XQCD: Elliptic flow and heavy quarks

XSYM: Heavy quarks in AdS/CFT

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- Heavy quarks: Jorge Casalderrey-Solana, DT; hep-th/0701123
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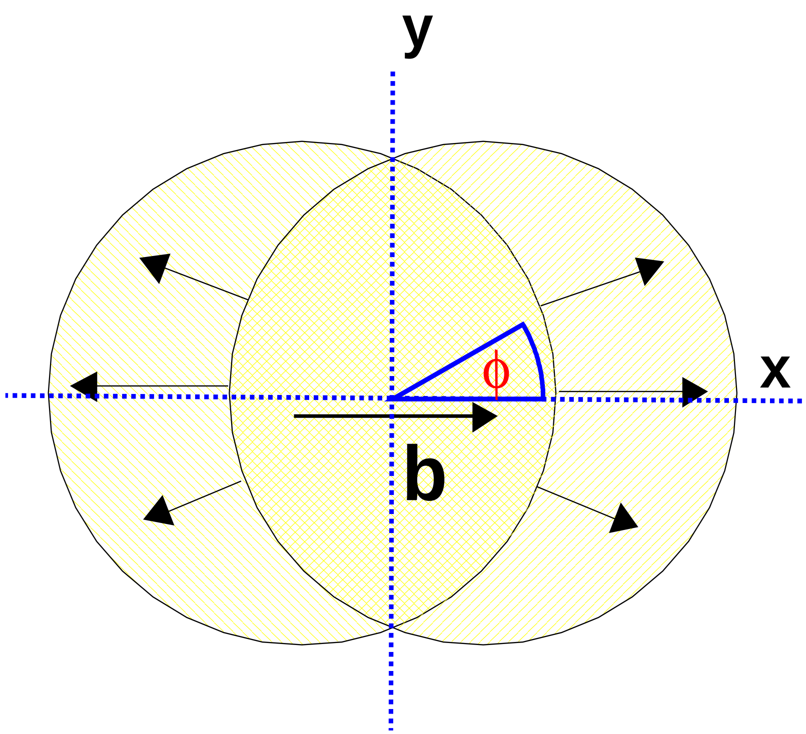
- Heavy quarks: Jorge Casalderrey-Solana, D. T. Son; In progress

Observation:



There is a large momentum anisotropy:

$$v_2 \equiv \frac{\left\langle p_x^2 - p_y^2 \right\rangle}{\left\langle p_x^2 + p_y^2 \right\rangle} \approx 20\%$$

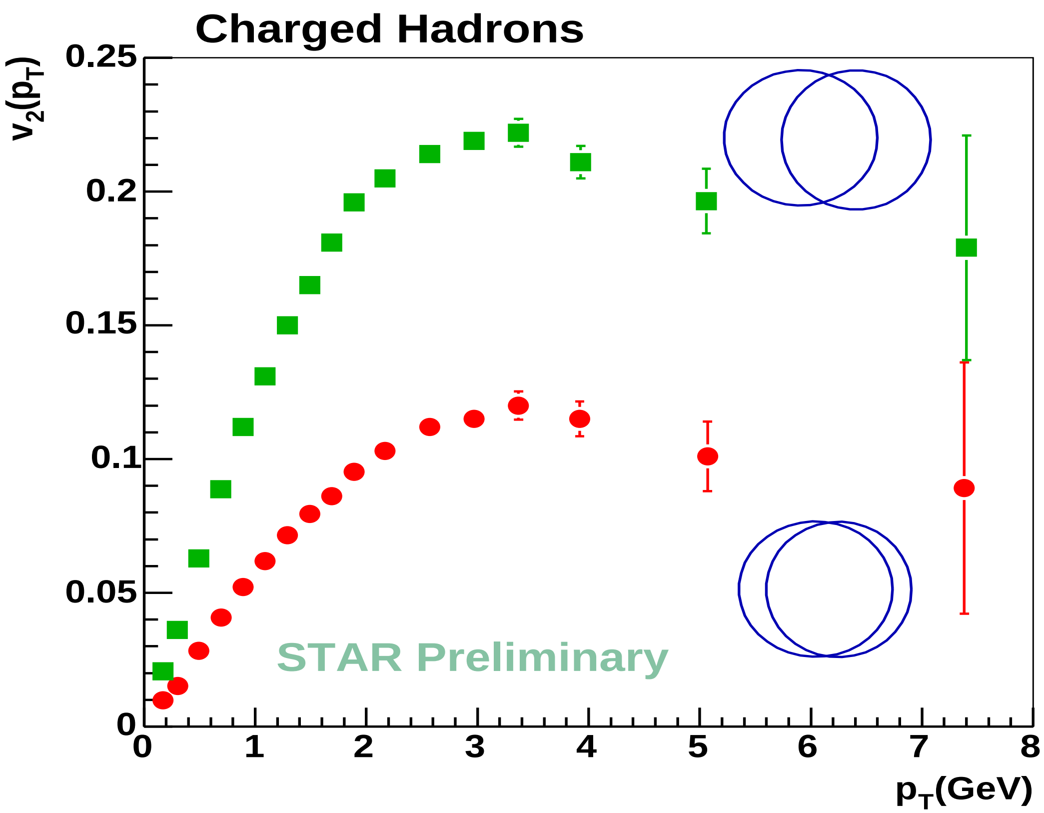
Interpretation

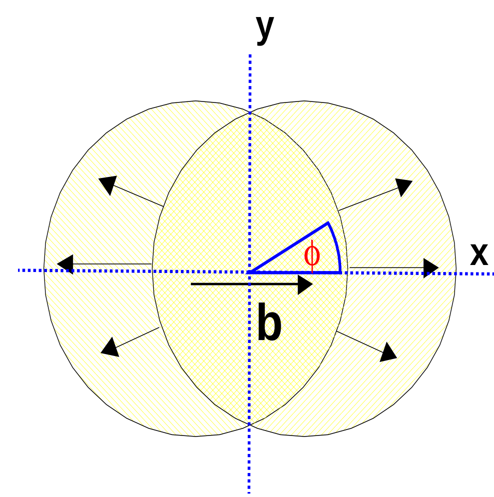
- The medium responds as a fluid to differences in X and Y pressure gradients

Hydro models “work”

Data on Elliptic Flow:

$$\frac{1}{p_T}\frac{dN}{dp_T\,d\phi} = \frac{1}{p_T}\frac{dN}{dp_T}\left(1 + 2\,v_2(p_T)\cos(2\,\phi) + ...\right)$$

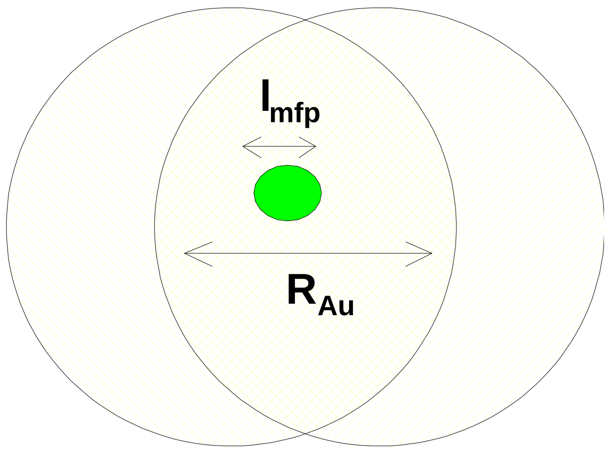




$$\text{X:Y} = (1 + \underbrace{2v_2}_{\sim 0.4} : 1 - \underbrace{2v_2}_{\sim 0.4})$$

Elliptic flow is large X:Y $\sim 2.0 : 1$

Hydrodynamics:



- For hydrodynamics need:

$$\frac{\ell_{\rm mfp}}{R_{\rm Au}} \ll 1$$

- How to define $\ell_{\rm mfp}$?

$$\ell_{\rm mfp} \sim \frac{\eta}{e+p} \qquad e+p = sT$$

Condition:

$$\underbrace{\frac{\eta}{s}}_{\text{Medium Property } \sim 1/\alpha_s^2} \times \underbrace{\frac{1}{R_{Au}T}}_{\text{Experimental Property } \sim 1/2} \ll 1$$

Need η/s small.

When is Hydrodynamics Valid?

- Go out of equilibrium when expansion rate is too fast

$$\tau_R \underbrace{\partial_\mu u^\mu}_{\frac{1}{V}\frac{dV}{dt}} \sim \frac{1}{2}$$

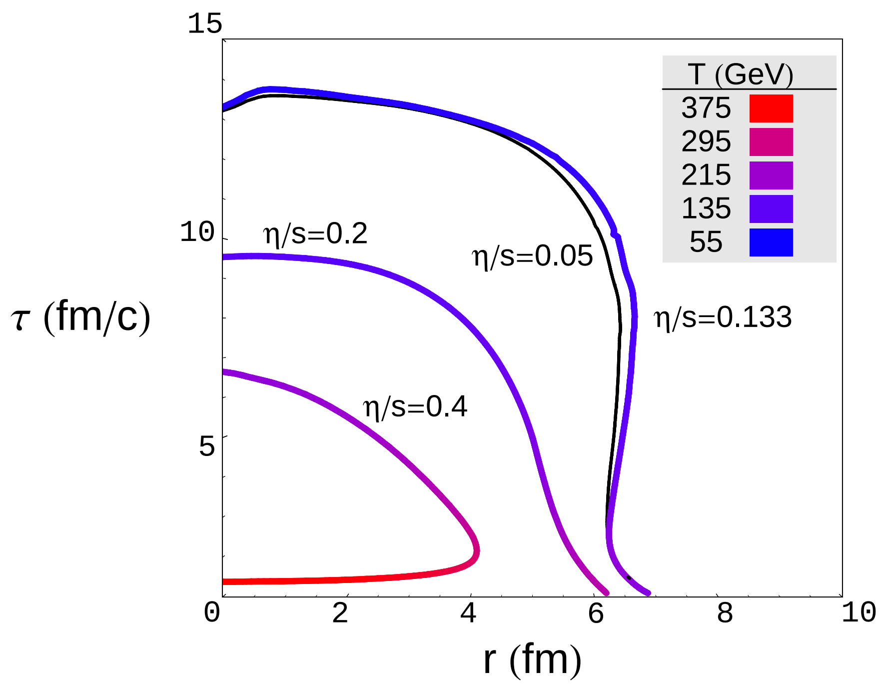
- The viscosity is related to the relaxation time

$$\frac{\eta}{e} \sim v_{\rm th}^2 \tau_R \qquad\qquad p \sim e\, v_{\rm th}^2$$

- So the freezeout cirterion is

$$\frac{\eta}{p}\,\partial_\mu u^\mu \sim \frac{1}{2}$$

Hydrodynamic Simulations of Central Heavy Ion Collisions



Need η/s small to describe a large fraction of collision

Solving Navier Stokes

- The Navier Stokes equations

$$\partial_\mu T^{\mu\nu} = 0 \qquad\qquad T^{ij} = \underbrace{p\delta^{ij}}_{\text{equilibrium}} + \underbrace{\pi^{ij}}_{\text{correction}}$$

- The "first order" stress tensor instantly assumes a definite form.

