

# ISO Presentation

## Quantum Programming

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# Plan

## ① Introduction

## ② Problem statement

## ③ Quantum Computing

Classical computing parallel

Quantum Computing formalism

## ④ Implementing a Quantum Recommendation system

## ⑤ Errata

## ⑥ Conclusion

# Quantum Computing applied to Machine Learning

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- $m$  users can rate  $n$  products (binary voting)
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- Can we predict whether user  $j$  will like product  $k$ ? Can we **recommend** a product to this user?

## Mathematical description

### Binary recommendation system

Let  $\mathbf{P} \in \{0; 1\}^{m \times n}$ , whose coefficients are known with a given probability. A quantum recommendation system, given  $\hat{\mathbf{P}}$  the incomplete representation of  $\mathbf{P}$  and an user  $j$ , predicts  $k$  such that  $\mathbf{P}_{j,k} = 1$  with high probability.

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### Problem statement

How does the real-world implementation of the Quantum Recommendation System algorithm differs from its theoretic implementation?



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## Classical computing

- Operates on bit-strings 0110110001

## Classical computing

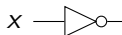
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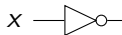
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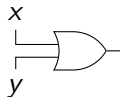
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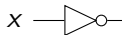
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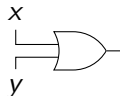
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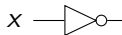


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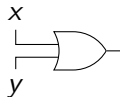
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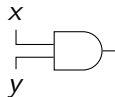
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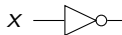
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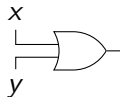


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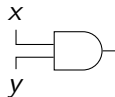
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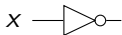


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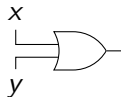
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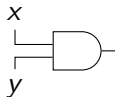
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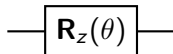
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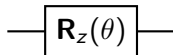


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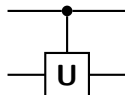


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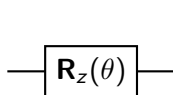
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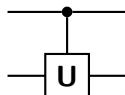
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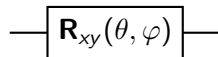
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(c) The  $R_{xy}(\theta, \varphi)$  gate

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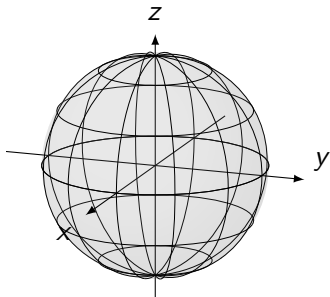
A qubits-string is called a **quantum register**.

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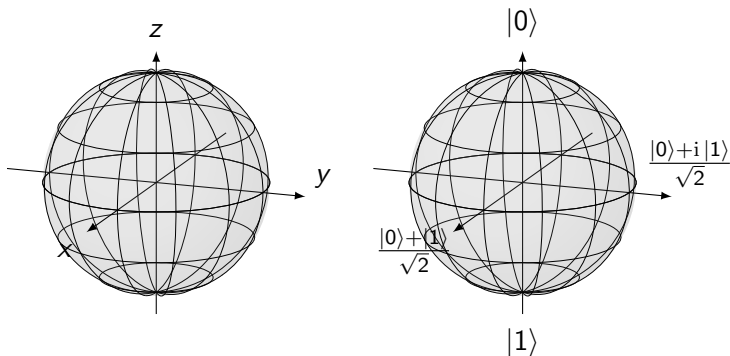
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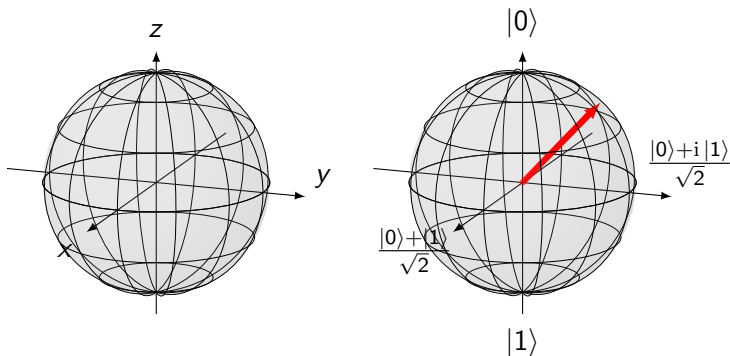
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Loading  $\mathbf{x}$  means creating  $|x\rangle$  from  $|0\rangle^{\otimes \lceil \log_2(n) \rceil}$  with a polylogarithmic number of gates in  $n^3$ .

