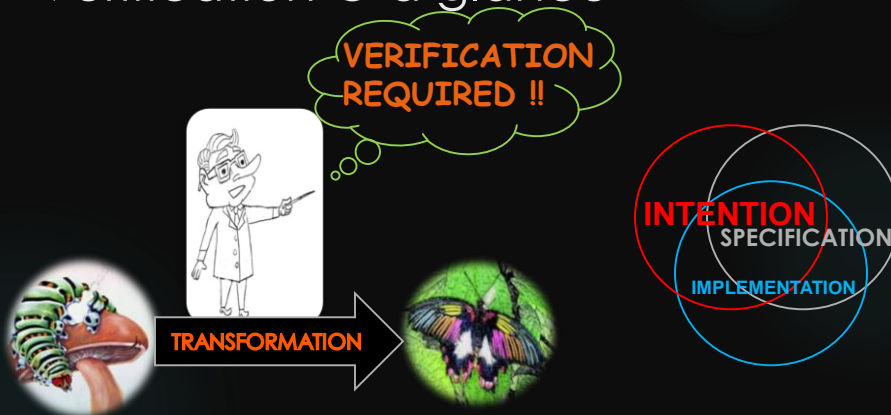


# Introduction to Formal verification

Prasenjit Biswas, Nvidia

## Verification @ a glance

2



### ➤ Sources of Bugs:

Mismatches in between **INTENTION** – SPECIFICATION – IMPLEMENTATION

➤ Validation: Am I Building A Correct System?

➤ Verification: Am I Building the system Correct ?

## Simulation – Two practical Examples

3

- ❑ Question :  
How long does it take to verify a 64 bit Floating Point Division Unit
- ❑ Answer :
  - There are  $(2^{64} \times 2^{64})$  : 2<sup>128</sup> test cases
  - At 1 test/us, it will take 10<sup>25</sup> years
- ❑ Question :  
How long does it take to verify a 256-bit RAM Memory Unit
- ❑ Answer :
  - There are  $2^{256} = 10^{80}$  bits to test
  - At 1 test/ps and using all matters in our galaxy to build computers of the size of a single electron, it will take 10<sup>10</sup> years to verify 0.05%!!!

## Functional Verification – Formal Methods

4

- ❑ Construct a computer based mathematical model of the system along with its random components
- ❑ Use mathematical reasoning to check functional properties of interest
  - ❑ Accurate results
    - ❑ Consideration of all cases is implicit
  - ❑ Sometimes is difficult and time consuming

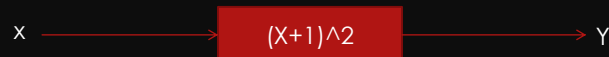
# Functional Verification – Formal Methods

5

- ❑ Construct a computer based **mathematical model** of the system along with its random components
- ❑ Use **mathematical reasoning** to check functional properties of interest
  - ❑ Accurate results
    - ❑ Consideration of all cases is implicit
  - ❑ Sometimes is difficult and time consuming

## Formal Methods - Example

6

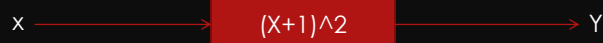


- ❑ Check if  $Y > X$  for the given system ( $X$  is a natural number)

Test vectors (X)	System output (Y)	$Y > X$
1	4	True
2	9	True
5	36	True
9	100	True
67	4624	True
1000	1002001	True
1000000	???	???

# Formal Methods - Example

7



- Check if  $Y > X$  for the given system ( $X$  is a natural number)

Steps		Properties
1	$Y > X$	Problem statement
2	$(X+1)^2 > X$	Implementation
3	$(X+1) \cdot (X+1) > X$	Definition of Square
4	$(X+1) \cdot X + (X+1) \cdot 1 > X$	Distributivity
5	$X \cdot X + 1 \cdot X + X \cdot 1 + 1 \cdot 1 > X$	Distributivity
6	$X \cdot X + X + X + 1 > X$	Multiplicative Identity
7	$X \cdot X + X + 1 + X > X$	Additive commutivity
8	$X \cdot X + X + 1 > 0$	Addition cancellation
9	<b>True</b>	Natural numbers $> 0$

8

## An Interesting bug from a real design

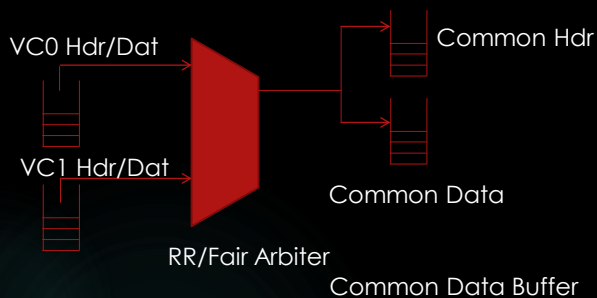
# Starvation

9

- ▶ What is this ?
  - ▶ A problem encountered in multitasking
  - ▶ A process is perpetually denied necessary resources
  - ▶ Without those resources, the intended task can never be finished
  - ▶ Remedy : Alteration of scheduling policy

## Starvation bug due to Shared resource

10



```
vc0/1_req_to_arbiter =
vc0/1_fifo_rd_req
& hdr_space_avail > 1
& vc0/1_data <= space_avail_in_dat_buffer ;
```

Arbiter grant Hdr for vc0/1 and then  
lock for corresponding data

### Starvation Case:

- vc0: big data pkt [lets say 20 beats] and vc1: small data pkt [lets say 1 beat]
- Common Data Buffer has <20 slots available [due to slow drain rate]
- vc0 will always get blocked due to the optimization

### Why Dynamic Verification Didn't able to catch this?

- Very corner scenario [slow drain rate, large pkt in vc0, continues small pkt in vc1]
- Deep inside the design



"But the car came on road very next day....."

