Effective Theories of Very Dense Matter:

From keV to GeV

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Motivation

There is a successful effective theory of fermionic many body systems
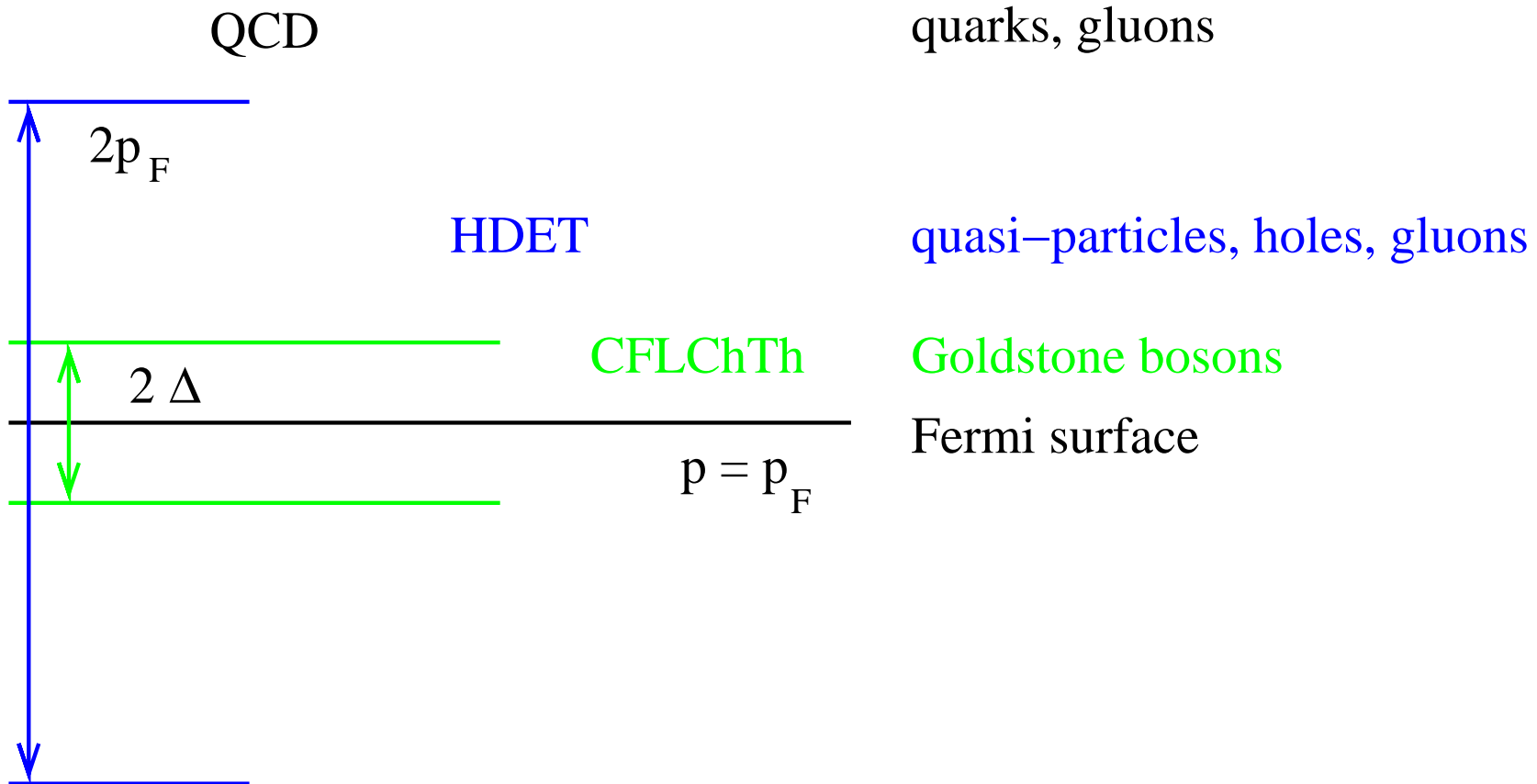
Landau Fermi-Liquid Theory

FLT theory: Quasi-particles near the Fermi surface. Interactions characterized by Fermi-liquid parameters.

Predicts collective modes, thermodynamics, transport, . . .

Gauge Theories: Unscreened long range forces
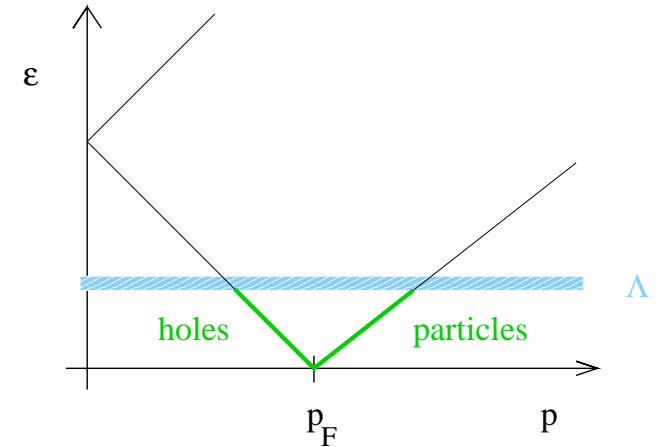
Does a quasi-particle EFT exist?

