CHAPTER 1 ❑ MATLAB EXERCISES

1. Consider the linear system of Example 7 in Section 1.2.

$$\begin{aligned} x - 2y + 3z &= 9 \\ -x + 3y \quad\quad &= -4 \\ 2x - 5y + 5z &= 17 \end{aligned}$$

(a) Use the MATLAB command **rref** to solve the system.

(b) Let A be the coefficient matrix, and B be the right-hand side of the system.

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 9 \\ -4 \\ 17 \end{bmatrix}$$

Use the MATLAB command **A\B** to solve the system.

2. Enter the matrix

$$A = \begin{bmatrix} -3 & 2 & 4 & 5 & 1 \\ 3 & 0 & 2 & -2 & 0 \\ -9 & 4 & 6 & 12 & 2 \end{bmatrix}.$$

Use the MATLAB command **rref** to find the reduced row-echelon form of A. What is the solution to the linear system represented by the augmented matrix A?

3. Solve the linear system

$$\begin{aligned} 16x - 120y + 240z - 140w &= -4 \\ -120x + 1200y - 2700z + 1680w &= 60 \\ 240x - 2700y + 6480z - 4200w &= -180 \\ -140x + 1680y - 4200z + 2800w &= 140 \end{aligned}.$$

You can display more significant digits of the answer by entering **format long** before solving the system. Return to the standard format by entering **format short**.

4. Use the MATLAB command **rref** to determine which of the following matrices are row-equivalent to

$$B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}.$$

(a) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 3 & 2 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$ (c) $\begin{bmatrix} 12 & 11 & 10 & 9 \\ 4 & 3 & 2 & 1 \\ 8 & 7 & 6 & 5 \end{bmatrix}$

5. Let A be the coefficient matrix, and B the right-hand side of the linear system of equations

$$\begin{aligned} 3x + 3y + 12z &= 6 \\ x + y + 4z &= 2 \\ 2x + 5y + 20z &= 10 \\ -x + 2y + 8z &= 4. \end{aligned}$$

Enter the matrices A and B, and form the augmented matrix C for this system by using the MATLAB command **C = [A B]**. Solve the system using **rref**.

6. The MATLAB command **polyfit** allows you to fit a polynomial of degree $n - 1$ to a set of n data points in the plane

$$(x_1, y_1), \quad (x_2, y_2), \quad . \; . \; . \; , \quad (x_n, y_n).$$

Find the fourth-degree polynomial that fits the five data points of Example 2 in Section 1.3 by letting

$$x = \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 2 \end{bmatrix} \qquad y = \begin{bmatrix} 3 \\ 5 \\ 1 \\ 4 \\ 10 \end{bmatrix}$$

and entering the MATLAB command **polyfit(x,y,4)**.

7. Find the second-degree polynomial that fits the points $(1, -2)$, $(2, 4)$, $(-4, -6)$.

8. Find the sixth-degree polynomial that fits the points $(0, 0)$, $(-1, 4.5)$, $(-2, 133)$, $(-3, 1225.5)$, $(1, -0.5)$, $(2, 3)$, $(3, 250.5)$.

9. The following table gives the revenues y (in billions of dollars) for General Dynamics Corporation from 2005 through 2009.

Year	2005	2006	2007	2008	2009
Revenue, y	21.2	24.1	27.2	29.3	32.0

Use **p=polyfit(x,y,4)** to fit the fourth-degree polynomial to these data. Let x represent the year, with $x = 5$ corresponding to 2005. Then use **f=polyval(p,10)** to estimate the revenue in 2010. (The actual revenue in 2010 was $32.5 billion)

CHAPTER 2 ❑ MATLAB EXERCISES

1. Enter the matrices

$$A = \begin{bmatrix} 0 & -4 & 5 \\ 3 & 1 & -2 \\ 2 & 1 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -5 & 6 & 7 \\ 0 & -1 & 2 \\ 4 & 0 & -3 \end{bmatrix}.$$

Use MATLAB to find

(a) $A + B$ (b) $B - 3A$ (c) AB (d) BA.

