National Sun Yat-sen University, Department of Mathematics

Reliability Analysis Assignment 3 (group)

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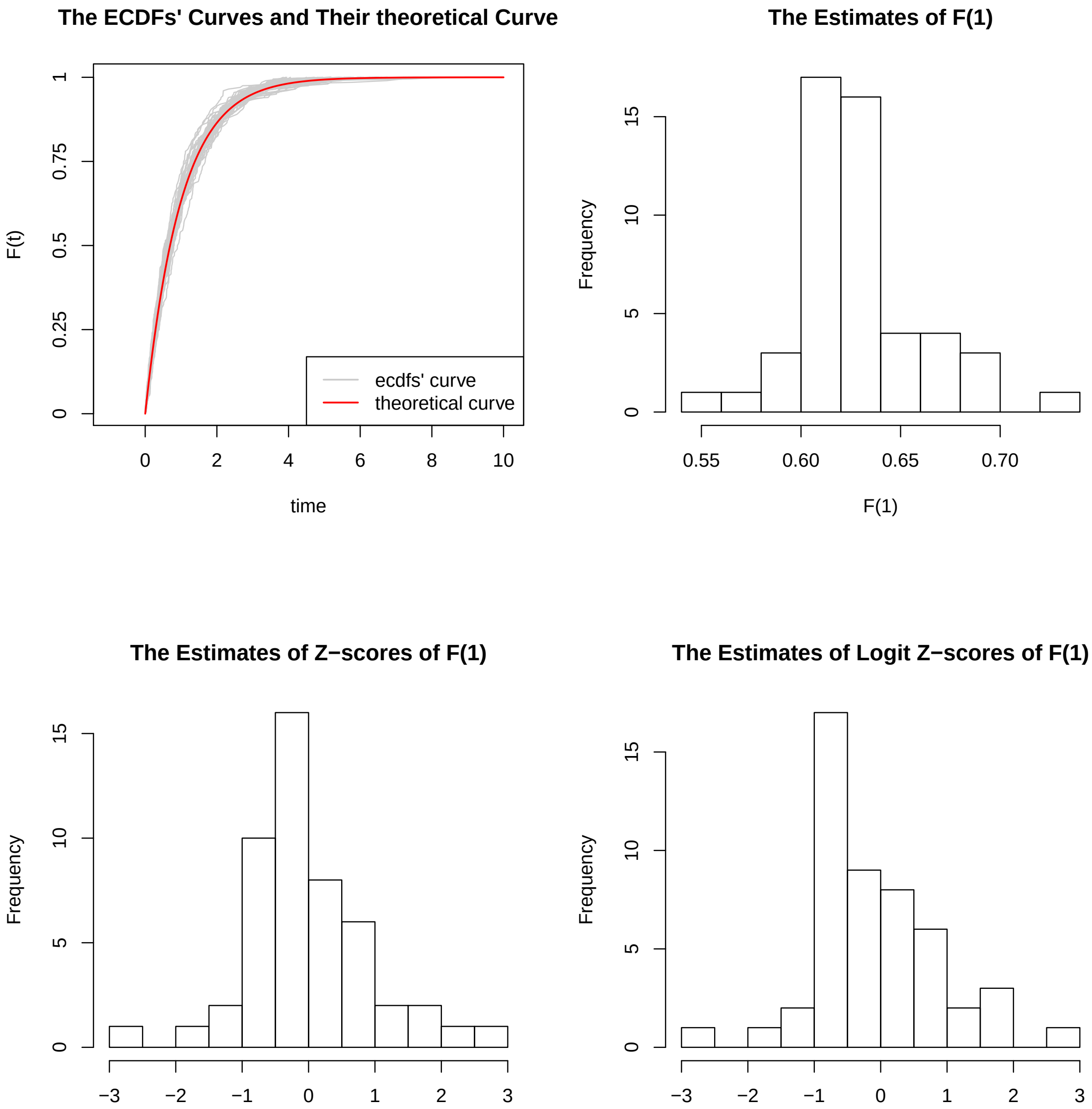
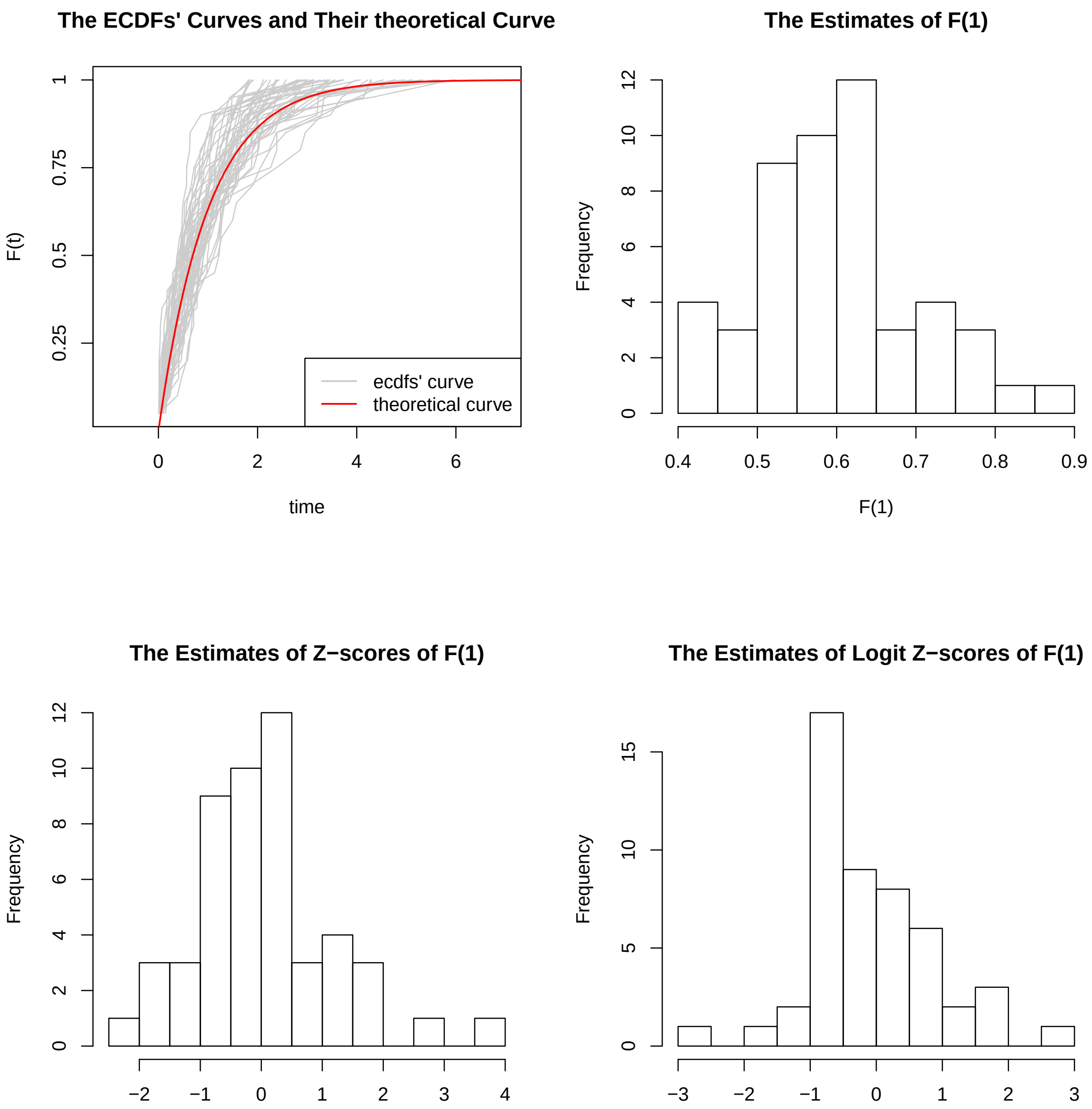
3.11

