Game Theory

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§1 Combinatorial Games

Definition 1.1. For our purposes, a **game** can be characterized by a set of states and transitions between the states, with the following properties:

- There are two players, sometimes called *Left* and *Right*, who take turns making moves (making a valid transition from one state to the next). Other than who moves first, the players are indistinguishable (i.e., they can make the same moves). This is a property of *impartial* games.
- The players have perfect information, and the game is entirely deterministic.
- If a player cannot make a move (i.e., there are no valid transitions out of the current state), they are declared the *loser*, and the other player is the *winner*.
- The game must end after a finite sequence of moves. There will always be a winner and a loser; there can be no draws.

As a consequence, the outcome of a game with optimal play is completely determined by the current state. A state is winning if there is at least one transition to a losing state, and a state is losing if it has no transitions, or every transition is to a winning state.

Because every game is finite, the game states can be modeled as a DAG, with directed edges representing transitions between states.