Nearly Perfect Fluidity in Cold Atomic Gases

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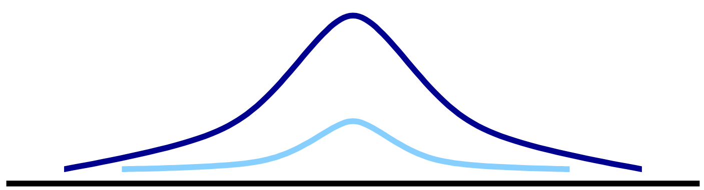
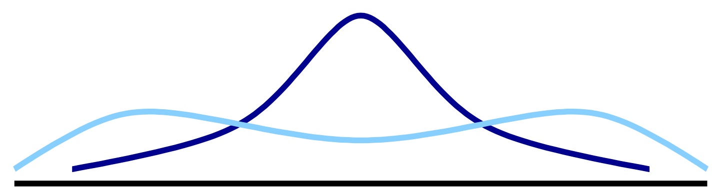
Fluids: Gases, Liquids, Plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



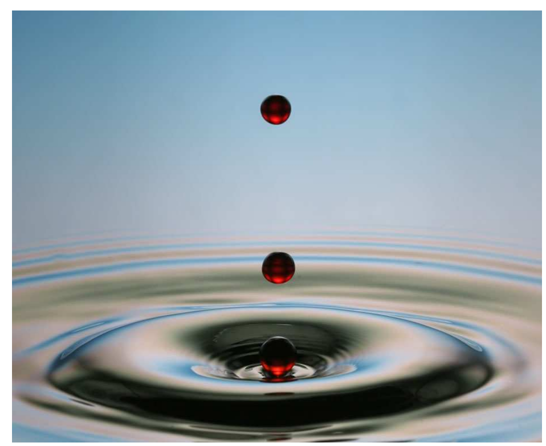
$\tau \sim \tau_{micro}$



$\tau \sim \lambda$

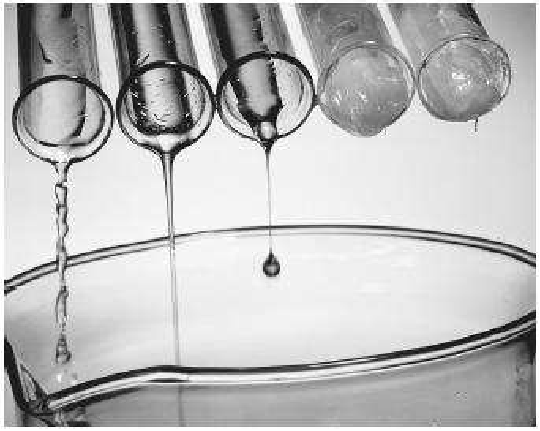
Historically: Water

$(\rho, \epsilon, \vec{\pi})$



Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum



$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$

Constitutive relations: Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k \right) + O(\partial^2)$$

reactive   dissipative   2nd order

Expansion $\Pi^0_{ij} \gg \delta\Pi^1_{ij} \gg \delta\Pi^2_{ij}$

Regime of applicability

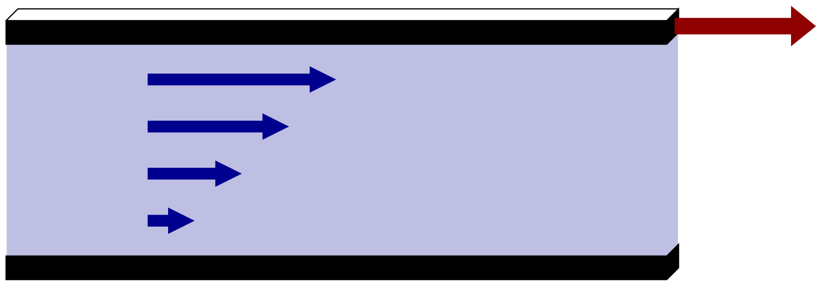
Expansion parameter $Re^{-1} = \dfrac{\eta(\partial v)}{\rho v^2} = \dfrac{\eta}{\rho L v} \ll 1$

$$Re^{-1} = \underbrace{\frac{\eta}{\hbar n}}_{\text{fluid property}} \times \underbrace{\frac{\hbar}{m v L}}_{\text{flow property}}$$

Consider $mvL \sim \hbar$: Hydrodynamics requires $\eta/(\hbar n) < 1$

Shear viscosity

Viscosity determines shear stress ("friction") in fluid flow

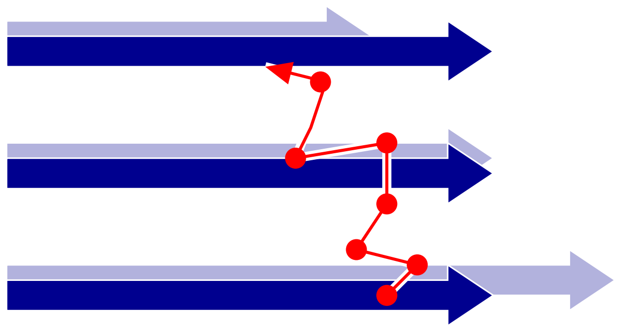


$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

Kinetic theory: conserved quantities carried by quasi-particles

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p + \vec{F}\cdot\vec{\nabla}_p f_p = C[f_p]$$

$$\eta \sim \frac{1}{3} n\,\bar{p}\,l_{mfp}$$



Dilute, weakly interacting gas: $l_{mfp} \sim 1/(n\sigma)$

$$\eta \sim \frac{1}{3}\frac{\bar{p}}{\sigma}$$

independent of density!

