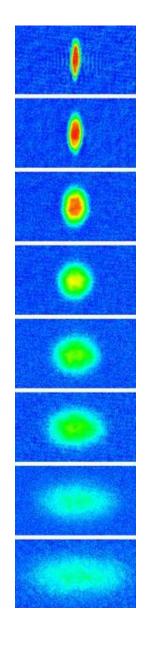
Strongly interacting quantum fluids:

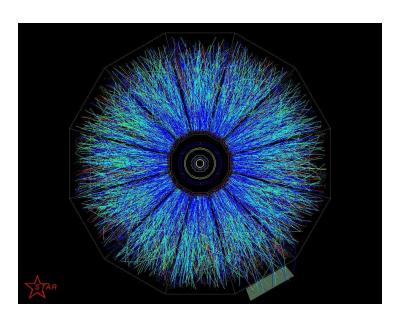
Experimental status

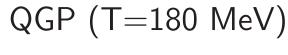
Thomas Schaefer

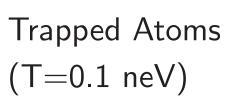
North Carolina State University

Perfect fluids: The contenders





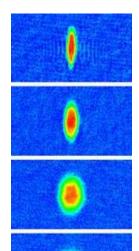


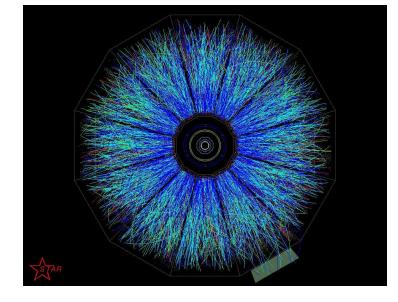




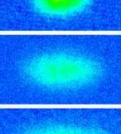
Liquid Helium (T=0.1 meV)

Perfect Fluids: The contenders





QGP
$$\eta = 5 \cdot 10^{11} Pa \cdot s$$



Trapped Atoms

$$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$$

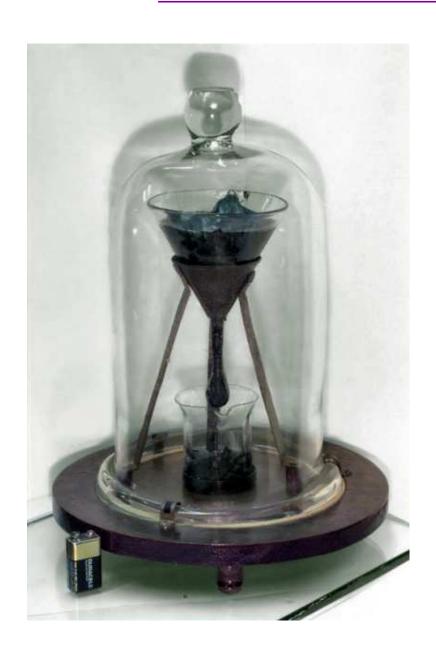


Liquid Helium

$$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$$

Consider ratios η/s

Perfect Fluids: Not a contender



Queensland pitch-drop experiment

1927-2011 (8 drops)

$$\eta = (2.3 \pm 0.5) \cdot 10^8 \, Pa \, s$$

I. Experiment (liquid helium)

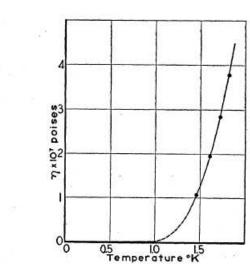
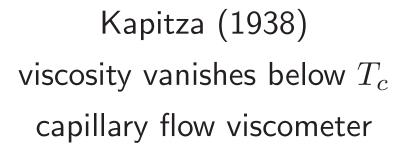
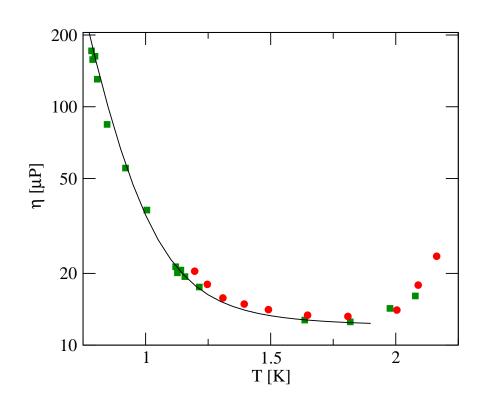


Fig. 1. The viscosity of liquid helium II measured by flow through a $10^{-4} \ \rm cm$ channel.

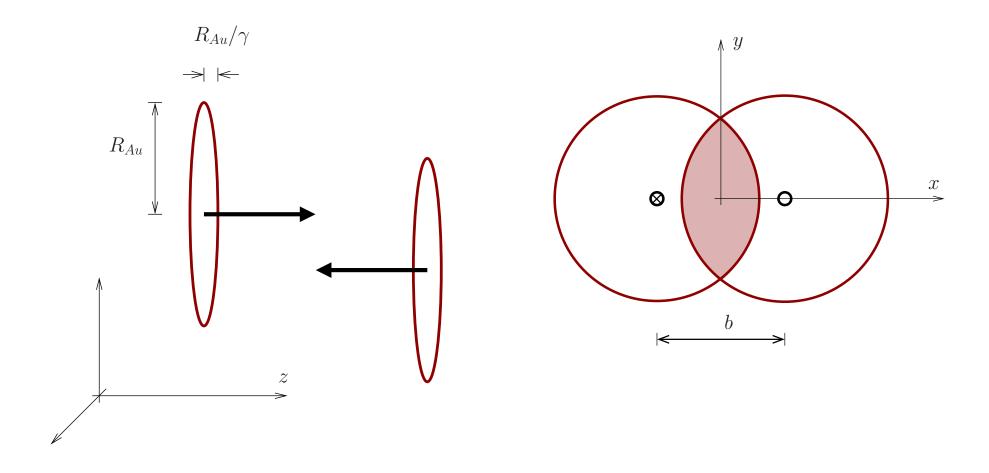




Hollis-Hallett (1955)
roton minimum, phonon rise
rotation viscometer

$$\eta/s \simeq 0.8 \, \hbar/k_B$$

II. Heavy ion collision: Geometry



rapidity:
$$y = \frac{1}{2} \log \left(\frac{E + p_z}{E - p_z} \right)$$

 $\begin{array}{c} \text{transverse} \\ \text{momentum} : \end{array} p_T^2 = p_x^2 + p_y^2$

