Formulas and Numerical Constants

De Broglie relations: De Broglie postulated the following relations between (E, p) and (λ, f)

$$E = hf, \qquad (E = \hbar\omega) \tag{1}$$
$$p = h/\lambda, \qquad (p = \hbar k) \tag{2}$$

where $\hbar = h/(2\pi)$. The most important dispersion relations are

$$E = pc \qquad \text{(light)}, \tag{3}$$
$$E = \frac{p^2}{2m} \qquad \text{(nonrelativistic matter)}. \tag{4}$$

Bohr's model: Bohr's model of hydrogen like atoms is based on the quantization condition $L = mvr = n\hbar$. The allowed energies and radii are

$$r_n = \frac{n^2 a_0}{Z}, \qquad a_0 = \frac{\hbar^2}{m_e k e^2}, \tag{5}$$
$$E_n = -\frac{Z^2 E_0}{n^2}, \qquad E_0 = \frac{m_e k^2 e^4}{2\hbar^2}, \tag{6}$$

where $k = 1/(4\pi\epsilon_0)$ is the Coulomb constant, e is the charge, and m_e is the mass of the electron. Z is the charge of the nucleus (in units of e). The constant a_0 is called the Bohr radius. The quantity

$$\alpha = \frac{ke^2}{\hbar c} \simeq \frac{1}{137} \tag{7}$$

is called the fine structure constant.

Schrödinger equation: The time-dependent and time-independent Schrödinger equations are

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right\}\psi(x,t), \tag{8}$$
$$E\psi(x) = \left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right\}\psi(x). \tag{9}$$

The wave function is related to the probability

$$P(x,t)\,dx = \psi^*(x,t)\psi(x,t)\,dx. \tag{10}$$

More generally, expectation values are given by

$$\langle f\rangle = \int dx\, f(x)\psi^*(x,t)\psi(x,t). \tag{11}$$

3d Schrödinger equation: Solutions of the Schrödinger equation for a potential with rotational symmetry have the form

$$\psi(r,\theta,\phi) = R_{nl}(r)Y_{lm}(\theta,\phi), \tag{12}$$

where Y_{lm} are the spherical harmonics, (l,m) label $L^2 = \hbar^2 l(l+1)$ and $L_z = \hbar m$ $(m \leq l)$, and $R_{nl}(r)$ is the radial wave function (labeled by the quantum number n). The ground state wave function of the hydrogen atom is

$$R_{10}(r) = \frac{2}{\sqrt{a_0^3}}\,e^{-r/a_0}, \qquad Y_{00}(\theta,\phi) = \frac{1}{\sqrt{4\pi}}, \tag{13}$$

where a_0 is the Bohr radius defined above.

Selection rules: Dipole transitions involving the emission or absorption of a photon are allowed if

$$\Delta m_l = \pm 1, 0 \quad \text{and} \quad \Delta l = \pm 1. \tag{14}$$

Vibrational and rotational energies: The energy levels of a one-dimensional harmonic oscillator are

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right), \tag{15}$$

where $\omega = \sqrt{k/m}$ and k is the spring constant. The energy levels of a rigid rotor are

$$E_l = \frac{\hbar^2}{2I}\,l(l+1), \tag{16}$$

where I is the moment of inertia.

Fermi gas: The Fermi energy is

$$E_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3} , \tag{17}$$

The average energy is $E_{av} = (3/5)E_F$, the Fermi temperature is $T_F = E_F/k_B$, and the Fermi velocity is $v_F = \sqrt{2E_F/m}$. The Fermi-Dirac distribution is

$$f(E) = \frac{1}{\exp((E - E_F)/(k_B T)) + 1} . \tag{18}$$

Conductivity: The resistivity is

$$\rho = \frac{m_e v_{av}}{e^2 n_e \lambda} , \tag{19}$$

where n_e is the density of electrons and v_{av} is the average velocity. The mean free path is $\lambda = 1/(n_I \sigma)$, where n_I is the density of scatterers and σ is the cross section.

Numerical Constants:

$$\begin{aligned}
k &= 1/(4\pi\epsilon_0) \ = \ 8.987 \cdot 10^9 \,\mathrm{N \cdot m^2 \cdot C^{-2}} \\
k_B &= 1.381 \times 10^{-23}\,\mathrm{J/K} = 8.617 \times 10^{-5}\,\mathrm{eV/K} \\
N_A &= 6.022 \times 10^{23} \\
h &= 6.626 \times 10^{-34}\,\mathrm{J \cdot s} \\
c &= 2.998 \times 10^8\,\mathrm{m/sec} \\
hc &= 1240\,\mathrm{eV \cdot nm} \\
\hbar c &= 197.33\,\mathrm{MeV \cdot fm} \\
E_0 &= 0.5\, m_e c^2 \alpha^2 = 13.6\,\mathrm{eV} \\
e &= 1.602 \times 10^{-19}\,\mathrm{C} \\
1\,\mathrm{cal} &= 4.186\,\mathrm{J} \\
1\,\mathrm{eV} &= 1.602 \times 10^{-19}\,\mathrm{J} \\
1\,u &= 1.661 \times 10^{-27}\,\mathrm{kg} = 931.49\,\mathrm{MeV}/c^2 \\
m_e c^2 &= 512\,\mathrm{keV} \\
m_e &= 9.109 \times 10^{-31}\,\mathrm{kg} \\
m_p c^2 &= 935\,\mathrm{MeV}
\end{aligned} \tag{20}$$