Properties of Unitary Fermi Gas from ε Expansion

Andrei Kryjevski

Washington University in St. Louis

arXiv:0712.2093 [nucl-th], arXiv:0804.2919 [nucl-th]

July 22, 2008

- Consider cold dilute Fermi gas made of 2 species (label them $\uparrow, \downarrow$)

$$n\,a^3 \gg 1,\ n\,r_0^3 \ll 1,$$

where n is the number density, a is the scattering length, r_0 is the effective range – Unitary Fermi Gas, e.g.

  – neutron gas $a \simeq -18$ fm, $r_0 \simeq 2.6$ fm in spin=0 channel

  – EXPERIMENTS with cold Fermion atoms in traps with tunable interactions

- No intrinsic scale parameter $\Rightarrow$ Universal Properties, Analytical description is difficult

- Progress in MC simulations but an analytical description is desirable

  – real time dynamics, insight

  – polarized gas (imbalance of pairing species $N_\uparrow \neq N_\downarrow$)

Scattering Length and all that

Scattering amplitude and cross-section

$$
\begin{aligned}
\psi &\simeq e^{ikz} + \frac{f(\theta)}{r} e^{ikr}, \; r \gg r_0, \; k^2/2m = E \\
f(\theta) &= \sum_{l=0}^{\infty} (2l+1) \; f_l \; \mathrm{P}_l(\cos\theta), \\
f_l &= \frac{e^{2i\delta_l} - 1}{2ik}, \\
\sigma &= \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2 \delta_l, \\
\delta_l &\propto k^{2l+1}
\end{aligned}
$$

s wave scattering dominates at low energies ($k \to 0$)

$$
\begin{aligned}
f_0 &= (k \cot \delta_0(k) - ik)^{-1} \simeq (-1/a + r_0 k^2/2 - ik)^{-1}, \\
\sigma_{l=0} &= 4\pi a^2
\end{aligned}
$$

- Unitarity$\Rightarrow$ optical theorem
  $\mathrm{Im} f(\theta = 0) = \frac{k}{4\pi} \sigma \Rightarrow \sigma = \frac{4\pi}{k^2} \equiv \sigma_{l=0 \; max}$ as $k \to 0, \; a \to \infty$
- Zero energy bound state $\Rightarrow \; a \to \infty$

The Lagrangian

Universality $\Rightarrow$ any short-range two-body interaction may be used, if $a = \infty$. Use local four-Fermi interaction, coupling c_0 ($\hbar = 1,\ T = 0$).

$$
\begin{aligned}
\mathcal{L}[\psi;\rho] &= \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} + \mu + \rho(x) \right) \psi \\
&+ c_0\, n_\uparrow\, n_\downarrow, \quad \mu = \mathrm{diag}(\mu_\uparrow,\, \mu_\downarrow), \\
& \quad \mathrm{Hubbard - Stratonovich} \Rightarrow
\end{aligned}
$$

$$
\mathcal{L}[\psi, \phi; \rho] = -\frac{1}{c_0}\phi^* \phi +
$$

$$
+ \quad \Psi^\dagger \left(i\partial_t + \frac{\sigma_3 \nabla^2}{2m} + (\mu + \rho(x))\sigma_3 + \delta\mu + \sigma_+ \phi + \sigma_- \phi^* \right) \Psi
$$

- $\phi(x) \propto \psi(x)_\uparrow\, \psi(x)_\downarrow$ auxiliary field, order parameter
- $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)^T$ Nambu-Gor'kov field
- $\sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$, $\sigma_{1,2,3}$ are Pauli matrices
- $\mu = (\mu_\uparrow + \mu_\downarrow)/2,\ \delta\mu = (\mu_\uparrow - \mu_\downarrow)/2$
- $a = \infty \Rightarrow$ in dim reg $1/c_0 = 0$ Nishida Son [06]

Hubbard-Stratonovich transformation

Introduce

$$\int \mathrm{D}\phi\, \mathrm{D}\phi^* \exp\left[-\frac{i}{c_0}\int \left(\phi^* - c_0\psi_\uparrow^\dagger\,\psi_\downarrow^\dagger\right)(\phi - c_0\psi_\downarrow\,\psi_\uparrow)\right] = \text{const},$$

$\Rightarrow$ 4 fermion interaction term is canceled,
Yukawa-like and $\frac{1}{c_0}\phi^*\phi$ terms are added.

