



UPPSALA  
UNIVERSITET



Uppsala  
Secure Learning  
and Control Lab

# Security Allocation in Networked Control Systems

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Dissertation for the degree of Licentiate

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STIFTELSEN för  
STRATEGISK FORSKNING



Swedish  
Research Council

# Outline

1

## Introduction

2

## Security in Networked Control Systems

3

## Problem Formulation

4

## Contributions

- Paper I
- Paper II
- Paper III
- Paper IV

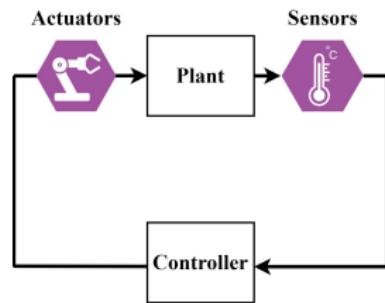
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## Conclusion and Future Work

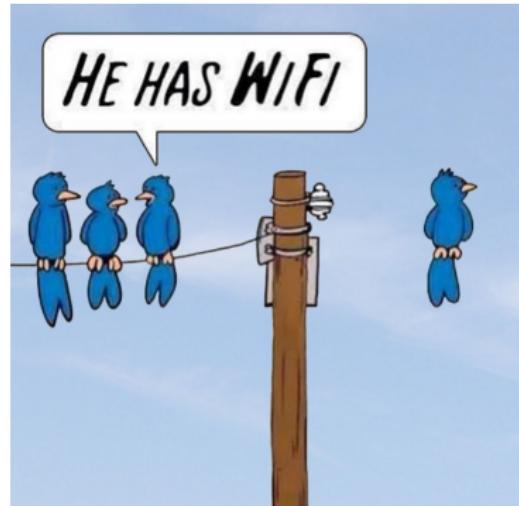
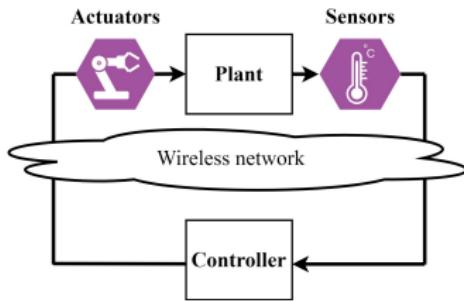
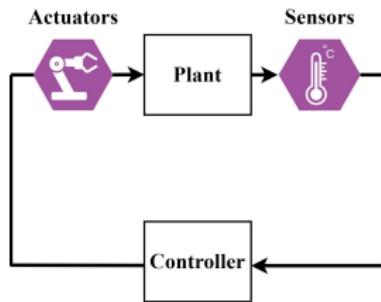
# Critical Infrastructure



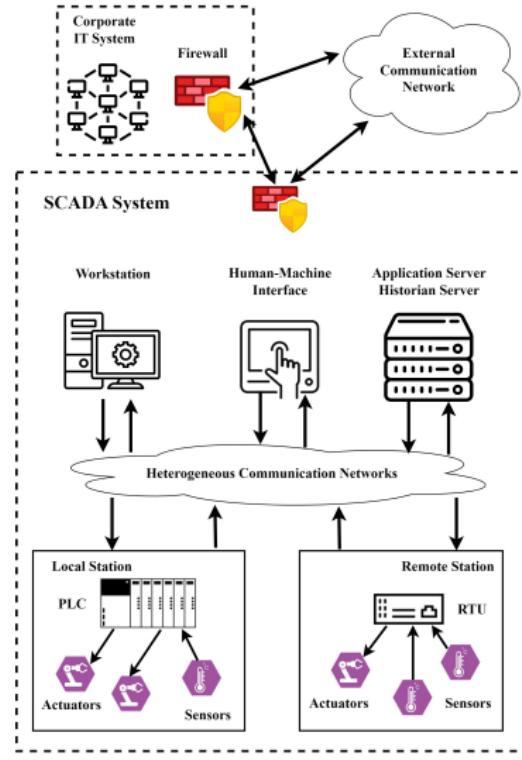
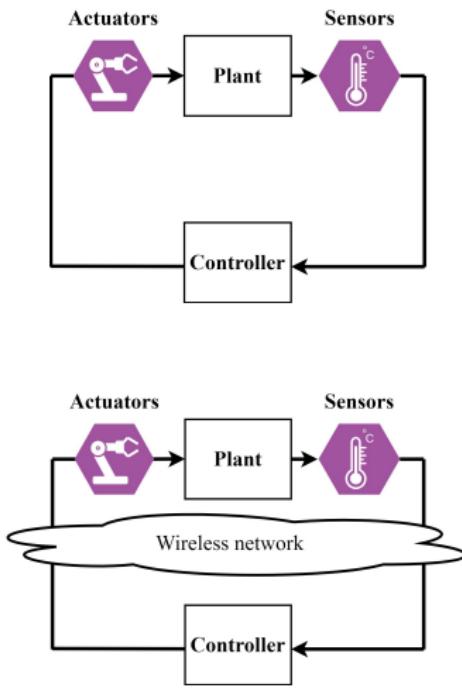
# Control of Critical Infrastructure



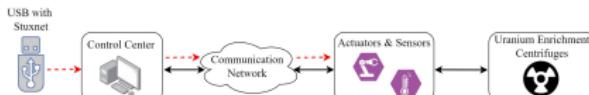
# Control of Critical Infrastructure



# Control of Critical Infrastructure



# Vulnerabilities in Critical Infrastructure



a) Management Access



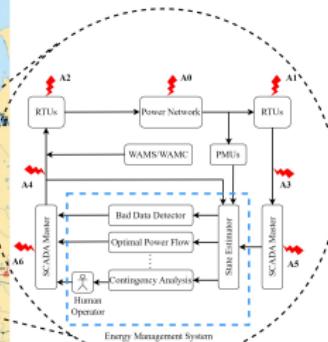
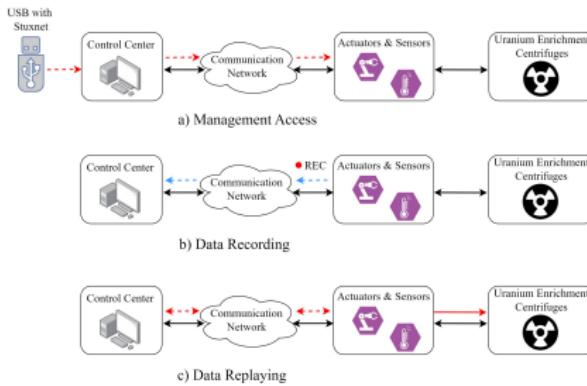
b) Data Recording



c) Data Replaying

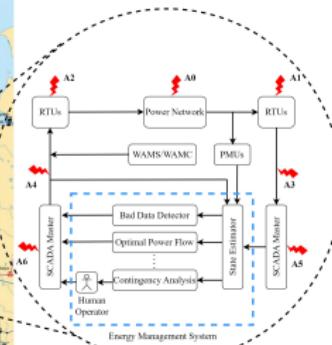
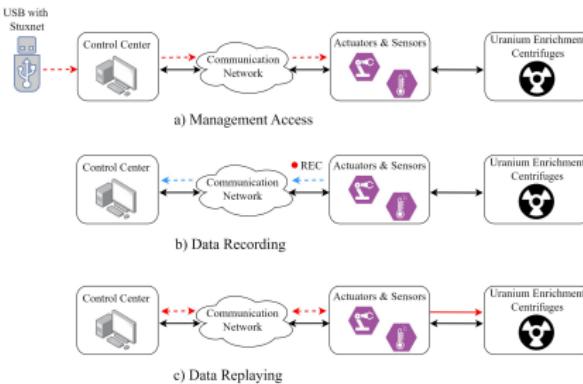
## Stuxnet

