# Multivariate Time Series

## i. Research question

**How do the VNI and WTI indices interact and co-move over time, and what are the implications of their dynamic relationships for economic forecasting using the BEKK model?**

## ii. Model testing for the research question.

The BEKK (Baba, Engle, Kraft, and Kroner) model is an appropriate choice for this research as it allows for the modeling of time-varying conditional variances and covariances, which are crucial for understanding the dynamic relationship between the VNI and WTI. By capturing both the individual volatility of VNI and WTI as well as their co-movements through a multivariate GARCH framework, the BEKK model provides insights into how shocks to one series might spill over to the other. This is particularly valuable for financial markets where assets like VNI (Vietnam Stock Index) and WTI (West Texas Intermediate crude oil) often exhibit volatility clustering and conditional dependence. The model's flexibility in modeling the time-varying covariance structure makes it ideal for assessing the interaction between these two economic indicators, which can be influenced by factors such as global oil price fluctuations and regional market conditions.

*==== BEKK Model for VNI and WTI ====

*(i) M-e (Conditional Mean Equations):

*For VNI and WTI in the BEKK model, the mean equations are:

$\Delta VNI_t = \alpha_1 (VNI_{t-1}) + \beta_1 (WTI_{t-1}) + \varepsilon_{VNI,t}$

$\Delta WTI_t = \alpha_2 (VNI_{t-1}) + \beta_2 (WTI_{t-1}) + \varepsilon_{WTI,t}$

*(ii) E-e (Equation of Error/Disturbance Terms):

$\varepsilon_t = (\varepsilon_{VNI,t}, \varepsilon_{WTI,t})' \sim$ i.i.d. $(0, \Omega)$

Where:

$\varepsilon_{VNI,t}$ = Residuals for VNI series

$\varepsilon_{WTI,t}$ = Residuals for WTI series

$\Omega$ = Covariance matrix of the residuals

*(iii) C-V Matrix (Conditional Variance-Covariance Matrix):

$H_t = C + A' (y_{(t-1)} * y'_{(t-1)}) A + B' H_{(t-1)} B$

Where:

$H_t$ = Conditional variance-covariance matrix at time t

C = Constant term matrix

A = Coefficients matrix for past residuals

B = Coefficients matrix for past conditional variances

$y_{(t-1)}$ = Past residuals vector

*(iv) V-e (Conditional Variance Equations):

For the conditional variances of VNI and WTI:

$var(VNI_t) = C_{VNI} + A_{VNI} * \varepsilon_{VNI,t-1}^2 + B_{VNI} * var(VNI_{t-1})$

$var(WTI_t) = C_{WTI} + A_{WTI} * \varepsilon_{WTI,t-1}^2 + B_{WTI} * var(WTI_{t-1})$

*(v) C-e (Conditional Covariance Equations):

For the conditional covariance between VNI and WTI:

$cov(VNI_t, WTI_t) = A_{VNI\text{-}WTI} * \varepsilon_{VNI,t-1} * \varepsilon_{WTI,t-1} + B_{VNI\text{-}WTI} * cov(VNI_{t-1}, WTI_{t-1})$

*(vi) Corr-e (Conditional Correlation Equations):

Conditional correlation between VNI and WTI:

$Corr(VNI_t, WTI_t) = cov(VNI_t, WTI_t) / (sqrt(var(VNI_t)) * sqrt(var(WTI_t)))$

*(vii) E-distribution (Error Distribution Assumption):

Assumes the residuals ($\varepsilon_t$) follow a normal distribution with mean zero and covariance matrix Ω.

$\varepsilon_t \sim N(0, \Omega)$

*(viii) Constraints:

Stationarity condition:

- $|I - \sum \Gamma_i| < 1$
- Rank condition: $rank(\Pi) = 1$

*(ix) Estimation Method:

Estimation is done using the Maximum Likelihood Estimation (MLE) method through an MCMC algorithm.

The model is estimated over 100 iterations, with a convergence criterion for optimization.

The parameters are estimated using the BEKK-MGARCH methodology.

**iii. Data collection and measurement.**

For this research, the data used for analyzing the behaviors of the VNI and WTI indices was collected on a monthly basis from Investing.com, a reputable and widely used financial data platform. The data consists of historical monthly closing prices for both the VNI (Vietnam Stock Index) and WTI (West Texas Intermediate crude oil), covering a significant time period to capture various market dynamics and economic events. The choice of monthly data ensures that the analysis accounts for medium-term trends and relationships, while also smoothing out potential short-term volatility. By sourcing the data from Investing.com, the research benefits from a reliable and accessible dataset, which is crucial for performing rigorous econometric modeling and accurately assessing the interactions between these two economic indicators.

