QCD at High Temperature

(Theory)

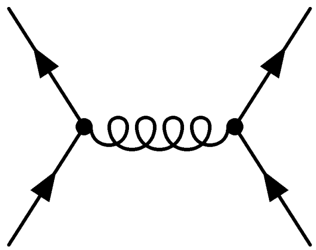
The High T Phase: Qualitative Argument

High T phase: Weakly interacting gas of quarks and gluons?

typical momenta $p \sim 3T$

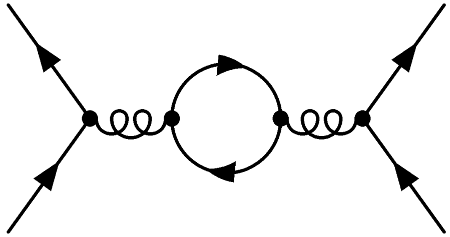
Large angle scattering involves large momentum transfer

effective coupling is small



Small angle scattering is screened (not anti-screened!)

coupling does not become large



Quark Gluon Plasma

Basic Thermodynamics

Massless particles, zero baryon density $(\zeta(3) = 1.2)$

$$n = g\frac{\zeta(3)}{\pi^2}T^3 \begin{cases} 1 \\ 3/4 \end{cases} \qquad \epsilon = g\frac{\pi^2}{30}T^4 \begin{cases} 1 & \text{bosons} \\ 7/8 & \text{fermions} \end{cases}$$

$$s/n = 2\pi^4/(45\zeta(3)) \simeq 3.6 \qquad P = \epsilon/3$$

massless quarks and gluons

$$g_{eff} = 2 \times 8 \times 1 + 4 \times 3 \times 2 \times 7/8 = 37$$

spin $\times$ color $\times$ boson $+$ spin $\times$ color $\times$ flavors $\times$ fermion

massless pions

$$g = (N_f^2 - 1) \times 1 = 3$$

First Approach: Bag Model

Low temperature: Pions

$$\epsilon = \frac{3\pi^2}{30} T^4 \qquad\qquad P = \frac{3\pi^2}{90} T^4$$

High temperature: Quarks and gluons

$$\epsilon = \frac{37\pi^2}{30} T^4 \qquad\qquad P = \frac{37\pi^2}{90} T^4$$

Include vacuum energy $T_{\mu\nu} = B g_{\mu\nu}$ (QCD cosmological constant)

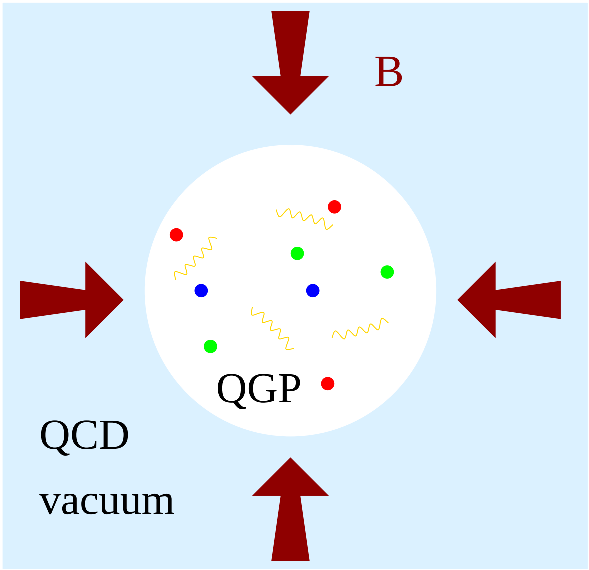
$$\epsilon_{vac} = -P_{vac} = -B \qquad\qquad \epsilon_{vac} = -\frac{b}{32} \langle \frac{\alpha}{\pi} G^2 \rangle \simeq -0.5 \text{ GeV/fm}^3$$

trace anomaly relation

Critical temperature: equate pressures

$$\frac{3\pi^2}{90}T^4 + B = \frac{37\pi^2}{90}T^4$$

$$T_c = \left(\frac{45B}{17\pi^2}\right)^{1/4} \simeq 180 \text{ MeV}$$



Pressure is continuous, but energy density jumps

$$\begin{aligned} \epsilon(T_c^-) &= \frac{3\pi^2}{30}T_c^4 \simeq 100 \text{ MeV/fm}^3 \\ \epsilon(T_c^+) &= \frac{37\pi^2}{30}T_c^4 + B \simeq 2000 \text{ MeV/fm}^3 \end{aligned}$$

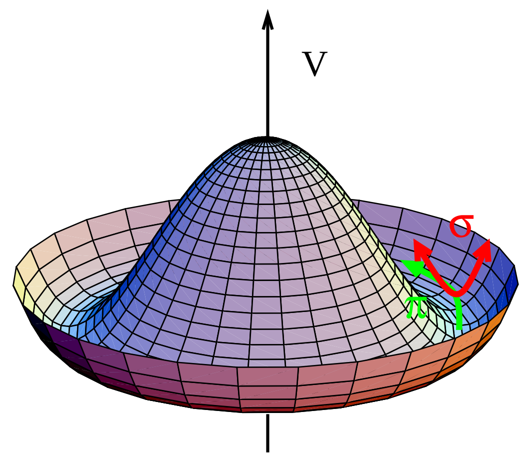
Second Approach: Sigma Model

Simple model based on linear representation of $SU(2)_L \times SU(2)_R$

$$\phi^a = (\sigma, \vec{\pi}) \qquad\qquad O(4) = SU(2)_L \times SU(2)_R$$

Chirally symmetric lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^a)^2 + V(\phi^a \phi^a)$$



$$V(\phi^a \phi^a) = -\frac{\mu^2}{2}(\phi^a \phi^a) + \frac{\lambda}{4}(\phi^a \phi^a)^2$$

Minimum of potential

$$\frac{\partial V}{\partial \phi^a} = \phi^a(-\mu^2 + \lambda \phi^a \phi^a) = 0 \qquad \phi_0^a = (\sigma_0, \vec{0}) \;\; \sigma_0^2 = \mu^2/\lambda \equiv f_\pi^2$$

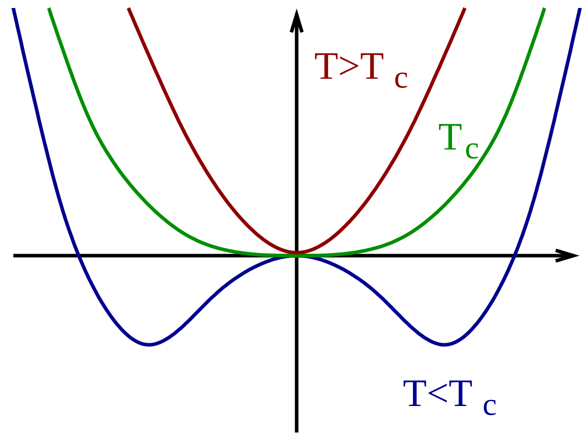
Direction fixed by explicit breaking $\mathcal{L}_{SB} = -c\sigma$

Thermal Fluctuations

Thermal averages

$$\vec{\pi}_T = 0$$

$$\sigma_T^2 = f_\pi^2 \left(1 - \frac{1}{f_\pi^2} \langle \tilde{\phi}^a \tilde{\phi}^a \rangle \right)$$



Gaussian fluctuations ($m = 0$)

$$\langle \tilde{\phi}^a \tilde{\phi}^a \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_k} \frac{1}{e^{\beta\omega_k} - 1} = \frac{T^2}{12}$$

Critical temperature (3 light d.o.f.)

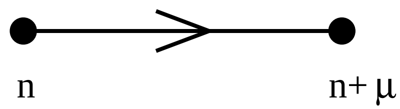
$$\sigma_T^2 = f_\pi^2 \left(1 - \frac{T^2}{3 f_\pi^2} \right) \qquad T_c = \sqrt{3} f_\pi \simeq 160 \text{ MeV}$$

Lattice QCD

Euclidean partition function

$$Z = \int dA_\mu d\psi \exp(-S) = \int dA_\mu \det(i\not{D}\,) \exp(-S_G)$$

Lattice discretization:



$$U_\mu(n) = \exp(igaA_\mu(n))$$

$$D_\mu \phi \;\rightarrow\; \frac{1}{a}[U_\mu(n)\phi(n+\mu) - \phi(n)]$$

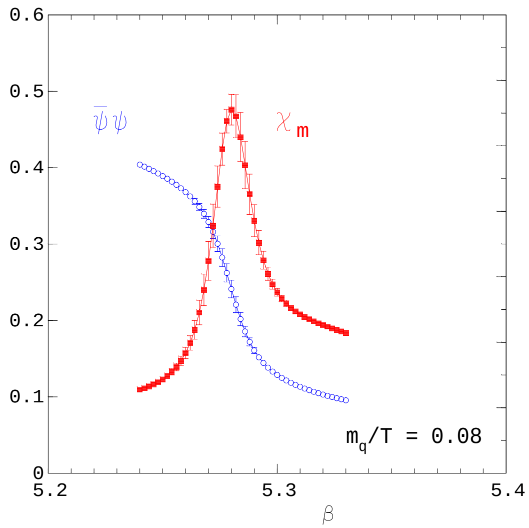
$$(G^a_{\mu\nu})^2 \;\rightarrow\; \frac{1}{a^4}\mathrm{Tr}[U_\mu(n)U_\nu(n+\mu)U_{-\mu}(n+\mu+\nu)U_{-\nu}(n+\nu) - 1]$$

