

# Collusion-resistant fingerprinting schemes

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# Problem 1 - Illegal redistribution

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## Half-Life 2 code leaked online

The makers of the eagerly awaited Half-Life 2 have appealed for help to track down who leaked the source code of the game on the internet.

The software is not the full game but contains core information about it.

Valve, the makers of Half-Life 2, said the leak followed a concerted hacking effort on the company's computers over a number of months.

The new game had originally been due for release at the end of September before being knocked back to Christmas.

But the leak of the source code of Half-Life 2 has raised fears of a further postponement.

Developers Valve have confirmed that the software that has appeared on the net is indeed the computer code behind the game.



The game is eagerly awaited by fans



## Harry Potter film excerpt leaked online

A 36-minute clip of the latest Harry Potter movie has been leaked online ahead of its international release.

Warner Bros said it was "working actively" to remove the video, which it said was "stolen and illegally posted" on file-sharing websites on Tuesday.

"We are vigorously investigating this matter and will prosecute those involved to the full extent of the law," it added in a statement.



Harry Potter and the Deathly Hallows Part 1 is out in cinemas this week

## Huge WikiLeaks release shows US 'ignored Iraq torture'

Wikileaks has released almost 400,000 secret US military logs, which suggest US commanders ignored evidence of torture by the Iraqi authorities.

The documents also suggest "hundreds" of civilians were killed at US military checkpoints after the invasion in 2003.

And the files show the US kept records of civilian deaths, despite previously denying it. The death toll was put at 109,000, of whom 66,081 were civilians.

## Miljoenennota uitgelekt via RTL

Uitgegeven: 15 september 2007 19:59

Laatst gewijzigd: 15 september 2007 21:02

DEN HAAG - De miljoenennota is zaterdag uitgelekt via RTL Nieuws. Evenals in enkele voorgaande jaren slaagde de redactie van het programma erin het stuk te pakken te krijgen voor Prinsjesdag.



Sinds vrijdag beschikken de fractievoorzitters in de Eerste en de Tweede Kamer over een exemplaar onder embargo. De andere Kamerleden krijgen pas dinsdagochtend een embargo-exemplaar, evenals de pers.

Dinsdagmiddag presenteert minister Wouter Bos van Financiën het stuk in de Tweede Kamer. Vorige jaar lekte de miljoenennota niet uit. Toen werden er geen embargo-exemplaren verstrekt.

# Solution 1 - Embed watermarks

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Add unique fingerprints (watermarks) to each copy.

**PvdA-Kamerlid Paul Tang bekent lekken begroting**

Uitgegeven: 14 september 2009 16:20  
Laatst gewijzigd: 15 september 2009 14:48

DEN HAAG - Tweede Kamerlid en financieel woordvoerder Paul Tang van de PvdA heeft bekend dat hij begrotingscijfers voor 2010 heeft gelekt aan RTL Nieuws.

 Tang heeft hiervorig maandag zijn excuses aangeboden aan Kamervoorzitter Gerd Verbeet. Hij zal ook zijn verontschuldigingen aanbieden aan premier Jan Peter Balkenende.

Het fractiebestuur keurt het lekken af en straf Tang daarvoor. Het Kamerlid mag een maand lang niet het woord voeren op zijn beleidsterrein.

 **GeenStijl**

De website [GeenStijl](#) ontdekte bij uitvergrooting van de gelekte stukken op de website van RTL Nieuws dat het watermerk 'PvdA' naar voren kwam.

Daarop bekende Tang dat hij Macro-economische Verkenningen aan RTL heeft gegeven.

Tabel 1.1 Kennisgevens voor Nederland, 2006-2010

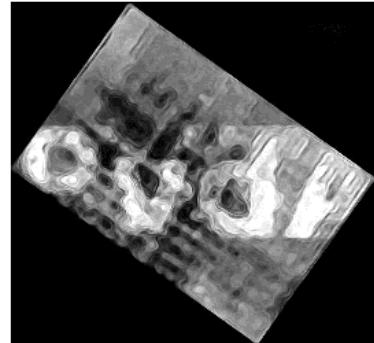
	2006 <sup>a</sup>	2007	2008	2009	2010	
natuurlijke concurrerende markten in % per jaar						
Relevante wettelijk domein	0,5	6,2	1,1	-14,6	-2,6	
Prijselastischheidsefficiëntie	3,5	1,9	4,5	-8,6	-1,6	
Concurrentieprijs	4,5	1,9	4,3	-1%	-1	
Oekapitaal (real gross in miljarden euro)	85,4	72,5	95,9	18	16	
Bruikbaar bedrijf (miljard euro)	1,26	1,87	1,47	1,37	1,40	
Lage rente (leven in %)	3,6	4,3	4,9	3%	4	
Volumen bestedingen in buitenlandse handel						
Bruikbaar handelsproduct (bhp)	3,4	3,6	2,0	-4%	0	
Concurrentie (handelsdiensten)	-0,2	(2,7)	1,7	1,0	-2%	-5%
Overheidsondersteuning	0,9	(2,7)	3,6	2,5	2%	1%
Bruto investeringen bedrijven (exclusief woningbouw)	9,7	5,2	7,9	-14%	-6%	
Uitvoer van producten (exclusief energie)	9,8	6,0	10,0	-19%	2	
w.t.b. voor eigen geproduceerde energie	4,7	5,0	1,0	-14%	1%	
Importenergie	14,1	10,9	3,6	-32%	4%	
Import van goederen	10,0	6,4	3,7	-31%	1%	
Prijzen, lenen en kooprechten						
Prijselastischheidsefficiëntie (exclusief energie)	1,2	1,6	2,0	-2%	-1%	
Prijsverminderingsefficiëntie <sup>b</sup>	1,2	-1,4	0,2	1%	0	
Concurrentieprijsvermindering (cp)	1,1	1,6	2,9	1	1	
Concurrentieprijsverhoging (cp)	4,0	1,6	1,5	-5%	1%	
Latensem per arbeidsterpunt in sector	2,6	(2,8)	3,3	3%	2%	
Koepelrecht, mediasia uit handelsdiensten	2,6	2,2	-0,1	1%	-1%	
Aanbiedmarkten						
Receptiecapaciteit (personen)	1,2	(0,9)	1,6	1,6	0	
Werktuinen berekeningssleutel	2,6	(2,0)	2,6	2,1	-1%	-2%
Werktuinen berekeningssleutel (leven in %)	5,6	4,5	3,9	5%	8	
Werktuinen berekeningssleutel (leven in dot personen)	413	344	304	405	616	
Marktdeel%						
Produktie	4,8	4,7	2,1	-5%	-1%	
Arbeidsproductiviteit	2,7	(2,8)	1,9	0,9	-3%	5%
Werkgelegenheid in arbeidssector	1,9	(1,8)	2,7	1,2	-5%	-5%
Prijsverminderingsefficiëntie waarde	-0,5	0,3	1,8	-4%	1	
Productie arbeidsuren	3,2	(3,1)	2,9	2,2	-5%	1%
natuurlijke concurrerende markten in %						
Acces en kennisvaardigheden	77,8	78,4	79,0	81%	79%	
Werkzaamheden	14,1	14,6	13,2	3%	12	
Collectieve financiering						
EML-krediet (% bhp)	0,5	0,2	0,7	-4,6	-6,2	
EML-krediet (% bhp)	47,4	45,5	56,2	93%	65,0	
Collectieve kosten (% bhp)	39,0	38,9	38,1	38,3	38,0	

<sup>a</sup> Cijfers tussen haakjes zijn berekend voor de financiële sector en wijzigd als gevolg van de invulling van de wet vla en zw.

<sup>b</sup> Concurrentieprijsvermindering via concurrentieprijsverhoging.

<sup>c</sup> Indirect arbeid, oog, gezichtsbescherming en orenbescherming.

<sup>d</sup> Werktuinen in Mees tot, maar zonder enkel werktuinen voor bewerkingsoptimalisatie.

