The Phases of QCD

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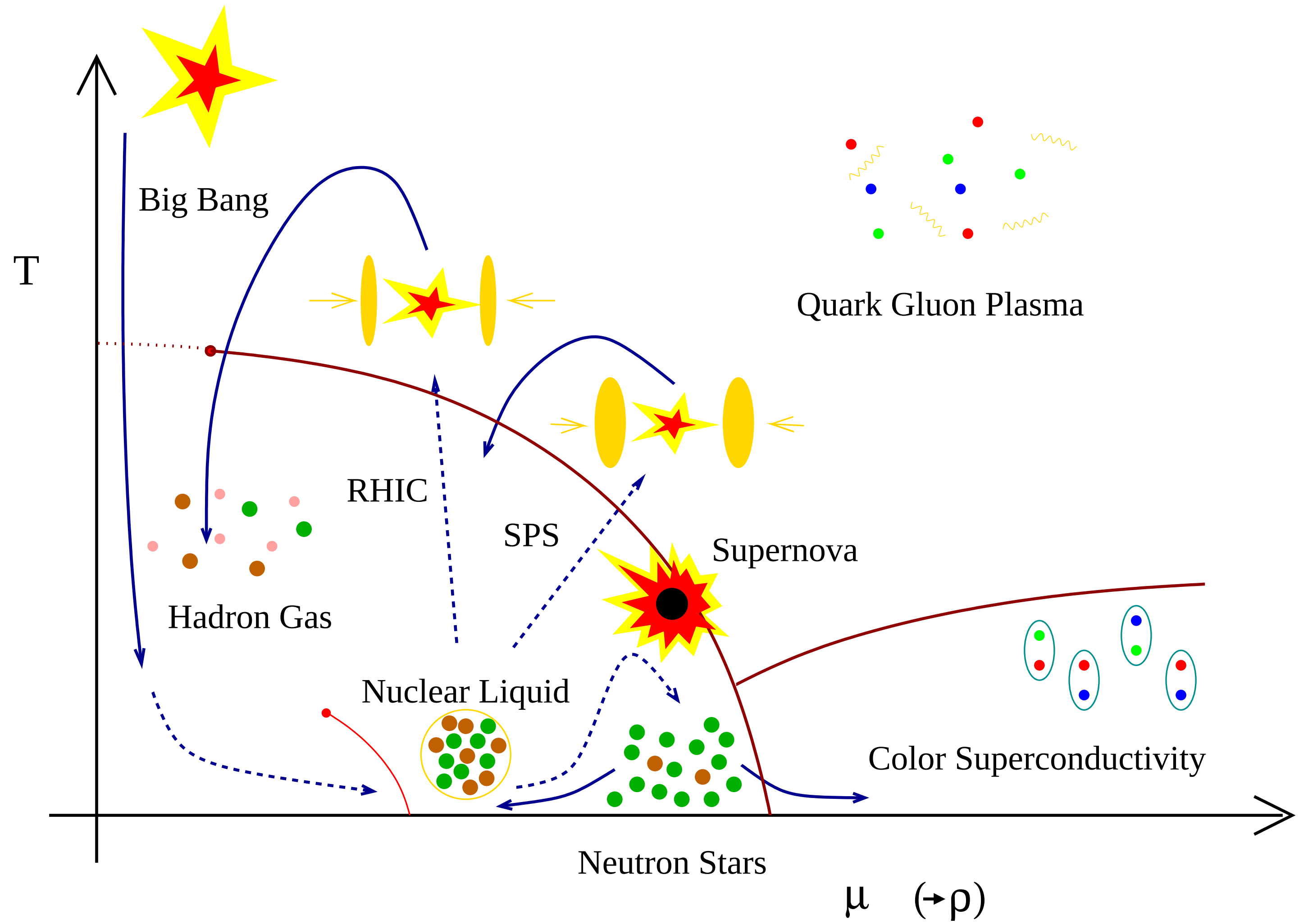
Plan of the lectures

1. QCD and States of Matter
2. The High Temperature Phase: Theory
3. Exploring QCD at High Temperature: Experiment
4. QCD at High Baryon Density: Quark Matter

QCD Phase Diagram



Why do we care?

Different phases of QCD occur in the universe

Neutron Stars, Big Bang

Exploring the phase diagram is important to understanding the phase that we happen to live in

Structure of hadrons is determined by the structure of the vacuum

Need to understand how vacuum can be modified

QCD simplifies in extreme environments

Study QCD matter in a regime where quarks and gluons

are the correct degrees of freedom

What is QCD? What is a Phase of QCD?

What is a Phase Diagram?

Phase Diagram: Equilibrium state as a function of thermodynamic (or other: m_q, N_c, N_f, B) variables.

$$\text{Here}: \quad \Omega(T,\mu,V) = -VP(\mu,T)$$

- Other choices of independent variables: $G(P,T,N),\ldots$
- At $T=0$ have $\mu = E(N_q+1) - E(N_q)$.
- In real experiments control parameters are more complicated (beam energy E_{cm}, impact parameter b ($\rightarrow N_{ch}$), system size A).

What is QCD (Quantum Chromo Dynamics)?

Elementary fields: Quarks Gluons

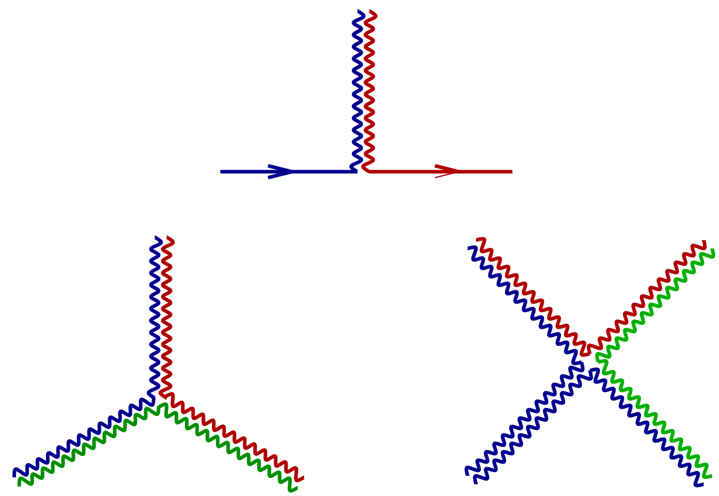
$$(q_\alpha)^a_f \begin{cases} \text{color} & a = 1, \ldots, 3 \\ \text{spin} & \alpha = 1, 2 \\ \text{flavor} & f = u, d, s, c, b, t \end{cases} \qquad A^a_\mu \begin{cases} \text{color} & a = 1, \ldots, 8 \\ \text{spin} & \epsilon^{\pm}_\mu \end{cases}$$

