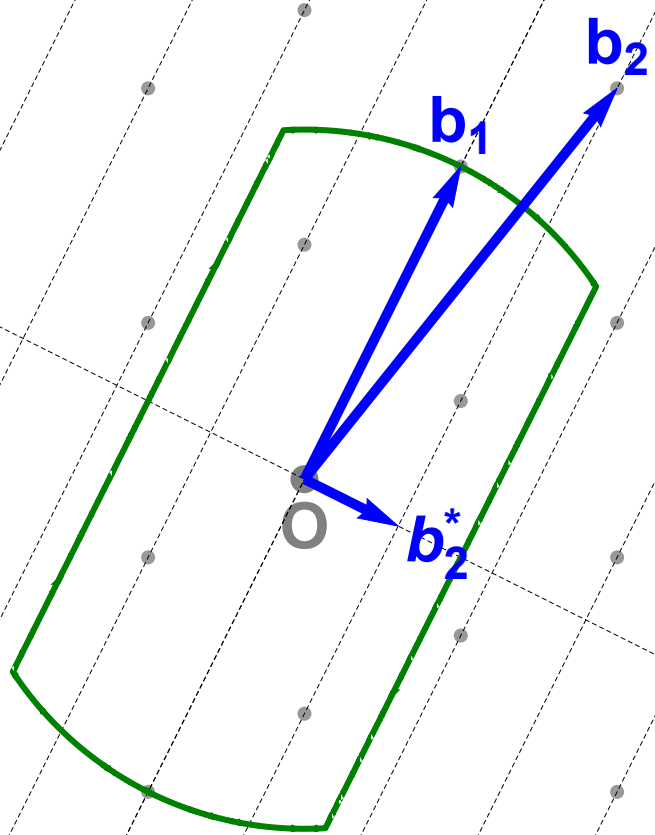

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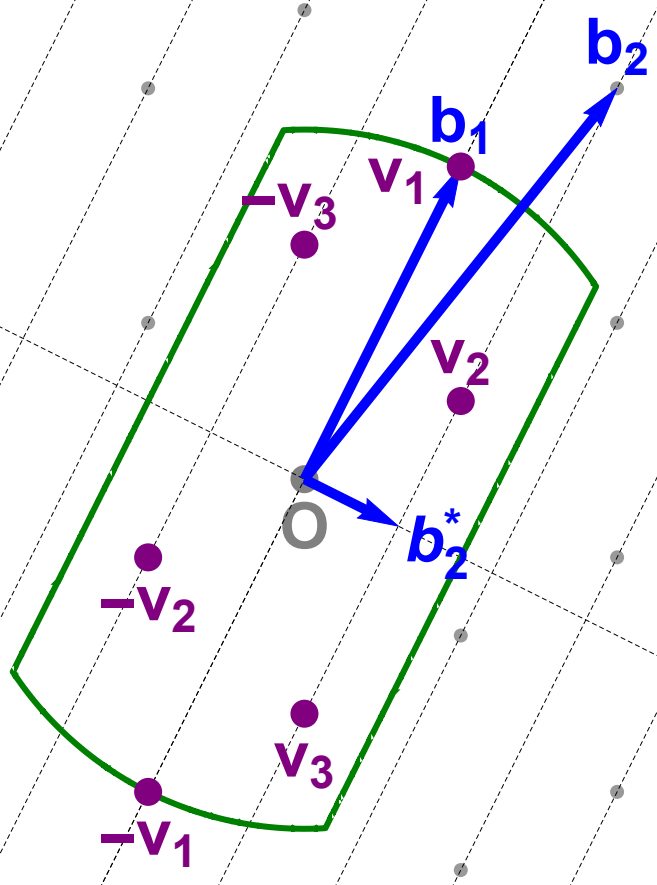

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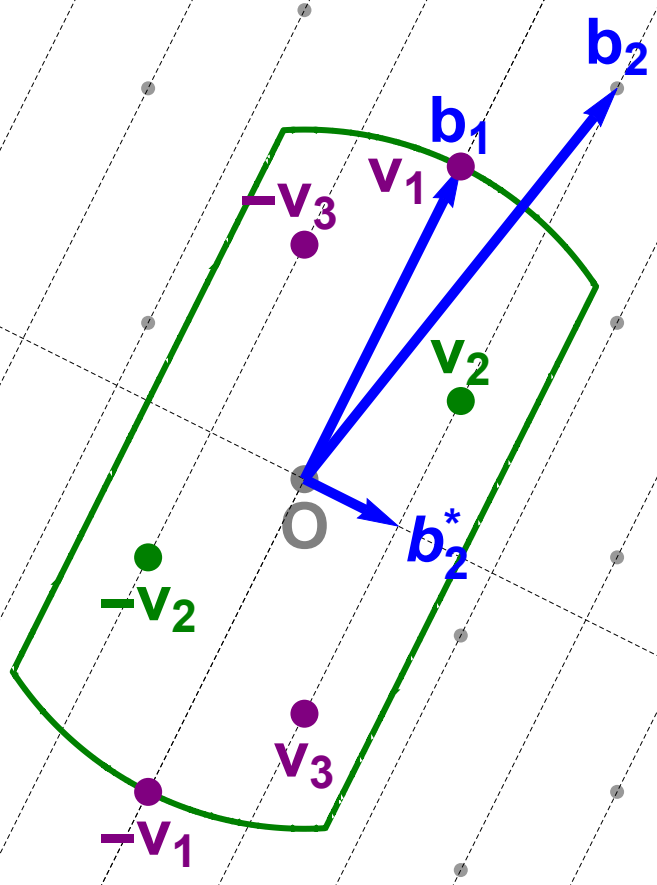

Exact SVP algorithms

Lattice enumeration

- Memory efficient
- Fincke-Pohst enumeration: $2^{O(n^2)}$ time
- Kannan enumeration: $2^{O(n \log n)}$ time
- Practical speedups with pruning heuristics

Nguyen-Vidick sieve

1. Sample a list L of random lattice vectors

Nguyen-Vidick sieve

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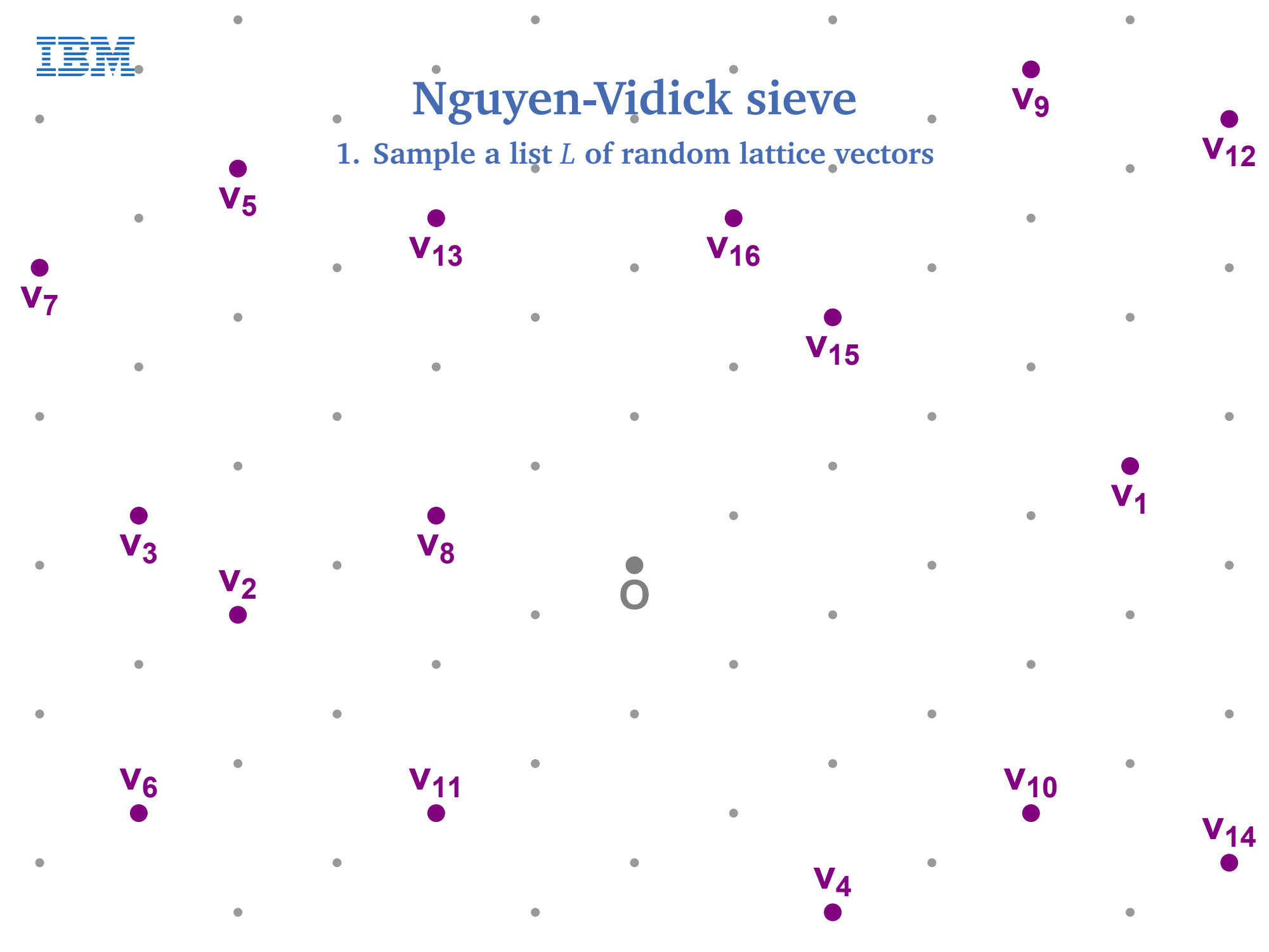

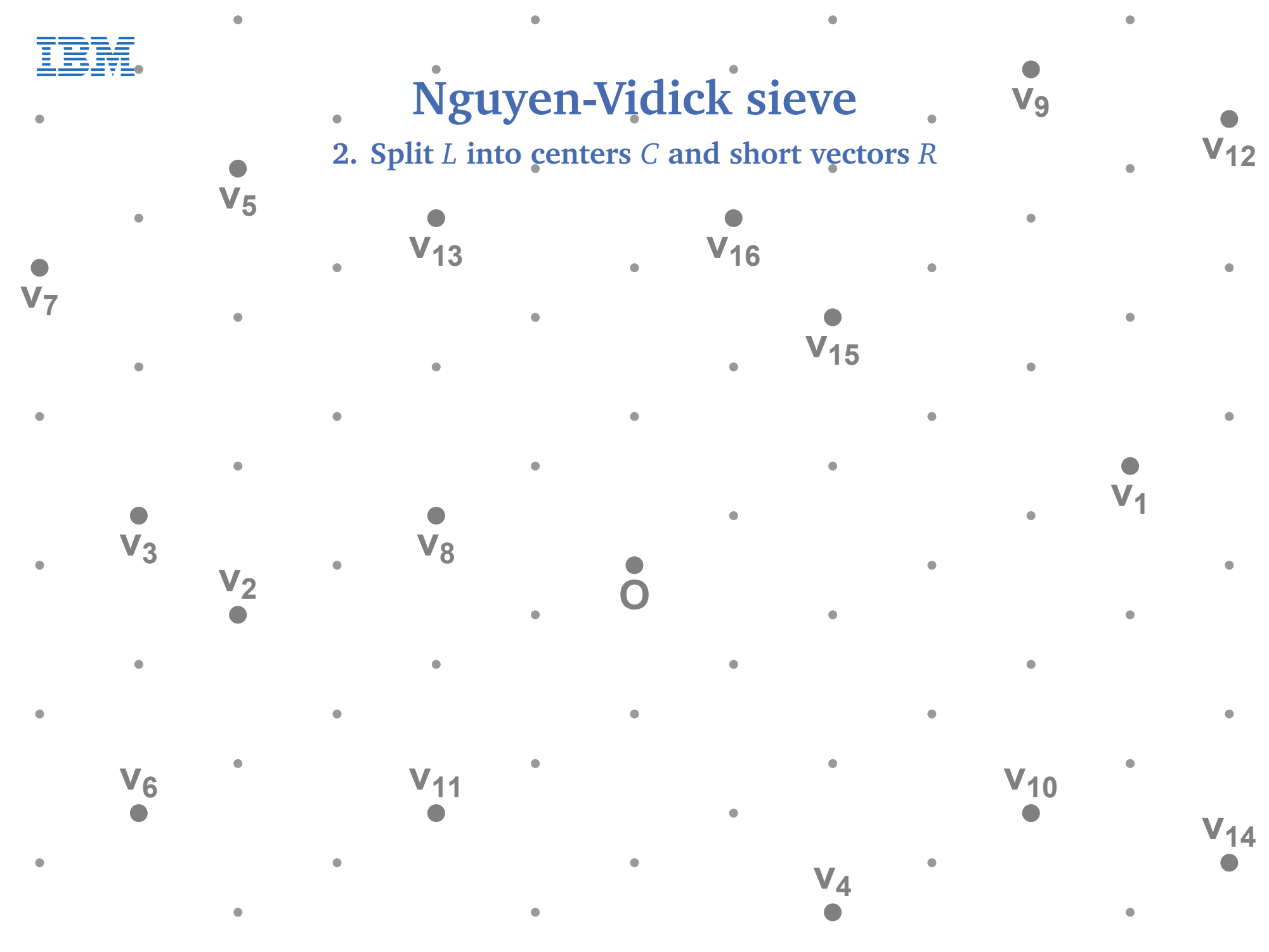
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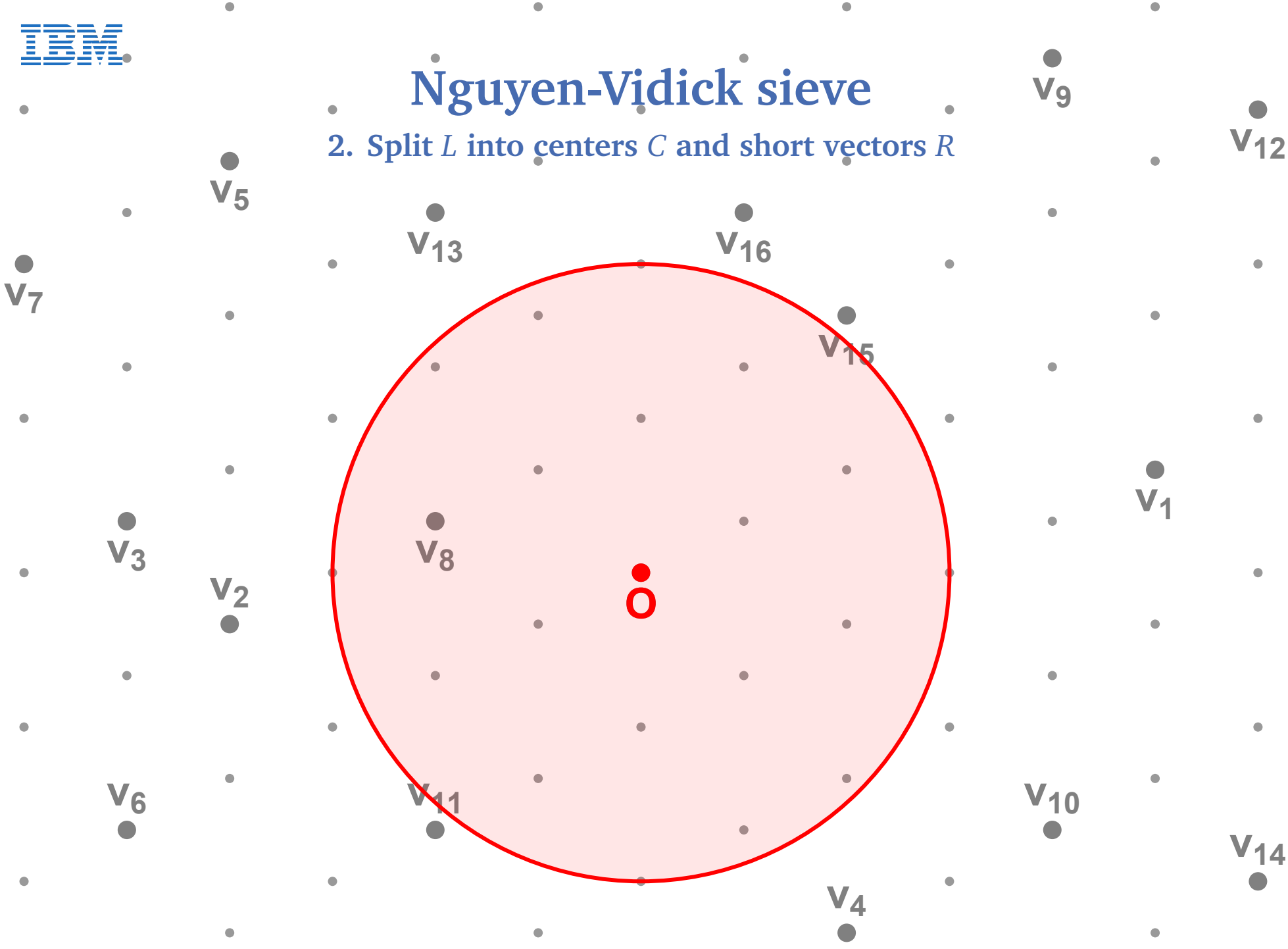
Nguyen-Vidick sieve
2. Split L into centers C and short vectors R
v_9
v_{12}
v_5
v_{13}
v_{16}
v_7
v_{15}
v_1
v_3
v_8
v_2
O
v_6
v_{11}
v_{10}
v_{14}
v_4

