# Graded-index shape optimization for delay spread reduction in optical fiber: A study case

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***Abstract*—In this work, different refractive index variation functions are studied in order to determine the behaviour that allows reducing the propagation delay for multi-mode optical fibre. The variation of the exponent of classical variation functions is studied. Additionally, variation functions are proposed that allow reducing the maximum propagation differences for the specific case of this study. Additionally, optimization of polynomial parameters is performed through computational optimization tools. From the study carried out, it is concluded the possibility of reducing the propagation delay by varying an intrinsic parameter of the optical fibre as it corresponds to its graded - index.**

***Index Terms*—Optical-fiber, graded-index, refractive index, propagation delay**

## I. Introduction

The increasing demand in data transmission rates associated in a large percentage to the increase of Internet usage has promoted the use of optical fibre as a replacement for legacy technology as coaxial cables or UTP because it advantages in terms of low attenuation, great potential for high-speed data transmission and immunity to electromagnetic interference.

In this context, Multimode fibre (MMF) has emerged as a preferential choice for local area networks, mainly due to its advantages in term of fabrication, and installation, with helps in reducing the total system operation cost [1].

In this article, we developed a study of propagation delay in multimode fibres that use graded-index (*GRIN*) in their construction. This study is made in order to reduce the propagation delay associated with the difference between the refractive index at the core of the fibre and its cladding.

The propagation delay corresponds to the difference in the time that two rays, one travelling directly across the axis (or centre) of the fibre and the other travelling in a smooth curve because of the initial inclination of the light beam, reach the same coordinate across the longitudinal axis of the fibre.

It should be noted that in this work a constant refraction index is considered for different wavelengths. An analysis of pulse broadening considering the wavelength dependent variations is presented in [2].

This article is organized as follows. In section **I**, a brief introduction to the context and problem to work was made. Next, section **II** presents the base analysis developed for a specific optical fiber. Subsequently, section **III** presents the methodology to be followed for the work of the problem previously proposed. Section **IV** presents the main results derived from the development of the methodology. Finally, section **V** presents the main conclusions of this work.

## II. Baseline Analysis

On the first place, in order to compare the performance of the used methodology with the existing scenario, it is performed a baseline analysis, using the previously used structure for the refractive index of a Graded-Index Optical-Fiber (*GRIN*).

The variation of the refractive index used in the analysis is based on [1] using an $\alpha$ value of two, i.e. A quadratic behaviour. The variation of the refractive index is presented in equation (1)

$$n(r) = \begin{cases} n_1[1 - a\Delta(|r|/a)^{\alpha}]^{1/2} & for \quad |r| < a \\ \\ n_1[1-2\Delta]^{1/2} & for \quad |r| > a \end{cases} \tag{1}$$

Where:

- $r$: Represents the distance from the core at which $n$ is being evaluated.
- $a$: Corresponds to the optical fiber width.
- $n_1$ and $n_2$ Indicates the core and cladding refractive index, respectively.
- $\Delta$ Represents the refractive index contrast between the fiber's core and cladding given by $\Delta = \frac{1}{2}[\frac{n_1^2 - n_2^2}{n_1^2}]$

It has to be remarked that in opposite to [1] it is considered that the refractive index is independent to the wavelength, in order to study only the optimization of GRIN profile in a general way and no with the dependence of the wavelength. The variation of refractive index used for the base case is presented in 1.

Several incidence angles are tested in order to determine the critical *angle*, i.e. The maximum angle that assures that the light rays will be confined in the optical fibre. The critical angle of this configuration is 14.5 degrees, as shown in figure 2.

Using the previously determined critical angle the delay spread is calculated for several points. As shown in figure 3 the delay spread is increased as a function of the distance travelled by the light beam with fluctuations around the drawn line. This fluctuation's amplitude is shown in figure 4. The objective of this work is related to the reduction of this fluctuation.

