Progress in Traitor Tracing with Applications to Group Testing

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Outline

Problem: Illegal redistribution

User	Copyrighted content															
Antonino	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Boris	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Caroline	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
David	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Eve	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Fred	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Gábor	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Henry	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...

Problem: Illegal redistribution

User	Copyrighted content																
Antonino	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Boris	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Caroline	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
David	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Eve	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Fred	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Gábor	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Henry	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Copy	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...

Solution: Embed fingerprints

User	Copyrighted content (fingerprinted)																
Antonino	0	1	1	1	0	0	1	1	1	0	1	1	0	1	0	0	...
Boris	0	1	1	1	0	1	0	1	1	0	1	1	1	1	1	0	...
Caroline	0	1	0	1	0	1	0	1	1	0	0	1	1	0	1	0	...
David	0	1	1	1	0	0	0	1	1	0	1	1	0	0	0	0	...
Eve	0	1	0	1	0	1	0	1	1	0	1	1	1	0	0	0	...
Fred	0	1	0	1	0	0	1	1	1	0	0	1	0	1	0	0	...
Gábor	0	1	1	1	0	1	1	1	1	0	1	1	0	0	1	0	...
Henry	0	1	0	1	0	1	1	1	1	0	0	1	0	1	1	0	...

Solution: Embed fingerprints

User	Copyrighted content (fingerprinted)															
Antonino	0	1	1	1	0	0	1	1	1	0	1	1	0	1	0	0	...
Boris	0	1	1	1	0	1	0	1	1	0	1	1	1	1	1	0	...
Caroline	0	1	0	1	0	1	0	1	1	0	0	1	1	0	1	0	...
David	0	1	1	1	0	0	0	1	1	0	1	1	0	0	0	0	...
Eve	0	1	0	1	0	1	0	1	1	0	1	1	1	0	0	0	...
Fred	0	1	0	1	0	0	1	1	1	0	0	1	0	1	0	0	...
Gábor	0	1	1	1	0	1	1	1	1	0	1	1	0	0	1	0	...
Henry	0	1	0	1	0	1	1	1	1	0	0	1	0	1	1	0	...
Copy	0	1	0	1	0	1	0	1	1	0	1	1	1	0	0	0	...

TU/e

Solution: Embed fingerprints

User	Copyrighted content (fingerprinted)															
Antonino	0	1	**1**	1	0	**0**	**1**	1	1	0	**1**	1	**0**	**1**	**0**	0	...
Boris	0	1	**1**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**1**	**1**	0	...
Caroline	0	1	**0**	1	0	**1**	**0**	1	1	0	**0**	1	**1**	**0**	**1**	0	...
David	0	1	**1**	1	0	**0**	**0**	1	1	0	**1**	1	**0**	**0**	**0**	0	...
Eve	0	1	**0**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**0**	**0**	0	...
Fred	0	1	**0**	1	0	**0**	**1**	1	1	0	**0**	1	**0**	**1**	**0**	0	...
Gábor	0	1	**1**	1	0	**1**	**1**	1	1	0	**1**	1	**0**	**0**	**1**	0	...
Henry	0	1	**0**	1	0	**1**	**1**	1	1	0	**0**	1	**0**	**1**	**1**	0	...
Copy	0	1	**0**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**0**	**0**	0	...

Solution: Embed fingerprints

User	Copyrighted content (fingerprinted)															
Antonino	0	1	**1**	1	0	**0**	**1**	1	1	0	**1**	1	**0**	**1**	**0**	0	...
Boris	0	1	**1**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**1**	**1**	0	...
Caroline	0	1	**0**	1	0	**1**	**0**	1	1	0	**0**	1	**1**	**0**	**1**	0	...
David	0	1	**1**	1	0	**0**	**0**	1	1	0	**1**	1	**0**	**0**	**0**	0	...
Eve	0	1	**0**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**0**	**0**	0	...
Fred	0	1	**0**	1	0	**0**	**1**	1	1	0	**0**	1	**0**	**1**	**0**	0	...
Gábor	0	1	**1**	1	0	**1**	**1**	1	1	0	**1**	1	**0**	**0**	**1**	0	...
Henry	0	1	**0**	1	0	**1**	**1**	1	1	0	**0**	1	**0**	**1**	**1**	0	...
Copy	0	1	**0**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**0**	**0**	0	...

Problem: Collusion attacks

User	Copyrighted content (fingerprinted)																
Antonino	0	1	1	1	0	0	1	1	1	0	1	1	0	1	0	0	...
Boris	0	1	1	1	0	1	0	1	1	0	1	1	1	1	1	0	...
Caroline	0	1	0	1	0	1	0	1	1	0	0	1	1	0	1	0	...
David	0	1	1	1	0	0	0	1	1	0	1	1	0	0	0	0	...
Eve	0	1	0	1	0	1	0	1	1	0	1	1	1	0	0	0	...
Fred	0	1	0	1	0	0	1	1	1	0	0	1	0	1	0	0	...
Gábor	0	1	1	1	0	1	1	1	1	0	1	1	0	0	1	0	...
Henry	0	1	0	1	0	1	1	1	1	0	0	1	0	1	1	0	...

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Problem: Collusion attacks

User	Copyrighted content (fingerprinted)															
Antonino	0	1	1	1	0	0	1	1	1	0	1	1	0	1	0	0 ...
Boris	0	1	1	1	0	1	0	1	1	0	1	1	1	1	1	0 ...
Caroline	0	1	0	1	0	1	0	1	1	0	0	1	1	0	1	0 ...
David	0	1	1	1	0	0	0	1	1	0	1	1	0	0	0	0 ...
Eve	0	1	0	1	0	1	0	1	1	0	1	1	1	0	0	0 ...
Fred	0	1	0	1	0	0	1	1	1	0	0	1	0	1	0	0 ...
Gábor	0	1	1	1	0	1	1	1	1	0	1	1	0	0	1	0 ...
Henry	0	1	0	1	0	1	1	1	1	0	0	1	0	1	1	0 ...
Copy	0	1	1	1	0	1	0	1	1	0	1	1	0	1	0	0 ...

TU/e

Problem: Collusion attacks

User	Copyrighted content (fingerprinted)															
Antonino	0	1	**1**	1	0	**0**	**1**	1	1	0	**1**	1	**0**	**1**	**0**	0 ...
Boris	0	1	**1**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**1**	**1**	0 ...
Caroline	0	1	**0**	1	0	**1**	**0**	1	1	0	**0**	1	**1**	**0**	**1**	0 ...
David	0	1	**1**	1	0	**0**	**0**	1	1	0	**1**	1	**0**	**0**	**0**	0 ...
Eve	0	1	**0**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**0**	**0**	0 ...
Fred	0	1	**0**	1	0	**0**	**1**	1	1	0	**0**	1	**0**	**1**	**0**	0 ...
Gábor	0	1	**1**	1	0	**1**	**1**	1	1	0	**1**	1	**0**	**0**	**1**	0 ...
Henry	0	1	**0**	1	0	**1**	**1**	1	1	0	**0**	1	**0**	**1**	**1**	0 ...
Copy	0	1	**1**	1	0	**1**	**0**	1	1	0	**1**	1	**0**	**1**	**0**	0 ...