Bjorken expansion

Experimental observation: At high energy $(\Delta y \to \infty)$ rapidity distributions of produced particles (in both pp and AA) are "flat"

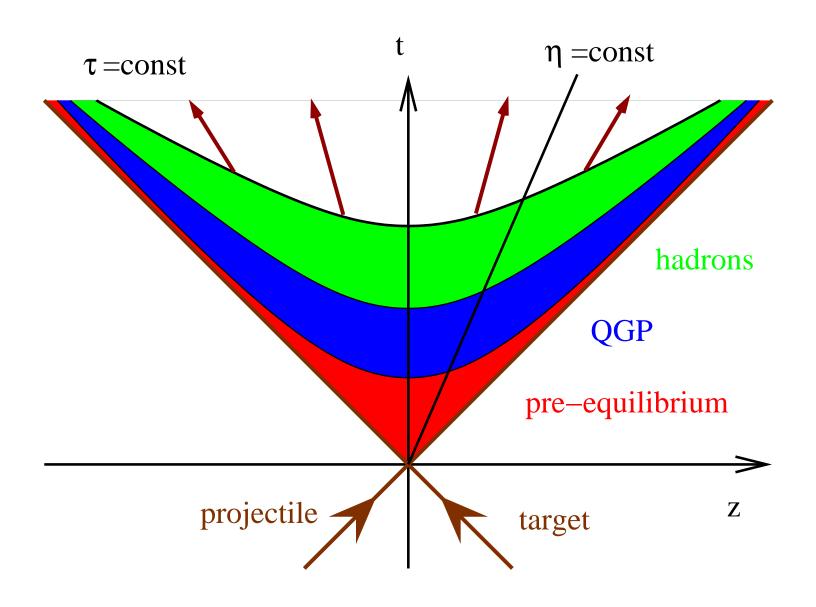
$$\frac{dN}{dy} \simeq const$$

Physics depends on proper time $\tau = \sqrt{t^2 - z^2}$, not on y

All comoving (v = z/t) observers are equivalent

Analogous to Hubble expansion

Bjorken expansion



Bjorken expansion: Hydrodynamics

Boost invariant expansion

$$u^{\mu} = \gamma(1, 0, 0, v_z) = (t/\tau, 0, 0, z/\tau)$$

solves Euler equation (no longitudinal acceleration)

$$\partial^{\mu}(su_{\mu}) = 0 \qquad \Rightarrow \qquad \frac{d}{d\tau} \left[\tau s(\tau)\right] = 0$$

Solution for ideal Bj hydrodynamics

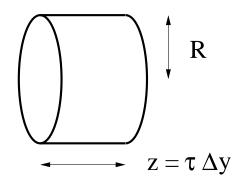
$$s(\tau) = \frac{s_0 \tau_0}{\tau} \qquad T = \frac{const}{\tau^{1/3}}$$

Exact boost invariance, no transverse expansion, no dissipation, ...

Numerical estimates

Total entropy in rapidity interval $[y, y + \Delta y]$

$$S = s\pi R^2 z = s\pi R^2 \tau \Delta y = (s_0 \tau_0) \pi R^2 \Delta y$$
$$s_0 \tau_0 = \frac{1}{\pi R^2} \frac{S}{\Delta y}$$



Use $S/N \simeq 3.6$

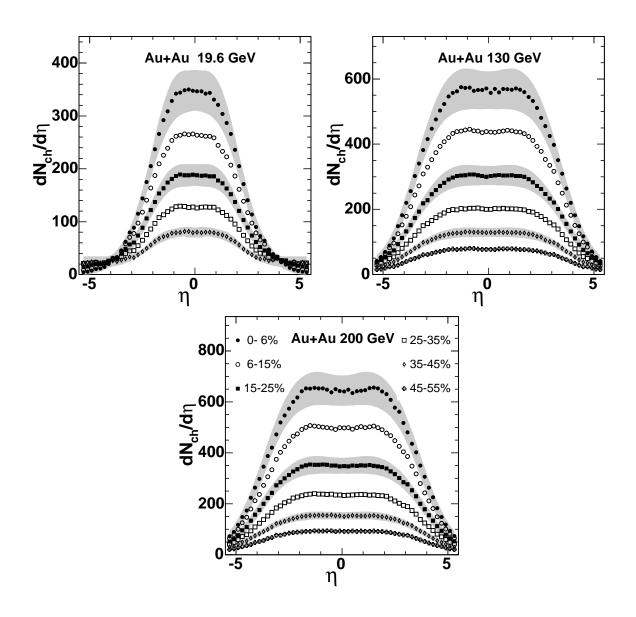
$$s_0 = \frac{3.6}{\pi R^2 \tau_0} \left(\frac{dN}{dy}\right)$$
 Bj estimate
$$\epsilon_0 = \frac{1}{\pi R^2 \tau_0} \left(\frac{dE_T}{dy}\right)$$

