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Hybrid Lifting Wavelet-Like Transform for Solution of Electromagnetic Integral Equation *

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A hybrid lifting wavelet-like transform scheme is successfully applied to the solution of electric field integral equation using Rao–Wilton–Glisson basis functions. To speed up the matrix transform process, the lifting scheme is adopted. Numerical examples of different three-dimensional perfectly electric conducting objects are considered. Compared with the method of moments, the proposed matrix transform scheme can save considerable CPU time and memory.

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The solution of electric field integral equation (EFIE) by method of moments (MOM) has been a very useful tool for accurately predicting the radar cross section (RCS) of arbitrarily shaped three-dimensional (3D) perfectly electric conducting (PEC) objects.[1] The traditional MOM produces dense linear systems with N unknowns, where N grows with the electrical size of the scattering object. The complexity of solving the linear equation by iterative method is $O(N^2)$, and the cost of one iteration is dominated by the matrix-vector multiplication (MVM). For years, various approaches have been developed to decrease the complexity of MVM, and generally can be divided into two groups: one is to avoid the direct computation of the impedance matrix, and the other is some matrix transform algorithms. Among the former is the fast multipole method (FMM)[2] and the adaptive integral method (AIM),[3] and wavelet matrix transform method is included in the latter.[4,5]

Recently, wavelet analysis[6,7] has attracted a great deal of attention not only in the field of signal processing, but also in electromagnetic (EM) analysis.[8] It has been shown that the use of wavelets in MOM constitutes effective means to obtain sparse matrix equations yielding accurate solutions for EM problems.[9,10]

However, these earlier works are confined to the analysis of two-dimensional (2-D) problems, or to wire or microstrip problems in which the current direction is one-dimensional (1-D). Meanwhile, preprocessing matrices must be constructed to operate the wavelet transform, which causes auxiliary memories consumed.

In this Letter, a hybrid wavelet transform method performed by a lifting scheme is proposed to reach a sparse linear system from the electromagnetic scattering problem of arbitrarily shaped 3-D objects, which can accelerate the solution of the impedance matrix equation and save much memory.

Consider an arbitrarily shaped 3-D conducting object illuminated by an incident field $\boldsymbol{E}^{\rm inc}(\boldsymbol{r})$, the EFIE is given by

$$\boldsymbol{n} \times L(J) = \boldsymbol{n} \times \boldsymbol{E}^{\rm inc}(\boldsymbol{r}), \quad r \in S, \tag{1}$$

where $\boldsymbol{J}(\boldsymbol{r})$ denotes the unknown surface current density and the integral operator $\boldsymbol{L}$ is defined by

$$L(\boldsymbol{J}) = jk\eta \iint_S [\boldsymbol{J}(\boldsymbol{r'})g(\boldsymbol{r}, \boldsymbol{r'}) + \frac{1}{k^2}\nabla' \cdot \boldsymbol{J}(\boldsymbol{r'})\nabla g(\boldsymbol{r}, \boldsymbol{r'})]dS' \tag{2}$$

in which S denotes the surface of the conducting object, k is the free-space wavenumber, η is the free-space wave impedance, $\boldsymbol{n}$ is an outwardly directed normal, and $\boldsymbol{g}(\boldsymbol{r}, \boldsymbol{r'})$ is the free-space Green's function.

By MOM with the Rao–Wilton–Glisson (RWG) basis functions, the EFIE reduces to

$$\boldsymbol{ZI} = \boldsymbol{V}, \tag{3}$$

where $\boldsymbol{Z}$ is the impedance matrix with size N; $\boldsymbol{I}$ is the unknown current coefficients vector; and $\boldsymbol{V}$ is the excitation vector.

In the traditional wavelet matrix transform method, the transform is carried out by the sparse wavelet transform matrix which can be constructed from finite impulse response (FIR) filters,[11] then the system of matrix equation (3) can be expressed as

$$\boldsymbol{Z'I'} = \boldsymbol{V'}, \tag{4}$$

where $\boldsymbol{Z'} = \boldsymbol{WZW'}$, $\boldsymbol{I'} = \boldsymbol{WI}$, $\boldsymbol{V'} = \boldsymbol{WV}$, and $\boldsymbol{WW'} = \boldsymbol{W'W} = \boldsymbol{u}$, with $\boldsymbol{u}$ being an identity matrix. Note that in the case of the orthogonal wavelets we have $\boldsymbol{W'} = \boldsymbol{W}^T$, where T stands for the transpose of a matrix. For a given threshold value, Eq. (4)

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becomes a sparse matrix equation which can be efficiently solved by a sparse solver.

