ABSTRACT

This research examines the dynamic relationship between Ethereum (ETH), a leading cryptocurrency, and gold (XAU), a traditional commodity, using advanced econometric models, specifically the DCC-GARCH and TVAR frameworks. The study focuses on understanding the volatility spillovers, co-movements, and time-varying correlations between these two assets over time. The DCC-GARCH model is applied to capture the dynamic conditional correlation and volatility between Ethereum and gold, while the TVAR model is used to explore non-linear relationships and threshold effects. By analyzing daily returns data from January 2020 to March 2025 for Ethereum and July 2021 to March 2025 for gold, the research investigates how their interactions change under different market conditions, including periods of heightened volatility and market stress.

The results reveal that while Ethereum exhibits high volatility compared to gold, the two assets show periods of increased correlation, particularly during times of market uncertainty. The TVAR model identifies three distinct regimes, reflecting changes in market behavior based on the threshold value of Ethereum returns. These findings have important implications for portfolio management and risk diversification, suggesting that including both assets in a portfolio can offer diversification benefits, particularly in volatile market conditions. This research contributes to the growing body of literature on cryptocurrency-commodity interactions and provides valuable insights for investors and policymakers seeking to understand the potential roles of digital assets like Ethereum in traditional financial markets.

CHAPTER 3: DATA AND METHODS

3.1. Data collection

For the analysis of the relationship between Ethereum (ETH) and gold (XAU), two critical datasets were collected: the daily closing prices of Ethereum (ETH) and the spot price of gold in the U.S. market (XAU/USD). These data series provide the foundation for understanding the dynamic relationship between the digital asset, Ethereum, and the traditional commodity, gold. Both assets serve as important benchmarks for investors, offering valuable insights into their respective market behaviors and their interdependencies over time.

Ethereum (ETH)

Ethereum is a decentralized blockchain platform introduced in 2015 by Vitalik Buterin, which allows for the creation of smart contracts and decentralized applications (DApps). Ethereum's native cryptocurrency, Ether (ETH), is used to power these applications and as a medium of exchange within the Ethereum network. The ETH data used in this study includes daily closing prices, sourced from a reliable financial data provider, https://vn.investing.com/crypto/Ethereum/eth-usd, which offers real-time market data on cryptocurrencies. The time frame for this dataset spans from January 6, 2020, to March 22, 2025, ensuring that the analysis captures both periods of high volatility and relative stability in the cryptocurrency market.

Ethereum's flexibility and innovation have made it a key player in the blockchain ecosystem, and its volatility characteristics are essential for understanding its relationship with traditional assets such as gold. This dataset is particularly useful in understanding the market's behavior during times of market uncertainty and the growing influence of digital currencies in global financial markets. The decision to use daily data for Ethereum was made to accurately capture the fluctuations and trends in its returns, which can be highly volatile, particularly in the short term.

The Ethereum price data is critical for evaluating its role as an alternative investment in relation to traditional commodities. Given its speculative nature, ETH's price movements are often influenced by technological advancements, regulatory developments, and market sentiment. The use of daily price data allows the analysis to examine the short-term volatility dynamics and assess how Ethereum responds to broader market shifts, particularly in times of economic turbulence or market crises.

Gold (XAU)

Gold, traditionally viewed as a safe-haven asset, is widely traded in global markets, with the spot price of gold in USD (XAU/USD) serving as a critical indicator of its value. The gold price data used in this study also comes from https://vn.investing.com/currencies/xau-usd , covering the daily spot price of gold from July 3, 2021, to March 21, 2025. This time frame provides a comprehensive dataset for analyzing how gold prices have reacted to changes in the macroeconomic environment, such as inflationary pressures, shifts in interest rates, and geopolitical events.

The spot price of gold is a crucial measure of gold's value in financial markets and is used for various purposes, including trading, investment, and hedging. The decision to use XAU/USD data stems from gold's global importance as a hedge against inflation and currency devaluation, particularly in the context of the traditional financial system. Gold has been considered a stable asset, especially during times of economic instability, and its price is less volatile compared to digital assets like Ethereum. However, this does not imply a lack of fluctuations, gold still experiences significant price movements, particularly during times of economic crisis or market shocks.

The use of daily data for gold, similar to Ethereum, ensures that the analysis captures the short-term volatility and longer-term trends in gold prices. By comparing the price dynamics of Ethereum and gold, this study seeks to assess how the two assets behave during periods of market stress, and whether they can offer diversification benefits when held together in an investment portfolio.

The datasets selected for this study are crucial for understanding the complex relationship between cryptocurrencies and traditional commodities. The daily price data for Ethereum and gold provides a solid foundation for analyzing their volatility dynamics, co-movements, and threshold effects, helping to inform investment strategies and risk management practices. By examining the behavior of these two asset classes, this research will contribute to the growing body of literature on the role of cryptocurrencies in financial markets and their interaction with traditional assets like gold. The methodology outlined in this chapter, coupled with the data collected, will facilitate a deeper understanding of the interdependencies between these assets and their implications for global financial markets.

3.2. Theoretical model

3.1. DCC-GARCH approach

In this section, the theoretical framework for the Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH) model is introduced. This model allows us to investigate the dynamic relationship between Ethereum (ETH) and gold (XAU) returns, considering time-varying volatility and correlation. The DCC-GARCH approach, proposed by Engle (2002), is widely used in financial econometrics to model the co-movements and volatility spillovers between asset returns.

The DCC-GARCH model consists of several key components, which include the conditional mean equation, the error equation, the conditional variance-covariance matrix, the conditional variance equations, and the conditional correlation equation. Below, we outline each component of the DCC-GARCH model applied to the analysis of ETH and XAU returns.

The conditional mean equation describes the expected returns of Ethereum and gold at time t:

$r\_{ETH,t} = \mu_1 + \varepsilon_{1,t}$

$r\_{XAU,t} = \mu_2 + \varepsilon_{2,t}$

where

$\boldsymbol{\mu_1}$, $\boldsymbol{\mu_2}$ are constants, possibly zero if de-meaned.

This simple model assumes that the returns of both assets are modeled as deviations from a constant mean, with no autoregressive moving average (ARMA) structure involved. Therefore, after pre-whitening, the returns are assumed to be zero-mean.

The error equation in the DCC-GARCH model is given by:

$$\varepsilon_t = H_t^{0.5} * z_t$$

where

ε_t is the vector of residuals

H_t is the conditional variance-covariance matrix

$z_t \sim$ i.i.d $N(0, I)$ — standardized shocks

The conditional variance-covariance matrix is defined as:

$$H_t = D_t * R_t * D_t$$

where

D_t is diagonal matrix of conditional standard deviations: $\text{diag}(\sqrt{h_{1\_t}}, \sqrt{h_{2\_t}})$

R_t is time-varying conditional correlation matrix

So, H_t captures both variances and correlation dynamics

The matrix Ht captures both the time-varying volatilities of Ethereum and gold as well as the correlations between their returns, allowing for a more comprehensive understanding of the joint behavior of these two assets over time.

