The gradient The gradient for f(x,y)
Understanding the gradient conceptutally

- The gradient represents the slope of a function.
- The gradient points in the direction of the greatest rate of increase of the function, and its' magnitude is the slope in that direction.

Gradient Vectors
$\nabla f(x,y,z) = [\frac{df(x,y,z)}{dx}, \frac{df(x,y,z)}{dy}, \frac{df(x,y,z)}{dz}]$
The Jacobian Matrix
Gradient vectors organize the partial derivatives for a scalar function. If we have multiple functions, we use the Jacobian
$\mathbf{J} = \begin{bmatrix} \nabla f(x,y) \\ \nabla g(x,y) \end{bmatrix} = \begin{bmatrix} \frac{df(x,y)}{dx} & \frac{df(x,y)}{dy} \\ \frac{dg(x,y)}{dx} & \frac{dg(x,y)}{dy} \end{bmatrix}$
The general case

$$f(x,y,z) \to f(x) where\ x = \begin{bmatrix} x1 \\ x2 \\ . \\ . \\ . \\ x_n \end{bmatrix}$$

$$\mathbf{J} = \begin{bmatrix} \nabla f_1(x,y) \\ \nabla f_2(x,y) \\ ... \\ \nabla f_n(x,y) \end{bmatrix} = \begin{bmatrix} \frac{df_1(\mathbf{x})}{dx_1} & \frac{df_1(\mathbf{x})}{dx_2} & ... & \frac{df_1(\mathbf{x})}{dx_n} \\ \frac{df_2(\mathbf{x})}{dx_1} & \frac{df_2(\mathbf{x})}{dx_2} & ... & \frac{df_2(\mathbf{x})}{dx_n} \\ ... \\ \frac{df_m(\mathbf{x})}{dx_1} & \frac{df_m(\mathbf{x})}{dx_2} & ... & \frac{df_m(\mathbf{x})}{dx_n} \end{bmatrix}$$