# Vulnerabilities in Critical Infrastructure



Stuxnet

# Vulnerabilities in Critical Infrastructure



Stuxnet

## Motivation

Critical Infrastructure should be protected actively

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2 Security in Networked Control Systems

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4 Contributions

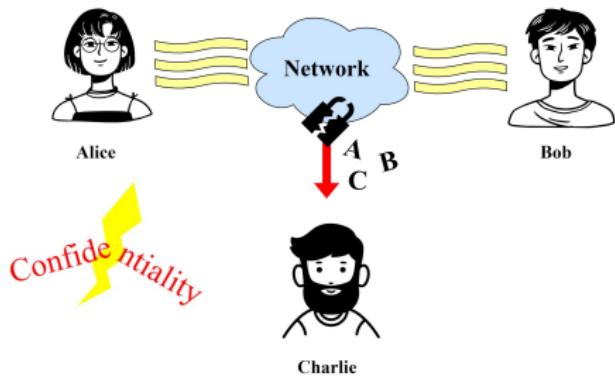
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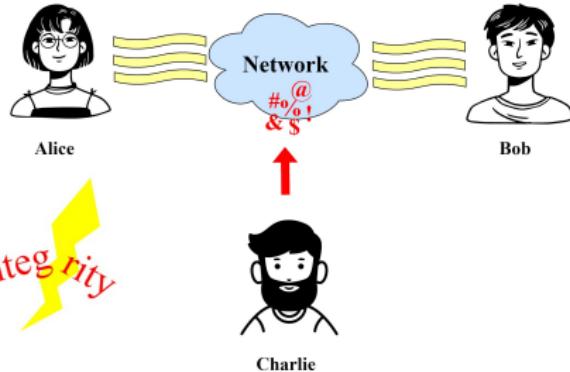
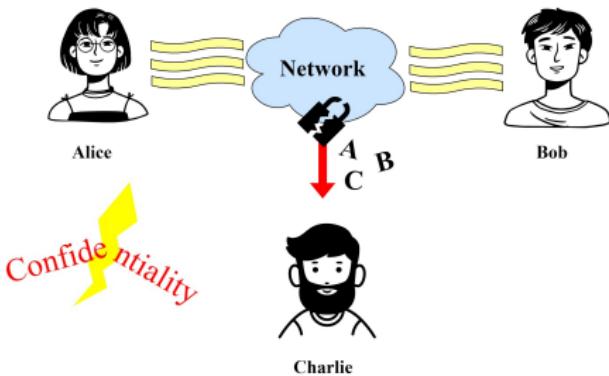
# Security Triad and Threats



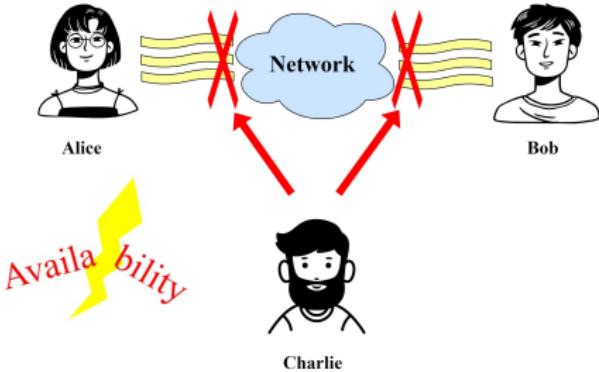
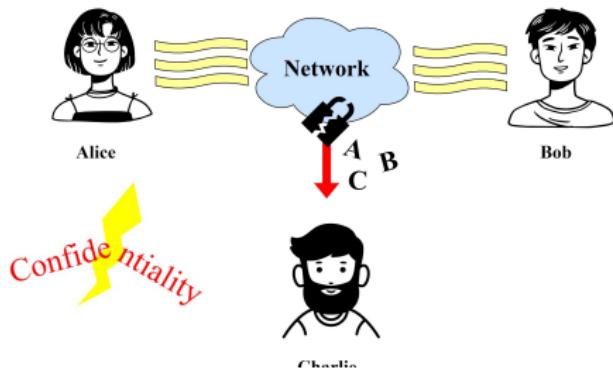
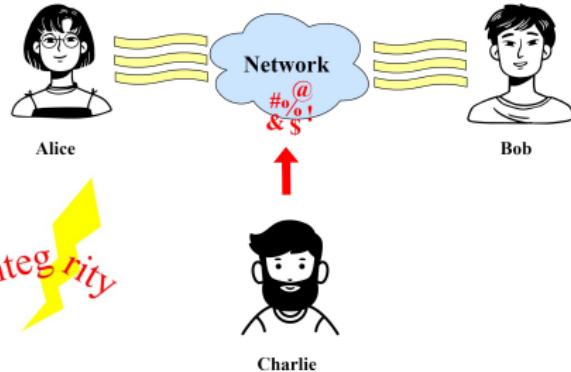
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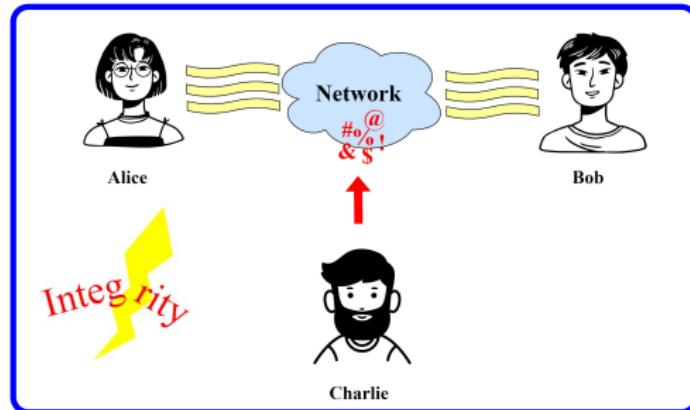
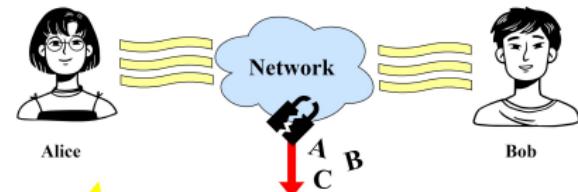
# Security Triad and Threats



