The current issue and full text archive of this journal is available on Emerald Insight at:
www.emeraldinsight.com/0307-4358.htm

Mutual fund efficiency and tradeoffs in the production of risk and return

Michael Devaney
Department of Economics and Finance, Southeast Missouri State University, Cape Girardeau, Missouri, USA
Thibaut Morillon
Department of Finance, University of Missouri, Columbia, Missouri, USA, and
William Weber
Department of Economics and Finance, Southeast Missouri State University, Cape Girardeau, Missouri, USA

Received 12 May 2015
Revised 14 September 2015
Accepted 3 November 2015

Abstract

Purpose – The purpose of this paper is to estimate the performance of 188 mutual funds relative to the risk/return frontier accounting for the transaction costs of producing a portfolio of investments.

Design/methodology/approach – The directional output distance function is used to estimate mutual fund performance. The method allows the data to define a frontier of return and risk accounting for the transaction costs associated with securities management and production of risky returns. Proxies for the transaction costs of producing a portfolio of securities include the turnover ratio, load, expense ratio, and net asset value. The estimates of mutual fund performance are bootstrapped to account for the unknown data generating process. By comparing each mutual fund's performance relative to the capital market line the authors determine how the fund should adjust their portfolio in regard to risk and return in order to maximize the inefficiency adjusted Sharpe ratio.

Findings – The bootstrapped estimates indicate that the average mutual fund could simultaneously expand return and contract risk by 3.2 percent if it were to operate on the efficient frontier. After projecting each mutual fund's return and risk to the efficient frontier the authors find that a majority of the mutual funds should reduce risk to be consistent with the capital market line.

Originality/value – Many researchers have used data envelopment analysis to estimate a piecewise linear frontier of risk and return to measure mutual fund performance. To the authors' knowledge the research is the first to use a twice-differentiable quadratic directional distance function to measure the managerial performance and risk/return tradeoff of mutual funds.

Keywords Efficiency, Directional distance function, Risk/return

Paper type Research paper

1. Introduction

The capital asset pricing model (CAPM) is often used to evaluate mutual fund performance using measures such as the Sharpe ratio, Treynor's index, and Jensen's α (Bodie *et al.*, 2011). Early criticism of the CAPM focussed on the non-normality of return data and the single factor (market returns) used to determine required fund returns. Recent studies have accounted for time-varying risk premiums by using generalized autoregressive, conditional heteroskedasticity estimation methods and have employed models that account for factors such as Tobin's q, the small-firm effect, and dividends in addition to market returns. However, the CAPM still relies on obtaining estimates of returns from the market portfolio which is seldom, if ever observed. Instead, researchers tend to use proxies to the market portfolio such as the S&P 500. However, high turnover in the firms that define the S&P 500 suggests that the index is subject to

Managerial Finance
Vol. 42 No. 3, 2016
pp. 225-243

0307-4358
DOI 10.1108/MF-05-2015-0142

survivorship bias. In addition, the S&P 500 index includes only US companies and yet, the US share of world stock market capitalization has declined from 45 percent to approximately 35 percent in the last ten years and the trend is expected to continue (Haslett, 2010; Quandl, 2015). Moreover, in addition to accounting for international stocks, the true market index should also include the returns to real estate, human capital, housing, and other investments that are not included in traditional stock market indexes. Furthermore, active investment strategies, such as those used by hedge funds and mutual funds, imply that investment management is more of a production process relative to passive investing. Even the minimal investment requirements and expense ratios of exchange traded funds (ETF) suggests that there are transaction costs associated with constructing a portfolio of securities. Luck might also be a factor that can bias measures of mutual fund performance (Barras *et al.*, 2010). Furthermore, recent work in behavioral finance and economics suggests that investors and even professional fund managers can be subject to behavioral biases which can make managerial performance difficult to measure. Finally, a wealth of studies (see Berger and Humphrey, 1997 for a review) have examined the production performance of various types of financial institutions and found widespread technical and allocative inefficiencies. Thus, performance measures that assume that the observed market index lies on the return/risk frontier will be another source of bias.

Grossman and Stiglitz (1980) argue that superior investment company returns do not occur spontaneously but are produced as a consequence of superior asset selection and/or the timing of transactions. Accordingly, in this paper we use the tools of production theory to offer some insight into the asset pricing problem of investment company returns. Our approach allows mutual funds to be inefficient and compares fund performance not to some unobservable ideal, but to actual best practice. From a production standpoint, the most efficient mutual fund managers employ the best-practice technology and occupy an observed production frontier. A well-known result of microeconomic theory indicates that the slope of the production frontier measures the opportunity cost of expanding one output in terms of another. In equilibrium, the relative prices of the two goods reflect the opportunity cost of production. We apply this result to the return-risk frontier controlling for the transaction costs of mutual fund portfolio construction. We utilize a frontier-projection pricing approach to measure the efficiency and risk/return of 188 US mutual funds that operated in the period 2010-2014. Rather than estimate returns using the standard CAPM we estimate a quasi-Markowitz (1952, 1959) efficient frontier using the directional output distance function accounting for the transactions costs of generating a portfolio of risk and return. Borrowing from both finance and production theory, we model a mutual fund's five year annualized return and standard deviation as desirable and undesirable outputs. Mutual fund inputs include the expense ratio, front and deferred loads, 12b-1 expenses, and net asset value. Although the objective of most funds is to occupy the frontier, each fund's location *vis-à-vis* other funds will be determined by its risk/return and input mix. Our method allows us to compare the standard Sharpe ratio (risk-adjusted excess return) of the fund with a Sharpe ratio adjusted for transaction costs and production inefficiency.

The next section is a brief review of the literature on the measurement of mutual fund risk/return performance. In Section 3 we demonstrate how the directional output distance function is used to measure output technical efficiency when return and risk are produced using selected inputs. The derivatives of this distance function give the tradeoff between return and risk along the frontier. Section 4 presents the deterministic method and functional form used to estimate the directional output distance function. In Section 5 we describe the data and present the empirical results. The final section summarizes our method and results.

2. Measuring mutual fund performance

Measurement of mutual fund performance has been the subject of criticism in the finance literature. The theoretical critique relates to the efficacy of proxies for the market portfolio in the application of the CAPM as well as assumptions regarding transaction costs and investor rationality. Elton *et al.* (1993), Carhart (1997), and Blake *et al.* (1993) attempt to address the shortcomings of the market portfolio proxy in mutual fund performance studies by including additional benchmarks such as small capitalization indices and bond indices. Elton *et al.* (1993) demonstrate how mutual fund performance studies based on a single benchmark like the S&P 500 can be biased by sub-period distortions, such as the small-firm effect. To control for this bias they estimate a multi-factor version of the single index model that includes the excess return on the S&P 500, excess return on a small cap index and excess return on a bond market index. After controlling for relative performance of three asset classes they find that mutual fund managers as a group do not appear to beat passive index strategies. Thus, performance is worse for firms with high expense and turnover ratios, since those firms do not increase return enough to justify the higher costs.

Similar to Elton *et al.* (1993), Carhart (1997) examines the issue of persistence in mutual fund returns in a multi-factor model. Carhart uses a four-factor extension of the index model in which the four benchmark portfolios are the S&P 500 index and portfolios based on book-to-market ratio, size, and prior year stock market returns. He finds some persistence in excess returns but attributes much of it to expenses and transactions costs rather than gross returns. Hendricks *et al.* (1993) also find evidence of persistent underperformance among the weakest performing mutual funds.

