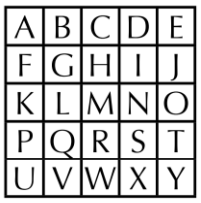
Question 3: *Unlock*



A security system consists of a 5×5 keypad of buttons. Each button has a light which can be either *off*, *dim* or *bright*. When a button is pressed its light moves to the next state (*off*→*dim*, *dim*→*bright* and *bright*→*off*) as do the lights on any buttons touching it horizontally and vertically. The system is *unlocked* when all the lights are *off*.

The current lighting can be represented by a string, in alphabetical order, where a lowercase letter indicates the corresponding button is *dim* and an uppercase letter indicates it is *bright*. Lights that are *off* do not have a letter in the string.

For example, if all the buttons are *off* and button R is pressed, the lighting is `mqrsw`. If T is now pressed it becomes `moqrStwy`. Pressing S will make it `mnoqRTwxy`.

3(a) [24 marks]

Write a program to determine a sequence of button presses to *unlock* the system.

Your program should read in a string of between 1 and 12 *distinct* letters (inclusive) in alphabetical order, representing the current lighting of the security system.

You should output a string (in alphabetical order) that indicates a sequence of buttons which can be pressed to unlock the system, or output `IMPOSSIBLE` if it cannot be unlocked. A button can be pressed at most twice; a lowercase letter in your string will indicate that the corresponding button should be pressed once and an uppercase letter indicating that it should be pressed twice.

If there are multiple solutions you are only required to print out a single solution.

Sample run

```
mnoqRTwxy
RST
```

Alternative answer

```
mnoqRTwxy
aCeFhJprSUwY
```

3(b) [2 marks]

What is the lighting of the system if the system is unlocked and then B is pressed, followed by I and O? (Each button being pressed once.)

3(c) [4 marks]

Starting from an unlocked system, in how many ways can three different buttons be pressed once each, in alphabetical order, so that no light becomes bright?

3(d) [5 marks]

Suppose that the system has a configuration of lighting and that a particular sequence of button presses will unlock the system. Is it possible for the same sequence of button presses to unlock the system when it is in a different configuration? Justify your answer.

Total Marks: 100

End of BIO 2013 Round One paper