Revisiting the strong coupling limit of lattice QCD

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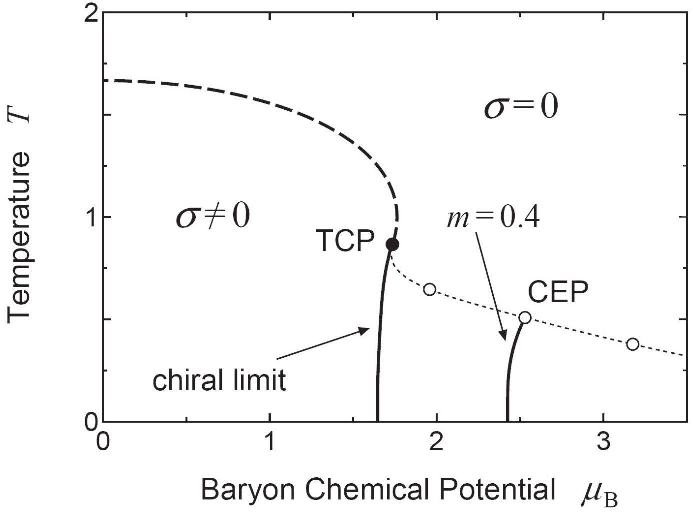
Motivation

- 25^+ years of analytic predictions:
  80's: Kluberg-Stern et al., Kawamoto-Smit, Damgaard-Kawamoto
  $$T_c(\mu=0)=5/3,\ \ \mu_c(T=0)=0.66$$
  90's: Petersson et al., $1/g^2$ corrections
  00's: detailed (μ,T) phase diagram: Nishida, Kawamoto,...
  08: Ohnishi, Münster & Philipsen,...
  How accurate is mean-field ($1/d$) approximation?

- Almost no Monte Carlo crosschecks:
  89: Karsch-Mütter $\longrightarrow$ MDP formalism $\longrightarrow$ $\mu_c(T=0)\sim 0.63$
  92: Karsch et al. $T_c(\mu=0)\approx 1.40$
  99: Azcoiti et al., MDP ergodicity ??
  06: PdF-Kim, HMC $\longrightarrow$ hadron spectrum $\sim 2\%$ of mean-field

Can one trust the details of analytic phase-diagram predictions?

Phase diagram according to Nishida (2004)



Very similar to conjectured phase diagram of $N_f = 2$ QCD

Strong coupling $SU(3)$ with staggered quarks

$Z = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi \exp(-\bar{\psi}(\not{D}(U)+m)\psi)$, no plaquette term ($\beta = 0$)

- One KS fermion field (ie. 4 “tastes"): 6 d.o.f. per site
- $\not{D}(U) = \frac{1}{2}\sum_{x,\nu}\eta_\nu(x)(U_\nu(x) - U_\nu^\dagger(x-\hat{\nu}))$, $\eta_\nu(x) = (-)^{x_1+..+x_{\nu-1}}$
- Chemical potential $\mu \quad \longrightarrow \exp(\pm a\mu)U_{\pm 4}$

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$$Z = \int \mathcal{D}U \; \det(\not{D}(U)+m) \quad \rightarrow \quad \text{HMC, etc...}$$

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- Alternative 2: integrate over links — Rossi & Wolff

$\longrightarrow$ Color singlet degrees of freedom:

- Monomer (meson $\bar\psi\psi$) $M(x) \in \{0,1,2,3\}$
- Dimer (meson hopping), non-oriented $n_\nu(x) \in \{0,1,2,3\}$
- Baryon hopping, oriented $\bar{B}B_\nu(x) \in \{0,1\}$ $\longrightarrow$ self-avoiding loops C

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$$Z(m,\mu) = \sum_{\{M,n_\nu,C\}} \prod_x \frac{3!}{M(x)!} m^{M(x)} \prod_{x,\nu} \frac{(3-n_\nu(x))!}{3!n_\nu(x)!} \prod_{\text{loops } C} \rho(C)$$

with constraint $(M + \sum_{\pm\nu} n_\nu)(x) = 3 \ \forall x \notin \{C\}$

MDP Monte Carlo

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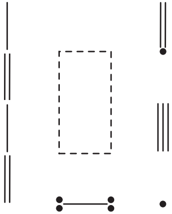
3 difficulties:

