

Extreme QCD 2008

Raleigh, North Carolina State University, July 21–23 2008

The Phases of QCD in the T, N_f Plane

Maria Paola Lombardo

Istituto Nazionale di Fisica Nucleare
Laboratori Nazionali di Frascati
Roma, Italia

In collaboration with *Albert Deuzmann* and *Elisabetta Pallante*

0804.2905[hep-lat]

Talks by E. Pallante and A. Deuzemann at Lattice08 - Beyond QCD Session.

PROPERTIES:

Chiral Symmetry

Screening, confinement

Asymptotic freedom

Scale Invariance

OUTLINE:

- The Banks-Zacs scenario
- The conformal window
- Early Lattice Studies
- Introducing temperature
- Results
- Discussions

HISTORY : THE BANKS ZACS scenario

The Lagrangian of an $SU(N)$ gauge theory is:

$$\mathcal{L} = \bar{\psi}(i \not{\partial} + g(\mu) \not{A}^a T^a)\psi + \frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu}$$

The renormalization group (RG) equation for the running coupling is:

$$\mu \frac{\partial}{\partial \mu} \alpha(\mu) = \beta(\alpha) \equiv -b \alpha^2(\mu) - c \alpha^3(\mu) - d \alpha^4(\mu) - \dots ,$$

where $\alpha(\mu) = g^2(\mu)/4\pi$.

With the N_f fermions in the fundamental representation

$$b = \frac{1}{6\pi} (11N - 2N_f)$$

$$c = \frac{1}{24\pi^2} \left(34N^2 - 10NN_f - 3\frac{N^2 - 1}{N}N_f \right) .$$

- The theory is asymptotically free if $b > 0$:

$$N_f < \frac{11}{2}N$$

- At two loops, the theory has an infrared stable, non-trivial fixed point if $b > 0$ and $c < 0$.

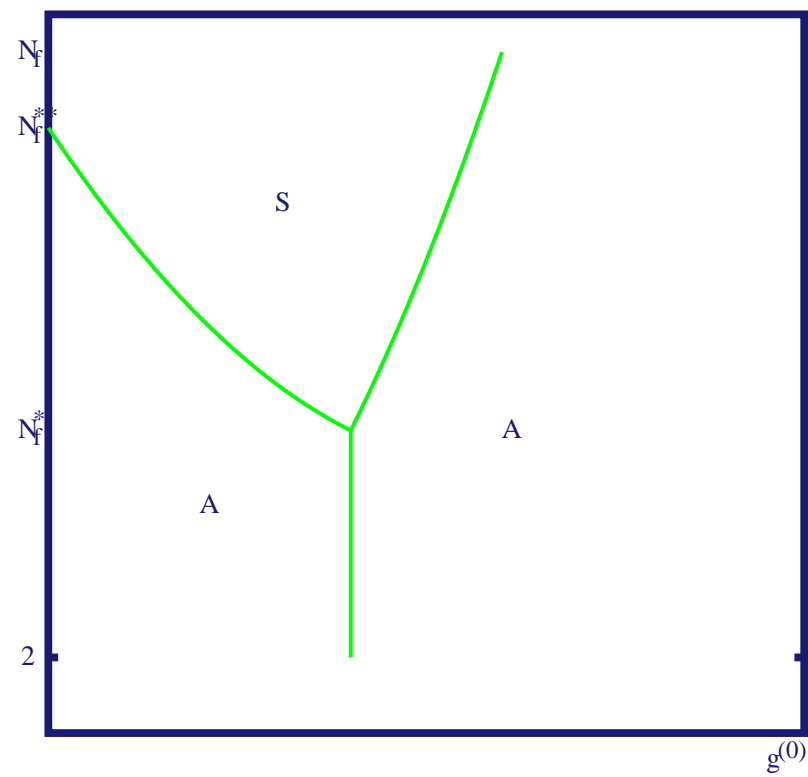
$$\frac{34N^3}{13N^2 + 3} < N_f < \frac{11}{2}N$$

$$N_f^* < N_f < N_f^{**}$$

- In this case the fixed point is at

$$\alpha_* = -\frac{b}{c}.$$

- The coefficients b and c are scheme-independent : if a zero, α_* , of the β function exists at two loops, it exists to any order in perturbation theory.



PHASES OF QCD -- BANKS ZACS

THE DISCOVERY OF THE CONFORMAL WINDOW OF QCD

Appelquist et al. 1997

CHIRAL SYMMETRY BREAKING NEEDS LARGE GAUGE COUPLING:

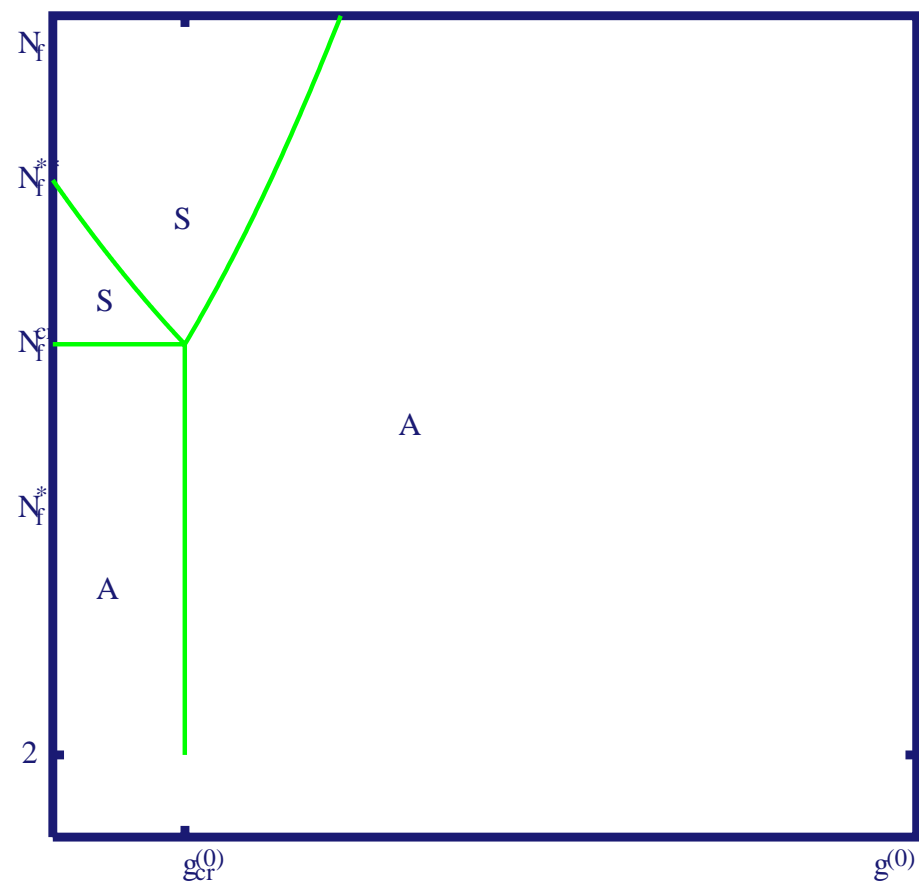
- Chiral symmetry breaking is possible when the gauge coupling exceeds a critical value

$$\alpha_c \equiv \frac{\pi}{3 C_2(R)} = \frac{2\pi N}{3(N^2 - 1)},$$

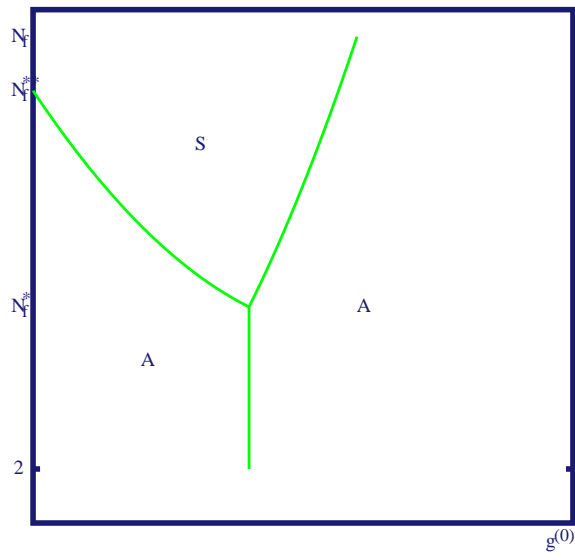
where $C_2(R)$ is the quadratic Casimir of the representation R .

