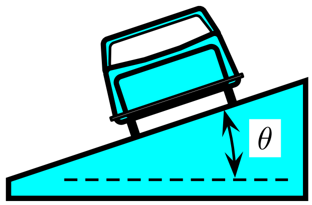
This print-out should have 17 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 (part 1 of 2) 5.0 points

A curve of radius 70 m is banked so that a 1040 kg car traveling at 60 km/h can round it even if the road is so icy that the coefficient of static friction is approximately zero.

The acceleration of gravity is 9.81 m/s^2 .



Find the minimum speed at which a car can travel around this curve without skidding if the coefficient of static friction between the road and the tires is 0.2.

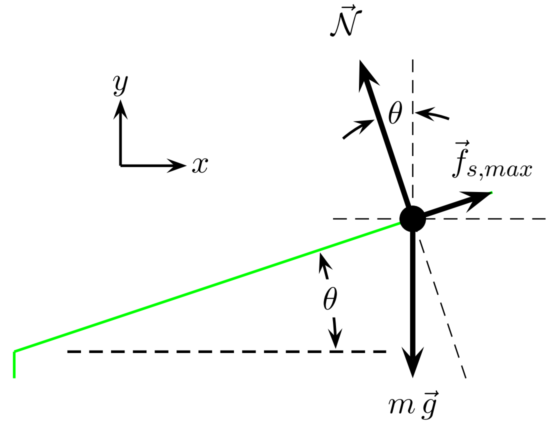
1. 10.5668
2. 10.2569
3. 5.03486
4. 5.5889
5. 3.09329
6. 11.2801
7. 13.6345
8. 11.3985
9. 9.30281
10. 12.7042

Correct answer: 11.3985 m/s.

Explanation:

$$\text{Let}: \quad m = 1040\ \text{kg}, \quad v = 60\ \text{km/h}, \quad \text{and} \quad g = 9.81\ \text{m/s}^2 .$$

Consider the forces acting on the car:



The static friction force up the incline balances the downward component of the car's weight and prevents it from sliding.

Applying $\sum \vec{F} = m\,\vec{a}$ to a car traveling around the curve when the coefficient of static friction is zero,

$$\sum F_x = \mathcal{N}\sin\theta = m\,\frac{v^2}{r} \quad \text{and}$$

$$\sum F_y = \mathcal{N}\cos\theta - m\,g = 0$$
$$\mathcal{N}\cos\theta = m\,g\,.$$

Dividing, we have

$$\tan\theta = \frac{v^2}{r\,g} = \frac{(60\ \text{km/h})^2}{(70\ \text{m})\,(9.81\ \text{m/s}^2)} \times \left(\frac{1000\ \text{m}}{\text{km}}\right)\left(\frac{\text{h}}{3600\ \text{s}}\right) = 0.404511\,,$$

so that

$$\theta = \tan^{-1}(0.404511) = 22.0239^\circ\,.$$

Applying $\sum \vec{F} = m\,\vec{a}$ to a car traveling at minimum speed,

$$\sum F_x = \mathcal{N}\sin\theta - f_s\cos\theta = m\,\frac{v^2}{r}$$
$$\mathcal{N}\sin\theta - \mu_s\,\mathcal{N}\cos\theta = m\,\frac{v^2}{r}$$
$$\mathcal{N}\,(\sin\theta - \mu_s\cos\theta) = m\,\frac{v^2}{r} \quad \text{and}$$

$$\sum F_y = \mathcal{N}\cos\theta + f_s\sin\theta - m\,g = 0$$
$$\mathcal{N}\cos\theta + \mu_s\,\mathcal{N}\sin\theta = m\,g$$
$$\mathcal{N}\,(\cos\theta + \mu_s\sin\theta) = m\,g\,.$$

