Progressive lattice sieving

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Lattices

What is a lattice?

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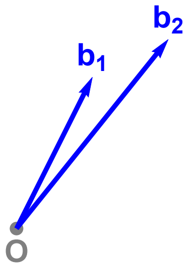

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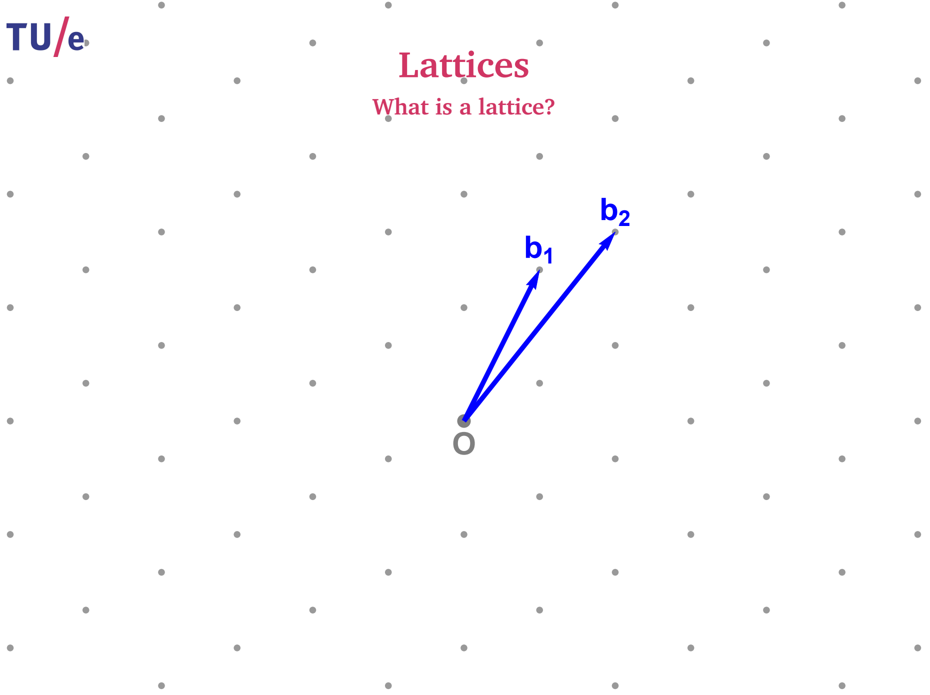
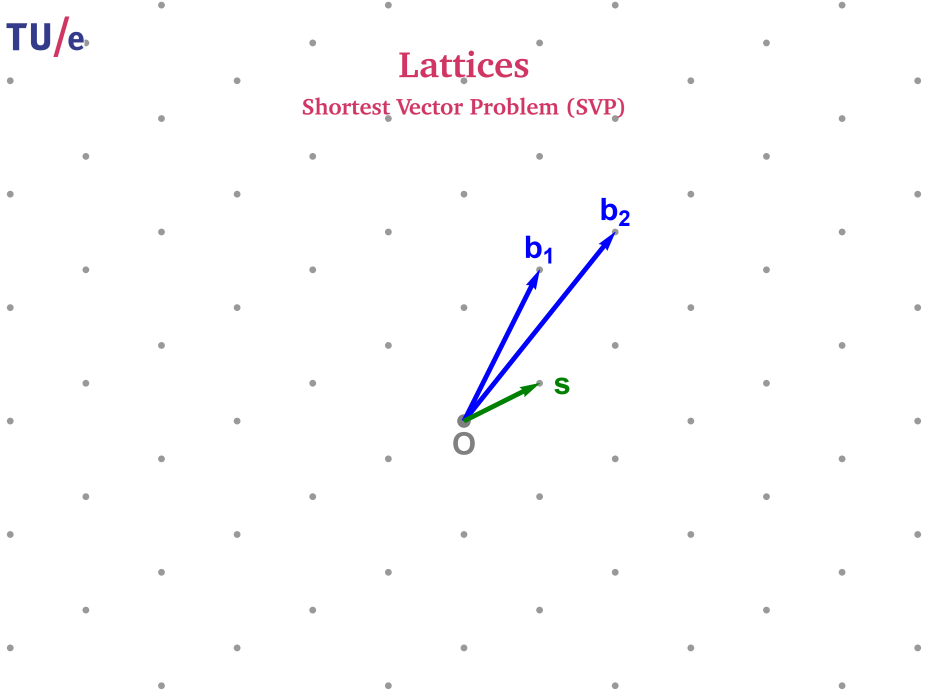


Lattices

Shortest Vector Problem (SVP)



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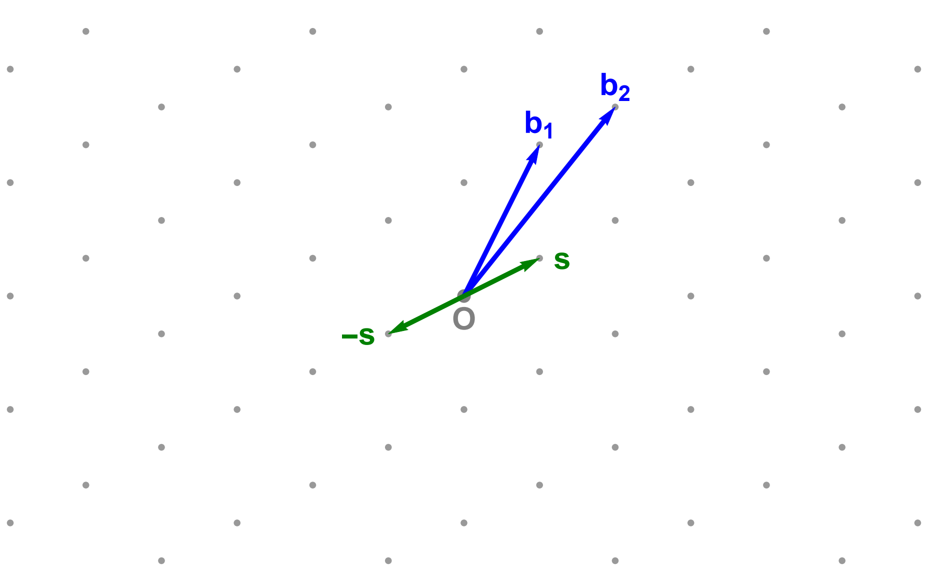


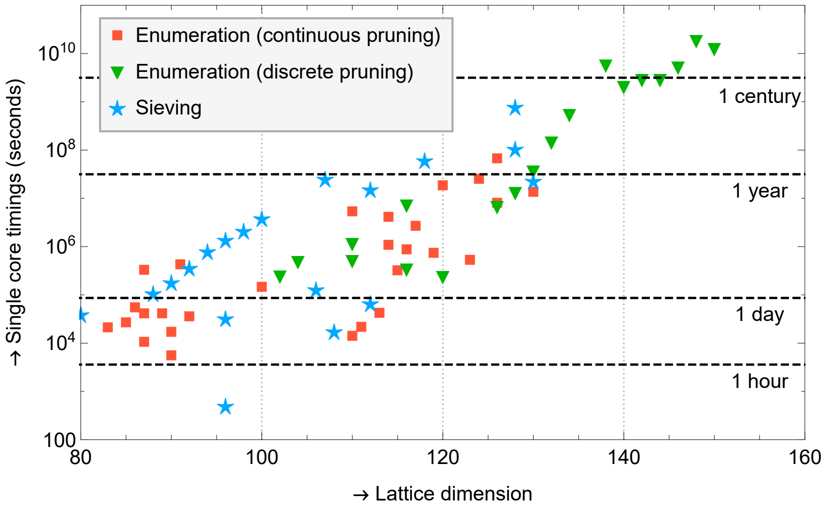
SVP hardness

Theory

	Algorithm	$\log_2$(Time)	$\log_2$(Space)
Proven SVP	Enumeration [Poh81, Kan83, ..., MW15, AN17]	$O(n \log n)$	$O(\log n)$
	AKS-sieve [AKS01, NV08, MV10, HPS11]	$3.398n$	$1.985n$
	ListSieve [MV10, MDB14]	$3.199n$	$1.327n$
	Birthday sieves [PS09, HPS11]	$2.465n$	$1.233n$
	Enumeration/DGS hybrid [CCL17]	$2.048n$	$0.500n$
	Voronoi cell algorithm [AEVZ02, MV10b]	$2.000n$	$1.000n$
	Quantum sieve [LMP13, LMP15]	$1.799n$	$1.286n$
	Quantum enum/DGS [CCL17]	$1.256n$	$\mathbf{0.500n}$
	Discrete Gaussian sampling [ADRS15, ADS15, AS18]	$\mathbf{1.000n}$	$1.000n$
Heuristic SVP	The Nguyen–Vidick sieve [NV08]	$0.415n$	$0.208n$
	The GaussSieve [MV10, ..., IKMT14, BNvdP16, YKYC17]	$0.415n$	$0.208n$
	Triple sieve [BLS16, HK17]	$0.396n$	$0.189n$
	Two-level sieve [WLTB11]	$0.384n$	$0.256n$
	Three-level sieve [ZPH13]	$0.3778n$	$0.283n$
	Overlattice sieve [BGJ14]	$0.3774n$	$0.293n$
	Triple sieve with NNS [HK17, HKL18]	$0.359n$	$\mathbf{0.189n}$
	Hyperplane LSH [Cha02, Laa15, ..., LM18, Duc18]	$0.337n$	$0.337n$
	Graph-based NNS [EPY99, DCL11, MPLK14, Laa18]	$0.327n$	$0.282n$
	Hypercube LSH [TT07, Laa17]	$0.322n$	$0.322n$
	Quantum sieve [LMP13, LMP15]	$0.312n$	$0.208n$
	May–Ozerov NNS [MO15, BGJ15]	$0.311n$	$0.311n$
	Spherical LSH [AINR14, LdW15]	$0.298n$	$0.298n$
	Cross-polytope LSH [TT07, AILRS15, BL16, KW17]	$0.298n$	$0.298n$
	Spherical LSF [BDGL16, MLB17, ALRW17, Chr17]	$\mathbf{0.292n}$	$0.292n$
	Quantum NNS sieve [LMP15, Laa16]	$\mathbf{0.265n}$	$0.265n$

SVP hardness

Practice [SVP17]



SVP hardness

NIST submissions

Title	Si	En	Submitters
CRYSTALS–Dilithium	•		**Lyubashevsky**, Ducas, Kiltz, Lepoint, Schwabe, Seiler, Stehlé
CRYSTALS–Kyber	•		**Schwabe**, Avanzi, Bos, Ducas, Kiltz, Lepoint, Lyubashevsky, Schanck, ...
Ding Key Exchange	•		**Ding**, Takagi, Gao, Wang
(R.)EMBLEM	•		**Seo**, Park, Lee, Kim, Lee
FALCON	•		**Prest**, Fouque, Hoffstein, Kirchner, Lyubashevsky, Pornin, Ricosset, ...
FrodoKEM	•		**Naehrig**, Alkim, Bos, Ducas, Easterbrook, LaMacchia, Longa, Mironov, ...
Giophantus	•		**Akiyama**, Goto, Okumura, Takagi, Nuida, Hanaoka, Shimizu, Ikematsu
HILA5	•		**Saarinen**
KCL	•		**Zhao**, Jin, Gong, Sui
KINDI	•		**El Bansarkhani**
LAC	•		**Lu**, Liu, Jia, Xue, He, Zhang
LIMA	•		**Smart**, Albrecht, Lindell, Orsini, Osheter, Paterson, Peer
Lizard	•		**Cheon**, Park, Lee, Kim, Song, Hong, Kim, Kim, Hong, Yun, Kim, Park, ...
LOTUS		•	**Phong**, Hayashi, Aono, Moriai
NewHope	•		**Pöppelmann**, Alkim, Avanzi, Bos, Ducas, De La Piedra, Schwabe, Stebila
NTRUEncrypt	◦	◦	**Zhang**, Chen, Hoffstein, Whyte
NTRU-HRSS-KEM	•		**Schanck**, Hülsing, Rijneveld, Schwabe
NTRU Prime		•	**Bernstein**, Chuengsatiansup, Lange, Van Vredendaal
pqNTRUSign	◦	◦	**Zhang**, Chen, Hoffstein, Whyte
qTESLA	•		**Bindel**, Akleylek, Alkim, Barreto, Buchmann, Eaton, Gutoski, Krämer, ...
Round2	•		**Garcia-Morchon**, Zhang, Bhattacharya, Rietman, Tolhuizen, Torre-Arce
SABER	•		**D'Anvers**, Karmakar, Roy, Vercauteren
Three Bears	•		**Hamburg**
Titanium	•		**Steinfeld**, Sakzad, Zhao
Totals:	**21**	**3**	**Total: 24 proposals estimate SVP hardness with sieving/enumeration**

*Not included in this overview: Compact LWE, DRS, Mersenne, Odd Manhattan, Ramstake, ...