- Thus we would expect that when N_f is decreased below the value N_f^c at which $\alpha_* = \alpha_c$, the theory undergoes a transition to a phase where chiral symmetry is spontaneously broken.

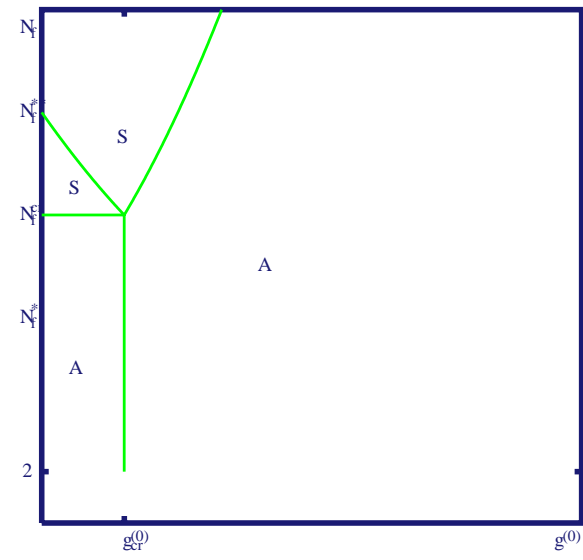
$$N_f^c = N \left(\frac{100N^2 - 66}{25N^2 - 15} \right).$$



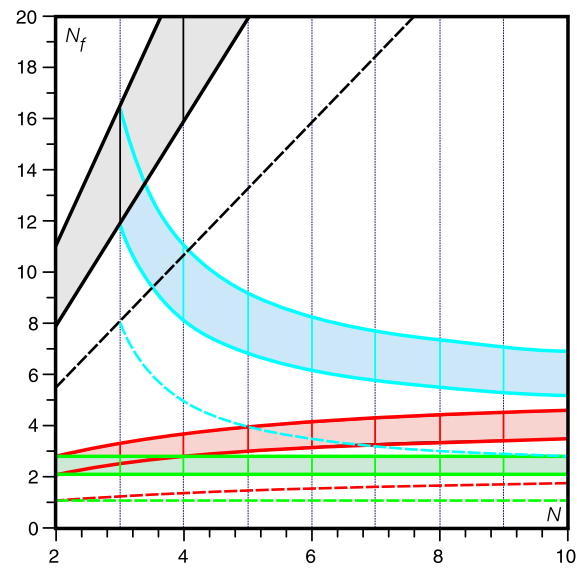
PHASES OF QCD - THE CONFORMAL WINDOW



PHASES OF QCD -- BANKS ZACS



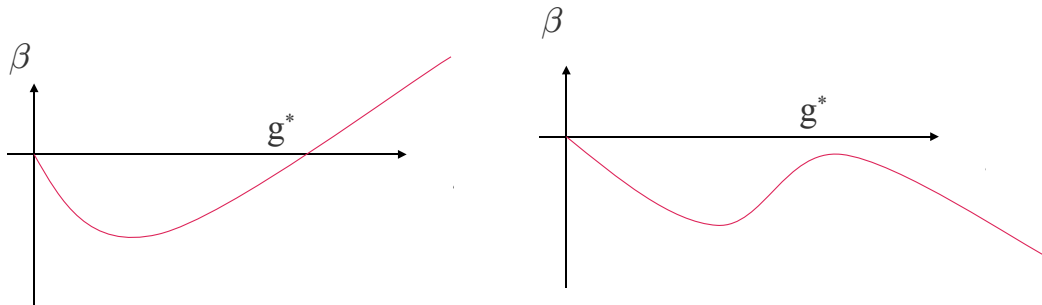
PHASES OF QCD - THE CONFORMAL WINDOW



Phases of QCD in the N_f, N_c plane
D. Dietrich, F. Sannino 2006; D. Dietrich xQCD07

Adding flavors to QCD: Conformal Window

- An IR fixed point can emerge already in the two-loop β function as you increase the number N_f of fermions. (Gross and Wilczek, Banks and Zaks, ...)



○ **Conformal:** $N_f > N_f^*$

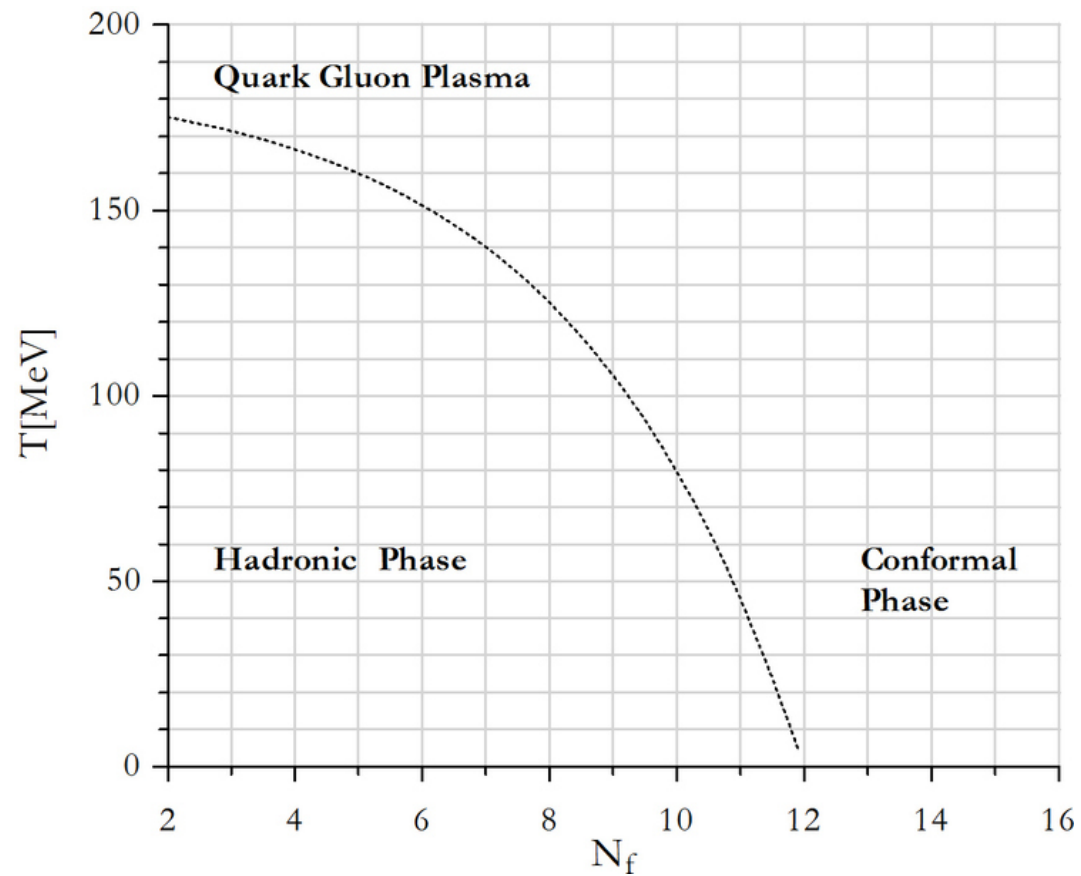
- Long distance (IR) Conformal theory.
- Chiral symmetry $SU(N_f) \times SU(N_f)$
- No Confinement
- But asymptotically free in the UV.

