NATIONAL SUN YAT-SEN UNIVERSITY, DEPARTMENT OF MATHEMATICS

Reliability Analysis Assignment 3 (personal)

Kuan-I Chung

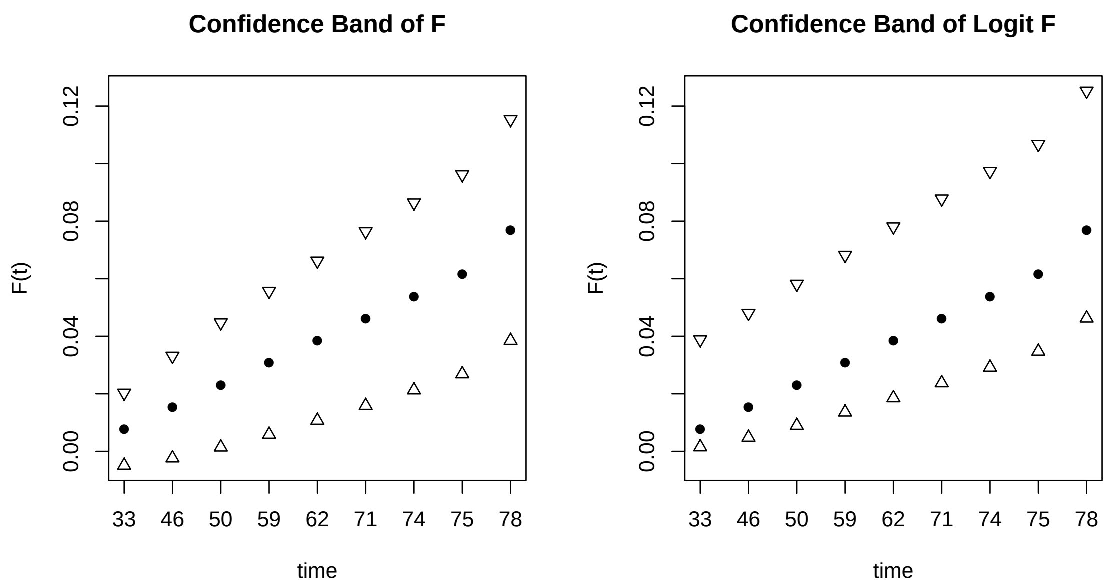
2017.04.06

3.4

i	t_i	$\widehat{F}(t_i)$	$\widehat{F}_{(3.7)}(t_i)$	$\widehat{se}_{\widehat{F}(t_i)}$	$C.I._L$	$C.I._U$	$Logit\ C.I._L$	$Logit\ C.I._U$
1	33	0.0077	0.0077	0.0077	−0.0049	0.0203	0.0015	0.0388
2	46	0.0154	0.0154	0.0108	−0.0024	0.0331	0.0048	0.0480
3	50	0.0231	0.0231	0.0132	0.0014	0.0447	0.0090	0.0582
4	59	0.0308	0.0308	0.0151	0.0059	0.0557	0.0136	0.0682
5	62	0.0385	0.0385	0.0169	0.0107	0.0662	0.0185	0.0781
6	71	0.0462	0.0462	0.0184	0.0159	0.0764	0.0238	0.0878
7	74	0.0538	0.0538	0.0198	0.0213	0.0864	0.0292	0.0973
8	75	0.0615	0.0615	0.0211	0.0269	0.0962	0.0347	0.1068
9	78	0.0769	0.0769	0.0234	0.0385	0.1154	0.0463	0.1253

(a) The answer is shown above.

(b) The answer is shown above. Some of the lower bounds of the original confidence intervals is less than 0, and it is unreasonable. Thus, we provides a logit-transformed confidence intervals, and the two methods are shown in the following figure. We could noticed that the logit-transformed confidence band is wider but more reasonable than the original one.



(c) The answer is shown above. In the first three failures, the two methods have the same estimate.

(d) The experimenter might not monitor failure occurring throughout the experiment. Also, the experiment lasted only a little time. These made us unable to predict the failure rate after 78 thousand cycles.

(e) That would be not general enough.

(f) If we know the order of the starting time of the each links, than we will be able to consider the censoring effect. An that would help us predict the failure rates more precisely.