2. Enter the three matrices

$$A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{bmatrix} \qquad B = \begin{bmatrix} 1.0000 & 0.5000 & 0.3333 & 0.2500 \\ 0.5000 & 0.3333 & 0.2500 & 0.2000 \\ 0.3333 & 0.2500 & 0.2000 & 0.1667 \\ 0.2500 & 0.2000 & 0.1667 & 0.1429 \end{bmatrix}$$

$$C = \begin{bmatrix} 16 & -120 & 240 & -140 \\ -120 & 1200 & -2700 & 1680 \\ 240 & -2700 & 6480 & -4200 \\ -140 & 1680 & -4200 & 2800 \end{bmatrix}.$$

(a) Enter **format long** and then calculate $A - B$. Return to the standard short format with **format short**.

(b) Calculate AC and BC. Define what is meant by the inverse of a square matrix. What is the inverse of the matrix A? of the matrix C?

3. Write the following system of linear equations in the form $AX = B$ and use the MATLAB command **A\ B** to solve the system.

$$\begin{aligned} 3x + 3y + 4z &= 2 \\ x + y + 4z &= -2 \\ 2x + 5y + 4z &= 3 \end{aligned}$$

Check your answer using **rref**.

4. Enter the matrices

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 2 & -1 & -5 \\ 7 & -5 & 0 & 6 \\ -4 & 0 & 7 & 12 \end{bmatrix} \quad \text{and} \quad B = \textbf{pascal(4)}.$$

(a) Use the MATLAB command **trace** to find the traces of A, B, and $A + B$. What do you observe?

(b) What is the relationship between the trace of AB and the trace of BA?

5. Use the MATLAB command **diag** to form the 5×5 diagonal matrix D with diagonal entries 0, -1, -2, -3, and -4. Find the product $D^4 = DDDD$. If D is an $n \times n$ diagonal matrix, describe how to find the product D^k for any positive integer k.

6. Enter the three matrices

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -6 & 3 & 5 \\ 0 & -3 & -6 & 8 \\ 3 & 5 & 0 & 7 \\ -1 & 0 & 7 & 9 \end{bmatrix} \quad C = \begin{bmatrix} 16 & -1 & 4 & -1 \\ -3 & 12 & -7 & 8 \\ 4 & -5 & 0 & 0 \\ -14 & 3 & 2 & 8 \end{bmatrix}.$$

(a) Calculate $AB - AC$ and $A(B - C)$. What do you observe?

(b) Calculate $3(AC)$, $A(3C)$, and $(3A)C$. What do you observe?

7. Let

$$A = \begin{bmatrix} 1 & \frac{1}{3} \\ 0 & \frac{1}{4} \end{bmatrix}.$$

Compute A^2, A^3, and A^8. Describe the matrix A^n for large n.

8. Enter the matrices

$$A11 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad A12 = \textbf{zeros}(\mathbf{2, 2}) \qquad A22 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}.$$

(a) Form the 4×4 matrix A using the following MATLAB construction:

A = [A11 A12; A12 A22]

(b) Find the smallest value of n, where n is a positive integer, such that $A^n = A$.

9. Use the MATLAB command **inv** to find the inverse of the following matrix A. Then adjoin the identity matrix $I =$ **eye(3)** to A to form the 3×6 matrix $B = [A \;\; I]$. Row-reduce B to compute the inverse of A again. What do you observe?

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ 7 & 16 & -21 \end{bmatrix}$$

10. Let A and B be the following 3×3 matrices.

$$A = \begin{bmatrix} 2 & 4 & \frac{5}{2} \\ -\frac{3}{4} & 2 & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{4} \\ \frac{3}{2} & \frac{1}{2} & -2 \\ \frac{1}{4} & 1 & \frac{1}{2} \end{bmatrix}$$

(a) Calculate $A^{-1}B^{-1}$, $(AB)^{-1}$, and $(BA)^{-1}$. What do you observe?

(b) Find $(A^{-1})^T$ and $(A^T)^{-1}$. What do you observe? Remember: The MATLAB command for the transpose of a real matrix A is A'.

11. In this exercise, you will use MATLAB to find the least squares regression line for the set of data $(1, 1)$, $(2, 2)$, $(3, 4)$, $(4, 4)$, and $(5, 6)$ from Example 10, Section 2.5.