SVP hardness

Overview

Problem: How hard is SVP in high dimensions?

- Two main approaches: *enumeration* and *sieving*
  - Enumeration: memory-efficient, asymptotically slow
  - Sieving: memory-intensive, asymptotically fast
- Theoretically (large n): sieving > enumeration
- Practically (small n): enumeration > sieving
- NIST submissions: (mostly) sieving

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Problem: Can sieving still be improved?

- Theoretically: Probably not... [BDGL16, ALRW17, HKL18]
- Practically: Yes! **(this work)**, [Duc18]

GaussSieve

1. Generate random lattice vectors

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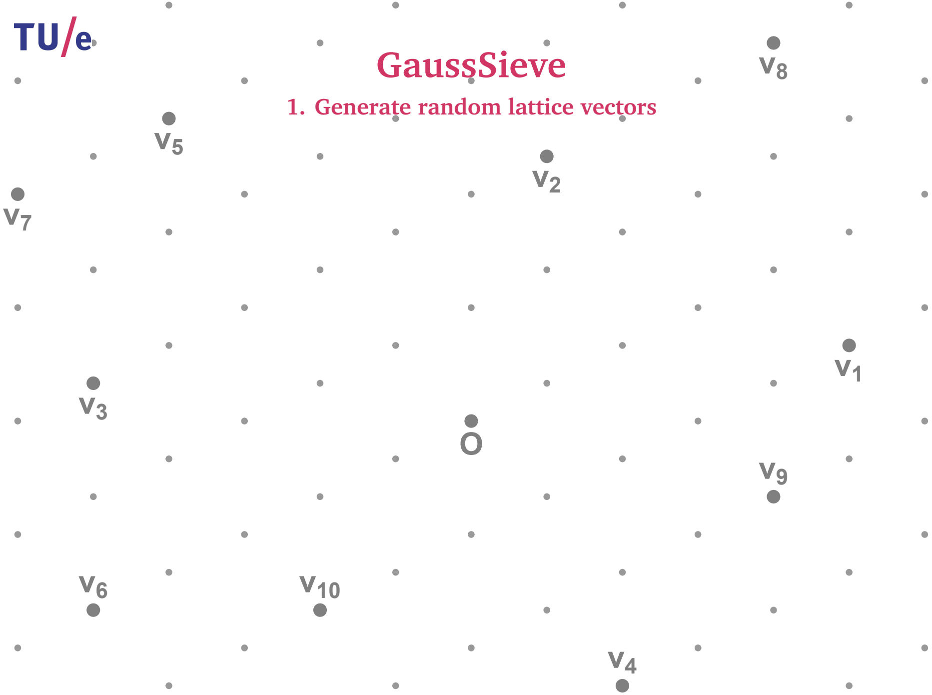
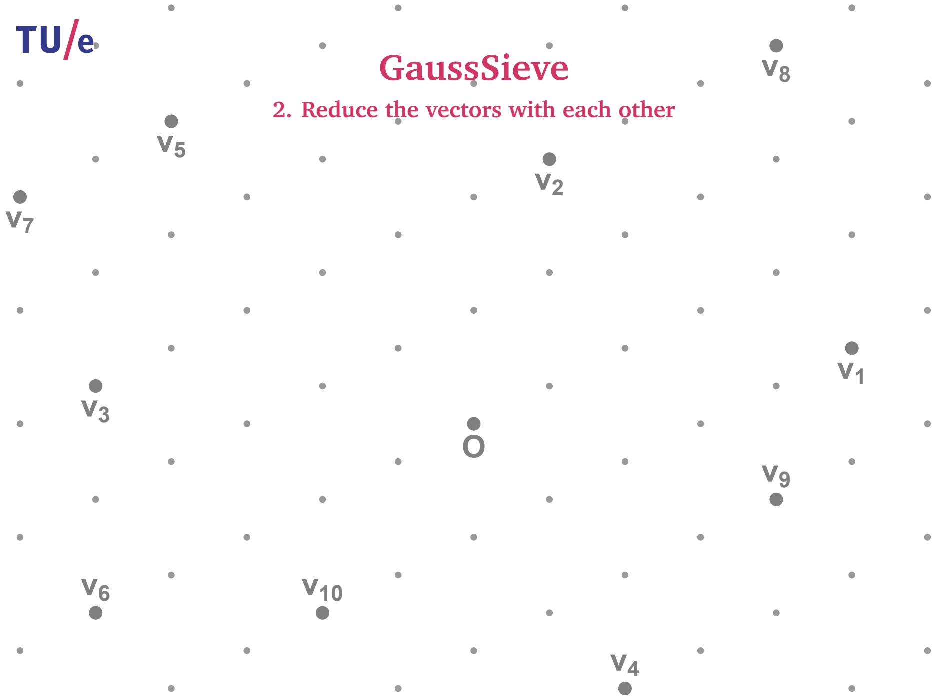


GaussSieve

2. Reduce the vectors with each other



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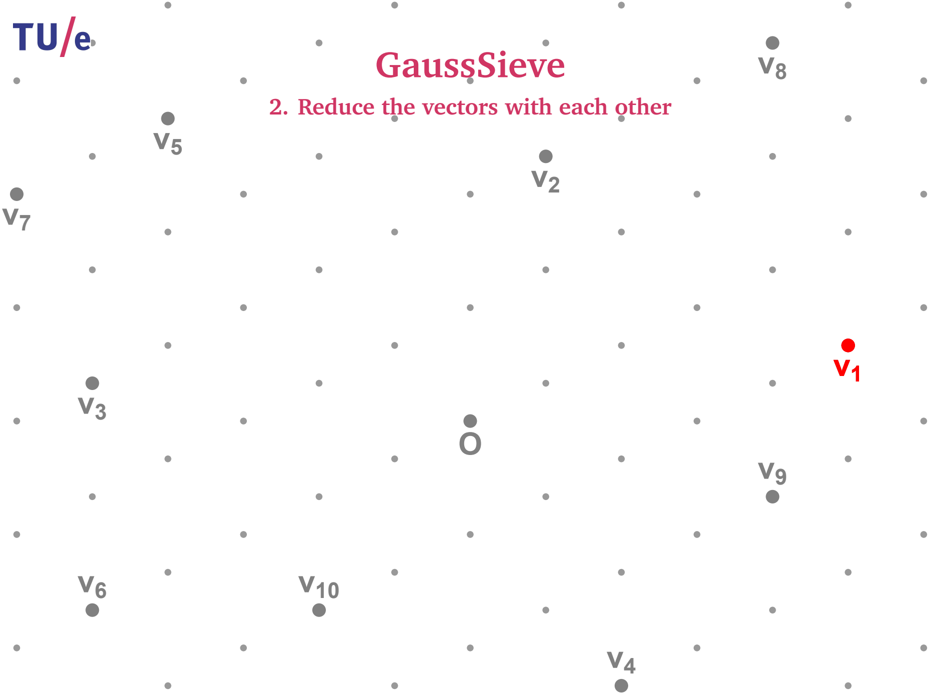

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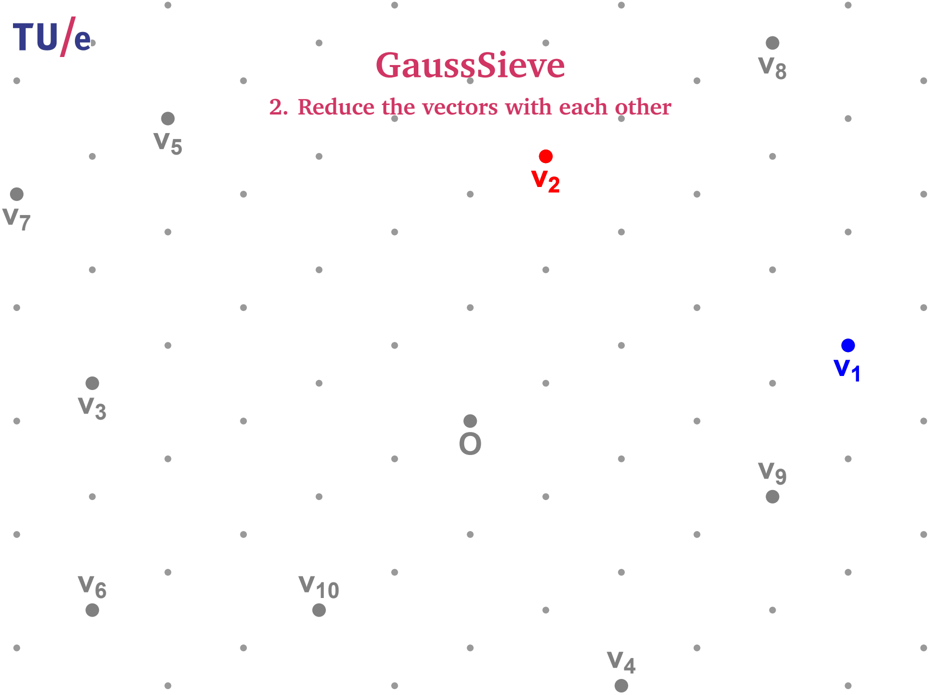

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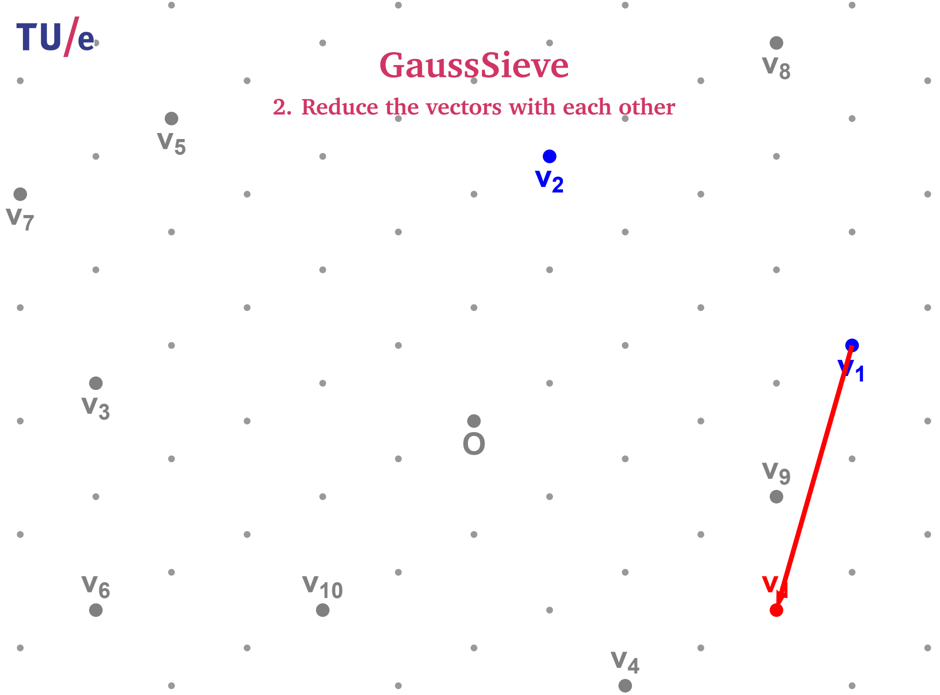

