The chiral critical surface of QCD for $\frac{\mu}{T} \lesssim 1$

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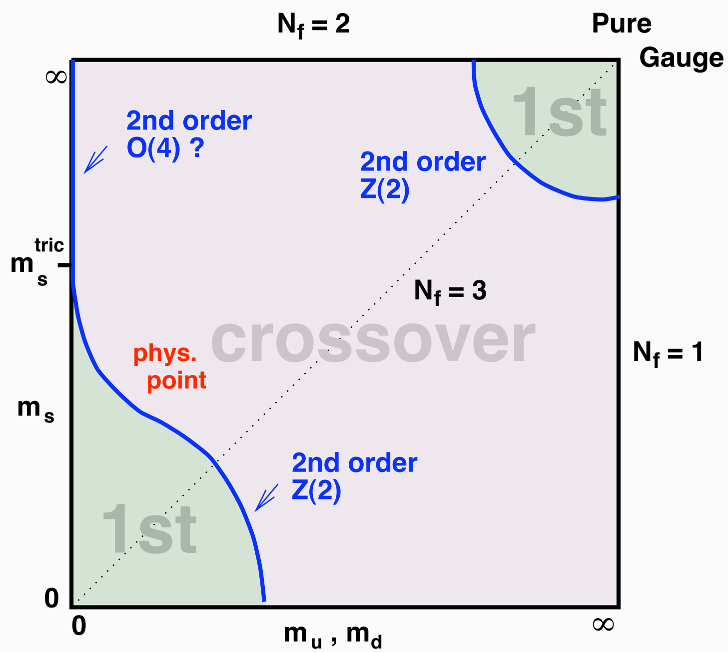
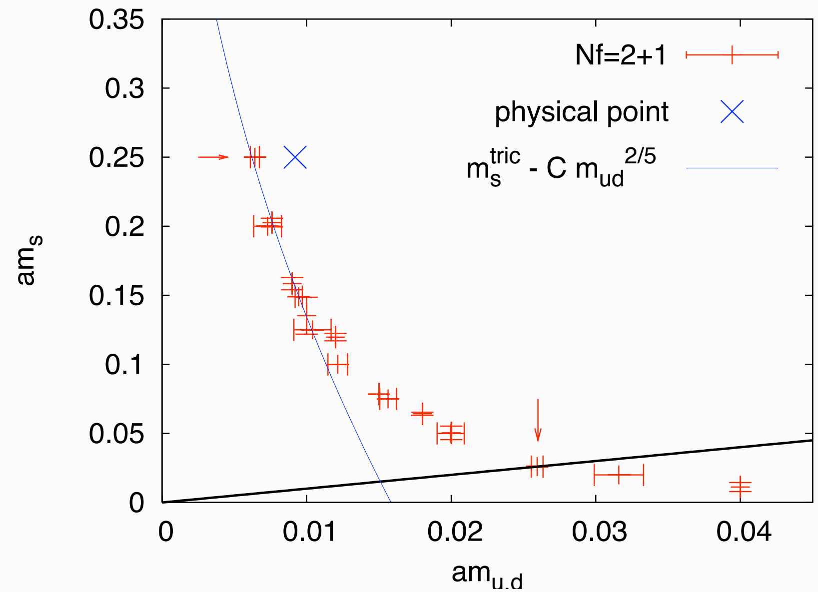
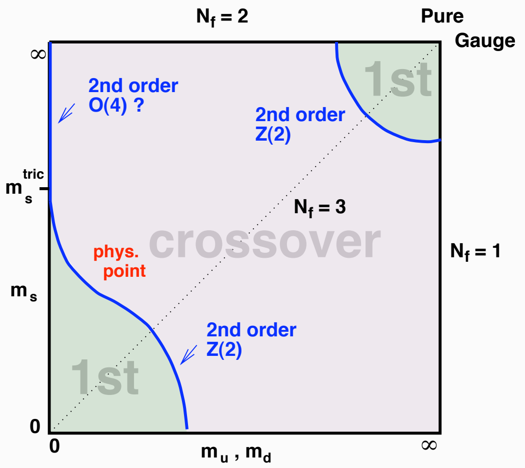
The situation at zero density

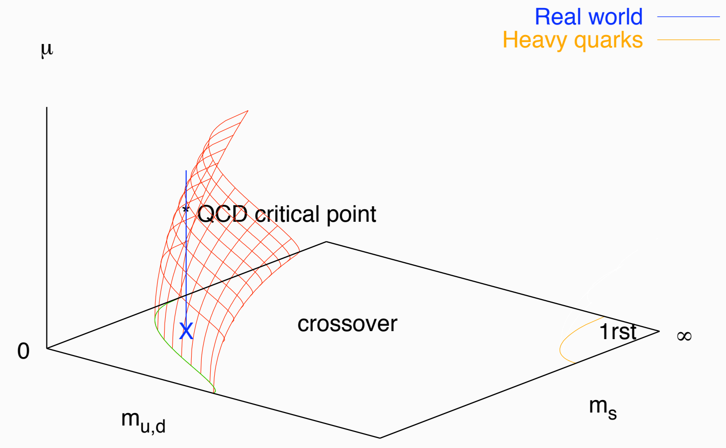


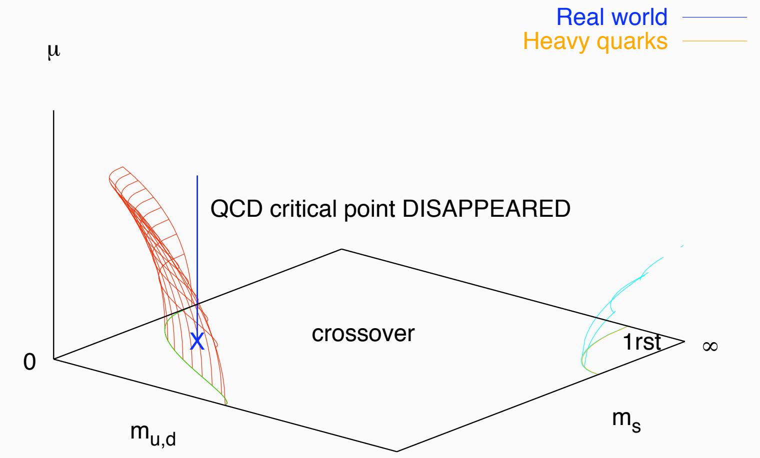


- $N_f = 2, m = 0$: $N_t = 4$, still not settled ax. U(1) anomaly — DiGiacomo et al. 05; Kogut, Sinclair 06; Chandrasekharan, Mehta 07
- phys. point: crossover in continuum — Aoki et al. 06
- chiral critical line: two points on $N_t = 4$ — de Forcrand, O.P. 07; $N_t = 6$ — de Forcrand, O.P. 07; Endrodi et al. 07