Solution: Collusion-resistant schemes

User	Copyrighted content (fingerprinted)															
Antonino	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Boris	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Caroline	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
David	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Eve	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Fred	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Gábor	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Henry	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Copy	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...

Solution: Collusion-resistant schemes

User	Copyrighted content (fingerprinted)															
Antonino	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Boris	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Caroline	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
David	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Eve	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Fred	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Gábor	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Henry	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Copy	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...

1. An algorithm to construct collusion-resistant codes

Solution: Collusion-resistant schemes

User	Copyrighted content (fingerprinted)															
Antonino	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Boris	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Caroline	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
David	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Eve	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Fred	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Gábor	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Henry	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Copy	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...

1. An algorithm to construct collusion-resistant codes
2. An algorithm to trace pirate copies to colluders

Solution: Collusion-resistant schemes

User	Copyrighted content (fingerprinted)							
Antonino	?	?	?	?	?	?	?	...
Boris	?	?	?	?	?	?	?	...
Caroline	?	?	?	?	?	?	?	...
David	?	?	?	?	?	?	?	...
Eve	?	?	?	?	?	?	?	...
Fred	?	?	?	?	?	?	?	...
Gábor	?	?	?	?	?	?	?	...
Henry	?	?	?	?	?	?	?	...
Copy	?	?	?	?	?	?	?	...

1. An algorithm to construct collusion-resistant codes
2. An algorithm to trace pirate copies to colluders

Solution: Collusion-resistant schemes

User	Copyrighted content (fingerprinted)	
Antonino		...
Boris		...
Caroline	$X \in \{0,1\}^{n \times \ell}$	...
David		...
Eve		...
Fred		...
Gábor		...
Henry		...
Copy	$y \in \{0,1\}^{\ell}$	...

1. An algorithm to construct collusion-resistant codes
2. An algorithm to trace pirate copies to colluders

Solution: Collusion-resistant schemes

User	Copyrighted content (fingerprinted)	
Antonino		...
Boris		...
Caroline	$X \in \{0,1\}^{n \times \ell}$	...
David		...
Eve		...
Fred	$\left(\text{[Tar03]: } \ell = L \cdot c^2 \log n/\varepsilon\right)$	...
Gábor		...
Henry		...
Copy	$y \in \{0,1\}^{\ell}$	...

1. An algorithm to construct collusion-resistant codes
2. An algorithm to trace pirate copies to colluders

Solution: Collusion-resistant schemes

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Solution: Collusion-resistant schemes

1. An algorithm to construct collusion-resistant codes

2. An algorithm to trace pirate copies to colluders

The Tardos scheme: Overview

1. An algorithm to construct collusion-resistant codes

2. An algorithm to trace pirate copies to colluders

The Tardos scheme: Overview

1. An algorithm to construct collusion-resistant codes
   1a. For each segment i, generate $p_i \sim F$.
      - Many values of p_i close to 0 and 1.
      - Hide choice of p_i from pirates.

2. An algorithm to trace pirate copies to colluders

The Tardos scheme: Overview

1. An algorithm to construct collusion-resistant codes
   1a. For each segment i, generate $p_i \sim F$.
      - Many values of p_i close to 0 and 1.
      - Hide choice of p_i from pirates.

   1b. For each segment i, user j, choose $X_{j,i} = 1$ with prob. p_i.
2. An algorithm to trace pirate copies to colluders

TU/e

The Tardos scheme: Overview

1. An algorithm to construct collusion-resistant codes
   1a. For each segment i, generate $p_i \sim F$.
      - Many values of p_i close to 0 and 1.
      - Hide choice of p_i from pirates.

   1b. For each segment i, user j, choose $X_{j,i} = 1$ with prob. p_i.
2. An algorithm to trace pirate copies to colluders
   2a. For each segment i, user j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.
      - Positive scores ($S_{j,i} > 0$) for matches ($X_{j,i} = y_i$).
      - Negative scores ($S_{j,i} < 0$) for differences ($X_{j,i} \neq y_i$).
      - Large scores ($|S_{j,i}| \gg 0$) for rare events.

The Tardos scheme: Overview

1. An algorithm to construct collusion-resistant codes
   1a. For each segment i, generate $p_i \sim F$.
      - Many values of p_i close to 0 and 1.
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2. An algorithm to trace pirate copies to colluders
   2a. For each segment i, user j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.
      - Positive scores ($S_{j,i} > 0$) for matches ($X_{j,i} = y_i$).
      - Negative scores ($S_{j,i} < 0$) for differences ($X_{j,i} \neq y_i$).
      - Large scores ($|S_{j,i}| \gg 0$) for rare events.

   2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

The Tardos scheme: Codewords

p_i	p_1	p_2	p_3	p_4	p_5	$\cdots$	p_{1200}
Antonino	$X_{1,1}$	$X_{1,2}$	$X_{1,3}$	$X_{1,4}$	$X_{1,5}$	$\cdots$	$X_{1,1208}$
Boris	$X_{2,1}$	$X_{2,2}$	$X_{2,3}$	$X_{2,4}$	$X_{2,5}$	$\cdots$	$X_{2,1208}$
Caroline	$X_{3,1}$	$X_{3,2}$	$X_{3,3}$	$X_{3,4}$	$X_{3,5}$	$\cdots$	$X_{3,1208}$
David	$X_{4,1}$	$X_{4,2}$	$X_{4,3}$	$X_{4,4}$	$X_{4,5}$	$\cdots$	$X_{4,1208}$
Eve	$X_{5,1}$	$X_{5,2}$	$X_{5,3}$	$X_{5,4}$	$X_{5,5}$	$\cdots$	$X_{5,1208}$
Fred	$X_{6,1}$	$X_{6,2}$	$X_{6,3}$	$X_{6,4}$	$X_{6,5}$	$\cdots$	$X_{6,1208}$
Gábor	$X_{7,1}$	$X_{7,2}$	$X_{7,3}$	$X_{7,4}$	$X_{7,5}$	$\cdots$	$X_{7,1208}$
Henry	$X_{8,1}$	$X_{8,2}$	$X_{8,3}$	$X_{8,4}$	$X_{8,5}$	$\cdots$	$X_{8,1208}$
Copy	y_1	y_2	y_3	y_4	y_5	$\cdots$	y_{1208}

The Tardos scheme: Codewords

1a. For each segment i, generate $p_i \sim F$.

p_i	p_1	p_2	p_3	p_4	p_5	$\cdots$	p_{1200}
Antonino	$X_{1,1}$	$X_{1,2}$	$X_{1,3}$	$X_{1,4}$	$X_{1,5}$	$\cdots$	$X_{1,1208}$
Boris	$X_{2,1}$	$X_{2,2}$	$X_{2,3}$	$X_{2,4}$	$X_{2,5}$	$\cdots$	$X_{2,1208}$
Caroline	$X_{3,1}$	$X_{3,2}$	$X_{3,3}$	$X_{3,4}$	$X_{3,5}$	$\cdots$	$X_{3,1208}$
David	$X_{4,1}$	$X_{4,2}$	$X_{4,3}$	$X_{4,4}$	$X_{4,5}$	$\cdots$	$X_{4,1208}$
Eve	$X_{5,1}$	$X_{5,2}$	$X_{5,3}$	$X_{5,4}$	$X_{5,5}$	$\cdots$	$X_{5,1208}$
Fred	$X_{6,1}$	$X_{6,2}$	$X_{6,3}$	$X_{6,4}$	$X_{6,5}$	$\cdots$	$X_{6,1208}$
Gábor	$X_{7,1}$	$X_{7,2}$	$X_{7,3}$	$X_{7,4}$	$X_{7,5}$	$\cdots$	$X_{7,1208}$
Henry	$X_{8,1}$	$X_{8,2}$	$X_{8,3}$	$X_{8,4}$	$X_{8,5}$	$\cdots$	$X_{8,1208}$
Copy	y_1	y_2	y_3	y_4	y_5	$\cdots$	y_{1208}

TU/e

The Tardos scheme: Codewords

1a. For each segment i, generate $p_i \sim F$.

