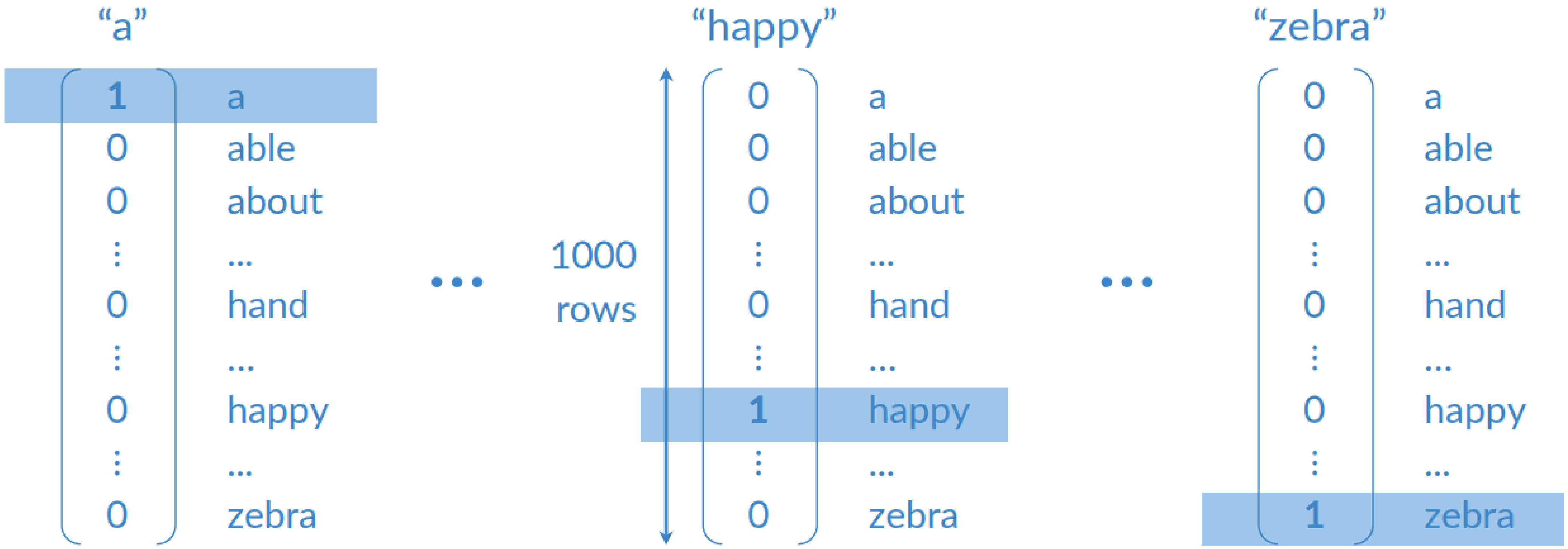
Vector Semantics & Embeddings

Basic Word Representation

Integers

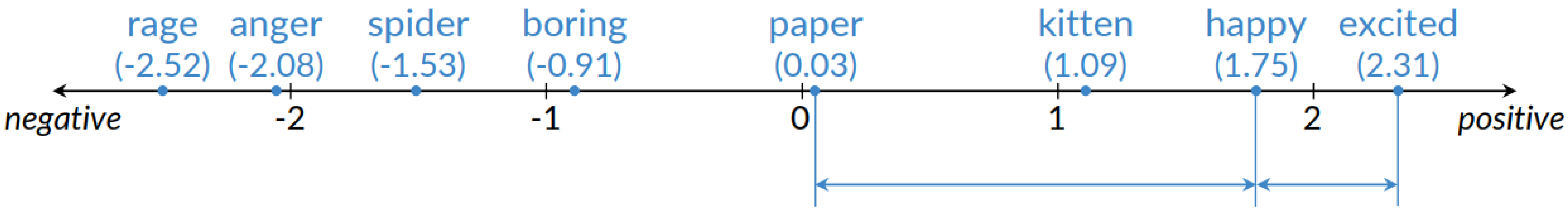
Word	Number
a	1
able	2
about	3
...	...
hand	615
...	...
happy	621
...	...
zebra	1000

