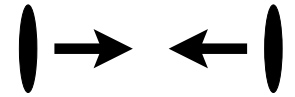


QCD at High Temperature

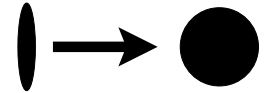
(Experiment)

Kinematics

CMS: $s = (p_1 + p_2)^2 = 4E_{CM}^2$



Lab: $p_1 = (m, 0)$ $p_2 = (E_L, p_z) = (E_L, \sqrt{E_L^2 - m^2})$



$$s = (m + E_L)^2 - (E_L^2 - m^2) = 2m(E_L + m) \quad E_{CM} = \sqrt{mE_L/2}$$

SPS : 200 GeV (LAB)

$$E_{CM} = 10 \text{ GeV} \quad \gamma = 10$$

RHIC : 100 GeV (CMS)

$$E_{CM} = 100 \text{ GeV} \quad \gamma = 100$$

LHC : 2.75 TeV (CMS)

$$E_{CM} = 2.75 \text{ TeV} \quad \gamma = 2750$$

Rapidity:

$$y = \frac{1}{2} \log \left(\frac{E + p_z}{E - p_z} \right)$$

$$\text{SPS : } \Delta y = 6 \quad \text{RHIC : } \Delta y = 10.6 \quad \text{LHC : } \Delta y = 17.3$$

Bjorken Expansion

Experimental observation: At high energy ($\Delta y \rightarrow \infty$) rapidity distributions of produced particles (in both pp and AA) are “flat”

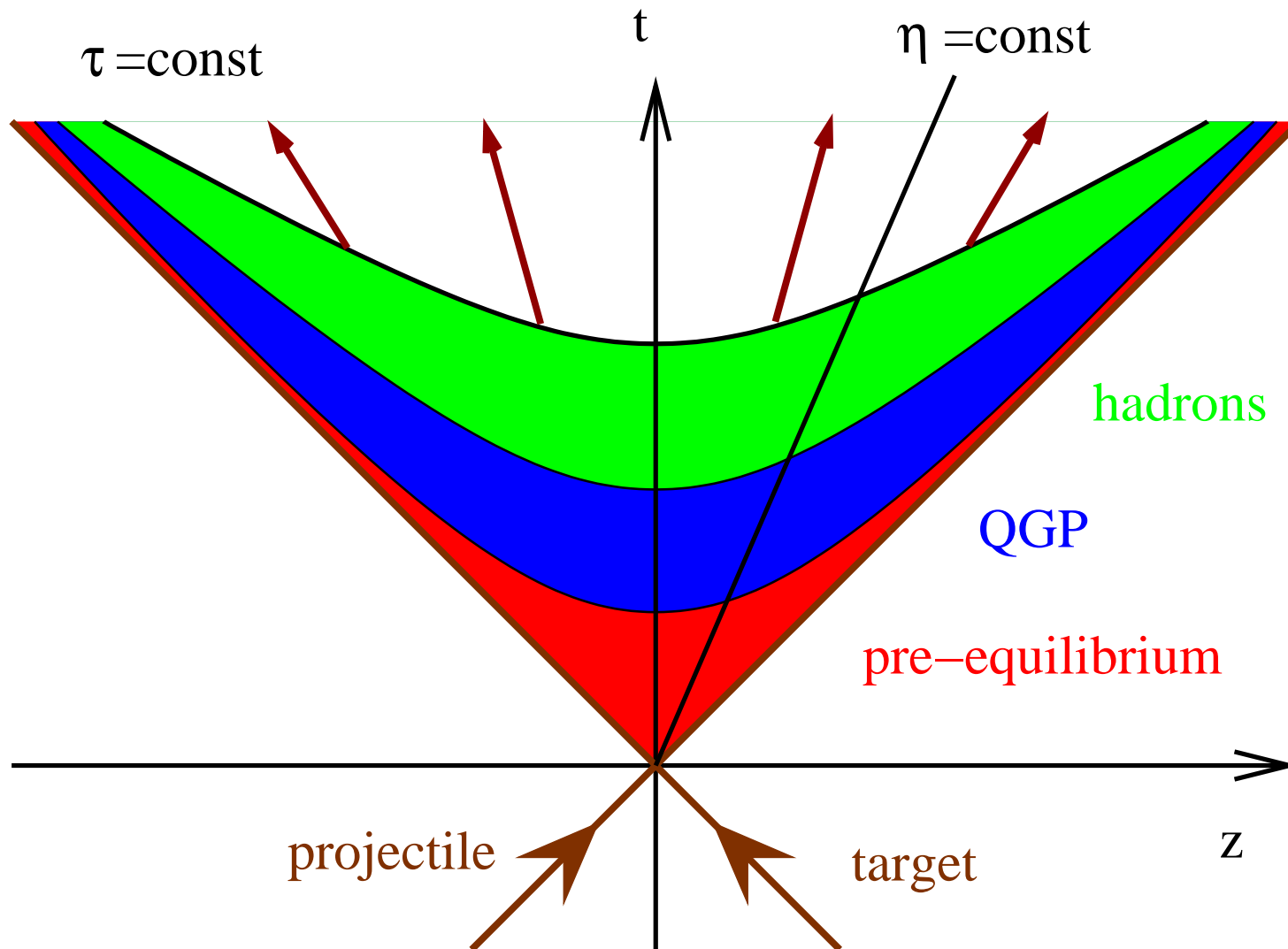
$$\frac{dN}{dy} \simeq \text{const}$$

Physics depends on proper time $\tau = \sqrt{t^2 - z^2}$, not on y

All comoving ($v = z/t$) observers are equivalent

Analogous to Hubble expansion

Bjorken Expansion



Bjorken Expansion: Hydrodynamics

Consider perfect relativistic fluid; 4-velocity $u_\mu = (1, \vec{v})\gamma$

$$T_{\mu\nu} = (\epsilon + P)u_\mu u_\nu - P g_{\mu\nu}$$

Hydro = Conservation Laws ($\partial^\mu T_{\mu\nu} = 0$) + Equ. of State ($P = P(\epsilon)$)

$$\partial^\mu T_{\mu\nu} = (\partial^\mu \epsilon + \partial^\mu P)u_\mu u_\nu + (\epsilon + P)((\partial^\mu u_\mu)u_\nu + u_\mu \partial^\mu u_\nu) - \partial_\nu P = 0$$

Contract with u_ν , use $u^2 = 1$

$$(\partial^\mu \epsilon + \partial^\mu P)u_\mu + (\epsilon + P)\partial^\mu u_\mu - u^\nu \partial_\nu P = 0$$

$$u_\mu \partial^\mu \epsilon + (\epsilon + P)\partial^\mu u_\mu = 0$$

Thermodynamic relations

$$d\epsilon = Tds$$

$$\epsilon + P = Ts$$

Hydrodynamic equations

$$u^\mu (T \partial_\mu s) + (Ts) \partial^\mu u_\mu = 0$$

$$\boxed{\partial_\mu (s u^\mu) = 0} \quad \text{isentropic expansion}$$

Variables: $t = \tau \cosh \alpha$, $z = \tau \sinh \alpha$. $\Rightarrow u_\mu = (\cosh \alpha, 0, 0, \sinh \alpha)$

$$\partial^\mu (s u_\mu) = 0 \quad \Rightarrow \quad \frac{d}{d\tau} [\tau s(\tau)] = 0$$

Solution for ideal Bj hydrodynamics

$$\boxed{s(\tau) = \frac{s_0 \tau_0}{\tau}} \quad T = \frac{\text{const}}{\tau^{1/3}}$$

Exact boost invariance, no transverse expansion, no dissipation, ...

Numerical Estimates

Total entropy in rapidity interval $[y, y + \Delta y]$

$$S = s\pi R^2 z = s\pi R^2 \tau \Delta y = (s_0 \tau_0) \pi R^2 \Delta y$$

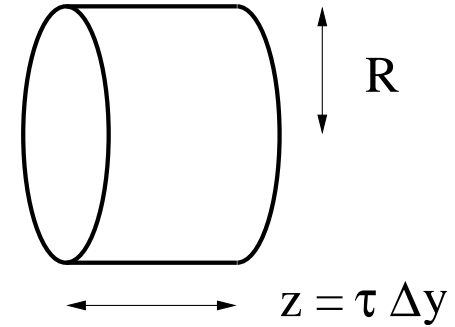
$$s_0 \tau_0 = \frac{1}{\pi R^2} \frac{S}{\Delta y}$$

Use $S/N \simeq 3.6$

$$s_0 = \frac{3.6}{\pi R^2 \tau_0} \left(\frac{dN}{dy} \right) \quad \text{Bj estimate}$$

$$\epsilon_0 = \frac{1}{\pi R^2 \tau_0} \left(\frac{dE_T}{dy} \right)$$

Depends on initial time τ_0



RHIC: Au-Au collisions ($\sqrt{s} = 200$ GeV)

$$\frac{dN}{dy} \simeq 998 \quad \tau_0 = 1 \text{ fm} \quad s_0 \simeq 33 \text{ fm}^{-3}$$

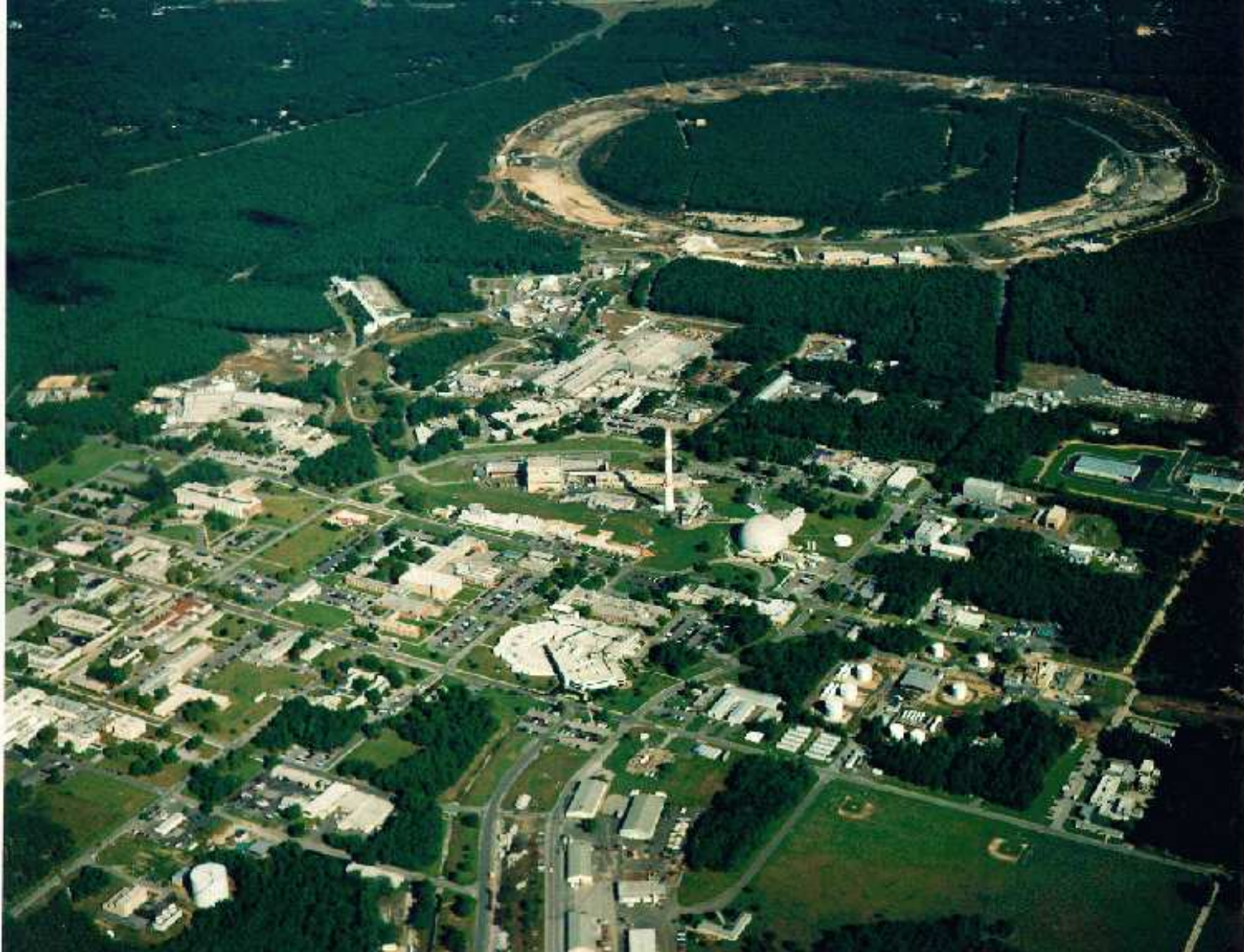
Use QGP equation of state $s = 2g\pi^2 T^3/45$