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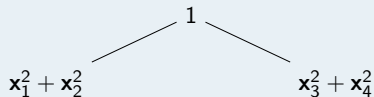
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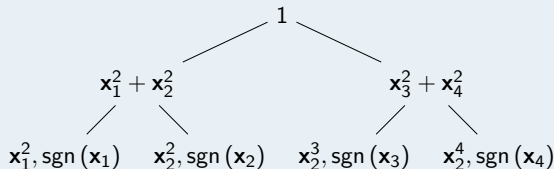
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It is possible, using QRAM, to load  $|x\rangle$  using  $2^k$  controlled rotations at level  $k$  of the QRAM tree in  $O(n)$ .

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$$\mathbf{L} |\theta_k\rangle |x\rangle |\text{target}\rangle = \begin{cases} |\theta_k\rangle |x\rangle \mathbf{R}_y(\theta_k) |\text{target}\rangle & \text{if } |x\rangle = |k\rangle \\ |\theta_k\rangle |x\rangle |\text{target}\rangle & \text{otherwise} \end{cases}.$$

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### The $z$ -rotation

Given an angle  $\theta$ , the rotation around the  $z$ -axis of the Bloch sphere is given by the gate:

$$\mathbf{R}_z(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}.$$

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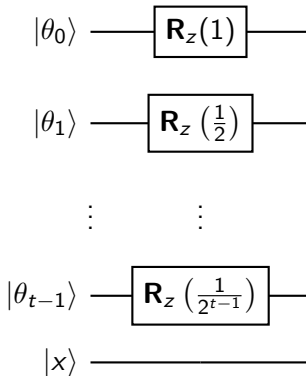
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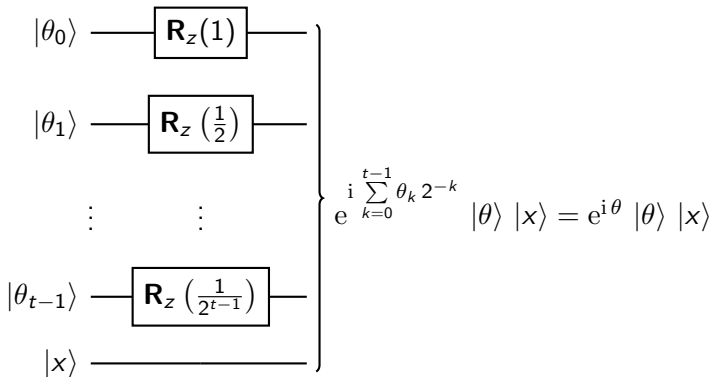
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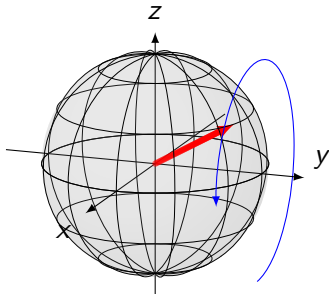
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Is it possible to transform a y-rotation into a z-rotation of the same angle?

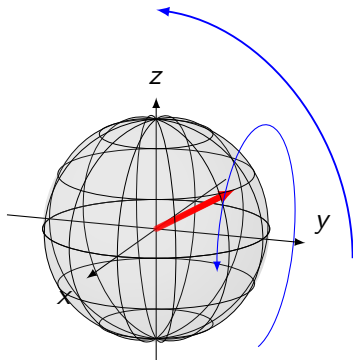


## Converting a $z$ -rotation to a $y$ -rotation



**Figure:** An example of a transformation of a  $y$ -rotation into a  $z$ -rotation

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- Error-correction, number of qubits,  $\dots$

## References I

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