One-hot vectors



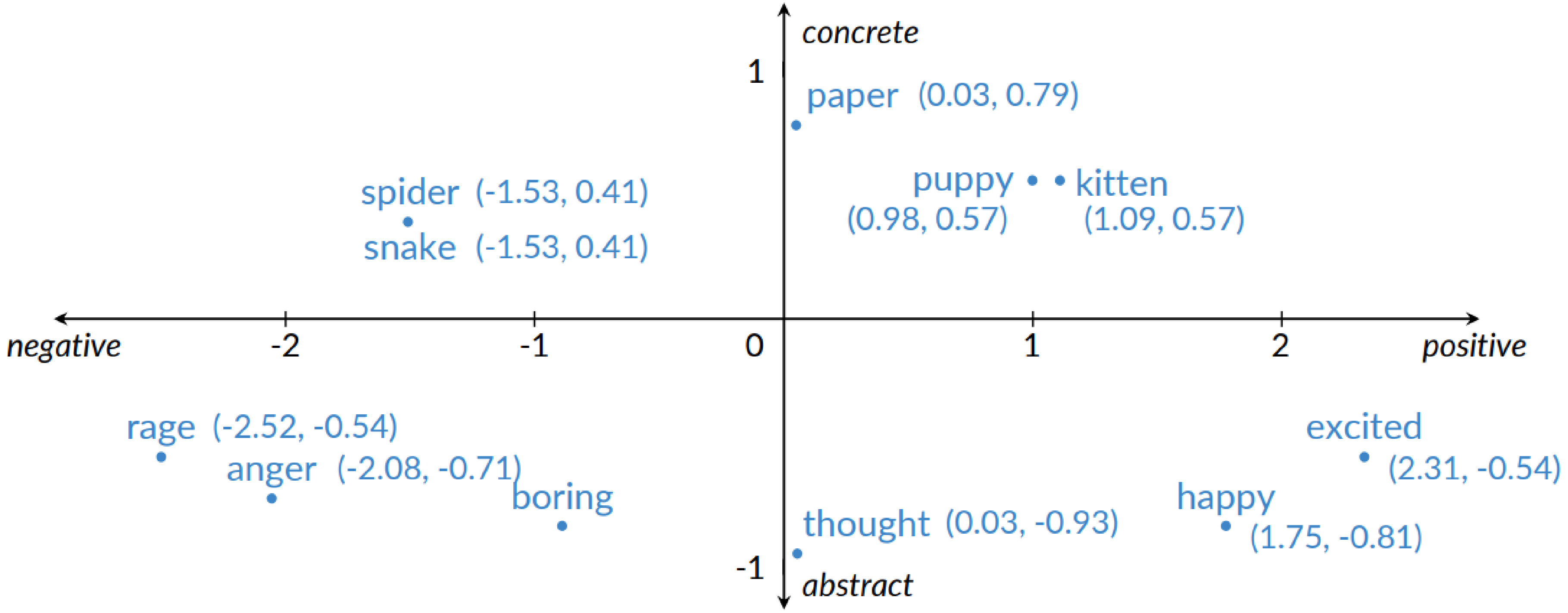
Word-Embedding

Meaning as vectors



Word-Embedding

Meaning as vectors



We'll discuss 2 kinds of embeddings

tf-idf

- Information Retrieval workhorse!
- A common baseline model
- **Sparse** vectors
- Words are represented by (a simple function of) the **counts** of nearby words

Word2vec

- **Dense** vectors
- Representation is created by training a classifier to **predict** whether a word is likely to appear nearby
- Later we'll discuss extensions called **contextual embeddings**

Vector Semantics & Embeddings

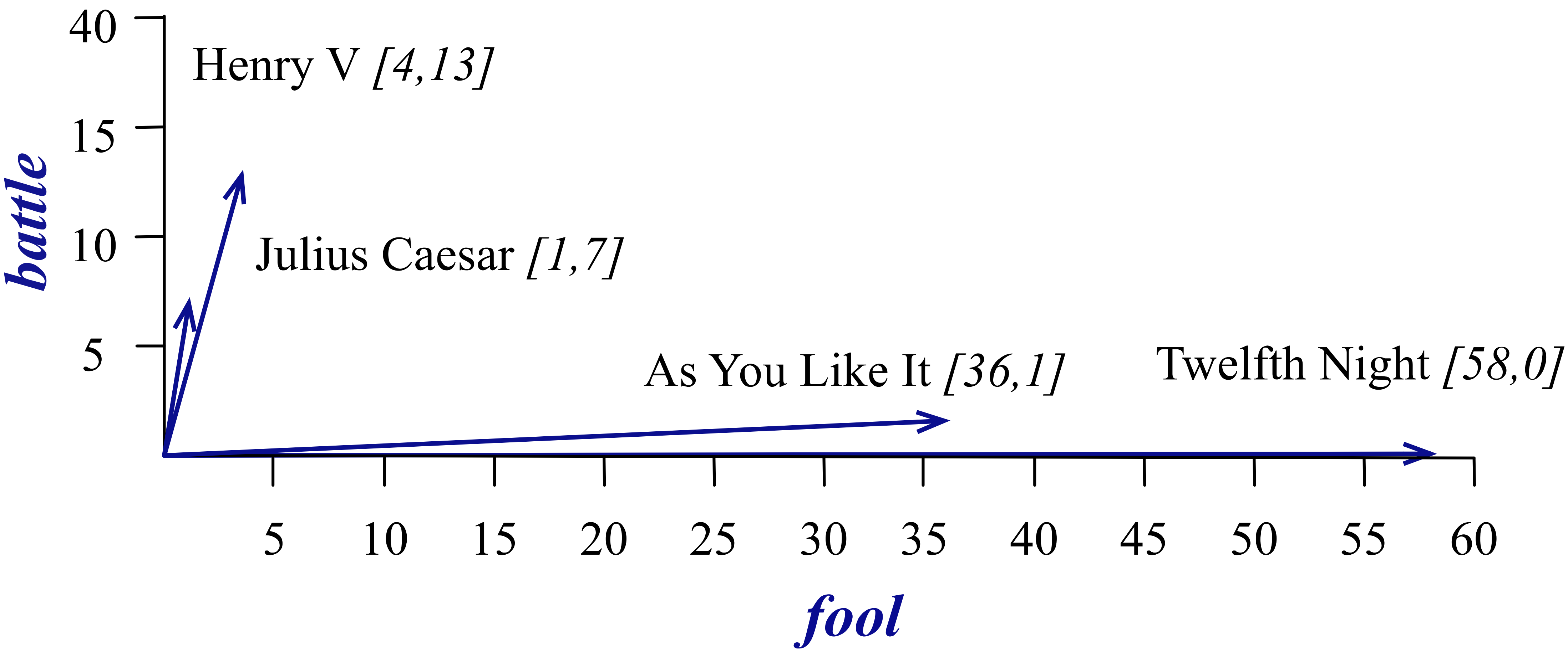
Words and Vectors

Term-document matrix

Each document is represented by a vector of words

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

Visualizing document vectors



Vectors are the basis of information retrieval

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

Vectors are similar for the two comedies

But comedies are different than the other two

Comedies have more *fools* and *wit* and fewer *battles*.

Idea for word meaning: Words can be vectors too!!!