The conditional variance equations for each asset are given by:

$$h_{i,t} = \omega_i + \alpha_i * \varepsilon^2_{i,t-1} + \beta_i * h_{i,t-1}$$

where

ω_i: constant (long-run volatility)

α_i: ARCH effect (short-term shock)

β_i: GARCH effect (volatility persistence)

The ARCH effect accounts for the impact of recent market shocks on current volatility, while the GARCH effect reflects how volatility from previous periods influences the current period's volatility.

The conditional correlation equation is updated dynamically and is given by:

$$Q_t = (1 - a - b) * \bar{Q} + a * (z_{t-1} * z'_{t-1}) + b * Q_{t-1}$$

where

Q_t: intermediate covariance matrix of standardized residuals

$\bar{Q}$: unconditional correlation matrix of z_t

a and b: DCC parameters (should satisfy $a + b < 1$)

This equation models the dynamic changes in the correlation between the returns of Ethereum and gold, capturing how their relationship evolves over time based on the shocks observed in the returns.

3.2. TVAR model approach

The Threshold Vector Autoregression (TVAR) model is a non-linear econometric model used to capture regime-dependent relationships between multiple time series. This approach allows for different dynamic behaviors (or regimes) based on the values of a threshold variable. In the context of this study, the TVAR model is applied to understand the relationship between Ethereum (ETH) and gold (XAU), considering the potential for different market regimes. Specifically, a Bivariate TVAR(2) model with three threshold regimes is used, where the threshold variable is the lagged value of Ethereum (ETH) returns, and the dynamics of ETH and XAU returns are modeled for each regime.

The model is specified as follows:

$$y_t = [\, ETH_t \,,\, XAU_t \,]'$$

where

Z_{t-1} = threshold variable = ETH_{t-1}

with

Regime 1: $ETH_{t-1} \leq q1$

Regime 2: $q1 < ETH_{t-1} \leq q2$

Regime 3: $ETH_{t-1} > q2$

Here, q1 and q2 are threshold values that divide the observed data into different regimes based on the lagged return of Ethereum. The idea behind using the TVAR model is to allow for different relationships between Ethereum and gold during different market conditions, as indicated by the threshold values. Each regime captures a distinct market behavior, which might reflect periods of stability, high volatility, or extreme market events.

For each regime rrr (where r=1,2,3r = 1, 2, 3r=1,2,3), the conditional mean equation is specified as, For each regime r = 1,2,3:

$$y_t = \Phi_0^{(r)} + \Phi_1^{(r)} y_{t-1} + \Phi_2^{(r)} y_{t-2} + \varepsilon_t^{(r)}$$

where:

$\Phi_0^{(r)}$ = Intercept vector for regime r

$\Phi_j^{(r)}$ = Coefficient matrices (2x2) for lag j in regime r

$\varepsilon_t^{(r)} \sim N(0, \Sigma^{(r)})$ = regime-specific shock

For each regime r, the conditional mean equations for Ethereum and gold are given by:

Conditional Mean Equations:

$$ETH_t = \alpha_{0r} + \alpha_{11r} * ETH_{t-1} + \alpha_{12r} * XAU_{t-1} + \alpha_{21r} * ETH_{t-2} + \alpha_{22r} * XAU_{t-2} + \varepsilon 1_t^{(r)}$$

$$XAU_t = \beta_{0r} + \beta_{11r} * ETH_{t-1} + \beta_{12r} * XAU_{t-1} + \beta_{21r} * ETH_{t-2} + \beta_{22r} * XAU_{t-2} + \varepsilon 2_t^{(r)}$$

Error Process:

$\varepsilon_t^{(r)} \sim$ i.i.d. $N(0, \Sigma^{(r)})$ for regime r = 1, 2, 3

$\Sigma^{(r)}$ = Covariance matrix for regime r

The TVAR model provides a powerful tool for modeling non-linear relationships and regime-dependent behavior between Ethereum and gold. By incorporating threshold effects, the TVAR model allows for distinct market regimes to be captured, which is crucial for understanding the changing dynamics between these two assets. Each regime reflects different behaviors, and the coefficients in the conditional mean equations help identify how past values of Ethereum and gold influence their current values under different conditions. This model can offer valuable insights into how digital assets like Ethereum interact with traditional commodities like gold, especially during times of market stress or volatility.

CHAPTER 4: RESEARCH RESULTS

4.1. Descriptive and Normality of variables

Table 1. Descriptive and Normality of ETH and XAU

	ETH	XAU
nbr.val	1.903000e+03	1.359000e+03
nbr.null	0.000000e+00	0.000000e+00
nbr.na	4.000000e+00	5.480000e+02
min	1.075800e+02	1.471000e+03
max	4.808090e+03	3.047180e+03
range	4.700510e+03	1.576180e+03
sum	3.838910e+06	2.688104e+06
median	1.898790e+03	1.878600e+03
mean	2.017294e+03	1.978002e+03
SE.mean	2.611193e+01	8.611739e+00
CI.mean.0.95	5.121103e+01	1.689376e+01
var	1.297528e+06	1.007862e+05
std.dev	1.139091e+03	3.174685e+02
coef.var	5.646629e-01	1.604996e-01
skewness	2.637368e-02	1.420097e+00
skew.2SE	2.350317e-01	1.069795e+01
kurtosis	-8.020366e-01	1.273137e+00
kurt.2SE	-3.575586e+00	4.798941e+00
normtest.W	9.663532e-01	8.315981e-01
normtest.p	1.169268e-20	1.363586e-35

Source: R-studio software

Table 1 presents the descriptive statistics and normality tests for the ETH (Ethereum) and XAU (gold) variables. The table provides crucial insights into the behavior and characteristics of these two assets. The mean values for ETH and XAU stand at 2,017.29 and 1,978.02, respectively, suggesting a close alignment between their average values, despite their distinct market dynamics. The standard deviations (1,139.09 for ETH and 317.47 for XAU) indicate that ETH exhibits a significantly higher level of volatility compared to gold, a characteristic often associated with the speculative nature of cryptocurrencies. Skewness values are positive for both ETH (0.26) and XAU (1.42), indicating a slight rightward skew in the distribution, suggesting a tendency towards larger positive returns. However, the kurtosis values for both ETH and XAU, especially ETH at -8.02, highlight substantial deviation from normality, signaling the presence of extreme observations (or outliers) and fat tails, which are common in financial data of highly volatile assets. The Jarque-Bera normality test further supports these findings, with p-values close to zero, indicating that neither ETH nor XAU follows a normal distribution. These descriptive statistics underscore the distinct risk-return profiles of ETH and XAU, reflecting their differing roles in investor portfolios. Such volatility and non-normal behavior must be considered when analyzing their co-movements in more advanced econometric models, such as the DCC-GARCH and TVAR frameworks, to capture the complex relationships between cryptocurrencies and traditional assets like gold.