2 body scattering amplitude (T matrix) is given by the geometric series of particle-particle bubble diagrams. Summation $\Rightarrow$

$$T(p_0, \vec{p})^{-1} = \frac{1}{c_0} - \int_{\vec{k}} \frac{1}{2\epsilon_k - p_0 + \frac{\epsilon_p}{2} - i\delta}$$

At the threshold $p_0 = \vec{p} = 0$ the integral vanishes in dim reg $\Rightarrow$ as $a^{-1} \propto T(0,0)^{-1}$ $a = \infty$ limit corresponds to $c_0 = \infty$, $1/c_0 = 0$

The Role of Dimensionality

Nussinov Nussinov [2004]

- Schrodinger equation for $E = 0$ ($\Rightarrow a = \infty$), $l = 0$ 2 body bound state in d spatial dimensions

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}\, r^2} - \frac{(d-1)}{r}\frac{\mathrm{d}}{\mathrm{d}\, r}\right) \mathrm{R}(r) = 0,\ r > r_0,\ r_0 \ll n^{-1/d}$$

- The solution is $\mathrm{R}(r) \propto r^{2-d}$ and the probability density $\rho(r) \propto r^{d-1}|\mathrm{R(r)}|^2 \propto r^{-d+3}$ $\Rightarrow$ for $d \geq 4$ the two body bound state is strongly peaked within the range of the potential $\Rightarrow$

- For $d \geq 4$ the ground state of the unitary Fermi gas may consist of tightly bound weakly interacting spin=0 dimers. The conjecture has been confirmed by Nishida Son [2006].

- Set $d = 4 - \varepsilon$ and reach $d = 3$ by doing perturbation theory in $\varepsilon \Rightarrow \varepsilon$ expansion.

Bosons at Low energy

- Integrate out fermions, set $\phi(x) = \phi_0 + g\,\varphi(x)$ where $\langle\phi(x)\rangle = \phi_0$, $d = 4 - \varepsilon$

$$\mathrm{S}(\varphi) = -i\,\mathrm{tr}\log\begin{pmatrix} i\,\partial_t + \frac{\nabla^2}{2\,m} + \mu & \phi_0 + g\,\varphi^*(x) \\ \phi_0 + g\,\varphi(x) & i\,\partial_t - \frac{\nabla^2}{2\,m} - \mu \end{pmatrix}$$

- Expand $\mathrm{S}(\varphi)$ in $g\,\varphi(k)$, ε and $k \Rightarrow$ low energy $\mathcal{L}(\varphi)$

$$\mathrm{S}_{eff}(\varphi) = -i\,\mathrm{tr}\log(\phi_0) + \int_p \frac{g^2 m^2}{8\,\pi^2 \varepsilon} \varphi_p^* (p_0 - \frac{p^2}{4\,m} + 2\,\mu)\varphi_p + \mathcal{O}(g^2)$$

$$g = \frac{(8\pi^2\varepsilon)^{1/2}}{m}\left(\frac{m\phi_0}{2\pi}\right)^{\epsilon/4} \Rightarrow$$

canonical kinetic term $\mathcal{O}(1) \Rightarrow$ boson propagator.

- At $\varepsilon \to 0$ the low energy ($\omega < \phi_0$) free bosons with mass $2\,m$ and charge 2.

- At finite ε fermions interact by exchanging bosons: $g\ \Psi^\dagger \sigma_+ \varphi \Psi$ +h.c. Behaves as a renormalizable theory. Expansion in $\varepsilon = 1$.

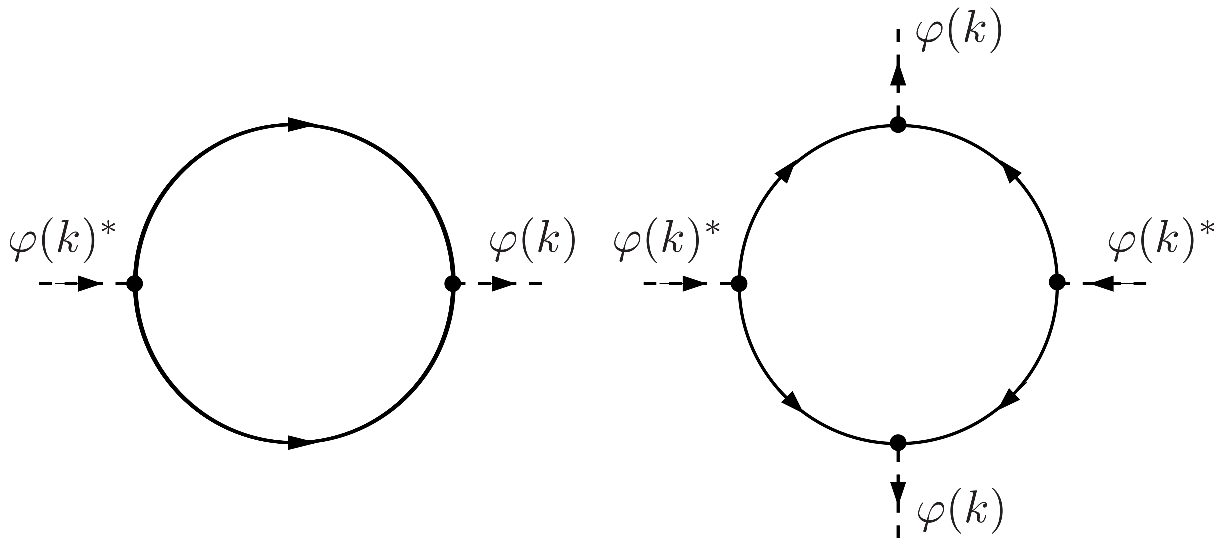

Figure 1: Diagrams that produce operators $g^2\,k^2\,|\varphi(k)|^2 \sim \varepsilon^{-1}$ and $g^2\,k^4\,|\varphi(k)|^2, g^4\,k^4\,|\varphi(k)|^4 \sim 1$. The NG fermion propagators (in the background of ϕ_0) are solid lines, the $\varphi(k)$ insertions are the dashed lines.

