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FOREIGN TECHNOLOGY DIVISION

THEORY OF IRREGULAR WAVEGUIDES WITH SLOWLY CHANGING PARAMETERS

by

B.Z. Katsenelenbaum

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By: B.Z. Katsenelenbaum

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Table of Contents

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

THEORY OF IRREGULAR WAVEGUIDES WITH SLOWLY CHANGING PARAMETERS.

B. Z. Katsenelenbaum.

Page 2.

Monograph contains the systematic presentation of the method of calculation of fields in irregular radiowaveguides and acoustic waveguides. Are examined the bent waveguides, tapered welds and waveguides, filled by the material whose parameters are changed along the line, which in particular contain the compensating and matching inserts. Are studied in detail the curvatures of a large radius, flat changes in the section, slow change in the parameters of material.

Monograph is designed for scientific workers, graduate students and students of the old courses, which are occupied by electrodynamic calculations and adjacent questions of mathematical physics, and also to specialists, workers in waveguide communication/connection.

Responsible editor of the assoc. member of the AS USSR V. I. Siforov.

Page 3.

Introduction.

§ 1. Content of monograph.

1. Theory of regular waveguides, i.e., rectilinear waveguides of constant section, is developed at present very fully. There are also many monographs and textbooks, in which in systematic form are set forth the methods of calculation of the effect of different irregularities in rectangular waveguide on the propagated in it fundamental wave. These irregular cell/elements - the coupling elements and tuning - bear usually the local character: they are concentrated in the region of the order of the transverse size/dimensions of rectangular waveguide. These cell/elements provide agreement, i.e., the absence of considerable reflection, in the relatively narrow frequency band. In this frequency band, as a rule, wave of the highest types be propagated they cannot.

In the last 5-10 years appeared the problem of developing of the plumbing, capable of passing the very broadband, order of one octave. An example of this problem is the problem of long-distance

communication in circular waveguide on wave H_{01} [1]. The matching plumbing in such waveguides no longer can be, as a rule, local tuning cell/elements - diaphragm, stubs, etc. Typical irregular cell/element in this waveguide is the steady waveguide transition whose length is great in comparison with the transverse size/dimensions of waveguide. The parameters of this waveguide are slowly changing functions of one of the coordinates. To this class of systems they are related, for example, the bent waveguide, whose bending radius is great in comparison with cross section, or long transition between two waveguides of different sections, long joining, etc. Waveguide cell/elements of such type possess large band coverage.

Page 4.

In broadband plumbing together with fundamental, useful wave, can be propagated, as a rule, also the waves of other types, the so-called parasitic waves. On the irregular cell/elements of these circuits, not only must not occur the noticeable reflection of the incident wave, but must not be also the considerable transformation of the fundamental wave into parasitic ones. Usually most essential proves to be the second requirement - smallness of losses to transformation.

The calculation of irregular waveguides with the slowly changing

parameters required the development of special mathematical methods. To what extent is great interest in this problem, it is evident from the fact that in last/latter 5-6 years in our and foreign press appeared about 60 theoretical works on this theme, and to 1954-1955 to it were devoted a total of several various articles.

In the most typical cases electrodynamic problem is placed as follows: in the irregular section, which connects two generally speaking different regular ones of waveguide, falls any wave from single amplitude; it is necessary to calculate the amplitudes of all waves, which diverge into both of sides from irregular section. The totality of these all composite amplitudes occasionally referred to as the scattering matrix of irregular section. Great interest is of, it goes without saying, also inverse problem - identification of the parameters, which ensure the smallest ones in the assigned frequency band of the loss to transformation.

Present monograph is dedicated to systematic presentation of one of the methods of the solution of the class of the electrodynamic problems, the so-called method of cross sections indicated.

2. Monograph consists of six chapters. Chapter I contains the presentation of the auxiliary method of small heterogeneities. Any how conveniently complex irregularity of waveguide (retaining the

topological structure of its section) can be considered as the imposition of three fundamental types of irregularities - waveguide bend, change in the properties of the filling waveguide medium and change in its cross section. In three paragraphs of chapter I, are investigated the elementary irregularities of each of these three types, i.e., the fracture of waveguide on small angle, small abrupt change in the filling, small step. The determination of the wave amplitudes, scattered on elementary irregularity, makes it possible to incidentally determine tolerances for the production of waveguide lines.

Page 5.

The waveguide, bent to final angle, can be treated as the maximum form of waveguide with many small fractures. In exactly the same manner waveguide with alternating/variable filling is the limit of waveguide with laminar filling, and waveguide with alternating/variable section - a limit of stepped waveguide. The analysis of an elementary irregularity of any type makes it possible therefore in a number of cases to calculate the amplitudes of waves, scattered on a final (not small) irregularity of this type. This calculation method is not completely strict, but it possesses the specific physical clarity, but in many problems it makes it possible even to obtain complete solution. It can be considered as the

physical interpretation of the mathematical apparatus of the fundamental method, named us by the method of cross sections.

This method is presented in chapter II. Its fundamental idea lies in the fact that the field in any section of irregular section is represented in the form of the infinite sum of the fields of waves of both of directions, capable of being propagated of the so-called waveguide of comparison - in the regular waveguide of the same section and with the same distribution of electrical and magnetic permeability over section. The coefficients of this expansion are the functions of longitudinal coordinate and satisfy the infinite system of the ordinary differential equations of the first order. The investigation of irregular waveguide, i.e., three-dimensional electrodynamic problem, is reduced, thus, to the two-dimensional problem of the fields of waves in regular waveguide and to one-dimensional problem - to the solution of the system of ordinary differential equations.

The greatest difficulties during the application/use of a method of cross sections appear for waveguides with alternating/variable section. Fields in irregular waveguide satisfy other boundary conditions, than fields in the regular waveguides of the same section, and the row/series, comprised on the fields of these waves, on the duct/contour, which limits section, generally speaking, not

converge to the functions which they must represent. Therefore solution is constructed first for the waveguide of the constant section, filled by medium with the continuous distribution of dielectric constant $\varepsilon(x, y, z)$, then is realized transition to the discontinuous distribution during which the part of the waveguide remains empty (see Fig. 10, page 74), and then - to composite ε when $|\varepsilon| \to \infty$. In this way it is possible to avoid process/operations with the unevenly converging series which cannot be piecemeal differentiated.

Page 6.

The basic values, which characterize heterogeneity, are coupling coefficients - coefficients in the system of differential equations for wave amplitudes. The properties of these coefficients are investigated in detail. These coefficients can be also found from the matrix elements of scattering from a small irregularity, calculated in chapter I. The method of cross sections gives explicit expressions for coupling coefficients for any irregularity, in particular for the combined irregularity.

In application to the steady irregularities, in which the parameters, which characterize waveguide, are changed slowly, to easy solve the system of differential equations and to find explicit

expression for the wave amplitudes, scattered by the irregular section: it it is analyzed thoroughly in detail in chapter II. There are two special cases, examined in chapter III, when this solution - even for very steady irregularities - becomes complicated. In § 11 and 12 this chapter is investigated the case, when in tapered weld is the so-called critically section, i.e., the section, which separate/liberates, at this frequency, the region of the propagation of any wave from the region where it be propagated cannot. Near this section the coupling coefficients during how conveniently slow change in the parameters of waveguide become high values, and usual methods of the type of Wentzel - Kramers - Brillouin (WKB) prove to be inapplicable. It is establish/installed, in particular, the end condition, equivalent to the presence of critical section and which makes it possible to be limited to the solution of differential equations in the region, distant from critical section.

In § 14 of Chapter III is examined the second special case - incidence in the wave on the fracture of waveguide when the frequency is close to the critical frequency of the excitable parasitic wave. In this case, appear the resonance effects, and the amplitude of this parasitic wave can become relative to greater. These effects it depends substantially on the conductivity of the material of wall. For their analysis it is necessary to utilize the expression for a wave number in waveguide with imperfect walls, used, in particular,

in any nearness to critical frequency; this expression is derived in § 15.

In chapters IV and V the developed method is used to concrete/specific/actual waveguide systems. In chapter IV, are presented the problems of irregular waveguides with rectilinear axis, in chapter V, - about the bent waveguides. In certain cases is given also solution of the reverse problem of the optimum form of transition, curvature or compensating insert. The material of these chapters does not bear reference character, it must only illustrate the possibilities of method and the contemporary state of a question.

Page 7.

In chapter VI formalism of theory is transferred to the case of the irregular acoustic waveguides, rectilinear and bent. In some ratio/relations this transference proved to be not so trivial, as this it was possible to assume, on the basis of usual relationships between problems for vector and scalar fields.

§ 2. Survey of literature.

Is published at the present time about 100 works on the theme, formulated in the name of the book. At the end of the book, is given

the list of these works, led approximately to the middle of 1960; the articles, published up to 1950, in it barely are reflected.

In this paragraph we will give very short survey/coverage of different methods, which were being applied during the solution of the problems of this class. Articles are grouped according to the used in them method, that it is somewhat arbitrarily, but it is the very convenient method of the classification of material. We isolated four methods, and the last of them (we call its method of cross sections) let us dismantle/select in somewhat more detail.

In this survey/coverage are not included the articles of the author and work, which adjoin them, this material is presented in book itself.

1. Method of join of fields lies in the fact that field in regular and irregular parts of waveguides is represented in the form of sum of waves, capable of existing in both waveguides, and from requirement of continuity of fields is system of linear algebraic equations for coefficients of these sums. If irregularity is small, then problem contains series expansion parameter, and system can be solved in general form. Method is limited by the condition so that the field in irregular waveguide would have sufficiently simple structure.

For rectilinear waveguides the method makes it possible to calculate transition to cone with small flare angle or (for rectangular waveguides) to small expansion. By this method were found the coefficients of reflection of wave from the expansion of rectangular waveguide - works of Levin [2] [1], Piefke [3] and Solymar [4].

FOOTNOTE [1]. The part of Levin's results [2] is erroneous; see [3] or § 16 this book. ENDFOOTNOTE.

Equivalent method was used by N. P. Mar'in [5] for determining the conversion factors and reflection during the expansion of rectangular waveguide in E-plane. To circular waveguides, the method is used by Solymar [6, 7], which found the conversion factor of wave H_{01} into H_{02}, and Tanaka [8] whose formulas make it possible to find the conversion factors and reflection during incidence in any wave.

In the waveguide, bent on circular arc, it is possible to introduce the so-called their own waves and the complete field to present in the form of the superposition of such their own waves of the bent waveguide. Utilizing this expansion, Jouguet [9] by the method of join solved the problem of coupling of rectilinear and bent

(with constant curvature) the waveguides of rectangular cross section.

2. Conformal transformations were widely used by P. Ye. Krasnushkin [10] for calculation of flat/plane waveguides. In this method the complex boundary of irregular waveguide is converted into two parallel lines. The wave equation, which describes field in waveguide, in this case becomes complicated and acquires this form, as if within these parallel lines was arrange/located inhomogeneous medium. The means of this heterogeneity in a known manner is connected with the function, which realizes conformal transformation. Rice [11], B. L. Rozhdestvenskiy [12] and N. P. Mar'in [13], applying different reception/procedures for the solution of wave equation with variable coefficients, they examined by this sequence method of problems. B. L. Rozhdestvenskiy and D. N. Chetayev [14] used him to the problem of the creation of the matching transitions with dielectric filling. Conformal transformation was used also by L. A. Weinstein in article [15]; in this work the problem of flat irregularity in flat/plane waveguide was solved with the enlistment of variation principles.

The method of conformal transformations can be, apparently, it is generalized to rectilinear circular waveguides, although this causes its essential complication. For more complex problems, for

example for rectangular waveguide, which is expanded simultaneously in two planes, it is virtually unsuitable.

3. In many works is utilized coordinate system in which walls of waveguide coincide with one of coordinate surfaces. During the construction of this system in the general case (i.e., not for flat/plane waveguides) it is not possible to apply conformal transformation and it is necessary to resort to special reception/procedures.

Jouguet [16] examined by this method the waveguide bend of round cross-section throughout the circumference of the large radius r. In the introduced to them system of coordinates of the equation of Maxwell, acquire supplementary in comparison with Cartesian system term/component/addends, proportional to curvature. These term/component/addends have a character of outside currents, created by the transmitted wave, and problem is reduced to the solution of the equations of Maxwell with right side. In [16] were found their own waves of the bent waveguide and it was obtained, in particular, fundamental in the theory of the circular waveguide result about transformation in the curvature of wave H_{01} into wave E_{11}.

Page 9.

However, the determination of the amplitudes of all other scattered waves, i.e., the values of order 1/r, was not produced; it requires still supplementary process/operations on the join of fields on the boundaries of the bent section. This same method adjoins Levin's article [17], in which are found propagation constant of their own waves of twisted and bent rectangular waveguides. In this article also is not produced the join of fields on the boundary of irregular section and are not determined the amplitudes of the scattered waves.

In works of A. G. Sveshnikov [18-20] and S. L. Viktorova and A. G. Sveshnikov [21] this method is far moved and used to the decision of the series of problems of more common/general/total type - of waveguide bends in three-dimensional curve with a simultaneous slow change in his cross section. The special feature/peculiarity of these works is the use of regular methods of solving the nonhomogeneous equations of Maxwell, to whom is reduced the problem. Which follow work of Sveshnikov [22, 23] even more greatly expand the possibilities of applying this apparatus. In the introduced to them nonorthogonal curvilinear coordinate system, the surface of the assigned waveguide is converted into the surface of the cylinder of a single radius. The solution of the equations of Maxwell in this system is conducted as in [21], by the method, which adjoins the method of cross sections; field searches for in the form of

row/series on the products of Bessel functions to trigonometric functions, and for the coefficients of this expansion is establish/installed the system of ordinary differential equations, which contains, in particular, the metric coefficients of the adopted system of coordinates.

Another fruitful idea for the calculation of rectilinear tapered welds, base of which is also the introduction of the special coordinate system, was proposed in the article of V. L. Pokrovskiy, F. R. Ulinich, S. K. Savvinykh [24] about flat/plane waveguide. According to this method is introduced the coordinate system, orthogonal with an accuracy to the square of the mean angle of the slope/inclination of generatrix. This value is series expansion parameter of problem, and the wave equation, recorded in this coordinate system, is decompose/expanded in row/series from this parameter. Are solved the equations of zero and first order. In zero order is obtained homogeneous equation, in the first - heterogeneous. The solutions search for according to method WKB. The coefficients of reflection and transformation depend on the character of the function, which describes the form of generatrix; the order of the smallness of these values is determined by degree of smoothness of this function.

Page 10.

The method of article [24] was recently used to the sufficiently wide circle of questions [24-31]. Were designed infinite horns, transitions between two waveguides, waveguides with forming, described both the analytic functions and functions whose one of the derivatives suffers disruption; were investigated problems with critical sections. Were examined in essence waveguides flat/plane [24-29], also, with circular section [30]; in work [31] was made the attempt to calculate by this method also the waveguides of rectangular cross section.

4. Is published at present about 50 articles, in which are develop/processed and are applied to specific problems diverse variants of method of cross sections; large part of these works appeared after 1955. The fundamental idea of this method lies in the fact that the field in irregular waveguide is represented in the form of the superposition of the fields of waves, which exist in simpler waveguides. The coefficients of these superpositions satisfy the system of ordinary differential equations. From the solution of this system, are determined the wave amplitudes, scattered by irregular section.

The first ideas in this plan/layout belong to G. V. Kisun'ko

[(for example, see his monograph [32]), and A. L. Gutman's article [33-36] and B. F. Yemelin [37] they develop these ideas. Outside boundary there was first Stevenson's article [38]; however, the energetic/energy application/use of this method was begun there only after the appearance of an article of Schelkunoff [39].

In Stevenson's work [38] were examined rectilinear tapered welds. Field was expressed as six functions each of which was decompose/expanded according to the membrane/diaphragm functions of electrical and magnetic waves in the regular waveguide of this section. For the coefficients of these expansions, was establish/installed the system of the ordinary differential second order equations, which then were investigated according to method WKB. The obtained mathematical apparatus proved to be very complex and bulky. The only finished to end/lead attempt to use it to specific problems for determining the field, scattered by an irregular section, is made in Leonard's article and yen [40]. In this work are calculated the coefficients of reflection of several waves from coupling of rectilinear circular waveguide with cone and from expansion in rectangular waveguide; formulas for rectangular waveguide are accurate, but formulas for a circular waveguide proved to be erroneous.

Page 11.

In the article of Schelkunoff [39] the method of cross sections is proposed anew and is illustrated based on the examples of that expanding and bent on the circular arc of flat/plane waveguides. In Heyn's work [41] is also produced field expansion of waves in the bent waveguide of constant section in terms of the fields of waves in rectilinear waveguides. In both these articles for the coefficients of expansion, are establish/installed differential first-order equations; however, the solutions of these differential equations and expression for the amplitudes of the scattered waves do not bring.

In the article of Unger [42] the apparatus, proposed by Schelkunoff, is used to the specific problem of the symmetrical transition between two circular waveguides, on which falls wave H_{01}. As variables are accepted the wave amplitudes of both of directions, but not Fourier coefficients field expansion as in [39] and [41], and are obtained explicit expressions for the amplitudes of the scattered forward waves H_0. In the article of Iiguchi [43] it is made the attempt to calculate combined transition from rectangular to the circular waveguide.

In the articles of Morgan [44], Shimizu [45] and Oguchi and Kato [46] the same method are found differential equations for wave

amplitudes in curvature and are determined . ~ coupling coefficients of wave H_{01} with H_{m1} and E_{11} in the bent circular waveguide. In the article of Andreasen [47], that develops the work of Morgan [44], are calculated the coupling coefficients of wave H_{11} with waves E_{01} and H_{21} in the circular waveguide and wave H_{10} with several waves in the waveguide of square section. In the articles of Unger [48] and Andreasen [49] was solved for the first time the problem of the curvature of alternating/variable curvature. However, the method, used by the authors, forced them to be restricted to the case when curvature in all points was final, i.e., to exclude the fractures of the axes whose application/use, as it seemed, they make it possible to design shortest curvatures.

Apparently independent of Schelkunoff, but by approximately the same method examined rectilinear tapered welds Reiter [50]. His results were then used to the calculation of concrete/specific/actual systems in the article of Solymar [51], that gave to them most convenient form, and in the works of Schnetzler [52, 53], which calculated the joining of the waveguide of square section and simple transition from rectangular waveguide to circular.

In the works of Gutman [33-36] also are investigated rectilinear tapered welds. Their special feature/peculiarity in comparison with the works of Schelkunoff [39] and Reiter [50] and the subsequent

works, which are based on [39] and [50], is the introduction of the special coordinate system and another method of the conclusion/derivation of differential equations. The article of Yemelin [37] generalizes method to the case when simultaneously it changes both the form of section and the direction of axis.

Page 12.

The article of G. Ya. Lyubarskiy and A. Ya. Pevzner [54] can be also considered as the version of the method of cross sections. In it the field is record/written in the form of row/series, but not on the fields of waves in regular waveguide, but on the fields of waves in the cone, formed by tangents to the duct/contour of this section. For the coefficients of expansion, is establish/installed the system of differential equations; the low parameter in this system proves to be not the angle of the slope of generatrix, but derivative this angle. If in irregular section the angle of the slope of generatrix slowly is changed with coordinate z, then it is possible to obtain integral expression for the wave amplitudes, scattered by this section, and for the validity of the approach/approximation of the first order the angle must not be compulsorily small. However, this is reached because of the very essential complication of expressions for coupling coefficients. Method is applicable only for flat/plane or cylindrical waveguides, and in the article indicated it is not yet

obtained concrete/specific/actual results, although is solved not the electrodynamic, but simpler acoustic problem.

The first work on waveguides with variable filling was the article of Schelkunoff [55]. In it, strictly speaking, consecutively were examined only regular waveguides in which ε and μ the filling medium they depend on coordinates in cross section x, y and do not depend on the longitudinal coordinate z. Field in such waveguides was decompose/expanded in row/series on fields in empty waveguide, and for the coefficients of this expansion was establish/installed the system of differential first-order equations. In this case, each term of expansion represented field in very simple system, on even in regular waveguide the field was described by infinite system of equations. The same idea in the work of Morgan [44] was used to the bent waveguide, and in V. B. Brodskiy's article [56] - to the rectilinear waveguide of constant section, in which ε and μ they depend on all three coordinates.

There are, it goes without saying, works, which according to the methods used only with large stipulations can be referred to one of four groups indicated. In them is utilized any method, specially developed for this problem. Such works include, for example, already mentioned article of Weinstein [15] or of article of Barlow [57, 58] and Marie [59]; in [57-59] are investigated the conditions of the

fact that this wave (for example, wave H_{01} in the circular waveguide) is its own wave for a curvature. In the formulation of the problem of Barlow's article, they are close to the article of Rozhdestvenskiy and Chetayev [14].

Page 13.

As can be seen from this survey/coverage, energetically are develop/processed at present several directions. Each of them can be connected with the specific work, in which were laid the bases of the corresponding method. Such works we consider the book of Kusin'ko [32] and article of Schelkunoff [39], Sveshnikov [18], Pokrovskiy, etc. [24] and Reiter [50].

In present monograph we do not give the presentation of these directions, we do not investigate a question concerning strictness, validity and completeness of the obtained results, and also we do not compare these directions between themselves and with the direction, which were being developed in author's articles, and will be restricted to given short survey/coverage. This book is written in essence based on materials of author's articles [60-78], published during the years 1953-1961; in it used also several works, which develop the results of these articles, and are given author's some unpublished results. According to our opinion, as a whole in these

works is given the relatively complete theory of a question. However, wherever this it was necessary for the completeness of presentation, we included the results also other authors.

In the text of the book, are given the references to all used works, eliminating articles [60-78]. We did not refer to the works, in which the results, which were being contained in [60-78] or other articles, used in the book, were obtained simultaneously or later than in these articles. However, in those all cases when any results were obtained by other authors earlier, in text were given the corresponding references.

§ 3. Regular waveguide.

1. In this paragraph are summarized fundamental properties of waves in regular waveguides and are given designations which we used. Presentation in it most of all adjoins Weinstein's monograph [79].

By regular ones we call the rectilinear waveguides whose all properties are not changed along the axis of waveguide. In the theory of irregular waveguides by us will be necessary the examination of the regular waveguides of very general view. We let us assume that the cross section is limited by the arbitrary locked duct/contour and that the properties of the medium, which fills waveguide, are changed

in cross section. The waveguides of comparison will be, generally speaking, precisely such regular waveguides.

Are examined the steady solutions of the equations of Maxwell

$$\operatorname{rot} E = -ik\mu H; \quad (3.1a)$$

$$\operatorname{rot} H = ik\varepsilon E. \quad (3.1b)$$

Page 14.

Dependence on time is accepted in the form $e^{i\omega t}$, k - wave number in void, equal to ω/c, where c - speed of light. Is applied the Gaussian system of unity. Dielectric and magnetic constants ε and μ - dimensionless quantities, equal to one in void. Unless otherwise stated, then ε and μ - real scalar quantities.

In (3.1), are not introduced outside currents. It is assumed that they are located out of the sections in question and that the fields are excited by the waves, falling on these sections.

With exception only § 13 and 14, the metallic walls of waveguides are considered as ideally carrying out, so that on them

$$E \cdot t = 0, \quad (3.2)$$

where t - tangential to surface vector. Since is allow/assumed

arbitrary dependence ε on coordinates in cross section, then (3.2) does not limit the class of the examined waveguides.

Let us introduce the Cartesian coordinate system, x, y, z, and the coordinate axis, parallel to the axis of waveguide, let us call/name Z-axis. In cross section instead of the Cartesian coordinates x, y, can be introduced the polar coordinates ρ, β (β is counted off from x axis or is used any other two-dimensional reference grid.

The field of any wave in regular waveguide depends on z means of factor e^{-ihz}, where h - wave number in waveguide. At frequency is higher than the critical for a wave of this type $h^2>0$, at the critical frequency $h^2=0$, at the frequency of lower than critical $h^2<0$.

We will use also given dimensionless wave number $\hbar=h/k$. For the propagated waves $0\leqslant\hbar\leqslant1$. At high frequencies $\hbar$ it is close to unity, at critical frequency $\hbar=0$.

Let us introduce complete wave system, capable of existing in this irregular waveguide, including both the those propagating ($h^2>0$) and those not running ($h^2\leqslant0$). Let us label these waves by index j, which takes all values *t -∞ to +∞.

The fields of these waves let us designate $\hat{E}^j(x, y, z)$ and $\hat{H}^j(x, y, z)$. Let us introduce designations E^j, H^j for that factor in the fields of these waves, which depends only on x, y, i.e., let us designate

$$\hat{E}^j(x, y, z) = E^j(x, y)e^{-ih_j z}, \quad \hat{H}^j(x, y, z) = H^j(x, y)e^{-ih_j z}. \quad (3.3)$$

Page 15.

Vectors E^j, H^j they satisfy six equations, which are obtained with substitution (3.3) into the equations of Maxwell (3.1):

$$\begin{gathered}\frac{\partial E_z^j}{\partial y} + ih_j E_y^j = -ik\mu H_x^j; \quad \frac{\partial E_z^j}{\partial x} + ih_j E_x^j = ik\mu H_y^j;\\ \frac{\partial E_y^j}{\partial x} - \frac{\partial E_x^j}{\partial y} = -ik\mu H_z^j;\end{gathered} \quad (3.4a)$$

$$\begin{gathered}\frac{\partial H_z^j}{\partial y} + ih_j H_z^j = ik\varepsilon E_x^j; \quad \frac{\partial H_z^j}{\partial x} + ih_j H_x^j = -ik\varepsilon E_y^j;\\ \frac{\partial H_y^j}{\partial x} - \frac{\partial H_x^j}{\partial y} = ik\varepsilon H_z^j.\end{gathered} \quad (3.4b)$$

Each wave can be propagated (when $h_j^2 > 0$) or attenuate (when $h_j^2 < 0$) in two opposite directions. To two waves, which are distinguished only by direction of propagation, we will appropriate the indices, equal in magnitude and opposite on sign. If we in this case assume

$$h_{-j} = -h_j, \quad (3.5)$$

then relationship/ratios (3.3) will be valid for waves of both of directions. Let us agree to consider that if $h_j^2>0$, the with j>0 wave number is positive $h_j>0$, and if $h_j^2<0$, then with j>0 $h_j=-i|h_j|$. Then waves with positive indices will be propagated in the positive direction of Z-axis, of wave with negative indices - in the negative direction of Z-axis. The first we will call/name direct waves, the second - reverse/inverse. The components of direct/straight and backward waves we will connect with the relationship/ratios

$$\begin{gathered} E_x^{-j}=-E_x^j,\ E_y^{-j}=-E_y^j,\ E_z^{-j}=E_z^j; \\ H_x^{-j}=H_x^j,\ H_y^{-j}=H_y^j,\ H_z^{-j}=-H_z^j. \end{gathered} \tag{3.6}$$

In this case, is provided invariance (3.4) relative to the sign of index - if the fields of direct wave satisfy equations (3.4), then, according to (3.5) and (3.6), them they will satisfy also the fields of backward wave. Would be possible and another communication/connection, differing from (3.6) exchange E and H.

Page 16.

The fields of different waves are orthogonal between themselves. The conditions of orthogonality we will record in such a way that it would be correct during any combination of the signs of the indices of different waves, in particular for two counter waves of one and the same type. The application/use of a condition of orthogonality in

this form and invariance (3.4) relative to the sign of index will make it possible subsequently not to make any special stipulations relative to the direction of propagation of waves.

According to formula (79.11) from [79], the integral

$$\int\{[H^{m}E^{l}]_z+[H^{l}E^{m}]_z\}\,dS\equiv \equiv\int(E_y^lH_x^m-E_x^lH_y^m-E_x^mH_y^l+E_y^mH_x^l)\,dS \tag{3.7}$$

is equal to zero, if $h_m \neq h_l$. In (3.7), as it is everywhere lower, integral $\int \ldots dS$ is undertaken according to the cross section of waveguide. Equality zero (3.7) it is easy to obtain also directly from (3.4), (3.2). As is known, it is retained during replacement (3.2) by Leontovich's condition.

We will take for fields E^l, H^l the following standardization:

$$\int(E_y^lH_x^l-E_x^lH_y^l)\,dS=kh_l. \tag{3.8}$$

Right side (3.8) is selected by such shape, in order to standardization condition for the membrane/diaphragm functions [see below (3.16)] it assumed the simplest and customary form.

The condition of orthogonality and (3.8) it is possible to record in the form one condition

$$\int (E_y^j H_x^m - E_x^j H_y^m - E_x^m H_y^j + E_y^m H_x^j) dS = 2kh_j \delta_{mj}, \qquad (3.9)$$

where $\delta_{mj}=0$ when $m \neq j$, $\delta_{jj}=1$. By equations (3.4) and (3.9) the fields of normal waves in regular waveguide are determined unambiguously (with an accuracy to sign).

The energy flow, transferred through the cross section by the normal wave of number j (with an accuracy to common/general/total to all waves of unessential factor) is equal for real ones h_j is simple h_j. We will speak, that the wave amplitude is equal to P_j, if its field essence $E=P_j\hat{E}^j$, $H=P_j\hat{H}^j$; in this case energy flow is equal, again with an accuracy to this factor,

$$|P_j|^2 h_j. \qquad (3.10)$$

Speaking below about energy losses to transformation into the wave of any number, we always will imply the ratio/relation of values (3.10), calculated for this wave and for the incident wave.

Page 17.

2. If $\varepsilon=1$, $\mu=1$, i.e., for empty waveguides, field E^j, H^j they can be expressed through two membrane/diaphragm functions $\psi^j(x, y)$ and $\varphi^j(x, y)$; these functions are proportional to longitudinal components of magnetic and electric vectors of hertz. Functions ψ^j

and φ^j satisfy the identical equations

$$\nabla^2\psi^j + \alpha_j^2\psi^j = 0; \qquad (3.11a)$$

$$\nabla^2\varphi^j + \alpha_j^2\varphi^j = 0, \qquad (3.11b)$$

where ∇^2 the two-dimensional (in the variables x, y) operator of Laplace. Values α_j essence the eigenvalues of equation (3.11) under boundary boundary conditions of the cross section

$$\frac{\partial\psi^j}{\partial n} = 0; \qquad (3.12a)$$

$$\varphi^j = 0 \qquad (3.12b)$$

Here, as is everywhere lower, n - external normal to the duct/contour of the section of regular waveguide. Between values α_j^2, depending only on the geometry of cross section, and by wave numbers k and h_j there is relationship

$$h_j^2 = k^2 - \alpha_j^2. \qquad (3.13)$$

The eigenvalues of systems (3.11a), (3.12a) and (3.11b), (3.12b), generally speaking, do not coincide.

There is division into magnetic and electrical waves. The fields of magnetic waves are expressed as function ψ^j

$$E_x^j = -ik\frac{\partial\psi^j}{\partial y};\ E_y^j = ik\frac{\partial\psi^j}{\partial x};\ E_z^j = 0;$$
$$H_x^j = -ih_j\frac{\partial\psi^j}{\partial x};\ H_y^j = -ih_j\frac{\partial\psi^j}{\partial y};\ H_z^j = \alpha_j^2\psi^j. \qquad (3.14a)$$

The fields of electrical waves are expressed as function φ^j

$$E_x^j = -ih_j \frac{\partial\varphi^j}{\partial x};\quad E_y^j = -ih_j \frac{\partial\varphi^j}{\partial y};\quad E_z^j = \alpha_j^2\varphi^j; \qquad (3.14b)$$

$$H_x^j = ik\frac{\partial\varphi^j}{\partial y};\quad H_y^j = -ik\frac{\partial\varphi^j}{\partial x};\quad H_z^j = 0.$$

Page 18.

We will assume

$$\varphi^{-j} = \varphi^j,\ \psi^{-j} = -\psi^j. \qquad (3.15)$$

Then relationship/ratics (3.14) are also invariant relative to the sign of index.

Expressions (3.14) satisfy, as it is easy to check, the condition of orthogonality; equality zerc integrals (3.7) with i≠j it follows from the normal conditions of the orthogonality of different solutions of systems (3.11), (3.12). The accepted for fields E^j, H^j standardization leads for functions ψ^j and φ^j to the conditions

$$\int(\nabla\psi^j)^2 dS = 1,\quad \int(\nabla\varphi^j)^2 dS = 1, \qquad (3.16)$$

where ∇ the operator of two-dimensional gradient; to usually more simply apply equivalent ones, according to (3.12), the condition

$$\alpha_j^2\int(\psi^j)^2 dS = 1,\quad \alpha_j^2\int(\varphi^j)^2 dS = 1. \qquad (3.17)$$

Thus ψ^j and φ^j they are dimensionless quantities.

Sometimes it manages also for the filled waveguides - under the special laws of filling - to express the fields through two membrane/diaphragm functions. In this case, generally speaking, the components of their own waves are expressed as both membrane/diaphragm functions.

3. Let us extract function ψ^l and φ^l for waveguides of round and rectangular cross sections. For circular waveguides

$$\psi^l = N^l J_n(\alpha_{nq}\rho)^{\cos}_{\sin} n\beta;\ \varphi^l = M^l J_n(\alpha_{nq}\rho)^{\cos}_{\sin} n\beta. \qquad (3.18)$$

Here (n, g) - the number of wave (H_{nq} or respectively E_{nq}), J_n - Bessel function. Eigenvalues α_{nq} are connected with a radius of waveguide by the relationship/ratios

$$\alpha_{nq} = \frac{\mu_{nq}}{a}; \qquad (3.19a)$$

$$\alpha_{nq} = \frac{\nu_{nq}}{a}, \qquad (3.19b)$$

relating, correspondingly, to H-waves and to E-waves.

Page 19.

Here μ_{nq} there is the q positive root of equation $J_n'(\mu)=0$ and ν_{nq}— the q positive root of equation $J_n(\nu)=0$. We will use obvious

designations μ_j and ν_j. Normalizing factors N^l and M^l are determined from (3.16):

$$N^l = \sqrt{\frac{2}{\pi \epsilon_n}} \frac{1}{\sqrt{\mu_j^2 - n^2} \cdot J_n(\mu_j)}, \quad M^l = \sqrt{\frac{2}{\pi \epsilon_n}} \frac{1}{\nu_j J_n'(\nu_j)}, \quad (3.20)$$

where $\epsilon_n = 1 + \delta_{on}$, i.e., $\epsilon_0 = 2$, $\epsilon_n = 1$ with n=0.

The system of coordinates x, y for describing the field in rectangular waveguide we will arrange/locate so that the origin of coordinates would coincide with the apex/vertex of rectangle, axis x - with it is wide and y axis - with narrow by the sides of rectangle (Fig. 1). Then

$$\begin{aligned} \psi^l \cdot\ & N^l \cos\frac{\pi n x}{a} \cos\frac{\pi q y}{b}; \\ \varphi^l =\ & M^l \cdot \sin\frac{\pi n x}{a} \sin\frac{\pi q y}{b}, \end{aligned} \quad (3.21)$$

where a, b - respectively wide and narrow sides. Eigenvalues H_{nq} of waves and E_{nq} waves are expressed by the one and the same formula

$$\alpha_{nq}^2 = \pi^2 \left(\frac{n^2}{a^2} + \frac{q^2}{b^2}\right). \quad (3.22)$$

Normalizing factors are equal to

$$N^l = M^l = \sqrt{\frac{1}{ab\epsilon_n\epsilon_q} \frac{2}{\alpha_j}}. \quad (3.23)$$

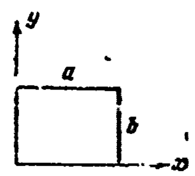

Fig. 1. Section of rectangular waveguide.

End Section.

Page 20.

Chapter 1.

METHOD OF SMALL HETEROGENEITIES.

The simplest heterogeneity is a small heterogeneity, concentrated in low region and which leads to a small change in the properties of waveguide. There are three fundamental types of heterogeneities - curvature, change in the properties of filling and change in the section. The appropriate small heterogeneities they are fracture to small angle, small jump ε and μ of the filling medium and a small step. In three paragraphs of this chapter, are solved the electrodynamic problems of wave dissipation, which fall to such small heterogeneities - for coupling of two semi-infinite regular waveguides, which are characterized by either the direction of axis or by the filling medium, or by the duct/contour of cross section. As it proves to be, in certain cases the analysis of the obtained solutions makes it possible to find also the field, scattered by the irregular section of finite length with a sizable change in the

parameters.

§4. Fracture of waveguide. Bent waveguide as the limit of waveguide with a large number of fractures.

1. One of the basic types of irregular waveguide is the bent waveguide of constant section. Simplest case of this curvature - curvature to small angle. We will begin from the computation of the fields of waves, scattered at this elementary curvature. AA and BB - two planes, which limit curvature and perpendicular to both regular waveguides, connected by curvature (Fig. 2). Let us place angle $\Delta\vartheta$ between these planes by so/such small that the distance between the corresponding to each other points of planes AA and BB would be little in comparison with wavelength.

Page 21.

This curvature of small electrical length we will call fracture. It suffices to examine the flat/plane curvature, since a small three-dimensional/space curvature is the imposition of a small flat/plane curvature and twisting on small angle, but a small twisting is a special case of the strain, examine/considered into §6. Wave amplitudes, scattered at a small three-dimensional/space curvature, can be obtained by the addition of amplitudes scattered on two small strains indicated.

Let on fracture fall to the left the wave of number m(m>0) from

single amplitude, i.e., the wave whose fields are equal to $\hat{E}^m$, $\hat{H}^m$. Complete field of section AA(z=0) consists of the field of the incident wave and sum of the fields of backward waves. It can be recorded in the form

$$E = E^m + \sum_{j=1}^{\infty} P_{-j} E^{-j};\ H = H^m + \sum_{j=1}^{\infty} P_{-j} H^{-j}. \tag{4.1}$$

Of section BB, the field consists only of direct waves

$$E = \sum_{j=1}^{\infty} P_j E^j,\ H = \sum_{j=1}^{\infty} P_j H^j. \tag{4.2}$$

Amplitudes P_{-j}, P_j are the unknown values.

Let us begin from the determination of the amplitudes of direct waves in the second waveguide. Multiplying E, H in (4.2) vector on H^j, E^j, by store/adding up, integrating by cross section and utilizing a condition of orthogonality (3.9), we will obtain

$$P_j = \frac{1}{2kh_j}\int([H^j E]_z + [HE^j]_z)\, dS. \tag{4.3}$$

Confronting in (4.3) vectors E, H they are related to section BB. It is expressed from through vectors E, H in section AA. It is oriented x axis of the Cartesian coordinate system in the cross section of waveguide in such a way that it would indicate the center of curvature. Y axis is directed toward drawing, so that set of three (x, y, z) - is right. Any two corresponding to each other points in sections AA and BB, in reference to the coordinate system in one of the sections, for example in section AA, have in it the identical coordinates y; coordinates x differ to the members of order, $(\Delta\vartheta)^2$, coordinates z - to value Δz, proportional $\Delta\vartheta$.

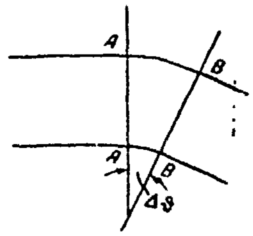

Fig. 2. Fracture of waveguide.

Page 22.

Difference in coordinates x we will be able tc disregard, since we should it will be necessary determine the amplitudes of the scattered waves only with an accuracy down tc the terms of the first degree in $\Delta\vartheta$.

Between the transverse components of fields E, H in sections AA and BB, there is the relationship/ratio

$$H_x\Big|_{BB} = H_x + \Delta z\frac{\partial H_x}{\partial z} - \Delta\vartheta\cdot H_z\Big|_{AA};\quad H_y\Big|_{BB} = H_y + \Delta z\frac{\partial H_y}{\partial z}\Big|_{AA}. \quad (4.4)$$

The same second relationship/ratio, which contains components E, we do not extract. First two terms in (4.3) are the first members of Taylor series. Third term/component/addend corresponds to the rotation of the coordinate system in BB relative to the coordinate system in AA. Value Δz in (4.4) is equal to $r\Delta\vartheta$, where r - a distance from center of curvature to this point in section; it is different

for the different points of section.

Let us present (4.4) in (4.3). In this case, in (4.4) under fields E, H it is possible to understand not complete field (4.1) in section AA, that contains incident and backward waves, but only E^m, H^m, i.e. only field of the incident wave. Really/actually, with the substitution of the fields of backward waves $P_{-j}E^{-j}$, $P_{-j}H^{-j}$ $(j>0)$ into first addend (4.4) appropriate terms in (4.3) they will fall out as a result of the condition of orthogonality, and with the substitution of the fields of these waves in the second addend (4.4) corresponding terms in (4.3) will give different from zero ones add/composed; however, they will be order $(\Delta\vartheta)^2$, and the members of this order we disregard. Thus, in (4.3) it is possible to substitute, for H, field with the components

$$H_x = H_x^m - \Delta\vartheta\,(ih_m r H_x^m + H_z^m),\ H_y = H_y^m - \Delta\vartheta \cdot ih_m r H_y^m, \quad (4.5)$$

and the analogous field E. Of this, consists the special feature/peculiarity of expression (4.3).

Substituting (4.5) in (4.3), we will obtain for the unknown values P_j the following expressions, valid, according to preceding/previous, with an accuracy down to the terms of order $\Delta\vartheta$ inclusively:

$$P_j = \delta_{jm} + \Delta\vartheta \cdot F_{jm}, \quad (4.6)$$

where the coefficients F_{jm} are equal to

$$F_{lm} = \frac{1}{2kh_m}\left\{-ih_m\int r\,(E_y^l H_x^m - E_x^l H_y^m - E_x^m H_y^l + E_y^m H_x^l)\,dS - \right.$$
$$\left. - \int (E_y^l H_z^m - E_z^m H_y^l)\,dS\right\}. \qquad (4.7)$$

Page 23.

Thus the amplitude of the scattered waves with indices j≠m they are proportional, as one would expect, the angle of fracture Δϑ.

The amplitude of the transmitted wave of number m differs from unity, i.e., from the amplitude of the incident wave, by the value of the order-Δϑ; this is equivalent so that with the passage of fracture the incident wave acquires the factor

$$e^{-iq_m \Delta \vartheta}, \qquad (4.8)$$

where $q_m = iF_{mm}$, i.e.

$$q_m = \frac{1}{k}\int r\,(E_y^m H_x^m - E_x^m H_y^m)\,dS - \frac{i}{2kh_m}\int (E_y^m H_z^m - E_z^m H_y^m)\,dS. \quad (4.9)$$

With real h_m value q_m is also real. Formula (4.8) gives, thus, phase change of the wave, which passed through the fracture of waveguide.

In the bent part of the waveguide always it is possible to conduct certain line, parallel to generatrix, thus, in such a way that phase change $q_m \Delta\vartheta$ would be equal to the propagation constant h_m, of the multiplied by differential arc of this line dl_m. In other

words, in the bent part the wave is propagated with the same phase rate, as in rectilinear, if we count off the distance between planes AA and BB along the arc of the specific line. Is strict this correctly only for infinitesimal nodes; in final curvatures, as we will see further, the phase rate depends also on transformation into waves of another type. Generally speaking, for different waves of line l_m they are different, but, for example, for a circular waveguide they for all waves coincide as is shown the calculation (see §22), with the axis of waveguide.

For determining the amplitudes of backward waves, which are propagated in left waveguide, let us use the same reception/procedure, as during the derivation of formula (4.6). Let us multiply (4.1) on H^{-l}, E^{-l} and let us perform the same transformations, as during conclusion/derivation (4.3). So we will obtain the expression

$$P_{-l}=\frac{1}{2kh_{-l}}\int\{[H^{-l}E]_z+[HE^{-l}]_z\}\,dS. \qquad (4.10)$$

Vectors E, H in (4.10) are related to section AA. We express them through vectors E, H in section BB; the corresponding formulas, analogous (4.4), they take the form

$$H_x\Big|_{AA}=H_x-\Delta z\frac{\partial H_x}{\partial z}+\Delta 0 H_z\Big|_{BB};\quad H_y\Big|_{AA}=H_y-\Delta z\frac{\partial H_y}{\partial z}\Big|_{BB}, \qquad (4.11)$$

where Δz has the same sense, that in (4.4).

Page 24.

With substitution (4.11) in (4.10) it is possible in (4.11) to omit first term, since, according to (4.2), it contains only the fields of waves with the positive indices which drop out with substitution in (4.10). In remaining term/component/addends (4.11) it is possible with an accuracy down to the terms of order $\Delta\vartheta$ to inclusively omit the fields of all waves, except the wave of number m, and its amplitude to take as equal to unity. Producing further the same transformations, as during conclusion/derivation (4.6), we will obtain

$$P_{-j} = -\Delta\theta\, F_{-jm}. \qquad (4.12)$$

Coefficient F_{-jm} is obtained from F_{jm} (4.7) by the formal replacement j on -j. Formulas (4.6), (4.12) and (4.7) solve stated in this point/item problem of scattering on a small fracture. The investigation of coefficients F_{jm} we will produce further during the development of the general method of cross sections.

2. Obtained above results make it possible in number of cases to find wave amplitudes, arising with passage of curvature with final angle ϑ_0. Let us replace for this the mentally bent waveguide with waveguide with numerous small fractures, i.e., by the waveguide, which consists of the rectilinear cuts, connect/joined together at small angles. The parasitic wave of any fixed/recorded index j at

output is obtained by the addition of the amplitudes of the "elementary" waves of this index, generatrices on each fracture. Results for a curvature are obtained by passage to the limit from sum to integral. Analogous reasoning is applied, as is known, in the theory of long lines during the derivation of formulas for a reflection coefficient from line with alternating/variable wave impedance; elementary heterogeneity in this problem is a small jump of wave impedance.

The amplitude of the elementary parasitic wave of number j≠m, which arose in the section of the bent waveguide, situated between ϑ and ϑ+Δϑ, is equal, according to (4.6), value $F_{jm}\Delta\vartheta$, multiplied by wave amplitude of number m (incident wave) at this point. According to (4.8), this last/latter amplitude is equal to $e^{-i\int_0^{\vartheta} q_m\, d\vartheta} = e^{-ih_m l_m}$, if we count off arc length l_m from the beginning of curvature. After reaching the end/lead of the bending, parasitic wave in turn, acquires supplementary phase $e^{-i\int_{\vartheta}^{\vartheta_0} q_j\, d\vartheta} = e^{-ih_j[l_j(\vartheta_0)-l_j(\vartheta)]}$. Thus, amplitude of elementary parasitic wave at output will be equal to

$$F_{jm}\Delta\vartheta e^{-ih_j l_j(\vartheta_0)}\cdot e^{-i[h_m l_m(\vartheta)-h_j l_j(\vartheta)]}.$$

Page 25.

The total amplitude of parasitic wave at output is equal, with an accuracy to unessential factor $e^{-ih_j l_j(\vartheta_0)}$, to value

$$F_{jm}\int_0^{\vartheta_0} e^{-i[h_m l_m(\vartheta)-h_j l_j(\vartheta)]}\, d\vartheta. \qquad (4.13)$$

For the amplitude of backward wave, will be obtained the same expression with replacement F_{jm} and F_{-jm} and h_j on $h_{-j}=-h_j$.

In row/series the case integral (4.13) can be calculated in an elementary manner, for example, for a bending with the constant radius when $l_m(\vartheta)$ and $l_j(\vartheta)$ they are proportional ϑ. For a bending with a variable radius, if we for simplicity assume $l_m(\vartheta)=l_j(\vartheta)$ and to call radius r distance from center of curvature to the line, along which is counted off the length l, integral (4.13) can be recorded in the form

$$F_{jm}\int_0^L e^{-i(h_m-h_j)l}\frac{dl}{r(l)}, \qquad (4.14)$$

where L - length of bending.

The produced higher analysis of the propagation of waves in bending by the final angle, which proceeds from the solution of the problem of a small bending (fracture), possesses the specific physical clarity. With this approach is explained the sense of coefficient F_{jm}, it becomes clear the structure of exponential factor in (4.13). According to this point of view, the formation of parasitic waves at the bend consists of two processes: from the formation of the elementary waves on fracture and from the addition of these elementary waves with appropriate phase change. This point of view is highly useful during the qualitative analysis of concrete/specific/actual systems. The method, with the aid of which

was obtained explicit expression (4.7) for F_{lm}, it allows, being based on expansion (4.4), to isolate two factors, calling the regeneration of waves in fracture and bending. The first - dissimilarity of the distance between the appropriate points of two sections, turned relative to each other on certain angle. The second factor bears vector character - the transverse components of fields in certain section prove to be connected with longitudinal components in the adjacent, turned section.

On the other hand, strict derivation of formulas of type (4.13) from (4.6) proves to be sufficiently laborious, more complex than their direct obtaining from the equations of Maxwell.

Page 26.

The developed in last/latter point/item method is limited by the condition that energy of the fundamental wave is not changed noticeably with the passage of entire bending and that it is possible to disregard secondary interaction of parasitic waves with each other and with the fundamental wave. This condition in any case requires so that the radius of curvature would be great, kr>>1. Furthermore, the perturbation method, used during the solution of the problem of fracture, proves to be insufficient near the critical frequency when $h_l \to 0$ and $F_{lm} \to \infty$ and even in row/series the case. Therefore the

total analysis of bending to final angle we will produce below, being based on the system of differential equations for coefficients P_i, considered as functions from ϑ. This system also can be derived from (4.6); however, simpler and more common/general/total is the application/use of a method of cross sections, developed in chapter II.

§5. Small jump dielectric and of magnetic filling. Waveguide with alternating/variable filling as the limit of waveguide with laminar filling.

1. Second fundamental type of irregular waveguide is rectilinear waveguide of constant section, in which ε and μ its filling medium they are changed along waveguide, so that $\varepsilon=\varepsilon(x, y, z)$, $\mu=\mu(x, y, z)$. To this type of irregularities they are related, in particular, different inserts with alternating/variable section. Usually in them ε (or μ) - discontinuous function; however, common/general/total results to is simpler obtain, counting first ε and μ by continuous functions and completing then the appropriate passages to the limit.

The simplest case of this irregularity is a small jump ε and μ in any section, i.e., articulation of two semi-infinite regular waveguides with close values ε and μ. Let us find wave amplitudes, scattered on this jump. Let us begin from the determination of the

amplitudes of direct waves.

Let when $z<0$ $\varepsilon=\varepsilon(x, y)-\Delta\varepsilon$, $\mu=\mu(x, y)-\Delta\mu$, when $z>0$ $\varepsilon=\varepsilon(x, y)$, $\mu=\mu(x, y)$, where $\Delta\varepsilon$ and $\Delta\mu$ - low values, which are, generally speaking, functions from x and y (Fig. 3a).

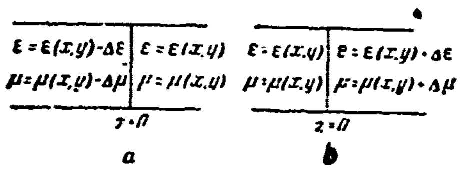

Fig. 3. Small jump of the properties of medium.

Page 27.

Falls to the left the wave of number m from single amplitude, it is necessary to find the amplitudes of direct waves, which exit from the place of coupling. This problem is solved below by the method, which are the version of the method of the slight disturbances; disturbance/perturbation is change ε and μ in left waveguide to low values $-\Delta\varepsilon, -\Delta\mu$.

When $\Delta\varepsilon=0$, $\Delta\mu=0$ undisturbed field in entire waveguide equal to the field incident wave $\hat{E}^m, \hat{H}^m$. When $\Delta\varepsilon\neq0$ $\Delta\mu\neq0$ appear supplementary fields ΔE and ΔH. These fields satisfy the equations, which are obtained during the variation of the equations of Maxwell (3.19, written for $\hat{E}^m, \hat{H}^m$

$$\operatorname{rot}\Delta E+ik\mu\Delta H=ik\Delta\mu\hat{H}^m,\ \operatorname{rot}\Delta H-ik\varepsilon\Delta E=-ik\Delta\varepsilon\hat{E}^m. \quad (5.1)$$

According to (5.1), fields ΔE and ΔH are created by the magnetic currents whose density is proportional $\Delta\mu\hat{H}^m$, and by the electric currents whose density is proportional $\Delta\varepsilon\cdot\hat{E}^m$. These exciting currents

are arrange/located with z<0. It is necessary to determine the fields, excited with z>0.

With z=+0 field ΔE and ΔH they take the form

$$\Delta E = \sum_{j=1}^{\infty} (P_j - \delta_{mj}) E^j, \quad \Delta H = \sum_{j=1}^{\infty} (P_j - \delta_{mj}) H^j, \qquad (5.2)$$

where P_j - the unknown amplitudes of direct waves.

According to (3.7) and (3.6), the condition of orthogonality (3.9) can be recorded in the form

$$\int \{[H^{-j}E^m]_z - [H^m E^{-j}]_z\} dS = 2kh_j\delta_{jm}. \qquad (5.3)$$

Applying this formula, we will obtain that P_j (with j≠m) the integral expression, analogous (4.3):

$$P_j = \frac{1}{2kh_j} \int \{[H^{-j}\Delta E]_z - [\Delta H \cdot E^{-j}]_z\} dS. \qquad (5.4)$$

On the other hand, from (5.1) and the equations of Maxwell (3.1), recorded for fields $\hat{E}^{-j}, \hat{H}^{-j}$ in the undisturbed waveguide, follows

$$\operatorname{div} \{[\hat{H}^{-j}\Delta E] - [\Delta H \cdot \hat{E}^{-j}]\} = \\ = ik(\Delta\varepsilon \hat{E}^m \hat{E}^{-j} - \Delta\mu \cdot \hat{H}^m \hat{H}^{-j}). \qquad (5.5)$$

Let us integrate this equality with respect to entire space in which $\Delta\varepsilon \neq 0, \Delta\mu \neq 0$.

Page 23.

Let us consider in this case that according to boundary condition (3.2), which satisfy the fields E^{-j} and ΔE, the normal to metal components of vectors $[\hat{H}^{-j}\Delta E]$ and $[\Delta H \cdot \hat{E}^{-j}]$ are equal to zero. Let us introduce a small complexity of wave number k, what will ensure the

convergence of integral during integration on z of - ∞ to 0 and disappearance of the result of the substitution of lower limit. Thus for the unknown amplitudes of direct/straight parasitic waves (5.4) we will obtain

$$P_l=\frac{1}{2h_l(h_l-h_m)}\int(\Delta\varepsilon E^m E^{-l}-\Delta\mu H^m H^{-l})\,dS, \qquad (5.6)$$

where the integral it is undertaken with z=-0. The comparison of this expression with the usual formulas of the theory of excitation of waveguides emphasizes the physical sense of the right sides of equations (5.1).

During the determination of the amplitudes of backward waves, we let us assume that the disturbance/perturbation occurs in right waveguide, i.e., that when $z<0$ $\varepsilon=\varepsilon(x,\ y)$, $\mu=\mu(x,\ y)$, while when $z>0$ $\varepsilon=\varepsilon(x,\ y)+\Delta\varepsilon$, $\mu=\mu(x,\ y)+\Delta\mu$ (Fig. 3.b). In this case, $\Delta\varepsilon$ and $\Delta\mu$ they will have the same values, as is above, but, as for the definition of the amplitudes of direct waves, to calculate field will have only in the region, which does not contain sources, which, as is known, it is simpler. Repeating the same lining/calculations, as during the computation of the amplitudes of direct waves, and again utilizing (5.1), we will obtain

$$P_{-l}=\frac{-1}{2h_l(h_l+h_m)}\int(\Delta\varepsilon E^m E^l-\Delta\mu H^m H^l)\,dS. \qquad (5.7)$$

Two last/latter formulas solve stated problem of the fields, which appear during a small abrupt change in the properties of medium, that fills waveguide.

2. From formulas (5.6-5.7) it is possible, in particular, to obtain condition of fact that jump ε and μ will not cause appearance of parasitic waves of another type, than falling. Let us show first that, besides the condition of orthogonality (3.9), of field of different waves satisfy still another condition of orthogonality, which contains all three components of the fields:

$$\int (\varepsilon E^m E^j + \mu H^m H^j)\, dS = -2h_m^2 \delta_{mj}. \qquad (5.8)$$

This condition follows from the identity

$$\mathrm{div}\,\{[\hat{H}^m \hat{E}^j] + [\hat{H}^j \hat{E}^m]\} = 2ik(\varepsilon \hat{E}^m \hat{E}^j + \mu \hat{H}^m \hat{H}^j), \qquad (5.9)$$

which in turn, it is easy to obtain from the equations of Maxwell (3.1), of written for fields waves of numbers m and j.

Page 29.

In order to obtain from (5.9) condition (5.8), it is necessary in (5.9) to produce differentiation with respect to z, and then to integrate the obtained relationship/ratio the cross section and to utilize boundary condition (3.2) for E_m and E^j and the normal condition of orthogonality (3.9).

From comparison (5.6) and (5.7) with (5.8) it follows that if in all points of interface (z=0) is satisfied the condition

$$\Delta\varepsilon = A\cdot\varepsilon,\ \Delta\mu = -A\cdot\mu, \qquad (5.10)$$

where the constant A does not depend on x and y, then the phenomena of transformation will not be and will arise only reflected and past waves.

Condition (5.10) - sufficient, but not necessary. Let us note that it is more powerful than the requirement of equality the phase rate in both waveguides, which can be recorded in the form

$$\Delta(\varepsilon\mu)=0. \qquad (5.11)$$

This condition follows from (5.10). The absence of jump in phase rate, i.e., condition (5.11), generally speaking, is insufficient so that there would not be the transformations.

3. Before passing to examination of waveguides in which ε and μ essence arbitrary functions z, let us generalize (5.6) and (5.7) to case when transition between two semi-infinite waveguides with close values ε and μ occurs in low section of length $\Delta z (k\Delta z<<1)$. Analyzing the method which obtained these formulas, it is easy to ascertain that they remain valid also in this case. It is necessary only by $\Delta\varepsilon$ and $\Delta\mu$ to understand the complete value of difference ε and μ in both waveguides. For future reference it is convenient to replace under integral sign $\Delta\varepsilon$ by $\frac{\partial\varepsilon}{\partial z}\Delta z$ and $\Delta\mu$ - on $\frac{\partial\mu}{\partial z}\Delta z$. Let us introduce another designation

$$S_{jm}=\frac{1}{2h_j(h_j-h_m)}\int\left(\frac{\partial\varepsilon}{\partial z}E^mE^{-j}-\frac{\partial\mu}{\partial z}H^mH^{-j}\right)dS; \qquad (5.12)$$

then for the amplitudes of the direct/straight and reverse/inverse parasitic waves, scattered by this heterogeneity, we will obtain

$$P_j = S_{jm}\Delta z \ \ (j \neq m); \ \ P_{-j} = -S_{-jm}\Delta z, \qquad (5.13)$$

where S_{-jm} - is obtained from S_{jm} by the formal replacement j on -j. Strict derivation of these formulas will be given in the following chapter.