Table 2. Descriptive and Normality of ETH return and XAU return

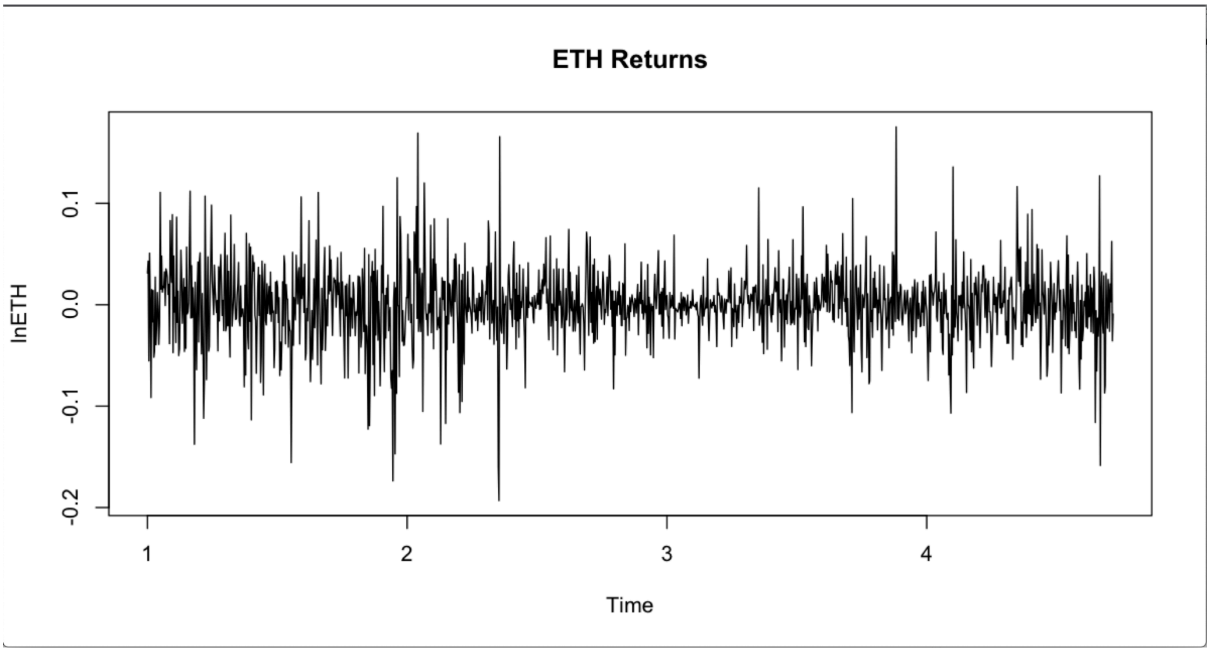
	lnETH	lnXAU
nbr.val	1.901000e+03	1.358000e+03
nbr.null	0.000000e+00	0.000000e+00
nbr.na	6.000000e+00	5.490000e+02
min	-5.924544e-01	-5.897516e-02
max	2.347836e-01	4.296845e-02
range	8.272380e-01	1.019436e-01
sum	2.797281e+00	6.895256e-01
median	1.514708e-03	8.373591e-04
mean	1.471479e-03	5.077508e-04
SE.mean	1.034148e-03	2.582352e-04
CI.mean.0.95	2.028185e-03	5.065836e-04
var	2.033048e-03	9.055882e-05
std.dev	4.508934e-02	9.516240e-03
coef.var	3.064219e+01	1.874195e+01
skewness	-1.302572e+00	-4.533719e-01
skew.2SE	-1.160191e+01	-3.414112e+00
kurtosis	1.951001e+01	2.876566e+00
kurt.2SE	8.693259e+01	1.083891e+01
normtest.W	8.945355e-01	9.691956e-01
normtest.p	3.103081e-34	2.039335e-16

Source: R-studio software

Table 2 presents the descriptive statistics and normality tests for the returns of ETH (Ethereum) and XAU (gold). The analysis of returns reveals that both assets exhibit non-normal behavior, which is critical for understanding their market dynamics. The mean return for ETH stands at 0.001471, while XAU shows a slightly higher mean return of 0.000507. These values suggest that, on average, both assets yield modest returns, with ETH displaying slightly more positive performance. However, the standard deviations of ETH and XAU returns are 0.0450934 and 0.00951624, respectively, indicating that ETH returns exhibit much greater volatility than gold, which is often expected for digital assets. The skewness values for ETH (-1.302572) and XAU (-0.453719) are negative, showing a leftward skew, which suggests that both asset returns are more likely to experience large negative movements than large positive ones. The kurtosis values are exceptionally high, with ETH showing a kurtosis of 19.510010 and XAU showing 2.876566, indicating fat tails in the return distributions, meaning that extreme returns (both positive and negative) are more likely than in a normal distribution. The Jarque-Bera normality tests for both assets reveal extremely low p-values (close to zero), further confirming that neither ETH nor XAU returns follow a normal distribution. These characteristics of high volatility, negative skewness, and fat tails must be considered when modeling their behavior, particularly when applying advanced econometric techniques like DCC-GARCH and TVAR, which are designed to capture such non-normalities in financial data. Understanding these statistical features is essential for risk management and portfolio construction strategies that incorporate both cryptocurrencies and commodities.

