Entropy and Viscosity of Strongly Coupled Quantum Fluids

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Paradigms: “weak” vs “strong” QGP

wQGP	sQGP
quasi-particles ($\omega \gg \Gamma$)	no quasi-particles
$s = s_0(1 + O(g^2) + \ldots)$	$s = s_0 \times const$
kinetic description	no kinetic description
poor fluid	hydrodynamic behavior
$\eta/s \sim 1/(\alpha_s^2 \log(\alpha_s^{-1}))$	$\eta/s \sim 1/(4\pi)$

Transport more useful than thermodynamics

in distinguishing “w” from “s”?

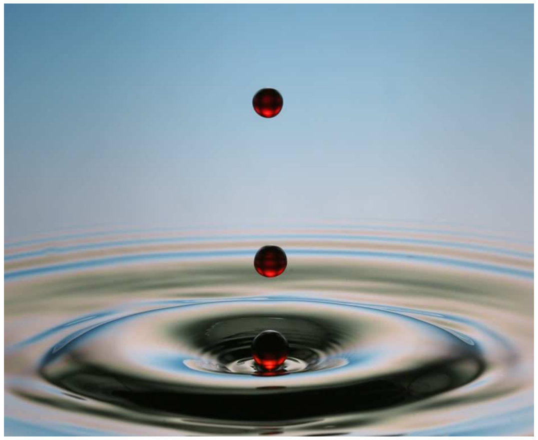
Hydrodynamics

Long-wavelength, low-frequency dynamics of conserved or spontaneoulsy broken symmetry variables.

$\tau \sim \tau_{micro}$ $\qquad$ $\tau \sim \lambda^{-1}$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



Example: Simple Fluid

Continuity equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

Euler (Navier-Stokes) equation

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0$$

Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right) + \ldots$$

reactive dissipative

Kinetic Theory

Quasi-Particles ($\gamma \ll \omega$): introduce distribution function $f_p(x,t)$

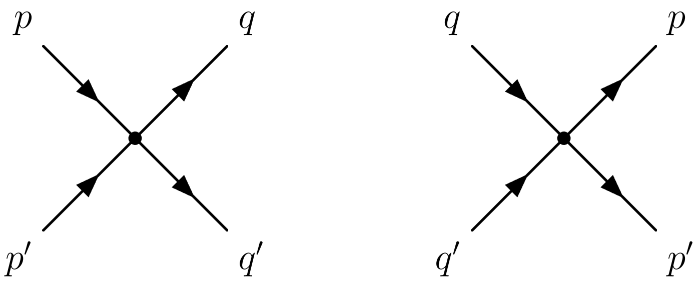
$$\rho = \int \frac{d^3p}{E_p}\, m f_p \qquad \Pi_{ij} = \int d^3p\, \frac{p_i p_j}{E_p}\, f_p,$$

Boltzmann equation

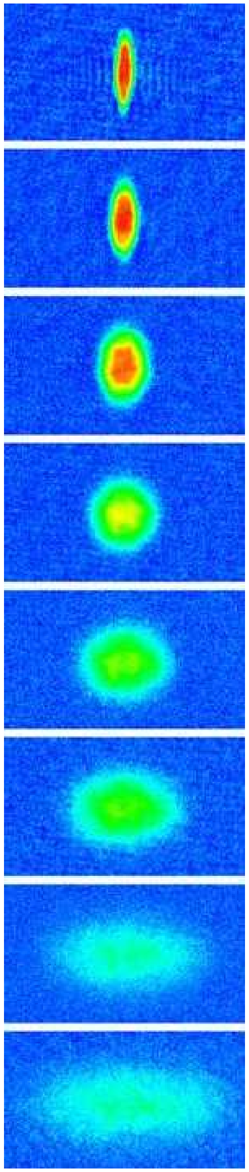
$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p + \vec{F}\cdot\vec{\nabla}_p f_p = C[f_p]$$

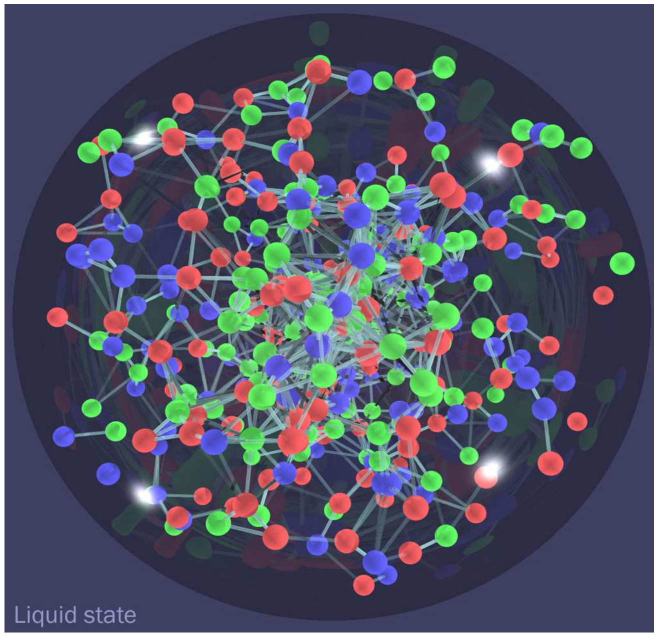
Collision term $C[f_p] = C_{gain} - C_{loss}$

$$C_{loss} = \int dp'dqdq'\; f_p f_{p'} w(p,p';q,q') \qquad C_{gain} = \ldots$$

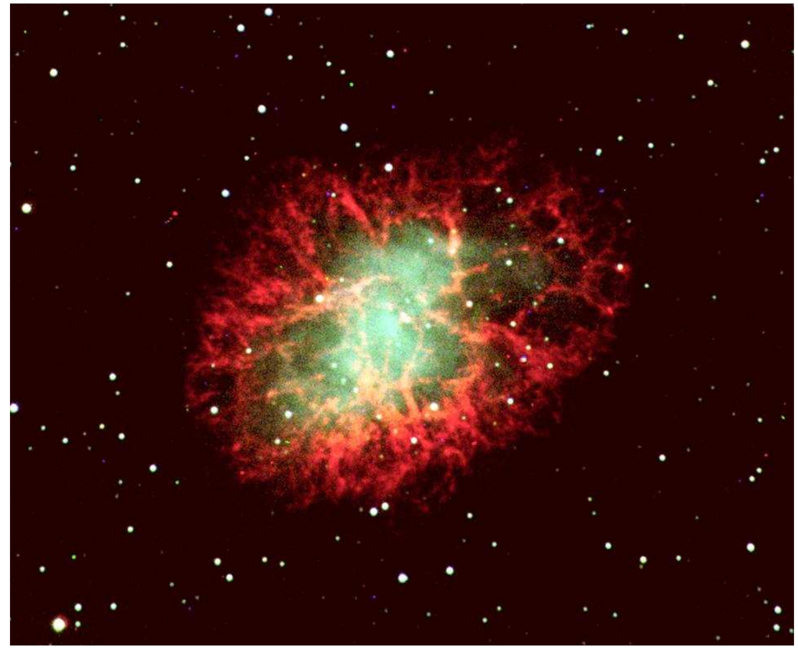


Perfect Fluids





sQGP (T=180 MeV)



