Outline

1. Raw Data
2. Frequency Distribution - Histograms
3. Cumulative Frequency Distribution
4. Measures of Central Tendency
5. Mean, Median, Mode
6. Measures of Dispersion
7. Range, IQR, Standard Deviation,coefficient of variation
8. Normal distribution, Chebyshev Rule.
9. Five number summary, boxplots, QQ plots, Quantile plot, scatter plot.
10. Visualization: scatter plot matrix.
11. Correlation analysis

Data versus Information

When analysts are bewildered by plethora of data, which do not make any sense on the surface of it, they are looking for methods to classify data that would convey meaning. The idea here is to help them draw the right conclusion. Data needs to be arranged into information.

Raw Data

Raw Data represent numbers and facts in the original format in which the data have been collected. We need to convert the raw data into information for decision making.

Frequency Distribution

In simple terms, frequency distribution is a summarized table in which raw data are arranged into classes and frequencies.

Frequency distribution focuses on classifying raw data into information. It is a widely used data reduction technique in descriptive statistics.

Histogram

Histogram (also known as frequency histogram) is a snapshot of the frequency distribution.

Histogram is a graphical representation of the frequency distribution in which the X-axis represents the classes and the Y-axis represents the frequencies in bars.

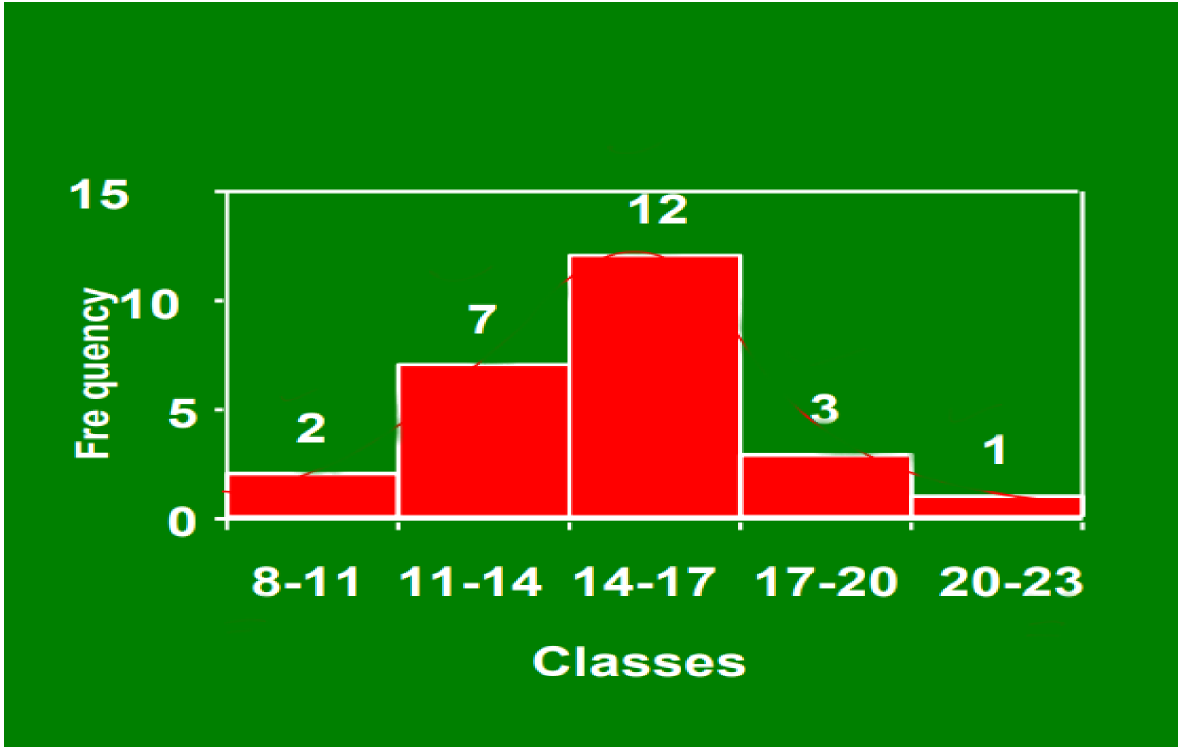
Histogram depicts the pattern of the distribution emerging from the characteristic being measured.

Histogram - Example

The inspection records of a hose assembly operation revealed a high level of rejection. An analysis of the records showed that the "leaks" were a major contributing factor to the problem. It was decided to investigate the hose clamping operation. The hose clamping force (torque) was measured on twenty five assemblies. (Figures in foot-pounds). The data are given below: Draw the frequency histogram and comment.

