ISO Presentation

Quantum Programming

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Quantum Computing applied to Machine Learning

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- Can we predict whether user j will like product k? Can we **recommend** a product to this user?

Mathematical description

Binary recommendation system

Let $\mathbf{P} \in \{0\,;\,1\}^{m\times n}$, whose coefficients are known with a given probability. A quantum recommendation system, given $\hat{\mathbf{P}}$ the incomplete respresentation of $\mathbf{P}$ and an user j, predicts k such that $\mathbf{P}_{j,k} = 1$ with high probability.

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Problem statement

How does the real-world implementation of the Quantum Recommendation System algorithm differs from its theoretic implementation?

Plan

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Classical computing

- Operates on bit-strings 0110110001

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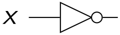

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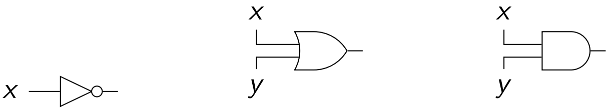

(a) NOT gate **(b)** OR gate **(c)** AND gate

Figure: Classical gates

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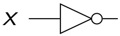

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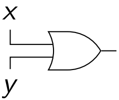

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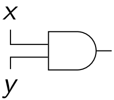

(c) AND gate

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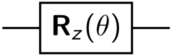

(a) The $\mathbf{R}_z(\theta)$ gate

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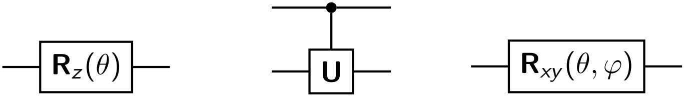

(a) The $\mathbf{R}_z(\theta)$ gate **(b)** A controlled gate **(c)** The $\mathbf{R}_{xy}(\theta, \varphi)$ gate

Figure: Quantum gates

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A qubits-string is called a **quantum register**.

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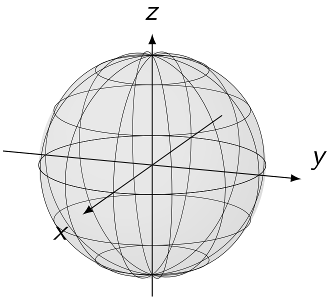

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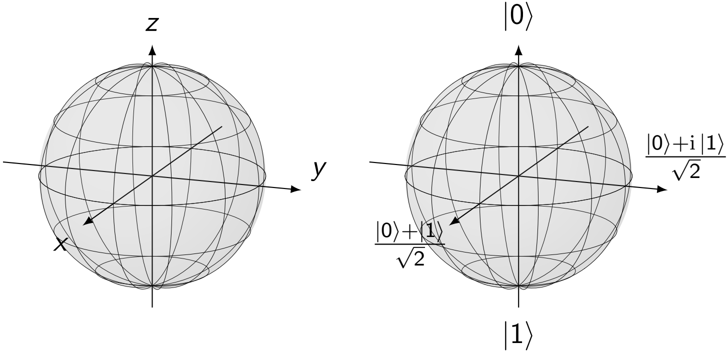

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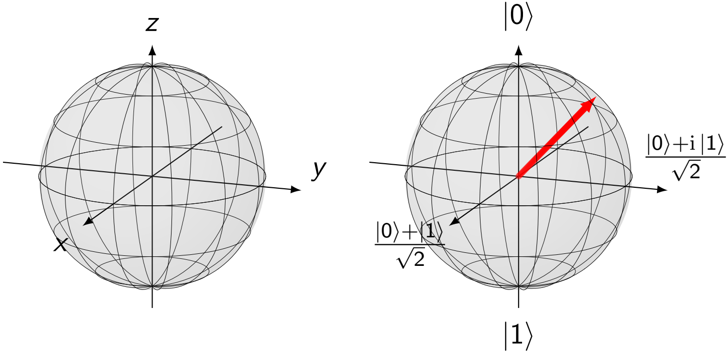

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- Linearity allows to apply operations on several qubits-string at a time
- Possibility to get a probabilistic view of a qubit at the price of forcing it into a certain state. Measurement destroys information.

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Loading $\mathbf{x}$ means creating $|x\rangle$ from $|0\rangle^{\otimes \lceil \log_2(n) \rceil}$ with a polylogarithmic number of gates[3].

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[3] Prakash, "Quantum Algorithms for Linear Algebra and Machine Learning."

QRAM[4]

A QRAM is a binary tree whose leaves stores the coefficients of a vector $\mathbf{x}$ and whose nodes stores the sum of its leaves values.

