SOCIETY OF ACTUARIES

EXAM P   PROBABILITY

EXAM P SAMPLE QUESTIONS

This set of sample questions includes those published on the probability topic for use with previous versions of this examination. Questions from previous versions of this document that are not relevant for the syllabus effective with the September 2022 administration have been deleted. The questions have been renumbered. Unless indicated below, no questions have been added to the version published for use with exams through July 2022.

Some of the questions in this study note are taken from past SOA examinations.

These questions are representative of the types of questions that might be asked of candidates sitting for the Probability (P) Exam. These questions are intended to represent the depth of understanding required of candidates. The distribution of questions by topic is not intended to represent the distribution of questions on future exams.

Questions 271-287 were added July 2022.
Questions 288-319 were added August 2022.

1. A survey of a group's viewing habits over the last year revealed the following information:

      (i)   28% watched gymnastics
      (ii)   29% watched baseball
      (iii)   19% watched soccer
      (iv)   14% watched gymnastics and baseball
      (v)   12% watched baseball and soccer
      (vi)   10% watched gymnastics and soccer
      (vii)   8% watched all three sports.

Calculate the percentage of the group that watched none of the three sports during the last year.

(A)   24%
(B)   36%
(C)   41%
(D)   52%
(E)   60%

2. The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35%. Of those coming to a PCP's office, 30% are referred to specialists and 40% require lab work.

Calculate the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

(A)   0.05
(B)   0.12
(C)   0.18
(D)   0.25
(E)   0.35

3. You are given $P[A \cup B] = 0.7$ and $P[A \cup B'] = 0.9$.

Calculate $P[A]$.

(A)   0.2
(B)   0.3
(C)   0.4
(D)   0.6
(E)   0.8

4. An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44.

Calculate the number of blue balls in the second urn.

(A)　　4
(B)　　20
(C)　　24
(D)　　44
(E)　　64

5. An auto insurance company has 10,000 policyholders. Each policyholder is classified as

　　(i)　　young or old;
　　(ii)　　male or female; and
　　(iii)　　married or single.

Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males.

Calculate the number of the company's policyholders who are young, female, and single.

(A)　　280
(B)　　423
(C)　　486
(D)　　880
(E)　　896

6. A public health researcher examines the medical records of a group of 937 men who died in 1999 and discovers that 210 of the men died from causes related to heart disease. Moreover, 312 of the 937 men had at least one parent who suffered from heart disease, and, of these 312 men, 102 died from causes related to heart disease.

Calculate the probability that a man randomly selected from this group died of causes related to heart disease, given that neither of his parents suffered from heart disease.

(A)　　0.115
(B)　　0.173
(C)　　0.224
(D)　　0.327
(E)　　0.514

7. An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year and 60% of policyholders who have only a homeowners policy will renew next year. The company estimates that 80% of policyholders who have both an auto policy and a homeowners policy will renew at least one of those policies next year.

Company records show that 65% of policyholders have an auto policy, 50% of policyholders have a homeowners policy, and 15% of policyholders have both an auto policy and a homeowners policy.

Using the company's estimates, calculate the percentage of policyholders that will renew at least one policy next year.

(A) 20%
(B) 29%
(C) 41%
(D) 53%
(E) 70%

8. Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist.

Calculate the probability that a randomly chosen member of this group visits a physical therapist.

(A) 0.26
(B) 0.38
(C) 0.40
(D) 0.48
(E) 0.62

9.   An insurance company examines its pool of auto insurance customers and gathers the following information:

        (i)   All customers insure at least one car.
        (ii)   70% of the customers insure more than one car.
        (iii)   20% of the customers insure a sports car.
        (iv)   Of those customers who insure more than one car, 15% insure a sports car.

Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

(A)   0.13
(B)   0.21
(C)   0.24
(D)   0.25
(E)   0.30

10.   An actuary studying the insurance preferences of automobile owners makes the following conclusions:
        (i)   An automobile owner is twice as likely to purchase collision coverage as disability coverage.
        (ii)   The event that an automobile owner purchases collision coverage is independent of the event that he or she purchases disability coverage.
        (iii)   The probability that an automobile owner purchases both collision and disability coverages is 0.15.

Calculate the probability that an automobile owner purchases neither collision nor disability coverage.

(A)   0.18
(B)   0.33
(C)   0.48
(D)   0.67
(E)   0.82

11. A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:

> (i)   14% have high blood pressure.
> (ii)   22% have low blood pressure.
> (iii)   15% have an irregular heartbeat.
> (iv)   Of those with an irregular heartbeat, one-third have high blood pressure.
> (v)   Of those with normal blood pressure, one-eighth have an irregular heartbeat.

Calculate the portion of the patients selected who have a regular heartbeat and low blood pressure.

(A)   2%
(B)   5%
(C)   8%
(D)   9%
(E)   20%

12. An actuary is studying the prevalence of three health risk factors, denoted by A, B, and C, within a population of women. For each of the three factors, the probability is 0.1 that a woman in the population has only this risk factor (and no others). For any two of the three factors, the probability is 0.12 that she has exactly these two risk factors (but not the other). The probability that a woman has all three risk factors, given that she has A and B, is 1/3.

Calculate the probability that a woman has none of the three risk factors, given that she does not have risk factor A.

(A)   0.280
(B)   0.311
(C)   0.467
(D)   0.484
(E)   0.700

13. In modeling the number of claims filed by an individual under an automobile policy during a three-year period, an actuary makes the simplifying assumption that for all integers $n \geq 0$, $p(n+1) = 0.2\,p(n)$ where $p(n)$ represents the probability that the policyholder files n claims during the period.

Under this assumption, calculate the probability that a policyholder files more than one claim during the period.

(A)   0.04
(B)   0.16
(C)   0.20
(D)   0.80
(E)   0.96

14. An insurer offers a health plan to the employees of a large company. As part of this plan, the individual employees may choose exactly two of the supplementary coverages A, B, and C, or they may choose no supplementary coverage. The proportions of the company's employees that choose coverages A, B, and C are 1/4, 1/3, and 5/12 respectively.

Calculate the probability that a randomly chosen employee will choose no supplementary coverage.

(A)   0
(B)   47/144
(C)   1/2
(D)   97/144
(E)   7/9

15. An insurance company determines that N, the number of claims received in a week, is a random variable with $P[N = n] = \dfrac{1}{2^{n+1}}$ where $n \geq 0$. The company also determines that the number of claims received in a given week is independent of the number of claims received in any other week.

Calculate the probability that exactly seven claims will be received during a given two-week period.

(A)   1/256
(B)   1/128
(C)   7/512
(D)   1/64
(E)   1/32

16. An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims.

Calculate the probability that a claim submitted to the insurance company includes operating room charges.

(A)   0.10
(B)   0.20
(C)   0.25
(D)   0.40
(E)   0.80

17. Two instruments are used to measure the height, h, of a tower. The error made by the less accurate instrument is normally distributed with mean 0 and standard deviation $0.0056h$. The error made by the more accurate instrument is normally distributed with mean 0 and standard deviation $0.0044h$.

The errors from the two instruments are independent of each other.

Calculate the probability that the average value of the two measurements is within $0.005h$ of the height of the tower.

(A)   0.38
(B)   0.47
(C)   0.68
(D)   0.84
(E)   0.90

18.   An auto insurance company insures drivers of all ages. An actuary compiled the following statistics on the company's insured drivers:

Age of Driver	Probability of Accident	Portion of Company's Insured Drivers
16-20	0.06	0.08
21-30	0.03	0.15
31-65	0.02	0.49
66-99	0.04	0.28

A randomly selected driver that the company insures has an accident.

Calculate the probability that the driver was age 16-20.

(A)   0.13
(B)   0.16
(C)   0.19
(D)   0.23
(E)   0.40

19.   An insurance company issues life insurance policies in three separate categories: standard, preferred, and ultra-preferred. Of the company's policyholders, 50% are standard, 40% are preferred, and 10% are ultra-preferred. Each standard policyholder has probability 0.010 of dying in the next year, each preferred policyholder has probability 0.005 of dying in the next year, and each ultra-preferred policyholder has probability 0.001 of dying in the next year.

A policyholder dies in the next year.

Calculate the probability that the deceased policyholder was ultra-preferred.

(A)   0.0001
(B)   0.0010
(C)   0.0071
(D)   0.0141
(E)   0.2817

20. Upon arrival at a hospital's emergency room, patients are categorized according to their condition as critical, serious, or stable. In the past year:

      (i)   10% of the emergency room patients were critical;
      (ii)   30% of the emergency room patients were serious;
      (iii)   the rest of the emergency room patients were stable;
      (iv)   40% of the critical patients died;
      (vi)   10% of the serious patients died; and
      (vii) 1% of the stable patients died.

Given that a patient survived, calculate the probability that the patient was categorized as serious upon arrival.

(A)   0.06
(B)   0.29
(C)   0.30
(D)   0.39
(E)   0.64

21. A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers.

Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers.

A randomly selected participant from the study died during the five-year period.

Calculate the probability that the participant was a heavy smoker.

(A)   0.20
(B)   0.25
(C)   0.35
(D)   0.42
(E)   0.57

22. An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are:

Type of driver	Percentage of all drivers	Probability of at least one collision
Teen	8%	0.15
Young adult	16%	0.08
Midlife	45%	0.04
Senior	31%	0.05
Total	100%	

Given that a driver has been involved in at least one collision in the past year, calculate the probability that the driver is a young adult driver.

(A) 0.06
(B) 0.16
(C) 0.19
(D) 0.22
(E) 0.25

23. The number of injury claims per month is modeled by a random variable N with

$$P[N = n] = \frac{1}{(n+1)(n+2)}, \text{ for nonnegative integers, } n.$$

Calculate the probability of at least one claim during a particular month, given that there have been at most four claims during that month.

(A) 1/3
(B) 2/5
(C) 1/2
(D) 3/5
(E) 5/6

24. A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not actually present. One percent of the population actually has the disease.

Calculate the probability that a person actually has the disease given that the test indicates the presence of the disease.

(A) 0.324
(B) 0.657
(C) 0.945
(D) 0.950
(E) 0.995

25. The probability that a randomly chosen male has a blood circulation problem is 0.25. Males who have a blood circulation problem are twice as likely to be smokers as those who do not have a blood circulation problem.

Calculate the probability that a male has a blood circulation problem, given that he is a smoker.

(A) 1/4
(B) 1/3
(C) 2/5
(D) 1/2
(E) 2/3

26. A study of automobile accidents produced the following data:

Model year	Proportion of all vehicles	Probability of involvement in an accident
2014	0.16	0.05
2013	0.18	0.02
2012	0.20	0.03
Other	0.46	0.04

An automobile from one of the model years 2014, 2013, and 2012 was involved in an accident.

Calculate the probability that the model year of this automobile is 2014.

(A)   0.22
(B)   0.30
(C)   0.33
(D)   0.45
(E)   0.50

27. A hospital receives 1/5 of its flu vaccine shipments from Company X and the remainder of its shipments from other companies. Each shipment contains a very large number of vaccine vials.

For Company X's shipments, 10% of the vials are ineffective. For every other company, 2% of the vials are ineffective. The hospital tests 30 randomly selected vials from a shipment and finds that one vial is ineffective.

Calculate the probability that this shipment came from Company X.

(A)   0.10
(B)   0.14
(C)   0.37
(D)   0.63
(E)   0.86

28. The number of days that elapse between the beginning of a calendar year and the moment a high-risk driver is involved in an accident is exponentially distributed. An insurance company expects that 30% of high-risk drivers will be involved in an accident during the first 50 days of a calendar year.

Calculate the portion of high-risk drivers are expected to be involved in an accident during the first 80 days of a calendar year.

(A) 0.15
(B) 0.34
(C) 0.43
(D) 0.57
(E) 0.66

29. An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims.

The number of claims filed has a Poisson distribution.

Calculate the variance of the number of claims filed.

(A) $\dfrac{1}{\sqrt{3}}$

(B) 1

(C) $\sqrt{2}$

(D) 2

(E) 4

30. A company establishes a fund of 120 from which it wants to pay an amount, C, to any of its 20 employees who achieve a high performance level during the coming year. Each employee has a 2% chance of achieving a high performance level during the coming year. The events of different employees achieving a high performance level during the coming year are mutually independent.

Calculate the maximum value of C for which the probability is less than 1% that the fund will be inadequate to cover all payments for high performance.

(A)   24
(B)   30
(C)   40
(D)   60
(E) 120

31. A large pool of adults earning their first driver's license includes 50% low-risk drivers, 30% moderate-risk drivers, and 20% high-risk drivers. Because these drivers have no prior driving record, an insurance company considers each driver to be randomly selected from the pool.

This month, the insurance company writes four new policies for adults earning their first driver's license.

Calculate the probability that these four will contain at least two more high-risk drivers than low-risk drivers.

(A) 0.006
(B) 0.012
(C) 0.018
(D) 0.049
(E) 0.073

32. The loss due to a fire in a commercial building is modeled by a random variable X with density function

$$f(x) = \begin{cases} 0.005(20-x), & 0 < x < 20 \\ 0, & \text{otherwise.} \end{cases}$$

Given that a fire loss exceeds 8, calculate the probability that it exceeds 16.

(A) 1/25
(B) 1/9
(C) 1/8
(D) 1/3
(E) 3/7

33. The lifetime of a machine part has a continuous distribution on the interval (0, 40) with probability density function $f(x)$, where $f(x)$ is proportional to $(10 + x)^{-2}$ on the interval.

Calculate the probability that the lifetime of the machine part is less than 6.

(A) 0.04
(B) 0.15
(C) 0.47
(D) 0.53
(E) 0.94

34. A group insurance policy covers the medical claims of the employees of a small company. The value, V, of the claims made in one year is described by

$$V = 100{,}000Y$$

where Y is a random variable with density function

$$f(y) = \begin{cases} k(1-y)^4, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant.

Calculate the conditional probability that V exceeds 40,000, given that V exceeds 10,000.

(A) 0.08
(B) 0.13
(C) 0.17
(D) 0.20
(E) 0.51

35. The lifetime of a printer costing 200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, a one-half refund if it fails during the second year, and no refund for failure after the second year.

Calculate the expected total amount of refunds from the sale of 100 printers.

(A)   6,321
(B)   7,358
(C)   7,869
(D) 10,256
(E) 12,642

36. An insurance company insures a large number of homes. The insured value, X, of a randomly selected home is assumed to follow a distribution with density function

$$f(x) = \begin{cases} 3x^{-4}, & x > 1 \\ 0, & \text{otherwise.} \end{cases}$$

Given that a randomly selected home is insured for at least 1.5, calculate the probability that it is insured for less than 2.

(A)   0.578
(B)   0.684
(C)   0.704
(D)   0.829
(E)   0.875

37. A company prices its hurricane insurance using the following assumptions:

(i)   In any calendar year, there can be at most one hurricane.
(ii)   In any calendar year, the probability of a hurricane is 0.05.
(iii)   The numbers of hurricanes in different calendar years are mutually independent.

Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

(A)   0.06
(B)   0.19
(C)   0.38
(D)   0.62
(E)   0.92

38. An insurance policy pays for a random loss X subject to a deductible of C, where $0 < C < 1$. The loss amount is modeled as a continuous random variable with density function

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Given a random loss X, the probability that the insurance payment is less than 0.5 is equal to 0.64.

Calculate C.

(A)   0.1
(B)   0.3
(C)   0.4
(D)   0.6
(E)   0.8

39. A study is being conducted in which the health of two independent groups of ten policyholders is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants).

Calculate the probability that at least nine participants complete the study in one of the two groups, but not in both groups?

(A)   0.096
(B)   0.192
(C)   0.235
(D)   0.376
(E)   0.469

40. For Company A there is a 60% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean 10,000 and standard deviation 2,000.

For Company B there is a 70% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean 9,000 and standard deviation 2,000.

The total claim amounts of the two companies are independent.

Calculate the probability that, in the coming year, Company B's total claim amount will exceed Company A's total claim amount.

(A)   0.180
(B)   0.185
(C)   0.217
(D)   0.223
(E)   0.240

41. A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. The probability that one or more accidents will occur during any given month is 0.60. The numbers of accidents that occur in different months are mutually independent.

Calculate the probability that there will be at least four months in which no accidents occur before the fourth month in which at least one accident occurs.

(A)   0.01
(B)   0.12
(C)   0.23
(D)   0.29
(E)   0.41

42. An insurance policy pays 100 per day for up to three days of hospitalization and 50 per day for each day of hospitalization thereafter.

The number of days of hospitalization, *X*, is a discrete random variable with probability function

$$P[X = k] = \begin{cases} \dfrac{6-k}{15}, & k = 1, 2, 3, 4, 5 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the expected payment for hospitalization under this policy.

(A) 123
(B) 210
(C) 220
(D) 270
(E) 367

43. Let *X* be a continuous random variable with density function

$$f(x) = \begin{cases} \dfrac{|x|}{10}, & -2 \le x \le 4 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected value of *X*.

(A) 1/5
(B) 3/5
(C) 1
(D) 28/15
(E) 12/5

44. A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$.

Calculate $E(X)$.