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18
Antonino	$X_{1,1}$	$X_{1,2}$	$X_{1,3}$	$X_{1,4}$	$X_{1,5}$	...	$X_{1,1208}$
Boris	$X_{2,1}$	$X_{2,2}$	$X_{2,3}$	$X_{2,4}$	$X_{2,5}$	...	$X_{2,1208}$
Caroline	$X_{3,1}$	$X_{3,2}$	$X_{3,3}$	$X_{3,4}$	$X_{3,5}$	...	$X_{3,1208}$
David	$X_{4,1}$	$X_{4,2}$	$X_{4,3}$	$X_{4,4}$	$X_{4,5}$	...	$X_{4,1208}$
Eve	$X_{5,1}$	$X_{5,2}$	$X_{5,3}$	$X_{5,4}$	$X_{5,5}$	...	$X_{5,1208}$
Fred	$X_{6,1}$	$X_{6,2}$	$X_{6,3}$	$X_{6,4}$	$X_{6,5}$	...	$X_{6,1208}$
Gábor	$X_{7,1}$	$X_{7,2}$	$X_{7,3}$	$X_{7,4}$	$X_{7,5}$	...	$X_{7,1208}$
Henry	$X_{8,1}$	$X_{8,2}$	$X_{8,3}$	$X_{8,4}$	$X_{8,5}$	...	$X_{8,1208}$
Copy	y_1	y_2	y_3	y_4	y_5	...	y_{1208}

The Tardos scheme: Codewords

1b. For each segment i, user j, choose $X_{j,i} = 1$ with prob. p_i.

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18
Antonino	$X_{1,1}$	$X_{1,2}$	$X_{1,3}$	$X_{1,4}$	$X_{1,5}$	...	$X_{1,1208}$
Boris	$X_{2,1}$	$X_{2,2}$	$X_{2,3}$	$X_{2,4}$	$X_{2,5}$	...	$X_{2,1208}$
Caroline	$X_{3,1}$	$X_{3,2}$	$X_{3,3}$	$X_{3,4}$	$X_{3,5}$	...	$X_{3,1208}$
David	$X_{4,1}$	$X_{4,2}$	$X_{4,3}$	$X_{4,4}$	$X_{4,5}$	...	$X_{4,1208}$
Eve	$X_{5,1}$	$X_{5,2}$	$X_{5,3}$	$X_{5,4}$	$X_{5,5}$	...	$X_{5,1208}$
Fred	$X_{6,1}$	$X_{6,2}$	$X_{6,3}$	$X_{6,4}$	$X_{6,5}$	...	$X_{6,1208}$
Gábor	$X_{7,1}$	$X_{7,2}$	$X_{7,3}$	$X_{7,4}$	$X_{7,5}$	...	$X_{7,1208}$
Henry	$X_{8,1}$	$X_{8,2}$	$X_{8,3}$	$X_{8,4}$	$X_{8,5}$	...	$X_{8,1208}$
Copy	y_1	y_2	y_3	y_4	y_5	...	y_{1208}

TU/e

The Tardos scheme: Codewords

1b. For each segment i, user j, choose $X_{j,i} = 1$ with prob. p_i.

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18
Antonino	0	0	1	1	1	...	0
Boris	1	0	1	1	1	...	1
Caroline	1	0	0	1	0	...	0
David	0	0	1	1	1	...	0
Eve	0	0	1	0	1	...	0
Fred	1	0	1	0	1	...	0
Gábor	0	0	1	0	1	...	0
Henry	1	0	0	0	1	...	0
Copy	y_1	y_2	y_3	y_4	y_5	...	y_{1208}

The Tardos scheme: Coalition

Pirates get their versions, ...

p_i	.	.	.	.	.	...	.
Antonino	.	.	.	.	.	...	.
Boris	.	.	.	.	.	...	.
Caroline	1	0	0	1	0	...	0
David	.	.	.	.	.	...	.
Eve	0	0	1	0	1	...	0
Fred	.	.	.	.	.	...	.
Gábor	.	.	.	.	.	...	.
Henry	1	0	0	0	1	...	0
Copy	y_1	y_2	y_3	y_4	y_5	...	y_{1208}

$$\text{Coalition} = \{\text{Caroline, Eve, Henry}\}$$

The Tardos scheme: Coalition

Pirates get their versions, compare them ...

p_i	.	.	.	.	.	...	.
Antonino	.	.	.	.	.	...	.
Boris	.	.	.	.	.	...	.
Caroline	**1**	0	**0**	**1**	**0**	...	0
David	.	.	.	.	.	...	.
Eve	**0**	0	**1**	**0**	**1**	...	0
Fred	.	.	.	.	.	...	.
Gábor	.	.	.	.	.	...	.
Henry	**1**	0	**0**	**0**	**1**	...	0
Copy	y_1	y_2	y_3	y_4	y_5	...	y_{1208}

Coalition $=$ {Caroline, Eve, Henry}

The Tardos scheme: Coalition

Pirates get their versions, compare them and make a copy.

p_i	.	.	.	.	.	...	.
Antonino	.	.	.	.	.	...	.
Boris	.	.	.	.	.	...	.
Caroline	**1**	0	**0**	**1**	**0**	...	0
David	.	.	.	.	.	...	.
Eve	**0**	0	**1**	**0**	**1**	...	0
Fred	.	.	.	.	.	...	.
Gábor	.	.	.	.	.	...	.
Henry	**1**	0	**0**	**0**	**1**	...	0
Copy	**0**	0	**0**	**1**	**1**	...	0

Coalition $=$ {Caroline, Eve, Henry}

The Tardos scheme: Scores

The copy is distributed and detected by the tracer.

