CS428 (Advanced Operating Systems) / MA500 (Geometric Foundations of Data Analysis – Classical Techniques) – Assignment 1

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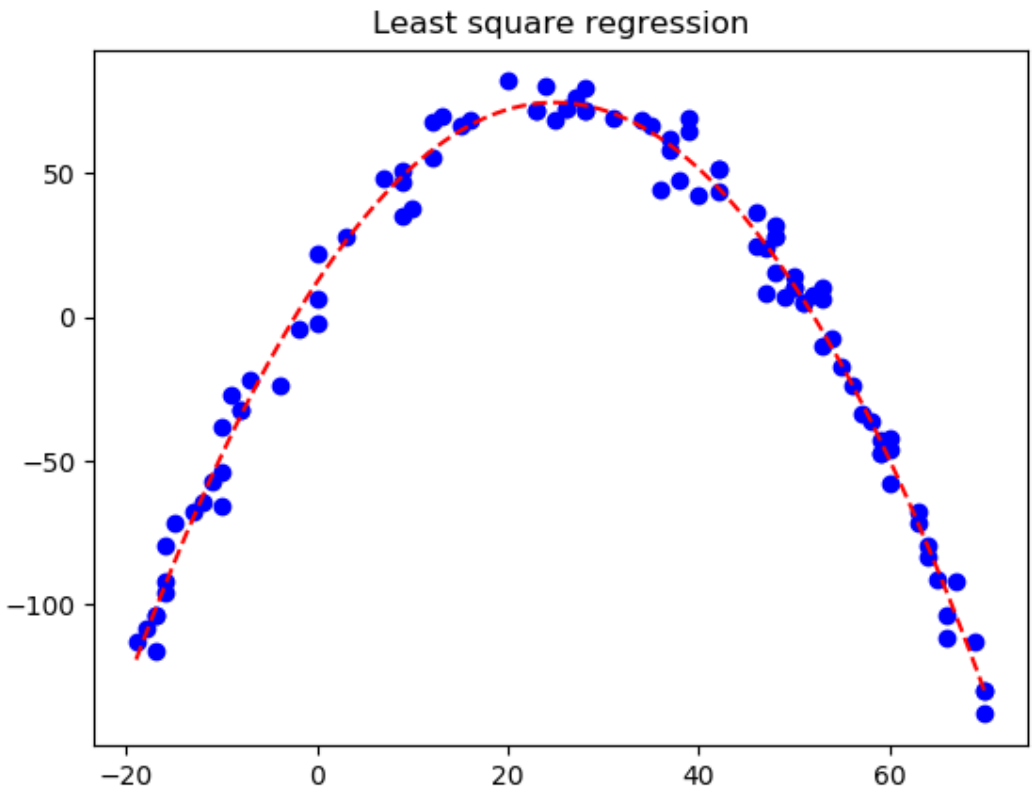
Part 1 Outputs:

Least Square estimator is:

$$y = b_0 + b_1 x + b_0 x^2$$

For the model:

$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \epsilon_i$$



In general, the line of best fit is

$$y = b_0 + b_1 x$$

With

$$\sum_{y=1}^{n} y_i = m\, b_0 + b_1 \sum x_i$$

And

$$\sum x_i y_i = b_0 \sum x_i + b_1 \sum x_i^2$$

SSR:

$$\sum (\hat{y}_i - \bar{y})^2$$

Define the Fitted Value as:

$$\hat{y}_i = b_0 + b_1 x_i$$

And the residual as:

$$e_i = y_i - \hat{y}_i$$

These values are to also be used in Part 2.

My calculated b0 values are [12.34270521 5.00610241 -0.10068145]

SSR/SSTo = 0.5234696113226226

1 - SSE/SSTo = 0.52772458233925

Both these values are = to R^2. For the most part they are similar values

Part 1 Code:

```
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#CS428/MA500 Homework 1

import math
import pylab
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

# Open the data file in a read format
f = open('data.txt', 'r')

# yi = b0+b1*xi+b2*xi^2+error(i)

