MP307 Practical 2017/2018: Discrete Population Models

This practical uses the Maple worksheet `discrete.mw` that may be downloded from the **MP307 Blackboard** pages.

Notice

Solutions to **all** questions marked with **(*)** has to be shown (and explained) to the instructor at the practicals in order to get 4% that count towards the overall mark.

1. (*) **Geometric Model for Populations.** Consider the geometric population model $P_{n+1} = (1+r)P_n$ for $n = 0, 1, \ldots$ with solution $P_n = P_0 e^{sn}$ for $s = \log(1+r)$ as a model of Sweden's population from 1750 to 1960 and the US population from 1790 to 1990. The data for this are given in the worksheet `discrete.mw`. Explain what you see.

2. (*) **The Non-Linear Verhulst Model.** The general Verhulst model of population growth is as follows
$$P_{n+1} = \frac{1+r}{1+\frac{rP_n}{K}} P_n$$
where $P_n \to K$ as $n \to \infty$. For the choice of parameters $P_0 = 2.79$, $r = 0.35$ and $K = 300$ compare the Verhulst model to the US population from 1790 to 1990. Explain what you see.

3. (*) **Population Model with Age Distribution.** Consider the human population divided into three age groups $0-14$, $15-39$ and ≤ 40 with population size P_i , $i = 1, 2, 3$ and yearly birth rates of 0, 0.06 and 0 and death rates of 0.005, 0.01 and 0.015, respectively.

   (a) Show that $\mathbf{P}(t+1) = \mathbf{AP}(t)$ where
$$\mathbf{P} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0.92867 & 0.06 & 0 \\ 0.0663 & 0.9504 & 0 \\ 0 & 0.0396 & 0.985 \end{bmatrix}$$

   (b) Find the largest eigenvalue of $\mathbf{A}$ and describe the long-term behaviour of this system.

   (c) Consider the growth of the population with the following initial populations. How long does it take for the total population to double in each case?

      i. $P_1 = 200$, $P_2 = P_3 = 400$.

      ii. $P_1 = 400$, $P_2 = P_3 = 300$.

      iii. $P_1 = 200$, $P_2 = 500$ and $P_3 = 300$.

      Explain what you see.

4. (*) **Chaotic Models.** Consider the Ricker model of Salmon population size as follows

$$P_{n+1} = re^{-\frac{P_n}{K}} P_n$$

where r is the rate and K is another parameter. For simplicity choose $K = 500$ and consider the behaviour of this system for

(a) $r < 1$, with initial population of 1000.

(b) $1 < r < e^2$, for initial population 100, 500 and 1000.

(c) $r > e^2$, for initial population 100, 500 and 1000.

Explain what you see.