8	13	15	10	16
11	14	11	14	20
15	16	12	15	13
12	13	16	17	17
14	14	14	18	15

Histogram Example Solution

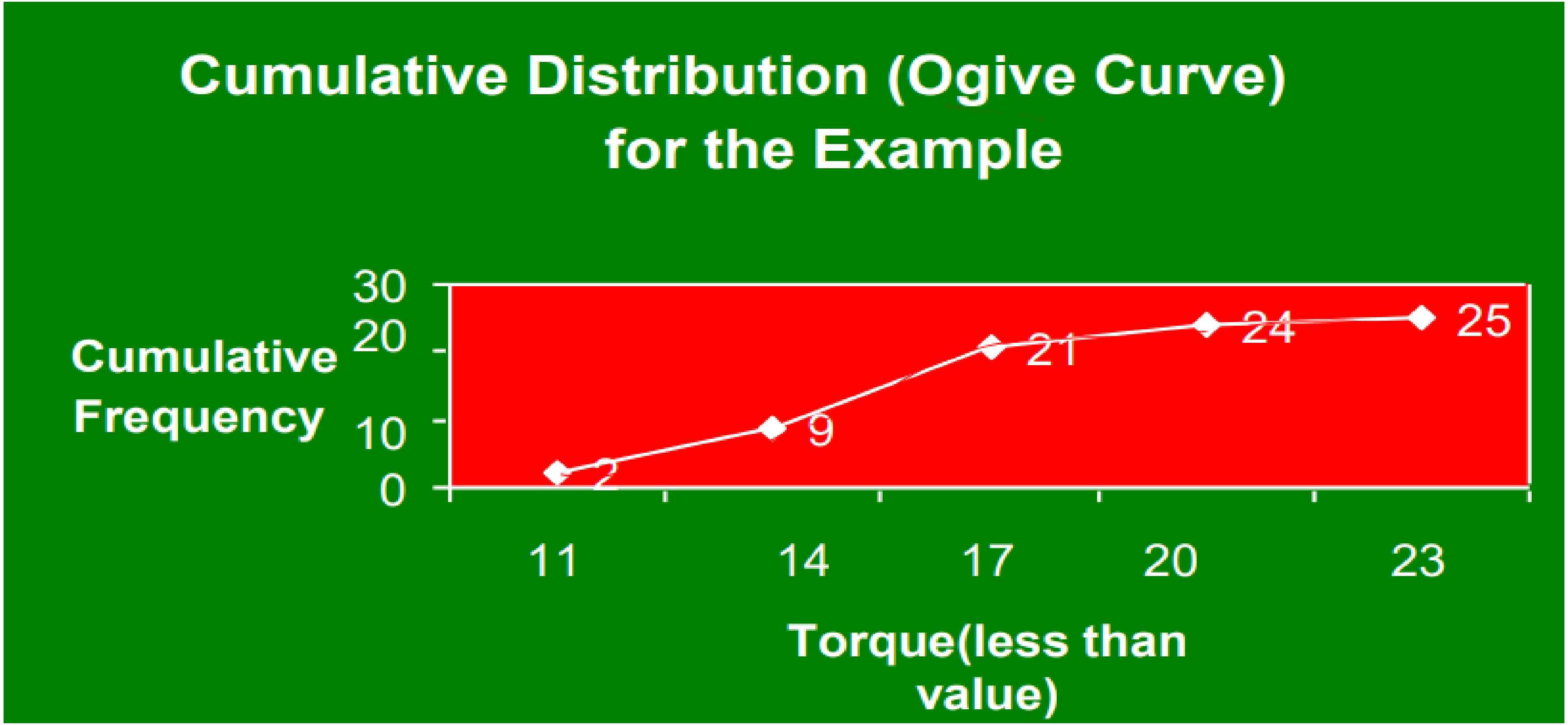


Cumulative Frequency Distribution

A type of frequency distribution that shows how many observations are above or below the lower boundaries of the classes. You can formulate the following from the previous example of hose clamping force(torque).

Class	Frequency	Relative Frequency	Cumulative Frequency	Cumulative Relative Frequency
8 - 11	2	0.08	2	0.08
11 - 14	7	0.28	9	0.36
14 - 17	12	0.48	21	0.84
17 - 20	3	0.12	24	0.96
20 - 23	1	0.04	25	1.00
Total	25	1.00		

Cumulative Distribution Function



What is Central Tendency?

Whenever you measure things of the same kind, a fairly large number of such measurements will tend to cluster around the middle value. Such a value is called a measure of "Central Tendency". The other terms that are used synonymously are "Measures of Location", or "Statistical Averages".

Arithmetic Mean

Arithmetic Mean (called mean) is defined as the sum of all observations in a data set divided by the total number of observations. For example, consider a data set containing the following observations:

In symbolic form mean is given by $\bar{X} = \frac{\Sigma X}{n}$

X = Arithmetic Mean

$\sum X$ = Indicates sum all X values in the data set

n = Total number of observations(Sample Size)

Arithmetic Mean -Example

The inner diameter of a particular grade of tire based on 5 sample measurements are as follows: (figures in millimeters)

565, 570, 572, 568, 585

Applying the formula $\bar{X} = \frac{\Sigma X}{n}$

We get mean = (565+570+572+568+585)/5 =572

Caution: Arithmetic Mean is affected by extreme values or fluctuations in sampling. It is not the best average to use when the data set contains extreme values (Very high or very low values).

Median

Median is the middle most observation when you arrange data in ascending order of magnitude. Median is such that 50% of the observations are above the median and 50% of the observations are below the median.

Median is a very useful measure for ranked data in the context of consumer preferences and rating. It is not affected by extreme values (greater resistance to outliers)

Median = $\frac{n+1}{2}$ th value of ranked data

n = Number of observations in the sample

Median - Example

Marks obtained by 7 students in Computer Science Exam are given below: Compute the median.

45 40 60 80 90 65 55

Arranging the data after ranking gives

90 80 65 60 55 45 40

Median = (n+1)/2 th value in this set = (7+1)/2 th observation= 4th observation=60

Hence Median = 60 for this problem.

Mode

Mode is that value which occurs most often. It has the maximum frequency of occurrence. Mode also has resistance to outliers.

Mode is a very useful measure when you want to keep in the inventory, the most popular shirt in terms of collar size during festive season.

Mode - Example

The life in the number of hours of 10 flashlight batteries is as follows: Find the mode.

340 350 340 340 320 340 330 330
340 350

340 occurs five times. Hence, mode=340.

Comparison of Mean, Median, Mode

Mean	Median	Mode
Defined as the arithmetic average of all observations in the data set.	Defined as the middle value in the data set arranged in ascending or descending order.	Defined as the most frequently occurring value in the distribution; it has the largest frequency.
Requires measurement on all observations.	Does not require measurement on all observations	Does not require measurement on all observations
Uniquely and comprehensively defined.	Cannot be uniquely determined under all conditions.	Not uniquely defined for multi-modal situations.

Comparison of Mean, Median, Mode Cont.

Mean	Median	Mode
Affected by extreme values. Can be treated algebraically. That is, the means of several groups can be combined.	Not affected by extreme values. Cannot be treated algebraically. That is, Medians of several groups cannot be combined.	Not affected by extreme values. Cannot be treated algebraically. That is, Modes of several groups cannot be combined.

Measures of Dispersion

In simple terms, measures of dispersion indicate how large the spread of the distribution is around the central tendency. It answers unambiguously the question " What is the magnitude of departure from the average value for different groups having identical averages?".

Range

Range is the simplest of all measures of dispersion. It is calculated as the difference between maximum and minimum value in the data set.

$$\text{Range} = X_{\text{Maximum}} - X_{\text{Minimum}}$$

Range - Example

Example for Computing Range

The following data represent the percentage return on investment for 10 mutual funds per annum. Calculate Range.

12, 14, 11, 18, 10.5, 11.3, 12, 14, 11, 9

Range = $\mathbf{X}_{\mathbf{Maximum}} - \mathbf{X}_{\mathbf{Minimum}}$ = 18 – 9=9

Caution: If one of the components of the range namely the maximum value or minimum value becomes an extreme value, then the range should not be used.

Inter Quartile Range(IQR)

IQR= Range computed on middle 50% of the observations after eliminating the highest and lowest 25% of observations in a data set that is arranged in ascending order. IQR is less affected by outliers.

$IQR = Q_3 - Q_1$

Inter Quartile Range - Example

The following data represent the annual percentage returns of 9 mutual funds.

Data Set: 12, 14, 11, 18, 10.5, 12, 14, 11, 9

Arranging in ascending order, the data set becomes

9, 10.5, 11, 11, 12, 12, 14, 14, 18

$IQR = Q_3 - Q_1 = 14 - 10.75 = 3.25$

Standard Deviation

To define standard deviation, you need to define another term called variance. In simple terms, standard deviation is the square root of variance.

