# Fractals and Chaos Writing Assignment 1

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**Abstract**

A paper explaining the similarities and differences between atmospheric dynamics and population dynamics.

## Introduction

In this world, there is a mathematical phenomenon called "Chaos" or the "Butterfly Effect." This occurs when the initial conditions to a mathematical equation are highly sensitive, changing the end result drastically, due to a small change in the conditions at the beginning [1]. The concept of chaos is ever so prominent in several fields that involve math, more specifically meteorology and biology. Of course, these seem to be two very different fields of study, however, they actually do indeed share similarities when it comes to chaotic behavior. Concepts such as mathematical equations, physical representations, and how they both turn into chaos. Together, they show how chaos can affect situations ranging from the predicting of weather (atmospheric dynamics), studied by Ed Lorenz, to the fluctuant behavior of populations (population dynamics), studied by Robert May.

## The Similarities

These two major concepts that involve chaotic behavior seem to be quite different from each other. The study of weather patterns doesn't sound like it would be related to how populations rise and fall at all. However, they do indeed share similar aspects, first of which is math. In order to determine both of these behaviors, mathematical equations have to be used in order to describe what these two are trying to prove. Not only that, but they both had figures that are used to show the behavior such as the "Lorenz Attractor" [figure 1] and the "cobweb plot" [figure 2]. The actual purpose of the Lorenz Attractor is that it shows the chaotic behavior of warm water and cool water circulating in a non-linear motion. The cobweb plot is supposed to represent the rise and fall of populations with the line function $f(x) = x$ showing a point of equilibrium. This brings up another similarity, which is the actual intention of the functions that were created to show this. They were needed to be simplified in order to make easier predictions for the behavior that will unfold. The behavior, as we discussed before, is chaotic. Both of the concepts that Lorenz and May wanted to convey turn into chaos as they progress.

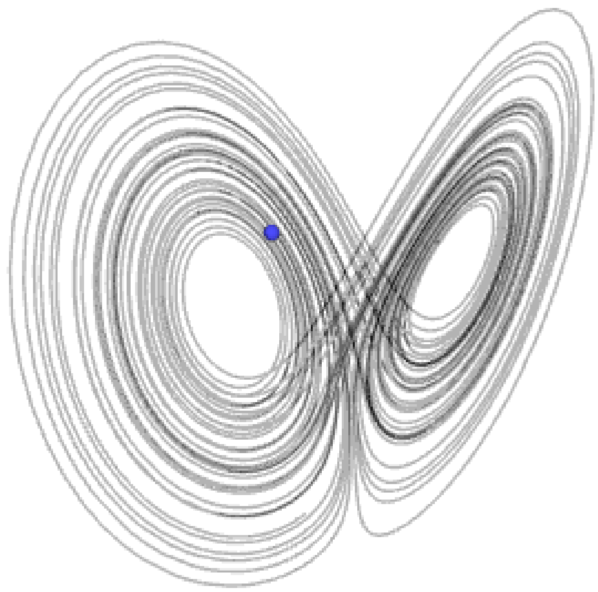

Figure 1: The Lorenz attractor [3]

## The Differences

Now to go with more of our intuition. Meteorology and biology seem VERY different. Even though there are the similarities listed above, as we go more specifically, they tend to show their differences. The cobweb plot in figure 2 shows a periodic orbit, while the Lorenz attractor is not, due to its non-linear math behind it. In other words, the Lorenz attractor goes back and forth on the axes while the cobweb plot has periods going from 0 to infinity. With this, it shows that the Lorenz attractor's time varies continuously as it moves around in the same space while the cobweb plot's time varies discretely. With the actual equations themselves, they vary in complexity and amount. The Lorenz attractor's functions are:

$$x' = \sigma(y - x)$$
$$y' = x(\rho - z) - y$$
$$z' = xy - \beta z,$$

while May uses a logistic function:

$$f(x) = ax(1 - x)$$

which is usually illustrated along with the linear function:

$$f(x) = x$$

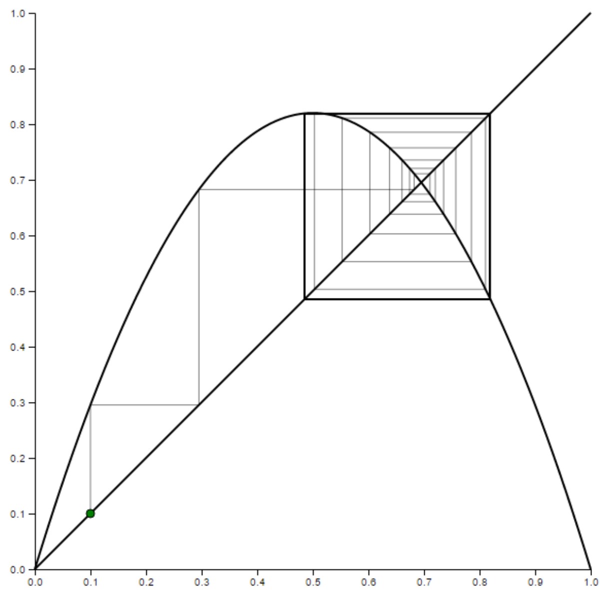


Figure 2: Cobweb Plot with the function $3.280x(1 - x)$ [4]

With the logistic function, it can actually be iterated, which shows a representation of a population over time, which is shown in figure 3, for example. Although they both show chaotic behavior, one is more simple than the other, doing the same thing with the manipulation of initial values.

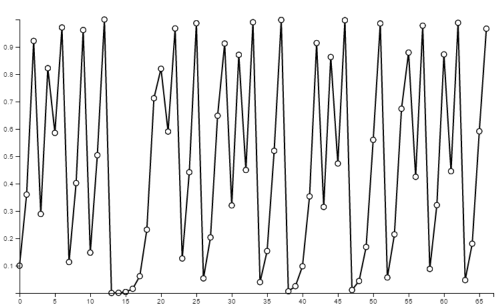

Figure 3: 66 iterations of the logistic function: $f(x) = 4x(1 - x)$ [5]

## Conclusion

In conclusion, although both Lorenz's representation of chaos with atmospheric dynamics and May's chaotic way of showing population dynamics have similar general themes, as they get more specific with how they work, they become more and more different. They both are trying to show the same idea that chaotic behavior ensues when there are small manipulations to initial values, creating extreme results. With this, they tried to show these phenomenons through means of natural patterns, such as actual isolated populations and pots of warm and cool water flowing around because of the heat.

## References


1. James Gleick, Chaos: Making a New Science. Penguin Books, New York, NY 1987.

2. Mark's Math: The Lorenz Attractor
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3. Mark's Math: Cobweb Plots
$http://marksmath.org/classes/Fall2016FractalLAC/demos/cobwebs/cobweb_tool.html$

4. Mark's Math: Iteration with Javascript
$http://marksmath.org/classes/Fall2016FractalLAC/demos/basic_iteration/iteration_loop.html$

5. Fractals and Chaos Discourse
$http://discourse2.marksmath.org/$