$n = 200$



The ECDFs' Curves and Their theoretical Curve
F(t)
0
0.25
0.5
0.75
1
0
2
4
6
8
10
time
ecdfs' curve
theoretical curve
The Estimates of F(1)
Frequency
0
5
10
15
0.55
0.60
0.65
0.70
F(1)
The Estimates of Z−scores of F(1)
Frequency
0
5
10
15
−3
−2
−1
0
1
2
3
The Estimates of Logit Z−scores of F(1)
Frequency
0
5
10
15
−3
−2
−1
0
1
2
3

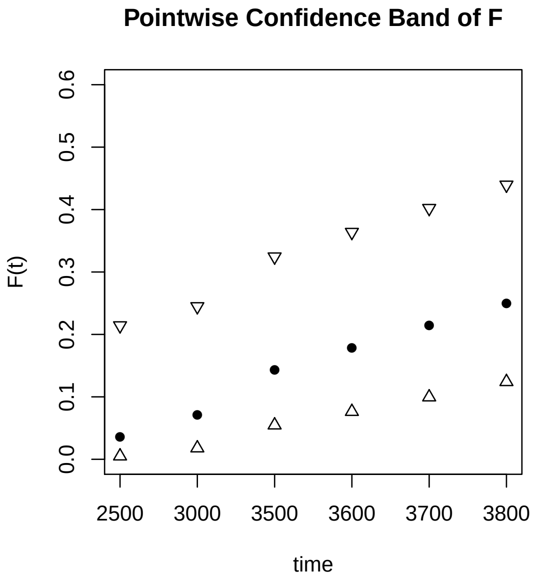
$n = 20$

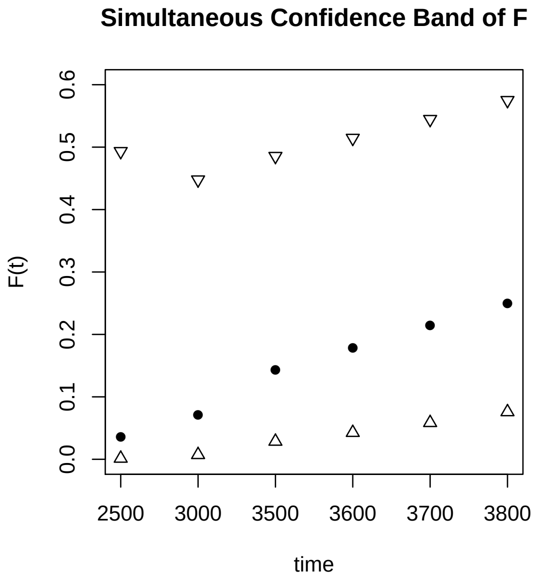


The ECDFs converge to the theoretical curve as the sample size n increases. Also, $\widehat{F}(1)$, $Z_{\widehat{F}(1)}$ and $Z_{logit(\widehat{F}(1))}$ with $n = 200$ are more like normal distributions than those with $n = 20$.

3.12

i	t_i	$\widehat{F}(t_i)$	$\widehat{se}_{\widehat{F}(t_i)}$	$Logit\ C.I._L$	$Logit\ C.I._U$	$Simu\ C.I._L$	$Simu\ C.I._U$
1	2500	0.0357	0.0351	0.0050	0.2142	0.0014	0.4933
2	3000	0.0714	0.0487	0.0179	0.2448	0.0072	0.4478
3	3500	0.1429	0.0661	0.0547	0.3245	0.0286	0.4855
4	3600	0.1786	0.0724	0.0763	0.3638	0.0427	0.5145
5	3700	0.2143	0.0775	0.0996	0.4021	0.0585	0.5447
6	3800	0.2500	0.0818	0.1241	0.4395	0.0759	0.5750





(a) The experiment checked at particular time points, this set is a multiply-censored data.

(b) (c) (d) (e)
The answer is shown in the table and the figures above.