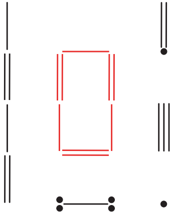
- sign of $\prod_C \rho(C)$:

  associate $\pm$ baryon loops with (1212.. & 2121..) polymer loops

  weight: $\pm \cosh\frac{\mu}{T}$ $+1$ $\longrightarrow$ much milder sign problem

  MDP ensemble — Karsch & Mütter





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Solved with **worm algorithm** (Prokof'eev & Svistunov)

Worm algorithm for MDP

Here for chiral limit $m = 0$ (no monomers: $M(x) = 0\ \forall x$)

- Break a dimer bond and introduce a pair of adjacent monomers $M(x), M(y)$
- Choose among neighbours of y by local heatbath and move $M(y)$ there
  heatbath: sampling of 2-point function $\frac{1}{Z_{||}} M(x)M(y)\exp(-S_{||})$
- Keep moving "head" y until $y \to x$, ie. "worm closes" $\to$ new configuration in $Z_{||}$

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Global change obtained from sequence of local updates

Each local step gives information on 2-point function

Very close to Adams & Chandrasekharan for $U(N)$

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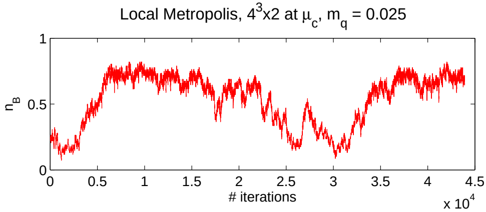
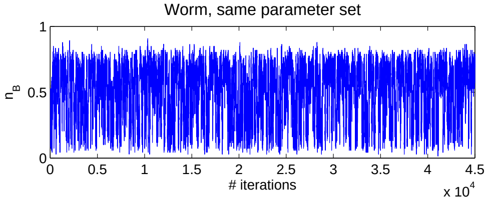
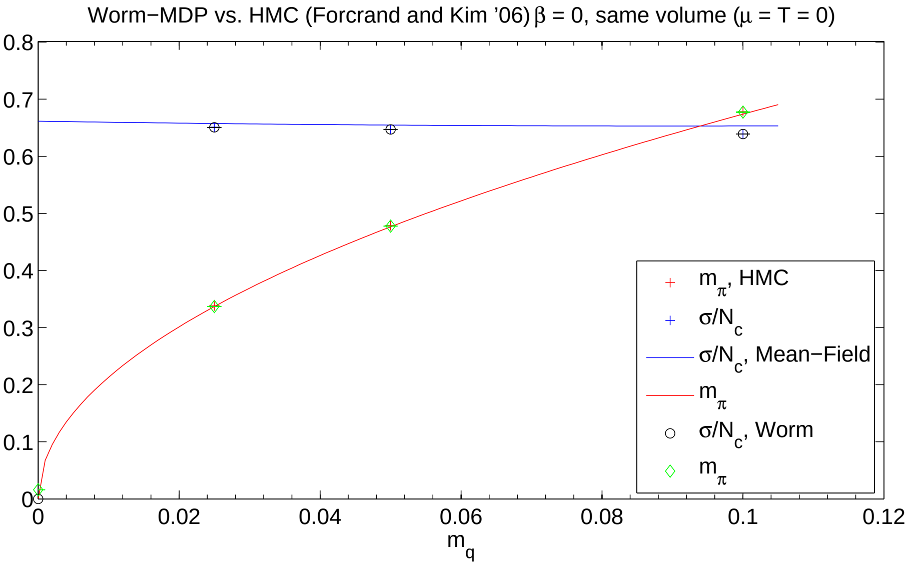