Depends on initial time au_0

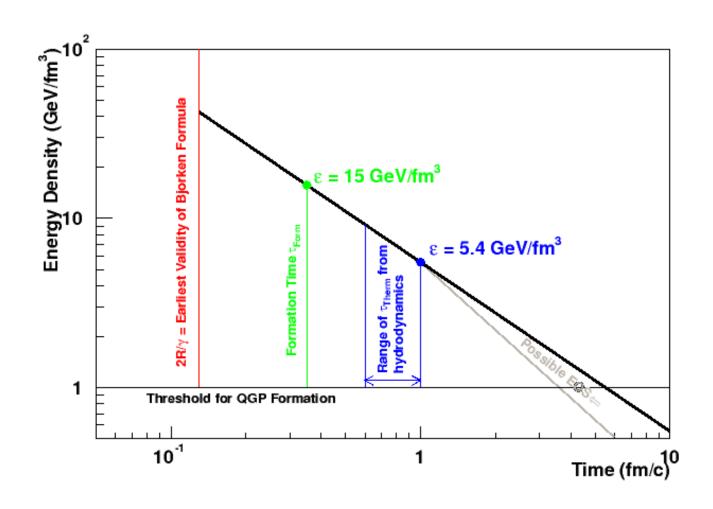
BNL and RHIC



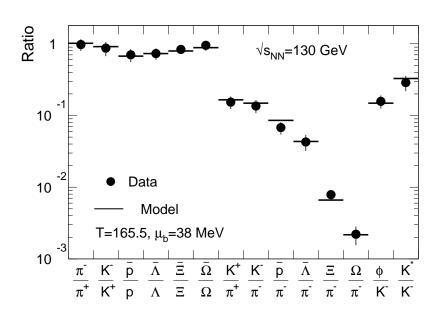
Multiplicities

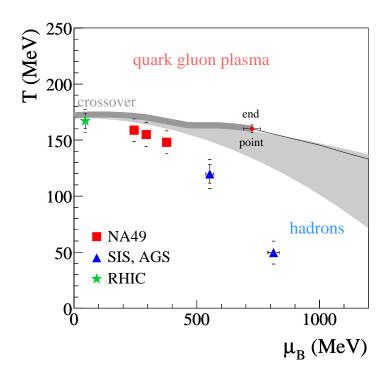


Bjorken expansion



Chemical equilibrium at freezeout

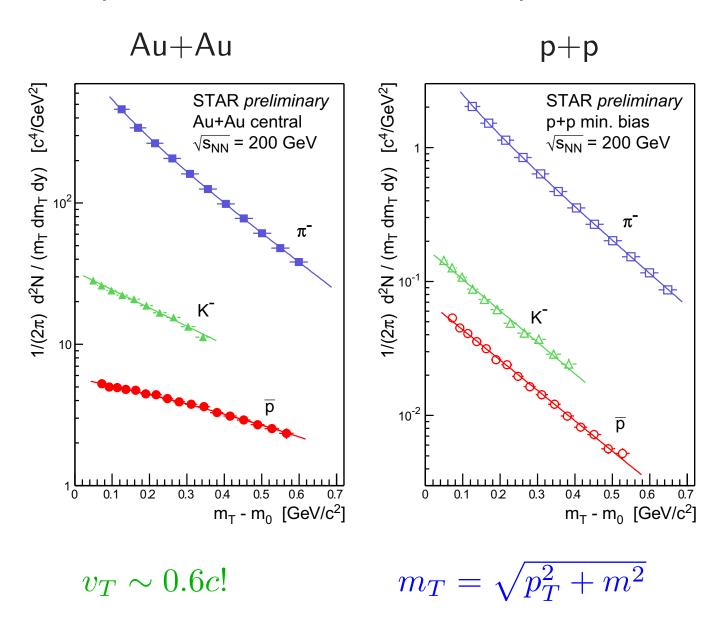




Andronic et al. (2006)

Collective behavior: Radial flow

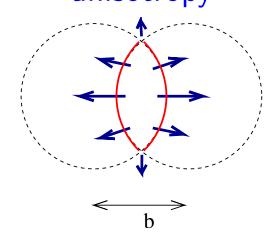
Radial expansion leads to blue-shifted spectra in Au+Au

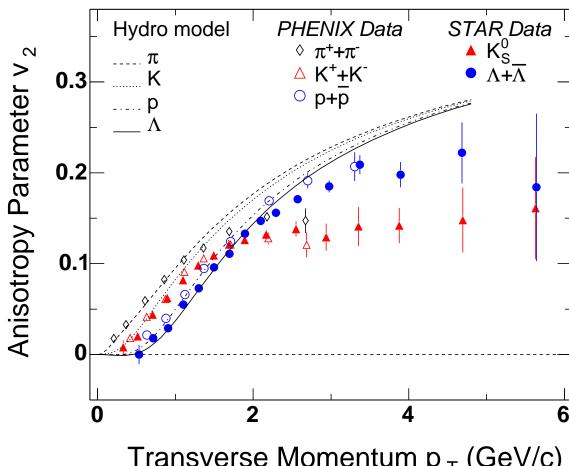


Collective behavior: Elliptic flow

Hydrodynamic expansion converts coordinate space anisotropy

to momentum space anisotropy



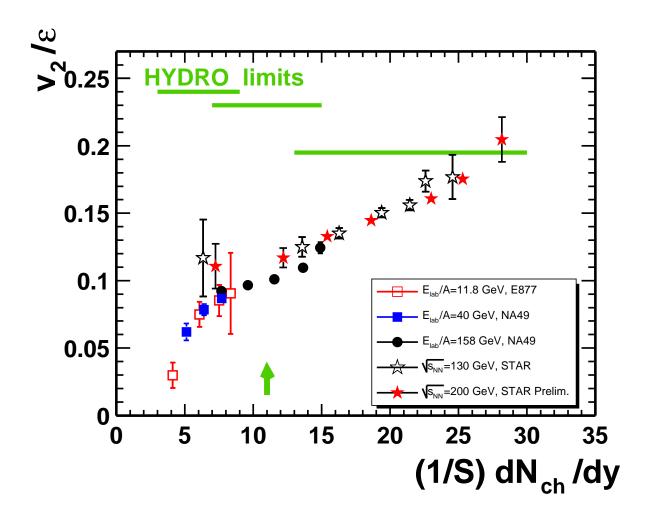


Transverse Momentum p_T (GeV/c)

source: U. Heinz (2005)

$$p_0 \left. \frac{dN}{d^3p} \right|_{p_z=0} = v_0(p_\perp) \left(1 + 2v_2(p_\perp) \cos(2\phi) + \ldots \right)$$