The method we employ in this paper to measure mutual fund performance is in the spirit of Luenberger's (2001) projection pricing method, but differs in several ways. First, while Luenberger's pricing vector is a portfolio of returns and standard deviations that has minimum distance from the origin to the Markowitz frontier, we estimate the tradeoff between return and risk for each mutual fund by finding the distance between the mutual fund's observed risk and return relative to a quasi-Markowitz efficient frontier for a pre-specified directional vector. The calculated distance of observed return and risk to the frontier of return and risk serves as a measure of inefficiency and indicates the simultaneous increase in return and decline in risk that is feasible if the mutual fund were to operate efficiently. Second, we account for the fact that scarce resources must be harnessed to construct and manage a portfolio of financial assets. Third, while Luenberger uses the minimum norm portfolio to determine an implied risk-free return, our approach estimates the intercept of the capital market line adjusted for transaction costs and compares that intercept to the observed risk-free rate. Thus, we can also determine if a given amount of return and risk is allocatively efficient relative to the unobserved market portfolio.

Even if the processing of information were perfect, individual investors might utilize the information to make biased or irrational decisions. A large behavioral finance literature has determined that these biases influence the way investors frame questions and make decisions (DeBondt and Thaler, 1987; Kahneman and Tversky, 1979; Odean, 1998). It is generally assumed that the potential for behavioral bias is less likely among professional investment managers than it is for individual investors. Goetzmann and Ibbotson (1994) examine the performance of mutual fund managers and find evidence that management expertise can result in persistent superior performance, a finding in conflict with the efficient markets hypothesis. In contrast, Malkiel (1995) concludes that mutual fund managers are unable to outperform the market index (S&P 500) on a risk-adjusted basis.

Numerous researchers have used standard data envelopment analysis (DEA) methods to measure output or input performance of mutual funds or hedge funds. Murthi *et al.* (1997), Sengupta (2000), and Basso and Funari (2001) measure the Farrell (1957) output efficiency of mutual funds using nonparametric DEA. They model risk as an input in the production of mutual fund returns and measure the expansion of returns holding risk constant. Similarly Eling (2006) uses DEA and various specifications of risk as an input and return as an output to measure hedge fund efficiency. As inputs Eling (2006) includes the risk measures of standard deviation of returns, the lower partial moments of the return distribution, β, value at risk, and drawdown factors. As measures of hedge fund outputs Eling (2006) includes excess return (arithmetic and geometric), skewness, and the higher partial moments of the return distribution.

One criticism of risk as a fixed input is that financial market theory typically interprets risk/return as joint by-products that are generated simultaneously as a consequence of the investment process. In a statistical sense it is not the case that the second moment of the return distribution (risk) determines the first moment of the distribution (return). Instead, all moments of the return distribution occur simultaneously as a consequence of the fund manager's decision about which stocks to include in the fund's portfolio. Devaney and Weber (2005) use DEA to measure real estate investment trust (REIT) inefficiency in which risk is an undesirable output and return is a desirable output. In a similar vein, Fukuyama and Weber (2014) use nonperforming loans of banks as an undesirable output that captures credit risk.

Rubio *et al.* (2012) compare the efficiency of Islamic and international mutual funds and find that despite their lower degree of diversification, Islamic mutual funds outperformed their international counterparts from 1999 to 2011. Patari *et al.* (2012) use DEA for a sample of Finnish non-financial stocks and find that stocks in the top quartile portfolio outperformed market portfolios. For 167 managed futures funds Glawischnig and Sommersguter-Reichmann (2010) compare DEA-based performance indexes with traditional financial performance indexes and conclude that although the single number performance index offered by DEA is useful, it cannot replace traditional financial indicators. In contrast, Brandouya *et al.* (2015) conclude that frontier-based mutual fund performance ratings allows one to design better investment strategies and policies relative to traditional rating systems.

Lamb and Tee (2012a, b) examine monthly returns for 30 hedge funds during 2000-2004 accounting for the skewness and kurtosis of fund returns, as well as risk and return. Basso and Funari (2014) examine the size of mutual funds as a potential source of bias in DEA models. Using data on over 260 European mutual funds they find no linear correlation between fund size and performance, although they find a positive and significant rank correlation between fund size and performance indicating the potential for scale economies in mutual fund production.

A series of papers analyze mutual fund or individual stock performance using the Luenberger (1992) shortage function or benefit function. Briec *et al.* (2004) analyze mutual fund performance in producing return and risk. In addition to risk and return Briec *et al.* (2007), Kerstens *et al.* (2011a), and Briec *et al.* (2013) account for skewness as a desirable property of a risky portfolio of assets. Briec and Kerstens (2009) extend Morey and Morey's (1999) analysis of 26 mutual funds by incorporating time discounting into the shortage function framework. Briec and Kerstens (2010) analyze 30 blue chip stocks traded on the London exchange assuming that investors prefer to maximize the odd moments (mean and standardized skewness) of the return distribution and simultaneously minimize the even moments (variance and standardized kurtosis). Kerstens *et al.* (2011b) account for odd and even moments of the return distribution and also estimate mutual fund performance under different assumptions about returns to scale and whether or not the production frontier satisfies convexity or is non-convex.

Rather than adding one or more additional benchmarks, our approach utilizes data on a sample of mutual funds reported by Morningstar to construct the efficient return/risk frontier using production theory. In addition to risk as the undesirable output and return as the desirable output, inputs are measured by net asset value, expense ratio, turnover, and load. Instead of using DEA to construct the production frontier, we use a deterministic estimation method, which allows the simultaneous evaluation of inefficiency and the return/risk tradeoff. Gregoriou *et al.* (2005) argue that the non-normal returns of commodity trading advisors cause linear factor models to overstate Sharpe ratios. Instead, they use DEA to examine the efficiency of commodity trading advisors. We extend the work of Gregoriou *et al.* (2005) and construct Sharpe ratios that are adjusted for the level of mutual fund inefficiency. In addition, our method allows computation of the return-risk tradeoff. We then use knowledge of the return-risk tradeoff and the risk-free interest rate to calculate whether mutual funds should take more or less risk.

Most mutual fund risk/return studies model risk in the framework of the CAPM which identifies two types of risks associated with an investment in mutual fund j. Systematic risk reflects the co-movement of mutual fund returns with the market portfolio, the mutual fund's β (β_j), and the volatility of the market portfolio (σ_M), while unsystematic risk (σ_{ej}^2) is specific to the mutual fund. Since unsystematic risk can be diversified away, investors are only compensated for systematic risk. When systematic risk and unsystematic risk are independent, the total risk of mutual fund j equals the sum of systematic and unsystematic risk:

$$\sigma_j^2 = \beta_j^2 \sigma_M^2 + \sigma_{ej}^2.$$

In theory, the market portfolio consists of all risky assets including stocks, bonds, real estate (Ibbotson *et al.*, 1985), human capital, etc., and by definition does not contain any unsystematic or diversifiable risk[1]. In practice, market portfolio proxies in mutual fund performance studies likely retain significant diversifiable risk relative to the true market portfolio. We argue that despite its empirical convenience, a priori, the S&P 500 may not be a better approximation of the true market portfolio than any of other indices tracked by financial service firms.

Accordingly, we use the standard deviation of mutual fund returns (σ_j) as our measure of risk. Both Modigliani and Modigliani (1997) and the Sharpe ratio focus on total volatility as a measure of risk. Engle *et al.* (1987) demonstrate that the relation between the variance and the maturity premium in the debt market depends on the utility functions of the agents and the supply conditions of assets. Empirical research finds evidence of both a positive and negative tradeoff between return and total risk. French *et al.* (1987) find a positive tradeoff between return and variance as did Campbell and Hentschel (1992). Devaney (2001) finds a positive tradeoff between returns and own conditional variances for REIT indices. Devaney and Weber (2005) conclude that efficiency measures based on variance are better at predicting out of sample returns to individual REITs than those derived from β. Baillie and DeGennaro (1990) conclude that the tradeoff parameter between variance and return is insignificant for many of the portfolios in their study while Glosten *et al.* (1993) find some support for a negative tradeoff between return and volatility for some individual stocks. In general, empirical research suggests variance or standard deviation is a better measure of risk for portfolios than it is for individual securities.