Consistency check with HMC



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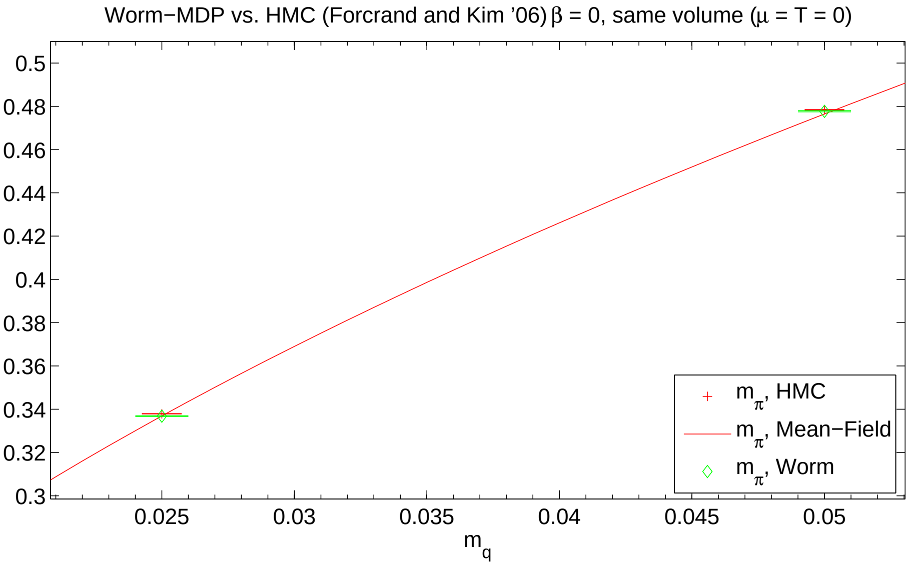
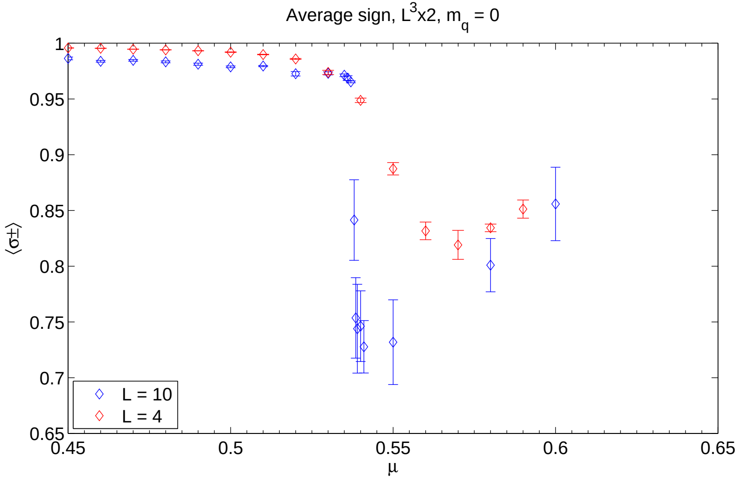


Sign problem?

Worst case $m = 0$:



Can reach $\sim 16^3 \times 4 \; \forall \mu$, ie. adequate

Transition $T = 0, \mu = \mu_c$

Puzzle:

- Mean-field baryon mass is ≈ 3 $\Rightarrow$ expect $\mu_c = \frac{1}{3} F_B(T = 0) \approx 1$
- Mean-field estimate $\mu_c \sim 0.55 - 0.66$ much smaller

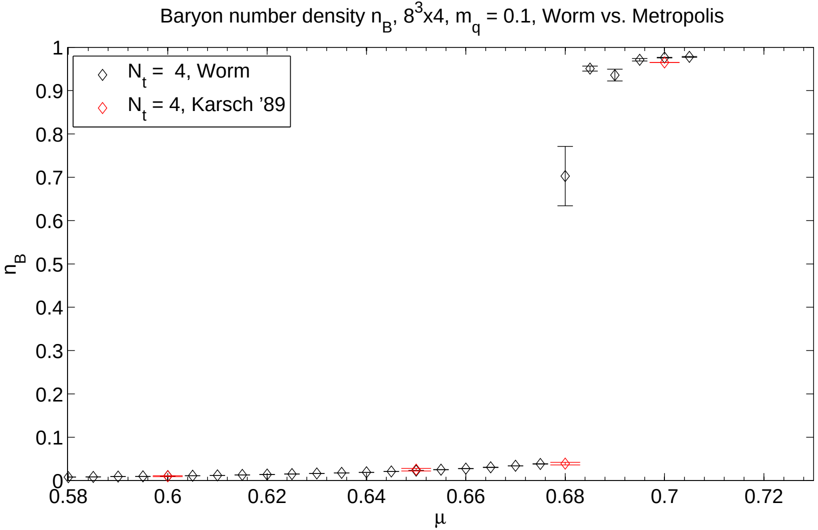
- Baryon mass ≈ 3 checked by HMC — PdF & Kim
- $\mu_c \approx 0.63$ checked by Karsch & Mütter *for* $T = 1/4$ *only*

Explanation ?

