Intersections of nuclear physics and cold atom physics

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About this talk

Jochen was part of the “golden” era of nuclear many-body physics and nuclear collective motion at Stony Brook, Juelich and Illinois.

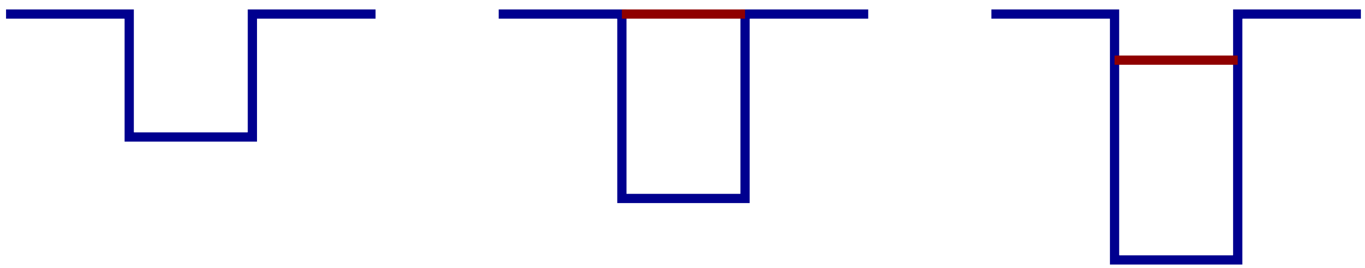
In retrospect, the perfect bench mark problem for many of these problems is the unitary Fermi gas.

The unitary Fermi gas represents the crossover between BCS and BEC behavior. The BCS/BEC crossover was studied as far back as Eagles (1969), Leggett (1980, at Illinois), Nozieres and Schmitt-Rink (1985).

But the full significance of the unitary Fermi gas was not fully grasped unitil it was experimentally realized by O’Hara et al. in 2002.

Unitarity limit

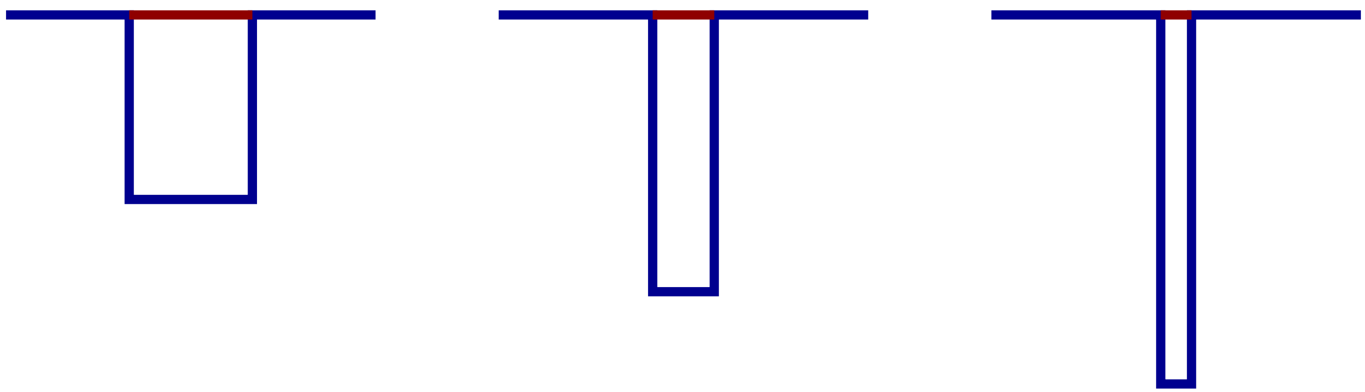
Consider simple square well potential



$a < 0$ $\qquad$ $a = \infty,\ \epsilon_B = 0$ $\qquad$ $a > 0,\ \epsilon_B > 0$

Unitarity limit

Now take the range to zero, keeping $\epsilon_B \simeq 0$

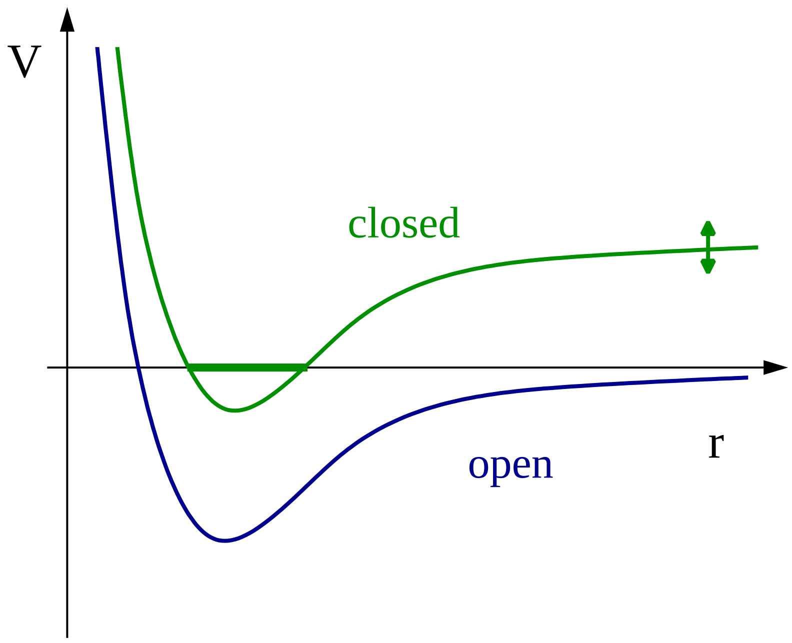


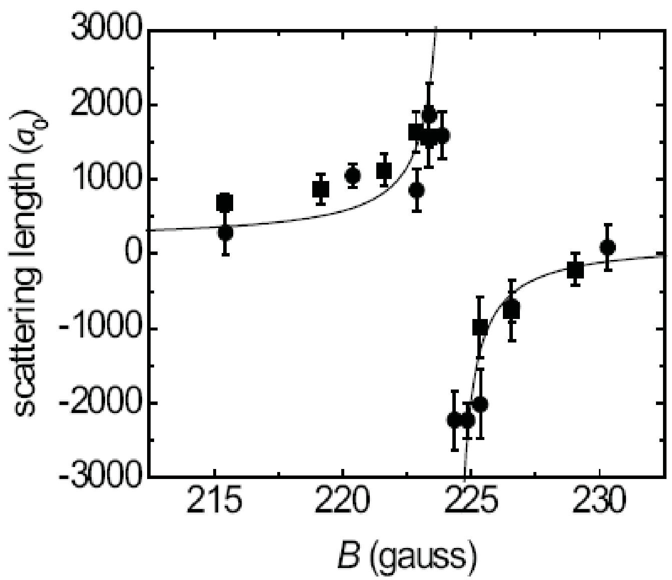
Universal relations

$$\mathcal{T} = \frac{1}{ik + 1/a} \qquad \epsilon_B = \frac{1}{2ma^2} \qquad \psi_B \sim \frac{1}{\sqrt{a}r} \exp(-r/a)$$

