

Question 3: Block-chain

A set of children's blocks, each illustrated with a single different letter, have been chained together in a line. They have been arranged so that it is not possible to find three (not necessarily adjacent) letters, from left to right, that are in alphabetical order.

For example, if there are four blocks (A, B, C and D) the possible block-chains are:

ADCB	BADC	BDAC	BDCA	CADB
CBAD	CBDA	CDAB	CDBA	DACB
DBAC	DBCA	DCAB	DCBA	

3(a) [24 marks]

Write a program that enumerates block-chains.

Your program should input a single integer l ($1 \leq l \leq 19$) indicating that the blocks are illustrated with the first l letters of the alphabet, followed by a word p of between 1 and l uppercase letters indicating (in order) the leftmost letters of the block chain. p will only contain letters taken from the first l letters of the alphabet and will not contain any duplicates.

You should output a single integer giving the number of possible block-chains that begin with p .

Sample run

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4 CB
2
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3(b) [2 marks]

List the valid block-chains containing the blocks B, I and O.

3(c) [4 marks]

A block-chain containing the first n ($1 \leq n \leq 13$) letters of the alphabet is attached to the left of a block-chain containing the last m ($1 \leq m \leq 13$) letters of the alphabet, forming a new block-chain. What were the original block-chains? Justify your answer.

3(d) [5 marks]

Suppose all the valid block-chains containing the first 19 letters of the alphabet are sorted into alphabetical order. Which one comes first? Which one comes 1,000,000,000th?