Example for Standard Deviation

The following data represent the percentage return on investment for 10 mutual funds per annum. Calculate the sample standard deviation.

12, 14, 11, 18, 10.5, 11.3, 12, 14, 11, 9

Solution for the Example

A	B	C	D
1			
2	X	$x - \bar{x}$	$(x - \bar{x})^2$
3	12	-0.28	0.08
4	14	1.72	2.96
5	11	-1.28	1.64
6	18	5.72	32.72
7	10.5	-1.78	3.17
8	11.3	-0.98	0.96
9	12	-0.28	0.08
10	14	1.72	2.96
11	11	-1.28	1.64
12	9	-3.28	10.76
13	Mean =		56.96
14	12.28	Variance =	6.33
15		Standard Deviation =	2.52

Standard Deviation Formula

Coefficient of Variation (Relative Dispersion)

Coefficient of Variation (CV) is defined as the ratio of Standard Deviation to Mean. In symbolic form

CV = $\frac{S}{\bar{X}}$ for the sample data and = $\frac{\sigma}{\mu}$ for the population

Coefficient of Variation Example

Consider two SalesPersons working in the same territory.

The sales performance of these two in the context of selling PCs are given below. Comment on the results.

Sales Person 1

Mean Sales (One year average)

50 units

Standard Deviation

5 units

Sales Person 2

Mean Sales (One year average)

75 units

Standard deviation

25 units

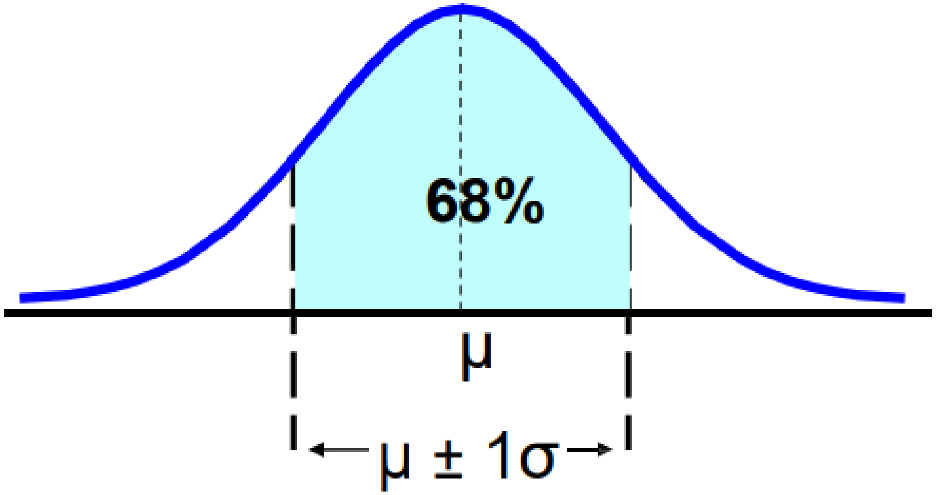
Interpretation for the Example

The CV is 5/50 =0.10 or 10% for the Sales Person 1 and 25/75=0.33 or 33% for sales Person 2.

The moral of the story is "don't get carried away by by averages. Consider variation (“risk”).

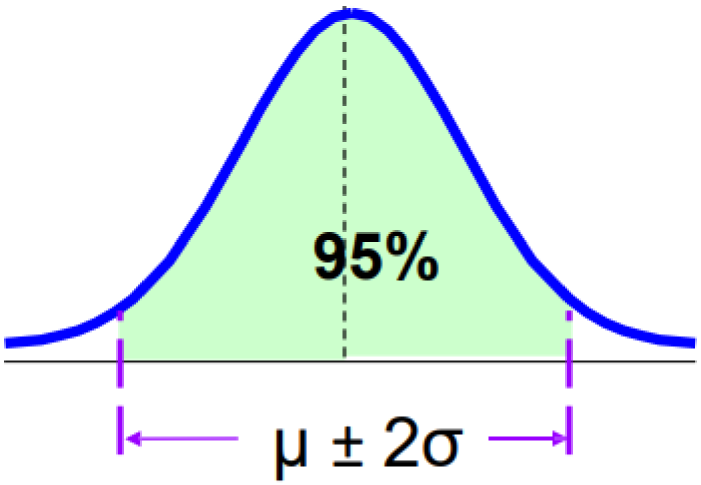
The Empirical Rule

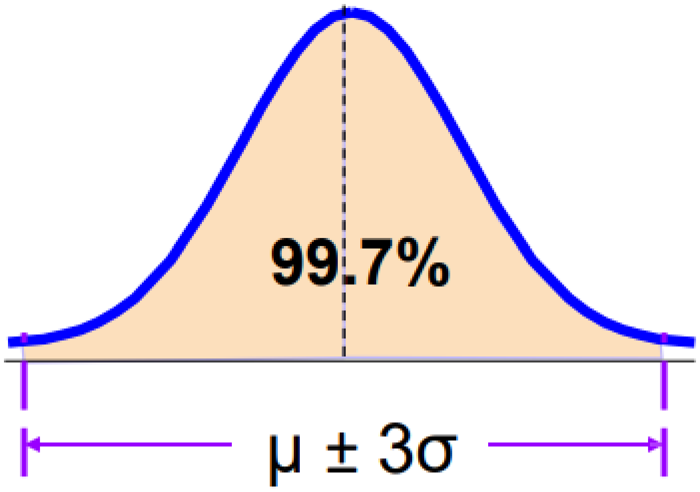
- The empirical rule approximates the variation of data in a bell-shaped distribution
- Approximately 68% of the data in a bell shaped distribution is within 1 standard deviation of the mean or **$\mu \pm 1\sigma$**



The Empirical Rule

- Approximately 95% of the data in a bell-shaped distribution lies within two standard deviations of the mean, or $\mu \pm 2\sigma$
- Approximately 99.7% of the data in a bell-shaped distribution lies within three standard deviations of the mean, or $\mu \pm 3\sigma$





Chebyshev Rule

Regardless of how the data are distributed, at least (1 - 1/k2) x 100% of the values will fall within k standard deviations of the mean (for k > 1).

