MP305 Practical 2017/2018 - Network Flows II

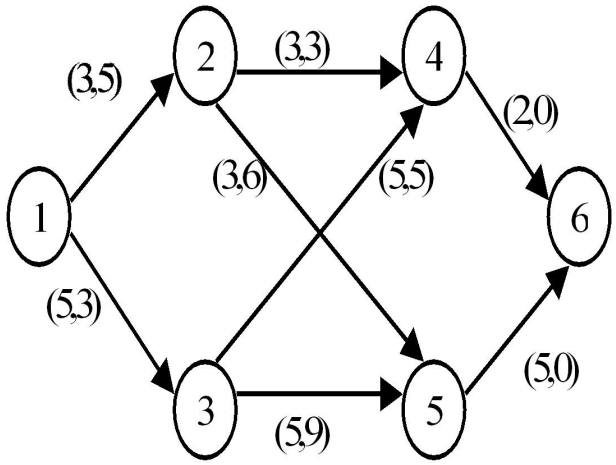
The Maple procedures that perform the maximal network flow algorithm is found by opening up the Maple worksheet `Network.mw`

This file may be downloded from the **MP305 Blackboard** web page.

An explanation is given there of all the procedures used. Network nodes are assumed to be labelled from 1 to some maximum value `NV` where node 1 is the source and node `NV` is the sink. The network arcs are then described as a set of two element lists e.g. `G:={[1,2],[1,3],[3,5],..}` etc. The capacity, flow and cost of each arc `[i,j]` is `c[i,j]`, `phi[i,j]`and `l[i,j]` resp. The main procedure is `Iterate(G)` which performs one iteration of the algorithm.

Solutions to **all** questions with **(*)** have to be shown (and explained) to the instructor at the practicals in order to get 5% that count towards the overall mark.

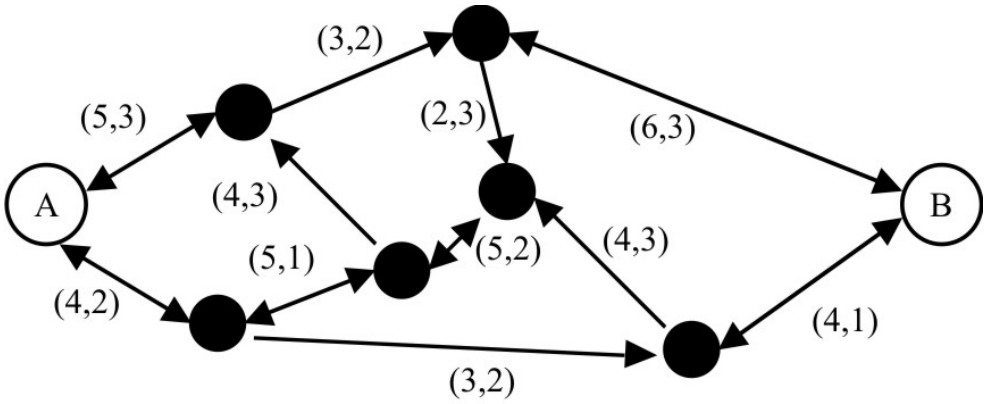
1. Find the maximal flow for minimal cost for the network below where (capacity,cost) is shown:



This is the example discussed in class. You may read in the data for the example from the Maple worksheet `net2.mw`

Find the incremental network, its capacities and costs at each iteration.

2. (*) A road network is shown below with the capacity and time taken per car on each road indicated. Find the maximal flow through the network for minimal total travel time from A to B. Compare this to flow from B to A.



Note: You can use the network you created in lab Network Flows I, Question 3.

3. (*) 5 students A, B, C, D, E are assigned 5 projects P_1, P_2, P_3, P_4, P_5 by 4 supervisors S_1, S_2, S_3, S_4, each of whom have the expertise to supervise a number of projects but can only be assigned **at most** 2 students. Furthermore, each project can only be assigned to one student. The students rank the projects in order of preference from 1 to 5 as shown.

Project	Supervisor	A	B	C	D	E
P_1	S_1, S_2, S_3	4	3	4	2	1
P_2	S_2, S_4	2	1	2	5	4
P_3	S_2, S_3, S_4	5	5	3	4	3
P_4	S_1, S_3	3	2	1	3	2
P_5	S_1, S_2, S_4	1	4	5	1	5

(a) Construct a network flow model with appropriate capacities and costs for this system. Find the best assignment of projects by minimizing the overall cost for maximal flow.

(b) Do you think that the assignment you find is a fair one for all the students? Modify the flow model accordingly, if necessary, to find a fairer assignment.

4. **(*)** A soft drinks firm buys fruit at the beginning of each month i at a cost per 100kg of p_i in units of €1000. The firm can store up 2000kg of fruit at any given time but the cost in units €1000 of refrigeration per month per 100kg is r_i. The consumption requirements are c_i per month in 100kg units. Based on last year's figures the following estimates have been made:

i	Jan	Feb	Mar	April	May	June	July	Aug	Sept	Oct	Nov	Dec
p_i	18	17	17	15	12	8	7	6	9	12	14	17
r_i	1	1	2	2	3	5	6	6	5	3	2	1
c_i	9	6	6	7	11	14	16	18	15	10	7	6

   (a) Find the best purchasing schedule starting from January based on these estimates assuming that the firm has no fruit in storage on Jan 1st. How much will fruit cost for the year and how much will be spent on refrigeration?

   (b) Suppose that the firm has 500kg of fruit in storage at the beginning of the year. What is the best purchasing schedule that ensures that 500kg are again in storage at the very end of the year?