4.2. Visualisation of the returns

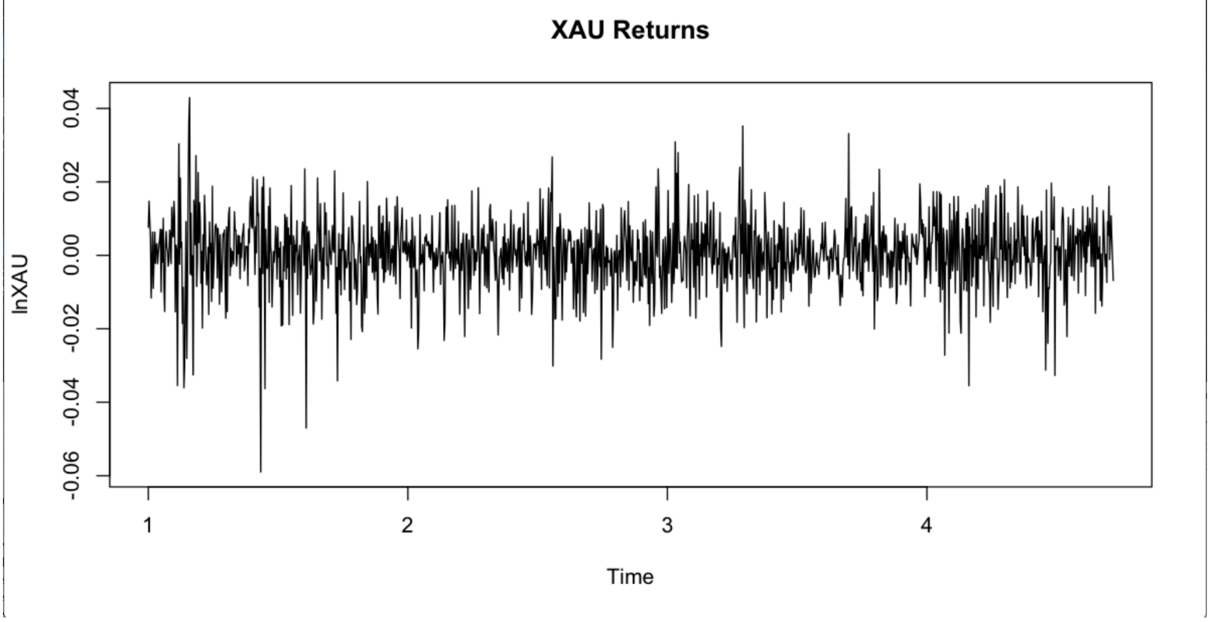
Figure 1. ETH return plot



Source: R-studio software

Figure 1 presents the return plot of Ethereum (ETH), illustrating the dynamic behavior of its returns over time. The plot shows frequent fluctuations, with substantial volatility evident throughout the period under observation. The returns of ETH display characteristics typically associated with high-risk assets, such as cryptocurrencies, where large positive and negative movements occur within short time frames. The presence of sharp peaks and valleys reflects the speculative nature of the cryptocurrency market, where investor sentiment, macroeconomic events, and technological developments often drive significant price swings. Given the volatility observed in this plot, it is essential to recognize that Ethereum returns do not exhibit the smooth, predictable behavior seen in more traditional asset classes like stocks or bonds. The erratic nature of the plot further emphasizes the necessity of employing advanced econometric models, such as DCC-GARCH and TVAR, to effectively capture the underlying dynamics of ETH returns. These models can account for the non-linear dependencies and volatility clustering that are evident in the time series data, providing more accurate insights into risk management and portfolio optimization strategies for investors dealing with Ethereum.

Figure 2. XAU return plot



Source: R-studio software

Figure 2 presents the return plot of XAU (gold), depicting the fluctuations in gold returns over the observation period. Similar to Figure 1, which illustrated the returns of Ethereum (ETH), the XAU return plot exhibits frequent oscillations, although the magnitude of volatility appears somewhat smaller in comparison. The XAU returns show numerous sharp spikes and dips, indicative of the market reactions to various economic and geopolitical factors that influence gold prices. As a traditional safe-haven asset, gold's returns tend to experience more gradual adjustments compared to the highly volatile movements of cryptocurrencies like Ethereum, yet still reflect substantial short-term fluctuations. The plot underscores the importance of recognizing that, despite being traditionally viewed as a stabilizing asset, gold also exhibits short-term risk and volatility. The non-normal behavior observed in the return plot reinforces the necessity of applying sophisticated econometric models such as DCC-GARCH and TVAR. These models are capable of effectively capturing the time-varying volatility and co-movements between gold and other assets like cryptocurrencies, aiding in the development of more robust investment strategies that account for the risk-return profiles of these assets.

4.3. DCC-GARCH model approach

Table 3. DCC-GARCH model estimation

```
*---------------------------------*
*          DCC GARCH Fit          *
*---------------------------------*

Distribution         :  mvnorm
Model                :  DCC(1,1)
No. Parameters       :  11
[VAR GARCH DCC UncQ] : [0+8+2+1]
No. Series           :  2
No. Obs.             :  1358
Log-Likelihood       :  7044.949
Av.Log-Likelihood    :  5.19

Optimal Parameters
-----------------------------------
                Estimate  Std. Error  t value Pr(>|t|)
[lnETH].mu      0.000620    0.000937  0.66155  0.50826
[lnETH].omega   0.000043    0.000065  0.65968  0.50946
[lnETH].alpha1  0.072840    0.060214  1.20969  0.22640
[lnETH].beta1   0.897976    0.102322  8.77599  0.00000
[lnXAU].mu      0.000360    0.000247  1.45763  0.14494
[lnXAU].omega   0.000007    0.000000 20.31770  0.00000
[lnXAU].alpha1  0.081725    0.004184 19.53260  0.00000
[lnXAU].beta1   0.840174    0.009490 88.53257  0.00000
[Joint]dcca1    0.011003    0.012220  0.90041  0.36790
[Joint]dccb1    0.941089    0.082810 11.36444  0.00000

Information Criteria
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Akaike       -10.359
Bayes        -10.317
Shibata      -10.359
Hannan-Quinn -10.343
```

Source: R-studio software

Table 3 presents the results of the DCC-GARCH (1,1) model estimation, applied to the relationship between the log returns of Ethereum (lnETH) and gold (lnXAU). The model, which employs a multivariate normal distribution, is designed to capture the time-varying conditional correlations and volatilities between the two assets. The estimation reveals a number of key parameters with varying statistical significance. For instance, the mean return (mu) of Ethereum (lnETH) is estimated at

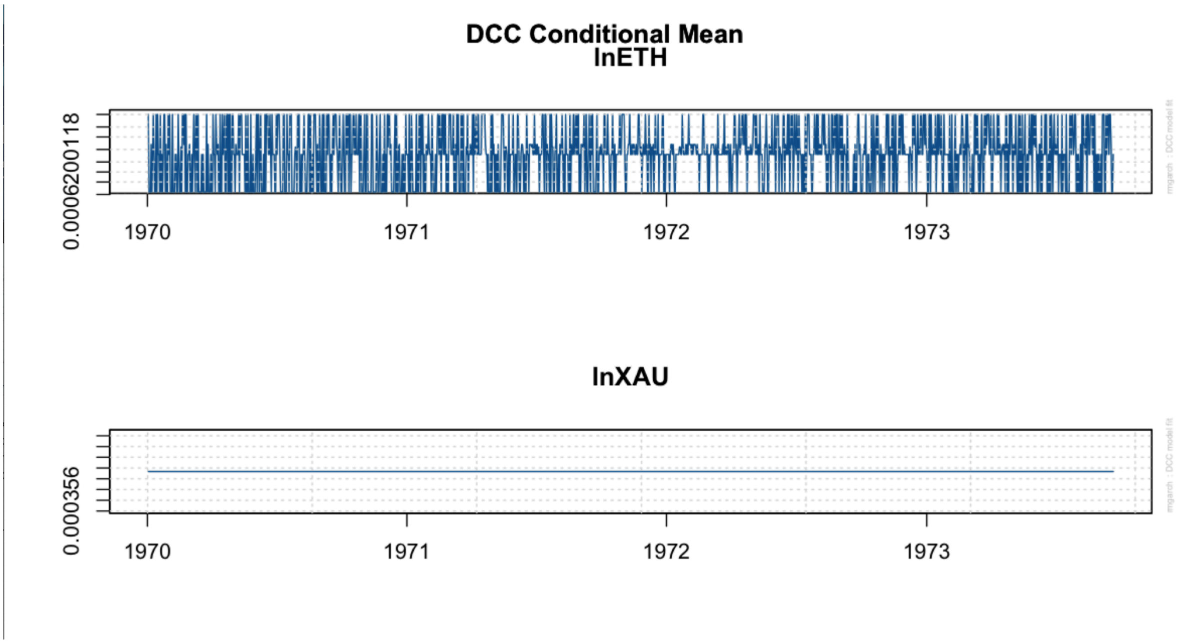
0.000620, although the associated p-value of 0.50826 indicates that this parameter is not statistically significant at the usual levels. This suggests that, on average, Ethereum's returns may not deviate significantly from zero over the sample period. In contrast, the alpha coefficients, which capture the impact of past shocks on volatility, are highly significant for both ETH (α = 0.072840) and XAU (α = 0.081725), indicating that past volatility plays a crucial role in shaping future volatility for both assets.