Dynamics: Generalized Maxwell (Yang-Mills) + Dirac theory

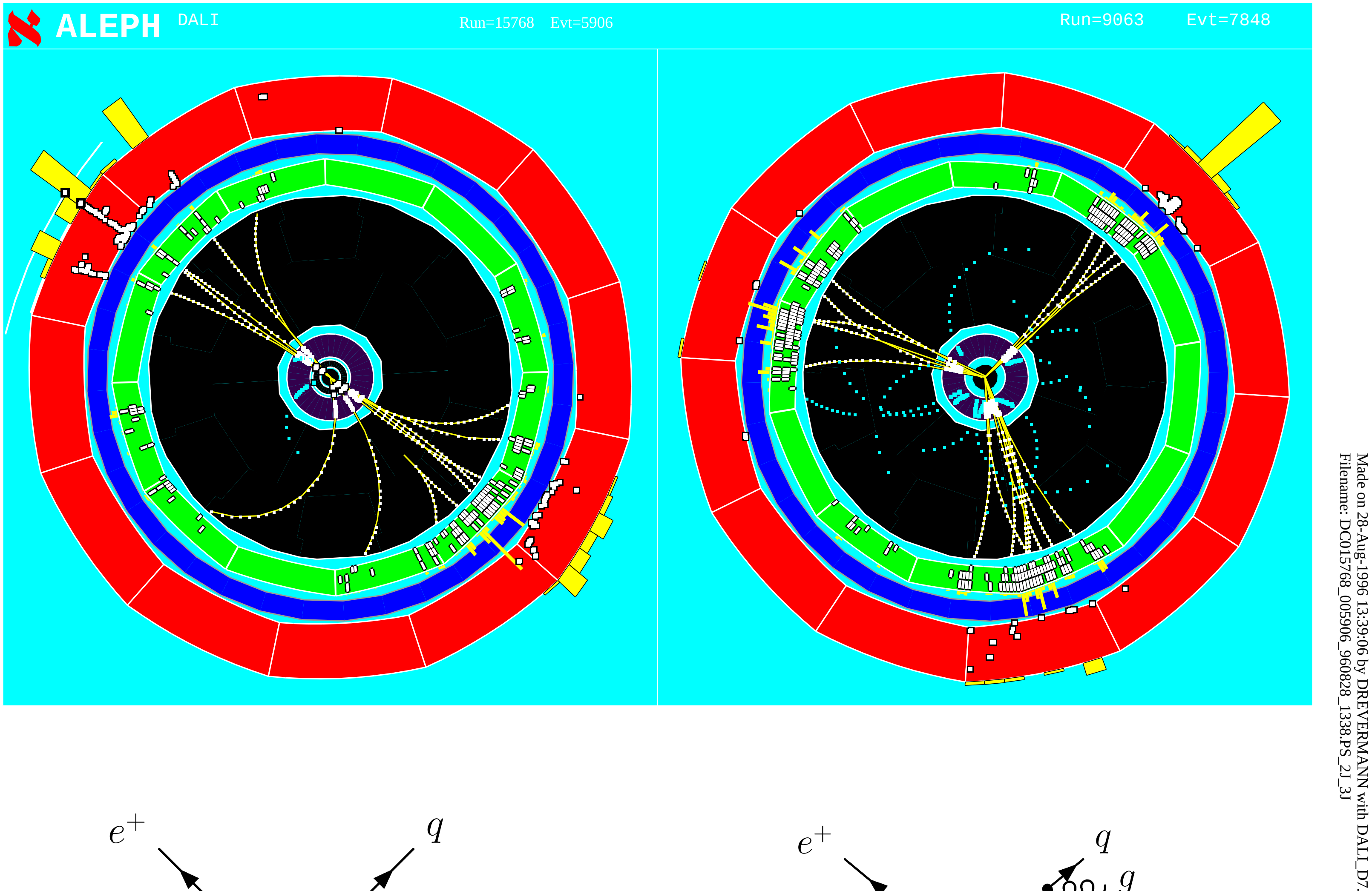
$$\boxed{\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}}$$

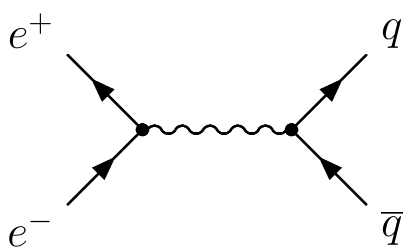
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

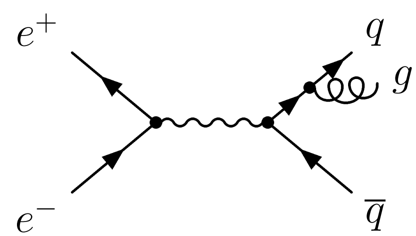
$$i\not{D} q = \gamma^\mu \left(i\partial_\mu + g A^a_\mu t^a \right) q$$



“Seeing” Quarks and Gluons



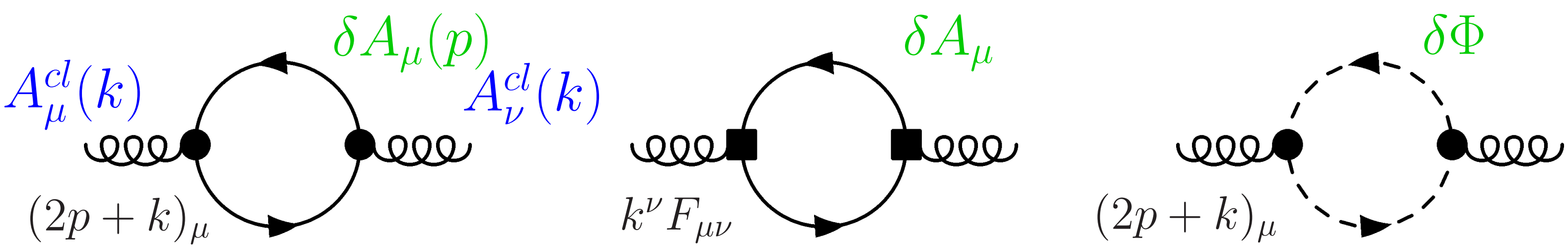




Asymptotic Freedom

Classical field A_μ^{cl}. Modification due to quantum fluctuations:

$$A_\mu = A_\mu^{cl} + \delta A_\mu \qquad \frac{1}{g^2} F_{cl}^2 \to \left(\frac{1}{g^2} + c \log\left(\frac{k^2}{\mu^2}\right)\right) F_{cl}^2$$

