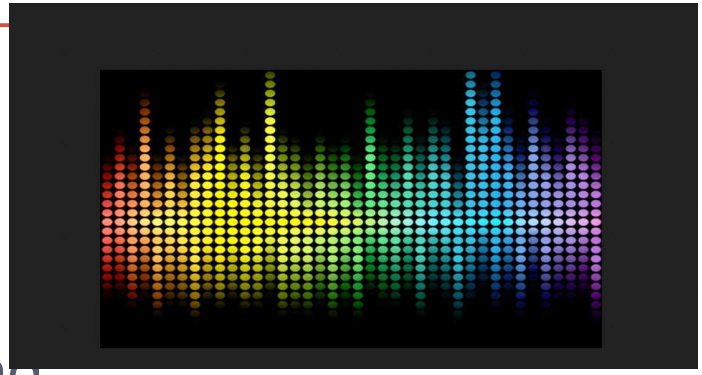
DIGITAL IMAGE PROCESSING



Lecture 2

Basics of Image Processing

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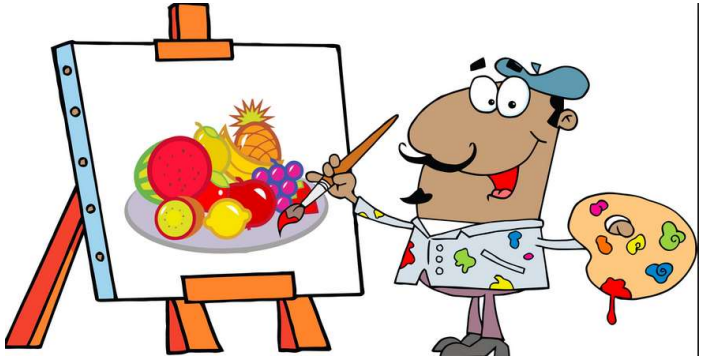

The Makeover of My First Image

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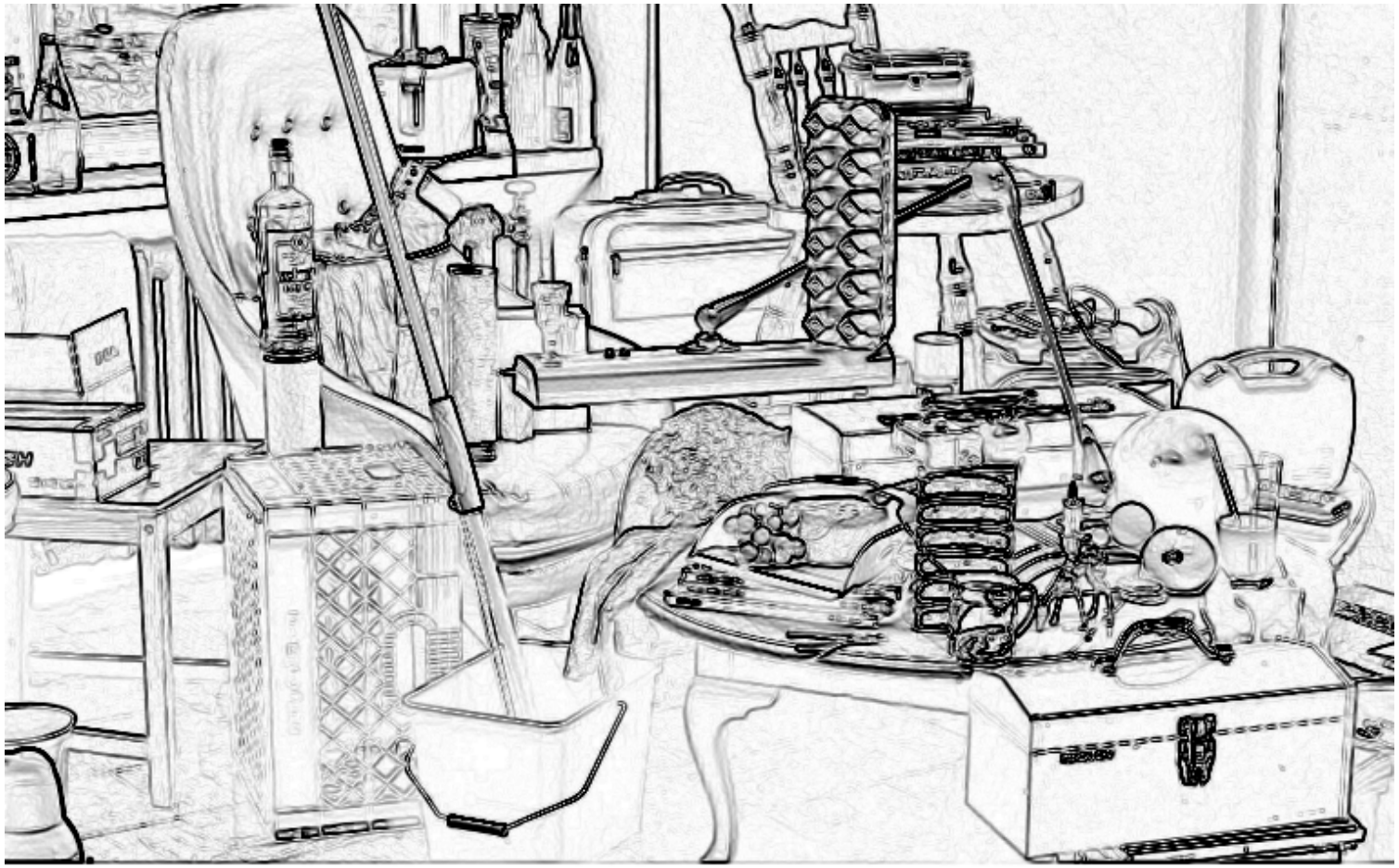


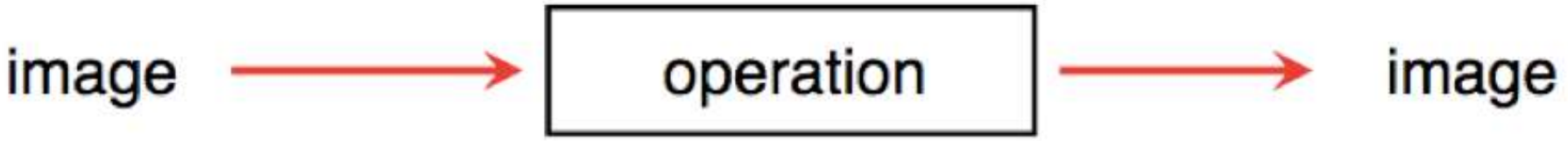
Image processing vs. Image detection

Image processing paradigm

In contrast the image detection paradigm is

for example, features might be edges, lines, points, faces, …
i.e. geometry or objects (but not an image)

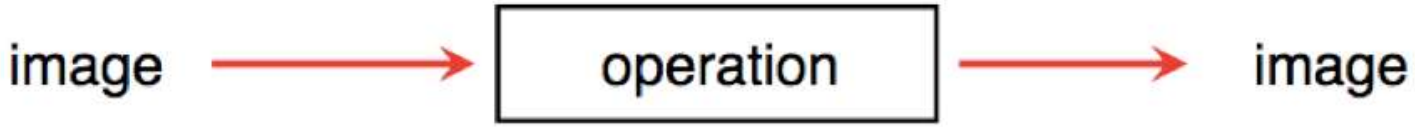
The basics of image Processing



- Sampling and Quantization
- Pointwise/Pixelwise operation
  - Histogram manipulation
  - Temporal averaging/median
- Spatial filters (linear and non-linear)
  - Box, Gaussian, Derivative

Not in this class

This class



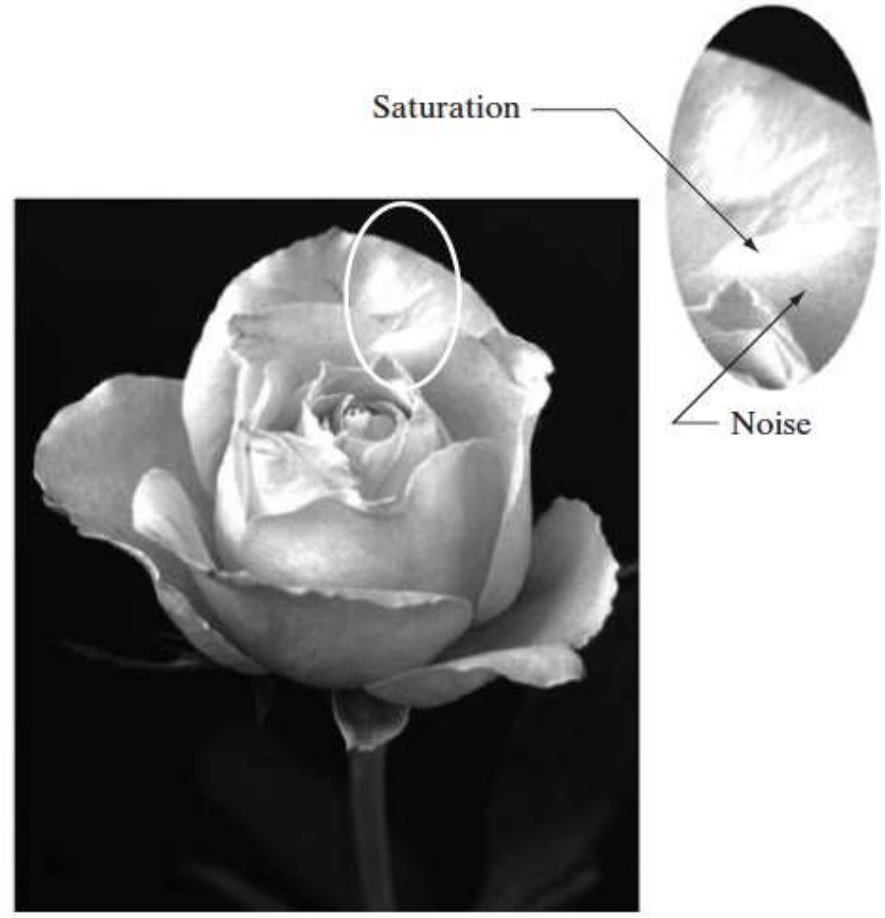
- Image basics
  - Two types of quantization
- Point operations
  - Histogram equalization
  - Temporal averaging
- Spatial filters (linear)
  - Noise reduction, sharpening
  - Separability

Zisserman

Image Processing – Why?

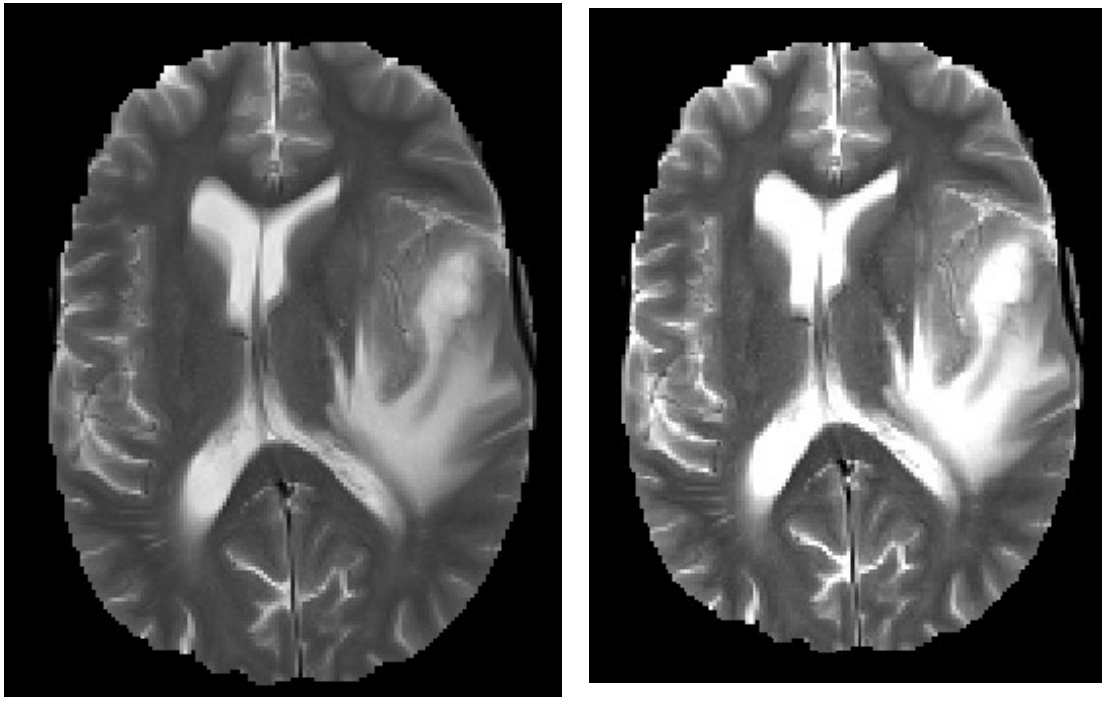
- Main Objective: Image Enhancement

- Why?
  - Further processing
  - Aesthetic



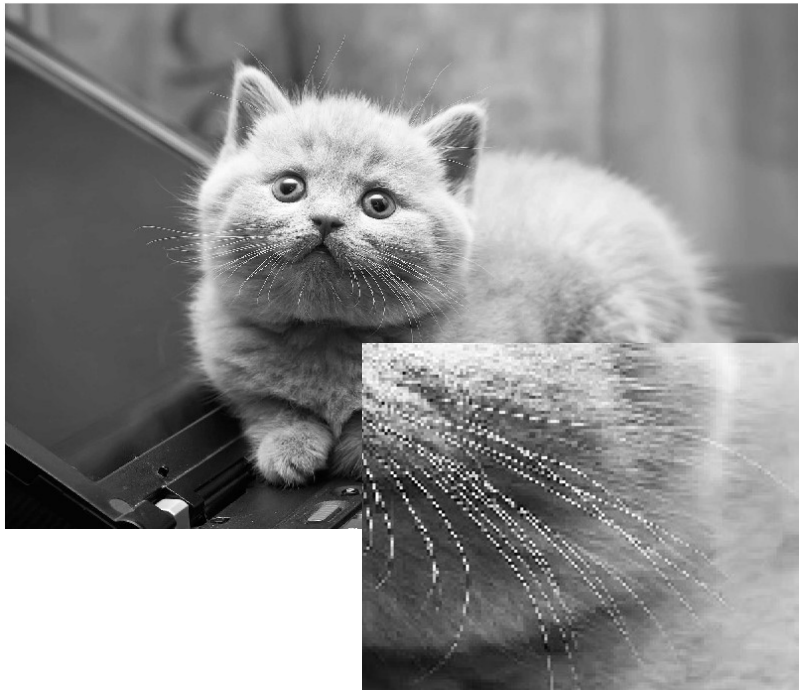
What is image enhancement?

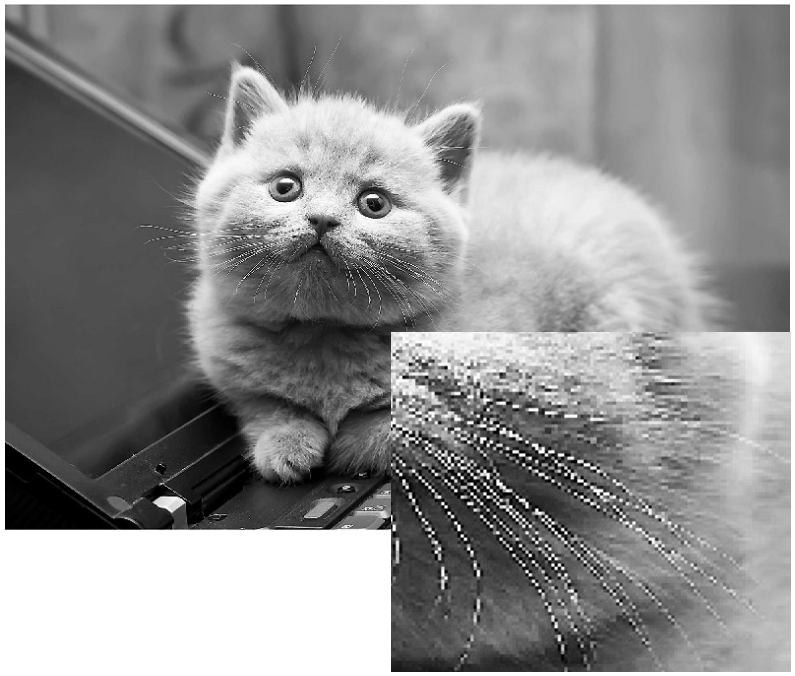
- How to improve contrast ?



What is image enhancement?

- How to improve contrast ?
- How to sharpen edges?





Original image: http://www.rd.com/advice/pets/how-to-decode-your-cats-behavior/

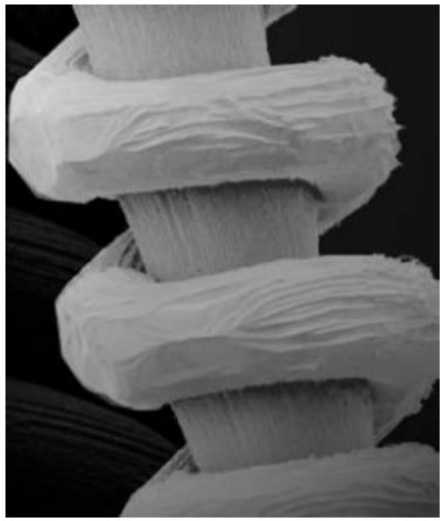
What is image enhancement?

- How to improve contrast ?
- How to sharpen edges?
- How to reduce noise?

https://leegihan.wordpress.com/category/the-best-noise-reduction/

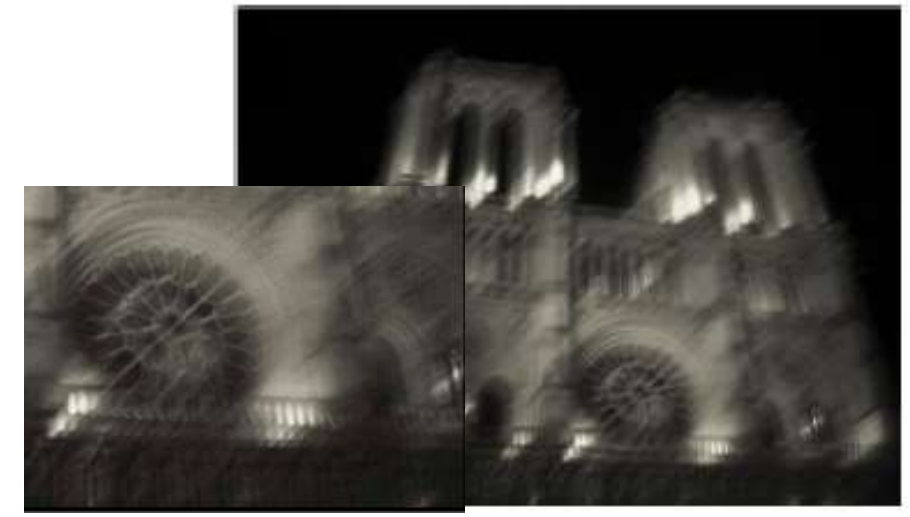
What is image enhancement?

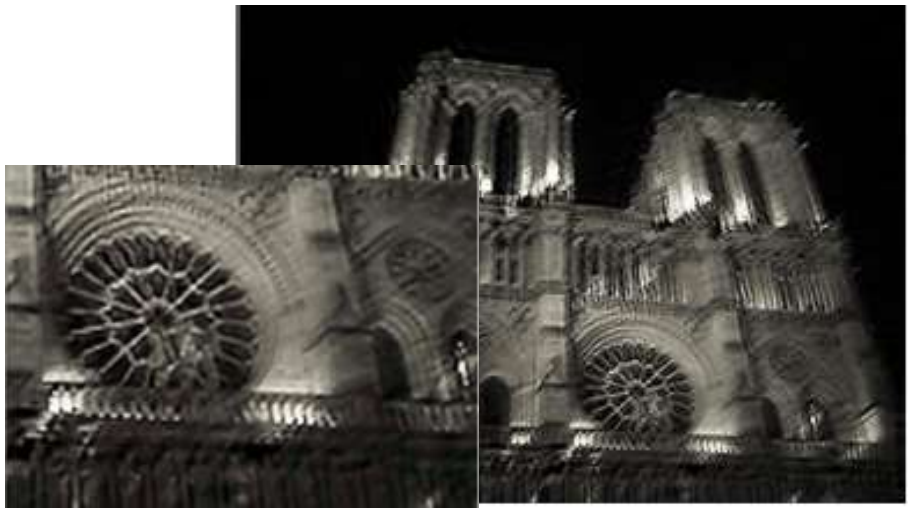
- How to improve contrast ?
- How to sharpen edges?
- How to reduce noise?
- How to remove shadows ?




