Recent Progress in QC$_2$D at High Baryon Density

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- BEC vs. BCS
- Equation of State
- Evidence for deconfinement?

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QC$_2$D – the large N_c^{-1} limit

QCD with gauge group SU(2) and non-zero quark chemical potential μ has a real functional measure; it is the simplest dense matter system with long-range interactions amenable to study with standard LGT methods.

Since q and $\bar{q}$ live in equivalent reps. of the color group, chiral multiplets contain both $q\bar{q}$ mesons and qq baryons. For $m_\pi \ll m_\rho$ the behaviour as μ is varied can be studied using chiral perturbation theory (χPT)

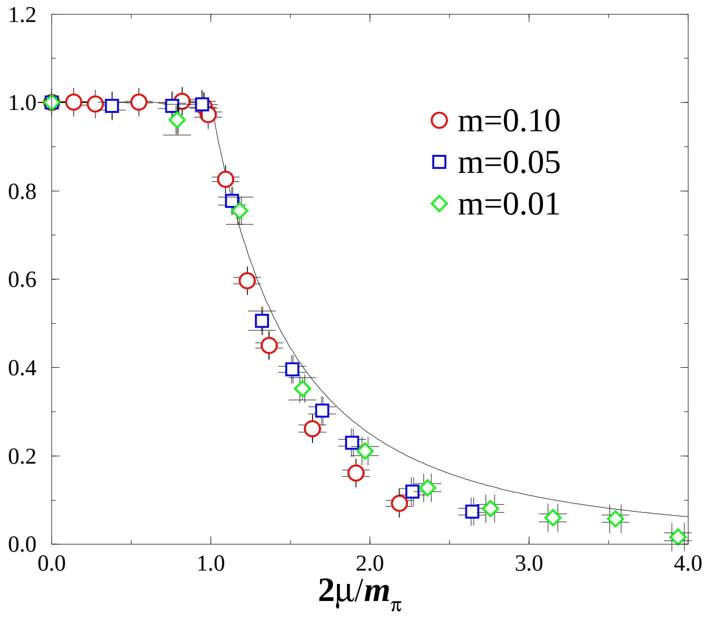
Key result: for $\mu \geq \mu_o = \frac{1}{2} m_\pi$ a baryon charge density develops, $n_q > 0$, along with a gauge invariant superfluid condensate $\langle qq \rangle \neq 0$. For $\mu \gtrsim \mu_o$, the system is a dilute Bose Einstein Condensate (BEC) consisting of weakly interacting scalar qq baryons.

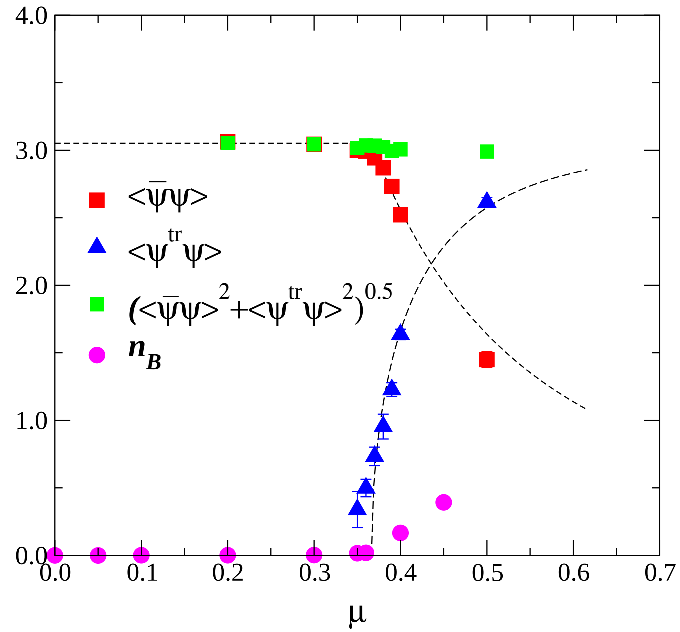
Quantitatively, for $\mu \gtrsim \mu_o$ χPT predicts

$$\frac{\langle\bar\psi\psi\rangle}{\langle\bar\psi\psi\rangle_0} = \left(\frac{\mu_o}{\mu}\right)^2; \quad n_q = 8N_f f_\pi^2 \mu \left(1 - \frac{\mu_o^4}{\mu^4}\right); \quad \frac{\langle qq\rangle}{\langle\bar\psi\psi\rangle_0} = \sqrt{1 - \left(\frac{\mu_o}{\mu}\right)^4}$$

[Kogut, Stephanov, Toublan, Verbaarschot & Zhitnitsky, Nucl.Phys.B582(2000)477]

confirmed by QC$_2$D simulations with staggered fermions





[SJH, I. Montvay, S.E. Morrison, M. Oevers, L. Scorzato J.I. Skullerud, Eur.Phys.J.C17(2000)285, *ibid* C22(2001)451]

Thermodynamics at $T = 0$ from χPT

quark number density $n_{\chi PT} = 8N_f f_\pi^2 \mu \left(1 - \frac{\mu_o^4}{\mu^4}\right)$ [KSTVZ]

pressure $p_{\chi PT} = -\frac{\Omega}{V} = \int_{\mu_o}^{\mu} n_q d\mu = 4N_f f_\pi^2 \left(\mu^2 + \frac{\mu_o^4}{\mu^2} - 2\mu_o^2\right)$

energy density $\varepsilon_{\chi PT} = -p + \mu n_q = 4N_f f_\pi^2 \left(\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 2\mu_o^2\right)$

conformal anomaly

$$(T_{\mu\mu})_{\chi PT} = \varepsilon - 3p = 8N_f f_\pi^2 \left(-\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 4\mu_o^2\right)$$

NB $(T_{\mu\mu})_{\chi PT} < 0$ for $\mu > \sqrt{3}\mu_o$

speed of sound $v_{\chi PT} = \sqrt{\frac{\partial p}{\partial \varepsilon}} = \left(\frac{1 - \frac{\mu_o^4}{\mu^4}}{1 + 3\frac{\mu_o^4}{\mu^4}}\right)^{\frac{1}{2}}$