○ **Walking:** $N_f < N_f^*$, but close to N_f^*

- Spontaneous breaking of chiral symmetry $SU(N_f) \times SU(N_f)$
- Confinement
- Spontaneous breaking of an approximate (IR) conformal symmetry

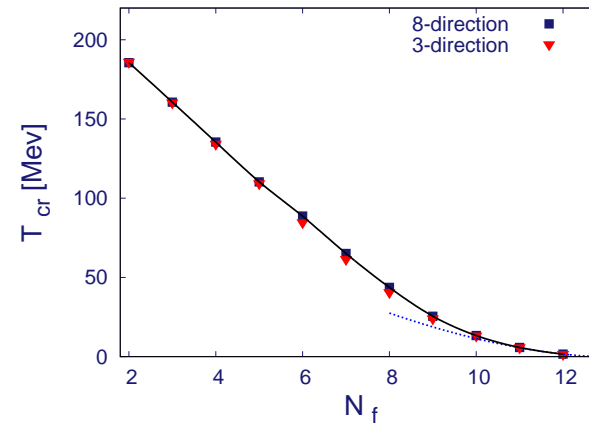
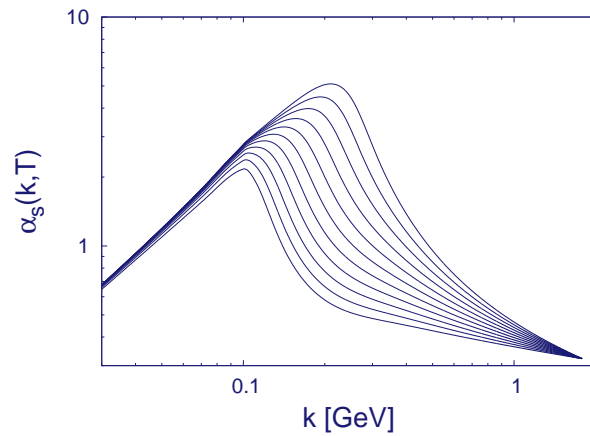
From R. Brower

ONE MORE DIMENSION : TEMPERATURE



Qualitative graph of the phase diagram in the T, N_f plane.

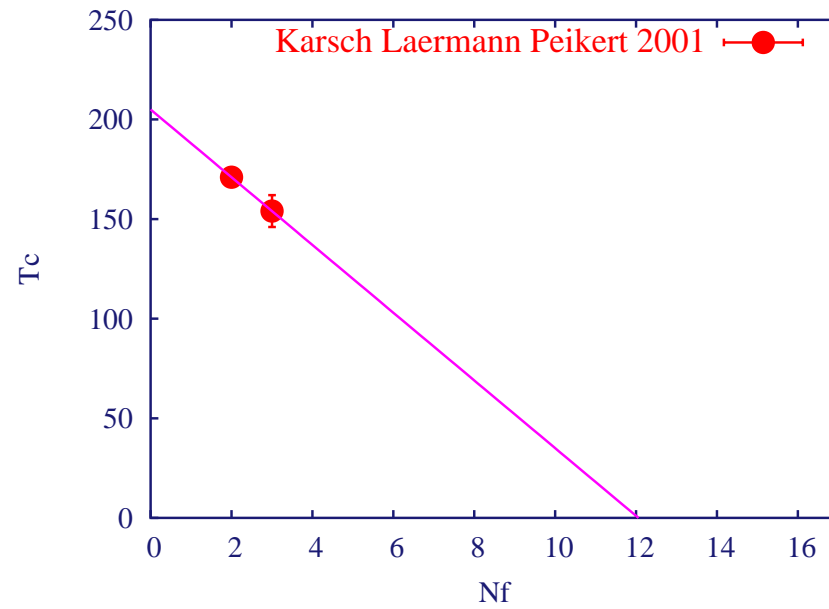
QUANTITATIVE ESTIMATES OF THE CHIRAL BOUNDARY FROM RG



Braun, Gies 2007; Gies at xQCD07

Scale fixed by $\alpha_s(\tau)$

Just for fun....



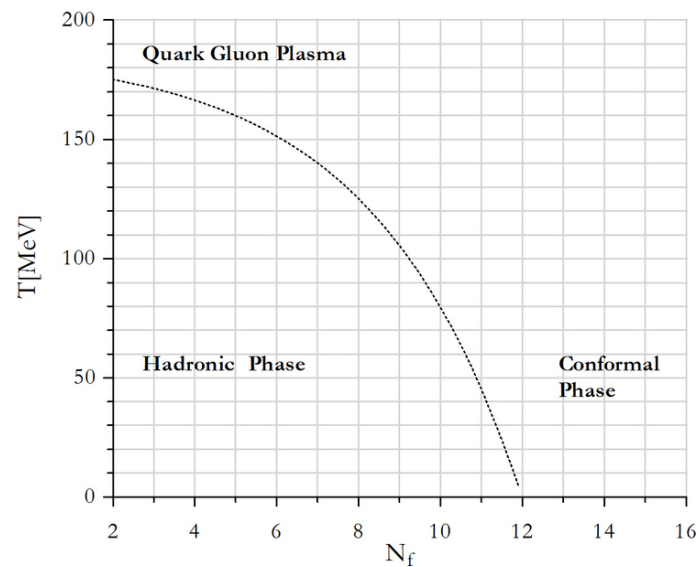
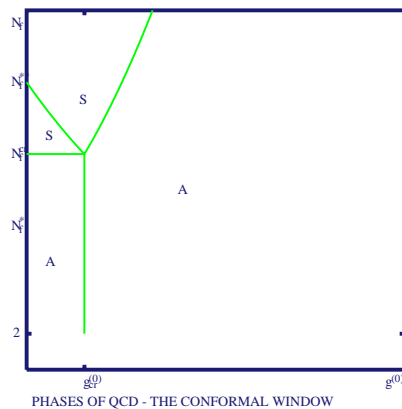
LATTICE STUDIES

- Fukugita et al. :1988 Pioneering study at $N_f = 10$ and $N_f = 12$ at small volumes (mainly $8^3 \times 4$) and relatively heavy masses, reporting interesting evidence for a persistence of a first order thermal phase transition at $N_f = 10$ and discussing its sensitivity to fermion masses.
- Columbia: 1992
Subsequent work by the Columbia collaboration suggested a rich structure of the phase diagram, dominated by lattice artifacts and was not completely conclusive.
- Damgaard et al.: 1996
Consider $N_f = 16$, the largest value of N_f still compatible with asymptotic freedom, and confirm that chiral symmetry is not broken at weak coupling by observing the expected bulk transition at intermediate coupling. First attempt at a direct estimate of the beta function
- Iwasaki et al: 2003
A value as low as $N_f = 6$ should be set as the lower limit of the conformal window
Commonly held opinion that the strong coupling limit of QCD breaks chiral symmetry, irrespectively of N_f is challenged
- Appelquist et al. 2007
Calculation of the β function. $N_f = 8$ is still confining in the continuum limit. N_f^C , the threshold of the conformal window, should be close to 12.
- Nice review by J. Kogut at the Lattice Strong Dynamics meeting, May2008

THE PHASE DIAGRAM in the N_f , T plane

Deuzemann, Pallante, MpL 2008

NB: adjusting temperature = tuning the coupling!



