Progress in Traitor Tracing with Applications to Group Testing

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Outline

Introduction

The Tardos scheme

A capacity-achieving score function

The dynamic Tardos scheme

Applications to group testing

Problem: Illegal redistribution

User	Copyrighted content																
Antonino	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Boris	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Caroline	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
David	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Eve	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Fred	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Gábor	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Henry	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...

Problem: Illegal redistribution

User	Copyrighted content																
Antonino	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Boris	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Caroline	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
David	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Eve	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Fred	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Gábor	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Henry	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...
Copy	0	1	1	1	0	0	1	1	1	0	1	1	0	0	1	0	...

Solution: Embed fingerprints

User	Copyrighted content (fingerprinted)																
Antonino	0	1	1	1	0	0	1	1	1	0	1	1	0	1	0	0	...
Boris	0	1	1	1	0	1	0	1	1	0	1	1	1	1	1	0	...
Caroline	0	1	0	1	0	1	0	1	1	0	0	1	1	0	1	0	...
David	0	1	1	1	0	0	0	1	1	0	1	1	0	0	0	0	...
Eve	0	1	0	1	0	1	0	1	1	0	1	1	1	0	0	0	...
Fred	0	1	0	1	0	0	1	1	1	0	0	1	0	1	0	0	...
Gábor	0	1	1	1	0	1	1	1	1	0	1	1	0	0	1	0	...
Henry	0	1	0	1	0	1	1	1	1	0	0	1	0	1	1	0	...

Solution: Embed fingerprints

User	Copyrighted content (fingerprinted)																
Antonino	0	1	1	1	0	0	1	1	1	0	1	1	0	1	0	0	...
Boris	0	1	1	1	0	1	0	1	1	0	1	1	1	1	1	0	...
Caroline	0	1	0	1	0	1	0	1	1	0	0	1	1	0	1	0	...
David	0	1	1	1	0	0	0	1	1	0	1	1	0	0	0	0	...
Eve	0	1	0	1	0	1	0	1	1	0	1	1	1	0	0	0	...
Fred	0	1	0	1	0	0	1	1	1	0	0	1	0	1	0	0	...
Gábor	0	1	1	1	0	1	1	1	1	0	1	1	0	0	1	0	...
Henry	0	1	0	1	0	1	1	1	1	0	0	1	0	1	1	0	...
Copy	0	1	0	1	0	1	0	1	1	0	1	1	1	0	0	0	...

Solution: Embed fingerprints

User	Copyrighted content (fingerprinted)																
Antonino	0	1	**1**	1	0	**0**	**1**	1	1	0	**1**	1	**0**	**1**	**0**	0	...
Boris	0	1	**1**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**1**	**1**	0	...
Caroline	0	1	**0**	1	0	**1**	**0**	1	1	0	**0**	1	**1**	**0**	**1**	0	...
David	0	1	**1**	1	0	**0**	**0**	1	1	0	**1**	1	**0**	**0**	**0**	0	...
Eve	0	1	**0**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**0**	**0**	0	...
Fred	0	1	**0**	1	0	**0**	**1**	1	1	0	**0**	1	**0**	**1**	**0**	0	...
Gábor	0	1	**1**	1	0	**1**	**1**	1	1	0	**1**	1	**0**	**0**	**1**	0	...
Henry	0	1	**0**	1	0	**1**	**1**	1	1	0	**0**	1	**0**	**1**	**1**	0	...
Copy	0	1	**0**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**0**	**0**	0	...

TU/e

Solution: Embed fingerprints

User	Copyrighted content (fingerprinted)															
Antonino	0	1	**1**	1	0	**0**	**1**	1	1	0	**1**	1	**0**	**1**	**0**	0	...
Boris	0	1	**1**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**1**	**1**	0	...
Caroline	0	1	**0**	1	0	**1**	**0**	1	1	0	**0**	1	**1**	**0**	**1**	0	...
David	0	1	**1**	1	0	**0**	**0**	1	1	0	**1**	1	**0**	**0**	**0**	0	...
Eve	0	1	**0**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**0**	**0**	0	...
Fred	0	1	**0**	1	0	**0**	**1**	1	1	0	**0**	1	**0**	**1**	**0**	0	...
Gábor	0	1	**1**	1	0	**1**	**1**	1	1	0	**1**	1	**0**	**0**	**1**	0	...
Henry	0	1	**0**	1	0	**1**	**1**	1	1	0	**0**	1	**0**	**1**	**1**	0	...
Copy	0	1	**0**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**0**	**0**	0	...

Problem: Collusion attacks

User	Copyrighted content (fingerprinted)															
Antonino	0	1	1	1	0	0	1	1	1	0	1	1	0	1	0	0 ...
Boris	0	1	1	1	0	1	0	1	1	0	1	1	1	1	1	0 ...
Caroline	0	1	0	1	0	1	0	1	1	0	0	1	1	0	1	0 ...
David	0	1	1	1	0	0	0	1	1	0	1	1	0	0	0	0 ...
Eve	0	1	0	1	0	1	0	1	1	0	1	1	1	0	0	0 ...
Fred	0	1	0	1	0	0	1	1	1	0	0	1	0	1	0	0 ...
Gábor	0	1	1	1	0	1	1	1	1	0	1	1	0	0	1	0 ...
Henry	0	1	0	1	0	1	1	1	1	0	0	1	0	1	1	0 ...

Problem: Collusion attacks

User	Copyrighted content (fingerprinted)															
Antonino	0	1	1	1	0	0	1	1	1	0	1	1	0	1	0	0 ...
Boris	0	1	1	1	0	1	0	1	1	0	1	1	1	1	1	0 ...
Caroline	0	1	0	1	0	1	0	1	1	0	0	1	1	0	1	0 ...
David	0	1	1	1	0	0	0	1	1	0	1	1	0	0	0	0 ...
Eve	0	1	0	1	0	1	0	1	1	0	1	1	1	0	0	0 ...
Fred	0	1	0	1	0	0	1	1	1	0	0	1	0	1	0	0 ...
Gábor	0	1	1	1	0	1	1	1	1	0	1	1	0	0	1	0 ...
Henry	0	1	0	1	0	1	1	1	1	0	0	1	0	1	1	0 ...
Copy	0	1	1	1	0	1	0	1	1	0	1	1	0	1	0	0 ...

Problem: Collusion attacks

User	Copyrighted content (fingerprinted)																
Antonino	0	1	**1**	1	0	**0**	**1**	1	1	0	**1**	1	**0**	**1**	**0**	0	...
Boris	0	1	**1**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**1**	**1**	0	...
Caroline	0	1	**0**	1	0	**1**	**0**	1	1	0	**0**	1	**1**	**0**	**1**	0	...
David	0	1	**1**	1	0	**0**	**0**	1	1	0	**1**	1	**0**	**0**	**0**	0	...
Eve	0	1	**0**	1	0	**1**	**0**	1	1	0	**1**	1	**1**	**0**	**0**	0	...
Fred	0	1	**0**	1	0	**0**	**1**	1	1	0	**0**	1	**0**	**1**	**0**	0	...
Gábor	0	1	**1**	1	0	**1**	**1**	1	1	0	**1**	1	**0**	**0**	**1**	0	...
Henry	0	1	**0**	1	0	**1**	**1**	1	1	0	**0**	1	**0**	**1**	**1**	0	...
Copy	0	1	**1**	1	0	**1**	**0**	1	1	0	**1**	1	**0**	**1**	**0**	0	...