$$T_0 \simeq 240 \text{ MeV} \quad \epsilon_0 \simeq (5 - 6) \text{ GeV/fm}^3$$

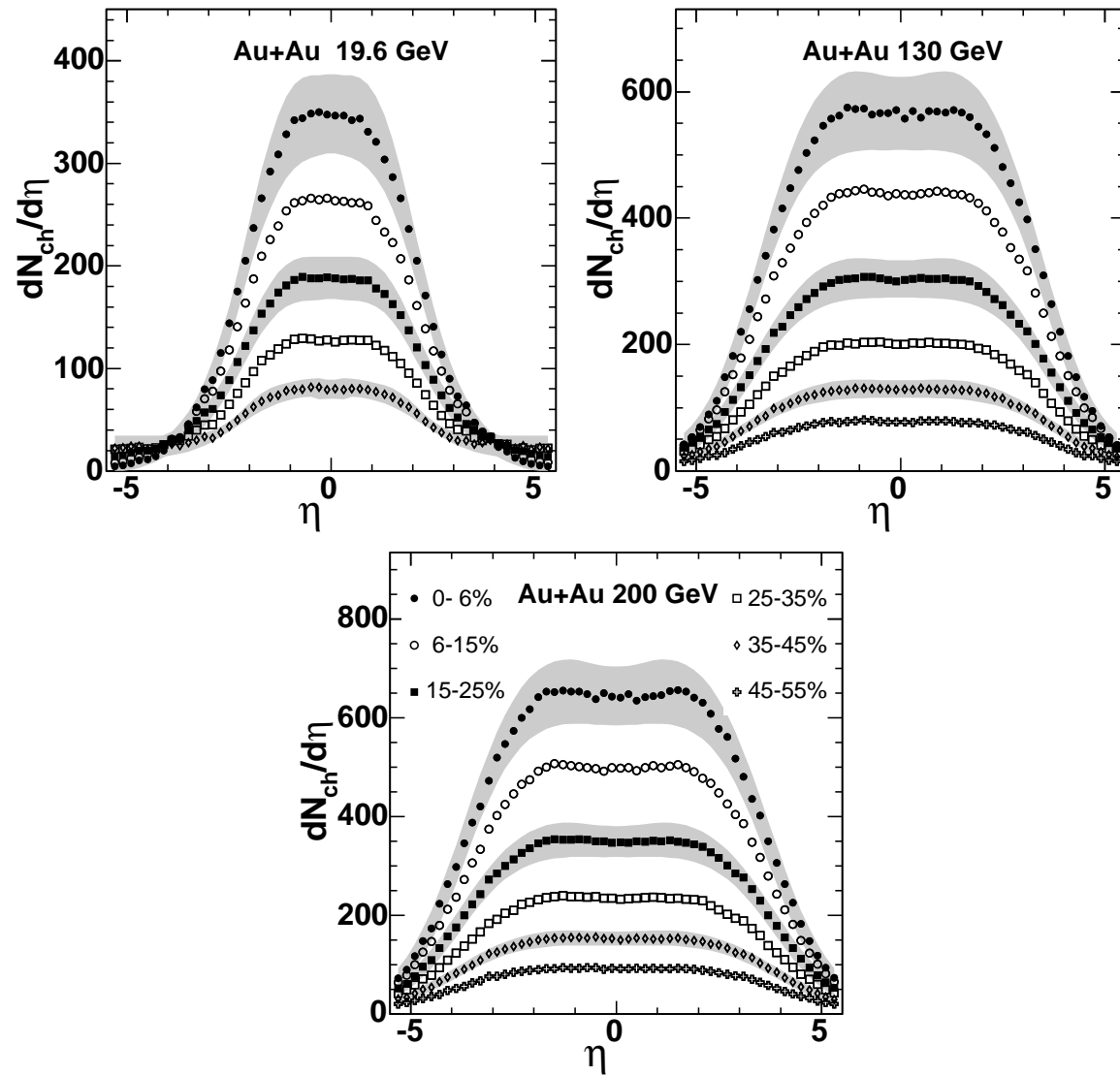
LHC: Factor ~ 2 in multiplicity

$$T_0 \simeq 300 \text{ MeV} \quad \epsilon_0 \simeq 15 \text{ GeV/fm}^3$$

BNL and RHIC

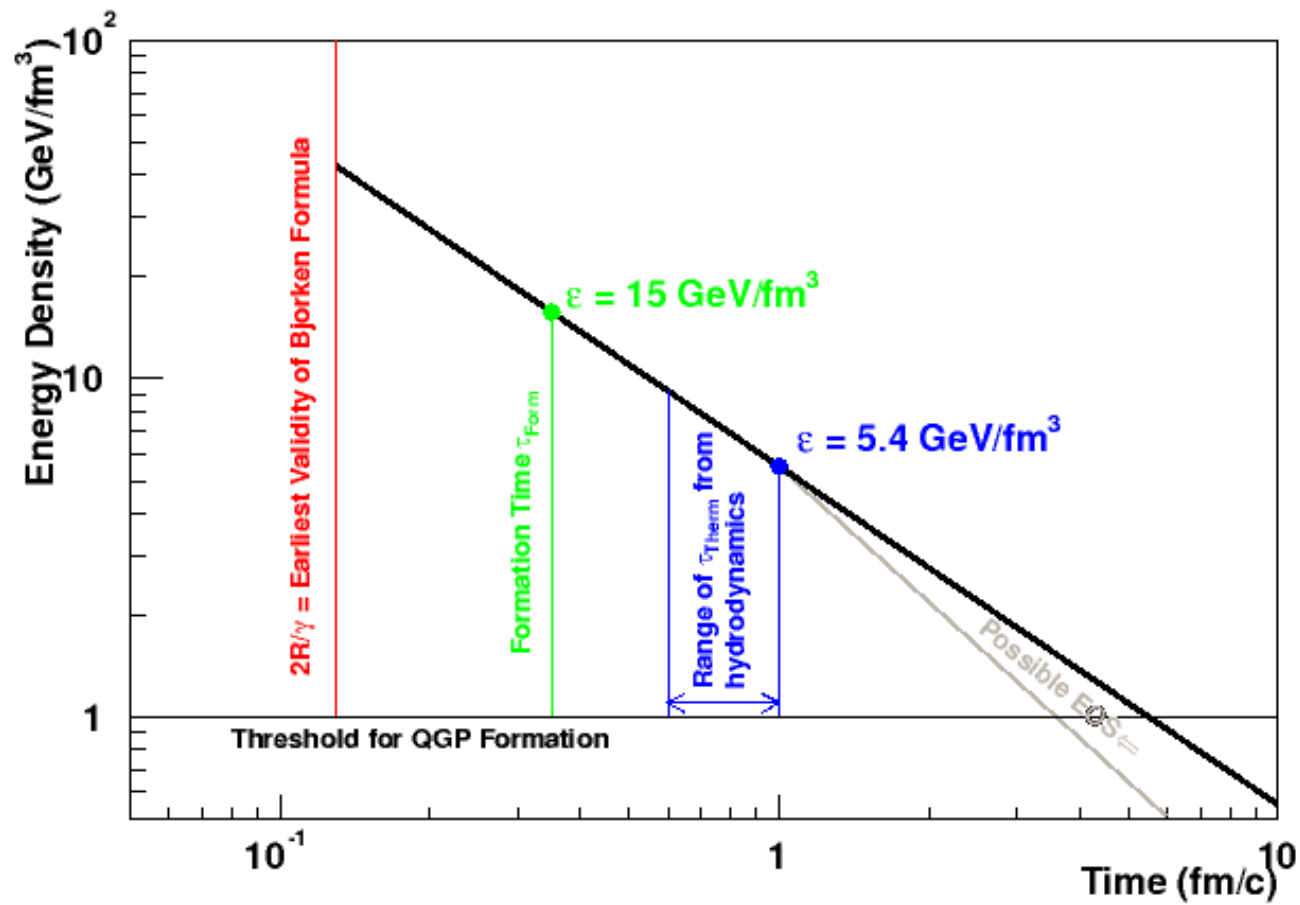


Multiplicities



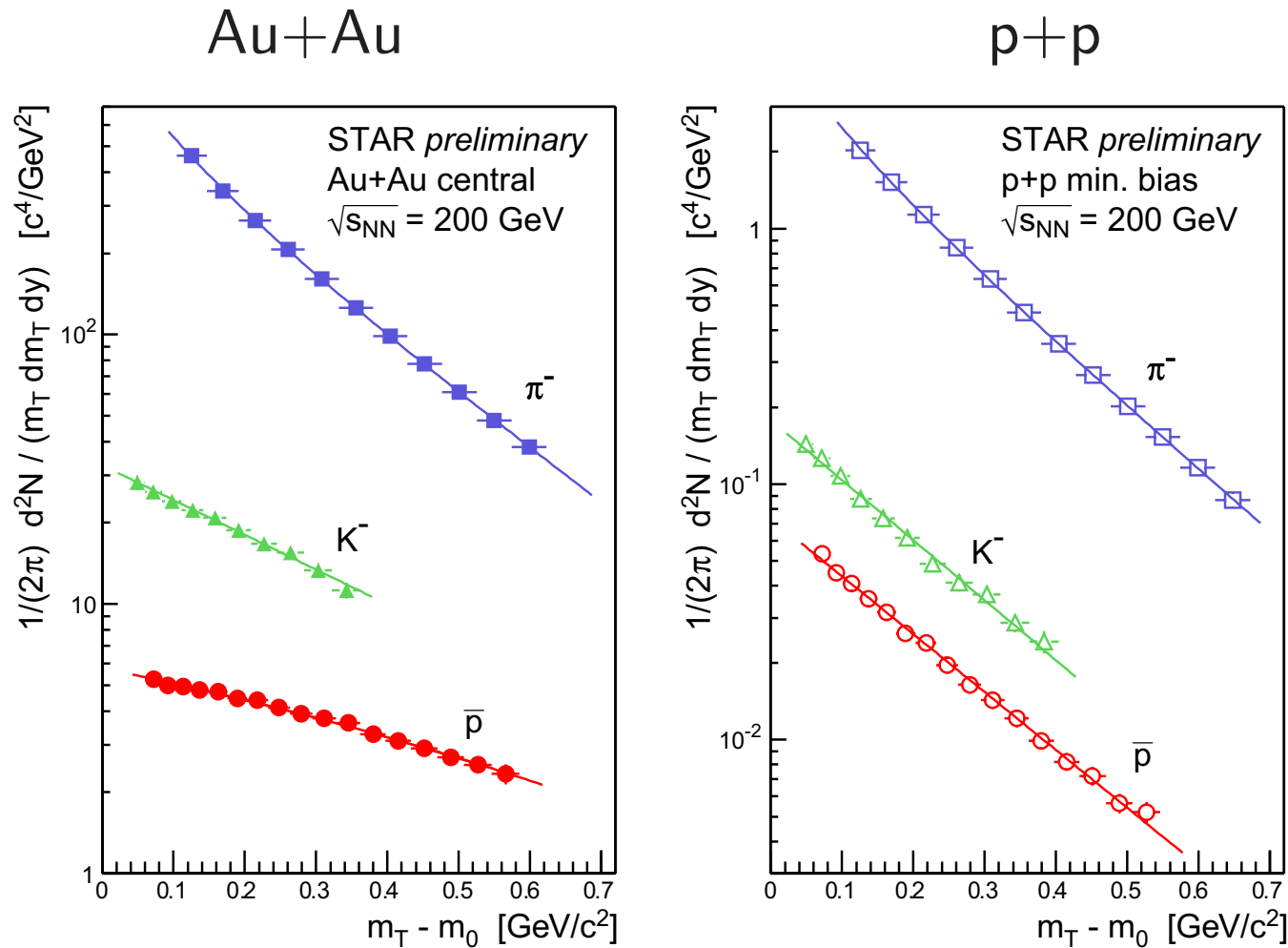
Phobos White Paper (2005)

