

# Swarthmore College

cout << 1/0 << endl;

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# Contents

int dfs(int i) {

int low = val[i] = ++time, x; z.push\_back(i);

for(int e : gr[i]) if (!comp[e])

 $x = z.back(); z.pop_back();$ 

if (low == val[i]) do {

comp[x] = low;

low = min(low, val[e] ?: dfs(e));

1 Templates 1	if (values[x>>1] == -1) values[x>>1] = x&1;	return FS; };
	<pre>while (x != i);</pre>	,,
2 Math Hints 24	<pre>return val[i] = low; }</pre>	AdvancedHash.cpp
Tompletos (1)	,	ll base1[MX], base2[MX];
$\overline{\text{Templates}} $ (1)	bool solve() {	int base:
	values.assign(N, -1);	
$\underset{\text{a73310, 66 lines}}{2\text{sat.cpp}}$	val.assign(2*N, 0); comp = val;	const 11 p1 = MOD;
/* kactl 2-SAT Solver	FOR(i,0,2*N) if (!comp[i]) dfs(i); FOR(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;	const 11 p2 = MOD+2;
* Negated variables are represented by $bit$ -inversions (\	return 1;	// 76
$texttt\{ \setminus tilde\{ \}x\} $ ).	}	// If you don't need to
* Usage:	};	// only maintain val1 and // $get() = val1 * p2 + ve$
* TwoSat ts(number of boolean variables);		struct hsh {
* $ts.either(0, \tilde3); // \ Var \ 0 \ is \ true \ or \ var \ 3 \ is false$	3dHull.cpp	11 val1, val2;
* $ts.setValue(2)$ ; // $Var 2 is true$	"Point3D.h" de4d99, 49 lines	vl valls, val2s;
* $ts.atMostOne(\{0, \forall tilde1, 2\}); // \le 1 \text{ of } vars 0, \forall tilde1$	typedef Point3D <double> P3;</double>	vl nums;
and 2 are true		hsh() {
* ts.solve(); // Returns true iff it is solvable	struct PR {	val1 = 0;
* ts.values[0N-1] holds the assigned values to the vars	<pre>void ins(int x) { (a == -1 ? a : b) = x; }</pre>	val2 = 0; val1s.pb(0);
*/	<pre>void rem(int x) { (a == x ? a : b) = -1; }</pre>	valls.pb(0);
, m, o, , , (	int cnt() { return (a != -1) + (b != -1); }	}
<pre>struct TwoSat {   int N;</pre>	int a, b; };	
vector <vi> qr;</vi>		<pre>void push_back(ll v)</pre>
vi values; $//$ $0 = false$ , $1 = true$	struct F { P3 q; int a, b, c; };	v++;
. , ,		val1 *= base; val
$TwoSat(int n = 0) : N(n), gr(2*n) \{ \}$	<pre>vector<f> hull3d(const vector<p3>&amp; A) {</p3></f></pre>	val2 *= base; val
	assert(sz(A) >= 4);	valls.pb(vall);
int addVar() { // (optional)	vector <vector<pr>&gt; E(sz(A), vector<pr>(sz(A), {-1, -1}));</pr></vector<pr>	val2s.pb(val2);
<pre>gr.emplace_back(); gr.emplace_back();</pre>	<pre>#define E(x,y) E[f.x][f.y] vector<f> FS;</f></pre>	nums.pb(v);
return N++;	auto mf = [&] (int i, int j, int k, int l) {	}
}	P3 q = (A[j] - A[i]).cross((A[k] - A[i]));	
	if (q.dot(A[1]) > q.dot(A[i]))	ll get(int L, int R)
<pre>void either(int f, int j) {</pre>	q = q * -1;	11 A = (val1s[R+1
$f = \max(2*f, -1-2*f);$	F f{q, i, j, k};	+ p1) % p1; 11 B = (val2s[R+3
j = max(2*j, -1-2*j); $gr[f].push_back(j^1);$	<pre>E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i); FS.push_back(f);</pre>	+ p2) % p2;
<pre>gr[i].push_back(f 1); gr[j].push_back(f^1);</pre>	};	return A*p2+B;
}	FOR(i,0,4) rep(j,i+1,4) rep(k,j+1,4)	}
<pre>void setValue(int x) { either(x, x); }</pre>	mf(i, j, k, 6 - i - j - k);	};
word at Magt One (genet wis li) ( // (entional)	EOD/; / oz /A)) (	void prepHash() {
<pre>void atMostOne(const vi&amp; li) { // (optional)   if (sz(li) &lt;= 1) return;</pre>	FOR(i, 4, sz(A)) {     FOR(j, 0, sz(FS)) {	base = uid(MOD/5, MOI
int cur = $\sim$ li[0];	F f = FS[j];	
FOR(i,2,sz(li)) {	if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {	base1[0] = 1; base2[0
<pre>int next = addVar();</pre>	E(a,b).rem(f.c);	FOR(i, 1, MX) {
either(cur, ~li[i]);	E(a,c).rem(f.b);	base1[i] = (base2 base2[i] = (base2
either(cur, next);	E(b,c).rem(f.a);	}
<pre>either(~li[i], next);</pre>	swap(FS[j], FS.back());	}
<pre>cur = ~next; }</pre>	<pre>FS.pop_back(); }</pre>	
either(cur, ~li[1]);	}	AhoCorasick.cpp
}	int nw = sz(FS);	l — — — — — — — — — — — — — — — — — — —
	FOR(j,0,nw) {	// NOTE: val/num variable
vi val, comp, z; int time = 0;	F f = FS[j];	// and dfs/compute method

c);

```
838740, 46 lines
                         query arbitrary ranges,
                         nd val2 to save space.
                         al2
                         il1 += v; val1 %= p1;
                         112 += v; val2 %= p2;
                          1] - (valls[L] * base1[R-L+1]) % p1
                         1] - (val2s[L] * base2[R-L+1]) % p2
                         D/2);
                         [0] = 1;
                          1[i-1] * base) % p1;
                         2[i-1] * base) % p2;
                                                    b658a8, 80 lines
                         les in v, cut argument to add\_string,
// and dfs/compute methods may be unnecessary
struct AhoCorasick {
    static const int K = 26;
    struct V {
        int nxt[K];
        bool leaf = false;
        int p = -1;
```

 $A[it.c] - A[it.a]).dot(it.q) \le 0) swap(it.c, it.b);$ 

#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.

C(a, b, c); C(a, c, b); C(b, c, a);

trav(it, FS) if ((A[it.b] - A[it.a]).cross(

```
char pch;
   int link = -1;
   int go[K];
   11 \text{ val} = -1;
   11 \text{ num} = 0;
   V(int p=-1, char ch='$') : p(p), pch(ch) {
        fill(begin(nxt), end(nxt), -1);
        fill(begin(go), end(go), -1);
};
vector<V> t:
void init() {
   V v; t.pb(v);
void add_string(string const& s, int cnt) {
   int v = 0;
   trav(ch, s) {
       int c = ch - 'a';
        if (t[v].nxt[c] == -1) {
           t[v].nxt[c] = t.size();
            t.emplace_back(v, ch);
        v = t[v].nxt[c];
    t[v].leaf = true;
    t[v].num = cnt;
11 dfs(int v) {
   if (t[v].val != -1) {
        return t[v].val;
   11 \text{ ans} = t[v].num;
    ans += dfs(get link(v));
    return t[v].val = ans;
// sets value for each node to sum of values
// over suffix links
void compute() {
   t[0].val = 0;
   FOR(i, 1, sz(t)) {
       dfs(i);
int get_link(int v) {
   if (t[v].link == -1) {
        if (v == 0 || t[v].p == 0)
            t[v].link = 0;
            t[v].link = go(get_link(t[v].p), t[v].pch);
    return t[v].link;
int go(int v, char ch) {
   int c = ch - 'a';
   if (t[v].go[c] == -1) {
        if (t[v].nxt[c] != -1)
           t[v].go[c] = t[v].nxt[c];
```

```
t[v].qo[c] = v == 0 ? 0 : qo(qet_link(v), ch
                     ):
        return t[v].go[c];
};
Angle.cpp
                                                    0f0602, 35 lines
struct Angle {
 int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t};
  int half() const {
   assert(x || v);
   return y < 0 \mid \mid (y == 0 \&\& x < 0);
  Angle t90() const { return \{-y, x, t + (half() \&\& x \ge 0)\}
  Angle t180() const { return \{-x, -y, t + half()\}; }
 Angle t360() const { return \{x, y, t + 1\}; }
bool operator<(Angle a, Angle b) {</pre>
  // add a. dist2() and b. dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (11)b.x) <</pre>
         make_tuple(b.t, b.half(), a.x * (11)b.y);
// Given two points, this calculates the smallest angle
// them, i.e., the angle that covers the defined line
     seament.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
 if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
 if (a.t180() < r) r.t--;
 return r.t180() < a ? r.t360() : r;
Angle angleDiff(Angle a, Angle b) { // angle \ b - angle \ a}
 int tu = b.t - a.t; a.t = b.t;
 return \{a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)
      };
Basis.cpp
                                                    95f6d3, 27 lines
const int D; //length of masks
11 basis[D]; // basis[i] keeps the mask of the vector whose
    f value is i
int bs = 0; //basis\ size
void insertVector(ll mask) {
```

for (int i = 0; i < D; i++) {

**if** (!basis[i]) {

if ((mask & 111 << i) == 0) continue;

```
basis[i] = mask;
      ++bs;
      return;
    mask ^= basis[i];
bool inSpan(ll mask) {
  for (int i = 0; i < D; i++) {
   if ((mask & 111 << i) == 0) continue;
   mask ^= basis[i];
 return mask == 0;
Berlekamp-Massey.cpp
                                                   335c7c, 44 lines
vector<int> berlekamp_massey(const vector<int> &a) {
 vector<int> v, last; // v is the answer, 0-based, p is
       the module
  int k = -1, delta = 0;
  for (int i = 0; i < (int)a.size(); i++) {
    int tmp = 0;
    for (int j = 0; j < (int)v.size(); j++)
     tmp = (tmp + (long long)a[i - j - 1] * v[j]) % p;
    if (a[i] == tmp) continue;
    if (k < 0) {
     k = i;
      delta = (a[i] - tmp + p) % p;
     v = vector < int > (i + 1);
      continue:
    vector<int> u = v;
    int val = (long long)(a[i] - tmp + p) * power(delta, p -
    if (v.size() < last.size() + i - k) v.resize(last.size()</pre>
          + i - k);
    (v[i - k - 1] += val) %= p;
    for (int j = 0; j < (int) last.size(); <math>j++) {
     v[i - k + j] = (v[i - k + j] - (long long)val * last[j]
      if (v[i - k + j] < 0) v[i - k + j] += p;
    if ((int)u.size() - i < (int)last.size() - k) {</pre>
     last = u;
     k = i;
      delta = a[i] - tmp;
     if (delta < 0) delta += p;
```

for (auto &x : v) x = (p - x) % p;

v.insert(v.begin(), 1);

```
BinSearchSegtree.cpp
                                                  853b41, 54 lines
const 11 identity = 0;
const 11 SZ = 131072;
11 sum[2*SZ], lazy[2*SZ];
ll combine(ll A, ll B) {
    return A+B;
11 combineUpd(11 A, 11 B) {
    return A+B;
void push(int index, 11 L, 11 R) {
    sum[index] = combineUpd(sum[index], lazy[index]);
    if (L != R) lazy[2*index] = combineUpd(lazy[2*index],
         lazy[index]), lazy[2*index+1] = combineUpd(lazy[2*
         index+1], lazy[index]);
    lazy[index] = identity;
void pull(int index) {
    sum[index] = combine(sum[2*index], sum[2*index+1]);
bool checkCondition(int index) {
    //FILL THIS IN
11 query(int lo = 0, int hi = SZ-1, int index = 1, ll L = 0,
     11 R = SZ-1) { //returns first node satisfying con
    push (index, L, R);
    if (lo > R || L > hi) return -1;
    bool condition = checkCondition(index);
    if (L == R) {
        return (condition ? L : -1);
    int M = (L+R) / 2;
    if (checkCondition(2*index)) {
        return query(lo, hi, 2*index, L, M);
    return query(lo, hi, 2*index+1, M+1, R);
void update(int lo, int hi, ll increase, int index = 1, ll L
     = 0, 11 R = SZ-1) {
    push(index, L, R);
    if (hi < L || R < lo) return;
    if (lo <= L && R <= hi) {
        lazy[index] = increase;
        push (index, L, R);
        return;
    int M = (L+R) / 2;
    update(lo, hi, increase, 2*index, L, M); update(lo, hi,
         increase, 2*index+1, M+1, R);
    pull(index);
```

return v; // \$\forall i, \sum\_{\{j}=0\} ^ m a\_{-\{i-j\}} v\_{-j}=

```
BipartiteMatching.cpp
                                                   7a6c4b, 24 lines
//Storing the graph
vector<int> g[maxn];
//Storing whether we have visited a node
bool vis[maxn];
 //Storing the vertex matched to
int match[maxn];
bool hungarian(int u) {
  for (int i = 0; i < q[u].size(); ++i){
    int v = q[u][i];
    if (!vis[v]){
      vis[v] = true;
      if (!match[v] || hungarian(match[v])){
        match[u] = v; match[v] = u; return true;
  return false;
//in main: call hungarian for each vertex on one side
for (int i = 1; i <= n1; ++i) {
    memset(vis, false, sizeof vis);
    if (hungarian(i)) ans++; //if we can match i
BipartiteMatchingWithWeights.cpp
                                                    c21d5f, 74 lines
11 q[maxn][maxn];
11 fx[maxn], fy[maxn], a[maxn], b[maxn], slack[maxn], pre[
    maxnl:
bool visx[maxn], visy[maxn];
int q[maxn];
int n;
void augment(int v){
 if (!v) return; fy[v] = pre[v]; augment(fx[pre[v]]); fx[fy
       [v] = v;
void bfs(int source){
  memset (visx, 0, sizeof visx);
  memset (visy, 0, sizeof visy);
  memset(slack, 127, sizeof slack);
  int head, tail; head = tail = 1;
  g[tail] = source;
  while (true) {
    while (head <= tail) {</pre>
      int u = q[head++];
      visx[u] = true;
      for (int v = 1; v \le n; ++v) {
        if (!visy[v]){
          if (a[u] + b[v] == g[u][v])
            visv[v] = true; pre[v] = u;
            if (!fy[v]){
              augment(v); return;
            q[++tail] = fy[v];continue;
          if (slack[v] > a[u] + b[v] - g[u][v])
            slack[v] = a[u] + b[v] - q[u][v];
            pre[v] = u;
```

```
11 d = inf;
    for (int i = 1;i <= n;++i) {</pre>
     if (!visy[i]) d = min(d, slack[i]);
    for (int i = 1;i <= n;++i) {</pre>
     if (visx[i]) a[i] -= d;
     if (visy[i]) b[i] += d;
     else slack[i] -= d;
    for (int v = 1; v \le n; ++v) {
      if (!visy[v] && !slack[v]){
        visy[v] = true;
        if (!fy[v]){
          augment (v);
          return;
        q[++tail] = fy[v];
11 km(){
 for (int i = 1; i \le n; ++i) {
   a[i] = -inf;
   b[i] = 0;
    for (int j = 1; j \le n; ++j) a[i] = max(a[i], q[i][j]);
  memset(fx, 0, sizeof fx);
  memset(fy, 0, sizeof fy);
  for (int i = 1; i \le n; ++i) bfs(i);
 11 \text{ ans} = 0;
  for (int i = 1; i \le n; ++i) ans += a[i] + b[i];
  //vertex i on left is matched to g2[i][fx[i]] * fx[i]
  //g2[a][b]=1 iff exists edge ab
  return ans;
BIT.cpp
                                                    992848, 14 lines
const int maxn = 30005;
int n, bit[maxn];
void add(int i, int x) {
   for (; i <= n; i += i & (-i))
        bit[i] += x;
int sum(int i) {
   int r = 0;
    for (; i; i -= i & (-i)) {
       r += bit[i];
    return r;
BlockCut.cpp
                                                   512e87, 63 lines
// note: just need dfs if all you need is cutpoints or BCCs
// if all you need is BCCs, ignore id
// if all you need is cutpoints, ignore stk, id, comps
// can add LCA on top of this to check if a path from a-b
     that
// does not visit c exists
```

```
// assumes graph is simple; must dfs multiple times if not
    connected
// be careful about handling isolated vertices
vector<vi> graph(MX), comps;
vi stk, num(MX), lo(MX), is_cp(MX), id(MX);
int ct = 0:
void dfs(int v, int p) {
    num[v] = lo[v] = ++ct;
    if (sz(graph[v]) == 0) {
        comps.pb({v});
        return;
    stk.pb(v);
    trav(a, graph[v]) {
       if (a == p) continue;
       if (num[a]) {
            lo[v] = min(lo[v], num[a]);
       } else {
            dfs(a, v);
            lo[v] = min(lo[v], lo[a]);
            if (lo[a] >= num[v]) {
                is_{cp}[v] = (num[v] > 1 || num[a] > 2);
                comps.pb({v});
                while (comps.back().back() != a) {
                    comps.back().pb(stk.back());
                    stk.pop_back();
vector<vi> bct;
void build tree() {
    int nid = 0;
    FOR(i, N) {
        if (is_cp[i]) {
            id[i] = nid++;
            bct.pb({});
    trav(comp, comps) {
       int v = nid++;
       bct.pb({});
       trav(u, comp) {
            if (!is_cp[u]) {
               id[u] = v;
            } else {
                bct[v].pb(id[u]);
                bct[id[u]].pb(v);
```