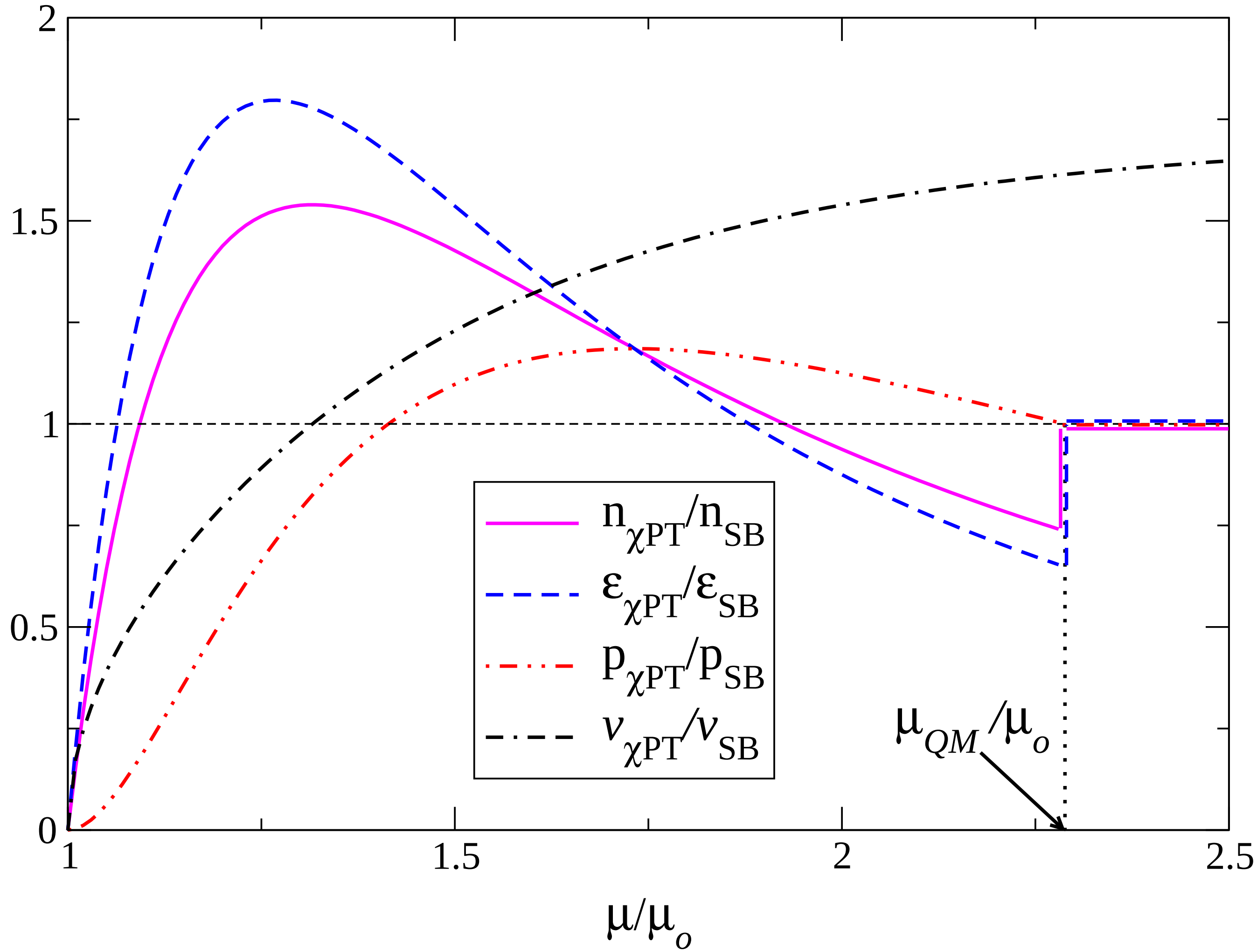
This is to be contrasted with another paradigm for cold dense matter, namely a degenerate system of weakly interacting quarks populating a Fermi sphere up to some maximum momentum $k_F \approx E_F = \mu$

$$\Rightarrow \quad n_{SB} = \frac{N_f N_c}{3\pi^2}\mu^3; \quad \varepsilon_{SB} = 3p_{SB} = \frac{N_f N_c}{4\pi^2}\mu^4;$$

$$(T_{\mu\mu})_{SB} = 0; \quad v_{SB} = \frac{1}{\sqrt{3}}$$

Superfluidity arises from condensation of diquark Cooper pairs from within a layer of thickness Δ centred on the Fermi surface:

$$\Rightarrow \quad \langle qq \rangle \propto \Delta\mu^2$$



By equating free energies, we naively predict a first order deconfining transition from BEC to quark matter;
eg. for $f_\pi^2 = N_c/6\pi^2, \qquad \mu_{QM} \approx 2.3\mu_o$.

Simulation Details ($N_f = 2$ Wilson flavors)

Initial runs used a $8^3 \times 16$ lattice with parameters
$\beta = 1.7$, $\kappa = 0.1780$ (Wilson gauge action)

$\Rightarrow a = 0.220$ fm, $m_\pi a = 0.79(1)$, $m_\pi / m_\rho = 0.779(4)$

Now have data from a matched $12^3 \times 24$ lattice with
$\beta = 1.9$, $\kappa = 0.1680$

$\Rightarrow a = 0.15$ fm, $m_\pi a = 0.68(1)$, $m_\pi / m_\rho = 0.80(1)$

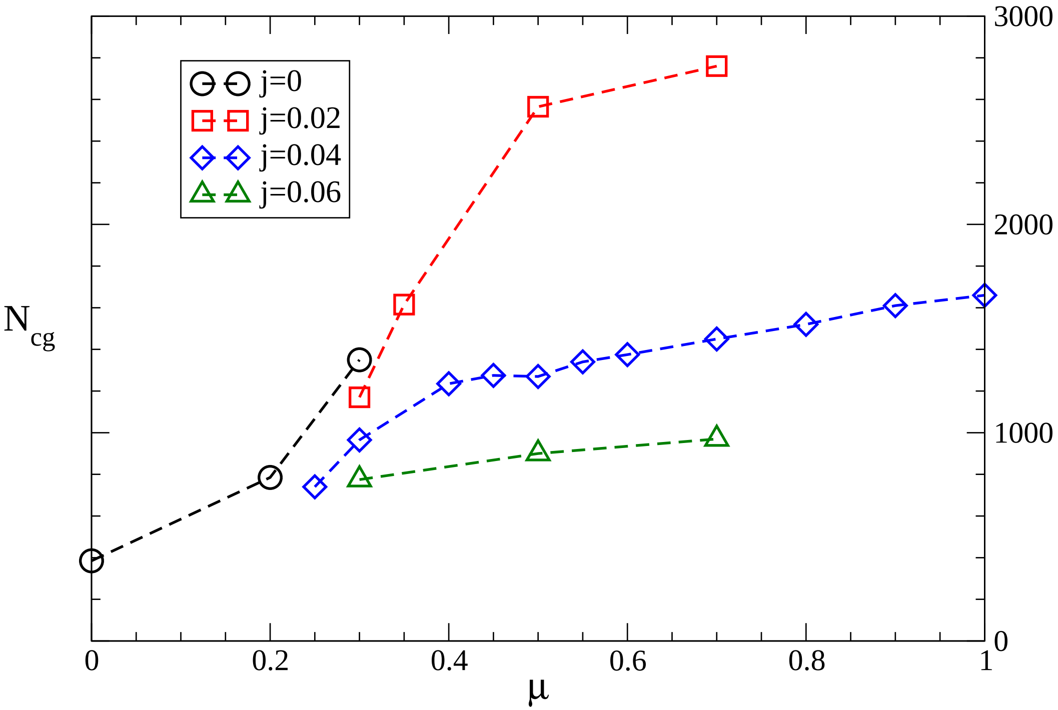
$\Rightarrow T \approx 60$MeV in both cases