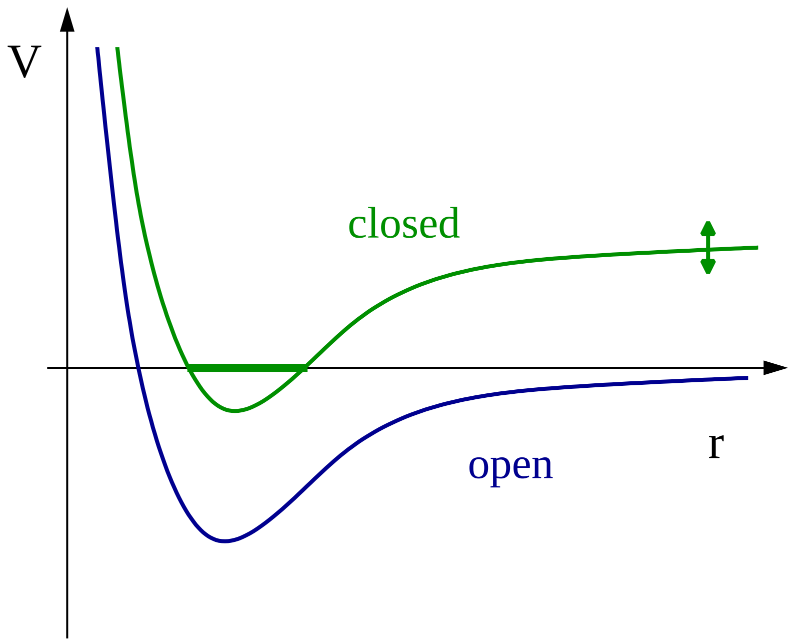
Neutron Matter (T=1 MeV)

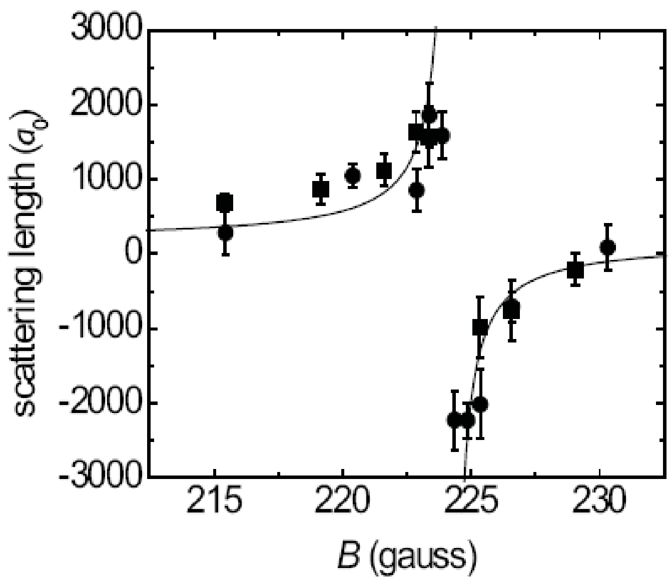
Trapped Atoms

(T=0.1 neV)

Designer Fluids

Atomic gas with two spin states: "$\uparrow$" and "$\downarrow$"





Feshbach resonance
$$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

"Unitarity" limit $a \to \infty$
$$\sigma = \frac{4\pi}{k^2}$$

Why are these systems interesting?

System is intrinsically quantum mechanical

cross section saturates unitarity bound

Scale (and conformally) invariant at unitarity

$$\frac{E}{A} = \xi \left(\frac{E}{A}\right)_0, \qquad \text{OPE, Holography, \dots}$$

System is strongly coupled but dilute

$$(k_F a) \to \infty \qquad (k_F r) \to 0$$

Strong hydrodynamic elliptic flow observed experimentally

I. EOS, Quasi-Particles

Microscopic Effective Field Theory

Effective field theory for pointlike, non-relativistic fermions

$$\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \left[(\psi\psi)^\dagger (\psi \overleftrightarrow{\nabla}^2 \psi) + h.c. \right] + \ldots$$

Match to effective range expansion

$$C_0 = \frac{4\pi a}{M}, \qquad C_2 = \frac{4\pi a^2}{M} \frac{r}{2}$$

Unitarity limit $C_0 \to \infty$, $C_2 \to 0$

Scattering amplitude

$$\mathcal{T} = \frac{4\pi}{m} \frac{1}{1/a - ik} \to \frac{4\pi}{imk}$$

Perturbative at high energy (temperature)

Low Energy Effective Lagrangian

Fermions are paired $\langle\psi\psi\rangle \neq 0$. Energy gap

$$\omega \sim \Delta \sim E_F$$

Low energy degrees of freedom: phase of condensate

$$\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$$

Effective lagrangian

$$\mathcal{L} = f^2\left(\dot{\varphi}^2 - v^2(\vec{\nabla}\varphi)^2\right) + \ldots$$

Effective Lagrangian

Low energy ($\omega < \Delta \sim E_F$) effective lagrangian

$$\mathcal{L} = c_0 m^{3/2} X^{5/2} + c_1 m^{1/2} \frac{(\vec{\nabla} X)^2}{\sqrt{X}} + \frac{c_2}{\sqrt{m}} \left[\left(\nabla^2 \varphi\right)^2 - 9m \nabla^2 A_0 \right] \sqrt{X}$$

$$X = \mu - A_0 - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}$$

variables

φ: phase $\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$

μ: chemical potential

A_0: gauge potential

constrained by

$U(1)$ invariance

Galilean invariance

Scale invariance

Conformal invariance

Greiter et al. (1989); Son, Wingate (2005)

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Effective lagrangian determines

Coupling to external fields

Energy density functional

Phonon interactions

Superfluid hydrodynamics

Non-perturbative physics in $c_0, c_1, c_2, \ldots$

Use epsilon ($\epsilon = d - 4$) expansion

Upper and lower critical dimension

Zero energy bound state for arbitrary d

$$\psi''(r) + \frac{d-1}{r}\psi'(r) = 0 \qquad (r > r_0)$$

d=2: Arbitrarily weak attractive potential has a bound state

free fermions: $\mu = E_F$

d=4: Bound state wave function $\psi \sim 1/r^{d-2}$. Pairs do not overlap

free bosons: $\mu = 0$

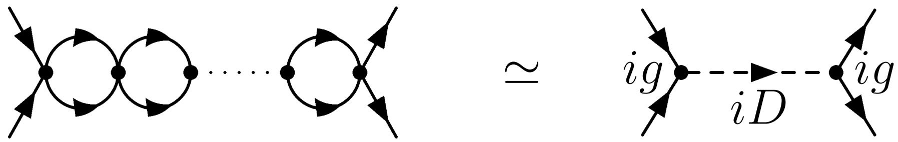
Conclude $\xi = \mu/E_F \sim 1/2$?

