Nonlinear Multivariate Time Series

i. Research question

How do Ethereum (ETH) and West Texas Intermediate (WTI) oil prices interact over time, and what are the implications of their dynamic relationships and regime changes using the TVECM model ?

ii. Model testing for the research question

The Threshold Vector Error Correction Model (TVECM) is particularly suitable for analyzing the behavior of ETH and WTI returns due to its ability to capture non-linear dynamics in time series data that may exhibit different regimes of behavior. In financial markets, such as those involving cryptocurrencies (ETH) and commodities (WTI), the relationship between variables often changes during periods of market stress or significant price movements. TVECM accounts for these regime switches by modeling different short-run dynamics in each regime, allowing for the identification of threshold values that separate the data into distinct regimes. This flexibility is crucial for understanding how ETH and WTI react to different market conditions, such as fluctuations in market sentiment or external shocks. The TVECM not only captures the long-term equilibrium relationship between ETH and WTI but also provides insight into the short-term adjustments and the speed of convergence toward equilibrium, making it an effective tool for understanding the complex interactions between these variables in a non-linear context.

Equations of TVECM model:

$$\ln ETH\_t = \alpha\_1 * (\ln ETH\_{t-1} - \lambda \ln WTI\_{t-1}) + \gamma\_11 \, \Delta \ln ETH\_{t-1} + \gamma\_12 \, \Delta \ln ETH\_{t-2} + \varepsilon\_{ETH,t}$$

$$\ln WTI\_t = \alpha\_2 * (\ln ETH\_{t-1} - \lambda \ln WTI\_{t-1}) + \gamma\_21 \, \Delta \ln ETH\_{t-1} + \gamma\_22 \, \Delta \ln WTI\_{t-1} + \varepsilon\_{WTI,t}$$

where

$$\varepsilon\_t = (\varepsilon\_{ETH,t}, \varepsilon\_{WTI,t})' \sim i.i.d. \, (0, \Omega).$$

iii. Data collection and measurement

The data for this research was collected from Investing.com, a reputable financial platform providing comprehensive market data. Specifically, daily time series data for ETH (Ethereum) and WTI (West Texas Intermediate) were retrieved to

analyze their dynamic relationships over time. The data spans from the beginning of 2018 to the most recent available date, providing a robust sample for examining the interaction between the cryptocurrency market (ETH) and the oil market (WTI). Daily frequency allows for capturing the short-term fluctuations and volatility inherent in these markets, offering valuable insights into the short-run dynamics, regime changes, and long-term equilibrium relationships between the two assets. This high-frequency data ensures that the analysis reflects the most up-to-date market conditions, enhancing the relevance and accuracy of the research findings.

iv. Descriptive analysis.

```
> stat.desc(data[, c("lnETH","lnWTI")], norm = TRUE)
                 lnETH       lnWTI
nbr.val       1.901e+03  1.610e+03
nbr.null      0.000e+00  6.000e+00
nbr.na        6.000e+00  2.970e+02
min          -5.925e-01 -4.046e-01
max           2.348e-01  4.035e-01
range         8.272e-01  8.082e-01
sum           2.797e+00  1.090e-01
median        1.515e-03  1.122e-03
mean          1.471e-03  6.773e-05
SE.mean       1.034e-03  7.489e-04
CI.mean.0.95  2.028e-03  1.469e-03
var           2.033e-03  9.029e-04
std.dev       4.509e-02  3.005e-02
coef.var      3.064e+01  4.436e+02
skewness     -1.303e+00 -1.473e+00
skew.2SE     -1.160e+01 -1.208e+01
kurtosis      1.951e+01  5.615e+01
kurt.2SE      8.693e+01  2.303e+02
normtest.W    8.945e-01  7.351e-01
normtest.p    3.103e-34  6.774e-45
```

The descriptive statistics for the natural logarithms of ETH and WTI returns provide valuable insights into the data's distribution, volatility, and dynamics. The sample consists of 1,901 observations for ETH and 1,610 for WTI, with some missing values for both series, particularly for WTI. The data for ETH and WTI displays notable skewness, with ETH showing a leftward skew (-1.303) and WTI showing a

stronger leftward skew (-1.473), suggesting the presence of large negative returns in both series. The kurtosis values indicate leptokurtic distributions for both, with ETH having a kurtosis of 19.51 and WTI having an even higher kurtosis of 56.15, implying that the returns exhibit more extreme outliers compared to a normal distribution. The mean return for ETH is slightly positive (0.001471), while WTI shows a much lower mean (0.00006773), indicating a weaker overall trend. The standard deviations are also substantial, with ETH's volatility higher (0.04509) compared to WTI (0.03005), reflecting greater fluctuations in ETH returns. Furthermore, the p-values from the normality test (W-statistics) for both series suggest significant deviations from normality, confirming the need for nonlinear models like TVECM to capture the underlying dynamics of these time series. These statistics set the stage for the subsequent analysis using the TVECM model, which is particularly suited to explore the potential nonlinear relationships and threshold effects between ETH and WTI prices.

