FLORIDA STATE UNIVERSITY

Assignment 3: Equivalence Relations

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1 Exercise

Determine whether or not the below relations on the set $A = \{0, 1, 2, 3\}$ are equivalence relations. If not, state all missing properties.

$$R = \{(0,0),(1,1),(2,2),(2,1),(1,2)\}$$

$$R = \{(3,1),(2,1),(1,2),(0,0),(2,2),(3,3),(1,1)\}$$

$$R = \{(0,0),(1,1),(2,2),(3,3)\}$$

1.1 Solution

(a)

$$R = \{(0,0),(1,1),(2,2),(2,1),(1,2)\}$$

R is symmetric and transitive but not reflexive due to the fact that $(0,0) \not\in R$.

(b)

$$R = \{(3,1),(2,1),(1,2),(0,0),(2,2),(3,3),(1,1)\}$$

R is reflexive but not symmetric due to the fact that $(3,1) \in R$ but $(1,3) \not\in R$. Nor is it transitive because $(3,1) \in R \wedge (1,2) \in R$ but $(3,2) \not\in R$.

(c)

$$R = \{(0,0),(1,1),(2,2),(3,3)\}$$

R is reflexive, symmetric, and transitive because it is the identity relation on A.

2 EXERCISE

Determine whether or not the below relations are equivalnce relations, if not state all properties.

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_R = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

2.1 SOLUTION

For each of these let M_R be treated as a zero indexed matrix as in most programming languages.
(a)

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

M_R is not reflexive due to the fact that $M_R[1][1] = 0$.
M_R is not symmetric due to the fact that $M_R[1][0] = 1 \wedge M_R[0][1] = 0$.
M_R is not transitive due to the fact that $M_R[1][2] = 1 \wedge M_R[2][1] = 1 \wedge M_R[1][1] = 0$.
(b)

$$M_R = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

M_R is not reflexive due to the fact that $M_R[1][1] = 0$.
M_R is not transitive due to the fact that $M_R[1][0] = 1 \wedge M_R[0][1] = 1 \wedge M_R[1][1] = 0$.
M_R is symmetric.

3 EXERCISE

What are the equivalence classes of the below bit strings for the equivalence relation, R, on the set of all bit strings of length 2 or more, where R consists of all pairs (x, y) such that x and y are bit strings of length 2 or more that agree (are the same) except possibly (i.e. they may differ) in their first 2 bits. (You may describe the elements in the sets instead of listing them all out).
(a) 1001 (b) 10 (c) 10111

3.1 Solution

(a) 1001

$$\{1001\} = \{ab01 | \forall a, b \in \{0,1\}\}$$

(b) 10

$$\{10\} = \{ab | \forall a, b \in \{0,1\}\}$$

(c) 10111

$$\{10111\} = \{ab111 | \forall a, b \in \{0,1\}\}$$

4 Exercise

4) Which of these collections of subsets are partitions of $A = \{a, b, c, d, e\}$? If not, why?
(a) $\{a, b\}, \{c\}, \{d\}$
(b) $\{a\}, \{b, d\}, \{c\}, \emptyset$
(c) $\{a, b, d\}, \{c, d\}, \{e\}$

4.1 Solution

(a)

$$\{a, b\}, \{c\}, \{d\}$$

While the subsets might be disjoint and non empty, the union of them is not A.
(b)

$$\{a\}, \{b, d\}, \{c\}, \emptyset$$

They are disjoint, but one of them is empty and their union is also not A.
(c)

$$\{a, b, d\}, \{c, d\}, \{e\}$$

This time, they are empty and their union is A, but d is in two sets making them not disjoint.

5 Exercise

List the ordered pairs in the equivalence relations produced by the following partitions of $A = \{a, b, c\}$.
(a) $\{\{a\}, \{b\}, \{c\}\}$
(b) $\{\{c\}, \{a, b\}\}$

5.1 Solution

(a)

$$\{\{a\},\{b\},\{c\}\}$$

This is the identity relation:

$$\{(a,a),(b,b),(c,c)\}$$

(b)

$$\{\{c\},\{a,b\}\}$$

This is not the identity relation but close:

$$\{(a,a),(b,b)(a,b),(b,a),(c,c)\}$$