Shear viscosity

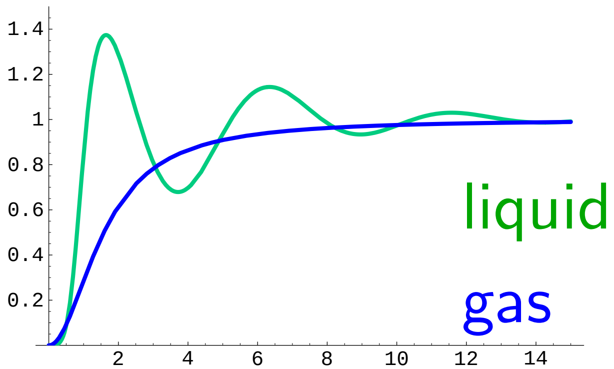
non-interacting gas ($\sigma \to 0$): $\eta \to \infty$

non-interacting and hydro limit ($T \to \infty$) limit do not commute

strongly interacting gas: $\dfrac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar$

but: kinetic theory not reliable!

what happens if the gas condenses into a liquid?



Eyring, Frenkel:

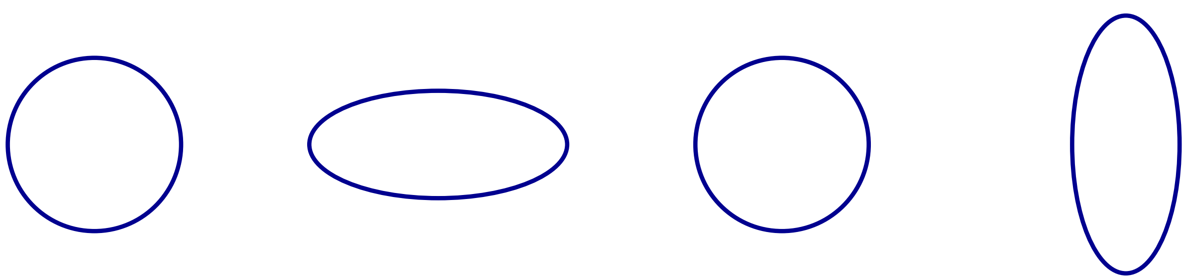
$$\eta \simeq hn \exp(E/T) \geq hn$$

Holographic Duals: Transport Properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature	$\Leftrightarrow$	Hawking temperature
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \qquad g_{\mu\nu} = g^0_{\mu\nu} + \gamma_{\mu\nu}$$



Holographic Duals: Transport Properties

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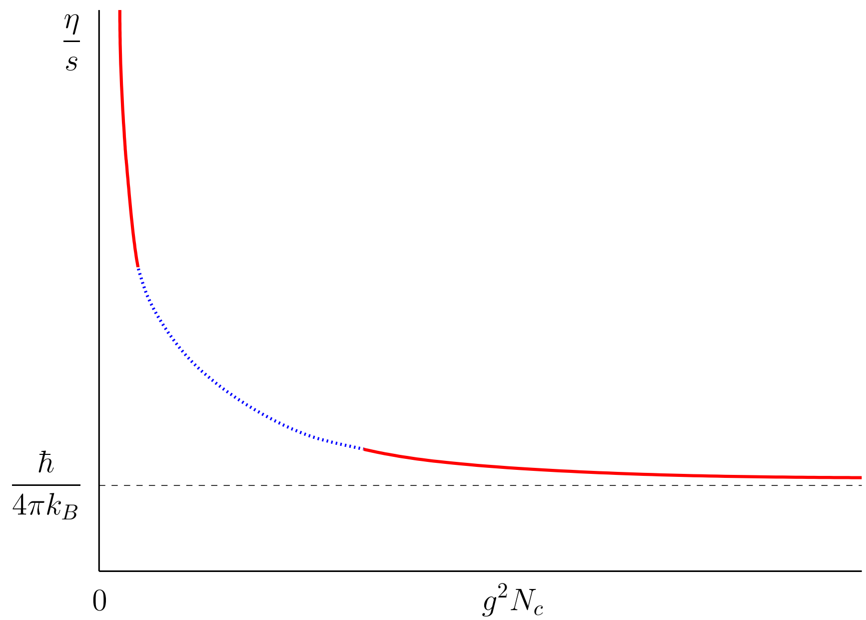
Strong coupling limit

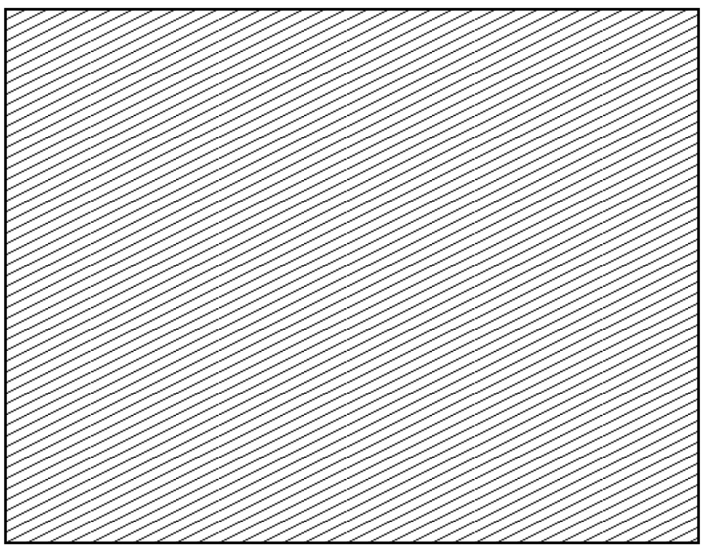
$$\boxed{\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}}$$

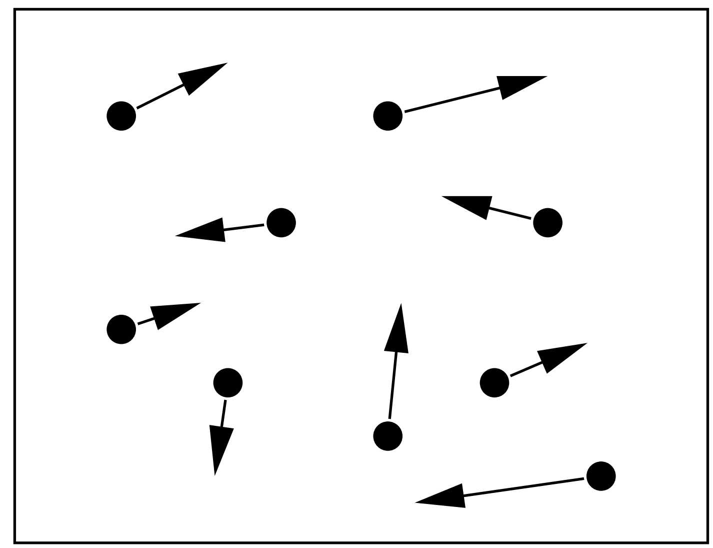
Son and Starinets (2001)



Strong coupling limit universal? Provides lower bound for all theories?