Finite density: chiral critical line $\longrightarrow$ critical surface

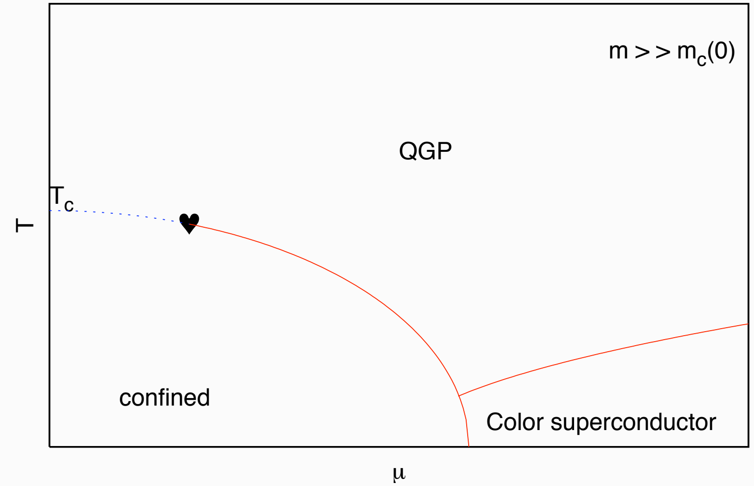


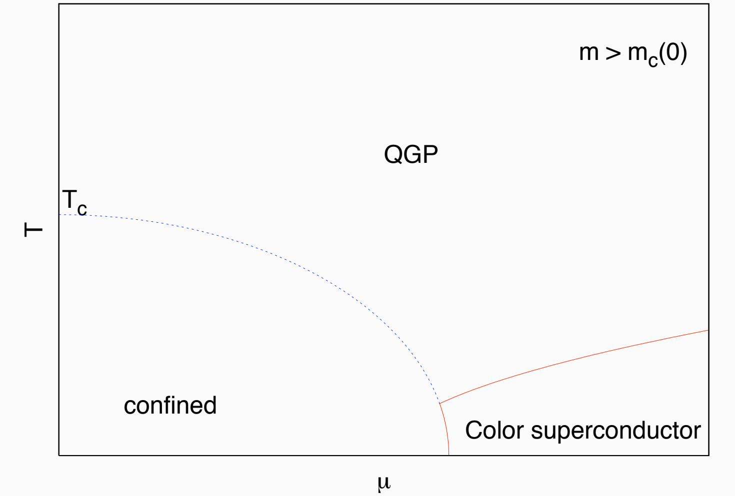




$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

$c_1 > 0$ $\qquad$ $c_1 < 0$



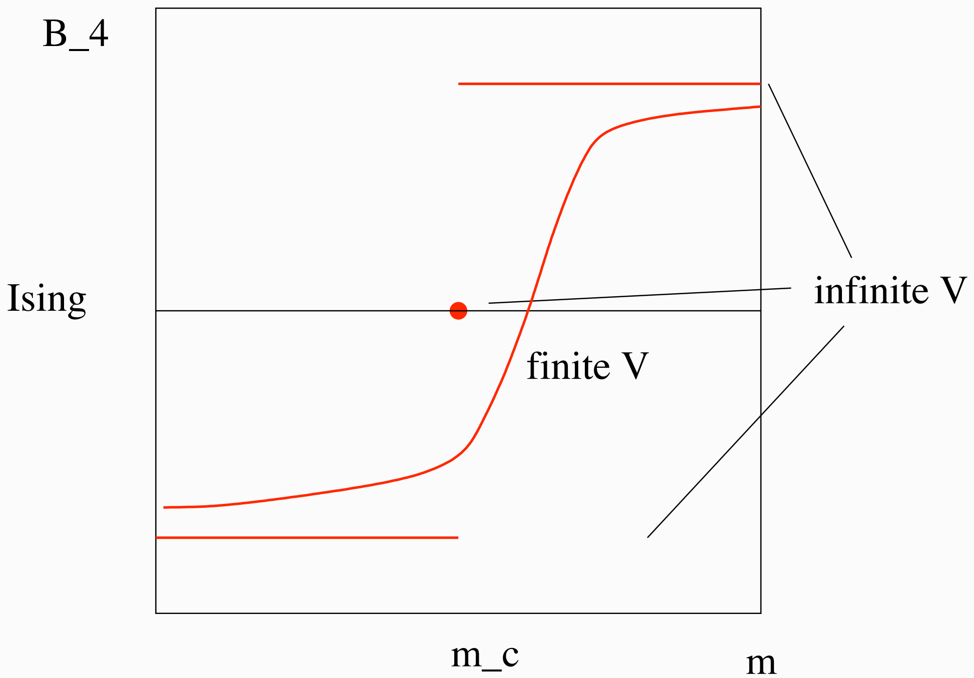


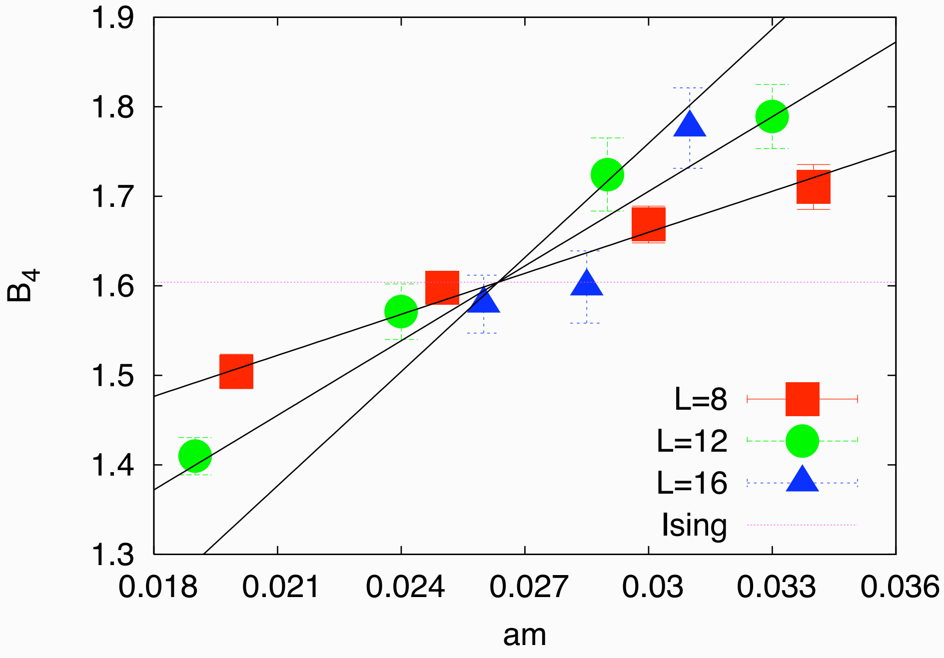
How to identify the critical surface

$$B_4(\bar\psi\psi) \equiv \frac{\langle(\delta\bar\psi\psi)^4\rangle}{\langle(\delta\bar\psi\psi)^2\rangle^2} \quad \xrightarrow{V\to\infty} \quad \begin{cases} 1.604 & \text{3d Ising} \\ 1 & \text{first-order} \\ 3 & \text{crossover} \end{cases}$$

$\mu = 0$:

$$B_4(m,L) = 1.604 + bL^{1/\nu}(m - m_0^c), \quad \nu = 0.63$$





Observable expanded about chiral critical point

$$B_4(am, a\mu) = 1.604 + \sum_{k,l=1} b_{kl}\,(am - am_0^c)^k (a\mu)^{2l}$$

$$\begin{aligned}
c_1' &= \frac{d\,am^c}{d(a\mu)^2} = -\frac{\partial B_4}{\partial (a\mu)^2}\left(\frac{\partial B_4}{\partial am}\right)^{-1} = -\frac{b_{01}}{b_{10}}\ , \\
c_2' &= \frac{d^2\,am^c}{d[(a\mu)^2]^2} = \ldots = -\frac{b_{02}}{b_{10}} + \frac{b_{01}b_{11}}{b_{10}^2}
\end{aligned}$$

continuum conversion: (requires beta-function)

$$\begin{aligned}
c_1 &= \frac{\pi^2}{N_t^2}\frac{c_1'}{am_0^c} + \frac{1}{T_c(m_0^c,0)}\frac{dT_c(m^c(\mu),\mu)}{d(\mu/\pi T)^2}, \\