# Bridge.cpp

a44485, 33 lines

int n; // number of nodes vector<vector<int>> adj; // adjacency list of graph

```
vector<bool> visited;
vector<int> tin, low;
int timer;
void dfs(int v, int p = -1) {
   visited[v] = true;
   tin[v] = low[v] = timer++;
   for (int to : adj[v]) {
       if (to == p) continue;
       if (visited[to]) {
            low[v] = min(low[v], tin[to]);
       } else {
            dfs(to, v);
            low[v] = min(low[v], low[to]);
            if (low[to] > tin[v])
                IS_BRIDGE(v, to);
   }
void find_bridges() {
   timer = 0;
   visited.assign(n, false);
   tin.assign(n, -1);
   low.assign(n, -1);
   for (int i = 0; i < n; ++i) {
       if (!visited[i])
            dfs(i);
```

#### CentroidDecomp.cpp

```
b8fb4<u>8, 54 lines</u>
struct CentroidDecomposition {
 vector<set<int>> tree; // it 's not vector<vector<int>>!
 vector<int> dad;
 vector<int> sub;
   vector<int> dep;
 CentroidDecomposition(vector<set<int>> &tree) : tree(tree)
   int n = tree.size();
   dad.resize(n);
   sub.resize(n);
       dep.resize(n);
   build(0, -1);
 void build(int u, int p) {
   int n = dfs(u, p); // find the size of each subtree
   int centroid = dfs(u, p, n); // find the centroid
       if (p == -1) {
            dep[centroid] = 0;
            dep[centroid] = dep[p] + 1;
   if (p == -1) p = centroid; // dad of root is the root
        itself
   dad[centroid] = p;
   // for each tree resulting from the removal of the
        while (!tree[centroid].emptv()) {
            int v = *(tree[centroid].begin());
      tree[centroid].erase(v); // remove the edge to
           disconnect
```

```
tree[v].erase(centroid); // the component from the
         tree
    build(v, centroid);
      }
int dfs(int u, int p) {
 sub[u] = 1;
  for (auto v : tree[u])
   if (v != p) sub[u] += dfs(v, u);
  return sub[u];
int dfs(int u, int p, int n) {
  for (auto v : tree[u])
   if (v != p \text{ and } sub[v] > n/2) \text{ return } dfs(v, u, n);
int operator[](int i) {
  return dad[i];
```

#### ChordalGraph.cpp

78d46d, 44 lines

```
//Maximum Cardinality Search
while (cur) {
 p[cur] = h[nww];
  rnk[p[cur]] = cur;
 h[nww] = nxt[h[nww]];
 lst[h[nww]] = 0;
 lst[p[cur]] = nxt[p[cur]] = 0;
 tf[p[cur]] = true;
  for (vector<int>::iterator it = G[p[cur]].begin(); it != G
      [p[cur]].end();
      it++)
    if (!tf[*it]) {
     if (h[deg[*it]] == *it) h[deg[*it]] = nxt[*it];
     nxt[lst[*it]] = nxt[*it];
     lst[nxt[*it]] = lst[*it];
     lst[*it] = nxt[*it] = 0;
     deg[*it]++;
     nxt[*it] = h[deg[*it]];
     lst[h[deq[*it]]] = *it;
     h[deq[*it]] = *it;
 if (h[nww + 1]) nww++;
  while (nww && !h[nww]) nww--;
//Checking if a sequence is a perfect elimination ordering
jud = true;
for (int i = 1; i <= n; i++) {
 for (vector<int>::iterator it = G[p[i]].begin(); it != G[p
      [i]].end(); it++)
   if (rnk[p[i]] < rnk[*it]) {</pre>
     s[++cur] = *it;
     if (rnk[s[cur]] < rnk[s[1]]) swap(s[1], s[cur]);</pre>
```

```
}
for (int j = 2; j <= cur; j++)
    if (!st[s[1]].count(s[j])) {
        jud = false;
        break;
    }
}
if (!jud)
    printf("Imperfect\n");
else
    printf("Perfect\n");
</pre>
```

#### CircleIntersection.cpp

#### CircleLine.cpp

#### CirclePolygonIntersection.cpp

```
"../../content/geometry/Point.h"
                                                    3e5102, 19 lines
typedef Point < double > P;
#define arg(p, g) atan2(p.cross(g), p.dot(g))
double circlePoly(P c, double r, vector<P> ps) {
 auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
   P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det))
    if (t < 0 \mid \mid 1 \le s) return arg(p, q) * r2;
   P u = p + d * s, v = p + d * t;
    return arg(p, u) * r2 + u.cross(v)/2 + arg(v, g) * r2;
  auto sum = 0.0;
 FOR(i, 0, sz(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
```

#### CircleTangents.cpp

```
"Point.h" b0153d, 13 lines

template < class P >
vector < pair < P, P >> tangents (P c1, double r1, P c2, double r2
    ) {
    P d = c2 - c1;
    double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
    if (d2 == 0 || h2 < 0) return {};
    vector < pair < P, P >> out;
    for (double sign : {-1, 1}) {
        P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
        out.push_back({c1 + v * r1, c2 + v * r2});
    }
    if (h2 == 0) out.pop_back();
    return out;
}
```

#### Circumcircle.cpp

#### ClosestPair.cpp

```
"Point.h"
                                                   d31bbf, 17 lines
typedef Point<11> P:
pair<P, P> closest(vector<P> v) {
  assert (sz(v) > 1);
  set<P> S:
  sort(all(v), [](P a, P b) { return a.y < b.y; });</pre>
  pair<11, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  trav(p, v) {
   P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y \le p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d
    for (; lo != hi; ++lo)
      ret = min(ret, \{(*lo - p).dist2(), \{*lo, p\}\});
   S.insert(p);
  return ret.second;
```

# ${\bf ConvexHull.cpp}$

```
// WARNING: May include multiple copies of the same point if
    duplicates are included in input
// Returns convex hull in clockwise order

struct pt {
    ld x, y;
};

bool cmp(pt a, pt b) {
    return a.x < b.x || (a.x == b.x && a.y < b.y);
```

```
bool cw(pt a, pt b, pt c) {
   return a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y) < 0;
bool ccw(pt a, pt b, pt c) {
   return a.x*(b.y-c.y)+b.x*(c.y-a.y)+c.x*(a.y-b.y) > 0;
void convex_hull(vector<pt>& a) {
   if (sz(a) == 1)
       return;
   sort(all(a), &cmp);
   pt p1 = a[0], p2 = a.back();
   vector<pt> up, down;
   up.pb(p1);
   down.pb(p1);
   FOR(i, 1, sz(a)) {
       if (i == sz(a) - 1 \mid | cw(p1, a[i], p2)) {
           while (sz(up) \ge 2 \&\& !cw(up[sz(up)-2], up[sz(up)
                )-1], a[i]))
                up.pop_back();
            up.pb(a[i]);
       if (i == sz(a) - 1 \mid | ccw(p1, a[i], p2)) {
            while (sz(down) >= 2 \&\& !ccw(down[sz(down)-2],
                down[sz(down)-1], a[i])
                down.pop_back();
            down.pb(a[i]);
   for (int i = 0; i < sz(up); i++)
        a.push_back(up[i]);
    for (int i = sz(down) - 2; i > 0; i--)
        a.push_back(down[i]);
```

#### ConvexHullTrick.cpp

875d66 26 lines

```
//This represents those relying on j only; i.e intercept
inline 11 y_axis(int j){
   return dp[j] + a * s[j] * s[j] - b * s[j];
//This represents those relying on both i and j; i.e slope
inline ll x_axis(int j){
   return s[j];
inline ld getSlope(int j,int k){
   ld y = y_axis(k) - y_axis(j);
   1d x = x_axis(k) - x_axis(j);
    return v / x;
//s stores prefix sum
int head = 1;int tail = 1;
\alpha[head] = 0;
for (int i = 1; i \le n; i++) {
 dp[i] = a*s[i]*s[i] + b*s[i] + c;
 while (head < tail && getSlope(g[head],g[head+1]) >= 2*a*s
      [i]) head++;
 if (head <= tail) {
   int j = q[head];
    dp[i] = max(dp[i], dp[j] + a *(s[i] - s[j]) * (s[i] - s[
        j]) + b*(s[i] - s[j]) + c);
```

q[++tail] = i;

#### CRT DelaunayTriangulation Dinic DSU EulerPath

```
CRT.cpp
                                                   118857, 32 lines
//each is x \mod p_i = a_i
11 p[maxn], a[maxn];
//for quickmult see pollard rho
11 exgcd(11 x, 11 v, 11 & a, 11 & b) {
   if (y == 0) {
       a = 1;b = 0;return x;
    11 d = exgcd(y, x%y, a, b);
   11 temp = a; a = b; b = temp - (x / y) * b;
    return d;
int first_nontrivial = 0;
ll current p :
11 sol = 0; //this is the solution
for (int i = 1; i <= n; i++) {
 if (p[i] != 1) {
    first nontrivial = i;
    current_p = p[i]; sol = a[i];
   break;
for (int i = first nontrivial+1; i <= n; i++) {</pre>
 11 x,y;
 if (p[i] == 1) continue;
  11 d = exqcd(current p, p[i], x, v);
  ll r = ((a[i] - sol) % p[i] + p[i]) % p[i];
 ll temp = quickmult(x, r / d,p[i] / d);
  sol = sol + current_p * temp;
 current p = current p / d * p[i];
 sol = (sol % current p + current p) % current p;
Delaunay Triangulation.cpp
"Point.h", "3dHull.h"
                                                   d173fc, 10 lines
template<class P, class F>
void delaunay(vector<P>& ps, F trifun) {
 if (sz(ps) == 3) { int d = (ps[0].cross(ps[1], ps[2]) < 0)
    trifun(0,1+d,2-d);}
 vector<P3> p3;
  trav(p, ps) p3.emplace back(p.x, p.y, p.dist2());
 if (sz(ps) > 3) trav(t, hull3d(p3)) if ((p3[t.b]-p3[t.a]).
      cross(p3[t.c]-p3[t.a]).dot(P3(0,0,1)) < 0)
```

while (head < tail && getSlope(g[tail],i) >= getSlope(g[

tail-1],q[tail])) tail--;

# Dinic.cop $//Complexity: O(E*V^2)$

trifun(t.a, t.c, t.b);

```
//from https://cp-algorithms.com/graph/dinic.html
struct Edge {
  int v, u;
   11 \text{ cap, flow} = 0;
   Edge(int v, int u, ll cap) : v(v), u(u), cap(cap) {}
};
```

681177, 83 lines

```
struct Dinic {
   const 11 flow inf = 1e18;
   vector<Edge> edges;
  vector<vi> adi;
  int n, m = 0;
  int s, t;
  vi lev, ptr;
  queue<int> q:
  Dinic(int n, int s, int t) : n(n), s(s), t(t) {
      adj.resize(n);
      lev.resize(n);
      ptr.resize(n);
  void add_edge(int v, int u, ll cap) {
      edges.emplace back(v, u, cap);
      edges.emplace_back(u, v, 0);
      adj[v].push_back(m);
      adj[u].push_back(m + 1);
      m += 2;
  bool bfs() {
      while (!q.empty()) {
           int v = q.front();
           trav(id, adj[v]) {
               if (edges[id].cap - edges[id].flow < 1)</pre>
                   continue;
               if (lev[edges[id].u] != -1)
                   continue;
               lev[edges[id].u] = lev[v] + 1;
               q.push(edges[id].u);
       return lev[t] != -1;
  11 dfs(int v, 11 pu) {
      if (pu == 0)
           return 0:
       if (v == t)
           return pu;
       for (int& cid = ptr[v]; cid < sz(adj[v]); cid++) {</pre>
           int id = adi[v][cid];
           int u = edges[id].u;
           if (lev[v] + 1 != lev[u] || edges[id].cap - edges
                [id].flow < 1)
               continue:
           11 tr = dfs(u, min(pu, edges[id].cap - edges[id].
                flow));
           if (tr == 0)
               continue:
           edges[id].flow += tr;
           edges[id ^ 1].flow -= tr;
           return tr;
       return 0:
  1.1 flow() {
      11 f = 0;
       while (true) {
           fill(all(lev), -1);
```

```
f += pu;
      return f;
};
DSU.cpp
                                                  89f1c6, 19 lines
int parent[MX], si[MX];
void init(int N) {
   FOR(i, N) parent[i] = i, si[i] = 0;
int get(int x) {
   if (parent[x] != x) parent[x] = get(parent[x]);
    return parent[x];
void unify(int x, int y) {
   x = get(x); y = get(y);
   if (x == v) return;
   if (si[x] < si[y]) swap(x, y);
   if (si[x] == si[y]) si[x]++;
   parent[y] = x;
EulerPath.cpp
                                                  274951, 25 lines
int N, M;
vector<vpi> graph(MX); //{ed, edNum}
vector<vpi::iterator> its(MX);
vector<bool> used(MX);
vpi eulerPath(int r) {
   FOR(i, N) its[i] = begin(graph[i]);
   FOR(i, M) used[i] = false;
   vpi ans, s\{\{r, -1\}\};
   int lst = -1;
   while (sz(s)) {
        int x = s.back().f; auto &it = its[x], en = end(
            graph[x]);
       while (it != en && used[it->s]) it++;
       if (it == en) {
           if (lst != -1 && lst != x) return {};
            ans.pb(s.back()); s.pop_back(); if (sz(s)) lst =
                 s.back().f;
       } else {
           s.pb(*it);
           used[it->s] = 1;
   } // Returns path in reverse order if graph is directed.
    if (sz(ans) != M+1) return {};
    return ans;
```

lev[s] = 0;

break:

fill(all(ptr), 0);

while (ll pu = dfs(s, flow\_inf)) {

q.push(s); if (!bfs()) "Point.h"

#### FastDelaunay FastHashTable FFT GaussElim

#### FastDelaunay.cpp

```
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t ll1; // (can be ll if coords are < 2e4)
P arb (LLONG_MAX, LLONG_MAX); // not equal to any other point
struct Quad {
 bool mark; Q o, rot; P p;
 P F() { return r()->p; }
 Q r() { return rot->rot; }
 Q prev() { return rot->o->rot; }
 Q next() { return r()->prev(); }
};
bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
 111 p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
 return p.cross(a,b) *C + p.cross(b,c) *A + p.cross(c,a) *B >
      0;
Q makeEdge(P orig, P dest) {
 Q q[] = \{new Quad\{0,0,0,oriq\}, new Quad\{0,0,0,arb\},
           new Quad{0,0,0,dest}, new Quad{0,0,0,arb}};
   q[i] -> o = q[-i \& 3], q[i] -> rot = q[(i+1) \& 3];
 return *q;
void splice(Q a, Q b) {
 swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
Q connect(Q a, Q b) {
 Q = makeEdge(a->F(), b->p);
 splice(q, a->next());
 splice(q->r(), b);
 return q;
pair<Q,Q> rec(const vector<P>& s) {
 if (sz(s) <= 3) {
    Q = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back())
   if (sz(s) == 2) return { a, a->r() };
    splice(a->r(), b);
   auto side = s[0].cross(s[1], s[2]);
   0 c = side ? connect(b, a) : 0;
   return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
 O A, B, ra, rb;
 int half = sz(s) / 2;
 tie(ra, A) = rec({all(s) - half});
 tie(B, rb) = rec({sz(s) - half + all(s)});
 while ((B->p.cross(H(A)) < 0 \&& (A = A->next())) | |
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
 O base = connect(B->r(), A);
 if (A->p == ra->p) ra = base->r();
 if (B->p == rb->p) rb = base;
#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
     Q t = e->dir; \
      splice(e, e->prev()); \
```

```
splice(e->r(), e->r()->prev()); \
     e = t; \
   }
 for (;;) {
   DEL(LC, base->r(), o); DEL(RC, base, prev());
   if (!valid(LC) && !valid(RC)) break;
   if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
     base = connect(RC, base->r());
     base = connect(base->r(), LC->r());
 return { ra, rb };
vector<P> triangulate(vector<P> pts) {
 sort(all(pts)); assert(unique(all(pts)) == pts.end());
 if (sz(pts) < 2) return {};
 Q e = rec(pts).first;
 vector<Q> q = \{e\};
 int qi = 0;
 while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { O c = e; do { c->mark = 1; pts.push back(c->p)
 q.push_back(c->r()); c = c->next(); } while (c != e); }
 ADD; pts.clear();
 while (qi < sz(q)) if (!(e = q[qi++]) -> mark) ADD;
 return pts;
```

#### FastHashTable.cpp

1c46ca, 88 lines

```
<ext/pb_ds/assoc_container.hpp>
                                                    f39118, 19 lines
using namespace __gnu_pbds;
    static uint64_t splitmix64(uint64_t x) {
        // http://xorshift.di.unimi.it/splitmix64.c
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    size_t operator()(uint64_t x) const {
        static const uint64_t FIXED_RANDOM = chrono::
             steady clock::now().time since epoch().count();
        return splitmix64(x + FIXED_RANDOM);
};
template<typename T> using pb_set = gp_hash_table<T,</pre>
     null_type, chash>;
template<typename T, typename U> using pb_map =
     gp hash table<T, U, chash>;
```

