

Question 3: Juggl(ug)ing

A cookery set contains several jugs of **different** sizes, each of which has a known capacity in oz (fluid ounces). There are no graduations on the jugs, so the only way to measure the liquid is by filling up a jug completely, pouring as much liquid as possible from one jug into another or by emptying a jug. A *step* consists of performing one of these three operations.

Initially all the jugs are empty. To measure n oz it is necessary to perform a sequence of steps and finish with exactly n oz in one of the jugs.

For example, suppose we have two jugs: jug A holds 3 oz and jug B holds 5 oz.

- If we fill up jug B and then pour as much as possible from jug B into jug A, we would have (after 2 steps) 3 oz in jug A and 2 oz in jug B. This is one way to measure 2 oz.
- If we now empty jug A, pour the 2 oz from jug B into jug A, fill jug B and finally pour as much as possible from jug B into jug A, we would finish (after 6 steps) with 3 oz in jug A and 4 oz in jug B. We have now measured 4 oz.

3(a) [23 marks]

Write a program that, given the capacities of several jugs, determines the shortest number of steps necessary to measure n oz.

The input will consist of two lines. The first line will contain two integers j ($1 \leq j \leq 3$) then n ($1 \leq n \leq 250$), indicating the number of jugs and the required amount to measure respectively. The second line will contain j integers, each between 1 and 250 inclusive, indicating the capacity of the jugs.

Sample run

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2 4
3 5
6
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You should output a single integer giving the minimum number of steps necessary to measure n oz.

Your program will only be asked to measure amounts that are possible.

3(b) [2 marks]

Given two jugs A and B, whose capacities are 3 oz and 8 oz respectively, show how to measure 2 oz in 4 steps.

3(c) [5 marks]

Two cookery sets that can measure the same values and no others are said to be *equivalent*. For example, the pair of jugs of capacity 5 oz and 9 oz, and the pair 2 oz and 9 oz can both measure every value from 1 oz to 9 oz. Every cookery set is equivalent to itself and the order of the jugs in a set does not matter.

Consider the cookery set containing three jugs whose capacities are 6 oz, 18 oz and 20 oz. How many equivalent cookery sets are there that contain two jugs? How many are there that contain three jugs?

3(d) [5 marks]

Is it possible to find a cookery set, containing no 1 oz capacity jugs, with a sequence of steps that leaves 1 oz in all the jugs simultaneously? Justify your answer.