Although capital market investors might be unable to hold the true market portfolio they are increasingly able to combine a wide variety of capital market proxies (both stocks and bonds) as represented by mutual fund indices. From 1990 to 2009, total financial assets in bond, income, and equity market mutual funds grew from \$608.4 to \$6,667.4 billion (Saunders and Cornett 2011). ETFs grew from \$102 billion in 2002 to \$1,675 billion in 2013 (Investment Company Factbook, 2014) and is evidence of investor demand for liquid capital market proxies. Although indexed mutual funds and ETFs tend to slightly under-perform the indices they track due to fund fees, expenses, rebalancing, dividend reinvestment, and tracking error, they provide investors a convenient means for holding a particular capital market sector and diversifying risk. Thus, not only does the S&P 500 market proxy not correlate well with broader market measures, it also fails to reflect the transactions costs associated with producing market portfolios of assets.

3. Method

We assume that mutual fund managers produce a desirable output of return (r) and an undesirable output of risk (σ), through the use of various inputs, represented by $x = (x_1, \ldots, x_N)$. We let the output possibility set, $P(x)$, represent the technology or process of generating return and risk, where:

$$P(x) = \{(r, \sigma): \quad x \text{ can produce}(r, \sigma)\}. \tag{1}$$

Following Chambers *et al.* (1996, 1998) we assume that $P(x)$ is a convex, closed, and continuous set that satisfies strong disposability of r, weak disposability of σ, and null-jointness. Strong disposability implies that if $(r, \sigma) \in P(x)$, then for $r' \leqslant r$, $(r', \sigma) \in P(x)$. Weak disposability of risk implies that if $(r, \sigma) \in P(x)$ and $0 \leqslant \theta \leqslant 1$, then $(\theta r, \theta\sigma) \in P(x)$. Weak disposability allows us to model the assertion that there is an opportunity cost of reducing risk: some return must also be foregone. Finally, null-jointness suggests that if mutual fund managers take no risk, they will earn no excess return; that is, if $(r, \sigma) \in P(x)$ and $\sigma = 0$ then $r = 0$.

We are interested in two aspects of the performance of mutual funds. First, do mutual fund managers choose a portfolio that allows them to operate on the return-risk frontier given inputs? Second, what is the frontier tradeoff between return and risk? To address these questions we use the directional output distance function of Chambers *et al.* (1996, 1998) who extend Luenberger's (1992) consumer benefit function to the producer's side of the market. The directional output distance function scales the observed values of mutual fund return and risk, given inputs, to the return-risk output possibility frontier, $P(x)$. The directional vector, $g = (g_r, g_\sigma)$determines how a mutual fund's observed return and risk are scaled to the $P(x)$ frontier. The directional output distance function is defined as:

$$\vec{D}_o(x, r, \sigma; g_r, g_\sigma) = \max\{\beta: \ (r + \beta g_r, \ \sigma - \beta g_\sigma) \in P(x)\}. \tag{2}$$

The directional output distance function seeks the maximum expansion of return and simultaneous contraction of risk for the directional scaling vector $g = (g_r, g_\sigma)$.

The solution to the directional output distance function, β^*, is a measure of technical inefficiency. When $\vec{D}_o(x, r, \sigma; g_r, g_\sigma) = \beta^* = 0$ the mutual fund occupies the return-risk frontier and is efficient in that it cannot add to return and simultaneously reduce risk. Combinations of (r, σ) such that $\vec{D}_o(x, r, \sigma; g_r, g_\sigma) > 0$ indicates the mutual fund operates inside the frontier and is inefficient with higher values of $\vec{D}_o(x, r, \sigma; g_r, g_\sigma)$ indicating greater inefficiency.

Figure 1 illustrates the output possibility set ($P(x)$) and the directional output distance function defined on that set. Strong disposability of return is satisfied in that for any combination of return-risk (r, σ), one can move downward in a vertical direction and still be in $P(x)$. Weak disposability is satisfied in that for any combination of (r, σ), a proportional contraction along a ray from the origin to the (r, σ) point is feasible. Consider the directional vector $g = (1, 1)$. In this case the directional output distance function gives the maximum unit expansion of return and simultaneous unit contraction of risk that is feasible given inputs, x. For the mutual fund represented by the combination of (r, σ) given by point A, the directional distance function scales (r, σ) in a northwesterly direction back to the frontier at point B. Other directional vectors might also be chosen. For instance, when $g = (1, 0)$, risk (σ) is held constant and the directional output distance function gives the maximum unit expansion of return. For the mutual fund represented by A, return is scaled in a northerly direction to point C. For a different direction, such as $g = (0, 1)$, return (at point A) is held constant and the directional output distance function scales risk in a westerly direction back to point E.

The function $\vec{D}_o(x, r, \sigma;\ g_r,\ g_\sigma)$inherits its properties from the output possibility set, $P(x)$. These properties include:

(i) $\vec{D}_o(x, r, \sigma; g_r, g_\sigma) \geqslant 0$ if and only if$(r, \sigma) \in P(x)$

(ii) $\vec{D}_o(x, r, \sigma; g_r, g_\sigma) \leqslant \vec{D}_o(x, r', \sigma; g_r, g_\sigma)$ for $(r', \sigma) \leqslant (r, \sigma) \in P(x)$

(iii) $\vec{D}_o(x, r, \sigma; g_r, g_\sigma) \leqslant \vec{D}_o(x, r, \sigma'; g_r, g_\sigma)$ for $(r, \sigma) \leqslant (r, \sigma') \in P(x)$

(iv) $\vec{D}_o(x, r + \alpha g_r, \sigma - \alpha g_\sigma; g_r, g_\sigma) = \vec{D}_o(x, r, \sigma; g_r, g_\sigma) - \alpha.$ (3)

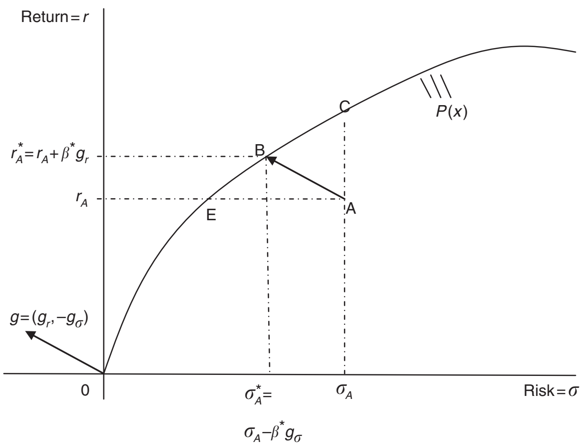

Figure 1.
Output possibility set, $P(x)$, and directional output distance function

Property 3(i) implies that for feasible combinations of (r, σ) the directional output distance function must be non-negative. Properties 3(ii) and 3(iii) are monotonicity properties which indicate that inefficiency does not decrease if a mutual fund generates a lower return or a higher risk holding inputs constant. Finally, 3(iv) is the translation property. The translation property implies that if αg_r is added to return and αg_σ is subtracted from risk, inefficiency will decline by α. The translation property is useful if the return of a mutual fund is negative. By an appropriate choice of α so that $r+\alpha g_r$ and $\sigma-\alpha g_\sigma$ are positive, the researcher can avoid the problems associated with arbitrary transformations to make negative output values positive.

The directional output distance function can also be used to evaluate the tradeoff between return and risk. Let $dr/d\sigma$ indicate the increase in return for a one unit increase in risk. We compute the partial total differential of Equation (2) and evaluate it on the frontier (i.e. at $\vec{D}_o(x, r, \sigma;\ g_r,\ g_\sigma) = 0$) to obtain:

$$d\vec{D}_o(x, r, \sigma; g_r, g_\sigma) = \sum_n \frac{\partial \vec{D}_o}{\partial x_n} dx_n + \frac{\partial \vec{D}_o}{\partial r} dr + \frac{\partial \vec{D}_o}{\partial \sigma} d\sigma = 0. \tag{4}$$

To keep the mutual fund on the $P(x)$ frontier we set $dx_n = 0$ for all n, and then rearrange Equation (4) to obtain:

$$\frac{\partial \vec{D}_o}{\partial r} dr = -\frac{\partial \vec{D}_o}{\partial \sigma} d\sigma \quad \text{or} \quad \frac{dr}{d\sigma} = -\frac{\partial \vec{D}_o/\partial \sigma}{\partial \vec{D}_o/\partial r}. \tag{5}$$

Thus, Equation (5) allows us to estimate the frontier return-risk tradeoff. In addition, by referencing the capital market line of the CAPM, we can determine whether the mutual fund has chosen an efficient allocation (mix) of return and risk.