# Security Triad and Threats

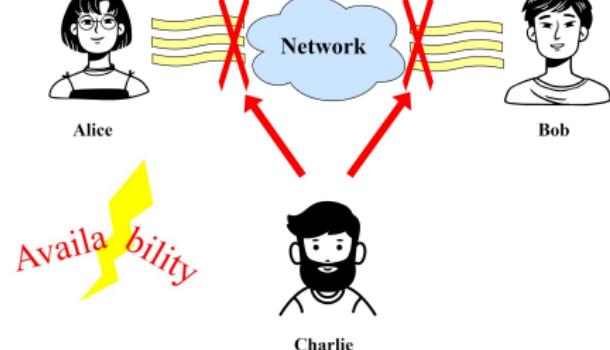


# Security Triad and Threats



Confidentiality

Charlie



Availability

Charlie

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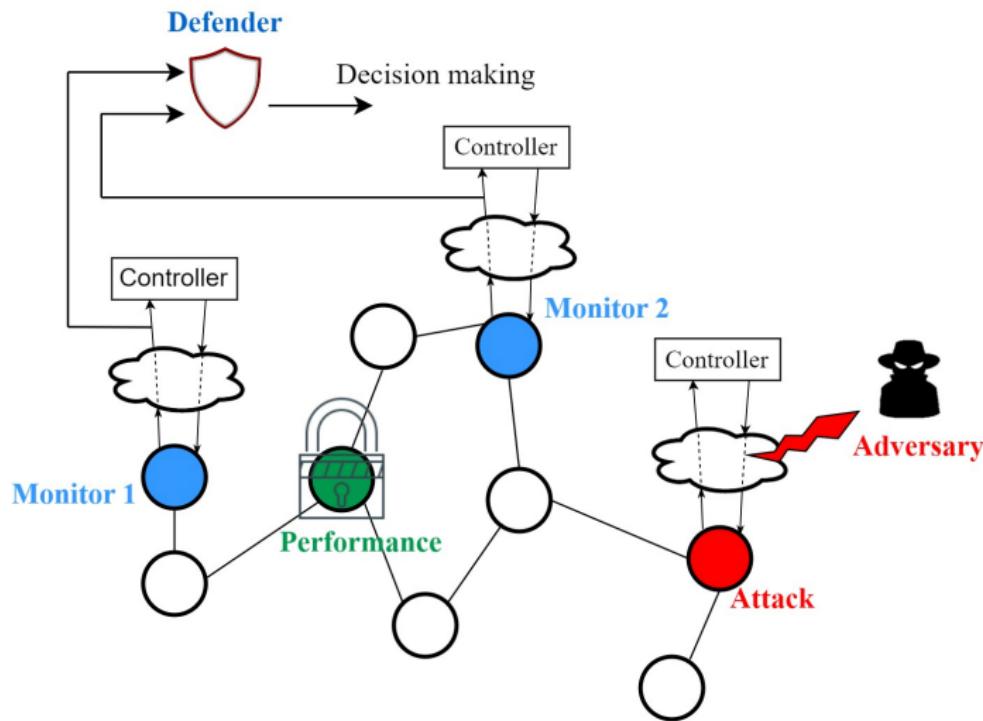
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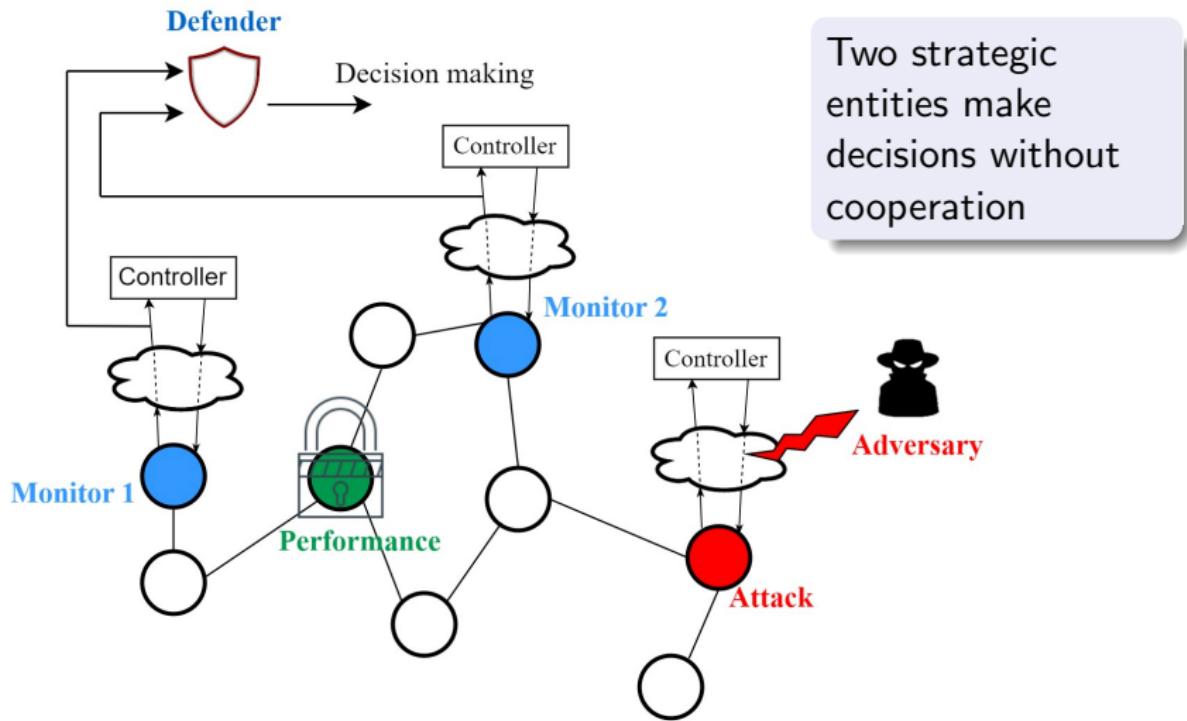
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# Problem description



# Problem description



# Problem analysis



Defender



Adversary

- Purpose: protect the system
- Purpose: attack the system

# Problem analysis



Defender



Adversary

- Purpose: protect the system
  - Action: monitor what?
- Purpose: attack the system
  - Action: attack what?

# Problem analysis



Defender



Adversary

- Purpose: protect the system
  - Action: monitor what?
- Purpose: attack the system
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Action order:

- 1) Make decisions simultaneously
- 2) The defender goes first

# Problem analysis



Defender



Adversary

- Purpose: protect the system
- Action: monitor what?