Epsilon Expansion

- Perturbation theory treats interactions as small perturbations about a known and simple state (e.g. non-interacting particles). Observables are represented by power series in interaction parameter, in our case $\kappa = k_F a$: $\mathcal{O} = \sum_n \mathcal{O}_n \kappa^n$
- When $k_F a \gg 1$ perturbation theory is unreliable. Have to look for an alternative expansion parameter $\Rightarrow$
- For space dimension $d = 4$ the ground state of the unitary Fermi gas consists of tightly bound non interacting spin=0
  bosons (dimers/molecules).
- Set $d = 4 - \varepsilon$ and reach $d = 3$ by doing perturbation theory in ε $\Rightarrow$ An observable $\mathcal{O} = \sum_n \mathcal{O}_n \varepsilon^n$
- Expansion parameter $\varepsilon = 1$, useful if the series is well-behaved. Convergence improvement techniques may be applied (e.g., Borel transformation, Pade approximation, etc).

Effective Potential to NLO in ε

$$e^{-i\int \mathrm{V}_{\mathrm{eff}}} = \int \mathrm{D}\varphi\, \mathrm{D}\varphi^*\, \det \left(\mathrm{G}^{-1} + \begin{pmatrix} \mu & g\varphi^*(x) \\ g\varphi(x) & -\mu \end{pmatrix} \right) \Big|_{1\mathrm{PI}},$$

$$\mathrm{G}^{-1} = \begin{pmatrix} i\,\partial_t + \frac{\nabla^2}{2\,m} & \phi_0 \\ \phi_0 & i\,\partial_t - \frac{\nabla^2}{2\,m} \end{pmatrix},$$

$$\mathrm{D}^{-1} = i\,\partial_t + \frac{\nabla^2}{4\,m},\ d = 4 - \varepsilon,\ g \sim \varepsilon^{1/2}$$

$$\mathrm{V}_{\mathrm{eff}} =$$

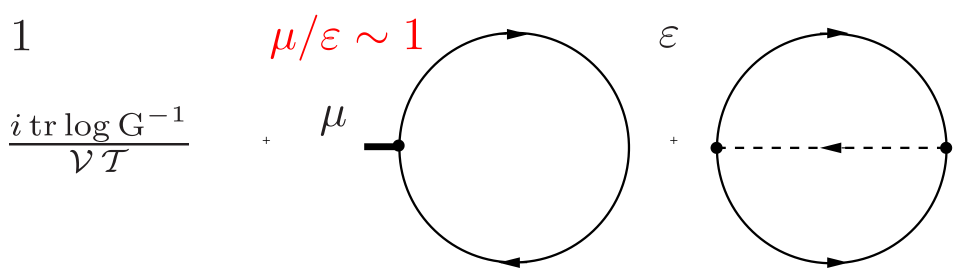

Figure 2: Diagrams that contribute to $\mathrm{V}_{\mathrm{eff}}$ to NLO in ε. The NG fermion propagators (in the background of ϕ_0) are solid lines, the dashed line is the boson propagator. $\phi_0 = 2\,\frac{\mu}{\varepsilon}\,(1 + \varepsilon(3\,C - 1 + \mathrm{Log}\,2))$ Nishida Son[06]

Some results ($\mu_\uparrow = \mu_\downarrow$)

Energy $E/N = \xi \frac{d}{d+2}\epsilon_F$ at $T = 0$ $\xi \equiv \mu/\epsilon_F(n(\mu))$

Fermionic quasiparticle energy

$E(\mathbf{p}) = \Delta + \frac{(\epsilon_{\mathbf{p}} - \epsilon_0)^2}{2\,\phi_0}$, $\epsilon_{\mathbf{p}} = \mathbf{p}^2/2\,m$, $\phi_0 \propto \langle \psi_\uparrow \psi_\downarrow \rangle$

$\mathcal{V} = \mathcal{V}_0 + \mathcal{V}_1 \varepsilon^1 + ... \Rightarrow \varepsilon = 1$

	NLO $\varepsilon = 1$	Borel–Pade	MC
ξ	0.475	0.367 NNLO	0.329, 0.4, 0.449
ϵ_0/μ	2		1.9
Δ/μ	1.31		1.2
T_c/ϵ_F	.249	0.183 NLO	.14 – .25

Table 1:

ε: Nishida Son, Nishida, Arnold Drut Son

MC: Carlson Reddy, Bulgac et al, Burovski et al, Akkineni et al.,Lee, Schaefer, Lee

Fermion-dimer, dimer-dimer scattering to NLO, good agreement with expt., simpler than Faddeev eq. Rupak [06]

Experiment $\xi = 0.46 \pm .04,\ 0.51 \pm .05$ Rice, Duke

Will the trend hold for other observable quantities at NLO?