Solution: Collusion-resistant schemes

User	Copyrighted content (fingerprinted)															
Antonino	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Boris	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Caroline	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
David	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Eve	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Fred	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Gábor	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Henry	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Copy	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...

Solution: Collusion-resistant schemes

User	Copyrighted content (fingerprinted)															
Antonino	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Boris	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Caroline	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
David	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Eve	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Fred	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Gábor	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Henry	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Copy	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...

1. An algorithm to construct collusion-resistant codes

Solution: Collusion-resistant schemes

User	Copyrighted content (fingerprinted)															
Antonino	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Boris	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Caroline	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
David	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Eve	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Fred	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Gábor	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Henry	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...
Copy	0	1	?	1	0	?	?	1	1	0	?	1	?	?	?	0	...

1. An algorithm to construct collusion-resistant codes
2. An algorithm to trace pirate copies to colluders

Solution: Collusion-resistant schemes

User	Copyrighted content (fingerprinted)								
Antonino	?	?	?	?	?	?	?	...
Boris	?	?	?	?	?	?	?	...
Caroline	?	?	?	?	?	?	?	...
David	?	?	?	?	?	?	?	...
Eve	?	?	?	?	?	?	?	...
Fred	?	?	?	?	?	?	?	...
Gábor	?	?	?	?	?	?	?	...
Henry	?	?	?	?	?	?	?	...
Copy	?	?	?	?	?	?	?	...

1. An algorithm to construct collusion-resistant codes
2. An algorithm to trace pirate copies to colluders

Solution: Collusion-resistant schemes

User	Copyrighted content (fingerprinted)	
Antonino		...
Boris		...
Caroline	$X \in \{0,1\}^{n \times \ell}$	...
David		...
Eve		...
Fred		...
Gábor		...
Henry		...
Copy	$y \in \{0,1\}^{\ell}$	...

1. An algorithm to construct collusion-resistant codes
2. An algorithm to trace pirate copies to colluders

Solution: Collusion-resistant schemes

User	Copyrighted content (fingerprinted)	
Antonino		...
Boris		...
Caroline	$X \in \{0,1\}^{n \times \ell}$	...
David		...
Eve		...
Fred	$\left(\text{[Tar03]: } \ell = L \cdot c^2 \log n / \varepsilon\right)$	...
Gábor		...
Henry		...
Copy	$y \in \{0,1\}^{\ell}$	...

1. An algorithm to construct collusion-resistant codes
2. An algorithm to trace pirate copies to colluders

Solution: Collusion-resistant schemes

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Solution: Collusion-resistant schemes

1. An algorithm to construct collusion-resistant codes

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The Tardos scheme: Overview

1. An algorithm to construct collusion-resistant codes

2. An algorithm to trace pirate copies to colluders

The Tardos scheme: Overview

1. An algorithm to construct collusion-resistant codes
   1a. For each segment i, generate $p_i \sim F$.
      - Many values of p_i close to 0 and 1.
      - Hide choice of p_i from pirates.

2. An algorithm to trace pirate copies to colluders

The Tardos scheme: Overview

1. An algorithm to construct collusion-resistant codes
   1a. For each segment i, generate $p_i \sim F$.
      - Many values of p_i close to 0 and 1.
      - Hide choice of p_i from pirates.

   1b. For each segment i, user j, choose $X_{j,i} = 1$ with prob. p_i.
2. An algorithm to trace pirate copies to colluders

The Tardos scheme: Overview

1. An algorithm to construct collusion-resistant codes
   - 1a. For each segment i, generate $p_i \sim F$.
     - Many values of p_i close to 0 and 1.
     - Hide choice of p_i from pirates.
   - 1b. For each segment i, user j, choose $X_{j,i} = 1$ with prob. p_i.
2. An algorithm to trace pirate copies to colluders
   - 2a. For each segment i, user j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.
     - Positive scores ($S_{j,i} > 0$) for matches ($X_{j,i} = y_i$).
     - Negative scores ($S_{j,i} < 0$) for differences ($X_{j,i} \neq y_i$).
     - Large scores ($|S_{j,i}| \gg 0$) for rare events.

The Tardos scheme: Overview

1. An algorithm to construct collusion-resistant codes
   1a. For each segment i, generate $p_i \sim F$.
      - Many values of p_i close to 0 and 1.
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   1b. For each segment i, user j, choose $X_{j,i} = 1$ with prob. p_i.
2. An algorithm to trace pirate copies to colluders
   2a. For each segment i, user j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.
      - Positive scores ($S_{j,i} > 0$) for matches ($X_{j,i} = y_i$).
      - Negative scores ($S_{j,i} < 0$) for differences ($X_{j,i} \neq y_i$).
      - Large scores ($|S_{j,i}| \gg 0$) for rare events.

   2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

The Tardos scheme: Codewords

p_i	p_1	p_2	p_3	p_4	p_5	$\cdots$	p_{1200}
Antonino	$X_{1,1}$	$X_{1,2}$	$X_{1,3}$	$X_{1,4}$	$X_{1,5}$	$\cdots$	$X_{1,1208}$
Boris	$X_{2,1}$	$X_{2,2}$	$X_{2,3}$	$X_{2,4}$	$X_{2,5}$	$\cdots$	$X_{2,1208}$
Caroline	$X_{3,1}$	$X_{3,2}$	$X_{3,3}$	$X_{3,4}$	$X_{3,5}$	$\cdots$	$X_{3,1208}$
David	$X_{4,1}$	$X_{4,2}$	$X_{4,3}$	$X_{4,4}$	$X_{4,5}$	$\cdots$	$X_{4,1208}$
Eve	$X_{5,1}$	$X_{5,2}$	$X_{5,3}$	$X_{5,4}$	$X_{5,5}$	$\cdots$	$X_{5,1208}$
Fred	$X_{6,1}$	$X_{6,2}$	$X_{6,3}$	$X_{6,4}$	$X_{6,5}$	$\cdots$	$X_{6,1208}$
Gábor	$X_{7,1}$	$X_{7,2}$	$X_{7,3}$	$X_{7,4}$	$X_{7,5}$	$\cdots$	$X_{7,1208}$
Henry	$X_{8,1}$	$X_{8,2}$	$X_{8,3}$	$X_{8,4}$	$X_{8,5}$	$\cdots$	$X_{8,1208}$
Copy	y_1	y_2	y_3	y_4	y_5	$\cdots$	y_{1208}

The Tardos scheme: Codewords

1a. For each segment i, generate $p_i \sim F$.