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18
Antonino	0	0	1	1	1	...	0
Boris	1	0	1	1	1	...	1
Caroline	1	0	0	1	0	...	0
David	0	0	1	1	1	...	0
Eve	0	0	1	0	1	...	0
Fred	1	0	1	0	1	...	0
Gábor	0	0	1	0	1	...	0
Henry	1	0	0	0	1	...	0
Copy	0	0	0	1	1	...	0

Coalition = {Caroline, Eve, Henry}

The Tardos scheme: Scores

2a. For each segment i, user j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18
Antonino	0	0	1	1	1	...	0
Boris	1	0	1	1	1	...	1
Caroline	1	0	0	1	0	...	0
David	0	0	1	1	1	...	0
Eve	0	0	1	0	1	...	0
Fred	1	0	1	0	1	...	0
Gábor	0	0	1	0	1	...	0
Henry	1	0	0	0	1	...	0
Copy	0	0	0	1	1	...	0

Coalition = {Caroline, Eve, Henry}

The Tardos scheme: Scores

2a. For each segment i, user j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18
Antonino	+0.5	+0.2	−0.4	+0.5	+0.1	...	+0.5
Boris	−2.0	+0.2	−0.4	+0.5	+0.1	...	−2.1
Caroline	−2.0	+0.2	+2.7	+0.5	−7.2	...	+0.5
David	+0.5	+0.2	−0.4	+0.5	+0.1	...	+0.5
Eve	+0.5	+0.2	−0.4	−1.9	+0.1	...	+0.5
Fred	−2.0	+0.2	−0.4	−1.9	+0.1	...	+0.5
Gábor	+0.5	+0.2	−0.4	−1.9	+0.1	...	+0.5
Henry	−2.0	+0.2	+2.7	−1.9	+0.1	...	+0.5
Copy	0	0	0	1	1	...	0

Coalition = {Caroline, Eve, Henry}

TU/e

The Tardos scheme: Scores

2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18	$\sum_i S_{j,i}$
Antonino	+0.5	+0.2	−0.4	+0.5	+0.1	...	+0.5	0
Boris	−2.0	+0.2	−0.4	+0.5	+0.1	...	−2.1	0
Caroline	−2.0	+0.2	+2.7	+0.5	−7.2	...	+0.5	0
David	+0.5	+0.2	−0.4	+0.5	+0.1	...	+0.5	0
Eve	+0.5	+0.2	−0.4	−1.9	+0.1	...	+0.5	0
Fred	−2.0	+0.2	−0.4	−1.9	+0.1	...	+0.5	0
Gábor	+0.5	+0.2	−0.4	−1.9	+0.1	...	+0.5	0
Henry	−2.0	+0.2	+2.7	−1.9	+0.1	...	+0.5	0
Copy	0	0	0	1	1	...	0	

Coalition = {Caroline, Eve, Henry}

The Tardos scheme: Scores

2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18	$\sum_i S_{j,i}$
Antonino	$+0.5$	$+0.2$	-0.4	$+0.5$	$+0.1$	...	$+0.5$	$+14$
Boris	-2.0	$+0.2$	-0.4	$+0.5$	$+0.1$	...	-2.1	-19
Caroline	-2.0	$+0.2$	$+2.7$	$+0.5$	-7.2	...	$+0.5$	$+291$
David	$+0.5$	$+0.2$	-0.4	$+0.5$	$+0.1$	...	$+0.5$	$+29$
Eve	$+0.5$	$+0.2$	-0.4	-1.9	$+0.1$	...	$+0.5$	$+292$
Fred	-2.0	$+0.2$	-0.4	-1.9	$+0.1$	...	$+0.5$	-53
Gábor	$+0.5$	$+0.2$	-0.4	-1.9	$+0.1$	...	$+0.5$	-42
Henry	-2.0	$+0.2$	$+2.7$	-1.9	$+0.1$	...	$+0.5$	$+269$
Copy	0	0	0	1	1	...	0	

Coalition $=$ {Caroline, Eve, Henry}

TU/e

The Tardos scheme: Scores

2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18	$\sum_i S_{j,i}$
Antonino	$+0.5$	$+0.2$	-0.4	$+0.5$	$+0.1$	...	$+0.5$	$+14$
Boris	-2.0	$+0.2$	-0.4	$+0.5$	$+0.1$	...	-2.1	-19
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Fred	-2.0	$+0.2$	-0.4	-1.9	$+0.1$	...	$+0.5$	-53
Gábor	$+0.5$	$+0.2$	-0.4	-1.9	$+0.1$	...	$+0.5$	-42
Henry	-2.0	$+0.2$	$+2.7$	-1.9	$+0.1$	...	$+0.5$	$+269$
Copy	0	0	0	1	1	...	0	

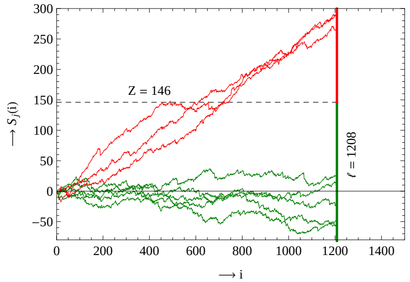
$$\text{Coalition} = \{\text{Caroline, Eve, Henry}\}$$

$$\text{Accused} = \{\text{Caroline, Eve, Henry}\}$$

The Tardos scheme: Scores

2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18	$\sum_i S_{j,i}$
Antonino								$+14$
Boris								-19
Caroline								$+291$
David								$+29$
Eve								$+292$
Fred								-53
Gábor								-42
Henry								$+269$
Copy	0	0	0	1	1	...	0	



Coalition = {Caroline, Eve, Henry}

Accused = {Caroline, Eve, Henry}

The Tardos scheme: Overview

1. An algorithm to construct collusion-resistant codes
   - 1a. For each segment i, generate $p_i \sim F$.
     - Many values of p_i close to 0 and 1.
     - Hide choice of p_i from pirates.
   - 1b. For each segment i, user j, choose $X_{j,i} = 1$ with prob. p_i.
2. An algorithm to trace pirate copies to colluders
   - 2a. For each segment i, user j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.
     - Positive scores ($S_{j,i} > 0$) for matches ($X_{j,i} = y_i$).
     - Negative scores ($S_{j,i} < 0$) for differences ($X_{j,i} \neq y_i$).
     - Large scores ($|S_{j,i}| \gg 0$) for rare events.
   - 2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

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1. An algorithm to construct collusion-resistant codes
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      - Negative scores ($S_{j,i} < 0$) for differences ($X_{j,i} \neq y_i$).
      - Large scores ($|S_{j,i}| \gg 0$) for rare events.

   2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

What is the best choice for F and g?

The Tardos scheme: Improvements

Table: Constructive upper bounds on L (large c asymptotics).

	[Tar03]	[S+06]	[N+07]	[BT08]	[S+08]	[N+09]	[LdW12]	[LdW13]	[O+13]
L :	100	39.5	20.6	19.7	9.9	5.4	4.9	4.9*	2
g :	g^{Tar}	g^{Tar}	g^{Tar}	g^{Tar}	g^{sym}	g^{sym}	g^{sym}	g^{sym}	g^{int}
F :	F^{arc}	F^{arc}	F^{*}	F^{arc}	F^{arc}	F^{*}	F^{arc}	F^{*}	F^{arc}

The Tardos scheme: Improvements

Table: Constructive upper bounds on L (large c asymptotics).

	[Tar03]	[S+06]	[N+07]	[BT08]	[S+08]	[N+09]	[LdW12]	[LdW13]	[O+13]
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g :	g^{Tar}	g^{Tar}	g^{Tar}	g^{Tar}	g^{sym}	g^{sym}	g^{sym}	g^{sym}	g^{int}
F :	F^{arc}	F^{arc}	F^{*}	F^{arc}	F^{arc}	F^{*}	F^{arc}	F^{*}	F^{arc}

$$g^{\mathrm{Tar}}(X_{j,i}, y_i, p_i) = \begin{cases} 0, & \text{if } X_{j,i} = 0, y_i = 0, \\ -\sqrt{p_i/(1-p_i)}, & \text{if } X_{j,i} = 0, y_i = 1, \\ 0, & \text{if } X_{j,i} = 1, y_i = 0, \\ +\sqrt{(1-p_i)/p_i}, & \text{if } X_{j,i} = 1, y_i = 1. \end{cases}$$

The Tardos scheme: Improvements

Table: Constructive upper bounds on L (large c asymptotics).

