In Search of the Perfect Fluid

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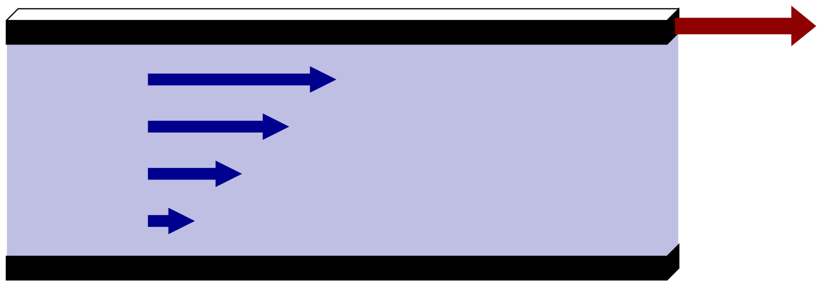
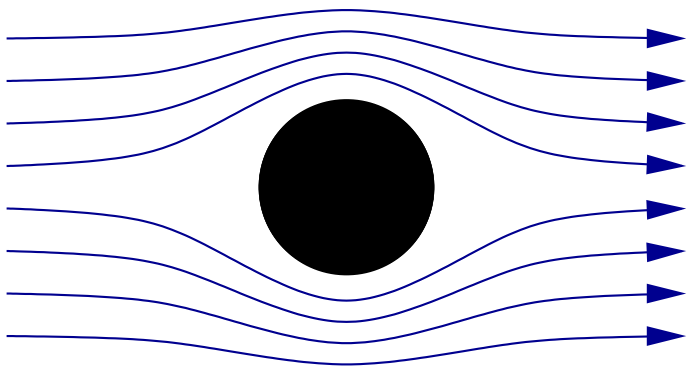
Measures of Perfection

Viscosity determines shear stress ("friction") in fluid flow



$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

Dimensionless measure of shear stress: Reynolds number



$$Re = \frac{n}{\eta} \times mvr$$

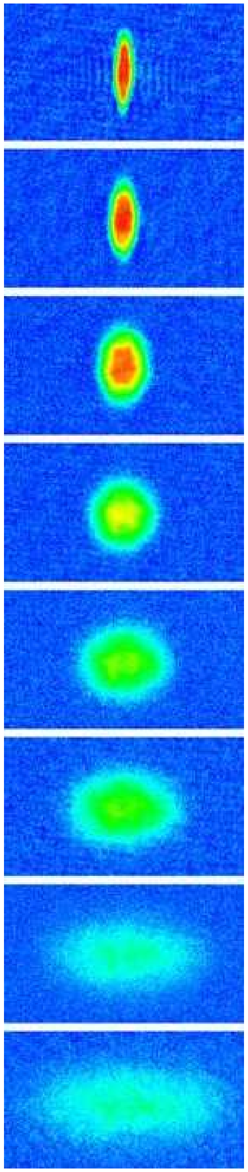
fluid property — flow property

- $[\eta/n] = \hbar$
- Relativistic systems $Re = \dfrac{s}{\eta} \times \tau T$

There are good reasons to expect that η/s is bounded from below by some constant (possibly, $1/(4\pi)$) times $\hbar/k_B$.

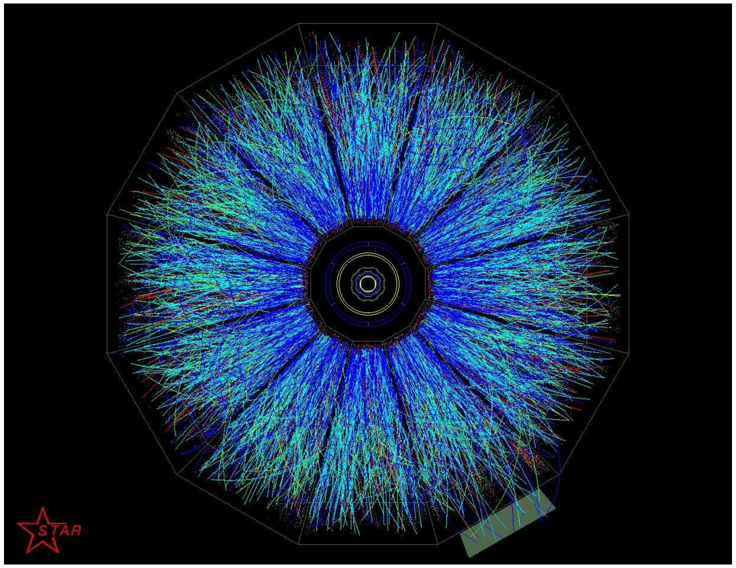
A fluid that saturates the bound is a “perfect fluid”.

Perfect Fluids: The contenders

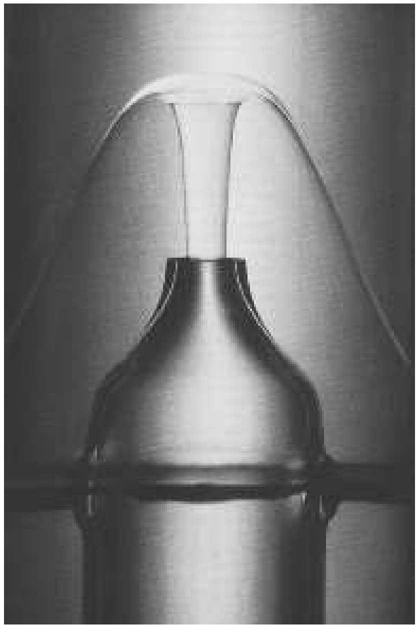


Trapped Atoms

(T=0.1 neV)



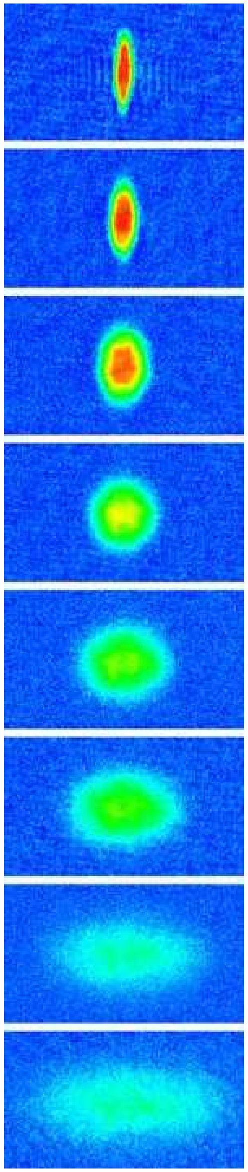
QGP (T=180 MeV)