- Purpose: attack the system
- Action: attack what?

Action order:

1) Make decisions simultaneously

2) The defender goes first

Non-cooperative  
two-player game



# Problem analysis



Defender



Adversary

- Purpose: protect the system
- Action: monitor what?

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Action order:

- 1) Make decisions simultaneously
- 2) The defender goes first

## Problem variations

- ▷ System models
- ▷ Resources & knowledge
- ▷ Action order

# Problem variations



Defender



Adversary

Performance  $\rho$ 

## Paper I

- Certain LFO
- Performance  $\rho$  is fixed
- Def./Adv. chooses one
- Take actions simultaneously

## Paper II

- Uncertain LFO
- Performance  $\rho$  is fixed
- Def./Adv. chooses one
- Take actions simultaneously

## Paper III

- Certain LSO
- Performance  $\rho$  is fixed
- Def./Adv. chooses one
- Take actions simultaneously

## Paper IV

- Certain LFO
- Performance  $\rho$  is uncertain
- Adv. chooses one, Def. chooses several
- Def. takes action firstly

# Problem variations



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# Problem formulation

- Undirected connected graph  $\mathcal{G}$  with  $N$  nodes

$$\begin{aligned}\dot{x}_i(t) &= A_i x_i(t) + b \tilde{u}_i(t), \\ y_i(t) &= c^\top x_i(t).\end{aligned}$$

# Problem formulation

- Undirected connected graph  $\mathcal{G}$  with  $N$  nodes

$$\begin{aligned}\dot{x}_i(t) &= A_i x_i(t) + b \tilde{u}_i(t), \\ y_i(t) &= c^\top x_i(t).\end{aligned}$$

- Local performance:  $\|y_\rho\|_{\mathcal{L}_2[0,T]}^2 = \frac{1}{T} \int_0^T |y_\rho(t)|^2 dt$

# Problem formulation

- Undirected connected graph  $\mathcal{G}$  with  $N$  nodes

$$\dot{x}_i(t) = A_i x_i(t) + b \tilde{u}_i(t),$$

$$y_i(t) = c^\top x_i(t).$$

Adversary  
chooses node  $a$

- Local performance:  $\|y_\rho\|_{\mathcal{L}_2[0,T]}^2 = \frac{1}{T} \int_0^T |y_\rho(t)|^2 dt$

- Healthy/attacked local controller

$$\tilde{u}_i(t) = \underbrace{\sum_{j \in \mathcal{N}_i} \phi_{ij}(x_i, x_j)}_{\text{healthy}} + \begin{cases} 0, & \text{if } i \neq a \\ \zeta(t), & \text{if } i \equiv a \end{cases}$$

$\Rightarrow$  Closed-loop system:  $\dot{x}(t) = Ax(t) + b \otimes e_a \zeta(t)$

# Problem formulation (Cont.)

- Closed-loop system:

$$\dot{x}(t) = Ax(t) + b \otimes e_a \zeta(t), \quad x(0) = 0$$

# Problem formulation (Cont.)

- Closed-loop system:

$$\dot{x}(t) = Ax(t) + b \otimes e_a \zeta(t), \quad x(0) = 0$$

- The defender can choose several nodes  $\mathcal{M} = \{m_1, m_2, \dots, m_{|\mathcal{M}|}\}$

$$y_{m_1}(t) = e_{m_1}^\top x(t), \quad y_{m_2}(t) = e_{m_2}^\top x(t), \quad \dots \quad y_{|\mathcal{M}|}(t) = e_{|\mathcal{M}|}^\top x(t).$$

- Monitor outputs such that at least

$$\|y_{m_k}\|_{\mathcal{L}_2}^2 = \frac{1}{T} \int_0^T |y_{m_k}(t)|^2 dt > \delta_{m_k} \quad \Rightarrow \quad \text{Attack is detected!!!}$$

# Problem formulation (Cont.)

- Closed-loop system:

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- Adversary's purpose: stay stealthy

$$\|y_{m_k}\|_{\mathcal{L}_2}^2 = \frac{1}{T} \int_0^T |y_{m_k}(t)|^2 dt \leq \delta_{m_k} \quad \forall m_k \in \mathcal{M}$$

$\Rightarrow$  Stealthy False Data Injection Attacks (Stealthy FDI Attacks)

Attack      Monitor      Challenges

$$\begin{array}{ccc}
 J_{\rho}(a, \mathcal{M}) & \triangleq & \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. state}} \|y_{\rho}\|_{\mathcal{L}_2}^2 \\
 \downarrow & & \swarrow \\
 \text{Performance} & & \text{s.t.} & \quad \|y_{m_k}\|_{\mathcal{L}_2}^2 \leq \delta_{m_k}, \quad \forall m_k \in \mathcal{M}
 \end{array} \tag{1}$$

Attack	Monitor	Challenges
$\downarrow$ $J_p(a, \mathcal{M}) \triangleq \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. state}} \ y_p\ _{\mathcal{L}_2}^2$ $\uparrow$	$\leftarrow$ $\text{s.t. } \ y_{m_k}\ _{\mathcal{L}_2}^2 \leq \delta_{m_k}, \forall m_k \in \mathcal{M}$	
The defender minimizes $J_p(a, \mathcal{M})$	The adversary maximizes $J_p(a, \mathcal{M})$	(1)

**Attack**      **Monitor**      **Challenges**

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The defender  
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The adversary  
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$$\min_{\mathcal{M} \subset \mathcal{V}} \quad \max_{a \in \mathcal{V}_{-\rho}} \quad J_{\rho}(a, \mathcal{M})$$

**Attack**      **Monitor**      **Challenges**

$$\begin{array}{ccc}
 J_{\rho}(a, \mathcal{M}) & \triangleq & \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. state}} \|y_{\rho}\|_{\mathcal{L}_2}^2 \\
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Combinatorial optimization problem

Attack      Monitor      Challenges

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Combinatorial optimization problem

$\Rightarrow$  Computational burden

# Attack Monitor Challenges

$$\begin{array}{ccc}
 \text{Attack} & \text{Monitor} & \\
 \downarrow & \swarrow & \\
 J_p(a, \mathcal{M}) & \triangleq & \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. state}} \|y_p\|_{\mathcal{L}_2}^2 \\
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Combinatorial optimization problem

$\Rightarrow$  Computational burden

Shrink defender action space  $\mathcal{M} \subset \mathbb{D} \subset \mathcal{V}$

# Attack Monitor Challenges

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Combinatorial optimization problem

$\Rightarrow$  Computational burden

Shrink defender action space  $\mathcal{M} \subset \mathbb{D} \subset \mathcal{V}$

$\Rightarrow$  Efficiently allocate  
defense resources

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$\mathbb{D}$  guarantees the boundedness of (1)

