$$\underset{X}{\arg\min} \quad \frac{1}{2}\|Y - DX\|_2^2 + \beta\|X\|_1$$

$$\underset{X}{\arg\min} \quad \frac{1}{2}\|Y - DX\|_2^2 + \beta\|C\|_1$$
$$s.t.\ X = C$$

$$\underset{X,C}{\arg\min} \quad \frac{1}{2}\|Y - DX\|_2^2 + \beta\|C\|_1 + \frac{\mu}{2}\|X - C\|_2^2 + L^T(X - C)$$

$$\underset{X,C}{\arg\min} \quad \frac{1}{2}\|Y - DX\|_2^2 + \beta\|C\|_1 + \frac{\mu}{2}\|C - X\|_2^2 + L^T(X - C)$$

$$\mathrm{E}(X, C) = \frac{1}{2}\|Y - DX\|_2^2 + \beta\|C\|_1 + \frac{\mu}{2}\|X - C\|_2^2 + L^T(X - C)$$

$$\mathrm{E}(X, C) = \frac{1}{2}\|Y - DX\|_2^2 + \beta\|C\|_1 + \frac{\mu}{2}\|C - X\|_2^2 + L^T(X - C)$$

Solve for X:

$$\frac{\partial E}{\partial X} = -D^T(Y - DX) + \mu(X - C) + L = 0$$

$$X = \left(D^T D + \mu I\right)^{-1}\left(D^T Y + \mu C - L\right)$$

Solve for C:

$$\frac{\partial E}{\partial C} = \frac{\partial}{\partial C}\beta\|C\|_1 + \mu(C - X) - L = 0$$

$$C = sign\left(X + \frac{L}{\mu}\right) \cdot \max\left(\left|X + \frac{L}{\mu}\right| - \frac{\beta}{\mu}, 0\right)$$

Update Multiplier:

$$L^{k+1} = L^k + \mu(X - C)$$

Update Penalty:

$$\mu^{k+1} = \min\left(\tau \cdot \mu^k, \mu_{\max}\right)$$