NATIONAL SUN YAT-SEN UNIVERSITY, DEPARTMENT OF MATHEMATICS

Reliability Analysis Assignment 3 (personal)

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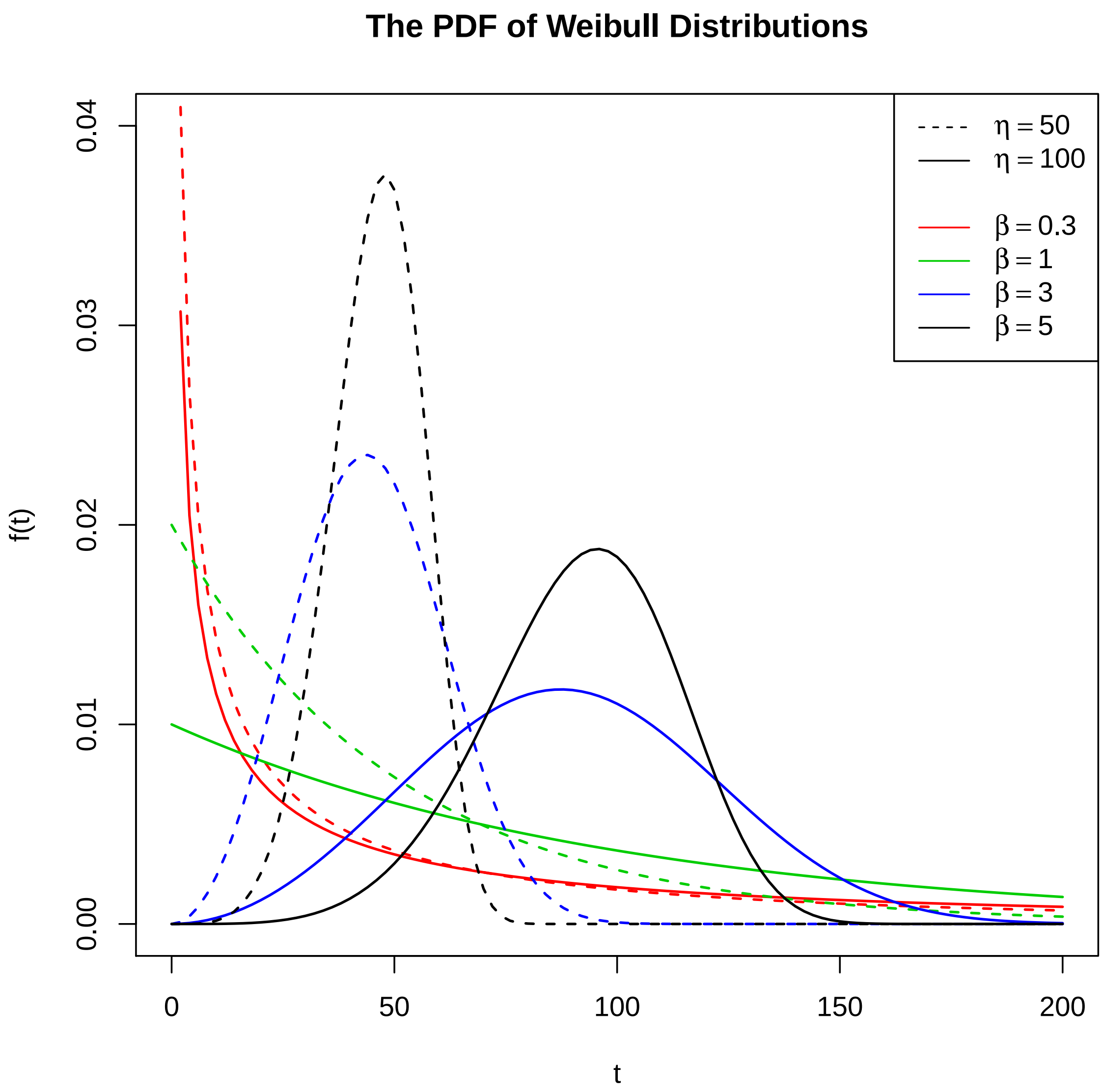
4.15

(a) For $\sigma = \frac{1}{\beta}$ and $\mu = log(\eta)$, coefficient of variation,

$$\gamma_2 = \frac{\sigma}{\mu} = \frac{1}{\beta log(\eta)} \quad .$$

(b)

γ_2	$\beta = 0.5$	$\beta = 1$	$\beta = 3$	$\beta = 5$
$\eta = 50$	0.511	0.256	0.085	0.051
$\eta = 100$	0.434	0.217	0.072	0.043



(c) For the fixed η, $F(t)$ converges to 0 and γ_2 increases as β increases.