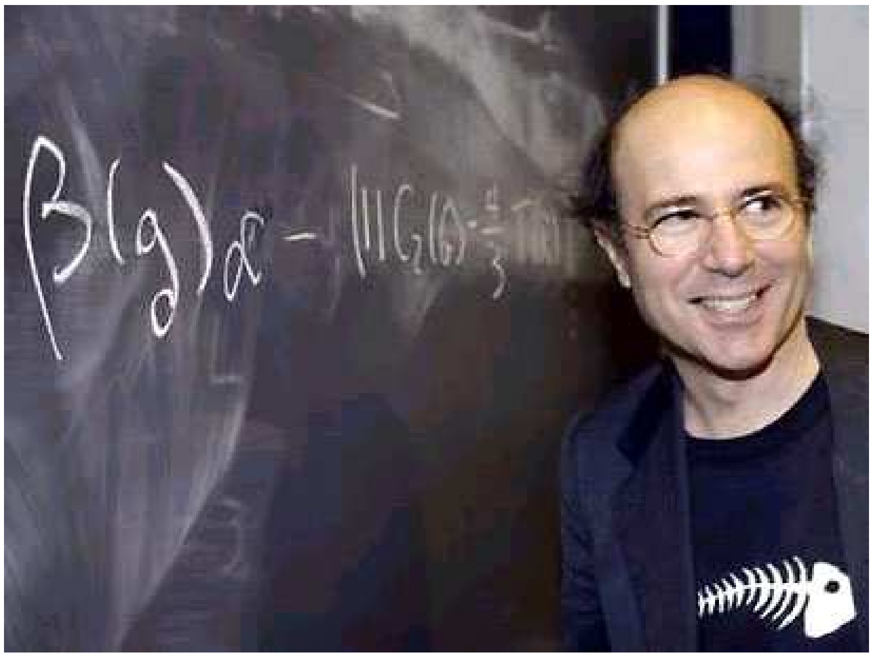


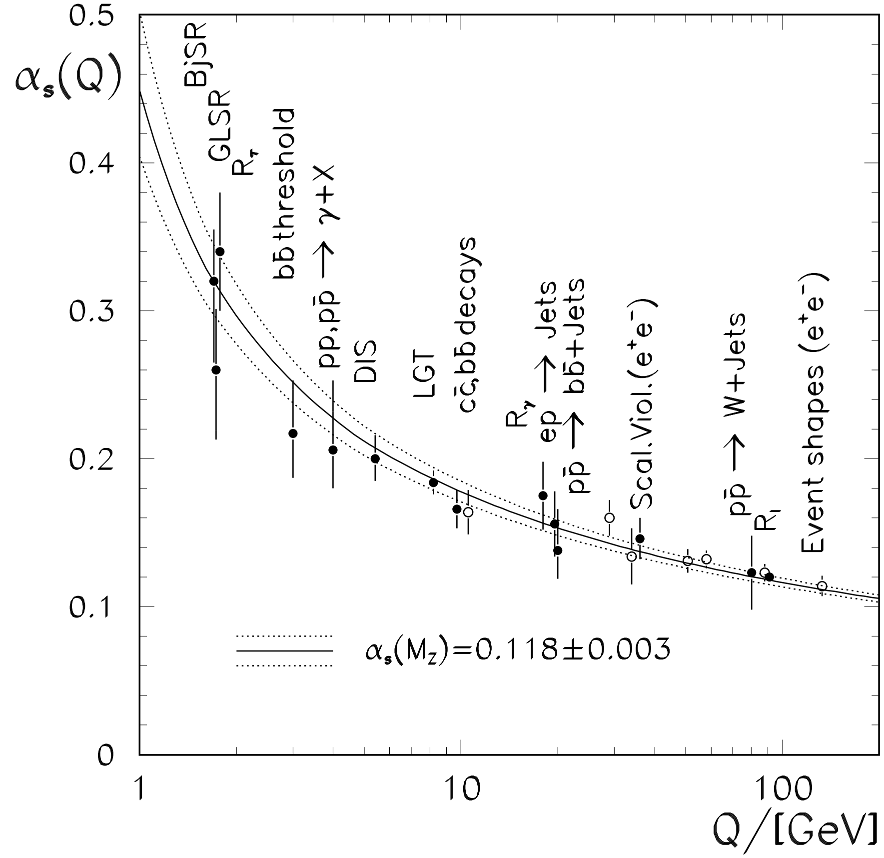
dielectric $\epsilon > 1$ paramagnetic $\mu > 1$ dielectric $\epsilon > 1$

$$\mu\epsilon = 1 \;\Rightarrow\; \epsilon < 1$$

$$\beta(g) = \frac{\partial g}{\partial \log(\mu)} = \frac{g^3}{(4\pi)^2}\left\{ \left[\frac{1}{3} - 4\right] N_c + \frac{2}{3} N_f \right\} < 0$$

Running Coupling Constant





About Units

Consider QCD Lite*

The lagrangian has a coupling constant, g, but no scale.