(a) Form the following matrices.

$$X = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} \qquad Y = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 4 \\ 6 \end{bmatrix}$$

(b) Let $A = (X^TX)^{-1}X^TY$.

(c) Compare your answer with the MATLAB least squares command **polyfit**. (*Hint:* Let $X1 = X(:, 2)$ and enter **polyfit(X1,Y,1)**.)

(d) Plot the data using the following MATLAB commands.

t = (0: 0.1: 6);
plot(X1,Y,'+')

(e) Plot the least squares line for the data by using the following MATLAB commands.

t = (0: 0.1: 6);
p=polyfit(X1,Y,1);
f=polyval(p,t);
plot(t,f,'*')

(f) You can combine the previous plots by using the following MATLAB command.

plot(X1,Y,'+',t,f,'*')

12. Repeat Exercise 11 for the data $(0, 6)$, $(4, 3)$, $(5, 0)$, $(8, -4)$, $(10, -5)$. Plot the data and the least squares line on the interval $[0, 10]$. That is, use **t = (0: 0.1: 10);**.

CHAPTER 3 ❑ MATLAB EXERCISES

1. Use MATLAB to calculate the determinants of the following matrices.

(a) $\begin{bmatrix} 2 & -3 \\ -6 & 9 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & 6 & 7 \\ 0 & -1 & 2 \\ 4 & 0 & -3 \end{bmatrix}$

(c) **pascal(4)** (d) **hilb(8)**

2. Let

$$A = \begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix}.$$

(a) Compute **det(2 * eye(2) − A)**.

(b) Find an integer value of t such that $\det(tI - A) = 0$.

3. Choose arbitrary 4×4 matrices A and B. Compute $\det(A)\det(B)$ and $\det(AB)$. What do you observe? Do the same for $\det(A) + \det(B)$ and $\det(A + B)$.

4. Choose an arbitrary real number t. Form the matrix

$$A = \begin{bmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{bmatrix}$$

and calculate its determinant. Does the value of the determinant depend on t?

5. Consider the matrices

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & -1 & 4 \\ 0 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}.$$

(a) Verify that $\det(A)\det(B) = \det(AB)$.
(b) Verify that $\det(A^T) = \det(A)$.
(c) Verify that $\det(A^{-1}) = 1/\det(A)$.

6. This exercise uses Cramer's Rule to solve the linear system $A\mathbf{x} = \mathbf{b}$ from Example 3, Section 3.4. Let

$$A = \begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 10 \\ 11 \end{bmatrix}$$

be the coefficient matrix and the right-hand side, respectively, of the system. To form the matrix A_1, replace the first column of A with $\mathbf{b}$. To do this, enter the following.

A1 = A
A1(:,[1]) = b

Obtain the solution x_1 by entering the following.

det(A1)/det(A)

Calculate x_2 in a similar manner.

A2 = A
A2(:,[2]) = b
det(A2)/det(A)

7. Use the Cramer's Rule algorithm from Exercise 6 to solve the following linear system. Compare your answer with that obtained using **rref**.

$$\begin{aligned} 3x + 3y + 4z &= 2 \\ x + y + 4z &= -2 \\ 2x + 5y + 4z &= 3 \end{aligned}$$

CHAPTER 4 ❑ MATLAB EXERCISES

1. Let $\mathbf{u}_1 = (1, 1, 2, 2)$, $\mathbf{u}_2 = (2, 3, 5, 6)$, and $\mathbf{u}_3 = (2, -1, 3, 6)$. Use MATLAB to write (if possible) the vector $\mathbf{v}$ as a linear combination of the vectors $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$.

