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1. Pseudocode:

//input: n, a positive integer, and a two-dimensional matrix W of integer values

//output: a message stating either “ring” for ring topology, “star” for star topology, “complete” for fully connected mesh, or “neither” if the weight matrix does not represent either of these three topologies.

//n is the number of node; count is the tracking number of edges of individual node

int n, count, center
bool cond

//Reading the input is NOT part of the algorithm so the timer starts after reading the input. Depending on platform so the implementation of this section varies.
star the timer now.

cond←true //assuming the graph is a ring

//In ring graph, each node only has exact 2 edges

for i←0 to i<n do //starting exhaustive search
 count←0 //initializing count to before
 //checking nodes

 for j←0 to j<n do //checking value in each node
 if W[i][j] > 0 and W[i][j] < 100
 count++

 //if any node has more or less than 2 edges, then the graph is not a ring

 if count != 2
 cond←false //graph is not ring
 i←n //exiting the outer loop

if cond = true do
 Print “The graph is ring”
 stop the timer now.
 print the elapse time
 terminate the program

cond←true //assuming the graph is a star
center←0 //center = 0 meaning that there is no
 //center node yet

//In star graph, the center node has (n-1) edges while others have only 1 edge

```
for i←0 to i<n do
    count←0
    for j←0 to j<n do
        if W[i][j] > 0 and W[i][j] < 100
            count++
    if count != 1 and center = 1
        cond←false
        i←n
```

//if a node has n-1 edges and there is no center node yet,
if count = n-1 and center = 0 //the center node is found
center←1

//if the is one center node and the condition of star is met

```
if cond = true and center = 1
    Print "The graph is star"
    stop the timer
    print the elapse time
    terminate the program
```

```
cond = true;                                //assuming the graph is a connected
                                              //MESH
```

//In a fully connected MESH, all nodes have (n-1) edges

```
for i←0 to i<n do
    count = 0
    for j←0 to j<n do
        if W[i][j] > 0 and W[i][j] < 100
            count++
    if count != (n-1)
        cond←false
if cond = true do
    Print "The graph is fully connected MESH"
```

```
else do
    // because we check all the condition to this point, the
    // graph is neither one type.
    Print "The graph is neither type"
```

```
stop the timer now
print the elapse time
terminate the program
```

2. Analysis of the pseudocode:

$$\begin{aligned}
& \sum_{i=0}^n \sum_{j=0}^n 1 + \sum_{i=0}^n \sum_{j=0}^n 1 + \sum_{i=0}^n \sum_{j=0}^n 1 = 3 \sum_{i=0}^n \sum_{j=0}^n 1 = 3 \sum_{i=0}^n (n-0+1) = 3 \sum_{i=0}^n (n+1) = 3 \sum_{i=0}^n n + 3 \sum_{i=0}^n 1 \\
& = 3n \sum_{i=0}^n 1 + 3 \sum_{i=0}^n 1 = 3n(n-0+1) + 3(n-0+1) = 3n(n+1) + 3(n+1) = 3n^2 + 3n + 3n + 3 \\
& = 3n^2 + 6n + 3 \in O(n^2)
\end{aligned}$$

3. Prove the efficiency:

$$3n^2 + 6n + 3 \leq 3n^2 + 6n^2 + 3 \leq 3n^2 + 6n^2 + 3n^2 \leq 12n^2 \quad \forall n \geq 1$$

Therefore, $c = 12$ and $n_0 = 1 \Rightarrow O(n^2)$