Nearly Perfect Fluidity:

From Atoms to Quarks

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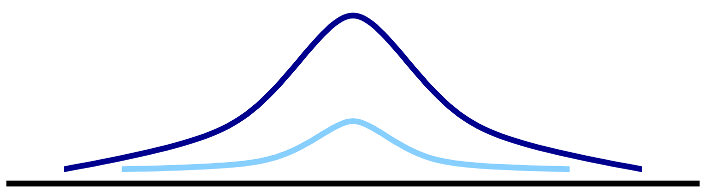
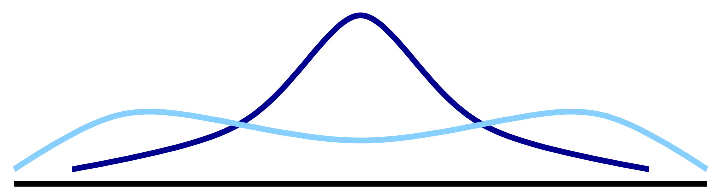
See T. Schaefer, D. Teaney, "Perfect Fluidity" [arXiv:0904.3107]

Fluids: Gases, Liquids, Plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



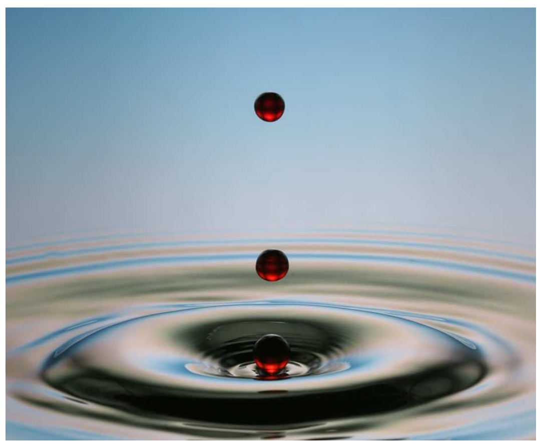
$\tau \sim \tau_{micro}$



$\tau \sim \lambda$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



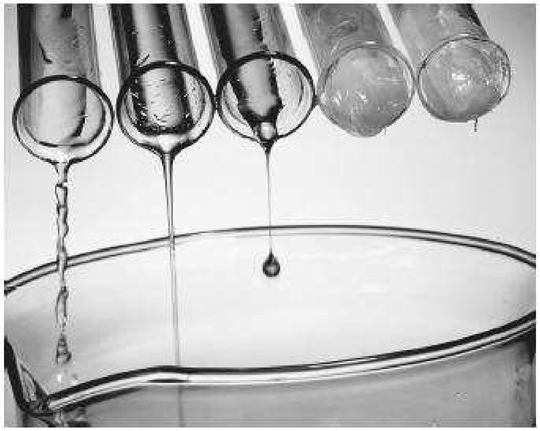
Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$



Constitutive relations: Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + O(\partial^2)$$

reactive dissipative 2nd order

Expansion $\Pi^0_{ij} \gg \delta\Pi^1_{ij} \gg \delta\Pi^2_{ij}$

Regime of applicability

Expansion parameter $Re^{-1} = \frac{\eta(\partial v)}{\rho v^2} = \frac{\eta}{\rho L v} \ll 1$

$$Re = \frac{\hbar n}{\eta} \times \frac{m v L}{\hbar}$$

fluid property $\qquad$ flow property

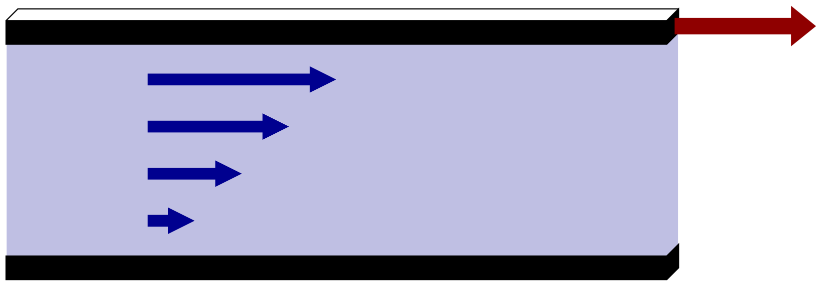
Kinetic theory estimate: $\eta \sim n p l_{mfp}$

$$Re^{-1} = \frac{v}{c_s} Kn \qquad Kn = \frac{l_{mfp}}{L}$$

expansion parameter $Kn \ll 1$

Shear viscosity

Viscosity determines shear stress ("friction") in fluid flow

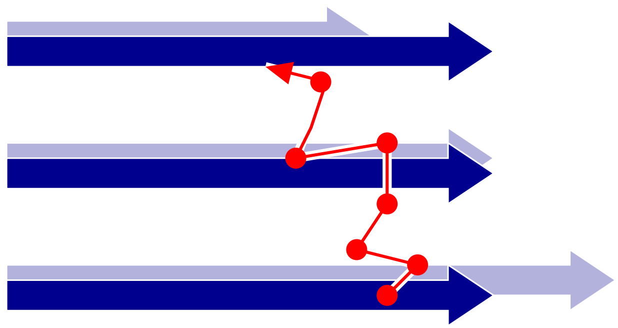


$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

Kinetic theory: conserved quantities carried by quasi-particles

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p + \vec{F}\cdot\vec{\nabla}_p f_p = C[f_p]$$

$$\eta \sim \frac{1}{3} n\,\bar{p}\,l_{mfp}$$



Dilute, weakly interacting gas: $l_{mfp} \sim 1/(n\sigma)$

$$\eta \sim \frac{1}{3}\frac{\bar{p}}{\sigma}$$

independent of density!

Shear viscosity

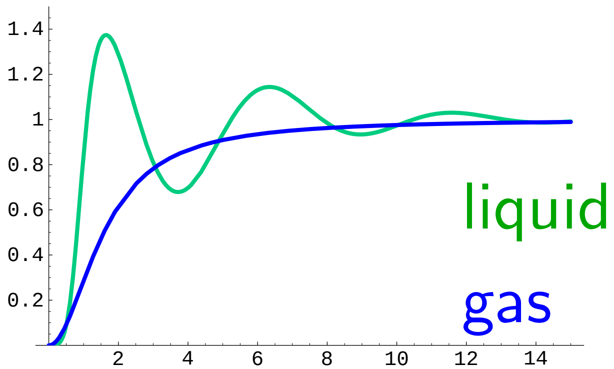
non-interacting gas ($\sigma \to 0$): $\boxed{\eta \to \infty}$

non-interacting and hydro limit ($T \to \infty$) limit do not commute

strongly interacting gas: $\boxed{\frac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar}$

but: kinetic theory not reliable!

what happens if the gas condenses into a liquid?



