ISO Presentation

Quantum Programming

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Quantum Computing applied to Machine Learning

- Goal: exponential speed-up

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- Not all ratings are known
- Can we predict whether user j will like product k?

Mathematical description

Binary recommendation system

Let $\mathbf{P} \in \{0\,;\,1\}^{m \times n}$, whose coefficients are known with a given probability. A quantum recommendation system, given $\hat{\mathbf{P}}$ the incomplete respresentation of $\mathbf{P}$ and an user j, predicts k such that $\mathbf{P}_{j,k} = 1$ with high probability.

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Impossible problem without assumptions

Problem assumptions

The low-rank assumption

Let $\mathbf{P}$ be the unknown preference matrix. For a given precision parameter ε, there exists a rank-k matrix $\mathbf{T}$ such that:

$$\|\mathbf{P} - \mathbf{T}\|_F \leqslant \varepsilon \|\mathbf{P}\|_F \,.$$

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$\implies$ there are k types of typical users.

Problem initialization

$\mathbf{P}$ can be sampled into $\hat{\mathbf{P}}$ such that $\hat{\mathbf{P}}$ holds the low-rank assumption. Hence, ε is assumed to be known.

A Quantum Recommendation System

- Introduced in 2016[3]

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Problem statement

How does the real-world implementation of the Quantum Recommendation System algorithm differs from its theoretic implementation?

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Classical computing

- Operates on bit-strings 0110110001

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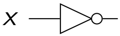

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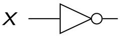

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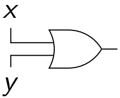

(b) OR gate

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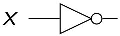

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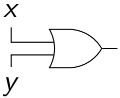

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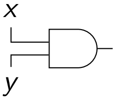

(c) AND gate

Figure: Classical gates

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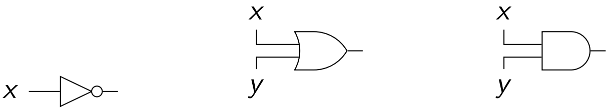

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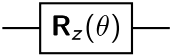

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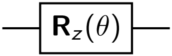

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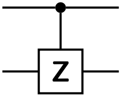

(b) The $\mathrm{C} - \mathbf{Z}$ gate

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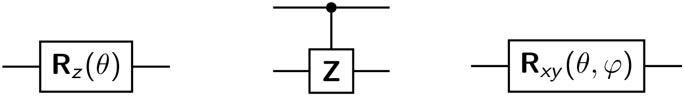

(a) The $\mathbf{R}_z(\theta)$ gate **(b)** The $\mathrm{C} - \mathbf{Z}$ gate **(c)** The $\mathbf{R}_{xy}(\theta, \varphi)$ gate

Figure: Quantum gates

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A qubits-string is called a **quantum register**. It is mathematically described as the tensor product of these qubits, that is:

$$|q_1\rangle |q_2\rangle \cdots |q_n\rangle = \bigotimes_{j=1}^{n} |q_k\rangle .$$

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A qubit $|q\rangle$ can be represented on the so-called Bloch sphere:

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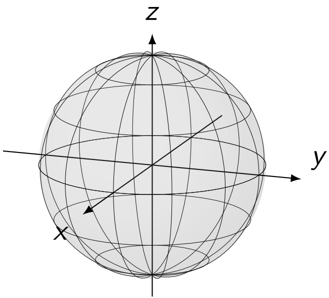

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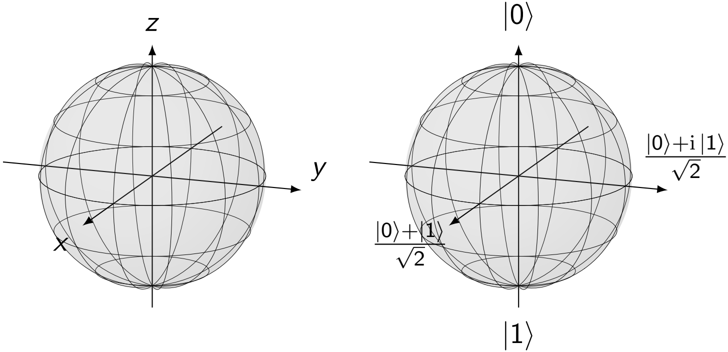

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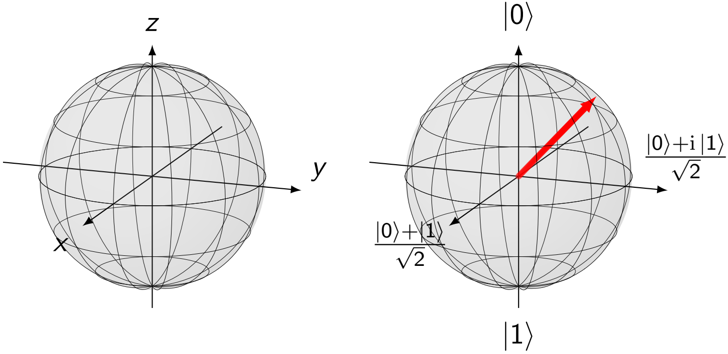

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- Linearity allows to apply operations on several qubits-string at a time
- Possibility to get a probabilistic view of a qubit at the price of forcing it into a certain state. Measurement destroys information.