What is image enhancement?

- How to improve contrast ?
- How to sharpen edges?
- How to reduce noise?
- How to remove shadows ?
- How to do deblurring?





What is image enhancement?

- How to improve contrast ?
- How to sharpen edges?
- How to reduce noise?
- How to remove shadows ?
- How to do deblurring?
- How to do impainting?

Images as functions

- We can think of an **image** as a function, f ,from $\mathbb{R}^2 \to \mathbb{R}$:
  - $f(x, y)$ gives the **intensity** at position (x, y)
  - Realistically, we expect the image only to be defined over a rectangle, with a finite range:

$$f : [a, b] \times [c, d] \to [0, 1]$$

- A color image is just three functions pasted together. We can write this as a "vector-valued" function:

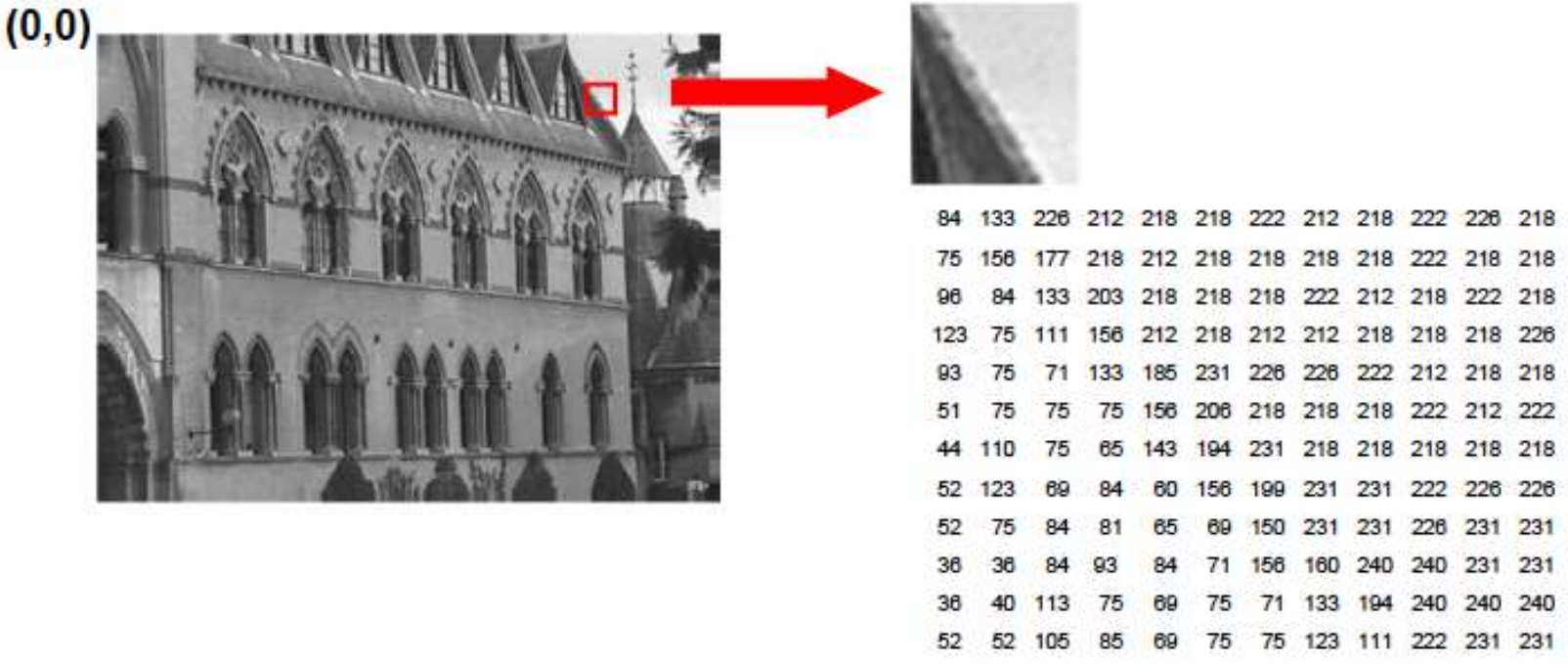
$$f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}$$

Sampling and Quantization

A monochrome image is an array of values.

There are two types of discretization involved:

1. Spatial sampling (pixels -'picture elements'), and
2. Intensity quantization (grey level value).



Intensity quantization

2 levels - binary

4 levels

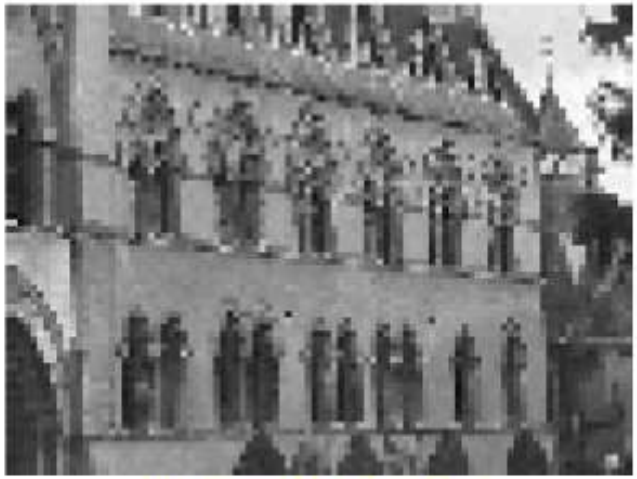
8 levels

256 levels – 1 byte

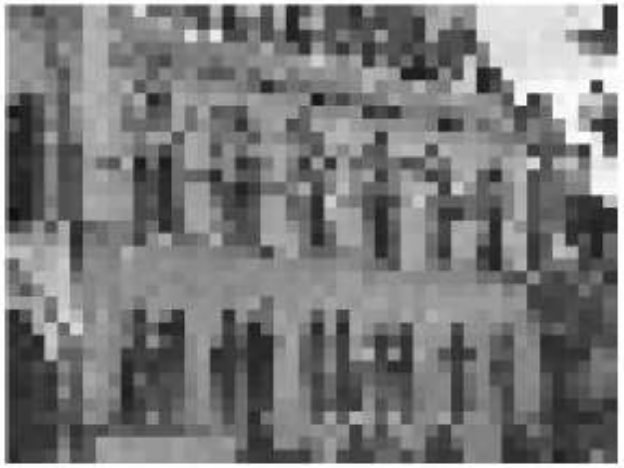
Spatial sampling

384 x 288 pixels

192 x 144 pixels



92 x 72 pixels



48 x 36 pixels

Some numbers

Quantization:

- Often 8 bits per pixel (0-255, 2^8 = 256 levels) for monochrome
- 24 bits per pixel for colour (8 bits for each of Red Green Blue)
- Medical images 12 bits (4096 levels) or 16 bits (65536 levels)

Size

- Cameras typically 4K x 3K pixels or far more
- Satellite images 10 -100K pixels width

• 3D images

- e.g. Magnetic Resonance Images
- Videos (2D + time)

Shanghai Skyline - Stitched from 12,000 photos

273 G pixels

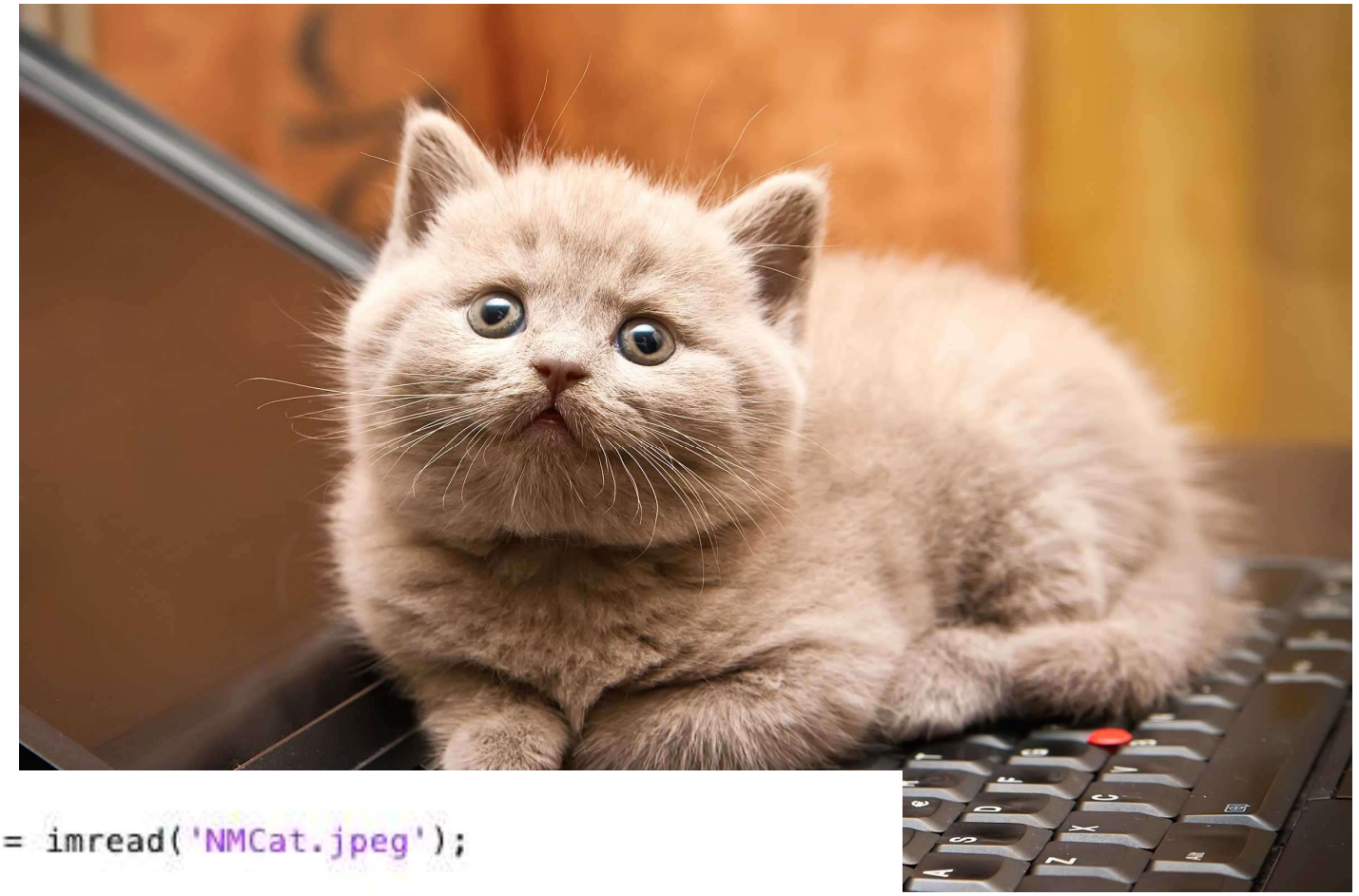
http://gigapan.com

Google Art Project

resolution 30,000 × 23,756 pixels

https://www.google.com/culturalinstitute/about/artproject/

Not-My-Cat Image



```
>>
>> Irgb = imread('NMCat.jpeg');
>> whos
  Name          Size                     Bytes  Class

  Irgb       1600x2400x3                11520000  uint8
```

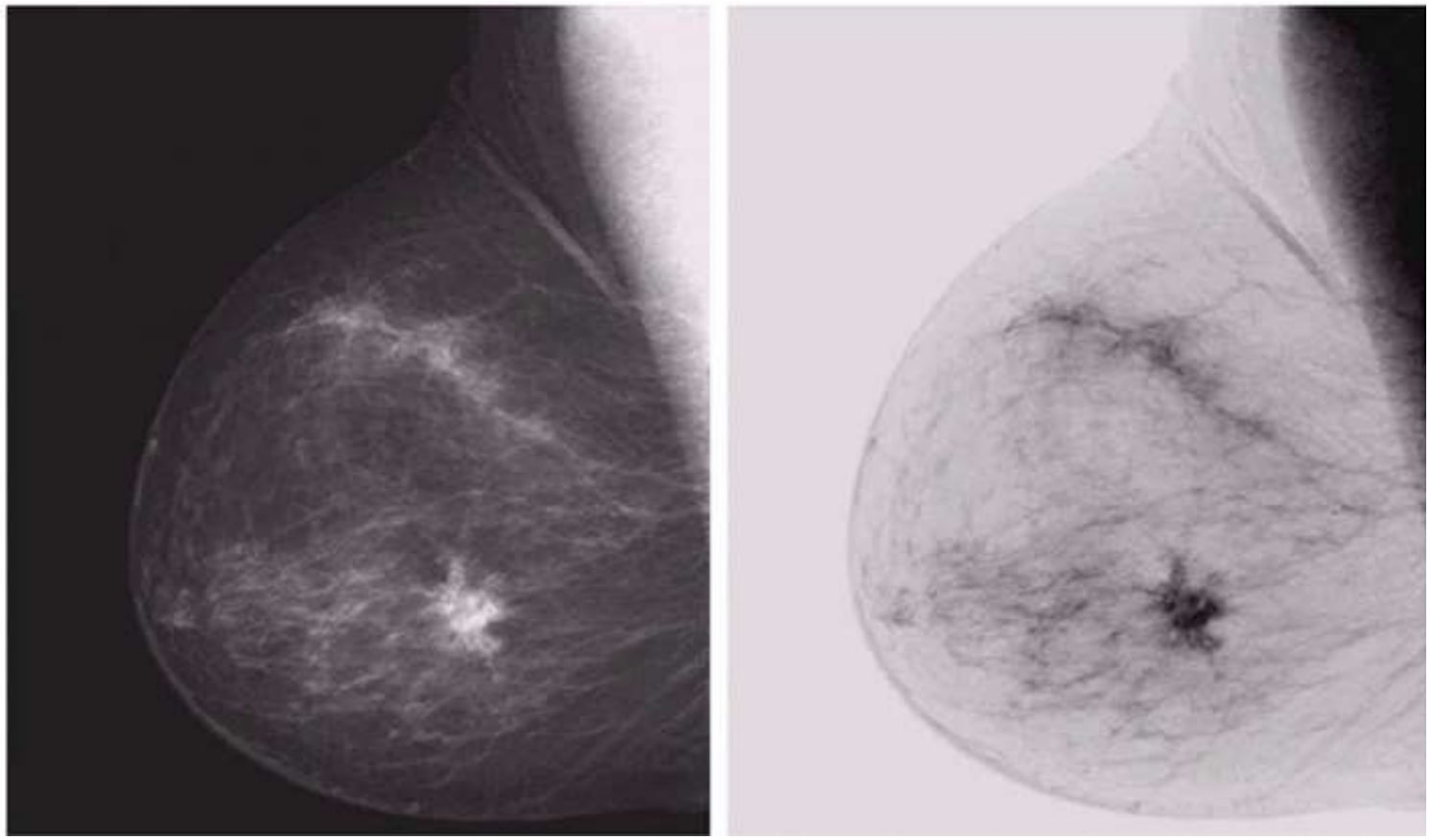
Point (Pixelwise) Operation

$$i_n = \text{function}(i)$$

new intensity

original intensity

Pixelwise operation: Negative



Pixelwise operation: power low transformation

$$i_n = i^{\gamma}$$

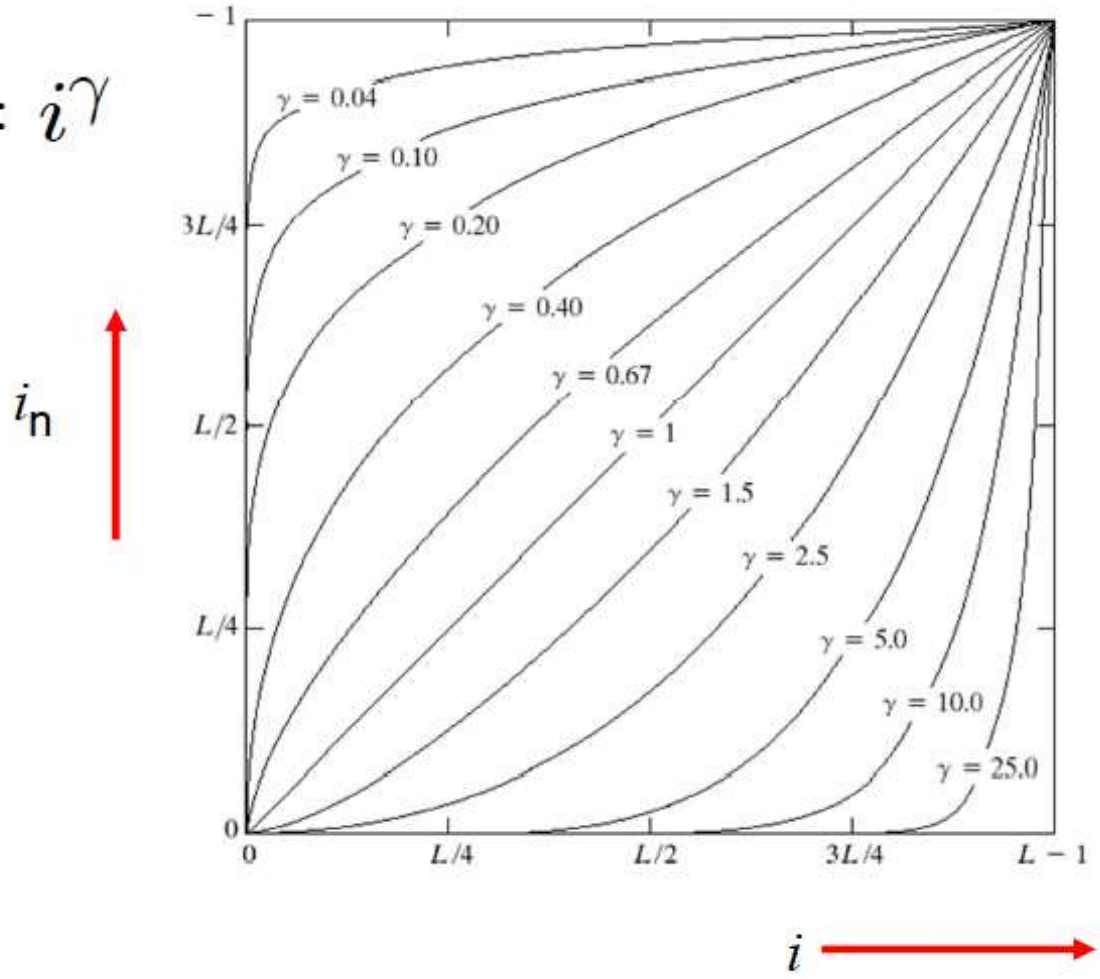
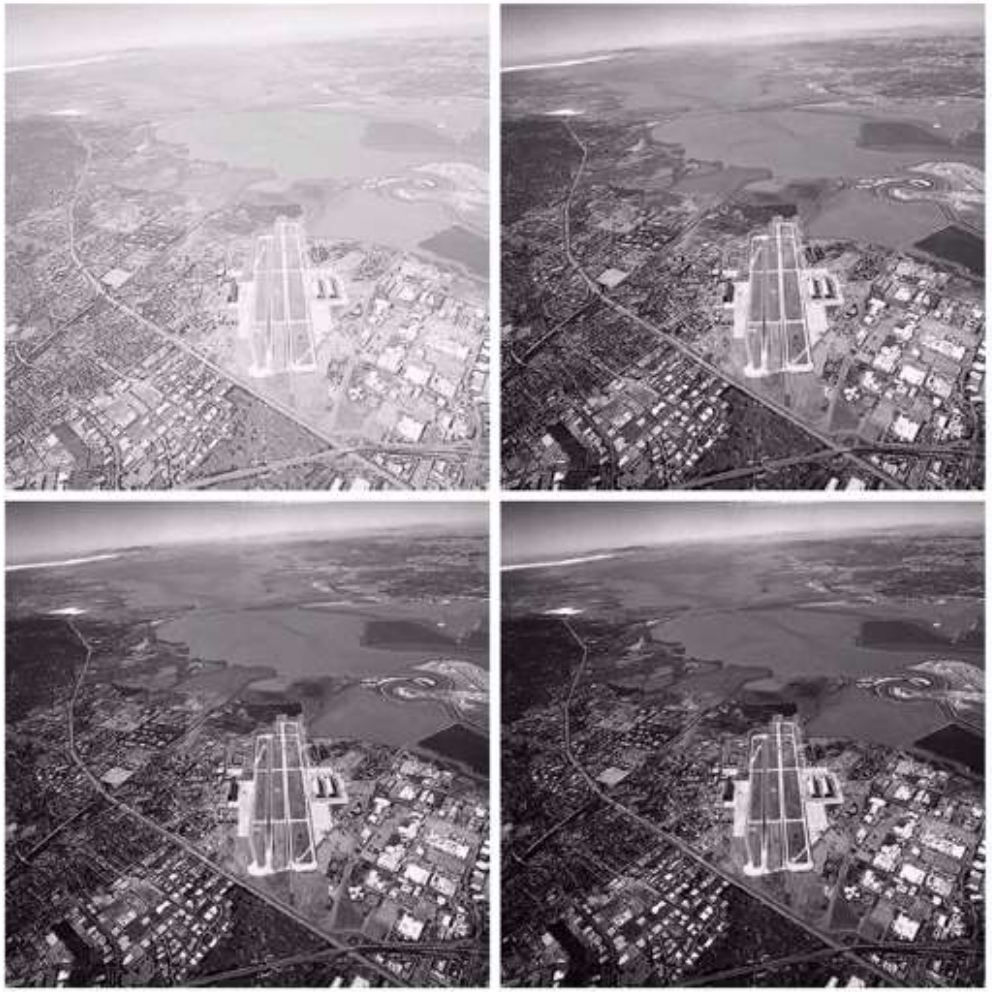