Elliptic flow II: Multiplicity scaling



source: U. Heinz (2005)

Viscous Corrections

Longitudinal expansion: Bj expansion solves Navier-Stokes equation

$$\frac{1}{s}\frac{ds}{d\tau} = -\frac{1}{\tau}\left(1 - \frac{\frac{4}{3}\eta + \zeta}{sT\tau}\right)$$

Viscous corrections small if $\frac{4}{3}\frac{\eta}{s} + \frac{\zeta}{s} \ll (T\tau)$

$$\frac{4}{3}\frac{\eta}{s} + \frac{\zeta}{s} \ll (T\tau)$$

early
$$T\tau \sim \tau^{2/3}$$
 $\eta/s \sim const$ $\eta/s < \tau_0 T_0$

$$/s \sim const$$
 $\eta/s < au_0 T_0$

late
$$T\tau \sim const$$
 $\eta \sim T/\sigma$ $\tau^2/\sigma < 1$

$$\tau^2/\sigma < 1$$

Hydro valid for $\tau \in [\tau_0, \tau_{fr}]$

Viscous corrections to T_{ij} (radial expansion)

$$T_{zz} = P - \frac{4}{3} \frac{\eta}{\tau}$$
 $T_{xx} = T_{yy} = P + \frac{2}{3} \frac{\eta}{\tau}$

increases radial flow (central collision)

decreases elliptic flow (peripheral collision)

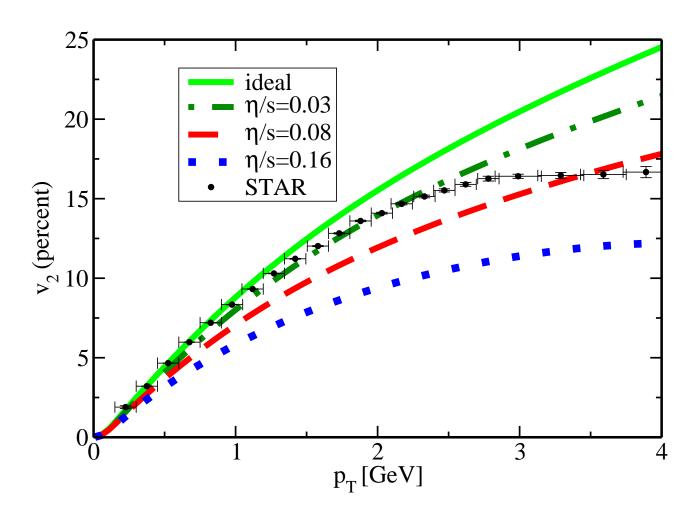
Modification of distribution function

$$\delta f = \frac{3}{8} \frac{\Gamma_s}{T^2} f_0 (1 + f_0) p_\alpha p_\beta \nabla^{\langle \alpha} u^{\beta \rangle}$$

Correction to spectrum grows with p_{\perp}^2

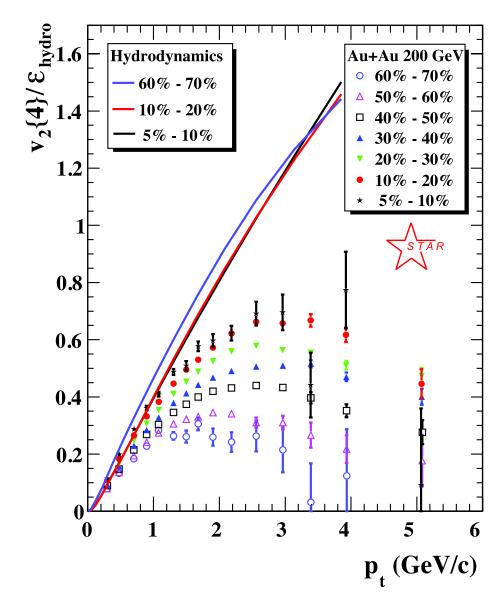
$$\frac{\delta(dN)}{dN_0} = \frac{\Gamma_s}{4\tau_f} \left(\frac{p_\perp}{T}\right)^2$$

Elliptic flow III: Viscous effects



Romatschke (2007), Teaney (2003)

Elliptic flow IV: Systematic trends



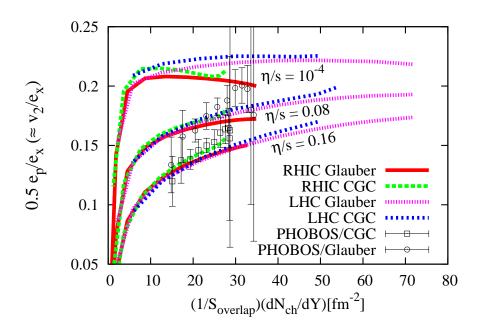
Deviation from ideal hydro

increases for more peripheral events

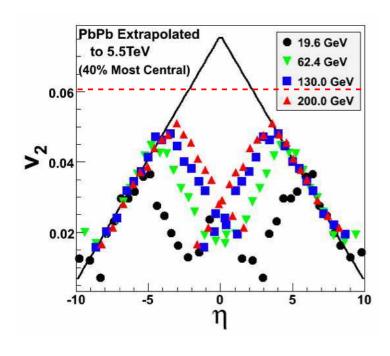
increases with p_{\perp}

source: R. Snellings (STAR)

Elliptic flow V: Predictions for LHC



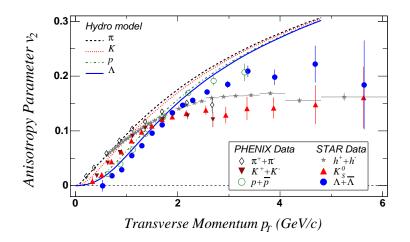
Romatschke, Luzum (2009)