p_i	p_1	p_2	p_3	p_4	p_5	$\cdots$	p_{1200}
Antonino	$X_{1,1}$	$X_{1,2}$	$X_{1,3}$	$X_{1,4}$	$X_{1,5}$	$\cdots$	$X_{1,1208}$
Boris	$X_{2,1}$	$X_{2,2}$	$X_{2,3}$	$X_{2,4}$	$X_{2,5}$	$\cdots$	$X_{2,1208}$
Caroline	$X_{3,1}$	$X_{3,2}$	$X_{3,3}$	$X_{3,4}$	$X_{3,5}$	$\cdots$	$X_{3,1208}$
David	$X_{4,1}$	$X_{4,2}$	$X_{4,3}$	$X_{4,4}$	$X_{4,5}$	$\cdots$	$X_{4,1208}$
Eve	$X_{5,1}$	$X_{5,2}$	$X_{5,3}$	$X_{5,4}$	$X_{5,5}$	$\cdots$	$X_{5,1208}$
Fred	$X_{6,1}$	$X_{6,2}$	$X_{6,3}$	$X_{6,4}$	$X_{6,5}$	$\cdots$	$X_{6,1208}$
Gábor	$X_{7,1}$	$X_{7,2}$	$X_{7,3}$	$X_{7,4}$	$X_{7,5}$	$\cdots$	$X_{7,1208}$
Henry	$X_{8,1}$	$X_{8,2}$	$X_{8,3}$	$X_{8,4}$	$X_{8,5}$	$\cdots$	$X_{8,1208}$
Copy	y_1	y_2	y_3	y_4	y_5	$\cdots$	y_{1208}

The Tardos scheme: Codewords

1a. For each segment i, generate $p_i \sim F$.

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18
Antonino	$X_{1,1}$	$X_{1,2}$	$X_{1,3}$	$X_{1,4}$	$X_{1,5}$	...	$X_{1,1208}$
Boris	$X_{2,1}$	$X_{2,2}$	$X_{2,3}$	$X_{2,4}$	$X_{2,5}$	...	$X_{2,1208}$
Caroline	$X_{3,1}$	$X_{3,2}$	$X_{3,3}$	$X_{3,4}$	$X_{3,5}$	...	$X_{3,1208}$
David	$X_{4,1}$	$X_{4,2}$	$X_{4,3}$	$X_{4,4}$	$X_{4,5}$	...	$X_{4,1208}$
Eve	$X_{5,1}$	$X_{5,2}$	$X_{5,3}$	$X_{5,4}$	$X_{5,5}$	...	$X_{5,1208}$
Fred	$X_{6,1}$	$X_{6,2}$	$X_{6,3}$	$X_{6,4}$	$X_{6,5}$	...	$X_{6,1208}$
Gábor	$X_{7,1}$	$X_{7,2}$	$X_{7,3}$	$X_{7,4}$	$X_{7,5}$	...	$X_{7,1208}$
Henry	$X_{8,1}$	$X_{8,2}$	$X_{8,3}$	$X_{8,4}$	$X_{8,5}$	...	$X_{8,1208}$
Copy	y_1	y_2	y_3	y_4	y_5	...	y_{1208}

The Tardos scheme: Codewords

1b. For each segment i, user j, choose $X_{j,i} = 1$ with prob. p_i.

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18
Antonino	$X_{1,1}$	$X_{1,2}$	$X_{1,3}$	$X_{1,4}$	$X_{1,5}$	...	$X_{1,1208}$
Boris	$X_{2,1}$	$X_{2,2}$	$X_{2,3}$	$X_{2,4}$	$X_{2,5}$	...	$X_{2,1208}$
Caroline	$X_{3,1}$	$X_{3,2}$	$X_{3,3}$	$X_{3,4}$	$X_{3,5}$	...	$X_{3,1208}$
David	$X_{4,1}$	$X_{4,2}$	$X_{4,3}$	$X_{4,4}$	$X_{4,5}$	...	$X_{4,1208}$
Eve	$X_{5,1}$	$X_{5,2}$	$X_{5,3}$	$X_{5,4}$	$X_{5,5}$	...	$X_{5,1208}$
Fred	$X_{6,1}$	$X_{6,2}$	$X_{6,3}$	$X_{6,4}$	$X_{6,5}$	...	$X_{6,1208}$
Gábor	$X_{7,1}$	$X_{7,2}$	$X_{7,3}$	$X_{7,4}$	$X_{7,5}$	...	$X_{7,1208}$
Henry	$X_{8,1}$	$X_{8,2}$	$X_{8,3}$	$X_{8,4}$	$X_{8,5}$	...	$X_{8,1208}$
Copy	y_1	y_2	y_3	y_4	y_5	...	y_{1208}

The Tardos scheme: Codewords

1b. For each segment i, user j, choose $X_{j,i} = 1$ with prob. p_i.

p_i	0.20	0.05	0.88	0.79	0.98	…	0.18
Antonino	0	0	1	1	1	…	0
Boris	1	0	1	1	1	…	1
Caroline	1	0	0	1	0	…	0
David	0	0	1	1	1	…	0
Eve	0	0	1	0	1	…	0
Fred	1	0	1	0	1	…	0
Gábor	0	0	1	0	1	…	0
Henry	1	0	0	0	1	…	0
Copy	y_1	y_2	y_3	y_4	y_5	…	y_{1208}

The Tardos scheme: Coalition

Pirates get their versions, ...

p_i	.	.	.	.	.	...	.
Antonino	.	.	.	.	.	...	.
Boris	.	.	.	.	.	...	.
Caroline	1	0	0	1	0	...	0
David	.	.	.	.	.	...	.
Eve	0	0	1	0	1	...	0
Fred	.	.	.	.	.	...	.
Gábor	.	.	.	.	.	...	.
Henry	1	0	0	0	1	...	0
Copy	y_1	y_2	y_3	y_4	y_5	...	y_{1208}

$$\text{Coalition} = \{\text{Caroline}, \text{Eve}, \text{Henry}\}$$

The Tardos scheme: Coalition

Pirates get their versions, compare them ...

p_i	.	.	.	.	.	...	.
Antonino	.	.	.	.	.	...	.
Boris	.	.	.	.	.	...	.
Caroline	**1**	0	**0**	**1**	**0**	...	0
David	.	.	.	.	.	...	.
Eve	**0**	0	**1**	**0**	**1**	...	0
Fred	.	.	.	.	.	...	.
Gábor	.	.	.	.	.	...	.
Henry	**1**	0	**0**	**0**	**1**	...	0
Copy	y_1	y_2	y_3	y_4	y_5	...	y_{1208}

Coalition $= \{$Caroline, Eve, Henry$\}$

The Tardos scheme: Coalition

Pirates get their versions, compare them and make a copy.

p_i	.	.	.	.	.	...	.
Antonino	.	.	.	.	.	...	.
Boris	.	.	.	.	.	...	.
Caroline	**1**	0	**0**	**1**	**0**	...	0
David	.	.	.	.	.	...	.
Eve	**0**	0	**1**	**0**	**1**	...	0
Fred	.	.	.	.	.	...	.
Gábor	.	.	.	.	.	...	.
Henry	**1**	0	**0**	**0**	**1**	...	0
Copy	**0**	0	**0**	**1**	**1**	...	0

Coalition = {Caroline, Eve, Henry}

The Tardos scheme: Scores

The copy is distributed and detected by the tracer.