Kinetics vs No-Kinetics





AdS/CFT low viscosity goo

kinetic liquid

Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

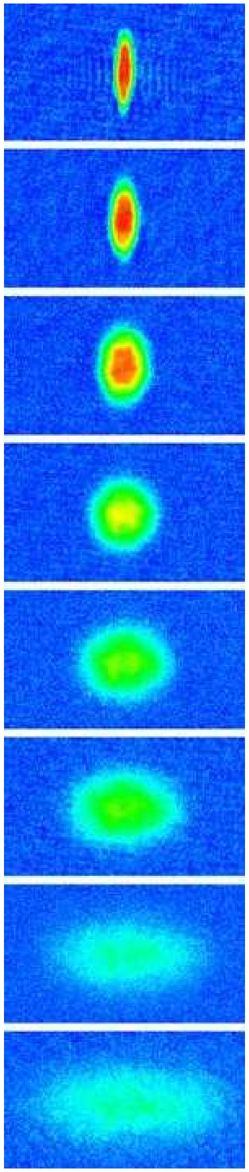
Bound is incompatible with weak coupling and kinetic theory

strong interactions, no quasi-particles

Model system has conformal invariance (essential?)

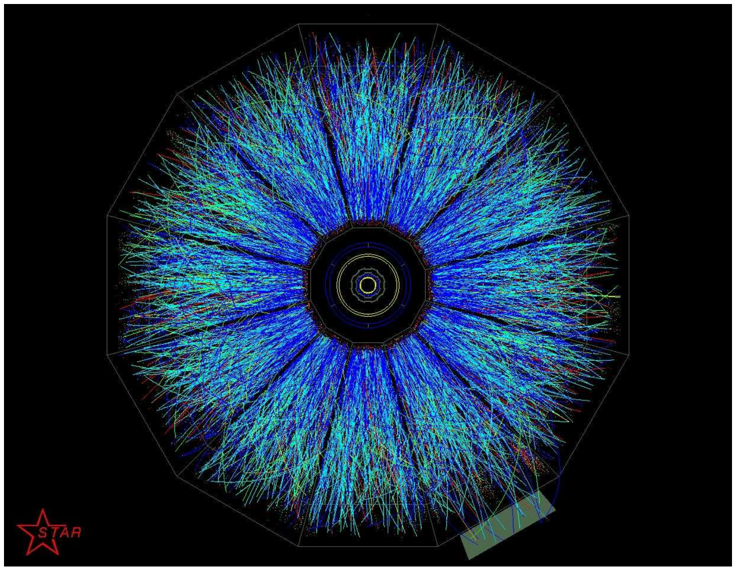
(Almost) scale invariant systems

Perfect Fluids: The contenders

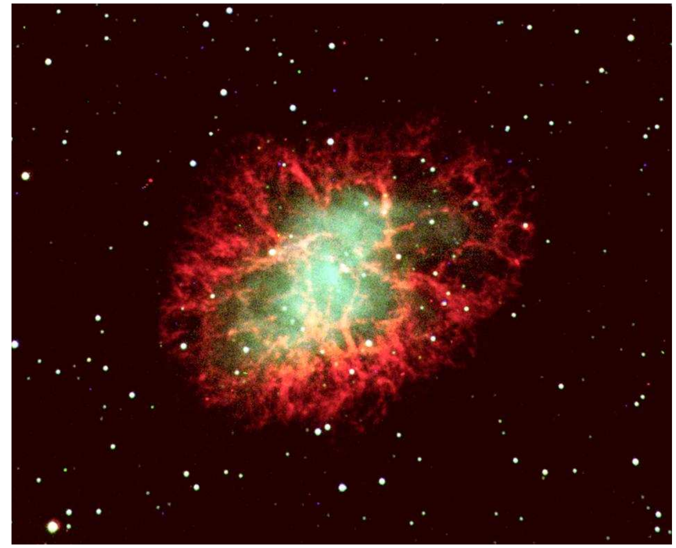


trapped atoms

(T=0.1 neV)



QGP (T=180 MeV)

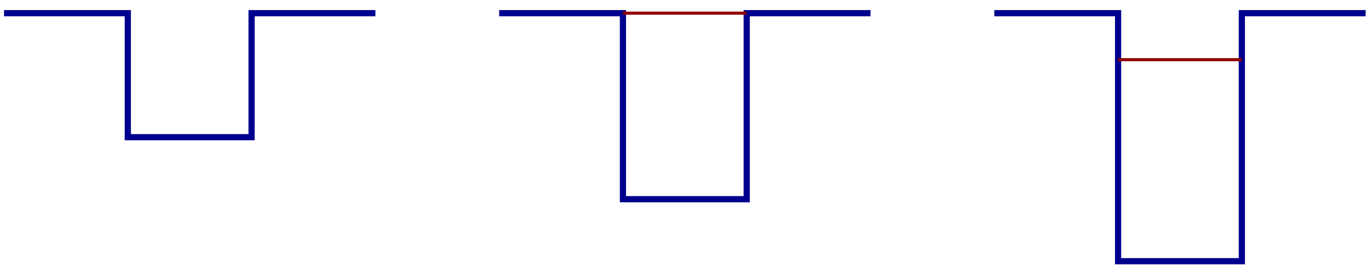


neutron matter

(T=1 MeV)