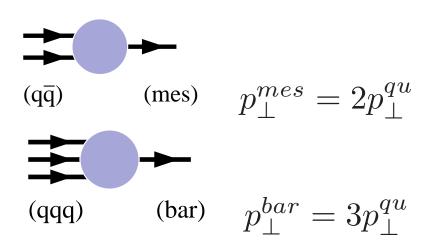


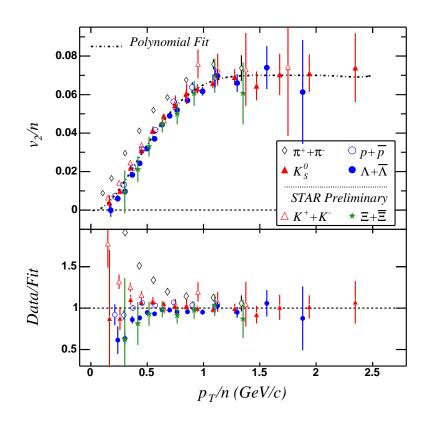
Busza (QM 2009)

Elliptic flow VI: Recombination

"quark number" scaling of elliptic flow

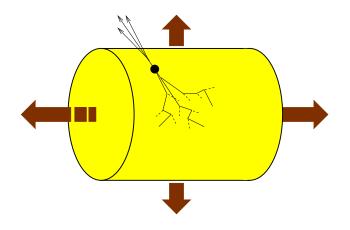


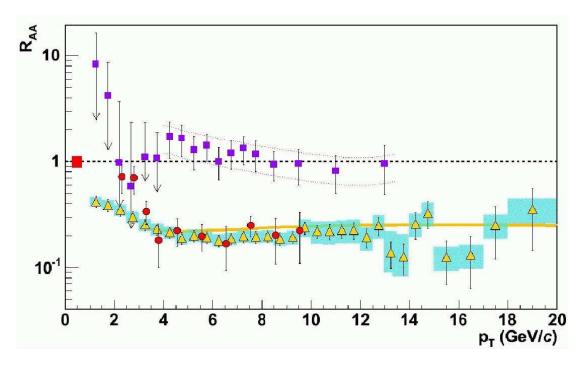




Jet quenching

$$R_{AA} = \frac{n_{AA}}{N_{coll} n_{pp}}$$

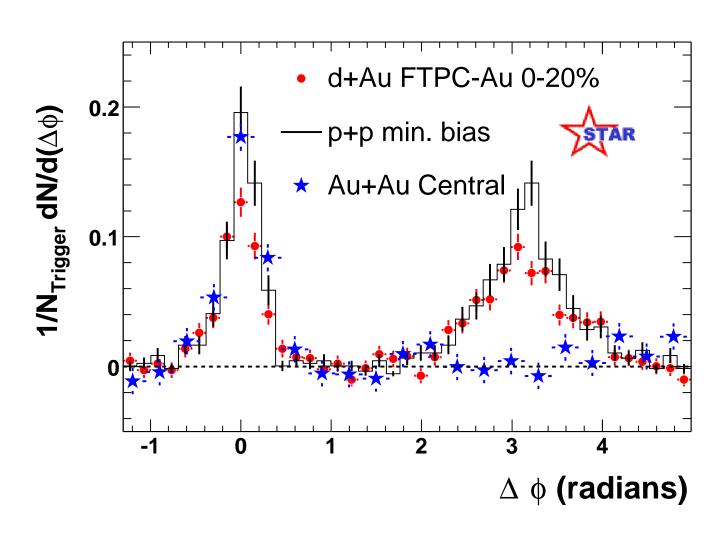




source: Akiba [Phenix] (2006)

Jet quenching II

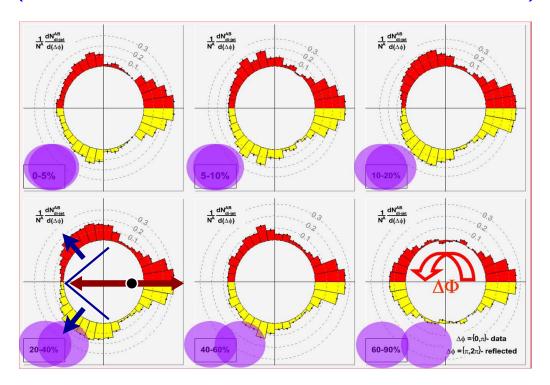
Disappearance of away-side jet



source: Star White Paper (2005)

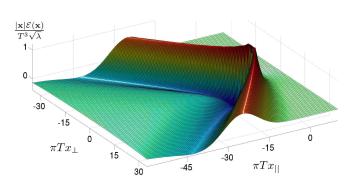
Jet quenching III: The Mach cone

azimuthal multiplicity $dN/d\phi$ (high energy trigger particle at $\phi = 0$) in $\mathcal{N} = 4$ plasma



source: Phenix (PRL, 2006), W. Zajc (2007)

wake of a fast quark

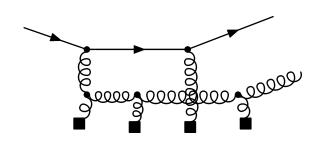


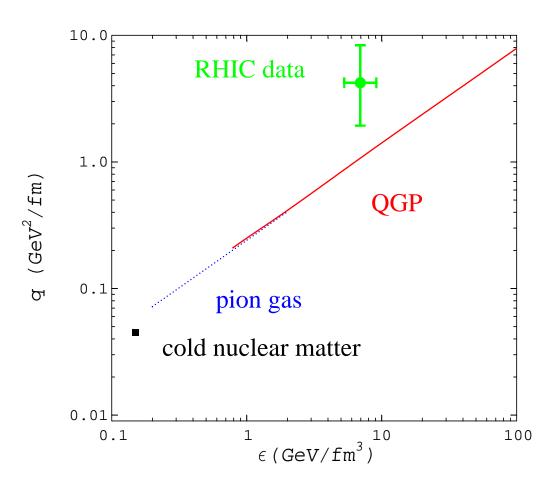
Chesler and Yaffe (2007)

Jet quenching: Theory

energy loss governed by

$$\hat{q} = \rho \int q_{\perp}^2 dq_{\perp}^2 \frac{d\sigma}{dq_{\perp}^2}$$





larger than pQCD predicts? relation to η ? ($\hat{q} \sim 1/\eta$?)

also: large energy loss of heavy quarks

Where are (were) we?