Now in the irregular section of length L ε and μ essence arbitrary functions not only from x and y, but also from z.

Page 30.

This section can be represented in that consisting of the row/series of short regular waveguides in which ε and μ they differ to low values $\Delta\varepsilon(x, y)$, $\Delta\mu(x, y)$. The amplitudes of the parasitic waves, which are formed on each jump, are given by formula (5.13). Complete wave amplitude, scattered in entire irregular section, is obtained by the addition of all elementary parasitic waves taking into account phase change. This calculation method is valid, it goes without saying, only when the secondary transformations of the parasitic and incident waves can be disregarded. This, in any case, it can be fulfilled only for waveguides with the slowly changing parameters, i.e., for the

waveguides, in which the change ε and μ at a distance of the order of the wavelength or linear dimensions of cross section is small in comparison with values ε and μ, and far from the points at which $h_l=0$. Analogous reasoning is applied in the theory of the propagation of waves in infinite inhomogeneous medium (L. M. Brekhovskikh, [80]).

The propagation of the fundamental wave from the beginning of the irregular section z=0 to the layer, which lies between z and z+Δz, and the propagation of formed on this layer parasitic wave from it to output of the irregular section (i.e. to section z=L for direct waves and z=0 for reverse/inverse ones) occurs somewhat more complex than in the analogous problem of the bent waveguide, since wave numbers h_m and h_l in the case in question depend on z. From the representation of irregular waveguide as about the limit of laminar it follows that phase change with the passage of any section is equal to integral $\int h_l dz$, undertaken on this section, i.e., a difference in the values of functions $\gamma_l(z)$, which we will determine by the equation

$$\gamma_l = \int_0^z h_l dz, \qquad (5.14)$$

at the end and beginning of section. The module/modulus of the wave amplitude, which is propagated along waveguide, is changed in such a way that the constant would remain the flow of its energy; according to (3.10), in this case $P_l \sim 1/\sqrt{h_l}$. Repeating the same reasoning which brought us to (4.14), we will obtain for the amplitudes P_l of the direct/straight parasitic waves, which appear during incidence in

the wave of number m, when $P_m = 1$, expression ($j \neq m$)

$$\sqrt{\frac{h\ (0)}{h_j\ L)}}\int_0^L S_{jm} \sqrt{h_j/h_m}\, e^{-i(\gamma_m-\gamma_j)}\, dz, \qquad (5.15)$$

where is again lowered unessential phase factor.

Page 31.

The amplitude of backward wave is obtained by replacement in (5.15) γ_j on γ_{-j}, S_{jm} on $-S_{-jm}$ and $h_j(L)$ on $h_j(0)$. In contrast to analogous coefficient F_{jm} in the problem of bending, the coefficient S_{jm} depends on z and therefore it cannot be removed as integral sign.

The analysis of expressions (5.15) and of entering in them coefficients S_{jm} we will produce further after we will obtain by their although less demonstrative, stricter and serial mode. Let us note here only that by a sufficient condition of the fact that during propagation in waveguide with alternating/variable filling will not occur the phenomena of transformation, will be the condition

$$\frac{\partial \varepsilon}{\partial z} = A\varepsilon, \quad \frac{\partial \mu}{\partial z} = -A\mu \ (A \text{ не зависит от } x, y), \qquad (5.16)$$

Key: (1). A does not depend on x, y).

analogous (5.10). This condition is again more powerful than the requirement of the constancy of phase rate $\partial(\varepsilon\mu)/\partial z = 0$. Condition (5.16) in integral form means that there is this function $\Phi(z)$, that

$$\varepsilon(x, y, z) = \varepsilon(x, y, z_0)\frac{\Phi(z)}{\Phi(z_0)};$$

$$\mu(x, y, z) = \mu(x, y, z_0)\frac{\Phi(z_0)}{\Phi(z)}. \qquad (5.17)$$

§6. Small step. Rectilinear irregular waveguide as the limit of stepped waveguide.

1. Third and most complex basic type irregular waveguide is rectilinear tapered weld. The simplest, "elementary" irregularity of this type to calculation by which we pass, is step in waveguide, i.e., the connection of two waveguides with close cross sections.

Let with z<0 the section of waveguide be limited by duct/contour C_-, when z>0 - by duct/contour C_+. Duct/contours C_- and C_+ can be characterized by in terms of position (as with shift or joining of waveguides), value or form. As the characteristic of step serves the function δ(s), equal regarding the distance between the appropriate points of duct/contours C_- and C_+; it depends on coordinate s, calculated along the duct/contour of cross section. Since we examine only small steps, then there is no need strict mathematical determination what points of two duct/contours we call appropriate. For a small step $|\delta|$, it is small in comparison with wavelength and with the size/dimensions of cross section.

Page 32.

We will place $\delta(s)>0$, if at the particular point transition from C_- to C_+ indicates motion to the side of standard into metal, i.e., the expansion of waveguide, and $\delta<0$, if occurs reverse/inverse condition (Fig. 4). With $\delta\equiv0$ both of waveguides, they are identical (it is equally arrange/located); difference δ from zero it is the disturbance/perturbation, calling the appearance of waves, scattered from joint. The special feature/peculiarity of this disturbance/perturbation of mathematical sense lies in the fact that it is related not to equation, but to the position of the surface on which are placed the boundary conditions. Disturbance/perturbation, i.e., small strain of the surface of metal, is transition from one surface, on which is fulfilled boundary condition (3.2), to another, on which is satisfied the same condition.

For the calculation of field distortion, it is expedient to replace the strain of surface, on which it is correct (3.2), by a small change in this condition on the undisturbed surface. Let us establish/install this new boundary condition, equivalent strain, which let us call/name equivalent boundary condition.

Let before strain the surface occupy position S_0 (Fig. 5a) (n - standard into metal), after strain - position S, and let these surfaces be only close, but also they compose small angle. The distance between the appropriate points S and S_0 let us designate by letter l. Sign l is accepted by such, that during the location of surfaces, shown on Fig. 5a, $l<0$. With second possible location S and S_0, appropriate Fig. 5b, $l>0$.

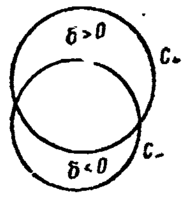

Fig. 4. Step in waveguide.

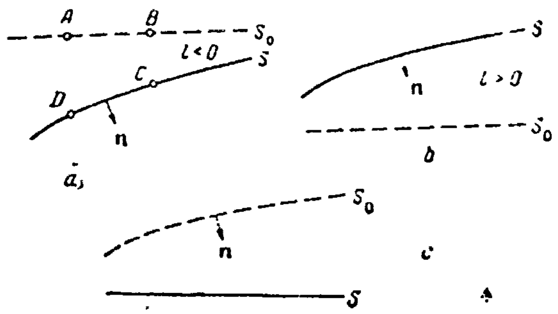

Fig. 5. To conclusion/derivation of equivalent boundary condition (6.1).

Page 33.

Tangential to S_0 component E on S_0 is small, it is determined by currents and charges in the section, close to the point in question, in other words, for E there is a local boundary condition. After using the integral form the first equation of Maxwell (3.1) (with $\mu=1$) to duct/contour ABCD and taking into account that for S correctly usual boundary condition (3.2), we will obtain the unknown equivalent boundary condition

$$Et = ikl\,nHt + t\,\mathrm{grad}\,(E_n l), \qquad (6.1)$$

where t - is unit vector, tangential to S_0. This condition makes following sense: in the field, limited S_0, the field, created by any sources, will under equivalent boundary condition (6.1) on the undisturbed surface S_0 have the same value, as under usual boundary condition (3.2) on the disturbed surface of S.

Fig. 5a corresponds $l<0$. It is easy to show that the form of equivalent boundary condition (6.1) will not change during sign change l. In order to obtain (6.1) for the location of surfaces, datum in Fig. 5b, it is necessary to first establish/install boundary condition for S in the auxiliary problem, depicted in Fig. 5c, and then to pass to the conditions, which correspond to Fig. 5b. In this way it is possible to show which (6.1) is correct with any sign l.

The replacement of strain by boundary condition (6.1) introduces into calculation some conditional surface magnetic currents. The field distortion, connected with strain, from a formal point of view proves to be that caused by these currents.

2. let us use equivalent boundary condition (6.1) to it is calculated wave amplitudes, scattered on step. During the computation of the amplitudes of direct waves $P_i (j \neq m)$ we will consider strain the divergence of the surface of left waveguide (z<0) from the continuation of the surface of right waveguide (z>0); then the

magnetic curren s, calling parasitic field will be arrange/located the regions where searches for this field. In this case, is obvious $l(z,s)=\delta(s)$ when $z<0$, $l=0$ with z>0. To the right in (6.1), stand the undisturbed fields, equal to $\hat{E}^m$, $\hat{H}^m$.

Boundary condition (6.1) is correct only, where surfaces S and S_0 are not only close, but also comprise small angles. Near step this condition is not fulfilled; therefore in the vicinity of order δ of section z=0 the tangential component E on S_0 is different from (6.1). Because of this the obtained below formulas will contain the error for order δ^2. We will determine the amplitude of the scattered waves only in the first order on δ.

Amplitudes P_i we will find by the same formula (4.3), which they used in the problem of a small bending.

Page 34.

For determining entering in (4.3) integral from section let us integrate over entire space of left waveguide the equality

$$\operatorname{div}([\hat{H}^{-i}E]-[HE^{-i}])=0, \qquad (6.2)$$

which it is easy to obtain from the equations of Maxwell (3.1). In (6.2) E, H - complete field; on surface S_0 whose cross section, regarding, despite all z limited by duct/contour C_+,E satisfies

condition (6.1), into which it is possible to substitute field $\hat{E}^m, \hat{H}^m$. During integration and during conclusion/derivation (5.6), let us introduce the small complexity $\varkappa$. Field $\hat{E}^{-l}, \hat{H}^{-l}$ satisfies for S_0 normal condition (3.2); therefore

$$P_l = \frac{-1}{2kh_j} \oint [\hat{H}^{-l} E]_n \, ds dz. \qquad (6.3)$$

It is necessary to calculate this integral, utilizing boundary condition (6.1). The direction or unit vector s we will select in such a way that the vectors n, s and the unit vector in direction z would form the right-handed triad. The entering in (6.3) components fields E to S_0 will be, according to (6.1),

$$E_z = ik\delta\hat{H}_s^m + \frac{\partial}{\partial z}(\hat{E}_n^m \delta), \; E_s = ik\delta\hat{H}_z^m + \frac{\partial}{\partial s}(\hat{E}_n^m \delta). \qquad (6.4)$$

Let us substitute (6.4) in (6.3); adding and deducting the appropriate terms, let us isolate under integral term/component/addends

$$\frac{\partial}{\partial z}(\hat{E}_n^m \hat{H}_s^{-l} \delta) - \frac{\partial}{\partial s}(\hat{E}_n^m \hat{H}_z^{-l} \delta). \qquad (6.5)$$

During integration these term/component/addends disappear: the first due to condition $\delta=0$ with $z=0$, the second as a result of the periodicity of all functions from s. Since the dependence of integrand in (6.3) on z is known, then it is possible to produce integration for z. Utilizing another equation of Maxwell for the field of wave of number - j, we will obtain the unknown expression for the amplitudes of direct waves, scattered by the step

$$P_l = \frac{-1}{2h_j (h_j - h_m)} \oint \delta(s) (E_n^m E_n^{-l} + H_z^m H_z^{-l} + H_s^m H_s^{-l}) ds. \quad (6.6)$$

It it is possible to record, taking into account conditions (3.6),

which connect the fields of counter waves, in the more convenient form:

$$P_j = \frac{1}{2h_j(h_j - h_m)} \oint \delta(s)(E_n^m E_n^j + H_z^m H_z^j - H_s^m H_s^j)\,ds. \quad (6.7)$$

The amplitudes of backward waves are found analogously.

Page 35.

As surface S_0, one should despite all z take in this case the surface of left waveguide and to set/assume $l(z)=0$ with z<0, $l(z,l)=-\delta(s)$ with z>0, i.e., to examine as disturbance/perturbation the divergence of the surface of right waveguide from the continuation of the surface of left.

Repeating the same lining/calculations, we will obtain

$$P_{-j} = \frac{1}{2h_j(h_j + h_m)} \oint \delta(s)(E_n^m E_n^j + H_z^m H_z^j + H_s^m H_s^j)\,ds. \quad (6.8)$$

Wave amplitudes, scattered by step, are expressed, thus, integrals, undertaken on the duct/contour of the section of waveguide; they contain the height/altitude of step to the first degree. On step will arise those all waves for which with datum $\delta(s)$ integrals (6.6)-(6.8) are different from zero, i.e., all waves which are excited by magnetic currents (6.1), created by the transmitted wave of number m.

3. As example of application/use of obtained formulas, let us

examine simplest problem of incidence in symmetrical magnetic wave on step in circular waveguide. In symmetrical magnetic wave $H_s = 0$,

$E_n = 0$, therefore will arise only magnetic waves. The character of the dependence of the fields of these waves on azimuthal angle is determined by the fact, what components strongly are represented in the expansion of function δ(s) in Fourier series

$$\delta(s) \quad \mathrm{Re}\sum_{n=0}^{\infty} \delta_n e^{ins/a}. \qquad (6.9)$$

From (6.7), (6.8) it follows that the wave amplitude H_{nq} will be proportional $|\delta_n|$. For symmetrical step $\delta_n = 0$ with $n \neq 0$, will arise only the waves H_{0q}, amplitude of which it is proportional $|\delta_0|$. This step is formed during coupling of the waveguides of different radii with common axis. In expansion (6.9) the coefficient δ_0 is different from zero in those all cases when the sectional areas of the joined waveguides are different, and in this case in stray field are present waves H_{0q}. If with butting are somewhat displaced the centers of sections, and radii are equal, then (6.9) it will consist of one term/component/addend (n=1), $|\delta_1|$ it will be equal to the shift of axes, and will arise only waves H_{1q}. If the cross section of one waveguide is a circle, and another - ellipse with semi-axes $a+\Delta_1$, $a-\Delta_2$, then will arise waves H_{0q} and H_{2q}, moreover wave amplitudes H_{0q} are proportional $|\delta_0| = |\Delta_1 - \Delta_2| \; 2$, and wave amplitudes H_{2q} are proportional $|\delta_2| = (\Delta_1 + \Delta)\ /2$.

Page 36.

The common/general/total expression for energy losses during incidence in the wave H_{01} is easy to find from the preceding/previous formulas and formulas §3. Simple lining/calculations give

$$\frac{\mu_{01}^4}{4}\frac{|\delta_0|^2}{h_1^4 a^6}+\sum_{n=0}^{\infty}\frac{|\delta_n|^2}{a^2}\sum_q \epsilon_n\left(\frac{\mu_{01}\mu_n}{\mu_{nq}^2-\mu_{01}^2}\right)\frac{1}{1-\frac{n^2}{\mu_{nq}^2}}\left(\frac{h_1}{h_{nq}}+\frac{h_{nq}}{h_1}\right), \quad (6.10)$$

where h_1 - wave number of incident wave. Addition in (6.10) is conducted on all propagated waves. In (6.10), are taken into account waves of both of directions. The energy, taken away by each direct wave, $|h_{nq}+h_1|^2/|h_{nq}-h_1|^2$ times is more than the energy, taken away by backward wave of the same number.

We will not here investigate in detail expressions (6.10). Let us note only that at the high frequencies when the phase rates of all waves are close to each other, it is possible out of the very narrow resonance regions in which $|h_{nq}|$ is small, to replace last/latter factor in (6.10) by number with 2. In this case, the losses will not virtually depend on frequency.

This result, as it will be shown into §9, has general character, it is valid for any waveguide. It it is possible to connect with the fact that at high frequencies the passage by the waveguide wave of

step is analogous with incidence/drop at the angle of Brillouin of plane wave on step on plane. The amplitude of diffracted on this step wave, as it is easy to show, is proportional to the ratio of the height/altitude of step to the wavelength, multiplied by the sine of slip angle. At high frequencies the sine of slip angle is of the order λ/a, so that the amplitude of the diffracted wave does not depend on λ.

The numerical values of the internal sum in (6.10) at high frequencies depend only on the azimuthal number n; for rough estimates these sums it is possible to replace by numbers for 4, 7, 19, 122, 12, ... ζ_r, correspondingly, n=0, 1, 2, 3, 4. These numbers can be considered as statistical weights, with which into expression for losses enter the average/mean values of the squares of Fourier coefficients function $\delta(s)$. These results make it possible to establish/install tolerances for the accuracy of the butting of waveguide sections.

4. There is single bond between P_{-m} and Δh_m, i.e. between wave amplitude of number m, reflected from step, and change Δh_m in wave number of this wave upon transfer from left to right waveguide. Let us establish this communication/connection.

A change in the wave number simply can be obtained, applying

formula (6.2) to the fields of the wave of number m in right waveguide ($\hat{E}^m$, $\hat{H}^m$) and the wave of number - m in left waveguide ($\hat{E}^{-m}$, $\hat{H}^{-m}$).

Page 37.

In the left waveguide of the field of the second wave, they satisfy simple boundary conditions (3.2), the fields of the first - to equivalent boundary conditions (6.1), moreover l for all z is equal $-\delta(s)$. Isolating in (6.2) derivative by z and integrating by the region, limited by duct/contour C_-, we will obtain

$$\Delta h = \frac{1}{2ikh_m} \oint [H^{-m}E^m]_n\, ds. \qquad (6.11)$$

After substituting then (6.1) and retaining everywhere only first order on Δh_m and δ, let us find expression for changing the wave number upon transfer through the small step

$$\Delta h_m = \frac{1}{2h_m} \oint \delta\,[-(H_s^m)^2 + (H_z^m)^2 - (E_n^m)^2]\, ds. \qquad (6.12)$$

Comparing this with reflection amplitude P_{-m} (6.8), we will obtain the unknown communication/connection

$$P_{-m} = \frac{\Delta h_m}{2h_m} + \frac{1}{2h_m^2} \oint \delta\,(H_s^m)^2\, ds. \qquad (6.13)$$

In a number of cases, the computation of the coefficient of reflection P_{-m} in formula (6.13) is simpler than on general formula (6.8). For example, in the field of wave H_{0q} in by circle waveguide there are no longitudinal currents, $H_s = 0$, and $P_{-m} = \Delta h_m / 2h_m$ with any form of step, i.e., any function $\delta(s)$. If wave number in both

waveguides is equal, $\Delta h_m = 0$, for example, during coupling of two identical displaced relative to each other waveguides, then P_{-m} is expressed only through the integral of the square of longitudinal current. For example, for the asymmetric wave, which falls to this joint in the circular waveguide, it is easy to find the dependence of reflection coefficient from the polarization of wave, etc.

For magnetic and electrical waves second term in formula (6.13) is expressed as functions ψ^m and φ^m. According to (3.14), the corresponding formulas take the form

$$
\begin{aligned}
P_{-m} &= \frac{\Delta h_m}{2h_m} - \frac{1}{2}\oint \delta \left(\frac{\partial \psi^m}{\partial s}\right)^2 ds; \\
P_{-m} &= \frac{\Delta h_m}{2h_m} - \frac{k^2}{2h_m^2}\oint \delta \left(\frac{\partial \varphi^m}{\partial s}\right)^2 ds.
\end{aligned}
\qquad (6.14)
$$

First term also can be, according to (6.12) and (3.14), expressed by ψ^m or φ^m.

Page 38.

Let us note that the appropriate formulas can be obtained directly from two-dimensional equations, determining a change in the eigenvalue α_m of problems (3.11a, 3.12a) or (3.11b), (3.12b) during small strain of boundaries. Communication/connection Δh^m with $\Delta \alpha^m$ is obtained by differentiation of relationship/ratio (3.13).

5. Let us pass to determination of wave amplitudes, scattered by finite segment of tapered weld. Let us note, in the first place, that formulas (6.7), (6.8) will remain valid and in such a case, when connection occurs not in one plane, but it occupies certain low region Δz (Δz•k<<1). Really/actually, in entire derivation of these formulas, nothing in this case will change. Repeating the analogous reasoning of the preceding/previous paragraph, let us replace δ(s) for product $\nu(s)\cdot\Delta z$, where as can be seen from simple geometric considerations, $\nu(s)$ - a slope tangent toward Z-axis of straight line, that connects the appropriate points of duct/contours C_- and C_+. This straight line composes with C_- and C_+ the angle, close to π/2. Value $\nu(s)$ has the same sign, as δ(s). After carrying Δz as integral sign, we will obtain for $P_j (j \neq m)$ and P_{-j} the formula which can be recorded in the form

$$P_j = S_{jm}\Delta z, \quad P_{-j} = -S_{-jm}\Delta z. \qquad (6.15)$$

Here, according to (6.7) and (6.8), through S_{jm} is designated value

$$S_{jm} = \frac{1}{2h_j(h_j - h_m)} \oint \nu(s)(E_n^m E_n^j + H_z^m H_z^j - H_s^m H_s^j)\,ds, \quad (6.16)$$

and S_{-jm} is obtained from S_{jm} by the formal replacement j on -j.

We used for coefficients (6.16) the same designation, as for coefficients (5.12) in the problem of the jump of dielectric and magnetic constant; as it will be shown into §9, between them there are simple correlations. During this recording (6.15) it is completely identical (to 5.13).

Let be now given irregular tapered weld. Let us replace this waveguide with the stepped waveguide, shown on Fig. 6a or Fig. 6b; result in both cases will be identical. Value $v(s)$ depends now on z, it, obviously, is equal to the tangent of the angle, formed with Z-axis of tangent to metal, perpendicular to the duct/contour of cross section. The determination of the wave amplitudes, scattered on a heterogeneity of the type in question, is conducted by the same in the accuracy diagram, on which these values were determined in the preceding/previous paragraph. Formulas (5.14) and (5.15) for the wave amplitudes, scattered in finite segment with alternating/variable filling, were obtained from formula (5.13) for the wave amplitudes, scattered on a small jump of filling. Formulas (6.15) are identical with formulas (5.13); therefore formulas (5.14), (5.15) will remain valid for tapered weld, if we now by S_{im} understand value (6.16).

Page 39.

It remains valid also observation about limitedness of the method of "small heterogeneities" in application to large heterogeneities and about the difficulties, which appear during a strict derivation of formulas (5.14)-(5.15) from (5.13). Therefore the analysis of these formulas and coefficients S_{im} we will transfer into §9, where they will be obtained by stricter path.

In chapter I, are found the wave amplitudes, scattered on small irregularities of each of three fundamental types, imposition of which is any irregularity in waveguide. For irregularities not of small, final, the formation parasitic waves can be represented as the result of the imposition (with appropriate phase change and change in the amplitude, which retains energy) of the parasitic waves, which are formed on the row/series of the small irregularities to which can be broken this large irregularity. This method is not completely strict, in certain cases generally inapplicable, but it possesses the specific physical clarity and makes it possible simpler entire to obtain the most interesting characteristics of irregular section.

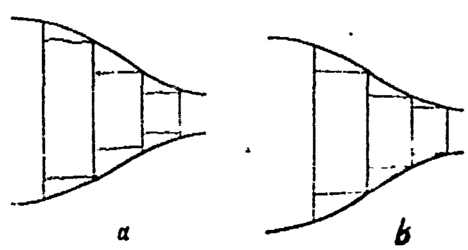

Fig. 6a, b - stepped waveguide.

Page 40.

Chapter II.

METHOD OF CROSS SECTIONS.

The fundamental idea of the method, named us by the method of cross sections, lies in the fact that in any section of the irregular waveguide of field they are represented in the form of superpositions of the fields of waves of both of directions, which exist in the auxiliary rectilinear regular waveguide of the same section and with the same distribution ε and μ over section. The coefficients of this superposition satisfy the system of the ordinary differential equations of the first order. Common/general/total of the problems of the determination of fields in irregular waveguide is reduced in this case to the problem of the fields of waves in regular waveguide and to the solution of the syst ı or ordinary differential equations. Below we will use method to the fundamental types of irregular waveguides and to the general case of the combined irregularity.

§7. Calculation of the bent waveguides by the method of cross sections.

1. Let us begin from establishment of fundamental equations for fields in waveguide, bent on circular arc. Let to bending the cut of waveguide in question have for entire extent/elongation constant (and besides arbitrary) section. Let us bend it on circular arc. In this case, all generatrices of rectilinear waveguide will become the circumferences whose centers lie/rest on one straight line. Let us accept this straight line for y axis of cylindrical coordinates; other two coordinates will be r and ϑ. Planes ϑ=const intersect waveguide with respect to the same duct/contour, identical for all ϑ, which limited the cross sections of rectilinear waveguide. In the plane ϑ=const of coordinate r and y, is formed the grid of Cartesian coordinates; x axis it is directed toward center of curvature, so that x is identical s $-r$, $\partial/\partial x \equiv -\partial/\partial r$.

Page 41.

Let us examine any section ϑ=const; let us construct the rectilinear waveguide, passing through the duct/contour of this section and perpendicular to it. Fields in this waveguide of comparison and in the bent waveguide satisfy on the duct/contour of section the identical boundary conditions E•t=0, since tangents to

both waveguides coincide. If we present field in the bent waveguide in the form of the superposition of the fields, capable of existing in the rectilinear waveguide of the same section, with the coefficients, which depend on ϑ, then each term of this superposition will satisfy boundary condition (3.1).

Let us assume

$$E_r = -\sum_{m=1}^{\infty} Q_m E_x^m,\ E_y = \sum_{m=1}^{\infty} Q_m E_y^m;$$
$$H_r = -\sum_{m=1}^{\infty} R_m H_x^m,\ H_y = \sum_{m=1}^{\infty} R_m H_y^m, \quad (7.1)$$

i.e. let us decompose the transverse components of field in the bent waveguide on the transverse components of the fields of the waves of all types (one direction), capable of existing in the waveguide of comparison. Coefficients Q_m, R_m depend on ϑ. Let us introduce then new coefficients $P_m(\vartheta)$ and $P_{-m}(\vartheta)$ by the equations

$$P_m - P_{-m} = Q_m,\ P_m + P_{-m} = R_m. \quad (7.2)$$

According to accepted by us communication/connection (3.6) between the fields of direct/straight and backward waves, expansion (7.1) can be recorded in the form

$$E_r = -P_\nu E_x^\nu,\ E_y = P_\nu E_y^\nu;\ H_r = -P_\nu H_x^\nu,\ H_y = P_\nu H_y^\nu. \quad (7.3)$$

Here, as everywhere subsequently, on the repeated index ν is conducted addition from $\nu=-\infty$ to $\nu=+\infty$. In detailed recording, for example, first formula (7.3) indicates

$$E_r(r,\ \vartheta,\ y)=-\sum_{\nu=-\infty}^{+\infty}P_\nu(\vartheta)E_x^\nu(x,\ y). \qquad (7.4)$$

Expansion (7.3), in contrast to (7.1), contains waves of both of directions; in this case, the coefficients in field expansion E and H prove to be identical.

Page 42.

The functions according to which is produced the expansion in (7.3), satisfy the same boundary conditions that and fields E, H. Row/series (7.3) allow/assume term-by-term differentiation. Let us substitute them into two equations of Maxwell

$$\frac{\partial E_r}{\partial y}-\frac{\partial E_y}{\partial r}=-ik\mu H_0;\ \frac{\partial H_r}{\partial y}-\frac{\partial H_y}{\partial r}=ik\varepsilon E_0. \qquad (7.5)$$

From comparison with last/latter equations in (3.4) it follows that for the longitudinal components E and H there is expansion, analogous (7.3), with the same coefficients P_m:

$$E_\vartheta=P_\nu E_z^\nu;\ H_\vartheta=P_\nu H_z^\nu. \qquad (7.6)$$

Let us install now system of equations for $P_m(\vartheta)$. The substitution of expansions (7.4) and (7.6) into the remaining four equations of Maxwell

$$\frac{\partial E_y}{\partial \theta} - r\frac{\partial E_\theta}{\partial y} = -ik\mu r H_r;\ \frac{\partial}{\partial r}(rE_\theta) - \frac{\partial E_r}{\partial \theta} = -ik\mu r H_y;$$

$$\frac{\partial H_y}{\partial \theta} - r\frac{\partial H_\theta}{\partial y} = ik\varepsilon r E_r;\ \frac{\partial}{\partial r}(rH_\theta) - \frac{\partial H_r}{\partial \theta} = ik\varepsilon r E_y \quad (7.7)$$

gives

$$P'_\nu E^\nu_x = -P_\nu E^\nu_z - iP_\nu h_\nu E^\nu_x r;\ P'_\nu E^\nu_y = -iP_\nu h_\nu E^\nu_y r;$$

$$P'_\nu H^\nu_x = -P_\nu H^\nu_z - iP_\nu h_\nu H^\nu_x r;\ P'_\nu H^\nu_y = -iP_\nu h_\nu E^\nu_y r. \quad (7.8)$$

Here prime indicates derivative on ϑ.

Let us multiply these equations on - H^j_y, H^j_x, E^j_y and $-E^j_x$, where j - any whole number, positive or negative, let us add and let us integrate over section. Utilizing a condition of orthogonality (3.9), we will obtain for P'_j expression in the form of linear combination from P_m

$$P'_j = F_{j\nu} P_\nu. \quad (7.9)$$

Since this expression is correct for all j, then (7.9) is the unknown system of differential first-order equations for coefficients $P_j(\vartheta)$.

For coefficients F_{jm} during this calculation in (7.9) is obtained the expression, which coincides with the coefficients F_{jm} (4.7), found from the problem of a small fracture of waveguide.

Page 43.

According to expansions (7.3), (7.6), value $P_j(\vartheta)$ can be considered as the composite wave amplitude of number m. It replaces factor $e^{-ih_j z}$ in rectilinear waveguide; however, it changes along waveguide with more complex shape. According to (7.9), change $P_j(\vartheta)$ with change ϑ depends on amplitude of all waves, which exist with datum ϑ. The amplitude of each wave P_m participates in the education/formation of value P_j' in by the fact of larger degree, the greater the coefficient F_{jm}. We will call/name values F_{jm} coupling coefficients between the waves of numbers j and m. Multiplying (7.9) on $\Delta\vartheta$, mu we can give to this equation the following sense: the amplitude of any wave with the passage of small angle $\Delta\vartheta$ changes to the value, proportional $\Delta\vartheta$, and this value is composed of the changes, obliged to the effect of all waves. The action of each wave on P_j all the more, the greater its amplitude and the greater the coupling coefficient between these waves. In this respect (7.9) it is the generalization of equalities (4.6) and (4.12), which are obtained, as we will show, from (7.9) on the assumption that in this section it is possible to disregard all waves, except one. According to (7.9), the propagation of waves on irregular section is the analog of the propagation of waves along the system of the connected lines with space coupling.

2. Let us establish/install end conditions for $P_j(\vartheta)$ on boundaries of bent section. Let the bending in question according to

circular arc be joined with $\vartheta=0$ and with $\vartheta=\vartheta_0$ with two rectilinear waveguides, and from left waveguide on bending falls the wave of number m of single amplitude.

From the orthogonality of the fields of different waves and continuity of the transverse components E and H, it follows that the values of coefficients $P_j(0)$ in the beginning and end/lead of the bending are equal to the amplitudes of the corresponding waves in rectilinear part. In other words, $P_j(\vartheta)$ they are continuous upon transfer through the boundaries of the bent section. End conditions must provide the absence of other incoming to bending waves, except that falling. These conditions take the form

$$P_m(0)=1,\ P_j(0)=0 \overset{(1)}{\text{при}}\ j>0,\ j\neq m;\ P_j(\vartheta_0)=0 \overset{(1)}{\text{при}}\ j<0. \tag{7.10}$$

Key: (1). with.

System (7.9) and (7.10) is complete, from it it is possible to find all amplitudes $P_j(\vartheta)$. The amplitudes of backward waves, which exit into left waveguide, are equal to $P_j(0)$ $(j<0)$, while the amplitudes of direct waves, which exit into right waveguide, are equal to $P_j(\vartheta_0)(j>0)$; the determination of these amplitudes does not require supplementary process/operations on the join of fields.

Page 44.

Since coefficients F_{lm} in system (7.9) do not depend on ϑ, the solution of this system can be searched for by the methods, borrowed from the theory of final systems with constant coefficients. Retaining in (7.9) certain finite number of variables N, we will obtain the system of a finite number of equations whose solution is reduced to the solution of the algebraic characteristic equation of order N. During satisfaction of some conditions, the solution of infinite system (7.9)-(7.10) will be obtained with $N\to\infty$. We will not here base and develop this method; when problem does not contain any series expansion parameter, the actual solution of system (7.9)-(7.10) is expedient, probably, to produce is direct on with equations (7.9) whose structure is adapted to direct/straight numerical calculations, in particular to calculation in electronic computers. Certain difficulty into such calculations introduces the fact that for the part of the variables of condition (7.10) they are placed at one end/lead of the interval, for a part - on other; however always, apparently, can be used the known reception/procedure, which reduces stated problem to several analogous ones, to which the conditions of type (7.10) are placed for all variables at one and the same end/lead of the interval.

3. let us examine in somewhat more detail coupling coefficients,

explicit expression for which is given to §4. When $m \neq j$ F_{jm} it does not depend on the radius of curvature which enters into first term in (4.7). This follows from the comparison of this term/component/addend with the condition of orthogonality (3.9). Coupling coefficients depend, it goes without saying, from the position of center of curvature relative to the section of waveguide, since on this depends the orientation of x and y axes in section.

Coupling coefficients satisfy the condition

$$F_{-j,-m} = -F_{jm}, \qquad (7.11)$$

directly escape/ensuing from (4.7) and (3.6).

They are connected also by reciprocal relations. It is simpler anything to establish/install these relationship/ratios, utilizing independence of the integral

$$\int \{E_r^{(1)}H_y^{(2)} - E_y^{(1)}H_r^{(2)} - E_r^{(2)}H_y^{(1)} + E_y^{(2)}H_r^{(1)}\}\, dS, \qquad (7.12)$$

undertaken according to section ϑ=const, of value ϑ. In (7.12) $E^{(1)}, H^{(1)}$ and $E^{(2)}, H^{(2)}$ - two arbitrary field in bending, created by the sources, arrange/located out of bending. Independence (7.12) from ϑ follows of Lorenz's lemma, used to the region, limited by two any sections ϑ=const. Let us record the components of the first and second field in relationship/ratio (7.12) in the form of expansions (7.3) with different amplitudes $P_l^{(1)}$ and $P_l^{(2)}$ let us substitute in (7.12).

Page 45.

After the simple transformations, which use again a condition of orthogonality, we will obtain for integral (7.12) expression $h_\nu P_\nu^{(1)} P_{-\nu}^{(2)}$. Derivative on ϑ of this value is equal, according to (7.9), to the dual sum

$$(-F_{\nu\mu}h_\nu + F_{-\mu,-\nu}h_{-\mu})P_{-\nu}^{(1)}P_\mu^{(2)}, \qquad (7.13)$$

where the addition is conducted on V and on μ. This value must be equal to zero for any fields $E^{(1)}, H^{(1)}$ and $E^{(2)}, H^{(2)}$, any functions $P_l^{(1)}$ and $P_l^{(2)}$; consequently, for any V and μ must be equal to zero bracket in (7.13). Utilizing still (7.11), we will obtain the unknown condition, which connects F_{jm} and F_{mj},

$$h_m F_{mj} = h_j F_{jm}. \qquad (7.14)$$

This relationship/ratio makes it possible to simplify expression (4.7) for F_{jm}. According to (7.14), between the integrals, which participate in (4.7), there is the following communication/connection:

$$i(h_l - h_m)\int r(E_y^l H_x^m - E_x^l H_y^m + E_y^m H_x^l - E_x^m H_y^l)\,dS = \\ = \int (E_y^l H_z^m - E_z^m H_y^l - E_y^m H_z^l + E_z^l H_y^m)\,dS. \qquad (7.15)$$

This formula can be obtained, it goes without saying, it is direct from equations (3.4) and boundary conditions (3.2) via elementary ones, although bulky, computations. Eliminating with the aid of

(7.15) the first of two integrals, which participate in (4.7), we will obtain for coupling coefficients the symmetrical expression

$$F_{lm} = \frac{1}{2kh_l(h_l - h_m)}(h_m B^{ml} - h_l B^{lm}), \qquad (7.16)$$

where through B^{ml} is designated the integral

$$B^{ml} = \int (E_y^m H_z^l - E_z^l H_y^m) dS. \qquad (7.17)$$

All these results are used not only for waveguides with the ideal walls for which correctly boundary condition (3.2), but also for waveguides with the well carrying out walls, since during the replacement of condition (3.2) by Leontovich boundary condition the condition of orthogonality (3.9) is retained.

The actual computation of coupling coefficients for the waveguides of rectangular and round cross-sections will be produced in chapter V. Here we will only give expressions for F_{lm} through membrane/diaphragm functions ψ and Φ. These expressions are used in those all cases when it is possible to use formulas (3.14), first of all for waveguides without filling.

Page 46.

Let us introduce the following designations for the integrals according to cross section, which appear with substitution (3.14) into integral (7.17):

$$\int \psi^j \frac{\partial \psi^m}{\partial x} dS = K^{jm};\ \int \varphi^j \frac{\partial \varphi^m}{\partial x} dS = C^{jm};\ \int \psi^j \frac{\partial \varphi^m}{\partial y} dS = M^{jm}. \quad (7.18)$$

These values do not contain frequency, they depend only on the transmission mode, form of cross section and direction in center of curvature in this section. They do not depend on radius of curvature and are of the order of the linear dimensions of section. It is easy to show that from boundary condition for membrane/diaphragm function ϕ on the duct/contour of section follow two relationship/ratios:

$$C^{mj} = -C^{jm};\ \int \varphi^m \frac{\partial \psi^j}{\partial y} dS = -M^{jm}. \quad (7.19)$$

Substituting in (7.16) field expression through membrane/diaphragm functions, we will obtain that if m and j - magnetic waves, then

$$F_{jm} = \frac{i}{2h_j(h_j - h_m)}(h_m \alpha_j^2 K^{jm} - h_j \alpha_m^2 K^{mj}). \quad (7.20)$$

If both of electrical type waves, then

$$F_{jm} = \frac{i}{2h_j(h_j - h_m)}(h_m \alpha_j^2 + h_j \alpha_m^2) C^{jm}. \quad (7.21)$$

Finally, if j - are magnetic, and m - electrical of wave, then

$$F_{jm} = \frac{ik(h_j + h_m)}{2h_j} M^{jm}. \quad (7.22)$$

Coupling coefficient for the fourth possibility of the combination of waves easily is obtained from last/latter formula and condition of reciprocity (7.14).

According to formulas (7.20)-(7.22), the coupling coefficient between two waves of one direction is more than between the same waves, which are propagated in opposite directions.

4. There is two special cases, in which it is easy to obtain explicit solution of system (7.9)-(7.10); them it is possible to call/name cases of loose coupling. For their examination it is expedient instead of $P_j(\vartheta)$ to introduce the new variables $p_j(\vartheta)$, after determining by their equality

$$P_j(\vartheta) = p_j(\vartheta)\, e^{-iq_j\vartheta}, \qquad (7.23)$$

where coefficient $q_j = iF_{jj}$ it is given in (4.9).

Page 47.

Values $p_j(\vartheta)$, in contrast to $P_j(\vartheta)$, we will call/name the given amplitudes. In these variables the system of equations (7.9) will take the form

$$p_j' = F_{j\nu} p_\nu (1 - \delta_{j\nu})\, e^{-i(q_\nu - q_j)\vartheta}. \qquad (7.24)$$

The special feature/peculiarity of the given coefficients p_j is the absence of diagonal cell/elements in the matrix/die of system of equations for them.

If to the right in (7.24) it is possible to disregard the amplitudes of all waves, besides the incident wave of number m, then

the solution of system (7.24) is obtained in an explicit form. The first case when this is possible, is examined into §4 fracture, i.e., the bending for which $\vartheta_0 \ll 1$. Then from (7.24) or (7.9) for direct/straight and backward waves immediately follow expressions (4.6) and (4.12), in which only one should replace $\Delta\vartheta$ by ϑ_0. For the wave of number m from (7.24) it follows that p_{mm}=const=1, so that a change in the phase with passage of fracture is determined by the exponential factor, isolated in (7.23). Let us examine this factor in somewhat more detail.

In contrast to F_{jm} with $m \neq j$, q_j' contains the radius of curvature. Ratio/relation q_j/h_j can be call/named radius of curvature for the wave of number j. This value

$$r_j = q_j / h_j \qquad (7.25)$$

will be, generally speaking, it is different for different waves. However, for example, for a circular waveguide with any symmetrical over section distribution ε and μ, as is shown calculation, r_j is identical for all waves and is equal to distance from center of curvature to the axis of waveguide. The same is valid for rectangular waveguides (without filling or with uniform filling).

The common/general/total expression for r_j is obtained from (4.9) and (7.25). For magnetic and electrical waves this expression takes respectively the form

$$r_j = \int r\,(\nabla\psi^j)^2 dS + \frac{\alpha_j^2}{h_j^2} K^{jj};\ r_j = \int r\,(\nabla\psi^j)^2 dS. \qquad (7.26)$$

If we, as into §4, count off distance l_j between two cross sections according to the arc of radius r_j, $dl_j = r_j d\vartheta$, then exponential curve in (7.23) can be recorded in the form $e^{-ih_j l_j}$. For a circular waveguide, as it is shown, arc length is counted off along the axis of waveguide.

Page 48.

Thus, the exponential factor in (7.23), which describes a change in the phase (and in the amplitude, if h_j - it is composite or imaginary), it corresponds to the propagation of wave with wave number h_j along arc l_j.

The second case for which it is easy to give the solution of system (7.24), is bending with large radius of curvature. It is simpler anything to obtain the necessary solution, having first assumed that with large radius of curvature it is possible in (7.24) to drop/omit to the right all term/component/addends, except term with j=m, that corresponds to the incident wave, and then to check that obtained in this case solution satisfies this assumption. Assumption this is based on what with large radius of curvature the

exponential factor in (7.24) is the rapidly changing function of angle; therefore the average/mean value of derivatives p_l' little and value of variables p_l they must be close to those values which they have on boundary of the region. System of equations (7.24) takes in this case the form

$$p_m' = 0;\ p_j' = F_{jm} p_m e^{-i(q_m - q_j)},\ j \neq m. \qquad (7.27)$$

Its solution under the conditions (7.10), which are record/written for variables $p_l(\vartheta)$ accurately in the same manner as for for variables $P_j(\vartheta)$, has the form

$$p_m(\vartheta) = 1; \qquad (7.28a)$$

$$p_l(\vartheta_0) = F_{jm}\ \frac{\sin(q_m - q_j)\,\vartheta_0/2}{(q_m - q_j)/2} - e^{-i(q_m - q_j)\vartheta_0/2},\ j > 0,\ j \neq m; \quad (7.28b)$$

$$p_l(0) = -F_{jm} \frac{\sin(q_m - q_l)\,\vartheta_0/2}{(q_m - q_j)/2} e^{-i(q_m - q_j)\vartheta_0/2},\ j < 0. \quad (7.28c)$$

The first of these equalities means that a change in the amplitude $P_m(\vartheta)$ of the incident wave is described by the same factor $e^{-ih_m l_m}$, as in rectilinear waveguide. Energy of this wave (with real h_m) does not change. This is correct only in that approach/approximation, in which is obtained system (7.27). As we will see into §22, in the following order on curvature, the phase and the amplitude of the incident wave at the output of bending with a large radius depend, although to small degree, from the phenomenon of transformation into other waves.

Formulas (7.28b, c) give the amplitudes of straight lines ($p_j(\vartheta_0)$, $j>0$) and reverse/inverse ($p_j(0)$, $j<0$) the scattered parasitic waves. Proposing also that r_j for all j one and the same and that $h_m>0$, and h_j really, let us record these formulas in the more convenient form:

$$p_j(\vartheta_0)=F_{jm}\frac{\sin(h_m-h_j)r\vartheta_0/2}{(h_m-h_j)r/2}e^{-i(h_m-h_j)r\vartheta_0/2}\quad j>0;\quad (7.29a)$$

$$p_j(0)=-F_{jm}\frac{\sin(h_m+|h_j|)r\vartheta_0/2}{(h_m+|h_j|)r/2}e^{-i(h_m+|h_j|)r\vartheta_0/2}\quad <0.\ (7.29b)$$

Page 49.

Applicability condition for entire solution (7.28-7.29) is the smallness of amplitudes (7.29). In the denominator of these formulas, will cost the radius of curvature; therefore for sufficiently large bending radii, this approach/approximation will be valid. Applicability condition for these formulas can be recorded in the form

$$\left|F_{jm}\frac{1}{(h_m\mp|h_j|)\,r}\right|\ll 1.\qquad (7.30)$$

For small angles, it is more precise, when $(h_m\mp|h_j|)r\vartheta_0\ll 1$, formulas (7.29) pass into formulas (4.6), (4.12), that relate to fracture. For direct waves the bending with small ϑ_0 behaves as fracture, if a difference in its electrical lengths for both of waves is small, while for reverse/inverse ones - it is small the sum of electrical

lengths; for backward waves the requirement for smallness ϑ_0 is considerably more powerful.

Thus, (7.28), (7.29) with with large radii of curvature it is correct for any angles ϑ_0, and for small ϑ_0 - with any r, and with small r it passes into formulas for fracture (4.6), (4.12). It is obvious that (7.28) it can be also obtained - with an accuracy to the unessential phase factor, omitted during conclusion/derivation into §4, from formula (4.13), derived via elementary reasonings.

The amplitude of parasitic waves at the output of bending is the nonmonotonic function of angle of curvature. This is explained by the interference character the formation of parasitic wave, in detail examined with this point view into §4. The arguments of sines in (7.29) make simple physical sense - they are equal to half-difference (for direct waves) and to half-sum (for reverse/inverse ones) of phase change of the corresponding waves at entire bending. Let us note that for any fixed/recorded frequency it is possible in limits of accuracy (7.28) to turn into zero amplitude of any parasitic wave, after selecting angle ϑ_0 by such, so that the formed in different parts of the bending parasitic waves would completely extinguish each other.

The amplitudes of direct waves (7.29a) are much more than the

amplitudes of reverse/inverse ones (7.29b), since for them, in firs place are much more coupling coefficients, in the second place, confronting under integral in (4.13) exponential factor is considerably the less rapidly changing function.

Page 50.

Therefore p_l for direct waves contains in denominator a difference in the absolute values of wave numbers $h_m - |h_l|$, and for backward waves - a considerably larger value, their sum $h_m + |h_l|$. Usually it is possible to disregard energy loss to transformation into reverse/inverse parasitic waves. This cannot be made only in frequency region, in which $|h_l|$ is small; appearing in this case conditions will be examined into §13.

Coupling coefficient for backward wave of the same number, i.e., for the wave F_{-mm}, reflected, for electrical waves is equal to zero, since $C^{mm} = 0$, and is equal to

$$F_{-mm} = \frac{-i\varkappa_m^2}{2h_m} K^{mm} \tag{7.31}$$

for magnetic waves. If $F_{-mm} = 0$, then is equal to zero also coefficient of reflection of this wave from bending, $p_{-m}(0) = 0$. Thus, reflection coefficient for any electrical wave is equal to zero. For magnetic waves in the waveguides of round and rectangular cross section $K^{mm} = 0$, and the reflection coefficient is also equal to zero, although

there are forms of the waveguides in which $K^{mn}\neq 0$. it goes without saying, from condition $F_{-mm}=0$ follows the equality of zero coefficients of reflection $\rho_{-m}(0)$ only in the first order in 1/r (for a bending with large radius of curvature) or on ϑ_0 (for curvature small angle). Other parasitic waves, which were being formed at the bend, prove to be, generally speaking, connected with the wave of number -m, and amplitude $\rho_{-m}(0)$ is different from zero, but it will be order $1/r^2_0$ or ϑ^2_0.

Inequality (7.30), which gives applicability condition for solution (7.29), will render/show also disrupted for how conveniently large r, if for any j $h_j=h_m$ and in this case $F_{jm}\neq 0$ [1].

FOOTNOTE [1]. Formula (7.16) for a coupling coefficient takes the indefinite form when $|h_j-h_m|\to 0$. In this case it is necessary to use formula (4.7), from which, in particular, it follows that F_{jm} remains final when $|h_j-h_m|\to 0$. ENDFOOTNOTE.

This is so-called case of confluence when two waves, connected with bending, have identical phase rates. In this case, all elementary parasitic waves, which are formed in the various sections of bending, at any point of inflection store/add up with one and the same phase, and, how not was great r, the result of this addition will be for final ϑ_0 finite quantity. In this case it is not possible to

disregard the reverse/inverse effect of the wave of number j on the wave of number m; the given amplitude of this wave is changed, i.e., proves to be inaccurate also equality (7.28a). This situation appears, in particular, with passage by wave H_{01} of bending in circular waveguide, and based on this specific example we studied in §22. The apparatus, which is based on system (7.9) or (7.27), makes it possible to thoroughly investigate not only the case of precise expression $h_j = h_m$, but also the general case when propagation constant (generally speaking - composite) are close to each other.

Page 51.

5. Fundamental results of this paragraph easily are generalized to bending with alternating/variable curvature. In this case, in equations (7.9), (7.27) as variable it is convenient to take not angle ϑ, but arc length l_j. For a reduction in the recording, let us assume that for all j l_j is counted off on by one and the same friend, and let us omit index in l_j. Equations (7.9) and (7.27) take the form

$$\frac{dP_j}{dl} = \frac{F_{j\nu}}{r} P_\nu; \quad \frac{dp_j}{dl} = \frac{F_{j\nu}}{r} p_\nu (1 - \delta_{\nu j}) e^{-i(h_\nu - h_j)l}, \qquad (7.32)$$

the communication/connection between P_j and p_j will be

$$P_j = p_j e^{-ih_j l}. \qquad (7.33)$$

In this form of equation, they are used also for the case of bending with alternating/variable curvature, when r depends on l.

Really/actually, during coupling of two sections with constant, but by different values of r of amplitude p_l they remain continuous upon transfer through the boundary of sections. this follows from the same reasoning which was used for the conclusion/derivation of end condition, i.e., from the requirement of the continuity of fields and from the condition of orthogonality. Continuous will be also the given amplitudes p_l. In each section of amplitude, they satisfy equations (7.32); therefore these equations will be valid also for entire bent waveguide, which consists of two sections, if we in them count r different in both sections. Generalizing this reasoning to a larger number of sections and passing to limit, we will obtain that r in (7.31) can be any, including disruptive, by function l.

It would be possible to give also the direct analytical proof of the validity of systems (7.32), after introducing not cylindrical coordinate system, but more common/general/total system in which l it would be one of the coordinates; this system was proposed, for example, in [19]. The given above reasoning more distinctly emphasizes the physical sense of the fundamental fact, which makes it possible to generalize (7.32) to the case of a variable radius - a local character of coupling coefficient, i.e., the independence transformation from curvature at adjacent points. therefore in (7.32) does not enter the derivative of curvature.

Equations (7.32) are valid also for straight portions ($r=\infty$), which they give simply $p_j'=0$, p_j = const or $P_j(l)=P_j(0)e^{-ih_jl}$. If waveguid they will bend on space curve, then the orientation of the Cartes: coordinate system in section is changed along waveguide, and therefore coupling coefficients F_{jm} also prove to be function l.

Page 52.

For large radii of curvature (more precise, when the amplitud of all parasitic waves are small) also it is possible to give the general solution of system (7.32). Again as for the bending of a constant radius, it is possible in right Part Two system (7.32) tc reject/throw all term/component/addends, except member, who corresponds to the incident wave, and to set/assume $p_m(\vartheta)=\text{const}=1$. T amplitudes of the parasitic waves, scattered by bending, are founc this case from the equation

$$\frac{dp_j}{dl}=\frac{F_{jm}}{r}e^{-i(h_m-h_j)l}. \tag{7.34}$$

Its solution under boundary conditions (7.10) takes the form

$$\begin{aligned} p_j(\vartheta_0)&=\int_0^L\frac{F_{jm}}{r}e^{-i(h_m-h_j)l}\,dl, \quad j>0,\ j\neq i;\\ p_j(0)&=-\int_0^L\frac{F_{jm}}{r}e^{-i(h_m-h_j)l}\,dl, \quad j<0. \end{aligned} \tag{7.35}$$

This solution is understood identically with the solution, found i §4.

Under confluence ($h_j = h_m$) or conditions, close to degeneration ($|h_j - h_m| L \leqslant 1$), in system (7.32), just as in system (7.24), it is necessary even with large radii of curvature to retain to the righ several term/component/addends and to solve simultaneously several equations of this system (see §22).

For the constants r and F_{jm} solution (7.35) passes in (7.28). However, computation p_j from integral expression (7.35) in an explicit form is possible also somewhat more general case. Specifically,, let $r(l)$ slowly to changed with l and has at entire bending one and the same order of magnitude, but at its end/leads experience/tested disruption and it goes to infinity, so that at t points, hich occurs coupling the bent and rectilinear waveguic the curvature of the bent waveguide it has the same order of smallness, as at entire bending, and by jump is turned into zero. Integrating in this case (7.35) in parts

$$p_j = \pm \frac{i}{h_m - h_j} \left| \frac{F_{jm}}{r} e^{-i(h_m - h_j)l} \right|_0^L \mp$$

$$\mp \frac{i}{h_m - h_j} \int_0^L e^{-i(h_m - h_j)l} \frac{d}{dl} \left(\frac{F_{jm}}{r} \right) dl, \qquad (7.36)$$

we we can reject/throw second term, which contains the derivative curvature and product from curvature for twisting.

Page 53.

Reject/thrown term/component/addend has, generally speaking, the same order of smallness, as add/composed, which appear with preservation/retention/maintaining in (7.32) all members of sum. The obtained binomial formula is the generalization of expressions (7.29).

Thus, in this case in the old order of the amplitude of parasitic waves they explicitly depend only on the values of curvature and coupling coefficients at the points of the discontinuity of curvature. This does not indicate, it goes without saying that the formation of parasitic waves occurs only in these only at these points - it occurs at entire bending, but the points of the discontinuity of curvature are isolated by the fact that near them is considerably attenuate/weakened the mutual compensation the waves, which were being formed in different sections. We still will return to the problem of the computation of integral (7.35) in chapter V. Analogous expression we will encounter during the computation of the wave amplitudes, scattered on the waveguide transitions of alternating/variable section.

If the mean curvature of the bending of a variable radius is not small, then it is necessary to apply numerical methods. System (7.32) allow/assumes comparatively simple calculation on computers. With this better to operate not with $p_j(\vartheta)$, but with the variables $P_j(\vartheta)$.

6. Field in waveguide, bent with constant curvature, can be described also in terms of its own waves, i.e., waves, which are propagated along bending without regeneration. Its own can be considered as some linear combinations of the waves of rectilinear waveguide. they they can be obtained from common/general/total system (7.9) during supplementary requirement, so that all amplitudes $P_j(\vartheta)$ would depend on ϑ according to the one and the same law

$$P_j(\vartheta) = P_j(0)\, e^{-iq\vartheta}. \quad (7.37)$$

Substitution (7.37) in (7.9) reduces to the system of algebraic equations for $P_j(0)$:

$$(F_{j\nu} + iq\,\delta_{j\nu})\, P_\nu(0) = 0. \quad (7.38)$$

This system has solution, if q equal to one of the roots characteristic equation:

$$\mathrm{Det}\,|F_{jm} + iq\,\delta_{jm}| = 0. \quad (7.39)$$

Each root of this equation corresponds to one its own wave. The relationship/ratios between coefficients $P_j(0)$ for each their own wave are determined from system (7.38) after the substitution into it of the corresponding root.

Knowing the fields of the inherent flexural waves, it is possible then to determine the amplitude of the scattered waves, joining fields on the boundaries of the bent part; this will give the system of algebraic equations for the amplitudes of its own waves in bending and the amplitudes of the outgoing waves in rectilinear waveguides.

Page 54.

This procedure, generally speaking, is somewhat more complex than described above the direct/straight determination of the amplitudes of the scattered waves; however, in some problems (for example, see last/latter point/item §22) the apparatus of its own waves proves to be convenient.

The application/use of a concept of its own waves in the case of the bending of alternating/variable curvature is suggested in Juan Khun-tsz' article [81]. In any section it is possible to introduce the system of the waves which would be their own waves of the waveguide, bent with the constant curvature, equal to the curvature of this section. These waves are not not depended, between in the curvature. It is easy to find the regular method of determining this coefficients in (7.9).

Let us designate the matrix/die, which leads matrix/die F_{lm} to the diagonal form, which contains only the roots of equation (7.39), by letter O; the amplitudes of the natural waves, which we will designate W_j, are obtained in any cross section of P_j by the action

of the matrix/die of O^{-1}. In the matrix recording

$$W = O^{-1}P, \qquad (7.40)$$

where by W and P - matrix/dies, which consist of one column, with cell/elements W_i and P_i. Amplitudes W_i satisfy the system of differential equations, which is easy to derive from (7.9) and (7.40):

$$W' = NW; \qquad (7.41a)$$

$$N = O^{-1}FO - O^{-1}O'. \qquad (7.41b)$$

If bending radius is constant, then the constant$ i.e., do not depend on ϑ, also all matrix elements F and diagonal matrix/die, i.e., their own waves are not connected. In [81] it is shown, that for the bending of the alternating/variable curvature when second term in N is excellent from zero, this term/component/addend does not contain diagonal terms; its cell/elements are proportional to derivative of curvature, and precisely they describe the communication/connection between normal waves. If curvature is changed slowly, then the system of differential equations (7.41a) contains the low parameter and can be solved in an explicit form.

0, the second term $(-O^{-1}O')$ in matrix N -

Transition from one variables P_i to the next W_i is actually introduction to the method of the cross sections of other waveguides of comparison. Instead of rectilinear the waveguide, tangential to this bent, the waveguide of comparison undertook the waveguide, bent on the circular arc of curvature. Therefore series expansion parameter of problem becomes not curvature, but it derivative; this

is reached, however, by the essential complication of apparatus, conduct of matrix/die N instead of F, variables W_j instead of P_j, and so forth.

Page 55.

The analogous generalization of method in the theory of the rectilinear waveguides of constant section was suggested by G. Lyubarskiy and A. Povzner [54], that accepted as the waveguide of comparison cone. However, the application/use of such waveguides of comparison is possible only in some particular problems, but the use of an apparatus, based to (7.40) and (7.41), in principle it is possible always, wh 's known the initial matrix/die F.

Let us make on conclusion of this paragraph one observation, which relates not to waveguide, but to the toroidal cavity resonators, which are obtained from the bent in circumference waveguides of constant section. If a radius of this circumference is very great in comparison with the size/dimensions of section, then the fields of their own waves of such endovibrators coincide with the fields of the traveling waves in the rectilinear waveguide, and natural frequencies are found from obvious asymptotic condition, according to which the electrical length of cavity resonator rh_j is equal to $2\pi/n$, where n=0, +-1, +-2, ..., and h_j— wave number in

rectilinear waveguide. With any radius of curvature, natural frequencies can be found from (7.39), if we in this relationship/ratio assume $q=2\pi n$ and to consider it as equation for frequency. In particular, with the large radii of curvature in this way it is possible to find explicit expression for first-order correction, proportional to curvature, to the asymptotic value natural frequency. Asymptotic value is obtained from (7.39) neglecting of the coupling coefficients between different waves, i.e., by all nondiagonal elements in (7.39).

