14. QUARK MODEL

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14.1. Quantum numbers of the quarks

Quarks are strongly interacting fermions with spin 1/2 and, by convention, positive parity. Then antiquarks have negative parity. Quarks have the additive baryon number 1/3, antiquarks -1/3. Table 14.1 gives the other additive quantum numbers (flavors) for the three generations of quarks. They are related to the charge Q (in units of the elementary charge e) through the generalized Gell-Mann-Nishijima formula

$$\mathsf{Q} = \mathsf{I}_z + \frac{\mathcal{B} + \mathsf{S} + \mathsf{C} + \mathsf{B} + \mathsf{T}}{2} , \tag{14.1}$$

where $\mathcal{B}$ is the baryon number. The convention is that the *flavor* of a quark (I_z, S, C, B, or T) has the same sign as its *charge* Q. With this convention, any flavor carried by a charged meson has the same sign as its charge, e.g. the strangeness of the K^+ is $+1$, the bottomness of the B^+ is $+1$, and the charm and strangeness of the D_s^- are each -1. Antiquarks have the opposite flavor signs.

Table 14.1: Additive quantum numbers of the quarks.

Property \ Quark	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	-1	0	0	0
C – charm	0	0	0	$+1$	0	0
B – bottomness	0	0	0	0	-1	0
T – topness	0	0	0	0	0	$+1$

14.2. Mesons

Mesons have baryon number $\mathcal{B} = 0$. In the quark model they are $q\overline{q}\,'$ bound states of quarks q and antiquarks $\overline{q}\,'$ (the flavors of q and q' may be different). If the orbital angular momentum of the $q\overline{q}\,'$ state is ℓ, then the parity P is $(-1)^{\ell+1}$. The meson spin J is given by the usual relation $|\ell - s| < J < |\ell + s|$ where s is 0 (antiparallel quark spins) or 1 (parallel quark spins). The charge conjugation, or C-parity $C = (-1)^{\ell+s}$, is defined only for the $q\bar{q}$ states made of quarks and their own antiquarks. The C-parity can be generalized to the G-parity $G = (-1)^{I+\ell+s}$ for mesons made of quarks and their own antiquarks (isospin $\mathsf{I}_z = 0$) and for the charged $u\bar{d}$ and $d\bar{u}$ states (isospin $\mathsf{I} = 1$).

The mesons are classified in J^{PC} multiplets. The $\ell = 0$ states are the pseudoscalars (0^{-+}) and the vectors (1^{--}). The orbital excitations $\ell = 1$ are the scalars (0^{++}), the axial vectors

(1^{++}) and (1^{+-}), and the tensors (2^{++}). Assignments for many of the known mesons are given in Tables 14.2 and 14.3. Radial excitations are denoted by the principal quantum number n. The very short lifetime of the t quark makes it likely that bound state hadrons containing t quarks and/or antiquarks do not exist.

States in the natural spin-parity series $P = (-1)^J$ must, according to the above, have $s = 1$ and hence $CP = +1$. Thus mesons with natural spin-parity and $CP = -1$ (0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, etc) are forbidden in the $q\bar{q}'$ model. The $J^{PC} = 0^{--}$ state is forbidden as well. Mesons with such *exotic* quantum numbers may exist, but would lie outside the $q\bar{q}'$ model (see section below on exotic mesons).

Following SU(3) the nine possible $q\bar{q}'$ combinations containing the light u, d, and s quarks are grouped into an octet and a singlet of light quark mesons:

$$\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1} \ . \tag{14.2}$$

A fourth quark such as charm c can be included by extending SU(3) to SU(4). However, SU(4) is badly broken owing to the much heavier c quark. Nevertheless, in an SU(4) classification the sixteen mesons are grouped into a 15-plet and a singlet:

$$\mathbf{4} \otimes \overline{\mathbf{4}} = \mathbf{15} \oplus \mathbf{1} \ . \tag{14.3}$$

The *weight diagrams* for the ground-state pseudoscalar (0^{-+}) and vector (1^{--}) mesons are depicted in Fig. 14.1. The light quark mesons are members of nonets building the middle plane in Fig. 14.1(a) and (b).

Isoscalar states with the same J^{PC} will mix but mixing between the two light quark mesons and the much heavier charm or bottom states are generally assumed to be negligible. In the following we shall use the generic names a for the $I = 1$, K for the $I = 1/2$, f and f' for the $I = 0$ members of the light quark nonets. Thus the physical isoscalars are mixtures of the SU(3) wave function ψ_8 and ψ_1:

$$f' = \psi_8 \, \cos\theta - \psi_1 \, \sin\theta \ , \tag{14.4}$$

$$f = \psi_8 \, \sin\theta + \psi_1 \, \cos\theta \ , \tag{14.5}$$

where θ is the nonet mixing angle and

$$\psi_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \ , \tag{14.6}$$

$$\psi_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \ . \tag{14.7}$$

The mixing angle has to be determined experimentally.