In which direction are we going?

Needed control over lattice artifacts

THE ACTION

- Improved Kogut-Susskind fermion action, the “Asqtad” action which removes lattice artifacts up to $O(a^2 g^2)$
- one-loop Symanzik improved and tadpole improved gauge action.

$$S = -\frac{N_f}{4} \text{Tr} \ln M(am, U, u_0) + S_{gauge} ,$$

where

$$S_{gauge} = \sum_{i=p,r,pg} \beta_i(g^2) \sum_{\mathcal{C} \in \mathcal{S}_i} \text{Re}(1 - U(\mathcal{C})) ,$$

with couplings defined as

$$\beta_p \quad \equiv \quad \beta = 10/g_0^2$$

$$\beta_r \quad = \quad -\frac{\beta}{20u_0^2}(1 + 0.4805\alpha_s)$$

$$\beta_{pg} \quad = \quad -\frac{\beta}{u_0^2}0.03325\alpha_s ,$$

$$\alpha_s \quad = \quad -4 \log u_0/3.0684$$

THE SIMULATIONS

- Slightly modified version of the publicly available MILC code.
- RHMC simulations (arbitrary number of fermions)
- $N_t = 6, 12$ Check of asymptotic scaling
- $N_s = 12, 20, 24$ Control of Finite size effects
- Monte Carlo trajectories of total length $\tau = 0.3$ to 0.4
- time step $\delta\tau = 0.003$ to 0.007
- Use Blue Gene in Amsterdam

OBSERVABLES

- Real part of the Polyakov loop

$$L \equiv \frac{1}{3N_s^3} \sum_{\vec{x}} \text{Re Tr} \prod_{x_4=1}^{N_t} U_4(\vec{x}, x_4)$$

- Chiral Condensate

$$a^3 \langle \bar{\psi} \psi \rangle = \frac{N_f}{4N_s^3 N_t} \langle \text{Tr} [M^{-1}] \rangle ,$$

- The chiral susceptibility

$$\chi = \partial \langle \bar{\psi} \psi \rangle / \partial m = \chi_{\text{conn}} + \chi_{\text{disc}}$$

,

$$\begin{aligned} a^2 \chi_{\text{conn}} &= -\frac{N_f}{4N_s^3 N_t} \langle \text{Tr} [(MM)^{-1}] \rangle \\ a^2 \chi_{\text{disc}} &= \frac{N_f^2}{16N_s^3 N_t} \left[\langle \text{Tr} [M^{-1}]^2 \rangle - \langle \text{Tr} [M^{-1}] \rangle^2 \right] , \end{aligned} \tag{1}$$

WARD IDENTITIES

The susceptibilities;

$$\chi_\sigma \equiv \chi = \frac{\partial \langle \bar{\psi} \psi \rangle}{\partial m} = \chi_{\text{conn}} + \chi_{\text{disc}}$$

and

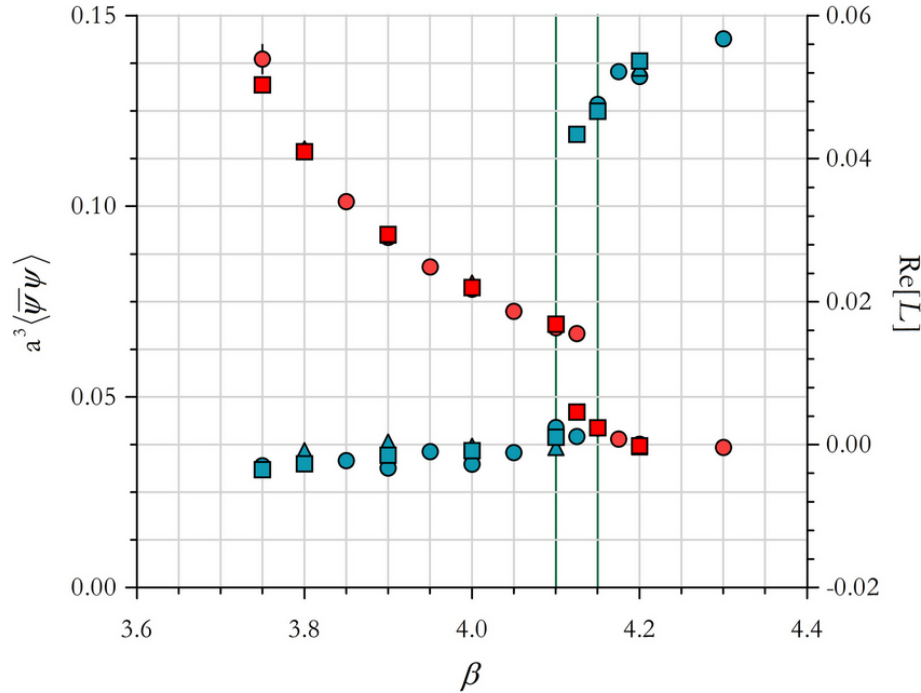
$$\chi_\pi = \frac{\langle \bar{\psi} \psi \rangle}{m} .$$

are related through Ward identities (Kocic, Kogut MpL 1992) to the spacetime volume integral of the scalar (σ) and pseudoscalar (π) propagators

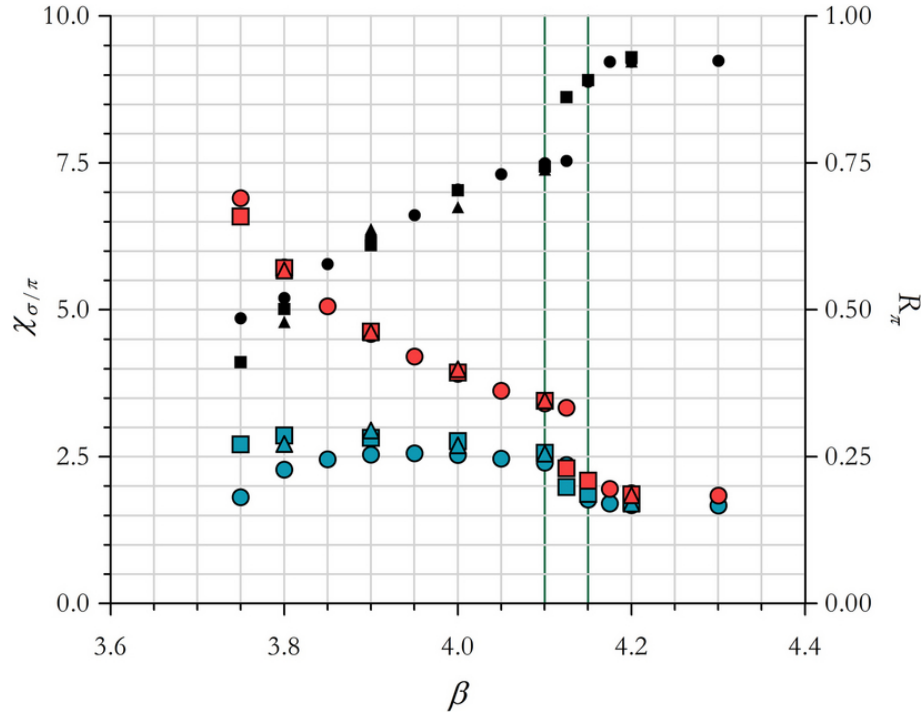
$$\chi_{\sigma,\pi} = \int d^4x G_{\sigma,\pi}(x) ,$$

thus implying that they should become degenerate when chiral symmetry is restored, following the degeneracy of the chiral partners.