Try expansion around $d = 4$ or $d = 2$?

Nussinov & Nussinov (2004)

Epsilon Expansion

EFT version: Compute scattering amplitude ($d = 4 - \epsilon$)



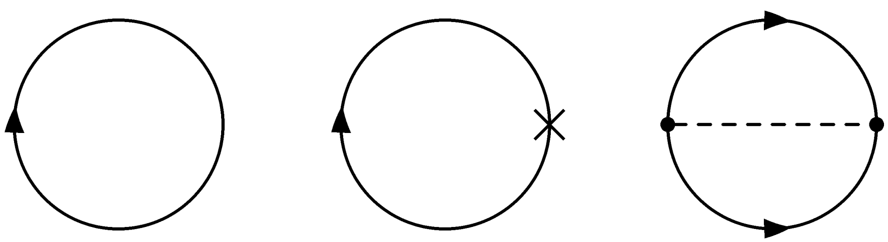
$$T = \frac{1}{\Gamma\left(1 - \frac{d}{2}\right)} \left(\frac{m}{4\pi}\right)^{-d/2} \left(-p_0 + \frac{\epsilon_p}{2}\right)^{1-d/2} \simeq \frac{8\pi^2 \epsilon}{m^2} \frac{i}{p_0 + \frac{\epsilon_p}{2} + i\delta}$$

$$g^2 \equiv \frac{8\pi^2 \epsilon}{m^2} \qquad D(p_0, p) = \frac{i}{p_0 + \frac{\epsilon_p}{2} + i\delta}$$

Weakly interacting bosons and fermions

Matching Calculations

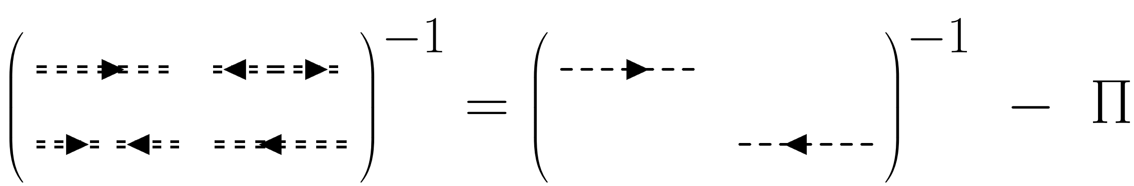
Effective potential



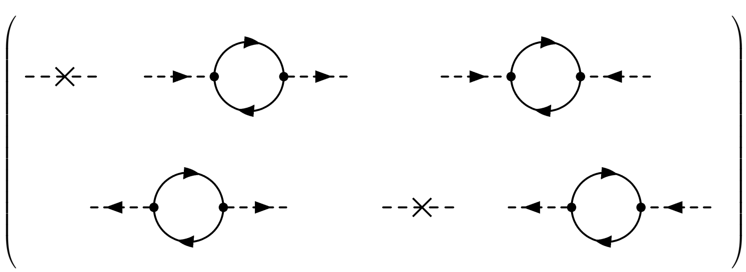
$O(1)$ $\qquad$ $O(1)$ $\qquad$ $O(\epsilon)$

$$P = \#(2m)^{d/2}\mu^{d/2+1}$$

Phonon Propagator



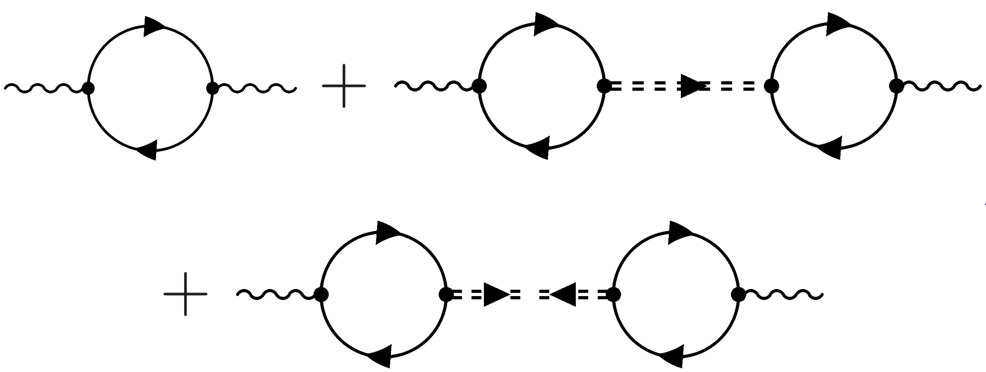
$$-\Pi =$$



$$\omega = c_s p \left\{ 1 + \# \left(\frac{p^2}{m\mu} \right) + \ldots \right\}$$

Matching (continued)

Static susceptibility $\qquad \chi(q) = \int d^3x\, e^{iqx} \langle \psi^\dagger\psi(x)\psi^\dagger\psi(0)\rangle$