#### FFT.cpp

**Description:** fft(a) computes  $\hat{f}(k) = \sum_{x} a[x] \exp(2\pi i \cdot kx/N)$  for all k. N must be a power of 2. Useful for convolution: conv (a, b) = c, where  $c[x] = \sum a[i]b[x-i]$ . For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if  $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$  (in practice 10<sup>16</sup>; higher for random inputs). Otherwise, use NTT/FFTMod. **Time:**  $O(N \log N)$  with  $N = |A| + |B| (\sim 1s \text{ for } N = 2^{22})$ 

```
typedef complex<double> C;
```

```
R.resize(n); rt.resize(n);
   auto x = polar(1.0L, acos(-1.0L) / k);
   FOR(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
 vi rev(n);
 FOR(i, 0, n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
 FOR(i, 0, n) if (i < rev[i]) swap(a[i], a[rev[i]]);
 for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) FOR(j,0,k) {
     Cz = rt[j+k] * a[i+j+k]; // (25\% faster if hand-
           rolled)
     a[i + j + k] = a[i + j] - z;
     a[i + j] += z;
vector<double> conv(const vector<double>& a, const vector<
    double>& b) {
  if (a.empty() || b.empty()) return {};
 vector<double> res(sz(a) + sz(b) - 1);
 int L = 32 - \underline{\quad} builtin_clz(sz(res)), n = 1 << L;
 vector<C> in(n), out(n);
  copy(all(a), begin(in));
 FOR(i,0,sz(b)) in[i].imag(b[i]);
  fft(in);
  for (C& x : in) x *= x;
  FOR(i, 0, n) out[i] = in[-i & (n - 1)] - conj(in[i]);
 FOR(i, 0, sz(res)) res[i] = imag(out[i]) / (4 * n);
 return res:
GaussElim.cpp
int h = 0, k = 0;
while (h < sz(coef) && k < sz(coef[0])) {
```

int n = sz(a),  $L = 31 - _builtin_clz(n)$ ;

for (static int k = 2; k < n; k \*= 2) {

static vector<complex<long double>> R(2, 1);

static vector<C> rt(2, 1); // (^ 10% faster if double)

void fft(vector<C>& a) {

369f57, 24 lines

```
int i_max = h; ld max_val = abs(coef[h][k]);
FOR(j, h+1, sz(coef)) if (ckmax(max_val, abs(coef[j][k])
     )) i max = j;
if (abs(coef[i_max][k]) < 1e-9) {</pre>
    k++;
} else {
    FOR(i, sz(coef[0])) {
        swap(coef[h][i], coef[i_max][i]);
    ld inVal = (ld) 1 / coef[h][k];
    FOR(i, sz(coef[0])) {
        coef[h][i] *= inVal;
    FOR(i, sz(coef)) {
        if (i == h) continue;
        ld cur = coef[i][k];
        FOR(j, sz(coef[0])) {
            coef[i][j] -= cur * coef[h][j];
    h++; k++;
```

#### GeneralMatching.cpp

53f4fc, 78 lines

```
//belong is a DSU; unit = union
int n, match[N], next[N], mark[N], vis[N], Q[N];
std::vector<int> e[N];
int rear;
int LCA(int x, int y){
 static int t = 0; t++;
 while (true) {
   if (x != -1) {
     x = findb(x);
     if (vis[x] == t) return x;
     vis[x] = t;
     if (match[x] != -1) x = next[match[x]];
     else x = -1:
    std::swap(x, y);
void group(int a, int p){
   while (a != p) {
     int b = match[a], c = next[b];
      if (findb(c) != p) next[c] = b;
      if (mark[b] == 2) mark[Q[rear++] = b] = 1;
     if (mark[c] == 2) mark[Q[rear++] = c] = 1;
     unit(a, b); unit(b, c);
     a = c;
void aug(int s){
 for (int i = 0; i < n; i++)
   next[i] = -1, belong[i] = i, mark[i] = 0, vis[i] = -1;
 mark[s] = 1;
 0[0] = s; rear = 1;
  for (int front = 0; match[s] == -1 && front < rear; front
    int x = Q[front];
    for (int i = 0; i < (int)e[x].size(); i++){
     int y = e[x][i];
     if (match[x] == y) continue;
     if (findb(x) == findb(y)) continue;
      if (mark[y] == 2) continue;
      if (mark[y] == 1){
       int r = LCA(x, y);
       if (findb(x) != r) next[x] = y;
       if (findb(y) != r) next[y] = x;
       group(x, r);
       group(y, r);
      else if (match[y] == -1) {
       next[y] = x;
        for (int u = y; u != -1; ) {
         int v = next[u];
         int mv = match[v];
         match[v] = u, match[u] = v; u = mv;
       break;
      else{
       next[v] = x;
       mark[Q[rear++] = match[y]] = 1;
       mark[y] = 2;
```

```
} } } // the graph is stored as e[N] and g[N] // for (int i = 0; i < n; i++) match[i] = -1; // for (int i = 0; i < n; i++) if (match[i] == -1) aug(i); // int tot = 0; i < n; i++){ // for (int i = 0; i < n; i++){ // matched pairs = tot/2 // printf("%d\n", tot/2); // for (int i = 0; i < n; i++){ // printf("%d\n", match[i] + 1); // }
```

#### GeometrySnippets.cpp

f647cc, 86 lines

```
They are from KACTL, KTH's Team Reference Document
//check point in convex hull
typedef Point<11> P;
bool inHull(const vector<P>& 1, P p, bool strict = true) {
 int a = 1, b = sz(1) - 1, r = !strict;
 if (sz(1) < 3) return r && onSegment(1[0], 1.back(), p);</pre>
 if (sideOf(1[0], 1[a], 1[b]) > 0) swap(a, b);
 if (sideOf(1[0], 1[a], p) >= r || sideOf(1[0], 1[b], p) <=
    return false;
  while (abs(a - b) > 1) {
   int c = (a + b) / 2;
    (sideOf(1[0], 1[c], p) > 0 ? b : a) = c;
  return sqn(l[a].cross(l[b], p)) < r;</pre>
//center of mass of polygon
typedef Point < double > P;
P polygonCenter(const vector<P>& v) {
 P res(0, 0); double A = 0;
 for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
   res = res + (v[i] + v[j]) * v[j].cross(v[i]);
   A += v[j].cross(v[i]);
 return res / A / 3;
// Returns a vector with the vertices of a polygon with
    everything to the left
// of the line going from s to e cut away.
ypedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
 vector<P> res:
 FOR(i,0,sz(polv)) {
   P cur = poly[i], prev = i ? poly[i-1] : poly.back();
   bool side = s.cross(e, cur) < 0;</pre>
   if (side != (s.cross(e, prev) < 0))</pre>
      res.push back(lineInter(s, e, cur, prev).second);
      res.push_back(cur);
  return res;
```

```
//volumn of polyhedron, with face outwards
template<class V, class L>
double signed_poly_volume(const V& p, const L& trilist) {
 double v = 0;
 trav(i, trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
 return v / 6;
//intersection of two lines
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
 auto d = (e1 - s1).cross(e2 - s2);
 if (d == 0) // if parallel
   return \{-(s1.cross(e1, s2) == 0), P(0, 0)\};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return \{1, (s1 * p + e1 * q) / d\};
// Returns where p is as seen from s towards e. 1/0/-1
     $\Leftrightarrow$ left/on line/right.
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }
template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
 auto a = (e-s).cross(p-s);
 double l = (e-s).dist()*eps;
 return (a > 1) - (a < -1);
//f is longitude, t is latitude
double sphericalDistance(double f1, double t1,
 double f2, double t2, double radius) {
 double dx = \sin(t2) \cdot \cos(f2) - \sin(t1) \cdot \cos(f1);
 double dy = \sin(t2) \cdot \sin(f2) - \sin(t1) \cdot \sin(f1);
 double dz = cos(t2) - cos(t1);
 double d = sqrt(dx*dx + dy*dy + dz*dz);
 return radius*2*asin(d/2);
```

#### HalfPlaneIntersection.cpp

80d545, 86 lines

```
// Basic half-plane struct.
struct Halfplane {
    // 'p' is a passing point of the line and 'pq' is the
         direction vector of the line.
    Point p, pg;
    long double angle;
    Halfplane() {}
    Halfplane (const Point& a, const Point& b) : p(a), pq(b -
        angle = atan21(pq.y, pq.x);
    // Check if point 'r' is outside this half-plane.
    // Every half-plane allows the region to the LEFT of its
         line.
    bool out(const Point& r) {
        return cross(pg, r - p) < -eps;
    // Comparator for sorting.
   bool operator < (const Halfplane& e) const {
        return angle < e.angle;
```

```
// Intersection point of the lines of two half-planes.
         It is assumed they're never parallel.
    friend Point inter(const Halfplane& s, const Halfplane&
        long double alpha = cross((t.p - s.p), t.pq) / cross
             (s.pq, t.pq);
        return s.p + (s.pq * alpha);
};
// Actual algorithm
vector<Point> hp_intersect(vector<Halfplane>& H) {
    Point box[4] = { // Bounding box in CCW order
        Point(inf, inf),
       Point (-inf, inf),
        Point (-inf, -inf),
        Point(inf, -inf)
    for (int i = 0; i < 4; i++) { // Add bounding box half—
        Halfplane aux(box[i], box[(i+1) % 4]);
        H.push_back(aux);
    // Sort by angle and start algorithm
    sort(H.begin(), H.end());
    deque<Halfplane> dq;
    int len = 0:
    for(int i = 0; i < int(H.size()); i++) {</pre>
        // Remove from the back of the deque while last half
             -plane is redundant
        while (len > 1 && H[i].out(inter(dg[len-1], dg[len
             -2]))) {
            dg.pop back();
            --len:
        // Remove from the front of the deque while first
             half-plane is redundant
        while (len > 1 && H[i].out(inter(dg[0], dg[1]))) {
            dq.pop_front();
            --len;
        // Special case check: Parallel half-planes
        if (len > 0 && fabsl(cross(H[i].pq, dq[len-1].pq)) <</pre>
            // Opposite parallel half-planes that ended up
                 checked against each other.
            if (dot(H[i].pq, dq[len-1].pq) < 0.0)</pre>
                return vector<Point>();
            // Same direction half-plane: keep only the
                 leftmost\ half-plane.
            if (H[i].out(dq[len-1].p)) {
                dq.pop_back();
                --len:
            else continue;
        // Add new half-plane
        dq.push_back(H[i]);
        ++len;
    // Final cleanup: Check half-planes at the front against
          the back and vice-versa
    while (len > 2 \&\& dq[0].out(inter(dq[len-1], dq[len-2]))
        ) {
        dq.pop_back();
```

```
while (len > 2 && dq[len-1].out(inter(dq[0], dq[1]))) {
    dq.pop_front();
    --len;
// Report empty intersection if necessary
if (len < 3) return vector<Point>();
// Reconstruct the convex polygon from the remaining
     half-planes.
vector<Point> ret(len);
for (int i = 0; i+1 < len; i++) {
    ret[i] = inter(dq[i], dq[i+1]);
ret.back() = inter(dq[len-1], dq[0]);
return ret;
```

#### HopcroftKarp.cpp

Time:  $\mathcal{O}\left(\sqrt{V}E\right)$ 

cur.swap(next);

**Description:** Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and btoa should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. btoa[i] will be the match for vertex i on the right side, or -1 if it's not matched.

```
Usage: vi btoa(m, -1); hopcroftKarp(q, btoa);
```

```
bool dfs(int a, int L, vector<vi>& g, vi& btoa, vi& A, vi& B
    ) {
 if (A[a] != L) return 0;
 A[a] = -1;
  for (int b : q[a]) if (B[b] == L + 1) {
   B[b] = 0;
   if (btoa[b] == -1 \mid | dfs(btoa[b], L + 1, g, btoa, A, B))
      return btoa[b] = a, 1;
 return 0;
int hopcroftKarp(vector<vi>& q, vi& btoa) {
 int res = 0;
 vi A(g.size()), B(btoa.size()), cur, next;
  for (;;) {
   fill(all(A), 0);
   fill(all(B), 0);
   cur.clear();
    for (int a : btoa) if (a != -1) A[a] = -1;
   FOR(a, 0, sz(q)) if (A[a] == 0) cur.push_back(a);
    for (int lay = 1;; lay++) {
     bool islast = 0;
      next.clear();
      for (int a : cur) for (int b : g[a]) {
       if (btoa[b] == -1) {
          B[b] = lay;
          islast = 1;
        else if (btoa[b] != a && !B[b]) {
          B[b] = lav;
          next.push_back(btoa[b]);
      if (islast) break;
      if (next.empty()) return res;
      for (int a : next) A[a] = lay;
```

```
FOR(a, 0, sz(q))
  res += dfs(a, 0, g, btoa, A, B);
```

### HullDiameter.cpp

0c6e60, 12 lines

```
typedef Point<11> P;
array<P, 2> hullDiameter(vector<P> S) {
 int n = sz(S), j = n < 2 ? 0 : 1;
 pair<11, array<P, 2>> res({0, {S[0], S[0]}});
 FOR(i,0,j)
   for (;; j = (j + 1) % n) {
      res = \max(res, \{(S[i] - S[j]).dist2(), \{S[i], S[j]\}\});
      if ((S[(j+1) % n] - S[j]).cross(S[i+1] - S[i]) >=
        break;
  return res.second;
```

#### InsidePolygon.cpp

fd40d6, 11 lines

```
"Point.h", "OnSegment.h", "SegmentDistance.h"
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
 int cnt = 0, n = sz(p);
 FOR(i,0,n) {
   P q = p[(i + 1) % n];
   if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) \le eps) return ! strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) >
        0;
  return cnt;
```

#### InsidePolygonFast.cpp

96e771, 52 lines

```
bool lexComp(const pt &1, const pt &r) {
   return 1.x < r.x || (1.x == r.x && 1.v < r.v);
int sgn(long long val) { return val > 0 ? 1 : (val == 0 ? 0) }
   : -1); }
vector<pt> seq;
pt translation;
bool pointInTriangle(pt a, pt b, pt c, pt point) {
   long long s1 = abs(a.cross(b, c));
    long long s2 = abs(point.cross(a, b)) + abs(point.cross(
        b, c)) + abs(point.cross(c, a));
   return s1 == s2;
void prepare(vector<pt> &points) {
   n = points.size();
   int pos = 0;
   for (int i = 1; i < n; i++) {
        if (lexComp(points[i], points[pos]))
           pos = i;
   rotate(points.begin(), points.begin() + pos, points.end
         ());
    seq.resize(n);
```

# IntervalContainer KdTree KMP KnuthOptimization

```
for (int i = 0; i < n; i++)
       seq[i] = points[i + 1] - points[0];
    translation = points[0];
bool pointInConvexPolygon(pt point) {
    point = point - translation;
    if (seq[0].cross(point) != 1 &&
            sgn(seq[0].cross(point)) != sgn(seq[0].cross(seq
                 [n - 1])))
        return false;
    if (seg[n - 1].cross(point) != 0 &&
            sgn(seg[n-1].cross(point)) != sgn(seg[n-1].
                 cross(seq[0])))
        return false;
    if (seq[0].cross(point) == 0)
        return seq[0].sqrLen() >= point.sqrLen();
    int 1 = 0, r = n - 1;
    while (r - 1 > 1) {
       int mid = (1 + r) / 2;
        int pos = mid;
        if (seq[pos].cross(point) >= 0)
           1 = mid;
        else
            r = mid;
    int pos = 1:
    return pointInTriangle(seq[pos], seq[pos + 1], pt(0, 0),
         point);
```

#### IntervalContainer.cpp

**Description:** Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).

```
Time: \mathcal{O}(\log N)
                                                   edce47, 23 lines
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
 if (L == R) return is.end();
 auto it = is.lower bound({L, R}), before = it;
 while (it != is.end() && it->first <= R) {</pre>
   R = max(R, it->second);
   before = it = is.erase(it);
  if (it != is.begin() && (--it)->second >= L) {
   L = min(L, it->first);
   R = max(R, it->second);
   is.erase(it);
  return is.insert(before, {L,R});
void removeInterval(set<pii>& is, int L, int R) {
 if (L == R) return;
  auto it = addInterval(is, L, R);
 auto r2 = it->second;
 if (it->first == L) is.erase(it);
 else (int&)it->second = L;
 if (R != r2) is.emplace (R, r2);
```

```
KdTree.cpp
Description: KD-tree (2d, can be extended to 3d)
"Point.h"
                                                   bac5b0, 63 lines
typedef long long T;
typedef Point<T> P;
const T INF = numeric limits<T>::max();
bool on_x(const P& a, const P& b) { return a.x < b.x; }</pre>
bool on_y(const P& a, const P& b) { return a.y < b.y; }</pre>
struct Node {
 P pt; // if this is a leaf, the single point in it
 T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;
  T distance (const P& p) { // min squared distance to a
   T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
   T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  Node (vector<P>&& vp) : pt(vp[0]) {
   for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    if (vp.size() > 1) {
      // split on x if width >= height (not ideal...)
      sort(all(vp), x1 - x0 >= v1 - v0 ? on x : on y);
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle
      int half = sz(vp)/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
struct KDTree {
  Node* root:
  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}
  pair<T, P> search (Node *node, const P& p) {
   if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p = node \rightarrow pt) return \{INF, P()\};
      return make_pair((p - node->pt).dist2(), node->pt);
    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);
    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)</pre>
      best = min(best, search(s, p));
    return best;
  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
```

pair<T, P> nearest(const P& p) {

```
return search(root, p);
};
KMP.cpp
                                                    7ef32f, 13 lines
vector<int> prefix_function(string s) {
    int n = sz(s);
    vector<int> pi(n);
    for (int i = 1; i < n; i++) {
        int j = pi[i-1];
        while (i > 0 \&\& s[i] != s[i])
           j = pi[j-1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    return pi;
KnuthOptimization.cpp
                                                   a5b59d 38 lines
/*Class1 : Interval DP: f_{\{l,r\}} = min_{\{k=l\}}^{r-1} f_{\{l,k\}+f_{-1}\}}
     \{k+1,r\} + w(l,r)
weights w(l,r) satisfying the following inequality:
(1) For any l \le l' \le r' \le r, we have w(l', r') \le w(l, r).
(2) (The important one): For any l1 \le l2 \le r1 \le r2, we
w(l1,r1) + w(l2,r2) \le w(l1,r2) + w(l2,r1).
for (int len = 2; len <= n; ++len) // Enumerate Interval
     Length
  for (int l = 1, r = len; r <= n; ++1, ++r) {
    // Enumerate Intervals of Length Len
    f[1][r] = INF;
    for (int k = m[1][r - 1]; k \le m[1 + 1][r]; ++k)
     if (f[1][r] > f[1][k] + f[k + 1][r] + w(1, r)) {
        f[1][r] = f[1][k] + f[k + 1][r] + w(1, r); //Update
        m[1][r] = k; // Update Decision Point
/*Class2: 2D DP, f_{i,j} = min_{k} = j_{f_{i,j}} + w(k,j)
Where 1 \le i \le n, 1 \le j \le m
*/
int n;
long long C(int i, int j);
vector<long long> dp_before(n), dp_cur(n);
// compute dp\_cur[l], ... dp\_cur[r] (inclusive)
//Call compute for each possible i.
void compute(int 1, int r, int opt1, int optr) {
 if (1 > r) return;
 int mid = (1 + r) >> 1;
 pair<long long, int> best = {INF, -1};
  for (int k = optl; k <= min(mid, optr); k++) {</pre>
   best = min(best, {dp before[k] + C(k, mid), k});
 dp_cur[mid] = best.first;
  int opt = best.second;
```

compute(1, mid - 1, opt1, opt);

compute(mid + 1, r, opt, optr);

