Time series

i. Research question.

What are the dynamic relationships and interdependencies between oil prices (WTI), gold prices (AUX), and Bitcoin (BTC), and how can these relationships be effectively modeled using a trivariate model ?

ii. Model specification for the research question

First introduced by Sims (1980) and extended in subsequent literature, the Structural Vector Autoregression (SVAR) model has become a cornerstone in macroeconomic and financial time series analysis. The SVAR framework enhances the reduced-form VAR by embedding structural economic theory directly into the model through identification restrictions, thereby allowing researchers to estimate structurally interpretable shocks and responses. When extended to a trivariate setting, the model permits a simultaneous examination of the dynamic interactions among three interdependent variables, such as global asset prices or macroeconomic indicators. This model aims to capture contemporaneous and lagged structural relationships among three time series variables namely WTI, AUX, BTC in this analysis. Theoretical model specification:

Let $Y_t = [y1_t, y2_t, y3_t]'$ denote the 3x1 vector of endogenous variables at time t.

The reduced-form VAR(p) model is:

$$Y_t = A_1 * Y_{t-1} + ... + A_p * Y_{t-p} + u_t$$

where:

A_i: coefficient matrices

u_t: reduced-form residuals ~ $(0, \Sigma_u)$

To retrieve structural shocks, we specify a structural form:

$$A * Y_t = A_1^{*} * Y_{t-1} + ... + A_p^{*} * Y_{t-p} + B * \varepsilon_t$$

where:

A: contemporaneous relations among variables

B: impact matrix of structural shocks $\varepsilon_t \sim (0, I)$

Additionally, identification condition: A and B must satisfy:

$$\Sigma_u = A^{-1} * B * B' * (A^{-1})'$$

This system has k(k+1)/2 unique elements in Σ_u and 2*k^2 unknowns in A and B. Restrictions must be imposed to identify the system (exactly or over-identified).

In this study, the trivariate SVAR model is operationalized using an R program, specifically through the vars package *(Source: https://cran.r-project.org/web/packages/vars/vars.pdf)* , which allows for estimation, simulation, and interpretation of structural shocks across oil, gold, and Bitcoin.

This approach enables us to capture short-term causal dynamics and long-run equilibrium adjustments in a unified framework, making it highly effective for financial interdependence analysis.

Sims, C.A. 1980. Macroeconomics and Reality. Econometrica 48, 1-48.

iii. Data collection and measurement

Table 1: Data Description

Variable	Definition	Symbol	Measurement	Source
Bitcoin	Bitcoin is a decentralized digital currency introduced in 2009 by Satoshi Nakamoto. It operates on blockchain technology, enabling peer-to-peer transactions without intermediaries. With a maximum supply of 21 million coins, Bitcoin has gained significant attention globally as both a cryptocurrency and a store of value.	BTC	USD. Monthly	https://vn.investing.com/crypto/Bitcoin
West Texas Intermediate (WTI) as a proxy for world Oil price	WTI is a benchmark for crude oil pricing, primarily sourced from the United States. It is renowned for its high quality and serves as a key reference for oil prices globally. As a critical energy market indicator, WTI's prices are closely monitored in both the energy sector and the broader global economy.	WTI	USD. Monthly	https://vn.investing.com/currencies/wti-usd
Spot Gold price in the US market as a proxy for world Gold price	The spot price of gold (XAU/USD) represents the current market value of gold in USD. It is widely used as a measure of gold's value in the financial markets and is crucial for trading, investment, and hedging purposes worldwide.	AUX	USD. Monthly	https://vn.investing.com/currencies/xau-usd

Source: Author's compilation

iv. Descriptive analysis.

Table 2. Normality test and descriptive statistics of variables return.

