Chiral Symmetry Restoration, Deconfinement and Dressed Polyakov Loops

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The question we want to address:

- At zero temperature QCD shows two characteristic features:
  - Quarks are confined.
  - Chiral symmetry is broken: $\langle\overline{\psi}\psi\rangle \neq 0$.

- QCD has a finite temperature transition where:
  - Quarks become deconfined.
  - Chiral symmetry is restored: $\langle\overline{\psi}\psi\rangle = 0$.

Is there an underlying mechanism that links the two key features of QCD?

A possible approach

- Confinement and chiral symmetry breaking both should leave a trace in properties of the Dirac operator D, since D^{-1} describes the propagation of quarks.

- For chiral symmetry breaking the Banks-Casher formula connects the order parameter $\langle \overline{\psi} \psi \rangle$ to IR properties of the Dirac spectrum.

- Concerning confinement it is not even clear where to look in the spectrum, in the UV or the IR part.

- Maybe through analyzing spectral properties of D one can find a link between confinement and chiral symmetry breaking.

- The lattice formulation provides a suitable framework (rigorously defined) which allows for both, analytical and numerical approaches.

Some literature

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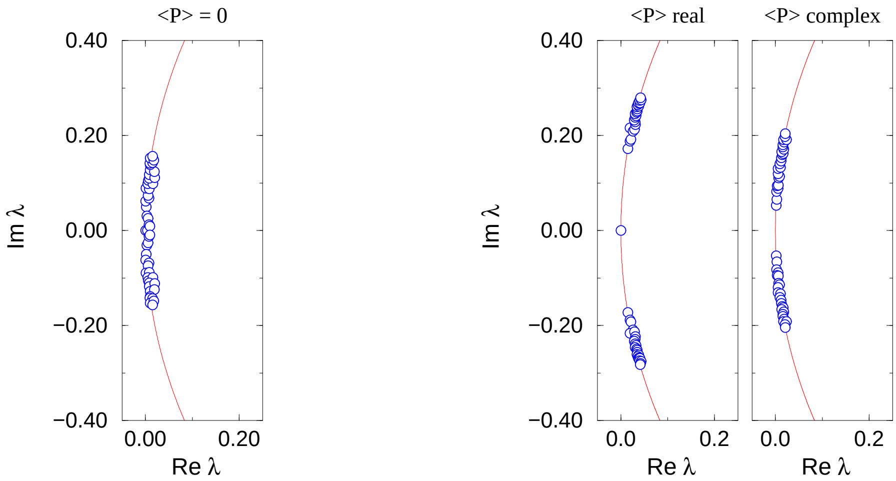
Chiral symmetry breaking and Dirac spectrum

- The Banks Casher formula relates the chiral condensate to the spectral density of the Dirac operator at the origin.

$$\langle\, \overline{\psi}\, \psi\, \rangle = -\,\pi\, \rho(0)$$

- At the QCD phase transition a gap opens up in the spectrum and the chiral condensate vanishes.



Center symmetry and Polyakov loops

- The gauge action is invariant under center transformations ($z \in Z_3$):

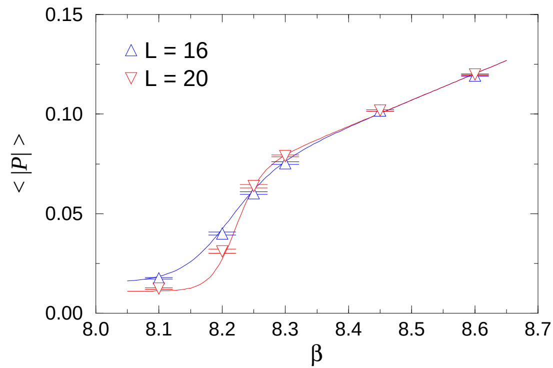
$$U_4(x) \;\to\; z\,U_4(x) \qquad\qquad \forall\; x_4 = t_0$$

- The deconfinement transition of pure gauge theory can be described as spontaneous breaking of the center symmetry.