LazySegtree.cpp

```
8ed0ff, 45 lines
const 11 identity = 0;
const 11 SZ = 131072;
11 sum[2*SZ], lazv[2*SZ];
ll combine(ll A, ll B) {
    return A+B;
ll combineUpd(ll A, ll B) {
    return A+B;
void push(int index, ll L, ll R) {
    sum[index] = combineUpd(sum[index], lazy[index]);
    if (L != R) lazy[2*index] = combineUpd(lazy[2*index],
         lazy[index]), lazy[2*index+1] = combineUpd(lazy[2*
         index+1], lazy[index]);
    lazy[index] = identity;
}
void pull(int index) {
    sum[index] = combine(sum[2*index], sum[2*index+1]);
ll query(int lo, int hi, int index = 1, ll L = 0, ll R = SZ
    -1) {
    push (index, L, R);
    if (lo > R || L > hi) return identity;
    if (lo <= L && R <= hi) return sum[index];</pre>
    int M = (L+R) / 2:
    return combine (query (lo, hi, 2*index, L, M), query (lo,
         hi, 2*index+1, M+1, R));
void update(int lo, int hi, ll increase, int index = 1, ll L
     = 0, 11 R = SZ-1) {
    push(index, L, R);
    if (hi < L || R < lo) return;
    if (lo <= L && R <= hi) {
        lazy[index] = increase;
        push (index, L, R);
        return:
    int M = (L+R) / 2;
    update(lo, hi, increase, 2*index, L, M); update(lo, hi,
         increase, 2*index+1, M+1, R);
    pull(index);
```

# LCA.cpp

e1efce, 52 lines const int L; //SET THIS TO CEIL(LOG(MX\_N)) int N: int anc[MX][L]; int dep[MX]; vector<vi> graph(MX); int jmp(int x, int d) { FOR(i, L) { if (d&(1<<i)) { x = anc[x][i];

```
return x;
int lca(int a, int b) {
    if (dep[a] < dep[b]) {</pre>
        swap(a, b);
    a = jmp(a, dep[a] - dep[b]);
    if (a == b) return a;
    FORd(i, L) {
        if (anc[a][i] != anc[b][i]) {
            a = anc[a][i];
            b = anc[b][i];
    return anc[a][0];
void dfs(int v, int p) {
    anc[v][0] = p;
    trav(a, graph[v]) {
        if (a == p) continue;
        dep[a] = dep[v] + 1;
        dfs(a, v);
void prep() {
    FOR(i, N) FOR(j, L) anc[i][j] = -1;
    dep[0] = 0;
    dfs(0, -1);
    FOR(j, 1, L) {
        FOR(i, N) {
            if (anc[i][j-1] != -1) {
                anc[i][j] = anc[anc[i][j-1]][j-1];
```

# LCT.cpp

1809e5, 116 lines

```
struct rec
   int ls, rs, p; //ls = left son; rs = right son; p =
        parent
   uint siz; //siz = size of the subtree
   uint key, sum; //sum: sum of weights in the subtree
   uint mult, add; //two lazy tags
   bool rev; //denote whether this segment has been
        reverted
rec splay[maxn];
void clear(){
   splay[0].p = splay[0].ls = splay[0].rs = splay[0].rev =
        splay[0].key = splay[0].sum = 0;
   splay[0].siz = 0;
void update(int x){
   clear();
   splay[x].sum = splay[splay[x].ls].sum + splay[splay[x].
        rs].sum + splay[x].key;
```

```
splay[x].sum %= modi;
    splay[x].siz = splay[splay[x].ls].siz + splay[splay[x].
        rsl.siz + 1;
    splay[x].siz %= modi;
void zig(int x){
    int y = splay[x].p, z = splay[y].p;
    if (y == splay[z].ls) splay[z].ls = x;
    else if (y == splay[z].rs) splay[z].rs = x;
    splay[x].p = z;
    // Switch Is and rs for zag.
    if (splay[x].rs) splay[splay[x].rs].p = y;
    splay[y].ls = splay[x].rs;
    splay[x].rs = y;
    splay[y].p = x;
    update(y);
bool is_root(int x){
    return x != splay[splay[x].p].ls && x != splay[splay[x].
        pl.rs;
void rev(int x) {
    if (!x)return;
    swap(splay[x].ls, splay[x].rs);
    splav[x].rev ^= true;
void pushdown(int x) {
    if (splay[x].rev) {
        rev(splay[x].ls);
        rev(splay[x].rs);
        splav[x].rev = false;
    //Todo: Push lazy tags here.
void set_root(int x) {
    static int q[maxn];
    static int top;
    int i;
    for (i = x; !is_root(i); i = splay[i].p) {
        q[++top] = i;
    q[++top] = i;
    while (top) {
        pushdown(q[top--]);
    while (!is root(x)){
        int y = splay[x].p;
        if (is_root(v)){
            if (x == splay[y].ls) zig(x); else zag(x);
        else{
            int z = splav[v].p;
            if (y == splay[z].ls){
                if (x == splay[y].ls) zig(y), zig(x);
                else zaq(x), ziq(x);
            else{
                if (x == splay[y].rs) zag(y), zag(x);
                else zig(x), zag(x);
    update(x);
//this is a special operation on LCT
```

```
void access(int x)
    for (int t = 0; x; t = x, x = splay[x].p) {
        set_root(x);
        splay[x].rs = t;
        update(x);
//we will make x be the new root of the tree it belongs to
void makeroot(int x) {access(x); set_root(x); rev(x);}
void split(int x, int y) {makeroot(x);access(y);set_root(y);}
//link vertex x and vertex y
void link(int x, int y) {makeroot(x); makeroot(y); splay[x].p =
//cut the edge between x and y
void cut(int x, int v){
    split(x, y);
    splay[y].ls = splay[x].p = 0;
    update(y);
//find the root; x connected with y IFF findroot(x) =
    findroot(y)
int findroot(int x){
    access(x);
    set_root(x);
    while (splay[x].ls) {
        pushdown(x);
        x = splay[x].ls;
    set root(x);
    return x:
//Adding\ edge\ between\ u\ and\ v:\ link(u,\ v);
//Removing edge between u and v: cut(u1, v1);
//Adding vertices on route between u and v by c :
/* split(u, v);
   calc(v, 1, c):*/
//Query the sum on route from u to v: split(u1,v1) print(
    splay[v1].sum);
```

### LeftistTree.cpp

7d92ca, 21 lines

```
struct node{
    node *1, *r;
    //key is the priority
    int key, id;
    //distanct to the leftist child - it is used to maintain
          the properties of the lefitst tree
    int dist:
    int rdist(){return (r==NULL)?0:r->dist;}
    int ldist() {return (l==NULL)?0:1->dist;}
node* merge(node*1,node*r)
    if (1 == NULL) return r;
    if (r == NULL) return 1;
    //we want to make sure the root has the smallest key
    if (1->key > r->key) swap(1,r);
    1->r = merge(1->r,r);
    //maintain the properties of the leftist tree
    if (1->ldist() < 1->rdist()) swap(1->1,1->r);
    1->dist = 1->rdist()+1;
    return 1;
```

```
LinearTransformation.cpp
Description:
```

```
Apply the linear transformation (translation, rotation and
scalin which takes line p0-p1 to line q0-q1 to point r.
```

typedef Point < double > P; P linearTransformation(const P& p0, const P& p1, const P& q0, const P& q1, const P& r) { P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq)); return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();

#### LineDistance.cpp

"Point.h"

```
"Point.h"
                                                   f6bf6b, 7 lines
Returns the signed distance between point p and the line
    containing points a and b. Positive value on left side
     and negative on right as seen from a towards b. a=b
     gives nan. P is supposed to be Point<T> or Point3D<T>
     where T is e.g. double or long long. It uses products in
     intermediate steps so watch out for overflow if using
     int or long long. Using Point3D will always give a non-
     negative distance. For Point3D, call .dist on the result
     of the cross product.
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
 return (double) (b-a).cross(p-a)/(b-a).dist();
```

#### LineHullIntersection.cpp Time: $\mathcal{O}(N + Q \log n)$

```
"Point.h"
                                                   f78f76, 39 lines
typedef array<P, 2> Line;
#define cmp(i, j) sqn(dir.perp().cross(poly[(i)%n]-poly[(j)%n
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
int extrVertex(vector<P>& poly, P dir) {
 int n = sz(poly), lo = 0, hi = n;
 if (extr(0)) return 0;
  while (lo + 1 < hi) {
   int m = (lo + hi) / 2;
   if (extr(m)) return m;
    int 1s = cmp(1o + 1, 1o), ms = cmp(m + 1, m);
    (ls < ms \mid | (ls == ms \&\& ls == cmp(lo, m)) ? hi : lo) =
 return lo;
#define cmpL(i) sqn(line[0].cross(poly[i], line[1]))
array<int, 2> lineHull(Line line, vector<P> polv) {
  int endA = extrVertex(poly, (line[0] - line[1]).perp());
  int endB = extrVertex(poly, (line[1] - line[0]).perp());
 if (cmpL(endA) < 0 || cmpL(endB) > 0)
   return {-1, -1};
  arrav<int, 2> res:
  FOR(i,0,2) {
   int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
```

```
int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
    (cmpL(m) == cmpL(endB) ? lo : hi) = m;
 res[i] = (lo + !cmpL(hi)) % n;
 swap (endA, endB);
if (res[0] == res[1]) return {res[0], -1};
if (!cmpL(res[0]) && !cmpL(res[1]))
 switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
   case 0: return {res[0], res[0]};
   case 2: return {res[1], res[1]};
return res;
```

#### LineContainer.cpp

03a306, 6 lines

8ec1c7, 34 lines

```
/* Author: KACTL Line Container
* Description: Container where you can add lines of the
     form kx+m, and query maximum values at points x.
 * Useful for dynamic programming (''convex hull trick'').
* Time: O(\setminus log\ N) */
struct Line {
 mutable 11 k, m, p;
 bool operator<(const Line& o) const { return k < o.k; }
 bool operator<(ll x) const { return p < x; }</pre>
struct LineContainer : multiset<Line, less<>>> {
 // (for doubles, use inf = 1/.0, div(a,b) = a/b)
 static const 11 inf = LLONG MAX;
 11 div(11 a, 11 b) { // floored division
   return a / b - ((a ^ b) < 0 && a % b); }
 bool isect(iterator x, iterator y) {
   if (y == end()) return x \rightarrow p = inf, 0;
   if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
   else x->p = div(y->m - x->m, x->k - y->k);
   return x->p >= y->p;
  void add(ll k, ll m) {
   auto z = insert(\{k, m, 0\}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
   if (x != begin() \&\& isect(--x, y)) isect(x, y = erase(y))
    while ((y = x) != begin() \&\& (--x)->p >= y->p)
     isect(x, erase(y));
 11 query(11 x) {
   assert(!empty());
   auto 1 = *lower_bound(x);
   return 1.k * x + 1.m;
};
```

#### Manacher.cpp

2fddb0, 28 lines

```
vector<int> manacher odd(string s) {
 int n = s.size();
 s = "$" + s + "^";
 vector < int > p(n + 2);
 int 1 = 1, r = 1;
 for(int i = 1; i <= n; i++) {
   p[i] = max(0, min(r - i, p[1 + (r - i)]));
    while (s[i - p[i]] == s[i + p[i]]) {
     p[i]++;
```

#### MinCostMaxFlow MinCostMaxFlowPR

```
if(i + p[i] > r) {
     l = i - p[i], r = i + p[i];
 return vector<int>(begin(p) + 1, end(p) - 1);
vector<int> manacher(string s) {
 string t;
 for(auto c: s) {
   t += string("#") + c;
 auto res = manacher_odd(t + "#");
 for (auto& x: res) x--;
 return vector<int>(begin(res) + 1, end(res) - 1);
// returns array P of length 2N-1, p[i] = length of longest
    odd/even palindrome
// abcbcba: 1 0 1 0 3 0 7 0 3 0 1 0 1
```

#### MinCostMaxFlow.cpp

**Description:** Min-cost max-flow. cap[i][j] != cap[j][i] is allowed; double edges are not. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.

Time: Approximately  $\mathcal{O}(E^2)$ 

```
<ext/pb_ds/priority_queue.hpp>
                                                   45af93, 80 lines
const 11 INF = numeric_limits<11>::max() / 4;
typedef vector<11> VL;
using pii = pair<int, int>;
struct MCMF {
 int N;
 vector<vi> ed, red;
 vector<VL> cap, flow, cost;
 vi seen;
 VL dist, pi;
 vector<pii> par;
 MCMF(int N):
   N(N), ed(N), red(N), cap(N, VL(N)), flow(cap), cost(cap)
    seen(N), dist(N), pi(N), par(N) {}
 void addEdge(int from, int to, ll cap, ll cost) {
   this->cap[from][to] = cap;
    this->cost[from][to] = cost;
   ed[from].push_back(to);
   red[to].push back(from);
 void path(int s) {
   fill(all(seen), 0);
    fill(all(dist), INF);
    dist[s] = 0; ll di;
    __gnu_pbds::priority_queue<pair<11, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({0, s});
    auto relax = [&](int i, ll cap, ll cost, int dir) {
     11 val = di - pi[i] + cost;
     if (cap && val < dist[i]) {</pre>
       dist[i] = val;
```

```
par[i] = {s, dir};
        if (its[i] == q.end()) its[i] = q.push({-dist[i], i}
        else q.modify(its[i], {-dist[i], i});
   };
   while (!q.empty()) {
      s = q.top().second; q.pop();
     seen[s] = 1; di = dist[s] + pi[s];
      for (int i : ed[s]) if (!seen[i])
        relax(i, cap[s][i] - flow[s][i], cost[s][i], 1);
      for (int i : red[s]) if (!seen[i])
        relax(i, flow[i][s], -cost[i][s], 0);
   FOR(i, 0, N) pi[i] = min(pi[i] + dist[i], INF);
  pair<11, 11> maxflow(int s, int t) {
   11 \text{ totflow} = 0, \text{ totcost} = 0;
   while (path(s), seen[t]) {
     11 fl = INF;
      for (int p,r,x = t; tie(p,r) = par[x], x != s; x = p)
        fl = min(fl, r ? cap[p][x] - flow[p][x] : flow[x][p]
      totflow += fl;
      for (int p,r,x = t; tie(p,r) = par[x], x != s; x = p)
        if (r) flow[p][x] += fl;
        else flow[x][p] -= fl;
   FOR(i, 0, N) FOR(j, 0, N) totcost += cost[i][j] * flow[i][j]
   return {totflow, totcost};
 // If some costs can be negative, call this before maxflow
 void setpi(int s) { // (otherwise, leave this out)
   fill(all(pi), INF); pi[s] = 0;
   int it = N, ch = 1; 11 v;
   while (ch-- && it--)
      FOR(i, 0, N) if (pi[i] != INF)
        for (int to : ed[i]) if (cap[i][to])
          if ((v = pi[i] + cost[i][to]) < pi[to])</pre>
            pi[to] = v, ch = 1;
   assert(it >= 0); // negative cost cycle
};
MinCostMaxFlowPR.cpp
                                                   9f8191, 160 lines
// Push-Relabel implementation of the cost-scaling algorithm
// Runs in O(\langle max\_flow \rangle * log(V * max\_edge\_cost)) = <math>O(V^3)
     * log(V * C))
// Operates on integers
template<typename flow t = int, typename cost t = int>
```