Dividing, we have

$$\frac{v_{min}^2}{r\,g} = \frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}$$

$$v_{min} = \sqrt{\frac{r\,g\,(\sin\theta - \mu_s\cos\theta)}{\cos\theta + \mu_s\sin\theta}}$$

$$= \sqrt{\frac{r\,g\,(\tan\theta - \mu_s)}{1 + \mu_s\,\tan\theta)}}$$

$$= \sqrt{\frac{(70\text{ m})\,(9.81\text{ m/s}^2)\,(0.404511 - 0.2)}{1 + 0.2\,(0.404511)}}$$

$$= \boxed{11.3985\text{ m/s}}.$$

002 (part 2 of 2) 5.0 points

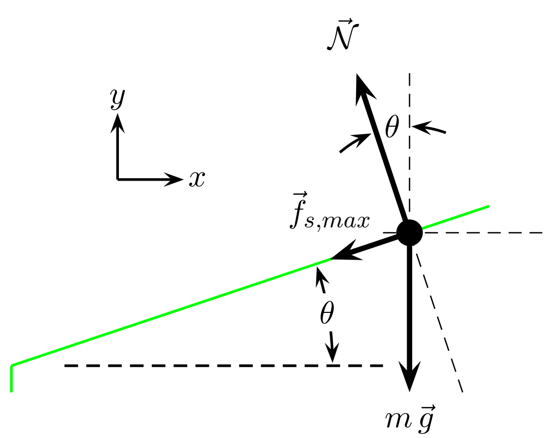
Find the maximum speed under the same conditions.

1. 33.0925
2. 23.465
3. 23.9244
4. 21.63
5. 27.5218
6. 37.7604
7. 21.2522
8. 20.2428
9. 25.673
10. 35.8718

Correct answer: 21.2522 m/s.

Explanation:

Consider the forces acting on the car:



The static friction force points in the opposite direction since the tendency of the car moving at the maximum safe speed is to slide toward outside of the curve.

Applying $\sum\vec{F} = m\,\vec{a}$ to a car traveling at maximum speed,

$$\sum F_x = \mathcal{N}\sin\theta + f_s\cos\theta = m\,\frac{v^2}{r}$$

$$\mathcal{N}\sin\theta + \mu_s\,\mathcal{N}\cos\theta = m\,\frac{v^2}{r}$$

$$\mathcal{N}\,(\sin\theta + \mu_s\cos\theta) = m\,\frac{v^2}{r} \quad \text{and}$$

$$\sum F_y = \mathcal{N}\cos\theta - f_s\sin\theta - m\,g = 0$$

$$\mathcal{N}\cos\theta - \mu_s\,\mathcal{N}\sin\theta = m\,g$$

$$\mathcal{N}\,(\cos\theta - \mu_s\sin\theta) = m\,g$$

Dividing, we have

$$\frac{v_{max}^2}{r\,g} = \frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}$$

$$v_{max} = \sqrt{\frac{r\,g\,(\sin\theta + \mu_s\cos\theta)}{\cos\theta - \mu_s\sin\theta}}$$

$$= \sqrt{\frac{r\,g\,(\tan\theta + \mu_s)}{1 - \mu_s\tan\theta}}$$

$$= \sqrt{\frac{(70\text{ m})\,(9.81\text{ m/s}^2)\,(0.404511 + 0.2)}{1 - 0.2\,(0.404511)}}$$

$$= \boxed{21.2522\text{ m/s}}.$$

003 (part 1 of 2) 5.0 points

Given: $G = 6.67259 \times 10^{-11}\text{ N m}^2/\text{kg}^2$.

A 1190 kg geosynchronous satellite orbits a planet similar to Earth at a radius 1.98×10^5 km from the planet's center. Its angular speed at this radius is the same as the rotational speed of the Earth, and so they appear stationary in the sky. That is, the period of the satellite is 24 h.

What is the force acting on this satellite?

1. 1396.06
2. 1541.91
3. 1246.08
4. 449.374
5. 1448.15
6. 508.108
7. 1121.16
8. 1013.47

9. 934.964
10. 1218.95

Correct answer: 1246.08 N.

Explanation:

$$\text{Let :}\quad G = 6.67259 \times 10^{-11}\ \text{N m}^2/\text{kg}^2\,,$$
$$M_{satellite} = 1190\ \text{kg}\,,\quad \text{and}$$
$$T = 86400\ \text{s}\,.$$

Solution:

$$\begin{aligned} v &= \frac{2\,\pi\,r}{T} \\ F &= M_{satellite}\,\frac{v^2}{r} \\ &= M_{satellite}\,\frac{4\,\pi^2\,r}{T^2} \qquad (1) \\ &= (1190\ \text{kg})\,\frac{4\,\pi^2\,(1.98\times 10^8\ \text{m})}{(86400\ \text{s})^2} \\ &= 1246.08\ \text{N}\,. \end{aligned}$$

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004 (part 2 of 2) 5.0 points

What is the mass of this planet?