3.8

$$L(F(t_i)) = \binom{n}{\sum_{j=1}^{i} d_j} (F(t_i))^{\sum_{j=1}^{i} d_j} (1-F(t_i))^{n-\sum_{j=1}^{i} d_j}$$

$$\Rightarrow \; logL(F(t_i)) = log\binom{n}{\sum_{j=1}^{i} d_j} + \left(\sum_{j=1}^{i} d_j\right) logF(t_i) + \left(n - \sum_{j=1}^{i} d_j\right) log(1-F(t_i))$$

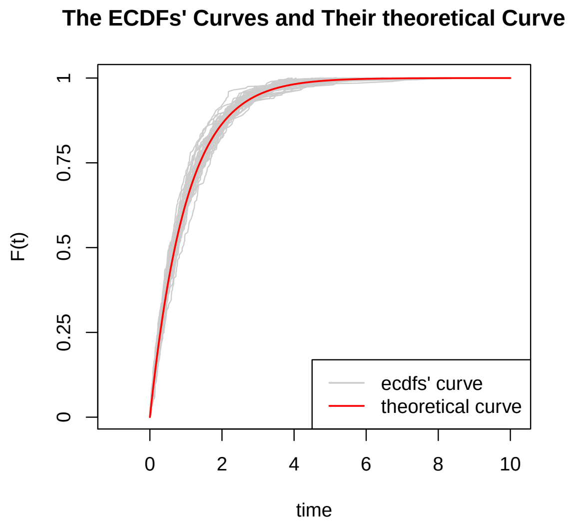
Let $\dfrac{\partial logL(F(t_i))}{\partial F(t_i)} = \dfrac{\sum_{j=1}^{i} d_j}{F(t_i)} + \dfrac{n-\sum_{j=1}^{i} d_j}{1-F(t_i)} = 0 \quad \Rightarrow \quad F(t_i) = \dfrac{\sum_{j=1}^{i} d_j}{n}$

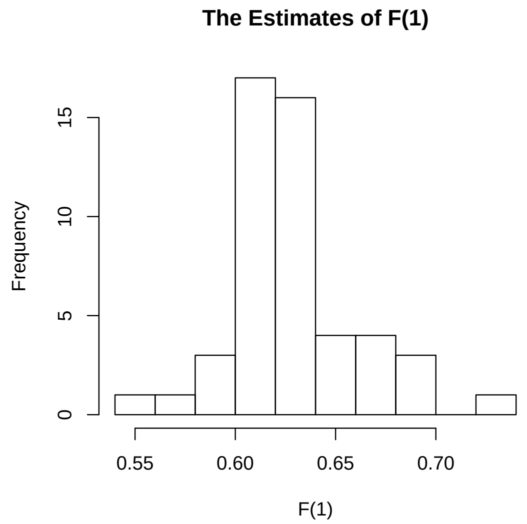
$$\because \; \left.\frac{\partial^2 logL(F(t_i))}{\partial F(t_i)^2}\right|_{F(t_i)=\frac{\sum_{j=1}^{i} d_j}{n}} < 0$$

$\therefore \; \hat{F}(t_i) = \dfrac{\sum_{j=1}^{i} d_j}{n}$ is the maximum likelihood estimate.