Feshbach resonances

Atomic gas with two spin states: "↑" and "↓"



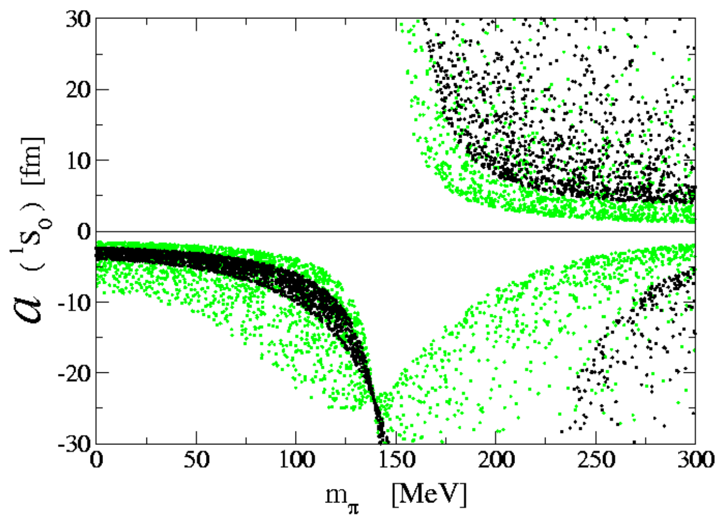


Feshbach resonance $$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

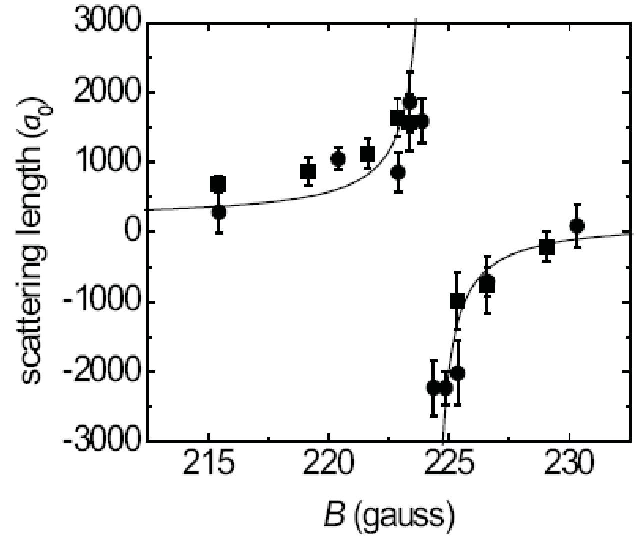
"Unitarity" limit $a \to \infty$ $$\sigma = \frac{4\pi}{k^2}$$

Universality

Neutron Matter



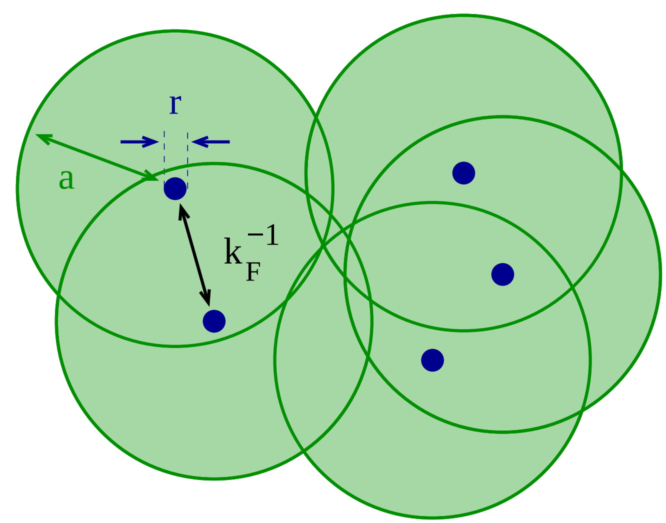
Feshbach Resonance in ^{6}Li



What do these systems have in common?

dilute: $r\rho^{1/3} \ll 1$

strongly correlated: $a\rho^{1/3} \gg 1$



Dilute Fermi gas: field theory

Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

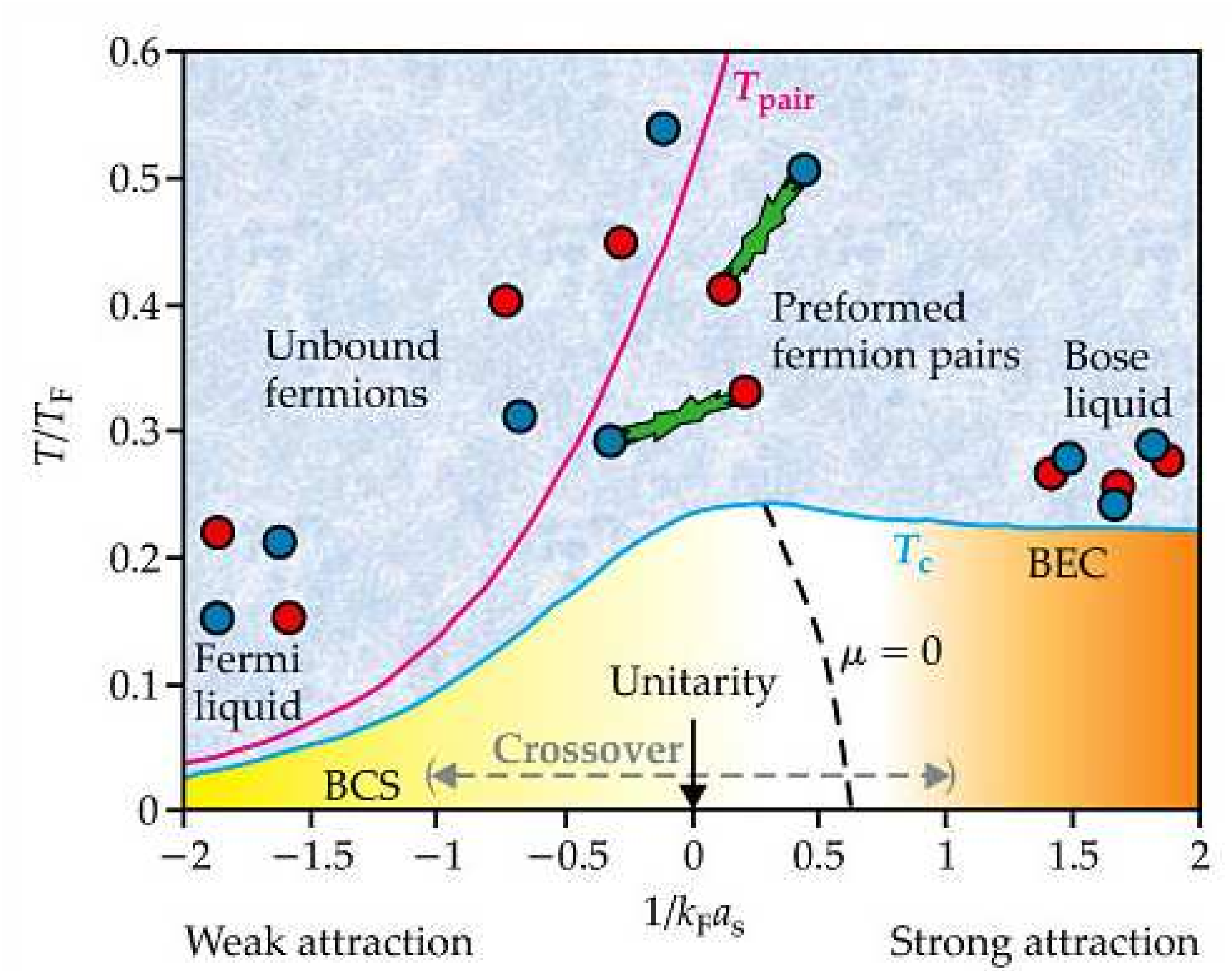
Unitary limit: $a \to \infty$, $\sigma \to 4\pi/k^2$ ($C_0 \to \infty$)

