Nearly Perfect Fluidity:
From Cold Atoms to Hot Quarks

Thomas Schaefer, North Carolina State University

RHIC serves the perfect fluid

Experiments at RHIC and the LHC are consistent with the idea that a thermalized plasma is produced, and that the equation of state is that of a weakly coupled gas of quarks and gluons.

But: Transport properties of the system (primarily viscosity and energy loss) are in dramatic disagreement with expectations for a weakly coupled QGP. The plasma must be very strongly coupled.

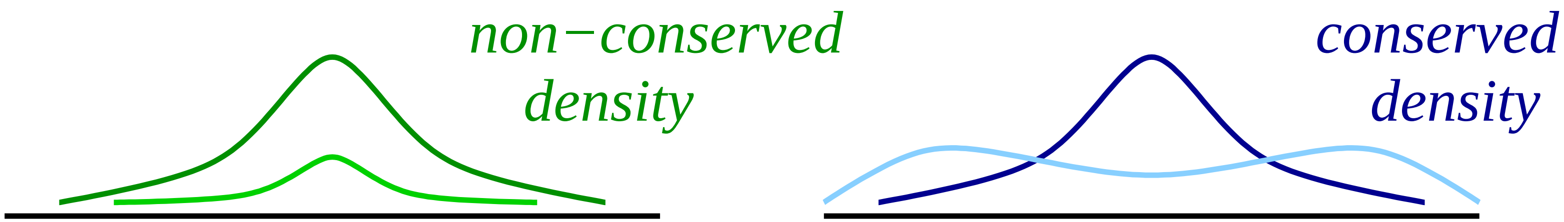
In this talk I will try to explain this statement, review the current evidence, and put the results in a broader perspective (by comparing with another strongly coupled fluid, the dilute atomic Fermi gas at "unitarity").

Hydrodynamics

Hydrodynamics (undergraduate version): Newton's law for continuous, deformable media.

Fluids: Gases, liquids, plasmas, . . .

Hydrodynamics (postmodern): Effective theory of non-equilibrium long-wavelength, low-frequency dynamics of any many-body system.



$\tau \sim \tau_{micro}$ $\tau \sim \lambda$

$\tau \gg \tau_{micro}$: Dynamics of conserved charges.

Water: $(\rho, \epsilon, \vec{\pi})$

Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0 \qquad \frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{\jmath}^{\,\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \nabla_j \Pi_{ij} = 0$$

Constitutive relations: Stress tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\nabla_i v_j + \nabla_j v_i - \frac{2}{3}\delta_{ij} \nabla_k v_k \right) + O(\nabla^2)$$

reactive dissipative 2nd order

Expansion $\Pi^0_{ij} \gg \delta\Pi^1_{ij} \gg \delta\Pi^2_{ij}$

Note: Full expression $\Pi^1_{ij} = \eta \nabla_{\langle i} v_{j \rangle} + \zeta \delta_{ij} \nabla \cdot v$ and $(\jmath^{\epsilon}_i)^1 = -\kappa \nabla_i T$

Regime of applicability

Expansion parameter $Re^{-1} = \frac{\eta(\partial v)}{\rho v^2} = \frac{\eta}{\rho L v} \ll 1$

-1

$$\frac{1}{Re} = \underbrace{\frac{\eta}{\hbar n}}_{\text{fluid property}} \times \underbrace{\frac{\hbar}{m v L}}_{\text{flow property}}$$

Bath tub :	$mvL \gg \hbar$	hydro reliable
Heavy ions :	$mvL \sim \hbar$	need $\eta < \hbar n$

Note: Bacteria swim in the regime $Re^{-1} \gg 1$ but $Ma^2 \cdot Re^{-1} \ll 1$.

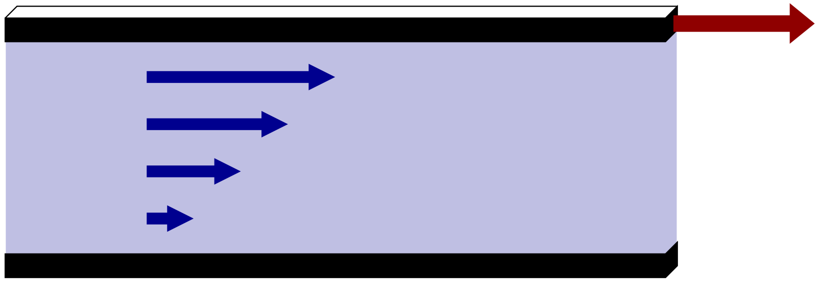
Shear viscosity and friction

Momentum conservation at $O(\nabla v)$

$$\rho\left(\frac{\partial}{\partial t}\vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right) = -\vec{\nabla}P + \eta\nabla^2\vec{v}$$

Navier-Stokes equation

Viscosity determines shear stress ("friction") in fluid flow



$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

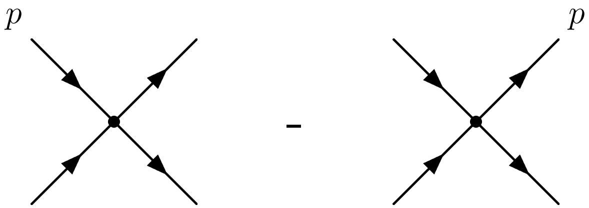
Kinetic theory

Kinetic theory: conserved quantities carried by quasi-particles.

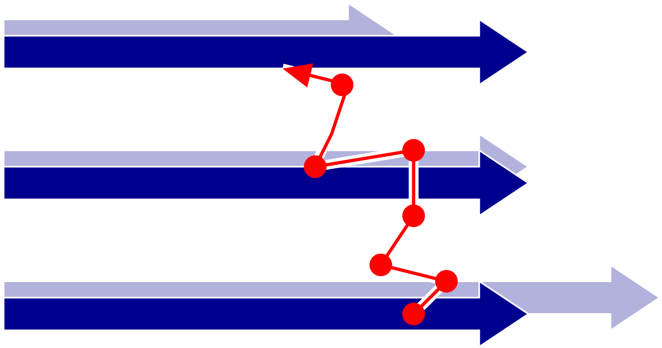
Quasi-particles described by distribution functions $f(x,p,t)$.

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p + \vec{F}\cdot\vec{\nabla}_p f_p = -C[f_p]$$

$$C[f_p] \quad =$$



Shear viscosity corresponds to momentum diffusion



$$\eta \sim \frac{1}{3} n\,\bar{p}\,l_{mfp}$$

Shear viscosity: Additional properties

Weakly interacting gas, $l_{mfp} \sim \dfrac{1}{n\sigma}$:

$$\boxed{\eta \sim \frac{1}{3}\frac{\bar{p}}{\sigma}}$$

shear viscosity independent of density

Non-interacting gas ($\sigma \to 0$):

$$\boxed{\eta \to \infty}$$

non-interacting and hydro limit ($T \to \infty$) limit do not commute

strongly interacting gas:

$$\boxed{\frac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar}$$

but: kinetic theory not reliable!

Historical digression: Mott's minimal conductivity

(Sir) Nevill Mott predicted that the metal-insulator transition cannot be continuous; there is a minimal conductivity.

Conduction in Non-crystalline Systems

IX. The Minimum Metallic Conductivity

By N. F. Mott

Cavendish Laboratory, Cambridge

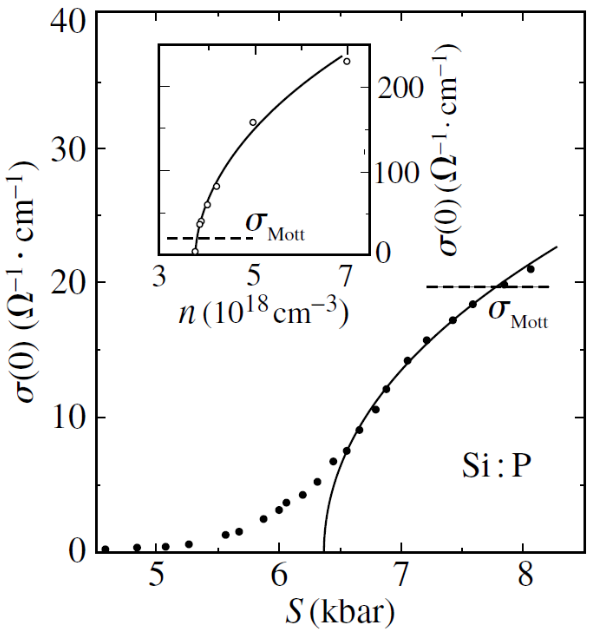
[Received 27 July 1972]

$$\frac{\sigma}{n^{1/3}} \geq \frac{1}{(3\pi^2)^{2/3}} \frac{e^2}{\hbar}$$

This idea is not correct,

the metal-insulator transition can

be continuous.



Historical digression: Minimal shear viscosity

Danielewicz & Gyulassy argue that the shear viscosity cannot be zero.

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Dissipative phenomena in quark-gluon plasmas

P. Danielewicz* and M. Gyulassy
Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 12 April 1984; revised manuscript received 24 September 1984)

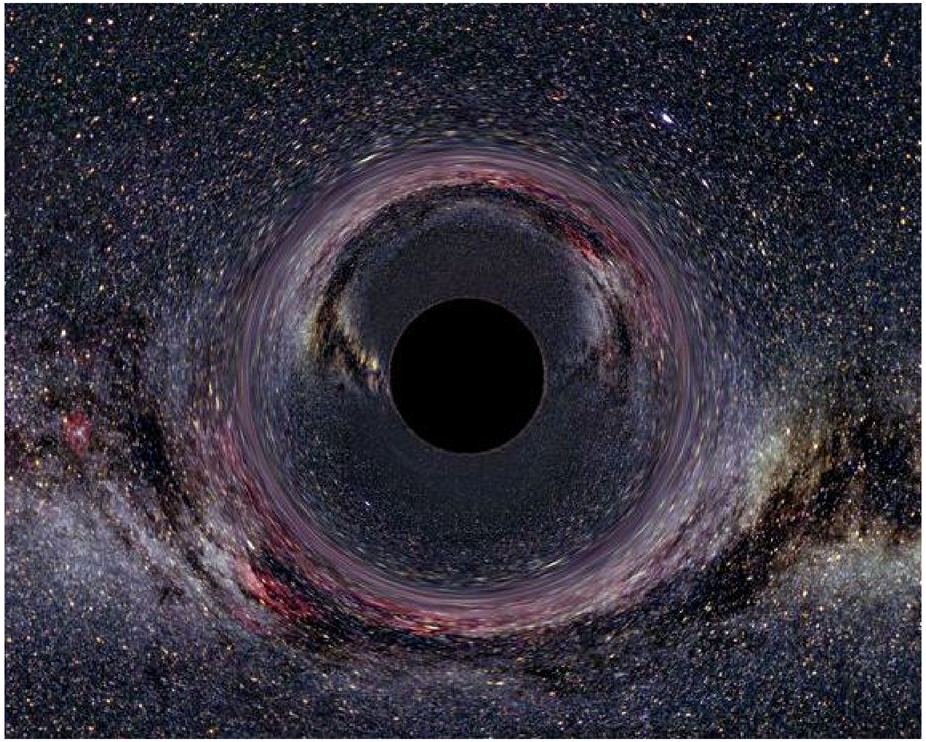
than $\langle p \rangle^{-1}$. Requiring $\lambda_i \gtrsim \langle p \rangle_i^{-1}$ leads to the lower bound

$$\eta \gtrsim \tfrac{1}{3} n \ , \tag{3.3}$$

where $n = \sum n_i$ is the total density of quanta. What seems amazing about (3.3) is that it is independent of dynamical details. There is a finite viscosity regardless of