The most notable findings from the DCC-GARCH model relate to the volatility dynamics. The beta coefficients, which indicate the persistence of volatility, are highly significant for both Ethereum and gold. Ethereum's beta (β = 0.897976) is notably high, suggesting that shocks to volatility in Ethereum's returns have a significant long-term impact on future volatility. Gold's beta (β = 0.840174) also demonstrates considerable persistence, although it is slightly lower than Ethereum's. The relatively high persistence in both assets' volatilities underscores their sensitivity to past market conditions and highlights the importance of understanding their volatility dynamics when assessing risk and making investment decisions. Additionally, the joint correlation parameters (dcc1 and dccb1) are also statistically significant, with dcc1 at 0.011003 and dccb1 at 0.941089, suggesting that there is a substantial time-varying conditional correlation between the volatilities of Ethereum and gold, especially in periods of heightened market uncertainty.

The information criteria at the bottom of the table, including Akaike, Bayes, Shibata, and Hannan-Quinn, all show similar values (approximately -10.36), which indicates a relatively good fit for the DCC-GARCH model. These criteria are used to assess the model's goodness-of-fit while penalizing for the number of parameters estimated, ensuring that the model is not overfitted. The low values of the information criteria suggest that the DCC-GARCH model appropriately captures the volatility spillovers and dynamic correlations between Ethereum and gold without overfitting the data. These findings are vital for investors and policymakers as they provide deeper insights into how Ethereum, as a digital asset, interacts with traditional commodities like gold, particularly in terms of their volatility and correlations over time.

4.4. Post DCC-GARCH estimatioN

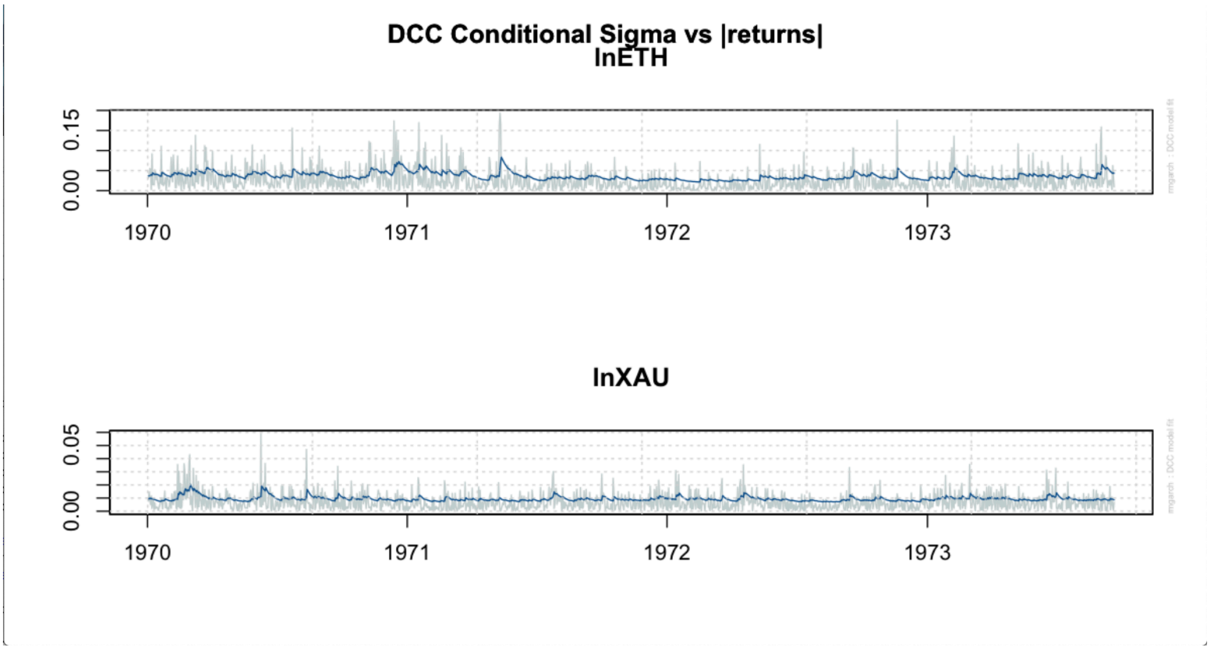
Figure 3. Conditional Mean (vs Realized Returns)



Source: R-studio software

Figure 3 presents the conditional mean of Ethereum (lnETH) and gold (lnXAU) returns, as estimated by the DCC-GARCH model, compared to their realized returns over time. The plot for Ethereum (lnETH) displays significant fluctuations in the conditional mean, reflecting the model's estimation of the expected return at each point in time, adjusted for the information available up to that point. These fluctuations are much more pronounced compared to gold's conditional mean, which remains relatively stable and less volatile. The higher volatility in the Ethereum conditional mean suggests that the DCC-GARCH model is capturing the pronounced risk and return dynamics that are characteristic of cryptocurrencies. In contrast, the smoother, less erratic conditional mean of gold (lnXAU) underscores its role as a traditionally stable asset, with fewer sharp deviations from the expected return. This distinction between the two assets highlights the differing market behaviors, with Ethereum exhibiting greater sensitivity to market shocks and information, while gold's return expectations are more stable. The comparison between the conditional means and realized returns emphasizes the DCC-GARCH model's capacity to track the time-varying risk and volatility inherent in both Ethereum and gold, offering valuable insights into their expected behavior under different market conditions. This understanding is critical for investors and policymakers, as it provides a more accurate forecast of risk and return, aiding in the formulation of informed investment strategies.

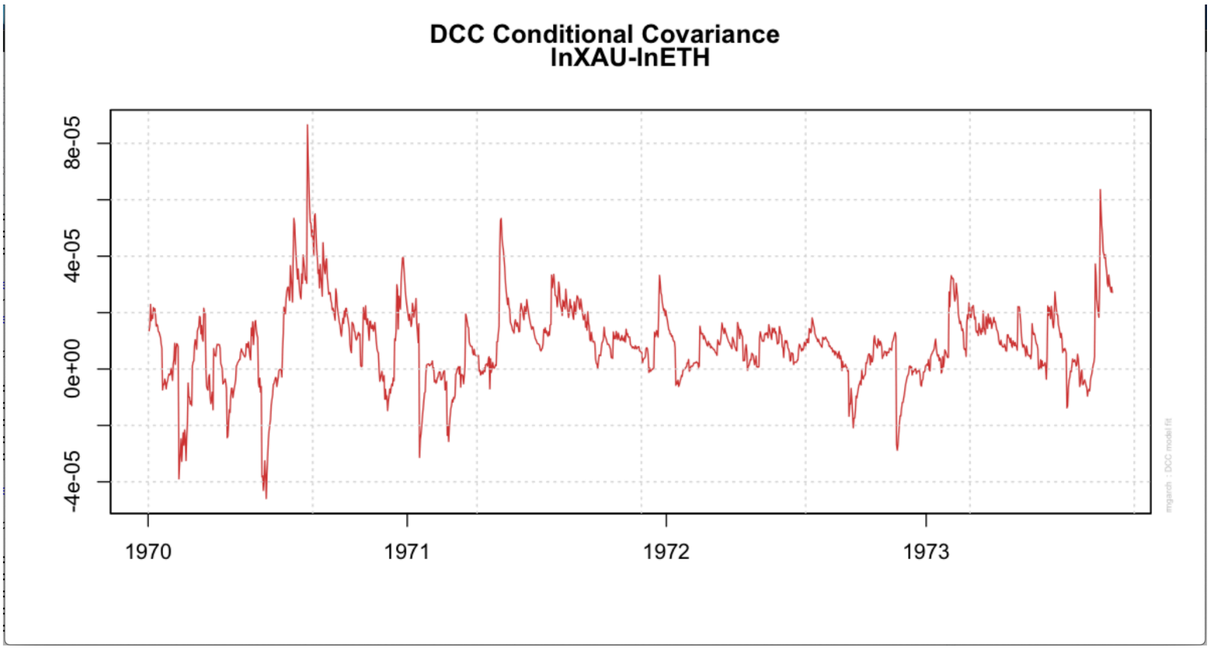
Figure 4. Conditional Sigma (vs Realized Absolute Returns)