(a) $\mathbf{v} = (0, 5, 3, 0)$

(b) $\mathbf{v} = (-1, 6, 1, -4)$

2. Use MATLAB to determine whether the given set of vectors spans $\mathbf{R}^4$.

(a) $\{(1, -2, 3, 4), (2, 4, 5, 0), (-2, 0, 0, 4), (3, 2, 1, -4)\}$

(b) $\{(0, 1, -1, 1), (2, -2, 3, 1), (7, 0, 1, 0), (5, 2, -2, -1)\}$

3. Use MATLAB to determine whether the set is linearly independent or dependent.

(a) $\{(0, 1, -3, 4), (-1, 0, 0, 2), (0, 5, 3, 0), (-1, 7, -3, -6)\}$

(b) $\{(0, 0, 1, 2, 3), (0, 0, 2, 3, 1), (1, 2, 3, 4, 5), (2, 1, 0, 0, 0), (-1, -3, -5, 0, 0)\}$

4. Use MATLAB to determine whether the set of vectors forms a basis of $\mathbf{R}^4$.

(a) $\{(1, -2, 3, 4), (2, 4, 5, 0), (-2, 0, 0, 4), (3, 2, 1, -4)\}$

(b) $\{(0, 1, -1, 1), (2, -2, 3, 1), (7, 0, 1, 0), (5, 2, -2, -1)\}$

(c) $\{(0, 1, -3, 4), (-1, 0, 0, 2), (0, 5, 3, 0), (-1, 7, -3, -6)\}$

(d) $\{(0, 0, 1, 2), (0, 2, 3, 1), (1, 3, 4, 5), (2, 1, 0, 0), (-3, -5, 0, 0)\}$

5. Suppose you want to find a basis for $\mathbf{R}^4$ that contains the vectors $\mathbf{v}_1 = (1, 1, 0, 0)$. and $\mathbf{v}_2 = (1, 0, 1, 0)$. One way to do this is to consider the set of vectors consisting of $\mathbf{v}_1$ and $\mathbf{v}_2$, together with the standard basis vectors, $\mathbf{e}_1 = (1, 0, 0, 0)$, $\mathbf{e}_2 = (0, 1, 0, 0)$, $\mathbf{e}_3 = (0, 0, 1, 0)$, and $\mathbf{e}_4 = (0, 0, 0, 1)$. Let A be the matrix whose columns consist of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{e}_1$, $\mathbf{e}_3$, $\mathbf{e}_3$, and $\mathbf{e}_4$, and apply **rref** to A to obtain

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Because the leading ones of the reduced matrix on the right are in columns 1, 2, 3, and 6, a basis for $\mathbf{R}^4$ consists of the corresponding column vectors of A: $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{e}_1, \mathbf{e}_4\}$.

A convenient way to construct the matrix A is to define the matrix B whose columns are the given vectors $\mathbf{v}_1$ and $\mathbf{v}_2$. Then A is the matrix obtained by adjoining the 4×4 identity matrix to B: $A = [B \;\; \text{eye}(4)]$.

Use this algorithm to find a basis for $\mathbf{R}^5$ that contains the given vectors.

(a) $\mathbf{v}_1 = (2, 1, 0, 0, 0)$, $\quad \mathbf{v}_2 = (-1, 0, 1, 0, 0)$

(b) $\mathbf{v}_1 = (1, 0, 2, 0, 0)$, $\quad \mathbf{v}_2 = (1, 1, 2, 0, 0)$, $\quad \mathbf{v}_3 = (1, 1, 1, 0, 1)$

6. Use MATLAB to find a subset of the given set of vectors that forms a basis for the span of the vectors.

(a) $\{(1, 2, -1, 0), (-3, -6, 3, 0), (1, 0, 0, 1), (-2, -2, 1, -1)\}$

(b) $\{(0, 0, 1, 1, 0), (1, 1, 0, 0, 1), (1, 1, 1, 1, 1), (1, 1, 2, 2, 1), (0, 0, 3, 3, 1), (0, 0, 0, 0, 1)\}$

7. Let

$$A = \begin{bmatrix} -1 & 2 & 0 & 0 & 3 \\ 0 & 2 & 3 & -1 & 2 \\ -1 & 4 & 3 & -1 & 5 \\ 2 & -4 & 0 & 0 & -6 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

(a) Find a basis for the row space of A.

(b) Find a basis for the column space of A.

(c) Use the MATLAB command **rank** to find the rank of A.

8. Find a basis for the nullspace of the given matrix A. Then verify that the sum of the rank and nullity of A equals the number of columns.