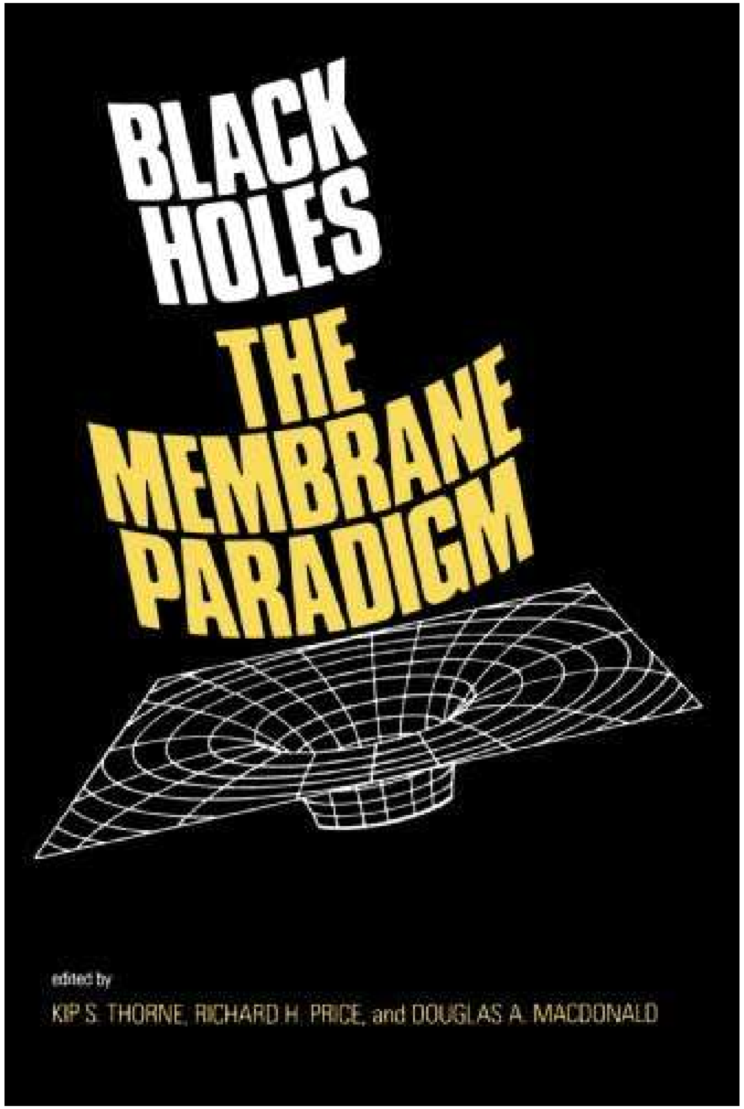
Is this idea correct?

And now for something completely different . . .



This is an irreversible process, $\Delta S > 0$.

And now for something completely different . . .



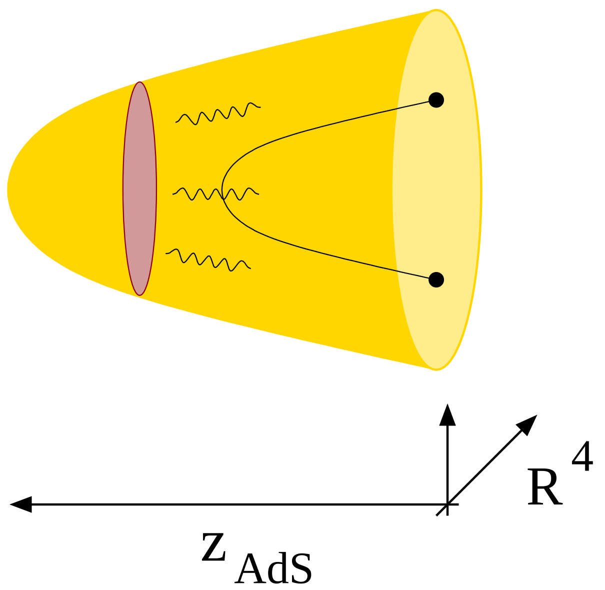
Ringdown can be described in terms of stretched horizon that behaves as a sheared fluid

$$\eta = \frac{s}{4\pi}$$

Note: Unusual thermodynamics, e.g. ζ, $C < 0$.

Idea can be made precise using the "AdS/CFT correspondence"

Strongly coupled thermal field theory on R^4	$\Leftrightarrow$	Weakly coupled string theory on AdS_5 black hole
CFT temperature	$\Leftrightarrow$	Hawking temperature of black hole
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon



Holographic duals: Transport properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy

$\sim$ area of event horizon

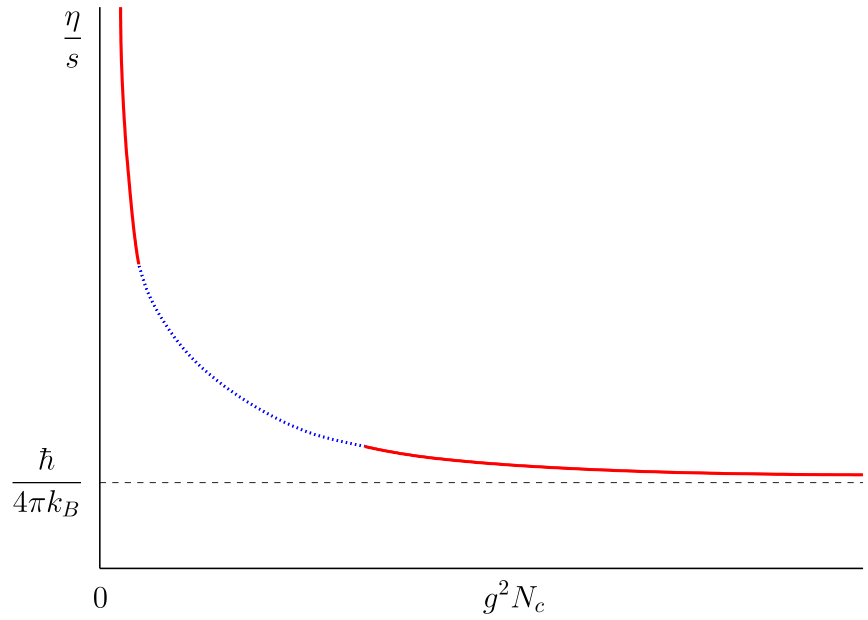
shear viscosity $\Leftrightarrow$ Graviton absorption cross section

$\sim$ area of event horizon

Strong coupling limit

$$\boxed{\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}}$$

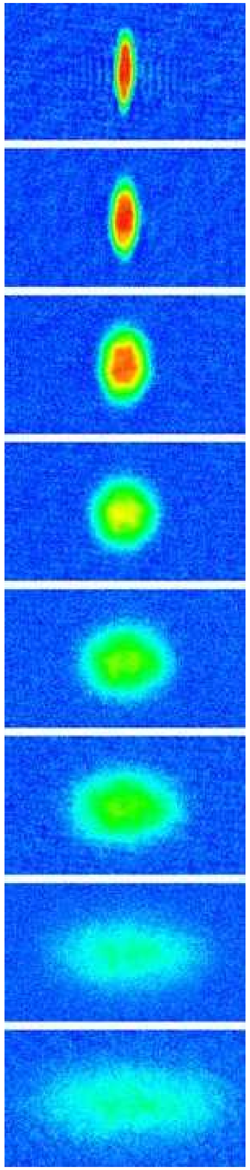
Son and Starinets (2001)

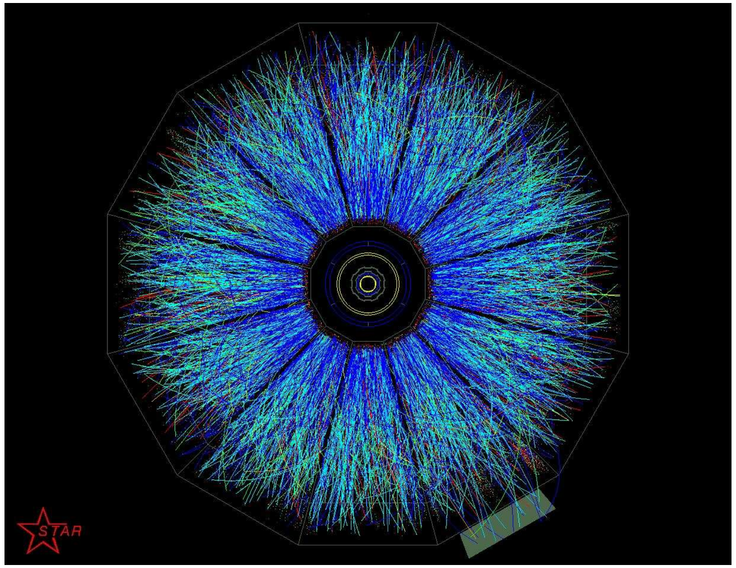


Strong coupling limit universal? Provides lower bound for all theories?

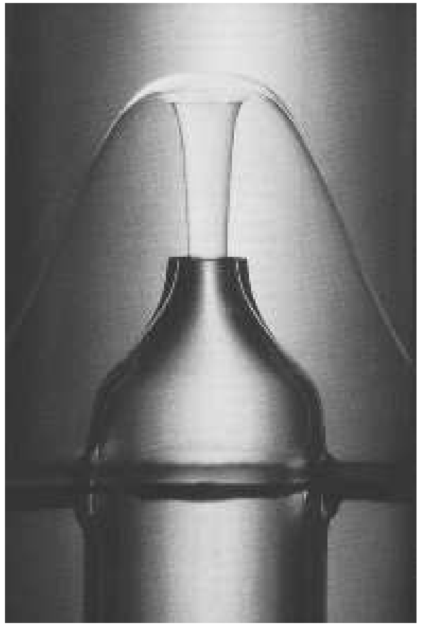
Answer appears to be no; e.g. theories with higher derivative gravity duals.