After renormalization g becomes scale dependent

g is traded for a scale parameter Λ

Λ is the only scale, the QCD "standard kilogram"

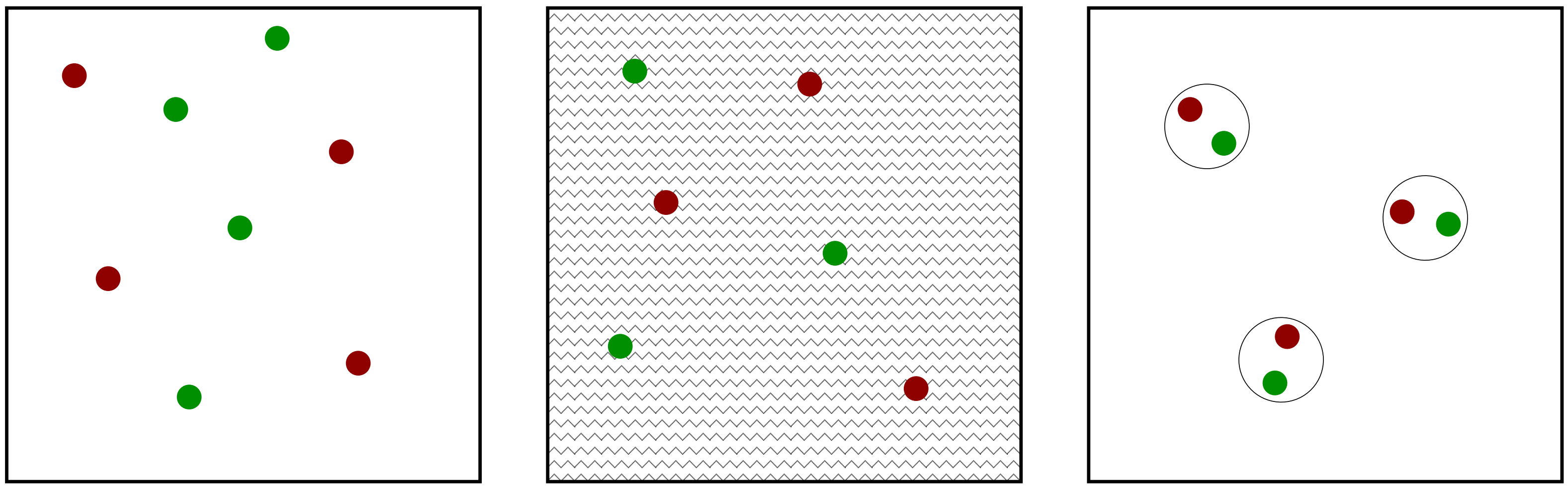
QCD Lite is a parameter free theory

Standard units: $\Lambda_{QCD} \simeq 200\,\mathrm{MeV} \simeq 1\,\mathrm{fm}^{-1}$

*QCD Lite is QCD in the limit $m_q \to 0$, $m_Q \to \infty$

What is a Phase of QCD? Phases of Gauge Theories

Coulomb | Higgs | Confinement

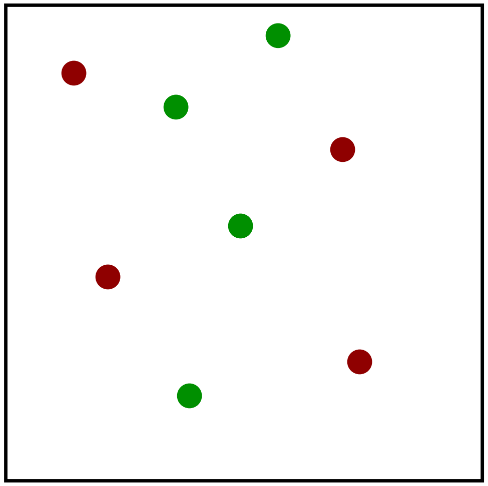


$$V(r) \sim -\frac{e^2}{r} \qquad\qquad V(r) \sim -\frac{e^{-mr}}{r} \qquad\qquad V(r) \sim kr$$

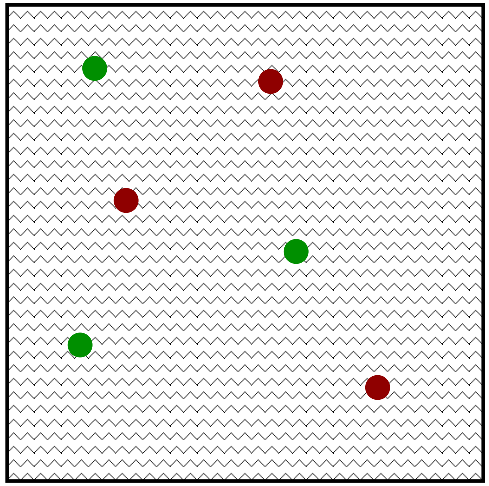
Standard Model: $U(1) \times SU(2) \times SU(3)$

What is a Phase of QCD? Phases of Gauge Theories

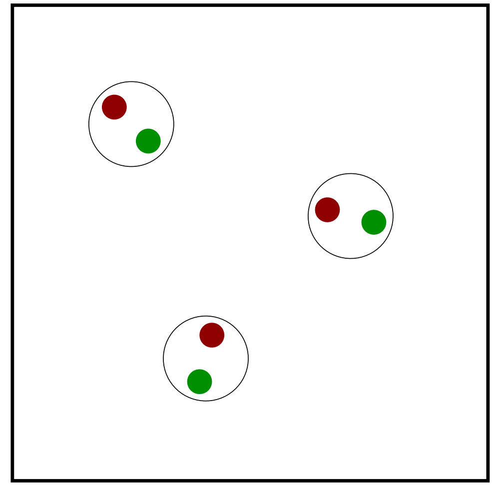
Coulomb



Higgs



Confinement



$$V(r) \sim -\frac{e^2}{r}$$

$$V(r) \sim -\frac{e^{-mr}}{r}$$

$$V(r) \sim kr$$

QCD: High T phase

High μ phase

Low T, μ phase

Gauge Symmetry

Local gauge symmetry $U(x) \in SU(3)_c$

$$\begin{aligned} \psi &\to U\psi & D_\mu\psi \to UD_\mu\psi \\ A_\mu &\to UA_\mu U^\dagger + iU\partial_\mu U^\dagger & F_{\mu\nu} \to UF_{\mu\nu}U^\dagger \end{aligned}$$

Gauge "symmetries" (redundance) cannot be broken

Gauge symmetries can be realized in different modes

	Coulomb	Higgs	confined
d.o.f:	2 (massless)	3 (massive)	3 (massive)

Distinction between Higgs and confinement phase not always sharp

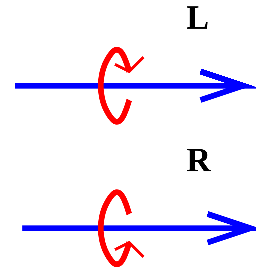
Phases of Matter: Symmetries

phase	order param	broken symmetry	rigidity phenomenon	Goldstone boson
crystal	ρ_k	translations	rigid	phonon
magnet	$\vec{M}$	rotations	hysteresis	magnon
superfluid	$\langle\Phi\rangle$	particle number	supercurrent	phonon
supercond.	$\langle\psi\psi\rangle$	gauge symmetry	supercurrent	none (Higgs)
χsb	$\langle\bar{\psi}\psi\rangle$	chiral symmetry	axial current	pion