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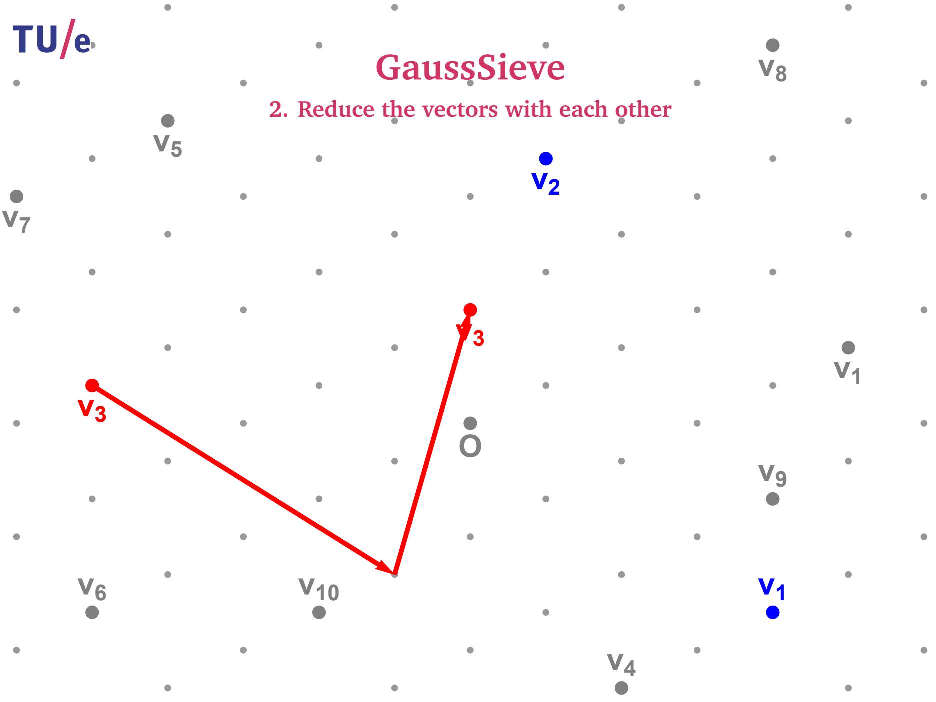

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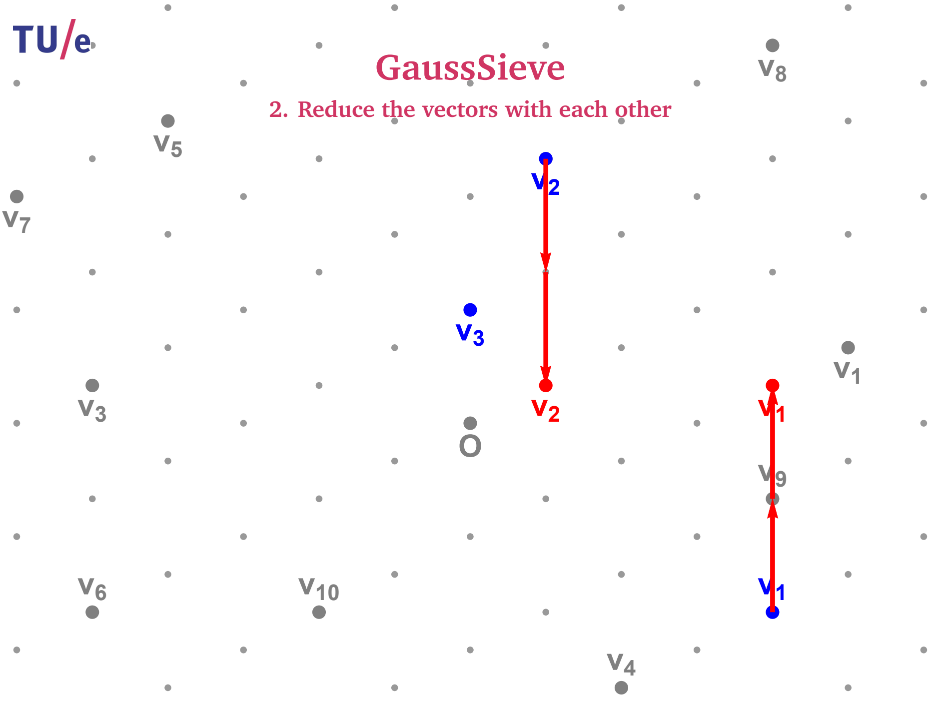

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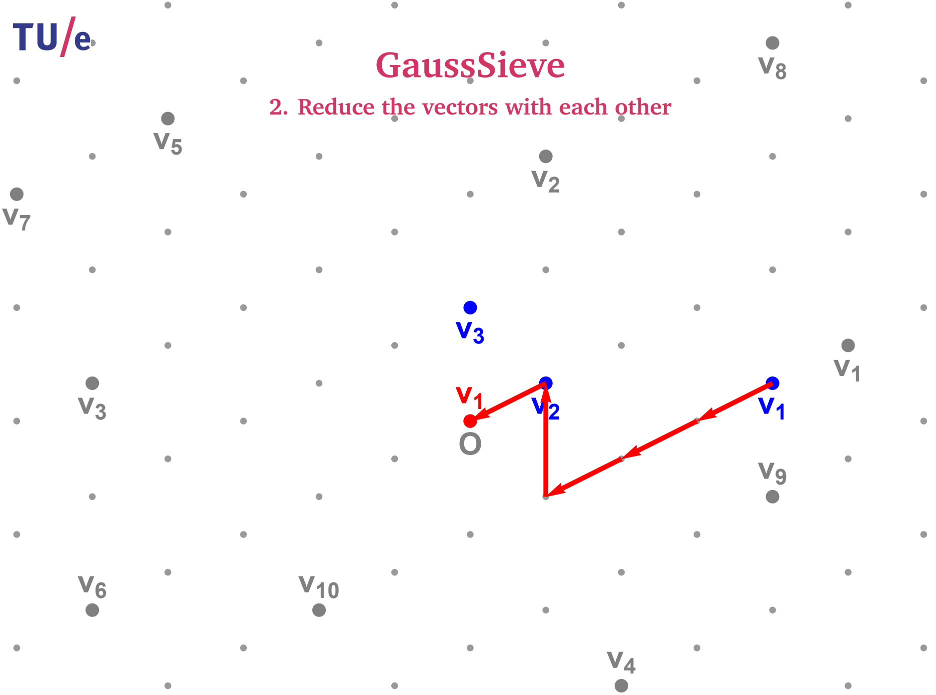

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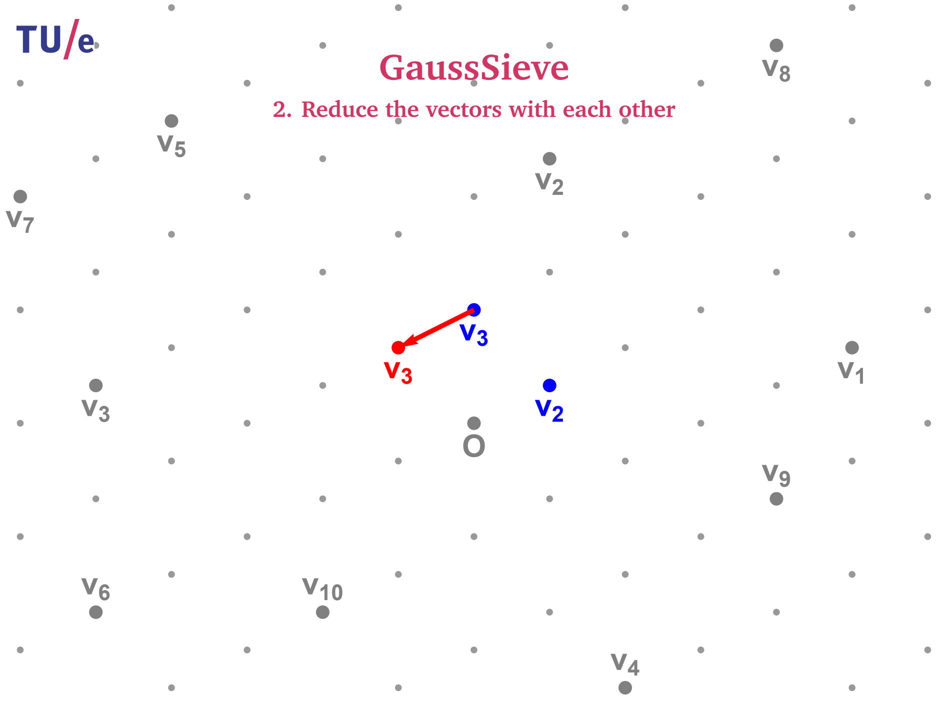

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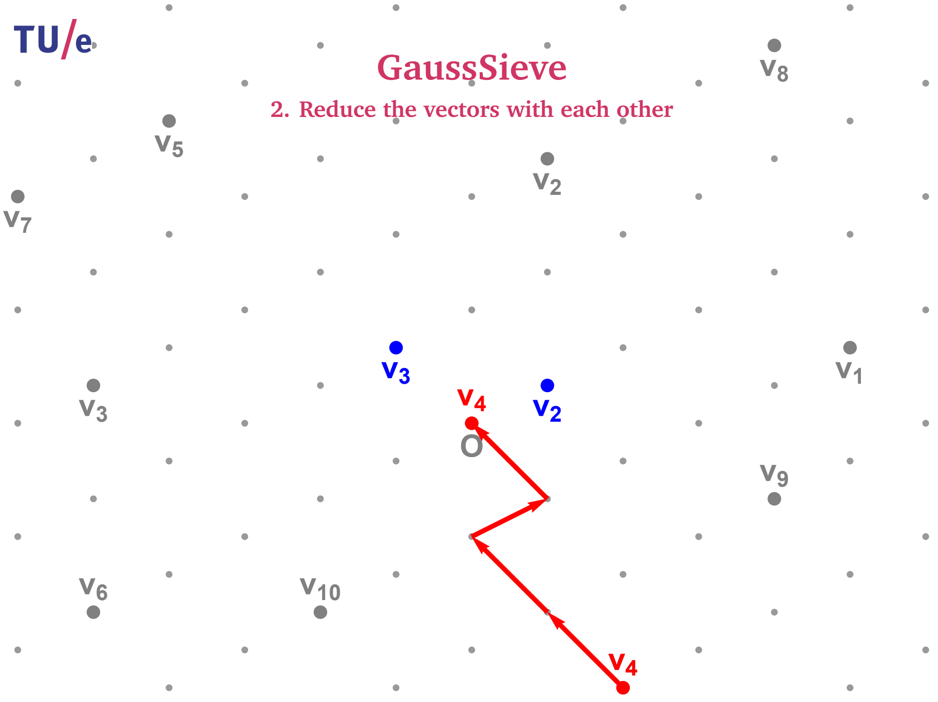