Effective Field Theories

QCD   quarks, gluons

$2p_F$

HDET   quasi–particles, holes, gluons

CFLChTh   Goldstone bosons

$2\,\Delta$

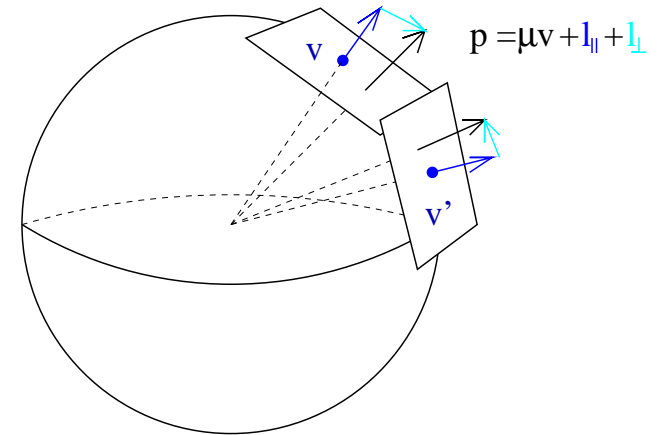$p = p_F$ Fermi surface

High Density Effective Theory

Quasi-particles (holes)

$$E_\pm = -\mu \pm \sqrt{\vec{p}^2 + m^2} \simeq -\mu \pm |\vec{p}|$$

Effective field theory on v-patches

$$\psi_{v\pm} = e^{-i\mu v \cdot x} \left(\frac{1 \pm \vec{\alpha} \cdot \vec{v}}{2} \right) \psi$$
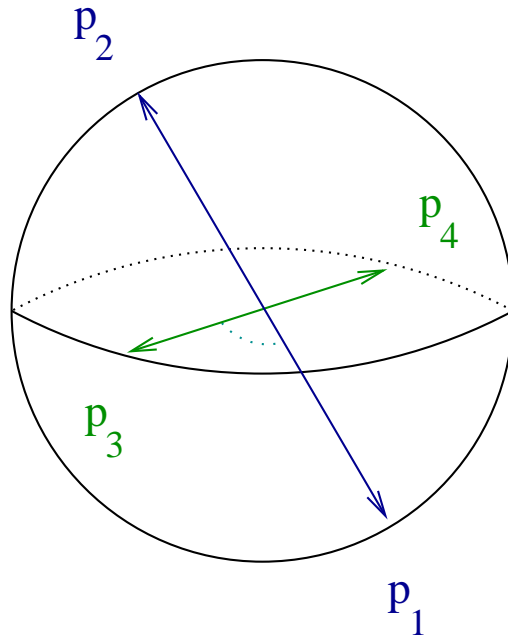

Effective lagrangian for ψ_{v+}

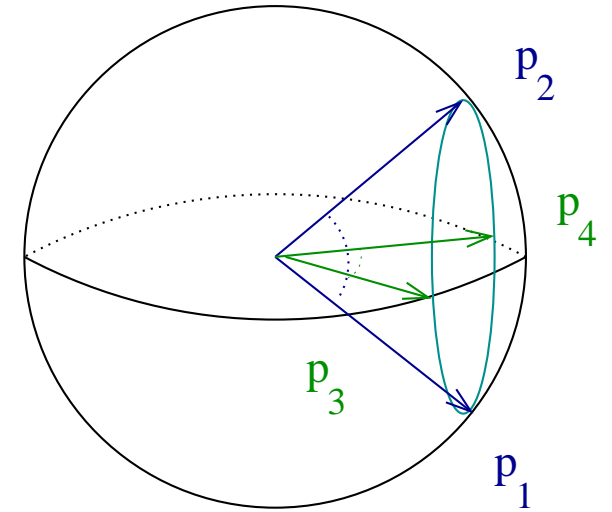$$\mathcal{L} = \sum_v \psi_v^\dagger (iv \cdot D) \psi_v - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + O(1/\mu)$$

