Dungeons and Doorkeys

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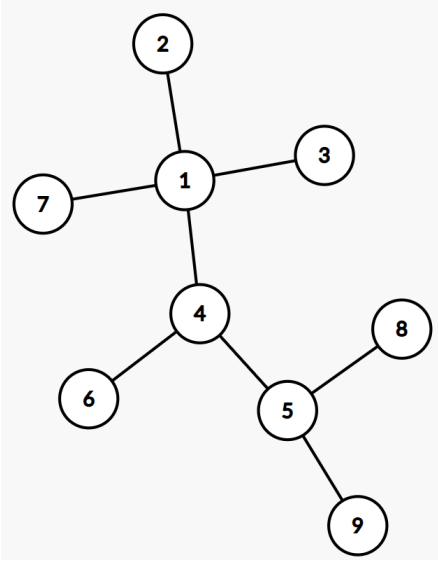
C++ — 2 SEC — 512 MB

Far below the guild-house, nestled amongst the bedrock, is the Dastardly Dungeon of the Guild of Sorcerers. Many an unfortunate soul has been held here against their will, tormented by the Feathers of Tickling and the Steel Pipes of Singing. It is said that some of the deeper rooms, pitch black and silent, have the lingering smells of the aftershave that was once worn by the resident "Entertainers".

These days, the dungeon is used as a museum, displaying all sorts of magical exhibits, from sawing a bunny in half to pulling a young lady out of a hat. Late at night (5:30 sharp), the museum is cleared and the dungeon is locked up. Each room in the dungeon must be locked individually to prevent the magic from leaking out into the corridors. Afterall, no one wants a repeat of the Ace of Spades Incident.



The locking of the dungeons is the responsibility of the (aptly named) Doorkeeper. Their job is to walk around the dungeon (at 5:30 sharp), locking each room behind them. Once a room is locked, they cannot enter it again.

The Dastardly Dungeon is comprised of **n** rooms, numbered from 1 to **n**. These are connected by **n**-1 corridors. With the aid of magic, the Doorkeeper can enter and exit the dungeon from any of the rooms, but they must walk around the dungeon itself. The Doorkeeper is very particular, and so it takes them exactly 1 minute to walk along a corridor (and a negligible amount of time to cross or lock a room) The dungeon is rather large, so, for the sake of time and their safety, the they would like to spend as little time in the dungeon as possible.

INPUT You will be given an integer **n**, denoting the number of rooms in the dungeon. This will be followed by **n** lines of two integers, **a** and **b**, denoting that rooms **a** and **b** are connected by a corridor.

$3 \leq \mathbf{n} \leq 1{,}000{,}000$

OUTPUT Output a single integer, the shortest time the Doorkeeper needs to spend in the dungeon to lock every single room.

SAMPLE For example, consider the dungeon layout shown above. Starting in room 3, the Doorkeeper can take the following route, spending 12 minutes in the dungeon:

Lock 3, move to 1, move to 2, lock 2, move to 1, move to 7, lock 7, move to 1, lock 1, move to 4, move to 6, lock 6, move to 4, lock 4, move to 5, move to 8, lock 8, move to 5, lock 5, move to 9, lock 9.

This is the shortest length of time that the Doorkeeper must spend in the dungeon to lock every room.

INPUT	OUTPUT
3 1 2 2 3	2
10 2 1 3 2 4 2 5 4 6 3 7 1 8 1 9 5 10 2	12
6 6 1 6 2 6 3 6 4 6 5	8