- The Polyakov loop transforms non-trivially and is an order parameter.

$$L(\vec{x}) \;=\; \mathrm{tr}_c \prod_{t=1}^{N_t} U_4(\vec{x},t)$$

$$L(\vec{x}) \;\to\; z\,L(\vec{x})$$



The Dirac operator on the lattice (here staggered, Wilson works too)

- Discretized Dirac operator on the lattice

$$D \;=\; \frac{1}{2a} \sum_{\mu=1}^{4} \gamma_\mu(x) \left[U_\mu(x)\, \delta_{x+\hat{\mu}\,,\,y} \;-\; U_\mu(x-\hat{\mu})^\dagger\, \delta_{x-\hat{\mu}\,,\,y} \right]$$

- The gauge links

$$U_\mu(x) \;=\; e^{i\,a\,A_\mu(x)}$$

- are the objects we need for the Polyakov loop

$$L(\vec{x}) \;=\; \mathrm{tr}_c \prod_{t=1}^{N_t} U_4(\vec{x},t)$$

- The gauge links appear in hopping terms that connect nearest neighbors on the lattice.

Fermion propagators and loops

- The chiral condensate has an expansion in terms of loops:

$$\langle\overline{\psi}\psi\rangle = -\frac{1}{V}\mathrm{Tr}[m+D]^{-1} = -\frac{1}{mV}\sum_{k=0}^{\infty}\frac{(-1)^k}{m^k}\mathrm{Tr}\left[\,D^k\,\right]$$

$$= -\frac{1}{mV}\sum_{l\in\mathcal{L}}\frac{s(l)}{(2am)^{|l|}}\;\mathrm{Tr}_c\prod_{(x,\mu)\in l}U_\mu(x)$$

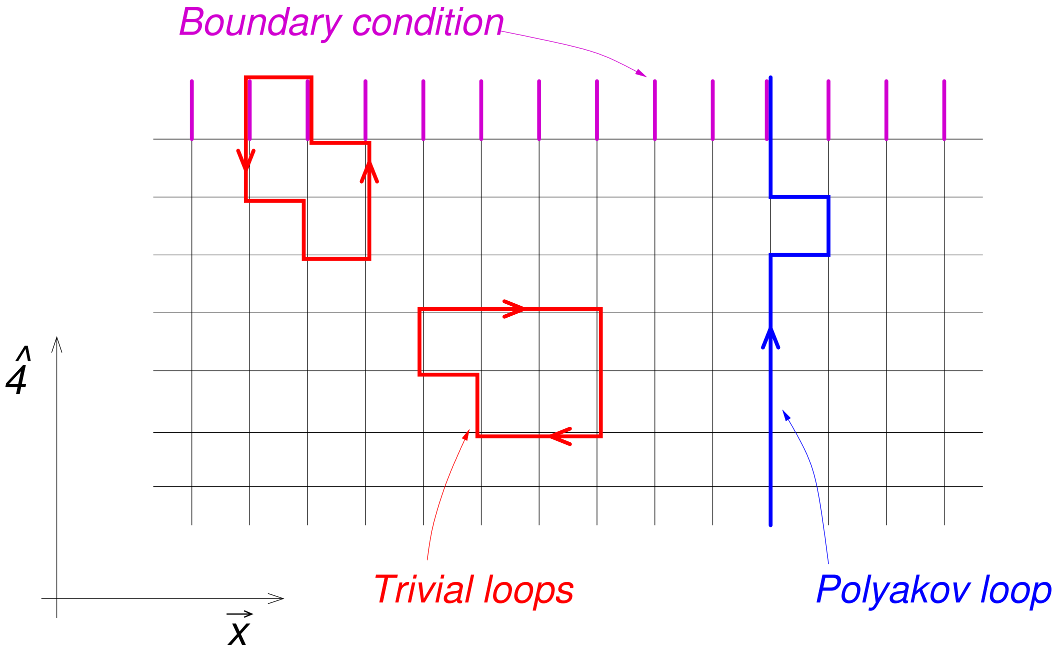
- A change of the temporal boundary conditions

$$U_4(\vec{x},N_t) \longrightarrow z\,U_4(\vec{x},N_t) \quad , \qquad z = e^{i\varphi} \in \mathsf{U}(1)$$

affects only loops that wind non-trivially around compact time.