Image Enhancement

original



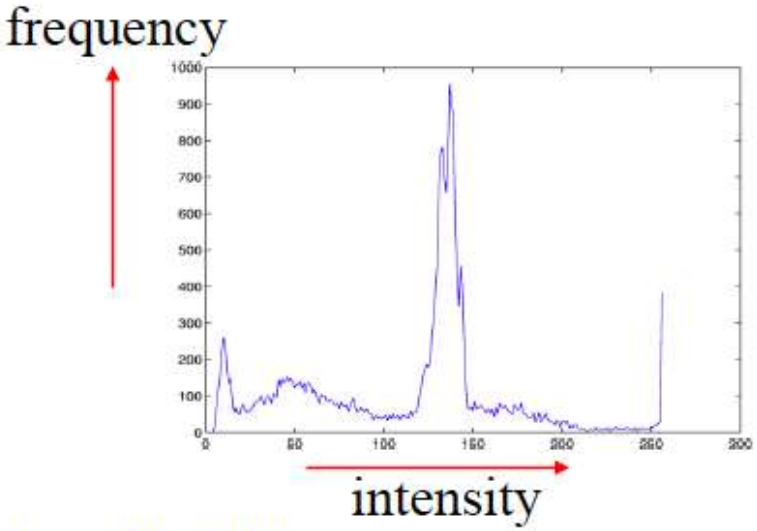
$\gamma = 3$

$\gamma = 4$

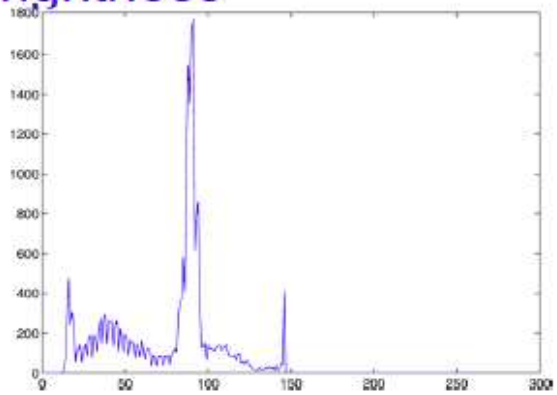
$\gamma = 5$

Point operation: Histograms

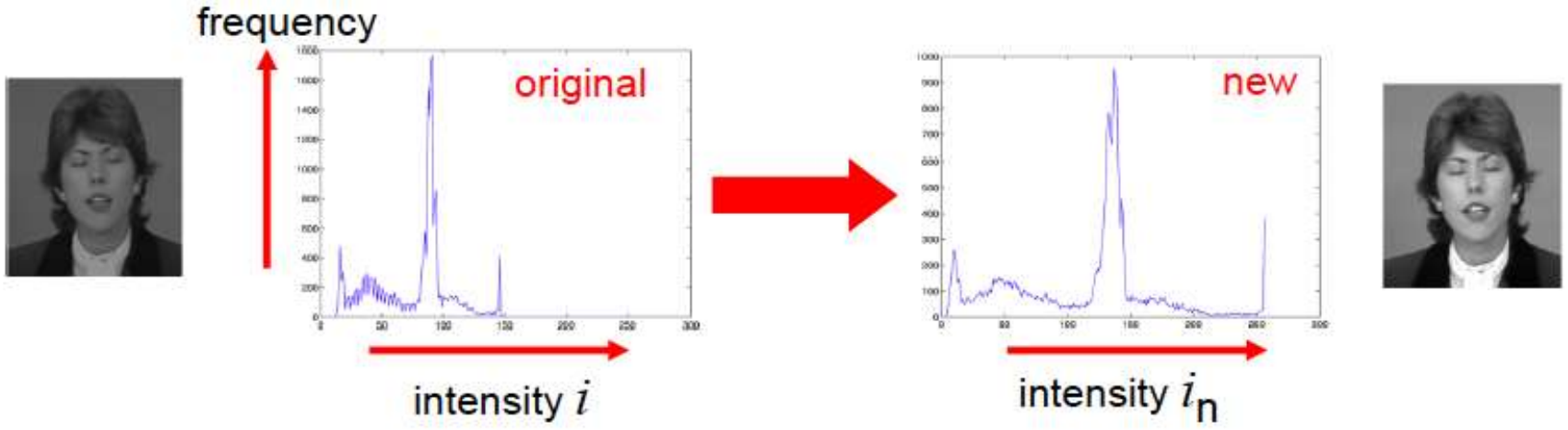
- image intensity histogram



- histogram for image with reduced brightness

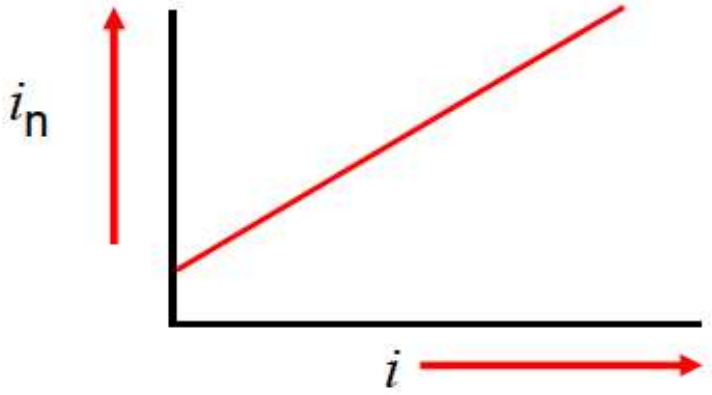


Point Operation: Intensity Map

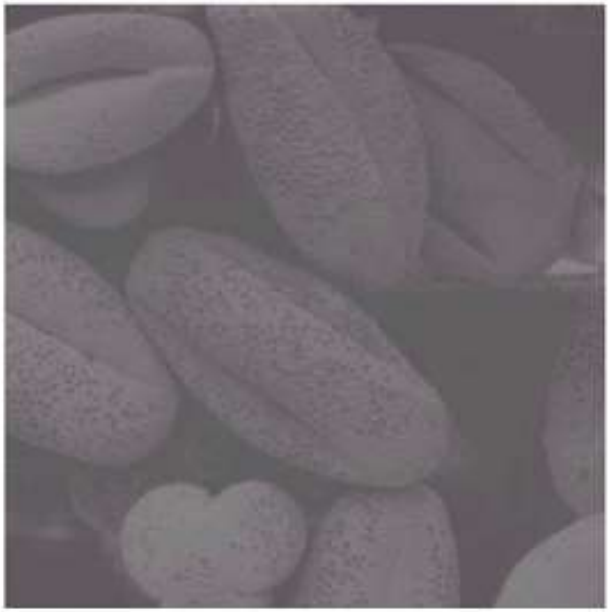


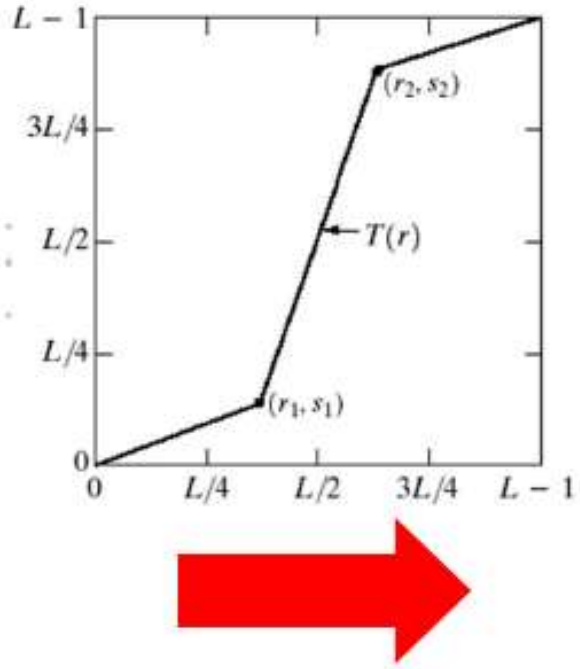
- change contrast and brightness by transforming pixel intensities
- assign the same new intensity value to all pixels having a given original value
- e.g. an affine (linear) map:

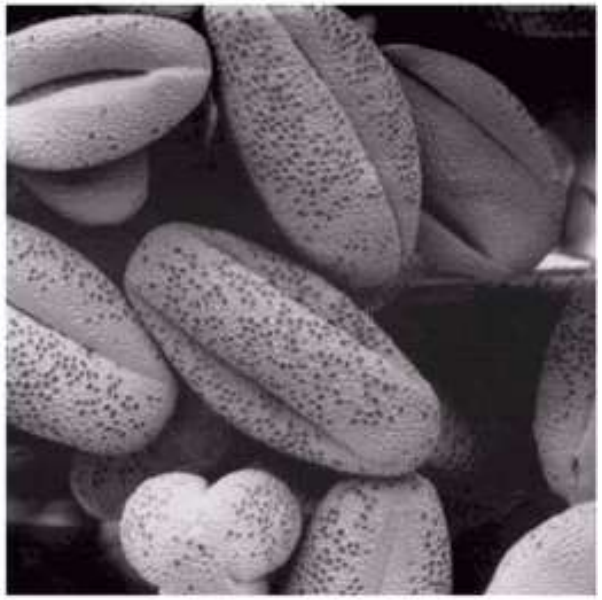
$$i_n = \alpha i + \beta$$



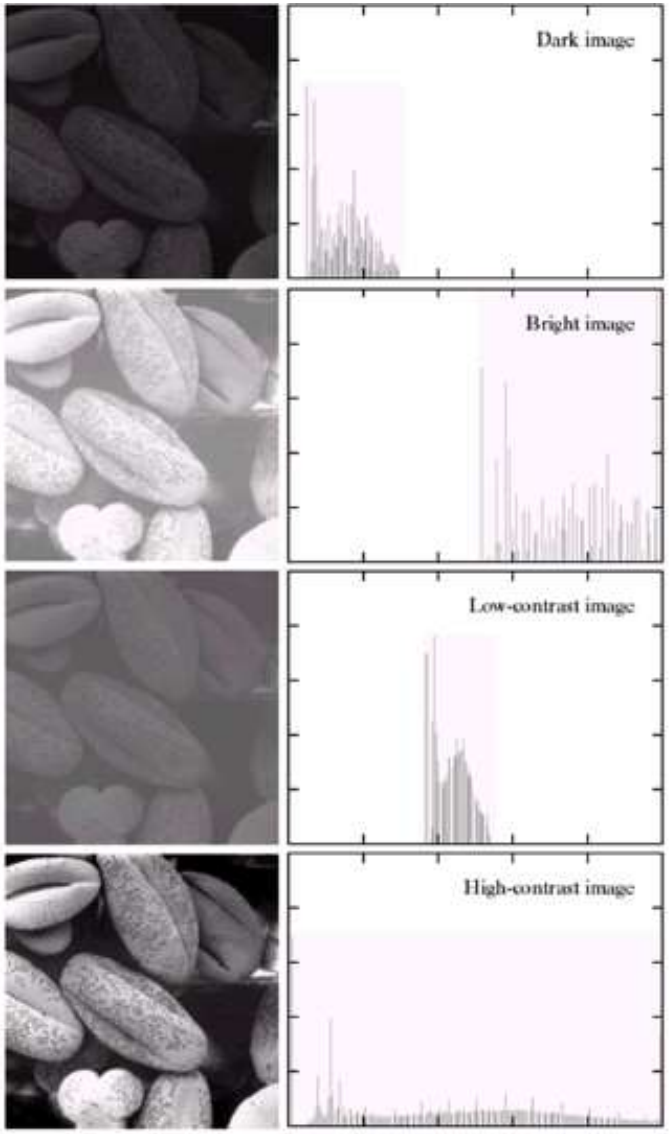
Contrast Stretching

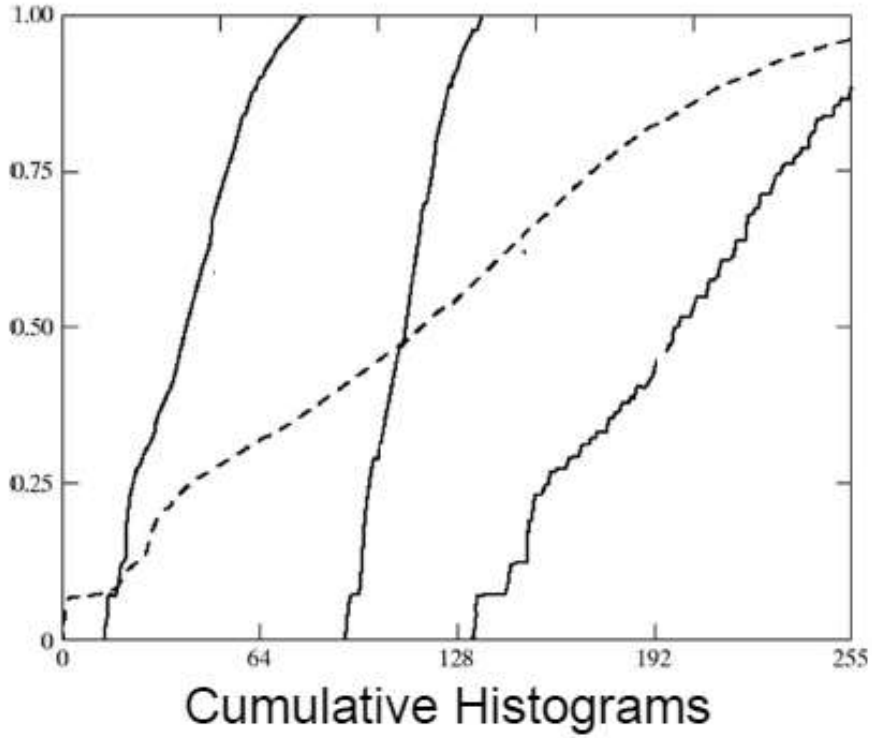




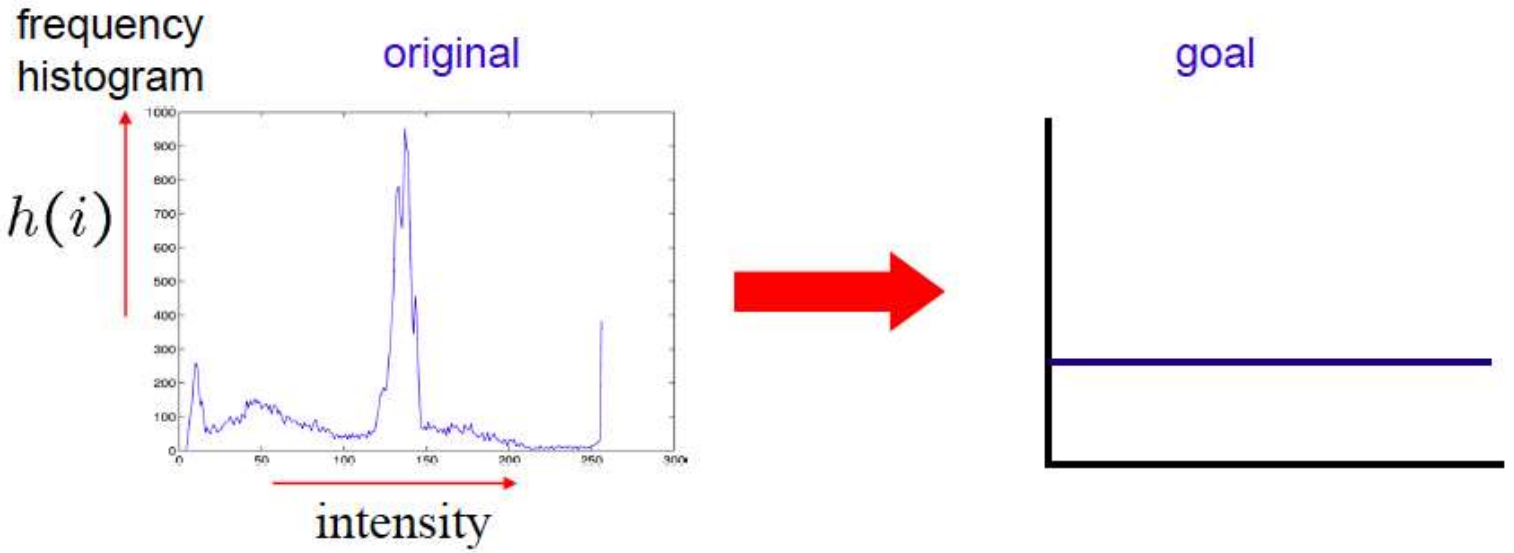


Cumulative Histograms

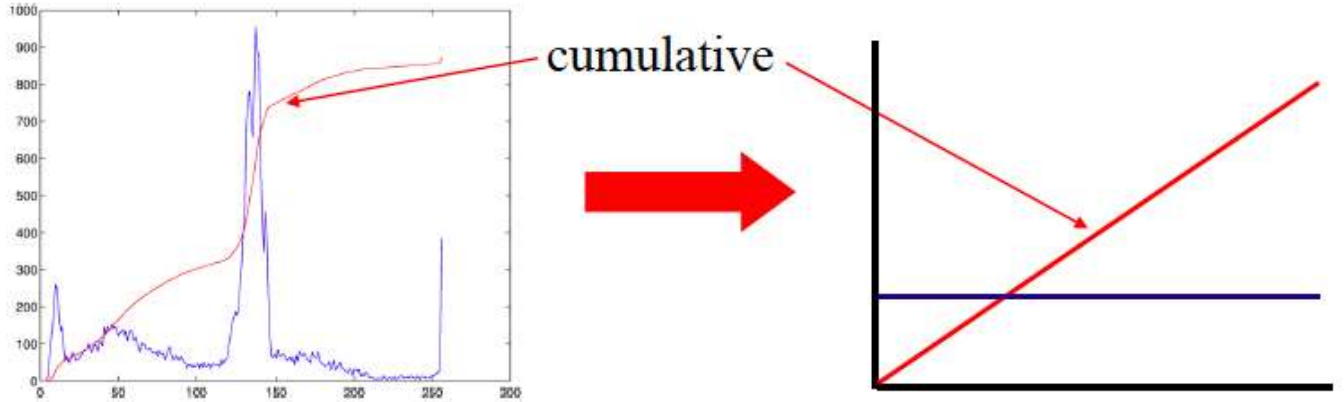




Histogram Equalization

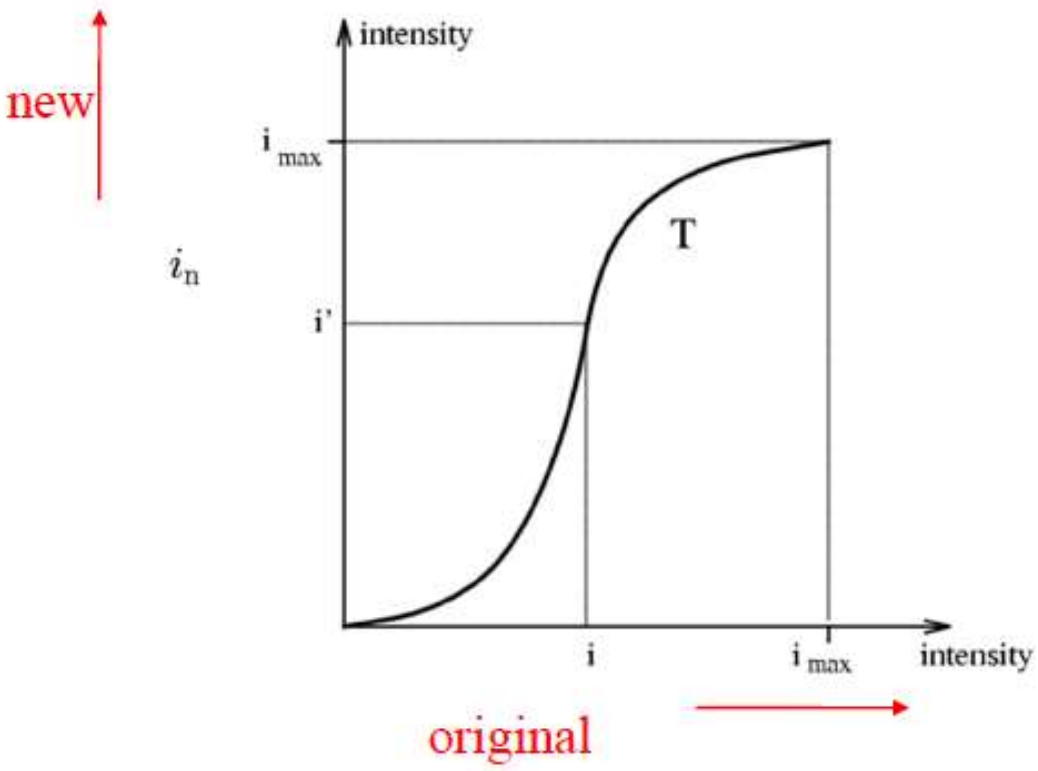


- use cumulative distribution $C(i) = \int_0^i h(u)\,du$ as a measure



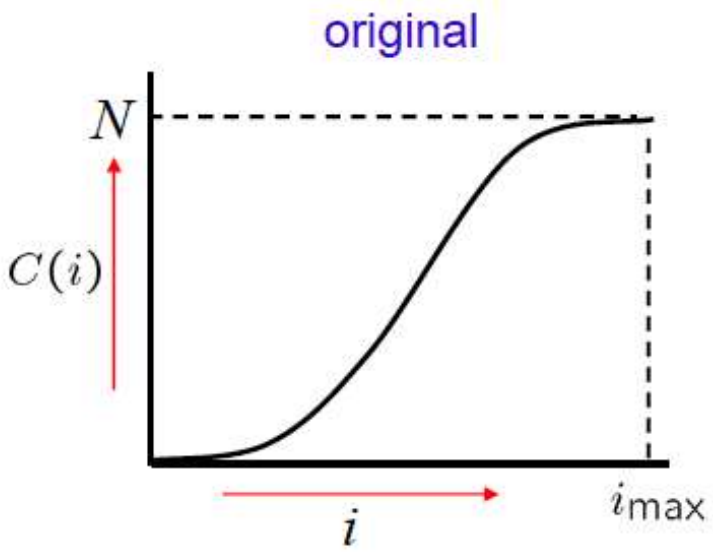
Histogram Equalization

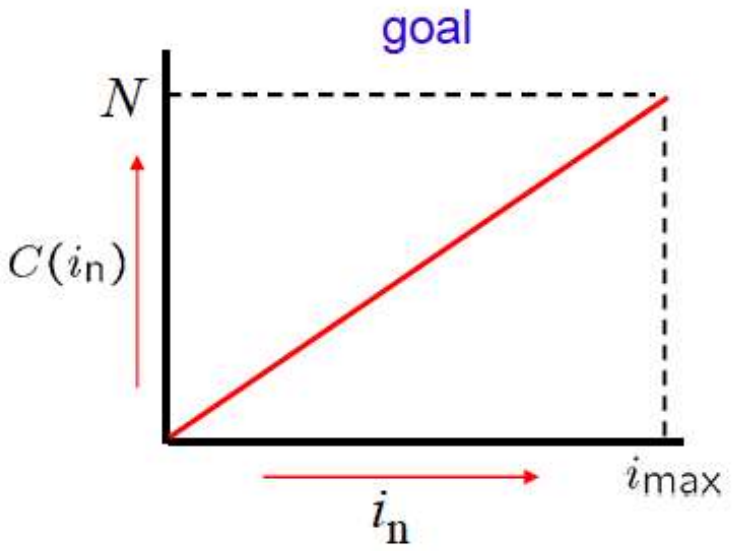
Objective: apply a monotonic map $T(i)$ to the intensities so that the intensity histogram is less peaked (flattened)



Histogram Equalization (flatening)

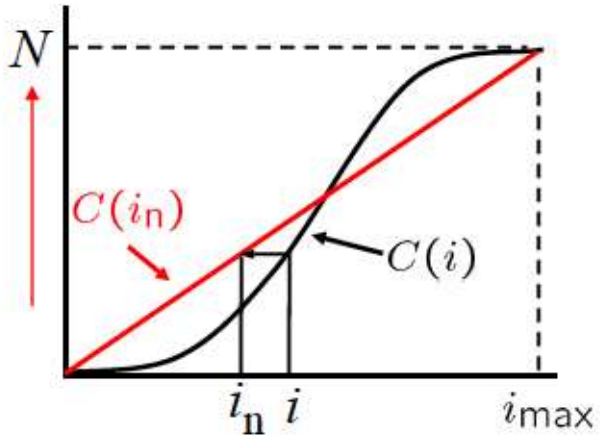
$$C(i) = \int_0^i h(u)\,du \qquad C(i_{\max}) = \#\text{pixels} = N$$





$$C(i_{\text{n}}) = \frac{N}{i_{\max}} i_{\text{n}}$$

Histogram Equalization



$$C(i) = C(i_n) = \frac{N}{i_{max}} i_n$$

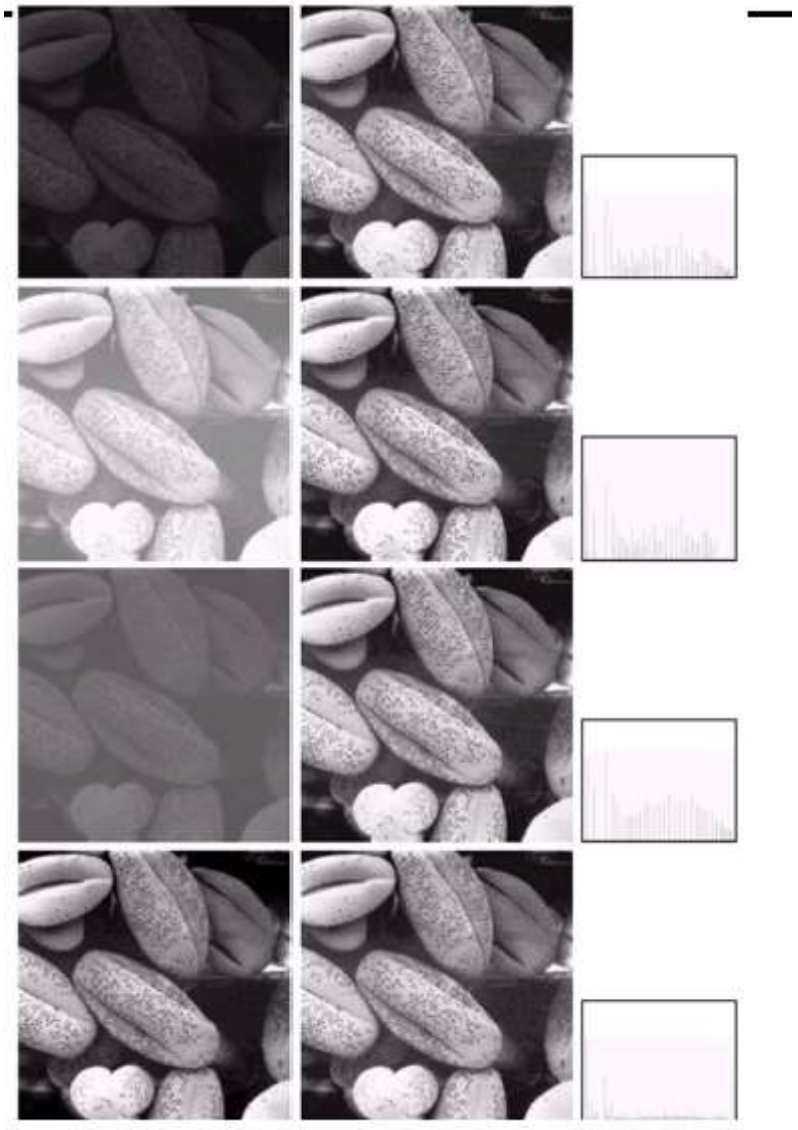
$$i_n = \frac{i_{max}}{N} C(i)$$

Algorithm:

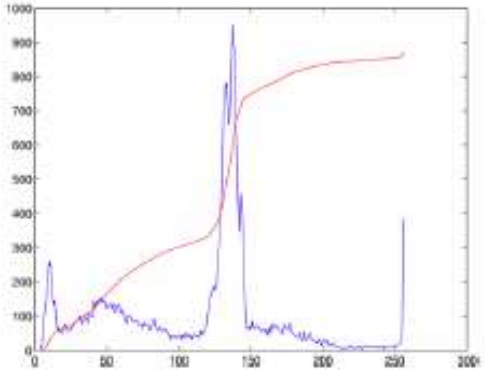
- compute the cumulative probability distribution $C(i)$ from the intensity histogram
- map pixel intensities as

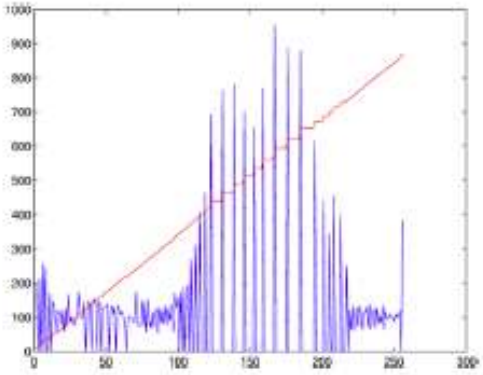
$$i_n = T(i) \quad \text{where} \quad T(i) = \frac{i_{max}}{N} C(i)$$

Histogram Equalization



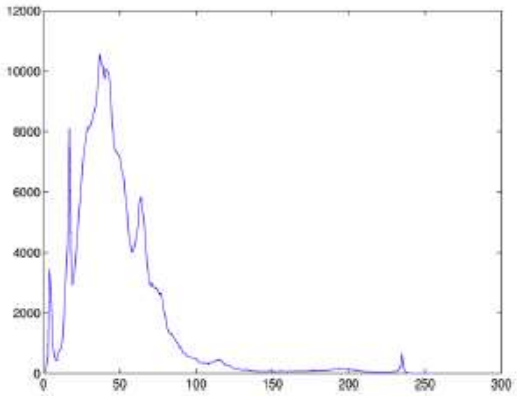
Histogram Equalization

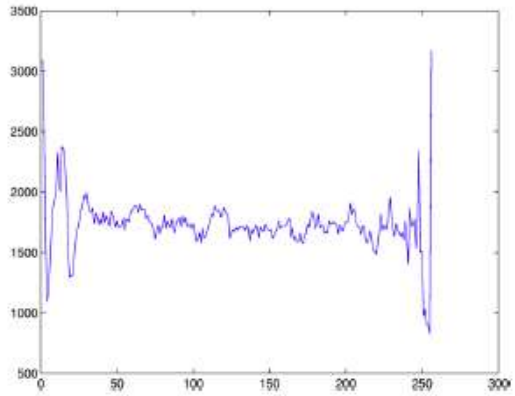




- some values which are quite close together are spread out so that minor variations are more visible
- some values which were originally different are reassigned to the same value, so information is lost
- note, histogram is not flat

Histogram Equalization



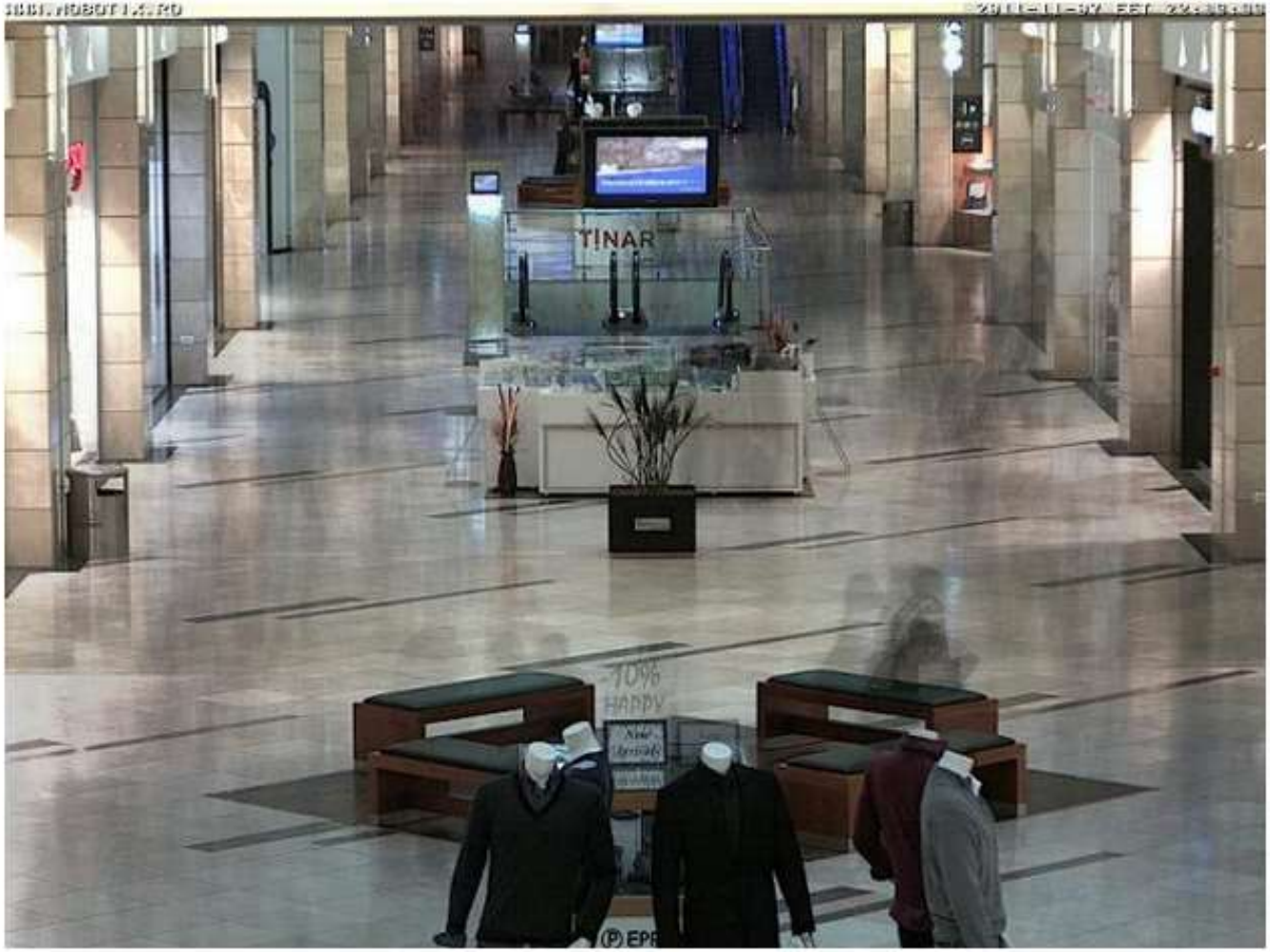


- intervals without many corresponding pixels are compressed

Temporal Averaging

Temporal Averaging

Temporal Averaging



Temporally Averaged from 70 Images

Spatial Operation

$$i_{\mathsf{n}}(x, y) = \text{function}(i \in \text{spatial neighbourhood}(x, y))$$

new intensity

original intensity

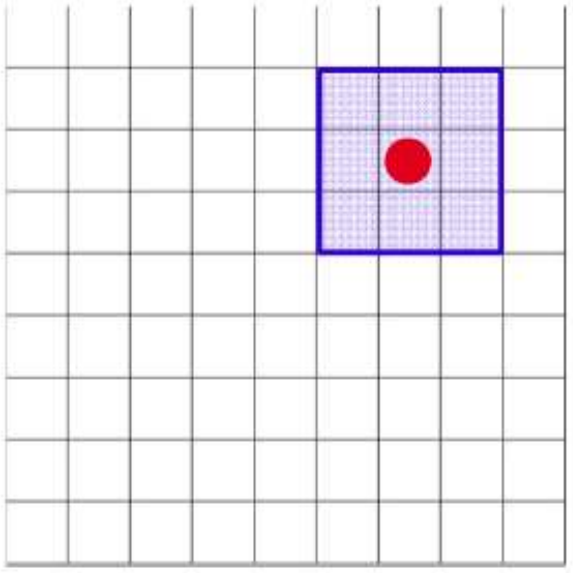


Image Filtering

- Image filtering: compute function of local neighborhood at each position

- Linear filtering: function is a weighted sum/difference of pixel values

- Many applications:
  - Enhance images
    - Denoise, resize, increase contrast, etc.
  - Extract information from images
    - Texture, edges, distinctive points, etc.
  - Detect patterns
    - Template matching

Image filtering

- Compute function of local neighborhood at each position

```
h=output        f=filter    I=image
```

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Image filtering

Example: box filter

$$f[\cdot,\cdot]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

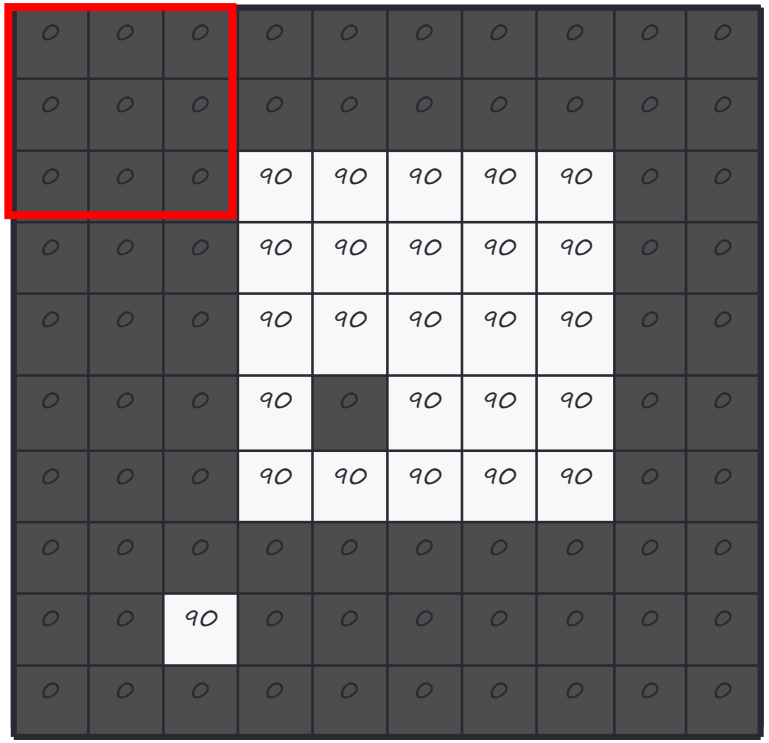
Slide credit: David Lowe (UBC)