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

battle is "the kind of word that occurs in Julius Caesar and Henry V"

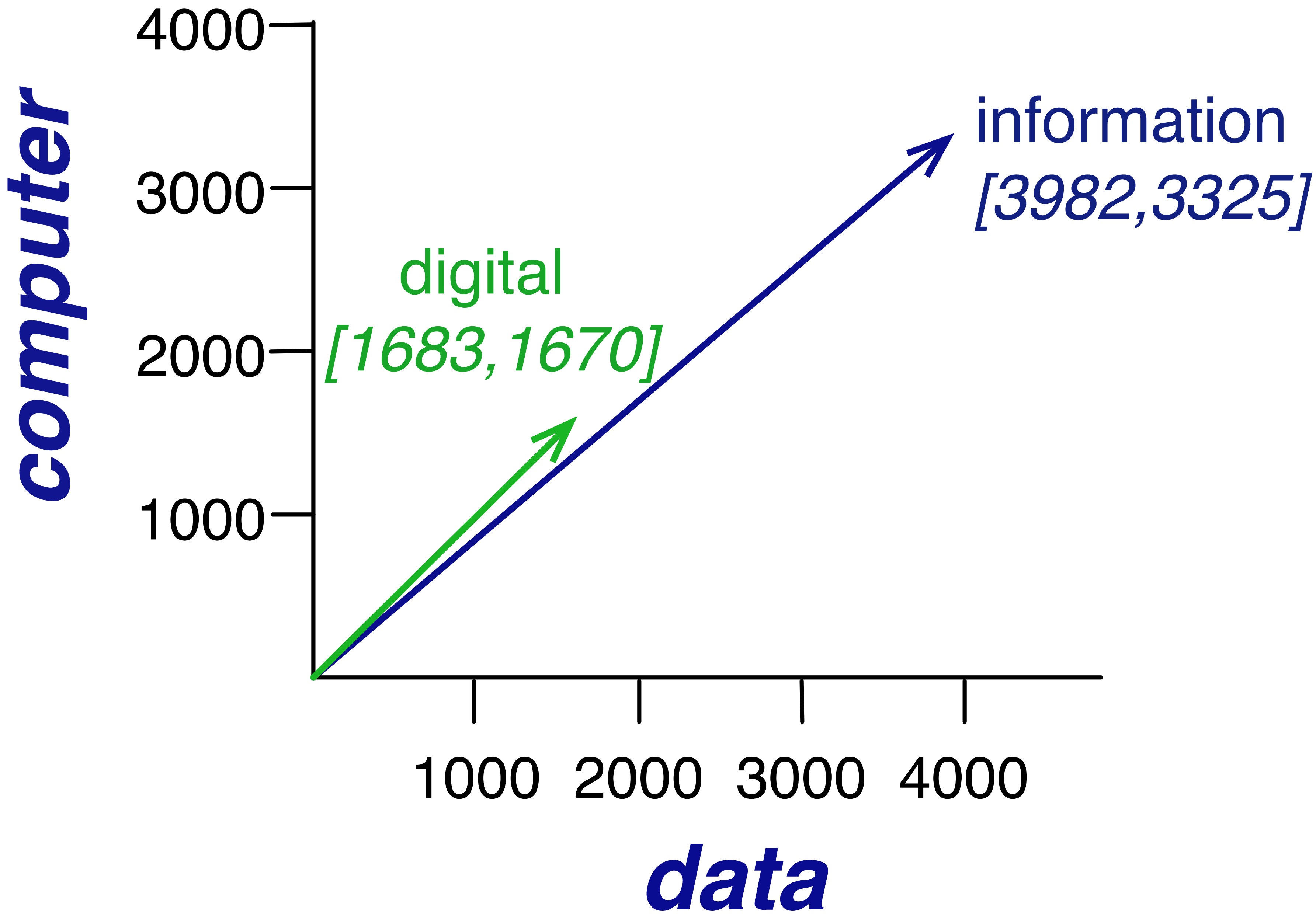
fool is "the kind of word that occurs in comedies, especially Twelfth Night"

More common: word-word matrix (or "term-context matrix")

Two **words** are similar in meaning if their context vectors are similar

is traditionally followed by	**cherry**	pie, a traditional dessert
often mixed, such as	**strawberry**	rhubarb pie. Apple pie
computer peripherals and personal	**digital**	assistants. These devices usually
a computer. This includes	**information**	available on the internet

	aardvark	...	**computer**	**data**	**result**	**pie**	**sugar**	...
cherry	0	...	2	8	9	442	25	...
strawberry	0	...	0	0	1	60	19	...
digital	0	...	1670	1683	85	5	4	...
information	0	...	3325	3982	378	5	13	...



4000
3000
2000
1000
computer
information
[3982,3325]
digital
[1683,1670]
1000 2000 3000 4000
data

Vector Semantics & Embeddings

Cosine for computing word similarity

Computing word similarity: Dot product and cosine

The dot product between two vectors is a scalar:

$$\text{dot product}(\mathbf{v}, \mathbf{w}) = \mathbf{v} \cdot \mathbf{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \ldots + v_N w_N$$

The dot product tends to be high when the two vectors have large values in the same dimensions

Dot product can thus be a useful similarity metric between vectors

Problem with raw dot-product

Dot product favors long vectors

Dot product is higher if a vector is longer (has higher values in many dimension)

Vector length:

$$|\mathbf{v}| = \sqrt{\sum_{i=1}^{N} v_i^2}$$

Frequent words (of, the, you) have long vectors (since they occur many times with other words).