This limit is smooth: HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$

$$\mathcal{L} = \Psi^\dagger \left[i\partial_0 + \sigma_3 \frac{\vec{\nabla}^2}{2m} \right] \Psi + \left(\Psi^\dagger \sigma_+ \Psi \phi + h.c. \right) - \frac{1}{C_0} \phi^* \phi \ ,$$

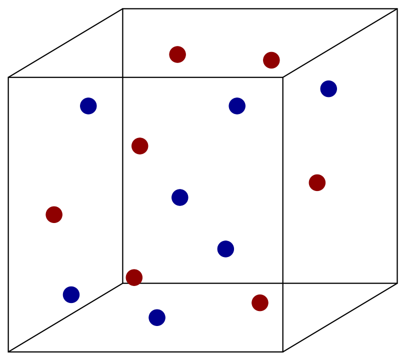
Low T ($T < T_c \sim \mu$): Pairing and superfluidity

Dilute Fermi gas: BCS-BEC crossover



Intersection I: Many body physics/equation of state

Free fermi gas at zero temperature



$$\frac{E}{N} = \frac{3}{5}\frac{k_F^2}{2m} \qquad \frac{N}{V} = \frac{k_F^3}{3\pi^2}$$

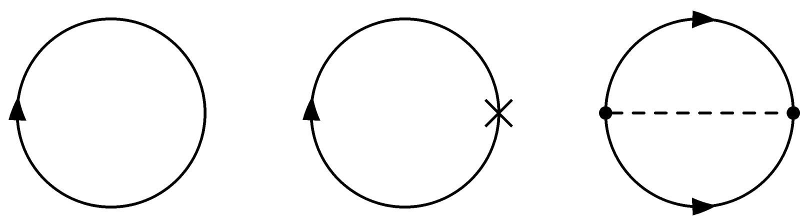
Consider unitarity limit ($a \to \infty$, $r \to 0$)

$$\frac{E}{N} = \xi\,\frac{3}{5}\frac{k_F^2}{2m} \qquad k_F \equiv (3\pi^2 N/V)^{1/3}$$

Prize problem (George Bertsch, 1998): Determine ξ

Similar problems: $\Delta = \alpha\epsilon_F$, $\qquad k_B T_c = \beta\epsilon_F$

Analytic work: Epsilon expansion

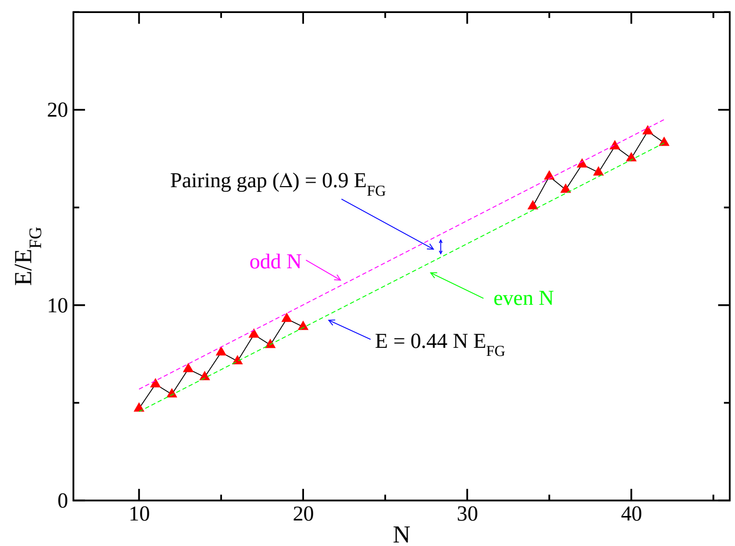


$O(1)$ $O(1)$ $O(\epsilon)$

$$\xi = \frac{1}{2}\epsilon^{3/2} + \frac{1}{16}\epsilon^{5/2}\ln\epsilon - 0.0246\,\epsilon^{5/2} + \ldots$$