RESULTS at $N_t = 6$

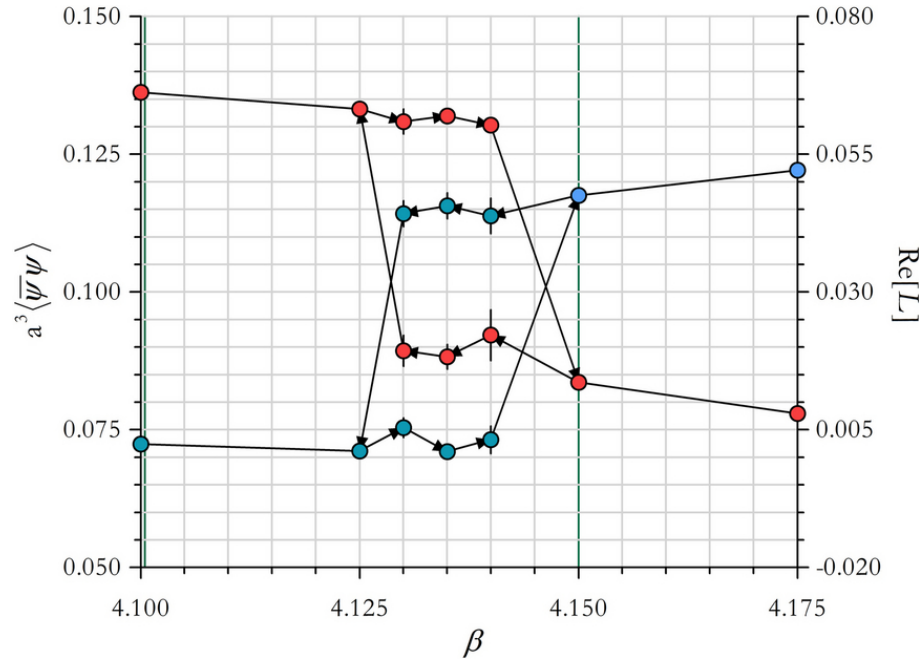


The chiral condensate (red) and Polyakov loop (blue) as a function of the lattice coupling β . Measurements were done for three different spatial extents of the lattice N_s : 12 (\circ), 20 (\triangle) and 24 (\square). The critical region has been indicated by vertical lines.

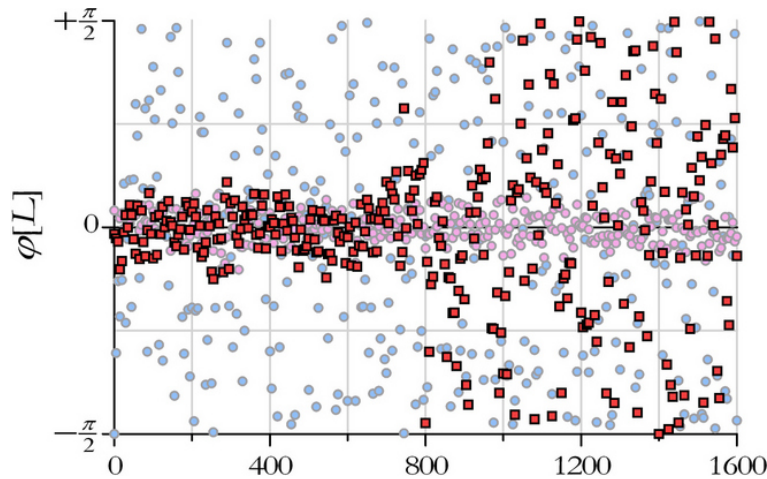


The scalar χ_σ (blue) and pseudoscalar χ_π (red) chiral susceptibilities as a function of the lattice coupling β , confirming the degeneracy of the chiral partners in the symmetric phase. Measurements are shown for three different spatial extents of the lattice N_s : 12 (\bigcirc), 20 (\triangle) and 24(\square). The cumulant R_π is also shown (black, top of figure). The critical region has been indicated by vertical lines.

THE CRITICAL REGION



Hysteresis effect for the chiral condensate (red) and Polyakov loop (blue) within the critical region, data are for the spatial extent $N_s = 12$. Arrows indicate the β of the configurations involved, the vertical lines indicate the estimate of the broadest critical region which covers the jump of the chiral condensate and Polyakov loop at all spatial volumes and the hysteresis cycle.



Monte Carlo history for $\beta = 4.1$ and $N_s = 24$, at the threshold. The distribution flattens around zero in the deconfined phase, while it is randomly distributed between $\pm \frac{\pi}{2}$ in the confined phase. The same histories are shown for runs below and above the critical region (at $\beta = 4.0$ and 4.2) in lighter colours. The points are enumerated according to their trajectory number, disregarding the first 100 trajectories after the cold start for each.

SUMMARY $N_t=6$

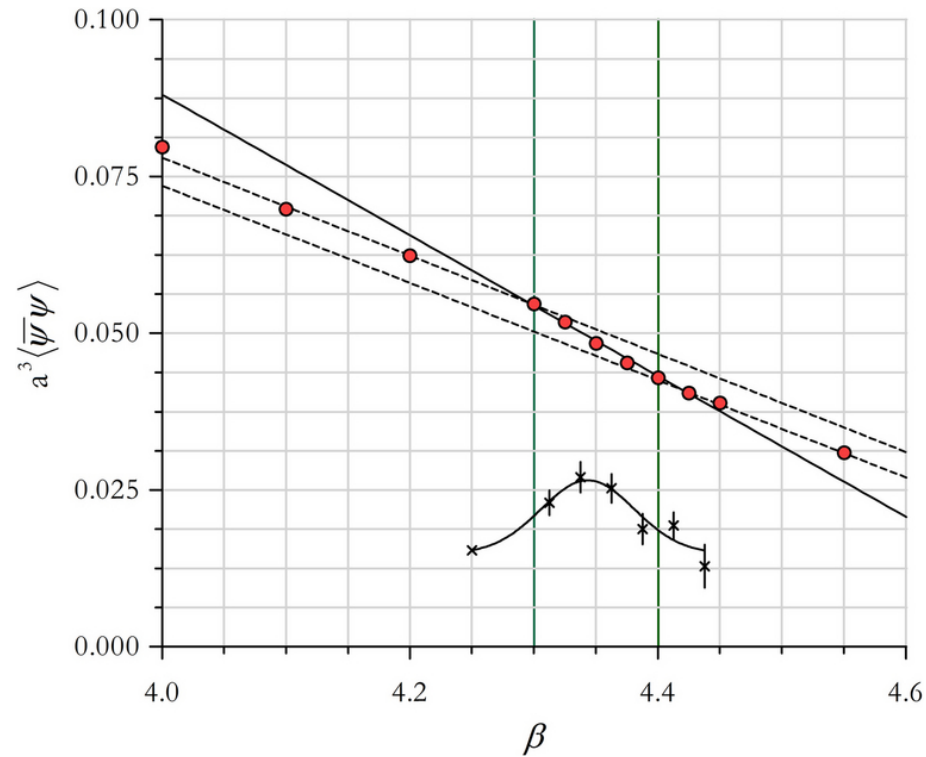
- All data at $N_t = 6$ point towards a first order phase transition.
- The location of the jump of the chiral condensate and the Polyakov loop at all three spatial volumes considered, and the location of the hysteresis cycle at the smallest spatial extent, allows us to give the estimate of the broadest critical region, to be $4.10 < \beta_c < 4.15$.
- We take the occurrence of the jump of the chiral condensate and the Polyakov loop at our largest spatial volume $N_s = 24$, as the best indicator of the critical point, giving $\beta_c(N_t = 6, N_s = \infty) = 4.1125 \pm 0.0125$ corresponding to the upper and lower bounds

$$4.10 < \beta_c(N_t = 6, N_s = \infty) < 4.125$$

.