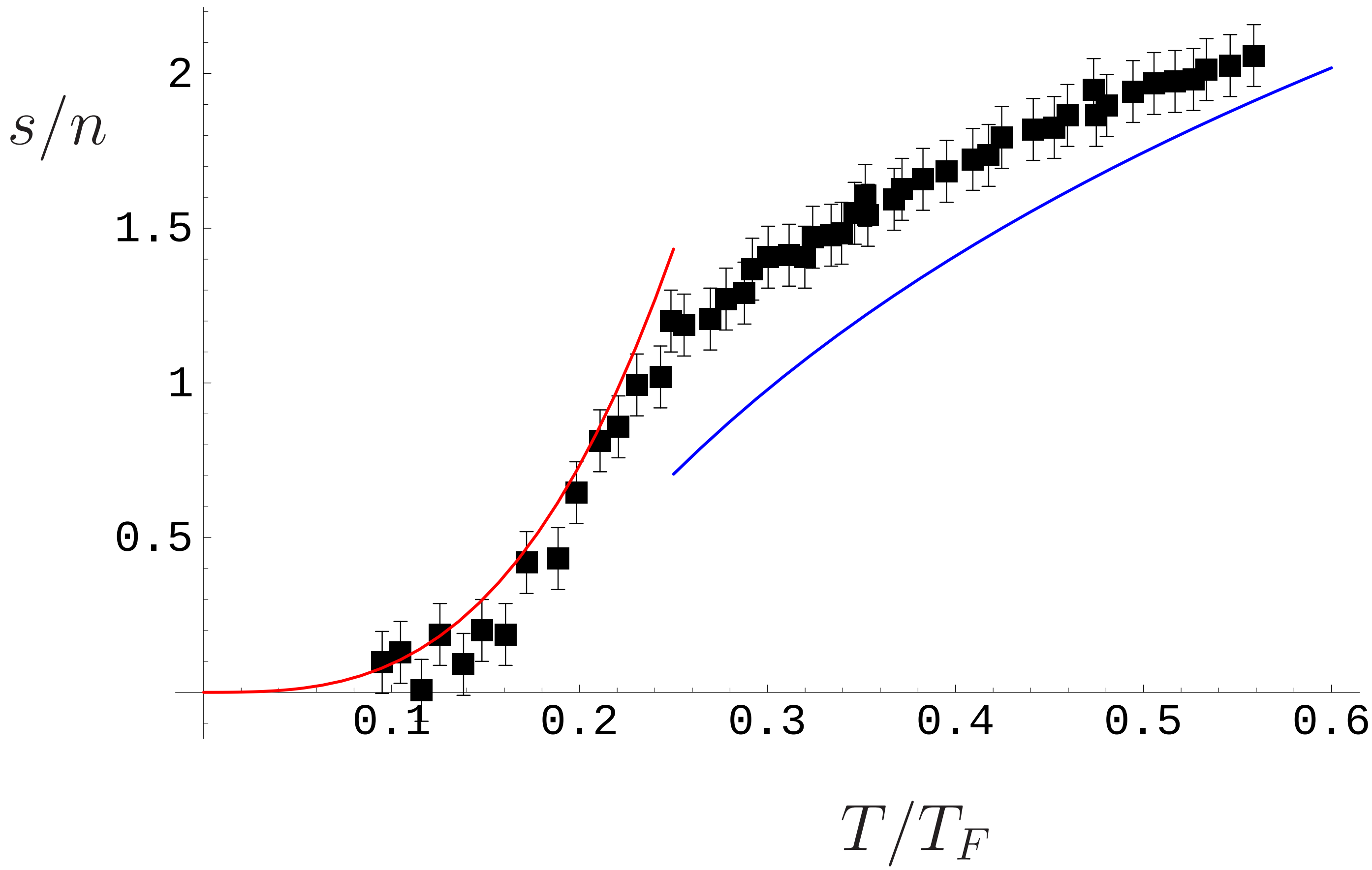
$$\chi(q) = \chi(0)\left\{1 - \#\left(\frac{q^2}{m\mu}\right) + \ldots\right\}$$

Nishida, Son (2007), Rupak, Schaefer (2008)

Match $P, \omega(q), \chi(q)$ to c_0, c_1, c_2

$$c_0 = 3.1c_0^{free}, \qquad c_1/c_0 = 1/8, \qquad c_2/c_0 = 0$$

Application: Entropy

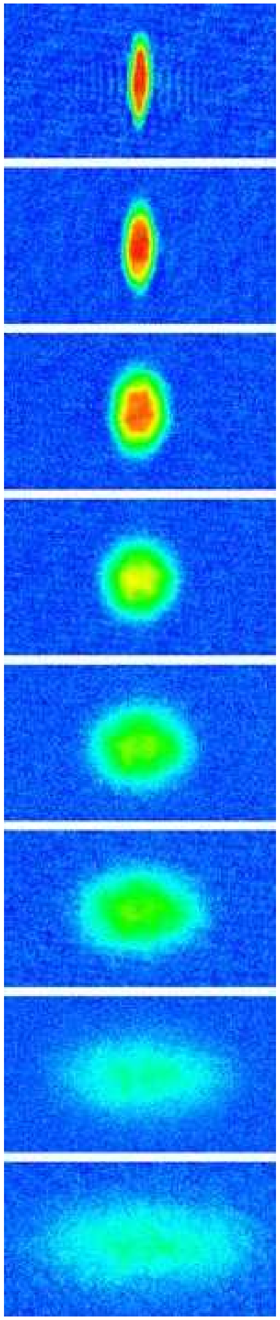


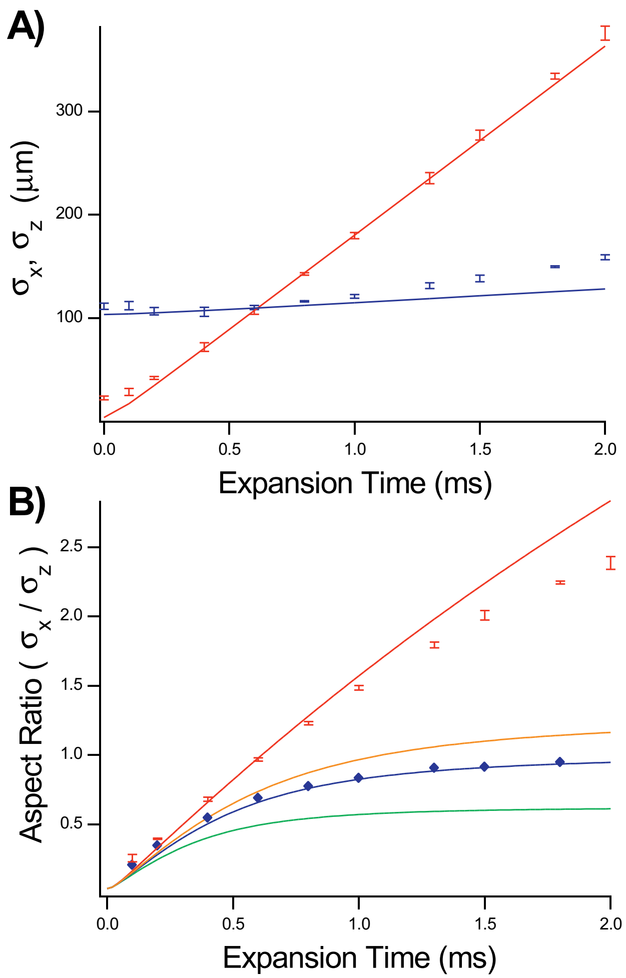
$$s = \frac{11\pi^2}{90}\frac{T^3}{v_s^3}$$

$$s = \frac{2\sqrt{2}}{3\pi^2}(mT_F)^{3/2}\log\left(\frac{T^{3/2}}{T_F^{3/2}}\right)$$