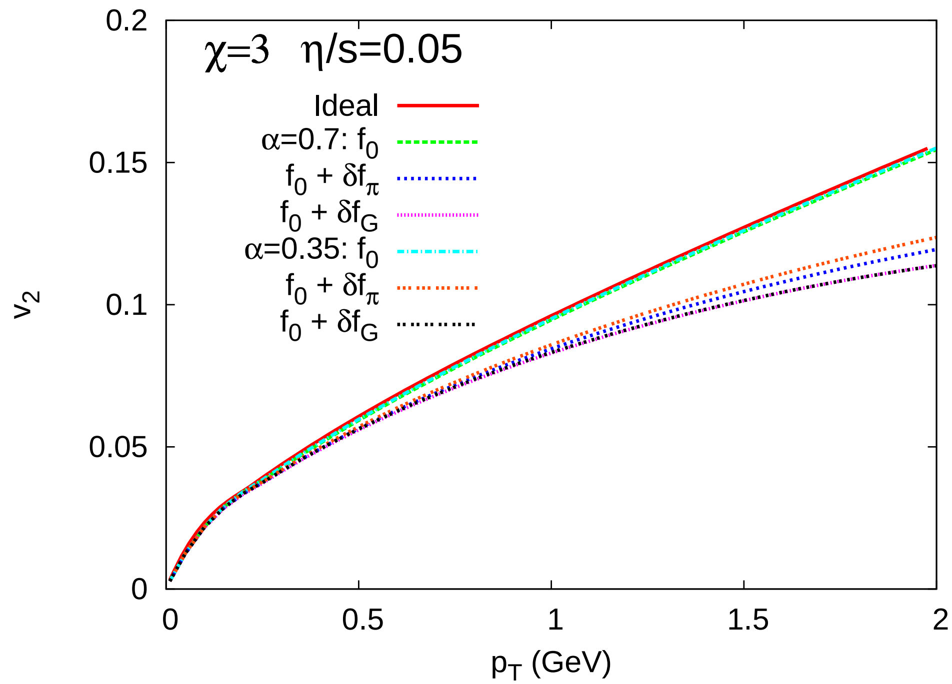
$$\begin{aligned} \pi^{ij} &= -\eta\left(\partial^i v^j + \partial^j v^i - \frac{2}{3}\delta^{ij}\partial\cdot v\right) \\ O(\epsilon) &= O(\epsilon) \end{aligned}$$

- Can make "second order" models which relax to the correct form

$$\begin{aligned} -\tau_R\,\partial_t\pi^{ij} &= \pi^{ij} + \eta\left(\partial^i v^j + \partial^j v^i - \frac{2}{3}\delta^{ij}\partial\cdot v\right) \\ O(\epsilon^2) &= O(\epsilon) + O(\epsilon) \end{aligned}$$

Can solve these models

Independent of second derivative terms



Gradient expansion is working. Temperature is a good concept.

Worse at larger viscosities and larger p_T

Running Viscous Hydro

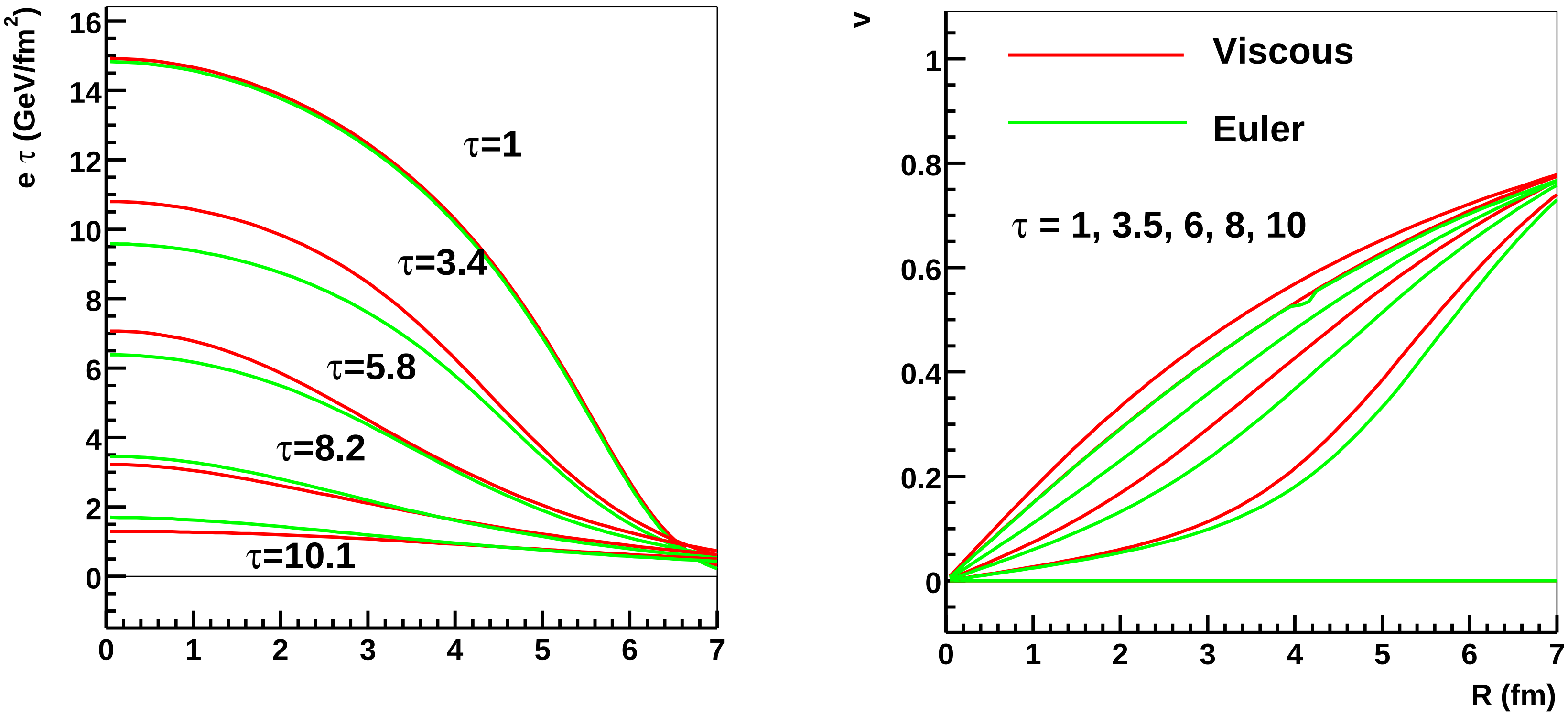
- Run the evolution and monitor the viscous terms
- When the viscous term is about half of the pressure:
  - T^{ij} is not asymptotic with $\eta \langle \partial^i v^j \rangle$

Freezeout is signaled by the equations.

- Kinetic theory distribution Functions modified
  - With viscosity $T^{\mu\nu} \longrightarrow T_0^{\mu} + \delta T^{\mu\nu}$ so $f \longrightarrow f_0 + \delta f$.

Maximum p_T is also signaled by the equations.

Bjorken Solution with transverse expansion: $(\eta/s = 0.2)$



Viscous corrections do NOT integrate to give an O(1) change to the flow.

Viscous corrections to the distribution function $f_o \to f_o + \delta f$

- Corrections to thermal distribution function $f_0 \to f_0 + \delta f$
  - Must be proportional to strains
  - Must be a scalar
  - General form in rest frame and ansatz

$$\delta f = F(|\mathbf{p}|)p^i p^j \pi_{ij} \Longrightarrow \delta f \propto f_0\, p^i p^j \pi_{ij}$$

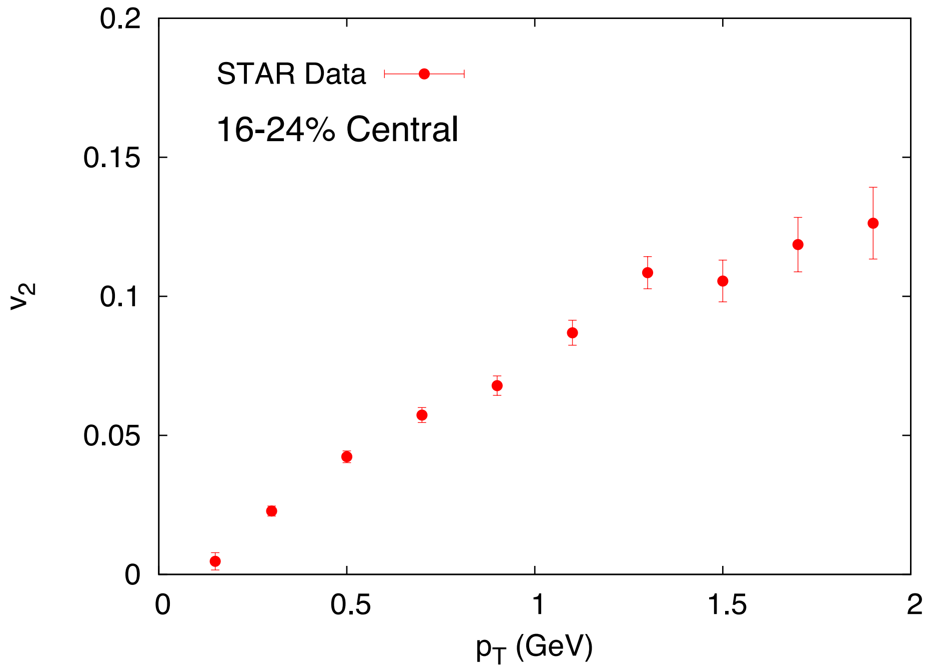
  - Can fix the constant

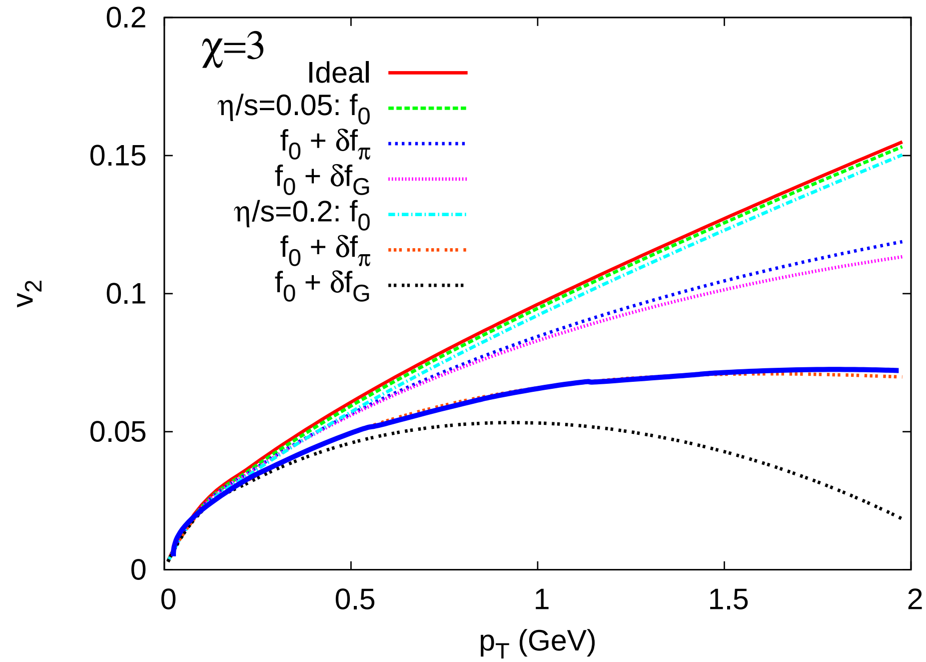
$$p\delta^{ij} + \pi^{ij} = \int \frac{d^3p}{(2\pi)^3} \frac{p^i p^j}{E_{\mathbf{p}}} (f_0 + \delta f)$$

