Homework 6, due 10-13

In class we introduced product wave functions

$$\uparrow\uparrow = \chi_\uparrow(1)\chi_\uparrow(2) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Spin operators act on product spin wave functions as follows

$$\sigma_{x,1}\sigma_{y,2}\chi_\uparrow(1)\chi_\uparrow(2) = [\sigma_x\chi_\uparrow(1)]\,[\sigma_y\chi_\uparrow(2)]\,.$$

Expectation values are defined as

$$(\chi_\uparrow(1)\chi_\uparrow(2))^\dagger\sigma_{x,1}\sigma_{y,2}(\chi_\uparrow(1)\chi_\uparrow(2)) = \left[\chi_\uparrow^\dagger(1)\sigma_x\chi_\uparrow(1)\right]\left[\chi_\uparrow^\dagger(2)\sigma_y\chi_\uparrow(2)\right].$$

In class we argued that $\chi_{A,S}$

$$\chi_{A,S} = \frac{1}{\sqrt{2}}\left(\uparrow\downarrow \mp \downarrow\uparrow\right)$$

have spin zero and one, respectively. Check this statement explicitely by computing

$$\vec{S}^2\chi_{A,S}$$

where

$$\vec{S} = \vec{S}_1 + \vec{S}_2 = \frac{\hbar}{2}\left(\vec{\sigma}_1 + \vec{\sigma}_2\right).$$

Use you result to compute the expectation value of $\vec{S}_1 \cdot \vec{S}_2$ in the spin zero and one states,

$$\chi_A^\dagger(\vec{S}_1 \cdot \vec{S}_2)\chi_A = ?$$

$$\chi_S^\dagger(\vec{S}_1 \cdot \vec{S}_2)\chi_S = ?$$