};

```
struct mcSFlow{
    struct Edge{
        cost t c;
        flow t f;
        int to, rev;
```

f(\_f), to(\_to), rev(\_rev){}

Edge(int \_to, cost\_t \_c, flow\_t \_f, int \_rev):c(\_c),

```
const cost t INFCOST = numeric limits<cost t>::max()/2;
const cost t INFFLOW = numeric limits<flow t>::max()/2;
cost_t epsilon;
int N, S, T;
vector<vector<Edge> > G;
vector<unsigned int> isEngueued, state;
mcSFlow(int _N, int _S, int _T):epsilon(0), N(_N), S(_S)
     , T(_T), G(_N){}
void add_edge(int a, int b, cost_t cost, flow_t cap){
    if (a==b) {assert (cost>=0); return;}
    cost *=N; // to preserve integer-values
    epsilon = max(epsilon, abs(cost));
    assert (a>=0&&a<N&&b>=0&&b<N);
    G[a].emplace_back(b, cost, cap, G[b].size());
    G[b].emplace_back(a, -cost, 0, G[a].size()-1);
\verb|flow_t calc_max_flow()| {\it // Dinic max-flow}|
    vector<flow_t> dist(N), state(N);
    vector<Edge*> path(N);
    auto cmp = [](Edge*a, Edge*b){return a->f < b->f;};
    flow_t addFlow, retflow=0;;
        fill(dist.begin(), dist.end(), -1);
        dist[S]=0;
        auto head = state.begin(), tail = state.begin();
        for(*tail++ = S; head!=tail; ++head) {
            for (Edge const&e:G[*head]) {
                if (e.f && dist[e.to] == -1) {
                    dist[e.to] = dist[*head]+1;
                    *tail++=e.to;
        addFlow = 0;
        fill(state.begin(), state.end(), 0);
        auto top = path.begin();
        Edge dummy(S, 0, INFFLOW, -1);
        *top++ = &dummy;
        while(top != path.begin()){
            int n = (*prev(top))->to;
                 auto next_top = min_element(path.begin()
                     , top, cmp);
                flow_t flow = (*next_top)->f;
                while (--top!=path.begin()) {
                    Edge &e=**top, &f=G[e.to][e.rev];
                    e.f-=flow;
                    f.f+=flow;
                addFlow=1:
                retflow+=flow;
                top = next top;
                continue:
            for(int &i=state[n], i_max = G[n].size(),
                 need = dist[n]+1;;++i){
                if (i==i_max) {
                    dist[n]=-1;
                    --top;
                    break;
                if(dist[G[n][i].to] == need && G[n][i].f
                     *top++ = &G[n][i];
                    break;
```

```
}while(addFlow);
    return retflow;
vector<flow_t> excess;
vector<cost_t> h;
void push(Edge &e, flow_t amt) {
   if(e.f < amt) amt=e.f;</pre>
   e.f-=amt;
    excess[e.to]+=amt;
    G[e.to][e.rev].f+=amt;
    excess[G[e.to][e.rev].to]-=amt;
void relabel(int vertex) {
    cost t newHeight = -INFCOST;
    for (unsigned int i=0; i < G[vertex].size(); ++i) {</pre>
        Edge const&e = G[vertex][i];
        if(e.f && newHeight < h[e.to]-e.c){
            newHeight = h[e.to] - e.c;
            state[vertex] = i;
    h[vertex] = newHeight - epsilon;
const int scale=2;
pair<flow_t, cost_t> minCostFlow() {
    cost_t retCost = 0;
    for (int i=0; i<N; ++i) {</pre>
        for (Edge &e:G[i]) {
            retCost += e.c*(e.f);
    //find feasible flow
    flow t retFlow = calc max flow();
    excess.resize(N); h.resize(N);
    queue<int> q;
    isEngueued.assign(N, 0); state.assign(N,0);
    for(;epsilon;epsilon>>=scale){
        fill(state.begin(), state.end(), 0);
        for(int i=0;i<N;++i)</pre>
            for (auto &e:G[i])
                if(h[i] + e.c - h[e.to] < 0 && e.f) push
                      (e. e.f):
        for (int i=0; i<N; ++i) {</pre>
            if(excess[i]>0){
                q.push(i);
                isEnqueued[i]=1;
        while(!q.empty()){
            int cur=q.front();q.pop();
            isEnqueued[cur]=0;
            // discharge
            while (excess[cur]>0) {
                if(state[cur] == G[cur].size()){
                     relabel(cur);
                for(unsigned int &i=state[cur], max_i =
                     G[cur].size();i<max_i;++i){
                     Edge &e=G[cur][i];
                    if(h[cur] + e.c - h[e.to] < 0){
                         push(e, excess[cur]);
```

```
if(excess[e.to]>0 && isEngueued[
                                   e.tol == 0) {
                                  q.push(e.to);
                                  isEnqueued[e.to]=1;
                              if(excess[cur]==0) break;
            if(epsilon>1 && epsilon>>scale==0){
                 epsilon = 1<<scale;
        for (int i=0; i < N; ++i) {</pre>
            for(Edge &e:G[i]) {
                 retCost -= e.c*(e.f);
        return make_pair(retFlow, retCost/2/N);
    flow_t getFlow(Edge const &e) {
        return G[e.to][e.rev].f;
};
```

```
Minimum Enclosing Circle.cpp
                                                   6d8e96, 44 lines
point a[maxn];
void getCenter2(point a,point b,point & c)
    c.x = (a.x+b.x)/2;
    c.y = (a.y+b.y)/2;
void getCenter3(point a,point b,point c,point &d)
    double a1 = b.x-a.x, b1 = b.y-a.y, c1 = (a1*a1+b1*b1)/2;
    double a2 = c.x-a.x, b2 = c.y-a.y, c2 = (a2*a2+b2*b2)/2;
    double de = (a1 * b2 - b1 * a2);
    d.x = a.x + (c1 * b2 - c2 * b1)/de;
    d.y = a.y + (a1 * c2 - a2 * c1)/de;
    //randomP_shuffle before using
    radius = 0;
    center = a[0];
    for (int i = 1; i < n; ++i)
        if (!isIn(a[i], center, radius))
            radius = 0;
            center = a[i];
            for (int j = 0; j < i; ++j)
                if (!isIn(a[j], center, radius))
                    getCenter2(a[i], a[j], center);
                    radius = dis(a[i],center);
                    for (int k = 0; k < j; ++k) if (!isIn(a[k
                         l, center, radius))
                        getCenter3(a[i], a[j], a[k], center)
                        radius = dis(a[k],center);
```

```
//printf("\%.2lf\n\%.2lf\n", radius, center.x, center.y)
printf("%.31f\n", radius);
return 0:
```

#### MinimumVertexCover.cpp

**Description:** Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

```
"DFSMatching.h"
                                                  8c9aed, 20 lines
vi cover(vector<vi>& q, int n, int m) {
 vi match(m, -1);
 int res = dfsMatching(g, match);
 vector<bool> lfound(n, true), seen(m);
  for (int it : match) if (it != -1) lfound[it] = false;
  vi q, cover;
 FOR(i,0,n) if (lfound[i]) q.push_back(i);
  while (!q.empty()) {
   int i = q.back(); q.pop_back();
   lfound[i] = 1;
    for (int e : q[i]) if (!seen[e] && match[e] != -1) {
     seen[e] = true;
     q.push_back(match[e]);
  FOR(i,0,n) if (!lfound[i]) cover.push_back(i);
 FOR(i,0,m) if (seen[i]) cover.push_back(n+i);
  assert(sz(cover) == res);
  return cover:
```

#### MinkowskiSum.cpp

```
18ebb1, 31 lines
void reorder_polygon(vector<pt> & P) {
    size_t pos = 0;
    for(size_t i = 1; i < P.size(); i++){</pre>
        if(P[i].y < P[pos].y \mid | (P[i].y == P[pos].y && P[i].
            x < P[pos].x)
    rotate(P.begin(), P.begin() + pos, P.end());
vector<pt> minkowski (vector<pt> P, vector<pt> Q) {
    // the first vertex must be the lowest
    reorder_polygon(P);
    reorder_polygon(Q);
    // we must ensure cyclic indexing
   P.push_back(P[0]);
   P.push back(P[1]);
    Q.push_back(Q[0]);
    0.push back(0[1]);
    // main part
    vector<pt> result;
    size t i = 0, j = 0;
    while(i < P.size() - 2 || j < Q.size() - 2) {
        result.push_back(P[i] + Q[j]);
        auto cross = (P[i + 1] - P[i]).cross(Q[j + 1] - Q[j
            1);
        if(cross >= 0)
```

#### Mo NTT orderedset PlanarPointLocation

```
++i;
        if(cross <= 0)
            ++j;
    return result;
Mo.cpp
                                                   aab82d, 45 lines
void remove(idx); // TODO: remove value at idx from data
    structure
void add(idx);
                   // TODO: add value at idx from data
    structure
int get_answer(); // TODO: extract the current answer of
    the data structure
int block_size;
struct Ouerv {
    int l. r. idx:
    bool operator<(Query other) const
        return make_pair(l / block_size, r) <</pre>
               make pair(other.1 / block size, other.r);
};
vector<int> mo_s_algorithm(vector<Query> queries) {
    vector<int> answers(queries.size());
    sort(queries.begin(), queries.end());
    // TODO: initialize data structure
    int cur_1 = 0;
    int cur_r = -1;
    // invariant: data structure will always reflect the
         range [cur_l, cur_r]
    for (Query q : queries) {
        while (cur_1 > q.1) {
            cur_1--;
            add(cur_1);
        while (cur_r < q.r) {</pre>
            cur_r++;
            add(cur_r);
        while (cur_1 < q.1) {
            remove(cur_l);
            cur_1++;
        while (cur_r > q.r) {
            remove(cur_r);
            cur_r--;
        answers[q.idx] = get_answer();
    return answers;
NTT.cpp
                                                   f921a6, 38 lines
const 11 MOD = (119 << 23) + 1, root = 62; // = 998244353
```

```
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 <<
// and 483 \ll 21 (same root). The last two are > 10^9.
```

```
11 modExp(ll a, ll b) {
 ll res = 1;
 for (; b; a = (a * a) % MOD, b >>= 1)
   if (b & 1) res = (res * a) % MOD;
 return res;
void ntt(vl &a) {
  int n = sz(a), L = 31 - \underline{builtin_clz(n)};
  static vl rt(2, 1);
  for (static int k = 2, s = 2; k < n; k *= 2, s++) {
    rt.resize(n);
    ll z[] = \{1, modExp(root, MOD >> s)\};
    FOR(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % MOD;
  vi rev(n);
  FOR(i, 0, n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  FOR(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) FOR(j,k) {
      ll z = rt[j + k] * a[i + j + k] % MOD, &ai = a[i + j];
      a[i + j + k] = ai - z + (z > ai ? MOD: 0);
      ai += (ai + z >= MOD ? z - MOD : z);
vl conv(const vl &a, const vl &b) {
  if (a.empty() || b.empty()) return {};
  int s = sz(a) + sz(b) - 1, B = 32 - _builtin_clz(s), n = _builtin_clz(s)
  int inv = modExp(n, MOD - 2);
  vl L(a), R(b), out(n);
  L.resize(n), R.resize(n);
  ntt(L), ntt(R);
  FOR(i,n) \text{ out}[-i \& (n-1)] = (11)L[i] * R[i] % MOD * inv %
        MOD;
  return {out.begin(), out.begin() + s};
orderedset.cpp
```

```
647225, 6 lines
<ext/pb_ds/assoc_container.hpp>
using namespace gnu pbds;
template<typename T>
using ordered_set = tree<T, null_type, less<T>, rb_tree_tag,
      tree_order_statistics_node_update>;
/* \ order\_of\_key(k), how many elements < k */
/* find_by_order(k), k'th element (from 0) */
```

#### PlanarPointLocation.cpp

2512ba, 199 lines

```
* CP algorithm point point_location
 * This implementation assumes that the subdivision is
      correctly stored inside a DCEL
  and the outer face is numbered -1.
  For each query a pair (1,i) is returned if point is
       strictly inside face number i.
 (0,i) returned if point lies on the edge number i.
bool edge_cmp(Edge* edge1, Edge* edge2)
    const pt a = edge1->1, b = edge1->r;
    const pt c = edge2 -> 1, d = edge2 -> r;
    int val = sgn(a.cross(b, c)) + sgn(a.cross(b, d));
```

```
if (val != 0)
        return val > 0;
   val = sgn(c.cross(d, a)) + sgn(c.cross(d, b));
   return val < 0;
enum EventType { DEL = 2, ADD = 3, GET = 1, VERT = 0 };
struct Event {
   EventType type;
   int pos;
   bool operator<(const Event& event) const { return type <
         event.type; }
};
vector<Edge*> sweepline(vector<Edge*> planar, vector<pt>
    queries)
   using pt_type = decltype(pt::x);
    // collect all x-coordinates
   auto s =
        set<pt_type, std::function<bool(const pt_type&,
            const pt_type&) >> (lt);
    for (pt p : queries)
        s.insert(p.x);
    for (Edge* e : planar) {
        s.insert(e->1.x);
        s.insert(e->r.x);
    // map all x-coordinates to ids
    int cid = 0;
   auto id =
        map<pt_type, int, std::function<bool(const pt_type&,</pre>
             const pt type&)>>(
    for (auto x : s)
        id[x] = cid++;
    // create events
    auto t = set<Edge*, decltype(*edge_cmp)>(edge_cmp);
    auto vert cmp = [](const pair<pt type, int>& 1,
                       const pair<pt_type, int>& r) {
        if (!eq(l.first, r.first))
            return lt(l.first, r.first);
        return 1.second < r.second;
   };
    auto vert = set<pair<pt_type, int>, decltype(vert_cmp)>(
        vert cmp);
    vector<vector<Event>> events(cid);
    for (int i = 0; i < (int) queries.size(); i++) {
        int x = id[queries[i].x];
        events[x].push_back(Event{GET, i});
    for (int i = 0; i < (int)planar.size(); <math>i++) {
        int lx = id[planar[i]->1.x], rx = id[planar[i]->r.x
            ];
        if (lx > rx) {
            swap(lx, rx);
            swap(planar[i]->1, planar[i]->r);
       if (1x == rx) {
            events[lx].push_back(Event{VERT, i});
        } else {
```

```
events[lx].push back(Event{ADD, i});
        events[rx].push_back(Event{DEL, i});
// perform sweep line algorithm
vector<Edge*> ans(queries.size(), nullptr);
for (int x = 0; x < cid; x++) {
   sort(events[x].begin(), events[x].end());
   vert.clear();
    for (Event event : events[x]) {
        if (event.type == DEL) {
            t.erase(planar[event.pos]);
        if (event.type == VERT) {
           vert.insert(make_pair(
                min(planar[event.pos]->1.y, planar[event
                     .pos]->r.y),
                event.pos));
        if (event.type == ADD) {
           t.insert(planar[event.pos]);
        if (event.type == GET) {
            auto jt = vert.upper_bound(
                make_pair(queries[event.pos].y, planar.
                     size()));
            if (jt != vert.begin()) {
                --jt;
                int i = jt->second;
                if (ge(max(planar[i]->1.v, planar[i]->r.
                       queries[event.pos].v)) {
                    ans[event.pos] = planar[i];
                    continue:
            Edge* e = new Edge;
            e->1 = e->r = queries[event.pos];
            auto it = t.upper_bound(e);
            if (it != t.begin())
                ans[event.pos] = \star (--it);
            delete e:
    for (Event event : events[x]) {
        if (event.type != GET)
            continue;
        if (ans[event.pos] != nullptr &&
            eq(ans[event.pos]->1.x, ans[event.pos]->r.x)
            continue:
        Edge* e = new Edge;
        e->1 = e->r = queries[event.pos];
        auto it = t.upper bound(e);
       delete e;
        if (it == t.begin())
            e = nullptr;
            e = *(--it);
        if (ans[event.pos] == nullptr) {
            ans[event.pos] = e;
            continue;
```

```
if (e == nullptr)
                continue;
            if (e == ans[event.pos])
                continue:
            if (id[ans[event.pos]->r.x] == x) {
                if (id[e->1.x] == x) {
                    if (gt(e->1.y, ans[event.pos]->r.y))
                        ans[event.pos] = e;
            } else {
                ans[event.pos] = e;
    return ans;
struct DCEL {
    struct Edge {
        pt origin;
        Edge* nxt = nullptr;
        Edge* twin = nullptr;
        int face;
    vector<Edge*> body;
vector<pair<int, int>> point_location(DCEL planar, vector<pt
    > gueries)
    vector<pair<int, int>> ans(queries.size());
    vector<Edge*> planar2;
    map<intptr_t, int> pos;
    map<intptr_t, int> added_on;
    int n = planar.body.size();
    for (int i = 0; i < n; i++) {
        if (planar.body[i]->face > planar.body[i]->twin->
             face)
            continue;
        Edge* e = new Edge;
        e->1 = planar.body[i]->origin;
        e->r = planar.bodv[i]->twin->origin;
        added on [(intptr t)e] = i;
        pos[(intptr_t)e] =
            lt(planar.body[i]->origin.x, planar.body[i]->
                 twin->origin.x)
                ? planar.bodv[i]->face
                : planar.body[i]->twin->face;
        planar2.push back(e);
    auto res = sweepline(planar2, queries);
    for (int i = 0; i < (int) queries.size(); i++) {</pre>
        if (res[i] == nullptr) {
            ans[i] = make_pair(1, -1);
            continue:
        pt p = queries[i];
        pt 1 = res[i] -> 1, r = res[i] -> r;
        if (eq(p.cross(1, r), 0) && le(p.dot(1, r), 0)) {
            ans[i] = make_pair(0, added_on[(intptr_t)res[i
                 ]]);
            continue;
        ans[i] = make_pair(1, pos[(intptr_t)res[i]]);
```

```
for (auto e : planar2)
   delete e;
return ans;
```

```
PersistentSegtree.cpp
                                                  ed1804, 86 lines
//Define the node of a persistent segment tree
struct node{
   int l.r.sum;
//the persistent segment tree. Warning: Check memory limit
    before using persistent segment tree!
node tree[maxn*32];
//Storing the root of versions of segment tree
int head[maxn];
//allocate next position. You can implement in a way that
    support garbage collection.
int nextPos(){
   static int ct:return ++ct:
//Building the first version of our segmetn tree
void build(int cur, int 1, int r) {
 tree[cur].sum = 0;
 tree[cur].1 = nextPos();
 tree[cur].r = nextPos();
 if (1 == r) {
   tree[cur].1 = tree[cur].r = 0;
 else{
   int mid = (1+r)>>1;
   build(tree[cur].1,1,mid);
   build(tree[cur].r,mid+1,r);
//This function is: currently we are at node cur, which is a
     node in the latest version of segment tree
//we want to make modifications based on some past segment
    tree, and the corresponding node in the last version is
    at last
//we want to add 1 at position key
void modify(int cur, int last, int key, int l, int r){
 //this is creating the node for our latest version
 tree[cur].sum = tree[last].sum;
 tree[cur].l = nextPos();
 tree[cur].r = nextPos();
 if (1 == r) {
    //base case:add on current version of our segment tree
   tree[cur].sum++;
   tree[cur].1 = tree[cur].r = 0;
 else{
   int mid = (1+r) >> 1;
    if (kev <= mid) {
      //we are going to modify in the left part, so we can
          reuse the right child
      tree[cur].r = tree[last].r;
      modify(tree[cur].1, tree[last].1, key,1, mid);
      //update information for the current version of
          segment tree
      tree[cur].sum++;
```

```
else
      tree[cur].l = tree[last].l;
     modify(tree[cur].r, tree[last].r, key,mid+1, r);
      tree[cur].sum++;
int query(int cur,int last,int l,int r,int k){
 if (1 == r) return 1;
 int mid = (1+r) \gg 1;
  //notice the subtraction here - we want too see the
       dfiffernce between today's version and old versions.
  int ct = tree[tree[cur].1].sum - tree[tree[last].1].sum ;
  //if there are to many larger than mid, the k-th element
       should be in the left
  if (ct >= k) {
   return query(tree[cur].1, tree[last].1, 1, mid, k);
  //otherwise, the k-th element should be in the right
    return query(tree[cur].r, tree[last].r, mid+1, r, k-ct);
//Build segment tree to support queres k-th element in a
     subinterval
void build(int n) {
  for (int i = 0; i \le n; ++i) {
   head[i] = nextPos();
 build(head[0], 1, n);
 for (int i = 1; i \le n; ++i) {
   modify(head[i], head[i-1], c[i], 1, n);
/*Query the k-th element in [l,r]:
printf("%d\n", a[query(head[r], head[l-1], 1, n, k)].key); */
```