1. 6.15224e+26
2. 6.43615e+26
3. 5.87681e+26
4. 6.34056e+26
5. 6.0595e+26
6. 5.78686e+26
7. 5.60972e+26
8. 6.24593e+26
9. 5.69783e+26
10. 5.96769e+26

Correct answer: 6.15224×10^{26} kg.

Explanation:

Using the general gravitation law and Eq. 1, we have

$$\begin{aligned} F &= G\,\frac{M_{satellite}\,M_{planet}}{r^2} \\ &= M_{satellite}\,\frac{4\,\pi^2\,r}{T^2} \end{aligned}$$

$$G\,\frac{M_{satellite}\,M_{planet}}{r^2} = M_{satellite}\,\frac{4\,\pi^2\,r}{T^2}\,,\quad \text{so}$$

$$\begin{aligned} M_{planet} &= \frac{4\,\pi^2}{G\,T^2}\,r^3 \\ &= \frac{4\,\pi^2}{(6.67259\times 10^{-11}\ \text{N m}^2/\text{kg}^2)} \\ &\quad\times \frac{(1.98\times 10^8\ \text{m})^3}{(86400\ \text{s})^2} \\ &= 6.15224\times 10^{26}\ \text{kg}\,. \end{aligned}$$

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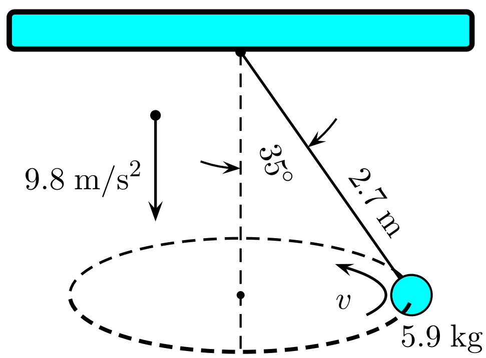
005 (part 1 of 2) 5.0 points

A small metal ball is suspended from the ceiling by a thread of negligible mass. The ball is then set in motion in a horizontal circle so that the thread describes a cone.

The acceleration of gravity is 9.8 m/s^2 .



What is the speed of the ball when it is in circular motion?

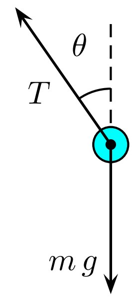
1. 1.67508
2. 1.52235
3. 2.05392
4. 2.50552
5. 3.2599
6. 2.31843
7. 2.19732
8. 2.55083
9. 1.93349
10. 2.96002

Correct answer: 3.2599 m/s.

Explanation:

$$\text{Let :}\quad \ell = 2.7\ \text{m}\,,$$
$$\theta = 35^\circ\,,$$
$$g = 9.8\ \text{m/s}^2\,,\quad \text{and}$$
$$m = 5.9\ \text{kg}\,.$$

Use the free body diagram below.



The tension on the string can be decomposed into a vertical component which balances the weight of the ball and a horizontal component which causes the centripetal acceleration, $a_{centrip}$ that keeps the ball on its horizontal circular path at radius $r = \ell \sin\theta$.

If T is the magnitude of the tension in the string, then

$$T_{vertical} = T\cos\theta = m\,g \qquad (1)$$

and

$$T_{horiz} = m\,a_{centrip}$$

or

$$T\sin\theta = \frac{m\,v_{ball}^2}{\ell\sin\theta}\,. \qquad (2)$$

Solving (1) for T yields

$$T = \frac{m\,g}{\cos\theta} \qquad (3)$$

and substituting (3) into (2) gives

$$m\,g\tan\theta = \frac{m\,v_{ball}^2}{\ell\sin\theta}\,.$$

Solving for v yields

$$\begin{aligned} v &= \sqrt{g\,\ell\tan\theta\sin\theta} \\ &= \sqrt{(9.8\text{ m/s}^2)\,(2.7\text{ m})\tan 35^\circ\sin 35^\circ} \\ &= 3.2599\text{ m/s}\,. \end{aligned}$$

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006 (part 2 of 2) 5.0 points

How long does it take T_{period} for the ball to rotate once around the axis?

1. 2.98491
2. 3.37764
3. 2.92041
4. 3.05338
5. 3.30952
6. 3.41447
7. 3.25115
8. 2.64828
9. 2.20354
10. 2.01351

Correct answer: 2.98491 s.