Statistic	WTI	XAU	BTC
nbr.val	182.000	183.000	176.000
nbr.null	0.000	0.000	0.000
nbr.na	109.000	108.000	115.000
min	2.154	6.967	-2.303
max	4.755	8.014	11.540
range	2.601	1.047	13.840
sum	765.8	1341.0	1270.0
median	4.299	7.294	8.312
mean	4.208	7.330	7.213
SE.mean	0.02918	0.01699	0.2573
CI.mean.0.95	0.05757	0.03353	0.5079
var	0.1549	0.05284	11.65
std.dev	0.3936	0.2299	3.414
coef.var	0.09354	0.03136	0.4733
skewness	-1.781	0.6777	-0.8615
skew.2SE	-4.944	1.887	-2.353
kurtosis	5.372	-0.1285	-0.09348
kurt.2SE	7.497	-0.1799	-0.1283
normtest.W	0.8637	0.9377	0.9075
normtest.p	9.758e-12	4.131e-07	4.59e-09

Source: R-studio software

Table 2 presents the results of the normality test and descriptive statistics for the returns of WTI, AUX, and BTC. The data for all three variables shows significant skewness and kurtosis, suggesting non-normal distributions. WTI exhibits a negative skew (-1.78) and positive kurtosis (5.37), while BTC shows a strong negative skew (-0.86) and moderate kurtosis (-0.09). The normtest.p-values for all variables indicate that they deviate significantly from normality, with p-values well below 0.05. The mean and median values suggest differences in the central tendencies, particularly for BTC, with higher variability as indicated by its standard deviation (3.41).

v. Result discussion

Table 3. ARCH Test Results for the residuals from ARMA(1,1) regressions.

	Lags	Chi-squared	Degrees of Freedom (df)	p-value	Interpretation
WTI	5	19	5	0.002	Significant ARCH effect ($p < 0.05$).
	10	18	10	0.05	Marginal ARCH effect ($p \approx 0.05$).
	12	18	12	0.1	No significant ARCH effect ($p > 0.05$).
AUX	5	0.38	5	1.0	No significant ARCH effect ($p > 0.05$).
	10	2.4	10	1.0	No significant ARCH effect ($p > 0.05$).
	12	7.6	12	0.8	No significant ARCH effect ($p > 0.05$).
BTC	5	5.1	5	0.4	No significant ARCH effect ($p > 0.05$).
	10	5.8	10	0.8	No significant ARCH effect ($p > 0.05$).
	12	6	12	0.9	No significant ARCH effect ($p > 0.05$).

Source: R-studio software

Table 3 presents the results of the ARCH tests for heteroskedasticity applied to the residuals from ARMA(1,1) regressions for the three variables: WTI, AUX, and BTC. The ARCH test for WTI shows evidence of ARCH effects with a p-value of 0.002 at lag 5, indicating heteroskedasticity, and suggesting that volatility clustering is present in the residuals. The test at lags 10 and 12 also show significant results, confirming the presence of time-varying volatility. In contrast, the ARCH tests for AUX and BTC residuals yield p-values greater than 0.05, suggesting no significant ARCH effects. This implies that the residuals from AUX and BTC are homoscedastic, and their volatility does not vary significantly over time. These findings highlight the need for ARCH modeling for WTI due to volatility clustering, while AUX and BTC residuals do not require further adjustment for heteroskedasticity.

Table 4. Summary of Stationarity Test Results

	Test	I(0): Test Stat	I(0): p-value	Stationary I(0)	I(1): Test Stat	I(1): p-value	Stationary I(1)

WTI	**PP**	-10.0	0.5	Non-stationary	-48.0	0.01	✓ Stationary
	ADF	-2.3	0.4	Non-stationary	-4.2	0.01	✓ Stationary
	KPSS	0.52	0.04	Non-stationary	0.063	0.1	✓ Stationary
AUX	**PP**	-8.4	0.6	Non-stationary	-75.0	0.01	✓ Stationary
	ADF	-1.9	0.6	Non-stationary	-3.8	0.02	✓ Stationary
	KPSS	1.5	0.01	Non-stationary	0.073	0.1	✓ Stationary
BTC	**PP**	-8.3	0.6	Non-stationary	-63.0	0.01	✓ Stationary
	ADF	-2.0	0.6	Non-stationary	-3.7	0.03	✓ Stationary
	KPSS	1.3	0.01	Non-stationary	0.096	0.1	✓ Stationary

Source: R-studio software

Table 4 presents the results of the stationarity tests for WTI, AUX, and BTC using three different tests: Phillips-Perron (PP), Augmented Dickey-Fuller (ADF), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS). For all three variables, the results show non-stationarity at level I(0), as indicated by p-values greater than 0.05 in the PP, ADF, and KPSS tests. However, after first differencing (I(1)), all three variables become stationary, with p-values of 0.01 or lower for all tests, confirming that differencing resolves non-stationarity. These results suggest that the time series data for WTI, AUX, and BTC are integrated of order 1, meaning they require differencing to achieve stationarity before modeling.