c_2 &= c_1^2 + \left(\frac{\pi}{N_t}\right)^4\left(\frac{c_2'}{am_0^c} - \frac{c_1'^2}{(am_0^c)^2}\right) - \frac{1}{T_c^2(m_0^c,0)}\left(\frac{dT_c(m^c(\mu),\mu)}{d(\mu/\pi T)^2}\right)^2 \\
&\quad + \frac{1}{T_c^2(m_0^c,0)}\frac{d^2T_c(m^c(\mu),\mu)}{d[(\mu/\pi T)^2]^2}.
\end{aligned}$$

Two methods to extract Taylor coefficients

I. Calculate at imaginary chem. potential, fit to truncated polynomial

$$\langle O \rangle = \sum_n^N c_n \left(\frac{\mu_i}{\pi T}\right)^{2n} \Rightarrow \mu_i \longrightarrow i\mu_i$$

de Forcrand, O.P., JHEP 07: ($N_t = 4$) $c_1 < 0$ exotic scenario!

contradicts expectations, Fodor,Katz 04

Truncation errors, potentially dangerous: cf. Cea et al. 08 for T_c, SU(2)

$$\begin{aligned} O &= o_0 - o_1 \left(\frac{\mu_i}{\pi T}\right)^2 + o_2 \left(\frac{\mu_i}{\pi T}\right)^4 - o_3 \left(\frac{\mu_i}{\pi T}\right)^6 + \ldots \\ O &= o_0 + o_1 \left(\frac{\mu_r}{\pi T}\right)^2 + o_2 \left(\frac{\mu_r}{\pi T}\right)^4 + o_3 \left(\frac{\mu_r}{\pi T}\right)^6 + \ldots \end{aligned}$$

Finite order fits “average” over higher terms

II. Calculate Taylor coefficients directly

Bielefeld-Swansea; Gavai, Gupta; MILC:

express derivatives by traces of non-local operators, f(det M),
evaluate by stochastic estimators ➡ delicate cancellations!