find

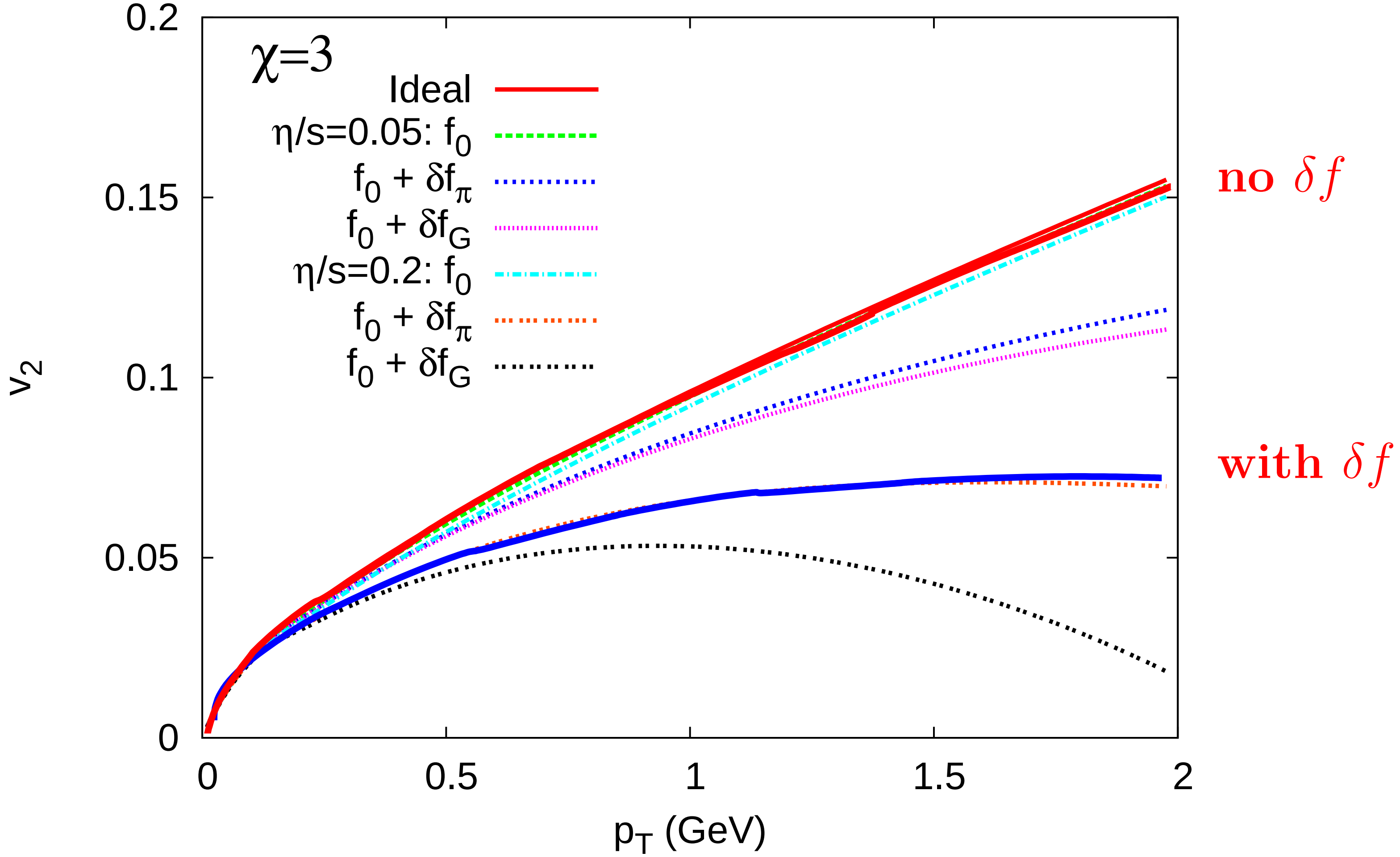
$$\delta f = \frac{1}{2(e+p)T^2} f_o\, p^i p^j \pi_{ij}$$

Viscous Hydro Results:

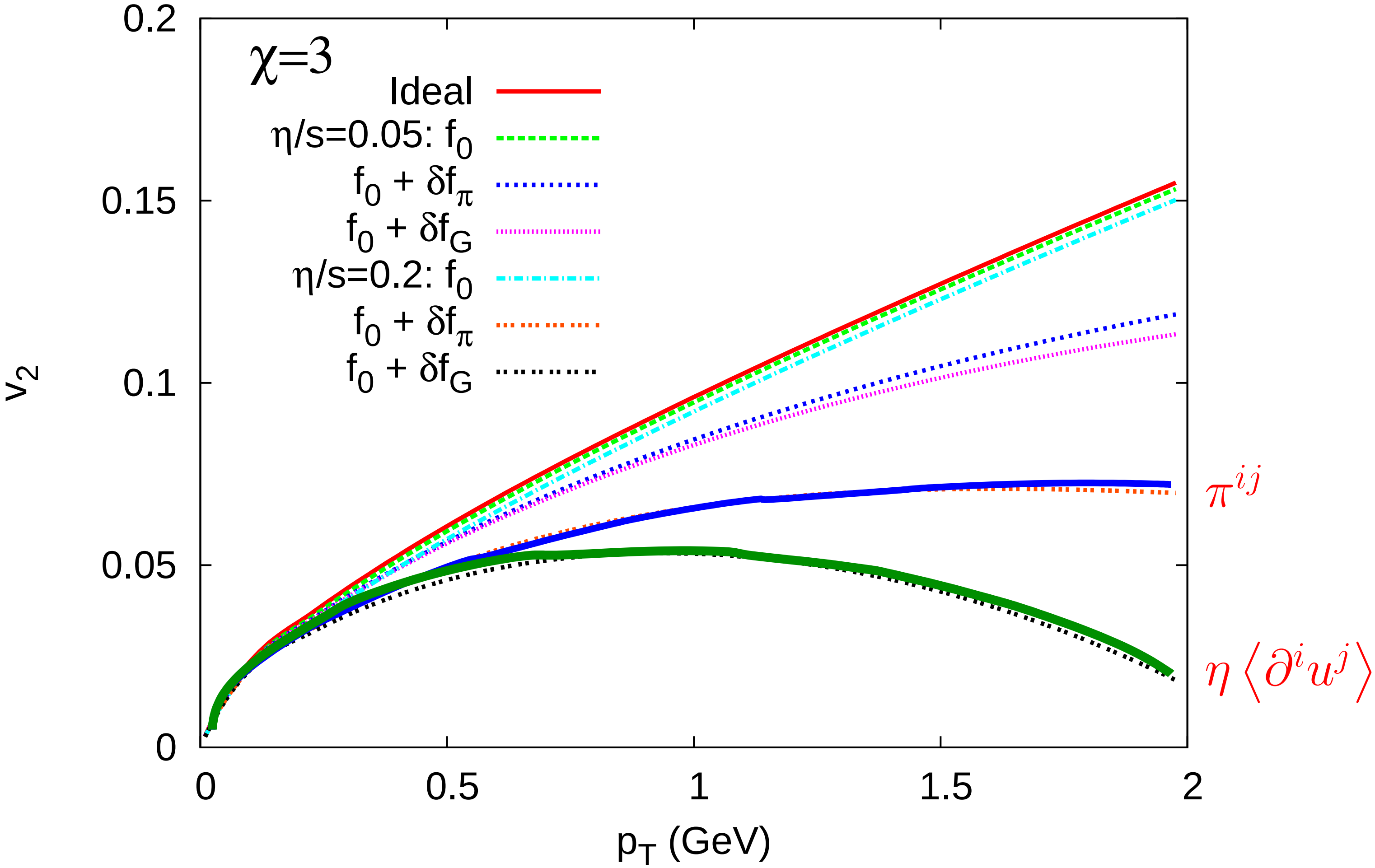




Effect of modifying the distribution function $\eta/s = 0.2$

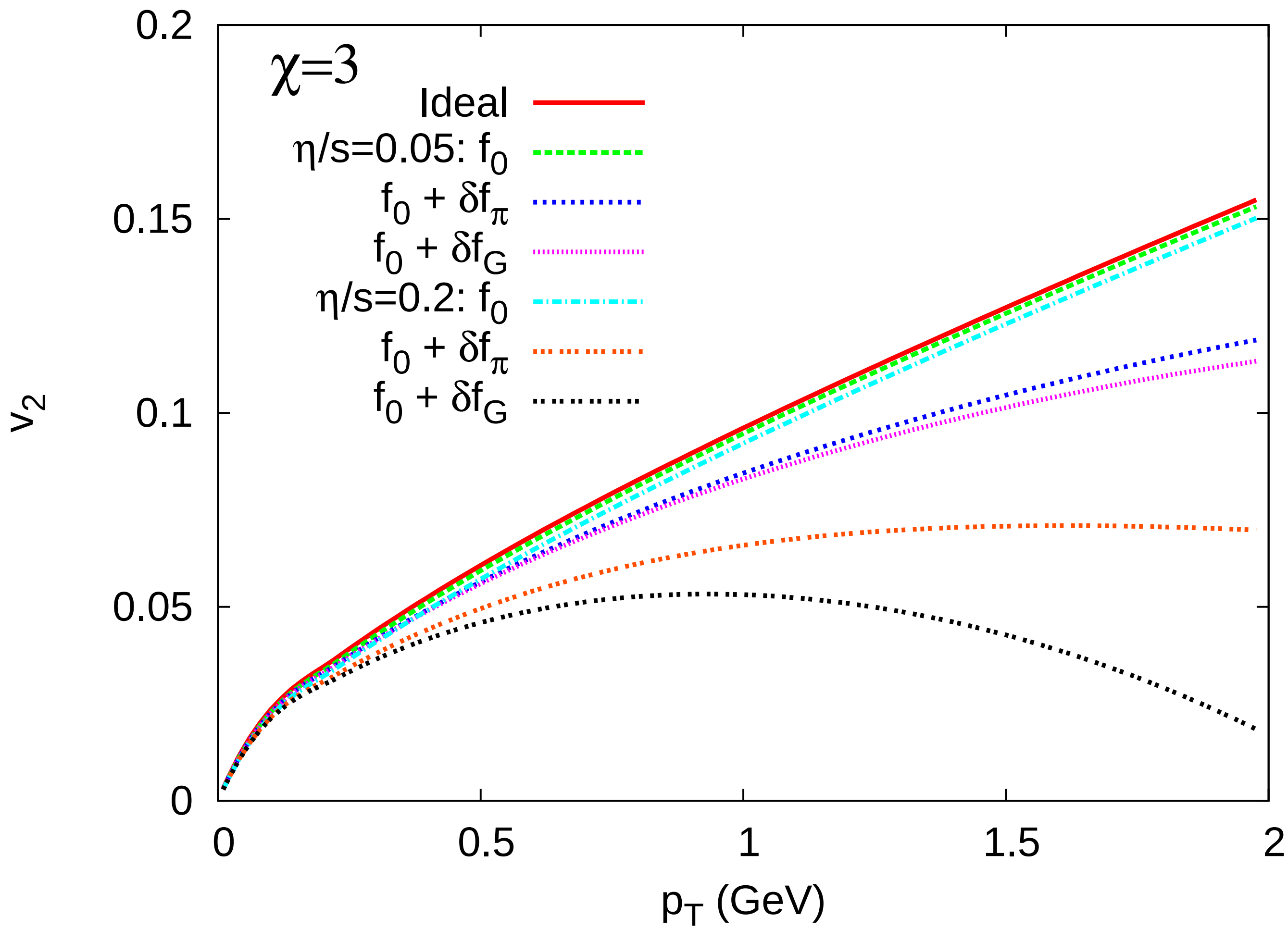


Estimate the uncertainty between first order and "some" second order



$$\pi^{ij} \simeq \eta \left\langle \partial^i v^j \right\rangle + O(\epsilon^2)$$

Compare to $\eta/s = 0.05$

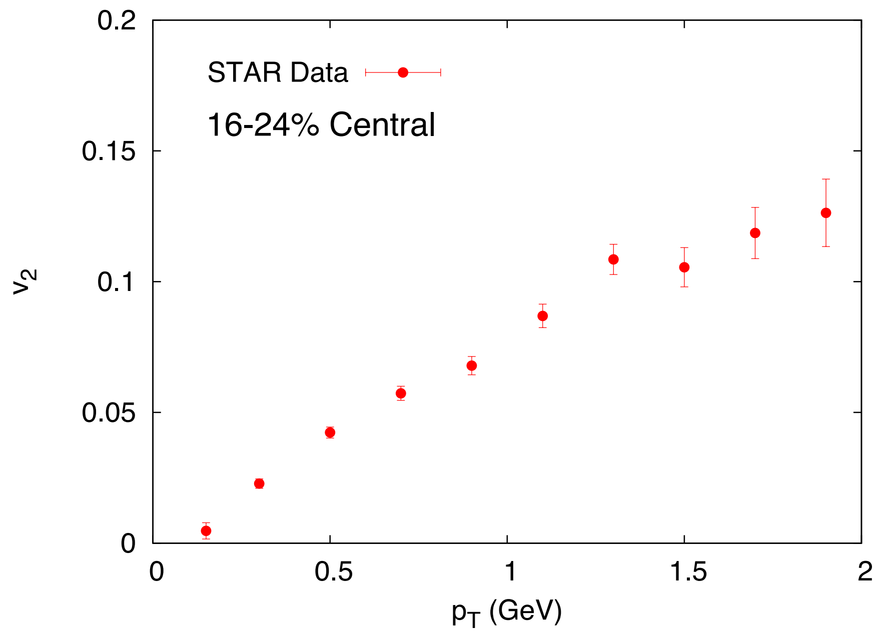


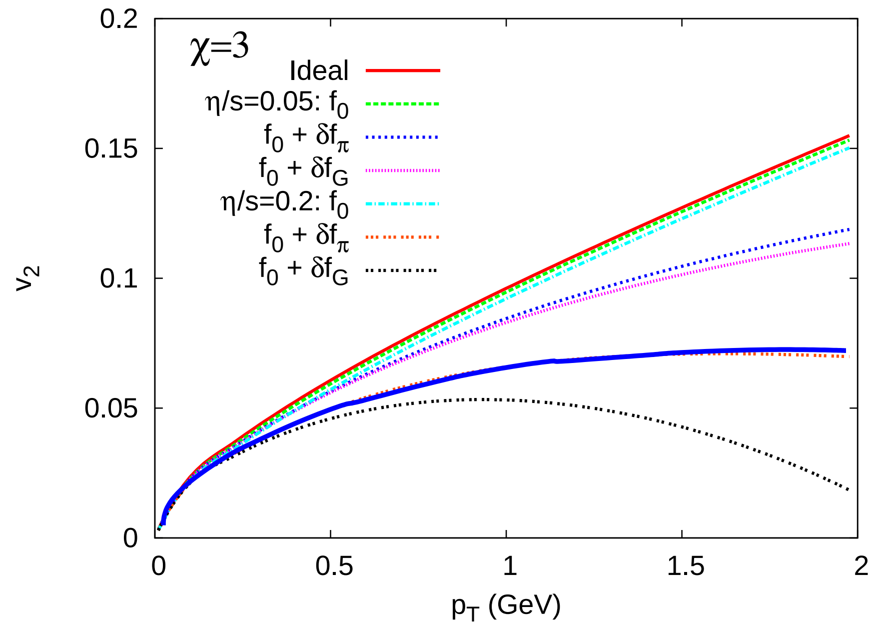
π^{ij}

$\eta \langle \partial^i u^j \rangle$

$$\pi^{ij} \simeq \eta \left\langle \partial^i v^j \right\rangle + O(\epsilon^2)$$

Viscous Hydro Results:





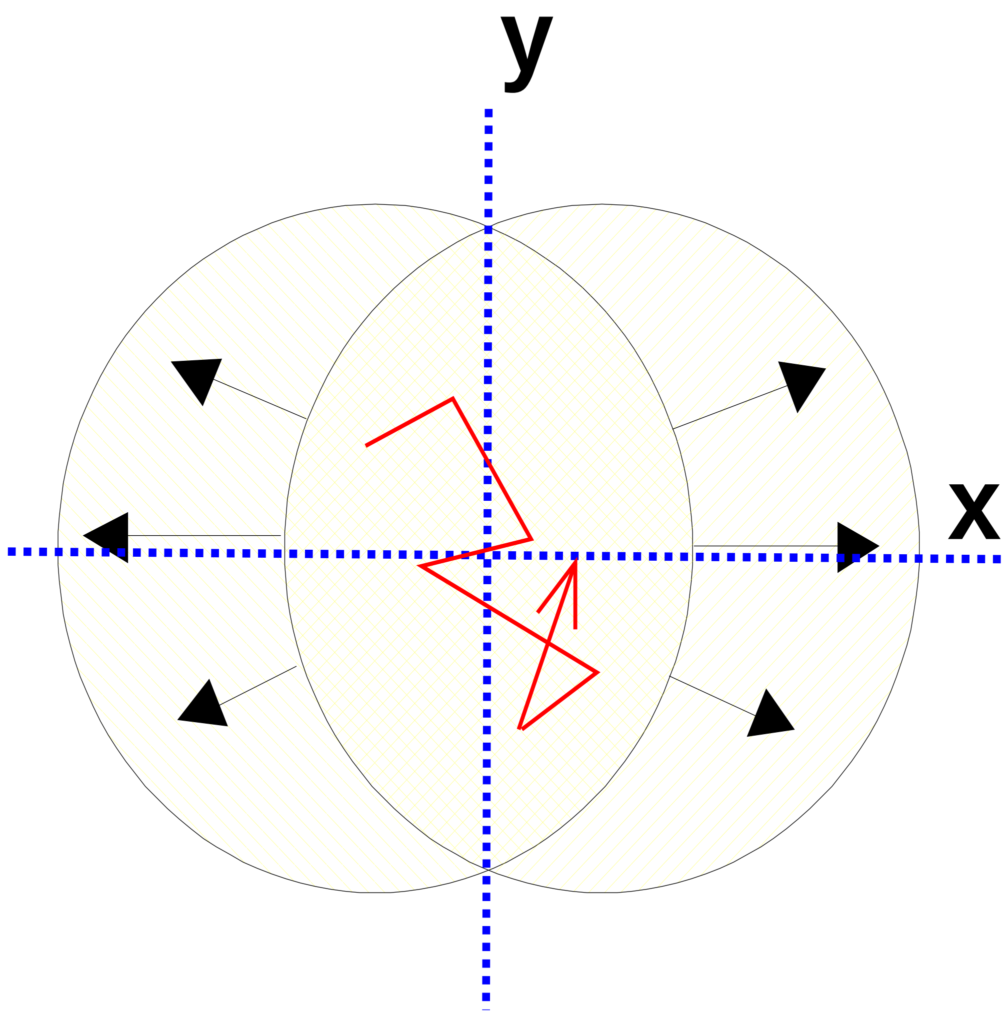
- To get anywhere close to the data need:

$$\eta/s \sim \frac{1}{4\pi}$$

- The hydrodynamic results are under relatively good control when

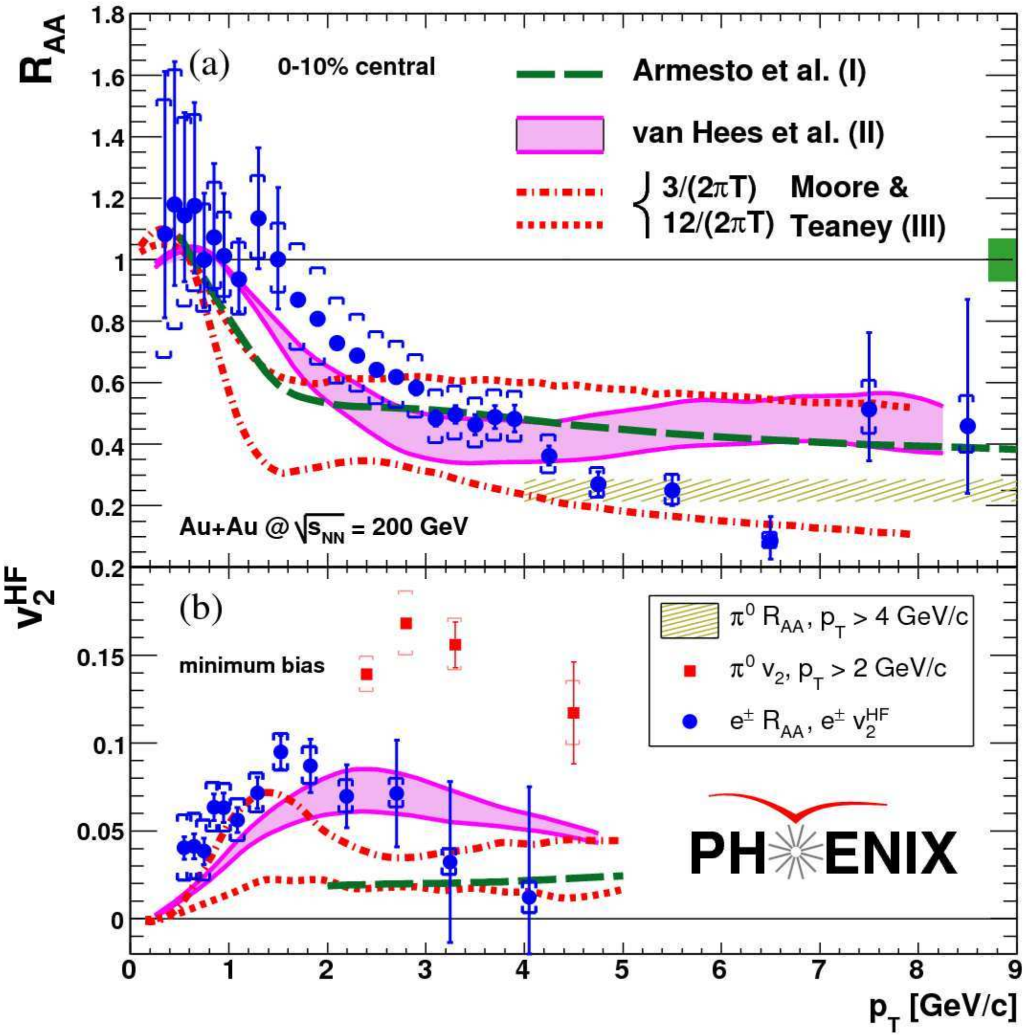
$$\eta/s \sim \frac{1}{4\pi}$$

Heavy Quarks at RHIC and AdS/CFT



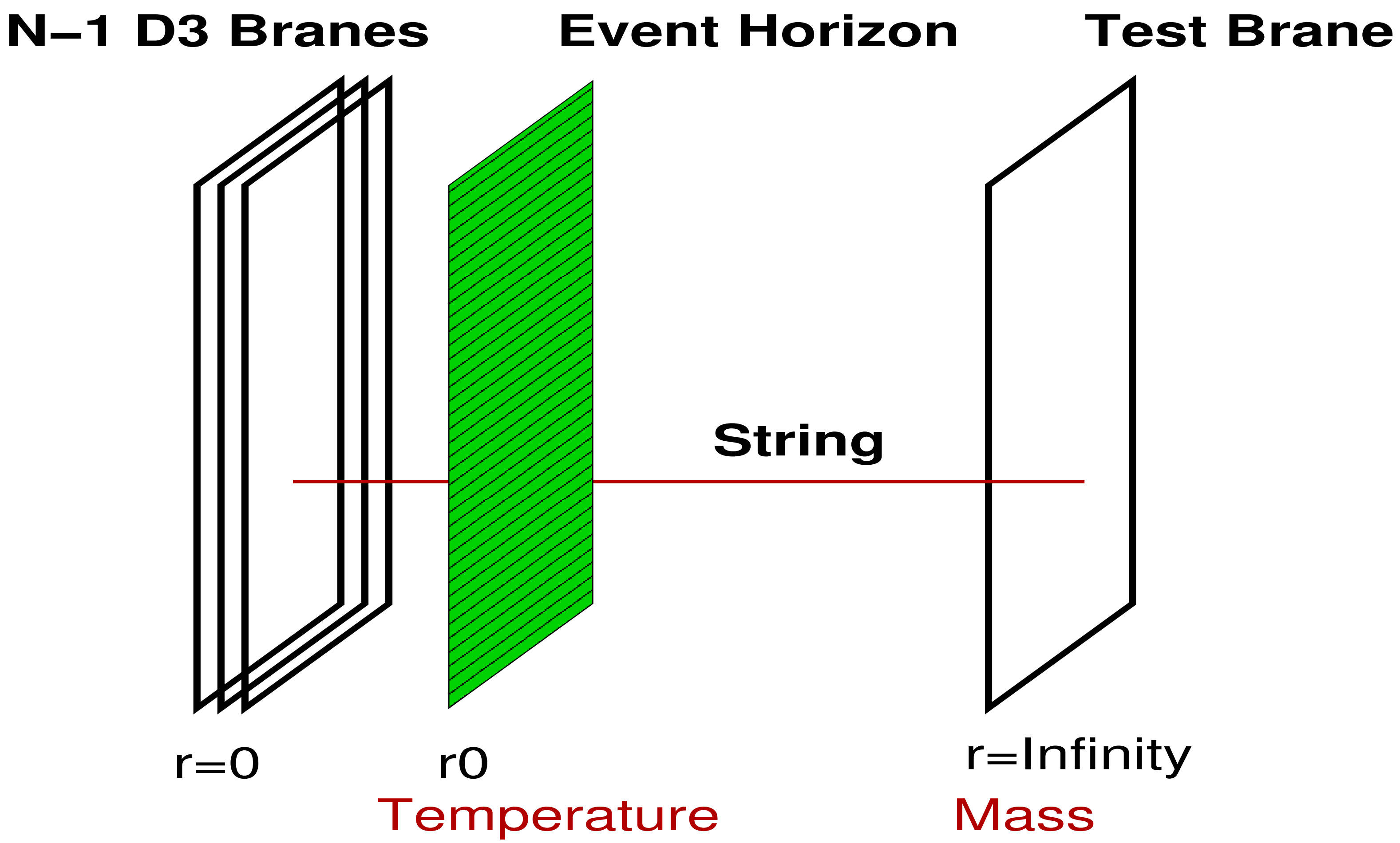
View heavy quark energy loss as Brownian Motion

Experimental Motivation:



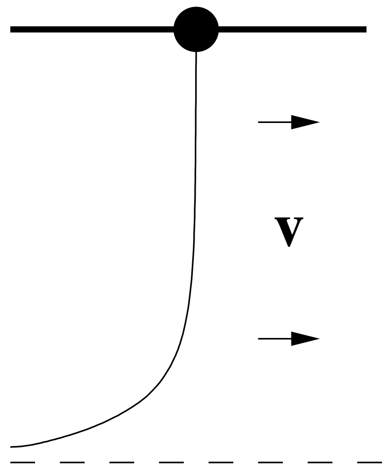
$$D \lesssim 6/(2\pi T)$$

Theoretical Motivation:



The quark doesn’t move. Where is the noise in “standard” AdS/CFT?

Moving Quarks in AdS/CFT: (HKKKY; S. Gubser)



1. No transverse acceleration!

2. No photon emission for example

Need to find the noise in AdS/CFT

Langevin description of heavy quark thermalization:

- Write down an equation of motion for the heavy quarks.

$$\begin{aligned} \frac{dx}{dt} &= \frac{p}{M} \\ \frac{dp}{dt} &= -\underbrace{\eta_D p}_{\text{Drag}} + \underbrace{\xi(t)}_{\text{Random Force}} \end{aligned}$$

- The drag and the random force are related

$$\langle \xi_i(t)\xi_j(t')\rangle = \frac{\kappa}{3}\delta_{ij}\,\delta(t-t') \qquad\qquad \eta_D = \frac{\kappa}{2MT}$$

$\kappa =$ Mean Squared Momentum Transfer per Time

- People computed the coefficients κ and η

Want to see the whole brownian process!

Langevin in Quantum Mechanics

$$M\frac{d^2x}{dt} + \underbrace{\frac{\kappa}{2T}\,\dot{x}}_{\text{Drag}} = \underbrace{\xi}_{\text{Noise}}$$

- Consider a heavy particle coupled to bath a force on the contour

$$Z_Q = \left\langle \int Dx_1 Dx_2\, e^{i\int \frac{1}{2}Mv_1^2 - i\int \frac{1}{2}Mv_2^2}\; e^{i\int dt_1 F_1 x_1}\; e^{-i\int dt_2 F_2 x_2} \right\rangle_{\text{Bath}}$$

- The force term is small compared to the inertia

$$\left\langle e^{i\int dt_1 F_1 x_1}\; e^{-i\int dt_2 F_2 x_2} \right\rangle_{\text{bath}} \simeq e^{-\frac{1}{2}\int dt\, dt'\, x_a(t)\, \langle F_a(t) F_b(t')\rangle\, x_b(t')}$$

- Stir the soup:
  - Now switch vars to ave and diff: $\bar{X} = (x_1 + x_2)/2 \;\; \Delta X = x_1 - x_2$
  - Do the path integral over difference

Result Generalized Langevin

$$M_Q \frac{d^2 \bar{X}}{dt^2} + \int^t \underbrace{G_R(t-t')}_{\text{Drag}} \bar{X}(t') \quad = \quad \underbrace{\xi}_{\text{Noise}}$$