13

WHAT IS  
THIS !!!



We have covered  
@#\$\$ paths,  
!~\*& Houses .....  
Found \$%#& nos.  
of yellow Cars .....



**Moral of The Story:**

"Program testing is very effective way to show the presence of  
**BUGs**, but it is hopelessly inadequate to show their absence " --  
Edsger W. Dijkstra

14

Welcome to the world of  
Formal Verification

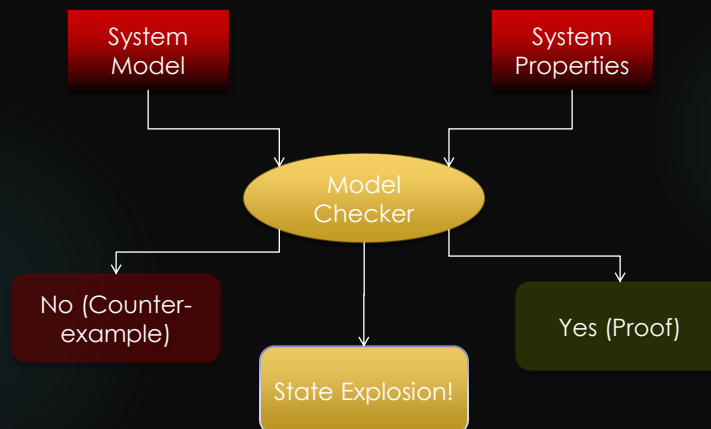
# Most widely used Formal Methods

15

- ❑ Model Checking
- ❑ Equivalence checking
- ❑ Theorem proving

## Simplistic Description

16





# The Truth about Model Checking

17

- Can
  - prove interesting properties
    - No deadlock
    - Arbitration
    - No overflow
    - ...
  - find complex bugs
  - handle unit level complexity
- Suffers from
  - state explosion
  - false alarms
- Requires
  - good knowledge about the design
  - well-defined specification
- Available tools @ industry
  - Jasper (Jasper)
  - Magellan (Synopsys)
  - IFV (Cadence)

# Equivalence Checking

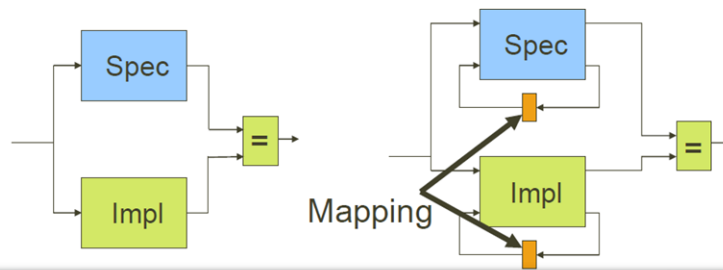
18

- Between two levels of abstraction
  - Algorithmic vs. timed model
  - Transactions vs. RTL
  - RTL vs. Gate-level
  - ...
- Is not restricted to
  - Formality
  - RTL vs. gate-level
- Can handle
  - Equivalence between C and RTL
  - Clocked vs. functional models
  - Mathematical operations (add, multiply, subtract, etc.)
- Requires
  - Well-defined observation function
  - Constrained coding of the high level models
- Available tools @ industry
  - Hector (Synopsys)
  - Slec (Calypso)

# Combinational Equivalence checking

19

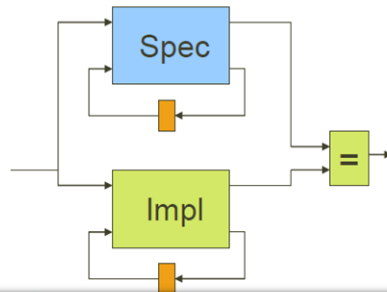
- Both designs are combinational, functions of outputs must match.
- If registers are present, must have one-to-one register mapping.



# Cycle accurate equivalence

20

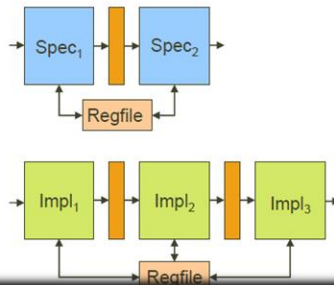
- Classical sequential equivalence
- Start execution in initial state, apply same inputs
- Outputs must match in every clock cycle
- Variants: Sequential hardware equivalence (Pixley)  
Safe replaceability (Singhal, Pixley, Aziz, Brayton)



# Pipelined Equivalence

21

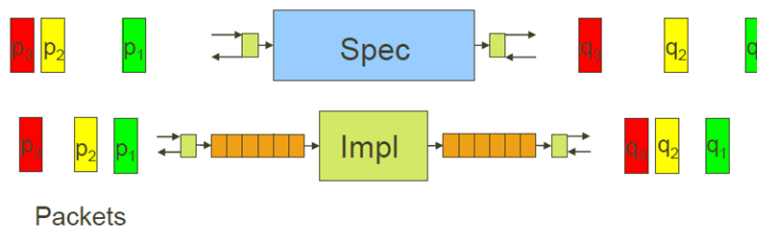
- Both designs are pipelined, but may have different number of stages
- Start in initial state
- Check results of each pipeline execution
- Pipeline can have forwarding logic, register files



# Stream based Equivalence

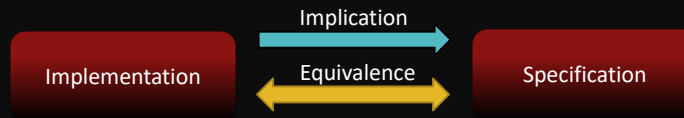
22

- Handshaking protocol on inputs and outputs
- Input data packets may arrive temporally uncorrelated in both designs
- Output data may be generated temporally uncorrelated
- Only order and content of output packets is checked