Bjorken Expansion



Collective Behavior: Radial Flow

Radial expansion leads to blue-shifted spectra in Au+Au



$$v_T \sim 0.6c!$$

$$m_T = \sqrt{p_T^2 + m^2}$$

Elliptic Flow

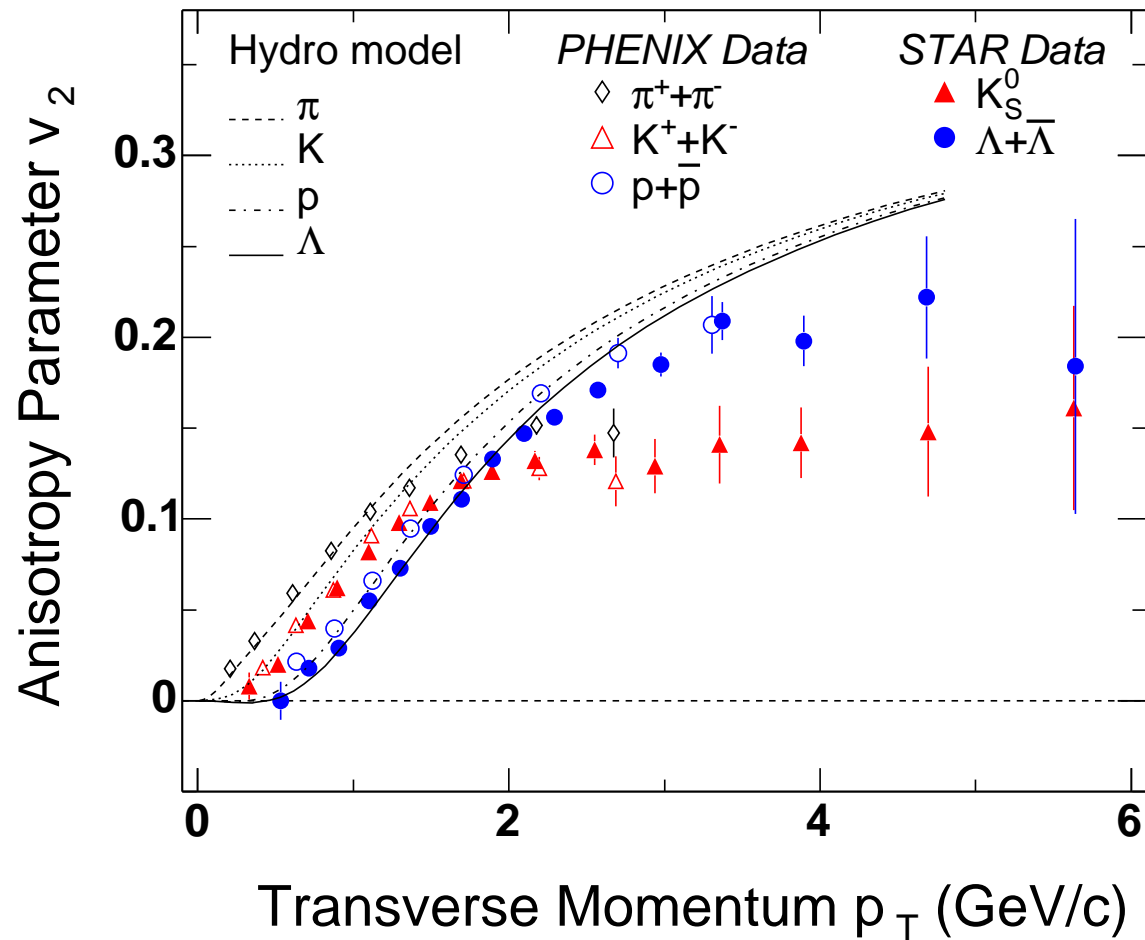
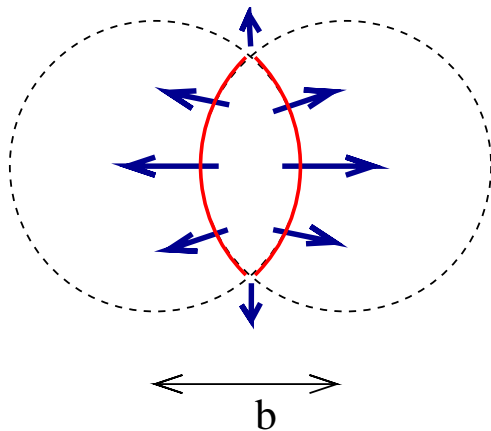
Hydrodynamic
expansion converts

coordinate space

anisotropy

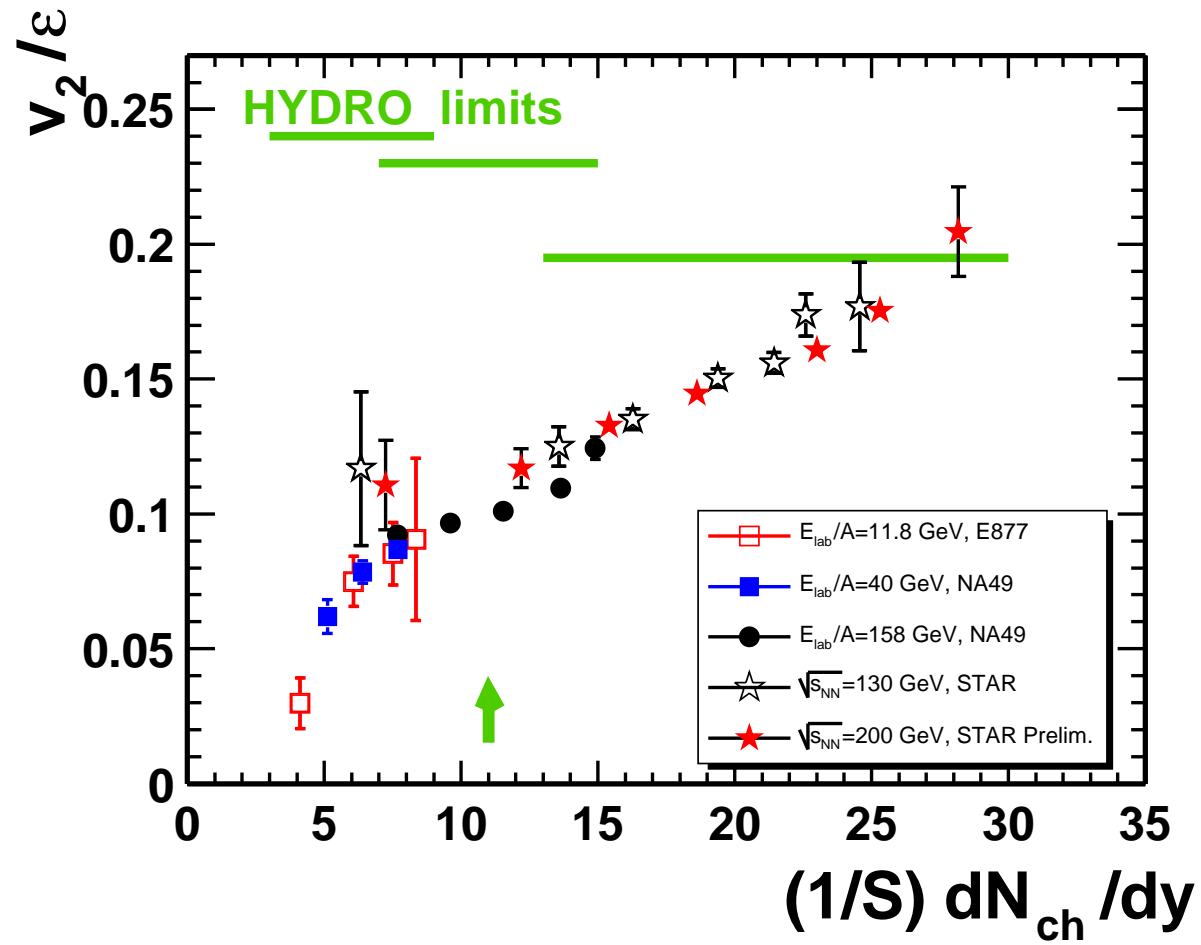
to momentum space

anisotropy



source: U. Heinz (2005)

Elliptic Flow II



source: U. Heinz (2005)

Viscous Corrections

Longitudinal expansion: Bj expansion solves Navier-Stokes equation

entropy equation

$$\frac{1}{s} \frac{ds}{d\tau} = -\frac{1}{\tau} \left(1 - \frac{\frac{4}{3}\eta + \zeta}{sT\tau} \right)$$

Viscous corrections small if $\frac{4}{3} \frac{\eta}{s} + \frac{\zeta}{s} \ll (T\tau)$

early $T\tau \sim \tau^{2/3}$ $\eta/s \sim \text{const}$ $\eta/s < \tau_0 T_0$

late $T\tau \sim \text{const}$ $\eta \sim T/\sigma$ $\tau^2/\sigma < 1$

Hydro valid for $\tau \in [\tau_0, \tau_{fr}]$

Viscous corrections to T_{ij} (radial expansion)

$$T_{zz} = P - \frac{4}{3} \frac{\eta}{\tau} \quad T_{xx} = T_{yy} = P + \frac{2}{3} \frac{\eta}{\tau}$$

increases radial flow (central collision)

decreases elliptic flow (peripheral collision)

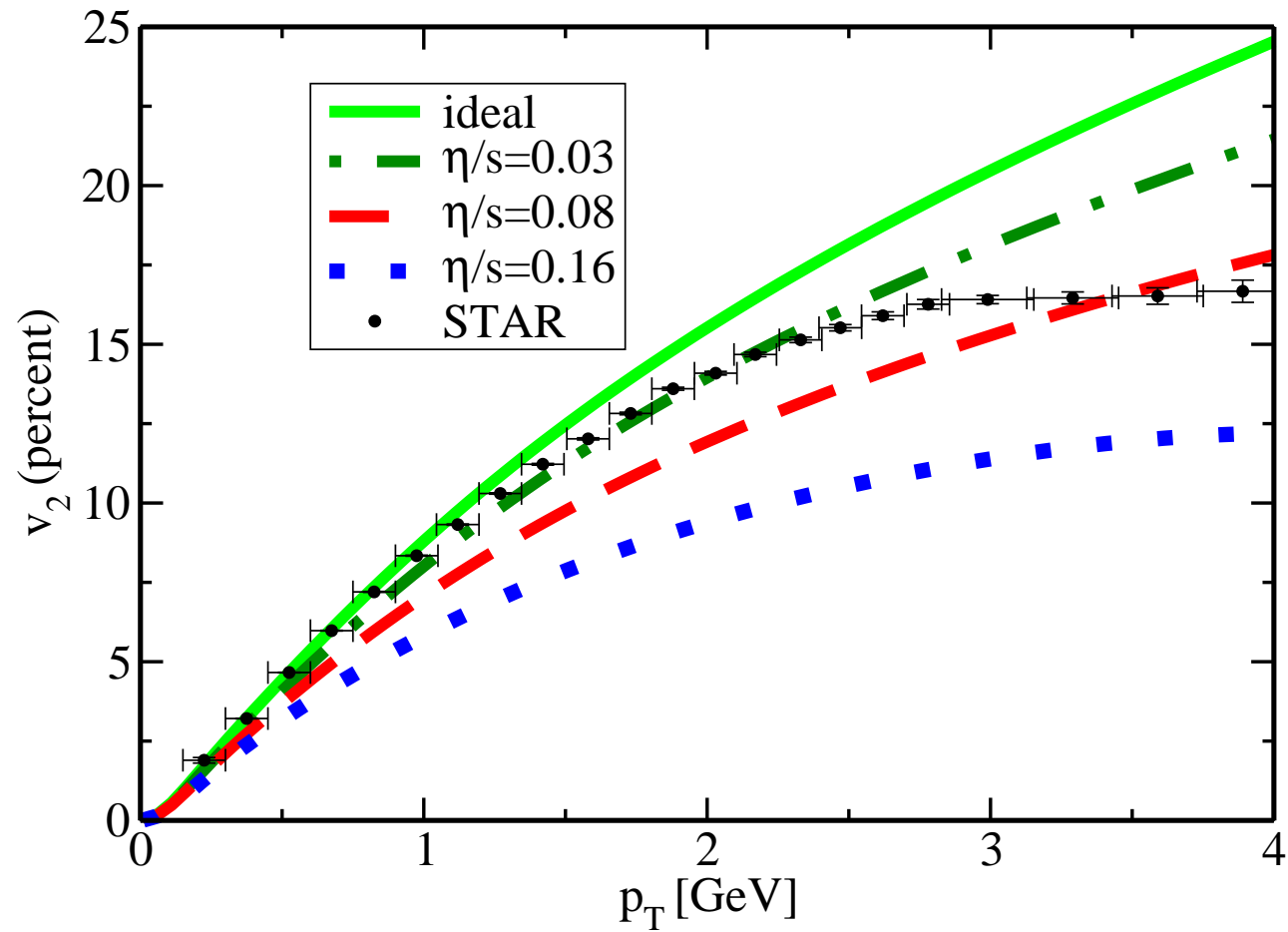
Modification of distribution function

$$\delta f = \frac{3}{8} \frac{\Gamma_s}{T^2} f_0 (1 + f_0) p_\alpha p_\beta \nabla^{\langle \alpha} u^{\beta \rangle}$$

