

# Quick Python Intro

A *very* quick crash-course in the basics of Python, broadly covering

- Libraries (NumPy and Matplotlib, specifically)

- Variables

- Whitespace

- Array slicing and assignment

## Step 1: 1-D Linear Convection

$$\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0 \quad (1)$$

### Math

This section introduces the reader to the “parts“ of the PDE and how to discretize them.

- Introduce the idea of a grid

- Expand equation using the definition of a derivative

- Discretize into small chunks

- Re-arrange to solve for  $u_i^{n+1}$

- Initial and boundary conditions

### Python

- Importing libraries

- Assigning variables

- Basic 2D plotting

- Simple for-loops

YouTube videos on order of convergence, truncation error, etc...

## Step 2: 1-D Nonlinear Convection

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = 0 \quad (2)$$

### Math

- Introduce non-linear PDE equation

- Expand equation using definition of derivative

- Discretize

- Solve for  $u_i^{n+1}$

## CFL Condition

A short side-trip into the CFL condition, order of convergence and blowing things up.

## Math

The Courant number

Explanation of blow-up behavior when wave travels a distance  $> dx$  during a time  $dt$

## Python

Quick introduction to defining a function to use code repeatedly

## Step 3: 1-D Diffusion

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2} \quad (3)$$

## Math

Introduce diffusion equation

Discretize 2nd order derivative using Taylor series expansion

Discretize time derivative using def. of derivative

## Python

Nothing new, still no functions being used (yet)

## Step 4: 1-D Burgers' Equation

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2} \quad (4)$$

## Math

Introduce Burgers' Equation

Note that it is combination of diffusion and non-linear convection

Introduce different I.C. and B.C. for periodic behavior

e.g. What does  $u_{i+1}^n$  mean at the end of the frame?

## Python

Introduce Sympy

Pretty printing

Symbolic solving of derivatives

Usage of Lambdify to make solutions 'accessible' to Numpy

Matplotlib

Plotting multiple lines per plot

Setting line styles

Legends

## Array Operations

Another brief interlude to introduce handling calculations with array operations instead of iterating over the entire array.

## Python

Array operations, slicing and copying

Note about using the `%%timeit` magic function to compare performance

## Step 5: 2D Linear Convection

### Math

Introduction to 2D grid

Extension of current discretization rules into  $i, j$  flatland

Discretize 2D equation and solve for unknown

### Python

meshgrid

Axes3D

surf and wireframe plots

Demonstration that nested for-loop results and array operations results are the same

## Step 6: 2D Nonlinear Convection

### Math

Introduction of coupled PDEs

Discretization of two equations

Solving for both  $u_{i,j}^{n+1}$  and  $v_{i,j}^{n+1}$

## Step 7: 2D Diffusion

### Math

Introduction to 2D diffusion equation

Discretization of equation, etc...

### Python

Nothing new, although functions are used to display results (probably should switch this over to `jsanim`)

## Step 8: 2D Burgers'