- Problem with $m \to 0$ or $T \to 0$ extrapolation of MC data ?
- Or nuclear attraction $\sim 1/3$ baryon mass!

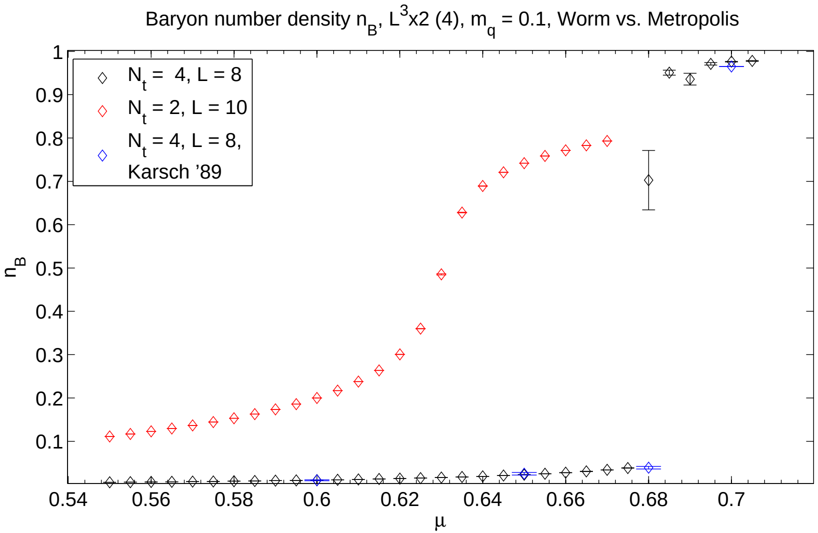
Check with $m = 0$, $T \approx 0$ worm simulations

Consistency check with Karsch & Mütter



Agreement except at $\mu = 0.68 \sim \mu_c$ $\leftrightarrow$ ergodicity of local update

Reducing the quark mass



As $m \longrightarrow 0$, μ_c decreases and transition becomes stronger

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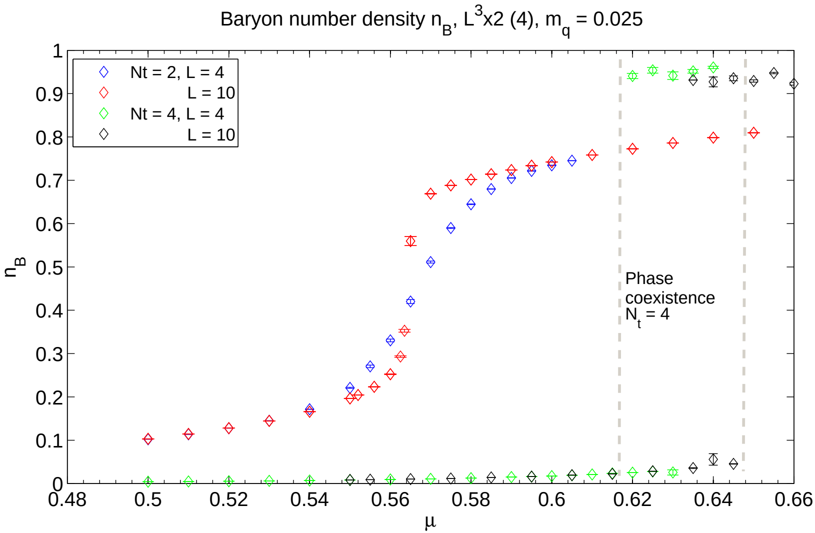