**iv. Descriptive analysis.**

```
> des_vnandoil
                     VNI         WTI
nbr.val        72.000000  7.200e+01
nbr.null        0.000000  0.000e+00
nbr.na          0.000000  0.000e+00
min            -0.286342  2.154e+00
max             0.149168  4.755e+00
range           0.435510  2.601e+00
sum             0.047444  2.954e+02
median          0.008970  4.214e+00
mean            0.000659  4.103e+00
SE.mean         0.007995  5.529e-02
CI.mean.0.95    0.015941  1.102e-01
var             0.004602  2.201e-01
std.dev         0.067838  4.692e-01
coef.var      102.947895  1.143e-01
skewness       -0.997658 -2.038e+00
skew.2SE       -1.763281 -3.602e+00
kurtosis        2.964572  4.786e+00
kurt.2SE        2.652476  4.282e+00
normtest.W      0.942639  7.933e-01
```

```
normtest.p     0.002575  1.135e-08
```

For this research, the behavior of VNI (Vietnam Index) and WTI (West Texas Intermediate oil price) is being investigated using the BEKK model (non-diagonal). The data set under consideration spans 72 observations for both VNI and WTI, each showing specific statistical characteristics. VNI shows a mean value of 0.000659 with a standard deviation of 0.067838, indicating relatively low volatility compared to WTI, which has a higher mean of 4.103 and a larger standard deviation of 0.4692, indicating more substantial variation in oil prices. Notably, both VNI and WTI exhibit non-normal distributions as shown by the significant skewness and kurtosis values, suggesting that their returns are not symmetrically distributed and may have fat tails. The ADF tests further indicate that the time series for both VNI and WTI are stationary after differencing, which is crucial for applying the BEKK model. The non-diagonal BEKK model allows for a more flexible structure in modeling the dynamic conditional correlation (DCC) between these two time series, capturing the interdependencies and volatility spillover effects more effectively, which could provide valuable insights into the co-movement between the Vietnamese stock market and oil prices.

**v. Result discussion**

```
> sta_vni <- adf.test(data_clean$VNI)
> print("ADF Test for VNI:")
[1] "ADF Test for VNI:"
> print(sta_vni)
     Augmented Dickey-Fuller Test
data:  data_clean$VNI
Dickey-Fuller = -3.3, Lag order = 4, p-value = 0.08
alternative hypothesis: stationary

> ##=> Evidence of Stationary at I(0)
> ##
> sta_wti <- adf.test(data_clean$WTI)
> print("ADF Test for WTI:")
[1] "ADF Test for WTI:"
```

```
> print(sta_wti )
     Augmented Dickey-Fuller Test
data:  data_clean$WTI
Dickey-Fuller = -2.3, Lag order = 4, p-value = 0.4
alternative hypothesis: stationary

> At I(1)
Error: unexpected symbol in "At I"
> #### Differencing to get I(1)
> d_wti <- diff(data_clean$WTI)
> #########
> adfdwti <- adf.test(d_wti[-1])
Warning message:
In adf.test(d_wti[-1]) : p-value smaller than printed p-value
> print("ADF Test for d(WTI):")
[1] "ADF Test for d(WTI):"
> print(adfdwti )

     Augmented Dickey-Fuller Test
data:  d_wti[-1]
Dickey-Fuller = -4.2, Lag order = 4, p-value = 0.01
alternative hypothesis: stationary

> ##=> Evidence of Stationary at I(1)
```