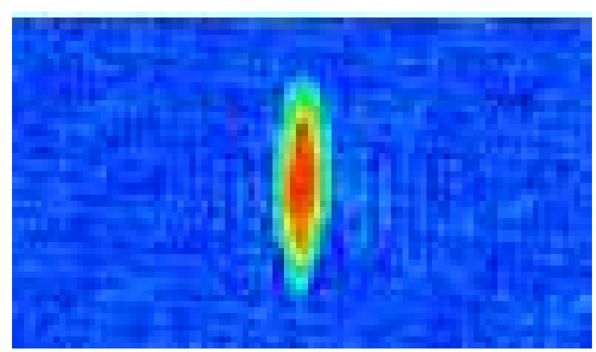
$$\xi(\epsilon = 1) = 0.475$$

Green function MC



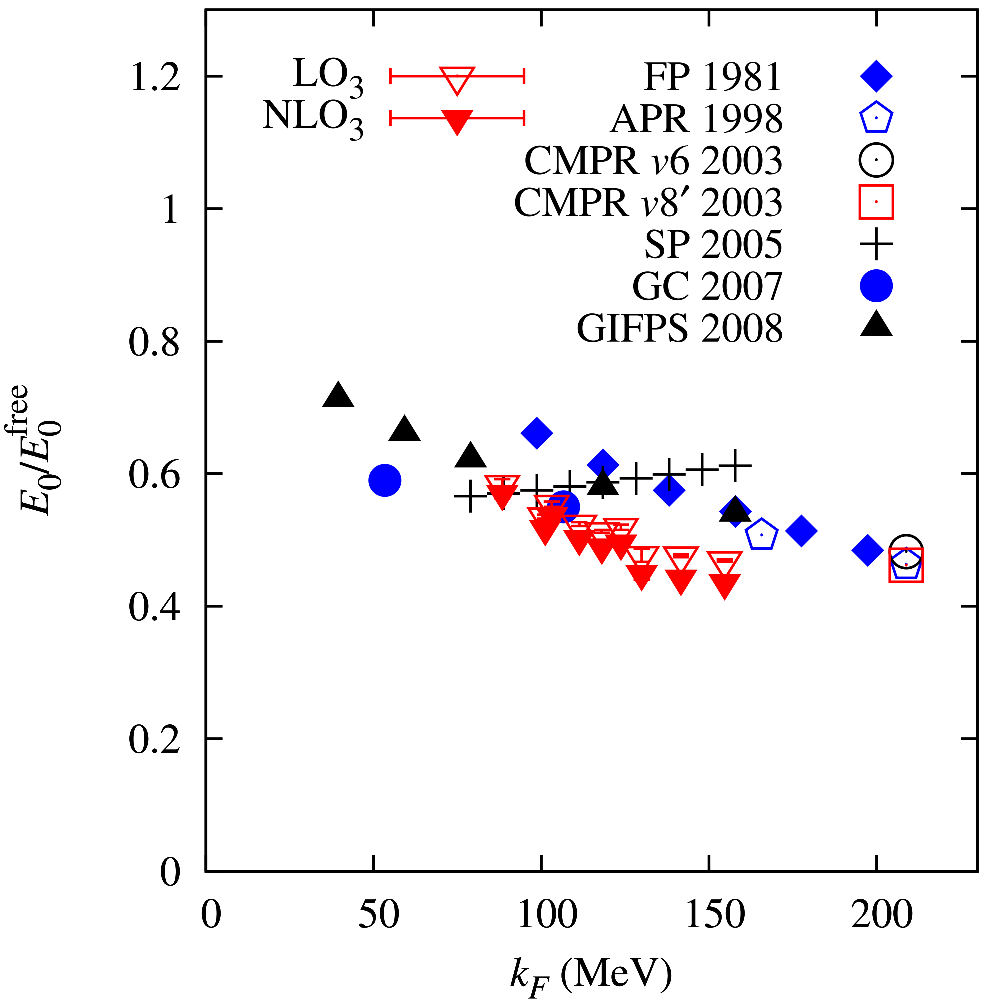
$\xi = 0.40$-0.44 (Carlson et al.)

Experiment



$\xi = 0.38(2)$ (Luo, Thomas)

Neutron matter with realistic interactions



Results close to unitary limit (for $k_F|a| > 10$).

Corrections tend to cancel (range effects, p-waves, 3-body).

Approach to unitarity and Tan's "contact"

Small $k_F a$:
$$\frac{E}{E_0} = 1 - \frac{10}{9\pi}(k_F a) + \ldots$$

Large $k_F a$:
$$\frac{E}{E_0} = \xi + \frac{\zeta}{k_F a} + \ldots$$

where $\zeta \simeq 0.9$ is related to the "contact"

$$n_\sigma(k) \to \frac{\mathcal{C}}{k^4} \qquad\qquad \zeta = \frac{5\pi}{2}\frac{\mathcal{C}}{k_F^4}$$

Contact controls many short distance properties, for example

$$\eta(\omega) \sim \frac{\mathcal{C}}{5\sqrt{m\omega}}$$

Density Functionals

Gradient terms (from epsilon expansion)

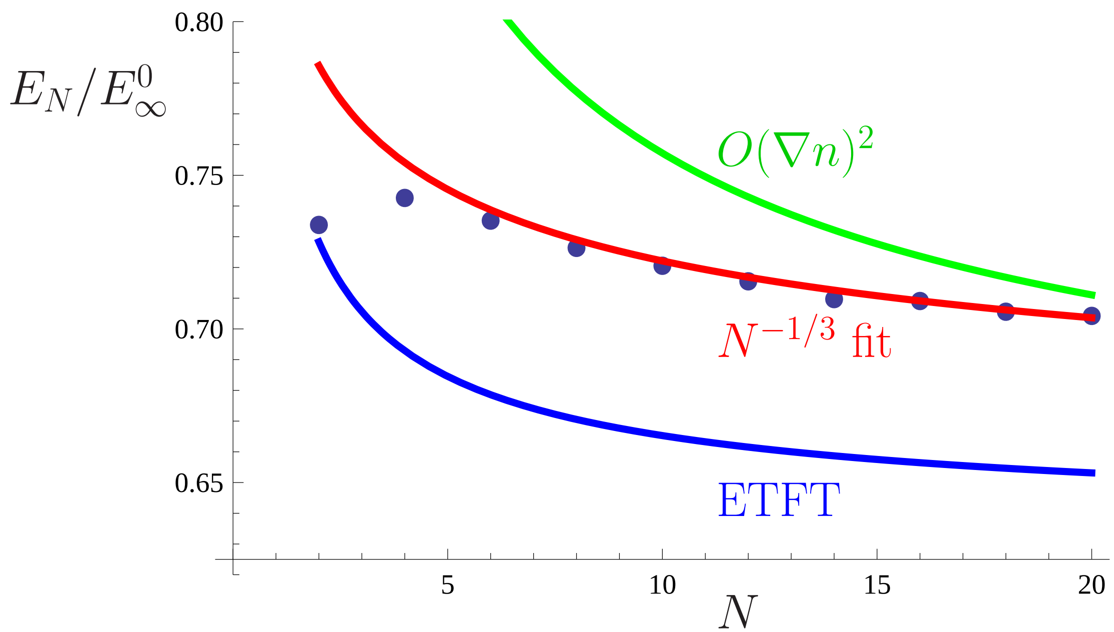
$$\mathcal{E}(x) = n(x)V(x) + 1.364\,\frac{n(x)^{5/3}}{m} + 0.032\,\frac{(\nabla n(x))^2}{mn(x)} + O(\nabla^4 n)$$

free Fermi gas: $(1.364 \to 2.871)$ $(0.032 \to 0.014)$

consider $V(x) = \frac{1}{2}m\omega^2 x^2$

$$\lim_{N\to\infty} \frac{E_N}{E_N^0} = \sqrt{\xi} \simeq 0.63$$

evidence for large surface effects



Blume et al., see also Bulgac (SFLDA), Gandolfi et al.