# Theorem Proving

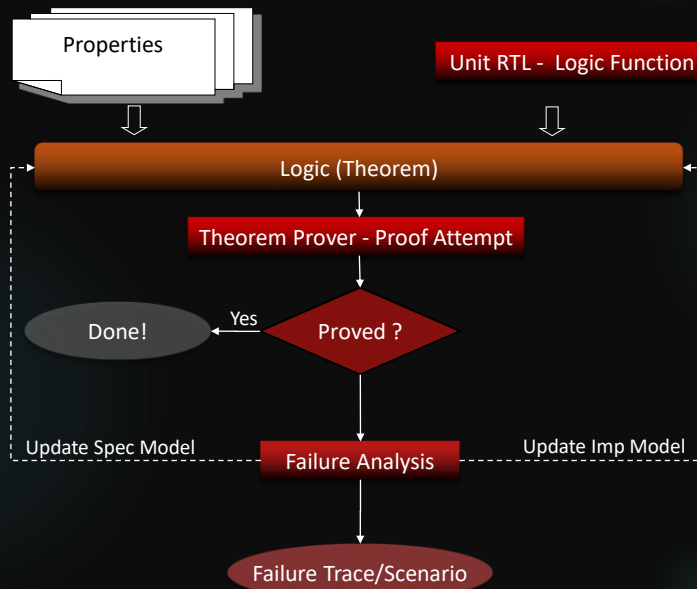
23



- Implementation and specification expressed as *formulas in a formal logic*
- Required relationship (logical equivalence/logical implication) described as a *theorem to be proven within the context of a proof calculus*
- A proof system: set of axioms and inference rules (simplification, rewriting, induction, etc.)

# Theorem Proving Methodology

24



# Fundamental operations

25

- ▶ For consistency, will use Verilog-like notation:
  - AND:  $a \& b$
  - OR:  $a \mid b$
  - NOT:  $\sim a$
- ▶ Sometimes AND represented as multiplication, and OR as addition
  - ▶ Like arithmetic, except  $1+1 == 1$
- ▶ Implication:  $a \rightarrow b$ 
  - ▶ Same as:  $\sim a \mid b$
  - ▶ Terms:  $a$  is the *antecedent*,  $b$  is the *consequent*

# Basic Boolean Identities

26

- ▶ Commutative, Associative
  - ▶  $a \& b == b \& a$
  - ▶  $a \& (b \& c) == (a \& b) \& c$
- ▶ Distributive both ways
  - ▶  $a \& (b \mid c) == (a \& b) \mid (a \& c)$
  - ▶  $a \mid (b \& c) == (a \mid b) \& (a \mid c)$
- ▶ Idempotence:  $(a \& a) == a$ ,  $(a \mid a) == a$
- ▶ DeMorgan
  - ▶  $\sim(a \& b) = \sim a \mid \sim b$
  - ▶  $\sim(a \mid b) = \sim a \& \sim b$

# Implication relationships

27

$a \rightarrow b$

- ▶ Converse:  $b \rightarrow a$
- ▶ Inverse:  $\sim a \rightarrow \sim b$
- ▶ Contrapositive:  $\sim b \rightarrow \sim a$

Which pairs are identical in truth value?

- ▶ Can be useful when restating for FV
- ▶ Use  $\models$  ("logically entails") symbol as distinct from implication when appropriate
  - $(a \rightarrow b) \models (\sim b \rightarrow \sim a)$
  - $(a \rightarrow b), (b \rightarrow c) \models (a \rightarrow c)$ , transitive rule
  - $(a \rightarrow c), (b \rightarrow c) \models ((a \mid b) \rightarrow c)$ , disjunctive rule

# What is a Proof?

28

- ▶ Apply sequence of inference rules
- ▶ Example:
  - ▶ Known:
    - S1:  $a$ , S2:  $(a \rightarrow b)$ , S3:  $(d \rightarrow \sim b)$
  - ▶ Prove:  $\sim d$ 
    - ▶ C1:  $S1, S2 \models b$
    - ▶ C2:  $S3 \models (\sim d \mid \sim b)$
    - ▶ C3:  $C1, D3 \models \sim d$
- ▶ Known:
  - S1: Texas A & M University is awesome,
  - S2: (Awesome university  $\rightarrow$  Alumni network "is" powerful),
  - S3: (Former students "failed to do" great for themselves and the community  $\rightarrow$  Alumni network "is not" powerful))
- ▶ Prove: Former A&M students "have done" great or themselves and the community
  - ▶ C1:  $S1, S2 \models$  Alumni network of A&M "is" powerful
  - ▶ C2:  $S3 \models$  (Former A&M students "have done" great or themselves and the community  $\mid \mid$  Alumni network of A&M "is not" powerful)
  - ▶ C3:  $C1, D3 \models$  Former A&M students "have done" great for themselves and the community (**Proved**)

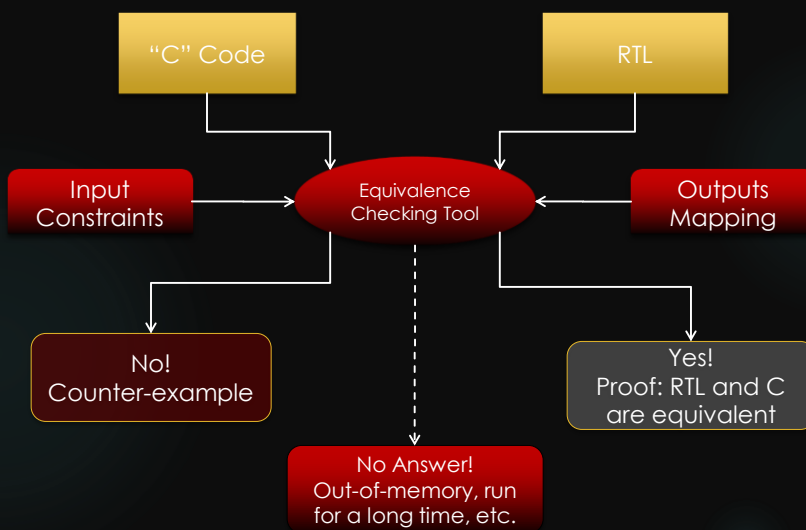
# The Truth about Theorem Proving

29

- Can
  - prove complex properties
    - Computation
    - Error analysis
    - Protocols
    - ...
  - handle low level complexity
- Suffers from
  - Interactions with user
  - Limited support for hardware verification
- Requires
  - good knowledge about the design
  - know the proof before you mechanize it
  - expertise-level in using the provers
- Available tools
  - HOL (U. of Cambridge)