Effective Lagrangian

- Effective action

$$\Gamma[\Phi(x),\mu] = -\Omega[J(x),\mu] - \left(\int_x J^*(x)\,\Phi(x) + h.c.\right),$$

where

$$\frac{\delta}{\delta J^*(x)}\Omega[J(x),\mu] = -\Phi(x)$$

and

$$\mathrm{Exp}\,(-i\,\Omega[J,\mu]) = \int \mathrm{D}\phi\,\mathrm{D}\phi^*\,\mathrm{Det}\,\mathcal{M}(\phi)\,e^{i\int J^*\phi + J\phi^*},$$

where

$$\mathcal{M} = \begin{pmatrix} i\,\partial_t + \frac{\nabla^2}{2\,m} + \mu_\uparrow & \phi^*(x) \\ \phi(x) & i\,\partial_t - \frac{\nabla^2}{2\,m} - \mu_\downarrow \end{pmatrix}$$

-

$$\frac{\delta}{\delta\Phi(x)}\mathrm{L}_{eff}[\Phi(x),\mu,\delta\mu] = 0$$

$$\mathrm{F}(\mu,\delta\mu) = -\int \mathrm{d}^d x\,\mathcal{L}_{eff}[\hat{\Phi}(x),\mu,\delta\mu]$$

$$\mathrm{N}_1 + \mathrm{N}_2 = -\frac{\partial\mathrm{F}(\mu,\delta\mu)}{\partial\mu} \qquad \mathrm{N}_1 - \mathrm{N}_2 = -\frac{\partial\mathrm{F}(\mu,\delta\mu)}{\partial\delta\mu}$$

Effective Lagrangian to NLO in ε

The effective potential Nishida and Son [2006]

$$
\begin{aligned}
&\mathrm{V}_{\mathrm{eff}}(\Phi(\mathrm{x}),\mu) = \\
= \quad &\left(\frac{m\,|\Phi(x)|}{2\pi}\right)^{d/2} \frac{|\Phi(x)|}{3}\left[1+\frac{7-3(\gamma+\ln 2)}{6}\,\varepsilon - 3\,C\varepsilon\right] - \\
- \quad &\left(\frac{m\,|\Phi(x)|}{2\pi}\right)^{d/2} \frac{\mu}{\varepsilon}\left[1+\frac{1-2(\gamma-\ln 2)}{4}\,\varepsilon\right]
\end{aligned}
$$

$\gamma \approx 0.57722$, is the Euler-Mascheroni constant and $C \approx 0.14424$.

To describe non-homogeneous phenomena need $\mathrm{V}_{\mathrm{eff}}(\phi) \Rightarrow \mathrm{L}_{\mathrm{eff}}(\Phi(\mathrm{x}))$. Will attempt a derivative expansion.

$$
\begin{aligned}
&\mathrm{V}_{\mathrm{eff}}(\Phi(\mathrm{x}),\mu) \Rightarrow \mathrm{L}_{\mathrm{eff}}[\Phi(\mathrm{x}),\mu] = -\mathrm{V}_{\mathrm{eff}} + \partial\Phi(\mathrm{x}), \\
&\frac{\delta \mathrm{L}_{eff}}{\delta\Phi(x)} = 0 \Rightarrow \mathrm{F}(\mu) = -\int_x \mathcal{L}_{eff}[\hat{\Phi}(x),\mu] \;\Rightarrow \mathrm{N} = -\frac{\partial \mathrm{F}(\mu)}{\partial\mu}
\end{aligned}
$$

LO DERIVATIVE TERMS

- Gauge $U(1)$ particle symmetry $\Rightarrow$ a more general theory

$$\mathrm{L}_{kin} = \psi^\dagger\,(i\,\partial_t - A_0)\,\psi - \frac{1}{2\,m}\,(\vec{\nabla}\psi^\dagger - i\,\vec{A}\,\psi^\dagger)\cdot(\vec{\nabla}\psi + i\,\vec{A}\,\psi)$$

$$\psi \rightarrow e^{i\alpha(x)}\psi,\ \Phi \rightarrow e^{2i\alpha(x)}\Phi$$

$$A_0 \rightarrow A_0 - \partial_t\alpha(x),\ A_i \rightarrow A_i - \partial_i\alpha(x)$$

Set $A_0 = -\mu,\ \vec{A} = 0,\ \alpha(x) = \mathrm{const}$ to return to the original theory.