- Fourier transformation of φ allows one to project to the equivalence class of loops that wind exactly once: *Dressed Polyakov Loops*

Graphical representation



Dual chiral condensate = dressed Polykov loop

- Fourier transformation with respect to the boundary condition connects the order parameters for confinement and for chiral symmetry breaking:

$$\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} = \int_0^{2\pi} \frac{d\varphi\, e^{-i\varphi}}{2\pi} \left\langle \overline{\psi}\psi \right\rangle_\varphi \quad = \quad \frac{1}{mV} \sum_{l \in \mathcal{L}_{\mathbf{1}}} \frac{s(l)}{(2am)^{|l|}} \left\langle \mathsf{Tr}_c \prod_{(x,\mu)\in l} U_\mu(x) \right\rangle$$

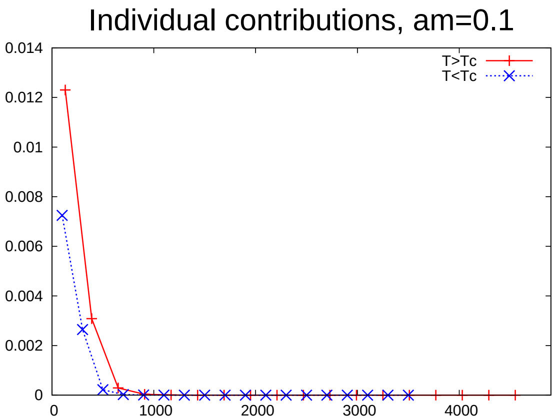
$$= \quad -\int_0^{2\pi} \frac{d\varphi\, e^{-i\varphi}}{2\pi\, V} \sum_k \left\langle \frac{1}{m+\lambda_\varphi^{(k)}} \right\rangle_\varphi$$

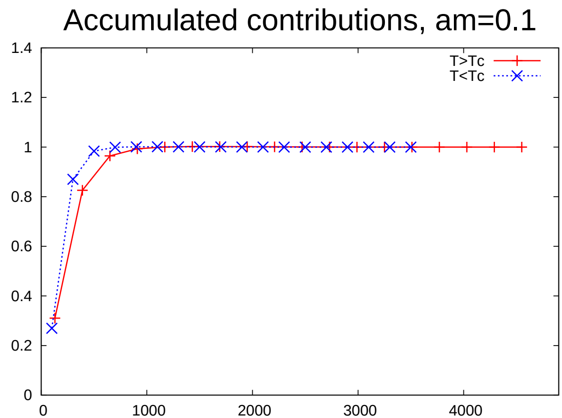
- The representation as a spectral sum of Dirac eigenvalues allows one to study the role of IR and UV eigenmodes for the mechanisms of confinement and chiral symmetry breaking.

Outline of the numerical tests

- We analyze quenched SU(3) configurations below and above T_c.
- Above T_c the gauge configurations are classified with respect to the phase of the Polyakov loop to mimic center symmetry breaking on a finite volume.
- Complete spectra of the staggered Dirac operator are calculated for 8 or 16 values of the boundary angle φ.
- The φ-integration is implemented with Simpson's rule.