# C to RTL Equivalence Checking

30



## Example : The FIFO (Data transfer block)

31



Constraints

```
// If a FIFO is full, then there shouldn't be any further writes
env_no_write_when_full: assume property ((full |-> !write_en));

// If a FIFO empty, then there shouldn't be any further reads
env_no_read_when_empty: assume property ((empty |-> !read_en));
```

Assertions

```
// FIFO cannot have full and empty asserted at the same time
fifo_no_full_and_empty: assert property (!(full && empty));

// FIFO must keep full asserted until a read occurs
fifo_remain_full_until_read:...((full & !read_en) |=> full));

// FIFO must keep empty asserted until a write occurs
fifo_remain_empty_until_write:...((empty & !write_en) |=> empty));
```

## Fire Up The Model Checker

32

FormalVerifier

Status: Not Scheduled

RTL Automatic UserDefined

Type	Property Name	Trigger	Status
C	env_no_write_when_full		
C	env_no_read_when_empty		
A	fifo_no_full_and_empty	? Not_Run	? Not_Run
A	fifo_remain_full_until_read	? Not_Run	? Not_Run
A	fifo_remain_empty_until_wr	? Not_Run	? Not_Run

Specification Cover

Console

```
FormalVerifier> constraint -add *env_* -r
FormalVerifier> assert -add *fifo_* -r
FormalVerifier> prove
```

\*Source: Cadence IFV Training Material



# Analyze One At A Time

33

FormalVerifier

Status: Running

RTL Automatic UserDefined

Type	Property Name	Trigger	Status
C	env_no_write_when_full		
C	env_no_read_when_empty		
A	fifo_no_full_and_empty		Pass
A	fifo_remain_full_until_read	? Not_Run	? Not_Run
A	fifo_remain_empty_until_wr	? Not_Run	? Not_Run

Specification Cover

Console

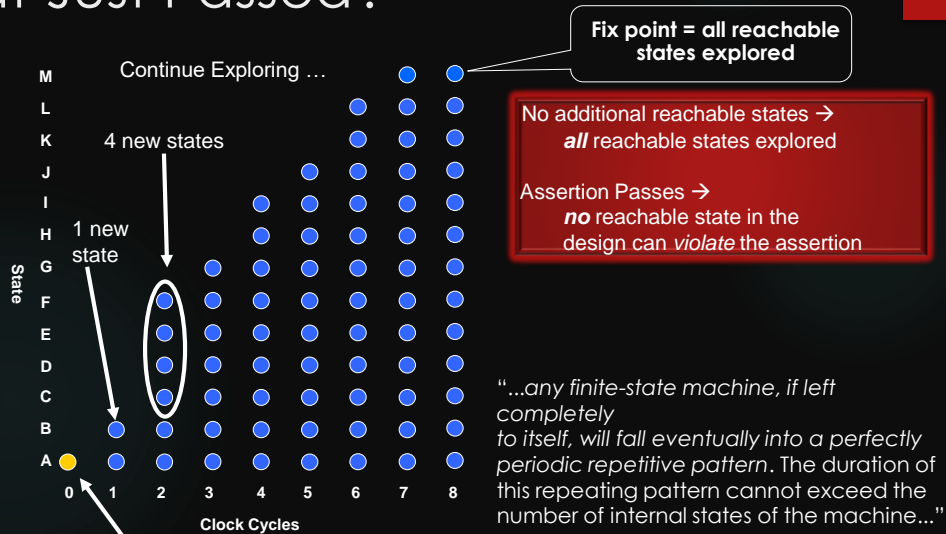
```

FormalVerifier> constraint -add *env* -r
FormalVerifier> assert -add *fifo* -r
FormalVerifier> prove
    fifo_no_full_and_empty : Pass
  
```

\*Source: Cadence IFV Training Material

# What Just Passed?

34



\*Source: Cadence IFV Training Material

# Point Of Failure

35

FormalVerifier

Status: Running

RTL: Automatic UserDefined

Type	Property Name	Trigger	Status
C	env_no_write_when_full		
C	env_no_read_when_empty		
A	fifo_no_full_and_empty		Pass
A	fifo_remain_full_until_read	Pass	Fail (20)
A	fifo_remain_empty_until_wr	? Not_Run	? Not_Run