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18
Antonino	0	0	1	1	1	...	0
Boris	1	0	1	1	1	...	1
Caroline	1	0	0	1	0	...	0
David	0	0	1	1	1	...	0
Eve	0	0	1	0	1	...	0
Fred	1	0	1	0	1	...	0
Gábor	0	0	1	0	1	...	0
Henry	1	0	0	0	1	...	0
Copy	0	0	0	1	1	...	0

Coalition = {Caroline, Eve, Henry}

The Tardos scheme: Scores

2a. For each segment i, user j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18
Antonino	0	0	1	1	1	...	0
Boris	1	0	1	1	1	...	1
Caroline	1	0	0	1	0	...	0
David	0	0	1	1	1	...	0
Eve	0	0	1	0	1	...	0
Fred	1	0	1	0	1	...	0
Gábor	0	0	1	0	1	...	0
Henry	1	0	0	0	1	...	0
Copy	0	0	0	1	1	...	0

Coalition $= \{$Caroline, Eve, Henry$\}$

The Tardos scheme: Scores

2a. For each segment i, user j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18
Antonino	$+0.5$	$+0.2$	-0.4	$+0.5$	$+0.1$	...	$+0.5$
Boris	-2.0	$+0.2$	-0.4	$+0.5$	$+0.1$	...	-2.1
Caroline	-2.0	$+0.2$	$+2.7$	$+0.5$	-7.2	...	$+0.5$
David	$+0.5$	$+0.2$	-0.4	$+0.5$	$+0.1$	...	$+0.5$
Eve	$+0.5$	$+0.2$	-0.4	-1.9	$+0.1$	...	$+0.5$
Fred	-2.0	$+0.2$	-0.4	-1.9	$+0.1$	...	$+0.5$
Gábor	$+0.5$	$+0.2$	-0.4	-1.9	$+0.1$	...	$+0.5$
Henry	-2.0	$+0.2$	$+2.7$	-1.9	$+0.1$	...	$+0.5$
Copy	0	0	0	1	1	...	0

Coalition = {Caroline, Eve, Henry}

The Tardos scheme: Scores

2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18	$\sum_i S_{j,i}$
Antonino	$+0.5$	$+0.2$	-0.4	$+0.5$	$+0.1$	...	$+0.5$	0
Boris	-2.0	$+0.2$	-0.4	$+0.5$	$+0.1$	...	-2.1	0
Caroline	-2.0	$+0.2$	$+2.7$	$+0.5$	-7.2	...	$+0.5$	0
David	$+0.5$	$+0.2$	-0.4	$+0.5$	$+0.1$	...	$+0.5$	0
Eve	$+0.5$	$+0.2$	-0.4	-1.9	$+0.1$	...	$+0.5$	0
Fred	-2.0	$+0.2$	-0.4	-1.9	$+0.1$	...	$+0.5$	0
Gábor	$+0.5$	$+0.2$	-0.4	-1.9	$+0.1$	...	$+0.5$	0
Henry	-2.0	$+0.2$	$+2.7$	-1.9	$+0.1$	...	$+0.5$	0
Copy	0	0	0	1	1	...	0	

Coalition = {Caroline, Eve, Henry}

The Tardos scheme: Scores

2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18	$\sum_i S_{j,i}$
Antonino	+0.5	+0.2	−0.4	+0.5	+0.1	...	+0.5	+14
Boris	−2.0	+0.2	−0.4	+0.5	+0.1	...	−2.1	−19
Caroline	−2.0	+0.2	+2.7	+0.5	−7.2	...	+0.5	+291
David	+0.5	+0.2	−0.4	+0.5	+0.1	...	+0.5	+29
Eve	+0.5	+0.2	−0.4	−1.9	+0.1	...	+0.5	+292
Fred	−2.0	+0.2	−0.4	−1.9	+0.1	...	+0.5	−53
Gábor	+0.5	+0.2	−0.4	−1.9	+0.1	...	+0.5	−42
Henry	−2.0	+0.2	+2.7	−1.9	+0.1	...	+0.5	+269
Copy	0	0	0	1	1	...	0	

Coalition = {Caroline, Eve, Henry}

The Tardos scheme: Scores

2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18	$\sum_i S_{j,i}$
Antonino	+0.5	+0.2	−0.4	+0.5	+0.1	...	+0.5	+14
Boris	−2.0	+0.2	−0.4	+0.5	+0.1	...	−2.1	−19
Caroline	−2.0	+0.2	+2.7	+0.5	−7.2	...	+0.5	+291
David	+0.5	+0.2	−0.4	+0.5	+0.1	...	+0.5	+29
Eve	+0.5	+0.2	−0.4	−1.9	+0.1	...	+0.5	+292
Fred	−2.0	+0.2	−0.4	−1.9	+0.1	...	+0.5	−53
Gábor	+0.5	+0.2	−0.4	−1.9	+0.1	...	+0.5	−42
Henry	−2.0	+0.2	+2.7	−1.9	+0.1	...	+0.5	+269
Copy	0	0	0	1	1	...	0	

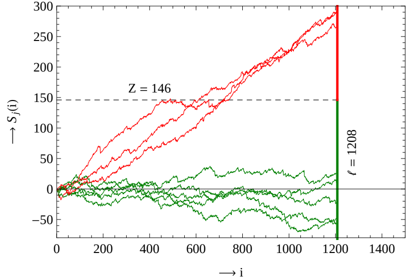
Coalition = {Caroline, Eve, Henry}

Accused = {Caroline, Eve, Henry}

The Tardos scheme: Scores

2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

p_i	0.20	0.05	0.88	0.79	0.98	...	0.18	$\sum_i S_{j,i}$
Antonino								$+14$
Boris								-19
Caroline								$+291$
David								$+29$
Eve								$+292$
Fred								-53
Gábor								-42
Henry								$+269$
Copy	0	0	0	1	1	...	0	



Coalition $=$ {Caroline, Eve, Henry}

Accused $=$ {Caroline, Eve, Henry}

The Tardos scheme: Overview

1. An algorithm to construct collusion-resistant codes
   1a. For each segment i, generate $p_i \sim F$.
      - Many values of p_i close to 0 and 1.
      - Hide choice of p_i from pirates.

   1b. For each segment i, user j, choose $X_{j,i} = 1$ with prob. p_i.
2. An algorithm to trace pirate copies to colluders
   2a. For each segment i, user j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.
      - Positive scores ($S_{j,i} > 0$) for matches ($X_{j,i} = y_i$).
      - Negative scores ($S_{j,i} < 0$) for differences ($X_{j,i} \neq y_i$).
      - Large scores ($|S_{j,i}| \gg 0$) for rare events.

   2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

The Tardos scheme: Overview

1. An algorithm to construct collusion-resistant codes

   1a. For each segment i, generate $p_i \sim F$.