Image Filtering

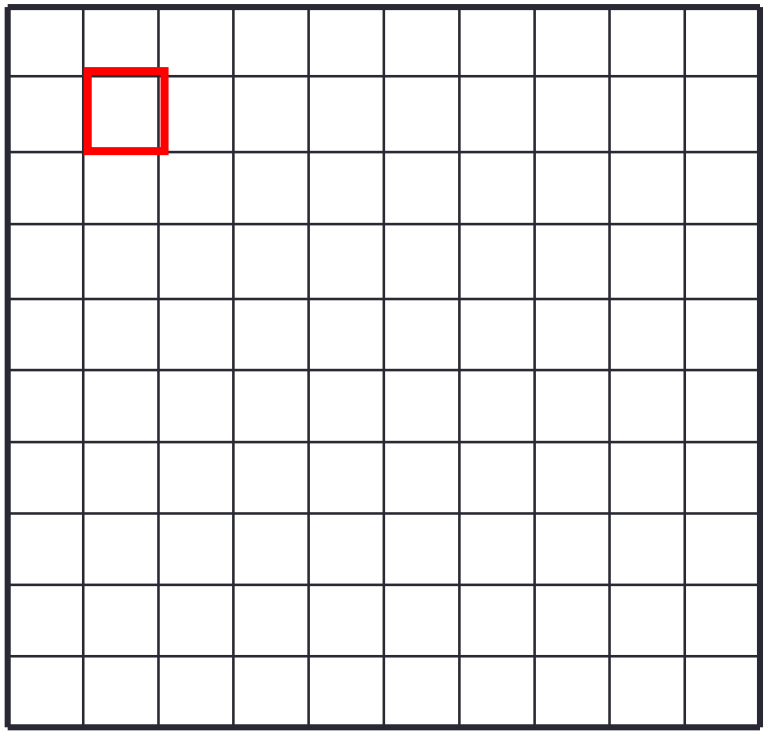
$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$



$h[.,.]$



$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$f[\cdot,\cdot]$ $\frac{1}{9}$

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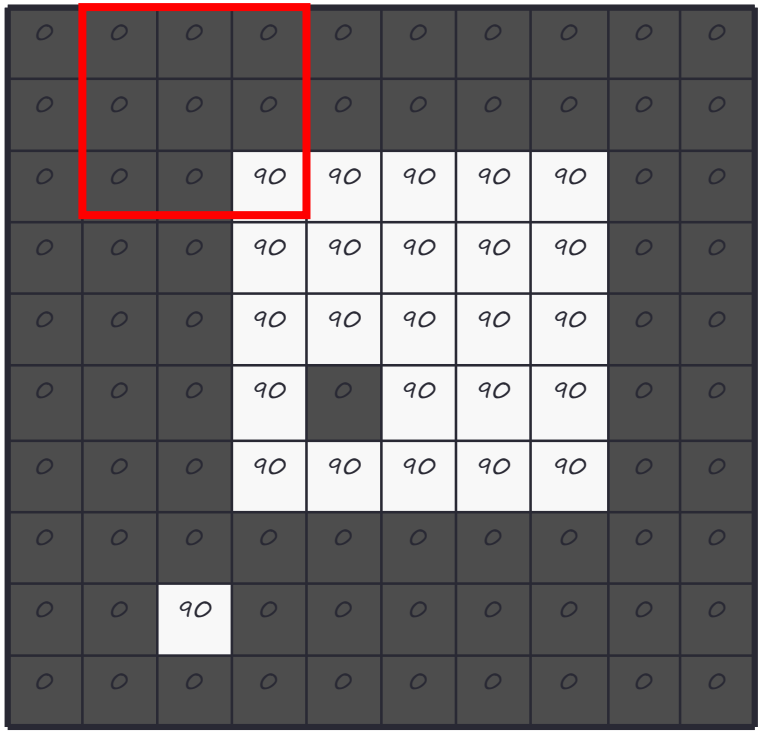
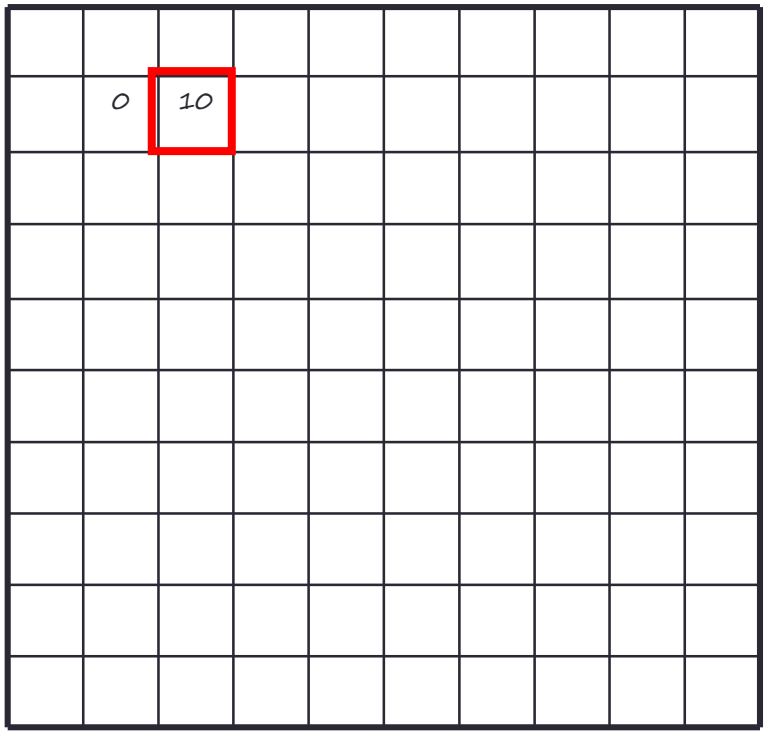
1	1	1

$I[.,.]$



$h[.,.]$



$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Image Filtering

$f[\cdot,\cdot]$ $\frac{1}{9}$

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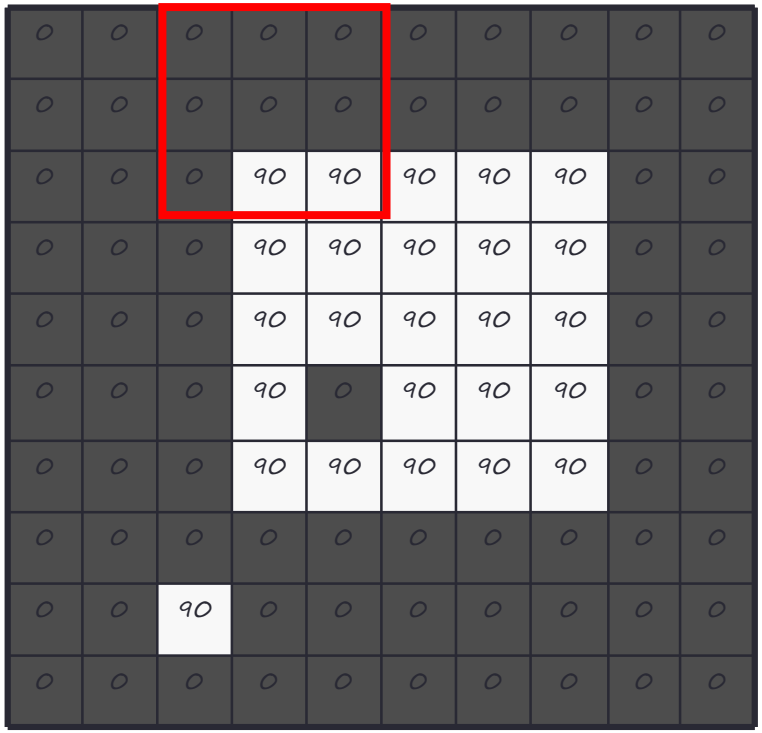

$h[.,.]$

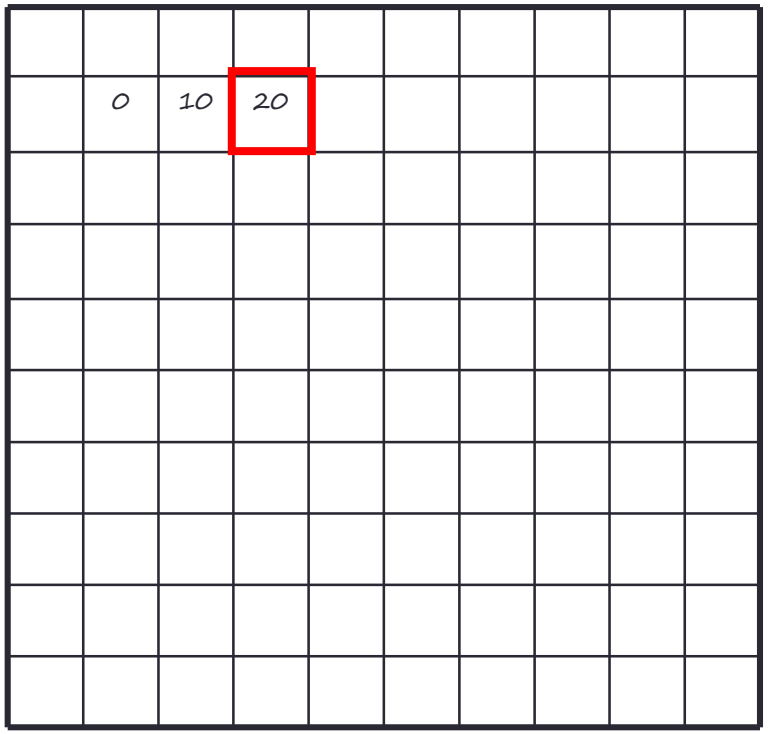

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Image Filtering

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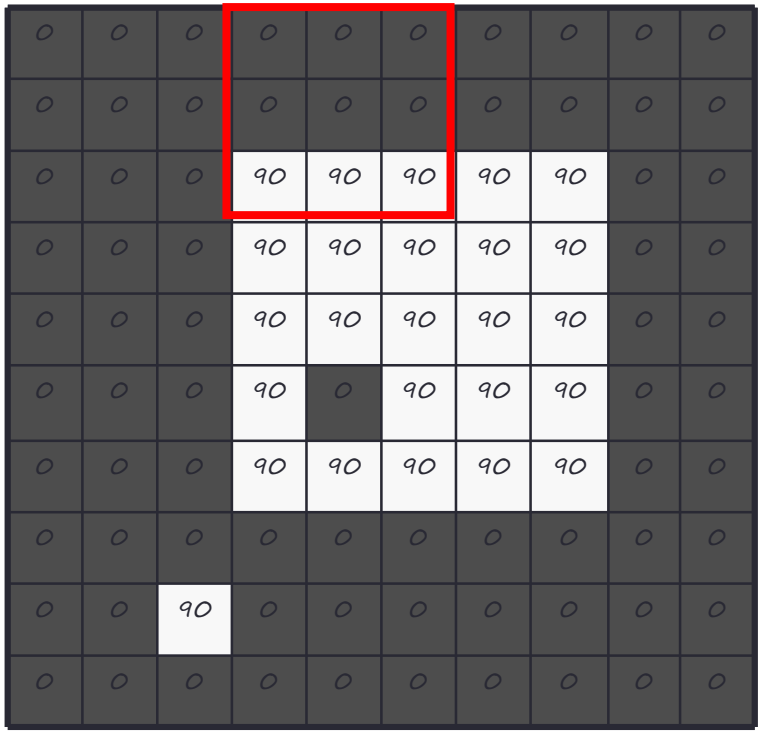
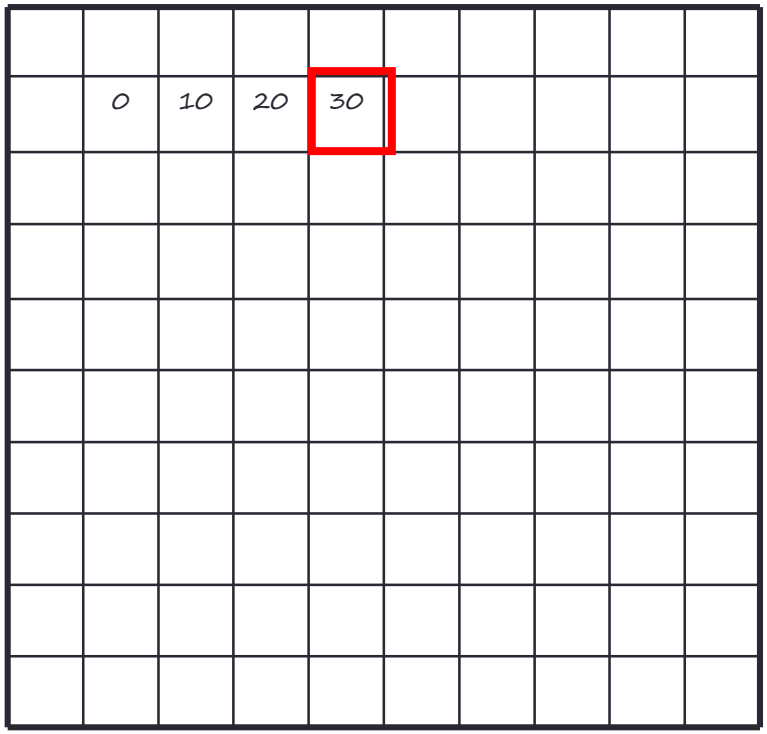


$h[.,.]$



$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Image Filtering

$f[\cdot,\cdot]$ $\frac{1}{9}$

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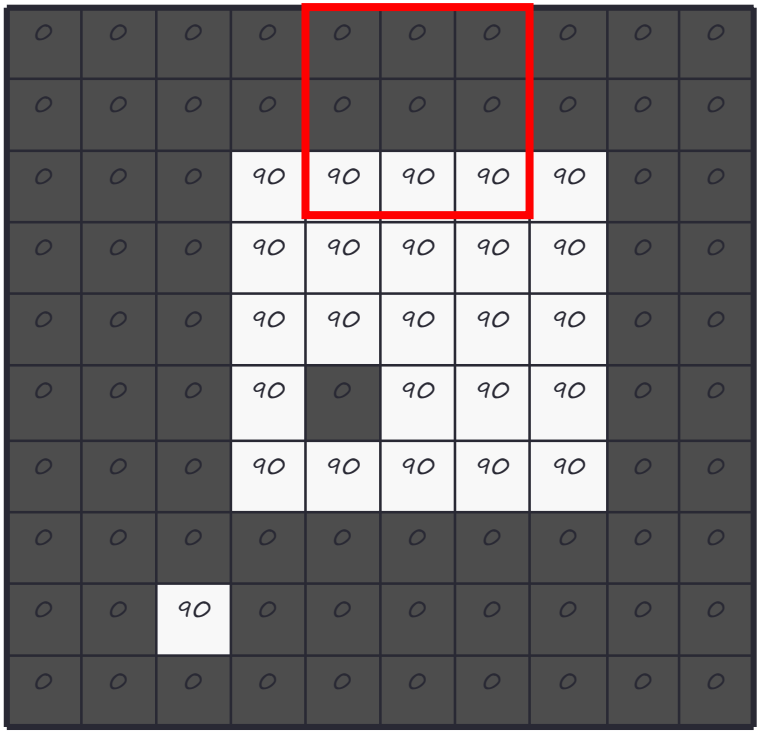
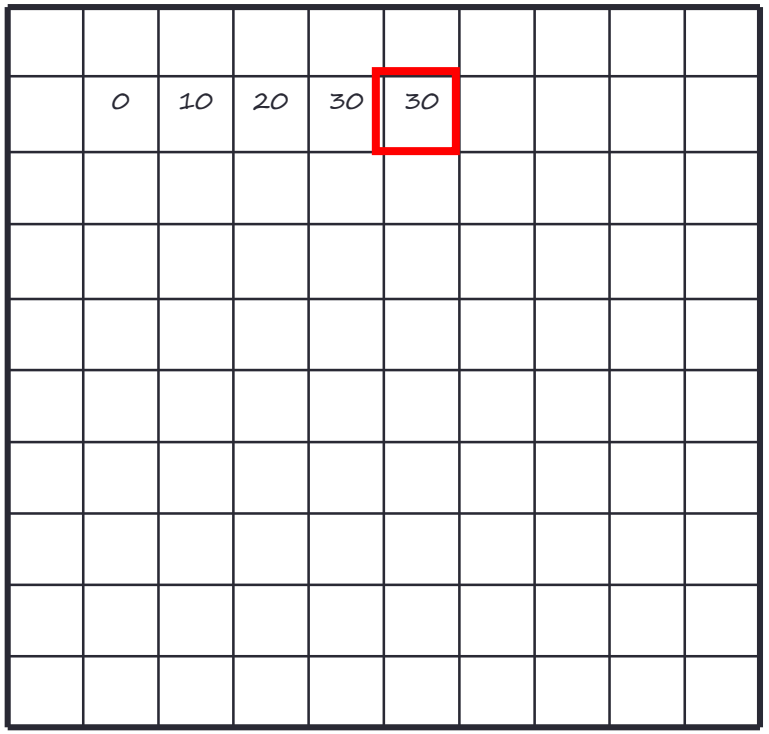
1	1	1

$I[.,.]$



$h[.,.]$



$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Image filtering

$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				
				?					

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

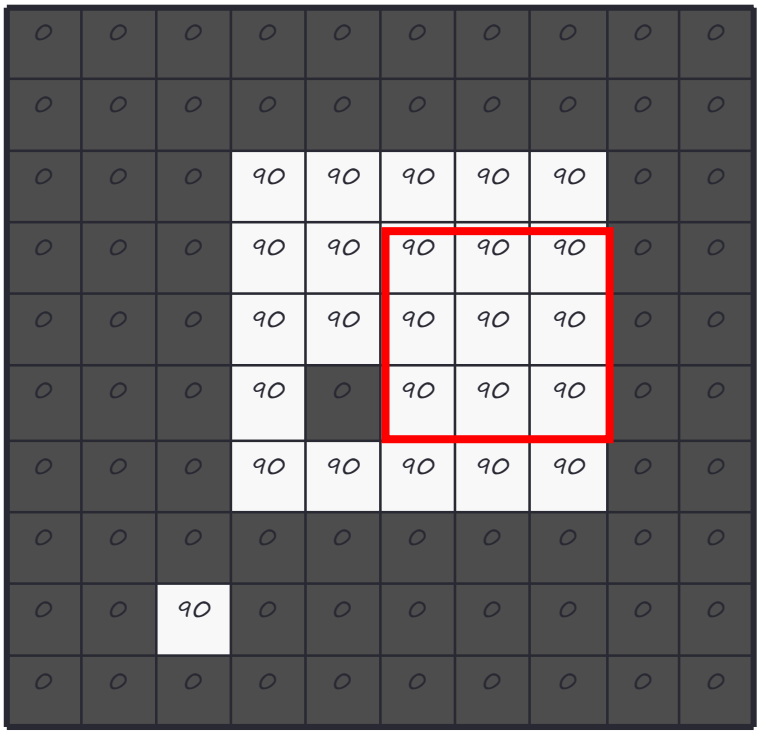
Credit: S. Seitz

Image filtering

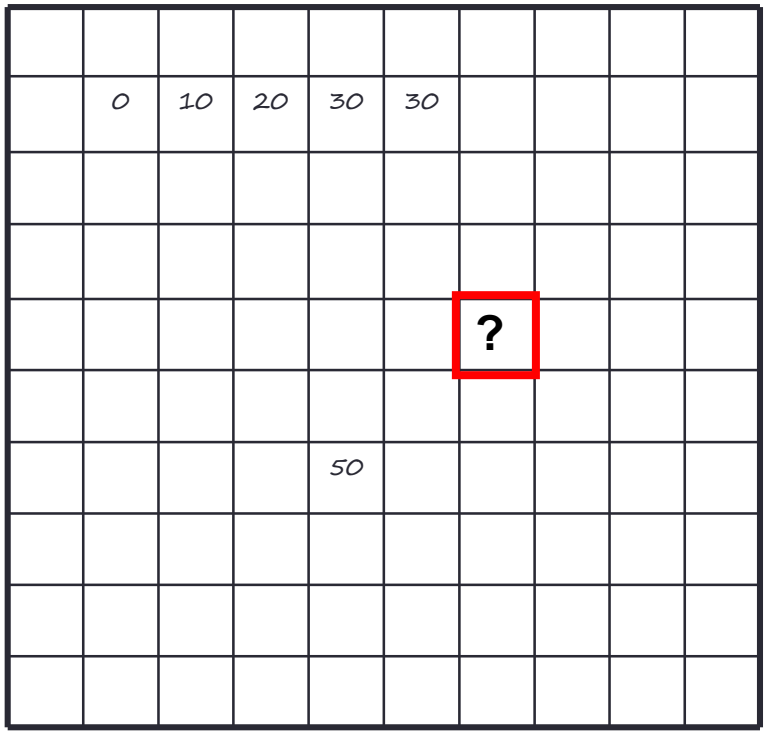
$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$