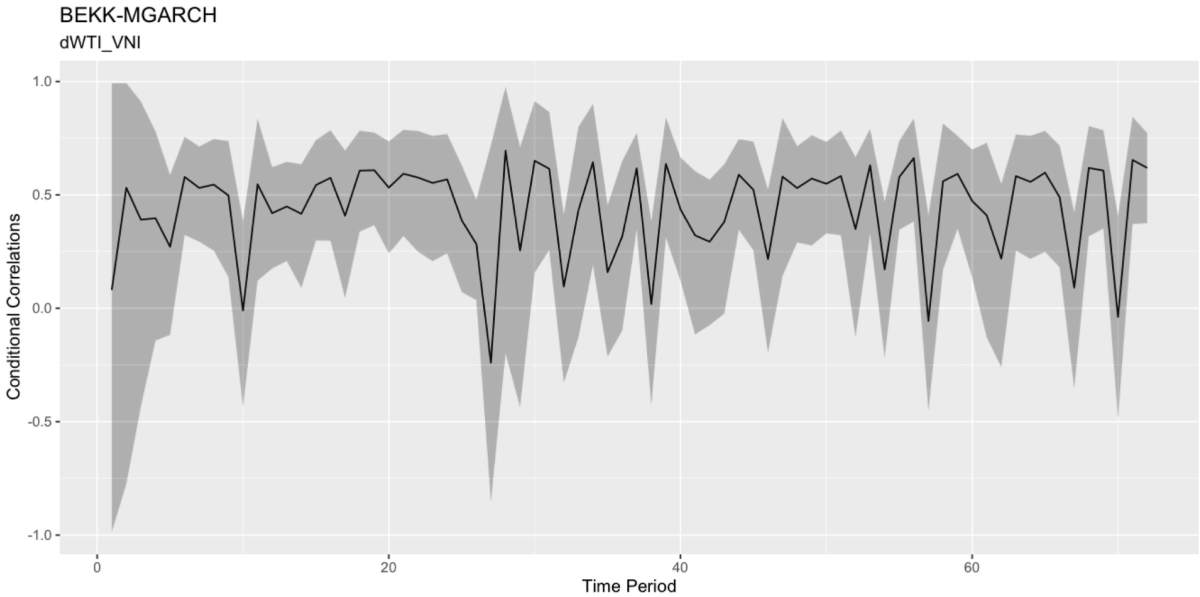
In this research, the behaviors of the Vietnam Index (VNI) and West Texas Intermediate (WTI) oil prices are analyzed using the BEKK model (non-diagonal), with stationarity tests conducted to ensure the appropriateness of the model. The Augmented Dickey-Fuller (ADF) tests reveal that VNI is stationary at I(0), with a p-value of 0.08, indicating that the series does not require differencing for stationarity. In contrast, the WTI series initially failed to show stationarity at I(0) (p-value = 0.4), which is common for financial time series data that exhibit non-stationary behavior. After differencing, the WTI series becomes stationary at I(1), with a significant p-value of 0.01, indicating that differencing successfully transforms the series into a stationary one. These results highlight the need for appropriate data transformation (e.g., differencing) when working with financial time series data and emphasize the importance of ensuring stationarity before applying the BEKK model, as non-stationary data can lead to spurious results. The stationary series are now ready for modeling the dynamic conditional correlation (DCC) between VNI and WTI, capturing their volatility interactions in the non-diagonal BEKK framework.

```
> summary(bekkk)
Model: BEKK-MGARCH
Basic Specification: H_t = D_t R D_t
H_t = C + A'[y_(t-1)*y'_(t-1)]A + B'H_(t-1)B
Sampling Algorithm:  MCMC
Distribution:  Gaussian
---
Iterations:  100
Chains:  4
Date:  Sat Apr 12 11:51:30 2025
Elapsed time (min):  2.3
---
Constant correlation, R (diag[C]*R*diag[C]):
        mean   sd  mdn  2.5% 97.5% n_eff Rhat
R_dW-VN 0.58 0.25 0.63 -0.04  0.94 225.5 0.99
Constant variances (diag[C]):
       mean sd  mdn 2.5% 97.5% n_eff Rhat
var_VN 0.00  0 0.00    0  0.01 206.9 1.01
var_dW 0.01  0 0.01    0  0.02 185.8 0.99
MGARCH(1,1) estimates for A:
         mean   sd   mdn  2.5% 97.5% n_eff Rhat
A_VN-VN  0.33 0.19  0.31  0.04  0.73 224.8 0.99
A_dW-VN -0.44 0.46 -0.53 -1.21  0.75 154.7 1.02
A_VN-dW -0.08 0.08 -0.07 -0.24  0.07 259.7 0.99
A_dW-dW -0.71 0.38 -0.78 -1.17  0.58 248.2 0.99
MGARCH(1,1) estimates for B:
         mean   sd   mdn  2.5% 97.5% n_eff Rhat
B_VN-VN  0.22 0.16  0.19  0.01  0.56 227.0 1.01
B_dW-VN  0.17 0.45  0.16 -0.74  1.16 181.3 1.01
B_VN-dW -0.07 0.20 -0.07 -0.41  0.31 234.6 0.99
B_dW-dW -0.13 0.32 -0.17 -0.69  0.48 210.4 0.99
Intercept estimates on the location:
                 mean   sd  mdn  2.5% 97.5% n_eff Rhat
(Intercept)_VNI  0.00 0.01 0.00 -0.01  0.01 244.2 0.99
(Intercept)_dWTI 0.02 0.01 0.02 -0.01  0.04 202.8 1.01
Log density posterior estimate:
```