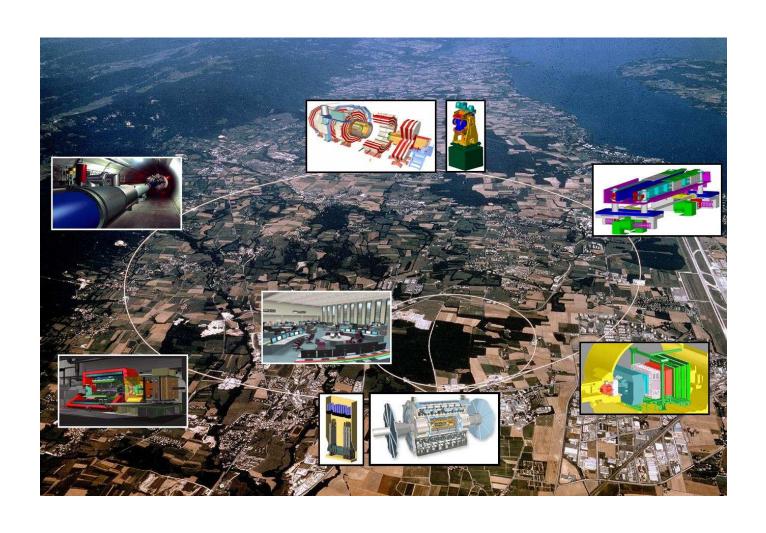
observe almost ideal fluid behavior, initial conditions well above critical energy density.

systematics require $0.1 < \eta/s < 0.4$; more studies needed, LHC elliptic flow will be very interesting.

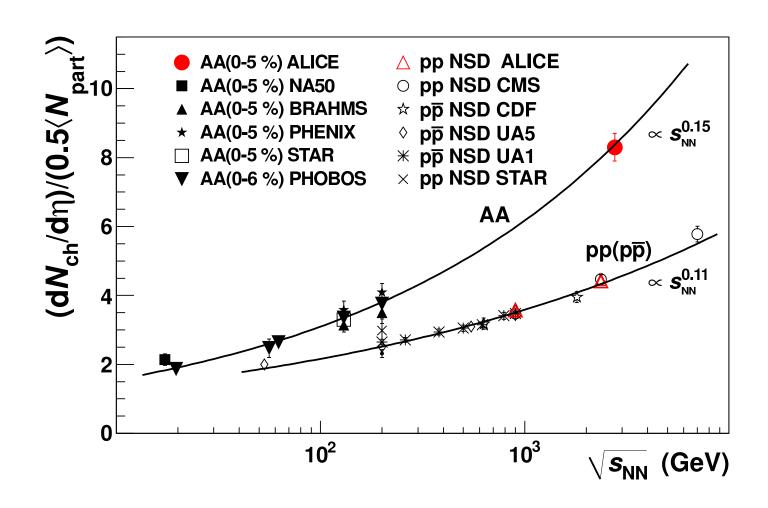
jet quenching large; very detailed studies under way. LHC will provide unprecedented range.

heavy flavors: large energy loss seen, flavor studies (c/b) under way.

LHC: Pb+Pb $\sqrt{s_{NN}}=2.76~{\rm TeV}$



Alice results: Multiplicity scaling with energy



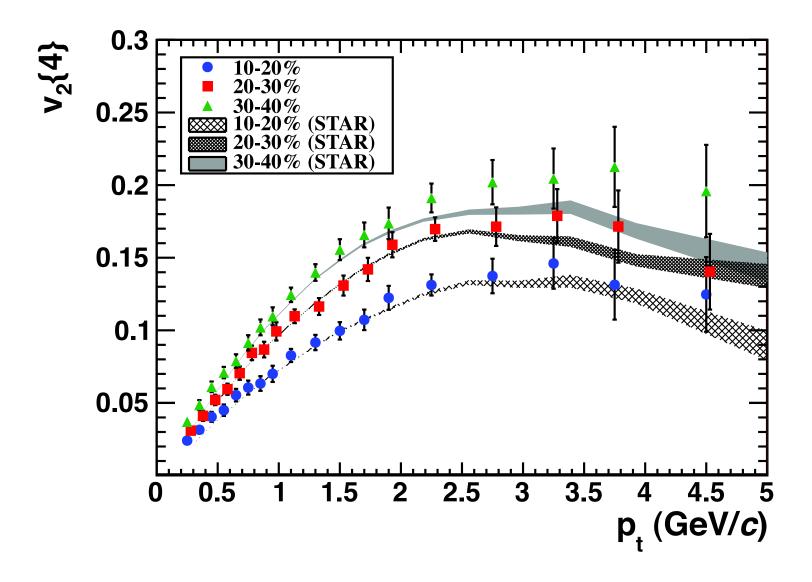
What does it mean?

Factor 2.2 in multiplicity: factor 2.85 in energy density, factor 1.3 in temperature (at fixed τ_0)

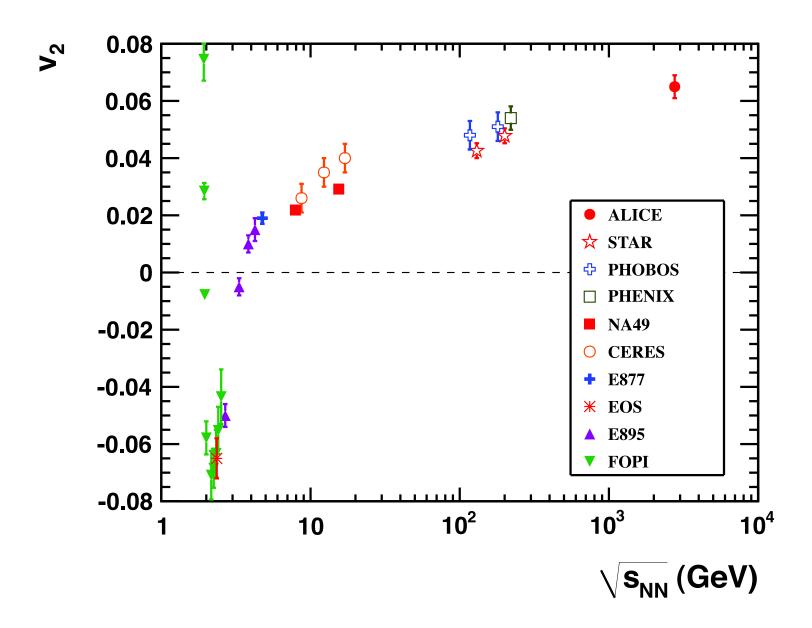
 $AA \neq pp$: extra multiplicity per participant pair.