Intersection II: Pairing

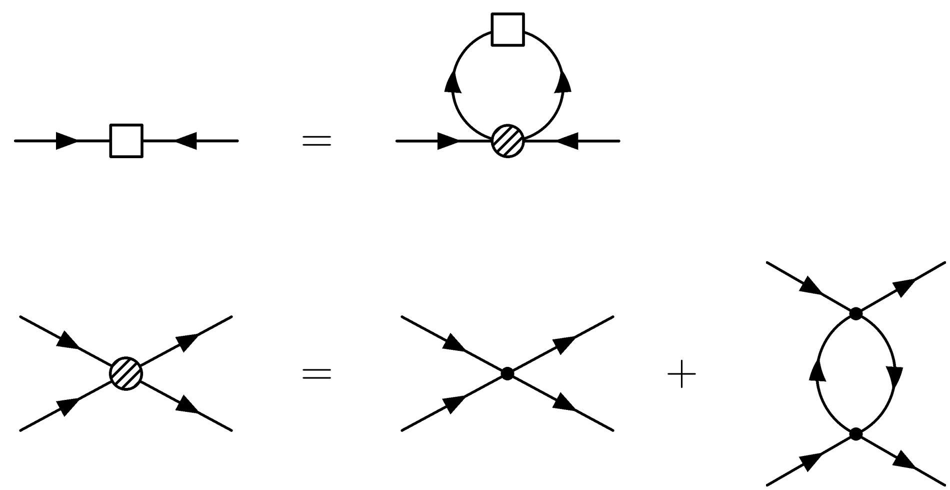
Numerical results (Carlson & Reddy, Burovsky et al.)

$$\Delta = 0.48 E_F \qquad\qquad T_c = 0.15 E_F$$

Gap remarkably close to extrapolated BCS+Gorkov result

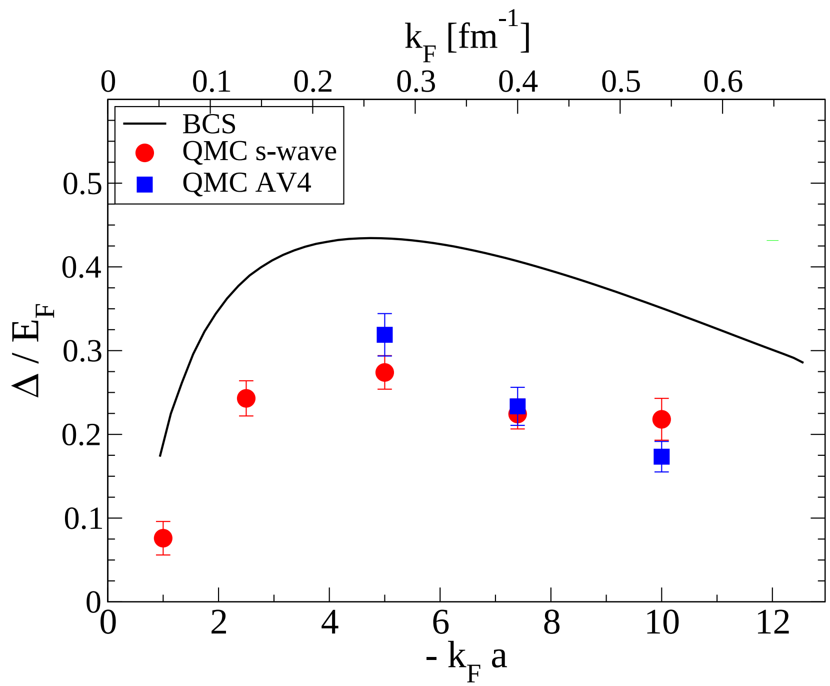
$$\Delta = \frac{8E_F}{(4e)^{1/3}e^2}\exp\left(-\frac{\pi}{2k_F|a|}\right)$$