de Forcrand, Kim, O.P. (LAT07):

$$\frac{dO}{d(a\mu)^2} = \lim_{(a\mu)^2 \to 0} \frac{O(a\mu) - O(0)}{(a\mu)^2}$$

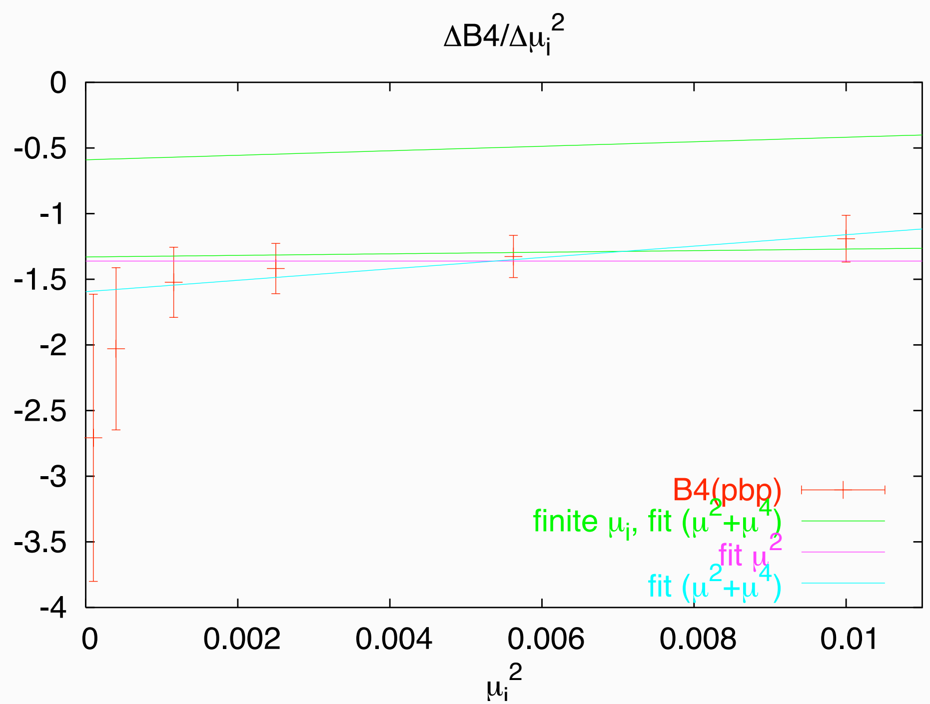
- evaluate by "infinitesimal" reweighting, no overlap problem, correlated errors drop out of observables
- reweight in imaginary direction: reweighting factor real
- compute reweighting factor by stochastic estimator
- numerically very efficient

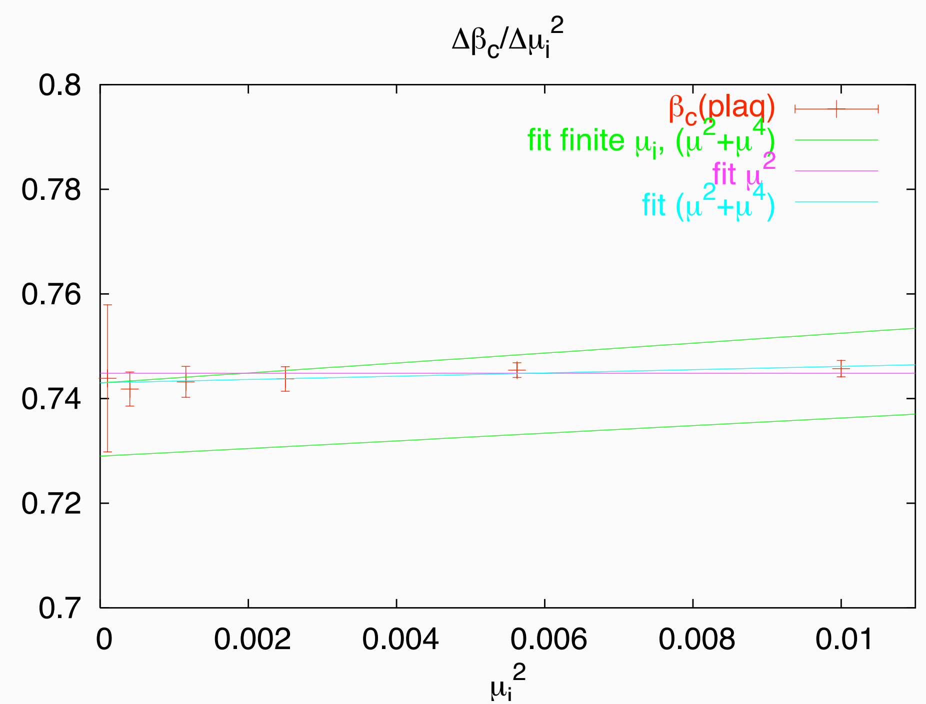
Numerical results for $N_f = 3, N_t = 4$

unimproved staggered fermions, RHMC algorithm

Method I: $8^3 \times 4, 42$ pairs $(am, a\mu)$ > 20 million traj., 18 unconstrained dof's in fits

Method II: $8^3, 12^3 \times 4$ $m_\pi L \gtrsim 3, 4.5$ > 5 million, 0.5 million traj.