Simple saturation works better than improved saturation.

Alice flow



Flow excitation function



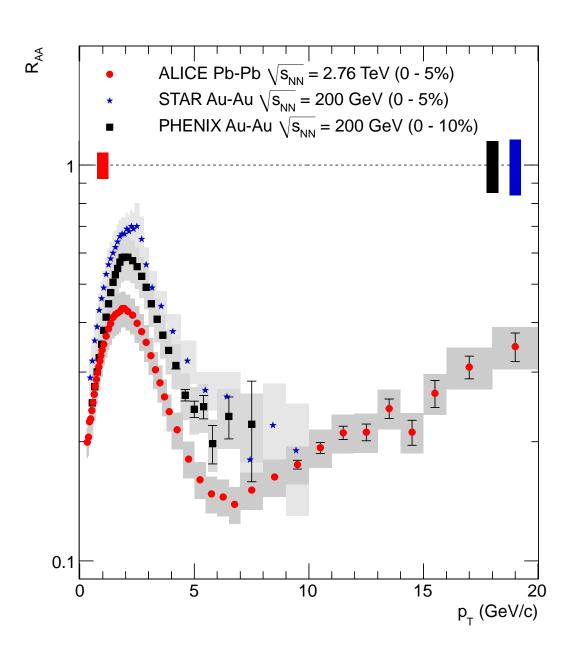
What does it mean?

Hydro rules! RHIC data not an accident.

Differential v_2 exactly equal to RHIC (!?)

Integrated v_2 somewhat high: mean p_T increase? acceptance?

Alice jet quenching

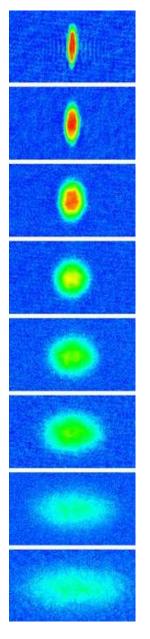


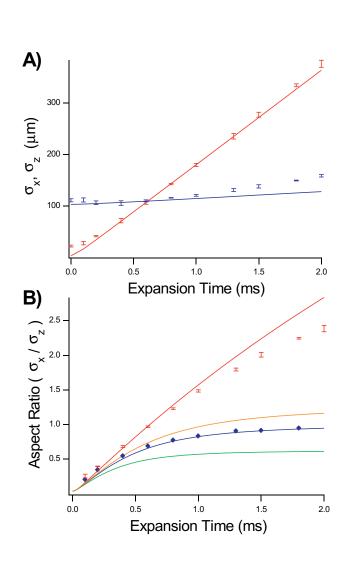
What does it mean?

Suppresion at 10 GeV same as RHIC (!??)

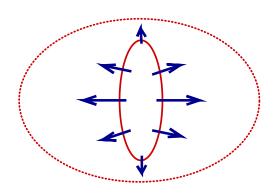
But: p_T dependence no longer flat, agrees with predictions (expect factorization as $p_T \to \infty$).

III: Almost ideal fluid dynamics in cold atomic gases





Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

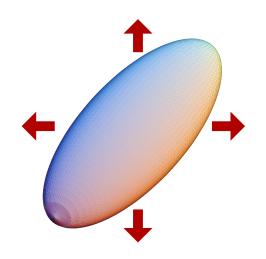


O'Hara et al. (2002)

Almost ideal fluid dynamics: Collective modes

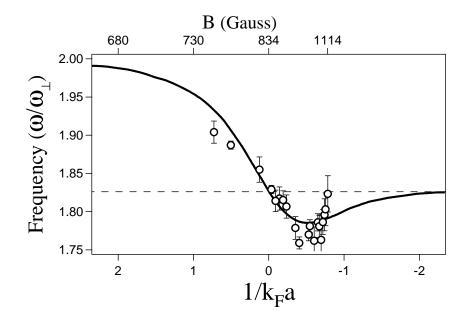
Radial breathing mode

Ideal fluid hydrodynamics $(P \sim n^{5/3})$



$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{\vec{\nabla}P}{mn} - \frac{\vec{\nabla}V}{m}$$



Hydro frequency at unitarity

$$\omega = \sqrt{\frac{10}{3}} \, \omega_{\perp}$$

experiment: Kinast et al. (2005)

Dissipation (scaling flows)

Energy dissipation (η, ζ, κ) : shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{1}{2} \int d^3x \, \eta(x) \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right)^2$$
$$- \int d^3x \, \zeta(x) \left(\partial_i v_i \right)^2 - \frac{1}{T} \int d^3x \, \kappa(x) \left(\partial_i T \right)^2$$

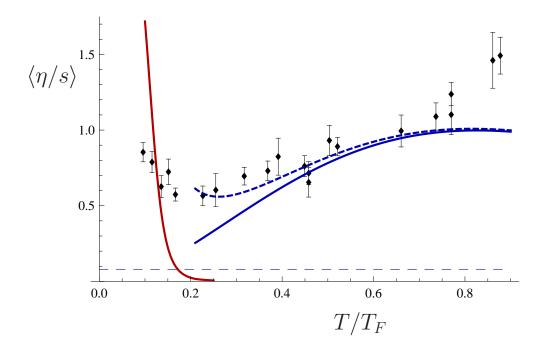
Have $\zeta = 0$ and T(x) = const. Universality implies

$$\eta(x) = s(x) \alpha_s \left(\frac{T}{\mu(x)}\right)$$
$$\int d^3x \, \eta(x) = S\langle \alpha_s \rangle$$