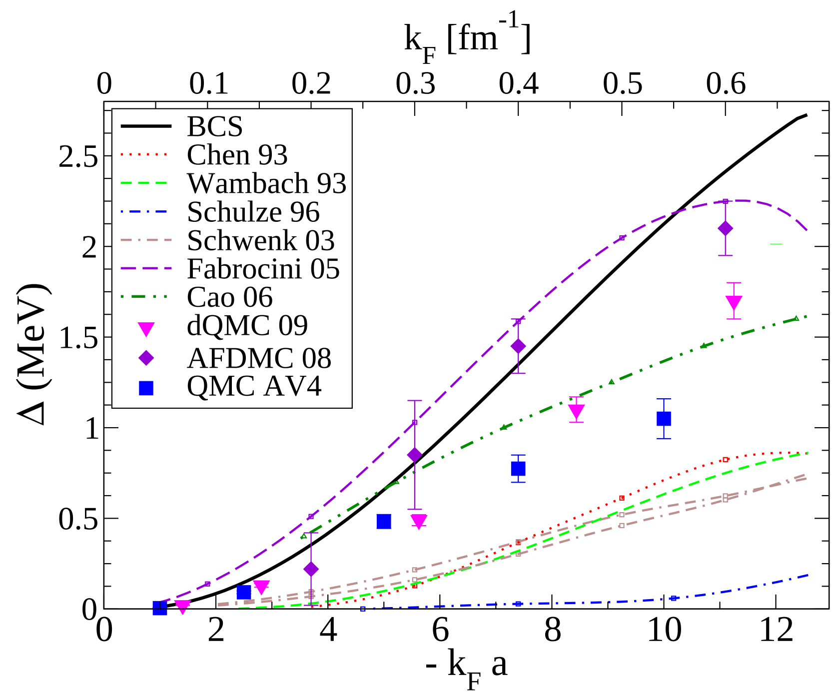
$$\Delta(a\to\infty) = 0.49 E_F$$



Gorkov (induced interaction) crucial, reduces gap by $\sim 1/2$

Pairing gap with realistic interactions



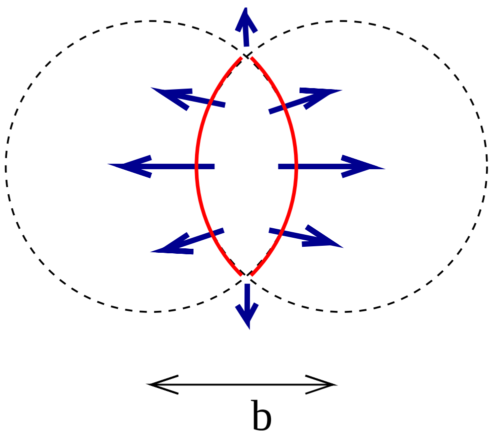


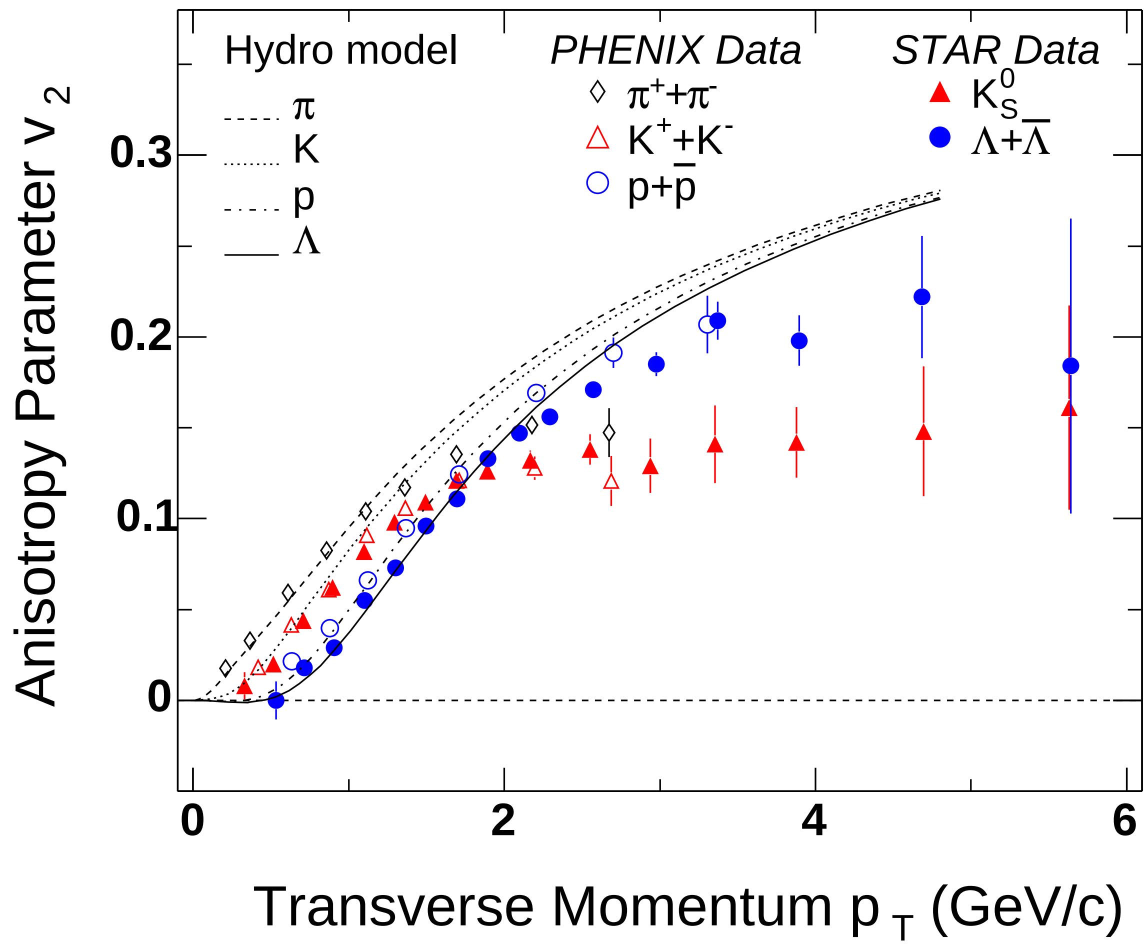
Range corrections important, Δ smaller than in unitary limit.

But: QMC gaps larger than previous estimates.

Intersection III: Elliptic flow (QGP)

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy



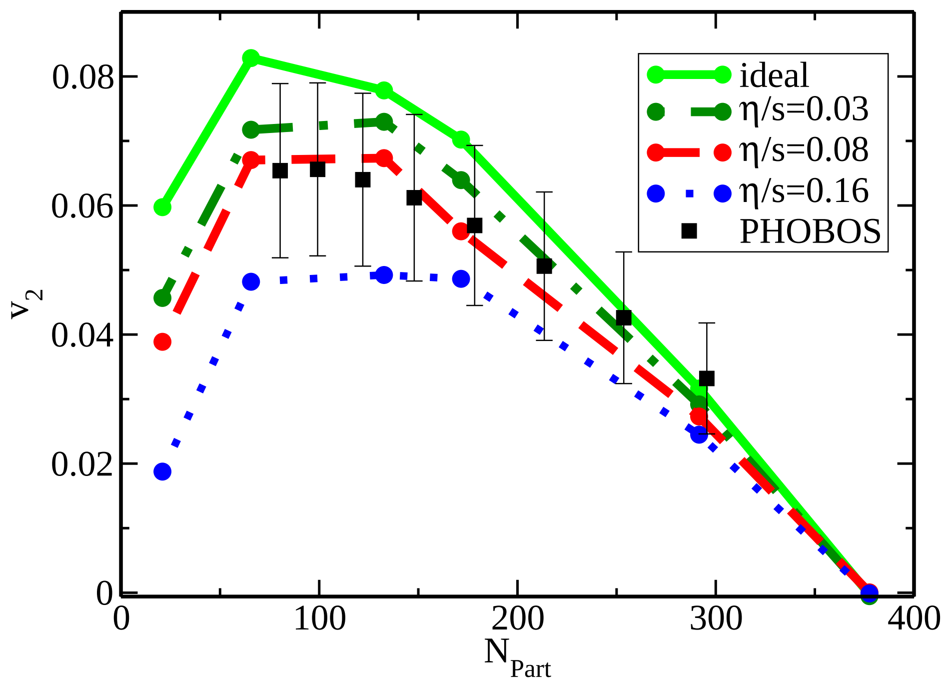


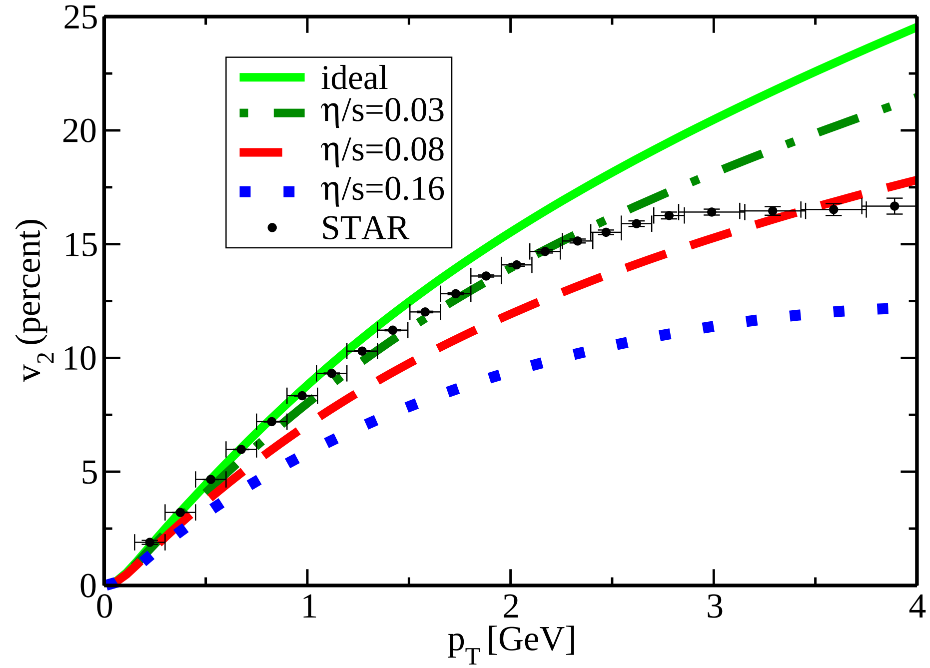
source: U. Heinz (2005)

$$p_0 \left. \frac{dN}{d^3p} \right|_{p_z=0} = v_0(p_\perp)\,(1 + 2v_2(p_\perp)\cos(2\phi) + \ldots)$$

Viscosity and elliptic flow

Viscous effects increase with impact parameter and p_T.