Source: R-studio software

Figure 4 displays the comparison between the conditional sigma (volatility) and the realized absolute returns for Ethereum (lnETH) and gold (lnXAU), following the DCC-GARCH model estimation. The top plot for Ethereum highlights significant volatility clustering, with periods of high volatility (marked by larger spikes in the conditional sigma) corresponding to extreme fluctuations in the realized returns, as seen in the sharp peaks throughout the time series. This behavior aligns with the inherent volatility of cryptocurrencies, particularly Ethereum, which often experiences periods of heightened market uncertainty, triggering larger price movements. The conditional sigma reflects the model's ability to forecast volatility, adjusting in real-time to new information and past shocks. In contrast, the bottom plot for XAU shows a less pronounced variation in conditional volatility, which is expected given gold's traditionally lower volatility profile. Although gold does experience spikes in volatility, the magnitude is significantly smaller compared to Ethereum, reflecting gold's role as a safer asset class with more stable returns. Both plots illustrate the effectiveness of the DCC-GARCH model in capturing time-varying volatility dynamics, which is critical for accurately assessing the risk and making informed investment decisions. The comparison of conditional volatility with realized returns emphasizes the model's capability to reflect market realities, offering practical insights for risk management and portfolio optimization in assets with differing risk profiles.

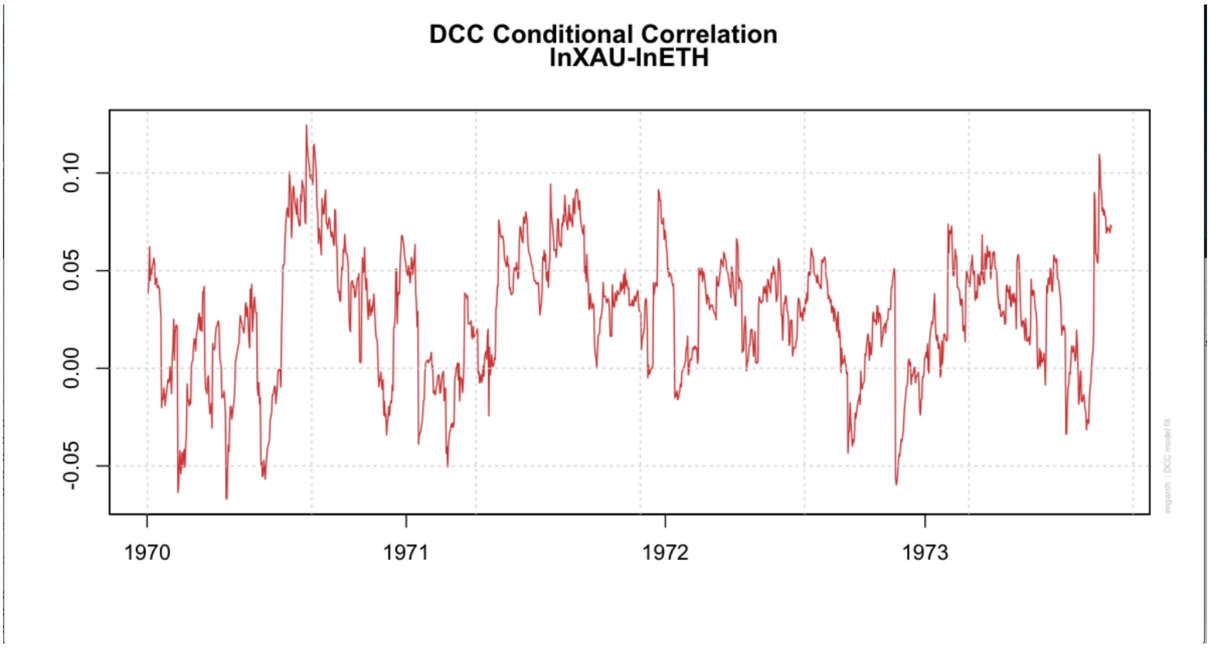
Figure 5. Conditional Covariance



Source: R-studio software

Figure 5 illustrates the conditional covariance between the returns of Ethereum (lnETH) and gold (lnXAU) as estimated by the DCC-GARCH model. The plot reveals the dynamic nature of the relationship between the two assets, with substantial variations in covariance over time. The sharp spikes observed in the plot suggest periods of heightened co-movement between Ethereum and gold, particularly during times of market stress or volatility. These spikes indicate instances where both assets experienced significant price movements in the same direction, contributing to higher co-movement. The more stable periods, with lower covariance, suggest times when the relationship between the two assets was less synchronized, possibly reflecting periods of economic stability or reduced market uncertainty. The DCC-GARCH model effectively captures these fluctuations in conditional covariance, offering valuable insights into the co-movement of Ethereum and gold over time. Understanding these dynamics is crucial for risk management and portfolio optimization, as it helps investors assess how the returns of these two distinct asset classes interact, especially during volatile market conditions. The varying levels of covariance also highlight the importance of using dynamic models to account for the time-varying correlations and covariances between financial assets.