Eyring, Frenkel:

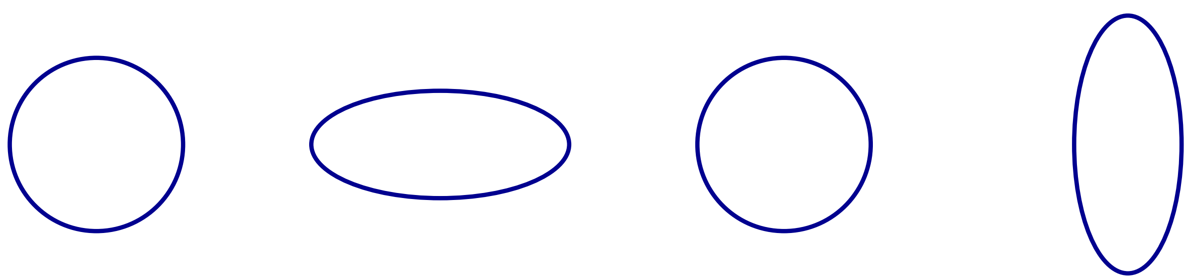
$$\boxed{\eta \simeq hn \exp(E/T) \geq hn}$$

Holographic Duals: Transport Properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature	$\Leftrightarrow$	Hawking temperature
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \qquad\qquad g_{\mu\nu} = g^0_{\mu\nu} + \gamma_{\mu\nu}$$



Holographic Duals: Transport Properties

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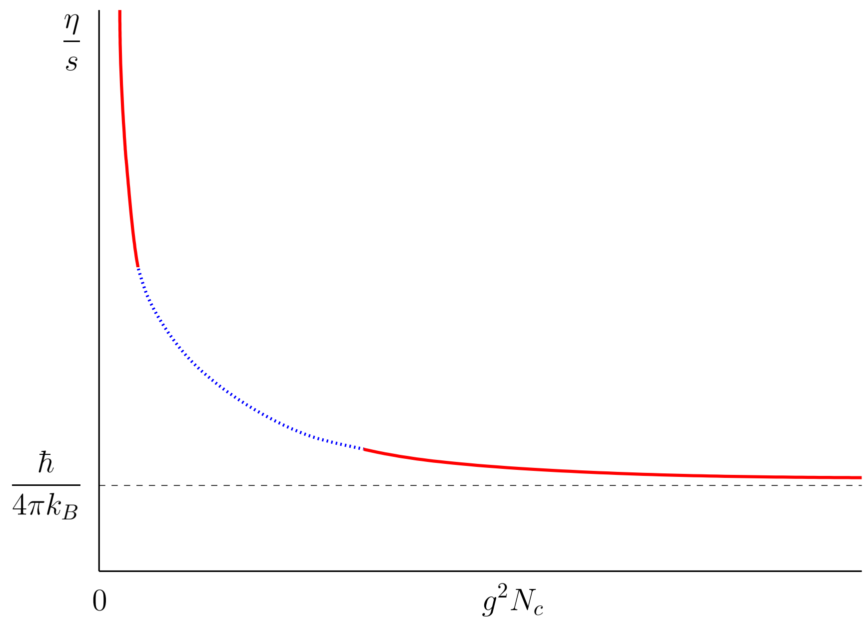
CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy $\sim$ area of event horizon

shear viscosity $\Leftrightarrow$ Graviton absorption cross section $\sim$ area of event horizon

Strong coupling limit

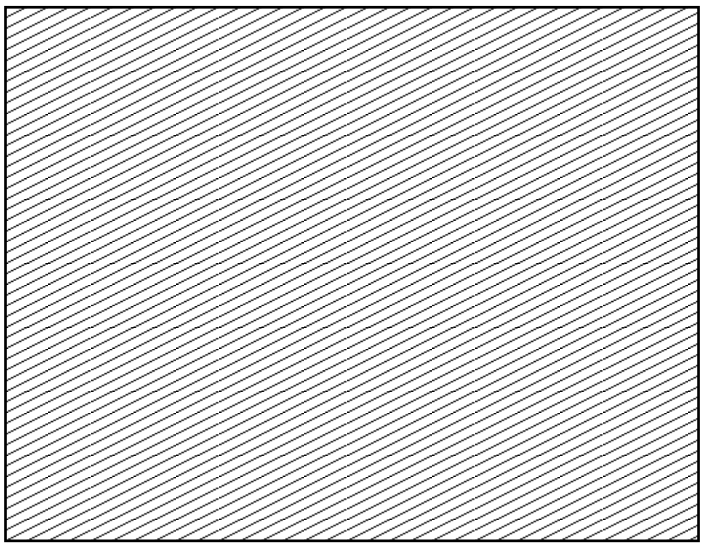
$$\boxed{\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}}$$

Son and Starinets (2001)

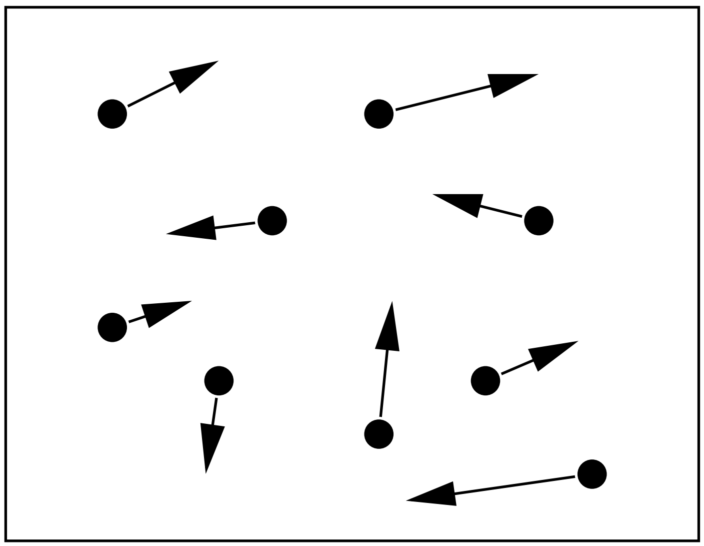


Strong coupling limit universal? Provides lower bound for all theories?

Kinetics vs No-Kinetics



AdS/CFT low viscosity goo



pQCD kinetic plasma

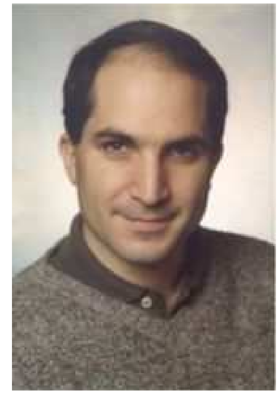
Effective theories for fluids (Here: Weak coupling QCD)

$$\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p = C[f_p] \qquad (\omega < T)$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \qquad (\omega < g^4 T)$$

Effective theories (Strong coupling)