- Note no $F^2_{\mu\nu}$ for A field

- Gauge invariance $\Rightarrow$ LO derivatives in $\mathrm{L}_{eff}[\Phi(x),\mu]$

$$Z_1(|\Phi|)\,\Phi^*(i\,\partial_t - 2\,A_0)\,\Phi - Z_2(|\Phi|)(\vec{\nabla}\Phi^* - 2\,i\,\vec{A}\,\Phi^*)\cdot(\vec{\nabla}\Phi + 2\,i\,\vec{A}\,\Phi)$$

$$\mathrm{V}_{eff}(\phi,-\mu,0) \Rightarrow \mathrm{V}_{eff}(\phi,A_0,\vec{A}) \ \Rightarrow\ Z_1,\ Z_2$$

NLO Effective Lagrangian ($\mu_\uparrow = \mu_\downarrow$)

For $x \geq (m\,\mu)^{-1/2} \propto$ int. part. separation, $t \geq \mu^{-1}$

$$\begin{aligned}
\mathcal{L}_{eff} &= \left[\left(\Phi^*\, i\,\partial_t\,\Phi - \frac{1}{4\,m}|\vec{\nabla}\Phi|^2\right)\frac{1}{2\,|\Phi|^2\,\varepsilon} + \frac{\mu}{\varepsilon}\right] \times \\
&\times \left(1 + \frac{1 - 2(\gamma_E - \ln 2)}{4}\,\varepsilon\right)\left(\frac{m\,|\Phi|}{2\pi}\right)^{d/2} - \\
&- \left(\frac{m\,|\Phi|}{2\pi}\right)^{d/2}\frac{|\Phi|}{3}\left[1 + \frac{7 - 3(\gamma_E + \ln 2)}{6}\,\varepsilon - 3\,C\varepsilon\right]
\end{aligned}$$

where $\gamma_E \approx 0.57722$, $C \approx 0.14424$

Higher derivative terms are $\mathcal{O}(\varepsilon^2)$

$\mathcal{L}_{eff}$ on $\Phi = \rho\exp 2\,i\,\beta \Rightarrow$ LO phonon terms,

$$\mathcal{L}_{NGB} = \frac{1}{2}\,\frac{\partial\,n}{\partial\,\mu}\,\left(\partial_t\beta\right)^2 - \frac{n\,|\vec{\nabla}\beta(\vec{x})|^2}{2\,m},$$

where n is the equilibrium number density Son Stephanov[05].

THE SMOKING GUN OF SUPERFLUIDITY

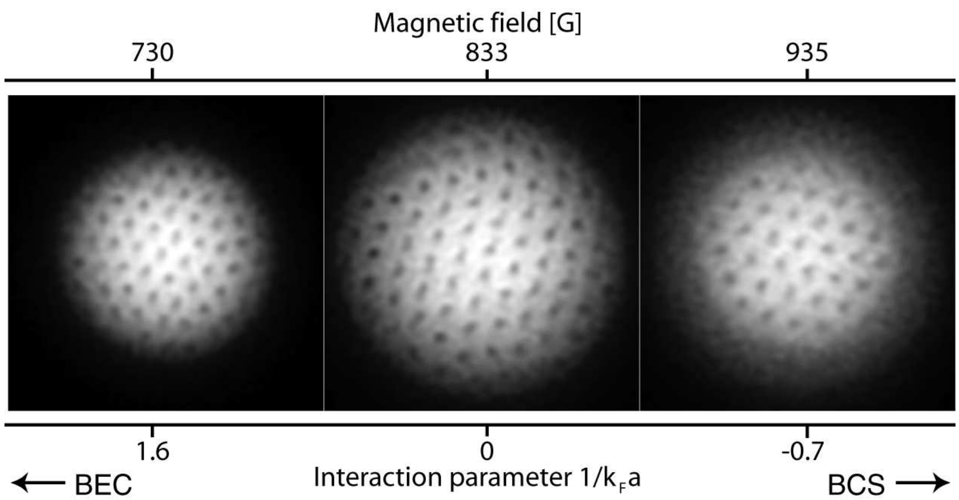

Figure 3: Vortex lattices observed by Ketterle's group, MIT [05]

Vortex Profile

Particle number $U(1)$ is spont. broken $\Rightarrow$ stable vortex configurations observed. A single vortex configuration of unit winding number $\Phi(\vec{x}) = \rho(r)\, e^{i\,\theta}$ with $\vec{x} = \{r, \theta, ...\}$.