§8. Calculation of waveguides with the alternating/variable filling with the method of cross sections.

1. Let us examine rectilinear waveguide of constant section, in which ε and μ essence of function of all three coordinates. At first we will assume that ε and μ by continuous functions. For describing the field in this waveguide, it is necessary, according to the method of cross sections, to decompose field in any section z=const in row/series of Fourier's type on the fields of waves in the waveguide of comparison. The waveguide of comparison in this problem is the regular waveguide, in which ε and μ in all sections they are the same functions from x and y, as in this section of irregular waveguide. The waveguides of comparison for different z are different. Coordinate system in the waveguide of comparison let us

designate through x, y, ζ. The fields of their own waves in the waveguide of comparison depend, as usual, from ζ, entering the exponential factor $e^{-ih_j\zeta}$, and also on z, since on z depends the form of the function $\varepsilon(x, y)$ and $\mu(x, y)$.

Page 56.

Let us assume, it is analogous how we this made in the problem of the bent waveguide,

$$E_x = \sum_{m=1}^{\infty} Q_m E_x^m, \quad E_y = \sum_{m=1}^{\infty} Q_m E_y^m;$$
$$H_x = \sum_{m=1}^{\infty} R_m H_x^m, \quad H_y = \sum_{m=1}^{\infty} R_m H_y^m, \qquad (8.1)$$

where the coefficients Q_m, R_m are functions from z. Let us introduce then variables $P_m(z)$ and $P_{-m}(z)$ by conditions (7.2). Then these expansions can be recorded in the form

$$E_x = P_\nu E_x^\nu, \; E_y = P_\nu E_y^\nu; \; H_x = P_\nu H_x^\nu, \; H_y = P_\nu H_y^\nu, \qquad (8.2)$$

Fields E, H satisfy on the walls of irregular waveguide the same boundary conditions (3.2) that and field E^m, H^m on the walls of the waveguides of comparison; expansions (8.2) can be piecemeal differentiated. Substituting (8.2) in two of the equations of Maxwell and comparing result with two last/latter formulas in (3.4a) and (3.4b), we will obtain for longitudinal components the same expansions, as for transverse ones:

$$E_z = P_\nu E_z^\nu, \; H_z = P_\nu H_z^\nu. \qquad (8.3)$$

Thus, $P_j(z)$ can be considered as wave amplitudes, which exist in this irregular waveguide.

Let us pass to the conclusion/derivation of equations by which they satisfy function $P_j(z)$. Let us substitute for this row/series (8.2), (8.3) into four those remaining the equation of Maxwell. Utilizing several times (3.4), after simple transformations we will obtain four equations:

$$\begin{aligned} &E_x^{\nu}(P_{\nu}' + ih_{\nu}P_{\nu}) = -P_{\nu}E_x^{\nu\prime};\ E_y^{\nu}(P_{\nu}' + ih_{\nu}P_{\nu}) = -P_{\nu}E_y^{\nu\prime};\\ &H_x^{\nu}(P_{\nu}' + ih_{\nu}P_{\nu}) = -P_{\nu}H_x^{\nu\prime};\ H_y^{\nu}(P_{\nu}' + ih_{\nu}P_{\nu}) = -P_{\nu}H_y^{\nu\prime}. \end{aligned} \tag{8.4}$$

Here prime indicates derivative on z.

In order to obtain equation for P_j', let us multiply (8.4) respectively on $-H_x^j$, H_y^j, E_x^j and $-E_y^j$, let us add and let us integrate over cross section. Taking into account the condition of orthogonality, we will obtain expression for P_j' in the form of linear combination from P_m, which it is possible to record in the form

$$P_j' + ih_jP_j = S_{j\nu}P_{\nu}. \tag{8.5}$$

Page 57.

This expression is correct for any j; therefore (8.5) it is the unknown system of the ordinary differential equations of the first

order for $P_j(z)$.

We call values S_{jm} coupling coefficients, since they describe interaction of waves, obliged of the irregularity of waveguide. For them is obtained the following expression:

$$S_{jm} = \frac{1}{2kh_j} \int (E_x^j H_y^{m'} - E_y^j H_x^{m'} + E_x^{m'} H_y^j - E_y^{m'} H_x^j)\, dS. \qquad (8.6)$$

End conditions for P_j are located as in the problem of the bent waveguide, from considerations about the continuity of fields upon transfer from irregular to regular waveguides. If irregular section with a length of L is joined with z=0 and z=L with two regular waveguides and from side of left waveguide on it falls the wave of number m from single amplitude, then these conditions take the form

$$P_m(0) = 1,\ P_j(0) = 1 \text{ при (when) } j > 0,\ j \neq m;\ P_j(L) = 0 \text{ при (when) } j < 0. \quad (8.7)$$

The physical sense of system (8.5) is such as systems (7.9) and interpretation, given to system (7.9) at the end of the first point/item §7, it can be almost literally (only with replacement Δϑ on Δz) it is used to (8.5).

2. From common expression (8.6) for coupling coefficients it is possible to find several simple correlations. Compare, in the first place, (8.6) with the formula which is obtained during differentiation with respect to z of equality (3.8)

$$\int (E_x^m H_y^m - E_y^m H_x^m)\, dS = kh_m. \qquad (8.8)$$

In this case, differentiate one should only integrand, since range of

integration for all z one and the same. In this way we find

$$S_{mm} = -h_m'/2h_m. \qquad (8.9)$$

Differentiating then for z the condition of orthogonality (3.9) and expressing the obtained integrals through coupling coefficients according to (8.6), let us find reciprocal relation for coupling coefficients

$$h_m S_{mj} = -h_j S_{jm} \quad (m \neq j). \qquad (8.10)$$

Two last/latter formulas can be, it goes without saying, obtained, using only system of equations (8.5), in the same way as was obtained relationship/ratio (7.14).

Page 58.

One should only consider that, according to (8.6), during a change in direction of both of waves value S_{jm}, in contrast to (7.11), does not reverse the sign, i.e., $S_{-j,-m} = S_{jm}$.

Relationship/ratios (8.9) and (8.10) make it possible to check that system (8.5) satisfies the requirements, which escape/ensue from law of conservation of energy. According to the law of conservation of energy, for any field in irregular waveguide with real-valued ε and μ must be constant the sum

$$\sum_j |P_j|^2 h_j, \qquad (8.11)$$

where the addition goes over to all propagated (h_j — really) waves

of both of directions. We convert derivative of (8.11) on z, after replacing P_l' according to equation (8.5). Then this derivative will take the form

$$\sum_l [h_l' + h_l(S_{ll} + S_{ll}^*)] |P_l|^2 + \sum_{l \neq m}\sum [h_l S_{lm}^* + h_m S_{ml}] P_l P_m^*. \quad (8.12)$$

This value must be equal to zero with any P_l, therefore, must be equal to zero brackets in both sums. Since S_{lm} with real ones ε, μ, h_l and h_m is also real, this the requirement is fulfilled according to (8.9) and (8.10).

Let us note that in exactly the same manner it is possible to show that the system of equations (7.9), that describes field in the bent waveguide of constant section, also satisfies the law of conservation of energy. In this case, the equality zero brackets in (8.12) follows from reciprocal relation (7.14); it is necessary still to bear in mind, that for the propagated without attenuation waves the coupling coefficients F_{lm} are pure imaginary and that into this to problem $h_l' = 0$.

3. In specific problems of using expression (8.6) it is usually inconveniently, and below we will find several other formulas for S_{lm}, after obtaining by their transformation of fundamental expression (8.6).

In the regular waveguide in which ε and μ they do not depend

on z, i.e., when $\partial\varepsilon/\partial z=0$, $\partial\mu/\partial z=0$, all coupling coefficients are equal to zero. Let us find such expression for S_{jm}, which, in contrast to (8.6), will contain derivatives on z of ε and μ, but not from fields.

Let us record the equations of Maxwell in the variables x, y, ζ for fields $\hat{H}^m$ and $\hat{E}^m$ in the waveguide of comparison. In these equations E^m, H^m, h_m, and also $\varepsilon(x, y)$, μ(x, y) they depend on z, i.e., from that, to which regular waveguide these values are related. Let us differentiate these equations for z:

Page 59.

$$\mathrm{rot}\,\hat{E}^{m\prime}=-ik\mu\hat{H}^{m\prime}-ik\mu'\hat{H}^m;\ \mathrm{rot}\,\hat{H}^{m\prime}=ik\varepsilon\hat{E}^{m\prime}+ik\varepsilon'\hat{E}^m. \quad (8.13)$$

This differentiation indicates transition from one waveguide of comparison to another, close. For example,

$$(\hat{H}^m)'=(H^{m\prime}-i\zeta h_m' H^m)e^{-ih_m\zeta}. \quad (8.14)$$

Let us multiply equations (8.13) on - $\hat{H}^{-j}$, $\hat{E}^{-j}$, and the equation of Maxwell (3.1), written for a wave numbers (-j), will multiply on $(\hat{H}^m)'$ and $(\hat{E}^m)'$ let us add results. We will obtain the equality

$$\mathrm{div}\,\{[\hat{H}^{-j}(\hat{E}^m)']-[(\hat{H}^m)'\hat{E}^{-j}]\}=ik(\mu'\hat{H}^m\hat{H}^{-j}-\varepsilon'\hat{E}^m\hat{E}^{-j}). \quad (8.15)$$

Let us isolate to the left derived on ζ and let us integrate (8.15) with respect to section, after accepting $j\neq m$. On metal $(\hat{E}_m)'$ it satisfies the same boundary condition (3.2) that and $\hat{E}^m$, therefore will remain to the left only the integral of derivative on ζ from

ζ-th the component of the vector, from which is undertaken the divergence. With substitution $(\hat{H}^m)'$, $(\hat{E}^m)'$ second terms in (8.14) will fall out due to the condition of orthogonality, and the remaining to the left integral will be identical with integral in (8.6). This transformation is led for a coupling coefficient (8.6) to the expression

$$S_{jm}=\frac{1}{2h_j(h_j-h_m)}\int\left(\frac{\partial\varepsilon}{\partial z}E^{-j}E^{m}-\frac{\partial\mu}{\partial z}H^{-j}H^{m}\right)dS \quad (8.16a)$$

or, which is the same thing, to the expression

$$S_{jm}=\frac{1}{2h_j(h_m-h_j)}\int\left[\frac{\partial\varepsilon}{\partial z}(E_x^jE_x^m+E_y^jE_y^m-E_z^jE_z^m)+\right.$$
$$\left.+\frac{\partial\mu}{\partial z}(H_x^jH_x^m+H_y^jH_y^m-H_z^jH_z^m)\right]dS. \quad (8.16b)$$

The presence in the denominator of the obtained formula of difference h_j-h_m does not mean that the coupling coefficient of two waves becomes very large, if propagation constant of these waves converge. According to (8.6), S_{jm} becomes large only with small h_j, but when $|h_j-h_m|\to 0$ S_{jm} it remains final. At close values h_j and h_m is low the integral, which stands in the numerator of expression (8.16). This remains valid and for the row/series of other expressions for S_{jm}, obtained is below, also containing difference h_j-h_m in the denominator (see §9).

Expression (8.16) for coupling coefficients is identical with expression (5.12), found by the examination of small heterogeneities.

On the other hand, applying system (8.5), (8.7) to a small heterogeneity, it is easy to obtain formula (5.6), (5.7) for the amplitudes of the parasitic waves, scattered by this heterogeneity.

Analogously can be obtained expression for h_m'. Let us assume for this into equations (8.15) j=m. Dependence on ζ on the left side of the equation is determined only by second term/component/addend in (8.14). Isolating newly derived according to ζ and taking into account the condition of orthogonality, let us find

$$h_m' = \frac{1}{2h_m}\int(\varepsilon' E^m E^{-m} - \mu' H^m H^{-m})\,dS. \qquad (8.17)$$

4. Expression (8.16) for coupling coefficients we utilize as an intermediate result in order to find another expression for S_{jm}, most convenient under the normal conditions, when ε and μ they are the piecewise constant functions of coordinates and ε' and μ' they are different from zero only on the boundary of the region, occupied with material.

In order to calculate S_{jm} for this case, let us replace first interface with the thin layer, in which ε and μ they pass from the values which they have in material, to $\varepsilon=1$, $\mu=1$. Integral in (8.16) is taken only on this layer (shaded in Fig. 7). Of this, consists the essential difference for expression (8.16) from (8.6), in which and in this case the integral is taken according to entire section. In order not to complicate recording, let us suppose that there is only

one interface. Result naturally can be generalized in the case when such interfaces several.

Let us introduce local system of coordinates s, n in the thin layer on which is conducted the integration in (8.16), thus, in such a way that the unit vector s would be tangent to the duct/contour, on which they intersect interface and cross section, and unit vector n it was normal to s, it lie/rested at the cross-sectional flow and it was directed from area in which $\varepsilon=1$, $\mu=1$, into material. Vector n, s the unit vector, parallel to axis z, they must form the right-handed triad. In order to connect $d\varepsilon/dz$ with $d\varepsilon/dn$ and $d\mu/dx$ with $d\mu/dn$, one should first express these values through gradients ε and μ.

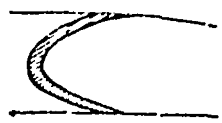

Fig. 7. Transition layer near interface.

Page 61.

The geometric examination which we lower, it gives

$$\frac{\partial\varepsilon}{\partial z} = -\nu\frac{\partial\varepsilon}{\partial n}, \quad \frac{\partial\mu}{\partial z} = -\nu\frac{\partial\mu}{\partial n}, \qquad (8.18)$$

where ν is equal to the tangent of angle α between Z-axis and tangent to the interface, perpendicular to the duct/contour of cross section. Relationship/ratios (8.18) are illustrated in Fig. 8 for the simplest case when vectors n, grad ε and Z-axis lie/rest at one plane.

In this case, according to Fig. 8, it will simply be

$$\frac{\partial\varepsilon}{\partial z} = -|\operatorname{grad}\varepsilon|\sin\alpha; \quad \frac{\partial\varepsilon}{\partial n} = |\operatorname{grad}\varepsilon|\cos\alpha, \qquad (8.19)$$

whence it follows (8.18). In the general case the factors with grad ε in (8.19) take the more complex form, but their sense is always equal $-\nu$.

Strictly speaking, (8.18) it is correct only in the limit, to which we will pass below, bracing the thin layer where occur changes ε and μ, into surface (interface); in this case, grad ε and grad

μ become perpendicular to this surface. Before passage to the limit ν in (8.18) - certain value, which has in limit the geometric value indicated.

Substituting (8.18) in expression (8.16) for a coupling coefficient, we will obtain

$$S_{jm}=\frac{1}{2h_j(h_j-h_m)}\oint\nu\left\{\int\left[\frac{\partial\varepsilon}{\partial n}(E_n^jE_n^m+E_s^jE_s^m-\right.\right.$$
$$\left.\left.-E_z^jE_z^m)+\frac{\partial\mu}{\partial n}(H_n^jH_n^m+H_s^jH_s^m-H_z^jH_z^m)\right]dn\right\}ds. \quad (8.20)$$

Let us recall that those participating in (8.20) fields are related to the regular waveguides of comparison; the characteristic of irregular waveguide is now only value ν. The cross section of the waveguides of comparison consists of two regions with constant values ε and μ and their fine/thin dividing transition region in which ε and μ they depend on x and y.

In transition layer tangential components E^m, H^m and normal components εE^m, μH^m are continuous, so that

$$\begin{aligned}&E_n^m(n)=E_n^m/\varepsilon(n);\ E_s^m(n)=E_s^m,\ E_z^m(n)=E_z^m;\\&H_n^m(n)=H_n^m/\mu(n);\ H_s^m(n)=H_s^m,\ H_z^m(n)=H_z^m\end{aligned}\quad (8.21)$$

and also is related to the fields of the wave of number j.

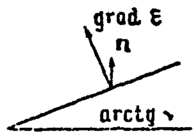

Fig. 8. To the conclusion/derivation of relationship/ratio (8.18).

Page 62.

Here $E_n^j(n)$ and so forth - field in transition layer; to the right values E_n^m confronting and so forth - value of fields on interface from its that side, where $\varepsilon=1, \mu=1$. Formulas (8.21) that are more precise the less the transition layer. They are derive/concluded from the integral form of the equations of Maxwell in the same way as usual boundary conditions of the section of two dielectrics.

After substituting (8.21) in (8.20), it is possible in (8.20) to produce actual integration for n. After this S_{jm} will contain only integral, undertaken on duct/contour S.

In this way is obtained the unknown expression for a coupling coefficient

$$S_{jm} = \frac{1}{2h_j(h_j - h_m)} \int \nu(s) \Big[(\varepsilon - 1)(E_s^j E_s^m - E_z^j E_z^m) - \left(\frac{1}{\varepsilon} - 1\right) E_n^j E_n^m + (\mu - 1)(H_s^j H_s^m - H_z^j H_z^m) - \left(\frac{1}{\mu} - 1\right) H_n^j H_n^m \Big] ds. \quad (8.22)$$

Here ε and μ - permeability value in material.

For computing the coupling coefficient in formula (8.22) it suffices, thus, to know the fields of waves on the boundary of material in regular waveguides and the function $v(s)$, which characterizes the divergence of the waveguide in question in this section from regular.

For a regular waveguide $v=0$, all coupling coefficients are equal to zero.

Expression (8.22) will be assumed as the basis of analysis in the following paragraph.

Formula (8.22) especially is conveniently used, if ε and μ they are close to unity, since substitution in (8.22) fields in the empty waveguide immediately gives the first terms of expansion S_{jm} according to degrees $(\varepsilon-1)$ and $(\mu-1)$.

To the form, analogous (8.22) perhaps it goes without saying, is given also formula (8.17) for a derivative of wave number.

Formulated in the beginning of this paragraph continuity condition of functions $\varepsilon(x, y, z)$ and $\mu(x, y, z)$ ensured convergence of series (8.1)-(8.2) in all points of cross section. It made permissible term-by-term differentiation of these row/series and led to system (8.5) and values S_{jm}, given by formula (8.6). However, further transformation of expressions (8.6), that led for disruptive ones ε and μ to expression (8.22), none is connected with this condition.

Page 63.

Accurately also system (8.5) for coefficients in expansions (8.2), not depending on the degree of the smoothness of functions ε and μ, it remains valid in all stages of passage to the limit from continuous ones to discontinuous functions ε and μ. It will be valid also in limit for discontinuous functions, although row/series (8.2) and (8.3) during this passage to the limit can cease to converge on discontinuity surfaces ε and μ they will become unevenly converging. Situation is here analogous by that with which we are encountered during the usual conclusion of conditions for electromagnetic fields on the interface of two media. These

conditions are also establish/installed first by applying the equations of Haxwell, valid for continuous media, to thin transition layer; the subsequent passage to the limit does not disrupt these conditions, which are satisfied in entire process of passage to the limit.

This reasoning will allow us in the following paragraph to continue the transformation of expression for coupling coefficients and to pass in expression (8.22) to the limiting case of infinite value $|\varepsilon|$.

Passage to the limit to the case of disruptive ones ε and μ we produced above in formula (8.16). This same transition it would be possible to produce and in initial expression (8.6). Obtained in this case expressions for coupling coefficients would be, however, it is considerably more complex than formula (8.22). They would contain, in the first place, the contour integral, analogous (8.22). This integral appears from integration on the thin transition layer where the field gradients are very great and that participating in (8.6) derivatives have as derivatives in formula (8.16), character of δ-functions. In the second place, in these expressions would be preserved integrals of the same type, that (8.6), undertaken according to entire section, since in (8.6) integrands were different from zero in entire section. During passage to the limit $|\varepsilon|\to\infty$

which we will produce in the following paragraph, with this method of calculation in expression for S_{lm} would be preserved integrals of both of types, and integral according to section would contain derivatives on z of membrane/diaphragm functions. Obtained in this way expressions would be considerably more complexly than obtained lower formulas (9.2) or (9.5), and we will not give them. Let us note only that, utilizing formulas (9.11) of the following paragraph, it would be possible algebraically to demonstrate the identity of these formulas for the coupling coefficients and obtained below formulas (9.5).

5. In this point/item we will give, following K. A. Barsukova's work [82], generalization of formalism, developed above, to case when waveguide is filled by material with tensor parameters.

Page 64.

Let the medium be characterized by the hermitian tensors of the dielectric and magnetic constants

$$E = \begin{pmatrix} \varepsilon_1 & -i\varepsilon_2 & 0 \\ i\varepsilon_2 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}, \quad M = \begin{pmatrix} \mu_1 & -i\mu_2 & 0 \\ i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_2 \end{pmatrix}. \qquad (8.23)$$

The condition of orthogonality for fields in waveguide we accept in the form of equality zero integrals

$$\int(E_y^m H_x^{l*} - E_x^m H_y^{l*} + H_x^m E_y^{l*} - H_y^m E_x^{l*})\,dS. \qquad (8.24)$$

with j≠m. It goes without saying, with scalar ones ε and μ condition (3.9) also could be replaced by condition (8.24), but the accepted by us condition is simpler. We standardize fields in such a way that with j=m integral (8.23) would be equal to $2kh_m$.

All reasonings, leading to the conclusion/derivation of system of equations (8.5), in this case will be preserved. Instead of formula (8.6) for a coupling coefficient will be obtained the expression

$$S_{lm} = \frac{1}{2kh_l}\int\{[E^{m*}H^{l}]_z + [E^{l}H^{m*}]_z\}\,dS. \qquad (8.25)$$

From this expression it is possible to pass by the same transformations as produced in the preceding/previous point/item, to expression for S_{lm}, that is the generalization of formula (8.16):

$$S_{lm} = \frac{1}{2h_l(h_l - h_m)}\int\left\{E^{m*}\frac{\partial \mathrm{E}}{\partial z}E^{l} + H^{m*}\frac{\partial \mathrm{M}}{\partial z}H^{l}\right\}dS. \quad (8.26)$$

Derivative of tensor is defined, as usual, as the tensor whose cell/elements are equal to derivatives of the cell/elements of this tensor.

If the components of tensors E and M are the piecewise constant functions, then expression (8.26) also can be brought to contour integral. Field expressions in transition layer will satisfy relationship/ratios, somewhat more complex, than (8.21):

$$E_s^m(n) = E_s^m;\ E_z^m(n) = E_z^m;\ E_n^m(n) = \frac{1}{\varepsilon_1(n)} E_n^m - i\frac{\varepsilon_2(n)}{\varepsilon_1(n)} E_s^m, \quad (8.27)$$

where are preserved the designations of formula (8.21). The same formula for components H^m we do not extract.

Page 65.

Integrating (8.26) on n on transition layer, let us find the expression for a coupling coefficient, which contains only fields on the boundary of the introduced in waveguide body

$$S_{jm} = \frac{1}{2h_j(h_m - h_j)} \oint \nu(s) \Big[\Big(\frac{\varepsilon_1^2 - \varepsilon_2^2}{\varepsilon_1} - 1 \Big) E_s^j E_s^{m*} + \frac{\varepsilon_1 - 1}{\varepsilon_1} E_n^j E_n^{m*} + i\frac{\varepsilon_2}{\varepsilon_1}(E_s^j E_n^{m*} - E_n^j E_s^{m*}) + (\varepsilon_3 - 1) E_z^j E_z^{m*} + \Big(\frac{\mu_1^2 - \mu_2^2}{\mu_1} - 1 \Big) H_s^j H_s^{m*} + \frac{\mu_1 - 1}{\mu_1} H_n^j H_n^{m*} + i\frac{\mu_2}{\mu_1}(H_s^j H_n^{m*} - H_n^j H_s^{m*}) + (\mu_3 - 1) H_z^j H_z^{m*} \Big] ds. \quad (8.28)$$

The results of this point/item can be used, for example, with the examination of the phenomenon of the transformation of the electromagnetic waves during different ferrite connection/inclusions.

6. Let us pass to application/use of obtained system of differential equations (8.5) for amplitudes $P_m(z)$. In the following paragraph it will be shown, that this same the system describes field in tapered weld. Therefore all obtained below results will be used also to such waveguides. Coupling coefficients for them also are

expressed in the form of the contour integrals, which contain the angle of the slope of generatrices to Z-axis.

Let us examine the case of loose coupling, i.e., the case when all coupling coefficients are low. This occurs, if along waveguide its parameters are changed slowly - is low value ν_0, equal to the average/mean value of the tangent of generatrix, or generally ε and μ little they are changed at the distances of the order of the linear dimensions of section. The exceptional case when during a slow change in the parameters coupling coefficients are great due to the smallness of the confronting in denominator wave number, will be examined in the following chapter.

As in the preceding/previous paragraph, let us give first system of equations (8.5) to this form that its matrix/die would not contain diagonal terms. Let us introduce the newly given amplitudes $p_j(z)$, after determining by their now equality

Page 66.

$$P_j(z) = \sqrt{\frac{h_j(0)}{h_j(z)}}\, p_j(z)\, e^{-i\gamma_j};\quad \gamma_j(z) = \int_0^z h_j dz. \qquad (8.29)$$

Substituting (8.29) in system (8.5) and taking into account relationship/ratio (8.9) between S_{jj} and h_j', we will obtain for the given amplitudes system of equations

$$p_j'(z) = \sqrt{\frac{h_\nu(0)}{h_j(0)}}\, S_{j\nu} \sqrt{\frac{h_j(z)}{h_\nu(z)}}\, (1 - \delta_{j\nu})\, p_j e^{-i(\gamma_\nu - \gamma_j)}. \quad (8.30)$$

End conditions for $p_j(z)$, according to conditions (8.7) and introduced in (8.29) to factors, take the form

$$p_m(0)=1,\ p_j(0)=0 \text{ when } j>0,\ j\neq m;$$
$$p_j(L)=0 \text{ when } j<0. \tag{8.31}$$

Repeating the considerations, presented before formula (7.34), let us preserve in (8.30) to the right only amplitude of the incident wave. Then $p_m = 0$, i.e.

$$p_m(z) = 1,\ P_m(z) = \sqrt{\frac{h_m(0)}{h_m(z)}}\, e^{-i\gamma_m}. \qquad (8.32)$$

From comparison with expression (3.10) for the energy, transferred by wave, it follows that in the first order energy of the incident wave does not change along waveguide.

Substituting (8.32) in (8.30), we will obtain for $j \neq m$ the equation

$$p_j' = \sqrt{\frac{h_m(0)}{h_j(0)}}\, S_{jm} \sqrt{\frac{h_j(z)}{h_m(z)}}\, e^{-i(\gamma_m - \gamma_j)}. \qquad (8.33)$$

From it and end conditions (8.3) let us find the amplitudes of the scattered parasitic waves

$$p_j = \pm \sqrt{\frac{h_m(0)}{h_j(0)}} \int_0^L S_{jm} \sqrt{\frac{h_j}{h_m}}\, e^{-i(\gamma_m - \gamma_j)}\, dz. \qquad (8.34)$$

With upper sign formula (8.34) gives $p_j(L)$ with j>0, $j \neq m$, with lower it gives $p_j(0)$ with j<0. The square modulus of the integral, which stands in (8.34), gives energy of the scattered parasitic wave, divided into energy of the incident wave. The physical sense of different cell/elements of formula (8.34) was discussed into §5, where it was obtained by other means.

Formula (8.34) together with the obtained above different

expressions for coupling coefficients reduces stated problem of the calculation of waveguide with the slowly changing parameters to quadrature.

Page 67.

7. Approximate solutions (8.32), (8.34) it is possible to derive of precise system (8.30) somewhat more formal path. Let us expand for this p_j in row/series according to the degrees of the low parameter of the problem

$$p_j = p_j^{(0)} + \nu_0 p_j^{(1)} + \dots \quad (8.35)$$

The character of system (8.30) makes it possible trivially to produce the separation of the members of different orders of smallness. All coupling coefficients have order ν_0, so that $S_{jm} = \nu_0 \bar{S}_{jm}$, where $\bar{S}_{jm}$ is not contained already low factor. With substitution (8.35) into system (8.30) we will obtain

$$p_j^{(0)\prime} = 0, \; p_j^{(1)\prime} = \sqrt{\frac{h_\nu(0)}{h_j(0)}} \bar{S}_{j\nu} \sqrt{\frac{h_j(z)}{h_\nu(z)}} (1 - \delta_{j\nu}) p_j^{(0)} e^{-i(\gamma_\nu - \gamma_j)}. \quad (8.36)$$

The first of these equations gives $p_j^{(0)} = 0$ with $j \neq m$, $p_m^{(0)} = 1$. From the second equation follows that $p_m^{(1)} = 0$ and the formula for $p_j^{(1)}$, identical (8.34).

This reasoning can, however, prove to be erroneous. The problem in question contains, besides the low parameter ν_0, also the high

parameter - length of section L, and in condition total variation in the properties of waveguide, proportional to product $\nu_0 L$, it is not considered small. Therefore it can seem that expression (8.34) for $p_l^{(1)}(z)$, giving low value in the beginning of section, with large z ceases to be small. This really/actually occurs during the degeneration when the propagation constant of any wave is close to the propagation constant of the incident wave. In such cases exponential factor under integral in (8.34) will not be the rapidly changing function from z, and integral in (8.34) will not be low value. Used in this case calculation method we will examine into §15 based on the example of the twisted waveguide. Analogous conditions we met in the theory of the waveguides, bent on curved small curvature. Applicability condition for entire approach/approximation of "loose coupling" is therefore not simply smallness ν_0 or analogous values, but smallness of all amplitudes, given (8.34).[1].

FOOTNOTE [1]. It is more precise, small in comparison with unity must be all expressions of type (8.34) with substitution instead of the constant upper limit of L of any value z from the interval in question. ENDFOOTNOTE.

This is reasoning, in particular, it explains, why to more conveniently examine the given amplitudes $p_l(z)$ it is not possible the same expansion to produce directly with system (8.5) for $P_l(z)$. The

right side of the equation for P_j' contains term/component/addend, proportional P_l.

Page 68.

With the substitution of the expansion of amplitudes P_l, analogous (8.35), this term/component/addend (with $j \neq m$) would be the second order of smallness (on v_0). However, during integration on long section, it would give in $P_l(z)$, as it is easy to check, the contribution of the first order on v_0. Thus, for obtaining the correct result it would be necessary during integration for long section to unite the members of the different order of smallness. Transition from one variables P_j to the next p_j allows, applying expansion according to degrees v_0, to construct the solution which considerably more slowly diverges during advance along waveguide. Physically this is connected with the fact that the solution $p_m(z)=1$ satisfies, as we indicated, to the law of conservation of energy.

There is a specific analogy between transition from $P_j(z)$ to the given amplitudes $p_j(z)$ and known transformations, produced in the theory of the propagation of waves in the inhomogeneous media (for example, see [80], §16) when deriving the equations of geometric optics.

8. In chapter V, we will use results of this paragraph to calculation of row/series of concrete/specific/actual systems. Let us here note only three special cases when formulas for the amplitudes of the scattered waves take simpler form. Let, in the first place, the coupling coefficient S_{lm} in entire irregular section have one and the same order of magnitude, and at the end/leads of the section with z=0, z=L jump takes zero values. This can, for example, occur, if the slope tangent of generatrix $v(s, z)$ in (8.22) little is changed in interval 0<z<L. Then integral in expression (8.34) can be calculated by integration and to reject/throw in parts integral term. [1].

FOOTNOTE [1]. If (8.37) it is equal to zero (at any frequency), then to calculate p_l according to (8.34) is impossible, as the remaining integral term there will be the same (the second on v_0) order, that also third add/composed in (8.35), not considered in (8.34). This consideration is related also to the third case, mentioned at the end of this point/item. ENDFOOTNOTE.

In this case, we disregard the values of the order of the square of average/mean value $v(s, z)$ and order by derivative v on z. For p_l is obtained in this way explicit expression in the form of the binomial formula, analogous (7.35):

$$p_l = \mp i\left(\frac{S_{lm}}{h_m - h_l}\right)_{z=0} \pm i\sqrt{\frac{h_m(0)\,h_l(L)}{h_l(0)\,h_m(L)}}\left(\frac{S_{lm}}{h_m - h_l}\right)_{z=L} \times$$
$$\times e^{-[\gamma_m(L) - \gamma_l(L)]}. \quad (8.37)$$

Here upper sign is related to direct waves, lower - to reverse/inverse ones.

Page 69.

As for the bent waveguide, the amplitude of direct/straight parasitic waves it is more than the amplitude of reverse/inverse ones, since for direct waves the oscillatory factor under integral in (8.34) is the less rapidly changing function, than for reverse/inverse ones. However, for rectilinear waveguides this difference in amplitudes is less, since very coupling coefficients S_{lm}, as we will see below, have, generally speaking, one and the same order of magnitude for direct/straight and backward waves, but coupling coefficients F_{lm} in bending considerably more for direct waves, than for reverse/inverse ones.

Integration in parts, produced during conclusion/derivation (8.37), isolated the values of coupling coefficient at the salient points of generatrix. At these points occurs less complete, than in other sections, the compensation the parasitic waves, which are

formed along irregular section. The second case, for the first time examined in Yu. M. Isaenko's article [83], by whom it is possible to produce the approximative integration in expression (8.34), is connected with the character of a change of the rapidly oscillatory exponential factor in (8.34). The derivative of the exponential in integral (8.34) is equal to $i(h_j-h_m)$. If in any section the propagation constant of the appearing direct/straight parasitic wave $h_j(z)$ is equal to the propagation constant of the fundamental wave $h_m(z)$

$$h_j(z)=h_m(z), \qquad (8.38)$$

the for integral (8.34) corresponding value z gives the point of steady state. In the vicinity of this point, exponential factor changes more slowly than in other regions of irregular section, compensation also is attenuate/weakened; entire integral (8.34) proves to be the approximately equal to the integral, undertaken on this vicinity, and the value or the functions, which stand by factor with exponent, simply they will be carried as integral sign.

Integral (8.34) for direct waves can be recorded in the form

$$p_j=\int_0^L \Phi(z)e^{-i\sigma\psi(z)}dz, \qquad (8.39)$$

where Φ equal to all product preexponential factors in (8.34), and the function ψ, which is everywhere of the order of one, is determined by the equality

$$\psi=\frac{\gamma_m(z)-\gamma_j(z)}{\gamma_m(L)-\gamma_j(L)}. \qquad (8.40)$$

In this case, is isolated the high parameter of problem σ, equal to a

difference in phase change of two these waves in entire irregular section

$$\sigma = \gamma_m(L) - \gamma_l(L). \qquad (8.41)$$

Page 70.

If in region 0<z<L there is equation $\psi'=0$, i.e., the root of equation (8.38), then integral (8.39) can be calculated according to the method of steady state, utilizing condition $|\sigma|\gg 1$. Leading term in expansion (8.39) according to reverse/inverse degrees σ is of the order $\sigma^{-1/2}$. He is equal to

$$p_j = e^{-i[\gamma_m(z_s)-\gamma_j(z_s)-\pi/4]}\Phi(z_s)\sqrt{\frac{2\pi}{h_m'(z_s)-h_j'(z_s)}}, \quad (8.42)$$

where $z=z_s$ - a root of equation (8.38). The reject/thrown terms will be of the order $\sigma^{-3/2}$.

Expression (8.42) contains low factor $\Phi(z_s)$, however, in the denominator of this expression, also will cost low value - root from derivative on z. Therefore, when in irregular section there is points of steady state, i.e., the points, at which is satisfied condition (8.38), then the amplitude of the corresponding parasitic wave proves to be comparatively large, proportional to square root of the angle of the slope of generatrix, i.e., from series expansion parameter of problem. Wave amplitudes for which equation (8.38) does not have roots, will be less, they contain slope angle to the first degree. In

article [83] formula (8.42) is used to the determination of the amplitudes of the parasitic waves, which appear in special complex waveguide transition.

The third case, in some ratio/relations opposite to the first, with which also it is possible to calculate integrals (8.34), appears, if the functions, which describe the surface of irregular waveguide, are continuous together with all their derivatives. This case is examined in the article of Pokrovskiy, etc. [26], in which are analyzed thoroughly the integrals of the same type as (8.34). As that is accepted in analogous quantum-mechanical problems, these integrals are calculated by the steepest descent method in the plane of a complex variable; they prove to be in this case exponentially low values, order e^{-C/ν_0}, where number C depends on the concrete/specific/actual form of integral (8.34). The application/use of this computational procedure is connected with definite difficulty, since under actual conditions the surface of metal is not described analytic function. This is procedure, apparently, it can prove to be essential in the theory of natural ones, for example atmospheric, waveguides.

9. In all constructions of this paragraph, we assumed that field $E^m(x, y)$, $H^m(x, y)$ and wave numbers h_m of its own waves in regular waveguides with heterogeneous filling, i.e., in waveguides in which

$\varepsilon = \varepsilon(x, y)$, $\mu = \mu(x, y)$, to us were known.

Page 71.

In a number of cases, however, the determination of these fields is independent and complex problem. Is presented briefly a method which makes it possible to solve it to any degree of accuracy.

In contrast to the expansions which we used above and let us use for the extent/elongation of an entire monograph, field in the filled waveguide it is possible to decompose on the fields of waves in the same empty waveguide. This expansion was for the first time suggested by Shchelkunov [55] and it is used by Morgan [44]; however, the developed by them apparatus is extremely complex; considerably more effective the same expansion was used by Brodskiy [56]. Method presented below makes it possible to obtain results, virtually equivalent to results articles [56], by more direct/straight and simpler method.

Let us introduce for fields and wave numbers of its own waves in the regular empty waveguide of designation E^{0m}, H^{0m}, h_m^0. These values satisfy equations (3.4) with $\varepsilon = 1$, $\mu = 1$. The transverse components of fields in the filled waveguide of the same section let us decompose in the row/series

$$E_x = P_\nu E_x^{0\nu},\ E_y = P_\nu E_y^{0\nu};\ H_x = P_\nu H_x^{0\nu},\ H_y = P_\nu H_y^{0\nu}, \quad (8.43a)$$

where $P_l = P_l(z)$. After substituting these expansions into two of the six equations of Maxwell, we will obtain for the longitudinal components of the expansion

$$E_z = \frac{1}{\varepsilon} P_\nu E_z^{0\nu},\ H_z = \frac{1}{\mu} P_\nu H_z^{0\nu}, \qquad (8.43b)$$

characterizing by from (8.43a) cofactors $1/\varepsilon$ and $1/\mu$. The presence of these factors shows, by the way that the complete field in the filled waveguide cannot be, strictly speaking, it is decomposed on the fields of waves in the empty waveguide. According to formulas (8.2-8.3), this expansion in terms of the fields of waves in waveguide with the same accurately section exists.

After substituting (8.43) into the remaining four equations of Maxwell, we will obtain four equations of type (8.4). One of them takes the form

$$P'_\nu E_x^{0\nu} = P_\nu \left\{ - ik\mu H_y^{0\nu} + \frac{\partial}{\partial x}\left(\frac{1}{\varepsilon} E_z^{0\nu}\right)\right\}, \qquad (8.44)$$

three others we we do not extract. Using further the condition of orthogonality for fields in the empty waveguide, we will obtain for variables $P_l(z)$ the system of ordinary differential equations with the constant coefficients

$$P'_l = M_{l\nu} P_\nu. \qquad (8.45)$$

Page 72.

Values M_{lm} characterize the communication/connection between waves in the empty waveguide, obliged to difference ε and μ from unity. After simple transformations, utilizing, in particular, the theorem of Stokes and boundary condition for E, we will obtain from (8.44)

$$M_{lm} = \frac{i}{2h_l^*} \int \Big\{ \frac{1}{\varepsilon} E_z^{0l} E_z^{0m} + \frac{1}{\mu} H_z^{0l} H_z^{0m} + \\ + \varepsilon (E_x^{0l} E_x^{0m} + E_y^{0l} E_y^{0m}) + \mu (H_x^{0l} H_x^{0m} + H_y^{0l} H_y^{0m}) \Big\} dS. \quad (8.46)$$

Formulas (8.45-8.46) make the same sense, as formula (19-20) of article [56]. The definition of coefficients M_{lm} in this method does not require as in [55] and [44], the solution of infinite system of equations.

The fields of their own waves in waveguide with $\varepsilon = \varepsilon(x, y), \mu = \mu(x, y)$ are obtained from requirement so that the solutions of equation (8.45) would satisfy the condition

$$P_l(z) = P_l(0) e^{-ihz}. \quad (8.47)$$

Substitution (8.47) in (8.45) reduces as for the bent waveguides, to the infinite system of homogeneous algebraic equations for $P_l(0)$

$$(h\delta_{l\nu} - iM_{l\nu}) P_\nu(0) = 0. \quad (8.48)$$

The wave numbers of their own waves h are, thus, roots of equation [sr (7. 39)]

$$\mathrm{Det} \, | M_{lm} + ih\delta_{lm} | = 0. \quad (8.49)$$

Equations (8.48) and (8.49) are analogous to equations (7.38) and (7.39). After solving (8.49), we for each root of h will find from (8.48) and the condition for standardization (3.8) of amplitudes $P_j(0)$, which correspond to this their own wave.

If $\varepsilon=1$, $\mu=1$, then $M_{jm}=-ih_j^0\delta_{jm}$, the matrix/die of system (8.48) will be diagonal, and the roots of equation (8.49) will be all numbers h_j^0. If $\varepsilon=\text{const}\neq 1$, $\mu=\text{const}\neq 1$, then they are different from zero only those coefficients M_{jm}, in which j=+-m. That standing in (8.49) determinant decomposes into the product of the determinants of the second order, so that wave numbers will be located from quadratic equations; it is easy to confirm that these wave numbers are equal to $\pm((h_j^0)^2+k^2(\varepsilon\mu-1))^{1/2}$, as this follows from elementary considerations.

The effectiveness of the method of determining the field of its own waves presented depends on the order of magnitude of nondiagonal elements M_{jm} and of rate of their decrease during distance from diagonal.

Page 73.

If, for example, $|\varepsilon(x, y)-1|$ and $|\mu(x, y)-1|$ they are small, then will

be low all nondiagonal coefficients M_{lm}, and matrix/die in (8.48) will take almost diagonal form. The solution of equation (8.49) can be in this case in old order on $|\varepsilon-1|$ and $|\mu-1|$ obtained in explicit form.

Let us note in conclusion still following. Expansion (8.43) and system of equations (8.45) describe field also in the irregular waveguide of constant section, i.e., when $\varepsilon(x,y,z)$ and μ(x, y, z), and in this sense they have the same value, as system (8.5). During the use of system (8.45) for irregular waveguides in which now M_{lm} they depend on z, variables $P_l(z)$ must satisfy not condition (8.47), but to other any end conditions. If waveguide with irregular filling is connected from both of sides with the empty waveguides, then end conditions take form (8.7). In the more general case end conditions for (8.45), in turn, are determined from the system of the algebraic equations, which are obtained from the requirement of the continuity of fields at the end/leads of the irregular section.

The question concerning that, which of two systems of equations - (8.5) or (8.45) - expedient to apply in any specific problem, in the final analysis it is determined by the structure of field in this irregular waveguide. In the broadband well matched equipment/devices field in any section it is usually close to field in the regular waveguide of the same section, and therefore during the analysis of such equipment/devices to more conveniently use system of equations

(8.5): in it will be small the coupling coefficients and, as a rule, also all amplitudes P_l, except one or two. Entire examination of §7 would be very hinder/hampered during the use of system (8.45); field would be described by a large number of amplitudes P_l, strongly connected. It would be to use this system and to the common/general/total constructions of the following paragraph. Most probable, it, as a rule, it is expedient to utilize only for the determination of their own waves in waveguides with heterogeneous filling.

However, in last/latter point/item §22, we will examine equipment/device in which the field is close to field in the empty regular waveguide; during the analysis of this bending, the application/use of expansions of type (8.43) will lead to target/purpose somewhat faster than the use of expansions of type (8.2-8.3).

Page 74.

§9. Calculation of tapered welds by the method of cross sections.

1. Direct application/use of method of cross sections to tapered welds is connected with definite difficulty, which consists of the fact that in row/series of type (8.1) of field E, H and of field

E^m, H^m they satisfy different boundary conditions. Therefore on the duct/contours of cross sections, these row/series not converge to the functions which they must represent. Differentiation of such row/series brings, as is known, to the certain complications.

From a physical point of view, the difficulty consists in the fact that the representation of field in the form of row/series is not applicable near metal, i.e., precisely, where must be placed boundary conditions.

In order not to operate with the unevenly converging series, we will produce computations by the fol'-'wing diagram, which rests on the results of the preceding/previous paragraph: compare this tapered weld (Fig. 9) with the auxiliary waveguide of constant section (Fig. 10), by the filled material with constants ε and μ in such a way that free from material there remains only the region, which corresponds to interior of this tapered weld. In other words, let us enter this waveguide in the waveguide of constant and larger section and will fill with material with $\varepsilon=\mathrm{const}\neq 1$, $\mu=\mathrm{const}\neq 1$ the region between the boundaries of both waveguides. For this large waveguide with irregular filling, we can, using the results of the preceding/previous paragraph, to write expansions (8.2), (8.3) and system of equations (8.5) for the coefficients of expansion. For that discontinuous distribution of material, which we assigned in large

waveguide, coupling coefficients are expressed by formula (8.22). These results are valid for any value ε and μ material.

Let us then increase the dielectric constant ε of material, making it in this case composite. All above formulas will remain valid, will be they valid also in passage to the limit $|\varepsilon|\to\infty$. Upon this transfer auxiliary waveguide (Fig. 10) becomes identical to this tapered weld (Fig. 9). Thus, formulas (8.2), (8.3), (8.5), (8.7), (8.9), (8.10), derived for a waveguide with alternating/variable filling, and all formulas and results of three last/latter point/items §8 prove to be directly used also for a waveguide with alternating/variable section.

Fig. 9. Tapered weld.

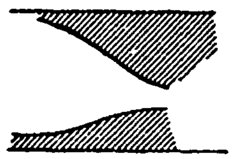

Fig. 10. Auxiliary waveguide of constant section.

Page 75.

As far as formula is concerned (8.22) for coupling coefficients, in it appears an indeterminancy/uncertainty of type $\infty 0$, since when $|\varepsilon| \to \infty$ components E_s^m and E_z^m on duct/contour, i.e., on the boundary of material, will vanish. In order to find explicit expression for a coupling coefficient in tapered weld, let us produce in (8.22) consecutive passage to the limit.

When $|\varepsilon| \to \infty$ fields E_s^m, E_z^m and H_n^m will vanish. In this case, according to Leontovich boundary condition, between tangential components of electrical and magnetic fields there is the communication/connection:

$$E_s^m = \sqrt{\frac{\mu}{\varepsilon}} H_z^m; \quad E_z^m = -\sqrt{\frac{\mu}{\varepsilon}} H_s^m;$$
$$E_s^l = \sqrt{\frac{\mu}{\varepsilon}} H_z^l; \quad E_z^l = -\sqrt{\frac{\mu}{\varepsilon}} H_s^l. \qquad (9.1)$$

Let us recall that relationship/ratios (9.1) are related to fields in the regular waveguides of the comparisons for which normal to the surface coincides with n.

After substituting (9.1) into formula (8.22), we will obtain under integral the expression in which it is already easy to pass to limit $|\varepsilon| \to \infty$. In this way is obtained the following formula for a coupling coefficient in tapered weld:

$$S_{lm} = \frac{1}{2h_l(h_l - h_m)} \oint \nu(s)(E_n^l E_n^m + H_z^l H_z^m - H_s^l H_s^m)\, ds. \qquad (9.2)$$

As one would expect, the value of magnetic permeability of material fell out from the resultant expression. Let us note that this it would be possible to use, in order by the shortest path to derive formula (9.2), operating with scalar, but not vector expressions. For this, it was necessary to consider that in waveguide with alternating/variable filling (Fig. 10), introduced in the beginning of paragraph, $\mu(x, y, z)$ is equal to $1/\varepsilon(x, y, z)$ Then and in the waveguides of comparison with heterogeneous filling product $\varepsilon \cdot \mu$ would be constantly. In such waveguides, as it is easy to show, fields are expressed as two scalar functions ψ and ϕ on the

formulas, analogous (3.14), and there is as in the empty waveguides, division into E- and H-waves. Functions ψ and ϕ satisfy the self-adjoint equations of type (3.11), that also generating the complete system of eigenfunctions and, etc.

Formula (9.2) expresses the coupling coefficient through currents and charges, which appear of the passage of the waves of numbers j and m on the walls of the regular waveguide of comparison.

Page 76.

for in question in this paragraph irregular tapered welds, the waveguide of comparison is the empty waveguide with the section, that also the section of irregular waveguide with the given z.

Completing the same dual passage to the limit (to discontinuous distribution ε and to $|\varepsilon|\to\infty$) in formula (8.17), is easy to find also for a derivative of wave number expression in the form of the contour integral

$$h'_m = \frac{1}{2h_m} \oint \nu(s) [(E_n^m)^2 + (H_z^m)^2 - (H_s^m)^2] ds. \qquad (9.3)$$

Formulas (9.2) and (9.3) coincide with formulas (6.16) and (6.12), obtained in §6 more elementary methods. On the other hand, applying (8.5) and (8.7) to a small irregularity, it is easy to

obtain formulas (6.7) and (6.8).

From expressions (9.2) and (9.3) is easy to obtain the relationship/ratio between $S_{-m,m}$ and h'_m

$$S_{-m,m}=-\frac{h'_m}{2h_m}-\frac{1}{2h_m^2}\oint \nu(s)(H_s^m)^2 ds, \qquad (9.4)$$

being another recording of formula (6.13).

In the empty waveguide of field, they can be expressed through membrane/diaphragm functions. Substituting expressions (3.14) in (9.2), we will obtain formulas for coupling coefficients; in chapter VI, these formulas are applied for concrete/specific/actual computations. Formula (9.5a) is related to that case when both of magnetic type waves, (9.5b) - when they are electrical types both, and (9.5c) - when the wave m - magnetic type number, but the wave of number j - electrical. According to reciprocal relation (8.10), the fourth possible case separately examined not must not be.

$$S_{jm}=\frac{1}{2h_j(h_j-h_m)}\oint \nu(s)\left[\alpha_j^2\alpha_m^2\psi^j\psi^m+(h_jh_m-k^2)\frac{\partial\psi^j}{\partial s}\frac{\partial\psi^m}{\partial s}\right]ds; \qquad (9.5a)$$

$$S_{jm}=\frac{k^2-h_jh_m}{2h_j(h_j-h_m)}\oint \nu(s)\frac{\partial\varphi^j}{\partial n}\cdot\frac{\partial\varphi^m}{\partial n}ds; \qquad (9.5b)$$

$$S_{jm}=-\frac{k}{2h_j}\oint \nu(s)\frac{\partial\varphi^j}{\partial n}\cdot\frac{\partial\psi^m}{\partial s}ds. \qquad (9.5c)$$

In these formulas n - the standard, directed into metal, s is selected so that n, s and unit vector, directed along the axis z,

would form the right-handed triad.

Page 77.

For a derivative of wave number for magnetic and electrical waves, it will be, according to (9.3),

$$h'_m = \frac{\alpha_m^2}{2h_m} \oint \nu(s) \left[\alpha_m^2 (\psi^m)^2 - \left(\frac{\partial \psi^m}{\partial s}\right)^2\right] ds; \qquad (9.6a)$$

$$h'_m = \frac{\alpha_m^2}{2h_m} \oint \nu(s) \left(\frac{\partial \varphi^m}{\partial n}\right)^2 ds. \qquad (9.6b)$$

In the same form, analogous (6.13), can be recorded also formula (9.4).

2. Coupling coefficients depend on frequency, since in (9.5) enter k and h, moreover dependence this for direct/straight (j<0) and reverse/inverse (j<0) waves is different. However, of high frequencies all coefficients of coupling (9.5) on frequency do not virtually depend. High we call the frequencies at which the phase rates of the waves in question are close to the speed of light, i.e., $\bar{h}_m \approx 1, \bar{h}_j \approx 1$. Utilizing the identity

$$\frac{h_m h_j - k^2}{h_j - h_m} = \frac{h_m \alpha_j^2 + h_j \alpha_m^2}{\alpha_j^2 - \alpha_m^2}, \qquad (9.7)$$

escape/ensuing from determination of h (3.13), it is easy for S_{jm} when $\bar{h}_m \approx 1, \bar{h}_j \approx 1$ to write the expressions, corresponding to three formulas (9.5):

$$S_{lm}=\frac{1}{2(\alpha_m^2-\alpha_l^2)}\oint\nu(s)\left[\alpha_l^2\alpha_m^2\psi^l\psi^m-(\alpha_l^2+\alpha_m^2)\frac{\partial\psi^l}{\partial s}\cdot\frac{\partial\psi^m}{\partial s}\right]ds; \quad (9.8a)$$

$$S_{lm}=\frac{\alpha_m^2\pm\alpha_l^2}{2(\alpha_m^2-\alpha_l^2)}\oint\nu(s)\frac{\partial\varphi^m}{\partial n}\frac{\partial\varphi^l}{\partial n}ds; \quad (9.8b)$$

$$S_{lm}=\mp\frac{1}{2}\oint\nu(s)\frac{\partial\varphi^l}{\partial n}\frac{\partial\psi^m}{\partial s}ds. \quad (9.8c)$$

These expressions do not contain frequency. Two last/latter formulas (9.8b) and (9.8c), are recorded both for straight lines and for backward waves; upper sign in them is related to direct waves, lower - to reverse/inverse ones. Formula (9.8a) is written for direct waves. For backward waves in it, one should drop/omit first term and replace ratio/relation $(\alpha_m^2+\alpha_l^2)/(\alpha_m^2-\alpha_l^2)$ with unity.

Page 78.

If one of the magnetic waves is symmetrical, i.e., $\partial\psi/\partial s\equiv 0$, then in (9.5a) is present only first term/component/addend. For reverse/inverse magnetic waves in this case, on (9.8a) will be obtained the zero value of coupling coefficient. It is necessary, however, to bear in mind, that (9.8) is only the old (not depending on frequency) term in expansions S_{lm} according to the negative degrees of frequency. For reverse/inverse symmetrical magnetic waves old will be the following term, and for S_{lm} it is necessary to use the expression

$$S_{lm}=\frac{\alpha_l^2\alpha_m^2}{4k^2}\oint\nu(s)\psi^m\psi^l ds. \quad (9.9)$$

If we do not consider this exceptional case, then with an increase in the frequency coupling coefficients approach the finite values, different for direct/straight and backward waves and by order of value by equal to v_0/a, where a - order of the linear dimensions of the section of waveguide. For magnetic symmetrical waves and in some other special cases the coupling coefficient of backward waves of high frequencies vanishes as v_0/k^2a^3.

From independence S_{lm} from frequency for high frequencies (and far from resonance frequencies) follows also the independence of losses on step from frequency. In §6 this independence was shown based on the example of symmetrical magnetic wave in the circular waveguide. Really/actually, from the comparison of expressions (6.7), (6.8) for amplitudes P_l scattered by the step of waves and expressions (9.2) for a coupling coefficient S_{lm} it follows that P_l is obtained from S_{lm} by replacement under integral sign $v(s)$ to the height/altitude of step δ(s). Consequently, at the high frequencies of amplitude P_l also they do not depend on frequency; they are of the order δ/a. Do not depend on frequency also the total relative energy losses on step, equal, according to (3.10),

$$\sum_l |P_l|^2 h_l/h_m, \qquad (9.10)$$

where the addition goes over all those propagating waves. A number of members of sum (9.10), true, increases with an increase in the frequency, but usually S_{jm} rapidly decreases with an increase in the number [see, for example, (16.1), (16.2)]. The large part of the energy is taken away by waves with the small index (it is more precise - with number j, by close to m), row/series (9.10) converges well and its sum little changes with a change in the number of term/component/addends. The energy losses of any wave in the waveguide of arbitrary section have at high frequencies an order of ratio δ^2/a^2.

Page 79.

3. If in any section of irregular waveguide eigenvalues α of two waves coincide, i.e., $\alpha_m = \alpha_l$, then in this section coincide wave numbers, and denominators of expressions (9.5a) and (9.5b) for coupling coefficients are turned into zero. Simultaneously disappear the numerators of these formulas, and they acquire indefinite form 0/0. This section is, for example, in the steady converter of wave H_{10} in rectangular waveguide into a wave of the type H_{01} in the circular waveguide. Usually the analytical expressions, to which in specific problems are given formulas (9.5), they do not have the indefinite form or indeterminancy/uncertainty in them is opened by elementary shape; however, we all the same will give the

common/general/total transformation of formula for S_{lm} to the form, used also in such special sections.

For the derivation of the corresponding formula from (9.5a) we will use two auxiliary identities, which connect the undertaken according to section and on the duct/contour of section integrals of the membrane functions of magnetic waves. These identities take the form

$$\oint \nu \frac{\partial\psi^m}{\partial s}\frac{\partial\psi^l}{\partial s}ds + \alpha_l^2\int \psi^{m'}\cdot\psi^l dS + \alpha_m^2\int \psi^m\psi^{l'}dS = 0;$$

$$\oint \nu\psi^m\psi^l ds + \int \psi^{m'}\cdot\psi^l dS + \int \psi^m\psi^{l'}dS = 0. \qquad (9.11a)$$

It is easy to obtain them, differentiating on z of the condition of the orthogonality

$$\int \nabla\psi^m\nabla\psi^l dS = 0, \quad \int \psi^m\psi^l dS = 0, \qquad (9.12)$$

converting result on Green's formula and utilizing the differential equation and a boundary condition for functions ψ. Replacing in (9.5a) contour integrals on (9.11a), we will obtain for the coupling coefficients of two magnetic waves the expression

$$S_{lm} = \frac{1}{2h_l}\left\{\alpha_m^2 h_l\int \psi^m\psi^{l'}dS - \alpha_l^2 h_m\int \psi^{m'}\cdot\psi^l dS\right\}, \qquad (9.13a)$$

not having the already indefinite form when $\alpha_m - \alpha_l \to 0$.

Page 80.

The analogous transformation of formulas (9.5b) is based on the

auxiliary identities

$$\oint \nu \frac{\partial \varphi^m}{\partial n} \frac{\partial \varphi^l}{\partial n} ds + \alpha_m^2 \int \varphi^{m'} \varphi^l dS + \alpha_l^2 \int \varphi^m \varphi^{l'} dS = 0;$$

$$\int \varphi^{m'} \varphi^l dS + \int \varphi^m \varphi^{l'} dS = 0, \qquad (9.11b)$$

obtaining in the same way as (9.11a), from equation (3.11). This transformation is led for the coupling coefficient of two electrical waves to the expression

$$S_{lm} = -\frac{1}{2h_l}(h_m \alpha_l^2 + h_l \alpha_m^2) \int \varphi^{m'} \varphi^l dS. \qquad (9.13b)$$

Formulas (9.13) confirm that the coupling coefficients can become infinity only when $|h_l| \to 0$. However, for specific calculations they are considerably less convenient, than formula (9.5), since contain derivatives on z of membrane/diaphragm functions.