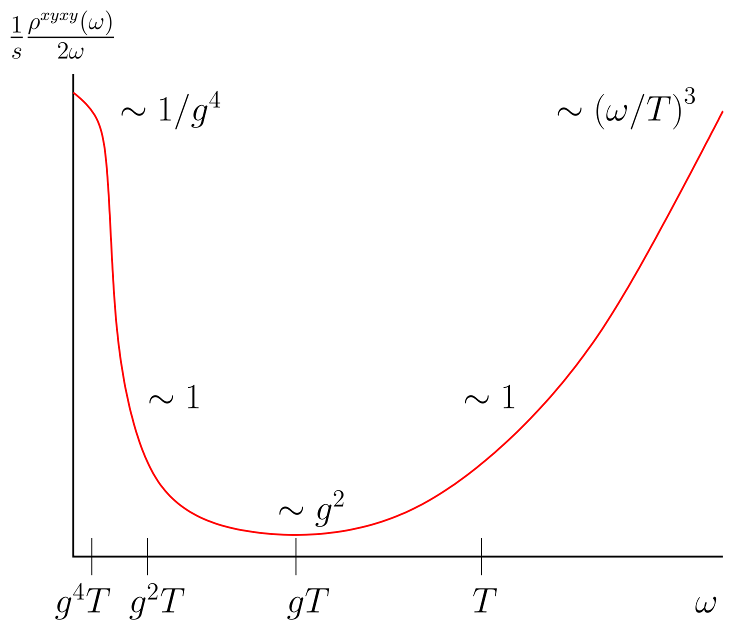


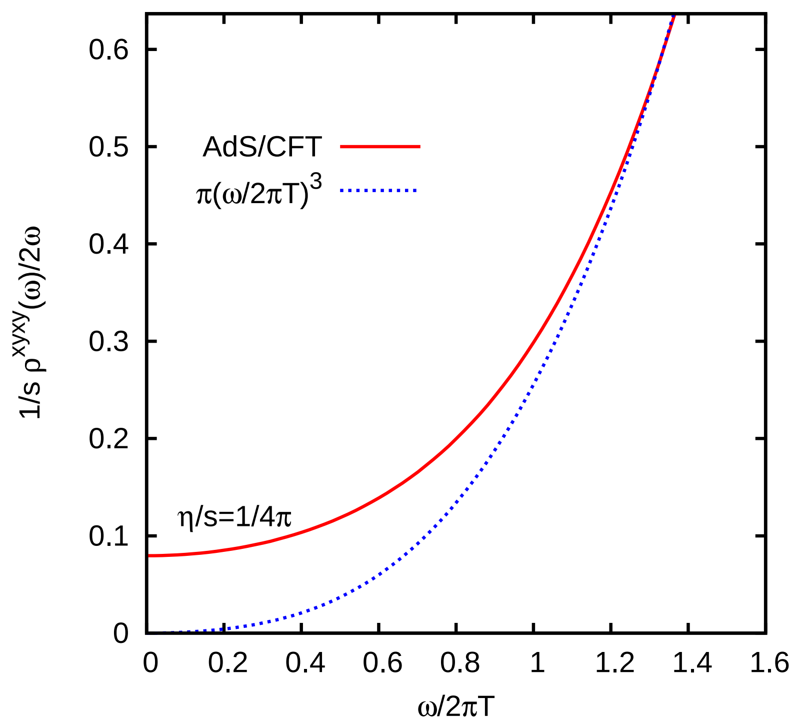
$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \ldots \;\Leftrightarrow\; S = \frac{1}{2\kappa_5^2}\int d^5x \sqrt{-g}\mathcal{R} + \ldots$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \quad (\omega < T)$$

Kinetics vs No-Kinetics

Spectral function $\rho(\omega) = \mathrm{Im} G_R(\omega, 0)$ associated with T_{xy}

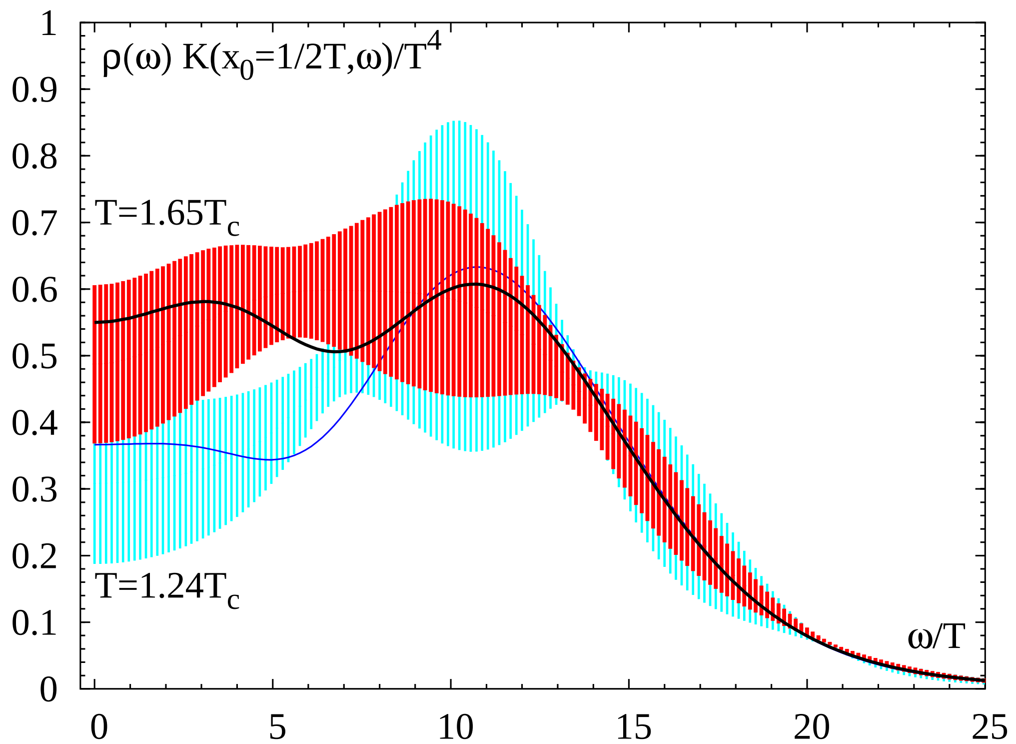




weak coupling QCD — strong coupling AdS/CFT

transport peak vs no transport peak

Spectral function (lattice QCD)



T	1.02 T_c	1.24 T_c	1.65 T_c
η/s		0.102(56)	0.134(33)
ζ/s	0.73(3)	0.065(17)	0.008(7)

H. Meyer (2007)

Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

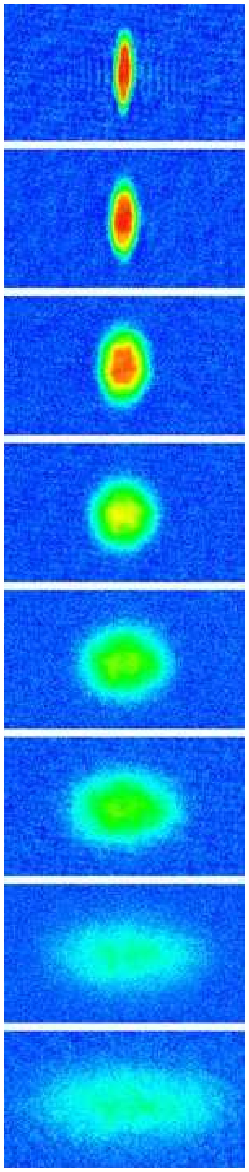
Bound is incompatible with weak coupling and kinetic theory

strong interactions, no quasi-particles

Model system has conformal invariance (essential?)

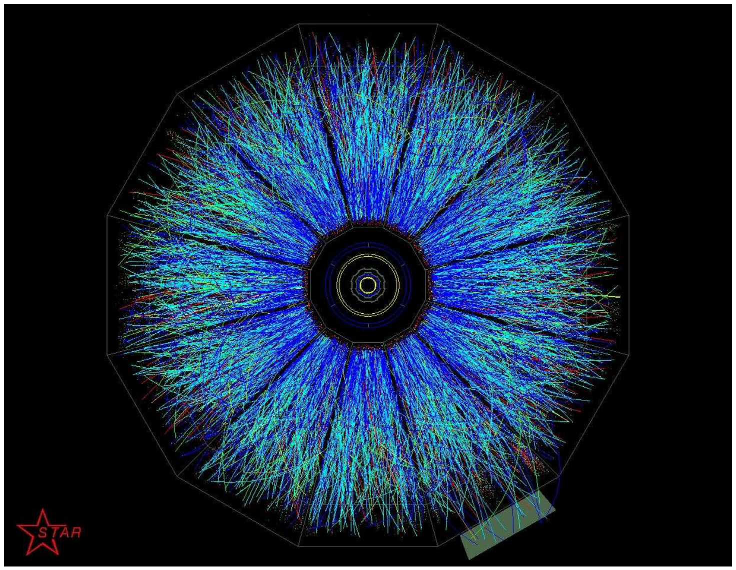
(Almost) scale invariant systems

Perfect Fluids: The contenders



Trapped Atoms

(T=0.1 neV)