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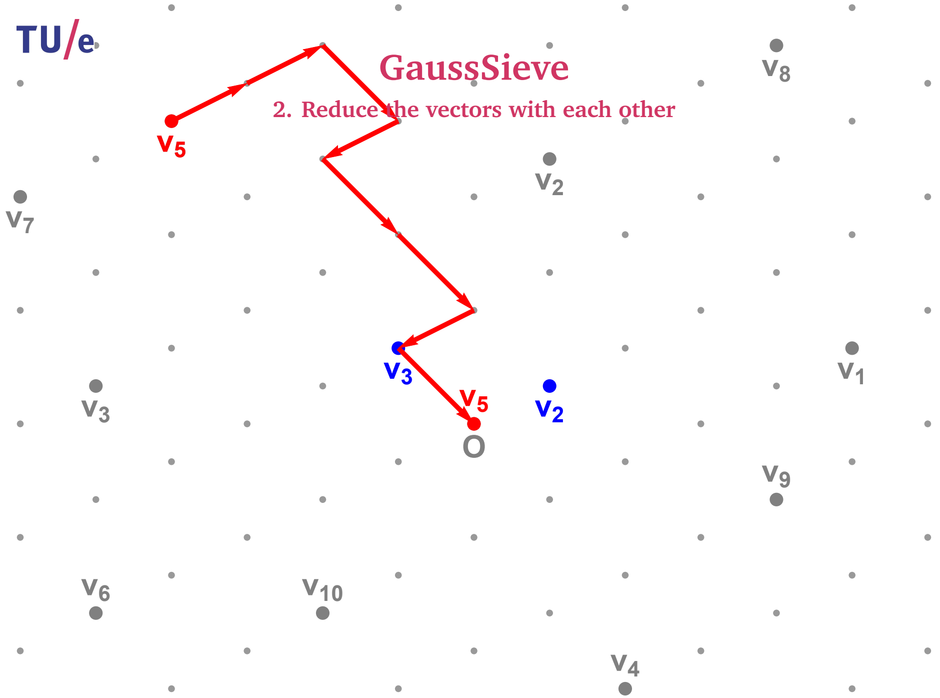

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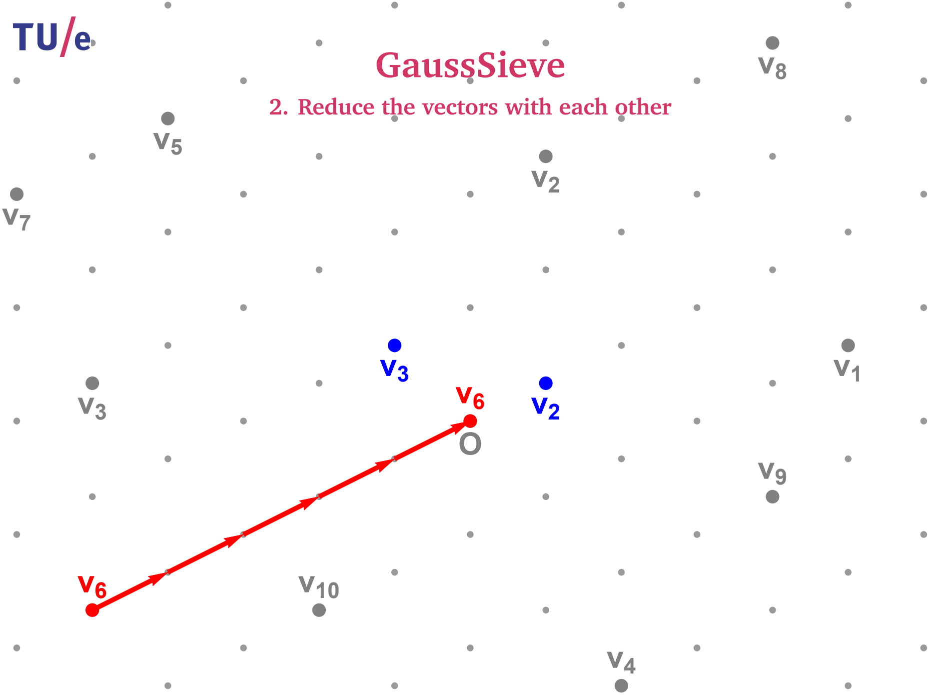

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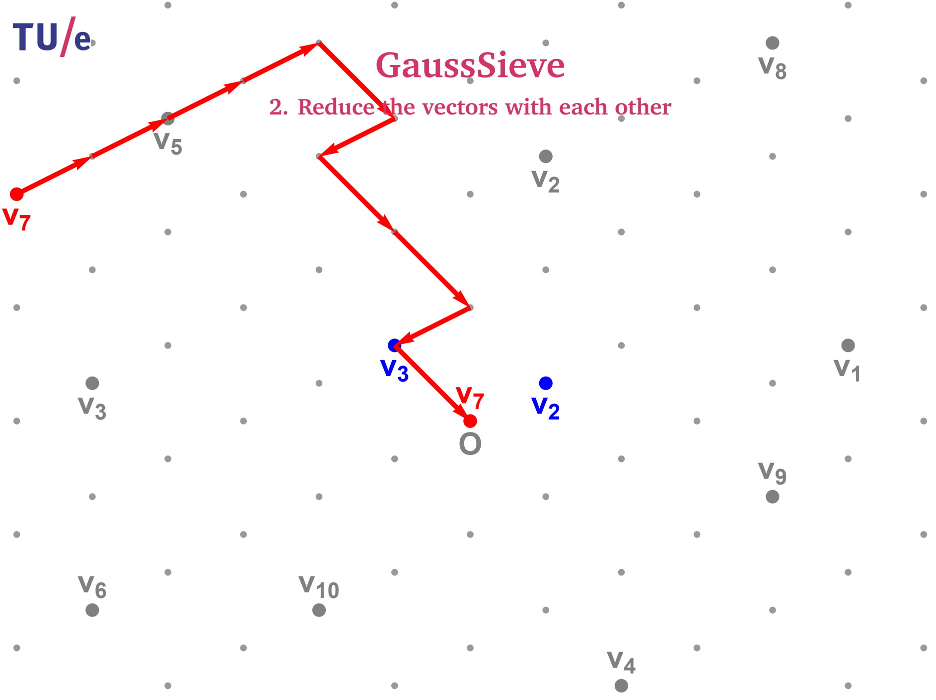

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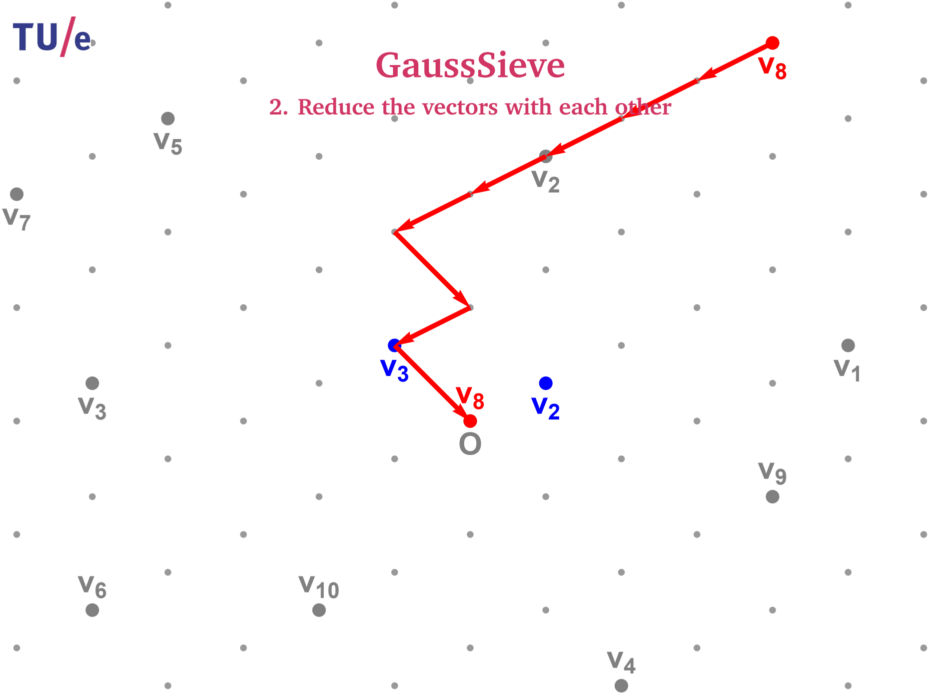

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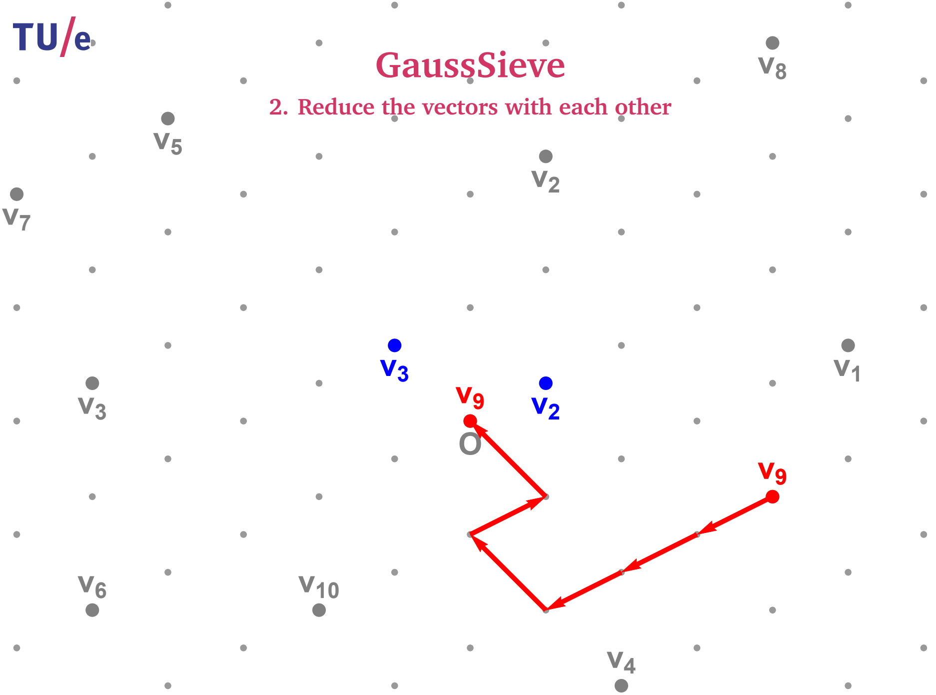

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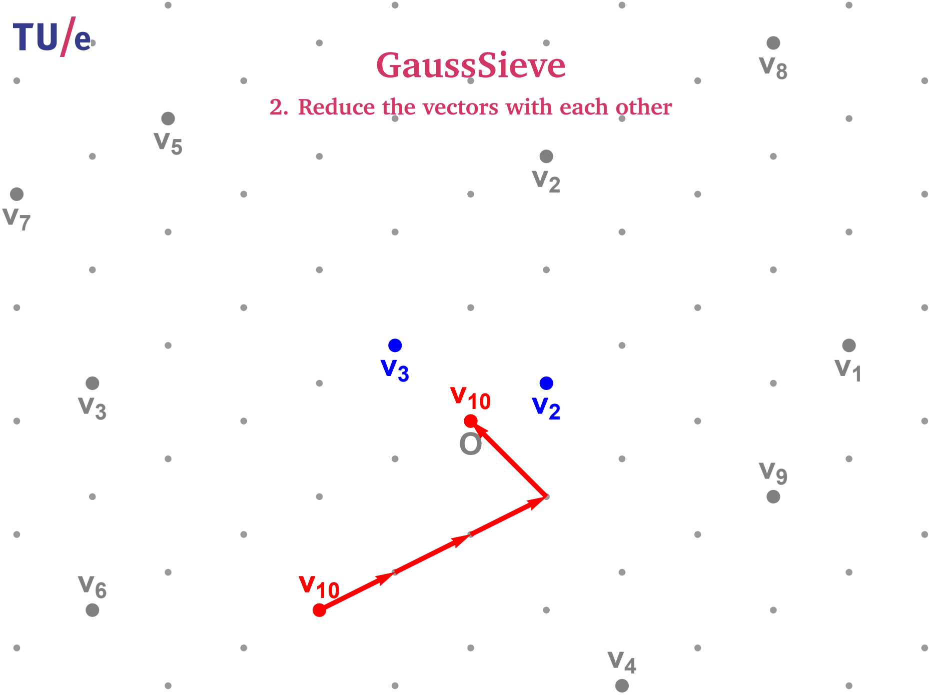