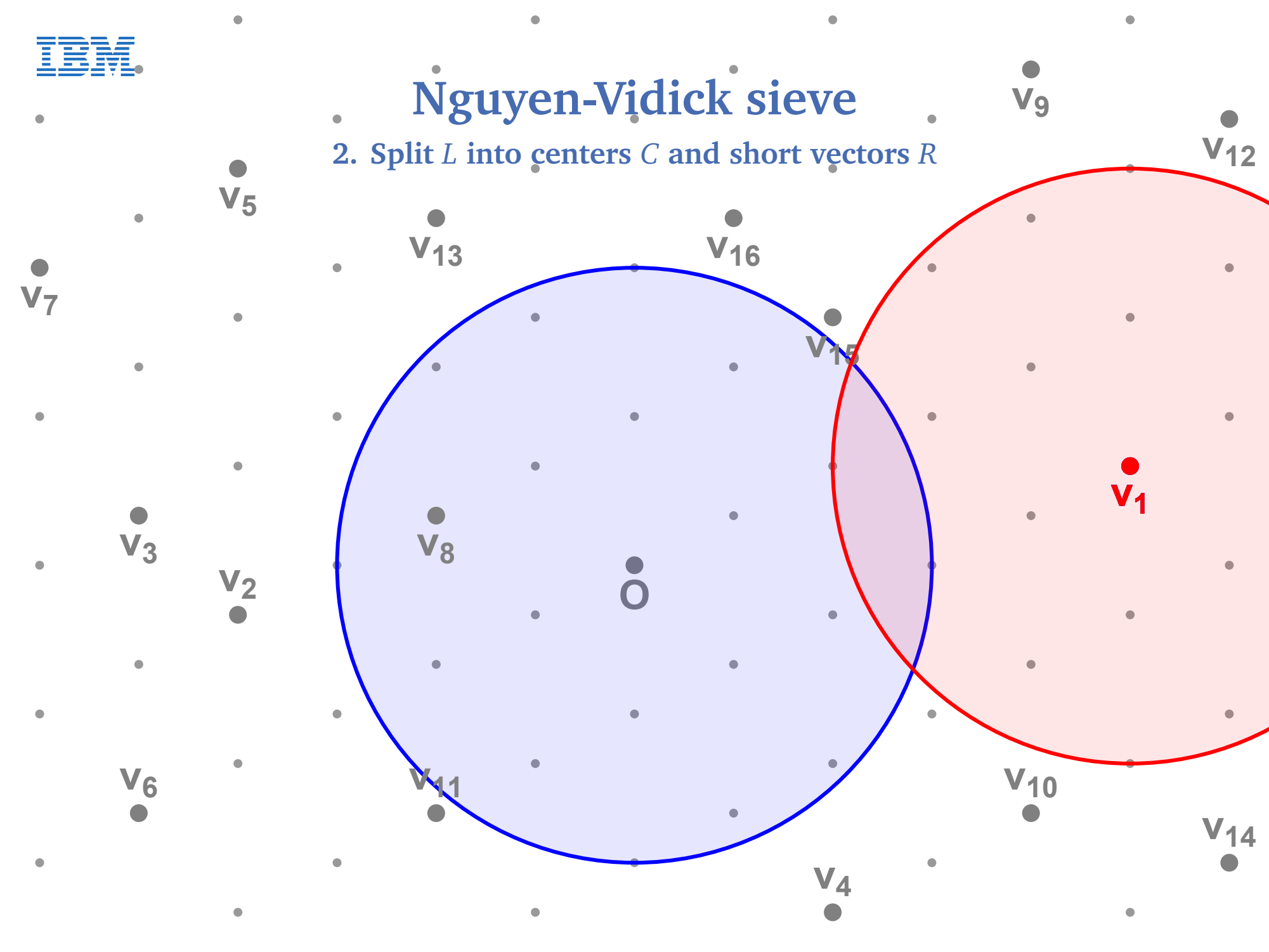
Nguyen-Vidick sieve

2. Split L into centers C and short vectors R



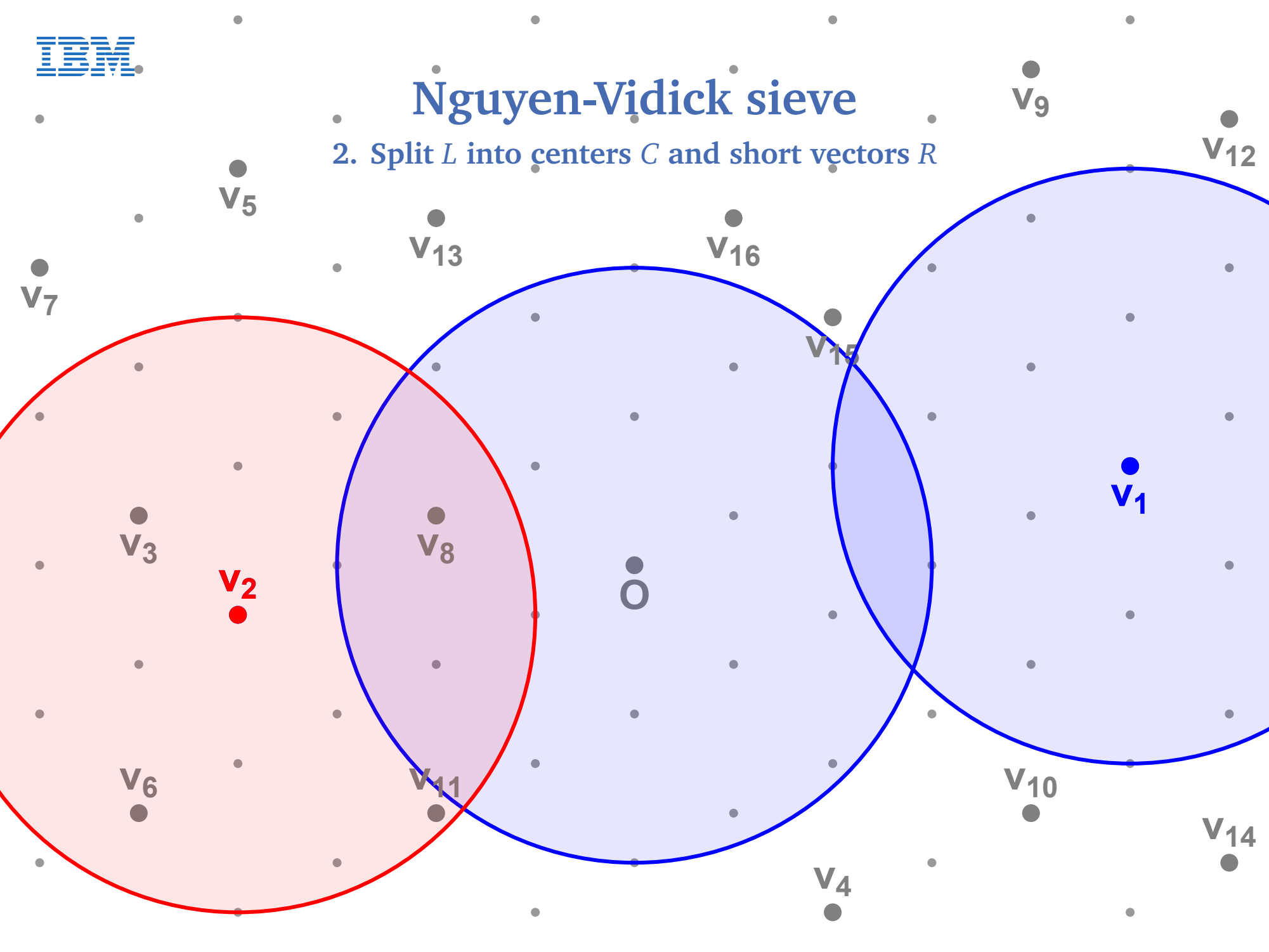
Nguyen-Vidick sieve

2. Split L into centers C and short vectors R



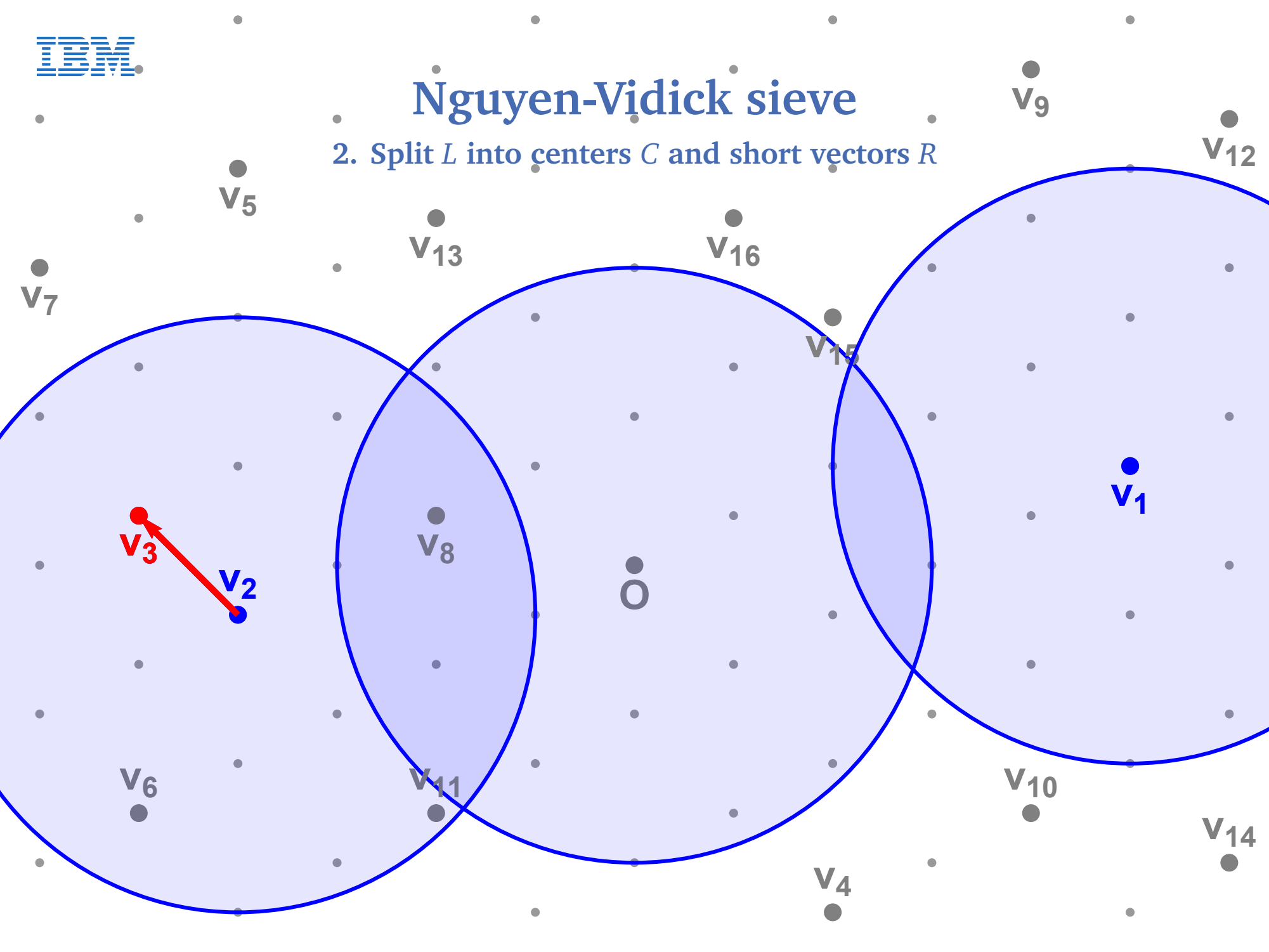
Nguyen-Vidick sieve

2. Split L into centers C and short vectors R



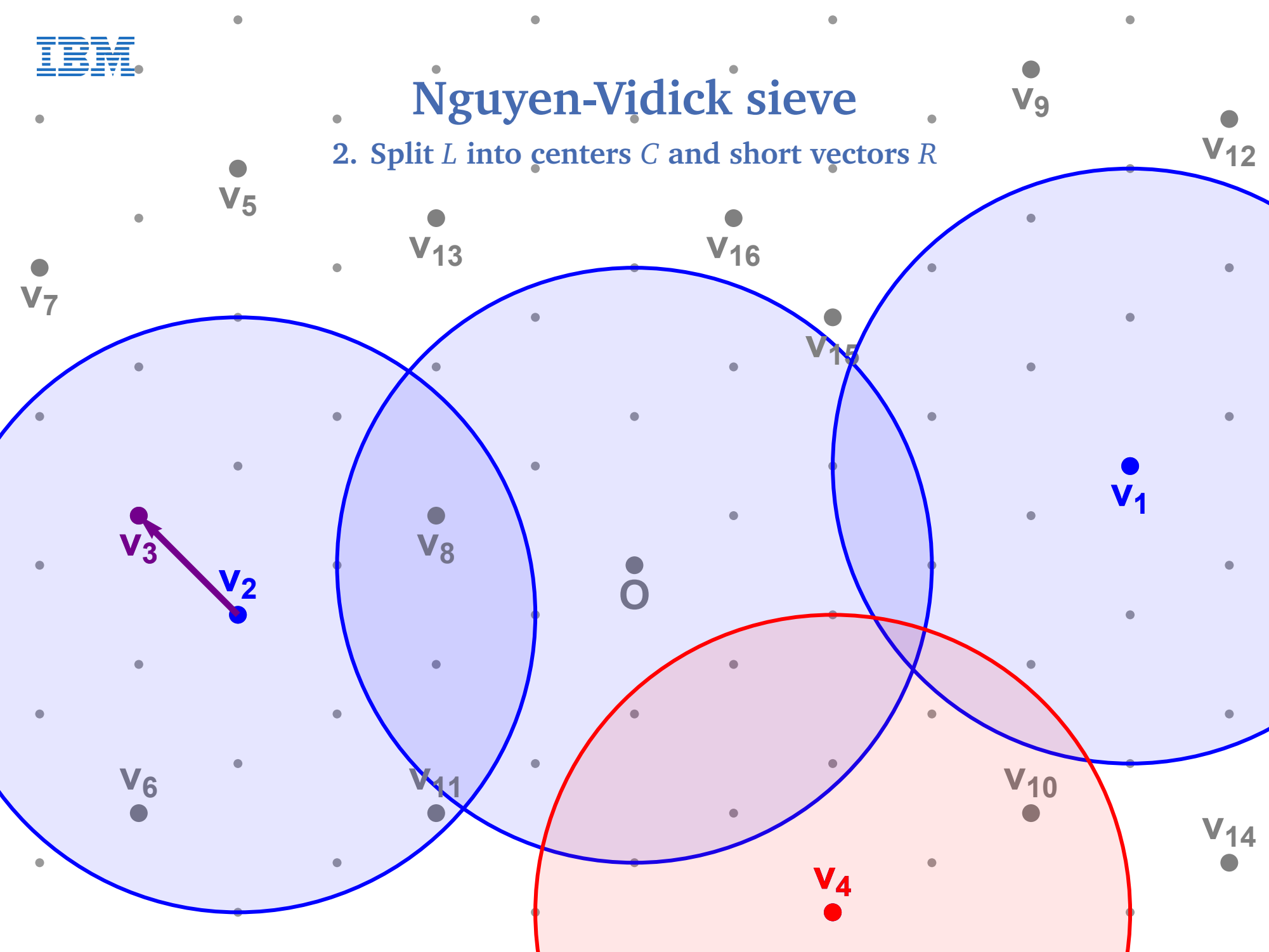
Nguyen-Vidick sieve

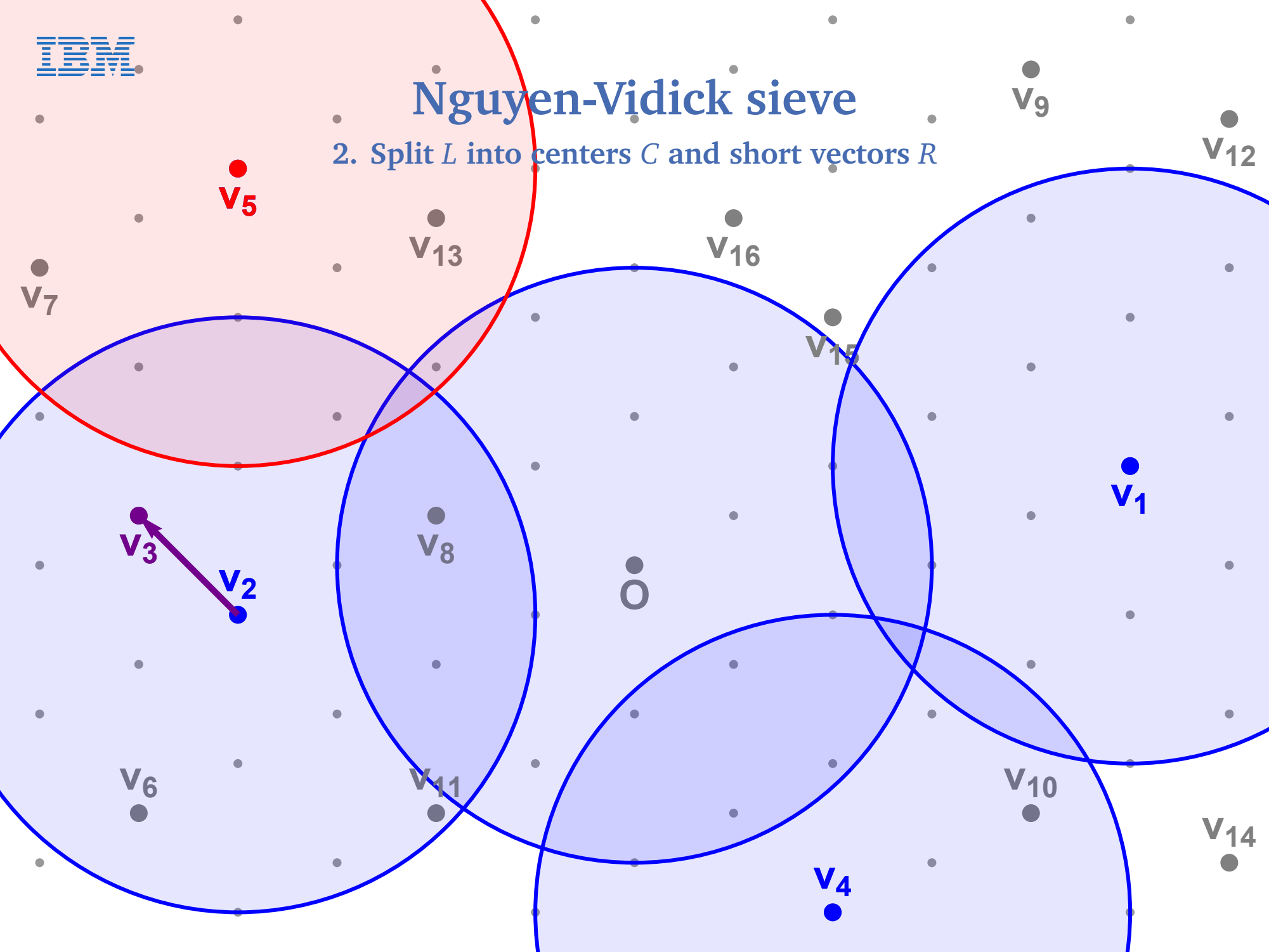
2. Split L into centers C and short vectors R



Nguyen-Vidick sieve

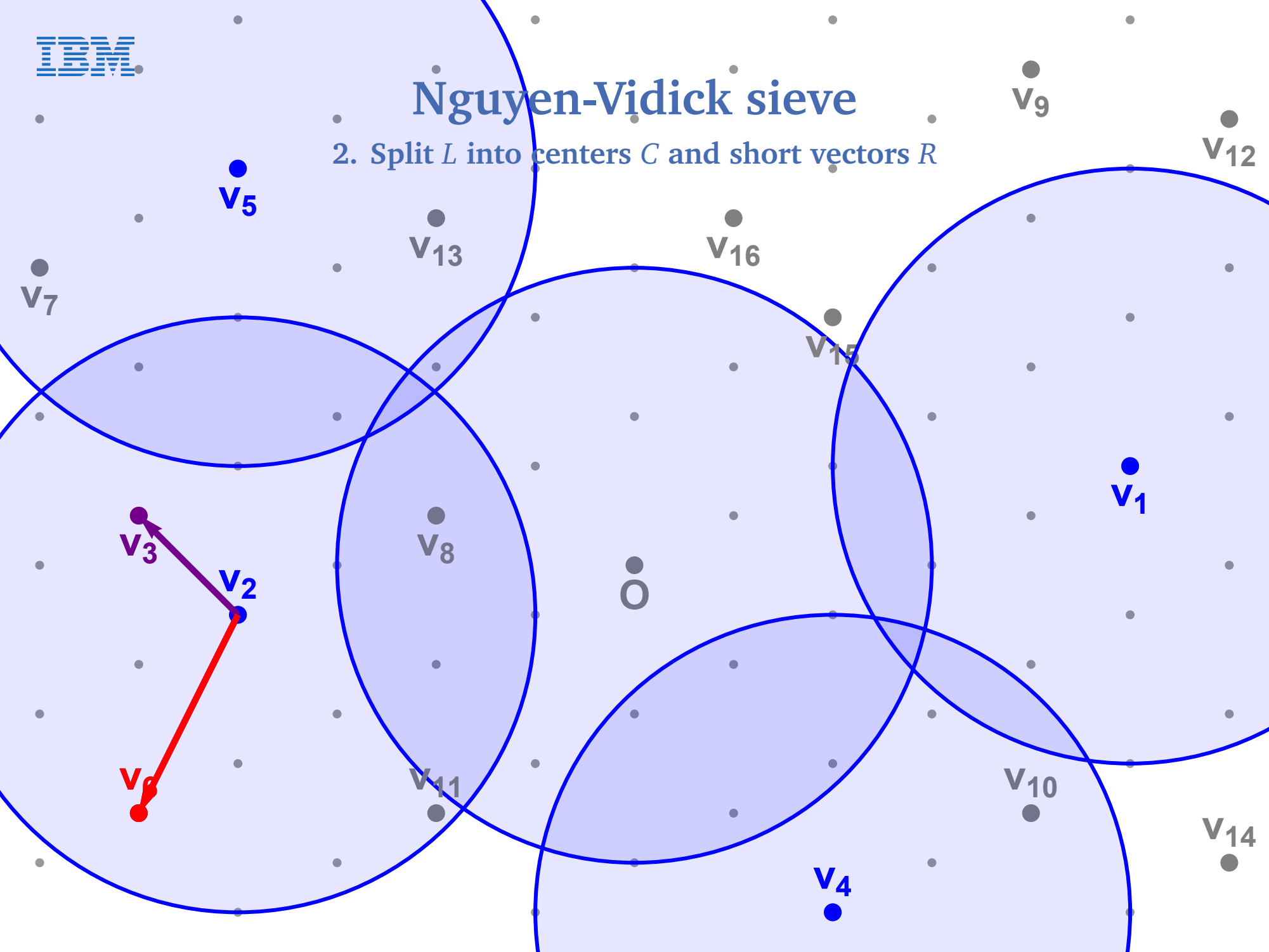
2. Split L into centers C and short vectors R




IBM
Nguyen-Vidick sieve
2. Split L into centers C and short vectors R
v_5
v_{13}
v_7
v_{16}
v_9
v_{12}
v_{15}
v_1
v_3
v_2
v_8
O
v_6
v_{11}
v_{10}
v_{14}
v_4

Nguyen-Vidick sieve

2. Split L into centers C and short vectors R



Nguyen-Vidick sieve

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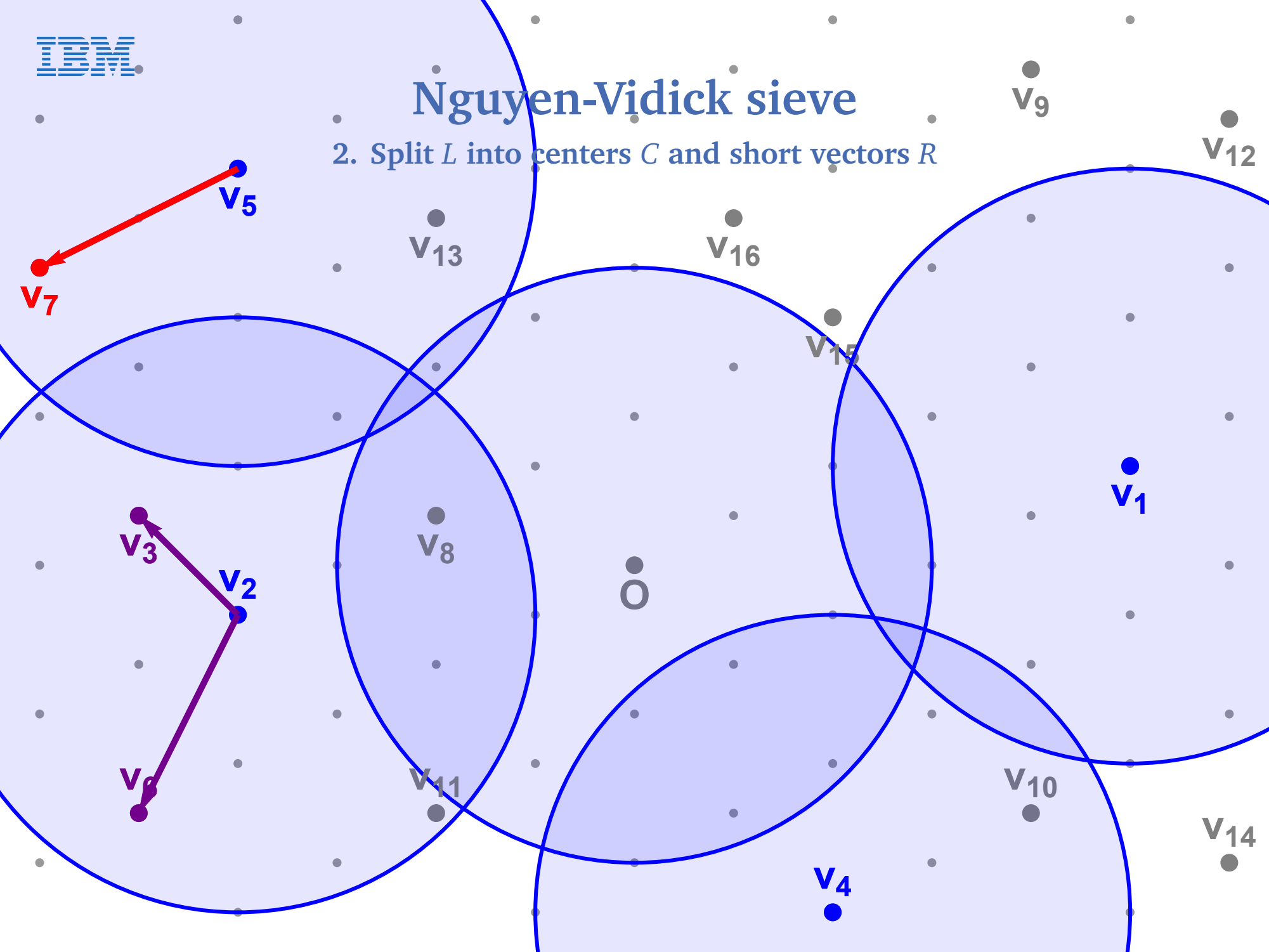

Nguyen-Vidick sieve

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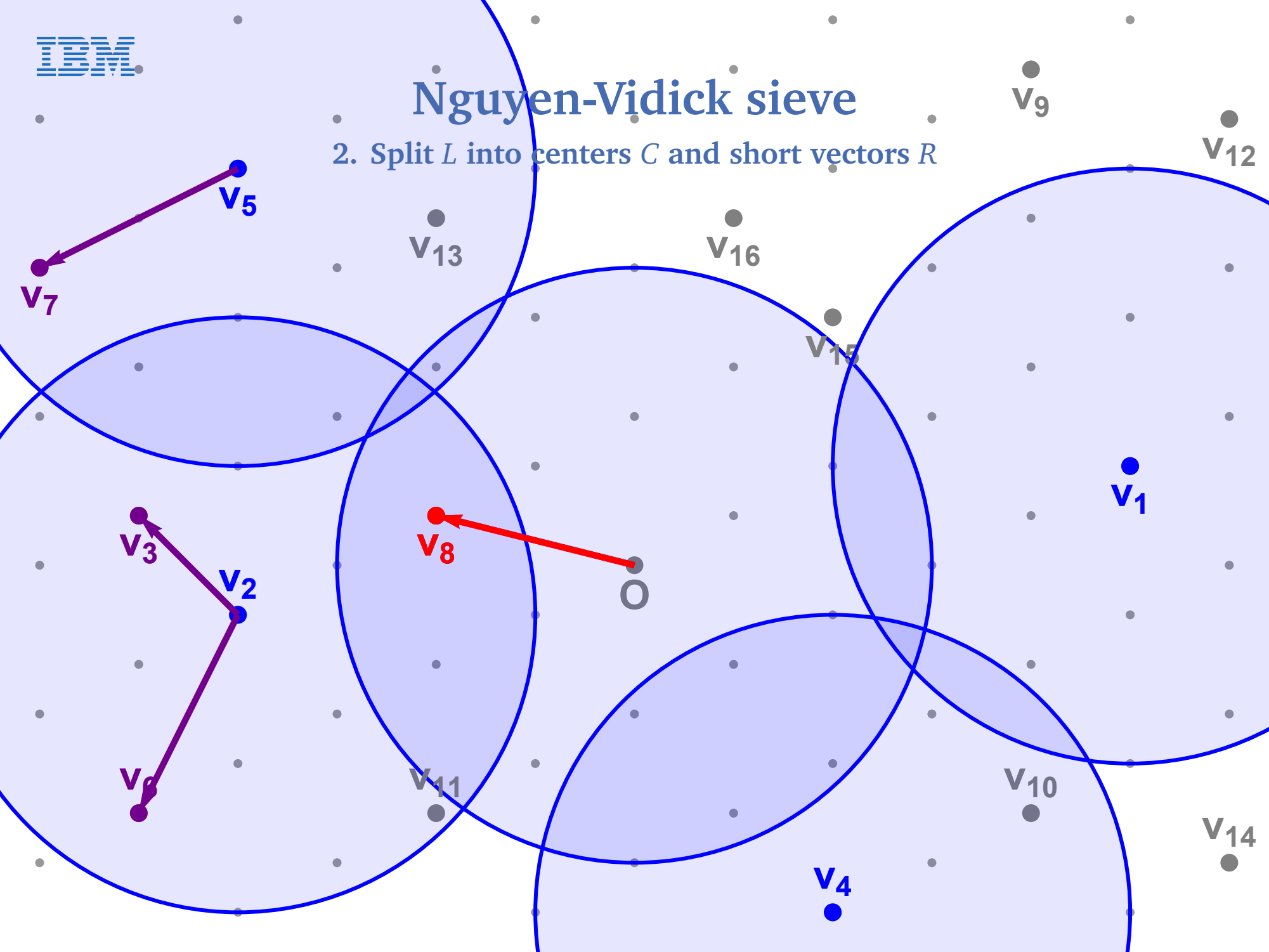

Nguyen-Vidick sieve

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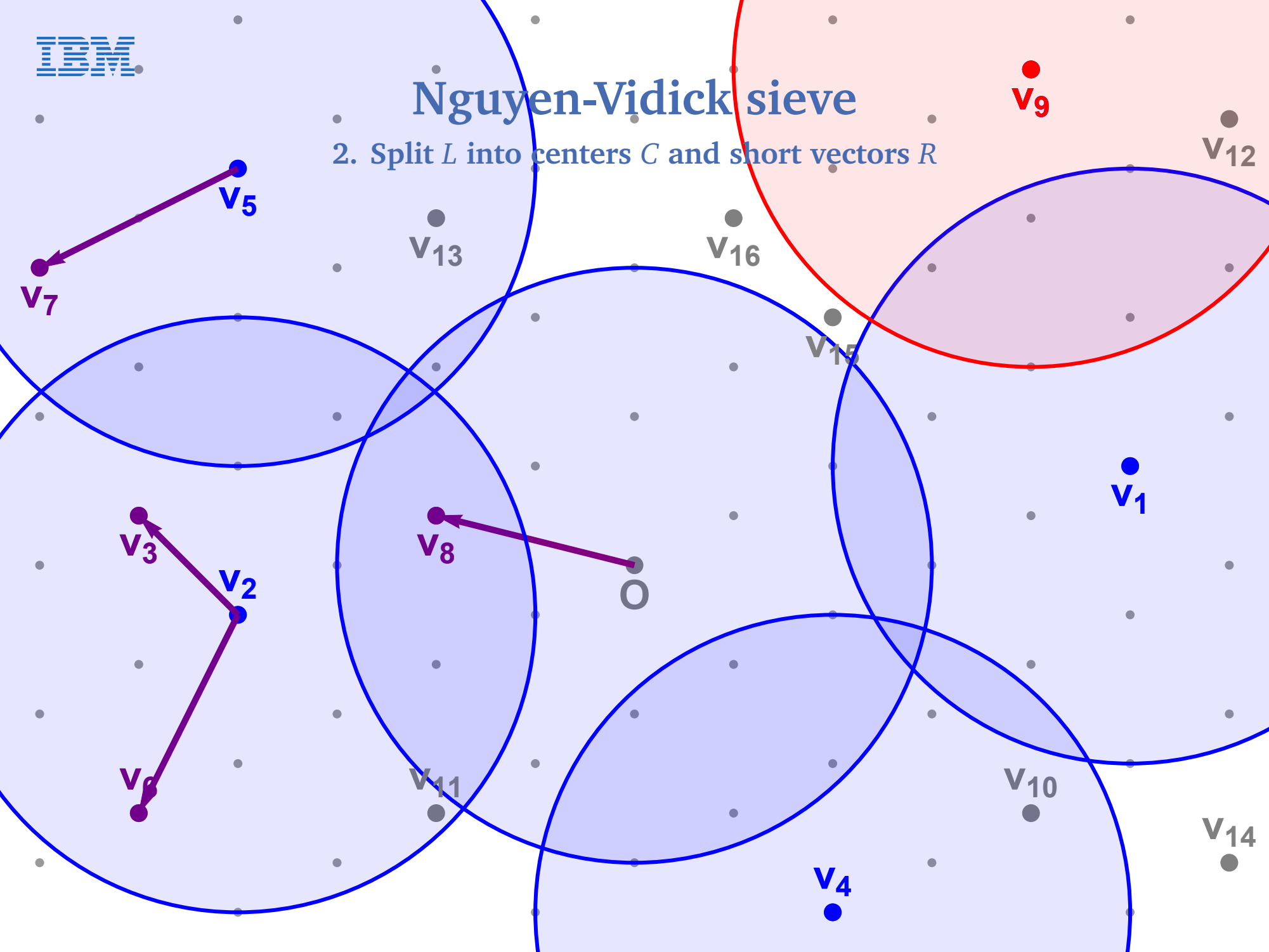

