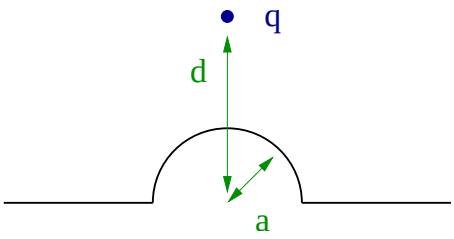
Homework 6, due 9-30

1. A conductor has the shape of an infinite conducting plane except for a spherical boss of radius a. A charge q is placed above the center of the boss at a distance d from the plane. Compute the force on the charge. (Hint: Imagine that the plate is absent, and that there is only the charge q and a conducting sphere of radius a. How does the conducting plane change the problem?)



2. (The method of conformal mappings) Consider a complex function $w(z) = \Phi(x + iy) + i\chi(x + iy)$.

   (a) Show that if $w(z)$ is an analytic function ($w(x, y)$ satisfies the Cauchy-Riemann differential equations) then both Φ and χ satisfy the two-dimensional Laplace equation.

   (b) Show: If Φ is interpreted as the electrostatic potential then lines of constant χ represents the lines of force. Also show that the lines $\Phi = const$ and $\chi = const$ are orthogonal.

   (c) Consider $w(z) = -2\log(z)$. What is the physical situation represented by $\Phi(x, y) = \mathrm{Re}w(z)$?

   (d) Find a complex function $w(z)$ that represents two parallel line charges $\pm\lambda$ in three dimensions. Calculate the surface charge density induced by a line charge located at $x = a$, $y = z = 0$ on a conducting plane at $x = 0$.