Homework 5, due 9-29

In class we introduced the spin operator $\vec{S} = (\hbar/2)\vec{\sigma}$ where $\vec{\sigma}$ are the Pauli-matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

We also introduced the vectors $\chi_\uparrow = (1, 0)$ and $\chi_\downarrow = (0, 1)$ which satisfy

$$S_z \chi_{\uparrow,\downarrow} = \pm \frac{\hbar}{2} \chi_{\uparrow,\downarrow}.$$

Find the vectors $\chi(S_x, \uparrow, \downarrow)$ and $\chi(S_y, \uparrow, \downarrow)$ that satisfy

$$S_x \chi(S_x, \uparrow, \downarrow) = \pm \frac{\hbar}{2} \chi(S_x, \uparrow, \downarrow),$$

$$S_y \chi(S_y, \uparrow, \downarrow) = \pm \frac{\hbar}{2} \chi(S_y, \uparrow, \downarrow).$$

What is the expectation value of S_z in the states $\chi(S_x, \uparrow)$ and $\chi(S_x, \downarrow)$?