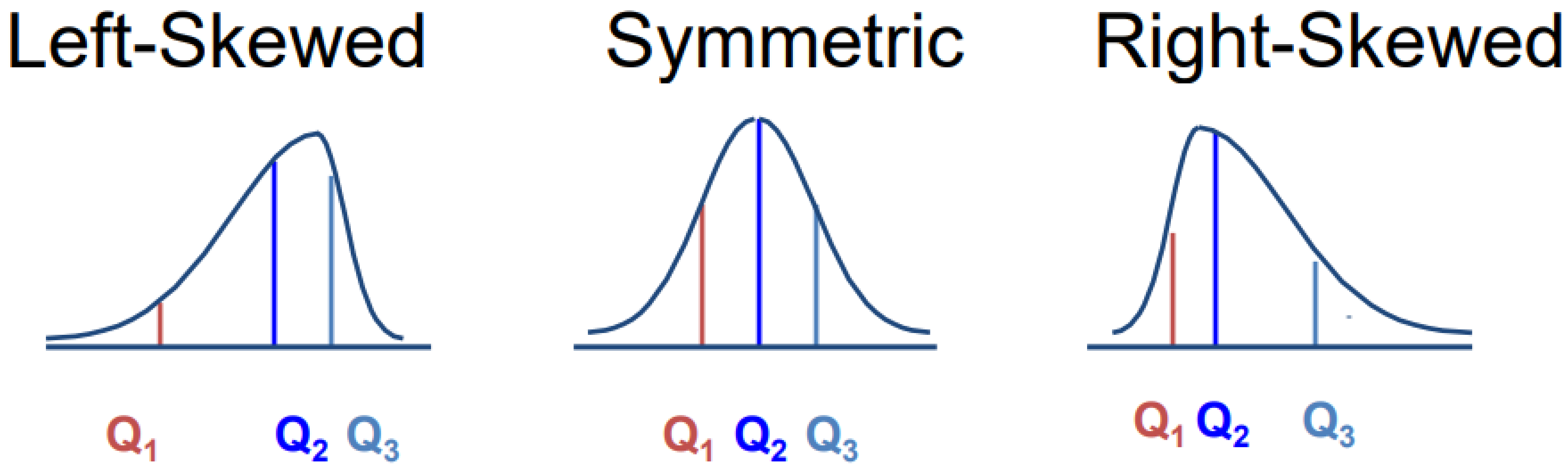
For Example, when k=2, at least 75% of the values of any data set will be within **$\mu \pm 2\sigma$**.

The Five Number Summary

The five numbers that help describe the center, spread and shape of data are:

- $X_{smallest}$
- First Quartile (Q_1)
- Median (Q_2)
- Third Quartile (Q_3)
- $X_{largest}$

Distribution Shape

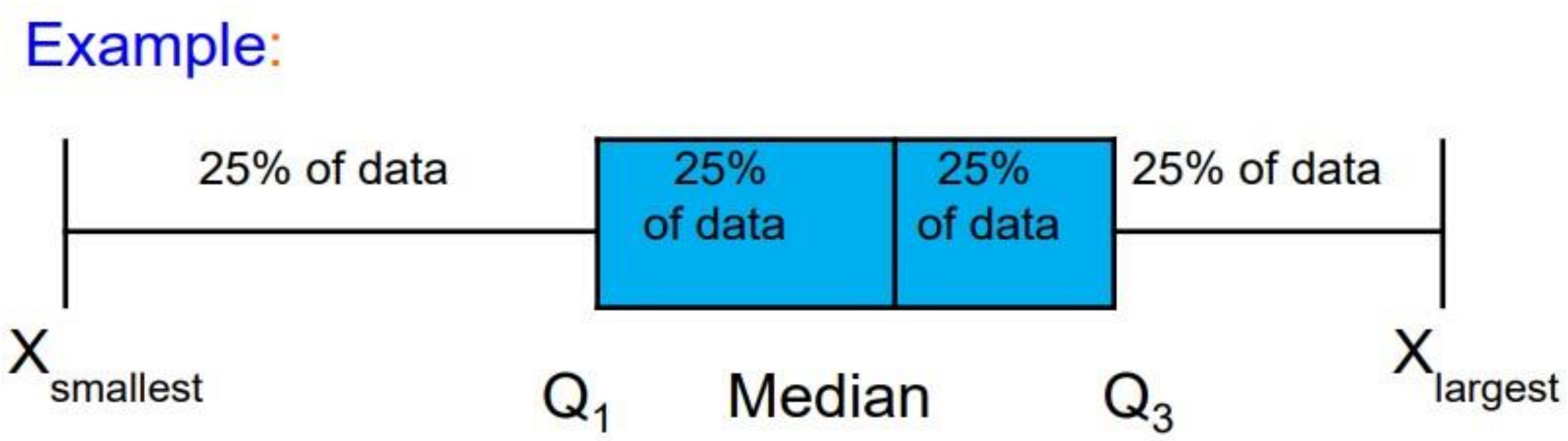


Relationships among the five-number summary and distribution shape

Left-Skewed	Symmetric	Right-Skewed
$\text{Median} - X_{smallest}$ > $X_{largest} - \text{Median}$	$\text{Median} - X_{smallest}$ ≈ $X_{largest} - \text{Median}$	$\text{Median} - X_{smallest}$ < $X_{largest} - \text{Median}$
$Q_1 - X_{smallest}$ > $X_{largest} - Q_3$	$Q_1 - X_{smallest}$ ≈ $X_{largest} - Q_3$	$Q_1 - X_{smallest}$ < $X_{largest} - Q_3$
$\text{Median} - Q_1$ > $Q_3 - \text{Median}$	$\text{Median} - Q_1$ ≈ $Q_3 - \text{Median}$	$\text{Median} - Q_1$ < $Q_3 - \text{Median}$

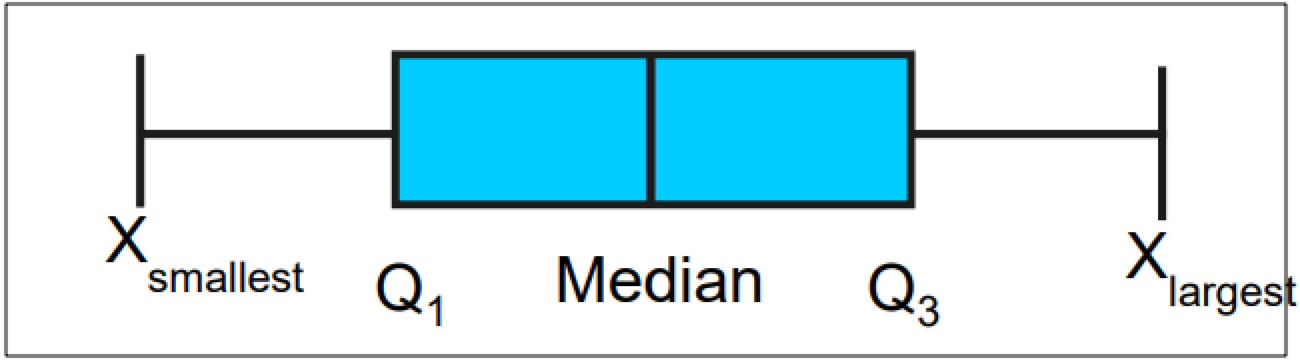
Five Number Summary and The Boxplot

The Boxplot: A Graphical display of the data based on the five-number summary:



