Question 3: *Upside-Down*

An *upside-down* number is an integer where the i^{th} digit from the left plus the i^{th} digit from the right is always equal to 10. For example 13579 is an upside-down number since 1+9 = 10, 3+7 = 10 and (since 5 is both the 3rd digit from the left and from the right) 5+5 = 10.

The first few upside-down numbers, in numerical order, are 5, 19, 28, 37, … , 82, 91, 159, …

3(a) [24 marks]

Write a program to determine the n^{th} upside-down number (in numerical order).

The input will consist of a single integer n ($1 \leq n \leq 2^{31}$). You should output a single integer giving the n^{th} upside-down number.

Sample run

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11
159
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3(b) [2 marks]

Consider all the different 9 digit numbers that use each of the digits 1, … , 9 once each. How many of these are upside-down numbers?

3(c) [3 marks]

How many digits are in the 1,000,000,000,000,000,000th upside-down number?

3(d) [6 marks]

Are there more upside-down numbers with 1000 digits that contain at least one 5, or more upside-down numbers with 1001 digits that contain at least one 5? Justify your answer.

Total Marks: 100

End of BIO 2011 Round One paper