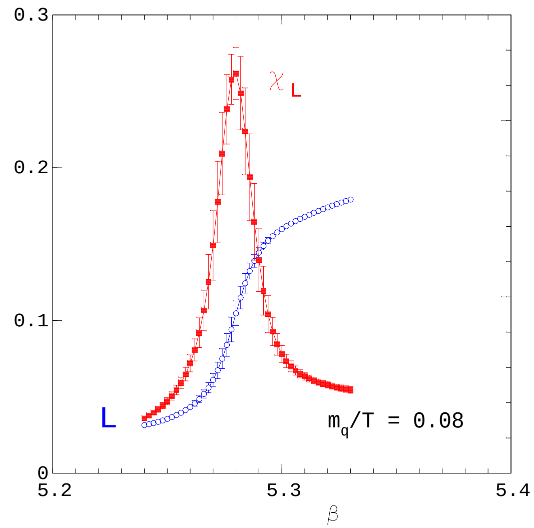
Monte Carlo:

$$\int dA_\mu \; e^{-S} \rightarrow \{U^{(1)}_\mu(n), U^{(2)}_\mu(n), \ldots\}$$

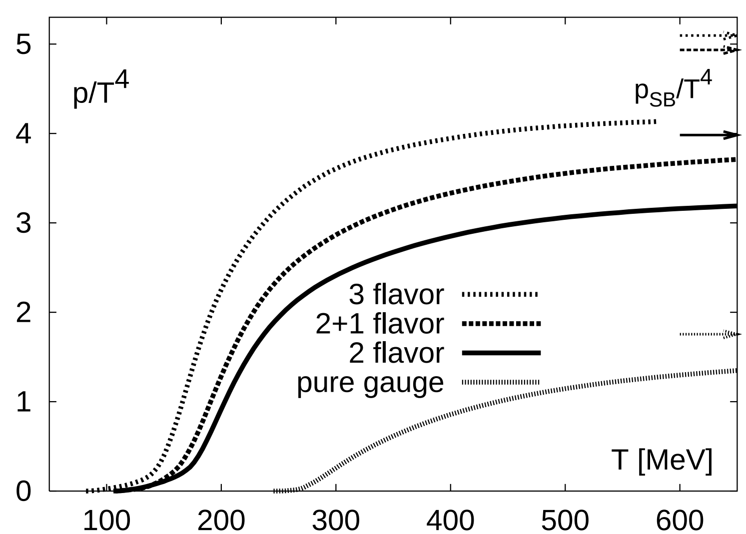
Lattice Results

order parameters

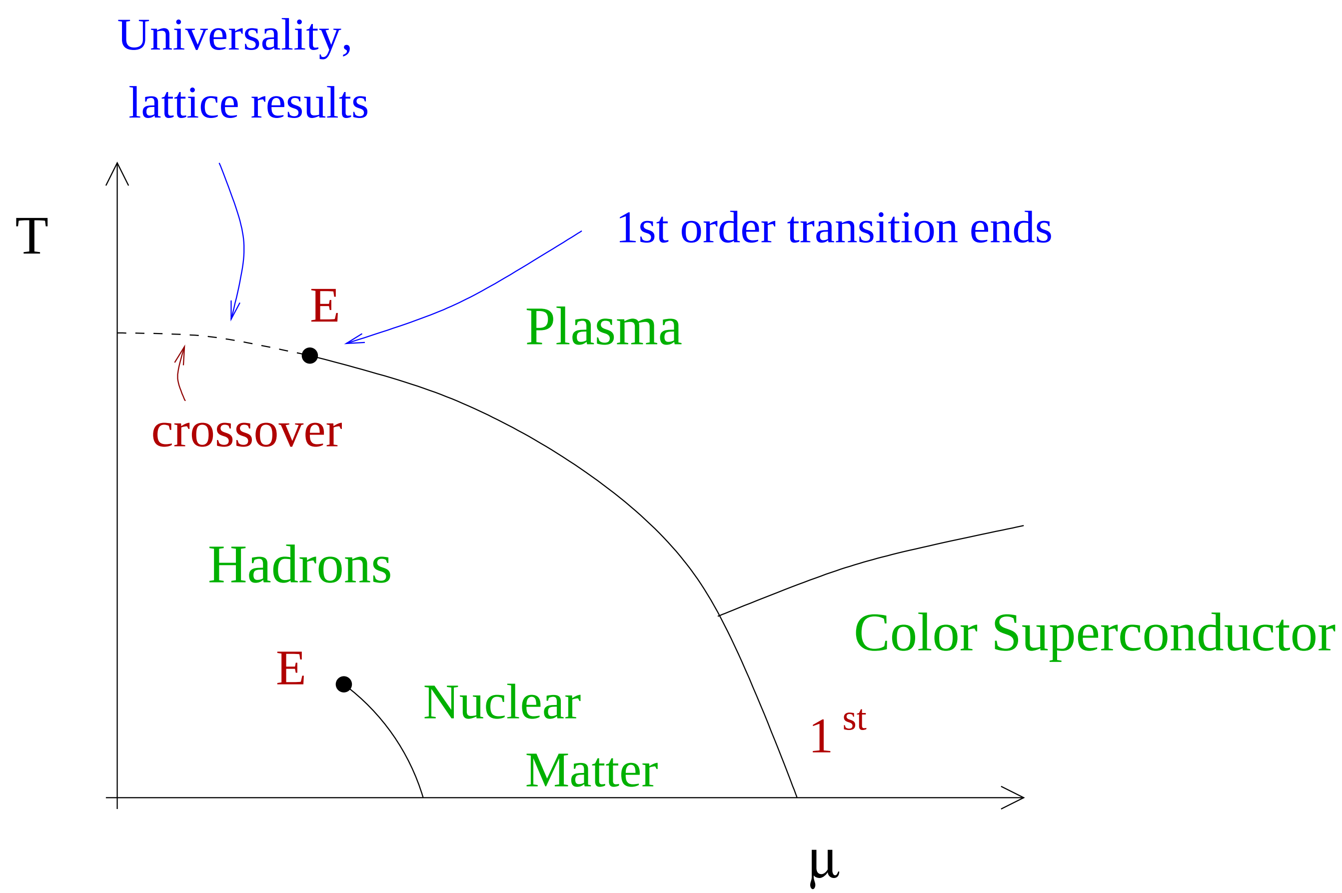




equation of state



Phase Diagram: First Version



critical endpoint (E) persists even if $m \neq 0$

Weakly coupled QGP

Basic object: Partition function

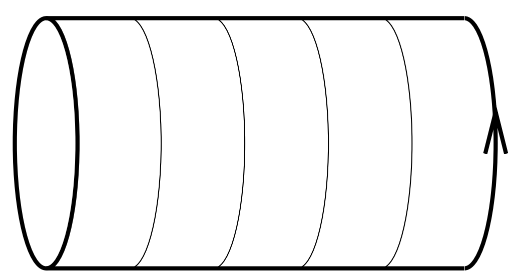
$$Z = \mathrm{Tr}[e^{-\beta H}], \quad \beta = 1/T \qquad\qquad F = T \log(Z)$$

Basic trick

$$Z = \mathrm{Tr}[e^{-i(-i\beta)H}] \qquad\qquad \text{imaginary time evolution}$$

Path integral representation ($\tau = it$)

$$Z = \int dA_\mu d\psi \exp\left(- \int_0^\beta d\tau \int d^3x \; \mathcal{L}_E \right)$$



$$A_\mu(\vec{x}, \beta) = A_\mu(\vec{x}, 0); \; \psi(\vec{x}, \beta) = -\psi(\vec{x}, 0)$$

Fourier representation

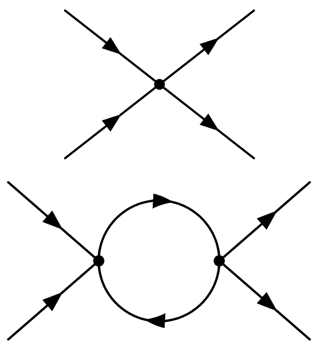
$$A_\mu(\vec{x}, \tau) = \sum_n \int d^3k A^n_\mu(\vec{k}) e^{i(\vec{k}\vec{x} + \omega_n \tau)}$$