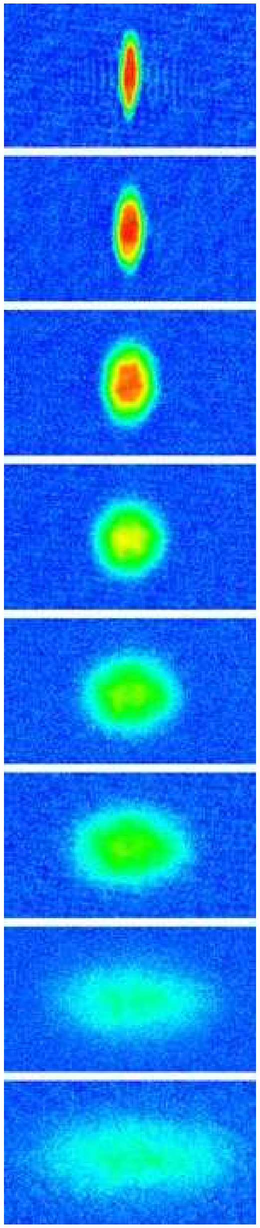


Romatschke (2007), see also Teaney (2003)

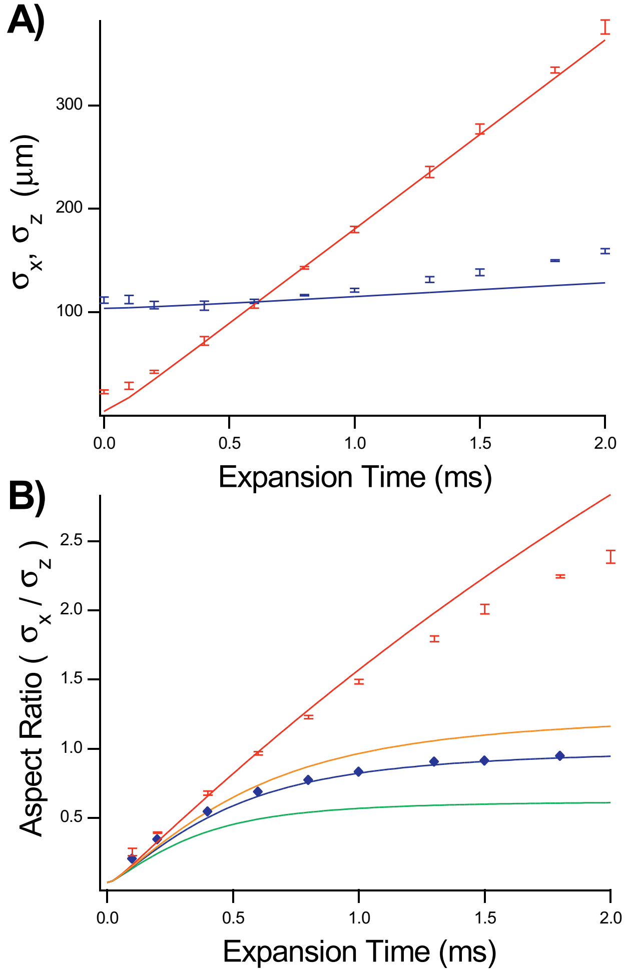
Many questions: Dependence on initial conditions, freeze out, etc.

conservative bound $\boxed{\frac{\eta}{s} < 0.4}$

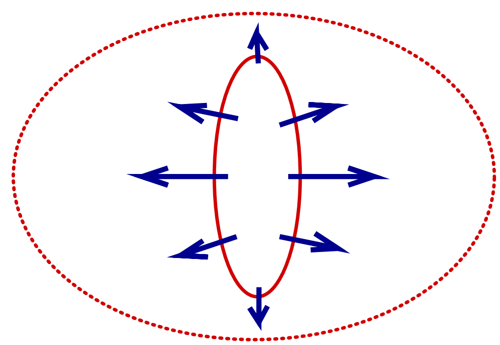
Almost ideal fluid dynamics (cold gases)



O'Hara et al. (2002)

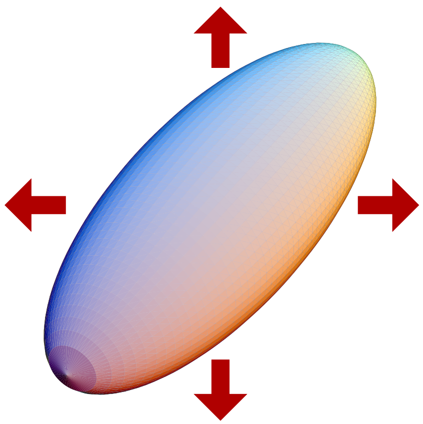


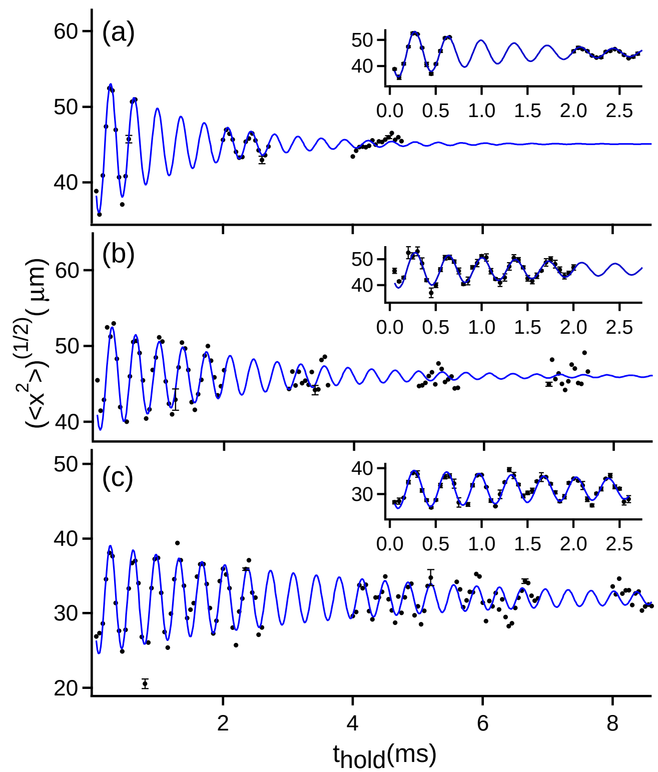
Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy



Collective oscillations

Radial breathing mode





Ideal fluid hydrodynamics $(P = \frac{2}{3}\mathcal{E})$

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} = -\frac{\vec{\nabla} P}{mn} - \frac{\vec{\nabla} V}{m}$$

Hydro frequency at unitarity

$$\omega = \sqrt{\frac{10}{3}}\,\omega_\perp$$

Damping small, depends on T/T_F.