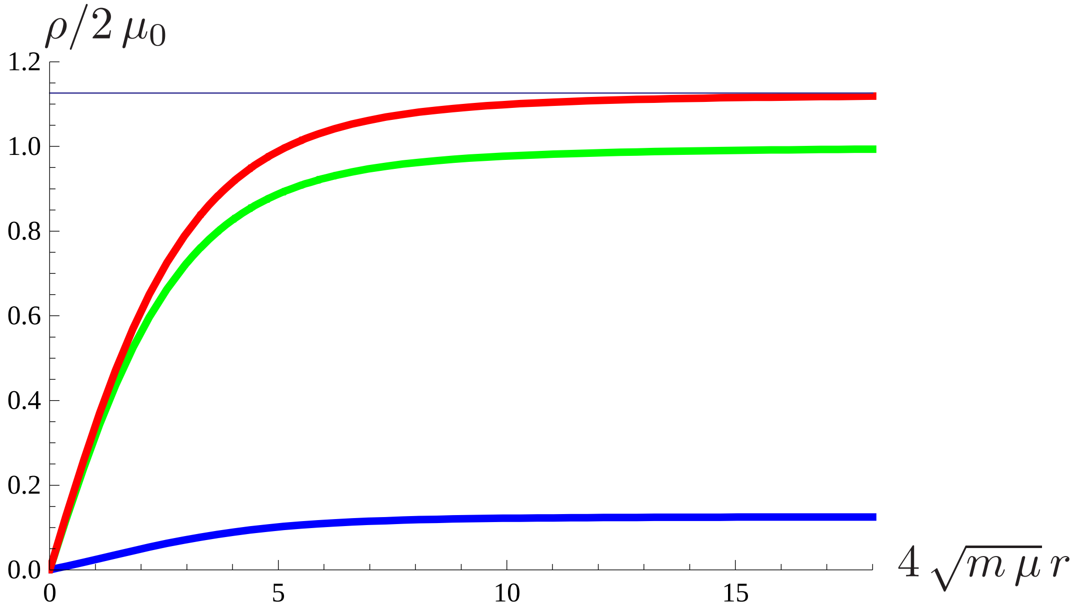

Figure 4: The single vortex profile. The LO – red, the NLO – green, LO + ε NLO with $\varepsilon = 1$ – blue curve ($\mu_0 = \frac{\mu}{\varepsilon} \sim \phi_0 \sim 1$). The typical size, $\rho(r_0) = \rho(r = \infty)/2$, $r_0 = .43,\ .45\sqrt{m\,\mu}$ – LO, NLO, respectively. $r_0 k_F = 0.86,\ 0.92$ – LO, NLO, respectively. Bulgac Yu [03]. Spherical trap: $r_0 \simeq 0.25\, L\, N^{-1/3}$, L is the radius of the trap, $d = 3$.

Polarized gas ($\delta\mu \neq 0$)

As imbalance between pairing species is increased s.f. - normal phase transition is expected and has been observed.

$\mu = (\mu_\uparrow + \mu_\downarrow)/2,\ \delta\mu = (\mu_\uparrow - \mu_\downarrow)/2$

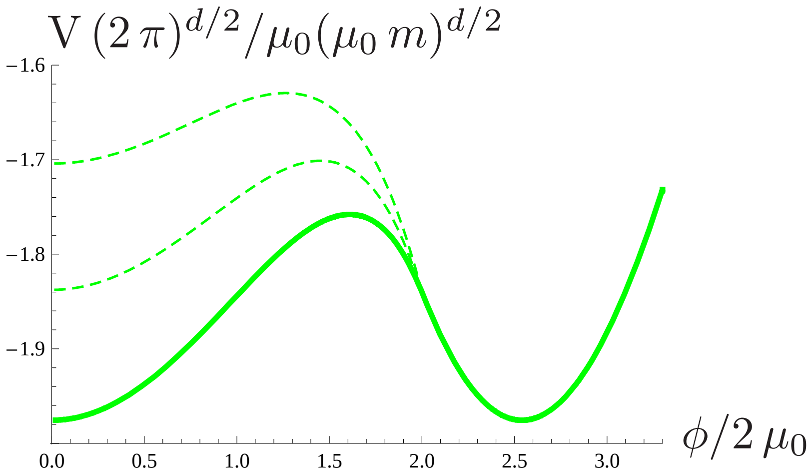

Figure 5: Effective potential of the bulk medium V at $\delta\mu \simeq \delta\mu_c = 2\mu_0\,(1 - 0.4672\varepsilon)\,,\ \mu_0 = \mu/\varepsilon \sim 1$ Rupak, Schaefer, AK [06] as a function of the order parameter ϕ. Shown is the NLO in ε result with $\varepsilon = 1$.

$\mathcal{L}_{eff}[\Phi(x), \mu, \delta\mu]$ for polarized gas in derivative expansion.