	[Tar03]	[S+06]	[N+07]	[BT08]	[S+08]	[N+09]	[LdW12]	[LdW13]	[O+13]
L :	100	39.5	20.6	19.7	9.9	5.4	4.9	4.9*	2
g :	g^{Tar}	g^{Tar}	g^{Tar}	g^{Tar}	g^{sym}	g^{sym}	g^{sym}	g^{sym}	g^{int}
F :	F^{arc}	F^{arc}	F^{*}	F^{arc}	F^{arc}	F^{*}	F^{arc}	F^{*}	F^{arc}

$$g^{\mathrm{sym}}(X_{j,i}, y_i, p_i) = \begin{cases} +\sqrt{p_i/(1-p_i)}, & \text{if } X_{j,i} = 0, y_i = 0, \\ -\sqrt{p_i/(1-p_i)}, & \text{if } X_{j,i} = 0, y_i = 1, \\ -\sqrt{(1-p_i)/p_i}, & \text{if } X_{j,i} = 1, y_i = 0, \\ +\sqrt{(1-p_i)/p_i}, & \text{if } X_{j,i} = 1, y_i = 1. \end{cases}$$

TU/e

The Tardos scheme: Improvements

Table: Constructive upper bounds on L (large c asymptotics).

	[Tar03]	[S+06]	[N+07]	[BT08]	[S+08]	[N+09]	[LdW12]	[LdW13]	[O+13]
L :	100	39.5	20.6	19.7	9.9	5.4	4.9	4.9*	2
g :	g^{Tar}	g^{Tar}	g^{Tar}	g^{Tar}	g^{sym}	g^{sym}	g^{sym}	g^{sym}	g^{int}
F :	F^{arc}	F^{arc}	F^{*}	F^{arc}	F^{arc}	F^{*}	F^{arc}	F^{*}	F^{arc}

$$g^{\mathrm{int}}(X_{j,i}, y_i, p_i) = \begin{cases} +p_i/(1-p_i), & \text{if } X_{j,i} = 0, y_i = 0, \\ -1, & \text{if } X_{j,i} = 0, y_i = 1, \\ -1, & \text{if } X_{j,i} = 1, y_i = 0, \\ +(1-p_i)/p_i, & \text{if } X_{j,i} = 1, y_i = 1. \end{cases}$$

The Tardos scheme: Comparison

Table: Constructive upper bounds on L (large c asymptotics).

	[Tar03]	[S+06]	[N+07]	[BT08]	[S+08]	[N+09]	[LdW12]	[LdW13]	[O+13]
L :	100	39.5	20.6	19.7	9.9	5.4	4.9	4.9^*	2
g :	g^{Tar}	g^{Tar}	g^{Tar}	g^{Tar}	g^{sym}	g^{sym}	g^{sym}	g^{sym}	g^{int}
F :	F^{arc}	F^{arc}	F^*	F^{arc}	F^{arc}	F^*	F^{arc}	F^*	F^{arc}

Table: Information-theoretic lower bounds on L (large c asymptotics).

	[Tar03]	[AT09]	[HM09]	[HM12]
L :	$\Omega(1)$	"2"	2?	2
F :	?	?	F^{arc}?	F^{arc}
ρ :	?	?	int?	int

The Tardos scheme: Comparison

Table: Constructive upper bounds on L (large c asymptotics).

	[Tar03]	[S+06]	[N+07]	[BT08]	[S+08]	[N+09]	[LdW12]	[LdW13]	[O+13]
L :	100	39.5	20.6	19.7	9.9	5.4	4.9	4.9^*	**2**
g :	g^{Tar}	g^{Tar}	g^{Tar}	g^{Tar}	g^{sym}	g^{sym}	g^{sym}	g^{sym}	g^{int}
F :	F^{arc}	F^{arc}	F^*	F^{arc}	F^{arc}	F^*	F^{arc}	F^*	F^{arc}

Table: Information-theoretic lower bounds on L (large c asymptotics).

	[Tar03]	[AT09]	[HM09]	[HM12]
L :	$\Omega(1)$	"2"	2?	**2**
F :	?	?	F^{arc}?	F^{arc}
ρ :	?	?	int?	int

Efficient capacity-achieving construction

Main result: Using the score function $g = g^{\mathrm{int}}$ given by

$$g^{\mathrm{int}}(X_{j,i}, y_i, p_i) = \begin{cases} +p_i/(1-p_i), & \text{if } X_{j,i} = 0, y_i = 0, \\ -1, & \text{if } X_{j,i} = 0, y_i = 1, \\ -1, & \text{if } X_{j,i} = 1, y_i = 0, \\ +(1-p_i)/p_i, & \text{if } X_{j,i} = 1, y_i = 1. \end{cases}$$

and the distribution function $F = F^{\mathrm{arc}}$ given by

$$F^{\mathrm{arc}}(p_i) = \frac{2}{\pi} \arcsin \sqrt{p_i}$$

we get an efficient construction that achieves capacity for large c.

Static vs. Dynamic

Previous slides dealt with the *static setting*:

- First generate X, then y, then trace.
- Static content: video on demand, software, ...
- Optimized Tardos scheme achieves capacity.

In some practical scenarios, we have a *dynamic setting*:

- $X_1, y_1, X_2, y_2, \ldots, X_\ell, y_\ell$.
- Dynamic content: live streams of baseball, football, ...
- More efficient schemes?
- What is the capacity?

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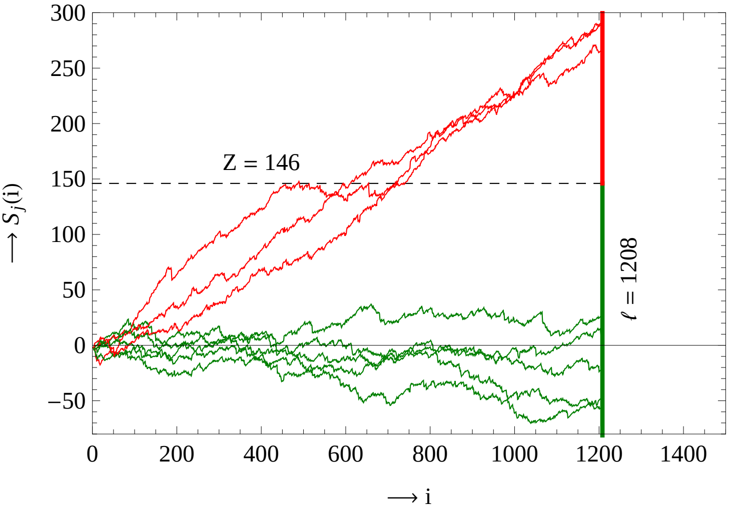
- Dynamic content: live streams of baseball, football, ...
- More efficient schemes?
- What is the capacity?