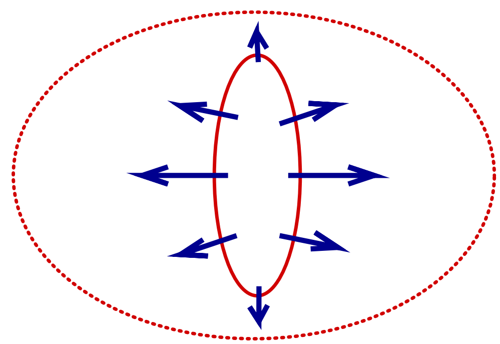
II. Transport Properties

Elliptic Flow



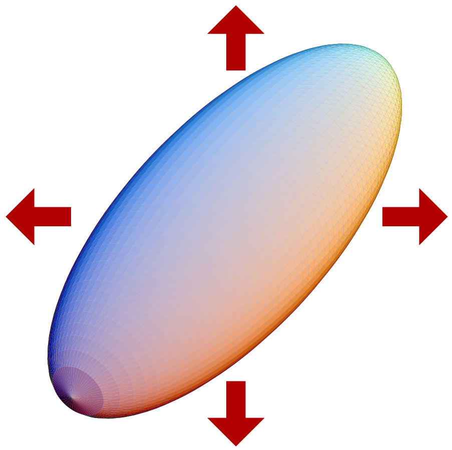


Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

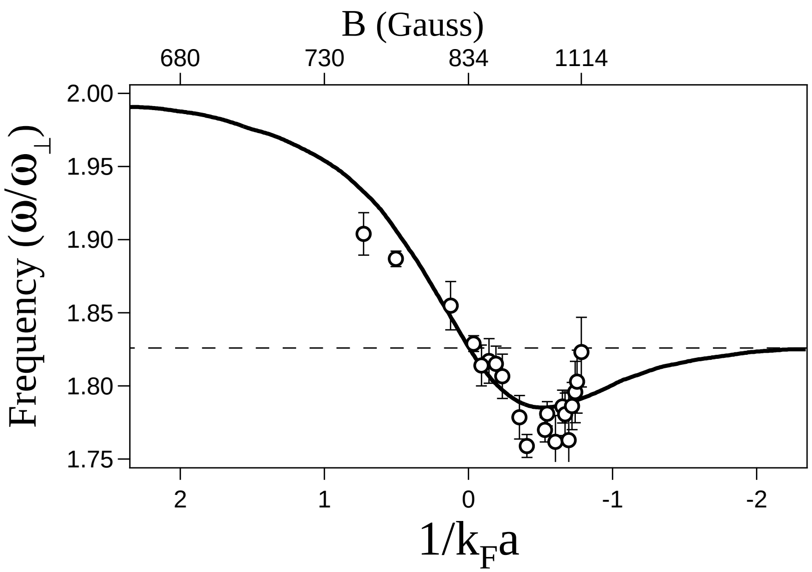


Collective Modes

Radial breathing mode



Kinast et al. (2005)

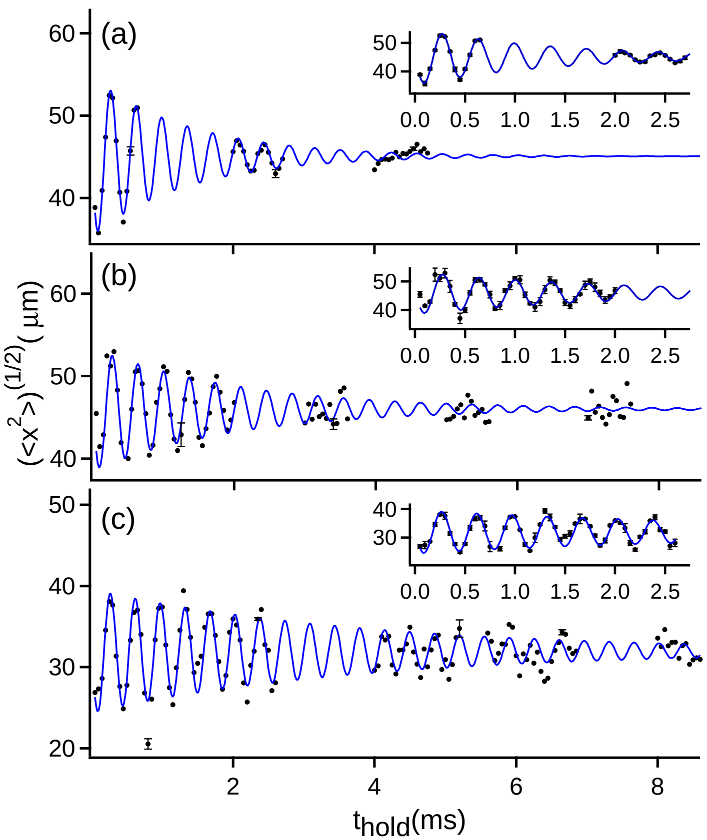


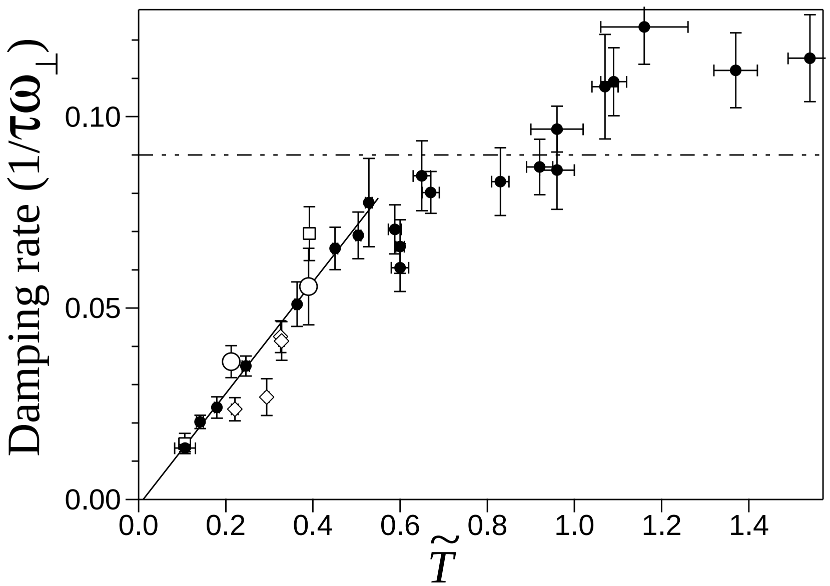
Ideal fluid hydrodynamics, equation of state $P \sim n^{5/3}$

$$\begin{aligned} \frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{v}) &= 0 \\ \frac{\partial \vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} &= -\frac{1}{mn}\vec{\nabla}P - \frac{1}{m}\vec{\nabla}V \end{aligned}$$

$$\omega = \sqrt{\frac{10}{3}}\omega_\perp$$

Damping of Collective Excitations





$T/T_F = (0.5, 0.33, 0.17)$

$\tau\omega$: decay time $\times$ trap frequency

Kinast et al. (2005)