So dot product overly favors frequent words

Alternative: cosine for computing word similarity

$$\text{cosine}(\vec{v},\vec{w}) = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

Based on the definition of the dot product between two vectors a and b

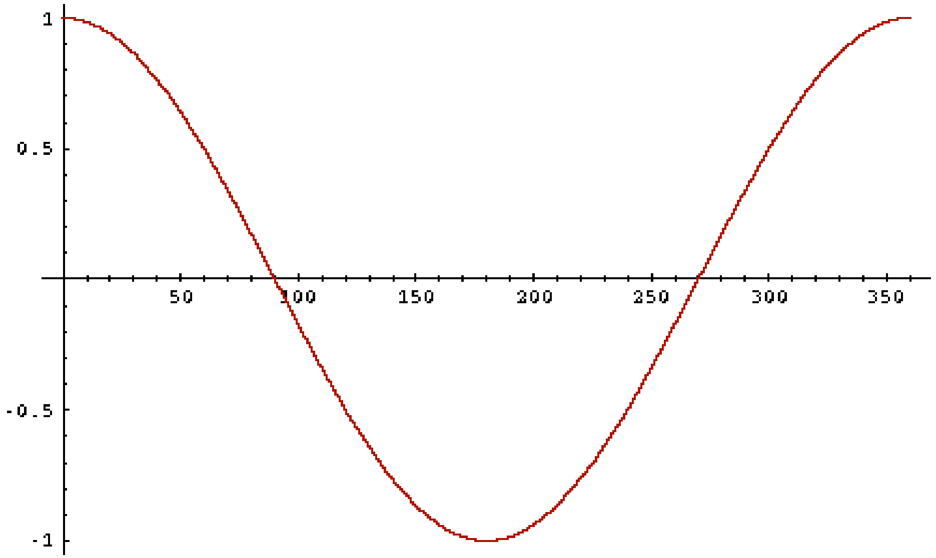
$$\begin{aligned}\mathbf{a}\cdot\mathbf{b} &= |\mathbf{a}||\mathbf{b}|\cos\theta \\ \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|} &= \cos\theta\end{aligned}$$

Cosine as a similarity metric

-1: vectors point in opposite directions

+1: vectors point in same directions

0: vectors are orthogonal



But since raw frequency values are non-negative, the cosine for term-term matrix vectors ranges from 0–1

Cosine examples

$$\cos(\vec{v},\vec{w}) = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|} = \frac{\vec{v}}{|\vec{v}|}\cdot\frac{\vec{w}}{|\vec{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

	pie	data	computer
cherry	442	8	2
digital	5	1683	1670
information	5	3982	3325

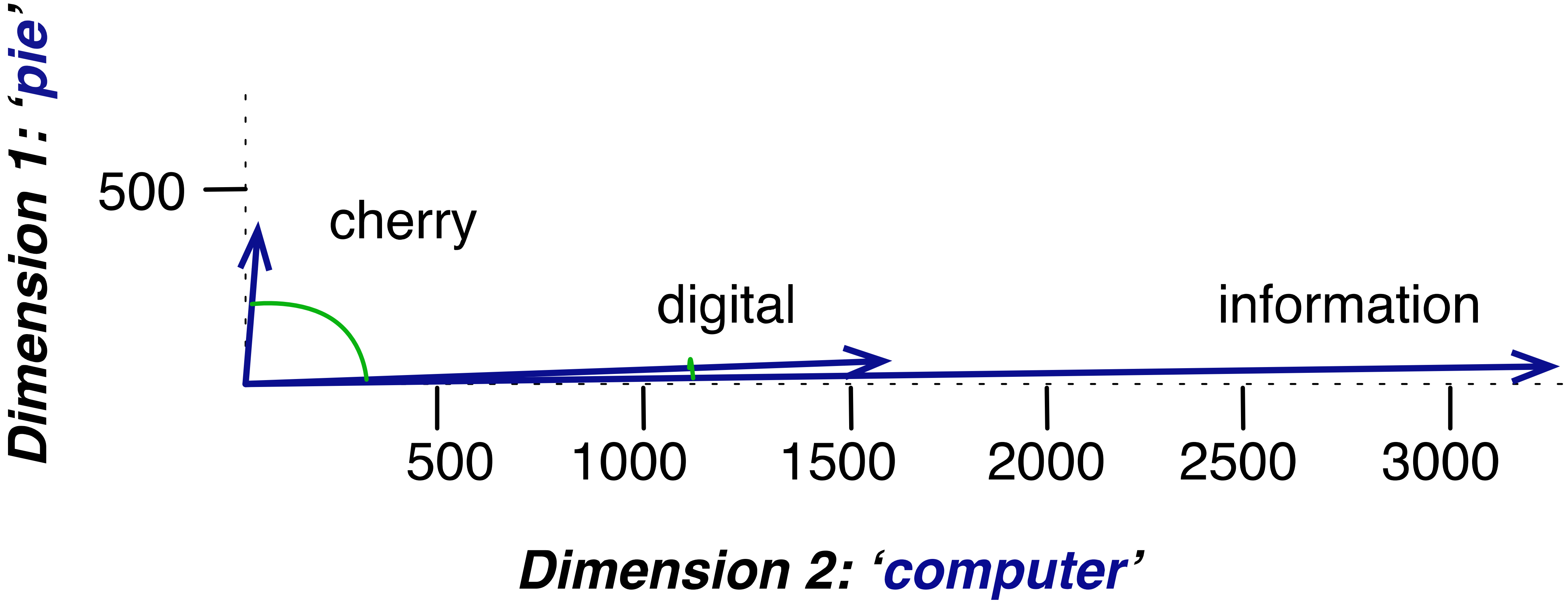
$$\cos(\text{cherry},\text{information}) = $$

$$\frac{442*5+8*3982+2*3325}{\sqrt{442^2+8^2+2^2}\sqrt{5^2+3982^2+3325^2}} = .017$$

$$\cos(\text{digital},\text{information}) = $$

$$\frac{5*5+1683*3982+1670*3325}{\sqrt{5^2+1683^2+1670^2}\sqrt{5^2+3982^2+3325^2}} = .996$$

Visualizing cosines (well, angles)



Vector Semantics & Embeddings

TF-IDF

But raw frequency is a bad representation

- The co-occurrence matrices we have seen represent each cell by word frequencies.
- Frequency is clearly useful; if *sugar* appears a lot near *apricot*, that's useful information.
- But overly frequent words like *the*, *it,* or *they* are not very informative about the context
- It's a paradox! How can we balance these two conflicting constraints?