We will use formulas (9.13) in order to connect between themselves coefficients F_{lm} and S_{lm}. The strain of waveguide, which we considered as fracture (Fig. 11), it is possible to treat also as the special case of section change, with which the duct/contour of the waveguide is displaced in the direction of x-axis in the distance, proportional to coordinate z. This waveguide is related to the type of the waveguides of alternating/variable cross section with the fracture of generatrix, and the amplitudes of the scattered waves can be found from formula (8.37), in which, obviously, will have to preserve only first term/component/addend. On the other hand, these

amplitudes, according to (4.6), are proportional F_{jm}. Thus, coupling coefficients F_{jm} can be expressed by the coupling coefficients S_{jm}, calculated for the special strain of Fig. 11.

Comparing formulas (8.37) and (4.5), we will obtain (with $j \neq m$)

$$F_{jm} = \frac{-i}{h_m - h_j} \frac{S_{jm}}{\Delta\theta}. \qquad (9.14)$$

Let us calculate S_{jm} according to formulas (9.13). Let us introduce for this the system of coordinates $\bar{x}$, $\bar{y}$, rigidly circuital of waveguide. It is obvious that

$$\bar{x} = x - z \cdot \Delta\vartheta, \quad \bar{y} = y. \qquad (9.15)$$

Those leading in (9.13) derivatives on z let us replace with derivatives on $\bar{x}$:

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial \bar{x}} \cdot \frac{\partial \bar{x}}{\partial z} = -\Delta\vartheta \frac{\partial}{\partial \bar{x}}. \qquad (9.16)$$

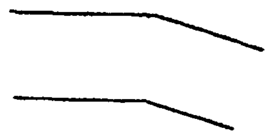

Fig. 11. Break of waveguide as change in its section.

Page 81.

We will begin with expression (9.13a). It it is possible to record in the form

$$S_{lm} = \frac{-\Delta\theta}{2h_l}\left\{h_l\varkappa_m^2\int\psi^m\frac{\partial\psi^l}{\partial x}dS - h_m\varkappa_l^2\int\psi^l\frac{\partial\psi^m}{\partial x}dS\right\}. \quad (9.17)$$

Entering this formula integrals according to section are identical with appropriate by the integrals, determined in (7.18), and coupling coefficient S_{lm} for this special means of strain proves to be equal to

$$S_{lm} = -\frac{\Delta\theta}{2h_l}\{h_l\varkappa_m^2K^{ml} - h_m\varkappa_l^2K^{lm}\}. \quad (9.18)$$

Substituting this in (9.14), let us find for a coefficient F_{lm} the expression, which coincides with (7.20).

For two electrical waves, accordingly (9.13b) and (9.16), it will be

$$S_{lm} = \frac{\Delta\theta}{2h_l}(h_m\varkappa_l^2 + h_l\varkappa_m^2)\int\varphi^l\frac{\partial\varphi^m}{\partial x}dS. \quad (9.19)$$

Substituting in equation (9.14) and using designation (7.18), we for F_{jm} will obtain the expression, identical with (7.21).

Thus it would be possible to check expression (7.22) for the coupling coefficient of magnetic and electrical wave.

The calculation conducted only does not serve as testing formulas (7.20-7.21), but it makes it possible also to explain, why for direct waves F_{jm} it is more than for reverse/inverse ones. From this point of view, the fracture is not the elementary, but extended heterogeneity, in which are essential the phenomena of the interference between the elementary parasitic waves, which arose at different points. But in such heterogeneities, as it was noted, the amplitudes of direct waves are greater than the amplitude of reverse/inverse ones. With this is connected the appearance in the denominator of formula (9.14) - and therefore even in the denominator of formulas (7.20-7.21) - difference in the wave numbers.

Page 82.

§10. Calculation of the general case of irregular waveguide by the method of cross sections.

The general view of irregularity lies in the fact that the axis of waveguide changes its direction and simultaneously they change both properties of the which fills waveguide medium and form or position of cross section. From the point of view of the theory of small heterogeneities, developed in the preceding/previous chapter, the elementary heterogeneity of general view is composed of they are elementary heterogeneities of three types to those examined into §4-6. In a certain sense are summarized, as we will see, and the coupling coefficients; necessary in this case to bear in mind, that the coefficients F_{lm} and S_{lm} have different different dimensionalities: F_{lm} is designed per angular unit, and S_{lm} - per the unit of length.

1. Simply is analyzed case of rectilinear tapered weld with alternating/variable filling (ε and μ), since coupling coefficients, obliged to both heterogeneities, only do not have identical geometric nature, but, according to two preceding/previous paragraphs, they have common/general/total analytical expression. In order to find formula for S_{lm} in this case, let us enter how in §9, this tapered weld into the larger waveguide of constant section let us fill space between the surfaces of waveguides with medium with parameters ε_0, μ_0. Coupling coefficient in this auxiliary waveguide is given by formula

(8.16). Howeve., in contrast to the analogous integral, examined into §9, integrand not is equal to zero not only within narrow transition layer near the boundary of material with parameters ε_0, μ_0, but also on entire plane of cross-section of initial waveguide. Completing then passage to the limit $|\varepsilon_0|\to\infty$, we will obtain for a coupling coefficient the sum of two expressions - integral (8.16), undertaken in cross section initial irregular waveguide, and integral of type (9.2) obtained from integration on transition layer.

This contour integral will differ from formula (9.2), valid, when within waveguide $\varepsilon=1$, $\mu=1$, by factor ε with first term of integrand and by factor μ with two others term/component/addends. This is connected with the fact that in (8.21) into formulas for $E_n^m(n)$ and $H_n^m(n)$ will enter the additional factors ε and μ, but in (8.20) ε and μ on the boundary of the region of integration for n different from unity. Formula (8.22) of signs therefore the form

$$S_{jm}=\frac{1}{2h_j(h_j-h_m)}\oint\nu(s)\Big((\varepsilon_0-\varepsilon)(E_s^jE_s^m-E_z^jE_z^m)-$$
$$-\left(\frac{\varepsilon^2}{\varepsilon_0}-\varepsilon\right)E_n^jE_n^m+(\mu_0-\mu)(H_s^jH_s^m-H_z^jH_z^m)-$$
$$-\left(\frac{\mu^2}{\mu_0}-\mu\right)H_n^jH_n^m\Big)\,ds, \qquad (10.1)$$

where the field they are related to that part of the interface in which the permeability have values ε and μ.

Page 83.

After passage to the limit $|\varepsilon| \to \infty$ will be obtained the formula, which differs from (9.2) by the mentioned cofactors under integral.

Thus, coupling coefficient will prove to be consisting of two term/component/addends, obligeds with respect to a change in the section and to a change in the filling. Since usually irregular filling is realized in the form of certain dielectric body with sharp interface, introduced into waveguide, then for that part of the coupling coefficient, that depends on irregular filling, it is also expedient to use formula (8.22). In order not to complicate recording, we let us assume that in dielectric body $\varepsilon \neq 1$, but $\mu=1$, but between the boundary of dielectric and metallic walls of waveguide $\varepsilon = 1,\ \mu = 1$. Then total coupling coefficient is equal to

$$S_{lm} = \frac{1}{2h_l(h_l - h_m)} \left\{ \oint \nu(s) (E_n^l E_n^m + H_z^l H_z^m - \right.$$
$$- H_s^l H_s^m) ds + \oint \nu(s) \Big[(\varepsilon - 1)(E_s^l E_s^m - E_z^l E_z^m) -$$
$$\left. - \left(\frac{1}{\varepsilon} - 1\right) E_n^l E_n^m \Big] ds \right\}. \qquad (10.2)$$

The functions $\nu(s)$, which stand in both integrals, are different; in the first integral $\nu(s)$ is determined the slope/inclination of generatrix of metal, the secondly - slope/inclination of the forming dielectric insert. Fields are undertaken on that side of the interface on which $\varepsilon = 1,\ \mu = 1$. It is easy to write also more general

formula for that case when in the material of insert is different from zero also magnetic permeability, but in the medium, which fills space between the walls of waveguide and insert, $\varepsilon \neq 1$, $\mu = 1$.

Formula (10.2) is the mathematical basis of the calculation of the compensating dielectric lenses in rectilinear waveguides and it will be used into §18.

2. Coupling coefficient for heterogeneity, examined in preceding/previous point/item, alternating/variable section and alternating/variable filling - could be in general form is immediately recorded in the form of one formula (8.6). In this case, it should be implied that surface integral in (8.6) contains also the contour integral on the duct/contours of the disruptions on which the integrands have special feature/peculiarities. This representation is inconvenient for concrete/specific/actual calculations, but subsequently the examination of the bent waveguides, it will facilitate to us total analysis.

Let us examine the bent waveguide with heterogeneous filling. Repeating the considerations of last/latter point/item, we will be restricted first to the waveguide of constant section with heterogeneous filling. Let us begin from bending according to circular arc.

Page 84.

We should it will be repeat formal calculations §7, after considering that now fields E^l, H^l depend on angle ϑ, since on the position of section ϑ=const it depends water of functions $\varepsilon(x, y)$, $\mu(x, y)$ in the regular waveguides of comparison.

Let us introduce expansions (7.3), (7.6) into four equations of Maxwell (7.70. Together (7.8) we will obtain the more complex formulas:

$$
\begin{aligned}
P'_\nu E^\nu_x &= -P_\nu E^\nu_z - iP_\nu h_\nu E^\nu_x r - P_\nu E^{\nu'}_x;\\
P'_\nu H^\nu_x &= -P_\nu H^\nu_z - iP_\nu h_\nu H^\nu_x r - P_\nu H^{\nu'}_x;\\
P'_\nu E^\nu_y &= -iP_\nu h_\nu E^\nu_y r - P_\nu E^{\nu'}_y;\\
P'_\nu H^\nu_y &= -iP_\nu h_\nu H^\nu_y r - P_\nu H^{\nu'}_y.
\end{aligned}
\qquad (10.3)
$$

Here and everywhere in this paragraph prime, as into §7, it indicates derivative according to angle ϑ. Utilizing further a condition of orthogonality for isolation/liberation by to the left derived P'_l, we will obtain

$$P'_j = (F_{j\nu} + T_{j\nu}) P_\nu. \qquad (10.4)$$

Here F_{lm} there is the same coefficient (4.7), and through T_{lm} are designated values

$$T_{lm} = \frac{1}{2kh_l}\int (E^l_x H^{m'}_y - E^l_y H^{m'}_x + E^{m'}_x H^l_y - E^m_y H^{l'}_x)\, dS. \qquad (10.5)$$

In contrast to (8.6), in (10.5) under integral will cost the derivatives on ϑ.

Equation (10.4) occurs for all values of j. Therefore (10.4) it is the unknown system of the differential equations, which describe field in the irregular waveguide or general view. End conditions for system (10.4) coincide with (7.10).

Applying the same method, as during conclusion/derivation (7.14), is easy to show that for a matrix/die in (10.4) from reciprocity theorem escape/ensues the condition

$$h_j(F_{-j,-m}+T_{-j,-m})+h_m(F_{mj}+T_{mj})=0. \qquad (10.6)$$

This relationship/ratio can be, it goes without saying, it is obtained simply from relationship/ratios (7.14) and (8.10), which remain valid for each of two addend total coupling coefficient in (10.4).

Page 85.

Coefficient F_{jm} can be again recorded in symmetrical form (7.16), but further transformation to form (7.20)-(7.22) is impossible, since in the regular waveguides of the comparison of our

problem - waveguides with heterogeneous of section dielectric filling - fields, as a rule, are not expressed as membrane/diaphragm functions.

We will not transform coefficient T_{lm} (10.5) to the form, analogous (8.16), although this transformation allows, as we saw, to obtain convenient expressions for a coupling coefficient in waveguide with sharp interfaces and with alternating/variable section. In general form the obtained formulas are very bulky. We will be restricted to the transformation which can be produced when the radius of curvature is great on the cross section with the linear dimensions of cross section. Bearing in mind that $\partial/\partial\vartheta = r\partial/\partial z$, it is possible in T_{lm} to remove with this certain average/mean value of r as integral sign. Then

$$T_{lm} = rS_{lm}, \qquad (10.7)$$

where S_{lm} given in (8.6) can be recorded in the form (8.16). Now we can remove/take limitation - constancy of section, superimposed in the beginning of this point/item. Further transition to discontinuous distribution ε and μ, to tapered welds and general case of tapered weld with alternating/variable filling is conducted in the same way as it is above, and it leads for S_{lm} in (10.7) to formulas (8.22) or (10.2).

Let us pass further to variables $p_l(\vartheta)$, for which the matrix/die

of equations does not contain diagonal terms. We will be restricted again to the conditions, under which it is correct (10.7). According to the determination of the radius of curvature of break (7.25) and relationship/ratios (8.9) and (10.7), the diagonal members of the matrix/die of system (10.4) take the form

$$F_{jj} = -irh_j, \quad T_{jj} = -rh_j'/2h_j. \qquad (10.8)$$

For simplicity of recording here it is below placed, that the radii of curvature (7.25) for all waves are equal to each other. Taking into account (10.8), we will obtain that variables $p_j(\vartheta)$ must be determined analogous (8.29) by the equalities

$$P_j(\vartheta) = \sqrt{\frac{h_j(0)}{h_j(\vartheta)}}\, p_j(\vartheta)\, e^{-i\gamma_j}; \quad \gamma_j = r\int_0^{\vartheta} h_j d\vartheta. \qquad (10.9)$$

System of equations for these variables will be analogous (8.30)

$$p_j'(\vartheta) = \sqrt{\frac{h_\nu(0)}{h_j(0)}}(F_{j\nu} + rS_{j\nu})\sqrt{\frac{h_j(\vartheta)}{h_\nu(\vartheta)}}\, p_\nu (1 - \delta_{j\nu})\, e^{-i(\gamma_\nu - \gamma_j)}. \qquad (10.10)$$

Page 86.

Now it is easy to pass to bending with a variable radius of curvature of r. For this, as into §7, it is necessary as the independent variable to accept instead of the angle ϑ the arc length l, calculated along the bent waveguide. In this variable the system of equations (10.10) of signs the form

$$\frac{\partial p_j(l)}{\partial l} = \sqrt{\frac{h_\nu(0)}{h_j(0)}}\left(\frac{F_{j\nu}}{r} + S_{j\nu}\right)\sqrt{\frac{h_j(l)}{h_\nu(0)}}(1 - \delta_{j\nu})\, e^{-i(\gamma_\nu - \gamma_j)}, \qquad (10.11)$$

value γ_l will become equal to phase change at length l

$$\gamma_l = \int_0^l h_l dl. \qquad (10.12)$$

To system (10.11) it is the unknown generalization of systems (7.32) for the bent waveguide or constant section and (8.30) for a rectilinear irregular waveguide. In the first of them, it passes when $h_j(l) \equiv h_j(0), S_{jm}(l) \equiv 0$, the secondly - with r → ∞. End condition for system (10.11), according to (7.10) and accepted in (10.9) standardization, coincides with (8.31).

System (10.11), (8.31) together with the given above different expressions for coupling coefficients describes field in the most general case of irregularity in waveguide. Its application/use in the case of loose coupling leads to the same in accuracy results, as for system (8.30), which describes field in rectilinear waveguide. If in entire irregular section the amplitude of the incident wave is much more than the amplitudes of other waves, i.e., is applicable approach/approximation (8.32), then the amplitudes of all parasitic waves are given by formula (8.34), in which, however, coupling coefficient S_{jm} must be replaced by the diagram

$$S_{jm} \to S_{jm} + \frac{F_{jm}}{r}. \qquad (10.13)$$

If are realized the conditions by which the amplitudes of parasitic waves can be expressed by binomial formula (8.37), then the same formula will be valid, also, in the general case in question, if

we in (8.37) replace (10.13). In other words, the supplementary field distortion, caused by bending, can be examined formally by the same apparatus, as the effect of the irregularities, which do not change the direction of axis, if we consider that the bending introduces the additional constraint, characterized by coupling coefficient F_{lm}/r.

Page 87.

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This additivity of the results, which relate to the irregularities of different types, is strictly valid only under the conditions when it is correct (10.7). For bendings with small radius of the curvature when (10.7) is not a place, coefficient T_{ln} also can be, as we already noted, was converted to the form, analogous (10.7), with specific by value r, but in this case, T_{lm}/r will be excellent from (8.6). In other words, bending will not only introduce second term in (10.13), but change the first, so that the coupling coefficient, obliged, for example, to section change, will be in the bent waveguide somewhat different, than in rectilinear. We will not give the appropriate formulas, in the first place, because they are sufficiently bulky, but mainly because the most interesting results are obtained usually for a loose coupling or, in any case, for large radii of curvature, when it is applicable (10.7)and, consequently, also (10.13).

It goes without saying, during the impositions of small heterogeneities, the coupling coefficients are also additive. There this is simply the result of the common/general/total additivity law of the slight disturbances. The amplitudes of the parasitic waves, which arose in the irregular section by length Δz, on which simultaneously occurs the fracture to angle Δϑ and change in the properties, described by matrix/die S_{lm}, are equal, according to (4.6), (5.13)

$$F_{lm}\Delta\vartheta + S_{lm}\Delta z. \qquad (10.14)$$

The main result of this point/item is formula (10.13) and explanation of the conditions for its applicability. This formula together with (10.2) can be placed as the basis of the mathematical analysis of the work of equipment/devices, in which is utilized the effect of the mutual compensation for the heterogeneities of different types.

For the fundamental types of irregular waveguide and for common type irregular waveguide are establish/installed the systems of differential equations for wave amplitudes, are found different expressions of the coupling coefficients, obliged to different irregularities, is establish/installed their additivity. In the loose coupling when the properties of waveguide are changed along waveguide slowly, is found expression for the amplitudes of parasitic waves in the form of the integral, undertaken along irregular section. In

certain cases there are explicit expressions for these amplitudes into which enter only the values of the parameters, which relate to the end/leads of the irregular section.

Page 88.

Chapter III.

CRITICAL SECTIONS. Resonance frequencies.

If the properties of waveguide are changed slowly, then coupling coefficients are, generally speaking, low values, systems of equations (7.9) and (8.5) are reduced for each parasitic wave to one equation (if there is no degeneration) and the solutions they take form (7.35), (8.34) or with respect (7.37). If, however, propagation constant h_l is low or equal to zero, then even during a slow change in the parameters for very small v_0 and a/r value S_{lm} and $F_{lm/r}$ they will not be small. For the rectilinear waveguides in which h_l is different in different sections, I could exist the so-called critical sections in which at this frequency $h_l=0$. Near these sections $|h_l|$ it will be little. For the bent waveguides of constant section $|h_l|$ there can be little only in narrow frequency range, near that of the called resonance frequency at which $h_l=0$. In this chapter will be examined the special conditions which appear in the presence of critical section or near resonance frequency.

§11. Reflection and the passage of wave in the presence of critical section.

1. In any section $z=\tilde{z}$ (by the sign ~ let us supply values, which relate to critical section) for certain j eigenvalue is equal to wave number in free space $\alpha_j = k$, so that $h_j(\tilde{z}) = 0$; then for all or almost all m coefficients S_{jm} are turned with $z=\tilde{z}$ into infinity, and near $z = \tilde{z}$ S_{jm} are taken high values. In equations for P_j and P_{-j} system (8.5) entire or almost all coefficients go to infinity, and solution (8.34) becomes inaccurate.

Page 89.

The incident wave is reflected from critical section, which from a mathematical point of view indicates the close coupling between direct/straight and backward waves. Near critical section the separation of field in the field of direct/straight and backward waves does not correspond to the physical picture of phenomenon and becomes inconvenient for calculations.

Amplitudes P_j really/actually can become any section into infinity, since final must be full of field E, H, but not individual

terms in expansions (8.2), (8.3) these fields into the fields of waves of both of directions. Let us return to variables Q_j, R_j, connected with P_j and P_{-j} relationship/ratios (7.2). From the form of row/series (8.1) it follows that, for example, for the magnetic waves for which, according to (3.14), in critical section $H_x^j = 0,\ H_y^j = 0$, the coefficient R_j into critical section can become infinity, and coefficient Q_j it must everywhere remain final. Therefore in the critical section of amplitude P_j and P_{-j} for magnetic waves, they can go to infinity, but in such a way that their difference Q_j would be final. Accurately the same, for electrical waves P_j and P_{-j} they can go to infinity, but their sum R_j must remain final.

Variables $Q_j(z)$, $R_j(z)$ satisfy the system of the differential equations

$$Q_j' + ih_jR_j = \sum_{m=1}^{\infty} Q_m (S_{jm} - S_{-jm}); \qquad (11.1a)$$

$$R_j' + ih_jQ_j = \sum_{m=1}^{\infty} R_m (S_{jm} + S_{-jm}). \qquad (11.1b)$$

which it is easy to obtain from equations (8.5) for variables $P_j(z)$ and from (7.2). However, for our purposes system (11.1) is not directly used; in order to utilize it, for example, for the magnetic wave of number j, it was necessary to still find the limit of product h_jR_j near critical section. Let us pass therefore to the systems of

equations of the second order. Differentiating equations (11.1) for z, we will obtain the system

$$Q_j'' + h_j^2 Q_j = -i\sum_{m=1}^{\infty} R_m \{h_j (S_{jm} + S_{-jm}) +$$

$$+ h_m (S_{jm} - S_{-jm})\} (1 - \delta_{jm}) + \sum_{m=1} Q_m (S_{jm} - S_{-jm})' +$$

$$+ \sum\sum_{m,q=1}^{\infty} Q_q (S_{jm} - S_{-jm})(S_{mq} - S_{-mq}); \qquad (11.2a)$$

$$R_j'' + h_j^2 R_j = -i\sum_{m=1}^{\infty} Q_m \{h_j (S_{jm} - S_{-jm}) +$$

$$+ h_m (S_{jm} + S_{-jm})\} (1 - \delta_{jm}) + \sum_{m=1}^{\infty} R_m (S_{jm} + S_{-jm})' +$$

$$+ \sum\sum_{m,q=1}^{\infty} R_q (S_{jm} + S_{-jm})(S_{mq} + S_{-mq}). \qquad (11.2b)$$

Page 90.

In contrast to equations (11.1), the equation for Q_j system (11.2) does not contain variable R_j, and equation for R_j does not contain Q_j. Their essential deficiency/lack - appearance in them of derivatives of coupling coefficients. Near the fracture of generatrices and generally in the regions where $v(s,z)$ very is rapid varies, these products are great, and the application/use of systems (11.2) leads to serious complications.

Therefore we below will limit the field of waveguide in which

for describing the field are utilized second order equations, by vicinity of critical section.

Coupling coefficients S_{jm} possess the following properties: if the wave of number j is a magnetic wave, then for any wave of number m of value

$$S_{jm}-S_{-jm},\ h_j(S_{jm}+S_{-jm}) \qquad (11.3)$$

they are regular when $h_j\to 0$ and, furthermore, is not contained the first degree h_j, so that all derivatives these values are also regular when $h_j\to 0$; if the wave of number j is an electrical wave, then together with all their derivatives they are regular with any m in the critical section of value

$$S_{jm}+S_{-jm},\ h_j(S_{jm}-S_{-jm}). \qquad (11.4)$$

This property it is easy to demonstrate, after calculating values (11.3) (11.4) according to formulas (9.5).

The combinations of coupling coefficients (11.3) and (11.4) are coefficients in equations (11.2). ~~Thus, for magnetic waves right side (11.2a).~~ Thus, for magnetic waves right side (11.2a) is regular everywhere, including the vicinity of critical section, and for electrical waves right sides has no special features (11.2b).

Page 91.

Now we can write the equations of the first for v_0 order, valid everywhere, including vicinity of the critical section:

$$Q_l'' + h_l^2 Q_l = G_l, \; G_l = -iR_m (h_l (S_{lm} + S_{-lm}) + + h_m (S_{lm} - S_{-lm}) (1 - \delta_{ml}); \quad (11.5a)$$

$$R_l'' + h_l^2 R_l = G_l, \; G_l = -iQ_m (h_l (S_{lm} - S_{-lm}) + + h_m (S_{lm} + S_{-lm})) (1 - \delta_{ml}). \quad (11.5b)$$

Equation (11.5a) describes magnetic waves, (11.5b) - electrical. In (11.5), are reject/thrown term/component/addends of order v_0^2 and of derivative v'; reject/thrown term/component/addends do not have special feature/peculiarities near critical section.

2. In this paragraph we will examine only field of incident wave, after supposing that in critical section is turned into zero wave number h_m of precisely this wave. Us it will first of all interest the case when critical section is close at the beginning of narrow waveguide.

Parasitic waves, created by the incident wave, it would be possible then to determine by the method, developed in the preceding/previous chapter, substituting in formulas of type (8.30)

for (8.32) the solution, found in this paragraph. conditions, that appear if the critical cross section will be not for falling, but for a parasitic wave, require independent examination which will be produced in the following paragraph.

Let us examine the becoming narrow waveguide; near critical section the derivative of wave number in this waveguide is negative, $\tilde{h}'_m < 0$. Wave falls from the side of wide waveguide. Let for certainty it belong to magnetic type, so that it is alternating/variable $Q_m(z)$ it is everywhere final. Its field we will find from equation (11.5a)

$$Q''_m + h^2_m Q_m = 0 \qquad (11.6)$$

and equations (11.1a), in which to the right it is also necessary to retain only one term/component/addend:

$$Q'_m + ih_m R_m = Q_m (S_{mm} - S_{-mm}). \qquad (11.7)$$

The rejected terms in (11.7), as in (11.6) they are despite all z smalls of the second order.

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Page 94.

Coefficient in (11.18) - a large number, since $A^{-4/3} \sim \tilde{\nu}^{-2/3} \gg 1$.

Solution (11.16) is correct in the region in which it is possible to use expansion (11.11), approximately when $h_m^{\prime}\ll 1$, when $|t|\ll A^{-2/3}$. This region overlaps with the region where is satisfied the condition (11.10), which can be recorded in the form |t|>>1. Therefore (11.16) must with t>>1 pass into the solutions, obtained in the approach/approximation of geometric optics. Keeping in mind this fact, we will use solution (11.16) in order to establish/install the communication/connection between P_m and P_{-m} on the boundary of region (11.10). This communication/connection can be considered as the end condition, equivalent to the presence of critical section and arranged/located after it narrow waveguide. The use of this equivalent end condition will make it possible to be restricted below to the examination of the region, distant from critical section. Analogous idea was proposed by L. M. Brekhovskiy and I. D. Ivanov in [84] for another problem, reducing to the same equation (11.6).

If critical section is found at the end of the transition of "that smoothed", i.e., where $\nu=0$, then $h_m^{\prime}(\tilde{z})=0$, and expansion (11.11) begins from higher degree (z-$\tilde{z}$). The solution is expressed in this case not through Airy's functions, but through other special functions, for example through the cylindrical functions whose order in a known manner is connected with the order of degree (z-$\tilde{z}$); the

general method of calculation remains in this case is valid. If $h'_m(\tilde{z})$ although not is equal to zero, it is very small, then in (11.11) it is necessary to retain two members.

Below we will assume that $h'_m(\tilde{z}) \neq 0$ and that it is possible to be restricted to expansion (11.11). Therefore some of that obtained are below in this paragraph of results, namely those results, which are related to the case when distance $\frac{|\tilde{z}-L|}{\Lambda}$ is small or certainly, they are valid only under the supplementary assumption that coupling ir ular waveguide. with narrow waveguide is not smoothed.

4. Let us begin from determination of reflection coefficient for case when wave does not penetrate narrow waveguide (Fig. 12). Reflection coefficient in module/modulus is equal in this case to one; let us search for its phase. Let us introduce variable δ(z), after determining by its condition

$$P_{-m}/P_m = e^{i\delta}. \qquad (11.19)$$

Page 95.

The unknown phase of the wave reflected at input is a value δ of z=0, δ(0). It is easy to show that, according to equations (8.5), in which it is necessary to preserve only terms, which contain P_m and P_{-m}, δ satisfies the nonlinear equation

$$\delta' - 2h - 2S_{-mm} \sin\delta = 0. \qquad (11.20)$$

In the region of the applicability of geometric optics during satisfaction of condition (11.10) third term/component/addend in (11.20) is small and it it is possible to reject/throw; integral of this value to interval is also low, since it contains the oscillatory factor. In order to determine $\delta(0)$ from obtained in this case equation

$$\delta' - 2h = 0, \qquad (11.21)$$

one should still find end condition for δ. This can be made, after calculating ratio/relation P_{-m}/P_{m} near boundary of the region (11.10) according to (11.8) and general solution (11.16). Simple calculation gives

$$e^{i\delta} = \frac{[\dot{u} + (t^{1/2} + B)\,u] + \frac{N}{M}\,[\dot{v} + (t^{1/2} + B)\,v]}{[\dot{u} + (-t^{1/2} + B)\,u] + \frac{N}{M}\,[\dot{v} + (-t^{1/2} + B)\,v]}, \qquad (11.22)$$

also, in it to substitute expression (11.17) for ratio N/M.

In order to find equivalent end condition for equation (11.21), it is necessary in (11.22) to assume t>>1. In the which interests us region $z<\tilde{z}$, the variable t is negative, and one should utilize asymptotic formulas for u(t) and V(t) at the high negative values of t. After producing the appropriate computations, let us find the

asymptotic value of function δ, i.e., leading terms in expansion δ (11.22) according to the reverse/inverse degrees of $|t|$:

$$\delta = -2\alpha + \delta_0. \tag{11.23}$$

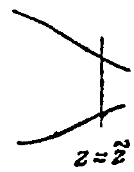


Fig. 12. Critical section in tapered weld.

Page 96.

Here δ_0 - constant, i.e., not depending on t value, determined by the equation

$$e^{i\delta_0} = \frac{N/M - i}{N/M + i}, \qquad (11.24)$$

and α - value, entering asymptotic formulae for Airy's functions at the high negative value of argument and equal to

$$\alpha = \frac{2}{3}(-t)^{3/2} + \pi/4. \qquad (11.25)$$

Function (11.23) satisfies, as it is easy to check, equation (11.21). The unknown solution this equation, which converts into complete solution (11.22), must during approach/approximation to critical section pass into solution (11.23). With the formal substitution t=0, this solution takes value $\delta_0-\pi/2$. This value is the equivalent end condition for equation (11.21), which must satisfy the solution of this equation in order in the region, which adjoins the

critical section, to pass into solution (11.16).

Thus, the solution of equation (11.20) in that interesting us region exists

$$\delta(z) = 2\int\limits_{\tilde{z}}^{z} h_m(z)\,dz - \pi/2 + \delta_0 \qquad (11.26)$$

and the unknown phase of reflection coefficient is equal to

$$\delta(0) = -2\tilde{\gamma}_m - \pi/2 + \delta_0. \qquad (11.27)$$

The physical sense of first term is evident - this is phase change with the passage of wave from z=0 to critical section and vice versa, calculated in the approach/approximation of geometric optics. It it is easy to find for any concrete/specific/actual form of the dependence of eigenvalue from the position of section, i.e., for any function $\alpha_m(z)$. For example, for a cone value $1/\alpha_m$ is a linear function from z

$$\frac{1}{\alpha_m(z)} = \frac{1}{\alpha_m(0)} - \frac{1}{C}z, \quad C = \mathrm{const},\ C > 0, \qquad (11.28)$$

and simple lining/calculations give

$$\tilde{\gamma}_m = C(g - \operatorname{arc\,tg} g); \quad g = \frac{h_m(0)}{\alpha_m(0)}. \qquad (11.29)$$

Value of δ_0 in (11.27) depends on the distance between the critical section and the beginning of narrow waveguide. According to (11.24) and (11.17), δ_0 depends on the difference in the

frequencies, which is determining the parameter t_L (11.18).

Page 97.

It gives the correction to the phase of the wave reflected, connected with the presence of narrow waveguide. Although on narrow waveguide energy not propagated, field exists also in region $z>\tilde{z}$, exponentially decreasing during removal from critical section, and the structure of waveguide in this region affects the phase of reflected wave. The effect of narrow waveguide, however, is noticeable only with final ones t_L; when $t_L \gg 1$, but virtually already when $t_L \sim 1$, δ_0 is very small, and the phase of reflected wave differs from the phase, calculated in the approach/approximation of geometric optics, in terms of constant term $-\pi/2$. In this field seemingly completely it does not reach the beginning of narrow waveguide. Let us note that in the problem, of a normal incidence in the electromagnetic wave on the ionosphere and in the row/series of other problems, which are reduced to the same equation (11.6) under condition (11.11), it appears, as is known, the same in value supplementary phase shift.

The precise form of the function $\delta_0(t_L)$ depends, according to (11.24) and (11.17), on the type of wave and character of waveguide. Figure 13 value $-\delta_0$ depicts in the form of function from t_L for waves H_{0n} in circular waveguide. For these waves, according to (9.4),

(9.5), $S_{-m,m}=S_{mm}$, and addend B in (11.17) is absent. However, this term/component/addend is substantial only for very small ones t_L, and the curve of Fig. 13 for $t_L \geqslant$ 0.4-0.5 correctly transmits dependence - of δ_0 on t_L for all waves.

The determination of the phase of reflection coefficient for electrical waves is conducted by the same in accuracy diagram. We will not give these computations, let us note only, that when $t_L \gg 1$ the reflection coefficient for electrical waves has a phase $-2\tilde{\gamma}_m$ $+\pi/2$. This same result will be obtained in the next paragraph. It will be there shown, that when $t_L \gg 1$ the phase of the coefficient of reflection of magnetic waves is equal to $-2\tilde{\gamma}_m-\pi/2$, as this is obtained from (11.27), even if the beginning of narrow waveguide lie/rests so/such far from critical section, that the application/use of expansion (11.11) with z=L, produced is above, it is already illegal.

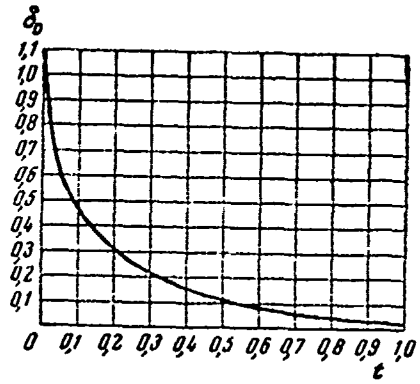


Fig. 13. Correction to the phase of reflection coefficient, caused by the effect of narrow waveguide.

Page 98.

5. Let us now move on to determination of reflection coefficient for case when wave penetrates narrow waveguide, so that h_m^2 everywhere is positive $(k>\alpha_m(L))$ and, strictly speaking, there is no critical section, but value of difference $k-\alpha_m(L)$ and of wave number in narrow waveguide is very small $(h_m \ll 1)$, so that coupling coefficients near beginning of narrow waveguide are high values. In this case, the reflection coefficient can accept the values of the order of one. To this case also it is possible to use developed above apparatus; it is necessary to only consider that the critical section is found on the continuation of irregular waveguide (Fig. 14), i.e., that $\tilde{z}>L$.

Let us introduce new variable $\rho(z)=P_{-m}/P_{m}\cdot e^{-2i\tau_m}$. It satisfies, as it is easy to check, the equation of Riccati

$$\rho' = -S_{-mm}e^{-2i\tau}\left(1-\rho^2 e^{4i\tau_m}\right). \qquad (11.30)$$

If $|t_L|$ is great, then, as we will see below, for all $|\rho|<1$ in (11.30) it is possible to reject/throw last/latter term/component/addend, and the coefficient of reflection $\rho(0)$ is equal to

$$\rho(0) = -\int_0^L S_{-mm}e^{-2i\tau_m}\,dz. \qquad (11.31)$$

This solution, it goes without saying, coincides with solution (8.34) (for j=-m), which it is possible to use, if on entire transition is correct the approach/approximation of geometric optics.

Formula (11.31) gives for $\rho(0)$ the low value of order Meanwhile from equation (11.30) in zero-order for series expansion parameter it follows only $\rho'(z)=0$. Therefore, in $\rho(z)$ can participate also constant add/composed, which has, generally speaking, zero order. In (11.31) this term/component/addend no, but, as we now will show, with small or final ones $|t_L|$ it proves to be essential. The value of this constant term/component/addend one should search for in the same way as in the preceding/previous point/item searched for

equivalent boundary condition for equation (11.21). it is necessary to write explicit expression for $\rho(z)$ through functions u(t) and to the $v(t)$, analogous complete expression for $\delta(z)$ (11.22), and in this expression to pass to large negative t. This there will be value $\rho(z)$ in the region, common/general/total for (11.10) and (11.11). Value this, such as to check, does not depend on z, has zero order on v_0 and, according to preceding/previous, it is retained in entire region (11.10), including z=0. Using obviously the communication/connection between $\rho(z)$ and $\delta(z)$ and formulas (11.23), (11.24), we will obtain for this constant term/component/addend, which we will designate ρ_0, the value

$$\rho_0 = \frac{N/M - i}{N/M + i} e^{-2i(\eta_m + a)}. \quad (11.32)$$

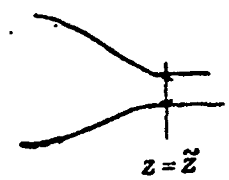


Fig. 14. "critical section" in narrow regular waveguide.

Page 99.

Ratio N/M is found in (11.17); in contrast to the preceding/previous point/item the entering it value t_L is negative.

The character of dependence of ρ_0 on t_L is various, according to (11.17), for different waveguides. Figure 15 gives curve/graph of $|\rho_0|$ in function $-t_L$ for the waves H_{0n} in the circular waveguide, for which (11.17) somewhat is simplified; however common/general/total variation of ρ_0 on a difference in the frequencies, entering in (11.18), it is identical for all waves.

Than flatter form has transient waveguide near its narrow end/lead, the fact for the fixed/recorded difference in the frequencies will be less reflection coefficient. With decrease $v(L)$ the frequency region, in which $|\rho_0|$ passes from large ones to low values, becomes narrow. If, for example, in circular waveguide the

angle between the forming and Z-axis at the end of the transient waveguide is equal to $5°40'(\nu(z)=-0.1)$, then for wave $H_{01}|\rho_0|$ it is changed from 1 (for t=0) to 0.5 (with $t_L=-0{,}086$) during change $[k-a_m(L)]/k$ from 0 to 0.006, i.e., with drift (increase) operating frequencies from the critical frequency of narrow waveguide to 0.6o/o. At double smaller angle (when $\nu(L)=-0{,}05$) the same value $|\rho_0|=0.5$ is reached during the deviation of the frequency in all of 0.4o/o, and the frequency deviation of 0.6o/o causes in this case decrease in $|\rho_0|$ to value of 0.4.

With growth $|t_L|$ when $-t_L\gg 1$ ρ_0 it decreases and becomes the low value:

$$\rho_0=\frac{1}{8(-t_L)^{3/2}}e^{-2i\eta_m(L)}. \qquad (11.33)$$

This value no longer zero, but first order on ν. However, this expression does not pass accurately in (11.31). In order this transition to ensure, it is necessary to find also the second term in the expansion of ρ_0 according to degrees ν. We will not carry out these computations, which are reduced to the determination of the following terms of the expansion of solution (11.22) and of equation (11.30), and let us give only result for a special case of round cone and wave H_{01}.

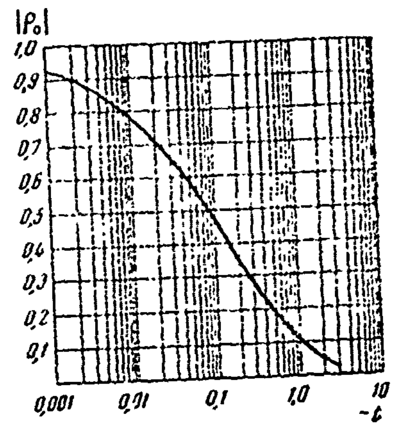


Fig. 15. Coefficient of reflection of wave H_{01}.

Page 100.

The refined value of reflection coefficient in this case exists

$$\rho(0)=\rho_0-\frac{i\nu}{4\mu_m}\left\{\frac{1+\rho_0^2}{g^2}-\rho_0\left(\frac{1}{3g^3}-\frac{1}{g}-\operatorname{arctg} g\right)\right\}. \quad (11.34)$$

This expression already passes in (11.31), if this last/latter expression is recorded in the form of binomial formula (8.37). In the second correction term in (11.34) it is necessary to retain with this, only first term/component/addend.

first term in (11.34), i.e., term ρ_0, will be usually considerably more than term/component/addend $-i\nu/(4\mu g^3)$, although both they with $|t_L|\gg 1$ one and the same (the first) order on ν. Therefore

simple formula $\rho(0) = \rho_0$ practically gives good results not only with final ones, but also with small $|\rho_0|$ when $|\rho(0)|$, according to (11.34), is proportional $\nu(L)$.

Let us note on conclusion of this paragraph, that for a cone, i.e., with the validity of condition (11.28), equation (11.6) in an entire region $0 \leqslant z \leqslant L$ has, as is easy to show, the exact solution:

$$\alpha_m^{1/2} Z_p(Ck/\alpha_m), \qquad (11.35)$$

where Z_p - any cylindrical function and $p^2=C^2+1/4$. however, since argument and system of function Z_p are great in comparison with unity, then during actual computations will have to apply asymptotic representations Z_p. In the region where value

$$2^{1/3}(p - Ck/\alpha_m)(Ck/\alpha_m)^{-1/3} \qquad (11.36)$$

is final, Z_p is expressed, as is known, through Airy's functions from argument (11.36). It is easy to show that, where value (11.36) is final, it coincides with by the variable t (11.13). Therefore even for the cone when there is explicit solution of equation (11.6), in region final t immediately to expediently represent the solution in the form of Airy's functions, and in region large $|t|$ - in the form of the linear combination of functions $h_m^{-1/2} e^{\pm i \tau_m}$. This same method we will preserve in the following paragraph.

§12. Wave development of parasitic type in the presence of critical section for this wave.

1. Let is now with $z=\tilde{z}$ arranged/located critical section of any of forming parasitic waves $h_l(\tilde{z})=0$, and incident wave can be propagated in narrow waveguide.

Page 101.

Let us find under these conditions the amplitude of the parasitic wave, which exits into wide waveguide.

By us they will be necessary for this solution of the equation

$$Q_l'' + h_l^2 Q_l = 0 \qquad (12.1)$$

in entire irregular waveguide. V(z) and U(z) - two solutions (12.1), calibrated, so that their Wronskian determinant would be equal to unity

$$U'V - V'U = 1. \qquad (12.2)$$

Let us determine by their in such a way that near the critical section where is correct expansion (11.11), they would be proportional to Airy's functions u(t) and V(t). From condition (12.2) it follows that the proportionality factor must be equal to $A^{-1/3}$, so

that with small and final t

$$U(z) = A^{-1/4} u(t); \quad (12.3a)$$

$$V(z) = A^{-1/4} v(t), \quad (12.3b)$$

where A and t are determined in (11.12) and (11.13) with the replacement of index m on j.

In region large $|t|$ of function U(z) and V(z) they must be determined in such a way that with final $|t|$ would be provided analytical transition in (12.3). Substituting in (12.3) the asymptotic value of Airy's functions and expressing the variable t through h_j and $\gamma_j - \tilde{\gamma}_j$, we will obtain that for $z<\tilde{z}$, i.e., with real h_j, functions U(z) and V(z) must be determined by the conditions:

$$U(z) = h_j^{-1/2} \cos(-\gamma_j + \tilde{\gamma}_j + \pi/4); \quad (12.4a)$$

$$V(z) = h_j^{-1/2} \sin(-\gamma_j + \tilde{\gamma}_j + \pi/4). \quad (12.4b)$$

With $z>\tilde{z}$, when $h_j = -i|h^j|$, we assume

$$U(z) = (ih_j)^{-1/2} e^{i(\gamma_j - \tilde{\gamma}_j)}; \quad (12.5a)$$

$$V(z) = (ih_j)^{-1/2} e^{-i(\gamma_j - \tilde{\gamma}_j)}. \quad (12.5b)$$

With distance into supercritical area, i.e., with an increase in difference $z-\tilde{z}$, function U(z) rapidly grow/rises, in V(z) it decreases.

Page 102.

Far from critical section these functions are, according to (12.4),

(12.5) the linear combinations of solutions $h_l^{-1/2} e^{\pm i\eta_l}$, near critical section they are transformed into final solutions (12.3). The regions where U and V can be represented in the form (12.3) and where they take form (12.4) and (12.5), they overlap.

In this paragraph we will immediately assume that the critical section is located so/such far from the beginning of narrow waveguide, that the field of parasitic type wave completely does not reach the narrow waveguide. In this case, as it will render/show, function U(z) will fall out from final results, field will be completely described by function V(z).

2. Returning to problem of preceding/previous paragraph, we will find again, utilizing introduced function V(z), phase of wave, reflected from critical section. For a magnetic wave the field is described by equation (11.6). With $z>\tilde{z}$ the field must not grow/rise; under the made assumption about the fact that the beginning of narrow waveguide is arrange/located far from critical section, this requirement replaces boundary condition with z=L. the solution of equation (11.6) in this case is function CV, where the constant C can be determined from condition $P_m(0)=1$. Near the beginning of the wide waveguide z=0, function V(z) takes form (12.4b), so that

$$Q_m = \frac{C}{2i}\left\{e^{i(-\eta_m+\tilde{\eta}_m+\pi/4)} - e^{-i(-\eta_m+\tilde{\eta}_m+\pi/4)}\right\}. \qquad (12.6)$$

This expression must be substituted into formulas (11.8), in which it is possible to drop/omit term/component/addend $S_{mm}-S_{-mm}$, essential only near critical section, i.e., in small ones $|h_m|$. Thus, it is possible to show which first term in (12.6) is equal to $P_m(z)$, and the second is equal to $P_{-m}(z)$, which (12.6)

corresponds to

representation Q_m in the form of difference $Q_m = P_m - P_{-m}$. It is hence easy to determine C and unknown coefficient of reflection

$$P_{-m}(0) = e^{-2i(\widetilde{\gamma}_m+\pi/4)}. \qquad (12.7)$$

This formula coincides with the result of the preceding/previous paragraph: $\delta(0) = -2\widetilde{\gamma}_m - \pi/2$ when $t_L \gg 1$, which thus is generalized also in the case when the d ance between the critical section and the beginning of narrow wa guide is great.

For electrical waves the field is described by equation (11.5b)

$$R_m'' + h_m^2 R_m = 0, \qquad (12.8)$$

solution of which is again function CV, and far from critical section $R_m(z)$ has the same form (12.6).

Page 103.

However, R_m is equal to sum, but not difference in amplitude (7.2);

therefore second term in (12.6) corresponds $-P_{-m}(z)$, and

$$P_{-m}(0)=e^{-2i(\widetilde{\gamma}_m-\pi/4)}. \qquad (12.9)$$

The phase of electrical wave differs from the phase of magnetic wave only in terms of sign of constant term/component/addend $\pi/2$, which is added to geometric phase change $-2\widetilde{\gamma}_m$.

Let us compare these results with those, which would be, if reflection proceeded not from critical section, but from the metallic partition/baffle, supplied across waveguide. On this partition/baffle there would be $P_{-m}=P_m$. Having this in form, it is possible conditionally formulas (12.7) and (12.9) to treat thus: the phase of the wave, reflected from critical section, coincides with the phase of wave, reflected from metallic mirror in waveguide, if this mirror is displaced relative to critical section to one eighth of wavelength; for magnetic waves is displaced it must for critical section, for electrical ones - towards the incident wave. The conditionality of this formulation lies in the fact that the comparison of supplementary phase shift +-π/4 in (12.7) and (12.9) and the geometric shift of mirror assumes that the phase rate is final and little it is changed, that near critical section is not a place.

3. Let us return to fundamental problem of present paragraph -

study of field of parasitic wave. For certainty the parasitic wave of number j - magnetic type in question, so that its field is described by equation (11.5a). As in the preceding paragraph, we utilize this equation in order to establish/install end condition for first-order equations (8.5) or (8.30) - the condition, equivalent to the presence of critical section. Applying this end condition, it will be possible to in the larger part of the irregular waveguide use first-order equations (8.5). These equations are considerably simpler than the equation of second order (11.2), and, which is especially important, during the isolation/liberation in them of the first-order terms of smallness on the basis of ν_0 do not appear any difficulties near the fractures of generatrix or generally in the regions where $\nu(z)$ rapidly is changed with z.

The solution of nonhomogeneous equation (11.5a) can be, according to condition (12.2), recorded in the form

$$Q_j = U\int_L^z VG_j dz - V\int_C^z UG_j dz. \qquad (12.10)$$

Page 104.

The selection of lower limit in the first integral is defined by requirement, in order to in supercritical area, i.e., when $z \to L$, $Q_j(z)$ not increased, as grow/rises U(z). Lower limit in the second integral we will leave not defined.

Far from critical section in old on v_0 order Q_j and R_j they are connected, accordingly (11.1a), by the equation

$$Q_j' + ih_jR_j = Q_m(S_{jm} - S_{-jm}). \qquad (12.11)$$

From this equation and from (7.2) it is possible to exclude R_j and thus to express $P_j(z)$ and $P_{-j}(z)$ through $Q_j(z)$ and $Q_j'(z)$. Analogous (with 11.8) we have

$$\begin{aligned} P_j(z) &= \frac{i}{2h_j}[Q_j' + ih_jQ_j - (S_{jm} - S_{-jm})Q_m]; \\ P_{-j}(z) &= \frac{i}{2h_j}[Q_j' - ih_jQ_j - (S_{jm} - S_{-jm})Q_m]. \end{aligned} \qquad (12.12)$$

For obtaining the unknown equivalent end condition, let us substitute (12.11) in (12.12) and will exclude the second integral in (12.11), i.e., let us establish such algebraic communication/connection between $P_j(z)$ and $P_{-j}(z)$, which does not contain by the arbitrary constant C. The obtained communication/connection between $P_j(z)$ and $P_{-j}(z)$ is valid everywhere; we will use it to points, which are located far from critical section. For such points function V(z) is given in (12.4b). Appearing during this calculation expressions $ih_jV \pm V'$ are equal to

$$ih_jV \pm V' = \mp h_j^{1/2} e^{\pm i(\gamma_j - \tilde{\gamma}_j - \pi/4)}. \qquad (12.13)$$

Thus is establish/installed the communication/connection, which exists far from the critical section between $P_j(z)$ and $P_{-j}(z)$:

$$h_j^{1/2}P_j(z)\,e^{-i(\gamma_j+\tilde{\gamma}_j+\pi/4)}-h_j^{1/2}P_{-j}(z)\,e^{i(-\gamma_j+\tilde{\gamma}_j+\pi/4)}=$$
$$=\int\limits_L^z VG_j dz-VQ_m(S_{jm}-S_{-jm}). \qquad (12.14)$$

entering in (12.14) function $Q_m(z)$ and $R_m(z)$, of the describing the field fundamental wave, are sufficient to determine in old order. In this order the fundamental wave passes without distortions, $P_{-m}(z)\equiv 0$, and

$$R_m(z)=Q_m(z)=\sqrt{\frac{h_m(0)}{h_m(z)}}e^{-i\gamma_m}. \qquad (12.15)$$

Page 105.

Let us give end condition (12.14) more convenient, more symmetrical form. We will use the fact that for any function f(z), not having special feature/peculiarities in range from given z to L, is correct the identity

$$fVe^{-i\gamma_m}=\int_L^z f(V'-ih_mV)\,e^{-i\gamma_m}dz+\int_L^z f'\cdot Ve^{-i\gamma_m}dz, \quad (12.16)$$

in which is still placed V(L)=0; it easy to demonstrate by integration in parts. Applying (12.16) to second term in (12.14) and reject/throwing last/latter term/component/addend in (12.16), which has higher order of smallness, it is possible to record right side (12.14) in the form of one integral expression.

In this way is obtained the unknown symmetrical form of equivalent end condition, most convenient both for the physical analysis and for the concrete/specific/actual calculations

$$P_j(z)\,e^{-i(\gamma_j-\widetilde{\gamma}_j-\pi/4)}-P_{-j}(z)\,e^{-i(\gamma_j-\widetilde{\gamma}_j-\pi/4)}=$$
$$=\sqrt{\frac{h_m(0)}{h_j(z)}}\int_z^L\frac{e^{-i\gamma_m}}{\sqrt{h_m}}\{S_{jm}(ih_jV+V')+S_{-jm}(ih_jV-V')\}\,dz. \quad (12.17)$$

Integrand contains coupling coefficients in such combinations (11.3), which, as shown in § 11, do not have special feature/peculiarities in an entire range of integration and, in particular, in critical section. Value z in this formula can be any, it only must lie/rest at the region, in which it is correct (12.4b).

By the same method it is possible to find end condition for all other possible cases. If j - electrical type wave, then near critical section it is necessary to ut.``ze equation (11.5b); if the wave of number m falls from the side o narrow waveguide, then in an obvious manner it is modified solution (12.15)and, etc.

4. Field of parasitic wave can be, thus, it is found of first-order equations (8.5) and from end condition in the form (12.17), of equivalent to presence critical section. The second end condition takes form $P_i(0)=0$, it provides the absence of the incident wave of this type.

Page 106.

The solution of these equations for any z out of rejection region (and with $z<\tilde{z}$) under condition $P_j(0)=0$ will be analogous (with 8.34):

$$P_j(z) = \sqrt{\frac{h_m(0)}{h_j(z)}}\, e^{-i\gamma_j} \int_0^z S_{jm} \sqrt{\frac{h_j}{h_m}}\, e^{-i(\gamma_m-\gamma_j)}\, dz;$$

$$P_{-j}(z) = \sqrt{\frac{h_m(0)}{h_j(z)}}\, e^{-i\gamma_j} \int_0^z S_{-jm} \sqrt{\frac{h_j}{h_m}}\, e^{-i(\gamma_m+\gamma_j)}\, dz + $$

$$+ \sqrt{\frac{h_j(0)}{h_j(z)}}\, e^{i\gamma_j} \cdot P_{-j}(0). \qquad (12.18)$$

Substituting this solution under end condition (12.17), we find $P_{-j}(0)$ - the unknown amplitude of the parasitic wave, which exits in wide waveguide:

$$P_{-j}(0) = \sqrt{\frac{h_m(0)}{h_j(0)}} \Big\{ e^{-2i(\tilde{\gamma}_j+\pi/4)} \int_0^z S_{jm} \sqrt{\frac{h_j}{h_m}}\, e^{-i(\gamma_m-\gamma_j)}\, dz - $$

$$- \int_0^z S_{-jm} \sqrt{\frac{h_j}{h_m}}\, e^{-i(\gamma_m+\gamma_j)}\, dz - $$

$$- e^{-i(\tilde{\gamma}+\pi/4)} \int_z^L \frac{e^{-i\gamma_m}}{\sqrt{h_m}} [S_{jm}(ih_jV + V') + S_{-jm}(ih_jV - V')]\, dz \Big\}. \qquad (12.19)$$

Value z in (12.19) also can be any, provided point z did not lie/rest near critical section. Really/actually, factors when S_{jm} and S_{-jm} in last/latter integral coincide with the appropriate factors in the first two integrals, and during change z the sum of all

integrals, entering in (12.19), it does not change.

In spite of certain conditionality of representation $P_{-j}(0)$ in formula (12.19) in the form of the sum of three term/component/addends, the conditionality, connected with arbitrariness in the selection of point z - this representation makes completely specific physical sense, prompted by results of § 5 and 6. In each cut of irregular waveguide from z=0 to section z, which participates in (12.19), under the action of the transmitted wave of number m occurs shaping of direct/straight and backward waves of number j. All parasitic waves, which go in opposite direction, are summarized in section z=0, and the result of this addition, according to (5.15) or (8.34), it coincides with second term in (12.19). Direct/straight parasitic waves reach before critical section and are reflected from it, acquiring the same supplementary phase factor (12.7)as during incidence in this wave on irregular waveguide from without; so it is formed first term in (12.19). Finally, last/latter term/component/addend (12.19) is the result of shaping of the field of parasitic wave near critical section.

Page 107.

In this region the separation on direct/straight and backward waves does not have special sense (see analogous observation in [80],

§17.6), and the education/formation of parasitic wave occurs from more complex laws, than found in Chapter II.

Under the normal conditions of the amplitude of direct/straight parasitic waves it is more than the amplitudes of reverse/inverse ones, (§ 8), and if z in (12.19) is selected not very far from critical section, then greatest is first term; third term/component/addend one must take into account only in the exceptional cases. The proposed interpretation of formula (12.19) makes it possible for other possible cases to find two fundamental term/component/addends in the field of parasitic wave, which exits into wide waveguide, without solvin equations (11.5) and without establish/installing equivalent end condition. For example, if the exciting wave falls from the side of the narrow waveguide, arrange/located to the left of critical section, then direct/straight, but backward waves will acquire supplementary phase factor (12.7) or (12.9), that corresponds to the reflection of parasitic wave from critical section.

5. Let us use obtained results to waveguides in which ν (s, z) has despite all z one and the same order, and with z=-0 as jump is taken zero value, i.e., to waveguides with fracture of generatrix. For a reduction in the recording, we will assume how in § 8, that these fractures are arrange/located only on the end/leads of the

irregular waveguide.

In § 8 we obtained for the wave amplitudes, scattered on such irregular waveguides, formula (8.37), which was used, if not critical section.

From the preceding/previous examination it follows that for the field of parasitic wave in this case must be obtained an expression of type (8.37), in which, it goes without saying, there will not be term/component/addend, pertaining to narrow waveguide, but there will be the terms, which correspond both to direct waves, which were reflected from critical section, and to backward waves.

For further conversions is convenient to record $P_{-l}(0)$ in the form one integral. This expression is obtained, if we in solution (12.19) or under end condition (12.17) assume z=0:

$$P_{-j}(0) = -\sqrt{\frac{h_m(0)}{h_j(0)}}\, e^{-i(\tilde{\gamma}_j + \pi/4)} \int_0^L \frac{e^{-i\gamma_m}}{\sqrt{h_m}} \times$$

$$\times [S_{jm}(ih_jV + V') + S_{-jm}(ih_jV - V')]\, dz. \qquad (12.20)$$

Page 108.