   - Many values of p_i close to 0 and 1.
   - Hide choice of p_i from pirates.

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   - Positive scores ($S_{j,i} > 0$) for matches ($X_{j,i} = y_i$).
   - Negative scores ($S_{j,i} < 0$) for differences ($X_{j,i} \neq y_i$).
   - Large scores ($|S_{j,i}| \gg 0$) for rare events.

   2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

What is the best choice for F and g?

The Tardos scheme: Improvements

Table: Constructive upper bounds on L (large c asymptotics).

	[Tar03]	[S+06]	[N+07]	[BT08]	[S+08]	[N+09]	[LdW12]	[LdW13]	[O+13]
L :	100	39.5	20.6	19.7	9.9	5.4	4.9	4.9*	2
g :	g^{Tar}	g^{Tar}	g^{Tar}	g^{Tar}	g^{sym}	g^{sym}	g^{sym}	g^{sym}	g^{int}
F :	F^{arc}	F^{arc}	F^{*}	F^{arc}	F^{arc}	F^{*}	F^{arc}	F^{*}	F^{arc}

The Tardos scheme: Improvements

Table: Constructive upper bounds on L (large c asymptotics).

	[Tar03]	[S+06]	[N+07]	[BT08]	[S+08]	[N+09]	[LdW12]	[LdW13]	[O+13]
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g :	g^{Tar}	g^{Tar}	g^{Tar}	g^{Tar}	g^{sym}	g^{sym}	g^{sym}	g^{sym}	g^{int}
F :	F^{arc}	F^{arc}	F^{*}	F^{arc}	F^{arc}	F^{*}	F^{arc}	F^{*}	F^{arc}

$$g^{\mathrm{Tar}}(X_{j,i}, y_i, p_i) = \begin{cases} 0, & \text{if } X_{j,i} = 0, y_i = 0, \\ -\sqrt{p_i/(1-p_i)}, & \text{if } X_{j,i} = 0, y_i = 1, \\ 0, & \text{if } X_{j,i} = 1, y_i = 0, \\ +\sqrt{(1-p_i)/p_i}, & \text{if } X_{j,i} = 1, y_i = 1. \end{cases}$$

The Tardos scheme: Improvements

Table: Constructive upper bounds on L (large c asymptotics).

	[Tar03]	[S+06]	[N+07]	[BT08]	[S+08]	[N+09]	[LdW12]	[LdW13]	[O+13]
$L:$	100	39.5	20.6	19.7	9.9	5.4	4.9	4.9^*	2
$g:$	g^{Tar}	g^{Tar}	g^{Tar}	g^{Tar}	g^{sym}	g^{sym}	g^{sym}	g^{sym}	g^{int}
$F:$	F^{arc}	F^{arc}	F^*	F^{arc}	F^{arc}	F^*	F^{arc}	F^*	F^{arc}

$$g^{\mathrm{sym}}(X_{j,i}, y_i, p_i) = \begin{cases} +\sqrt{p_i/(1-p_i)}, & \text{if } X_{j,i} = 0, y_i = 0, \\ -\sqrt{p_i/(1-p_i)}, & \text{if } X_{j,i} = 0, y_i = 1, \\ -\sqrt{(1-p_i)/p_i}, & \text{if } X_{j,i} = 1, y_i = 0, \\ +\sqrt{(1-p_i)/p_i}, & \text{if } X_{j,i} = 1, y_i = 1. \end{cases}$$

The Tardos scheme: Improvements

Table: Constructive upper bounds on L (large c asymptotics).

	[Tar03]	[S+06]	[N+07]	[BT08]	[S+08]	[N+09]	[LdW12]	[LdW13]	[O+13]
L :	100	39.5	20.6	19.7	9.9	5.4	4.9	4.9^*	2
g :	g^{Tar}	g^{Tar}	g^{Tar}	g^{Tar}	g^{sym}	g^{sym}	g^{sym}	g^{sym}	g^{int}
F :	F^{arc}	F^{arc}	F^*	F^{arc}	F^{arc}	F^*	F^{arc}	F^*	F^{arc}

$$g^{\text{int}}(X_{j,i}, y_i, p_i) = \begin{cases} +p_i/(1-p_i), & \text{if } X_{j,i} = 0, y_i = 0, \\ -1, & \text{if } X_{j,i} = 0, y_i = 1, \\ -1, & \text{if } X_{j,i} = 1, y_i = 0, \\ +(1-p_i)/p_i, & \text{if } X_{j,i} = 1, y_i = 1. \end{cases}$$

The Tardos scheme: Comparison

Table: Constructive upper bounds on L (large c asymptotics).

	[Tar03]	[S+06]	[N+07]	[BT08]	[S+08]	[N+09]	[LdW12]	[LdW13]	[O+13]
L :	100	39.5	20.6	19.7	9.9	5.4	4.9	4.9^*	2
g :	g^{Tar}	g^{Tar}	g^{Tar}	g^{Tar}	g^{sym}	g^{sym}	g^{sym}	g^{sym}	g^{int}
F :	F^{arc}	F^{arc}	F^*	F^{arc}	F^{arc}	F^*	F^{arc}	F^*	F^{arc}

Table: Information-theoretic lower bounds on L (large c asymptotics).

	[Tar03]	[AT09]	[HM09]	[HM12]
L :	$\Omega(1)$	"2"	2?	2
F :	?	?	F^{arc}?	F^{arc}
ρ :	?	?	int?	int

The Tardos scheme: Comparison

Table: Constructive upper bounds on L (large c asymptotics).

	[Tar03]	[S+06]	[N+07]	[BT08]	[S+08]	[N+09]	[LdW12]	[LdW13]	[O+13]
L :	100	39.5	20.6	19.7	9.9	5.4	4.9	4.9^*	**2**
g :	g^{Tar}	g^{Tar}	g^{Tar}	g^{Tar}	g^{sym}	g^{sym}	g^{sym}	g^{sym}	g^{int}
F :	F^{arc}	F^{arc}	F^*	F^{arc}	F^{arc}	F^*	F^{arc}	F^*	F^{arc}

Table: Information-theoretic lower bounds on L (large c asymptotics).

	[Tar03]	[AT09]	[HM09]	[HM12]
L :	$\Omega(1)$	"2"	2?	**2**
F :	?	?	F^{arc}?	F^{arc}
ρ :	?	?	int?	int

Efficient capacity-achieving construction

Main result: Using the score function $g = g^{\mathrm{int}}$ given by

$$g^{\mathrm{int}}(X_{j,i}, y_i, p_i) = \begin{cases} +p_i/(1-p_i), & \text{if } X_{j,i} = 0, y_i = 0, \\ -1, & \text{if } X_{j,i} = 0, y_i = 1, \\ -1, & \text{if } X_{j,i} = 1, y_i = 0, \\ +(1-p_i)/p_i, & \text{if } X_{j,i} = 1, y_i = 1. \end{cases}$$

and the distribution function $F = F^{\mathrm{arc}}$ given by

$$F^{\mathrm{arc}}(p_i) = \frac{2}{\pi} \arcsin \sqrt{p_i}$$

we get an efficient construction that achieves capacity for large c.