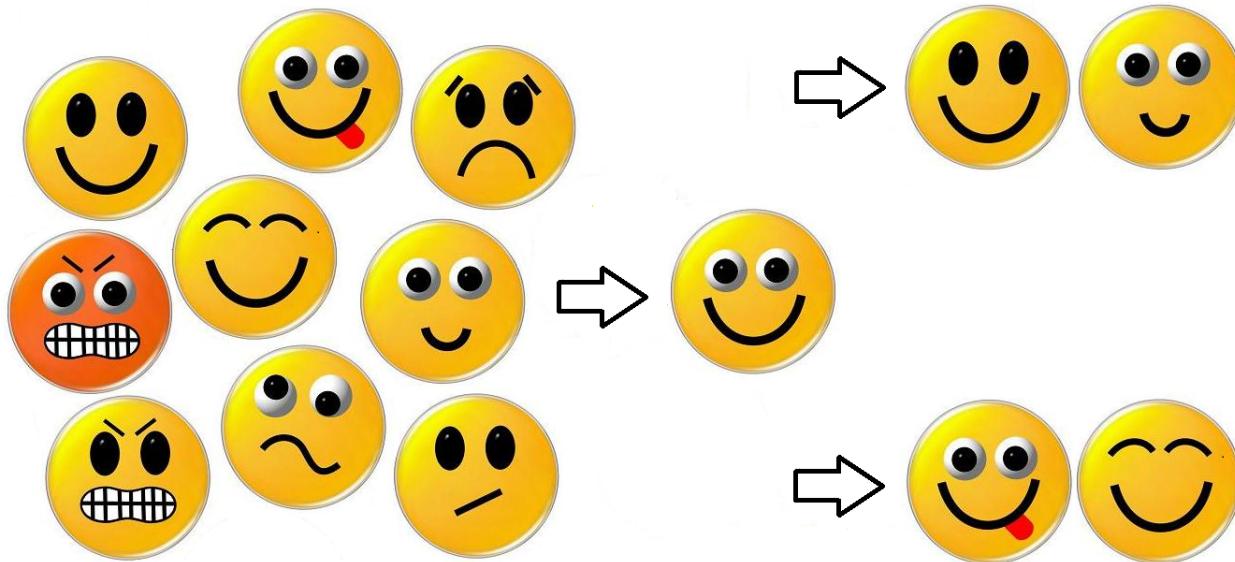


 **PvdA**

This works only if it is hard to detect, edit and/or remove the watermarks.

# Problem 2 - Collusion-attacks

Colluders compare their copies, searching for differences. Since their data is the same, the differences must be part of the watermark.



Colluders can then detect and edit that part of the fingerprint, making it hard to trace them.



What makes the problem so hard?

- If the fingerprints are very different, then it is easy for colluders to detect and edit big parts of the watermark
- If the fingerprints are very similar, then it is hard to distinguish between users and get accurate accusations

But using smart mathematical techniques, we can construct fingerprinting schemes resistant against collusion attacks.

- Set of users  $U = \{1, \dots, n\}$
- Coalition or traitors  $C = \{j_1, \dots, j_c\} \subseteq U$
- Fingerprinting code  $\mathcal{X}$ : Codewords (vectors) over some alphabet  $Q$ 
  - Alphabet size:  $q$  symbols ( $|Q| = q$ )
  - Codelength:  $\ell$  positions ( $\vec{x} \in Q^\ell$ )
  - Cardinality:  $n$  users ( $\mathcal{X} = \{\vec{x}_1, \dots, \vec{x}_n\}$ )
- Code  $\mathcal{X}$  in matrix form:  $X \in Q^{n \times \ell}$

$$X = \begin{pmatrix} \leftarrow & \vec{x}_1 & \rightarrow \\ & \vdots & \\ \leftarrow & \vec{x}_n & \rightarrow \end{pmatrix} \text{ e.g. } X = \begin{pmatrix} 0 & 2 & 1 & 1 & 2 & 1 & 3 \\ 3 & 1 & 2 & 0 & 0 & 0 & 2 \\ 3 & 3 & 2 & 0 & 1 & 0 & 1 \\ 2 & 3 & 1 & 2 & 2 & 2 & 1 \end{pmatrix} \in \{0, \dots, 3\}^{4 \times 7}$$

- Coalition generates forgery  $\vec{y}$  using some pirate strategy  $\rho$

Assumptions on what pirates can do:

- If a coalition sees symbols  $S \subseteq Q$  on position  $i$  ( $|S| > 1$ ), then...
  - Restricted digit model:  $\vec{y}_i \in S$ .
  - Arbitrary digit model:  $\vec{y}_i \in Q$ .
  - Allowing erasures:  $\vec{y}_i \in S \cup \{?\}$  (or  $\vec{y}_i \in Q \cup \{?\}$ )
  - Binary alphabet: All equivalent
- Marking Assumption: If  $S = \{\sigma\}$  then  $\vec{y}_i = \sigma$
- Secret embedding of fingerprints in data is perfect (not our problem)

Example:

$$\begin{aligned} X(C) &= \begin{pmatrix} 0 & 2 & \mathbf{1} & 1 & \mathbf{2} & 1 & 3 \\ 2 & 3 & \mathbf{1} & 2 & \mathbf{2} & 2 & 1 \end{pmatrix} \\ \vec{y} &= (0 \ 3 \ \mathbf{1} \ 2 \ \mathbf{2} \ 1 \ 3) \end{aligned}$$

Besides these assumptions, pirates can do anything they want. Suppose  $q = 2$  and  $0 < k < c$  is the number of ones seen by  $C$  ( $k = 0$  or  $k = c$ : Marking Assumption).

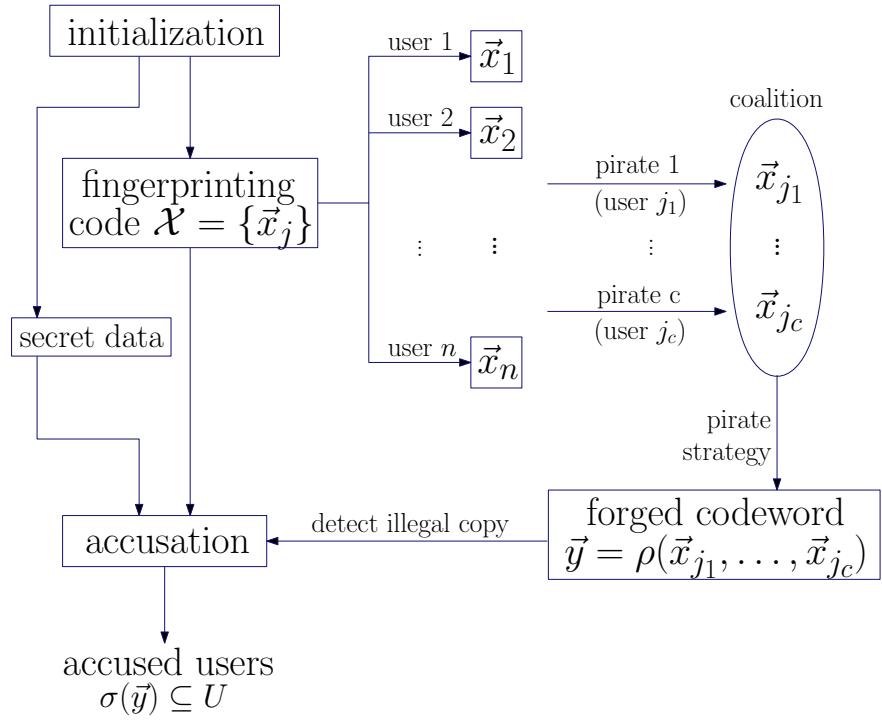
- Random:  $y_i \in_R \{0, 1\}$
- Always 1:  $y_i = 1$
- Majority:  $y_i = 1$  if  $k > c/2$  and  $y_i = 0$  if  $k < c/2$ 
  - Majority/one: If  $k = c/2$ ,  $y_i = 1$
  - Majority/first: If  $k = c/2$ ,  $y_i = \sigma_1$
  - Majority/random: If  $k = c/2$ ,  $y_i \in_R \{0, 1\}$
- Minority:  $y_i = 1$  if  $k < c/2$  and  $y_i = 0$  if  $k > c/2$ 
  - Minority/...
- Interleaving:  $\mathbb{P}[y_i = 1] = k/c$  (i.e.  $y_i \in_R \{\sigma_1, \dots, \sigma_c\}$ )
- Scapegoat:  $y_i = \sigma_1$

The scheme should be secure against all attacks.