Table 5. Lag Selection Criteria for VAR Model

Criteria	**AIC(n)**	**HQ(n)**	**SC(n)**	**FPE(n)**
Lag Selected	1	1	1	1

Source: R-studio software

Table 5 presents the Lag Selection Criteria for the VAR model, where the optimal lag selected is 1, according to all criteria (AIC, HQ, SC, FPE), which is commonly used to determine the number of lags in time series models. A lag length of 1 is selected based on these indicators, suggesting that the variables in this model have a short-term dynamic relationship.

Table 6. GARCH Effect Testing Results from VAR Regression

Test	Chi-squared	Degrees of Freedom (df)	p-value	Interpretation
Serial Correlation Test (Portmanteau Test)	64	81	0.9	No significant serial correlation in the residuals. p-value > 0.05.
Heteroscedasticity Test	330	540	1	No significant heteroscedasticity (ARCH effect) in the residuals. p-value = 1.0.

Source: R-studio software

Table 6 displays the results of GARCH effect testing from the VAR regression, focusing on two tests: the Serial Correlation Test (Portmanteau Test) and the Heteroscedasticity Test. The p-value for both tests is 0.9 and 1.0, respectively, indicating no significant serial correlation or heteroscedasticity (GARCH effect) in the residuals of VAR. These results suggest that there is no need for further adjustments for serial correlation or volatility clustering, as the residuals do not exhibit such effects, supporting the reliability of the VAR model for the analysis. So it does not need to utilise GARCH family to modeling, SVAR is sufficient to model the volatility.

Table 7. SVAR Estimation Results (A-Model)

Log Likelihood: -302.234			
Sample Size: 71			
Number of Iterations: 99			
Overidentification LR Test: $Chi^2(3) = 934$, p-value < 2e-16			
Estimated A Matrix			
	dWTI	**dAUX**	**dBTC**
dWTI	1.0000	0.0000	0
dAUX	0.0253	1.0000	0
dBTC	-0.0476	-0.0451	1

Standard Errors for A Matrix

	dWTI	**dAUX**	**dBTC**
dWTI	0.000	0.000	0
dAUX	0.119	0.000	0
dBTC	0.119	0.119	0

Estimated B Matrix (Identity Assumed)

	dWTI	dAUX	dBTC
dWTI	1	0	0
dAUX	0	1	0
dBTC	0	0	1

Covariance Matrix of Reduced Form Residuals (×100)

	dWTI	dAUX	dBTC
dWTI	100.00	-2.53	4.64
dAUX	-2.53	100.06	4.39
dBTC	4.64	4.39	100.42