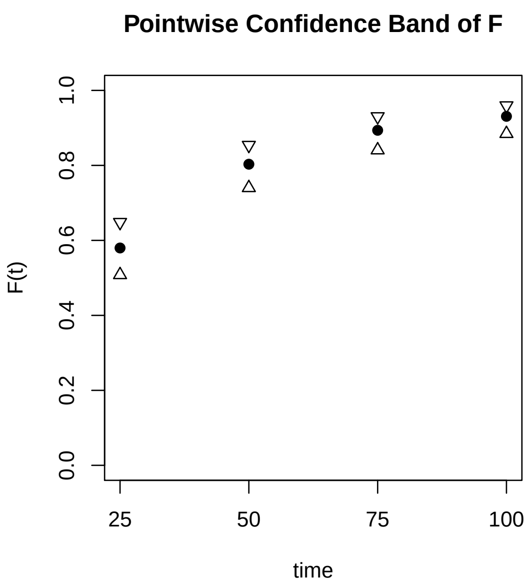
(f) Pointwise confidence interval is for only "one" particular t_i .Simultaneous confidence band quantify sampling uncertainty simultaneously over a range of t_i's, $t_i \in (a, b)$. Therefore, simultaneous confidence band is wider than pointwise one at the same level.

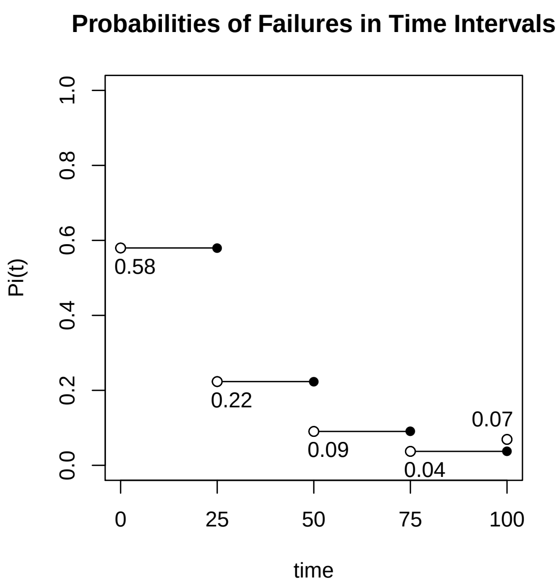
3.17

i	t_i	$\widehat{F}(t_i)$	$\widehat{se}_{\widehat{F}(t_i)}$	$C.I._L$	$C.I._U$
1	1	0.0133	0.0066	0.0050	0.0350
2	2	0.0384	0.0128	0.0198	0.0730
3	3	0.0582	0.0187	0.0307	0.1076

3.19

i	t_i	$p(t_i)$	$\widehat{F}(t_i)$	$C.I._L$	$C.I._U$
1	25	0.5798	0.5798	0.5081	0.6483
2	50	0.5316	0.8032	0.7402	0.8539
3	75	0.4595	0.8936	0.8409	0.9303
4	100	0.3500	0.9309	0.8846	0.9594