Correction to spectrum grows with p_\perp^2

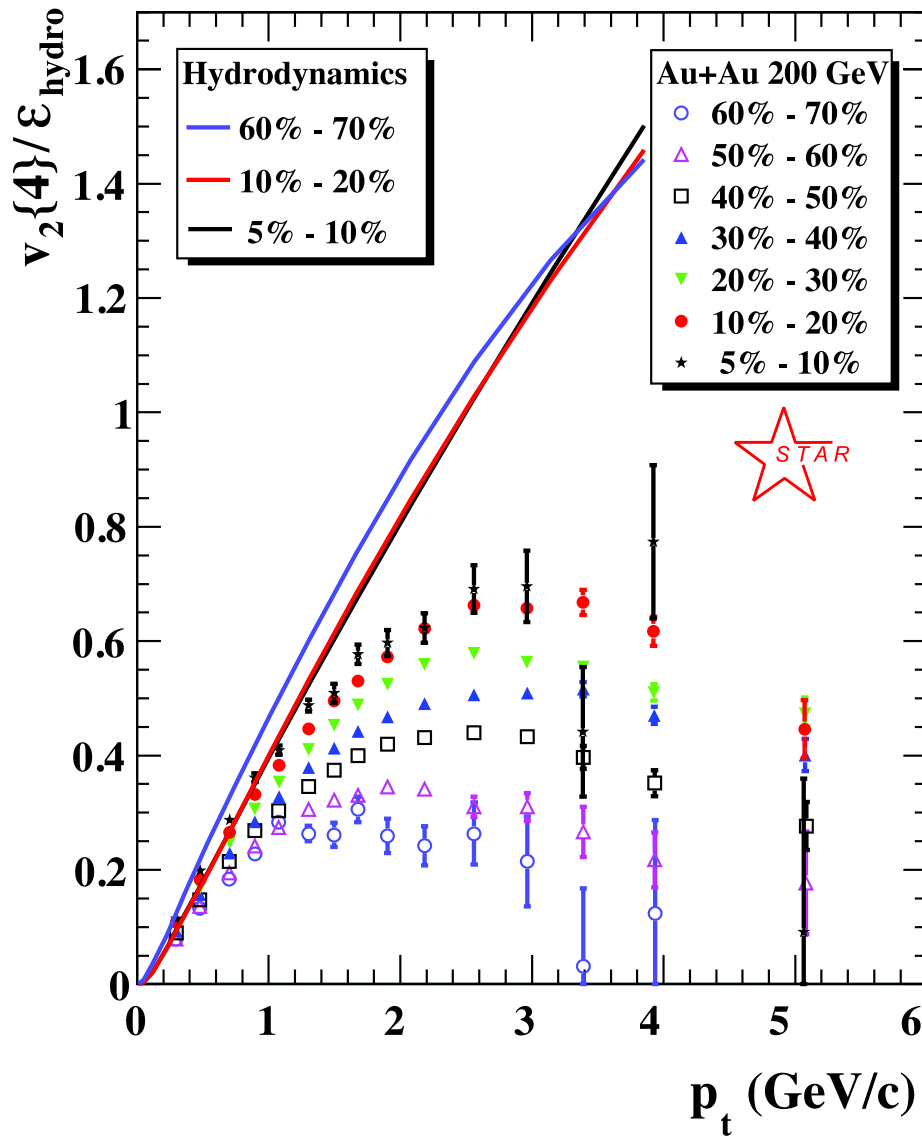
$$\frac{\delta(dN)}{dN_0} = \frac{\Gamma_s}{4\tau_f} \left(\frac{p_\perp}{T} \right)^2$$

Elliptic flow III: Viscous effects



Romatschke (2007), Teaney (2003)

Elliptic flow IV: Systematic trends



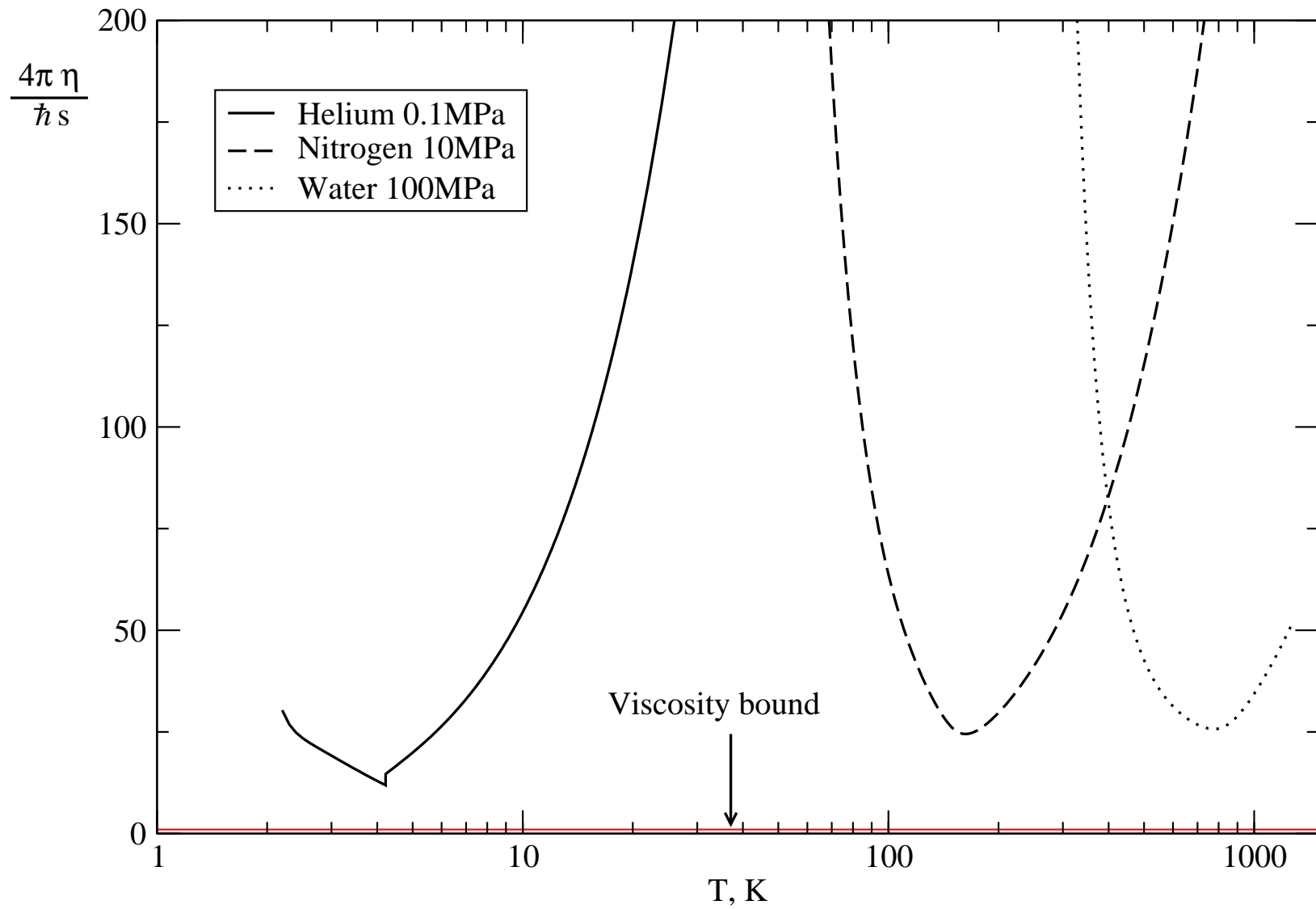
source: R. Snellings (STAR)

Deviation from ideal hydro
increases for more peripheral
events
increases with p_{\perp}

A (Most) Perfect Fluid?

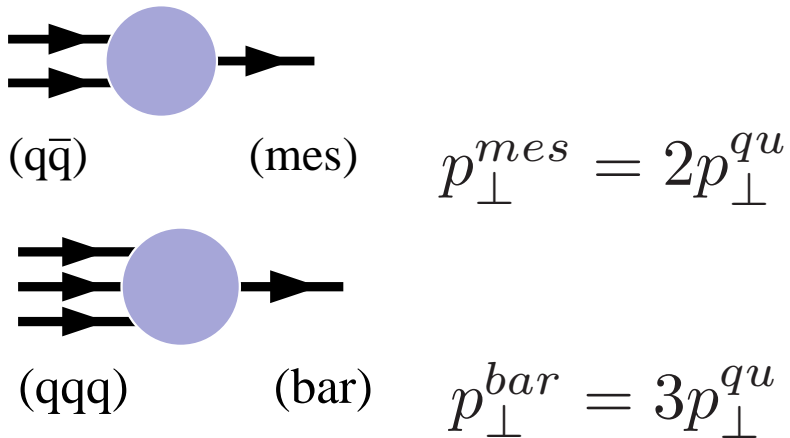
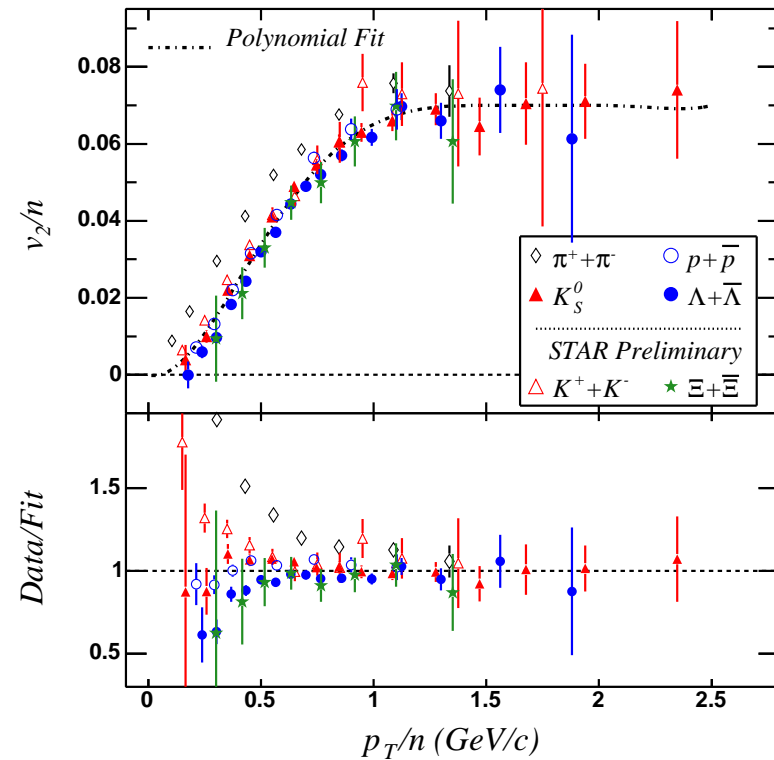
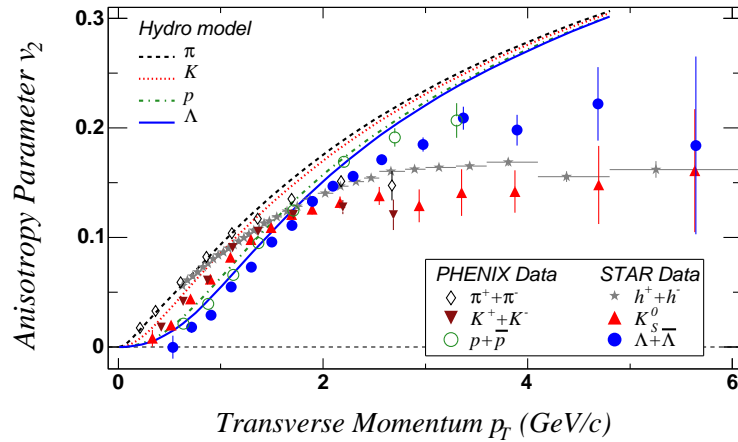


A (Most) Perfect Fluid?