These mixing relations are often rewritten to exhibit the $u\bar{u} + d\bar{d}$ and $s\bar{s}$ components which decouple for the "ideal" mixing angle θ_i such that $\tan\theta_i = 1/\sqrt{2}$ (or θ_i=35.3°). Defining $\alpha = \theta + 54.7°$, one obtains the physical isoscalar in the flavor basis

$$f' = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\cos\alpha - s\bar{s} \, \sin\alpha \ , \tag{14.8}$$

and its orthogonal partner f (replace α by $\alpha - 90°$). Thus for ideal mixing ($\alpha_i = 90°$) the f' becomes pure $s\bar{s}$ and the f pure $u\bar{u} + d\bar{d}$. The mixing angle θ can be derived from the mass relation

$$\tan\theta = \frac{4m_K - m_a - 3m_{f'}}{2\sqrt{2}(m_a - m_K)} \ , \tag{14.9}$$

which also determines its sign or, alternatively, from

$$\tan^2\theta = \frac{4m_K - m_a - 3m_{f'}}{-4m_K + m_a + 3m_f} \ . \tag{14.10}$$

Eliminating θ from these equations leads to the sum rule [1]

$$(m_f + m_{f'})(4m_K - m_a) - 3m_f m_{f'} = 8m_K^2 - 8m_K m_a + 3m_a^2. \tag{14.11}$$

This relation is verified for the ground-state vector mesons. We identify the $\phi(1020)$ with the f' and the $\omega(783)$ with the f. Thus

$$\phi(1020) = \psi_8 \cos\theta_V - \psi_1 \sin\theta_V \ , \tag{14.12}$$

$$\omega(782) = \psi_8 \sin\theta_V + \psi_1 \cos\theta_V \ , \tag{14.13}$$

with the vector mixing angle $\theta_V = 35°$ from Eq. (14.9), very close to ideal mixing. Thus $\phi(1020)$ is nearly pure $s\bar{s}$. For ideal mixing Eq. (14.9) and Eq. (14.10) lead to the relations

$$m_K = \frac{m_f + m_{f'}}{2} \ , \quad m_a = m_f \ , \tag{14.14}$$

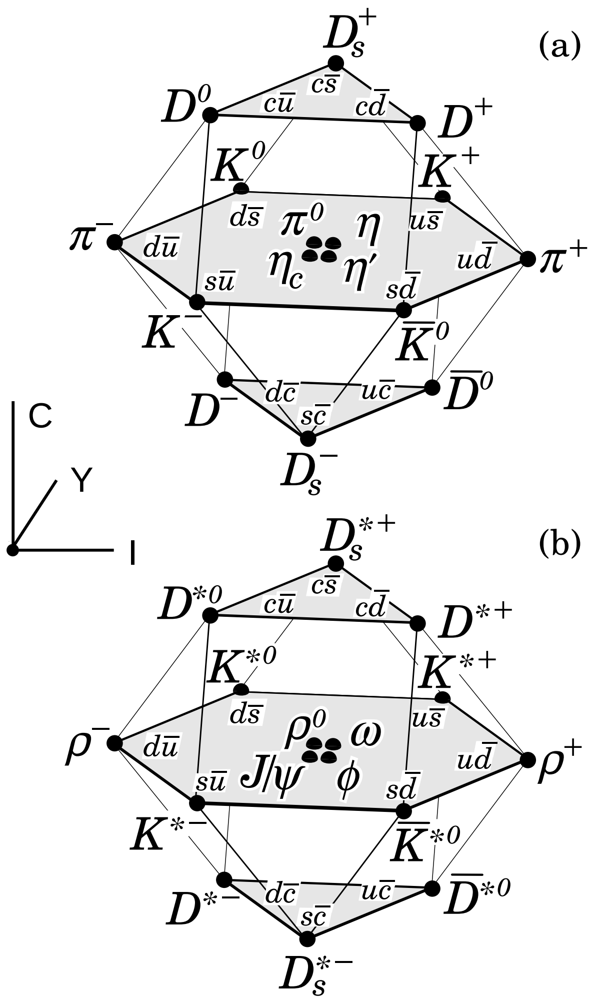

Figure 14.1: SU(4) weight diagram showing the 16-plets for the pseudoscalar (a) and vector mesons (b) made of the u, d, s and c quarks as a function of isospin I, charm C and hypercharge $\mathsf{Y} = \mathsf{S}+\mathcal{B} - \frac{\mathsf{C}}{3}$. The nonets of light mesons occupy the central planes to which the $c\bar{c}$ states have been added.

which are satisfied for the vector mesons. However, for the pseudoscalar (and scalar mesons) Eq. (14.11) is satisfied only approximately. Then Eq. (14.9) and Eq. (14.10) lead to somewhat different values for the mixing angle. Identifying the η with the f' one gets

$$\eta = \psi_8 \cos\theta_P - \psi_1 \sin\theta_P \ , \tag{14.15}$$

$$\eta' = \psi_8 \sin\theta_P + \psi_1 \cos\theta_P \ . \tag{14.16}$$

Following chiral perturbation theory the meson masses in the mass formulae (Eq. (14.9)) and (Eq. (14.10)) should be replaced by their squares. Table 14.2 lists the mixing angle $\theta_{\rm lin}$ from Eq. (14.10) and the corresponding $\theta_{\rm quad}$ obtained by replacing the meson masses by their squares throughout.

Table 14.2: Suggested $q\overline{q}$ quark-model assignments for some of the observed light mesons. Mesons in bold face are included in the Meson Summary Table. The wave functions f and f' are given in the text. The singlet-octet mixing angles from the quadratic and linear mass formulae are also given for some of the nonets. The classification of the 0^{++} mesons is tentative and the mixing angle uncertain due to large uncertainties in some of the masses. The $f_0(1500)$ in the Meson Summary Table is not in this table as it is hard to accommodate in the scalar nonet. The light scalars $a_0(980)$, $f_0(980)$ and $f_0(600)$ are often considered as meson-meson resonances or four-quark states and are therefore not included in the table. See the "Note on Non-$q\overline{q}$ Mesons" at the end of the Meson Listings.