To counter IR fluctuations and to maintain ergodicity, we introduce a diquark source $j\kappa(-\bar\psi_1 C\gamma_5\tau_2\bar\psi_2^{tr} + \psi_2^{tr} C\gamma_5\tau_2\psi_1)$

So far have accumulated roughly 300 trajectories of mean length 0.5 on $8^3 \times 16$ and 500 trajectories on $12^3 \times 24$

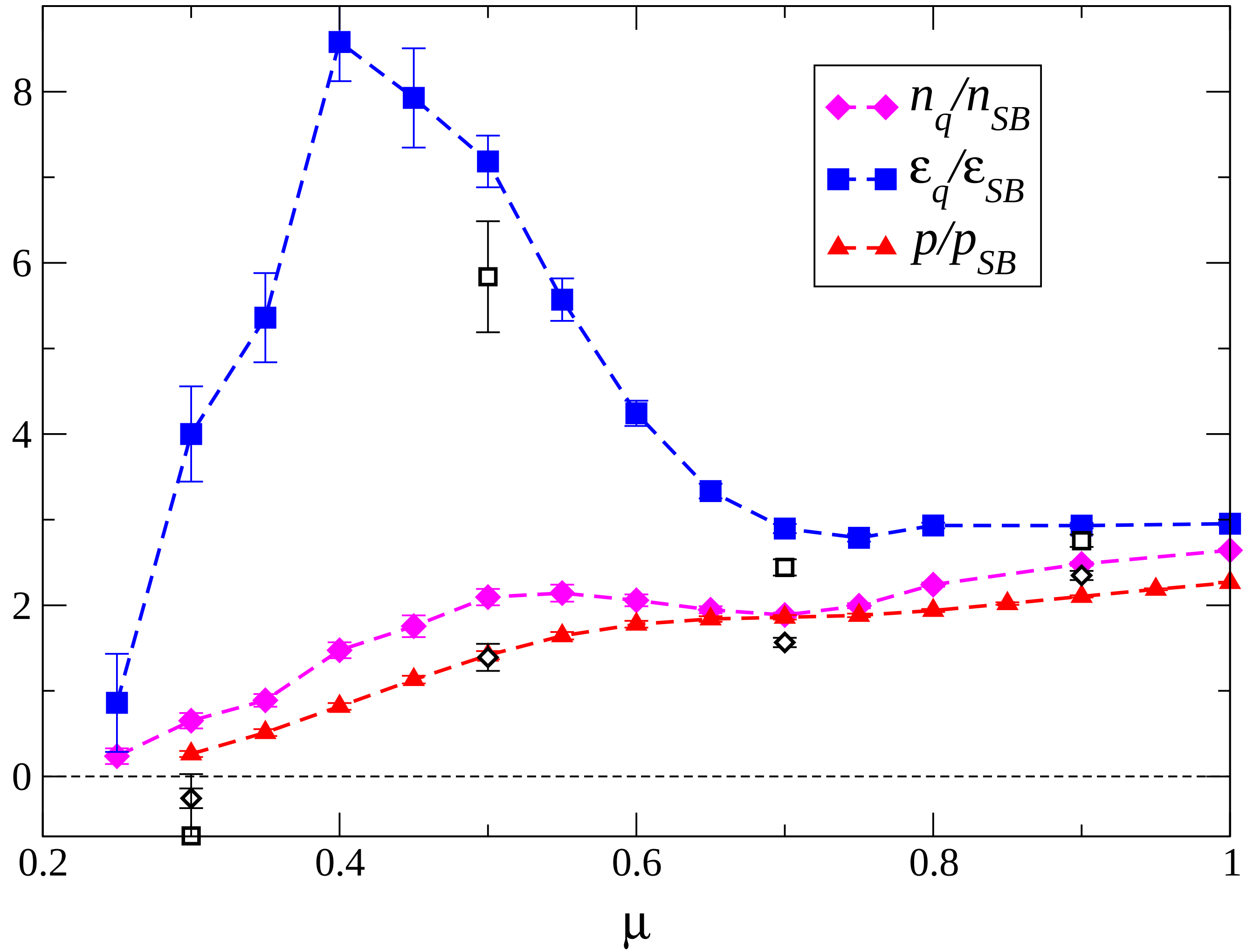
SJH, Kim, Skullerud, Eur.Phys. J. C48 (2006)193