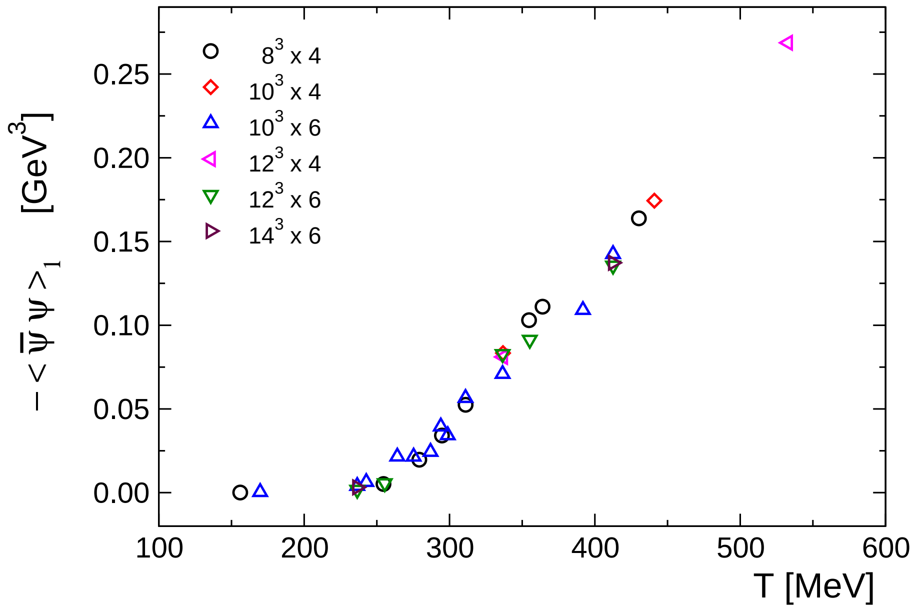
The Dressed Polyakov Loop is dominated by IR modes





$$-\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} = \sum_k \frac{1}{2\pi V}\int_0^{2\pi} d\varphi\, e^{-i\varphi} \left\langle \frac{1}{m+\lambda_\varphi^{(k)}} \right\rangle_\varphi$$

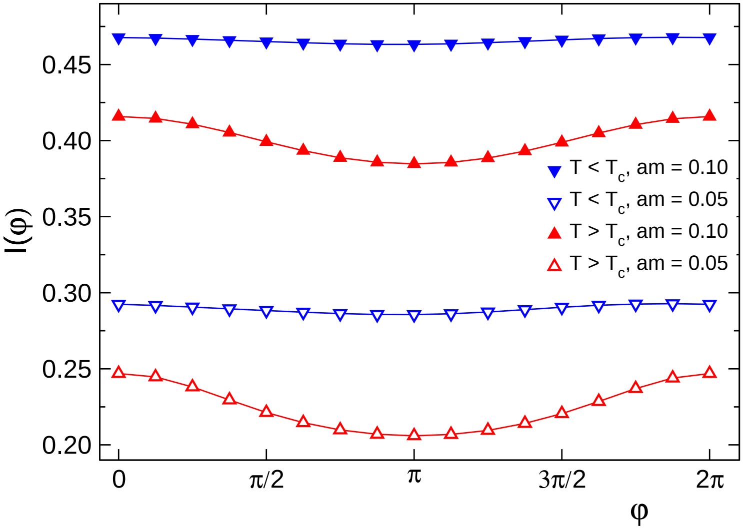
The Dressed Polyakov Loop is an order parameter



Results from different lattices fall on a universal curve.

$\longrightarrow$ Good scaling and renormalization properties.

Spectral properties at the phase transition



$$I(\varphi) = \frac{1}{V} \sum_k \left\langle \frac{1}{m + \lambda_\varphi^{(k)}} \right\rangle$$

The confined and deconfined phases give rise to a different response of the IR part of the Dirac spectrum to changing boundary conditions.

Generalization of the Banks-Casher formula

- Having identified the connection between spectral properties and the dressed Polyakov loops, we can now formulate the physical picture in terms of a generalized Banks-Casher relation.

- Performing $\lim_{m \to 0} \lim_{V \to \infty}$ we find:

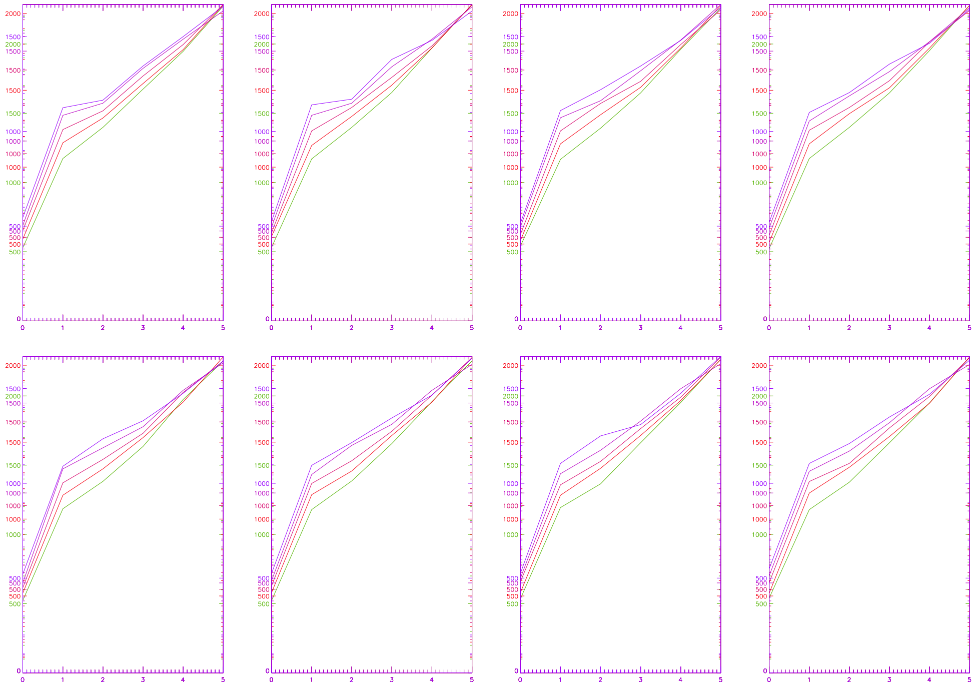
$$-\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} = \frac{1}{2} \int_0^{2\pi} d\varphi \, e^{-i\varphi} \, \rho(0)_\varphi$$

- How does the spectral density $\rho(0)_\varphi$ at the origin have to behave as a function of φ such that:

$$-\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} = 0 \qquad \text{below } T_c$$

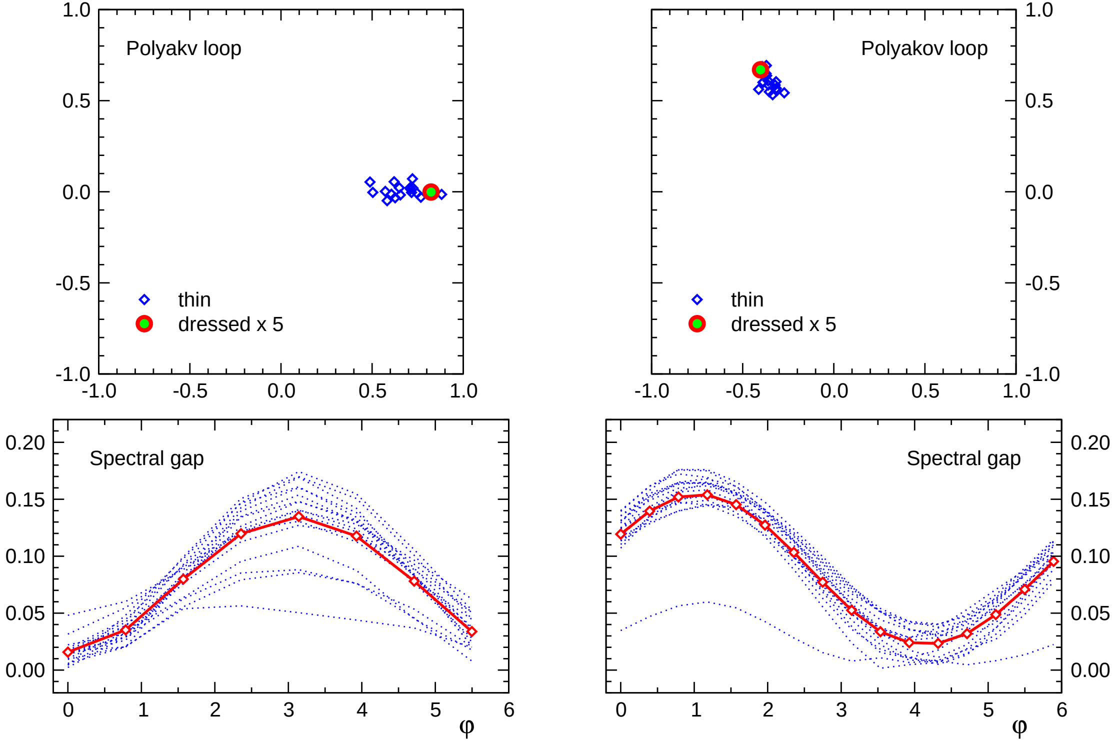
$$-\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} > 0 \qquad \text{above } T_c$$

Spectral density below T_c



Below T_c the spectral density $\rho(0)_\varphi$ is independent of the boundary angle φ.