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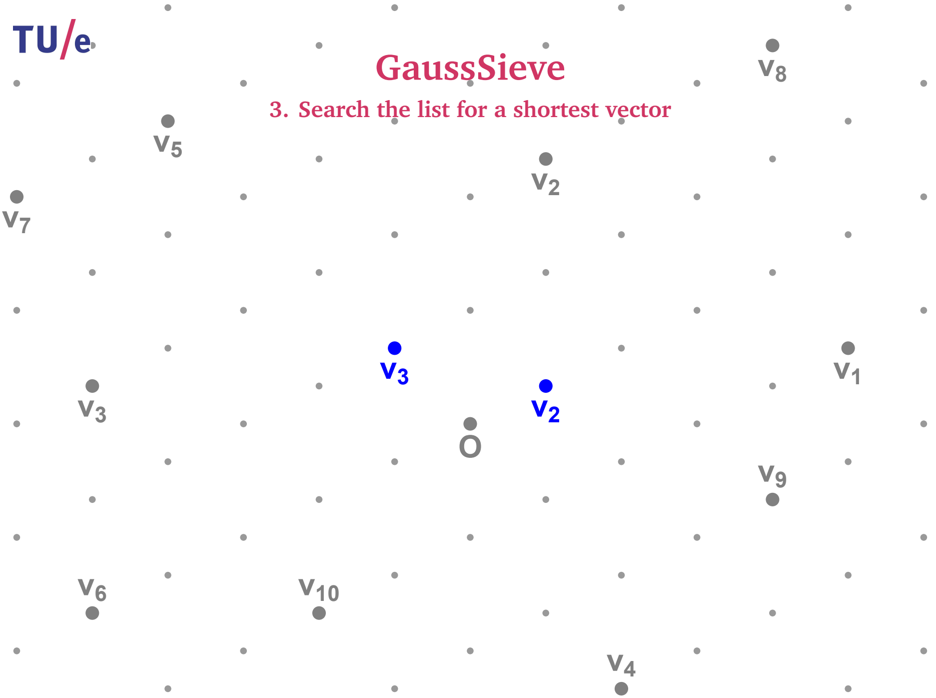

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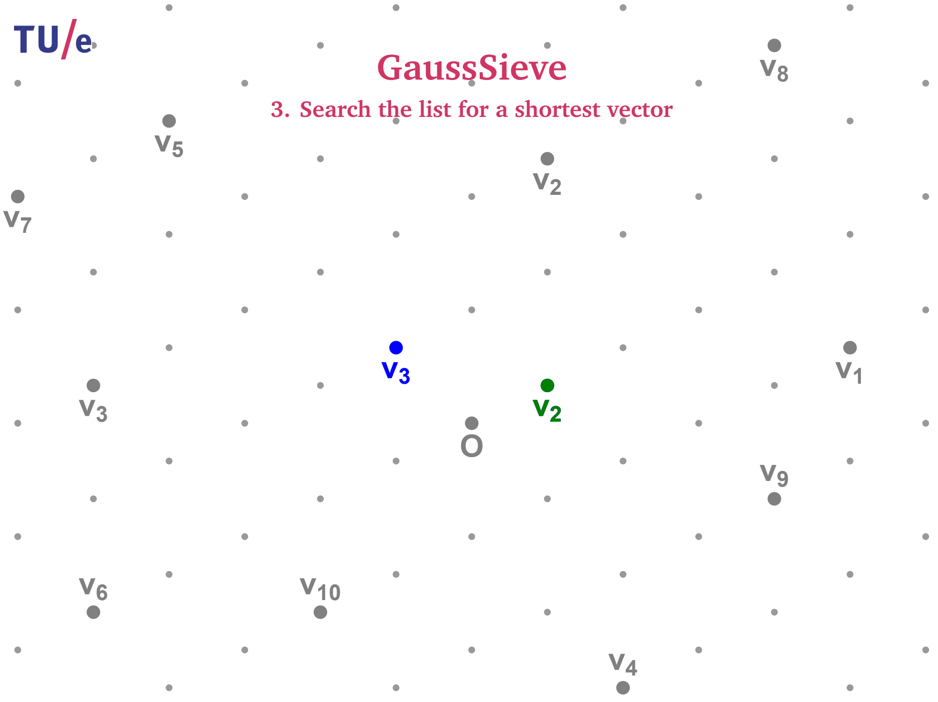

ProGaussSieve

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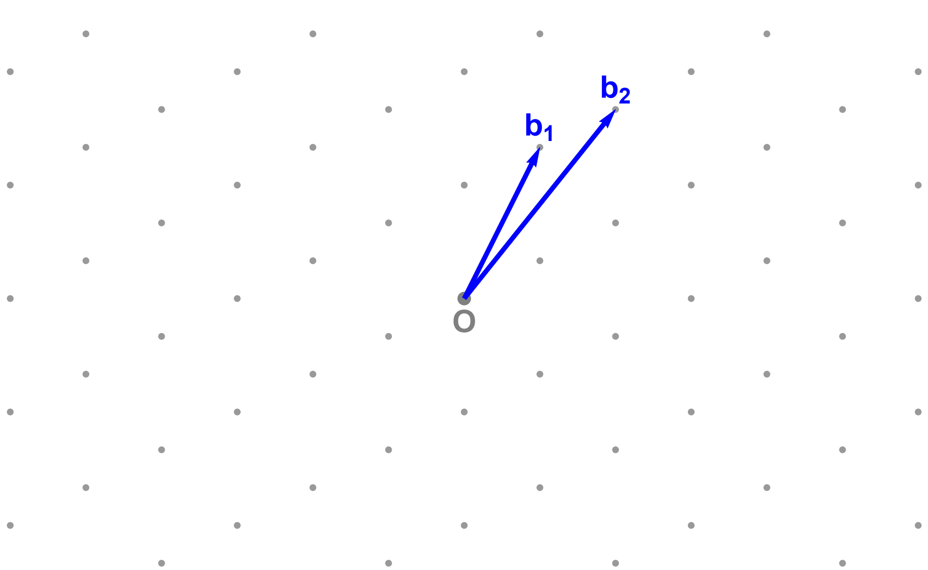

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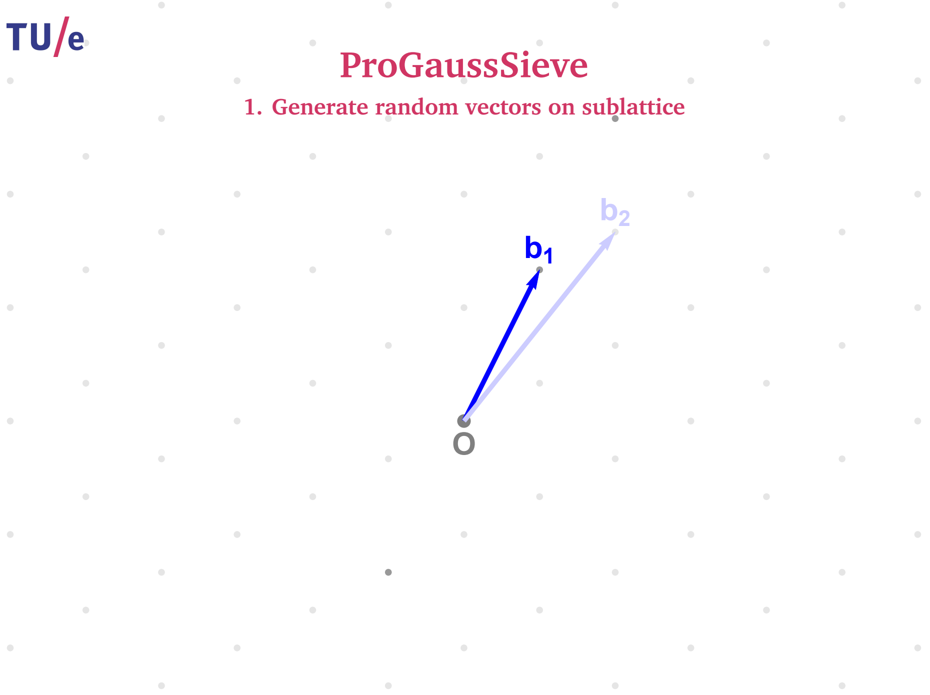

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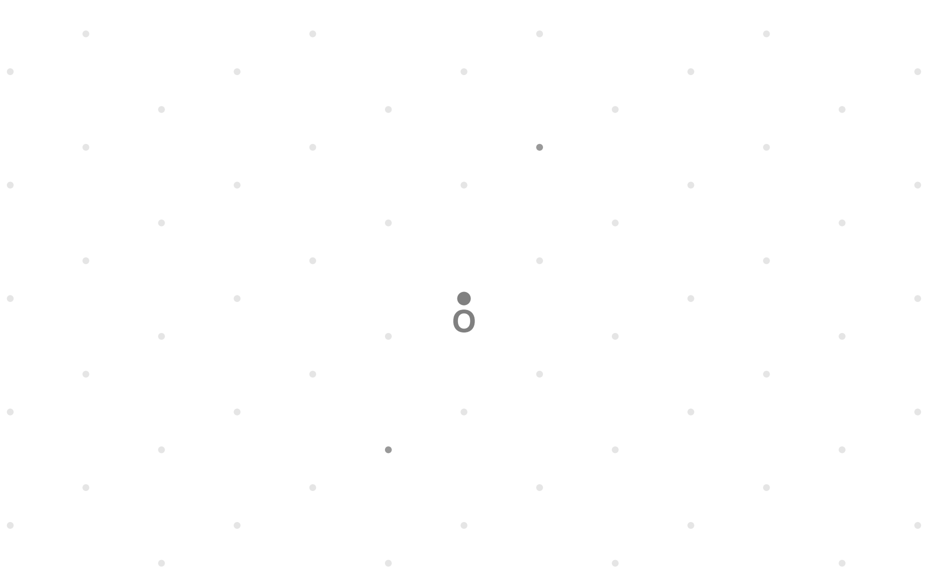

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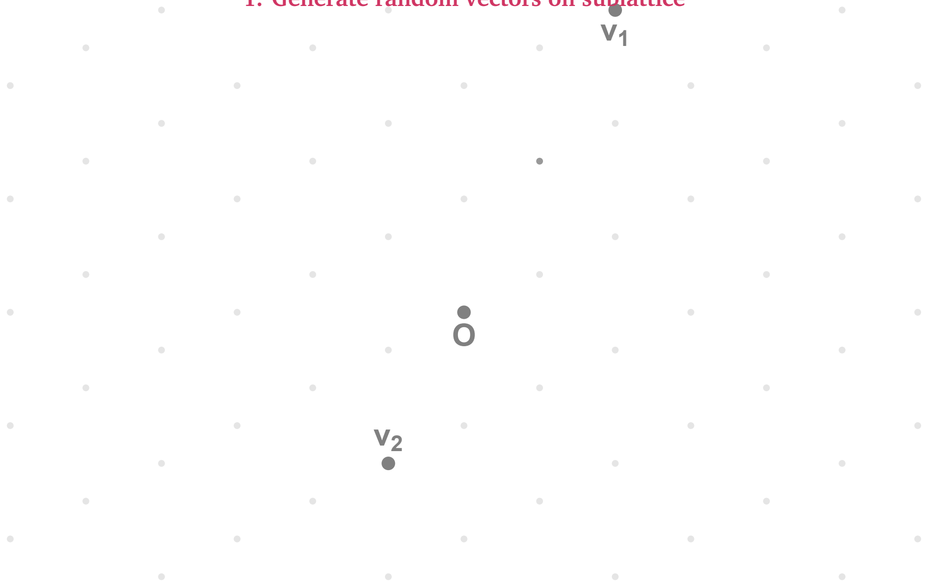