RESULTS at $N_t = 12$

$$T_c = \frac{1}{a(\beta_c)N_t}$$



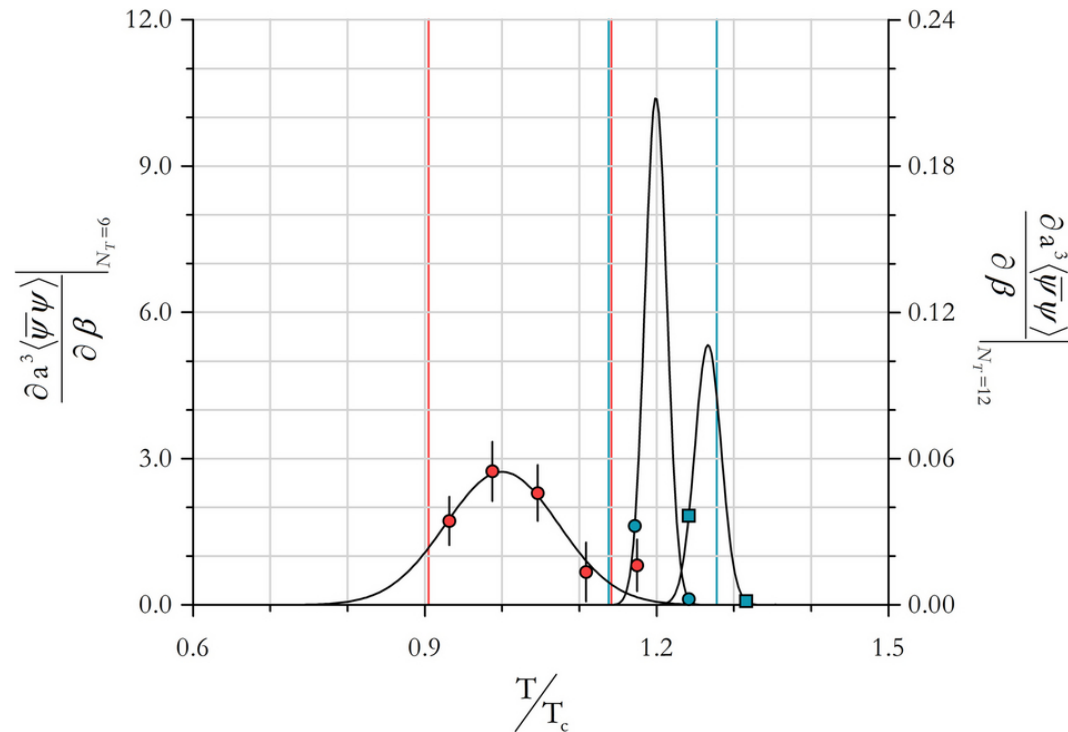
The chiral condensate at $N_t = 12$ and $N_s = 24$ in lattice units as a function of the lattice coupling β . Best fit curves are superimposed and the vertical lines indicate the critical region

Asymptotic Scaling of the Critical Temperature

- Results at $N_t = 12$ and $N_t = 6$ strongly suggest the occurrence of a first order thermal transition.
- To confirm this an asymptotic scaling analysis is needed in order to verify that we are actually measuring a critical temperature in the continuum real world.
- Given a physical temperature T_c , we expect the scaling relation

$$N_t R(g_c(N_t)) = \text{const}$$

- Solving for $N_t = 6$ and $N_t = 12$, we can predict $\beta_c(g_c(N_t = 6))$ by knowing $\beta_c(g_c(N_t = 12))$.
- By using $\beta_c(N_t = 12) = 4.34 \pm 0.04$ we obtain the prediction $\beta_c(N_t = 6) = 4.04 \pm 0.04$, which deviates by less than 2% from the lattice determined β_c .
- We also verified that a rescaling of the effective coupling $\beta \rightarrow \beta u_0^{-4}$ improves the prediction by only 0.5% indicating that our effective coupling prescription is in fact already accounting for all the improvement



‘Scaling plot’ for the finite difference approximation to the absolute value of the first derivative of the chiral condensate as a function of T/T_c , determined using the effective coupling $\beta = 6/g^2$ in the RG formula. Data at $N_t = 6$, $N_s = 24$, 12 (blue) are compared with data at $N_t = 12$, $N_s = 24$ (red). The point $T/T_c = 1$ corresponds to $\beta_c(N_t = 12) = 4.34$.

IN SUMMARY...

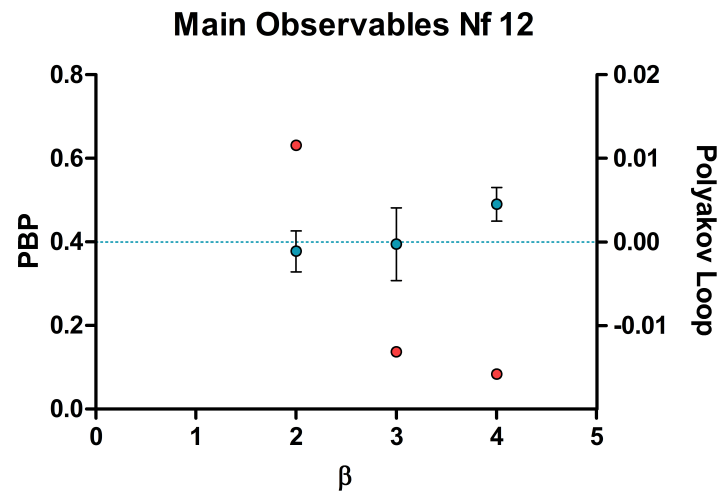
$$\beta_c(N_f = 8, N_t = 6, N_s = \infty) = 4.1125 \pm 0.0125$$

$$4.1 \geq \beta_c(N_f = 8, N_t = 6, N_s = \infty) \leq 4.125$$

$$\beta_c(N_f = 8, N_t = 12, N_s = 24) = 4.34 \pm 0.04$$

$$4.30 \geq \beta_c(N_f = 8, N_t = 12, N_s = 24) \leq 4.38$$

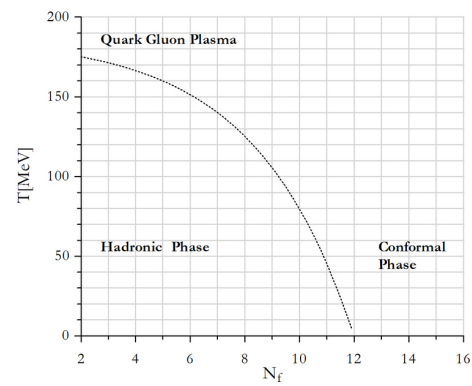
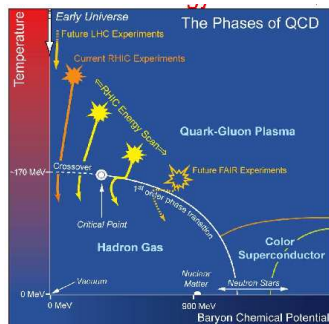
Asymptotic scaling within 20% of the central values of the critical temperature, or, satisfied within error, consistent with the occurrence of a true thermal transition. at $N_f = 8$, placing $N_f = 8$ below the threshold for a conformal window.