v. Result discussion

```
> coint_testing <- ca.jo(bivarii, type = "trace", K = 2, ecdet = "const")
> summary(coint_testing)
######################
# Johansen-Procedure #
######################

Test type: trace statistic , without linear trend and constant in
cointegration


Eigenvalues (lambda):
[1] 0.3495482 0.3346159 0.0000000


Values of teststatistic and critical values of test:


           test 10pct  5pct  1pct
r <= 1 |  654.68  7.52  9.24 12.97
r = 0  | 1345.83 17.85 19.96 24.60


Eigenvectors, normalised to first column:
 (These are the cointegration relations)
```

```
            lnETH.l2      lnWTI.l2     constant
lnETH.l2   1.00000000   1.000000000   1.00000000
lnWTI.l2   4.25286792  -0.806780616   0.06070256
constant  -0.00224767   0.001152593  29.41686518


Weights W:
(This is the loading matrix)


          lnETH.l2     lnWTI.l2       constant
lnETH.d  -0.1675962   -0.8615985    -1.206538e-22
lnWTI.d  -0.2051443    0.1924805     1.214718e-21
```

The Johansen cointegration test results indicate a significant relationship between ETH and WTI returns, as confirmed by the trace statistic and eigenvalues. The test reveals that there is one cointegration equation, as indicated by the test statistic for the hypothesis r≤1r , which exceeds the critical value at the 1% significance level. The eigenvalues of 0.3495 and 0.3346 suggest the presence of a strong cointegrating relationship, particularly between the lagged values of ETH and WTI. The estimated cointegration equation, normalized to the first column, shows that the long-run relationship between ETH and WTI is inversely related, with the weight on WTI significantly negative (-0.8068), implying a negative relationship in their equilibrium state. The constant term, indicating the long-term equilibrium level, is quite high at 29.42. Furthermore, the loading matrix reveals how deviations from equilibrium are corrected over time, with ETH's return showing a stronger adjustment coefficient (-0.8616) compared to WTI's (-0.1925), suggesting that ETH's return adjusts more rapidly to restore equilibrium. These findings suggest that both ETH and WTI share a long-term relationship, and their short-term dynamics are characterized by adjustments to this equilibrium. This supports the use of the TVECM model to further explore the non-linear and threshold effects in the relationship between ETH and WTI, especially when considering their possible regime-dependent behavior.

```
> test1<-TVECM.HStest(bivarii, lag=1, intercept=TRUE, nboot=100)
> test1
## Test of linear versus threshold cointegration of Hansen and Seo (2002)
```

```
##

Test Statistic:     23.72174   (Maximized for threshold value: 0.05536441 )
P-Value:      0.01         ( Fixed regressor bootstrap )
```