The capital market line is written as:

$$r_j = r_f + \frac{E(r_M) - r_f}{\sigma_M} \sigma_j \tag{6}$$

where the subscript j represents asset j, r_f is the risk-free return, $E(r_m)$ is the expected value of the market return, σ_M is the standard deviation of the market portfolio and σ_j is the standard deviation of the jth asset. In Figure 2 we illustrate how to implement Equation (5) and the risk-free rate, r_f, to determine whether the jth mutual fund has chosen an efficient combination of return and risk. Consider the mutual fund represented by point A. The directional output distance function scales the return and risk for fund A to the output possibility set at point B. Given fund A's observed return and risk, r_A and σ_A, the reduction in inefficiency will allow the fund to have a return-risk profile of $\left(r_A + \beta^* \times g_r, \sigma_A - \beta^* \times g_\sigma\right) = \left(r_A^*, \sigma_A^*\right)$. The slope of $P(x)$ at point B is calculated using Equation (5). Once $dr_A/d\sigma_A$ is known, we calculate the vertical intercept, call it $\overline{r_A}$, by moving along the line with slope $dr_A/d\sigma_A$ from point B to the vertical axis. That is:

$$\overline{r_A} = r_A^* - \left(\frac{dr_A}{d\sigma_A}\right) \sigma_A^* \tag{7}$$

If $\overline{r_A} < r_f$, the mutual fund could improve its performance by taking greater risk, moving to the northeast along $P(x)$. However, if $\overline{r_A} > r_f$, the mutual fund could improve performance by moving to the southwest along $P(x)$. Finally, if $\overline{r_A} = r_f$, the mutual

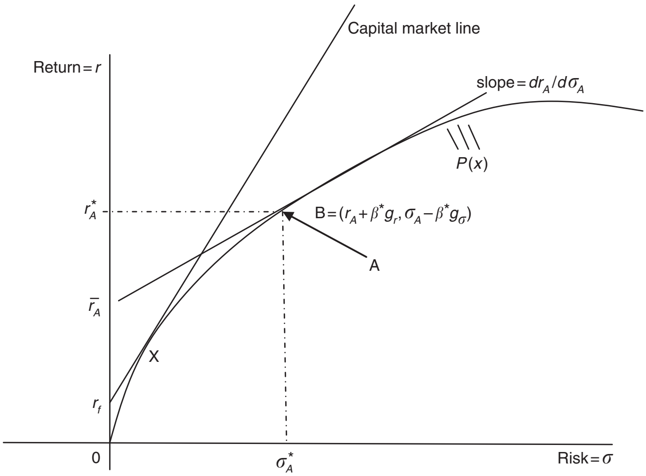

Figure 2.
Comparison of the return-risk tradeoff with the capital market line

fund has chosen the optimal combination of return and risk. In Figure 2, the capital market line is tangent with the return-risk possibility set at X, with $\overline{r_A} > r_f$, suggesting that the fund represented by A could improve performance by choosing a portfolio with less risk and less return.

The Sharpe ratio for fund j is a measure of its risk-adjusted excess return:

$$\text{Sharpe}_j = \frac{r_j - r_f}{\sigma_j}. \tag{8}$$

An outcome of the CAPM is that allocatively and technically efficient portfolios of assets lie along the capital market line and have the highest possible Sharpe ratio. Technically inefficient funds operate below the return-risk frontier and have a Sharpe ratio that is less than their potential. We adjust the Sharpe ratio for fund inefficiency while accounting for the transactions costs of production by adding measured inefficiency to actual returns and subtracting measured inefficiency from actual risk. The adjusted Sharpe ratio takes the form:

$$\text{Adjusted Sharpe}_j = \frac{r_j + \overrightarrow{D}_o\left(x^j, r^j, \sigma^j; 1, 1\right) \times g_r - r_f}{\sigma_j - \overrightarrow{D}_o\left(x^j, r^j, \sigma^j; 1, 1\right) \times g_\sigma}. \tag{9}$$

In the next section we present a quadratic functional form for the directional output distance function, discuss the parameter restrictions implied by our assumptions, and describe our estimation method.

4. Parametric specification

Although DEA can be used to estimate the directional output distance function, the DEA piecewise linear representation of the output possibility set, $P(x)$, is not continuously differentiable, so we cannot evaluate Equation (5). Instead, we use a quadratic form for the directional output distance function and estimate it using the deterministic method of Aigner and Chu (1968). The quadratic form is differentiable and can be restricted to satisfy feasibility, monotonicity, and the translation property set forth in Equation (3) (Färe *et al.*, 2005). Taking the directional vector to be $g=(1, 1)$ the quadratic form of the directional output distance function is:

$$\vec{D}_o(x_j, r_j, \sigma_j; 1, 1) = \alpha_o + \sum_{n=1}^{N} \alpha_n x_{nj} + \frac{1}{2}\sum_{n=1}^{N}\sum_{n'=1}^{N} \alpha_{nn'} x_{nj} x_{n'j} + \beta_1 r_j + \gamma_1 \sigma_j + \frac{1}{2}\beta_{11} r_j^2$$
$$+ \frac{1}{2}\gamma_{11}\sigma_j^2 + \mu_{11} r_j \sigma_j + \sum_{n=1}^{N} \delta_n x_{nj} r_j + \sum_{n=1}^{N} \upsilon_n x_{nj} \sigma_j. \quad (10)$$

Given $g=(1, 1)$, we impose the translation property via the restrictions:

$$\beta_1 - \gamma_1 = -1, \quad \beta_{11} = \gamma_{11} = \mu_{11}, \quad \delta_n - \upsilon_n = 0, \quad n = 1, \ldots, N \quad (11)$$

and symmetry conditions given by:

$$\alpha_{nn'} = \alpha_{n'n}, n, n' = 1, \ldots, N, n \neq n'. \quad (12)$$

We assume that there are $j=1,\ldots,J$ mutual funds and use the deterministic method of Aigner and Chu (1968) to estimate the parameters α_o, α_n, α_{nn}, β_1, β_{11}, γ_1, γ_{11}, μ_{11}, δ_n, and υ_n by solving the following minimization problem:

$$\min \sum_{j=1}^{J} \left[\vec{D}_o(x_j, r_j, \sigma_j; 1, 1) - 0\right] \quad \text{subject to}$$

$$\text{(i)} \quad \vec{D}_o(x_j, r_j, \sigma_j; 1, 1) \geqslant 0, \quad j = 1, \ldots, J,$$
$$\text{(ii)} \quad \partial\vec{D}_o(x_j, r_j, \sigma_j; \ 1, 1)/\partial r \leqslant 0, \quad j = 1, \ldots, J,$$
$$\text{(iii)} \quad \partial\vec{D}_o(x_j, r_j, \sigma_j \ \ 1, 1)/\partial \sigma \geqslant 0, \quad j = 1, \ldots, J,$$
$$\text{(iv)} \quad \beta_1 - \gamma_1 = -1, \quad \beta_{11} = \gamma_{11} = \mu_{11}, \quad \delta_n - \upsilon_n = 0, \ n = 1, \ldots, N,$$
$$\text{(v)} \quad \alpha_{nn'} = \alpha_{n'n}, n, n' = 1, \ldots, N, n \neq n'. \quad (13)$$

The problem set forth in Equation (13) chooses the parameters of the quadratic directional distance function Equation (10) by minimizing the deviation of the estimated distance functions for each mutual fund from its frontier value of zero. The constraints associated with (12(i)) require each mutual fund's return-risk to be feasible. The function $\vec{D}_o(x_j, r_j, \sigma_j; 1, 1)$serves as a measure of inefficiency. Given the directional vector of $g=(1,1)$, the directional output distance function gives the maximum unit expansion in return and unit contraction in risk that is feasible given $P(x)$. The monotonicity constraint in (12(ii)) requires that if return increases, holding risk and inputs constant, mutual fund inefficiency will not increase. The monotonicity constraint in (12(iii)) requires that if risk decreases, holding return and inputs constant, mutual fund inefficiency will not increase. Taken together, (12(ii)) and (12(iii)) constrain the tradeoff between return and risk, $dr/d\sigma$, to be non-negative. Finally, the constraints in (12(iv)) impose the translation property and the constraints in (12(v)) impose symmetry.