#### Point3d.cpp

8058ae, 32 lines

```
template<class T> struct Point3D {
 typedef Point3D P;
 typedef const P& R;
 T x, y, z;
 explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {
 bool operator<(R p) const {
   return tie(x, y, z) < tie(p.x, p.y, p.z); }</pre>
 bool operator == (R p) const {
   return tie(x, y, z) == tie(p.x, p.y, p.z); }
 P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
 P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
 P operator*(T d) const { return P(x*d, y*d, z*d); }
 P operator/(T d) const { return P(x/d, y/d, z/d); }
 T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
 P cross(R p) const {
   return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
 T dist2() const { return x*x + y*y + z*z; }
 double dist() const { return sqrt((double)dist2()); }
 //Azimuthal angle (longitude) to x-axis in interval [-pi,
 double phi() const { return atan2(y, x); }
```

```
//Zenith angle (latitude) to the z-axis in interval [0, pi
 double theta() const { return atan2(sqrt(x*x+y*y),z); }
 P unit() const { return *this/(T)dist(); } //makes dist()
  //returns unit vector normal to *this and p
 P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
 P rotate(double angle, P axis) const {
   double s = sin(angle), c = cos(angle); Pu = axis.unit()
   return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
};
Point.cop
                                                   ec47f9, 93 lines
template <class T> int sqn(T x) \{ return (x > 0) - (x < 0) \}
template<class T>
struct Point {
 typedef Point P;
 Тх, у;
 explicit Point (T x=0, T y=0) : x(x), y(y) {}
 bool operator (P p) const \{ return tie(x,y) < tie(p.x,p.y) \}
 bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y)
      ; }
 P operator+(P p) const { return P(x+p.x, y+p.y); }
 P operator-(P p) const { return P(x-p.x, y-p.y); }
 P operator*(T d) const { return P(x*d, y*d); }
 P operator/(T d) const { return P(x/d, y/d); }
 T dot(P p) const { return x*p.x + y*p.y; }
 T cross(P p) const { return x*p.y - y*p.x; }
 T cross(P a, P b) const { return (a-*this).cross(b-*this);
 T dist2() const { return x*x + y*y; }
 double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
 P unit() const { return *this/dist(); } // makes dist()=1
 P perp() const { return P(-y, x); } // rotates +90 degrees
 P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
 P rotate(double a) const {
   return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
 friend ostream& operator<<(ostream& os, P p) {</pre>
   return os << "(" << p.x << "," << p.y << ")"; }
};
//Line distance
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
 return (double) (b-a).cross(p-a)/(b-a).dist();
//LineProjectionReflection
template<class P>
P lineProj(P a, P b, P p, bool refl=false) {
 return p - v.perp() * (1+refl) *v.cross(p-a) /v.dist2();
//Point-Segment Distance
typedef Point < double > P;
```

```
double segDist(P& s, P& e, P& p) {
 if (s==e) return (p-s).dist();
 auto d = (e-s).dist2(), t = min(d, max(.0, (p-s).dot(e-s)));
 return ((p-s)*d-(e-s)*t).dist()/d;
//Segment-Segment Distance
double TwoSegMinDist(P A, P B, P C, P D)
    return min(min(segDist(A,B,C),segDist(A,B,D)),
               min(segDist(C,D,A),segDist(C,D,B)));
//On Segment
template < class P > bool on Segment (P s, P e, P p) {
 return p.cross(s, e) == 0 \&\& (s - p).dot(e - p) <= 0;
//Segment Intersection
/*If a unique intersection point between the line segments
     going from s1 to e1 and from s2 to e2 exists then it is
     returned.
If no intersection point exists an empty vector is returned.
      If infinitely many exist a vector with 2 elements is
     returned, containing the endpoints of the common line
     segment.
The wrong position will be returned if P is Point<ll> and
     the intersection point does not have integer coordinates
Products of three coordinates are used in intermediate steps
      so watch out for overflow if using int or long long.*/
template < class P > vector < P > segInter (P a, P b, P c, P d) {
 auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sqn(oa) * sqn(ob) < 0 && sqn(oc) * sqn(od) < 0)
    return { (a * ob - b * oa) / (ob - oa) };
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {all(s)};
//The notations from here might be different
//Find the symmetric point of point p about line p1p2
Point SymPoint (Point p. Line 1)
   Point result;
    double a=1.p2.x-1.p1.x;
    double b=1.p2.v-1.p1.v;
    double t = ((p.x-1.p1.x)*a+(p.y-1.p1.y)*b)/(a*a+b*b);
    result.x=2*1.p1.x+2*a*t-p.x;
    result.y=2*1.p1.y+2*b*t-p.y;
    return result;
```

#### PollardRho.cpp

d9508f, 82 lines

```
//Quick\ Multiplication - Calculate\ x\ *\ y\ mod\ modi
     efficiently
//where x and y is in long long range
11 quickmult(ll x, ll y, ll p){
```

# PolygonUnion Polynomial

```
11 \text{ temp} = x * y - ((11)((long double)x / p * y + 0.5)) *
    return (temp < 0) ? temp + p : temp;</pre>
//Prime Test via Miller-Rabin
bool prime_test(ll p){
    static int tests[] = {2, 3, 5, 7, 11, 13, 17, 19, 23,
         29, 31, 37};
    int r = 0:
    11 b = p - 1;
    if (p == 2) return true;
    if (p == 1 || (p & 1) == 0) return false;
    while ((b \& 1) == 0) {
       r++;
        b >>= 1;
    11 d = (p - 1) / (111 << r);
    for (int i = 0; i < 12; i++) {
       if (p == tests[i]) {
            return true;
        11 x = quickpow2(tests[i], d, p);
        for (int j = 1; j \le r; j++) {
            11 y = quickmult(x, x, p);
            if (y == 1 \&\& x != 1 \&\& x != p - 1) return false
                ;
            x = y;
        if (x != 1) return false;
    return true;
//We will store factors in a global variable to save time
map<11, int> factors;
ll gcd(ll x, ll y) {
    if (y == 0) return x;
    return gcd(y, x % y);
l get_next(ll x, ll addi,ll modi){
    return (quickmult(x, x, modi) + addi);
//find a prime factor of n based on the seed, if we cannot
     find it return -1
11 rho_find(ll n, ll seed, ll addi){
    11 a = seed:
    11 b = get_next(seed, addi, n);
    while (a != b) {
        11 p = gcd(abs(a - b), n);
        if (p > 1) {
            if (p == n) return -1;
            return p;
        a = get_next(a, addi, n);
        b = get_next(get_next(b, addi, n), addi, n);
    return -1;
//factorizing n via pollard rho
void pollard_rho(ll n) {
```

```
if (n == 1) {
        return:
   if (prime_test(n)){
        factors[n]++;
        return;
   11 p = -1;
   while (p == -1) {
        11 \text{ addi} = \text{rand}() \% (n - 1) + 1;
        p = rho_find(n, rand() % (n - 1) + 1, addi);
       if (p != -1) {
            pollard_rho(p);
            pollard_rho(n / p);
            return;
PolygonUnion.cpp
"Point.h", "sideOf.h"
                                                   b73c5a, 39 lines
/* KACTL Polygon Union
* Description: Calculates the area of the union of $n$
     polygons (not necessarily
 * convex). The points within each polygon must be given in
     CCW order.
 * (Epsilon checks may optionally be added to sideOf/sqn,
      but shouldn't be needed.)
 * Time: $O(N^2)$, where $N$ is the total number of points*/
typedef Point < double > P;
double rat(P a, P b) { return sqn(b.x) ? a.x/b.x : a.y/b.y;
double polyUnion(vector<vector<P>>& poly) {
 double ret = 0;
 FOR(i, 0, sz(poly)) rep(v, 0, sz(poly[i])) {
   P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
   vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
   FOR(j, 0, sz(poly)) if (i != j) {
     FOR(u, 0, sz(poly[j])) {
       P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])
        int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
        if (sc != sd) {
          double sa = C.cross(D, A), sb = C.cross(D, B);
          if (\min(sc, sd) < 0)
            segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
        } else if (!sc && !sd && j<i && sqn((B-A).dot(D-C))
          segs.emplace back(rat(C - A, B - A), 1);
          segs.emplace_back(rat(D - A, B - A), -1);
```

for (auto& s : seqs) s.first = min(max(s.first, 0.0),

if (!cnt) sum += segs[j].first - segs[j - 1].first;

}

sort(all(segs));

double sum = 0;

FOR(i,1,sz(seqs)) {

int cnt = segs[0].second;

cnt += segs[i].second;

ret += A.cross(B) \* sum;

```
return ret / 2;
Polynomial.cpp
                                                  92b62a, 87 lines
constexpr int maxn = 262144;
constexpr int mod = 998244353;
using i64 = long long;
using poly_t = int[maxn];
using poly = int *const;
//Find f^{-1} \mod x^n.
void polyinv(const poly &h, const int n, poly &f) {
 /* f = 1 / h = f_{-0} (2 - f_{-0} h) */
  static poly_t inv_t;
  std::fill(f, f + n + n, 0);
  f[0] = fpow(h[0], mod - 2);
  for (int t = 2; t <= n; t <<= 1) {
    const int t2 = t \ll 1;
    std::copy(h, h + t, inv_t);
    std::fill(inv_t + t, inv_t + t2, 0);
   DFT(f, t2);
   DFT(inv_t, t2);
    for (int i = 0; i != t2; ++i)
     f[i] = (i64) f[i] * sub(2, (i64) f[i] * inv_t[i] % mod)
          % mod;
    IDFT(f, t2);
    std::fill(f + t, f + t2, 0);
//Find h(x) such that h^2(x) = f(x) \mod x^{deg}.
inline void sqrt(int deg, int *f, int *h) {
 if (deg == 1) {
   h[0] = 1;
    return;
  sart(deq + 1 >> 1, f, h);
  int len = 1;
  while (len < deg * 2) { // doubling
   len \star= 2;
  fill(q, q + len, 0);
  inv(deg, h, g);
  copy(f, f + deq, t);
  fill(t + deg, t + len, 0);
  NTT(t, len, 1);
  NTT(q, len, 1);
  NTT(h, len, 1);
  for (int i = 0; i < len; i++) {
   h[i] = (long long)1 * inv2 *
           ((long long)1 * h[i] % mod + (long long)1 * q[i]
                * t[i] % mod) % mod;
 NTT(h, len, -1);
 fill(h + deg, h + len, 0);
/*This is Fast Walsh Transformation
Goal: Given A, B, compute C_{-i} = \sum_{j=1}^{n} A_{-j}B_{-k}
? is or, and, xor*/
void FWT(int *f,int pd) {
```

## PushRelabel QuasiExgcdSum QuickPhiSum

```
for (int d=1; d<n; d<<=1)</pre>
         for (int m=d<<1,i=0;i<n;i+=m)</pre>
             for (int j=0; j<d; j++) {</pre>
                 int x=f[i+j],y=f[i+j+d];
                 if (pd==0) f[i+j+d] = (x+y) p; // or
                 if (pd==1) f[i+j]=(x+y)%p; // and
                  if(pd==2) f[i+j]=(x+y) p, f[i+j+d]=(x-y+p) p;
                        //xor
void IFWT(int *f,int pd){
    for (int d=1; d<n; d<<=1)
         for (int m=d<<1, i=0; i<n; i+=m)</pre>
             for (int j=0; j<d; j++) {
                 int x=f[i+j], y=f[i+j+d];
                  if (pd==0) f [i+j+d] = (y-x+p) p; //OR
                 if(pd==1) f[i+j]=(x-y+p)%p;; // AND
                 if(pd==2) f[i+j]=111*(x+y)*inv*p, f[i+j+d]=1
                       11*(x-y+p)*inv%p; //XOR
void solve_or()
    memcpy(a, A, sizeof a);
    memcpy(b,B,sizeof b);
    FWT(a,0); FWT(b,0);
    for (int i=0;i<n;i++)</pre>
         a[i]=111*a[i]*b[i]%p;
    IFWT(a, 0);
```

#### PushRelabel.cpp

Description: Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

Time:  $\mathcal{O}\left(V^2\sqrt{E}\right)$ 

be9cf6, 48 lines

```
struct PushRelabel {
 struct Edge {
   int dest, back;
   11 f, c;
 vector<vector<Edge>> g;
 vector<ll> ec;
 vector<Edge*> cur;
 vector<vi> hs; vi H;
 PushRelabel(int n): q(n), ec(n), cur(n), hs(2*n), H(n) {}
 void addEdge(int s, int t, ll cap, ll rcap=0) {
   if (s == t) return;
   g[s].push_back({t, sz(g[t]), 0, cap});
   g[t].push_back({s, sz(g[s])-1, 0, rcap});
 void addFlow(Edge& e, ll f) {
   Edge &back = g[e.dest][e.back];
   if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
   e.f += f; e.c -= f; ec[e.dest] += f;
   back.f -= f; back.c += f; ec[back.dest] -= f;
 11 calc(int s, int t) {
   int v = sz(g); H[s] = v; ec[t] = 1;
   vi co(2*v); co[0] = v-1;
   FOR(i,0,v) cur[i] = g[i].data();
```

```
for (Edge& e : g[s]) addFlow(e, e.c);
  for (int hi = 0;;) {
     while (hs[hi].empty()) if (!hi--) return -ec[s];
     int u = hs[hi].back(); hs[hi].pop_back();
     while (ec[u] > 0) // discharge u
       if (cur[u] == g[u].data() + sz(g[u])) {
         H[u] = 1e9;
         for (Edge& e : g[u]) if (e.c && H[u] > H[e.dest
            H[u] = H[e.dest]+1, cur[u] = &e;
         if (++co[H[u]], !--co[hi] && hi < v)
           FOR(i, 0, v) if (hi < H[i] && H[i] < v)
              --co[H[i]], H[i] = v + 1;
         hi = H[u];
       } else if (\operatorname{cur}[u] \rightarrow \operatorname{c \&\& H}[u] == \operatorname{H}[\operatorname{cur}[u] \rightarrow \operatorname{dest}] + 1)
         addFlow(*cur[u], min(ec[u], cur[u]->c));
       else ++cur[u];
bool leftOfMinCut(int a) { return H[a] >= sz(q); }
```

#### QuasiExgcdSum.cpp

```
b1006b, 60 lines
Using Quasi_Exqcd to sum
f(a,b,c,n) = sum_{\{i=0\}}^n \setminus floor\{(ai+b)/c\}
q(a,b,c,n) = sum_{\{i=0\}} \hat{n} \setminus \{loor i_{\{(ai+b)/c\}}\}
h(a,b,c,n) = sum_{\{i=0\}} n \left(\frac{1}{n} \left(\frac{ai+b}{c}\right)^2\right)
all are done under mod p
struct rec{
    ll f, g, h;
//add, sub, quickpow omitted
11 \text{ inv2} = \text{quickpow}(2, \text{modi}-2);
11 \text{ inv6} = \text{quickpow}(6, \text{ modi} - 2);
rec solve(ll a, ll b, ll c, ll n) {
  rec ans;
  if (a == 0) {
    ans.f = (b / c) * (n + 1) % modi;
    ans.q = (b / c) * (n + 1) % modi * n % modi * inv2 %
    ans.h = (b / c) * (b / c) % modi * (n+1) % modi;
  ans.f = ans.q = ans.h = 0;
  if (a >= c || b >= c) {
    rec temp = solve(a % c, b % c, c, n);
    add(ans.f, (a/c)*n%modi*(n+1)%modi*inv2%modi);
    add(ans.f, (b/c)*(n+1)%modi);
    add(ans.f, temp.f);
    add(ans.g, (a/c)*n%modi*(n+1)%modi*
    ((2*n+1)%modi)%modi*inv6%modi);
    add(ans.g, (b/c)*n%modi*(n+1)%modi*inv2 % modi);
    add(ans.q, temp.q);
    add(ans.h, (a/c)*(a/c)%modi*n%modi*
    (n+1) %modi * ((2 * n + 1) %modi) %modi * inv6% modi);
    add(ans.h, (b/c)*(b/c)*modi*(n+1)*modi);
    add(ans.h, (a/c)*(b/c)*modi*n*modi*(n+1)*modi);
    add(ans.h, temp.h);
```

```
add(ans.h, 2LL * (a/c)%modi*temp.g%modi);
 add(ans.h, 2LL * (b/c)%modi*temp.f%modi);
  return ans;
if (a < c && b < c) {
 11 m = (a * n + b) / c;
 rec temp = solve(c, c - b - 1, a, m - 1);
 ans.f = n * m % modi;
 sub(ans.f, temp.f);
 ans.q = n * (n + 1) % modi * m % modi;
 sub(ans.g, temp.f);
 sub(ans.q, temp.h);
 ans.g = ans.g * inv2 % modi;
 ans.h = n * m % modi * (m + 1) % modi;
 sub(ans.h, 2LL * temp.g % modi);
 sub(ans.h, 2LL * temp.f % modi);
 sub(ans.h, ans.f);
 return ans;
return ans;
```