Computer effort



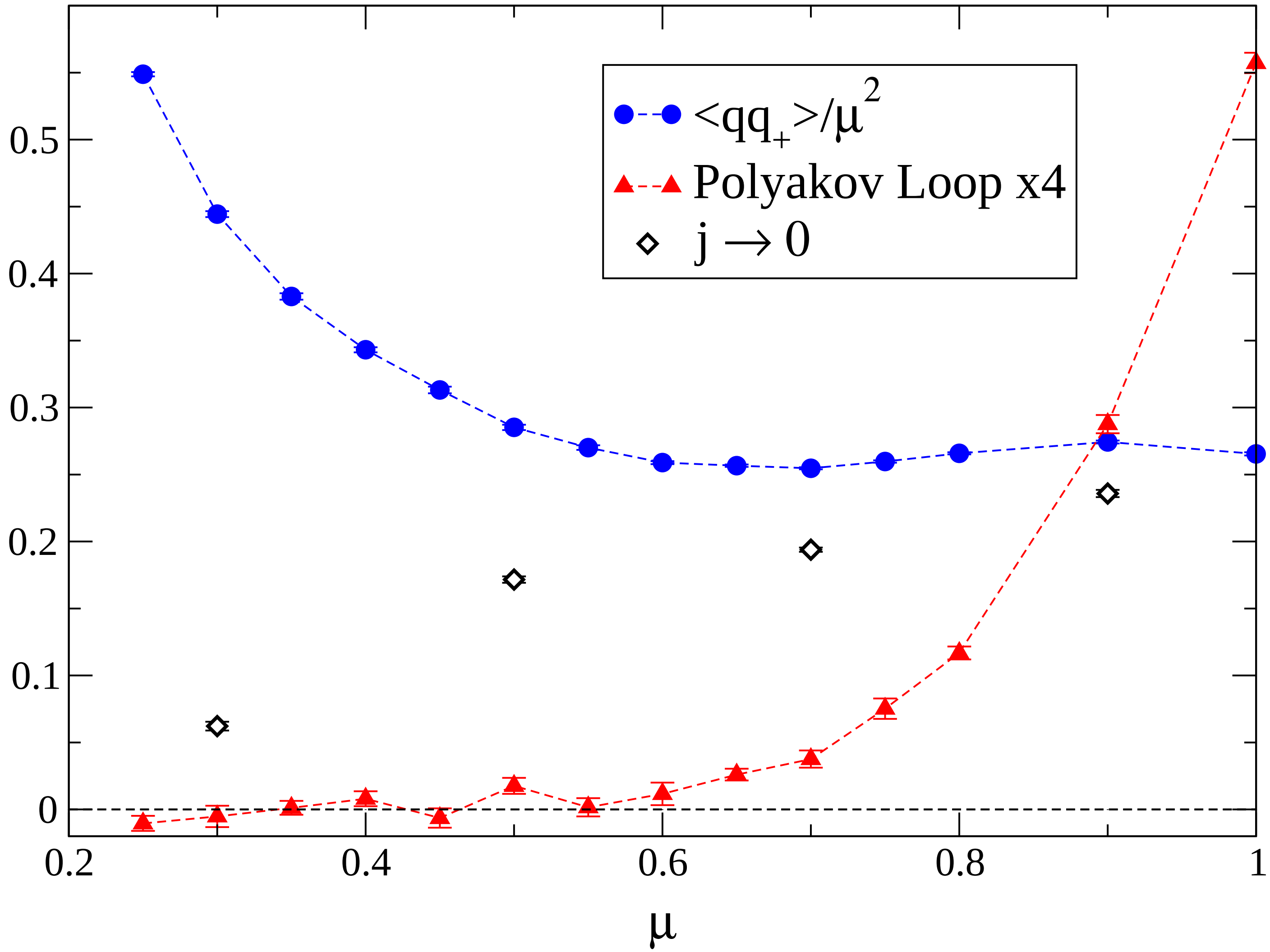
The number of `congrad` iterations required for convergence during HMC guidance rises as μ increases $\Leftrightarrow$ accumulation of small eigenvalues of M.

Equation of State ($8^3 \times 16$)



Open symbols denote $j \to 0$ extrapolation

Evidence for Deconfinement at $\mu a \simeq 0.65$



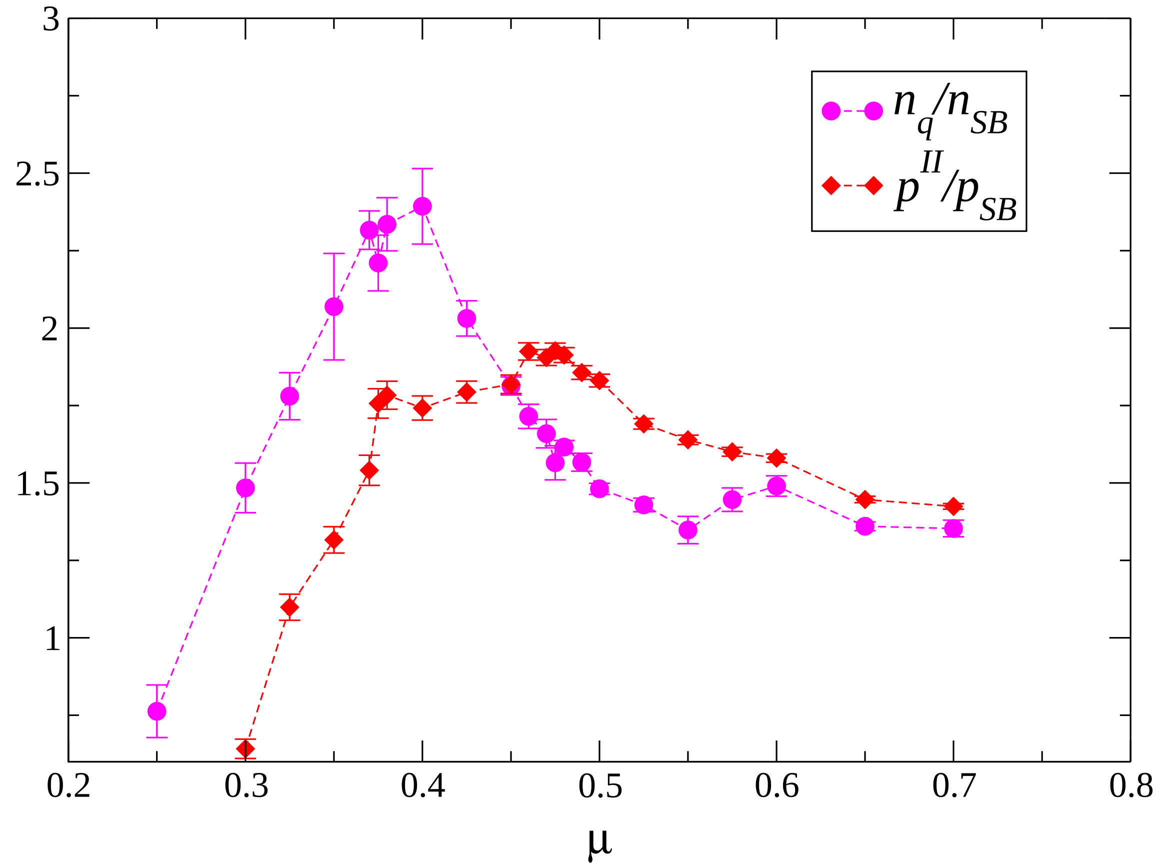
We conclude there is a transition from confined bosonic "nuclear matter" to degenerate fermionic "quark matter" at $\mu_{QM}a \approx 0.65$. Both phases are superfluid, but for $\mu > \mu_{QM}$ the scaling is that expected of a degenerate system.

In condensed matter parlance we are observing a BEC/BCS crossover.

What is the nature of the transition between these two régimes?