Once $\boldsymbol{I}'$ is solved, one can obtain $\boldsymbol{I}$ by

$$\boldsymbol{I} = \boldsymbol{W}'\boldsymbol{W}\boldsymbol{I} = \boldsymbol{W}'\boldsymbol{I}'. \tag{5}$$

To speed up the matrix transform process and to reduce the memory allocated for the preprocessing matrices, we introduce the lifting scheme to accelerate the traditional wavelet matrix transform method without auxiliary memories. It was proven in Ref. [12] that the lifting algorithm is twice as fast as the standard algorithm, and in the lifting scheme, one does not need to construct wavelet matrices, but to operate the impedance matrix directly.

The FIR wavelet transform can be viewed as subband transform using FIR filters. The forward transform uses two analysis filters $\tilde{h}$ (low pass) and $\tilde{g}$ (high pass), followed by subsampling, while the inverse transform firstly upsamples and then uses two synthesis filters h (low pass) and g (high pass). The perfect reconstruction (PR) property is defined by

$$\begin{aligned}\tilde{h}(z^{-1})h(z) + \tilde{g}(z^{-1})g(z) &= 2,\\ \tilde{h}(-z^{-1})h(z) + \tilde{g}(-z^{-1})g(z) &= 0.\end{aligned} \tag{6}$$

The polyphase representation of filter h is given by

$$h(z) = h_e(z^2) + z^{-1}h_o(z^2), \tag{7}$$

where $h_e(z) = \sum h_{2k}z^{-k}$, $h_o(z) = \sum h_{2k+1}z^{-k}$, and the superscripts e and o denote, respectively, even component and odd component of the filter. The polyphase representation of other filters can be obtained similarly.

Constructed by the synthesis filters and analysis filters, the polyphase matrix and its dual matrix are defined as

$$\boldsymbol{P}(z) = \begin{bmatrix} h_e(z) & g_e(z) \\ h_o(z) & g_o(z) \end{bmatrix}, \tag{8}$$

$$\tilde{\boldsymbol{P}}(z) = \begin{bmatrix} \tilde{h}_e(z) & \tilde{g}_e(z) \\ \tilde{h}_o(z) & \tilde{g}_o(z) \end{bmatrix}, \tag{9}$$

Then the PR condition can be rewritten as

$$\boldsymbol{P}(z)\tilde{\boldsymbol{P}}(z^{-1})^T = \boldsymbol{u}, \tag{10}$$

where $\boldsymbol{u}$ is the identity matrix. For orthogonal wavelet transform, $\tilde{h}(z) = h(z)$, $\tilde{g}(z) = g(z)$, and $\tilde{\boldsymbol{P}} = \boldsymbol{P}(z)$.

The problem of finding an FIR wavelet transform thus amounts to finding a matrix $\boldsymbol{P}(z)$. Once we have such a matrix, the dual matrix and other filters for the wavelet transforms follow immediately. From Eq. (10) it follows that

$$\tilde{h}_e = g_o(z^{-1}), \quad \tilde{h}_o(z) = -g_e(z^{-1}), \tag{11a}$$

$$\tilde{g}_e(z) = -h_o(z^{-1}), \quad \tilde{g}_o(z) = h_e(z^{-1}). \tag{11b}$$

Substituting Eqs. (11a) and (11b) into the polyphase representation of filters, one can obtain that

$$\tilde{g}(z) = z^{-1}h(-z^{-1}), \quad \tilde{h}(z) = -z^{-1}g(-z^{-1}). \tag{12}$$

According to the Euclidean algorithm, the polyphase matrices produced by the complementary filter pairs can always be factored into lifting steps.[12] The forward wavelet transform can be factored through lifting the polyphase matrix and its dual matrix, and there always exist Laurent polynomials $\boldsymbol{s}_i(z)$ and $t_i(z)$ for $1 \le i \le M$ and a nonzero constant K so that

$$\boldsymbol{P}(z) = \prod_{i=1}^{M} \begin{bmatrix} 1 & s_i(z) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ t_i(z) & 1 \end{bmatrix} \begin{bmatrix} K & 0 \\ 0 & 1/K \end{bmatrix}, \tag{13}$$

$$\tilde{\boldsymbol{P}}(z) = \prod_{i=1}^{M} \begin{bmatrix} 1 & 0 \\ -s_i(z^{-1}) & 1 \end{bmatrix} \begin{bmatrix} 1 & -t_i(z^{-1}) \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1/K & 0 \\ 0 & K \end{bmatrix}, \tag{14}$$