#### QuickPhiSum.cpp

024bac, 29 lines

This algorithm concerns efficient evaluation of sum of number theoric functions like phi or mu. We know that using Eulerian sieve, we can only archieve O(n)

time complexity. What we are doing is to archieve  $O(n^{2/3})$  time complexity. The example program shows how to evaluate sum of phi and sum

```
of mu efficientyl.
For smaller n (n less than (N^{2/3})), we use calculate them
     as usual.
For larger n, see getphi and getmu
//See Sieve for more technical details.
//When i is prime, phi[i] = i-1, mu[i] = -1; Otherwise,
     inside the inner loop, let p = primes[j].
//Then it follows phi[p*i] = phi[i] * (p-1); mu[p*i] = -mu[i]
//finally, when i \% p == 0, phi[p*i] = phi[i]*p and mu[p*i]
     = 0;
ll getphi(ll n)
  if (n <= m) return phi[n];</pre>
 if (phi_cheat.find(n) != phi_cheat.end()) return phi_cheat
 11 ans = (11) n*(n + 1) / 2; //this is <math>\sum_{i=1}^{n} \sum_{j=1}^{n} sum_{-j}
       \{d \mid n\} \setminus phi(d)
  //when getting mu, ans = 1
  ll last;
  for (11 i = 2; i \le n; i = last + 1)
    last = n / (n / i);
    ans -= (last-i+1) *getphi(n / i);
 phi_cheat[n] = ans;
 return ans;
```

#### Random RootNonTree Sam Simplex

```
Random.cpp

#define uid(a, b) uniform_int_distribution<int>(a, b) (rng)
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
count());
```

### ${\bf RootNonTree.cpp}$

9d4bb6, 38 lines

```
void dfs(int u){
  static int top;
  stk[++top] = u;
  for (int i = 0; i < g[u].size(); i++){
    int v = q[u][i];
    if (v != p[u]) {
        p[v] = u;
        dfs(v);
        if (siz[u] + siz[v] >= magic) {
          siz[u] = 0;
          tot_cols++;
          cap[tot cols] = u;
          while (stk[top] != u) {
            col[stk[top--]] = tot cols:
        else siz[u] += siz[v];
  siz[u]++;
void paint(int u,int c){
    if (col[u]) c = col[u];
    else col[u] = c;
    for (int i = 0; i < g[u].size(); i++){
        int v = q[u][i];
        if (v != p[u]) {
            paint (v,c);
//actual blokcing; magic = block size
dfs(1);
if (!tot cols) {
    cap[++tot\_cols] = 1;
paint(1,tot_cols);
```

#### Sam.cpp

<cstdio>, <cstdlib>, <cstring>, <algorithm>

6196dc, 58 lines

```
using namespace std;

typedef long long l1;

const int maxn = 510000;
const int sigma = 26;

struct edge
{
    int v, next;
};

edge g[maxn << 1];
int trie[maxn << 1][sigma], fa[maxn << 1], maxi[maxn << 1],
    sizia[maxn << 1];
char str[maxn];</pre>
```

```
int head[maxn << 1];</pre>
int siz.last;
void insert(int u, int v)
    static int id;
    g[++id].v = v;
    g[id].next = head[u];
    head[u] = id;
//This is the core of SAM
void add(int id)
    int p = last;
    int np = last = ++siz;
    sizia[np] = 1;
    maxi[np] = maxi[p] + 1;
    while (p && !trie[p][id]) {
        trie[p][id] = np;
        p = fa[p];
    if (!p){
        fa[np] = 1;
    else{
        int q = trie[p][id];
        if (maxi[p] + 1 == maxi[q]) {
            fa[np] = q;
        else{
            int nq = ++siz;
            maxi[nq] = maxi[p] + 1;
            memcpy(trie[nq], trie[q], sizeof trie[q]);
            fa[nq] = fa[q];
            fa[np] = fa[q] = nq;
            while (trie[p][id] == q){
                trie[p][id] = nq;
                p = fa[p];
```

#### Simplex.cpp

**Description:** Solves a general linear maximization problem: maximize  $c^Tx$  subject to  $Ax \leq b, \ x \geq 0$ . Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of  $c^Tx$  otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that x=0 is viable.

```
Usage: vvd A = \{\{1,-1\}, \{-1,1\}, \{-1,-2\}\}; vd b = \{1,1,-4\}, c = \{-1,-1\}, x; T val = LPSolver(A, b, c).solve(x);
```

**Time:**  $\mathcal{O}(NM \cdot \#pivots)$ , where a pivot may be e.g. an edge relaxation.  $\mathcal{O}\left(2^{N}\right)$  in the general case.

```
int m, n; // # contraints, # variables
vi N. B:
vvd D;
LPSolver(const vvd& A, const vd& b, const vd& c) :
  m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2), vd(n+2)) {
    FOR(i,m) FOR(j,n) D[i][j] = A[i][j];
    FOR(i,m) {
      B[i] = n+i, D[i][n] = -1, D[i][n+1] = b[i];
      // B[i]: add basic variable for each constraint,
           convert inegs to egs
      //D[i][n]: artificial variable for testing
           feasibility
    FOR(j,n) {
      N[j] = j; // non-basic variables, all zero
      D[m][j] = -c[j]; // minimize -c^T x
    N[n] = -1; D[m+1][n] = 1;
void pivot (int r, int s) { // r = row, c = column
  1d *a = D[r].data(), inv = 1/a[s];
  FOR(i, m+2) if (i != r && abs(D[i][s]) > eps) {
    ld *b = D[i].data(), binv = b[s]*inv;
    FOR(j,n+2) b[j] -= a[j]*binv; // make column
         corresponding to s all zeroes
    b[s] = a[s] * binv; // swap N[s] with B[r]
  // equation corresponding to r scaled so x_r coefficient
  FOR(j, n+2) if (j != s) D[r][j] *= inv;
  FOR(i,m+2) if (i != r) D[i][s] *= -inv;
  D[r][s] = inv; swap(B[r], N[s]); // swap basic w/ non-
       basic
bool simplex(int phase) {
  int x = m + phase - 1;
  while (1) {
    int s = -1; FOR(j, n+1) if (N[j] != -phase) ltj(D[x]);
         // find most negative col for nonbasic variable
    if (D[x][s] >= -eps) return true; // can't get better
         sol by increasing non-basic variable, terminate
    int r = -1:
    FOR(i,m) {
      if (D[i][s] <= eps) continue;
      if (r == -1 \mid | mp(D[i][n+1] / D[i][s], B[i])
             < mp(D[r][n+1] / D[r][s], B[r])) r = i;
      // find smallest positive ratio, aka max we can
           increase nonbasic variable
    if (r == -1) return false; // increase N/s/ infinitely
          \rightarrow unbounded
    pivot(r,s);
ld solve(vd &x) {
  int r = 0; FOR(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
  if (D[r][n+1] < -eps) { // x=0 not feasible, run simplex
        to find smth feasible
    pivot(r, n); //N[n] = -1 is artificial variable,
         initially set to smth large
    if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;</pre>
    //D[m+1][n+1] is max possible value of the negation
         of
```

```
// artificial variable, optimal value should be zero
      // if exists feasible solution
      FOR(i,m) if (B[i] == -1) { // ?
       int s = 0; FOR(j, 1, n+1) Itj(D[i]);
       pivot(i,s);
    bool ok = simplex(1); x = vd(n);
    FOR(i,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
};
SCC.cpp
                                                   a6b342, 37 lines
vector < vector<int> > g, gr; //g stores graph, gr stores
    graph transposed
vector<bool> used;
vector<int> order, component;
void dfs1 (int v) {
    used[v] = true;
    for (size_t i=0; i<g[v].size(); ++i)</pre>
        if (!used[ q[v][i] ])
            dfsl (a[v][i]);
    order.push_back (v);
void dfs2 (int v) {
    used[v] = true;
    component.push_back (v);
    for (size_t i=0; i<gr[v].size(); ++i)</pre>
        if (!used[ gr[v][i] ])
            dfs2 (gr[v][i]);
void findSCCs() {
    order.clear();
    used.assign (n, false);
    for (int i=0; i<n; ++i)</pre>
       if (!used[i])
           dfs1 (i);
    used.assign (n, false);
    for (int i=0; i<n; ++i) {
       int v = order[n-1-i];
        if (!used[v]) {
            dfs2 (v);
            //SCC FOUND, DO SOMETHING
            component.clear();
Schreier-Sims.cpp
                                                  a288c1, 102 lines
// time complexity : O(n^2 \log^3 |G| + t \log |G|)
// memory complexity : O(n^2 \log |G| + tn)
// t : number of generators
//|G|: group size, obviously \ll (n!)
vector<int> inv(vector<int>& p){
  vector<int> ret(p.size());
 for (int i = 0; i < p.size(); i++) ret[p[i]] = i;</pre>
  return ret:
```

```
vector<int> operator * (vector<int>& a, vector<int>& b ) {
  vector<int> ret(a.size());
  for (int i = 0 ; i < a.size(); i++) ret[i] = b[a[i]];</pre>
  return ret:
// a group contains all subset products of generators
struct Group {
  int n, m;
  vector<vector<int>> lookup;
  vector<vector<int>>> buckets, ibuckets;
  int yo(vector<int> p, bool add_to_group = 1){
    n = buckets.size();
    for (int i = 0; i < n; i++) {
      int res = lookup[i][p[i]];
      if (res == -1 ) {
        if (add to group) {
          buckets[i].push_back(p);
          ibuckets[i].push_back(inv(p));
          lookup[i][p[i]] = buckets[i].size() - 1;
        return i;
      p = p * ibuckets[i][res];
    return -1;
 ll size() {
    ll ret = 1;
    for (int i = 0; i < n; i++) ret *= buckets[i].size();</pre>
    return ret:
  bool in group (vector<int> g) { return yo(g, false) == -1;
  Group(vector<vector<int>> &gen, int _n){
   n = n, m = gen.size(); //m permutations of size <math>n, 0
        indexed
    lookup.resize(n);
    buckets.resize(n);
    ibuckets.resize(n);
    for (int i = 0; i < n; i++) {
      lookup[i].resize(n);
      fill(lookup[i].begin(), lookup[i].end(), -1);
    vector<int> id(n);
    for (int i = 0; i < n; i++) id[i] = i;
    for (int i = 0; i < n; i++) {
      buckets[i].push_back(id);
      ibuckets[i].push_back(id);
      lookup[i][i] = 0;
    for (int i = 0; i < m; i++) vo(qen[i]);
    queue<pair<pair<int, int>,pair<int, int>>> q;
    for (int i = 0; i < n; i++) {
      for (int j = i; j < n; j++) {
        for (int k = 0; k < buckets[i].size(); k++) {
          for (int 1 = 0; 1 < buckets[j].size(); 1++) {</pre>
            q.push({pair<int, int>(i, k), pair<int, int>(j,
                1) });
    while(!q.empty()) {
      pair<int, int> a = q.front().first;
```

```
pair<int, int> b = q.front().second;
      () gog.p
      int res = yo(buckets[a.first][a.second] * buckets[b.
           first][b.second]);
      if (res == -1) continue;
      pair<int, int> cur(res, (int)buckets[res].size() - 1);
      for (int i = 0; i < n; i ++) {
        for (int j = 0; j < (int)buckets[i].size(); ++j){
          if (i <= res) q.push(make_pair(pair<int, int>(i ,
               j), cur));
          if (res <= i) q.push(make_pair(cur, pair<int, int</pre>
               >(i, j)));
int32_t main() {
 ios_base::sync_with_stdio(0);
 cin.tie(0);
 int k, n; cin >> k >> n;
 vector<vector<int>> a;
  while (k--) {
  vector<int> v;
  for (int i = 0; i < n; i++) {
    int x; cin >> x;
     v.push_back(x - 1);
  a.push_back(v);
 Group q(a, n);
  cout << q.size() << '\n';
 return 0;
Segtree.cpp
                                                   4515af, 20 lines
const 11 SZ = 262144; //set this to power of two
11 seg[2*SZ];
11 combine(11 a, 11 b) { return max(a, b); }
void build() { FORd(i,SZ) seg[i] = combine(seg[2*i],seg[2*i
    +1]); }
void update(int p, ll value) {
    for (seg[p += SZ] = value; p > 1; p >>= 1)
        seg[p>>1] = combine(seg[(p|1)^1], seg[p|1]);
ll query(int l, int r) { // sum on interval [l, r]
   11 \text{ resL} = 0, \text{ resR} = 0; \text{ r++;}
    for (1 += SZ, r += SZ; 1 < r; 1 >>= 1, r >>= 1) {
        if (1&1) resL = combine(resL, seg[1++]);
        if (r&1) resR = combine(seq[--r], resR);
    return combine(resL, resR);
Sieve.cpp
                                                   559ed1, 14 lines
vi primes, leastFac;
```

void compPrimes(int N) {

leastFac.resize(N, 0);

leastFac[0] = 1; leastFac[1] = 1;

#### Simpson stress SuffixArray SuffixTree

```
FOR(i, 2, N) {
 if (leastFac[i] == 0) {
   primes.pb(i);
   leastFac[i] = i;
  for (int j = 0; j < sz(primes) && i*primes[j] < N &&</pre>
      primes[j] <= leastFac[i]; j++) {</pre>
    leastFac[i*primes[j]] = primes[j];
```

#### Simpson.cpp

88ae08, 20 lines

```
This is a template for solving simpson integration
We are going to integrate \frac{frac\{cx+d\}\{ax+b\}}{over} over L and R
ld simpson(ld lower, ld upper){
    ld mid = (lower + upper) / 2;
    return (f(lower) + 4 * f(mid) + f(upper)) * (upper-lower
         ) / 6;
ld simpson_integration(ld lower, ld upper, ld target) {
    ld mid = (lower + upper) / 2;
    ld left sum = simpson(lower, mid);
    ld right sum = simpson(mid, upper);
    if (fabs(left sum + right sum - target) < eps){</pre>
        return left sum + right sum;
    return simpson integration(lower, mid, left sum) +
         simpson integration (mid, upper, right sum);
//Call: simpson_integration(lower, upper, simpson(lower,
     upper)) << endl;
```

#### stress.sh

95d57c, 22 lines

```
#!/usr/bin/env bash
for ((testNum=0;testNum<$4;testNum++))</pre>
    ./$3 > input
    ./$2 < input > outSlow
    ./$1 < input > outWrong
    H1= 'md5sum outWrong'
    H2= 'md5sum outSlow'
   if !(cmp -s "outWrong" "outSlow")
        echo "Error found!"
       echo "Input:"
        cat input
        echo "Wrong Output:"
        cat outWrong
        echo "Slow Output:"
        cat outSlow
        exit
    fi
done
echo Passed $4 tests
```

```
SuffixArray.cpp
                                                52732a, 74 lines
vector<int> sort_cyclic_shifts(string const& s) {
    int n = s.size();
   const int alphabet = 256;
    vector<int> p(n), c(n), cnt(max(alphabet, n), 0);
    for (int i = 0; i < n; i++)
       cnt[s[i]]++;
    for (int i = 1; i < alphabet; i++)
       cnt[i] += cnt[i-1];
    for (int i = 0; i < n; i++)
       p[--cnt[s[i]]] = i;
   0 = [[0]q]
    int classes = 1;
   for (int i = 1; i < n; i++) {
       if (s[p[i]] != s[p[i-1]])
           classes++;
       c[p[i]] = classes - 1;
   vector<int> pn(n), cn(n);
    for (int h = 0; (1 << h) < n; ++h) {
        for (int i = 0; i < n; i++) {
           pn[i] = p[i] - (1 << h);
           if (pn[i] < 0)
               pn[i] += n;
        fill(cnt.begin(), cnt.begin() + classes, 0);
       for (int i = 0; i < n; i++)
           cnt[c[pn[i]]]++;
       for (int i = 1; i < classes; i++)
           cnt[i] += cnt[i-1];
        for (int i = n-1; i >= 0; i--)
           p[--cnt[c[pn[i]]]] = pn[i];
       cn[p[0]] = 0;
       classes = 1;
       for (int i = 1; i < n; i++) {
           pair<int, int> cur = \{c[p[i]], c[(p[i] + (1 << h))\}
                )) % nl}:
           << h)) % n]};
           if (cur != prev)
               ++classes:
           cn[p[i]] = classes - 1;
       c.swap(cn);
   return p;
vector<int> suffix array construction(string s) {
   vector<int> sorted shifts = sort cyclic shifts(s);
   sorted_shifts.erase(sorted_shifts.begin());
   return sorted shifts:
vector<int> lcp construction(string const& s, vector<int>
    const& p) {
   int n = s.size();
   vector<int> rank(n, 0);
   for (int i = 0; i < n; i++)
       rank[p[i]] = i;
   int k = 0:
   vector<int> lcp(n-1, 0);
   for (int i = 0; i < n; i++) {
```

```
if (rank[i] == n - 1) {
       k = 0;
        continue;
    int j = p[rank[i] + 1];
    while (i + k < n \&\& j + k < n \&\& s[i+k] == s[j+k])
   lcp[rank[i]] = k;
    if (k)
        k--:
return lcp;
```

#### SuffixTree.cpp

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).