$n\ ^{2s+1}\ell_J$	J^{PC}	I = 1 ud, $\overline{u}d$, $\frac{1}{\sqrt{2}}(d\overline{d} - u\overline{u})$	I = $\frac{1}{2}$ $u\overline{s}$, $d\overline{s}$; $\overline{d}s$, $-\overline{u}s$	I = 0 f'	I = 0 f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\ ^1S_0$	0^{-+}	$\boldsymbol{\pi}$	$\boldsymbol{K}$	$\boldsymbol{\eta}$	$\boldsymbol{\eta'(958)}$	−11.5	−24.6
$1\ ^3S_1$	1^{--}	$\boldsymbol{\rho(770)}$	$\boldsymbol{K^*(892)}$	$\boldsymbol{\phi(1020)}$	$\boldsymbol{\omega(782)}$	38.7	36.0
$1\ ^1P_1$	1^{+-}	$\boldsymbol{b_1(1235)}$	$\boldsymbol{K_{1B}}$†	$h_1(1380)$	$\boldsymbol{h_1(1170)}$		
$1\ ^3P_0$	0^{++}	$\boldsymbol{a_0(1450)}$	$\boldsymbol{K_0^*(1430)}$	$\boldsymbol{f_0(1710)}$	$\boldsymbol{f_0(1370)}$		
$1\ ^3P_1$	1^{++}	$\boldsymbol{a_1(1260)}$	$\boldsymbol{K_{1A}}$†	$\boldsymbol{f_1(1420)}$	$\boldsymbol{f_1(1285)}$		
$1\ ^3P_2$	2^{++}	$\boldsymbol{a_2(1320)}$	$\boldsymbol{K_2^*(1430)}$	$\boldsymbol{f_2'(1525)}$	$\boldsymbol{f_2(1270)}$	29.6	28.0
$1\ ^1D_2$	2^{-+}	$\boldsymbol{\pi_2(1670)}$	$\boldsymbol{K_2(1770)}$†	$\eta_2(1870)$	$\boldsymbol{\eta_2(1645)}$		
$1\ ^3D_1$	1^{--}	$\boldsymbol{\rho(1700)}$	$\boldsymbol{K^*(1680)}$‡		$\boldsymbol{\omega(1650)}$		
$1\ ^3D_2$	2^{--}		$\boldsymbol{K_2(1820)}$‡				
$1\ ^3D_3$	3^{--}	$\boldsymbol{\rho_3(1690)}$	$\boldsymbol{K_3^*(1780)}$	$\boldsymbol{\phi_3(1850)}$	$\boldsymbol{\omega_3(1670)}$	32.0	31.0
$1\ ^3F_4$	4^{++}	$\boldsymbol{a_4(2040)}$	$\boldsymbol{K_4^*(2045)}$		$\boldsymbol{f_4(2050)}$		
$1\ ^3G_5$	5^{--}	$\rho_5(2350)$					
$1\ ^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\ ^1S_0$	0^{-+}	$\boldsymbol{\pi(1300)}$	$K(1460)$	$\boldsymbol{\eta(1475)}$	$\boldsymbol{\eta(1295)}$	−22.4	−22.6
$2\ ^3S_1$	1^{--}	$\boldsymbol{\rho(1450)}$	$\boldsymbol{K^*(1410)}$‡	$\boldsymbol{\phi(1680)}$	$\boldsymbol{\omega(1420)}$		

† The $1^{+\pm}$ and $2^{-\pm}$ isospin $\frac{1}{2}$ states mix. In particular, the K_{1A} and K_{1B} are nearly equal (45°) mixtures of the $K_1(1270)$ and $K_1(1400)$.

‡ The $K^*(1410)$ could be replaced by the $K^*(1680)$ as the $2\ ^3S_1$ state.

Table 14.3: $q\overline{q}$ quark-model assignments for the observed heavy mesons. Mesons in bold face are included in the Meson Summary Table.

$n\ ^{2s+1}\ell_J$	J^{PC}	I = 0 $c\overline{c}$	I = 0 $b\overline{b}$	I = $\frac{1}{2}$ $c\overline{u}$, $c\overline{d}$; $\overline{c}u$, $\overline{c}d$	I = 0 $c\overline{s}$; $\overline{c}s$	I = $\frac{1}{2}$ $b\overline{u}$, $b\overline{d}$; $\overline{b}u$, $\overline{b}d$	I = 0 $b\overline{s}$; $\overline{b}s$	I = 0 $b\overline{c}$; $\overline{b}c$
$1\ ^1S_0$	0^{-+}	$\boldsymbol{\eta_c(1S)}$	$\eta_b(1S)$	$\boldsymbol{D}$	$\boldsymbol{D_s^{\pm}}$	$\boldsymbol{B}$	$\boldsymbol{B_s}$	$\boldsymbol{B_c^{\pm}}$
$1\ ^3S_1$	1^{--}	$\boldsymbol{J/\psi(1S)}$	$\boldsymbol{\Upsilon(1S)}$	$\boldsymbol{D^*}$	$\boldsymbol{D_s^{*\pm}}$	$\boldsymbol{B^*}$	$B_s^{*\pm}$	
$1\ ^1P_1$	1^{+-}	$h_c(1P)$		$\boldsymbol{D_1(2420)}$	$\boldsymbol{D_{s1}(2536)^{\pm}}$			
$1\ ^3P_0$	0^{++}	$\boldsymbol{\chi_{c0}(1P)}$	$\boldsymbol{\chi_{b0}(1P)}$		$\boldsymbol{D_{sJ}^*(2317)^{\pm\dagger}}$			
$1\ ^3P_1$	1^{++}	$\boldsymbol{\chi_{c1}(1P)}$	$\boldsymbol{\chi_{b1}(1P)}$		$\boldsymbol{D_{sJ}^*(2460)^{\pm\dagger}}$			
$1\ ^3P_2$	2^{++}	$\boldsymbol{\chi_{c2}(1P)}$	$\boldsymbol{\chi_{b2}(1P)}$	$\boldsymbol{D_2(2460)}$	$\boldsymbol{D_{s2}^*(2573)^{\pm}}$			
$1\ ^3D_1$	1^{--}	$\boldsymbol{\psi(3770)}$						
$2\ ^1S_0$	0^{-+}	$\boldsymbol{\eta_c(2S)}$						
$2\ ^3S_1$	1^{--}	$\boldsymbol{\psi(2S)}$	$\boldsymbol{\Upsilon(2S)}$					
$2\ ^3P_{0,1,2}$	$0^{++}, 1^{++}, 2^{++}$		$\boldsymbol{\chi_{b\,0,1,2}(2P)}$					

