Question 3: *Prime Connections*

A *prime number* is a whole number, greater than 1, that can only be divided by itself and the number 1. Two prime numbers are *connected* if the difference between them is 2^n for some whole number $n \geq 0$; e.g. possible differences are 1, 2, 4, 8, 16, 32, …

A *path* is a sequence of (at least two) prime numbers, without repetition, where adjacent numbers in the sequence are *connected*. If the first number in the sequence is p and the last number is q then we say the path is *between* p and q.

The *length* of a path is the total number of prime numbers used. There may be multiple paths between two prime numbers; the lengths of these paths may be different.

For example:

- 13 is connected to 5 ($13 - 5 = 8 = 2^3$), 5 is connected to 3 ($5 - 3 = 2 = 2^1$) and 3 is connected to 2 ($3 - 2 = 1 = 2^0$);
- As 13 and 5 are connected there is a path between them (13—5) whose length is 2;
- There is a path from 13 to 2 (13—5—3—2) whose length is 4;
- There is a longer path from 13 to 2 (13—17—19—3—2) whose length is 5.

You will be given an upper limit on the primes you are allowed to use. For example, if the limit was 18 then the path 13—17—19—3—2 would *not* be permitted as it includes a prime above this limit.

3(a) [27 marks]

Write a program to determine the length of the *shortest* path between two primes.

Your program should input three integers in order: l ($4 \leq l \leq 2^{24}$) indicating the highest value you are allowed to use, followed by the primes p then q ($2 \leq p < q < l$). You will only be given input where there is a path between p and q using values below l.

You should output the length of the shortest path.

Sample run

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100 2 13
4
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3(b) [2 marks]

How many different *paths* are there between 2 and 19 with a upper limit of 20?

3(c) [3 marks]

How many pairs of *connected* primes are there with an upper limit of 250,000?
(Reversing the order of the primes does *not* count as a different pair.)

3(d) [3 marks]

Suppose that there are two (different) paths of length n between p and q, and that both of these paths contain exactly the same primes. What can you say about the length of the shortest path between p and q?

Total Marks: 100

End of BIO 2016 Round One paper