Specification Cover

Console

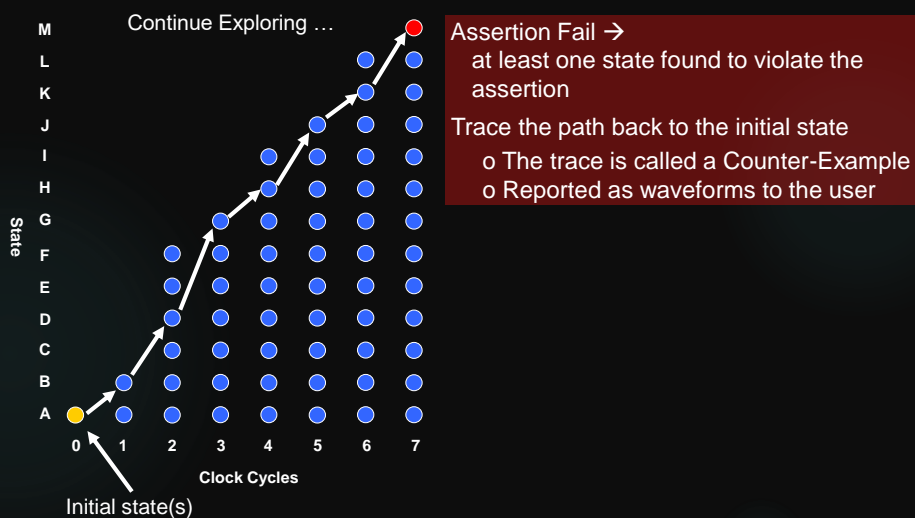
```

FormalVerifier> constraint -add *env_* -r
FormalVerifier> assert -add *fifo_* -r
FormalVerifier> prove
    fifo_no_full_and_empty : Pass
    fifo_remain_full_until_read : Fail (20)
  
```

\*Source: Cadence IFV Training Material

# What Just Failed?

36



\*Source: Cadence IFV Training Material

# Analyzing Further...

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FormalVerifier

Design Browser

- FormalVerifier
  - fifo

Status Finished

RTL Automatic UserDefined

Type	Property Name	Trigger	Status
C	env_no_write_when_full		
C	env_no_read_when_empty		
A	fifo_no_full_and_empty		Pass
A	fifo_remain_full_until_read	Pass	Fail (20)
A	fifo_remain_empty_until_wr	Pass (8)	Explored (22)

Specification Cover

Console

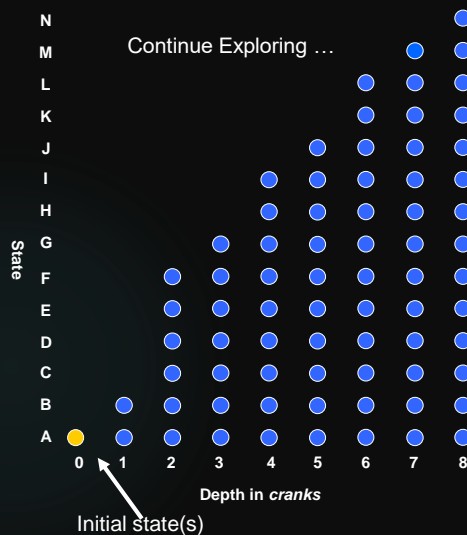
```

FormalVerifier> constraint -add *env_* -r
FormalVerifier> assert -add *fifo_* -r
FormalVerifier> prove
    fifo_no_full_and_empty : Pass
    fifo_remain_full_until_read : Fail (20)
    fifo_remain_empty_until_write : Explored (22) - Trigger : Pa
  
```

\*Source: Cadence IFV Training Material

# Until We Cannot Anymore

38

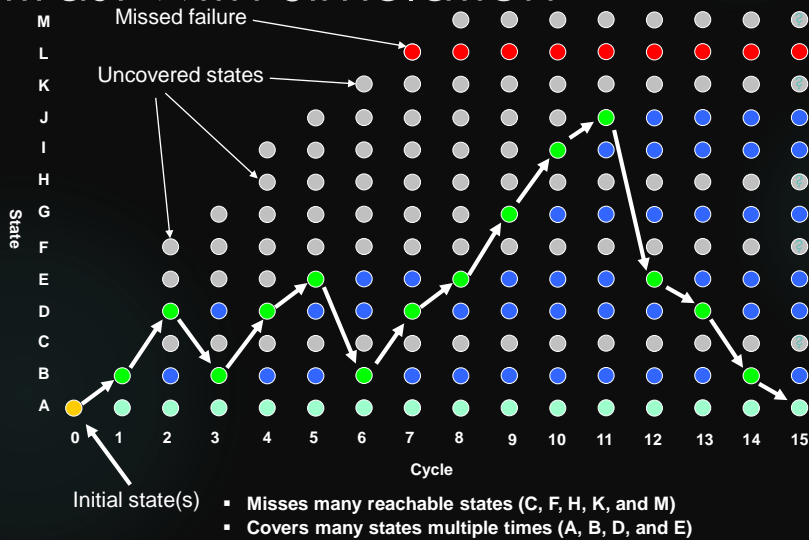


Additional state space still being reached implies that the proof is not complete

Effort (time-out) is defined by user  
Result is explored with reported depth. Up to that depth, no state is found to *violate* the assertion

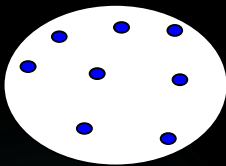
# Contrast With Simulation

39



# Motivation for Formal Verification

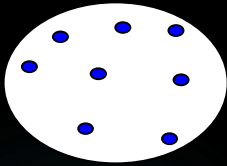
40



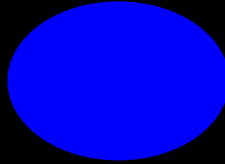
Simulation: spot coverage of design space

# Motivation for Formal Verification

41



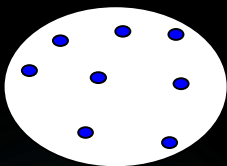
Simulation: spot coverage of design space



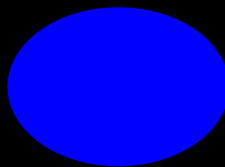
Formal Verification (ideal case): full coverage of design space