Matsubara frequencies

$$\omega_n = 2\pi n T \qquad \text{bosons}$$

$$\omega_n = (2n+1)\pi T \qquad \text{fermions}$$

Feynman rules: Euclidean QCD with discrete energies



$$T \sum_n \int \frac{d^3p}{(2\pi)^3}$$

$$(2\pi)^3 \delta^3\left(\sum \vec{p}_i\right) \delta_{\sum n_i}$$

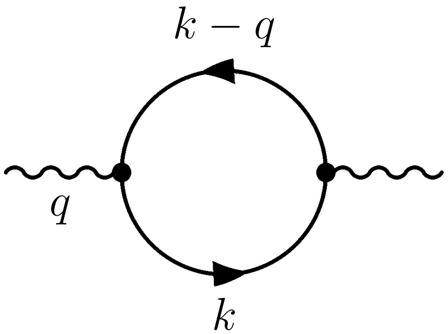
Typical Matsubara Sums

$$\sum_k \frac{1}{x^2+k^2} = \frac{2\pi}{x}\left(\frac{1}{2} + \frac{1}{e^{2\pi x}-1}\right) \qquad \text{bosons}$$

$$\sum_k \frac{1}{x^2+(2k+1)^2} = \frac{\pi}{x}\left(\frac{1}{2} - \frac{1}{e^{\pi x}+1}\right) \qquad \text{fermions}$$

Gluon Polarization Tensor

Warmup: Photon polarization function $\Pi_{\mu\nu}$



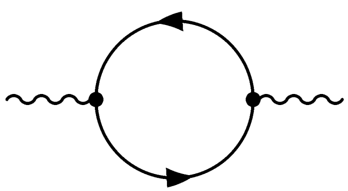
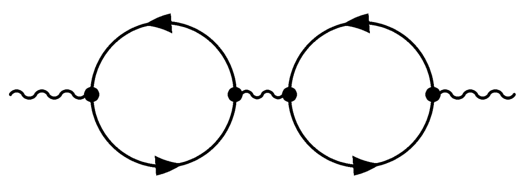
$$= e^2 T \sum_n \int \frac{d^3k}{(2\pi)^3} \mathrm{tr}[\gamma_\mu \not{k} \gamma_\nu (\not{k} - \not{q})] \Delta(k) \Delta(k-q)$$

Hard Thermal Loop (HTL) limit ($q \ll k \sim T$)

$$\Pi_{\mu\nu} = 2m^2 \int \frac{d\Omega}{4\pi} \Big(\frac{i\omega \hat{K}_\mu \hat{K}_\nu}{q \cdot \hat{K}} + \delta_{\mu 4} \delta_{\nu 4} \Big) \qquad \hat{K} = (-i, \hat{k})$$

$2m^2 = \frac{1}{3} e^2 T^2$ Debye mass

Significance of $\Pi_{\mu\nu}$

$D_{\mu\nu} =$  $+$  $+ \ \cdots = \dfrac{1}{(D^0_{\mu\nu})^{-1} + \Pi_{\mu\nu}}$

$D_{00}(\omega = 0, \vec{q})$ determines static potential

$$V(r) = e \int \frac{d^3q}{(2\pi)^3} \frac{e^{iqr}}{\vec{q}^2 + \Pi_{00}} \simeq -\frac{e}{r} \exp(-m_D r)$$

screened Coulomb potential

D_{ij} determines magnetic interaction

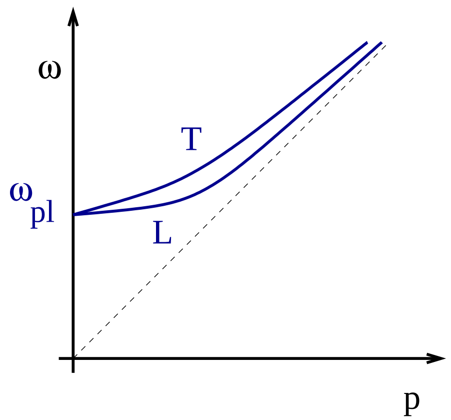
$$\Pi_{ii}(\omega \to 0, 0) = 0$$

no magnetic screening

$$\mathrm{Im}\Pi_{ii}(\omega, q) \sim \frac{\omega}{q} m_D^2 \Theta(q - \omega)$$

Landau damping

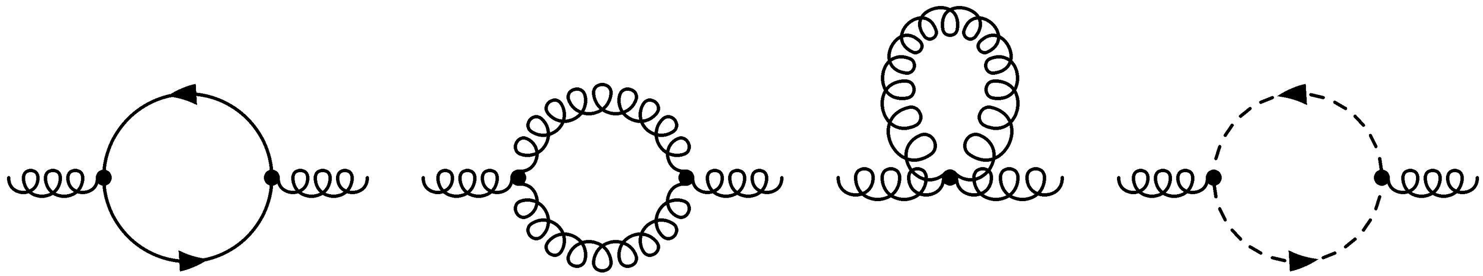
Poles of propagator: Plasmon dispersion relation



$$\text{pole}: \; D^{-1}_{L,T}(\omega, q) = 0$$

$$q \to 0: \quad \omega_L^2 = \omega_T^2 = \frac{1}{3} m_D^2$$

QCD looks more complicated



same result as QED with $m_D^2 = g^2T^2(1 + N_f/6)$

Conclusion: Perturbative Quark Gluon Plasma

quasi-quarks and quasi-gluons

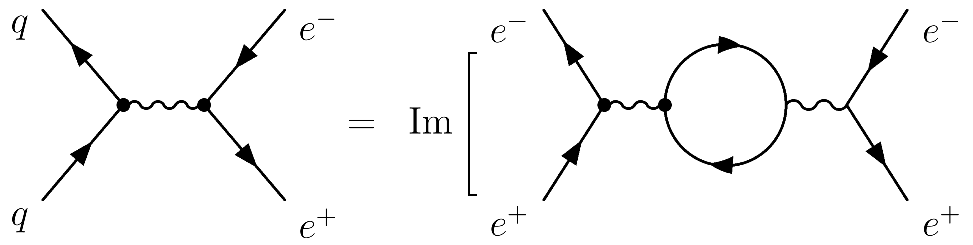
typical energies, momenta $\omega, p \sim T$

effective masses $m \sim gT$, width $\gamma \sim g^2T$

Note that $\gamma \ll \omega$ (long lived quasi-particles)

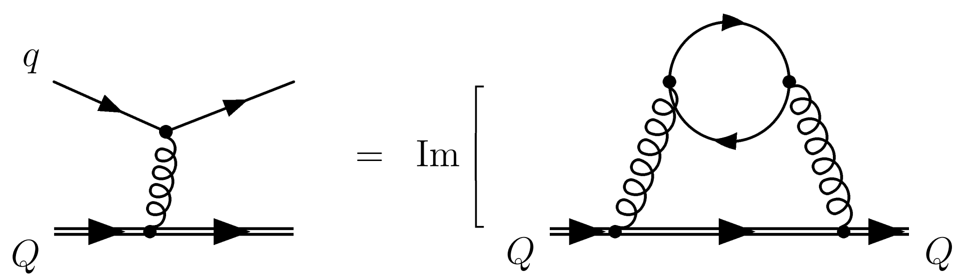
Physical Applications

Dilepton production



$$\frac{dR}{d^4q} = \frac{\alpha^2}{48\pi^2}\Big(12\sum_q e_q^2\Big)e^{-E/T}$$

Collisional energy loss



$$\frac{dE}{dx} = \frac{8\pi}{3}\alpha_s^2 T^2\Big(1+\frac{N_f}{6}\Big)\log\Big(c\frac{\sqrt{ET}}{m_D}\Big) \qquad E \gg M^2/T$$

$E = 20$ GeV: $dE/dx \simeq 0.3\ \mathrm{GeV/fm}$ for c, b quarks

note: for light quarks radiative energy loss dominates

Kinetic Theory

Quasi-Particles ($\gamma \ll \omega$): introduce distribution function $f_p(x,t)$