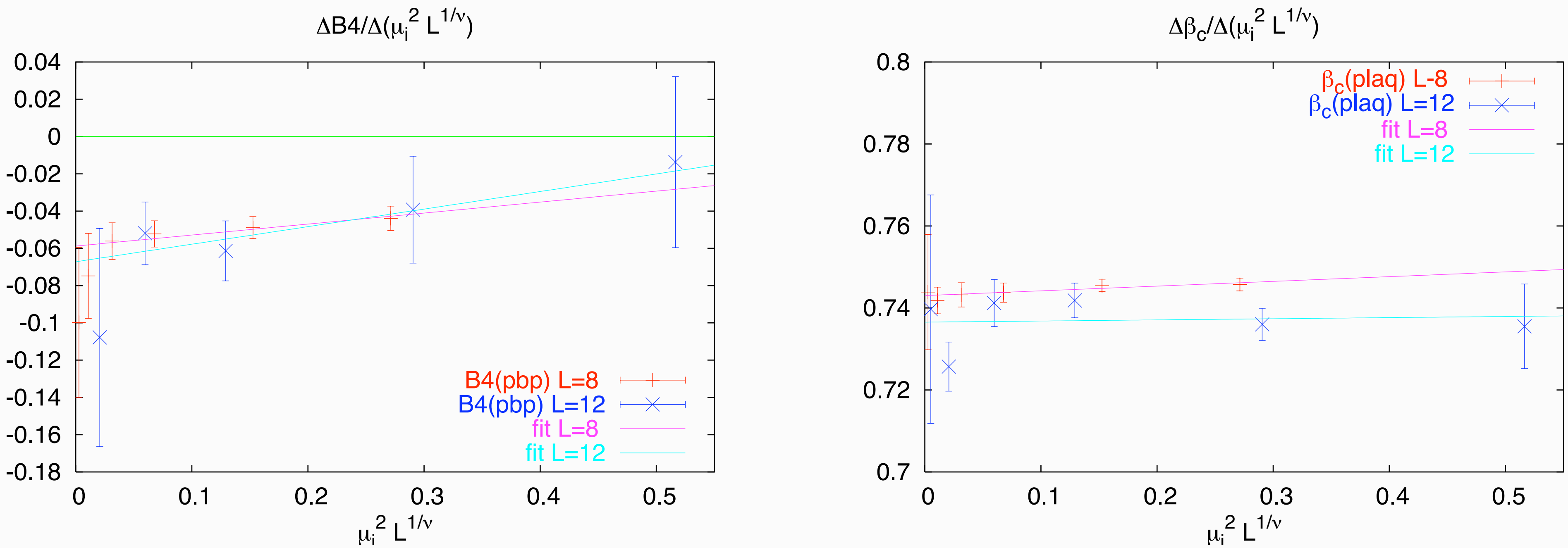




Mutually consistent; significant NLO-contribution!

Finite size scaling

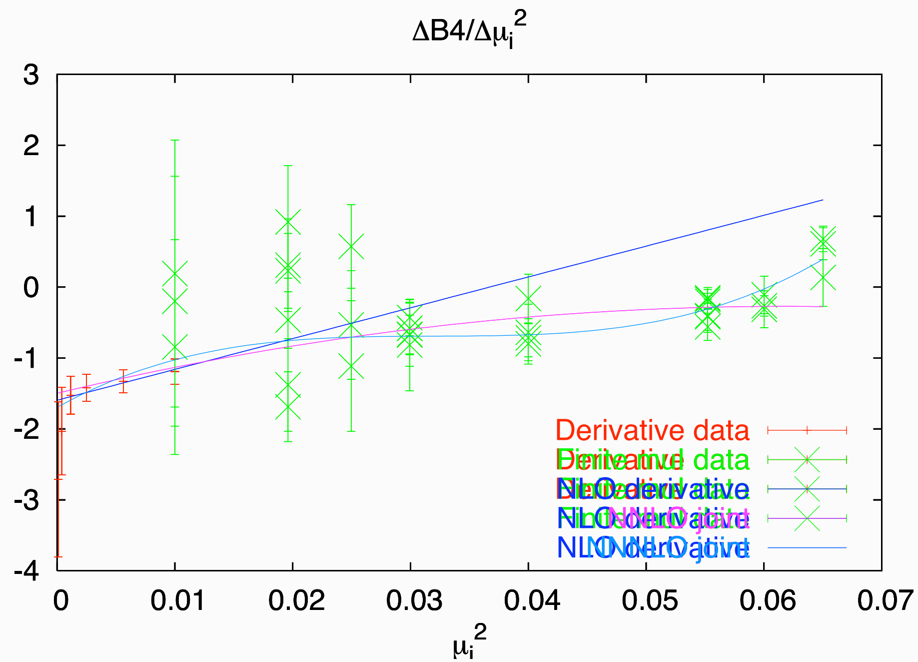
scaling: each term $\propto L^{1/\nu}; \nu = 0.63$



LO and NLO close to thermodynamic limit

Combination of both methods

complementary techniques to extract coefficients ➡ combine!



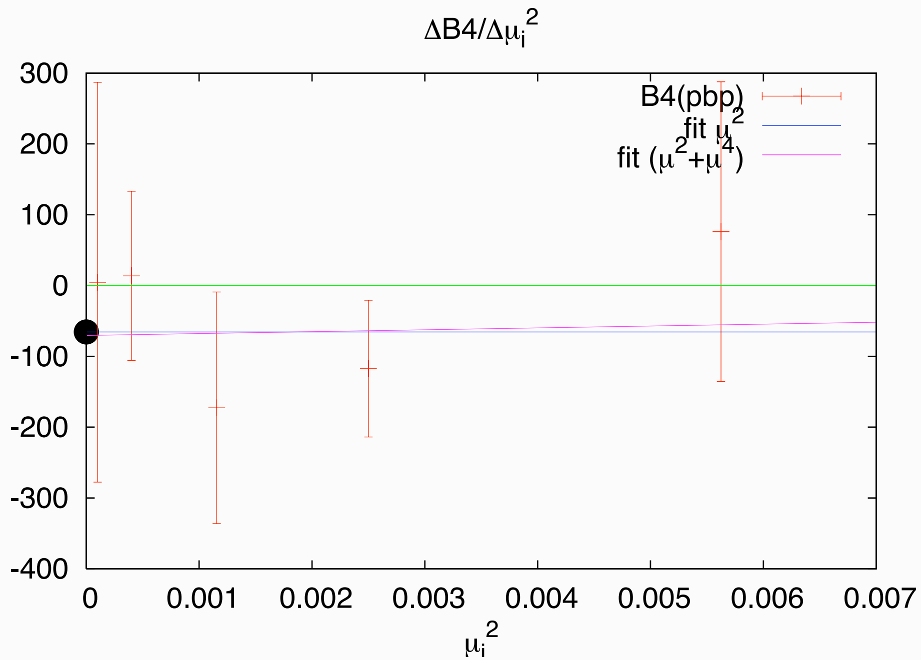
$$\frac{B_4(\mu_i) - B_4(0)}{\mu_i^2} = \underbrace{b_1}_{<0} + \underbrace{b_2}_{>0}\mu_i^2 + \underbrace{b_3}_{<0}\mu_i^4 + \underbrace{b_4}_{>0}\mu_i^6$$