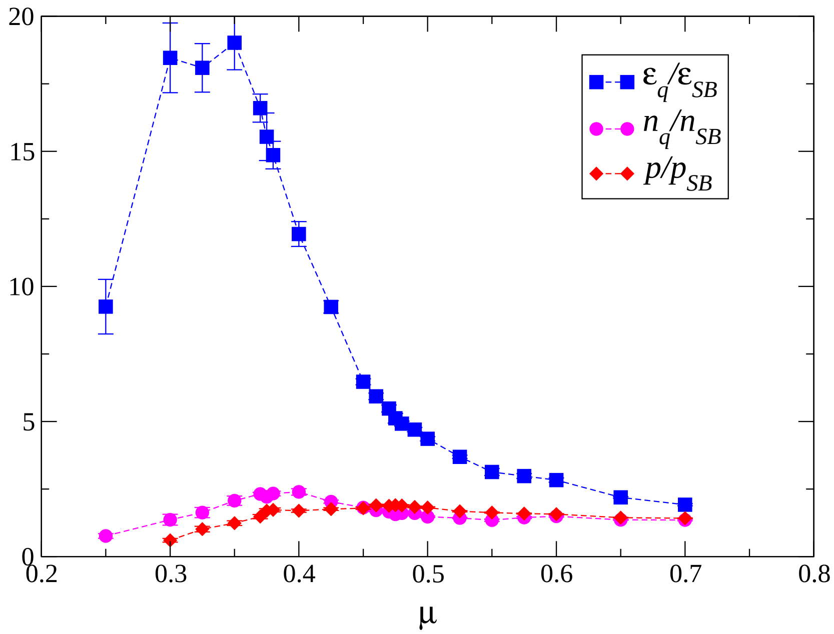
Does it coincide with deconfinement?

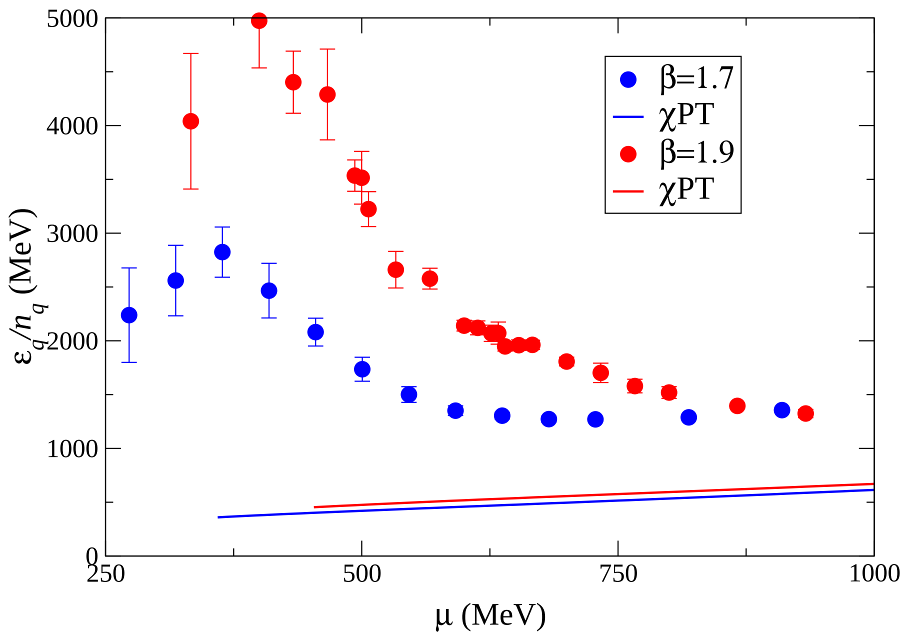
Towards the continuum limit on $12^3 \times 24$...



Identify onset transition at $\mu_o a \approx 0.35$ and a transition/crossover to quark matter at $\mu_{QM} a \approx 0.5$ i.e. with $\mu_q \approx 670\,$MeV, $n_q \approx 5\,\text{fm}^{-3}$

Quark Energy Density



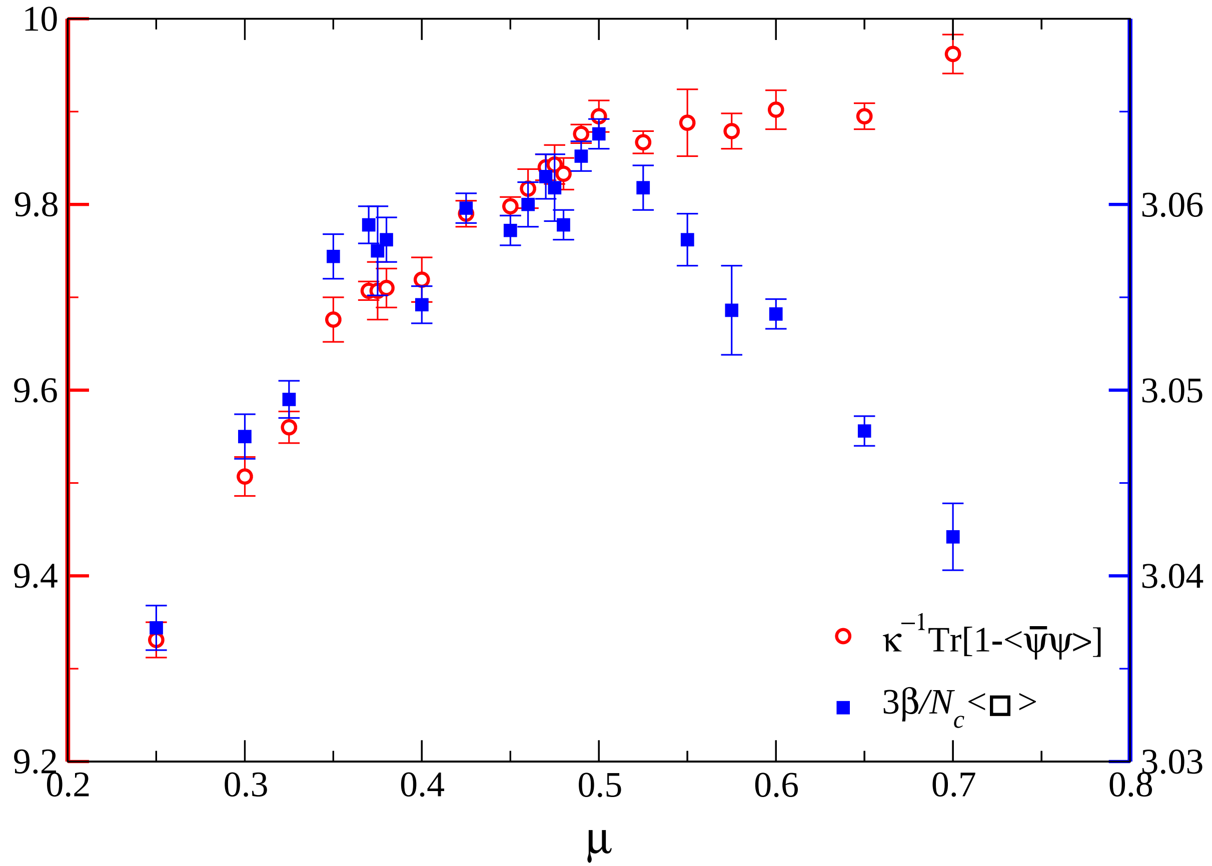


In contrast to χPT prediction, (unrenormalised) quark energy density ε_q greatly exceeds SB value as $\mu \searrow \mu_{o+}$