† The masses of these states are considerably smaller than most theoretical predictions. They have also been considered as four-quark states (See the "Note on Non-$q\overline{q}$ Mesons" at the end of the Meson Listings).

The pseudoscalar mixing angle θ_P can also be measured by comparing the partial widths for radiative J/ψ decay into a vector and a pseudoscalar [2], radiative $\phi(1020)$ decay into η and η' [3], or $\bar{p}p$ annihilation at rest into a pair of vector and pseudoscalar or into two pseudoscalars [4,5]. One obtains a mixing angle between $-10°$ and $-20°$.

The nonet mixing angles can be measured in $\gamma\gamma$ collisions, e.g. for the 0^{-+}, 0^{++} and 2^{++} nonets. In the quark model the coupling of neutral mesons to two photons is proportional to $\sum_i Q_i^2$, where Q_i is the charge of the i-th quark. The 2γ partial width of an isoscalar meson with mass m is then given in terms of the mixing angle α by

$$\Gamma_{2\gamma} = C(5\cos\alpha - \sqrt{2}\sin\alpha)^2 m^3 \ , \tag{14.17}$$

for f' and f ($\alpha \to \alpha - 90°$). The coupling C may depend on the meson mass. It is often assumed to be a constant in the nonet. For the isovector a one then finds $\Gamma_{2\gamma} = 9\ C\ m^3$. Thus the members of an ideally mixed nonet couple to 2γ with partial widths in the ratios $f : f' : a = 25 : 2 : 9$. For tensor mesons one finds from the ratios of the measured 2γ partial widths for the $f_2(1270)$ and $f_2'(1525)$ mesons a mixing angle α_T of $(81\pm 1)°$, or $\theta_T = (27 \pm 1)°$, in accord with the linear mass formula. For the pseudoscalars one finds from the ratios of partial widths $\Gamma(\eta' \to 2\gamma)/\Gamma(\eta \to 2\gamma)$ a mixing angle $\theta_P = (-18 \pm 2)°$ while the ratio $\Gamma(\eta' \to 2\gamma)/\Gamma(\pi^0 \to 2\gamma)$ leads to $\sim -24\ °$. SU(3) breaking effects for pseudoscalars are discussed in Ref. 6.

Table 14.4: SU(3) couplings γ^2 for quarkonium decays as a function on nonet mixing angle α, up to a common multiplicative factor C ($\phi \equiv 54.7° + \theta_P$).

Isospin	Decay channel	γ^2
0	$\pi\pi$	$3\cos^2\alpha$
	$K\overline{K}$	$(\cos\alpha - \sqrt{2}\sin\alpha)^2$
	$\eta\eta$	$(\cos\alpha\cos^2\phi - \sqrt{2}\sin\alpha\sin^2\phi)^2$
	$\eta\eta'$	$\frac{1}{2}\sin^2 2\phi\ (\cos\alpha + \sqrt{2}\sin\alpha)^2$
1	$\eta\pi$	$2\cos^2\phi$
	$\eta'\pi$	$2\sin^2\phi$
	$K\overline{K}$	1
$\frac{1}{2}$	$K\pi$	$\frac{3}{2}$
	$K\eta$	$(\sin\phi - \frac{\cos\phi}{\sqrt{2}})^2$
	$K\eta'$	$(\cos\phi + \frac{\sin\phi}{\sqrt{2}})^2$

The partial width for the decay of a scalar or a tensor meson into a pair of pseudoscalar mesons is model dependent. Following Ref. 7,

$$\Gamma = C \times \gamma^2 \times |F(q)|^2 \times q \ . \tag{14.18}$$

C is a nonet constant, q the momentum of the decay products, $F(q)$ a form factor and γ^2 the SU(3) coupling. The model-dependent formfactor may be written as

$$|F(q)|^2 = q^{2\ell} \times \exp(-\frac{q^2}{8\beta^2}), \tag{14.19}$$

where ℓ is the relative angular momentum between the decay products. The decay of a $q\bar{q}$ meson into a pair of mesons involves the creation of a $q\bar{q}$ pair from the vacuum and SU(3) symmetry assumes that the matrix elements for the creation of $s\bar{s}$, $u\bar{u}$ and $d\bar{d}$ pairs are equal. The couplings γ^2 are given in Table 14.4 and their dependence upon the mixing angle α is shown in Fig. 14.2 for isoscalar decays. The generalization to unequal $s\bar{s}$, $u\bar{u}$ and $d\bar{d}$ couplings is given in Ref. 7. An excellent fit to the tensor meson decay widths is obtained assuming SU(3) symmetry, with $\beta \simeq 0.5$ GeV/c, $\theta_V \simeq 26\,^\circ$ and $\theta_P \simeq$ -17 $^\circ$ [7].