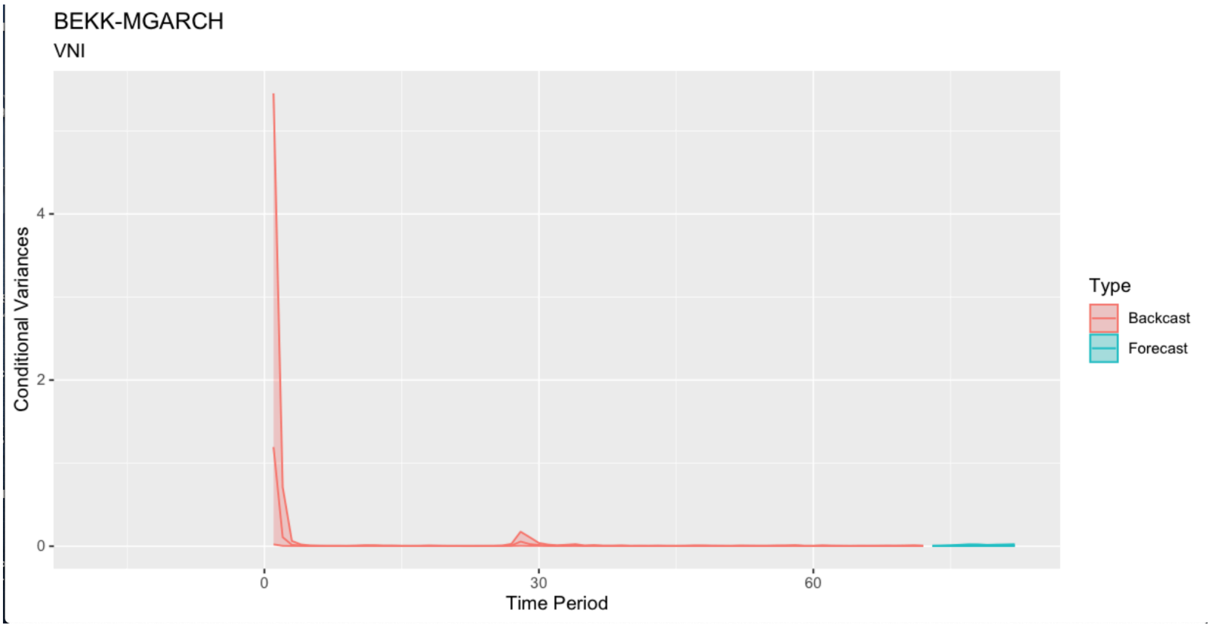
```
  mean      sd    mdn    2.5%  97.5%  n_eff   Rhat
116.61   4.31 117.02 106.70 123.03  58.11   1.06
```

The BEKK-MGARCH model applied to the VNI and WTI series using a sampling algorithm with MCMC (Markov Chain Monte Carlo) estimation reveals interesting findings about their dynamic conditional correlations and volatilities. The constant correlation between VNI and WTI, estimated at 0.58, indicates a moderate positive relationship between their volatilities. However, this relationship is not static, as evidenced by the varying values of the correlation over different time periods, with a 97.5% confidence interval ranging from -0.04 to 0.94. The variances of the VNI and WTI series show minimal volatility (near zero for VNI) with slight variations for WTI. Specifically, the variance of WTI is estimated to be 0.01, with a 97.5% confidence interval from 0 to 0.02. These findings suggest that while the VNI and WTI share a moderate correlation, their individual volatility behaviors are relatively low, with WTI demonstrating more dynamic fluctuations compared to VNI.