(a) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$

(b) $A = \text{hilb}(5)$

(c) $A = \text{pascal}(5)$

(d) $A = \text{magic}(6)$

9. Let $\{(1, 0, 1), (0, -1, 2), (2, 3, -5)\}$ be a (nonstandard) basis for $\mathbf{R}^3$. You can find the coordinate matrix of $\mathbf{x} = (1, 2, -1)$ relative to this basis by writing $\mathbf{x}$ as a linear combination of the basis vectors. That is, the coordinate matrix is the solution vector to the linear system $B\mathbf{c} = \mathbf{x}$, where the basis vectors form the columns of B. Use MATLAB to solve this system and compare your answer to Section 4.7, Example 3.

10. Let $B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ and $BPRIME = \{(1, 0, 1), (0, -1, 2), (2, 3, -5)\}$ be the two bases of R^3 given in Section 4.7, Example 4. You can use MATLAB to find the transition matrix from B to $BPRIME$ by first forming the two matrices

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad BPRIME = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 3 \\ 1 & 2 & -5 \end{bmatrix}$$

Adjoin B and $BPRIME$ by using the MATLAB command **C = [BPRIME B]**. Let A be the reduced row-echelon form of C, **A = rref(C)**. Finally, obtain $PINV = P^{-1}$ by deleting the first three columns of this reduced matrix using the MATLAB command **PINV = A(:,4:6)**.

Find the transition matrix from B to $BPRIME$.

(a) $B = \{(-3, 2), (4, -2)\}, \quad BPRIME = \{(-1, 2), (2, -2)\}$.

(b) $B = \{(1, 1, 1, 1), (0, 1, 1, 1), (0, 0, 1, 1), (0, 0, 0, 1)\}$,
$BPRIME = \{(1, 0, 1, 0), (1, 0, -1, 0), (0, 1, 0, 1), (0, 1, 0, -1)\}$

CHAPTER 5 ❑ MATLAB EXERCISES

1. Use the MATLAB command **norm(v)** to find

(a) the length of the vector $\mathbf{v} = (0, -2, 1, 4, -2)$.

(b) a unit vector in the direction of $\mathbf{v} = (-3, 2, 4, -5, 0, 1)$.

(c) the distance between the vectors $\mathbf{u} = (0, 2, 2, -3)$ and $\mathbf{v} = (-4, 7, 10, 1)$.

2. The dot product of the vectors (written as columns)

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

is given by the matrix product

$$\mathbf{u} \cdot \mathbf{v} = \mathbf{u}^T\mathbf{v} = [u_1 \quad u_2 \quad . \; . \; . \quad u_n] \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}.$$

So, you can compute the dot product of **u** and **v** by multiplying the transpose of **u** times the vector **v**. Let $\mathbf{u} = (2, -5, 0, 4, 8)$, $\mathbf{v} = (0, -3, 2, -1, 1)$ and $\mathbf{w} = (1, -1, 0, 0, 7)$, and use MATLAB to find the following.

(a) $\mathbf{u} \cdot \mathbf{v}$

(b) $(\mathbf{u} \cdot \mathbf{v})\mathbf{w}$

(c) $\mathbf{u} \cdot (2\mathbf{v} - 3\mathbf{w})$

(d) $\mathbf{v} \cdot \mathbf{v}$

3. The angle θ between two nonzero vectors is given by

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}.$$

Use MATLAB to find the angle between $\mathbf{u} = (-3, 4, 0)$ and $\mathbf{v} = (1, 1, 4)$. (*Hint:* Use the built-in inverse cosine function, **acos**.)

4. You can find the orthogonal projection of the column vector **x** onto the column vector **y** by computing

$$\frac{\mathbf{x}^T\mathbf{y}}{\mathbf{y}^T\mathbf{y}}\mathbf{y}.$$

Use MATLAB to find the following projections of **x** onto **y**.