5. Data and results

To implement the method for evaluating mutual fund performance we selected a sample of US domestic mutual funds from Morningstar. To get a broad representation of funds in the sample we first identified ten different Morningstar style categories: large value, large blend, large growth, mid-cap growth, small growth, aggressive allocation, diversified emerging markets, foreign large growth, moderate allocation, and tactical allocation. For inclusion in the sample funds had to operate during the period 2010-2014 with complete data on the inputs and outputs. Funds that entered or exited during the five year period were not eligible to be sampled and thus, our sample is subject to survival bias. However, if the funds that exited were inefficient, those funds would not have influenced the choice of parameters of the quadratic directional distance function, since the directional distance function binds ($\vec{D}(x, r, \sigma; g_r, g_\sigma) = 0$) only for those funds that occupy the frontier. We took a random sample of 20 funds in each style category except for tactical allocation, where only eight mutual funds with complete data were found. Mutual fund return (y)and risk (b) equal the five-year annual return and standard deviation of the fund adjusted for dividends. We use each fund's turnover ratio (x_1), load ratio (x_2), expense ratio (x_3), and net asset value (x_4) to represent the inputs used to produce mutual fund return and risk. The turnover ratio refers to the number of times the fund's assets are traded in the previous year, with a low turnover ratio indicating a more passive strategy. The load ratio includes the current load, deferred load, and 12b-1 distribution fees. Other expenses include fees for shareholder expenses such as toll-free phone communications, computerized account services, website services, recordkeeping, printing, and mailing (Investment Company Factbook, 2014).

Similar to the degrees of freedom problem in classical regression, DEA models can suffer from the curse of dimensionality when a small number of producers each use a large number of inputs to produce many different outputs. When this problem occurs many of the producers are likely to operate on the production frontier because there are few producers for comparison. To remedy the problem researchers (see, e.g. Eling, 2006) sometimes use principal components analysis to reduce the number of inputs and outputs to a more tractable number or have chosen inputs with the lowest correlations for use in the analysis[2]. Given 188 mutual funds that use four inputs to produce return and risk, the curse of dimensionality is less prevalent in our data. Furthermore, use of principal components analysis would unnecessarily complicate the interpretation of the calculated slope of the return/risk frontier and the adjusted Sharpe ratio.

Descriptive statistics on the output and input variables are provided in Table I. Average annual returns during the five-year period were 11.2 percent with risk equal to 14.5 percent. Returns range from a low of −0.9 percent to a high of 21.5 percent.

	Mean	SD	Minimum	Maximum
Annual return $= r$	0.112	0.048	−0.009	0.215
Risk $= \sigma$	0.145	0.033	0.042	0.231
Turnover $= x_1$	0.636	0.547	0.000	3.740
Expense $= x_2$	0.011	0.004	0.001	0.031
Load $= x_3$	0.037	0.027	0.000	0.061
Net assets[a] $= x_4$	2,829	6,846	9.61	72,530
Actual Sharpe ratio	0.915	0.347	−0.397	1.696

Note: [a]Net assets are in millions of US dollars

Table I. Descriptive statistics, 188 mutual funds, January 2010 to December 2014

The average fund has \$2,829 million in net assets and 63.6 percent of those assets turned over in 2014. Average annual expenses are 1.1 percent and the load averages 3.7 percent. Table II reports the means and standard deviations of the inputs and outputs by investment category. The 20 funds in the small growth category have the highest return of 17.5 percent and the second highest level of risk at 17.9 percent. The 20 funds in the foreign large growth category have the smallest adjusted annual return, 7.1 percent. The greatest risk occurs for funds in the diversified emerging markets category, while funds in the tactical allocation category exhibit the least risk. Aggressive allocation funds had the lowest turnover (0.499) while the tactical allocation funds had the greatest turnover (1.076). Net assets range from \$23 million for funds in the tactical allocation category to \$6.459 billion for funds in the large blend category.

Because of the negative return of one fund we use the translation property and add 0.01 to each fund's annual return and subtract 0.01 from each fund's annual risk before estimating the model given by Equation (13). Given our choice of vector, $g = (1,1)$, and the translation property given by Equation (3) this transformation means that $\vec{D}_o(x, r, \sigma; 1, 1) = \vec{D}_o(x, r+0.01 \times 1, \sigma-0.01 \times 1; 1, 1)+0.01$. Thus, our transformation is grounded in the theory and properties of the directional output distance function. We estimate the function $\vec{D}_o(x, r+0.01 \times 1, \sigma-0.01 \times 1; 1, 1)$ but report the estimates of $\vec{D}_o(x, r, \sigma; 1, 1)$ for each mutual fund.

The coefficient estimates from the deterministic method found by Equation (13) are reported in Table III. These coefficients, along with the observed values of x, r, and σ for each mutual fund give an estimate of inefficiency $(\vec{D}_o(x_j, r_j, \sigma_j; 1, 1))$and allow calculation of the tradeoff between return and risk ($dr_j/d\sigma_j$). The deterministic method provides an estimate of the distance of each mutual fund's observed inputs and outputs to an estimated frontier, but not to the true, but unobservable frontier. As such, the estimates depend on the particular sample of observed mutual funds and are subject to sampling variation.

To analyze the sensitivity of the nonparametric estimates we employ the bootstrap of Simar and Wilson (1998) which has been recently employed by Bostian and Herlihy (2014) for a quadratic directional distance function estimated using the Aigner and Chu (1968) method. By repeated random sampling the bootstrap provides a way of estimating

Table II. Descriptive statistics for outputs/inputs by investment category (standard deviations)

Category[a]	Annual return $= r$	Risk $= \sigma$	Turnover $= x_1$	Expense ratio $= x_2$	Load $= x_3$	Net assets[b] $= x_4$
1	0.137 (0.032)	0.136 (0.017)	0.552 (0.381)	0.011 (0.003)	0.039 (0.026)	4,137 (7,272)
2	0.148 (0.019)	0.134 (0.010)	0.449 (0.288)	0.009 (0.003)	0.038 (0.029)	6,459 (16,226)
3	0.158 (0.015)	0.149 (0.012)	0.640 (0.375)	0.010 (0.002)	0.039 (0.029)	2,121 (1,836)
4	0.160 (0.031)	0.169 (0.022)	0.750 (0.525)	0.013 (0.004)	0.037 (0.027)	1,755 (2,895)
5	0.175 (0.025)	0.179 (0.014)	0.724 (0.513)	0.011 (0.003)	0.031 (0.029)	1,973 (2,886)
6	0.110 (0.017)	0.121 (0.016)	0.499 (0.870)	0.009 (0.006)	0.037 (0.028)	1,149 (4,040)
7	0.042 (0.024)	0.184 (0.016)	0.505 (0.289)	0.015 (0.002)	0.026 (0.029)	3,475 (6,077)
8	0.071 (0.021)	0.165 (0.016)	0.737 (0.579)	0.014 (0.003)	0.045 (0.024)	2,697 (5,186)
9	0.102 (0.010)	0.095 (0.010)	0.696 (0.618)	0.010 (0.004)	0.048 (0.022)	2,703 (5,967)
10	0.079 (0.024)	0.088 (0.028)	1.076 (0.869)	0.010 (0.003)	0.028 (0.029)	23 (317)