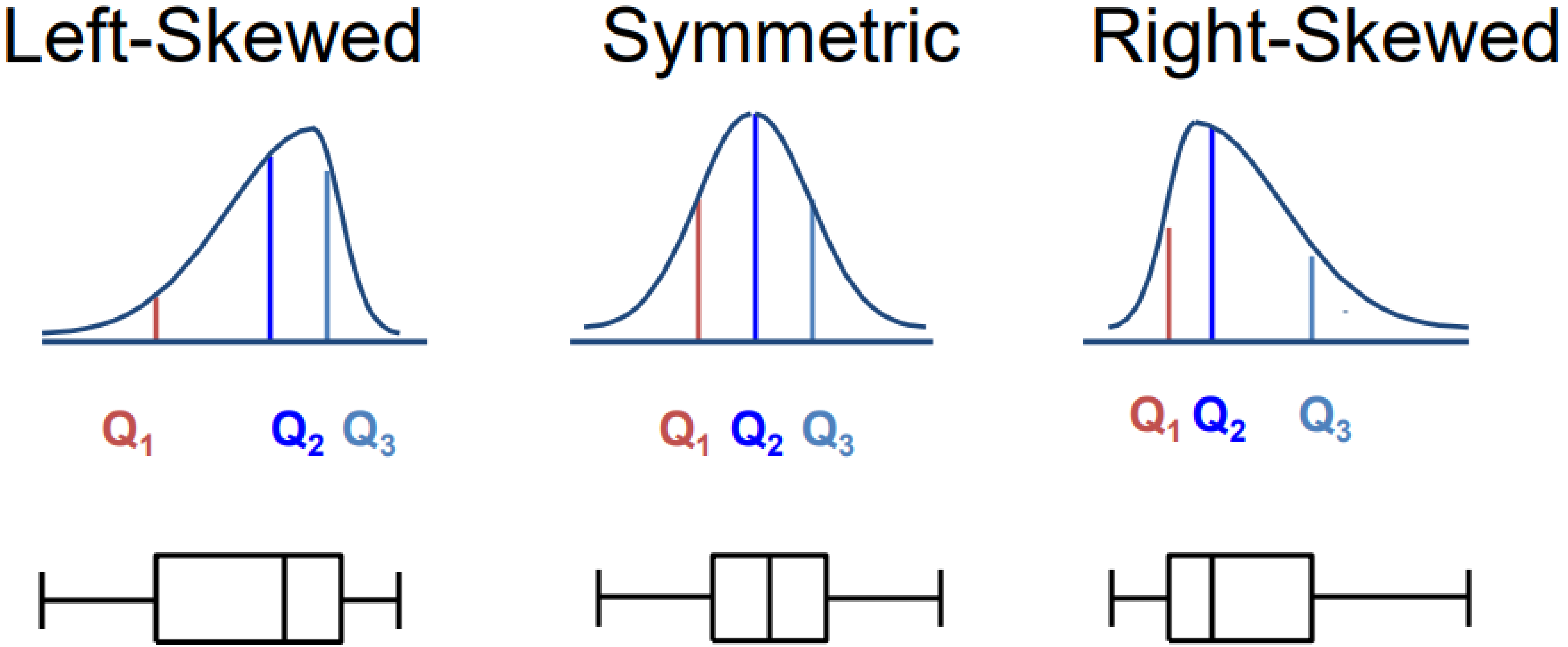
Five Number Summary: Shape of Boxplots

- If data are symmetric around the median then the box and central line are centered between the endpoints.



- A Boxplot can be shown in either a vertical or horizontal orientation.

Distribution Shape and The Boxplot



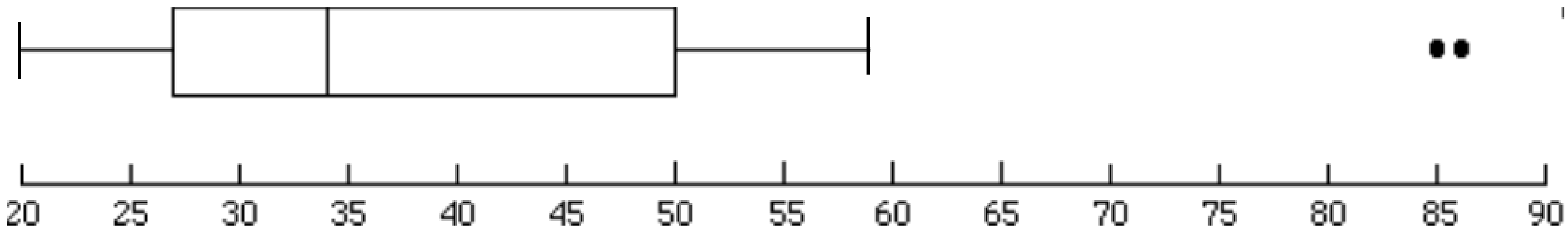
Boxplot Example

The data are right skewed in the following plot.

Box plot example showing an outlier

The boxplot below of the same data shows the outlier value of 27 plotted separately.

A value is considered an outlier if it is more than 1.5 times the interquartile range below Q_1 or above Q_3.



Graphic Displays of Basic Statistical Descriptions

Boxplot: graphic display of five-number summary

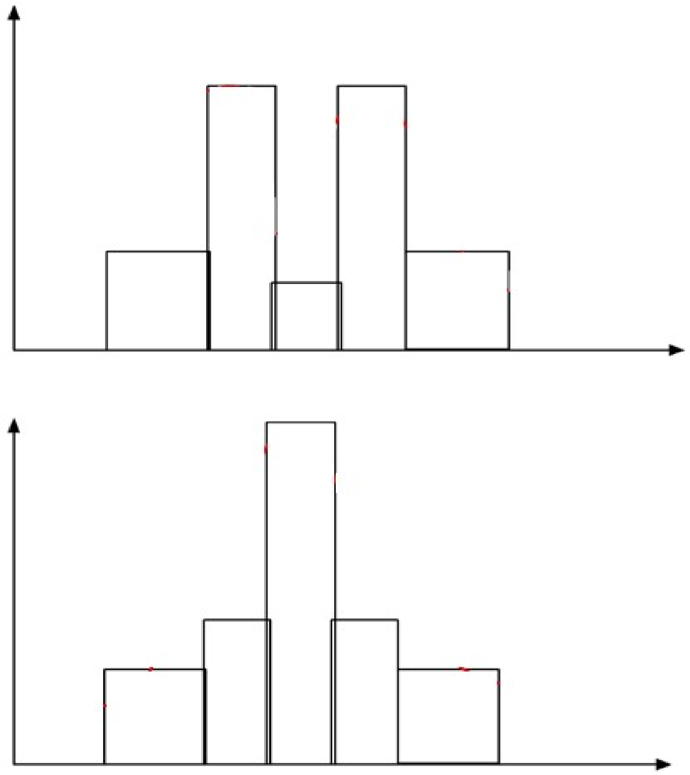
Histogram: x-axis are values, y-axis repres. frequencies