Two common solutions for word weighting

tf-idf: tf-idf value for word t in document d:

$$w_{t,d} = \mathrm{tf}_{t,d} \times \mathrm{idf}_t$$

Words like "the" or "it" have very low idf

PMI: (Pointwise mutual information)

- **PMI**$(w_1, w_2) = log \frac{p(w_1,w_2)}{p(w_1)p(w_2)}$

See if words like "good" appear more often with "great" than we would expect by chance

Term frequency (tf)

$\mathrm{tf}_{t,d} = \mathrm{count}(t,d)$

Instead of using raw count, we squash a bit:

$\mathrm{tf}_{t,d} = \log_{10}(\mathrm{count}(t,d)+1)$

Document frequency (df)

df_t *is* the number of documents *t* occurs in.

(note this is not collection frequency: total count across all documents)

"*Romeo*" is very distinctive for one Shakespeare play:

	Collection Frequency	**Document Frequency**
Romeo	113	1
action	113	31

Inverse document frequency (idf)

$$\mathrm{idf}_t = \log_{10}\left(\frac{N}{\mathrm{df}_t}\right)$$

N is the total number of documents in the collection

Word	**df**	**idf**
Romeo	1	1.57
salad	2	1.27
Falstaff	4	0.967
forest	12	0.489
battle	21	0.246
wit	34	0.037
fool	36	0.012
good	37	0
sweet	37	0

What is a document?

Could be a play or a Wikipedia article

But for the purposes of tf-idf, documents can be **anything**; we often call each paragraph a document!

Final tf-idf weighted value for a word

$$w_{t,d} = \text{tf}_{t,d} \times \text{idf}_t$$

Raw counts:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

tf-idf:

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	0.074	0	0.22	0.28
good	0	0	0	0
fool	0.019	0.021	0.0036	0.0083
wit	0.049	0.044	0.018	0.022

Vector Semantics & Embeddings

PPMI

Pointwise Mutual Information

Pointwise mutual information:

Do events x and y co-occur more than if they were independent?

$$\text{PMI}(X,Y) = \log_2 \frac{P(x,y)}{P(x)P(y)}$$

PMI between two words: (Church & Hanks 1989)

Do words x and y co-occur more than if they were independent?

$$\text{PMI}(word_1, word_2) = \log_2 \frac{P(word_1, word_2)}{P(word_1)P(word_2)}$$

Positive Pointwise Mutual Information

- PMI ranges from $-\infty$ to $+\infty$
- But the negative values are problematic
  - Things are co-occurring **less than** we expect by chance
  - Unreliable without enormous corpora
    - Imagine w1 and w2 whose probability is each 10^{-6}
    - Hard to be sure p(w1,w2) is significantly different than 10^{-12}
  - Plus it's not clear people are good at "unrelatedness"
- So we just replace negative PMI values by 0
- Positive PMI (**PPMI**) between word1 and word2:

$$\mathrm{PPMI}(word_1, word_2) = \max\left(\log_2 \frac{P(word_1, word_2)}{P(word_1)P(word_2)}, 0\right)$$

Computing PPMI on a term-context matrix

Matrix F with W rows (words) and C columns (contexts)

f_{ij} is # of times w_i occurs in context c_j

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}} \qquad p_{i*} = \frac{\sum_{j=1}^{C} f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}} \qquad p_{*j} = \frac{\sum_{i=1}^{W} f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

	computer	data	result	pie	sugar	count(w)
cherry	2	8	9	442	25	486
strawberry	0	0	1	60	19	80
digital	1670	1683	85	5	4	3447
information	3325	3982	378	5	13	7703
count(context)	4997	5673	473	512	61	11716

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*}p_{*j}} \qquad ppmi_{ij} = \begin{cases} pmi_{ij} & \text{if } pmi_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W} \sum_{j=1}^{C} f_{ij}}$$

	computer	data	result	pie	sugar	count(w)
cherry	2	8	9	442	25	486
strawberry	0	0	1	60	19	80
digital	1670	1683	85	5	4	3447
information	3325	3982	378	5	13	7703
count(context)	4997	5673	473	512	61	11716

p(w=information,c=data) = 3982/111716 = .3399

p(w=information) = 7703/11716 = .6575

p(c=data) = 5673/11716 = .4842

$$p(w_i) = \frac{\sum_{j=1}^{C} f_{ij}}{N} \qquad p(c_j) = \frac{\sum_{i=1}^{W} f_{ij}}{N}$$

	p(w,context)					p(w)
	computer	**data**	**result**	**pie**	**sugar**	**p(w)**
cherry	0.0002	0.0007	0.0008	0.0377	0.0021	0.0415
strawberry	0.0000	0.0000	0.0001	0.0051	0.0016	0.0068
digital	0.1425	0.1436	0.0073	0.0004	0.0003	0.2942
information	0.2838	0.3399	0.0323	0.0004	0.0011	0.6575
p(context)	0.4265	0.4842	0.0404	0.0437	0.0052	

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*}p_{*j}}$$

	p(w,context)					p(w)
	computer	**data**	**result**	**pie**	**sugar**	**p(w)**
cherry	0.0002	0.0007	0.0008	0.0377	0.0021	0.0415
strawberry	0.0000	0.0000	0.0001	0.0051	0.0016	0.0068
digital	0.1425	0.1436	0.0073	0.0004	0.0003	0.2942
information	0.2838	0.3399	0.0323	0.0004	0.0011	0.6575
p(context)	0.4265	0.4842	0.0404	0.0437	0.0052	

pmi(information,data) = $\log_2$ (.3399 / (.6575*.4842)) = .0944

Resulting PPMI matrix (negatives replaced by 0)

	computer	**data**	**result**	**pie**	**sugar**
cherry	0	0	0	4.38	3.30
strawberry	0	0	0	4.10	5.51
digital	0.18	0.01	0	0	0
information	0.02	0.09	0.28	0	0

Weighting PMI

PMI is biased toward infrequent events

- Very rare words have very high PMI values

Two solutions:

- Give rare words slightly higher probabilities
- Use add-one smoothing (which has a similar effect)