Chiral Symmetry

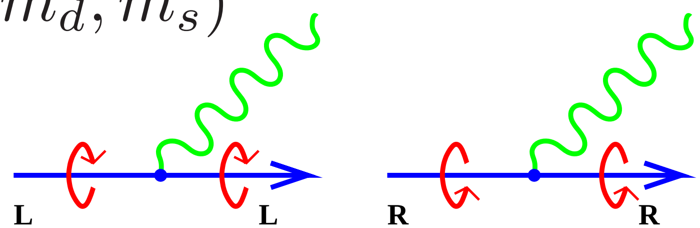
Define left and right handed fields

$$\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi$$

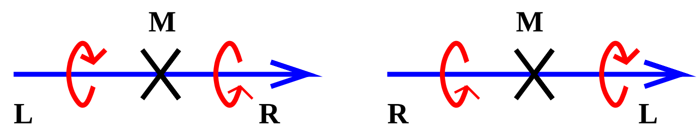


Fermionic lagrangian, $M = \mathrm{diag}(m_u, m_d, m_s)$

$$\mathcal{L} \;=\; \bar{\psi}_L(i\not{D})\psi_L + \bar{\psi}_R(i\not{D})\psi_R$$



$$+\,\bar{\psi}_L M \psi_R + \bar{\psi}_R M \psi_L$$



$M = 0$: Chiral symmetry $(L, R) \in SU(3)_L \times SU(3)_R$

$$\psi_L \to L\psi_L, \qquad\qquad \psi_R \to R\psi_R$$

Chiral Symmetry Breaking

Chiral symmetry is spontaneously broken

$$\langle \bar{\psi}_L^f \psi_R^g + \bar{\psi}_L^f \psi_R^g \rangle \simeq -(230\,\mathrm{MeV})^3\ \delta^{fg}$$

$$SU(3)_L \times SU(3)_R \to SU(3)_V \qquad\qquad (G \to H)$$

Consequences: dynamical mass generation $m_Q = 300\,\mathrm{MeV} \gg m_q$

$$m_N = 890\,\mathrm{MeV} + 45\,\mathrm{MeV} \qquad\qquad (\mathrm{QCD},\ 95\%) + (\mathrm{Higgs},\ 5\%)$$

Goldstone Bosons: Consider broken generator Q_5^a

$$[H, Q_5^a] = 0 \qquad Q_5^a|0\rangle = |\pi^a\rangle \qquad\qquad H|\pi^a\rangle = HQ_5^a|0\rangle = Q_5^a H|0\rangle = 0$$

Low Energy Effective Lagrangian

Low energy degrees of freedom: Goldstone modes

$$U(x) = \exp(i\pi^a \lambda^a / f_\pi)$$

Effective lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + (B\mathrm{Tr}[MU] + h.c.) + \ldots$$

controls

Goldstone boson scattering

Coupling to external currents

Quark mass dependence

Symmetries of the QCD Vacuum: Summary

Local $SU(3)$ gauge symmetry

confined: $V(r) \sim kr$

Chiral $SU(3)_L \times SU(3)_R$ symmetry

spontaneously broken to $SU(3)_V$

Axial $U(1)_A$ symmetry

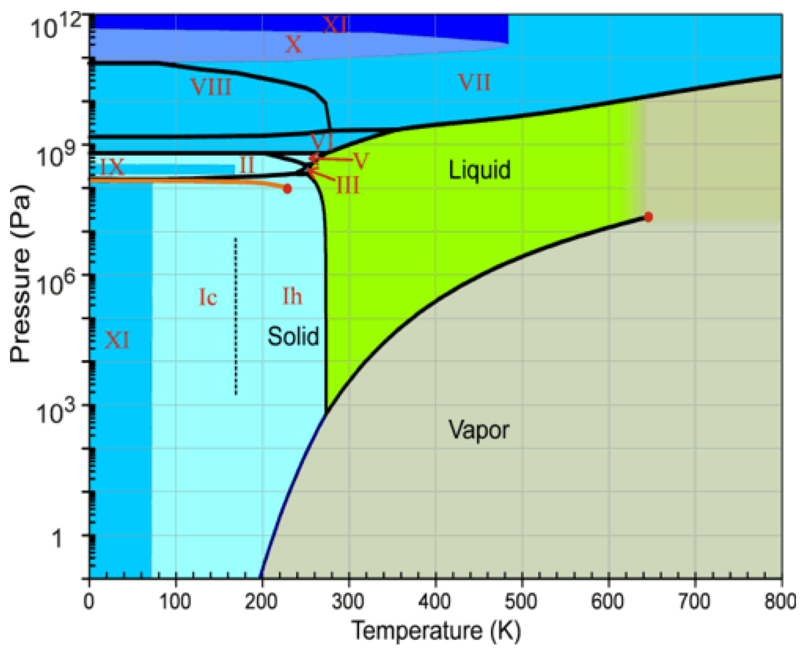
$$\text{anomalous}: \qquad \partial_\mu A^0_\mu = \frac{N_f}{16\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$$

Vectorial $U(1)_B$ symmetry

unbroken: $B = \int d^3x\, \psi^\dagger \psi$ conserved

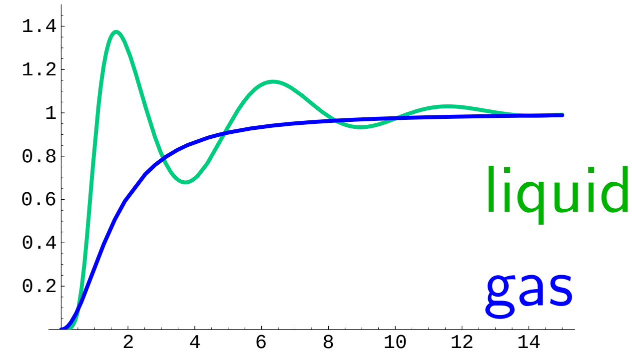
Transitions without change of symmetry: Liquid-Gas