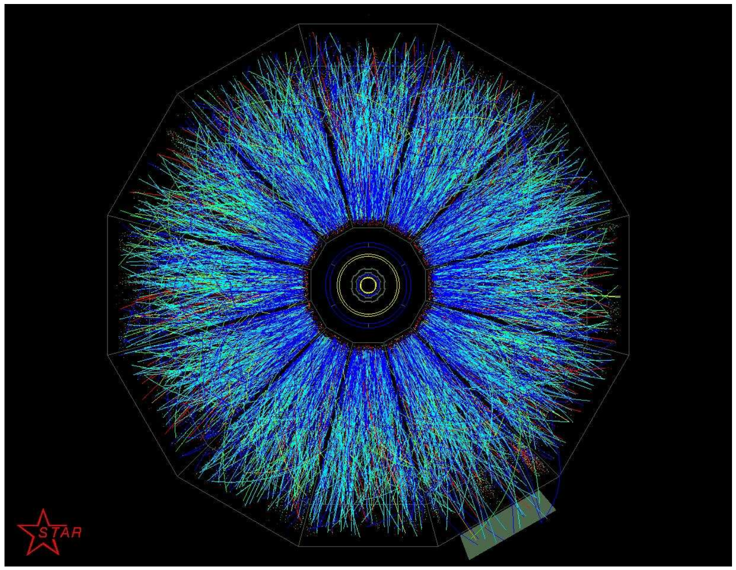


Liquid Helium

(T=0.1 meV)

Perfect Fluids: The contenders





QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$

Liquid Helium

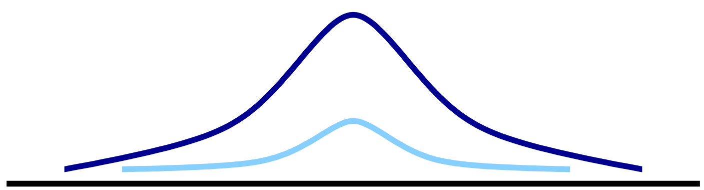
$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

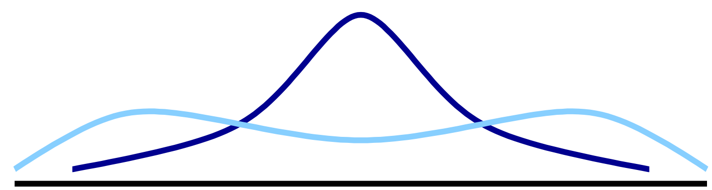
η / s

Fluids: Gases, Liquids, Plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



$\tau \sim \tau_{micro}$



$\tau \sim \lambda^{-1}$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$

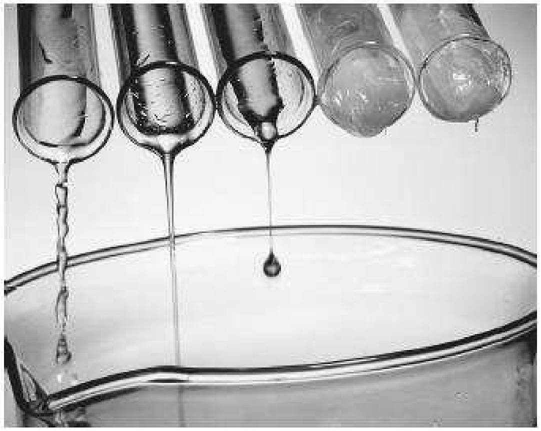
Example: Simple Fluid

Continuity equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

Euler (Navier-Stokes) equation

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0$$



Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right) + \ldots$$

reactive dissipative

Kinetic Theory

Quasi-Particles ($\gamma \ll \omega$): introduce distribution function $f_p(x,t)$

$$T_{ij} = \int d^3p \, \frac{p_i p_j}{E_p} f_p,$$

Boltzmann equation

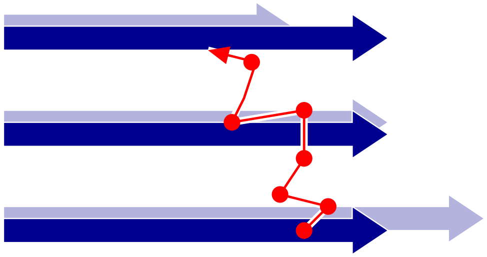
$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = C[f_p]$$

Collision term $C[f_p]$

Linearized theory (Chapman-Enskog): $f_p = f_p^0(1 + \chi_p/T)$

suitable for transport coefficients

shear viscosity $\chi_p = g_p p_x p_y \partial_x v_y$



Viscosity Bound: Rough Argument

Kinetic theory estimate of shear viscosity

$$\eta \sim \frac{1}{3} n \, \bar{p} \, l_{mfp} \qquad\qquad (\mathrm{Note:} \; l_{mfp} \sim 1/(n\sigma))$$

Normalize to density. Uncertainty relation implies

$$\frac{\eta}{n} \sim \bar{p} \, l_{mfp} \geq \hbar$$

Also: $s \sim k_B n$ and $\eta/s \geq \hbar/k_B$

Validity of kinetic theory as $\bar{p} \, l_{mfp} \sim \hbar$?

Effective Theories for QCD fluids (Weak Coupling)

$$\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

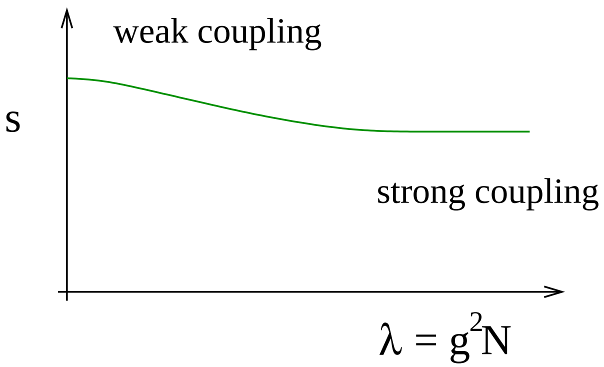
$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p = C[f_p] \qquad (\omega < T)$$

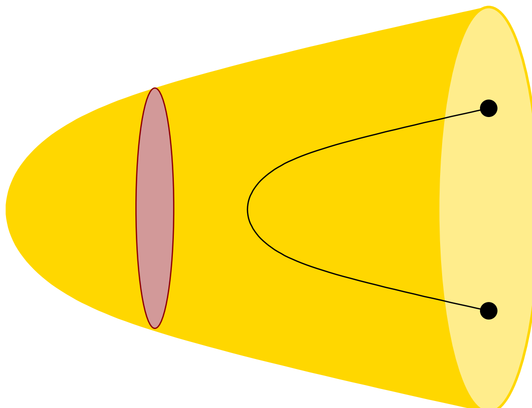
$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \qquad (\omega < g^4 T)$$

Holographic Duals at Finite Temperature

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature	$\Leftrightarrow$	Hawking temperature of black hole
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon





$$s(\lambda \to \infty) = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s(\lambda = 0)$$

Gubser and Klebanov (1996)

Holographic Duals: Transport Properties

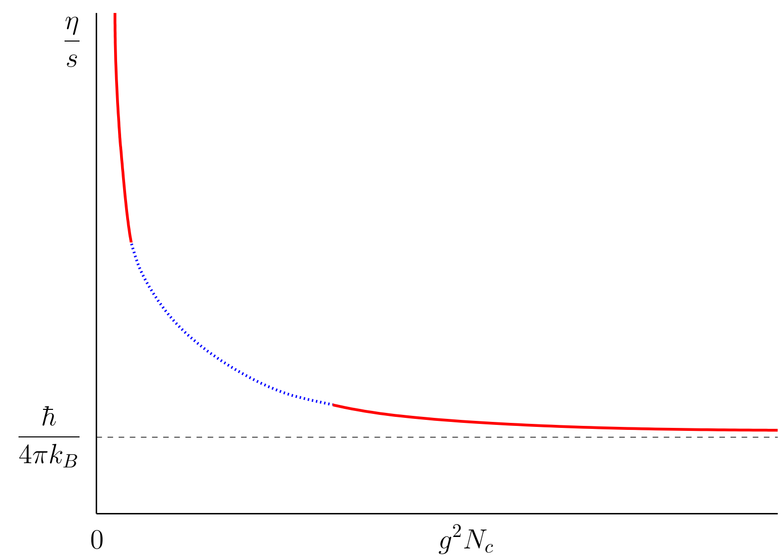
Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets (2001)



Strong coupling limit universal? Provides lower bound for all theories?