(A) $2 + \dfrac{1}{3}e^{-6}$

(B) $2 - 2e^{-2/3} + 5e^{-4/3}$

(C) 3

(D) $2 + 3e^{-2/3}$

(E) 5

45. A piece of equipment is being insured against early failure. The time from purchase until failure of the equipment is exponentially distributed with mean 10 years. The insurance will pay an amount x if the equipment fails during the first year, and it will pay $0.5x$ if failure occurs during the second or third year. If failure occurs after the first three years, no payment will be made.

Calculate x such that the expected payment made under this insurance is 1000.

(A) 3858
(B) 4449
(C) 5382
(D) 5644
(E) 7235

46. An insurance policy on an electrical device pays a benefit of 4000 if the device fails during the first year. The amount of the benefit decreases by 1000 each successive year until it reaches 0. If the device has not failed by the beginning of any given year, the probability of failure during that year is 0.4.

Calculate the expected benefit under this policy.

(A) 2234
(B) 2400
(C) 2500
(D) 2667
(E) 2694

47.   A company buys a policy to insure its revenue in the event of major snowstorms that shut down business. The policy pays nothing for the first such snowstorm of the year and 10,000 for each one thereafter, until the end of the year. The number of major snowstorms per year that shut down business is assumed to have a Poisson distribution with mean 1.5.

Calculate the expected amount paid to the company under this policy during a one-year period.

(A)   2,769
(B)   5,000
(C)   7,231
(D)   8,347
(E)   10,578

48.   A manufacturer's annual losses follow a distribution with density function

$$f(x) = \begin{cases} \dfrac{2.5(0.6)^{2.5}}{x^{3.5}}, & x > 0.6 \\ 0, & \text{otherwise.} \end{cases}$$

To cover its losses, the manufacturer purchases an insurance policy with an annual deductible of 2.

Calculate the mean of the manufacturer's annual losses not paid by the insurance policy.

(A)   0.84
(B)   0.88
(C)   0.93
(D)   0.95
(E)   1.00

49. An insurance company sells a one-year automobile policy with a deductible of 2. The probability that the insured will incur a loss is 0.05. If there is a loss, the probability of a loss of amount N is K/N, for $N = 1, \ldots, 5$ and K a constant. These are the only possible loss amounts and no more than one loss can occur.

Calculate the expected payment for this policy.

(A) 0.031
(B) 0.066
(C) 0.072
(D) 0.110
(E) 0.150

50. An insurance policy reimburses a loss up to a benefit limit of 10. The policyholder's loss, Y, follows a distribution with density function:

$$f(y) = \begin{cases} 2y^{-3}, & y > 1 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected value of the benefit paid under the insurance policy.

(A) 1.0
(B) 1.3
(C) 1.8
(D) 1.9
(E) 2.0

51. An auto insurance company insures an automobile worth 15,000 for one year under a policy with a 1,000 deductible. During the policy year there is a 0.04 chance of partial damage to the car and a 0.02 chance of a total loss of the car. If there is partial damage to the car, the amount X of damage (in thousands) follows a distribution with density function

$$f(x) = \begin{cases} 0.5003e^{-x/2}, & 0 < x < 15 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected claim payment.

(A) 320
(B) 328
(C) 352
(D) 380
(E) 540

52. An insurance company's monthly claims are modeled by a continuous, positive random variable X, whose probability density function is proportional to $(1 + x)^{-4}$, for $0 < x < \infty$. Calculate the company's expected monthly claims.

(A)   1/6
(B)   1/3
(C)   1/2
(D)   1
(E)   3

53. An insurance policy is written to cover a loss, X, where X has a uniform distribution on [0, 1000]. The policy has a deductible, d, and the expected payment under the policy is 25% of what it would be with no deductible.

Calculate d.

(A)   250
(B)   375
(C)   500
(D)   625
(E)   750

54. An insurer's annual weather-related loss, X, is a random variable with density function

$$f(x) = \begin{cases} \dfrac{2.5(200)^{2.5}}{x^{3.5}}, & x > 200 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the difference between the 30th and 70th percentiles of X.

(A)   35
(B)   93
(C)   124
(D)   231
(E)   298

55. A recent study indicates that the annual cost of maintaining and repairing a car in a town in Ontario averages 200 with a variance of 260.

A tax of 20% is introduced on all items associated with the maintenance and repair of cars (i.e., everything is made 20% more expensive).

Calculate the variance of the annual cost of maintaining and repairing a car after the tax is introduced.

(A) 208
(B) 260
(C) 270
(D) 312
(E) 374

56. A random variable X has the cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 1 \\ \dfrac{x^2 - 2x + 2}{2}, & 1 \leq x < 2 \\ 1, & x \geq 2. \end{cases}$$

Calculate the variance of X.

(A) 7/72
(B) 1/8
(C) 5/36
(D) 4/3
(E) 23/12

57. The warranty on a machine specifies that it will be replaced at failure or age 4, whichever occurs first. The machine's age at failure, X, has density function

$$f(x) = \begin{cases} 1/5, & 0 < x < 5 \\ 0, & \text{otherwise.} \end{cases}$$

Let Y be the age of the machine at the time of replacement.

Calculate the variance of Y.

(A)   1.3
(B)   1.4
(C)   1.7
(D)   2.1
(E)   7.5

58. A probability distribution of the claim sizes for an auto insurance policy is given in the table below:

Claim Size	Probability
20	0.15
30	0.10
40	0.05
50	0.20
60	0.10
70	0.10
80	0.30

Calculate the percentage of claims that are within one standard deviation of the mean claim size.

(A)   45%
(B)   55%
(C)   68%
(D)   85%
(E)   100%

59. The owner of an automobile insures it against damage by purchasing an insurance policy with a deductible of 250. In the event that the automobile is damaged, repair costs can be modeled by a uniform random variable on the interval (0, 1500).

Calculate the standard deviation of the insurance payment in the event that the automobile is damaged.

(A) 361
(B) 403
(C) 433
(D) 464
(E) 521

60. A baseball team has scheduled its opening game for April 1. If it rains on April 1, the game is postponed and will be played on the next day that it does not rain. The team purchases insurance against rain. The policy will pay 1000 for each day, up to 2 days, that the opening game is postponed.

The insurance company determines that the number of consecutive days of rain beginning on April 1 is a Poisson random variable with mean 0.6.

Calculate the standard deviation of the amount the insurance company will have to pay.

(A) 668
(B) 699
(C) 775
(D) 817
(E) 904

61. An insurance policy reimburses dental expense, X, up to a maximum benefit of 250. The probability density function for X is:

$$f(x) = \begin{cases} ce^{-0.004x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

where c is a constant.

Calculate the median benefit for this policy.

(A)   161
(B)   165
(C)   173
(D)   182
(E)   250

62. The time to failure of a component in an electronic device has an exponential distribution with a median of four hours.

Calculate the probability that the component will work without failing for at least five hours.

(A)   0.07
(B)   0.29
(C)   0.38
(D)   0.42
(E)   0.57

63. An insurance company sells an auto insurance policy that covers losses incurred by a policyholder, subject to a deductible of 100. Losses incurred follow an exponential distribution with mean 300.

Calculate the 95th percentile of losses that exceed the deductible.

(A)   600
(B)   700
(C)   800
(D)   900
(E)   1000

64. Claim amounts for wind damage to insured homes are mutually independent random variables with common density function

$$f(x) = \begin{cases} \dfrac{3}{x^4}, & x > 1 \\ 0, & \text{otherwise,} \end{cases}$$

where x is the amount of a claim in thousands.

Suppose 3 such claims will be made.

Calculate the expected value of the largest of the three claims.

(A)   2025
(B)   2700
(C)   3232
(D)   3375
(E)   4500

65. A charity receives 2025 contributions. Contributions are assumed to be mutually independent and identically distributed with mean 3125 and standard deviation 250.

Calculate the approximate 90$^{\text{th}}$ percentile for the distribution of the total contributions received.

(A)   6,328,000
(B)   6,338,000
(C)   6,343,000
(D)   6,784,000
(E)   6,977,000

66. Claims filed under auto insurance policies follow a normal distribution with mean 19,400 and standard deviation 5,000.

Calculate the probability that the average of 25 randomly selected claims exceeds 20,000.

(A)   0.01
(B)   0.15
(C)   0.27
(D)   0.33
(E)   0.45

67. An insurance company issues 1250 vision care insurance policies. The number of claims filed by a policyholder under a vision care insurance policy during one year is a Poisson random variable with mean 2. Assume the numbers of claims filed by different policyholders are mutually independent.

Calculate the approximate probability that there is a total of between 2450 and 2600 claims during a one-year period?

(A)   0.68
(B)   0.82
(C)   0.87
(D)   0.95
(E)   1.00

68. A company manufactures a brand of light bulb with a lifetime in months that is normally distributed with mean 3 and variance 1. A consumer buys a number of these bulbs with the intention of replacing them successively as they burn out. The light bulbs have mutually independent lifetimes.

Calculate the smallest number of bulbs to be purchased so that the succession of light bulbs produces light for at least 40 months with probability at least 0.9772.

(A)   14
(B)   16
(C)   20
(D)   40
(E)   55

69. Let X and Y be the number of hours that a randomly selected person watches movies and sporting events, respectively, during a three-month period. The following information is known about X and Y:

$$E(X) = 50, E(Y) = 20, Var(X) = 50, Var(Y) = 30, Cov(X,Y) = 10.$$

The totals of hours that different individuals watch movies and sporting events during the three months are mutually independent.

One hundred people are randomly selected and observed for these three months. Let T be the total number of hours that these one hundred people watch movies or sporting events during this three-month period.

Approximate the value of $P[T < 7100]$.

(A)   0.62
(B)   0.84
(C)   0.87
(D)   0.92
(E)   0.97

70. The total claim amount for a health insurance policy follows a distribution with density function

$$f(x) = \frac{1}{1000} e^{-(x/1000)}, \ x > 0.$$

The premium for the policy is set at the expected total claim amount plus 100.

If 100 policies are sold, calculate the approximate probability that the insurance company will have claims exceeding the premiums collected.

(A)   0.001
(B)   0.159
(C)   0.333
(D)   0.407
(E)   0.460

71. A city has just added 100 new female recruits to its police force. The city will provide a pension to each new hire who remains with the force until retirement. In addition, if the new hire is married at the time of her retirement, a second pension will be provided for her husband. A consulting actuary makes the following assumptions:

> (i) Each new recruit has a 0.4 probability of remaining with the police force until retirement.

> (ii) Given that a new recruit reaches retirement with the police force, the probability that she is not married at the time of retirement is 0.25.

> (iii) The events of different new hires reaching retirement and the events of different new hires being married at retirement are all mutually independent events.

Calculate the probability that the city will provide at most 90 pensions to the 100 new hires and their husbands.

(A) 0.60
(B) 0.67
(C) 0.75
(D) 0.93
(E) 0.99

72. In an analysis of healthcare data, ages have been rounded to the nearest multiple of 5 years. The difference between the true age and the rounded age is assumed to be uniformly distributed on the interval from ‑2.5 years to 2.5 years. The healthcare data are based on a random sample of 48 people.

Calculate the approximate probability that the mean of the rounded ages is within 0.25 years of the mean of the true ages.

(A) 0.14
(B) 0.38
(C) 0.57
(D) 0.77
(E) 0.88

73. The waiting time for the first claim from a good driver and the waiting time for the first claim from a bad driver are independent and follow exponential distributions with means 6 years and 3 years, respectively.

Calculate the probability that the first claim from a good driver will be filed within 3 years and the first claim from a bad driver will be filed within 2 years.

(A) $\dfrac{1}{18}\left(1-e^{-2/3}-e^{-1/2}+e^{-7/6}\right)$

(B) $\dfrac{1}{18}e^{-7/6}$

(C) $1-e^{-2/3}-e^{-1/2}+e^{-7/6}$

(D) $1-e^{-2/3}-e^{-1/2}+e^{-1/3}$

(E) $1-\dfrac{1}{3}e^{-2/3}-\dfrac{1}{6}e^{-1/2}+\dfrac{1}{18}e^{-7/6}$

74. A tour operator has a bus that can accommodate 20 tourists. The operator knows that tourists may not show up, so he sells 21 tickets. The probability that an individual tourist will not show up is 0.02, independent of all other tourists.

Each ticket costs 50, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat is not available, the tour operator has to pay 100 (ticket cost + 50 penalty) to the tourist.

Calculate the expected revenue of the tour operator.

(A) 955
(B) 962
(C) 967
(D) 976
(E) 985

75. An insurance policy pays a total medical benefit consisting of two parts for each claim. Let X represent the part of the benefit that is paid to the surgeon, and let Y represent the part that is paid to the hospital. The variance of X is 5000, the variance of Y is 10,000, and the variance of the total benefit, $X + Y$, is 17,000.

Due to increasing medical costs, the company that issues the policy decides to increase X by a flat amount of 100 per claim and to increase Y by 10% per claim.

Calculate the variance of the total benefit after these revisions have been made.

(A)   18,200
(B)   18,800
(C)   19,300
(D)   19,520
(E)   20,670

76. A car dealership sells 0, 1, or 2 luxury cars on any day. When selling a car, the dealer also tries to persuade the customer to buy an extended warranty for the car. Let X denote the number of luxury cars sold in a given day, and let Y denote the number of extended warranties sold.

$$P[X = 0, Y = 0] = 1/6$$
$$P[X = 1, Y = 0] = 1/12$$
$$P[X = 1, Y = 1] = 1/6$$
$$P[X = 2, Y = 0] = 1/12$$
$$P[X = 2, Y = 1] = 1/3$$
$$P[X = 2, Y = 2] = 1/6$$

Calculate the variance of X.

(A)   0.47
(B)   0.58
(C)   0.83
(D)   1.42
(E)   2.58

77. The profit for a new product is given by $Z = 3X - Y - 5$. X and Y are independent random variables with $\text{Var}(X) = 1$ and $\text{Var}(Y) = 2$.

Calculate $\text{Var}(Z)$.

(A)   1
(B)   5
(C)   7
(D)   11
(E)   16

78. A company has two electric generators. The time until failure for each generator follows an exponential distribution with mean 10. The company will begin using the second generator immediately after the first one fails.

Calculate the variance of the total time that the generators produce electricity.

(A)   10
(B)   20
(C)   50
(D)   100
(E)   200

79. In a small metropolitan area, annual losses due to storm, fire, and theft are assumed to be mutually independent, exponentially distributed random variables with respective means 1.0, 1.5, and 2.4.

Calculate the probability that the maximum of these losses exceeds 3.

(A)   0.002
(B)   0.050
(C)   0.159
(D)   0.287
(E)   0.414

80. Let X denote the size of a surgical claim and let Y denote the size of the associated hospital claim. An actuary is using a model in which
$E(X) = 5, E(X^2) = 27.4, E(Y) = 7, E(Y^2) = 51.4, Var(X + Y) = 8.$

Let $C_1 = X + Y$ denote the size of the combined claims before the application of a 20% surcharge on the hospital portion of the claim, and let C_2 denote the size of the combined claims after the application of that surcharge.

Calculate $Cov(C_1, C_2)$.

(A)   8.80
(B)   9.60
(C)   9.76
(D)   11.52
(E)   12.32

81. Two life insurance policies, each with a death benefit of 10,000 and a one-time premium of 500, are sold to a married couple, one for each person. The policies will expire at the end of the tenth year. The probability that only the wife will survive at least ten years is 0.025, the probability that only the husband will survive at least ten years is 0.01, and the probability that both of them will survive at least ten years is 0.96.

Calculate the expected excess of premiums over claims, given that the husband survives at least ten years.

(A)   350
(B)   385
(C)   397
(D)   870
(E)   897

82. A diagnostic test for the presence of a disease has two possible outcomes: 1 for disease present and 0 for disease not present. Let X denote the disease state (0 or 1) of a patient, and let Y denote the outcome of the diagnostic test. The joint probability function of X and Y is given by:

$$P[X = 0, Y = 0] = 0.800$$
$$P[X = 1, Y = 0] = 0.050$$
$$P[X = 0, Y = 1] = 0.025$$
$$P[X = 1, Y = 1] = 0.125$$

Calculate $\text{Var}(Y \mid X = 1)$.

(A)   0.13
(B)   0.15
(C)   0.20
(D)   0.51
(E)   0.71

83. An actuary determines that the annual number of tornadoes in counties P and Q are jointly distributed as follows:

		Annual number of tornadoes in county Q			
		0	1	2	3
Annual number	0	0.12	0.06	0.05	0.02
of tornadoes	1	0.13	0.15	0.12	0.03
in county P	2	0.05	0.15	0.10	0.02

Calculate the conditional variance of the annual number of tornadoes in county Q, given that there are no tornadoes in county P.

(A)   0.51
(B)   0.84
(C)   0.88
(D)   0.99
(E)   1.76

84. You are given the following information about N, the annual number of claims for a randomly selected insured:

$$P(N = 0) = \frac{1}{2}, \quad P(N = 1) = \frac{1}{3}, \quad P(N > 1) = \frac{1}{6}.$$

Let S denote the total annual claim amount for an insured. When $N = 1$, S is exponentially distributed with mean 5. When $N > 1$, S is exponentially distributed with mean 8.