5.1

(a)

$$\begin{aligned} F(t;\boldsymbol{\theta}) &= \frac{1}{2}F(t;\theta_1) + \frac{1}{2}F(t;\theta_1) \\ &= \frac{1}{2}\left(1 - exp(-t)\right) + \frac{1}{2}\left(1 - exp\left(-\frac{t}{5}\right)\right) \\ &= 1 - \frac{1}{2}\left(exp(-t) + exp\left(-\frac{t}{5}\right)\right) \end{aligned}$$

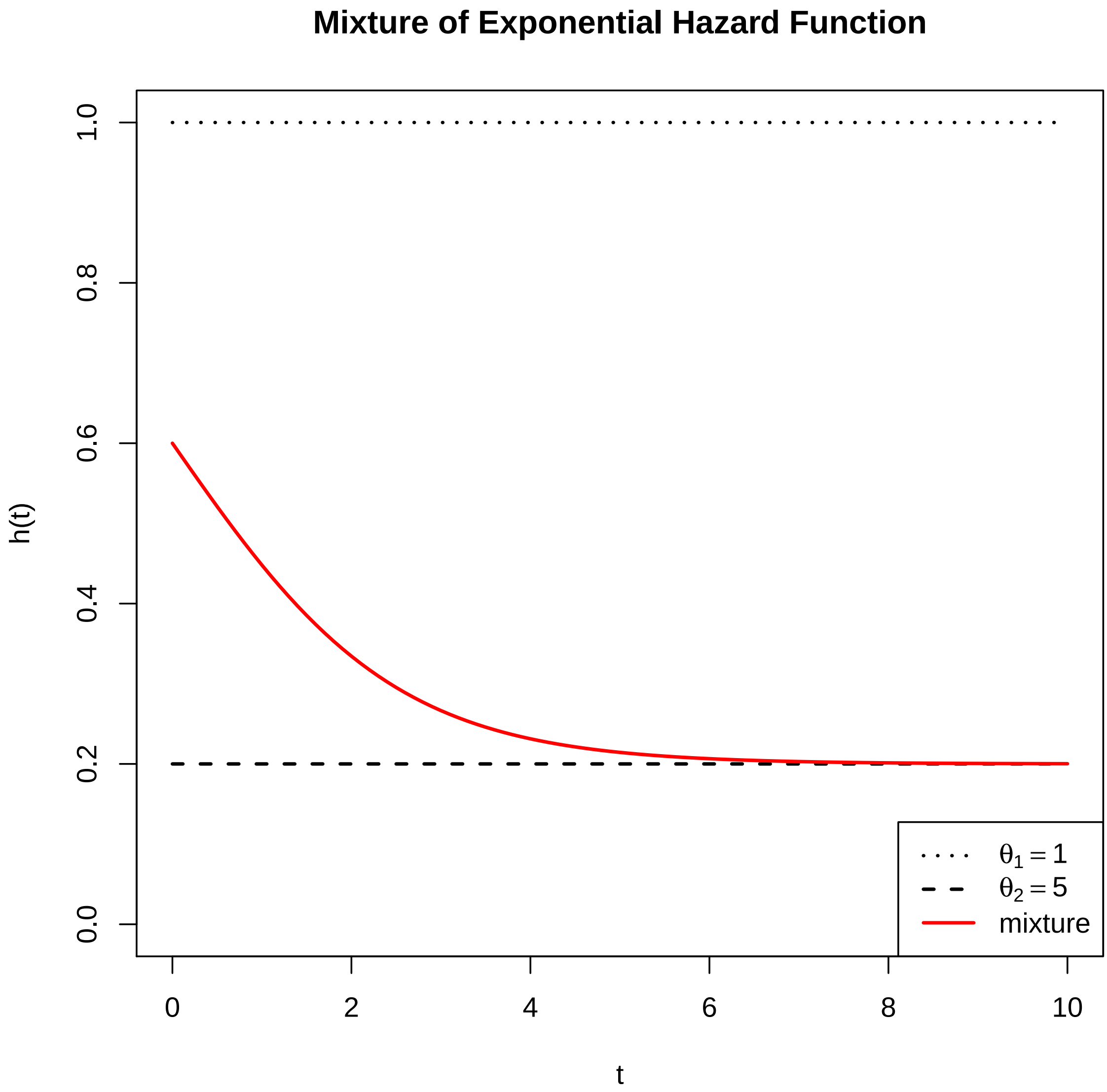
(b)

$$f(t;\boldsymbol{\theta}) = \frac{d}{dt}F(t;\boldsymbol{\theta}) = \frac{1}{2}\left(exp(-t) + \frac{1}{5}exp\left(-\frac{t}{5}\right)\right)$$

