A NEW ECCENTRIC GEOMAGNETIC DIPOLE TO GIVE THE CORRECT DIP POLE LOCATIONS

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Summary In this paper, we describe a new eccentric dipole model of the Earth's magnetic field. The constraints under which the conventional eccentric dipole model is defined result in predicted dip pole locations that differ significantly from the measured locations. Here, we give a preliminary exposition of a new dipole model which, because it is constrained by the observed dip pole locations, overcomes this problem.

1. INTRODUCTION

It is customary to describe the geomagnetic field as the gradient of scalar potential V, which is expressed as a series expansion of orthogonal spherical harmonics [1]. Ideally, this is an infinite series in terms of coefficients g_n^m and h_n^m, where m runs from 0 to n and n runs from 1 to ∞. In practice, the series is typically truncated at $n=10$ or $n=12$.

Truncated at $n=1$, the series includes terms containing only g_1^0, g_1^1 and h_1^1. This gives the field of a dipole centred at the geographic centre of the earth, having an axis inclined with respect to the geographic axis. This is the conventional *centred dipole model*. Truncating the series at $n=2$ produces a higher order approximation, retaining the 8 coefficients g_1^0, g_1^1, g_2^0, g_2^1, g_2^2 and h_1^1, h_2^1, h_2^2. This is the conventional *eccentric dipole model*,

which has been the subject of papers by Schmidt [2], Bartels [3] and Fraser-Smith [4]. This eccentric dipole is restricted insofar as its axis is parallel with that of the centred dipole. One consequence of this is that the associated dip poles are quite far from the measured dip pole positions. Since the dip poles are such a prominent and measurable feature of the geomagnetic field, we felt that it might be of value to introduce another eccentric dipole model which gives the correct dip pole locations. To reproduce dip poles at any two specified locations, there are an infinite number of points at which the dipole can be situated. However, by introducing an additional constraint - that the dipole position be equidistant from the two dip poles – a unique solution for the dipole's position and orientation is obtained. We give here a preliminary exposition of this development. It remains for the future to establish how accurately other features of the geomagnetic field are reproduced by this new dipole model.

2. DEVELOPMENT OF THEORY

At point S in Fig. 1, the magnetic flux density vector is readily calculated as

$$\boldsymbol{B} = k\{3r\sin\theta(r\cos\theta - d),$$
$$r^2(1 - 3\cos 2\theta) - 2d^2 + 4dr\cos\theta\}$$

(1)

where

$$k = \frac{m\mu_0}{4\pi[r^2 + d^2 - 2dr\cos\theta]^{5/2}}$$

(2)

We now consider that S is in fact the Southern dip pole and N the northern dip pole. At S therefore, the flux is along the radius vector $\boldsymbol{r} = r(\cos\theta, \sin\theta)$. Therefore, S satisfies the relations

$$k.3r\sin\theta(r\cos\theta - d) = \gamma r\cos\theta$$

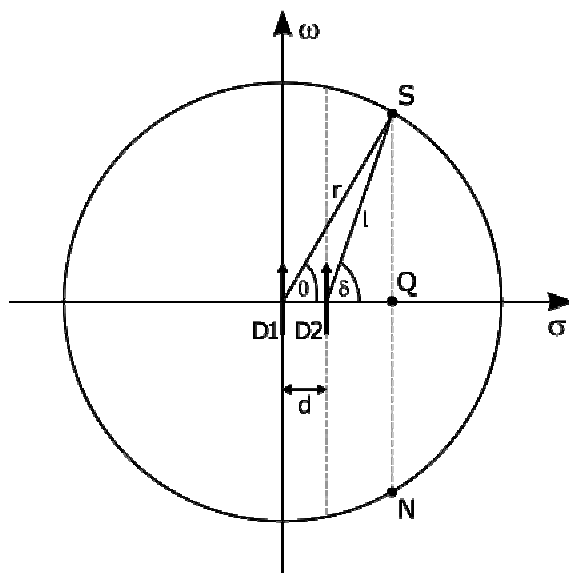$$k\frac{[r^2(1 - 3\cos 2\theta) - 2d^2 + 4dr\cos\theta]}{2} = \gamma r\sin\theta$$

(3)

Fig. 1. A cross-section of the Earth containing the dip poles S and N and the geographic centre. The proposed eccentric dipole, D2, is aligned parallel to SN (the line joining the dip poles) and located at a distance d from the geographic centre.