```

```
# Initialise x & y arrays, along with error and beta value

Xs, Ys= [], []
b0 = [1.0, 1.0, 1.0]

def func(x, a, b, c):
    return a +  b*x + c*x**2


# For-loop
# Iterates through lines in file
# If the number is an x, it's added to the Xi array
# If the number is a y, it's added to the Yi Array
for l in f:
    # Split the lines up by their = sign
    # print(l)
    sp = l.split('=')

    # Splits the x value by its ',' character
    xVal= sp[1].split(',')

    # Adds corresponding x & y values in the line ot the arrays
    Xs.append(float(xVal[0]))
    Ys.append(float(sp[2]))

# Closes file
f.close()

# Changing X and Y data to numpy arrays
Xs = np.array(Xs)
Ys = np.array(Ys)
# Creating an arbitrary sigma array filled with ones to indicate error values
sigma = np.ones((100), dtype=float)


# Calculating the b0, b1 & b2 values
linear = np.linspace(Xs.min(), Xs.max(), 100)
B, _ = curve_fit(func, Xs, Ys, b0, sigma)
y = func(linear, *B)


# Plotting & drawing the graph

#Plotting the points
plt.plot(Xs,Ys, 'bo', label='Data')
#Plotting the line of best fit
plt.plot(linear, y, 'r--', label='Fit')
plt.title('Least square regression')
#Drawing graph
plt.show()

print(B)

# Defining yHat - The fitted value
def yHat(i):
    return b0[0] + b0[1]*Xs[i]

# Defining yMean - The sample mean
yTotal = 0

for i in Ys:
    yTotal += i
yMean = (1/len(Ys))*(yTotal)

# Defining SSTO - The Total Sum of Squares
SSTO = 0
```

```
for i in Ys:
    SSTO += (i - yMean)**2

# Defining SSE - The Error Sum of Squares
SSE = 0
count = 0
for j in Ys:
    SSE += (j - (yHat(count)))**2
    count+=1

# Defining SSR - The Regression Sum of Squares
count = 0
SSR = 0
for k in Ys:
    SSR += ((yHat(count)) - yMean)**2
    count+=1

# Showcasing that r^2 = SSR/SSTO = (SSE/SSTO)-1 (Which, they largely are)
print('SSR/SSTo = ', SSR/SSTO)
print('1 - SSE/SSTo = ', (SSE/SSTO)-1)
```

Part 2 Outputs:

b0, b1, b2: [3.36508341 4.06510383 4.72071285]

MSE:

11133.1

F* value:

[-0.20144135 -1.38350205 -0.10066079 -0.51791505 -0.61085113 -0.08646888
 -2.46765651 -0.36431901 -0.81971778 -0.11593057]

Define the Fitted Value as:

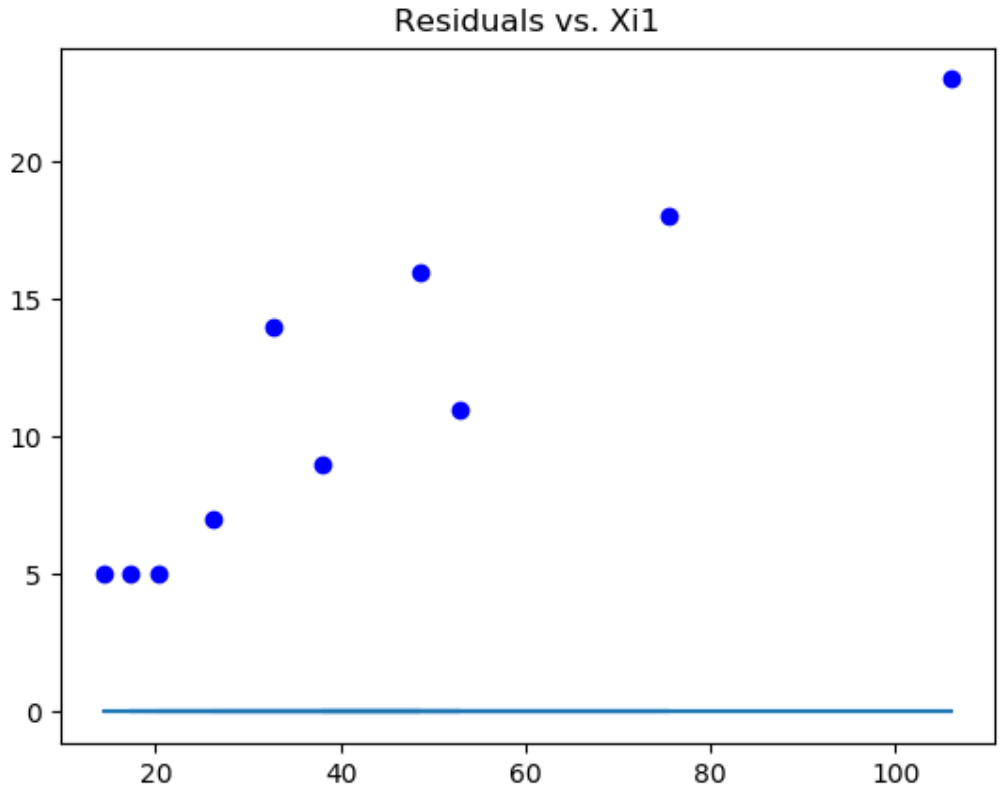
$$\hat{y}_i = b_0 + b_1 x_i$$

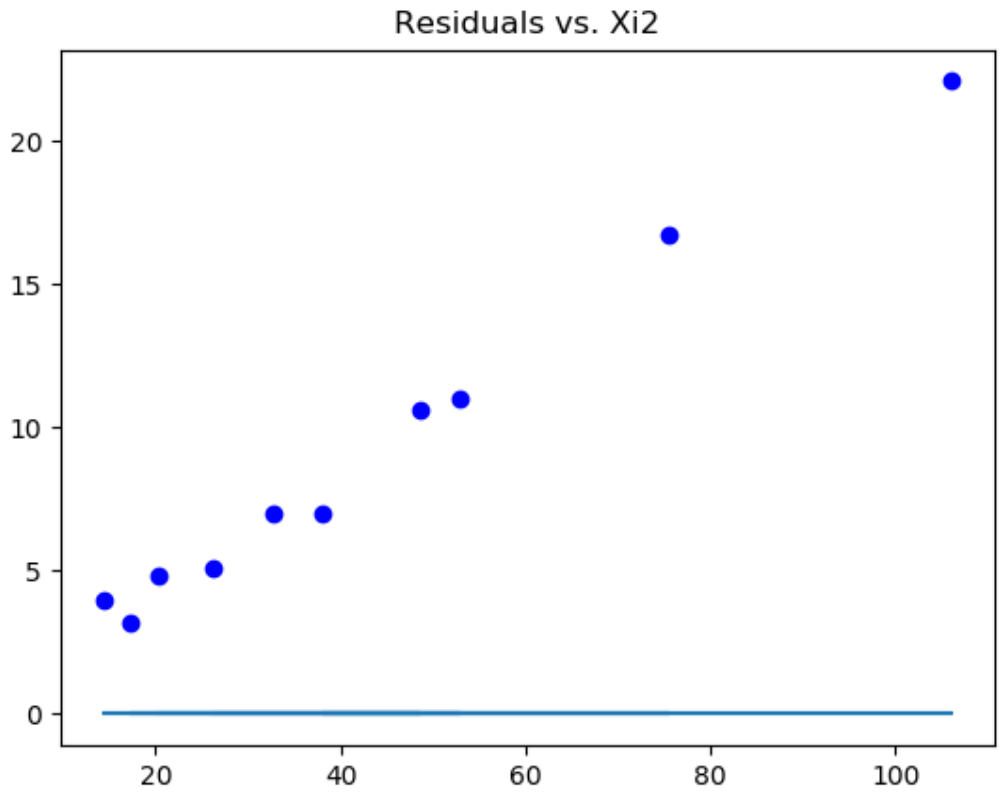
Define the residual as:

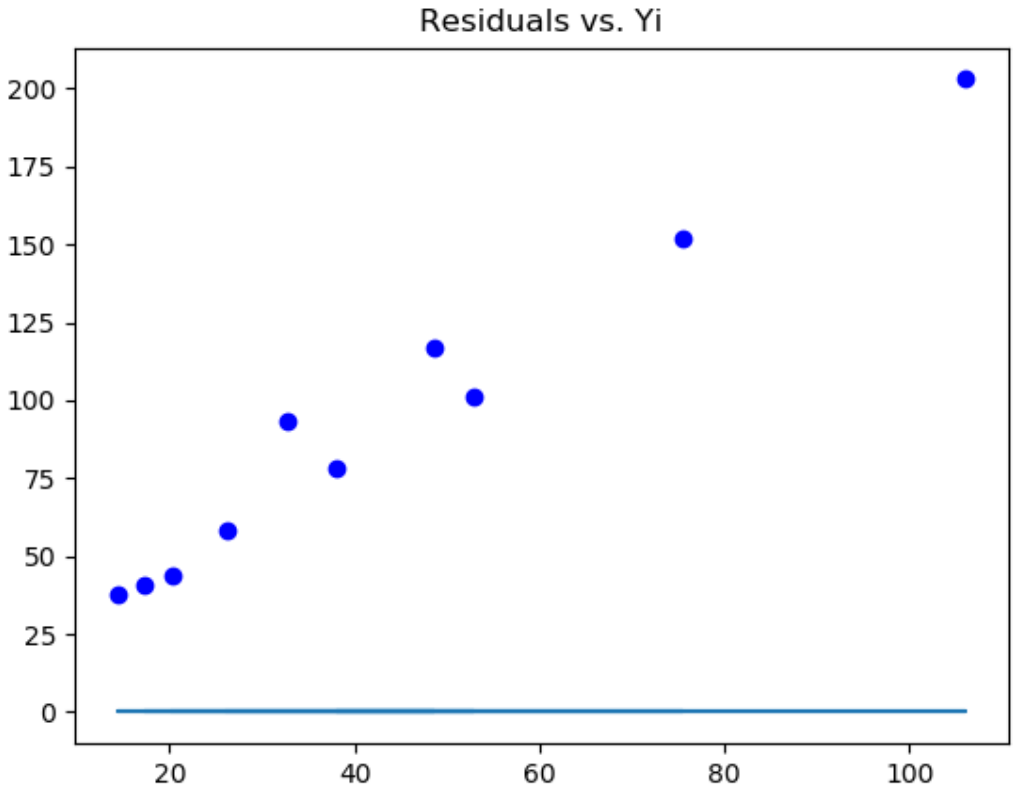
$$e_i = y_i - \hat{y}_i$$

Estimated Covariance matrix:

$$s^2(B) = MSE * (x^t x)^{-1}$$


Residuals vs. Xi1
20
15
10
5
0
20
40
60
80
100


Residuals vs. Xi2
20
15
10
5
0
20
40
60
80
100



To obtain the residuals, plot each $\hat{y}i$, Xi1, & Xi2 against e_i. The residuals seem to be independent (as there is no systematic deviation) and not dependent on the level of $\hat{y}$ or the xi1 or xi2 values. You can conclude that all values are independent in N(0, δ^2).

Part 2 Code:

```
# Assignment 1 - Question 2

#Student ID: 15315901
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#CS428/MA500 Homework 1

import math
import pylab
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit, leastsq
from scipy.stats import t

# Defining data as np arrays
Xi1=np.array([7, 18, 5, 14, 11, 5, 23, 9, 16, 5])
Xi2=np.array([5.11, 16.70, 3.20, 7.00, 11.00, 4.00, 22.10, 7.00, 10.60, 4.80])
Yi=np.array([58, 152, 41, 93, 101, 38, 203, 78, 117, 44])
b0=np.array([0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0])
bZ = np.array([0.0,0.0,0.0])

# Model definition
def model(Beta, Xi1, Xi2, Error, i):
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    return Beta[0] + Beta[1]*Xi1[i] + Beta[2]*Xi2[i] + Error[i]

# Defining the least square estimator
def leastSquareEstimator(b,x, x2):
    return b[0] + b[1]*x[1] + b[2]*x2[2]

# Defining the least square estimator
def func(X, a, b, c):
    Xi1, Xi2 = X
    return a + b*Xi1 + c*Xi2


# Calculating the b0, b1 & b2 values
B = (curve_fit(func, (Xi1, Xi2), Yi, p0=bZ))
print('b0, b1, b2: ', B[0])
B0 = B[0]


# T value with 12 degrees of freedom & a significange level of 0.05
T = t.ppf(0.975, 12)
P = 3

#MSR = (1/p-1)(yTranspose*y - bTranspose * xTranspose * y)


# Defining yHat - The fitted value
def yHat(i):
    return B0[0] + B0[1]*Xi1[i]

# Defining e(i) - The residual value
def e(i):
    return (Yi[i] - yHat(i))

# Defining yMean - The sample mean
yTotal = 0

for i in Yi:
    yTotal += i

yMean = (1/len(Yi))*(yTotal)

# Defining SSTO - The Total Sum of Squares
SSTO = 0
for i in Yi:
    SSTO += (i - yMean)**2

# Defining SSE - The Error Sum of Squares
SSE = 0
count = 0
for j in Yi:
    SSE += (j - float(yHat(count)))**2
    count+=1

# Defining SSR - The Regression Sum of Squares
count = 0
SSR = 0
for k in Yi:
    SSR += (float(yHat(count)) - yMean)**2
    count+=1

# Defining MSR
MSR = (1/P-1) * (Yi.transpose()*Yi - b0.transpose()*Xi1.transpose()*Yi)

# Obtaining residuals
resVals=[]
for i in range(len(Yi)):
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    resVals.append(e(i))

# Plotting the residuals
plt.plot(resVals, Yi, 'bo', label='Yi')
plt.plot(resVals, b0)
plt.title('Residuals vs. Yi')
plt.show()

plt.plot(resVals, Xi1, 'bo', label='Yi')
plt.plot(resVals, b0)
plt.title('Residuals vs. Xi1')
plt.show()

plt.plot(resVals, Xi2, 'bo', label='Yi')
plt.plot(resVals, b0)
plt.title('Residuals vs. Xi2')
plt.show()

# Printing MSE
print('MSE:')
MSE = SSE/len(Yi)-P
print(MSE)

# Printing Fstar value as found in the notes
print('F* value:')
FStar = MSR/MSE
print(FStar)
```