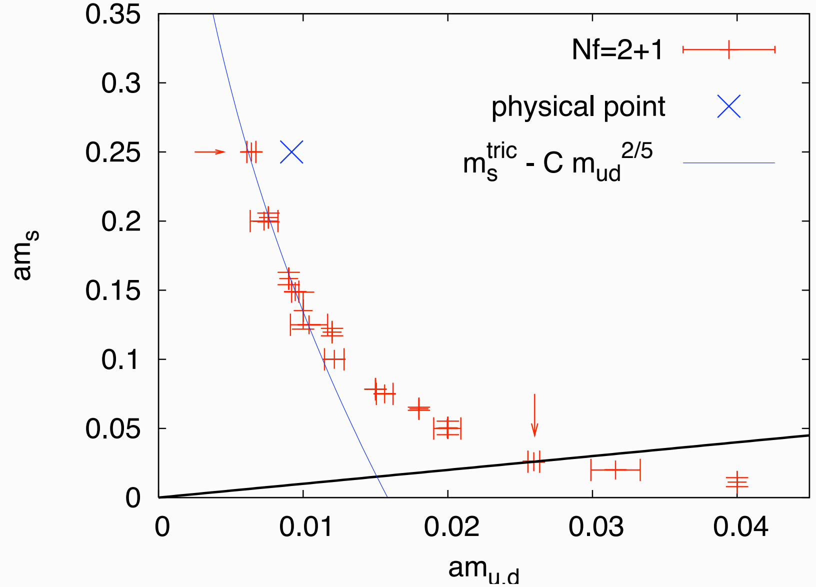
Converting to the continuum

- fits to imag. mu alone
$$\frac{m_c(\mu)}{m_c(0)} = 1 - 2.1(7)\left(\frac{\mu}{\pi T}\right)^2 - 9(5)\left(\frac{\mu}{\pi T}\right)^4 + \ldots$$
- derivatives alone
$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T}\right)^2 - 40(22)\left(\frac{\mu}{\pi T}\right)^4 + \ldots$$
- combined, b3=0
$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T}\right)^2 - 20(5)\left(\frac{\mu}{\pi T}\right)^4 + \ldots$$
- combined, b3 released
$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T}\right)^2 - 43(20)\left(\frac{\mu}{\pi T}\right)^4 - \ldots$$

Higher order corrections reinforce shrinking of first order region!

Non-degenerate fermion masses, $N_f = 2+1, N_t = 4$





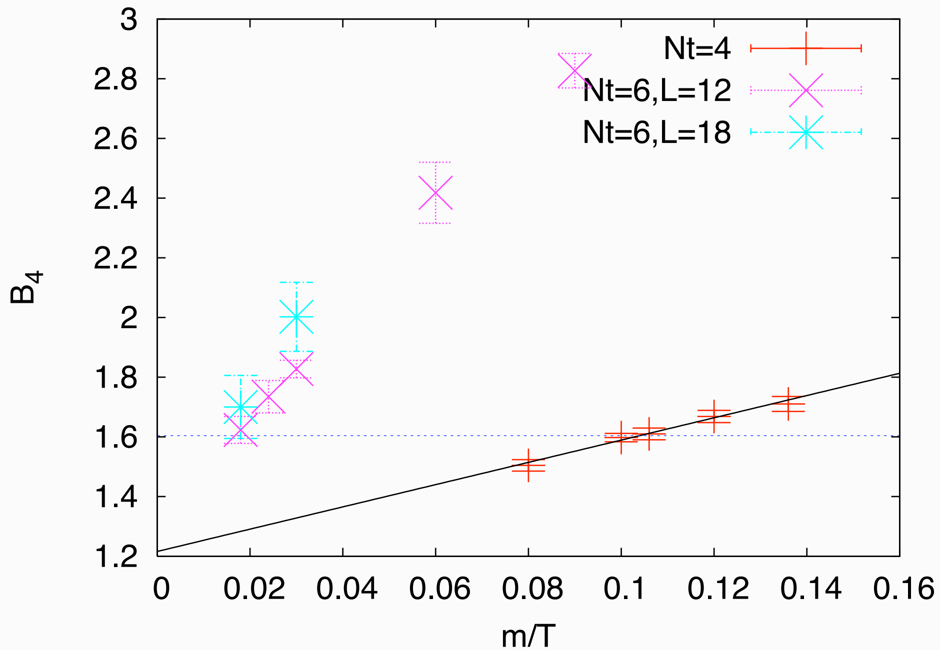
$16^3 \times 4, am_s = 0.25, am_{u,d} = 0.005, m_\pi L \sim 3$ lighter than in nature 350k traj.

$b_1 = -66(41)\ (\mu^2 \text{ fit}), \quad b_1 = -71(75)\ (\mu^2 + \mu^4 \text{ fit})$

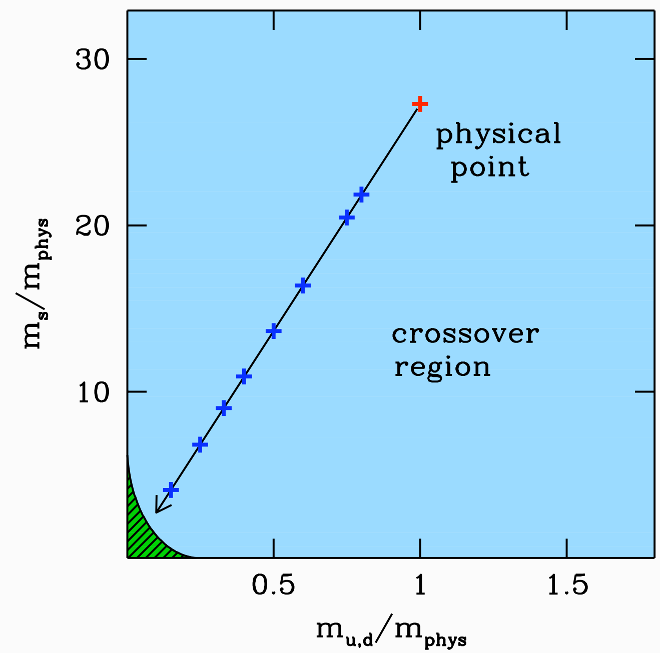
$$\frac{m_c(\mu)}{m_c(0)} = 1 - 80(50) \left(\frac{\mu}{\pi T}\right)^2 - \ldots$$

not conclusive yet

Towards the continuum limit $N_f = 3, N_t = 6, \mu = 0$



de Forcrand, Kim, O.P. (LAT07)