Notes: [a]Categories-1 = large value, 2 = large blended, 3 = large growth, 4 = mid-cap growth, 5 = small growth, 6 = aggressive allocation, 7 = diversified emerging markets, 8 = foreign large growth, 9 = moderate allocation, 10 = tactical allocation; [b]net assets are in millions of US dollars

Coefficient	Variable	Deterministic estimate	Bootstrapped estimate	Bootstrapped SE
α_0	Constant	0.01584	0.00885	0.00176
α_1	x_1	−0.00072	0.00089	0.00022
α_2	x_2	0.00000	0.22463	0.03882
α_3	x_3	0.00000	0.04773	0.00641
α_4	x_4	0.00056	0.00011	0.00003
α_{11}	x_1^2	−0.00038	−0.00025	0.00004
α_{12}	x_1x_2	0.00000	0.00260	0.00340
α_{13}	x_1x_3	0.00000	0.00237	0.00085
α_{14}	x_1x_4	0.00028	0.00021	0.00003
α_{22}	x_2^2	0.00000	−3.88856	0.88628
α_{23}	x_2x_3	0.00000	−0.02432	0.11783
α_{24}	x_2x_4	0.00028	0.00553	0.00100
α_{33}	x_3^2	0.00000	−0.06767	0.04032
α_{34}	x_3x_4	0.00000	0.00040	0.00013
α_{44}	x_4^2	0.00000	0.00000	0.00000
β_1	r	−0.48275	−0.40116	0.01272
$\gamma_1 = 1+\beta_1$	σ	0.51725	0.59884	0.01272
β_{11}	r^2	0.20097	−0.07330	0.04601
γ_{11}	σ^2	0.20097	−0.07330	0.04601
$\mu = \beta_{11} = \gamma_{11}$	$r\sigma$	0.20097	−0.07330	0.04601
δ_1	x_1r	0.00823	0.00171	0.00084
δ_2	x_2r	0.00000	−0.10348	0.10355
δ_3	x_3r	0.00000	−0.06731	0.01784
δ_4	x_4r	−0.00155	−0.00003	0.00011
$\nu_1 = \delta_1$	$x_1\sigma$	0.00823	0.00171	0.00084
$\nu_2 = \delta_2$	$x_2\sigma$	0.00000	−0.10348	0.10355
$\nu_3 = \delta_3$	$x_3\sigma$	0.00000	−0.06731	0.01784
$\nu_4 = \delta_4$	$x_4\sigma$	−0.00155	−0.00003	0.00011

Table III. Parameter estimates of the quadratic directional output distance function

the sensitivity of each mutual fund's distance to the frontier. Let $\hat{\theta}$ be the vector of parameter estimates found as the solution to Equation (13). This vector of parameter estimates is combined with each mutual fund's observed inputs (x_k) and outputs (r_k, σ_k) to obtain an estimate of the distance to the estimated frontier, $\hat{\vec{D}}_o(x_k, r_k, \sigma_k; 1, 1) = \hat{\vec{D}}_o^k$. The estimated parameters also estimate the shadow price ratio given by Equation (5).

The true, but unknown data generating process generates the random sample of observations (x, r, σ) for each mutual fund via some process F. The bootstrap approach uses the empirical distribution function to estimate the data generating process by $\hat{F}$, by taking $s = 1, \ldots, S$ bootstrap samples from the observed data.

The bootstrap process proceeds as follows: we estimate Equation (13) and obtain $\hat{\vec{D}}_o^k$ for each of the $k = 1, \ldots, K$ mutual funds. The frontier values of risk (σ^*) and return (r^*) are found as $\sigma_k^* = (\sigma_k - \hat{\vec{D}}_o^k \times 1)$ and $r_k^* = (r_k + \hat{\vec{D}}_o^k \times 1)$. For the sth bootstrap iteration we use the empirical distribution function of $k = 1, \ldots, K$ distance function estimates to draw with replacement with probability K^{-1} a distance estimate which we call $\hat{\vec{D}}_o^{ks}$ for mutual fund k for iteration s. Given this random draw we construct a sample of pseudo-outputs, $\sigma_k^s = \sigma_k^* + \hat{\vec{D}}_o^{ks}$ and $r_k^s = r_k^s - \hat{\vec{D}}_o^{ks}$. Then, given the pseudo-outputs and actual inputs we re-estimate Equation (13) and obtain a new vector of parameter estimates $\hat{\theta}^s$. We correct for the bias in the estimates following Simar and Wilson (1998).

The bootstrapped parameter estimates are reported alongside the deterministic estimates in Table III. We use these parameter estimates to calculate the level of inefficiency and the shadow price of risk and to determine whether a particular mutual fund should take more or less risk which are reported for the entire sample in Table IV and by investment category in Table V. From Table IV we see that average inefficiency is approximately the same for the deterministic method (0.033) and the bootstrapped estimates (0.032). These estimates indicate that on average, a mutual fund should be able to increase annual return by 0.032 and reduce the standard deviation of return by 0.032 given the inputs it employs. Five mutual funds operate on the deterministic frontier. The five frontier funds include one fund in each of the following style categories: large value, large blended, aggressive allocation, moderate allocation, and tactical allocation.

For the bootstrap we calculate the 2.5-97.5 percentile range of estimates for the directional distance function for each mutual fund and then catalog a mutual fund as

Table IV. Inefficiency and the risk-return tradeoff

	Deterministic estimate	Bootstrapped estimate
$\vec{D}_o(x_j, r_j, \sigma_j; 1, 1)$	0.033 (0.028)	0.032 (0.028)
Number of frontier firms	5	82
Adjusted Sharpe ratio	2.606 (102.57)	1.386 (0.103)
$\frac{dr}{d\sigma}$	1.337 (0.089)	1.399 (0.493)
Proportion of funds that should take more risk	0.016	0.31
Proportion of funds that should take less risk	0.984	0.69

Table V. Estimates by investment fund category

Category[a]	Deterministic estimate $\vec{D}(x, r, \sigma; 1, 1)$	Bootstrapped estimate $\vec{D}(x, r, \sigma; 1, 1)$	Bootstrapped no. on frontier	Actual Sharpe	Bootstrapped inefficiency adjusted Sharpe	No. of more risk
1	0.021 (0.020)	0.032 (0.028)	6	0.929 (0.327)	1.385 (0.072)	5
2	0.015 (0.010)	0.032 (0.028)	10	0.941 (0.325)	1.381 (0.110)	7
3	0.019 (0.007)	0.033 (0.028)	7	0.961 (0.308)	1.376 (0.055)	12
4	0.028 (0.020)	0.033 (0.028)	6	0.981 (0.299)	1.389 (0.065)	14
5	0.030 (0.013)	0.031 (0.027)	12	1.016 (0.282)	1.380 (0.062)	18
6	0.022 (0.013)	0.032 (0.027)	14	0.868 (0.358)	1.362 (0.102)	1
7	0.088 (0.016)	0.033 (0.028)	5	0.872 (0.366)	1.404 (0.090)	0
8	0.065 (0.014)	0.033 (0.027)	4	0.890 (0.353)	1.406 (0.079)	1
9	0.012 (0.007)	0.032 (0.028)	13	0.836 (0.393)	1.391 (0.106)	0
10	0.018 (0.014)	0.031 (0.028)	5	0.773 (0.434)	1.395 (0.289)	1

Notes: [a]Categories-1 = large value, 2 = large blended, 3-large growth, 4 = mid-cap growth, 5 = small growth, 6 = aggressive allocation, 7 = diversified emerging markets, 8 = foreign large growth, 9 = moderate allocation, 10 = tactical allocation operating on the frontier if that range includes zero. As reported in Table V, under the bootstrap the number of frontier mutual funds increases to 82. The style categories with the most frontier funds are aggressive allocation (14), moderate allocation (13), small growth (13), and large blended (10). In addition, the tactical allocation category places five out of eight funds on the bootstrapped frontier. The style categories with the fewest frontier funds are foreign large growth (4), diversified emerging markets (5), mid-cap growth (6), and large growth (7).