1. Drag = retarded force force correlator

$$G_R(t) = \theta(t) \left\langle [F(t), F(0)] \right\rangle$$

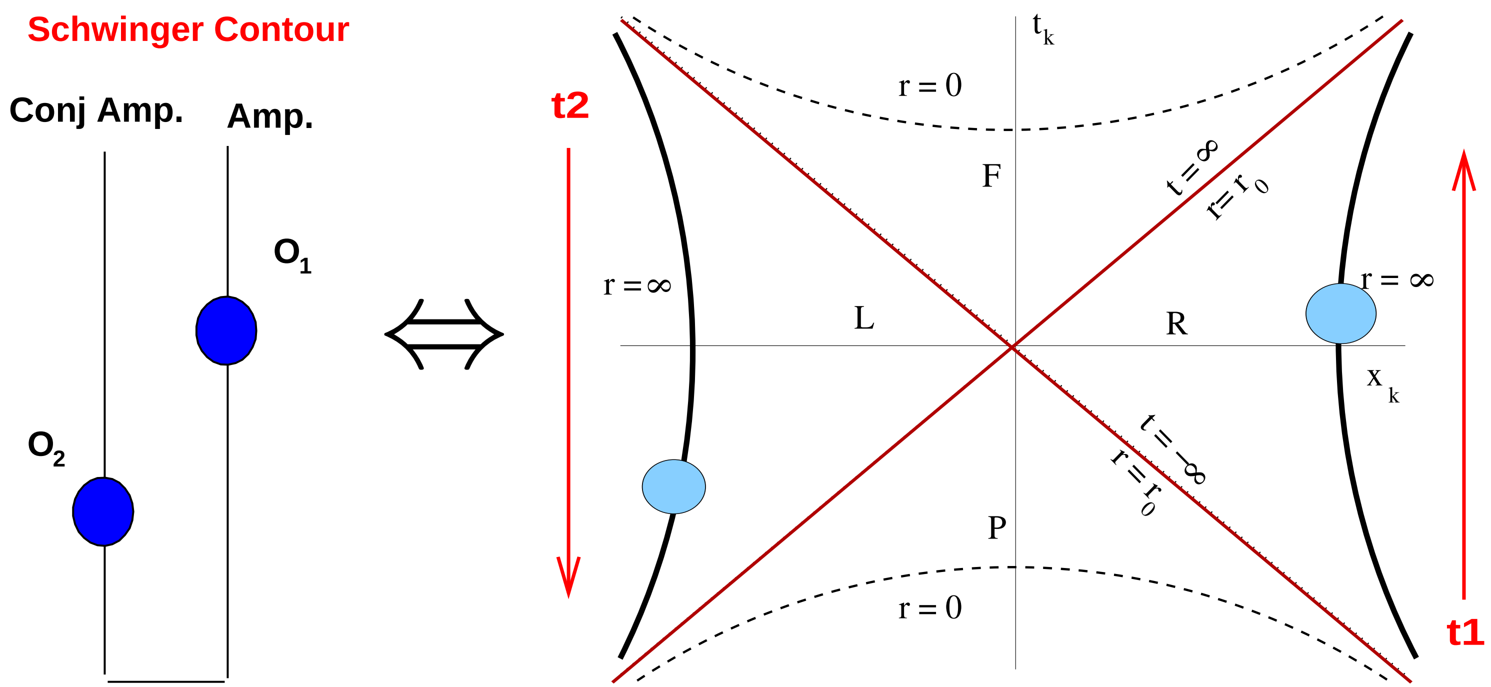
2. Noise = symmetrized force-force correlator

$$\langle \xi(t)\xi(0) \rangle = \langle \{F(t), F(0)\} \rangle$$

AdS/CFT in the Kruskal Plane

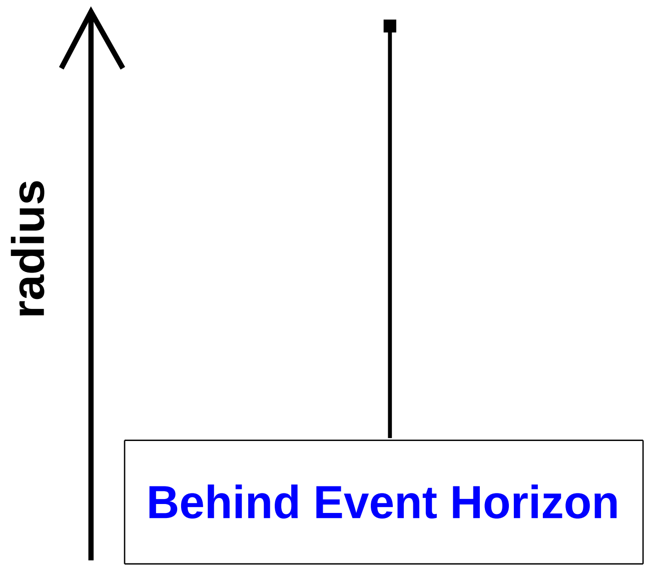
- Source fields for ”1” and ”2” operators live on the right and left quadrants

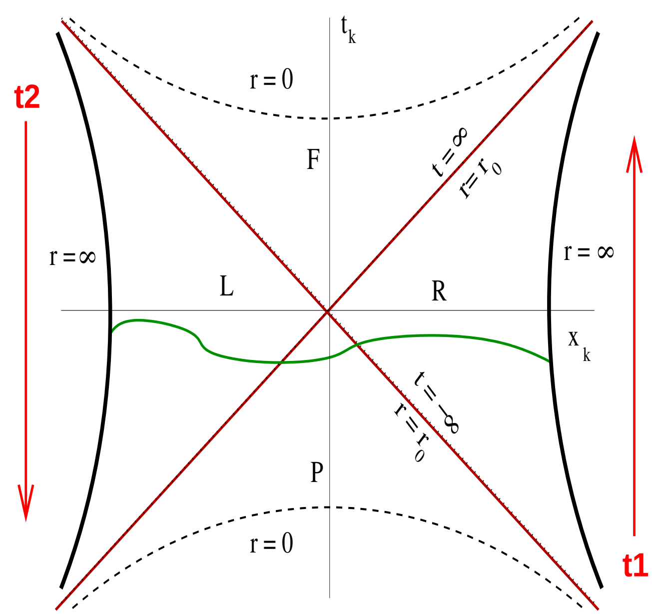
$$\mathcal{O}_1\ ,\ \mathcal{O}_2 \Leftrightarrow \phi_1\ ,\ \phi_2$$



$$\left\langle e^{i\int dt_1\ \phi_1\mathcal{O}_1} e^{-i\int dt_2\ \phi_2\mathcal{O}_2}\right\rangle_{SYM} = e^{iS[\phi_1,\phi_2]}$$

(t, r) Observer vs. Kruskall Observer

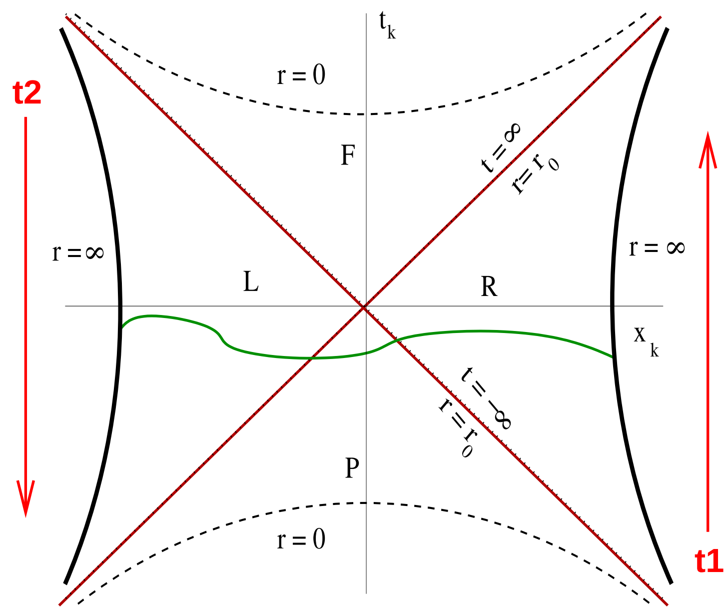




Integrating out the Bulk

- The real time partition function of string for small fluctuations

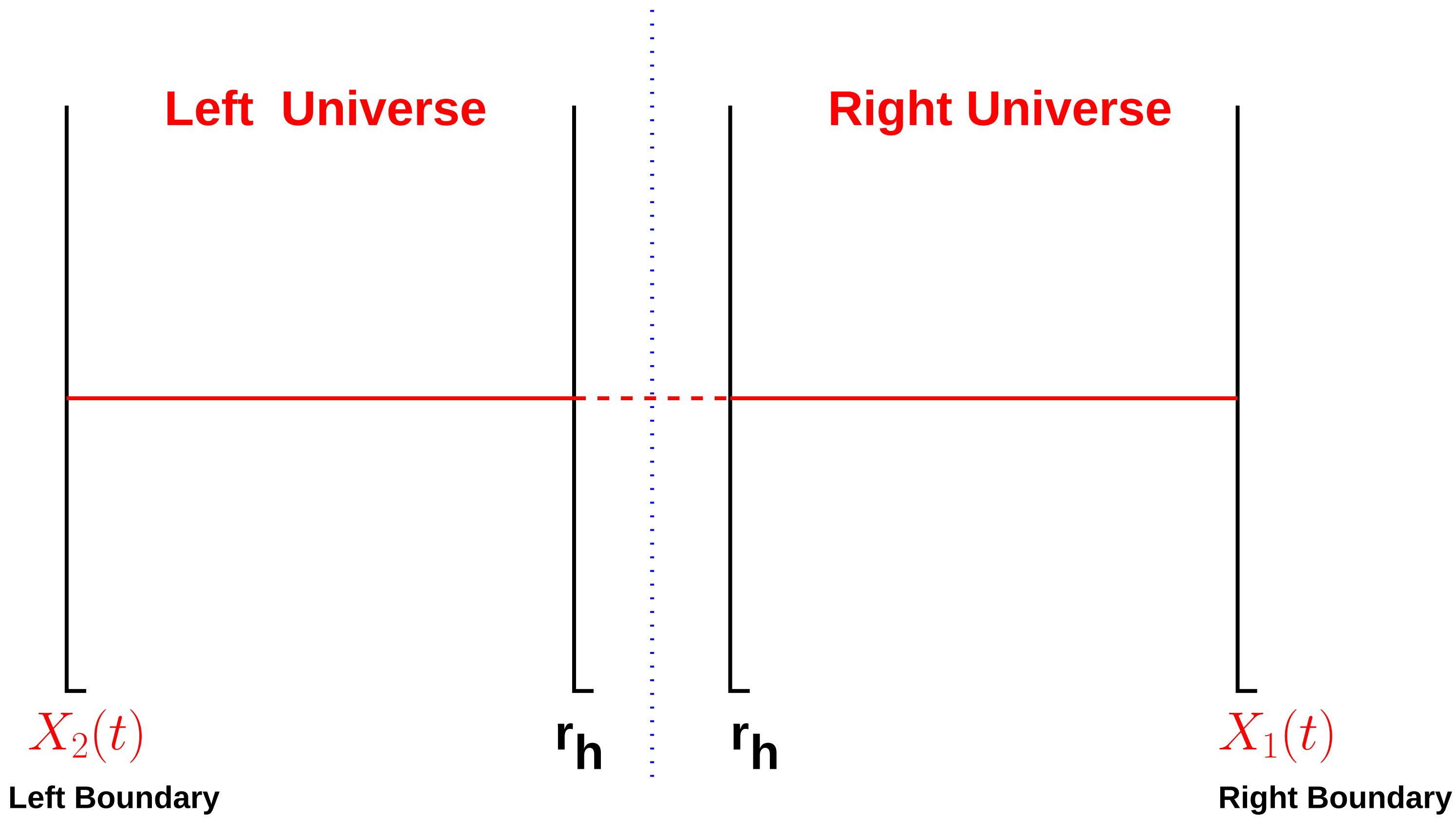
$$Z = \int \prod_{t_1} d\mathbf{X}_1(t_1) \prod d\mathbf{X}_2(t_2) \prod_{t,z} d\mathbf{x}_1(t,z) d\mathbf{x}_2(t,z) e^{iS_{NG}}$$