Phase diagram of water

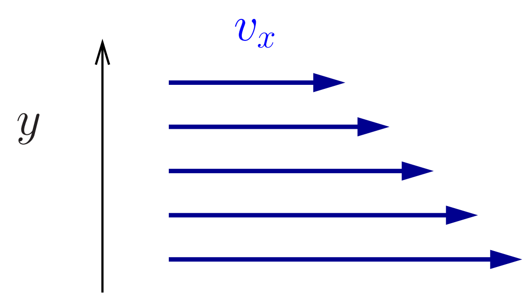


Characteristics of a liquid

Pair correlation function



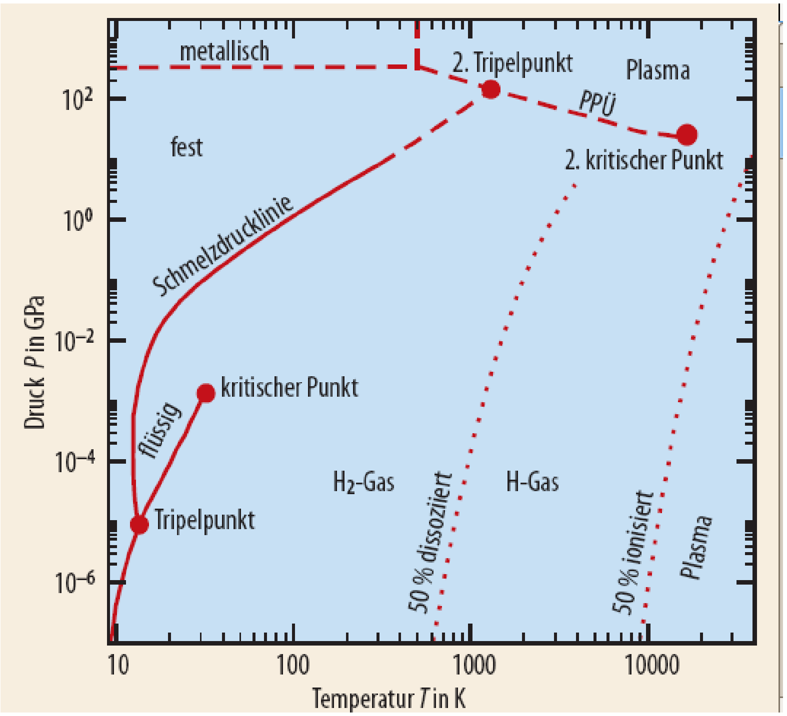
Good fluid: low viscosity



$$F_x = \eta A \frac{\partial v_x}{\partial y}$$

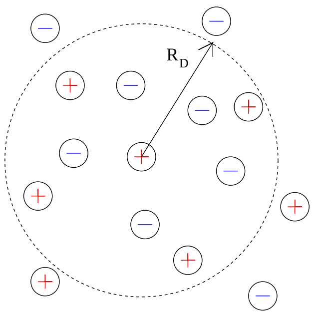
Transitions without change of symmetry: Gas-Plasma