Four Quark Operators

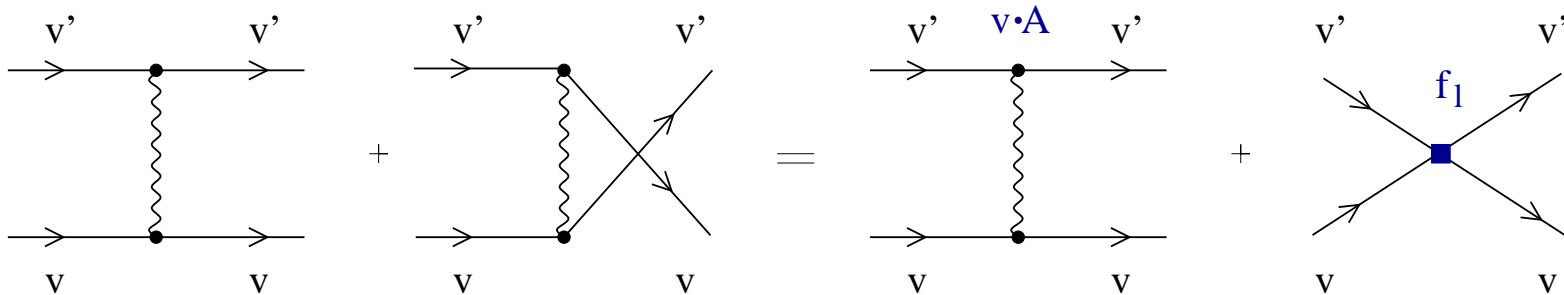quark-quark scattering

$(v_1, v_2) \to (v_3, v_4)$

BCS

Landau

$$\mathcal{L}_{BCS} = \frac{1}{\mu^2} \sum V_l^{\Gamma\Gamma'} R_l^{\Gamma\Gamma'}(\vec{v} \cdot \vec{v}') \left(\psi_v \Gamma \psi_{-v} \right) \left(\psi_{v'}^\dagger \Gamma' \psi_{-v'}^\dagger \right),$$

$$\mathcal{L}_{FL} = \frac{1}{\mu^2} \sum F_l^{\Gamma\Gamma'}(\phi) R_l^{\Gamma\Gamma'}(\vec{v} \cdot \vec{v}') \left(\psi_v \Gamma \psi_{v'} \right) \left(\psi_{\tilde{v}}^\dagger \Gamma' \psi_{\tilde{v}'}^\dagger \right)$$

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Four Fermion Operators: Matching

Match scattering amplitudes on Fermi surface: forward scattering
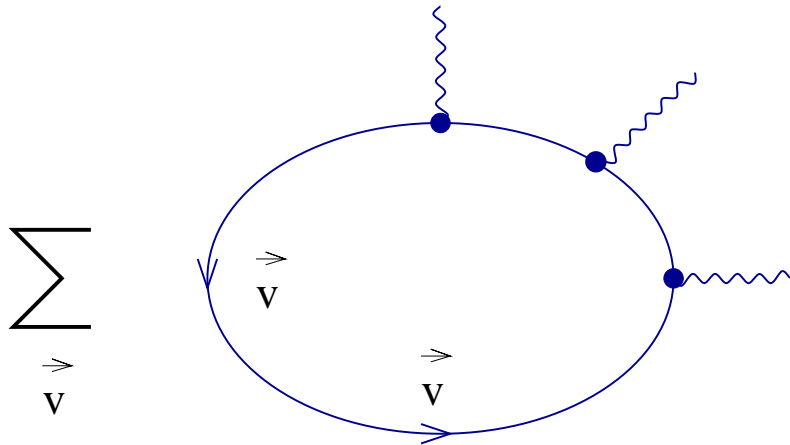

Color-flavor-spin symmetric terms

$$f_0^s = \frac{C_F}{4 N_c N_f} \frac{g^2}{p_F^2}, \quad f_i^s = 0 \;\; (i > 1)$$

Power Counting

Naive power counting

$$\mathcal{L} = \hat{\mathcal{L}} \left(\psi, \psi^\dagger, \frac{D_{\|}}{\mu}, \frac{D_\perp}{\mu}, \frac{\bar{D}_{\|}}{\mu}, \frac{m}{\mu} \right)$$
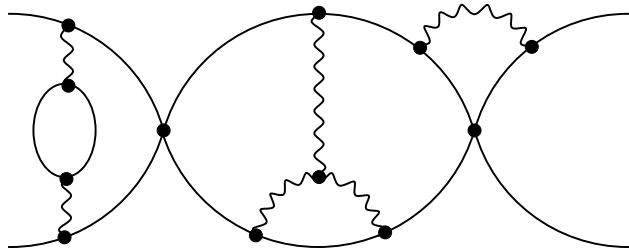
Problem: hard loops

$$\frac{1}{2\pi} \sum_{\vec{v}} \int \frac{d^2 l_\perp}{(2\pi)^2} = \frac{\mu^2}{2\pi^2} \int \frac{d\Omega}{4\pi}.$$

Modified Power Counting: $\mathcal{A} \sim l^\delta$

$$\delta = \sum_i \left[(k-4)V_k^S + (k-2-f_k)V_k^H \right] + E_Q + 4 - 2N_C$$
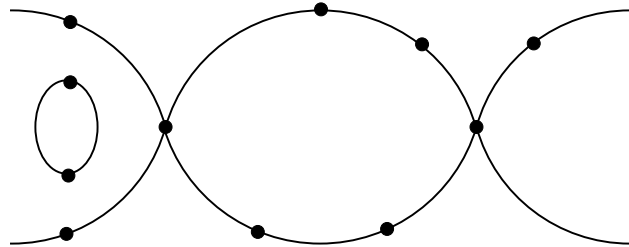

V_k^S soft vertices of $O(l^k)$

V_k^H hard vertices of $O(l^k)$

f_k number of fermion fields in V_k^H

E_Q external quark lines

N_C connected graphs in hard graph

- resum quark loops in gluon n-pt fcts

- four quark operators are leading order

- six quark operators (etc) are suppressed

Effective Theory for $l \sim g\mu$

$$\mathcal{L} = \psi_v^\dagger (iv \cdot D)\psi_v + f_0^s (\psi_v^\dagger \psi_v)(\psi_{v'}^\dagger \psi_{v'}) - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \mathcal{L}_{HDL}$$