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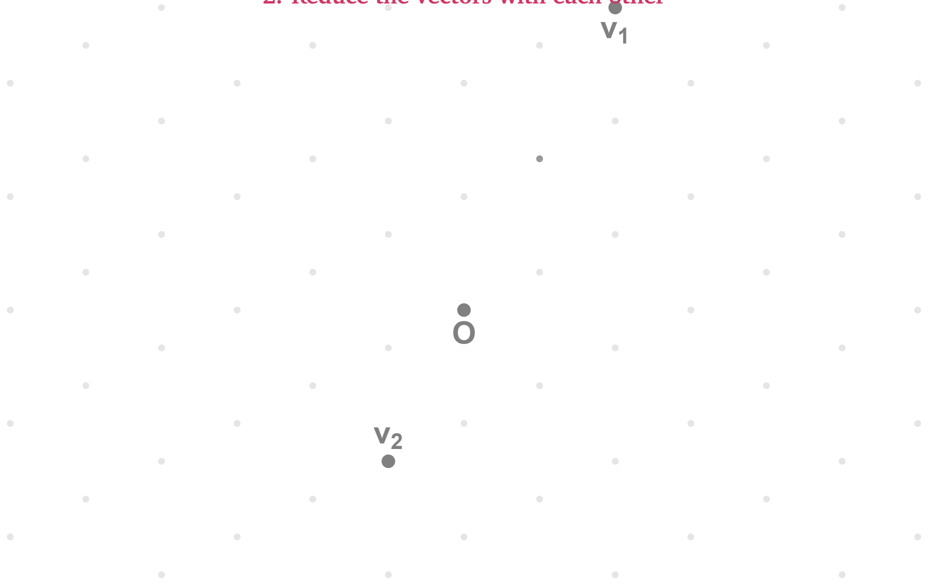

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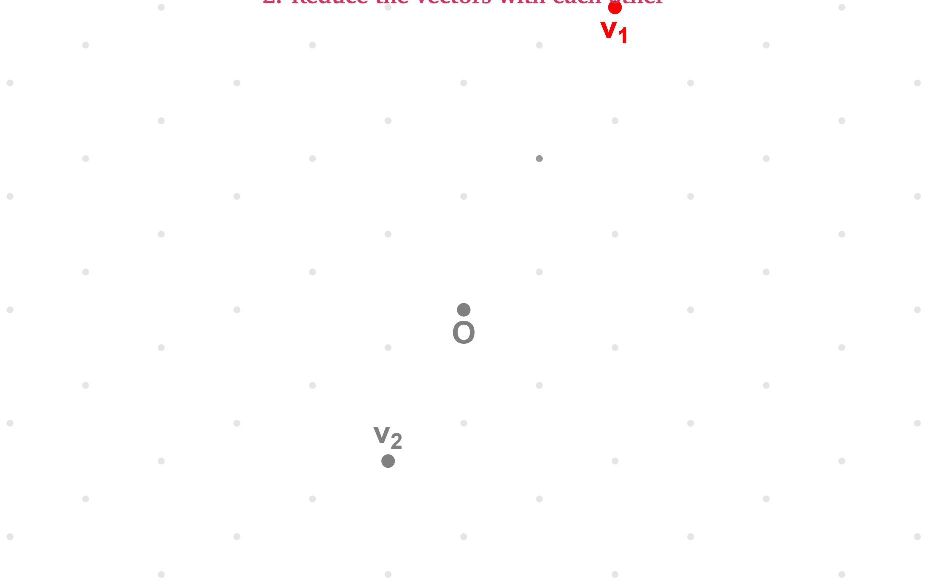

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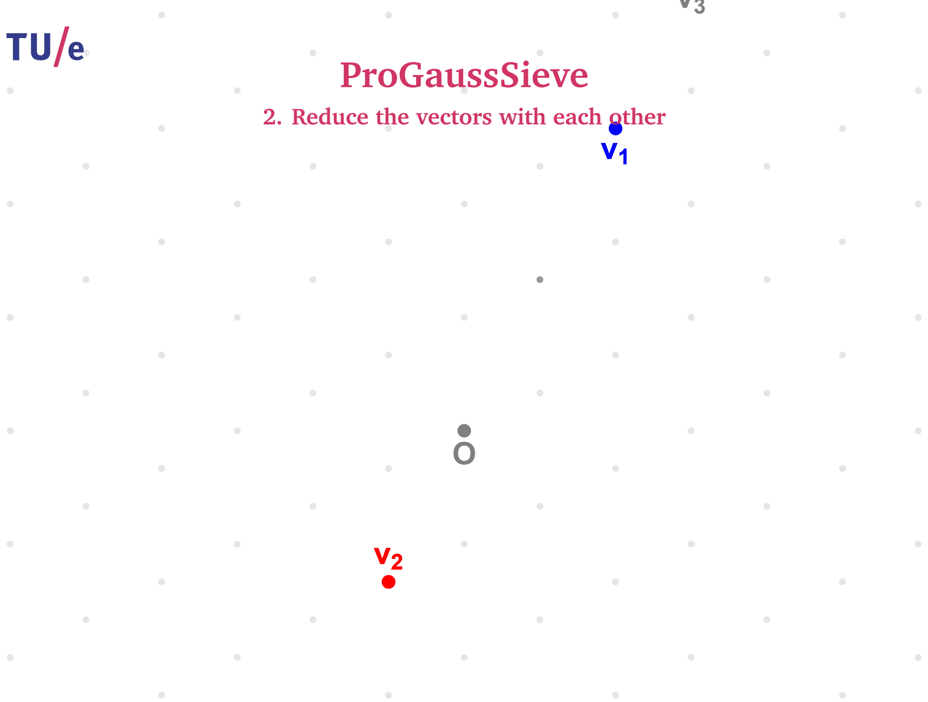

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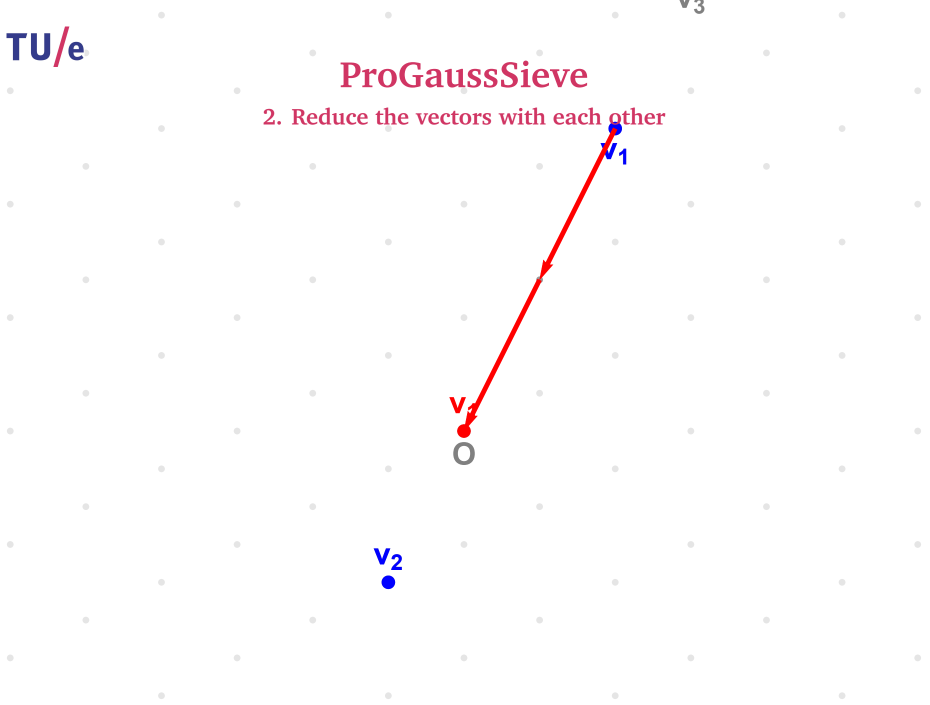

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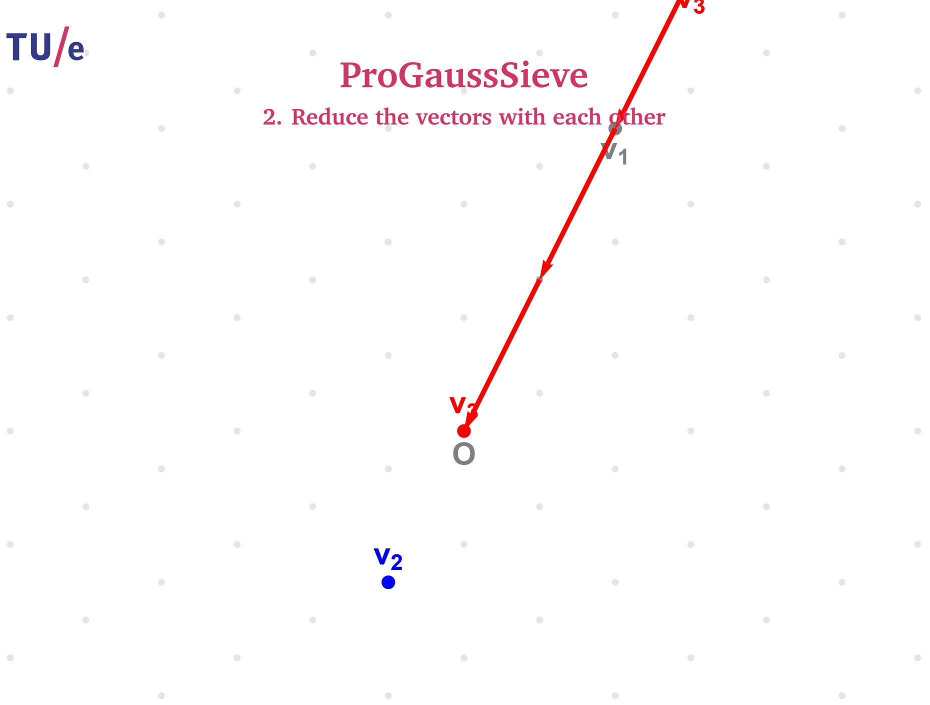

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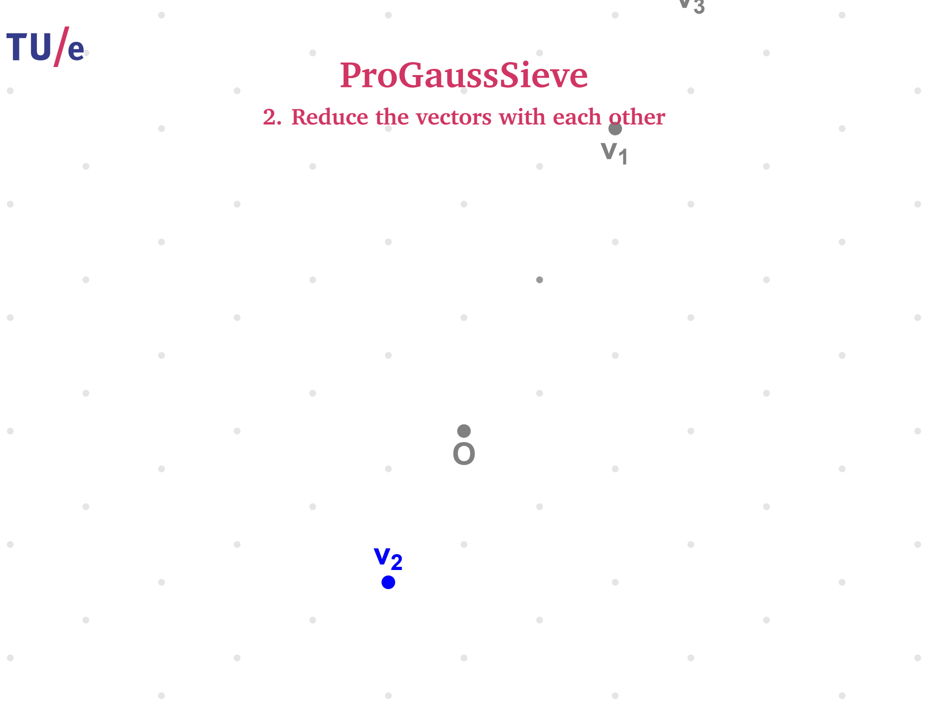