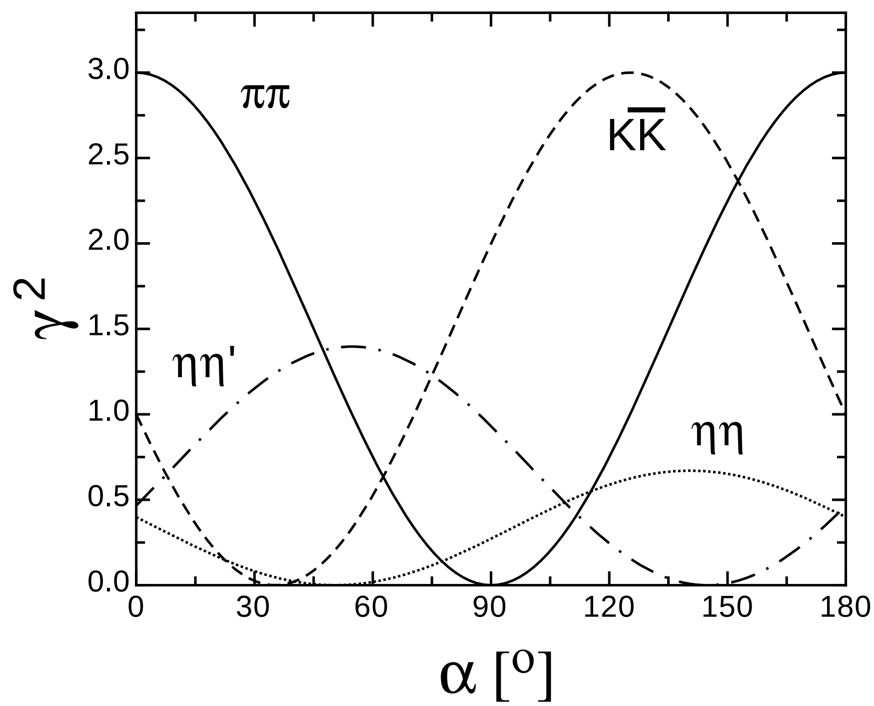

Figure 14.2: SU(3) couplings as a function of mixing angle α for isoscalar decays, up to a common multiplicative factor C and for $\theta_P = -17.3°$ (from Ref. 4).

14.3. Exotic mesons

The existence of a light nonet composed of four quarks with masses below 1 GeV was suggested a long time ago [8]. Coupling two triplets of light quarks u, d and s one obtains nine states, of which the six symmetric (uu, dd, ss, $ud+du$, $us+su, ds+sd$) form the six dimensional representation **6**, while the three antisymmetric ($ud-du$, $us-su$, $ds-sd$) form the three dimensional representation $\bar{\mathbf{3}}$ of SU(3):

$$\mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \bar{\mathbf{3}}\ . \tag{14.20}$$

Combining with spin and color and requiring antisymmetry, one finds that the most deeply bound diquark (and hence the lightest) is the one in the $\bar{\mathbf{3}}$ and spin singlet state. The combinination of the diquark with an antidiquark in the **3** representation then gives a light nonet of four-quark scalar states. Letting the number of strange quarks determine the mass splitting one obtains a mass inverted spectrum with a light isosinglet ($ud\bar{u}\bar{d}$), a medium heavy isodublet (e.g. $ud\bar{s}\bar{d}$) and a heavy isotriplet (e.g. $ds\bar{u}\bar{s}$) + isosinglet (e.g. $us\bar{u}\bar{s}$). It is then tempting to identify the lightest state with the $f_0(600)$, and the heaviest states with the $a_0(980)$, and $f_0(980)$. Then the meson with strangeness $\kappa(800)$ would lie in between.

QCD predicts the existence of isoscalar mesons which contain only gluons, the glueballs. The ground state glueball is predicted by lattice gauge theories to be 0^{++}, the first excited state 2^{++}. Errors on the mass predictions are large. Ref. 9 predicts a mass of about 1600 MeV for the ground state with an uncertainty of 160 MeV. As an example for the glueball mass spectrum we show in Figure 14.3 a recent calculation from the lattice [10]. The first excited state has a mass of about 2.4 GeV and the lightest glueball with exotic quantum numbers (2^{+-}) has a mass of about 4 GeV.