Weighting PMI: Giving rare context words slightly higher probability

Raise the context probabilities to $\alpha = 0.75$:

$$\text{PPMI}_\alpha(w,c) = \max(\log_2 \frac{P(w,c)}{P(w)P_\alpha(c)}, 0)$$

$$P_\alpha(c) = \frac{count(c)^\alpha}{\sum_c count(c)^\alpha}$$

This helps because $P_\alpha(c) > P(c)$ for rare *c*

Consider two events, P(a) = .99 and P(b)=.01

$$P_\alpha(a) = \frac{.99^{.75}}{.99^{.75} + .01^{.75}} = .97 \quad P_\alpha(b) = \frac{.01^{.75}}{.01^{.75} + .01^{.75}} = .03$$

Vector Semantics & Embeddings

Word2vec

Sparse versus dense vectors

tf-idf (or PMI) vectors are

- **long** (length |V|= 20,000 to 50,000)
- **sparse** (most elements are zero)

Alternative: learn vectors which are

- **short** (length 50-1000)
- **dense** (most elements are non-zero)

Sparse versus dense vectors

Why dense vectors?

- Short vectors may be easier to use as **features** in machine learning (fewer weights to tune)
- Dense vectors may **generalize** better than explicit counts
- Dense vectors may do better at capturing synonymy:
  - *car* and *automobile* are synonyms; but are distinct dimensions
    - a word with *car* as a neighbor and a word with *automobile* as a neighbor should be similar, but aren't
- **In practice, they work better**

Common methods for getting short dense vectors

"Neural Language Model"-inspired models

- Word2vec (skipgram, CBOW), GloVe

Singular Value Decomposition (SVD)

- A special case of this is called LSA – Latent Semantic Analysis

Alternative to these "static embeddings":

- Contextual Embeddings (ELMo, BERT)
- Compute distinct embeddings for a word in its context
- Separate embeddings for each token of a word

Simple static embeddings you can download!

Word2vec (Mikolov et al)

https://code.google.com/archive/p/word2vec/

GloVe (Pennington, Socher, Manning)

http://nlp.stanford.edu/projects/glove/

Word2vec

Popular embedding method

Very fast to train

Code available on the web

Idea: **predict** rather than **count**

Word2vec provides various options. We'll do:

skip-gram with negative sampling (SGNS)

Word2vec

Instead of **counting** how often each word *w* occurs near "*apricot*"

- Train a classifier on a binary **prediction** task:
  - Is *w* likely to show up near "*apricot*"?

We don’t actually care about this task

- But we'll take the learned classifier weights as the word embeddings

Big idea: **self-supervision**:

- A word c that occurs near apricot in the corpus cats as the gold "correct answer" for supervised learning
- No need for human labels
- Bengio et al. (2003); Collobert et al. (2011)

Approach: predict if candidate word *c* is a "neighbor"

1. Treat the target word *t* and a neighboring context word *c* as **positive examples**.
2. Randomly sample other words in the lexicon to get negative examples
3. Use logistic regression to train a classifier to distinguish those two cases
4. Use the learned weights as the embeddings

Skip-Gram Training Data

Assume a +/- 2 word window, given training sentence:

…lemon, a [tablespoon of apricot jam, a] pinch…

c1 c2 [target] c3 c4

Skip-Gram Classifier

(assuming a +/- 2 word window)

…lemon, a [tablespoon of apricot jam, a] pinch…

c1 c2 [target] c3 c4

Goal: train a classifier that is given a candidate (**w**ord, **c**ontext) pair

(apricot, jam)

(apricot, aardvark)

…

And assigns each pair a probability:

$P(+|w, c)$

$P(-|w, c) = 1 - P(+|w, c)$

Similarity is computed from dot product

Remember: two vectors are similar if they have a high dot product

- Cosine is just a normalized dot product

So:

- Similarity(w,c) $\propto w \cdot c$

We'll need to normalize to get a probability

- (cosine isn't a probability either)

Turning dot products into probabilities

Sim(w,c) ≈ $w \cdot c$

To turn this into a probability

We'll use the sigmoid from logistic regression:

$$P(+|w,c) = \sigma(c \cdot w) = \frac{1}{1+\exp(-c \cdot w)}$$

$$\begin{aligned} P(-|w,c) &= 1 - P(+|w,c) \\ &= \sigma(-c \cdot w) = \frac{1}{1+\exp(c \cdot w)} \end{aligned}$$

How Skip-Gram Classifier computes $P(+|w, c)$

$$P(+|w,c) = \sigma(c \cdot w) = \frac{1}{1+\exp(-c \cdot w)}$$

This is for one context word, but we have lots of context words.
We'll assume independence and just multiply them:

$$P(+|w,c_{1:L}) = \prod_{i=1}^{L} \sigma(c_i \cdot w)$$

$$\log P(+|w,c_{1:L}) = \sum_{i=1}^{L} \log \sigma(c_i \cdot w)$$

Skip-gram classifier: summary

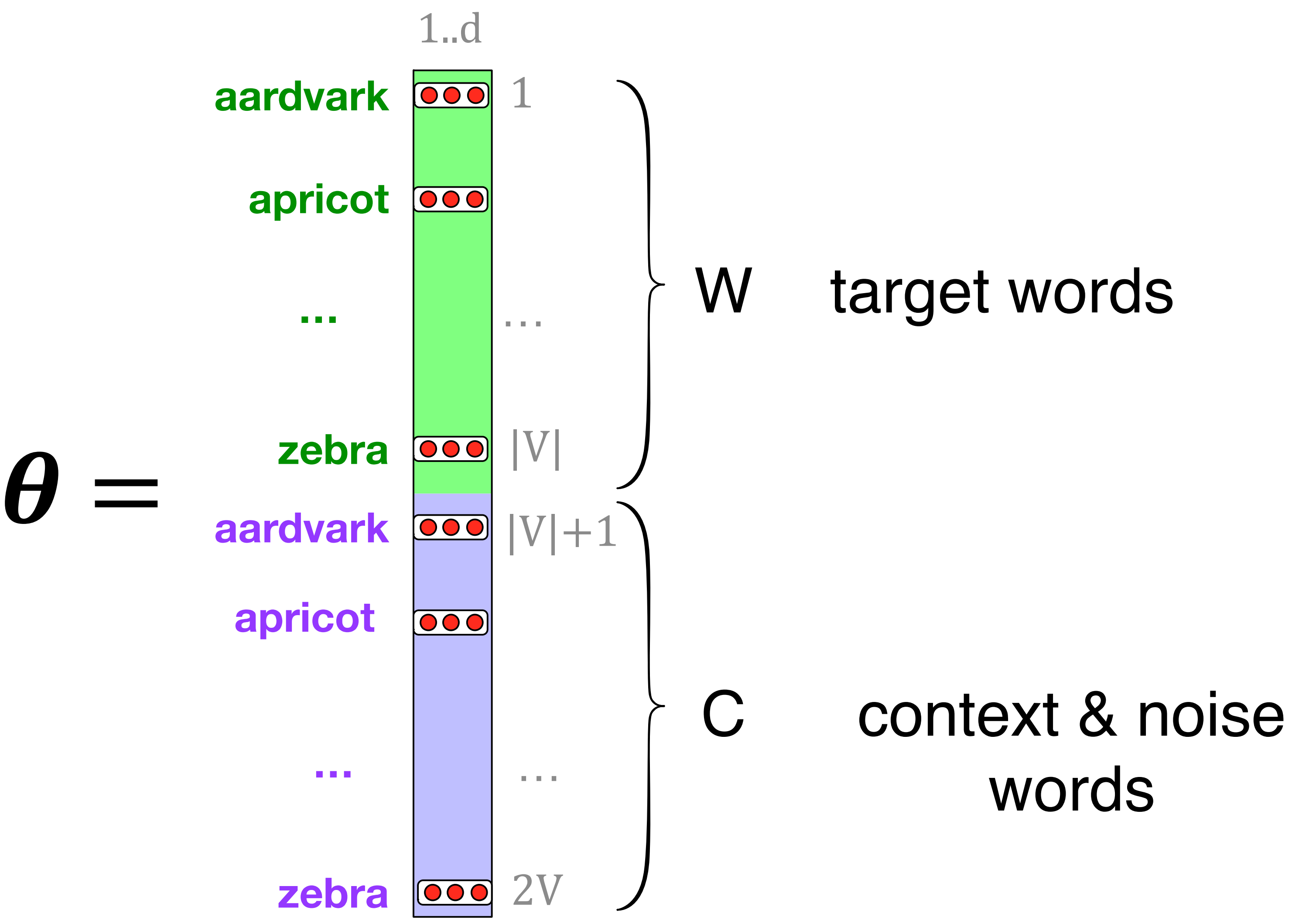
A probabilistic classifier, given

- a test target word w
- its context window of L words $c_{1:L}$

Estimates probability that w occurs in this window based on similarity of w (embeddings) to $c_{1:L}$ (embeddings).

To compute this, we just need embeddings for all the words.

These embeddings we'll need: a set for w, a set for c



Vector Semantics & Embeddings

Word2vec: Learning the embeddings

Skip-Gram Training data

…lemon, a [tablespoon of apricot jam, a] pinch…

c1 c2 [target] c3 c4

positive examples +

t	c
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

Skip-Gram Training data

...lemon, a [tablespoon of apricot jam, a] pinch...

c1 c2 [target] c3 c4

positive examples +

t	c
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

For each positive example we'll grab k negative examples, sampling by frequency

Skip-Gram Training data

…lemon, a [tablespoon of apricot jam, a] pinch…

c1 c2 [target] c3 c4

positive examples +

t	c
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

negative examples -

t	c	t	c
apricot	aardvark	apricot	seven
apricot	my	apricot	forever
apricot	where	apricot	dear
apricot	coaxial	apricot	if

Word2vec: how to learn vectors

Given the set of positive and negative training instances, and an initial set of embedding vectors

The goal of learning is to adjust those word vectors such that we:

- **Maximize** the similarity of the target word, context word pairs (w , c_{pos}) drawn from the positive data
- **Minimize** the similarity of the (w , c_{neg}) pairs drawn from the negative data.

Loss function for one *w with* c_{pos} , c_{neg1} ...c_{negk}

Maximize the similarity of the target with the actual context words, and minimize the similarity of the target with the *k* negative sampled non-neighbor words.

$$
\begin{aligned}
L_{\mathrm{CE}} &= -\log\left[P(+|w,c_{pos})\prod_{i=1}^{k}P(-|w,c_{neg_i})\right] \\
&= -\left[\log P(+|w,c_{pos})+\sum_{i=1}^{k}\log P(-|w,c_{neg_i})\right] \\
&= -\left[\log P(+|w,c_{pos})+\sum_{i=1}^{k}\log\left(1-P(+|w,c_{neg_i})\right)\right] \\
&= -\left[\log\sigma(c_{pos}\cdot w)+\sum_{i=1}^{k}\log\sigma(-c_{neg_i}\cdot w)\right]
\end{aligned}
$$