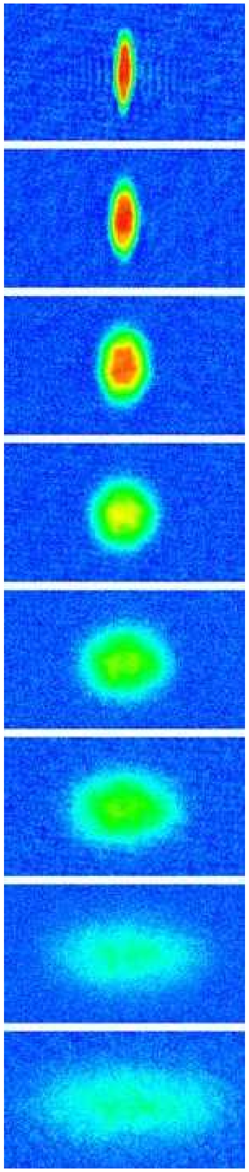


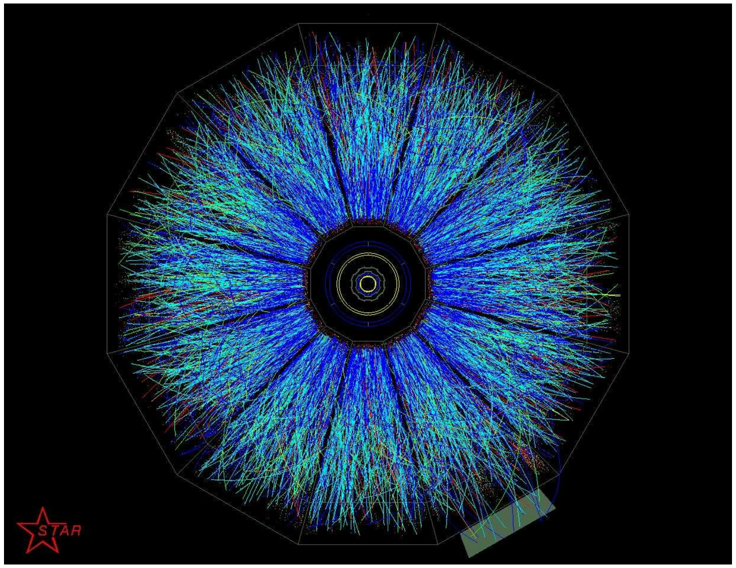
QGP (T=180 MeV)

Liquid Helium

(T=0.1 meV)

Perfect Fluids: The contenders

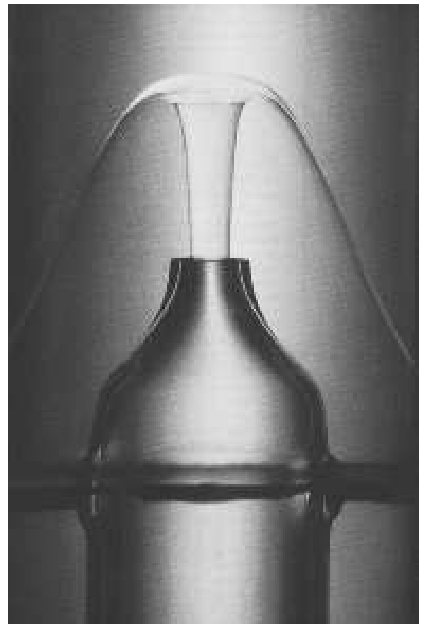




QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$



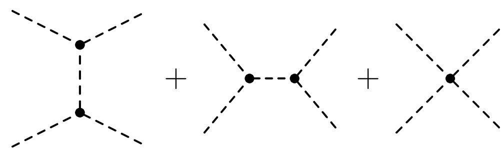
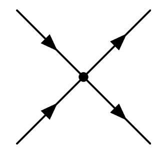
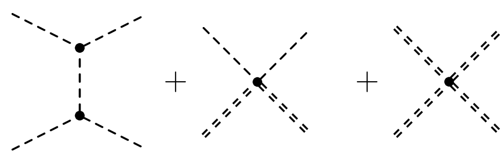
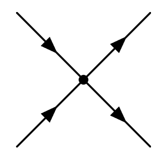
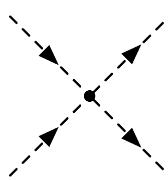
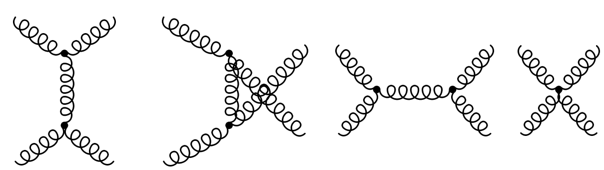
Liquid Helium