# Attack Monitor Challenges

$$\begin{array}{ccc}
 \text{Attack} & \text{Monitor} & \\
 \downarrow & \swarrow & \\
 J_p(a, \mathcal{M}) & \stackrel{\triangle}{=} & \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. state}} \|y_p\|_{\mathcal{L}_2}^2 \\
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$\Rightarrow$  Efficiently allocate defense resources

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**Main contributions**  
Characterize  $\mathbb{D}$

# Preliminary results

Boundedness of the worst-case  
impact of stealthy FDI attacks

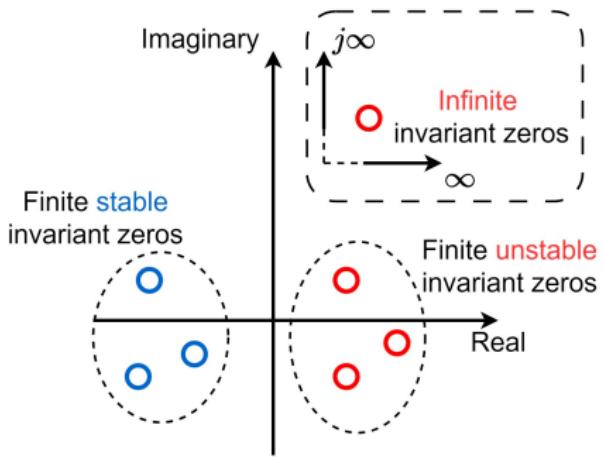


Invariant zeros

# Preliminary results

Boundedness of the worst-case impact of stealthy FDI attacks

↔ Invariant zeros

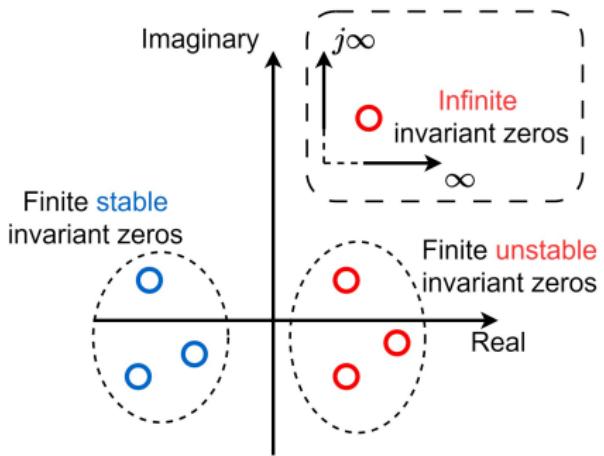


# Preliminary results

Boundedness of the worst-case impact of stealthy FDI attacks

$\Leftrightarrow$  Invariant zeros

$$\begin{aligned} \dot{x}(t) &= Ax(t) + b \otimes e_a \zeta(t), \\ y_\rho(t) &= e_\rho^\top x(t), \\ y_{m_k}(t) &= e_{m_k}^\top x(t), \quad \forall m_k \in \mathcal{M} \end{aligned}$$



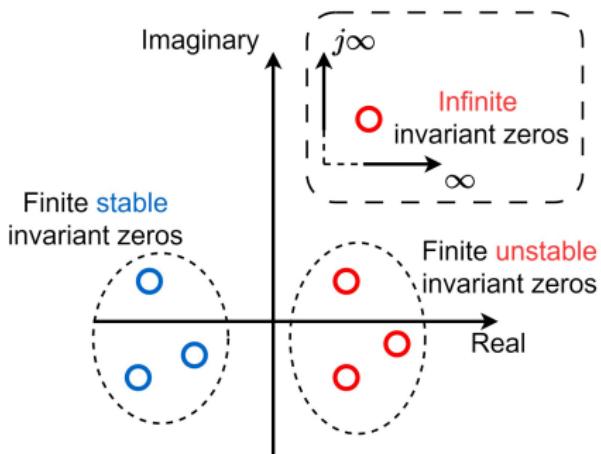
- Systems  $\Sigma_\rho = (A, b \otimes e_a, e_\rho^\top, 0)$  and  $\Sigma_{m_k} = (A, b \otimes e_a, e_{m_k}^\top, 0)$
- $$J_\rho(\mathbf{a}, \mathcal{M}) \triangleq \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. state}} \|y_\rho\|_{\mathcal{L}_2}^2$$
- s.t.       $\|y_{m_k}\|_{\mathcal{L}_2}^2 \leq \delta_{m_k}, \quad \forall m_k \in \mathcal{M}$

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Boundedness of the worst-case impact of stealthy FDI attacks

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- Systems  $\Sigma_\rho = (A, b \otimes e_a, e_\rho^\top, 0)$  and  $\Sigma_{m_k} = (A, b \otimes e_a, e_{m_k}^\top, 0)$
- $$J_\rho(a, \mathcal{M}) \triangleq \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. state}} \|y_\rho\|_{\mathcal{L}_2}^2$$
- $$\text{s.t.} \quad \|y_{m_k}\|_{\mathcal{L}_2}^2 \leq \delta_{m_k}, \quad \forall m_k \in \mathcal{M}$$
- At least  $\Sigma_{m_k}$ , its  $\lambda_{m_k}$  ( $\text{Re}[\lambda_{m_k}] > 0$ ) is also invariant zero of  $\Sigma_\rho$ ,  
if, and only if,  $J_\rho(a, \mathcal{M}) < \infty$

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# Paper I - Problem variation



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# Challenges

- Unweighted graph  $\mathcal{G}$  with  $N$  vertices, certain Laplacian matrix  $L$

$$\dot{x}(t) = -Lx(t) + e_a \zeta(t),$$

$$y_\rho(t) = e_\rho^\top x(t),$$

$$y_m(t) = e_m^\top x(t) \quad (\mathcal{M} = \{m\}).$$

- Worst-case impact of stealthy FDI attacks

$$\begin{aligned} J_\rho(a, m) &\triangleq \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. state}} \|y_\rho\|_{\mathcal{L}_2}^2 \\ &\text{s.t.} \quad \|y_m\|_{\mathcal{L}_2}^2 \leq \delta_m \end{aligned}$$

# Challenges

- Unweighted graph  $\mathcal{G}$  with  $N$  vertices, certain Laplacian matrix  $L$

$$\dot{x}(t) = -Lx(t) + e_a \zeta(t),$$

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No finite unstable  
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1. J. A. Torres & S. Roy, "Graph-theoretic analysis of network input-output processes: Zero structure and its implications on remote feedback control", *Automatica*, 2015