Effective Theories (Strong coupling)

$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \ldots \;\Leftrightarrow\; S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\mathcal{R} + \ldots$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \;\; (\omega < T)$$

Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

Bound is incompatible with weak coupling and kinetic theory

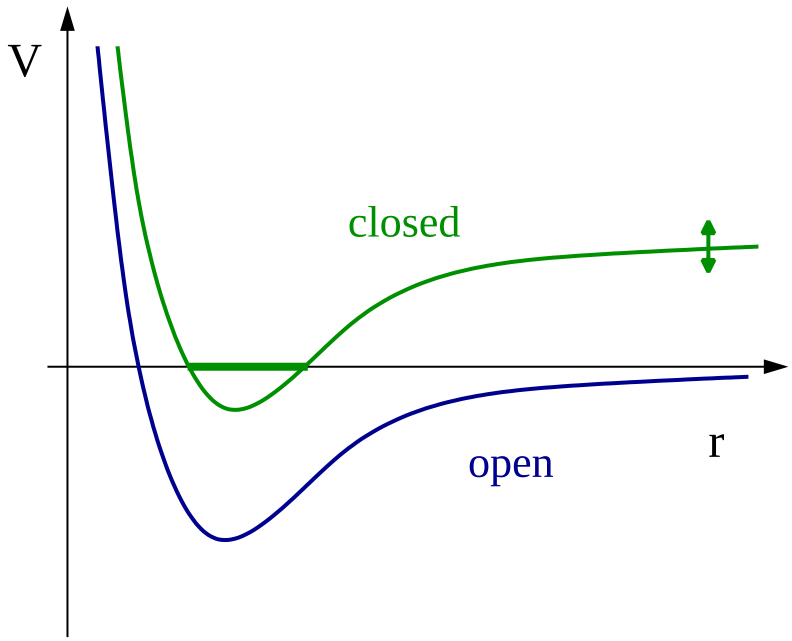
strong interactions, no quasi-particles

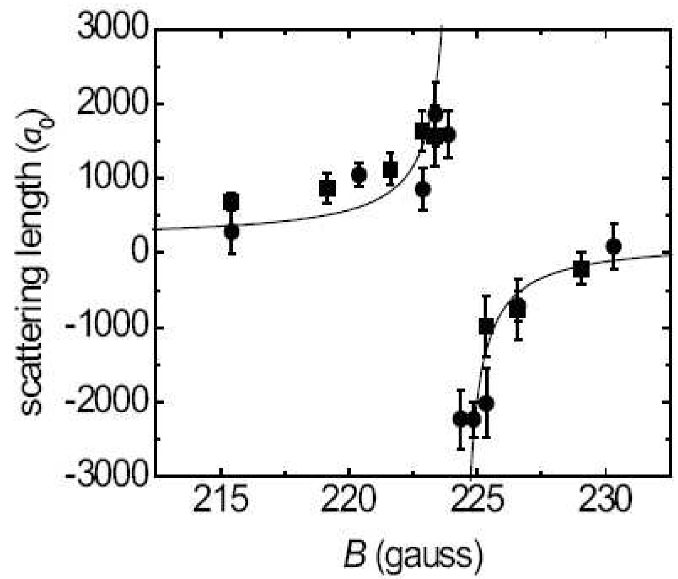
Model system has conformal invariance (essential?)

(Almost) scale invariant systems

I. Unitary Fermi Gas

Atomic gas with two spin states: "↑" and "↓"





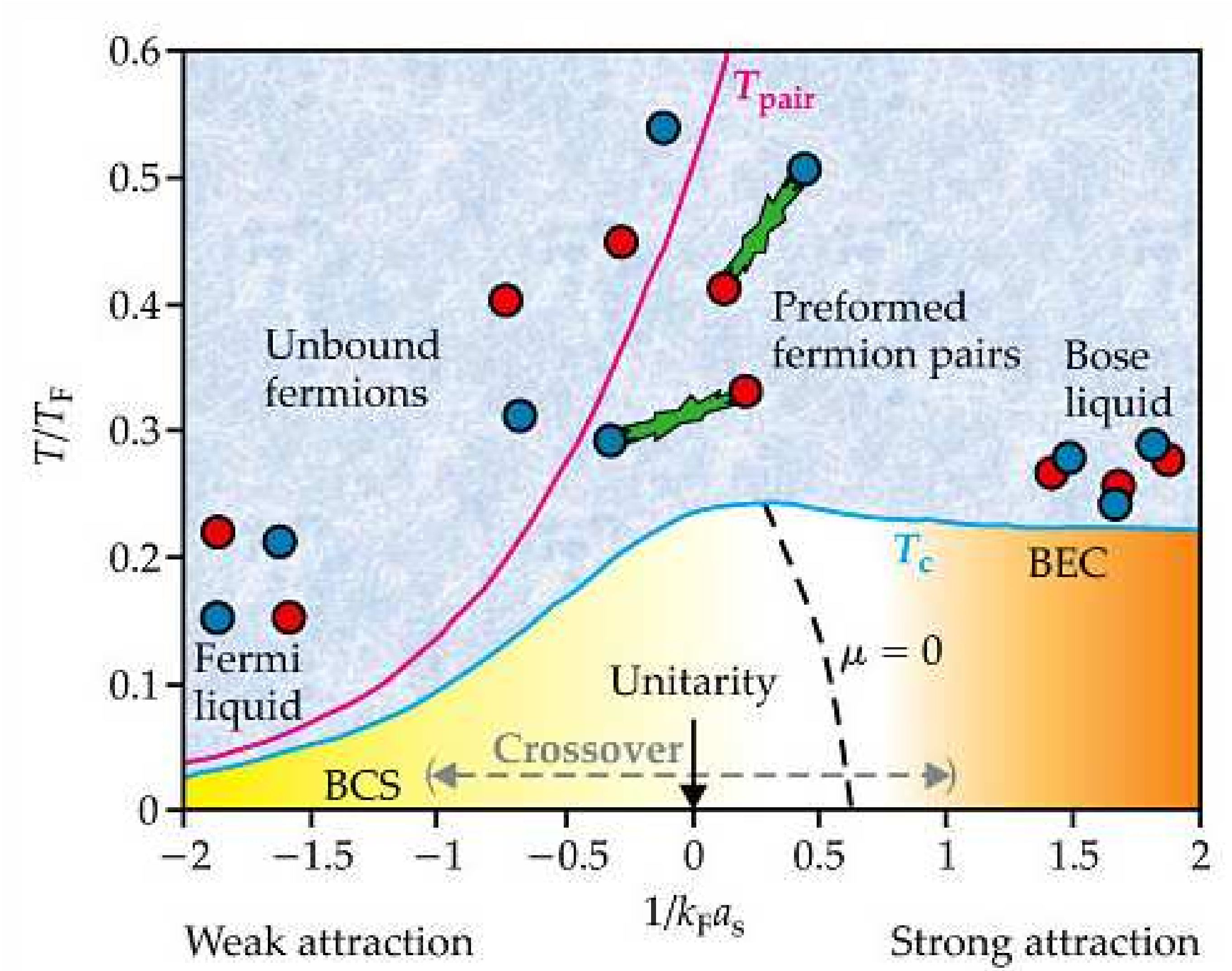
Feshbach resonance

$$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

"Unitarity" limit $a \to \infty$

$$\sigma = \frac{4\pi}{k^2}$$

Fermi Gas at Unitarity: Phase Diagram



Fermi gas at unitarity: Field Theory

Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

Unitary limit: $a \to \infty$, $\sigma \to 4\pi/k^2$ ($C_0 \to \infty$)