$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

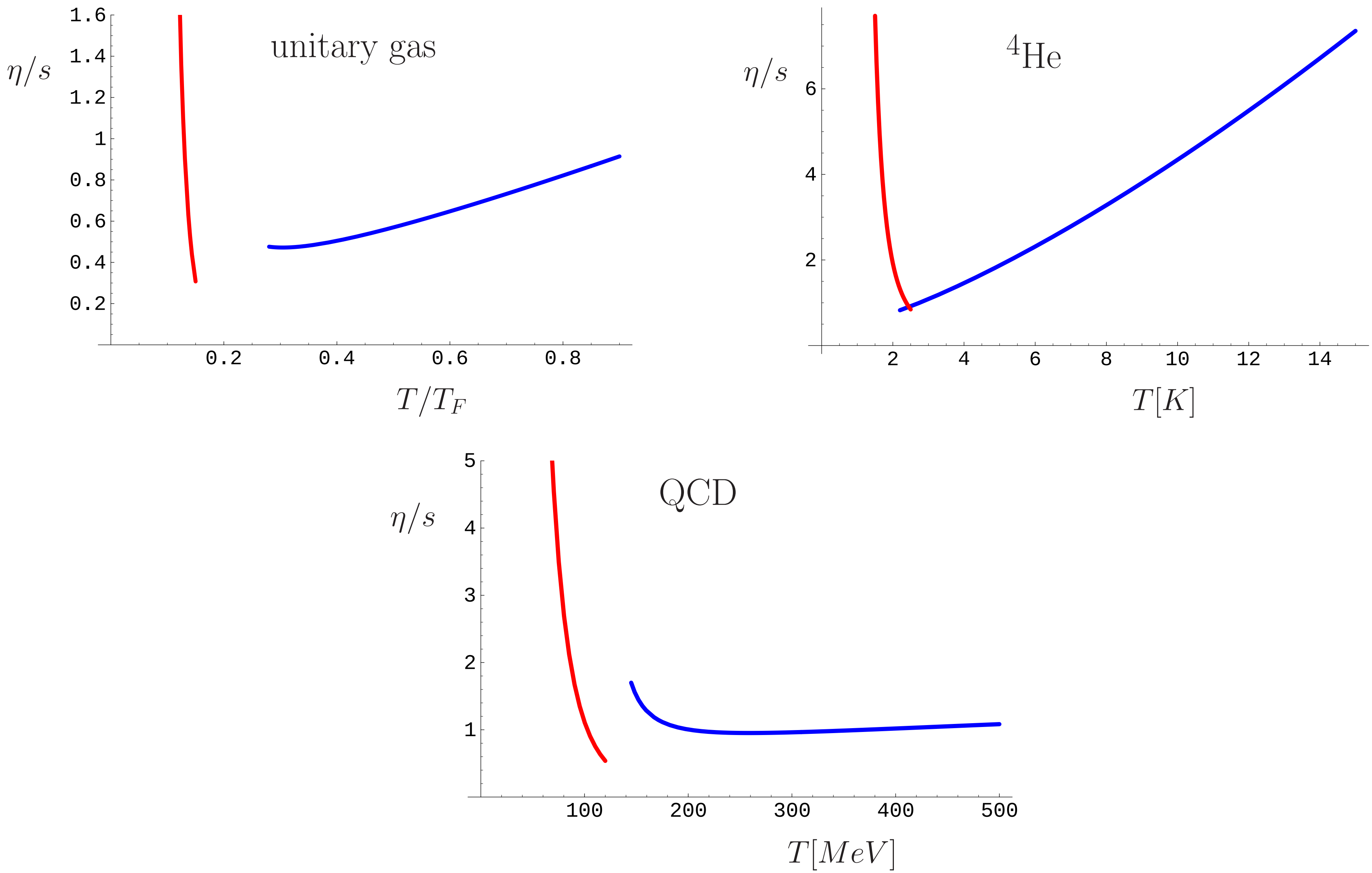
Consider ratios

η/s

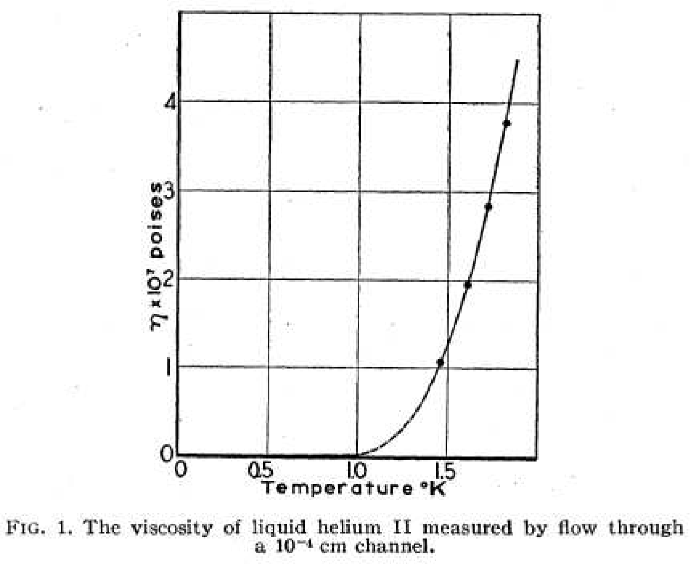
Kinetic Theory: Quasiparticles

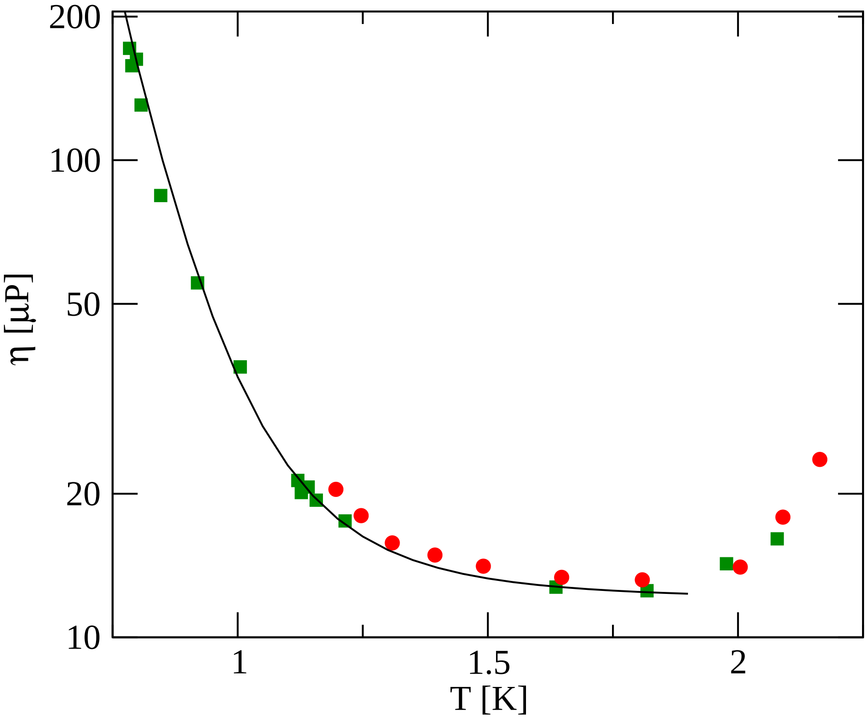
	low temperature	high temperature
unitary gas	phonons 	atoms 
helium	phonons, rotons 	atoms 
QCD	pions 	quarks, gluons 

Theory Summary



I. Experiment (Liquid Helium)





Kapitza (1938)

viscosity vanishes below T_c

capillary flow viscometer

Hollis-Hallett (1955)

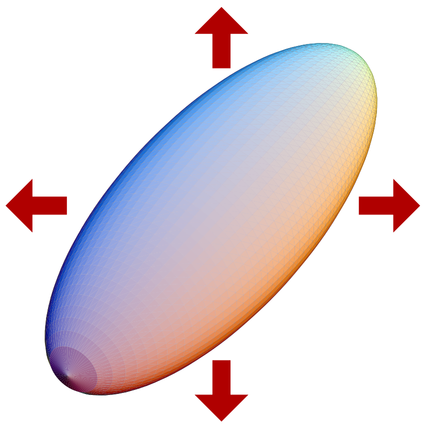
roton minimum, phonon rise

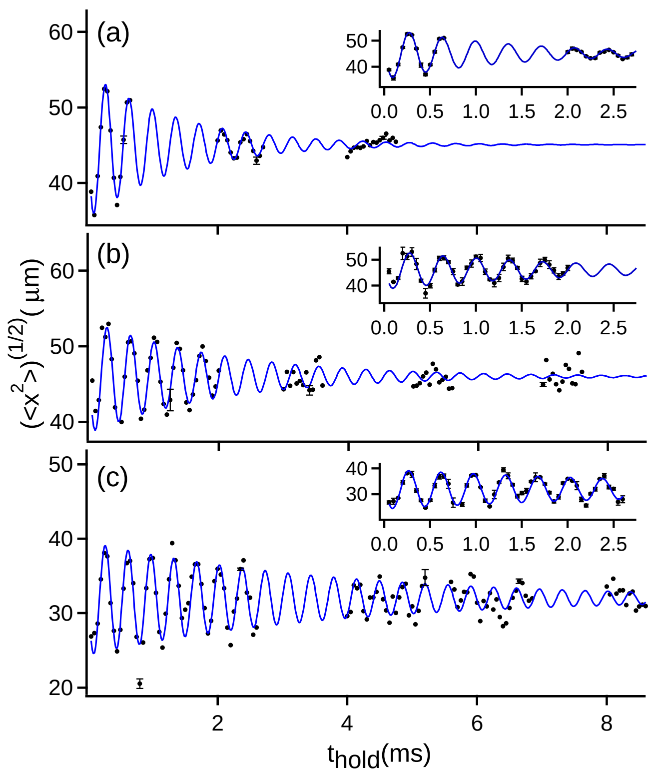
rotation viscometer

$$\eta/s \simeq 0.8\,\hbar/k_B$$

II. Hydrodynamics (Cold atoms)

Radial breathing mode





Ideal fluid hydrodynamics ($P \sim n^{5/3}$)