$\mathrm{V_{eff}}(\Phi(\mathrm{x}), \mu, \delta\mu) \Rightarrow \mathrm{L_{eff}}[\Phi(\mathrm{x}), \mu, \delta\mu] = -\mathrm{V_{eff}}(\Phi(\mathrm{x}), \mu, \delta\mu) + \partial\Phi(\mathrm{x})'\mathrm{s}$

S.F.-NORMAL PHASE INTERFACE

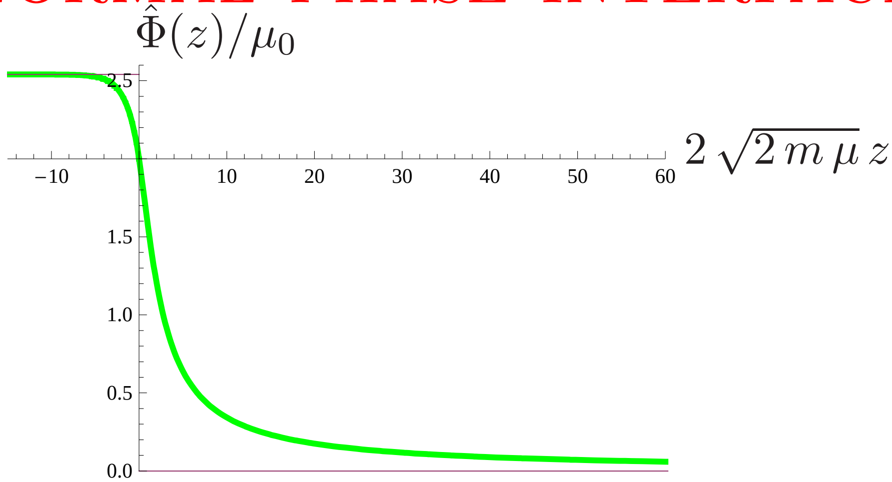

Figure 6: The superfluid-normal phase interface from the NLO effective Lagrangian with $\varepsilon = 1$.

The superfluid-normal phase surface energy is then

$$\sigma = -\int \mathrm{d}z \left(\mathcal{L}_{eff}[\hat{\Phi}(z), \mu, \delta\mu_c] - \mathcal{L}_{eff}[\hat{\Phi}(z=\infty), \mu, \delta\mu_c]\right)$$
$$\simeq 0.81 \sqrt{\frac{\mu_0}{m}} \left(\frac{\mu_0\, m}{2\pi}\right)^{d/2}, \;\; \mu_0 = \mu/\varepsilon \sim 1.$$

$$\int_S \sigma \;\; \simeq \;\; \int_S \frac{n^{4/3}}{2\,m}\,\mathrm{s}, \;\; \mathrm{s} = 0.28\; N^{-1/3}, \;\; d = 3.$$

$N = 10^7$ $\mathrm{s} = 0.0013$; $N = 5 \times 10^5$ $\mathrm{s} = 0.0034$ vs $\mathrm{s} = 0.001$ De Silva Mueller [06]

Density Correlation Function

Generating functional

$$Z[\rho(x)] = \int D\psi\, D\psi^\dagger\, \mathrm{Exp}\; i \int_x \mathcal{L}(\psi; \rho(x))$$

$\Rightarrow$ the correlation function

$$\begin{aligned} S(x) &= -i\langle 0|T\, \psi^\dagger(x)\psi(x)\, \psi^\dagger(0)\psi(0)|0\rangle \\ &= i\, \frac{\delta^2}{\delta\,\rho(x)\,\delta\,\rho(0)} \log Z(\rho(x))|_{\rho=0} \end{aligned}$$

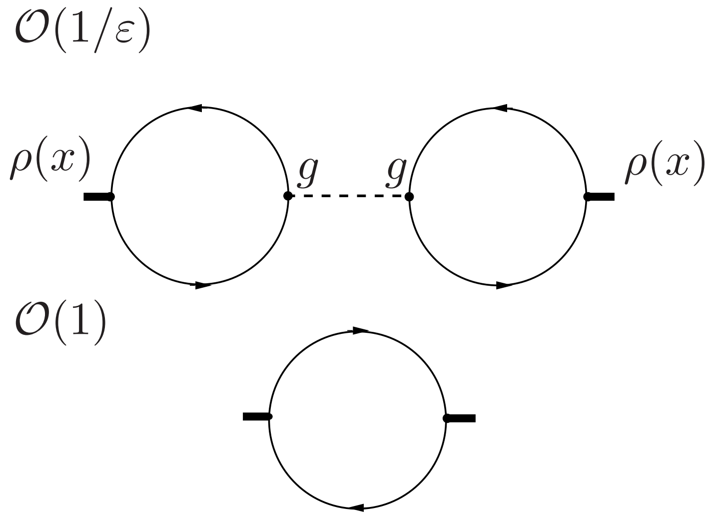

Figure 7: Feynman diagrams relevant for the calculation of the density correlation function to NLO in ε. $\epsilon_{\mathbf{p}}$ and p_0 are treated as $\mathcal{O}(1)$. Solid lines are fermions in s.f. background, dashed line is the dressed boson propagator. Coupling $g = \frac{(8\pi^2\varepsilon)^{1/2}}{m}\left(\frac{m\phi_0}{2\pi}\right)^{\epsilon/4}$.