This limit is smooth (HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$)

$$\mathcal{L} = \Psi^\dagger \left[i\partial_0 + \sigma_3 \frac{\vec{\nabla}^2}{2m} \right] \Psi + \left(\Psi^\dagger \sigma_+ \Psi \phi + h.c. \right) - \frac{1}{C_0} \phi^* \phi \ ,$$

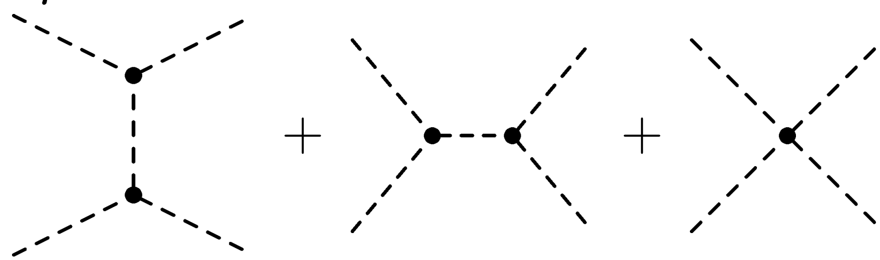
Low T ($T < T_c \sim \mu$): Pairing and superfluidity

Low T: Phonons Goldstone boson $\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$

$$\mathcal{L} = c_0 m^{3/2}\left(\mu - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}\right)^{5/2} + \ldots$$

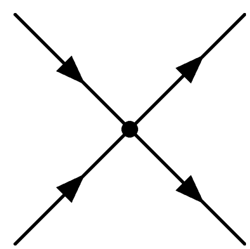
Viscosity dominated by $\varphi + \varphi \to \varphi + \varphi$

$$\eta = A\frac{\xi^5}{c_s^3}\frac{T_F^8}{T^5}$$



High T: Atoms Cross section regularized by thermal momentum

$$\eta = \frac{15}{32\sqrt{2}}(mT)^{3/2}$$



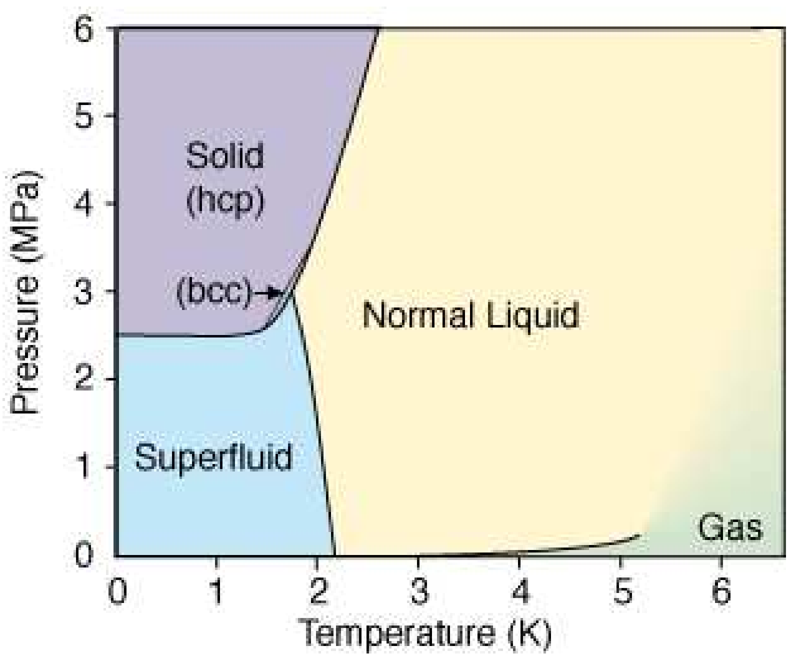
II. Liquid Helium

Bosons, van der Waals + short range repulsion

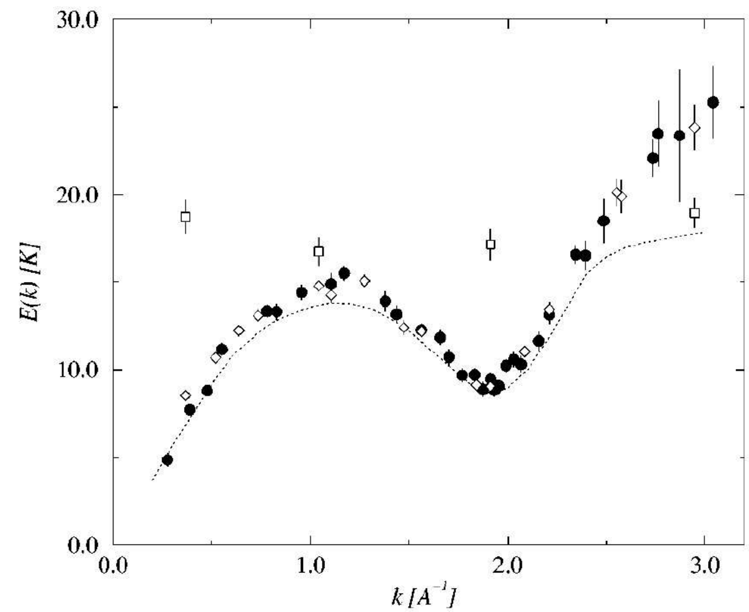
$$S = \int \Phi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \Phi + \int \int \left(\Phi^\dagger \Phi \right) V(x-y) \left(\Phi^\dagger \Phi \right)$$

with $V(x) = V_{sr}(x) - c_6/x^6$. Note: $a = 189 a_0 \gg a_0$

Phase Diagram



Excitations



Low T: Phonons and Rotons Effective lagrangian

$$\mathcal{L} = \varphi^*(\partial_0^2 - v^2)\varphi + i\lambda\dot{\varphi}(\vec{\nabla}\varphi)^2 + \ldots$$

$$+ \varphi^*_{R,v}(i\partial_0 - \Delta)\varphi_{R,v} + c_0(\varphi^*_{R,v}\varphi_{R,v})^2 + \ldots$$