(c)

$$h(t;\boldsymbol{\theta}) = \frac{f(t;\boldsymbol{\theta})}{1-F(t;\boldsymbol{\theta})} = \frac{exp(-t) + \frac{1}{5}exp\left(-\frac{t}{5}\right)}{exp(-t) + exp\left(-\frac{t}{5}\right)}$$

(d)



(e) It is a strictly decreasing convex curve ranges from 0.2 to 1.

(f) Its hazard is strictly decreasing and converges to 0.2.

5.8

(a)

$$X \sim Gamma(\alpha, \beta) \text{ and } Y \sim Exp(X)$$

$$\begin{aligned}
f_Y(y) &= \int_{x=0}^{\infty} f_{Y|X}(y \mid x) f_X(x)\, dx \\
&= \int_{x=0}^{\infty} x e^{-xy} \frac{\beta^{\alpha} x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}\, dx \\
&= \frac{\beta^{\alpha}}{\Gamma(\alpha)} \int_{x=0}^{\infty} x^{\alpha} e^{-(y+\beta)x}\, dx \\
&= \frac{\beta^{\alpha}\Gamma(\alpha+1)}{\Gamma(\alpha)(y+\beta)^{\alpha+1}} \int_{x=0}^{\infty} \frac{(y+\beta)^{\alpha+1} x^{\alpha} e^{-(y+\beta)x}}{\Gamma(\alpha+1)}\, dx \\
&= \frac{\alpha\beta^{\alpha}}{(y+\beta)^{\alpha+1}} \\
F(t) &= \int_0^t \frac{\alpha\beta^{\alpha}}{(y+\beta)^{\alpha+1}}\, dy \\
&= \alpha\beta^{\alpha} \frac{1}{-\alpha} \left(y+\beta\right)^{-\alpha-1} \Big|_0^t \\
&= 1 - \beta^{\alpha}\left(t+\beta\right)^{-\alpha} \\
&= 1 - \frac{1}{\left(1+\frac{t}{\beta}\right)^{\alpha}} = 1 - \frac{1}{(1+\theta t)^{\kappa}} \text{ , where } \theta = \frac{1}{\beta} \text{ and } \kappa = \alpha
\end{aligned}$$

(b)

$$f(t;\theta,\kappa) = \frac{d}{dt} F(t;\theta,\kappa) = \kappa\theta\,(1+\theta t)^{-\kappa-1} \text{, where } t > 0.$$

(c)

$$h(t;\theta,\kappa) = \frac{f(t;\theta,\kappa)}{1-F(t;\theta,\kappa)} = \frac{\kappa\theta}{1+\theta t} \text{, where } t > 0.$$

5.12

(a) Let T_1, T_2, ..., $T_m \sim Weilbull(\mu_i, \sigma)$, $i = 1, 2, ..., m$.

$$Y = min(T_1, T_2, ..., T_m) = T_{(i)}$$

$$F(t) = 1 - exp\left[-\left(\frac{t}{\mu_i}\right)^{\sigma}\right]$$

$$S(t) = 1 - F(t) = exp\left[-\left(\frac{t}{\mu_i}\right)^{\sigma}\right]$$

$$
\begin{aligned}
S_Y(y) &= P(Y \geq y) = P(T_{(1)} \geq y) \\
&= P(T_{(1)} \geq y, T_{(2)} \geq y, ..., T_{(m)} \geq y) \\
&= P(T_{(1)} \geq y)P(T_{(2)} \geq y), ..., P(T_{(m)} \geq y) \\
&= exp\left[-\left(\frac{t}{\mu_1}\right)^{\sigma}\right] \cdot exp\left[-\left(\frac{t}{\mu_2}\right)^{\sigma}\right] \cdots exp\left[-\left(\frac{t}{\mu_m}\right)^{\sigma}\right] \\
&= exp\left[-\sum_{i=1}^{m}\left(\frac{t}{\mu_i}\right)^{\sigma}\right]
\end{aligned}
$$

$$
F(y) = 1 - S(y) = 1 - exp\left[-\sum_{i=1}^{m}\left(\frac{t}{\mu_i}\right)^{\sigma}\right]
$$

Hence, $T_{(1)}$ has a Weilbull distribution with shape parameter σ and scale parameter $\sum_{i=1}^{m} \frac{1}{\mu_i}$.