Nguyen-Vidick sieve

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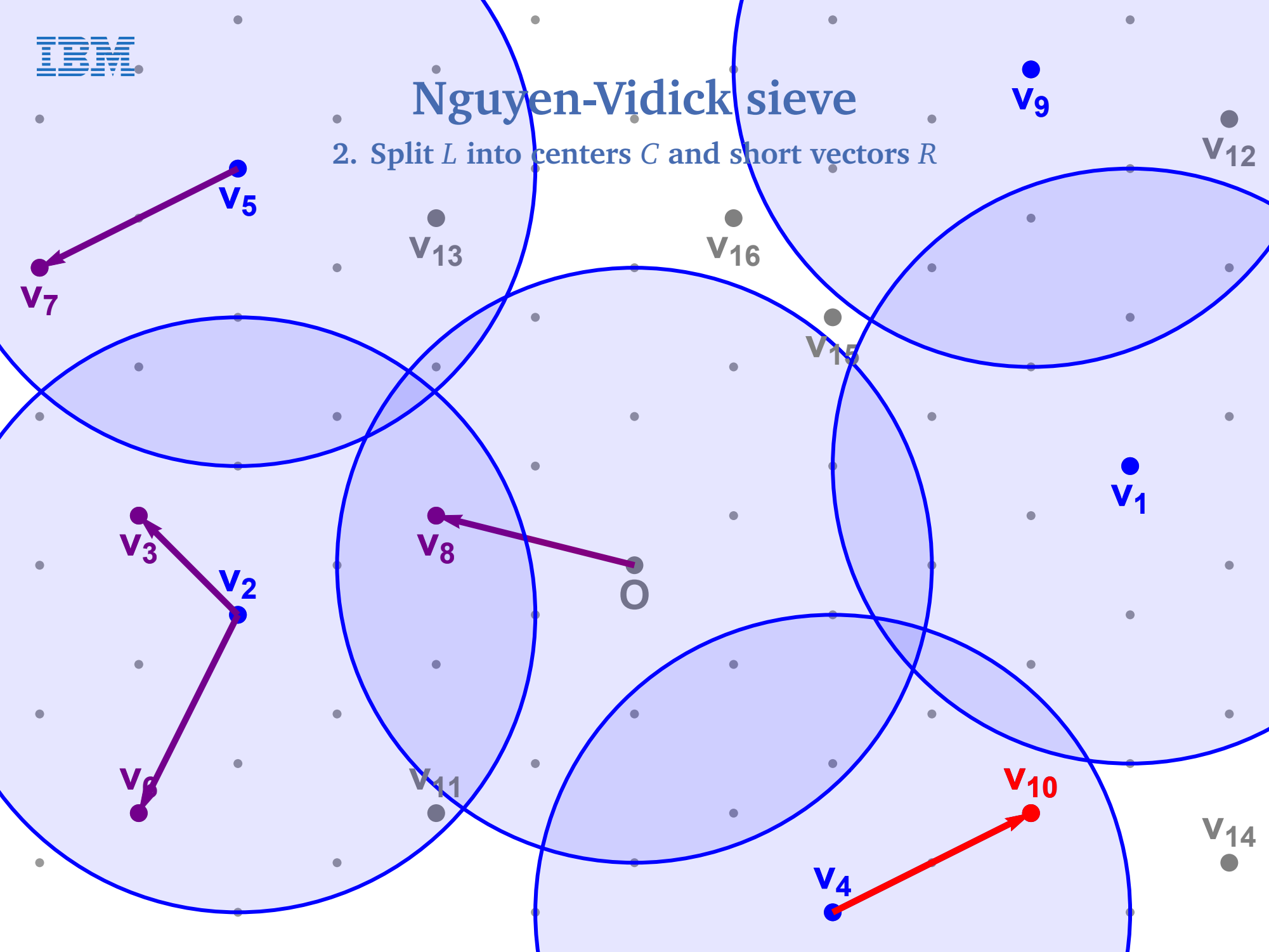

Nguyen-Vidick sieve

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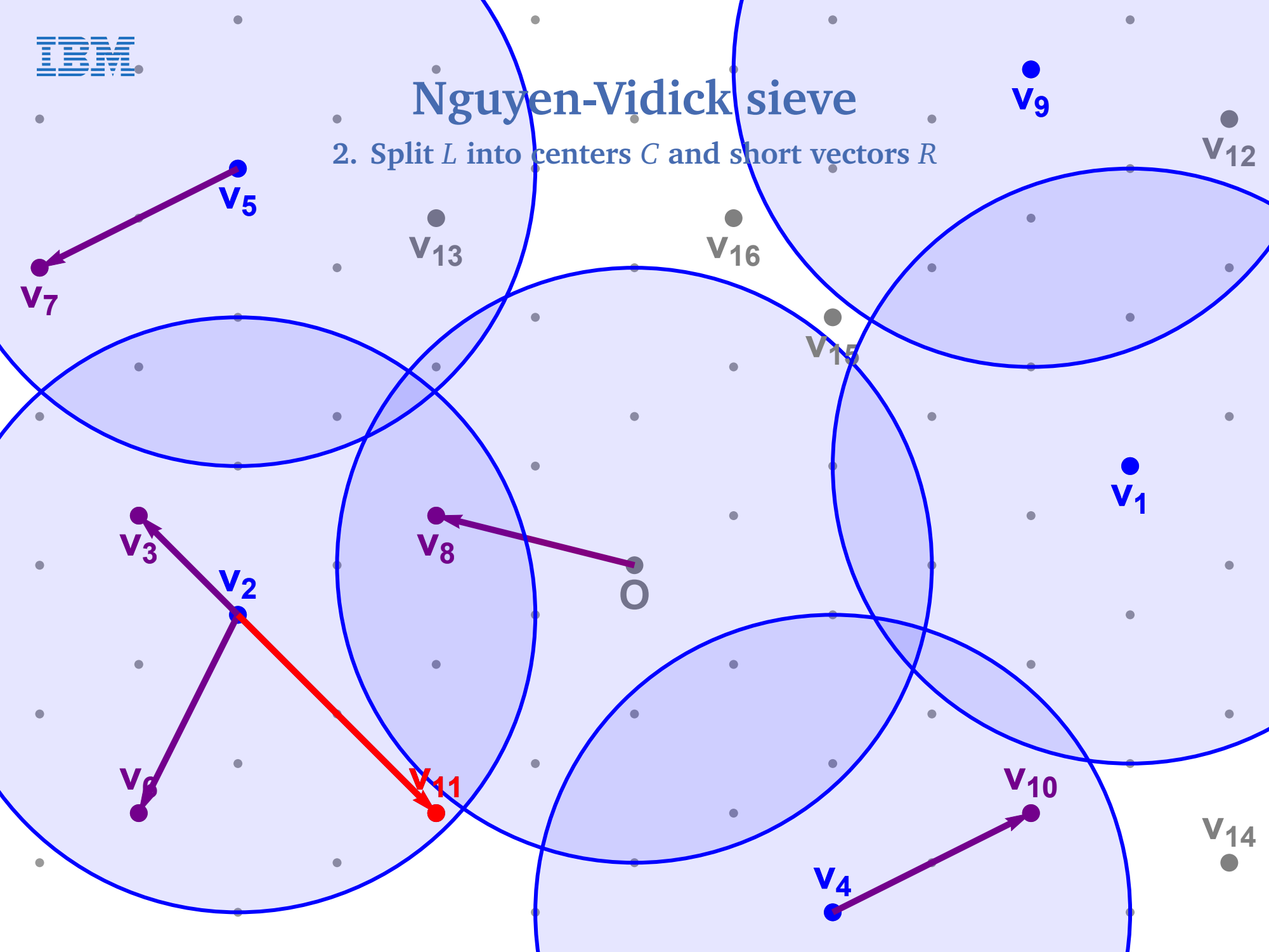

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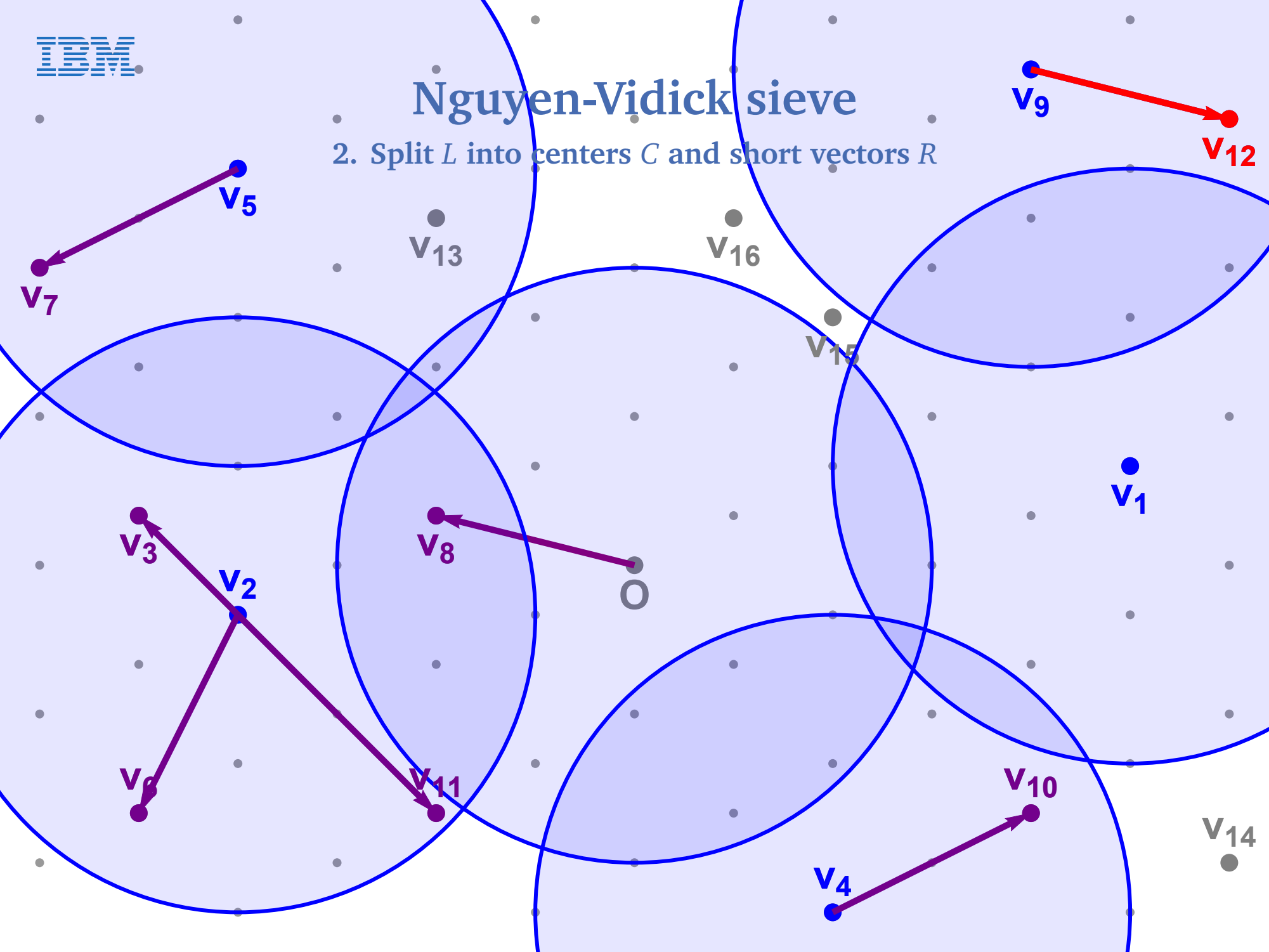

Nguyen-Vidick sieve

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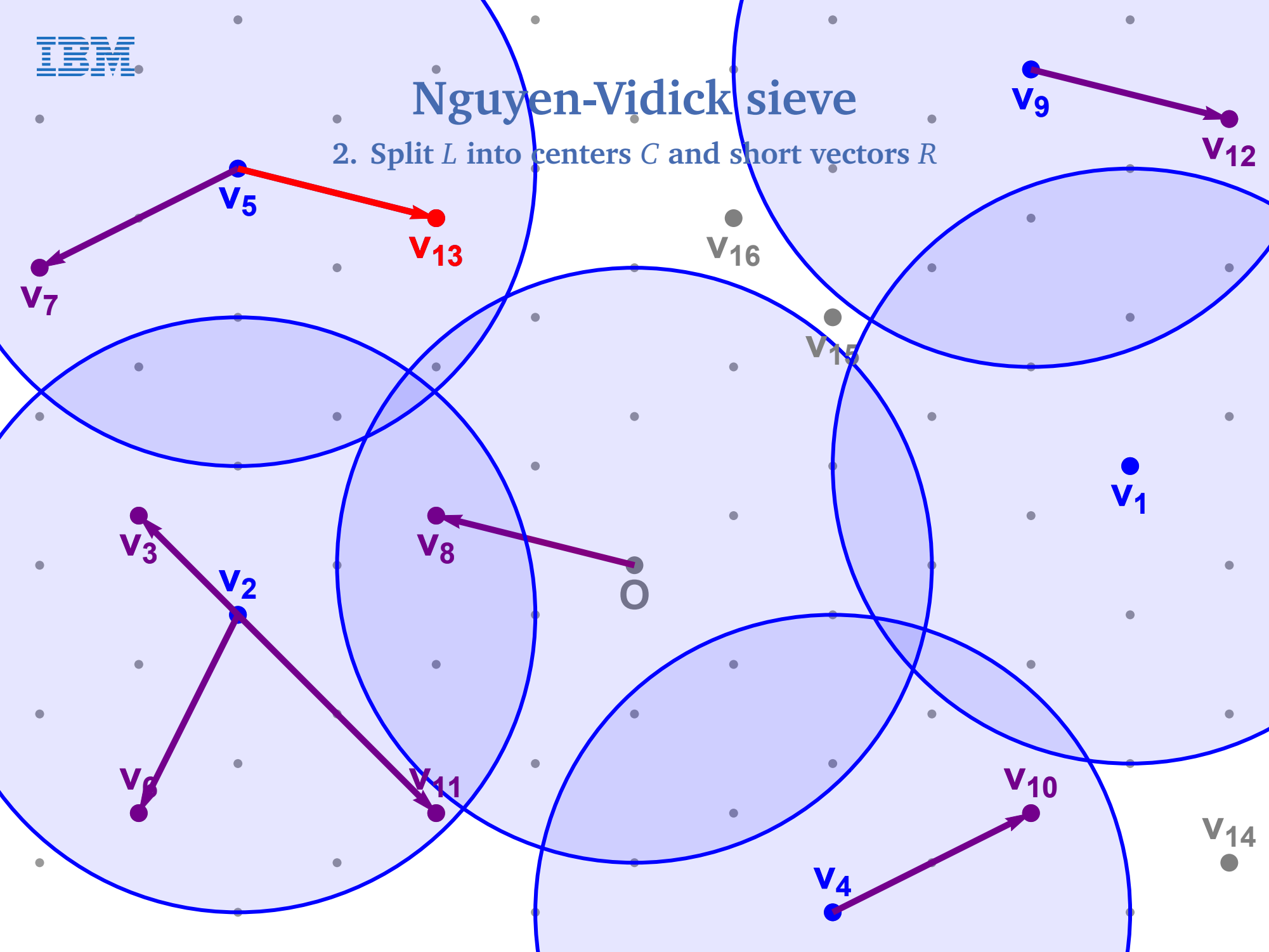

Nguyen-Vidick sieve

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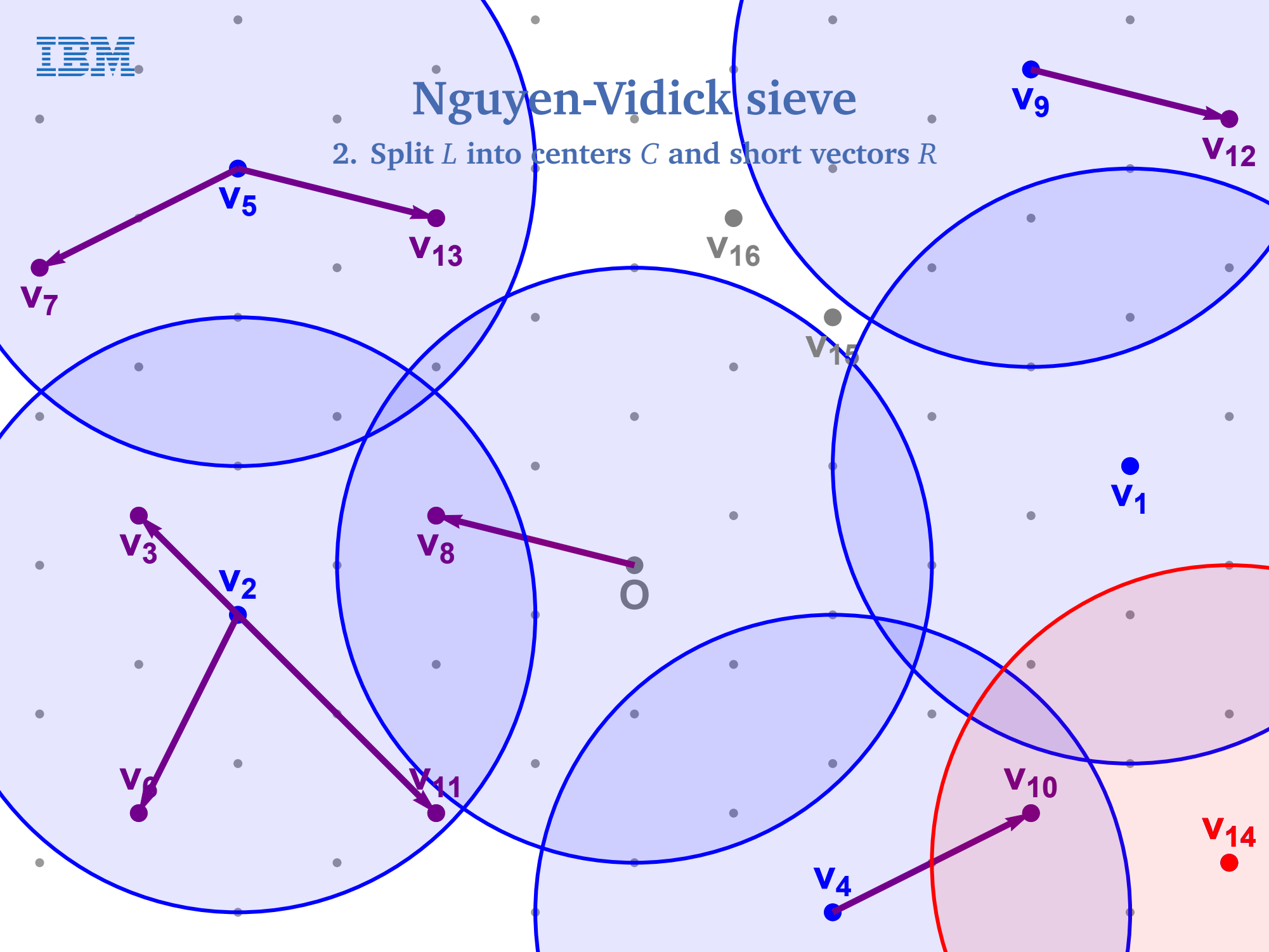

Nguyen-Vidick sieve

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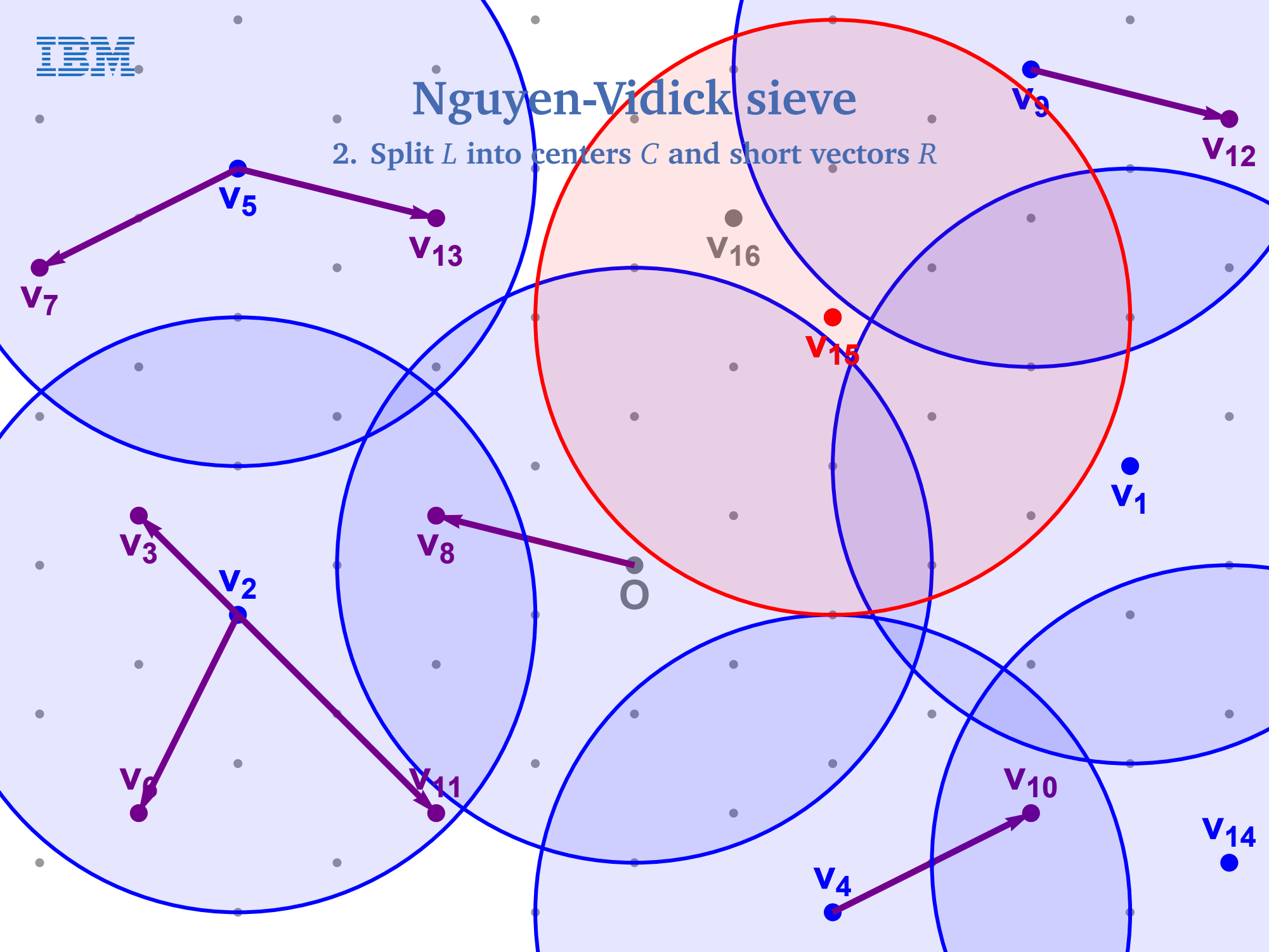