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \sum_v G_{\mu\alpha}^a \frac{v^\alpha v^\beta}{(v \cdot D)^2} G_{\mu\beta}^b$$
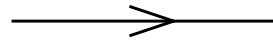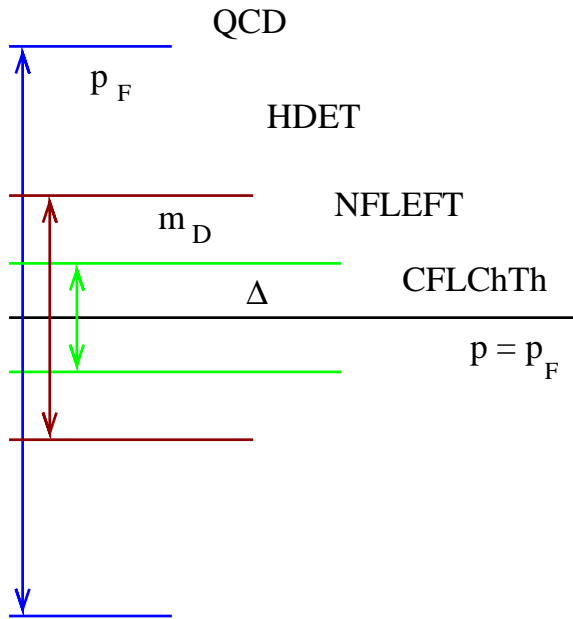
Transverse gauge boson propagator

$$D_{ij}(k) = \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{k_0^2 - \vec{k}^2 + i\eta |k_0|/|\vec{k}|},$$

Scaling of gluon momenta

$$|\vec{k}| \sim k_0^{1/3} \eta^{2/3} \gg k_0 \quad \text{gluons are very spacelike}$$

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Effective Theory for $l < g\mu$

$$\frac{i}{p_0 - p_{||} - p_\perp^2/(2\mu)}$$

QCD

p_F

HDET

NFLEFT

m_D

Δ

CFLChTh

$p = p_F$

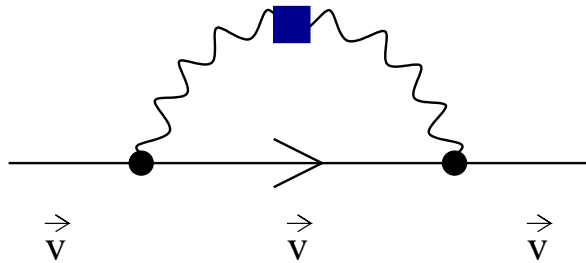$$\frac{i\delta_{ij}k_\perp}{k_\perp^3 - i\eta k_0}$$
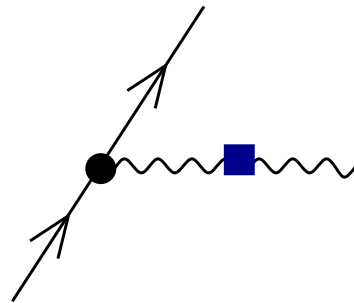
$$igv_i$$

Low energy expansion contains fractional powers $\omega^{1/3}$

Quasi-Quarks at Large Density:

Non-Fermi Liquid Effects

Loop Corrections

$$\Sigma(\omega) \simeq \frac{g^2 C_F}{12\pi^2} \omega \log\left(\frac{\Lambda}{\omega}\right)$$
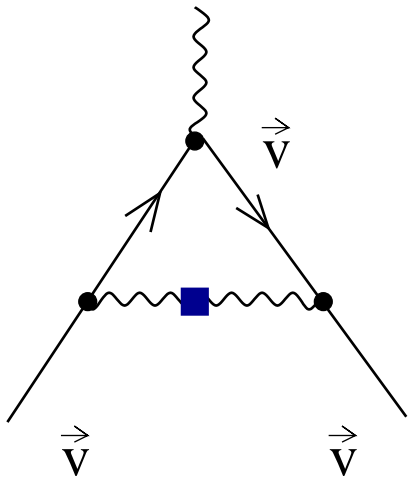
$$\Gamma_\alpha = \frac{g^3 C_F v_\alpha}{12\pi^2} \log\left(\frac{\Lambda}{\omega}\right) \quad \text{time like}$$
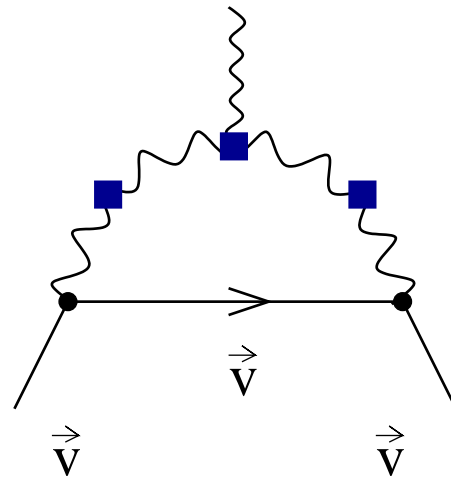
$$\Gamma_\alpha = O(g^3) \quad \text{space like}$$