# Motivation for Formal Verification

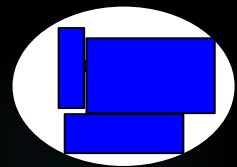
42



Simulation: spot coverage of design space



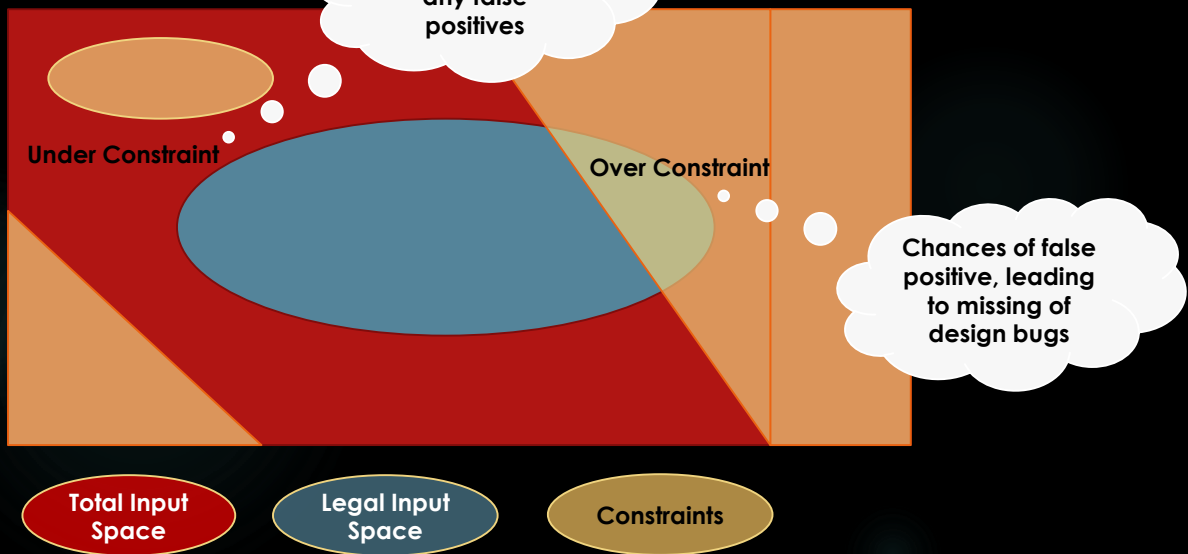
Formal Verification (ideal case): full coverage of design space



Formal Verification (real life): full coverage in some areas

# Coverage

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# Mechanics Informs Thought

44

Problem: How do I verify the fairness of a round-robin arbiter?

Simulation:

- I think I will write a behavioral predictor for the arbitration
- Need to think of test scenarios to exercise the DUT
  - Client A requests twice, while B requests once
  - All clients request simultaneously and keep requesting ...
- How many clients can hang off the arbiter again?
- Aaargh! I think I will just randomize the inputs

Formal :

- How do I come up with invariants for fairness?
- Example: If there are  $N$  agents, and an agent is requesting, it must get a grant within  $N$  grants from the arbiter. Must prove this for all the agents in the system.

# Introducing the ARBITER

45

□ mem-arbiter (input  $r1$ ,  $r2$ ,  $clk$ , output  $g1, g2$ )

• Specification/requirements :

1. Request line  $r1$  has higher priority than request line  $r2$ . Whenever  $r1$  goes high, the grant line  $g1$  must be asserted for the next two cycles.
2. None of the request lines are high, the arbiter parks the grant on  $g2$  in the next cycle.
3. The grant lines,  $g1$  and  $g2$ , are mutually exclusive.

# Properties in temporal logic

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□ Request line  $r1$  has higher priority than request line  $r2$ . Whenever  $r1$  goes high, the grant line  $g1$  must be asserted for the next two cycles.

- $G [r1 \rightarrow Xg1 \wedge XXg1]$
- $r1 \Rightarrow g1 \ \#\#1 \ g1$

□ When none of the request lines are high, the arbiter parks the grant on  $g2$  in the next cycle.

- $G [\neg r1 \wedge \neg r2 \rightarrow Xg2]$
- $!r1 \ \&\& \ !r2 \Rightarrow g2$

□ The grant lines,  $g1$  and  $g2$ , are mutually exclusive.

- $G [\neg g1 \vee \neg g2]$
- $!g1 \ || \ !g2$



Is it consistent ?

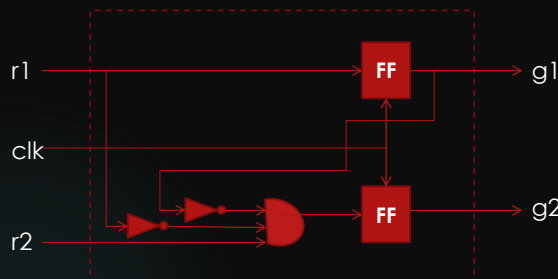
# Remove it ...

47

- $G [\neg g1 \rightarrow g2]$
- $!g1 \rightarrow g2$
  
- Have I written enough properties?
  1.  $G [r1 \rightarrow Xg1 \wedge XXg1]$  i.e.  $r1 \Rightarrow g1 \#1 g1$
  2.  $G [\neg g1 \rightarrow g2]$  i.e.  $!g1 \rightarrow g2$
  3.  $G [\neg g1 \vee \neg g2]$  i.e.  $!(g1) || !(g2)$
  
- Does the spec cover any behavior where  $g1$  is required to be high?
  - YES
- Does it enforce  $g2$  to be high?
  - NO
- Should any extra property be added?
  - $G [\neg r1 \wedge X\neg r1 \rightarrow XX\neg g1]$
  - $!r1 \#1 !r1 \Rightarrow !g1$