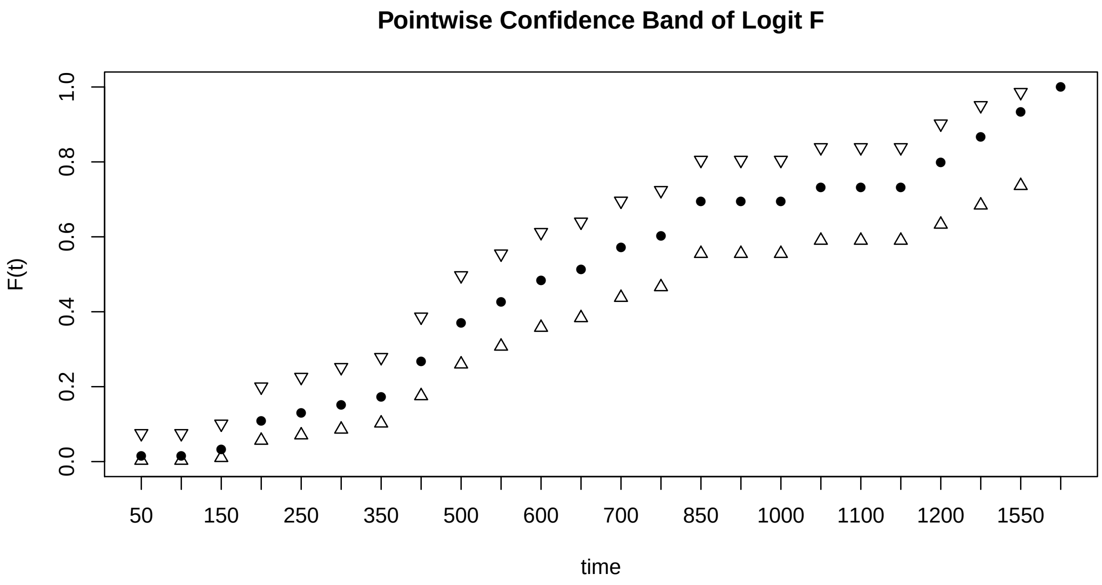


(a) (b) (c) (d)
The answer is shown in the table and the figures above.

(e) The failure rate was high initially, however it decreased as the time went by.

3.21 All the answers are shown in the table and the figure.

i	t_i	$\widehat{F}(t_i)$	$\widehat{se}_{\widehat{F}(t_i)}$	$C.I._L$	$C.I._U$
1	50	0.0153	0.0152	0.0029	0.0752
2	100	0.0153	0.0152	0.0029	0.0752
3	150	0.0330	0.0230	0.0103	0.1006
4	200	0.1089	0.0421	0.0564	0.1997
5	250	0.1296	0.0460	0.0708	0.2254
6	300	0.1511	0.0496	0.0861	0.2516
7	350	0.1734	0.0531	0.1024	0.2784
8	400	0.2679	0.0647	0.1754	0.3864
9	500	0.3706	0.0732	0.2600	0.4967
10	550	0.4266	0.0767	0.3076	0.5547
11	600	0.4839	0.0790	0.3579	0.6121
12	650	0.5126	0.0796	0.3837	0.6399
13	700	0.5717	0.0802	0.4378	0.6958
14	750	0.6023	0.0801	0.4663	0.7241
15	850	0.6940	0.0772	0.5551	0.8049
16	900	0.6940	0.0772	0.5551	0.8049
17	1000	0.6940	0.0772	0.5551	0.8049
18	1050	0.7323	0.0764	0.5903	0.8386
19	1100	0.7323	0.0764	0.5903	0.8386
20	1150	0.7323	0.0764	0.5903	0.8386
21	1200	0.7992	0.0815	0.6332	0.9018
22	1350	0.8661	0.0771	0.6844	0.9508
23	1550	0.9331	0.0610	0.7364	0.9858
24	1700	1.0000	NA	NA	NA



3.22

(a)

$$
\begin{aligned}
L(\pi) &= \pi_1^{d_1} \cdot \pi_2^{d_2} \cdots \pi_m^{d_m} \cdot (\pi_2 + \cdots + \pi_{m+1})^{r_1} \cdot (\pi_3 + \cdots + \pi_{m+1})^{r_2} \cdots (\pi_m + \pi_{m+1})^{r_{m-1}} \cdot \pi_{m+1}^{r_m} \\
&= \pi_1^{d_1} \cdot \pi_2^{d_2} \cdots \pi_m^{d_m} \cdot S(t_1)^{r_1} \cdot S(t_2)^{r_2} \cdots S(t_m)^{r_m}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\because \ \ p_i &= \frac{\pi_i}{S(t_{i-1})}, \ \forall \ i = 1, 2, \ldots, m \\
L(p) &= \left(p_1 \cdot S(t_0)\right)^{d_1} \cdot \left(p_2 \cdot S(t_1)\right)^{d_2} \cdots \left(p_m \cdot S(t_{m-1})\right)^{d_m} \cdot S(t_1)^{r_1} \cdot S(t_2)^{r_2} \cdots S(t_m)^{r_m} \\
&= p_1^{d_1} \cdot p_2^{d_2} \cdots p_m^{d_m} \cdot S(t_1)^{r_1+d_2} \cdot S(t_2)^{r_2+d_3} \cdots S(t_{m-1})^{r_{m-1}+d_m} \cdot S(t_m)^{r_m} \\
&= p_1^{d_1} \cdot p_2^{d_2} \cdots p_m^{d_m} \cdot (1-p_1)^{r_1+d_2} \cdot \left((1-p_1)(1-p_2)\right)^{r_2+d_3} \cdots \\
&\quad \cdots \left((1-p_1)(1-p_2)\cdots(1-p_{m-1})\right)^{r_{m-1}+d_m} \cdot \left((1-p_1)(1-p_2)\cdots(1-p_m)\right)^{r_m} \\
&= p_1^{d_1} \cdot p_2^{d_2} \cdots p_m^{d_m} \cdot (1-p_1)^{r_m + \sum_{i=1}^{m-1}(d_{i+1}+r_i)} \cdot (1-p_2)^{r_m + \sum_{i=2}^{m-1}(d_{i+1}+r_i)} \cdots \\
&\quad \cdots (1-p_{m-1})^{r_m + \sum_{i=m-1}^{m-1}(d_{i+1}+r_i)} \cdot (1-p_m)^{r_m} \\
&= p_1^{d_1} \cdot p_2^{d_2} \cdots p_m^{d_m} \cdot (1-p_1)^{n_1-d_1} \cdot (1-p_2)^{n_2-d_2} \cdots (1-p_m)^{n_m-d_m} \\
&= \prod_{j=1}^{m} p_j^{d_j} (1-p_j)^{n_j-d_j}
\end{aligned}
$$