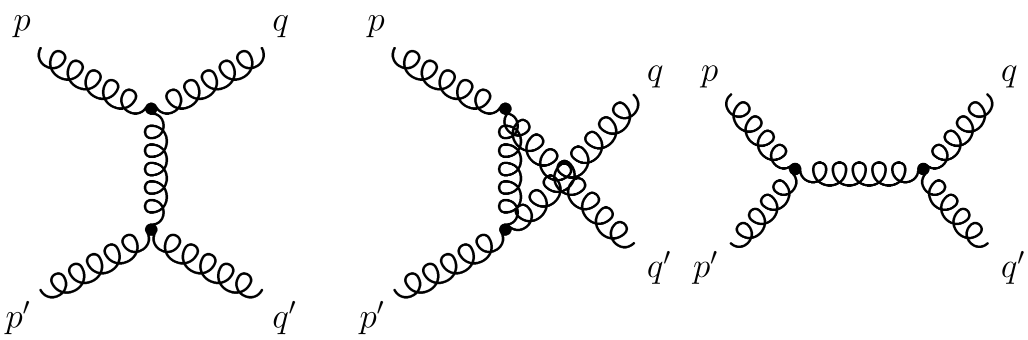
$$N = \int \frac{d^3p}{E_p}\, f_p \qquad\qquad T_{ij} = \int d^3p\, \frac{p_i p_j}{E_p} f_p,$$

Boltzmann equation

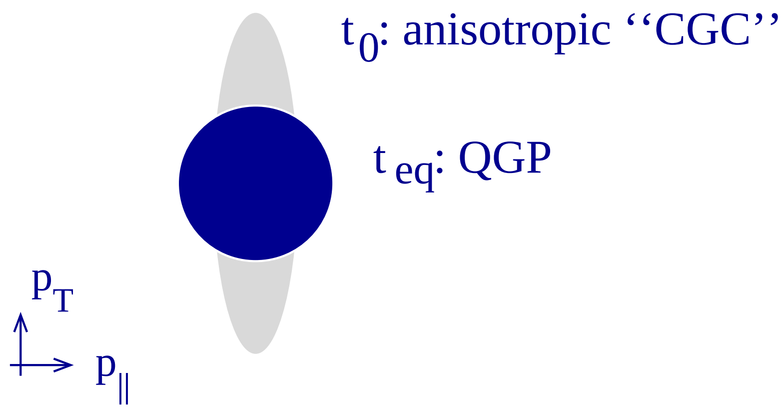
$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p + \vec{F}\cdot\vec{\nabla}_p f_p = C[f_p]$$

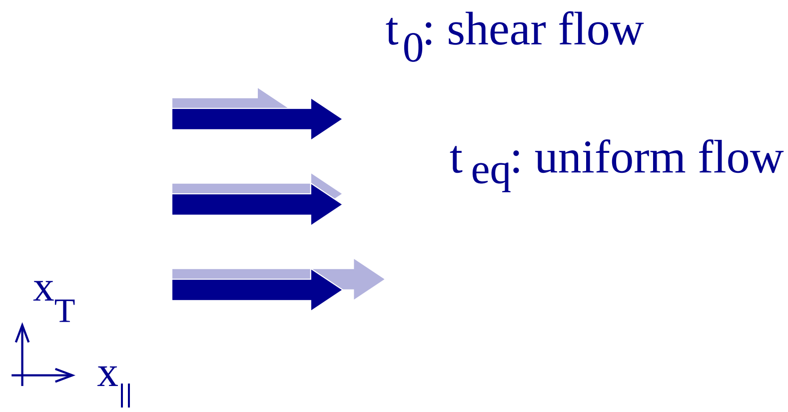
Collision term $C[f_p] = C_{gain} - C_{loss}$

$$C_{loss} = \int dp'dqdq'\; f_p f_{p'} w(p,p';q,q') \qquad\qquad C_{gain} = \ldots$$



Applications: Equilibration, transport coefficients, . . .





Linearized theory (Chapman-Enskog): $f_p = f_p^0(1 + \chi_p/T)$

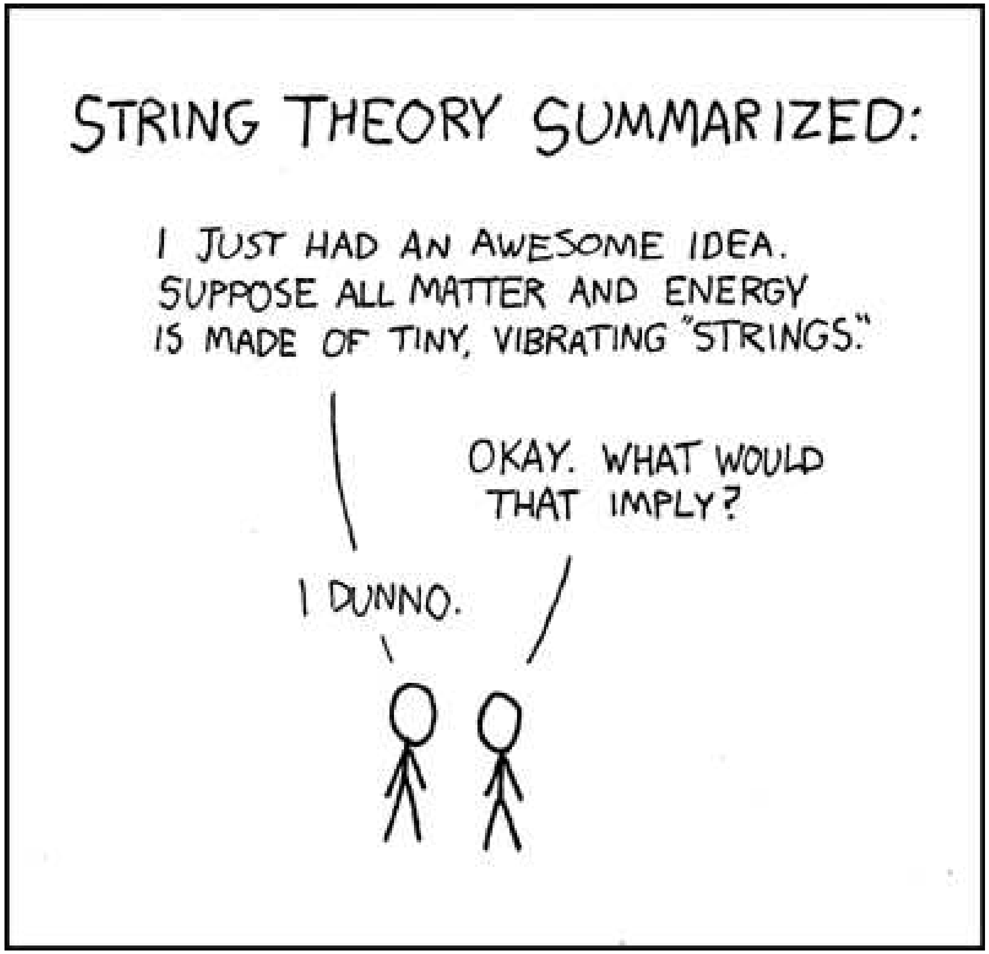
suitable for transport coefficients

Example: shear viscosity $\chi_p = g_p p_i p_j v_{ij}$ $(v_{ij} = \partial_i v_j + \partial_j v_i - trace)$

$$\eta \geq \frac{\langle \chi | X \rangle^2}{\langle \chi | C | \chi \rangle} \qquad \langle \chi | X \rangle = \int d^3p \, f_p^0 \, (\chi_p \cdot p_i p_j v_{ij})$$

$$QCD \qquad \eta = \frac{0.34 T^3}{\alpha_s^2 \log(1/\alpha_s)}$$

And now for something completely different . . .