Explanation:

Basic Concept:

$$d = v\,t\,.$$

Solution: Because the tangential speed of the ball around the circle is constant, we have

$$v_{ball} = \frac{s}{T_{period}}\,.$$

s is the distance the ball travels in one revolution, which is the perimeter of the circle of radius $\ell\sin\theta$, therefore, we have

$$s = 2\,\pi\,\ell\sin\theta\,.$$

Equating both expressions for v_{ball}, we have

$$\frac{2\,\pi\,\ell\sin\theta}{T_{period}} = v_{ball} = \sqrt{g\,\ell\tan\theta\sin\theta}$$

$$\frac{2\,\pi\,\ell\sin\theta}{T_{period}} = \sqrt{\frac{g\,\ell\sin^2\theta}{\cos\theta}}$$

$$\begin{aligned} T_{period} &= 2\,\pi\sqrt{\frac{\ell\cos\theta}{g}} \\ &= 2\,\pi\sqrt{\frac{(2.7\text{ m})\cos 35^\circ}{9.8\text{ m/s}^2}} \\ &= 2.98491\text{ s}\,. \end{aligned}$$

as $\sin\theta$ cancels.

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007 (part 1 of 2) 5.0 points

A 1460 kg car starts from rest and accelerates uniformly to 15.4 m/s in 10.3 s.

Find the average power developed by the engine. Assume that air resistance remains constant at 325 N during this time.

1. 25.886
2. 8.87588

3. 20.6983
4. 23.8914
5. 33.8636
6. 9.79143
7. 20.4896
8. 21.6525
9. 19.423
10. 15.6726

Correct answer: 25.886 hp.

Explanation:

$$\begin{aligned}\text{Let :}\quad m &= 1460\text{ kg}\,,\\ v_i &= 0\text{ m/s}\,,\\ v_f &= 15.4\text{ m/s}\,,\quad\text{and}\\ \Delta t &= 10.3\text{ s}\,.\end{aligned}$$

The acceleration of the car is

$$\begin{aligned}a &= \frac{v_f - v_i}{\Delta t} = \frac{v_f}{\Delta t}\\ &= \frac{15.4\text{ m/s}}{10.3\text{ s}} = 1.49515\text{ m/s}^2\end{aligned}$$

and the constant forward force due to the engine is found from

$$\begin{aligned}\sum F &= F_{engine} - F_{air} = m\,a\\ F_{engine} &= F_{air} + m\,a\\ &= 325\text{ N} + (1460\text{ kg})\left(1.49515\text{ m/s}^2\right)\\ &= 2507.91\text{ N}\,.\end{aligned}$$

The average velocity of the car during this interval is

$$v_{av} = \frac{v_f + v_i}{2}\,,$$

so the average power output is

$$\begin{aligned}P &= F_{engine}\,v_{av} = F_{engine}\left(\frac{v_f}{2}\right)\\ &= (2507.91\text{ N})\left(\frac{15.4\text{ m/s}}{2}\right)\left(\frac{1\text{ hp}}{746\text{ W}}\right)\\ &= \boxed{25.886\text{ hp}}\,.\end{aligned}$$

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008 (part 2 of 2) 5.0 points

Find the instantaneous power output of the engine at $t = 10.3$ s just before the car stops accelerating.

1. 36.3973
2. 55.5458
3. 78.4665
4. 21.3808
5. 37.9926
6. 51.7719
7. 23.908
8. 17.6548
9. 35.7584
10. 28.1676

Correct answer: 51.7719 hp.

Explanation:

The instantaneous velocity is 15.4 m/s and the instantaneous power output of the engine is

$$\begin{aligned}P &= F_{engine}\,v_f\\ &= (2507.91\text{ N})(15.4\text{ m/s})\left(\frac{1\text{ hp}}{746\text{ W}}\right)\\ &= \boxed{51.7719\text{ hp}}\,.\end{aligned}$$

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009 10.0 points

A satellite circles planet Roton every 7.9 h in an orbit having a radius of 8.7×10^6 m.

If the radius of Roton is 5.829×10^6 m, what is the magnitude of the free-fall acceleration on the surface of Roton?

1. 25.6162
2. 13.4482
3. 8.90913
4. 5.34326
5. 7.93301
6. 2.76297
7. 6.34291
8. 0.945954
9. 9.62417
10. 1.49399

Correct answer: 0.945954 m/s^2.