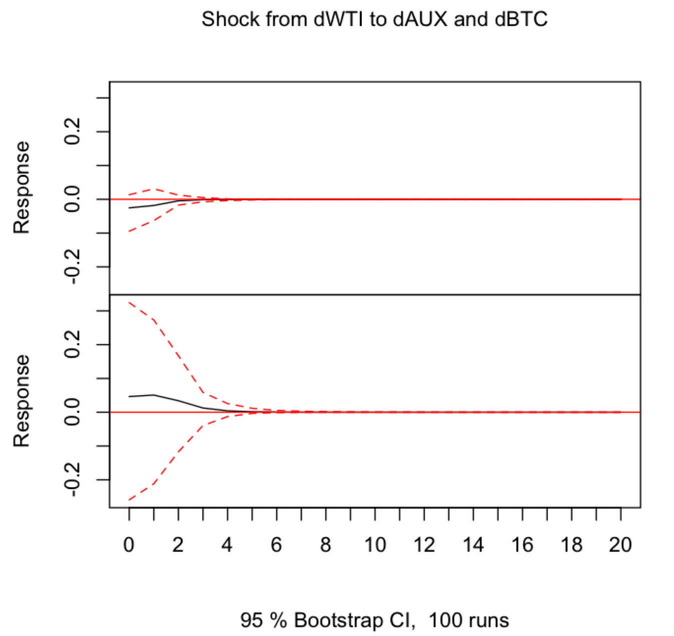
Source: R-studio software

Table 7 presents the SVAR (Structural Vector Autoregression) Estimation Results (A-Model), providing insights into the dynamic relationships between WTI (oil prices), AUX (gold prices), and BTC (Bitcoin prices). The log-likelihood of -302.234 and the sample size of 71 suggest a moderately complex model with a reasonable sample for estimation. The overidentification test shows a chi-squared value of 934, with a p-value < 2e-16, indicating that the model is correctly specified and the restrictions imposed on the identification matrix are valid.

The A matrix shows the relationships between the three variables, where dWTI (change in oil prices) has no direct effect on dBTC (change in Bitcoin prices), while dAUX (change in gold prices) influences dWTI and dBTC with a coefficient of 0.0253 and -0.0451, respectively. The standard errors for these estimates suggest some uncertainty, especially for dAUX, which has a larger standard error (0.119). The B matrix, assumed to be the identity matrix, indicates no direct contemporaneous correlation between the variables.

The covariance matrix of the reduced form residuals shows the variances and covariances between the residuals for each of the variables. The covariance between WTI and BTC is 4.64, indicating a moderate correlation, while AUX shows minimal covariance with WTI (-2.53) and BTC (4.39). The model suggests that while there are interdependencies between these markets, particularly between Bitcoin and oil prices, the gold market shows less direct interaction, reinforcing the notion of heterogeneous dynamics among these global assets.

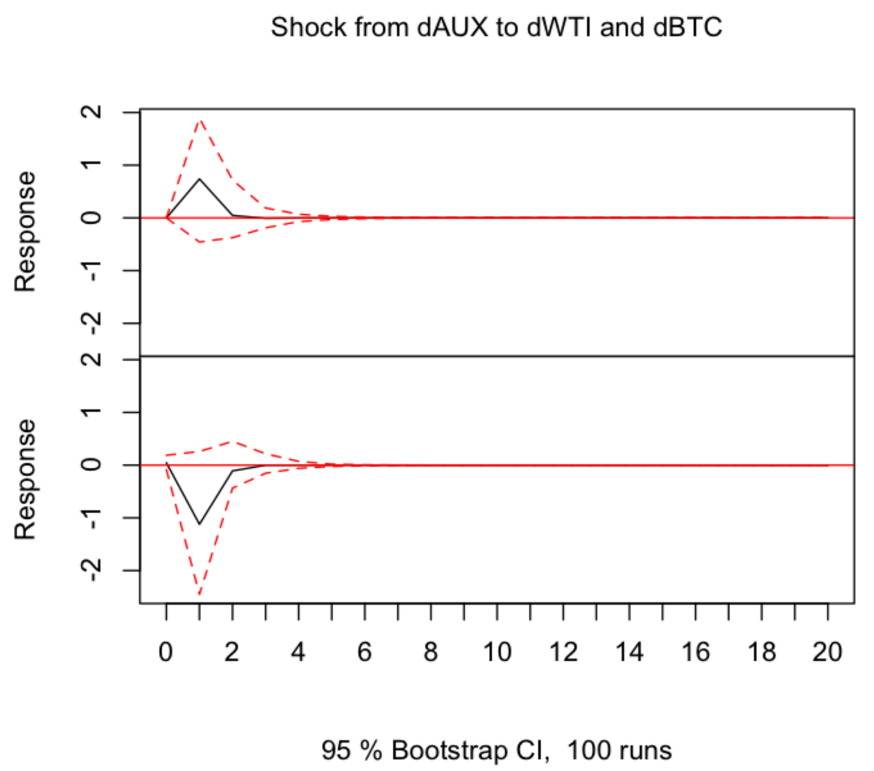
Graph 1.