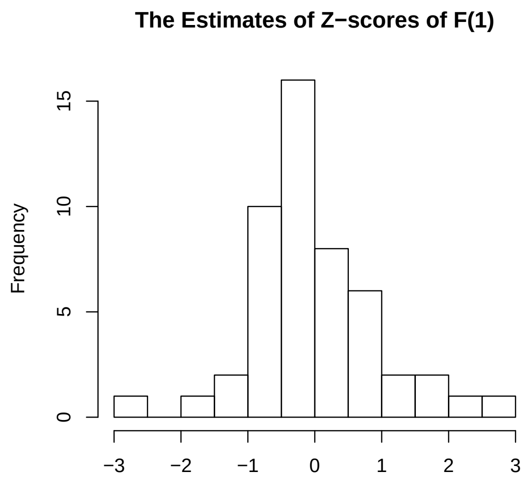
3.10

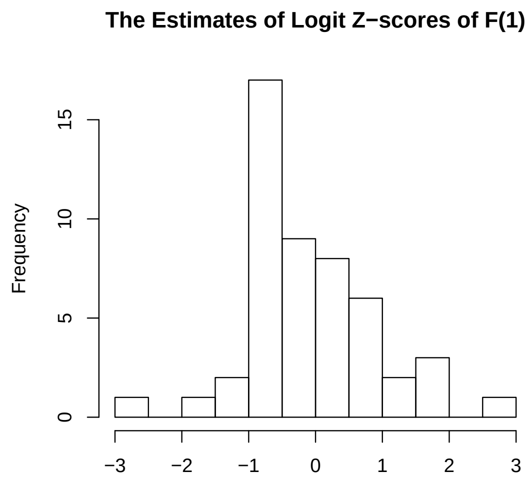
(a) and (b)





(c) and (d)





(e) The original estimates of $Z-$scores of F(1) are more like a normal distribution.

3.14

By applying the $\delta-$method: $Var(g(\widehat{\theta})) = \left(\frac{dg}{d\theta}\right)^2 Var(\widehat{\theta})$

Let $g(F) = logit(F) = log(\frac{F}{1-F})$, then $\frac{dg}{dF} = \frac{1-F}{F}\frac{(1-F)+F}{(1-F)^2} = \frac{1}{F(1-F)}$

$$\Rightarrow \quad Var(logit(\widehat{F})) = \left(\frac{1}{F(1-F)}\right)^2 Var(\widehat{F})$$

$$\Rightarrow \quad se_{logit(\widehat{F})} = \sqrt{Var(logit(\widehat{F}))} = \frac{se_{\widehat{F}}}{F(1-F)}$$

$$\Rightarrow \quad \widehat{se}_{logit(\widehat{F})} = \frac{\widehat{se}_{\widehat{F}}}{\widehat{F}(1-\widehat{F})}$$

By (3.15), $Z_{logit(\widehat{F})} = \frac{logit(\widehat{F}) - logit(F)}{se_{logit(\widehat{F})}} \sim N(0,1)$

$$Pr\left(\left|\frac{logit(\widehat{F}) - logit(F)}{se_{logit(\widehat{F})}}\right| \le Z_{1-\frac{\alpha}{2}}\right) = 1-\alpha$$

$$\Rightarrow \quad -Z_{1-\frac{\alpha}{2}} \le \frac{logit(\widehat{F}) - logit(F)}{se_{logit(\widehat{F})}} \le Z_{1-\frac{\alpha}{2}}$$

$$\Rightarrow \quad logit(F) \in \left[logit(\widehat{F}) - Z_{1-\frac{\alpha}{2}}\, se_{logit(\widehat{F})} \ ,\ logit(\widehat{F}) + Z_{1-\frac{\alpha}{2}}\, se_{logit(\widehat{F})}\right]$$

For $logit^{-1}(v) = \frac{1}{1+exp(-v)}$,

$$F(t) \in \left[\frac{1}{1+exp\left(-(logit(\widehat{F}) - Z_{1-\frac{\alpha}{2}}\, se_{logit(\widehat{F})})\right)} \ ,\ \frac{1}{1+exp\left(-(logit(\widehat{F}) + Z_{1-\frac{\alpha}{2}}\, se_{logit(\widehat{F})})\right)}\right]$$

$$= \left[\frac{1}{1+\frac{1-\widehat{F}}{\widehat{F}}exp\left(Z_{1-\frac{\alpha}{2}}\frac{se_{\widehat{F}}}{F(1-F)}\right)} \ ,\ \frac{1}{1+\frac{1-\widehat{F}}{\widehat{F}}exp\left(-Z_{1-\frac{\alpha}{2}}\frac{se_{\widehat{F}}}{F(1-F)}\right)}\right]$$

$$= \left[\frac{\widehat{F}}{\widehat{F}+(1-\widehat{F})w} \ ,\ \frac{\widehat{F}}{\widehat{F}+(1-\widehat{F})w^{-1}}\right] \quad \text{,where } w = exp\left(Z_{1-\frac{\alpha}{2}}\frac{se_{\widehat{F}}}{F(1-F)}\right)$$