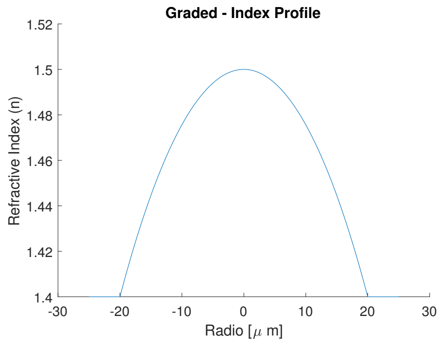


Fig. 1. Refractive index for the base case.

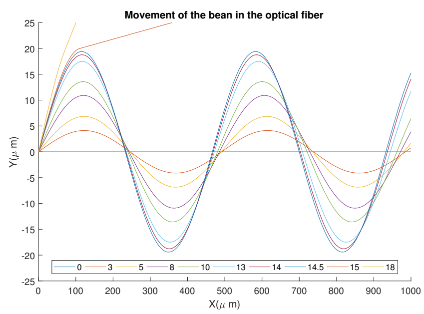


Fig. 2. Critical angle determination for base case.

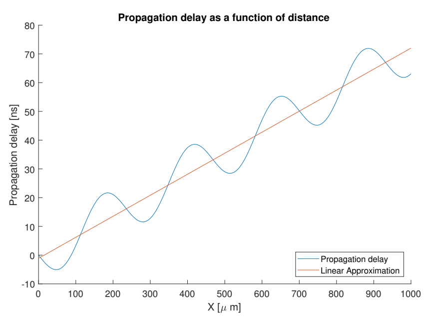


Fig. 3. Delay spread of the original variation of $n$

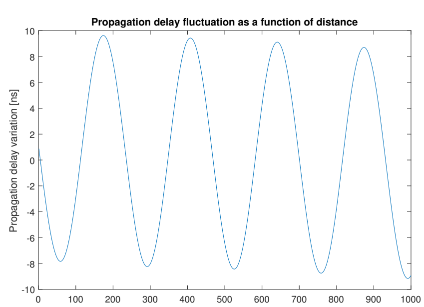


Fig. 4. Variation of delay spread in the original variation.

## III. Methodology

In this section, we present the methodology used for the development of the study of refractive index variations in the order in which the tests were performed.

As a comparison method of the different functions to be tested, both the propagation delay as a function of distance plot and the area enclosed by this curve are used, considering a maximum distance of 1000 micrometres for both criteria.

### A. Variation of $\alpha$ parameter.

First, a variation of the $\alpha$ exponent in equation (1) is performed, in order to analyze the effect that different types of decay have on the propagation delay.

First, a variation of the exponent is performed, in order to analyze the effect that different types of decay have on the propagation delay. In the study made we tested values of two, three, four, and five that correspond to polynomials of grade two, three, four, and five, respectively. The shape of the different variations of the refractive index are presented in figure 5

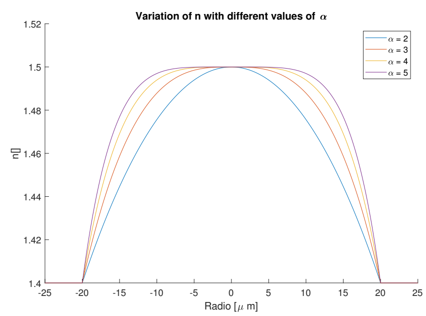


Fig. 5. Refractive index variation with different values of $\alpha$

### B. Custom functions.

Then we tested two custom variations of n defined by polynomials of grade three and four defined by the function *polyfit* of Matlab and a set of points chosen by the study of the propagation delay as a function of the distance from the centre of the optical fibre named $y$. This analysis will be presented in the next section “Results”.

### C. Optimization of parameters.

Finally, we make optimization of the parameters of the best polynomial found in the study of the custom defined functions. This optimization is made using the function *fminsearch* of Matlab using as the initial guess the parameters used in the previous sections. The cost function corresponds to the area of variation of enclosed propagation delay and The distance for the determination of the enclosed area is 300 $\mu m$.