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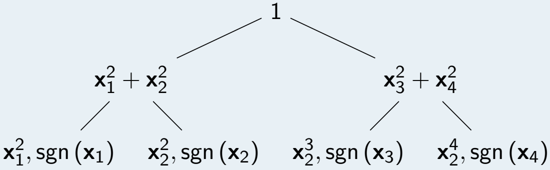

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- Goal: parallelize the execution of the rotations using superposition

Parallel execution

Parallel execution of controlled rotations

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QRAM function assumption

It is possible, using QRAM, to design a gate $\mathbf{L}_k$ such that:

$$\mathbf{L}_k \; |k\rangle \; |0\rangle^{\otimes t} = |k\rangle \; \left|\overline{\theta_k}\right\rangle$$

where $\overline{\theta_k}$ is the best t-bits approximation of θ_k.

A simpler problem

Parallel execution of rotations

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The z-rotation

Given an angle θ, the rotation around the z-axis of the Bloch sphere is given by the gate:

$$\mathbf{R}_z(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\,\theta} \end{pmatrix}\ .$$

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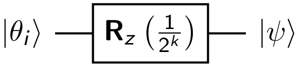

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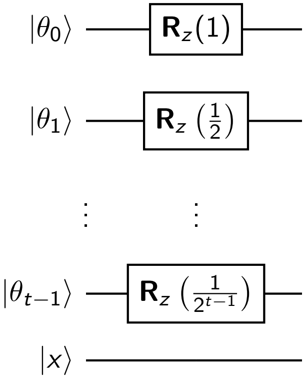

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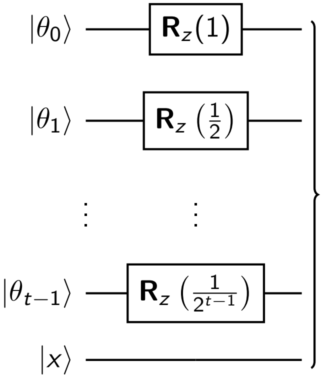

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Is it possible to transform a y-rotation into a z-rotation of the same angle?

Converting a z-rotation to a y-rotation

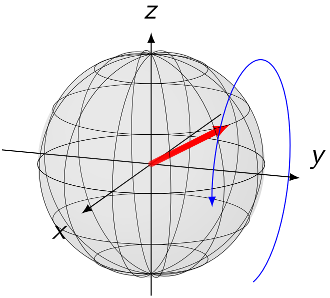
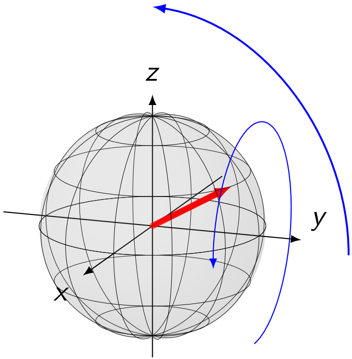


Figure: An exemple of a transformation of a y-rotation into a z-rotation

Converting a z-rotation to a y-rotation



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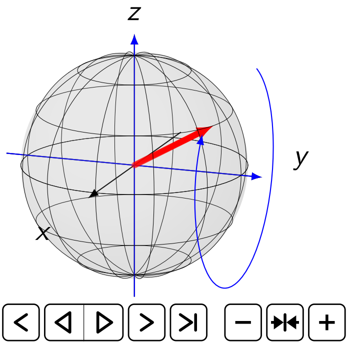

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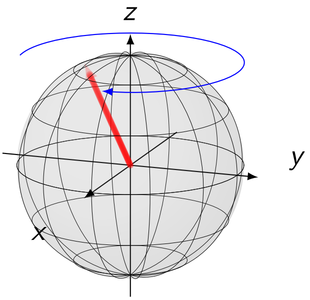

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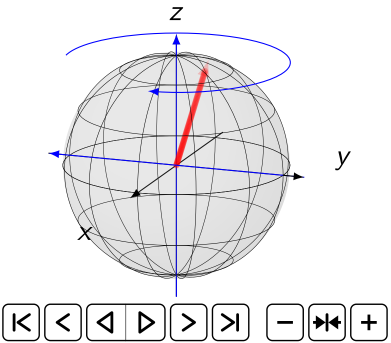

Figure: An exemple of a transformation of a y-rotation into a z-rotation

Plan

Errata

- It **is** possible to apply QPE without measuring

Errata

- It **is** possible to apply QPE without measuring (the error analysis is complex though)

Plan

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Conclusion

- QPE, strings comparison

Conclusion

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- Difference between real-world implementation and theoretic implementation

Conclusion

- QPE, strings comparison
- Difference between real-world implementation and theoretic implementation (dequantized algorithms)

References I

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