Spectral gap above T_c



Spectral gap depends on the relative phase between b.c. and Polyakov loop.

Emerging picture for the generalized Banks-Casher formula

- The spectral density at the origin, $\rho(0)_\varphi$, behaves as (θ denotes the phase of the Polyakov loop):

$$\rho(0)_\varphi \quad = \quad \text{const} \qquad\qquad \text{below}\, T_c$$

$$\rho(0)_\varphi \quad \propto \quad \delta(\,\varphi + \theta\,) \qquad\qquad \text{above}\, T_c$$

- The dual chiral condensate is given by:

$$-\,\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} \;=\; \frac{1}{2}\int_0^{2\pi} d\varphi\, e^{-i\varphi}\,\rho(0)_\varphi$$

- And behaves correctly as:

$$-\,\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} \quad = \quad 0 \qquad\qquad \text{below}\, T_c$$

$$-\widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} \quad = \quad \rho_0\, \exp(i\theta) \qquad\qquad \text{above}\, T_c$$

A small digression: The centerless gauge group G_2

- What role does the center play in our picture?
  $\Rightarrow$ Study a gauge group with trivial center.

- Analyze the Dirac spectrum and its response to changing boundary conditions using quenched G_2 configurations.

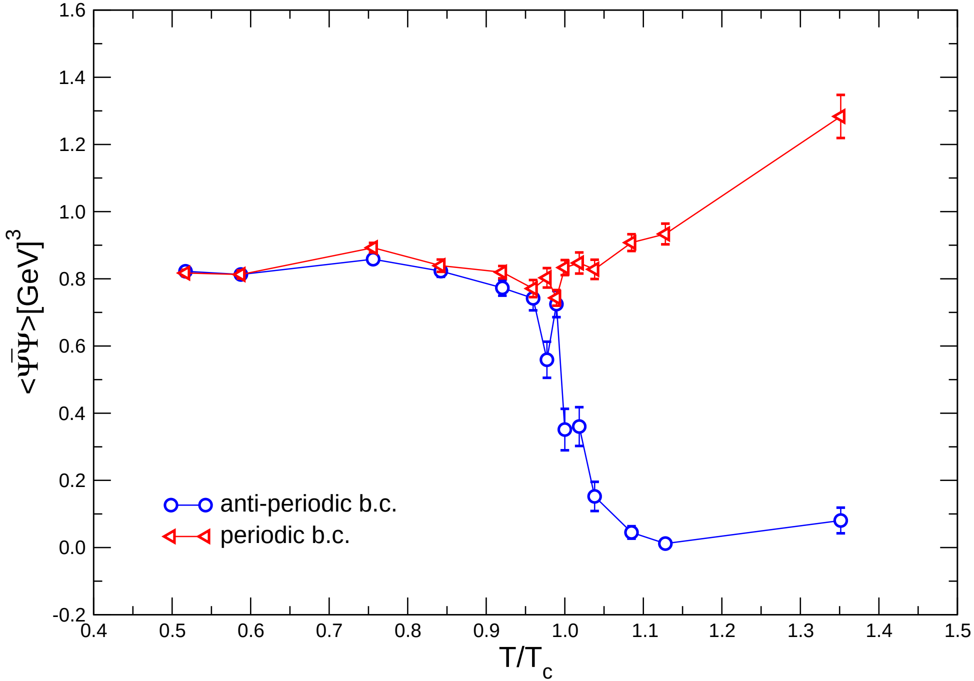
- Preliminary results on small lattices. (J. Danzer, A. Maas, C.G.)