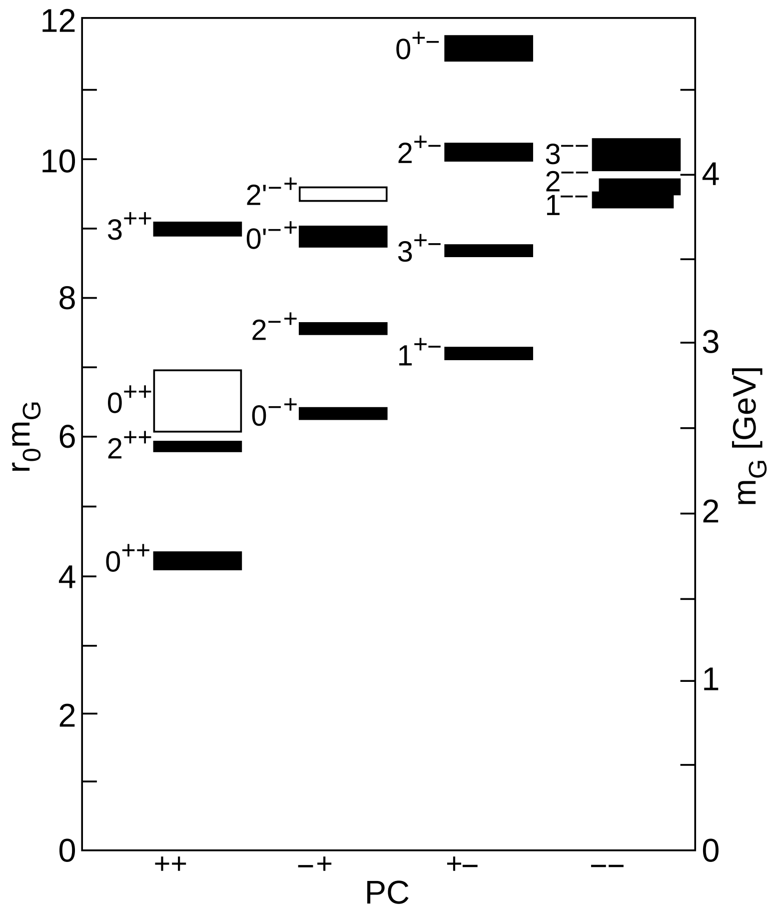

Figure 14.3: Predicted glueball mass spectrum from the lattice (from Ref. 10).

Lattice calculations assume that the quark masses are infinite and neglect $q\bar{q}$ loops. However, one expects that glueballs will mix with nearby $q\bar{q}$ states of the same quantum numbers [7,11]. For example, the two isoscalar 0^{++} mesons will mix with the pure ground state glueball to generate the observed physical states $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. Experimental evidence is mounting that the $f_0(1500)$ has considerable affinity for glue and that the $f_0(1370)$ and $f_0(1710)$ have large $u\bar{u} + d\bar{d}$ and $s\bar{s}$ components, respectively (See the "Note on Non-$q\bar{q}$ Mesons" at the end of the Meson Listings and Ref. 12).

Mesons made of $q\bar{q}$ pairs bound by excited gluons g, the hybrid states $q\bar{q}g$, are also predicted. They should lie in the 1.9 GeV mass region, according to gluon flux tube models [13]. Lattice QCD also predicts the lightest hybrid, an exotic 1^{-+}, at a mass of 1.9 GeV [14]. However, the bag model predicts four nonets, among them an exotic 1^{-+} around 1.4 GeV [15]. Most hybrids are rather broad but some can be as narrow as 100 MeV. There are so far two prominent candidates for exotic states with quantum numbers 1^{-+}, the $\pi_1(1400)$ and $\pi_1(1600)$, which could be hybrids or four-quark states (See the "Note on Non-$q\overline{q}$ Mesons" at the end of the Meson Listings and Ref. 12).

14.4. Baryons: *qqq* states

All the established baryons are apparently 3-quark (qqq) states, and each such state is an SU(3) color singlet, a completely antisymmetric state of the three possible colors. Since the quarks are fermions, the state function for any baryon must be antisymmetric under interchange of any two equal-mass quarks (up and down quarks in the limit of isospin symmetry). Thus the state function may be written as

$$|\,qqq\,\rangle_A = |\,\text{color}\,\rangle_A \times |\,\text{space, spin, flavor}\,\rangle_S \ , \tag{14.21}$$

where the subscripts S and A indicate symmetry or antisymmetry under interchange of any two of the equal-mass quarks. Note the contrast with the state function for the three nucleons in ^{3}H or ^{3}He:

$$|\,NNN\,\rangle_A = |\,\text{space, spin, isospin}\,\rangle_A \ . \tag{14.22}$$

This difference has major implications for internal structure, magnetic moments, *etc.* (For a nice discussion, see Ref. 16.)

The "ordinary" baryons are made up of u, d, and s quarks. The three flavors imply an approximate flavor SU(3), which requires that baryons made of these quarks belong to the multiplets on the right side of

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_M \oplus \mathbf{8}_M \oplus \mathbf{1}_A \tag{14.23}$$

(see Sec. 37, on "SU(n) Multiplets and Young Diagrams"). Here the subscripts indicate symmetric, mixed-symmetry, or antisymmetric states under interchange of any two quarks. The **1** is a uds state (Λ_1) and the octet contains a similar state (Λ_8). If these have the same spin and parity they can mix. An example is the mainly octet D_{03} $\Lambda(1690)$ and mainly singlet D_{03} $\Lambda(1520)$. In the ground state multiplet, the SU(3) flavor singlet Λ is forbidden by Fermi statistics. The mixing formalism is the same as for η-η' or ϕ-ω (see above), except that for baryons the mass M instead of M^2 is used. Section 36, on "SU(3) Isoscalar Factors and Representation Matrices", shows how relative decay rates in, say, $\mathbf{10} \to \mathbf{8} \otimes \mathbf{8}$ decays may be calculated. A summary of results of fits to the observed baryon masses and decay rates for the best-known SU(3) multiplets is given in Appendix II of our 1982 edition [17].

The addition of the c quark to the light quarks extends the flavor symmetry to SU(4). Figures 14.4(a) and 14.4(b) show the (badly broken) SU(4) baryon multiplets that have as their bottom levels an SU(3) octet, such as the octet that includes the nucleon, or an SU(3) decuplet, such as the decuplet that includes the $\Delta(1232)$. All the particles in a given SU(4) multiplet have the same spin and parity. The charmed baryons are discussed in more detail in the "Note on Charmed Baryons" in the Particle Listings. The addition of a b quark extends the flavor symmetry to SU(5); it would require four dimensions to draw the multiplets.