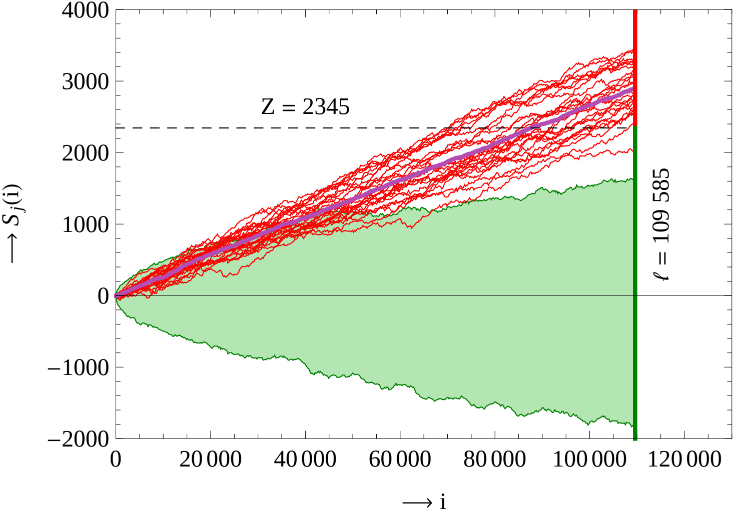
The static Tardos scheme

Earlier example, with $n = 8$ users and $c = 3$ colluders.



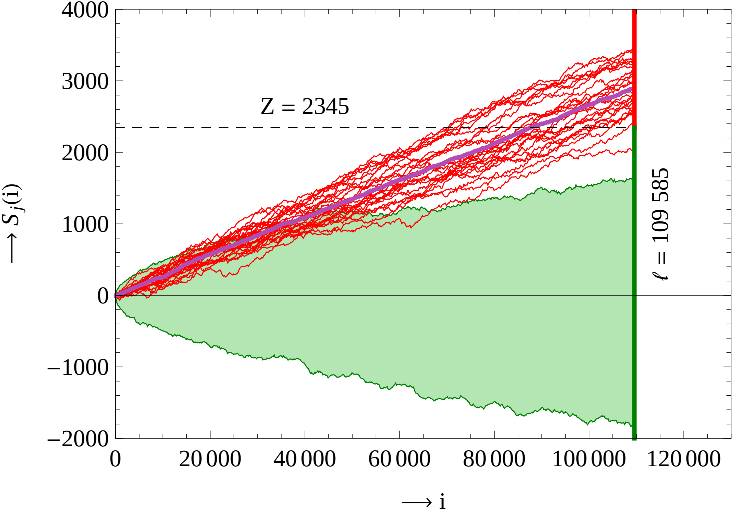
The static Tardos scheme

Bigger example, with $n = 10^6$ users and $c = 25$ colluders.



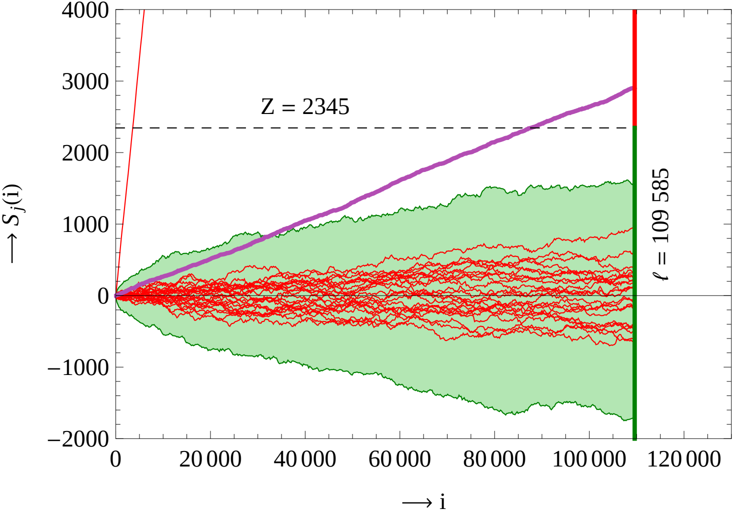
The static Tardos scheme

Catch many, sometimes.



The static Tardos scheme

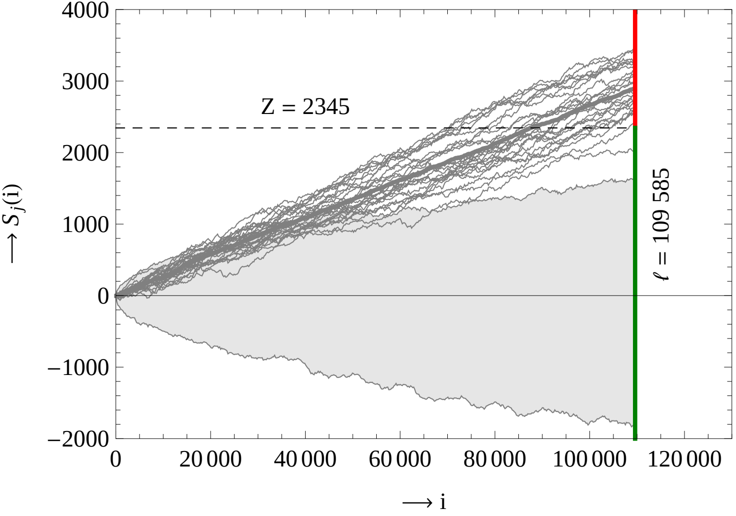
Catch many, sometimes. Catch one, worst-case.



TU/e

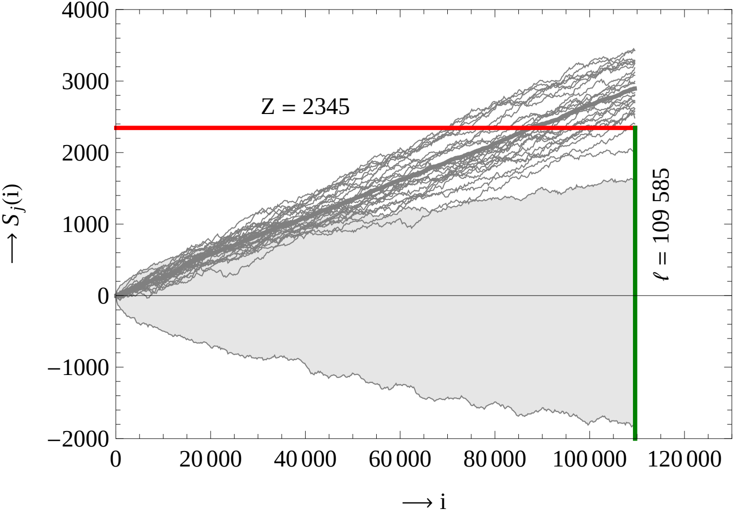
The static Tardos scheme

Instead of disconnecting pirates at the end...



The dynamic Tardos scheme

Instead of disconnecting pirates at the end, disconnect them asap!



