

Define

$$J(\theta) = -\frac{1}{m} \sum_{i=1}^m (y^{(i)} \log(h_{\theta}(x^{(i)})) + (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)})))$$

where $y^{(i)} \in \{0, 1\}$, $x^{(i)} = (x_0^{(i)}, x_1^{(i)}, \dots, x_n^{(i)})^T$ is a $(n+1)$ -dimensional vector with $x_0^{(i)} = 1$, $\theta = (\theta_0, \theta_1, \dots, \theta_n)^T$ is a $(n+1)$ -dimensional vector, $h_{\theta}(x^{(i)}) = 1/(1 + e^{-\theta^T x^{(i)}})$, and the superscript T indicates the vector transpose. Prove

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

for $j = 0, 1, \dots, n$.