Nguyen-Vidick sieve

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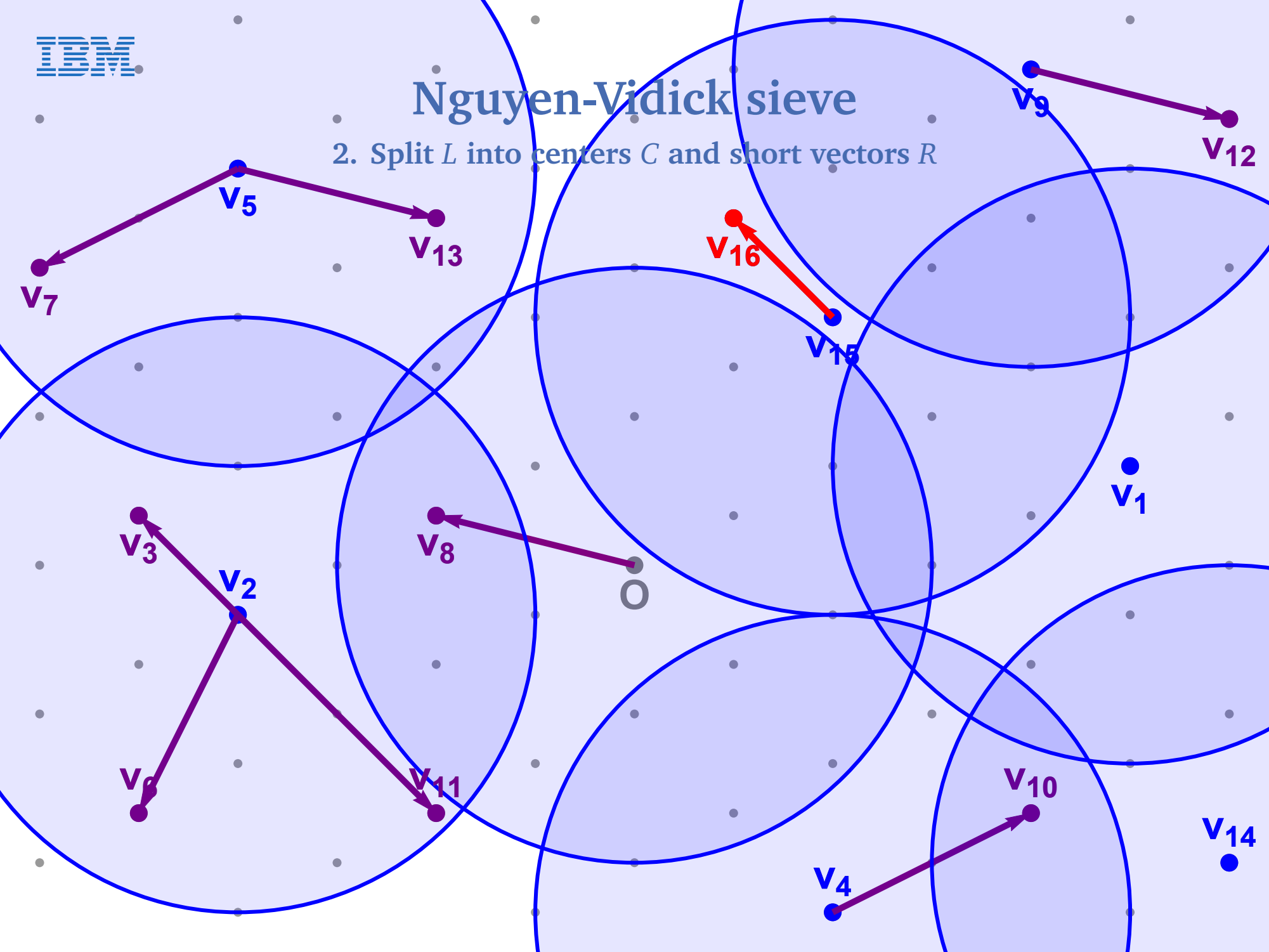

Nguyen-Vidick sieve

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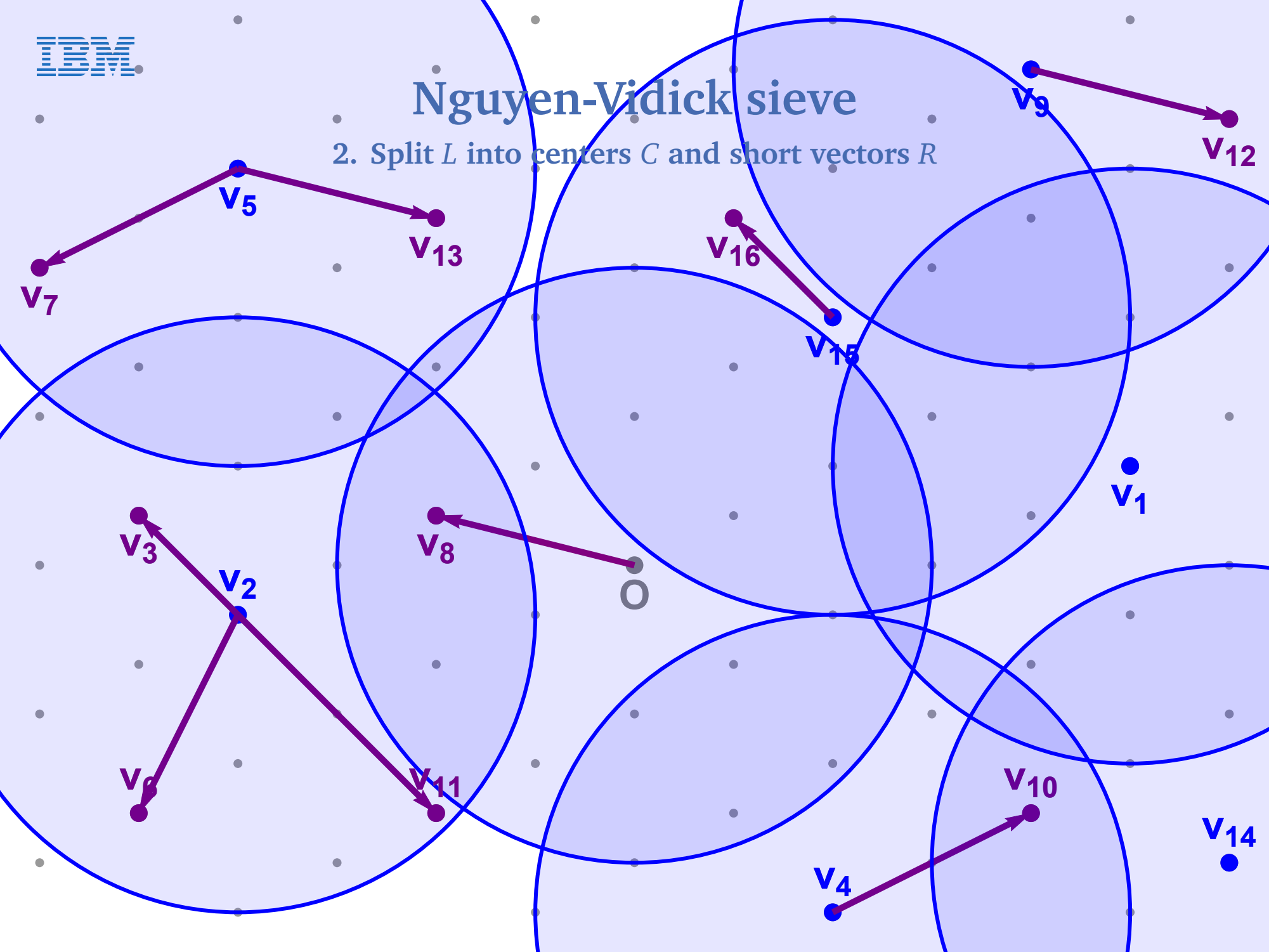
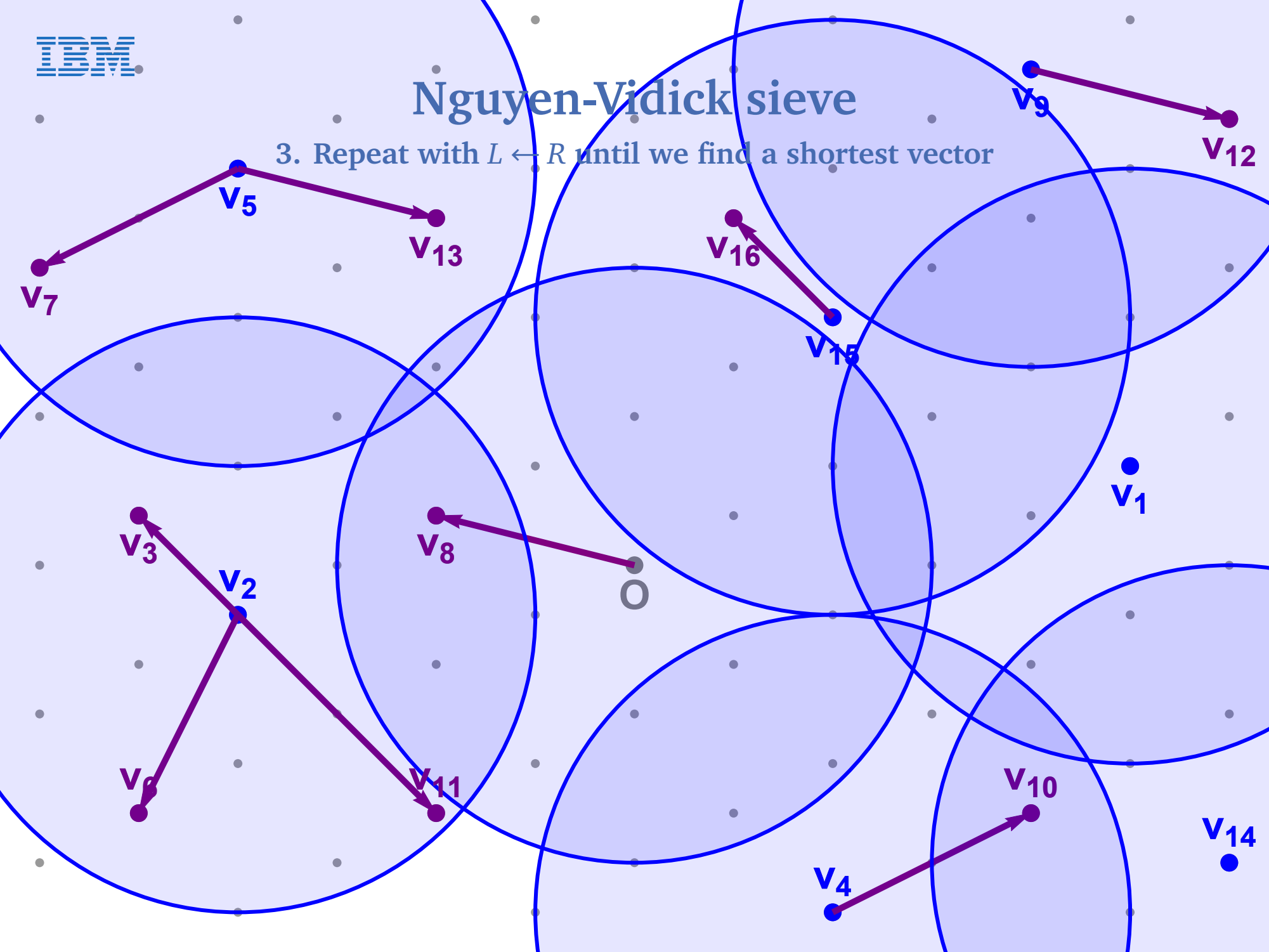


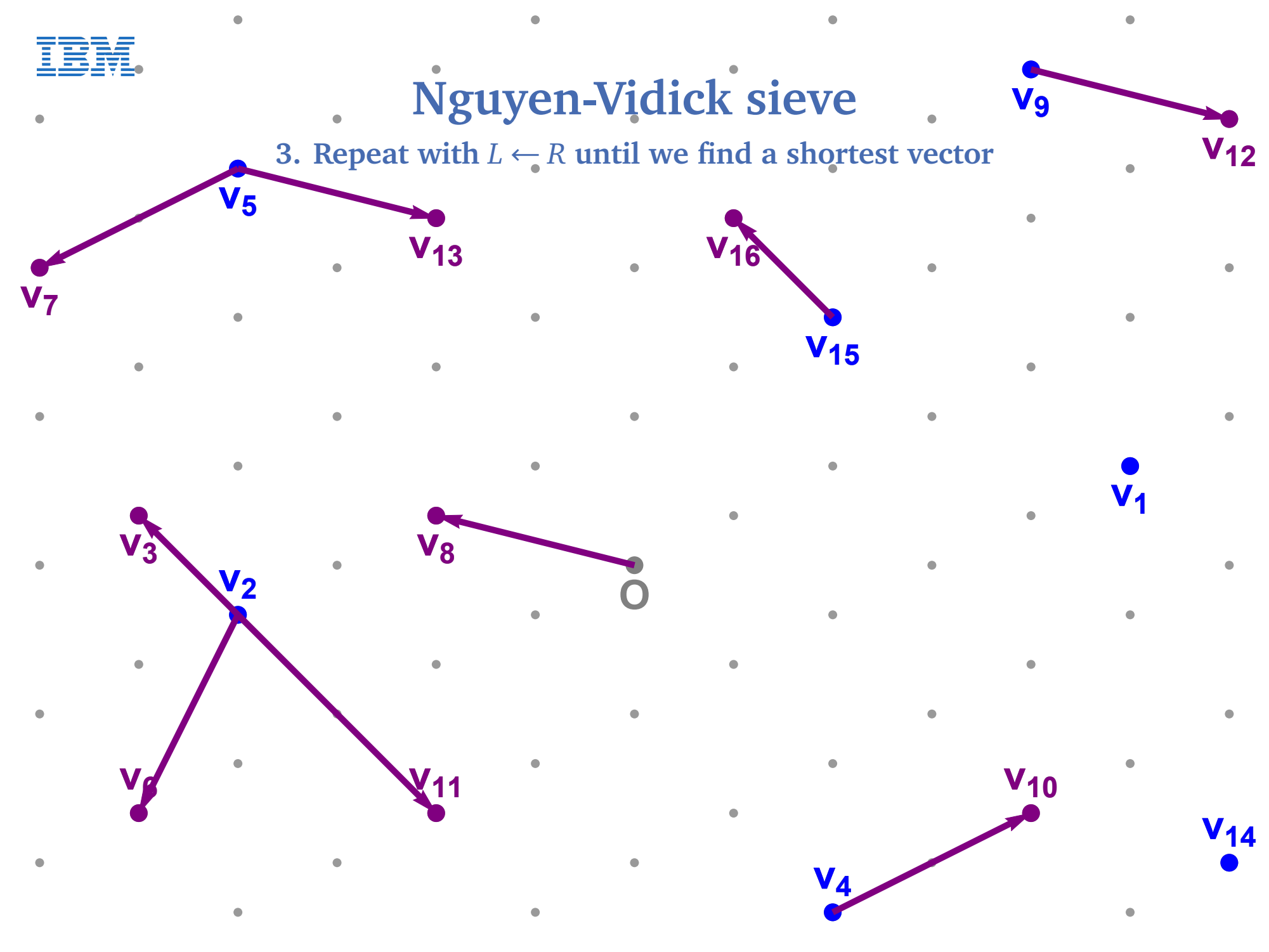
Nguyen-Vidick sieve

3. Repeat with $L \hookleftarrow R$ until we find a shortest vector



Nguyen-Vidick sieve

3. Repeat with $L \leftarrow R$ until we find a shortest vector



Nguyen-Vidick sieve

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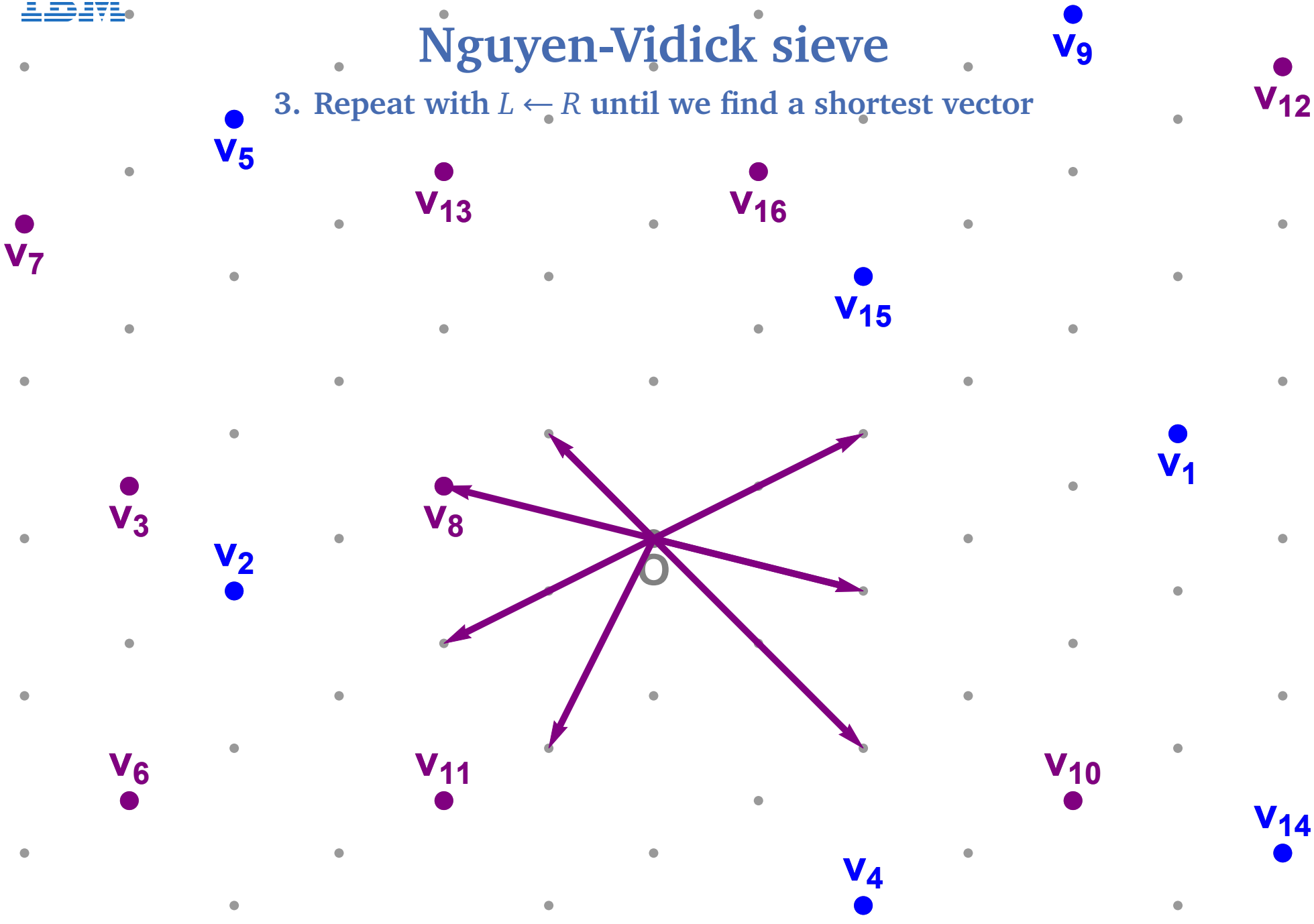

Nguyen-Vidick sieve

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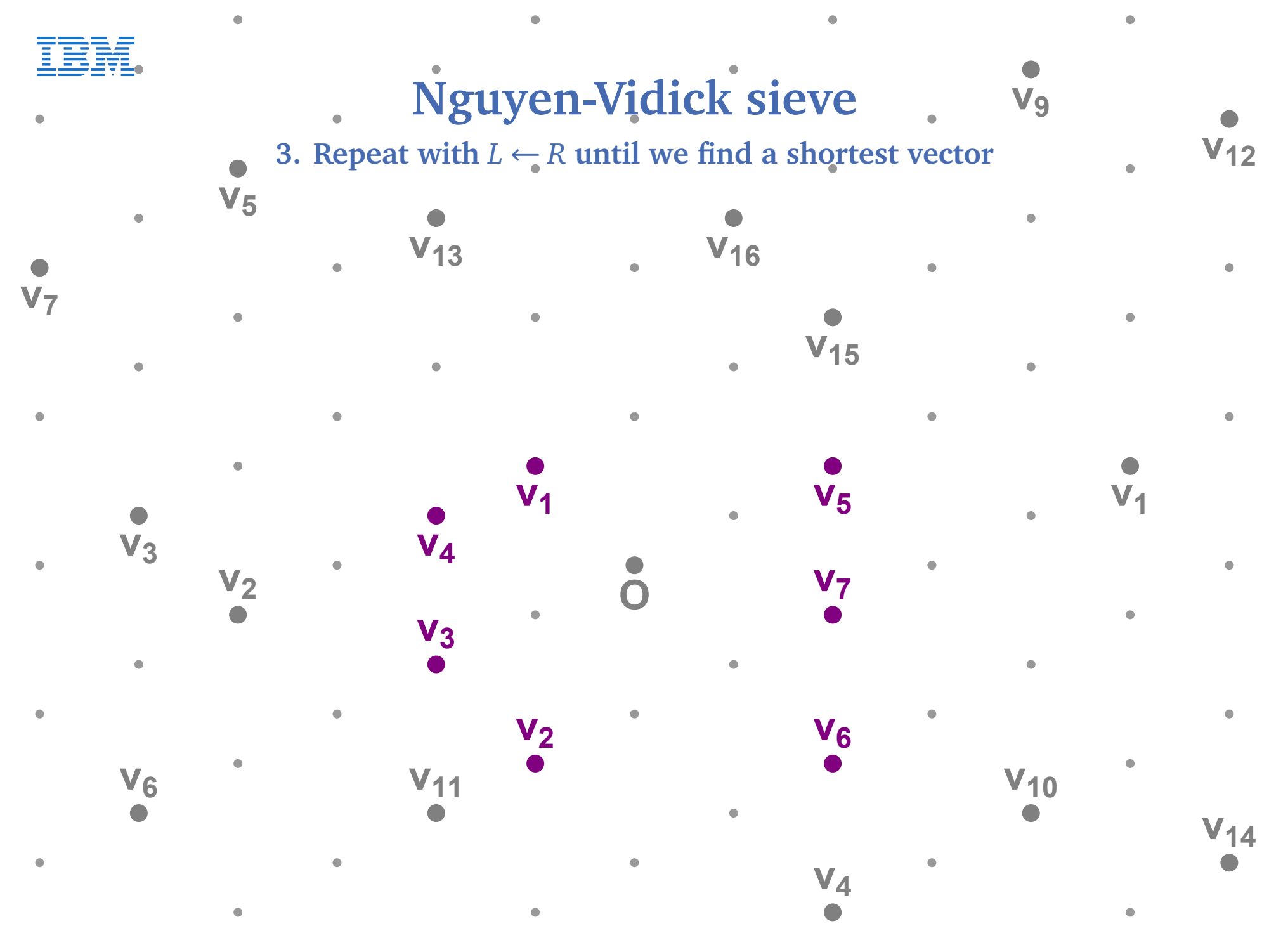