According to (12.13), in the larger part of the range of integration in (12.20) products $(ih_lV \pm V')e^{\mp i\gamma_l}$ is not contained the

rapidly oscillatory factors. Bearing this in mind, let us break integral in (12.20) into two parts, which contain respectively S_{lm} and S_{-lm}, let us isolate in both of these addend products and let us integrate in parts. First term, for example, will take in this case this form:

$$\int_0^L \frac{e^{-i\gamma_m}}{\sqrt{h_m}} S_{lm}(ih_lV + V')\,dz = -i\left\{\frac{(ih_lV + V')S_{lm}}{\sqrt{h_m}(h_m - h_l)}\right\}_{z=0} - $$
$$-i\int_0^L e^{-(\gamma_m-\gamma_l)}\left\{\frac{e^{-i\gamma_l}(ih_lV + V')}{\sqrt{h_m}(h_m - h_l)}\right\}' dz, \qquad (12.21)$$

where it is placed still V(L)=0 and V'(L)=0. Replacing with z=0 $ih_lV \pm V'$ on formula (12.13), we will obtain from (12.21) and the same second expression, which contains S_{-lm}, formula for the unknown amplitude of backward wave

$$P_{-l}(0) = -i\left\{\frac{S_{lm}}{h_m - h_l}\right\}_{z=0} e^{-2i(\tilde{\gamma}_l+\pi/4)} + i\left\{\frac{S_{-lm}}{h_m + h_l}\right\}_{z=0} + I. \qquad (12.22)$$

The integral term I, which is obtained of the second addend formulas of type (12.21) we will extract below.

First two term in (12.22) they make simple physical sense. One should compare them with formula (8.37), valid in the absence of critical section. First term corresponds to direct waves, which were reflected from critical section, the second - to backward waves. The obtained formula can be considered as illustration of that

interpretation forming the parasitic field in the presence of critical section that was given in the preceding/previous point/item. This interpretation allows, applying formula (8.34) and expressions for reflection coefficients from critical section, to obtain formulas of type (12.22).

From this point of view, integral I describes the effect of the region, which adjoins the critical section. Bulky, but elementary conversions, are which we let us lower, lead to the expression

$$I = i\sqrt{\frac{h_m(0)}{h_j(0)}}\, e^{-i(\tilde{\gamma}_m+\pi/4)}\cdot$$
$$\cdot\int_0^L e^{-i\gamma_m}\left\{\left[i\,\frac{h_m h_j(S_{jm}+S_{-jm})+h_j^2(S_{jm}-S_{-jm})}{\sqrt{h_m}(h_m^2-h_j^2)}\right]' V+\right.$$
$$\left.+\left[\frac{h_j(S_{jm}+S_{-jm})+h_m(S_{jm}-S_{-jm})}{\sqrt{h_m}(h_m^2-h_j^2)}\right]' V'\right\} ds. \qquad (12.23)$$

Page 109.

In these conversions is used equation $V''+h_j^2V=0$. Furthermore in order not to complicate a question concerning the convergence of the integrals, which appear in intermediate conversions, it is convenient to consider wave number in void k composite; in the resultant expression, it goes without saying, k - really/actually. During analysis (12.23) it is substantial to bear in mind, that the coupling coefficients enter into integrand only in the form of combinations (11.3). These combinations are not only final in critical section,

but also is not contained first degree h_l. Therefore integrands in (12.23) are final everywhere, including critical section. The coefficients of V and V' in (12.23) will be of order ν_0^2 or ν'. Because V and V', as it follows from (12.3b) and (11.12), somewhat grow/rise near critical section, the order of integral I will be, apparently, somewhat below, however, since first two term in (12.22) have order ν_0, then in all cases during not very precise calculations by third term/component/addend in (12.22) it is possible to disregard.

Thus we obtained that for waveguides with the fracture of generatrix the amplitude of parasitic wave depends only on the values, which relate to salient point; this result, obtained in § 8 for a waveguide without critical section, is valid and in the general case. As noted above, this does not contradict so that the formation of parasitic waves bears nonlocal character, i.e., it occurs on entire irregular waveguide.

The explicit expressions of coupling coefficients for the waveguides of rectangular and round cross-sections are given in § 16.

§ 13. Fracture of waveguide. Frequency is close to the critical frequency of generatrix of parasitic wave.

1. Second case, which requires special examination, is resonance, which attacks in bent waveguide of constant section during approach/approximation of operating frequency to critical frequency of any of parasitic waves, which are formed at the bend. From a mathematical point of view, the special feature/peculiarity of this case lies in the fact that the coupling coefficients F_{jm} go to infinity when $k \to \alpha_j$, $h_j \to 0$, and therefore solutions (4.6), (4.14) and (7.35) become inapplicable. Us will interest first of all the physical picture of phenomenon, and therefore we will be restricted below to the examination of fracture, i.e., curvature to small angle ϑ_0, $\vartheta_0 \ll 1$. Resonance phenomena for a fracture are simpler than for a curvature to final angle, and they are expressed in some ratio/relations even more powerful.

Page 110.

Furthermore, in this case they can be completely investigated in general form.

The resonance, which appears during small strain, in particular with fracture, in certain sense has the same character, as during the excitation of waveguide by the outside current at critical frequency of forming wave. With given outside current, as is known, the amplitude of the excitable wave becomes very large, how not was small

the exciting current, and for the correct formulation of the problem it is necessary to consider the reverse/inverse action of the appearing wave on the exciting cell/element. In the case of curvature or fracture the analog of the exciting cell/element is the incident wave. For example, the field of the waves, which exit into right waveguide in Fig. 2, can be considered as the field, created by the currents of the incident wave, flowing on left waveguide. Therefore it is logical that in this case the resonance effects will be revealed first of all in the fact that the field of the incident wave will be strongly changed, to be more precise, that will arise the wave reflected with large amplitude.

We will begin analysis in the assumption that the walls of waveguide are ideally carrying out. As we will see, this will not cause the appearance of any infinity - resonance phenomena in waveguides can be strictly investigated without taking into account losses. However, although the account to conductivity and does not lead to qualitatively new phenomena, it all the same is completely necessary during the quantitative estimate/evaluation of the amplitudes of the appearing waves.

A single work, in which was examined analogous problem, was Jouguet's article [9]. In it were located the wave amplitudes scattered during incidence in wave H_{n0}, on coupling of two H_{p0}

semi-infinite rectangular waveguides - rectilinear and bent with small curvature in H-plane (Fig. 41), and was for the first time investigated resonance. Jouguet examined only the case when frequency strictly coincides with the critical frequency of the formed parasite, and was not considered the final conductivity of walls.

2. As in first paragraphs of this chapter, let us use variables Q_j and R_j connected with amplitudes P_j and P_{-j} by relationship/ratios (7.2). They satisfy system of equations

$$\frac{dQ_l}{d\vartheta}=\sum_{m=1}^{\infty}(F_{lm}-F_{-lm})R_m;$$

$$\frac{dR_l}{d\vartheta}=\sum_{m=1}^{\infty}(F_{lm}+F_{-lm})Q_m, \qquad (13.1)$$

which it is easy to obtain from system (7.9).

Page 111.

Far from resonance conditions the incident wave, as shown in § 7, barely is distorted, and for each number j in system (13.1) it is necessary to preserve two equations - for variables R_j and Q_j After solving these equations, it is possible then to find $P_l(\vartheta)$ and $P_{-l}(\vartheta)$; the obtained formulas will be, it goes without saying, to coincide with (7.28). In resonance frequency region, one must take into

account also a change in the field of the incident wave and to simultaneously examine four equations:

$$\begin{aligned}
\frac{dQ_j}{d\theta} &= (F_{jm} - F_{-jm}) R_m + (F_{jj} - F_{-jj}) R_j;\\
\frac{dQ_m}{d\theta} &= (F_{mm} - F_{-mm}) R_m + (F_{mj} - F_{-mj}) R_j;\\
\frac{dR_j}{d\theta} &= (F_{jm} + F_{-jm}) Q_m + (F_{jj} + F_{-jj}) Q_j;\\
\frac{dR_m}{d\theta} &= (F_{mm} + F_{-mm}) Q_m + (F_{mj} + F_{-mj}) Q_j;
\end{aligned} \qquad (13.2)$$

as in (11.5), remaining waves it is possible not to take into consideration.

When $k \to \alpha_j$, $h_j \to 0$ the coupling coefficients F_{jm} approach infinity. It is easy to show, using, for example, formulas (7.16), what these coefficients possess the following properties, analogous to the properties of coefficients S_{jm}: torrents j - magnetic wave, first with any m difference $F_{jm} - F_{-jm}$ does not go to infinity, but if j - electrical wave, then does not have special feature/peculiarities sum $F_{jm} + F_{-jm}$. Therefore in system of equations (13.2) only one coefficient goes to infinity when $h_j \to 0$; in this respect system (13.1) more convenient than reference system (7.9). All special feature/peculiarities in the coefficients of these equations, as one would expect from the considerations, given in the beginning § 11, will disappear, if we for a magnetic wave introduce instead of R_j

new variable $R_l h_l$, which, as Q_l, will not go to infinity when $h_l \to 0$; for electrical waves one should introduce variable $Q_l h_l$.

We will be restricted below to the examination of curvature to small angle, i.e., let us assume $\vartheta_0 << 1$. Let us note here only that for a curvature to final angle are used the usual methods of solving the system of equations with constant coefficients; the solution for each of the variables is sum four exponent of type $e^{p\vartheta}$. The analysis of the characteristic equation of this system shows that all its roots p remain finite when $h_l \to 0$.

Page 112.

Let us begin from the limiting case when frequency is exactly equal to critical, $h_l = 0$. Let us assume for a definition that both j and m - magnetic waves; qualitatively all results are retained during other possible combinations. Let us accept even for a reduction in the recording, that $K^{ll} = 0$, where integrals K^{im} are determined in (7.18); according to § 7 this equality usually is fulfilled. System of equations for variables Q_m, R_m, Q_l and $h_l R_l$ let us solve resolution in row/series according to the degrees of low value of ϑ_0. For the leading terms of terms of these expansions, calculation gives

$$R_m = 2\left(1 - \frac{\sigma}{\vartheta_0}\right) + O(\vartheta_0);\ Q_m = O(\vartheta_0);$$

$$R_l h_l = O(\vartheta_0^2);\ Q_l = \frac{2i h_m}{kK^{lm}} \cdot \frac{1}{\vartheta_0} + O(1). \tag{13.3}$$

In the bent part, according to the latter from these equalities, the field of parasitic wave takes the very large values, order $1/\vartheta_0$. According to (13.3), in old on ϑ_0 order $P_{-m}(0) \doteq 1$, $P_m(\vartheta_0) = 0$, the incident wave completely is reflected. How conveniently small fracture leads to the total reflection of the incident wave at frequency, in the accuracy of the coinciding with critical frequency forming parasitic wave. We will see below, that under actual conditions this effect is considerably attenuate/weakened.

Calculating the following terms in expansions (13.3) according to degrees ϑ_0, it is possible to find more exact expressions for the coefficients of reflection and passage

$$|P_{-m}(0)| = 1 - \frac{1}{18}|F_{mm}|^2 \vartheta_0^2 + O(\vartheta_0^3);$$
$$|P_m(\vartheta_0)| = \frac{1}{3}|F_{mm}|\vartheta_0 + O(\vartheta_0^2). \tag{13.4}$$

These formulas are valid, while is small product $|F_{mm}|\vartheta_0$, i.e., for a curvature with little electrical length.

Far from critical frequency the reflection coefficient has,

according to results of § 7, an order ϑ^2_0, and transmission coefficient differs from unity by the members of order ϑ^2_0. In order to trace transition from (13.4) to these conditions, it is necessary to solve the system of four equations (13.2) for R_m, Q_m, R_j and Q_j, utilizing simultaneously smallness of two parameters - ϑ_0 and $\hbar_j$. Expanding variables in row/series from the low parameters, it is necessary to assume the first, that $\vartheta_0/\hbar_j \ll 1$, since $\vartheta_0/\hbar_j \cong 1$, and finally that $\vartheta_0/\hbar_j \gg 1$. In this case, it proves to be that in old order for Q_m and Q_j are obtained in all three cases the identical analytical expressions.

Page 113.

The which interest us amplitudes of the scattered waves it is possible, utilizing (7.10), to express only through Q_m and Q_j, therefore and for them they are obtained the expressions, valid during any relationship/ratio between ϑ_0 and $\hbar_j$:

$$P_{-m}(0)=\frac{\tau}{1+\tau},\ P_m(1)=\frac{1}{1+\tau}; \qquad (13.5a)$$

$$P_j(1)=-P_{-j}(0)=-i\,\frac{kK^{jm}\vartheta_0/2\hbar_j}{1+\tau}. \qquad (13.5b)$$

Here through τ is designated they are essential in entire analysis the parameter

$$\tau=\frac{k^2(K^{jm})^2\,\vartheta_0^2}{4\hbar_m\hbar_j}. \qquad (13.6)$$

It is proportional to the ratio/relation of two low values $\vartheta_0^2/\hbar'$.

Energy of the reflected and transmitted fundamental wave, divided into energy of the incident wave, is equal, correspondingly,

$$|P_{-m}(0)|^2=\frac{|\tau|^2}{|1+\tau|^2},\ |P_m(\vartheta_0)|=\frac{1}{|1+\tau|^2}. \qquad (13.7)$$

If frequency is lower than the critical, i.e., $\hbar_j^2<0$, then parasitic waves, it goes without saying, is not taken away energy; when $\hbar_j^2>0$ both parasitic waves take away equal energy content:

$$|P_j(\vartheta_0)|^2\hbar_j/\hbar_m=|P_{-j}(0)|^2\hbar_j/\hbar_m=\frac{\tau}{(1+\tau)^2}. \qquad (13.8)$$

It is easy to check that entire/all taken away energy is equal to incident energy both at the frequency of higher than the critical $(\hbar_j^2>0,\ |\tau|=\tau)$, and at the frequency of lower than the critical $(\hbar_j^2<0,\ |\tau|=-i\tau)$.

Formulas (13.5) make it possible to trace the onset of resonance phenomena during the approach/approximation of frequency to critical, i.e., when value $|\hbar_j|$ decreases, and the parameter τ thereby increases. As yet $|\tau|\ll 1$, there is no resonance, strictly speaking, still; the amplitude of the fundamental wave does not change with the passage of fracture, but the amplitudes of parasitic waves (13.5b)

are given as before by formulas (4.6), (4.12), with which, according to expression (7.20), formula (13.5b) coincid·s with $\tau<<1$. approach/approximation to critical frequency with $\tau<<1$ manifests itself only in the fact that the amplitudes of straight line and by the reverse/inverse of parasitic waves are compared between themselves; these amplitudes can become final ones, even greater; however, the taken away by parasitic waves energy is small.

Page 114.

Is small also, accordingly (13.5a), the effect of these waves on the incident wave; this, actually, it is explained by the fact that the coupling coefficient F_{ml} is final, and angle ϑ_0 is small.

Resonance begins when $|\tau|\sim1$. With this appears the effective disturbance/perturbation of the transmitted wave and simultaneously becomes final the energy, taken away by parasitic waves. The field of parasitic wave will be in this case very large, order τ/ϑ_0. Greatest it will be with precise resonance $h_l=0, |\tau|\to\infty$, when on (13.5b) or last/latter formula (13.3), which will agree with (13.5b), this field inversely proportional to ϑ_0; in this case entire/all energy of the incident wave is reflected. Parasitic waves take away maximum energy content with $\tau=1$; in this case, 1/4 incident energies it is reflected, 1/4 pass also on 1/4 it is scattered by direct/straight

and reverse/inverse parasitic waves.

3. In real waveguides condition of powerful resonance practically never is satisfied. As a result of the fact that the walls of waveguide possess final conductivity, h_l is complex quantity, not at what frequency non-vanishing. Complete expression for a wave number in the waveguide with imperfect walls whose conclusion/derivation in order not to break presentation, we will transfer into the following paragraph, it shows that the minimum value which can accept $|h_l|$, is reached at frequency somewhat lower than the critical and equally for the magnetic waves

$$|h_l|_{\min} = k\left\{\frac{d}{2}\oint(\psi^l)^2\,ds\right\}^{1/4} \qquad (13.9a)$$

where d the thickness of skin-layer. For the electrical waves

$$|h_l|_{\min} = \left\{\frac{d}{2}\oint\left(\frac{\partial\varphi^l}{\partial n}\right)^2 ds\right\}^{1/4}. \qquad (13.9b)$$

In (13.9) for reduction of recording, magnetic permeability of the material of wall is placed to equal unity. By order of value $|h_l|_{\min}$ it is equal $(d/a)^{1/4}$.

Entire mathematical apparatus, developed in the preceding/previous point/item, remains valid also upon consideration to conductivity. However, at given one ϑ_0 parameter $|\tau|$ cannot be more than

$$|\tau|_{\text{макс}} = \frac{k^2(K^{lm})^2\vartheta_0^2}{4h_m|h_l|_{\text{мин}}}. \qquad (13.10)$$

Page 115.

This value is proportional $\vartheta_0^2(a/d)^{1/2}$. The numerical coefficient depends on the section of waveguide and numbers of waves. For example, if on the fracture of the waveguide of circular section falls wave H_{01} (wave of number m), and frequency coincides with the critical frequency of the forming wave H_{12} (wave of number j), then, substituting (3.18) and (3.20) in (7.18) and (13.9a) we will obtain

$$K^{lm} = -\frac{\sqrt{2}\mu_m\mu_l}{k\sqrt{\mu_m^2-1}}\cdot\frac{1}{\mu_j^2-\mu_m^2}; \quad \oint(\psi^l)^2 ds = \frac{2a}{\mu_j^2-1}. \qquad (13.11)$$

If we even for simplicity assume $h_m = 1$, then

$$|\tau|_{\text{макс}} = \frac{\mu_l}{2\sqrt{\mu_j^2-1}}\cdot\frac{\mu_m^2}{(\mu_j^2-\mu_m^2)^2}\sqrt{\frac{a}{d}}\cdot\vartheta_0 = 0{,}04\sqrt{\frac{a}{d}}\,\vartheta_0^2. \qquad (13.12)$$

For waves H_{1q}, $q>0$, the numerical coefficient in (3.12) will be still less. Let us accept for ratio a/d value $2.5\cdot10^4$, which corresponds to the conductivity of copper and $\lambda\sim1$ cm, a~1 cm. Then

$$|\tau|_{\text{макс}} = 5\vartheta_0^2. \qquad (13.13)$$

Since all calculations of the preceding/previous point/item were produced on the assumption that $\vartheta_0 \ll 1$, then $|\tau_{\text{макс}}|$ is always small in

comparison with one. Very strongly resonance is expressed be it cannot, but it all the same is noticeable.

Accordingly (13.5a), the coefficient of reflection $|P_{-m}(0)|_{макс}$ of small ones $|\tau|_{макс}$ is equal to $|\tau|_{макс}$, i.e., to the value, given by formula (13.10), and, for example, even with $\vartheta_0=1/5$ reaches only value $|P_{-m}(0)|_{макс}=0,2$. The amplitude of parasitic waves, accordingly (13.5b), will be more than reflection amplitude, since it is proportional to the first degree ϑ_0, but not ϑ^2_0 as $P_{-m}(0)$. Its maximum value is equal

$$|P_{-l}(0)|=|P_l(\vartheta_0)|=\frac{1}{\sqrt{2}}\frac{\mu_m}{\mu_l^2-\mu_m^2}\sqrt{\frac{a}{d}}\vartheta_0=0,2\sqrt{\frac{a}{d}}\vartheta_0. \quad (13.14)$$

For value $a/d\ |P_l(\vartheta_0)|\cong 40\vartheta_0$ accepted - to the value which even for small angles can become large in comparison with unity.

The frequency dependence of the coefficient of reflection and amplitudes of parasitic waves is determined with small ones $|\tau|$ by factor $1/|h_l|$.

Page 116.

This frequency dependence is sharper, the less is d/a, but, as always in waveguides, resonance peaks are sufficiently wide. According to (14.12), value $|h_l|$ increases in $\sqrt{2}$ the time, which corresponds to

the same decrease of the field of parasitic waves and is reflected wave, with frequency drift to value

$$\frac{\Delta k}{k} = \frac{d}{4} k^2 \oint (\psi')^2 ds \qquad (13.15)$$

(for magnetic waves). For wave H_{12}, this gives, according to (13.11), $\Delta k/k \sim 0.5$ d/a. With $a/d=2.5 \cdot 10^4$ $\Delta k/k=2 \cdot 10^{-5}$, which for $\lambda=8$ mm gives the half-width of the order of megahertz.

Let us note on conclusion of this paragraph, that the always existing under actual conditions dissimilarity of cross section at different points of waveguide also attenuate/weakens the resonance, which attacks during the coincidence of operating frequency with the critical frequency of the waveguide of constant section. For coarse-p of the evaluation of this effect, it is possible to count that value $|h_j|^2_{\min}$, which, according to (13.9), is proportional d/a, contains one additional term/component/addend, equal to the root-mean-square relative spread of the linear dimensions of cross section. In other words, the inconstancy of cross section throughout its effect on resonance phenomena is equivalent to certain increase in the thickness of skin-layer.

§ 14. Wave number in waveguide with imperfect walls.

1. In this paragraph we will derive formula for wave number in

waveguide whose walls possess large, but final conductivity which we applied in preceding/previous paragraph. Since in the which interests us frequency region wave number h_l very small, then we will not be able to use the method, used for an analogous problem in § 6, or the conventional energy method, used only at not too low and real values h_l.

For electrical waves the corresponding results were obtained for the first time - not completely strict method - in the book of Kisunko [32]. For waves in circular waveguide, the common/general/total expression for a wave number was found by Ya. L. Alpert [85].

In waveguide with imperfect walls, boundary condition on metal (3.2) is replaced by the boundary condition of Leontovich according to which the tangential to metal components of fields are connected by the relationship/ratios

$$E_s = wH_z, \ E_z = -wH_s. \qquad (14.1)$$

Page 117.

These relationship/ratios are valid with an accuracy down to the terms first-order in $|w|$ inclusively. Here w - so-called wave impedance of walls, $w=\sqrt{\mu/\varepsilon}$, where μ and ε - the parameters of the material of walls. In metal dielectric constant ε - a large

imaginary number, magnetic permeability μ can be considered real, and, introducing the thickness of skin-layer d, we will obtain

$$w = \frac{1+i}{2} \mu k d. \qquad (14.2)$$

Module/modulus w - low value. We will search for wave number h progressive scanning of fields in row/series in terms of $|w|$. We will be restricted to the case when there is no degeneration, i.e., let us assume that to each eigenvalue α^2 in waveguide with ideal walls corresponds magnetic or electrical type only one wave.

Since in this paragraph is examined the regular waveguide, in which there is not passage of waves, then we will be able to omit index in wave number, in membrane/diaphragm functions, etc. The values, which relate in to waveguide with ideal walls, let us designate 0.

After recording the participating under boundary condition (14.1) components of the fields through membrane/diaphragm functions ψ and ϕ for formulas (3.14), we will obtain the conditions, which connect the values of these functions on the duct/contour of the section:

$$-ih\frac{\partial\varphi}{\partial s} + ik\frac{\partial\psi}{\partial n} = w\alpha^2\psi; \quad \alpha^2\varphi = w\left(ik\frac{\partial\varphi}{\partial n} + ih\frac{\partial\psi}{\partial s}\right). \quad (14.3)$$

From this condition, in particular, immediately follows known result

about the fact that the division of waves into electrical ones ($\psi \equiv 0$) and magnetic ($\phi \equiv 0$) possibly only in waveguides with ideal walls, i.e., with w=0, or in symmetrical fields, with d/ds≡0; in the general case the solution contains both of functions ψ and ϕ.

Functions ψ and φ satisfy identical equations (3.11). From these equations and boundary conditions (14.3) it is necessary to find these functions and eigenvalue α^2. Wave number h is connected with α condition (3.13).

For an eigenvalue it is possible to write resolution in row/series according to degrees of w

$$\alpha^2 = \alpha_0^2 + w\alpha^2{}_1 + \cdots \qquad (14.4)$$

In the same row/series is decompose/expanded the square of the wave number

$$h^2 = h_0^2 - w\alpha^2{}_1 + \cdots, \quad h_0^2 = k^2 - \alpha_0^2. \qquad (14.5)$$

Page 118.

It is necessary to find the second term of this expansion, i.e., coefficient $\alpha^2{}_1$. However, the leading term of this expansion $h^2{}_0$ at critical frequency, i.e., when $\kappa = \alpha_0$, is equal to zero, and near critical frequency it will be let us compare with the following term. Therefore, obvious expansion of wave number, that follows from expansion (14.5)

$$h=\sqrt{h_0^2-w\alpha_1^2+\cdots}=h_0-w\frac{\alpha_1^2}{2h_0}+\cdots, \qquad (14.6)$$

not by Budde applicably for all frequencies, including the vicinity of critical frequency. From (14.3) it follows that both of functions ψ and ϕ, also are not decomposed simultaneously in row/series according to degrees of w which would be used everywhere; this does not have place also for the components of fields. Therefore the entering under condition (14.3) values cannot be directly expanded in row/series according to degrees of w.

In order to use the method of successive approximations to system (3.11), (14.3) and to obtain the solutions, valid also in the vicinity of critical frequency, we utilize an artificial reception/procedure, introducing also auxiliary functions, so that under the boundary conditions would participate only the square of wave number. For simplification in the recording of the convenient for waves magnetic and electrical type to introduce different auxiliary functions.

2. Let us begin from magnetic waves. Let us multiply the second equality in (14.3) by h and will introduce auxiliary function $\Phi=h\varphi$. Functions Φ and ψ let us expand in row/series, taking into account that as a result of the nondegeneracy the electrical

waves in zero order on w are absent, $\Phi_0=0$, so that expansion of begins from the member of order w.

$$\psi=\psi_0+w\psi_1+\ldots,\ \Phi=w\Phi_1+\ldots \qquad (14.7)$$

Let us introduce expansions (14.4), (14.5) and (14.7) under equations (3.11) and boundary conditions (14.3). Dividing orders, we will obtain in zero on w order

$$\nabla^2\psi_0+\alpha_0^2\psi_0=0; \qquad (14.8a)$$

$$\left.\frac{\partial\psi_0}{\partial n}\right|_C=0. \qquad (14.8b)$$

and in the first order

$$\nabla^2\psi_1+k^2\psi_1=-\alpha^2{}_1\psi_0; \qquad (14.9a)$$

$$\left.\frac{\partial\psi_1}{\partial n}\right|_C=-\frac{i\alpha_0^2}{k}\psi_0+\frac{1}{k}\left.\frac{\partial\Phi_1}{\partial s}\right|_C,\ \Phi_1|_C=\frac{ih_0^2}{\alpha_0^2}\cdot\left.\frac{\partial\psi_0}{\partial s}\right|_C. \qquad (14.9b)$$

Page 119.

eliminating the hence boundary values of function Φ_1, we will obtain boundary condition for ψ_1, the only solution of zero-order system

$$\left.\frac{\partial\psi_1}{\partial n}\right|_C=\frac{-i\alpha_0^2}{k}\psi_0+\frac{ih_0^2}{k\alpha_0^2}\cdot\left.\frac{\partial^2\psi_0}{\partial s^2}\right|_C. \qquad (14.10)$$

The unknown value $\alpha^2{}_1$ is determined from requirement so that equation (14.9a) with boundary condition (14.10) would have final solution when system (14.8) has nontrivial solution. In order to find value $\alpha^2{}_1$, let us multiply equation (14.8a) on ψ_1, equation (14.9a) - on ψ_0, let us deduct, will integrate over cross section and convert

integral on Green's formula. After substituting then into contour integral boundary conditions (14.8b) and (14.10), after producing in it integration in parts and after considering the conditions for standardization (3.16), we will obtain

$$\alpha^2{}_1 = \frac{i}{k}\left\{\alpha_0^4 \oint \psi_0^2 ds + h_0^2 \oint \left(\frac{\partial \psi_0}{\partial s}\right)^2 ds. \qquad (14.11)$$

After calculating $\alpha^2{}_1$, we, according to (14.5), find two members in expansion h^2 according to degrees of w. After substituting value of w on (14.2), we will obtain

$$h^2 = h_0^2 + (1-i)M + 0(|w|^2), \qquad (14.12)$$

where through M is designated value

$$M = \frac{\mu d}{2}\left\{\alpha_0^4 \oint \psi_0^2 ds + h_0^2 \oint \left(\frac{\partial \psi_0}{\partial s}\right)^2 ds\right\}. \qquad (14.13)$$

small value $|h|$, considered as the function of frequency, is reached at $h^2{}_0=-M$ and it is equal

$$|h|_{\text{мин}} = \sqrt{M}, \qquad (14.14)$$

moreover in M it is possible in this case to retain only first term/component/addend, obliged to longitudinal current, and to set/assume α=k. So is obtained formula (13.9a).

Page 120.

Far from critical frequency, with $h^2{}_0>0$, $h^2{}_0>>M$, (14.12) is led to expression for the attenuation factor

$$h'' = M/2h_0, \qquad (14.15a)$$

coinciding after the substitution of value M (14.13) with the formula which is obtained by usual energy method.

The phase speed v=c/Re h is equal to

$$v = v_0(1 - M/2h_0^2). \qquad (14.15b)$$

It is somewhat less than for the waveguide of the same section with ideal walls. Introduction to final conductivity on effect on phase speed of the equivalently to the expansion linear dimensions of section to value $M/2\alpha^2{}_0$, proportional to the thickness of skin-layer.

At frequency lower than the critical, if $h_0^2 < 0,\ |h_0|^2 \gg M,$ equation (14.12) gives

$$h = -i\left(|h_0| - \frac{M}{2|h_0|}\right) + \frac{M}{2|h_0|}. \qquad (14.16)$$

The second term in this formula proportional to energy flow, which is propagated in waveguide with imperfect walls with frequency lower than the critical.

Transition from (14.15) to (14.16) is described by formula (14.12), which makes it possible to give explicit expression for Re h and Im h with any relationship between $h^2{}_0$ and M; Re h and Im h are found from the system

$$(\mathrm{Re}\,h)^2 - (\mathrm{Im}\,h)^2 = h_0^2 + M; \quad 2(\mathrm{Re}\,h)(\mathrm{Im}\,h) = M. \qquad (14.17)$$

For example, with $k=\alpha_0$, i.e., at critical frequency, when $h_0=0$, $h=\sqrt{M}(1.10-i0.46)$. Generally wave number h, which far from critical frequency, according to (14.12), differs from h_0 by terms of order $k|w|$, in immediate proximity to the critical frequency of Budde of order $k|w|^{1/2}$.

We will not here determine the disturbance, experience/tested by membrane/diaphragm functions and fields upon transfer from ideal ones to imperfect walls. It easily is found from system (14.9). Despite all frequencies ψ_1 there will be order w; ϕ Budde order $wh^2{}_0/kh$, so that, for example, sense E_z/H_z far from the critical frequency there will be of order w. Near critical frequency, with small $|h_0|$, that compose of fields, connected with electrical wave (in particular, component E_z), will be very small.

Calculating thus two members in the resolution of wave number for electrical waves, one should instead of function ψ introduce function $\Psi=h\psi$ and to expand in row/series according to degrees of w of function ϕ and Ψ. Function ϕ_1 satisfies the system

$$\nabla^2\varphi_1+\alpha_0^2\varphi_1=-\alpha^2{}_1\varphi_0;\ \varphi_1|_C=\frac{ik}{\alpha_0^2}\cdot\frac{\partial\varphi_0}{\partial n}. \qquad (14.18)$$

Page 121.

From the condition of the solvability of this system, it is located $\alpha^2{}_1$:

$$\alpha^2{}_1=ik\oint\left(\frac{\partial\varphi_0}{\partial n}\right)ds. \qquad (14.19)$$

All formulas (14.12), (14.14)-(14.17), obtained are above for magnetic waves, they are transferred, thus, to electrical waves during replacement of M in (14.13) to the value

$$M = \frac{\mu k^2 d}{2} \oint \left(\frac{\partial \varphi_0}{\partial n}\right)^2 ds. \qquad (14.20)$$

Despite all frequencies Φ_1 it will be of order w; ψ there will be order $wh^2{}_0/kh$, and at the critical frequency of components H_z it disappears.

For two cases for which the approach/approximation of loose coupling even for very small or flat irregularities proves to be insufficient (in the presence of critical section and at resonance frequency), are developed the methods of solving the system of differential equations for wave amplitudes. They are calculated the reflection coefficient during incidence in the wave in critical section, the amplitude of the parasitic wave, scattered by irregular waveguide in the presence of critical section for this wave, amplitudes of the waves, scattered on the fracture of the waveguide of constant section at frequency, as to close as desired to the critical frequency of the appearing parasitic wave. In connection with last/latter problem is found the expression for a composite

propagation constant in waveguide with imperfect walls, valid despite all frequencies, including the vicinity of critical frequency.

Page 122.

Chapter IV.

RECTILINEAR IRREGULAR WAVEGUIDES.

In this chapter the developed above mathematical apparatus is applied to the solution of several specific problems of the propagation of waves into the rectilinear irregular special. Examples are chosen which illustrate the various peculiarities of the method of transverse sections.

§ 15. Twisted waveguides.

1. As the first let us examine joining. Joining is the special case of changing the cross section, with which are changed not the form or value, but only position or cross section. In any section duct/contour C has one and the same form; the position of duct/contour is determined by certain angle Φ, composed by fixed/recorded relative to this duct/contour direction with x-axis of motionless system of coordinates x, y, z. Angle Φ depends on z,

$\Phi=\Phi(z)$. We will begin from the computation of coupling coefficients in the dense waveguides; will be examined the waveguides of rectangular cross section and the waveguides whose section is ellipse with small eccentricity.

During the computation of function $v(s)$, which enters into formulas (9.5), we will not be turned to the bulky methods of differential geometry, but we utilize an auxiliary reception/procedure, being based on relationship/ratio $v(s)\Delta z=\delta(s)$ between the function $v(s)$ and the height/altitude of step $\delta(s)$ in certain auxiliary stepped waveguide which in limit during the decrease of the height/altitude of each step will pass into this irregular waveguide. Let us find first distance $\delta(s)$ between the appropriate points of two duct/contours, turned relative to each other to angle $\Delta\Phi$, and then let us divide $\delta(s)$ on Δz and let us pass to limit $\Delta z\to 0$. In this case, $\delta(s)$ it is necessary to determine, it goes without saying, only with an accuracy down to the terms first-order in $\Delta\Phi$.

Page 123.

The axes of the coordinate system, rigidly connected with cross section and which rotate together with it during motion along waveguide, let us designate $\bar{x}$, $\bar{y}$. For the waveguide of rectangular

cross section, it is directed these axes along the sides of rectangle, as shown in Fig. 16a; in the same figure is represented the second section, turned to angle $\Delta\Phi$. From obvious geometric considerations it follows that, for example, on lower side, with $\bar{y}=0$

$$\delta(\bar{x}) = \Delta\Phi\left(\frac{a}{2}-\bar{x}\right)+O[(\Delta\Phi)^2]. \qquad (15.1)$$

In this case, it is assumed, that the joining leaves motionless the central point of section. If motionless was, for example apex/vertex $\bar{x}=0$, $\bar{y}=0$ (Fig. 16b), then instead of (15.1) it would be

$$\delta(\bar{x}) = \Delta\Phi\cdot\bar{x}+O[(\Delta\Phi)^2]. \qquad (15.2)$$

In this case, would change the computed further coupling coefficients. For certainty we will produce calculation for a joining, appropriate Fig. 16a. After writing for other three sides of expression for $\delta(s)$, analogous (15.1), and after passing to $v(s)$, we will obtain

$$\begin{aligned}&\bar{y}=0 \quad v(s)=\left(\frac{a}{2}-\bar{x}\right)\Phi'; \quad \bar{x}=a \quad v(s)=\left(\frac{b}{2}-\bar{y}\right)\Phi';\\ &\bar{y}=b \quad v(s)=\left(\bar{x}-\frac{a}{2}\right)\Phi'; \quad \bar{x}=0 \quad v(s)=\left(\bar{y}-\frac{b}{2}\right)\Phi',\end{aligned} \qquad (15.3)$$

where it is marked $\Phi'=d\Phi/dz$.

The computation of coefficients S_{jm} in (9.5) for any waves is reduced now to elementary quadratures. We will be restricted to the case when the incident wave (numbers m) is principle wave of rectangular waveguide, i.e., wave H_{10} with membrane/diaphragm function (3.21)

$$\psi^m = N^m\cdot\cos\frac{\pi x}{a}. \qquad (15.4)$$

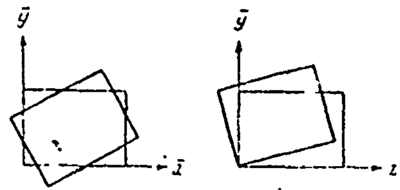


Fig. 16. a, b) the joining of the waveguide of rectangular cross section.

Page 124.

There is greatest interest in the communication/connection of this wave with wave H_{01}, which possesses perpendicular polarization; its membrane/diaphragm function takes the form

$$\psi^{l} = N^{l} \cdot \cos \frac{\pi y}{b}. \tag{15.5}$$

Elementary calculation according to (9.5a) and (15.3-15.5) gives

$$S_{lm} = -\frac{8}{\pi^2} \frac{h_l + h_m}{2h_l} \Phi'. \tag{15.6}$$

Let us note that this formula can be also obtained, calculating S_{lm} not according to (9.5), but directly leaving from (8.6), moreover in (8.6) after the passage to the limit, discussed at the end of the fourth point/item of § 8, does not appear supplementary contour integral. This supplementary contour integral, which contains, as it is easy to show, charges and longitudinal currents, for two waves H_{01}

and H_{01} in question it is equal to zero. This is explained by the fact that both charges and longitudinal currents for these waves they are formed in different sections of duct/contour, so that the communication/connection of these waves they are formed in different sections of duct/contour, so that the communication/connection of these waves with each other is realized only by transverse currents. On this same reason in (9.5a) is absent in this case second term. Let us note that for this very reason in this case, just as for waves H_{0q}, in circular waveguide, coupling coefficient for the waves of opposite directions is proportional to a difference in the propagation constants, as that follows from (15.6), and it is considerably less than for direct waves.

Coupling coefficient with backward wave of the same type H_{10} as falling, is simplest to calculate, applying formula (9.4). Since for a joining wave number does not change, the add/composed h_m' in (9.4) will not be. Integral term is also equal to zero, since on opposite sides of section $\bar{y}=0$ and $\bar{y}=b$ $v(s)$ has opposite signs; therefore $S_{-mm}=0$. Reflection coefficient from the section, twisted to small angle $\Delta\Phi$, is proportional, thus, $(\Delta\Phi)^2$; for a long joining with the slowly changing angle Φ it is proportional $(\Phi')^2$. It is easy to check further that the joining does not connect wave H_{10} with waves H_{n0}. Coupling coefficient with other waves it is also easy to find from (9.5) and (15.3).

In the examination of the waveguides of elliptical cross section, we will be restricted to waveguides with small eccentricity, close to circular waveguides; this will allow us not to introduce elliptical functions and to obtain all results in elementary form.

Page 125.

With small eccentricity the duct/contour of the section in polar coordinates ρ, β can be recorded in the form (Fig. 17)

$$\rho = a + l\cos 2\beta, \quad (15.7)$$

where positive value l, $l \ll a$, is connected with excentricity e by relationship/ratio $e_2=4l/a$. The equation of the turned duct/contour is obtained by replacement in (1.57) vectorial angle β on $\beta - \Delta\Phi$. With an accuracy down to the terms first-order in $\Delta\Phi$ inclusively $\delta(s)$ is equal to a difference in the radius-vectors, which relate to one and the same β for the turned and unturned duct/contours. Passing to $\nu(s)$, i.e., after dividing $\delta(s)$ on Δz and set/assuming $\Delta z \to 0$, we will obtain

$$\nu(s) = 2l\sin 2\beta \cdot \Phi'. \quad (15.8)$$

Let the incident wave be close to wave H_{11} in circular waveguide, polarized along transverse. With $l \ll a$ it is possible for computing the coupling coefficients not to consider in expression for the membrane/diaphragm functions of the disturbance/perturbation,

connected with the difference for section from circular, and to set/assume

$$\psi^m = N^m J_1(\alpha_m \rho)\sin\beta, \qquad (15.9)$$

where N^m is given in (3.20). This approximation for membrane/diaphragm function it is possible to use everywhere, including the points of duct/contour, because the duct/contours of ellipse (15.7) and of circle $\rho=a$ are not only close, but also comprise everywhere small angles; the difference for the eigenvalue α_m of wave in slightly elliptical waveguide from eigenvalue μ_m/a (3.19) for a circular waveguide will prove to be essential.

The wave, close to wave H_{11} with the lattice-type polarization, has the membrane/diaphragm function

$$\psi' = N' J_1(\alpha_j \beta)\cos\beta. \qquad (15.10)$$

The coupling coefficient between these two waves, computed on (9.5) and three last/latter formulas, is equal to

$$S_{jm} = \frac{l}{a}\cdot\frac{\alpha_m}{\alpha_m-\alpha_j}\,\frac{\mu_m^2+1}{\mu_m^2-1}\,\Phi'. \qquad (15.11)$$

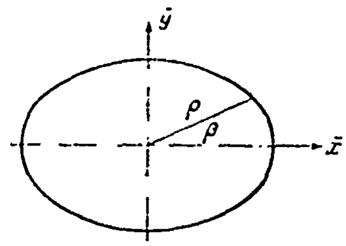


Fig. 17. Section of elliptical waveguide.

Page 126.

We convert this expression, after substituting the value of a difference in eigenvalues of two H_{11} - waves with the lattice-type polarization in slightly elliptical waveguide. This difference is proportional to l, it is easy to find, applying, for example, formula (6.12). It is necessary only during calculation to consider that in (6.12) δ(s) indicates the distance between the duct/contours of two ellipses, turned relative to each other on π/2, so that $\delta=-2l\cos 2\beta$. Elementary calculation, which we lower, gives

$$\alpha_m - \alpha_l = \frac{\mu_m^2+1}{\mu_m^2-1}\cdot\frac{l}{a}. \qquad (15.12)$$

Thus, coupling coefficient between two H_{11} waves, obliged to joining, is equal to simply

$$S_{lm} = \Phi'. \qquad (15.13)$$

It is easy to show, being based to (15.8), that in old order on l/a the joining does not connect direct/straight and backward waves in

slightly elliptical waveguide. Wave H_{11} with membrane/diaphragm function (15.9) is connected with all H_{1q} -waves of perpendicular polarization and with those E_{1q} -waves for which $\varphi^{i} \sim \sin\beta$, but only for wave H_{11} with membrane/diaphragm function (15.10) S_{lm} will not be low value, in spite of the presence of factor l in (15.8).

2. Coupling coefficients (15.6) and (15.13) are proportional to derivative of angle Φ along the length. If a change in the angle at length a or λ is small, then communication/connection is small. For a small, loose coupling of the amplitude of parasitic waves, they are found in (8.34): this formula is valid, if amplitude p_m falling/incident wave is not changed noticeably along transition, i.e., if is correct solution (8.32). There are, however, conditions, under which, how not were small the coupling coefficients, the amplitude of the incident wave all the same noticeably changes and approach/approximation (8.32), (8.34) proves to be inapplicable. This can occur near degeneration, to be more precise, when propagation constant of two waves, connected with strain, are so/such close, that a difference in phase change of both of waves in entire irregular section σ is small or final. For the joining

$$\sigma = (h_m - h_l) L, \qquad (15.14)$$

where L - length of joining.

Page 127.

In this case, the amplitude of parasitic wave takes finite values. From a physical point of view, this is explained by the fact that the elementary parasitic waves, which are formed in the individual sections of irregular waveguide (§ 5), store/add up with each other with a very small phase difference. In turn, the opposite effect of this parasitic wave on that falling of wave changes the amplitude of this latter. We will study this question based on the example of the twisted waveguide; in the theory of the curvature of the circular waveguide, we will clash with it once more.

Let us start the examination of the conditions, which occur during complete (σ=0) or incomplete degeneration, from the study of the passage of wave H_{10} in the joining of rectangular waveguide. Waves H_{01} and H_{10} on the joining of rectangular waveguide. Waves H_{01} and H_{10} will be in it completely confluent, if its section becomes square (a=b).

The field of these waves is described by system of equations (8.30) for the given amplitudes. In order to describe conditions near degeneration, it is necessary in this system to preserve two equations, the relating to direct waves numbers m and j

$$p_m' = -\frac{h_j}{h_m} S_{jm} e^{-i(\gamma_j - \gamma_m)} p_j,\quad p_j' = S_{jm} e^{i(\gamma_j - \gamma_m)} p_m. \quad (15.15)$$

During the writing of this system, are taken into account the conditions of reciprocity (8.10).

Bearing in mind that S_{jm} is proportional Φ', it is convenient as the independent variable to accept instead of z pitch angle Φ. After the substitution of expression (15.6) for S_{jm} equation (15.15) they take the form

$$\frac{dp_m}{d\Phi} = \sqrt{\frac{h_j}{h_m}} A p_j e^{-i(\gamma_j - \gamma_m)};$$
$$\frac{dp_j}{d\Phi} = -\sqrt{\frac{h_m}{h_j}} A p_m e^{i(\gamma_j - \gamma_m)}, \qquad (15.16)$$

where through A is designated value

$$A = \frac{8}{\pi^2} \frac{h_m + h_j}{2\sqrt{h_m h_j}}. \qquad (15.17)$$

End conditions for system (15.16), according to (8.31), take form $p_m(0) = 1,\ p_j(0) = 0.$

Exponential factors in (15.16), are proportional to z, since h_m and h_j they do not depend on z. The solution of system (15.16) depends on the form of joining, i.e., from the form of the function $\Phi(z)$. Let us examine for certainty the uniform helical joining in which Φ is proportional to z:

$$\Phi = \theta \cdot \frac{z}{L}. \qquad (15.18)$$

Page 128.

Here θ - complete angle of rotation. In this case, in (15.16) it is possible to substitute

$$\gamma_l - \gamma_m = q\Phi, \qquad (15.19)$$

where parameter q, which plays the significant role in further analysis, it is equal to the ratio of complete phase change σ to complete angle of rotation θ:

$$q = \frac{\sigma}{\theta}. \qquad (15.20)$$

According to (15.14), parameter q can be also defined as the ratio/relation to a difference in the propagation constants $(h_m - h_l)$ to angle of rotation to the unit of length θ/L. The first of these values characterizes local, i.e., not depending on the length of joining, nearness to degeneration conditions, the second - local rate of change in the properties of waveguide.

Low values $|q|$ mean that the conditions are close to degeneration, q=0 corresponds to the complete degeneration which can be only in square waveguide. If $|q| \gg 1$, then, as we now will see, degeneration is virtually removed. Thus, the effect of strain is determined not by angle of rotation per the unit of length θ/L - to

this value are proportional coupling coefficients, but by the relationship/ratio between it and difference Δh. This relationship/ratio is characterized by parameter q.

The solution of system (15.16) for the given amplitudes p_m and p_j under condition (15.19) is located by elementary shape. The value of peak-tos-peak amplitude P_m and P_j at the end of the joining, i.e., with z=L, $\Phi=0$, is determined then from (8.29). These amplitudes prove to be equal to

$$P_m=\left(\cos\varkappa+\frac{i\sigma}{2\varkappa}\sin\varkappa\right)e^{-i\frac{h_m+h_j}{2}L};$$

$$P_j=-\frac{A0}{\varkappa}\sin\varkappa\cdot e^{-i\frac{h_m+h_j}{2}L}. \qquad (15.21)$$

Here through $\varkappa$ is designated value

$$\varkappa=0\sqrt{A^2+q^2/4}. \qquad (15.22)$$

Let us assume first in solution (15.21) q=0. Then

$$P_m=\cos\frac{8}{\pi^2}\Phi;\quad P_j=-\sin\frac{8}{\pi^2}\Phi. \qquad (15.23)$$

Page 129.

In proportion to advance along the twisted waveguide, the wave energy completely is pumped over from wave H_{10} into wave H_{01} and vice versa, complete transition occurs through each $\pi^3/16$ radian, i.e., through

111°. It is easy to show, being based to (15.16), that this result, valid for a square waveguide (i.e. when $\gamma_l - \gamma_m = 0$). it is retained with any dependence $\Phi(z)$.

With final ones $|q|$ the amplitude of parasitic wave is always lower than unity, occurs the periodic pumping over of the part of the energy. The maximum energy, transferred by wave H_{01}, is equal to

$$\frac{A^2\theta^2}{A^2\theta^2 + \frac{\sigma^2}{4}}. \qquad (15.24)$$

finally, when $|q| \gg 1$, when total phase change at entire length of joining σ is great in comparison with common pitch angle θ^*, the amplitude of parasitic wave is equal to

$$|P_j| = \left|\frac{2A}{q}\sin x\right|. \qquad (15.25)$$

FOOTNOTE [1]. This case is examined for the first time in the article of Sveshnikov [20]. ENDFOOTNOTE.

This result is obtained also during computation on formula (8.34), which relates to the case of loose coupling. Thus when a difference in the propagation constants is sufficiently great (more precise, when $|q| \gg 1$). the amplitude of parasitic wave is inversely proportional to this difference.

The disturbance/perturbation, caused by twisting, can be characterized not only by the amplitude of parasitic wave with perpendicular polarization, but also by change of the propagation constant of the incident wave. The phase of fundamental wave H_{10} at output it is easy to find from solution (15.21). After dividing it to the length of section L, we will obtain the value which can be call/named the effective propagation constant of the incident wave in the twisted section. When $|q| \gg 1$, $|\sigma| \gg 1$, far from degeneration conditions, this value is equal to

$$h_m + A^2 \frac{\theta^2}{L^2} \cdot \frac{1}{h_m - h_j}. \qquad (15.26)$$

Generally speaking, the changes in the propagation constant, proportional $1/L^2$, will arise due to communication/connection with all waves, including even with the nonrunning waves. However, from formula (15.26) it is evident that most essential will be the effect of those waves whose propagation constant are close to h_m and for which will be low the coupling coefficient.

Page 130.

Virtually almost always, when a change of the effective constant of the propagation of wave in joining not is very small, for its computation it suffices to consider only interaction with wave H_{01}.

Formula (15.26) can be obtained also by other means. For this, it is necessary to examine the so-called its own waves of the twisted waveguide, i.e., the waves, which are propagated along this waveguide without distortion. They are defined as such solutions of systems (15.16), for which $P_m(z)$ and $P_l(z)$ they have identical dependence on z. Their own waves do not satisfy usual end conditions, and with incidence/drop from the side of the regular waveguide of wave H_{10} in joining will arise all their own waves.

The propagation constant of one of its own waves, as it is easy to find from (15.16), is equal to

$$h_m + \frac{h_m - h_l}{2}\{\sqrt{1 + 4A^2/q^2} - 1\}. \qquad (15.27)$$

Since this formula is obtained from the system of two equations (15.16), then it is also valid only on the assumption that it suffices to consider only communication/connection of two waves.

When $|q| \to \infty$, in the absence of communication/connection, value (15.27) approaches the propagation constant h_m of wave H_{10}, and the corresponding its own wave passes into this wave. When $|q| \gg 1$ is rubbed the difference between its own waves and waves, which appear in irregular section during incidence on it in the wave from regular waveguide, and propagation constant (15.27) passes in this case into effective propagation constant (15.26).

If the section of waveguide is close to square $|a-b|\ll a$, but length L is so/such great, which $|\sigma|\gg 1$, $|q|\gg 1$ is correct (15.26), then for an effective propagation constant is obtained the simple expression:

$$h_m\left(1+\frac{256}{\pi^4}\cdot\frac{a^2}{L^2}\cdot\frac{a}{a-b}\right).^{*} \qquad (15.28^{*})$$

FOOTNOTE [1]. In this expression passes with a-b<<a formula (45) of Levin's article [17], in which were determined propagation constant of their own waves far from degeneration conditions. Formula (15.27) is the generalization of Levin's formula [17(45)], valid with any q, if only it is possible to be restricted to interaction of two waves. ENDFOOTNOTE.

3. Let us examine now twisted slightly elliptical waveguide and communication/connection of two waves H_{11} polarized on larger and to minor axes of ellipse. We will be restricted again to helical joining for which pitch angle Φ is proportional to z (15.18). Amplitudes of both of waves in this waveguide are described by the same formulas (15.21), in which, according to (15.13), it is necessary now to assume $A_j=-1$.

Page 131.

At the end of the twisted section two waves of the type H_{11} with

mutually perpendicular polarization and some difference of phases form elliptically polarized wave. The total field at output is convenient to describe in the terms of the so-called ellipse of polarization. The determination of its cell/elements from amplitudes and phases of oscillation/vibrations in two mutually perpendicular directions which are given in (15.21), is conducted on usual optical formulas. The axes of this ellipse of polarization are equal to

$$|E_{\text{мин}}| = \sqrt{\frac{1-T}{2}}; \; |E_{\text{макс}}| = \sqrt{\frac{1+T}{2}};$$

$$T = \sqrt{1 - \frac{\theta^2\sigma^2}{\varkappa^4}\sin^4\varkappa}. \qquad (15.29)$$

The angle θ_0 of the rotation of the transverse of polarization relative to the direction of the polarization of the incident wave is found from the equation

$$\operatorname{tg} 2(\theta_0 - \theta) = -\frac{\varkappa \sin 2\varkappa}{\theta\left(\cos 2\varkappa + \frac{q^2}{4}\right)}. \qquad (15.30)$$

Another characteristic of the disturbance/perturbation, caused by joining, is the component of complete field at output in the direction, which composes a right angle with the direction of the polarization of the incident wave. With an accuracy to factor $e^{-i(h_m+h'_j)L/2}$ this value is equal to

$$B = -\frac{\theta}{\varkappa}\sin\varkappa\cos\theta + \sin\theta\cos\varkappa + i\frac{\delta}{2\varkappa}\sin\varkappa\sin\theta. \qquad (15.31)$$

Last/latter three formulas give the complete description of field at the output of irregular section and they make it possible to analyze thoroughly the disturbance/perturbation, caused by joining.

In Fig. 18 they are represented $|E_{мин}|$ (solid lines) and $|E_{макс}|$ (broken lines) in the function of complete pitch angle for several values σ, and in Fig. 19, value $|B|$.

With small $|\sigma|$ ($|\sigma|\ll 1$) the angle of rotation of the transverse of polarization θ_0 is small, order $\sigma^2|E_{макс}|\sim 1$, and $|E_{мин}|$ is in effect equal to $|B|$. Value B in this case is equal to

$$B=\frac{i\sigma}{2}\cdot\frac{\sin^2\theta}{\theta}. \qquad (15.32)$$

Page 132.

Under these conditions the incident field is disturbed very little, and since waveguide itself is also close to circular, then the value of the component of perpendicular polarization (15.32) can be obtained also by the simple methods of perturbation theory. For this, it is necessary the twisted waveguide to present as the sequence of the regular slightly elliptical waveguides of the lengths Δz, whose major axes are turned to angle $\Phi(z)$ relative to the direction of the polarization of the transmitted wave, and this direction and amplitude of the transmitted wave to consider constant. The value of the component of field in perpendicular direction, forming with the passage of each section of slightly elliptical

waveguide, is easy to find from usual optical formulas, decompose/expanding the incident wave to two waves, polarized along the axes of ellipse, and taking into account a difference in phase change of these waves (for example, see the article of Sandsmark [86]). The resulting value B the components of field in perpendicular direction at output is obtained by the addition of these small components, moreover due to condition $|\sigma| \ll 1$ it is not necessary to consider a supplementary phase difference during propagation from cell/element z to the end/lead of the section. In this way with any form of the function $\Phi(z)$ it is obtained

$$B = \frac{i(h_j - h_m)}{2} \int_0^L \sin 2\Phi \, dz. \qquad (15.33)$$

For helical joining (15.33) it passes in (15.21).

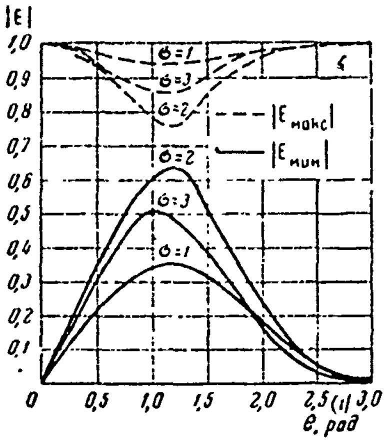


Fig. 18.

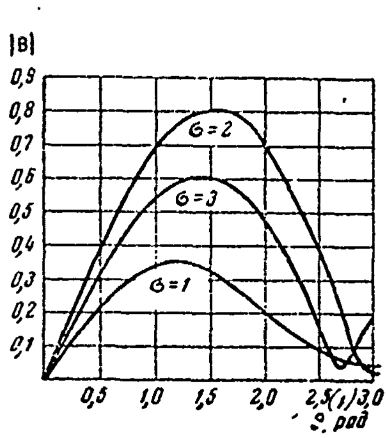


Fig. 19.

Fig. 18. Axes of the ellipse of polarization.

Key: (1). rad.

Fig. 19. Depolarisation at the end of joining.

Key: (1). rad.

Page 133.

Formula (15.33) can be used for statistical approach to the problem, when the direction of the axes of ellipse in each section is random function from z. Total depolarized effect of any irregular section is proportional, according to (15.33), the average on this section from $\sin 2\Phi(z)$.

According to (15.32), for a helical joining with that fix/recorded σ, $|\sigma|\ll 1$, value $|B|$ increases with growth θ to value of θ=67°, when it takes maximum value $0{,}42\,|\sigma|$. This value only for 15o/o is less than maximum value $|B|$ with given one σ, equal, according to (15.33), $0{,}5\,|\sigma|$. Maximum value is reached for regular elliptical waveguide whose axis is turned relative to the direction of polarization of incident wave on $\phi=\pi/4$.

At the finite (not small) value of total phase change σ, the character of field at the output of irregular section depends on parameter q. Generally, if q=0, then the isolation/liberation of two mutually perpendicularly polarized waves becomes conditional, total field in any section coincides with the field of the incident wave. With $|q|\ll 1$ (and finite value $|\sigma|$) the communication/connection between both waves is great, occurs noticeable energy transfer between them, almost completely compensating for the rotation of the plane of the polarization of these waves, so that the plane of the polarization of total field barely changes.

Only at finite (not small) value of both of parameters of problem σ and q the passing field strongly is agitated and formula (1533) and (15.32), obtained from the theory of the slight

disturbances, they prove to be inaccurate. At sizable $|\sigma|$ and finite values $|q|$ is especially noticeable the elliptical polarization of field at output. With growth $|q|$ decreases the communication/connection between two waves, caused by joining. When $|q|\gg 1$ the wave remains almost plane-polarized, its depolarisation is small

$$|E_{\text{мин}}/E_{\text{макс}}| = 2\sin^2\varkappa/|q| \ll 1. \qquad (15.34)$$

The plane of polarization is turned to angle, only by a little smaller than the angle of rotation θ of the axis of the section of waveguide. Finally, when $|q|\to\infty$ wave strictly follows the screw/propeller, just as in rectangular waveguide it is far from degeneration.

Let us note on conclusion of this paragraph, which on formulas (9.5) and (15.8) is easy to also find coupling coefficients between any two waves in slightly elliptical waveguide. For example, for two waves, analogous E_{11} to the waves in the waveguide of round section, polarized along the axes of the ellipse of section, coupling coefficient, as for two waves H_{11}, it is equal to Φ'. Therefore during incidence in wave E_{11} on the joining of slightly elliptical waveguide, will be fulfilled the same relationship/ratios, as in the examined above problem of an incidence in wave H_{11}.