# Property Verification

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```

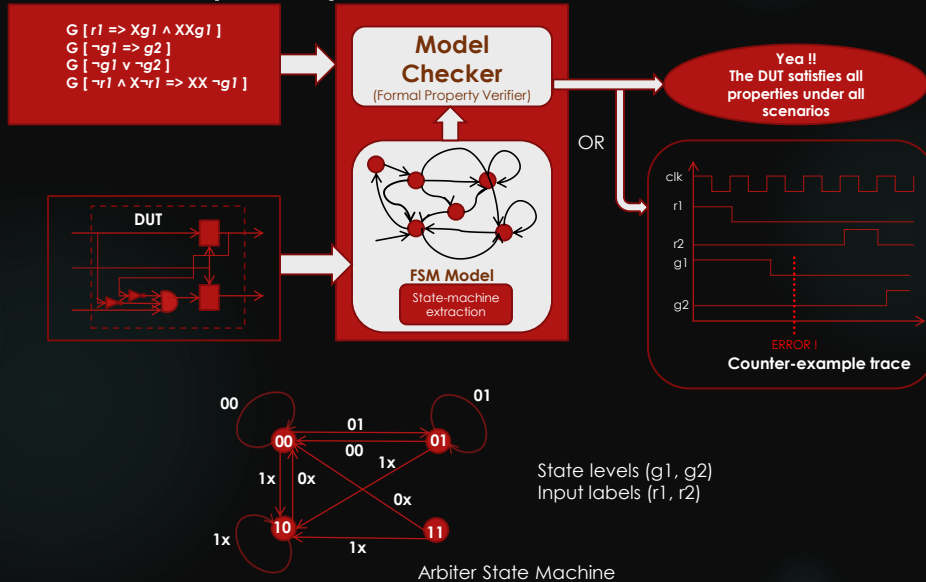
module arbiter( r1, r2, g1, g2, clk )
input clk, r1, r2;
output g1, g2;
reg g1, g2;
always @( posedge clk )
begin
  g2 <= r2 & ~r1 & ~g1;
  g1 <= r1;
end
endmodule

```



# Formal Property Verification Platform

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## Types of Properties

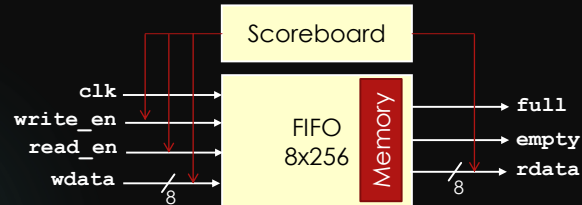
50

- ▶ **Safety** : "Something bad won't happen."  
 $G(\sim (\text{grant} \ \& \ \text{busy}))$
- ▶ **Liveness** : "Something good will happen."  
 $F(\text{grant})$
- ▶ **Fairness** : "Something happens infinitely often."  
 $G(F(!\text{busy}))$ 
  - ▶ Usually considered subset of liveness

# Down The Data Path – FIFO revisited

51

Problem: How do I verify the data integrity of my FIFO DUT?



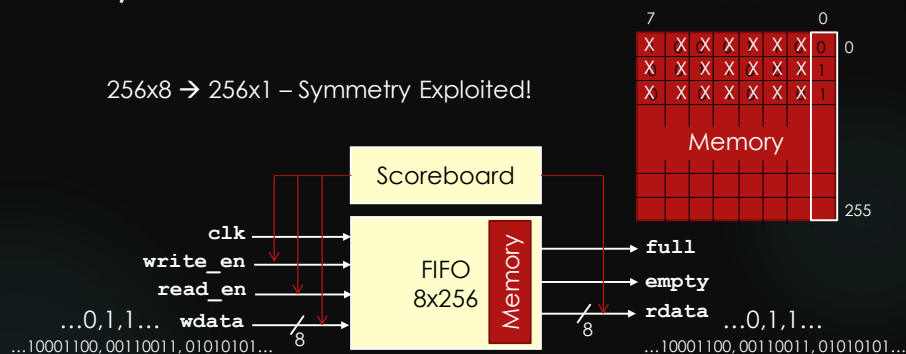
Problem Characterization: What is data integrity for the FIFO?

- The DUT should not corrupt any data
- The DUT should not drop any data
- The DUT should not duplicate any data
- The DUT should not re-order the data

# Cut, Cut, Cut

52

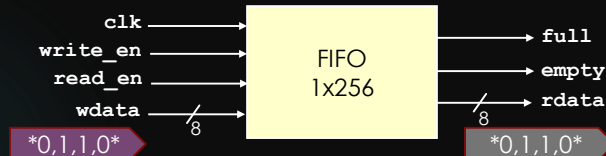
256x8 → 256x1 – Symmetry Exploited!



# The basic Abstractions

53

Key Insight: A *suitably aligned* input sequence of 0s, followed by two consecutive 1s, followed by 0s can detect *all* of the integrity errors



Problem Characterization: What is data integrity for the FIFO?

- The DUT should not corrupt the data
- The DUT should not drop any data
- The DUT should not duplicate any data
- The DUT should not re-order the data

`*0,1,0*``*0,1,0*``*0,1,1,0*``*0,1,0,1,0*`

# Enumerating The Alignment

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Applying the idea in simulation would involve enumerating the alignment

Test-1 : 1,1  
 Test-2 : 0,1,1  
 Test-3 : 0,1,1,0  
 Test-4 : 0,0,1,1,0 ...

Applying the idea in formal would involve...

Two invariants specify the pattern .....

- The *first time* we see 1, it must be followed by another 1
- We must not see more than two 1s

# Assume-Assert Duality

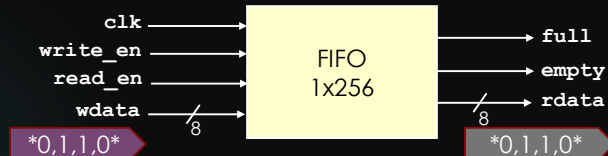
55

## Assume:

- The first time we write 1 we must write 1 the next time
- We must not write more than two 1s

## Assert:

- The first time we read 1 we must read 1 the next time
- We must not read more than two 1s



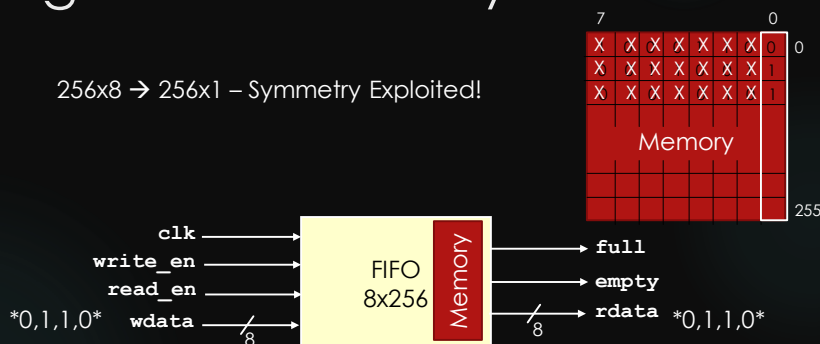
## Application :

- Asynchronous FIFOs can (and should) be verified this way
- Generalized to almost every data-transport design you can think of including
  - Memory controllers
  - DMA controllers
  - Bridges and cross-bars etc.

# Anything Else Cut-worthy?

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256x8 → 256x1 – Symmetry Exploited!

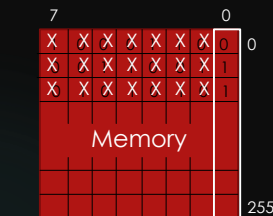


## Key Insights

- The memory array is not very interesting i.e. can be assumed correct
- We really, only need to track the 2 consecutive 1s

# And... Cut

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256x8 → 2x8!

Each time we write a 1 into the RAM we record the address

Write 0 to Address 0x???

Don't Care 0 0 0 0 1 1 0 0

Write 1 to Address 0xD

0 0 0 0 1 1 0 1 A

Write 1 to Address 0xC

0 0 0 0 1 1 0 0 B

Each time we read we compare with the recorded addresses. Send 1 on match, else 0.

Read from Address X