To calculate the Sharpe ratio we use the five-year (2010-2014) average of the one month T-bill rate as our estimate of the risk-free return: $r_f = 0.0007$. In addition, by comparing this risk-free return with the calculated vertical intercept from Equation (7) we can determine whether a mutual fund should take more or less risk. The mean of the actual Sharpe ratio reported in Table I is 0.915 and ranges from −0.397 to 1.696. On average, mutual funds in the small growth category have the highest actual Sharpe ratio (1.016) and funds in the tactical allocation category have the lowest Sharpe ratio (0.773). For the deterministic method the inefficiency adjusted Sharpe ratio calculated from Equation (9) is 2.606. In contrast, when the estimates are bootstrapped, the inefficiency adjusted Sharpe ratio is only 1.386. Furthermore, while funds in the small growth category have the highest average actual Sharpe ratio, those same funds have the third lowest inefficiency adjusted Sharpe ratio at 1.38, just slightly above funds in the aggressive allocation category at 1.362 and funds in the large growth category at 1.376. Mutual funds in the categories of diversified emerging markets and foreign large growth have the largest average inefficiency adjusted Sharpe ratios, 1.404 and 1.406, respectively.

Returning to Table VI, when the average mutual fund is projected to the frontier the slope of the frontier is approximately the same for the two methods: 1.337 for the deterministic estimates vs 1.399 for the bootstrap. However, the number of firms that should take more risk increases from only 1.6 percent for the deterministic method to 31 percent under the bootstrap. Still, for the period 2010-2014, an important result is that a majority of mutual funds should take less risk. For the bootstrapped estimates a majority of mutual funds in the categories large growth, mid-cap growth, and small growth could improve performance relative to the capital market line by taking more risk. In the other seven categories a majority of funds could improve performance relative to the capital market line by taking less risk. All of the funds in the categories of diversified emerging markets and moderate allocation could increase their performance relative to the capital market line by taking less risk.

6. Summary

In this paper we use the directional output distance function to model the five-year adjusted return/risk performance of 188 mutual funds during 2010-2014 as reported by Morningstar. Our approach allows mutual funds to operate off the efficient frontier by accounting for inefficiency and the transaction costs associated with the production of a mutual fund portfolio. Furthermore, our method does not require knowledge of the unobservable market portfolio and partly mitigates the market proxy problem identified by Roll (1977).

Using deterministic and bootstrapped estimates of the directional output distance function we find that if mutual funds were to adopt the best practice technology they could expand return and reduce risk by approximately 3.2 percent. We also estimate the frontier tradeoff between return and risk and compare it to the tradeoff suggested by the capital market line. The estimates indicate that after mutual funds are projected to the efficient frontier a majority of funds could improve performance relative to the capital market line by taking less risk and earning a lower return.

Notes

1. Roll (1977) argues that tests of the CAPM are in reality tests of *M*'s mean-variance efficiency. Because the true *M* can never be totally observed, the efficiency of *M* is not testable. If the proxy for M is mean-variance efficient, then it occupies the efficient frontier *ex post*. The finding that betas calculated against the mean-variance efficient proxy are linearly related to the proxy portfolio risk and return is a mathematical tautology. If the proxy used for *M* is not *ex post* mean-variance efficient, then empirical results derived from the CAPM are meaningless.
2. The Spearman rank correlation coefficients between the inputs (*p*-values in parentheses) are 0.26(0.01) for turnover and expense ratio, 0.14 (0.06) for turnover and load, −0.26 (.01) for turnover and net assets, 0.19 (0.01) for expense ratio and load, −0.34 (0.01) for expense ratio and net assets, and −0.05 (0.47) for load and net assets.

References

Aigner, D.J. and Chu, S.J. (1968), "On estimating the industry production function", *American Economic Review*, Vol. 58 No. 4, pp. 826-839.

Baillie, R. and DeGennaro, R. (1990), "Stock returns and volatility", *Journal of Financial and Quantitative Analysis*, Vol. 25 No. 2, pp. 203-214.

Barras, L., Scaillet, O. and Wermers, R. (2010), "False discoveries in mutual fund performance: measuring luck in estimated alphas", *Journal of Finance*, Vol. LXV No. 1, pp. 179-216.

Basso, A. and Funari, S. (2001), "A data envelopment analysis approach to measure the mutual fund performance", *European Journal of Operational Research*, Vol. 135 No. 3, pp. 477-492.

Basso, A. and Funari, S. (2014), "The role of fund size in the performance of mutual funds assessed with DEA models", in Perna, C. and Sibillo, M. (Eds), *Mathematical and Statistical Methods for Actuarial Sciences and Finance*, Springer International Publishing, Cham Switzerland, pp. 21-26.

Berger, A.N. and Humphrey, D.B. (1997), "Efficiency of financial institutions: international survey and directions for future research", *European Journal of Operations Research*, Vol. 98 No. 2, pp. 175-212.

Blake, C., Elton, E. and Gruber, M. (1993), "The performance of bond mutual funds", *Journal of Business*, Vol. 66 No. 3, pp. 371-404.

Bodie, Z., Kane, A. and Marcus, A. (2011), *Investments* 9th ed., McGraw-Hill Irwin, New York, NY.

Bostian, M.B. and Herlihy, A.T. (2014), "Valuing tradeoffs between agricultural production and wetland condition in the US Mid-Atlantic region", *Ecological Economics*, Vol. 105, pp. 284-291.

Brandouya, O., Kerstens, K. and VandeWoestyne, I. (2015), "Frontier based vs traditional mutual fund ratings: a first back testing analysis", *European Journal of Operational Research*, Vol. 242 No. 1, pp. 332-342.

Briec, W. and Kerstens, K. (2009), "Multi-horizon Markowitz portfolio performance appraisals: a general approach", *Omega*, Vol. 37 No. 1, pp. 50-62.

Briec, W. and Kerstens, K. (2010), "Portfolio selection in multidimensional general and partial moment space", *Journal of Economic Dynamics and Control*, Vol. 34 No. 4, pp. 636-656.

Briec, W., Kerstens, K. and Jokung, O. (2007), "Mean-variance-skewness portfolio performance gauging: a general shortage function and dual approach", *Management Science*, Vol. 53 No. 1, pp. 135-149.

Briec, W., Kerstens, K. and Lesourd, J.B. (2004), "Single-period Markowitz portfolio selection, performance gauging, and duality: a variation on the Luenberger shortage function", *Journal of Optimization Theory and Applications*, Vol. 120 No. 1, pp. 1-27.

Briec, W., Kerstens, K. and Van de Woestyne, I. (2013), "Portfolio selection with skewness: a comparison of methods and a generalized one fund result", *European Journal of Operational Research*, Vol. 230 No. 2, pp. 412-421.

Campbell, J. and Hentschel, L. (1992), "No new is good news: an asymmetric model of changing volatility in stock returns", *Journal of Financial Economics*, Vol. 31 No. 3, pp. 281-318.

Carhart, M.M. (1997), "On persistence in mutual fund performance", *Journal of Finance*, Vol. 52 No. 1, pp. 57-82.

Chambers, R., Chung, Y. and Färe, R. (1996), "Benefit and distance functions", *Journal of Economic Theory*, Vol. 70 No. 2, pp. 407-419.

Chambers, R., Chung, Y. and Färe, R. (1998), "Profit, directional distance functions, and Nerlovian efficiency", *Journal of Optimization Theory and Applications*, Vol. 98 No. 2, pp. 351-364.

