NATIONAL SUN YAT-SEN UNIVERSITY, DEPARTMENT OF MATHEMATICS

Reliability Analysis Assignment 5 (group)

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$$L(\eta,\beta) = \left[\prod_{i=1}^{r}\frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_i}{\eta}\right)^{\beta}}\right]\left[e^{-\left(\frac{t_c}{\eta}\right)^{\beta}}\right]^{n-r}$$

$$logL(\eta,\beta) = \sum_{i=1}^{r} log\left[\frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1} e^{-\left(\frac{t_i}{\eta}\right)^{\beta}}\right] - (n-r)\left(\frac{t_c}{\eta}\right)^{\beta}$$

$$= \sum_{i=1}^{r} log\frac{\beta}{\eta} + (\beta-1)\sum_{i=1}^{r} log\left(\frac{t_i}{\eta}\right) - \sum_{i=1}^{r}\left(\frac{t_i}{\eta}\right)^{\beta} - (n-r)\left(\frac{t_c}{\eta}\right)^{\beta}$$

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(a) Given β, Let

$$\frac{dlogL}{d\eta} = \sum_{i=1}^{r}\frac{\eta}{\beta}\cdot\frac{-\beta}{\eta^2} + (\beta-1)\sum_{i=1}^{r}\frac{\eta}{t_i}\cdot\frac{-t_i}{\eta^2} + \sum_{i=1}^{r}\beta\left(\frac{t_i}{\eta}\right)^{\beta-1}\cdot\frac{t_i}{\eta^2} + (n-r)\beta\left(\frac{t_c}{\eta}\right)^{\beta-1}\cdot\frac{t_c}{\eta}$$

$$= -\frac{r}{\eta} + (\beta-1)\left(-\frac{r}{\eta}\right) + \sum_{i=1}^{r}\frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta} + (n-r)\frac{\beta}{\eta}\left(\frac{t_c}{\eta}\right)^{\beta} = 0$$

$$\Rightarrow \frac{-\beta r}{\eta} + \sum_{i=1}^{r}\frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta} + (n-r)\frac{\beta}{\eta}\left(\frac{t_c}{\eta}\right)^{\beta} = 0$$

$$\Rightarrow -r + \sum_{i=1}^{r}\left(\frac{t_i}{\eta}\right)^{\beta} + (n-r)\left(\frac{t_c}{\eta}\right)^{\beta} = 0$$

$$\Rightarrow \sum_{i=1}^{r}\left(\frac{t_i}{\eta}\right)^{\beta} + (n-r)\left(\frac{t_c}{\eta}\right)^{\beta} = r$$

$$\Rightarrow \frac{1}{\eta^{\beta}}\sum_{i=1}^{r} {t_i}^{\beta} + \frac{n-r}{\eta^{\beta}}\cdot {t_c}^{\beta} = r$$

$$\Rightarrow \widehat{\eta} = \sqrt[\beta]{\frac{\sum_{i=1}^{r} {t_i}^{\beta} + (n-r){t_c}^{\beta}}{r}}$$

For

$$\left.\frac{d^2 logL}{d\eta^2}\right|_{\eta=\widehat{\eta}} < 0,$$

$$\widehat{\eta}_{mle} = \sqrt[\beta]{\frac{\sum_{i=1}^{r} {t_i}^{\beta} + (n-r){t_c}^{\beta}}{r}}$$

(b) When $\beta = 1$, it becomes an exponential distribution.

$$\widehat{\eta}_{mle} = \frac{\sum_{i=1}^{r} t_i + (n-r)t_c}{r} = \frac{TTT}{r}$$