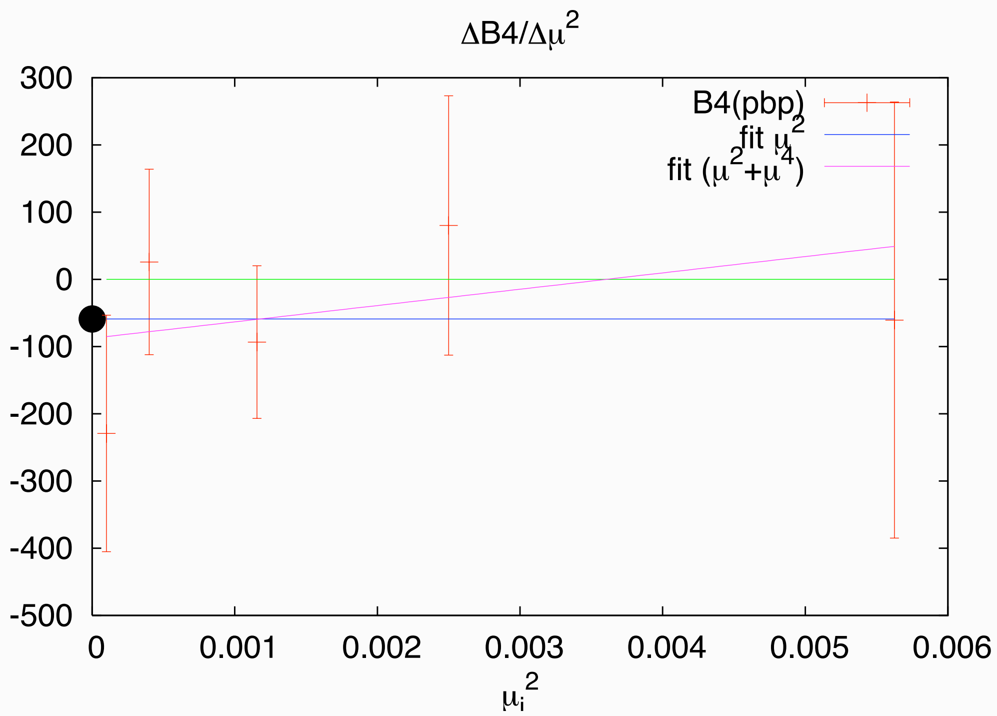


Endrodi et al. (LAT07)

$$\frac{m_\pi^c(N_t = 4)}{m_\pi^c(N_t = 6)} \approx 1.77 \approx \sqrt{3}$$

Towards the continuum limit $N_f = 3, N_t = 6, \mu \neq 0$

$\frac{\partial B_4}{\partial am}$ easy, from fits to m-dependence $\frac{\partial B_4}{\partial (a\mu)^2}$ hard, finite differences



$18^3 \times 6, am = 0.003$

120k trajectories

$$b_1 = -58(49)\ (\mu^2 \text{ fit}), \quad b_1 = -88(75)\ (\mu^2 + \mu^4 \text{ fit})$$

$$\frac{m_c(\mu)}{m_c(0)} = 1 - 28(23)\left(\frac{\mu}{\pi T}\right)^2 - \ldots$$

Assume $c_1 = +18$ (+two sigma) $\frac{m_c(\mu = T)}{m_c(0)} \approx 3$

still no critical point!