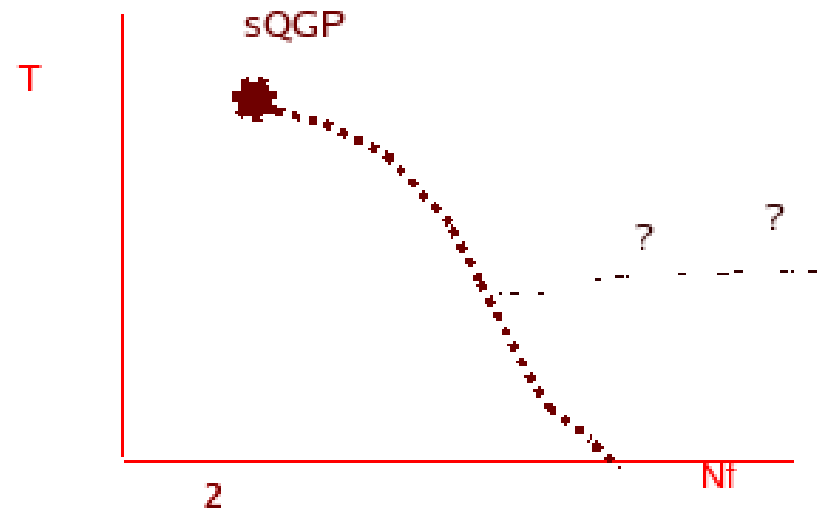
NEXT STEPS

- Precision measurements at $N_f=12$, $N_f=14$, $N_f = 4$
- N_f^c by linear extrapolation (lower bound)
- N_f^c by direct study chiral symmetry realizations
- Full reconstruction of the phase diagram
- Critical behaviour at the chiral/conformal transition
- Anomalous dimensions.
- Large N_f vs Quenched QED4
- Physics of the conformal window: spectrum of technifermions; composite Higgs.
- A large parameter space... challenges for thermodynamics →

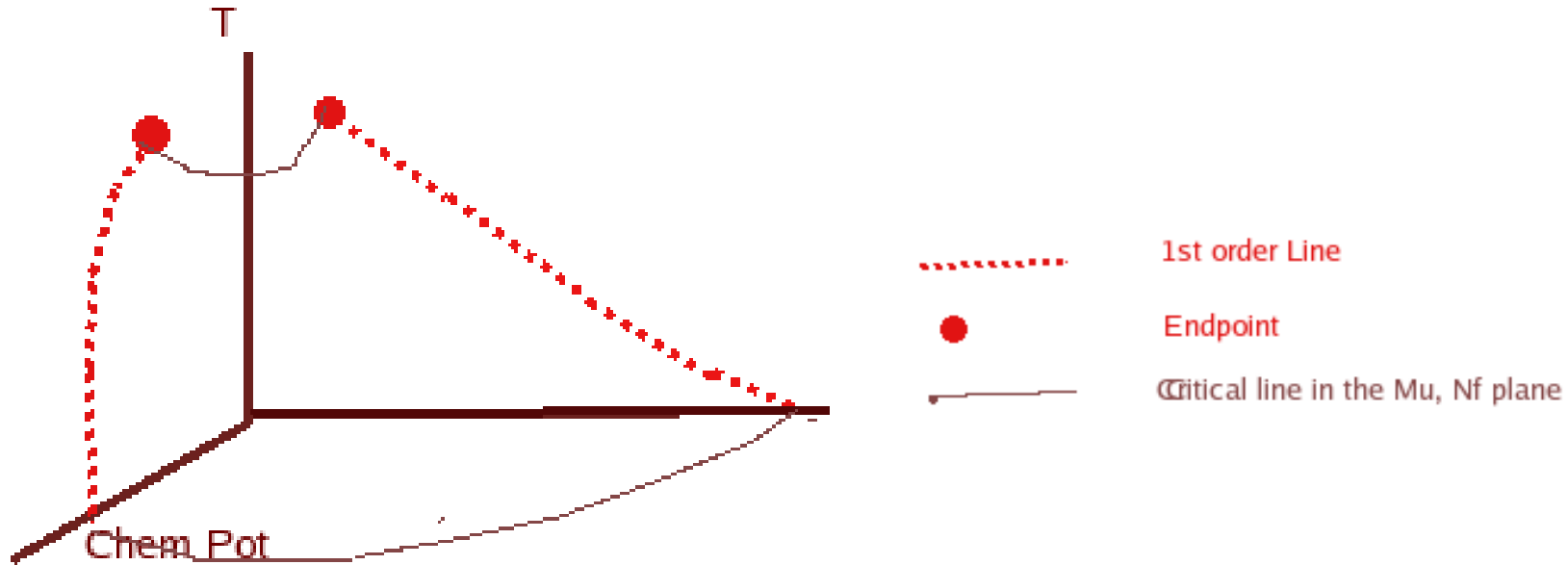
N_f vs μ_B ?



- Endpoint of the chiral boundary in the T, f plane?
- Nature of the crossover from conformal phase at low T to sQGP to QGP..



The Phase diagram in T, μ_B, N_f Space?



BACKUP SLIDES

- The RG equation for the running coupling can be written as

$$b \log \left(\frac{q}{\mu} \right) = \frac{1}{\alpha} - \frac{1}{\alpha(\mu)} - \frac{1}{\alpha_*} \log \left(\frac{\alpha (\alpha(\mu) - \alpha_*)}{\alpha(\mu) (\alpha - \alpha_*)} \right) ,$$

where $\alpha = \alpha(q)$.

- For $\alpha, \alpha(\mu) < \alpha_*$ we can introduce a scale defined by

$$\Lambda = \mu \exp \left[\frac{-1}{b \alpha_*} \log \left(\frac{\alpha_* - \alpha(\mu)}{\alpha(\mu)} \right) - \frac{1}{b \alpha(\mu)} \right] ,$$

so that

$$\frac{1}{\alpha} = b \log \left(\frac{q}{\Lambda} \right) + \frac{1}{\alpha_*} \log \left(\frac{\alpha}{\alpha_* - \alpha} \right) .$$

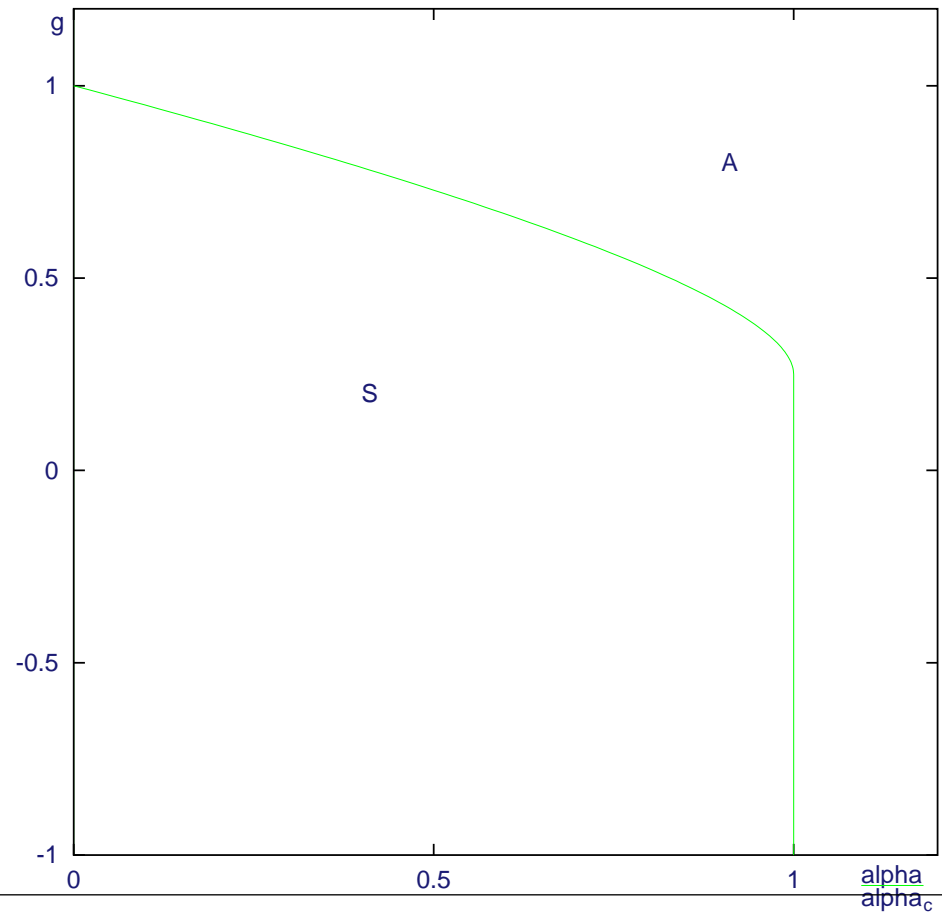
- Then for $q \gg \Lambda$ the running coupling displays the usual perturbative behavior:

$$\alpha \approx \frac{1}{b \log \left(\frac{q}{\Lambda} \right)} ,$$

- For $q \ll \Lambda$ it approaches the fixed point α_* :

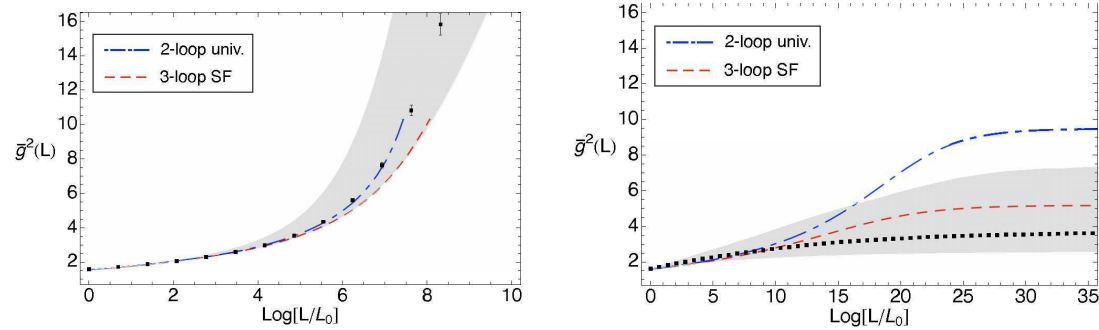
$$\alpha \approx \frac{\alpha_*}{1 + \frac{1}{e} \left(\frac{q}{\Lambda} \right)^{b\alpha_*}} .$$

QED / Gauged NJL Model: example of SSB at strong coupling



Conformal Window for Lattice:

○ **Schrödinger Functional Results:** (coupling constant is determined by response of Action to applied E fields and the beta function by step scaling)



Compare $N_f=8$ and 12 staggered quarks

Applequist, Fleming and Neil arXiv:0712.0609 Type B USQCD allocation 2007

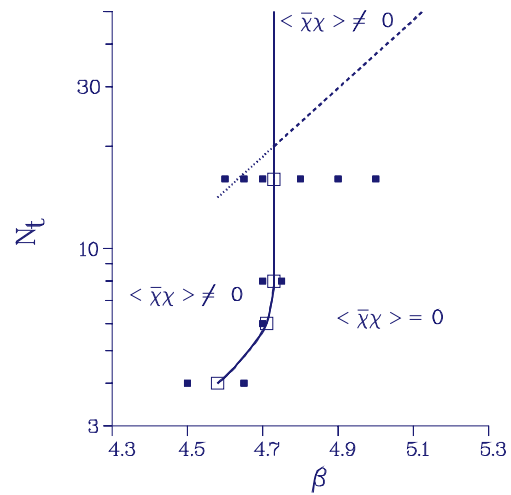
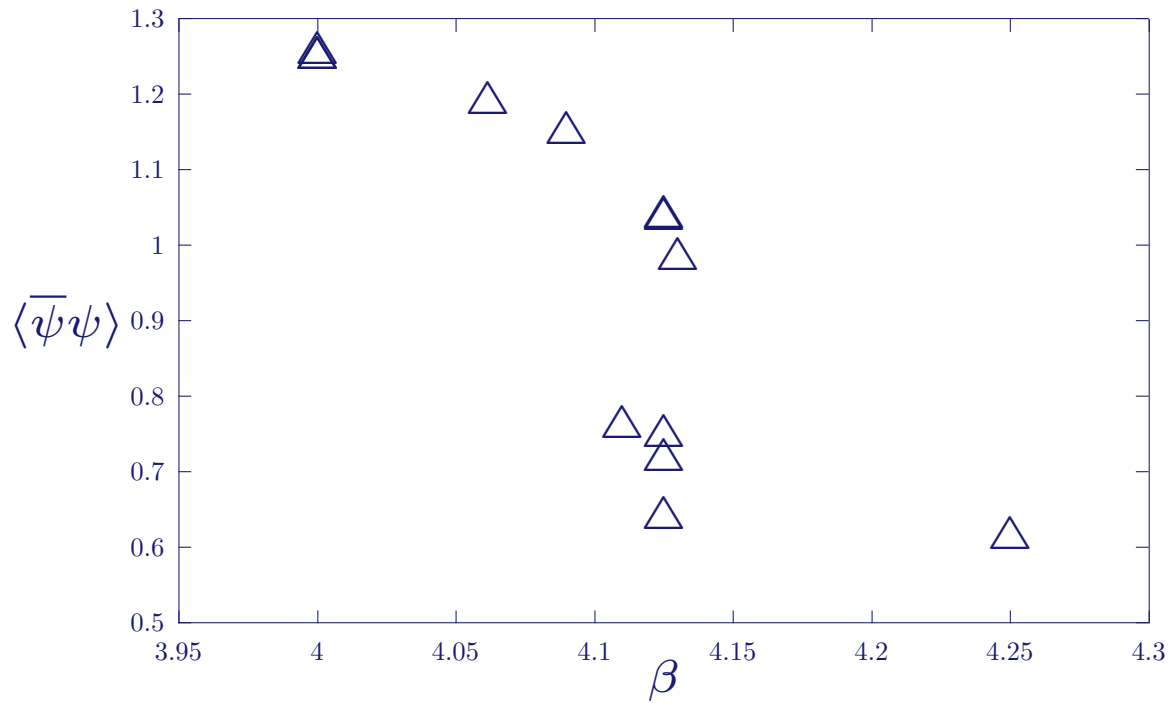


Figure 1: A phase diagram in the $\beta - N_t$ plane for eight-flavor QCD in infinite spatial volume consistent with the results presented here. N_t is the temporal extent of the lattice and $\beta = 6/g^2$ is the lattice-coupling strength. The solid line, becoming vertical for $N_t \geq 8$ locates a “zero-temperature”, first-order transition—a lattice artifact. The dashed line suggests a possible, continuum finite-temperature transition that occurs in the weak-coupling phase. The system shows chiral symmetry to the right of and below this dashed line while we speculate that chiral symmetry will be spontaneously broken to the left of and above this line. The solid squares label parameter values where we have performed simulations, while the open squares locate critical values.



Damgaard et al. 1996: The β

dependence of the chiral order parameter for lattice sizes $8^3 \times 16$, 12^4 and $6^3 \times 16$. The quark mass is $am_q = 0.1$.

The error bars are smaller than the plotting symbols.