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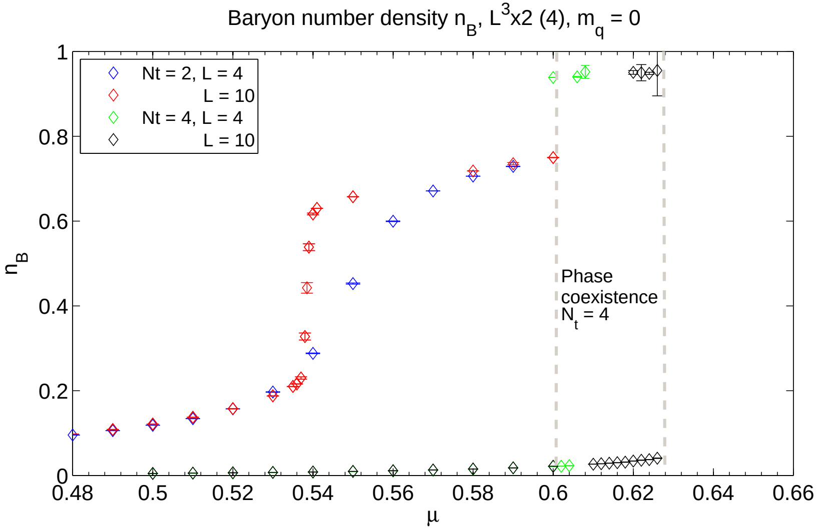

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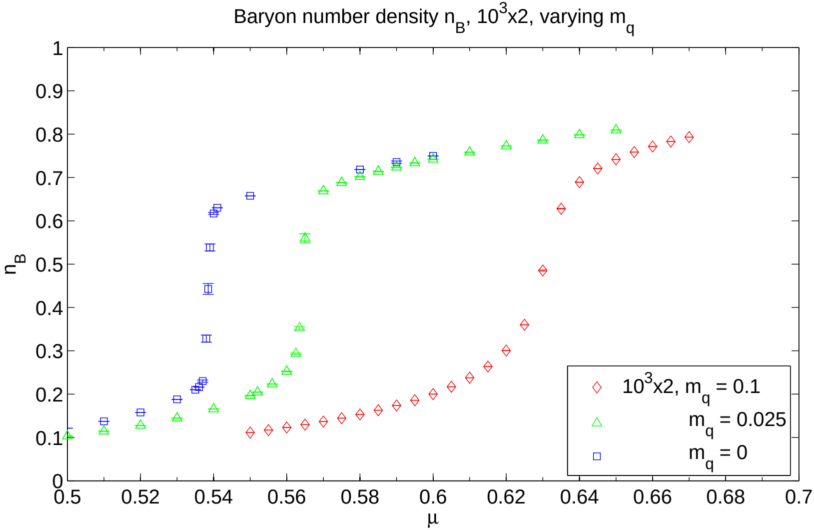
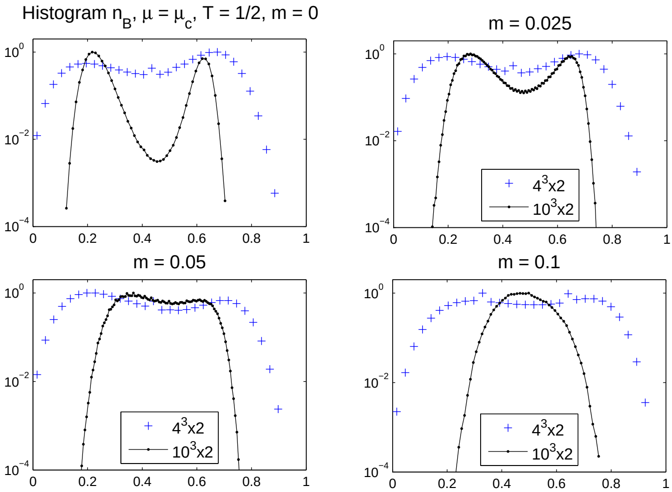
Varying the mass at fixed $T = 1/2$



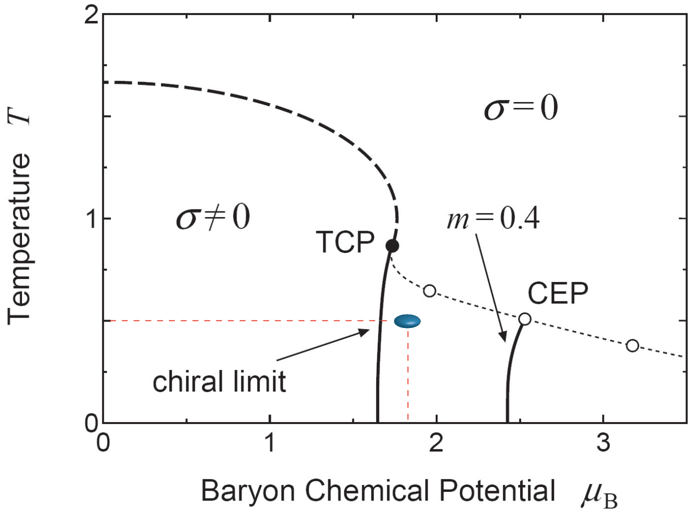
From first-order ($m = 0$) to crossover ($m = 0.1$) $\Rightarrow$ critical mass m_c ?

