Reliability Analysis Assignment 2 (personal)

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2.4

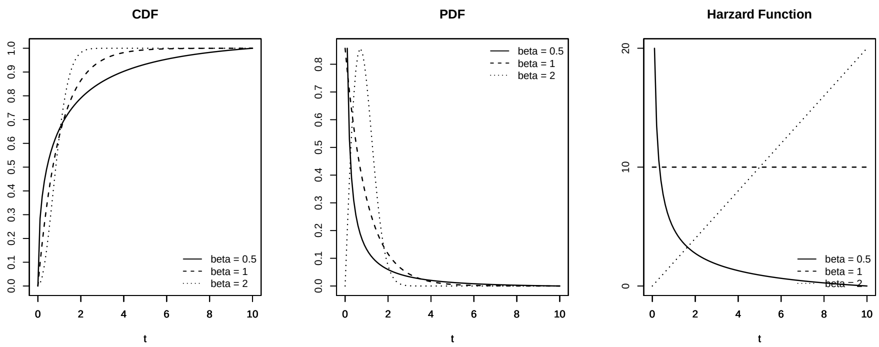
(a)

$$f(t) = \frac{d}{dt}F(t) = -exp\{-(\frac{t}{\eta})^{\beta}\}(-\beta(\frac{t}{\eta})^{\beta-1}\frac{1}{\eta}) = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}exp\{-(\frac{t}{\eta})^{\beta}\}$$

(b)

$$h(t) = \frac{f(t)}{S(t)} = \frac{f(t)}{1-F(t)} = \frac{\frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}exp\{-(\frac{t}{\eta})^{\beta}\}}{1-(1-exp\{-(\frac{t}{\eta})^{\beta}\})} = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}$$

(c)



2.7 $h(t) = 75 \cdot 10^{-9} \Rightarrow (75 \cdot 10^{-9})(20 \cdot 1500)(8760 \cdot 2) = 39.42$ (units)

2.13

(i) Given $f(t)$

$$F(t) = \int_0^t f(x)dx$$
$$S(t) = 1 - F(t) = 1 - \int_0^t f(x)dx$$
$$h(t) = \frac{f(t)}{1-F(t)} = \frac{f(t)}{1 - \int_0^t f(x)dx}$$
$$H(t) = -log(1-F(t)) = -log\left(1 - \int_0^t f(x)dx\right)$$

(ii) Given $F(t)$

$$f(t) = \frac{d}{dt}F(t)$$
$$S(t) = 1 - F(t)$$
$$h(t) = \frac{\frac{d}{dt}F(t)}{1-F(t)}$$
$$H(t) = -log(1-F(t))$$

(iii) Given $S(t)$

$$f(t) = \frac{d}{dt}F(t) = \frac{d}{dt}(1-S(t))$$
$$F(t) = 1 - S(t)$$
$$h(t) = \frac{\frac{d}{dt}(1-S(t))}{S(t)}$$
$$H(t) = -log(S(t))$$

(iv) Given $h(t)$

$$[l]H(t) = \int_0^t h(x)dx$$
$$S(t) = exp(-H(t)) = exp\left(-\int_0^t h(x)dx\right)$$
$$F(t) = 1 - exp(-H(t)) = 1 - exp\left(-\int_0^t h(x)dx\right)$$
$$f(t) = \frac{d}{dt}F(t) = exp\left(-\int_0^t h(x)dx\right) \cdot h(t)$$

(v) Given $H(t)$

$$
\begin{aligned}
&h(t) = \frac{d}{dt}H(t) \\
&S(t) = exp\{-H(t)\} \\
&F(t) = 1 - exp\{-H(t)\} \\
&f(t) = \frac{d}{dt}F(t) = exp\{-H(t)\} \cdot (H(t))
\end{aligned}
$$

2.14

(a)

$$
\begin{aligned}
&Pr(t_{i-1} < T \leq t_i | T > t_{i-1}) \\
=&\frac{Pr(t_{i-1} < T \leq t_i,\ T > t_{i-1})}{Pr(T > t_{i-1})} = \frac{Pr(t_{i-1} < T \leq t_i)}{1 - Pr(T > t_{i-1}))} \\
=&\frac{F(t_i) - F(t_{i-1})}{1 - F(t_{i-1})} = \frac{\pi_i}{S(t_{i-1})}
\end{aligned}
$$