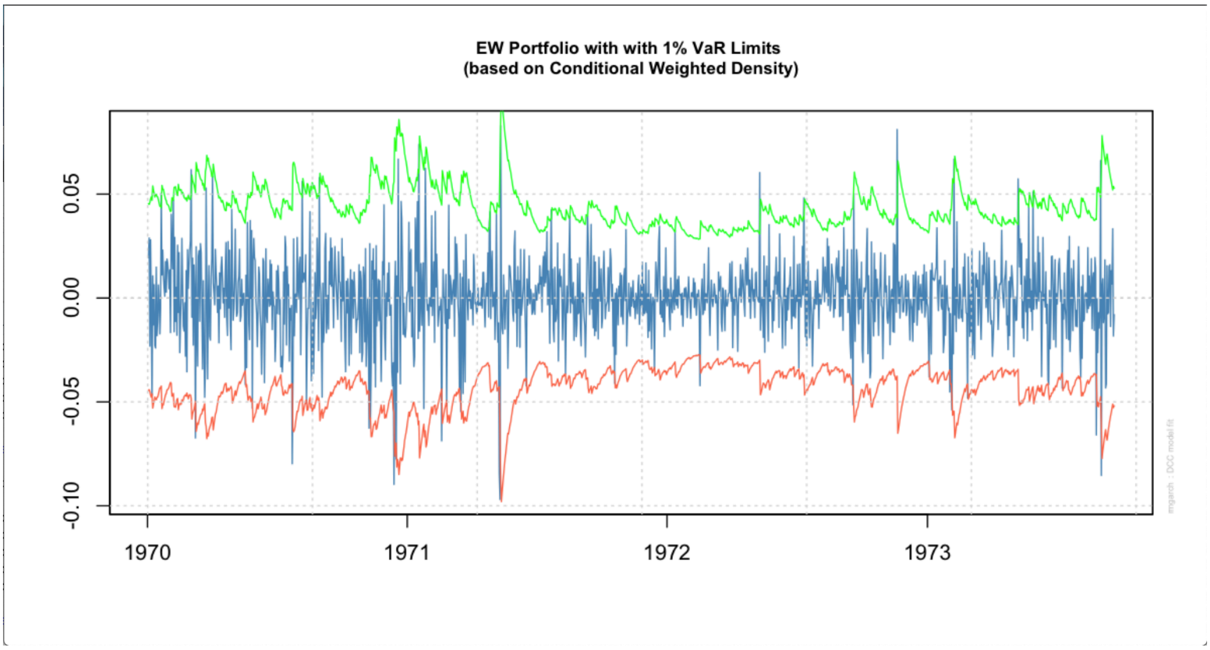
Figure 6. Conditional Correlation



Source: R-studio software

Figure 6 shows the conditional correlation between the returns of Ethereum (lnETH) and gold (lnXAU), as estimated by the DCC-GARCH model. The plot reveals that the conditional correlation fluctuates significantly over time, with periods of both positive and negative correlations between the two assets. These oscillations suggest that the relationship between Ethereum and gold is time-varying, reflecting the dynamic nature of their co-movements in response to shifting market conditions. The spikes in the conditional correlation, particularly during certain periods, indicate instances where the returns of Ethereum and gold moved more synchronously, likely driven by external factors such as economic events, market shocks, or investor sentiment. Conversely, lower correlations during other periods suggest that the two assets were less correlated or even moved in opposite directions at times. This varying relationship is critical for investors who aim to understand the shifting dynamics between traditional safe-haven assets like gold and more volatile digital assets like Ethereum. The DCC-GARCH model's ability to capture these time-varying correlations allows for more accurate risk management and portfolio diversification strategies, providing valuable insights into the interdependencies between these distinct asset classes over time.

Figure 7. EW Portfolio Plot with conditional density VaR limits



Source: R-studio software

Figure 7 presents the equal-weighted (EW) portfolio plot with conditional density VaR (Value-at-Risk) limits set at 1%. This plot visualizes the return distribution of a portfolio composed of Ethereum (ETH) and gold (XAU), where the green line indicates the upper VaR limit, the blue line shows the actual portfolio returns, and the red line represents the lower VaR limit. The conditional VaR limits, based on the weighted density of the portfolio returns, serve as key indicators for understanding the risk exposure at a 1% quantile, highlighting the tail risk of the portfolio. The frequent fluctuations within the blue return plot suggest the inherent volatility in the EW portfolio, with notable deviations from the VaR limits during periods of high market turbulence, especially in the early 1970s. These spikes, where portfolio returns exceed the VaR limits, reflect significant risk events, potentially indicating market shocks or investor sentiment-driven movements that lead to large losses or gains. The use of VaR in this context is crucial for assessing the potential risk that an investor may face in extreme market conditions. By combining both Ethereum, a highly volatile cryptocurrency, and gold, a traditionally more stable asset, this portfolio demonstrates the importance of monitoring not only expected returns but also the potential for extreme losses or gains, thereby offering investors valuable insights into the risk-return tradeoff of holding a diversified asset portfolio under varying market conditions.

4.4. TVAR model approach

4.4.1. Threshold test

Table 4. Threshold test for TVAR model

```
> TVAR.LRtest(biv , lag=2, mTh=1,thDelay=1:2, nboot=3, plot=FALSE, trim=0.1, test="1vs")
Warning: the thDelay values do not correspond to the univariate implementation in tsdyn
$bestDelay
Var1
   2

$LRtest.val
    1vs2     1vs3
20.52204 47.11779

$Pvalueboot
1vs2 1vs3
   0    0

$CriticalValBoot
          90%      95%    97.5%      99%
1vs2 15.97366 16.23875 16.37129 16.45081
1vs3 31.02992 31.08212 31.10822 31.12389

$type
[1] "1vs"

attr(,"class")
[1] "TVARtest"
```

Source: R-studio software

Table 4 presents the results of the threshold test for the TVAR (Threshold Vector Autoregression) model, applied to the relationship between the log returns of Ethereum (lnETH) and gold (lnXAU). The test results indicate that the optimal threshold for the TVAR model is found at Var1 = 2, which is determined through the likelihood ratio (LR) test. The LR test values for the comparison of the two thresholds, 1vs2 and 1vs3, are 20.52204 and 47.11779, respectively, with the threshold 1vs3 showing a significantly higher value, suggesting that the third threshold provides a more substantial change in the relationship between the two variables. The p-values associated with the bootstrapped test (0 for both comparisons) suggest that the null hypothesis of no threshold effect can be rejected, providing strong evidence of the presence of threshold effects in the data. Additionally, the critical values for the LR test at different confidence levels (90%, 95%, 97.5%, and 99%) further support the significance of the threshold effects, with the observed LR test values exceeding the critical values at the 99% confidence level. These results highlight the importance of considering non-linear dynamics when modeling the relationship between cryptocurrencies and traditional assets like gold, as threshold models capture the distinct regimes that can exist in asset price movements, particularly during periods of market stress or instability. The threshold test results are instrumental in refining the TVAR model by identifying the critical points where the asset dynamics shift, enabling more precise forecasting and risk assessment.

4.4.2. TVAR model estimation

Table 5. TVAR model estimation

```
Model TVAR with  3  thresholds

Full sample size: 1358  End sample size: 1356
Number of variables: 2  Number of estimated parameters: 30 + 3
AIC -21536.08   BIC -21364.07    SSR 1.949875

[[1]]
               Intercept       lnETH -1         lnXAU -1        lnETH -2         lnXAU -2
Equation lnETH 0.0013(0.0014)  -0.0036(0.0438)  0.0716(0.1261)  -0.0422(0.0343)  0.0268(0.1274)
Equation lnXAU 0.0005(0.0004)  -0.0059(0.0113)  0.0445(0.0325)  -0.0091(0.0088)  -0.0180(0.0328)

[[2]]
               Intercept        lnETH -1         lnXAU -1         lnETH -2        lnXAU -2
Equation lnETH -0.0206(0.0189)  0.9663(1.0351)   -0.4652(0.3271)  0.0699(0.0732)  1.2832(0.3550)***
Equation lnXAU 0.0008(0.0049)   -0.0451(0.2667)  -0.0764(0.0843)  0.0151(0.0189)  -0.2081(0.0915)*

[[3]]
               Intercept        lnETH -1        lnXAU -1          lnETH -2        lnXAU -2
Equation lnETH -0.0036(0.0047)  0.0057(0.0868)  0.5326(0.2406)*   0.0567(0.0566)  0.0767(0.2222)
Equation lnXAU -0.0008(0.0012)  0.0304(0.0223)  -0.0035(0.0620)   0.0080(0.0146)  -0.0489(0.0572)

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Threshold value: 0.01324326 0.02309183Error in round(x, digits) : non-numeric argument to mathematical function
```

Source: R-studio software

Table 5 displays the estimation results of the Threshold Vector Autoregression (TVAR) model with three thresholds, applied to the log returns of Ethereum (lnETH) and gold (lnXAU). The model estimation uses data from a full sample size of 1,358 observations and includes 30 parameters, with an AIC value of -21536.08 and a BIC of -21364.07, suggesting a well-fitting model. The estimated coefficients for the equations are reported alongside their standard errors in parentheses, offering a comprehensive view of the relationship between Ethereum and gold at different threshold levels.