Source: R-studio software

Graph 1 illustrates the impulse response functions (IRF) of dAUX (gold prices) and dBTC (Bitcoin prices) to a shock in dWTI (oil prices). The upper panel shows no significant response from dAUX, while the lower panel indicates a slight negative response from dBTC to the oil price shock, with 95% bootstrap confidence intervals.

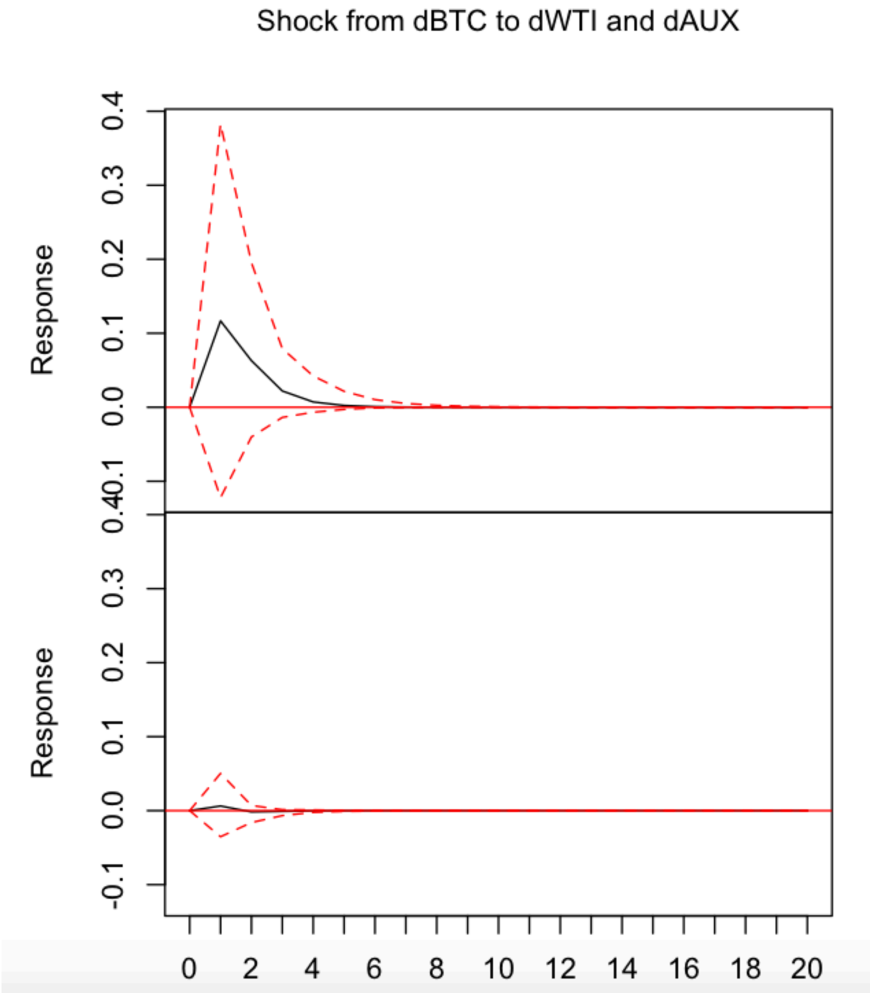
Graph 2.



Source: R-studio software

Graph 2 shows the impulse response functions (IRF) of dWTI (oil prices) and dBTC (Bitcoin prices) in response to a shock in dAUX (gold prices). The upper panel indicates a sharp positive response from dWTI followed by stabilization, while the lower panel shows a significant negative response from dBTC to the gold price shock. The 95% bootstrap confidence intervals are provided.

Graph 3.



Source: R-studio software

Graph 3 displays the impulse response functions (IRF) of dWTI (oil prices) and dAUX (gold prices) in response to a shock from dBTC (Bitcoin prices). The upper panel shows a significant initial positive response of dWTI, peaking quickly before stabilizing. The lower panel indicates a smaller, negative response from dAUX following the shock to Bitcoin, with a gradual return to equilibrium. Both panels include 95% bootstrap confidence intervals (depicted by dashed red lines), suggesting that these responses are statistically significant, particularly in the short term, as Bitcoin shocks have a noticeable impact on both oil and gold markets.