Calculate $P(4 < S < 8)$.

(A) 0.04
(B) 0.08
(C) 0.12
(D) 0.24
(E) 0.25

85. Under an insurance policy, a maximum of five claims may be filed per year by a policyholder. Let $p(n)$ be the probability that a policyholder files n claims during a given year, where $n = 0,1,2,3,4,5$. An actuary makes the following observations:

       i) $p(n) \geq p(n+1)$ for $n = 0, 1, 2, 3, 4$.

      ii)   The difference between $p(n)$ and $p(n+1)$ is the same for $n = 0,1,2,3,4$.

     iii)   Exactly 40% of policyholders file fewer than two claims during a given year.

Calculate the probability that a random policyholder will file more than three claims during a given year.

(A) 0.14
(B) 0.16
(C) 0.27
(D) 0.29
(E) 0.33

86. The amounts of automobile losses reported to an insurance company are mutually independent, and each loss is uniformly distributed between 0 and 20,000. The company covers each such loss subject to a deductible of 5,000.

Calculate the probability that the total payout on 200 reported losses is between 1,000,000 and 1,200,000.

(A)   0.0803
(B)   0.1051
(C)   0.1799
(D)   0.8201
(E)   0.8575

87. An insurance agent offers his clients auto insurance, homeowners insurance and renters insurance. The purchase of homeowners insurance and the purchase of renters insurance are mutually exclusive. The profile of the agent's clients is as follows:
i)   17% of the clients have none of these three products.
ii)   64% of the clients have auto insurance.
iii)   Twice as many of the clients have homeowners insurance as have renters insurance.
iv)   35% of the clients have two of these three products.
v)   11% of the clients have homeowners insurance, but not auto insurance.

Calculate the percentage of the agent's clients that have both auto and renters insurance.

(A)   7%
(B)   10%
(C)   16%
(D)   25%
(E)   28%

88. The cumulative distribution function for health care costs experienced by a policyholder is modeled by the function

$$F(x) = \begin{cases} 1 - e^{-\frac{x}{100}}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

The policy has a deductible of 20. An insurer reimburses the policyholder for 100% of health care costs between 20 and 120. Health care costs above 120 are reimbursed at 50%.

Let G be the cumulative distribution function of reimbursements given that the reimbursement is positive.

Calculate $G(115)$.

(A)   0.683
(B)   0.727
(C)   0.741
(D)   0.757
(E)   0.777

89. Let N_1 and N_2 represent the numbers of claims submitted to a life insurance company in April and May, respectively. The joint probability function of N_1 and N_2 is

$$p(n_1, n_2) = \begin{cases} \dfrac{3}{4}\left(\dfrac{1}{4}\right)^{n_1-1} e^{-n_1}\left(1 - e^{-n_1}\right)^{n_2-1}, & n_1 = 1,2,3,..., \ n_2 = 1,2,3,... \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected number of claims that will be submitted to the company in May, given that exactly 2 claims were submitted in April.

(A) $\dfrac{3}{16}\left(e^2 - 1\right)$

(B) $\dfrac{3}{16}e^2$

(C) $\dfrac{3e}{4 - e}$

(D) $e^2 - 1$

(E) e^2

90. A store has 80 modems in its inventory, 30 coming from Source A and the remainder from Source B. Of the modems in inventory from Source A, 20% are defective. Of the modems in inventory from Source B, 8% are defective.

Calculate the probability that exactly two out of a sample of five modems selected without replacement from the store's inventory are defective.

(A) 0.010
(B) 0.078
(C) 0.102
(D) 0.105
(E) 0.125

91. A man purchases a life insurance policy on his 40th birthday. The policy will pay 5000 if he dies before his 50th birthday and will pay 0 otherwise. The length of lifetime, in years from birth, of a male born the same year as the insured has the cumulative distribution function

$$F(t) = \begin{cases} 0, & t \leq 0 \\ 1 - \exp\left(\dfrac{1 - 1.1^t}{1000}\right), & t > 0. \end{cases}$$

Calculate the expected payment under this policy.

(A) 333
(B) 348
(C) 421
(D) 549
(E) 574

92. A mattress store sells only king, queen and twin-size mattresses. Sales records at the store indicate that the number of queen-size mattresses sold is one-fourth the number of king and twin-size mattresses combined. Records also indicate that three times as many king-size mattresses are sold as twin-size mattresses.

Calculate the probability that the next mattress sold is either king or queen-size.

(A) 0.12
(B) 0.15
(C) 0.80
(D) 0.85
(E) 0.95

93. The number of workplace injuries, *N*, occurring in a factory on any given day is Poisson distributed with mean λ. The parameter λ is a random variable that is determined by the level of activity in the factory, and is uniformly distributed on the interval [0, 3].

Calculate Var(*N*).

(A) λ
(B) 2λ
(C)   0.75
(D)   1.50
(E)   2.25

94. A fair die is rolled repeatedly. Let *X* be the number of rolls needed to obtain a 5 and *Y* the number of rolls needed to obtain a 6.

Calculate $E(X \mid Y = 2)$.

(A)   5.0
(B)   5.2
(C)   6.0
(D)   6.6
(E)   6.8

95. A driver and a passenger are in a car accident. Each of them independently has probability 0.3 of being hospitalized. When a hospitalization occurs, the loss is uniformly distributed on [0, 1]. When two hospitalizations occur, the losses are independent.

Calculate the expected number of people in the car who are hospitalized, given that the total loss due to hospitalizations from the accident is less than 1.

(A)   0.510
(B)   0.534
(C)   0.600
(D)   0.628
(E)   0.800

96. Each time a hurricane arrives, a new home has a 0.4 probability of experiencing damage. The occurrences of damage in different hurricanes are mutually independent.

Calculate the mode of the number of hurricanes it takes for the home to experience damage from two hurricanes.

(A)   2
(B)   3
(C)   4
(D)   5
(E)   6

97. Thirty items are arranged in a 6-by-5 array as shown.

A_1	A_2	A_3	A_4	A_5
A_6	A_7	A_8	A_9	A_{10}
A_{11}	A_{12}	A_{13}	A_{14}	A_{15}
A_{16}	A_{17}	A_{18}	A_{19}	A_{20}
A_{21}	A_{22}	A_{23}	A_{24}	A_{25}
A_{26}	A_{27}	A_{28}	A_{29}	A_{30}

Calculate the number of ways to form a set of three distinct items such that no two of the selected items are in the same row or same column.

(A)   200
(B)   760
(C)   1200
(D)   4560
(E)   7200

98. An auto insurance company is implementing a new bonus system. In each month, if a policyholder does not have an accident, he or she will receive a cash-back bonus of 5 from the insurer.

Among the 1,000 policyholders of the auto insurance company, 400 are classified as low-risk drivers and 600 are classified as high-risk drivers.

In each month, the probability of zero accidents for high-risk drivers is 0.80 and the probability of zero accidents for low-risk drivers is 0.90.

Calculate the expected bonus payment from the insurer to the 1000 policyholders in one year.

(A)   48,000
(B)   50,400
(C)   51,000
(D)   54,000
(E)   60,000

99. The probability that a member of a certain class of homeowners with liability and property coverage will file a liability claim is 0.04, and the probability that a member of this class will file a property claim is 0.10. The probability that a member of this class will file a liability claim but not a property claim is 0.01.

Calculate the probability that a randomly selected member of this class of homeowners will not file a claim of either type.

(A)   0.850
(B)   0.860
(C)   0.864
(D)   0.870
(E)   0.890

100. A survey of 100 TV viewers revealed that over the last year:

i)   34 watched CBS.
ii)   15 watched NBC.
iii)   10 watched ABC.
iv)   7 watched CBS and NBC.
v)   6 watched CBS and ABC.
vi)   5 watched NBC and ABC.
vii)   4 watched CBS, NBC, and ABC.
viii)   18 watched HGTV, and of these, none watched CBS, NBC, or ABC.

Calculate how many of the 100 TV viewers did not watch any of the four channels (CBS, NBC, ABC or HGTV).

(A)   1
(B)   37
(C)   45
(D)   55
(E)   82

101. The amount of a claim that a car insurance company pays out follows an exponential distribution. By imposing a deductible of d, the insurance company reduces the expected claim payment by 10%.

Calculate the percentage reduction on the variance of the claim payment.

(A)   1%
(B)   5%
(C)   10%
(D)   20%
(E)   25%

102. The number of hurricanes that will hit a certain house in the next ten years is Poisson distributed with mean 4.

Each hurricane results in a loss that is exponentially distributed with mean 1000. Losses are mutually independent and independent of the number of hurricanes.

Calculate the variance of the total loss due to hurricanes hitting this house in the next ten years.

(A)   4,000,000
(B)   4,004,000
(C)   8,000,000
(D)   16,000,000
(E)   20,000,000

103. A motorist makes three driving errors, each independently resulting in an accident with probability 0.25.

Each accident results in a loss that is exponentially distributed with mean 0.80. Losses are mutually independent and independent of the number of accidents.

The motorist's insurer reimburses 70% of each loss due to an accident.

Calculate the variance of the total unreimbursed loss the motorist experiences due to accidents resulting from these driving errors.

(A)   0.0432
(B)   0.0756
(C)   0.1782
(D)   0.2520
(E)   0.4116

104. An automobile insurance company issues a one-year policy with a deductible of 500. The probability is 0.8 that the insured automobile has no accident and 0.0 that the automobile has more than one accident. If there is an accident, the loss before application of the deductible is exponentially distributed with mean 3000.

Calculate the 95th percentile of the insurance company payout on this policy.

(A)   3466
(B)   3659
(C)   4159
(D)   8487
(E)   8987

105. From 27 pieces of luggage, an airline luggage handler damages a random sample of four.

The probability that exactly one of the damaged pieces of luggage is insured is twice the probability that none of the damaged pieces are insured.

Calculate the probability that exactly two of the four damaged pieces are insured.

(A) 0.06
(B) 0.13
(C) 0.27
(D) 0.30
(E) 0.31

106. Automobile policies are separated into two groups: low-risk and high-risk. Actuary Rahul examines low-risk policies, continuing until a policy with a claim is found and then stopping. Actuary Toby follows the same procedure with high-risk policies. Each low-risk policy has a 10% probability of having a claim. Each high-risk policy has a 20% probability of having a claim. The claim statuses of polices are mutually independent.

Calculate the probability that Actuary Rahul examines fewer policies than Actuary Toby.

(A) 0.2857
(B) 0.3214
(C) 0.3333
(D) 0.3571
(E) 0.4000

107. Let X represent the number of customers arriving during the morning hours and let Y represent the number of customers arriving during the afternoon hours at a diner.
You are given:
i) X and Y are Poisson distributed.
ii)   The first moment of X is less than the first moment of Y by 8.
iii)   The second moment of X is 60% of the second moment of Y.

Calculate the variance of Y.

(A) 4
(B) 12
(C) 16
(D) 27
(E) 35

108. In a certain game of chance, a square board with area 1 is colored with sectors of either red or blue. A player, who cannot see the board, must specify a point on the board by giving an x-coordinate and a y-coordinate. The player wins the game if the specified point is in a blue sector. The game can be arranged with any number of red sectors, and the red sectors are designed so that

$$R_i = \left(\frac{9}{20}\right)^i, \text{ where } R_i \text{ is the area of the } i^{th} \text{ red sector.}$$

Calculate the minimum number of red sectors that makes the chance of a player winning less than 20%.

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

109. Automobile claim amounts are modeled by a uniform distribution on the interval [0, 10,000]. Actuary A reports X, the claim amount divided by 1000. Actuary B reports Y, which is X rounded to the nearest integer from 0 to 10.

Calculate the absolute value of the difference between the 4th moment of X and the 4th moment of Y.

(A) 0
(B) 33
(C) 296
(D) 303
(E) 533

110.   The probability of x losses occurring in year 1 is $(0.5)^{x+1}$ for $x = 0, 1, 2, \ldots$.

The probability of y losses in year 2 given x losses in year 1 is given by the table:

Number of losses in year 1 (x)	Number of losses in year 2 (y) given x losses in year 1				
	0	1	2	3	4+
0	0.60	0.25	0.05	0.05	0.05
1	0.45	0.30	0.10	0.10	0.05
2	0.25	0.30	0.20	0.20	0.05
3	0.15	0.20	0.20	0.30	0.15
4+	0.05	0.15	0.25	0.35	0.20

Calculate the probability of exactly 2 losses in 2 years.
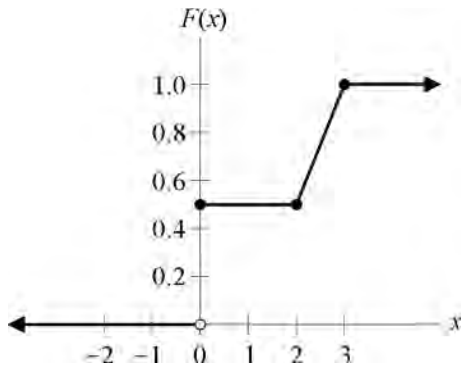
(A)   0.025
(B)   0.031
(C)   0.075
(D)   0.100
(E)   0.131

111.   Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \dfrac{p-1}{x^p}, & x > 1 \\ 0, & \text{otherwise} \end{cases}$$

Calculate the value of p such that $E(X) = 2$.

(A)   1
(B)   2.5
(C)   3
(D)   5
(E)   There is no such p.

112. The figure below shows the cumulative distribution function of a random variable, X.

Calculate $E(X)$.

(A)   0.00
(B)   0.50
(C)   1.00
(D)   1.25
(E)   2.50

113. Two fair dice are rolled. Let X be the absolute value of the difference between the two numbers on the dice.

Calculate the probability that $X < 3$.

(A)   2/9
(B)   1/3
(C)   4/9
(D)   5/9
(E)   2/3

114. An actuary analyzes a company's annual personal auto claims, M, and annual commercial auto claims, N. The analysis reveals that $\text{Var}(M) = 1600$, $\text{Var}(N) = 900$, and the correlation between M and N is 0.64.

Calculate $\text{Var}(M + N)$.

(A)   768
(B)   2500
(C)   3268
(D)   4036
(E)   4420

115. An auto insurance policy has a deductible of 1 and a maximum claim payment of 5. Auto loss amounts follow an exponential distribution with mean 2.

Calculate the expected claim payment made for an auto loss.

(A) $0.5e^{-2} - 0.5e^{-12}$

(B) $2e^{-\frac{1}{2}} - 7e^{-3}$

(C) $2e^{-\frac{1}{2}} - 2e^{-3}$

(D) $2e^{-\frac{1}{2}}$

(E) $3e^{-\frac{1}{2}} - 2e^{-3}$

116. A student takes a multiple-choice test with 40 questions. The probability that the student answers a given question correctly is 0.5, independent of all other questions. The probability that the student answers more than N questions correctly is greater than 0.10. The probability that the student answers more than $N + 1$ questions correctly is less than 0.10.

Calculate N using a normal approximation with the continuity correction.

(A) 23
(B) 25
(C) 32
(D) 33
(E) 35

117. In each of the months June, July, and August, the number of accidents occurring in that month is modeled by a Poisson random variable with mean 1. In each of the other 9 months of the year, the number of accidents occurring is modeled by a Poisson random variable with mean 0.5. Assume that these 12 random variables are mutually independent.

Calculate the probability that exactly two accidents occur in July through November.

(A) 0.084
(B) 0.185
(C) 0.251
(D) 0.257
(E) 0.271

118. An airport purchases an insurance policy to offset costs associated with excessive amounts of snowfall. For every full ten inches of snow in excess of 40 inches during the winter season, the insurer pays the airport 300 up to a policy maximum of 700.

The following table shows the probability function for the random variable X of annual (winter season) snowfall, in inches, at the airport.

Inches	[0,20)	[20,30)	[30,40)	[40,50)	[50,60)	[60,70)	[70,80)	[80,90)	[90,inf)
Probability	0.06	0.18	0.26	0.22	0.14	0.06	0.04	0.04	0.00

Calculate the standard deviation of the amount paid under the policy.

(A)   134
(B)   235
(C)   271
(D)   313
(E)   352

119. Damages to a car in a crash are modeled by a random variable with density function

$$f(x) = \begin{cases} c(x^2 - 60x + 800), & 0 < x < 20 \\ 0, & \text{otherwise} \end{cases}$$

where c is a constant.

A particular car is insured with a deductible of 2. This car was involved in a crash with resulting damages in excess of the deductible.

Calculate the probability that the damages exceeded 10.

(A)   0.12
(B)   0.16
(C)   0.20
(D)   0.26
(E)   0.78

120. Two fair dice, one red and one blue, are rolled.

Let A be the event that the number rolled on the red die is odd.
Let B be the event that the number rolled on the blue die is odd.
Let C be the event that the sum of the numbers rolled on the two dice is odd.

Determine which of the following is true.