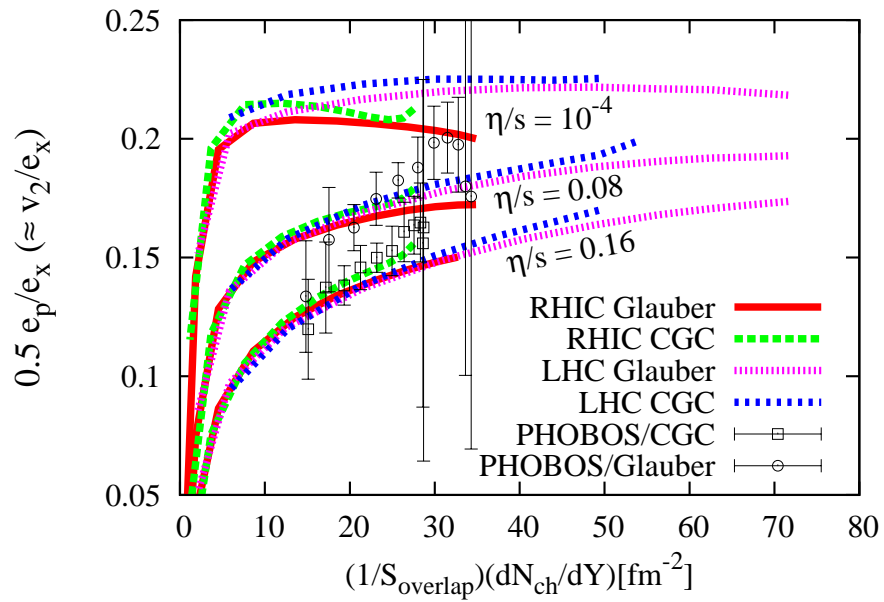


Elliptic Flow V: Recombination

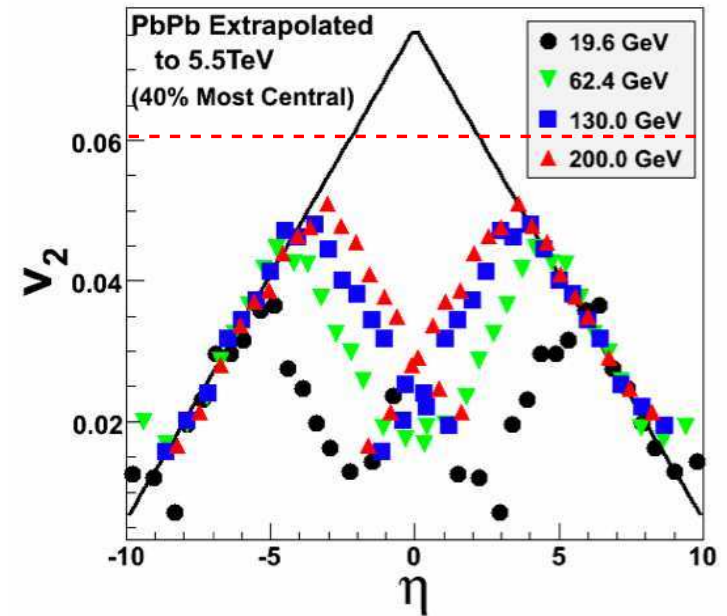
“quark number” scaling of elliptic flow



Elliptic flow VI: Predictions for LHC



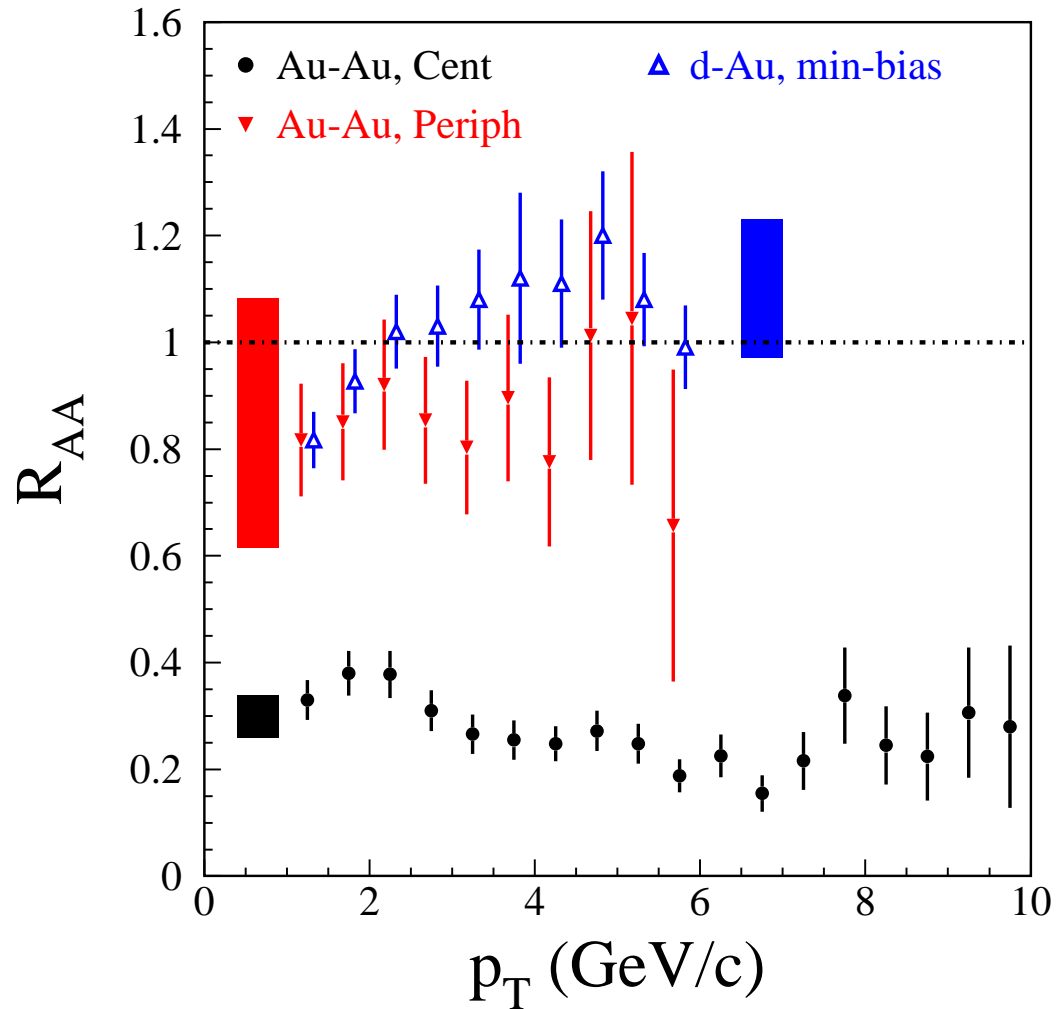
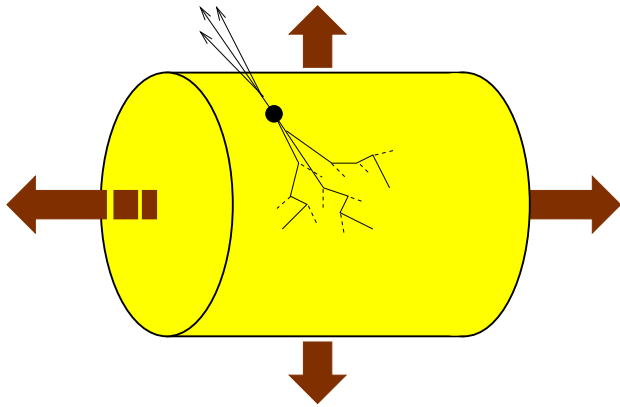
Romatschke, Luzum (2009)



Busza (QM 2009)

Jet Quenching

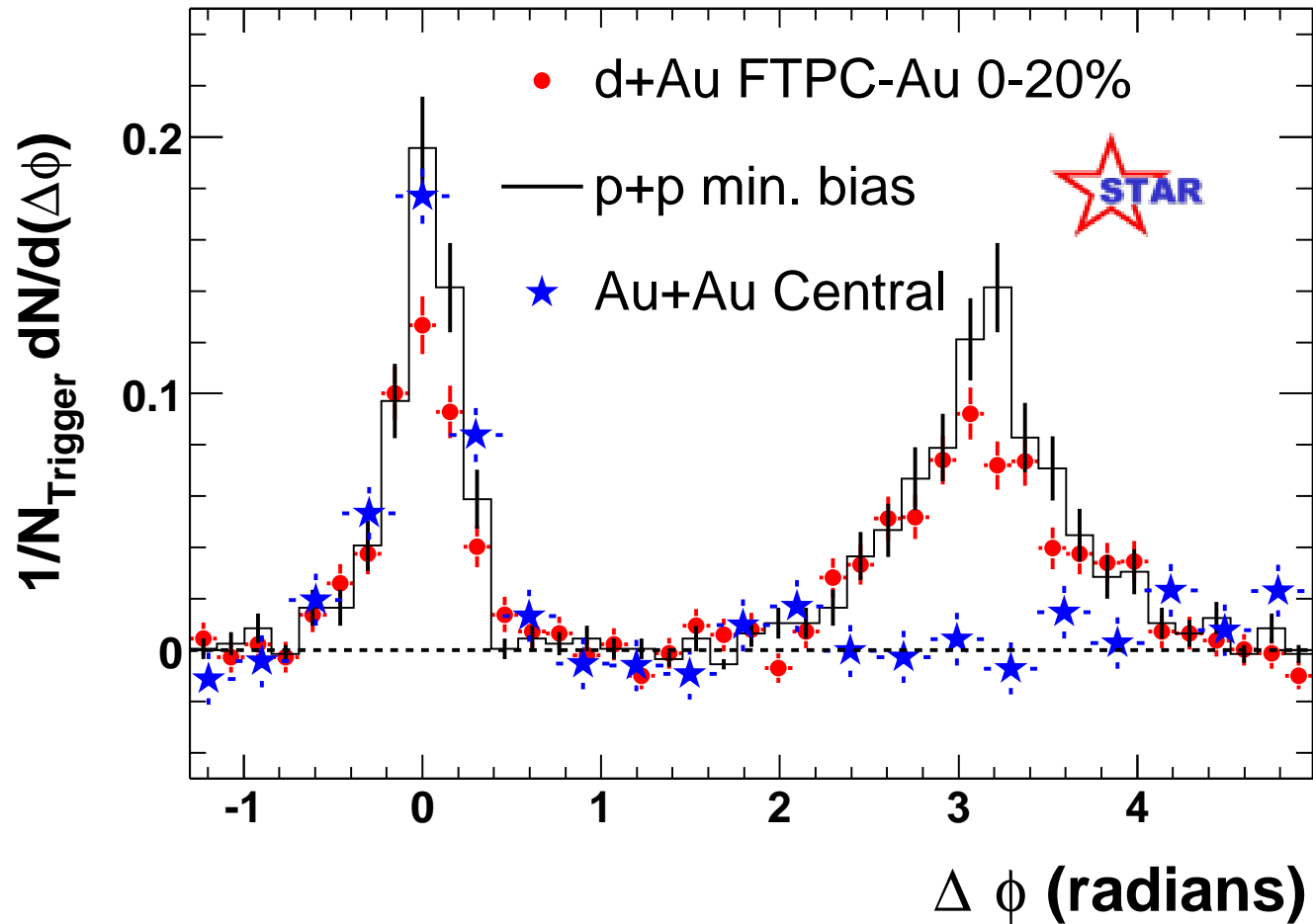
$$R_{AA} = \frac{n_{AA}}{N_{coll}n_{pp}}$$



source: Phenix White Paper (2005)

Jet Quenching II

Disappearance of away-side jet

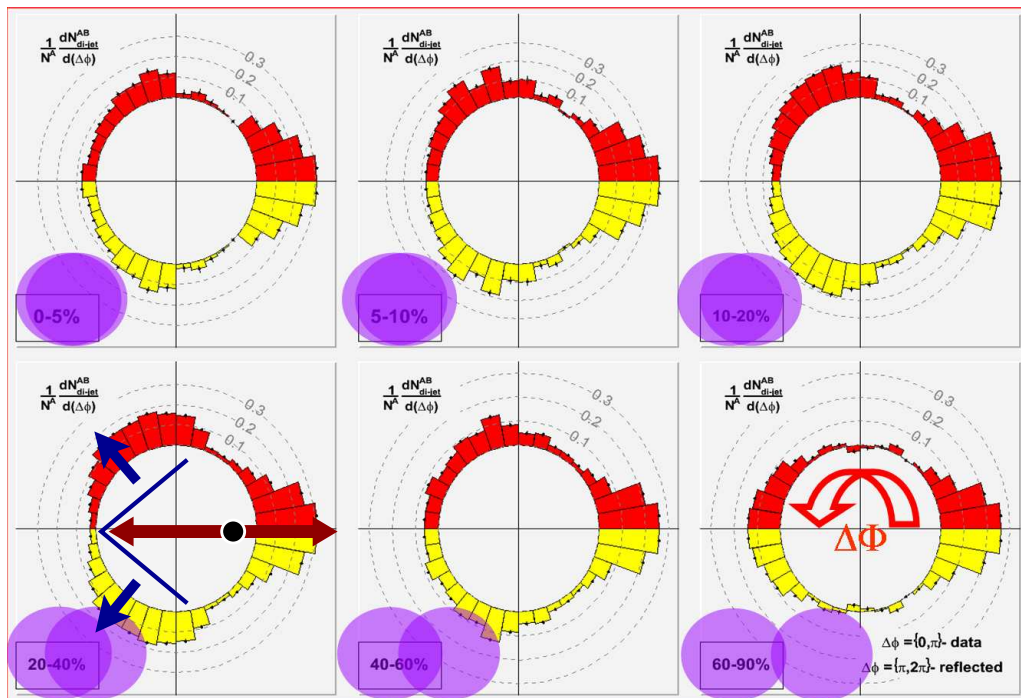


source: Star White Paper (2005)

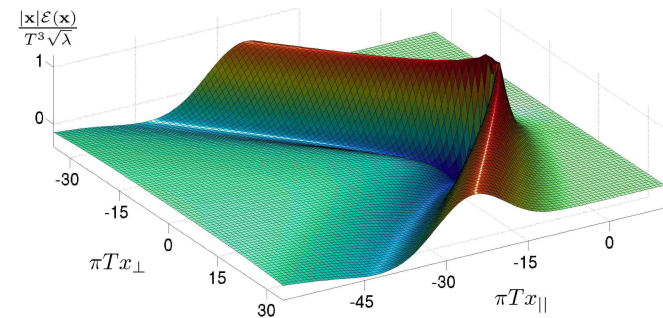
Jet Quenching III: The Mach Cone

azimuthal multiplicity $dN/d\phi$
(high energy trigger particle at $\phi = 0$)

wake of a fast quark
in $\mathcal{N} = 4$ plasma



source: Phenix (PRL, 2006), W. Zajc (2007)