Unitarity limit

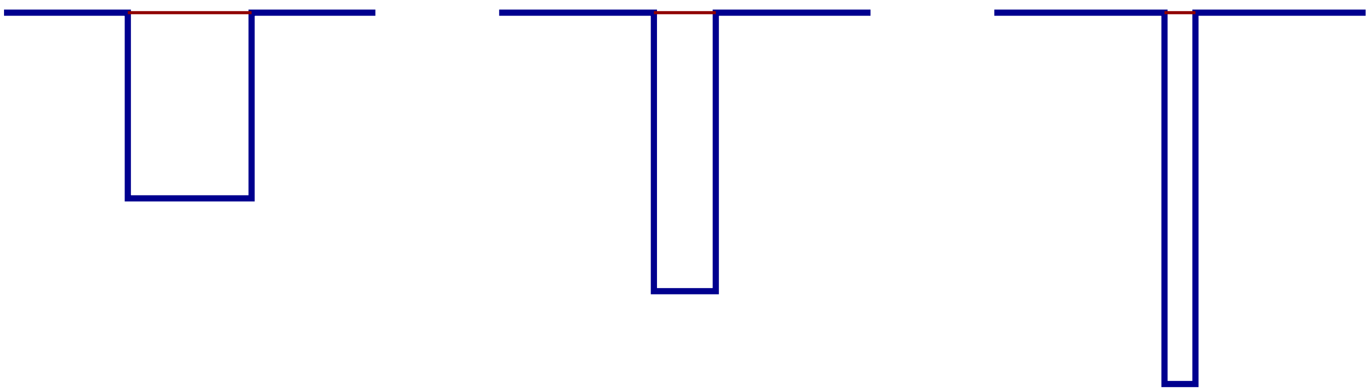
Consider simple square well potential



$a < 0$ $\qquad$ $a = \infty,\ \epsilon_B = 0$ $\qquad$ $a > 0,\ \epsilon_B > 0$

Unitarity limit

Now take the range to zero, keeping $\epsilon_B \simeq 0$



Universal relations

$$\mathcal{T} = \frac{1}{ik + 1/a} \qquad \epsilon_B = \frac{1}{2ma^2} \qquad \psi_B \sim \frac{1}{\sqrt{a}r}\exp(-r/a)$$

Equation of State

Universality implies that

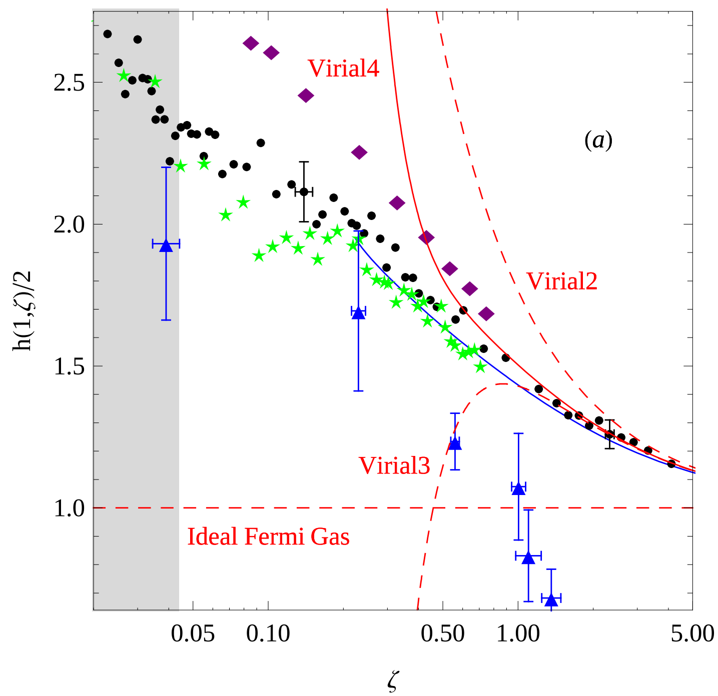
$$P(\mu, T) = P_0(\mu, T) f\left(\frac{\mu}{T}\right) \qquad \mathcal{E} = \frac{3}{2}P$$

At $T = 0$ have $P(\mu) = \xi^{-3/2} P_0(\mu)$ with $\xi \simeq 0.4$

Harmonic trap: $f(z)$ determined by (twice integrated) column density

$$P(\mu(x), T) = \frac{m\omega_\perp^2}{\pi}\tilde{n}(x)$$

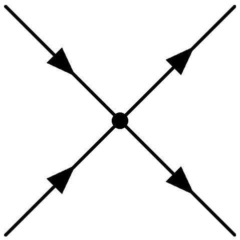
Nascimbene et al, Science (2010).



Kinetic theory

High T: Atoms Cross section regularized by thermal momentum

$$\eta = \frac{15}{32\sqrt{2}}(mT)^{3/2}$$



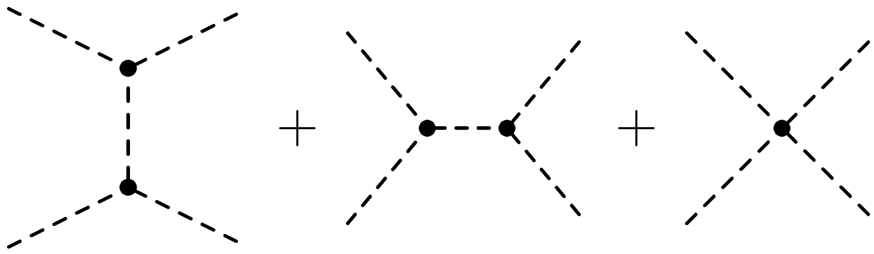
Bruun (2005)