- Resistancy against many colluders
- Resistancy against any pirate strategy
- Short codelength
  - Less redundant data
- Small alphabet
  - In practice: Bandwidth needed linear in alphabet size
- Avoid accusing innocent users
- Accuse at least one guilty user (preferably more)

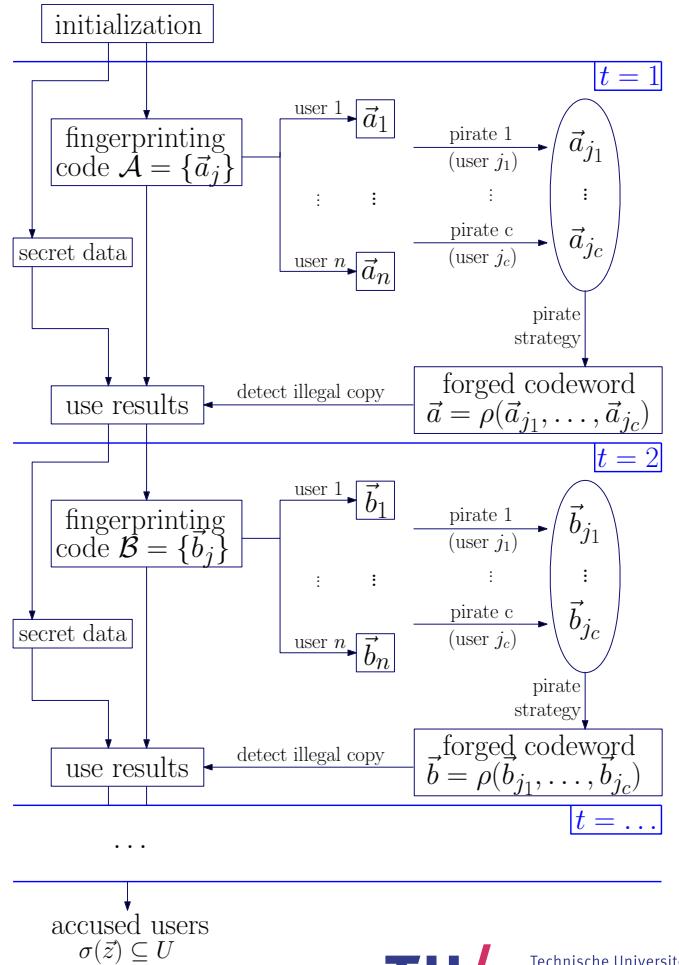
# Static schemes

- Send codewords
- Coalition produces some forgery
- Receive forgery
- Accuse certain users
- Advantages:
  - Many applications
  - Only one codeword
- Disadvantages:
  - More data needed
  - Catch few colluders



# Dynamic schemes

- Send first set of codewords
- Receive first forgery
- Send new codewords
  - ⋮
- Receive final forgery
- Accuse users based on all results
- Advantages:
  - Less total data needed
  - Possibly catch all colluders
- Disadvantages:
  - Only few applications
  - Computations required during the broadcast



Deterministic schemes: No error

- Always absolute certainty
- Soundness: Never accuse any innocent users
- Completeness: Always accuse at least one guilty user
- Always alphabet size  $q \geq c + 1$
- Works only in restricted digit model

Probabilistic schemes: Error bounded by  $\epsilon > 0$

- Small probability of error
- Soundness: Accuse no innocent users with probability at least  $1 - \epsilon$
- Completeness: Accuse a guilty user with probability at least  $1 - \epsilon$
- Decoupling  $\epsilon$  to  $\epsilon_1$  (Soundness) and  $\epsilon_2$  (Completeness):  $\epsilon_1 \ll \epsilon_2$
- Alphabet size  $q \geq 2$
- Works against any attack model

- Identifiable Parent Property: Always identify a "parent"
- Advantages:
  - No error, always absolute certainty
  - Only one codeword necessary
- Disadvantages:
  - Large alphabet size ( $q \geq c + 1$ , or even  $q \geq c^2$ )
  - Long codelength
- Lower bounds on codelength:
  - $\ell = \Omega(c \log(n/c) / \log(q))$  [Bla03b]
  - $\ell = \Omega(c^2 \log(n) / \log(q))$  [AS04]
- Upper bounds on codelength: (constructions)
  - $\ell = \mathcal{O}(c^2 \log(n) / \log(q))$  for  $q = \mathcal{O}(c^2 \log(n))$  [SSW01]
  - $\ell = \mathcal{O}(c^2 \log(n) / \log(q \cdot g(c)))$  for any  $q \geq c$ , for some  $g(c)$  [AS04]

Tetracode; Hamming code [HVLLT98] [BEN07]  
 Only non-trivial "beautiful" code [BEN07]

- $n = 9$  users
- $c = 2$  colluders
- $q = 3$  alphabet size
- $\ell = 4$  codelength

Why is it secure against 2 colluders?

$$X = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 \\ 1 & 0 & 1 & 2 \\ 1 & 1 & 2 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 2 & 1 \\ 2 & 1 & 0 & 2 \\ 2 & 2 & 1 & 0 \end{pmatrix}$$

- Every two codewords have distance 3
- Every word has distance  $\leq 1$  to exactly one codeword

$$\begin{aligned} \vec{a} &= (\mathbf{1}, 0, \mathbf{1}, 2) \\ \vec{b} &= (2, \mathbf{2}, \mathbf{1}, \mathbf{0}) \\ \vec{y} &= (1, 2, 1, 0) \\ \rightarrow d(\vec{y}, \vec{b}) &= 1 \end{aligned}$$

- If  $\mathcal{C} = (\ell, K, d)_q$  is an error-correcting code of cardinality  $n = K$  satisfying  $d > \ell(1 - 1/c^2)$ , then  $\mathcal{C}$  is a  $c$ -IPP-code. [SSW01] [SNW03]
- If  $q \geq \ell - 1$  and  $k = \lceil \ell/c^2 \rceil$ , then there exists a linear Reed-Solomon error-correcting code with parameters  $[\ell, k, d]_q$  satisfying  $d > \ell(1 - 1/c^2)$  of cardinality  $n = q^k = q^{\lceil \ell/c^2 \rceil}$ . [SSW01]
- If  $q \geq \ell - 1$ , then there exist  $c$ -IPP codes satisfying  $n = q^{\lceil \ell/c^2 \rceil}$ , i.e.  $\ell = \mathcal{O}(c^2 \log(n))$  and  $q = \mathcal{O}(c^2 \log(n))$ .

- Probabilistic static schemes: Static schemes with  $\epsilon > 0$  error
- Advantages:
  - Small alphabet size ( $q \geq 2$ )
  - Short codelength
- Disadvantages:
  - Small probability of error  $\epsilon$
- Lower bounds on codelength:
  - $\ell = \Omega(c \log(1/c\epsilon))$  for  $q = 2$  [BS98]
  - $\ell = \Omega(c^2 \log(1/\epsilon))$  for  $q = 2$  [Tar03]
  - $\ell \geq 1.38c^2 \log(1/\epsilon)$  for  $q = 2$  [HM09b]
- Upper bounds on codelength: (constructions)
  - $\ell = \mathcal{O}(c^4 \log(n/\epsilon) \log(1/\epsilon))$  for  $q = 2$  [BS98]
  - $\ell = 100c^2 \log(1/\epsilon)$  for  $q = 2$  [Tar03]
  - $\ell \approx 4.93c^2 \log(1/\epsilon)$  for  $q = 2$  and  $c \rightarrow \infty$  [SKC08]

# The Tardos scheme - Rough outline

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1. Initialization: Choose the codelength  $\ell(c, \epsilon)$  and parameters  $t(c), Z(c, \epsilon)$ , and choose probabilities  $p_i \sim F_t$ .
2. Codeword generation: Choose the symbols  $X_{ji}$  by  $X_{ji} \sim \text{Ber}(p_i)$ .