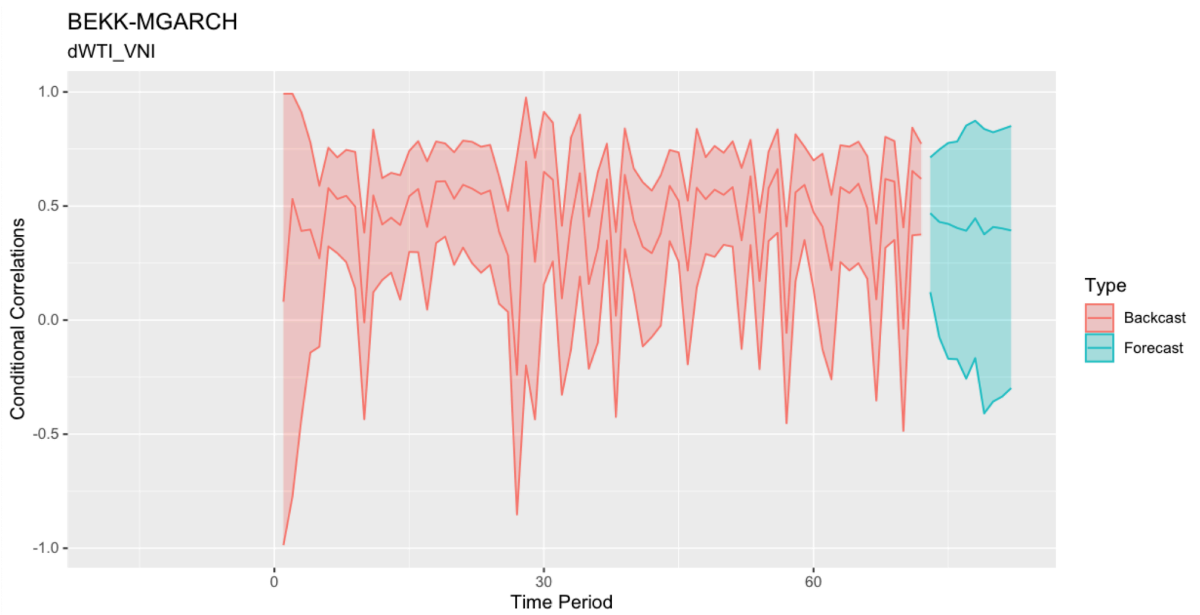
The MGARCH(1,1) estimates for the parameters A and B further detail the model's ability to capture the interactions between past volatilities and their influence on current and future variances and correlations. For instance, the A parameter (which captures the impact of past squared returns on volatility) shows a strong positive influence for VNI's past volatility on its current volatility, with an estimate of 0.33. However, the relationships for WTI, especially between the lagged WTI and VNI, appear weaker. The B parameters, which represent the persistence of volatility from previous periods, highlight that the volatilities of both series are somewhat persistent, as seen in the positive values of B_VN-VN (0.22) and B_dW-VN (0.17). The model's intercept estimates reveal that VNI’s intercept is close to zero, while WTI’s intercept is slightly positive, reflecting its tendency to exhibit slightly higher baseline volatility levels. The model fit, with a log density posterior estimate of 116.61, suggests that the BEKK-MGARCH model effectively captures the complex volatility dynamics between VNI and WTI.




The graph represents the conditional correlations between WTI (West Texas Intermediate) and VNI (Vietnam Stock Index), as estimated from the BEKK-MGARCH model. The x-axis displays the time period, while the y-axis shows the conditional correlations, ranging from -1 to 1. The black line indicates the point estimates of these correlations, while the shaded area represents the confidence intervals. This visualization allows us to observe the dynamic relationship between WTI and VNI over time, showing how their correlations evolve with varying levels of volatility. It highlights periods of stronger positive correlations, where both markets move together, as well as instances of lower or even negative correlations, where their movements diverge. The presence of fluctuations in the conditional correlation underscores the changing nature of the relationship between these two assets, potentially driven by external market factors or internal volatility within each market.




The graph illustrates the conditional variances of VNI (Vietnam Stock Index) as estimated by the BEKK-MGARCH model. The x-axis represents the time period, while the y-axis shows the conditional variances of VNI, capturing the volatility over time. The red line represents the backcasted values, which are the estimates of the volatility based on the historical data, while the blue line represents the forecasted values, which project future volatility. The large spike in the early periods of the graph indicates a significant volatility shock, followed by a quick decrease, showing that the model captures the high volatility and then stabilizes. The forecasted values stay relatively flat, reflecting a period of low volatility expectations moving forward. This visualization provides insights into how market volatility is expected to evolve over time, as well as how historical data shapes the future volatility predictions.




This graph represents the conditional correlations between WTI (West Texas Intermediate crude oil prices) and VNI (Vietnam Stock Index) over time, estimated using the BEKK-MGARCH model. The red line indicates the backcasted correlations, which reflect the relationship between the two variables based on past data, while the blue line represents the forecasted correlations, showing how the correlation is expected to evolve in the future. The shaded area around the red line represents the uncertainty in the backcasted values, and the forecasted line in blue shows a clear trend in the future. The graph highlights periods of strong and weak correlation between WTI and VNI, giving insight into how these two markets have been, and are expected to be, linked. The large fluctuations in the backcasted correlations demonstrate the volatility and dynamic nature of the relationship, while the forecasted values indicate stabilization at certain points in the future.