$\Rightarrow$

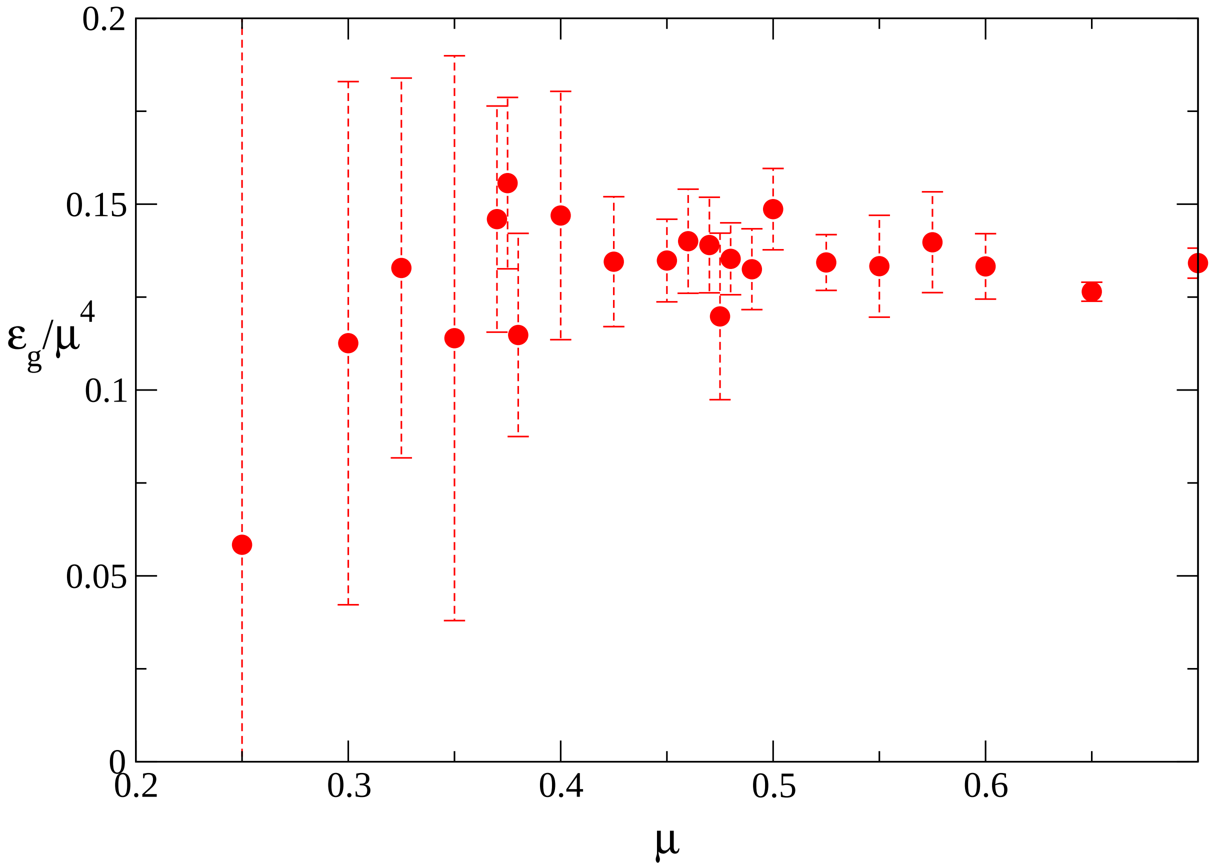
Energy per quark ε_q/n_q has shallow minimum for $\mu \gtrsim \mu_{QM}$

Conformal Anomaly $T_{\mu\mu}$



Quark and gluon contributions to $T_{\mu\mu}$ are qualitatively very similar below the transition at μ_{QM}, but diverge above

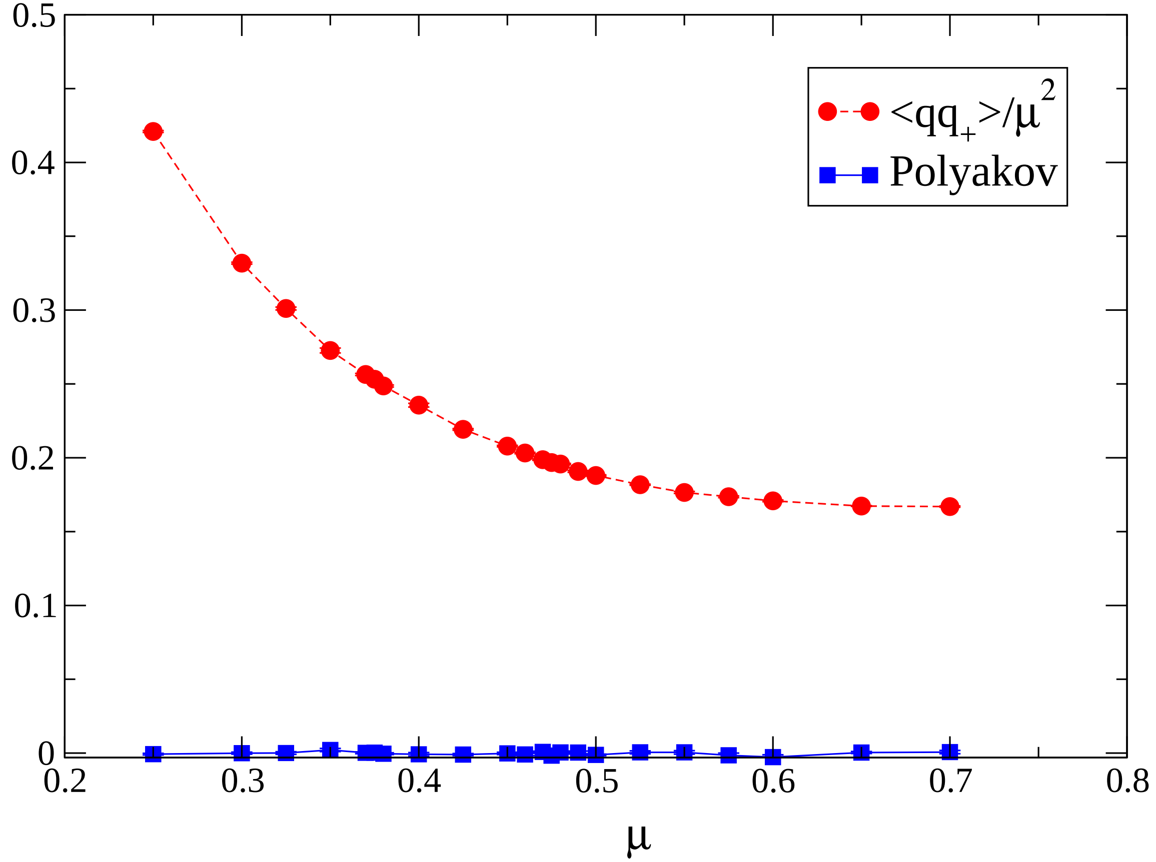
Gluon Energy Density



Gluon energy density $\propto \langle \Box_t - \Box_s \rangle$ scales according to dimensional analysis: ε_g/μ^4 constant over whole range of μ