$h[.,.]$



$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

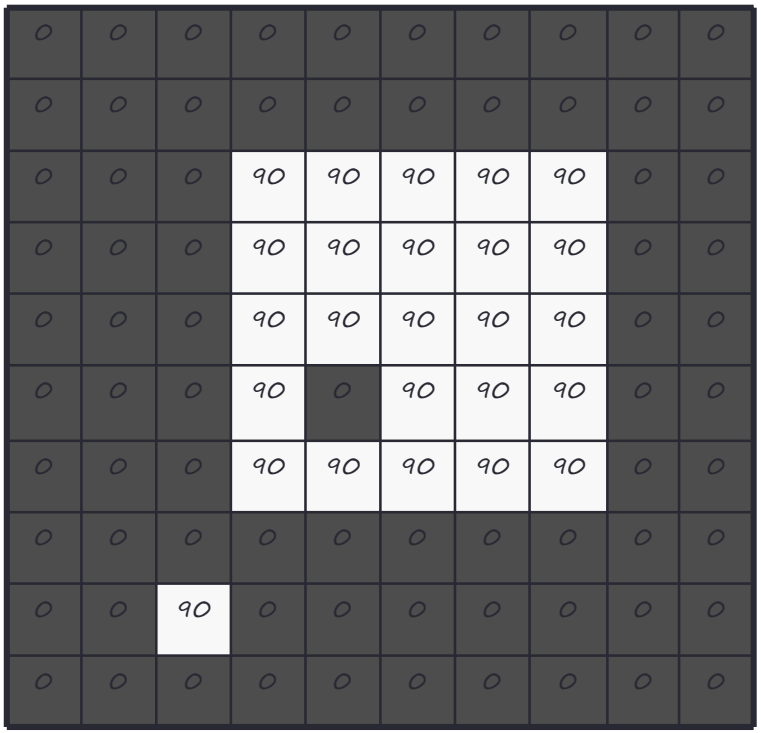
Credit: S. Seitz

Image filtering

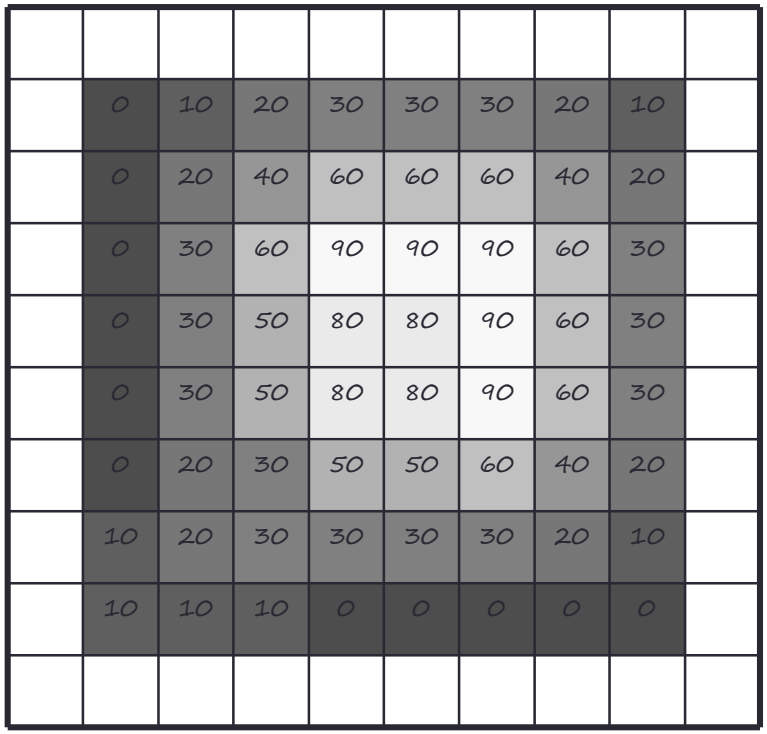
$f[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$



$h[.,.]$



$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Credit: S. Seitz

Box Filter

What does it do?

- Replaces each pixel with an average of its neighborhood

- Achieve smoothing effect (remove sharp features)

$$f[\cdot,\cdot]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

Box Filter

What does it do?

- Replaces each pixel with an average of its neighborhood
- Achieve smoothing effect (remove sharp features)
- Why does it sum to one?

$$f[\cdot,\cdot]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

Smoothing with box filter

Image filtering

- Compute function of local neighborhood at each position

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

- Really important!
  - Enhance images
    - Denoise, resize, increase contrast, etc.
  - Extract information from images
    - Texture, edges, distinctive points, etc.
  - Detect patterns
    - Template matching

Linear Filters

1.

0	0	0
0	1	0
0	0	0

2.

0	0	0
0	0	1
0	0	0

3.

1	0	-1
2	0	-2
1	0	-1

4.

0	0	0
0	2	0
0	0	0

$-\frac{1}{9}$

1	1	1
1	1	1
1	1	1

1. Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

?

Source: D. Lowe

1. Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

Filtered
(no change)

Source: D. Lowe

2. Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

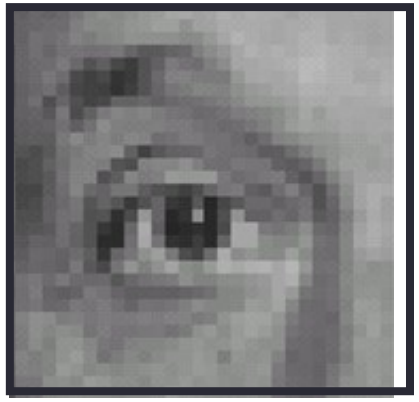
?

Source: D. Lowe

2. Practice with linear filters

Original

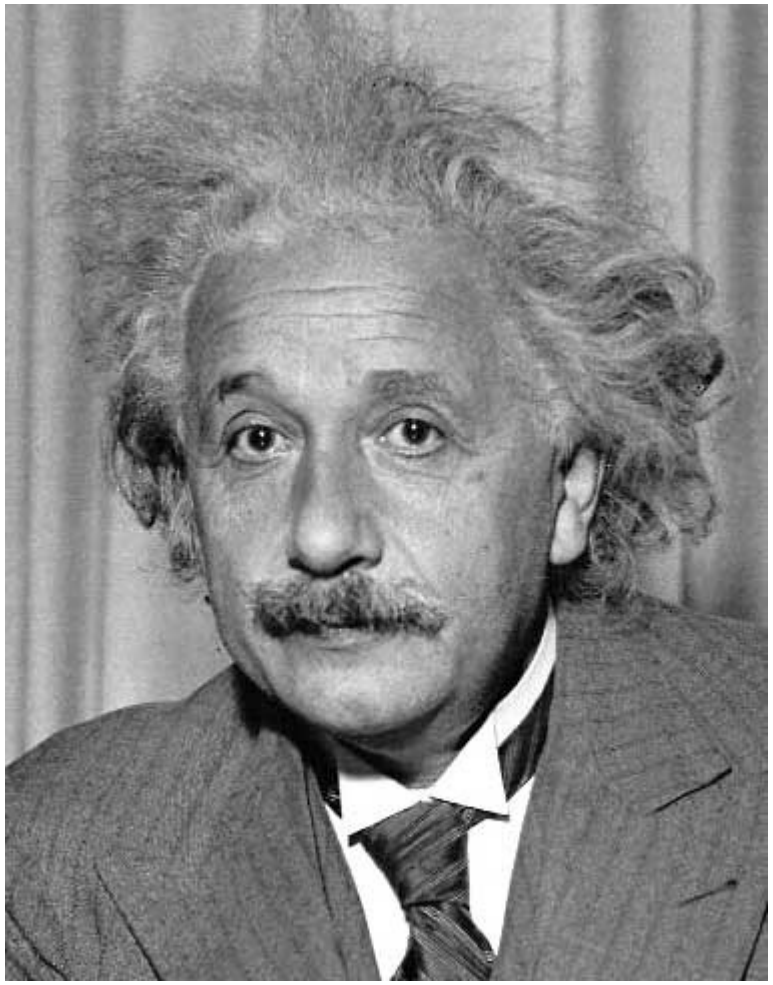
0	0	0
0	0	1
0	0	0



Shifted left
By 1 pixel

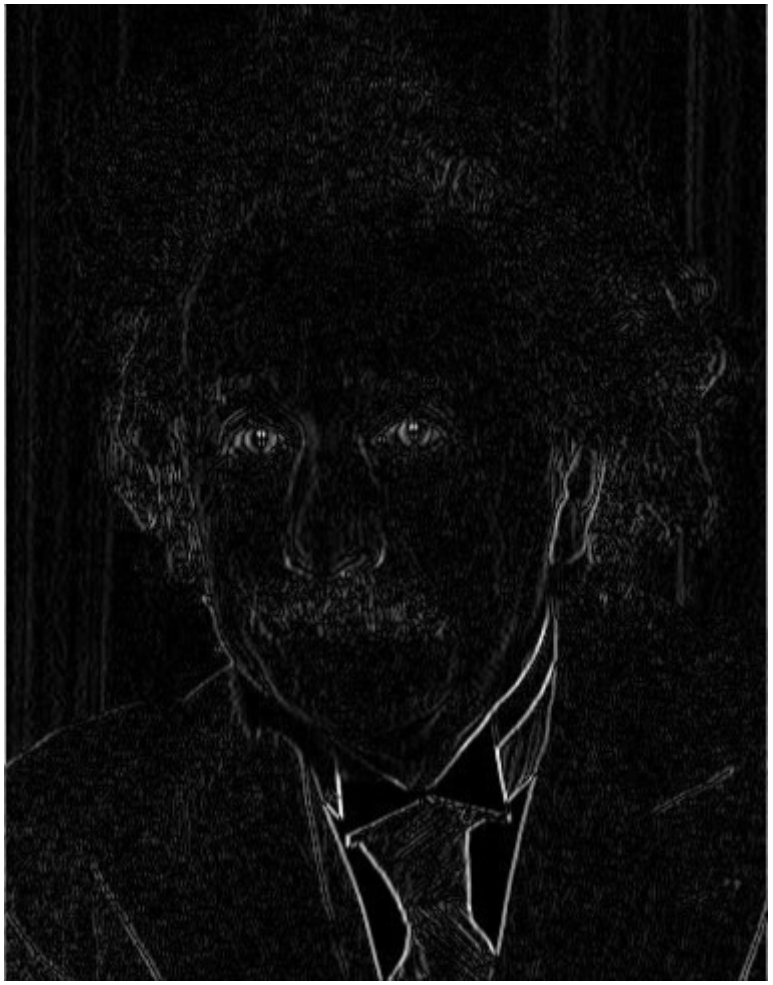
Source: D. Lowe

3. Practice with linear filters



1	0	-1
2	0	-2
1	0	-1

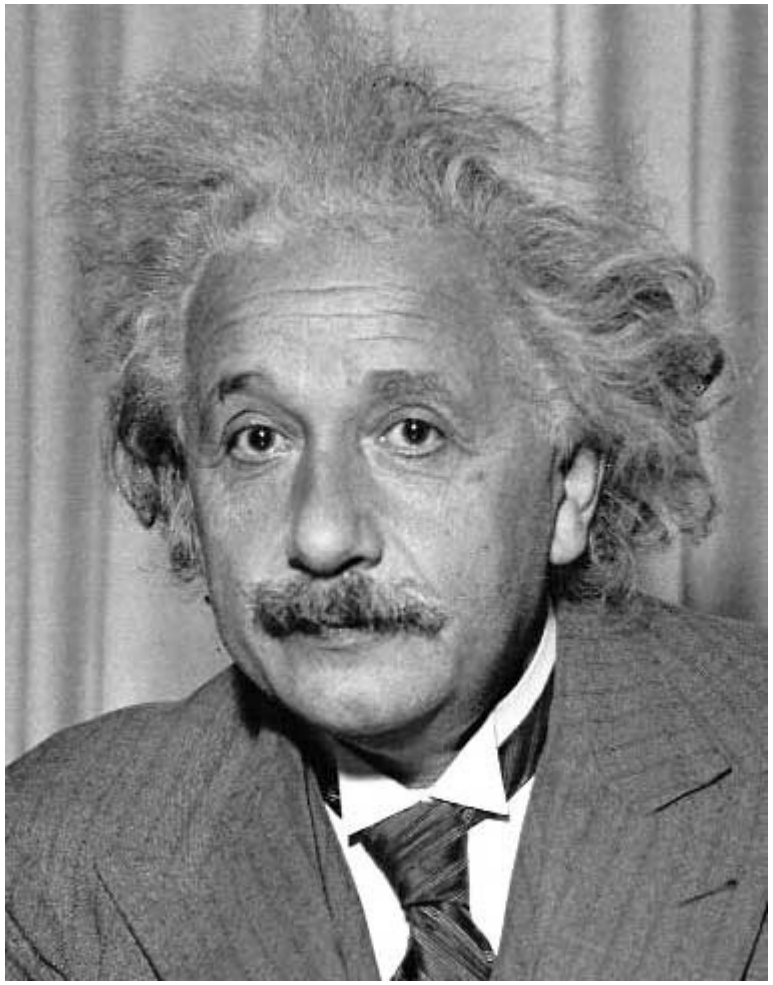
Sobel



Vertical Edge
(absolute value)

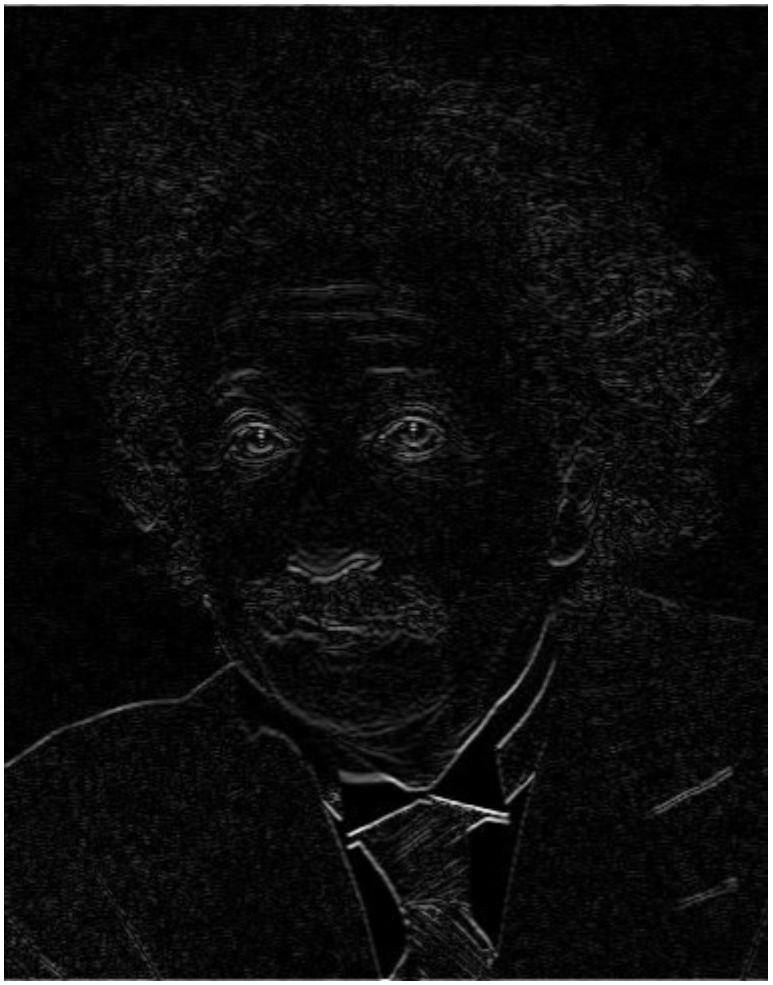
David Lowe

3. Practice with linear filters



1	2	1
0	0	0
-1	-2	-1

Sobel



Horizontal Edge
(absolute value)

David Lowe

4. Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(Note that filter sums to 1)

?

Source: D. Lowe

4. Practice with linear filters

Original

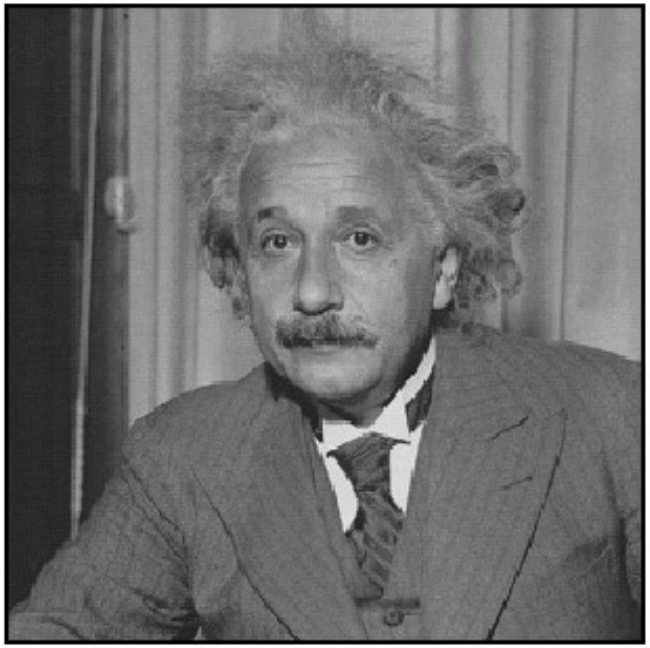
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Sharpening filter

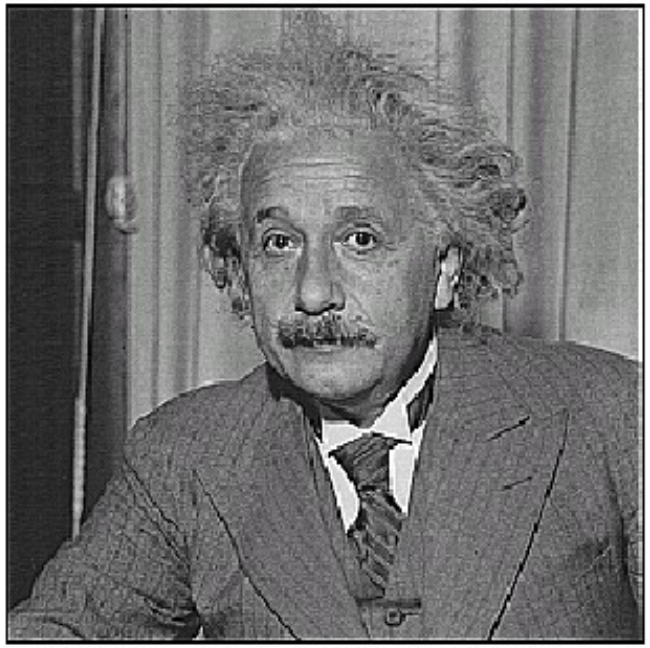
- Accentuates differences with local average

Source: D. Lowe

4. Practice with linear filters



before



after

Sharpening

Source: D. Lowe

Filtering: Correlation vs. Convolution

- 2d correlation

  `h=filter2(f,I);` or `h=imfilter(I,f);`

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Filtering: Correlation vs. Convolution

- 2d correlation

  `h=filter2(f,I);` or `h=imfilter(I,f);`

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

- 2d convolution

  `h=conv2(f,I);`

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m-k, n-l]$$

`conv2(I,f)` is the same as `filter2(rot90(f,2),I)`

Correlation and convolution are identical when the filter is symmetric.

Key properties of linear filters

Linearity:

$\texttt{imfilter(I, } f_1 + f_2\texttt{)} =$

$\texttt{imfilter(I,} f_1\texttt{)} + \texttt{imfilter(I,} f_2\texttt{)}$

Shift invariance: same behavior regardless of pixel location

`imfilter(I,shift(f)) = shift(imfilter(I,f))`

Any linear, shift-invariant operator can be represented as a convolution

Source: S. Lazebnik

Convolution properties

- Commutative: $a * b = b * a$
  - Conceptually no difference between filter and signal
  - But particular filtering implementations might break this equality, e.g., image edges

- Associative: $a * (b * c) = (a * b) * c$
  - Often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
  - This is equivalent to applying one filter: $a * (b_1 * b_2 * b_3)$
  - Correlation is _not_ associative (rotation effect)
  - Why important?