For the "ordinary" baryons (no c or b quark), flavor and spin may be combined in an approximate flavor-spin SU(6) in which the six basic states are $d\uparrow$, $d\downarrow$, $\cdots$, $s\downarrow$ ($\uparrow$, $\downarrow$ = spin up, down). Then the baryons belong to the multiplets on the right side of

$$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56}_S \oplus \mathbf{70}_M \oplus \mathbf{70}_M \oplus \mathbf{20}_A \ . \tag{14.24}$$

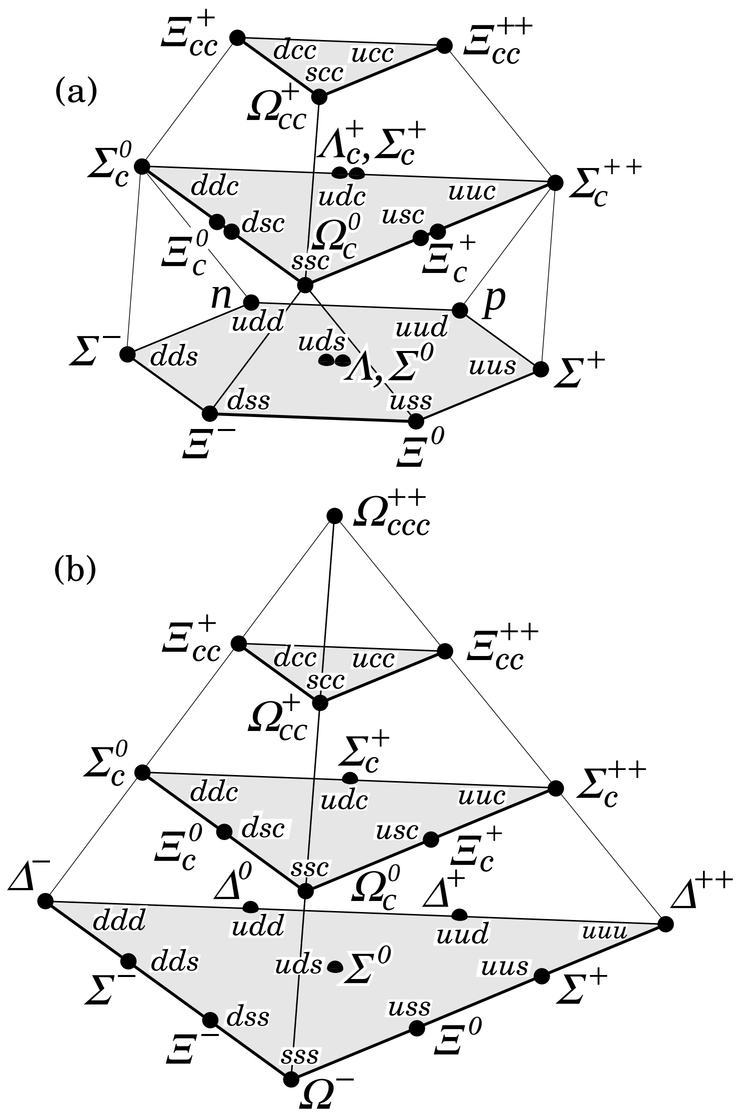

Figure 14.4: SU(4) multiplets of baryons made of u, d, s, and c quarks. (a) The 20-plet with an SU(3) octet. (b) The 20-plet with an SU(3) decuplet.

These SU(6) multiplets decompose into flavor SU(3) multiplets as follows:

$$\mathbf{56} = {}^4\mathbf{10} \oplus {}^2\mathbf{8} \tag{14.25a}$$

$$\mathbf{70} = {}^2\mathbf{10} \oplus {}^4\mathbf{8} \oplus {}^2\mathbf{8} \oplus {}^2\mathbf{1} \tag{14.25b}$$

$$\mathbf{20} = {}^2\mathbf{8} \oplus {}^4\mathbf{1} \ , \tag{14.25c}$$

where the superscript $(2S+1)$ gives the net spin S of the quarks for each particle in the SU(3) multiplet. The $J^P = 1/2^+$ octet containing the nucleon and the $J^P = 3/2^+$ decuplet containing the $\Delta(1232)$ together make up the "ground-state" 56-plet in which the orbital angular momenta between the quark pairs are zero (so that the spatial part of the state function is trivially symmetric). The **70** and **20** require some excitation of the spatial part of the state function in order to make the overall state function symmetric. States with nonzero orbital angular momenta are classified in SU(6)⊗O(3) supermultiplets. Physical baryons with the same quantum numbers do not belong to a single supermultiplet, since SU(6) is broken by spin-dependent interactions, differences in quark masses, *etc.* Nevertheless, the SU(6)⊗O(3) basis provides a suitable framework for describing baryon state functions.