Time:  $\mathcal{O}(26N)$ 

07bb84, 50 lines

```
struct SuffixTree {
 enum { N = 200010, ALPHA = 26 }; //N \sim 2*maxlen+10
 int toi(char c) { return c - 'a'; }
  string a; //v = cur \ node, q = cur \ position
 int t[N][ALPHA], 1[N], r[N], p[N], s[N], v=0, q=0, m=2;
 void ukkadd(int i, int c) { suff:
   if (r[v] <=q) {
     if (t[v][c]==-1) { t[v][c]=m; l[m]=i;
       p[m++]=v; v=s[v]; q=r[v]; goto suff; }
     v=t[v][c]; q=l[v];
    if (q==-1 || c==toi(a[q])) q++; else {
     l[m+1]=i; p[m+1]=m; l[m]=l[v]; r[m]=q;
     p[m]=p[v]; t[m][c]=m+1; t[m][toi(a[q])]=v;
     l[v]=q; p[v]=m; t[p[m]][toi(a[l[m]])]=m;
     v=s[p[m]]; q=l[m];
     while (q < r[m]) \{ v = t[v][toi(a[q])]; q + = r[v] - l[v]; \}
     if (q==r[m]) s[m]=v; else s[m]=m+2;
      q=r[v]-(q-r[m]); m+=2; qoto suff;
 }
  SuffixTree(string a) : a(a) {
   fill(r,r+N,sz(a));
   memset(s, 0, sizeof s);
   memset(t, -1, sizeof t);
    fill(t[1],t[1]+ALPHA,0);
   s[0] = 1; 1[0] = 1[1] = -1; r[0] = r[1] = p[0] = p[1] =
   FOR(i, 0, sz(a)) ukkadd(i, toi(a[i]));
 // example: find longest common substring (uses ALPHA =
 pii best;
 int lcs(int node, int i1, int i2, int olen) {
   if (l[node] <= i1 && i1 < r[node]) return 1;</pre>
   if (1[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - 1[node]) :
   FOR(c, 0, ALPHA) if (t[node][c] != -1)
```

```
mask |= lcs(t[node][c], i1, i2, len);
    if (mask == 3)
     best = max(best, {len, r[node] - len});
    return mask;
 static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2))
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
   return st.best;
};
```

#### Template.cpp

```
"bits/stdc++.h"
                                                     c20da9, 65 lines
using namespace std;
typedef long long 11;
typedef long double 1d; // change to double if appropriate
typedef pair<int, int> pi;
typedef pair<11, 11> pl;
typedef pair<ld, ld> pd;
typedef vector<int> vi;
typedef vector<ld> vd;
typedef vector<ll> vl;
typedef vector<pi> vpi;
typedef vector<pl> vpl;
#define FOR(i, a, b) for (int i = a; i < (b); i++)
#define FOR(i, a) for (int i = 0; i < (a); i++)
#define FORd(i, a, b) for (int i = (b) - 1; i >= (a); i--)
#define FORd(i, a) for (int i = (a) - 1; i \ge 0; i--)
#define trav(a, x) for (auto &a : x)
#define sz(x) (int)(x).size()
#define pb push_back
#define f first
#define s second
#define lb lower_bound
#define ub upper bound
#define all(x) x.begin(), x.end()
#define ins insert
const char nl = '\n';
void print(int x) {cerr << x;}</pre>
void __print(long x) {cerr << x;}</pre>
void __print(long long x) {cerr << x;}</pre>
void __print(unsigned x) {cerr << x;}</pre>
void __print(unsigned long x) {cerr << x;}</pre>
void print(unsigned long long x) {cerr << x;}</pre>
void __print(float x) {cerr << x;}</pre>
void __print(double x) {cerr << x;}</pre>
void print(long double x) {cerr << x;}</pre>
void __print(char x) {cerr << '\'' << x << '\'';}</pre>
void print(const char *x) {cerr << '\"' << x << '\"';}</pre>
void __print(const string &x) {cerr << '\"' << x << '\"';}</pre>
void print(bool x) {cerr << (x ? "true" : "false");}</pre>
template<typename T, typename V>
void print(const pair<T, V> &x) {cerr << '{';} print(x.</pre>
    first); cerr << ", "; __print(x.second); cerr << '}';}
template<tvpename T>
void __print(const T &x) {int f = 0; cerr << '{'; for (auto</pre>
     &i: x) cerr << (f++ ? ", " : ""), __print(i); cerr << "}
     ";}
```

```
void print() {cerr << "]\n";}</pre>
template <typename T, typename... V>
void _print(T t, V... v) {__print(t); if (sizeof...(v)) cerr
     << ", "; _print(v...);}
#ifdef DEBUG
#define dbg(x...) cerr << "\e[91m"<< func <<":"<< LINE
    <<" [" << #x << "] = ["; _print(x); cerr << "\e[39m" <<
    endl:
#else
#define dbg(x...)
#endif
void solve() {
int main() {
 ios_base::sync_with_stdio(0); cin.tie(0);
 solve();
 return 0;
```

#### Treap.cpp

396fd3, 50 lines

```
struct node
    node *ch[2]; //ch[0] = left \ child; \ ch[1] = right \ child;
    int ct,priority,size,key;
    int lsize(){return(ch[0] == NULL)?0:ch[0]->size;}
    int rsize() {return(ch[1] == NULL)?0:ch[1]->size;}
typedef node* tree;
void update(tree & o) {//this part depends on the actual info
      to maintain
    o->size = o->ct; o->size += o->lsize(); o->size += o->
        rsize():
void rotate (tree & o, int dir) { //dir = 0: left rotate
   tree temp = o->ch[dir^1]; o->ch[dir^1] = temp->ch[dir];
        temp->ch[dir] = o;
    update(o); update(temp); o = temp;
void insert(tree & o, int key) {
   if (o == NULL) {
        o = new node;
        o->size = o->ct = 1;o->priority = rand();o->ch[0]=o
            ->ch[1]=NULL;o->key=key;
    else if (key == o->key) {
        o->ct++;o->size++;
    else{
        int dir = (\text{key}<o->\text{key})?0:1;
        insert (o->ch[dir], key);
        if (o->ch[dir]->priority>o->priority) rotate(o,dir^
             1);
        update(o);
void remove(tree & o,int key) {
    if (key == o->key) {
        if (o->ct > 1) {
            o->ct--;o->size--;return;
```

else if (o->ch[0] ==NULL||o->ch[1] ==NULL) {

```
int d = (o -> ch[0] == NULL)?0:1;
        tree temp = o; o = o->ch[d^1]; delete temp;
   else{
        int d = (o->ch[0]->priority > o->ch[1]->priority)
             ?1:0:
        rotate(o,d); remove(o,key);
elset
    int d = (key<o->key)?0:1;
    remove (o->ch[d], key);
if (o) update(o);
```

#### validate.sh

26fb12, 22 lines

```
#!/usr/bin/env bash
for ((testNum=0;testNum<$4;testNum++))</pre>
    ./$3 > input
    ./$1 < input > out
    cat input out > data
    ./$2 < data > res
    result=$(cat res)
   if [ "${result:0:2}" != "OK" ];
        echo "Error found!"
        echo "Input:"
        cat input
        echo "Output:"
        echo "Validator Result:"
        exit
    fi
echo Passed $4 tests
```

#### Vimrc.cpp

13 lines

```
source $VIMRUNTIME/defaults.vim
set ts=4 sw=4 ai cin nu cino+=L0
syntax on
inoremap {<CR> {<CR>}<Esc>0
imap jk
                <Esc>
" Select region and then type : Hash to hash your selection.
" Useful for verifying that there aren't mistypes.
ca Hash w !cpp -dD -P -fpreprocessed \| tr -d '[:space:]' \
\| md5sum \| cut -c-6
autocmd filetype cpp nnoremap <F9> :w <bar> !build.sh %:r <
```

#### Voronoi.cpp

5f5301, 105 lines

```
// Source: http://web.mit.edu/~ecprice/acm/acm08/notebook.
    html#file7
#define MAXN 1024
#define INF 1000000
```

#### Z-algorithm hash build

```
//Voronoi diagrams: O(N^2*LogN)
//Convex hull: O(N*LogN)
typedef struct {
 int id;
 double x;
 double y;
 double ang;
} chp;
double x[MAXN], y[MAXN]; // Input points
chp inv[2*MAXN]; // Points after inversion (to be given to
    Convex Hull)
int vors;
int vor[MAXN]; // Set of points in convex hull;
               //starts at lefmost; last same as first!!
PT ans[MAXN][2];
int chpcmp(const void *aa, const void *bb) {
 double a = ((chp *)aa)->ang;
 double b = ((chp *)bb) -> ang;
 if (a<b) return -1;
 else if (a>b) return 1;
 else return 0; // Might be better to include a
                 // tie-breaker on distance, instead of the
                      cheap hack below
int orient(chp *a, chp *b, chp *c) {
 double s = a->x*(b->y-c->y) + b->x*(c->y-a->y) + c->x*(a->y-a->y)
  if (s>0) return 1;
 else if (s<0) return -1;
 else if (a->ang==b->ang && a->ang==c->ang) return -1; //
           //for points with same angles
 else return 0;
//the pt argument must have the points with precomputed
    angles (atan2()'s)
//with respect to a point on the inside (e.g. the center of
    mass)
int convexHull(int n, chp *pt, int *ans) {
 int i, j, st, anses=0;
 qsort(pt, n, sizeof(chp), chpcmp);
  for (i=0; i<n; i++) pt[n+i] = pt[i];
  for (i=1; i<n; i++) { // Pick leftmost (bottommost)</pre>
                        //point to make sure it's on the
                             convex hull
    if (pt[i].x<pt[st].x || (pt[i].x==pt[st].x && pt[i].y<pt</pre>
         [st].y) st = i;
  ans[anses++] = st;
  for (i=st+1; i<=st+n; i++) {</pre>
    for (j=anses-1; j; j--) {
      if (orient(pt+ans[j-1], pt+ans[j], pt+i)>=0) break;
      // Should change the above to strictly greater,
      // if you don't want points that lie on the side (not
           on a vertex) of the hull
      // If you really want them, you might also put an
           epsilon in orient
```

```
ans[j+1] = i;
   anses = j+2;
 for (i=0; i<anses; i++) ans[i] = pt[ans[i]].id;</pre>
 return anses:
int main(void) {
 int i, j, jj;
 double tmp;
  scanf("%d", &n);
  for (i=0; i<n; i++) scanf("%lf %lf", &x[i], &y[i]);</pre>
  for (i=0; i<n; i++) {
   x[n] = 2*(-INF)-x[i]; y[n] = y[i];
   x[n+1] = x[i]; y[n+1] = 2*INF-y[i];
   x[n+2] = 2*INF-x[i]; y[n+2] = y[i];
   x[n+3] = x[i]; y[n+3] = 2*(-INF)-y[i];
    for (j=0; j< n+4; j++) if (j!=i) {
      jj = j - (j>i);
      inv[jj].id = j;
      tmp = (x[j]-x[i])*(x[j]-x[i]) + (y[j]-y[i])*(y[j]-y[i])
      inv[jj].x = (x[j]-x[i])/tmp;
      inv[jj].y = (y[j]-y[i])/tmp;
      inv[jj].ang = atan2(inv[jj].y, inv[jj].x);
   vors = convexHull(n+3, inv, vor);
    // Build bisectors
    for (j=0; j<vors; j++) {</pre>
      ans[j][0].x = (x[i]+x[vor[j]])/2;
      ans[j][0].y = (y[i]+y[vor[<math>j]])/2;
      ans[j][1].x = ans[j][0].x - (v[vor[j]]-v[i]);
      ans[j][1].y = ans[j][0].y + (x[vor[j]]-x[i]);
   printf("Around (%lf, %lf)\n", x[i], y[i]);
    // List all intersections of the bisectors
    for (j=1; j<vors; j++) {
      vv = ComputeLineIntersection(ans[j-1][0], ans[j-1][1],
           ans[i][0], ans[i][1]);
      printf("%lf, %lf\n", vv.x, vv.y);
   printf("\n");
 return 0;
```

#### Z-algorithm.cpp

9dc088, 13 lines

```
vector<int> z_function(string s) {
    int n = sz(s);
    vector<int> z(n);
    for (int i = 1, 1 = 0, r = 0; i < n; ++i) {
        if (i <= r)
            z[i] = min (r - i + 1, z[i - 1]);
        while (i + z[i] < n && s[z[i]] == s[i + z[i]])
            ++z[i];
        if (i + z[i] - 1 > r)
            1 = i, r = i + z[i] - 1;
    }
    return z;
}
```

```
hash.sh
```

# Hashes a file, ignoring all whitespace and comments. Use for 
# verifying that code was correctly typed. 
cpp -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum |cut - c-6

build.sh 2 lines

#!/bin/bash q++ -lm -s -x c++ -Wall -Wextra -02 -std=c++20 -o \$1 \$1.cpp

# Math Hints (2)

Newton's Method:  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ 

Lagrange Multiplier: Let  $f: \mathbb{R}^n \to \mathbb{R}$  be the objective function,  $g: \mathbb{R}^n \to \mathbb{R}^c$  be the constraints function, let  $x^*$  be an optimal solution to the optimization problem such that  $Dg(x^*) = c$ : maximize f(x) subject to g(x) = 0. There there exists  $\lambda$  such that  $Df(x^*) = \lambda^* \top Dg(x^*)$ .

Burnside's Lemma: Let G be a finite group acting on set X. Let  $X^g$  denote the set of elements in X that are fixed by g. Then the number of orbits is given by

 $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|.$ 

Linear Time Inverses Modulo p: For  $i \ge 2$ ,  $i^{-1} = -\lfloor \frac{p}{i} \rfloor (p \bmod i)^{-1}$ .

Quadratic Residue:  $\left(\frac{a}{p}\right) = a^{(p-1)/2}$ . If  $x^2 \equiv a(\text{mod } p)$  has a solution, then  $\left(\frac{a}{p}\right) = 1$ .

 $LGV\ Lemma$ : Assume G=(V,E) is a DAG. Let  $\omega(P)$  be the product of edge weights on path P. Let  $e(u,v):=\sum_{P:u\to v}\omega(P)$  be the sum of  $\omega(P)$  for all paths from u to v. The set of sources  $A\subseteq V$  and set of sinks  $B\subseteq V$ . A collection of disjoint paths  $A\to B$  consists of n paths  $S_i$  such that  $S_i$  is a path from  $A_i$  to  $B_{\sigma(S)_i}$  such that for any  $i\neq j$ ,  $S_i$ 

and  $S_i$  does not share a common vertex. Then if we let

$$M = \begin{bmatrix} e(A_1, B_1) & e(A_1, B_2) & \cdots & e(A_1, B_n) \\ \vdots & \vdots & \ddots & \vdots \\ e(A_n, B_1) & e(A_n, B_2) & \cdots & e(A_n, B_n) \end{bmatrix}$$

then det  $M = \sum_{S:A\to B} \operatorname{sgn} \sigma(S) \prod_{i=1}^n \omega(S_i)$  where  $S:A\to B$  denotes a set of disjoint paths S from A to B.

Network Flow with Lower/Upper Bounds: Suppose the flow must satisfy  $b(u,v) \leq f(u,v) \leq c(u,v)$ , and have conservation of flows over vertices.

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**Variant 1:** No source/sink (i.e. flow at all vertices must be balanced), check if there is a feasible flow: create a graph G'. For any edge  $u \to v$ , add an edge with capacity c(u,v) - b(u,v). Now assume initially, at vertex u, the sum of capacities of edges into u minus the sum of capacities out of u is M. If M>0, add an edge from super source S to u with capacity M. If M<0, add an edge from u to the super sink T with capacity -M.

**Variant 2**: Feasible flow with sources and sinks: Add an edge from original sink t to source s with capacity  $+\infty$ .

**Variant 3**: Maximum flow with sources and sinks: first, check if there exists a feasible flow. Then augment using source s and sink t (i.e. run Dinic again with source s and sink t).

Variant 4: Minimum flow with sources and sinks: find the feasible flow first, remove the edge from sink to source (the current flow on the edge is the size of the original flow), and then run the maximum flow from the sink t to the source s to see how much we may get rid off from the original flow.

*Pick's Theorem*: Suppose a polygon has integer coordinates for all of its vertices; let i be the number of integer points that are interior to the polyon, b be the number of integer points on its boundary, the area of the polygon is  $A = i + \frac{b}{2} - 1$ .

Mobius Transformation/Circle Inversion: Mobius transformations  $f: \hat{\mathbb{C}} \to \hat{\mathbb{C}}$  are specified by  $f(z) = \frac{az+b}{cz+d}$ 

Dilworth's Theorem: For a partially ordered set S, the maximum size of an antichain is equal to the minimum number of chains (i.e. any two elements are comparable) required to cover S.

*LP Duality*: Suppose the primal linear program is given by maximize  $\mathbf{c}^{\top}\mathbf{x}$  subject to  $A\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0$ , the dual program is given by minimize  $\mathbf{b}^{\top}\mathbf{y}$  subject to  $A^{\top}\mathbf{y} \geq \mathbf{c}, \mathbf{y} \geq 0$ .

Mobius Inversion: If g, f are arithmetic functions satisfying  $g(n) = \sum_{d|n} f(d)$  for  $n \ge 1$ , then  $f(n) = \sum_{d|n} \mu(d)g(\frac{n}{d})$ .

Number of Points on Lattice Convex Polygon: A convex lattice polygon with coordinates in [0, N] has at most  $O(N^{2/3})$  points.

**Green's Theorem**: Let C be a positively oriented, smooth, simple closed curve and let D be the region bounded by C. If L and M are functions of (x,y) defined on an open region containing D and having continuous partial derivatives, then

$$\int_{C} (Ldx + Mdy) = \int_{D} (\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}) dx dy.$$

**Polynomial Division**: Suppose we are given polynomials f(x), g(x), and we want to write f(x) as f(x) = Q(x)g(x) + R(x). Let  $f^R(x) = x^{\deg f}f(\frac{1}{x})$  (i.e. reverse the coefficients of the polynomial). Let  $n = \deg f, m = \deg g$ . Then

$$f^R(x) \equiv Q^R(x)g^R(x) \mod x^{n-m+1}$$