## IV. Results

### A. Study of $\alpha$ variations.

From the variation of the $\alpha$ parameter, we determined the propagation delays as shown in figure 6. We also calculate

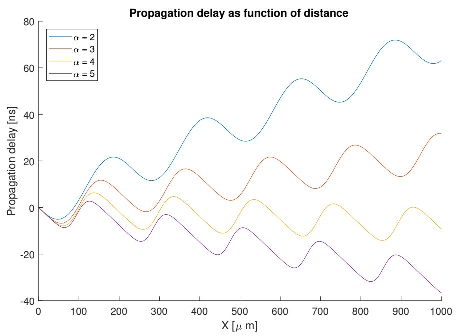


Fig. 6. Propagation delay variation with different values of $\alpha$

TABLE I
AREA OF PROPAGATION DELAY AS FUNCTION OF $\alpha$

| | $\alpha = 2$ | $\alpha = 3$ | $\alpha = 4$ | $\alpha = 5$ |
|---|---|---|---|---|
| **Area of propagation delay [ms * $\mu$ m]** | 0.0056 | 0.0051 | 0.0044 | 0.0040 |

the absolute area enclosed by the curve, these values are summarized in table I.

As it is observed from figure 4 and table 1, there does not exist a notorious difference between the studied values of the parameter alpha, graphically it is observed that the best performance is obtained with a grade four polynomial. However, the minimum area is provided by a grade five decay.

### *B. Study of custom functions.*

In order to study the distance of points to the core in which the propagation delay is maximum, a graph of distance to the nucleus in relation to the variation of the propagation delay is made. From this study, as shown in Figure 7, the minimum and maximum delay points are observed, with the maximum delay being at 13.7186 $\mu m$.

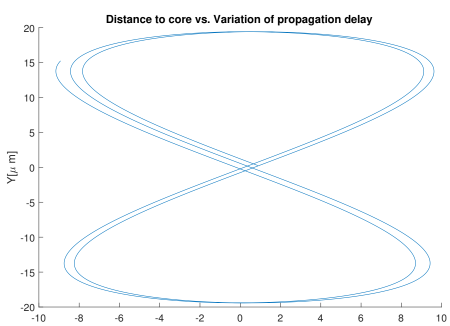


Fig. 7. Distance to core as function of propagation delay variation

The graph presented in figure 7 is used in order to propose variations of the refractive index that allow to decrease the propagation delay inside the optic fiber. To reduce the maximum delay variation, we propose a shape of refractive index that reduces the speed of the light beam at this point of maximum delay. In this way a *polyfit* is made using the coordinates presented in the table II. Due to the great increase in speed given by the large decrease of $n$, the experiments carried out show new maximums in delay spread, so we propose the use of 5 coefficients for a polynomial of grade 4, presented in the same table II.

TABLE II
COORDINATES USED FOR POLYNOMIAL DEFINITION

| | Coordinates 0 | Coordinates 1 | Coordinates 2 | Coordinates 3 | Coordinates 4 |
|---|---|---|---|---|---|
| **Grade 3** | - | $[0, n_{Core}]$ | $[\frac{13.7186}{2}, \frac{n_{Core}+n_{Clad}}{2}]$ | $[13.7186, n_{Core}]$ | $[a, n_{Clad}]$ |
| **Grade 4** | $[0, n_{Core}]$ | $[\frac{13.7186}{2}, \frac{n_{Core}+n_{Clad}}{2}]$ | $[10, \frac{2*(n_{Core}-n_{Clad}}{3} + n_{Clad}]$ | $[13.7186, n_{Core}]$ | $[a, n_{Clad}]$ |

The variations of refractive index given by the fitting of the given coordinates of table II are presented in figure 8 compared to the base function of $\alpha = 2$.