	Position 1	Position 2	...	Position $\ell$
Probability $p_i$	$p_1 \sim F_t$	$p_2 \sim F_t$	...	$p_\ell \sim F_t$
User 1	$X_{1,1} \sim \text{Ber}(p_1)$	$X_{1,2} \sim \text{Ber}(p_2)$	...	$X_{1,\ell} \sim \text{Ber}(p_\ell)$
User 2	$X_{2,1} \sim \text{Ber}(p_1)$	$X_{2,2} \sim \text{Ber}(p_2)$	...	$X_{2,\ell} \sim \text{Ber}(p_\ell)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
User $n$	$X_{n,1} \sim \text{Ber}(p_1)$	$X_{n,2} \sim \text{Ber}(p_2)$	...	$X_{n,\ell} \sim \text{Ber}(p_\ell)$

# The Tardos scheme - Rough outline

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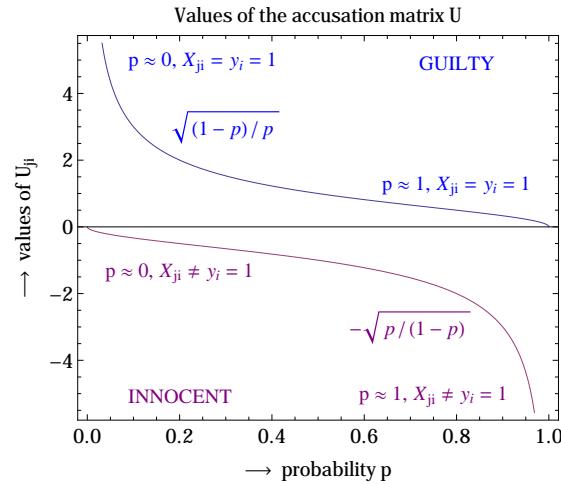
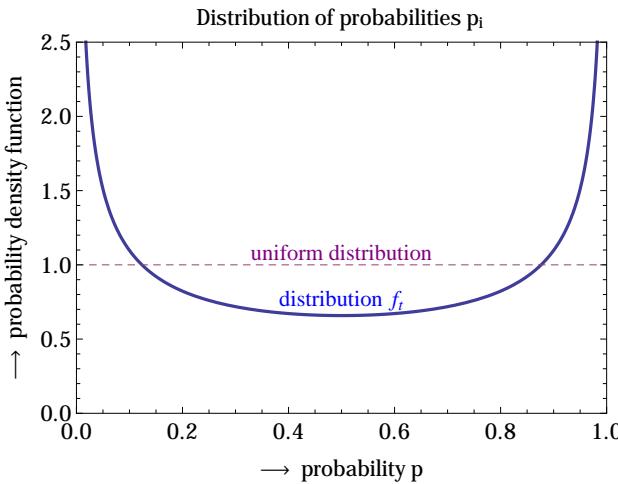
1. Initialization: Choose the codelength  $\ell(c, \epsilon)$  and parameters  $t(c), Z(c, \epsilon)$ , and choose probabilities  $p_i \sim F_t$ .
2. Codeword generation: Choose the symbols  $X_{ji}$  by  $X_{ji} \sim \text{Ber}(p_i)$ .

	Position 1	Position 2	...	Position $\ell$
Probability $p_i$	$p_1 = 0.03$	$p_2 = 0.81$	...	$p_\ell = 0.1$
User 1	$X_{1,1} = 0$	$X_{1,2} = 1$	...	$X_{1,\ell} = 0$
User 2	$X_{2,1} = 0$	$X_{2,2} = 0$	...	$X_{2,\ell} = 0$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
User $n$	$X_{n,1} = 0$	$X_{n,2} = 1$	...	$X_{n,\ell} = 1$

# The Tardos scheme - Rough outline

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1. Initialization: Choose the codelength  $\ell(c, \epsilon)$  and parameters  $t(c), Z(c, \epsilon)$ , and choose probabilities  $p_i \sim F_t$ .
2. Codeword generation: Choose the symbols  $X_{ji}$  by  $X_{ji} \sim \text{Ber}(p_i)$ .
3. Accusation (precomputation): Calculate the accusation matrix  $U$  by  $U_{ji} = +\sqrt{(1-p)/p}$  if  $X_{ji} = 1$  and  $U_{ji} = -\sqrt{p/(1-p)}$  if  $X_{ji} = 0$
4. Accusation (given forgery  $\vec{y}$ ): Accuse user  $j$  if  $S_j = (U\vec{y})_j > Z$ .



# The Tardos scheme - Dummy example

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Dummy parameters:  $n = 5, \ell = 6, Z = 1, \vec{p} = (0.8, 0.7, 0.2, 0.1, 0.5, 0.3)$

$$X = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad U \approx \begin{pmatrix} 0.5 & 0.7 & -0.5 & -0.3 & -1.0 & 1.5 \\ -2.0 & 0.7 & -0.5 & -0.3 & 1.0 & -0.7 \\ 0.5 & -1.5 & -0.5 & -0.3 & 1.0 & 1.5 \\ 0.5 & -1.5 & 2.0 & -0.3 & -1.0 & -0.7 \\ 0.5 & 0.7 & -0.5 & -0.3 & 1.0 & -0.7 \end{pmatrix}$$

Some examples of forgeries and accusations:

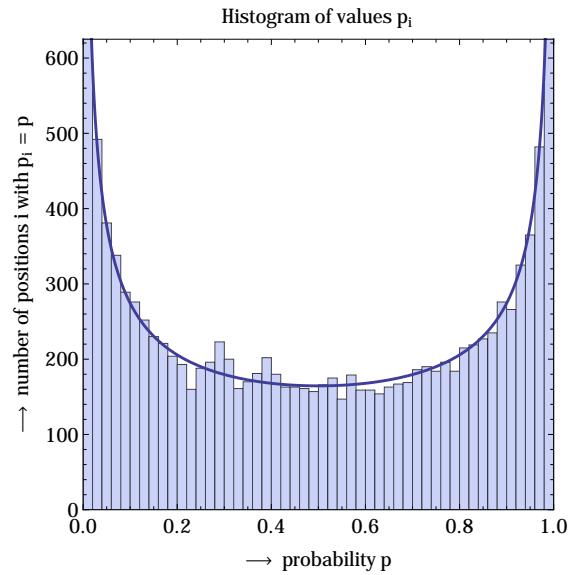
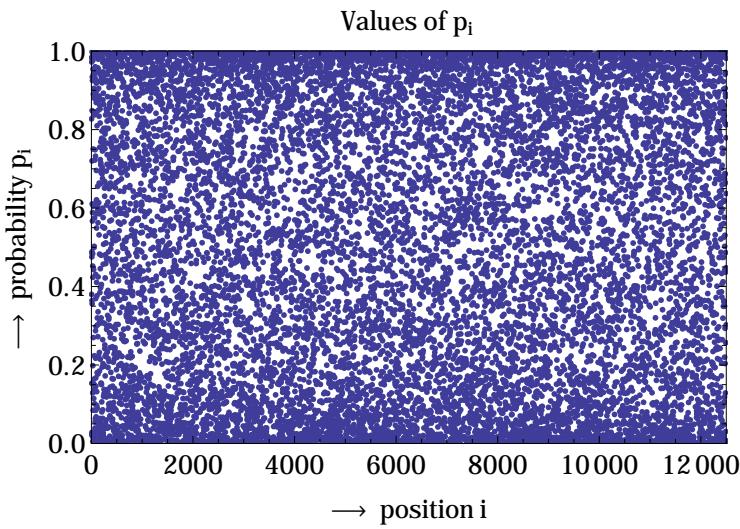
Forgery $\vec{y}$	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$\sigma(\vec{y})$	Comment
$(0, 1, 1, 0, 0, 1)$	<b>1.7</b>	-0.5	-0.5	-0.2	-0.5	{1}	
$(0, 1, 0, 0, 1, 0)$	-0.3	<b>1.7</b>	-0.5	-2.5	<b>1.7</b>	{2, 5}	
$(1, 0, 0, 1, 1, 0)$	-0.8	-1.3	<b>1.2</b>	-0.8	<b>1.2</b>	{3, 5}	impossible!
$(0, 0, 0, 0, 0, 0)$	0	0	0	0	0	$\emptyset$	always no one
$(1, 1, 1, 1, 1, 1)$	0.9	-1.8	0.7	-1.0	0.7	$\emptyset$	
$(1, 1, 1, 0, 1, 1)$	<b>1.2</b>	-1.5	<b>1.0</b>	-0.7	<b>1.0</b>	{1, 3, 5}	removed a 1
$(1, 1, 1, 0, 0, 1)$	<b>2.2</b>	-2.5	0.0	0.3	0.0	{1}	removed a 1
$(0, 0, 1, 0, 0, 0)$	-0.5	-0.5	-0.5	<b>2.0</b>	-0.5	{4}	user 4 accused