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right)\vec{v} = -\frac{\vec{\nabla} P}{mn} - \frac{\vec{\nabla} V}{m}$$

Hydro frequency at unitarity

$$\omega = \sqrt{\frac{10}{3}}\,\omega_\perp$$

Damping small, depends on T/T_F.

experiment: Kinast et al. (2005)

Viscous Hydrodynamics

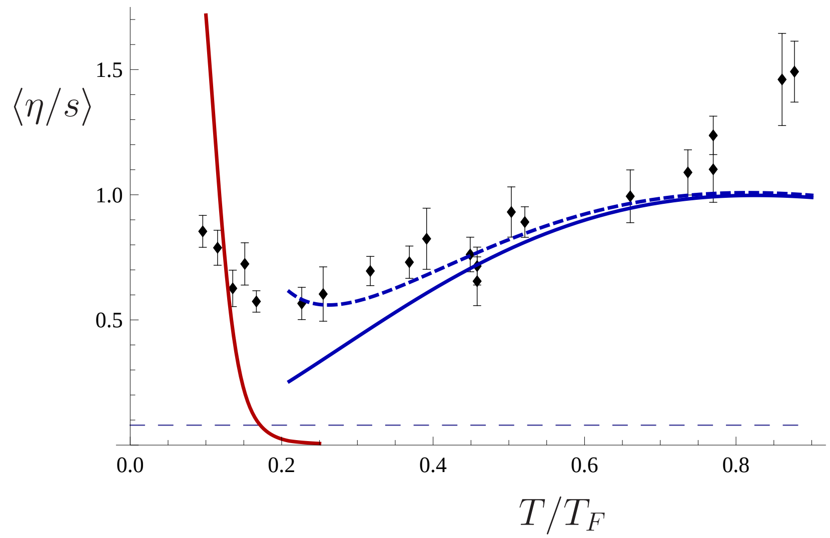
Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{1}{2}\int d^3x\, \eta(x) \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \int d^3x\, \zeta(x)\, (\partial_i v_i)^2 - \frac{1}{T}\int d^3x\, \kappa(x)\, (\partial_i T)^2$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

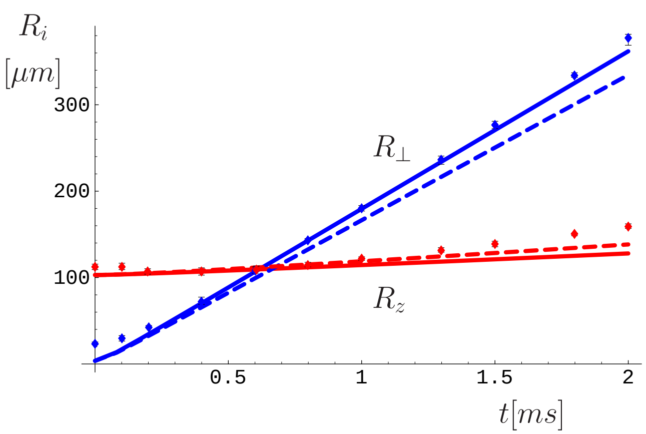
$$\frac{\eta}{s} = (3\lambda N)^{\frac{1}{3}} \frac{\Gamma}{\omega_\perp} \frac{E_0}{E_F} \frac{N}{S}$$

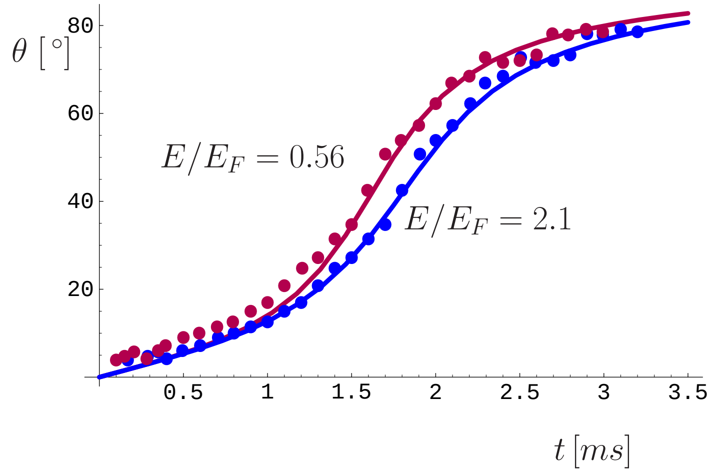


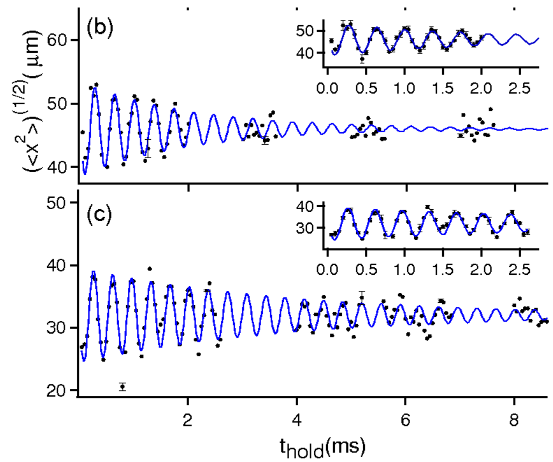
Schaefer (2007), see also Bruun, Smith

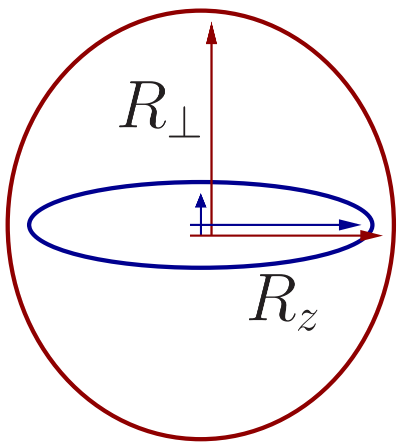
$T \ll T_F$ $\quad$ $T \gg T_F,\ \tau_R \simeq \eta/P$