experiment: Kinast et al. (2005)

Viscous hydrodynamics

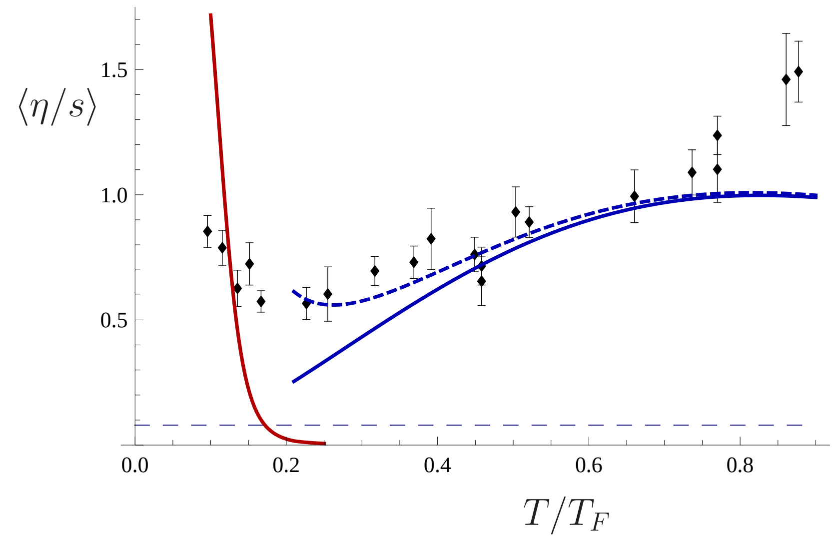
Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{1}{2}\int d^3x\,\eta(x)\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \int d^3x\,\zeta(x)\,(\partial_i v_i)^2 - \frac{1}{T}\int d^3x\,\kappa(x)\,(\partial_i T)^2$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

$$\boxed{\frac{\eta}{s} = (3\lambda N)^{\frac{1}{3}}\frac{\Gamma}{\omega_\perp}\frac{E_0}{E_F}\frac{N}{S}}$$

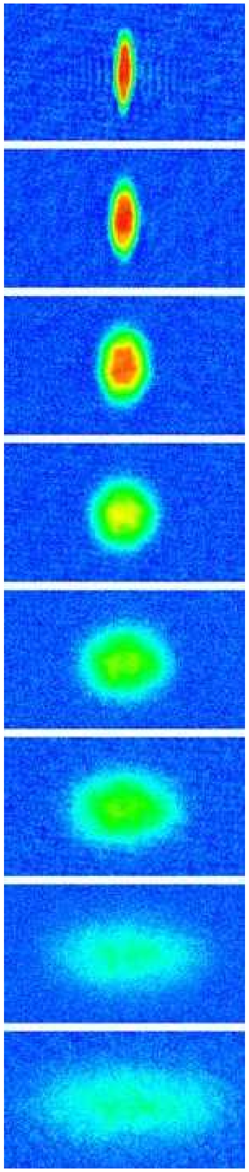


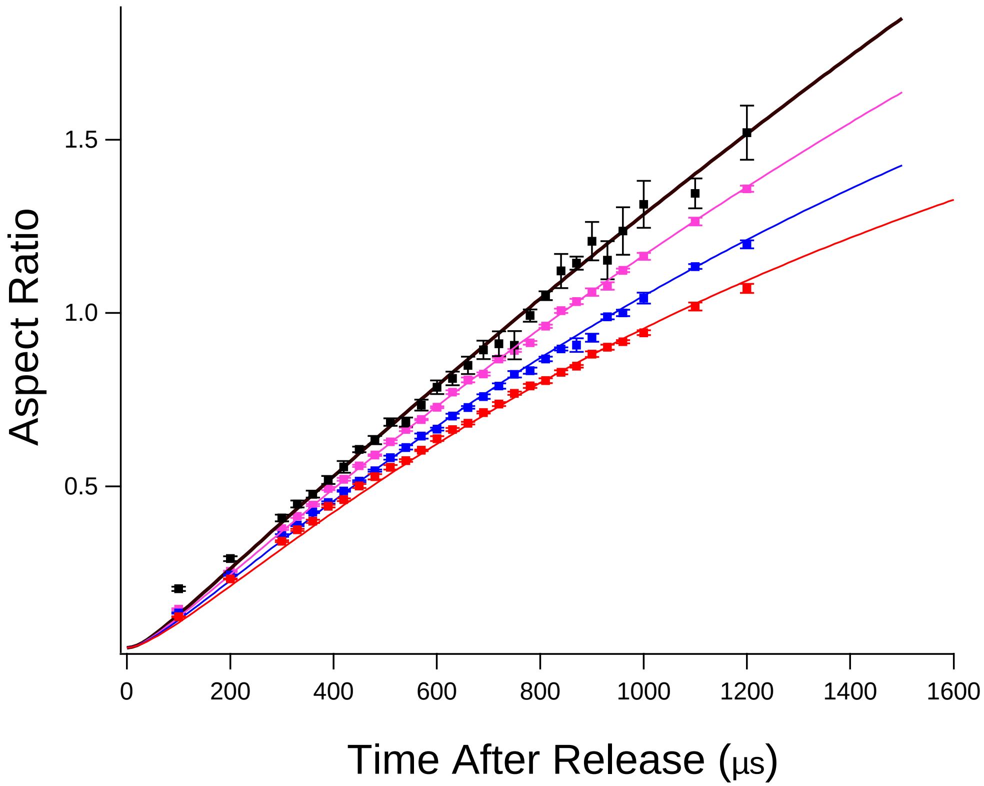
Schaefer (2007), see also Bruun, Smith

$T \ll T_F$ $\quad$ $T \gg T_F$, $\tau_R \simeq \eta/P$

Elliptic flow: High T limit

Quantum viscosity $\eta = \eta_0 \dfrac{(mT)^{3/2}}{\hbar^2}$





$\eta = \eta_0 (mT)^{3/2}$

$\tau_R = \eta / P$

fit: $\eta_0 = 0.33 \pm 0.04$

theory: $\eta_0 = \frac{15}{32\sqrt{\pi}} = 0.26$

Summary

Unitary Fermi gas has become "the" benchmark problem for many body methods (equation of state, pairing, DFT) in nuclear physics (at least for pure neutron matter).

Interesting (but maybe not quantitative) connections to the physics of quark matter and the quark gluon plasma, in particular nearly perfect fluidity.

Many questions: Universality of nearly perfect fluidity? Quasi-particle picture?

More intersections

Few body physics: Efimov effect, etc.

Several species: Three species (quark-hadron transition), four species (nuclear matter, SU(4) symmetry).

Finite polarization: critical $\delta\mu$, LOFF phase (relevant to stressed color superconductivity).

Rotating systems: Vortices (formation, pinning, etc.).

New ideas: gauge fields, role of dimensionality, AdS/NRCFT.