OISP 182 – Calculus 2 (MT 1005 – MT 1006). Midterm Review Answer (With Error!)

ATTENTION: THERE IS ONE ERROR IN EACH ANWSER. NEVERTHELESS, FINAL RESULT IS STILL RIGHT. STUDENT NEED TO FIND THESE ERRORS. DON'T COPY COMPLETE ANSWERS BELOW WITHOUT CORRECTING

1/ Given $z(x,y)=\sqrt{3x^2-5xy+2y^2}$, find $z_x(1;2)$, $z_y(1;2)$, $dz(1;2)$.

Ans: (With error) $\frac{\partial z}{\partial x}=\frac{u_x}{2\sqrt{u}}=\frac{6x-5y+4y}{2\sqrt{3x^2-5xy+2y^2}}$, $\frac{\partial z}{\partial y}=\frac{u_y}{2\sqrt{u}}=\frac{6x-5x+4y}{2\sqrt{3x^2-5xy+2y^2}}$

$\Rightarrow \frac{\partial z}{\partial x}(1;2)=-2$, $\frac{\partial z}{\partial y}(1;2)=\frac{3}{2}$, $dz(1;2) = -2dx+\frac{3}{2}dy$

2/ Consider z(x, y) = $\ln(2x+3y)$, $\begin{cases} x=x(u,v) \\ y=y(u,v) \end{cases}$. At $\begin{cases} u=u_0=4 \\ v=v_0=5 \end{cases}$ we have: $x=1$, $y=2$, $x_u=4$, $y_u=3$, $x_v=5$, $y_v=6$ find A = $4\frac{\partial z}{\partial u}+7\frac{\partial z}{\partial v}$ at $(u_0;v_0)$.

Ans: Chain rule: $\frac{\partial z}{\partial u}=z_x\cdot x_u+z_y\cdot y_u$, $\frac{\partial z}{\partial v}=z_x\cdot x_v+z_y\cdot y_v$.

(With error) $z_x=z_y=\frac{1}{2x+3y}$. Substitution $\Rightarrow \frac{\partial z}{\partial u}=\frac{17}{8}$, $\frac{\partial z}{\partial v}=\frac{7}{2}$ $\Rightarrow$ A = 33

3/ Using x hours of skilled labor and y hours of unskilled labor, a manufacturer can produce $Q(x,y) = 81x\cdot\sqrt[3]{y}$ units. Currently 32 hours of skilled labor and 27 hours of unskilled labor are being used. Suppose the manufacturer reduces the skilled labor level by 2 hours and increases the unskilled labor level by 4 hours. Use calculus to determine the approximate effect of these changes on production.

Ans: Evaluating partial derivatives $\Rightarrow$ $Q_x(32;27)=243$, $Q_y(32;27)=96$.

(With error) $\Delta Q\approx Q_x\Delta x+Q_y\Delta y$, where $\Delta x = 2$, $\Delta y = 4$ $\Rightarrow$ $\Delta Q \approx -102$: output will decrease about 102 unit.

4/ Study extrema of $z=x^3+y^2-xy$.

Ans: (With error) $\begin{cases} z_x=0 \\ z_y=0 \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases}$: A(0; 0), $\begin{cases} x=6 \\ y=12 \end{cases}$: B(6; 12).

Result: At A: No extrema. At B: local minimum.

5/ Find max, min of $f(x,y)=x^2+2y^2-x+2$ subject to constraint $x^2+y^2=4$.

Ans: (With error) L(x, y, λ) = $x^2+2y^2-x+2+\lambda(x^2+y^2)$.

$\begin{cases} L_x=0 \\ L_y=0 \\ x^2+y^2=4 \end{cases} \Rightarrow$ A$(2;0)$, B$(-2;0)$, C$\left(-\frac{1}{2};\frac{\sqrt{15}}{2}\right)$, D$\left(-\frac{1}{2};-\frac{\sqrt{15}}{2}\right)$.

Evaluate $f(x,y)$ at A, B, C, D $\Rightarrow$ max = $\frac{41}{4}$, min = 4.

6/ Find max, min of $f(x, y) = 2x + 3y - 1$ in the domain $x^2 + y^2 \le 9$

Ans: Find critical points:

Inside $\Rightarrow$ $x^2 + y^2 < 9$ $\Rightarrow \begin{cases} f_x = 2 = 0 \\ f_y = 3 = 0 \\ x^2 + y^2 < 9 \end{cases}$: No solution $\Rightarrow$ No critical points inside.

On boundary: $x^2 + y^2 = 9$ $\Rightarrow$ L(x, y, λ) = $2x + 3y - 1 + \lambda(x^2 + y^2 - 9)$.