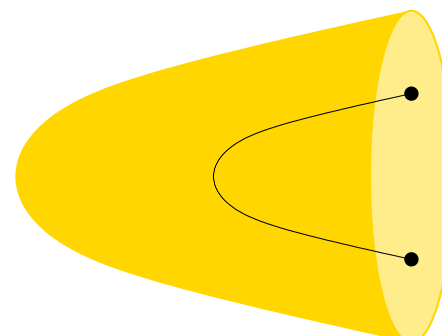
Gauge Theory at Strong Coupling: Holographic Duals

The AdS/CFT duality relates

large N_c (Conformal) gauge theory in 4 dimensions	$\Leftrightarrow$	string theory on 5 dimensional Anti-de Sitter space $\times S_5$
correlation fcts of gauge invariant operators	$\Leftrightarrow$	boundary correlation fcts of AdS fields

$$\langle \exp \int dx\ \phi_0 \mathcal{O} \rangle =$$

$$Z_{string}[\phi(\partial AdS) = \phi_0]$$



The correspondence is simplest at strong coupling $g^2 N_c$

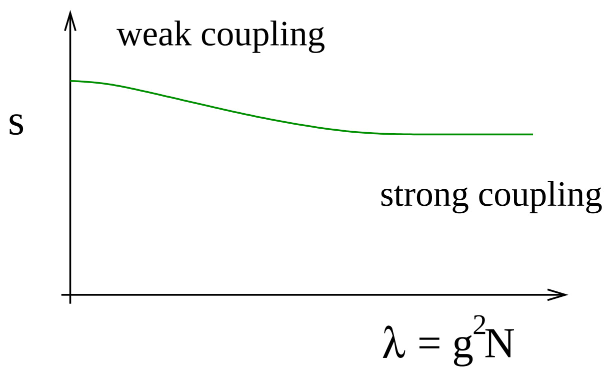
strongly coupled gauge theory $\Leftrightarrow$ classical string theory

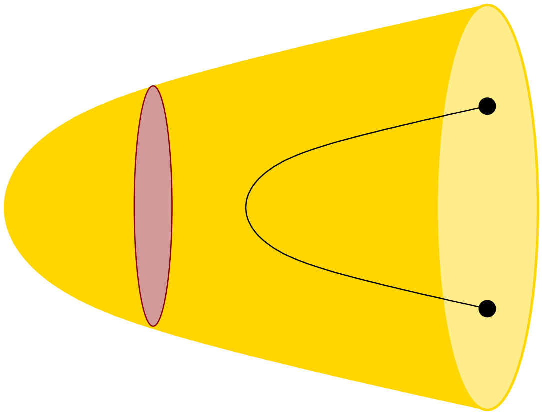
Holographic Duals at Finite Temperature

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature $\Leftrightarrow$ Hawking temperature of black hole

CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy $\sim$ area of event horizon



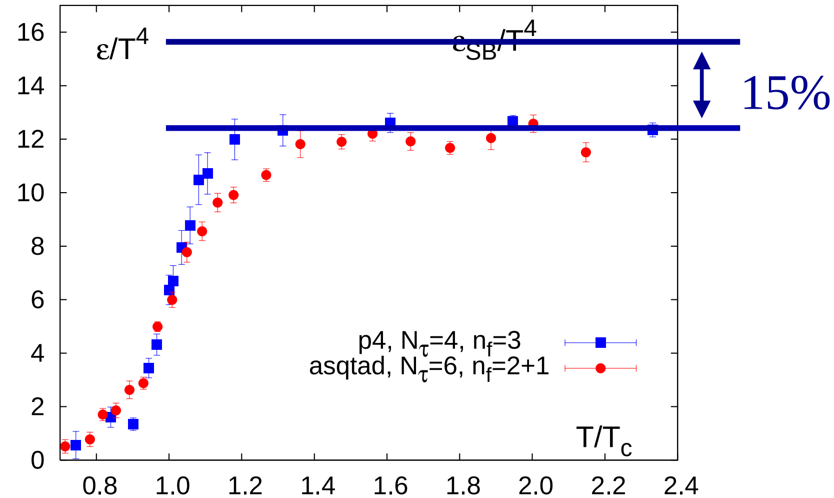


$$s(\lambda \to \infty) = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s(\lambda = 0)$$

Gubser and Klebanov

Relevance to QCD?

$\mathcal{N}=4$ QCD	QCD
gluons, gluinos [4], Higgses [6]	Matter content not relevant in QGP?
(all in adjoint representation)	approximately conformal for $T > T_c$?
exact conformal symmetry	
no chiral symmetry breaking	
no confinement	
no phase transition	



Ultimate goal: Find holographic dual of QCD

Holographic Duals: Transport Properties

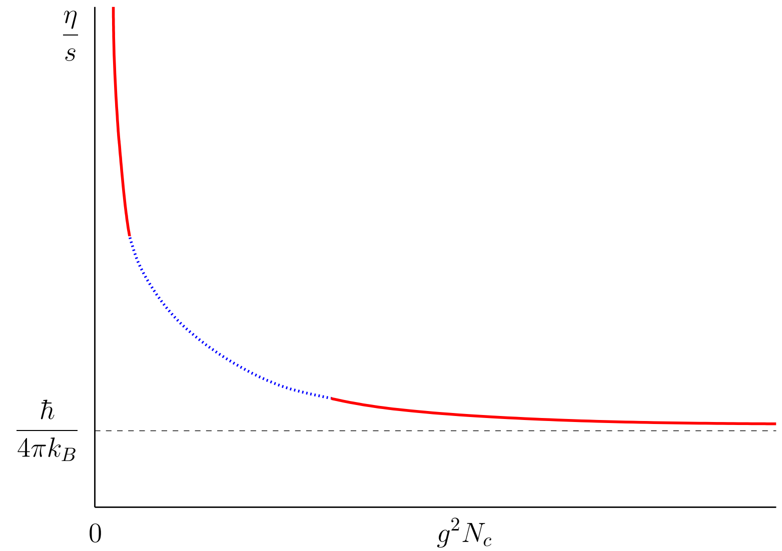
Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets



Strong coupling limit universal? Provides lower bound for all theories?

Summary (Theory)

Lattice QCD: single chiral and deconfinement crossover transition

$T_c \sim 185$ MeV, $\epsilon_{cr} \sim 1.5\,\mathrm{GeV/fm^3}$

Weakly coupled Quark Gluon Plasma

Quark and gluon quasi-particles, $\gamma \ll \omega$

Thermodynamics: Stefan-Boltzmann gas

Transport: long equilibration times, $\eta/s \simeq 1/\alpha_s^2 \gg 1$

Strongly coupled plasma

No quasi-particles, no kinetics, only hydrodynamics

Thermodynamics: Stefan-Boltzmann law

Transport: fast equilibration, $\eta/s \simeq 1/(4\pi) < 1$

Bonus Material:

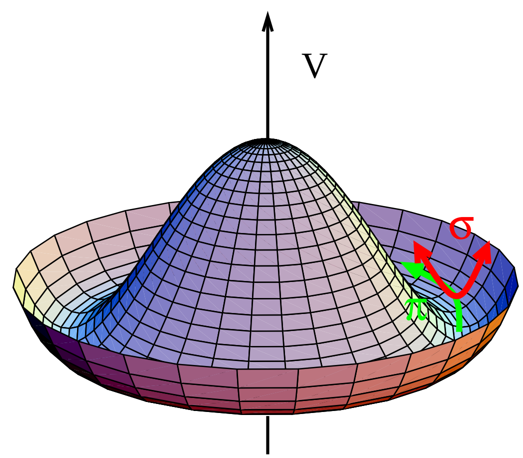
Sigma Model

Second Approach: Sigma Model

Simple model based on linear representation of $SU(2)_L \times SU(2)_R$

$$\phi^a = (\sigma, \vec{\pi}) \qquad\qquad O(4) = SU(2)_L \times SU(2)_R$$

Chirally symmetric lagrangian



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^a)^2 + V(\phi^a\phi^a)$$

$$V(\phi^a\phi^a) = -\frac{\mu^2}{2}(\phi^a\phi^a) + \frac{\lambda}{4}(\phi^a\phi^a)^2$$

Minimum of potential

$$\partial V/\partial\phi^a = \phi^a(-\mu^2 + \lambda\phi^a\phi^a) = 0 \qquad \phi_0^a = (\sigma_0, \vec{0}) \;\; \sigma_0^2 = \mu^2/\lambda$$

Direction fixed by explicit breaking $\mathcal{L}_{SB} = -c\sigma$

σ_0 related to pion decay constant

$$\vec{A}_\mu = \sigma\partial_\mu\vec{\pi} + \vec{\pi}\partial_\mu\sigma \simeq \sigma_0\partial_\mu\vec{\pi} \qquad\qquad \sigma_0 = f_\pi = 93 \text{ MeV}$$

Consider small oscillations. Equation of motion

$$\delta\mathcal{L}/\delta\phi^a = -\Box\phi^a - \partial V/\partial\phi^a = 0$$

Write $\phi^a = \phi_0^a + \delta\phi^a$

$$\begin{aligned}\Box(\delta\phi^a) &= (\phi_0^a + \delta\phi^a)\left(-\mu^2 + \lambda(\phi_0^a + \delta\phi^a)^2\right) \\ &= (-\mu^2 + \lambda\phi_0^a\phi_0^a)\phi_0^a + \left(-\mu^2 + 2\lambda\phi_0^a\phi_0^b + \lambda\delta^{ab}\phi_0^c\phi_0^c\right)\delta\phi^b + \ldots\end{aligned}$$

Split in (σ, π) components

$$\begin{aligned}\Box(\delta\sigma) &= \left(-\mu^2 + 3\lambda\sigma_0^2\right)\delta\sigma \qquad\qquad m_\sigma^2 = 2\mu^2 \\ \Box(\delta\vec{\pi}) &= \left(-\mu^2 + \lambda\sigma_0^2\right)\delta\vec{\pi} \qquad\qquad m_\pi^2 = 0\end{aligned}$$