Page 134.

In these two cases are distinct only relationship/ratios between a difference in the propagation constants $h_l - h_m$ and the amount of strain l/a. For two waves H_{11}, according to (15.12), it will be

$$h_l - h_m = 1{,}8 \frac{a_m}{h_m} \cdot \frac{l}{a^3}, \qquad (15.35)$$

while for two waves E_{11}, numerical coefficient in the same formula is equal to one.

§ 16. Transitions between circular ones or between rectangular waveguides of different ones section.

1. Let us examine education/formation of parasitic waves in irregular section of waveguide with changing round or rectangular cross section.

The amplitudes of parasitic waves are given by formulas (8.34), if this parasitic wave can be propagated in narrow waveguide, and by formulas (12.20), if we it are propagated cannot. If generatrix of waveguide has fractures at the end/leads of the irregular section, then for amplitude are obtained explicit expressions - binomial formulas (8.37) or (12.22). Coupling coefficients in general form are given in (9.5).

In this point/item are given expressions for the coupling coefficients S_{jm}, calculated according to formulas (9.5) for the waveguides of round and rectangular cross sections. As in (9.5), the first of the given below formulas (a) is related to the case when both of magnetic type waves, the second (b) - to the case when they electrical types both, and the third (c) - to the case when the wave m - magnetic type number, and the wave of number j - electrical. upper sign is related to direct waves, $h_j>0$, lower - to reverse/inverse ones, $h_j<0$. During the writing of these formulas, it was assumed that both waves in question are propagated without attenuation, i.e., that h_m and h_j they are real.

For circular waveguides

$$S_{jm}=\frac{\nu[\pm\mu_m^2\mu_j^2+(|h_j|h_m\mp k^2)n^2a^2]}{|h_j|(|h_j|\mp h_m)a^2\sqrt{\mu_j^2-n^2}\sqrt{\mu_m^2-n^2}}; \quad (16.1a)$$

$$S_{jm}=\frac{\nu(k^2\mp|h_j|h_m)}{|h_j|(|h_j|\mp h_m)a}; \quad (16.1b)$$

$$S_{jm}=\mp\frac{\nu kn}{|h_j|a\sqrt{\mu_m^2-n^2}}. \quad (16.1c)$$

Page 135.

According to (9.5), during incidence in any wave appear only

parasitic waves with the same first index n, which are determining the dependence of fields on azimutnal angle, that also in that falling they are water. With n=0 appear, it goes without saying, only waves of the same type, as falling. This fact is the illustration of general consideration, according to which under any law of a change in the radius of circular waveguide the symmetrical waves of electrical and magnetic types are propagated independently.

Coupling coefficients of any z proportional to ν - to slope tangent of generatrix with given z. However, expressions for the amplitudes P_i, by which we higher than referred, were valid only with small ν and with an accuracy down to the terms first-order in ν; applying in these expressions of the formula of this paragraph, it is possible therefore in them to consider it as ν equal to the angle between the forming and axis of waveguide.

For rectangular waveguides we will examine in order not to complicate recording, only such transitions in which both narrow walls in any section form to z-axis the identical angles $\mathrm{arctg}\,\nu_1$, but both wide-identical angles arctg ν_2. The value of narrow wall is equal to b, value of wide is equal to a, so that $\nu_1 = da/2dz$, $\nu_2 = db/2dz$.

In this transition the coupling coefficient of two waves is different from zero only when one of the indices of parasitic wave

coincides with corresponding index of the incident wave, and the second pair of indices has identical parity. In this case, the coupling coefficient with the amplitude of parasitic wave they are proportional ν_1, if in both waves coincides index q, which is determining the dependence of fields on the coordinate, parallel to narrow wall, i.e., the second index in (3.21); if coincide the first indices (n), then coefficient S_{jm} will be proportional ν_2. Therefore in given below formulas (16.2) it is necessary to retain either only first term/component/addend, proportional ν_1 (if $q_j=q_m$, and n_j+n_m - even number), or only second, proportional ν_2 (if $n_j=n_m$, and q_j+q_m - even number). Only coupling coefficients for backward waves contain both of angles. Thus, for instance, during incidence in wave $H_{10}(n_m=1, q_m=0)$ are formed waves $H_{30}, H_{50}, \ldots$ both of directions with the amplitudes, proportional ν_1, wave $H_{12}, H_{14}, \ldots$ $E_{12}, E_{14}, \ldots$ both of directions with the amplitudes, proportional ν_2, and the wave H_{10} reflected whose amplitude $\dot{P}_{-m}$ contains and ν_1 and ν_2.

Page 136.

Coupling coefficients are equal to

$$S_{jm} = \frac{N^j N^m}{2|h_j|(|h_j| \mp h_m)} \left\{ v_1 \left[\pm \alpha_j^2 \alpha_m^2 b \epsilon_m + (|h_j| h_m \mp k^2) \frac{\pi^2 m^2}{b} \right] + v_2 \left[\pm \alpha_j^2 \alpha_m^2 a \epsilon_n (|h_j| h_m \mp k^2) \frac{\pi^2 n^2}{a} \right] \right\}; \quad (16.2a)$$

$$S_{jm} = \frac{M^j M^m \pi^2 (k^2 \mp |h_j| h_m)}{2|h_j|(|h_j| \mp h_m)} \left(v_2 \frac{n_j n_m b}{a^2} + v_2 \frac{q_j q_m a}{b^2} \right); \quad (16.2b)$$

$$S_{jm} = \pm \frac{M^j N^m \pi^2 k}{2|h_j|} \left(v_1 \frac{q n_j}{a} + v_2 \frac{n q_j}{b} \right), \quad (16.2c)$$

where normalizing factors M and N are given in (3.23).

2. Let us examine in somewhat more detail question concerning reflection of fundamental wave H_{10} of rectangular waveguide from section with alternating/variable section. According to (8.34), the coefficient of reflection R is equal to

$$R = -\int_0^L S_{-mm} e^{-2i \int_0^z h_m dz} dz. \quad (16.3)$$

Coupling coefficient, accordingly (16.2a), can be recorded in the form

$$S_{-mm} = \frac{1}{2b} \frac{db}{dz} - \frac{\pi^2}{2a^3 h_m^2} \frac{da}{dz}. \quad (16.4)$$

It is easy to show that in this form the expression for S_{-mm} for wave H_{10} is correct during any change in the section during which is retained constant/invariable the direction of sides, in particular with the asymmetry of the section of relatively constant axis.

The coefficient of reflection of wave H_{10} from coupling of two rectangular waveguides with close sections, according to (16.4), are equal to

$$R = \frac{1}{2}\left(\frac{\Delta b}{b} - \frac{\Delta a}{a} \cdot \frac{\pi^2}{a^2 h_m^2}\right)^{*}. \qquad (16.5)$$

To formulas (16.3) and (16.5) it is possible to give the same form which have the formulas for a reflection coefficient in the heterogeneous long line, characterized by the wave impedance of W(z). As is known, in the same approach/approximation in which it is written (to 16.3), reflection coefficient from the section of long line with alternating/variable wave impedance is equal to

$$R = -\frac{1}{2}\int_0^L \frac{d \ln W}{dz} e^{-2i\int_0^z h_m dz} dz. \qquad (16.6)$$

Page 137.

Reflection coefficient from the connection of two lines wave impedance of which differ on ΔW, is equal to

$$R = \frac{\Delta W}{2W}. \qquad (16.7)$$

Formula (16.6) will pass into formulas (16.3) and (16.4), and simultaneously formula (16.7) will pass in (16.5), if we for the wave impedance of wave H_{10} in the waveguide of rectangular cross section accept the expression

$$W = C\frac{b}{h_m}, \qquad (16.8)$$

where C does not depend on a and b.

This expression is applicable, however, only during the computation of the coefficient of reflection. It, it goes without saying, cannot be applied, for example, to a question concerning the communication/connection between the current, the voltage and the energy flow, transferred in waveguide[1].

FOOTNOTE [1]. It is inadmissible also, as this sometimes is proposed (for example, see [87-90]), to use the usual in energy questions expression $W=Cb/ah$ for computation R concerning formulas (16.6) or

(16.7). Let us note that the existing experimental material [87-89] does not make it possible to determine dependence of W in (16.6-16.7) on a. ENDFOOTNOTE.

Let us note that expression (16.3) for the coefficient of reflection of any wave in any waveguide can be recorded in the form (16.6), after determining the wave impedance of W(z) by the relationship/ratio

$$\frac{d\ln W}{dz}=-\frac{1}{2}S_{-mm}. \qquad (16.9)$$

In this case, the reflection coefficient from coupling of two waveguides with close sections can be will be recorded in the form (16.7).

If generatrix of the irregular section of rectangular waveguide has a fracture, then integral expression (16.3) is reduced to a binomial formula of type (8.37). Let us extract first term of this formula

$$i\frac{S_{-mm}}{2h_m}, \qquad (16.10)$$

obliged to fracture in the beginning of transition. Sometimes in the literature this value is called "reflection coefficient from coupling". According to (16.4), this value is equal to

$$i\frac{1}{2h_m b}\nu_2-i\frac{\pi^2}{2h_m^2 a^3}\nu_1. \qquad (16.11)$$

Page 138.

First term in (16.11) describes the so-called "reflection from expansion in E-plane", the second - "reflection from expansion in the H-plane". Formula (16.11) was repeatedly obtained by different methods[1].

FOOTNOTE [1]. In Levin's book [2] is erroneously found the coefficient when ν_1 in (16.11); Pifke [3] indicated the reason for the error in [2]. ENDFOOTNOTE.

For a fundamental cable wave in flat/plane waveguide, the "reflection coefficient from expansion" (16.10) is equal, as it is easy to check, to first term in (16.11), in which it is necessary to replace h_m by k.

3. Let us give several examples of application/use of obtained in first point/item formulas for coupling coefficients. On this, to formulas it is possible to calculate the amplitudes of the scattered waves for any concrete/specific/actual system: the given below curve/graphs must only illustrate the general character of phenomenon.

Let us give first expression for phase factors $\gamma_1(z)$ in the transitions, in which generatrices are straight lines, i.e., ν does not depend on z.

For the round cone in which a radius of section changes according to the law $a(z)=a(0)+\nu z\,(\nu<0)$, simple computations give

$$\gamma_m(L)=C\{g_m(0)-\operatorname{arctg} g_m(0)-g_m(L)+\operatorname{arctg} g_m(L)\};$$
$$g_m(z)=\frac{h_m(z)}{\alpha_m(z)}. \qquad (16.12)$$

Here for magnetic waves coefficient $C=-\mu_m/\nu$, for electrical waves $C=-\nu_m/\nu$, where μ_m and ν_m - roots of Bessel function and it by the derivative (see §3). A special case of formula (16.12) it is (11.29); this formula is obtained from (16.12) when $h_m(L)=0$.

For rectangular waveguide with linear generator computation γ_m on (3.13), (3.22) in the general case (i.e. when $n_m\neq 0,\ q_m\neq 0,\ \nu_1\neq 0,\ \nu_2\neq 0$) it leads to elliptical integrals, and we carry it out will not. For wave H_{n0} h_m it does not depend on b, and it is correct (16.12) when $C=-\pi n/2\nu_1$; analogous result is valid for H_{0q}. If $\nu_1=0$ or $\nu_2=0$, for expansion only in one plane, is also correct - for all waves - (16.12) when $C=-\pi q/2\nu_2$ (when $\nu_1=0$) or $C=-\pi n/2\nu_1$ (when $\nu_2=0$).

Finally, formula (16.12) is accurate also for the transition in which all sections are similar to each other, i.e., a/b does not depend on z; in this case $C=-\pi\sqrt{n^2+a^2q^2/b^2}/2\nu_1$.

As the first example let us examine the transition from square waveguide (b=a) into rectangular with the ratio/relation of sides b/a=1/3.

Page 139.

Let us assume that changes only is indicated the side, $b(z)=b(0)+2\nu_2 z$, and wide side is constant, i.e., $\nu_1=0$ (Fig. 20). From the side of wide waveguide, falls wave H_{10}. Frequency lie/rests within the limits $\sqrt{5}\pi<ka<3\pi$. In this case, will arise waves E_{12} and H_{12}. Critical section for them at highest frequency, i.e., with $ka=3\pi$, lie/rests approximately at a distance of 3/5 from the beginning of wide waveguide; at the second end/lead of the range, with $ka=\sqrt{5}\pi$, it lie/rests at the very beginning of transition, and these waves are not formed. Wave amplitude H_{12}, which exits into wide waveguide, is calculated from (12.22) without integral term/component/addend. It is equal to

$$P_{-l}(0)=i\sqrt{\frac{2}{5}}\nu_2\left\{\frac{1}{a(h_m-h_l)}e^{-2i(\widetilde{\gamma}_m+\pi/4)}+\frac{1}{a(h_m+h_l)}\right\}_{z=0}. \tag{16.13}$$

Let us note that the coupling coefficient of wave H_{12} with H_{10} when $\nu_1=0$ does not contain in the denominator of wave number h_l wave H_{12} and does not go to infinity near the critical section where $h_l=0$. Therefore (16.13), in contrast to given below expression (16.14) for wave amplitude E_{12}, applicably with how conveniently small distance between the critical section and the

beginning of wide waveguide, i.e., with as small as desired $h_j(0)$. It goes without saying, other coupling coefficients are turned for this wave into infinity when $h_j \to 0$; in particular, $S_{-jj} \to \infty$, and entire examination §11 and 12 remains valid.

Wave amplitude E_{12} is equal to

$$P_{-j}(0) =$$
$$= -2i\sqrt{\frac{2}{5}}v_2\left\{\frac{k}{h_j}\left[\frac{1}{a(h_m - h_j)}e^{-2i(\widetilde{\gamma}_m - \pi/4)} + \frac{1}{a(h_m + h_j)}\right]\right\}_{z=0}. \tag{16.14}$$

This formula is valid, if $h_j(0)$ not is too small, otherwise it is not possible to disregard integral term/component/addend (12.23).

The module/moduli of amplitudes (16.13) and (16.14) when $v_2 = -0.1$ are represented in Fig. 21. The upper curve is related to wave E_{12}, its amplitude approximately two times of more than wave amplitude H_{12}.

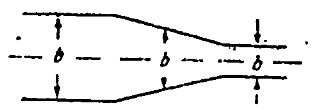
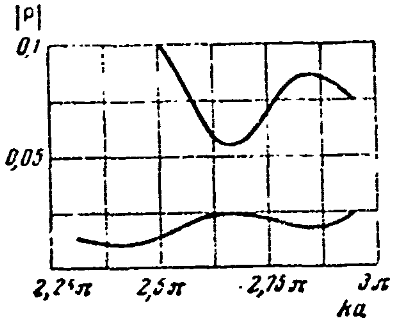


Fig. 20. Transition from wide waveguide to narrow.

Let us examine further the waveguide of circular section. The coefficient of reflection of wave H_{11} from direct/straight round cone $\rho = P_{-m}(0)$ is equal (in the absence of critical section for this wave)

$$\rho = \frac{i\nu}{4(\mu_m^2 - 1)} \left\{ \left[\frac{-\mu_m^4 + (k^2 + h_m^2) a^2}{h_m^3 a^3} \right]_{z=0} - \right.$$
$$\left. - \left[\frac{-\mu_m^4 + (k^2 + h_m^2) a^2}{h_m^3 a^3} \right]_{z=L} e^{-2i\gamma_m(L)} \right\}, \qquad (16.15)$$

where $\mu_m = 1{,}84$. Module/modulus ρ is represented in Fig. 22 when $\nu = -0{,}1$ in function ka (L) for two values of the ratio/relation of radii of waveguides. Curves have the oscillatory character, since, according to (16.15), ρ is equal to the sum of two interfering term/component/addends, that relate at the beginning and toward the end of the transition. However, the term, which corresponds to narrower end/lead, predominates, since in the denominator of formula (16.15) will cost the high degree of a radius. Of oscillations sufficiently rapid, since the phase of second term, is approximately equal to 2kL, rapidly it is changed with k.



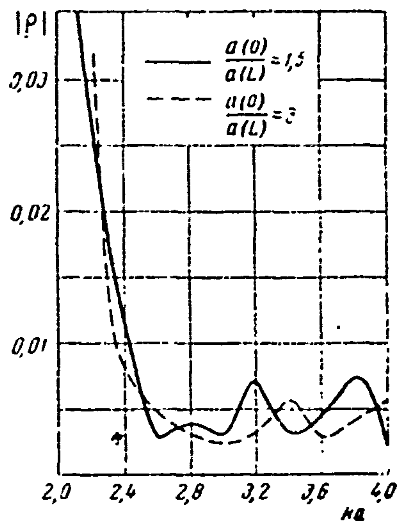


Fig. 21. Wave amplitude H_{12} and E_{12}.

Fig. 22. Coefficient of reflection of wave H_{11}.

Page 141.

The coefficient of reflection of wave H_{01} from any symmetrical transition in the absence of critical section can be calculated according to the integral formula

$$\rho = -\frac{1}{2}\int_0^L \frac{1}{h_m}\frac{dh_m}{dz} e^{-2i\int_0^z h_m dz} dz, \qquad (16.16)$$

of following from (8.34) and formula (9.4), in which for this wave is absent second term. For a cone, integrating (16.16) in parts and substituting the explicit value of wave number, we obtain after the

simple transformations

$$\rho = \frac{-i\nu\mu_m^2}{4}\left\{-\left(\frac{1}{h_m^3 a^3}\right)_{z=0} + \left(\frac{1}{h_m^3 a^3}\right)_{z=L} e^{-2i\Gamma_m(L)}\right\}, \quad (16.17)$$

where $\mu_m = 3{,}83$. The module/modulus of this value is represented in Fig. 23 as function ka(L) of value ka(L) =3.83 to ka(L)=5. The ratio/relation of radii of waveguides is accepted equal to two, a(0)/a(L)=2, the slope tangent of generatrix $\nu = -0{,}1$.

If product k•a(L), only a little exceeds μ_m, then formula (16.17), it goes without saying, not valid. In this case, it is necessary to use formulas (11.32) and (11.17). These formulas make it possible to trace change ρ with frequency from the low values, given by formula (16.16), or, which is the same thing, (11.31), to value $|\rho|$ =1. For wave H_{01}, parameter A in (11.18) is equal to $(-2\nu/\mu_m)^{1/3}$, the substitution of the value ν accepted gives A=0.23, and

$$-t_L = 3{,}74\,[k\cdot a(L) - \mu_m]. \quad (16.18)$$

The curves, computed on (11.32) (shaded line in Fig. 23) and on (16.17) (solid line) pass well into each other. Application/use of more complex formula (11.34) instead of (11.32) would give even better agreement.

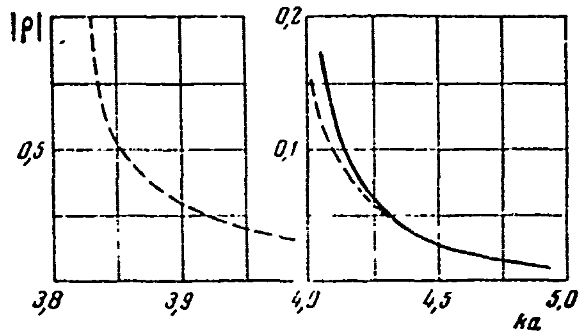


Fig. 23. Coefficient of reflection of wave H_{10}.

Page 142.

Fig. 24 is taken from Ii. B. Vaganova's article [91]. Curves give wave energy E_{11} in decibels with respect to energy of the incident wave H_{11} with the incidence in wave H_{11} on direct/straight round cone. Radii of two waveguides, connected by cone, are equal to with respect 1.5 and 2.5 cm. Upper curve is related to the case when $v=-0{,}05$ (length of cone it is equal to 20 cm), lower - to case $v=-0{,}11$ (length of cone it is equal to 9 cm). Curves are calculated according to formula (12.22) without integral term/component/addend. Points plotted/applied the experimental results: in the limits of measuring errors, they satisfactorily will agree with theoretical curves.

4. Formula (16.16) is valid also for determining coefficient of reflection of wave H_{01} from symmetrical dielectric transition, which

connects empty and that filled by dielectric transition connecting the parts of the circular waveguide which are not filled and folled with the dielectric. This follows from the comparison of expressions (8.17) for h'_m and (8.16), written for S_{-mm}. With constant magnetic permeability $\mu=1$, these expressions are characterized by common factor - $1/2h_m$ and sign, with which enters term/component/addend $(E_z^m)^2$. For symmetrical magnetic waves with symmetrical filling $E_z^m \equiv 0$, and therefore

$$S_{-mm} \approx -h'_m/2h_m. \qquad (16.19)$$

From this formula and common/general/total expression (8.34) follows expression (16.16) for a reflection coefficient from dielectric device.

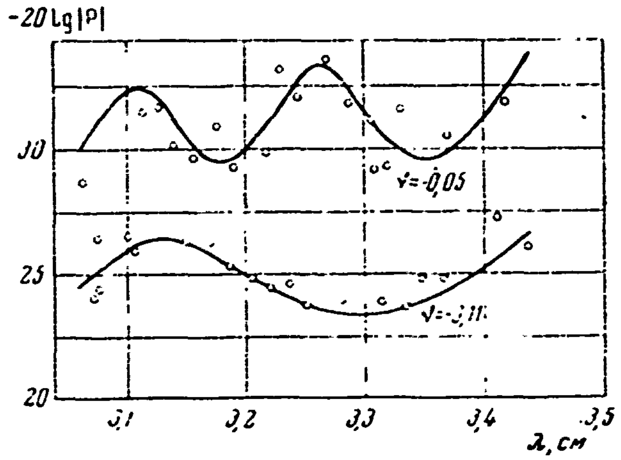
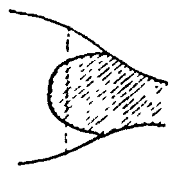


Fig. 24. Wave energy E_{11}.

Page 143.

The waveguide of comparison for the irregular waveguide of Fig. 25 is waveguide with the dielectric rod of Fig. 26. In the articles of Yu. N. Kazantseva [92] and Van Huan-Cjo [93] were determined the value h_m for wave H_{01} in this waveguide at the different values of a radius of rod b. According to obtained function $h_m(b)$ were calculated the coefficients of reflection ρ from the inserts of various forms; was investigated the dependence ρ on form and material of insert. Fig. 27 it is borrowed from article [92]. Curve/graph gives the frequency dependence of the coefficient of reflection of wave H_{01} from the polystyrene cone with a length of 10C mm in waveguide with a radius of 25 mm. By crosses are noted experimental results.

It should be noted that, although the angle of the slope of generatrix of cone on entire transition is constant, coupling coefficients for symmetrical magnetic waves vanish during approach/approximation to the foundation of cone. This follows from the fact that formula (8.22) contains for these waves only one to component of electric field E_s, which near metal is turned into zero. Therefore dielectric cone for waves H_{0q} is the smoothed transition i.e., the transition in which S_{lm} takes at end/leads zero value. Thus, to the problem, examine/considered in this point/item, formula (8.37) is not used.



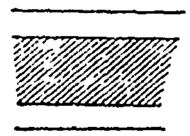


Fig. 25. Symmetrical dielectric transition.

Fig. 26. Waveguide of comparison to transition of Fig. 25.

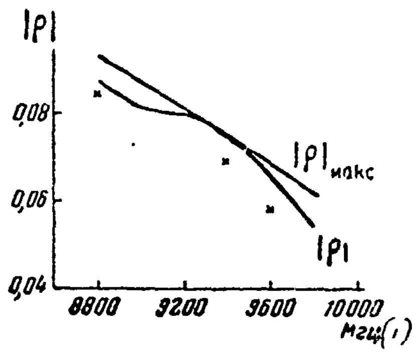


Fig. 27. Reflection coefficient from polystyrene cone.

Key: (1). MHz.

Page 144.

For other waves whose fields contain the normal to metal component of electric field E_n, this formula gives correct expression for the wave amplitudes, scattered on dielectric cone or by friend dielectric transition, since the values of coupling coefficients at the end/lead of the transition are different from zero. From (8.22) for these

coupling coefficients follows the simple expression

$$S_{jm}=\frac{1}{2h_j(h_j-h_m)}\frac{\varepsilon-1}{\varepsilon}\oint\limits_{c}\nu E_n^m E_n^j\,ds. \qquad (16.20)$$

In this formula enter the fields on the metal in waveguide, by pillar filled by dielectric, which easily are determined from the formulas, analogous (3.14).

§17. Smoothed transitions. Optimum transitions.

1. Losses to transformation into waves of parasitic types in waveguide transitions with fractures of generatrix, examined in first three point/items of preceding/previous paragraph, are frequently relatively they are great. For their decrease it is possible, generally speaking, to increase the length of transition. If on transition there are no points of steady state, then energy losses, according to (8.37), are proportional ν_0^2 and decrease with increase of L approximately inversely proportionally L^2. If be a point of steady state, then energy of parasitic waves decreases more slowly, approximately as 1/L. However, in a number of cases when it is necessary to ensure very low losses, this path, as we will see of the given below examples, is impossible: are necessary too great lengths. The second method consists of an improvement in the agreement of transient waveguide with regular waveguides. Without any calculations from (8.37), (12.22) and from the formulas of the preceding paragraph it is evident that the decrease of value ν at the end/leads of the

transition decreases the losses. Thus we come to the so-called smoothed transitions - transitions in which v at the end/leads of the transition it reaches zero or values, much smaller than value v at midpoints of transition. Strictly speaking, this property must possess coefficients of communication/connection, but if we do not consider a few exception/eliminations, analogous noted at the end of the preceding/previous paragraph, then the character of change S_{lm} is determined in by basic value v.

For such transitions or formula (8.37) and (12.22) it is already inapplicable, since during the derivation of these formulas it was assumed that the members of order v^2 and v' were small in comparison with $v(0)$ and $v(L)$, while for the smoothed transitions this does not occur. The calculation of the smoothed transitions must be performed on integral formulas (8.34) or (12.20).

Waveguide transitions must, as a rule, give low losses over a wide range of frequencies.

Page 145.

If this condition is not placed, then for the reduction of the amplitude of any wave it is possible to utilize an interference structure of the amplitude of the parasitic wave, which appears on

transition with the fractures of generatrix. For this, sufficient to select length L in such a way that both of term/component/addends in (8.37) would have on the assigned frequency opposite signs, and to establish between $\nu(0)$ and $\nu(L)$ this relationship/ratio, so that at this frequency these add/composed were equal in absolute value. For example, the coefficient of reflection of wave H_{01} from symmetrical transition between two circular waveguides will be, according to (16.17), equal to zero, if will be satisfied two conditions:

$$\gamma_m(L) = n\pi,\ n = 1,\ 2,\dots \left[\frac{\nu}{h_m^3 a^3}\right]_{z=0} = \left[\frac{\nu}{h_m^3 a^3}\right]_{z=L}. \quad (17.1)$$

According to the second condition, the angle of the fracture of generatrix must be at the wide end/lead of the transition considerably more than on narrow.

In order to ensure low losses to transformation into parasitic type direct waves, it is necessary, on the contrary, to make the angle of fracture at the wide end/lead of the transition is less than on narrow. Really/actually, to both end/leads must be, according to (8.37), are equal values $S_{jm}/(h_j - h_m)$, and since coupling coefficients have the order of ratio/relation ν/a, then $\nu(0)$ and $\nu(L)$ they must be related friend and to friend approximately as values $a(h_j - h_m) = \frac{1}{a}\frac{(a\varkappa_m)^2 - (a\varkappa_j)^2}{h_m + h_j}$, speaking in general terms, as 1/a. The difference in phase change of both of waves on entire transition, determined by formula (8.41), must be equal to π or to 2π, 3π and so

forth; if σ is small ($|\sigma|<\pi$), then compensation is impossible. as we will see below, the smccthed transitions, designed in such a way that the amplitudes of the forming direct/straight parasitic waves would be small over a wide range of frequencies, also we must be steadier from the side of wide waveguide, and value $|\sigma|$ for them must be sufficiently greatly.

Certain expansion of the frequency band can be reached, as showed Solimar [7, 94], during series connection of several cones with different aperture angles. Such systems approach, actually, the examined below smoothed transitions, in which the aperture angle is changed continuously.

It goes without saying, mutual compensation for both of term/component/addends in (8.37) or (12.22) indicates only that the amplitude of parasitic waves P_l will be equal to zero in the first order on v_0; in following order $P_l \neq 0$. In this case, it is necessary to keep in mind the consideration, noted in note on page 68.

2. During construction of waveguide transitions fundamental problem is selection of length and form of transition.

Page 146.

Analogous problem appears during the creation of the broadband matching sections between two long lines, for example between two coaxial lines with different wave impedance. This analogy we will use below during the determination of the form of waveguide transitions.

The coefficient of reflection ρ from non-uniform circuit with the alternating/variable wave impedance of W and constant phase rate is equal, according to (16.6),

$$\rho(\sigma) = -\frac{1}{2}\int_0^1 \frac{d\ln W}{d\zeta} e^{-i\sigma\zeta} d\zeta. \qquad (17.2)$$

Here W(ζ) - wave impedance, is alternating/variable of integration ζ=z/L, where L - length of transition;, in order to emphasize analogy with formulas for the amplitudes of the parasitic waves which we will record below in the same form, value 2kL is designated as zero σ. As we already noted into §16, formula (17.2) was valid in the same approach/approximation, in which were used expressions (8.34) and (12.20).

For equation (17.2) is investigated in detail inverse problem - problem of the determination of the optimum form of dependence W(ζ). In this setting function W(ζ) is determined from requirement so that beginning with certain value of the parameter σ $|\rho(\sigma)|$ there would be less than certain specific number $\rho_{\text{макс}}$, i.e., so that there would be

$$|\rho(\sigma)| < \rho_{\text{макс}} \text{ with } \sigma > \sigma_{\text{мин}}. \qquad (17.3).$$

Functions $W(\zeta)$ will be optimum, if for it $\rho_{макс}$ will be smallest, attainable with this $\sigma_{мин}$, or if $\sigma_{мин}$ will be smallest with that fix/recorded $\rho_{макс}$. Since σ is proportional L, then, selecting function $W(\zeta)$ optimum, we provide the smallest reflection coefficient of the smallest length.

Optimum in this sense function $W_{опт}(\zeta)$ was found in the article of Klopfenshteyn [95]. Between $\rho_{макс}$ and $\sigma_{мин}$ for $W_{опт}(\zeta)$ there is the single bond

$$\left|\frac{\rho_{макс}}{\frac{1}{2}\ln\frac{W(1)}{W(0)}}\right| = \frac{1}{\operatorname{ch}\frac{\sigma_{мин}}{2}}. \tag{17.4}$$

This function $W_{опт}(\zeta)$ we will use in the following chapter. In this paragraph during the determination of the form of generatrix of the waveguide transition, which gives sufficiently low losses to transformation, we accept other two functions $W(\zeta)$, proposed in article [96].

Page 147.

These functions solve optimum problem in the class of the functions, which have continuous derivative with $0\leq\zeta\leq1$ - in contrast to function $W_{опт}(\zeta)$, which has end/leads gaps. The waveguide transition, constructed

according to $W_{опт}(\zeta)$, would contain at end/leads the small steps.

The first of these functions corresponds $\sigma_{мин}=2{,}7\pi$. It is obtained from the equation

$$\frac{d\ln W(\zeta)}{d\zeta}=\sin\pi\zeta. \qquad (17.5)$$

the value of the left side of equality (17.4) for it is equal to 0.07, for $W_{опт}$ (on 17.4) this value is reached when $\sigma_{мин}=2{,}14\pi$, smaller by 20o/o. Figure 28a depicts the curve/graph of value

$$\left|\frac{\rho(\sigma)}{\frac{1}{2}\ln\frac{W(1)}{W(0)}}\right|=\left|\frac{\pi^2}{\pi^2-\sigma^2}\cos\frac{\sigma}{2}\right|, \qquad (17.6)$$

i.e. the standardized per unit value of integral (17.2), calculated according to this function W(ζ).

The second function W(ζ) corresponds only by a little to larger value $\sigma_{мин}$, $\sigma_{мин}=3\pi$, but the value of left side (17.4) for it is substantially less and it is equal to 0.03. For $W_{опт}(\zeta)$ this value is reached when $\sigma_{мин}$, which only to 11o/o is less. This function is found from the equation

$$\frac{d\ln W(\zeta)}{d\zeta}=(1-0{,}636\cos 2\pi\zeta). \qquad (17.7)$$

For this function

$$\left|\frac{\rho(\sigma)}{\frac{1}{2}\ln\frac{W(1)}{W(0)}}\right|=\left|\frac{\sin\frac{\sigma}{2}}{\frac{\sigma}{2}}\,\frac{4\pi^2-0{,}364\sigma^2}{4\pi^2-\sigma^2}\right|. \qquad (17.8)$$

The curve/graph of this value is represented in Fig. 28b.

3. Method of construction of smoothed waveguide transition, which gives low losses over a wide range of frequencies, we will dismantle/select based on example of symmetrical transition for wave H_{01} in circular waveguide. In this transition fundamental losses to the transformation are due to the formation of direct wave H_{02}. Let first the frequency be so high, that this wave can be propagated both in wide and in narrow waveguide, i.e., there is no critical section.

Page 148.

Its given amplitude $p_l(L)$ is given by formula (8.34). Substituting expression for a coupling coefficient (16.Ia), we will obtain

$$\sqrt{\frac{h_l(0)}{h_m(0)}}\, p_l(L) = -\frac{2\mu_m\mu_l}{\mu_l^2-\mu_m^2}\int_0^L \frac{h_m+h_l}{2\sqrt{h_m h_l}}\,\frac{a'}{a}\,e^{-i\int_0^z (h_m-h_l)\,dz}\,dz. \tag{17.9}$$

Here a(z) - the unknown function, which gives the form of generatrix. The square modulus of the value, which stands to the right, gives the ratio/relation the energy, taken away by wave H_{02}, to energy of the incident wave, i.e., loss to transformation into wave H_{02}.

Integral (17.9) it is possible to lead to the form, identical with (17.2), and to utilize for engineering the waveguide transitions

the mathematical apparatus, described in the preceding/previous point/item. Let us replace for this factor $(h_m+h_l)/2\sqrt{h_m h_l}$ with one; let us note that this superimposes on frequency somewhat weaker condition, than requirement $h_m\approx 1$, $h_l\approx 1$. Let us introduce the new variable

$$\zeta=\frac{\int_0^z (h_m-h_l)\,dz}{\int_0^L (h_m-h_l)\,dz}=\frac{1}{\sigma}\int_0^z (h_m-h_l)\,dz, \qquad (17.10)$$

then, substituting still $\mu_m=3{,}83$, $\mu_l=7{,}02$, we obtain the formula, which has the same form, that (17.2):

$$p_l(\sigma)=-1{,}56\int_0^1 \frac{d\ln a}{d\zeta}e^{-i\sigma\zeta}\,d\zeta. \qquad (17.11)$$

From comparison (17.11) with (17.2) it follows that if derivative of ln a(ζ), as the function in question by variable ζ, will be proportional to derivative of lnW(ζ), where W(ζ) is selected in such a way as to ensure the broadband agreement of two long lines with different wave impedance, then waveguide transition will give low losses to transformation into wave H_{02}.

For future reference it is convenient to introduce the parameter a_{cp} by the condition

$$a_{cp}^2=\int_0^1 \frac{h_m+h_l}{2}a^2(\zeta)\,d\zeta\cong\int_0^1 a^2(\zeta)\,d\zeta. \qquad (17.12)$$

Page 149.

This parameter does not virtually depend on frequency, has the dimensionality of length and lie/rests between a(0) and a(L). From (17.10) and (17.12) it follows that the parameter σ, which plays in the problem of waveguide transition the same role, as value 2kL in (17.2), it is connected with a_{cp} by the equation

$$\sigma = \frac{\mu_l^2 - \mu_m^2}{2} \frac{L}{k a_{cp}^2}, \qquad (17.13)$$

and differentials of variable ζ and z they are connected by the relationship/ratio

$$\frac{dz}{L} = \frac{h_m + h_l}{2} \cdot \frac{a^2}{a_{cp}^2} d\zeta \cong \frac{a^2}{a_{cp}^2} d\zeta. \qquad (17.14)$$

The determination of the unknown function a(z), that gives the airfoil/profile of transition, is conducted by the following diagram: are assigned values $\rho_{макс}$ the connected with it value $\sigma_{мин}$ and is selected corresponding to these values function W(ζ). Then it is found by a(ζ) from the condition that dlna/dζ is proportional dlnW/dζ. Appearing in this case two arbitrary constants - the proportionality factor between functions dlna/dζ and dlnW/dζ and

integration constant - are determined from requirement so that at the end/leads of the transition, with $\zeta=0$ and $\zeta=1$, a(ζ would take the assigned values. Then from formula (17.12) is calculated the parameter z_{cp} and on (17.14) is determined z/L as function of ζ. Thus, dependence of a on ratio z/L is obtained in parameter form.

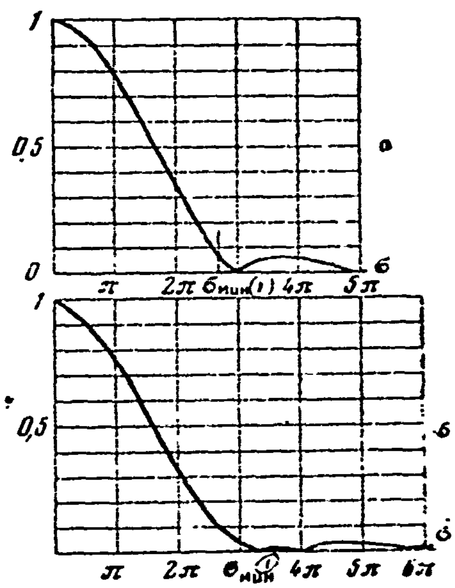


Fig. 28a, b - loss to conversion.

Key: (1). min.

Page 150.

The length of transition L is determined from requirement so that in entire assigned wavelength range the parameter σ would be the more than selected value $\sigma_{мин}$, i.e., from the equation

$$L = 0{,}73\,\sigma_{мин}\,\frac{a^2_{ср}}{\lambda_{мин}}\,, \qquad (17.15)$$

following from (17.13). Here $\lambda_{мин}$ corresponds to the short-wave

end/lead of the range.

The standardized/normalized amplitude of parasitic wave $|\rho_l(0)/1{,}56\ln q|$, where q is ratio/relation a(L)/a(0), is equal to the standardized/normalized coefficient of reflection

$$\left|\frac{\rho_l(\sigma)}{1{,}56\ln q}\right| = \left|\frac{\rho(\sigma)}{\frac{1}{2}\ln\frac{W(1)}{W(0)}}\right|, \qquad (17.16)$$

corresponding to the same function W(ζ). Finally, the parameter σ as the function of frequency is determined, according to (17.13) and (17.15), from the formula

$$\sigma = \sigma_{\text{мин}}\frac{\lambda}{\lambda_{\text{мин}}}. \qquad (17.17)$$

Let us note the essential difference between the problems, connected with integrals (17.2) and (17.11). In (17.2) the parameter σ, is inversely proportional λ. The action of the matching section of long line with alternating/variable wave impedance is limited from the side of long waves. Its length is selected from the condition

$$L = \frac{\sigma_{\text{мин}}}{4\pi}\cdot\lambda_{\text{макс}}. \qquad (17.18)$$

and despite all frequencies for which λ is less than the assigned value $\lambda_{\text{макс}}$ σ will be more than selected $\sigma_{\text{мин}}$, a $|\rho|$ - is less $|\rho_{\text{макс}}|$. For waveguide transition, on the contrary, σ directly proportional λ (17.15), the area of action of transition is limited from the side of short waves. In (17.15), enters the small assigned

value of wavelength; despite all the smaller frequencies of condition $\sigma > \sigma_{мин}$ and $|\rho| < \rho_{макс}$, they will be provided.

Difference this from a physical point of view is due so that in (17.2) is determined the amplitude of backward wave, σ is the sum of electrical lengths for direct/straight and backward wave; for each of these waves - and for their sum - electrical length increases with an increase in the frequency. In (17.11), is determined the amplitude of direct wave; σ - difference in the electrical lengths of two direct waves, and although h_m and h_l with frequency they increase, their difference decreases with frequency.

With an increase in the frequency, the phase speeds of all waves in waveguide converge with each other.

Page 151.

The amplitudes of parasitic type direct waves, which are formed on the irregular waveguide, in this case very high frequencies disappear the advantages of flat transitions in comparison with abrupt/steep ones. In mathematical sense this is developed in the fact that in the integrals of type (8.34) or of first term in (12.19) exponential factors is not provided sufficiently rapid sign change of integrand. These integrals will be in this case the order of product of v_0 by

L/a, i.e., will be finite quantities. It goes without saying, in this case, becomes already inapplicable all the approach/approximation of loose coupling.

4. Let us illustrate described calculation method, after selecting first function W(ζ) according to simple equation (17.5). Determining a(ζ) by diagram indicated above, we will obtain

$$a(\zeta)=\sqrt{a(0)\,a(L)}\,e^{-1/2\ln q\cdot\cos\pi\zeta}. \qquad (17.19)$$

Parameter a^2_{cp} is equal in this case

$$a^2_{cp}=a(0)\,a(L)\,I_0(\ln q), \qquad (17.20)$$

where I_0 - modified Bessel function. Dependence z/L on ζ is given in the integral form:

$$\frac{z}{L}=\frac{1}{I_0(\ln q)}\int\limits_0^{\zeta}e^{-\ln q\cos\pi\zeta}\,d\zeta. \qquad (17.21)$$

The length of transition, according to (17.15), must be not less than

$$L=3{,}1\,I_0(\ln q)\frac{a(0)\,a(L)}{\lambda_{\text{мин}}}. \qquad (17.22)$$

Calculating according to parametric dependence (17.19), (17.21) the derivatives da/da and d²a/dz², are easy to check that the obtained transition does not have fractures at end/lead and that

$$\frac{a''(0)}{a''(L)}=\frac{a^3(L)}{a^3(0)} \qquad (17.23)$$

i.e., that the slope of generatrix at narrow end/lead is considerably more than on wide.

Let us examine a numerical example.

Page 152.

Let radii of the coupled waveguides be equal to a(0)=24.4 mm and a(L)=9 mm and minimum wavelength in which must work the transition, it is equal to $\lambda_{мин}$=6 mm. The length of transition proves to be equal to L=144.5 to mm. Its airfoil/profile is represented on the appropriate scale in Fig. 29a. Wave amplitude H_{02} in the function λ is determined on Fig. 28a, or on formu 17.16) and (17.6). With $\lambda >> \lambda_{мин}$ the amplitude of parasitic wave p_j will be less than 0.11, which corresponds to losses to conversion less than - 19 dB.

Let us compare this smoothed transition with right cone (ν=const). For a cone with $\sigma >> 1$ integral (17.11) is equal to

$$|p_j(\sigma)| = 1{,}56\,\frac{1}{\sigma}\left|\frac{(1-q)^4}{q^2} + 4\,\frac{(1-q)^2}{q}\sin^2\frac{\sigma}{2}\right|^{1/2}. \qquad (17.24)$$

Let us replace $\sin^2\sigma/2$ in (17.24) consecutively per unit and for zero and will substitute explicit expression σ through L (17.13). Bearing

in mind that for cone a^2_{cp} =a(0)a(L), we will obtain for $\rho_j(\sigma)$ upper limit and from below, which to more conveniently use than by precise formula (17.24):

$$0{,}56\frac{[a(0)-a(L)]^2}{\lambda L}\leqslant|\rho_j(\sigma)|\leqslant 0{,}56\frac{|a^2(0)-a^2(L)|}{\lambda L}. \quad (17.25)$$

For the accepted in the preceding/previous example values of a(0), a(L) and λ_{MHH} condition $|\rho_j(\sigma)|$<0.11 will be reached at length L more than half meter; cone must be three times longer than the simplest smoothed transition.

Applying for the construction of transition function W(ζ), that satisfies somewhat more complex equation (17.7), we with the same values of the parameters of problem a(0), a(L) and λ_{MHH} will obtain transition with a length of 158 mm; at end/leads it will have small fractures. The airfoil/profile of this transition is represented in Fig. 29b. The losses upon this transfer, designed on formula (17.8), must be equal - 26 dB. The cone, which gives such losses, must have into length about meter.

The method of determining the optimum form of the smoothed matching waveguide presented assumes that the produced above transition from formula (17.9) to formula (17.11) does not lead to noticeable errors.

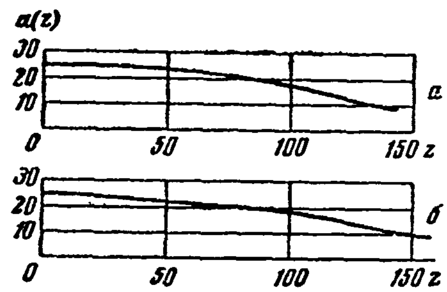


Fig. 29a, b - airfoil/profile of transition.

Page 153.

The degree of the legitimacy of this assumption is illustrated by the curve/graph of Fig. 30. On it is represented the standardized/normalized amplitude of the parasitic wave, which appears on the transition whose airfoil/profile is selected according to equation (17.7) at the value of the parameters: a(0) =9 mm, a(L)=30 mm, λ=6.6 mm. The length of this transition is equal to 193.7 mm, at end/leads generatrix composes angles by 5.4 and 1.6° with Z-axis. Plotted function, which gives the airfoil/profile of generatrix, is represented in Fig. 31. Solid line in Fig. 30 is computed on formula (17.8), it repeats the segment of a curve of Fig. 28b. Dash is found by the numerical integration of the system of two equations which are obtained during isolation/liberation in the system of equations (8.5) of two equations, which relate to

direct/straight and backward waves H_{02}. In each equation are preserved only addend, containing amplitudes of these two waves and wave amplitude H_{01}; the given wave amplitude H_{01} is considered constant/invariable. Thus, in this numerical calculation is taken into account also interaction of straight line and reverse/inverse will H_{02}, so that it gives results somewhat more precise, than integration for (17.9). The comparison of curves shows the effectiveness of the described above method of the selection of function a(z). Over a wide range, which reaches the wavelength with which in transition already appears critical section, losses will be only somewhat higher than on formula (17.8). However, detailed variation p_j on λ by this formula is not transmitted.

Dot-dash curve gives the amplitude of backward wave H_{02}. As it follows from common/general/total considerations, backward wave is very small.

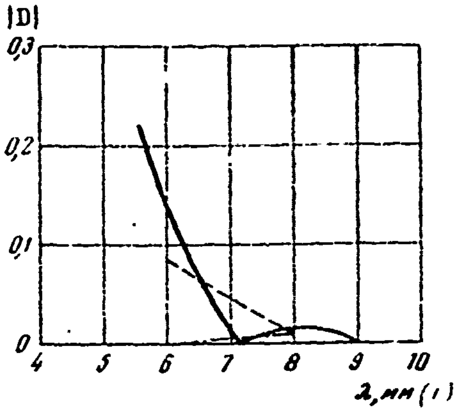


Fig. 30.

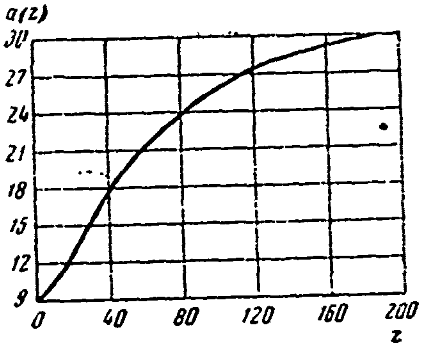


Fig. 31.

Fig. 30. Losses to conversion (to Fig. 31).

Key: (1). mm.

Fig. 31. Airfoil/profile of transition.

Page 154.

Let us note one auxiliary computational reception/procedure, used during numerical integration. It is connected with the fact that the end conditions for the amplitudes of direct/straight and backward waves (8.7) are placed at the different end/leads of the interval - such common/general/total property of system (8.5)-(8.7). End

conditions take the form (in variable ζ)

$$P_j(0) = 0;\ P_{-j}(1) = 0. \qquad (17.26)$$

Auxiliary reception/procedure lies in the fact that instead of the assigned heterogeneous system with end conditions (17.26) are examined other two systems with end conditions on one and the same end/lead of the interval:

$$P_j(1) = 1;\ P_{-j}(1) = 0. \qquad (17.27)$$

The first system coincides with given one, the second differs from it in terms of the absence of terms with a wave amplitude of H_{01}, i.e., is uniform. Solution of both of systems, i.e., the determination of functions $P_j^{(1)}(\zeta),\ P_{-j}^{(1)}(\zeta)$ and $P_j^{(2)}(\zeta),\ P_{-j}^{(2)}(\zeta)$ is two Cauchy problems, which, as is known, they are convenient for a machine calculation. The solutions of basic system are the linear combinations of solutions of both of Cauchy problems:

$$P_j = P_j^{(1)} - \frac{P_j^{(1)}(0)}{P_j^{(2)}(0)} P_j^{(2)};\ P_{-j} = P_{-j}^{(1)} - \frac{P_j^{(1)}(0)}{P_j^{(2)}(0)} P_{-j}^{(2)}. \qquad (17.28)$$

This auxiliary reception/procedure has the common/general/total value for the numerical solution of system (8.5), (8.7) or an analogous system for bent waveguide (7.32), (7.10).

5. Presented higher based on example of symmetrical magnetic wave in circular waveguide method of determining form of smoothed

transition can be used for waveguides of arbitrary section. According to results of § 9, on high frequencies, wher $\bar{h}_m \cong 1, \bar{h}_j \cong 1$ all coupling coefficients S_{jm} for direct waves do not depend on frequency and take form $C\nu/a$, where a - a characteristic size/dimension of section and C - constant. Formulas (8.34) for the amplitudes of direct waves at high frequency will acquire the following form:

$$p_j = C\int_0^L \frac{\nu}{a} e^{-i(\gamma_m - \gamma_j)} dz. \qquad (17.29)$$

Integral (17.29) also it is possible to lead to the form, identical with (17.2).

Let us introduce for this function f by the equation

$$\frac{df}{dz} = \frac{\nu}{a}. \qquad .0)$$

Page 155.

Then integral p_j will be

$$p_j = C\int_0^1 \frac{df}{d\zeta} e^{-i\sigma\zeta} d\zeta, \qquad (17.31)$$

where σ and ζ they have the same value, as in formula (17.10). The definition of function $f(\zeta)$ from the selected function $W(\zeta)$ is conducted in the same way as the determination of function lna in point/item 3. Knowing form of dependences $f(\alpha)$ and $f(\zeta)$, it is possible to find $a(\zeta)$. In order to find ratio z/L as function from ζ,

and then length of transition L, it is necessary to use the formulas

$$\frac{dz}{L}=\frac{1}{\int\limits_0^1 \frac{d\zeta}{\alpha_j^2-\alpha_m^2}}\frac{d\zeta}{\alpha_j^2-\alpha_m^2};\quad s=\frac{1}{\int\limits_0^1 \frac{d\zeta}{\alpha_j^2-\alpha_m^2}}\frac{L}{2k}, \quad (17.32)$$

being the obvious generalization of formulas (17.14), (17.13) and (17.12).

In comparison with the dismantle/selected above example of wave H_{02}, appears the supplementary difficulty, which consists of the need for determining function f. However, in certain cases this function is located by trivial shape. For example, for the joining, examined in § 15, f(z) is proportional to the angle of rotation of section F(z), and is easy to find the form of function F(z), that ensures (far from degeneration) low losses to conversion.

6. If in transition there is a critical section for parasitic wave, then form of generatrix, obtained by investigation of integral (17.11) or, in the general case, integral (17.29), can prove to be completely unsatisfactory. Let us return to the problem of wave H_{02} in symmetrical waveguide transition, and as illustration let us point out to the form of generatrix, presented in curved III Fig. 32. This form of airfoil/profile a(z) was found from method described above with the application/use of function, satisfying equation (17.7).

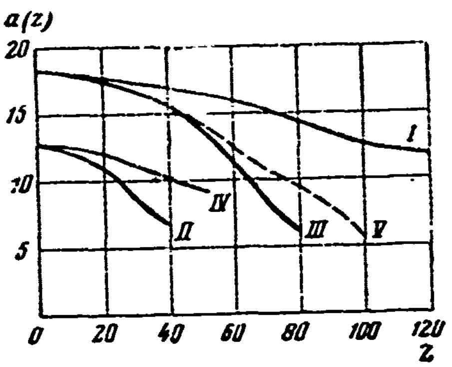


Fig. 32. Airfoil/profiles of transition.

Page 156.

Check computation, produced according to formula (12.19) for one point of range, gave inadmissibly high value (0.26) of the amplitude of parasitic wave. In other cases for the airfoil/profiles, presented in Fig. 32 by the curves I, II, IV, loss they proved to be small ($|P|<0.02$).

However it is possible thus to generalize method presented above of the determination of function a(z) so that it would give the satisfactory results also in the presence of critical section.

Of three addend formula (12.19) greatest is usually the first. With an accuracy to unessential factors, this term/component/addend

coincides with integral (17.9) - with the only difference that the the upper limit in it, which we will designate z_L, it is not equal to entire length of transition L, but somewhat less than the distance from wide end/lead to critical section. The replacement of integral (17.9) by integral (17.11) now is already not admitted, because factor $(h_m+h_l)/2\sqrt{h_m h_l}$ near critical section rapidly is changed.

The generalization of method lies in the fact that to form (17.2) is led not integral (17.11), but is direct integral (17.9), i.e., old term/component/addend in exact expression (12.19). For this, is introduced the new function F(a) by the equation

$$dF = \frac{da}{a}\frac{h_m+h_l}{2\sqrt{h_m h_l}}. \qquad (17.33)$$

With the introduction to this function and variable ζ (17.10) instead of z integral (17.9) it becomes identical to integral (17.2). Then it is assumed that F is proportional lnW(ζ), where W(ζ) satisfies, for example, equation (17.7). Proportionality factor is located just as in point/item 3. After determining the form of the function F(a) and F(ζ), it is possible in implicit form to find a(ζ). Dependence of z on ζ is found then from the equation, which generalizes (17.14):

$$\frac{dz}{z_L} = \frac{h_l+h_m}{2k}\frac{a^2(\zeta)\,d\zeta}{\int\limits_0^1 \frac{h_l+h_m}{2k}a^2(\zeta)\,d\zeta}. \qquad (17.34)$$

Thus is determined the length of transition before critical

section and the form of airfoil/profile by forming a(z). Dashed curve in Fig. 32 is the result of this construction. The amplitude of parasitic wave, calculated for this airfoil/profile by precise formula (12.19) taking into account all three term/component/addends, render/showed eight times less than for airfoil/profile III.

Page 157.

It goes without saying, and in the absence of critical section it is possible according to this method to investigate directly integral (17.9), however, apparently, sufficiently satisfactory results gives considerably more idle time construction, that begins from integral (17.11).

§ 18. Compensating inserts.

1. One of possible methods of decreasing distortion of field, caused by any irregularity, is introduction into waveguide of second irregularity, which must compensate for disturbance/perturbations, caused by first irregularity. 'tilizing that noted in § 10 additivity of coupling coefficients, it is possible in this way to attain the considerable decrease of the total coupling coefficient of the fundamental wave with most essential parasitic wave.

Let us examine this question based on the example of symmetrical waveguide transition for wave H_{01}, in which is introduced symmetrical dielectric insert, i.e., the dielectric insert/bushing, which possesses the symmetry of rotation and coaxial with waveguide. The total coupling coefficient, obliged to a change in the radii of both waveguide and insert, is calculated from formula (10.2). For waves H_{02} and H_{01}, it takes the form

$$S_{jm}=\frac{\pi}{h_j(h_j-h_m)}\left\{\frac{da}{dz}aH_z^j(a)H_z^m(a)+\frac{db}{dz}\cdot b(\varepsilon-1)E_\vartheta^j(b)E_\vartheta^m(b)\right\}, \tag{18.1}$$

where through a(z) is designated a radius of waveguide and through b(z) - a radius of insert, ε - dielectric constant of the material of insert. Formula (18.1) is obtained from (10.2) taking into account the fact that in the auxiliary waveguide which includes fields (18.1), for waves H_{0n} they are different from zero only components H_z, E_ϑ and H_r, and that integration on s in (10.2) is reduced to multiplication by the length of the duct/contour of cross section.

So that actions of both of heterogeneities average out, it is necessary that term/component/addends in (18.1) would be equal in magnitude and opposite on sign. Qualitative considerations about what form must for this have insert, can be obtained, examining the limiting case of very low value ε - 1. If ε - 1 << 1, then in (18.1) it is possible to substitute fields in the empty waveguide.

Page 158.

Expressing them on formulas (3.14a) and (3.18) through Bessel functions and them derivatives, we will obtain for a coupling coefficient

$$S_{jm} = -N^{j}N^{m}\frac{\pi\mu_m\mu_j}{h_j(h_j-h_m)a^2}\left\{-\frac{\mu_m\mu_j}{a}J_0(\mu_m)J_0(\mu_j)\frac{da}{dz}+\right.$$
$$\left.+(\varepsilon-1)k^2bJ_1\left(\mu_m\frac{b}{a}\right)J_1\left(\mu_j\frac{b}{a}\right)\frac{db}{dz}\right\}, \qquad (18.2)$$

where μ_m=3.83, μ_j=7.02. Factor with da/dz in (18.2) is positive. The sign of factor with db/dz coincides with the sign of value $J_1(\mu_m b/a)$, i.e., it is positive with $b<\mu_m/\mu_j\, a$=0.55a and is negative with b>0.55a. Thus, so that S_{jm} would be equal to zero, the derivatives db/dz and da/dz must have near the wall of waveguide one and the same sign, but near axis their signs must be opposite. This general character of insert is retained, probably, and at finite values ε-1. In this case, for determining the fields, entering formula (18.1), it is necessary to preliminarily solve transcendental equation and to determine wave numbers and fields of waves H_{01} and H_{02} in the regular waveguide of comparison (Fig. 26).

The analysis, produced in article [93], it showed, that, so that the coefficients of derivatives in (18.1) would have one and the same order of magnitude, product $kb\sqrt{\varepsilon-1}$ must be not very greatly, must be fulfilled the condition

$$kb\sqrt{\varepsilon-1}<2{,}4. \qquad (18.3)$$

At the high values of b or ε the field of wave H_{01} very strongly is concentrated near dielectric, currents on walls become very small, while in this case, it is difficult to attain mutual compensation for both of term/component/addends in formula (18.1).

Figure 33, borrowed from [93], schematically depicts the axial section of dielectric insert. The form of insert was found from the equation

$$S_{lm}=0, \qquad (18.4)$$

which under the assigned law a(z) is differential first-order equation for function in b(z).