Perfect Fluids: The contenders





QGP (T=180 MeV)



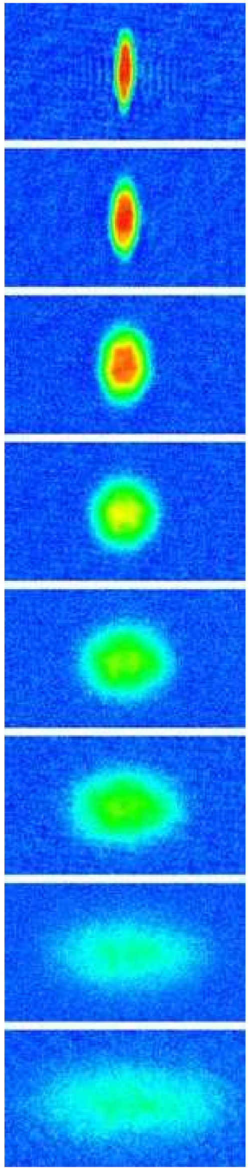
Liquid Helium

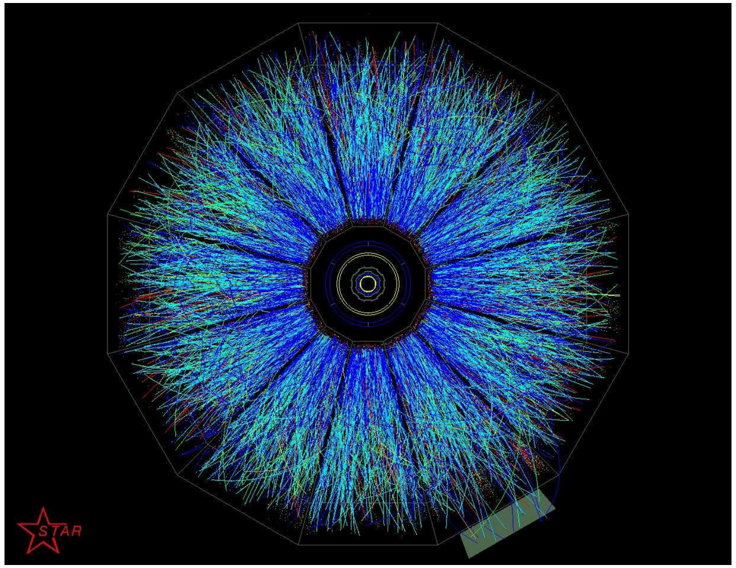
(T=0.1 meV)

Trapped Atoms

(T=0.1 neV)

Perfect Fluids: The contenders

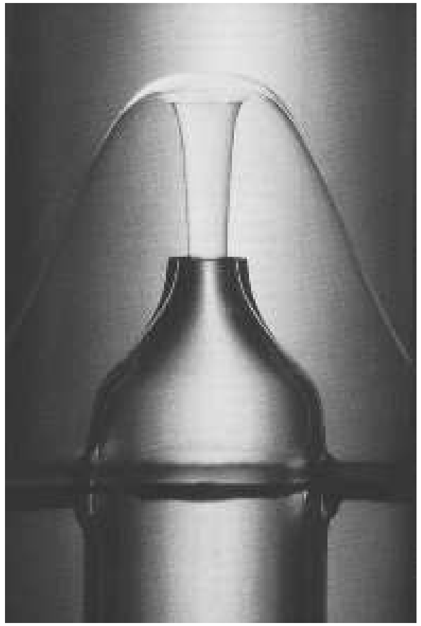




QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$



Liquid Helium

$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

η / s

Perfect Fluids: Not a contender

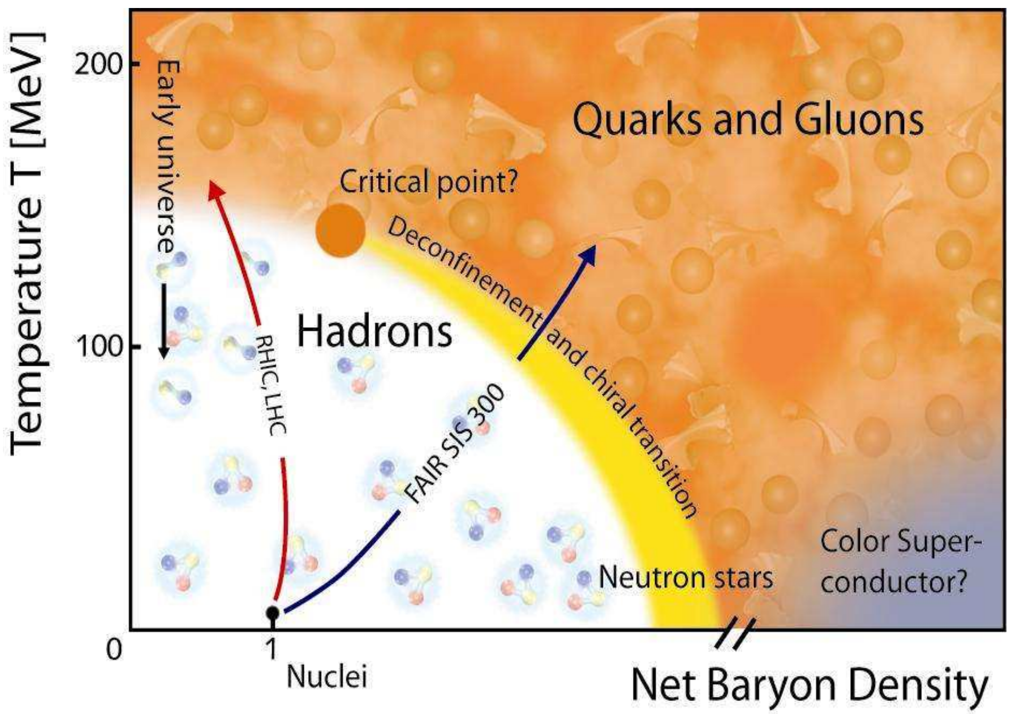
Queensland pitch-drop experiment

1927-2011 (8 drops)

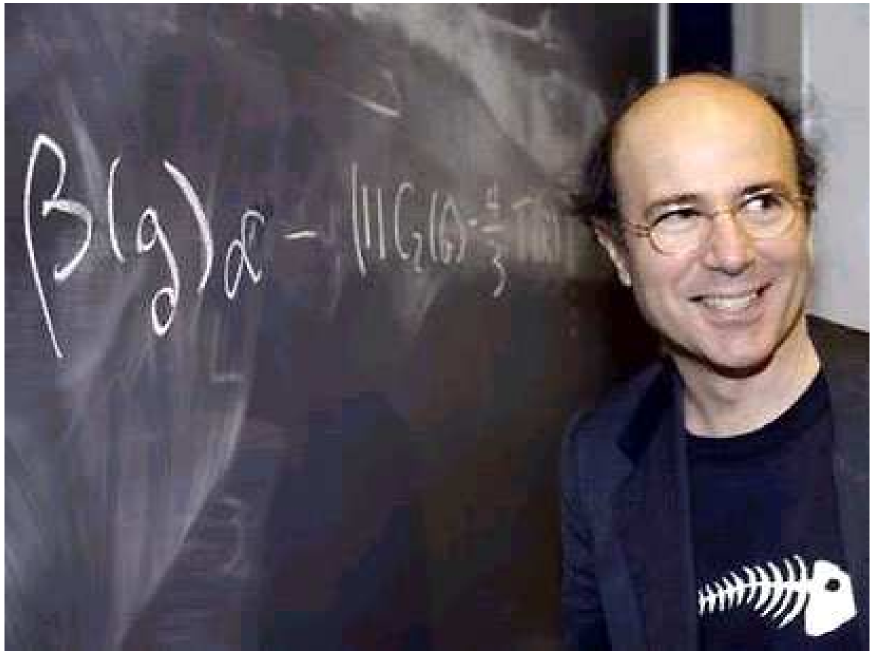
$$\eta = (2.3 \pm 0.5) \cdot 10^8 \; Pa\, s$$

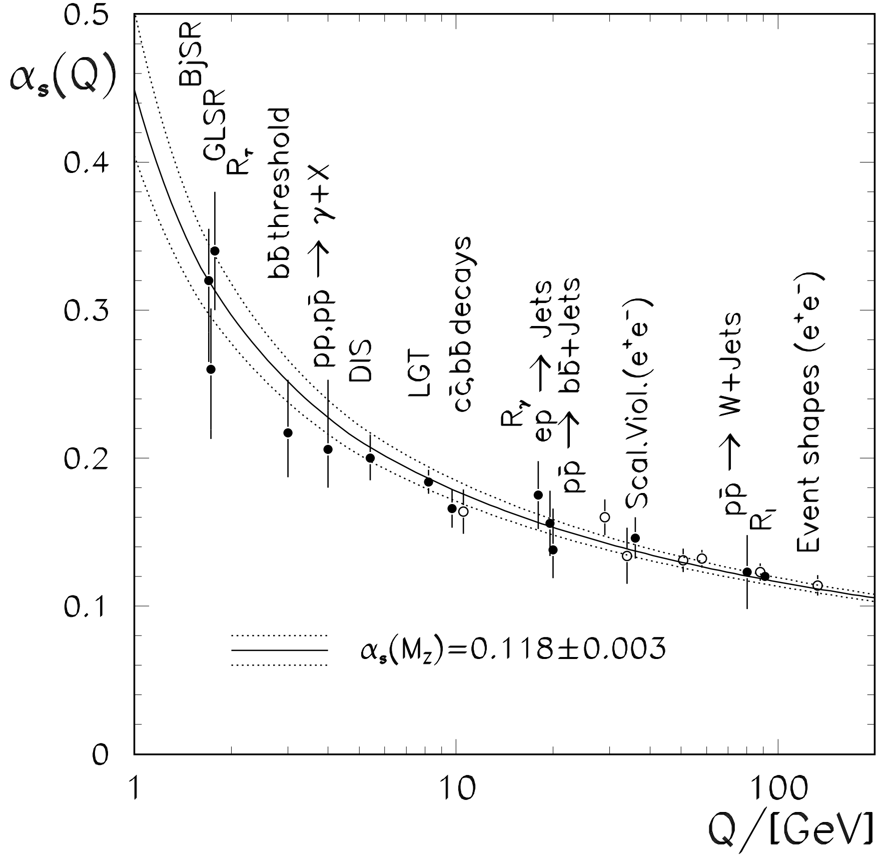
I. QCD and the Quark Gluon Plasma

$$\boxed{\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4g^2}G^a_{\mu\nu}G^a_{\mu\nu}}$$