Low T: Phonons Goldstone boson $\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$

$$\mathcal{L} = c_0 m^{3/2}\left(\mu - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}\right)^{5/2} + \ldots$$

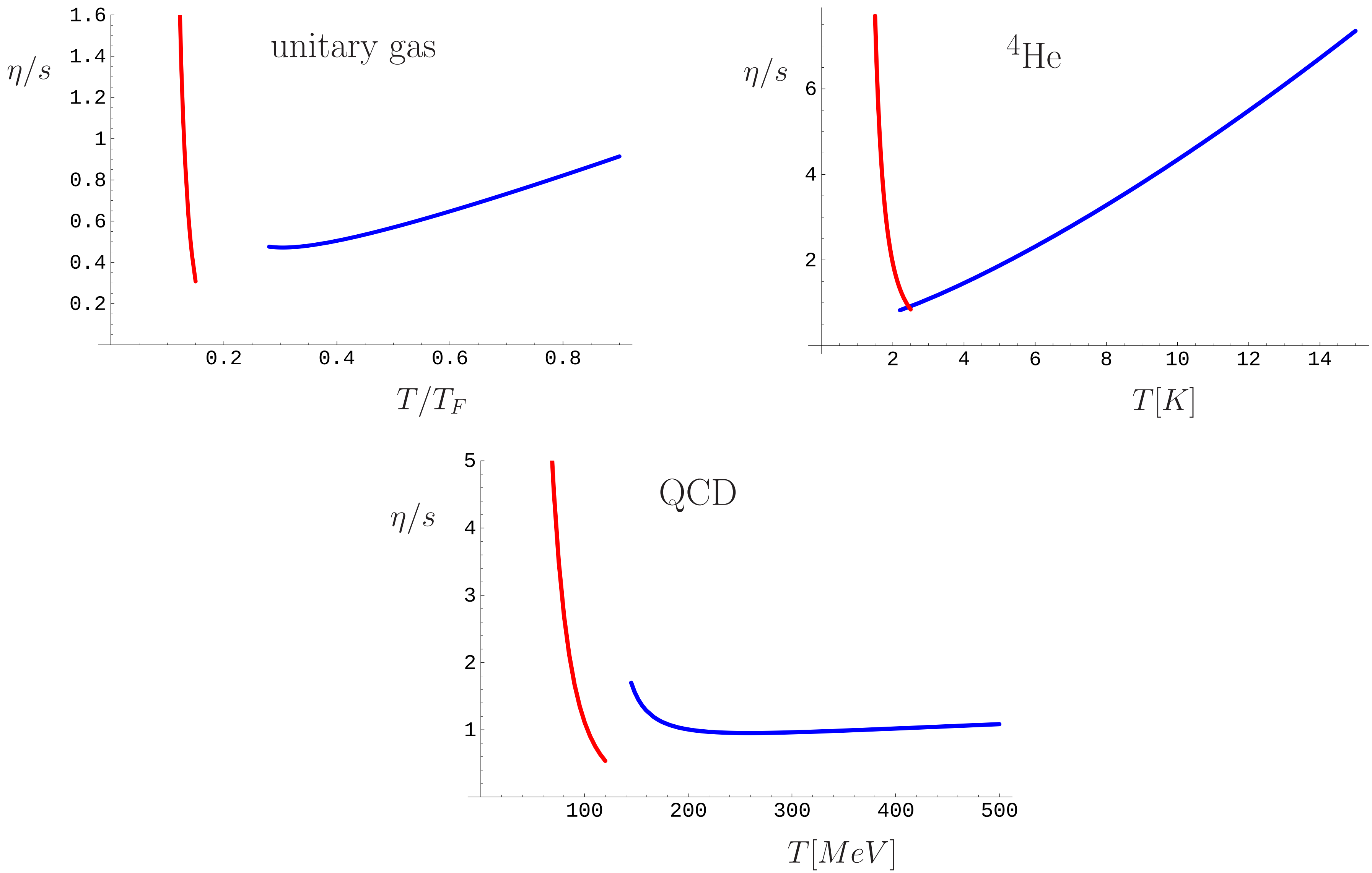
Viscosity dominated by $\varphi + \varphi \to \varphi + \varphi$

$$\eta = A\frac{\xi^5}{c_s^3}\frac{T_F^8}{T^5}$$



T.S., G.R. (2007)

Kinetic theory summary



Shear viscosity: sum rules

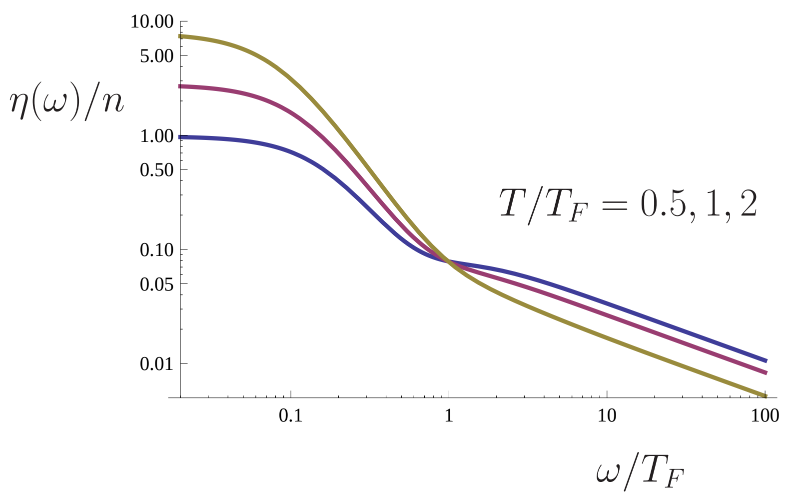
Taylor and Randeira proved the following sum rules