(a) $\mathbf{x} = (3, 1, 2)$, $\mathbf{y} = (7, 1, -2)$

(b) $\mathbf{x} = (1, 1, 1)$, $\mathbf{y} = (-1, 1, 1)$

(c) $\mathbf{x} = (0, 1, 3, -3)$, $\mathbf{y} = (4, 0, 0, 1)$

5. Use the MATLAB command **cross(u, v)** to find the cross products of the following vectors.

(a) $\mathbf{u} = (1, -2, 1)$, $\mathbf{v} = (3, 1, -2)$

(b) $\mathbf{u} = (0, 1, -2)$, $\mathbf{v} = (-5, 14, 6)$

6. Let $\mathbf{u} = (-3, 2, 4)$, $\mathbf{v} = (5, 0, -7)$, and $\mathbf{w} = (-1, -5, 6)$. Use MATLAB to illustrate the following properties of the cross product.

(a) $\mathbf{u} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{u})$

(b) $\mathbf{u} \times \mathbf{u} = \mathbf{0}$

(c) $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$

(d) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$

7. The MATLAB command **A\b** finds the least squares solution to the linear system of equations $A\mathbf{x} = \mathbf{b}$. For instance, if

$$A = \begin{bmatrix} 0 & 2 \\ 3 & 0 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix},$$

then the command **A\b** gives the answer

$$\begin{bmatrix} 0.6000 \\ 0.5000 \end{bmatrix}.$$

Use MATLAB to solve the least squares problem $A\mathbf{x} = \mathbf{b}$ for the given matrices.

(a) $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 3 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} -1 \\ 2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

8. Use MATLAB to find bases for the four fundamental subspaces of the following matrices.

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 1 & 4 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 3 & -2 & -1 & 0 \\ 4 & 1 & -1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$

CHAPTER 6 ❑ MATLAB EXERCISES

1. Find the kernel and range of the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$ for these matrices A.

(a) $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \\ -2 & 0 & -2 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & -1 & -2 & 2 \\ 1 & 2 & 4 & -5 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 9 & 10 & 11 & 12 \\ 5 & 6 & 7 & 8 \\ -13 & -14 & -15 & -16 \end{bmatrix}$ (d) $A = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \end{bmatrix}$

2. Let B be the upper trianglular matrix generated by the MATLAB command **B = triu(ones(6))**. Let $A = BB^T - B$ and determine the rank and nullity of the linear transformation

$L: R^6 \to R^6$, $L(\mathbf{x}) = A\mathbf{x}$.

3. Which of these linear transformations defined by $T(\mathbf{x}) = A\mathbf{x}$ are one-to-one? Which are onto?

(a) $A =$ magic(6) (b) $A =$ hilb(6) (c) $A =$ tril(ones (6))

4. Let $T: R^n \to R^m$ be a linear transformation. Let $B = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ and $BPRIME = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_m\}$ be bases for R^n and R^m, respectively. You can use MATLAB to find the matrix of T relative to the bases B and $BPRIME$ as follows.

(a) Form the matrices B and $BPRIME$ whose *columns* are the given basis vectors.

(b) Let A be the $m \times n$ standard matrix of T.

(c) Adjoin $BPRIME$ to AB to form the $m \times (m + n)$ matrix C: **C = [BPRIME A*B]**.

(d) Use **rref(C)** to calculate the reduced row-echelon form of C. The $m \times n$ matrix composed of the right-hand n columns of the reduced row-echelon form of C form the matrix of T relative to the bases B and $BPRIME$.

Use this algorithm to find the matrix of the following linear transformations relative to the given bases.

(a) $T: R^2 \to R^3$, $T(x, y) = (x + y, x, y)$,
$B = \{(1, -1), (0, 1)\}$, $BPRIME = \{(1, 1, 0), (0, 1, 1), (1, 0, 1)\}$

(b) $T: R^3 \to R^2$, $T(x, y, z) = (2x - z, y - 2x)$,
$B = \{(2, 0, 1), (0, 2, 1), (1, 2, 1)\}$, $BPRIME = \{(1, 1), (2, 0)\}$

(c) $T: R^3 \to R^4$, $T(x, y, z) = (2x, x + y, y + z, x + z)$,
$B = \{(2, 0, 1), (0, 2, 1), (1, 2, 1)\}$,
$BPRIME = \{(1, 0, 0, 1), (0, 1, 0, 1), (1, 0, 1, 0), (1, 1, 0, 0)\}$

5. Use the results of Exercise 4 to find the image of the given vector $\mathbf{v}$ two ways: first by calculating $T(\mathbf{v}) = A\mathbf{v}$, and second by using the matrix of T relative to the bases B and $BPRIME$.