We can obtain inverse wavelet transform factoring formulation by simply inverse the forward formulation, switch additions and subtractions, and switch multiplications and divisions

$$\boldsymbol{P}^{-1}(z) = \prod_{i=M}^{1} \begin{bmatrix} 1/K & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -t_i(z) & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -s_i(z) \\ 0 & 1 \end{bmatrix}, \tag{15}$$

$$\tilde{\boldsymbol{P}}^{-1}(z) = \prod_{i=M}^{1} \begin{bmatrix} K & 0 \\ 0 & 1/K \end{bmatrix} \begin{bmatrix} 1 & t_i(z^{-1}) \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ s_i(z^{-1}) & 1 \end{bmatrix}, \tag{16}$$

where $\boldsymbol{P}^{-1}(z)$ and $\tilde{\boldsymbol{P}}^{-1}(z)$ are the inverse polyphase matrix of $\boldsymbol{P}(z)$ and its dual matrix, respectively.

The lifting scheme can also be described by predict step and update step, which can be outlined in the following three basic operations.

Split: Divide the original data ($\boldsymbol{x}[n]$) into odd subsets ($\boldsymbol{x}_o[n]$) and even subsets ($\boldsymbol{x}_e[n]$)

$$\boldsymbol{x}_o[n] = \boldsymbol{x}[2n-1], \quad \boldsymbol{x}_e[n] = \boldsymbol{x}[2n]. \tag{17}$$

Predict: Generate high frequency component $\boldsymbol{d}[n]$ as the error in predicting odd subsets from even subsets using prediction operator Q

$$\boldsymbol{d}[n] = \boldsymbol{x}_o[n] - Q(\boldsymbol{x}_e[n]). \tag{18}$$

Update: Generate low frequency component $\boldsymbol{c}[n]$ as a coarse similarity to original signal by applying an update operator U to the wavelet coefficients and adding to even subsets

$$\boldsymbol{c}[n] = \boldsymbol{x}_e[n] + U(\boldsymbol{d}[n]). \quad (19)$$

The operators Q and U above can be deduced from the polyphase matrices.

When a matrix equation resulted from EM problems is considered, the numerical implementations with respect to the forward transform and inverse transform described in Eq. (4) can be implemented by the following steps:

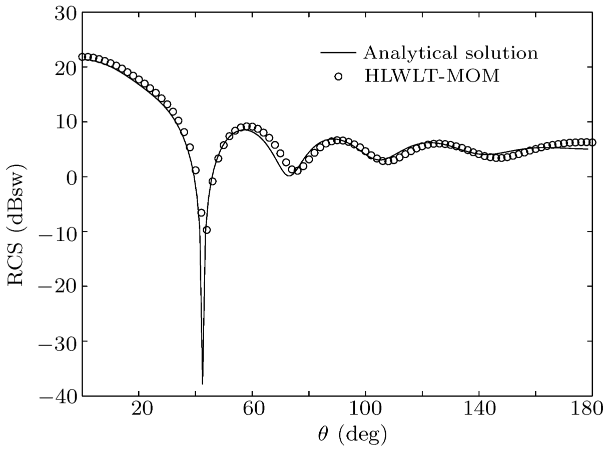

Fig. 1. E-plane bistatic RCS of a PEC sphere.

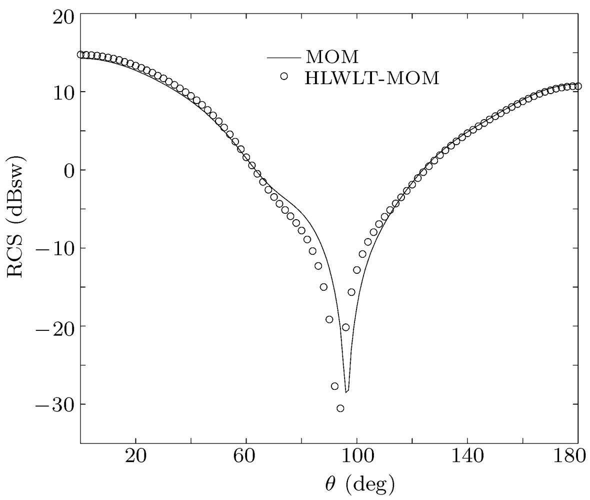

Fig. 2. E-plane bistatic RCS of a PEC cube.