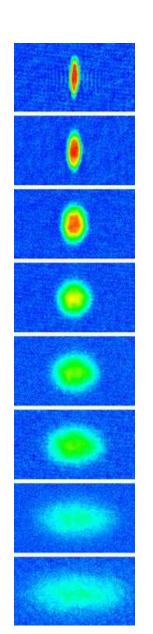
Collective modes: Small viscous correction exponentiates

$$a(t) = a_0 \cos(\omega t) \exp(-\Gamma t)$$

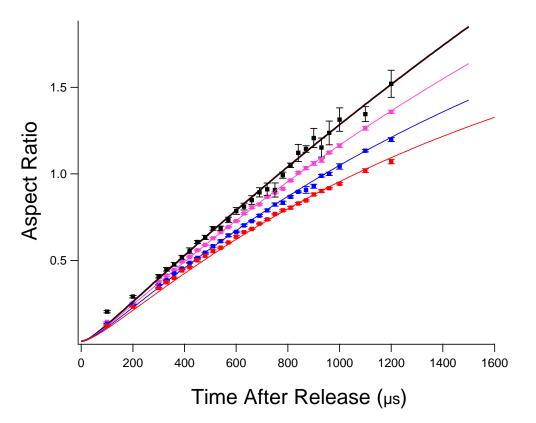
$$\langle \eta/s \rangle = (3N\lambda)^{1/3} \left(\frac{\Gamma}{\omega_{\perp}}\right) \left(\frac{E_0}{E_F}\right) \left(\frac{N}{S}\right)$$



Elliptic flow: High T limit



Quantum viscosity
$$\eta = \eta_0 \frac{(mT)^{3/2}}{\hbar^2}$$



$$\eta = \eta_0 (mT)^{3/2}$$

$$\tau_R = \eta/P$$

Cao et al., Science (2010)

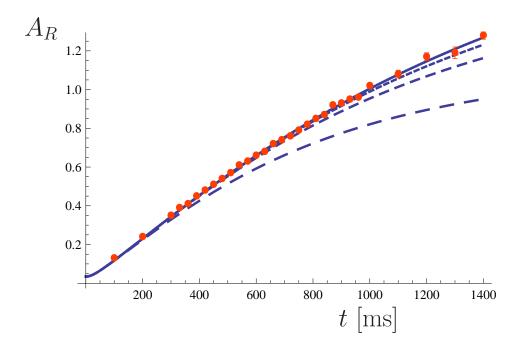
fit:
$$\eta_0 = 0.33 \pm 0.04$$

theory:
$$\eta_0 = \frac{15}{32\sqrt{\pi}} = 0.26$$

Elliptic flow: Freezeout?



at scale factor
$$b_{\perp}^{\!fr}=1,5,10,20$$

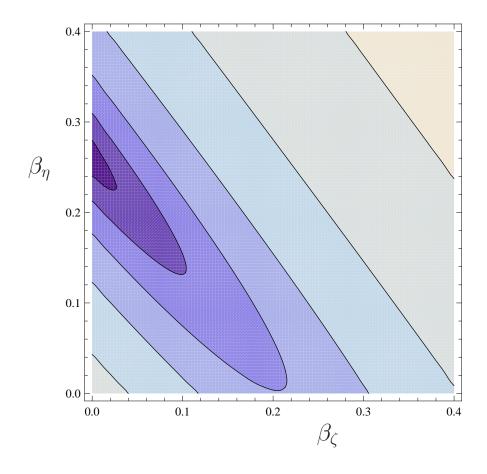


no freezeout seen in the data

Elliptic flow: Shear vs bulk viscosity

Dissipative hydro with both η, ζ

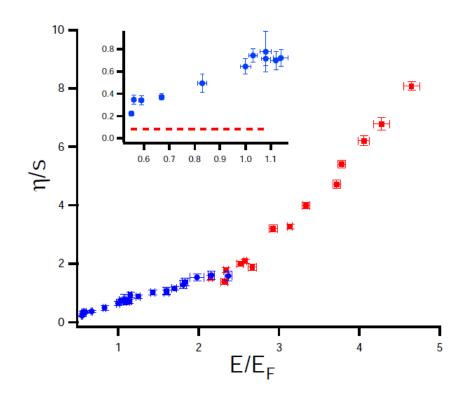
$$\beta_{\eta,\zeta} = \frac{(\eta,\zeta)}{n} \frac{E_F}{E} \frac{1}{(3\lambda N)^{1/3}}$$



$$\eta \gg \zeta$$

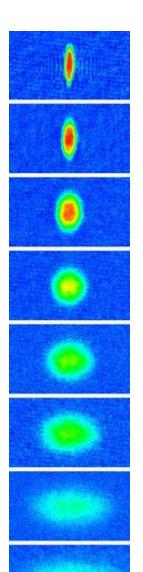
Viscosity to entropy density ratio

consider both collective modes (low T) and elliptic flow (high T)



Cao et al., Science (2010)

$$\eta/s \le 0.4$$



Where are we?

Hydro rules: Consistent explanation of expansion and collective mode data, no freezeout seen in the data.

Collective mode data gives $\langle \eta/s \rangle < 0.4$.

Local analysis requires second order hydro or hydro+kinetics.

The bottom-line

Remarkably, the best fluids that have been observed are the coldest and the hottest fluid ever created in the laboratory, cold atomic gases $(10^{-6} \rm K)$ and the quark gluon plasma $(10^{12} \rm K)$ at RHIC.

Both of these fluids come close to a bound on the shear viscosity that was first proposed based on calculations in string theory, involving nonequilibrium evolution of back holes in 5 (and more) dimensions.