ProGaussSieve

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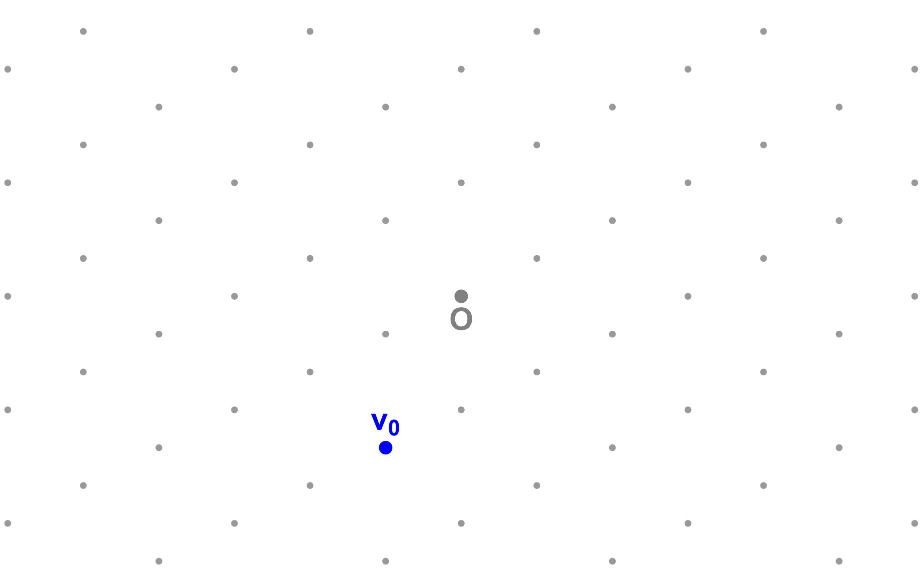

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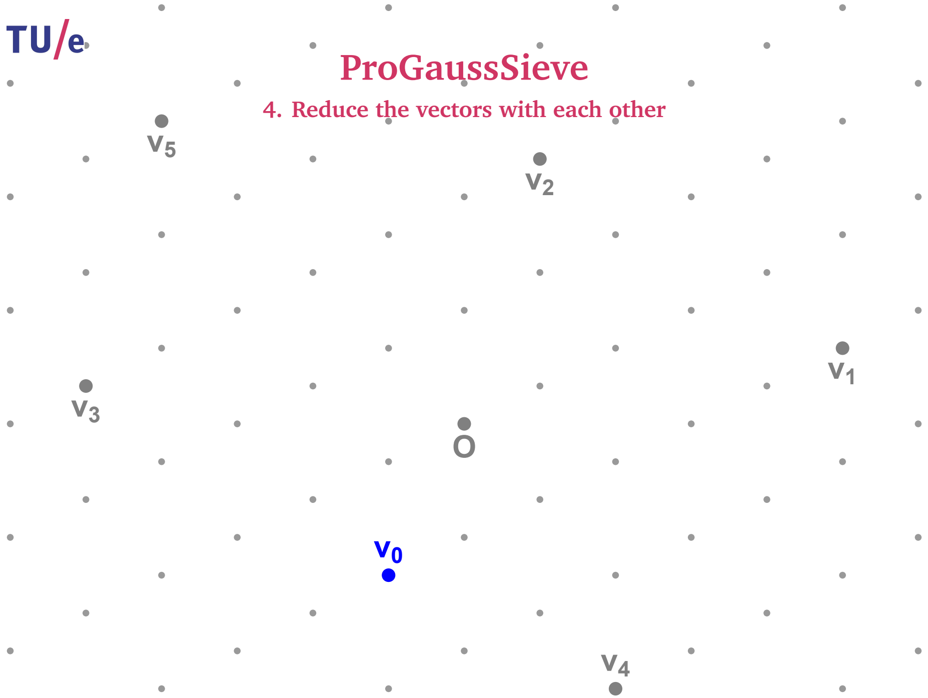

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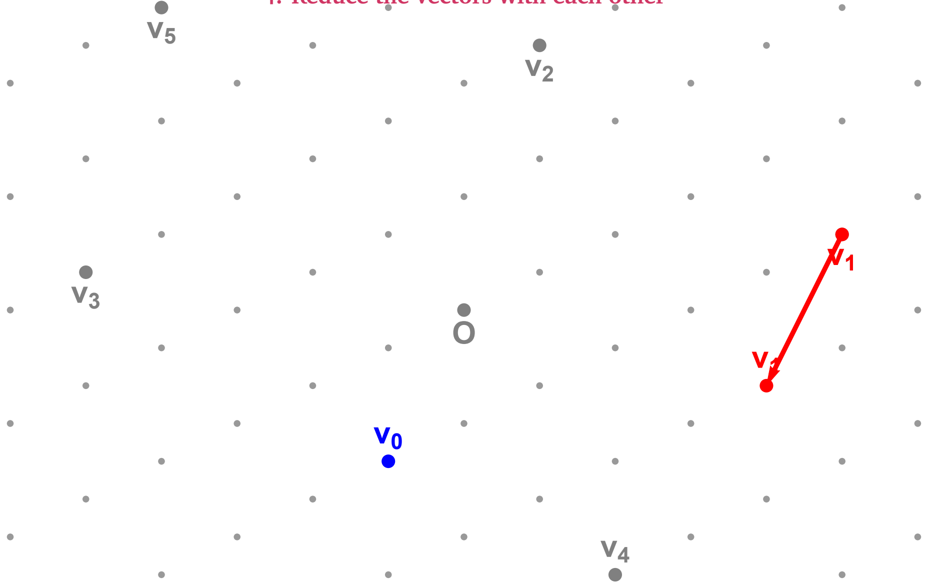

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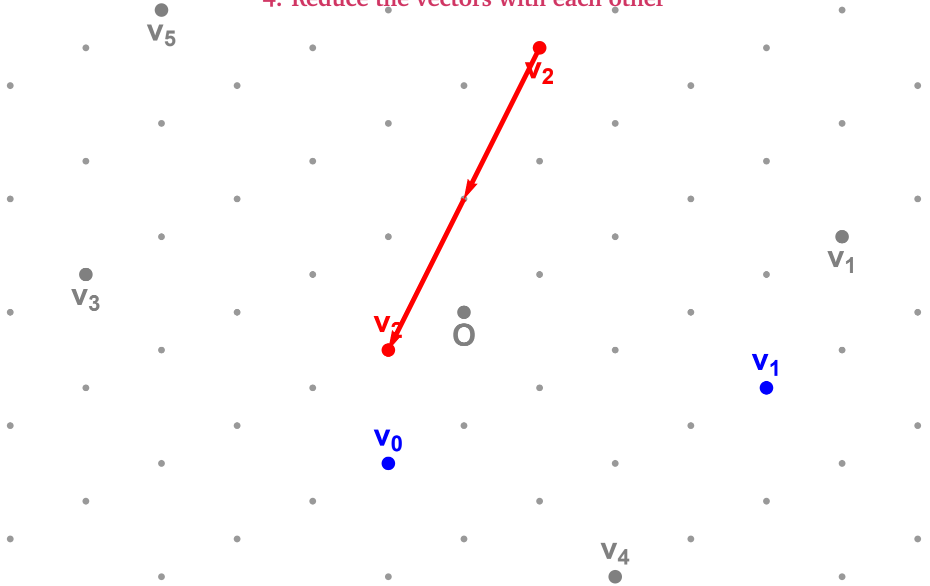

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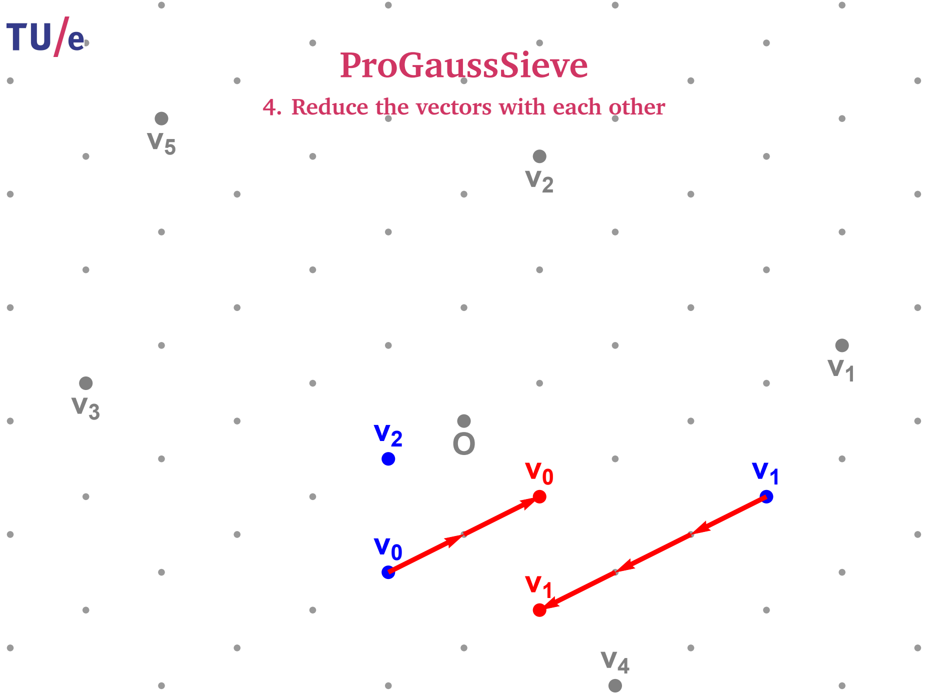

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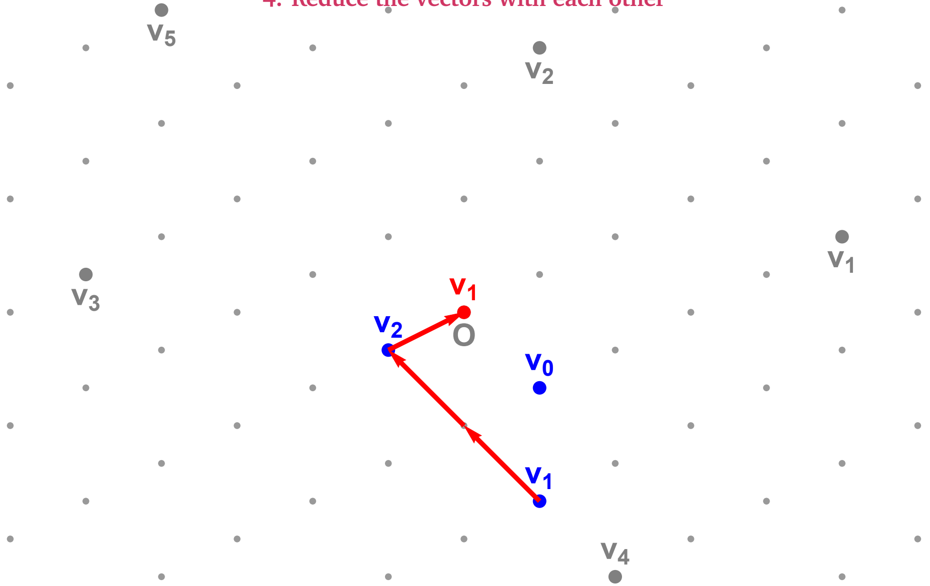

