Formulas and Numerical Constants

Lorentz transformation: The system S' is moving with velocity $(v_x, v_y, v_z) = (v, 0, 0)$ relative to the S system. The Lorentz transformations are

$$x' = \gamma(x - vt), \qquad y' = y, \qquad z' = z, \tag{1}$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right), \tag{2}$$

where $\gamma = 1/(1-\beta^2)^{1/2}$ and $\beta = v/c$. The inverse Lorentz transformation corresponds to $v \to -v$.

Velocity addition: An object moves with velocity (u_x, u_y, u_z) in the S-system. The components of the velocity in the S'-system are

$$u'_x = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}, \tag{3}$$

$$u'_y = \frac{u_y}{\gamma\left(1 - \frac{vu_x}{c^2}\right)}, \qquad u'_z = \frac{u_z}{\gamma\left(1 - \frac{vu_x}{c^2}\right)}. \tag{4}$$

The inverse transformation corresponds to $v \to -v$.

Relativistic kinematics: In the following m always refers to the rest mass of a particle

$$E^2 = p^2c^2 + m^2c^4, \tag{5}$$

$$E = \gamma mc^2 \qquad p = \gamma mv, \tag{6}$$

and $\gamma = (1 - v^2/c^2)^{-1/2}$. The four vector $(E, \vec{p}c)$ transforms under Lorentz transformations like the four vector $(ct, \vec{x})$:

$$p'_x = \gamma\left(p_x - vE/c^2\right), \qquad p'_y = p_y, \qquad p'_z = p_z, \tag{7}$$

$$E' = \gamma(E - vp_x), \tag{8}$$

De Broglie relations: De Broglie postulated the following relations between (E, p) and (λ, f)

$$E = hf, \qquad (E = \hbar\omega) \tag{9}$$

$$p = h/\lambda, \qquad (p = \hbar k) \tag{10}$$

where $\hbar = h/(2\pi)$. The most important dispersion relations are

$$E = pc \qquad \text{(light)}, \tag{11}$$

$$E = \frac{p^2}{2m} \qquad \text{(nonrelativistic matter)}. \tag{12}$$

Bohr's model: Bohr's model of hydrogen like atoms is based on the quantization condition $L = mvr = n\hbar$. The allowed energies and radii are

$$r_n = \frac{n^2 a_0}{Z}, \qquad a_0 = \frac{\hbar^2}{m_e k e^2}, \tag{13}$$

$$E_n = -\frac{Z^2 E_0}{n^2}, \qquad E_0 = \frac{m_e k^2 e^4}{2\hbar^2}, \tag{14}$$

where $k = 1/(4\pi\epsilon_0)$ is the Coulomb constant, e is the charge, and m_e is the mass of the electron. Z is the charge of the nucleus (in units of e). The constant a_0 is called the Bohr radius. The quantity

$$\alpha = \frac{ke^2}{\hbar c} \simeq \frac{1}{137} \tag{15}$$

is called the fine structure constant.

Schrödinger equation: The time-dependent and time-independent Schrödinger equations are

$$i\hbar \frac{\partial}{\partial t}\psi(x,t) = \left\{ -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) \right\} \psi(x,t), \tag{16}$$

$$E\psi(x) = \left\{ -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) \right\} \psi(x). \tag{17}$$

The wave function is related to the probability

$$P(x,t)\,dx = \psi^*(x,t)\psi(x,t)\,dx. \tag{18}$$

More generally, expectation values are given by

$$\langle f \rangle = \int dx\, f(x)\psi^*(x,t)\psi(x,t). \tag{19}$$

3d Schrödinger equation: Solutions of the Schrödinger equation for a potential with rotational symmetry have the form

$$\psi(r,\theta,\phi) = R_{nl}(r)Y_{lm}(\theta,\phi), \tag{20}$$

where Y_{lm} are the spherical harmonics, (l,m) label $L^2 = \hbar^2 l(l+1)$ and $L_z = \hbar m$ ($m \leq l$), and $R_{nl}(r)$ is the radial wave function (labeled by the quantum number n). The ground state wave function of the hydrogen atom is

$$R_{10}(r) = \frac{2}{\sqrt{a_0^3}}\, e^{-r/a_0}, \qquad Y_{00}(\theta,\phi) = \frac{1}{\sqrt{4\pi}}, \tag{21}$$

where a_0 is the Bohr radius defined above.

Selection rules: Dipole transitions involving the emission or absorption of a photon are allowed if

$$\Delta m_l = \pm 1, 0 \quad \text{and} \quad \Delta l = \pm 1. \tag{22}$$

Vibrational and rotational energies: The energy levels of a one-dimensional harmonic oscillator are

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right), \tag{23}$$

where $\omega = \sqrt{k/m}$ and k is the spring constant. The energy levels of a rigid rotor are

$$E_l = \frac{\hbar^2}{2I}\, l(l+1), \tag{24}$$

where I is the moment of inertia.

Fermi gas: The Fermi energy is

$$E_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}, \tag{25}$$

The average energy is $E_{av} = (3/5)E_F$, the Fermi temperature is $T_F = E_F/k_B$, and the Fermi velocity is $v_F = \sqrt{2E_F/m}$. The Fermi-Dirac distribution is

$$f(E) = \frac{1}{\exp((E-E_F)/(k_B T)) + 1}. \tag{26}$$

Standard model particles:

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} \qquad \begin{matrix} Q = 2/3 \\ Q = -1/3 \end{matrix} \tag{27}$$

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \quad \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \quad \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix} \qquad \begin{matrix} Q = -1 \\ Q = 0 \end{matrix} \tag{28}$$

Numerical Constants:

$$\begin{aligned}
k &= 1/(4\pi\epsilon_0) \;=\; 8.987 \cdot 10^9\,\mathrm{N \cdot m^2 \cdot C^{-2}} \\
k_B &= 1.381 \times 10^{-23}\,\mathrm{J/K} = 8.617 \times 10^{-5}\,\mathrm{eV/K} \\
N_A &= 6.022 \times 10^{23} \\
h &= 6.626 \times 10^{-34}\,\mathrm{J \cdot s} \\
c &= 2.998 \times 10^8\,\mathrm{m/sec} \\
hc &= 1240\,\mathrm{eV \cdot nm} \\
\hbar c &= 197.33\,\mathrm{MeV \cdot fm} \\
E_0 &= 0.5\, m_e c^2 \alpha^2 \;=\; 13.6\,\mathrm{eV} \\
\mu_B &= e\hbar/(2m_e) \;=\; 9.274 \times 10^{-24}\mathrm{J/T} \;=\; 5.788 \times 10^{-5}\mathrm{eV/T} \\
e &= 1.602 \times 10^{-19}\,\mathrm{C} \\
1\,\mathrm{cal} &= 4.186\,\mathrm{J} \\
1\,\mathrm{eV} &= 1.602 \times 10^{-19}\,\mathrm{J} \\
1\,u &= 1.661 \times 10^{-27}\,\mathrm{kg} = 931.49\,\mathrm{MeV}/c^2 \\
m_e c^2 &= 512\,\mathrm{keV} \\
m_e &= 9.109 \times 10^{-31}\,\mathrm{kg} \\
m_p c^2 &= 938.3\,\mathrm{MeV} \\
m_n c^2 &= 939.6\,\mathrm{MeV}
\end{aligned}$$