"Migdal's Theorem" for QCD

- self energy non-perturbative (?) for $\omega \sim \Lambda \exp(-9\pi^2/g^2)$

- no large logarithms in vertex $(k \gg k_4!)$

Renormalization Group

Renormalized parameters

$$\psi_{0,v} = Z^{1/2}\psi_v, \quad v_{0,F} = Z_F v_F, \quad g_0 = \frac{Z_g}{Z Z_F}g, \quad \alpha = \frac{g^2 v_F}{4\pi},$$

One-loop calculation $\left(\beta = \frac{\partial \alpha}{\partial \log \Lambda},\ \gamma = \frac{\partial \log Z}{\partial \log \Lambda},\ \gamma_F = \frac{\partial \log Z_F}{\partial \log \Lambda}\right)$

$$\gamma(\alpha) = -\gamma_F(\alpha) = \frac{4\alpha}{9\pi}, \qquad \beta(\alpha) = -\gamma_F(\alpha)\alpha \qquad \text{IR free !!}$$

RG equation

$$\left\{ \Lambda\frac{\partial}{\partial \Lambda} + \beta(\alpha)\frac{\partial}{\partial \alpha} - \gamma_F(\alpha)l_i\frac{\partial}{\partial l_i} + \frac{n}{2}\gamma(\alpha) \right\} G^{(n)}(\omega_i, l_i, \alpha) = 0,$$

$\gamma \ll 1$: RG equation can be solved exactly

$$S^{-1}(\omega, l) = \omega \left(1 + \gamma \log \left(\frac{\Lambda}{\omega} \right) \right) - v_F l$$

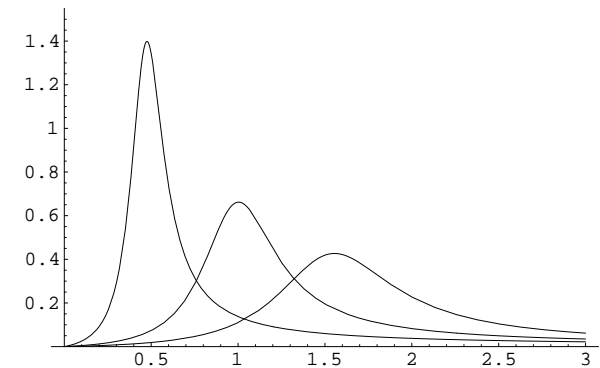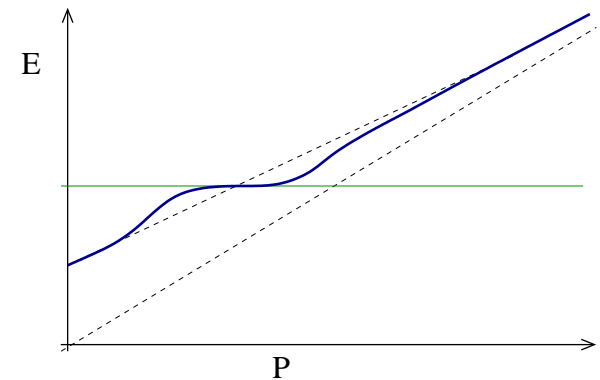no terms $\alpha^2 \log^2(\omega)$, etc.

- quasi-particle velocity vanishes as

$$v \sim \log(\Lambda/\omega)^{-1}$$

- anomalous term in the specific heat

$$c_v \sim \gamma T \log(T)$$

- enhanced corrections to the gap

$$\log(\mu/\Delta) = \log(\mu/\Delta_0)(1 - O(\gamma g))$$

Remnants of Fermi Liquid Behavior

Perturbative corrections to matching QCD→HDET

$$v_F = 1 - \frac{C_F \alpha_s}{2\pi}, \qquad \delta\mu = \frac{C_F \alpha_s}{\pi}\mu.$$

These coefficients satisfy Fermi-liquid relations (Baym&Chin)

$$v_F = \frac{p_F}{\mu} - \frac{N f_1^s}{3} \qquad p_F = \mu\left(1 - \frac{N f_0^s}{2}\right)$$