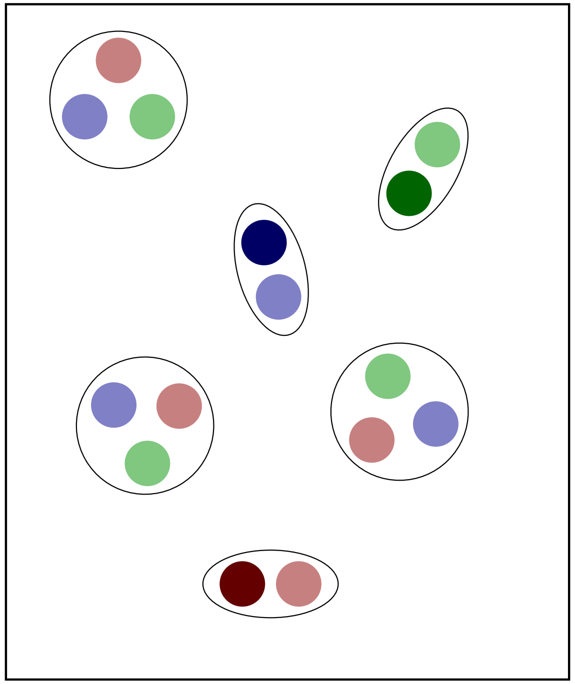


Running coupling constant

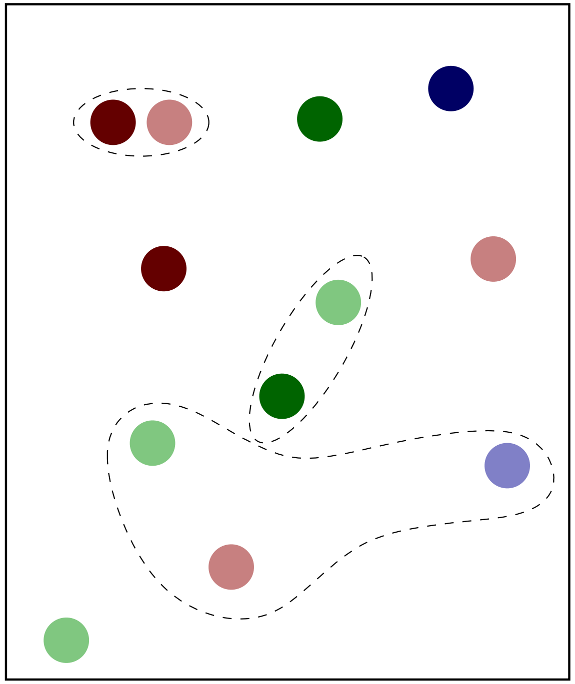




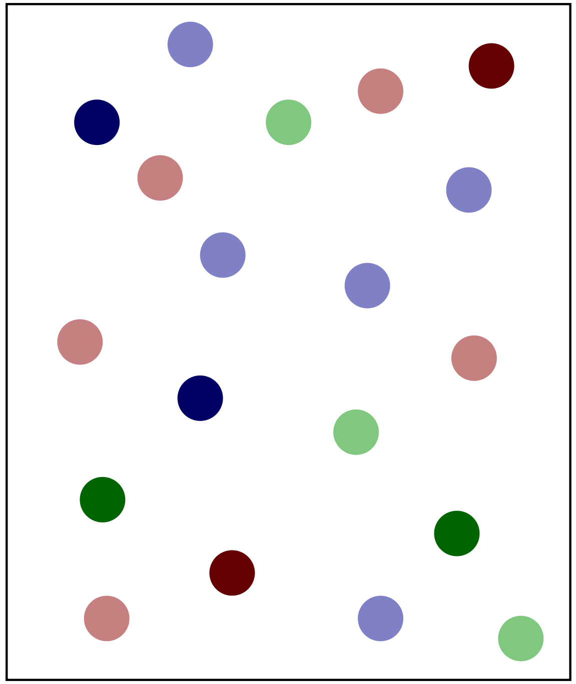
From hadrons to quarks



weakly coupled
hadron gas



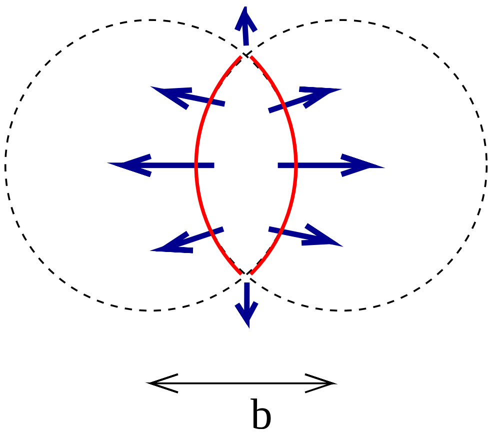
strongly correlated
fluid

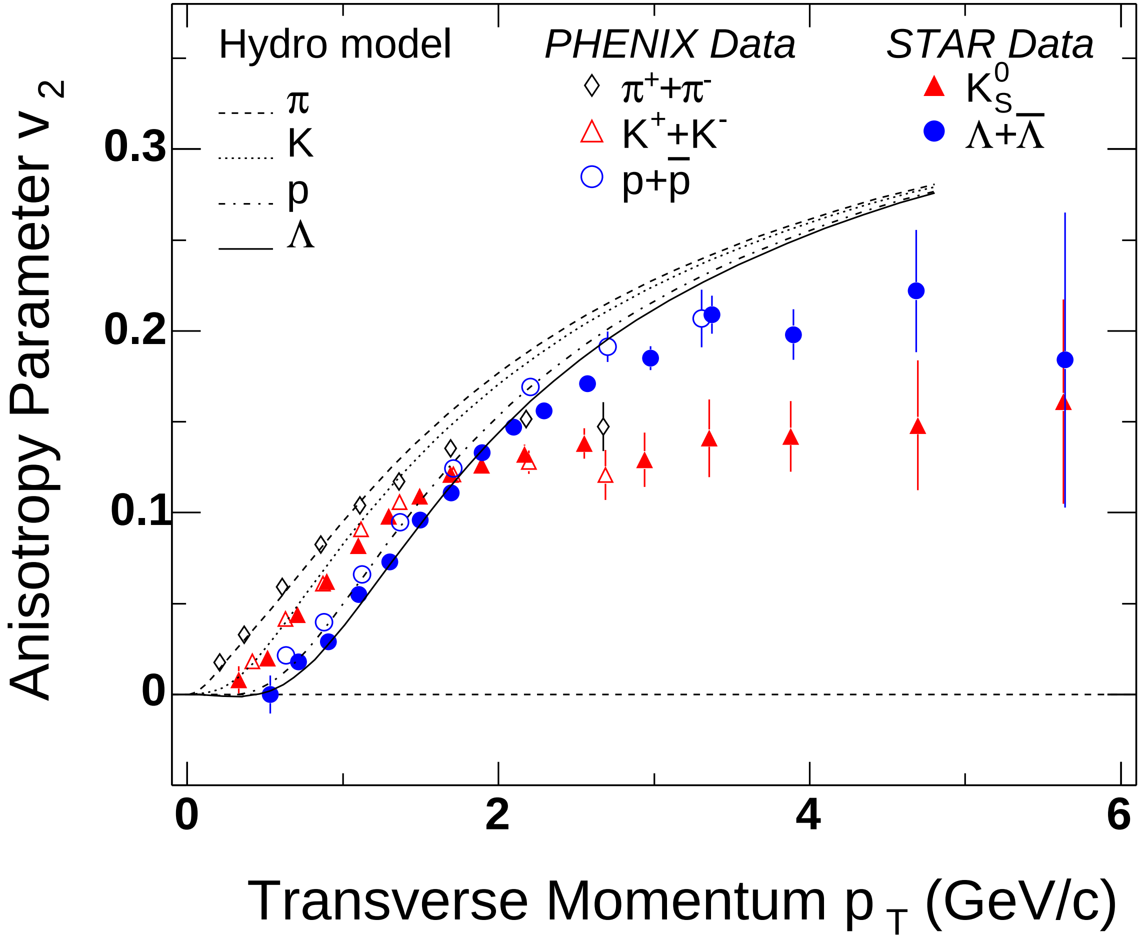


weakly coupled
quark gluon plasma

Elliptic Flow (QGP)

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

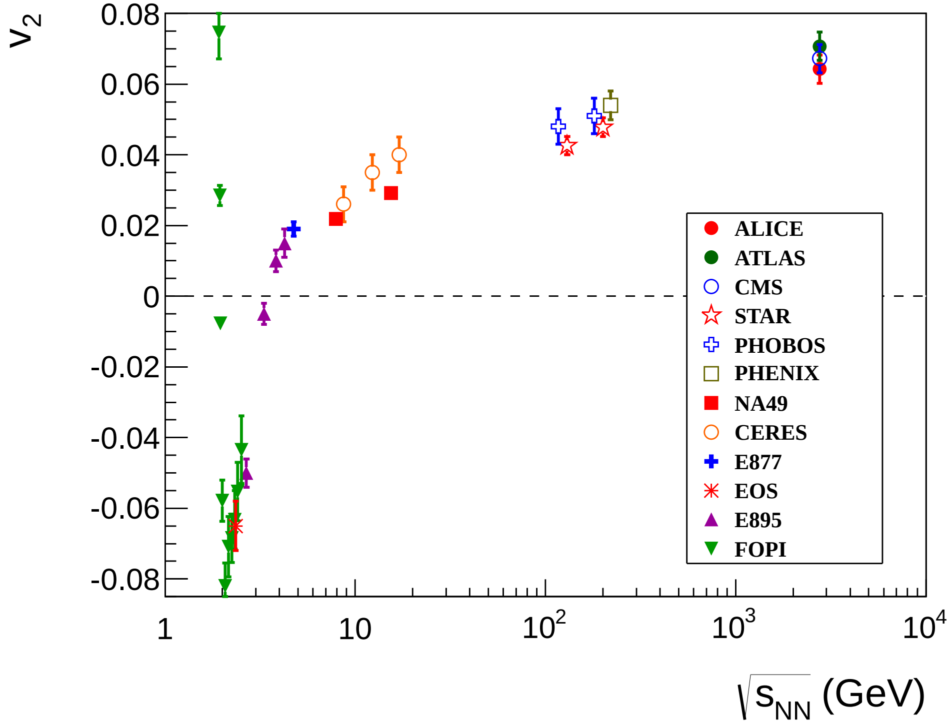




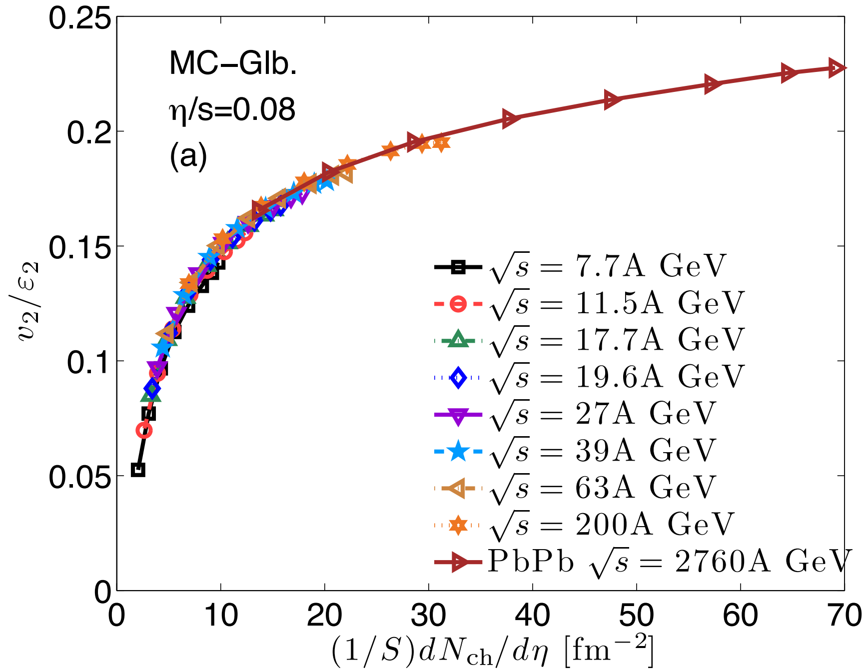
source: U. Heinz (2005)

$$p_0\,\frac{dN}{d^3p}\bigg|_{p_z=0} = v_0(p_\perp)\left(1+2v_2(p_\perp)\cos(2\phi)+\ldots\right)$$

Elliptic flow excitation function



Alice (2011)



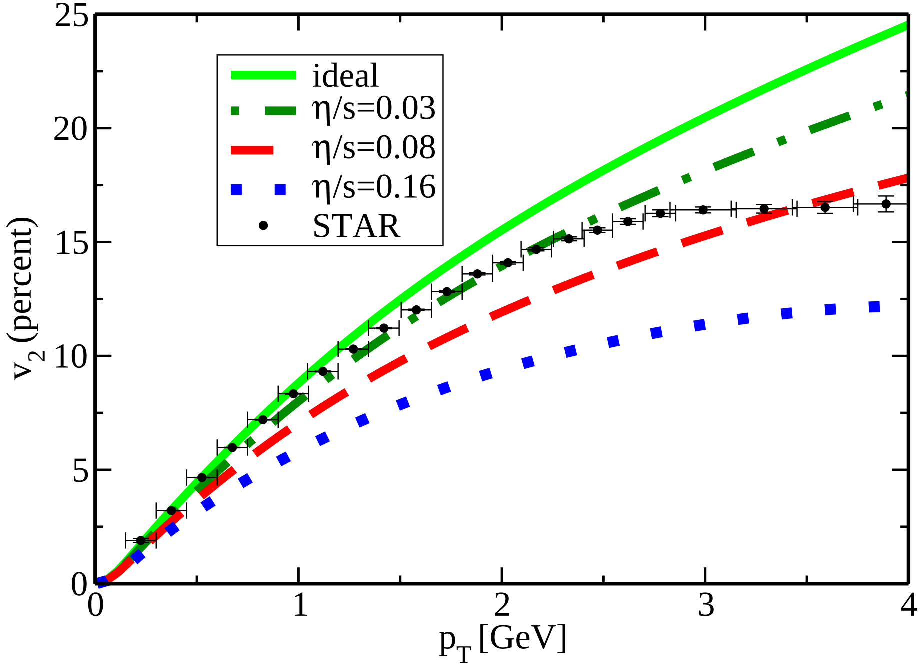
Shen, Heinz (2012)