- Finding so far:

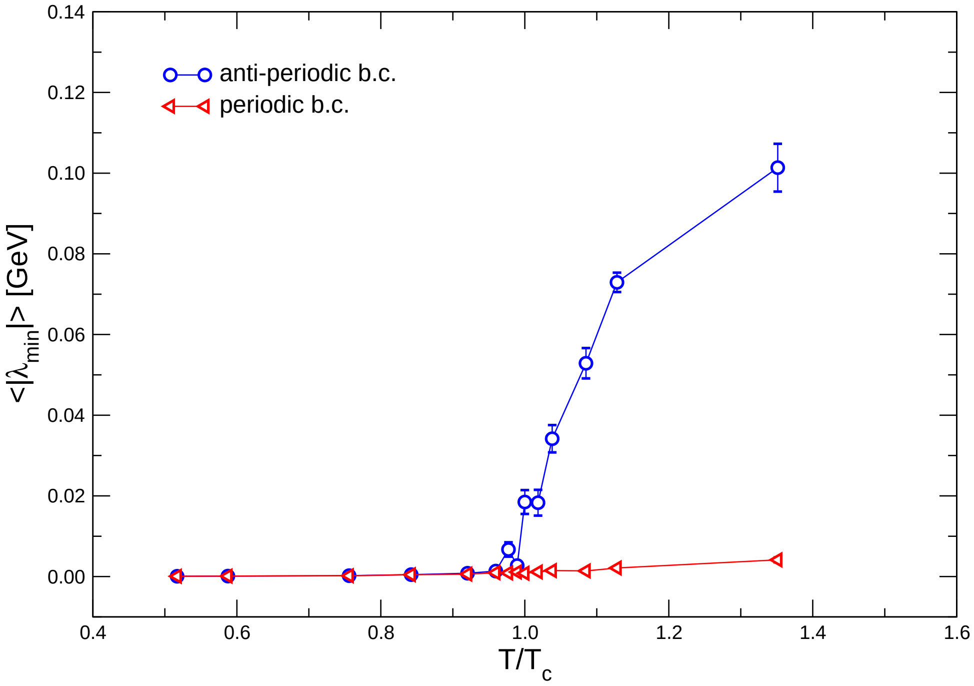
  Behavior is exactly the same as for SU(3) in the real Polyakov sector.

- Another piece of evidence that the picture developed here is universal are the recent (last week) results in SU(2): Bornyakov *et al.*

G_2 : Chiral condensate $(12^3 \times 6)$



G_2 : *Spectral gap* $(12^3 \times 6)$



Summary

- Fourier transforming the chiral condensate with respect to the fermionic boundary condition we define the *Dual Chiral Condensate*.
- The dual chiral condensate is an order parameter for center symmetry, interpreted as *Dressed Polyakov Loops*.
- The dual condensate can be represented as a spectral sum of Dirac eigenvalues which is dominated by the IR modes.
- At the phase transition the behavior of the low-lying eigenvalues changes:
    1. The chiral transition is signalled by a change from a non-zero to a vanishing density (Banks-Casher).
    2. The deconfinement transition is manifest in a different response of the eigenvalues to a change in the temporal boundary conditions.
- The center of the gauge group does not seem to play a major role.

Summary (continued)

- Most elegantly the results are expressed as a generalized Banks-Casher formula for the dual condensate:

$$- \widehat{\left\langle \overline{\psi}\psi \right\rangle}_{\mathbf{1}} \;=\; \frac{1}{2} \int_0^{2\pi} d\varphi \, e^{-i\varphi} \, \rho(0)_\varphi$$

1. In the confined phase we have a non-vanishing spectral density $\rho(0)_\varphi$ at the origin which is independent of the boundary conditions.
2. Above T_c the spectral gap has a non-trivial dependence on the phase between boundary condition and Polyakov loop and $\rho(0)_\varphi \propto \delta(\varphi + \theta)$.

Chiral symmetry breaking and confinement are, via a duality transformation, connected to closely related spectral properties of the IR Dirac spectrum.

Link between confinement and chiral symmetry breaking?