Calculation is carried out for waveguide transition in the form of direct/straight round cone by the length 240 mm, connecting two waveguides in radii 6 and 30 mm with ε=2.55 and λ=8 mm. Let us note that frequency dependence of both of term/component/addends in (18.1) is somewhat different; therefore the insert, which ensures at one any frequency complete compensation, at other frequencies will not give a strict fulfillment of equation (18.4), although it will be, generally speaking, lead to the noticeable decrease of coupling coefficient.

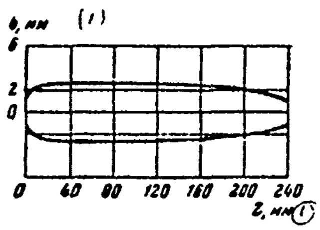


Fig. 33. Dielectric insert on [93].

Key: (1). mm.

Page 159.

The compensating dielectrically inserts for wave H_{02} possess one very intrinsic property. For symmetrical magnetic waves the coupling coefficients (10.2) for straight lines and for backward waves H_{02} become low values or are turned into zero simultaneously, since the curly brace in (18.1) enters by factor both in S_{jm}, and in S_{-jm}. In the presence of critical section, the amplitude of parasitic wave contains, according to (12.20) both S_{jm} and S_{-jm}. Therefore the compensating insert, in which is provided smallness S_{jm}, will also under these conditions provide a noticeable decrease in the amplitude of the appearing wave H_{02}. Under practical conditions the conversion of modes H_{01} in H_{02} is especially undesirable precisely in the presence of critical section for H_{02}, when can arise resonance spaces within the line of transmissions (for example, see the article of

King and Markatili [97]).

2. Application/use of matching transitions, described in preceding/previous paragraph, and introduction of compensating inserts is two different methods of decreasing losses to conversion. From a physical point of view, the difference lies in the fact that in the smoothed transitions is provided the mutual extinction of the parasitic waves, which appear in different sections, and in the compensated for transitions they extinguish each other of the waves, which appear on different irregularities in one and the same section.

The best results, which ensure agreement over a wide range of frequencies, which includes the frequencies, at which there are critical sections, they can be, probably, they are reached with combination of both of methods. The first attempt at the calculation of the corresponding insert is contained in the article van-Khuan'chzho [98]. In it are examined metallic inserts, so that entire system is coaxial waveguide with the alternating/variable diameters both of internal and external conductor. Coupling coefficient between waves H_{01} and H_{02} in this system, as it is easy to obtain, for example, from (10.2) or (16.1), is equal, it is analogous, (18.1)

$$S_{lm} = \frac{\pi}{h_l(h_l - h_m)}\left\{\frac{da}{dz} a H_z^l(a) H_z^m(a) + \frac{db}{dz} b H_z^l(b) H_z^m(b)\right\}, \quad (18.5)$$

where a(z) and b)z) - radii or external and internal conductors. The form of external conductor, i.e., function a(z), in [98] is considered given one, the form of internal conductor, i.e., function b(z), searches for from the condition so that the amplitude of parasitic wave (8.34) would be small.

Page 160.

For this, through the method of the preceding/previous paragraph is first located sufficient flat form of dependence of S_{jm} on z, and then from the corresponding differential equation is determined function b(z), this ensuring dependence $S_{jm}(z)$. Function $S_{jm}(z)$ is selected in such a way that near the critical section there would be $S_{jm}=0$.

The method of the construction of the compensated for transitions presented can be used to the row/series of other problems. As the compensating heterogeneous cell/element it is possible to utilize not only a dielectric or iron core, but also any rod with impedance, for example by that corrugated, by surface. Heterogeneity can consist either of a change in the radius of this rod or of a change in its surface impedance. In all cases it is substantial so that the supplementary heterogeneity would cause the communication/connection of the incident wave with the same parasitic

wave, as the fundamental heterogeneity, and did not cause the noticeable education/formation of other parasitic waves.

§ 19. Diffraction of plane wave on periodic surface.

1. There is identity between two electrodynamic problems, which relate - one to reflection and conversion of cable wave, which falls to the left to symmetrically becoming narrow end/lead of flat/plane waveguide of width d (Fig. 34), another - to reflection and scattering of plane wave of specific polarization, which falls normally to periodic (periodicity in one direction) metallic surface with the same airfoil/profile (Fig. 35). From the considerations of symmetry, it follows that on the horizontal planes, which exit to the left of the sharp apex/vertexes of the surface of Fig. 35, during diffraction do not appear the z-th of the component of electric field. The metallization of these surfaces will not agitate complete field in diffraction problem, which, thus, coincides with field in the waveguide of Fig. 34.

The solution of waveguide problem, i.e., the determination of the reflection amplitude and waves of the highest types, is in this case simultaneously the solution of diffraction problem. The amplitude of the cable wave reflected in waveguide is equal to reflection coefficient from periodic surface. The propagated

parasitic waves in waveguide will arise with $d>\lambda$; under this same condition appear the lateral diffraction spectra whose amplitudes are equal to the amplitudes of these waveguide waves. Thus, the diffraction problem, which corresponds to Fig. 35, also can be solved by cross sections.

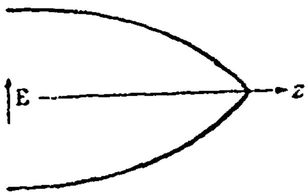


Fig. 34. Flat/plane waveguide.

Page 161.

We will examine in this paragraph the generalization of the method of cross sections, which makes it possible to use it for general problem of a normal incidence in the plane wave on periodic interface.

2. Let us begin how in chapter II, from dielectric medium with continuous distribution of dielectric constant $\varepsilon(x, y, z)$. For simplicity let us assume that ε - scalar value, generally speaking - composite, and that in entire space magnetic permeability is equal to unity. Let despite all z ε (x, y, z) be periodic function from x and y. Periods along the axes x and y, equal to a and b, are identical for all z, but the form of the function depends on z. With $z<0$ takes constant value ε_-, with $z>L$ $(L>0)$ - constant value ε_+. On transition layer falls to the left normally the wave (Fig. 36), it is

necessary to find field in entire space.

Let us introduce by analogy with the waveguides of comparison the medium of comparison, after defining it as the medium, in which dielectric constant ε is the same function from x and y, that also ε the transition layer with given z, and does not depend on the third coordinate ζ. In this medium of comparison, there is a system of its own waves of both of directions. The dependence of the fields of these waves on ζ is given by factor $e^{-ih_j\zeta}$. We will label their own waves by index j, which takes for direct waves positive value, and for reverse/inverse ones - negative. The fields of their own waves in the medium of comparison satisfy the equations of Maxwell and boundary boundary conditions of elementary rectangle from the side a and b.

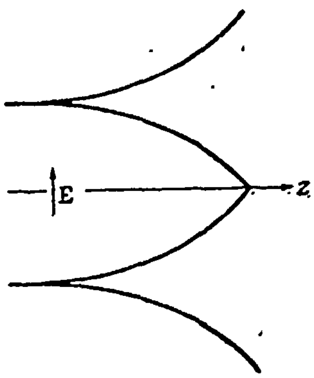


Fig. 35.

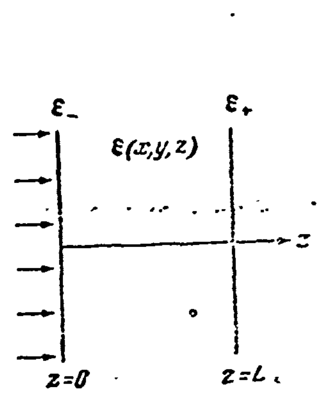


Fig. 36.

Fig. 35. Periodic metallic surface.

Fig. 36. Transient periodic layer.

Page 162.

These boundary conditions must ensure the periodicity of fields in the medium of comparison. They consist in the fact that on opposite sides of rectangle the tangential components must coincide; normal components in this case will be characterized by sign. Under such boundary conditions of the field of their own waves, they satisfy the same condition of orthogonality (3.7) as in waveguide. We

number them on (3.8).

With Z<0 and z>L auxiliary medium is uniform. The dependence of different components of the fields of its own waves on x and y is given by the functions

$$\frac{\sin}{\cos}\pi n\frac{x}{a}\cdot\frac{\sin}{\cos}\pi q\frac{y}{b}\quad, n, q = 0, 1\ldots \qquad (19.1)$$

In the transition region 0<z<L of the field of each wave, continuously they change with change z.

Let us decompose field at each point of transition layer in row/series on the fields of its own waves of the medium of comparison, which corresponds to given z. Let us designate the coefficients of expansion through $P_j^{\cdot}(z)$. They satisfy infinite system of equations (8.5), and coupling coefficients are given by the same formulas (8.6) or (8.16).

End conditions for system (8.5) are found from requirement so that with z=0 the field of the incident wave would coincide with field of one of the direct/straight its own waves, precisely, that wave whose fields do not depend on x and y. We will appropriate to it index j=1. End conditions coincide with (8.7) with m=1, they consist in the fact that with z=0 amplitude of all direct waves, except wave j=1, they are equal to zero, and $P_1(z)$ equal to the amplitude

incident wave. With z=L are equal to zero fields of all backward waves.

After solving system (8.5)-(8.7), we will find fields on planes z=0 and z=L. Simultaneously are determined fields in the uniform half-spaces z<0, z>L, since $P_{-1}(0)$ equal to reflection coefficient, $P_1(L)$ - coefficient of passage, and $P_j(0)$ with $j<-1$ and $P_j(L)$ with $j>1$ they give the amplitudes of the diffracted waves in left and right half-spaces.

Transition to the problem of diffraction on the periodic interface of two media with different values of dielectric constant ε_- and ε_+ (Fig. 37) is conducted in the same manner as for in § 8. In layer +0<z<L ε, it is piecewise constant function; considering it as the limit of continuous function, we we will again obtain for coupling coefficients expression (8.22).

Thus, three-dimensional diffraction problem is reduced to the computation of coupling coefficients and to system (8.5)-(8.7). If interface does not depend on one of the coordinates, then the determination of the fields of its own waves and coupling coefficients considerably is simplified.

Page 163.

As always in the method of cross sections, the solution of system (8.5) can be obtained in explicit integral form, if the parameters of medium are changed slowly. For a problem of diffraction on interface (Fig. 37) this means that the height of irregularities must be great in comparison with periods a and b. In this case, solution is given by formulas (8.34).

3. Let us examine now briefly problem of diffraction on periodic surface of metal. As in the theory of irregular waveguides, system of equations (8.5) remains valid upon transfer from dielectric to metal, and expressions for the coefficients of connection can be obtained by passage to the limit $|\varepsilon_{+}| \to \infty$; they are given by formulas (9.2) or (9.5). There is, however, one essential difference between problems, examined in § 9, and problem of diffraction. This is connected with the fact that the topological structure of the medium of comparison is changed by jump upon transfer through plane z=0. Therefore the fields of their own waves, generally speaking, are not continuous upon transfer through this plane. Formally the apparatus of the method of cross sections can be nevertheless it is used in entire space, if we consider that the coupling coefficients (8.6) are turned at point z=0 into infinity. It is simpler, however, to apply system of equations (8.5) only in region z>0 where all coupling coefficients

are final and where for them valid formulas § 9. End conditions (8.6), which assume the continuity of the fields of their own waves of z=0 and continuity of coefficients $P_j'(z)$, will be no longer used.

End conditions for (8.5) with z=+0 must be found from the auxiliary problem of diffraction on the boundary of half-space z>0, which regarding with all z>0 has the same structure, as the section of the periodic surface in question by plane z=+0. This auxiliary problem usually can be brought to the infinite system of algebraic equations. After finding from it the relationship between the amplitudes of various waves with z=+0 and z=-0, it is possible then to pass to system (8.5).

Thus, the application/use of a method of cross sections to the problem of diffraction on metallic surface is connected, generally speaking, with supplementary complexity.

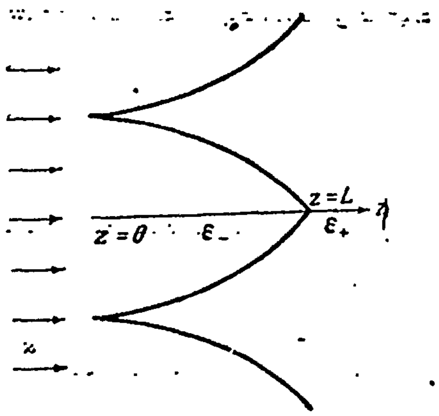


Fig. 37. Periodic dielectric surface.

Page 164.

It will not be only in the case, examined in the first point/item, since for this case corresponding auxiliary problem has the trivial solution; with that indicated in Fig. 36 polarization the field of the incident wave is not agitated by the system of planes of Fig. 38.

Upon transfer to metallic surface there are changed also end conditions with z=L. The condition, which ensures the absence of backward waves with z=L+0, will be replaced by the requirement of the finiteness of all amplitudes $P_j(z)$ with z=L. However, this end condition proves to be no more complicated than (8.7); it one should apply, it seems, also for closed waveguides of the type of Fig. 34.

In the theory of irregular waveguides, we also would encounter insufficiency of end conditions (8.7), torrents would examine the heterogeneities, changing the connection of sections. For solving such problems, for example, of the problem of metal cone in waveguide (Fig. 39), also one should first examine a question concerning coupling of two semi-infinite regular waveguides with the sections of different connectivity (Fig. 40) and only then apply system of equations (8.5).

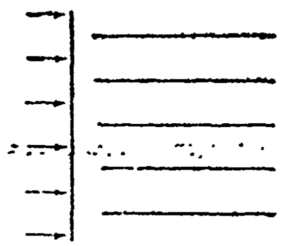


Fig. 38.

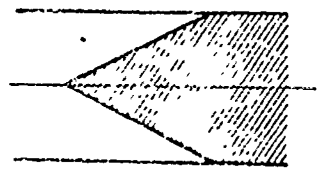


Fig. 39.

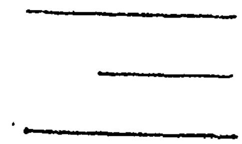


Fig. 40.

Fig. 38. System of parallel half-planes.

Fig. 39. Metallic cone in waveguide.

Fig. 40. Waveguide of comparison to Fig. 39.

[redacted].

Page 165.

Chapter V.

BENT WAVEGUIDES.

In this chapter we will use the method of cross sections to the calculation of the bent waveguides of rectangular and round sections.

§ 20. Bent waveguide of rectangular cross section.

1. During computation of coefficients of connection F_{lm} of two waves, we will be restricted to curvatures (discontinuities) whose axis is parallel to one of sides of section. For another location of axis, the determination F_{lm} from formulas (7.18) and (7.20-7.22) also is reduced to simple quadratures.

Fracture mutually connects only waves between indices of which are definite relationship/ratios. These relationships are analogous to those, with which are different from zero coupling coefficients

S_{jm}, those calculated in the first point/item § 16 for special type transitions. As it was noted in § 9, fracture is a special case of transition, it differs from the transition, examined in § 16, in terms of the direction of the angles, formed with Z-axis by the opposite walls of waveguide. Therefore the relationship/ratios between the indices of two waves with which F_{jm} not is equal to zero, differ from the appropriate relationship/ratios § 16 in terms of conditions, which relate to the parity of indices.

According to the condition, accepted in § 4 and 7, the axis is directed toward center of curvature, so that mutual location of cross section and axis of fracture (curvature) is given by Fig. 41 and 42. For Fig. 41 larger side of section is designated by letter a less - by letter b. In the case, which corresponds to Fig. 42, we will designate the length of the smaller side of section by letter a, the length of larger side - by letter b, so that in both cases will be preserved all designations of § 3. The dependence of fields on coordinate x is given by factors $\begin{smallmatrix}\sin\\ \cos\end{smallmatrix}\frac{\pi x}{a}n_m$ or $\begin{smallmatrix}\sin\\ \cos\end{smallmatrix}\frac{\pi x}{a}n_j$ for the waves of number m and j respectively; dependence on coordinate y is determined by factors $\begin{smallmatrix}\sin\\ \cos\end{smallmatrix}\frac{\pi y}{b}q_m$ or $\begin{smallmatrix}\sin\\ \cos\end{smallmatrix}\frac{\pi y}{b}q_j$.

Page 166.

Calculating integrals (7.18), it is easy to check that the necessary

conditions for difference F_{jm} from zero take the form

$$q_m = q_j, \quad n_m + n_j - \text{нечетное}^{(1)} \text{ число}. \qquad (20.1)$$

Key: (1). odd number.

In other words, fracture causes the appearance of waves, which have tighter dependence on the coordinate, parallel to the axis of fracture, that u'y the incident wave, and another parity of the index, which gives dependence on the coordinate, perpendicular to axis.

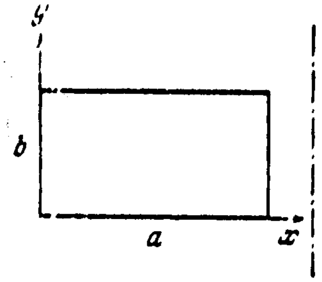
Computation on formulas (71.8), (7.22) and (3.21-3.23) is connected not with what complexities, and we will give only final results. Written below formula (20.2a) is related to the case when both of waves - magnetic type, formula (20.2b) - when they are electrical types both, and formula (20.2c) - to that case when m - magnetic wave, and j - electrical wave. Conditions (20.1) in all cases are considered carried out. We extract formulas in this form so as to emphasize that of the high frequencies when all given wave numbers h are close to unity, the coupling coefficients are proportional to the frequency

$$F_{jm} = -\frac{4i}{\pi^2} \cdot \frac{1}{\sqrt{\epsilon_{n_j}\epsilon_{n_m}}} \frac{h_m \alpha_j^2 n_m^2 + h_j \alpha_m^2 n_j^2}{h_j \alpha_j \alpha_m} \frac{h_j + h_m}{2} \frac{ka}{(n_j^2 - n_m^2)^2}; \qquad (20.2a)$$

$$F_{jm} = -\frac{4i}{\pi^2} \frac{h_m \alpha_j^2 + h_j \alpha_m^2}{h_j \alpha_j \alpha_m} \frac{h_j + h_m}{2} \frac{n_j \cdot n_m}{(n_j^2 - n_m^2)^2} ka; \qquad (20.2b)$$

$$F_{jm} = \frac{4i}{\sqrt{\epsilon_{n_m}}} \frac{h_j + h_m}{2h_j} \frac{q \cdot n_j}{n^2 - n_m^2} \frac{k}{\alpha_m \alpha_j b}. \qquad (20.2c)$$

Wave numbers α_m, α_j are determined by formula (3.22).



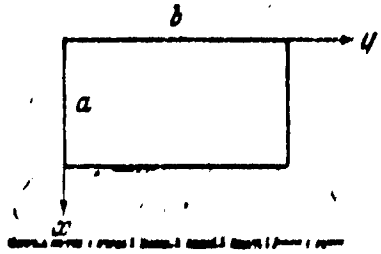


Fig. 41. Fracture in plane H.

Fig. 42. Fracture in plane E.

Page 167.

2. Let us use these general formulas to problem of incidence in fundamental wave on fracture or curvature.

With fracture in plane H (Fig. 41) main wave must be designated as H_{10}, so that $n_m = 1$, $q_m = 0$. Accordingly (20.2c), waves of the type E are not formed, will arise only waves H_{n0} where $n = n_l$ - even numbers. The coupling coefficient, computed in (20.2a), is equal to

$$F_{lm} = -\frac{8l}{\pi^2}\frac{n_l}{(n_l^2-1)^2}\frac{(h_m+h_l)^2}{4h_l}ka. \qquad (20.3)$$

Energy losses with fracture in plane H are determined virtually only by wave H_{20} $(n_l = 2)$. With $\pi \approx 1$ it takes away the energy, equal to approximately 0.032(ka) $^2\vartheta^2_0$, where ϑ_0 - angle of fracture in radians, and energy of the incident wave, as it is everywhere lower,

is accepted equal to unity. This comprises more than 99o/o entire energy, taken away by all H_{n0}-waves. Waves H_{40}, H_{60} and so forth one must take into account only near critical frequencies.

With curvature with large radius of curvature in plane H, the amplitude of appearing direct wave H_{20} is equal to

$$|P_j| = 1,2\cdot 10^{-2}\,\frac{(h_j+h_m)^3}{8h_j}(ka)^2\frac{a}{r}\left|\sin\frac{\sigma}{2}\right|. \qquad (20.4)$$

Here σ - difference in phase change of waves H_{10} and H_{20} at entire curvature. Counted off phase must be along the axis of waveguide, since for all waves value r_j, introduced by condition (7.25), is equal, as it is easy to check, to distance from the axis of curvature to the center of section.

If $|\sigma|$ not is small in comparison with unity and it is necessary to ensure low losses over a wide range of frequencies, then radius of curvature must be selected by such so that would be low value $1.4\cdot10^{-4}(ka)^4a^2/r^2$. If, for example, at the short-wave edge of range ka=12, then relative energy losses are equal to $3a^2/r^2$; in order to ensure losses are less - 30 dB, it is necessary to make the bending radius 55 times of more than the wide side a.

To fracture in plane E it corresponds to Fig. 42, and the designation of main wave will be H_{01}, so that $n_m=0$, $q_m=1$. According to (20.2), will arise the waves E_{nt} and H_{nt}, where $n\equiv n_j$ - odd number.

Coupling coefficients with waves E_{m1} are equal to

$$F_{jm} = \frac{2\sqrt{2}i}{\pi^2} \cdot \frac{h_j + h_m}{2h_j} \cdot \frac{1}{n_j\sqrt{1 + n_j^2 b^2/a^2}} kb. \qquad (20.5)$$

Coupling coefficients with waves H_{m1} are equal to

$$F_{jm} = -\frac{2\sqrt{2}i}{\pi^2} \cdot \frac{h_j + h_m}{2} \cdot \frac{1}{n_j^2\sqrt{1 + n_j^2 b^2/a^2}} ka. \qquad (20.6)$$

Page 168.

Let us recall that in two after their formulas b - the wide side of section and a - narrow side.

Far from the resonance frequencies of waves H_{31}, H_{51}, ..., E_{31}, E_{51} is sufficient to consider the energy, taken away by waves E_{11} and H_{11}. The ratio/relation the energy, taken away by wave E_{11}, to wave energy H_{11} is equal approximately to the square of the ratio of larger side to smaller. If the ratio/relation of the sides of the section is equal to two (b/a=2 in the designations of Fig. 42), then wave E_{11} takes away the energy, equal to approximately 0.017(k b) $^2\vartheta^2_0$, wave H_{11} - is approximately four times less. Total energy losses will be one and a half times less than with fracture in plane H to the same angle.

With curvature in plane E, the wave amplitude E_{11} is equal (with b/a=2)

$$|P_j| = 1{,}3 \cdot 10^{-2} \frac{(h_m + h_j)^2}{4h_j} (ka)^2 \frac{a}{r} \left|\sin\frac{\sigma}{2}\right|. \qquad (20.7)$$

According to (20.7) and (20.4), the losses to conversion and requirement for the value or bending radius at the fixed value ka for both of types of curvatures are very close.

Let us note on conclusion or this paragraph, that if the waveguide works in single-wave conditions/mode, i.e., frequency is not so great so that the parasitic waves could be propagated, then losses were due only to the reflected wave. Its amplitude is inversely proportional to the square of radius of curvature or (for a fracture) to the square of the angle of fracture. It can be found from the following diagram: from (7.27) are determined the amplitudes of the direct/straight and reverse/inverse parasitic waves, generated the transmitted main wave, and then from (7.24) - amplitude of the reverse/inverse main wave, generated these parasitic waves. For a reflection amplitude, is obtained in this case the expression in the form of row/series, however, since coupling coefficients rapidly decrease with an increase in the number of parasitic wave, then in computations will participate only several terms of row/series. This method, it goes without saying, is not limited only by rectangular waveguides and main wave.

§ 21. Bent circular waveguide. Wave H_{11}.

This paragraph is written in essence according to the results of N. P. Kerzhentseva's article [99].

Page 169.

1. Coupling coefficients of two waves in circular waveguide are nonzero only during specific ratios between indices of waves and between direction of their polarization. We will examine only such asymmetrical waves whose polarization, i.e., dependence on azimuthal angle β, is determined by factor $\sin n\beta$ or $\cos n\beta$ in expression for membrane/diaphragm function ψ or ϕ. Since waves of both of polarizations differently overcome fracture (or curvature), then during the propagation of wave in intermediate polarization, i.e., with factor $\cos n(\beta-\beta_0)$ in membrane/diaphragm function, it is necessary the field of this wave to present as the imposition of the fields of two waves of fundamental polarization and each term/component/addend to examine separately. Let us recall that the angle β is counted off from axis x, oriented to center of curvature, i.e., by that lying at the plane of bending.

The azimuthal indices of two waves, connected with fracture, must differ per unit, i.e., the necessary condition for difference

F_{lm} from zero is the equality

$$n_l = n_m \pm 1. \qquad (21.1)$$

When both of waves belong to one and the same type - magnetic or electrical, then F_{lm} is excellent from zero, only if membrane/diaphragm functions contain one and the same trigonometric function, i.e., both are proportional either to cos nβ or sin nβ. When one of the waves - magnetic type, and the second - electrical, then F_{lm} is excellent from zero, if membrane/diaphragm functions are proportional to different trigonometric functions. For the symmetrical waves n=0, and during the application/use of this rule it is necessary to consider that ψ or ϕ they contain factor cos nβ.

During satisfaction of two conditions indicated, superimposed to polarization and azimuthal dependence, coefficients F_{lm} are nonzero for any two waves. Exception/elimination is the combination of waves H_{0q_m} and E_{1q_l}, for which F_{lm} is excellent from zero only when $q_m = q_l$, and besides only for waves of one and the same direction.

The special character of this case is connected with the structure of given below formula (21.2c) and the fact that wave H_{0q} and E_{1q} degenerated.

Coupling coefficient for two magnetic waves is equal to

$$F_{lm} = -i\,\frac{h_m\mu_l^2(\mu_m^2 - n_l n_m) - h_l\mu_m^2(\mu_l^2 - n_l n_m)}{(\mu_l^2 - \mu_m^2)\sqrt{\mu_l^2 - n_l^2}\sqrt{\mu_m^2 - n_m^2}}\sqrt{\epsilon_{n_l}\epsilon_{n_m}} \times$$

$$\times \frac{h_l + h_m}{2h_l} ka. \qquad (21.2a)$$

Page 170.

For two electrical waves the coupling coefficient is equal to

$$F_{jm} = -i\frac{h_m \nu_j^2 + h_j \nu_m^2}{(\nu_j^2 - \nu_m^2)^2} \cdot \sqrt{\epsilon_{n_j}\epsilon_{n_m}} \cdot \frac{h_j + h_m}{2h_j} ka. \qquad (21.2b)$$

If wave m - is magnetic, and wave of number j - electrical, then

$$F_{jm} = -i\frac{n_m}{\sqrt{\mu_m^2 - n_m^2}\,(\mu_m^2 - \nu_j^2)}\sqrt{\epsilon_{n_j}}\frac{h_j + h_m}{2h_j} ka. \qquad (21.2c)$$

Last/latter formula is written for that case when

$$\psi^m \sim \cos n_m\beta, \ \varphi^j \sim \sin n_j\beta. \qquad (21.3)$$

But if

$$\psi^m \sim \sin n_m\beta, \ \varphi^j \sim \cos n_j\beta, \qquad (21.4)$$

then in (21.2c) it is necessary to change sign to reverse/inverse.

From (21.2c) it follows, in particular that wave $H_{0q}(n_m = 0)$ is not connected by fracture with E-waves. The special case they are waves E_{1q}, with the same value of q, for which $\nu_j = \mu_m$ and formula (21.2c) is not used. This case will be analyzed thoroughly in the following paragraph.

2. Let us examine in more detail wave H_{11}. Its membrane/diaphragm function takes the form

$$\psi^m = N^m \cdot J_1(\alpha_m \rho) \cos \beta \qquad (21.5)$$

or

$$\psi^m = N^m J_1(\alpha_m \rho) \sin \beta. \qquad (21.6)$$

Wave with membrane/diaphragm function (21.5) let us call/name the wave of the first polarization; its electric field in section is represented in Fig. 43. Wave with membrane/diaphragm function (21.6) let us call/name the wave of the second polarization; its field is given in Fig. 44. With incidence/drop on fracture, wave H_{11} of the first polarization causes the appearance of waves H_{0q}, of waves H_{2q} (for which $\psi' \sim \cos 2\beta$) and waves E_{2q} (for which $\psi' \sim \sin 2\beta$); wave H_{11} of the second polarization, correspondingly, it generates on the fracture of wave E_{0q}, $H_{2q}(\psi' \sim \sin 2\beta)$ and $E_{2q}(\varphi' \sim \cos 2\beta)$. However, coupling coefficients rapidly decrease with number q; therefore the large part of the energy of parasitic waves is taken away by waves H_{21} and H_{01} - for wave H_{11} of the first polarization and by waves E_{01} and H_{21} - for wave H_{11} of the second polarization. Table (21.1) gives energy losses (in o/o to that falling) on fracture in 1°.

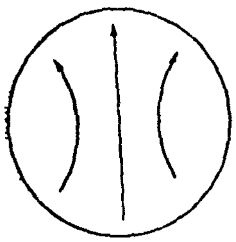
Page 171.

At the high frequencies when in formulas (21.2) it is possible to set/assume $h \approx 1$, the total energy losses of the wave of the first

polarization can be calculated according to the formula

$$0,14\,(ka)^2\,\vartheta_0^2 \qquad (21.7)$$

(ϑ_0 - in radians). Almost three fourths in this energy takes away wave H_{21}, about one fourth - wave H_{01}, to wave E_{21}, it is approximately 0.6o/o, remaining waves it is possible not to take into consideration.



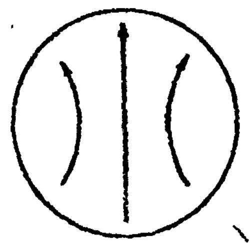


Fig. 43. Wave H_{11} of the first polarization.

Fig. 44. Wave H_{11} of second polarization.

Table 21.1. Losses of wave H_{11} on fracture in 1° (in o/o).

	l	ka									
		3	3,7	4,91	7,07	11,33	12,27	19,64	26,60	39,90	49,08
(1) 1-я поляризация	H_{21}	—	0,35	0,055	0,14	0,37	0,44	1,14	2,10	4,7	7,2
	H_{01}	—	—	0,015	0,04	0,12	0,15	0,39	0,73	1,65	2,5
	E_{21} (2)	—	—	—	0,001	0,003	0,004	0,01	0,02	0,04	C,06
	Всего . .	—	0,35	0,07	0,18	0,50	0,59	0,54	2,84	6,4	9,7
(1) 2-я поляризация	E_{01}	0,04	0,084	),10	),21	),54	0,64	1,63	3,01	6,7	10,2
	H_{21}	—	0,035	0,055	0,14	),37	0,44	1,14	2,10	4,7	7,2
	E_{21} (2)	—	—	—	0,001	0,003	0,004	0,01	0,02	0,04	0,06
	Всего . .	0,04	0,12	0,17	0,35	0,93	1,08	2,18	5,13	11,5	17,4

Key: (1). polarization. (2). Total

Page 172.

For the second polarization of energy loss on fracture, they are equal to

$$0{,}25\,(ka)^2\,\vartheta_0^2. \qquad (21.8)$$

Almost three fifth it falls to wave E_{01}, two fifth - to wave H_{21}.

3. With incidence/drop on bending of sick radius of loss to conversion for wave of first polarization, they prove to be approximately by an order lower than for wave of second polarization. Calculation according to formula (7.29) shows, for example, that so that in the broadband of the loss for the second polarization there would be less than 0.5 dB, bending radius must be, independent of angle of curvature, 50 times it is more than at least a radius of waveguide. For the second polarization it is sufficient so that there would be r/a>10.

If bending radius r is not very great, then amplitude of one of the parasitic waves can achieve the values of the order of one, while the amplitudes of all remaining parasitic waves will be still negligible. In this case, the given amplitude of the fundamental wave $p_m(\vartheta)$ will no longer be constant at entire bending, and formula (7.28-7.29) they will cease to be used. However, system (7.24) can be will be simplified, after preserving in it in the old system of two equations: for $p_m(\vartheta)$ and the given amplitude of the greatest parasitic wave $p_l(\vartheta)$. In this case, will be obtained the system of two

equations: after solving it, it will be possible to then establish/install the limits of the applicability of simpler solutions (7.28-7.29) and, if this is necessary, to calculate the amplitudes of remaining parasitic waves, retaining in the right side of each of equations (7.24) for these waves of two term/component/addends, which contain p_m and p_j. Analogous examination was by us carried out in § 15; there, however, us interested in essence the conditions, close to degeneration, which we in this paragraph be occupied will not be.

Let us illustrate this calculation, which is the refinement of the common/general/total calculation of the fourth point/item of § 7, based on the example of wave H_{11} of the second polarization. Main parasitic wave - the wave of number j - according to preceding/previous will be direct wave E_{01}. System of equations for $p_m(0)$ and $p_j(0)$ will be, according to (7.24),

$$\begin{aligned} p_m' &= F_{mj} p_j e^{-i(h_j - h_m)\vartheta}; \\ p_j' &= F_{jm} p_m e^{-i(h_m - h_j)\vartheta}. \end{aligned} \tag{21.9}$$

Since in this system are preserved only direct waves, then boundary conditions will be:

$$p_m(0) = 1; \quad p_j(0) = 0. \tag{21.10}$$

Page 173.

In order to record the solution in convenient for further analysis form, let us introduce critical angle $\vartheta_{\text{кр}}$, after determining by its formula

$$\vartheta_{\kappa p} = \frac{\pi}{r\,(h_m - h_j)}\,\frac{1}{W}; \quad W = \sqrt{1 - \frac{4F_{mj}\cdot F_{jm}}{r^2\,(h_m - h_j)^2}} \qquad (21.11)$$

The solution of system (21.9), (21.10) takes the form (sr of formula (15.21))

$$p_m = \left(\cos\frac{\pi\vartheta}{2\vartheta_{\kappa p}} - i\frac{1}{W}\sin\frac{\pi\vartheta}{2\vartheta_{\kappa p}}\right)e^{i\frac{1}{W}\frac{\pi\vartheta}{2\vartheta_{\kappa p}}}; \qquad (21.12a)$$

$$p_j = \frac{F_{jm}}{r\,(h_m - h_j)\,W/2}\sin\frac{\pi\vartheta}{2\vartheta_{\kappa p}}\,e^{-i\frac{1}{W}\frac{\pi\vartheta}{2\vartheta_{\kappa p}}}. \qquad (21.12b)$$

Formulas (21.12) are close, it goes without saying, to formulas (7.28), (7.29a) they pass in them with increase of r.

During change ϑ, occurs periodic energy transfer from the fundamental wave into parasitic and back; the period of this pumping is equal to $\vartheta_{\text{кр}}$. On (7.29a) the amplitude $|p_j|$ also periodically changes between the zero and maximum value. The period of this pumping differs from $\vartheta_{\text{кр}}$ (21.11) in terms of the absence of factor 1/W. Maximum value $|p_j|$, computed in formulas (7.29a) and (21.12b), also differs in terms of this factor.

The amplitude of the fundamental wave $|p_m|$, accordingly (21.12a), also periodically is changed from unity when $\vartheta = 0,\ 2\vartheta_{\text{кр}}$ and so forth to 1/W when $\vartheta = \vartheta_{\text{кр}},\ 3\vartheta_{\text{кр}}$ and so forth.

Thus, quantitative estimate/evaluation of error which appears with the disregard of the reverse/inverse action of parasitic wave on fundamental, serves the difference for factor W from unity. It is more precise, with $W-1<<1$ this reverse/inverse action causes a relative change in the maximum value of the amplitude of parasitic wave, it is equal

$$C=2\frac{|F_{lm}\cdot F_{ml}|}{r^2(h_m-h_l)^2}. \qquad (21.13)$$

This same to expression an equal relative change in the period of energy transfer.

Parameter C in the problem of bending makes approximately the same physical sense, that value $1/q^2$ for the twisted waveguide (§ 15).

Let us return to our an example - to waves H_{11} and E_{01}. For these waves value C is equal (with $\hbar\approx 1$)

$$C=0,2\frac{(ka)^4}{(r/a)^2}. \qquad (21.14)$$

Page 174.

If, for example, during the determination of the maximum power of wave E_{01} it is necessary to ensure accuracy into 10o/o, then formula (7.29a) it is possible to use with $r/a \geqslant 0.8\ (kA)^2$, but with

smaller r it is necessary to apply formula (21.12).

The higher the frequency, those more must be bending radius, so that it would be possible to set/assume C<<1 and to disregard the reverse/inverse action of parasitic wave on fundamental. This is explained to an increase in the coupling coefficients and by the approach of the propagation constants of different waves during an increase in the frequency.

With incidence/drop on the bending of wave H_{11} of the first polarization fundamental parasitic wave will be wave H_{21}. Parameter C (21.13) will differ from expression (21.14) in terms of the replacement of coefficient by 0.2 by 0.1.

It goes without saying, formulas (21.12) and condition C<<1, where C is determined by formula (21.13), they are used during the analysis of the waveguide bend of arbitrary section. For example, during incidence in wave H_{10} on the bending of rectangular waveguide in plane H the dominant role, according to (20.3), plays wave H_{20}, and parameter C for these waves is given by the same formula (21.14) (a - wide side of section), in which numerical coefficient is equal to $3 \cdot 10^{-4}$.

4. From formulas (7.28) $\rho_m(\vartheta)=1$, i.e. phase of wave is

determined only by factor $e^{-ih_m r\theta}$. Accordingly (21.12a), $p_m(\theta)$ has different from zero phases, and the phase of wave at output will not be equal to $h_m r \vartheta_0$. This can be treated as a change in the wave propogation constant of number m, connected with conversion into the parasitic wave of number j. With C<<1 this equivalent change in value h_m is equal, as it is easy to obtain from (21.12a),

$$\Delta h_m = -\frac{1}{2} C (h_m - h_j). \qquad (21.15)$$

This formula is analogous to formula (15.26) in the problem of the twisted waveguide of rectangular cross section far from degeneration conditions.

A change in the propagation constant is of the order $1/r^2$, and the value of supplementary phase there will be order 1/r.

In practical sense it is substantial, that the value of supplementary phase is different for waves H_{11} of both of polarizations. For the wave of the first polarization, supplementary phase is considerably less than for the wave of the second polarization. At high frequencies ($ħ \approx 1$) a phase difference of waves of both of polarizations is equal (in radians)

$$0{,}24 \frac{a}{r} \vartheta_0 (ka)^3. \qquad (21.16)$$

Page 174.

If wave H_{11} falls on bending in such a way that the plane of the

symmetry of wave is inclined toward the plane of bending at the angle, not equal to zero or $\pi/2$, then this difference in supplementary phases leads to the elliptical polarization of the transmitted wave H_{11}. This effect occurs also for a single-wave system, i.e., if ka<2.4 and wave E_{01} is not propagated. The analysis of formula (21.12a), in which in this case one should assume h_l pure imaginary ones, it shows that in single-wave system a phase difference is small and does not exceed several degrees. For example, with ka=2.2, r=10a and $\vartheta_0=\pi/2$ this phase difference is equal to 2.2°.

§ 22. Wave H_{01} in the bent circular waveguide.

1. Fracture connects wave H_{01} with waves H_{1q} of both of directions, polarized so that

$$\psi^l = N^l J_1(\alpha_l \rho)\cos\beta. \qquad (22.1)$$

Coupling coefficients are equal to

$$F_{lm} = i\frac{4\mu_l\mu_m}{(\mu_l^2-\mu_m^2)^2}\frac{(h_l+h_m)^2}{4h_l}\frac{1}{\sqrt{2(1-1/\mu_l^2)}}ka. \qquad (22.2)$$

Furthermore, it is different from zero coupling coefficients of wave H_{01} with wave E_{11} of the same direction, polarized so that

$$\varphi^l = M^l J_1(\alpha_l\rho)\cdot\sin\beta. \qquad (22.3)$$

Coupling coefficient is equal to

$$F_{lm} = -i\frac{ka}{\sqrt{2}\,\mu_l}. \qquad (22.4)$$

Coupling coefficient with wave E_{11}, for which $\varphi^l\sim\cos\beta$, is equal to

zero.

For a coaxial waveguide analogous formulas obtained O. Sh. Shushpanov [100]. The introduction of internal wiring does not remove/take degeneration between waves H_{01} and E_{11}.

Difference from zero coupling coefficients of wave H_{01} with wave E_{11} (and generally H_{0q} from E_{1q}) the same direction can be explained, relying on the considerations, given at the end of § 9, by the fact that these waves degenerated, i.e., possess identical phase speed. According to these considerations, the conversion on a small fracture is the imposition of two effects: conversion on a small step of special form (for displacement) and of addition of the parasitic waves, which were being formed along semi-infinite waveguide on such steps.

Page 176.

Since coupling coefficient S_{m} between the wave H_{0}, and all E-waves is equal to zero, then the amplitudes of E-waves, which arose on such steps, are very small (they are proportional to the square of the height/altitude of step) and their total amplitude also is very small. However, wave E_{11} possesses the same phase speed, as H_{01}, and all parasitic E_{11},- waves, which are formed on such steps, store/add

up in phase. Their total amplitude proves to be proportional to the angle of fracture, and this indicates that F_{jm} for waves H_{01} and E_{11} is different from zero.

2. Energy, scattered at the bend in the form of any H_{1q}- wave, in essence departs in forward direction. The ratio/relation to energy of backward wave to energy by straight line

$$\left(\frac{h_m - h_j}{h_m + h_j}\right)^4 \qquad (22.5)$$

always less than unity approaches unity only near the critical frequency of wave H_{1q} - special case, in detail investigated in § 13. The relative energy, taken away by direct/straight and reverse/inverse H_{1q}-waves, is equal to

$$\frac{8\mu_m^2\mu_j^4}{(\mu_j^2-\mu_m^2)^4(\mu_j^2-1)} \cdot \frac{(h_m+h_j)^4+(h_m-h_j)^4}{16h_m h_j}(ka)^2\vartheta_0^2. \qquad (22.6)$$

The energy, taken away by wave E_{11}, is equal to

$$\frac{1}{2\mu_m^2}\cdot(ka)^2\vartheta_0^2. \qquad (22.7)$$

In order to find entire lost energy, it is necessary (to 22.6) to sum up on all propagated waves H_{1q} and to add (22.7). Expression (22.7) is proportional to the square of parameter ka, and in (22.6), frequency enters, furthermore, in h_m and h_j, and with an increase in the frequency increases a number of members of type (22.6). During the approach/approximation of operating frequency to critical one of the waves of the type H_{1q} the energy, taken away by these waves, increases, and in immediate proximity to the critical frequency of

wave amplitude H_{1q} they become comparable with the amplitude of the incident wave (§ 13). Far from critical frequency, with $\hbar\approx 1$, total loss energies will be equal to $0.16(ka)^2\vartheta^2_0$ (ϑ_0 in radians). Three fifths of this energy takes away wave H_{12}, approximately on one fifth - waves E_{11} and H_{11}.

Given below Table 22.1 gives the values of relative losses on fracture in 1°, calculated according to (22.6) and (22.7). In row $H_{1q}(q=1,2,3)$ are given the energies, taken away by both H_{1q}- waves.

Page 177.

In brackets are shown the values which were given in Miller's article [101] [1].

FOOTNOTE [1]. This article contains no formulas according to the theory of fracture, in it are given only indicated numerical results. ENDFOOTNOTE.

Table 21.2 is borrowed from the work of M. V. Persikov, Yu. N. Kazantsev and A. I. Kozelev [102]. In the table are compared the measured and calculated according to the given formulas values of the ratio/relation to energy of the parasitic waves H_{11} and E_{11}, scattered forward, to energy of the incident wave H_{01} with a=25 mm,

λ=3.2 cm and by ϑ_0=2.3°. Results are given in decibels; measurements completely confirmed calculations.

Thus, great energy content is lost in the form of wave H_{12}. If, however, the electrical length of fracture increases (for example it increases r with constant ϑ_0), then the amplitudes of all waves decrease, according to (7.29), and wave amplitude E_{11} remains constant/invariable. Therefore with large bending radii with small ϑ_0 of loss in essence, they are connected with wave development E_{11}. An increase in the bending radius does not decrease amplitude by that appearing they are water E_{11}. Is especially essential this effect for the bendings on final angle ϑ_0 to examination of which we pass.

Table 22.1. Losses to conversion on fracture in 1° (in o/o).

i	ka								
	4,91	7,07	11,33	12,27	12,77	19,64	26,60	39,90	[illegible]
E_{11}	0,025	0,032	0,133	0,156	0,169	0,400	0,734	1,65	2,50
H_{11}	0,016	0,043	0,126	0,150	0,163	0,398	0,739	1,67	2,55
H_{12}	—	0,084	0,316 (0,34)	0,381	0,417 (0,41)	1,075	2,03 (2,0)	4,64 (4,8)	7,06
H_{13}	—	—	0,002	0,002	0,003	0,008	0,015	0,036	0,055
(1) Всего . . .	0,04	0,19	0,58	0,69	0,75	1,88	3,51	8,00	12,2

Key: (1). In all.

Table 22.2. Scattering on fracture.

H_{11}		E_{11}	
(1) Измеренное	(2) Расчетное	(1) Измеренное	(2) Расчетное
—28,2	—28,6	—28,3	—28,9

Key: (1). Measured. (2). Calculated.

Page 178.

3. Let r>>a, ϑ_0 - finite quantity; in practice greatest interest be of bendings on 90°. The cophasality of elementary waves E_{11}, which are formed in different sections of bending, leads to the fact that the wave amplitude E_{11} reaches finite values. As in § 15, for the investigation of the phenomenon of degeneration it is necessary to solve the system of two equations from common/general/total system

(7.24) - for amplitudes of direct waves $H_{01}(p_m(\vartheta))$ and $E_{11}(p_l(\vartheta))$. This system takes the form

$$p_m' = Fp_l; \quad p_l' = Fp_m. \qquad (22.8)$$

Here F - coupling coefficient (22.4) between waves H_{01} and E_{11}, moreover $F_{lm} = F_{ml}$, since $h_m = h_l$.

The solution of system (22.8) under obvious boundary conditions $p_m(0) = 1$, $p_l(0) = 0$ takes the form

$$p_m = \cos\frac{\pi\vartheta}{2\vartheta_c}; \qquad (22.9a)$$

$$p_l = -i\sin\frac{\pi\vartheta}{2\vartheta_c}. \qquad (22.9b)$$

Introduced here value

$$\vartheta_c = -i\frac{\pi}{2F} \qquad (22.10)$$

we call the critical angle of Jouguet. Angle ϑ_{cr}, introduced in the preceding/previous paragraph by condition (21.11), can be considered as generalization (22.10); (21.11) it passes in (22.10) when $h_l = h_m$.

According to (22.4)

$$\vartheta_c = \frac{\pi\mu_m}{\sqrt{2ka}}\ \text{рад}^{(1)} = 77{,}5\frac{\lambda}{a}\ \text{град}^{(2)}. \qquad (22.11)$$

Key: (1). rad. (2). deg.

Energy consistently passes from wave H_{01} to E_{11} and vice versa, complete pumping occurs when $\vartheta = \vartheta_c$. This result was for the first time obtained by Jouguet [16].

Thus, with final ones ϑ wave amplitude E_{11} stops the order of one, and wave amplitude H_{01} can noticeably differ from one. Therefore during the determination of the amplitudes of the parasitic waves, which appear together with wave E_{11}, at flat bending with final ϑ_0, it is necessary in right side (7.24) to substitute solution (22.9). The amplitude of any wave of index s is determined in this case of the equation

$$p_s' = F_{sm}\cos\frac{\pi\vartheta}{2\vartheta_c}e^{-i(h_m-h_s)r\vartheta}-iF_{sl}\sin\frac{\pi\vartheta}{2\vartheta_c}e^{-i(h_l-h_s)r\vartheta}. \quad (22.12)$$

Page 179.

Determination p_s is reduced to elementary quadratures. Already in the first order on 1/r will arise the parasitic waves with n≠1, obliged by communication/connection with wave E_{11}, for which it is different from zero coefficients F_{sl}.

4. Jouguet's result (22.9) was generalized by Viktorova and Sveshnikov [21], that examined incidence in wave H_{01} on three-dimensional/space bent circular waveguide. The axis of waveguide forms the helix, characterized by radius of curvature r and by twisting $\varkappa$. In [21] it is shown, that the wave amplitude H_{01} in this waveguide changes according to the law

$$p_m(\vartheta) = \frac{\varkappa^2 r^2}{\varkappa^2 r^2 + \frac{\pi^2}{4\theta_c^2}} + \frac{\frac{\pi^2}{4\theta_c^2}}{\varkappa^2 r^2 + \frac{\pi^2}{4\theta_c^2}} \cos\sqrt{\varkappa^2 r^2 + \frac{\pi^2}{4\theta_c^2}}\,\vartheta, \quad (22.13)$$

which is the generalization of formula (22.9a), valid for a flat/plane bending.

According to (22.13), for a nonplanar bending to any angle ϑ_0 it is possible (at one frequency) to ensure the absence of considerable conversion of modes H_{01} into E_{11}, after selecting twisting $\varkappa$ axis in such a way, that would be fulfilled the relationship/ratio

$$\sqrt{\varkappa^2 r^2 + \frac{\pi^2}{4\theta_c^2}}\,\vartheta_0 = 2\pi\cdot n \quad (n = 1, 2\ldots). \quad (22.14)$$

Formula (22.13) it is easy to obtain by overall diagram 7. Let us examine for this simultaneously three interacting with each other waves: wave H_{01} and two perpendicularly polarized waves E_{11}; the directions of the polarization of these waves let us consider the constants, for example horizontal and vertical. In that introduced in § 7 coordinate system, x axis is oriented toward center of curvature. In this coordinate system, the membrane/diaphragm functions of these two E_{11}- waves are proportional respectively

$$\sin(\beta - \beta_0) \text{ and } \cos(\beta - \beta_0), \quad (22.15)$$

where β_0 - angle in cross-sectional flow, composed by the direction of x axis with motionless direction, for example with the direction

of this axis in the beginning of bending. Angle β_0 is proportional to the twisting

$$\beta_0 = \varkappa r \vartheta. \qquad (22.16)$$

Page 180.

Let us designate the given wave amplitudes H_{01} and of two waves E_{11} with the angular dependence indicated respectively p_m, p_q and The coupling coefficient of the waves of numbers q and s with wave H_{01} they are proportional to integral of the product of trigonometric factor (22.15) on sinβ and they are equal to respectively

$$F \cos\beta_0 \text{ и } F \sin\beta_0, \qquad (22.17)$$

where F - a coupling coefficient (22.4) of wave H_{01} with wave E_1 (22.3), whose membrane/diaphragm function is proportional sinβ. Both of waves E_{11} are not directly connected; the wave numbers of all three waves coincide. System (7.24) for three variables $p_m(\vartheta)$, $p_q(\vartheta)$ and $p_s(\vartheta)$ takes the form

$$\frac{dp_m}{d\vartheta} = -\frac{i\pi}{2\vartheta_c}(p_q \cos\varkappa r\vartheta + p_s \sin\varkappa r\vartheta);$$
$$\frac{dp_q}{d\vartheta} = -\frac{i\pi}{2\vartheta_c} p_m \cos\varkappa r\vartheta; \quad \frac{dp_s}{d\vartheta} = -\frac{i\pi}{2\vartheta_c} p_m \sin\varkappa r\vartheta. \qquad (22.18)$$

Analogous system was by another method obtained in [21]. Function $p_m(\vartheta)$ in (22.13) is the solution of system (22.18) under obvious end conditions $p_m(0) = 1$, $p_q(0) = p_s(0) = 0$.

5. If it is necessary in broadband to ensure at the bend

relatively small conversion of modes H_{01} into E_{11}, then it is necessary by any path to remove/take degeneration, i.e., to create system, in which propagation constant of these waves do not coincide. One of the known methods of relieving the degeneration is transition from all-metal to spiral waveguide. The walls of this waveguide are formed by the wire, wound on the spiral (it is more precise - according to helix on cylinder) with very low pitch. After wire, with $\rho>a$, where a - radius of cylinder, is arrange/located dielectric layer, further - metallic waveguide (jacket). Spiral virtually completely shields the field of waves H_{0q}, and these waves they are propagated the same radius. The field of remaining waves penetrates beyond spiral; their propagation constant is different from value h_l in all-metal waveguide. Thus, in particular, it is possible to change value of h for wave E_{11}.

However, with this simultaneously change also coupling coefficients between different waves in this system in comparison with F_{lm} for an all-metal waveguide. In formulas of § 7 for F_{lm} one should now consider a change in the structure of all waves (except waves H_{0n}) in spiral waveguide in comparison with the waves, described in § 3. This calculation is produced in the articles of Unger [103] and Kerzhentseva [104].

Page 181.

W~ will give from these articles the numerical values of the coupling coefficient of wave H_{01} with E_{11}; for the all-metal waveguide F, is given in (22.4).

Curve/graph by 45 is borrowed from [103]. The module/modulus of coupling coefficient $|F|$ is represented in the function of the imaginary part ε'' of dielectric constant $\varepsilon=\varepsilon'-i\varepsilon''$ of the material, arrange/located between spirally and by external jacket. The curve a is related to case $\varepsilon'=4$, the curve b - to $\varepsilon'=\varepsilon''$; curves are calculated for ka=29.5. Table 22.3 is undertaken from [104]. In it $|F|$ is given for several values ka, ε' and ε'' when there is no external jacket, i.e., that dielectric fills entire space with ρ>a.

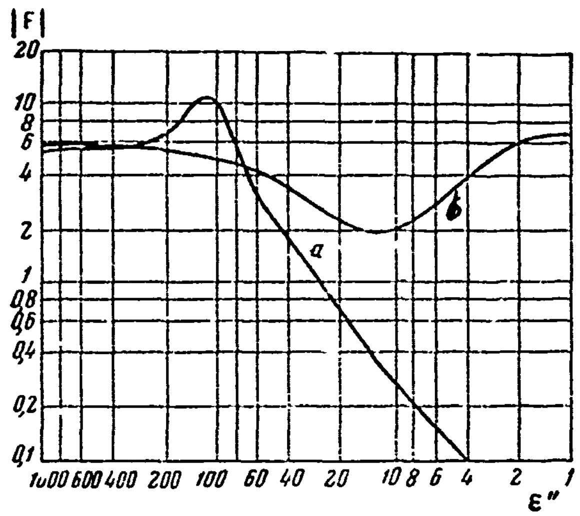


Fig. 45. Coupling coefficient in spiral waveguide in [101].

Table 22.3. Module/modulus of coupling cofficient.

ka	6,5	6,5	6,5	6,5	12,9	12,9	12,9	29,5	29,5	20,5	9,5	20,5	29,5	29,5
ε'	4	4	4	4	40	2	1	1000	64	25	10	1	4	4
ε''	250	64	10	1	40	2	1	1000	64	25	10	1	90	8
$\|F\|$	1,17	1,13	1,07	1,24	2,2	1,10	1,7	5,2	4,2	6,5	1,6	2,8	4,1	0,01
$\|F\|$ no (1) (22.4)	1,20				2,43			5,45						

Key. (1) on Page 182

In last/latter line corrected value $|F|$ for an all-metal waveguide on (22.4).

From these data it follows that the coupling coefficient in spiral waveguide can noticeably differ from (22.4), and during strict calculation of losses to conversion according to formulas of § 7 this difference one must take into account. The presence of jacket,

apparently, little affects value F, in any case at high values ε".

6. At high frequencies when parameter ka is great, propagation constant of all waves close to each other. In this case, the amplitudes of the direct/straight parasitic waves, which are formed in the bending of constant curvature, will be, accordingly (7.28b), it contains in denominator low value $\Delta h = h_i - h_m$; in order to ensure the sufficiently low level of losses to conversion, it is necessary to apply the bendings of very large length. It is possible, however, to decrease the length of bending with it was proposed by Unger [48] and was used in article [49]. However, used in these works mathematical apparatus not end/leads the small fractures; meanwhile precisely such bendings make it possible, as it turned out, to obtain the smallest losses at the smallest length. This was shown by Kerzhentseva in article [105]; in this point/item we presented the fundamental results of this work.

The amplitude of parasitic wave at the output of the bending of alternating/variable curvature is given by formula (7.35). We will record it in the form

$$p_i(\vartheta_0) = F_{im} \int_{-L/2}^{L/2} \frac{1}{r(l)} e^{-i\Delta h \cdot l} \, dl, \qquad (22.19)$$

after transferring the origin of coordinates into the middle of bending.

For selection r (l) let us examine first value

$$\frac{p_i(\vartheta_0)}{\vartheta_0 \cdot F_{im}} = \int\limits_{-L/2}^{L/2} \frac{1}{r(l)} e^{-i\Delta h \cdot l} dl. \qquad (22.20)$$

This value has the same analytical form, as the coefficient of reflection (17.2) from the section of long line with the alternating/variable wave impedance of W (ζ). The problem of finding of the optimum form of dependence $r(l)$ can be solved, utilizing the optimum forms of the function W (ζ). In contrast to the problems, examined in § 17, for bending it is expedient to utilize really/actually op' mum (maximum Chebyshev) function $W_{\text{опт.}}(\zeta)$, found by Klopfenshteyn [95].

Page 183.

Calculation shows that the curvature of bending must be changed according to the law

$$\frac{1}{r(l)} = \vartheta_0 \cdot \frac{1}{\operatorname{ch}\frac{\sigma_{\text{мин}}}{2}} \left\{ \frac{\sigma_{\text{мин}}}{2L} \cdot \frac{I_1\left(\sigma_{\text{мин}}\sqrt{1-(2l/L)^2/2}\right)}{\sqrt{1-(2l/L)^2}} + \right.$$

$$\left. + \frac{1}{2}\delta\left(l - \frac{L}{2}\right) + \frac{1}{2}\delta\left(l + \frac{L}{2}\right) \right\}; \left(-\frac{L}{2} \leqslant l \leqslant \frac{L}{2}\right). \quad (22.21)$$

Here I_1 - modified Bessel function, δ - delta function. Value $\sigma_{\text{мин}}$ is the parameter of bending; it determines the communication/connection

between the product $\sigma=L\cdot\Delta a$ and by value (22.20). For all values σ, greater than selected $\sigma_{\text{мин}}$ value $|p_j/\vartheta_0 F_{jm}|$ will be less $1/\text{ch}\frac{\sigma_{\text{мин}}}{2}$. If, for example $\sigma_{\text{мин}}$ is selected as being equal to 10.6, then value (22.20) will be less than 0.01.

For the bending, in which the curvature is changed according to (22.21), the amplitude of parasitic wave is equal to

$$|p_j| = |F_{jm}|\cdot\vartheta_0\frac{1}{\text{ch}\frac{\sigma_{\text{мин}}}{2}}\left|\cos\frac{\sqrt{\sigma^2-\sigma_{\text{мин}}^2}}{2}\right|. \qquad (22.22)$$

By the special feature/peculiarity of this bending is equality and equidistance of all maximums $|p_j|$, considered as function from σ.

