Homework 6, due 10-13 (Wednesday)

Quarks are fermions with spin and isospin 1/2. As usual, we will denote the two spin states by $|\uparrow\rangle$ and $|\downarrow\rangle$. The two isospin states are called up and down, $|u\rangle$ and $|d\rangle$. In addition to that, quarks carry a new quantum number called color. Color is different from spin and isospin in that the wave function is described by a three-dimensional vector. The three basis states are usually called red, blue and green, $|r\rangle$, $|b\rangle$ and $|g\rangle$.

The laws of QCD imply that boundstates of quarks have to be color neutral. This requirement is most easily satisfied by bound states of quarks and anti-quarks ($|r\rangle|\bar{r}\rangle + \ldots$) or boundstates of three quarks ($|r\rangle|b\rangle|g\rangle + \ldots$). In this problem set we will consider boundstates of only two quarks, called diquarks. These states are not color neutral, but they play an important role as building blocks for more complicated objects.

1. First, assume that color does not exist. What are the allowed spin-isospin quantum numbers of s-wave diquarks?

2. Take color into account. There are nine possible diquark color wave functions. Construct symmetric and anti-symmetric color wave functions. How many symmetric/anti-symmetric wave functions are there? What are the spin-isospin quantum numbers of color (anti)symmetric s-wave diquarks?

3. Assume that quarks interact through a potential of the form

$$V = V_0 + V_1(\vec{\tau}_1 \cdot \vec{\tau}_2),$$

   where V_0 is negative and V_1 is positive. What is the spin and isospin of the most tightly bound color anti-symmetric diquark?