DeBondt, W. and Thaler, R. (1987), "Further evidence on investor overreaction and stock market seasonality", *Journal of Finance*, Vol. 42 No. 3, pp. 557-581.

Devaney, M. (2001), "Time varying risk premia for real estate investment trusts: a GARCH-M model", *The Quarterly Review of Economics and Finance*, Vol. 41 No. 3, pp. 335-346.

Devaney, M. and Weber, W.L. (2005), "Efficiency, scale economies, and the risk/return performance of real estate investment trusts", *Journal of Real Estate Finance and Economics*, Vol. 31 No. 3, pp. 301-317.

Eling, M. (2006), "Performance measurement of hedge funds using data envelopment analysis", *Financial Markets and Portfolio Management*, Vol. 20 No. 4, pp. 442-471.

Elton, E., Gruber, M.J., Das, S. and Hlavka, M. (1993), "Efficiency with costly information: a reinterpretation of evidence from managed portfolios", *Review of Financial Studies*, Vol. 6, pp. 1-22.

Engle, R.F., Lilien, D. and Robins, R. (1987), "Estimating time varying risk premium in the term structure: the ARCH model", *Econometrica*, Vol. 55 No. 2, pp. 391-407.

Färe, R., Grosskopf, S., Noh, D.W. and Weber, W.L. (2005), "Characteristics of a polluting technology: theory and practice", *Journal of Econometrics*, Vol. 126 No. 2, pp. 469-492.

Farrell, M.J. (1957), "The measurement of production efficiency", *Royal Statistical Society*, Series t General, Vol. 120 No. 3, pp. 253-281.

French, K.G., Schwert, W. and Stambaugh, R.F. (1987), "Expected stock returns and volatility", *Journal of Financial Economics*, Vol. 19 No. 1, pp. 3-29.

Fukuyama, H. and Weber, W.L. (2014), "Two-stage network DEA with bad outputs", in Cook, W.D. and Zhu, J. (Eds), *Data Envelopment Analysis: A Handbook on the Modeling of Internal Structures and Networks*, Springer, New York, NY, pp. 451-474.

Glawischnig, M. and Sommersguter-Reichmann, M. (2010), "Assessing the performance of alternative investments using non-parametric efficiency measurement approaches: is it convincing?", *Journal of Banking and Finance*, Vol. 34 No. 2, pp. 295-303.

Glosten, L., Jagannathan, R. and Runkle, D. (1993), "On the relation between the expected value and the nominal excess returns on stock", *Journal of Finance*, Vol. 48 No. 5, pp. 1779-1801.

Goetzmann, W. and Ibbotson, R. (1994), "Do winners repeat? Predicting mutual fund performance", *Journal of Portfolio Management*, Vol. 20 No. 2, pp. 9-18.

Gregoriou, G.N., Rouah, F., Satchell, S. and Diz, F. (2005), "Simple and cross efficiency of CTAs using data envelopment analysis", *The European Journal of Finance*, Vol. 11 No. 5, pp. 393-409.

Grossman, S.J. and Stiglitz, J.E. (1980), "On the impossibility of informationally efficient markets", *American Economic Review*, Vol. 70 No. 3, pp. 393-408.

Haslett, W. Jr (Ed.) (2010), *Risk Management: Foundations for a Changing Financial World*, Chapter 47, Country Risk in Global Financial Management, CFA Investment Perspectives, Wiley, Hoboken, NJ.

Hendricks, D., Patel, J. and Zeckhauser, R. (1993), "Hot hands in mutual funds: short run persistence of relative performance, 1974-1988", *Journal of Finance*, Vol. 43 No. 1, pp. 93-130.

Ibbotson, R., Siegel, L. and Love, K. (1985), "World wealth: market values and returns", *Journal of Portfolio Management*, Vol. 12 No. 1, pp. 4-20.

Investment Company Factbook (2014), "A review of trends and activities in the US investment company industry", Investment Company Institute, available at: www.icifactbook.org/ (accessed March 1, 2015).

Kahneman, D. and Tversky, A. (1979), "Prospect theory: an analysis of choice under risk", *Econometrica*, Vol. 47 No. 2, pp. 263-291.

Kerstens, K., Mounir, A. and Van de Woestyne, I. (2011a), "Geometric representation of the mean-variance-skewness portfolio frontier based upon the shortage function", *European Journal of Operational Research*, Vol. 210 No. 1, pp. 81-94.

Kerstens, K., Mounir, A. and Van de Woestyne, I. (2011b), "Non-parametric frontier estimates of mutual fund performance using C- and L-moments: some specification tests", *Journal of Banking and Finance*, Vol. 35 No. 5, pp. 1190-1201.

Lamb, J. and Tee, K. (2012a), "Data envelopment analysis models of investment funds", *European Journal of Operational Research*, Vol. 216 No. 3, pp. 687-696.

Lamb, J. and Tee, K. (2012b), "Resampling DEA estimates of investment fund performance", *European Journal of Operational Research*, Vol. 223 No. 3, pp. 834-841.

Luenberger, D.G. (1992), "Benefit functions and duality", *Journal of Mathematical Economics*, Vol. 21 No. 5, pp. 461-486.

Luenberger, D.G. (2001), "Projection pricing", *Journal of Optimization Theory and Applications*, Vol. 1 No. 1, pp. 1-25.

Malkiel, B. (1995), "Returns from investing in equity mutual funds 1971-1991", *Journal of Finance*, Vol. 50 No. 2, pp. 549-572.

Markowitz, H. (1952), "Portfolio selection", *Journal of Finance*, Vol. 7 No. 1, pp. 77-91.

Markowitz, H. (1959), *Portfolio Selection: Efficient Diversification of Investments. Cowles Foundation for Research in Economics at Yale University, Monograph 16*, Yale University Press, New Haven, CT and London.

Modigliani, F. and Modigliani, L. (1997), "Risk adjusted performance", *Journal of Portfolio Management*, Vol. 23 No. 2, pp. 45-54.

Morey, M. and Morey, R. (1999), "Mutual fund performance appraisals: a multi-horizon perspective with endogenous benchmarking", *Omega*, Vol. 27 No. 2, pp. 241-258.

Murthi, B.P.S., Choi, Y.K. and Desai, P. (1997), "Efficiency of mutual funds and portfolio performance measurement: a nonparametric approach", *European Journal of Operational Research*, Vol. 98 No. 2, pp. 408-418.

Odean, T. (1998), "Are investors reluctant to realize their losses?", *Journal of Finance*, Vol. 53 No. 5, pp. 1775-1798.

Patari, E., Leivo, T. and Honkapuro, S. (2012), "Enhancement of equity portfolio performance using data envelopment analysis", *European Journal of Operational Research*, Vol. 220 No. 1, pp. 786-797.

Quandl (2015), "Stock market capitalization by country", available at: www.quandl.com/collections/economics/stock-market-capitalization-by-country (accessed September 4).

Roll, R. (1977), "A critique of asset pricing theory's tests: part I on the past and potential testability of the theory", *Journal of Financial Economics*, Vol. 4 No. 2, pp. 129-176.

Rubio, J., Hassan, M.K. and Merdad, H. (2012), "Non-parametric performance measurement of international and Islamic mutual funds", *Accounting Research Journal*, Vol. 25 No. 3, pp. 208-226.

Saunders, A. and Cornett, M. (2011), *Financial Institutions Management: A Risk Management Approach*, 7th ed., McGraw Hill, New York, NY.

Sengupta, J.K. (2000), *Dynamic and Stochastic Efficiency Analysis: Economics of Data Envelopment Analysis*, World Scientific Publishing Co. Pte. Ltd, NJ, Singapore, London and Hong Kong.

Simar, L. and Wilson, P.W. (1998), "Sensitivity analysis of efficiency scores: how to bootstrap in nonparametric frontier models", *Management Science*, Vol. 44 No. 1, pp. 49-61.

Corresponding author
William Weber can be contacted at: wlweber@semo.edu