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CS112 Exam

1. Formal definition

A one-headed double-banded Turing machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$, where Q, Σ and Γ are all finite sets and:

1) Q is the set containing the ~~sets~~ states
2) Σ is the input alphabet (which does not contain the blanks symbol)
3) Γ is the tape alphabet containing the blanks symbol and $\Sigma \subseteq \Gamma$
4) $q_0 \in Q$ is the start state
5) $q_{accept} \in Q$ is the accept state
6) $q_{reject} \in Q$ is the reject state and $q_{accept} \neq q_{reject}$
7) $\delta : Q \times \Gamma \times \Gamma \rightarrow Q \times \Gamma \times \Gamma \times \{L, R\}$ is the transition function

Intuitive computation description

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{accept}, q_{reject})$ be a one-headed double-banded Turing machine.

- At first, M receives a single input $w = w_1 \dots w_n \in \Sigma^*$, which is copied on the leftmost n squares of both tapes, while the rest of the tapes' content is blanks (filled with blanks symbols)

- Initially, the head points to the leftmost square of both tapes. Since Σ does not contain the blanks symbol, the first blanks symbol appearing on both tapes at the same position marks the end of the input string
- Once started, M computes according to the rules described by the transition function. M is similar to a ~~two-headed~~ double-headed double-banded Turing machine, where both heads are synchronized.
- If M ever tries to move its head to the left of the leftmost square of the tape, the head will stay in the same place for that move (the first square), even though the transition function indicates 'L'.
- The computation ~~procedure~~ proceeds until M enters either the accept or the reject state, at which point it halts. If neither occurs, M goes on forever.
- Changes occur in the current state, the current tapes' contents and the current head location during the computation.
- A ~~tuo~~ tuple consisting of these four items is called a configuration of the Turing machine. Configurations are often represented in the following way: For a state 'q' and ~~four letters~~ strings 's', 'u', 'v', 'w' over the tape alphabet Γ, with 's', 'u' belonging to the first tape and, 'v', 'w' belonging to the second tape and $|s|=|v|$, $|u|=|w|$, we write squ for the configuration where the current state vqw for the is q,

the first tape's current content is su, the second tape's ~~con~~ current content is vw and the current head location is the first symbol of 'u' and the first symbol of 'w'.

- For example, $\begin{matrix}10101 q_7 0111\\ 10101 q_7 0111\end{matrix}$ represents the configuration when both tapes are '101010111', the current state is q_7 and the head is currently on the third ~~b~~ '0' of both tapes.

Formal computation description

- Configuration C_1 yields configuration C_2 (defined according to the intuitive computation description) if the Turing Machine can legally go from C_1 directly to C_2. (in a single step).

- Suppose we have $'a', 'b', 'c', 'd', 'e', 'f' \in \Gamma$, $u, v, w, x \in \Gamma^*$ (~~cond~~ $|u| = |w|$, $|v| = |x|$) and states q_i and q_j.

C_1: $\begin{matrix}uaq_ibv\\ wdq_iex\end{matrix}$ and C_2: $\begin{matrix}uq_jacv\\ wq_jdfx\end{matrix}$ are two configurations

C_1 yields C_2 if $\delta(q_i, 'b', 'e') = (q_j, 'c', 'f', L)$

- Special cases occur when the head is at one of the ends of the input string.

- The starting configuration of M on input w is: $\begin{matrix}q_0w\\ q_0w\end{matrix}$

which indicates that M is in the start state q_0, with its head pointing to the leftmost position of the tapes

- An accepting configuration's state is q_{accept}
- A rejecting configuration's state is q_{reject}
- Accepting and rejecting configurations are halting and do not yield further configurations
- M accepts the input string $w \in \Sigma^*$ if a sequence of configurations $C_1, C_2, \ldots, C_k$ exists so that:
  1) C_1 is the start configuration of M on input w
  2) each C_i yields C_{i+1}
  3) C_k is an accepting configuration