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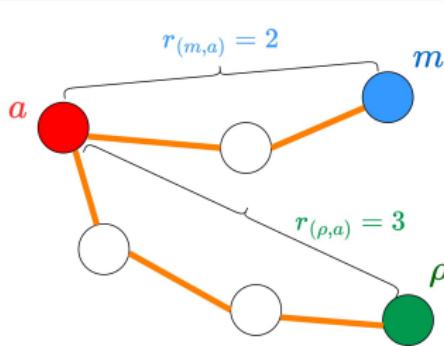
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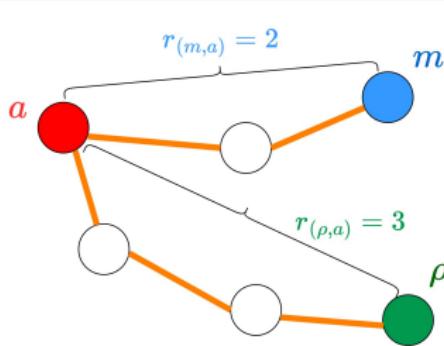
**Challenge**  
Infinite invariant zeros

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## Main results

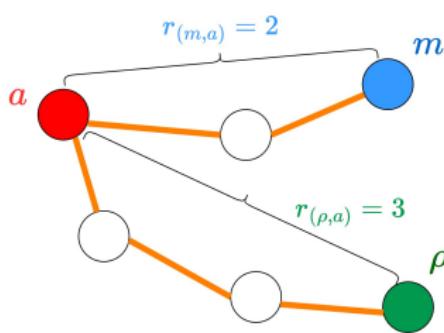
$\Sigma_m$ : output at  $m$ , relative degree  $r_{(m,a)}$   
 $\Sigma_\rho$ : output at  $\rho$ , relative degree  $r_{(\rho,a)}$



## Main results

$\Sigma_m$ : output at  $m$ , relative degree  $r_{(m,a)}$   
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# inf. inv. zero = relative degree



## Main results

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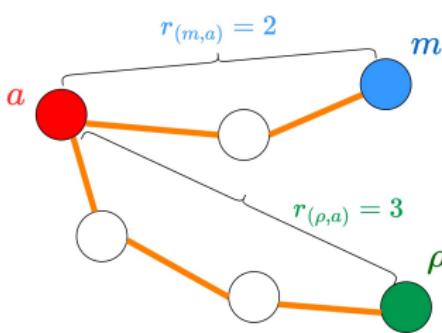
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### Theorem 1

# inf. inv. zero of  $\Sigma_m \leq$  # inf. inv. zero of  $\Sigma_\rho$

$$\Leftrightarrow r_{(m,a)} \leq r_{(\rho,a)} \Leftrightarrow J_\rho(a, m) < \infty$$



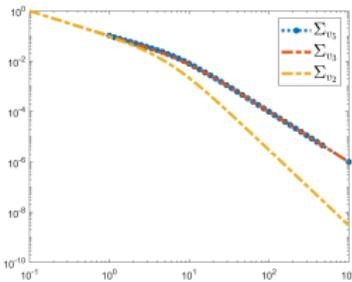
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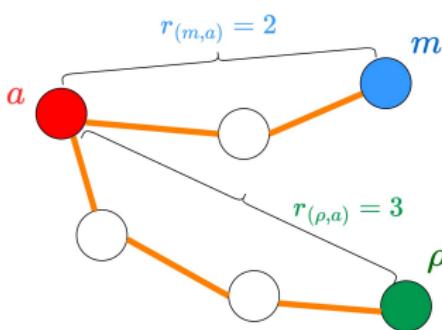
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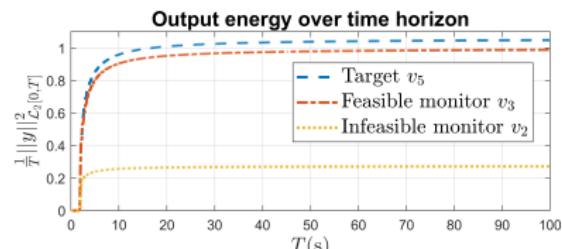
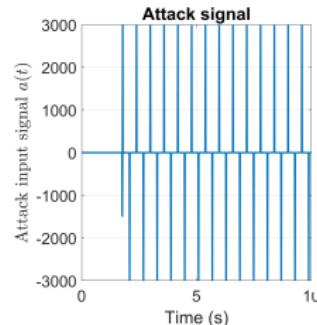
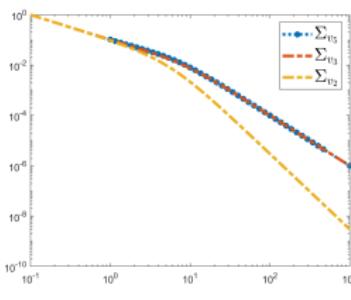
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# Outline

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# Paper II - Problem variation



Defender



Adversary

Performance  $\rho$ 

## Paper I

- Certain LFO
- Performance  $\rho$  is fixed
- Def./Adv. chooses one
- Take actions simultaneously

## Paper II

- Uncertain LFO
- Performance  $\rho$  is fixed
- Def./Adv. chooses one
- Take actions simultaneously

## Paper III

- Certain LSO
- Performance  $\rho$  is fixed
- Def./Adv. chooses one
- Take actions simultaneously

## Paper IV

- Certain LFO
- Performance  $\rho$  is uncertain
- Adv. chooses one, Def. chooses several
- Def. takes action first

# Challenges

- Uncertain weighted graph  $\mathcal{G}$  with  $N$  vertices, uncertain  $L^\Delta$

$$\dot{x}^\Delta(t) = -L^\Delta x^\Delta(t) + e_a \zeta(t),$$

$$y_\rho^\Delta(t) = e_\rho^\top x^\Delta(t),$$

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$$L^\Delta = \bar{L} + \Delta$$

$$\Delta \in \Omega$$

# Challenges

- Uncertain weighted graph  $\mathcal{G}$  with  $N$  vertices, uncertain  $L^\Delta$

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## Challenges

- 1) Finite unstable inv. zeros<sup>1</sup>
- 2) Infinite inv. zeros
- 3) Evaluate worst-case attack impact

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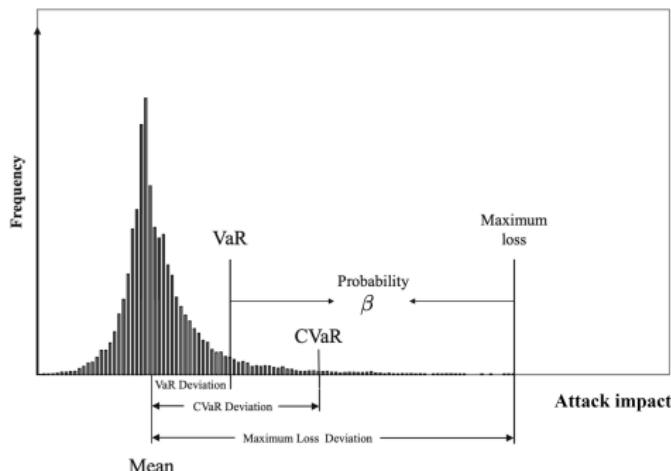
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# Value-at-Risk

$$\mathcal{J}_\rho(a, m) = \text{VaR}_{\beta, \Omega} \left[ \sup_{\zeta \in \mathcal{L}_{2e}} J_\rho(a, m; \Delta, \zeta) \right]$$