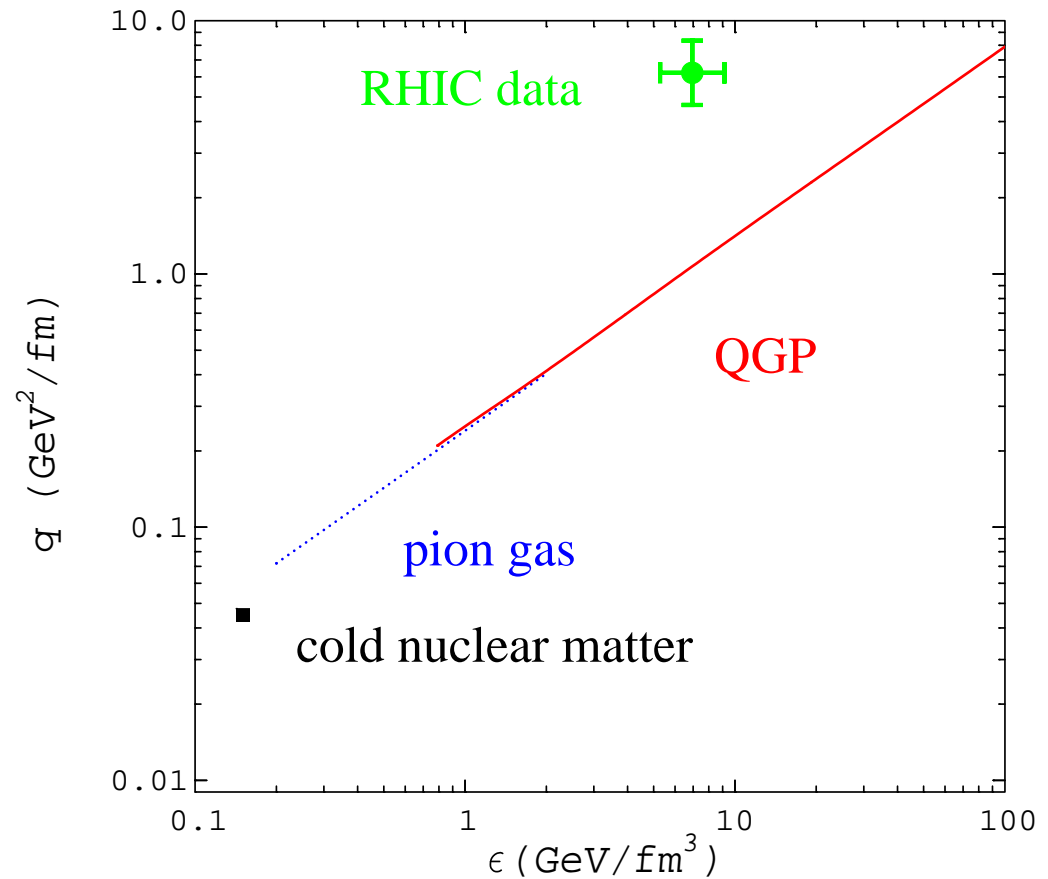
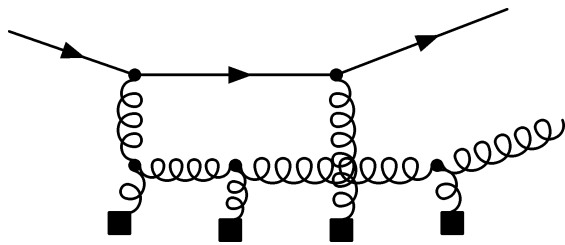


Chesler and Yaffe (2007)

Jet Quenching: Theory

energy loss governed by

$$\hat{q} = \rho \int q_{\perp}^2 dq_{\perp}^2 \frac{d\sigma}{dq_{\perp}^2}$$

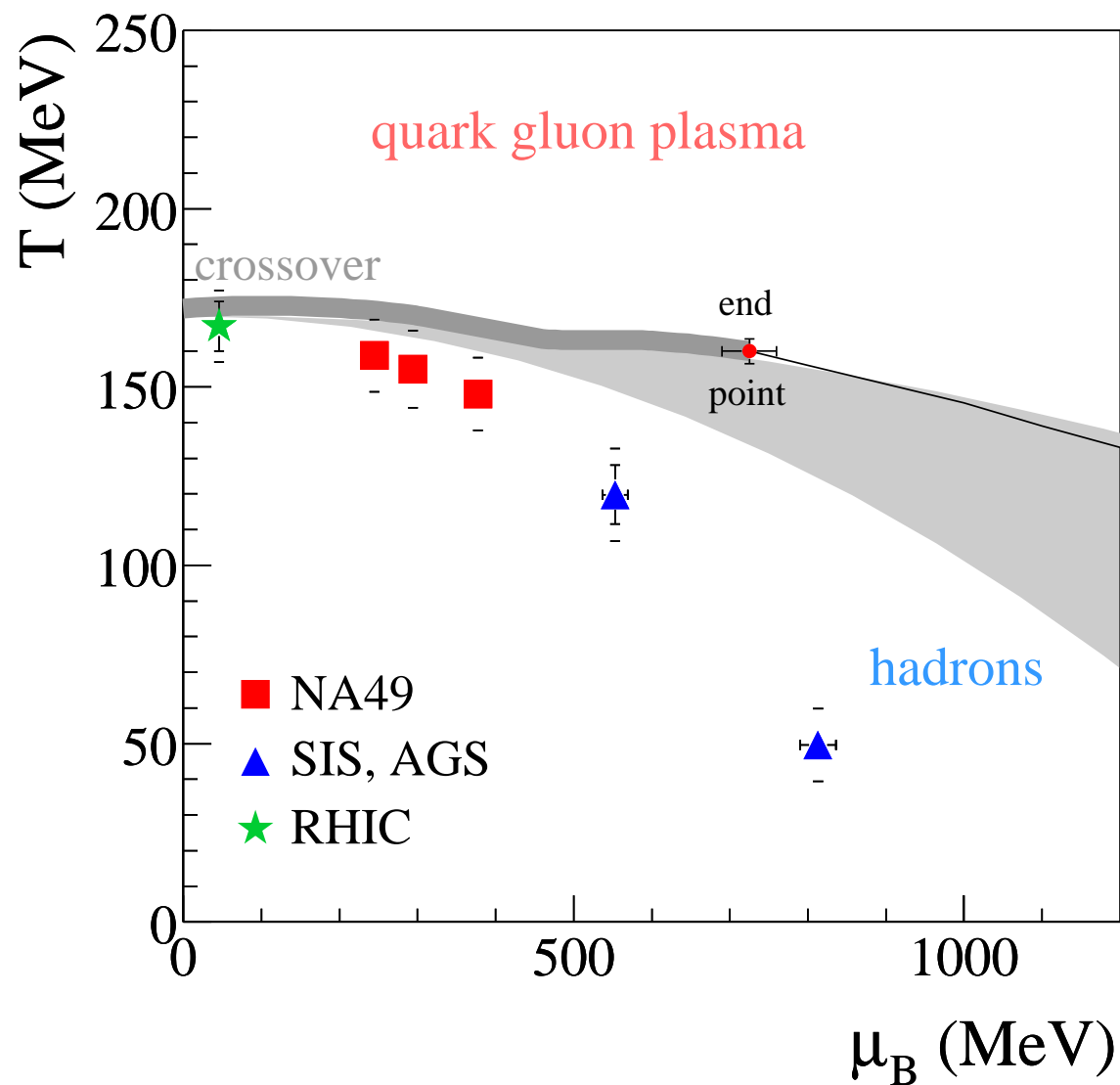


larger than pQCD predicts?

also: large energy loss of heavy quarks

[some recent doubts about \hat{q} , see P. Stankus seminar], source: R. Baier (2004)

Phase Diagram: Freezeout



Summary (Experiment)

Matter equilibrates quickly and behaves collectively

Little Bang, not little fizzle

Initial energy density in excess of 10 GeV/fm^3

Conditions for Plasma achieved

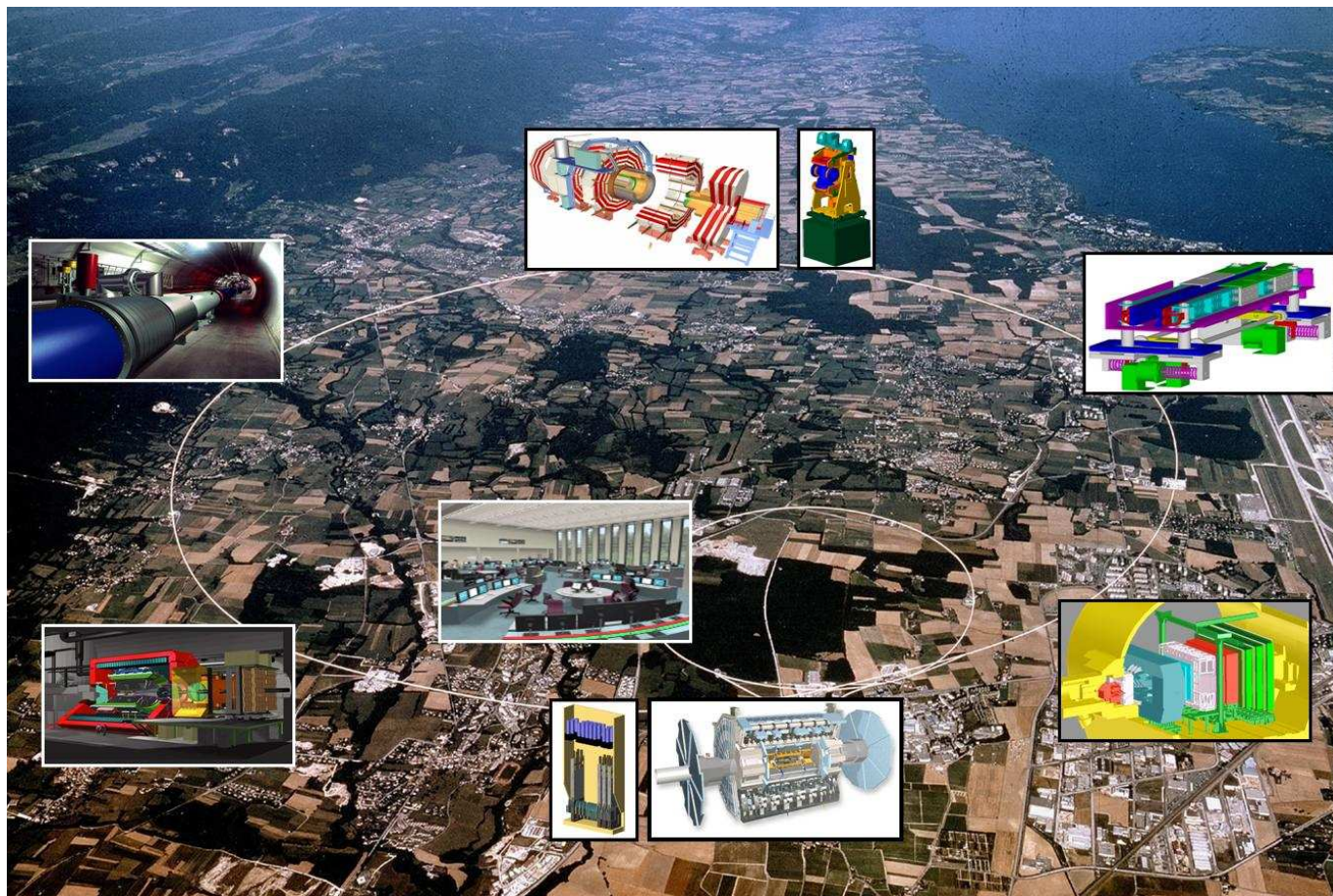
Evidence for strongly interacting Plasma (“sQGP”)