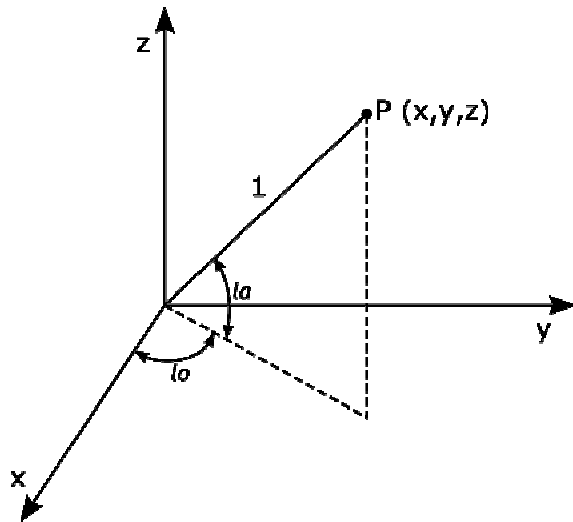*Fig. 2. Relationship between normalised Earth's spherical coordinates la (latitude) and lo (longitude) and the cartesian coordinates x, y and z.*

Eliminating k and γ between these equations leads after some manipulation to

$$\cos^2\theta + \left(\frac{r}{d} + \frac{d}{r}\right)\cos\theta - 3 = 0 \qquad (4)$$

We now use the simplified notation

$$e = \frac{d}{r}$$

where e is the (fractional) eccentricity of the dipole and

$$f = \cos\theta,$$

where f is the (fractional) eccentricity of the NS axis. Equation (4) can then be written

$$\frac{1}{e} + e = \frac{3}{f} - f \qquad (5)$$

In practice f is known (as we shall show below) from the positions of S and N measured on the Earth's surface and so, solving for the appropriate root of the resulting quadratic for e gives

$$e = \frac{1}{2f}\left[3 - f^2 - \sqrt{(9 - f^2)(1 - f^2)}\right] \quad (6)$$

This defines the eccentricity of our proposed dipole. Its axis is parallel to NS and lies in the plane containing N, S and the geographic center of the Earth.

The fractional eccentricity, f, of the NS axis is measured as follows. Normalising the radius of the Earth to unity, a point on the surface of the Earth may be specified in terms of its latitude, la, and longitude, lo, as shown in Fig 2.
In Fig. 2 the xz plane contains the great circle $0 - 180°$ longitude. The coordinates of P are

$$x = \cos la.\cos lo$$
$$y = \cos la.\sin lo \qquad (7)$$
$$z = \sin la$$

If we denote the coordinates of the north and south dip poles by (x_n, y_n, z_n) and (x_s, y_s, z_s) respectively, the coordinates of the centre point of NS, which corresponds to point Q in Fig. 1 are

$$\left(\frac{x_n + x_s}{2}, \frac{y_n + y_s}{2}, \frac{z_n + z_s}{2}\right)$$

Note that in our normalized coordinate system, f is equal to the distance of point Q from the origin.

$$f = \sqrt{\left(\frac{x_s + x_n}{2}\right)^2 + \left(\frac{y_s + y_n}{2}\right)^2 + \left(\frac{z_n + z_s}{2}\right)^2}$$
$$(8)$$

Scaling the coordinates of Q by e/f gives the coordinates of the dipole location as

$$(a, b, c) = \frac{e}{f}\left(\frac{x_n + x_s}{2}, \frac{y_n + y_s}{2}, \frac{z_n + z_s}{2}\right). \ (9)$$

The dipole axis is parallel to NS. Thus, if we define

$$(g, h, i) = (x_n - x_s, \ y_n - y_s, \ z_n - z_s), \ (10)$$

the dipole axis may be traced out parametrically as

$$(a + \lambda g, \ b + \lambda h, \ c + \lambda i), \qquad (11)$$

where λ is a real parameter.