The Hansen and Seo (2002) test for linear versus threshold cointegration provides compelling evidence for threshold cointegration between ETH and WTI returns, as indicated by the test statistic of 23.7217 and a p-value of 0.01. This suggests that a linear cointegration model is not sufficient to capture the dynamics between ETH and WTI, and the relationship between the two assets exhibits significant threshold behavior. The maximized threshold value of 0.0554 further emphasizes that the relationship between ETH and WTI is not constant but rather varies depending on the regime, reinforcing the appropriateness of a threshold model such as TVECM. The bootstrap p-value of 0.01, which is below the typical 5% significance level, further validates the existence of threshold cointegration. These findings underscore the importance of incorporating nonlinear dynamics and threshold effects in the analysis of ETH and WTI, providing more accurate and insightful modeling of their interdependence, particularly when market conditions shift across different regimes. This result aligns with the broader trend in financial econometrics, where nonlinear models like TVECM are increasingly used to better capture the complex interactions in asset prices.

```
> tvec <- TVECM(bivarii, nthresh=2,lag=1, ngridBeta=20, ngridTh=30,
plot=TRUE,trim=0.05, common="All")
9 (1.5%) points of the grid lead to regimes with percentage of observations < trim
and were not computed
Best threshold from first search -0.06292583
Best cointegrating value 0.1814589
There were  2  thresholds values which minimize the SSR in the conditional step,
the first one was taken
Second best (conditionnal on the first one) -0.06292583 -0.04093865       SSR
2.71321
There were  8  thresholds values which minimize the SSR in the iterative step, the
first one was taken
Second step best thresholds -0.06259126 -0.0420402               SSR 2.707365
There were 20 warnings (use warnings() to see them)
> print(tvec)
Model TVECM with  2  thresholds
```

```
$Bdown
                    ECT          Const      lnETH t -1 lnWTI t -1
Equation lnETH -1.30533482 -0.012613127  0.2381272954 -0.5359659
Equation lnWTI  0.05081317 -0.001145504 -0.0004876977 -0.4047015


$Bmiddle
                  ECT       Const lnETH t -1  lnWTI t -1
Equation lnETH -0.9487505 0.02254238  0.1642392 -0.02054729
Equation lnWTI  0.1206789 0.01011819  0.1489488 -0.82858125


$Bup
                   ECT          Const  lnETH t -1 lnWTI t -1
Equation lnETH -1.00368222 -3.736606e-04 -0.03336571 -0.1524175
Equation lnWTI  0.09436861 -3.950466e-06 -0.01846524 -0.4602646



Threshold value[1] -0.06259126 -0.04204020
> summary(tvec)
#############
###Model TVECM
#############
Full sample size: 1358    End sample size: 1356
Number of variables: 2    Number of estimated parameters 24
AIC -18865.08      BIC -18729.56      SSR 2.707365



Cointegrating vector: (1, - 0.1814589 )
$Bdown
              ECT                Const             lnETH t -1        lnWTI t -1
Equation lnETH -1.3053(2.9e-12)*** -0.0126(0.3879)  0.2381(0.0108)*
-0.5360(0.0007)***
Equation lnWTI 0.0508(0.6922)      -0.0011(0.9099)  -0.0005(0.9940)
-0.4047(0.0002)***


$Bmiddle
              ECT               Const            lnETH t -1       lnWTI t -1
Equation lnETH -0.9488(0.2201)   0.0225(0.5670)  0.1642(0.2063)  -0.0205(0.9107)
Equation lnWTI 0.1207(0.8218)    0.0101(0.7107)  0.1489(0.0982).
-0.8286(9.3e-11)***


$Bup
              ECT                Const            lnETH t -1       lnWTI t -1
Equation lnETH -1.0037(2.2e-79)*** -0.0004(0.7333)  -0.0334(0.2554)
```
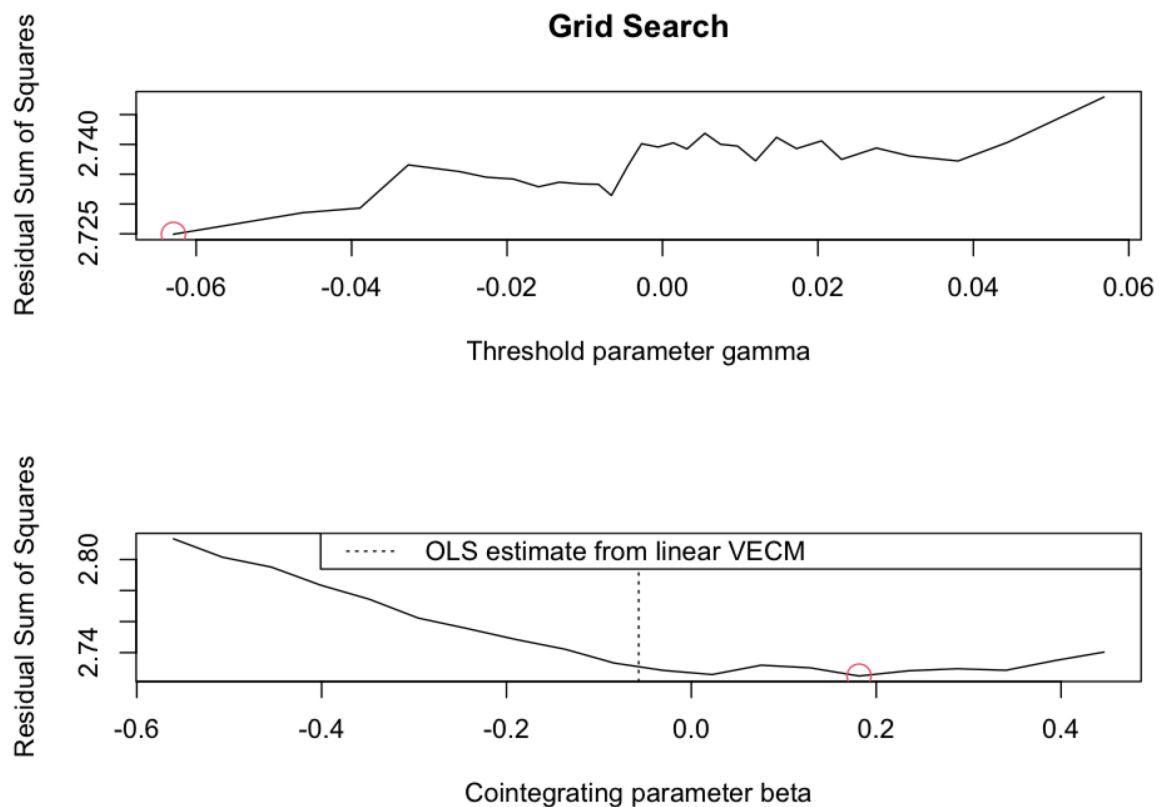
```
-0.1524(2.4e-05)***

Equation lnWTI 0.0944(0.0062)**    -4e-06(0.9959)    -0.0185(0.3636)

-0.4603(4.0e-68)***

Threshold

Values: -0.06259126 -0.0420402

Percentage of Observations in each regime 5.2% 5.1% 89.7%
```