Thermal Fluctuations

Write $\phi^a = \langle \phi^a \rangle + \tilde{\phi}^a$ where $\tilde{\phi}^a$ is a thermal fluctuation. Use

$$
\begin{aligned}
\langle \tilde{\phi}^a \rangle &= 0 \\
\langle \tilde{\phi}^a \tilde{\phi}^b \rangle &= (\delta^{ab}/4) \langle \tilde{\phi}^a \tilde{\phi}^a \rangle \\
\langle \tilde{\phi}^a \tilde{\phi}^b \tilde{\phi}^c \rangle &= 0
\end{aligned}
$$

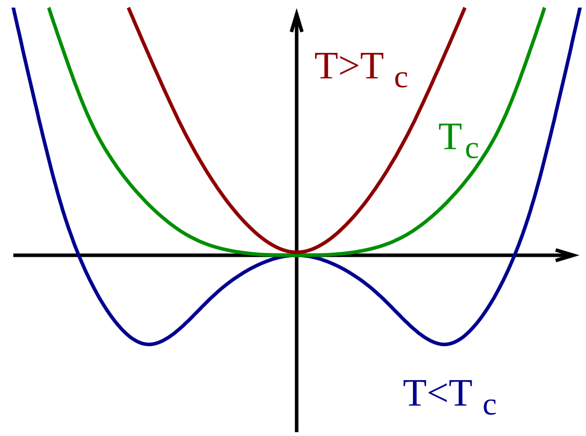
Equation of motion for $\langle \phi^a \rangle$ (use $1/N$)

$$
\begin{aligned}
\Box \langle \phi^a \rangle &= -\mu^2 \langle \phi^a \rangle + \lambda \langle \left(\langle \phi^a \rangle + \tilde{\phi}^a \right) \left(\langle \phi^b \rangle + \tilde{\phi}^b \right)^2 \rangle \\
&= -\mu^2 \langle \phi^a \rangle + \lambda \langle \phi^a \rangle \left[\langle \phi^b \rangle^2 + \langle \tilde{\phi}^b \tilde{\phi}^b \rangle \right]
\end{aligned}
$$

Fluctuations tend to restore symmetry

Thermal averages

$$\begin{aligned} \vec{\pi}_T &= 0 \\ \sigma_T^2 &= f_\pi^2 \left(1 - \frac{1}{f_\pi^2} \langle \tilde{\phi}^a \tilde{\phi}^a \rangle \right) \end{aligned}$$



Gaussian fluctuations ($m = 0$)

$$\langle \tilde{\phi}^a \tilde{\phi}^a \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_k} \frac{1}{e^{\beta\omega_k} - 1} = \frac{T^2}{12}$$

Critical temperature (3 light d.o.f.)

$$\sigma_T^2 = f_\pi^2 \left(1 - \frac{T^2}{3 f_\pi^2}\right) \qquad T_c = \sqrt{3} f_\pi \simeq 150 \text{ MeV}$$

Bonus Material:

Universality

Universality

Chiral phase transition might be continuous (2nd order)

Near T_c masses go to zero and correlation length diverges

Physics independent of microscopic details

Long distance behavior is universal

Only depends on symmetries of the order parameter

Landau-Ginzburg effective action

$$F = \int d^3x \left\{ \frac{1}{2}(\vec{\nabla}\phi^a)^2 + \frac{\mu^2}{2}(\phi^a\phi^a) + \frac{\lambda}{4}(\phi^a\phi^a)^2 + \ldots \right\}$$

Consider $\lambda > 0$, $\mu^2(T_c) = 0$

Universality

$SU(2)_L \times SU(2)_R$	QCD	$\equiv$	$O(4)$	magnet
$\langle\bar{\psi}\psi\rangle$	χ condensate		$\vec{M}$	magnetization
m_q	quark mass		H_3	magnetic field
$\vec{\pi}$	pions		$\vec{\phi}$	spin waves

Predictions

$$
\begin{aligned}
C &\sim t^{\alpha} & \alpha &= -0.19 \\
\langle\bar{\psi}\psi\rangle &\sim t^{\beta} & \beta &= 0.38 \\
m_{\pi} &\sim t^{\nu} & \nu &= 0.73
\end{aligned}
$$

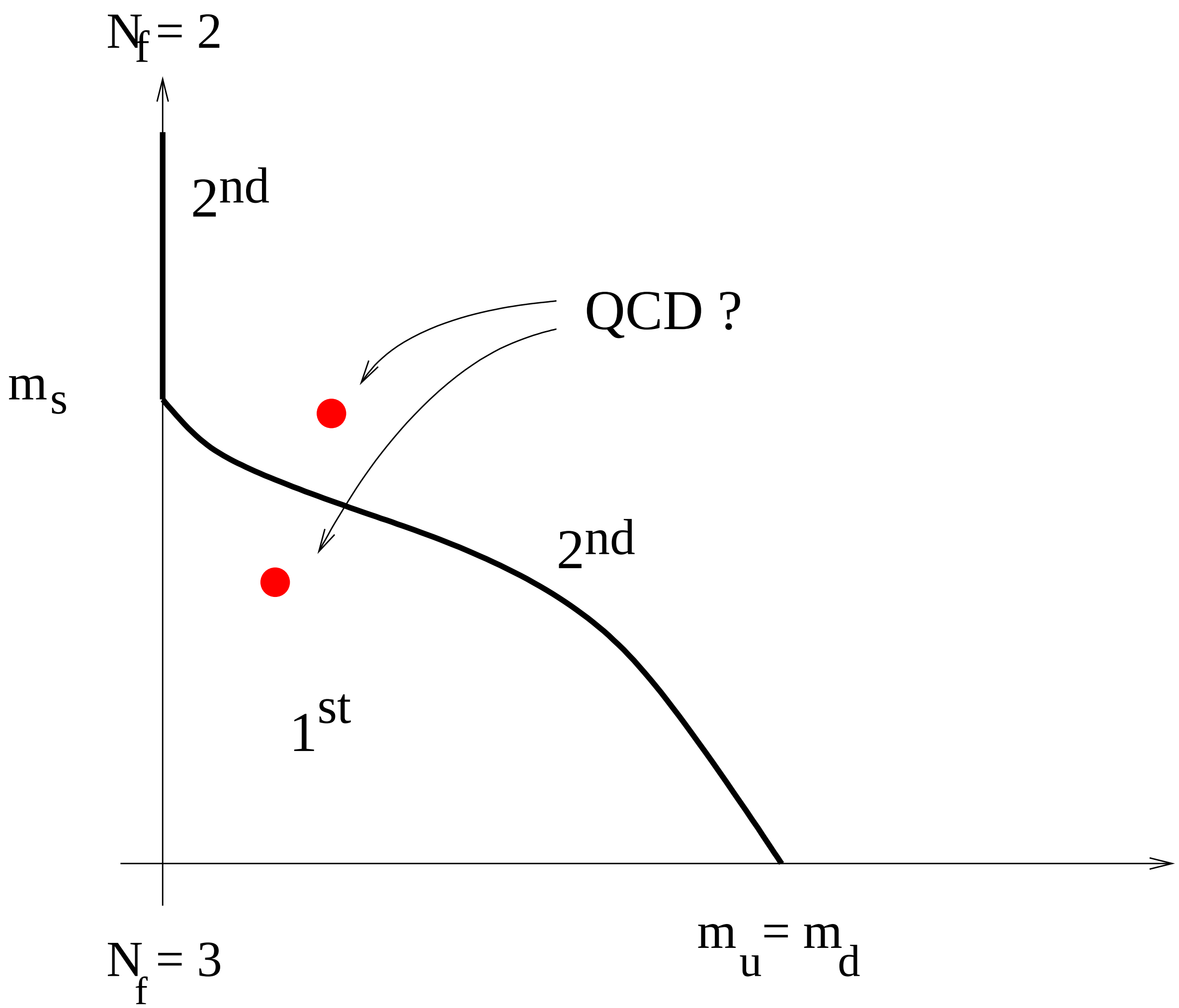
$t = (T - T_c)/T$

from ϵ expansion,

numerical simulations

$N_f = 3$: extra cubic invariant $\det(\phi)$, 2nd order transition unstable

$N_f = 3$ transition is 1st order

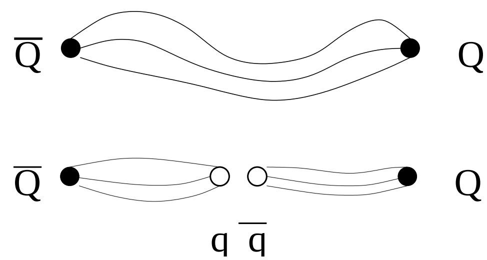


Universality: Confinement

Confinement characterized by heavy quark potential

$$V(r) \sim kr$$

$$k \sim 1 \text{ GeV/fm}$$

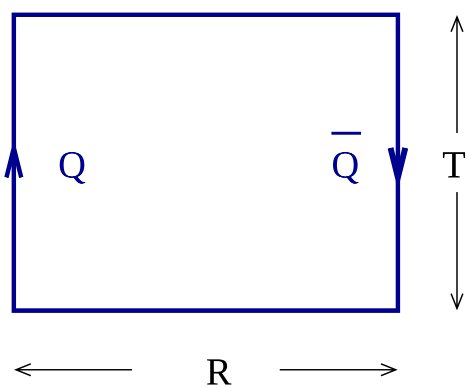


Propagator for heavy quark

$$\left(i\partial_0 + gA_0 + \vec{\alpha}(i\vec{\nabla} + g\vec{A}) + \gamma_0 M\right)\psi = 0$$

$$S(x,x') \simeq \exp\left(ig\int A_0 dt\right)\left(\frac{1+\gamma_0}{2}\right)e^{im(t-t')}\delta(\vec{x}-\vec{x}')$$