No sign of singular behaviour at $\mu = \mu_{QM}$

Order Parameters

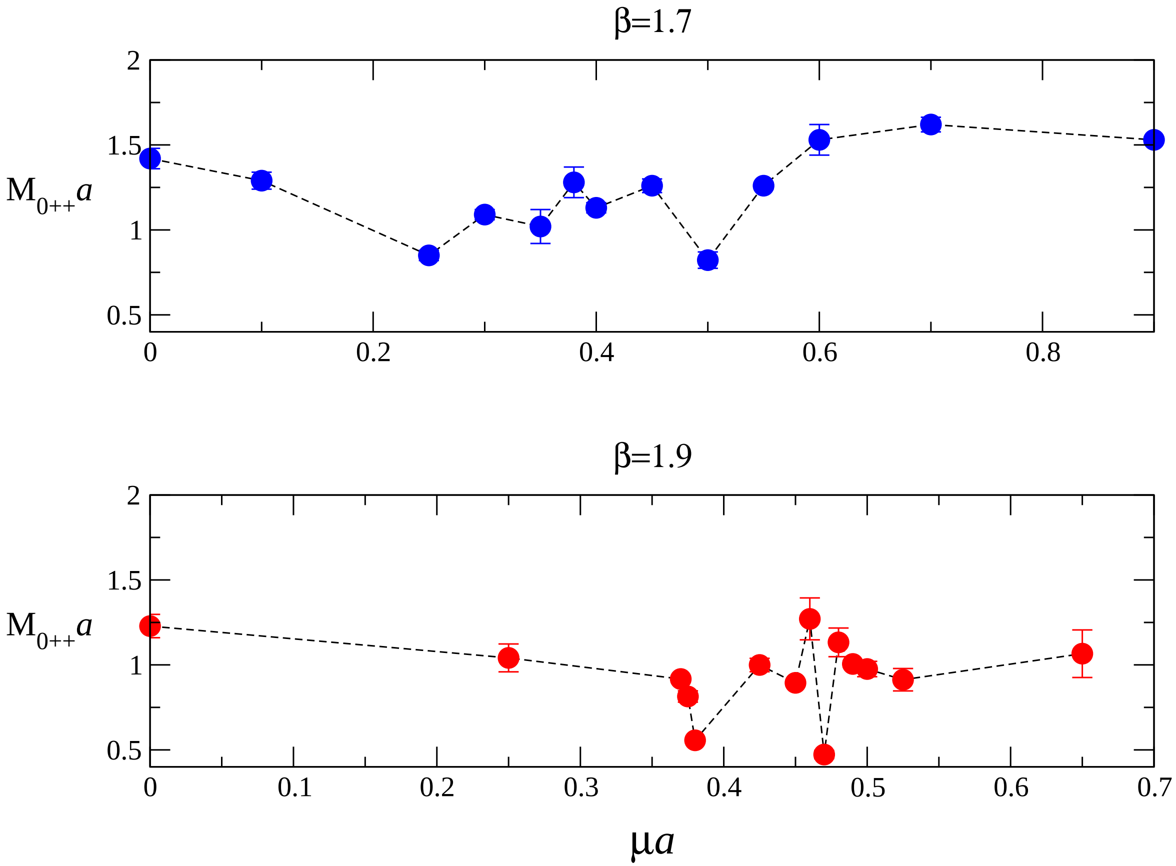


Superfluid condensate approaches BCS scaling as μ increases

Polyakov loop ≈ 0 throughout – no deconfinement!

Due to poor signal:noise? Or has it disappeared?

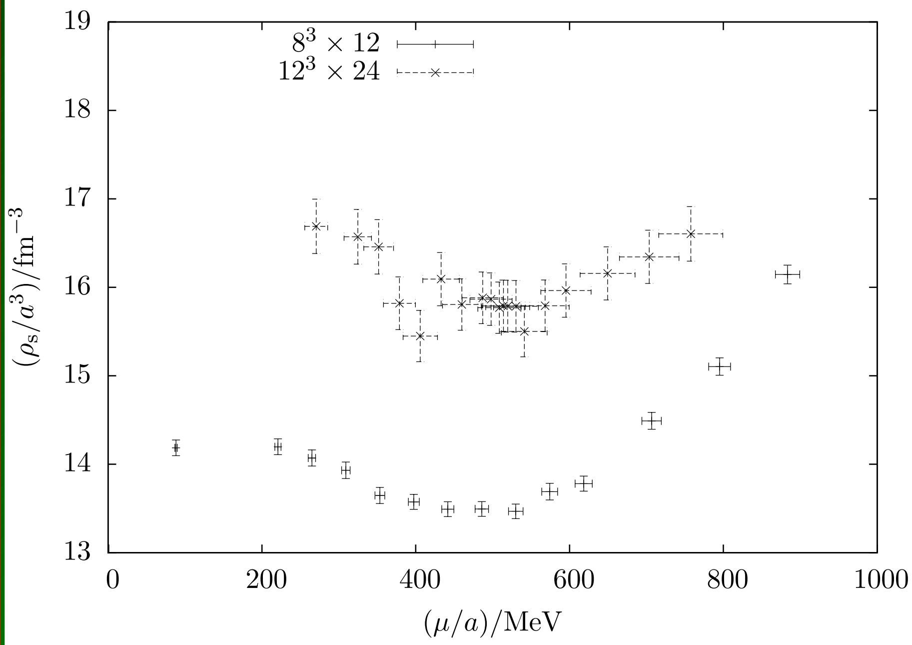
Glueball Mass (very preliminary)