Viscosity and Elliptic Flow

Viscous correction to v_2 (blast wave model)

$$\frac{\delta v_2}{v_2} = -\frac{1}{3}\frac{1}{\tau_f T_f}\left(\frac{\eta}{s}\right)\left(\frac{p_\perp}{T_f}\right)^2$$

Grows with $p_\perp$, decreases with system size



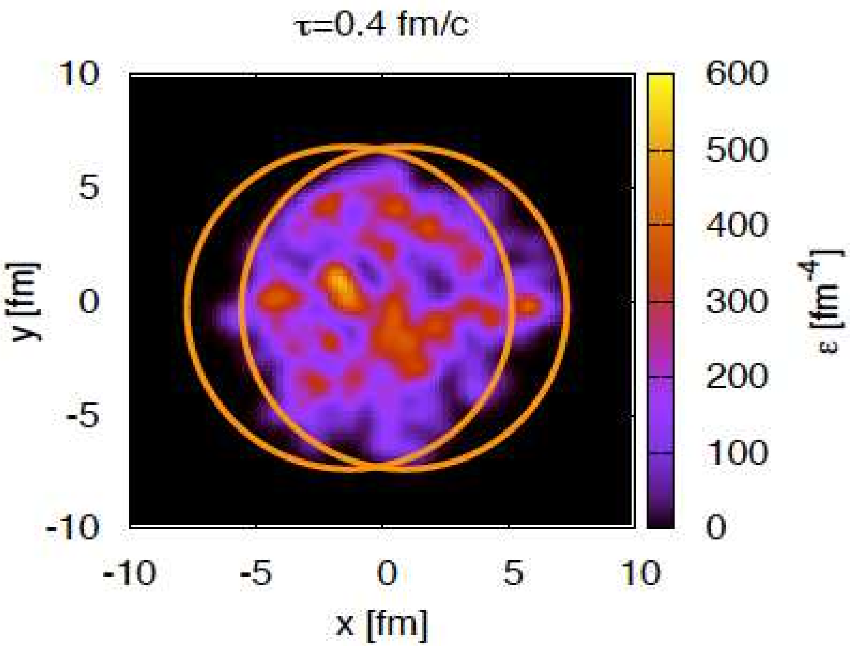
Romatschke (2007), Teaney (2003)

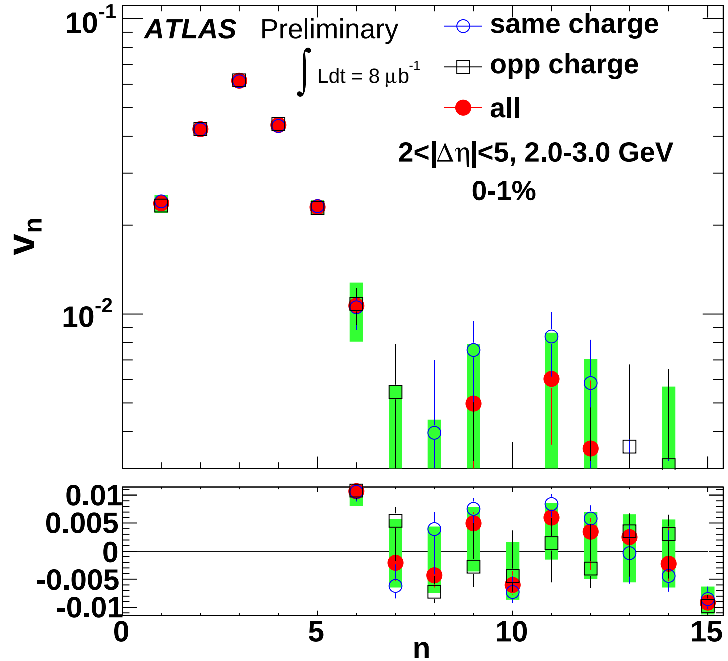
Many details: Dependence on initial conditions, freeze out, etc.

conservative bound $\boxed{\frac{\eta}{s} < 0.25}$

Higher moments of flow

Hydro converts moments of initial deformation to moments of flow





B. Schenke

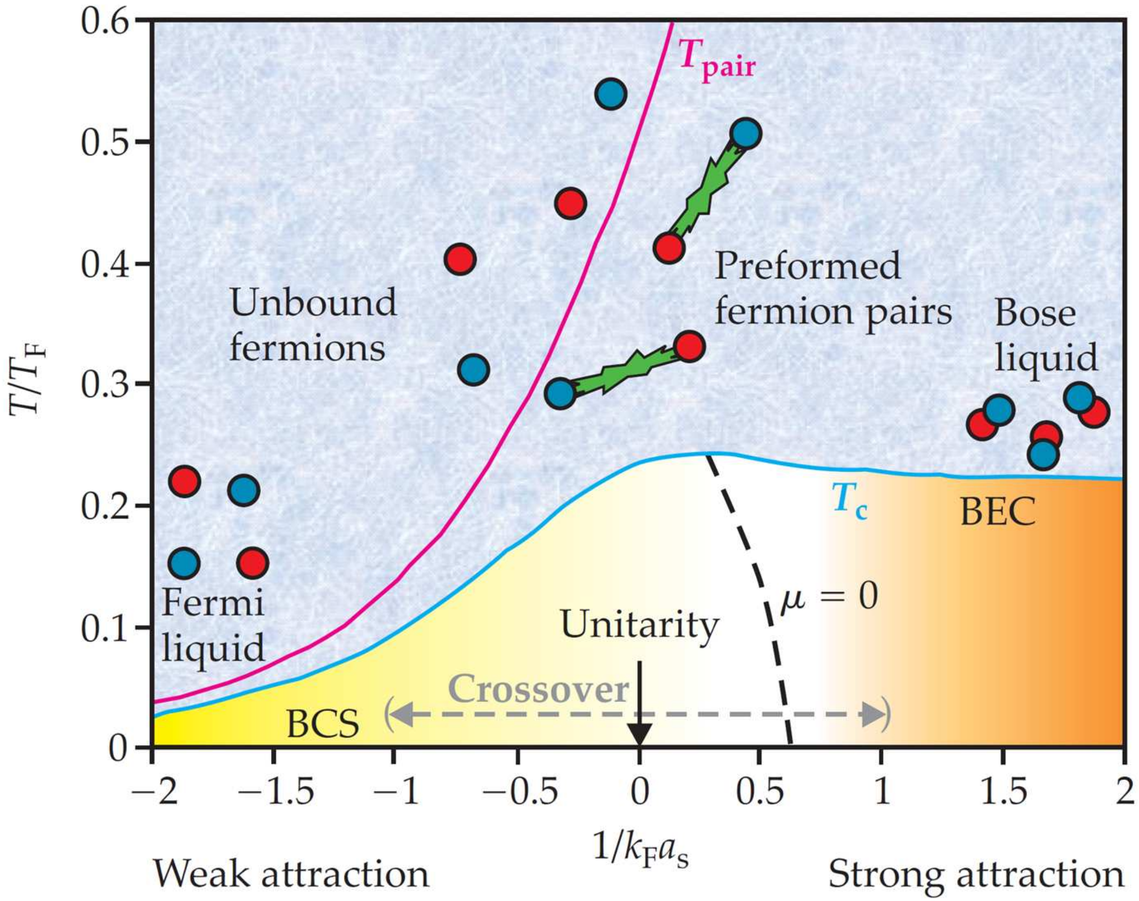
Atlas (J. Jia, QM 2011)

Glauber predicts flat initial spectrum $(n \geq 3)$. Observed flow spectrum consistent with sound attenuation

$$\delta T^{\mu\nu}(t) = \exp\left(-\frac{2}{3}\frac{\eta}{s}\frac{k^2 t}{T}\right)\delta T^{\mu\nu}(0)$$

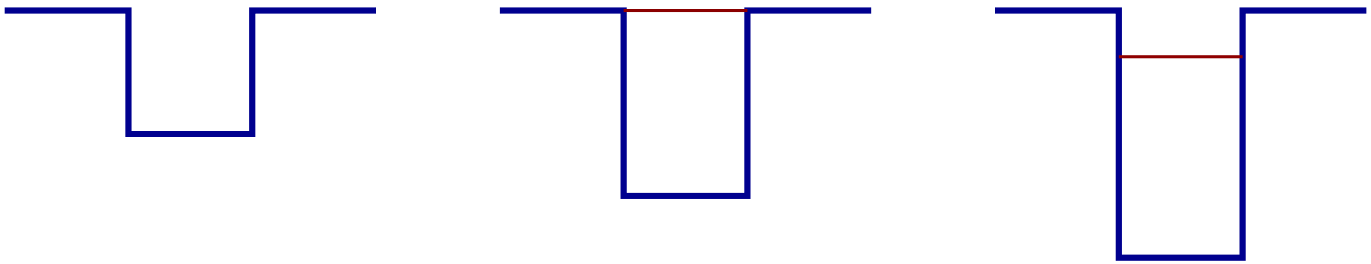
II. Dilute Fermi gas: BCS-BEC crossover

$$\boxed{\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2}$$



Unitarity limit

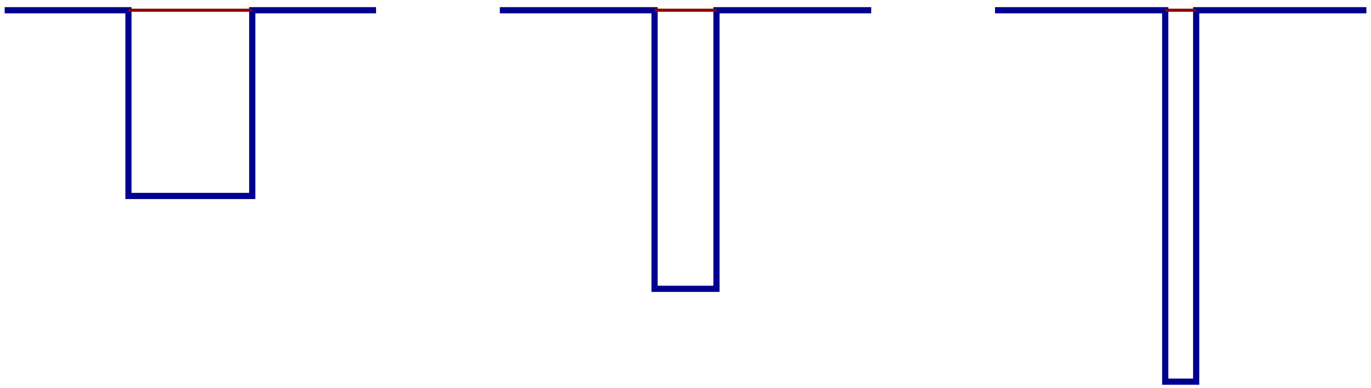
Consider simple square well potential



$a < 0$ $\qquad$ $a = \infty,\ \epsilon_B = 0$ $\qquad$ $a > 0,\ \epsilon_B > 0$

Unitarity limit

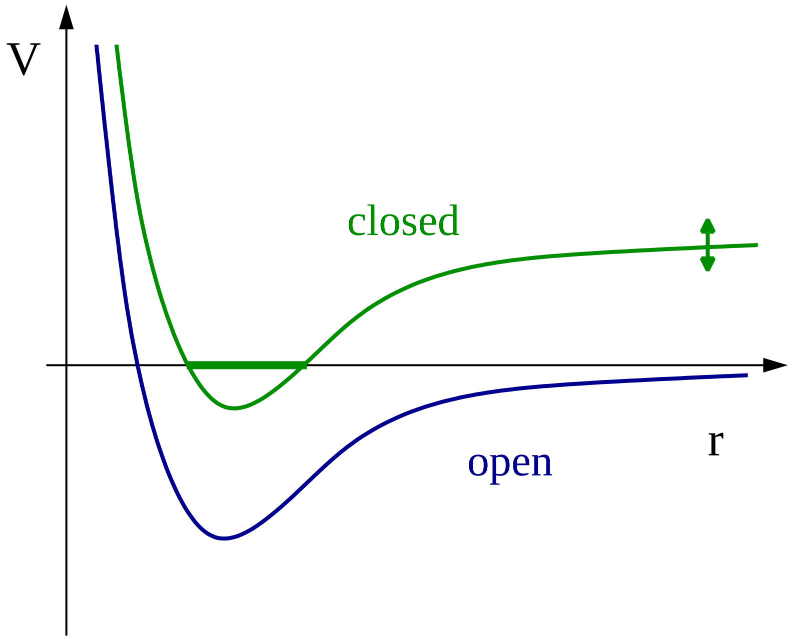
Now take the range to zero, keeping $\epsilon_B \simeq 0$