Quantile plot: each value x_i is paired with f_i indicating that approximately 100 f_i % of data are $\leq x_i$

Quantile-quantile (q-q) plot: graphs the quantiles of one univariate distribution against the corresponding quantiles of another

Scatter plot: each pair of values is a pair of coordinates and plotted as points in the plane

Histograms Often Tell More than Boxplots



The two histograms shown in the left may have the same boxplot representation

The same values for: min, Q1, median, Q3, max

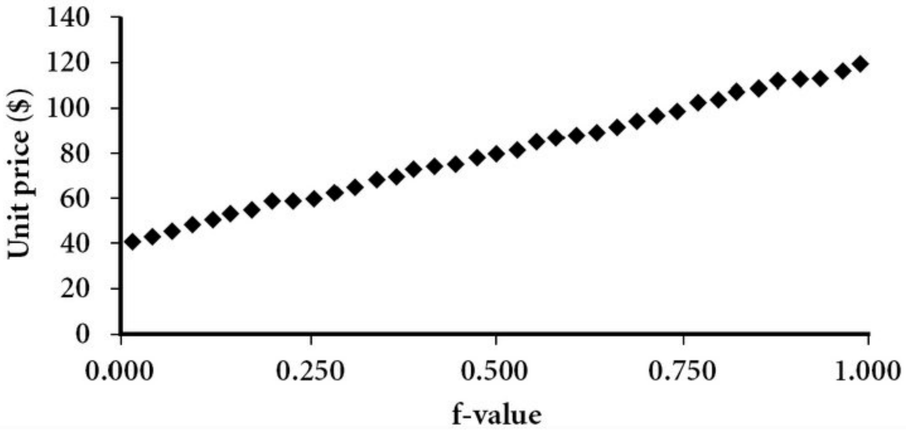
But they have rather different data distributions

Quantile Plot

Displays all of the data (allowing the user to assess both the overall behavior and unusual occurrences).

Plots **quantile** information.

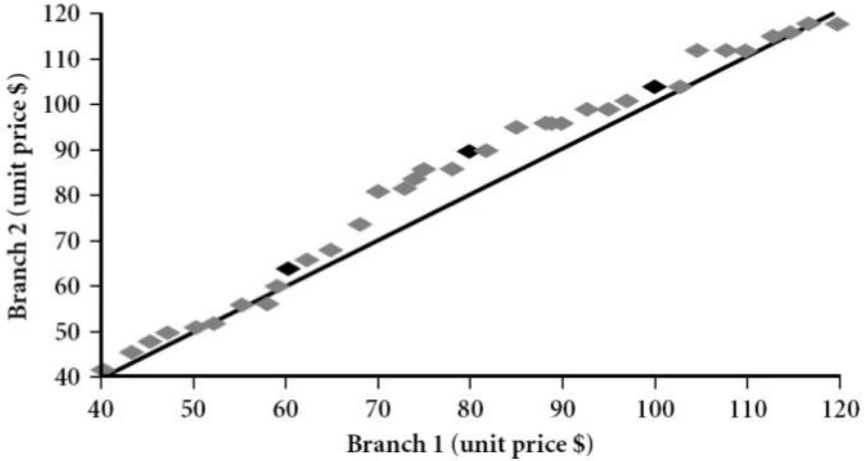
For a data x_i data sorted in increasing order, f_i indicates that approximately 100 f_i% of the data are below or equal to the value x_i.



Quantile-Quantile (Q-Q) Plot

Graphs the quantiles of one univariate distribution against the corresponding quantiles of another.

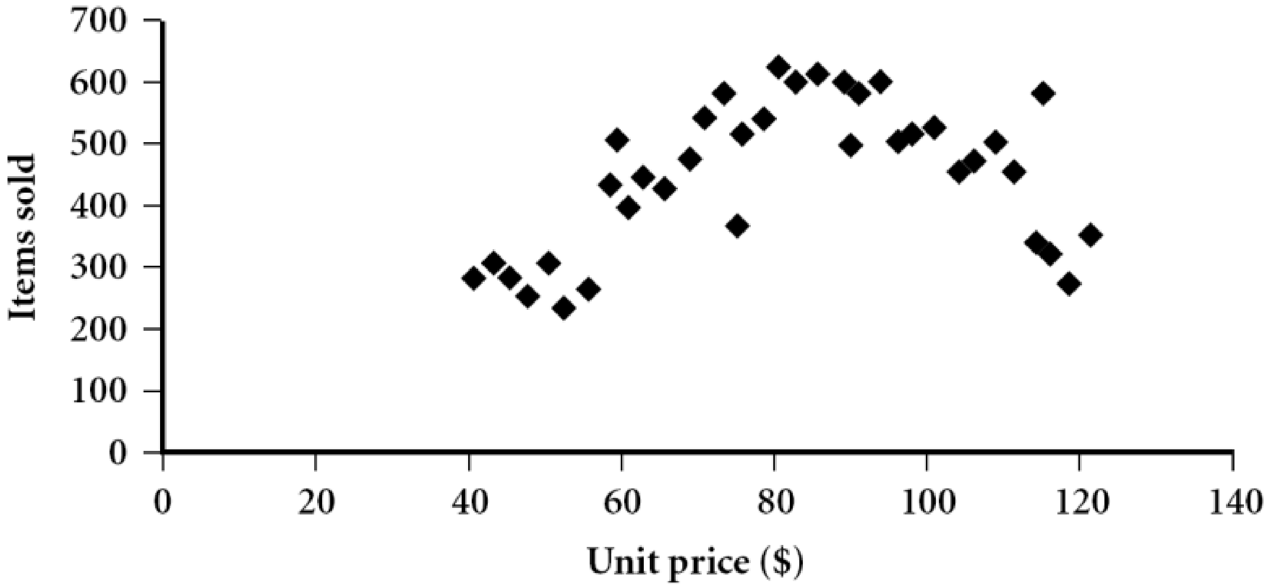
View: Is there is a shift in going from one distribution to another? Example shows unit price of items sold at Branch 1 vs. Branch 2 for each quantile. Unit prices of items sold at Branch 1 tend to be lower than those at Branch 2.