Nguyen-Vidick sieve

Overview

Hyperplane LSH

1. Sample a list L of random lattice vectors

Hyperplane LSH

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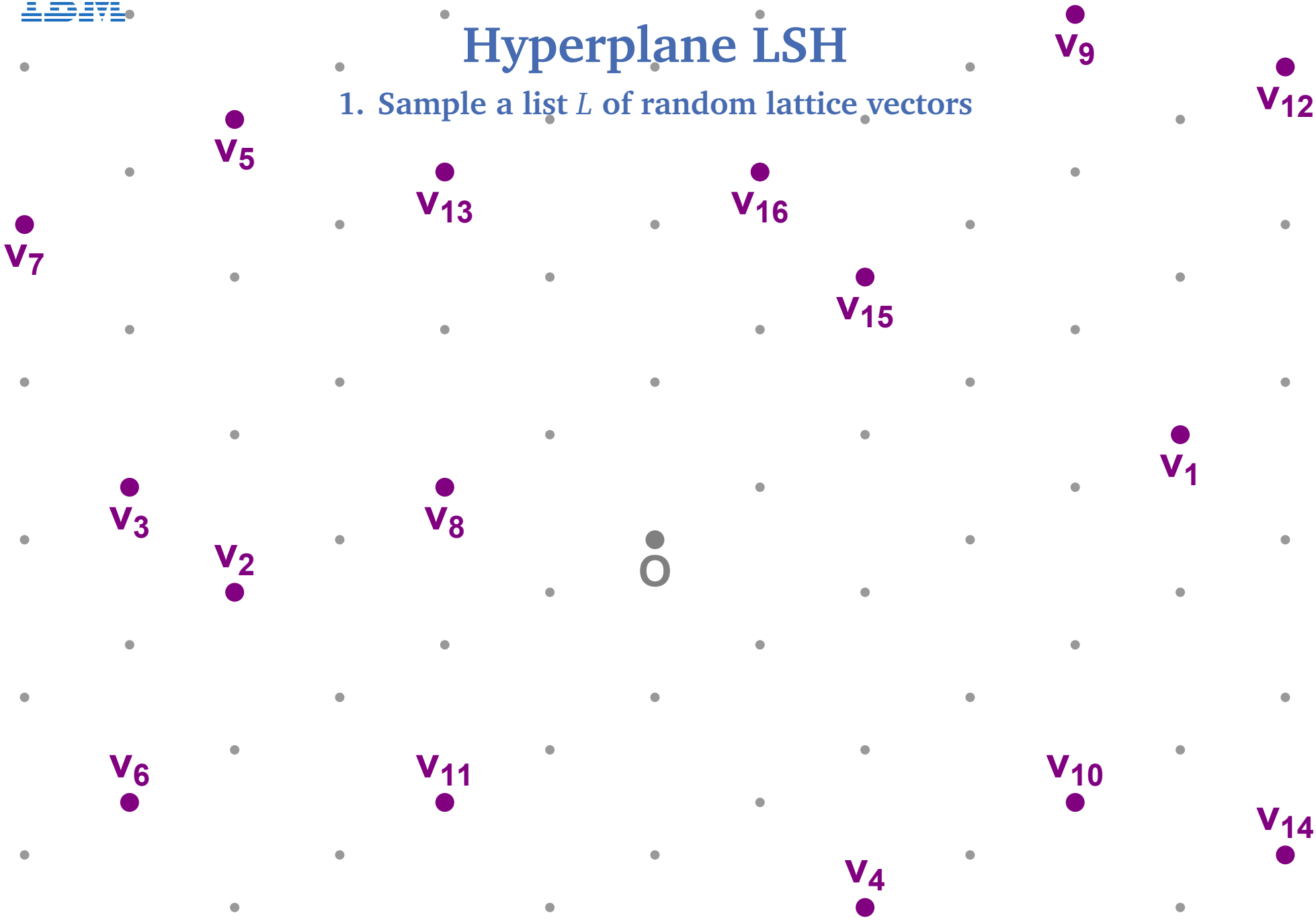
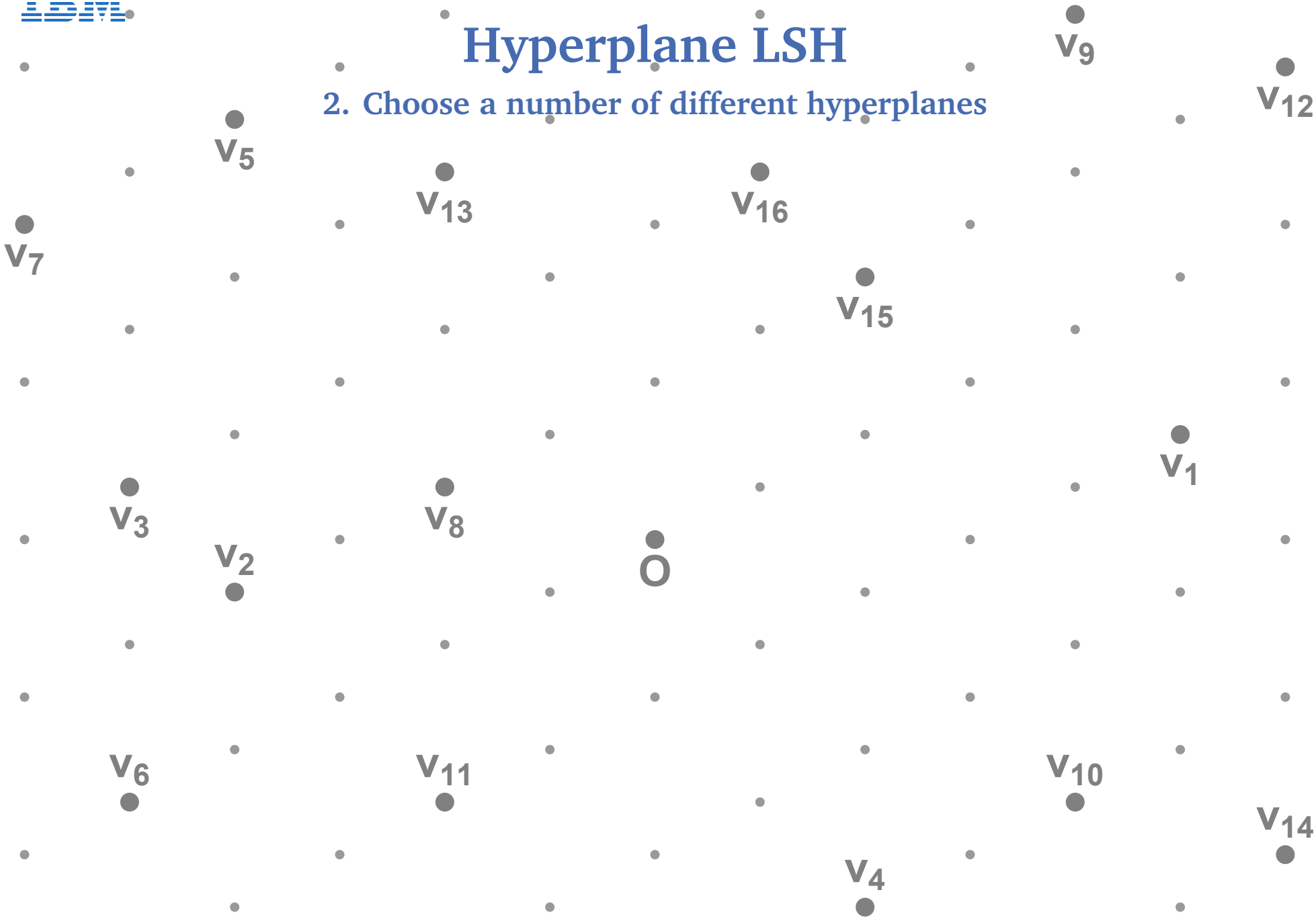


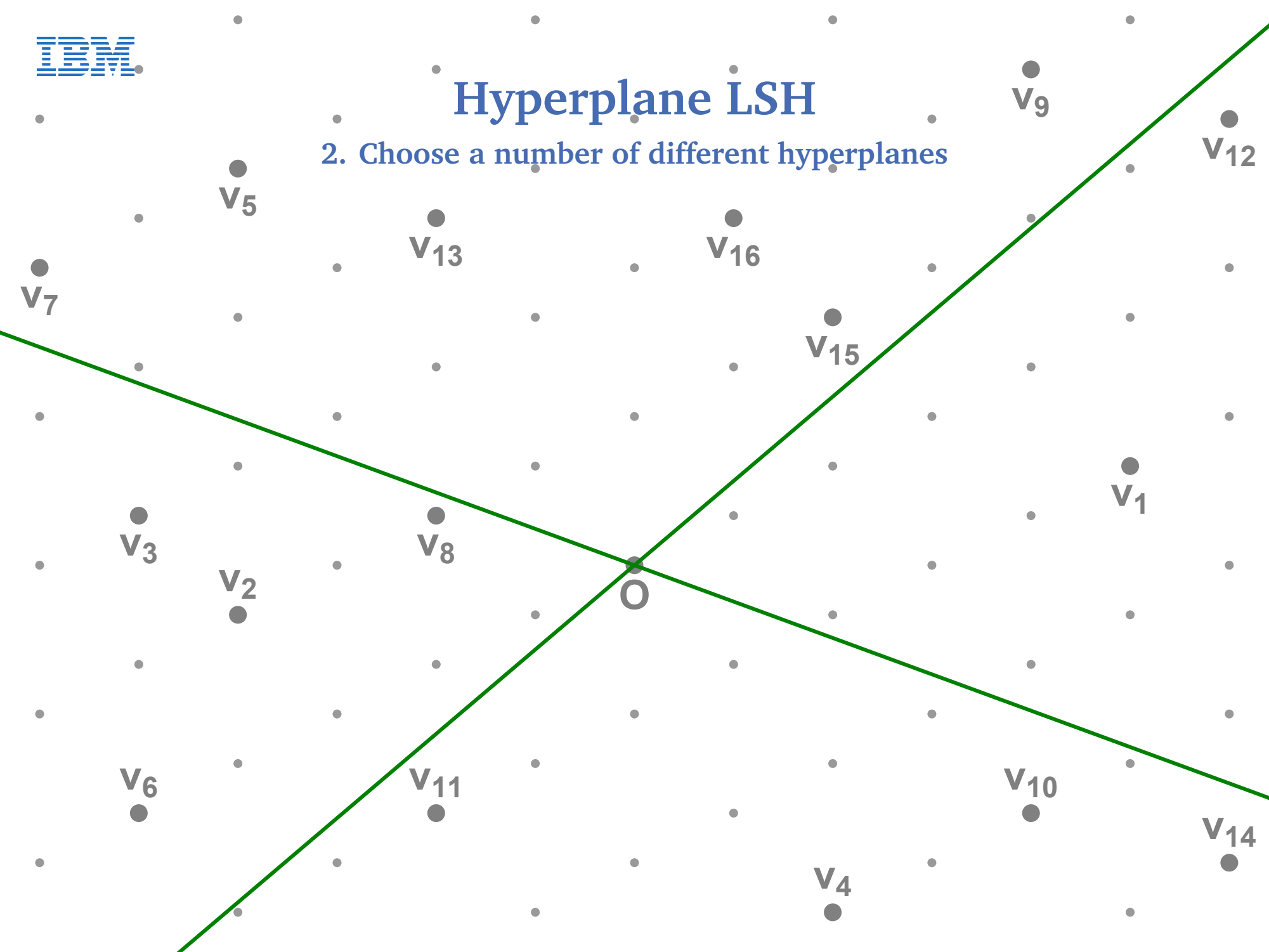
Hyperplane LSH

2. Choose a number of different hyperplanes



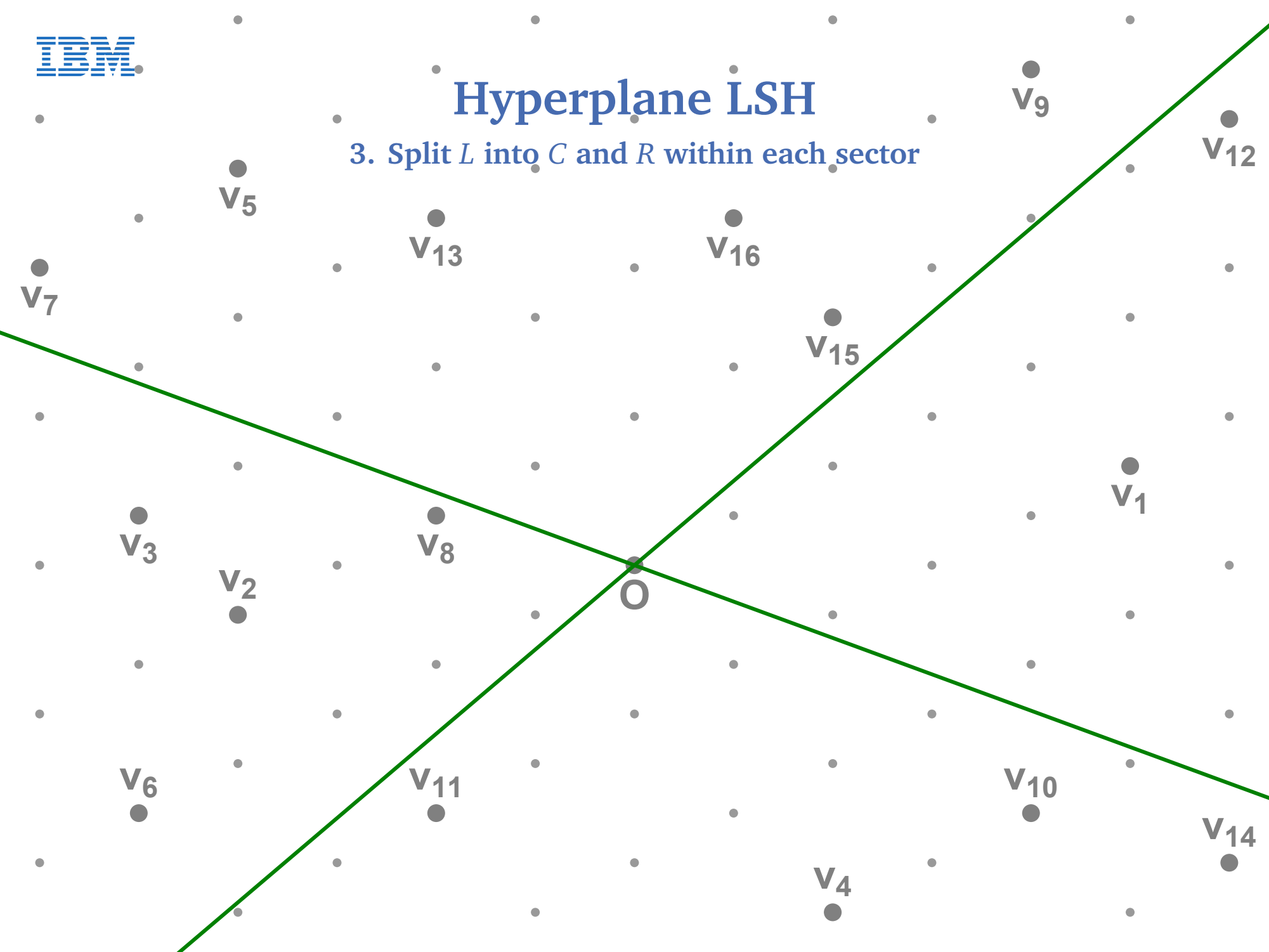
Hyperplane LSH

2. Choose a number of different hyperplanes



Hyperplane LSH

3. Split L into C and R within each sector



Hyperplane LSH

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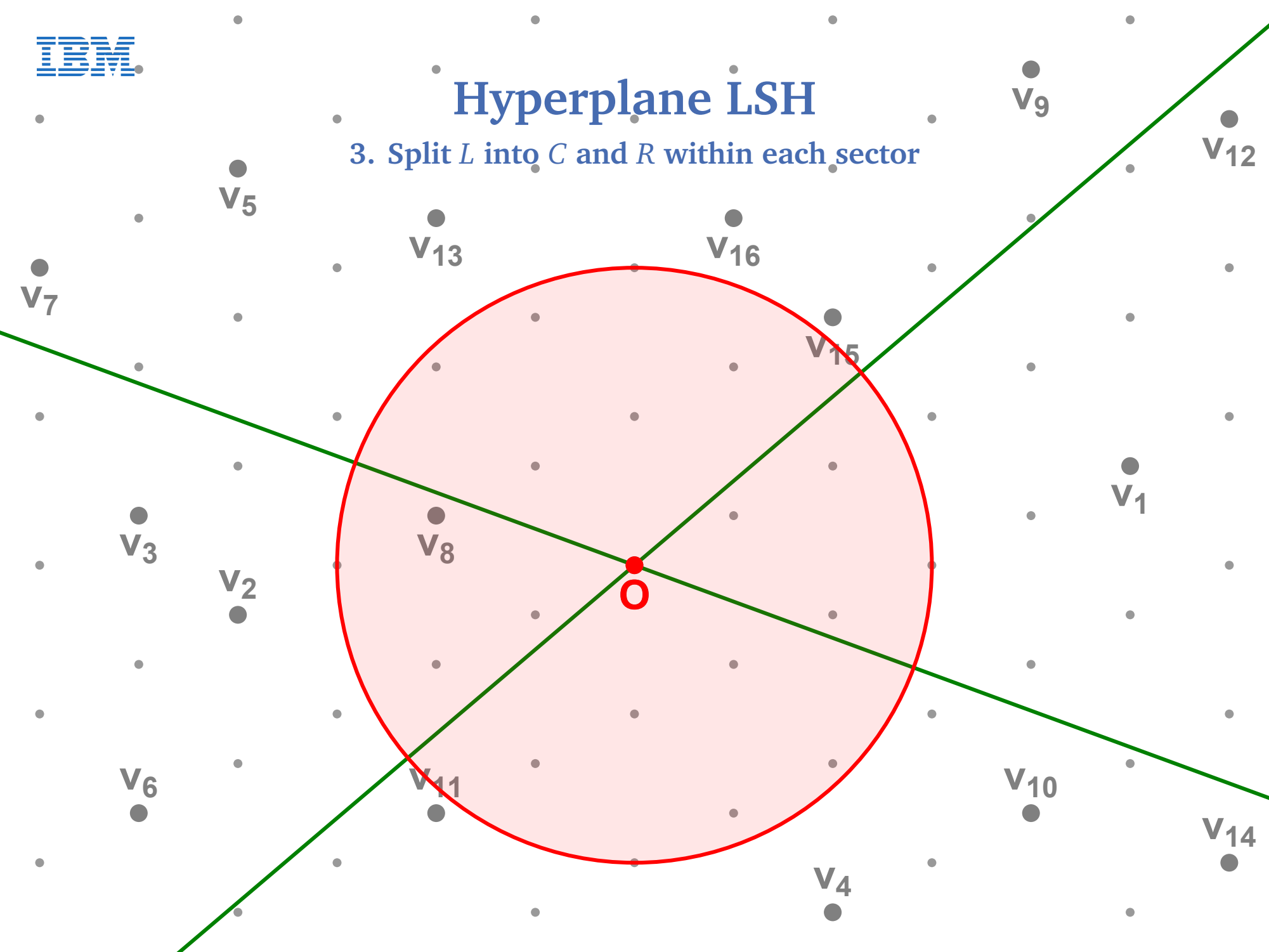

Hyperplane LSH

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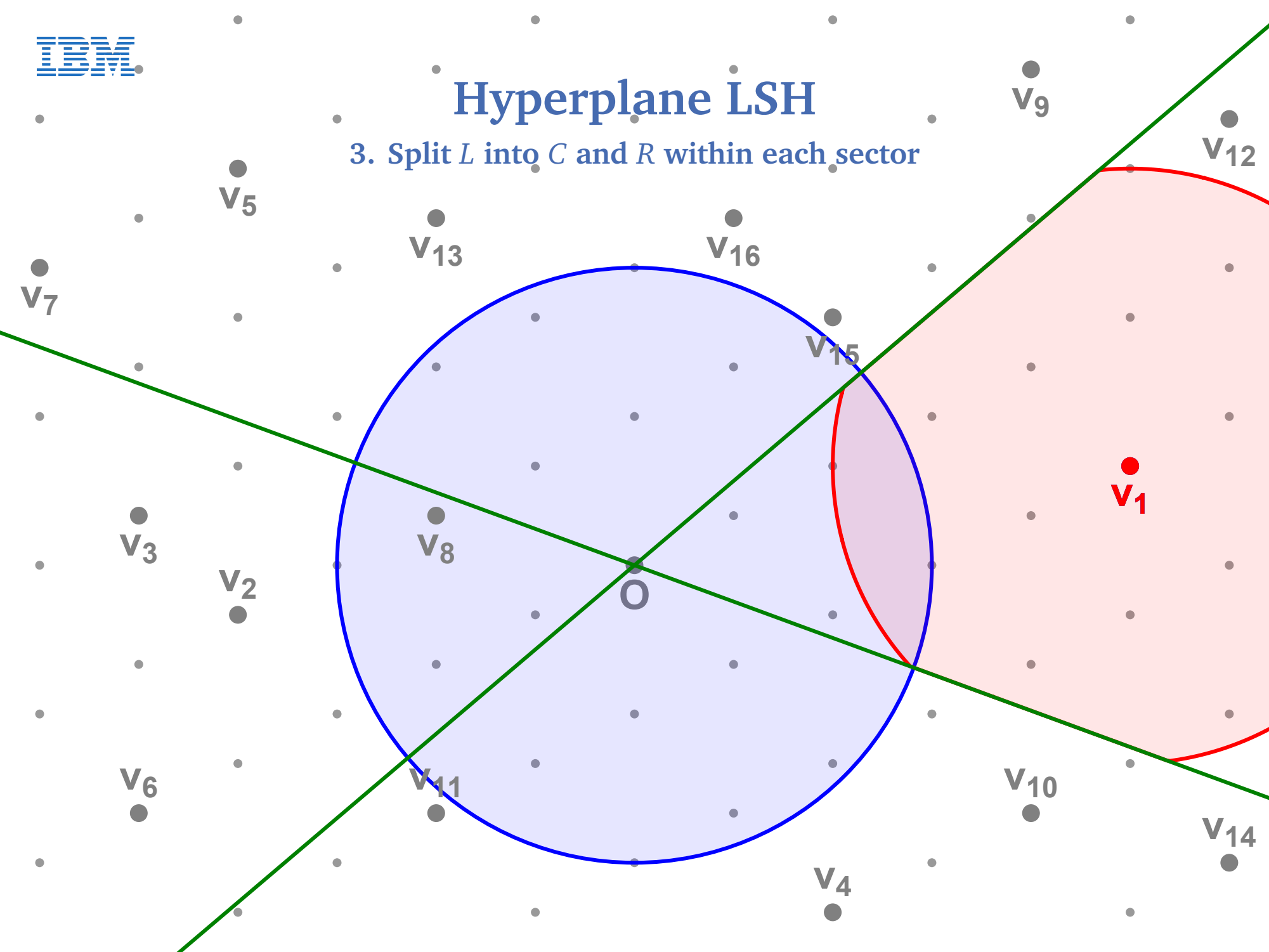

Hyperplane LSH

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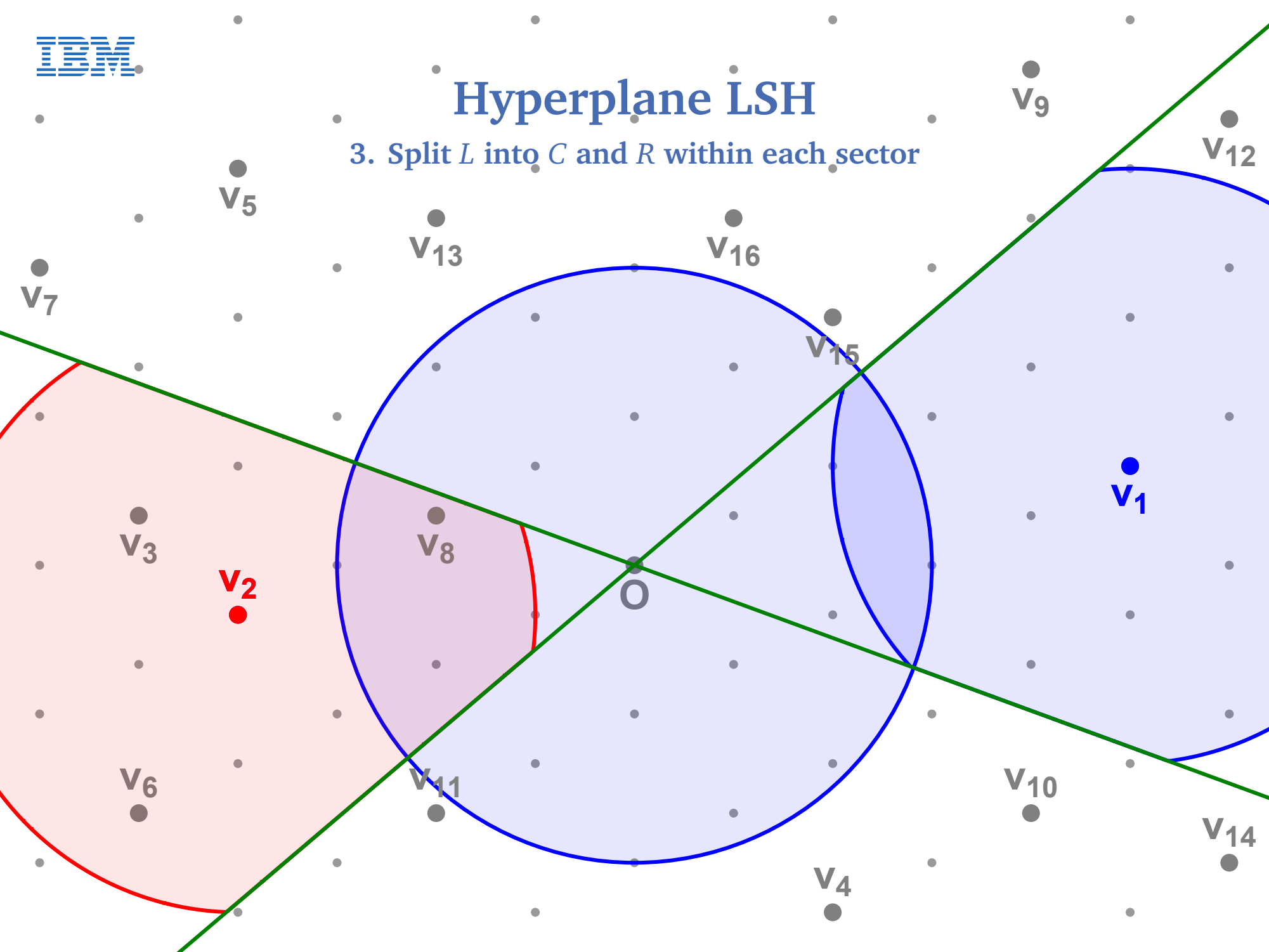