Fast equilibration $\tau_0 \ll 1 \text{ fm}$

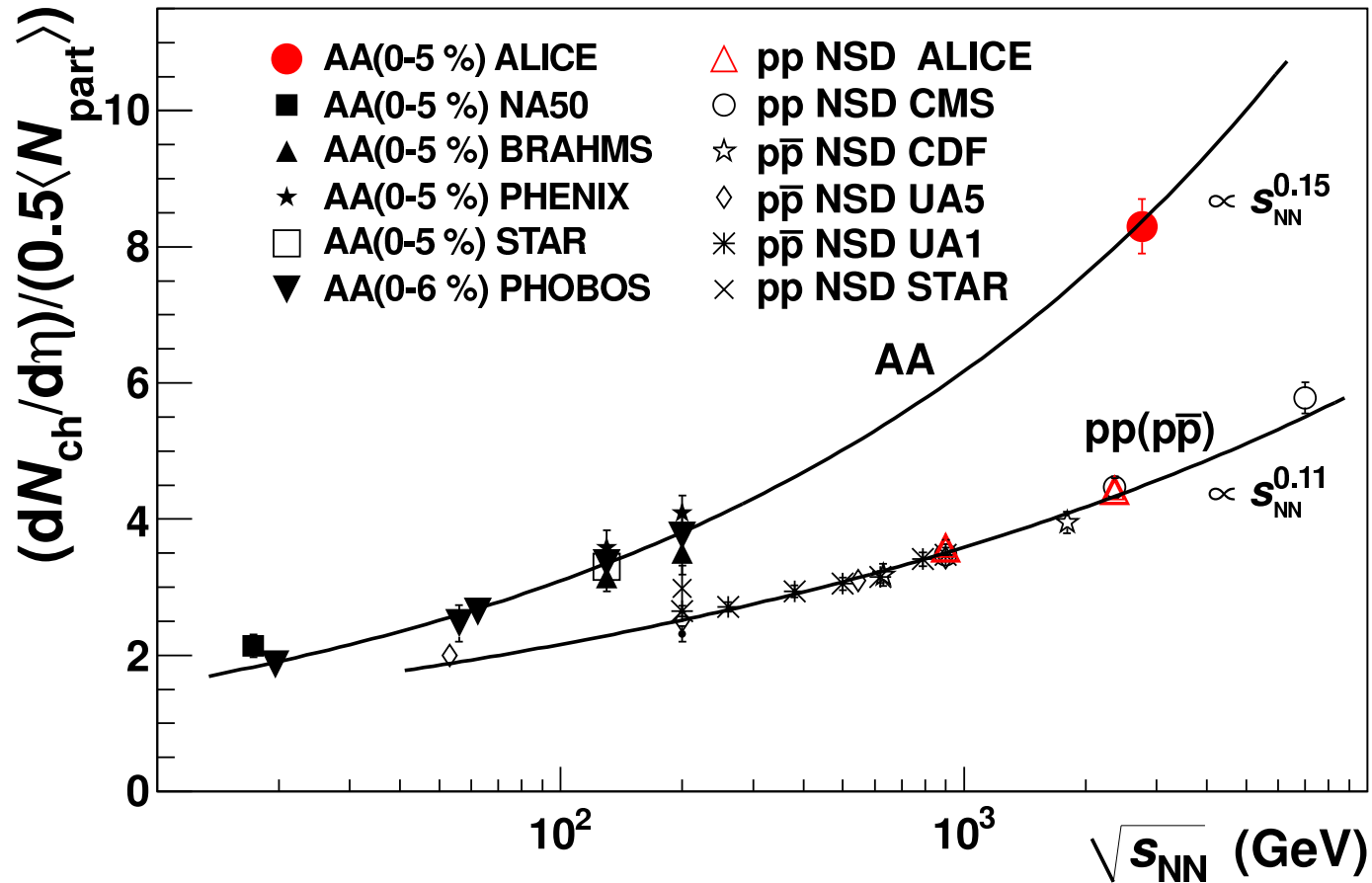
Large elliptic flow, “perfect fluid”

Strong energy loss of leading partons

LHC: Pb+Pb $\sqrt{s_{NN}} = 2.76$ TeV



Alice results: Multiplicity scaling with energy



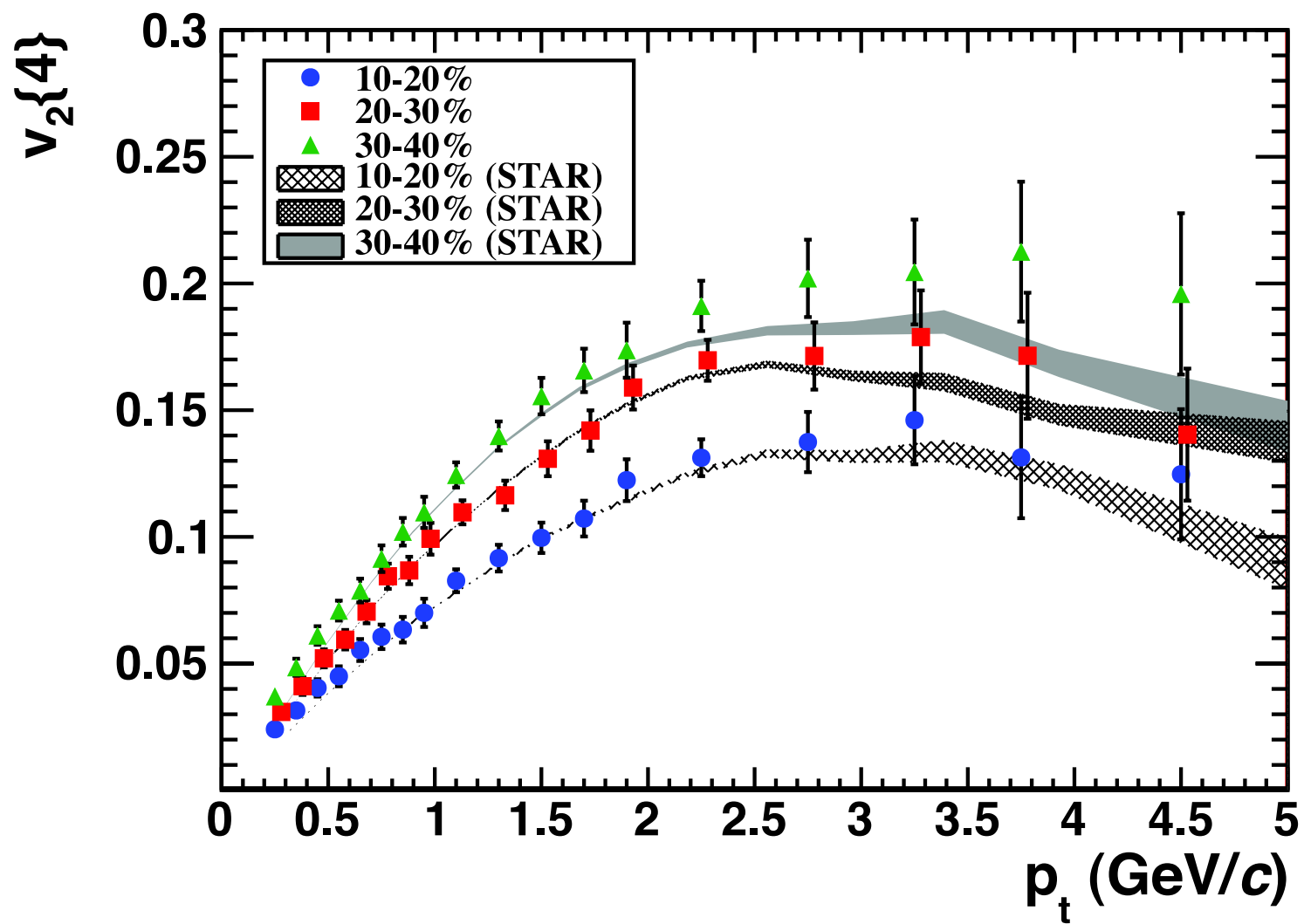
What does it mean?

Factor 2.2 in multiplicity: factor 2.85 in energy density, factor 1.3 in temperature (at fixed τ_0)

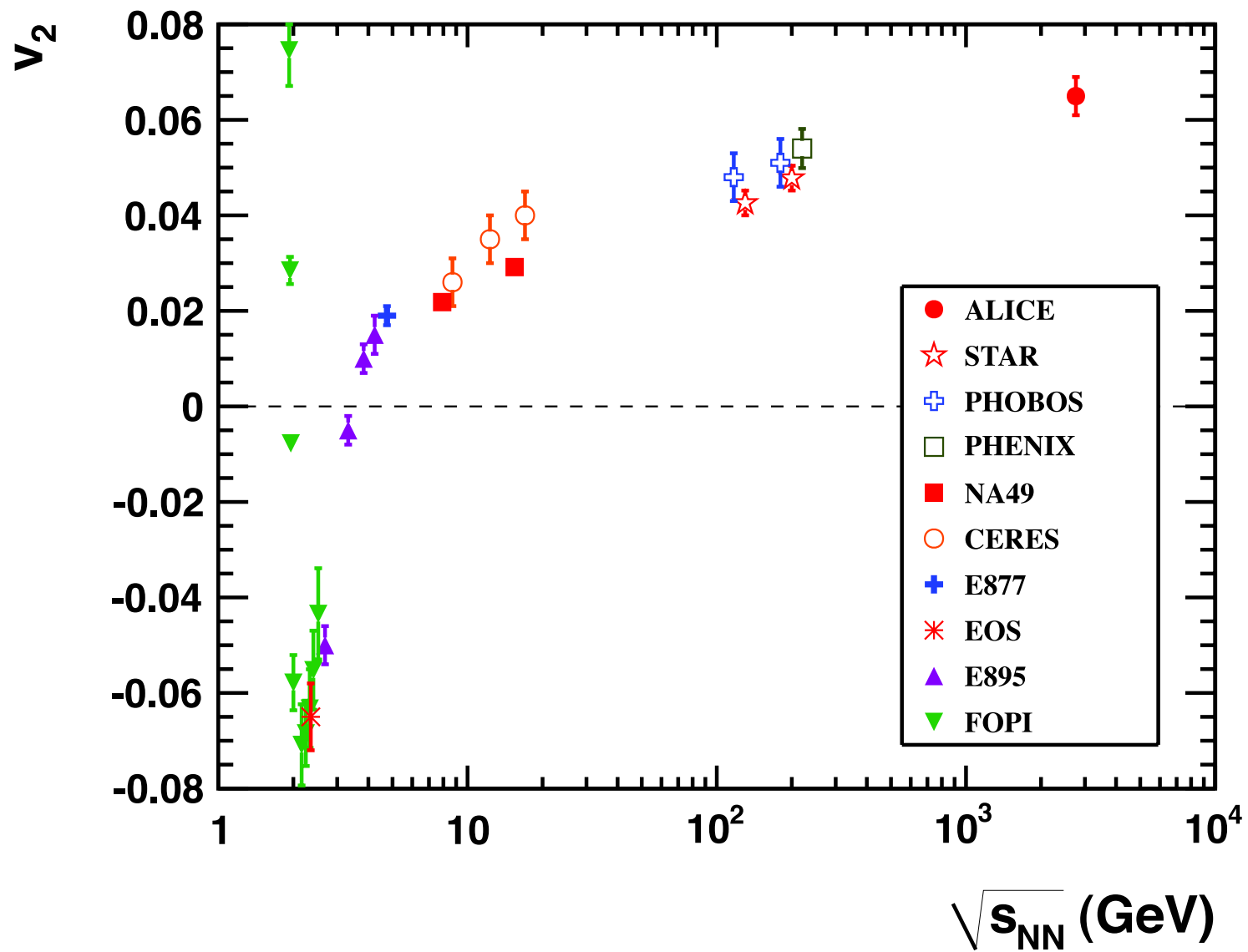
AA \neq pp: extra multiplicity per participant pair.

Simple saturation works better than improved saturation.

Alice flow



Flow excitation function



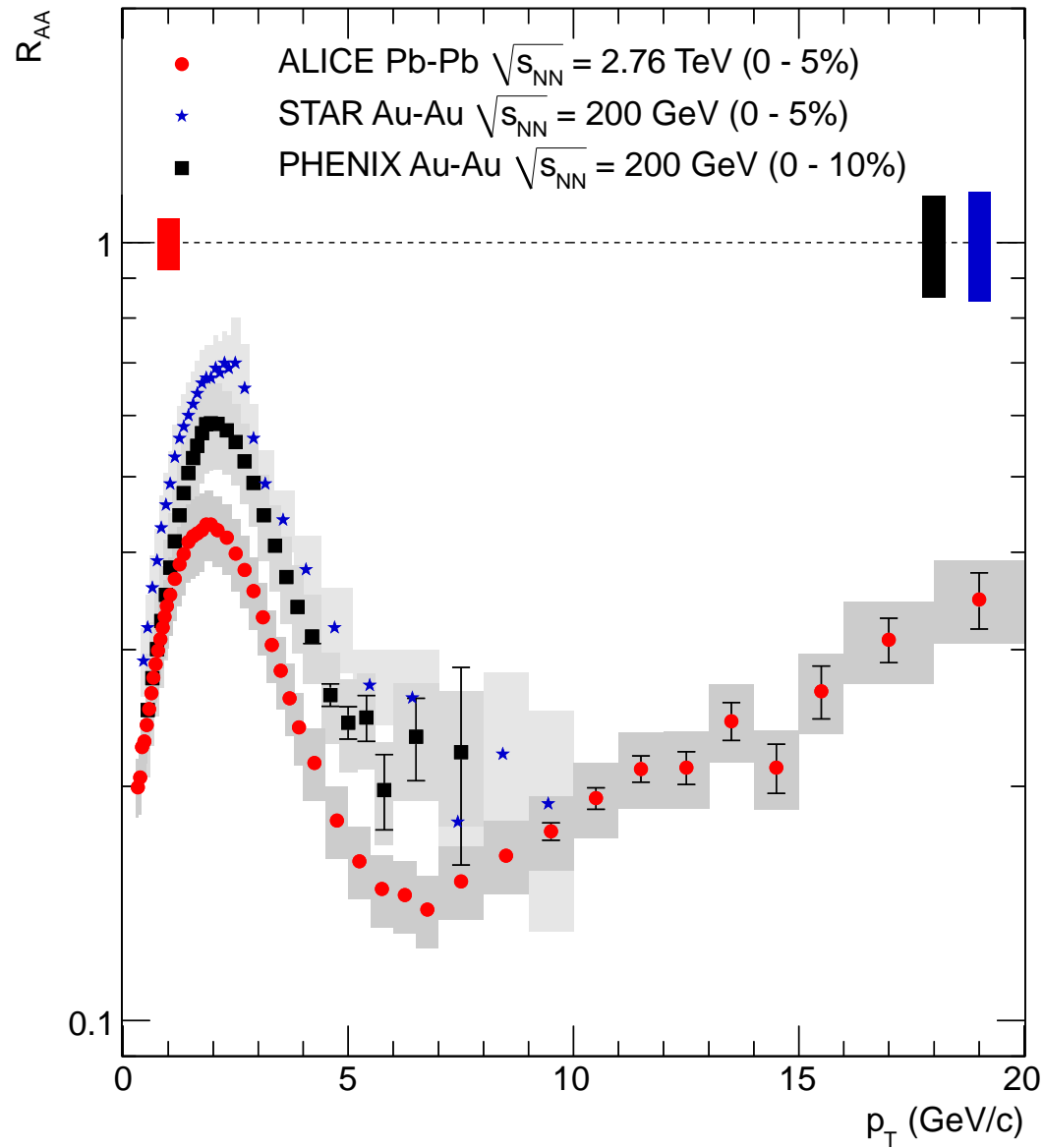
What does it mean?