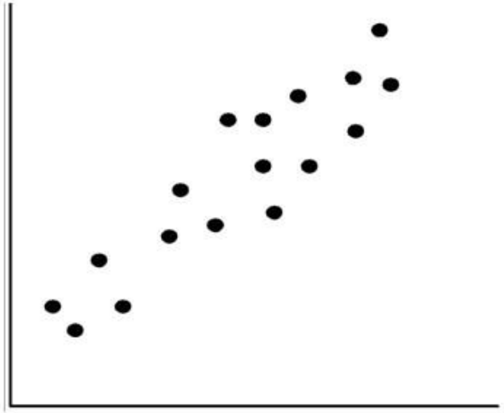
Scatter plot

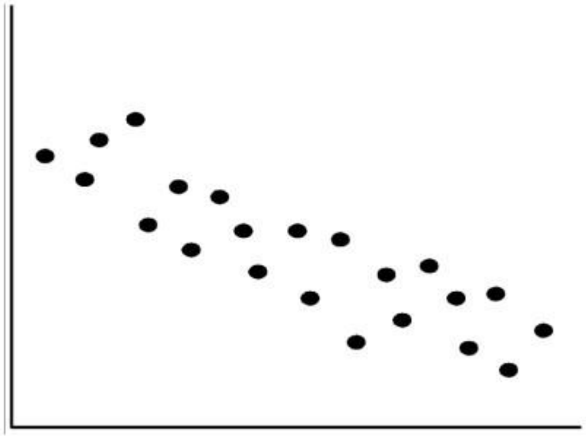
Provides a first look at bivariate data to see clusters of points, outliers, etc

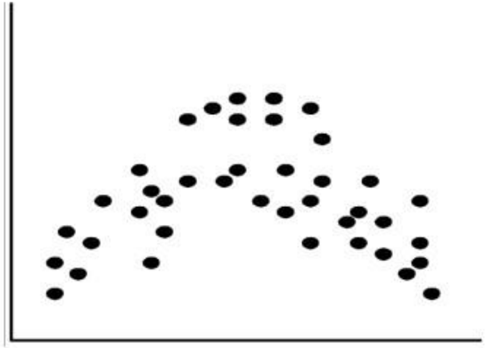
Each pair of values is treated as a pair of coordinates and plotted as points in the plane.



Positively and Negatively Correlated Data



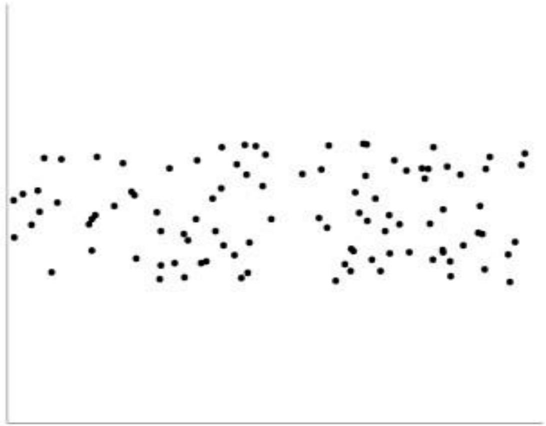


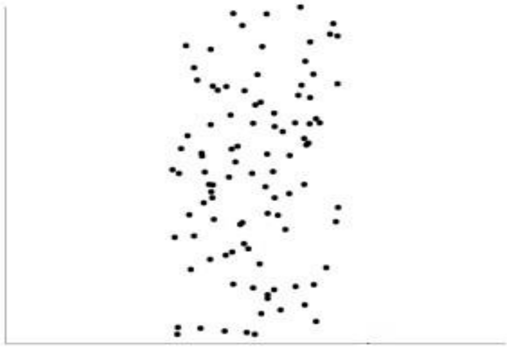


The left half fragment is positively correlated

The right half is negative correlated

Uncorrelated Data





Data Visualization

Why data visualization?

- Gain insight into an information space by mapping data onto graphical primitives
- Provide qualitative overview of large data sets
- Search for patterns, trends, structure, irregularities, relationships among data
- Help find interesting regions and suitable parameters for further quantitative analysis
- Provide a visual proof of computer representations derived

Categorization of visualization methods:

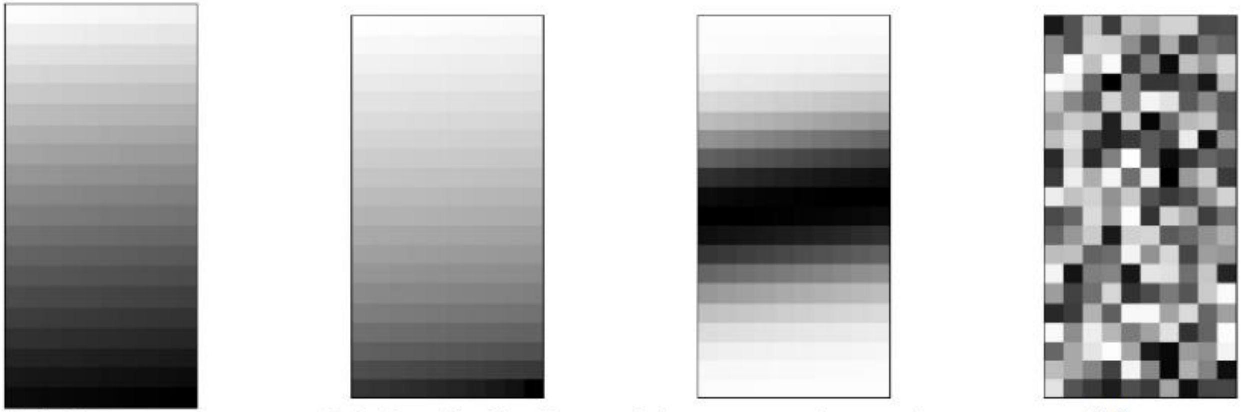
- Pixel-oriented visualization techniques
- Geometric projection visualization techniques
- Icon-based visualization techniques
- Hierarchical visualization techniques
- Visualizing complex data and relations

Pixel-Oriented Visualization Techniques

For a data set of m dimensions, create m windows on the screen, one for each dimension

The m dimension values of a record are mapped to m pixels at the corresponding positions in the windows