Shear viscosity

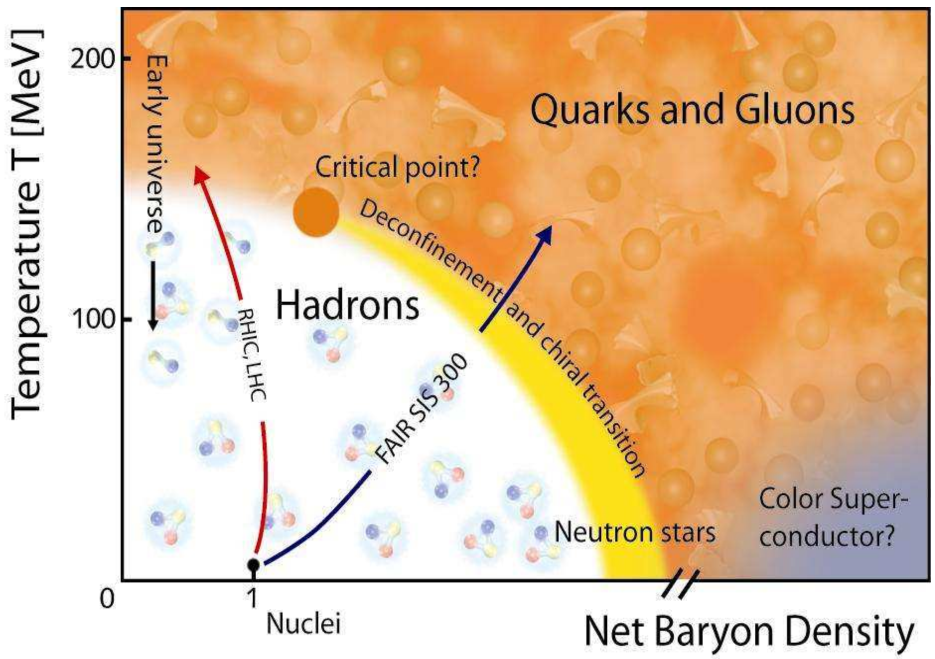
$$\eta = \eta_R + \frac{c_{Rph}}{\sqrt{T}}\, e^{\Delta/T} + \frac{c_{ph}}{T^5} + \ldots$$

High T: Atoms Viscosity governed by hard core ($V \sim 1/r^{12}$)

$$\eta = \eta_0 (T/T_0)^{2/3}$$

III. Quark Gluon Plasma

$$\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

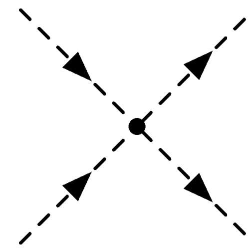


Low T: Pions Chiral perturbation theory

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + (B\mathrm{Tr}[MU] + h.c.) + \ldots$$

Viscosity dominated by $\pi\pi$ scattering

$$\eta = A\frac{f_\pi^4}{T}$$



High T: Quasi-Particles HTL theory (screening, damping, ...)

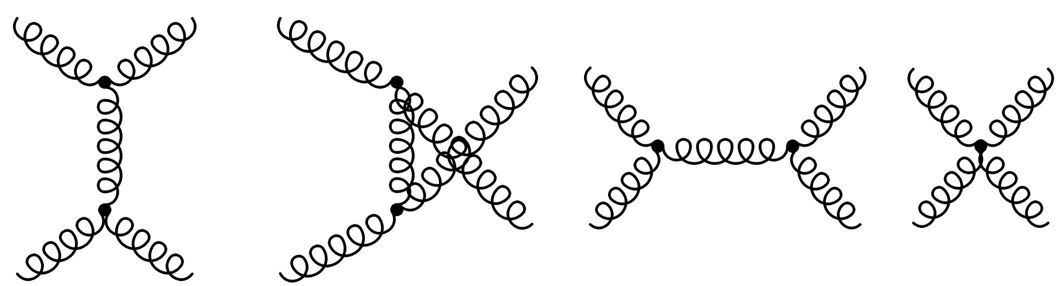
$$\mathcal{L}_{HTL} = \int d\Omega\, G^a_{\mu\alpha}\frac{v^\alpha v_\beta}{(v\cdot D)^2}G^{a,\mu\beta}$$