Fermi-liquid parameters

$$f_0^s = \frac{C_F}{4 N_c N_f}\frac{g^2}{p_F^2} \qquad f_1^s = 0$$

Application: Neutrino Emission

Quark Direct URCA: $\quad d \rightarrow u + e^- + \bar{\nu}, \quad u + e^- \rightarrow d + \bar{\nu}$
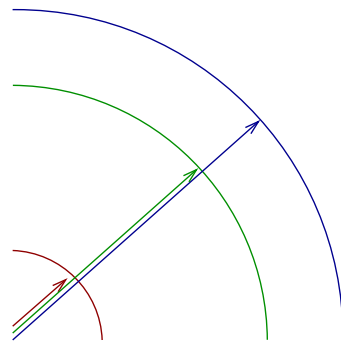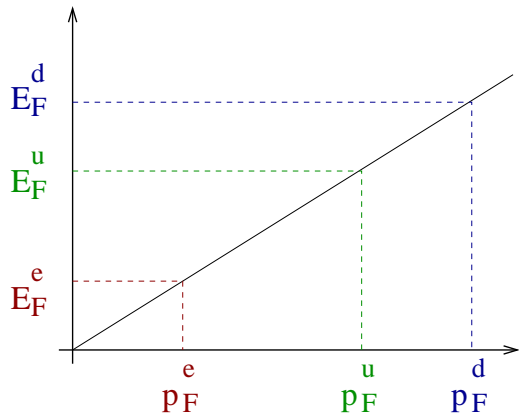
$$\epsilon \sim G_F T^7$$
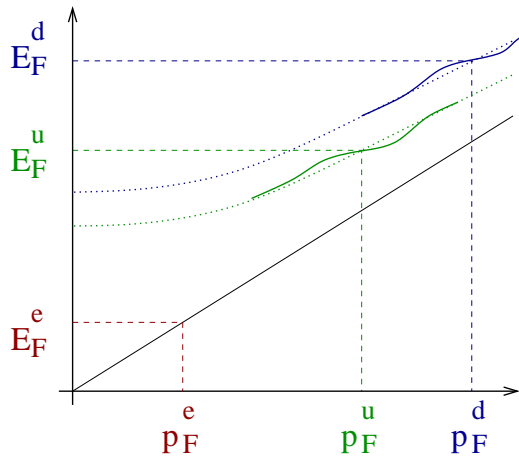
$$\epsilon \sim G_F \alpha_s^3 T^6 \log^2(T)$$

Quasi-Baryons at Large Density:

CFL Phase

Superconductivity

quark-quark scattering

$(\mu \gg \Lambda_{QCD})$
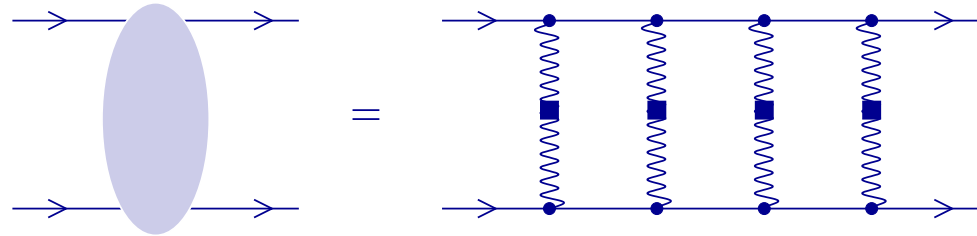
Gap equation: double logarithmic behavior

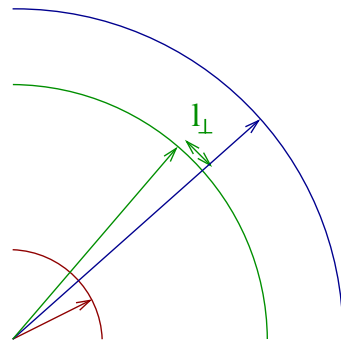$$\Delta(p_0) = \frac{g^2}{18\pi^2} \int dq_0 \left\{ \log\left(\frac{b_M}{|p_0 - q_0|}\right) + \ldots \right\} \frac{\Delta(q_0)}{\sqrt{q_0^2 + \Delta(q_0)^2}}$$

collinear log   BCS log

$$\Rightarrow \qquad \Delta_0 = 512\pi^4 \mu g^{-5} \exp\left(-\frac{\pi^2 + 4}{8}\right) \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

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CFL Phase

Consider $N_f = 3 \ (m_i = 0)$

$$\langle q_i^a q_j^b \rangle = \phi \, \epsilon^{abI} \epsilon_{ijI}$$

$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

$$\langle rb \rangle = \langle rg \rangle = \langle bg \rangle$$

Symmetry breaking pattern:

$$SU(3)_L \times SU(3)_R \times [SU(3)]_C$$

$$\times U(1) \rightarrow SU(3)_{C+F}$$

All quarks and gluons acquire a gap

$F_L \ C_L$ $C_R \ F_R$

$\langle q_L q_L \rangle$ $\langle q_R q_R \rangle$

Rotate left flavor   Compensate by rotating color

... have to rotate right flavor also !

$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$

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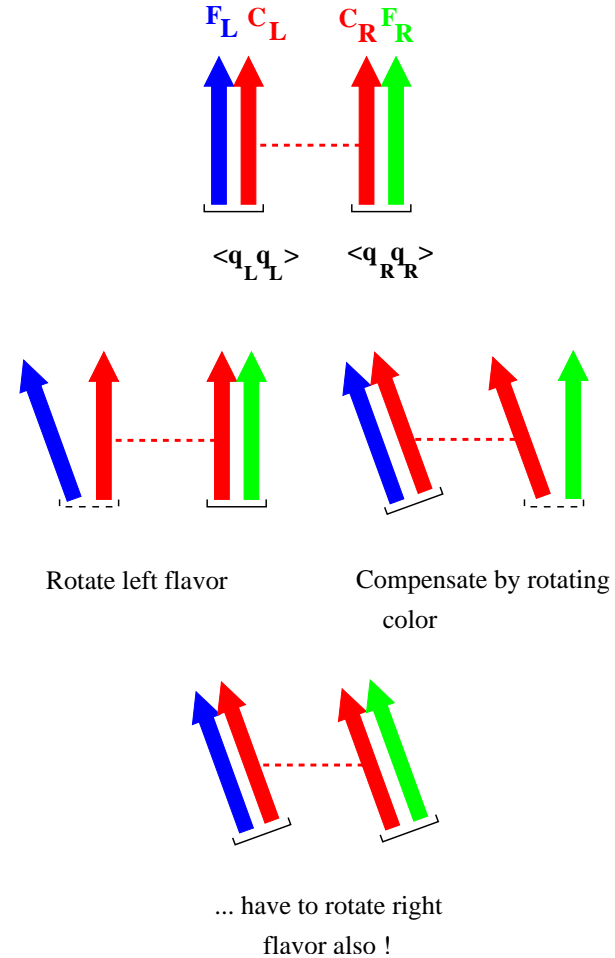
EFT in the CFL Phase