Viscous Hydrodynamics

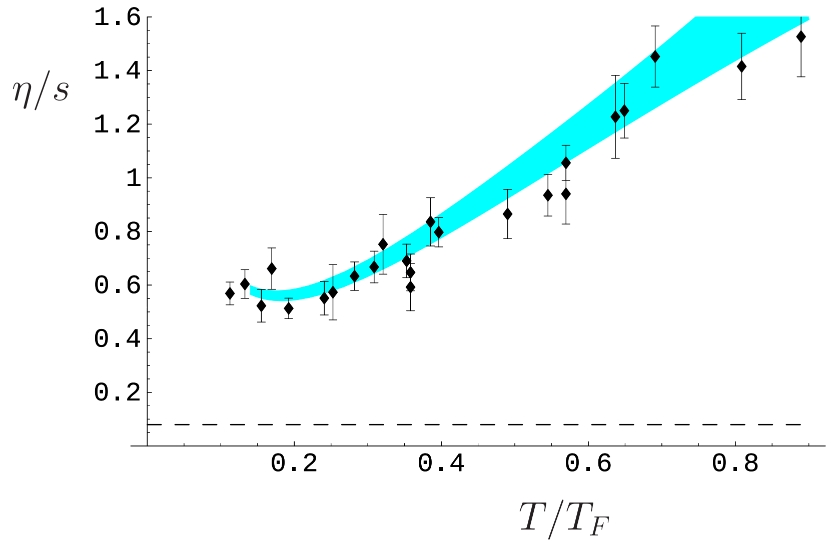
Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{\eta}{2}\int d^3x \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \zeta \int d^3x \, (\partial_i v_i)^2 - \frac{\kappa}{T}\int d^3x \, (\partial_i T)^2$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

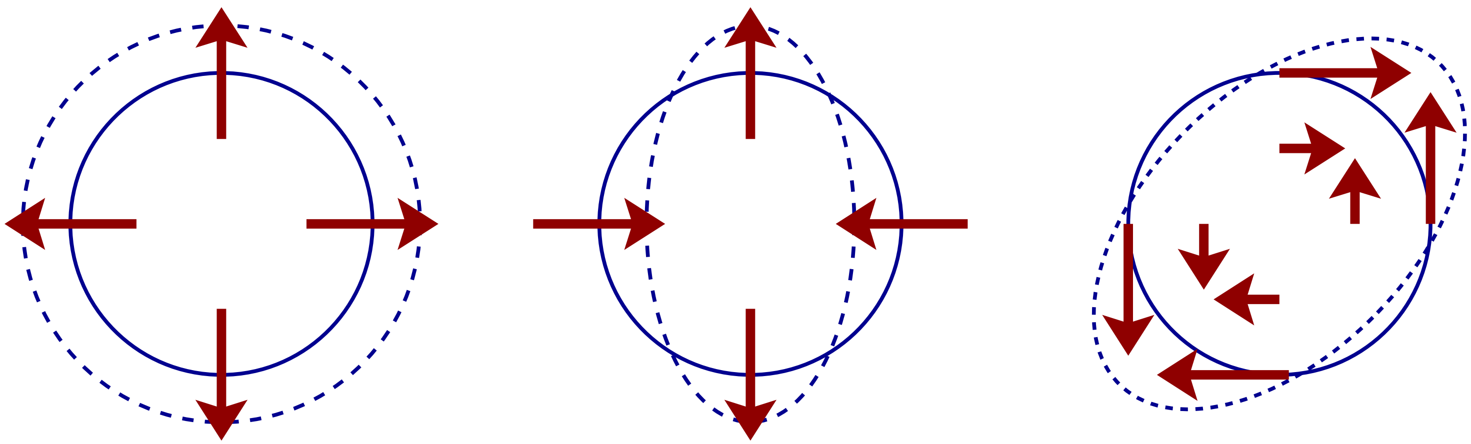
$$\frac{\eta}{s} = \frac{3}{4}\xi^{\frac{1}{2}}(3N)^{\frac{1}{3}}\frac{\Gamma}{\omega_\perp}\frac{\bar{\omega}}{\omega_\perp}\frac{N}{S}$$



see also Bruun, Smith, Gelman et al.

Damping dominated by shear viscosity?

Study dependence on flow pattern



Study particle number scaling

viscous hydro: $\Gamma \sim N^{-1/3}$ Boltzmann: $\Gamma \sim N^{1/3}$

Role of thermal conductivity?

suppressed for scaling flows: $\delta T \sim T(\delta n/n) \sim const \Rightarrow \nabla(\delta T) = 0$