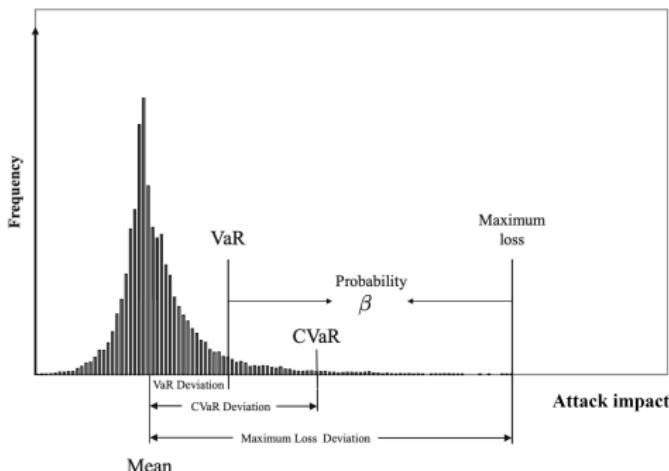
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## Theorem 1 & Lemma 2

$M_1$  values from  $\Omega$

Evaluate  $[M_1(1 - \beta_1)]$  values with  $\epsilon$  accuracy

$$M_1 \geq \frac{1}{2\epsilon_1^2} \log \frac{2}{\beta_1}$$

E.g.,  $\epsilon_1 = 0.06$ ,  $\beta_1 = 0.08$ ,

$$M_1 \geq 450 \Rightarrow 414 \text{ values}$$

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# Paper III - Problem variation



Defender



Adversary

Performance  $\rho$ 

## Paper I

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# Challenges

- **Main focus:** Power networks by linearized swing equations

$$m_i \ddot{p}_i(t) + h_i \dot{p}_i(t) = \sum_{j \in \mathcal{N}_i} \ell_{ij} (p_i(t) - p_j(t)) + \tilde{u}_i(t),$$

- Closed-loop system

$$\dot{x}(t) = Ax(t) + e_a \zeta(t),$$

$$y_i(t) = C_i x(t), \quad \forall i \in \mathcal{V},$$

$$y_\rho(t) = C_\rho x(t),$$

- Local performance:  $\|y_\rho\|_{\mathcal{L}_2[0,T]}^2 = \frac{1}{T} \int_0^T |y_\rho(t)|^2 dt$

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- At node  $m \in \mathcal{V}_{-\rho}$  where  $(A, C_m)$  is detectable,

Detector

$$\begin{aligned}\dot{\hat{x}}_m(t) &= A\hat{x}_m(t) + K_m \eta_m(t), \quad \hat{x}_m(0) = 0, \\ \eta_m(t) &= y_m(t) - C_m \hat{x}_d(t),\end{aligned}$$

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# Main results

- The worst-case impact of stealthy FDI attacks

$$J_{\rho}(a, m) \triangleq \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. states}} \|y_{\rho}\|_{\mathcal{L}_2}^2$$
$$\text{s.t.} \quad \|\eta_m\|_{\mathcal{L}_2}^2 \leq \delta$$

- Systems  $\Sigma_{\rho} = (A, e_a, C_{\rho}, 0)$  and  $\Sigma_m = (A, e_a, C_m, 0)$
- Denote  $r_{(\rho, a)}$  and  $r_{(m, a)}$  as the relative degrees of  $\Sigma_{\rho}$  and  $\Sigma_m$

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## Lemma 3 (choice of parameters)

Finite **unstable** invariant zeros  $\lambda_m$  of  $\Sigma_m$  can be excluded by proper local control parameters. Then,  $J_{\rho}(a, m) < \infty$ .

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## Theorem 3.1 (relative degree condition)

$$r_{(m, a)} \leq r_{(\rho, a)}$$

$$\Rightarrow J_{\rho}(a, m) < \infty$$

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# Paper IV - Problem variation



Defender



Adversary

Performance  $\rho$ 

## Paper I

- Certain LFO
- Performance  $\rho$  is **fixed**
- Def./Adv. chooses **one**
- Take actions **simultaneously**

## Paper III

- Certain LSO
- Performance  $\rho$  is fixed
- Def./Adv. chooses one
- Take actions simultaneously

## Paper II

- Uncertain LFO
- Performance  $\rho$  is fixed
- Def./Adv. chooses one
- Take actions simultaneously

## Paper IV

- **Certain LFO**
- Performance  $\rho$  is **uncertain**
- Adv. chooses one, Def. chooses **several**
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# Challenges

- Unweighted graph  $\mathcal{G}$  with  $N$  vertices, certain Laplacian matrix  $L$

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- Worst-case attack impact

$$J_\rho(a, \mathcal{M}) \triangleq \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. state}} \|y_\rho\|_{\mathcal{L}_2}^2$$

s.t.       $\|y_{m_k}\|_{\mathcal{L}_2}^2 \leq \delta_{m_k} \quad \forall m_k \in \mathcal{M}$

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Challenges

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# Players' strategies

Defender strategy

$$\mathcal{M}^* = \arg \min_{\mathcal{M} \subset \mathbb{D}} \text{Defense cost}|_{\mathbf{a}^*(\mathcal{M})}$$

$$\mathbf{a}^*(\mathcal{M}) = \arg \max_{\mathbf{a} \in \mathbb{A}} \text{Defense cost}$$

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Adversary response

$$\mathbf{a}^* = \arg \max_{\mathbf{a} \in \mathbb{A}} \text{Attack impact}|_{\mathcal{M}^*}$$

# Players' strategies

Defender strategy

$$\mathcal{M}^* = \arg \min_{\mathcal{M} \subset \mathbb{D}} \text{Defense cost}|_{a^*(\mathcal{M})}$$

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Defender

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Combinatorial optimization problem

Defender

Adversary



⇒ Computational burden

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## Combinatorial optimization problem

⇒ Computational burden

Shrink defender action space  $\mathcal{M} \subset \mathbb{D} \subset \mathcal{V}$

→ Efficiently allocate defense resources

1

▷ s.t. defense cost/attack impact  $< \infty$

# Players' strategies

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Combinatorial optimization problem

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Shrink defender action space  $\mathcal{M} \subset \mathbb{D} \subset \mathcal{V}$