Source: S. Lazebnik

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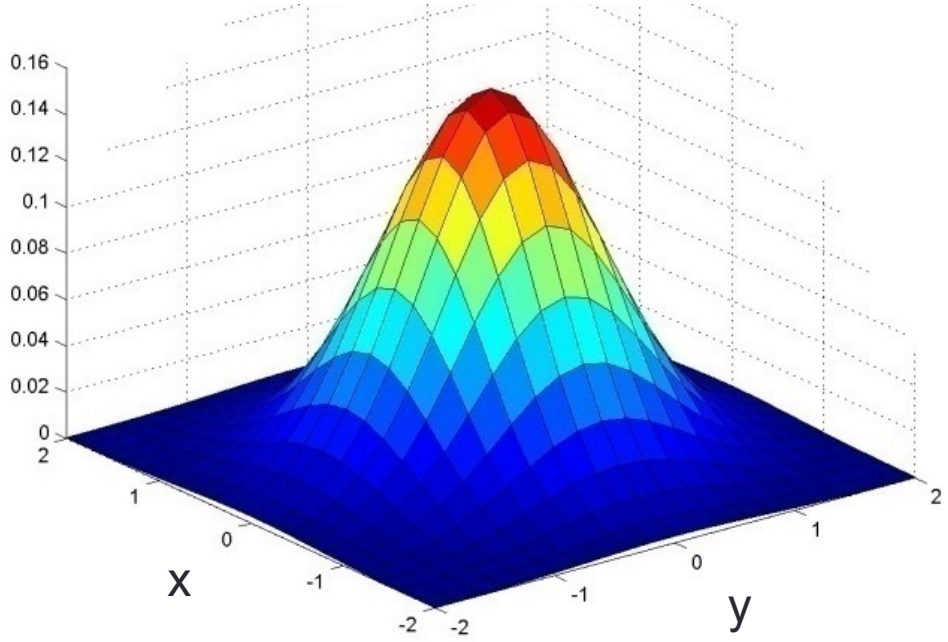
  - This is equivalent to applying one filter: $a * (b_1 * b_2 * b_3)$
  - Correlation is _not_ associative (rotation effect)
  - Why important?
- Distributes over addition: $a * (b + c) = (a * b) + (a * c)$
- Scalars factor out: $ka * b = a * kb = k (a * b)$
- Identity: unit impulse $e = [0, 0, 1, 0, 0]$, $a * e = a$

Source: S. Lazebnik

Important filter: Gaussian

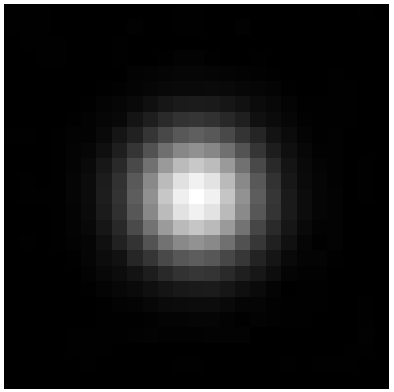
- Weight contributions of neighboring pixels by nearness



x

y					
	0.003	0.013	0.022	0.013	0.003
	0.013	0.059	0.097	0.059	0.013
	0.022	0.097	0.159	0.097	0.022
	0.013	0.059	0.097	0.059	0.013
	0.003	0.013	0.022	0.013	0.003

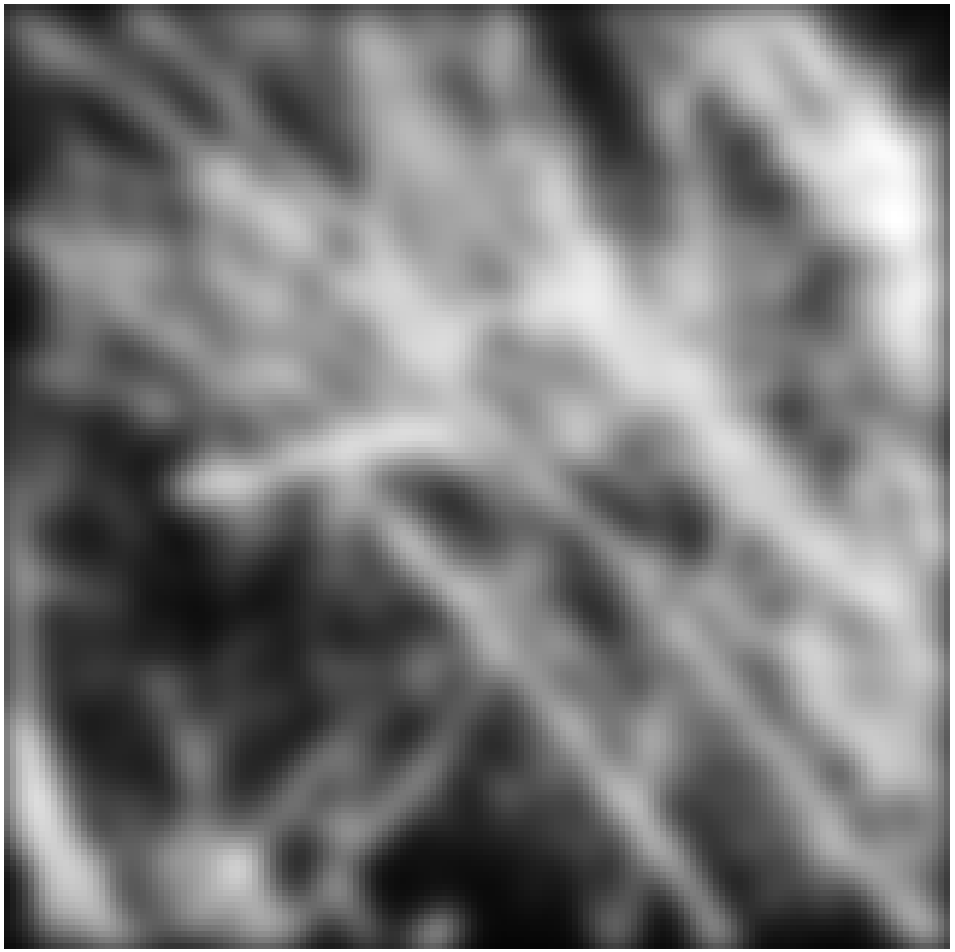
5 x 5, $\sigma = 1$



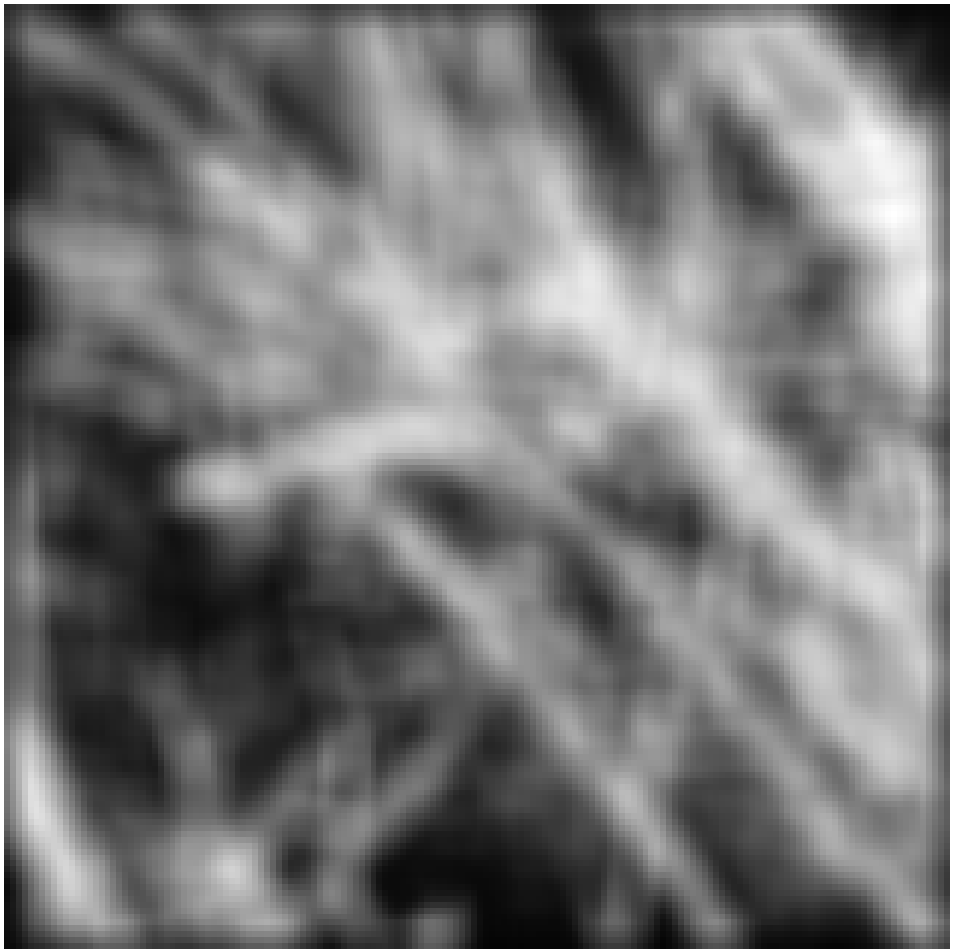
$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Slide credit: Christopher Rasmussen

Smoothing with Gaussian filter



Smoothing with box filter



Gaussian filters

- Remove “high-frequency” components from the image (low-pass filter)
  - Images become more smooth
- Convolution with self is another Gaussian
  - So can smooth with small-width kernel, repeat, and get same result as larger-width kernel would have
  - Convolving two times with Gaussian kernel of width σ is same as convolving once with kernel of width $\sigma\sqrt{2}$
- *Separable* kernel
  - Factors into product of two 1D Gaussians

Source: K. Grauman

Separability of the Gaussian filter

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{x^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{y^2}{2\sigma^2}}\right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

Separability example

2D convolution
(center location only)

1	2	1
2	4	2
1	2	1

$*$

2	3	3
3	5	5
4	4	6

65

The filter factors
into a product of 1D
filters:

1	2	1
2	4	2
1	2	1

=

1
2
1

x

1	2	1

Perform convolution
along rows:

1	2	1

*

2	3	3
3	5	5
4	4	6

=

	11	
	18	
	18	

Followed by convolution
along the remaining column:

1
2
1

*

	11	
	18	
	18	

=

	65	

Source: K. Grauman

Separability

- Why is separability useful in practice?

Separability

- Why is separability useful in practice?
- If K is width of convolution kernel:
  - 2D convolution = K^2 multiply-add operations
  - 2x 1D convolution: 2K multiply-add operations

Practical matters

How big should the filter be?

- Values at edges should be near zero
- Gaussians have infinite extent…
- Rule of thumb for Gaussian: set filter half-width to about 3 σ

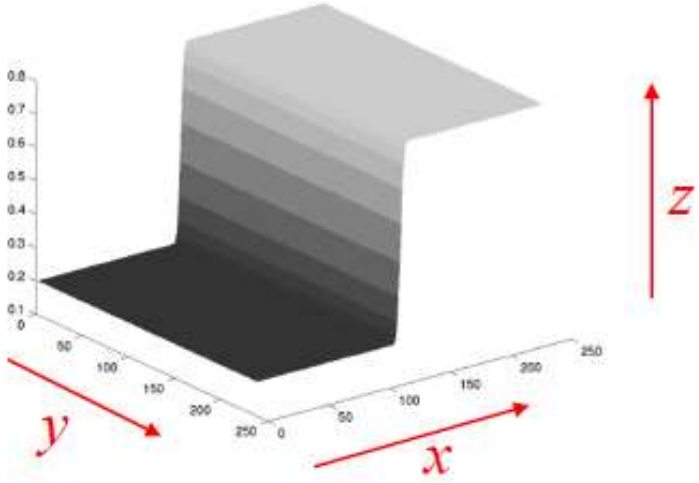
Practical matters

- What about near the edge?
  - the filter window falls off the edge of the image
  - need to extrapolate
  - methods:
    - clip filter (black)
    - wrap around
    - copy edge
    - reflect across edge

Source: S. Marschner

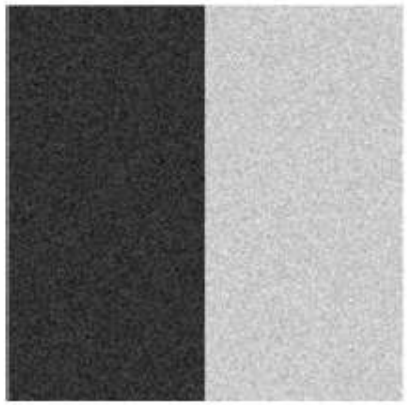
Computing Derivatives using Linear Filters

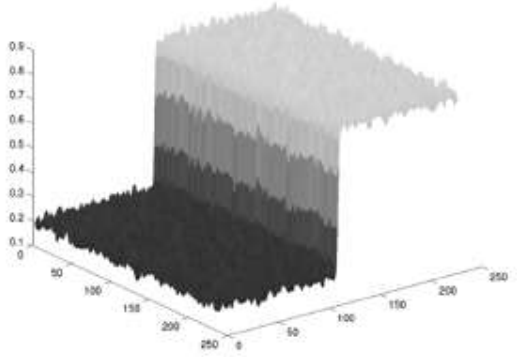
Think of the image as a surface with $z = f(x,y)$



Computing Derivatives using Linear Filters

Objective: compute gradient $\nabla f(x,y) = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y})$ of the image "surface"





- e.g. as a method to find the edge in the image
- there is the problem of noise ….

Computing Derivatives using Linear Filters

1D

Want $f'(x)$, use central difference

$$f'(i) = \frac{f_{i+1} - f_{i-1}}{2}$$

which is equivalent to the molecule $[-\frac{1}{2}, 0, \frac{1}{2}]$.

For a noisy signal the derivative amplifies the noise.

Solution?

Computing Derivatives using Linear Filters

1D

Want $f'(x)$, use central difference

$$f'(i) = \frac{f_{i+1} - f_{i-1}}{2}$$

which is equivalent to the molecule $[-\frac{1}{2}, 0, \frac{1}{2}]$.

For a noisy signal the derivative amplifies the noise.

Solution

- smooth with a Gaussian filter
- then differentiate

Computing Derivatives using Linear Filters

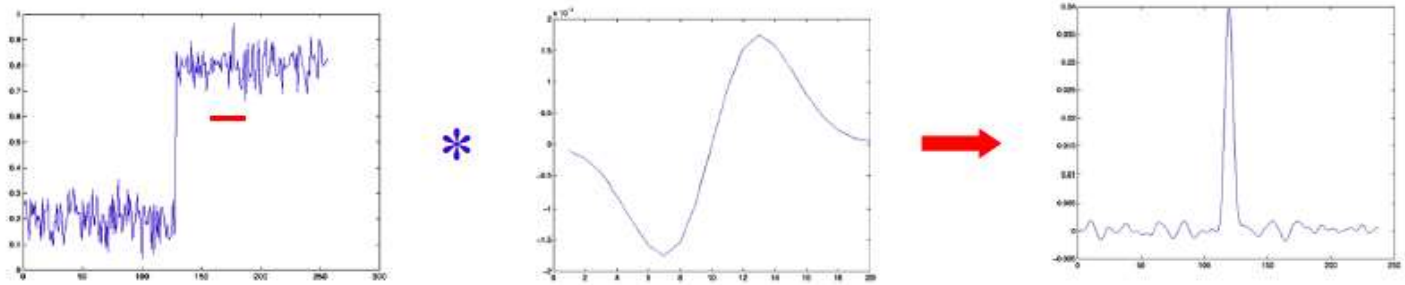
Differentiate smoothed signal $G(x) * f(x)$

$$\frac{d(G(x) * f(x))}{dx} = G(x) * \frac{df(x)}{dx}$$

$$= \frac{dG(x)}{dx} * f(x)$$

$$= \left(\frac{-x}{\sqrt{2\pi}\sigma^3} e^{-x^2/2\sigma^2} \right) * f(x)$$

Convolution with a derivative of Gaussian filter



e.g. for $\sigma = 1$ the molecule is

$[-0.0133, -0.1080, -0.2420, 0.0000, 0.2420, 0.1080, 0.0133]$.

Computing Derivatives using Linear Filters

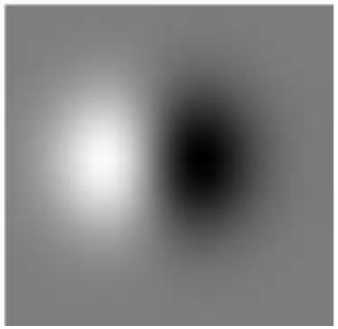
2D

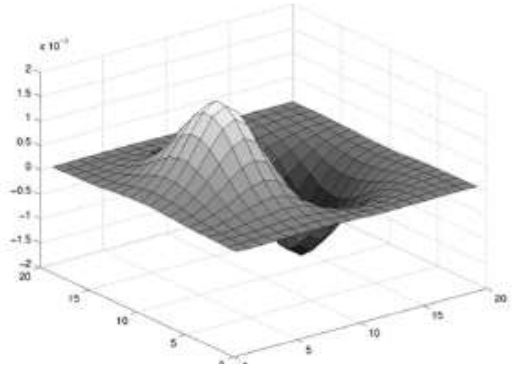
$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-r^2/2\sigma^2}$$

$$\frac{\partial}{\partial x} G(x,y) * f(x,y) = \left(\frac{-x}{2\pi\sigma^4} e^{-(x^2+y^2)/2\sigma^2} \right) * f(x,y)$$

$$= \left(\frac{-x}{\sqrt{2\pi}\sigma^3} e^{-x^2/2\sigma^2} \right) \times \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-y^2/2\sigma^2} \right) * f(x,y)$$

Filtering with a 1D derivative of Gaussian filter in x and a 1D Gaussian filter in y – it is a separable filter

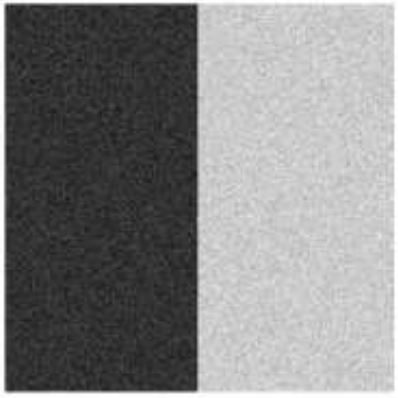




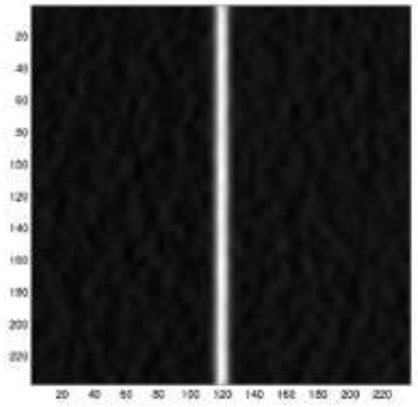
Computing Derivatives using Linear Filters

Example

filter with x and y derivatives of Gaussian to obtain directional image derivatives