quasi-particle width

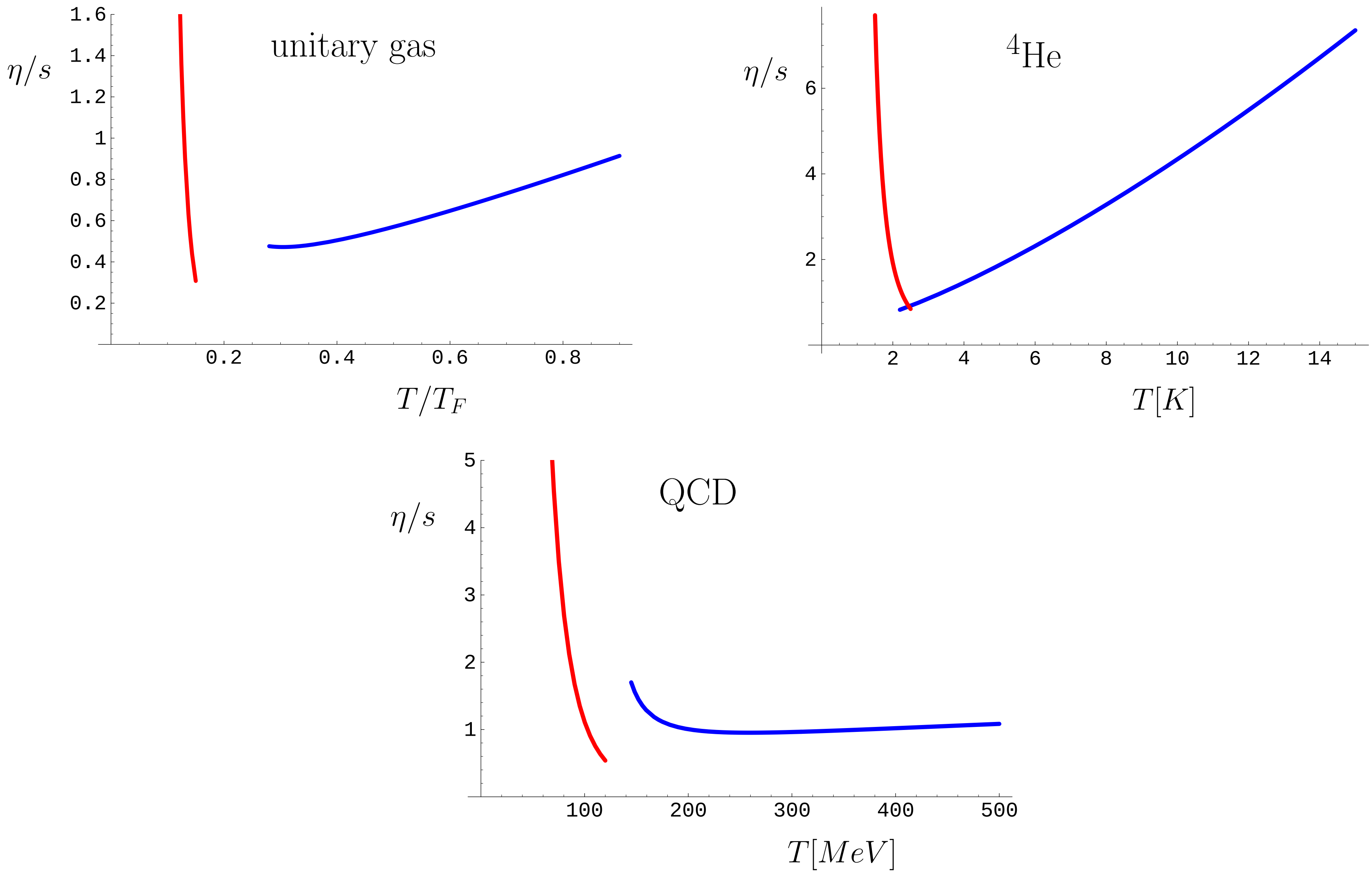
$$\gamma \sim g^2 T$$

Viscosity dominated by t-channel gluon exchange

$$\eta = \frac{27.13T^3}{g^4\log(2.7/g)}$$



Theory Summary



I. Experiment (Liquid Helium)

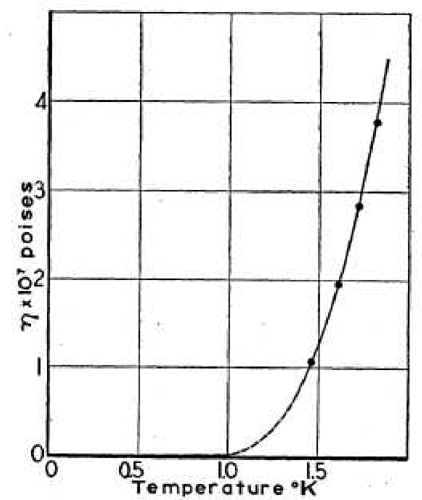


FIG. 1. The viscosity of liquid helium II measured by flow through a 10^{-4} cm channel.

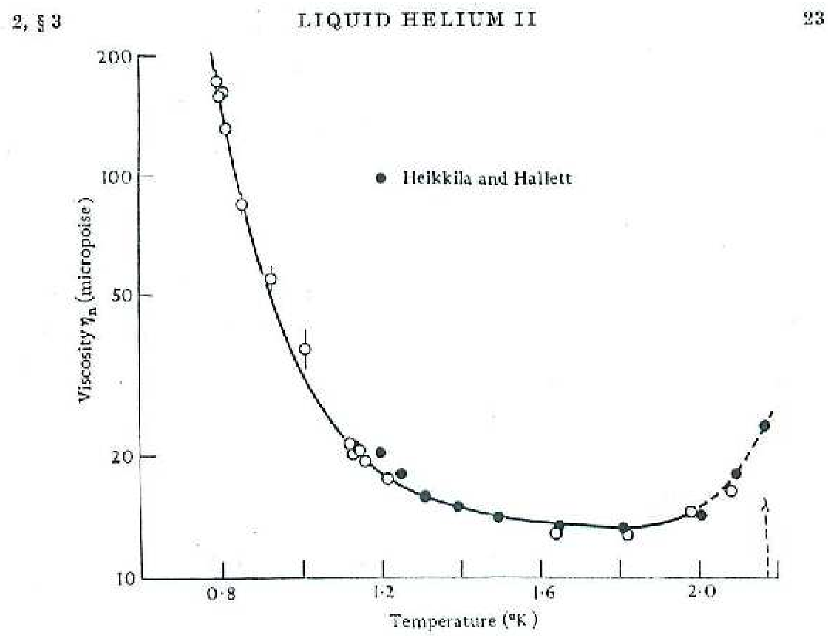


FIG. 11. The viscosity (η_n) of helium II as measured in a rotation viscometer (Woods and Hollis Hallett [50]). The full points show the earlier results of Heikkila and Hollis Hallett [51]).

Kapitza (1938)

viscosity vanishes below T_c

capillary flow viscometer

Hollis-Hallett (1955)

roton minimum, phonon rise

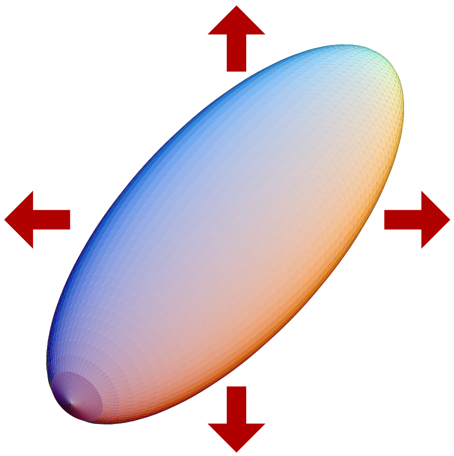
rotation viscometer

$$\eta/s \simeq 0.8\, \hbar/k_B$$

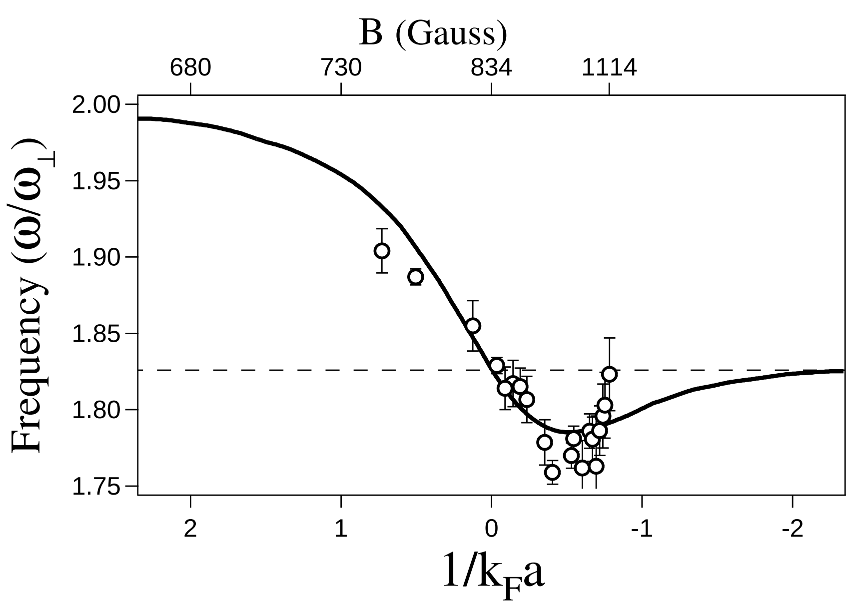
II. Collective Modes (Fermions)

Radial breathing mode

Ideal fluid hydrodynamics ($P \sim n^{5/3}$)