# The Tardos scheme - Real example

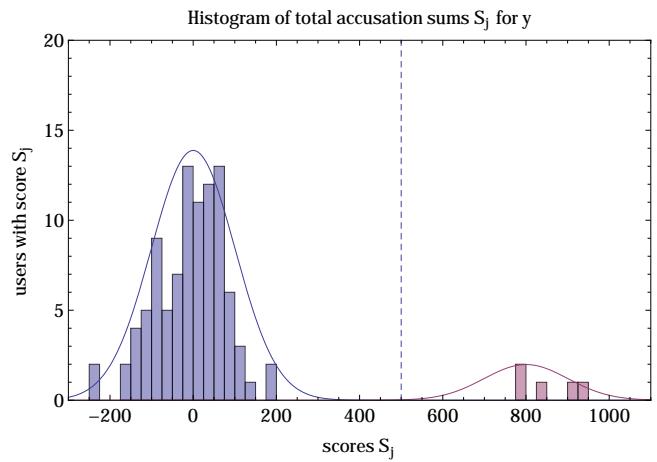
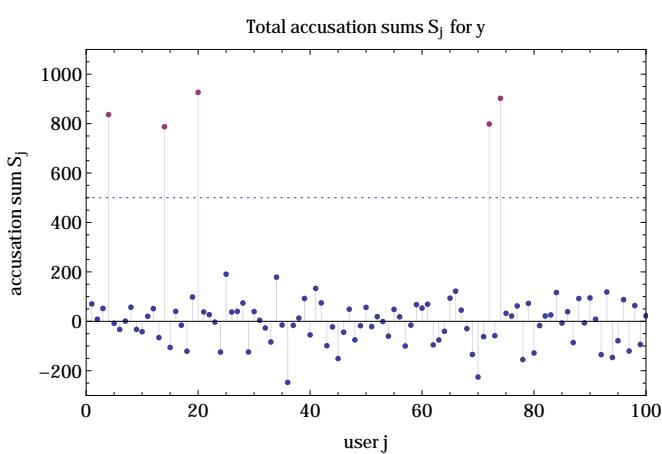
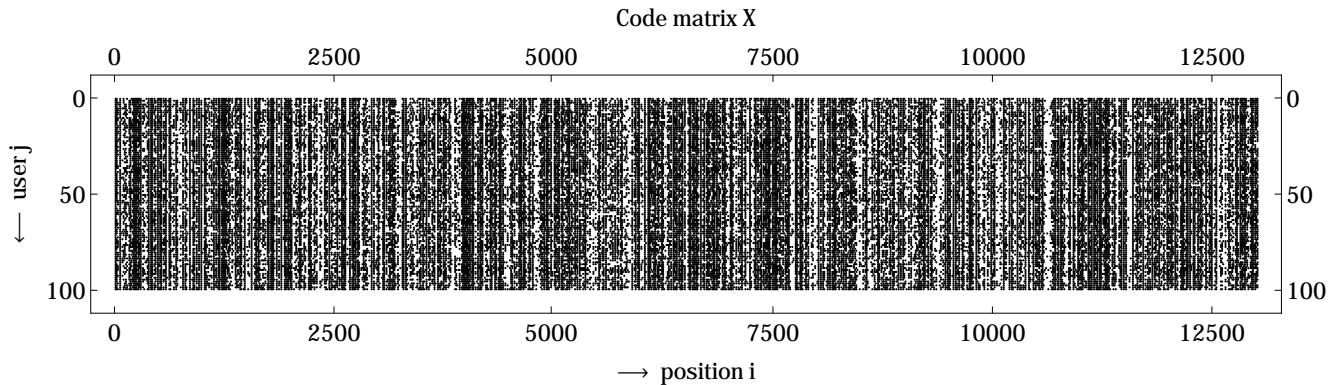
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Scheme parameters:  $\epsilon = e^{-5} \approx 0.0067$ ,  $c = 5$ ,  $n = 100 \Rightarrow \ell = 12500$ ,  $t = 1/1500$ ,  $Z = 500$ ,  $p_i \sim F_t$ ,  $X \in \{0, 1\}^{100 \times 12500}$ ,  $U \in \mathbb{R}^{100 \times 12500}$  (then  $U$  already contains 1.250.000 real numbers)

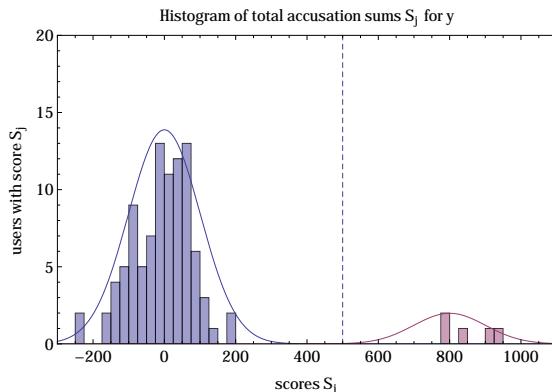
Simulations (interleaving attack): Select an arbitrary coalition, calculate  $\vec{y}$ , calculate  $\sigma(\vec{y})$  and see if the accusation worked



# The Tardos scheme - Real example



- Why are no innocent users accused?
  - All codewords are independent, so it is impossible to frame anyone
  - Positive and negative contributions outweigh each other
  - $S_j$  is roughly distributed as  $\mathcal{N}(0, \sqrt{\ell})$  while  $Z \gg \sqrt{\ell}$
- Why are guilty users accused?
  - On detectable positions, pirates cannot decrease  $S = \sum_{j \in C} S_j$
  - On undetectable positions,  $S$  definitely increases
  - $\frac{S}{c}$  is roughly distributed as  $\mathcal{N}(\tilde{\mu}\ell/c, \tilde{\sigma}^2\ell/c^2)$  while  $Z \ll \tilde{\mu}\ell/c$



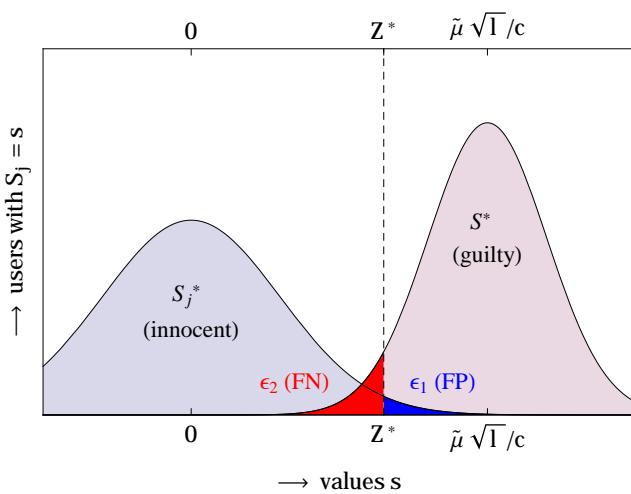
# The Tardos scheme - $\ell = \Theta(c^2)$ , $Z = \Theta(c)$

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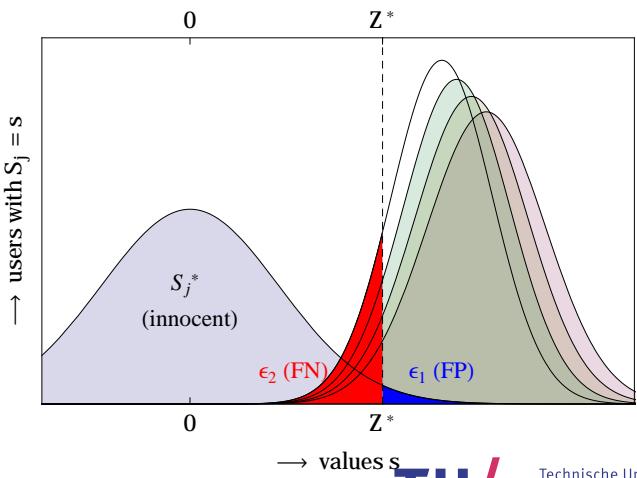
Suppose  $S_j^* = S_j/\sqrt{\ell} \sim \mathcal{N}(0, 1)$ ,  $S^* = \frac{S}{c}/\sqrt{\ell} \sim \mathcal{N}(\tilde{\mu}\sqrt{\ell}/c, \tilde{\sigma}^2/c^2)$ ,  $Z^* = Z/\sqrt{\ell}$ . Then we need  $\ell = \Theta(c^2)$  and  $Z = \Theta(c)$  for the scheme to work.