Explanation:

Basic Concepts: Newton's law of gravitation

$$F_g = G\,\frac{m_1\,m_2}{r^2}\,.$$

Kepler's third law

$$T^2 = \left(\frac{4\,\pi^2}{G\,M}\right) r^3 .$$

The free-fall acceleration a on the surface of the planet is the acceleration which a body in free fall will feel due to gravity

$$F_g = G\,\frac{M\,m}{R^2} = m\,a\,,$$

where M is the mass of planet Roton. This acceleration a is

$$a = G\,\frac{M}{R^2}\,, \qquad (1)$$

the number which is g on Earth. Here, however, the mass M is unknown, so we try to find this from the information given about the satellite. Use Kepler's third law for the period of the orbit

$$T^2 = \left(\frac{4\,\pi^2}{G\,M}\right) r^3 . \qquad (2)$$

By multiplying both sides with R^2 and comparing to equation (1), we can identify our a in the right hand side

$$T^2\,R^2 = \left(\frac{4\,\pi^2}{a}\right) r^3 .$$

If we solve for a, we obtain

$$a = \left(\frac{4\,\pi^2}{T^2\,R^2}\right) r^3 = 0.945954 \text{ m/s}^2$$

which is our answer. Although identifying a in this way is a "quick" way of solving the problem, we could just as well have calculated the planet mass M explicitly from equation (2) and inserted into equation (1).

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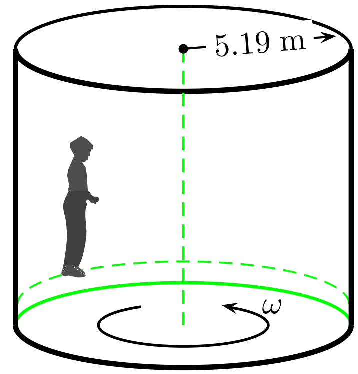
010 (part 1 of 2) 5.0 points

The coefficient of static friction between the person and the wall is 0.66 . The radius of the cylinder is 5.19 m .

The acceleration of gravity is 9.8 m/s^2 .

An amusement park ride consists of a large vertical cylinder that spins about its axis fast enough that any person inside is held up against the wall when the floor drops away.



What is the minimum *angular* velocity ω_{min} needed to keep the person from slipping downward?

1. 1.54612
2. 1.36308
3. 1.67093
4. 1.60317
5. 1.46506
6. 1.576
7. 1.51099
8. 1.69144
9. 1.41869
10. 1.62178

Correct answer: 1.69144 rad/s.

Explanation:

$$\text{Let :} \quad R = 5.19 \text{ m} \quad \text{and} \quad \mu = 0.66\,.$$

Basic Concepts: Centripetal force

$$F = \frac{m\,v^2}{r}\,.$$

Frictional force

$$f \le \mu\,\mathcal{N} = f_{max}\,.$$

Solution: The maximum force due to static friction is $f_{max} = \mu\,\mathcal{N}$, where $\mathcal{N}$ is the inward directed normal force exerted by the wall of the cylinder on the person. To support the person vertically, the *maximal* friction force must be larger than the force of gravity $m\,g$, so that the *actual* force, which is equal to or less than the maximum $\mu\,\mathcal{N}$, is allowed to take on the value $m\,g$ in the positive vertical direction. In other words, the "ceiling" $\mu\,\mathcal{N}$ on the frictional force has to be raised high enough to allow for the value $m\,g$. The normal force supplies the centripetal acceleration $\dfrac{v^2}{R}$ on the person, so from Newton's second law,

$$\mathcal{N} = \frac{mv^2}{R}\,.$$

Since

$$f_{max} = \mu\,\mathcal{N} = \frac{\mu\,m\,v^2}{R} \geq m\,g\,,$$

the minimum speed required to keep the person supported is at the limit of this inequality, which is

$$\frac{\mu\,m\,v_{min}^2}{R} = m\,g\,, \quad \text{or}$$

$$v_{min} = \sqrt{\frac{g\,R}{\mu}}\,.$$

From this we immediately find the angular speed

$$\begin{aligned}\omega_{min} &\equiv \frac{v_{min}}{R} \\ &= \sqrt{\frac{g}{\mu\,R}} \\ &= \sqrt{\frac{9.8\ \text{m/s}^2}{(0.66)\,(5.19\ \text{m})}} \\ &= 1.69144\ \text{rad/s}\,.\end{aligned}$$

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011 (part 2 of 2) 5.0 points

Suppose the person, whose mass is m, is being held up against the wall with an angular velocity of $\omega' = 2\,\omega_{min}$.