(A)   A, B, and C are not mutually independent, but each pair is independent.
(B)   A, B, and C are mutually independent.
(C)   Exactly one pair of the three events is independent.
(D)   Exactly two of the three pairs are independent.
(E)   No pair of the three events is independent.

121. An urn contains four fair dice. Two have faces numbered 1, 2, 3, 4, 5, and 6; one has faces numbered 2, 2, 4, 4, 6, and 6; and one has all six faces numbered 6. One of the dice is randomly selected from the urn and rolled. The same die is rolled a second time.

Calculate the probability that a 6 is rolled both times.

(A)   0.174
(B)   0.250
(C)   0.292
(D)   0.380
(E)   0.417

122. An insurance agent meets twelve potential customers independently, each of whom is equally likely to purchase an insurance product. Six are interested only in auto insurance, four are interested only in homeowners insurance, and two are interested only in life insurance.

The agent makes six sales.

Calculate the probability that two are for auto insurance, two are for homeowners insurance, and two are for life insurance.

(A)   0.001
(B)   0.024
(C)   0.069
(D)   0.097
(E)   0.500

123. A policyholder has probability 0.7 of having no claims, 0.2 of having exactly one claim, and 0.1 of having exactly two claims. Claim amounts are uniformly distributed on the interval [0, 60] and are independent. The insurer covers 100% of each claim.

Calculate the probability that the total benefit paid to the policyholder is 48 or less.

(A)   0.320
(B)   0.400
(C)   0.800
(D)   0.892
(E)   0.924

124. In a given region, the number of tornadoes in a one-week period is modeled by a Poisson distribution with mean 2. The numbers of tornadoes in different weeks are mutually independent.

Calculate the probability that fewer than four tornadoes occur in a three-week period.

(A)   0.13
(B)   0.15
(C)   0.29
(D)   0.43
(E)   0.86

125. An electronic system contains three cooling components that operate independently. The probability of each component's failure is 0.05. The system will overheat if and only if at least two components fail.

Calculate the probability that the system will overheat.

(A)   0.007
(B)   0.045
(C)   0.098
(D)   0.135
(E)   0.143

126. An insurance company's annual profit is normally distributed with mean 100 and variance 400.

Let Z be normally distributed with mean 0 and variance 1 and let F be the cumulative distribution function of Z.

Determine the probability that the company's profit in a year is at most 60, given that the profit in the year is positive.

(A) $1 - F(2)$

(B) $F(2)/F(5)$

(C) $[1 - F(2)]/F(5)$

(D) $[F(0.25) - F(0.1)]/F(0.25)$

(E) $[F(5) - F(2)]/F(5)$

127. In a group of health insurance policyholders, 20% have high blood pressure and 30% have high cholesterol. Of the policyholders with high blood pressure, 25% have high cholesterol.

A policyholder is randomly selected from the group.

Calculate the probability that a policyholder has high blood pressure, given that the policyholder has high cholesterol.

(A)   1/6
(B)   1/5
(C)   1/4
(D)   2/3
(E)   5/6

128. In a group of 25 factory workers, 20 are low-risk and five are high-risk.

Two of the 25 factory workers are randomly selected without replacement.

Calculate the probability that exactly one of the two selected factory workers is low-risk.

(A) 0.160
(B) 0.167
(C) 0.320
(D) 0.333
(E) 0.633

129. The proportion X of yearly dental claims that exceed 200 is a random variable with probability density function

$$f(x) = \begin{cases} 60x^3(1-x)^2, & 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate $\text{Var}[X/(1 - X)]$

(A) 149/900
(B) 10/7
(C) 6
(D) 8
(E) 10

130. This year, a medical insurance policyholder has probability 0.70 of having no emergency room visits, 0.85 of having no hospital stays, and 0.61 of having neither emergency room visits nor hospital stays

Calculate the probability that the policyholder has at least one emergency room visit and at least one hospital stay this year.

(A) 0.045
(B) 0.060
(C) 0.390
(D) 0.667
(E) 0.840

131. An insurer offers a travelers insurance policy. Losses under the policy are uniformly distributed on the interval [0, 5].

The insurer reimburses a policyholder for a loss up to a maximum of 4.

Determine the cumulative distribution function, F, of the benefit that the insurer pays a policyholder who experiences exactly one loss under the policy.

(A) $F(x) = \begin{cases} 0, & x < 0 \\ 0.20x, & 0 \leq x < 4 \\ 1 & x \geq 4 \end{cases}$

(B) $F(x) = \begin{cases} 0, & x < 0 \\ 0.20x, & 0 \leq x < 5 \\ 1 & x \geq 5 \end{cases}$

(C) $F(x) = \begin{cases} 0, & x < 0 \\ 0.25x, & 0 \leq x < 4 \\ 1 & x \geq 4 \end{cases}$

(D) $F(x) = \begin{cases} 0, & x < 0 \\ 0.25x, & 0 \leq x < 5 \\ 1 & x \geq 5 \end{cases}$

(E) $F(x) = \begin{cases} 0, & x < 1 \\ 0.25x, & 1 \leq x < 5 \\ 1 & x \geq 5 \end{cases}$

132. A company issues auto insurance policies. There are 900 insured individuals. Fifty-four percent of them are male. If a female is randomly selected from the 900, the probability she is over 25 years old is 0.43. There are 395 total insured individuals over 25 years old.

A person under 25 years old is randomly selected.

Calculate the probability that the person selected is male.

(A) 0.47
(B) 0.53
(C) 0.54
(D) 0.55
(E) 0.56

133. An insurance company insures red and green cars. An actuary compiles the following data:

Color of car	Red	Green
Number insured	300	700
Probability an accident occurs	0.10	0.05
Probability that the claim exceeds the deductible, given an accident occurs from this group	0.90	0.80

The actuary randomly picks a claim from all claims that exceed the deductible.

Calculate the probability that the claim is on a red car.

(A) 0.300
(B) 0.462
(C) 0.491
(D) 0.667
(E) 0.692

134. George and Paul play a betting game. Each chooses an integer from 1 to 20 (inclusive) at random. If the two numbers differ by more than 3, George wins the bet. Otherwise, Paul wins the bet.

Calculate the probability that Paul wins the bet.

(A) 0.27
(B) 0.32
(C) 0.40
(D) 0.48
(E) 0.66

135. A student takes an examination consisting of 20 true-false questions. The student knows the answer to *N* of the questions, which are answered correctly, and guesses the answers to the rest. The conditional probability that the student knows the answer to a question, given that the student answered it correctly, is 0.824..

Calculate *N*.

(A) 8
(B) 10
(C) 14
(D) 16
(E) 18

136. The minimum force required to break a particular type of cable is normally distributed with mean 12,432 and standard deviation 25. A random sample of 400 cables of this type is selected.

Calculate the probability that at least 349 of the selected cables will not break under a force of 12,400.

(A) 0.62
(B) 0.67
(C) 0.92
(D) 0.97
(E) 1.00

137. The number of policies that an agent sells has a Poisson distribution with modes at 2 and 3.

K is the smallest number such that the probability of selling more than *K* policies is less than 25%.

Calculate *K*.

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

138. Two fair dice are tossed. One die is red and one die is green.

Calculate the probability that the sum of the numbers on the two dice is an odd number given that the number that shows on the red die is larger than the number that shows on the green die.

(A) 1/4
(B) 5/12
(C) 3/7
(D) 1/2
(E) 3/5

139. In 1982 Abby's mother scored at the 93rd percentile in the math SAT exam. In 1982 the mean score was 503 and the variance of the scores was 9604.

In 2008 Abby took the math SAT and got the same numerical score as her mother had received 26 years before. In 2008 the mean score was 521 and the variance of the scores was 10,201.

Math SAT scores are normally distributed and stated in multiples of ten.

Calculate the percentile for Abby's score.

(A) 89th
(B) 90th
(C) 91st
(D) 92nd
(E) 93rd

140. A certain brand of refrigerator has a useful life that is normally distributed with mean 10 years and standard deviation 3 years. The useful lives of these refrigerators are independent.

Calculate the probability that the total useful life of two randomly selected refrigerators will exceed 1.9 times the useful life of a third randomly selected refrigerator.

(A) 0.407
(B) 0.444
(C) 0.556
(D) 0.593
(E) 0.604

141. Losses covered by a flood insurance policy are uniformly distributed on the interval [0, 2]. The insurer pays the amount of the loss in excess of a deductible d.

The probability that the insurer pays at least 1.20 on a random loss is 0.30.

Calculate the probability that the insurer pays at least 1.44 on a random loss.

(A) 0.06
(B) 0.16
(C) 0.18
(D) 0.20
(E) 0.28

142. The lifespan, in years, of a certain computer is exponentially distributed. The probability that its lifespan exceeds four years is 0.30.

Let $f(x)$ represent the density function of the computer's lifespan, in years, for $x > 0$.

Determine which of the following is an expression for $f(x)$.

(A) $1 - (0.3)^{-x/4}$

(B) $1 - (0.7)^{x/4}$

(C) $1 - (0.3)^{x/4}$

(D) $-\dfrac{\ln 0.7}{4}(0.7)^{x/4}$

(E) $-\dfrac{\ln 0.3}{4}(0.3)^{x/4}$

143. The lifetime of a light bulb has density function, f, where $f(x)$ is proportional to

$$\frac{x^2}{1+x^3}, \quad 0 < x < 5, \text{ and } 0, \text{ otherwise.}$$

Calculate the mode of this distribution.

(A) 0.00
(B) 0.79
(C) 1.26
(D) 4.42
(E) 5.00

144. An insurer's medical reimbursements have density function f, where $f(x)$ is proportional to

xe^{-x^2}, for $0 < x < 1$, and 0, otherwise.

Calculate the mode of the medical reimbursements.

(A)   0.00
(B)   0.50
(C)   0.71
(D)   0.84
(E)   1.00

145. A company has five employees on its health insurance plan. Each year, each employee independently has an 80% probability of no hospital admissions. If an employee requires one or more hospital admissions, the number of admissions is modeled by a geometric distribution with a mean of 1.50. The numbers of hospital admissions of different employees are mutually independent.

Each hospital admission costs 20,000.

Calculate the probability that the company's total hospital costs in a year are less than 50,000.

(A)   0.41
(B)   0.46
(C)   0.58
(D)   0.69
(E)   0.78

146. On any given day, a certain machine has either no malfunctions or exactly one malfunction. The probability of malfunction on any given day is 0.40. Machine malfunctions on different days are mutually independent.

Calculate the probability that the machine has its third malfunction on the fifth day, given that the machine has not had three malfunctions in the first three days.

(A)   0.064
(B)   0.138
(C)   0.148
(D   0.230
(E)   0.246

147. In a certain group of cancer patients, each patient's cancer is classified in exactly one of the following five stages: stage 0, stage 1, stage 2, stage 3, or stage 4.

i)   75% of the patients in the group have stage 2 or lower.
ii)   80% of the patients in the group have stage 1 or higher.
iii) 80% of the patients in the group have stage 0, 1, 3, or 4.

One patient from the group is randomly selected.

Calculate the probability that the selected patient's cancer is stage 1.

(A)   0.20
(B)   0.25
(C)   0.35
(D)   0.48
(E)   0.65

148. A car is new at the beginning of a calendar year. The time, in years, before the car experiences its first failure is exponentially distributed with mean 2.

Calculate the probability that the car experiences its first failure in the last quarter of some calendar year.

(A)   0.081
(B)   0.088
(C)   0.102
(D)   0.205
(E)   0.250

149. In a shipment of 20 packages, 7 packages are damaged. The packages are randomly inspected, one at a time, without replacement, until the fourth damaged package is discovered.

Calculate the probability that exactly 12 packages are inspected.

(A)   0.079
(B)   0.119
(C)   0.237
(D)   0.243
(E)   0.358

150. A theme park conducts a study of families that visit the park during a year. The fraction of such families of size m is $\dfrac{8-m}{28}$, $m = 1, 2, 3, 4, 5, 6$, and 7.

For a family of size m that visits the park, the number of members of the family that ride the roller coaster follows a discrete uniform distribution on the set $\{1, \ldots, m\}$.

Calculate the probability that a family visiting the park has exactly six members, given that exactly five members of the family ride the roller coaster.

(A)   0.17
(B)   0.21
(C)   0.24
(D)   0.28
(E)   0.31

151. The following information is given about a group of high-risk borrowers.

i)   Of all these borrowers, 30% defaulted on at least one student loan.
ii)   Of the borrowers who defaulted on at least one car loan, 40% defaulted on at least one student loan.
iii)   Of the borrowers who did not default on any student loans, 28% defaulted on at least one car loan.

A statistician randomly selects a borrower from this group and observes that the selected borrower defaulted on at least one student loan.

Calculate the probability that the selected borrower defaulted on at least one car loan.

(A)   0.33
(B)   0.40
(C)   0.44
(D)   0.65
(E)   0.72

152. An insurance company issues policies covering damage to automobiles. The amount of damage is modeled by a uniform distribution on [0, *b*].

The policy payout is subject to a deductible of *b*/10.

A policyholder experiences automobile damage.

Calculate the ratio of the standard deviation of the policy payout to the standard deviation of the amount of the damage.

(A)   0.8100
(B)   0.9000
(C)   0.9477
(D)   0.9487
(E)   0.9735

153. A policyholder purchases automobile insurance for two years. Define the following events:

> F = the policyholder has exactly one accident in year one.
> G = the policyholder has one or more accidents in year two.

Define the following events:

i)   The policyholder has exactly one accident in year one and has more than one accident in year two.
ii)   The policyholder has at least two accidents during the two-year period.
iii)   The policyholder has exactly one accident in year one and has at least one accident in year two.
iv)   The policyholder has exactly one accident in year one and has a total of two or more accidents in the two-year period.
v)   The policyholder has exactly one accident in year one and has more accidents in year two than in year one.

Determine the number of events from the above list of five that are the same as $F \cap G$.

(A)   None
(B)   Exactly one
(C)   Exactly two
(D)   Exactly three
(E)   All

154. An insurance company categorizes its policyholders into three mutually exclusive groups: high-risk, medium-risk, and low-risk. An internal study of the company showed that 45% of the policyholders are low-risk and 35% are medium-risk. The probability of death over the next year, given that a policyholder is high-risk is two times the probability of death of a medium-risk policyholder. The probability of death over the next year, given that a policyholder is medium-risk is three times the probability of death of a low-risk policyholder. The probability of death of a randomly selected policyholder, over the next year, is 0.009.

Calculate the probability of death of a policyholder over the next year, given that the policyholder is high-risk.

(A)   0.0025
(B)   0.0200
(C)   0.1215
(D)   0.2000
(E)   0.3750

155. A policy covers a gas furnace for one year. During that year, only one of three problems can occur:

i)   The igniter switch may need to be replaced at a cost of 60. There is a 0.10 probability of this.
ii)   The pilot light may need to be replaced at a cost of 200. There is a 0.05 probability of this.
iii)   The furnace may need to be replaced at a cost of 3000. There is a 0.01 probability of this.

Calculate the deductible that would produce an expected claim payment of 30.

(A)   100
(B)   At least 100 but less than 150
(C)   At least 150 but less than 200
(D)   At least 200 but less than 250
(E)   At least 250

156. On a block of ten houses, *k* are not insured. A tornado randomly damages three houses on the block.

The probability that none of the damaged houses are insured is 1/120.

Calculate the probability that at most one of the damaged houses is insured.

(A) 1/5
(B) 7/40
(C) 11/60
(D) 49/60
(E) 119/120

157. In a casino game, a gambler selects four different numbers from the first twelve positive integers. The casino then randomly draws nine numbers without replacement from the first twelve positive integers. The gambler wins the jackpot if the casino draws all four of the gambler's selected numbers.

Calculate the probability that the gambler wins the jackpot.

(A) 0.002
(B) 0.255
(C) 0.296
(D) 0.573
(E) 0.625

158. The number of days an employee is sick each month is modeled by a Poisson distribution with mean 1. The numbers of sick days in different months are mutually independent.

Calculate the probability that an employee is sick more than two days in a three-month period.

(A) 0.199
(B) 0.224
(C) 0.423
(D) 0.577
(E) 0.801

159. The number of traffic accidents per week at intersection Q has a Poisson distribution with mean 3. The number of traffic accidents per week at intersection R has a Poisson distribution with mean 1.5.

Let A be the probability that the number of accidents at intersection Q exceeds its mean. Let B be the corresponding probability for intersection R.

Calculate $B - A$.

(A) 0.00
(B) 0.09
(C) 0.13
(D) 0.19
(E) 0.31

160. Losses due to accidents at an amusement park are exponentially distributed. An insurance company offers the park owner two different policies, with different premiums, to insure against losses due to accidents at the park.

Policy A has a deductible of 1.44. For a random loss, the probability is 0.640 that under this policy, the insurer will pay some money to the park owner. Policy B has a deductible of d. For a random loss, the probability is 0.512 that under this policy, the insurer will pay some money to the park owner.

Calculate d.