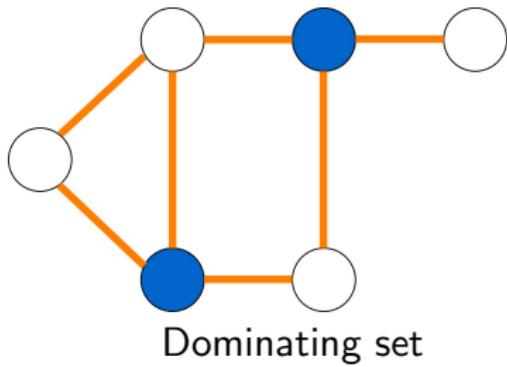


⇒ Efficiently allocate defense resources

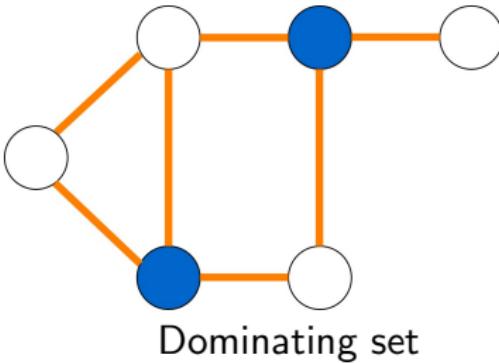
$\mathbb{D}$  s.t. defense cost/attack impact  $< \infty$

⇐ Characterize  $\mathbb{D}$

# Main results



# Main results



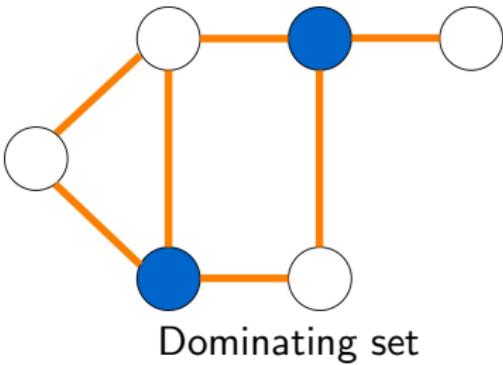
**Theorem 2 (necessary and sufficient condition)**

$\mathcal{M}$  is a dominating set

$\Leftrightarrow$  def. cost  $R(\mathbf{a}, \mathcal{M}) < \infty$

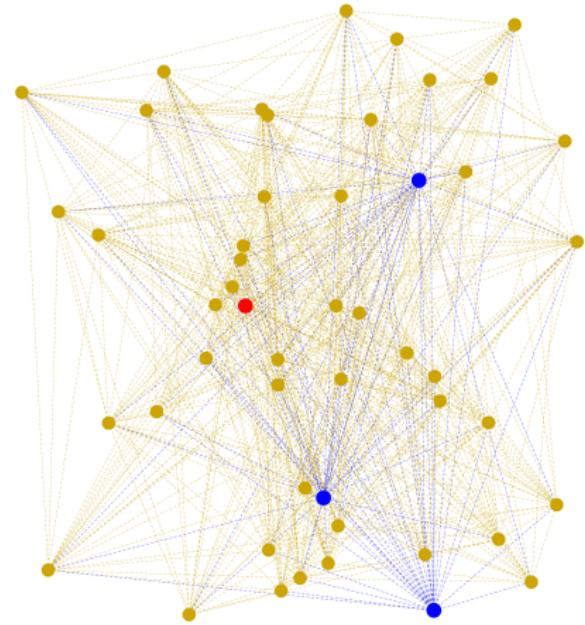
& attack impact  $Q(\mathbf{a}, \mathcal{M}) < \infty$

# Main results

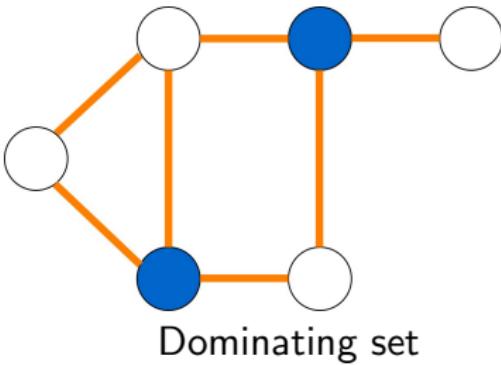


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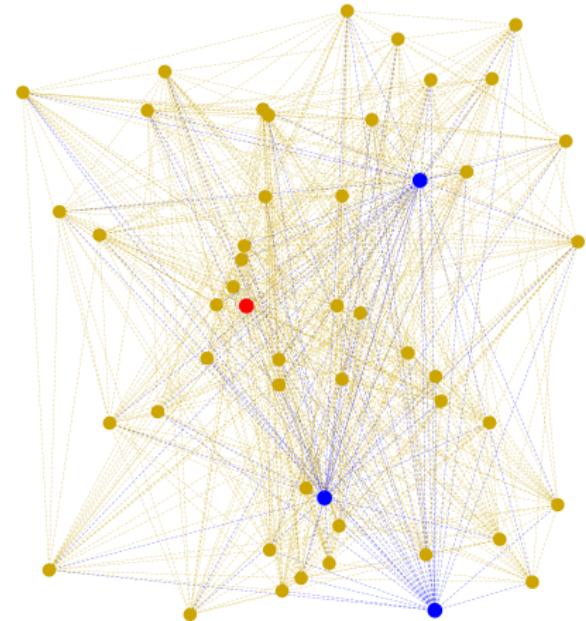


# Main results



**Theorem 2 (necessary and sufficient condition)**

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& attack impact  $Q(\mathbf{a}, \mathcal{M}) < \infty$



$$R(\mathbf{a}, \mathcal{M}) \leq 50.2456$$
$$Q(\mathbf{a}, \mathcal{M}) \leq 48.4235$$

# Outline

- 1 Introduction
- 2 Security in Networked Control Systems
- 3 Problem Formulation
- 4 Contributions
  - Paper I
  - Paper II
  - Paper III
  - Paper IV
- 5 Conclusion and Future Work

# Conclusion and Future Work

## This Licentiate thesis has

- ① considered several types of NCSs under attacks
- ② intensively investigated the worst-case impact of stealthy FDI attacks
- ③ found system- and graph-theoretic conditions
- ④ assisted the defender in allocating defense resources

# Conclusion and Future Work

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- ② intensively investigated the worst-case impact of stealthy FDI attacks
- ③ found system- and graph-theoretic conditions
- ④ assisted the defender in allocating defense resources

## Toward the PhD thesis, it will be extended to

- ① overcome combinatorial optimization problem
- ② consider uncompleted information
- ③ consider multiple adversaries
- ④ assist the defender in designing detectors
- ⑤ .....



Defender



Adversary

Performance  $\rho$ **Paper I**

- Certain LFO
- Performance  $\rho$  is fixed
- Def./Adv. chooses one
- Take actions simultaneously

**Paper II**

- Uncertain LFO
- Performance  $\rho$  is fixed
- Def./Adv. chooses one
- Take actions simultaneously

**Paper III**

- Certain LSO
- Performance  $\rho$  is fixed
- Def./Adv. chooses one
- Take actions simultaneously

**Paper IV**

- Certain LFO
- Performance  $\rho$  is uncertain
- Adv. chooses one, Def. chooses several
- Def. takes action first

Thanks for listening!!!