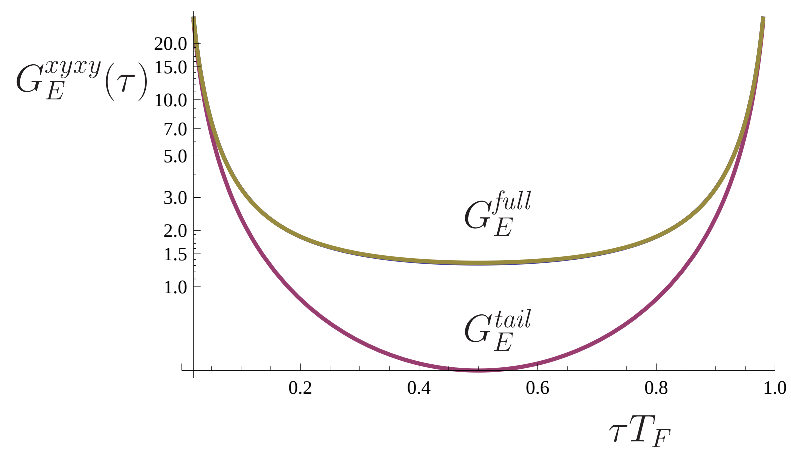
$$\frac{1}{\pi}\int dw\,\left[\eta(\omega)-\frac{C}{10\pi\sqrt{m\omega}}\right]=\frac{\mathcal{E}}{3}-\frac{C}{10\pi m a}$$

$$\frac{1}{\pi}\int dw\,\zeta(\omega)=\frac{1}{72\pi m a^2}\left(\frac{\partial C}{\partial a^{-1}}\right)$$

where C is Tan's contact, $\rho(k)\sim C/k^4$.

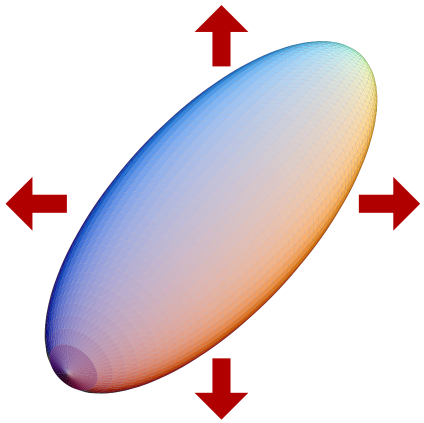
Sum rules constrain spectral fct and euclidean correlator

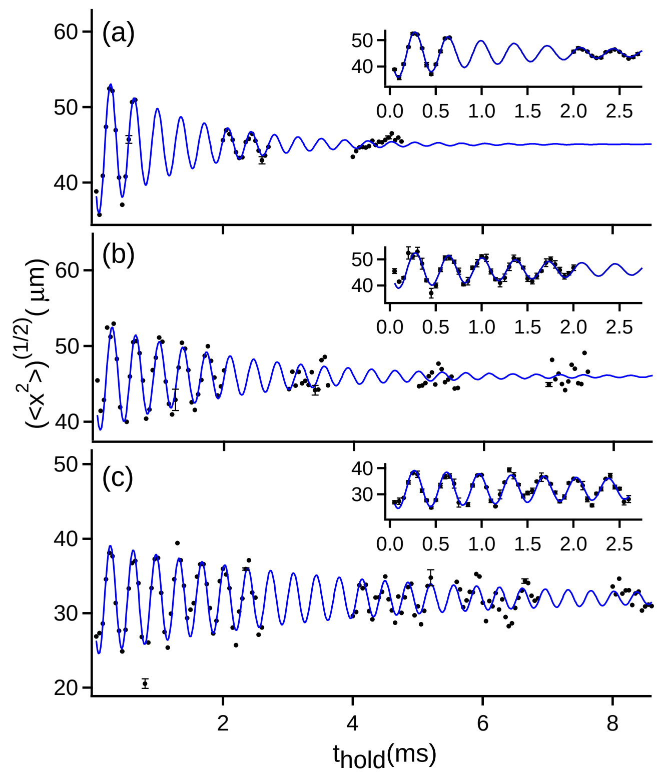




Hydrodynamics: Collective modes

Radial breathing mode





Ideal fluid hydrodynamics ($P \sim n^{5/3}$)

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right) \vec{v} = -\frac{\vec{\nabla} P}{mn} - \frac{\vec{\nabla} V}{m}$$

Hydro frequency at unitarity

$$\omega = \sqrt{\frac{10}{3}}\, \omega_\perp$$

Damping small, depends on T/T_F.

experiment: Kinast et al. (2005)

Viscous Hydrodynamics

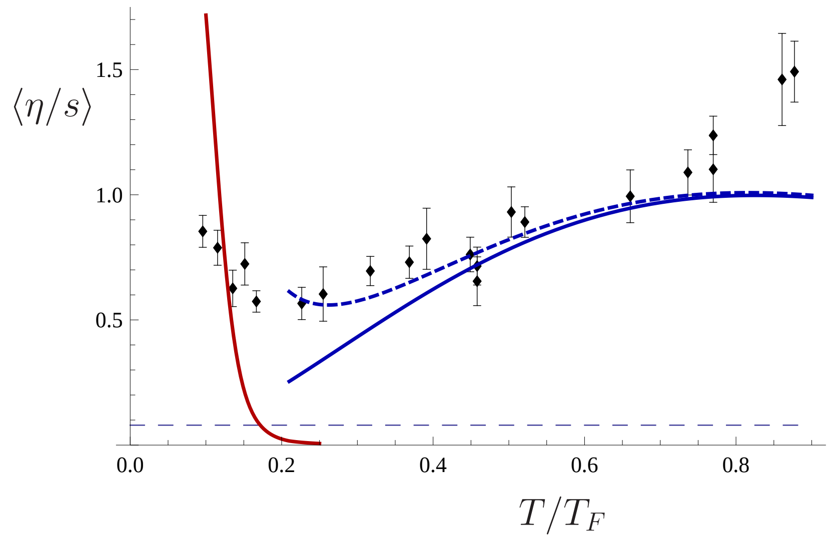
Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{1}{2}\int d^3x\,\eta(x)\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \int d^3x\,\zeta(x)\,(\partial_i v_i)^2 - \frac{1}{T}\int d^3x\,\kappa(x)\,(\partial_i T)^2$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