Potential related to Wilson loop



$$W(R,T) = \exp\left(ig\oint A_\mu dz_\mu\right)$$

Have $W(R,T) = \exp(-E \cdot T) = \exp(-V(R)T)$

$$W(R,T) \sim \exp(-kA) \qquad \text{Confinement} \equiv \text{AreaLaw}$$

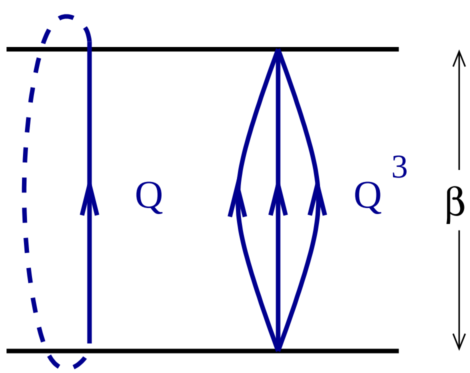
Local order parameter? Polyakov line

$$P(\vec{x}) = \frac{1}{N_c}\mathrm{Tr}[L(\vec{x})] = \frac{1}{N_c} P\mathrm{Tr}\left[\exp\left(ig\int_0^\beta A_0 dt\right)\right]$$

Naive Interpretation: $\langle P \rangle \sim \exp(-m_Q \beta)$

$$\langle P \rangle = 0 \qquad \text{confined} \qquad \langle P \rangle \neq 0 \qquad \text{deconfined}$$

Symmetry: Consider $L \to zL \quad z = \exp(2\pi k i/N_c) \in Z_{N_c}$



$$\mathrm{Tr}[L(\vec{x})] \to z\mathrm{Tr}[L(\vec{x})]$$
$$\mathrm{Tr}[L(\vec{x})^3] \to \mathrm{Tr}[L(\vec{x})^3]$$

Polyakov line: $P \to zP$

$$\langle P \rangle = 0 \qquad Z_{N_c} \text{ unbroken} \qquad T < T_c$$

$$\langle P \rangle \neq 0 \qquad Z_{N_c} \text{ broken} \qquad T > T_c$$

Landau-Ginzburg Theory (cubic invariant: $SU(3)$ only)

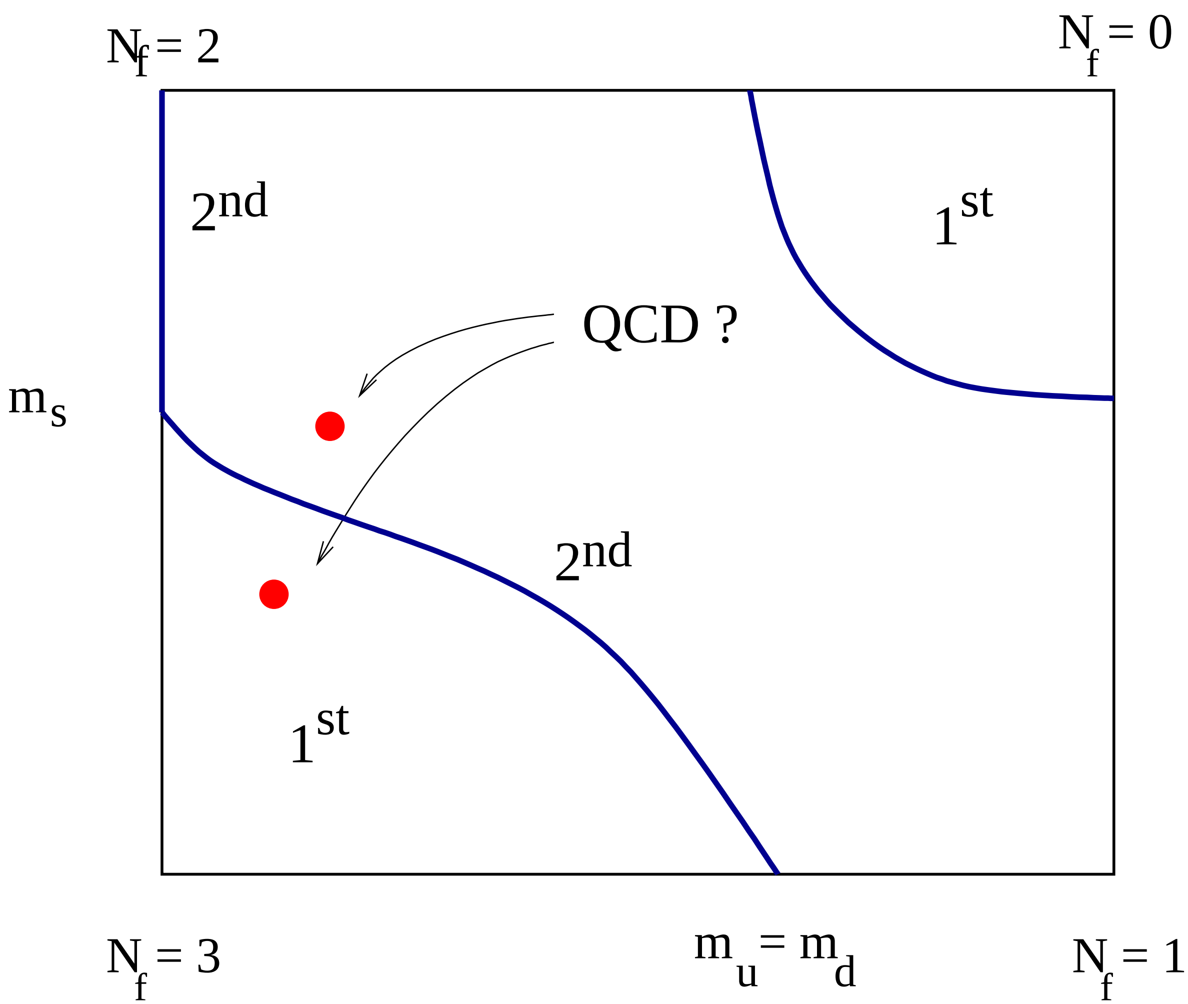
$$F = \int d^3x \left\{ \frac{1}{2} |\vec{\nabla} P|^2 + \mu^2 |P|^2 + g\mathrm{Re}(P^3) + \lambda |P|^4 + \ldots \right\}$$

Predictions

$SU(2)$-color: 2nd order

$SU(3)$-color: 1st order

Summary: Universality



Bonus Material:

Partition Function of Free Gas

Example: Free energy of non-interacting bosons

Partition function: $Z = \left[\det(p^2 + m^2)\right]^{-1/2}$

$$\log Z = -\frac{1}{2}\sum_n \log(\omega_n^2 + \omega^2) \qquad\qquad \omega^2 = \vec{p}^2 + m^2$$