The dynamic Tardos scheme

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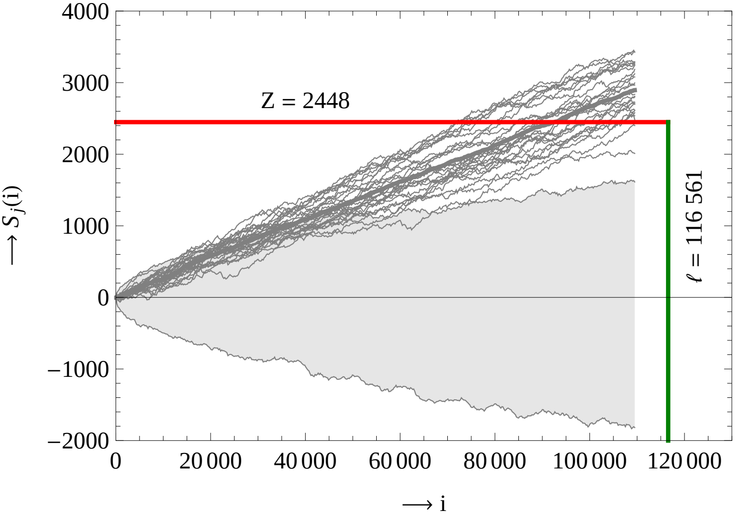
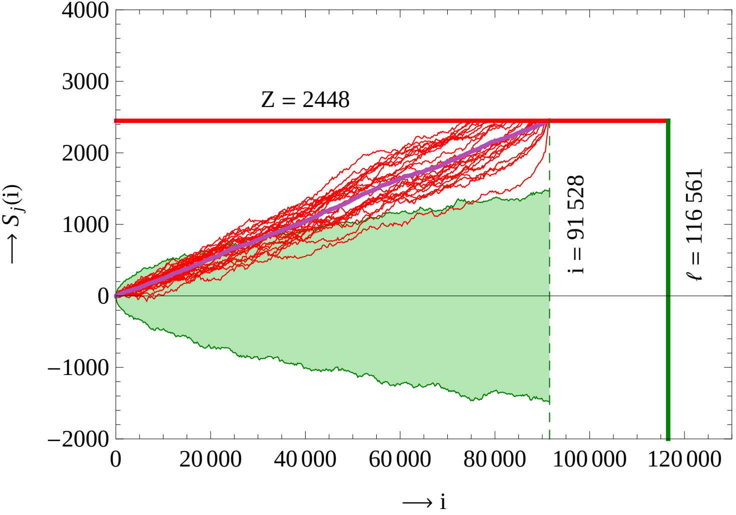


TU/e

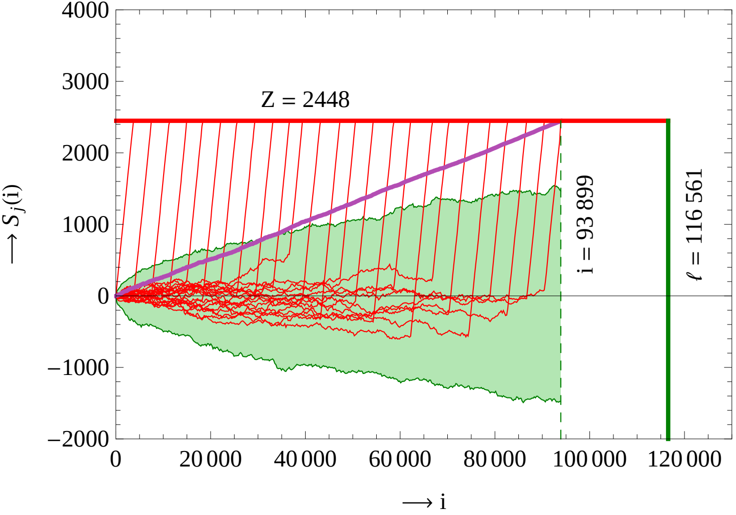
The dynamic Tardos scheme

Catch all, worst-case!



The dynamic Tardos scheme

Catch all, worst-case!



The dynamic Tardos scheme

Main result: By disconnecting users as soon as their scores are too large, we can catch *all pirates* in the worst-case with equivalent code lengths as in the static setting.

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In some practical scenarios, we have a *dynamic setting*:

- $X_1, y_1, X_2, y_2, \dots, X_\ell, y_\ell$.
- Dynamic content: live streams of cricket, football, ...
- More efficient schemes? The dynamic Tardos scheme!
- What is the capacity?

Static vs. Dynamic

Previous slides dealt with the *static setting*:

- First generate X, then y, then trace.
- Static content: video on demand, software, ...
- Optimized Tardos scheme achieves capacity.

In some practical scenarios, we have a *dynamic setting*:

- $X_1, y_1, X_2, y_2, \ldots, X_\ell, y_\ell$.
- Dynamic content: live streams of cricket, football, ...
- More efficient schemes? The dynamic Tardos scheme!
- What is the capacity?

Problem: Blood testing

User								
Antonino	1	0	0	0	0	0	0	0
Boris	0	1	0	0	0	0	0	0
Caroline	0	0	1	0	0	0	0	0
David	0	0	0	1	0	0	0	0
Eve	0	0	0	0	1	0	0	0
Fred	0	0	0	0	0	1	0	0
Gábor	0	0	0	0	0	0	1	0
Henry	0	0	0	0	0	0	0	1

Problem: Blood testing

User								
Antonino	1	0	0	0	0	0	0	0
Boris	0	1	0	0	0	0	0	0
Caroline	0	0	1	0	0	0	0	0
David	0	0	0	1	0	0	0	0
Eve	0	0	0	0	1	0	0	0
Fred	0	0	0	0	0	1	0	0
Gábor	0	0	0	0	0	0	1	0
Henry	0	0	0	0	0	0	0	1

Problem: Blood testing

User								
Antonino	1	0	0	0	0	0	0	0
Boris	0	1	0	0	0	0	0	0
Caroline	0	0	1	0	0	0	0	0
David	0	0	0	1	0	0	0	0
Eve	0	0	0	0	1	0	0	0
Fred	0	0	0	0	0	1	0	0
Gábor	0	0	0	0	0	0	1	0
Henry	0	0	0	0	0	0	0	1
Result	0	0	1	0	0	0	0	0

Solution: Using pools

User								
Antonino	0	0	0					
Boris	0	0	1					
Caroline	0	1	0					
David	0	1	1					
Eve	1	0	0					
Fred	1	0	1					
Gábor	1	1	0					
Henry	1	1	1					

Solution: Using pools

User								
Antonino	0	0	0					
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Fred	1	0	1					
Gábor	1	1	0					
Henry	1	1	1					
Result	0	1	0					

Problem: Several defectives

User								
Antonino	0	0	0					
Boris	0	0	1					
Caroline	0	1	0					
David	0	1	1					
Eve	1	0	0					
Fred	1	0	1					
Gábor	1	1	0					
Henry	1	1	1					

Problem: Several defectives

User								
Antonino	0	0	0					
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Henry	1	1	1					

Problem: Several defectives

User								
Antonino	0	0	0					
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Caroline	0	1	0					
David	0	1	1					
Eve	1	0	0					
Fred	1	0	1					
Gábor	1	1	0					
Henry	1	1	1					
Result	1	1	0					

Traitor tracing: All-1 attack

User								
Antonino	0	0	0					
Boris	0	0	1					
Caroline	0	1	0					
David	0	1	1					
Eve	1	0	0					
Fred	1	0	1					
Gábor	1	1	0					
Henry	1	1	1					
Result	1	1	0					