The magnitude of the frictional force between the person and the wall is

1. $F = \dfrac{1}{4}\,m\,g\,.$

2. $F = 3\,m\,g\,.$

3. $F = \dfrac{1}{3}\,m\,g\,.$

4. $F = 5\,m\,g\,.$

5. $F = \dfrac{1}{5}\,m\,g\,.$

6. $F = m\,g\,.$ **correct**

7. $F = 2\,m\,g\,.$

8. $F = 4\,m\,g\,.$

9. $F = \dfrac{1}{2}\,m\,g\,.$

Explanation:

As discussed above,

$$f \leq f_{max} = \mu\,\mathcal{N} \quad .$$

Once the angular velocity has increased past the minimum angular velocity ω_{min} required to keep the person pinned against the wall, there is no "incentive" for the force of friction to increase any more. Therefore, regardless of the maximum frictional force *allowed* by the angular speed, f stays at its value $m\,g$.

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012 10.0 points

A bucket full of water is rotated in a vertical circle of radius 1.22 m (the approximate length of a person's arm).

What must be the minimum speed of the pail at the top of the circle if no water is to spill out?

1. 2.9031
2. 2.46893
3. 2.84514
4. 2.0883
5. 2.0647
6. 3.62516
7. 2.2983
8. 3.45774
9. 3.8315
10. 3.95113

Correct answer: 3.45774 m/s.

Explanation:

If we analyze the forces on the water at the top of a vertical circle (using the notation F_{ji} for a two body force on $\hat{\imath}$ from $\hat{\jmath}$), we see that

$$\mathcal{N}_{pail,water} + \mathcal{W}_{earth,water} = m\,a_{centripetal}$$

$$= m\,\frac{v^2}{r}.$$

To minimize the velocity, we minimize the lhs of the equation. Since we can't change the weight of the water, we use the lowest Normal force that we can, 0. Then,

$$m\,g = m\,\frac{v^2}{r}$$

and so,

$$\begin{aligned} v &= \sqrt{g\,r} \\ &= \sqrt{(9.8\ \text{m/s}^2)\,(1.22\ \text{m})} \\ &= \boxed{3.45774\ \text{m/s}}. \end{aligned}$$

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013 10.0 points

A potential energy function for a two-dimensional force is of the form

$$\mathcal{U} = a\,x^3\,y + b\,x\,,$$

where $a = 11.85\ \text{J/m}^4$ and $b = -8\ \text{J/m}$.

Find the magnitude of the force that acts at the point (x, y) for $x = 1$ m, $y = 6$ m.

1. 1660860.0
2. 4225290.0
3. 4823740.0
4. 205.642
5. 138110.0
6. 151647.0
7. 195995.0
8. 3688040.0
9. 15035600.0
10. 6733620.0

Correct answer: 205.642 N.

Explanation:

Basic Concept: The force equals the negative gradiant of the potential. Thus, the general relation between the potential energy and the force it produces is

$$\vec{F} = -\nabla\vec{\mathcal{U}}\,,$$

or

$$F_x = -\frac{\partial\mathcal{U}}{dx}\,,$$

$$F_y = -\frac{\partial\mathcal{U}}{dy}\,, \quad \text{and}$$

$$F_z = -\frac{\partial\mathcal{U}}{dz}\,.$$

Solution: Therefore

$$\begin{aligned} F_x &= -\frac{\partial\mathcal{U}}{\partial x} \\ &= -b - 3\,a\,x^2\,y \\ &= -(-8\ \text{J/m}) \\ &\quad - 3\,(11.85\ \text{J/m}^4)\,(1\ \text{m})^2\,(6\ \text{m}) \\ &= -205.3\ \text{N}\,, \end{aligned} \tag{1}$$

and

$$\begin{aligned} F_y &= -\frac{\partial\mathcal{U}}{\partial y} \\ &= a\,x^3 \\ &= (11.85\ \text{J/m}^4)\,(1\ \text{m})^3 \\ &= 11.85\ \text{N}\,. \end{aligned} \tag{2}$$

Hence, using Eqs. (1) & (2), we have

$$\begin{aligned} F &= \sqrt{F_x^2 + F_y^2} \\ &= \sqrt{(-205.3\ \text{N})^2 + (11.85\ \text{N})^2} \\ &= 205.642\ \text{N}\,. \end{aligned}$$