Hydro rules! RHIC data not an accident.

Differential v_2 exactly equal to RHIC (!?)

Integrated v_2 somewhat high: mean p_T increase?
acceptance?

Alice jet quenching



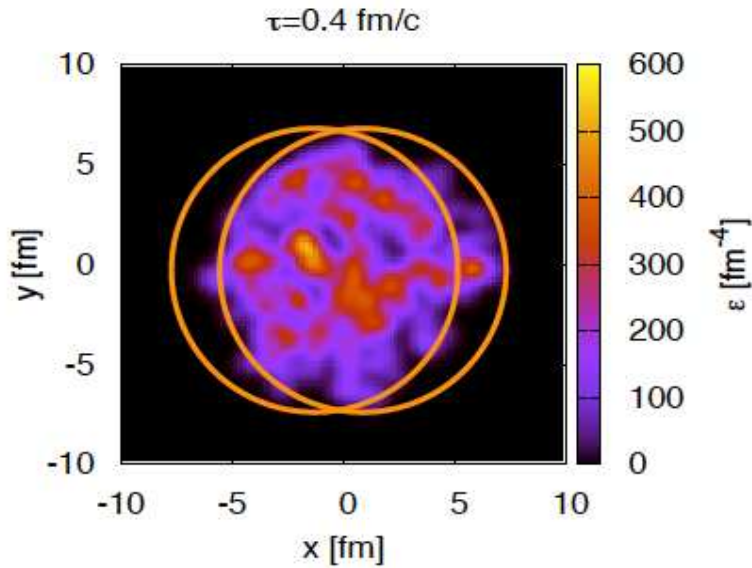
What does it mean?

Suppression at 10 GeV same as RHIC (!??)

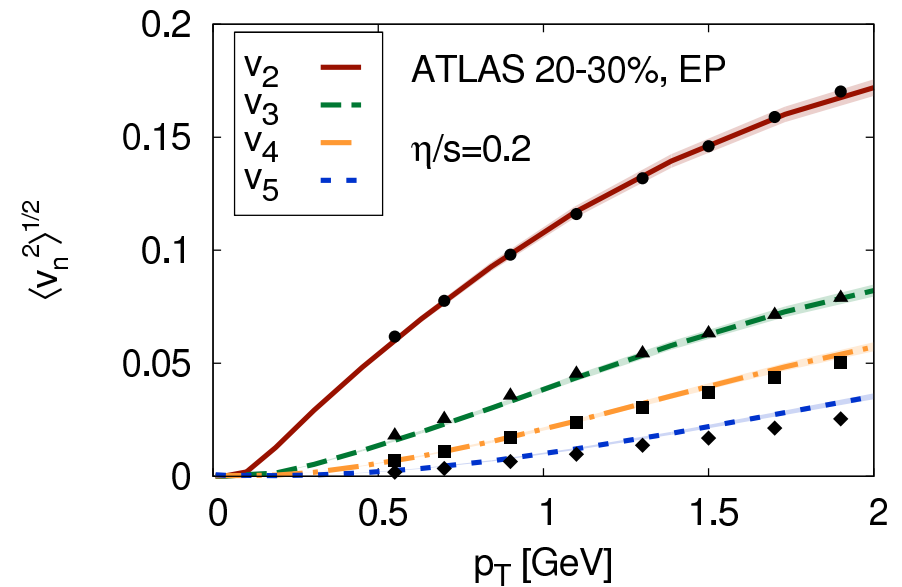
But: p_T dependence no longer flat, agrees with predictions (expect factorization as $p_T \rightarrow \infty$).

Frontier I: Initial conditions & higher moments of flow

Hydro converts moments of initial deformation to moments of flow



B. Schenke



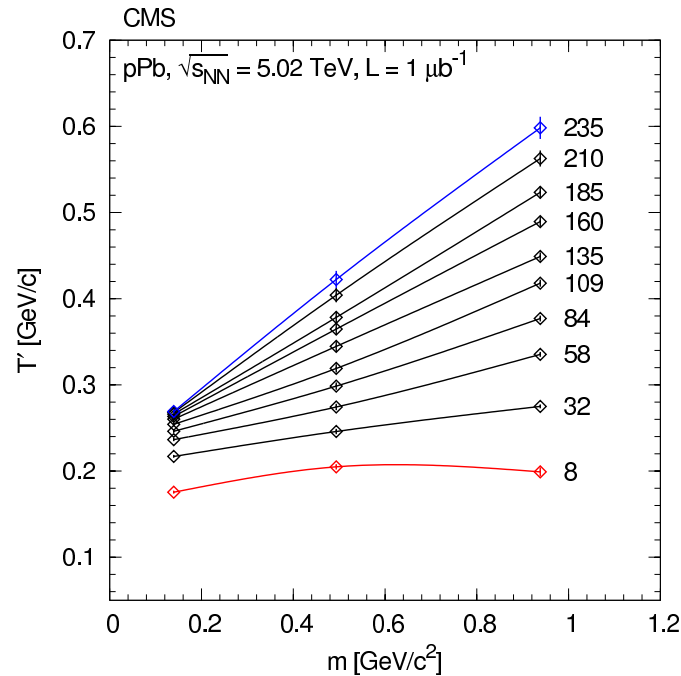
C. Gale et al.

Glauber predicts flat initial spectrum ($n \geq 3$). Observed flow spectrum consistent with sound attenuation

$$\delta T^{\mu\nu}(t) = \exp\left(-\frac{2}{3} \frac{\eta}{s} \frac{k^2 t}{T}\right) \delta T^{\mu\nu}(0)$$

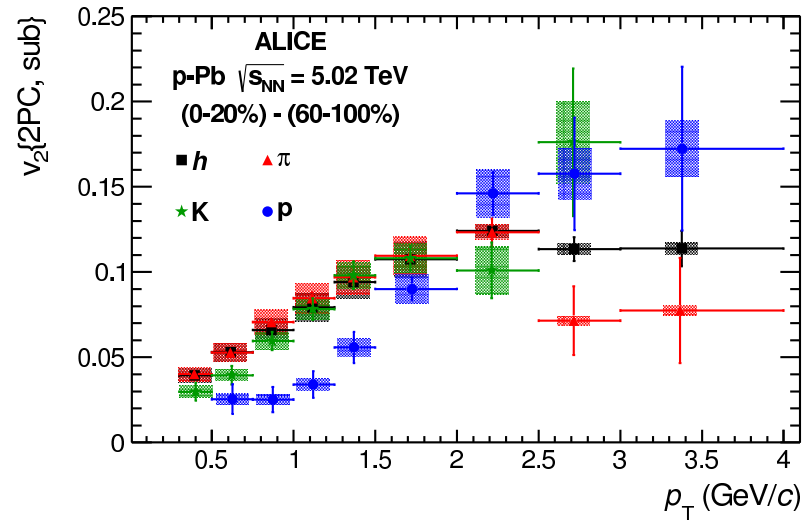
Frontier II: Everything flows (even p+Pb)

Signatures of collectivity in p+Pb collisions.



Mass ordering of mean p_T

CMS (2013)



Mass ordering of $v_2(p_T)$

Alice (2013)

Consistent with AA data for conformal hydro scaling

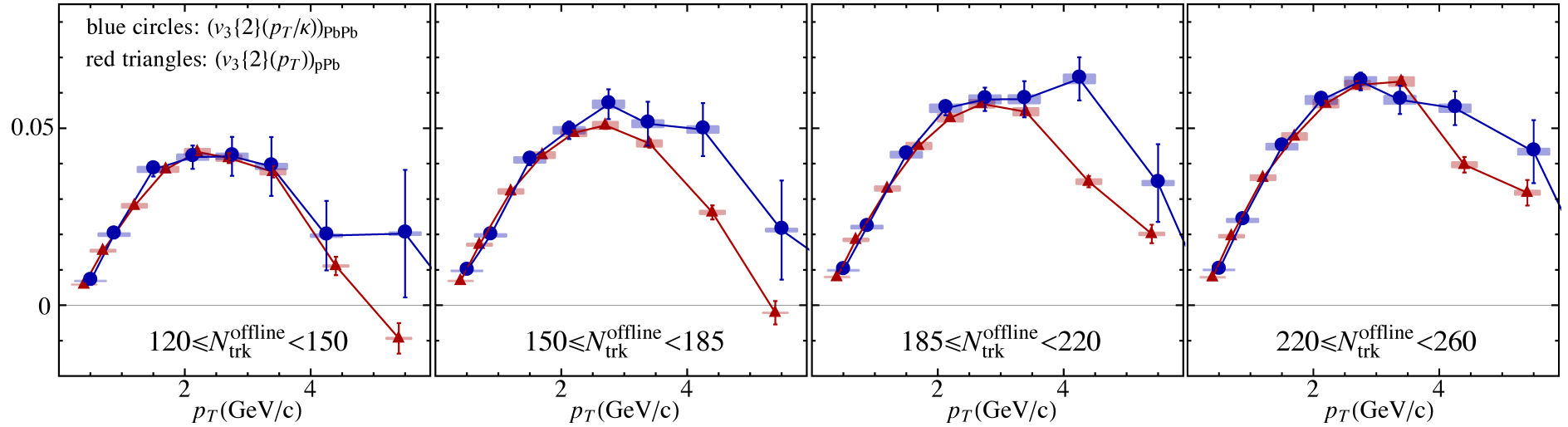
$$Kn^{-1} \sim \frac{c_s}{c} \frac{1}{S} \frac{dN}{dy}$$

non-conformal fluid

$$Kn^{-1} \sim \frac{c_s}{c} \frac{dN}{dy}$$

conformal fluid

Knudsen scaling: Compare pPb and PbPb

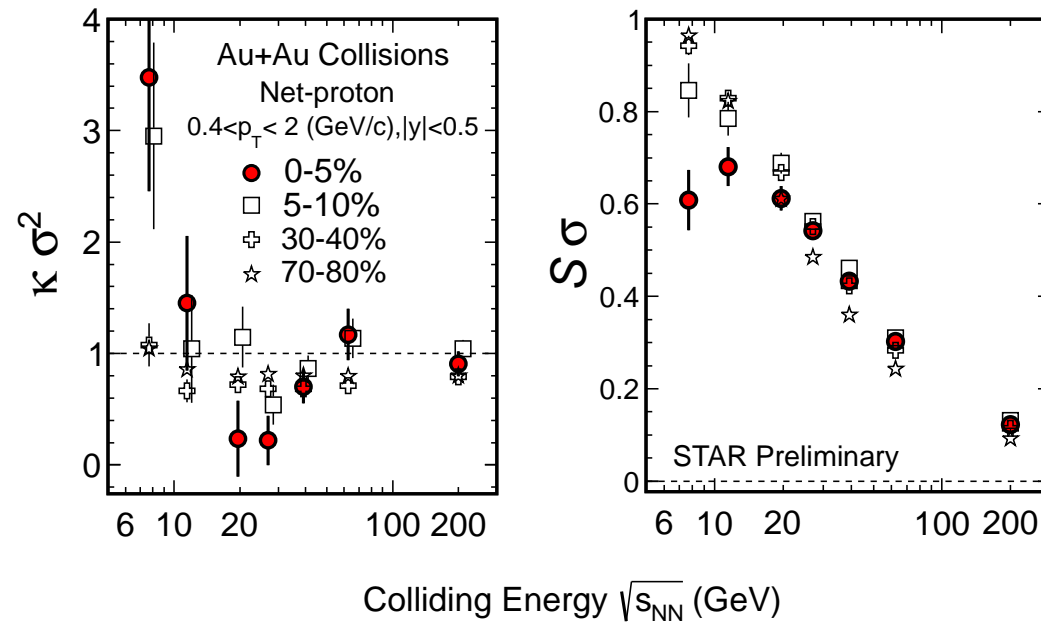


Triangular flow $v_3(p_T)$ in pPb (red) and PbPb (blue)

p_T dependence scaled by mean $\langle p_T \rangle$

Teaney, Basar (2014)

Frontier III: Fluctuations and the critical point



Net-Proton kurtosis and skewness from RHIC beam energy scan

Star Collaboration (2015)