(With error) $\begin{cases} L_x = 2 + 2\lambda x = 0 \\ L_y = 3 + 2\lambda y = 0 \\ x^2 + y^2 = 9 \end{cases} \Rightarrow x = -\dfrac{1}{\lambda},\ y = -\dfrac{3}{2\lambda} \Rightarrow$

$x^2 + y^2 = \dfrac{1}{\lambda^2} + \dfrac{9}{4\lambda^2} = 9 \Rightarrow \lambda = \dfrac{\sqrt{13}}{6}$ $\Rightarrow$ Critical point: $\left(-\dfrac{6}{\sqrt{13}}; -\dfrac{9}{\sqrt{13}}\right)$.

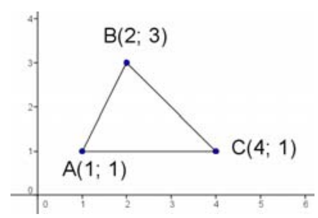
Evaluate f at critical points $\Rightarrow$ min = $-3\sqrt{13} - 1$, max = $3\sqrt{13} - 1$.

7/ Find the mass of the plate D with $\rho(x, y) = 2x + 3y$, when D is region bounded by $x = 2y^2$ and $x = y^2 + 4$.

Ans: Intercept equation: $2y^2 = y^2 + 4 \Leftrightarrow y = \pm 2 \Rightarrow$ D: $\begin{cases} -2 \le y \le 2 \\ 2y^2 \le x \le y^2 + 4 \end{cases}$

(With error) $\Rightarrow$ I = $\displaystyle\int_{2y^2}^{y^2+4} \int_{-2}^{2} (2x - 3y)\,dxdy = \frac{1024}{15}$.

8/ Find the total charge of the region D given in the next picture, when the charge density is distributed by $\sigma(x, y) = 2x^2 + 3y$.



Ans: The total charge = $\displaystyle\iint_D \sigma(xy)\,dA = \iint_D (2x^2 + 3y)\,dA$.

(With error) D: $\begin{cases} 1 \le x \le 3 \\ x_{AB} \le y \le x_{BC} \end{cases}$. AB: $x = \dfrac{y+1}{2}$, BC: $x = 5 - y$ $\Rightarrow$ Charge = 50.

9/ Evaluate $\displaystyle\iint_D e^{2\sqrt{x^2+y^2}}\,dA$ when D is in the upper half plane and bounded by the circle $x^2 + y^2 = 1$.

Ans: Polar coordinates $\Rightarrow$ D: $\begin{cases} 0 \le r \le 1 \\ 0 \le \varphi \le \pi \end{cases}$

(With error) $\Rightarrow$ I = $\displaystyle\int_0^1 \int_0^{\pi} e^{2r}\,d\varphi dr = \frac{\pi(e^2 + 1)}{4}$.

10/ Find the volume of the solid under the plane x + 2y – z = 0 and above the region bounded by y = x and y = x^4.

Ans: Solid (E): under the plane $z = x + 2y$ & above $z = 0 \Rightarrow 0 \le z \le x + 2y$.

Domain D: bounded by $\left| \begin{array}{l} y = x \\ y = x^4 \end{array} \right.$. Intercept equation: $x^4 = x \Rightarrow \left[\begin{array}{l} x = 0 \\ x = 1 \end{array} \right.$. Test with

$x = \frac{1}{2} \in (0;1)$: $\frac{1}{2^4} < \frac{1}{2} \Rightarrow x^4 \le x \ \forall \ x \in [0;1]$

(With error) $\Rightarrow$ D = $\begin{cases} 0 \le x \le 1 \\ x \le y \le x^4 \end{cases} \Rightarrow$ Volume V = $\int_0^1 \int_x^{x^4} [(x+2y) - 0] dydx = \frac{7}{18}$

11/ Find the volume of the solid under the surface z = xy and above the triangle with vertices (1; 1), (4; 1) and (1; 2).

Ans: (E): $0 \le z \le xy$. Sketch the triangle D $\Rightarrow \begin{cases} 1 \le x \le 4 \\ y_{AB} \le y \le y_{BC} \end{cases}$.

(With error) AB: $y = 1$, BC: $y = \frac{7+x}{3} \Rightarrow$ V = $\int_1^4 \int_1^{\frac{7+x}{3}} [(xy) - 0] dydx = \frac{31}{8}$

12/ Evaluate volume of the solid bounded by planes $z = y$, $z = 3y$ and inside the cylinder $x^2 + y^2 = 4y$.

Ans: D: $x^2 + y^2 \le 4y \Rightarrow$ V = $\iint_D (3y - y) dA$. Polar coordinates

(With error) $\Rightarrow$ D: $\begin{cases} 0 \le \varphi \le 2\pi \\ 0 \le r \le 4\sin\varphi \end{cases} \Rightarrow$ V = $\int_0^{2\pi} \int_0^{4\sin\varphi} 2r\sin\varphi \cdot r dr d\varphi = 2\pi$.