(A) 0.960
(B) 1.152
(C) 1.728
(D) 1.800
(E) 2.160

161. The distribution of the size of claims paid under an insurance policy has probability density function

$$f(x) = \begin{cases} cx^a, & 0 < x < 5 \\ 0, & \text{otherwise,} \end{cases}$$

Where $a > 0$ and $c > 0$.

For a randomly selected claim, the probability that the size of the claim is less than 3.75 is 0.4871.

Calculate the probability that the size of a randomly selected claim is greater than 4.

(A) 0.404
(B) 0.428
(C) 0.500
(D) 0.572
(E) 0.596

162. Company XYZ provides a warranty on a product that it produces. Each year, the number of warranty claims follows a Poisson distribution with mean c. The probability that no warranty claims are received in any given year is 0.60.

Company XYZ purchases an insurance policy that will reduce its overall warranty claim payment costs. The insurance policy will pay nothing for the first warranty claim received and 5000 for each claim thereafter until the end of the year.

Calculate the expected amount of annual insurance policy payments to Company XYZ.

(A) 554
(B) 872
(C) 1022
(D) 1354
(E) 1612

163. For a certain health insurance policy, losses are uniformly distributed on the interval [0, *b*]. The policy has a deductible of 180 and the expected value of the unreimbursed portion of a loss is 144.

Calculate *b*.

(A) 236
(B) 288
(C) 388
(D) 450
(E) 468

164. The working lifetime, in years, of a particular model of bread maker is normally distributed with mean 10 and variance 4.

Calculate the 12th percentile of the working lifetime, in years.

(A) 5.30
(B) 7.65
(C) 8.41
(D) 12.35
(E) 14.70

165. The profits of life insurance companies A and B are normally distributed with the same mean. The variance of company B's profit is 2.25 times the variance of company A's profit. The 14th percentile of company A's profit is the same as the *p*th percentile of company B's profit.

Calculate *p*.

(A) 5.3
(B) 9.3
(C) 21.0
(D) 23.6
(E) 31.6

166. The distribution of values of the retirement package offered by a company to new employees is modeled by the probability density function

$$f(x) = \begin{cases} \dfrac{1}{5}\, e^{-\frac{1}{5}(x-5)}, & x > 5 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the variance of the retirement package value for a new employee, given that the value is at least 10.

(A) 15
(B) 20
(C) 25
(D) 30
(E) 35

167. Insurance companies A and B each earn an annual profit that is normally distributed with the same positive mean. The standard deviation of company A's annual profit is one half of its mean.

In a given year, the probability that company B has a loss (negative profit) is 0.9 times the probability that company A has a loss.

Calculate the ratio of the standard deviation of company B's annual profit to the standard deviation of company A's annual profit.

(A) 0.49
(B) 0.90
(C) 0.98
(D) 1.11
(E) 1.71

168. Claim amounts at an insurance company are independent of one another. In year one, claim amounts are modeled by a normal random variable X with mean 100 and standard deviation 25. In year two, claim amounts are modeled by the random variable $Y = 1.04X + 5$.

Calculate the probability that a random sample of 25 claim amounts in year two average between 100 and 110.

(A) 0.48
(B) 0.53
(C) 0.54
(D) 0.67
(E) 0.68

169. An insurance company will cover losses incurred from tornadoes in a single calendar year. However, the insurer will only cover losses for a maximum of three separate tornadoes during this timeframe. Let X be the number of tornadoes that result in at least 50 million in losses, and let Y be the total number of tornadoes. The joint probability function for X and Y is

$$p(x, y) = \begin{cases} c(x + 2y), & \text{for } x = 0, 1, 2, 3, \ y = 0, 1, 2, 3, \ x \le y \\ 0, & \text{otherwise,} \end{cases}$$

where c is a constant.

Calculate the expected number of tornadoes that result in fewer than 50 million in losses.

(A) 0.19
(B) 0.28
(C) 0.76
(D) 1.00
(E) 1.10

170. At a polling booth, ballots are cast by ten voters, of whom three are Republicans, two are Democrats, and five are Independents. A local journalist interviews two of these voters, chosen randomly.

Calculate the expectation of the absolute value of the difference between the number of Republicans interviewed and the number of Democrats interviewed.

(A) 1/5
(B) 7/15
(C) 3/5
(D) 11/15
(E) 1

171. The random variables X and Y have joint probability function $p(x,y)$ for $x = 0,1$ and $y = 0,1,2$.

Suppose $3p(1,1) = p(1,2)$, and $p(1,1)$ maximizes the variance of XY.

Calculate the probability that X or Y is 0.

(A) 11/25
(B) 23/50
(C) 23/49
(D) 26/49
(E) 14/25

172. The number of severe storms that strike city J in a year follows a binomial distribution with $n = 5$ and $p = 0.6$. Given that m severe storms strike city J in a year, the number of severe storms that strike city K in the same year is m with probability 1/2, $m+1$ with probability 1/3, and $m+2$ with probability 1/6.

Calculate the expected number of severe storms that strike city J in a year during which 5 severe storms strike city K.

(A) 3.5
(B) 3.7
(C) 3.9
(D) 4.0
(E) 5.7

173. Let N denote the number of accidents occurring during one month on the northbound side of a highway and let S denote the number occurring on the southbound side.

Suppose that N and S are jointly distributed as indicated in the table.

$N \setminus S$	0	1	2	3 or more
0	0.04	0.06	0.10	0.04
1	0.10	0.18	0.08	0.03
2	0.12	0.06	0.05	0.02
3 or more	0.05	0.04	0.02	0.01

Calculate $\mathrm{Var}\,(N \mid N + S = 2)$.

(A)   0.48
(B)   0.55
(C)   0.67
(D)   0.91
(E)   1.25

174. An insurance company has an equal number of claims in each of three territories. In each territory, only three claim amounts are possible: 100, 500, and 1000. Based on the company's data, the probabilities of each claim amount are:

	Claim Amount		
	100	500	1000
Territory 1	0.90	0.08	0.02
Territory 2	0.80	0.11	0.09
Territory 3	0.70	0.20	0.10

Calculate the standard deviation of a randomly selected claim amount.

(A)   254
(B)   291
(C)   332
(D)   368
(E)   396

175. At the start of a week, a coal mine has a high-capacity storage bin that is half full. During the week, 20 loads of coal are added to the storage bin. Each load of coal has a volume that is normally distributed with mean 1.50 cubic yards and standard deviation 0.25 cubic yards.

During the same week, coal is removed from the storage bin and loaded into 4 railroad cars. The amount of coal loaded into each railroad car is normally distributed with mean 7.25 cubic yards and standard deviation 0.50 cubic yards.

The amounts added to the storage bin or removed from the storage bin are mutually independent.

Calculate the probability that the storage bin contains more coal at the end of the week than it had at the beginning of the week.

(A)   0.56
(B)   0.63
(C)   0.67
(D)   0.75
(E)   0.98

176. An insurance company insures a good driver and a bad driver on the same policy. The table below gives the probability of a given number of claims occurring for each of these drivers in the next ten years.

Number of claims	Probability for the good driver	Probability for the bad driver
0	0.5	0.2
1	0.3	0.3
2	0.2	0.4
3	0.0	0.1

The number of claims occurring for the two drivers are independent.

Calculate the mode of the distribution of the total number of claims occurring on this policy in the next ten years.

(A)   0
(B)   1
(C)   2
(D)   3
(E)   4

177. In a group of 15 health insurance policyholders diagnosed with cancer, each policyholder has probability 0.90 of receiving radiation and probability 0.40 of receiving chemotherapy. Radiation and chemotherapy treatments are independent events for each policyholder, and the treatments of different policyholders are mutually independent.

The policyholders in this group all have the same health insurance that pays 2 for radiation treatment and 3 for chemotherapy treatment.

Calculate the variance of the total amount the insurance company pays for the radiation and chemotherapy treatments for these 15 policyholders.

(A)   13.5
(B)   37.8
(C)   108.0
(D)   202.5
(E)   567.0

178. In a large population of patients, 20% have early stage cancer, 10% have advanced stage cancer, and the other 70% do not have cancer. Six patients from this population are randomly selected.

Calculate the expected number of selected patients with advanced stage cancer, given that at least one of the selected patients has early stage cancer.

(A)   0.403
(B)   0.500
(C)   0.547
(D)   0.600
(E)   0.625

179. Four distinct integers are chosen randomly and without replacement from the first twelve positive integers. Let X be the random variable representing the second largest of the four selected integers, and let p be the probability function for X.

Determine $p(x)$, for integer values of x, where $p(x) > 0$.

(A) $\dfrac{(x-1)(x-2)(12-x)}{990}$

(B) $\dfrac{(x-1)(x-2)(12-x)}{495}$

(C) $\dfrac{(x-1)(12-x)(11-x)}{495}$

(D) $\dfrac{(x-1)(12-x)(11-x)}{990}$

(E) $\dfrac{(10-x)(12-x)(11-x)}{990}$

180. An insurance policy covers losses incurred by a policyholder, subject to a deductible of 10,000. Incurred losses follow a normal distribution with mean 12,000 and standard deviation c. The probability that a loss is less than k is 0.9582, where k is a constant. Given that the loss exceeds the deductible, there is a probability of 0.9500 that it is less than k.

Calculate c.

(A)   2045
(B)   2267
(C)   2393
(D)   2505
(E)   2840

181. Losses covered by an insurance policy are modeled by a uniform distribution on the interval [0, 1000]. An insurance company reimburses losses in excess of a deductible of 250.

Calculate the difference between the median and the 20th percentile of the insurance company reimbursement, over all losses.

(A) 225
(B) 250
(C) 300
(D) 375
(E) 500

182. An insurance agent's files reveal the following facts about his policyholders:

    i)   243 own auto insurance.
    ii)   207 own homeowner insurance.
    iii)   55 own life insurance and homeowner insurance.
    iv)   96 own auto insurance and homeowner insurance.
    v)   32 own life insurance, auto insurance and homeowner insurance.
    vi)   76 more clients own only auto insurance than only life insurance.
    vii) 270 own only one of these three insurance products.

Calculate the total number of the agent's policyholders who own at least one of these three insurance products.

(A) 389
(B) 407
(C) 423
(D) 448
(E) 483

183. A profile of the investments owned by an agent's clients follows:

i)   228 own annuities.
ii)   220 own mutual funds.
iii)   98 own life insurance and mutual funds.
iv)   93 own annuities and mutual funds.
v)   16 own annuities, mutual funds, and life insurance.
vi)   45 more clients own only life insurance than own only annuities.
vii)   290 own only one type of investment (i.e., annuity, mutual fund, or life insurance).

Calculate the agent's total number of clients.

(A)   455
(B)   495
(C)   496
(D)   500
(E)   516

184. An actuary compiles the following information from a portfolio of 1000 homeowners insurance policies:

i)   130 policies insure three-bedroom homes.
ii)   280 policies insure one-story homes.
iii)   150 policies insure two-bath homes.
iv)   30 policies insure three-bedroom, two-bath homes.
v)   50 policies insure one-story, two-bath homes.
vi)   40 policies insure three-bedroom, one-story homes.
vii)   10 policies insure three-bedroom, one-story, two-bath homes.

Calculate the number of homeowners policies in the portfolio that insure neither one-story nor two-bath nor three-bedroom homes.

(A)   310
(B)   450
(C)   530
(D)   550
(E)   570

185. Each week, a subcommittee of four individuals is formed from among the members of a committee comprising seven individuals. Two subcommittee members are then assigned to lead the subcommittee, one as chair and the other as secretary.

Calculate the maximum number of consecutive weeks that can elapse without having the subcommittee contain four individuals who have previously served together with the same subcommittee chair.

(A)　　70
(B)　　140
(C)　　210
(D)　　420
(E)　　840

186. Bowl I contains eight red balls and six blue balls. Bowl II is empty. Four balls are selected at random, without replacement, and transferred from bowl I to bowl II. One ball is then selected at random from bowl II.

Calculate the conditional probability that two red balls and two blue balls were transferred from bowl I to bowl II, given that the ball selected from bowl II is blue.

(A)　　0.21
(B)　　0.24
(C)　　0.43
(D)　　0.49
(E)　　0.57

187. An actuary has done an analysis of all policies that cover two cars. 70% of the policies are of type A for both cars, and 30% of the policies are of type B for both cars. The number of claims on different cars across all policies are mutually independent. The distributions of the number of claims on a car are given in the following table.

Number of Claims	Type A	Type B
0	40%	25%
1	30%	25%
2	20%	25%
3	10%	25%

Four policies are selected at random.

Calculate the probability that exactly one of the four policies has the same number of claims on both covered cars.

(A) 0.104
(B) 0.250
(C) 0.285
(D) 0.417
(E) 0.739

188. A company sells two types of life insurance policies (P and Q) and one type of health insurance policy. A survey of potential customers revealed the following:

i) No survey participant wanted to purchase both life policies.
ii) Twice as many survey participants wanted to purchase life policy P as life policy Q.
iii) 45% of survey participants wanted to purchase the health policy.
iv) 18% of survey participants wanted to purchase only the health policy.
v) The event that a survey participant wanted to purchase the health policy was independent of the event that a survey participant wanted to purchase a life policy.

Calculate the probability that a randomly selected survey participant wanted to purchase exactly one policy.

(A) 0.51
(B) 0.60
(C) 0.69
(D) 0.73
(E) 0.78

189. A state is starting a lottery game. To enter this lottery, a player uses a machine that randomly selects six distinct numbers from among the first 30 positive integers. The lottery randomly selects six distinct numbers from the same 30 positive integers. A winning entry must match the same set of six numbers that the lottery selected.

The entry fee is 1, each winning entry receives a prize amount of 500,000, and all other entries receive no prize.

Calculate the probability that the state will lose money, given that 800,000 entries are purchased.

(A)   0.33
(B)   0.39
(C)   0.61
(D)   0.67
(E)   0.74

190. A life insurance company has found there is a 3% probability that a randomly selected application contains an error. Assume applications are mutually independent in this respect.

An auditor randomly selects 100 applications.

Calculate the probability that 95% or less of the selected applications are error-free.

(A)   0.08
(B)   0.10
(C)   0.13
(D)   0.15
(E)   0.18

191. Let A, B, and C be events such that $P[A] = 0.2$, $P[B] = 0.1$, and $P[C] = 0.3$. The events A and B are independent, the events B and C are independent, and the events A and C are mutually exclusive.

Calculate $P[A \cup B \cup C]$.

(A)   0.496
(B)   0.540
(C)   0.544
(D)   0.550
(E)   0.600

192. The annual numbers of thefts a homeowners insurance policyholder experiences are analyzed over three years.

Define the following events:

i)   A = the event that the policyholder experiences no thefts in the three years.
ii)   B = the event that the policyholder experiences at least one theft in the second year.
iii)   C = the event that the policyholder experiences exactly one theft in the first year.
iv)   D = the event that the policyholder experiences no thefts in the third year.
v)   E = the event that the policyholder experiences no thefts in the second year, and at least one theft in the third year.

Determine which three events satisfy the condition that the probability of their union equals the sum of their probabilities.

(A)   Events A, B, and E
(B)   Events A, C, and E
(C)   Events A, D, and E
(D)   Events B, C, and D
(E)   Events B, C, and E

193. Four letters to different insureds are prepared along with accompanying envelopes. The letters are put into the envelopes randomly.

Calculate the probability that at least one letter ends up in its accompanying envelope.

(A) 27/256
(B) 1/4
(C) 11/24
(D) 5/8
(E) 3/4

194. A health insurance policy covers visits to a doctor's office. Each visit costs 100. The annual deductible on the policy is 350. For a policy, the number of visits per year has the following probability distribution:

Number of Visits	0	1	2	3	4	5	6
Probability	0.60	0.15	0.10	0.08	0.04	0.02	0.01

A policy is selected at random from those where costs exceed the deductible.

Calculate the probability that this policyholder had exactly five office visits.

(A) 0.050
(B) 0.133
(C) 0.286
(D) 0.333
(E) 0.429

195. A machine has two parts labelled A and B. The probability that part A works for one year is 0.8 and the probability that part B works for one year is 0.6. The probability that at least one part works for one year is 0.9.

Calculate the probability that part B works for one year, given that part A works for one year.

(A) 1/2
(B) 3/5
(C) 5/8
(D) 3/4
(E) 5/6

196. Six claims are to be randomly selected from a group of thirteen different claims, which includes two workers compensation claims, four homeowners claims and seven auto claims.

Calculate the probability that the six claims selected will include one workers compensation claim, two homeowners claims and three auto claims.

(A)   0.025
(B)   0.107
(C)   0.153
(D)   0.245
(E)   0.643

197. A drawer contains four pairs of socks, with each pair a different color. One sock at a time is randomly drawn from the drawer until a matching pair is obtained.

Calculate the probability that the maximum number of draws is required.

(A)   0.0006
(B)   0.0095
(C)   0.0417
(D)   0.1429
(E)   0.2286

198. At a mortgage company, 60% of calls are answered by an attendant. The remaining 40% of callers leave their phone numbers. Of these 40%, 75% receive a return phone call the same day. The remaining 25% receive a return call the next day.

Of those who initially spoke to an attendant, 80% will apply for a mortgage. Of those who received a return call the same day, 60% will apply. Of those who received a return call the next day, 40% will apply.