(b) (i)

$$
\begin{aligned}
\pi_1 = Pr(t_0 < T \leq t_1) &= \frac{Pr(t_0 < T \leq t_1, T > t_0)}{Pr(T > t_0)} Pr(T > t_0) \\
&= Pr(t_0 < T \leq t_1 | T > t_0) \cdot 1 = p_1
\end{aligned}
$$

(ii)

$$
\begin{aligned}
\pi_i &= Pr(t_{i-1} < T \leq t_i) = \frac{Pr(t_{i-1} < T \leq t_i)}{Pr(T > t_{i-1})} Pr(T > t_{i-1}) \\
&= p_i \cdot \frac{Pr(T > t_{i-1})}{Pr(T > t_{i-2})} \cdot \frac{Pr(T > t_{i-2})}{Pr(T > t_{i-3})} \cdots \frac{Pr(T > t_i)}{Pr(T > t_0)} \cdot Pr(T > t_0) \\
&= p_i \cdot \frac{Pr(T > t_{i-2}) - Pr(t_{i-2} < T \leq t_{i-1})}{Pr(T > t_{i-2})} \cdots \frac{Pr(T > t_1) - Pr(t_1 < T \leq t_2)}{Pr(T > t_1)} \cdot Pr(T > t_0) \\
&= p_i \cdot (1 - p_{i-1}) \cdot (1 - p_{i-1}) \cdots (1 - p_1) \cdot 1 = p_i \prod_{j=1}^{i-1} (1 - p_j) \\
&\forall i = 1, 2, \ldots, m
\end{aligned}
$$

(iii)

$$
\begin{aligned}
\pi_{m+1} &= p_{m+1} \prod_{j=1}^{m+1-1} (1-p_j) \\
&= \frac{Pr(t_m < T \le t_{m+1})}{Pr(T > t_m)} \prod_{j=1}^{m} (1-p_j) \xlongequal{t_{m+1}\to\infty} \frac{Pr(T > t_m)}{Pr(T > t_m)} \prod_{j=1}^{m} (1-p_j) \\
&= \prod_{j=1}^{m} (1-p_j)
\end{aligned}
$$

(c)

$$
\begin{aligned}
p_i = \frac{F(t_i) - F(t_{i-1})}{1 - F(t_{i-1})} &= \frac{\pi_i}{S(t_{i-1})} \\
&\because \pi_i > 0 \quad \forall i = 1, \ldots, m+1 \\
&\therefore S(t_i) > 0 \quad i.e. F(t_i) < 1 \quad \forall i = 1, \ldots, m+1 \\
&\therefore p_i \neq 0 \wedge p_i \neq 1 \\
&\because p_i = Pr(t_{i-1} < T \le t_i | T > t_{i-1}) \\
&\therefore 0 \le p_i \le 1 \\
&\Rightarrow 0 < p_i < 1 \quad \forall i = 1, \ldots, m+1
\end{aligned}
$$

(d) By the statement (i), (ii) and (iii) in (b),

$$
\begin{aligned}
\pi_i &= Pr(t_{i-1} < T \le t_i) = \frac{Pr(t_{i-1} < T \le t_i)}{Pr(T > t_{i-1})} Pr(T > t_{i-1}) \\
&= p_i \cdot S(t_{i-1}) = p_i \prod_{j=1}^{i-1} (1-p_j) \quad \forall i = 1, \ldots, m+1 \\
&\Rightarrow S(t_i) = \prod_{j=1}^{i} (1-p_j) \quad \forall i = 1, \ldots, m+1
\end{aligned}
$$

2.16

$$
\begin{aligned}
&\because H(T) = -log(1 - F(T)) \\
&\quad Pr(H(T) \le t) = Pr(-log(1 - F(T)) \le t) \\
&= Pr(1 - F(T) \le e^{-t}) = Pr(F(T) \ge 1 - e^{-t}) \\
&= e^{-t}
\end{aligned}
$$

$\Rightarrow H(t)$ follows an exponential distribution.