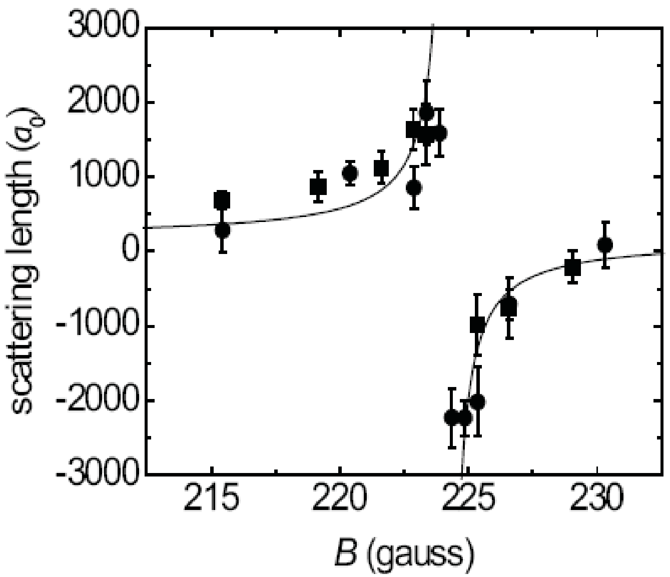


Universal scattering amplitude $\quad \mathcal{T} = \dfrac{1}{ik}$

Feshbach resonances

Atomic gas with two spin states: "$\uparrow$" and "$\downarrow$"

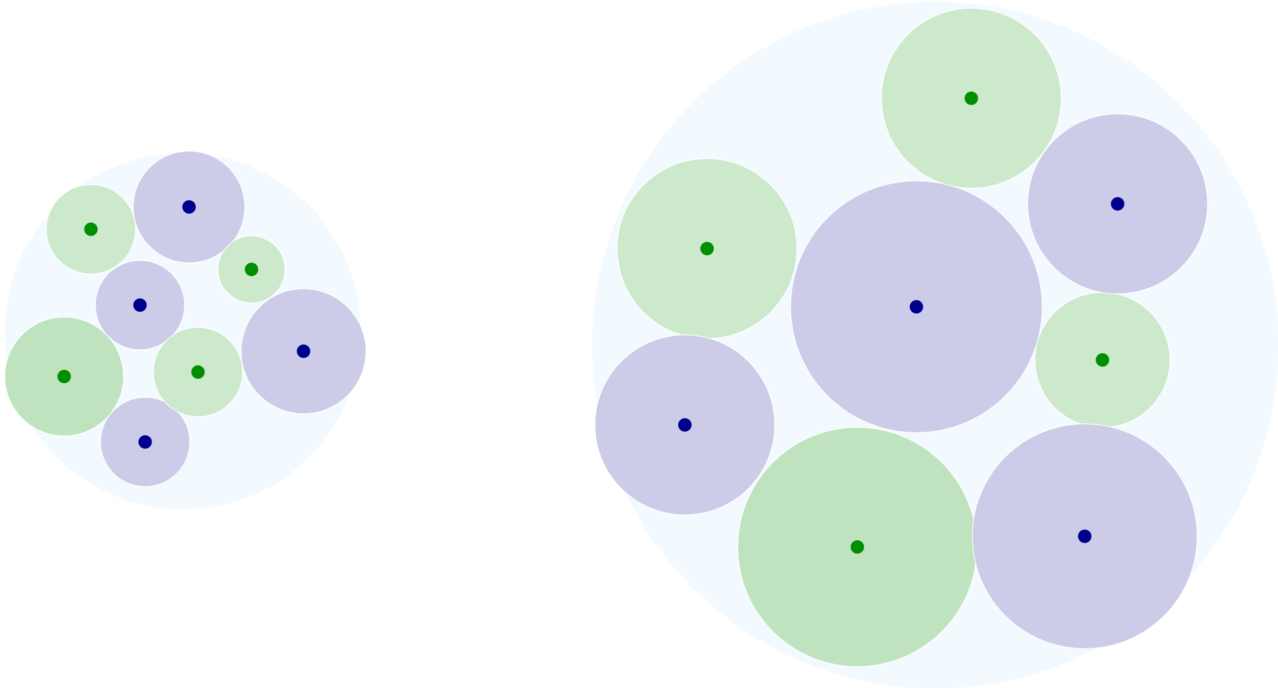




Feshbach resonance $$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

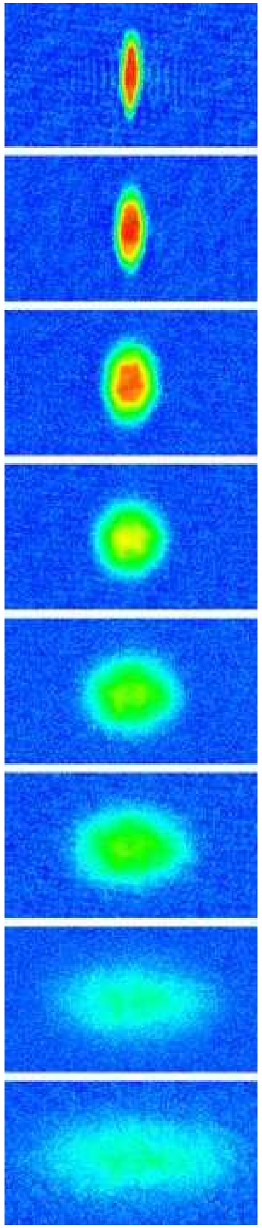
Universal fluid dynamics

Many body system: Effective cross section $\sigma_{tr} \sim n^{-2/3}$ (or $\sigma_{tr} \sim \lambda^2$)

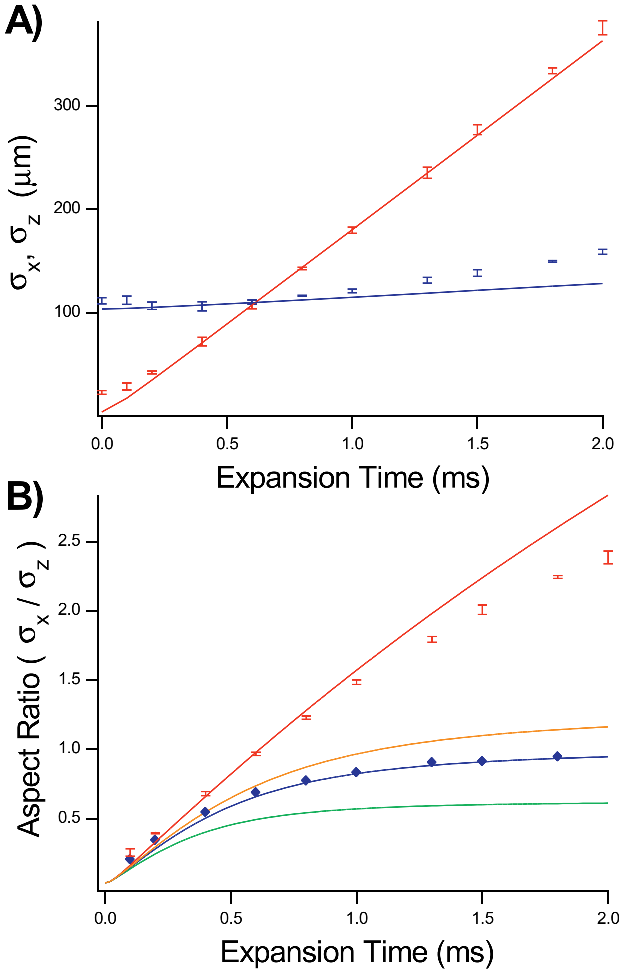


Systems remains hydrodynamic despite expansion

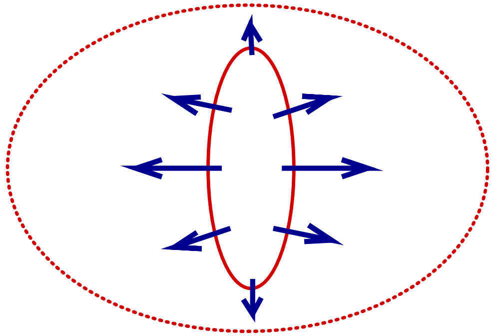
Almost ideal fluid dynamics



O'Hara et al. (2002)

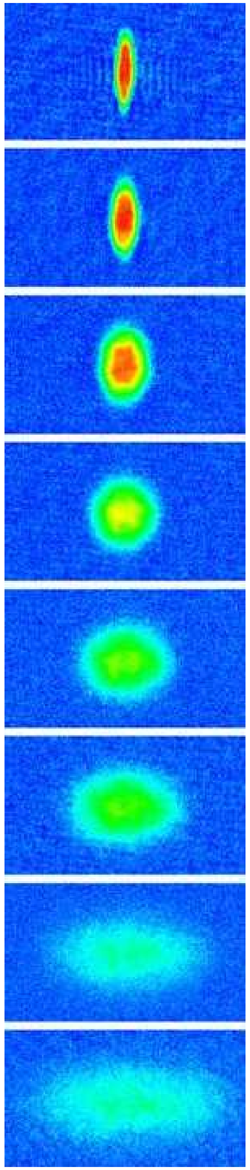


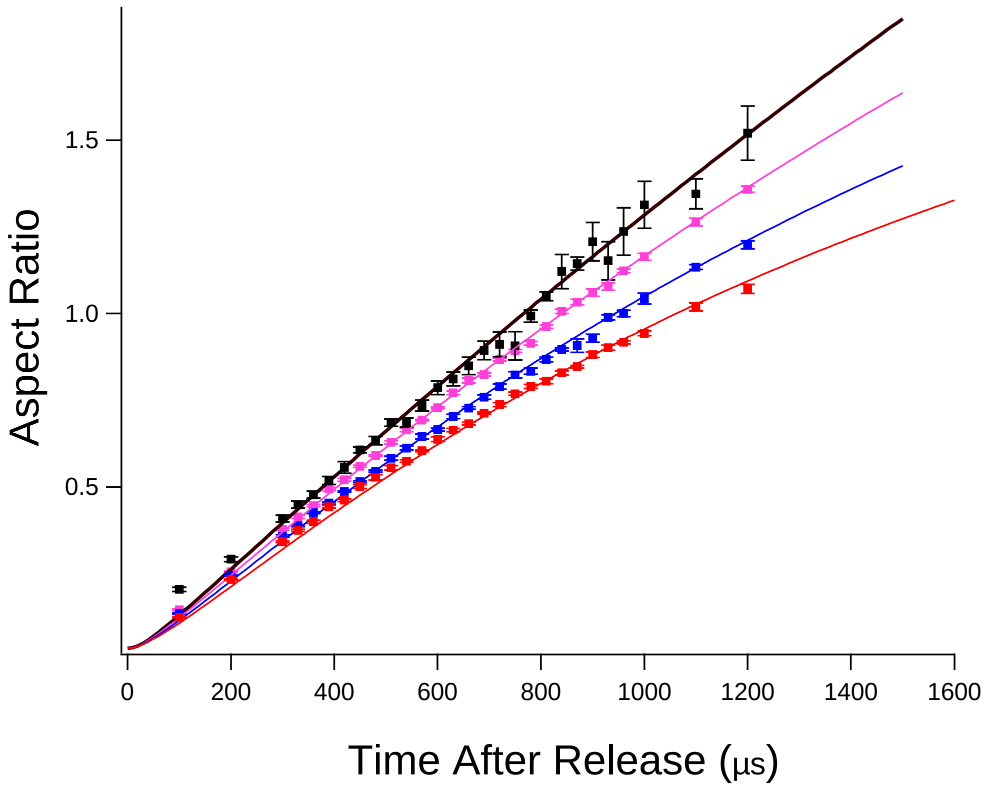
Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy



Elliptic flow: High T limit

Quantum viscosity $\eta = \eta_0 \frac{(mT)^{3/2}}{\hbar^2}$





$\eta = \eta_0 (mT)^{3/2}$

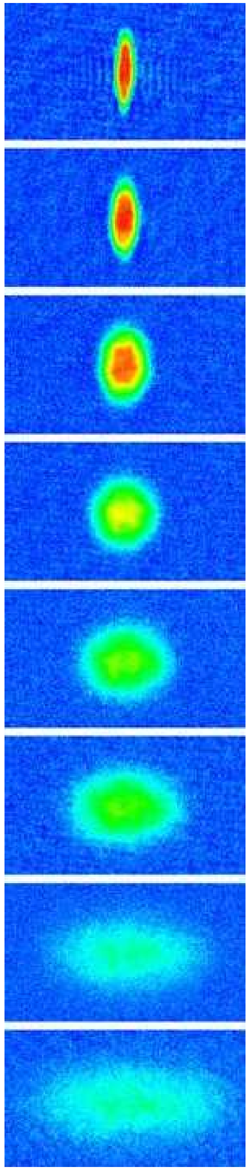
$\tau_R = \eta / P$

Cao, T.S. et al., Science (2010)

fit: $\eta_0 = 0.33 \pm 0.04$

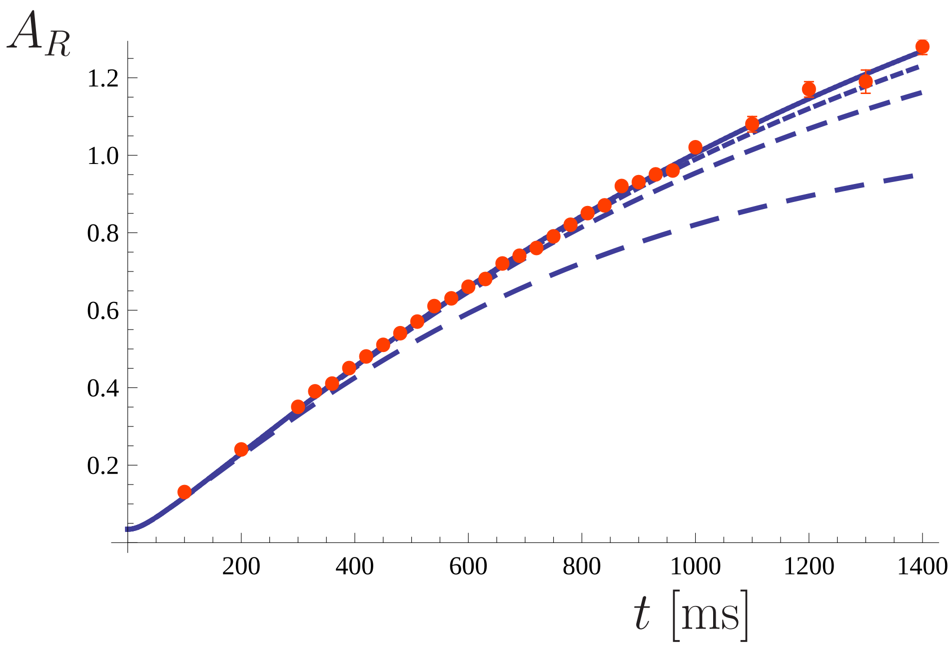
theory: $\eta_0 = \frac{15}{32\sqrt{\pi}} = 0.26$

Elliptic flow: Freezeout?



switch from hydro to (weakly collisional) kinetics

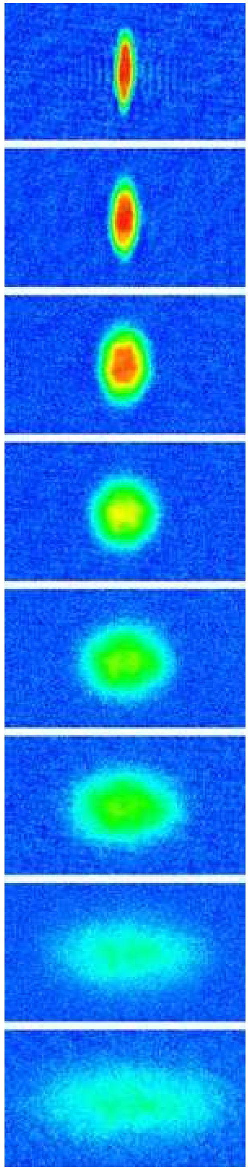
at scale factor $b_\perp^{fr} = 1, 5, 10, 20$



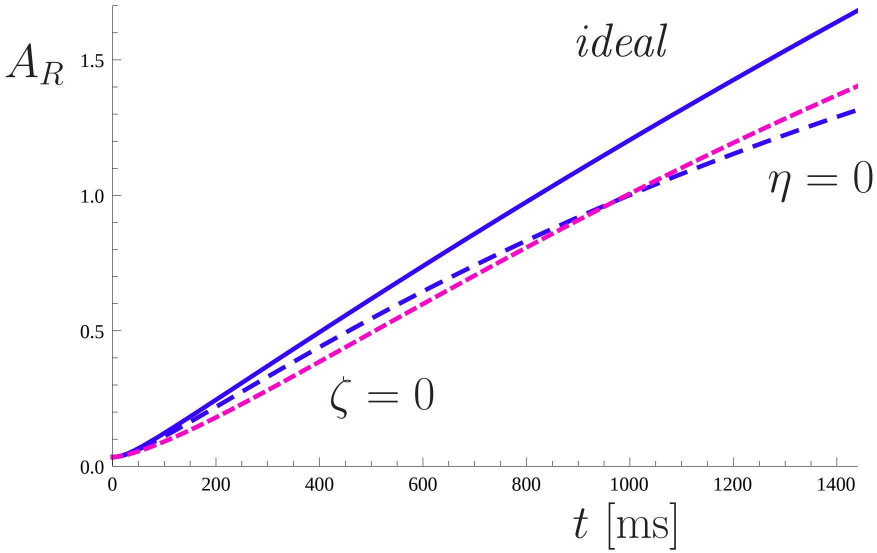
Dusling, Schaefer (2010)

no freezeout seen in the data

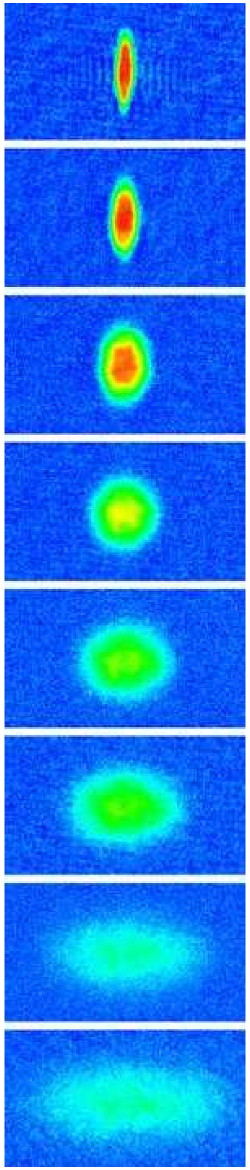
Elliptic flow: Shear vs bulk viscosity



Dissipative hydro with both η, ζ

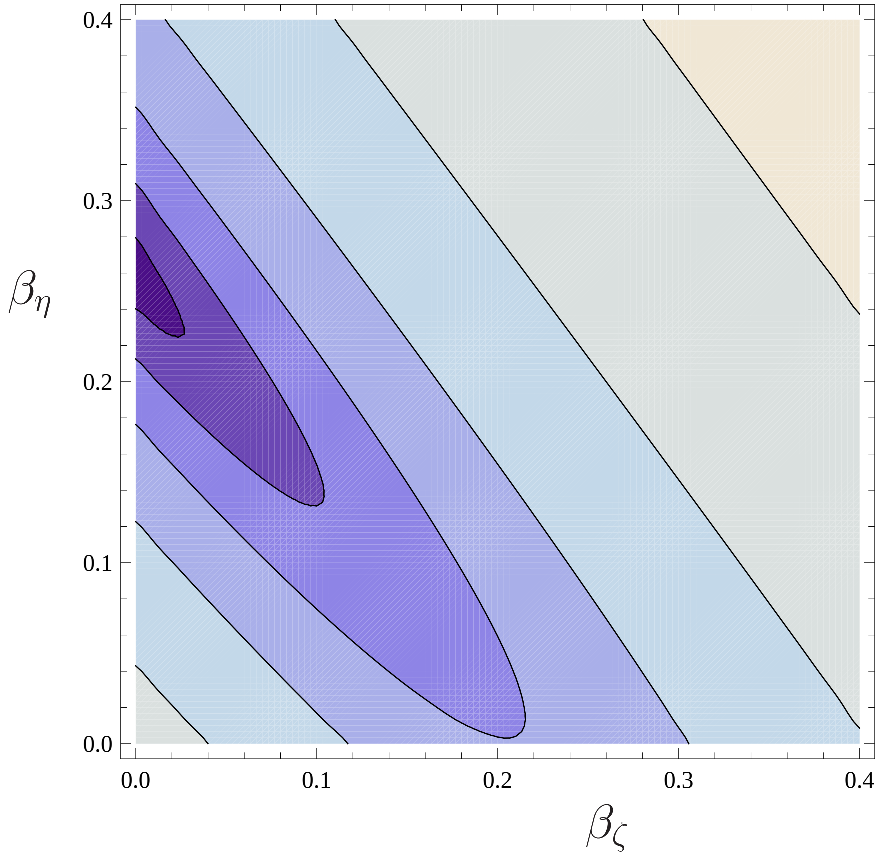


Elliptic flow: Shear vs bulk viscosity



Dissipative hydro with both η, ζ

$$\beta_{\eta,\zeta} = \frac{[\eta,\zeta]}{n}\frac{E_F}{E}\frac{1}{(3\lambda N)^{1/3}}$$