```
rdata[0] <= (X == A) ? 1 :
              (X == B) ? 1 :
              0;
```

# Compositional Reasoning

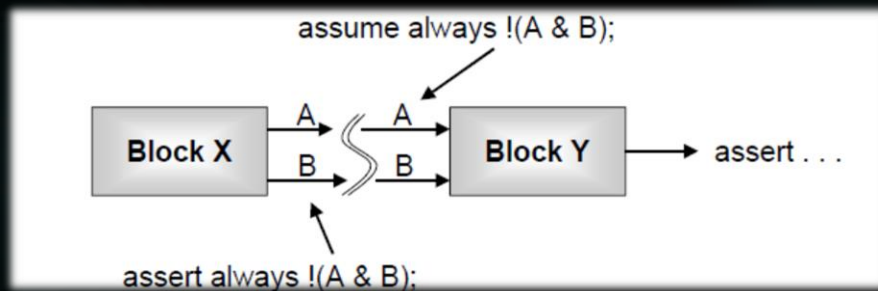
58

- ▶ Process of reducing an analysis of a larger concurrent system to reasoning about its individual functional pieces
- ▶ Effective for managing proof complexity and state explosion during a formal proof
- ▶ Transfers the burden of proof from the global component to the local functional component level
- ▶ Global properties can be inferred from independently verified functional component properties
- ▶ Example : Assume-Guarantee, Formal Abstraction (next slides)

# Assume-Guarantee

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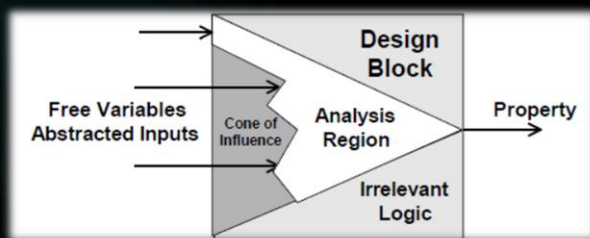
- ▶ Prove properties on a decomposed block using a set of assumptions about another neighboring block
- ▶ Prove these assumptions separately on the neighboring block



# Formal Abstraction

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- ▶ First prove properties on a subsection of the formal analysis block
- ▶ Then the driving logic for this subsection is abstracted
- ▶ The design logic is thus ignored in favor of the proved properties
- ▶ Key proponent :
  - ▶ If a property holds on the formal abstractions (the generalization), then it holds on the entire cone of influence (the actual design logic)



Remark : if a property fails on the formal abstraction, then it might be necessary to include additional logic into a larger analysis region, forming a new abstraction that eliminates the false negative.

# Some Formal Methods Myths

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- ❑ Formal methods can only be used by mathematicians
  - ❑ They are primarily based on mathematical concepts that is usually transparent to the user
- ❑ The reasoning process is itself prone to errors, so why bother?
  - ❑ We opt to reduce design bugs not eliminate them
- ❑ Using formal methods tends to slow the design process
  - ❑ The early detection of design bugs are allows us to speed up the overall design process

Welcome Questions  
from Green minds !!

Thank  
you

## Few References and acknowledgements :

- Formal HW Verification course by Prof. Sofiene Tahar
- Book : A Roadmap for Formal Property Verification by Prof. Pallab Dasgupta, Springer Publication
- DVCON paper "Guidelines for creating a FV testplan", authored by Harry Foster, Lawrence Loh, Bahman Rabi and Vigyan Singhal
- Introduction to FV talk @Nvidia-B'lore by Abhishek Datta
- Book : William K. Lam (2005) Hardware Design Verification: Simulation and Formal Method-Based Approaches
- Course Training Material
- Yogesh Mahajan, Deepanjan Roy, Mark Bezdany, Kalyan K, Abhishek Datta, Prosenjit Chatterjee (nVIDIA)