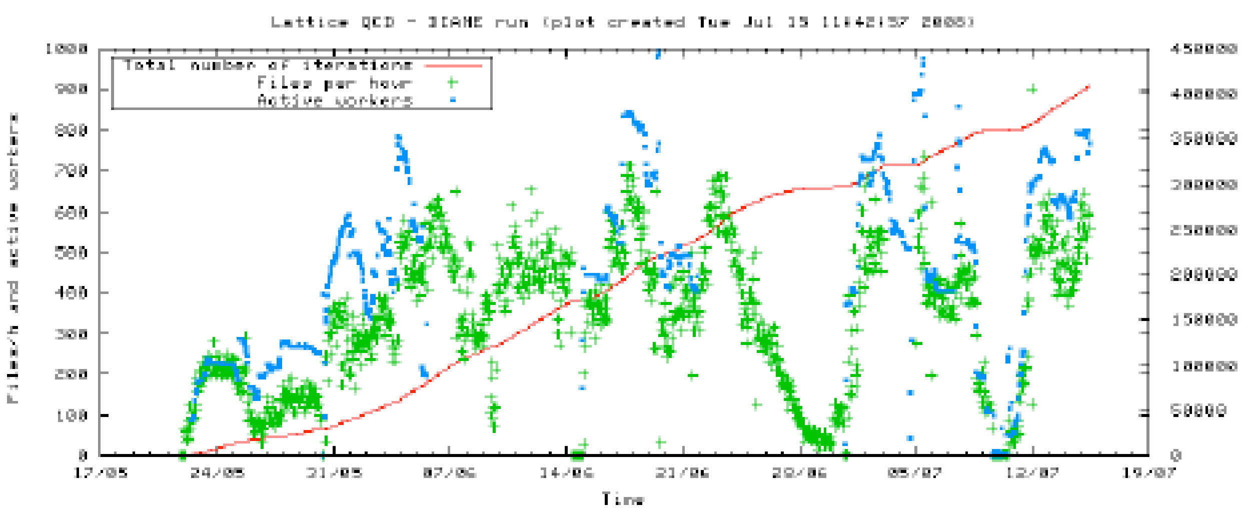
Conclusions

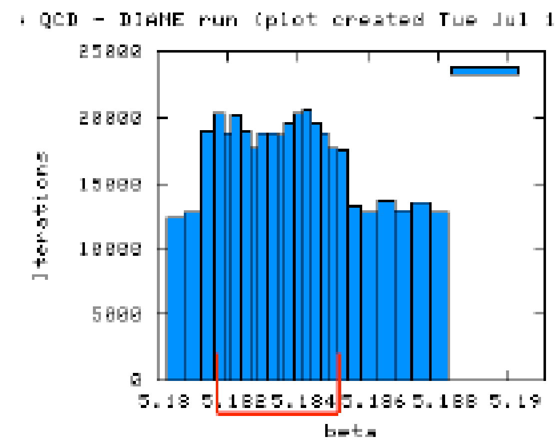
- $N_t = 4$: exotic scenario without chiral critical point established for $N_f = 3$
- reinforced by subleading terms in $\left(\frac{\mu}{T}\right)^2$
- so far no qualitative change for $N_f = 2+1$
- $N_t = 6$: sign undetermined so far, but curvature of crit. surface too small for critical point at $\mu \lesssim T_c$

- Caveat: all staggered, masses lighter than physical, rooting problems?

LQCD on the Computing Grid

- 725k trajectories (2 quark masses) in 2 months $\rightarrow$ 115 CPU years
- on average 700 CPUs active at all times
- 330k files = 3 TB of data transferred
- computing support provided by CERN IT/GS: *thanks a lot!*





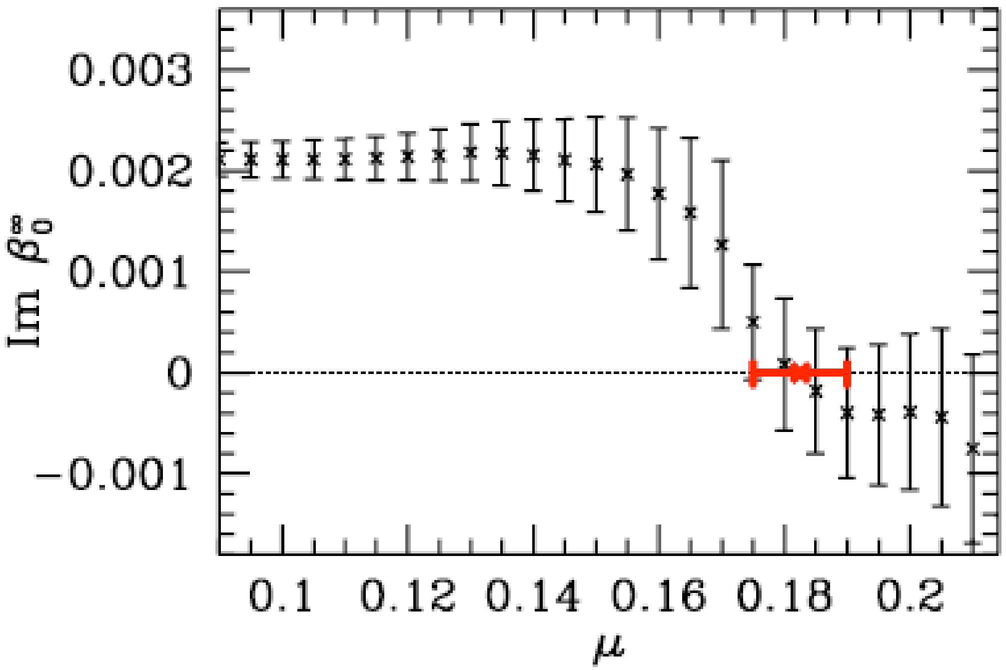
prioritized scheduling

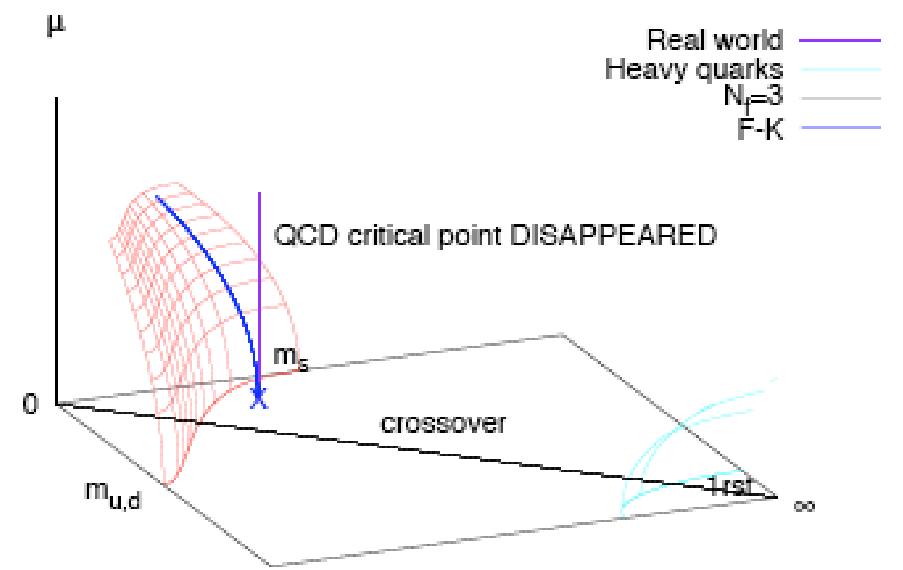
- calculations on EGEE Grid
- resources provided by CERN, CYFRONET (Poland), CSCS (Switzerland), NIKHEF (Holland) + 10 more across Europe

Contradiction with other lattice studies? ...not necessarily!

- Gavai & Gupta: $N_f = 2$ 'miles away'
- Fodor & Katz: $\{T_E, \mu_E\} = \{162(2), 120(13)\}$ MeV ?

  not same parameters, different systematics, lattice spacing effects





F&K keep (am_q) fixed, but $a(\mu)$ increases with μ

$\Rightarrow$unphysically light quarks at larger μ may cause the phase transition