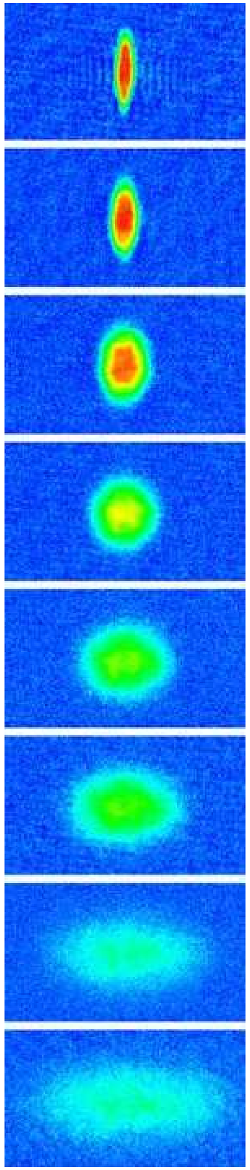
$\eta \gg \zeta$

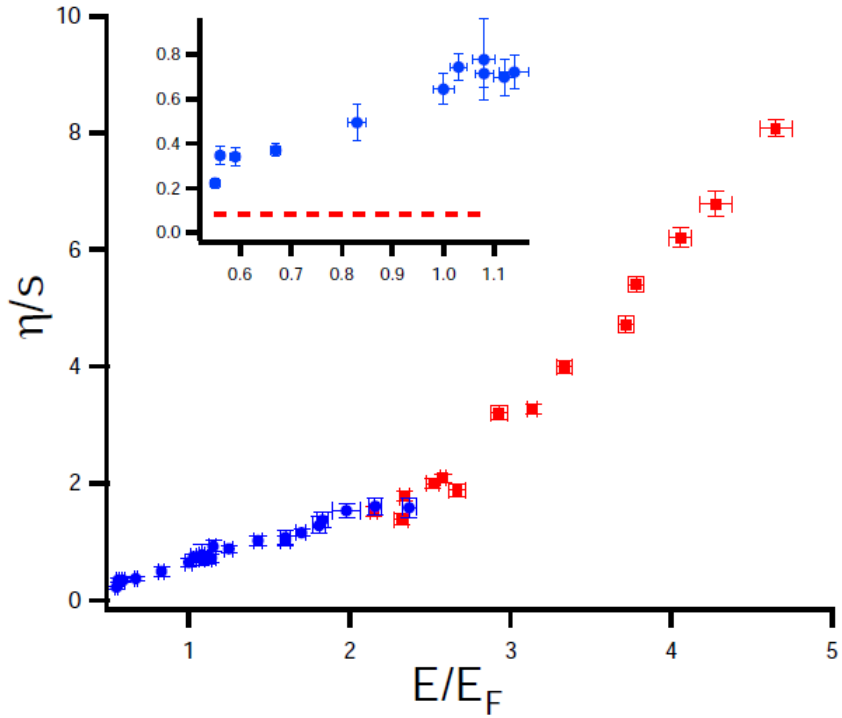
Dusling, Schaefer (2010)

Viscosity to entropy density ratio

consider both collective modes (low T)

and elliptic flow (high T)





Cao, T.S. et al., Science (2010)

$$\eta/s \leq 0.4$$

The bottom-line

Remarkably, the best fluids that have been observed are the *coldest* and the *hottest* fluid ever created in the laboratory, cold atomic gases (10^{-6}K) and the quark gluon plasma (10^{12}K) at RHIC.

Both of these fluids come close to a bound on the shear viscosity that was first proposed based on calculations in string theory, involving non-equilibrium evolution of back holes in 5 (and more) dimensions.

We still do not know whether there is a fundamental lower bound on η.

Outlook

Improved determinations of η/s for both the QGP and cold atomic gases. Need to unfold T, ρ dependence.

Work in progress.

Other transport properties: Bulk viscosity, diffusion constants, relaxation times, etc.

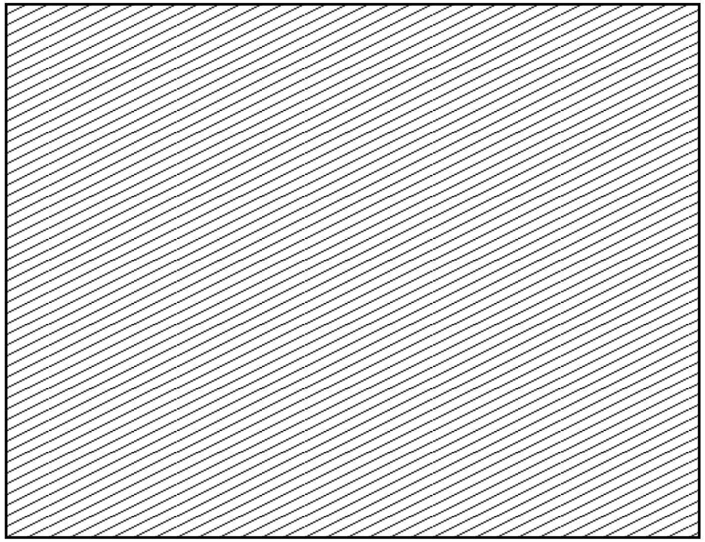
ζ (QGP), T.S., K. Dusling (2012), ζ (CAG) in progress.

Transport dominated by quasi-particles? How can we tell?

Possible path: Spectral fcts, see T.S. (2010), Drut et al.

Extra

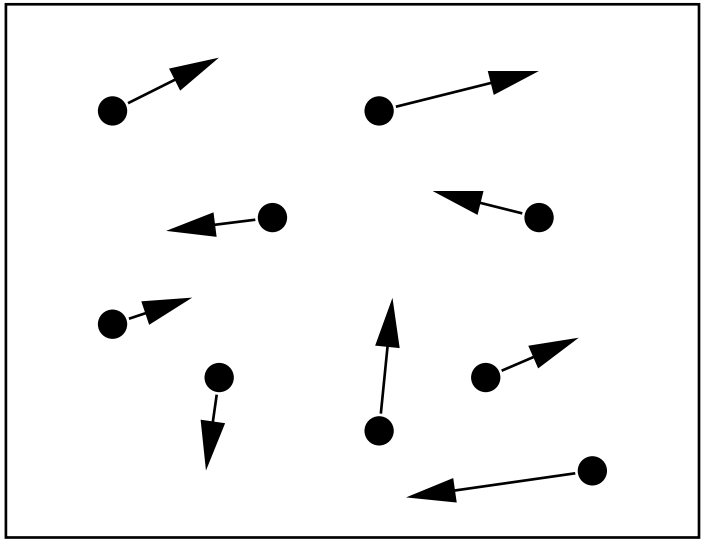
Kinetics vs no-kinetics



low viscosity goo

gravity dual

$\eta/s \simeq 1/(4\pi)$

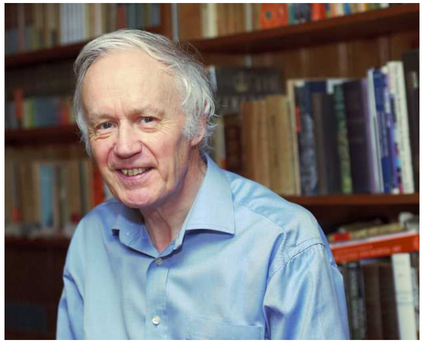


pQCD plasma

quasi-particles

$\eta/s \sim 1/\alpha_s^2 \gg 1$

Effective theories for fluids (Unitary Fermi Gas, $T > T_F$)



$$\mathcal{L} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p = C[f_p] \qquad \omega < T$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \quad \omega < T \frac{n}{\eta}$$

Effective theories (Strong coupling)

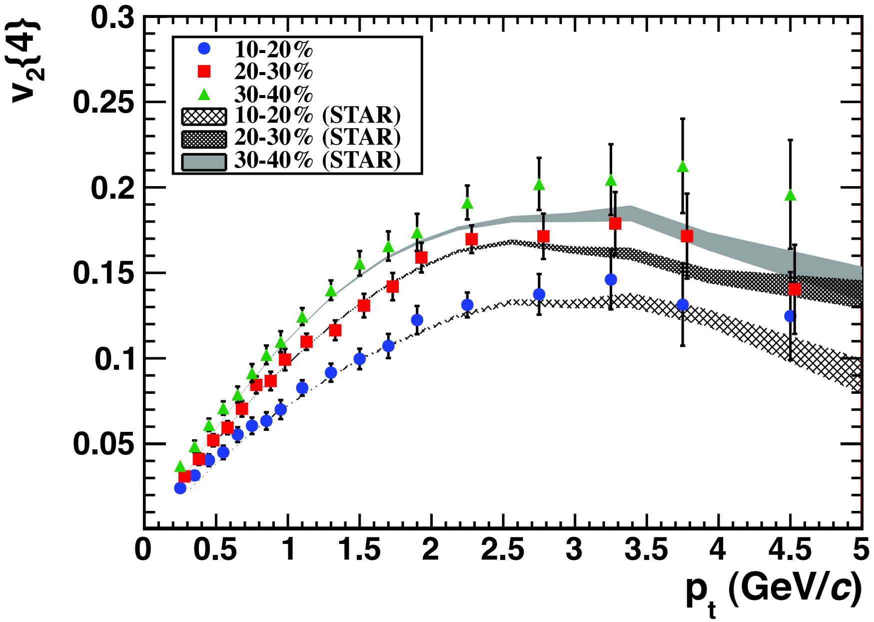
$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \ldots \quad \Leftrightarrow \quad S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\mathcal{R} + \ldots$$

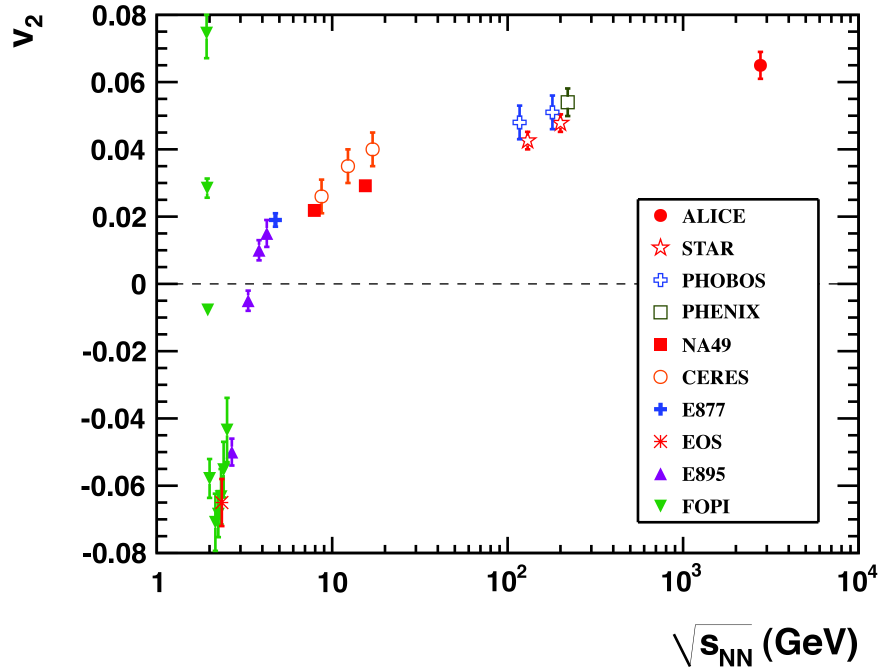
$$SO(d+2,2) \to Schr(d) \qquad\qquad AdS_{d+3} \to \mathcal{X}_{d+3}$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \quad (\omega < T)$$

Nearly perfect fluidity at the LHC?

Yes, but some questions remain.





Alice (2011)

Differential v_2 equal to RHIC

Coincidence? Freezeout?

Integrated v_2 somewhat high

Mean p_T increase?