- The integrals over the internal coordinates can be done and yield

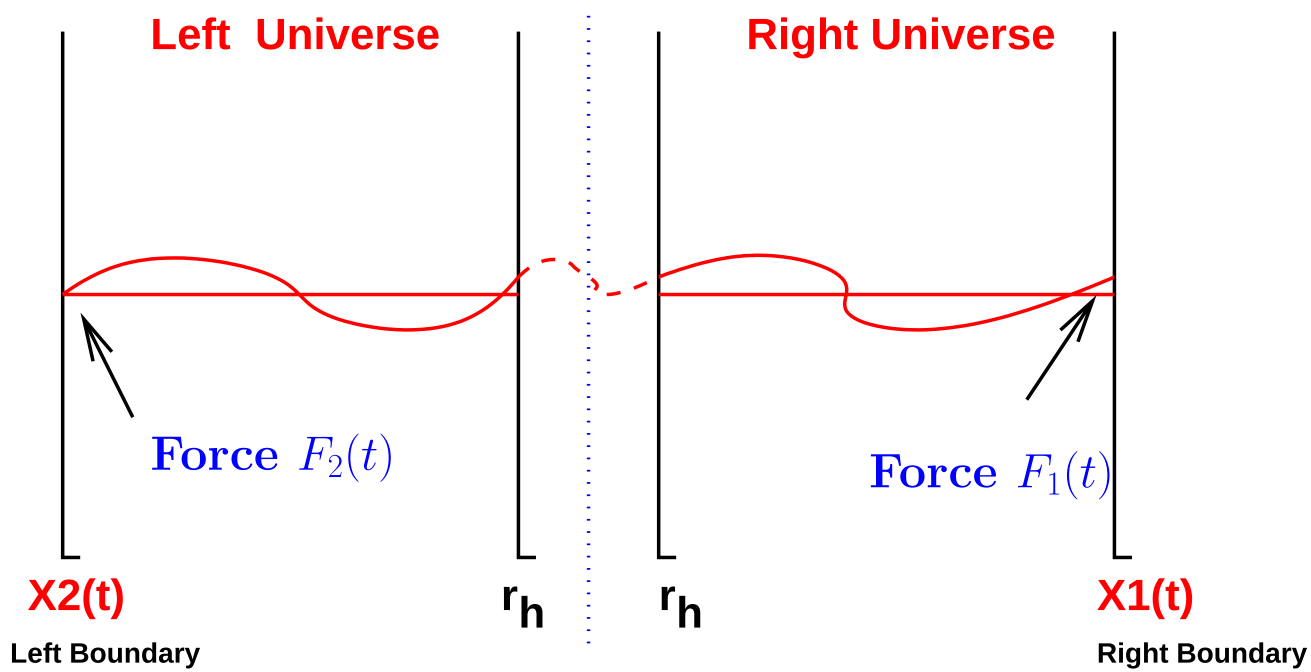
$$Z = \int DX_1 DX_2 \, e^{iS_{\rm eff}[X_{\rm cl}(X_1(t_1),X_2(t_2))]}$$

Strings and Fluctuations



$$S_{NG,\text{Left}} \sim \frac{R^2}{2\pi\ell_s^2}\int dt\,dr\left[1-\underbrace{\frac{1}{2}\left(\frac{\dot{\mathbf{x}}_{\|}^2}{f}-4fr^2\left(\mathbf{x}_{\|}'\right)^2\right)}_{\text{Quadratic Fluctuations}}\right]$$

Strings and Path Integrals



$$
\begin{aligned}
Z_{\rm str} &= \int \prod_{t_1} d\mathbf{X}_1(t_1) \prod d\mathbf{X}_2(t_2) \prod_{t,z} d\mathbf{x}_1(t,z) d\mathbf{x}_2(t,z) e^{iS_{NG}} \\
&= \int DX_1 DX_2 \, e^{iS_{\rm eff}[X_{\rm cl}(X_1(t_1),X_2(t_2))]}
\end{aligned}
$$

The effective action

$$
\begin{aligned}
iS_{\text{eff}} \quad = \quad & -\frac{1}{2}\int \frac{d\omega}{2\pi} \\
& + X_1(-\omega)\left[-iM_Q^0\omega^2 + \langle F_1F_1\rangle\right] X_1(\omega) \\
& + X_2(-\omega)\left[+iM_Q^0\omega^2 + \langle F_2F_2\rangle\right] X_2(\omega) \\
& - X_1(-\omega)\,\left[\langle F_1F_2\rangle\right]\, X_2(\omega) \\
& - X_2(-\omega)\,\left[\langle F_2F_1\rangle\right]\, X_1(\omega)
\end{aligned}
$$

where for example

$$\langle F_1F_1\rangle(\omega) \quad = \quad \text{Force-Force Correlator in 1}$$

$$\langle F_2F_2\rangle(\omega) \quad = \quad \text{Force-Force Correlator in 2}\ldots$$

$$\langle F_1F_2\rangle(\omega) \quad = \quad \text{Force-Force cross correlator}\ldots$$

Same as in Quantum Mechanics...

Result: Langevin with Memory

- Find the endpoint of the string obeys the expected Langevin equation

$$M_Q^0 \frac{d^2\mathbf{X}}{dt^2} + \int^t G_R(t-t')\mathbf{X}(t') = \xi$$

- To quadratic order the retarded green function is

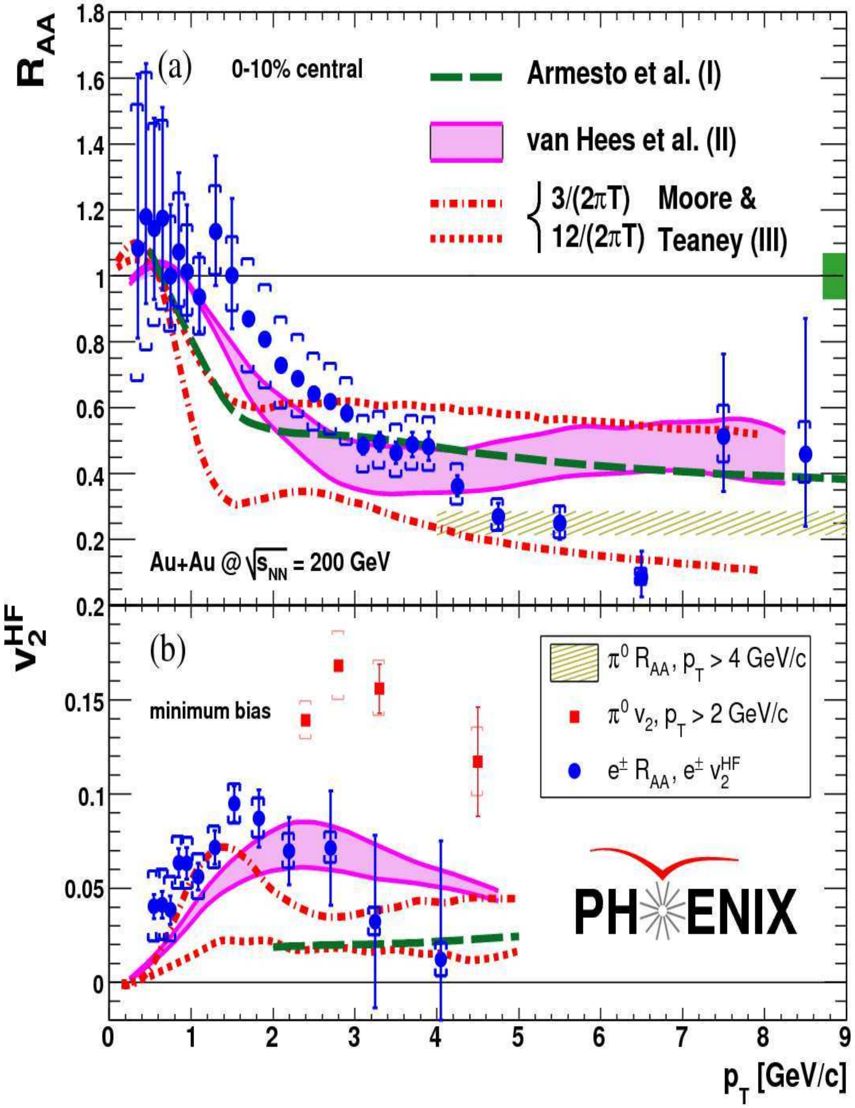
$$G_R(\omega) = \underbrace{(\Delta M)}_{\sqrt{\lambda}T/2}\,\omega^2 - i\omega \underbrace{\frac{\kappa}{2T}}_{\kappa=\sqrt{\lambda}\pi T^3}$$

- Then find the following effective equation of motion

$$\underbrace{M_{\rm kin}(T)}_{M-\Delta M}\,\frac{d^2\mathbf{X}}{dt^2} + \underbrace{\frac{\kappa}{2T}\frac{d\mathbf{X}}{dt}}_{\rm drag} = \xi$$

with the kinetic mass (Herzog et al '06)

$$M_{\rm kin}(T) = M_Q^0 - \frac{\sqrt{\lambda}T}{2}$$



Phenomenological Summary

$$D = \frac{2T^2}{\kappa} = \frac{2}{\sqrt{\lambda}\pi T}$$

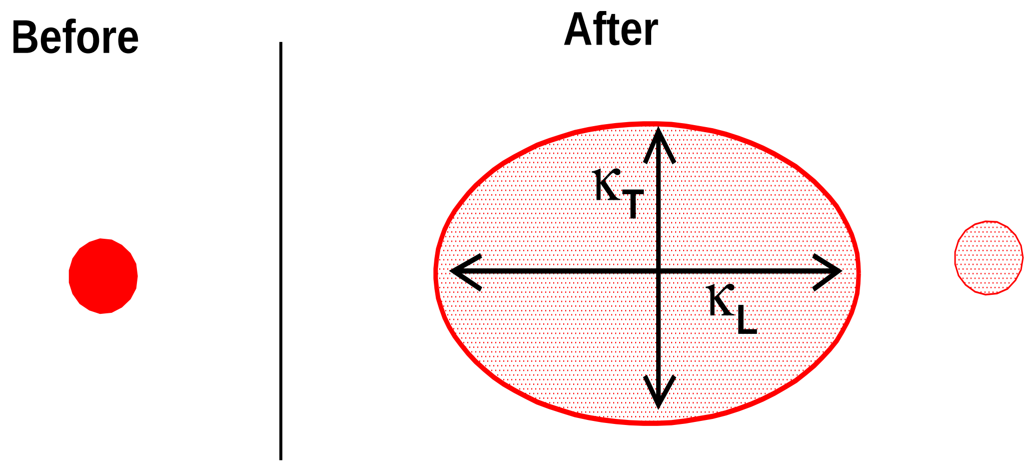
- Best QCD estimate from $\mathcal{N} = 4$

$$D_{QCD} \sim \frac{4 \div 8}{2\pi T}$$

- Weak Coupling best estimate

$$D_{QCD} \sim \frac{3 \div 6}{2\pi T}$$

Generalize to Relativistic Heavy Quarks

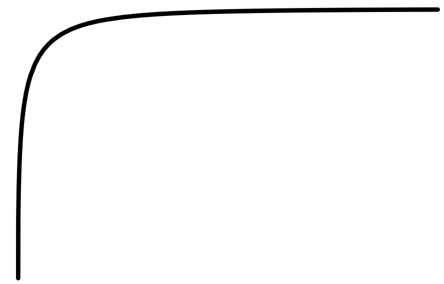


- Transverse Momentum Broadening of a heavy quark (analogous to $\hat{q}$)

$$
\begin{aligned}
\kappa_T(v) &= \text{Mean squared transverse momentum transfer per unit time} \\
\kappa_L(v) &= \text{Mean squared longitudinal momentum transfer per unit time}
\end{aligned}
$$

