Thomas Ngo Van Do CPSC335-04 Professor Avery Project 1

1. Pseudocode:

a. Left to right algorithm

//input: a positive integer n and a list of 2n disks of alternating colors dark-light, starting with dark //output: a list of 2n disks, the first n disks are light, the next n disks are dark, and an integer m representing the number of swaps to move the dark ones after the light ones

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\begin{array}{ll} m \leftarrow 0 & \text{//number of swaps} \\ \textbf{for } k \leftarrow 0 \textbf{ to } k < n \textbf{ do} \\ \textbf{for } i \leftarrow 0 \textbf{ to } i < 2n\text{-}1 \textbf{ do} \\ \textbf{if } disks[i] \text{ is } dark \textbf{ and } disks[i+1] \text{ is } light \\ swap \ disks[i] \text{ and } disks[i+1] \\ m++ \end{array}
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b. lawnmover algorithm

//input: a positive integer n and a list of 2n disks of alternating colors dark-light, starting with dark //output: a list of 2n disks, the first n disks are light, the next n disks are dark, and an integer m representing the number of swaps to move the dark ones after the light ones

$$\begin{array}{lll} m \leftarrow 0 & \text{//number of swaps} \\ \textbf{for } k \leftarrow 0 \textbf{ to } k < n/2 + 1 \textbf{ do} \\ \text{//Loop for going from left to right} \\ \textbf{for } i \leftarrow 0 \textbf{ to } i < 2n-1 \textbf{ do} \\ \textbf{if } \text{ disks}[i] \text{ is dark } \textbf{and } \text{ disks}[i+1] \text{ is light} \\ \text{swap } \text{ disks}[i] \text{ and } \text{ disks}[i+1] \\ m++ \\ \text{//Loop for going from right to left} \\ \textbf{for } i \leftarrow 2n-2 \textbf{ to } i > 0 \textbf{ do} \\ \textbf{if } \text{ disks}[i] \text{ is light } \textbf{and } \text{ disks}[i-1] \text{ is dark} \\ \text{swap } \text{ disks}[i] \text{ and } \text{ disks}[i-1] \\ m++ \\ \end{array}$$

2. Analysis:

a. Left to right algorithm

$$T(n) = \sum_{k=0}^{n-1} \sum_{i=0}^{2n-2} 1 = \sum_{k=0}^{n-1} (2n-2-0+1) = \sum_{k=0}^{n-1} (2n-1) = (2n-1) \sum_{k=0}^{n-1} 1$$

$$= (2n-1)(n-1-0+1) = n(2n-1) = 2n^2 - 1 \in O(n^2)$$

Proof:

$$2n^2-1\leq 2n^2 \forall n\in R$$

Therefore, c = 2 and n_0 is all real numbers

b. Lawnmover

$$T(n) = \sum_{k=0}^{n/2} \left(\sum_{i=0}^{2n-2} 1 + \sum_{i=1}^{2n-2} 1 \right) = \sum_{k=0}^{n/2} \left[(2n-2-0+1) + (2n-2-1+1) \right]$$

$$= \sum_{k=0}^{n/2} (2n-1+2n-2) = \sum_{k=0}^{n/2} (4n-3) = 4n \sum_{k=0}^{n/2} 1 - 3 \sum_{k=0}^{n/2} 1$$

$$= 4n \left(\frac{n}{2} - 0 + 1 \right) - 3 \left(\frac{n}{2} - 0 + 1 \right) = 2n^2 + 4n - \frac{3}{2}n - 3 = 2n^2 + \frac{5}{2}n - 3 \in O(n^2)$$

Proof:

$$2n^2 + \frac{5}{2}n - 3 \le 2n^2 + \frac{5}{2}n \le 2n^2 + \frac{5}{2}n^2 \le \frac{9}{2}n^2 \forall n \ge 1$$

Therefore, c = 9/2 and $n_0 = 1$