Phase diagram of hydrogen



Plasma Effects

Debye screening



$$V(r) = -\frac{e}{r} e^{-m_D r}$$

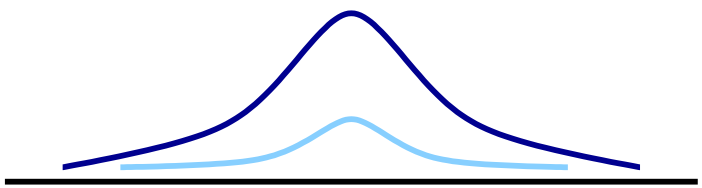
$$m_D^2 = \frac{4\pi e^2 n}{kT}$$

Plasma oscillations

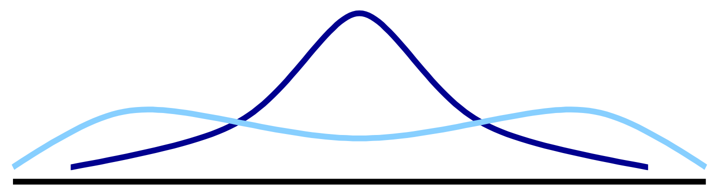
$$\omega_{pl} = \frac{4\pi e^2 n}{m}$$

Fluids: Gases, Liquids, Plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



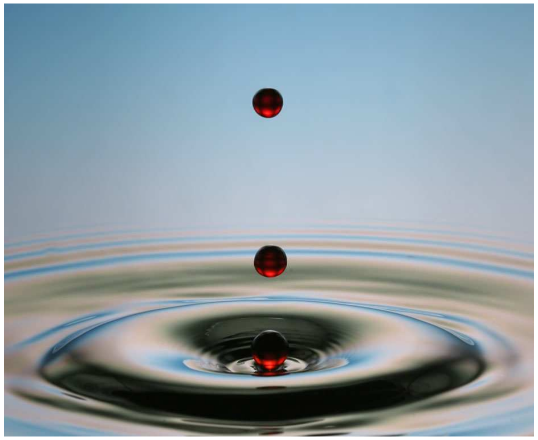
$\tau \sim \tau_{micro}$



$\tau \sim \lambda^{-1}$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



Example: Simple Fluid

Continuity equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

Euler (Navier-Stokes) equation

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0$$

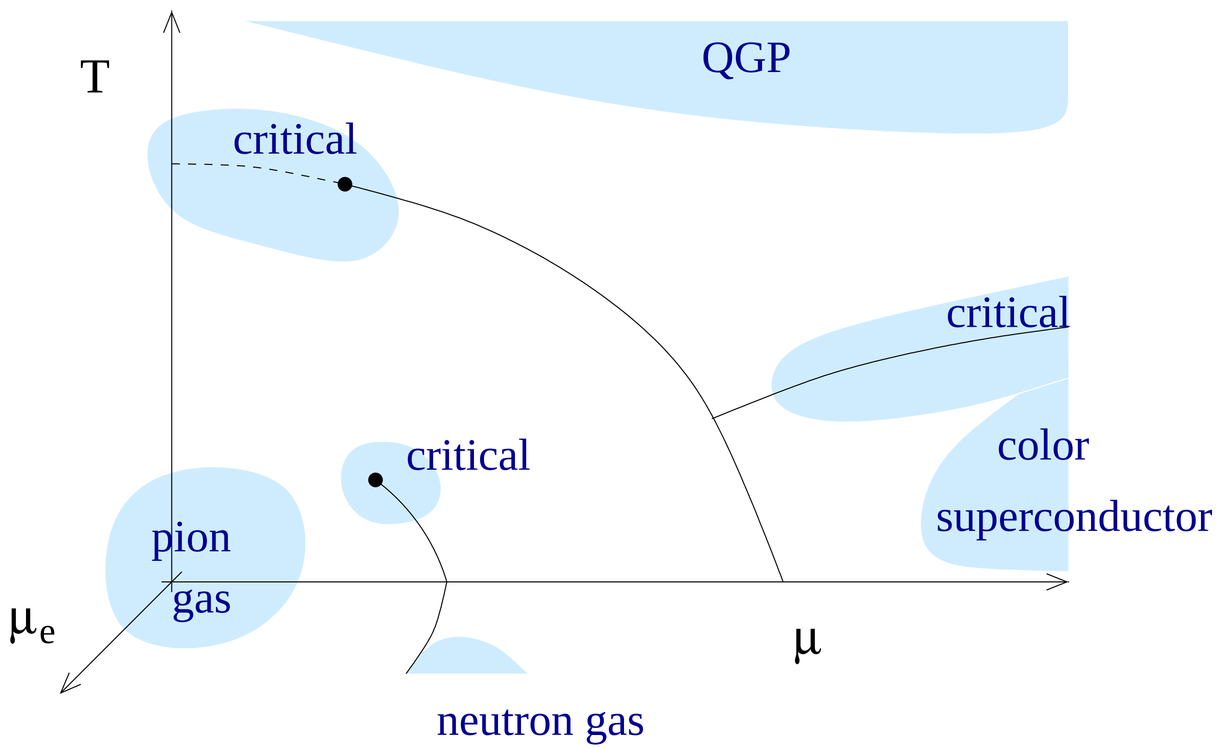
Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right) + \ldots$$

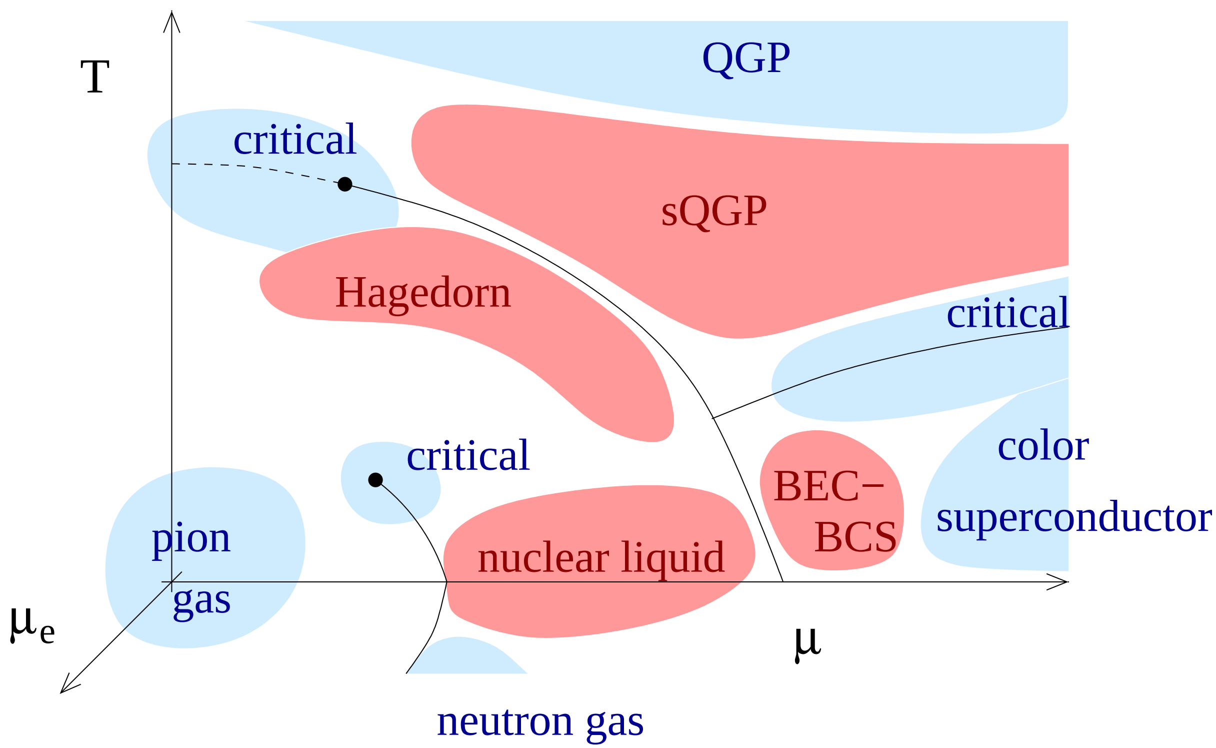
reactive dissipative

Approaching the Phase Diagram:

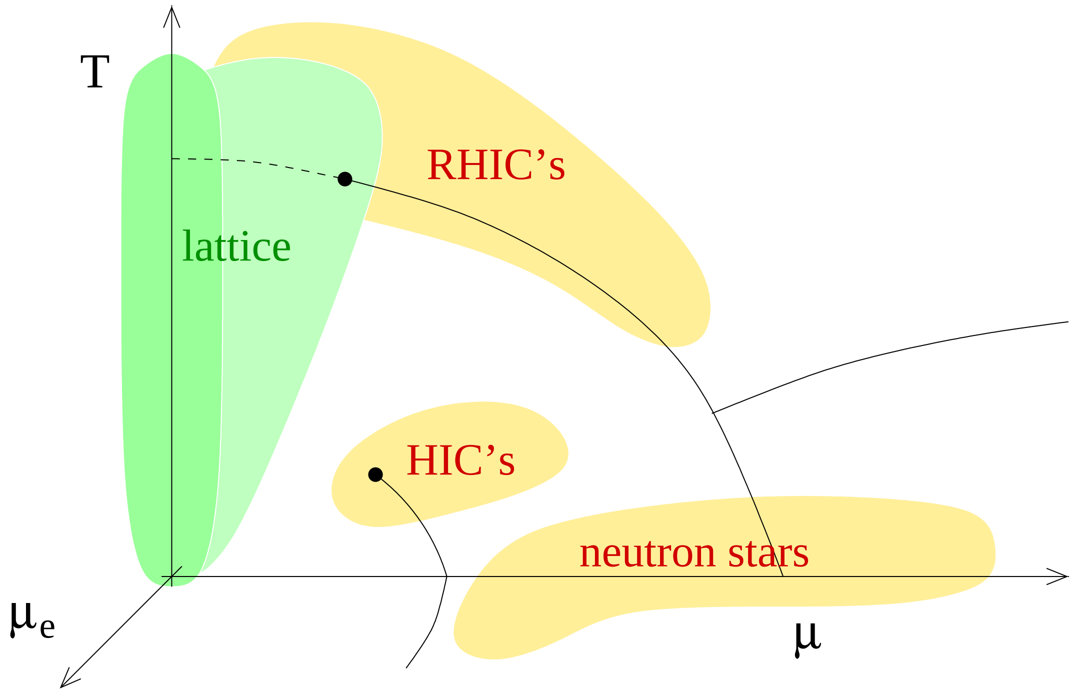
Symmetries and Weak Coupling Arguments



Approaching the Phase Diagram: Strongly Correlated Phases



Approaching the Phase Diagram:
Experiments and Numerical Simulations



Bonus Material:

Chiral Effective Field Theory

Low energy effective theory for the Goldstone modes

Step 1: Parameterize G/H =pseudoscalar GB's

$$U(x): \qquad U \to LUR^{\dagger} \qquad\qquad (L, R) \in SU(3)_L \times SU(3)_R$$

Vacuum $U^{fg} = \delta^{fg}$. Massless fluctuations (G/H)

$$U(x) = \exp(i\phi^a \lambda^a / f_\pi) \qquad\qquad \phi^a = (\pi^{\pm}, \pi^0, K^{\pm}, K^0, \bar{K}^0, \eta)$$

Step 2: Write most general G invariant effective lagrangian

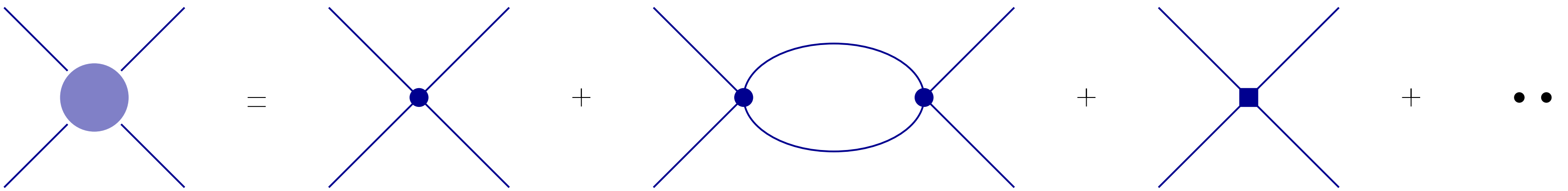
$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr}[\partial_\mu U \partial^\mu U^{\dagger}] + \ldots$$

Non-linear sigma model

Expand lagrangian ($SU(2)$ sector)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^a)^2 + \frac{1}{6f_\pi^2}\left[(\phi^a \partial_\mu \phi^a)^2 - (\phi^a)^2(\partial_\mu \phi^b)^2\right] + O\left(\frac{\partial^4}{f_\pi^4}\right)$$

Step 3: Low energy expansion (power counting)

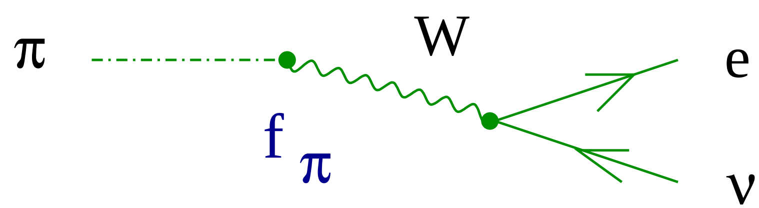


$$T_{\pi\pi} \quad = \quad O\left(k^2/f_\pi^2\right) \; + \qquad\qquad O\left((k^2/f_\pi^2)^2\right)$$

Relation to f_π: Couple weak gauge fields

$$\partial_\mu U \to (\partial_\mu + igW_\mu^\pm \tau^\mp)U$$

$$\mathcal{L} = gf_\pi W_\mu^\pm \partial^\mu \pi^\mp$$



Quark Masses

Non-zero quark masses: $\mathcal{L} = \bar{\psi}_L M \psi_R + \bar{\psi}_R M^\dagger \psi_R$

$$M \to L M R^\dagger \qquad\qquad \text{spurion field } M$$

Chiral lagrangian at leading order in M

$$\mathcal{L} = B\mathrm{Tr}[MU] + h.c.$$

Mass matrix $M = \mathrm{diag}(m_u, m_d m_s)$. Minimize effective potential

$$U_{vac} = 1, \qquad E_{vac} = -B\mathrm{Tr}[M] \qquad \langle\bar{\psi}\psi\rangle = -B$$

Expand around U_{vac}: pion mass

$$m_\pi^2 f_\pi^2 = (m_u + m_d)\langle\bar{\psi}\psi\rangle$$

Chiral expansion

$$\mathcal{L} = f_\pi^4 \left(\frac{\partial U}{\Lambda_\chi}\right)^m \left(\frac{m_\pi}{\Lambda_\chi}\right)^n \qquad \Lambda_\chi = 4\pi f_\pi$$

Bonus Material:

Remarks about χSB and Confinement

Notes

QCD with general N_f, N_c (with or without SUSY)

find theories without confinement and/or chiral symmetry breaking

QCD with $N_f = N_c = 3$

1. Confinement implies chiral symmetry breaking
2. Symmetry breaking pattern $SU(3)_L \times SU(3)_R \to SU(3)_V$ unique
3. Order parameter $\langle \bar{\psi}\psi \rangle \neq 0$

1. Follows from 't Hooft matching conditions

2. Proved in large N_C limit by Coleman and Witten

3. Kovner and Shifman showed that $\langle \bar{\psi}\psi \rangle = 0$, $\langle (\bar{\psi}\psi)^2 \rangle \neq 0$ violates Weingarten inequalities.

QCD Phase Diagram: N_c and N_f