- If  $\ell = o(c^2)$  then  $\mathbb{E}[S^*] \rightarrow 0$  and  $\text{Var}[S^*] \rightarrow 0$  as  $c \rightarrow \infty$
- If  $\ell = \Theta(c^2)$  then  $\mathbb{E}[S^*] \rightarrow L$  and  $\text{Var}[S^*] \rightarrow 0$  as  $c \rightarrow \infty$
- If  $Z = o(\sqrt{\ell})$  then  $Z^* \rightarrow 0 = \mathbb{E}[S_j^*]$  so  $\epsilon_1 \rightarrow 1/2$  which is bad
- If  $Z > \Omega(\sqrt{\ell})$  then  $Z^* \rightarrow \infty > \mathbb{E}[S^*]$  so  $\epsilon_2 \rightarrow 1$  which is bad

Distributions of  $S_j$  for innocent and guilty users



Distributions as  $c$  goes to infinity



Suggested improvements:

- Use symmetric accusation function instead of  $U$  [SVCT06]
- Tighten the analysis in the proof [SVCT06], [BT08]
- Use the Gaussian approximation to estimate error probabilities [SS10]
- Use a discrete optimal distribution  $F_t$  [NFH<sup>+</sup>09]

With these optimizations, the factor 100 has been reduced to less than 5 in the asymptotic case of  $c \rightarrow \infty$

# Intermezzo: Irdeto's scheme

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Irdeto's implementation: Uniformly random bits, accusation weights Hamming distance between the forgery and the codeword (simply count the number of matches), accuse if these weights are too large. This is a special case of Tardos' scheme with  $F \equiv 1/2$ . But is it safe?

Minority attack, 3 traitors:

Received symbols	Output	Matches	Differences	Increase in $S$
0, 0, 0	0	3	0	+3
0, 0, 1	1	1	2	-1
0, 1, 0	1	1	2	-1
0, 1, 1	0	1	2	-1
1, 0, 0	1	1	2	-1
1, 0, 1	0	1	2	-1
1, 1, 0	0	1	2	-1
1, 1, 1	1	3	0	+3
		12	12	Total: 0

# Intermezzo: Irdeto's scheme

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Irdeto's implementation: Uniformly random bits, accusation weights Hamming distance between the forgery and the codeword (simply count the number of matches), accuse if these weights are too large. This is a special case of Tardos' scheme with  $F \equiv 1/2$ . But is it safe?

Minority attack, 5 traitors:

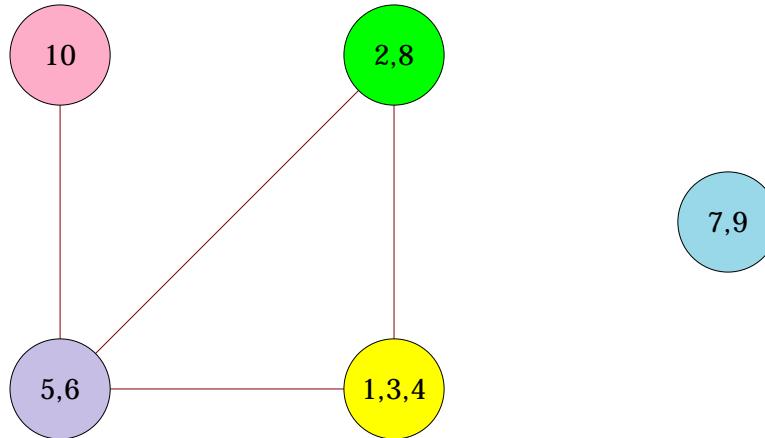
Received symbols	Output	Matches	Differences	Increase in $S$
0, 0, 0, 0, 0	0	5	0	+5
0, 0, 0, 0, 1	1	1	4	-3
0, 0, 0, 1, 0	1	1	4	-3
0, 0, 0, 1, 1	1	2	3	-1
⋮	⋮	⋮	⋮	⋮
1, 1, 1, 1, 0	0	1	4	-3
1, 1, 1, 1, 1	0	5	0	+5
		70	90	Total: -20

- Restricted digit model: only symbols of coalition allowed
- Advantages:
  - No error
  - Shorter time than length in static schemes
  - Catch all colluders with same effort
  - Works against any number of colluders;  $c$  need not be known
- Disadvantages:
  - Only works dynamically
  - Large alphabet size ( $q > c$ )
- Upper bounds on codelength, time: (constructions)
  - $q = 2c + 1, t \leq c \log(n) + c$  [FT01]
  - $q = c + 1, t = \mathcal{O}(c^3 \log(n))$  [BPS00]
  - $q = c + 1, t = \mathcal{O}(c^2 + c \log(n))$  [BPS00]

# General graph notation

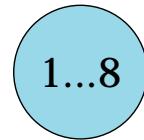
Graph description: Vertices (points)  $V$ , edges (lines)  $E$

- Vertices: Disjoint subsets of  $U$  (forms a partition of  $U$ )
- Edges: If  $S \sim T$  then  $S \cup T$  contains at least one pirate
- Vertex colors: Colors correspond to symbols
- A vertex  $S$  gets color  $c$  if all users in  $S$  get symbol  $c$
- At least  $c$  pirates  $\Leftrightarrow$  Any vertex cover has size at least  $c$



Example: 8 users, 2 traitors (users 2 and 5)

Colors seen by coalition: Blue, Blue

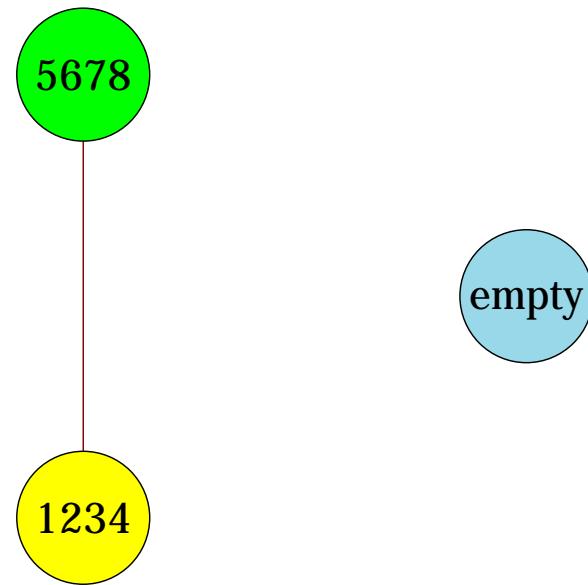


Output color: Blue

# The Fiat-Tassa scheme

Example: 8 users, 2 traitors (users 2 and 5)

Colors seen by coalition: Yellow, Green

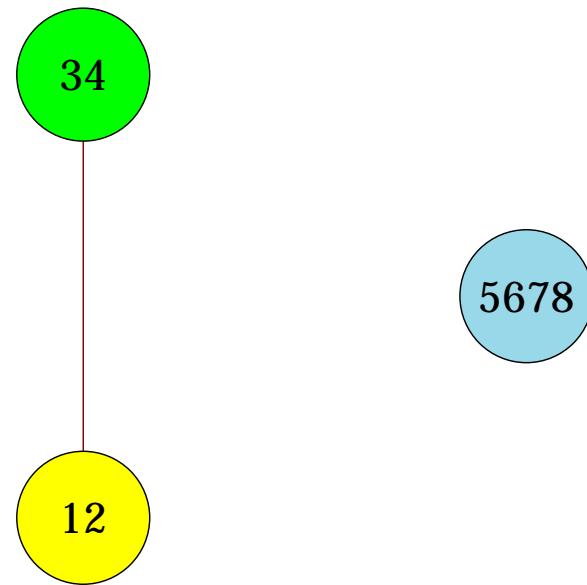


Output color: Yellow

# The Fiat-Tassa scheme

Example: 8 users, 2 traitors (users 2 and 5)

Colors seen by coalition: Yellow, Blue

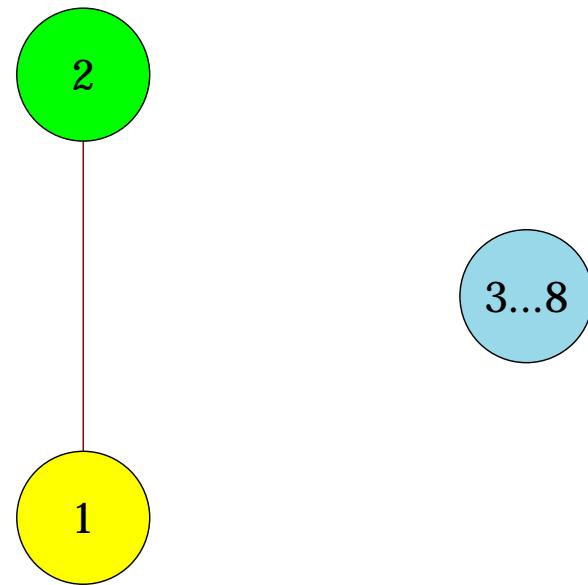


Output color: Yellow

# The Fiat-Tassa scheme

Example: 8 users, 2 traitors (users 2 and 5)