Kinetic Theory: Transport Coefficients

Quasi-Particles: Kinetic Theory

$$T_{ij} = \int d^3p \, \frac{p_i p_j}{E_p} f_p,$$

Boltzmann equation

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = C[f_p]$$

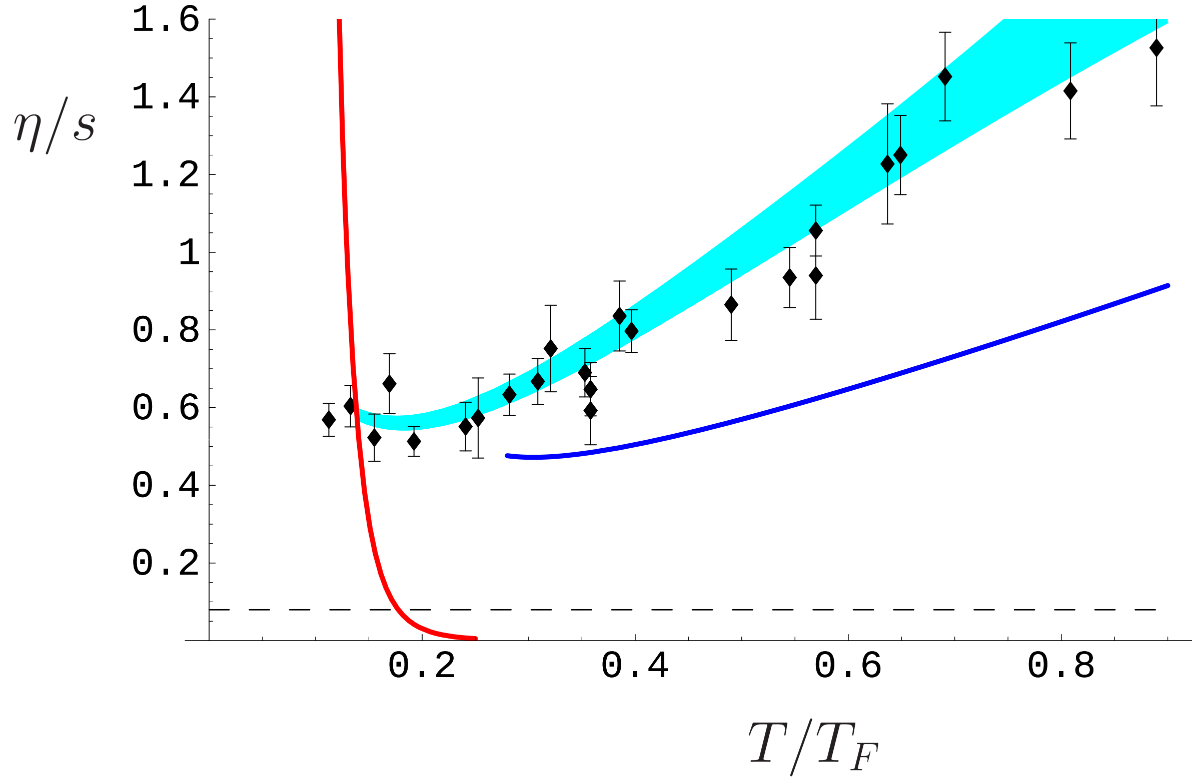
Linearized theory (Chapman-Enskog): $f_p = f_p^0 (1 + \chi_p / T)$

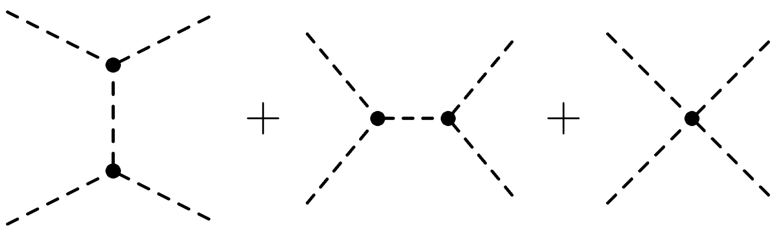
$$\eta \geq \frac{\langle \chi | X \rangle^2}{\langle \chi | C | \chi \rangle} \qquad \langle \chi | X \rangle = \int d^3p \, f_p^0 \, \chi_p \, p_{ij} v_{ij}$$

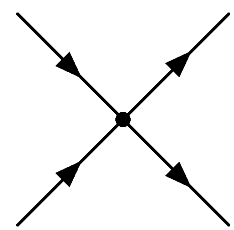
$$v_{ij} = v^2 \delta_{ij} - 3 v_i v_j$$

Low T: Phonons

High T: Atoms



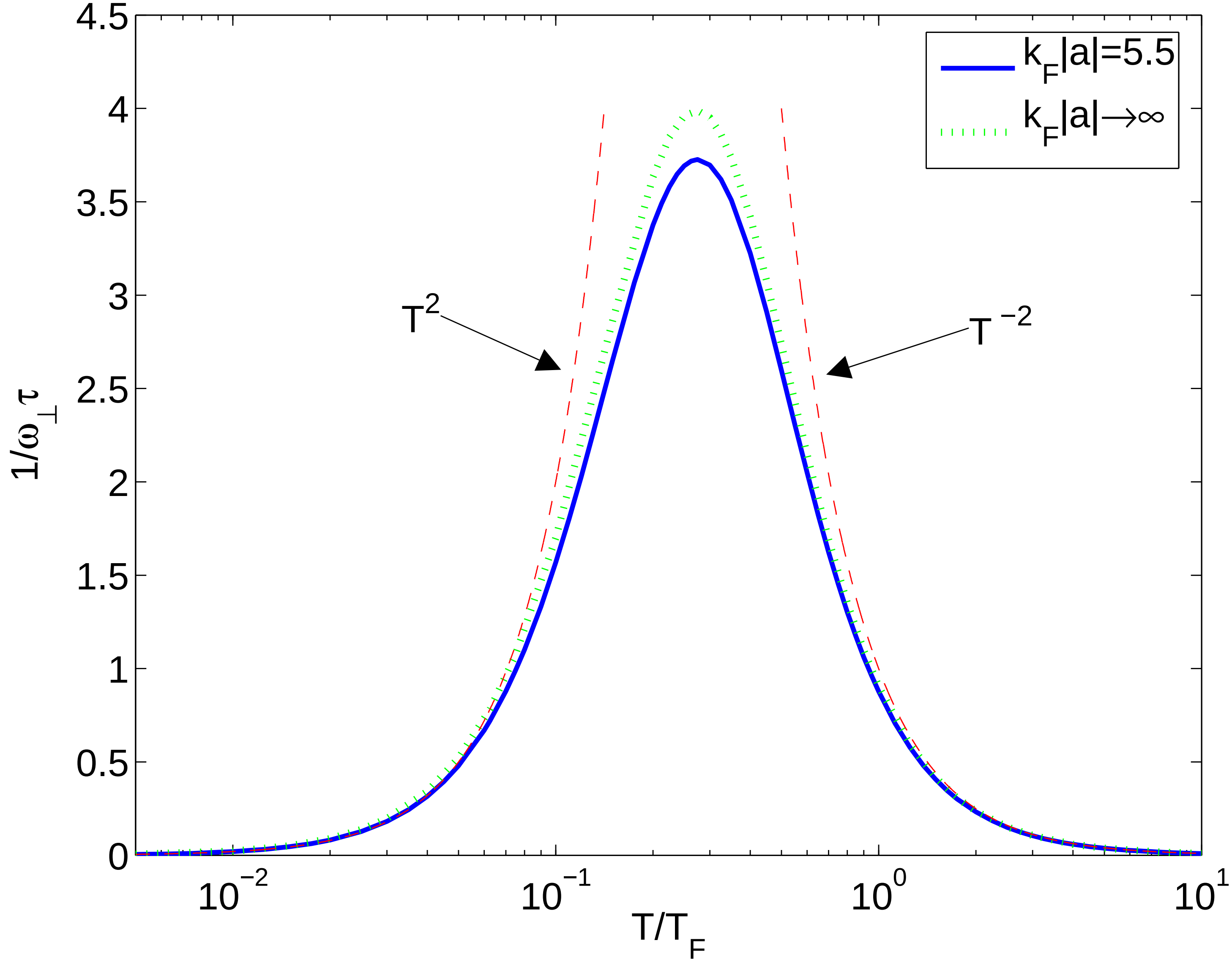




$$\frac{\eta}{s} \sim \left(\frac{T_F}{T}\right)^8$$

$$\frac{\eta}{s} \sim \left(\frac{T}{T_F}\right)^{3/2} \log\left(\frac{T}{T_F}\right)^{-1}$$