Consider HDET with a CFL gap term

$$\mathcal{L} = \mathrm{Tr}\left(\psi_L^\dagger(iv\cdot D)\psi_L\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(X^\dagger\psi_L X^\dagger\psi_L\right) - \kappa\left[\mathrm{Tr}\left(X^\dagger\psi_L\right)\right]^2\right\}$$

$$+ (L \leftrightarrow R, X \leftrightarrow Y)$$

$$\psi_L \to L\psi_L C^T, \ X \to LXC^T, \qquad \langle X\rangle = \langle Y\rangle = \mathbb{1}$$

Quark loops generate a kinetic term for X, Y

$$\mathcal{L} = -\frac{f_\pi^2}{2}\left\{\mathrm{Tr}\left((X^\dagger D_0 X)^2 + (Y^\dagger D_0 Y)^2\right)\right\} + \ldots$$

Integrate out gluons, identify low energy fields ($\xi = \Sigma^{1/2}$)

$$\Sigma = XY^\dagger \ \ [8]+[1] \ \mathrm{GBs}, \qquad N_L = \xi(\psi_L X^\dagger)\xi^\dagger \ \ [8]+[1] \ \mathrm{Baryons}$$

Effective theory: CFL(B)χPTh

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{f_\pi^2}{4} \left\{ \mathrm{Tr}\left(\nabla_0 \Sigma \nabla_0 \Sigma^\dagger\right) - v_\pi^2 \mathrm{Tr}\left(\nabla_i \Sigma \nabla_i \Sigma^\dagger\right) \right\} \\
& + \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) - D\,\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 \left\{\mathcal{A}_\mu, N\right\}\right) \\
& - F\,\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 \left[\mathcal{A}_\mu, N\right]\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\}
\end{aligned}
$$

with $D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N]$

$$
\begin{aligned}
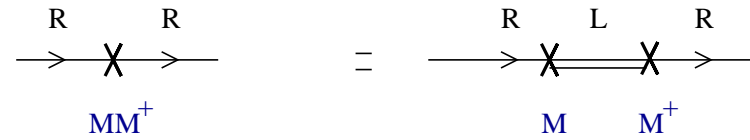\mathcal{V}_\mu &= -\frac{i}{2}\left(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi\right) \\
\mathcal{A}_\mu &= -\frac{i}{2}\xi\left(\partial_\mu \Sigma^\dagger\right)\xi
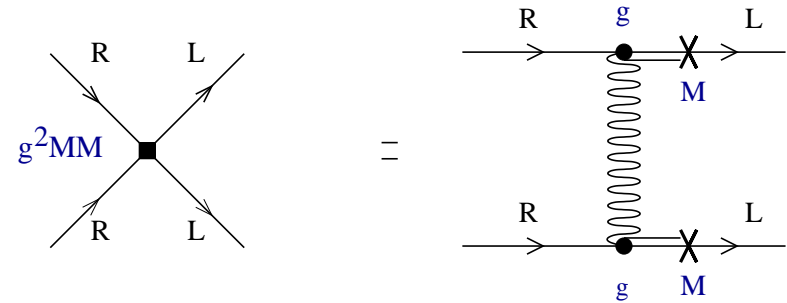\end{aligned}
$$

$$
f_\pi^2 = \frac{21 - 8\log 2}{18}\frac{\mu^2}{2\pi^2} \qquad v_\pi^2 = \frac{1}{3} \qquad D = F = \frac{1}{2}
$$

Mass Terms: Match HDET to QCD

$$\mathcal{L} = \psi_R^\dagger \frac{MM^\dagger}{2\mu}\psi_R + \psi_L^\dagger \frac{M^\dagger M}{2\mu}\psi_L$$

$$+ \frac{C}{\mu^2}(\psi_R^\dagger M\lambda^a \psi_L)(\psi_R^\dagger M\lambda^a \psi_L)$$

mass corrections to FL parameters $\hat{\mu}$ and $F^0(++ \rightarrow --)$

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Mass Terms: Match HDET to CFLχTh