Calculate the probability that a person initially spoke to an attendant, given that he or she applied for a mortgage.

(A)   0.06
(B)   0.26
(C)   0.48
(D)   0.60
(E)   0.69

199. An insurance company studies back injury claims from a manufacturing company. The insurance company finds that 40% of workers do no lifting on the job, 50% do moderate lifting and 10% do heavy lifting.

During a given year, the probability of filing a claim is 0.05 for a worker who does no lifting, 0.08 for a worker who does moderate lifting and 0.20 for a worker who does heavy lifting.

A worker is chosen randomly from among those who have filed a back injury claim.

Calculate the probability that the worker's job involves moderate or heavy lifting.

    (A)   0.75
    (B)   0.81
    (C)   0.85
    (D)   0.86
    (E)   0.89

200. The number of traffic accidents occurring on any given day in Coralville is Poisson distributed with mean 5. The probability that any such accident involves an uninsured driver is 0.25, independent of all other such accidents.

Calculate the probability that on a given day in Coralville there are no traffic accidents that involve an uninsured driver.

    (A)   0.007
    (B)   0.010
    (C)   0.124
    (D)   0.237
    (E)   0.287

201. A group of 100 patients is tested, one patient at a time, for three risk factors for a certain disease until either all patients have been tested or a patient tests positive for more than one of these three risk factors. For each risk factor, a patient tests positive with probability p, where $0 < p < 1$. The outcomes of the tests across all patients and all risk factors are mutually independent.

Determine an expression for the probability that exactly n patients are tested, where n is a positive integer less than 100.

(A) $\left[1-3p^2(1-p)\right]^{n-1}\left[3p^2(1-p)\right]$

(B) $\left[1-3p^2(1-p)-p^3\right]^{n-1}\left[3p^2(1-p)+p^3\right]$

(C) $\left[1-3p^2(1-p)-p^3\right]\left[3p^2(1-p)+p^3\right]^{n-1}$

(D) $n\left[1-3p^2(1-p)-p^3\right]^{n-1}\left[3p^2(1-p)+p^3\right]$

(E) $3\left[(1-p)^{n-1}p\right]^2\left[1-(1-p)^{n-1}p\right]+\left[(1-p)^{n-1}p\right]^3$

202. A representative of a market research firm contacts consumers by phone in order to conduct surveys. The specific consumer contacted by each phone call is randomly determined. The probability that a phone call produces a completed survey is 0.25.

Calculate the probability that more than three phone calls are required to produce one completed survey.

(A) 0.32
(B) 0.42
(C) 0.44
(D) 0.56
(E) 0.58

203. Four distinct integers are chosen randomly and without replacement from the first twelve positive integers. X is the random variable representing the second smallest of the four selected integers, and p is the probability function of X.

Determine $p(x)$ for $x = 2,3,\ldots,10$.

(A) $\quad \dfrac{(x-1)(11-x)(12-x)}{495}$

(B) $\quad \dfrac{(x-1)(11-x)(12-x)}{990}$

(C) $\quad \dfrac{(x-1)(x-2)(12-x)}{990}$

(D) $\quad \dfrac{(x-1)(x-2)(12-x)}{495}$

(E) $\quad \dfrac{(10-x)(11-x)(12-x)}{495}$

204. Losses due to burglary are exponentially distributed with mean 100.

The probability that a loss is between 40 and 50 equals the probability that a loss is between 60 and r, with $r > 60$.

Calculate r.

(A) 68.26
(B) 70.00
(C) 70.51
(D) 72.36
(E) 75.00

205. The time until the next car accident for a particular driver is exponentially distributed with a mean of 200 days.

Calculate the probability that the driver has no accidents in the next 365 days, but then has at least one accident in the 365-day period that follows this initial 365-day period.

(A)   0.026
(B)   0.135
(C)   0.161
(D)   0.704
(E)   0.839

206. The annual profit of a life insurance company is normally distributed.

The probability that the annual profit does not exceed 2000 is 0.7642. The probability that the annual profit does not exceed 3000 is 0.9066.

Calculate the probability that the annual profit does not exceed 1000.

(A)   0.1424
(B)   0.3022
(C)   0.5478
(D)   0.6218
(E)   0.7257

207. Individuals purchase both collision and liability insurance on their automobiles. The value of the insured's automobile is V. Assume the loss L on an automobile claim is a random variable with cumulative distribution function

$$F(l) = \begin{cases} \dfrac{3}{4}\left(\dfrac{l}{V}\right)^3, & 0 \le l < V \\[2mm] 1 - \dfrac{1}{10}e^{\frac{-(l-V)}{V}}, & V \le l. \end{cases}$$

Calculate the probability that the loss on a randomly selected claim is greater than the value of the automobile.

(A)   0.00
(B)   0.10
(C)   0.25
(D)   0.75
(E)   0.90

208. The lifetime of a machine part is exponentially distributed with a mean of five years.

Calculate the mean lifetime of the part, given that it survives less than ten years.

(A)   0.865
(B)   1.157
(C)   2.568
(D)   2.970
(E)   3.435

209. Let X be a random variable with density function

$$f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate $P[X \le 0.5 \mid X \le 1.0]$.

(A)   0.433
(B)   0.547
(C)   0.632
(D)   0.731
(E)   0.865

210. Events E and F are independent. $P[E] = 0.84$ and $P[F] = 0.65$.

Calculate the probability that exactly one of the two events occurs.

(A)   0.056
(B)   0.398
(C)   0.546
(D)   0.650
(E)   0.944

211. A flood insurance company determines that N, the number of claims received in a month, is a random variable with $P[N = n] = \dfrac{2}{3^{n+1}}$, for $n = 0, 1, 2, \ldots$. The numbers of claims received in different months are mutually independent.

Calculate the probability that more than three claims will be received during a consecutive two-month period, given that fewer than two claims were received in the first of the two months.

(A)   0.0062
(B)   0.0123
(C)   0.0139
(D)   0.0165
(E)   0.0185

212. Patients in a study are tested for sleep apnea, one at a time, until a patient is found to have this disease. Each patient independently has the same probability of having sleep apnea. Let r represent the probability that at least four patients are tested.

Determine the probability that at least twelve patients are tested given that at least four patients are tested.

(A) $r^{\frac{11}{3}}$

(B) r^3

(C) $r^{\frac{8}{3}}$

(D) r^2

(E) $r^{\frac{1}{3}}$

213. A factory tests 100 light bulbs for defects. The probability that a bulb is defective is 0.02. The occurrences of defects among the light bulbs are mutually independent events.

Calculate the probability that exactly two are defective given that the number of defective bulbs is two or fewer.

(A)   0.133
(B)   0.271
(C)   0.273
(D)   0.404
(E)   0.677

214. A certain town experiences an average of 5 tornadoes in any four year period. The number of years from now until the town experiences its next tornado as well as the number of years between tornados have identical exponential distributions and all such times are mutually independent

Calculate the median number of years from now until the town experiences its next tornado.

(A)   0.55
(B)   0.73
(C)   0.80
(D)   0.87
(E)   1.25

215. Losses under an insurance policy are exponentially distributed with mean 4. The deductible is 1 for each loss.

Calculate the median amount that the insurer pays a policyholder for a loss under the policy.

(A)   1.77
(B)   2.08
(C)   2.12
(D)   2.77
(E)   3.12

216. A company has purchased a policy that will compensate for the loss of revenue due to severe weather events. The policy pays 1000 for each severe weather event in a year after the first two such events in that year. The number of severe weather events per year has a Poisson distribution with mean 1.

Calculate the expected amount paid to this company in one year.

(A)   80
(B)   104
(C)   368
(D)   512
(E)   632

217. A company provides each of its employees with a death benefit of 100. The company purchases insurance that pays the cost of total death benefits in excess of 400 per year. The number of employees who will die during the year is a Poisson random variable with mean 2.

Calculate the expected annual cost to the company of providing the death benefits, excluding the cost of the insurance.

(A)   171
(B)   189
(C)   192
(D)   200
(E)   208

218. The number of burglaries occurring on Burlington Street during a one-year period is Poisson distributed with mean 1.

Calculate the expected number of burglaries on Burlington Street in a one-year period, given that there are at least two burglaries.

(A)   0.63
(B)   2.39
(C)   2.54
(D)   3.00
(E)   3.78

219. For a certain health insurance policy, losses are uniformly distributed on the interval [0, 450]. The policy has a deductible of d and the expected value of the unreimbursed portion of a loss is 56.

Calculate d.

(A)   60
(B)   87
(C)   112
(D)   169
(E)   224

220. A motorist just had an accident. The accident is minor with probability 0.75 and is otherwise major.

Let b be a positive constant. If the accident is minor, then the loss amount follows a uniform distribution on the interval [0, b]. If the accident is major, then the loss amount follows a uniform distribution on the interval [b, 3b].

The median loss amount due to this accident is 672.

Calculate the mean loss amount due to this accident.

(A)   392
(B)   512
(C)   672
(D)   882
(E)   1008

221. An insurance policy will reimburse only one claim per year.

For a random policyholder, there is a 20% probability of no loss in the next year, in which case the claim amount is 0. If a loss occurs in the next year, the claim amount is normally distributed with mean 1000 and standard deviation 400.

Calculate the median claim amount in the next year for a random policyholder.

(A)   663
(B)   790
(C)   873
(D)   994
(E)   1000

222. Losses incurred by a policyholder follow a normal distribution with mean 20,000 and standard deviation 4,500. The policy covers losses, subject to a deductible of 15,000.

Calculate the 95th percentile of losses that exceed the deductible.

(A)   27,400
(B)   27,700
(C)   28,100
(D)   28,400
(E)   28,800

223. A gun shop sells gunpowder. Monthly demand for gunpowder is normally distributed, averages 20 pounds, and has a standard deviation of 2 pounds. The shop manager wishes to stock gunpowder inventory at the beginning of each month so that there is only a 2% chance that the shop will run out of gunpowder (i.e., that demand will exceed inventory) in any given month.

Calculate the amount of gunpowder to stock in inventory, in pounds.

(A)   16
(B)   23
(C)   24
(D)   32
(E)   43

224. A large university will begin a 13-day period during which students may register for that semester's courses. Of those 13 days, the number of elapsed days before a randomly selected student registers has a continuous distribution with density function $f(t)$ that is symmetric about $t = 6.5$ and proportional to $1/(t + 1)$ between days 0 and 6.5.

A student registers at the 60th percentile of this distribution.

Calculate the number of elapsed days in the registration period for this student.

(A)   4.01
(B)   7.80
(C)   8.99
(D)   10.22
(E)   10.51

225. The loss L due to a boat accident is exponentially distributed.

Boat insurance policy A covers up to 1 unit for each loss. Boat insurance policy B covers up to 2 units for each loss.

The probability that a loss is fully covered under policy B is 1.9 times the probability that it is fully covered under policy A.

Calculate the variance of L.

(A)   0.1
(B)   0.4
(C)   2.4
(D)   9.5
(E)   90.1

226. Losses, *X*, under an insurance policy are exponentially distributed with mean 10. For each loss, the claim payment *Y* is equal to the amount of the loss in excess of a deductible $d > 0$.

Calculate Var(*Y*).

(A) 100 - *d*

(B) $(10-d)^2$

(C) $100e^{-d/10}$

(D) $100\left(2e^{-d/10} - e^{-d/5}\right)$

(E) $(10-d)^2\left(2e^{-d/10} - e^{-d/5}\right)$

227. For a certain insurance company, 10% of its policies are Type A, 50% are Type B, and 40% are Type C.

The annual number of claims for an individual Type A, Type B, and Type C policy follow Poisson distributions with respective means 1, 2, and 10.

Let *X* represent the annual number of claims of a randomly selected policy.

Calculate the variance of *X*.

(A) 5.10

(B) 16.09

(C) 21.19

(D) 42.10

(E) 47.20

228. The number of tornadoes in a given year follows a Poisson distribution with mean 3.

Calculate the variance of the number of tornadoes in a year given that at least one tornado occurs.

(A) 1.63

(B) 1.73

(C) 2.66

(D) 3.00

(E) 3.16

229. A government employee's yearly dental expense follows a uniform distribution on the interval from 200 to 1200. The government's primary dental plan reimburses an employee for up to 400 of dental expense incurred in a year, while a supplemental plan pays up to 500 of any remaining dental expense.

Let Y represent the yearly benefit paid by the supplemental plan to a government employee.

Calculate Var(Y).

(A)   20,833
(B)   26,042
(C)   41,042
(D)   53,333
(E)   83,333

230. Under a liability insurance policy, losses are uniformly distributed on $[0, b]$, where b is a positive constant. There is a deductible of $b/2$.

Calculate the ratio of the variance of the claim payment (greater than or equal to zero) from a given loss to the variance of the loss.

(A)   1:8
(B)   3:16
(C)   1:4
(D)   5:16
(E)   1:2

231. A company's annual profit, in billions, has a normal distribution with variance equal to the cube of its mean. The probability of an annual loss is 5%.

Calculate the company's expected annual profit.

(A)   370 million
(B)   520 million
(C)   780 million
(D)   950 million
(E)   1645 million

232. The number of claims X on a health insurance policy is a random variable with $E[X^2]=61$ and $E[(X-1)^2]=47$.

Calculate the standard deviation of the number of claims.

(A)   2.18
(B)   2.40
(C)   7.31
(D)   7.50
(E)   7.81

233. Ten cards from a deck of playing cards are in a box: two diamonds, three spades, and five hearts. Two cards are randomly selected without replacement.

Calculate the variance of the number of diamonds selected, given that no spade is selected.

(A)   0.24
(B)   0.28
(C)   0.32
(D)   0.34
(E)   0.41

234. The number of boating accidents a policyholder experiences this year is modeled by a Poisson random variable with variance 0.10.

An insurer reimburses only the first accident.

Let Y be the number of unreimbursed accidents the policyholder experiences this year and let p be the probability function of Y.

Determine $p(y)$.

(A) $p(y) = \begin{cases} 1.1e^{-0.1}, & y = 0 \\ \dfrac{(0.1)^{y+1}}{(y+1)!}e^{-0.1}, & y = 1,2,3,\ldots \end{cases}$

(B) $p(y) = \begin{cases} 1.1e^{-0.1}, & y = 0 \\ \dfrac{(0.1)^{y}}{y!}e^{-0.1} - 1, & y = 1,2,3,\ldots \end{cases}$

(C) $p(y) = \begin{cases} 1.1e^{-0.1}, & y = 0 \\ \dfrac{(0.1)^{y-1}}{(y-1)!}e^{-0.1}, & y = 1,2,3,\ldots \end{cases}$

(D) $p(y) = \begin{cases} \dfrac{(0.1)^{y+1}}{(y+1)!}e^{-0.1}, & y = 0,1,2,\ldots \end{cases}$

(E) $p(y) = \begin{cases} \dfrac{(0.1)^{y-1}}{(y-1)!}e^{-0.1}, & y = 0,1,2,\ldots \end{cases}$

235. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise,} \end{cases}$$

where $\lambda > 0$.

Let Y be the smallest integer greater than or equal to X.

Determine the probability function of Y.

(A) $\quad 1 - e^{-\lambda y}, \quad y = 1, 2, 3, \ldots$

(B) $\quad e^{-\lambda y}(e^{\lambda} - 1), \quad y = 1, 2, 3, \ldots$

(C) $\quad e^{-\lambda y}(1 - e^{-\lambda}), \quad y = 1, 2, 3, \ldots$

(D) $\quad \lambda e^{-\lambda y}, \quad y = 1, 2, 3, \ldots$

(E) $\quad \dfrac{e^{-\lambda} \lambda^{y}}{y!}, \quad y = 1, 2, 3, \ldots$

236. Batteries A and B have lifetimes that are independent and exponentially distributed with a common mean of m years. The probability that battery B outlasts battery A by more than one year is 0.33.

Calculate m.

(A) 0.42
(B) 0.59
(C) 0.90
(D) 1.80
(E) 2.41

237. A car and a bus arrive at a railroad crossing at times independently and uniformly distributed between 7:15 and 7:30. A train arrives at the crossing at 7:20 and halts traffic at the crossing for five minutes.

Calculate the probability that the waiting time of the car or the bus at the crossing exceeds three minutes.

(A) 0.25
(B) 0.27
(C) 0.36
(D) 0.40
(E) 0.56

238. Skateboarders A and B practice one difficult stunt until becoming injured while attempting the stunt. On each attempt, the probability of becoming injured is p, independent of the outcomes of all previous attempts.

Let $F(x, y)$ represent the probability that skateboarders A and B make no more than x and y attempts, respectively, where x and y are positive integers.

It is given that $F(2, 2) = 0.0441$.

Calculate $F(1, 5)$.

(A) 0.0093
(B) 0.0216
(C) 0.0495
(D) 0.0551
(E) 0.1112

239. The number of minor surgeries, X, and the number of major surgeries, Y, for a policyholder, this decade, has joint cumulative distribution function

$$F(x, y) = \left[1 - (0.5)^{x+1}\right]\left[1 - (0.2)^{y+1}\right],$$

for nonnegative integers x and y.