The results of the Threshold Vector Error Correction Model (TVECM) for ETH and WTI returns provide deep insights into the nonlinear dynamics and cointegration relationships between the two assets. The model identified two thresholds, with the first threshold value being -0.06259126 and the second -0.0420402, which are used to define the different regimes in the relationship. The model estimated three distinct regimes—down, middle, and up—each with different parameter values for the Error Correction Term (ECT) and the short-run dynamics represented by the lagged values of lnETH and lnWTI. Specifically, in the "down" regime, the ECT for lnETH is significantly negative (-1.305), indicating that a deviation from the long-run equilibrium is corrected with a significant speed in this regime, while the short-term effect of WTI returns on ETH returns is also significant. In the "middle" regime, the ECT for lnETH is less negative (-0.948), suggesting slower convergence to equilibrium, and the effect of WTI on ETH becomes less significant. The "up" regime shows a similar pattern, with the ECT for ETH being negative (-1.0037), but with a less pronounced effect of WTI on ETH, highlighting the asymmetric relationship between the assets across different market conditions.

The estimated parameters from the TVECM model further emphasize the differing influences of ETH and WTI under various regimes. For example, in the "down" regime, WTI has a significant negative impact on ETH returns, with a coefficient of -0.5360 (p-value < 0.001), while in the "middle" and "up" regimes, this relationship becomes less pronounced. The coefficients for the lagged values of lnETH and lnWTI also reveal how the past movements of these variables impact their current dynamics, with some coefficients being statistically significant across the regimes. Notably, the AIC (-18865.08) and SSR (2.707365) values, along with the BIC and model diagnostics, indicate a good fit for the model, and the high percentage of observations in the "up" regime (89.7%) suggests that most of the time, ETH and

WTI are in a regime with less volatility and more stable relationships. These findings provide valuable insights into how the relationship between ETH and WTI evolves under different market conditions, making the TVECM a powerful tool for modeling nonlinear interactions in financial markets.

The graph displays the results of a grid search for determining the optimal threshold parameter gamma and cointegrating parameter beta in the Threshold Vector Error Correction Model (TVECM). The upper plot shows the residual sum of squares (RSS) as a function of the threshold parameter gamma, with the lowest value of RSS occurring at around gamma = -0.0625, indicating the best threshold for the model. The plot also suggests that as gamma moves away from this optimal value, the RSS increases, highlighting the sensitivity of the model's fit to changes in the threshold parameter. The lower plot illustrates the RSS against the cointegrating parameter beta, with a clear downward trend as the value of beta increases. The OLS estimate from the linear VECM is marked, and the graph shows that the TVECM's residual sum of squares improves as the cointegrating parameter approaches the optimal value, reinforcing the importance of correctly identifying the cointegrating relationship

between the variables. This analysis provides crucial insights into the optimal settings for the TVECM, ensuring a more accurate model fit for ETH and WTI.