6.1

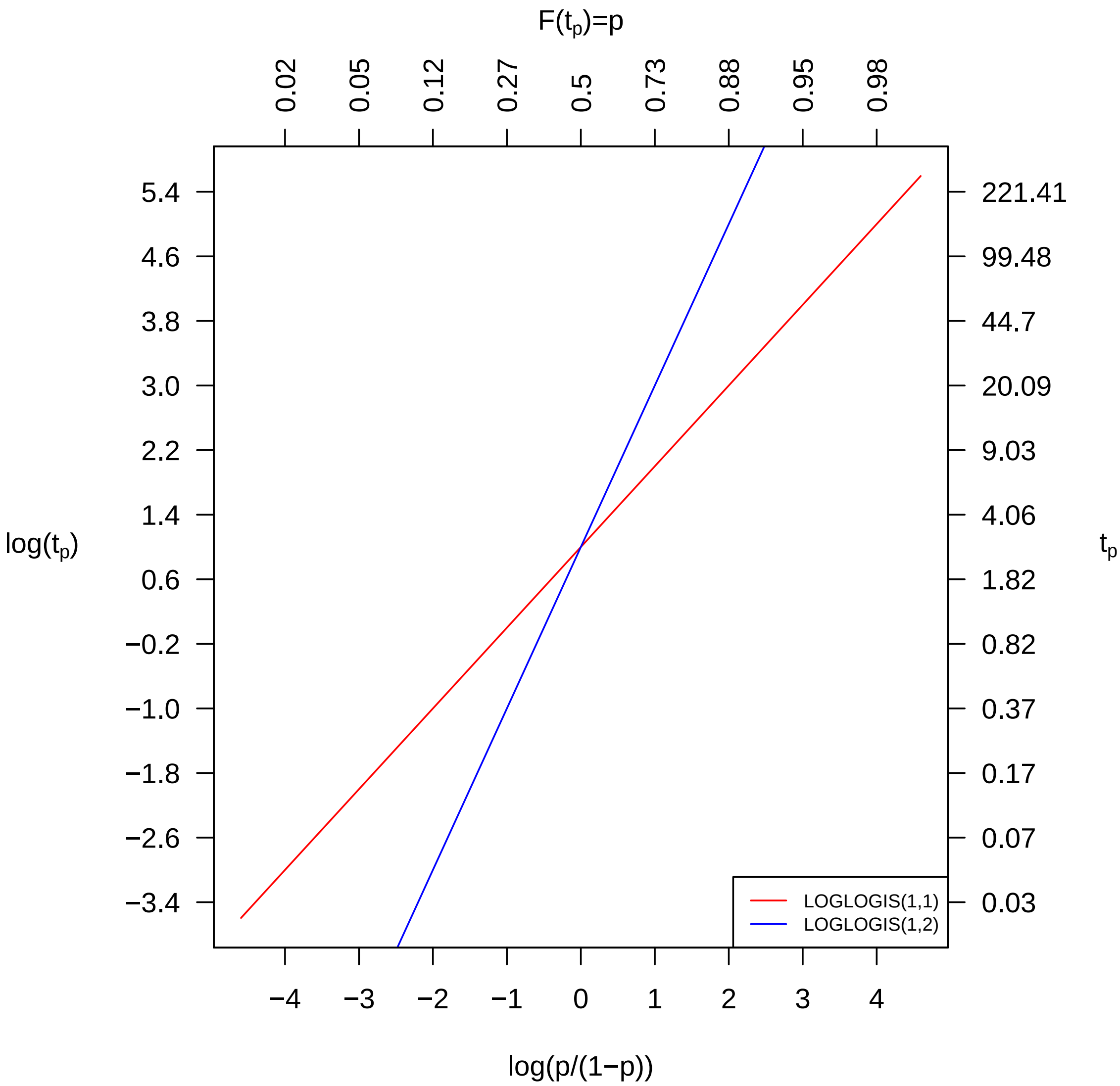
(a) Quantile:

$$
t_p = exp\left[\mu + \sigma\Phi^{-1}_{logis}(p)\right]
$$

$$
log(t_p) = \mu + \sigma\Phi^{-1}_{logis}(p)
$$

The plot of $log(t_p)$ versus $\Phi^{-1}_{logis}(p)$ is a straight line.

(b)



(c)

$$t_p = exp\left[\mu + \sigma\Phi^{-1}_{logis}(p)\right]$$