Firstly, factorization of $\boldsymbol{P}(z)$ for forward column transforms $(\boldsymbol{WZ}, \boldsymbol{WV})$. Secondly, factorization of for $\tilde{\boldsymbol{P}}(z)$ forward row transforms $(\boldsymbol{ZW}')$. Finally, factorization of $\boldsymbol{P}^{-1}(z)$ for solving $\boldsymbol{I}$ $(\boldsymbol{I} = \boldsymbol{W}^{-1}\boldsymbol{I}')$. The specific examples for implementing the steps above can be found in Refs. [12,13].

The hybrid wavelet transform algorithm can be described as $\boldsymbol{W} = \boldsymbol{W}_2\boldsymbol{W}_1$ and $\boldsymbol{W}' = \boldsymbol{W}_1'\boldsymbol{W}_2'$. In the following parts, the transform $\boldsymbol{Z}_1' = \boldsymbol{W}_1\boldsymbol{Z}\boldsymbol{W}_1'$ is accomplished by db8 wavelet; and $\boldsymbol{Z}' = \boldsymbol{W}_2\boldsymbol{Z}_1'\boldsymbol{W}_2'$ is operated by db97 wavelet. Obviously, the hybrid method will increase the CPU time needed for matrix transform, but fortunately, the lifting scheme can speed up the transform process by a factor of two, and hence the total computational complexity is not increased as compared with the traditional wavelet matrix transform method.

The proposed method is used to solve EM scattering problems and to show the validity of the hybrid lifting scheme. A solution of the resulting sparse linear systems is completed by using conjugate gradient (CG) method, and the sparsity of a matrix is defined as its percentage content of nonzero elements.

The first example is a PEC sphere with its radius of one wavelength, the surface of which is discretized into 1280 triangular elements resulting in 1920 unknown current coefficients. The bistatic RCS is calculated and the result is compared with the analytical solution, as shown in Fig. 1. The sparsity of the impedance matrix is 26.34% after transform. It takes the direct method by MOM 1327.4 s to obtain the solution, while the hybrid lifting scheme produces an accurate solution within 537.6 s.

As a second example, the scattering problem of a cube (1 cm×1 cm×1 cm) is calculated. The cube is illuminated by a pane wave propagating in the z direction and E-polarized in the x direction with frequency $f = 30$ GHz is considered. The cube is discretized with 1120 triangular subdomains resulting in 1680 current unknown coefficients. The sparsity of the impedance matrix is 25.64%. The total CPU time to obtain the bistatic RCS is 916.2 s for MOM and 417.4 s for hybrid lifting scheme, respectively.

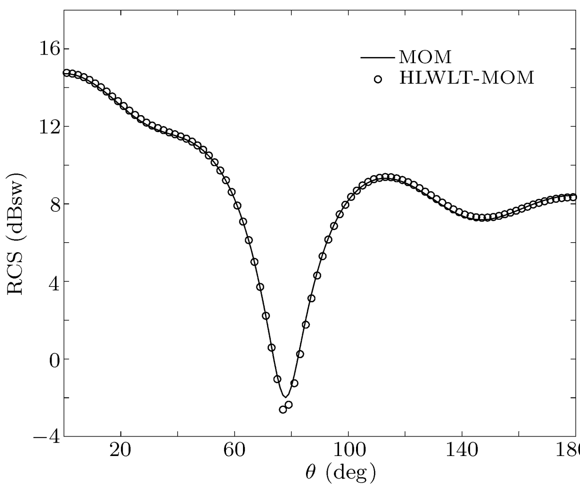

Fig. 3. E-plane bistatic RCS of a finite PEC cylinder.

Finally, a finite PEC cylinder parallel to the y-axis is considered. The diameter of the cylinder is chosen to be one wavelength, and the height is 1.5

wavelengths. The object is illuminated by a plane wave propagating in the z direction and E-polarized in the x direction. The surface of the object is discretized into 1152 triangular elements that result in 1728 unknown current coefficients. The sparsity of the impedance matrix obtained is 23.52%. The CPU time for solution of the problem is reduced from 1021.5 seconds to 438.1 s. The E-Plane bistatic RCS of the object is shown in Fig. 3.

A hybrid lifting wavelet-like transform method is proposed to obtain highly sparse moment-method matrices. The lifting scheme is used to speed up the hybrid method so as to make the total computational complexity the same as the traditional wavelet matrix transform method. Various numerical results show that the method proposed is highly efficient for analysis of EM scattering problems.

The choices of wavelet for differently shaped 3-D objects and the efficiency for other integral equations, especially for problems of multiple boundary conditions are topics for future research. With these topics addressed, the proposed method can be used as a novel matrix transform technique for EM problems.

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