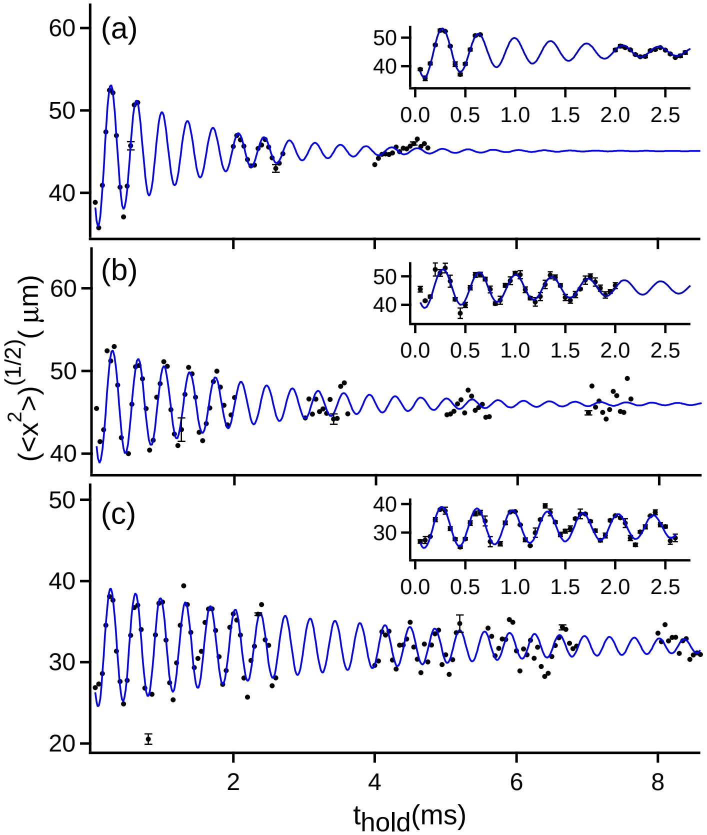
$$\begin{aligned} \frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) &= 0 \\ \frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right) \vec{v} &= -\frac{\vec{\nabla} P}{mn} - \frac{\vec{\nabla} V}{m} \end{aligned}$$

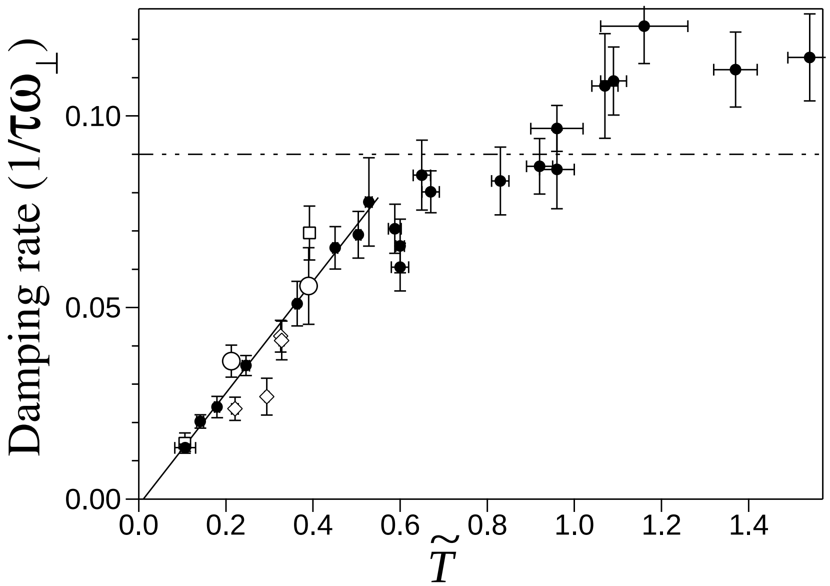


$$\omega = \sqrt{10/3}\,\omega_\perp$$

Kinast et al. (2005)

Damping of Collective Excitations





$$T/T_F = (0.5, 0.33, 0.17)$$

$\tau\omega$: decay time $\times$ trap frequency

Kinast et al. (2005)

Viscous Hydrodynamics

Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

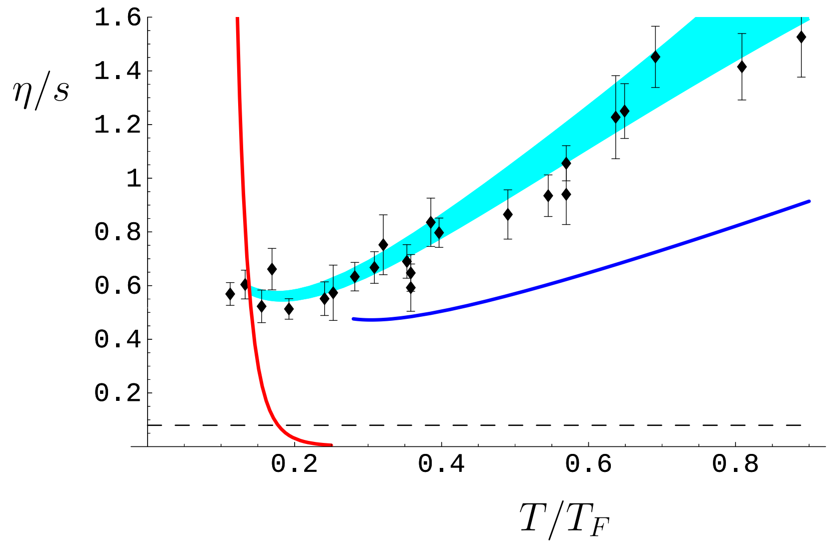
$$\dot{E} = -\frac{\eta}{2}\int d^3x\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2$$
$$-\zeta\int d^3x\,(\partial_i v_i)^2 - \frac{\kappa}{T}\int d^3x\,(\partial_i T)^2$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

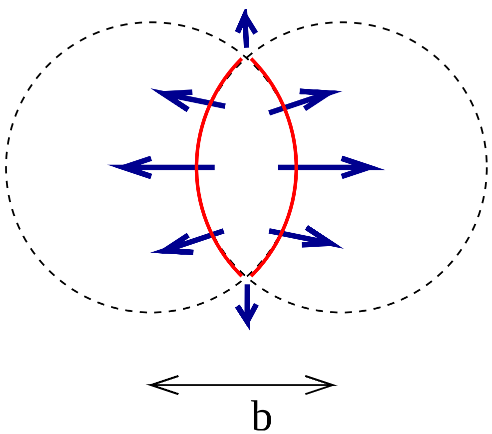
$$\frac{\eta}{s} = \frac{3}{4}\xi^{\frac{1}{2}}(3N)^{\frac{1}{3}}\frac{\Gamma}{\omega_\perp}\,\frac{\bar{\omega}}{\omega_\perp}\,\frac{N}{S}$$