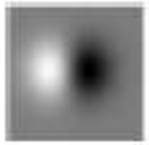
$$I_x = G_x(x,y) * I(x,y)$$



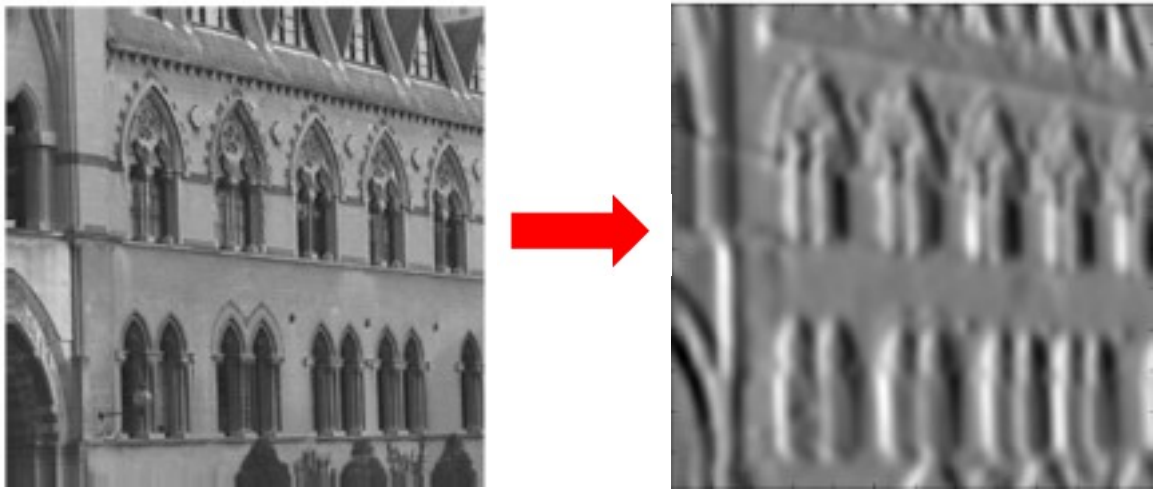
Computing Derivatives using Linear Filters

x-deriv

*

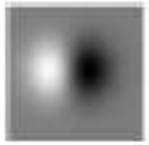


Computing Derivatives using Linear Filters

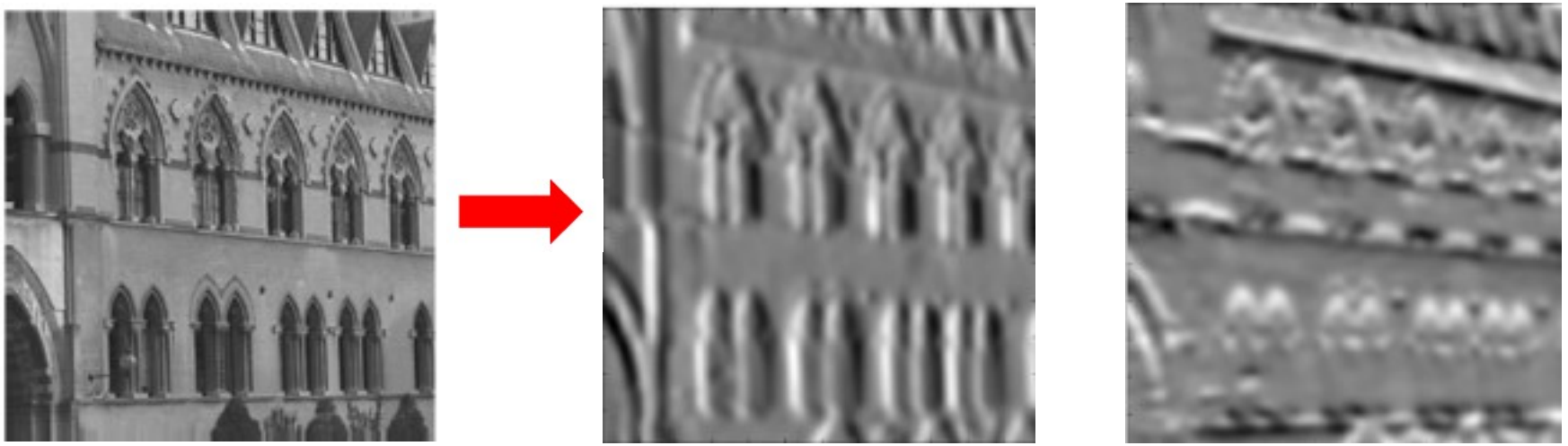


x-deriv

*



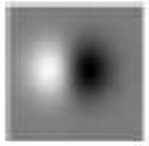
Computing Derivatives using Linear Filters



x-deriv

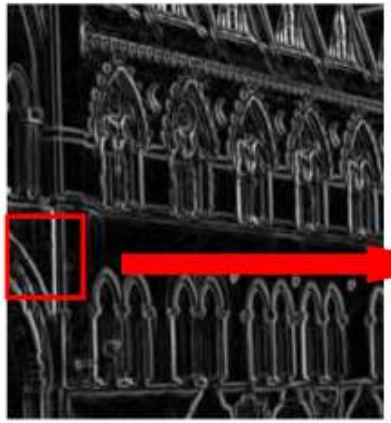
y-deriv

*



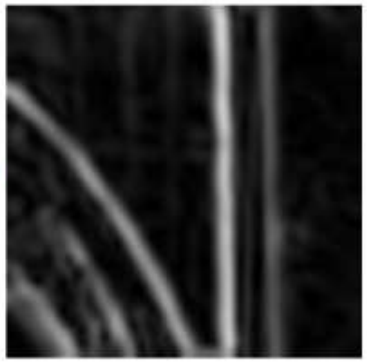
Computing Derivatives using Linear Filters

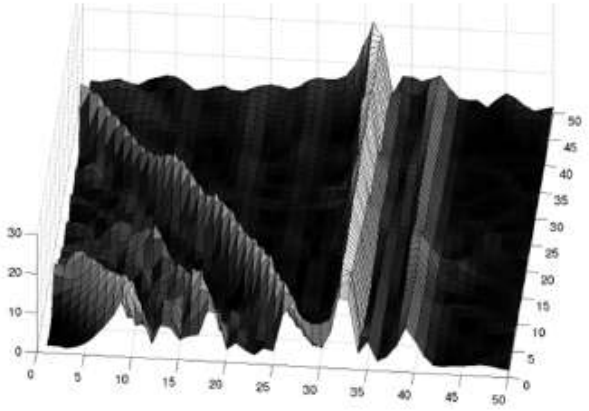
image



gradient magnitude

$$|\nabla(G(x,y) * I(x,y))|$$





A note about the Divergence

Let x, y be a 2D Cartesian coordinates

Let i, j be corresponding basis of unit vectors

The divergence of a continuously differential vector field

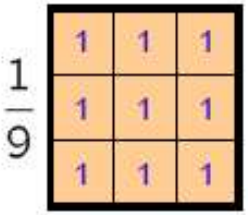
$$F = Ui + Vj$$

is defined as the (signed) scalar-valued function:

$$\mathrm{div} F = \nabla \cdot F = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}) \cdot (U, V) = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}$$

Filtering: summary

Linear filtering is a weighted sum/difference of pixel values



- Can smooth, sharpen, translate (among many other uses)
- Filtering in Matlab, e.g. to filter image f with h

```
g = filter2( h, f );
```

h=filter f=image

```
e.g. h = fspecial('gaussian');
```

Isotropic Diffusion

The diffusion equation is a general case of the heat equation that describes the density changes in a material undergoing diffusion over time. Isotropic diffusion, in image processing parlance, is an instance of the heat equation as a partial differential equation (PDE), given as:

$$\frac{\partial I}{\partial t} = \nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$$

where, I is the image and *t* is the time of evolution.

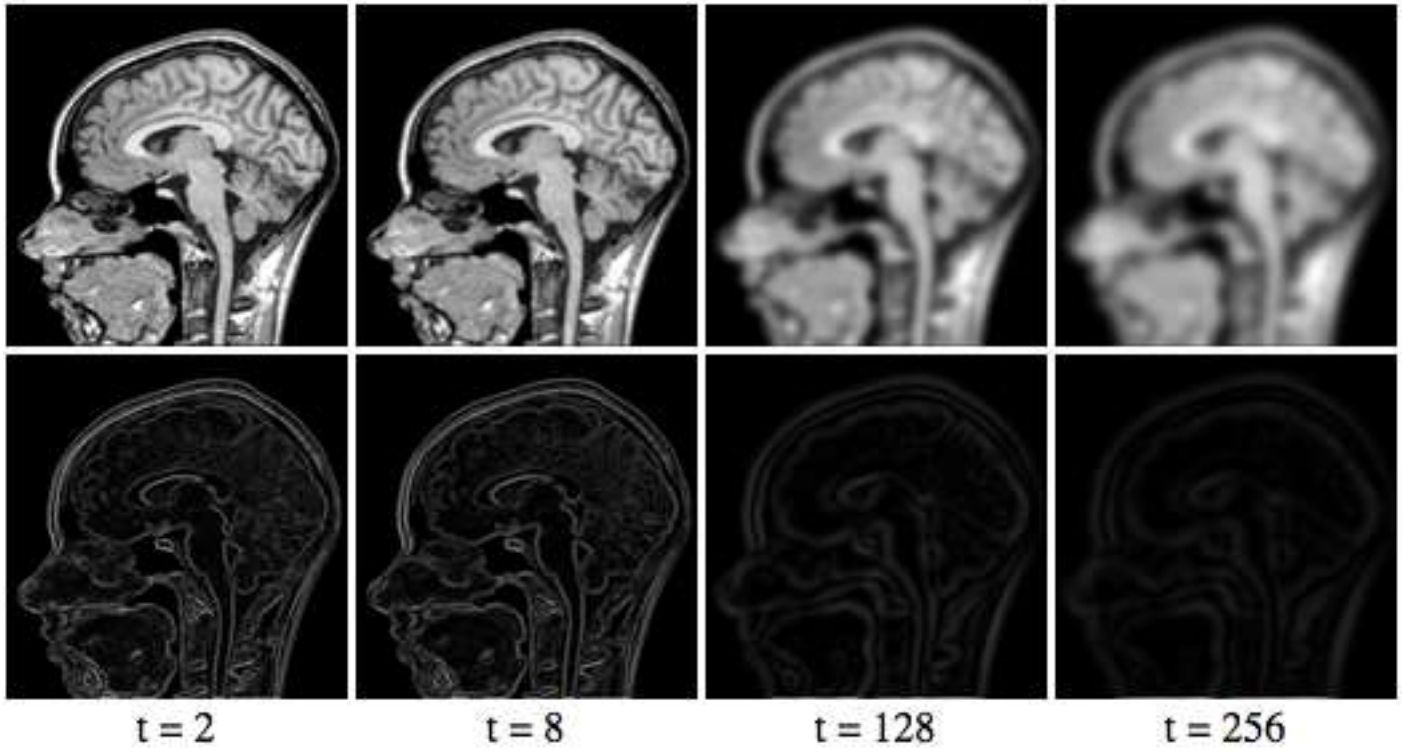
Isotropic Diffusion

Solving this for an image is equivalent to convolution with some Gaussian kernel.

In practice we iterate as follows:

$$I_{i,j}^{t+1} = I_{i,j}^{t} + \lambda \left[I_{i-1,j}^{t} + I_{i+1,j}^{t} + I_{i,j-1}^{t} + I_{i,j+1}^{t} - 4I_{i,j}^{t} \right]$$

Isotropic Diffusion



We can notice that while the diffusion process blurs the image considerably as the number of iterations increases, the edge information progressively degrades as well.

Anisotropic Diffusion: Perona-Malik

Perona & Malik introduce the flux function as a means to constrain the diffusion process to contiguous homogeneous regions, but not cross region boundaries. The heat equation (after appropriate expansion of terms) is thus modified to:

$$\frac{\partial I}{\partial t} = c(x, y, t)\Delta I + \nabla c \cdot \nabla I$$

where *c* is the proposed flux function which controls the rate of diffusion at any point in the image.

Anisotropic Diffusion: Perona-Malik

A choice of *c* such that it follows the gradient magnitude at the point enables us to restrain the diffusion process as we approach region boundaries. As we approach edges in the image, the flux function may trigger inverse diffusion and actually enhance the edges.

Anisotropic Diffusion: Perona-Malik

Perona & Malik suggest the following two flux functions:

$$c\left(||\nabla I||\right) = e^{-(||\nabla I||/K)^2}$$

$$c\left(||\nabla I||\right) = \frac{1}{1 + \left(\frac{||\nabla I||}{K}\right)^2}$$

Anisotropic Diffusion: Perona-Malik

The flux functions offer a trade-off between edge-preservation and blurring (smoothing) homogeneous regions. Both the functions are governed by the free parameter κ which determines the edge-strength to consider as a valid region boundary. Intuitively, a large value of κ will lead back into an isotropic-like solution.

We will experiment with both the flux functions in this report.

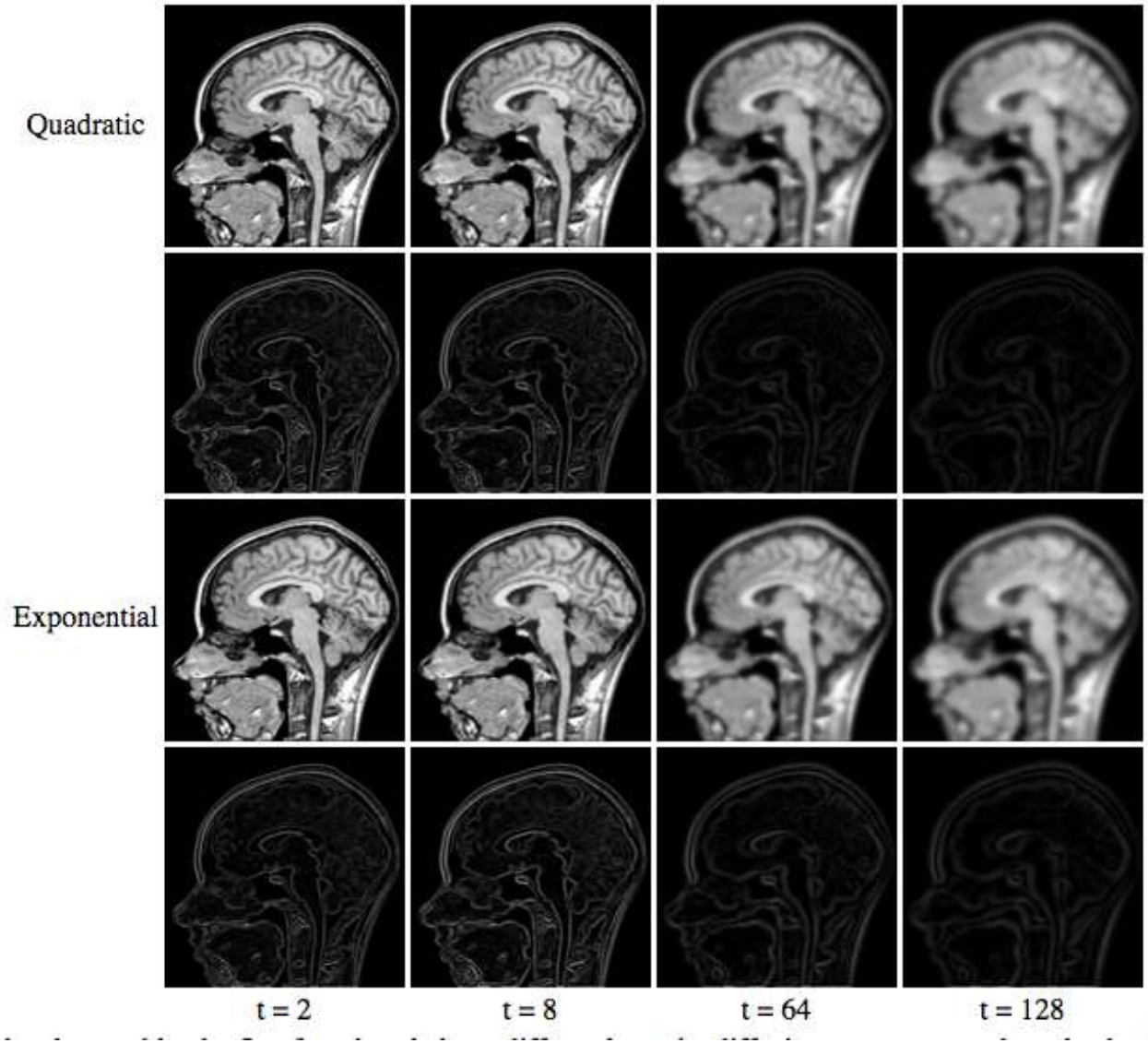
Anisotropic Diffusion: Perona-Malik

A discrete numerical solution can be derived for the anisotropic case as follows:

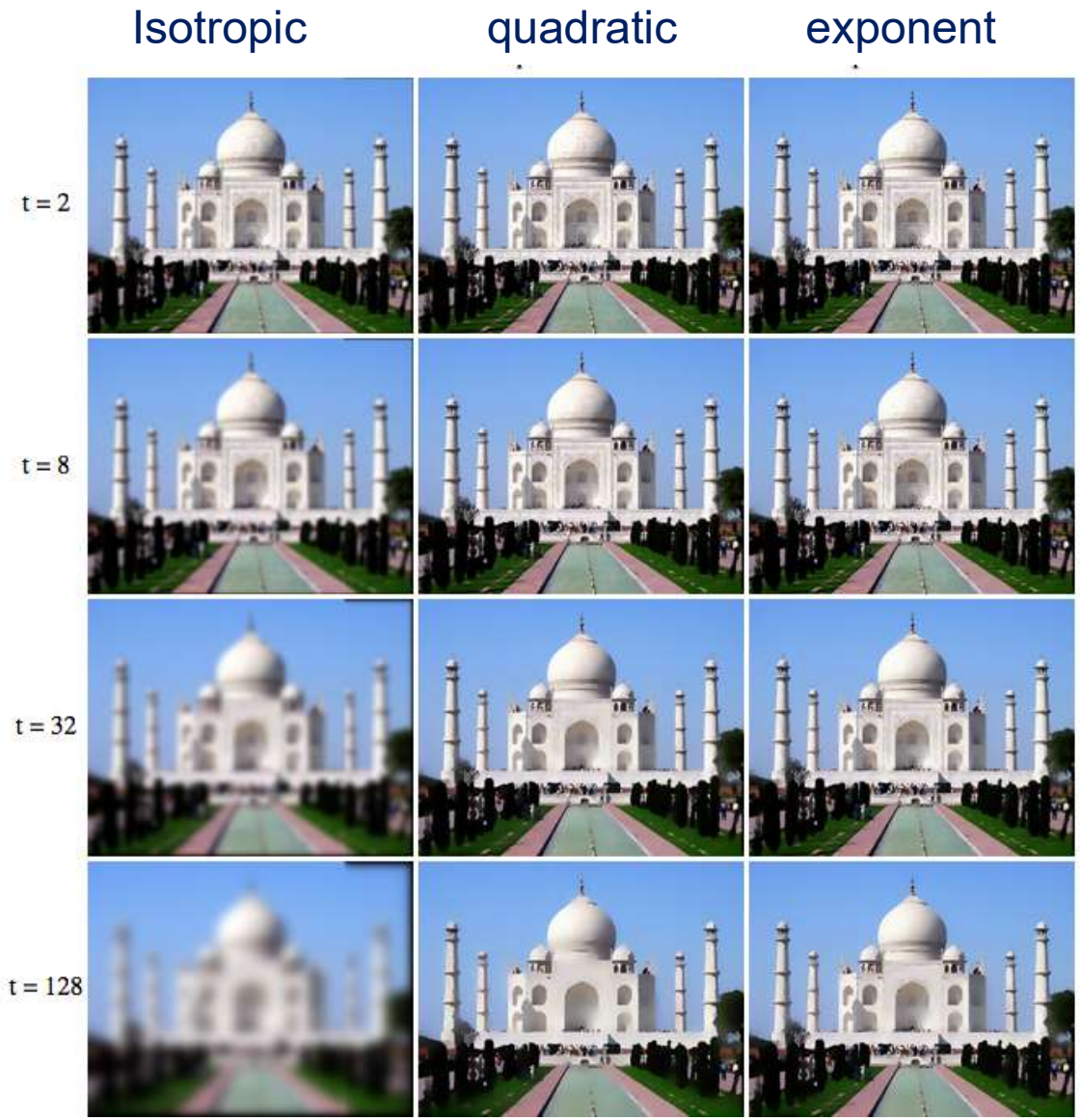
$$I_{i,j}^{t+1} = I_{i,j}^{t} + \lambda \left[c_N \cdot \nabla_N I + c_S \cdot \nabla_S I + c_E \cdot \nabla_E I + c_W \cdot \nabla_W I \right]_{i,j}^{t}$$

where {N,S,W,E} correspond to the pixel above, below, left and right of the pixel under consideration (i,j).

Anisotropic Diffusion: Perona-Malik



Anisotropic vs. Isotropic Diffusion



Bonus Question: Image Enhancement

- Take an image (any image, but preferably one's that needs enhancement) and enhance it.
- Use what learned in this class to do so
- Plot the "before" and "after"
- Plot its derivatives before and after
- Matlab code is needed
- 3 Best works in class get 1 bonus point

Next Class