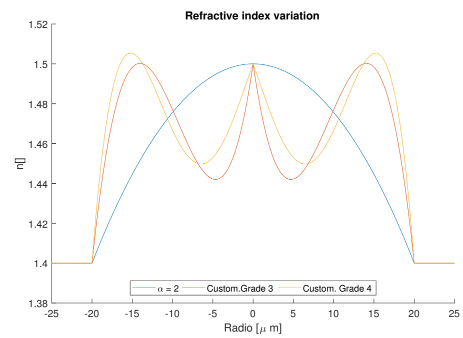


Fig. 8. Refractive index variation.

Given these distributions of the refractive index, the propagation delay is calculated for the conditions previously defined. The figure 9 shows the propagation delay for the polynomials defined in figure 8. From fig.9, is observed a decrease in the propagation delay, especially for the case of the polynomial of degree 4.

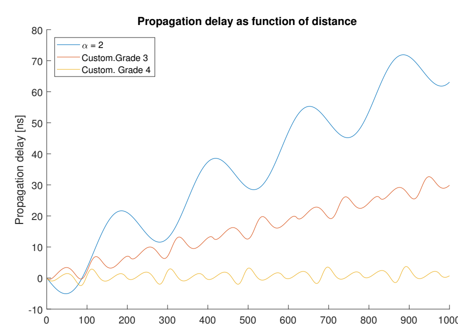


Fig. 9. Propagation delay with custom functions

To study differences on the variations of the propagation delay we follow the same procedure used on section **II** in order to plot the oscillatory component of the delay spread which is presented in figure 10. From figure 10 it can be noticed that the custom polynomial of grade 4 has a smaller amplitude when compared to the baseline and custom polynomial of grade 3. This can be confirmed with the results of enclosed area which is presented on table III.

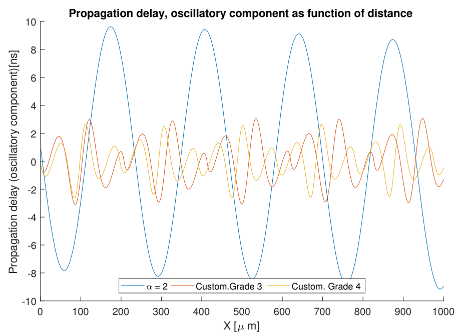


Fig. 10. Propagation delay, oscillatory component with custom functions

TABLE III
AREA OF VARIATION OF PROPAGATION DELAY

| | $\alpha = 2$ | **Custom. Grade 3** | **Custom. Grade 3** |
|---|---|---|---|
| **Area of propagation delay [ms * $\mu$ m]** | 0.0056 | 0.0013 | 0.0010 |

## C. *Optimization of parameters.*

Given that the polynomial that represents better performance corresponds to the adjustment of degree four, it is sought to explore parameter values that allow further reduction of the enclosed area, for this an optimization without restriction is performed so that the cost function corresponds to the area of variation of enclosed propagation delay and the optimization parameters corresponds to the five variables of a polynomial of fourth degree.

The evolution of the cost function is presented in figure 11 where the optimization algorithm stops at 35 iterations given the parameters settings.

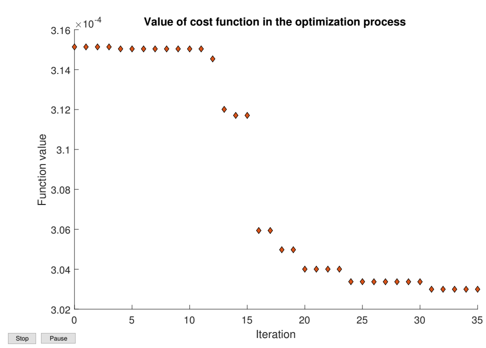


Fig. 11. Value of cost function. Optimization of parameters.

The variation of refractive index given by the optimization of the parameter of a fourth degree polynomial is presented in figure 8 compared to the base function of $\alpha = 2$ and the custom functions of presented in the previous section.