Calculate the probability that the policyholder experiences exactly three minor surgeries and exactly three major surgeries this decade.

(A)   0.00004
(B)   0.00040
(C)   0.03244
(D)   0.06800
(E)   0.12440

240. A company provides a death benefit of 50,000 for each of its 1000 employees. There is a 1.4% chance that any one employee will die next year, independent of all other employees. The company establishes a fund such that the probability is at least 0.99 that the fund will cover next year's death benefits.

Calculate, using the Central Limit Theorem, the smallest amount of money, rounded to the nearest 50 thousand, that the company must put into the fund.

(A)   750,000
(B)   850,000
(C)   1,050,000
(D)   1,150,000
(E)   1,400,000

241. An investor invests 100 dollars in a stock. Each month, the investment has probability 0.5 of increasing by 1.10 dollars and probability 0.5 of decreasing by 0.90 dollars. The changes in price in different months are mutually independent.

Calculate the probability that the investment has a value greater than 91 dollars at the end of month 100.

(A) 0.63
(B) 0.75
(C) 0.82
(D) 0.94
(E) 0.97

242. Let X denote the loss amount sustained by an insurance company's policyholder in an auto collision. Let Z denote the portion of X that the insurance company will have to pay. An actuary determines that X and Z are independent with respective density and probability functions

$$f(x) = \begin{cases} (1/8)e^{-x/8}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

and

$$P[Z = z] = \begin{cases} 0.45, & z = 1 \\ 0.55, & z = 0. \end{cases}$$

Calculate the variance of the insurance company's claim payment ZX.

(A) 13.0
(B) 15.8
(C) 28.8
(D) 35.2
(E) 44.6

243. A couple takes out a medical insurance policy that reimburses them for days of work missed due to illness. Let X and Y denote the number of days missed during a given month by the wife and husband, respectively. The policy pays a monthly benefit of 50 times the maximum of X and Y, subject to a benefit limit of 100. X and Y are independent, each with a discrete uniform distribution on the set $\{0,1,2,3,4\}$.

Calculate the expected monthly benefit for missed days of work that is paid to the couple.

(A) 70
(B) 90
(C) 92
(D) 95
(E) 140

244. The table below shows the joint probability function of a sailor's number of boating accidents and number of hospitalizations from these accidents this year.

		Number of Hospitalizations from Accidents			
		0	1	2	3
Number of Accidents	0	0.700			
	1	0.150	0.050		
	2	0.060	0.020	0.010	
	3	0.005	0.002	0.002	0.001

Calculate the sailor's expected number of hospitalizations from boating accidents this year.

(A) 0.085
(B) 0.099
(C) 0.410
(D) 1.000
(E) 1.500

245. On Main Street, a driver's speed just before an accident is uniformly distributed on [5, 20]. Given the speed, the resulting loss from the accident is exponentially distributed with mean equal to three times the speed.

Calculate the variance of a loss due to an accident on Main Street.

(A) 525
(B) 1463
(C) 1575
(D) 1632
(E) 1744

246. Let X be the annual number of hurricanes hitting Florida, and let Y be the annual number of hurricanes hitting Texas. X and Y are independent Poisson variables with respective means 1.70 and 2.30.

Calculate $\text{Var}(X - Y \mid X + Y = 3)$.

(A) 1.71
(B) 1.77
(C) 2.93
(D) 3.14
(E) 4.00

247. Random variables X and Y have joint distribution

	$X = 0$	$X = 1$	$X = 2$
$Y = 0$	1/15	a	2/15
$Y = 1$	a	b	a
$Y = 2$	2/15	a	1/15

Let a be the value that minimizes the variance of X.

Calculate the variance of Y.

(A) 2/5
(B) 8/15
(C) 16/25
(D) 2/3
(E) 7/10

248. Let X be a random variable that takes on the values -1, 0, and 1 with equal probabilities.

Let $Y = X^2$.

Which of the following is true?

    (A) $\text{Cov}(X, Y) > 0$; the random variables X and Y are dependent.
    (B) $\text{Cov}(X, Y) > 0$; the random variables X and Y are independent.
    (C) $\text{Cov}(X, Y) = 0$; the random variables X and Y are dependent.
    (D) $\text{Cov}(X, Y) = 0$; the random variables X and Y are independent.
    (E) $\text{Cov}(X, Y) < 0$; the random variables X and Y are dependent.

249. Losses follow an exponential distribution with mean 1. Two independent losses are observed.

Calculate the expected value of the smaller loss.

    (A)   0.25
    (B)   0.50
    (C)   0.75
    (D)   1.00
    (E)   1.50

250. A delivery service owns two cars that consume 15 and 30 miles per gallon. Fuel costs 3 per gallon. On any given business day, each car travels a number of miles that is independent of the other and is normally distributed with mean 25 miles and standard deviation 3 miles.

Calculate the probability that on any given business day, the total fuel cost to the delivery service will be less than 7.

    (A)   0.13
    (B)   0.23
    (C)   0.29
    (D)   0.38
    (E)   0.47

251. Two independent estimates are to be made on a building damaged by fire. Each estimate is normally distributed with mean $10b$ and variance b^2.

Calculate the probability that the first estimate is at least 20 percent higher than the second.

(A) 0.023
(B) 0.100
(C) 0.115
(D) 0.221
(E) 0.444

252. The independent random variables X and Y have the same mean. The coefficients of variation of X and Y are 3 and 4 respectively.

Calculate the coefficient of variation of $\frac{1}{2}(X+Y)$.

(A) 5/4
(B) 7/4
(C) 5/2
(D) 7/2
(E) 7

253. Points scored by a game participant can be modeled by $Z = 3X + 2Y - 5$. X and Y are independent random variables with Var $(X) = 3$ and Var $(Y) = 4$.

Calculate Var (Z).

(A) 12
(B) 17
(C) 38
(D) 43
(E) 68

254. An actuary is studying hurricane models. A year is classified as a high, medium, or low hurricane year with probabilities 0.1, 0.3, and 0.6, respectively. The numbers of hurricanes in high, medium, and low years follow Poisson distributions with means 20, 15, and 10, respectively.

Calculate the variance of the number of hurricanes in a randomly selected year.

(A)   11.25
(B)   12.50
(C)   12.94
(D)   13.42
(E)   23.75

255. A dental insurance company pays 100% of the cost of fillings and 70% of the cost of root canals. Fillings and root canals cost 50 and 500 each, respectively.

The tables below show the probability distributions of the annual number of fillings and annual number of root canals for each of the company's policyholders.

# of Fillings	0	1	2	3
Probability	0.60	0.20	0.15	0.05

# of Root Canals	0	1
Probability	0.80	0.20

Calculate the expected annual payment per policyholder for fillings and root canals.

(A)   90.00
(B)   102.50
(C)   132.50
(D)   250.00
(E)   400.00

256. A loss under a liability policy is modeled by an exponential distribution. The insurance company will cover the amount of that loss in excess of a deductible of 2000. The probability that the reimbursement is less than 6000, given that the loss exceeds the deductible, is 0.50.

Calculate the probability that the reimbursement is greater than 3000 but less than 9000, given that the loss exceeds the deductible.

(A)   0.28
(B)   0.35
(C)   0.50
(D)   0.65
(E)   0.72

257. Let X be the percentage score on a college-entrance exam for students who did not participate in an exam-preparation seminar. X is modeled by a uniform distribution on $[a, 100]$.

Let Y be the percentage score on a college-entrance exam for students who did participate in an exam-preparation seminar. Y is modeled by a uniform distribution on $[1.25a, 100]$.

It is given that $E(X^2) = \dfrac{19,600}{3}$.

Calculate the 80th percentile of Y.

(A)   80
(B)   85
(C)   90
(D)   92
(E)   95

258. In a study of driver safety, drivers were categorized according to three risk factors. Exactly 1000 drivers exhibited each individual risk factor. Also, for each of the risk factors, there were exactly 400 drivers exhibiting that risk factor and neither of the other two risk factors. Finally, there were exactly 300 drivers who exhibited all three risk factors and 500 who exhibited none of the three risk factors.

Calculate the number of drivers in the study.

(A) 2000
(B) 2300
(C) 2450
(D) 2750
(E) 3500

259. An insurance company examines its pool of auto insurance customers and gathers the following information:

i)   All customers insure at least one car.
ii)   64% of the customers insure more than one car.
iii) 20% of the customers insure a sports car.
iv)   Of those customers who insure more than one car, 15% insure a sports car.

Calculate the probability that a randomly selected customer insures exactly one car, and that the car is not a sports car.

(A) 0.16
(B) 0.19
(C) 0.26
(D) 0.29
(E) 0.31

260. An insurance company has found that 1% of all applicants for life insurance have diabetes.

Calculate the probability that five or fewer of 200 randomly selected applicants have diabetes.

(A)   0.85
(B)   0.88
(C)   0.91
(D)   0.95
(E)   0.98

261. The probability that an agent sells an insurance policy to a potential customer during a first appointment is 0.20. The events of selling an insurance policy to different potential customers during first appointments are mutually independent.

The agent has scheduled first appointments with five potential customers.

Calculate the probability that the agent sells an insurance policy during at least two of these appointments.

(A)   0.04
(B)   0.20
(C)   0.26
(D)   0.40
(E)   0.74

262. A manufacturer produces computers and releases them in shipments of 100. From a shipment of 100, the probability that exactly three computers are defective is twice the probability that exactly two computers are defective. The events that different computers are defective are mutually independent.

Calculate the probability that a randomly selected computer is defective.

(A)   0.040
(B)   0.042
(C)   0.058
(D)   0.060
(E)   0.072

263. In any 12-month period, the probability that a home is damaged by fire is 20% and the probability of a theft loss at a home is 30%. The occurrences of fire damage and theft loss are independent events.

Calculate the probability that a randomly selected home will either be damaged by fire or will have a theft loss, but not both, during the next year.

(A)   0.30
(B)   0.38
(C)   0.44
(D)   0.50
(E)   0.56

264. In one company, 30% of males and 20% of females contribute to a supplemental retirement plan. Furthermore, 45% of the company's employees are female.

Calculate the probability that a randomly selected employee is female, given that this employee contributes to a supplemental retirement plan.

(A)   0.09
(B)   0.23
(C)   0.35
(D)   0.45
(E)   0.55

265. A health insurer sells policies to residents of territory X and territory Y. Past claims experience indicates the following:

i)   20% of the total policyholders from territory X and territory Y combined filed no claims.
ii)   15% of the policyholders from territory X filed no claims.
iii)   40% of the policyholders from territory Y filed no claims.

Calculate the probability that a randomly selected policyholder was a resident of territory X, given that the policyholder filed no claims.

(A)   0.09
(B)   0.27
(C)   0.50
(D)   0.60
(E)   0.80

266. Claim amounts are independent random variables with probability density function

$$f(x) = \begin{cases} \dfrac{10}{x^2}, & \text{for } x > 10 \\[2em] 0, & \text{otherwise.} \end{cases}$$

Calculate the probability that the largest of three randomly selected claims is less than 25.

(A) $\dfrac{8}{125}$

(B) $\dfrac{12}{125}$

(C) $\dfrac{27}{125}$

(D) $\dfrac{2}{5}$

(E) $\dfrac{3}{5}$

267. The lifetime of a certain electronic device has an exponential distribution with mean 0.50.

Calculate the probability that the lifetime of the device is greater than 0.70, given that it is greater than 0.40.

(A)   0.203
(B)   0.247
(C)   0.449
(D)   0.549
(E)   0.861

268. A farmer purchases a five-year insurance policy that covers crop destruction due to hail. Over the five-year period, the farmer will receive a benefit of 20 for each year in which hail destroys his crop, subject to a maximum of three benefit payments. The probability that hail will destroy the farmer's crop in any given year is 0.5, independent of any other year.

Calculate the expected benefit that the farmer will receive over the five-year period.

(A)   30
(B)   34
(C)   40
(D)   46
(E)   50

269. An insurance company has two divisions, auto and property. Total annual claims, X, in the auto division follow a normal distribution with mean 10 and standard deviation 3. Total annual claims, Y, in the property division follow a normal distribution with mean 12 and standard deviation 4.

Assume that X and Y are independent.

Calculate the probability that total overall claims, $X + Y$, will not exceed 29.

(A)   0.61
(B)   0.69
(C)   0.78
(D)   0.84
(E)   0.92

270. An industrial company provides health insurance to employees located at four different plants. Health insurance costs at each plant are independent of the costs at any other plant. Plant managers have calculated the following statistics:

Plant	Average Cost	Standard Deviation
W	2	1.0
X	2	1.0
Y	5	1.5
Z	7	2.0

Calculate the standard deviation of total company health insurance costs.

(A) 1.4
(B) 2.1
(C) 2.9
(D) 5.5
(E) 6.3

271. In one company, 30% of men and 20% of women contribute to a supplemental retirement plan. Furthermore, 45% of the company's employees are women.

Calculate the probability that a randomly selected employee is a woman, given that this employee contributes to a supplemental retirement plan.

(A) 0.09
(B) 0.23
(C) 0.35
(D) 0.45
(E) 0.55

272. An insurance company has found that 1% of all applicants for life insurance have diabetes.

Calculate the probability that five or fewer of 200 randomly selected applicants have diabetes.

(A)   0.85
(B)   0.88
(C)   0.91
(D)   0.95
(E)   0.98

273. An industrial company provides health insurance to employees located at four different plants. Health insurance costs at each plant are independent of the costs at any other plant. Plant managers have calculated the following statistics:

Plant	Average Cost	Standard Deviation
P_1	2	1.0
P_2	2	1.0
P_3	5	1.5
P_4	7	2.0

Calculate the standard deviation of total company health insurance costs.

(A)   1.4
(B)   2.1
(C)   2.9
(D)   5.5
(E)   6.3

274. An insurance company examines its pool of auto insurance customers and gathers the following information:

    i)   All customers insure at least one car.
    ii)   62% of the customers insure more than one car.
    iii)   15% of the customers insure a sports car.
    iv)   Of those customers who insure more than one car, 20% insure a sports car.

Calculate the probability that a randomly selected customer insures exactly one car, and that the car is not a sports car.

    (A)   0.230
    (B)   0.260
    (C)   0.323
    (D)   0.354
    (E)   0.380

275. An insurance company examines its pool of auto insurance customers and gathers the following information:

    i)   All customers insure at least one car.
    ii)   64% of the customers insure more than one car.
    iii)   20% of the customers insure a sports car.
    iv)   Of those customers who insure more than one car, 15% insure a sports car.

Calculate the probability that a randomly selected customer insures exactly one car, and that the car is not a sports car.

    (A)   0.16
    (B)   0.19
    (C)   0.26
    (D)   0.29
    (E)   0.31

276. An insurance company has two divisions, auto and property. Total annual claims, X, in the auto division follow a normal distribution with mean 10 and standard deviation 3. Total annual claims, Y, in the property division follow a normal distribution with mean 12 and standard deviation 4.

Assume that X and Y are independent.

Calculate the probability that total overall claims, $X + Y$, will not exceed 29.

(A) 0.61
(B) 0.69
(C) 0.78
(D) 0.84
(E) 0.92

277. In a study of driver safety, drivers were categorized according to three risk factors. Exactly 1000 drivers exhibited each individual risk factor. Also, for each of the risk factors, there were exactly 400 drivers exhibiting that risk factor and neither of the other two risk factors. Finally, there were exactly 300 drivers who exhibited all three risk factors and 500 who exhibited none of the three risk factors.

Calculate the number of drivers in the study.

(A) 2000
(B) 2300
(C) 2450
(D) 2750
(E) 3500

278. A farmer purchases a five-year insurance policy that covers crop destruction due to hail. Over the five-year period, the farmer will receive a benefit of 20 for each year in which hail destroys his crop, subject to a maximum of three benefit payments. The probability that hail will destroy the farmer's crop in any given year is 0.5, independent of any other year.

Calculate the expected benefit that the farmer will receive over the five-year period.

(A)   30
(B)   34
(C)   40
(D)   46
(E)   50

279. The lifetime of a certain electronic device has an exponential distribution with mean 0.50.

Calculate the probability that the lifetime of the device is greater than 0.70, given that it is greater than 0.40.

(A)   0.203
(B)   0.247
(C)   0.449
(D)   0.549
(E)   0.861

280. The probability that an agent sells an insurance policy to a potential customer during a first appointment is 0.20. The events of selling an insurance policy to different potential customers during first appointments are mutually independent.

The agent has scheduled first appointments with five potential customers.

Calculate the probability that the agent sells an insurance policy during at least two of these appointments.

(A)   0.04
(B)   0.20
(C)   0.26
(D)   0.40
(E)   0.74

281. A health insurer sells policies to residents of territory A and territory B. Past claims experience indicates the following:

i) 20% of the total policyholders from territory A and territory B combined filed no claims.

ii) 15% of the policyholders from territory A filed no claims.

iii) 40% of the policyholders from territory B filed no claims.

Calculate the probability that a randomly selected policyholder was a resident of territory A, given that the policyholder filed no claims.