Kinetic term: $\psi_L^\dagger X_L \psi_L + \psi_R^\dagger X_R \psi_R$

$$
\begin{aligned}
D_0 N &= \partial_0 N + i[\Gamma_0, N], \quad \Gamma_0 = \mathcal{V}_0 + \frac{1}{2}\left(\xi X_R \xi^\dagger + \xi^\dagger X_L \xi\right) \\
\nabla_0 \Sigma &= \partial_0 \Sigma + i X_L \Sigma - i \Sigma X_R
\end{aligned}
$$

vector (axial) potentials

Contact term: $(\psi_R^\dagger M \psi_L)(\psi_R^\dagger M \psi_L)$

$$
\mathcal{L} = \frac{3\Delta^2}{4\pi^2}\left\{[\mathrm{Tr}(M\Sigma)]^2 - \mathrm{Tr}(M\Sigma M\Sigma)\right\}
$$

meson mass terms

Phase Structure and Spectrum

Phase structure determined by effective potential

$$V(\Sigma) = \frac{f_\pi^2}{2}\mathrm{Tr}\left(X_L\Sigma X_R\Sigma^\dagger\right) - A\mathrm{Tr}(M\Sigma^\dagger) - B_1\left[\mathrm{Tr}(M\Sigma^\dagger)\right]^2 + \ldots$$

$$V(\Sigma_0) \equiv min$$

Fermion spectrum determined by

$$\mathcal{L} = \mathrm{Tr}\left(N^\dagger iv^\mu D_\mu N\right) + \mathrm{Tr}\left(N^\dagger \gamma_5 \rho_A N\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\},$$
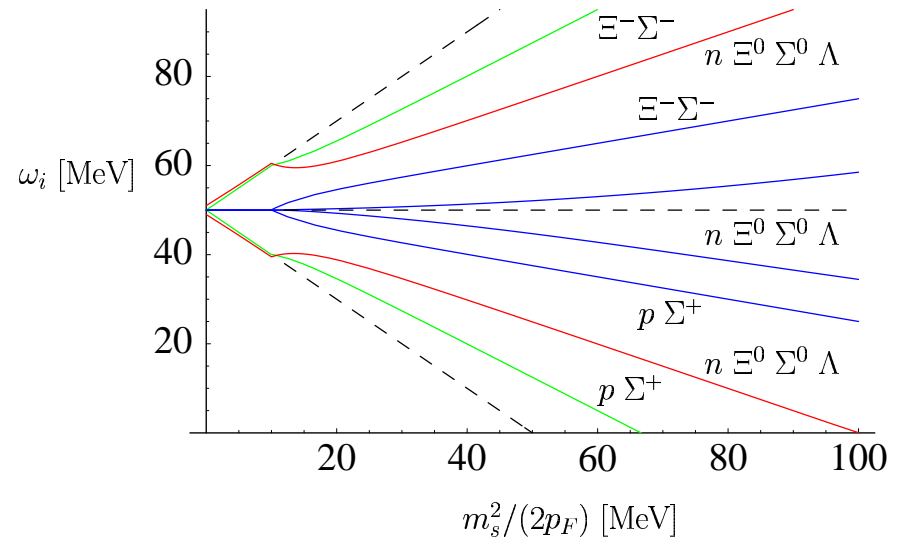
$$\rho_{V,A} = \frac{1}{2}\left\{\xi\frac{M^\dagger M}{2p_F}\xi^\dagger \pm \xi^\dagger\frac{MM^\dagger}{2p_F}\xi\right\} \qquad \xi = \sqrt{\Sigma_0}$$

Phase Structure and Spectrum
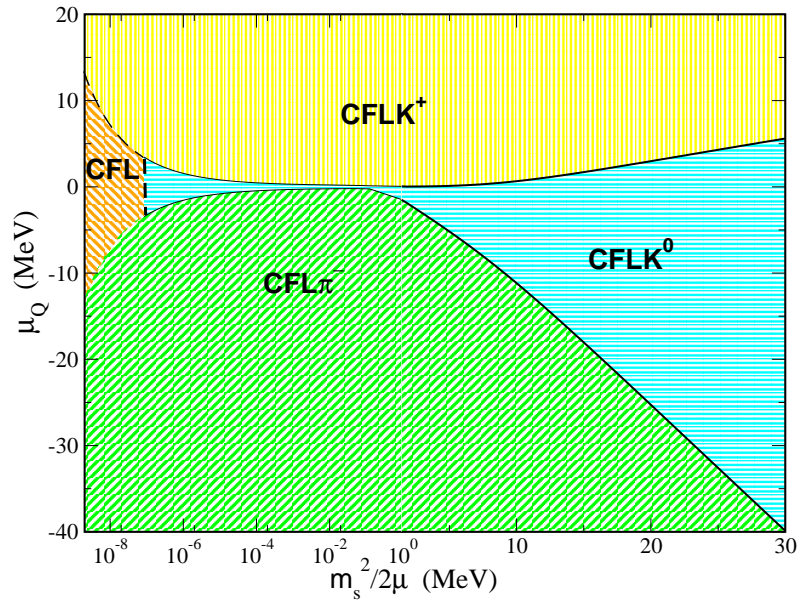
meson condensation: CFLK

reliable: yes!

gapless modes? (gCFLK)

reliable: not clear yet

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Summary

EFT/RG methods provide powerful tools

phase structure and spectrum at large density

Normal phase: Non-Fermi liquid behavior due to unscreened transverse gauge bosons

modified quasi-particles

but: no break-down of perturbation theory

Superfluid phase: effective chiral theory with calculable coefficients

kaon condensation, possibility of gapless modes