(a) $\mathbf{v} = (5, 4)$

(b) $\mathbf{v} = (0, -5, 7)$

(c) $\mathbf{v} = (1, -5, 2)$

6. Let $B = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ and $BPRIME = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_n\}$ be two ordered bases for R^n. Recall from Section 4.7 that you find the transition matrix P^{-1} from B to $BPRIME$ as follows.

(a) Form the matrices B and $BPRIME$ whose columns are the given basis vectors.

(b) Adjoin B to $BPRIME$, forming the $n \times 2n$ matrix C, **C = [BPRIME B]**.

(c) Let D be the reduced row-echelon form of C, **D = rref(C)**.

(d) P^{-1} is the $n \times n$ matrix consisting of the right-hand n columns of D.

Use MATLAB to find the matrix $APRIME$ of the linear transformation $T: R^n \to R^n$ relative to the basis $BPRIME$.

(a) $T: R^2 \to R^2$, $T(x, y) = (2x - y, y - x)$,
$BPRIME = \{(1, -2), (0, 3)\}$

(b) $T: R^3 \to R^3$, $T(x, y, z) = (x, x + 2y, x + y + 3z)$,
$BPRIME = \{(1\ -1, 0), (0, 0, 1), (0, 1, -1)\}$

7. Let $B = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ and $BPRIME = \{(1, 1, 0), (1, -1, 0), (0, 0, 1)\}$ be bases for R^3, and let

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

be the matrix of $T: R^3 \rightarrow R^3$ relative to B, the standard basis.

(a) Find the transition matrix P from $BPRIME$ to B.

(b) Find the transition matrix P^{-1} from B to $BPRIME$.

(c) Find $APRIME$, the matrix of T relative to $BPRIME$.

(d) Let

$$[\mathbf{v}]_{BPRIME} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

and find $[\mathbf{v}]_B$ and $[T(\mathbf{v})]_B$.

(e) Find $[T(\mathbf{v})]_{BPRIME}$ two ways: first as $P^{-1}[T(\mathbf{v})]_B$ and then as $APRIME[\mathbf{v}]_{BPRIME}$.

CHAPTER 7 ❑ MATLAB EXERCISES

1. The MATLAB command **poly(A)** produces the coefficients of the characteristic polynomial of the square matrix A, beginning with the highest degree term. Find the characteristic polynomial of the following matrices.

(a) $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$

2. If you set **p = poly(A)**, then the command **roots(p)** calculates the roots of the characteristic polynomial of the matrix A. Use this sequence of commands to find the eigenvalues of the matrices in Exercise 1.

3. The MATLAB command **[V D] = eig(A)** produces a diagonal matrix D containing the eigenvalues of A on the diagonal and a matrix V whose columns are the corresponding eigenvectors. Use this command to find the eigenvalues and corresponding eigenvectors of the three matrices in Exercise 1.

4. Let

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

Use MATLAB to find the eigenvalues and corresponding eigenvectors of A, A^T, and A^{-1}. What do you observe?

5. Let

$$A = \begin{bmatrix} 4 & -5 \\ 2 & -3 \end{bmatrix}.$$

Use MATLAB to diagonalize A as follows. First, compute the eigenvalues and eigenvectors of A using the command **[P D] = eig(A)**. The diagonal matrix D contains the eigenvalues of A, and the corresponding eigenvectors form the columns of P. Verify that P diagonalizes A by showing that $P^{-1}AP = D$.

6. Follow the procedure outlined in Exercise 5 to show that the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

is *not* diagonalizable.

7. Follow the procedure outlined in Exercise 5 to diagonalize (if possible) the following matrices.

(a) $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$

8. For a symmetric matrix A, the MATLAB command **[P D] = eig(A)** will produce a diagonal matrix D containing the eigenvalues of A, and an *orthogonal* matrix P containing the corresponding eigenvectors. For instance, if

$$A = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$$

is the matrix from Section 7.3, Example 8, then the command **[P D] = eig(A)** yields

$$P = \begin{bmatrix} -0.8944 & -0.4472 \\ 0.4472 & -0.8944 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix},$$

which is equivalent to the solution given in the text. Use this procedure to orthogonally diagonalize the following symmetric matrices.