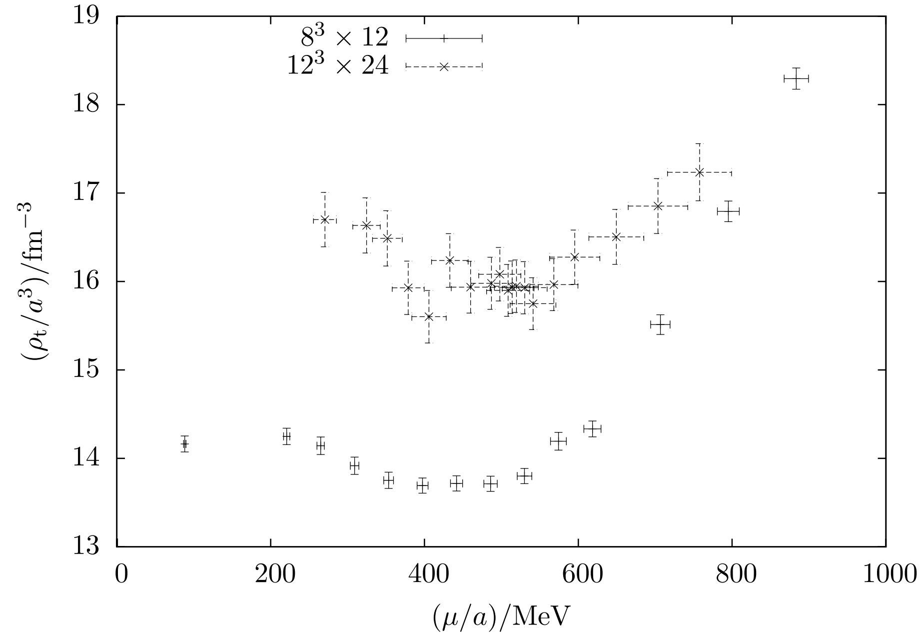


Scalar glueball decreases in mass in vicinity of both onset and deconfining transitions. We don't find any extended regions with $M_{0^{++}} \approx 0$.

[Cf. Lombardo, Paciello, Petrarca & Taglienti, `arXiv:0804.4863`]

Magnetic Monopoles in MAG Gauge

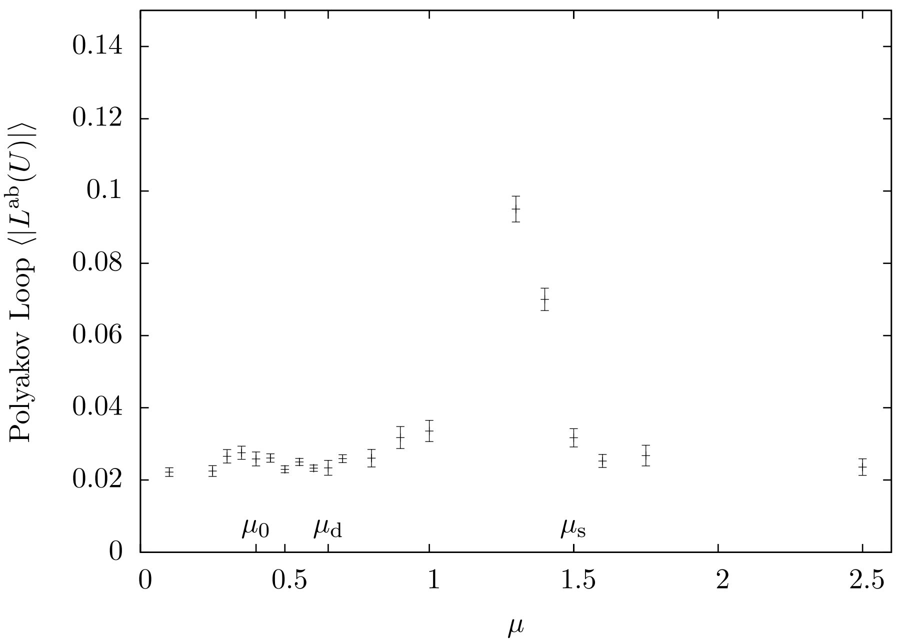


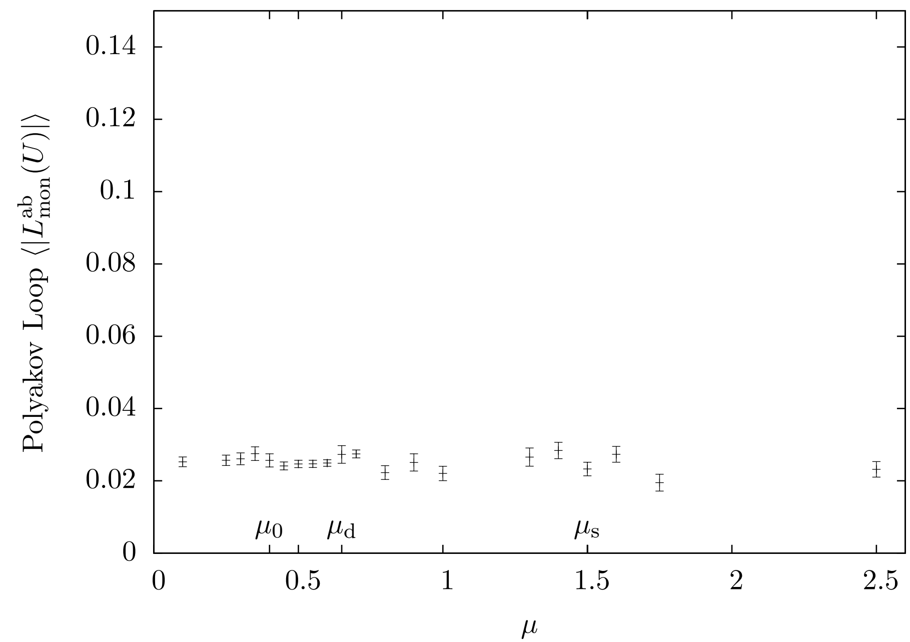


We have studied the distribution of magnetic monopoles found using the DeGrand-Toussaint procedure following abelian projection from the MAG gauge

The monopole densities ρ_s and ρ_t are approximately equal, and show structure both at $\mu \approx \mu_o$ and $\mu \approx \mu_{QM}$

The Polyakov Loop is Non-Abelian?





After abelian projection we can decompose the abelian link fields as $\theta_\mu = \theta_\mu^{\mathrm{mon}} + \theta_\mu^{\mathrm{phot}}$ with $\theta_{\mu x}^{\mathrm{mon}} \equiv -2\pi \sum_y (\partial^+ \partial^-)^{-1}_{xy} m_{\mu y}$

The abelian Polyakov loop on $8^3 \times 16$ is about 25% of the full loop, and remarkably, the signal completely vanishes in the $\{\theta^{\mathrm{mon}}\}$ configuration, showing that “deconfinement” is NOT due to abelian monopoles in this case

Summary & Outlook

- Thermodynamic observables support a bosonic superfluid at intermediate $\mu_o \lesssim \mu \lesssim \mu_{QM}$, and a degenerate system of quarks disrupted by Cooper pairing at large $\mu \gtrsim \mu_{QM}$
- Hint that bulk quark matter may be more stable energetically than predicted by χPT
- The glueball sector looks interesting but requires huge statistics
- Deconfinement in dense quark matter, if it occurs, does not resemble deconfinement at high temperature. No jump in ε_g, no rôle for abelian monopoles.

Could deconfinement be a lattice artifact?