MP307 Practical 2017/2018: Continuous Population Models

This practical uses the Maple worksheet `cont.mw` that can be downloaded from the MP307 Blackboard page.

Notice
Solutions to **all** questions marked with **(*)** has to be shown (and explained) to the instructor at the practicals in order to get 4% that count towards the overall mark.

1. **Verhulst Logistic Model.** Consider the logistic population model

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

for $r, K > 0$ as a model of the US population from 1790 to 1990. The data for this are given in the worksheet `cont.mw`.

2. (*) **Competitive Species.** Consider two species with population sizes P_1, P_2 with growth rates r_1, r_2 and limiting population sizes of K_1, K_2 where

$$\begin{aligned} \frac{dP_1}{dt} &= r_1 P_1 \left(1 - \frac{P_1 + P_2}{K_1}\right), \\ \frac{dP_2}{dt} &= r_2 P_2 \left(1 - \frac{P_1 + P_2}{K_2}\right). \end{aligned}$$

Analyse the behaviour of N_1, N_2 in the following cases, by plotting P_1, P_2 vs t and P_1 vs P_2:

(a) $r_1 = 1/10$, $r_2 = 1/10$, $K_1 = 100$, $K_2 = 50$ with $P_1(0) = 10$ and $P_2(0) = 15$.
(b) $r_1 = 1/10$, $r_2 = 1/10$, $K_1 = 100$, $K_2 = 50$ with $P_1(0) = 130$ and $P_2(0) = 200$.
(c) $r_1 = 1/10$, $r_2 = 1/100$, $K_1 = 40$, $K_2 = 50$ with $P_1(0) = 130$ and $P_2(0) = 20$.
(d) $r_1 = 1/10$, $r_2 = 1/100$, $K_1 = 50$, $K_2 = 60$ with $P_1(0) = 15$ and $P_2(0) = 10$.

3. (*) **Lotka-Volterra Predator/Prey System.** Consider a prey species with population size x and a predator species with population size y where

$$\begin{aligned} \frac{dx}{dt} &= x(a_1 - b_1 y), \\ \frac{dy}{dt} &= y(-a_2 + b_2 x), \end{aligned}$$

with $a_1, a_2, b_1, b_2 > 0$. Analyse the behaviour of the system for $a_1 = 3, a_2 = 5/2$, $b_1 = 2,\ b_2 = 1$ by plotting x, y vs t, and x vs y in the following cases:

(a) $x(0) = 1$ and $y(0) = 1$.
(b) $x(0) = 0.1 + 5/2$ and $y(0) = 0.1 + 3/2$. What behaviour do you observe?
(c) $x(0) = 1$ and $y(0) = 5$.