Density-density Correlator

Disclaimer: $\epsilon_{\mathbf{p}}$ and p_0 are treated as $\mathcal{O}(1)$.

$$
\begin{aligned}
\frac{1}{n} \quad & \mathrm{S}(p_0, \epsilon_{\mathbf{p}}) = \\
& \frac{2\,\epsilon_{\mathbf{p}}\left(1 - \frac{\varepsilon}{4}\right) - \frac{\varepsilon}{6\,\mu_0}\left(6\,p_0^2 + \epsilon_{\mathbf{p}}^2\right)\big) + \mathcal{O}\left(\varepsilon^2,\, \varepsilon\epsilon_{\mathbf{p}}^3/\mu_0^2\right)}{p_0^2 - \epsilon_{\mathbf{p}}(\mu + \frac{\epsilon_{\mathbf{p}}}{4}) + i\delta + \mathcal{O}\left(\varepsilon\epsilon_{\mathbf{p}}\mu\right)} - \\
- \quad & \frac{\varepsilon}{4\,\mu_0}\left(1 - \frac{\epsilon_{\mathbf{p}}}{6\,\mu_0} + \frac{p_0^2}{24\,\mu_0^2} + \frac{\epsilon_{\mathbf{p}}^2}{120\,\mu_0^2}\right) + \\
+ \quad & \mathcal{O}\left(\varepsilon^2,\, \varepsilon\frac{p0^3}{\mu_0^4},\, \varepsilon\frac{\epsilon_{\mathbf{p}}^3}{\mu_0^4}\right), \\
& 0 \le p_0 \le 2\Delta,\ \epsilon_{\mathbf{p}} = \frac{\mathbf{p}^2}{2m},\ \mu_0 = \frac{\mu}{\varepsilon},\ \delta = 0^+,
\end{aligned}
$$

$$
n = \left(\frac{m\,\mu_0}{2\,\pi}\right)^{d/2}\,\frac{4}{\varepsilon}\,\left[1 + \varepsilon\left(6\,C - \frac{\gamma_E}{2} + 2\log 2 - \frac{7}{4}\right)\right]
$$

p_0, ϵ_p expansion is not full to keep the pole structure

Dynamic structure factor

The dynamic structure factor, $\sigma(\omega, \epsilon_k)$

$$
\begin{aligned}
& \sigma(p_0, \epsilon_p) \equiv -\mathrm{Im}\, S(p_0, \epsilon_p) = \\
= \quad & \pi\, n \left[\frac{2\, \epsilon_p \left(1 - \frac{\varepsilon}{4}\right) - \frac{\varepsilon}{6\, \mu_0} \left(6\, p_0^2 + \epsilon_p^2\right)\right)}{2\, p_0} \right] \times \\
\times \quad & \delta \left(p_0 - \sqrt{\epsilon_p \left(\mu + \frac{\epsilon_p}{4} \right)} \right), \quad 0 \le p_0 \le 2\Delta
\end{aligned}
$$

Density-density Correlator: Checks

- The dispersion relation

$$S(\omega = 0, \epsilon_k) = \frac{2}{\pi} \int_0^\infty \mathrm{d}\omega \frac{\mathrm{Im}\, S(\omega, \epsilon_k)}{\omega}$$

 is satisfied to the NLO in ε and to $\mathcal{O}(\epsilon_k^2)$

- the compressibility sum rule

$$S(\omega = 0, \epsilon_k \to 0) = -\frac{n}{m\, c_s^2}$$

 holds to NLO in ε. $c_s^2 = \frac{\mu}{2\,m}\left[1 + \frac{\varepsilon}{4}\right]$. L.h.s. has LO c_s but r.h.s. - NLO c_s

- Energy weighted sum rule

$$-\frac{1}{\pi\, n} \int_0^\infty \mathrm{d}\omega\, \omega\, \mathrm{Im}\, S(\omega, \epsilon_p) = \epsilon_p$$

 gives

$$-\frac{1}{\pi\, n} \int_0^\infty \mathrm{d}\omega\, \omega\, \mathrm{Im}\, S(\omega, \epsilon_p) = \epsilon_p \left[\left(1 - \frac{\varepsilon}{4}\right) - \frac{5\, \varepsilon\, \epsilon_p}{24 \mu_0}\right]$$

 Note that it is sensitive to $\omega > 2\,\Delta$ not included

Summary

- ε expansion provides first principle description of Fermi gas near unitarity
- At NLO in ε several result agree well with MC results and experiment. Will this trend hold?
- Derived NLO $\mathcal{L}_{eff}$, calculated single vortex structure in Unitary Fermi Gas, sf/normal interface in imbalanced gas
- Calculated density correlation function $\rightarrow$ dynamic structure factor for energies below quasiparticle threshold

Work in Progress and Outlook

- NLO $\mathcal{L}_{eff} \Rightarrow$ vortex lattice, unitary Fermi gas in a periodic potential (optical lattice)
- Quasi-particle excitation energy and T_c to NNLO
- $< Tnn >$ for energies above quasiparticle threshold, spin density response
- Shear viscosity of unitary Fermi gas from ε expansion. Non-equilibrium properties of unitary Fermi gas.
- P-wave pairing in polarized gas Bulgac et al [06]