Static vs. Dynamic

Previous slides dealt with the *static setting*:

- First generate X, then y, then trace.
- Static content: video on demand, software, ...
- Optimized Tardos scheme achieves capacity.

In some practical scenarios, we have a *dynamic setting*:

- $X_1, y_1, X_2, y_2, \dots, X_\ell, y_\ell$.
- Dynamic content: live streams of baseball, football, ...
- More efficient schemes?
- What is the capacity?

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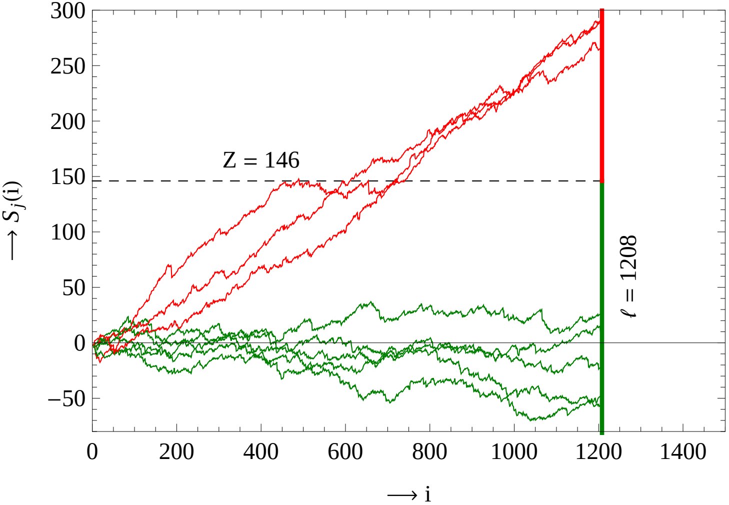
- Dynamic content: live streams of baseball, football, ...
- More efficient schemes?
- What is the capacity?

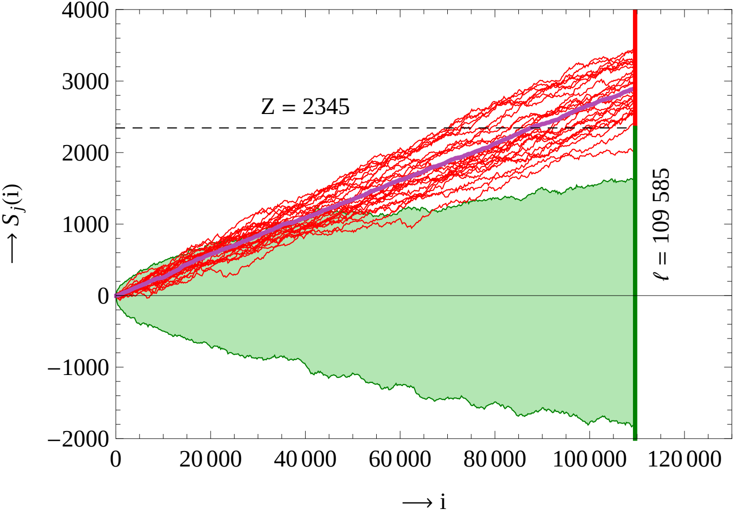
The static Tardos scheme

Earlier example, with $n = 8$ users and $c = 3$ colluders.



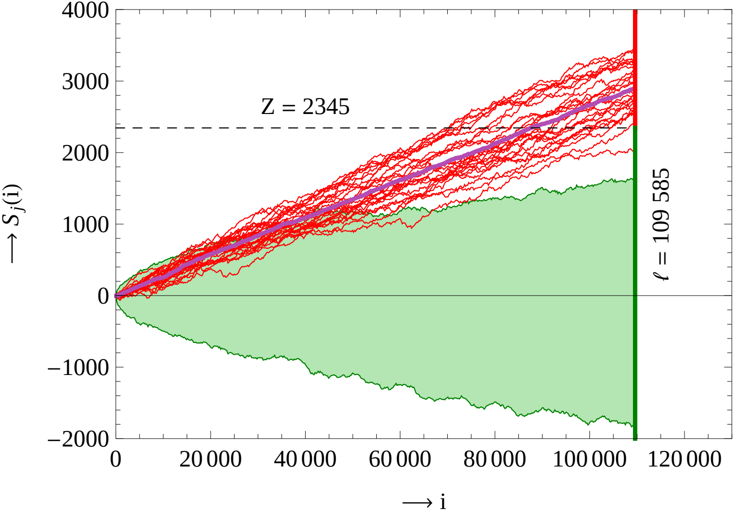
The static Tardos scheme

Bigger example, with $n = 10^6$ users and $c = 25$ colluders.



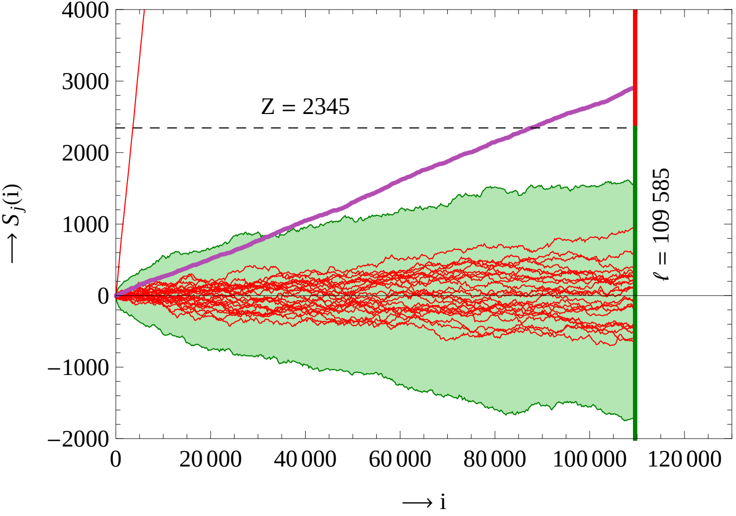
The static Tardos scheme

Catch many, sometimes.



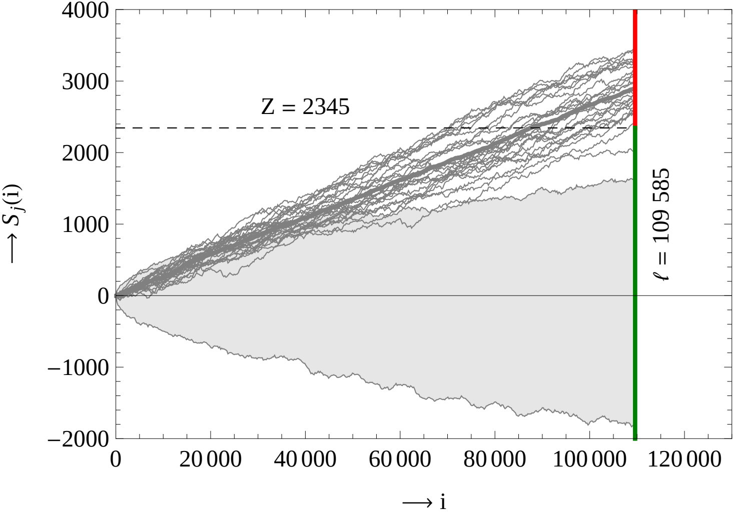
The static Tardos scheme

Catch many, sometimes. Catch one, worst-case.



