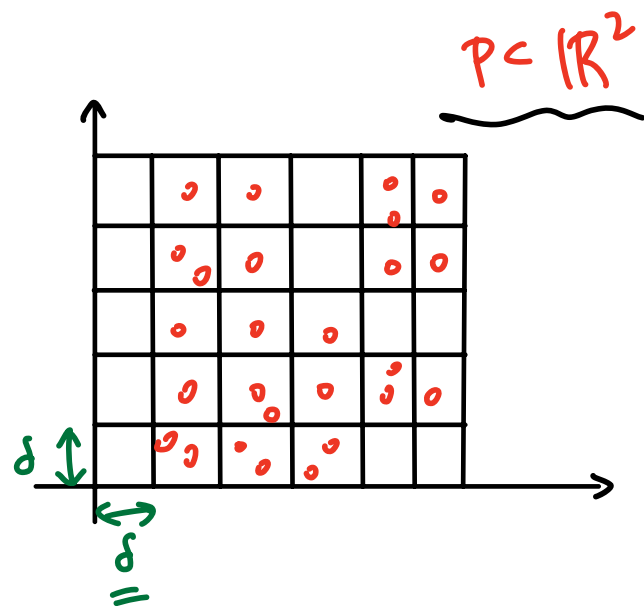


Grids

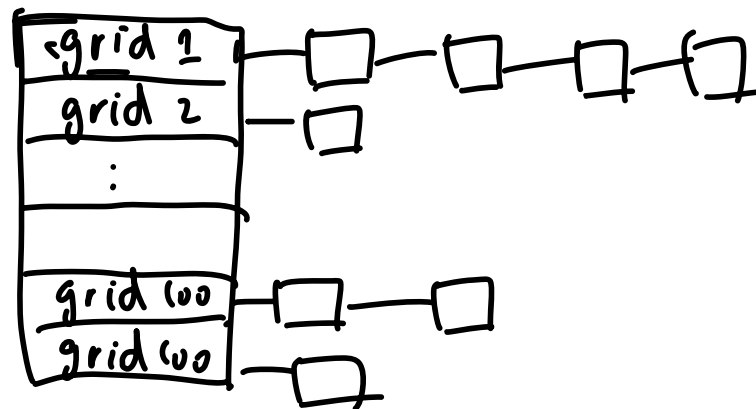
2025. 2. 28

δ -grids



Hash

$$\underbrace{(x, y)} \mapsto \left(\underbrace{\left\lfloor \frac{x}{\delta} \right\rfloor}_{\downarrow}, \underbrace{\left\lfloor \frac{y}{\delta} \right\rfloor}_{\downarrow} \right)$$



$O(1)$

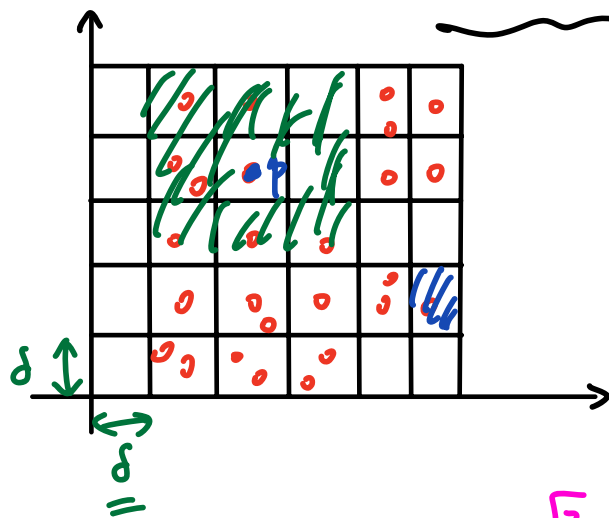
Lemma. $P \subset \mathbb{R}^2$, $\delta > 0$, \exists algorithm decide min-distance

$$\underline{d(P)} \begin{cases} < \delta \\ \geq \delta \end{cases} \quad \text{in time } O(n)$$

Idea. $\forall p \in P$, decide. $\exists q \in P$ s.t. $\|p - q\| < \delta$?

$$p. \quad B(p, \delta) \cap P = \emptyset$$

$P \subset \mathbb{R}^2$



If. $\boxed{\#} \geq \underline{100 \text{ points}}$, output $d(P) < \delta$ $O(1)$
 else $\forall q \in \boxed{\#}$, compute $\|p - q\|$ $O(1)$

If $\boxed{\#} \geq 100 \text{ points}$

$\exists \boxed{\#} \geq 10 \text{ points}$

$\exists \boxed{\#} \geq 2 \text{ points}$

$$\|u - v\| \leq \frac{\sqrt{2}}{2} \delta < \delta$$

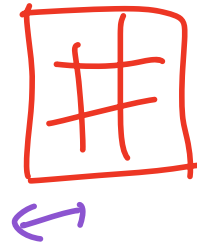
$d(P)?$

\exists algorithm compute $d(P)$

in expected $O(n)$

$\left\{ \begin{array}{l} \delta \ll d(P) \\ \delta \gg d(P) \end{array} \right.$

P



$$\frac{O(\Omega(n \log n))}{\Omega(n \log n)}$$

A blue double-headed vertical arrow is positioned below the fraction.

Idea: maintain $\delta \approx d(P)$