(A) 0.09
(B) 0.27
(C) 0.50
(D) 0.60
(E) 0.80

282. Claim amounts are independent random variables with probability density function

$$f(x) = \begin{cases} \dfrac{10}{x^2}, & \text{for } x > 10 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the probability that the largest of three randomly selected claims is less than 25.

(A) 0.064
(B) 0.096
(C) 0.216
(D) 0.400
(E) 0.600

283. In any twelve-month period, the probability that a home is damaged by fire is 20% and the probability of a theft loss at a home is 30%. The occurrences of fire damage and theft loss are independent events.

Calculate the probability that a randomly selected home will either be damaged by fire or will have a theft loss, but not both, during the next year.

(A) 0.30
(B) 0.38
(C) 0.44
(D) 0.50
(E) 0.56

284. An employer provides disability benefits to its employees for work-related and other injuries. The random variables X and Y denote the employer's annual expenditures for work-related and other injuries, respectively. An actuarial study reveals the following information about X and Y:

i)   The density of X is $f(x) = \dfrac{1}{20\sqrt{5}} e^{\frac{-x}{20\sqrt{5}}}$, for $x > 0$.

ii)   Var $(Y) = 12{,}500$.

iii)   The correlation between X and Y is 0.20.

Calculate the variance of the employer's total expenditures for work-related and other injuries.

(A) 12,500
(B) 13,500
(C) 15,500
(D) 16,500
(E) 18,972

285. Appraisals of the value of a necklace are uniformly distributed on the interval $[\theta - 3, \theta + 1]$, where θ is the actual price the owner paid for the necklace. Four mutually independent appraisals are obtained.

Let L denote the lowest of the four appraisals and H the highest.

Calculate $P[L < \theta < H]$.

(A)   0.152
(B)   0.188
(C)   0.600
(D)   0.680
(E)   0.996

286, Losses follow an exponential distribution with mean 1. Two independent losses are observed.

Calculate the probability that either of the losses is more than twice the other.

(A) $\dfrac{1}{6}$

(B) $\dfrac{1}{4}$

(C) $\dfrac{1}{3}$

(D) $\dfrac{1}{2}$

(E) $\dfrac{2}{3}$

287. A manufacturer produces computers and releases them in shipments of 100. From a shipment of 100, the probability that exactly three computers are defective is twice the probability that exactly two computers are defective. The events that different computers are defective are mutually independent.

Calculate the probability that a randomly selected computer is defective.

(A)   0.040
(B)   0.042
(C)   0.058
(D)   0.060
(E)   0.072

288. For a pregnant woman, a certain test will give the outcome "not pregnant" with probability 0.10. For a non-pregnant woman, the test will give the outcome "pregnant" with probability 0.20. Of women who take the test, 20% are pregnant.

Calculate the probability that a woman is pregnant, given her test outcome is "pregnant."

(A)   0.10
(B)   0.20
(C)   0.50
(D)   0.53
(E)   0.90

289. An airport owner purchases an insurance policy to offset costs associated with excessive amounts of snowfall. For every full ten inches of snow in excess of 40 inches during the winter season, the insurer pays the airport 200 up to a policy maximum of 500.

The following table shows a probability function for the random variable X of winter season snowfall, in inches, at the airport.

Inches of Snowfall (x)	$p(x)$
$0 \le x < 20$	0.06
$20 \le x < 30$	0.18
$30 \le x < 40$	0.26
$40 \le x < 50$	0.22
$50 \le x < 60$	0.14
$60 \le x < 70$	0.06
$70 \le x < 80$	0.04
$80 \le x < 90$	0.04
$90 \le x$	0.00

Calculate the standard deviation of the amount paid under the policy.

(A)   163.5
(B)   187.6
(C)   208.7
(D)   234.9
(E)   336.6

290. Let $X_1, \ldots, X_{100}$ be independent identically distributed random variables such that $P[X = 0] = P[X = 2] = 0.5$. Let $S = X_1 + \cdots + X_{100}$.

Calculate the approximate value of $P[S > 115]$.

(A)   0.005
(B)   0.067
(C)   0.144
(D)   0.147
(E)   0.440

291. Let X and Y be discrete random variables with joint probability function

$$p(x, y) = \begin{cases} 0.250, & \text{for } x = 0, y = 0 \\ 0.250, & \text{for } x = 1, y = 0 \\ 0.125, & \text{for } x = 0, y = 1 \\ 0.375, & \text{for } x = 1, y = 1. \end{cases}$$

Calculate Corr (X, Y), the correlation coefficient of X and Y.

(A)   0.06
(B)   0.23
(C)   0.26
(D)   0.38
(E)   0.63

292. Let X and Y be discrete random variables with joint probability function

$$p(x, y) = \begin{cases} \dfrac{1}{21}, & \text{for } x = 0,1,...,5 \text{ and } y = 0,..., x \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the variance of Y.

(A)   1.67
(B)   2.22
(C)   3.33
(D)   5.00
(E)   5.56

293. A company provides disability benefits to its employees. There are only two possible benefits: partial disability, costing the company 40, and total disability, costing the company 200. The company employs a number of married couples.

Let (X,Y) denote the company's disability costs for a randomly selected employed married couple. The joint probability function for (X,Y) is:

		x		
$p(x,y)$		0	40	200
	0	0.9729	0.0100	0.0020
y	40	0.0100	0.0020	0.0005
	200	0.0020	0.0005	0.0001

Calculate the standard deviation of the total disability cost $X + Y$ for the married couple.

(A) 12.3
(B) 13.8
(C) 15.7
(D) 16.6
(E) 19.8

294. The probability that the economy will improve, remain stable, or decline is:

State of the Economy	Probability
Improve	0.30
Remain stable	0.50
Decline	0.20

Prices for Stock X and Stock Y will change as follows:

State of the Economy	Stock X	Stock Y
Improve	Increase 18%	Increase 15%
Remain stable	Increase 8%	Increase 7%
Decline	Decrease 13%	Decrease 6%

Determine which of the following statements about the percentage price changes for Stock X and Stock Y is true.

(A) The percentage change for Stock X has a larger variance and a larger mean.
(B) The percentage change for Stock X has a larger variance and the means are equal.
(C) The percentage change for Stock X has a larger variance and a smaller mean.
(D) The variances are equal and the percentage change for Stock X has a larger mean.
(E) Both the variances and the means are equal.

295. A company is marketing an investment opportunity to four potential customers. The company believes that its probability of making a sale is 0.5 for each of the first three customers but that it is only 0.1 for the fourth customer. The customers' purchases are independent of one another.

Calculate the probability that at most two customers purchase the investment.

(A) 0.38
(B) 0.46
(C) 0.54
(D) 0.84
(E)   0.90

296. An actuary compiles the following information about a portfolio of life insurance policies:
i)   There are 150 more policies on males than there are on females.
ii)   There are 100 more policies on female nonsmokers than there are on male nonsmokers.
iii)   There are 350 policies on smokers.

Calculate the number of policies on female smokers within this portfolio.

(A)   50
(B)   100
(C)   200
(D)   250
(E)   300

297. The lifetime of a machine part has a continuous distribution on the interval $(0, 40)$ with probability density function f, where $f(x)$ is proportional to $(10 + x)^{-2}$.

Calculate the probability that the lifetime of the machine part is less than five.

(A)   0.03
(B)   0.13
(C)   0.42
(D)   0.58
(E)   0.97

298. The claim, *X*, for a dental insurance policy is a random variable with the following probability function:

x	P[X = x]
0	0.5
1	0.2
2	0.3

The premium for the policy is equal to 125% of the expected claim amount.

Calculate the approximate probability that the total claims on 76 independent policies exceed the total premium collected.

(A)   0.02
(B)   0.07
(C)   0.17
(D)   0.25
(E)   0.40

299. An insurance company categorizes its policyholders into three mutually exclusive groups. A study produced the following information:

Group	Number of policyholders	Probability a policyholder has no claims
A	20,000	70%
B	45,000	90%
C	35,000	50%

Within each group, the numbers of claims made by individual policyholders are mutually independent Poisson random variables.

Calculate the expected total number of claims, in thousands, made by the 100,000 policyholders.

(A)   21
(B)   28
(C)   36
(D)   64
(E)   72

300. A group of insurance policies have all been in force for at least three years. The insurance company plans to pay a dividend on each policy in the group that had no more than one claim incurred on it in the past three years. The number of claims incurred on a policy in any year follows a Poisson distribution with mean 0.288 and the number incurred in any year is independent of the number incurred in any other year.

Calculate the probability that a policy chosen at random from the group will receive a dividend.

(A)   0.01
(B)   0.36
(C)   0.42
(D)   0.54
(E)   0.79

301. An insurance company sells a one-year insurance policy that covers fire and theft losses. The variance of the number of fire losses is 5. The variance of the number of theft losses is 8. The covariance between the number of fire and theft losses is 3.

Calculate the variance of the total number of fire and theft losses covered by this policy.

(A)   7
(B)   10
(C)   13
(D)   16
(E)   19

302. The number of automobile accidents on any day in a city has a Poisson distribution with mean 4. The number of accidents on a given day is independent of the number of accidents on any other day.

Calculate the probability that at most one accident occurs in a three-day period.

(A) $13e^{-12}$
(B) $72e^{-12}$
(C) $85e^{-12}$
(D) $5e^{-4}$
(E) $13e^{-4}$

303. An experiment consists of tossing three fair coins and is deemed a success if the result is three heads or three tails. The experiment is repeated until a success occurs.

Calculate the probability that it takes exactly three experiments to obtain a success.

(A)   0.047
(B)   0.070
(C)   0.141
(D)   0.188
(E)   0.422

304. Companies P, Q, and R use routes that take their trucks through a common inspection checkpoint each day. The number of trucks for each company that pass the checkpoint each day is as follows:

Company	Number of Trucks
P	4
Q	3
R	2
Total	9

Calculate the probability that at least one of two randomly chosen trucks
is from Company P.

(A)   0.28
(B)   0.31
(C)   0.56
(D)   0.69
(E)   0.72

305. A company administers a typing test to screen applicants for a secretarial position. In order to pass the test, an applicant must complete the test in 50 minutes with no more than one error. Historical data reveals the following about the population of applicants:

i) The number of test errors follows a Poisson distribution with mean 3.
ii) The time required to complete the test follows a normal distribution with mean 45 and standard deviation 10.
iii) The number of errors and the time required to complete the test are independent.

Calculate the probability that an applicant chosen at random will pass the test.

(A) 0.10
(B) 0.14
(C) 0.19
(D) 0.84
(E) 0.89

306. An insurance company sells 40% of its renters policies to home renters and the remaining 60% to apartment renters. Among home renters, the time from policy purchase until policy cancellation has an exponential distribution with mean 4 years, and among apartment renters, it has an exponential distribution with mean 2 years.

Calculate the probability that the policyholder is a home renter, given that a renter still has a policy one year after purchase.

(A) 0.08
(B) 0.27
(C) 0.46
(D) 0.56
(E) 0.66

307. A company sells insurance policies for which benefit payments made to each policyholder are independently and identically normally distributed with mean 2475 and standard deviation 250.

Calculate the minimum number of policies that must be sold for there to be at least a 99% probability that the average benefit paid per policy will be no greater than 2500.

(A)   24
(B)   542
(C)   664
(D)   5815
(E)   6440

308. A life insurance policy pays 1000 upon the death of a policyholder provided that the policyholder survives at least one year but less than five years after purchasing the policy.

Let X denote the number of years that a policyholder survives after purchasing the policy with the following probabilities:

x	$P[X < x]$
1	0.05
2	0.12
3	0.21
4	0.33
5	0.48

Calculate the standard deviation of the payment made under this policy.

(A)   218
(B)   430
(C)   480
(D)   495
(E)   500

309. An insurer divides a city into three zones and assesses risks associated with fire loss as follows:

Zone	Probability of fire loss for a home in a given year	Percentage of insurer's fire policies
A	0.015	40%
B	0.011	35%
C	0.008	25%

Given that a fire loss occurs in a home covered by the insurer, calculate the probability that the home is in Zone A.

(A)   0.349
(B)   0.400
(C)   0.441
(D)   0.465
(E)   0.506

310. An insurer offers policies for which insured loss amounts follow a distribution with density function

$$f(x)=\begin{cases} \dfrac{x}{50}, & \text{for } 0 < x < 10 \\ 0, & \text{otherwise.} \end{cases}$$

Customers may choose one of two policies. Policy 1 has no deductible and a limit of 4, while Policy 2 has a deductible of 4 and no limit.

Given the occurrence of an insured loss, calculate the absolute value of the difference between the insurer's expected claim payments under Policies 1 and 2.

(A)   0.32
(B)   0.64
(C)   0.79
(D)   0.91
(E)   1.12

311. Employees of a large company all choose one of three levels of health insurance coverage, for which premiums, denoted by X, are 1, 2, and 3, respectively. Premiums are subject to a discount, denoted by Y, of 0 for smokers and 1 for non-smokers. The joint probability function of X and Y is given by

$$p(x, y) = \begin{cases} \dfrac{x^2 + y^2}{31}, & \text{for } x = 1,\ 2,\ 3 \text{ and } y = 0,\ 1 \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the variance of $X - Y$, the total premium paid by a randomly chosen employee.

(A)   0.20
(B)   0.69
(C)   0.74
(D)   1.90
(E)   2.65

312. An actuary determines the following regarding an individual auto policyholder:

i)   The probability that the auto policyholder will file a medical claim is 0.30.

ii)   The probability that the auto policyholder will file a property claim is 0.42.

iii)   The probability that the auto policyholder will file a medical claim or a property claim is 0.60.

Calculate the probability that the auto policyholder will file exactly one type of claim, given that the policyholder will not file both types of claims.

(A)   0.45
(B)   0.48
(C)   0.52
(D)   0.55
(E)   0.60

313. The probability that a homeowners policyholder reports a property claim in a year increases by 25% per year. Conversely, the probability that a homeowners policyholder reports a liability claim in a year decreases by 25% per year.

The probability that a homeowners policyholder reports both a property claim and a liability claim in Year 1 is 0.01. The event that a homeowners policyholder reports a property claim is independent of the event that the policyholder reports a liability claim.

Calculate the probability that a homeowners policyholder reports both a property claim and a liability claim in Year 9.

(A)   0.005
(B)   0.006
(C)   0.010
(D)   0.014
(E)   0.015

314. An auto insurance company tracks the experience of its first-year and multi-year policyholders separately. First-year policyholders account for 15% of the company's business while multi-year policyholders account for the rest.

The number of claims reported to the company in a year by a first-year policyholder follows a Poisson distribution with mean 0.50, while the number of claims reported to the company in a year by a multi-year policyholder follows a Poisson distribution with mean 0.20.

Calculate the probability that a policyholder is a first-year policyholder, given that the policyholder reports at least one claim in a year to the company.

(A)   0.246
(B)   0.277
(C)   0.306
(D)   0.476
(E)   0.685

315. The random variable $Y_1 = e^{X_1}$ characterizes an insurer's annual property losses, where X_1 is normally distributed with mean 16 and standard deviation 1.50. Similarly, the random variable $Y_2 = e^{X_2}$ characterizes the insurer's annual liability losses, where X_2 is normally distributed with mean 15 and standard deviation 2.

The insurer's annual property losses are independent of its annual liability losses.

Calculate the probability that, in a given year, the minimum of the insurer's property losses and liability losses exceeds e^{16}.

(A) 0.126
(B) 0.154
(C) 0.250
(D) 0.309
(E) 0.346

316. A health insurance company classifies applicants, depending on their health, into one of three categories: A, B, or C.

The following probabilities apply:

i)   P[A] = 5P[C]
ii)   P[B] = 4P[C]
iii) P[zero claims | A] = 0.1
iv)   P[zero claims | B] = 0.2
v)   P[zero claims | C] = 0.4

Calculate the probability that an insured was classified in category C, given that the insured had zero claims.

(A) 0.040
(B) 0.170
(C) 0.235
(D) 0.294
(E) 0.471

317. A five-year term insurance policy pays 25,000 if the insured dies in the first year. The benefit declines by 5000 per year for each of the next four years. In each of the five years covered by the policy, the probability of dying is 0.01, given that the insured is alive at the beginning of that year.

Calculate the expected benefit the insurance company will pay during the five-year term.

(A)   692
(B)   740
(C)   750
(D)   985
(E)   1225

318. Data on a certain pregnancy test show that a pregnant woman will test negative or not pregnant 10% of the time, while a non-pregnant woman will test positive 20% of the time.

Thirty percent of the women who take the test are pregnant.

Calculate the probability that a woman is pregnant given that her test outcome is positive.

(A)   0.18
(B)   0.30
(C)   0.66
(D)   0.82
(E)   0.90

319. A company is marketing an investment opportunity to four potential customers. The company believes that its probability of making a sale is 0.7 for each of the first three customers but that it is only 0.2 for the fourth customer. The customers' purchases are independent of one another.

Calculate the probability that at most two customers purchase the investment.

(A)   0.18
(B)   0.39
(C)   0.57
(D)   0.71
(E)   0.82