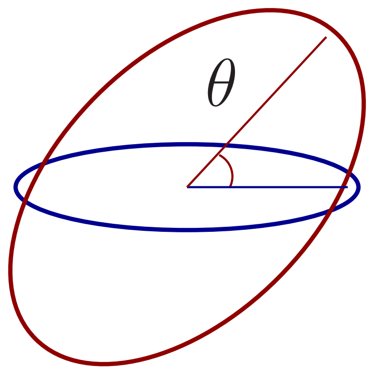
Dissipation

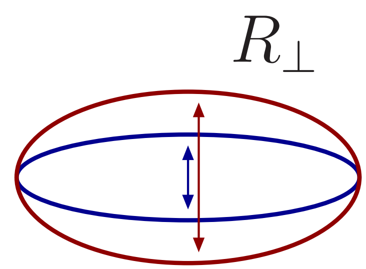






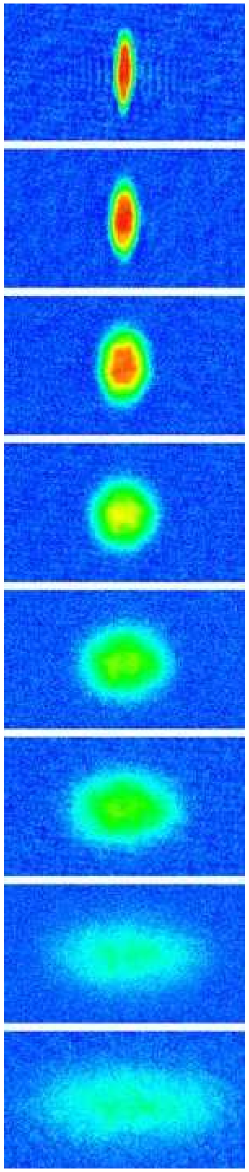




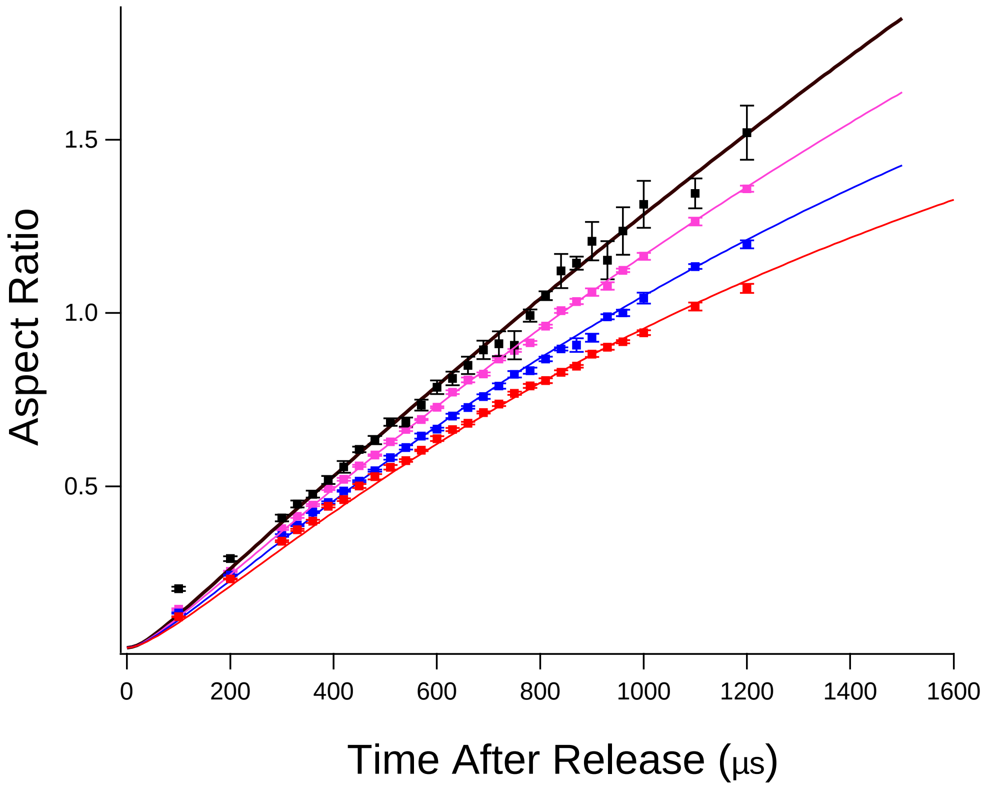


O'Hara et al (2002), Kinast et al (2005), Clancy et al (2007)

Elliptic flow: High T limit



Quantum viscosity $\eta = \eta_0 \frac{(mT)^{3/2}}{\hbar^2}$



$$\eta = \eta_0 (mT)^{3/2}$$

$$\tau_R = \eta / P$$

fit: $\eta_0 = 0.33 \pm 0.4$

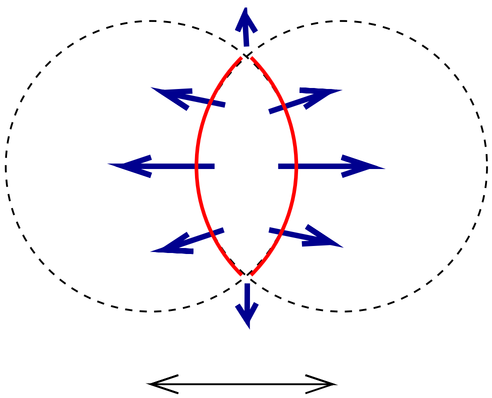
theory: $\eta_0 = \frac{15}{32\sqrt{\pi}} = 0.26$

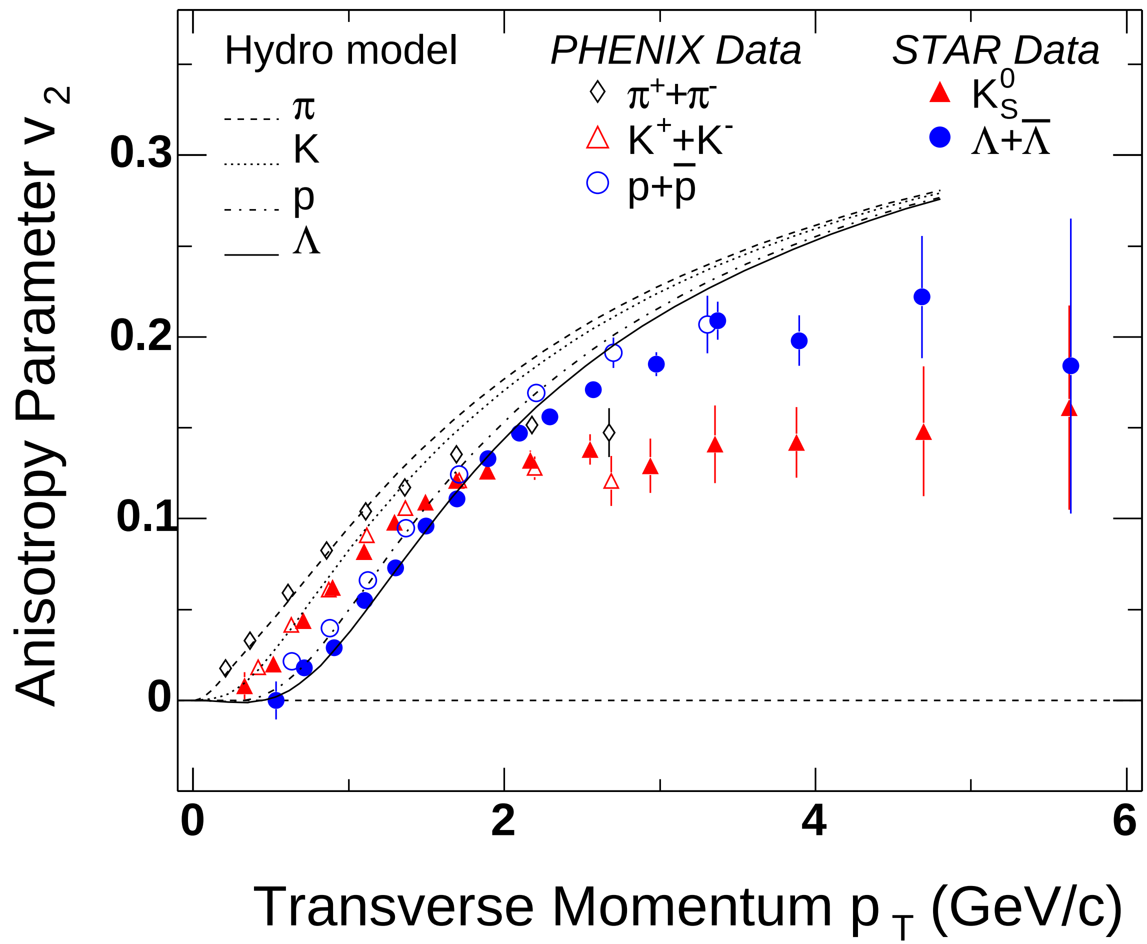
III. Elliptic Flow (QGP)