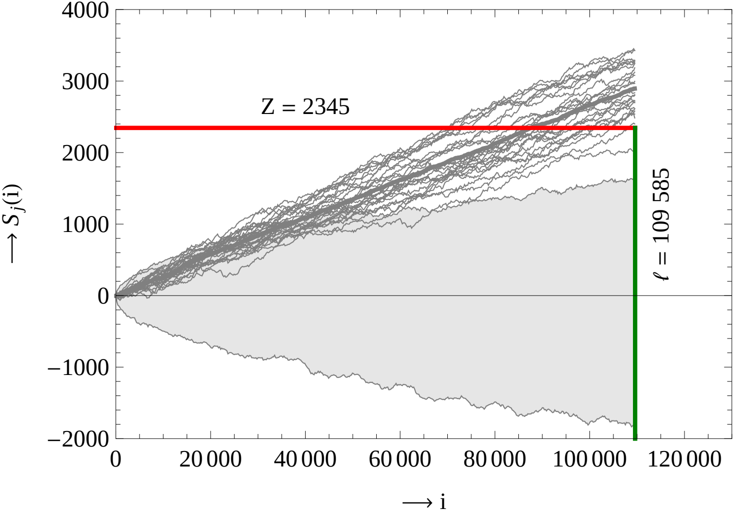
The static Tardos scheme

Instead of disconnecting pirates at the end...



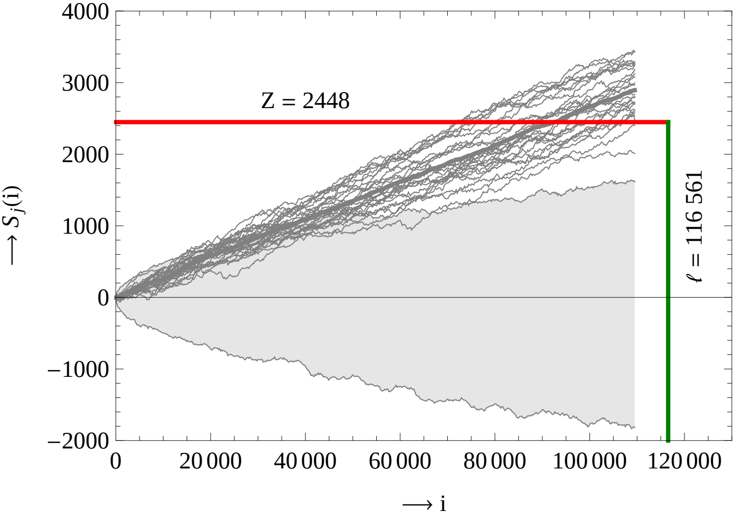
The dynamic Tardos scheme

Instead of disconnecting pirates at the end, disconnect them asap!



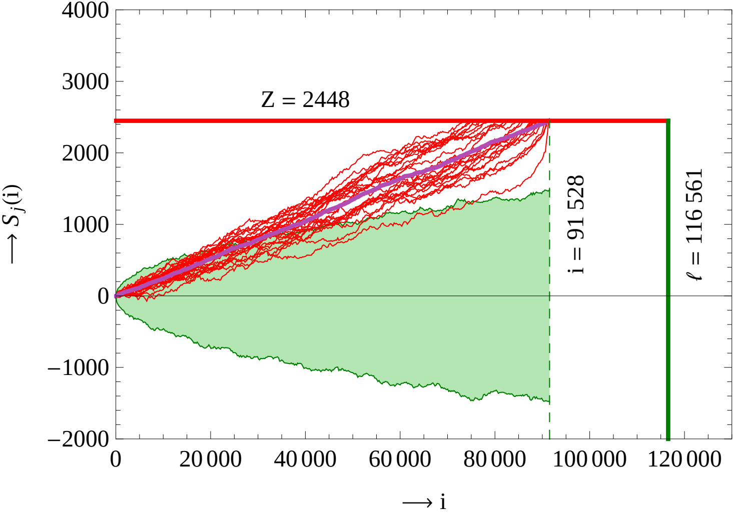
The dynamic Tardos scheme

Instead of disconnecting pirates at the end, disconnect them asap!



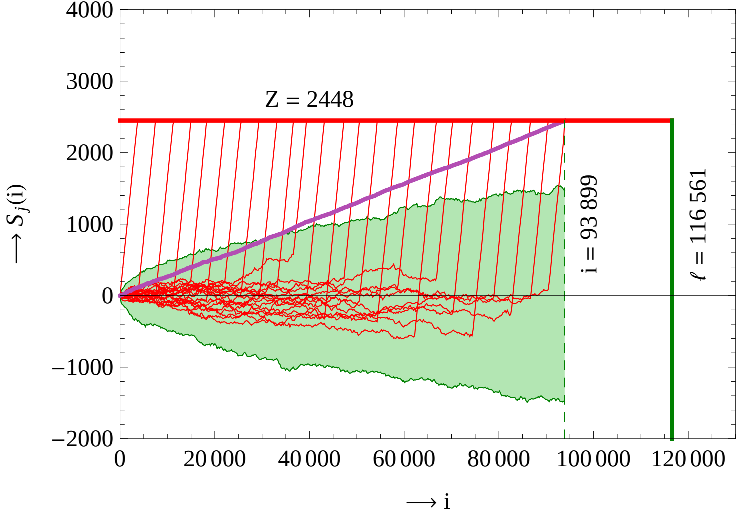
The dynamic Tardos scheme

Catch all, worst-case!



The dynamic Tardos scheme

Catch all, worst-case!



The dynamic Tardos scheme

Main result: By disconnecting users as soon as their scores are too large, we can catch *all pirates* in the worst-case with equivalent code lengths as in the static setting.

Static vs. Dynamic

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In some practical scenarios, we have a *dynamic setting*:

- $X_1, y_1, X_2, y_2, \ldots, X_\ell, y_\ell$.
- Dynamic content: live streams of cricket, football, ...
- More efficient schemes? The dynamic Tardos scheme!
- What is the capacity?

Static vs. Dynamic

Previous slides dealt with the *static setting*:

- First generate X, then y, then trace.
- Static content: video on demand, software, ...
- Optimized Tardos scheme achieves capacity.

In some practical scenarios, we have a *dynamic setting*:

- $X_1, y_1, X_2, y_2, \ldots, X_\ell, y_\ell$.
- Dynamic content: live streams of cricket, football, ...
- More efficient schemes? The dynamic Tardos scheme!
- What is the capacity?