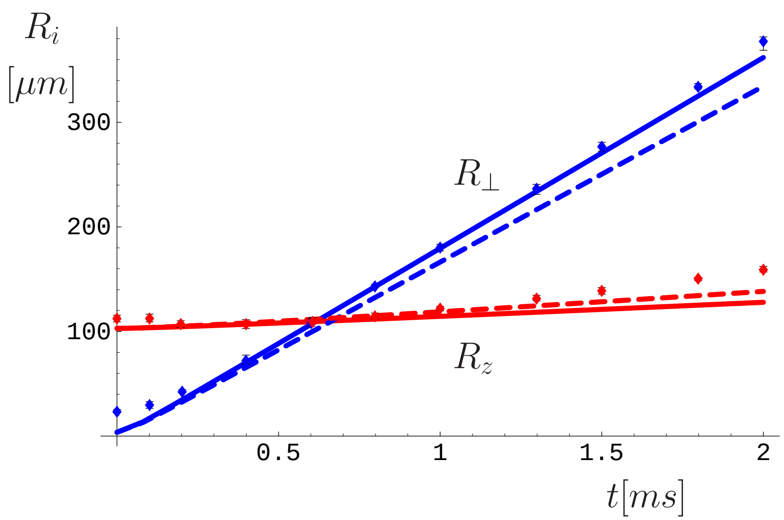
$$\boxed{\frac{\eta}{s} = (3\lambda N)^{\frac{1}{3}}\,\frac{\Gamma}{\omega_\perp}\,\frac{E_0}{E_F}\,\frac{N}{S}}$$

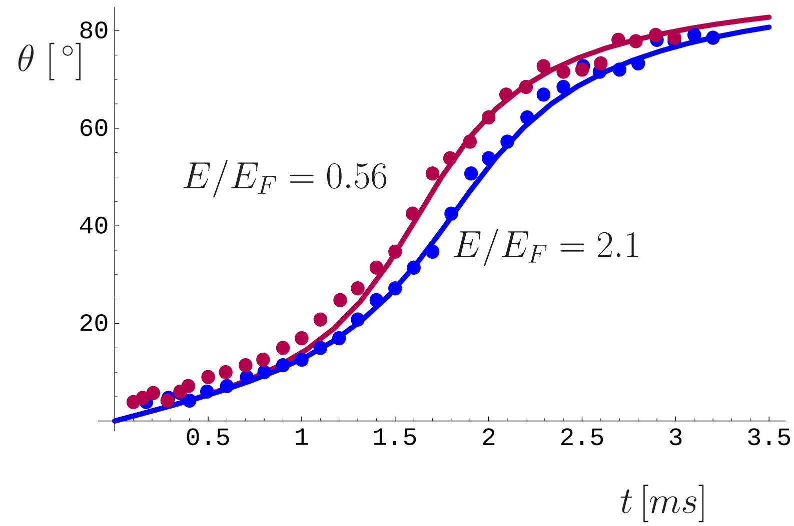


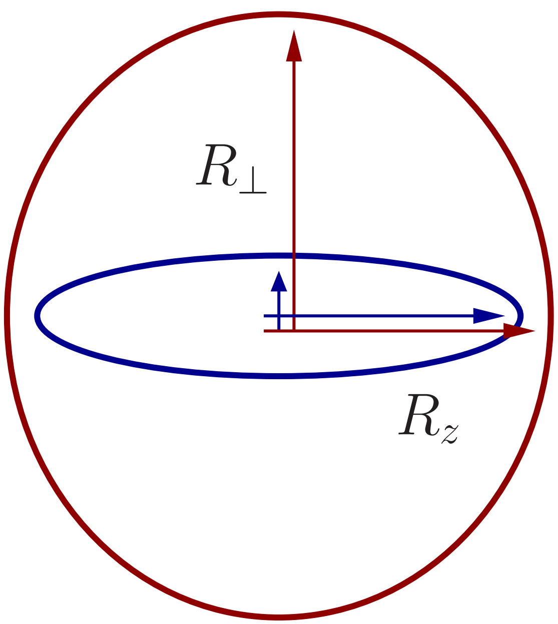
Schaefer (2007), see also Bruun, Smith

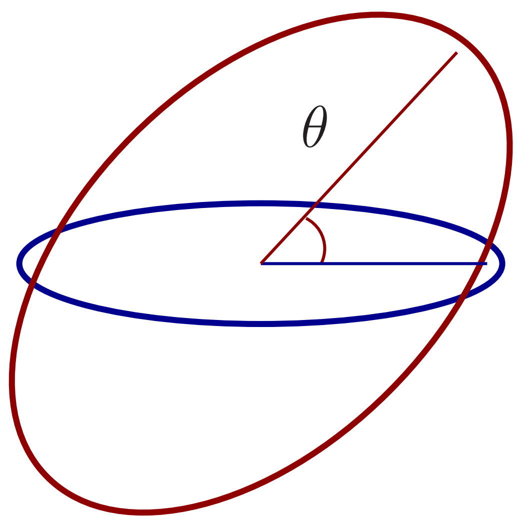
$T \ll T_F$ $\qquad$ $T \gg T_F$, $\tau_R \simeq \eta/P$

Hydrodynamics: Free expansion and rotation









O'Hara et al (2002), Clancy et al (2007)

Navier-Stokes equation

Option 1: Moment method

$$\int d^3x\, x_k\, (\rho \dot{v}_i + \ldots) = \int d^3x\, x_k\, (-\nabla_i P - \nabla_j \delta\Pi_{ij})$$

Integration by parts, only sensitive to $\langle \eta \rangle / E_0$.