Colors seen by coalition: Green, Blue

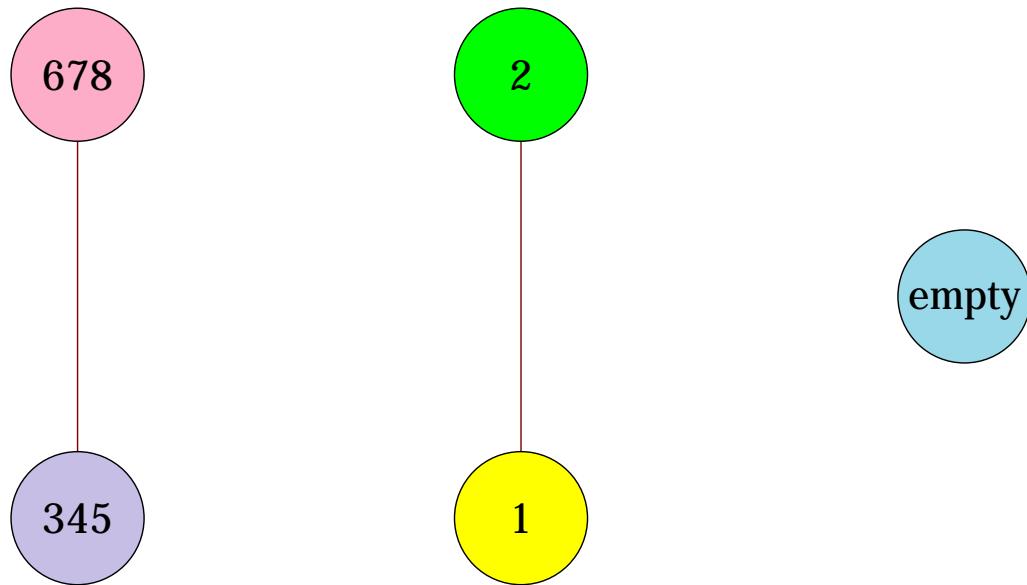


Output color: Blue

# The Fiat-Tassa scheme

Example: 8 users, 2 traitors (users 2 and 5)

Colors seen by coalition: Green, Purple

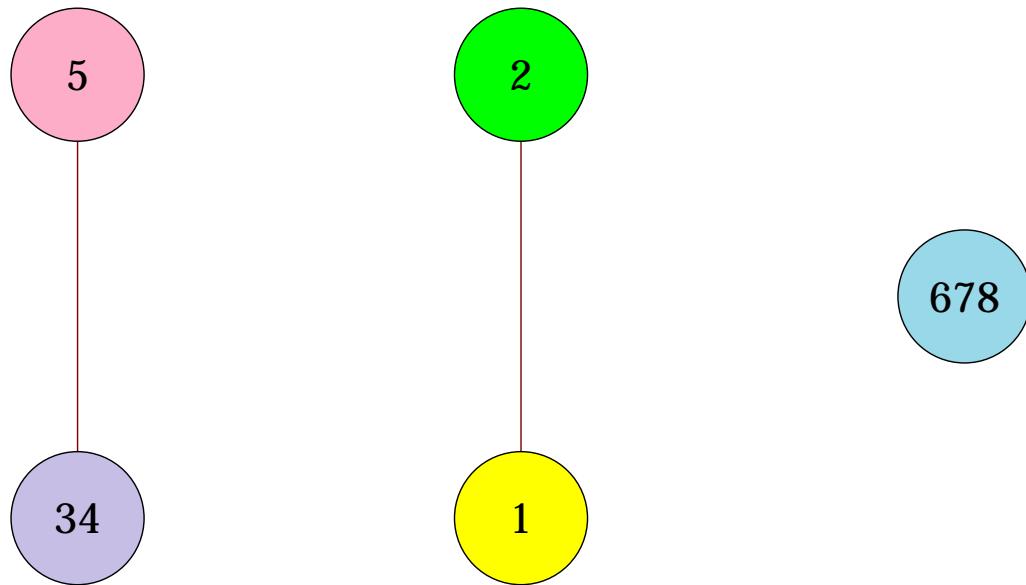


Output color: Purple

# The Fiat-Tassa scheme

Example: 8 users, 2 traitors (users 2 and 5)

Colors seen by coalition: Green, Rose

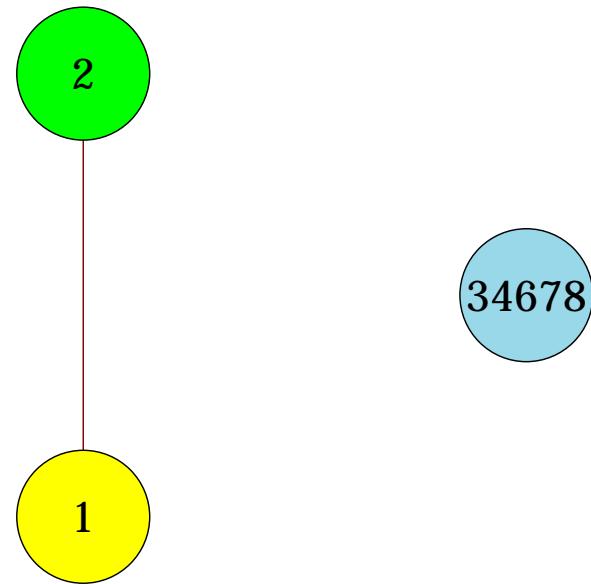


Output color: Rose

# The Fiat-Tassa scheme

Example: 8 users, 2 traitors (users 2 and 5)

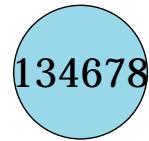
Colors seen by coalition: Green



Output color: Green

Example: 8 users, 2 traitors (users 2 and 5)

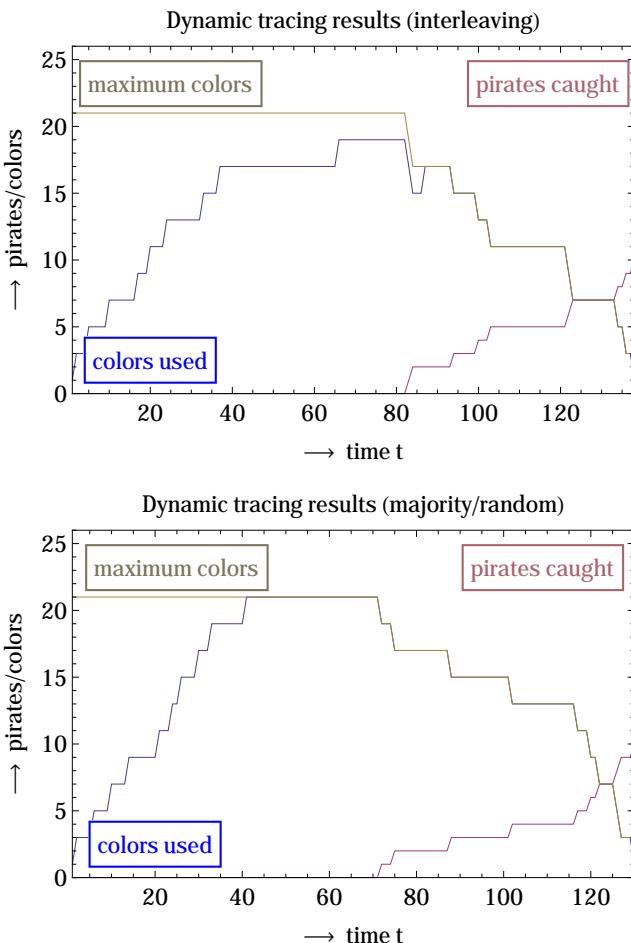
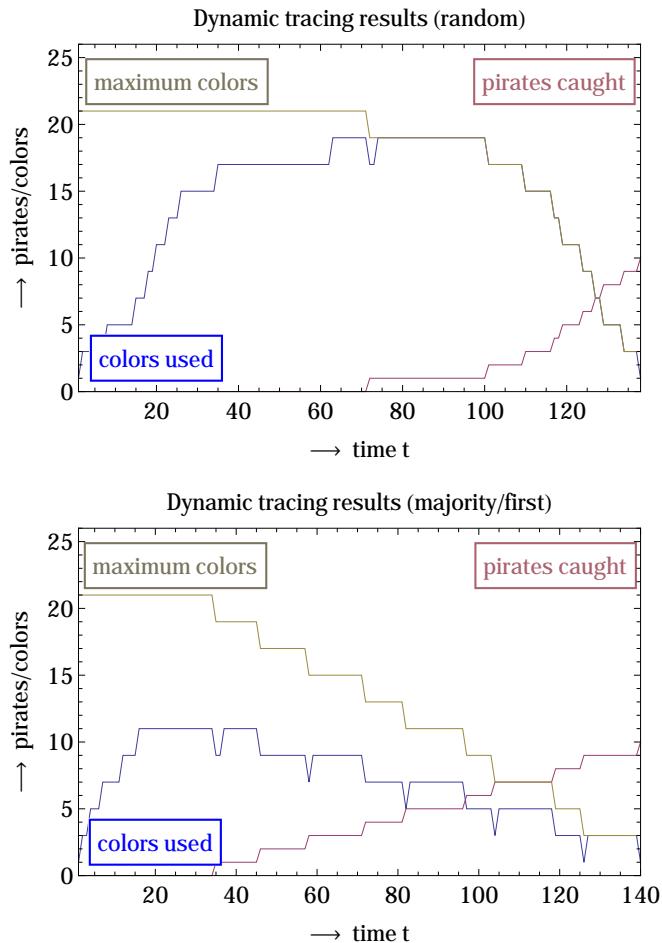
Colors seen by coalition: None