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Loading a vector as a quantum state

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Let $\mathbf{x} \in \mathbf{R}^n$ be a normalized vector. Then, its associated quantum state is:

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Loading $\mathbf{x}$ means creating $|x\rangle$ from $|0\rangle^{\otimes \lceil \log_2(n) \rceil}$ with a polylogarithmic number of gates[4].

[4] Prakash, "Quantum Algorithms for Linear Algebra and Machine Learning."

QRAM[5]

A QRAM is a binary tree whose leaves stores the coefficients of a vector $\mathbf{x}$ and whose nodes stores the sum of its leaves values.

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- Uses 2^k rotations gates around the y-axis at level k of the tree whose angle are determined using QRAM
- Goal: parallelize the execution of the rotations using superposition

Parallel execution

Parallel execution of controlled rotations

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QRAM function assumption

It is possible, using QRAM, to design a gate $\mathbf{L}_k$ such that:

$$\mathbf{L}_k \; |k\rangle \; |0\rangle^{\otimes t} = |k\rangle \; \left|\overline{\theta_k}\right\rangle$$

where $\overline{\theta_k}$ is the best t-bits approximation of θ_k.

A simpler problem

Parallel execution of rotations

It is possible to get the state $|\theta\rangle\ \mathrm{e}^{\mathrm{i}\,\theta}\ |x\rangle$ from the state $|\theta\rangle\ (\alpha\ |0\rangle + \beta\ |1\rangle)$

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The z-rotation

Given an angle θ, the rotation around the z-axis of the Bloch sphere is given by the gate:

$$\mathbf{R}_z(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{\mathrm{i}\,\theta} \end{pmatrix} \, .$$

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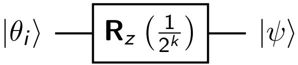

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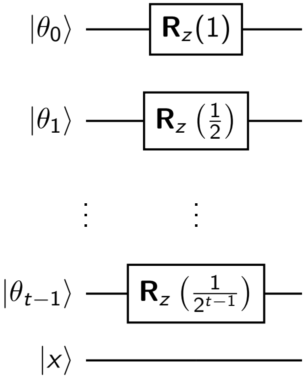

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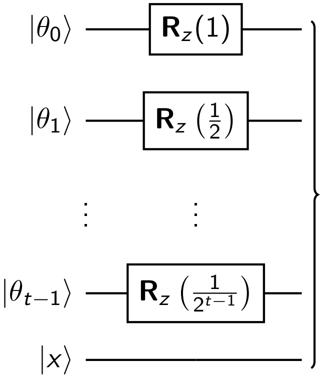

$$\mathrm{e}^{\mathrm{i} \sum_{k=0}^{t-1} \theta_k \, 2^{-k}} \, |\theta\rangle \, |x\rangle = \mathrm{e}^{\mathrm{i}\,\theta} \, |\theta\rangle \, |x\rangle$$

Figure: Parallel execution of z-rotations

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Is it possible to transform a y-rotation into a z-rotation of the same angle?

Converting a z-rotation to a y-rotation

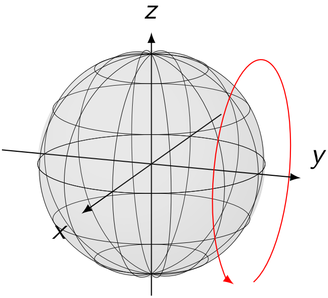
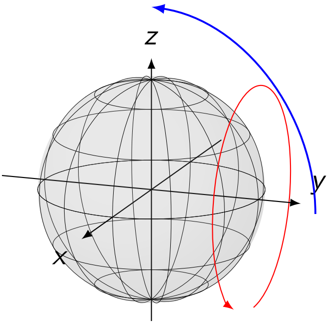


Figure: An exemple of a transformation of a y-rotation into a z-rotation

Converting a z-rotation to a y-rotation

Rotate the sphere



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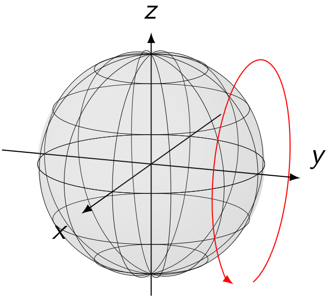

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Errata

- Patate

Conclusion

- Patate

References I

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