Option 2: Scaling ansatz for $\eta(\mu, T)$

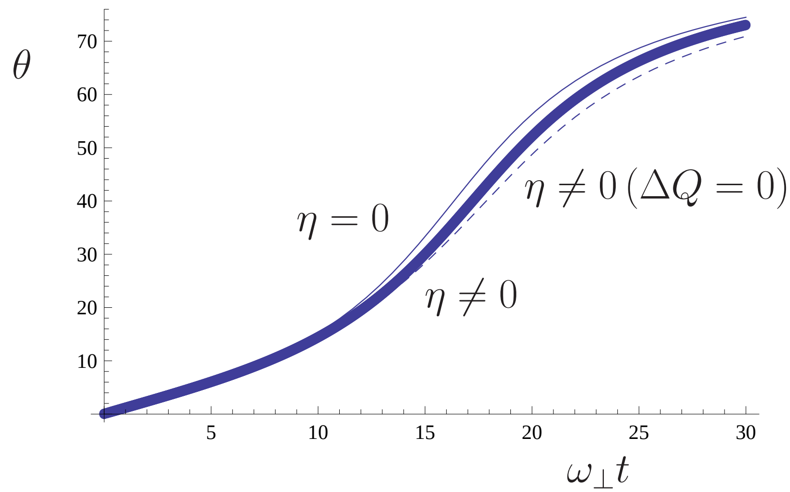
$$\eta(n,T) = \eta_0 (mT)^{3/2} + \eta_1 \frac{P(n,T)}{T}$$

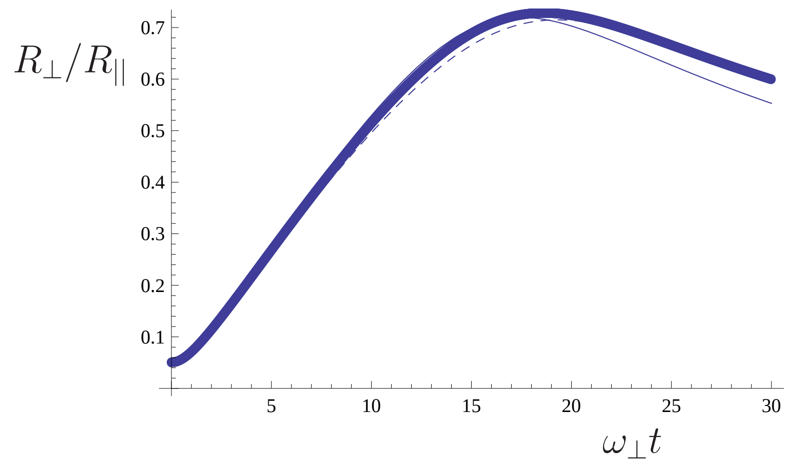
Option 3: Numerical solutions.

Issues: Options 1,2 ignore heat generated by dissipative effects as well as proper treatment of corona $\eta \sim T^{3/2}$ (integration by parts, etc.).

Navier-Stokes: Numerical results

Consider $\eta = \alpha_n n$. System parameters $\omega_z = 0.045\omega_\perp$, $\Omega = 0.4\omega_z$.





Reheating $\Delta Q = T\Delta S = \eta/2(\partial_i v_j + \ldots)^2$
counteracts viscous forces.

Scaling solution overestimates viscous effects by factor ~ 2.

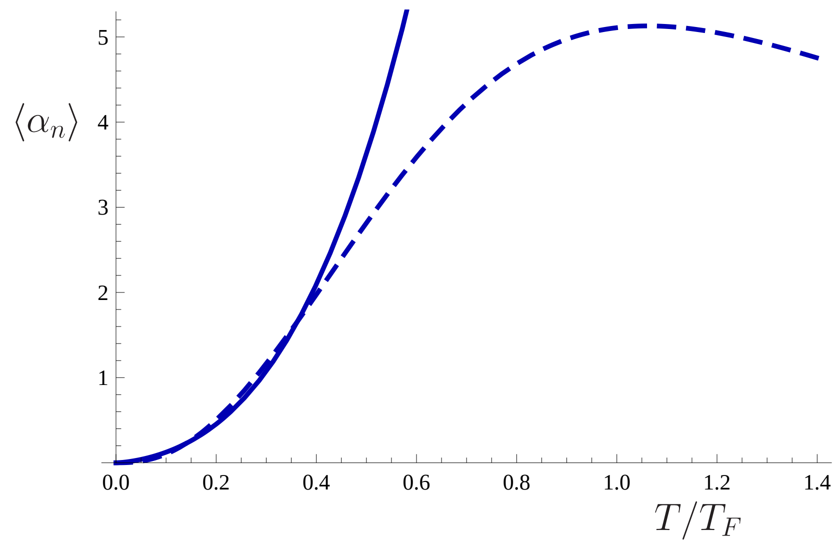
Relaxation Time Model

In real systems stress tensor does not relax to Navier-Stokes form instantaneously. Consider

$$\tau_R\left(\frac{\partial}{\partial t}+\vec{v}\cdot\vec{\nabla}\right)\delta\Pi_{ij}=\delta\Pi_{ij}-\eta\left(\partial_i v_j+\partial_j v_i-\frac{2}{3}\delta_{ij}\partial\cdot v\right)$$

In kinetic theory $\tau_R \simeq (\eta/n)\,T^{-1}$

- disspiation from $\eta\sim(mT)^{3/2}$: corona excerts drag force.
- find $\langle\alpha_n\rangle\sim T^3$
- system dependence



Outlook

The unitary Fermi gas is an important model system for other strongly correlated quantum fluids in nature (the quark gluon plasma, dilute neutron matter)

The equation of state has been determined to a few percent.

Transport properties are more difficult: Kinetic theory at $T \gg T_F$ and $T \ll T_F$. Sum rules constrain spectral fct at all T.

Experimental determination of transport properties: Collective modes give $\eta/s < 0.5$. Analysis of expanding systems still in progress. Requires full second order hydrodynamics.