

**Question 3: *Movie Magic***

The Hollywood studio *Greenlight Casting Couch* is producing its latest movie extravaganza. Each actor has a certain number of scenes to film and, to stay in character, each actor's scenes are to be filmed in order. There is also a hierarchy amongst the cast and it is important that, at all times, no actor has filmed more scenes than a more senior actor. The movie, a post-modernist black & white fairytale told in flashback, has no scene containing more than one actor.

The following is an example schedule for 2 actors. The numbers indicate the order in which the scenes are being shot, the top row showing those scenes for the senior actor and the bottom row showing those for the junior actor. Note that numbers increase from left to right (each actor's scenes are filmed in order) and from top to bottom (at all times the senior actor has filmed more scenes).

1	2	4
3	5	

This is only one of 5 possible schedules for this film. The other possible schedules are:

1	2	3	1	3	5	1	2	5	1	3	4
4	5		2	4		3	4		2	5	

**3(a) [ 25 marks ]**

Write a program which finds the number of possible schedules for a given movie.

The input will consist of two lines. The first will contain a single integer  $n$  ( $1 \leq n \leq 8$ ) indicating the number of actors. The second line will contain  $n$  integers (each between 1 and 8 inclusive) indicating the number of scenes for each actor, in order of seniority (most senior first).

*Sample run*

```
2
3 2
5
```

Your output should consist of a line giving the number of possible schedules. Every test case will have an answer between 1 and 1,000,000 inclusive.

**3(b) [ 3 marks ]**

Suppose the hierarchy condition was dropped (i.e. so the only condition is that each actor films their own scenes in order). If there are 2 actors, each with 2 scenes, how many possible schedules are there? How about if there are 3 actors, each with 3 scenes?

**3(c) [ 4 marks ]**

The most senior actor's first scene must be the first scene shot in every schedule. For what movies must the most junior actor's final scene always be the last scene shot? Justify your answer.

**3(d) [ 3 marks ]**

Support 3(a) was modified so the outputs had to be at least 1, but could be arbitrarily high. How many different inputs are valid for the modified question?