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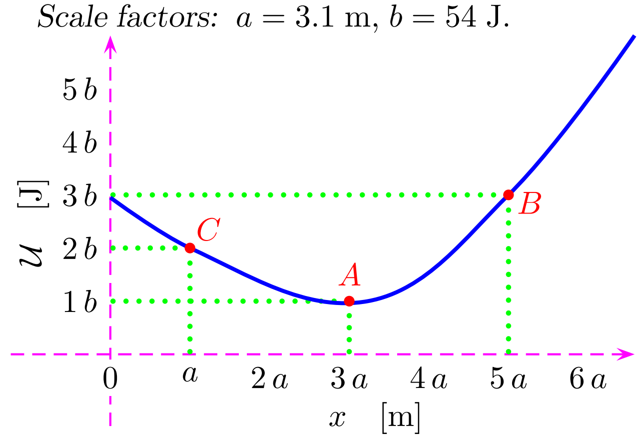
014 (part 1 of 3) 4.0 points

A particle moving along the x-axis has a potential energy $U(x)$, as shown in the accompanying graph.

Scale factors: $a = 3.1$ m, $b = 54$ J.



What is the force exerted on the particle when $x = 3\,a$ (point A, the lowest point on the curve)?

1. -5.80645 N

2. 5.80645 N

3. 17.4194 N

4. 0 **correct**

5. -17.4194 N

6. Undetermined, since the magnitude of the force is zero.

Explanation:

From the definition of Potential Energy, we know

$$F = -\frac{dU}{dx} .$$

In other words, the force on a particle at any particular point is given by the negative of the slope of the potential energy function at that point.

At $x = 3\,a$, the potential energy has a minimum; the slope of the function is zero at that point. Therefore there are no forces on a particle at that point. ($x = 3\,a$ is an equilibrium point.)

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015 (part 2 of 3) 3.0 points

What direction is the force exerted on the particle when $x = 5\,a$ (point B)?

1. Undetermined, since the magnitude of the force is zero.

2. To the right.

3. To the left. **correct**

Explanation:

If the potential energy is increasing, the force is negative and *vice versa*.

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016 (part 3 of 3) 3.0 points

If the particle has a mass of 7.8 kg and is released from rest at $x = 5\,a$, what is the particle's velocity when it reaches $x = a$?

1. 3.60098
2. 3.72104
3. 2.54133
4. 4.60977
5. 2.51312
6. 4.99056
7. 4.14039
8. 4.50146
9. 3.13928
10. 4.73

Correct answer: 3.72104 m/s.

Explanation:

Apply Conservation of Energy:

$$U_{init} + K_{init} = U_{final} + K_{final}$$
$$U_B + 0 = U_C + \frac{1}{2}\,m\,v^2$$

Solving for v gives:

$$\begin{aligned} v &= \sqrt{\frac{2}{m}\,(U_B - U_C)} \\ &= \sqrt{\frac{2}{m}\,(3\,b - 2\,b)} \\ &= \sqrt{\frac{2}{m}\,b} \\ &= \sqrt{\frac{2}{(7.8\text{ kg})}\,(54\text{ J})} \\ &= \boxed{3.72104\text{ m/s}} . \end{aligned}$$

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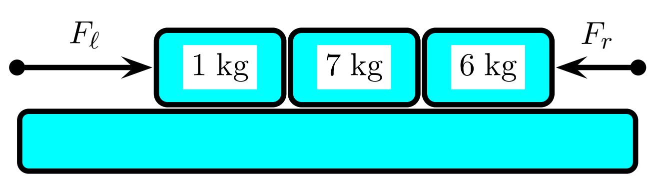
017 10.0 points

The horizontal surface on which the objects slide is frictionless.

The acceleration of gravity is 9.8 m/s^2 .



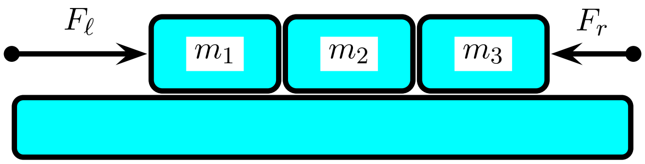
If $F_\ell = 15$ N and $F_r = 7$ N, what is the magnitude of the force exerted on the block with mass 7 kg by the block with mass 6 kg?