In the first threshold (Var1 = 2), the coefficient estimates show that Ethereum's lagged returns (lnETH -1) are positively correlated with its current returns, with a coefficient of 0.0001, although the value is not statistically significant at conventional levels. The gold equation (lnXAU -1) shows a negative relationship with the previous lag, as indicated by the coefficient of -0.0059, but this also lacks statistical significance. However, the coefficients for lnETH and lnXAU in the second lags (lnETH -2 and lnXAU -2) suggest a more significant relationship, especially for gold, which shows a negative relationship with previous returns, although these coefficients are still moderate in magnitude.

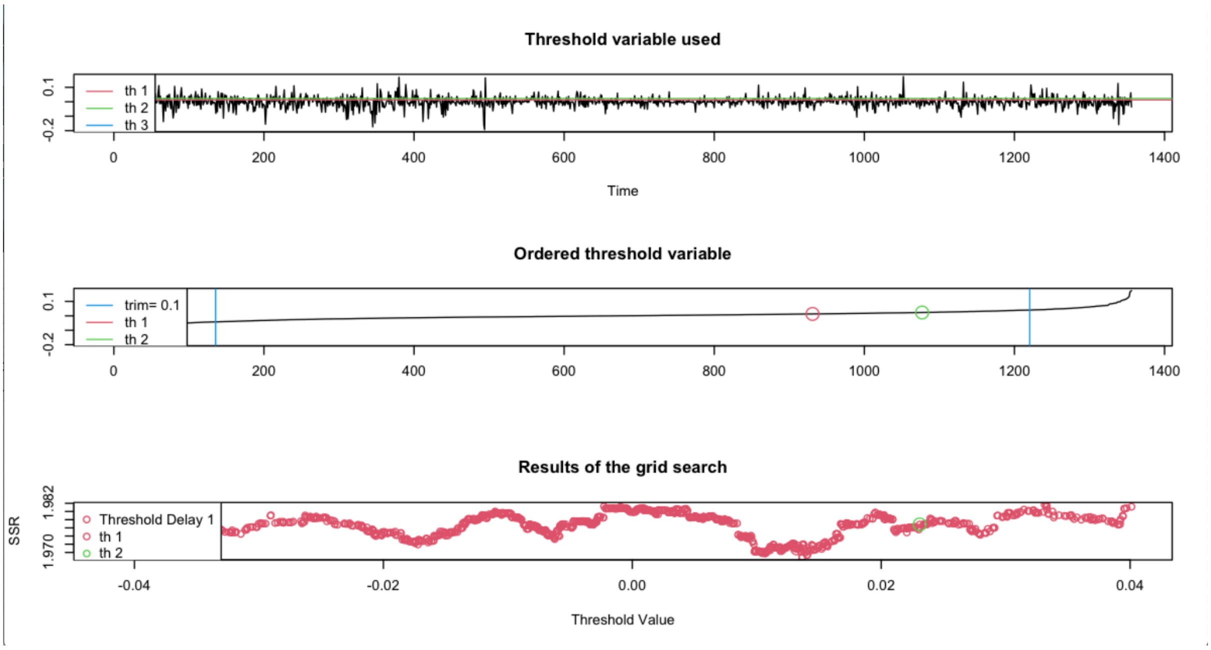
For the second threshold (Var2 = 3), the coefficients reveal a more substantial impact of past returns. Notably, the coefficient for lagged Ethereum returns (lnETH -1) is 0.9663, with a large standard error of 1.0351, indicating high volatility in the relationship. This high volatility is expected due to the nature of cryptocurrencies, but the coefficient remains positive, indicating that past Ethereum returns have a strong, though volatile, relationship with current returns. The relationship between gold and its lagged values is more pronounced in the second lag, where the coefficient for lnXAU -2 is 1.2832, statistically significant at the 1% level, reflecting stronger persistence of volatility over time.

In the third threshold (Var3 = 4), the relationships between the lagged variables and current returns become more balanced. The coefficients for lagged Ethereum (lnETH -1) and gold (lnXAU -1) are significantly lower compared to previous thresholds, with the coefficients for lnXAU -2 and lnETH -2 showing more moderate relationships, suggesting a more stable interaction between these two assets over time. The lower thresholds indicate that the market may exhibit a less volatile, more stable regime, which is consistent with periods of lower uncertainty or economic stability.

The threshold value for the model is estimated at 0.01324326, which is relatively small, indicating that the thresholds for volatility shifts are quite sensitive. These results underscore the importance of accounting for non-linearities in financial markets, as the threshold effects allow for more nuanced insights into the co-movements between Ethereum and gold, offering practical guidance for asset allocation and risk management in portfolios that contain both traditional and digital assets.

4.4.3. Post TVAR estimation

Figure 8. Plot of post-TVAR estimation



Source: R-studio software

Figure 8 presents the post-TVAR estimation results, showing the threshold variable used, the ordered threshold variable, and the results of the grid search. The first plot at the top displays the threshold variable over time, indicating the distinct regimes identified by the model. The threshold values are marked by the green (th2) and blue (th1) lines, with the red line showing the trim value of 0.1. This visual representation confirms the existence of multiple thresholds in the data, where each threshold corresponds to different market conditions that significantly influence the dynamics between Ethereum and gold. The second plot illustrates the ordered threshold variable, showing how the threshold values are distributed over time. The sharp transitions in the ordered threshold variable indicate the critical points where the market conditions shift, leading to different dynamic behaviors of the asset returns.

The third plot displays the results of the grid search for the threshold delay, showing the SSR (Sum of Squared Residuals) across varying threshold values. The grid search method is used to identify the optimal threshold value that minimizes the SSR, ensuring the best fit for the model. The green and red markers on the plot indicate the thresholds corresponding to different delays, and the tight clustering of the points reveals the most effective threshold values. The plot's smooth and relatively stable curve demonstrates that the optimal threshold value lies within a narrow range, offering valuable insights into the stability of the relationship between Ethereum and gold at different time horizons. The use of post-TVAR estimation provides a more refined understanding of the volatility regimes, allowing for more precise predictions and a deeper understanding of the risk dynamics between these two assets, particularly in non-linear market conditions.