The points of intersection of the eccentric dipole axis with the surface of the normalized Earth are defined by the condition

$$1 = (a + \lambda g)^2 + (b + \lambda h)^2 + (c + \lambda i)^2. \ (12)$$

This gives the equation

$$\lambda = \pm\sqrt{\frac{1 - (a^2 + b^2 + c^2)}{g^2 + h^2 + i^2}}, \qquad (13)$$

(where it is noted in the derivation that $ag + bh + ci \equiv 0$). The positive solution of λ corresponds to a new north "geomagnetic pole" and the negative solution of λ to the south. All that remains now is to use the equation x, y, z in reverse to find the find the latitude and longitude of the new geomagnetic fields.

3. EXAMPLE

Historic locations of S and N are summarized by Mandea and Dormy [5]. The latest measurements given by them are S, 2000 and N, 2001. We could use these in an example but we find in fact, that they are almost identical with values predicted by the full

geomagnetic series, and so we use the figures for 2006 predicted by the full series [6]:

$$N : la = 83.8°, lo = -122.0°$$
$$S : la = -64.5°, lo = 137.7°$$
(14)

These give the fractional center of our eccentric dipole as

$$(a,b,c) = (-0.063957, 0.033737, 0.01559)$$
(15)

and our new north and south geomagnetic poles are

$$N_g : la = 79.898°, lo = -66.669°$$
$$S_g : la = -72.424°, lo = 130.82°$$
(16)

4. DISCUSSION

By definition, our dipole model reproduces the experimentally observed positions of the dip poles. Other features that could be experimentally investigated include the predicted collinearity of the field at N_g and S_g, the new geomagnetic poles (where the dipole axis intersects the surface of the earth). Furthermore, there exists a unique circular locus of points at which the surface field due to the dipole is parallel to the dipole axis. This circle coincides with the intersection of a plane passing through the dipole position and normal to its axis with the earth's surface. Investigation of the correspondence between these predicted features and the observed field would be of interest. However, the inevitable effects of local perturbations should be expected.

Our dipole differs from the Schmidt-Bartels-Frazer-Smith eccentric dipole (1985 with 1984N and 1986S), but we note the drawback enshrined in Fraser-Smith's comment that "the ED dip poles do not closely conform to the observed dip pole points". We have at least shown how to define an eccentric dipole which overcomes this limitation. How accurate it is in other respects remains to be investigated.

Another noteworthy dipole model is that of Bochev [7]. His is an eccentric dipole, generated by minimizing the mean square error between the dipole field and the field as measured by 61 geomagnetical observatories. He tabulated the results for 1932, 1937, 1942, 1945, 1950, 1955 and 1960. To illustrate his results, we consider 1945, for which the measured dip pole locations were:

$$N : la = 73.9°, lo = -100.2°$$
$$S : la = -68.2°, lo = 144.5°$$
(17)

The corresponding normalised dipole positions given by Bochev are:

$$a = -0.0489, \ b = 0.0402, \ c = -0.0027.$$
(18)

This translates to an eccentricity $e = 0.0634$. The corresponding figures for our dipole are:

$$a = -0.0594, \ b = -0.0097, \ c = 0.0055$$
(19)

giving $e = 0.0605$. The normalised vector direction for Bochev's dipole is

$$(-0.0653, 0.1929, -0.9790)$$

and for our dipole is

$$(-0.1287, 0.2483, -0.9601)$$

The angle between these vectors is $4.9449°$. Although the position of Bochev's dipole is quite different to that of ours, the eccentricities are very close, and the angle between the dipole directions is small. Bochev's 61 geomagnetical observatories give a very sparse and geographically biased coverage of the Earth's magnetic field, perhaps accounting for Fraser-Smith's passing over his work in a single sentence. However, it would be very interesting to see his method repeated with the much more dense and widely dispersed measurements now available from satellites, which might move his diploe coordinates around closer to ours.

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