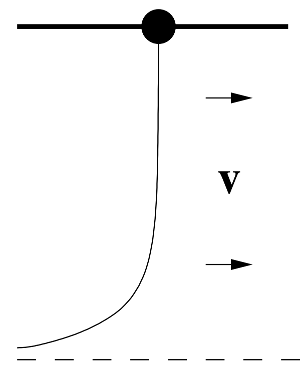
- Drag

$$\frac{dP}{dt} = -\eta(v)P + \xi_L(t) + \xi_T(t)$$



Finding the semi-classical string (Herzog et al and S. Gubser)

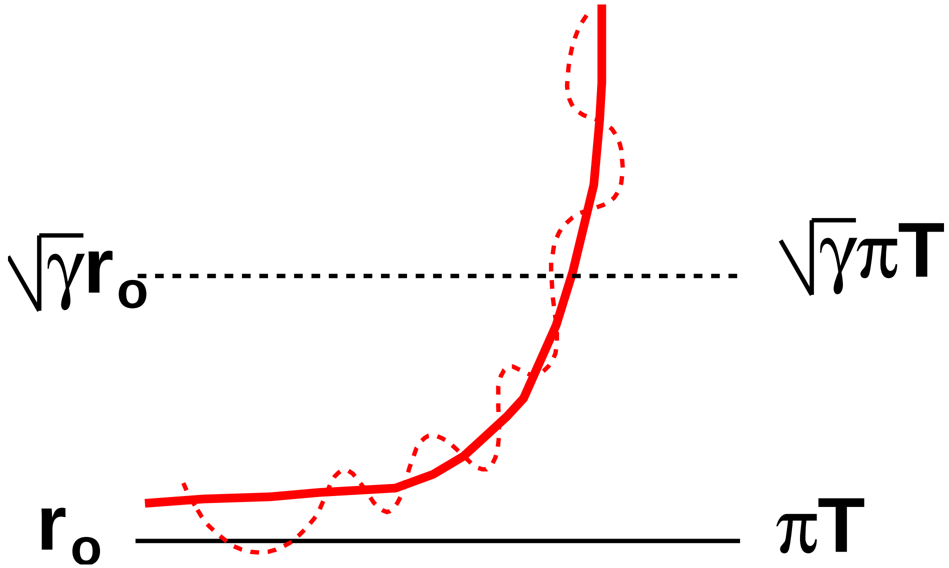
- Turn on an electric field to accelerate the quark



- A semiclassical string trails behind the quark

$$x_3 = vt + \frac{v}{2}\left[\arctan(z) - \operatorname{arctanh}(z)\right]$$

Quantum Mechanics of the Endpoint

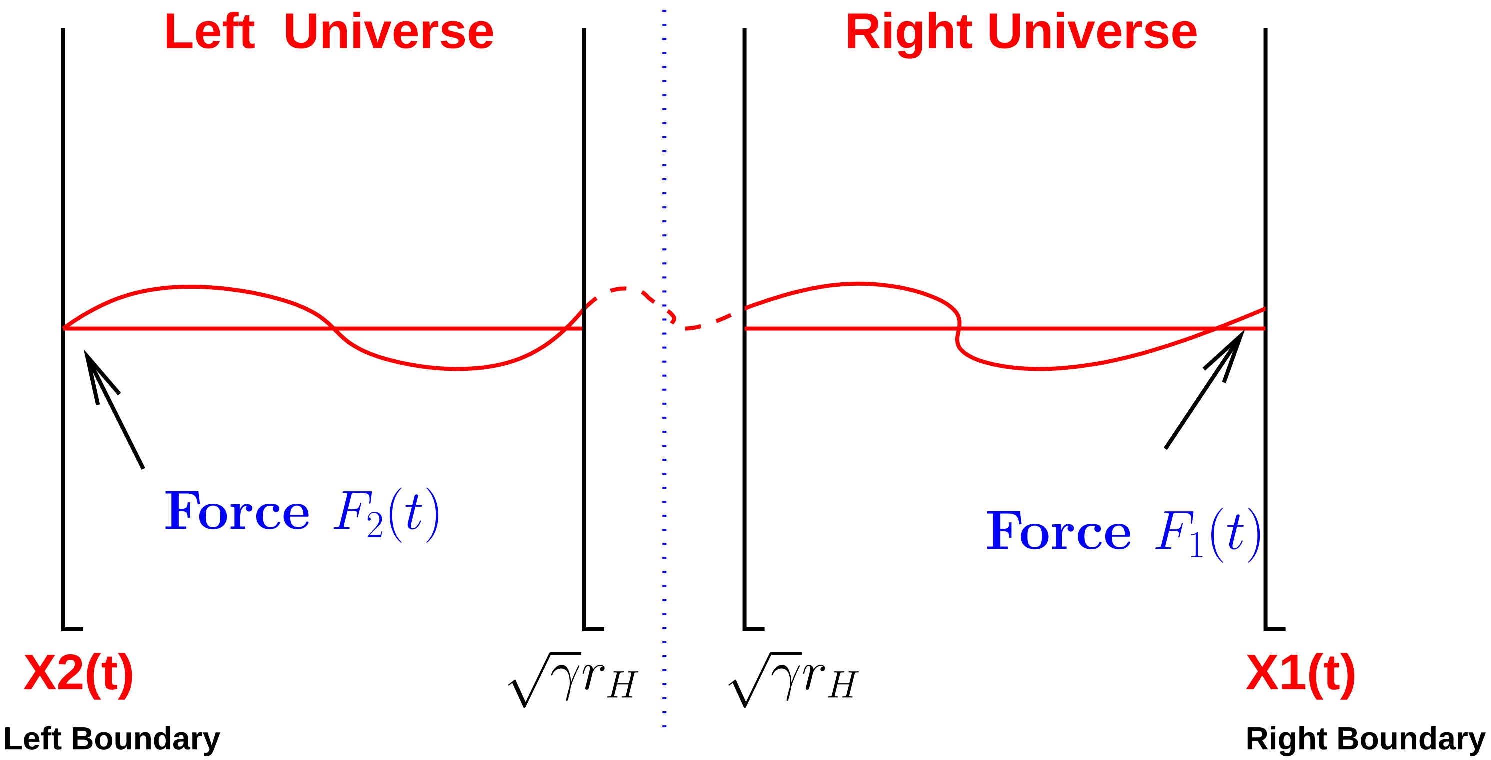


- There is a radius where the string exceeds the local speed of light

$$r_{\text{critical}} = \sqrt{\gamma} r_o$$

- Analogy with black holes can be made precise

Strings and Fluctuations



Effective Equation Motion

$$M_{\rm kin}(T)\frac{d(\gamma\mathbf{v})}{dt} = -\underbrace{\frac{\sqrt{\lambda}\pi T^2}{2}\gamma\mathbf{v}}_{\text{Drag}} + \underbrace{\xi^i(v)}_{\text{Noise}}$$

- Drag grows γ – Relaxation time independent of momentum $p = p_0 e^{-\eta t}$
- Fluctuations also grow with momentum

$$\begin{aligned}\kappa_T(v) &= \sqrt{\lambda}\,\pi\,T^3 \times \sqrt{\gamma} \\ \kappa_L(v) &= \sqrt{\lambda}\,\pi\,T^3 \times \gamma^{5/2} \Leftarrow \text{(Gubser)}\end{aligned}$$

- Effective Mass decreases with gamma

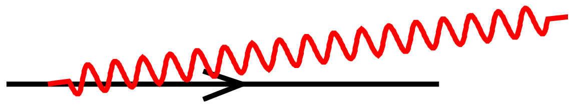
$$M_{\rm kin}(T) = M_Q^0 - \frac{\sqrt{\lambda}T}{2} \times \sqrt{\gamma}$$

Constraint on velocity: $\gamma \ll M_Q/\sqrt{\lambda T}$

Consequence of velocity constraint: No LPM

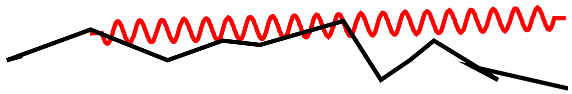
- Case 1: (Dead Cone) Photon decoheres because it moves faster than quark

$$t_{\rm decoh-v} \sim \frac{1}{\omega(1 - v\cos(\theta))} \sim \frac{\gamma^2}{\omega}$$



- Case 2: (LPM) Photon decoheres due to transverse momentum broadening

$$t_{\rm decoh-LPM} \sim \frac{E}{\sqrt{\hat{q}\,\omega}}$$



- But the quark stops in a finite time independent of momentum.

$$t_{\rm stop} \sim \frac{M}{\sqrt{\lambda}T^2}$$

- To see the LPM need

$$t_{\rm decoh-LPM} \ll t_{\rm decoh-v} \ll t_{\rm stop}$$

These conditions and the velocity constraint can't be simultaneously satisfied

Conclusions

- Quantum Mechanics of AdS_5 leads to thermal noise
  - Prototypical Example – Brownian Motion
  - Other examples – “Long Time” hydrodynamic tails
  - Necessary for thermalization ?

Order of limits Matters!

$$\begin{cases} \text{Time}, N_c \to \infty \\ \text{Time}, \sqrt{\lambda} \to \infty \end{cases}$$

- Saw some applications of AdS to heavy quark data
  - Thermal perturbation theory is poor. Quark and Gluons as Quasi-Particles?
  - AdS predictions are markedly different from perturbation theory.

More likely wrong than right! But maybe we should find out for sure