Output color: None

- Isolating a single traitor: At most  $t = \log_2(n)$  steps
- Tracing at least one traitor: At most  $t = \log_2(n) + 1$  steps
- Tracing all traitors: At most  $t = c \log_2(n) + c$  steps
- Colors needed (alphabet size): At most  $q = 2c + 1$  (2 for each traitor, 1 for not yet suspected users)
- Using certain pirate strategies, these bounds are also "often" achieved

# The Fiat-Tassa scheme - Simulations

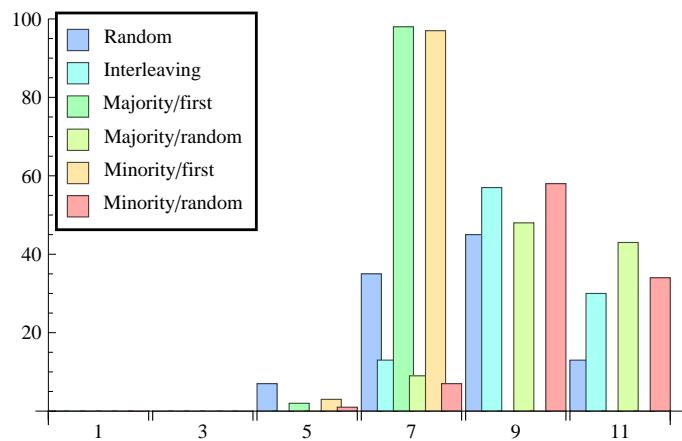
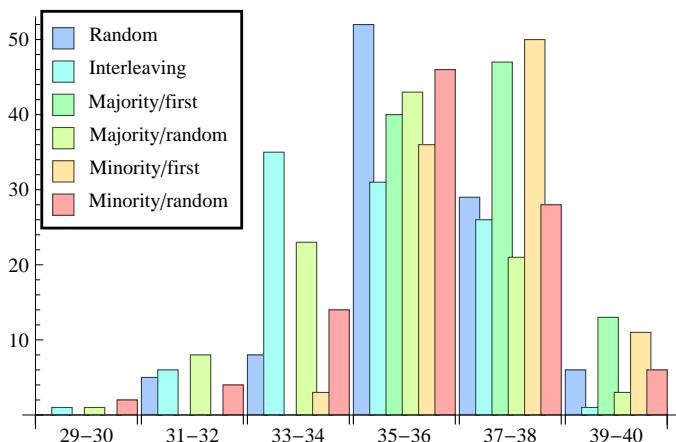


# The Fiat-Tassa scheme - Strategies

Compare strategies for  $n = 100, c = 5$  with 100 simulations.

Left: Time used for different pirate strategies.

Right: Maximum number of colors used during a tracing process.



Certain pairs of not yet connected vertices get the same color. If a received color belongs to two vertices, add an edge between the two vertices.

- Degree algorithm:  $t = \mathcal{O}(c^3 \log(n)), q = c + 1$ 
  - Keep adding edges until vertices get high enough degrees
  - If a vertex has degree  $d > c$ , then it must be guilty
  - Enough pairs of unconnected vertices always exist
  - Complication:  $c$  may not be known
- Clique algorithm:  $t = \mathcal{O}(c^3 \log(n)), q = c + 1$ 
  - Keep adding edges until cliques occur (clique: complete subgraph)
  - Any clique of size  $k$  contains at least  $k - 1$  traitors
  - At some point, traitors will be alone in a set and caught
- Optimal algorithm:  $t = \mathcal{O}(c^2 + c \log(n)), q = c + 1$ 
  - Very complicated extension of the clique algorithm

- Inner code of Fiat-Tassa scheme: IPP-code
  - $\mathcal{X} = \{0, 1, \dots, q - 1\}$
  - Constant codelength  $\ell = 1$
  - Maximum alphabet size  $q \leq 2c + 1$
- Replace with new inner code of hybrid scheme: Probabilistic code
  - $\mathcal{X} = \{\vec{x}_1, \dots, \vec{x}_q\}$
  - Maximum codelength  $\ell > 1$
  - Constant alphabet size  $q \geq 2$
- Advantage: Small alphabet size ( $q \geq 2$ )
- Disadvantages:
  - Errors stack up, so it's hard to bound the error probability
  - Longer codelength and time needed
- Upper bound on codelength, time: (constructions)
  - $q = 2, t \cdot \ell = \mathcal{O}(c^4 \log(1/\epsilon))$  [Tas05]

- Inner code of Tassa's hybrid scheme: Boneh-Shaw code
  - Maximum codelength  $\ell = \mathcal{O}(c^3 \log(1/\epsilon))$
  - Constant alphabet size  $q = 2$
- Total "effort" bounded by  $t \cdot \ell = \mathcal{O}(c^4 \log^2(1/\epsilon))$
- Tassa's analysis also gives no better bound than  $\mathcal{O}(c^4 \log^2(1/\epsilon))$

# Summary

Let  $\epsilon = \Theta(1/n)$  and  $k = \log(1/\epsilon) = \Theta(\log(n))$ . Then:

	$q$	$\epsilon$	$\ell$	$t$	$\ell \cdot t$
Det. static	$\Omega(c)$	0	$\Omega(c^2k)$	$\mathcal{O}(1)$	$\Omega(c^2k)$
- [SSW01]	$\mathcal{O}(c^2k)$	0	$\mathcal{O}(c^2k)$	1	$\mathcal{O}(c^2k)$
- [AS04]	$\mathcal{O}(c)$	0	$\mathcal{O}(c^2k)$	1	$\mathcal{O}(c^2k)$
Prob. static	$\mathcal{O}(1)$	$\Omega(\epsilon)$	$\Omega(c^2k)$	$\mathcal{O}(1)$	$\Omega(c^2k)$
- [BS98]	2	$\mathcal{O}(\epsilon)$	$\mathcal{O}(n^3k^2)$	1	$\mathcal{O}(n^3k^2)$
- [BS98]	2	$\mathcal{O}(\epsilon)$	$\mathcal{O}(c^4k^2)$	1	$\mathcal{O}(c^4k^2)$
- [Tar03]	2	$\mathcal{O}(\epsilon)$	$\mathcal{O}(c^2k)$	1	$\mathcal{O}(c^2k)$
Det. dynamic	$\Omega(c + \alpha)$	0	$\mathcal{O}(1)$	$\Omega(c^2/\alpha + ck)$	$\Omega(c^2/\alpha + ck)$
- [FT01]	$2c + 1$	0	1	$\mathcal{O}(ck)$	$\mathcal{O}(ck)$
- [BPS00]	$c + 1$	0	1	$\mathcal{O}(c^3k)$	$\mathcal{O}(c^3k)$
- [BPS00]	$c + 1$	0	1	$\mathcal{O}(c^2 + ck)$	$\mathcal{O}(c^2 + ck)$
Prob. dynamic	$\mathcal{O}(1)$	$\Omega(\epsilon)$	?	?	?
- [Tas05]	2	$\mathcal{O}(\epsilon)$	$\mathcal{O}(c^3k^2)$	$\mathcal{O}(ck)$	$\mathcal{O}(c^4k^3)$

- Investigate other options for a hybrid scheme
- Look at some more important papers
- Investigate practical implementation issues
- Consider the special (practical) case for  $c = 5 \dots 25$
- Run simulations with real values used in practice

# Questions

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Thank you for your attention!



Any questions?

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