It is useful to classify the baryons into bands that have the same number N of quanta of excitation. Each band consists of a number of supermultiplets, specified by (D, L_N^P), where D is the dimensionality of the SU(6) representation, L is the total quark orbital angular momentum, and P is the total parity. Supermultiplets contained in bands up to $N = 12$ are given in Ref. 18. The $N = 0$ band, which contains the nucleon and $\Delta(1232)$, consists only of the $(56,0_0^+)$ supermultiplet. The $N = 1$ band consists only of the $(70,1_1^-)$ multiplet and contains the negative-parity baryons with masses below about 1.9 GeV. The $N = 2$ band contains five supermultiplets: $(56,0_2^+)$, $(70,0_2^+)$, $(56,2_2^+)$, $(70,2_2^+)$, and $(20,1_2^+)$. Baryons belonging to the $(20,1_2^+)$ supermultiplet are not ever likely to be observed, since a coupling from the ground-state baryons requires a two-quark excitation. Selection rules are similarly responsible for the fact that many other baryon resonances have not been observed [19].

In Table 14.5, quark-model assignments are given for many of the established baryons whose SU(6)⊗O(3) compositions are relatively unmixed. We note that the unestablished resonances $\Sigma(1480)$, $\Sigma(1560)$, $\Sigma(1580)$, $\Sigma(1770)$, and $\Xi(1620)$ in our Baryon Particle Listings are too low in mass to be accommodated in most quark models [19,20].

The quark model for baryons is extensively reviewed in Ref. 21 and 22.

14.5. Dynamics

Many specific quark models exist, but most contain the same basic set of dynamical ingredients. These include:

i) A confining interaction, which is generally spin-independent.

ii) A spin-dependent interaction, modeled after the effects of gluon exchange in QCD. For example, in the S-wave states, there is a spin-spin hyperfine interaction of the form

$$H_{HF} = -\alpha_S M \sum_{i>j} (\vec{\sigma}\lambda_a)_i (\vec{\sigma}\lambda_a)_j \ , \tag{14.26}$$

where M is a constant with units of energy, λ_a $(a = 1, \cdots, 8,)$ is the set of SU(3) unitary spin matrices, defined in Sec. 36, on "SU(3) Isoscalar Factors and Representation Matrices," and the sum runs over constituent quarks or antiquarks. Spin-orbit interactions, although allowed, seem to be small.

iii) A strange quark mass somewhat larger than the up and down quark masses, in order to split the SU(3) multiplets.

iv) In the case of isoscalar mesons, an interaction for mixing $q\overline{q}$ configurations of different flavors ($e.g.$, $u\overline{u} \leftrightarrow d\overline{d} \leftrightarrow s\overline{s}$), in a manner which is generally chosen to be flavor independent.

These four ingredients provide the basic mechanisms that determine the hadron spectrum.

Table 14.5: Quark-model assignments for many of the known baryons in terms of a flavor-spin SU(6) basis. Only the dominant representation is listed. Assignments for some states, especially for the $\Lambda(1810)$, $\Lambda(2350)$, $\Xi(1820)$, and $\Xi(2030)$, are merely educated guesses. For assignments of the charmed baryons, see the "Note on Charmed Baryons" in the Particle Listings.

J^P	(D, L_N^P)	S	Octet members				Singlets
$1/2^+$	$(56,0_0^+)$	1/2	$N(939)$	$\Lambda(1116)$	$\Sigma(1193)$	$\Xi(1318)$	
$1/2^+$	$(56,0_2^+)$	1/2	$N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$	$\Xi(?)$	
$1/2^-$	$(70,1_1^-)$	1/2	$N(1535)$	$\Lambda(1670)$	$\Sigma(1620)$	$\Xi(?)$	$\Lambda(1405)$
$3/2^-$	$(70,1_1^-)$	1/2	$N(1520)$	$\Lambda(1690)$	$\Sigma(1670)$	$\Xi(1820)$	$\Lambda(1520)$
$1/2^-$	$(70,1_1^-)$	3/2	$N(1650)$	$\Lambda(1800)$	$\Sigma(1750)$	$\Xi(?)$	
$3/2^-$	$(70,1_1^-)$	3/2	$N(1700)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	
$5/2^-$	$(70,1_1^-)$	3/2	$N(1675)$	$\Lambda(1830)$	$\Sigma(1775)$	$\Xi(?)$	
$1/2^+$	$(70,0_2^+)$	1/2	$N(1710)$	$\Lambda(1810)$	$\Sigma(1880)$	$\Xi(?)$	$\Lambda(?)$
$3/2^+$	$(56,2_2^+)$	1/2	$N(1720)$	$\Lambda(1890)$	$\Sigma(?)$	$\Xi(?)$	
$5/2^+$	$(56,2_2^+)$	1/2	$N(1680)$	$\Lambda(1820)$	$\Sigma(1915)$	$\Xi(2030)$	
$7/2^-$	$(70,3_3^-)$	1/2	$N(2190)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	$\Lambda(2100)$
$9/2^-$	$(70,3_3^-)$	3/2	$N(2250)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	
$9/2^+$	$(56,4_4^+)$	1/2	$N(2220)$	$\Lambda(2350)$	$\Sigma(?)$	$\Xi(?)$	
			Decuplet members				
$3/2^+$	$(56,0_0^+)$	3/2	$\Delta(1232)$	$\Sigma(1385)$	$\Xi(1530)$	$\Omega(1672)$	
$1/2^-$	$(70,1_1^-)$	1/2	$\Delta(1620)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$	
$3/2^-$	$(70,1_1^-)$	1/2	$\Delta(1700)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$	
$5/2^+$	$(56,2_2^+)$	3/2	$\Delta(1905)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$	
$7/2^+$	$(56,2_2^+)$	3/2	$\Delta(1950)$	$\Sigma(2030)$	$\Xi(?)$	$\Omega(?)$	
$11/2^+$	$(56,4_4^+)$	3/2	$\Delta(2420)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$	

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