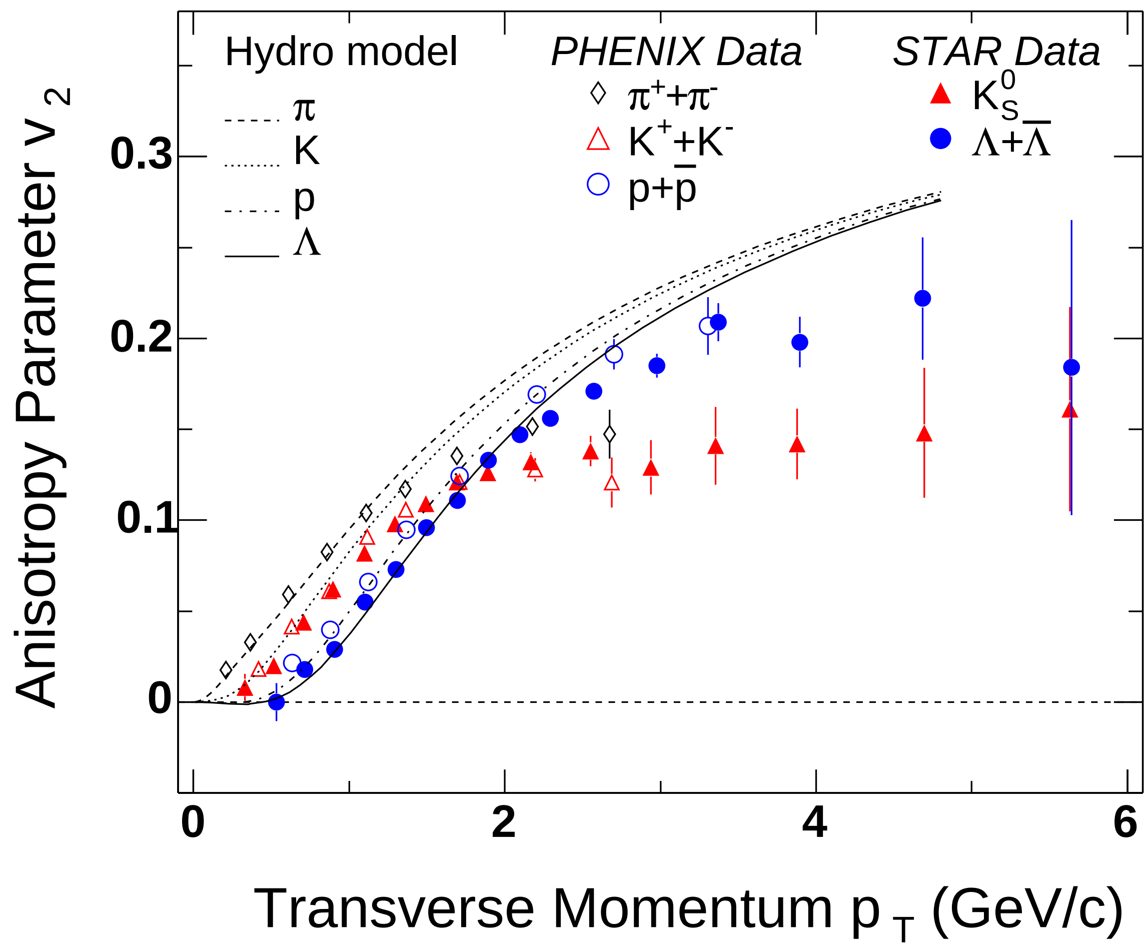
Schaefer (2007), see also Bruun, Smith



III. Elliptic Flow (QGP)

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy





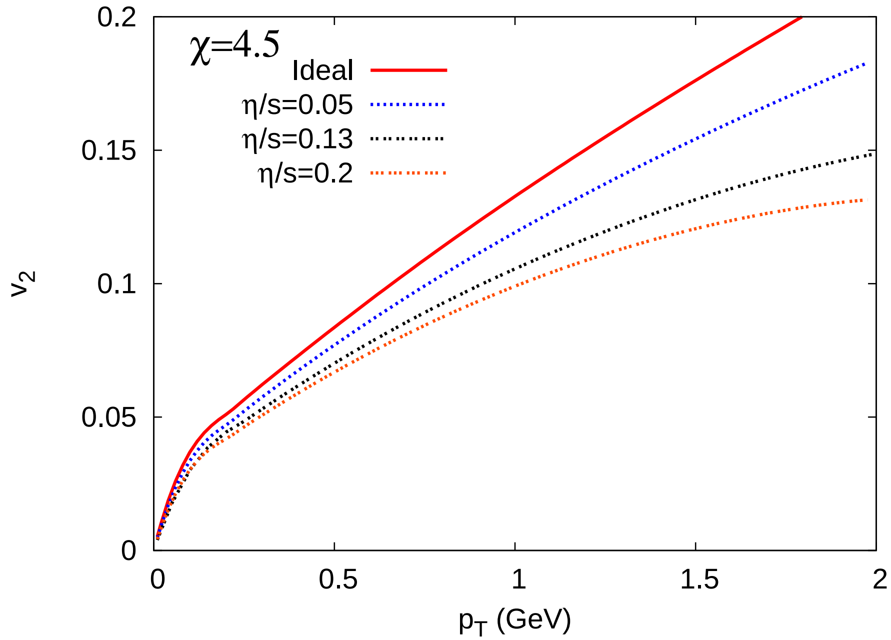
source: U. Heinz (2005)

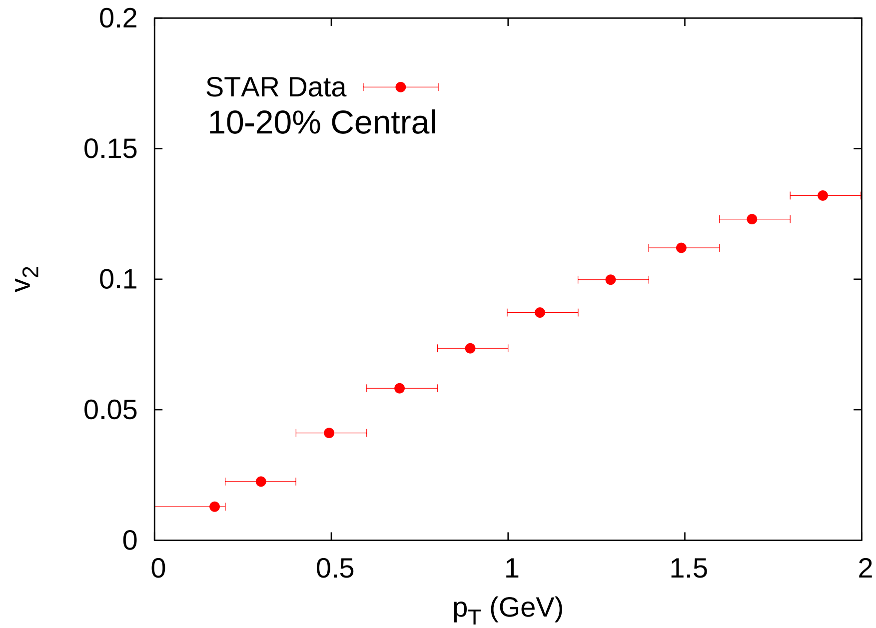
Viscosity and Elliptic Flow

Consistency condition $T_{\mu\nu} \gg \delta T_{\mu\nu}$ (applicability of Navier-Stokes) Very restrictive for $\tau < 1$ fm

$$\frac{\eta + \frac{4}{3}\zeta}{s} \ll \frac{3}{4}(\tau T)$$

Danielewicz, Gyulassy (1985)





Dusling, Teaney (2008), Romatschke (2007), Teaney (2003)

Many questions: Dependence on initial conditions, freeze out, etc.

Outlook

Too early to declare a winner.

$$\eta/s \simeq 0.8 \text{ (He)}, \qquad \eta/s \leq 0.5 \text{ (CA)}, \qquad \eta/s \leq 0.5 \text{ (QGP)}$$

Other experimental constraints (irrot flow ..), more analysis needed.

Kinetic theory: o.k. in He (all T), o.k. close to T_c in CA, QGP?

New theory tools: AdS/Cold Atom correspondence? Field theory approaches in cross over regime (large N, epsilon expansions, . . .)