Hyperplane LSH

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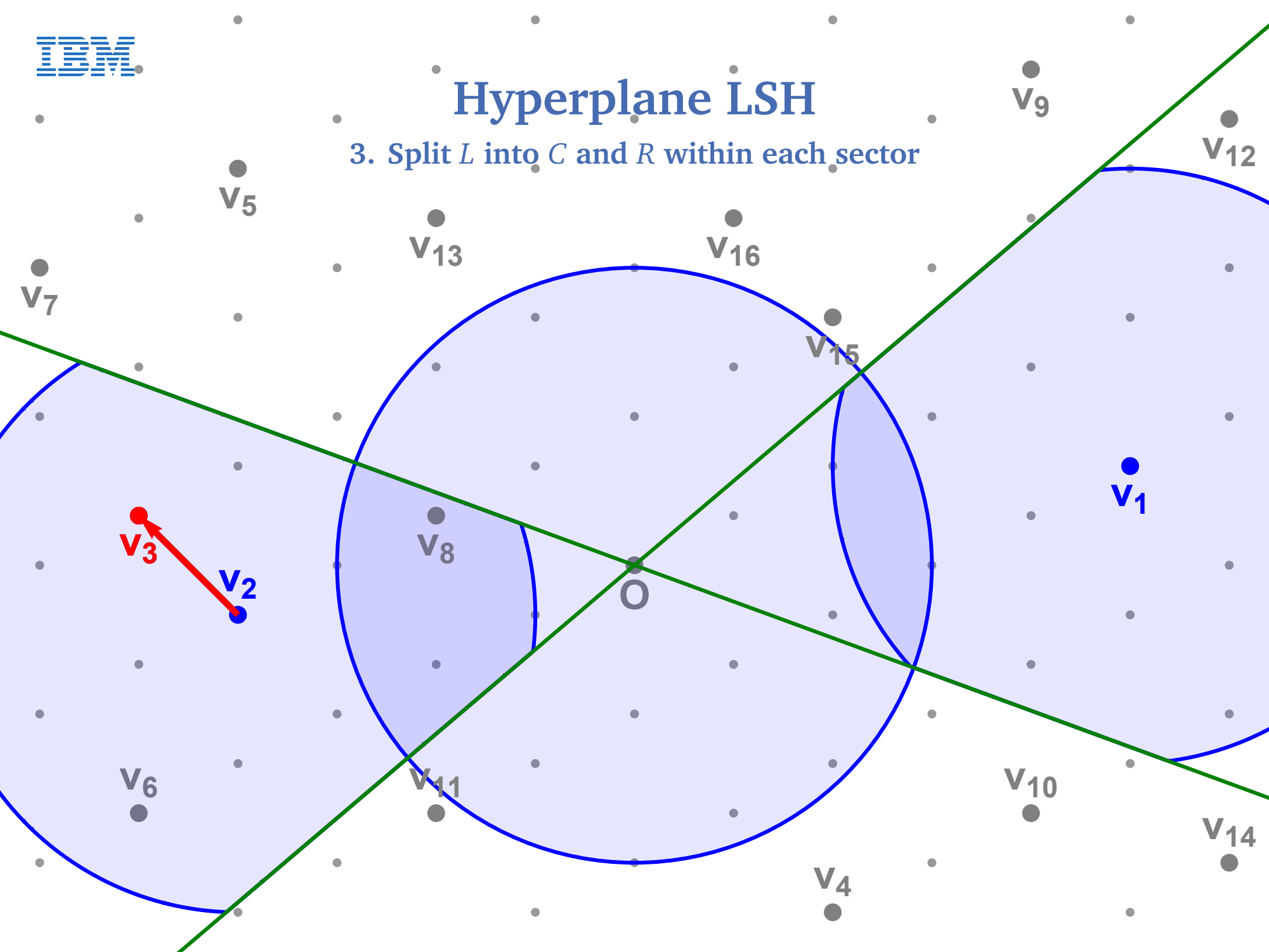

Hyperplane LSH

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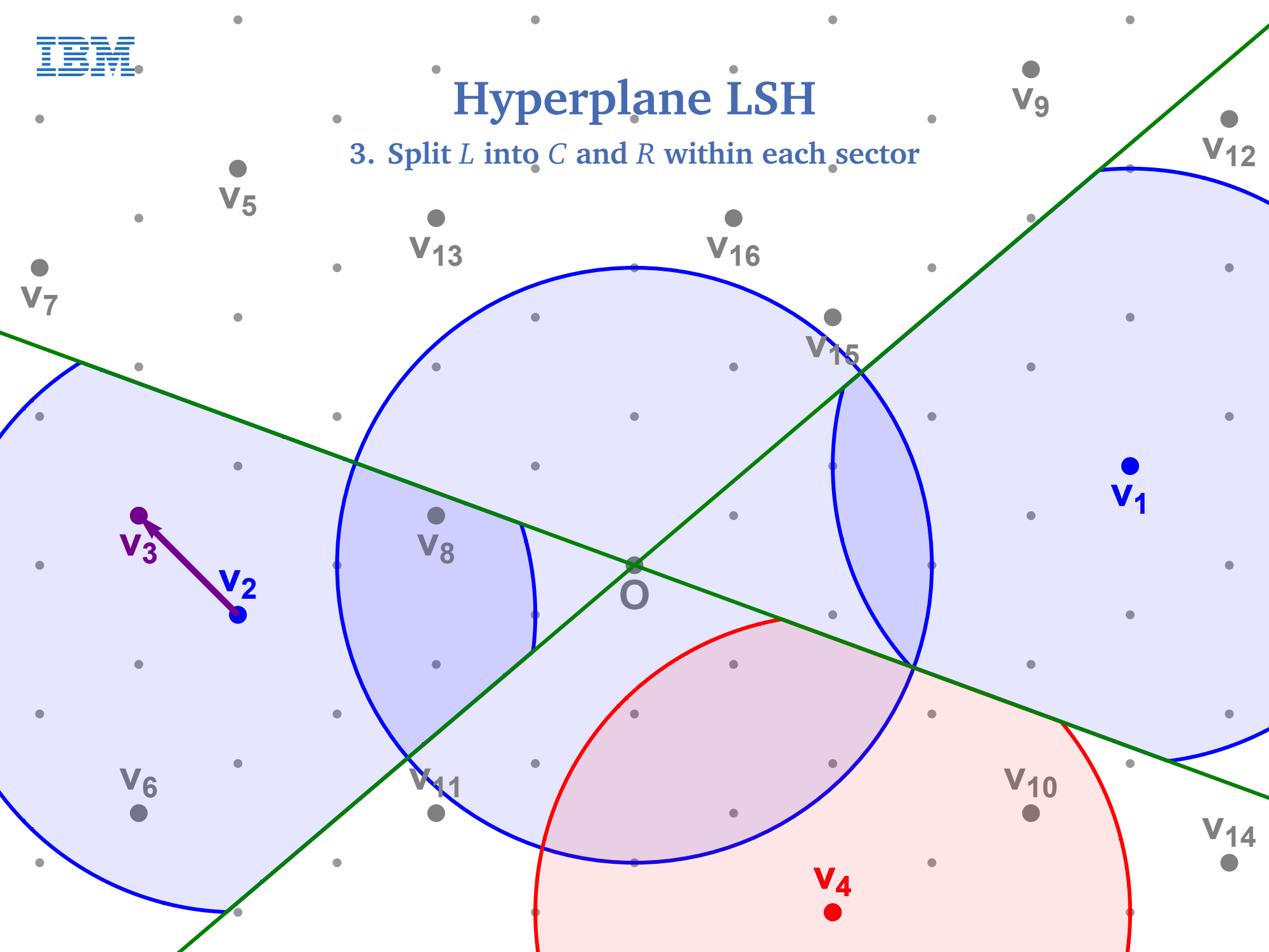

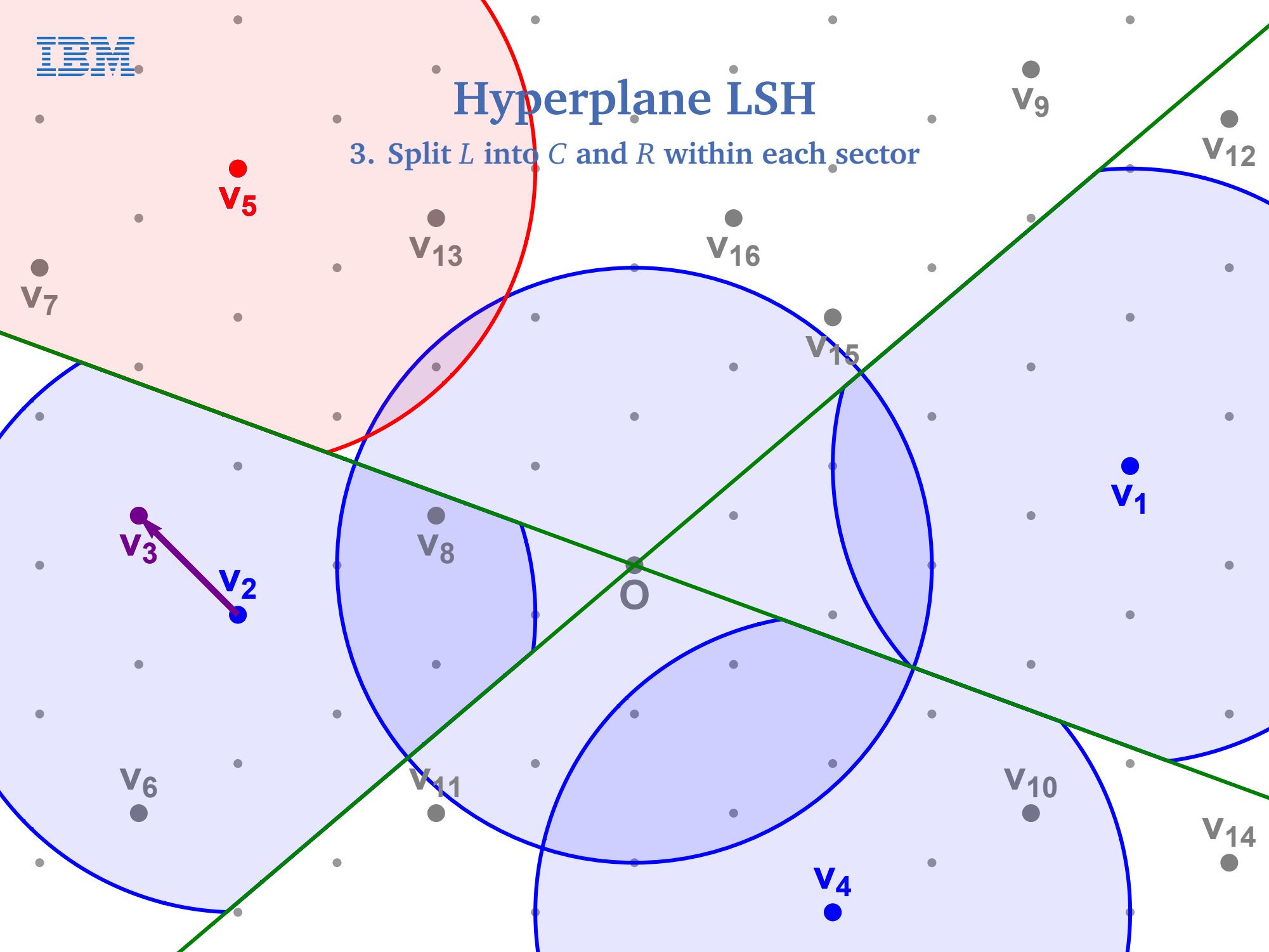
IBM
Hyperplane LSH
3. Split L into C and R within each sector
v5
v13
v16
v9
v12
v7
v15
v1
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O
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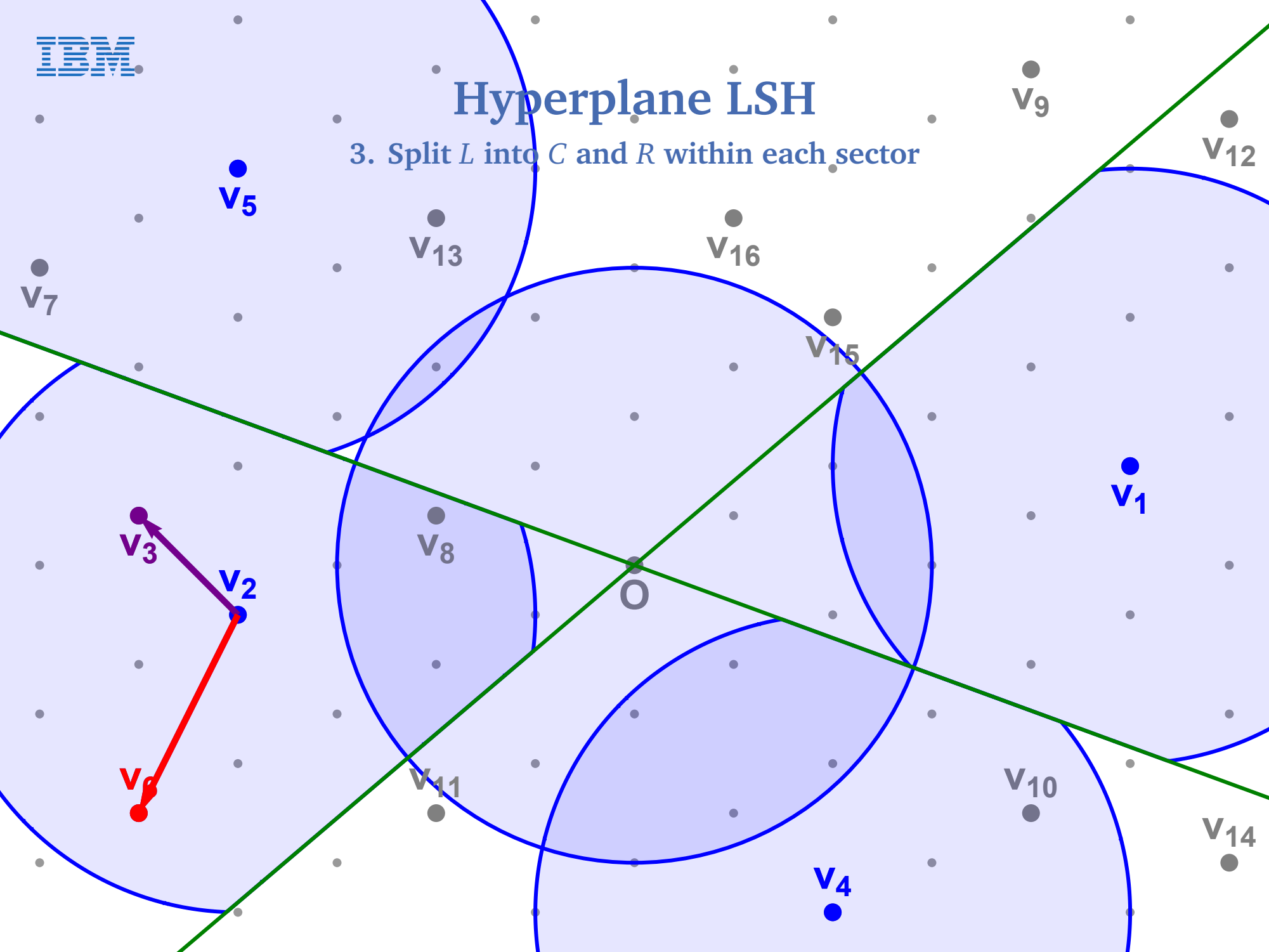
v10
v14
v4

Hyperplane LSH

3. Split L into C and R within each sector



Hyperplane LSH

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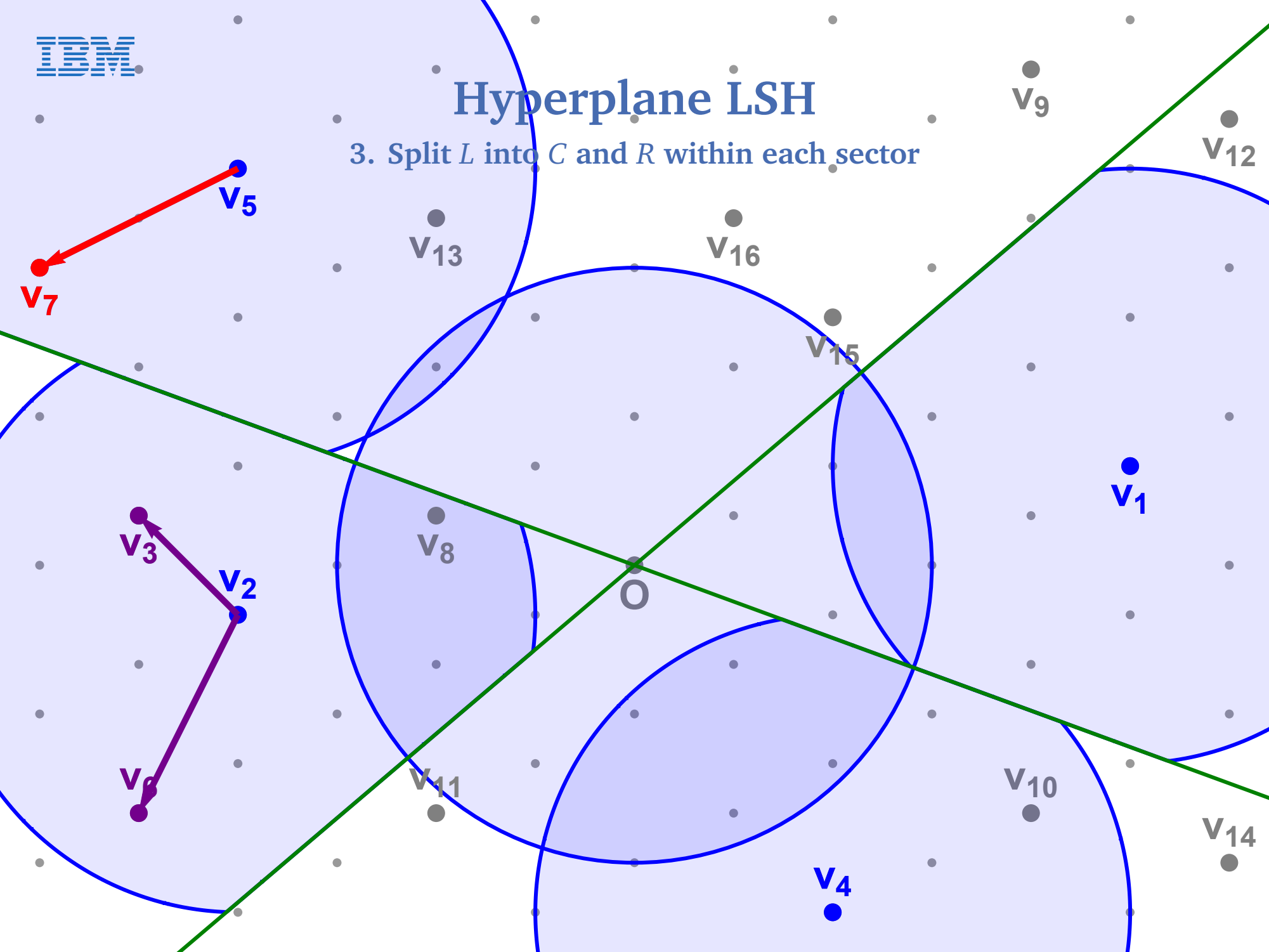

Hyperplane LSH

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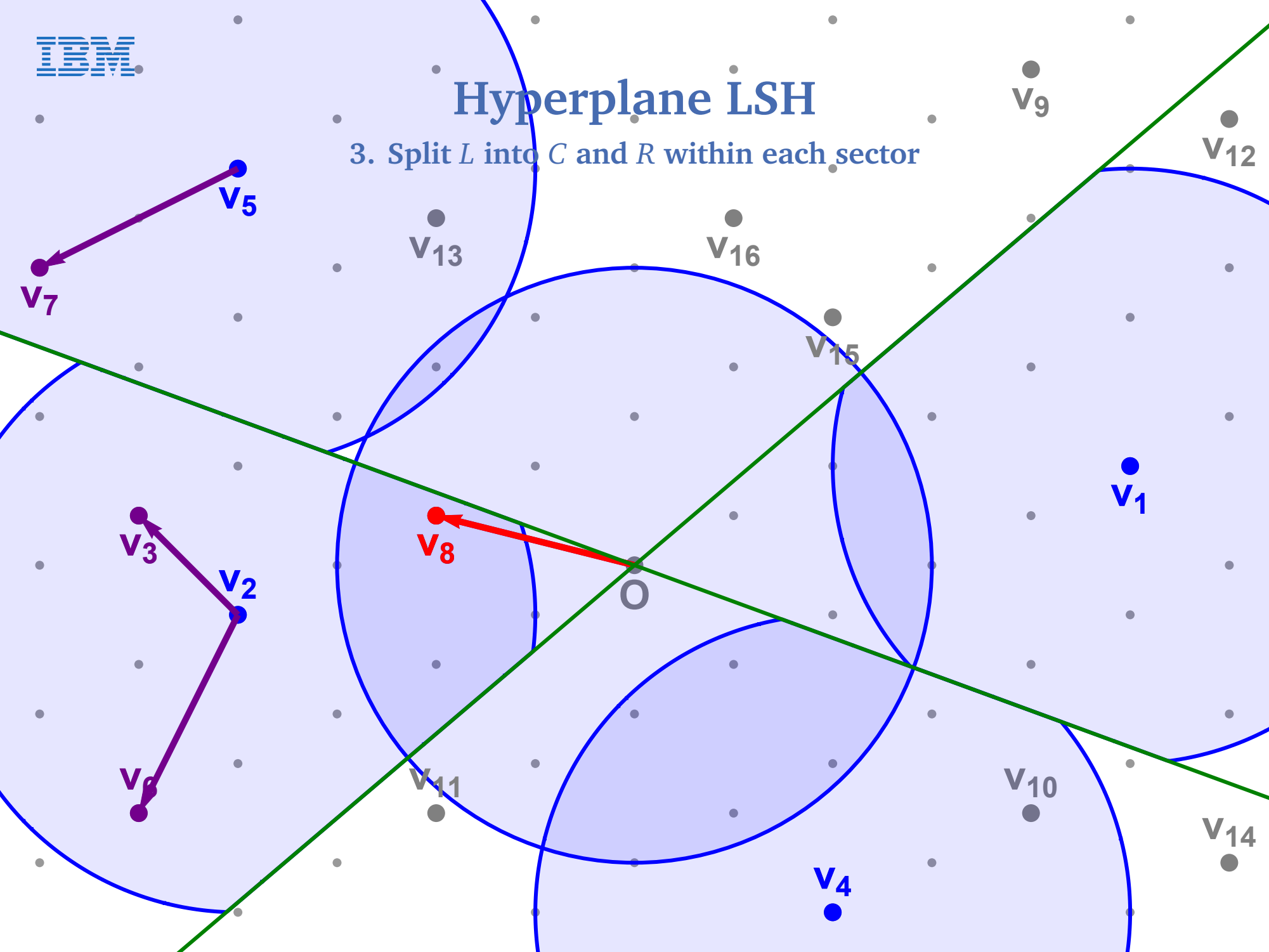

Hyperplane LSH

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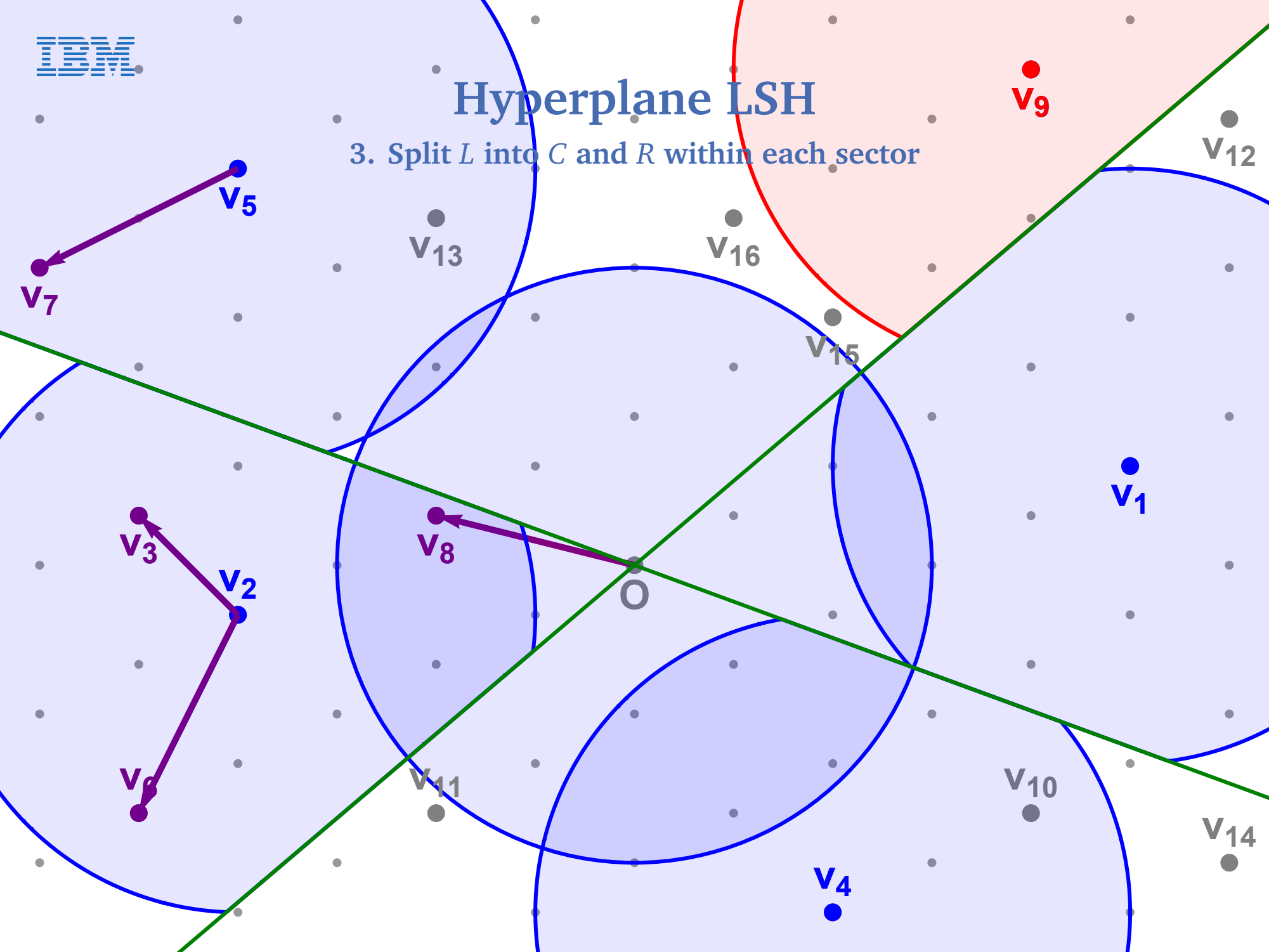