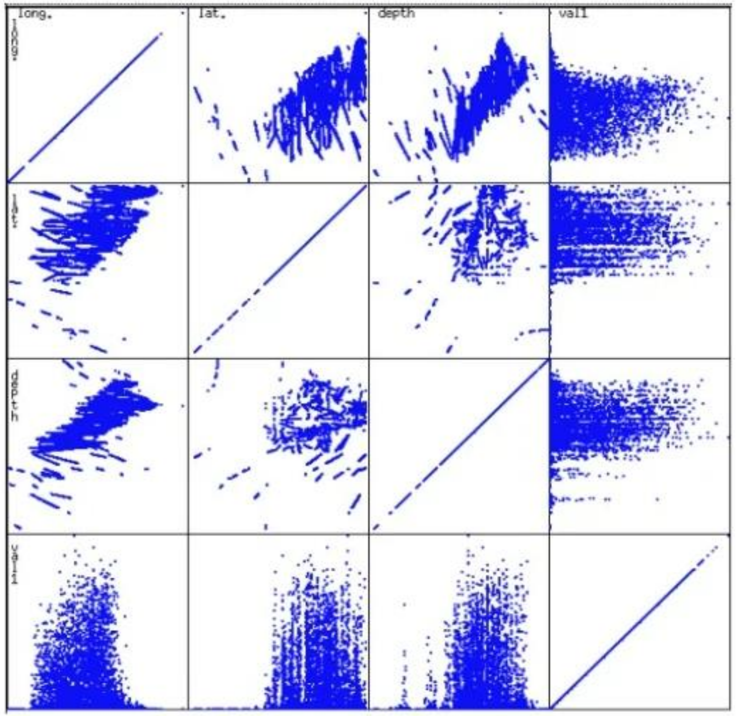
The colors of the pixels reflect the corresponding values.



(a) Income (b) Credit Limit (c) transaction volume (d) age

Scatterplot Matrices

Used by_ermission of M. Ward, Worcester Polytechnic Institute



Matrix of scatterplots (x-y-diagrams) of the k-dim. data [total of (k2/2-k) scatterplots]

Correlation Analysis (Nominal Data): ChiSquare Test

	Play chess	Not play chess	Sum (row)
Like science fiction	250(90)	200(360)	450
Not like science fiction	50(210)	1000(840)	1050
Sum(col.)	300	1200	1500

$$e_{ij} = \frac{count(A = a_i)\ X\ count(B = b_j)}{n}$$

$$e_{11} = \frac{count(male)\ X\ count(fiction)}{n} = \frac{300\ X\ 450}{1500} = 90$$

For this 2 × 2 table, the degrees of freedom are (2 − 1)(2 − 1) = 1. For 1 degree of freedom, the 2 value needed to reject the hypothesis at the 0.001 significance level is 10.828

X^2 (chi-square) calculation (numbers in parenthesis are expected counts calculated based on the data distribution in the two categories).

$$\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected}$$

$$\chi^2 = \frac{(250-90)^2}{90} + \frac{(50-210)^2}{210} + \frac{(200-360)^2}{360} + \frac{(1000-840)^2}{840} = 507.93$$

Shows that like_science_fiction and play_chess are correlated in the group.

Correlation Analysis (Numeric Data)

Correlation coefficient (also called Pearson's product moment coefficient)

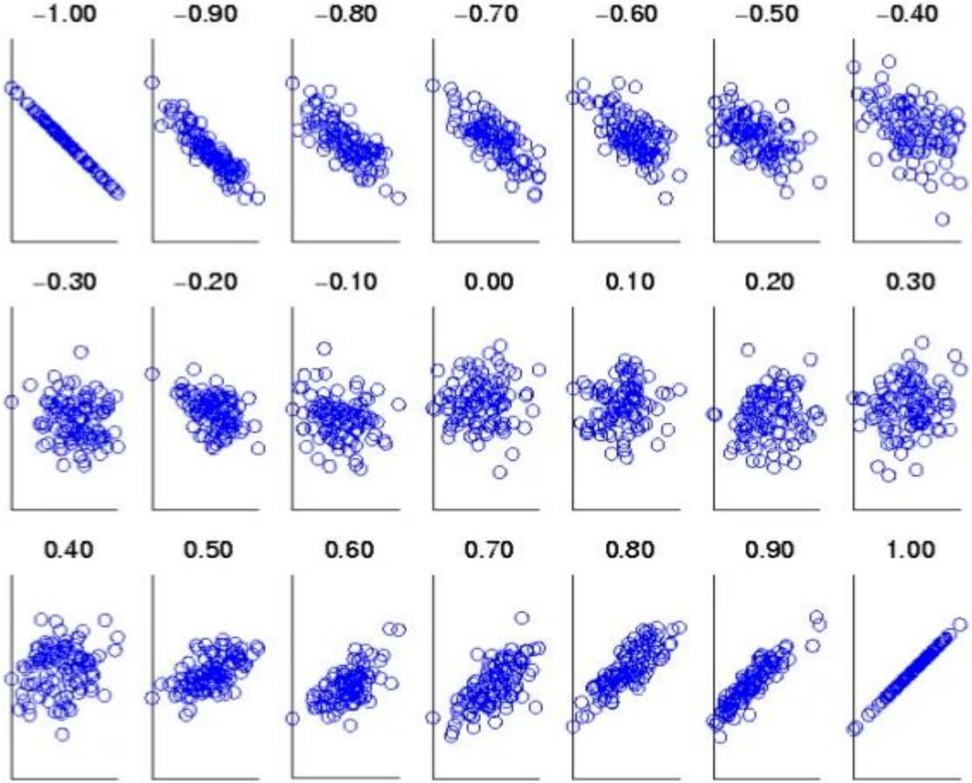
$$r_{A,B} = \frac{\sum_{i=1}^{n}(a_i - \bar{A})(b_i - \bar{B})}{(n-1)\sigma_A \sigma_B} = \frac{\sum_{i=1}^{n}(a_i b_i) - n\overline{AB}}{(n-1)\sigma_A \sigma_B}$$

where n is the number of tuples $\bar{A}$ and $\bar{B}$ are the respective means of A and B, σ_A and σ_B are the respective standard deviation of A and B, and $\Sigma(a_i b_i)$ is the sum of the AB cross-product.

If $r_{A,B} > 0$, A and B are positively correlated (A's values increase as B's). The higher, the stronger correlation.

$r_{A,B} = 0$: independent; $r_{A,B} < 0$: negatively correlated

Visually Evaluating Correlation



Scatter plots showing the similarity from – 1 to 1.

Summary

- Histograms
- Measures of central tendency: mean, mode, median
- Measures of dispersion: range, IQR, variance, std deviation, coefficient of variation.
- Normal distribution, Chebyshev Rule.
- Five number summary, boxplots, QQ plots, Quantile plot, scatter plot.
- Visualization: scatter plot matrix, parallel coordinates.
- Correlation analysis.