The Tardos scheme

1. An algorithm to construct collusion-resistant codes
   1a. For each segment i, generate $p_i \sim F$.

$$F(p_i) = \tfrac{2}{\pi} \arcsin \sqrt{p_i}$$

   1b. For each segment i, user j, choose $X_{j,i} = 1$ with prob. p_i.
2. An algorithm to trace pirate copies to colluders
   2a. For each segment i, user j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.

$$g(X_{j,i}, y_i, p_i) = \begin{cases} +p_i/(1-p_i), & \text{if } X_{j,i} = 0, y_i = 0, \\ -1, & \text{if } X_{j,i} = 0, y_i = 1, \\ -1, & \text{if } X_{j,i} = 1, y_i = 0, \\ +(1-p_i)/p_i, & \text{if } X_{j,i} = 1, y_i = 1. \end{cases}$$

   2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

TU/e

A group testing scheme

1. An algorithm to construct collusion-resistant codes
   1a. For each segment i, generate $p_i \sim F$.

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   2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

A group testing scheme: Terminology

1. An algorithm to construct the group testing matrix
   - 1a. For each test i, generate $p_i \sim F$.

$$F(p_i) = \frac{2}{\pi} \arcsin \sqrt{p_i}.$$

   - 1b. For each test i, person j, choose $X_{j,i} = 1$ with prob. p_i.
2. An algorithm to trace test results to defectives
   - 2a. For each test i, person j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.

$$g(X_{j,i}, y_i, p_i) = \begin{cases} +p_i/(1-p_i), & \text{if } X_{j,i} = 0, y_i = 0, \\ -1, & \text{if } X_{j,i} = 0, y_i = 1, \\ -1, & \text{if } X_{j,i} = 1, y_i = 0, \\ +(1-p_i)/p_i, & \text{if } X_{j,i} = 1, y_i = 1. \end{cases}$$

   - 2b. For each person j, mark person j iff $\sum_i S_{j,i}$ is "large".

A group testing scheme: Optimization

1. An algorithm to construct the group testing matrix
   - 1a. For each test i, generate $p_i \sim F$.

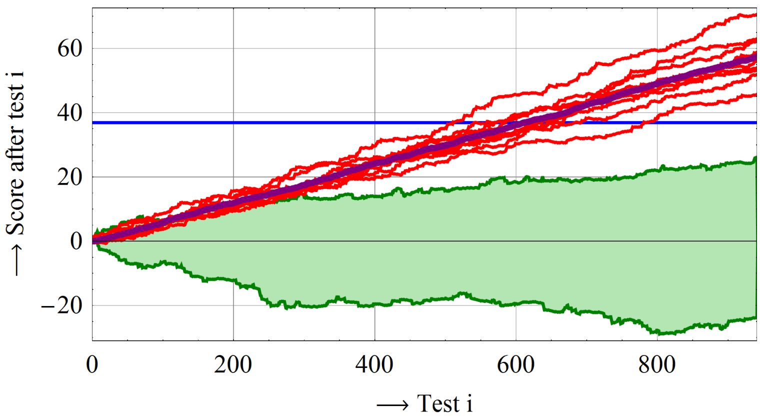
$$F(p_i) = H(p_i - p). \quad (p \approx \tfrac{1}{c})$$

   - 1b. For each test i, person j, choose $X_{j,i} = 1$ with prob. p_i.
2. An algorithm to trace test results to defectives
   - 2a. For each test i, person j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.

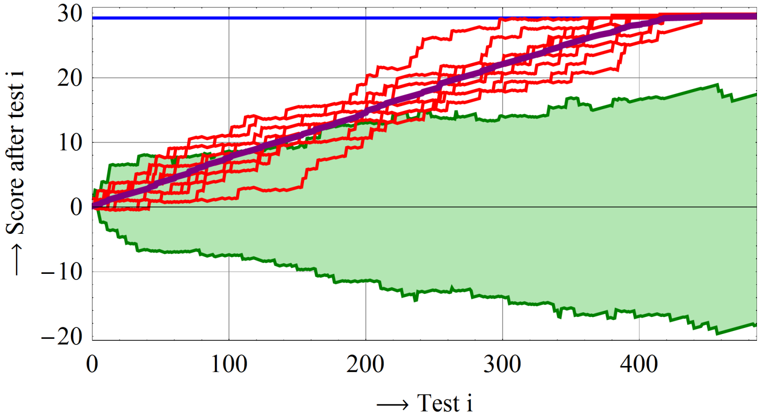
$$g(X_{j,i}, y_i, p_i) = \begin{cases} +p_i/(1-p_i), & \text{if } X_{j,i} = 0, y_i = 0, \\ -p_i(1-p_i)^{c-1}/(1-(1-p_i)^c), & \text{if } X_{j,i} = 0, y_i = 1, \\ -1, & \text{if } X_{j,i} = 1, y_i = 0, \\ +(1-p_i)^c/(1-(1-p_i)^c), & \text{if } X_{j,i} = 1, y_i = 1. \end{cases}$$

   - 2b. For each person j, mark person j iff $\sum_i S_{j,i}$ is "large".

A group testing scheme: Static



A group testing scheme: Dynamic



Applications to group testing

First result: Using the score function $g = g^{\mathrm{all}-1}$ given by

$$g^{\mathrm{all}-1}(X_{j,i}, y_i, p_i) = \begin{cases} +p_i/(1-p_i), & \text{if } X_{j,i} = 0, y_i = 0, \\ -p_i(1-p_i)^{c-1}/(1-(1-p_i)^c), & \text{if } X_{j,i} = 0, y_i = 1, \\ -1, & \text{if } X_{j,i} = 1, y_i = 0, \\ +(1-p_i)^c/(1-(1-p_i)^c), & \text{if } X_{j,i} = 1, y_i = 1. \end{cases}$$

and taking $p_i \equiv p \approx \frac{1}{c}$, we get a group testing scheme that uses

$$\ell \sim 2c \log n$$

group tests to find all c ill people with probability at least $1 - \varepsilon$.

Applications to group testing

Second result: By optimizing g and p, we get a framework for *arbitrary* group testing models:

- Noisy group testing, dilution: $\ell \sim 2c \log n/(1-r)$
- Noisy group testing, additive: $\ell \sim 2c \log n/(1-\sqrt{2r})$
- ...
- Threshold group testing, majority: $\ell \sim \pi c \log n$
- Threshold group testing, Bernoulli: $\ell \sim 4c \log n$
- Threshold group testing, linear gap: $\ell \sim 2c^2 \log n$
- ...

Applications to group testing

Second result: By optimizing g and p, we get a framework for *arbitrary* group testing models:

- Noisy group testing, dilution: $\ell \sim 2c \log n/(1-r)$
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- Threshold group testing, linear gap: $\ell \sim 2c^2 \log n$
- ...

We can combine this with the dynamic Tardos idea to get more efficient *adaptive* group testing schemes as well!

Conclusion

Progress in Traitor Tracing...

- Outline of the Tardos scheme
- A capacity-achieving score function
- A dynamic version of Tardos' scheme
- Open problem: Dynamic capacity?

...with Applications to Group Testing

- Construction based on the Tardos scheme
- Optimized bias (p) and score function
- Improves upon best known results
- Framework can be applied to other models
- Can be improved in the adaptive setting

Questions?