

Question 1: Roman Look-and-Say

The *Roman look-and-say* description of a string (of Is, Vs, Xs, Ls, Cs, Ds and Ms) is made by taking each block of adjacent identical letters and replacing it with the number of occurrences of that letter, given in *Roman numerals* (*), followed by the letter itself. A block of adjacent identical letters is never broken into smaller pieces before describing it.

For example:

- MMXX is described as “two Ms followed by two Xs”. Since two is II in Roman numerals, this is written as IIMXX;
- IIMXX is described as IIIIIMXX, which is “two Is, one M, two Is, one X”;
- IIIIIMXX is described as VIIMXX;
- It is *not* valid to describe III as, “two Is, one I” IIII.

Note that Roman look-and-say descriptions are *not* necessarily Roman numerals.

1(a) [25 marks]

Write a program that reads in a Roman numeral representing a number between 1 and 3999 inclusive, followed by n ($1 \leq n \leq 50$).

You should apply the *Roman look-and-say* description n times and then output the number of Is in the final description, followed by the number of Vs.

Sample run 1

MMXX 1
4 0

Sample run 2

MMXX 3
6 2

1(b) [2 marks]

How many Roman numerals (from 1 to 3999 inclusive) have a Roman look-and-say description that is also a Roman numeral? List these *descriptions*.

1(c) [4 marks]

The Roman look-and-say descriptions are generated for all the Roman numerals from 1 to 3999 (inclusive). How many distinct descriptions are there?

(*) *Roman numerals* are conventionally defined to represent numbers using seven letters: I=1, V=5, X=10, L=50, C=100, D=500 and M=1000. Numbers other than these are formed by placing letters together, from left to right, in descending order of size, and adding their values. The basic rule is to always use the biggest numeral possible (e.g. 15 is represented as XV but never as VVV, VX or XIIII).

Letters may not appear more than three times in a row, so there are six exceptions to these rules – the combinations IV, IX, XL, XC, CD and CM. In these cases a letter is placed before one of greater value and the smaller value is subtracted from the larger. E.g. CD = 400. These are the *only* exceptions so, for example, MIM is not valid.