(c)

$$log\left(L(p)\right) = \sum_{j=1}^{m}\left(d_j log(p_j) + (n_j - d_j) log(1 - p_j)\right)$$

$$\text{Let, } \quad \frac{\partial log\left(L(p)\right)}{\partial p_j} = \frac{d_j}{p_j} - \frac{n_j - d_j}{1 - p_j} = 0 \quad \Rightarrow \quad p_j = \frac{d_j}{n_j}$$

$$\because \quad \left.\frac{\partial^2 log\left(L(p)\right)}{\partial {p_j}^2}\right|_{p_j = \frac{d_j}{n_j}} < 0 \qquad \therefore \widehat{p}_j = \frac{d_j}{n_j}$$

(d)

$$S(t_i) = \prod_{j=1}^{i}(1 - p_j) \quad \Rightarrow \quad log(S(t_i)) = \sum_{j=1}^{i} log(1 - p_j) = \sum_{j=1}^{i} log(q_j)$$

$$\Rightarrow \quad \frac{1}{S(t_i)} \cdot \frac{\partial S(t_i)}{\partial q_j} = \frac{1}{q_j} \quad \Rightarrow \quad \frac{\partial S(t_i)}{\partial q_j} = \frac{S(t_i)}{q_j}$$

$$\because \widehat{S}(t_i) \approx S(t_i) + \sum_{j=1}^{i} \left.\frac{\partial S(t_i)}{\partial q_j}\right|_{q_j} (\widehat{q}_j - q_j) = S(t_i) + \sum_{j=1}^{i} \frac{\partial S(t_i)}{\partial q_j}(\widehat{q}_j - q_j)$$

$$\Rightarrow \quad Var\left(\widehat{S}(t_i)\right) = \sum_{j=1}^{i}\left(\frac{\partial S(t_i)}{\partial q_j}\right)^2 Var(\widehat{q}_j) = \sum_{j=1}^{i}\left(\frac{\partial S(t_i)}{\partial q_j}\right)^2 Var(1 - \widehat{p}_j)$$

$$= \sum_{j=1}^{i}\left(\frac{\partial S(t_i)}{\partial q_j}\right)^2 Var(\widehat{p}_j) = \sum_{j=1}^{i}\left(\frac{\partial S(t_i)}{\partial q_j}\right)^2 \frac{p_j(1 - p_j)}{n_j}$$

$$= (S(t_i))^2 \sum_{j=1}^{i}\left(\frac{1}{1 - p_j}\right)^2 \frac{p_j(1 - p_j)}{n_j} = (S(t_i))^2 \sum_{j=1}^{i} \frac{p_j}{n_j(1 - p_j)}$$

$$\therefore \widehat{Var}\left(\widehat{S}(t_i)\right) = (S(t_i))^2 \sum_{j=1}^{i} \frac{\widehat{p}_j}{n_j(1 - \widehat{p}_j)}$$