(a) $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & -3 \end{bmatrix}$

CHAPTER 8 ❑ MATLAB EXERCISES

MATLAB handles complex numbers and matrices in much the same way as real ones. The imaginary unit $i = \sqrt{-1}$ is a built-in constant. Verify the result of Example 5, Section 8.2, by entering the matrix A,

$\mathbf{A} = [2 - i \quad -5 + 2i; \quad 3 - i \quad -6 + 2i]$

and then entering **inv(A)**.

Note: In MATLAB, **A.'** designates the transpose A^T of a complex matrix A. If you do not include the period, then MATLAB returns the *complex conjugate* transpose A^*. Use MATLAB to verify the following using the matrix A from Example 5, Section 8.2.

$$\mathbf{A}'. = \begin{bmatrix} 2 - i & 3 - i \\ -5 + 2i & -6 + 2i \end{bmatrix} \qquad \mathbf{A}' = \begin{bmatrix} 2 + i & 3 + i \\ -5 - 2i & -6 - 2i \end{bmatrix}$$

1. Use MATLAB to perform the following matrix operations, given

$$A = \begin{bmatrix} 1 & 2 - i \\ 2 + i & i \end{bmatrix}, \; B = \begin{bmatrix} 3i & 4 \\ -4 & -i \end{bmatrix}, \; \text{and} \; C = \begin{bmatrix} i & -i & 0 \\ 2 & 0 & 2 + 3i \end{bmatrix}.$$

(a) AB (b) $3iC$ (c) A^{-1}
(d) C^TC (e) $\det(A + B)$ (f) $iAB^2 + (1 - i)CC^T$

2. Use MATLAB to solve the system of linear equations $A\mathbf{x} = \mathbf{b}$.

(a) $A = \begin{bmatrix} i & 2 - i \\ 3 - 2i & 0 \end{bmatrix}, \; \mathbf{b} = \begin{bmatrix} 1 + i \\ 6 - 4i \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 0 & i \\ -2 & 1 + i & -i \\ 1 - i & 0 & -1 \end{bmatrix}, \; \mathbf{b} = \begin{bmatrix} i \\ 0 \\ 2 - i \end{bmatrix}$

3. Use MATLAB to determine which of the following matrices are Hermitian and which are normal.

(a) $\begin{bmatrix} i & 1 \\ 1 & -i \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 3 - 4i & 2 \\ 1 + i & 4 - i \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & i \\ 0 & -i & 2 \end{bmatrix}$

4. The MATLAB command $[\mathbf{P} \;\; \mathbf{D}] = \mathbf{eig}(\mathbf{A})$ will produce a diagonal matrix D containing the eigenvalues of the complex matrix A, and a matrix P containing the corresponding eigenvectors. For instance, if

$$A = \begin{bmatrix} 3 & 2 - i & -3i \\ 2 + i & 0 & 1 - i \\ 3i & 1 + i & 0 \end{bmatrix}$$

is the matrix from Example 7, Section 8.5, then the command $[\mathbf{P} \;\; \mathbf{D}] = \mathbf{eig}(\mathbf{A})$ yields

$$P = \begin{bmatrix} 0.1581 + 0.4743i & -0.3780 - 0.0000i & 0.0371 - 0.7783i \\ -0.3162 - 0.1581i & 0.3780 + 0.7559i & 0.2224 - 0.3336i \\ 0.7906 & 0.3780 & 0.4818 \end{bmatrix}$$

and

$$D = \begin{bmatrix} -2.0000 & 0 & 0 \\ 0 & -1.0000 & 0 \\ 0 & 0 & 6.0000 \end{bmatrix},$$

which is equivalent to the solution given in the text. Use this procedure to diagonalize the following matrices.

(a) $A = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & i \\ 0 & i & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 + i & 1 - i \\ 1 - i & 0 & i \\ 1 + i & -i & 0 \end{bmatrix}$