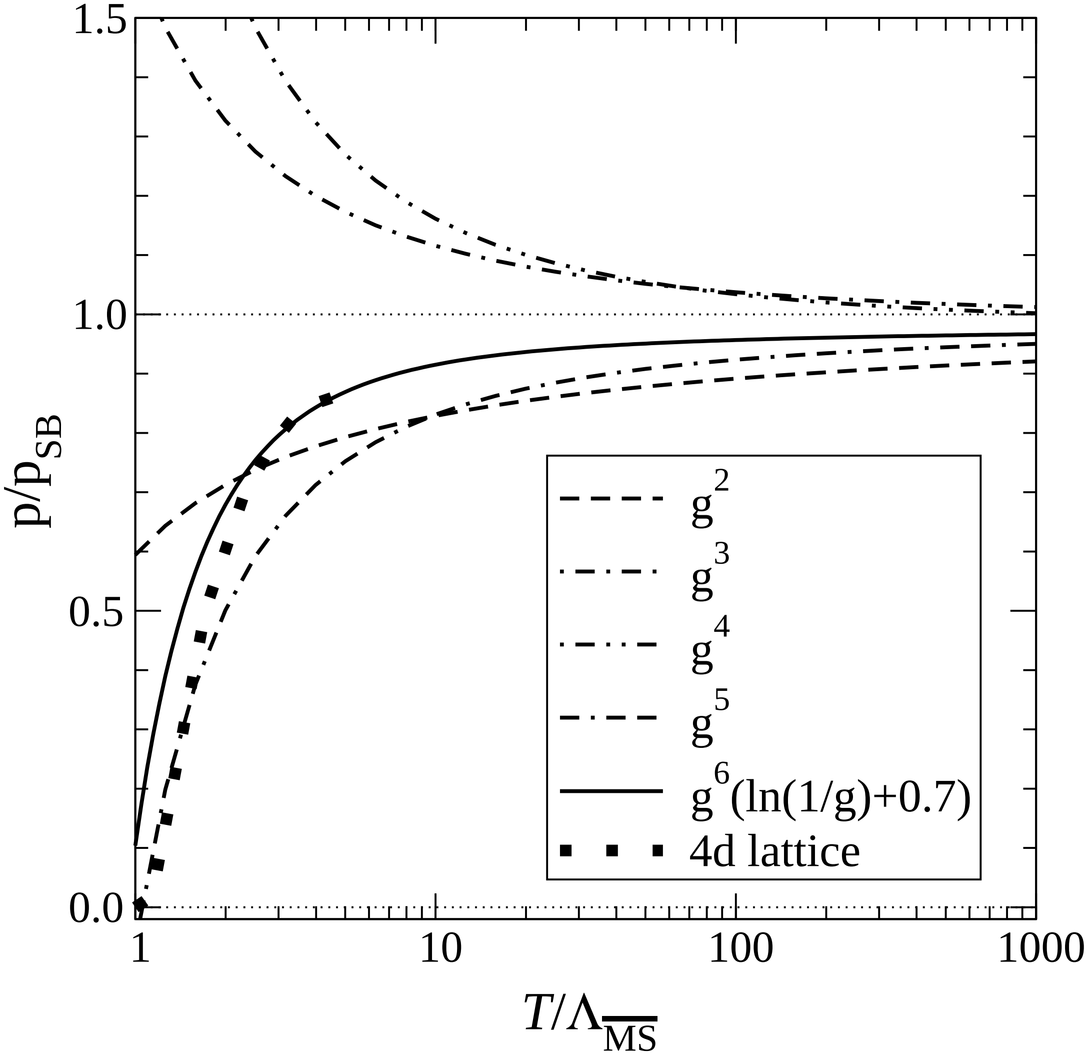
Consider derivative with respect to ω^2

$$\frac{d\log Z}{d\omega^2} = -\frac{1}{2}\sum_n \frac{1}{\omega_n^2 + \omega^2}$$

Use bosonic Matsubara sum and integrate back

$$-T\log Z = \frac{\omega}{2} + \frac{1}{\beta}\log\left(1 - e^{-\beta\omega}\right)$$

Weak Coupling Thermodynamics



Bonus Material:

Kinetic Theory and Shear Viscosity

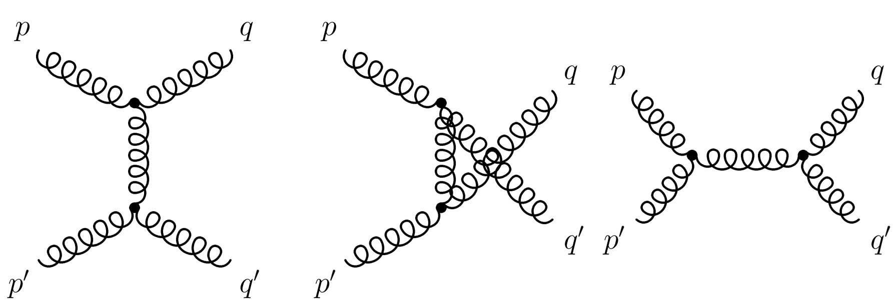
Kinetic Theory

Boltzmann equation

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = C[f_p]$$

Collision term $C[f_p] = C_{gain} - C_{loss}$

$$C_{loss} = \int dp' dq dq' \; f_p f_{p'} w(p, p'; q, q') \qquad C_{gain} = \ldots$$



$$C[f_p^0] = 0 \;\; (equ.)$$

Linearized theory (Chapman-Enskog): $f_p = f_p^0(1 + \chi_p/T)$

$$C[f_p] \equiv C_p \chi_p \qquad \text{linear collision operator}$$

Linear response to flow gradient

$$f_p = \exp(-(E_p - \vec{p}\cdot\vec{v})/(kT))$$

Drift term proportional to "driving term" $(v_{ij} = \partial_i v_j + \partial_j v_i - trace)$

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p \equiv X \qquad\qquad X \equiv p_i p_j v_{ij}$$

Boltzmann equation

$$C_p \chi_p = X \qquad\qquad \chi_p \equiv g_p p_i p_j v_{ij}$$

Viscosity $T_{ij} = T^0_{ij} + \eta v_{ij}$

$$\eta \sim \langle X|\chi\rangle \qquad\qquad \langle\chi|X\rangle = \int d^3p\, f^0_p\,(\chi_p \cdot p_i p_j v_{ij})$$

$$\eta \sim \langle\chi|C_p|\chi\rangle$$

Variational principle

$$\langle \chi_{var} | C_p | \chi_{var} \rangle \langle \chi | C_p | \chi \rangle \geq \langle \chi_{var} | C_p | \chi \rangle^2 = \langle \chi_{var} | X \rangle^2$$

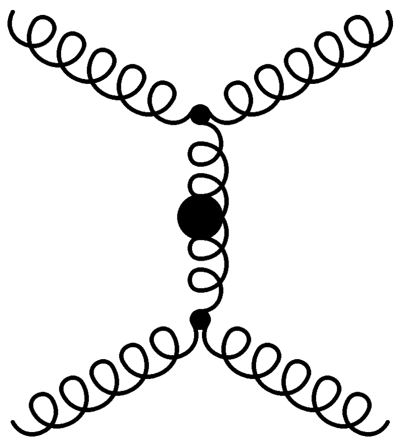
Variational bound

$$\eta \geq \frac{\langle \chi_{var} | X \rangle^2}{\langle \chi_{var} | C | \chi_{var} \rangle}$$

Best bound for $g_p \sim p^\alpha$ $(\alpha \simeq 0.1)$

$$\eta = \frac{0.34 T^3}{\alpha_s^2 \log(1/\alpha_s)}$$

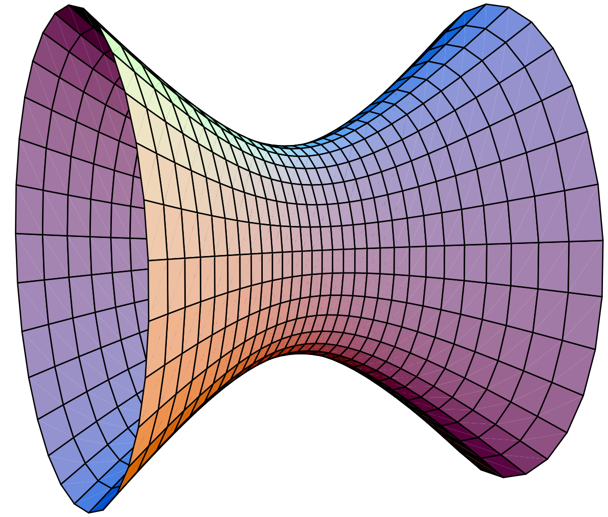
$\log(\alpha)$ from dynamic screening



Bonus Material:

AdS/CFT

Anti-DeSitter Space



Consider a hyperboloid embedded in 6-d euclidean space

$$-R^2 = \sum_{i=1,4} x_i^2 - x_0^2 - x_5^2$$

This is a space of constant negative curvature, and a solution of the Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{2} g_{\mu\nu} \Lambda$$

with negative cosmological constant. Isometries of AdS_5: $SO(4,2)$

Many possible choices of coordinates. Witten uses

$$ds^2 = \frac{1}{z^2} \left(-dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu \right)$$

$\mathcal{N} = 4$ Supersymmetric Yang-Mills Theory

Fields: Gluons, Gluinos, Higgses; all in the adjoint of $SU(N_c)$

$$\mathcal{L} = \frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\lambda}^a_A \sigma^\mu (D_\mu \lambda^A)^a + (D_\mu \Phi_{AB})^a (D_\mu \Phi^{AB})^a + \ldots$$

$$A^a_\mu \qquad \lambda^a_A \; (\bar{4}_R) \qquad \Phi^a_{AB} \; (6_R)$$

Global symmetries: Conformal and $SU(4)_R$

$$SO(4,2) \times SU(4)_R$$

Properties: Conformal $\beta(g) = 0$, extra scalars, no fundamental fermions, no chiral symmetry breaking, no confinement