Linearized Boltzmann



Clear disagreement with data for $a \to \infty$

Elliptic Flow

Free scaling expansion

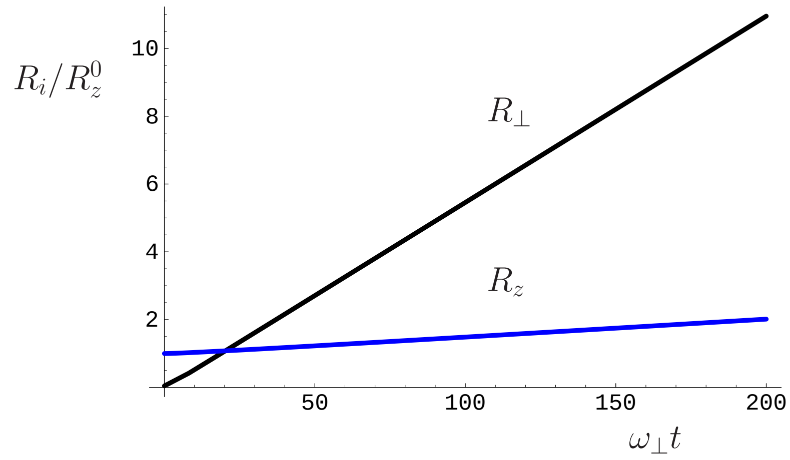
$$n(r_\perp, r_z) = \frac{1}{b_\perp^2 b_z} n_0 \left(\frac{r_\perp}{b_\perp}, \frac{r_z}{b_z} \right)$$

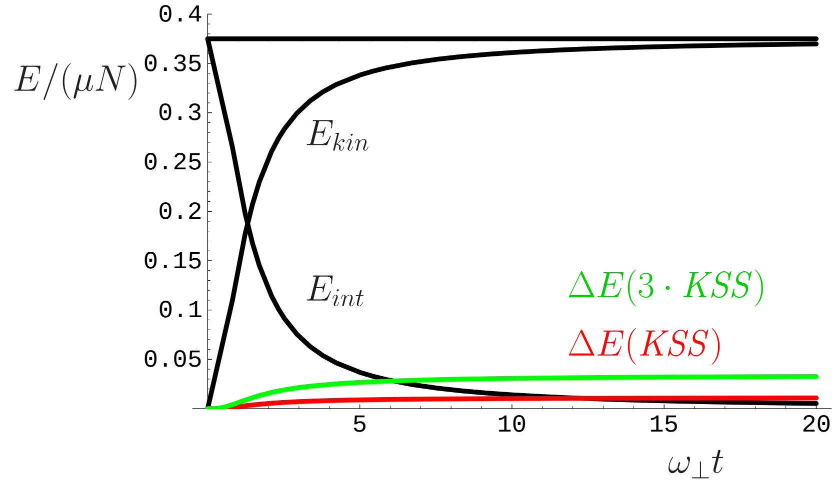
$$\ddot{b}_\perp = \frac{\omega_\perp^2}{b_\perp (b_\perp^2 b_z)^\gamma}$$

Viscous damping

$$\dot{E} = -\frac{4}{3} \left(\frac{\dot{b}_\perp}{b_\perp} - \frac{\dot{b}_z}{b_z} \right)^2 \int d^3x \, \eta(x)$$

$$\Delta E = \int dt \, \dot{E} \quad \text{converges quickly}$$

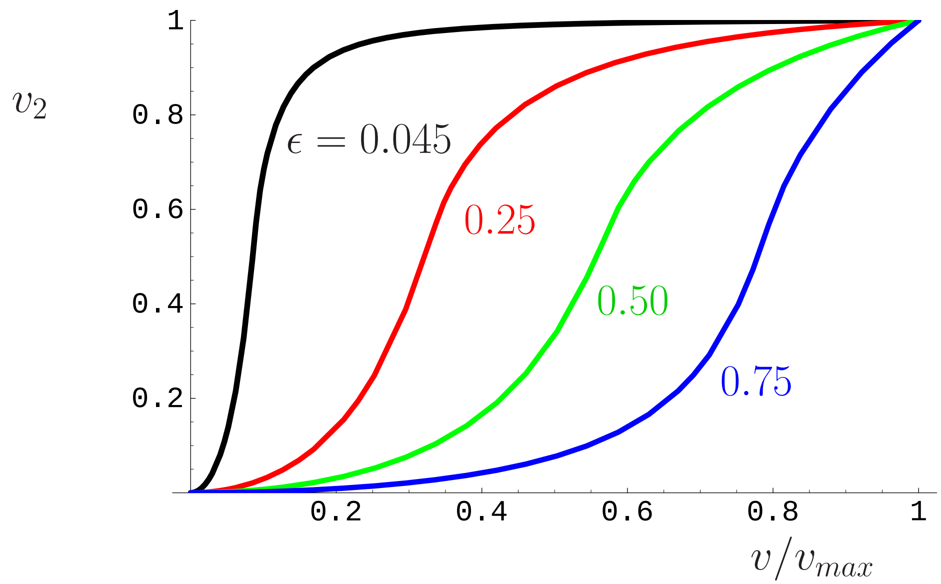




Elliptic Flow (cont)

Can define $v_2 = \langle \cos(2\phi) \rangle$ as in HI collisions

$$\epsilon = \frac{\langle 2z^2 - x^2 + y^2 \rangle}{\langle z^2 + x^2 + y^2 \rangle}$$



Can also sweep to BEC regime and simulate recombination models

Summary and Outlook

(Resummed) perturbative approaches, extrapolated to $T \sim T_F$, account for thermodynamics and transport in cold atomic gases.

Reliable methods for $T \sim T_F$?

Other experimental constraints: Observation of "irrotational flow"?

Other uses of conformal symmetry? OPE? Braaten, Platter (2008)

AdS/Cold Atom correspondence? Son (2008), Balasubramanian & McGreevy (2008)