Hyperplane LSH

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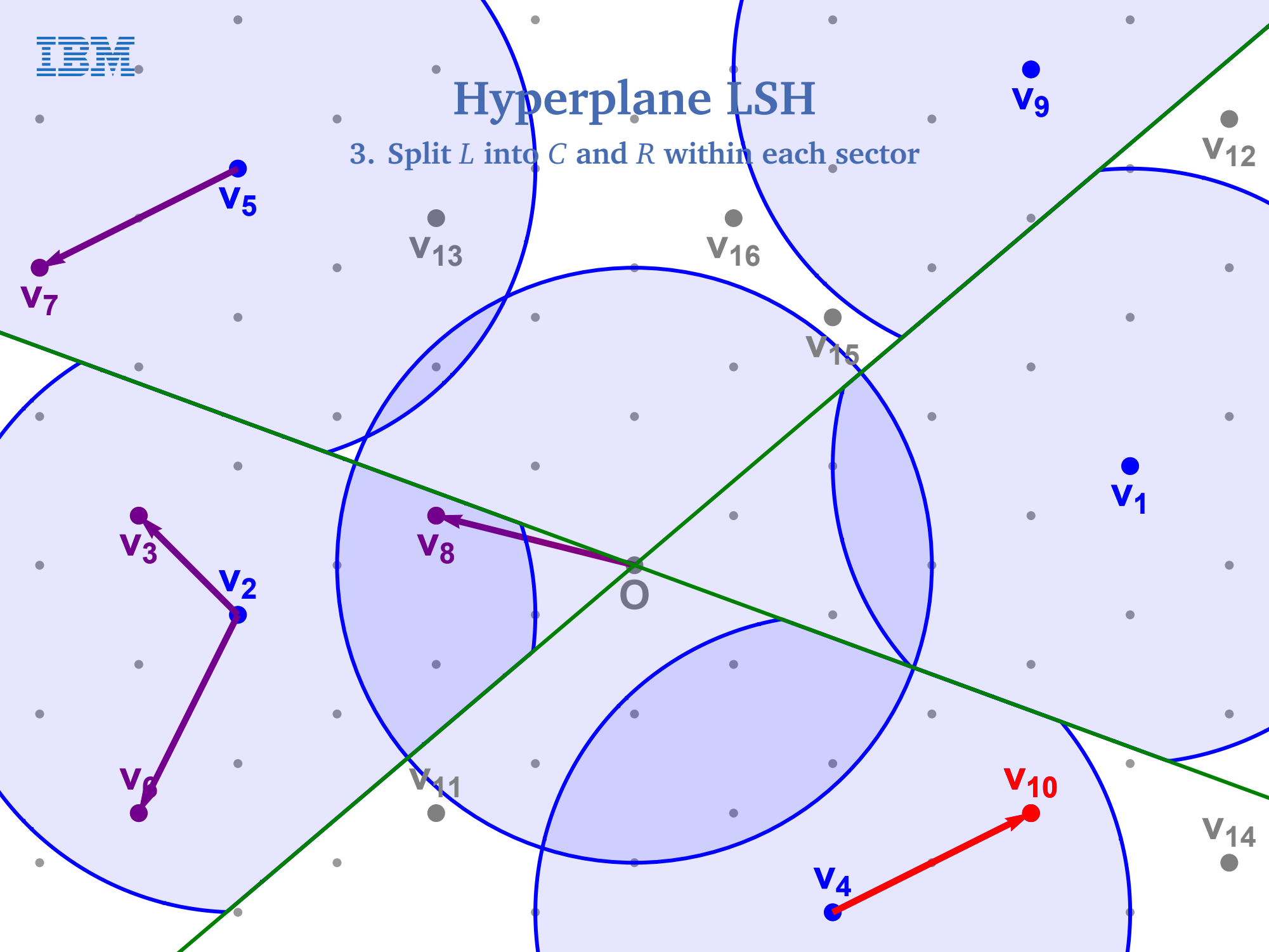

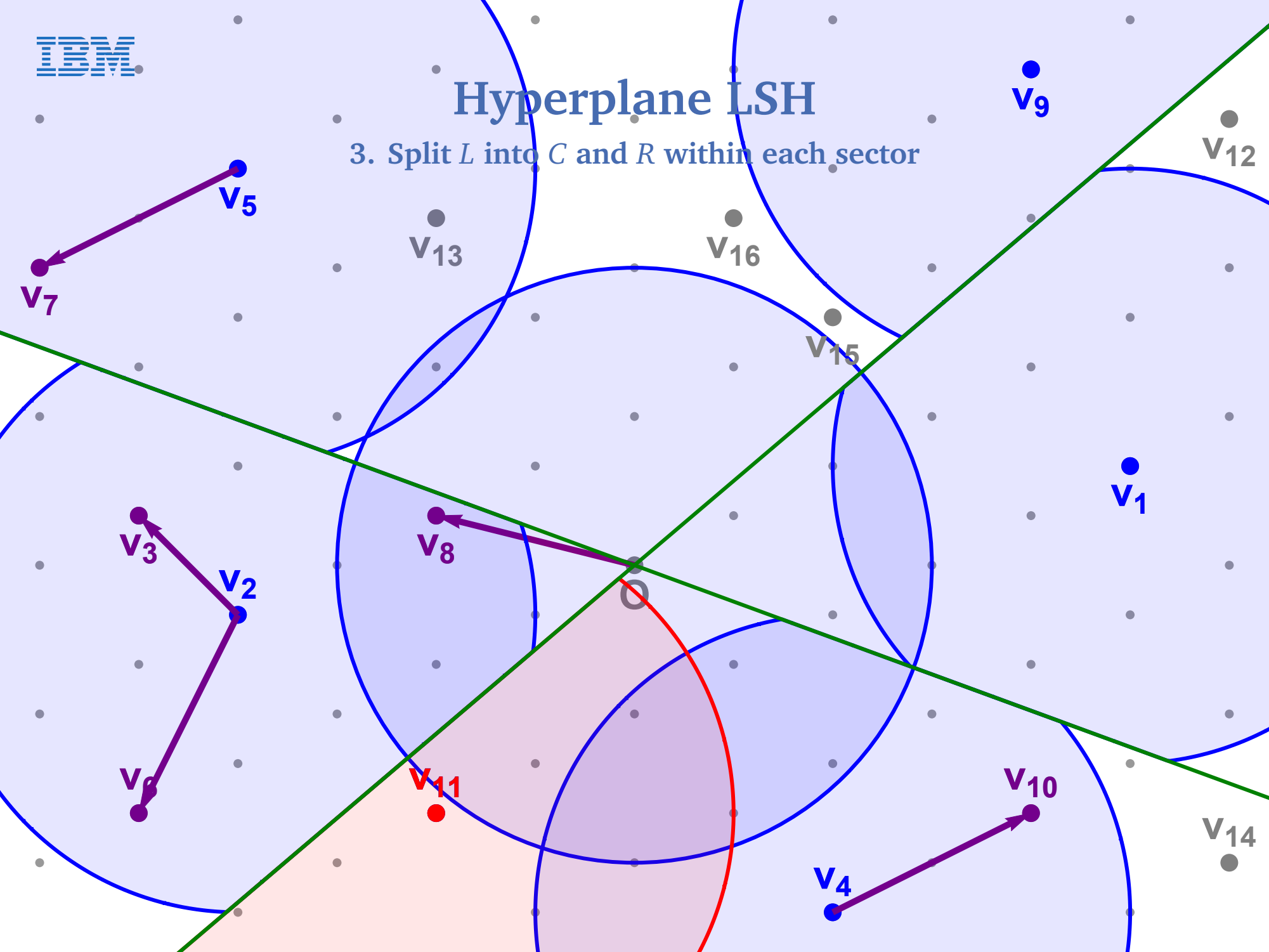
IBM
Hyperplane LSH
3. Split L into C and R within each sector
v9
v12
v5
v13
v16
v7
v15
v1
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v10
v14
v4

Hyperplane LSH

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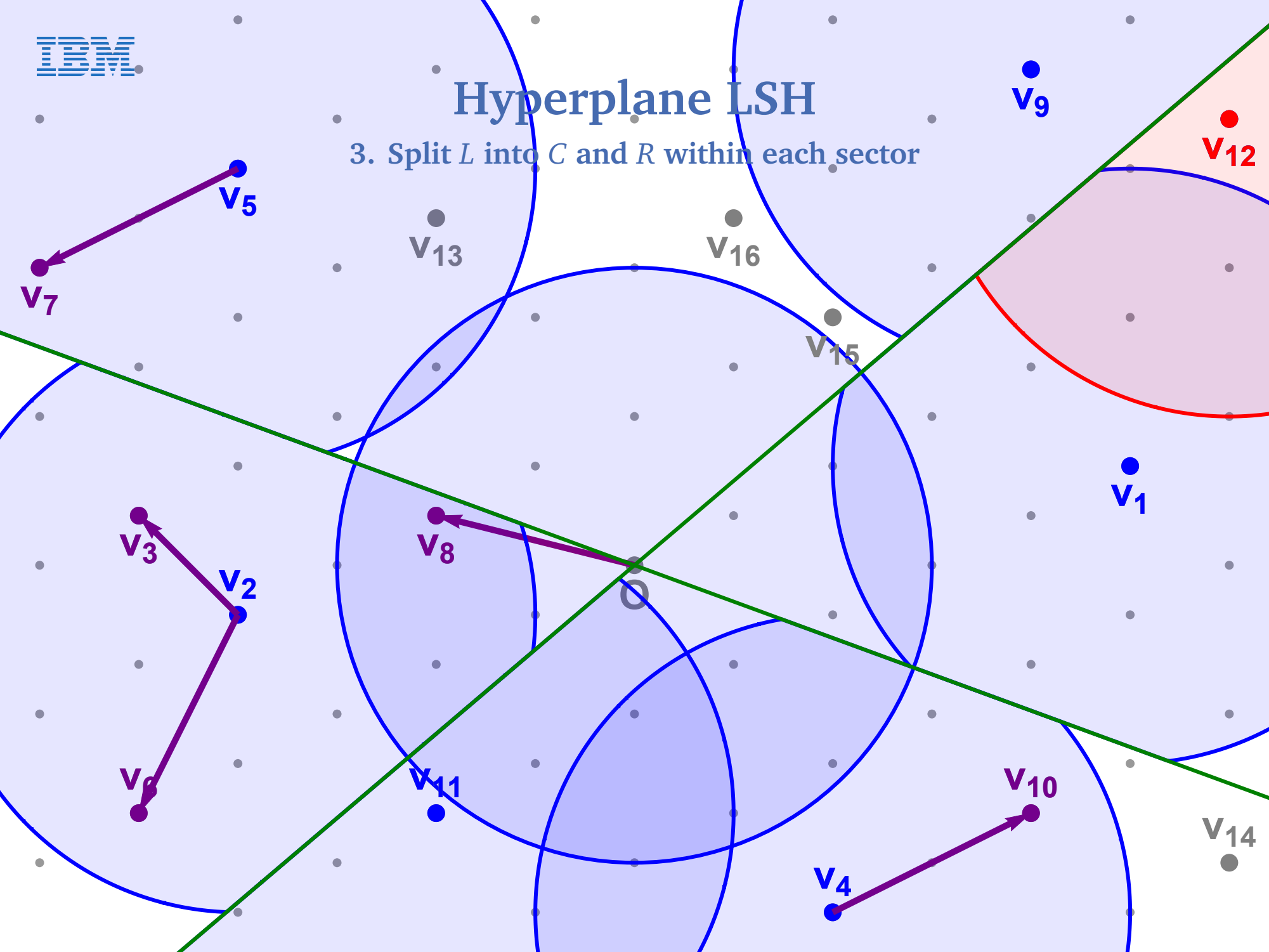

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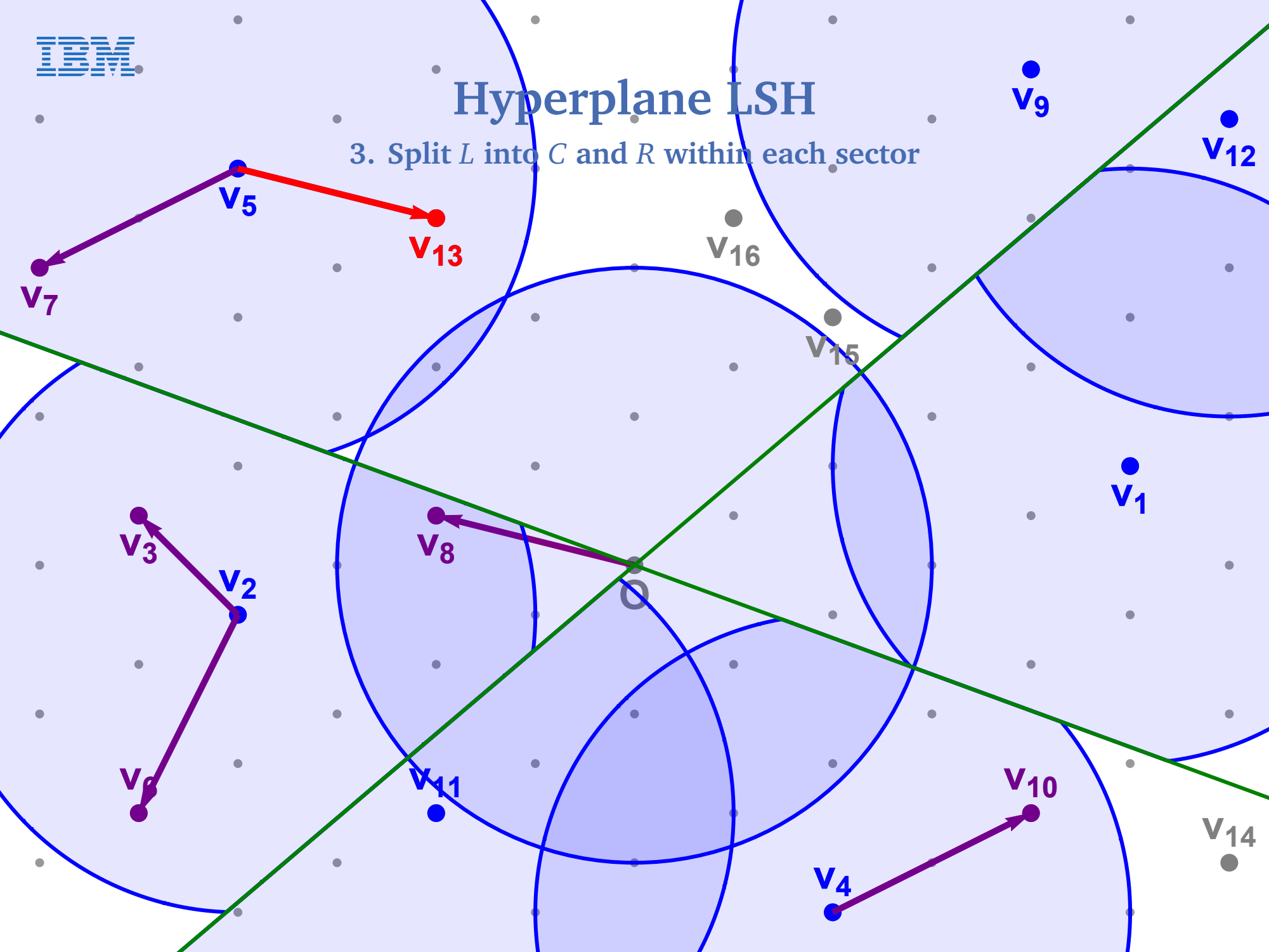

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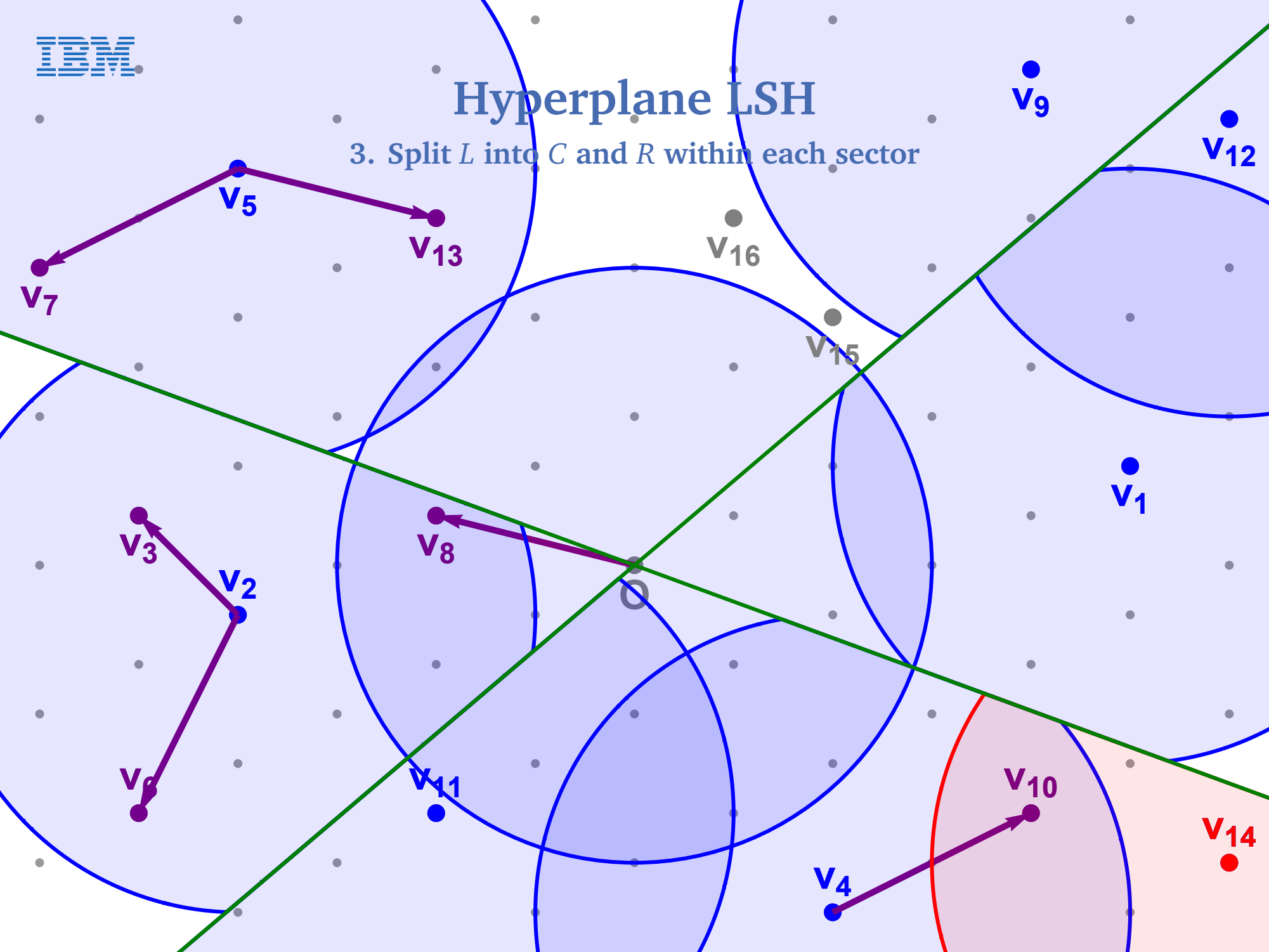

Hyperplane LSH

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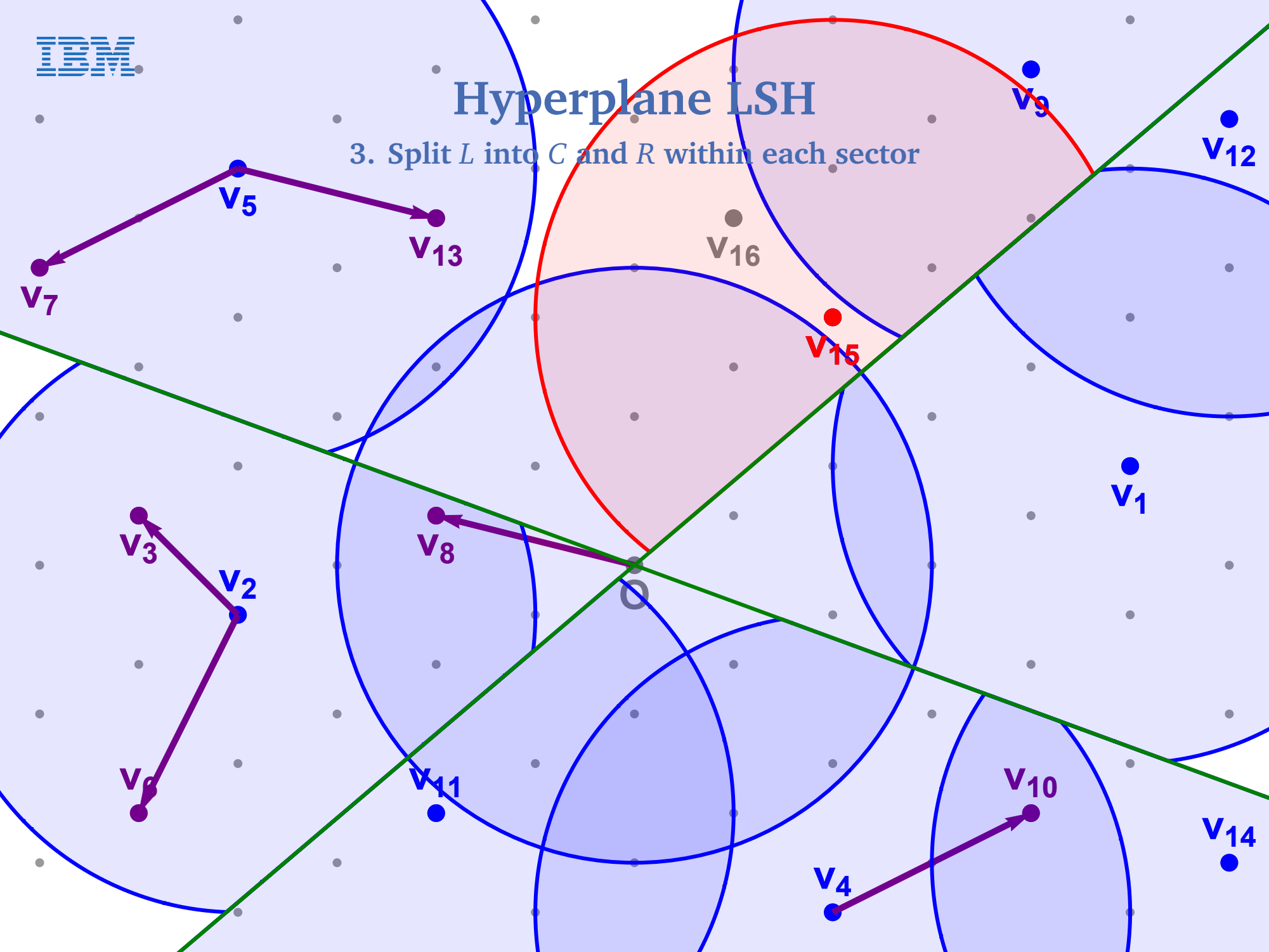

Hyperplane LSH

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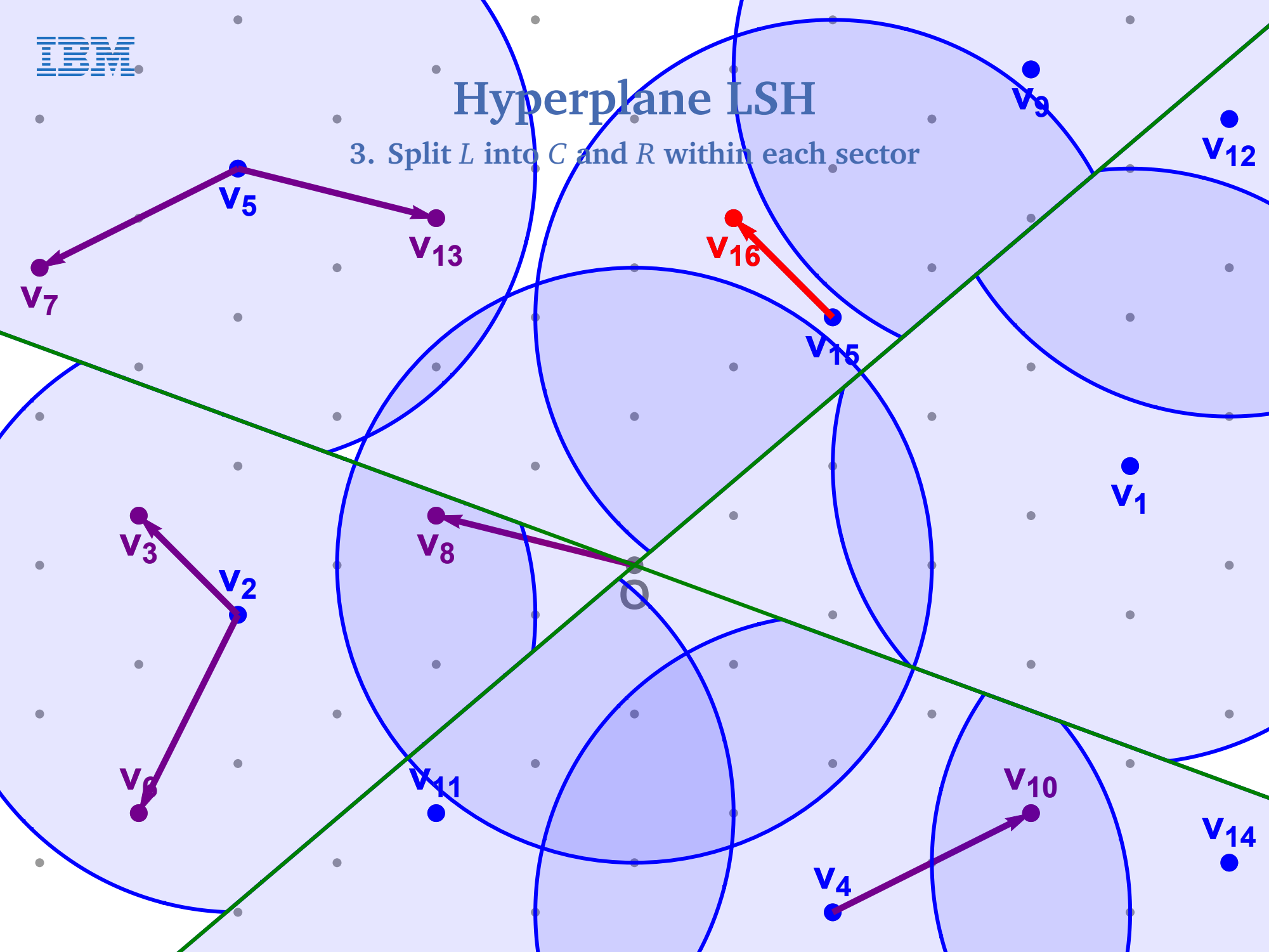