Learning the classifier

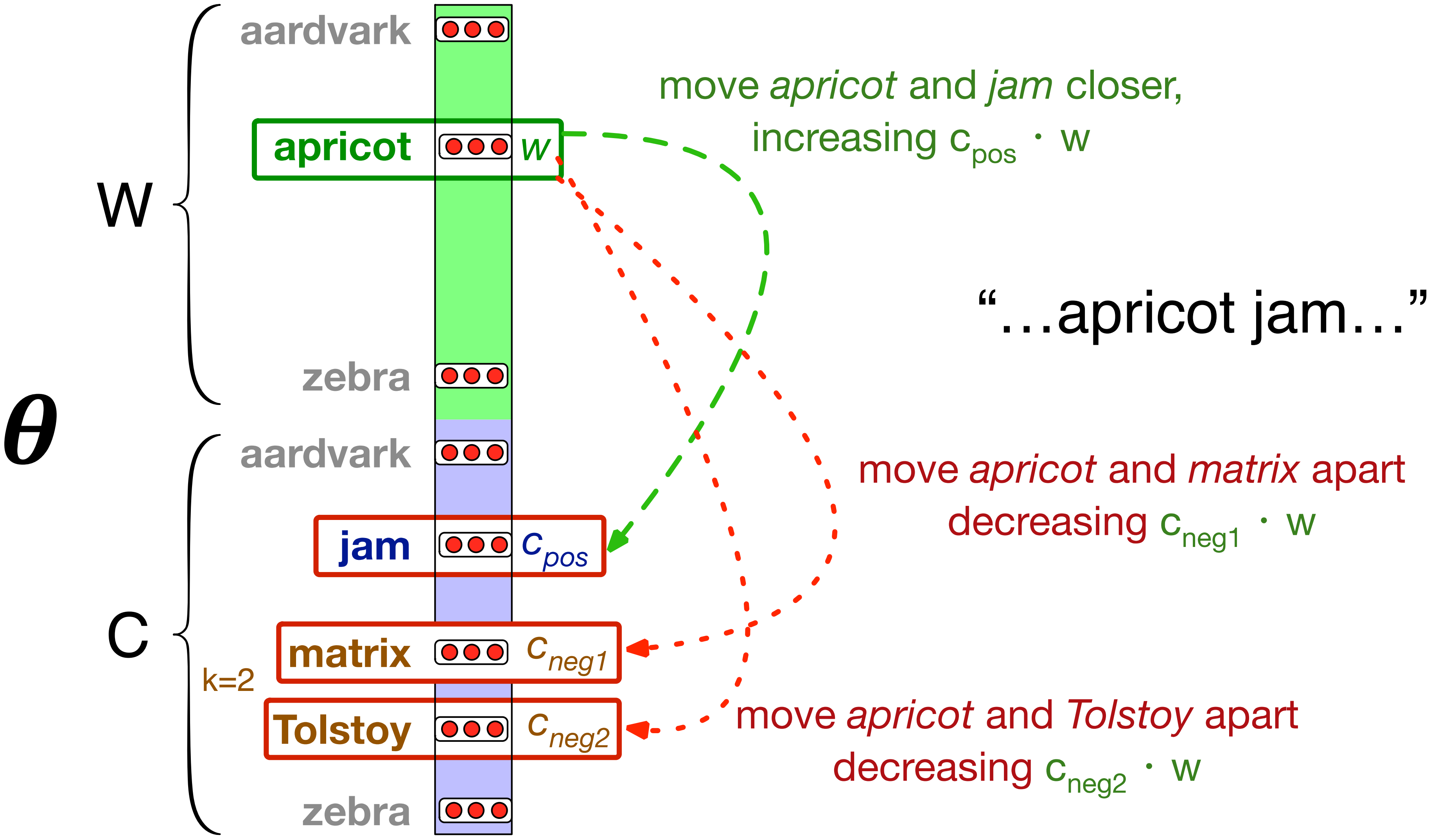
How to learn?

- Stochastic gradient descent!

We'll adjust the word weights to

- make the positive pairs more likely
- and the negative pairs less likely,
- over the entire training set.

Intuition of one step of gradient descent



Reminder: gradient descent

- At each step
  - Direction: We move in the reverse direction from the gradient of the loss function
  - Magnitude: we move the value of this gradient $\frac{d}{dw}L(f(x;w),y)$ weighted by a **learning rate** η
  - Higher learning rate means move *w* faster

$$w^{t+1} = w^{t} - h\frac{d}{dw}L(f(x;w),y)$$

The derivatives of the loss function

$$L_{CE} = -\left[\log \sigma(c_{pos} \cdot w) + \sum_{i=1}^{k} \log \sigma(-c_{neg_i} \cdot w)\right]$$

$$\frac{\partial L_{CE}}{\partial c_{pos}} = [\sigma(c_{pos} \cdot w) - 1]w$$

$$\frac{\partial L_{CE}}{\partial c_{neg}} = [\sigma(c_{neg} \cdot w)]w$$

$$\frac{\partial L_{CE}}{\partial w} = [\sigma(c_{pos} \cdot w) - 1]c_{pos} + \sum_{i=1}^{k} [\sigma(c_{\mathrm{neg}_i} \cdot w)]c_{neg_i}$$

Update equation in SGD

Start with randomly initialized C and W matrices, then incrementally do updates

$$
\begin{aligned}
c_{pos}^{t+1} &= c_{pos}^{t} - \eta[\sigma(c_{pos}^{t} \cdot w^{t}) - 1]w^{t} \\
c_{neg}^{t+1} &= c_{neg}^{t} - \eta[\sigma(c_{neg}^{t} \cdot w^{t})]w^{t} \\
w^{t+1} &= w^{t} - \eta\left[[\sigma(c_{pos} \cdot w^{t}) - 1]c_{pos} + \sum_{i=1}^{k}[\sigma(c_{\mathrm{neg}_i} \cdot w^{t})]c_{neg_i}\right]
\end{aligned}
$$

Two sets of embeddings

SGNS learns two sets of embeddings

Target embeddings matrix W

Context embedding matrix C

It's common to just add them together, representing word *i* as the vector $w_i + c_i$

Summary: How to learn word2vec (skip-gram) embeddings

Start with V random d-dimensional vectors as initial embeddings

Train a classifier based on embedding similarity

- Take a corpus and take pairs of words that co-occur as positive examples
- Take pairs of words that don't co-occur as negative examples
- Train the classifier to distinguish these by slowly adjusting all the embeddings to improve the classifier performance
- Throw away the classifier code and keep the embeddings.

CBOW