Quiz

1. Consider a particle of mass m subject to a constant force f in one dimension. The potential is $V(x) = -fx$.

   (a) Is the energy spectrum continuous? What are the allowed energy eigenvalues E? (5 points)

   (b) Calculate the energy eigenstates $\psi_E(p)$ in the momentum representation. Normalize the states according to $\langle E|E'\rangle = \delta(E - E')$. (10 points)

   (c) Calculate the propagator in the momentum representation (5 points),
   $$U(p, t; p', 0) = \langle p|e^{-\frac{i}{\hbar}Ht}|p'\rangle.$$

   (d) Calculate the coordinate space propagator (5 points),
   $$U(x, t; x', 0) = \langle x|e^{-\frac{i}{\hbar}Ht}|x'\rangle.$$

   (e) (Bonus) Show by explicit calculation that $U(x, t; x', 0)$ is of the form
   $$U(x, t; x', 0) = A(t)e^{\frac{i}{\hbar}S_{cl}}.$$
   What is $A(t)$? (5 points)

The Schrödinger equation is given by
$$i\hbar\frac{d}{dt}|\psi\rangle = H|\psi\rangle$$
The operators p, x satisfy $[p, x] = -i\hbar$. Then
$$\langle x|p\rangle = \frac{1}{\sqrt{2\pi}}e^{\frac{i}{\hbar}xp}$$
A few useful integrals
$$\int_{-\infty}^{\infty} dx\, e^{-\alpha x^2+\beta x} = \sqrt{\frac{\pi}{\alpha}}e^{\frac{\beta^2}{4\alpha}}, \qquad (\alpha, \beta,\ \text{complex})$$
$$\int_{-\infty}^{\infty} dp\, e^{ip(x-x')} = (2\pi)\delta(x - x')$$