Critical mass $m_c(T = 1/2)$?



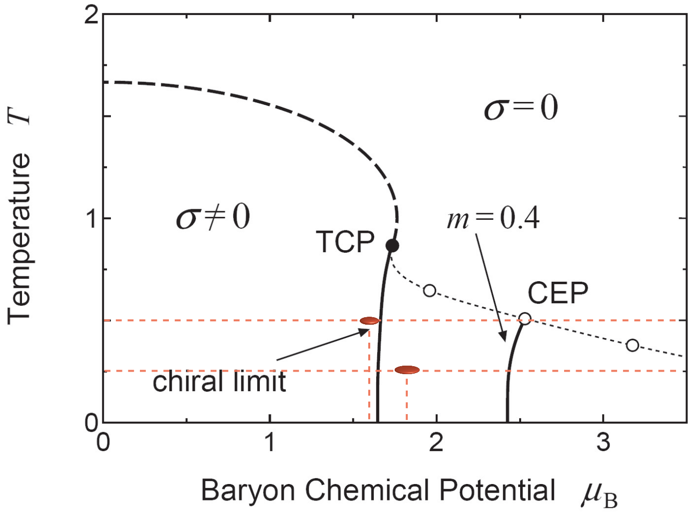
Critical mass $m_c(T = 1/2) \sim 0.05$

CEP: compare with Nishida (2004)



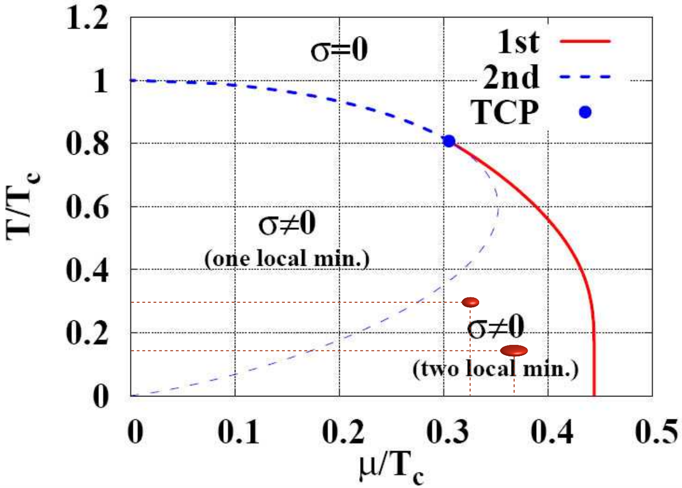
Qualitative agreement, but not quantitative

$m = 0$: compare $\mu_c(T = 1/2, T = 1/4)$ with Nishida (2004)



Qualitative agreement, but not quantitative

$m = 0$: compare $\mu_c(T = 1/2, T = 1/4)$ with Kawamoto (2005)



Take $T_c = 5/3$ (mean-field) [MC: 1.40 Karsch]

$\longrightarrow$ qualitative agreement, but not quantitative

Conclusions

Summary

- For $m = 0$, $\mu_c(T = 1/4) \approx 0.62$ ($< m_B/3$) and $\mu_c(T = 1/2) \approx 0.54$
- Critical end-point (not chiral) moves to larger μ as m increases

Outlook

- Improve systematics:
  - Multicanonical MC for first-order transition at low T
  - Asymmetry γ in Dirac coupling to vary T continuously
  - Check mean-field “scaling” $T = \gamma^2/N_t$
  - Compare real and imaginary μ
- Determine phase diagram:
  - Tricritical point for $m = 0$
  - Critical end-point as a function of m
- Extend to 2 KS fields:
  - Baryon no longer self-avoiding $\rightarrow$ $B\pi$ scattering etc..
  - Isospin μ