Problem: Blood testing

User								
Antonino	1	0	0	0	0	0	0	0
Boris	0	1	0	0	0	0	0	0
Caroline	0	0	1	0	0	0	0	0
David	0	0	0	1	0	0	0	0
Eve	0	0	0	0	1	0	0	0
Fred	0	0	0	0	0	1	0	0
Gábor	0	0	0	0	0	0	1	0
Henry	0	0	0	0	0	0	0	1

Problem: Blood testing

User								
Antonino	1	0	0	0	0	0	0	0
Boris	0	1	0	0	0	0	0	0
Caroline	0	0	1	0	0	0	0	0
David	0	0	0	1	0	0	0	0
Eve	0	0	0	0	1	0	0	0
Fred	0	0	0	0	0	1	0	0
Gábor	0	0	0	0	0	0	1	0
Henry	0	0	0	0	0	0	0	1

Problem: Blood testing

User								
Antonino	1	0	0	0	0	0	0	0
Boris	0	1	0	0	0	0	0	0
Caroline	0	0	1	0	0	0	0	0
David	0	0	0	1	0	0	0	0
Eve	0	0	0	0	1	0	0	0
Fred	0	0	0	0	0	1	0	0
Gábor	0	0	0	0	0	0	1	0
Henry	0	0	0	0	0	0	0	1
Result	0	0	1	0	0	0	0	0

Solution: Using pools

User								
Antonino	0	0	0					
Boris	0	0	1					
Caroline	0	1	0					
David	0	1	1					
Eve	1	0	0					
Fred	1	0	1					
Gábor	1	1	0					
Henry	1	1	1					

Solution: Using pools

User								
Antonino	0	0	0					
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Result	0	1	0					

Solution: Using pools

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Fred	1	0	1					
Gábor	1	1	0					
Henry	1	1	1					

Problem: Several defectives

User								
Antonino	0	0	0					
Boris	0	0	1					
Caroline	0	1	0					
David	0	1	1					
Eve	1	0	0					
Fred	1	0	1					
Gábor	1	1	0					
Henry	1	1	1					

Problem: Several defectives

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Antonino	0	0	0					
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Result	1	1	0					

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User								
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Gábor	1	1	0					
Henry	1	1	1					
Result	1	1	0					

1. Comparable to traitor tracing, but easier...

TU/e

Problem: Several defectives

User								
Antonino	0	0	0					
Boris	0	0	1					
Caroline	0	1	0					
David	0	1	1					
Eve	1	0	0					
Fred	1	0	1					
Gábor	1	1	0					
Henry	1	1	1					
Result	1	1	0					

1. Comparable to traitor tracing, but easier...
2. Fixed pirate strategy: Always output a 1.

The Tardos scheme

1. An algorithm to construct collusion-resistant codes
   1a. For each segment i, generate $p_i \sim F$.

$$F(p_i) = \tfrac{2}{\pi} \arcsin \sqrt{p_i}$$

   1b. For each segment i, user j, choose $X_{j,i} = 1$ with prob. p_i.
2. An algorithm to trace pirate copies to colluders
   2a. For each segment i, user j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.

$$g(X_{j,i}, y_i, p_i) = \begin{cases} +p_i/(1-p_i), & \text{if } X_{j,i} = 0, y_i = 0, \\ -1, & \text{if } X_{j,i} = 0, y_i = 1, \\ -1, & \text{if } X_{j,i} = 1, y_i = 0, \\ +(1-p_i)/p_i, & \text{if } X_{j,i} = 1, y_i = 1. \end{cases}$$

   2b. For each user j, accuse user j iff $\sum_i S_{j,i}$ is "large".

A group testing scheme

1. An algorithm to construct the group testing matrix
   1a. For each test i, generate $p_i \sim F$.

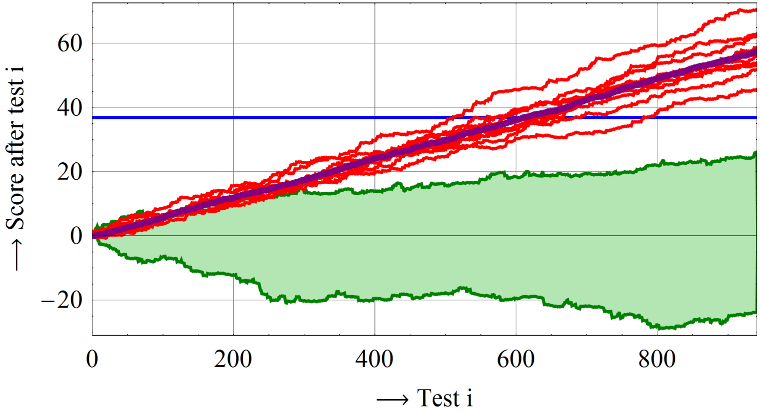
$$F^{\mathrm{arc}}(p_i) = H(p - p_0)$$

   1b. For each test i, person j, choose $X_{j,i} = 1$ with prob. p_i.
2. An algorithm to trace the test results to defectives
   2a. For each test i, person j, calculate $S_{j,i} = g(X_{j,i}, y_i, p_i)$.

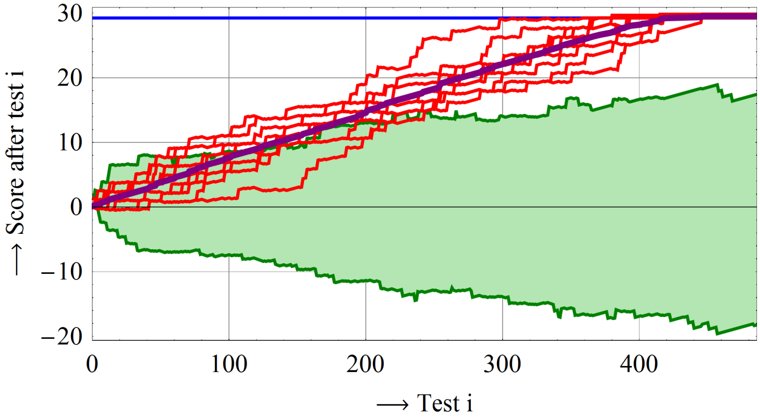
$$g(X_{j,i}, y_i, p_i) = \begin{cases} +p_i/(1-p_i), & \text{if } X_{j,i} = 0, y_i = 0, \\ -p_i(1-p_i)^{c-1}/(1-(1-p_i)^c), & \text{if } X_{j,i} = 0, y_i = 1, \\ -1, & \text{if } X_{j,i} = 1, y_i = 0, \\ +(1-p_i)^c/(1-(1-p_i)^c), & \text{if } X_{j,i} = 1, y_i = 1. \end{cases}$$

   2b. For each person j, mark him defective iff $\sum_i S_{j,i}$ is "large".

A group testing scheme: Static



A group testing scheme: Dynamic



Conclusion

Progress in Traitor Tracing...

- Outline of the Tardos scheme
- A capacity-achieving score function
- A dynamic version of Tardos' scheme
- Open problem: Dynamic capacity?

...with Applications to Group Testing

- Scheme based on traitor tracing
- Optimized bias p and score function g
- Improves upon best known results for large c

Questions?