Hydrodynamic expansion converts

coordinate space anisotropy

to momentum space anisotropy

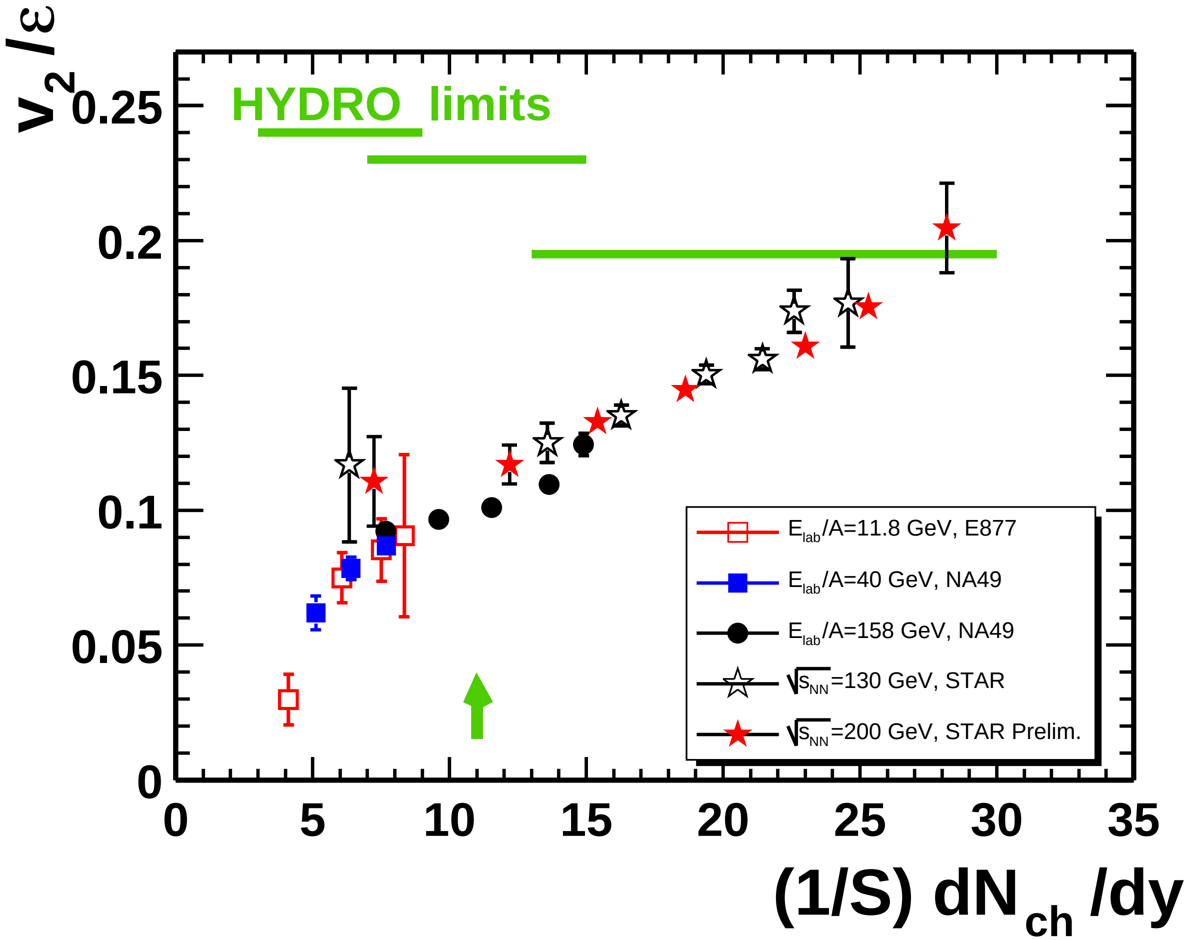




source: U. Heinz (2005)

$$p_0 \frac{dN}{d^3p}\bigg|_{p_z=0} = v_0(p_\perp)\,(1 + 2v_2(p_\perp)\cos(2\phi) + \ldots)$$

Elliptic flow: initial entropy scaling



source: U. Heinz (2005)

Viscosity and Elliptic Flow

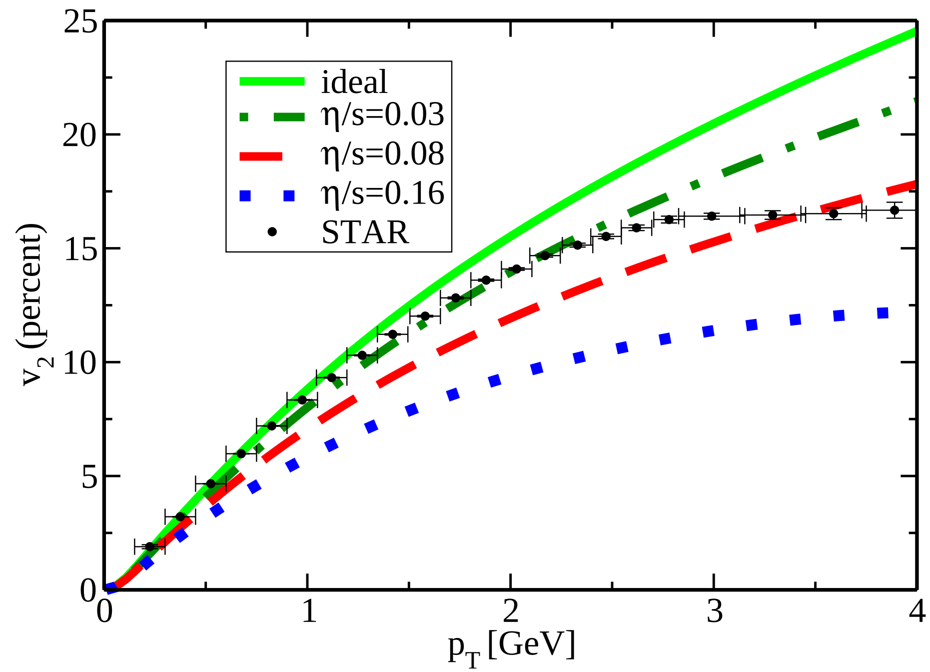
Consistency condition $T_{\mu\nu} \gg \delta T_{\mu\nu}$

(applicability of Navier-Stokes)

$$\frac{\eta + \frac{4}{3}\zeta}{s} \ll \frac{3}{4}(\tau T)$$

Danielewicz, Gyulassy (1985)

Very restrictive for $\tau < 1$ fm



Romatschke (2007), Teaney (2003)

Many questions: Dependence on initial conditions, freeze out, etc.

conservative bound $\boxed{\frac{\eta}{s} < 0.4}$

The bottom-line

Remarkably, the best fluids that have been observed are the *coldest* and the *hottest* fluid ever created in the laboratory, cold atomic gases (10^{-6}K) and the quark gluon plasma (10^{12}K) at RHIC.

Both of these fluids come close to a bound on the shear viscosity that was first proposed based on calculations in string theory, involving non-equilibrium evolution of back holes in 5 (and more) dimensions.

What does this have to do with confinement?

Are the excitations in the deconfined phase (just above T_c (quasi) quarks and (quasi) gluons?

This question can be addressed by looking for a transport peak.

If η/s is close to $1/(4\pi)$ the answer is probably no.