Hyperplane LSH

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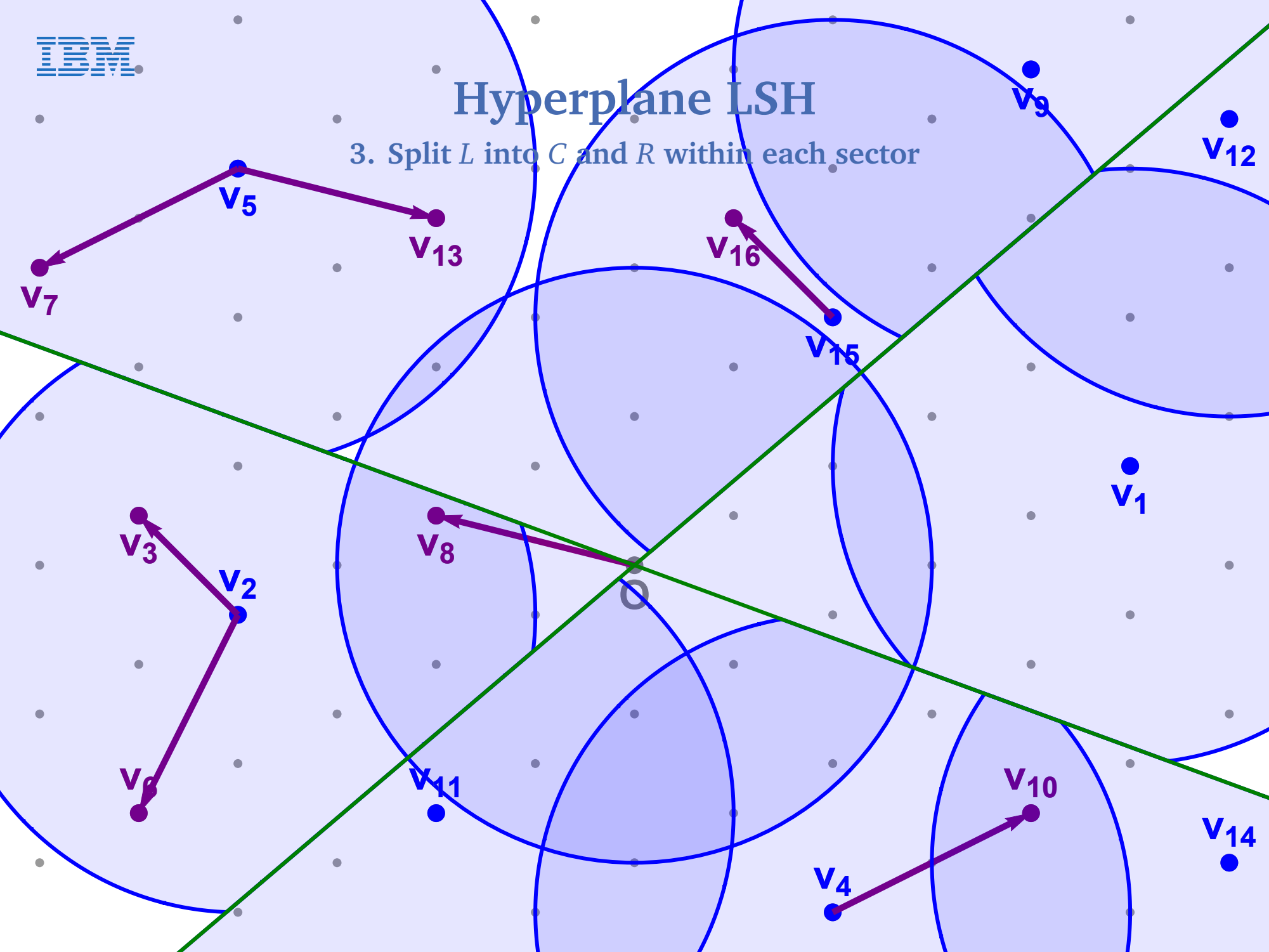
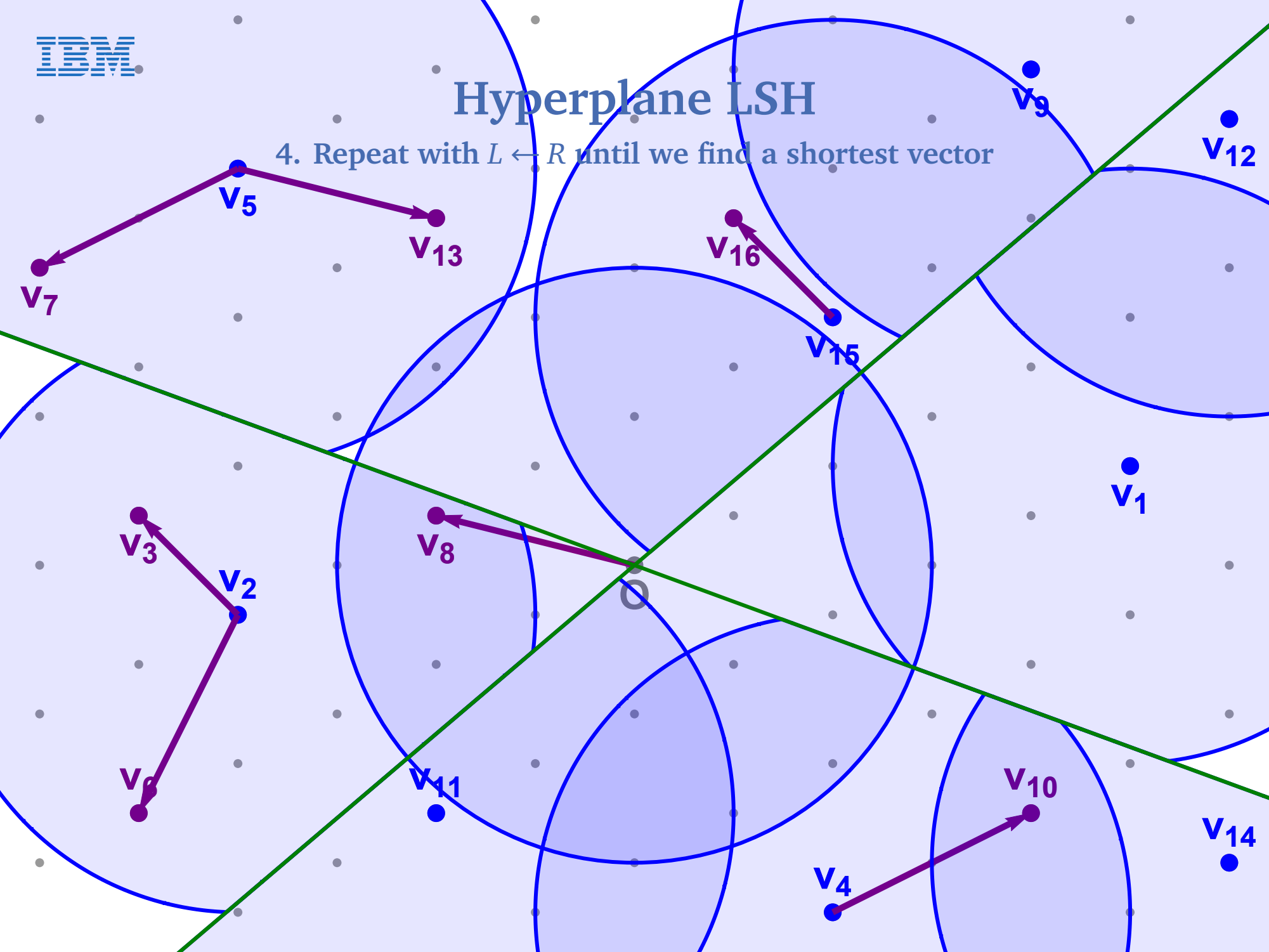


Hyperplane LSH

4. Repeat with $L \leftarrow R$ until we find a shortest vector



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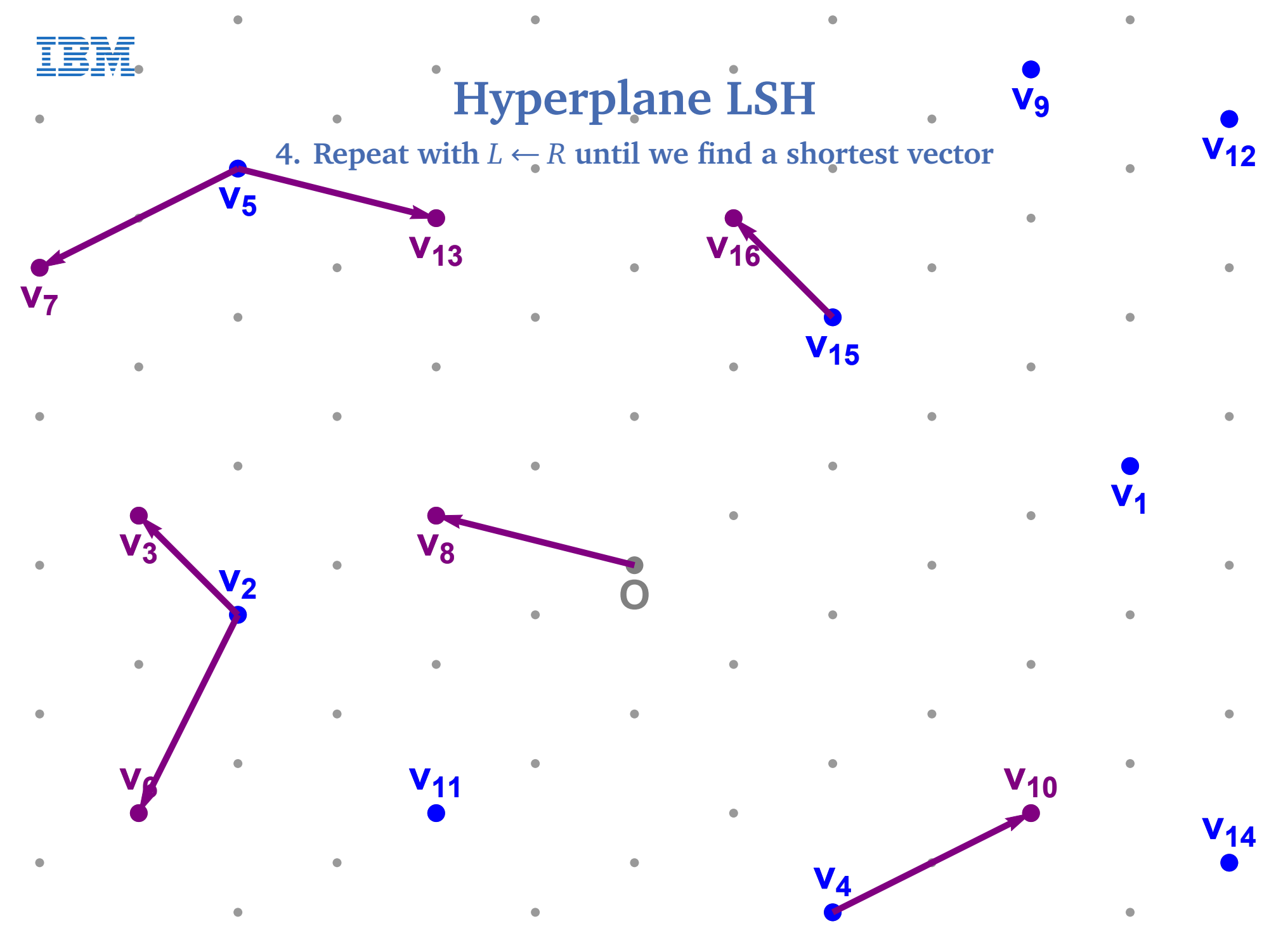

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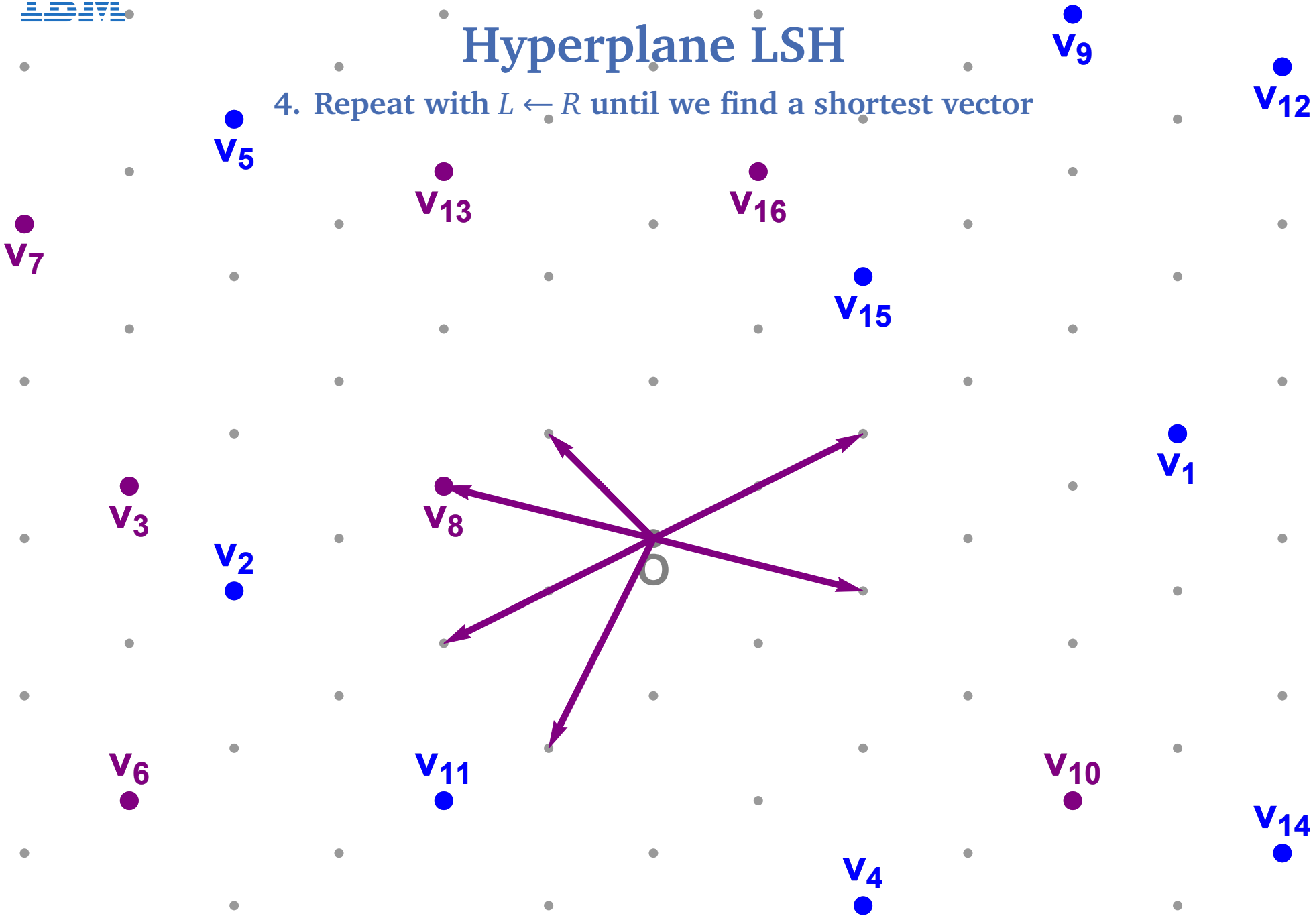
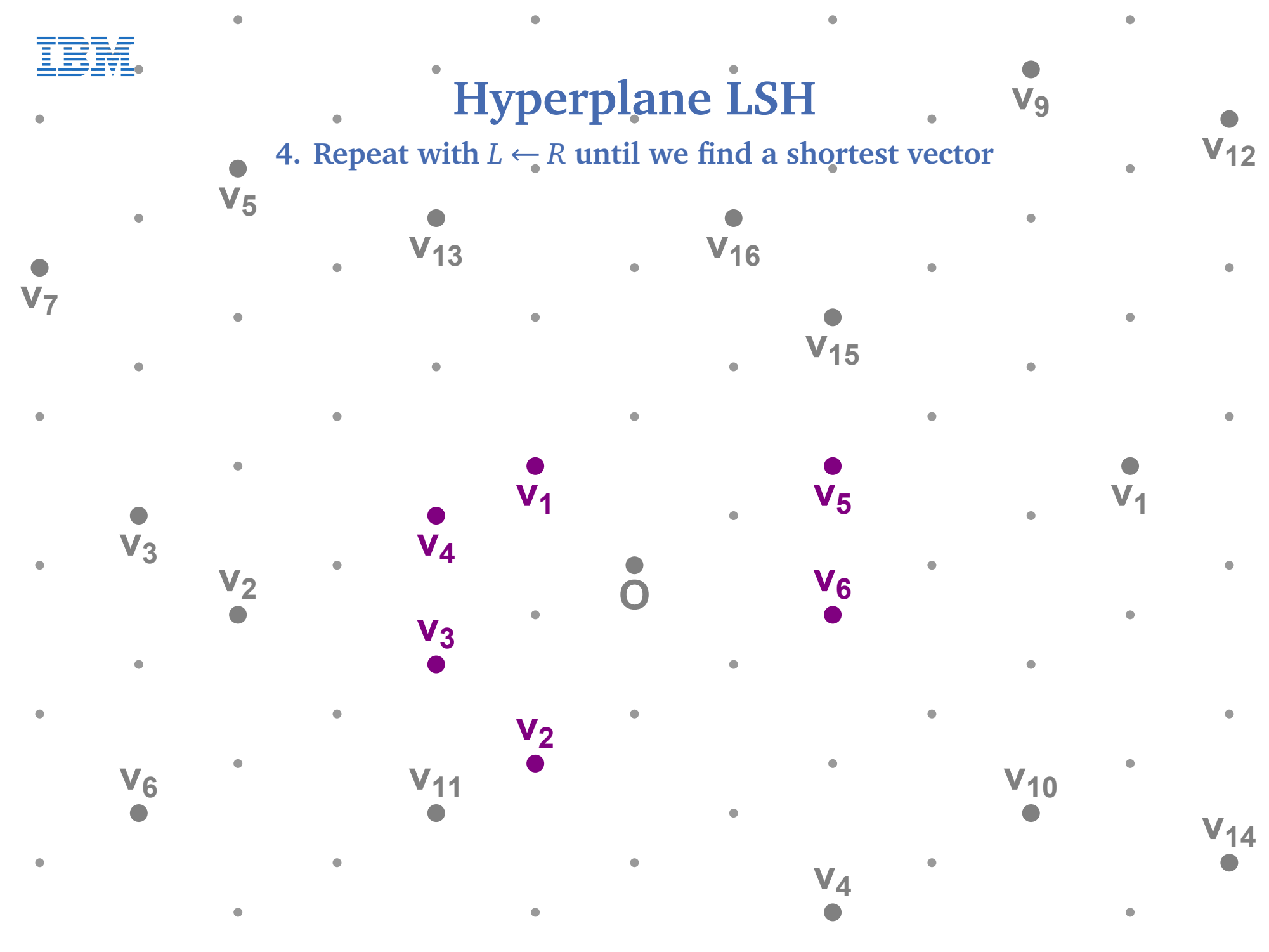


Hyperplane LSH

4. Repeat with $L \leftarrow R$ until we find a shortest vector



Exact SVP algorithms

Lattice enumeration

- Memory efficient
- Fincke-Pohst enumeration: $2^{O(n^2)}$ time
- Kannan enumeration: $2^{O(n \log n)}$ time
- Practical speedups with pruning heuristics

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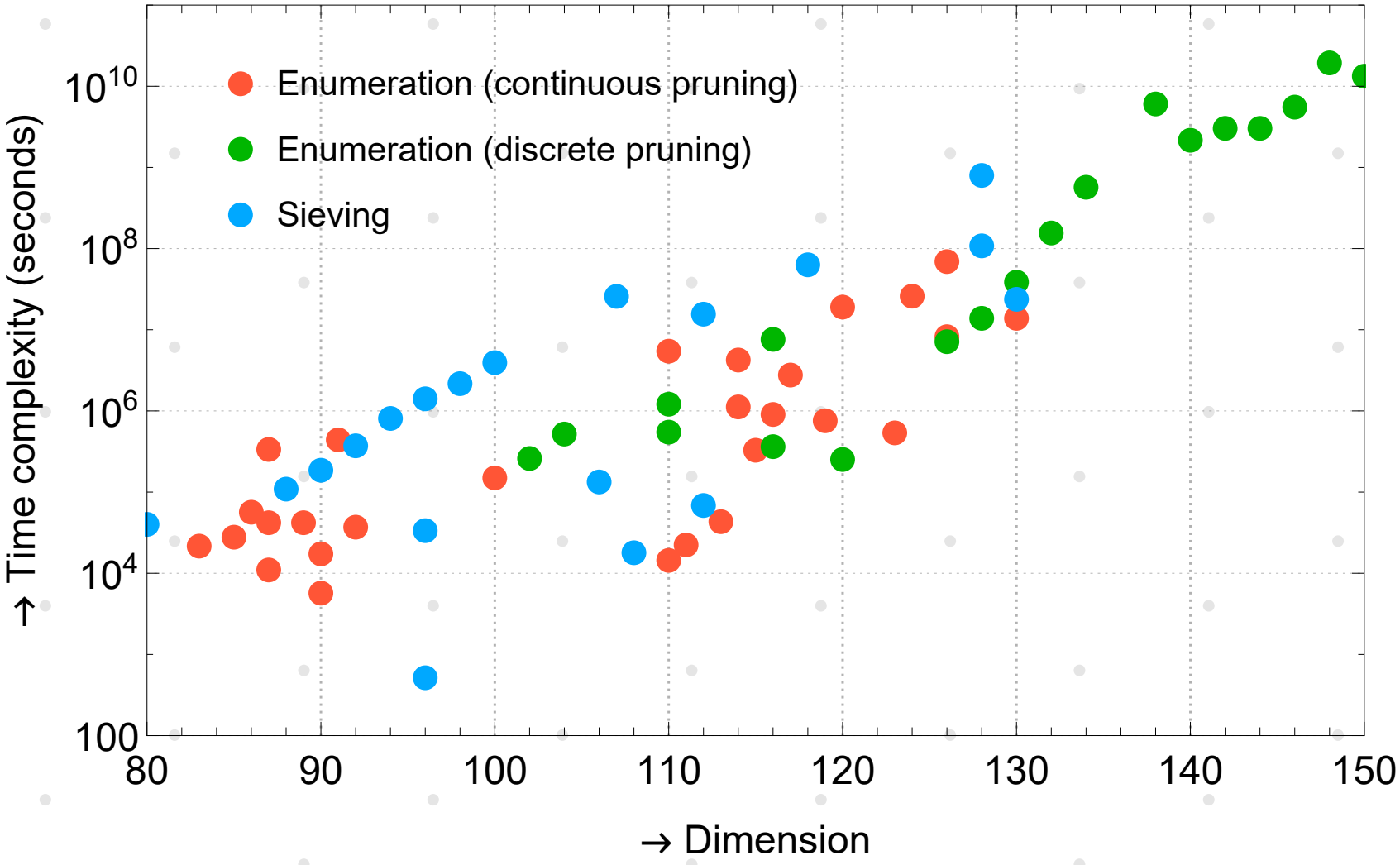
Lattice sieving

- Exponential memory footprint
- Asymptotically faster: $2^{O(n)}$ time
- Practical near neighbor speedups

SVP in practice



Summary

- Lattice-based crypto relies on hardness of finding short bases
- State-of-the-art basis reduction: BKZ with fast SVP subroutine
- Competitive SVP methods: enumeration and sieving
- Practice: Enumeration leads, sieving catching up

$$\text{SVP costs} \implies \text{BKZ costs} \implies \text{Security estimates} \implies \text{Parameters}$$

Questions?