1. 10.0
2. 9.58824
3. 9.13333
4. 8.42857
5. 6.35294
6. 11.1429

7. 8.75
8. 10.4286
9. 13.1429
10. 13.5714

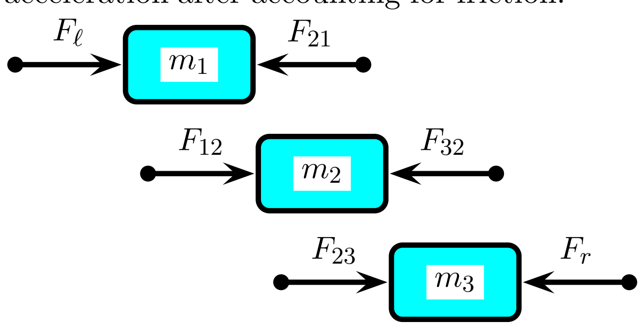
Correct answer: 10.4286 N.

Explanation:



$$\begin{aligned} \text{Given :} \quad \vec{F}_\ell &= +15 \text{ N } \hat{\imath}\,, \\ \vec{F}_r &= -7 \text{ N } \hat{\imath}\,, \\ m_1 &= 1 \text{ kg}\,, \\ m_2 &= 7 \text{ kg}\,, \\ m_3 &= 6 \text{ kg}\,, \quad \text{and} \\ g &= 9.8 \text{ m/s}^2\,. \end{aligned}$$

Note: F is acting on the combined mass of the three blocks, resulting in a common acceleration after accounting for friction.



Let F_ℓ, F_r, F_{32} represent the force exerted on the system from the right, from the left, and the force exerted on m_2 by m_3, respectively.

Note: $\vec{F}_{21} = -\vec{F}_{12}$ and $\vec{F}_{32} = -\vec{F}_{23}$, and $\|\vec{F}_{21}\| = \|\vec{F}_{12}\|$ and $\|\vec{F}_{32}\| = \|\vec{F}_{23}\|$, where $\|\vec{F}\| \equiv F$ is the magnitude of $\vec{F}$.

The sum of the forces acting on each block separately are

$$m_1\,a = +F_\ell - F_{21} = +F_\ell - F_{12} \quad (1)$$
$$m_2\,a = +F_{12} - F_{32} = +F_{12} - F_{23} \quad (2)$$
$$m_3\,a = +F_{23} - F_r \quad (3)$$

To find the acceleration we can treat the three masses as a single object or add the forces acting on each component of the system, Eqs. 1, 2, and 3.

$$F_\ell - F_r = (m_1 + m_2 + m_3)\,a$$

Solving for a, we have

$$\begin{aligned} a &= \frac{F_\ell - F_r}{m_1 + m_2 + m_3} \quad (4) \\ &= \frac{15 \text{ N} - 7 \text{ N}}{1 \text{ kg} + 7 \text{ kg} + 6 \text{ kg}} \\ &= 0.571429 \text{ m/s}^2\,. \end{aligned}$$

We can solve for F_{23} using Eq. 3 and substituting a from Eq. 4. The result is

$$\begin{aligned} F_{23} &= m_3\,a + F_r \quad (3) \\ &= \frac{m_3\,(F_\ell - F_r)}{m_1 + m_2 + m_3} \\ &\quad + \frac{F_r\,(m_1 + m_2 + m_3)}{m_1 + m_2 + m_3} \\ &= \frac{m_3\,F_\ell - (m_1 + m_2)\,F_r}{m_1 + m_2 + m_3} \quad (5) \\ &= \frac{(6 \text{ kg})\,(15 \text{ N})}{1 \text{ kg} + 7 \text{ kg} + 6 \text{ kg}} \\ &\quad + \frac{(1 \text{ kg} + 7 \text{ kg})\,(7 \text{ N})}{1 \text{ kg} + 7 \text{ kg} + 6 \text{ kg}} \\ &= 10.4286 \text{ N}\,. \end{aligned}$$

Alternative Solution: Using Eq. 3 and the numerical result of Eq. 4, we have

$$\begin{aligned} F_{23} &= m_3\,a + F_r \quad (3) \\ &= (6 \text{ kg})\,(0.571429 \text{ m/s}^2) + 7 \text{ N} \\ &= 10.4286 \text{ N}\,. \end{aligned}$$