The parameters of the optimized fourth-degree polynomial according to the reduction of the enclosed area are presented in equation (2) Where it is observed that the optimum parameters define a polynomial of grade 3 instead of grade 4.

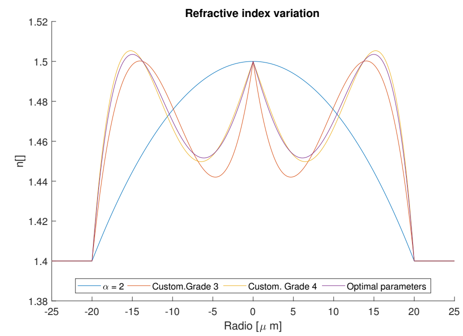


Fig. 12. Refractive index variation. Includes $n$ of optimal parameters.

$$n(r) = 0r^4 + 0.0002|r|^3 + 0.0002r^2 - 0.0134r + 1.5 \quad (2)$$

In order to make the comparison between the original variation function, the variation of the fourth degree presented in section 4 and the function with optimized parameters, a graph of the propagation delay as a function of the distance, with an initial angle of 14.5 degrees as in the previous results. The propagation delay is presented in figure 13. We also present the oscillatory component of the propagation delay, which is presented in figure 14.

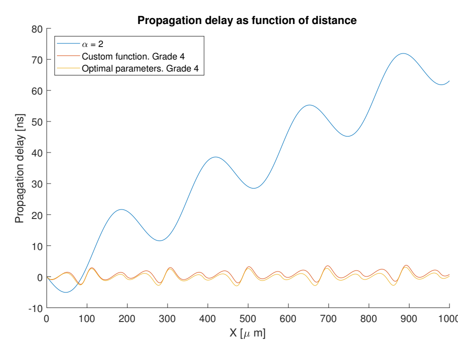


Fig. 13. Propagation delay as function of distance

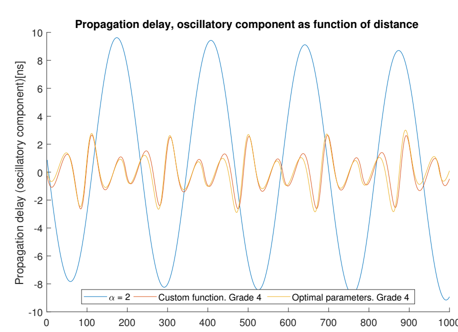


Fig. 14. Oscillatory component of propagation delay

From figures 13 and 14 it is observed that there are little differences between the optimized polynomial and the grade four polynomial, in order to see this in a more quantitative way, we perform an analysis of the enclosed area by the propagation delay using a distance of 1000 $\mu m$. The results are presented on table IV where we can observe a reduction in the propagation delay with respect to the custom grade 4 polynomial (2%) and a notorious difference from the baseline (82%).

TABLE IV
AREA OF PROPAGATION DELAY

| | $\alpha = 2$ | **Custom. Grade 4** | **Optimized polynomial** |
|---|---|---|---|
| **Area of propagation delay [ms * $\mu$ m]** | 0.005615 | 0.001023 | 0.001003 |

## V. CONCLUSION

From the work developed, the fulfilment of the objectives is observed first, since it was possible to find functions of variation of the refraction coefficient that allows reducing the propagation delay with respect to the base case. Being this significant decrease with respect to the initial condition, and close to 80% of reduction.

On the other hand, there is no significant difference between the use of polynomial grade 4 and the polynomial with optimized parameters, which could be caused by the low number of iterations of the optimizer. In this sense, a point of study would correspond to the performance of parameter tests with a greater number of iterations, in order to evaluate if it is possible to obtain a greater reduction, or else, this is limited.

## REFERENCES


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[2] R.Olshansky and D.B.Keck, “Pulse broadening in graded-index optical fibers”, *Applied Optics*, n$^o$2, vol.15, 1976