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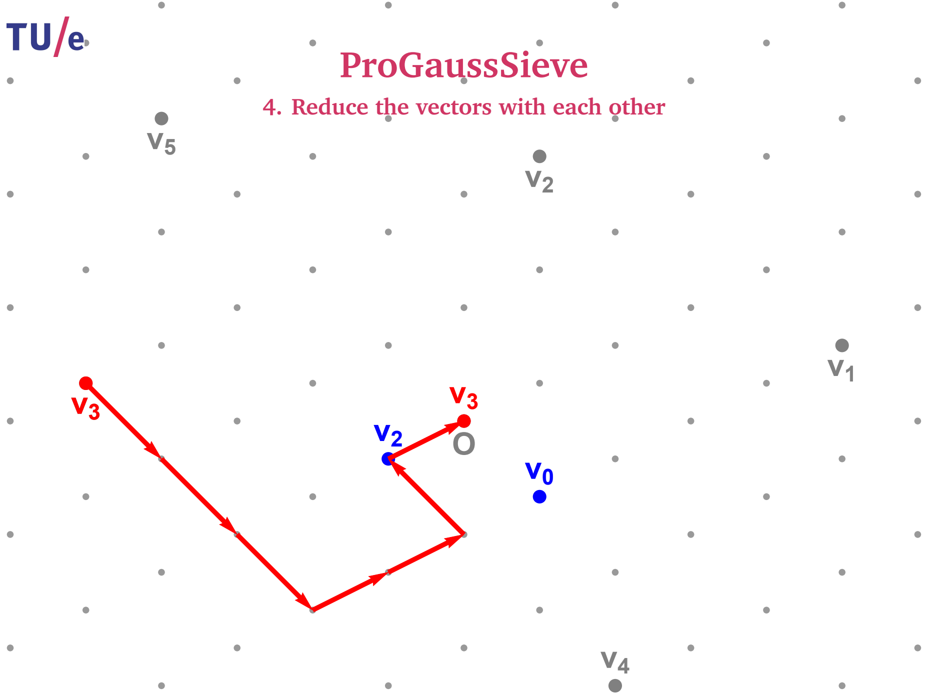

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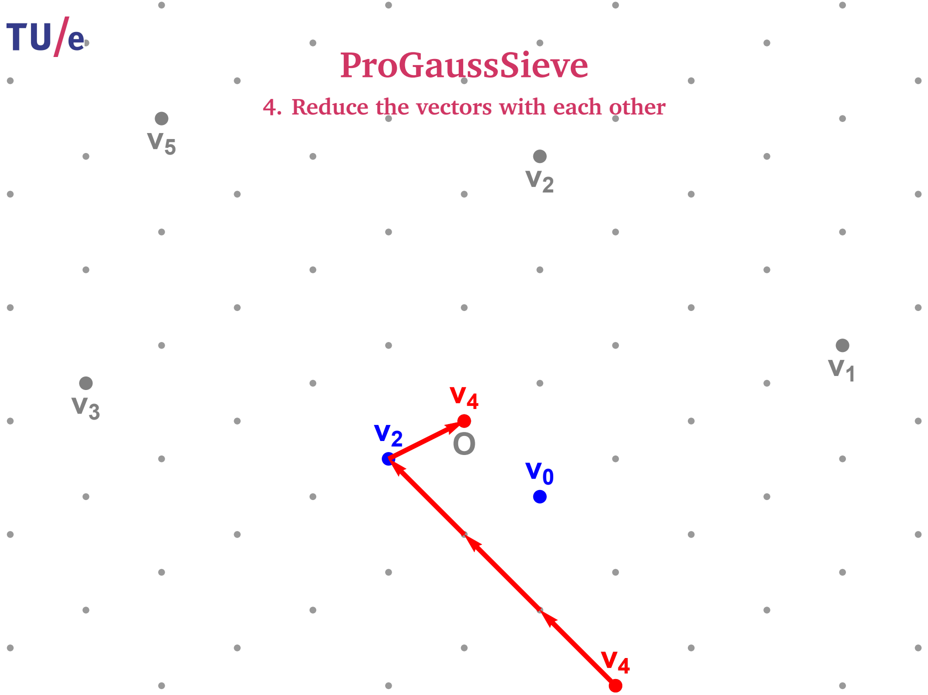

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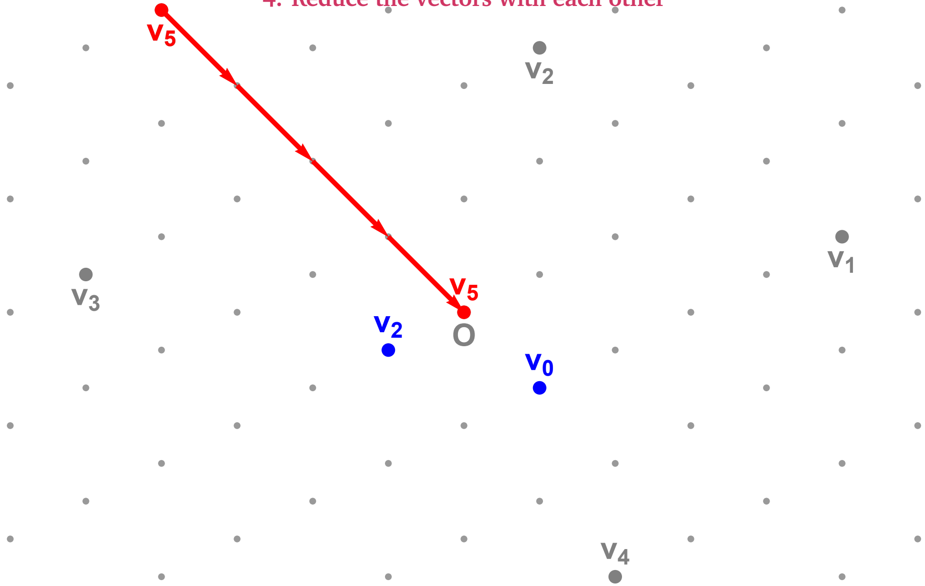

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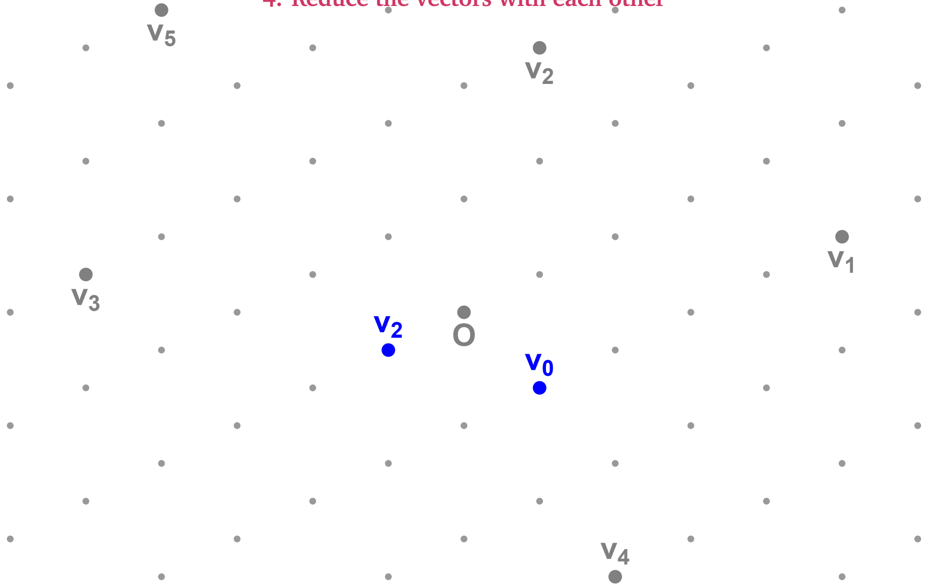
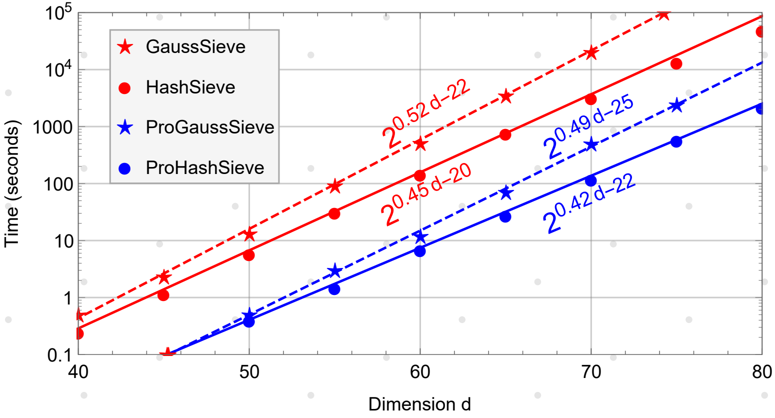


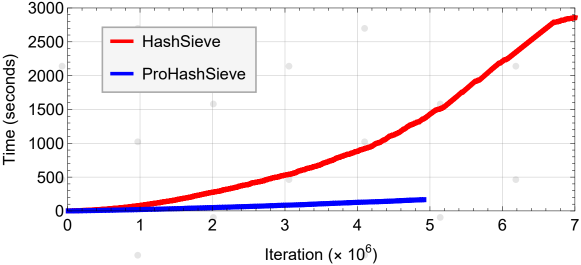
Progressive sieving

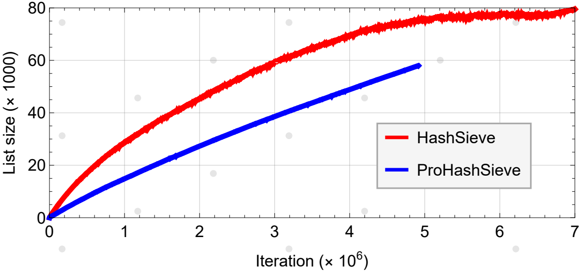
Time complexities



Progressive sieving

Execution profiles ($n = 70$)





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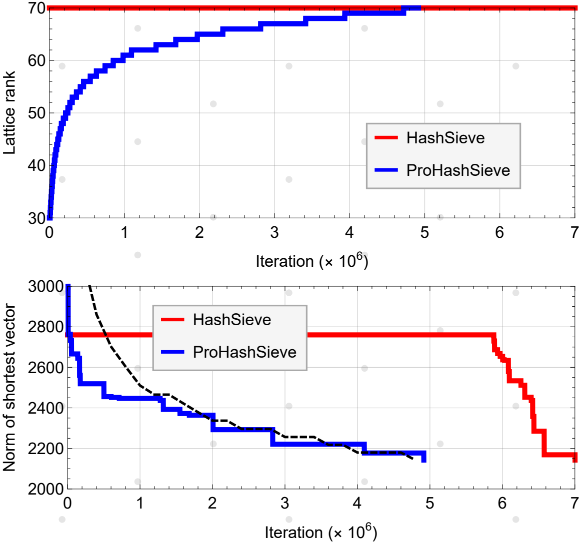

Progressive sieving

Effects of basis reduction ($n = 70$)

Exact SVP	⟵ **GaussSieve** ⟶			⟵ **HashSieve** ⟶		
	LLL	BKZ-10	BKZ-30	LLL	BKZ-10	BKZ-30
Standard sieving	19100	18100	16500	3300	3050	2900
Progressive sieving	595	440	390	165	125	115
Speedup factor	**32**×	**41**×	**42**×	**20**×	**24**×	**25**×

Approximate SVP	⟵ **GaussSieve** ⟶			⟵ **HashSieve** ⟶		
($\gamma = \mathbf{1.1}$)	LLL	BKZ-10	BKZ-30	LLL	BKZ-10	BKZ-30
Standard sieving	18500	17200	15600	3180	2960	2700
Progressive sieving	120	40	3	65	20	2
Speedup factor	**150**×	**400**×	**5000**×	**50**×	**150**×	**1000**×

Conclusion

Progressive lattice sieving

- Uses recursive approach (rank reduction)
- Finds approximate solutions faster
- Benefits more from reduced bases
- Better predictability
- Faster, using slightly less memory
- No theoretical/asymptotic improvements...
  - Best classical time: $(3/2)^{n/2+o(n)} \approx 2^{0.292n+o(n)}$
  - Best quantum time: $(13/9)^{n/2+o(n)} \approx 2^{0.265n+o(n)}$

Questions?