In Fig. 46 value $\frac{L}{r(l)}\cdot\frac{1}{\vartheta_0}$, proportional to curvature (22.21), is represented as function l/L for two values of parameter $\sigma_{\text{мин}}$ ($\sigma_{\text{мин}}=7.14$, flat curve, and $\sigma_{\text{мин}}=10.6$, steep curve). At the end/leads of the bending, there are small fractures, equal, according to (22.21), $\vartheta_0/2\,\text{ch}\frac{\sigma_{\text{мин}}}{2}$. The greatest curvature is reached in the middle of bending and is equal to

$$\frac{1}{r_{\text{мин}}(l)} = \frac{1}{L}\cdot\frac{\sigma_{\text{мин}}}{2\,\text{ch}\frac{\sigma_{\text{мин}}}{2}}I_1\left(\frac{\sigma_{\text{мин}}}{2}\right). \qquad (22.23)$$

Fig. 47 makes it possible to compare the lengths of three bendings, constructed according to different laws, but which give the identical maximum values of parameter (22.20), i.e., identical losses to conversion.

Page 184.

Solid line gives $|p_l/\vartheta_0 F_{lm}|$ depending on σ for the bending, constructed according to (22.21), dash - for the bending, proposed by Unger [48], in which $1/r(l)$ is changed linearly from zero to the greatest value in center, dot-dash - for a bending with constant curvature. Than the less permissible losses to conversion, the more the advantages it has a bending with small fractures, constructed according to (22.21). So, for $L\Delta h=20\,|p_l|$ it will be in this bending 400 times less than with the bending of the same length with linearly variable chamber $1/r(l)$.

For the bending, constructed according to (22.21), the maximum value of amplitudes p_l (ϑ_0) will change with frequency only due to a change in the coupling coefficient F_{lm}; change Δh, according to preceding/previous, it will not affect the maximum losses. In this case, the length of bending must be selected according to the requirement

$$L \geqslant \frac{\sigma_{\text{мин}}}{(\Delta h)_{\text{мин}}}, \qquad (22.24)$$

analogous to condition (17.15).

7. Another possible method of overcoming of bending is partial filling of cross section of bent waveguide with dielectric. This idea and calculation of the corresponding equipment/device for wave H_{01} are contained in the article of Morgan [44].

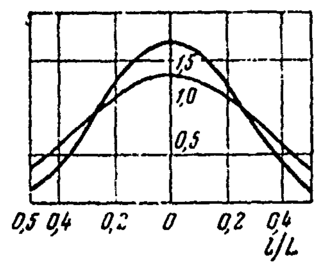


Fig. 46. Bending of alternating/variable curvature.

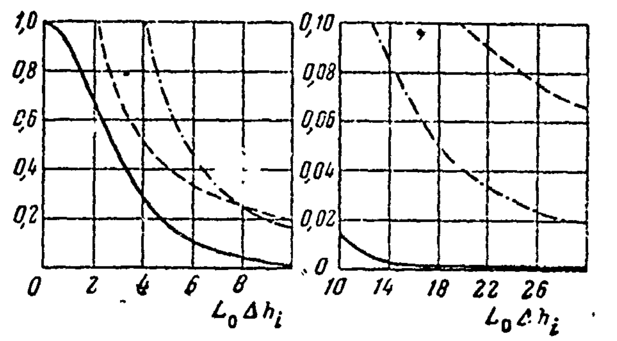


Fig. 47. Losses for bendings of different forms.

Page 185.

In this article the field in the bent waveguide with heterogeneous filling is represented in the form of the superposition of the fields of the waves of the empty rectilinear waveguide. The presence of heterogeneous filling and the bending are two reasons, which cause the **communication**/connection of these waves. By the selection of **function** $\hat{\varepsilon}(x, y)$ succeeds in ensuring equality to zero or the considerable decrease of the total coupling coefficients of wave H_{01}

with several waves, first of all with wave E_{11}. In [44] they are designed single-sector and three-sector compensators. In the first of them, is utilized one dielectric sector, situated from that side of section, that is nearer to the axis of bending, i.e., with $|\beta| > \pi/2$. The three-sector compensator contains three symmetrically arranged/located sectors, divided by sectorial interval/gaps; dielectric is also arrange/located when $|\beta| < \pi/2$.

In the designations of last/latter point/item of § 8, coupling coefficient between waves (E^{0n}, H^{0n}) and (E^{0n}, H^{0m}), obliged to heterogeneous filling, is determined by general formula (8.46); the coupling coefficient, caused by bending, is found in (22.2) and (22.4). For wave E_{11}, for example, the sum of these coefficients will be for single-sector compensator equal to zero, if its parameters ε and β_0 - the flare angle of sector - will be connected by the relationship/ratio

$$(\varepsilon - 1)\sin\frac{\beta_0}{2} = 1{,}53\,\frac{a}{r}. \qquad (22.25)$$

In this problem the application/use of expansion in terms of the fields of waves in empty waveguide, used in last/latter point/item of § 8, leads to target/purpose somewhat faster than expansion in terms of the fields of waves in regular waveguide with the same filling. This is explained by the fact that in this case the field in the filled bent waveguide is close to the field of wave H_{01} in the empty rectilinear waveguide, and precisely with such waveguides is connected bending. Wave H_{01} is its own wave of rectilinear part and

it is simultaneously close to its own flexural wave; therefore it overcomes bending with a small distortion.

By means of the complication of the form of dielectric it is possible to decrease the communication/connection of wave H_{01} with the row/series of other waves, besides E_{11}; however, with any selection of function $\varepsilon(x, y)$ it is not possible to ensure the absence of the communication/connection of wave H_{01} with all other waves and passage by it bending entirely without losses to conversion. The compensated for bendings, designed in [44], possess low losses; this is explained by the fact that for them it is accepted $|\varepsilon-1| \ll 1$. This limitation is superimposed both by the calculation method in [44] and by the need for ensuring low value ε'', i.e. small dielectric losses.

Page 186.

Calculation this same of equipment/device according to the method of § 7 required first of the determination of its own waves of the filled rectilinear waveguide; they it would be possible to find, for example, from the method of last/latter point/item of § 8. When $(\varepsilon-1) \ll 1$ and upon consideration to communication/connection H^0_{01} only with E^0_{11}, their own waves will be two linear combinations of these waves, close respectively to H^0_{01} and E^0_{11}. The passage of these its own waves according to bending is described by the system of two

differential equations of type (7.9). End conditions are found from requirement, so that the beginning of bending these their own waves they would form wave H^0_{01}. When $(\varepsilon-1)\ll 1$ this calculation somewhat more is bulky, than calculation in [44]; however at finite values $\varepsilon-1$ it, probably, it will be noticeably more simply.

Let us note finally that the impossibility to find this distribution $\varepsilon(x, y)$, so that the wave H_{01} in bending would not be distorted, from a physical point of view, it is possible, it is probable, to connect with the noted at the end of § 4 dependence between the longitudinal and transverse components of fields in the adjacent sections of the bent waveguide. With selection $\varepsilon(x, y)$ it would be possible to make even the phase speeds of all points of wave front in cross section and to ensure the rotation of front without distortions; however, will completely appear the supplementary components, caused by this dependence, and therefore unavoidably will arise the waves of other numbers. With an increase in the frequency, longitudinal components become relatively less essential, and losses to conversion in bending with heterogeneous filling can be made small ones.

Page 187.

Chapter VI.

IRREGULAR ACOUSTIC WAVEGUIDES.

The method of cross sections, developed above for radiowaveguides, can be used also during the investigation of the propagation of waves in irregular acoustic waveguides. In acoustics this method also reduces to the infinite system of ordinary differential equations for wave amplitudes.

For the first time for investigation of one type of waveguides - precisely tapered welds with rigid walls - the version of the method of cross sections was proposed by Stevenson [106]. In [106] for wave amplitudes was obtained the system of the differential second order equations; expressions for coefficients in this system are very complex, and this apparatus, apparently, was not used for the calculation of specific cases.

Below problem is reduced as in electrodynamics, to the system of

differential first-order equations. In this chapter are given the formulas for the coefficients of these equations, so-called coupling coefficients F_{lm} and S_{lm}, for different types of irregularities. In it is not examined the computation F_{lm} and S_{lm} for specific problems, the solution of system of equations for waveguides with the slowly changing parameters, the investigation of the special cases, the physical analysis of results, i.e., all that which composes the large part of the material of the preceding/previous chapters. For acoustic waveguides all this is conducted in much the same manner as for radiowaveguides. This chapter bears somewhat more formal and more concise character, than preceding/previous.

Page 188.

§23. Regular acoustic waveguides.

Acoustic field is described by sound pressure P and with a vibrational speed of V. With time/temporary dependence $e^{i\omega t}$ the fundamental equations of acoustics take the form

$$\operatorname{div} V = \frac{-i\omega}{\rho c^2} P; \qquad (23.1a)$$

$$\operatorname{grad} P = -i\omega\rho V, \qquad (23.1b)$$

where ρ - density of medium and c - the speed of propagation of sound in it. In common/general/total case these values depend on coordinates, $\rho=\rho(x,y,z)$ and $c=c(x,y,z)$. On the boundary of waveguide,

is fulfilled one of the two boundary conditions:

$$P = 0 \qquad (23.2a)$$

or

$$V_N = 0, \qquad (23.2b)$$

where N - a direction of standard. Condition (23.2a) corresponds to soft wall, condition (23.2b) - rigid.

Regular waveguides are called the media in which ρ and c do not depend on z and boundary conditions (23.2) are fulfilled on certain cylindrical surface, parallel to Z-axis and by that intersecting plane z=const according to the closed curve. We will examine the regular waveguides of general view, in which ρ and c depend arbitrarily on x and y.

1. Let us give necessary for further relationship/ratio between different values in acoustic waveguides. Let us begin with the case when density ρ is everywhere constant. In this case, it is convenient to use the potential Ψ, determined by the formula

$$V = -\operatorname{grad}\Psi. \qquad (23.3)$$

It satisfies the wave equation

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2} + \varkappa^2\Psi = 0, \qquad (23.4)$$

where $\varkappa = \omega/c$ — wave number. Boundary conditions for Ψ take the form

$$\Psi = 0 \tag{23.5a}$$

or

$$\frac{\partial \Psi}{\partial N} = 0. \tag{23.5b}$$

Let us recall that in the radiowaveguides of field in a number of cases also they can be expressed through the scalar functions, which satisfy wave equation (23.4); in symmetrical relative to axis waveguides it is sufficient even one function. However, conditions (3.2) for electromagnetic field on the boundary of irregular waveguides lead for these functions to the boundary conditions, different from (23.5).

Page 189.

For example, in the simplest problem of symmetrical m-gnetic waves in symmetrical tapered weld boundary condition for potential function exists $\partial\Psi/\partial n = 0$, where n (normal to the duct/contour of section) does not coincide with N in (23.5b). This difference under the boundary

conditions leads to the fact that for coupling coefficients in radiowaveguides and in acoustic waveguides are obtained different formulas, even if these coefficients are expressed as the same potential functions.

In the regular waveguides of comparison, we will replace as into §8, Z-axis by axis ζ. Wave number $\varkappa^2$ does not depend on ζ, and the solutions of wave equation are the functions

$$\psi'^m(x, y, \zeta) = \psi^m(x, y) e^{ih_m\zeta}, \quad m = 1, 2 \ldots \qquad (23.6)$$

Here ψ^m they satisfy the equation

$$\nabla^2\psi^m + (\varkappa^2 - h_m^2)\psi^m = 0. \qquad (23.7)$$

Boundary conditions (23.5) isolate the complete system of eigenfunctions ψ^m and of propagation constants h_m. As is known, function ψ^m, those corresponding to different indices, they are orthogonal between themselves:

$$\int \psi^j\psi^m dS = 0 \overset{(1)}{\text{при}} j \neq m \; (j > 0, \; m > 0). \qquad (23.8)$$

Key: (1). with.

We accept for them the standardization

$$\int (\psi^m)^2 dS = 1. \qquad (23.9)$$

different from the standardization of membrane/diaphragm functions

for radiowaveguides. Difference this is caused by existence in acoustic waveguides with the rigid walls of the solution $\psi = \text{const}$, which corresponds to the fundamental wave, for which standardization (3.16) is impossible.

Let us introduce, further eigenfunctions and wave numbers with negative indices, after defining them as into §3, by the condition

$$\psi^{-m} = -\psi^{m}; \qquad (23.10a)$$

$$h^{-m} = -h^{m}. \qquad (23.10b)$$

For the indices, which have different signs, condition of orthogonality in the form (23.8) will not be correct. The common/general/total recording of the conditions of orthogonality, which includes (23.8) and (23.9), which we will use subsequently, has the form

$$\int \psi^{l}\psi^{m} \, d\sigma = \delta_{lm} - \delta_{l,-m}. \qquad (23.11)$$

Page 190.

2. If density of medium, which fills waveguides, is not constant, then potential satisfies more complex equation, than (23.4). In this case more conveniently to operate directly with values P and V, similar to that as in radiowaveguides with heterogeneous filling is expedient to produce all computations in the components of fields E and H, but not in membrane/diaphragm

functions. This calculation was produced in V. V. Shevchenko's article [107]; all results of this chapter, obtained not from wave equation (23.4), but it is direct of the first-order equation (23.1), were borrowed from this article.

In the regular waveguides in which $\rho=\rho(x,y)$, $c=c(x,y)$, the solutions of the system (23.1) have the form

$$P^m(x, y, \zeta) = p^m(x, y)e^{-ih_m\zeta}\ (a);$$
$$V^m(x, y, \zeta) = v^m(x, y)e^{-ih_m\zeta}\ (b). \qquad (23.12)$$

Here the functions p^m and v_x^m, v_y^m, v_z^m satisfy, in accordance with (23.1), a system of four equations

$$\frac{\partial v_x^m}{\partial x}+\frac{\partial v_y^m}{\partial y}-ih_m v_z^m=\frac{-i\omega}{\rho c^2}p^m; \qquad (23.13a)$$

$$\frac{\partial p^m}{\partial x}=-i\omega\rho v_x^m;\ \frac{\partial p^m}{\partial y}=-i\omega\rho v_y^m;\ h_m p^m=\omega\rho v_z^m \qquad (23.13b)$$

and to boundary conditions which correspond (23.2a) or (23.2b).

System this will be invariant relative to the sign of index, if we assume together with (23.10b)

$$p^{-m}=-p^m;\ v_x^{-m}=-v_x^m;\ v_y^{-m}=-v_y^m;\ v_z^{-m}=v_z^m. \qquad (23.14)$$

If ρ=const, then always it is possible to assume ρ=1, and then p^m, v^m they are connected with eigenfunctions ψ^m by the relationship/ratios

$$p^m=i\omega\psi^m;\ v_x^m=-\partial\psi^m/\partial x;\ v_y^m=-\partial\psi^m/\partial y;\ v_z^m=ih_m\psi^m. \qquad (23.15)$$

Eigenfunctions p^m, v^m are orthogonal between themselves. For obtaining the condition of orthogonality, we form value

$$\operatorname{div}(P^m V^{-l}-P^{-l}V^m), \qquad (23.16)$$

where P^m, V^m and P^{-j}, V^{-j} — fields of two waves (23.12), of numbers m and -j.

Page 191.

According to (23.1), value (23.16) it is equal to zero. Let us integrate it with respect to cross section. Integral of

$$\frac{\partial}{\partial x}(P^m V^{-j}_x - P^{-j} V^m_x) + \frac{\partial}{\partial y}(P^m V^{-j}_y - P^{-j} V^m_y) \qquad (23.17)$$

can be converted according to the theorem of Gauss into integral on the duct/contour of section; it is equal to zero as a result of boundary conditions (23.2a) or (23.2b). Third term/component/addend in integral of (23.16), that contains derivative on ζ, after reduction on $e^{-i(h_m-h_j)\zeta}$ will take the form

$$-i(h_m-h_j)\int(p^m v_z^{-j} - p^{-j} v_z^m)\,dS. \qquad (23.18)$$

Thus, with j≠m the integral, which stands in (23.18), is equal to zero.

Let us accept for eigenfunctions (p^m, v^m) such standardization which with ρ=1, when it is correct (23.15), passes in (23.11). Then

the condition of orthogonality and standardization are recorded in the form

$$\int (\psi^m v_z^l + \psi^l v_z^m)\, dS = -2\omega h_l \delta_{lm}. \qquad (23.19)$$

§24. Bent acoustic waveguides.

1. We will begin from definition of coupling coefficient F_{lm} between two waves in bent waveguide with method of small heterogeneities, by examining as into §4, fracture of waveguide of constant section to small angle $\Delta\vartheta$. On fracture falls to the left the wave of number m from single amplitude, i.e., wave with potential $\psi^m e^{-ih_m z}$; potential in right wave takes the form

$$\Psi^r = \sum_{n>0} P_n \psi^n e^{-ih_n z}, \qquad (24.1)$$

and it is necessary to find amplitudes P_n. Let us connect between themselves the values of potential and its derivative in sections AA and BB (Fig. 2)

$$\Psi\Big|_{BB} = \Psi + r\frac{\partial\Psi}{\partial z}\Delta\vartheta\Big|_{AA}; \qquad (24.2)$$

$$\frac{\partial\Psi}{\partial z}\Big|_{BB} = \frac{\partial\Psi}{\partial z} + r\frac{\partial^2\Psi}{\partial z^2}\Delta\vartheta - \frac{\partial\Psi}{\partial r}\Delta\vartheta\Big|_{AA}.$$

Last/latter term/component/addend the secondly of these formulas corresponds to the rotation of Z-axis upon transfer from one section to adjacent, remaining term/component/addends are the first two members of Taylor series.

Page 192.

·· ·t· ı AA of value Ψ and $\partial\Psi/\partial z$ contain together with addend, that corresponds to the inciden' wave, also term/component/addends, which correspond to all reverse/inverse (reflected) waves. In order to exclude the unknown amplitudes of these backward waves, we will use both by expressions (24.2). Let us compose value

$$\frac{\partial\Psi}{\partial z}-ih_j\Psi. \qquad (24.3)$$

It, obviously, does not contain the field of backward wave of number j, but the field of direct wave of number j enters into it with amplitude $-2ih_jP_j$. Multiplying (24.3) on Ψ' and integrating by cross section, we as a result of the condition of orthogonality will exclude term/component/addends, which relate to the waves of all other numbers. Thus, determining P_j according to the formula

$$P_j=\frac{i}{2h_j}\int\left(\frac{\partial\Psi}{\partial z}-ih_j\Psi\right)dS, \qquad (24.4)$$

analogous to (4.3), where the integral is taken according to section AB, and expressing potential and its derivative on (24.2), we can into first terms in (24.2) substitute not entire potential, but only

potential of the incident wave. Since all the computation is conducted with an accuracy down to the terms first-order in $\Delta\vartheta$, then in term/component/addends (24.2), that contain factor $\Delta\vartheta$, also it is possible to substitute $\Psi = \psi^m e^{-ih_m z}$. Thus, $P_j (j>0)$ is located through formula (24.4), in which it is necessary to assume

$$\Psi = (1 - ih_m r\Delta\theta)\psi^m;$$
$$\frac{\partial\Psi}{\partial r} = (-ih_m - rh_m^2\Delta\theta)\psi^m - \Delta\theta\frac{\partial\psi^m}{\partial r}. \quad (24.5)$$

For P_j will be obtained the expression

$$P_j = \delta_{jm} + F_{jm}\Delta\theta. \quad (24.6)$$

Values F_{jm} are, as shown into §4, the unknown coupling coefficients. They are equal to

$$F_{jm} = \frac{-i}{2h_j}\left\{h_m(h_m + h_j)\int r\psi^m\psi^j dS + \int\frac{\partial\psi^m}{\partial r}\psi^j dS\right\}. \quad (24.7)$$

In a similar manner it would be possible to find the amplitudes of backward waves, i.e., value P_j with j<0, and to be convinced of the fact that formula (24.7) was valid with any sign of indices j and m.

Page 193.

2. Let us examine now waveguide bend according to circular arc and will install system of differential equations for P. those

considered as functions of angle ϑ, and expression for coefficients in this system, using formalism of method of cross sections. We will in this case proceed from wave equation for Ψ (23.4).

Let us expand for this Ψ and $\frac{1}{r}\frac{\partial\Psi}{\partial\vartheta}$ in row/series according to functions ψ^{m}:

$$\Psi = \sum_{n=0} Q_n \psi^n, \quad \frac{1}{r}\frac{\partial\Psi}{\partial\vartheta} = -i\sum_{n=0} h_n R_n \psi^n. \qquad (24.8)$$

The isolation/liberation of factor $-ih_n$ in the coefficients of the second from these expansions, it goes without saying, is arbitrary. Let us introduce instead $Q_n(\vartheta)$ and $R_n(\vartheta)$ new coefficients, $P_n(\vartheta)$ and $P_{-n}(\vartheta)$, after defining them just as in (7.2):

$$P_n + P_{-n} = Q_n; \; P_n - P_{-n} = R_n. \qquad (24.9)$$

Taking into account condition (23.10), it is possible now (24.8) to record in the form

$$\Psi = P_\nu \psi^\nu; \; (24.10a), \quad \frac{1}{r}\frac{\partial\Psi}{\partial\vartheta} = -ih_\nu P_\nu \psi^\nu. \qquad (24.10b)$$

where is implied summation over ν from $\nu = -\infty$ to $\nu = +\infty$.

Values $P_n(\vartheta)$ — amplitude of direct/straight and backward waves. On transition between two sections of different curvature, in particular at the end of the bending, during its coupling with

rectilinear ones: by waveguides, potential :ps and its normal Ψ derivative $\frac{1}{r}\frac{\partial\Psi}{\partial\vartheta}$ must be continuous. According to the condition of orthogonality for functions ψ^n, hence follows also the continuity of amplitudes $P_n(\vartheta)$. End conditions for $P_n(\vartheta)$ at the end/leads of the curved section coincide with (7.10).

In order to find system of equations for $P_n(\theta)$, let us substitute (24.10a) in (24.10b), and (24.10) - into wave equation (23.4), recorded in cylindrical coordinate system. Utilizing even more differential equation (23.7) for eigenfunctions ψ^n, we will obtain two expansions:

$$\frac{dP_\nu}{d\theta}\psi^\nu = -ih_\nu P_\nu r\psi^\nu; \quad \frac{dP_\nu}{d\theta}h_\nu\psi_\nu = -iP_\nu\left(h_\nu^2 r\psi^\nu + \frac{\partial\psi^\nu}{\partial r}\right). \quad (24.11)$$

Let us multiply both of expansions on ψ^l and will integrate over cross section. According to (23.10) and (23.11), the first of them will give in this case the expression for $\frac{d}{d\theta}(P_l - P_{-l})$, the second - for $\frac{d}{d\theta}(P_l + P_{-l})$.

Page 194.

From these two expressions we will obtain the unknown system of the differential equations

$$\frac{dP_l}{d\theta} = F_{l\nu}P_\nu. \quad (24.12)$$

where the expression for F_{jm} coincides with (24.7).

3. Formula (24.7) can be somewhat simplified, after excluding from it first integral and after leading it to symmetrical form, similar formula (4.7) was converted to form (7.16).

Let us show first that in acoustic waveguides the coupling coefficients F_{jm} also satisfy conditionally reciprocity (7.14).

For two any solutions of wave equation (23.4) $\Psi^{(1)}$, $\Psi^{(2)}$, it is obvious, is correct the identity

$$\operatorname{div}(\Psi^{(2)}\operatorname{grad}\Psi^{(1)} - \Psi^{(1)}\operatorname{grad}\Psi^{(2)}) = 0. \qquad (24.13)$$

Let us integrate it with respect to the region, included between the lateral surface of the curved waveguide and two cross sections ϑ=const, and we convert integral on the formula of Gauss. According to (23.5), the vector flux, which stands in (24.13) in brackets, through the lateral surface of waveguide is equal to zero, and therefore the integral

$$\int\left(\Psi^{(2)}\frac{\partial\Psi^{(1)}}{r\partial\vartheta} - \Psi^{(1)}\frac{\partial\Psi^{(2)}}{r\partial\vartheta}\right)dS, \qquad (24.14)$$

undertaken according to the section of waveguide, it does not depend on ϑ. The substitution of expansions (24.10) gives for this integral

of expression $2ih_\nu P_\nu^{(1)}P_{-\nu}^{(2)}$. Derivative this sum on ϑ is equal, according to (24.12),

$$2iP_\mu^{(1)}P_{-\nu}^{(2)}(h_\nu F_{\nu\mu} + h_\mu F_{-\mu,-\nu}) \qquad (24.15)$$

(is implied summation over μ and ν). Value (24.15) is equal identical to zero; therefore for any indices is equal to zero sum, which stands in brackets. Since, according to (24.7), during sign change of both of indices F_{lm} also reverse the sign, equality zero (24.15) mean that is correct the condition of reciprocity (7.14). Substituting under this condition expression (24.7), we will obtain (with j≠+-m)

$$\int r\psi^l\psi^m dS = \frac{1}{h_l^2 - h_m^2}\left\{\int \psi^l \frac{\partial\psi^m}{\partial r} dS - \int \psi^m \frac{\partial\psi^l}{\partial r} dS\right\}. \qquad (24.16)$$

Page 195.

Let us note that this identity could be obtained directly, utilizing only equations (23.7) and boundary conditions (23.5). For this, one ought not to have integrated over cross section function $\nabla[r(\psi^l\nabla\psi^m - \psi^m\nabla\psi^l)]$. According to (23.5), the integral of this function is equal to zero; on the other hand, it is easy to show, transforming it and utilizing (23.7), that it is equal to $(h_m^2 - h_l^2)r\psi^l\psi^m + \psi^l\partial\psi^m/\partial r - \psi^m\partial\psi^l/\partial r$. Hence again is obtained identity (24.16).

Replacing the first integral in (24.7), according to (24.16),

obtain the expressio for a coupling coefficient in the symmetrical form:

$$F_{jm} = \frac{i}{2h_j} \cdot \frac{1}{h_m - h_j} \cdot \left\{ h_m \int \psi^m \frac{\partial \psi^j}{\partial x} dS - h_j \int \psi^j \frac{\partial \psi^m}{\partial x} dS \right\}. \quad (24.17)$$

Here derivatives on r are replaced by derivatives on x, d/dr=-d/dx.

as is shown comparison with (24.7), this formula is valid also with j=-m. It is inapplicable only with j=m. Coefficient F_{jj} contains value r in an explicit form. Radius of curvature r_j for the wave of number j, introduced into §7 as radius of a circle along which the phase rate of this wave coincides with the phase rate in direct/straight waveguide, is equal to iF_{jj}/h_j, (and, according to (24.7), we obtain for this value the expression

$$r_j = \int r (\psi^j)^2 dS - \frac{1}{2h_j^2} \int \psi^j \frac{\partial \psi^j}{\partial x} dS. \quad (24.18)$$

For a waveguide with soft walls, i.e., under boundary condition (23.5a), there is the identity

$$\int \psi^j \frac{\partial \psi^m}{\partial x} dS + \int \psi^m \frac{\partial \psi^j}{\partial x} dS = 0, \quad (24.19)$$

analogous (7.19). In this case second term in (24.18) is absent, and formula (24.17) takes the simple form:

$$F_{jm} = \frac{i}{2h_j} \cdot \frac{h_m + h_j}{h_m - h_j} \int \psi^m \frac{\partial \psi^j}{\partial x} dS. \quad (24.20)$$

In this case, $F_{j,-j} = 0$ for any form of section.

Formulas for F_{lm} for acoustic waveguides (24.17) and (24.20) are analogous to formulas (7.20) and (7.21) for radio waveguides but, as already mentioned, they do not coincide with them. In both problems $\dot{F}_{lm}$ it contains the same integrals; however, these integrals enter with different coefficients.

Page 196.

§25. Acoustic waveguides with alternating/variable filling.

1. Let us begin from case of medium of constant density, when field is described by equation (23.4), in which $\varkappa$ $\varkappa$ (x,y,z).

In the waveguides of comparison, the wave number $\varkappa$ does not depend on ζ and with all ζ is the same function from x,y, as in irregular waveguide with fixed/recorded z. Eigenfunctions ψ^m and wave numbers h_m the waveguides of comparison depend on z.

Let us decompose Ψ and $\partial\Psi/\partial z$ in an irregular waveguide in row/series on ψ^n type (24.8) let us introduce variables $P_n(z)$ by conditions (24.9). Then we obtain the expansions

$$\Psi \quad P_N \psi^N, \tag{25.1a}$$

$$\frac{\partial\Psi}{\partial z} \quad ih_N P_N \psi^N \tag{25.1b}$$

Coefficients $P_n(z)$ are amplitudes of waves of both of directions and on the boundary of irregular section satisfy end conditions (8.7).

With support/socket (25.1a) in (25.1b) and (25.1) - into equation (23.4) are obtained two expansions:

$$(P_N' + ih_N P_N)\psi^N = -P_N \psi^{N'},$$
$$(P_N' + ih_N P_N)\psi^N = h_N P_N \psi^{N'} + h_N' P_N \psi^N. \qquad (25.2)$$

Equation (23.7) for ψ^N. prime here used indicates derivative on z.

Multiplying (25.2) on ψ^l, integrating by cross section and utilizing a condition of orthogonality (23.11), we will obtain the system of differential equations for amplitudes $P_l(z)$:

$$\frac{dP_l}{dz} + ih_l P_l = S_{lN} P_N. \qquad (25.3)$$

Coupling coefficients S_{lm} are equal to:

$$S_{lm} = -\frac{h_l + h_m}{2h_l} \int \psi^l \frac{\partial \psi^m}{\partial z} dS \text{ при } l \neq m, \qquad (25.4a)$$

$$S_{mm} = -\frac{1}{2h_m} \frac{dh_m}{dz}, \qquad (25.4b)$$

$$S_{-mm} = \frac{1}{2h_m} \frac{dh_m}{dz} \qquad (25.4c)$$

Key: (1). when.

Page 197.

Relationship/ratio (25.4b) coincides with (8.9), as we see below, it is correct in the more general case.

Formula (25.4c) in appendices to problem with a small change in the rate of medium means that the coefficient of reflection of any wave is proportional to a change in its propagation constant. As we will see below, in the general case when changes also density, there is not no this single bond between S_{-mm} and h_m'.

We convert formula (25.4a) in such a way that the coupling coefficients would contain clearly derived on z only that of wave number $\varkappa$, and not of eigenfunctions. Let us differentiate for this equation (23.7) for z:

$$\nabla^2\psi^{m\prime} + (\varkappa^{2\prime} - h_m^{2\prime})\psi^m + (\varkappa^2 - h_m^2)\psi^{m\prime} = 0. \qquad (25.5)$$

Let us multiply this equation by ψ^j, and equation (23.7), written for m=j, will multiply on $-\psi^{m\prime}$, we will add and integrate over cross section. Utilizing even boundary conditions (23.5), we will obtain

$$\int \varkappa^{2\prime}\psi^m\psi^j dS - h_m^{2\prime}\int \psi^m\psi^j dS = (h_m^2 - h_j^2)\int \psi^j\psi^{m\prime} dS. \qquad (25.6)$$

When $j \neq \pm m$ the factor when h'_m in (25.6) is equal to zero, and that participating in (25.4a) integral is expressed as the integral of derived wave number $\varkappa$. Substituting in (25.4a), we obtain

$$S_{jm} = \frac{1}{2h_j(h_j - h_m)} \int \frac{\partial \varkappa^2}{\partial z} \psi^m \psi^j dS \quad (j \neq m). \qquad (25.7)$$

Comparison with the following formula shows that (25.7) it is correct with j=-m.

Expression for a derivative of the constant of propagation through the derivative of wave number h_m is obtained from (25.6), if we in this formula assume j=+-1m. So we find

$$\frac{dh_m^2}{dz} = \int \frac{\partial \varkappa^2}{\partial z} (\psi^m)^2 dS. \qquad (25.8)$$

Two last/latter formulas are analogous to formulas (8.16) and (8.17), that express coupling coefficients and derivatives of constants expressing coupling coefficients and derivatives of propagation constants in radiowaveguides with the alternating/variable filling through derived in z of electrical and magnetic permeability media.

Page 198.

Formulas (25.7), (25.8) can be still converted for that case when medium consists of two mediums with the different values of the speed of sound, between which there is a sharp interface, so that $\varkappa^2$ is the piecewise constant function of coordinates. Repeating the corresponding considerations §8, we will obtain that in this case derived $\partial\varkappa^2/\partial z$ differs from zero only in infinitely thin layer adjoining the interface, and in this layer

$$\frac{\partial\varkappa^2}{\partial z} = -\nu\frac{\partial\varkappa^2}{\partial n}, \qquad (25.9)$$

where ν — tangent of the angle which forms with Z-axis the tangent to interface, perpendicular to intersection of interface with plane z=const, and n - a normal to interface in the waveguide of comparison, directed to side $\nabla\varkappa^2$ in this waveguide. Substituting (25.9) in (25.7) and (25.8), it is possible in these formulas to produce integration along direction n for transition layer: in this case, of the boundary conditions, follows that in layer necessary to consider it ψ^m as constant. In this way for S_{jm} and $h_m^{\prime}$ are obtained th· expressions, which contain the contour integral, undertaken along the line, on which the interface of two media intersects with plane z=const,

$$S_{jm} = \frac{1}{2h_j(h_j - h_m)} \oint \nu\psi^j\psi^m\cdot\Delta(\varkappa^2)\,ds \quad (j \neq m); \qquad (25.10a)$$

$$\frac{dh_m^2}{dz} = -\oint \nu(\psi^m)^2\Delta(\varkappa^2)\,ds. \qquad (25.10b)$$

In these formulas $\Delta(\varkappa^2)$ — difference in the value of wave numbers in both adjacent media. Formula (25.10a) is analogous to formula (8.22) for radio waveguides.

2. We will generalize the results of this paragraph for waveguides, in which both speed of sound and density of medium are alternating/variable value. Let us be in this case it proceeds from first-order equations (23.1) for pressure P and rate V.

Let us decompose P and V_z in row/series on p^ν and v_z^ν type (24.8) let us pass to variables $P_\nu(z)$. The expansions

$$P = P_\nu p^\nu, \qquad (25.11a)$$

$$V_z = P_\nu v_z^\nu \qquad (25.11b)$$

let us substitute into equation (23.1b) for components v_x and v_y. Keeping in mind first two of equations (23.1b), we will obtain for these components the same expansions:

$$V_x = P_\nu v_x^\nu, \quad V_y = P_\nu v_y^\nu. \qquad (25.12)$$

Page 194.

In order to install the system of differential equations for $P_l(z)$, we will substitute (25.11) into equation (23.1b) for V_z and (25.11), (25.12) - in (23.1a). Utilizing the also last/latter from relationship/ratios (23.13b) a relationship/ratio (23.13a), we will obtain two expansions:

$$(P_\nu' + ih_\nu P_\nu)\mathfrak{p}^\nu = -\mathfrak{p}^{\nu\prime} P_\nu;$$
$$(P_\nu' + ih_\nu P_\nu)v_z^\nu = -v_z^{\nu\prime} P_\nu. \qquad (25.13)$$

Multiplying these sums on v_z^l and $\mathfrak{p}^l$, store/adding up, integrating by the transverse cross section and utilizing a condition of orthogonality (23.19), we will obtain the unknown system of equations for $P_l(z)$:

$$P_l' + ih_l P_l = S_{l\nu} P_\nu; \qquad (25.14)$$
$$S_{lm} = \frac{1}{2}\int (\mathfrak{p}^{m\prime} v_z^l + v_z^{m\prime}\cdot \mathfrak{p}^l)\, dS. \qquad (25.15)$$

From the comparison of formula (25.15), written for j=-m, with the formula, which is obtained during differentiation with respect to z of the condition for standardization (23.19), it follows that and in this most general case is fulfilled equality (25.4b). Let us recall that this equality is the condition of the fact that upon transfer from variables $P_m(z)$ to by alternating/variable $p_m(z)$, specified condition (8.29), in the system of differential equations for these

new variables are absent tne diagonal terms. It has simple energy value.

The conversion of expression (25.15) to the form, which contains arbitrary ones on z only from the parameters of medium, is conducted by the same diagram which was used in point/item 3 of §8. Let us differentiate on z of equations (23.1), written for the wave of number m:

$$\operatorname{div} V^m = -\frac{i\omega}{\rho c^2} P^{m\prime} - \left(\frac{i\omega}{\rho c^2}\right)' P^m; \qquad (25.16)$$

$$\operatorname{grad} P^{m\prime} = -i\omega\rho V^{m\prime} - (i\omega\rho)' V^m.$$

Here, for example $P^{m\prime}$ is obtained during differentiation with respect to z of formula (23.12a), in which on z they depend p^m (x,y) and h_m (sr 8.14).

We form, further, value

$$\operatorname{div}\{P^{m\prime} V^{-j} - P^{-j} V^{m\prime}\}. \qquad (25.17a)$$

According to (23.1) and (25.16), it is equal to

$$\left(\frac{i\omega}{\rho c^2}\right)' \cdot P^m P^j - (i\omega\rho)' V^m V^{-j}. \qquad (25.17b)$$

Page 200.

Let us integrate (25.17) with respect to the transverse cross section of waveguide. Integral of term/component/addends, which contain two-dimensional (in the variables x, y) divergence, drops out after conversion on the formula of Gauss and substitution of boundary values (23.2). After substitution (23.12) and reduction, for the exponential factor $e^{-i(h_m - h_j)\zeta}$ we will obtain

$$(h_m - h_j)\int (p^{m'} \cdot v_z^j + p^j \cdot v_z^{m'})\,dS - h'_m \cdot 2\omega h_m \cdot \delta_{mj} = \omega \int \left[\left(\frac{1}{\rho c^2}\right)' \cdot p^m p^j + \rho' \, \mathbf{v}^m \mathbf{v}^{-j} \right] dS. \qquad (25.18)$$

In this way we obtain the unknown expressions for the coupling coefficient

$$S_{jm} = \frac{1}{2h_j (h_m - h_j)} \int \left[\left(\frac{1}{\rho c^2}\right)' p^m p^j + \rho' \, \mathbf{v}^m \mathbf{v}^{-j} \right] dS \quad (j \neq m) \qquad (25.19)$$

and of derivative of the wave number

$$h'_m = -\int \left[\left(\frac{1}{\rho c^2}\right)' (p^m)^2 + \rho' \mathbf{v}^m \mathbf{v}^{-m} \right] dS. \qquad (25.20)$$

With $\rho=1$ two last/latter formulas coincide in accordance with (25.7) and (25.8). These formulas, probably, can be placed as the basis of the theory of long natural waveguides.

Comparing (25.19) with (25.20), we find the common relationship/ratio between the coupling coefficient of straight line and of backward wave of one and the same number and a change in the

propagation constant:

$$S_{-mm} = -\frac{h'_m}{2h_m} - \frac{1}{2h^2_m} \cdot \int \tilde{\rho}' (v^m_z)^2. \qquad (25.21)$$

Simpler formula (25.4c) is valid only in waveguides with constant density.

Let us use finally formulas (25.19) and (25.20) to the problem of the medium, in which there are two regions with the different values of the parameters, between which there is a sharp interface, so that ρc^2 and ρ they are the piecewise constant functions of coordinates. In thin layer about the interface in which derived on z in (25.19) and (25.20) they are different from zero, they are replaced on formulas of type (25.9) by derivatives on n. From the continuity condition of pressure and normal component of rate, it follows that the change p^m and v^m in layer can be described, using the designations of formula (8.21) and assuming that in one of the media $\rho=1$, by the relationship/ratios

$$p^m(n) = p^m;\ v^m_n(n) = v^m_n;\ v^m_z(n) = v^m_z/\rho(n); \qquad (25.22)$$
$$v^m_s(n) = v^m_s/\rho(n).$$

Page 201.

Integrating along direction n in formulas (25.19) and (25.20), we will obtain respectively for coupling coefficients (with j=m)

$$S_{jm} = \frac{-1}{2h_j(h_m - h_j)} \oint \nu \left[p^m p^j \Delta\left(\frac{1}{\rho c^2}\right) - (v_z^m v_z^j - v_s^m v_s^j)\, \Delta\left(\frac{1}{\rho}\right. - \right.$$

$$\left. - v_n^m v_n^j \Delta(\rho) \right] ds \qquad (25.23)$$

and for a derivative of the wave number

$$h_m^{2\prime} = \oint \nu \left\{ (p^m)^2 \Delta\left(\frac{1}{\rho c^2}\right) - [(v_z^m)^2 - (v_s^m)^2]\, \Delta\left(\frac{1}{\rho}\right) - (v_n^m)^2 \Delta(\rho) \right\} ds. \qquad (25.24)$$

§26. Acoustic tapered welds.

1. Determination of coupling coefficients for tapered welds we will begin from auxiliary problem of scattering during small step during coupling of two semi-infinite waveguides with close sections. The value of step δ(efined as into §6, is considered small in comparison with all linear dimensions of problem. On step falls to the left the wave of number m(m>0) from single amplitude; let us search for amplitude P_j of the transmitted into right waveguide wave of number j(j>0).

Let us continue the surface of right waveguide into region z<0 and will consider this surface as undisturbed, but true surface of both of waveguides - as deformed. On the deformed surface are valid simple boundary conditions (23.5a) or (23.5b). Let us replace strain

is equivalent to the boundary condition which must be fulfilled on the close undeformed surface. After being steady this boundary condition, it will be possible to search for the solution of wave equation in the region, limited by the simple undeformed surface.

Equivalent boundary condition near the surface on which correctly boundary condition (23.5a), is given by M. A. Isakovich [108]. It is the simple consequence of the resolution of Taylor and takes the form

$$\Psi = l\frac{\partial\Psi}{\partial N}. \qquad (26.1)$$

Page 202.

Here l — distance between the deformed and undeformed surfaces, standard N is directed to wall, and $l>0$, if transition from the deformed surface to that not deformed occurs in direction N (Fig. 5).

If on true (deformed) surface is satisfied boundary condition (23.5b), then equivalent boundary condition for the function, which satisfies wave equation (23.4), has the form

$$\frac{\partial\Psi}{\partial N} = -k^2\Psi l - \frac{\partial}{\partial s}\left(l\frac{\partial\Psi}{\partial s}\right) - \frac{\partial}{\partial z}\left(l\frac{\partial\Psi}{\partial z}\right). \qquad (26.2)$$

This condition it is easy to obtain, integrating (23.4) by the low region, situated between to both surfaces, transforming integral of divergence on the formula of Gauss and utilizing (23.5b) and

smallness l. This condition can be recorded in another form, valid also for the functions, which do not satisfy wave equation, but this form contains the second derivative along the normal, which proves to be inconvenient in further calculations.

As (6.1), conditions (26.1) and (26.2) are disrupted in that part of the undisturbed surface, that composes final angle with the disturbed surface. In our problem this occurs in low field near joint, which leads to the error for order δ^2; however for determining the coupling coefficient, it suffices to find the amplitude of the scattered wave with an accuracy to δ.

Computation P_i is reduced to the determination of field in regular waveguide with boundary conditions (26.1) or (26.2), where by Ψ it is necessary to understand the potentials of the incident wave $\psi^m e^{-i\beta_m z}$ and $l = \delta(s)$ with z<0, $l = 0$ with z>0.

2. Let us examine first waveguide with soft walls, i.e., let us search for solution of equation (23.3), that satisfies on true boundary conditions (23.5a). Let us introduce auxiliary potential Φ - potential of the wave of number -j(j=m), that satisfies on the entire undeformed surface condition (23.5a). For two solutions of wave equation Ψ and of Φ, is correct the identity

$$\oint \left(\Phi \frac{\partial \Psi}{\partial N} - \Psi \frac{\partial \Phi}{\partial N} \right) dS = 0, \qquad (26.3)$$

where the integral is undertaken over any locked surface. We will propagate it over section z=+0 and the lateral surface of the undisturbed waveguide with z<0 let us assume that $\varkappa$ contains low imaginary component, which makes it possible to eliminate the effect of region z=-∞.

Page 203.

With z=+0 the potential Ψ contains the sum of the potentials of all direct waves; however, at the selected value of Φ in integral according to this section, will fall out (as a result of the condition of orthogonality) all term/component/addends, except one, and the integral will be equal to $2ih_jP_j$. In the integral over lateral surface, is present only second term/component/addend, in which Ψ is calculated from (26.1). The dependence of integrand on z is given by factor $e^{-i(h_m-h_j)z}$, and integration on z can be produced in an explicit form. Substituting in (26.3) and solving relative to the unknown value P_j, we obtain the solution of the auxiliary problem indicated:

$$P_j = \frac{1}{2h_j(h_j-h_m)} \oint \delta \frac{\partial\psi^m}{\partial n}\frac{\partial\psi^j}{\partial n}\, ds \quad (j>0,\ m>0,\ j\neq m). \quad (26.4)$$

In order from formula for $P_j(j>0)$ to obtain expression for a

coupling coefficient S_{jm} is must, as shown into §6, to replace under integral the height/altitude of step δ by the tangent of tangent inclination ν. Thus,

$$S_{jm} = \frac{1}{2h_j(h_j - h_m)} \oint \nu \frac{\partial \psi^m}{\partial n} \frac{\partial \psi^j}{\partial n} ds \quad (j \neq m). \qquad (26.5)$$

This expression we would obtain, if they would search for first the amplitude of backward wave, scattered on step, and then was utilized was second of relationship/ratios (6.15) and condition (23.10).

Let us find now amplitude P_j for a waveguide with rigid walls. For this, it is necessary in (26.3) to select Φ equal to the potential of the wave of number $-j(j \neq m)$, that satisfies on the entire undeformed surface condition (23.5b). In integral (26.3) over lateral surface second term is equal to zero, during the computation of first term, it is necessary to substitute for d ψ /dN expression (26.2). Utilizing the presence in (26.2) derivatives on the same to the variables s and z, on which is conducted the integration, it is possible by integration somewhat to simplify in parts the resultant expression. After replacing, further, in integral for P_j δ on ν, we will obtain the unknown expression for a coupling coefficient

$$S_{jm} = \frac{1}{2h_j(h_j - h_m)} \left[(\varkappa^2 - h_j h_m) \oint \nu \psi^m \psi^j ds - \right.$$
$$\left. - \oint \nu \frac{\partial \psi^m}{\partial s} \frac{\partial \psi^j}{\partial s} ds \right] \quad (j \neq m). \qquad (26.6)$$

This formula also is valid with any signs j and m.

Page 204.

Formulas (26.5) and (26.6) it is easy to generalize to the case when on the part of the wall of waveguide are satisfied conditions (23.5a), and on part - condition (23.5b).

3. Expressions (26.5) and (26.6) contain the same integrals, as formula for coupling coefficients in radiowaveguides for two electrical waves (9.5b) and for two magnetic waves (9.5a); however, with other coefficients. As we already noted, this difference is connected with the fact that for membrane/diaphragm functions in electrodynamics on the boundary of an irregular waveguide are satisfied the conditions, different from (23.5). However, expressions for a derivative of wave number in acoustic and electrodynamic problems agree, since these expressions can be obtained from formulas for a derivative of the eigenvalue of equations for membrane/diaphragm functions and are determined therefore the boundary conditions for these functions in regular waveguides, but these boundary conditions are identical in both problems. Expressions for h_m' in acoustic waveguides are obtained directly from formulas (9.6), into which it is necessary to introduce obvious changes due to difference in standardization (3.17) and (23.9). From formula (9.6b)

follows the expression for a derived wave number in tapered weld with the soft walls

$$h_m^{2'} = \oint \nu \left(\frac{\partial\psi^m}{\partial n}\right)^2 ds, \qquad (26.7)$$

and, according to (9.6a) for a waveguide with rigid walls it will be

$$h_m^{2'} = (\varkappa^2 - h_m^2) \oint \nu (\psi^m)^2 ds - \oint \nu \left(\frac{\partial\psi^m}{\partial s}\right)^2 ds. \qquad (26.8)$$

With both types of boundary conditions, the universal communication/connection between S_{mm} and h_m' (25.4b) is retained; however, relationship/ratio (25.4c), according to last/latter formulas, is correct only for waveguides with soft walls. For waveguides with rigid walls, is fulfilled the equation, analogous (9.4):

$$S_{-mm} = -\frac{h_m'}{2h_m} - \frac{1}{2} \oint \nu (\psi^m)^2 ds. \qquad (26.9)$$

In particular, for the fundamental wave $h_m = \varkappa = \text{const}$, i.e. $h_m' = 0$, and in (26.9) is retained only second term/component/addend. For this wave $(\psi^m)^2 = 1/S$, where S(z) - sectional area, and

$$S_{-mm} = -\frac{1}{2S} \oint \nu ds = -\frac{S'}{2S}. \qquad (26.10)$$

Page 205.

Hence is obtained known result, that the reflection coefficient from a small step is equal with respect to the modulus to the half of the

ratio/relation to the area of step to sectional area. The coefficient of reflection, fundamental wave from the section of long irregular waveguide with slowly changing section is equal, according to (8.34) and (26.10)

$$P_{-m} = -\frac{1}{2}\int_{0}^{L}\frac{d\ln S}{dz}e^{-2i\gamma z}dz. \qquad (26.11)$$

4. Obtained are above by method of small heterogeneities expressions for S_{jm} and h'_m it is possible to find also from results of preceding/previous paragraph. Repeating construction §9, one should for this compare this waveguide of the section, filled by media, interface between which coincides with the boundary of this irregular waveguide. Then it is necessary to produce passage to the limit to similar by the values of the parameters of external from these two media with which boundary conditions on the surface of this medium will coincide with boundary conditions on soft or rigid walls. Auxiliary waveguide coincides in this case with this tapered weld. Expansions (25.1) and system of differential equations (25.3) is retained, but in expressions (25.10), (25.23) and (25.24) will have to carry out a passage to the limit indicated.

In this case, will prove to be necessary to open an indeterminancy/uncertainty of the type $0\cdot\infty$. which it appears in these formulas during passage to the limit. In analogous by electrodynamic examination was conducted the passage to the limit $|\varepsilon|\to\infty$. and for

the expansion/disclosure of indeterminancy/uncertainty, was used the Leontovich boundary condition (9.1). The analcg of this condition for a soft wall is the boundary condition

$$\Psi = -\frac{i}{\varkappa}\frac{\partial\Psi}{\partial n}, \qquad (26.12)$$

which, as it is easy to check, approximately is fulfilled on the boundary of body with the high value of wave number $|\varkappa|$; wave number must be considered composite. Condition this bears local character, i.e., it does not depend on the structure of applied field. When $|\varkappa|\to\infty$ (26.12) passes in (23.5a). Thus, results for a waveguide of alternating/variable section with soft wall can be obtained on the assumption that in environment $|c|\to 0$, and the density of both of media is identical.

Page 206.

In this case it is possible to use to the formulas of the first point/item of the preceding/previous paragraph, to substitute under (25.10) boundary condition (26.12) and then to set/assume $|\varkappa|\to\infty$. It is easy to check that (25.10) in this case really/actually it will pass in (26.5) and (26.7).

The local boundary condition, in limit which converts in (23.5b), cannot be obtained on the assumption that the density is

everywhere constant. Really/actually, on one hand, so that the condition would be local wave number in medium must be composite and approach infinity, i.e., must be $|c|\to 0$. On the other hand, so that in the limit would be obtained boundary condition on rigid wall, must be $|\rho\cdot c|\to\infty$. Thus, density must approach infinity. Boundary condition takes the form

$$\frac{\partial\Psi}{\partial n} = -i\frac{\omega}{c\rho}\Psi, \qquad (26.13)$$

where the density of internal medium is placed to equal unity. When $|c\rho|\to\infty$ (26.13) passes in (23.5b).

In order to obtain expression for S_{lm} in tapered weld with rigid wall, it is necessary to apply the formulas of the second point/item of the preceding/previous paragraph. The analysis, produced in [107], it showed, that if we in (25.23) and (25.24) substitute (26.13) and then to place $|c\rho|\to\infty$, that for the coupling coefficients and derivative of wave number really/actually to be obtained formulas (26.6) and (26.8), found above by another method.

There is one additional difference in the mathematical apparatus of the method of cross sections for waveguides with soft and rigid walls. For waveguides with soft walls, the potentials Ψ and ψ^m in (25.1a) satisfy one and the same boundary condition, and to row/series (25.1a) the admissibly term-by-term application/use of an

operator ∇^2. In this way it is possible to again obtain formulas (25.3) and (25.4a); however, in (25.4a) integration in this case, it occurs according to the section of variable area. The conversion of an integral of such type to contour integral was produced into §9; during the standardization of functions ψ^m, by that accepted in present chapter, it will be

$$\int \psi^l \frac{\partial \psi^m}{\partial z} dS = \frac{1}{h_m^2 - h_l^2} \oint v \frac{\partial \psi^m}{\partial n} \frac{\partial \psi^l}{\partial n} ds, \qquad (26.14)$$

whence again follows formula (26.5).

For waveguides with rigid walls, the term-by-term application/use of an operator ∇^2 to row/series (25.1a) is inadmissible, since boundary conditions for Ψ and ψ^m in this case do not agree. Therefore the direct application/use of resolution of the unknown potential in row/series in terms of potentials in regular waveguides [106] leads to essential difficulties.

Page 207.

This chapter contains the construction of the formalism of the method of cross sections for irregular acoustic waveguides. In the bent waveguides the field is described by the system of equations (24.12), in which F_{lm} they are given in (24.17). For waveguides with

alternating/variable medium or alternating/variable section, the system of equations is given in (25.3), coupling coefficients take form (25.4), (25.7) and (25.19) or with respect (to 26.5) and (26.6). The theory of the combined irregular acoustic waveguides can be constructed analogously how this is done into §§10 and 18 for radiowaveguides.

§27. Conclusion.

The developed above mathematical apparatus makes it possible with single method to perform the calculation of the broad class of irregular waveguides, to compare between themselves different irregularities, to examine questions concerning their mutual compensation. Expression for the wave amplitudes, scattered in irregular sections, takes the simple analytical form; this facilitates the studies of the effect of different parameters of section on the value of the scattered energy. The comparison of elementary (small) and final irregularities makes it possible to give the simple physical treatment of the phenomena of scattering, which facilitates the qualitative analysis of different equipment/devices.

1. Further development of method of cross sections must first of all consist of application/use of this apparatus to calculation of row/series of concrete/specific/actual waveguide systems. In

proportion to development of wide-band waveguide technology, a number of problems, subjected to theoretical analysis, will increase, and these same problems will cause some expansion of the theoretical bases of method. In proportion to gaining of experience and, most importantly, as in calculations in of increasing degree it will be utilized machine technology, theory of multi-waveguide transitions it will acquire engineering character, similar this occurred in the theory of single-wave rectangular waveguide.

There is great interest also in the problems of relatively less wide-range waveguide transitions with the rapidly changing parameters - sharply curved, short matching sections between the waveguides of different sections, dielectric lenses, etc. Any method of the analysis of such systems will require numerical calculations.

Page 208.

In the method of cross sections, the problem is reduced to the system first-order of ordinary differential equations, which are in this case, for our opinion, sufficiently simple and common mathematical apparatus.

On one hand, by this method it is possible to design the propagation of waves in the unshielded systems, such, as one-wire

line with the ground wave, radio duct, etc. The disturbance/breakdown of uniformity in such systems, for example their bending, causes the emission/radiation of energy, so that complete field in irregular system contains not only plane, but also spherical or cylindrical waves. In the mathematical sense the presence of the continuous spectrum will lead to the appearance of contour integrals instead of the sums, which describe field in closed systems. Main station it will be in this case not the system of ordinary differential equations, but integrodifferential equations of the type

$$\frac{\partial P(j, z)}{\partial z} = \int S(j, \nu, z) P(\nu, z) d\nu, \qquad (27.1)$$

where P(j,z) - amplitude in the integral representation of the fields

$$E(x, y, z) = \int P(j, z) E(x, y, z, j) dj. \qquad (27.2)$$

For systems with the slowly changing parameters at the correct selection of functions E(x,y,z,j) the nucleus of equation (27.1) will contain by factor the low parameter, and equation (27.1) will be able to be solved by the method of successive approximations.

On the other hand, the method of cross sections let us use, probably, not only to the equations of electrodynamics and acoustics. Fundamental of this method completely elementary they bear, actually, geometric character. Therefore its mathematical apparatus can be used for a wide class of the systems of partial differential equations.

This method, for example, can be studied the problems of the propagation of thermal disturbance/perturbations or elastic vibrations along cylinder or layer with alternating/variable along the length properties, etc. Fundamental difficulty of each specific case consists in this case in the reasonable selection of functions on which is conducted the resolution, i.e., values of the type "E $E(x,y,z)$ in (8.2-8.3) or $E(x,y,z,j)$ in (27.2).

Page 209.

It follows, it goes without saying, to bear in mind, that as any other version of the method of Fourier, method of cross sections are limited by the requirement of the linearity of problem.

2. In waveguide problems method of cross sections becomes less effective, the greater number of equations must be simultaneously examined, i.e., the more waves with comparable amplitudes simultaneously it is propagated in waveguide. To those pores, while the plumbing is designed for the transfer of energy or the signal on one type wave, mathematical of the apparatus of the method of cross sections corresponds to the structure of electromagnetic field and is the most convenient means of studying this field. but if frequency is very great, and in line simultaneously there are many waves of the comparable between themselves amplitude, then field expansion in

row/series of type (8.2-8.3) becomes the artificial reception/procedure, which do not agree itself with the physical essence of transmitting of energy on waveguide; the effectiveness of the method of cross sections it decreases. Under these conditions the computation of the amplitude of each of the waveguide waves becomes very complex and at the same time unnecessary matter.

In this case, we enter into the region, which occupies the intermediate position between geometric optics and usual waveguide electrodynamics. Many equipment/devices, utilized in waveguides at very high frequencies, imitate appropriate optics - mirror, prism, lens, etc. A number of such equipment/devices recently rapidly increases [109-114]. Their theories yet does not exist; it is unclear even, in what concepts - ray/beams or waveguide waves - one should describe field in such systems. The method of cross sections can prove to be useful only with the initial approach to these problems; combining it with the methods of asymptotic addition, perhaps, be managed to install the physical and mathematical character of that apparatus which will have to create for the analysis of these systems. It is possible that more promising turns out the examination, which generalizes radiation treatment. In any case, the study of this intermediate quasi-optical case, i.e., geometric optics of wide waveguides, becomes one of the fundamental problems of waveguide electrodynamics.

Page 230.

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Page 214.

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C513 PICATINNY ARSENAL	1	FTD	
C535 AVIATION SYS COMD	1	CCN	1
C591 FSTC	5	ASD/FTD/NIIS	3
C619 MIA REDSTONE	1	NIA/PHS	1
D008 NISC	1	NIIS	2
H300 USAICE (USAREUR)	1		
P005 DOE	1		
P050 CIA/CRB/ADD/SD	2		
NAVORDSTA (50L)	1		
NASA/KSI	1		
AFIT/LD	1		
LLL/Code L-389	1		
NSA/1213/TDL	2		